As α^2 and $\Gamma_a \ll 1$,

$$S_{21} \simeq \frac{\epsilon^{-j\beta l}\alpha(1 - \Gamma_a)(1 - \Gamma_m)}{1 - \epsilon^{-j2\beta(l-x)}\Gamma_r\Gamma_m}. \qquad (2)$$

When the attenuator–reflector combination is moved along the guide, only x is varied, and the output exhibits a fluctuation having a maximum to minimum ratio w, where

$$w \simeq (1 + |\Gamma_r\Gamma_m|)/(1 - |\Gamma_r\Gamma_m|). \qquad (3)$$

Fig. 2 is a plot of this relation which represents the sensitivity of the match checker.

Equations (1)–(3) apply when the reflector is set to face the receiver port. When the reflector is in its second position, facing the source, Γ_r and Γ_s must be exchanged. Because w increases with Γ_m, the reflected voltage should be made large. However, a large mismatch reduces the transmitted signal, and also is difficult to keep constant during its travel along the guide; for these reasons $|\Gamma_m| \simeq 0.5$ has been used.

Superimposed upon w is a much smaller fluctuation which results from interactions between the source Γ_s and the attenuator Γ_a and also Γ_s and Γ_m through the attenuator, as shown by the third term in the denominator of (1). This fluctuation is reduced as α is made smaller until a value is reached which makes the two terms of the unwanted fluctuation roughly equal, i.e., $\Gamma_a \simeq \alpha^2\Gamma_m(1 - 2\Gamma_a)$. If $|\Gamma_a| = 0.03$ and $|\Gamma_m| = 0.5$, then $\alpha^2 \leq 0.064$ and the attenuation should be at least 12 dB.

The unwanted fluctuation is proportional to the reflection from the port facing the attenuator, not the port being tuned; therefore, to ensure that it remains small in comparison with the wanted fluctuation, the two ports should be tuned in turn.

It should be noted that as the tuning proceeds, and Γ_r and Γ_s approach zero, both wanted and unwanted fluctuations diminish; therefore the residual imperfections do not prevent a perfectly matched condition of Γ_r and Γ_s from being achieved.

In the theoretical treatment above it was assumed that Γ_m and α are constant; however, in practice, both vary slightly in magnitude during movement along the waveguide. It was found that for the X-band match checker the combination of these imperfections resulted in a standard deviation of w representing an uncertainty in Γ_r and Γ_s of 0.0008. This is attributed to lack of uniformity in the commercial waveguide used. If it is required to reduce residual reflections to this order of magnitude, lapped precision flanges must be used.

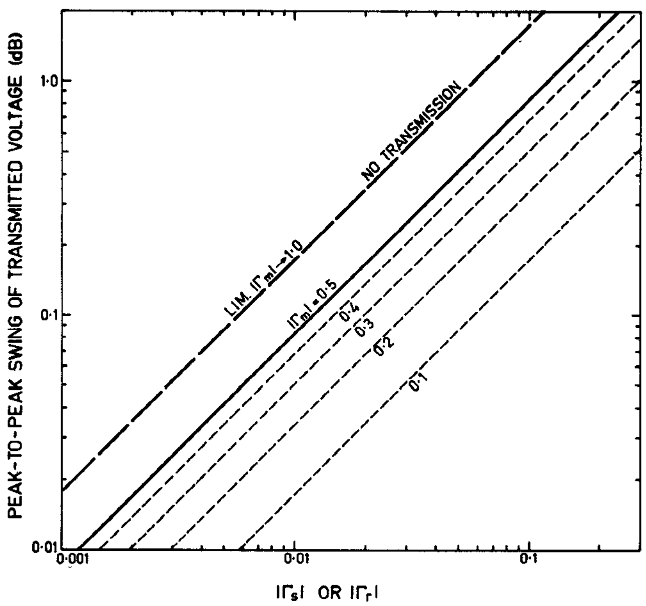

Fig. 2. The rate of maximum to minimum transmitted voltage produced by moving the reflector-and-attenuator combination along the waveguide. For example, if the reflector $|\Gamma_m| = 0.5$ is set facing the source, and the source mismatch $|\Gamma_s| = 0.01$, the indicated output will vary by $\sim$0.08 dB.

CONCLUSION

The match checker has proved to be a useful and practical device for tuning component parts of any transmission system whenever an accurate match is required. It is a portable passive device, requiring no tuning and thus may be used in the laboratory or in the field.

When it is inserted between ports to be matched, the variation in transmitted signal on sliding the reflector–attenuator combination is a measure of the mismatch of the port facing the reflector; thus no auxiliary equipment is required, as the original signal source and detector are used. Active and passive ports are checked by the same procedure, with active ports in the active condition, which distinguishes the match checker from slotted lines and reflectometers in their usual modes of operation.

As constant reflections caused by imperfection in the instrument do not in any way prevent an accurately matched condition from being achieved, it is suitable for obtaining the degree of match required for precision measurements.

ACKNOWLEDGMENT

The authors acknowledge the skilled assistance of P. Campbell who constructed the match checkers.

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Design Equations and Bandwidth of Loaded-Line Phase Shifters

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***Abstract*—Design equations for a loaded-line phase shifter are derived for any susceptance spacing. An analysis based on these relations shows that maximum bandwidth is obtained when the spacing between the switched susceptances is 90°.**

INTRODUCTION

The digital loaded-line phase shifter shown in Fig. 1(a) remains a popular and useful device for obtaining small-bit phase shifts up to approximately 45°. Recently, there has been some discussion over whether the optimum shunt susceptance separation θ should be 75 or 90°. Garver [1], on the basis of his lossless diode model, states that $\theta = 90°$, gives the widest bandwidth, while Opp and Hoffman [2] and Yahara [3], using lossy diodes find that smaller phase error, standing-wave ratio (SWR), and loss is achieved using $\theta = 75°$. A rigorous evaluation of the tradeoff between these performance parameters and bandwidth has been hindered by the

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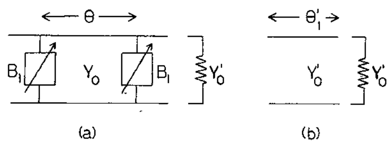

Fig. 1. (a) Physical loaded-line phase shifter. (b) Equivalent representation.

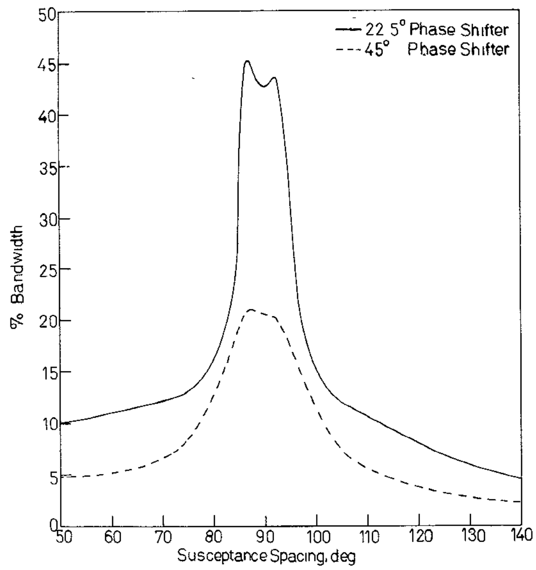

Fig. 2. Fractional bandwidth of the phase shifter using single lumped switched susceptances where SWR < 1.2 and the phase shift error <2°.

necessity of using iterative computer techniques to obtain the shunt susceptances for the given separation θ. Here, a noniterative design formula is presented which gives perfect match and zero phase shift error for a lossless diode for any shunt susceptance spacing at a given frequency. This formula not only gives design information from which bandwidth optimization can be performed, but it provides a method for comparing the bandwidths of phase shifters with different susceptance spacings.

THEORY

Under lossless conditions, the general expressions for Y_0' and θ_i' of the equivalent circuit in Fig. 1(b) are [4]

$$\theta_i' = \arccos\left[\cos\theta - (B_i/Y_0)\sin\theta\right], \qquad i = 1,2 \tag{1}$$

$$Y_0' = Y_0\left[1 - (B_i/Y_0)^2 + 2(B_i/Y_0)\cot\theta\right]^{1/2}, \qquad i = 1,2 \tag{2}$$

$$\psi = \theta_1' - \theta_2' \tag{3}$$

where ψ is the phase shift when the two shunt susceptances switch from B_1 to B_2. Opp and Hoffman [2] divide the solution of these equations into three classes of which only the last two special cases are seen to have a closed form solution. Class 1 is the general case where $B_1 \neq B_2 \neq 0$, class 2 is the case where B_i switches from 0 to some nonzero value, and class 3 is the case where $B_1 = -B_2$. Expressions (2) and (3) represent a set of three equations in the four unknowns (θ, Y_0, B_1, B_2) in terms of the given Y_0' (chosen to insure SWR = 1) and the phase shift ψ. Obviously, the solution for B_1, B_2, and Y_0 will have to be a function of θ. When (2) is solved for the two switched susceptances,

$$B_1 = Y_0\cot\theta + \left[Y_0^2\csc^2\theta - Y_0'^2\right]^{1/2} \tag{4}$$

$$B_2 = Y_0\cot\theta - \left[Y_0^2\csc^2\theta - Y_0'^2\right]^{1/2} \tag{5}$$

and the product, sum, and difference of B_1 and B_2 are substituted into

$$\tan(\psi/2) = \frac{\tan(\theta_1'/2) - \tan(\theta_2'/2)}{1 + \tan(\theta_1'/2)\tan(\theta_2'/2)}$$

an expression for the phase shift independent of B_i is obtained:

$$\tan(\psi/2) = \frac{\left[1 - (Y_0'/Y_0)^2\sin^2\theta\right]^{1/2}}{(Y_0'/Y_0)\sin\theta}. \tag{6}$$

Thus the solution is found for the unknown Y_0 from (6) and the shunt susceptance values B_i from (4) and (5):

$$Y_0 = Y_0'\sec(\psi/2)\sin\theta \tag{7}$$

$$B_1 = Y_0'\left[\sec(\psi/2)\cos\theta + \tan(\psi/2)\right] \tag{8}$$

$$B_2 = Y_0'\left[\sec(\psi/2)\cos\theta - \tan(\psi/2)\right]. \tag{9}$$

This reduces to the previously mentioned class 2 solution when $B_1 = 0$ and to the class 3 solution when $\theta = 90°$. Low SWR and low phase shift error at midband for any θ can be expected from a design based on (7)–(9) when low loss switching elements are used. These expressions also provide a good starting point for an iterative solution if maximum bandwidth is desired at the expense of midband phase accuracy and SWR.

DISCUSSION

With the derived expressions (7)–(9), a direct bandwidth comparison can be made for any susceptance spacing θ. If the bandwidth is defined by the SWR in both the forward and reverse bias states being less than 1.2 and the phase shift error being less than 2°, then the bandwidth as a function of θ is shown in Fig. 2. The susceptance in this case was assumed to be either a lumped capacitor or inductor of the value dictated by (8) and (9). For the 22.5° phase shifter the percentage bandwidth as a fraction of the center frequency

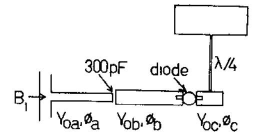

Fig. 3. Top view of one of the stripline stubs.

TABLE I
STUB DESIGN PARAMETERS FOR REALIZABLE PHASE SHIFTER IN FIG. 3

	$\theta = 90^\circ$		$\theta = 75^\circ$	
	ϕ	Y_0, mhos	ϕ	Y_0, mhos
A	76.24°	0.0105	102.7°	0.00986
B	30°	0.020	30°	0.020
C	20°	0.020	20°	0.020

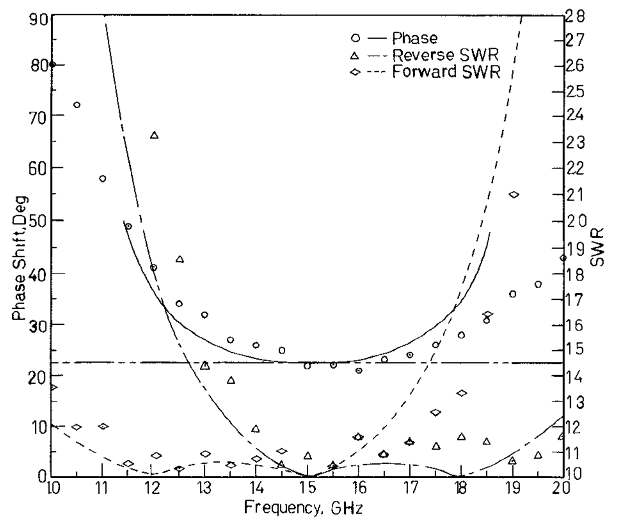

Fig. 4. The theoretical and experimental phase shift and SWR when the susceptance spacing is 90°.

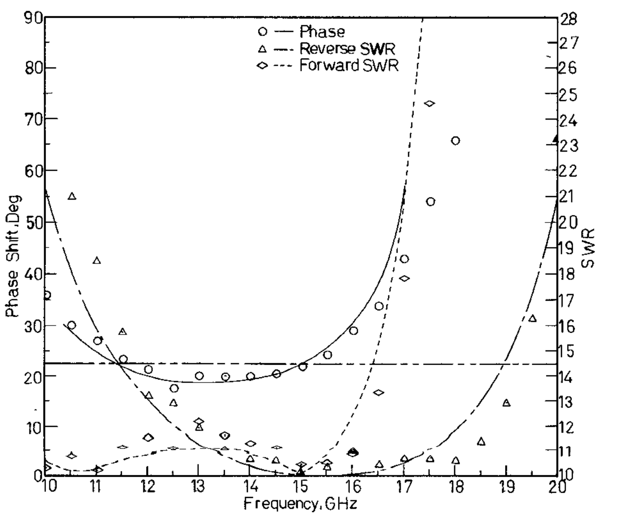

Fig. 5. The theoretical and experimental phase shift and SWR when the susceptance spacing is 75°.

drops from 43 percent, when $\theta = 90^\circ$, to 12 percent, when $\theta = 75^\circ$. Clearly, the 90° susceptance spacing phase shifter has superior bandwidth properties.

A test was made with a practical circuit to see if the bandwidth decreases as the susceptance spacing is reduced from 90°. Two 22.5° phase shifters were designed, analyzed, and built in stripline: one with $\theta = 90^\circ$ and the second with $\theta = 75^\circ$.

The first step in the design was to measure the impedance parameters of two shunt mounted pin diodes in the two bias states. To get the B_i required by (8) and (9), each diode was mounted in a stub circuit as shown in Fig. 3. Calculations for several stub configurations were made, each giving the required B_i, but one was chosen that had reasonably short line lengths and realizable characteristic admittances. Table I shows the design parameters used for the stubs, while Figs. 4 and 5 show the experimental and theoretical SWR and phase shift for the two phase shifters. Here the bandwidth decreased from 19.3 percent for $\theta = 90^\circ$ to 5.75 percent for $\theta = 75^\circ$. For both this circuit and the one using lumped reactances the bandwidth was reduced by a factor of approximately 3.5.

CONCLUSIONS

The commonly used spacing of $\theta = 90^\circ$ between shunt susceptances appears to offer the widest bandwidth available for a loaded-line phase shifter. Shortening θ to, say, 75°, implies that one of the required $|B_i|$ are larger than the $|B_i|$ when $\theta = 90^\circ$. Any bandwidth advantage gained through the shorter line length θ is overcome by the larger required shunt susceptance.

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Circularly Polarized Electric Field in Rectangular Waveguide

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Abstract—**The electric field in a waveguide partially filled with a low-loss slab of dielectric in the *H* plane presents a circularly polarized component at the air–dielectric interface over a limited frequency range. This effect could be used to improve the performance of nonreciprocal devices utilizing the gyroelectric effect in magnetized semiconductors.**

I. INTRODUCTION

Nonreciprocal wave propagation has been observed in rectangular waveguides loaded with semiconductor slabs at room temperature subjected to a dc-biasing magnetic field. Both E- and H-plane structures were tested, using, respectively, N-type InSb [1] and N-type silicon [2]. Devices such as circulators, isolators, and nonreciprocal

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