LECTURE NOTES FOR

EE 172

ELECTRICAL MACHINES

COURSE LECTURER: DR. PHILIP YAW OKYERE

Course Outline

EE 172 ELECTRICAL MACHINES (3 0 3)

Basic Laws of Electrical Machines: Faraday's law of electromagnetic inductions. Magnetic field, force on a current carrying wire, induced voltage in a conductor moving in a magnetic field.
D.C. Machines: Principles of operation, construction of DC machines, armature windings: lap and wave. DC generators: excitation, load characteristics and voltage regulations of separately excited, shunt wound, series wound and compound wound generators. Conditions for self-excitation of shunt-wound generator. DC motors: speed and torque, starting and speed control.
Transformers: Principle of single phase transformers, equivalent circuits, tests of transformers, parallel operation of transformer and performance characteristics. Three phase transformer: Connection methods for three phase transformers. Auto-transformers. Current and potential transformers.
Induction Machines: Production of rotating magnetic field by uniformly distributed three phase windings. Principles of operation and construction of induction motor. Definition of slip, equivalent circuit, Losses, efficiency and torque. Output characteristics, starting torque and maximum torque. Starting methods. Speed control. Braking of induction motor.
Special Electrical Machines: Single phase induction machine, stepper motor, hysteresis motor, reluctant motor, permanent magnet motors, brushless dc motor, universal motor.

Chapter One

Basic Laws of Electrical Machines

1. Faraday's law of electromagnetic induction

There are two related laws forming the basis of operation of electrical machines used to convert electrical energy to or from mechanical energy. One is the Faraday's law of electromagnetic induction known simply as the law of induction and the other is the law of interaction.

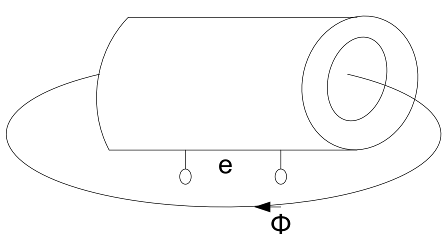
Let N be the number of turns of electric circuit or coil and ϕ the total flux linking the circuit or coil. The product $\lambda = N\phi$ is termed the flux linkage. If the flux linkage is made to change with time an emf is induced in the electric circuit. The instantaneous emf according to Faraday's law is given by

$$e = \frac{d\lambda}{dt} = N\frac{d\phi}{dt} \text{ volts} \tag{1}$$

Example 1

A variable flux $\phi(t) = 0.002\sin 120\pi t$ links a coil of 4000 turns. Calculate the instantaneous voltage induced in the coil.

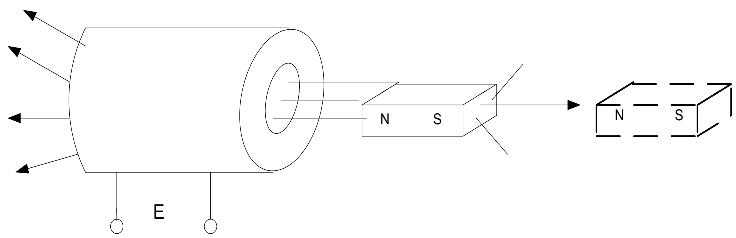
Solution



$$e = \frac{d\lambda}{dt} = \frac{d}{dt}\left(4000 \times 0.002\sin 120\pi t\right) = 960\pi\cos 120\pi t$$

Example 2

A coil of 2000 turns surrounds a flux of 5 mWb produced by a permanent magnet. The magnet is suddenly drawn away causing the flux inside the coil to drop to 2 mWb in 1/10 of a second. What is the average voltage induced?



Solution

$$\Delta\phi = \phi_2 - \phi_1 = 2 - 5 = -3\ mWb$$
$$E = N\frac{\Delta\phi}{\Delta t} = 2000\times\frac{3}{1000\times\frac{1}{10}} = 60V$$

Change of flux linkage in coil can occur in two ways:

(a) The flux has rate of change with time. See Example 1. Magnetic fields in engineering devices are mostly produced by electric currents. When the current changes, the field also changes.

(b) There is a relative motion of the coil and the flux. See Example 2.

Because the linkage flux changes in these two ways, we can state the Faraday's law in this form:

$$e = \frac{d\lambda(\theta,t)}{dt} = \frac{\partial\lambda}{\partial\theta}.\frac{d\theta}{dt} + \frac{\partial\lambda}{\partial t} = e_r + e_p \qquad (2)$$

$e_r = \frac{\partial\lambda}{\partial\theta}.\frac{d\theta}{dt}$ is called motional or rotational emf. This voltage when induced in a machine winding gives rise to mechanical / electrical power conversion.

$e_p = \frac{\partial\lambda}{\partial t}$ is called pulsational or transformer emf. It provides a means of electrical energy transfer between magnetically coupled windings as from primary to secondary windings in a transformer or from stator to rotor in a rotating machine.

2. Force on current–carrying conductor

A conductor of active length l metres and carrying a current i amps and lying in and perpendicular to the direction of a magnetic field B webers/m^2, experiences a mechanical force of magnitude:

$$f = Bil \text{ newtons} \qquad (3)$$

Example 3
A conductor 3 m long carrying a current of 200 A is placed in a magnetic field whose density is 0.5 Wb/ m^2. Calculate the force on the conductor if it is perpendicular to the field.

Solution:
$f = Bil = 0.5 \times 3 \times 200 = 300N$

The force expression given by (3) is also referred to as the law of interaction.

3. Voltage induced in a conductor moving in a magnetic field

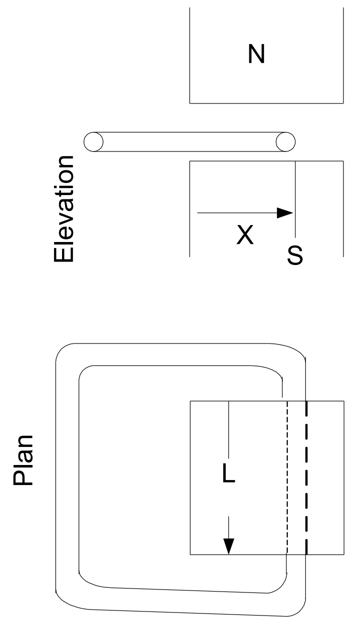


Fig. 1 Conductor in a magnetic field: plan and elevation

A single-turn coil is placed in a magnetic field of density B as shown in Fig.1. The flux linkage in the coil is $\lambda = Blx$. If we let the loop move horizontally to the right, say, the flux linkage changes at a rate:

$$\frac{d\lambda}{dt} = \frac{d}{dt}(Blx) = Bl\frac{dx}{dt}$$

By Faraday's law, this constitutes an induced emf. Specifically it is a motional emf. Using u for $\frac{dx}{dt}$, we obtain $e = Blu$ (4)

Alternatively,

$$e = \frac{\partial \lambda}{\partial x} . \frac{dx}{dt} = Blu$$

This voltage which is the result of change of flux linkage can also be attributed to the relative motion of conductor and a magnetic field.

The equation is referred to as the flux cutting rule. It can be stated as this: the emf in a single conductor of active length l metres which cuts across a magnetic field of density B webers per m^2 when moving at u m/s at right angles to the direction of the flux is given by $e = Blu$ volts.

In rotating electrical machines, the change of flux linkage is not clearly defined and it is therefore not easy to calculate the induced voltage with reference to the coils in the machines. The flux cutting rule which refers to the conductors rather than the coils themselves gives a more convenient method of calculating the voltage.

Example 4
The conductors of a large generator of length 2 m are moved at right angles across a magnetic field at a constant speed of 100 m/s. The flux density in the magnetic field is 0.6 Wb/ m^2. Calculate the emf induced in each conductor.

Solution
$E = Blu = 0.6 \times 2 \times 100 = 120\ V$

4. Direction of motional emf

It can be determined by applying Fleming's right hand rule: if the first finger of the right hand is pointed in the direction of the magnetic flux and the thumb in the direction of the motion as shown in the Fig. 2., then the second finger held at right angles to both the thumb and the first finger represents the direction of the emf. In the case where the conductor is stationary and the field moving, the conductor is assumed to move in the direction opposite to that of the field.

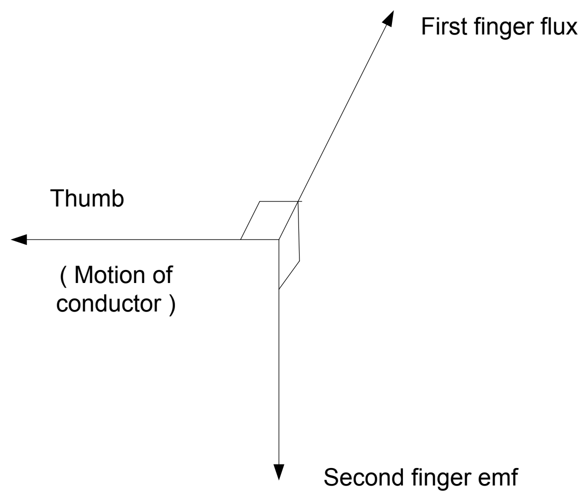


Fig. 2 Fleming's Right-hand Rule

5. The direction of force of interaction

It can be determined using the electromagnetic pictures in Fig. 3.

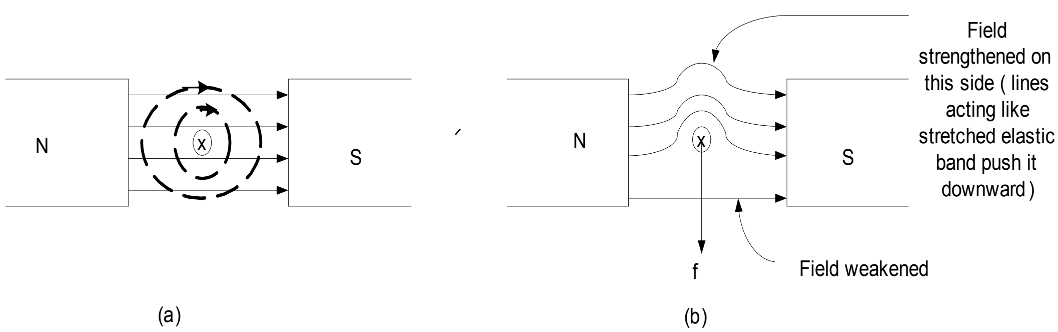


Fig. 3 Direction of force on current - carrying conductor lying in a magnetic field

6. Lenz's Law

It gives a method for determining the direction of induced emf. The induced emf according to this law will tend to set up a current whose magnetic field will oppose the motion or change of flux responsible for inducing that emf.

Example 5

The conductor c is being moved in the upward direction as shown in Fig 4. Use the Lenz's law to determine the direction of the induced emf in the conductor.

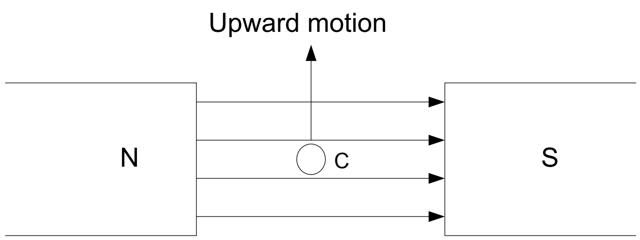


Fig. 4 See example 5

Solution

The upward motion of the conductor should produce an emf and current whose magnetic field by Lenz's law would oppose the upward motion of the conductor. The type of magnetic field which will oppose the motion of the conductor is obtained when the current in the conductor tend to flow into the paper.

Example 6

Fig. 5 shows a conductor lying in a magnetic field with a voltage applied across its ends. Use Lenz's law to determine the direction of the induced emf in the conductor.

Solution:

For the direction of the field and current shown, the force developed on the conductor is in upward direction. The force causes the conductor to move through the magnetic field, resulting in an induced emf. The induced emf according to Lenz's law will be in such a direction as to set up current whose magnetic field will oppose the motion responsible for inducing it. The direction is opposite to the current *I* and hence the applied voltage.

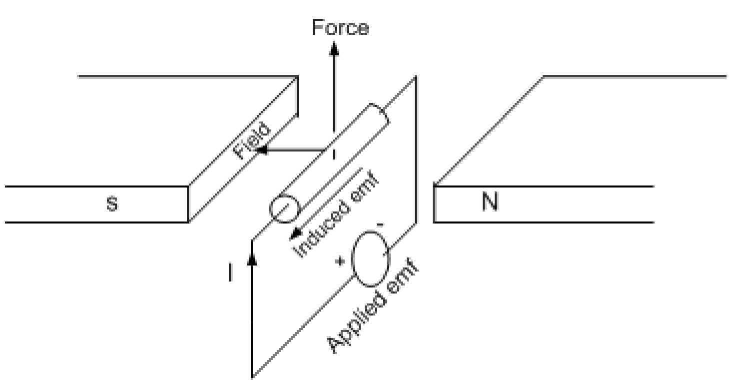


Fig. 5 See Example 6

7. Generator and motor actions

It is observed from Examples 5 and 6 that whenever motor action occurs, generator action is also developed and the converse is also true.

In a motor action (Example 6), the magnetic field produces the output torque and by inducing a counter emf, makes it possible for the machine to absorb power from the electric source to be converted into mechanical power.

In generator action (Example 5), the magnetic field induces the generated voltage, and by developing a counter torque, makes it possible for the machine to absorb power from the mechanical drive to be converted to electrical power

Chapter Two

DC Machines

1. Introduction

D.C. machine has two members. One (the 'field') is an electro- or permanent-magnet system providing the working magnetic field in an air gap. The other ('the armature') is an arrangement of current-carrying conductors so oriented as to develop interaction force and emf.

2. Elementary dc machine

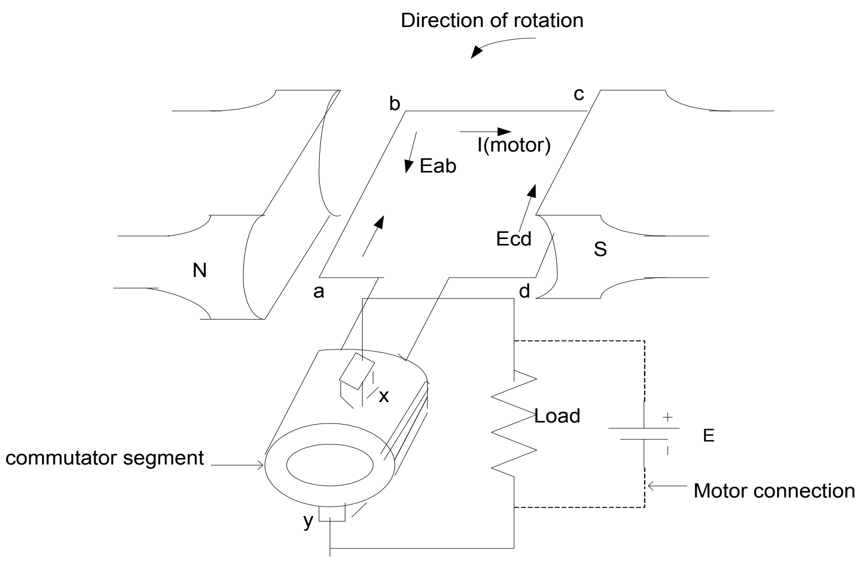


Fig. 1 Elementary dc machine

Fig. 1 shows an elementary dc machine

Generator action:

The rotation is due to an external driving torque. When the coil is in horizontal position as shown, the voltages E_{ab} and E_{cd} are positive. The voltage picked up by the two stationary brushes *x* and *y* is the sum of the two and the brush *x* is positive and *y* is negative. When the coil is in vertical position, no flux is cut and voltage induced in each conductor is zero. At this position, the brushes short the coil. When the coil moves beyond the vertical position, the direction of the voltages in the conductors reverses. At the same time the two commutator halves change contact from one brush to another. Thus brush *x* is always positive and brush *y* negative and the current in the external load always flows in the same direction.

Motor action:

With the current in the coil as shown, the forces on sides *ab* and *cd* are down and upward respectively. The two forces form a couple or a torque which causes the coil to rotate in the anticlockwise direction. In the vertical position, the coil current is cut off but the momentum of the coil; carries it past this position. When this occurs, the two commutator halves change contact from one brush to the other. This reverses the current in the coil and consequently the directions of the forces on the conductors. The coil thus continues to rotate in anticlockwise direction for so long as the current is passing.

3. **Construction**:

The general arrangement of a two-pole dc machine is shown in Fig.2. The rotor (or armature) consists of

(a) *The rotor shaft*: this imparts rotation to the armature core and winding.

(b) *The armature core*: It is constructed of iron laminations. The laminations insulated from one another, serve to reduce eddy-current loss. The core is keyed to the shaft and it provides a low reluctance magnetic path between the poles.

(c) *The armature winding*: This consists of coils insulated from each other and from the armature core. The coil sides are placed in slots firmly held in place by fiber slot sticks. If the armature current is below 10 A, round wire is used but for currents exceeding 20 A, rectangular conductors are preferred because they use the slot space better.

(d) *The commutator:* It consists of many tapered copper segments insulated from each other by mica sheets and clamped round the shaft of the machine. To each commutator is connected a coil junction. Hence the number of segments = the number of coils.

(e) *The brushes:* Multi-pole machines have as many brush sets as the number of poles. A brush set is composed of one or more brushes depending on the current that has to be carried. The brush sets are spaced at equal intervals around the commutator. The brushes are made of carbon because it has good electrical conductivity and its softness does not scratch the commutator.

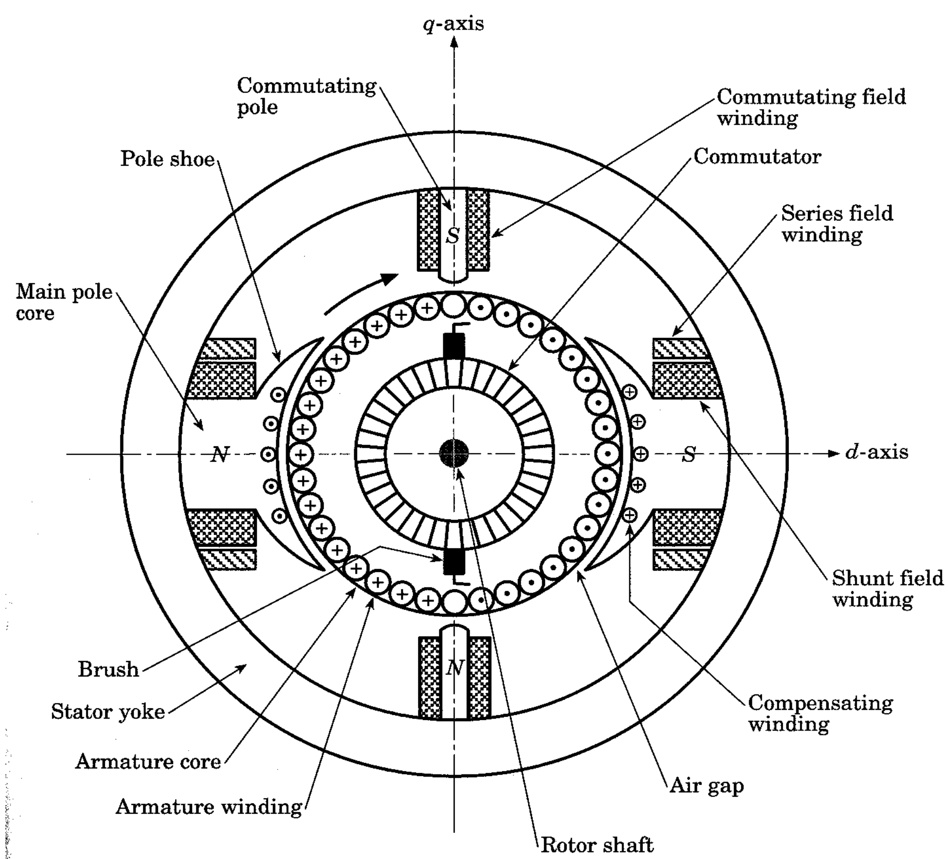


Fig. 2 General arrangement of a 2-pole dc machine

The stator of dc machine consists of

(a) *The yoke or cylindrical frame:* It is made up solid cast steel or iron. It provides a flux return path for the magnetic flux created by the field windings.
(b) *The field windings:* they are electromagnets. Their ampere-turns provide mmfs adequate to produce in the air gap, the flux needed to generate emfs in the conductors. The air gap is the short space between the pole faces and armature teeth under the poles. It ranges from about 1.5 to 5 mm as the machine rating increases from 1 kW to 100 kW. In some machines, the flux is created by permanent magnets.
(c) *The field poles*: they carry the field windings and are excited alternately N and S. The pole shoe is carried to spread the flux more uniformly. DC machine may have 2, 4, 6 or as many as 24 poles. The bigger the machine, the larger the number of poles it will have.

4. Induced voltage in armature conductor

Fig. 3 shows the variation of the emf generated in a conductor while it is moving through two pole pitches (A pole pitch is the distance between the centres of adjacent poles). The emf remains constant while the conductor is moving under a pole and then decreases rapidly to zero when it is midway between the pole tips of adjacent poles.

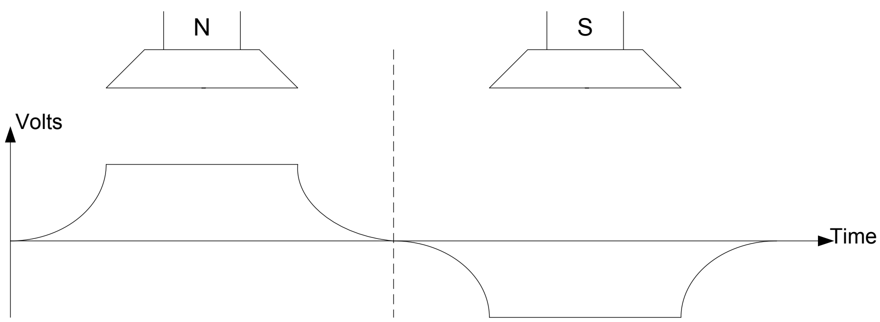


Fig. 3 Waveform of emf generation in a conductor

5. Induced voltage in dc armature windings

The average value of emf in a conductor when it is moving under a pole face is given by

$$E_c = B_{av} L u \tag{1}$$

where

B_{av} = mean flux density
L = active length of the conductor
u = the surface speed of the armature

$$B_{av} = \frac{Total\ flux\ under\ the\ poles}{Total\ surface\ area\ swept\ through\ by\ the\ active\ length\ of\ the\ conductor} = \frac{2p\phi}{2\pi r L} \tag{2}$$

where

r = radius of the armature core
ϕ = flux / pole

Let N = rotor speed in rev/min. Then $\omega = \frac{2\pi N}{60}$ and $u = \omega r = \frac{2\pi N r}{60}$ (3)

Substituting (2) and (3) into (1) yields

$$E_c = \frac{2Np\phi}{60}$$

If Z = total number of conductors in a winding and $2a$ = number of parallel paths.

Then the number of conductors in series per path $= \frac{Z}{2a}$ and the voltage induced between brushes is given by

$$E = E_c \times \frac{Z}{2a} = \frac{2Np\phi}{60} \times \frac{Z}{2a}$$

Or

$$E = \frac{N}{60} . \frac{p}{a} . Z\phi \tag{4}$$

6. Lap and wave winding

The number of parallel paths a machine is determined by the type of winding employed. Apart from a few special windings, armature windings can be divided into two groups depending upon the manner in which the front ends of a coil are joined to the commutator, namely: lap and wave windings.

Lap windings:

(a) The front-ends of a coil are bent inward to bring about the connection from one coil to another.

(b) Number of parallel paths = number of poles. Thus $a = p$

Wave winding:

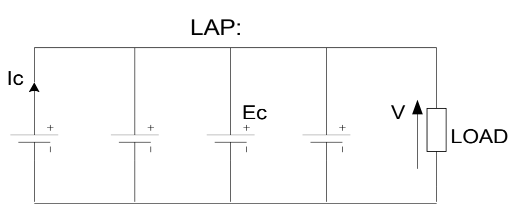
(a) The front ends are bent outward by about ½ a pole pitch.

(b) In this winding, all coils carrying current in the same direction at any instant are connected in series and since a coil has either its current in the clockwise or anticlockwise direction, the number of parallel paths = 2. Thus $a = 1$

7. Comparing lap and wave winding machines

For a given number of conductors and the same physical size and speed, the wave winding gives a higher terminal voltage and lower current than then lap winding. In general the lap winding is used for low-voltage, heavy-current machine.

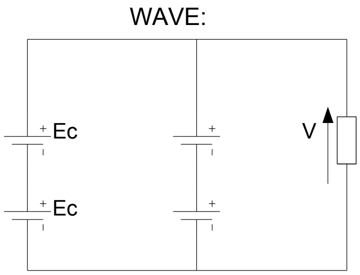
Consider a 4-pole machine having 4 conductors



For the lap, the terminal voltage V = voltage per conductor = E_c

Maximum current I_{max} = 4x Current rating of a conductor = $4I_c$

Maximum power = $4E_cI_c$



For the wave, $V = 2E_c$
$I_{max} = 2I_c$
and
Maximum power = $4E_cI_c$.

Example 1
A four-pole wave–connected armature has 51 slots with 12 conductors per slot and is driven at 900 rev/min. If the useful flux per pole is 25mWb, calculate the value of the generated emf.

Solution:
Total number of conductors, Z = 51 x 12 = 612; a = 1,; p = 2; N = 900 rev/min; ϕ = 0.025 Wb
Using (4), we have

$$E = \frac{900}{60}\times\frac{2}{1}\times 612\times 0.025$$
$$= 459 Volts$$

Example 2
An eight-pole lap–connected armature driven at 550 rev/min is required to generate 260 V. The useful flux per pole is about 0.05 Wb. If the armature has 120 slots, calculate a suitable number of conductors per slot.

Solution:
For an eight-pole lap winding, a = 4
Hence, from (4)

$260 = \frac{350}{60}\times\frac{4}{4}\times Z\times 0.05$. From which $Z = 890$ approximately

and the number of conductors per slot = 890/120 = 7.4 (approx.). This value must be an even number; hence 8 conductors per slot would be suitable. The flux corresponding to this would be $\frac{890}{8\times 120}\times 0.05 = 0.0464 Wb/pole$

Example 3:
An eight-pole armature is wound with 480 conductors. The magnetic flux and the speed are such that the average emf generated in each conductor is 2.2 V; each conductor is capable of carrying a full-load current of 100A. Calculate the terminal voltage on no load, the output current on full load and the total power generated on full load when the armature is (a) lap-connected and (b) wave-connected.

Solution:

(a) Lap connected

Number of parallel paths = number of poles = 8

Number of conductors / path = 480/8 = 60

Terminal voltage on no load = emf/conductor x (number of conductors/path)

= 2.2 x 60 = 132 V

Output current = full-load current per conductor x (number of parallel paths)= 100 x 8 = 800A

Total power generated = 800 x 132 = 105.6 kW

(b) Wave-connected

Number of parallel paths = 2

Number of conductors/path = 480/2 = 240

Terminal voltage on no load = 2.2 x 240 = 528V

Output current on full load = 100 x 2 =200A

Total power generated = 200 x 258 = 105.6 kW

8 D.C. Generators

8.1 Methods of excitation: there are two methods

(a) *Separate excitation*: the field current is obtained from a separate source such as battery

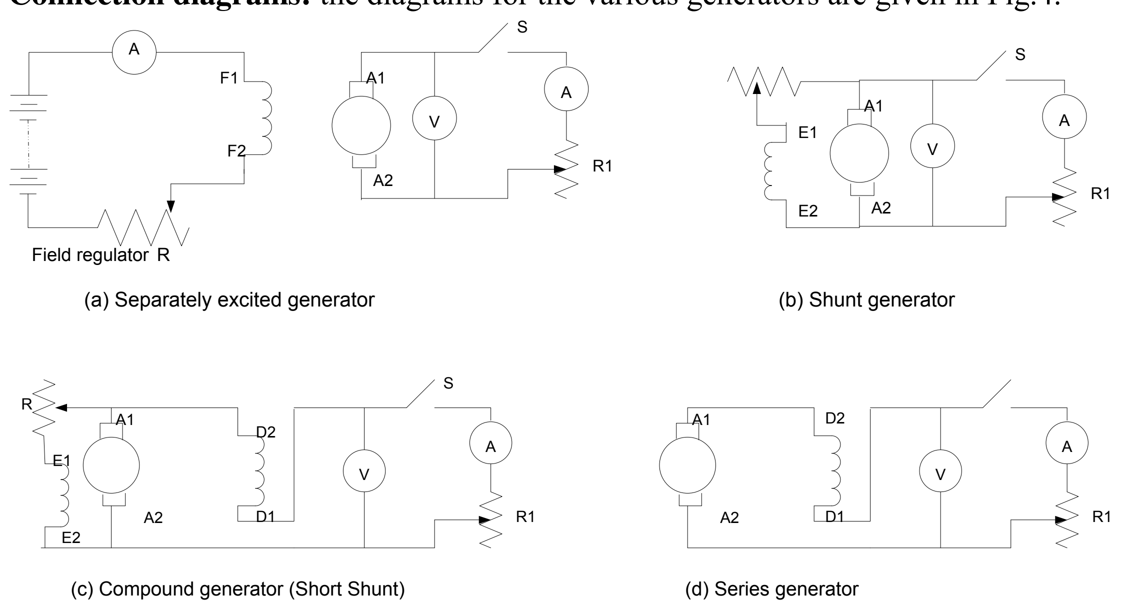
(b) *Self-excitation:* the field current is produced by the generator itself. The self-excited generators may be subdivided into 3 groups, namely:

(i) *shunt-wound generators:* the field winding is connected across the armature terminals.

(ii) *series-wound generators*: the field winding is connected in series with the armature winding

(iii) *compound-wound generators:* a combination of shunt and series windings.

8.2 Connection diagrams: the diagrams for the various generators are given in Fig.4.

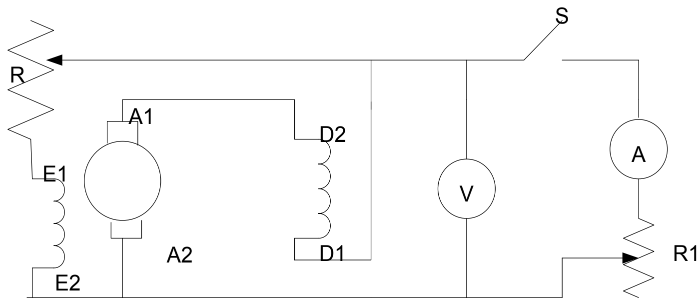


(a) Separately excited generator

(b) Shunt generator

(c) Compound generator (Short Shunt)

(d) Series generator



(e) Compound generator (Long Shunt)

Fig. 4 D.C. Generator connections

8.3 Separately excited generator on no load

The variation of the open-circuit voltage across the armature with the field current I_f at a given speed is shown in Fig. 5.

(a) The curve is called the open-circuit characteristic (OCC). The open-circuit curve for other dc machines is obtained by separately exciting the machine and driving the rotor by a prime mover.

(b) Since the open-circuit voltage *E* is proportional to ϕ at a given speed, the curve also shows how the flux varies with the field current. For this reason the curve is also referred to as the magnetization curve.

(c) The curve has three portions: a linear portion *oa*, the portion *ab* called the knee of the O.C. curve (in this portion the saturation of the iron begins to be important) and the portion *bc* (here, a large increase in the field current results in a small increases in flux)

(d) The rated voltage of a d.c. generator is usually a little above the knee of the curve if the curve is obtained at the rated speed.

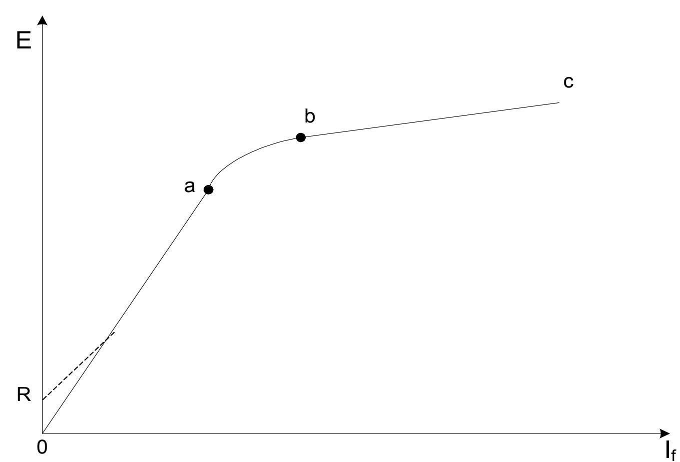


Fig. 5 Open-Circuit Characteristics of a dc generator

(e) If the machine has been used before and it is not demagnetised, the emf curve is found to follow the dotted line

(f) *oR* represents the emf generated by residual magnetism in the poles.

(g) At a given field current the generated voltage *E* is proportional to the speed *N*.

8.4 Shunt generator on no load

Refer to circuit in Fig.6.

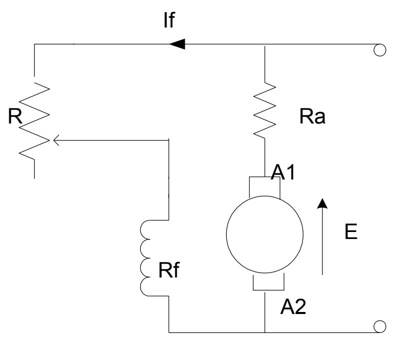


Fig. 6 Shunt generator on no load

Applying Ohm's law

$$E = R_s I_f \quad (5)$$

where $R_s = R + R_f + R_a$

We also know that at a given speed,

$$E = g(I_f) \quad (6)$$

where $g(I_f)$ is a nonlinear function given by the magnetization curve. The values of E and I_f are determined by the intersection of the curves described by (5 and (6). The steady state value of E and I_f are reached after a transient build–up process explained below with reference to Fig.7:

(a) At the rated (or a given speed), the voltage across the armature due to residual magnetism is E_1. But this voltage is also across R_s. Thus, the current which flows in the field circuit is I_1

(b) When I_1 flows in the circuit of the generator, an increase in mmf results which aids the residual magnetism in increasing the induced voltage to E_2 as shown in Fig. 7

(c) Voltage E_2 is now impressed across R_s causing a larger current I_2 to flow in the field circuit. An increased mmf due to I_2 produces generated voltage E_3

(d) The process continues until that point where the shunt resistance line crosses the magnetization curve. Here the process stops. The voltage induced, when impressed across R_s, produces a current flow that in turn produces an induced voltage of the same magnitude E_7 as shown in Fig. 7

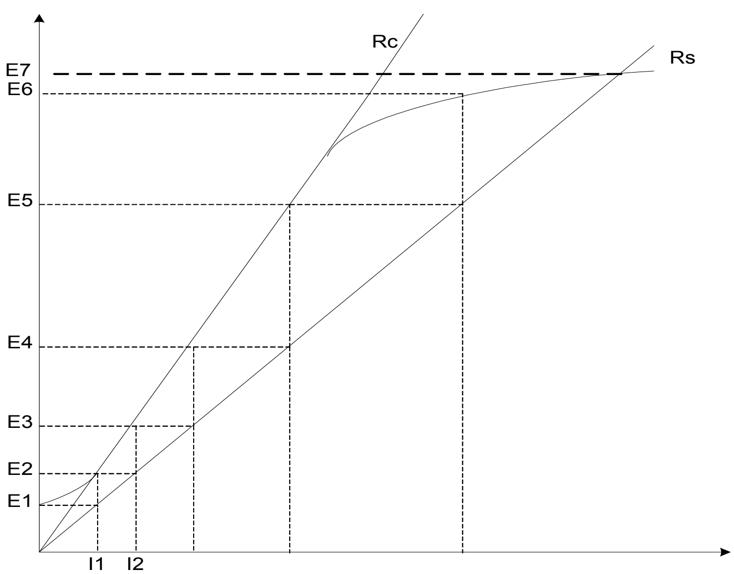


Fig. 7 The Build-Up of Self excited shunt generator

Example 4

A dc machine is connected as a shunt generator. When operated at rated speed of 1500 rev/min, the O.C. curve is approximated by

$$E = \frac{367 I_f}{0.233 + I_f}$$

The armature circuit resistance is 0.14 Ohm. The shunt field resistance is adjusted to be 459 ohms. Find the no load terminal voltage.

Solution:
At no load, $E \approx V$ (terminal voltage). Hence
$E = R_f I_f = 459 I_f$
Substituting this in the O.C. characteristic, we obtain $V = E = 260$ V and $I_f = 0.566$ A

8.5 Critical field circuit resistance
It is the field circuit resistance beyond which the shunt generator will fail to excite. The critical field circuit resistance, R_c, is shown as a tangent to the O.C. curve passing through the origin (See Fig.7. This resistance is proportional to the speed.

8.6 Reasons for failure of shunt generator to build up voltage
There are four reasons

(a) Lack of residual magnetism:

Causes:
Residual magnetism may be lost as a result of mechanical shock in shipment, excessive vibration, extreme heat and inactivity for long periods.

Remedy:
Separately excite the field for a few moments. Where possible, run the machine as a motor; its brushes being of the normal polarity and the direction of rotation also normal (a better method).

(b) Field circuit connections reversed with respect to the armature circuit.

Remedy:
Change over connections from the armature to the field circuit.

(c) Direction of rotation reversed: This also produces reversed armature connections with respect to the field.

Remedy:
Turn it in the right direction.

(d) Field circuit resistance higher than critical field resistance

Causes:
Broken lead in any part of the field circuit; too high a brush contact resistance (very rare); too large a resistance in the field regulator for the particular speed operation.

Example 5
A dc machine has the following details:
Four poles; shunt field resistance 80 Ω; armature resistance 0.4 Ω; number of armature conductors 400. The armature conductors are arranged in two parallel paths and the magnetization curve is as follows:

Field current (A)	0	0.5	1	1.5	2	2.5	3	4
Flux per pole (mWb)	1	6.5	12.8	17.5	20.5	22	23	24

Find the open-circuit voltage to which the machine will excite when driven as a generator at 750 rev/min.

Solution
The generated emf and hence the open-circuit characteristic can be derived from the given data:

$$E = \frac{N}{60}.\frac{p}{a}.Z\phi \Rightarrow 520 = \frac{750}{60}.\frac{2}{1}.400.\phi = 104\phi$$

Tabulate emf values:
Generated emf [V] 10 65 128 175 205 220 230 240
The emf may now be plotted against the field current. Plotting $E = 80I_f$ on the same axes gives an open-circuit terminal voltage of 228 V.

8.7 Load (or External) Characteristics of D.C. Generator

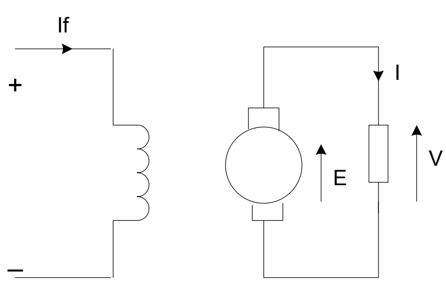
The external characteristic shows the variation of the terminal voltage with the load current at a given speed (usually the rated speed). When a generator is supplying power to a load the voltage-current curve of the load and the load curve of the generator can be plotted on the same graph to obtain the point of steady state operation which is the point of intersection of the two curves.

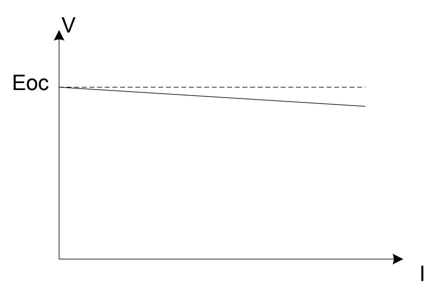
Separately-excited generator
The field current I_f is constant and speed is constant.

$$V = E - IR_a = E_{oc} - IR_a$$

where E_{oc} = open circuit emf and R_a = armature resistance or total resistance of armature circuit.

Voltage drop from no load to full load is usually less than 10 %



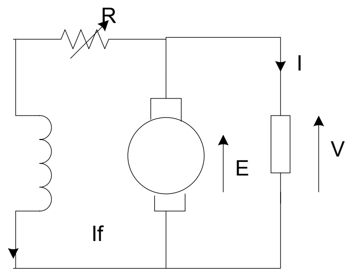


Generator circuit External characteristic

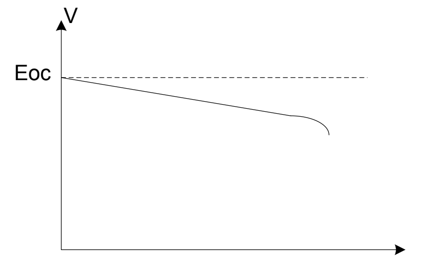
Shunt generator
Resistance in the field circuit is constant speed is also kept constant.
The fall in terminal voltage is due to
(a) I_aR_a drop and
(b) the reduction in field current.

The voltage drop due to the first results in a decreased field current which in turn results in a decreased air-gap flux ϕ and reduced emf, E. The fall in the terminal voltage is therefore more marked than with the separately excited generator. The voltage drop from no load to full load is about 15 %.

The field regulator can be used to regulate the terminal voltage V manually as the load current I changes



Generator circuit



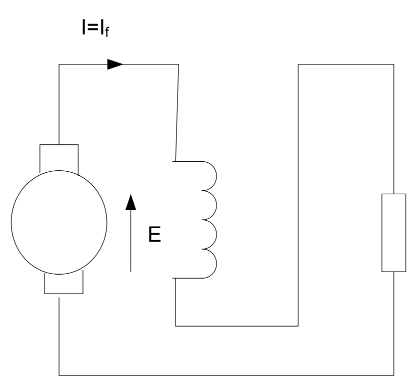
External characteristic

The external characteristic can be obtained following this procedure:
(a) For any value of V, calculate the field current $I_f = V/R_{sh}$ (R_{sh} = total resistance in the field circuit)
(b) From the O.C. curve, determine the corresponding generated emf, E
(c) Calculate the armature current, $I_a = (E - V)/R_a$
(d) Calculate the load current $I = I_a - I_f$

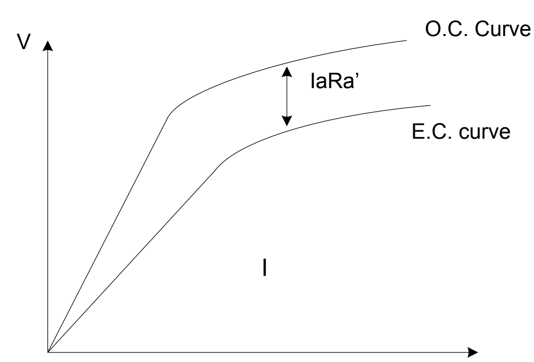
Series Generator

The external characteristic is readily deduced from O.C. curve by subtracting the total armature circuit resistance drop, $I_a R'_a$ (R'_a = total armature circuit resistance)

This generator is unsuitable when the voltage is to be maintained constant or approximately constant over a wide range of load current. Series generators are rarely used as self-excited generators.



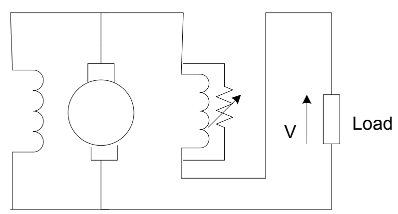
Generator circuit



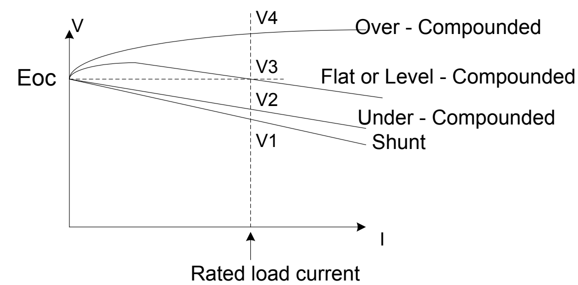
External characteristic (E.C.)

Compound generator

It is a shunt generator (i.e. the shunt field predominates) with an additional field winding in series with the armature to improve its external characteristics. The compounding is either cumulative (the two fields aid each other) or differential (the series field mmf opposes that of the shunt field). To achieve its internal purpose, compounding must be cumulative.



Generator circuit



External characteristics

The level-compounded generator has its terminal voltage practically constant from no load to full load. The over compounding is used when it is necessary to compensate not only for the armature circuit voltage drop but also for the drop in the feeder line between the generator and the load. The degree of compounding (over, level or under) may be adjusted by means of diverter connected across the series field. To obtain a very good approximation of the ohmic value of the diverter that will give any particular value, say V_2, the following steps are followed:

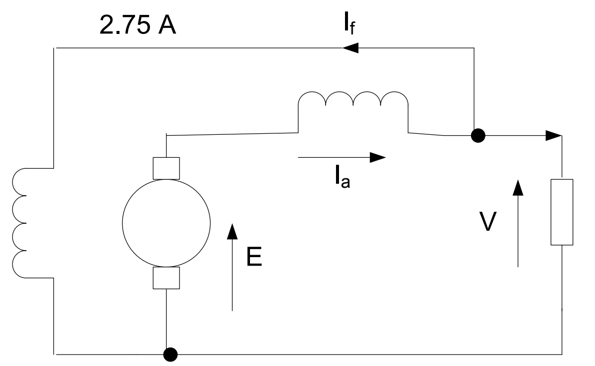
(a) Operate the machine as a shunt generator (i.e. series field not in circuit) at rated load I_{rated} and rated speed.
(b) Increase the field current I_f so as to raise the full load voltage from V_1 to V_2 and note the additional field ΔI_f required.
(c) Determine the series field current I_s required to produce the additional mmf: $N_s I_s = N_f \Delta I_f$ or $I_s = (N_f \Delta I_f)/N_s$ where N_f = number of turns of shunt field and Ns = number of turns of series field.
(d) Determine the value of the diverter:
Diverter current = load current – series field current: $I_d = I_{rated} - I_s$
Voltage drop across diverter = voltage drop across series field winding: $I_d R_d = I_s R_s$.
Thus
$R_d = (I_s R_s)/I_d$. R_d and R_s are the resistances of the diverter and the series field respectively.

Example 6

It is desired to level-compound a shunt generator so that the terminal voltage at full load of 100 A is equal to that at no load. When connected as a shunt generator the field current had to be increased from 2.75A to 3.3A in order to keep the terminal voltage the same at no load and full load. If the machine is to be connected long-shunt and the turns per pole on the shunt field is 1200, calculate the series turns per pole required. Neglect volt-drop in series winding.

The above machine, after its conversion, had a terminal voltage of 220 V at full-load current and the resistance of the armature; shunt and series fields were 0.12 Ω, 80 Ω and 0.08 Ω respectively. Calculate the value of the emf being generated under these conditions.

Solution



Additional mmf required $= N_f \Delta I_f = (3.3 - 2.75) \times 1200 = 660AT$

Current in series winding with full load $I_a = 100 + 2.75 = 102.75A$

Required number of series turns/pole $= \dfrac{6660}{102.75} = 6.42 \approx 7 turns$

With long-shunt, $E = V + I_a(R_a + R_s)$

$$I_a = I_f + I = \frac{V}{R_f} + I = \frac{220}{80} + 100 = 102.75\ A$$

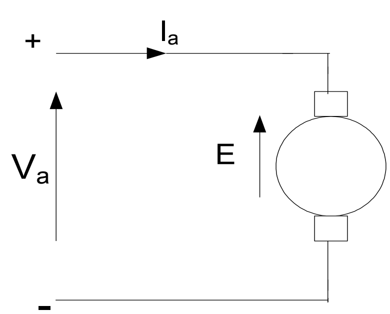
Therefore $E = 220 + 102.75(0.12 + 0.08) = 240.6\ V$

9.0 D.C. motors

The torque-speed characteristic of a motor must be adapted to the type of the load it has to drive. This requirement has given rise to three basic types of motors, namely, shunt motor, series motor and compound motor.

9.1 Torque of d.c. motor

Referring to the circuit below, the armature voltage equation is



$V_a = E + I_a R_a$

Where

V_a = voltage across the armature

I_a = armature current

E = back emf

From the equation, we can obtain

$V_a I_a = E I_a + I_a^2 R_a$

$V_a I_a$ represents the total electric power supplied to the armature and $I_a^2 R_a$ represents the loss due to the resistance of the armature circuit. The difference between these two quantities therefore represents the mechanical power developed by the armature.

If T_e is in newton-metres, then mechanical torque developed $= 2\pi T_e N/60$. Hence

$$\frac{2\pi T_e N}{60} = EI_a = \frac{N}{60}.\frac{P}{a}.Z\phi I_a$$

From which

$$T_e = \frac{1}{\pi}\frac{I_a}{2a}Zp\phi \qquad (5)$$

This expression holds equally for generators and motors.

9.2 Basic equations of dc machines (motors)

$$E = k_1 N\phi \qquad (6.a)$$

$$T_e = kI_a\phi \qquad (6.b)$$

Or

$$E = k\omega\phi \qquad (7.a)$$

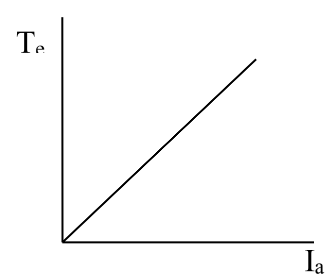
$$T_e = kI_a\phi \qquad (7.b)$$

9.3 Load characteristics of D.C. motors

(a) *Electromagnetic torque characteristic:* It shows the variation of T_e with armature current. The shape of the curve can be deduced from the fundamental torque equation $T_e = k\phi I_a$

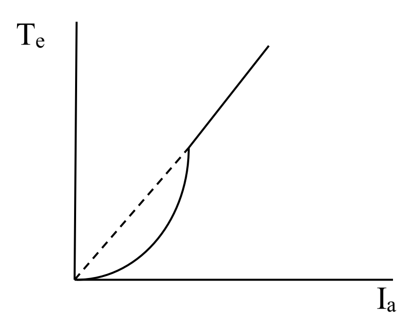
Shunt motor

The flux ϕ is constant and the torque equation becomes $T_e = k_2 I_a$ which is a linear relation



Series motor

When the magnetic circuit is not saturated ϕ is proportional to I_a and when saturated, $\phi \approx$ constant. Hence the initial portion of the torque-armature curve will be a parabola and ultimately it will merge into a straight line passing through the origin.



Compound motor

The basic torque equation is $Te = k(\phi_f + \phi_s)I_a$. The characteristic depends on the relative strengths of the two components.

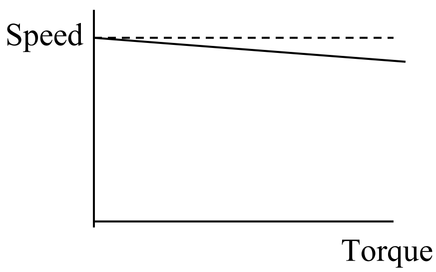
Fig.8 shows the torque-armature curves of dc motors of the same voltage, power and speed rating. For a given current below the full-load value the shunt motor exerts the largest torque, but for a given current above that value the series motor exerts the largest torque. The maximum permissible starting current is usually 1.5 to 2 times the full-load current. Consequently, where a large starting torque is required such as for hoists, cranes, electric trains etc, the series motor is the most suitable.

(a) *Speed-torque characteristics*: These characteristics can be derived from the two basic equations and the armature circuit voltage equation, $E = k\omega\phi$, $T_e = k\phi I_a$ and $E = V_a - I_a R$, where *R* includes external resistance in the armature circuit.

Shunt motor

$V_a = V$(applied voltage). Subtituting $E = k\omega\phi$, $I_a = T_e / k\phi$ and $V_a = V$ in the armature circuit voltage equation and rearranging, we obtain the relation between speed and torque as

$$\omega = \frac{V}{k\phi} - \frac{RT_e}{(k\phi)^2}$$



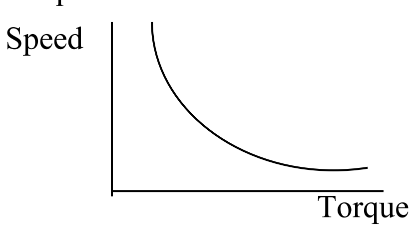
The difference between no-load and full-load speeds has typical values around 5% of full-load speed (In small machines the values are higher, (10 – 15%). Consequently, the dc shunt motor is considered a constant speed drive. A typical application is the lathe machine which requires constant speed and low starting torque.

Series motor

If the resistance in the armature circuit is neglected then $\omega = V/k\phi$ and if saturation is also neglected we obtain $\omega = V/k'I_a$. This and the torque equation lead to the following relation between speed and torque:

$$\omega = \frac{V}{\sqrt{Tek'}}$$

The speed of dc series motor varies widely with the torque. An increase of load calls for a decrease of speed. The series motor is considered a variable speed drive and its field of application is principally determined by this property. Typical cases are electric trains (runs uphill at a lower speed than on a level track), electric cranes and hoists (light loads are lifted quickly and heavy loads more slowly).



At no load the speed of a series motor may rise to dangerously high values. For this reason we never permit a series motor to operate at no load. Series motors are directly coupled or geared to a load as in hoists and cranes. (Note a series motor belt-coupled risks being run at no load). The extremely excessive speed at no load will give centrifugal forces which could tear the windings out of the armature and destroy the machine.

Fig. 8 shows the speed-torque curves of dc motors of the same voltage, power and speed rating.

Cumulative compound motors are widely used for individual drives of certain machines (eg lifts, winches etc) where a speed characteristic of the type possessed by the series motor is required; with the exception that the no-load speed must be limited to a safe value. The speed drop from no load to full load is generally between 10 and 30%.

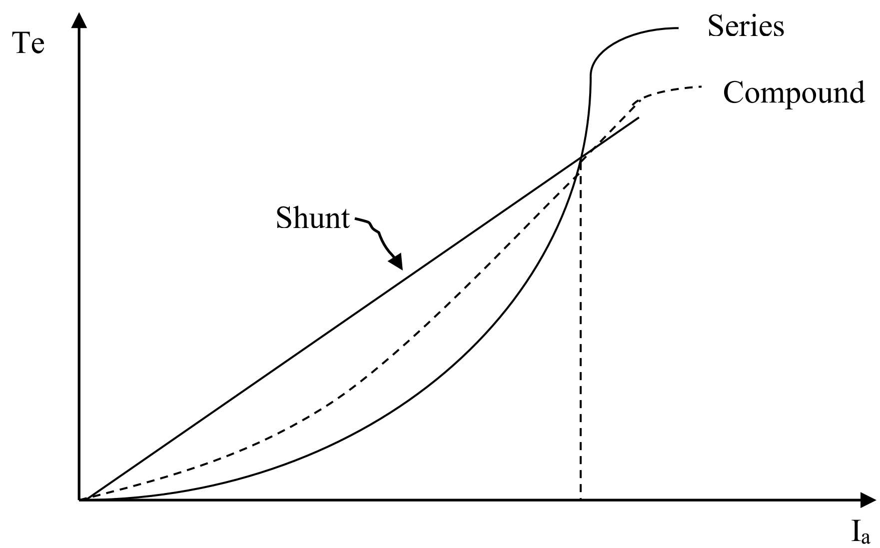


Fig. 8 Comparison of torque characteristics of dc motors

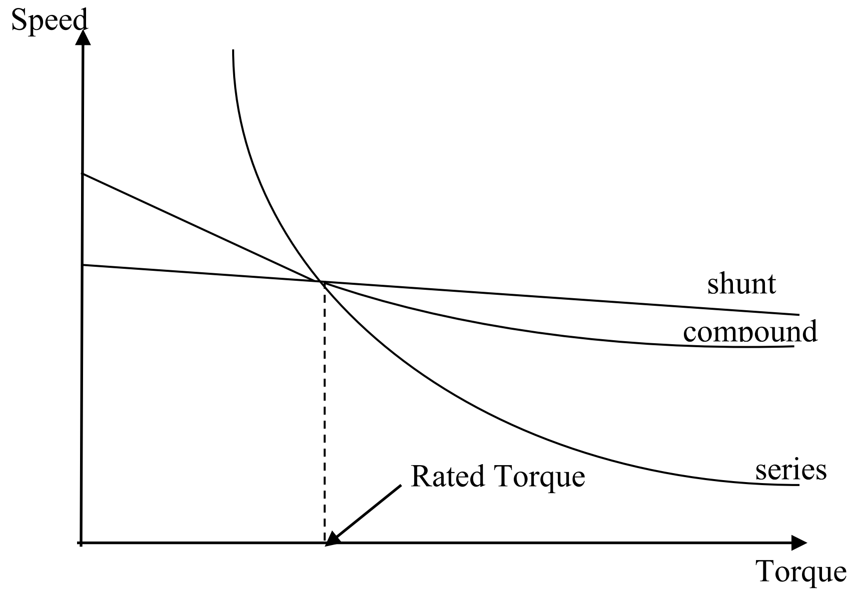


Fig. 9 Comparison of speed- torque curves of dc motors

Example 7
A shunt motor is running at 626 rev/min when taking an armature current of 50 A from a 440-V supply. The armature circuit has resistance of 0.28 Ω. If the flux is suddenly reduced by 5 % find
(a) the maximum value to which the current increases momentarily and the ratio of the corresponding torque to the initial torque
(b) the ultimate steady value of the armature current, assuming the torque due to the load to remain unaltered.

Solution
(a) Initial emf = 440 – 50 x 0.28 = 426V
Immediately after the flux is reduced by 5 %, i.e. before the speed has begun to increase, new emf = 426 x 0.95 = 404.7 V

Thus armature current $= \frac{V - E_{new}}{R_a} = \frac{440 - 404.7}{0.28} = 126A$

From $T_e = kI_a\phi$, we have

$$\frac{newtorque}{originaltorque} = \frac{newcurrent}{originalcurrent} \times \frac{new\,flux}{original\,flux} = \frac{126}{50} \times 0.95 = 2.394$$

Motor accelerates

(b) After the speed and current have attained steady values, the torque will have decreased to the original value, so that $newcurrent \times new\,flux = originalcurrent \times original\,flux$
Thus new armature current = 50 x 1/0.95 = 52.6A.

9.4 Starting of dc motor

Very small motors are started direct-on-line. For larger machines the armature resistance is small therefore applying full voltage to them when they are stationary results in a highly excessive armature current which can

(a) burn out the armature winding
(b) damage the commutator and brushes owing to heavy sparking
(c) overload the feeder
(d) snap off the shaft due to mechanical stock
(e) damage the driven equipment because of the sudden mechanical hammer blow.

The starting current must initially be limited to 1.5 to 2.5 times the rated current. If a variable-voltage supply is available (in some methods of speed control) the starting involves no additional equipment; but if the supply is at constant normal voltage it is necessary to absorb the excess by resistors in series with the armature. Robust metallic resistors arranged in series sections that can be cut out successively by manual or automatic operations are usually preferred.

An example of the manual starters is the faceplate starter. It is obsolete except for small motors up to 5 kW. Fig.10 shows the schematic diagram of a manual faceplate starter for a shunt motor. The current-limiting resistors are R_1, R_2, R_3 and R_4. Conducting arm 1 as it is pulled to the right by means of insulated handle 2 cuts out the resistors successively. Contact *M* is a dead contact. At this position, the motor circuit is open. The arm is held at the close position by a small electromagnet 4, which is in series with the shunt field. If the supply voltage should fail or the field excitation should be lost, the electromagnet releases the arm allowing it to return to its dead position, under the pull of spring 3. This safety feature prevents the motor from restarting unexpectedly when the supply voltage is re-established and also from being energized when the excitation is lost.

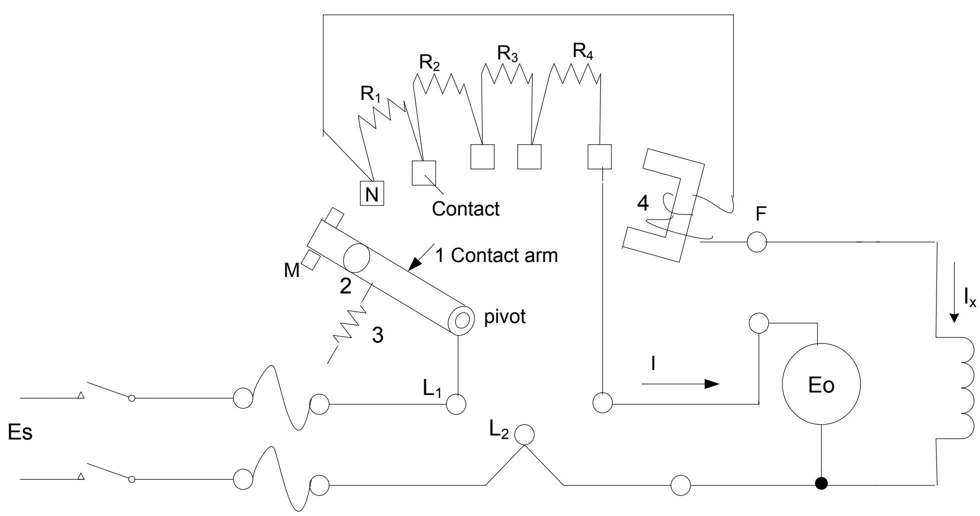


Fig. 10 Manual face-plate starter for a shunt motor

Example 8
A 240-V dc shunt motor has a full-load speed of 750 rev/min and a full-load armature current of 20A. The armature resistance is 1 **Ω**. The maximum armature current during the starting period is 30 A. Calculate the total resistance of the starter.

Solution
At starting the back emf = 0

Therefore $R_a + R_{st} = \frac{V}{I} = \frac{240}{30} = 8\ \Omega$

Therefore $R_{st} = 8 - 1 = 7\ \Omega$

9.5 Speed control of dc motor

The speed is given by the equation,

$$N = \frac{E}{k_1\phi} = \frac{V - I_a R_a}{k_1\phi}$$

Where V = the machine terminal voltage
R_a = armature resistance, but for series and compound motors R_a includes the resistance of the series filed winding.

According to the above equation, the speed of a dc motor can be changed by varying the flux, the terminal voltage or the resistance.

(a) Changing the armature resistance
An external variable resistor termed a controller is connected in series with the armature. For a given armature current, the larger the controller resistance in a circuit, the smaller the numerator of the speed equation and the lower, in consequence, is the speed. This method of speed control has the following disadvantages:
(i) The speed can be reduced below its base speed (i.e. speed when there is no external resistance).
(ii) It is relatively ineffective at no load
(iii) It is grossly wasteful of energy
(iv) The speed regulation is poor i.e. the speed may vary greatly with variation of load for a fixed setting of the controller. When used for example on shunt motor, the motor loses its constant speed property.

Its advantages are as follows:
(i) The method is simple
(ii) Speeds from zero upwards are easily obtainable

This method is only recommended for small motors because a lot of power is wasted in the controller and the overall efficiency is low.

(b) Changing the flux:
This method is frequently used when the motor has to run above its base speed. To control the flux a variable resistor termed the field regulator is connected in series with the shunt field winding in the case of shunt and compound motor and a variable resistor termed a

diverter is connected in parallel with the field winding in the case of series motor. Its disadvantages include the following:
(i) The speed can only be raised above its base value.
(ii) For shunt motors a high-speed/base-speed ratio of 3 to 1 is possible but when this speed range is exceeded instability and poor commutation result.

Advantages include the following:
(i) Method is economically sound.
(ii) The main motor characteristics are similar for all settings of the field strength. Thus working with full field or a reduced field, the shunt motor say maintains steady speed at all reasonable loads.

(c) Changing the motor terminal voltage:
The conventional method is known as the Ward-Leonard method. In this method the field winding of the motor is supplied from a constant–voltage source and the armature from a variable supply is shown in Fig.11. The control is effected by varying the excitation and hence the voltage of the generator. The potentiometer regulator indicated permits the variation of the exciting current from maximum to zero and in the reverse direction.

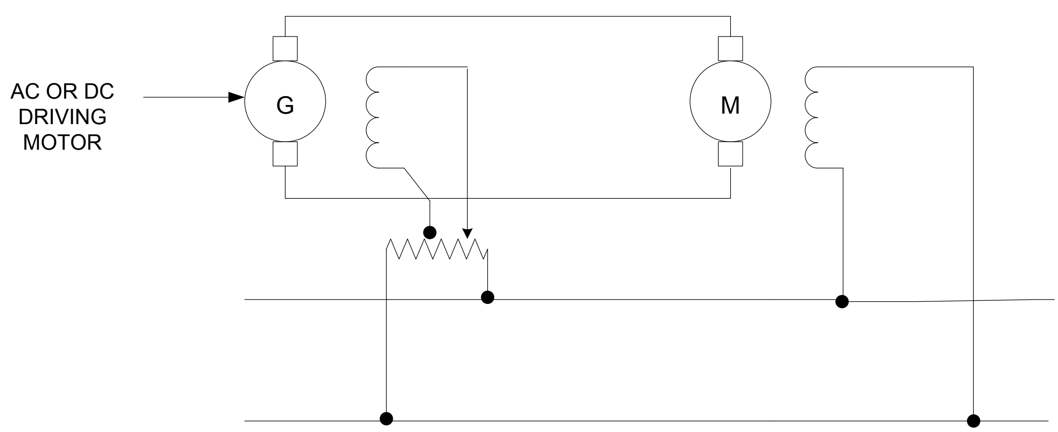


Fig. 11 Ward-Leonard speed control system

Its advantages include
(i) A wide range of speed from standstill to high speeds in either direction.
(ii) The main motor characteristics are similar.

Its disadvantage **i**s its high initial cost.

In modern installations high-power electronic converters are used to vary the armature terminal voltage. These are controlled rectifiers usually for shunt motors and chopper for the series motor. Ward-Leonard method of speed control is used for large reversing motors found in steel mills, high–rise elevators, mines and paper mills.

Example 9

A series motor runs from a 400 V direct current supply and has a total armature and series field resistance of 0.2 Ω. A variable resistance, *R*, is connected in series with the motor for speed adjustment. With a given load and *R = 0*, the current is 25 A and the speed 1000 rev/min. On another load with *R = 2* Ω, the current is 20 A. Calculate the new speed. Assume that the field flux is proportional to the current.

Solution

First condition: $E_1 = V - I_1R_1 = 400 - 25\times 0.2 = 395\,V$

Second condition $E_2 = V - I_2R_2 = 400 - 25\times 2.2 = 356\,V$

Now $E \propto \phi N \propto I_a N$. Therefore

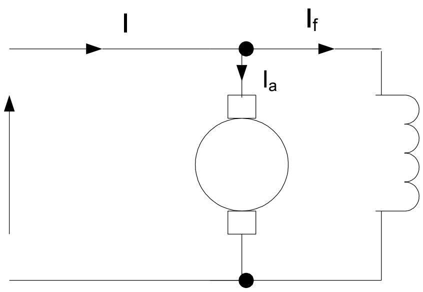
$$\frac{E_2}{E_1} = \frac{I_2N_2}{I_1N_1} \Rightarrow N_2 = \frac{E_2}{E_1}\times\frac{I_1}{I_2}\times N_1$$

$$N_2 = \frac{356}{395}\times\frac{25}{20}\times 1000 = 1128\,rev/\min$$

Example 10

A shunt motor, which has a field resistance of 200 Ω and an armature resistance of 0.8 Ω, takes 26 A from a 200 V supply when running at 500 rev/min on full load. In order to control the speed of the motor a 1.2 Ω resistor is connected in series with the armature. Calculate the speed at which the motor will run when supplying full-load torque.

Solution



$$I_f = \frac{200}{200} = 1\,A$$

Without external resistor

$I_{a1} = I - I_f = 26 - 1 = 25\,A$

And $E_1 = V - I_{a1}R_a = 200 - 25\times 0.8 = 180V$

With external resistor *R* connected:

The torque has to remain constant and the flux from the field is constant

Therefore $I_{a2} = I_{a1} = 25\ A$ and

$$E_2 = V - I_{a2}(R_a + R) = 200 - 25(0.8 + 1.2) = 150\,V$$

Now $E \propto \phi N$ and ϕ is constant.

Therefore $\frac{E_2}{E_1} = \frac{N_2}{N_1} \Rightarrow N_2 = N_1.\frac{E_2}{E_1} = 500\times\frac{150}{180} = 417\,rev/\min$

10. Efficiency of dc machines

The efficiency is calculated using the formula

$$\eta = \frac{Output}{Output + losses} = \frac{Input - Losses}{Input} = 1 - \frac{Losses}{Output + losses}$$

The power rating of machine corresponds to the useful output power, electrical or mechanical depending on whether the machine is a generator or motor.

In rotating machines both mechanical and electrical losses are produced. In dc machines, they are:

(a) Electrical losses

(i) Armature circuit copper loss ($I_a^2 R$). R = total armature circuit resistance.

(ii) Shunt field circuit copper loss ($V_f I_f$).

(iii) Brush contact loss (voltage contact drop x I_a). The value of the voltage contact drop depends on the type of brush and the pressure applied. When not given the usual value of 2 V is assumed.

(iv) Iron losses: They are produced in the armature core. They are due to hysterisis and eddy current. Iron losses depend upon flux density, the speed of rotation, the quality of the steel and the size of the armature. For a given machine, if the speed is constant then the iron loss varies approximately as (flux density)^2 and hence as the (applied voltage)^2

(b) Mechanical losses

(i) brush friction

(ii) bearing friction

(iii) windage

The friction losses depend on the design of the bearings, brushes and commutators. Windage losses depend on the speed, the design of the cooling fan and on the turbulence produced by revolving parts.

For a given machine, the mechanical loss is constant for a constant speed.

Example 11

A 100-kW, 460-V shunt generator was run as a motor on no load at its rated voltage and speed. The total current was 9.8 A including a shunt current of 2.7 A. The resistance of the armature circuit at normal working temp was 0.1 **Ω**. Calculate the efficiency at full load.

Solution

Output current at full load $100 \times 1000 / 460 = 217.5\ A$

Thus I_a at full load = $I + I_f$ = 217.5 + 2.7 = 220.2 A

Copper loss in armature circuit at full load = $(220.2)^2$ x 0.11 = 5325 W

Loss in shunt circuit = 2.7 x 460 = 1242 W

Armature current on no load as a motor = 9.8 – 2.7 = 7.1 A

Fixed losses (neglecting no load armature loss) = 7.1 x 460 = 3265 W

Input power at full load = output + losses = $100| + 5.325 + 1.242 + 3.265 = 109.832 kW$

Efficiency = $\dfrac{output}{input} = \dfrac{100}{109.832} = 0.9105 = 91.05\%$

Chapter Three

Transformers

1. Introduction

The transformer transfers electrical energy from one circuit to another via the medium of a pulsating magnetic field that links both circuits. The widespread development of ac power systems is principally due to the transformer. It enables us to produce and transmit power at economical voltages and to distribute it safely in factories and homes. In low-power low-current electronic and control circuits, it is used to provide impedance matching between a source and its load for maximum power transfer, to isolate one circuit from another, to isolate direct current while maintaining ac continuity between two circuits and to provide reduced ac voltages and currents for protection, metering, instrumentation and control.

2. Principle of operation of 1-phase transformers

The transformer is a straight-forward application of Faraday's Law of Electromagnetic Induction. Consider the general arrangement of a single-phase transformer shown in Fig. 1. An alternating voltage applied to coil 1, causes an alternating current to flow in the coil and this current produces an alternating flux in the iron core. A portion of the total flux links the second coil. The alternating flux induces a voltage in the second coil. If a load should be connected to the coil, this voltage would drive a current through it. Energy would then be transferred through the medium of magnetic field from coil 1 to coil 2. The combination of the two coils is called a transformer. The coil connected to the source is called the primary winding (or the primary) and the one connected to the load is called the secondary winding (or the secondary).

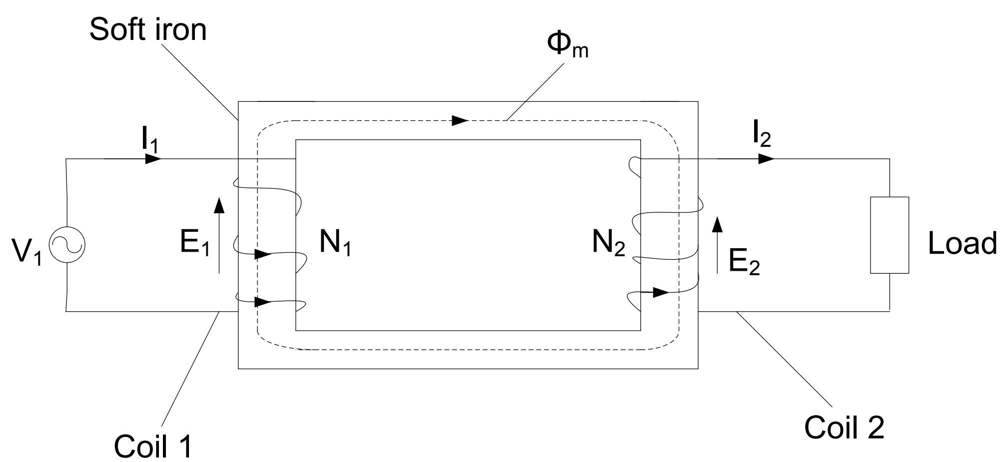
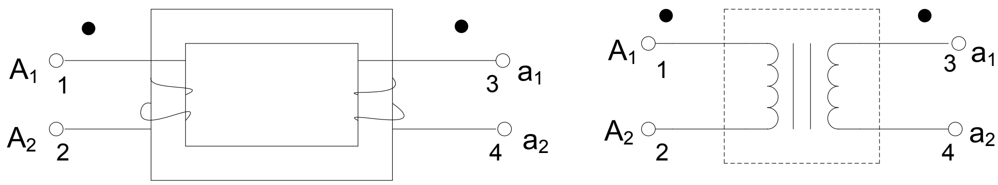


Fig. 1 An elementary transformer

3. Polarity and terminal markings of a transformer

Voltage E_1 is induced in coil 1 and voltage E_2 in coil 2. These voltages are in phase. Suppose at any given instant when the primary terminal 1 is positive with respect to primary terminal 2, the secondary terminal 3 is also positive with respect to secondary terminal 4 (See Fig. 2). Then terminals 1 and 3 are said to have the same polarity. To indicate that their polarities are the same, a dot is placed beside primary terminal 1 and secondary terminal 3. Alternatively, letters of the same suffix, A_1 (for the high-voltage winding) and a_1 (for the low-voltage winding) say can be used. Current I_1 entering coil 1 through the dotted terminal 1 and current I_2 entering coil 2 through the dotted terminal 3 create fluxes in the same direction.



(a) Magnetic Circuit (b) Electric Circuit

Fig. 2 Polarity and terminal markings

4.0 Ideal transformer

An ideal transformer has no losses, no leakage flux and its core is infinitely permeable. An ideal transformer is shown in Fig. 3. The mutual flux Φ_m is confined to the iron.

($I_1 = 0$ on no load)

Fig. 3 An ideal transformer

4.1 Properties of an ideal transformer

(a) Emf equation and voltage ratio (see Fig. 3): With the primary connected to an ac source V_1, an alternating flux Φ_m is produced in the core. Let the flux be expressed as

$$\Phi_m = \Phi_{\max} \sin \omega t \quad \text{webers} \tag{1}$$

The induced emf e_1 as indicated in the figure is given by

$$e_1 = \frac{d}{dt}\left(N_1 \Phi_m\right) = N_1 \frac{d}{dt}\left(\Phi_{\max} \sin \omega t\right) = N_1 \Phi_{\max} \omega \cos \omega t$$

Or

$$e_1 = \omega N_1 \Phi_{\max} \sin\left(\omega t + 90^\circ\right) \tag{2}$$

Hence

$$E_1 = \frac{\omega N_1 \Phi_{\max}}{\sqrt{2}} = \frac{2\pi f N_1}{\sqrt{2}} \Phi_{\max}$$

Or

$$E_1 = 4.44 f N_1 \Phi_{\max} \tag{3}$$

Similarly

$$E_2 = \frac{\omega N_2 \Phi_{\max}}{\sqrt{2}} = \frac{2\pi f N_2 \Phi_{\max}}{\sqrt{2}} = 4.44 f N_2 \Phi_{\max} \tag{4}$$

From (3) and (4), we obtain

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \tag{5}$$

The ratio $a = N_1/N_2$ is called the turns ratio. A step-up transformer has $a < 1$ and a step-down transformer has $a > 1$. In an ideal transformer, the applied voltage V_1 and the induced voltage E_1 must be identical. Hence we may write

$$V_1 = E_1 \tag{6.a}$$

And

$$V_1 = 4.44 f N_1 \Phi_{\max} \tag{6.b}$$

Equation (6.b) indicates that for a given frequency, number of turns and voltage, the peak flux $\Phi_{\max}$ must remain constant.

(b) Current ratio and power equation: On no load $I_1 = 0$. Now if a load is connected across the secondary terminals (i.e. switch *S* is closed) current I_2 flows through the load. This current produces *mmf* N_2I_2 which if it acted alone would by Lenz's law, cause the mutual flux to reduce. Since when V_1 is fixed the flux Φ_{max} is also fixed, the primary develops *mmf* N_1I_1 which is such that

$$N_1 I_1 = N_2 I_2 \tag{7.a}$$

Or

$$I_1 = I_2 / a \tag{7.b}$$

In ideal transformer the secondary voltage

$$V_2 = E_2 \tag{8}$$

remains constant since E_2 is fixed when the Φ_{max} is fixed. It can be deduced from above equations that for an ideal transformer

$$V_1 I_1 = V_2 I_2 \tag{9}$$

That is there are no reactive and active losses in an ideal transformer.

(c) Phasor diagram of an ideal transformer (Fig. 4)

(a) No load (b) Load is power factor lagging

Fig. 4 Phasor diagram of an ideal transformer

Example 1

An ideal transformer having 90 turns on the primary and 2250 turns on the secondary is connected to a 200-V, 50-Hz source. The load across the secondary draws a current of 2 A at a power factor of 0.80 lagging. Calculate (a) the rms value of the primary current (b) the flux linked by the secondary winding (c). Draw the phasor diagram.

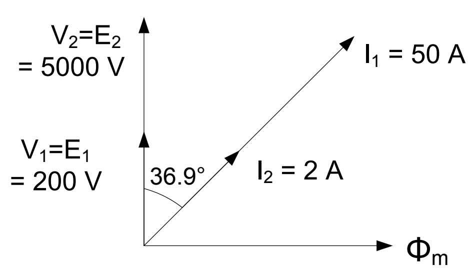
Solution

$$(a) \quad N_1 I_1 = N_2 I_2 \Rightarrow 90 I_1 = 2250 \times 2 \quad \Rightarrow I_1 = 50A$$

$$(b) \quad \Phi_{\max} = \frac{V_1}{4.44 f N_1} = \frac{200}{4.44 \times 50 \times 90} = 0.01 Wb$$

$$(c) \quad E_2 = \left(\frac{N_2}{N_1}\right) E_1 = 5000V$$

The phase angle between V_2 and I_2 is $\cos^{-1}(0.8) = 36.9°$



Example 2
A 200-kVA, 6600-V / 400-V, 50-Hz 1-ph transformer has 80 turns on the secondary. Calculate (a) the approximate values of the primary and secondary currents (b) the approximate number of primary turns and (c) the maximum value of the flux.

Solution

(a) Full-load primary current $\cong rated\ power / 1^e\ rated voltage = (200\times1000)/6600 = 30.3\,A$

and full-load secondary current $= rated\ power / 2^e\ rated voltage = (200\times1000)/400 = 500\,A$

$(b)\quad N_1 \cong (80\times6600)/400 = 1320$

$$(c)\quad \Phi_{max} = \frac{V_2}{4.44 f N_2} = \frac{400}{4.44\times50\times80} = 0.0225\,\text{Wb}$$

(d) Impedance ratio (see Fig. 5): The impedance seen by the source

$Z_e = V_1/I_1 = E_1/I_1 = aE_2/(I_2/a) = a^2\,E_2/I_2 = a^2 Z$

$\therefore Z_e = a^2 Z$ (10)

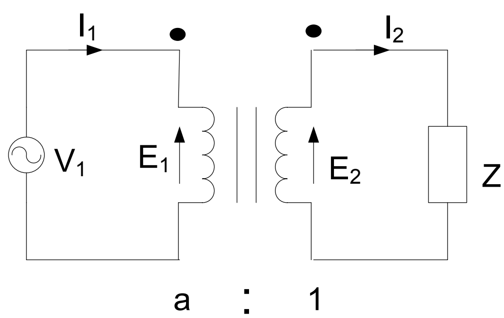


Fig. 5 impedance ratio

(e)Equivalent circuits of an ideal transformer: From (10), we can represent the transformer in Fig. 5 by equivalent circuit shown in Fig. 6.a. We may also write

$I_2 Z = E_2 = E_1/a = V_1/a$

Or

$$\frac{V_1}{a} = I_2 Z \qquad (11)$$

and then represent the transformer by an equivalent circuit shown in Fig. 6.b

(a) Circuit referred to the primary side (b) circuit referred to the secondary side

Fig. 6 Equivalent circuits of ideal transformer

Example 3
Calculate the voltage *V* and current *I* in the circuit of Fig. 7, knowing that the ideal transformer has a primary to secondary turns of 1:100 (i.e. a = 1/100).

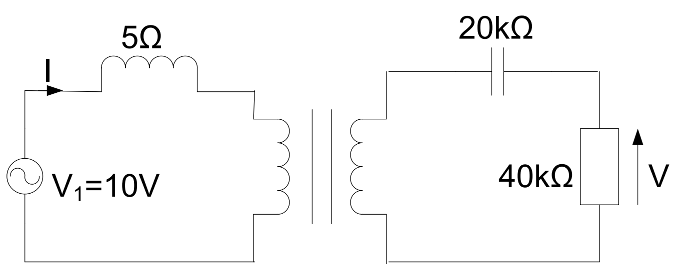
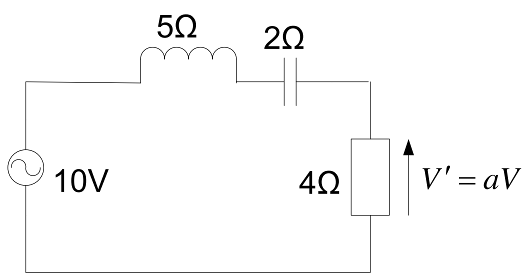


Fig. 7 See Ex. 3

Solution:
We shall shift all impedances to the primary side



$$Z_e = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{4^2 + 3^2} = 5\Omega$$

$$I = \frac{V_1}{Z_e} = \frac{10}{5} = 2\,A$$

$V' = IR = 2 \times 4 = 8V$. The actual voltage $V = \dfrac{V'}{a} = 100 \times 8 = 800\text{V}$

5.0 Practical single-phase transformer
The windings of a practical transformer have both resistance and leakage inductance. The core is also imperfect: it has a core loss and finite permeability. The core loss consists of hysteresis and eddy current loss.

5.1 Equivalent circuit of a practical transformer

The behaviour of a practical transformer may be conveniently considered by assuming it to be equivalent to an ideal transformer and then allowing for the imperfections of the actual transformer by means of additional circuits or impedances inserted between the supply and the primary winding and between the secondary winding and the load. The complete equivalent circuit of this transformer is shown in Fig. 8. R_1 and R_2 are resistances of the primary and secondary windings. X_1 and X_2 are the leakage reactances. The reactance X_m is such that it takes a reactive current I_m (i.e. the magnetizing current) of the actual transformer. The core loss is accounted for by the resistor R_m which takes the component I_p of the primary current.

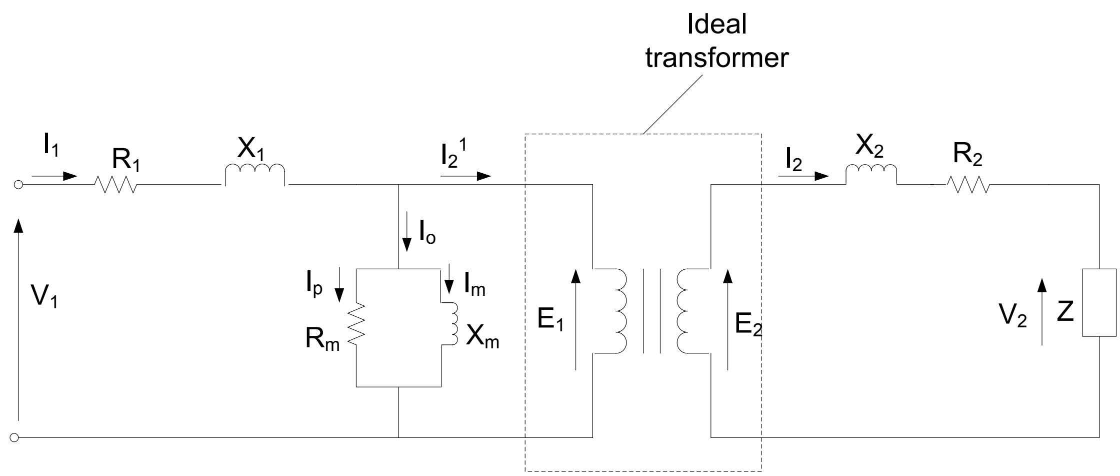


Fig. 8 Complete equivalent circuit of a practical transformer

Ideal transformer equations still apply: $I_2' N_1 = N_2 I_2$ and $E_1/E_2 = N_1/N_2$

5.2 Equivalent circuit referred to the primary side (Fig. 9)

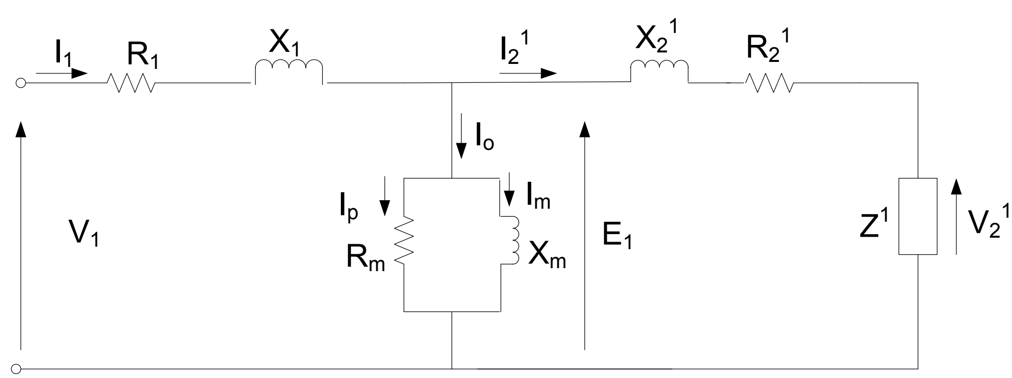


Fig. 9.Exact equivalent circuit referred to the primary side

Let $a = N_1/N_2$ (11)

Then

$$R_2' = \left(\frac{N_1}{N_2}\right)^2 R_2 = a^2 R_2 \text{ , etc.} \tag{12.a}$$

$$V_2' = \left(\frac{N_1}{N_2}\right) V_2 = aV_2 \qquad I_2' = \frac{I_2}{a} \; etc \tag{12.b}$$

It is worth noting that for a practical transformer, $R_2' \cong R_1 \quad and \quad X_2' \cong X_1$

5.3 Equivalent circuit referred to the secondary side (Fig. 10)

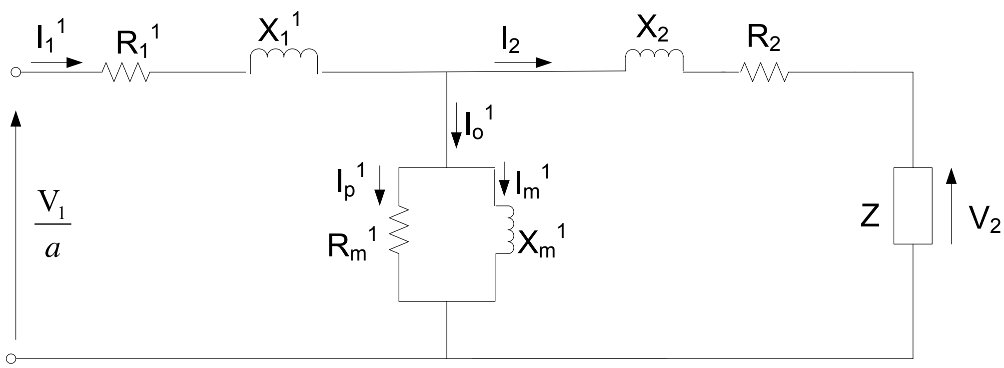


Fig. 10 Exact equivalent circuit referred to the secondary side

Let

$$a = \frac{N_1}{N_2}. \tag{13}$$

Then

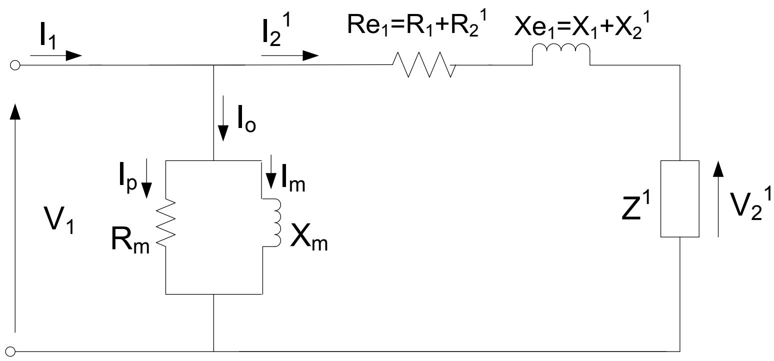
$$R_1' = \left(\frac{N_2}{N_1}\right)^2 R_1 = \frac{R_1}{a^2} \text{ , etc} \tag{14.a}$$

$$N_2 I_1' = N_1 I_1 \quad or \; I_p' = aI_1 \; etc.$$

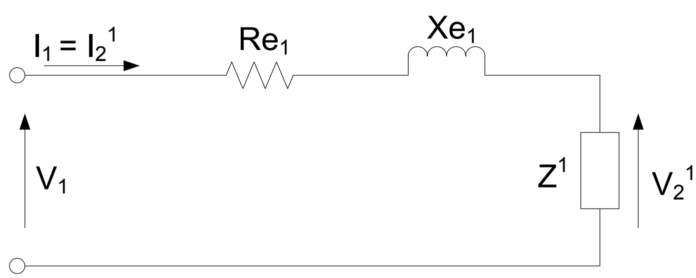
(14.b)

5.4 Approximate equivalent circuits

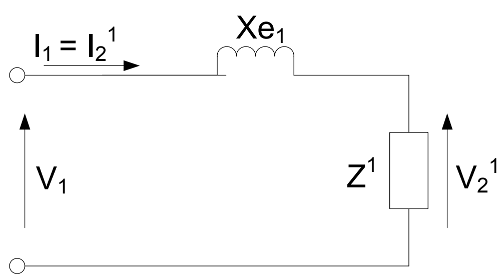
The exact equivalent circuit of the transformer is far more exact than needed for most practical applications. Consequently, we can simplify it to make calculations easier. Approximate equivalent circuits commonly used for power transformer calculations are given in Fig. 11



(a) Voltage drop in the primary leakage impedance due to exciting or no load current I_o is neglected.



(b) Exciting current is neglected entirely. Note that I_{1fl} (primary rated or full load current) is at least 20 times larger than I_o.



(c) For transformers above 500 kVA, X_e is at least 5 times greater than R_e. This circuit can be used to calculate voltage regulation of such transformers.

Fig. 11 Approximate equivalent circuits

5.5 Rating of transformers

To keep the transformer temperature at an acceptable level, limits are set to both the applied voltage (this determines the iron loss at a given frequency) and the current drawn by the load (this determines copper loss). The limits determine the rated voltage and rated current of transformers.

The power rating of transformer (S_{rated} = rated voltage x rated current) can be expressed in VA, kVA or MVA depending on the size of the transformer. The rated power, frequency and voltage are always shown on the name plate. In large transformers, the corresponding currents are also shown. We note that

$Rated\ kVA = V_{1r} \times I_{1fl} \times 10^{-3} = V_{2r} \times I_{2fl} \times 10^{-3} = V_{2n} \times I_{2fl} \times 10^{-3} = E_{2n} \times I_{2fl} \times 10^{-3}$

Where

V_{1r} = rated primary voltage

I_{1fl} = rated primary current = primary full load current

V_{2r} = rated secondary voltage = V_{2n} (no load secondary voltage corresponding to the rated primary voltage) = E_{2n} (no load induced secondary voltage corresponding to primary rated voltage)

I_{2fl} = rated secondary current = secondary full load current

5.6 The turns ratio

It is given by

$a = E_1 / E_2$

$E_1 \cong V_{1r}$ on no load because $I_o Z_1$ is very small. Since $\mathrm{E_2 = V_2}$ on no load

$$a \cong V_{1r} / V_{2r} \tag{15.a}$$

And $I_{1fl} = I_{2fl} / a$ (15.b)

Example 4

A transformer is rated 10 kVA, 2400 / 240 V, 60 Hz. The parameters for the approximate equivalent circuit of Fig. 11.a are $R_m = 80\ k\Omega$, $X_m = 35\ k\Omega$, $R_{e1} = 8.4\ \Omega$ and $X_{e1} = 13.7\ \Omega$. Determine the voltage to be applied to the primary to obtain the rated current in the secondary when the secondary terminal voltage is 240 V. What is the input power factor? The load power factor is 0.8 lagging.

Solution

$I_2' = I_{2fl} / a = I_{1fl} = 10000/2400 = 4.17\,A$

The power factor angle $\phi = \cos^{-1}(0.8) = 36.9°$

If we choose the load current $\boldsymbol{I_2}$ as the reference phasor, then $\boldsymbol{V}_2 = 240\angle 36.9°$

$\boldsymbol{V_2}' = a\boldsymbol{V_2} = 2400\angle 36.9° = 1920 + j1440$ volts

The voltage across the equivalent leakage impedance is

$\boldsymbol{Z_{e1} I_2}' = (8.4 + j13.7)(4.17 + j0) = 35 + j57$ volts

The primary voltage required

$\boldsymbol{V_1} = \boldsymbol{V_2}' + \boldsymbol{Z_{e1} I_2}' = (1920 + j1440) + (35 + j57) = 1955 + j1497 = 2462\angle 37.4°$ volts

$$\boldsymbol{I}_p = \frac{\boldsymbol{V_1}}{R_m} = \frac{1955 + j1497}{80k\Omega} = (24.4375 + j18.7125)\,\text{mA}$$

$$\boldsymbol{I}_m = \frac{\boldsymbol{V}_1}{jX_m} = \frac{1955 + j1497}{j35k\Omega} = (42.7714 - j55.8571)\,\text{mA}$$

$$\boldsymbol{I}_1 = \boldsymbol{I}_2' + \boldsymbol{I}_p + \boldsymbol{I}_m = (4.17 + j0) + (0.0244 + j0.0187) + (0.0427 - j0.0558)$$

$$= 4.2371 - j0.0371 = 4.237\angle -0.50°\ \text{A}$$

Phase angle between $\mathbf{V_1}$ and $\mathbf{I_1} = 37.4 - (-0.50) = 37.9°$

Input power factor $= \cos(37.9°) = 0.79$ lagging

Example 5

A 1-ph transformer operates from a 230-V supply. It has an equivalent resistance of 0.1 Ω and an equivalent leakage reactance of 0.5 Ω referred to the primary. The secondary is connected to a coil having a resistance of 200 Ω and a reactance of 100 Ω**.** Calculate the secondary terminal voltage. The secondary winding has four times as many turns as the primary.

Solution

Refer to approximate equivalent circuit of Fig. 11.b

$$a = \frac{N_1}{N_2} = \frac{1}{4}, \quad \mathbf{Z} = (200 + j100)\Omega, \quad \mathbf{Z}' = a^2(200 + j100) = 12.5 + j6.25\,\Omega$$

$$\text{Total impedance} = \mathbf{Z_{e1}} + \mathbf{Z}' = (0.1 + j0.5) + (12.5 + j6.25) = 12.6 + j6.75$$

$$\boldsymbol{I}_2' = \frac{\mathbf{V_1}}{\mathbf{Z_{e1}} + \mathbf{Z}'} = \frac{230\angle 0°}{12.6 + j6.75} \quad \textit{and} \quad \boldsymbol{V}_2' = \boldsymbol{I}_2'\mathbf{Z}' = \frac{230\angle 0°}{12.6 + j6.75}(12.5 + j6.25)$$

$$|\boldsymbol{V}_2'| = \frac{230 \times |12.5 + j6.25|}{|12.6 + j6.75|} = \frac{230 \times 13.9754}{14.2941} = 224.8719\ \text{volts}$$

$$V_2 = \frac{V_2'}{a} = 4 \times 224.8719 = 899\ \text{volts}$$

5.7 Definition of per-unit impedances

The leakage impedances Z_1 and Z_2 on the primary and secondary side are expressed in per unit as follows:

$$Z_{p.u.} = Z_1 \frac{I_{1fl}}{V_{1r}} = Z_1 \frac{S_{rated}}{V_{1r}^2} = \frac{Z_1}{Z_{1base}}$$

(16.a)

$$Z_{p.u.} = Z_2 \frac{I_{2fl}}{V_{2r}} = Z_2 \frac{S_{rated}}{V_{2r}^2} = \frac{Z_2}{Z_{2base}}$$

(16.b)

where

$$Z_{1base} = \frac{V_{1r}^2}{S_{rated}} \quad \textit{and} \quad Z_{2base} = \frac{V_{2r}^2}{S_{rated}}$$

(16.c)

The impedances are said to be expressed in per unit with reference to the bases V_{1r}, S_{rated} in the case of Z_1 and V_{2r}, S_{rated} in the case of Z_2. The total impedance of the transformer in per unit $= \hat{Z}_1 + \hat{Z}_2$.

Example 6
A single-phase transformer that is rated 3000 kVA, 69 kV / 4.16 kV, 60 Hz has an impedance of 8 percent. Calculate the total impedance of the transformer referred to (a) the primary side (b) the secondary side

Solution

$$(a) \quad Z_{1base} = \frac{V_r^2}{S_{rated}} = \frac{69^2 \times 10^6}{3000 \times 10^3} = 1587\Omega$$

$$Z_{e1} = \hat{Z}_{e1} \times Z_{1base} = 0.08 \times 1587 = 127\Omega$$

$$(b) \quad Z_{2base} = \frac{V_{2r}^2}{S_{rated}} = \frac{4.16^2 \times 10^6}{3000 \times 10^3} = 5.7685$$

$$Z_{e2} = \hat{Z}_{e2} \times Z_{2base} = 0.08 \times 5.7685 = 0.46\Omega$$

Alternatively

$$Z_{e2} = \left(\frac{N_2}{N_1} \right)^2 Z_{e1} = \left(\frac{4.16}{69} \right)^2 \times 127 = 0.46\Omega$$

5.8 Voltage regulation
With the primary voltage maintained constant, the secondary terminal voltage at no load differs from the secondary voltage under load. The voltage regulation or simply regulation of a transformer is the change in secondary voltage which occurs when the rated kVA output at a specified power factor is reduced to zero, with the primary voltage maintained constant. It is usually expressed as a percentage (called percentage regulation) or a fraction of the rated no-load terminal voltage (in per unit).

The equivalent circuit given in Fig. 11.b is used to calculate voltage regulation. The circuit may be either referred to the primary or the secondary side. The circuit in general form can thus be represented as shown in Fig. 12.

If the circuit is referred to the primary side, then

$$E = V_1 \ , \quad V = V_2' \ , \quad R_e = R_{e1} \ \ and \quad X_e = X_{e1}$$

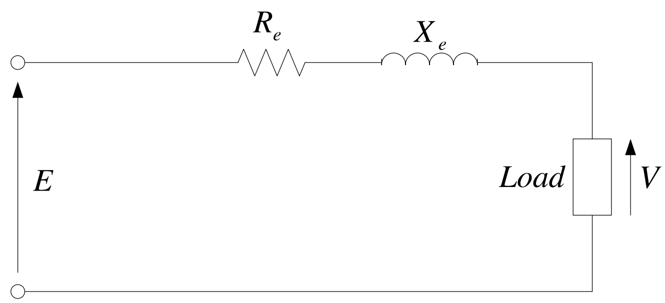


Fig. 12 Circuit for computing voltage regulation

Voltage regulation $\varepsilon = \dfrac{E - V}{E} = \dfrac{V_1 - V_2'}{V_1} \quad pu$ (17.a)

If the circuit is referred to the secondary side, then

$E = V_{2n} = \dfrac{V_1}{a}, \quad V = V_2, \quad R_e = R_{e2} \quad and \quad X_e = X_{e2}$

Voltage regulation $\varepsilon = \dfrac{V_{2n} - V_2}{V_{2n}} \quad pu$ (17.b)

It can be shown that $\varepsilon = \dfrac{V_1 - V_2'}{V_1} = \dfrac{V_{2n} - V_2}{V_{2n}}$ (18)

And $V_2 = [1 - \varepsilon]V_{2n}$ (19)

In general, let the load current be I lagging behind the load voltage V by ϕ. Then taking the load voltage as the reference phasor, we can write

$$\boldsymbol{E} = V(1 + j0) + (R_e + jX_e)(I\cos\phi - jI\sin\phi)$$
$$= (V + IR_e\cos\phi + IX_e\sin\phi) + j(IX_e\cos\phi - IR_e\sin\phi)$$

And hence

$$E = |\boldsymbol{E}| = \sqrt{(V + IR_e\cos\phi + IX_e\sin\phi)^2 + (IX_e\cos\phi - IR_e\sin\phi)^2}$$

The second term under the root is usually negligible except at low leading power factors. Considering the first term only gives

$$E = V + IR_e\cos\phi + IX_e\sin\phi$$

Or

$$E - V = IR_e\cos\phi + IX_e\sin\phi \qquad (20)$$

The angle ϕ is negative when current is leading and positive when current is lagging.

The voltage regulation is maximum when $\phi = \theta$ where

$$\theta = \tan^{-1}\left(\frac{X_e}{R_e}\right) \qquad \left[From\ \frac{d}{d\phi}(E-V) = IR_e(-\sin\phi) + IX_e\cos\phi = 0\right]$$

Hence $(E-V)_{\max} = IR_e\left(\frac{R_e}{Z_e}\right) + IX_e\left(\frac{X_e}{Z_e}\right) = I\left(\frac{R_e^{\,2}+X_e^{\,2}}{Z_e}\right) = I\,Z_e$ (21.a)

Where $Z_e = \sqrt{R_e^{\,2}+X_e^{\,2}}$ (21.b)

From (20)

$$\varepsilon = \frac{E-V}{E} = \left(\frac{I_{fl}}{I_{fl}}\right)\left(\frac{IR_e}{E}\right)\cos\varphi + \left(\frac{I_{fl}}{I_{fl}}\right)\left(\frac{IX_e}{E}\right)\sin\varphi$$

Or

$$\varepsilon = \frac{E-V}{E} = I_{p.u.}R_{p.u.}\cos\varphi + I_{p.u.}X_{p.u.}\sin\varphi \qquad (22)$$

Where

$I_{p.u.} = I/I_{fl}$ = current in per unit $R_{p.u.} = IR_e/E$ = resistance in per unit, etc

We note the following:

(i) Usually the quantities will be referred to the secondary side

(ii) With (22), it is not necessary to refer quantities from primary to secondary side, for per-unit values of primary and secondary impedances can be added directly

(iii) The equations are correct at any current or kVA and at rated current or kVA.

(iv) At rated current or kVA, $I_{p.u.}$=1.

(v) It is supposed that E is the rated voltage. When E is not the rated voltage, voltage regulation can still be calculated using (20).

Example 7

A 100-kVA 1-ph transformer has 400 turns on the primary and 80 turns on the secondary. The primary and secondary resistances are 0.3 Ω and 0.01 Ω respectively, and the corresponding leakage reactances are 1.1 Ω and 0.035 Ω respectively. The supply voltage is 2200 V. Calculate (a) the equivalent impedance referred to the primary circuit and (b) the voltage regulation and the secondary terminal voltage for full load having a power factor of (i) 0.8 lagging and (ii) 0.8 leading (c) the maximum voltage regulation

Solution

$(a)\quad R_{e1} = R_1 + (N_1/N_2)^2 R_2 = 0.3 + 0.01\times(400/80)^2 = 0.55\,\Omega$

$X_{e1} = 1.1 + 0.035\times(400/80)^2 = 1.975\,\Omega$

$Z_{e1} = \left[0.55^2 + 1.975^2\right]^{1/2} = 2.05\,\Omega$

$(b)\quad (i)\cos\phi = 0.8 \Rightarrow \sin\phi = 0.6$

$I_{1fl} = 100\times10^3/2200 = 45.45\ A$

$$\varepsilon = I_{1fl}\left(\frac{R_{e1}\cos\phi + X_{e1}\sin\phi}{V_{1r}}\right) = 45.45\left(\frac{0.55\times 0.8 + 1.975\times 0.6}{2200}\right) = 0.0336\,\text{pu}$$

$$V_{2n} = V_{1r}\times\frac{N_2}{N_1} = 2200\times\frac{80}{400} = 440\text{ V}$$

$$V_2 = V_{2n}(1-\varepsilon) = 440(1-0.0336) = 425.2\text{ V}$$

$$(ii)\ \varepsilon = I_{1fl}\left(\frac{R_{e1}\cos\phi - X_{e1}\sin\phi}{V_{1r}}\right) = 45.45\left(\frac{0.55\times 0.8 - 1.975\times 0.6}{2200}\right) = -0.0154\,\text{pu}$$

$$V_2 = 440[1-(-0.0154)] = 446.8\text{ V}$$

(c) maximum voltage regulation $\varepsilon_{max} = \frac{I_{1fl}Z_{e1}}{V_1} = \frac{45.45\times 2.05}{2200} = 0.0424$ pu.

Example 8

The primary and secondary windings of a 30-kVA, 6000-V / 230-V transformer have resistances of 10 Ω and 0.016 Ω respectively. The total reactance of the transformer referred to the primary is 23 Ω. Calculate the percentage regulation of the transformer when supplying full load current at a power factor of 0.8 lagging.

Solution

$$R_{e1} = R_1 + (N_1/N_2)^2 R_2 = 10 + 0.016\times(6000/230)^2 = 20.89\,\Omega$$

$$X_{e1} = 23\,\Omega$$

$$I_{1fl} = rated\ power/\ primary rated voltage = 30\times 10^3/6000 = 5\ A$$

$$\varepsilon = I_{1fl}\left(\frac{R_{e1}\cos\phi + X_{e1}\sin\phi}{V_{1r}}\right) = 5\left(\frac{20.89\times 0.8 + 23\times 0.6}{6000}\right) = 0.0254\ pu$$

Percent regulation = 2.54%

5.9 Transformer output

The transformer supply voltage and frequency are substantially constant. Therefore, the heating depends on the current taken by the load. Since the secondary voltage of the transformer is also substantially constant it means that the heating also depends on the load kVA. The transformer output is therefore usually quoted in kVA. The transformer load in kVA is given by

$$S = V_2 I_2 \times 10^{-3}\ \text{kVA} \qquad (23)$$

Where
V_2 = actual load voltage and
I_2 = actual load current.

When S is given and V_2 is unknown the load current can be estimated using the approximate equation

$$S \cong V_{2n} I_2 \times 10^{-3}\ \text{kVA} \qquad (24)$$

5.10 Efficiency

The losses which occur in a transformer on load are composed of

(i) Copper losses in primary and secondary windings, namely $I_1^2 R_1 + I_2^2 R_2 = I_1{}^2 R_{e1} = I_2{}^2 R_{e2}$

(ii) Iron losses in the core due to hysteresis and eddy currents. The iron losses depend on the peak value of the mutual flux Φ_m and frequency. It is therefore independent of load current if voltage and frequency are constant.

Let

P_i = the iron losses (fixed loss) in kW and

P_c = the copper loss with full-load S kVA in kW

Then the total loss at any load xS kVA at power factor $\cos\phi$ is $P_i + x^2 P_c$ and the efficiency is

$$\eta = \frac{output}{input} = \frac{xS\cos\phi}{xS\cos\phi + P_i + x^2 P_c} = \frac{S\cos\phi}{S\cos\phi + \left(\frac{P_i}{x} + xP_c\right)}$$

For a given power factor, the efficiency is maximum when the expression in brackets is a minimum. Hence for a maximum efficiency, we have

$$x = \sqrt{\frac{P_i}{P_c}} \qquad \left[From \quad \frac{d}{dx}\left(\frac{P_i}{x} + xP_c\right) = 0 \quad or \quad -\frac{P_i}{x^2} + P_c = 0 \quad or \quad P_i = x^2 P_c\right] \qquad (25)$$

i.e. efficiency is maximum when the copper loss, $x^2 P_c = P_i$, the fixed loss or iron losses. In other words, the efficiency is maximum when the load is such that its corresponding copper loss is equal to the iron losses.

The efficiency of a transformer is calculated using this form of efficiency equation:

$$\eta = 1 - \frac{losses}{losses + output} = 1 - \frac{P_i + x^2 P_c}{xS\cos\phi + P_i + x^2 P_c} \qquad (26)$$

Example 9

The primary and secondary windings of a 500-kVA transformer have resistances of 0.42 Ω and 0.0011 Ω respectively. The primary and secondary voltages are 6600 V and 400 V respectively and the iron loss is 2.9 kW. Calculate the efficiency on full load at a power factor of 0.8.

Solution

$$\text{Full-load secondary current } I_{2fl} = \frac{500 \times 1000}{400} = 1250\ A$$

$$Full\text{-}load\ primary\ current\ I_{1fl} = 500 \times \frac{1000}{6600} = 75.8\ A$$

$$\text{Secondary copper loss on full load} = I_{2fl}^2 R_2 = 1250^2 \times 0.0011 = 1720\ \text{W}$$

$$\text{Primary copper loss on full load} = I_{1fl}^2 R_1 = 75.8^2 \times 0.42 = 2415\ \text{W}$$

$$\text{Total copper loss on full load, } P_c = 1720 + 2415 = 4135\ \text{W} = 4.135\ \text{kW}$$

$$\text{Total loss on full load} = total\,copper\,loss + iron\,losses = 4.135 + 2.9 = 7.035\ \text{kW}$$

$$\text{Output power on full load at 0.8 pf} = S_{rated} \times load\ pf = 500 \times 0.8 = 400\ \text{kW}$$

$$Efficiency\ on\ full\ load = 1 - \frac{losses}{losses + output} = 1 - \frac{7.035}{400 + 7.035} = 0.9827 = 98.27\%$$

Example 10

Find the output, at which the efficiency of the transformer of example 11 is maximum and calculate its value assuming the power factor of the load to be 0.8.

Solution

Efficiency is maximum when the load is xS_{rated} such that $x = \sqrt{\dfrac{P_i}{P_c}} = \sqrt{\dfrac{2.9}{4.135}} = 0.837$

Therefore at maximum efficiency, the load kVA $= xS_{rated} = 0.837 \times 500 = 418.5$ kVA

$$\text{At maximum efficiency, total loss} = x^2 P_c + P_i = 2 \times P_i = 2 \times 2.9 = 5.8\ \text{kW}$$

$$\text{and output power at 0.8 pf} = xS_{rated} \times \cos\phi = 418.5 \times 0.8 = 334.8\ \text{kW}$$

$$\text{Therefore, maximum efficiency} = 1 - \frac{losses}{losses + output} = 1 - \frac{5.8}{5.8 + 334.8} = 0.983 = 98.30\%$$

Example 11

A 400-kVA transformer has an iron loss of 2 kW and the maximum efficiency at 0.8 pf occurs when the load is 240 kW. Calculate (a) the maximum efficiency at unity power factor and (b) the efficiency on full load at 0.71 power factor

Solution

(a) Total loss at maximum efficiency $= 2 \times 2 = 4$ kW

$$\text{Output kVA at maximum efficiency} = \frac{Output\ power\ in\ kW}{pf} = \frac{240}{0.8} = 300\ \text{kVA}$$

$$\text{Output power at maximum efficiency at unity pf} = 300 \times 1 = 300\ \text{kW}$$

$$\text{Maximum efficiency at unity pf} = 1 - \frac{losses}{losses + output} = 1 - \frac{4}{4 + 300} = 0.9868 = 98.68\%$$

(b) The fraction x of full load kVA at which the efficiency is max $= \dfrac{300}{400} = 0.75$

$$\text{Full load copper loss } P_c = \frac{P_i}{x^2} = \frac{2}{0.75^2} = 3.56\ \text{kW}$$

$$\therefore Full\ load\ efficiency\ at\ 0.71\ pf = 1 - \frac{P_i + P_c}{P_i + P_c + S\cos\phi}$$

$$= 1 - \frac{2 + 3.56}{2 + 3.56 + 400 \times 0.71} = 0.9808 = 98.08\%$$

5.11 Open-circuit and short-circuit tests on a transformer

These two tests enable the efficiency and voltage regulation to be calculated without actually loading the transformer.

(a) *Short-circuit test*: This test is used to determine the leakage impedance. During this test, one winding is short-circuited and a reduced voltage V_{sc} applied to the other to cause rated current to flow. The test circuit and the equivalent circuit are shown in Fig. 13.a and Fig. 13.b respectively. The magnetizing branch is neglected because its current under this condition is less than 1 % of the total.

Voltage V_{sc}, current I_{sc} and power P_{sc} measured by the instruments are used to make the following calculations.

$$(a)\ Z_{e1} = \frac{V_{sc}}{I_{sc}} \qquad (b)\ R_{e1} = \frac{P_{sc}}{I_{sc}^2} \qquad (c)\ X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2} \tag{27}$$

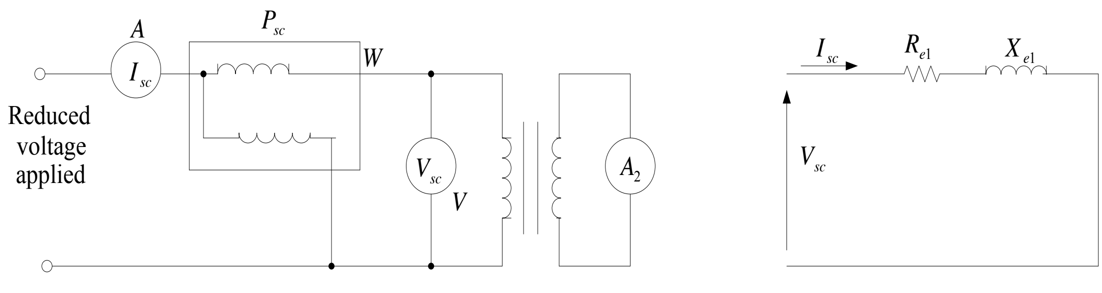
We note that the following:

(i) I_{sc} need not be the rated current since the equivalent circuit is linear. However, it is desirable that it should be near to the rated value so that stray losses (they are due to eddy currents set up in large section conductors, tank and metallic supports by leakage fluxes) are normal.

(ii) The supply could be fed to either winding. It is often convenient on the higher voltage transformers to supply the high-voltage winding, thus using a smaller current. V_{sc} which will be about 3-15 % of the rated value may also be more suitable for test facilities.

(iii) In laboratory experiments using small transformers, the instruments positions shown minimize measurement errors.

(iv) The circuit parameters obtained with (27) are referred to the side to which the test voltage is applied.



(a) Test circuit

(b) Equivalent circuit

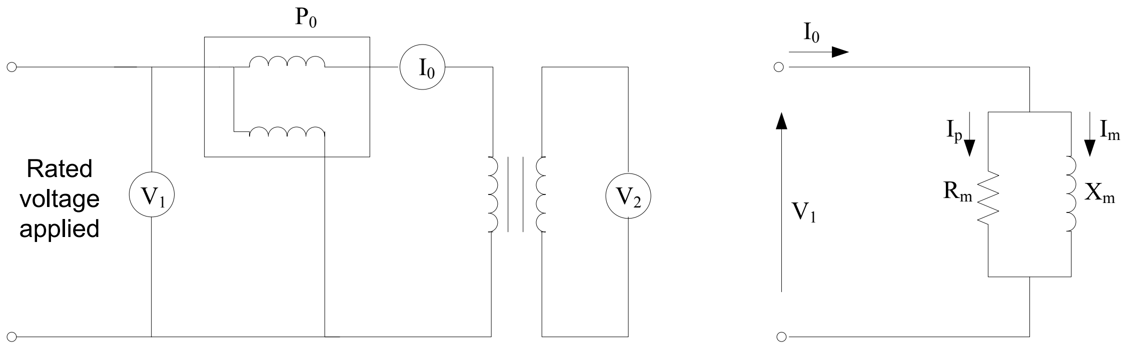
Fig. 13 Short-circuit test

(b) *Open-circuit test or no load test:* During this test, one winding is open-circuited and rated voltage at rated frequency is applied to the other. Quite often the low-voltage side is supplied to reduce the test voltage required for safety reasons. As with the short-circuit test, the equivalent circuit parameters will be referred to the side to which the test voltage is applied. The test circuit and equivalent circuit are shown in Fig. 14.a and b.

The following calculations can be made

$$(a) I_p = \frac{P_o}{V_1} \quad (b)\ I_m = \sqrt{I_o^{\,2} - I_p^{\,2}} \quad (c)\ R_m = \frac{V_1}{I_p} \quad (d)\ X_m = \frac{V_1}{I_m} \quad (e) a = \frac{V_1}{V_2} \qquad (28)$$

P_{sc} and P_o represent the full load copper loss and the core loss (or iron losses) respectively. They can be used directly to calculate efficiency.



(a) Test circuit (b) Equivalent circuit

Fig. 14 Open-circuit or no-load test

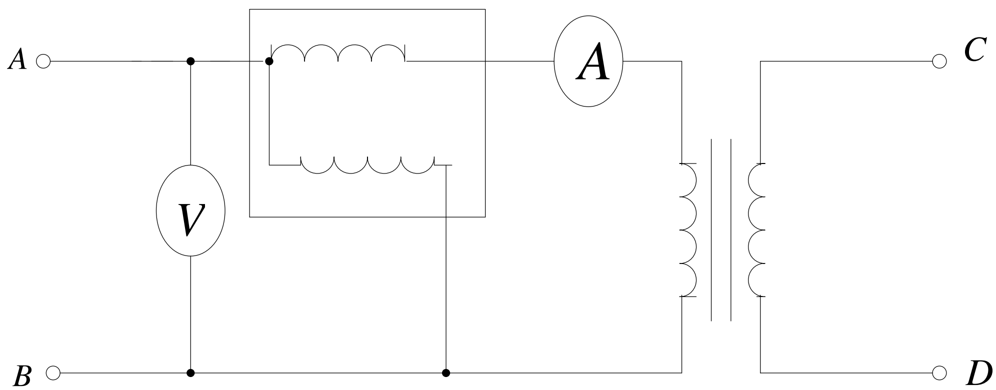
Example 12

The circuit shown below was used in a test on a 3-kVA transformer. A variable voltage supply of fixed frequency was connected to terminals A and B and two tests were performed:

(a) The voltage was raised to normal rated voltage and the meter reading were then 200 V, 24 W, 1.2 A
(b) The terminals C and D were short-circuited and the voltage was raised until the transformer full-load current was flowing. The meter readings were then 6.4 V, 28 W, 15 A

From the results of the tests, obtain:

(i) the no-load current and its power factor
(ii) the iron losses of the transformer at normal frequency and voltage
(iii)the full-load copper loss
(iv)the transformer resistances, R_{e1} and R_m and reactances X_{e1} and X_m
(v) the efficiency of the transformer at full load at a power factor of 0.8



Circuit diagram for Example 13

Solution

(*i*) No - load current $I_o = 1.2\,A$

(*ii*) Iron losses $P_i = P_o = 24\,W$

(*iii*) Full load copper loss $P_c = P_{sc} = 28\,W$

(iv) Open-circuit test calculations:

$$I_p = \frac{P_o}{V_1} = \frac{24}{200} = 0.12\,A \qquad I_m = \sqrt{I_o^{\,2} - I_p^{\,2}} = \sqrt{1.2^2 - 0.12^2} = 1.19\,A$$

$$R_m = \frac{V_1}{I_p} = \frac{200}{0.12} = 1.67\,k\Omega \qquad X_m = \frac{V_1}{I_m} = \frac{200}{1.19} = 168\Omega$$

Short-circuit test calculations:

$$Z_{e1} = \frac{V_{sc}}{I_{sc}} = \frac{6.4}{15} = 0.43\Omega \qquad R_{e1} = \frac{P_{sc}}{I_{sc}^2} = \frac{28}{15^2} = 0.12\Omega$$

$$X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2} = \sqrt{0.43^2 - 0.12^2} = 0.41\Omega$$

v) $$Efficiency = 1 - \frac{P_i + P_c}{P_i + P_c + S\cos\phi} = 1 - \frac{24 + 28}{24 + 28 + (3000 \times 0.8)} = 0.9788\,pu = 97.88\%$$

Example 13

A 10-kVA 1-ph transformer has a voltage ratio 1100 / 250 V. On no load and at normal voltage (1100 V) and frequency the input current is 0.75 A at a pf of 0.2 lagging. With the secondary short-circuited, full-load currents flow when the primary applied voltage is 77 V, the power input being 240 W. Calculate

(a) the transformer equivalent resistance and reactance referred to the secondary side

(b) the maximum value of the voltage regulation at full load and the load power factor at this regulation

(c) the percentage of full-load current at which the transformer has maximum efficiency

Solution

(a) Referring the circuit to the secondary side,

$$\frac{V_1}{a} = 77 \times \frac{250}{1100} = 17.5\,\mathrm{V}$$

$$I_{2fl} = \frac{S_{rated}}{V_{2n}} = \frac{10000}{250} = 40\,\text{A}$$

$$Z_{e2} = \frac{V_1}{a} \times \frac{1}{I_{2fl}} = \frac{17.5}{40} = 0.438\,\Omega,\ R_{e2} = \frac{P_{sc}}{I_{2fl}^2} = \frac{240}{40^2} = 0.15\,\Omega \text{ and}$$

$$X_{e2} = \sqrt{Z_{e2}^2 - R_{e2}^2} = \sqrt{0.438^2 - 0.15^2} = 0.41\,\Omega$$

(b) The maximum voltage drop at full load = $I_{2fl}Z_{e2} = 17.5\,\text{V}$

The maximum regulation occurs when $\phi = \theta$ or when

$$\cos\phi = \cos\theta = \frac{R_{e2}}{Z_{e2}} = \frac{0.15}{0.438} = 0.342 \text{lagging}$$

(c) Maximum efficiency occurs when $x^2 P_c = P_o$. Therefore

$$x = \sqrt{\frac{1100 \times 0.75 \times 0.2}{40^2 \times 0.15}} = 0.829 \text{ or } 82.9\%$$

5.12 Parallel Operation of single-phase transformers

Parallel connection of several transformers is widely used in electrical systems for the following reasons:

(a) In many cases, the amount of power to be transformed is greater than that which can be built into one transformer
(b) Frequently, the growth of load requires that the installed transformers supply an output greater than their total kVA capacity. Additional transformers are then installed to run in parallel with the existing transformers.
(c) It is sometimes found desirable to supply a load through two or more units in order to reduce the cost of the spare unit required to ensure continuity of service in case of damage.

The following conditions must be fulfilled when operating two or more single-phase transformers in parallel:

(a) The polarity should be the same. The polarity can be either right or wrong. A wrong polarity results in a severe short circuit. Terminals of the same markings are connected together to ensure correct polarity. See Fig. 15. If the polarity markings are either incorrect or not present, the polarity of the incoming transformer can be checked by connecting a voltmeter across the paralleling switch. The meter should read zero when the polarity is right.
(b) The voltage ratio should be the same. This is to avoid no-load circulating current and also over-loading on one transformer when the paralleled transformers are loaded.
(c) The per-unit impedances should be equal in magnitude and have the same angle. When they are equal in magnitude, the transformers share kVA loads in proportion to their respective ratings. If both their magnitudes and angles are the same they will not only share kVA loads in proportion to their respective ratings but also the combined load kVA will be the algebraic sum of the kVA carried by each transformer. If the angles are different, the resultant kVA capacity of the paralleled group will be slightly smaller than the sum of their individual ratings if none should be overloaded. It is not very necessary that the angles should be the same.

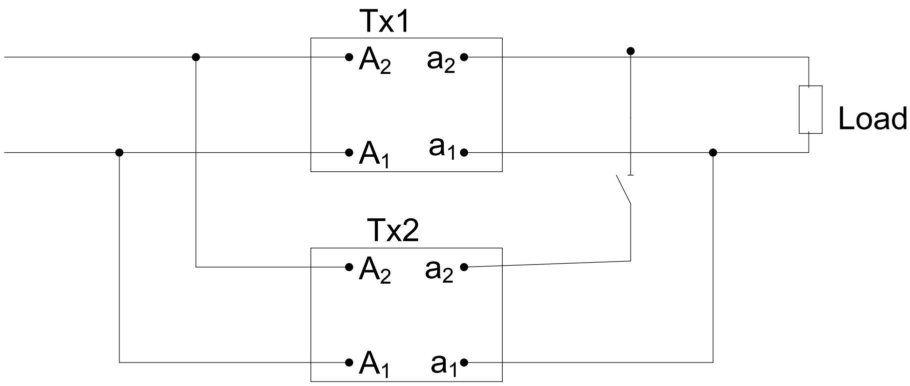


Fig. 15 *C*onnection ensuring correct polarity

5.13 Load sharing of parallel-connected transformers
The equivalent circuit of two transformers in parallel feeding a common load Z_L is shown in Fig. 16. The voltage ratios are supposed to be equal and the magnetizing branch is neglected. The circuit is referred to the secondary side but it may also be referred to the primary side.

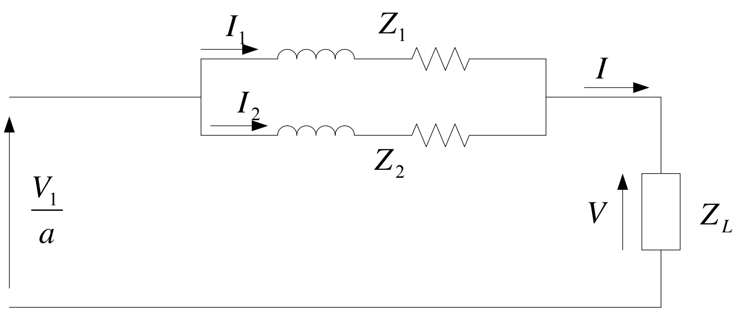


Fig. 16 Equivalent circuit of two transformers in parallel

Z_1 is the impedance of transformer 1 referred to the secondary side and Z_2 is the impedance of transformer 2 referred to the secondary side.

The voltage drops across the impedances are the same. Therefore

$$I_1 Z_1 = I_2 Z_2 = I \frac{Z_1 Z_2}{(Z_1 + Z_2)}$$

Or

$$I_1 = \frac{Z_2}{Z_1 + Z_2} I \quad and \quad I_2 = \frac{Z_1}{Z_1 + Z_2} I$$

Multiplying by the terminal voltage V gives

$$S_1 = S \frac{Z_2}{Z_1 + Z_2} \tag{29.a}$$

$$S_2 = S \frac{Z_1}{Z_1 + Z_2} \tag{29.b}$$

Where

$S_1 = VI_1 = \text{transformer 1 loading}$ (30.a)

$S_2 = VI_2 = \text{transformer 2 loading}$ (30.b)

$S = VI = \text{combined load}$ (30.c)

Equations (29) hold for per-unit impedances provided that all are expressed with reference to a common base power. The following equation can be used to obtain a new per unit value with reference to a new base power.

$$Z_{pu}^{new} = Z_{pu}^{old} \times \frac{S_{base}^{new}}{S_{base}^{old}} \tag{31}$$

Example 14

A 500-kVA transformer (Transformer 1) is connected in parallel with a 250-kVA transformer (Transformer 2). The secondary voltage of each is 400 V on no load. Find how they share a load of 750 kVA at power factor of 0.8 lagging if

$(a)\quad Z_1 = 0.01 + j0.05\ pu = 0.05099\angle 78.69°\ pu\ and$

$\qquad Z_2 = 0.015 + j0.04\ pu = 0.04272\angle 69.44°\ pu$

$(b)\quad Z_1 = 0.01 + j0.05\ pu = 0.05099\angle 78.69°\ pu\ and$

$\qquad Z_2 = 0.01 + j0.05\ pu = 0.05099\angle 78.69°\ pu$

$(c)\quad Z_1 = 0.01 + j0.05\ pu = 0.05099\angle 78.69°\ pu\ and$

$\qquad Z_2 = 0.025 + j0.0444 = 0.05095\angle 60.62°\ pu$

Solution

If all the impedances are referred to a base power of 500 kVA, then only the impedances of transformer 2 will change.

$$Z_{\text{pu}}^{\text{new}} = Z_{pu}^{old} \times \frac{S_{base}^{new}}{S_{base}^{old}} = Z_{pu}^{old} \times \frac{500}{250} = 2Z_{pu}^{old}$$

Case (a)

$Z_1 = 0.01 + j0.05 = 0.05099\angle 78.69°$ and

$Z_2 = 2(0.015 + j0.04) = (0.03 + j0.08) = 0.08544\angle 69.44°$

Further

$Z_1 + Z_2 = (0.01 + j0.05) + (0.03 + j0.08) = 0.04 + j0.13 = 0.1360\angle 72.90°$

Total kVA load $S = 750\angle -\cos^{-1} 0.8 = 750\angle -36.9°\,kVA$

$$S_1 = S \times \frac{Z_2}{Z_1 + Z_2} = \frac{750\angle -36.9° \times 0.08544\angle 69.44°}{0.136\angle 72.9°} = 471\angle -40.36°\,kVA$$

$$= 471\,kVA \text{ at power factor of } \cos 40.36° = 0.762 \text{ lagging}$$

Similarly

$$S_2 = \frac{750\angle -36.9° \times 0.05099\angle 78.69°}{0.136\angle 72.9°} = 281\angle -31.11°$$

$$= 281 kVA \text{ at power factor of } 0.856 \text{ lagging}$$

Remark: Transformer 1 with larger per-unit impedance is under-loaded whereas transformer 2 with lower per-unit impedance is overloaded.

Case (b)

$$Z_2 = 2(0.01 + j0.05) = 0.02 + j0.1 = 0.10198\angle 78.69° \ pu$$

$$Z_1 + Z_2 = 0.03 + j0.15 = 0.15297\angle 78.69° \ pu$$

$$S_1 = \frac{750\angle -36.9° \times 0.10198\angle 78.69°}{0.15297\angle 78.69°} = 500\angle -36.9° \ kVA$$

$$= 500 kVA \text{ at power factor of } 0.8 \text{ lagging}$$

$$S_2 = \frac{750\angle -36.9° \times 0.05099\angle 78.69°}{0.15297\angle 78.69°} = 250\angle -36.9° \ kVA$$

$$= 250 \text{kVA at power factor of } 0.8 \text{ laggging}$$

Remark: Load shared in proportion to transformer ratings. Arithmetic sum of loadings is equal to the combined load. A shorter approach can be used on recognizing that $Z_{1pu} = Z_{2pu}$

Case (c)

$$Z_2 = 2(0.025 + j0.0444) = 0.05 + j0.0888 = 0.1019\angle 60.62° \ pu$$

$$Z_1 + Z_2 = 0.06 + j0.1388 = 0.1512\angle 66.62°$$

$$S_1 = \frac{750\angle -36.9° \times 0.1019\angle 60.62°}{0.1512\angle 66.62°} = 505\angle -42.9°$$

$$= 505 kVA \text{ at power factor of } 0.732 \text{ lagging}$$

$$S_2 = \frac{750\angle -36.9° \times 0.05099\angle 78.69°}{0.1512\angle 66.62°} = 253\angle -24.82°$$

$$= 253 \text{kVA at power factor of } 0.91 \text{ lagging}$$

Remark: Transformers are slightly overloaded when the combined load is equal to the sum of individual kVAs

6.0 Three-phase transformers

These are required to transform 3-phase power. The three-phase transformer may be either of the following:

(a) A three-phase transformer bank: This consists of three identical single-phase transformers having their windings externally connected for three-phase working. The single-phase transformers retain all their basic single-phase properties such as current ratio, voltage ratio and the flux in the core. The kVA capacity of the bank is the sum of their individual ratings. See Fig.17.

(b) A three-phase transformer unit: This is a single unit of special construction for three-phase working. Modern large transformers are usually of the three-phase three-legged core type shown in Fig.18. A leg carries the primary and secondary windings of a phase. The windings are internally connected. For a given total capacity, 3-phase units are much cheaper in capital cost, lighter, smaller and more efficient.

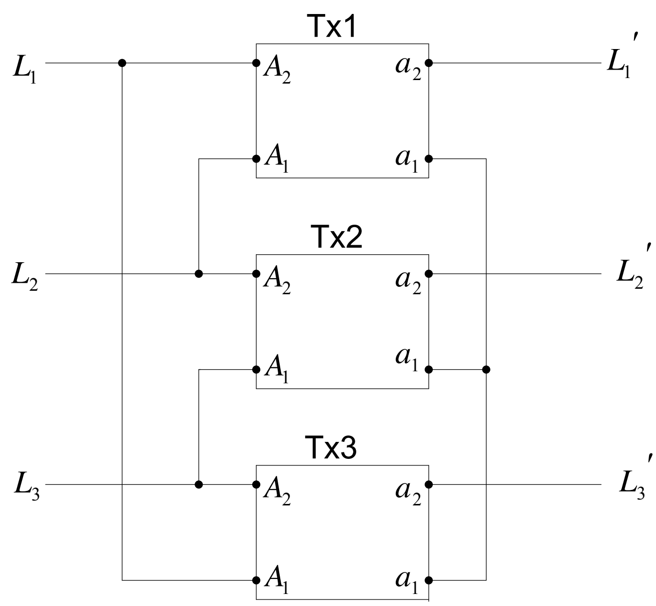


Fig. 9 Delta - Star connection of single - phase transformers

6.1 Winding arrangement

The three windings, primary or secondary, can be connected in three different ways:

(a) *Star connection*: For this connection $phase\,voltage = line\,voltage/\sqrt{3} \cong 58\%$ of the line voltage. This enables the insulation of the winding to be reduced to a minimum for a given supply voltage. *Line current = phase current*. It is the most economical connection for a high-voltage winding.

(b) *Delta connection*: *Phase voltage = Line voltage*. Therefore winding must be insulated for the full Line voltage. More turns are also required. With very high voltages a saving of 10 % may be achieved by using star-connection rather than delta connection on account of insulation. The saving is small, however, at voltages below 11 kV. For delta connection $phase\,current = line\,current/\sqrt{3}$ so the winding cross-sectional area is 58 % of that required for the star connection. Therefore it is the most economical for low-voltage winding.

(c) *Zigzag (or interconnected star) connection*: It is a modification of the star connection. Each phase winding is divided into two sections and placed on two different legs. The two sections are then connected in phase opposition. The zigzag connection is restricted to the low-voltage winding. 15 % more turns are required for a given phase terminal voltage compared with a normal star.

The three different winding arrangements give rise to several possible connection combinations: star-star, star-delta, star-zigzag, delta-star, delta-delta, etc

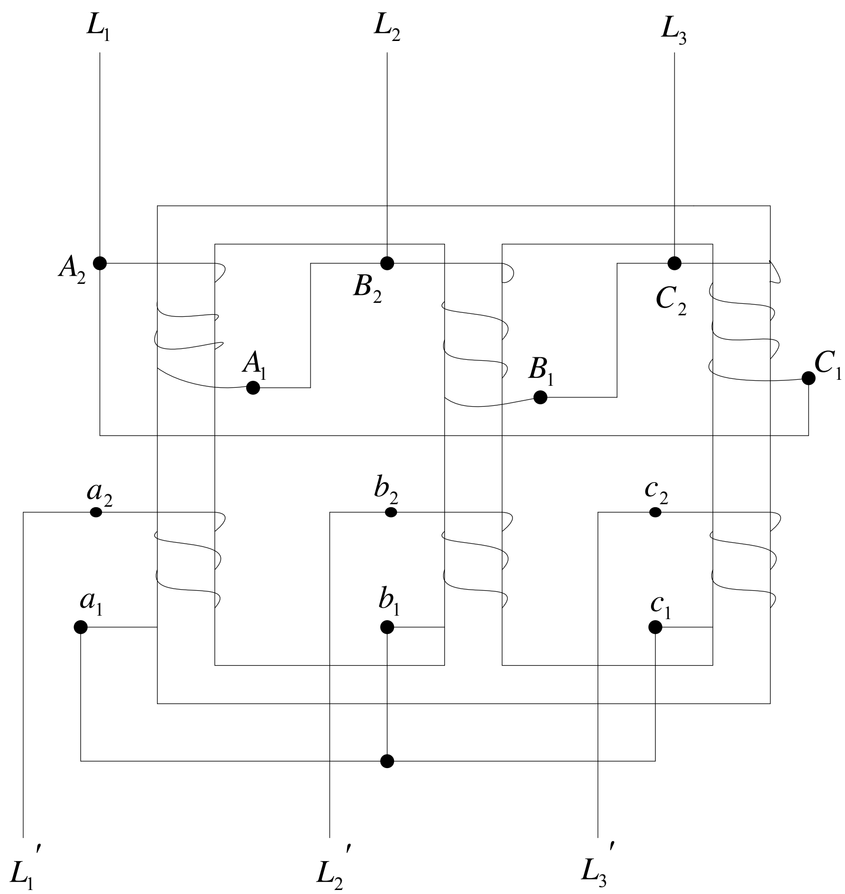
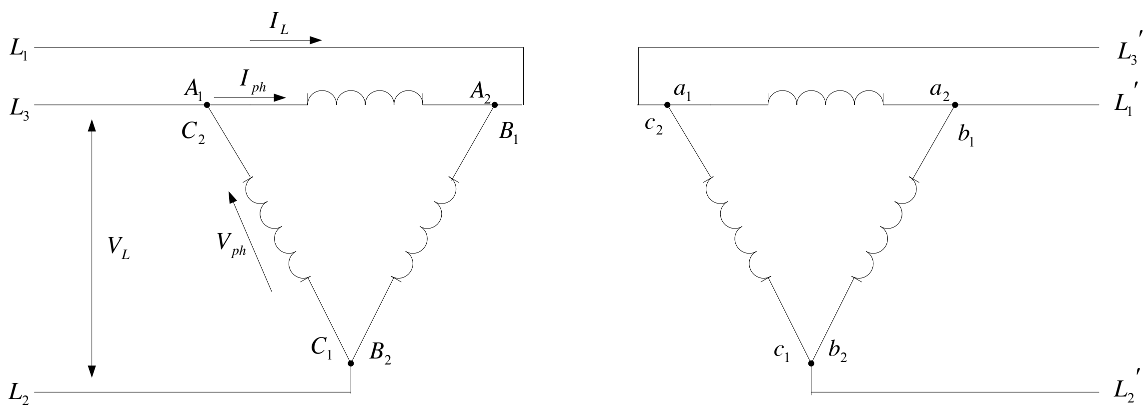


Fig. 18 Delta-Star connection of three-phase, three-legged core-type transformer

6.2Three-phase transformer connections

More common transformer connections are

(a) Delta-Delta connection (Fig 19). : This connection is economical for large low voltage transformer. This connection is not often used because there is no neutral point and a four-wire supply cannot be given. It is used in a 3-phase transformer bank but rarely in 3-phase transformer unit. It is possible to use this arrangement to provide 3-phase power with one transformer removed. This connection, known as open-delta or vee connection, can supply up to 57.7 % of the load capacity of the delta-delta connection.

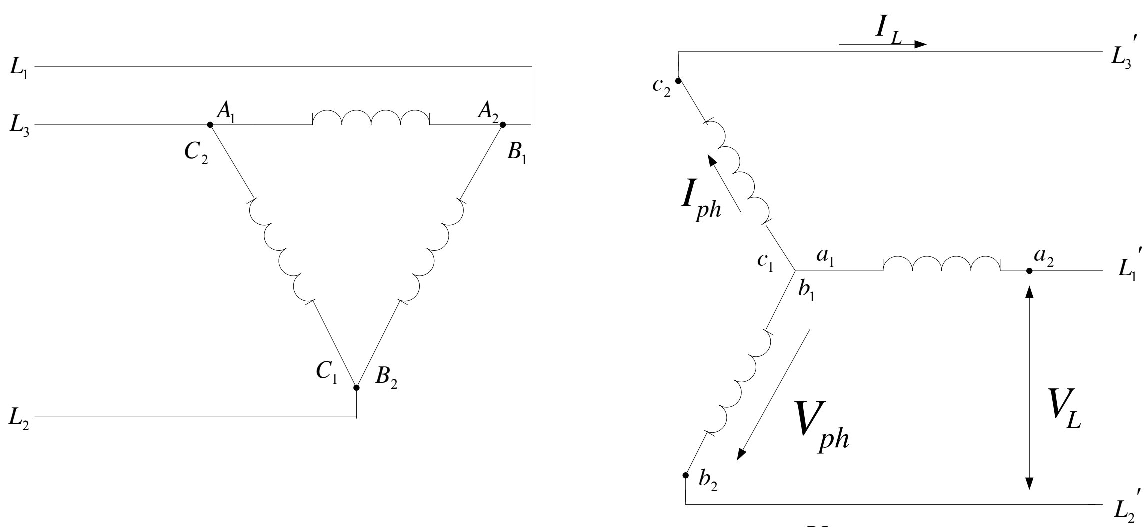


$$V_L = V_{ph}$$

$$I_L = \sqrt{3} I_{ph}$$

Fig. 19 Delta-delta connection

(b) Delta-Star (Fig. 20): It is commonly used to step up alternator voltage to transmission line voltage. Another common application is in distribution service where as a step-down transformer, the windings are not the most economical. The secondary star point can be earthed and a four-wire supply given.
(c) Star-Delta: There is no secondary neutral and four-wire supplies cannot be given. The main use is as a step-down transformer at the load end of transmission line.



$$V_{ph} = \frac{V_L}{\sqrt{3}} \qquad I_{ph} = I_L$$

Fig. 17 Delta-star connection

(d) Star-Star: This is economical for high-voltage transformer. However, if the secondary load is unbalanced, the neutral point will be displaced and the line-to-neutral voltages will become unequal. To stabilize the neutral, the neutral of the primary and the neutral of the source are connected together usually by way of ground. Another way is to provide a third

delta-connected winding called tertiary winding. Although it can be used to supply additional power, the tertiary winding generally has no external connection

Example 15
Three single-phase step-up transformers rated 40 MVA, 13.2 kV / 80 kV are connected in delta-star on a 13.2 kV transmission line. If they feed a 90 MVA load, calculate the following:
(a) the secondary line voltage
(b) the currents in the transformer windings
(c) the incoming and outgoing transmission line currents.
Assume transformers are ideal.

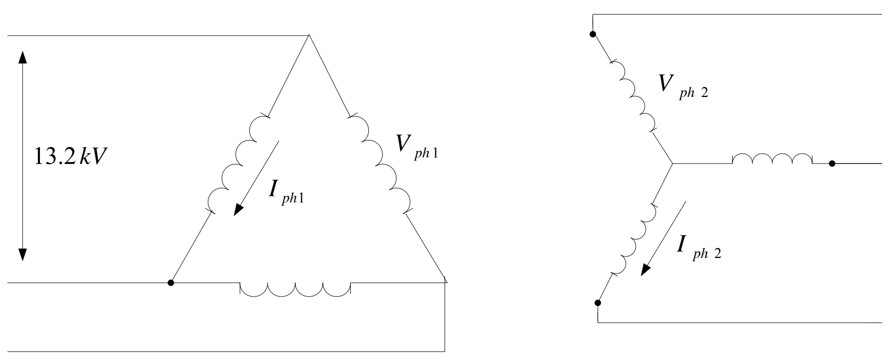
Solution

$$(a) \quad \frac{V_{ph2}}{V_{ph1}} = \frac{1}{a} = \frac{80}{13.2}$$

But $V_{ph1} = 13.2kV$ (transmission line voltage)

Therefore $V_{ph} = 80kV$

and the line voltage on the secondary side $= 80\sqrt{3} = 138kV$



$$(b) \quad \text{The load carried by each transformer} = \frac{90}{3} = 30MVA$$

$$\text{Current in the primary winding } I_{ph1} = \frac{30MVA}{13.2kV} = 2272A$$

$$\text{Current in the secondary winding } I_{ph2} = \frac{30MVA}{80kV} = 375A$$

$$(c) \quad \text{Current in incoming line} = \sqrt{3}I_{ph1} = 2272\sqrt{3} = 3932A$$

$$\text{Current in outgoing line} = I_{ph2} = I_L = 375A$$

Example 16
Three single-phase transformers have their primaries joined in delta to a 6600 V, three-phase, three-wire supply. Their secondaries are connected to give a three-phase, four-wire output at

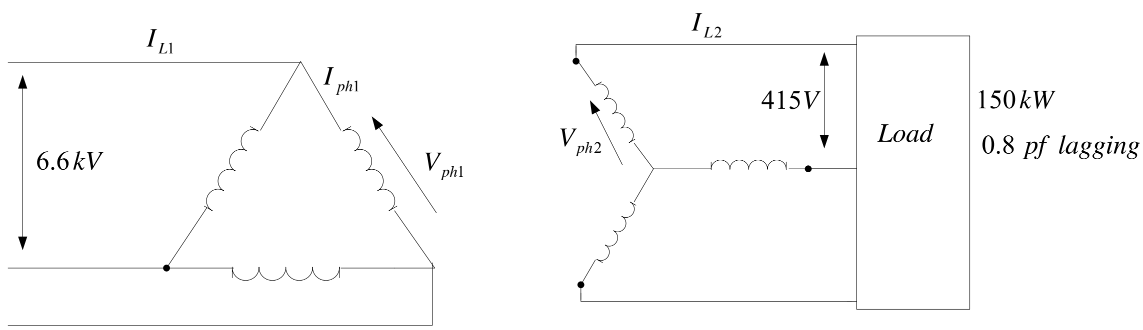
415 V across lines. The total load on the transformers is a balanced load of 150 kW at 0.8 pf lag. If the voltage per turn on the primaries is 4, find
(a) the number of turns on the primary winding and the secondary winding
(b) the currents and voltages in all windings and lines, including the neutral wire on the secondary side
(c) kVA load on each transformer

Assume transformers are ideal

Solution

$$(a)\quad \text{Primary turns per phase} = \frac{V_{ph1}}{volt/turn} = \frac{6600}{4} = 1650 turns$$

$$\text{Secondary turns per phase} = \frac{V_{ph2}}{volt/turn} = \frac{415}{\sqrt{3}\times 4} = 60 turns$$



$$(b)\quad \text{Secondary } I_{L2} = I_{ph2} = \frac{Power}{\sqrt{3}V_L\cos\phi} = \frac{150\times 10^3}{\sqrt{3}\times 415\times 0.8} = 261A$$

$I_N = 0\,A$ because load is balanced

$$I_{ph1} = \left(\frac{N_2 I_{ph2}}{N_1}\right) = \frac{60\times 261}{1650} = 9.5\,A$$

$I_L = 9.5\sqrt{3} = 16.4\,A$.

7.0 Autotransformers

An autotransformer has a single tapped winding which serves both primary and secondary functions as shown in Fig. 34. The circuit diagrams are shown in Fig. 35.

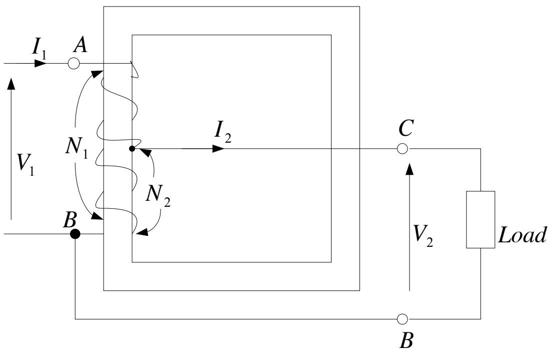


Fig.18.a Step-down autotransformer

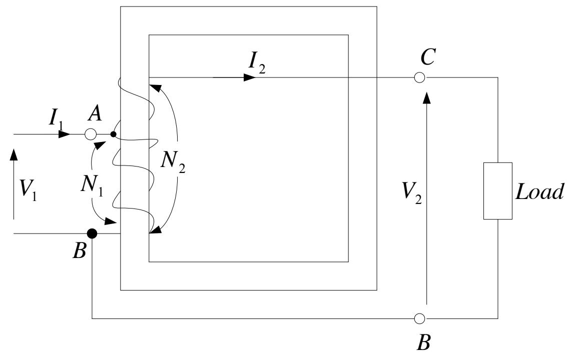


Fig.18.b Step-up autotransformer

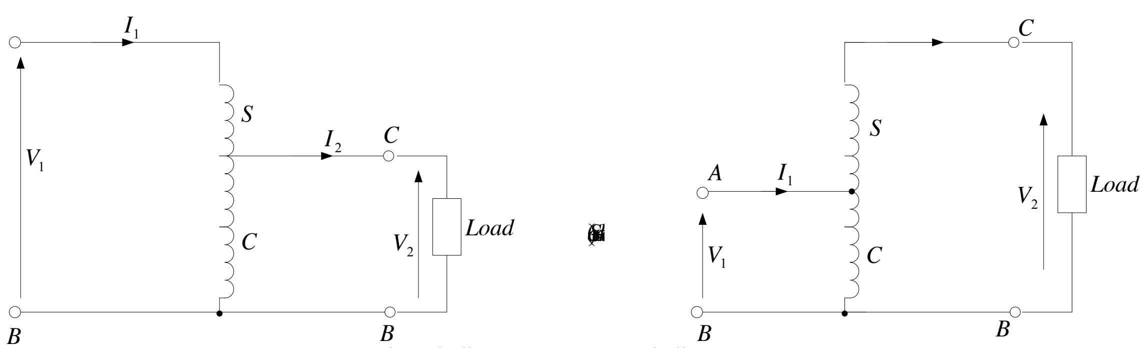
7.1 Autotransformer equations

If we neglect losses, leakage flux and magnetizing current then

$$n = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

7.2 Advantages and disadvantages of autotransformer over two-winding transformer

The main advantage gained in the use of autotransformer is the saving of copper. For a two-winding transformer and an autotransformer which can perform the same duty (they



S = series winding; C = common winding

Fig.18 Circuit diagrams of autotransformer

should have the same voltage per turn and therefore the same flux. We can also assume the same mean length per turn)

$$\frac{Volume\ of\ copper\ in\ autotransformer}{Volume\ of\ copper\ in\ two\text{-}winding\ transformer} = 1 - \frac{V_L}{V_H}$$

Or

$$\begin{array}{l} saving\ of\ copper\ effected\ by \\ using\ an\ autotransformer \end{array} = \frac{V_L}{V_H} \times (Volume\ of\ copper\ in\ the\ two\ winding\ transformer)$$

$Where$

$V_L = voltage\ on\ the\ low\ voltage\ side$

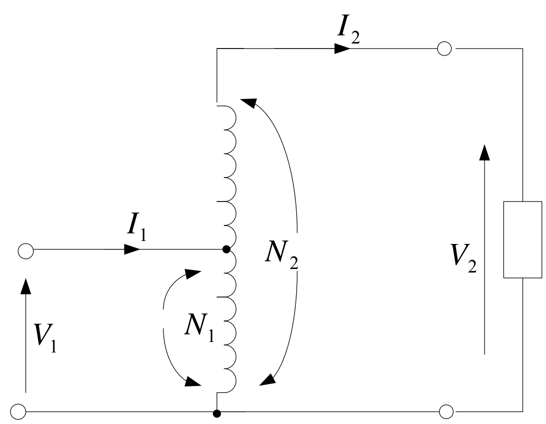
$V_H = voltage\ on\ high\ voltage\ side$

In practice, voltage ratios V_L/V_H less than about $1/3$ show little economic benefit over two-winding transformer because of other factors such as cost of insulation.

The main disadvantage is that the primary and secondary circuits are not isolated from each other.

Example 17
An autotransformer is required to step up a voltage from 220 to 250 V. The total number of turns is 2000. Determine (a) the position of the tapping point (b) the approximate value of the current in each part of the winding when the output is 10 kVA and (c) the economy in copper over the two winding transformer having the same peak flux and the same mean length per turn.

Solution:



$$(a)\quad \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{220}{250} \quad or \quad N_1 = \frac{220}{250} \times N_2 = \frac{220}{250} \times 2000 = 1760$$

Position is 240 turns from one end.

$$(b)\quad V_2 I_2 = 10 \times 10^3 \Rightarrow I_2 = \frac{10 \times 10^3}{250} = 40\,A$$

$$V_1 I_1 = 10 \times 10^3 \Rightarrow I_1 = \frac{10 \times 10^3}{220} = 45.45\,A$$

Therefore current in series winding is 40 A and current in common winding $= 45.45 - 40 = 5.45\,A$

$$(c)\quad Saving\,in\,copper = \frac{V_L}{V_H} = \frac{220}{250} = 0.88\ pu \quad or\ 88\%$$ of copper used in the two-winding transformer

7.3 Two-winding transformer connected as an autotransformer

A two-winding transformer can be changed into an autotransformer by connecting the primary and secondary windings in series. The following rules apply whenever a two-winding transformer is connected as autotransformer:

(a) the current in any winding should not exceed its current rating
(b) the voltage across any winding should not exceed its voltage rating
(c) rated current in one winding gives rise to rated current in the other
(d) rated voltage across one winding gives rise to rated voltage across the other
(e) if current in one winding flows from say A_2 to A_1, then current in the other winding must flow from a_1 to a_2 and vice versa
(f) the voltages add when terminals of opposite polarity (A_1 and a_2 or A_2 and a_1) are connected together by a jumper. The voltages subtract when A_1 and a_1 (or A_2 and a_2) are connected together.

Example 18

A two-winding single-phase transformer rated 15 kVA, 600 V / 120 V, 60 Hz. We wish to reconnect it as an autotransformer in three different ways to obtain three different voltage ratios:

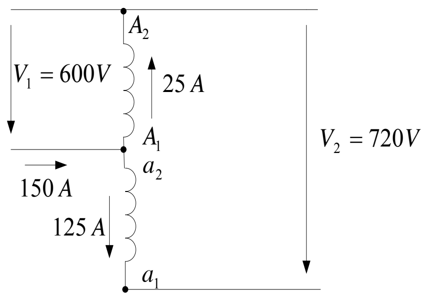
(a) 600 V primary to 480 V secondary
(b) 600 V primary to 720 V secondary
(c) 120 V primary to 480 V secondary

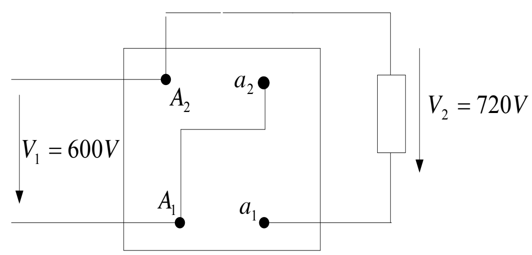
Calculate the maximum load the transformer can carry in each case.

Solution

(a) The secondary voltage 120 V must be subtracted from the primary voltage to obtain the 480 V.

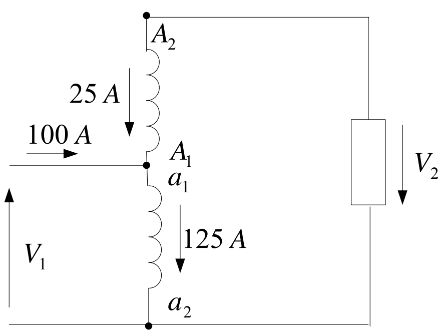
(b) The secondary voltage 120 V must be added to the 600 V to obtain 720 V

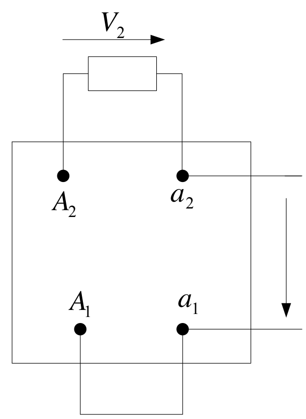




$$kVA = 600 \times 150 \times 10^{-3} = 90$$

(c) The 120 V becomes the primary of the autotransformer and the 120 V is subtracted from the 600 V to obtain its secondary





7.4 Applications of autotransformers

They are mainly used for

(a) variac

(b) interconnecting power systems that are operating at roughly the same voltage (eg. 132 kV, 275 kV, 400 kV) and

(c) starting squirrel-cage induction motors.

8.0 Instrument transformers

They are used in ac circuits to serve these purposes:

(a) to make possible the measurement of high voltages with low-voltage instruments or large currents with low current ammeters

(b) to insulate high voltage circuits being monitored from measuring circuit in order to protect the measuring apparatus and operator

(c) to energize relays for the operation of protective and automatic control devices

The load on the secondary of an instrument transformer is called its burden and is expressed in volt-amperes (VA). There are two types of instrument transformers: the voltage (or potential) transformer and the current transformer

8.1 The voltage or potential transformers (VTs or PTs)
The construction is similar to a power transformer. The primary is connected directly to the power circuit either between two phases or between a phase and ground and the secondary is connected to instruments and coils of relays. Sufficient insulation is provided between the primary and the secondary to withstand the full line voltage as well as the very high impulse voltage. Voltage transformers are designed to step down the primary voltage to a nominal or rated voltage of 110 V so that standard instruments and relays can be used. They introduce errors of two kinds into measurement being made: the ratio errors (the ratio between input and output voltages is not constant under all conditions of load) and the phase angle errors (the phase shift between input and output voltages is not zero). These errors are due to the exciting current and the equivalent series impedance of the transformer and they are kept low by using high quality iron (high permeability and low loss) and operating it at low flux densities so that the exciting current is very small. The resistance and reactance of the windings are also made very low.

Fig. 19 shows the circuit for a potential transformer. One terminal of the secondary winding is always earthed. The windings though insulated from each other, are connected invisibly together by distributed capacitance between them. By earthing one of the secondary terminals, the highest voltage between the secondary lines and earth can never rise above that of the secondary voltage.

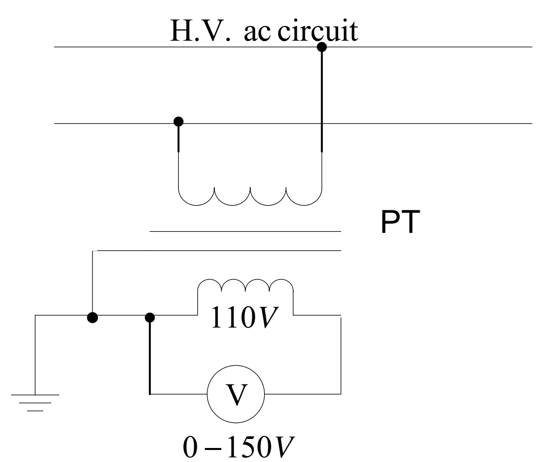


Fig. 19 Potential transformer installed on H.V. circuit

8.2 Current transformers (CTs)
Their primary consists of small number of turns connected in series with the power circuit load. The secondary consists of a larger number of turns and it is connected to ammeter, current coils of other instruments or current coils of relays. Current transformers have the ratio of primary to secondary current approximately constant. The nominal or rated secondary currents are usually 5 A or 1 A, irrespective of the primary current rating. The transformer ratio is usually stated to include the secondary current rating. Current transformers also introduce two errors in measurement: the ratio error (the ratio between primary and secondary currents is not constant) and phase angle error (the phase angle between the primary and secondary currents is not zero. The basic cause of ratio and phase angle errors is the exciting current. To keep the exciting current small, a high quality iron

operating at very low flux densities is used as in PTs. In Cts the secondary leakage impedance and impedances of the secondary leads and instruments should also be very low; for any increase in these impedances increases the core flux and therefore the exciting current. The tansformer is connected in the power circuit as shown in Fig. 20 As in the case of PT (and for the same reasons) one of the secondary terminals is always earthed

Current transformer secondary circuit must not be opened while current is flowing in the primary. Without opposing ampere-turns the line current, which may be 100 to 200 times the normal exciting current, becomes the exciting current. The iron core becomes.

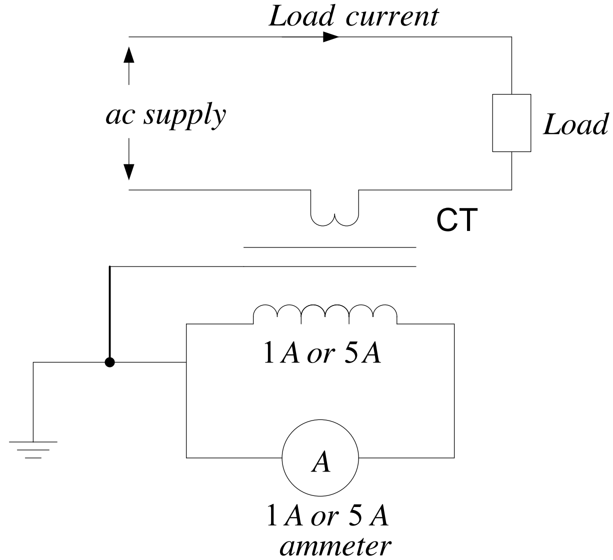


Fig. 20 Current transformer intalled in H.V. circuit

saturated and very high voltage spikes (several thousand volts) are induced across the open-circuited secondary. These voltages are dangerous to life and to the transformer insulation. The core when it becomes saturated can also cause excessive heating of the core and windings. Therefore when it is desired to remove a load from the secondary circuit, the secondary winding must first be short circuited.

When the line current exceeds 100 A we can sometimes use a toroidal or bar-primary (N_1=1) transformer shown in Fig. 21. It consists of a laminated ring-shaped core which carries the secondary winding. The primary is composed of a single conductor that simply passes through the centre of the ring as shown in the figure. Toroidal CTs are simple and inexpensive and are widely used in HV and MV indoor installations.

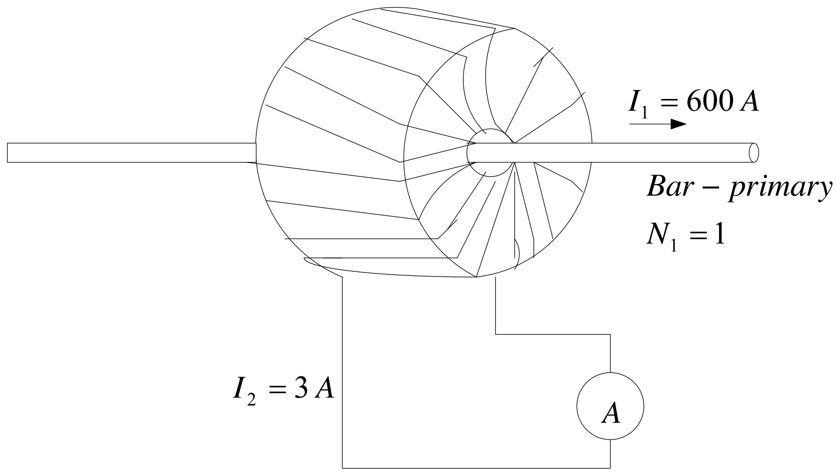


Fig. 21 Toroidal or bar-primary transformer having a ratio 1000A / 5A connected to measure a current in a line

Current transformers are also commonly used for the measurements of large currents even when the circuit voltage is not dangerously high. This avoids bringing heavy leads to the instrument panels. Whereas instrument CTs have to remain accurate up to 12 % rated current, protection CTs must retain proportionality up to 20 times normal full load

Example 19
A potential transformer rated 14400 V / 115 V and a current transformer rated 75 A / 5 A are used to measure the voltage and current in a transmission line. If the voltmeter indicates 111 V and the ammeter reads 3 A, calculate the voltage and current in the line.

Solution

$$The\ voltage\ on\ the\ line\ is\ \ V = 111 \times \frac{14400}{115} = 13900\,V$$

$$The\ current\ in\ the\ line\ is\ \ I = 3 \times \frac{75}{5} = 45\ A$$

Example 20
The toroidal current transformer of Fig. 38 has a ratio of 1000 A / 5 A. The line conductor carries a current of 600 A.
(a)Calculate the voltage across the secondary winding if the ammeter has an impedance of 0.15 Ω
(b) Calculate the voltage drop the transformer produces on the line conductor
(c) If the primary conductor is looped four times through the toroidal opening, calculate the new current ratio

Solution

$$(a)\ \ Current\ in\ the\ secondary\ I_2 = \frac{5}{1000} \times 600 = 3\ A$$

$$Voltage\ drop\ across\ the\ burden = 3 \times 0.15 = 0.45\,V$$

$(b)\ \ \dfrac{V_1}{V_2} = \dfrac{N_1}{N_2} = \dfrac{I_2}{I_1} \quad or \quad V_1 = \dfrac{I_2}{I_1} \times V_2 = \dfrac{5}{1000} \times 0.45 = 2.25\,mV$

$(c)\ \ I_2 N_2 = I_1 x1 = I_1' x4$

$This implies that\ I_1' = 250A$

$Therefore the\ new\ current\ ratio = 250A / 5A$

Chapter Four

3-phase Induction Motors

1 Introduction

The most widely used motors in the industry are the three phase induction motors. They are simple, robust, low-cost and easy to maintain.

2 Construction

A three-phase induction motor has two main parts: a stationary stator and a revolving rotor. The rotor is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm depending on the power rating of the motor.

(a) Stator: It consists of a steel frame which encloses a hollow, cylindrical core made up of stacked laminations. A number of slots are punched uniformly round the gap surface of the core. These slots carry the stator 3-phase winding.

(b) Rotor: There are two types:

Squirrel-cage rotor: On the gap surface of the rotor are also punched slots which contain bare copper or aluminum bars. The bars are short-circuited at each end by rings.

Wound or slip-ring rotor: The slots carry three-phase winding similar to the one on the stator. The windings are usually connected in star. The terminals are connected to three-slip rings which turn with the rotor. The revolving slip-rings and associated stationary brushes enable us to connect external resistors in series with the rotor windings. The rotor core in each case is also composed of laminations

3 Production of rotating magnetic field

Consider a simple stator having 3 coils, one per phase (Fig.1). The coils are geometrically spaced at 120 degrees from each other. This gives what is called a 3-phase winding which may be either connected in star or delta. Suppose each coil has 10 turns and the currents in them are coil a, $i_a = 10\sin\omega t$, coil b, $i_b = 10\sin(\omega t - 120^0)$ and coil c, $i_c = 10\sin(\omega t + 120^0)$

These currents produce magnetomotive forces which in turn create flux and thus make the stator appear as magnet. The axes of the fields produced by the coils when carrying positive currents are shown in Fig. 2. The fields are assumed to be sinusoidally distributed in space. As far as the rotor is concerned, three coils produce a single magnetic field having one broad North Pole and one broad South Pole. The combined magnetic field at the instants when $\omega t = 0^0, 90^o, 180^0, 270^0$ and 360^0 can be obtained as follows:

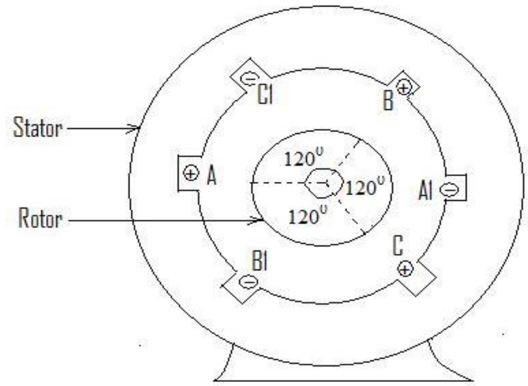


Fig. 1 Simple machine

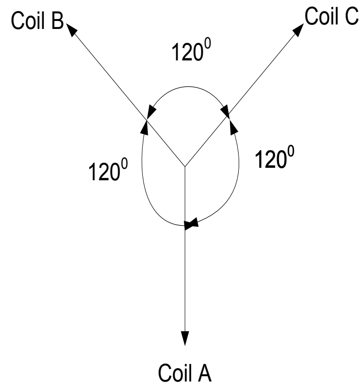


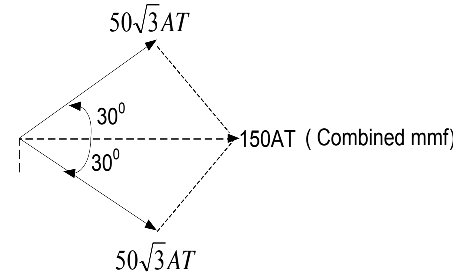
Fig. 2 Axes of mmf of coils A, B and C

(a) $\omega t = 0^0$

$i_a = 0, \quad mmF_a = 0$

$i_b = -5\sqrt{3}\ A, \quad mmF_b = -50\sqrt{3}\ AT$

$i_c = +5\sqrt{3}\ A, \quad mmF_c = +50\sqrt{3}\ AT$

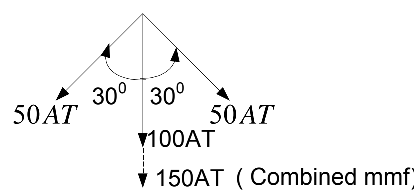


(b) $\omega t = 90^0$

$i_a = 10\,A, \quad mmF_a = 100\,AT$

$i_b = -5\ A, \quad mmF_b = -50\,AT$

$i_c = -5\ A, \quad mmF_c = -50\,AT$

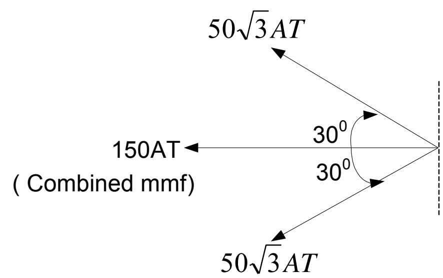


(c) $\omega t = 180^0$

$i_a = 0, \quad mmF_a = 0$

$i_b = 5\sqrt{3}\ A, \quad mmF_b = 50\sqrt{3}\ AT$

$i_c = -5\sqrt{3}\ A, \quad mmF_c = -50\sqrt{3}\ AT$

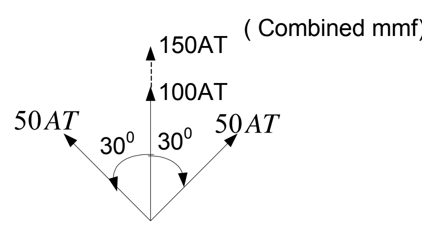


(d) $\omega t = 270^0$

$i_a = -10\ A, \quad mmF_a = -100\,AT$

$i_b = 5\ A, \quad mmF_b = 50\,AT$

$i_c = 5\ A, \quad mmF_c = 50\,AT$



(e) $\omega t = 360^0$ As when $\omega t = 0^0$

We make the following observations:

(a) The combined or resultant field is rotating

(b) The speed of rotation of the field $\omega_s = \omega$. In general for a 2p-pole machine, the speed of rotation called synchronous speed is given by

$$\omega_s = \frac{\omega}{p} rad/s \ or \ n_s = \frac{f}{p} \ rev/s \ or \ N_s = \frac{60f}{p} \ rev/\min$$

(c) The peak value of the combined field is constant and is equal to $\frac{3}{2}F_m$ where F_m = maximum peak value of the magnetic field produced by a phase coil. We note that peak value of the field produced by a phase is varying with time.

4. Principle of operation

When the stator 3-phase winding is supplied with 3-phase currents, a rotating magnetic flux density is produced in the airgap. The flux cuts the conductors or bars in the rotors. Emf generated in the bars circulates current in the bars consequently a force is exerted on the rotor which drags the rotor in the direction of the rotating flux.

5. Definition of slips

If the motor is wound as a 2p-pole machine, the rotating flux rotates at the speed

$$n_s = \frac{f}{p} \ rev/\sec \qquad (1)$$

where $f =$ the supply frequency

$p =$ number of pairs of poles

The speed n_s is called synchronous speed.

The slip, s of the machine is defined as

$$s = \frac{synchronous\ speed - rotor\ speed}{synchronous\ speed} = \frac{n_s - n_r}{n_s} \qquad (2)$$

6. Voltage and frequency induced in the rotor winding

Rotor frequency $f_r = sf$ (3)

Rotor voltage $E_r = sE_{oc}$ $(approximately)$ (4)

where $E_{oc} =$ open-circuit voltage induced in the rotor.

When $n_r = n_s$, $s = E_r = 0$ and there is no current in the rotor bars to produce the dragging force. Under these conditions, friction and windage would cause the motor to slow down. The rotor or motor speed is always less than the synchronous speed. On no load s is usually less than 0.001 and at full load s is usually less than 0.005 for large motors (1000 kW and more) and less than 0.03 for small motors (10 kW or less). For this reason induction motors are considered to be constant speed machines.

Example 1

An induction motor is excited by a 3-phase, 50-Hz source. If the full-load speed is 1440 rev/min calculate the slip.

Solution
The synchronous speed close to 1440 rev/min is 1500 rev/min. This is obtained if the machine is wound for four poles.

Check $N_s = \frac{60f}{p} = \frac{60 \times 50}{2} = 1500 rev/\min$

Consequently the slip is

$$s = \frac{n_s - n_r}{n_s} = \frac{1500 - 1440}{1500} = 0.04$$

7. Active power flow through induction motor

The power flow diagram shown in Fig.3 indicates what becomes of the active power P_e that flows into the stator of an induction motor.

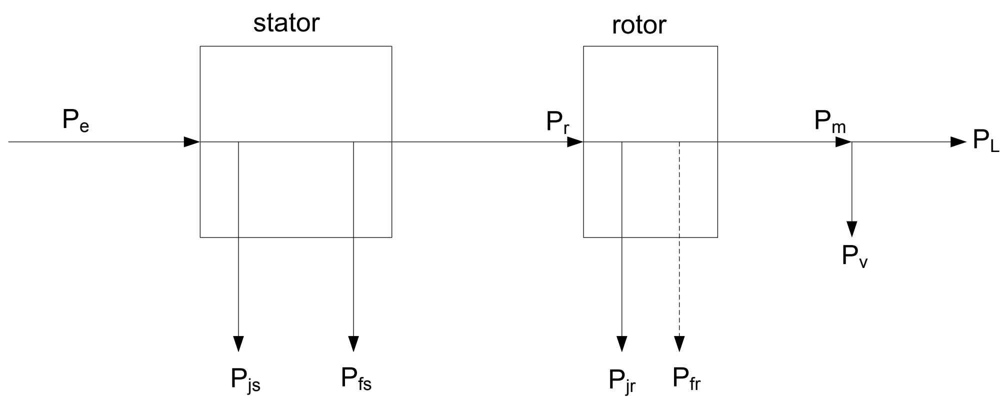


Fig. 3 Power flows diagram

$P_e = input\ power$

$P_{js} = stator\ copper\ loss$

$P_{fs} = stator\ iron\ loss$

$P_r = power\ transferred\ from\ stator\ to\ rotor$

$P_{jr} = rotor\ copper\ loss$

$P_{fr} = stator\ iron\ loss\ \ (negligible\ because\ f_r = 0)$

$P_m = gross\ output\ power$

$P_L = shaft\ power$

$P_v = friction\ and\ windage\ loss$

8. Torque and output power equations

The power transferred from the stator to the rotor and the gross mechanical power P_m are given by

$P_r = T_m \omega_s$

$P_m = T_m \omega_r$

where T_m is the gross torque developed by the motor. $T_m = T_e$ (electromagnetic torque). If the rotor iron loss is neglected then

$$P_{jr} = P_r - P_m = T_m \frac{\omega_s - \omega_r}{\omega_s} . \omega s = sP_r$$

ie

$$P_{jr} = sP_r \tag{5}$$

and

$$P_m = P_r - P_{jr} = (1-s)P_r \tag{6}$$

Example 2

The power supplied to a three-phase induction motor is 40 kW and the corresponding stator losses are 1.5 kW, calculate

(a) the total mechanical power developed and the rotor I^2R loss when the slip is 0.04 per unit.

(b) the output power of the motor if the friction and windage losses are 0.8 kW and

(c) the efficiency of the motor. Neglect the rotor iron loss.

Solution

(a) Input power to rotor, $P_r = 40 - 1.5 = 38.5 kW$

Rotor copper loss $P_{jr} = sP_r = 0.04 \times 38.5 = 1.54 kW$

Therefore the total mechanical power developed $P_m = P_r - P_{jr} = 38.5 - 1.54 = 36.96 kW$

(b) Output power of motor $P_L = P_m - P_v = 36.96 - 0.8 = 36.16 kW$

(c) Efficiency of motor $\eta = \frac{P_L}{P_e} = \frac{36.16}{40} = 0.904 pu = 90.4\%$

Example 3

If the speed of the motor of Example 2 is reduced to 40 % of its synchronous speed by means of external rotor resistors, calculate

(a) the total rotor I^2R loss and

(b) the efficiency, assuming the torque and the stator losses remain unaltered. Also assume that the increase in the iron loss is equal to the reduction in the friction and windage loss.

Solution

(a) *New slip*, $s = \frac{n_s - n_r}{n_s} = \frac{100 - 40}{100} = 0.6 pu$

and input power to rotor $P_r = 38.5 kW$ $(because\, torque\, is\, unaltered)$

(b) Total rotor copper loss $P_{jr} = sP_r = 0.6 \times 38.5 = 23.1 kW$

Total losses in rotor $= P_{jr} + P_v = 23.1 + 0.8 = 23.9 kW$

Therefore output power of motor $= P_r - total rotor losses = 38.5 - 23.9 = 14.6 kW$

$$Efficiency\ of\ motor = \frac{output\, power}{input\, power} = \frac{output\, power}{stator losses + P_r} = \frac{14.6}{40} = 0.365 pu = 36.5\%$$

9. Equivalent circuit

It is arrived at in the same way as for transformer. The circuit on the single-phase basis is shown in Fig. 4. Approximate circuits are given in Fig.5

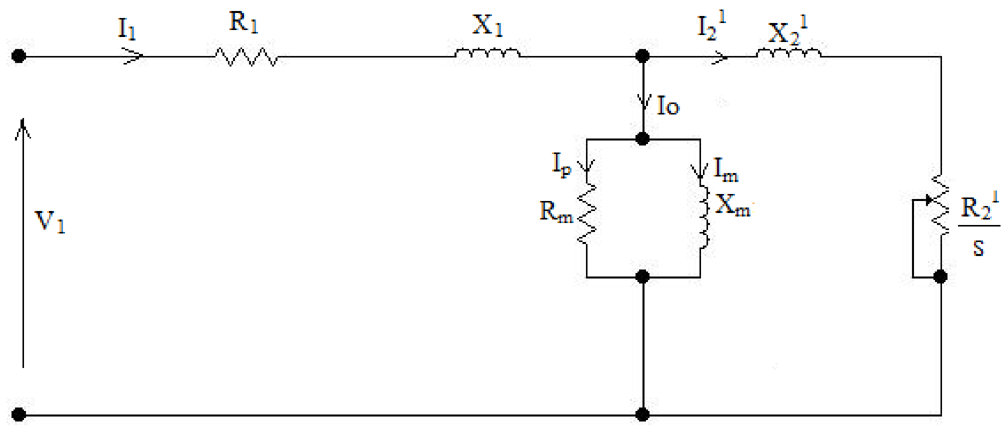
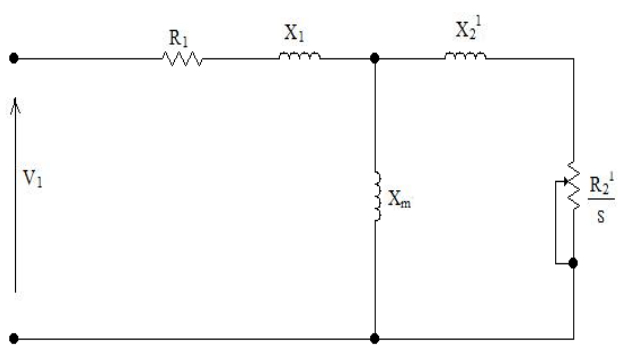
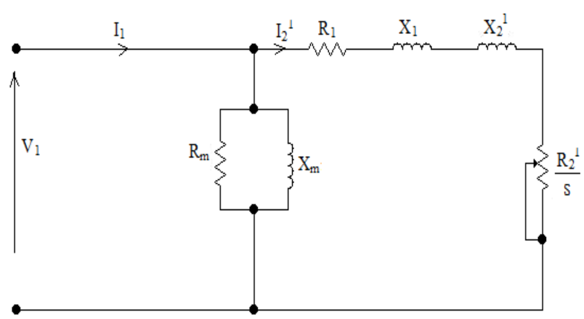


Fig.4 Exact Equivalent Circuit



The circuit is used on the usual assumption that the losses in the machine are constant.

Fig. 5a Approximate Circuit with R_m neglected



The voltage drop due to I_o flowing through the stator leakage impedance is appreciable. Its exclusion can give rise to errors of 10 % or more in some cases. It is acceptable for motors above 5 hp.

Fig. 5b Approximate Circuit with the magnetizing branch transferred to the terminals

Other approximations can be derived from the exact equivalent circuit by omitting R_1 if $R_1 << X_1$ *or* R_m *and* R_1 *or* R_1 *and* X_1. The last one supposes that E_{oc} (See equation 4) is constant. This gives very large errors at high values of slip.

10. Torque calculations

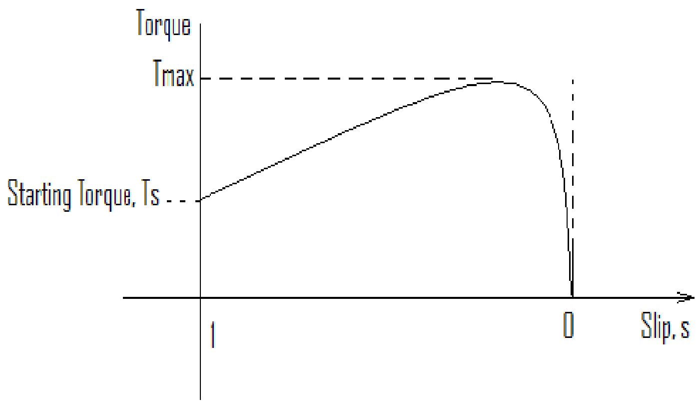
Consider the circuit shown in Fig. 5.b

$$Torque\,T_m = \frac{3[I_2']^2}{\omega_S}\frac{R_2'}{S}\;(from\,P_r = T_m\omega_s)$$

Or

$$T_m = \frac{3}{\omega_s}\frac{V_1^2}{\left[\left(R_1+\frac{R_2'}{s}\right)^2+\left(X_1+X_2'\right)^2\right]}\frac{R_2'}{s} \tag{7}$$

The general shape of the torque-slip curve for an induction motor is in Fig.6



T_{max} is called the breakdown torque
T_s is the starting torque.

Fig.6 General shape of the torque-slip curve for an induction motor

<u>Starting torque</u>: At starting $s = 1$ and

$$T_m = \frac{3}{\omega_s}\frac{V_1^2}{\left[\left(R_1+R_2'\right)^2+\left(X_1+X_2'\right)^2\right]}R_2' \tag{8}$$

<u>Maximum Torque</u>: The torque is maximum when the power dissipated in R_2'/s is maximum. Then by maximum power transfer theorem, maximum torque will occur when $\left|R_1 + jX\right| = R_2'/s$ where $X = X_1 + X_2'$. Thus the slip s_m, for maximum torque is given by

$$s_m = R_2' / \left(R_1^2 + X^2\right)^{1/2} \tag{9}$$

When this value of slip is substituted into the general equation for torque (7), the value of maximum torque T_{max} is obtained as

$$T_{max} = \frac{3V_1^2}{2\omega_s\left[R_1 + \sqrt{R_1^2 + X^2}\right]}$$
(10)

We observe that

(i) $s_m \propto$ rotor resistance but T_{max} is independent of it.

(ii) The speed of an induction motor can be controlled by varying the rotor resistance

(iii)The starting torque can be increased by increasing the rotor resistance.

(iv)

Typical torque-slip curves with variable rotor resistance are shown in Fig. 7

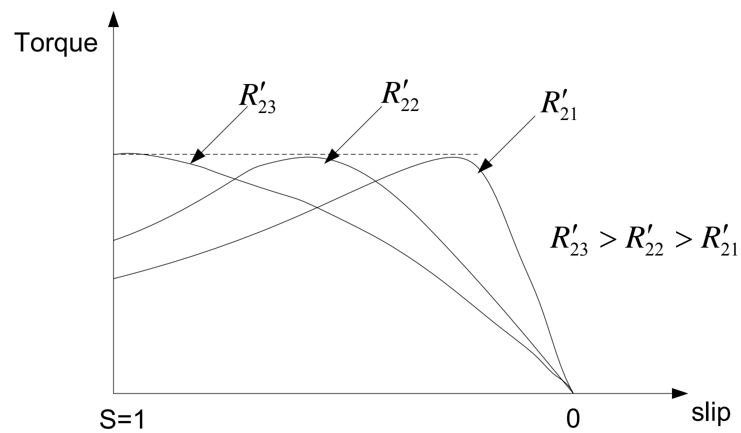


Fig. 7 Effect of rotor resistance on torque curve

Example 4

A 415-V, 3-phase, 6-pole, 50-Hz, star-connected slip ring induction motor has the following parameters in ohms per phase

$R_1 = 0.04$, $X_1 = 0.15$, $R'_2 = 0.05$, $X'_2 = 0.15$

Determine

(a) The stator current and the gross torque in Nm when the slip is 0.05 pu.

(b) The maximum gross torque, the slip at which it occurs and the gross output power under these conditions

(c) The value of external resistance to be inserted in the rotor circuit to produce the maximum torque at standstill. Neglect the magnetizing branch.

Solution

(a) Synchronous speed $\omega_s = \frac{2\pi f}{p} = \frac{2\pi \times 50}{3} = 104.7\, rad/s$

$$Total\ impedancde = (R_1 + \frac{R'_2}{s}) + (X_1 + X'_2) = \left(0.04 + \frac{0.05}{0.05}\right) + j(0.15 + 0.15)$$

$= 1.04 + j0.30 = 1.082\angle 16.09^0\,\Omega$

$V_{phase} = 415/\sqrt{3} = 240V$

Stator current $I = 240\angle 0/.082\angle 16.09 = 222\angle -16.09A$

Power into rotor $P_r = 3I^2\dfrac{R_2'}{s} = 3\times 222^2\left(\dfrac{0.05}{0.05}\right) = 147.8\ kW$

$Gross\ torque = \dfrac{P_r}{\omega_s} = \dfrac{147.8}{104.7} = 1.4\ kN$

(b) $T_{max} = \dfrac{3}{2}\times\dfrac{V_1^2}{\left[R_1 + \sqrt{R_1^2 + X^2}\right]\omega_s} = \dfrac{3}{2}\times\dfrac{240^2}{\left[0.04^2 + \sqrt{0.04^2 + 0.3^2}\right]}\times\dfrac{1}{104.7} = 2.41\ \text{kN}$

$s_m = R_2'/\left(R_1^2 + X^2\right)^{1/2} = 0.05/\left(0.04^2 + 0.3^2\right)^{1/2} = 0.16\ pu$

$Motor\ speed\omega_r = \omega(1-s) = 104.7\times(1-0.16) = 87.948 rad/s$

$Output\ power = T_{\max}\times\omega_r\ at \max imun slip = 2.41\times 87.948 = 212\ kW$

(c) At standstill $s = 1$. Therefore if the maximum occurs at standstill (or at starting), then

$1 = R_2'/\left(R_1^2 + X^2\right)^{1/2}$

or

$R_2' = \left(R_1^2 + X^2\right)^{1/2} = \sqrt{0.04^2 + 0.3^2} = 0.30\Omega$

Additional resistance required referred to the stator is $R_{ex}' = 0.30 - 0.05 = 0.25\ \Omega$

11. Effect of rotor resistance:

Increase in the rotor resistance:

(a) changes the torque-speed curve (see Fig.7)

(b) does not affect the breakdown torque but it occurs at a lower speed.

(c) reduces the starting current.

(d) increases the starting torque

12. Starting torque

With a motor having a low-resistance rotor, such as the usual type of cage rotor, the starting torque is small compared with the maximum torque available. A high rotor resistance produces a high starting torque. However, it also produces a rapid fall-off in speed with increasing load. Furthermore, because the slip which results is high, the rotor copper losses are high, the efficiency is low and the motor tends to get hot. When a high starting torque is required for a heavy starting duty such as accelerating a high inertia load from standstill to its running speed, the slip-ring is ideal (Fig.8). At starting the variable resistors are set to their highest value. As the motor accelerates, the resistance is gradually reduced until full-load speed is reached whereupon the brushes are short-circuited.

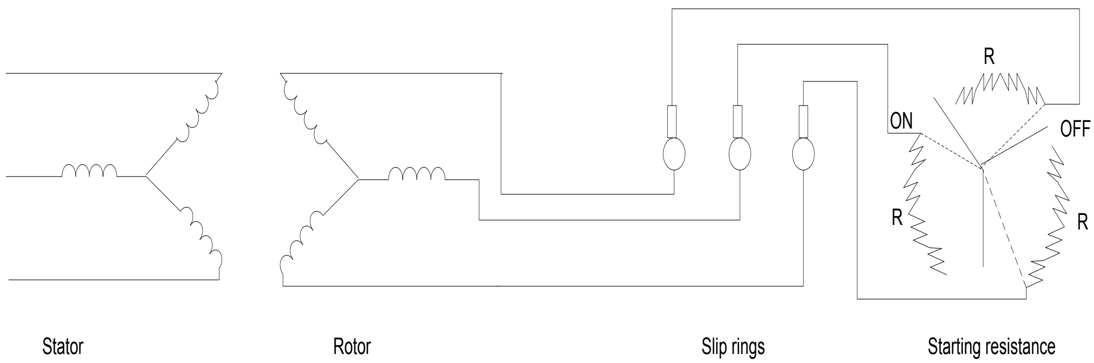


Fig. 8 Slip-ring induction motor

13. Comparison of cage and slip-ring motors

Advantages of squirrel-cage motors:

(i) Cheaper and more robust
(ii) Slightly higher efficiency and power factor
(iii)Explosion proof, since the absence of slip-rings and brushes eliminate risk of sparking

Advantages of slip-ring motors:

(i) The starting torque is much higher and the starting current much lower
(ii) The speed can be varied by means of external rotor resistor

14. Starting methods

Squirrel cage motors

Motors are started either by being switched directly across the supply or with a reduced voltage

(a) Direct-on-line starting: The initial current is high, about 5 to 6 times the full load current, and the power factor is low. This high low-power-factor current can produce a significant voltage drop which may affect other consumers connected to the same line. Consequently it is usual to start cage motors, except small machines, with reduced voltage. This method is simple and inexpensive.

(b) Reduced voltage starting: This method is used because
(i) The driven equipment has to be started very gradually to avoid damage to the equipment.
(ii) Direct-on-line starting produces too high starting current.

The methods available for reducing starting currents are
(i) *Stator impedance starting*: Three resistors or inductors (usually resistors) are connected in series with the stator during the start-up period. This method is used for smooth starting of small machines.
(ii) *Autotransformer starting*: A three-phase star-connected autotransformer is used to reduce the stator applied voltage. The autotransformers usually have taps to give output voltages of 0.8, 0.65 and 0.5 pu. A tap is selected to limit the starting current to a desired value. An autotransformer which reduces the voltage applied to the motor to a fraction x will

have motor starting torque reduced to $T_s = x^2 T_{sc}$ and the line starting current to $I_s = x^2 I_{sc}$ where T_{sc} and I_{sc} are what we obtain with direct-on-line starting.

(iii)*Star-delta starting*: The machine, designed for delta, has all the six stator leads brought out to the terminal box. The windings are connected in star during the starting period and in delta during normal running conditions. The phase voltage is reduced to a fraction $x = 1/\sqrt{3}$ and the motor behaves as if the autotransformer were employed, i.e., line starting current $I_s = \left(\frac{1}{\sqrt{3}}\right)^2 I_{sc} = \frac{1}{3} I_{sc}$ and starting torque $T_S = \frac{1}{3} T_{sc}$ This method is effective if the starting torque is adequate and cheap so long as the supply voltage does not exceed about 3 3kV(At voltages exceeding 3 kV, excessive number of stator turns will be needed for delta running).

Example 5

Calculate the relative values of (i)the starting torque and (ii)the starting current of a 3-phase cage-rotor induction motor when started by(a) direct switching(b) a star-delta starter and (c) an autotransformer having 40% tapping.

Solution

(i) Starting torque: $1 : \frac{1}{3} : 0.4^2 = 1 : 0.33 : 0.16$

(ii) Starting current $1 : \frac{1}{3} : 0.4^2 = 1 : 0.33 : 0.16$

The ratio between starting torque T_s and full-load torque T_{fl} is usefully expressed in terms of currents. Since the referred rotor current is approximately equal to the input current, and at starting $s = 1$

$$\frac{T_{sc}}{T_{fl}} = \frac{I_{sc}^2 R_2' / 1}{I_{fl}^2 R_2' / s_{fl}}$$

From which

$$T_{sc} = \left(\frac{I_{sc}}{I_{fl}}\right)^2 \times s_{fl} \times T_{fl}$$

In general

$$T_{sc} = x^2 \left(\frac{I_{sc}}{I_{fl}}\right)^2 \times s_{fl} \times T_{fl} \qquad (11)$$

Example 6

A 15-hp, 3-phase, 6-pole, 50-Hz, 400-V induction motor runs at 960 rev/min on full load. If it takes 80 A on D.O.L. switching, find the ratio of T_s and T_{fl} in the following cases:

(a) D.O.L. starting
(b) Star-delta starting
(c) Autotransformer starting with 60% tapping
(d) Stator resistance starter limiting the starting current to 50A

Take full-load power factor and efficiency to be 0.834 and 95.6% respectively

Solution

Output power $= 15 \times 746$

Full load current $= \dfrac{\text{output power}}{\sqrt{3} \times \text{Applied voltage} \times \text{efficiency} \times \text{pf}} = \dfrac{15 \times 746}{\sqrt{3} \times 400 \times 0.956 \times 0.834} = 20A$

$$N_s = \frac{60f}{p} = \frac{60 \times 50}{3} = 1000 \text{ rev/min}$$

Full-load slip $s_{fl} = \dfrac{(1000 - 960)}{1000} = 0.04$

(a) $\dfrac{T_s}{T_{fl}} = x^2 \left(\dfrac{I_{sc}}{I_{fl}} \right)^2 s_{fl}. = 1 \times \left(\dfrac{80}{20} \right)^2 \times 0.04 = 0.64$

(b) $\dfrac{T_s}{T_{fl}} = \left(\dfrac{1}{\sqrt{3}} \right)^2 \times \left(\dfrac{80}{20} \right)^2 \times 0.04 = 0.21$

(c) $\dfrac{Ts}{T_{fl}} = (0.6)^2 \times \left(\dfrac{80}{20} \right)^2 \times 0.04 = 0.23$

(e) In the stator resistance starter, the current reduces in proportion to the voltage,

$x = \left(\dfrac{50}{80} \right) = \dfrac{5}{8}$. Thus $\dfrac{T_s}{T_{fl}} = \left(\dfrac{5}{8} \right)^2 \times \left(\dfrac{80}{20} \right)^2 \times 0.04 = 0.25$

Slip-ring induction motor

The starting current is reduced by introducing additional rotor resistance during starting. The slip-ring motors are suitable for heavy, frequent starting and accelerating duty-cycles.

15. Speed Control

The speed n_r of an induction motor is given by $n_r = n_s (1 - s) = \left(\dfrac{f}{p} \right)(1 - s)$

This equation shows that the speed control can be achieved by varying the slip, the poles and the frequency

<u>Slip control</u>

The slip for a given torque can be varied in cage motors by changing the supply voltage and in a slip-ring type by inserting external rheostats in the rotor circuits

(i) *<u>Voltage control</u>*: The effect of stator voltage on torque–speed curve of a motor is shown in Fig. 9. The effectiveness of this method of speed control is affected by the torque speed characteristics of the load. The use of this method is practically limited to fan-type load. Typical application is for fans with two or more speeds. A high rotor resistance is necessary in practice to give a wide speed range. Owing to considerable slip losses this type of control is only feasible for motors rated below 20 hp.

Voltage controllers employed in practice are**:**

(a) Rheostat connected in series with the stator for motors of rating 1 kW or less
(b) Variable inductor instead of rheostat for more economical control
(c) Variable 3-phase autotransformers
(d) Electronic controllers (called phase voltage controllers)

The voltage control has the following advantage and disadvantages:

Advantage:
- It is simple

Disadvantages:
- It is wasteful in power
- It is wasteful in machine capacity
- It has poor speed regulation

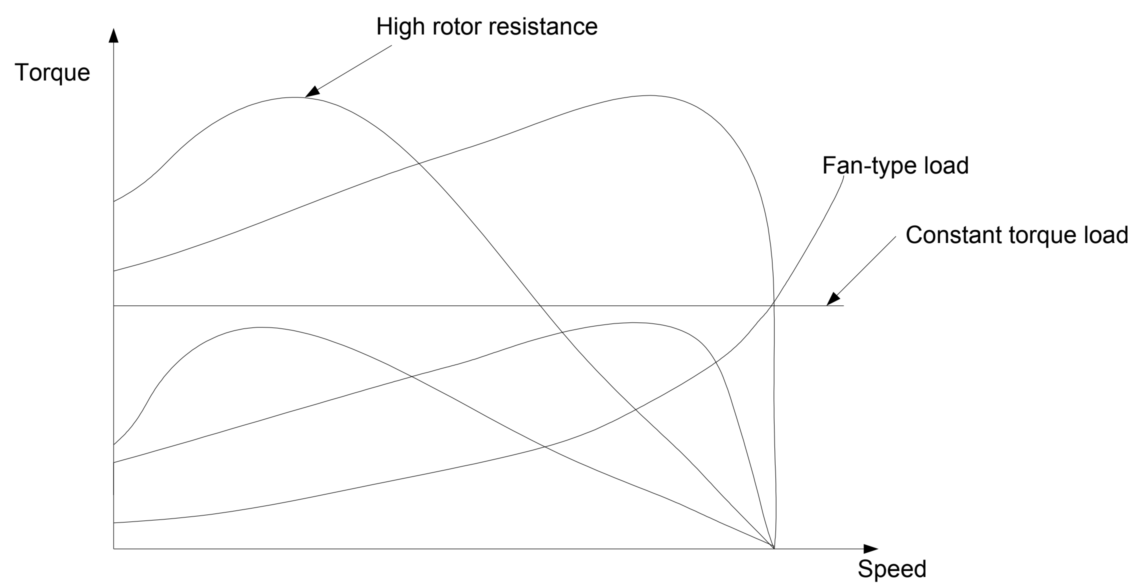
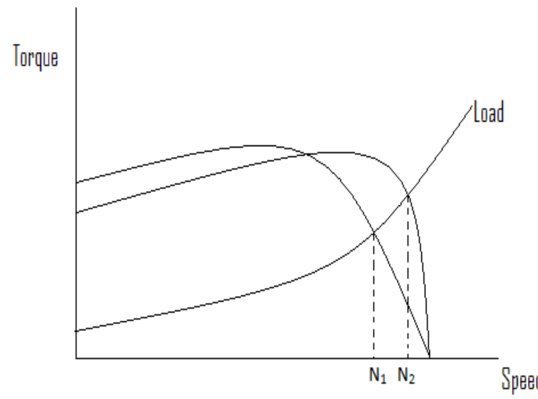


Fig. 9 Effect of voltage variation

(ii) *Rotor resistance control (see Fig. 10)*: This is suitable for slip-ring motor.
Advantage:
- It is simple

Disadvantages:
-It is used only to decrease speed
- It has poor speed regulation
- It is wasteful in power
- It is unsatisfactory for large speed changes



Example 7

A 3-phase, 415 V, 50 Hz, 4-pole induction motor drives a fan. At the rated torque, the motor runs at 1440 rev/min. What voltage must be applied to the motor so that it runs at 1080 rev/min?

Solution

From (7)

$$T_m = \frac{3}{\omega_s}\frac{V_1^2}{\left[\left(R_1+\frac{R_2'}{s}\right)^2+\left(X_1+X_2'\right)^2\right]}\frac{R_2'}{s}$$

In the operating portion of the torque-speed curve, s is very small and so $R_1 << R_2'/s$ and $X_1 + X_2' << R_2'/s$. Hence the torque equation can be approximated by

$$T_m = \frac{3}{\omega_s}\frac{V_1^2}{\left[\left(\frac{R_2'}{s}\right)^2\right]}\frac{R_2'}{s} = \frac{3}{\omega_s}\frac{V_1^2 s}{R_2'}$$

Hence

$$\frac{T_{m,new}}{T_{m,old}} = \left(\frac{V_{1,new}}{V_{1,old}}\right)^2 \times \left(\frac{R'_{2,old}}{R'_{2,new}}\right)\times\left(\frac{s_{new}}{s_{old}}\right) \quad (12)$$

From (12), if $R'_{2,new} = R'_{2,old}$ then

$$\frac{T_{m,new}}{T_{m,old}} = \left(\frac{V_{1,new}}{V_{1,old}}\right)^2 \times \left(\frac{s_{new}}{s_{old}}\right) \quad \text{Or} \quad \left(\frac{V_{1,new}}{V_{1,old}}\right)^2 = \left(\frac{s_{old}}{s_{new}}\right)\times\left(\frac{T_{m,new}}{T_{m,old}}\right)$$

For fan-type load, $\frac{T_{m,new}}{T_{m,old}} = \left(\frac{N_{r,new}}{N_{r,old}}\right)^2 = \left(\frac{1080}{1440}\right)^2 = 0.5625$

$$N_s = \frac{60f}{p} = \frac{60\times 50}{2} = 1500\text{rev/min}\,;\ s_{new} = \frac{1500-1080}{1500} = 0.28\,;\ s_{old} = \frac{1500-1440}{1500} = 0.04$$

Therefore

$$\frac{V_{1,new}}{V_{1,old}} = \sqrt{\left(\frac{s_{old}}{s_{new}}\right)\times\left(\frac{T_{m,new}}{T_{m,old}}\right)} = \sqrt{\left(\frac{0.04}{0.28}\right)\times(0.5625)} = 0.2835 \text{ and } V_{1,new} = 0.28\times 415 = 118\text{ V}$$

Example 8

A 3-phase, 208 V induction motor having a synchronous speed of 1200 rev/min, runs at 1140 rev/min when connected to a 215 V line. Calculate the speed if the voltage increases to 240 V. Assume the torque remains the same.

Solution:

$$s_{old} = \frac{N_s - N_r}{N_s} = \frac{1200-1140}{1200} = 0.05$$

$$s_{new} = \left(\frac{V_{1,old}}{V_{1,new}}\right)^2 \times s_{old} \quad \text{if } T_{m,new} = T_{m,old} \text{ and } R'_{2,new} = R'_{2,old}$$

Therefore

$$s_{new} = \left(\frac{215}{240}\right)^2 \times 0.05 = 0.04 \text{ and } N_{r,new} = N_s(1-s_{new}) = 1200(1-0.04) = 1152\text{rev/min}$$

Example 9

A 3-phase slip-ring induction motor has a rating of 110 kW, 1760 rev/min, 2.3 kV, 60 Hz. Three external resistors of 2 ohms are connected in star across the rotor slip-rings. Under these conditions the motor develops a torque of 300 Nm at a speed of 1000 rev/min.

(a) Calculate the speed for the torque of 400Nm

(b) Calculate the value of the external resistor so that the motor develops 10 kW at 200 rev/min.

Neglect the actual rotor resistance.

Solution

(a) For a rated speed of 1760 rev/min at 60 Hz, the synchronous speed must be 1800 rev/min. Hence $s_{old} = (1800-1000)/1800 = 0.444$

$T_{m,old} = 300$ Nm

All other conditions being fixed, we have for $T_{m.new} = 400\text{Nm}$ and

$$s_{new} = s_{old}\left(\frac{T_{m,new}}{T_{m,old}}\right) = 0.444\left(\frac{400}{300}\right) = 0.592$$

and

$$N_{r,new} = 1800(1-0.592) = 734\text{rev/min}$$

(b) $T_{m,new} = \dfrac{output\,power}{motor\,speed} = \dfrac{10\times 10^3}{2\pi\times\frac{200}{60}} = 477\text{Nm}$

$s_{new}\,(1800-200)/1800 = 0.889$

$$R_{r,new} = \left(\frac{s_{new}}{s_{old}}\right) \times \left(\frac{T_{m,old}}{T_{m,new}}\right) \times R_{r,old} = \left(\frac{0.889}{0.444}\right) \times \left(\frac{300}{477}\right) \times 2 = 2.5\Omega$$

Pole-changing

This method of speed control is applicable to cage motors. A cage motor may be provided with two or more stator winding arranged for different numbers of poles so as to achieve two or more synchronous speeds. A cage motor may also have a single stator winding, the connections of which can be changed to give two or three different number of poles.

The oldest single-stator pole-changing machine (invented over 60 years ago) has the stator designed so that the motor can operate at two different synchronous speeds in the ratios 2:1. Six leads are brought out of the stator and the speed is changed by simply changing the external stator connections (see Fig.11)

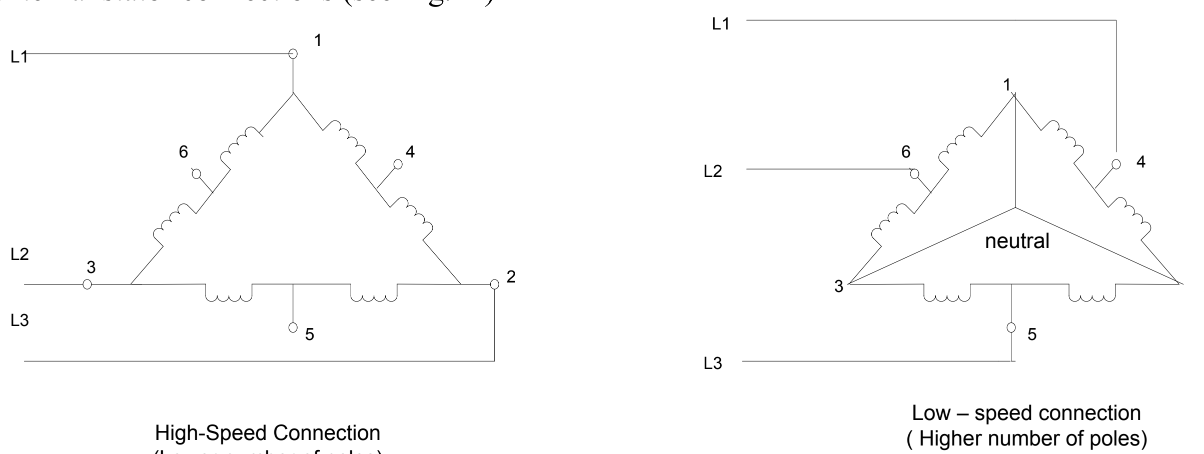


Fig. 11 Speed Control by Pole Changing

Frequency control

In practice the voltage is varied and is varied in the same proportion as the frequency so as to maintain a constant flux in the air gap. If both frequency and voltage are varied to maintain a constant flux in the air gap, the shape of the torque-speed curve remains the same, but its position along the speed axis shifts with frequency, Fig 12. Therefore for a given torque the slip speed in rev/min (not in *pu*) remains constant. This method permits a wide range of speed control. Maximum torque is available at all frequencies and with high efficiency since low-resistance cage motors can be used. In modern practice the frequency changer is a power electronic circuit able to develop the required frequency and voltage from a dc source (electronic circuit is an inverter) or more usually, from a fixed frequency ac mains supply (electronic circuit is a rectifier-inverter or a cycloconverter).

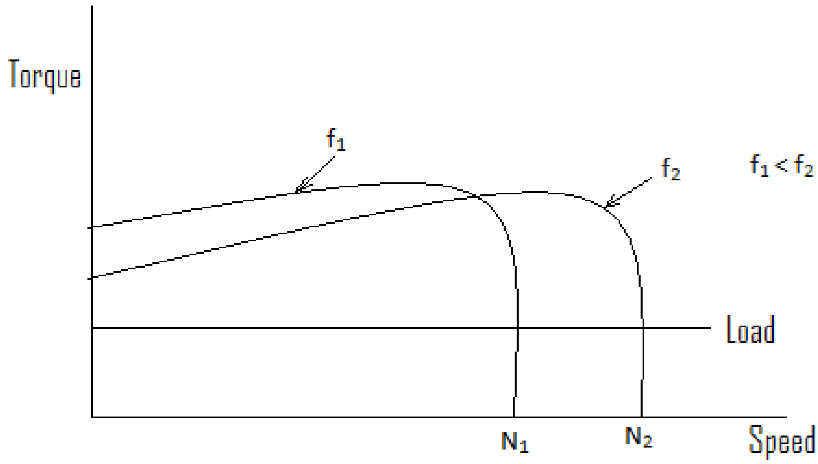


Fig.11 Effect of frequency and voltage change

Example 10
A standard 3-phase, 10 hp, 575 V, 1750 re/min, 60 Hz squirrel-cage induction motor develops a torque of 110 Nm at a speed of 1440 rev/min. If the motor is excited at a frequency of 25 Hz, calculate:
(a) the required stator voltage to maintain the same flux in the machine
(b) the new speed at a torque of 110 Nm

Solution
(a) $V = (25/60) \times 575 = 240\text{V}$
(b) $N_s = 1800\text{rev/min}$. Consequently, the slip speed at a torque of 110 Nm is
$N_{slip} = N_s - N_r = 1800 - 1440 = 360\text{rev/min.}$
This slip speed is the same for the same toque, irrespective of the frequency
The synchronous speed at 25 Hz $= \frac{25}{60} \times 1800 = 750\text{rev/min}$
The new speed at 110 Nm is $N_r = 750 - N_{slip} = 750 - 360 = 390$ rev/min

14. Braking of induction motors
Braking can be achieved using either electrical or electromechanical means. Methods of electrical braking include:
Plugging
Braking is achieved by simply interchanging two stator leads while the motor is running. This is the most inefficient method of braking because not only does the mechanical energy (K.E.) reconverted into heat in the rotor but also the electrical energy which is drawn from the supply is similarly wasted. Motors should not be plugged frequently because high rotor temperatures which result may melt the rotor bars or overheat the stator windings. It is necessary to disconnect the supply exactly at the instant when the motor stops otherwise it

will continue to move in the reverse direction. (Heat dissipated in rotor $= 3 \times \text{initial KE}$ when unloaded)

D.C. Injection

D.C. current is circulated in the stator winding through any two terminals immediately after all the three terminals have been disconnected from the 3-phase supply. The d.c. current produces stationary field as in the dc machines. When the rotor conductors sweep past the stationary field, an ac voltage is induced in them. The voltage produces an ac current and the resulting rotor copper losses are dissipated at the expense of the kinetic energy stored in the revolving parts. The motor finally comes to rest when all the kinetic energy has been dissipated as heat in the rotor. This method produces far less heat than plugging does(Heat $=$ Initial KE.).

Chapter Five

Special Electrical Machines

A Single-phase motors

1.0 Single-phase Induction motors

1.1 Introduction

Generally, single-phase motors are used when 3-phase power is not available. We find them mostly in household appliances and portable machine tools where they are of fractional horsepower ratings. There are many types on the market, each designed to meet a specific application.

1.2 Construction

It is very similar to the 3-phase squirrel-cage motor. It has a squirrel cage rotor, a main winding and an auxiliary (or start) winding. Without the auxiliary winding, motor at standstill behaves like a transformer having its secondary winding short-circuited (i.e. stationary field rather than revolving field is produced) hence the motor cannot self-start. The auxiliary winding is added to create a revolving field at standstill so that the motor can self-start. It is disconnected when the motor reaches a predetermined speed (about 70% of Ns) during the start-up. Special switches are used for this. An example is the speed sensitive centrifugal switch mounted on the motor shaft.

1.3 Motor starting torque

It is given by $T_s = kI_a I_m \sin\alpha$ where

I_m = locked-rotor or standstill current in the main winding

I_a = locked-rotor or standstill current in the auxiliary winding

α = phase angle between I_a and I_m

k = constant depending on the design of the motor.

1.4 Types of Single-phase induction motors

<u>Resistance split-phase (or simply split-phase) motor</u>

- The desired phase shift α is obtained by making the resistance of the auxiliary winding circuit high. This is achieved in practice by using fine wire for the auxiliary winding. Split-phase motors are the most popular single-phase motors on the market because they are cheap.
- Performance: Starting torque is moderate and starting current is usually 6 to 7 times the rated current. Operating efficiencies are around 65% and power factor is about 0.6 to 0.7.
- Ratings: Most of them are between 60 and 250W.
- Applications: They are used where moderate starting torque is required and where starting is no frequent. They drive fans, blowers, pumps, washing machines, dish washers, grinder, small bench drills and lathes.

Capacitor-start motor

- A large ac electrolytic capacitor is connected in series with the auxiliary winding to achieve the desired α. The main winding is just as the split-phase.
- Performance: Starting torque is high and starting current is 4 to 5 times the rated current. Running characteristic is the same as that of split–phase motor.
- Ratings: They usually range from 120 W to 7.5 kW.
- Typical values of capacitance: they vary from 20-30 μF for a 100-W motor up to 60-100 μF for 750 W.
- Applications: They are used where frequent starting or prolonged starting periods are required. They drive compressors, large fans, large washing machines, pumps and high inertia loads.

Permanent-split capacitor (or capacitor-run) motor

- A permanent duty capacitor (usually impregnated paper capacitor) is connected in series with the auxiliary winding which is left in circuit. The value of the capacitor may be chosen such that the motor acts as a true two-phase motor at full load. In this case the running performance is optimal: motor is quiet, and efficiency and power factor high. However, a weak starting torque compared to that of a capacitor-start motor is produced.
- The capacitor-run motors usually use a permanent duty capacitor of a compromise value to provide somewhat higher starting torque (about one-tenth of the best possible) at some sacrifice in running performance. The fractional horsepower ratings require about 2 to 20 μF capacitor.
- Performance: They operate at improved efficiency and power factor.
- Applications: They are used where quiet operation is required as in offices, classrooms, theaters, hospitals, studios, etc. They drive ceiling fans, business machines and good quality tape recorders. They drive also fans and blowers in heaters and air conditioners and refrigerator compressors.

Capacitor-run, capacitor start motor

- Optimum capacitance value for running is less than one-half of that for starting so this motor uses two different capacitors to achieve the two conflicting requirements. For 1-kW motor, the starting capacitor would be about 120 μF and the running capacitor 15-20 μF.
- Performance: Their full-load efficiency is about 80-85 % and power factor about 0.8 lagging.
- Applications: They are used for heavier loads such as compressors and reciprocating pumps.

Shaded-pole motor

- It is a split-phase induction motor but the construction is a bit different. See Fig.1. Rotor is squirrel-cage, stator is a salient-pole, and main winding consists of simple coils and the auxiliary is composed of copper or brass ring around a portion of each pole. The ring helps to produce a weak revolving field.
- Performance: Starting torque, operating efficiency (20 to 30 %) and power factor (0.5 to 0.7) are very low.

- Applications: They are very popular for ratings below 40 W. They drive tape recorders, turntables and small fans requiring little starting torque, business machines, hair dryers and cheap washing machines.

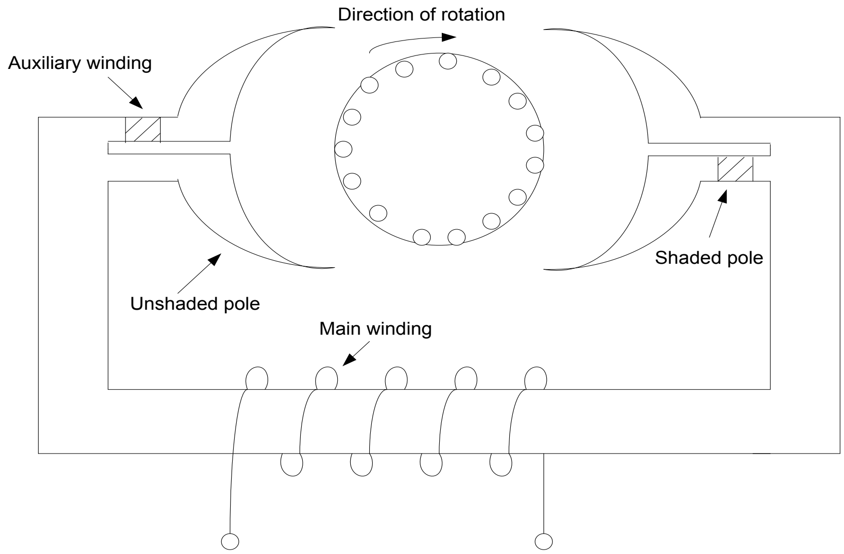


Fig. 1 Shaded-pole motor

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2.0 Universal motors

They are called universal motors because they can be operated from either ac or dc power supply.

(a) Construction: It is similar to dc series motor except that the stator is also laminated because the field flux is pulsating when the power supply is ac.

(b) Characteristics: Though small in size (its main advantage), these machines operate at high speeds and provide high starting torque. Typical operating speed, 3000 to 10000 rev/min.

(c) Performance: Full-load power factor is high, about 0.9 but efficiency is low, about 65 %.

(d) Ratings: They are built up to about 200 W.

(e) Disadvantages:
  - Motors have a high noise level.
  - They are not suitable for continuous-duty applications, mainly because of brush and commutator wear.

(f) Applications: Their ability to produce the highest output power per unit mass and per unit volume has made them popular for driving small machine tools and household appliances. They are used extensively to drive portable tools such as electric saws and drills, high speed vacuum cleaners, domestic food mixers, blenders, hand grinders and sewing machines.

3.0 Reluctance motors

It is a synchronous machine without field excitation. It uses any one of the normal single-phase induction motor windings, that is, split-phase, capacitor start, permanent-split capacitor, or shaded-pole arrangements. The rotor is squirrel-cage but this is milled out to create as many salient poles as there are stator poles. The milled-out sections increase the magnetic reluctance between the poles as compared to the low reluctance along the pole axis. The motor depends on this change of reluctance to produce a torque (called reluctance torque) hence the name reluctance motor. The reluctance motor starts up as a single-phase squirrel-cage motor but when it approaches synchronous speed, the salient poles lock with the revolving field, and so the motor runs at synchronous speed. Its drawback is that it cannot accelerate high inertia loads to synchronous speed. The reluctance motor is normally used in continuous-duty applications, such as electric clocks, other timing devices (such as timers), fans blowers, business machines, recorders, tape drives and small drive systems.

4.0 Hysteresis motors

The hysteresis motor is so named because it uses the phenomenon of hysteresis to produce mechanical torque. The rotor consists of a central nonmagnetic core upon which are mounted rings of magnetically hard material. The rings form a thin cylindrical shell of material whose iron losses consist mainly of hysteresis loss. The stator winding may be split-phase, capacitor start, permanent-split capacitor or shaded pole, although the permanent-split capacitor arrangement is most commonly used. The capacitor is selected to produce a vibration-free capacitor-run motor. When excited, the revolving field produced by the stator causes the rotor to accelerate until it reaches synchronous speed. The accelerating torque is essentially constant and its magnitude depends on the hysteresis energy dissipated in the rotor per turn. The hysteresis motor will achieve synchronous speed for any load it can accelerate, no matter how great the inertia. It is used in electric clocks and other precise timing devices. It is also used to drive tape decks, record-player turntables and other precision audio equipment for its constant-speed and accelerating torque (motors provide smooth starting torque reducing record slippage).

B Others

1. Permanent magnet motors

These are dc motors which use permanent magnets to create the working field.

(a) Advantages over the conventional dc motors: These include
  - Motors are smaller in size and their efficiency is higher
  - There is no risk of field failure.
  - It is possible to use long air gaps. This reduces armature circuit inductance and makes machines respond more quickly to changes in armature current.

(b) Disadvantages
  - Its only drawback is the relatively high cost of the magnets

(c) Applications: They are particularly advantageous for ratings below 5 hp. They are used in servo applications because of their low inertia and fast response.

2. Brushless dc motors(dc brushless motors)

Brushless dc motors are similar in speed-torque characteristic and application to brush-type dc permanent magnet motors. The motors differ, however, in construction and method of commutation. In these motors, the armature windings are located in the stator which permits direct connection of the power supply to the armature windings via electronic switches and thus eliminates stationary to moving contacts. The working field is provided by permanent magnets attached to the rotor shaft. The stator winding is generally three-phase, star or delta connected, although some stator windings are four phase. Motors powering small fans and other constant-speed equipment are often two phase. For the torque on the rotor to be always in the same direction, current must be directed to the armature windings at the right moment. Devices, sensing the rotor position, provide signals for the electronic switches supplying the stator windings to switch them in the proper sequence. In small inexpensive motors, inexpensive Hall effect sensors are used to sense the rotor position. Other position sensors are optical encoders and resolvers. The brushless dc motors are maintenance-free, pollution-free, much quieter and more reliable than the conventional dc motor. They are found in computers (they drive hard drives and blowers rated at 1 W, 12 V dc, 2500 rev/min used to cool components in a computer), CD and DVD players, and in anything else where efficiency and reliability are more important than price.

3. Stepper motors

Stepper motors are electrical motors that are driven by digital pulses rather than a continuously applied voltage. The motor as its name implies, moves the rotor through a precise angular step, clockwise or anticlockwise. Each step corresponds to a pulse that is supplied to one of its stator windings. If pulses are applied at a low repetition rate, the rotor will come to rest after each step. At high stepping rates (i.e. no. of steps/second), the motor can be made to move continuously at uniform speed. The motor is then said to be slewing. They are used when speed and position have to be precisely controlled. One of the most significant advantages of a stepper motor is its ability to be accurately controlled in an open loop system. Open loop control means no feedback information about position is needed. This type of control eliminates the need for expensive sensing and feedback devices such as optical encoders. Your position is known simply by keeping track of the input step pulses. Other advantages include

- Their performance is predictable and consistent.
- They are capable of rapid acceleration, deceleration and reversal.
- They are easily controlled using digital electronics, minicomputers, microprocessors and programmable logic controllers.
- They have non-cumulative position error.
- They are relatively maintenance free since the only wear items are the bearings

The main disadvantages include

- They have a low efficiency.
- They require relatively complex control systems.
- They are not completely silent when running. An audible hum is present, which is a direct function of the control pulse rate.

There are three main types**:**

<u>Variable reluctance stepper motors (VR)</u>

This type of stepper motor has been around for a long time. It is probably the easiest to understand from a structural point of view. Fig. 2 shows a typical four-phase (The stator coils

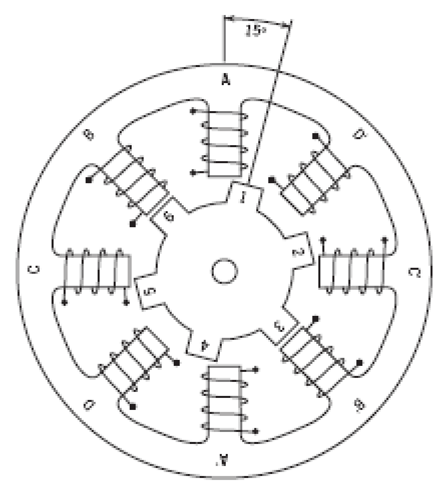

Fig. 2 Four-phase variable reluctance stepper motor

are energized in groups referred to as phases) variable reluctance stepper motor. The motor has eight stator poles and the rotor has six teeth or poles. The rotor is made of soft iron. When the stator windings are energized with dc current the poles become magnetized and the rotor being free to turn will move so as to minimize the length of the air gap.

Referring to Fig. 3, applying a dc voltage to phase ($A+, A'-$) will cause the rotor to turn so that the reluctance through the pole teeth at the 12 o'clock and 6 o'clock positions is minimized. If power is removed from the A phase (A, A'), and power is applied to the B phase ($B+, B'-$), then the rotor will turn 15° clockwise. Similarly, as phases C and D are energized in succession, the rotor will rotate 15° clockwise as each phase is energized. If the sequence of phase excitation is reversed, that is, A', D, C, B', A, the direction of rotation is reversed. They generally operate with step angles from 5° to 15° at relatively high step rates, and have no detent torque (detent torque is the holding torque when no current is flowing in the motor). The relationship among step angle ψ , rotor teeth and stator phases is given by

$$\psi = \frac{360^o}{rotorteeth \times stator\, phase}$$

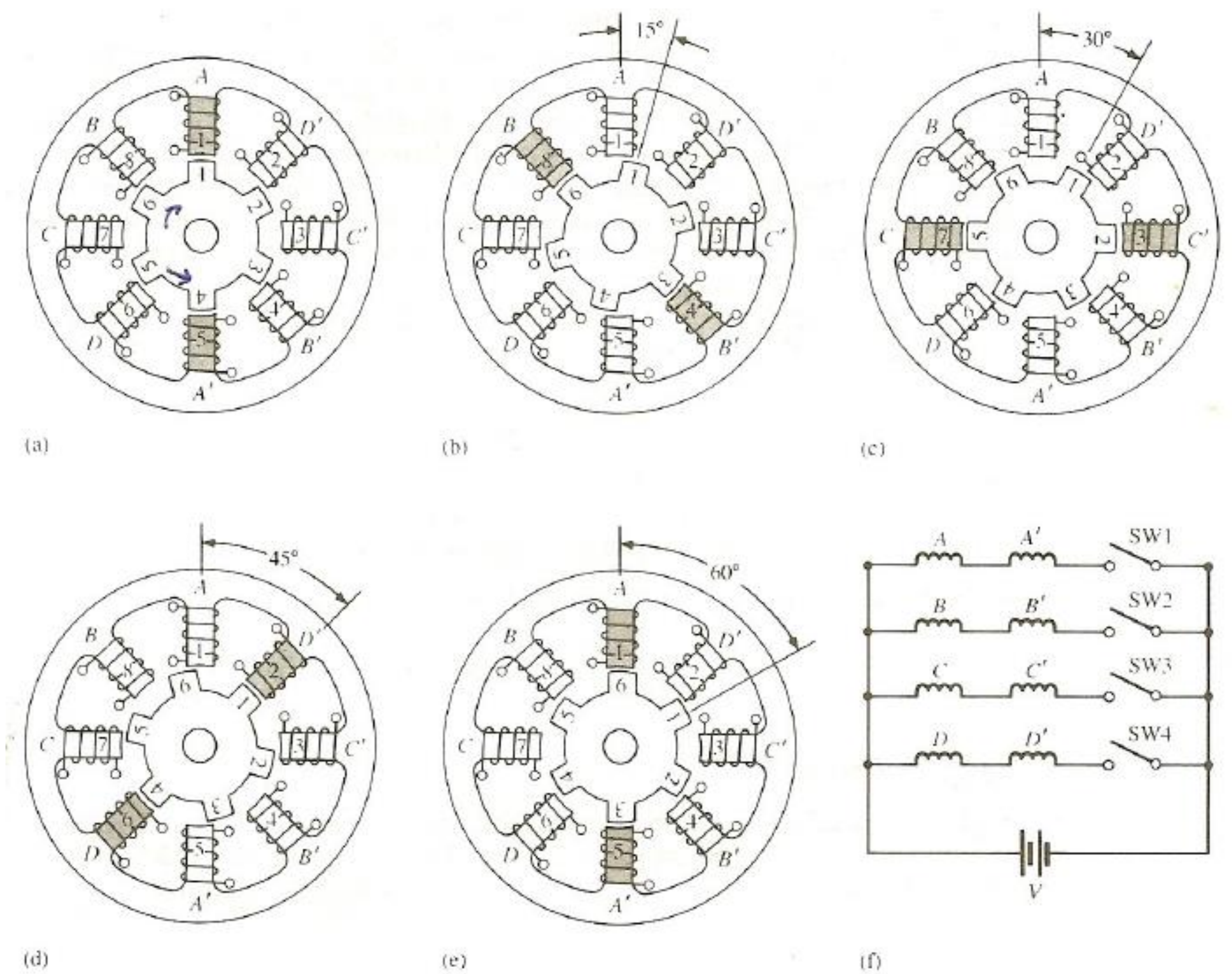

Fig. 3. Movement of the stepper motor rotor as current is pulsed to the stator. (a) Current is applied to the A and A’ windings, so the A winding is north, (b) Current is applied to B and B’windings, so the B winding is north, (c) Current is applied to the C and C’ windings, so the C winding is north, (d) Current is applied to the D and D’ windings so the D winding is north. (e) Current is applied to the A and A’ windings, so the A’ winding is north.

Permanent magnet stepper motor (PM)

They are similar to variable reluctance motors, except that the motors have permanent magnet rotors with no teeth and are magnetized perpendicular to the axis with the desired number of pole pairs. Due to the permanent magnets, the motor develops a detent torque which keeps the rotor in place even when no current flows in the stator windings. Typical step angles are 7.5^o to 15^o. They step at relatively low rates, but they exhibit high torque and good damping characteristics.

Hybrid stepper motors(HB)

They combine the best features of both the PM and VR type stepper motors. They have two identified soft iron armatures with teeth (or poles) mounted on the same shaft. The armatures are arranged such that the teeth on one armature are displaced from the teeth on the other by one-half of the tooth pitch (See Fig.4). A permanent magnet is sandwiched between the armatures making one armature act as a north pole and the other as a south pole. Their step angles are typically from 0.9^o to 3.6^o.

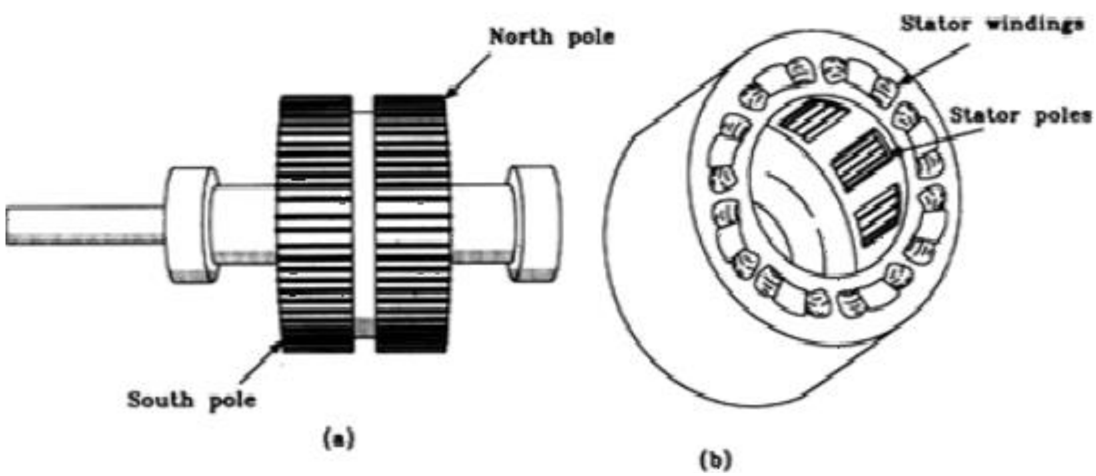

Fig. 4 Components of hybrid stepper motor (a) rotor (b) stator

The two most commonly used types are the permanent magnet and the hybrid types. For a given application, one should first evaluate the permanent magnet type as it is normally several times less expensive. If not the hybrid motor may be the right choice.

The correspondence between rotor steps and input pulses makes the stepper motor ideal device for digital control. Applications include quartz analogue clock and watches, printers (paper feed, print wheel) and graph plotters, head positioning in computer disc drives, robots, and numerically controlled machines tools. They are made in sizes ranging from milliwatts to tens of kilo-watts, and they are expected to replace conventional ac and dc machines in many control applications.

Exercises One

(1) State Lenz's Law and show by means of a sketch the direction of an induced emf in a wire moving across a magnetic field.

A wire of length 12 cm is moved at right angles across a magnetic field at a constant velocity of 2 metres per second. The flux density in the magnetic field is 20 mWb/m^2. Calculate the emf generated. [4.8 mV]

What force will be required to move this conductor if the emf causes a current of 10 A to flow in an external circuit? [0.024 N]

(2) A straight conductor 25 cm long is moved at right angles to a magnetic field of flux density 0.12 Wb/m^2 with velocity of 3 m/s. Calculate the magniyude of the emf induced in the conductor. [0.09 V]

If the above conductor forms part of a circuit of total resistance 0.2 Ω, calculate:

(a) the current flowing in the conductor, [0.45 A]
(b) the power dissipated in the circuit, [40.5 mW]
(c) the force required to produce the movement [13.5 mN]

Exercises Two

(1) The wave-connected armature of a four-pole dc generator is required to generate an emf of 520 V when driven at 660 rev/min. Calculate the flux per pole required if the armature has 144 slots with 2 coil sides per slot, each consisting of 3 turns.[27.3 mWb]
(2) A four-pole motor is fed at 440V and takes an armature current of 50 A. The resistance of the armature circuit is 0.28 Ω. The armature winding is wave-connected with 888 conductors and the useful flux per pole is 0.023 Wb. Calculate the speed. [626 rev/min]
(3) The following table relates to the open-circuit curve of a shunt generator running at 750 rev/min.

Generated voltage(V)	10	172	300	360	385	395
Field current(A)	0	1	2	3	4	5

Determine the no load terminal voltage if the field circuit resistance is 125 Ω. Find also the critical resistance of the shunt field circuit. (354 V)

If the speed is halved, what is the resultant terminal voltage? At the reduced speed, what value of the field circuit resistance will give a no-load terminal voltage of 175 V? [5 V, 64.5 Ω]

(4) The open-circuit characteristic of a shunt generator when separately excited and running at 1000 rev/min is given by

E	56	112	150	180	200	216	230 V
I_f	0.5	1.0	1.5	2.0	2.5	3.0	3.5 A

If the generator is shunt-connected and runs at 1100 rev/min with a total field resistance of 80 Ω, determine a) the no-load emf b) the output current when the terminal voltage is 200 V if the armature resistance is 0.1 Ω.

(5) A dc shunt-wound generator has the following open-circuit magnetisation curve at its rated speed:

E	180	340	450	500	550	570 V
I_f	0.5	1.0	1.5	2.0	3.0	4.0

The resistance of the field circuit is 200 ohms and that of the armature circuit 0.5 ohms. If the generator is driven at its rated speed, find the terminal voltage (i) on open-circuit, and (ii) when the armature current is 60 A. [540 V: 505 V]

(6) A 60-kW, 240-V short-shunt compound generator, operated as a shunt generator, required as increase in field current of 3 A to provide an over compounded voltage of 275 V at rated load current of 250 A. The shunt field has 200 turns per pole and the series field 5 turns per pole, with resistances of 240 Ω and 0.005 Ω respectively. Calculate the required diverter resistance.

(7) A shunt machine has armature and field resistances of 0.04 Ω and 100 Ω respectively. When connected to a 460-V dc supply and driven as a generator at 600 rev/min, it delivers 50 kW. Calculate its speed when running as a motor and taking 50 kW from the same supply.

Show that the direction of rotation of the machine as a generator and as a motor under these conditions is unchanged. [589 rev/min]

(8) How is the induced voltage of a separately excited dc generator affected if a) the speed increases? b) the field current is reduced?

(9) The terminal voltage of a shunt generator decreases with increasing load. Explain.

(10) Explain why the output voltage of an over compound generator increases as the load increases.

(11) What determines the magnitude and polarity of the back emf in a dc motor?

(12) Explain why the armature current of a shunt motor decreases as the motor accelerates.

(13) Why is starting resistor needed to bring a motor up to speed?

(14) Show one way to reverse the direction of rotation of a compound motor.

(15) What conditions must be fulfilled for the self-excitation of a dc shunt generator?

(16) Enumerate the losses in a dc shunt motor and explain how each loss is affected by a change of load.

(17) What is meant by the back emf of a dc motor? In a given motor, upon what factors does the emf depend?

(18) What is meant by the critical field resistance of a shunt generator?
(19) Give a brief explanation of why the current supplied to dc motor increases as the motor is loaded.
(20) Briefly discuss the merits and demerits of dc shunt motor speed control by adding resistance a) to the field circuit b) to the armature circuit.
(21) A motor is required to have high starting torque to operate a crane state which is the more suitable motor, a shunt wound or a series wound and give reasons for your choice.

Exercises Three

(1) A single-phase transformer has a primary winding with 1,500 turns and a secondary winding with 80 turns. If the primary winding is connected to a 2300-V, 50-Hz supply, calculate (a) the secondary voltage (b) the maximum value of the core flux. Neglect the primary impedance. [122.67 V: 0.0058 Wb]
(2) A single-phase 2,300/230-V, 500-kVA, 50-Hz transformer is tested with the secondary open-circuited. The following test results were obtained: V_1 = 2,300 V, I_o = 10.5 A, and P_o = 2,300 W. Calculate (a) the power factor (b) the core-loss current I_p (c) the magnetizing current I_m. [0.0952: 1.0 A: 10.45 A]
(3) A 150-kVA, 2,400/240-V, 50-Hz single-phase transformer has the following resistances and reactances: R_1 =0.225 Ω, X_{l1} = 0.525 Ω, R_2 = 0.00220 Ω and X_{l2} = 0.0445 Ω. Calculate the transformer equivalent values (a) referred to the primary (b) referred to the secondary. [0.445 Ω.: 0.970 Ω.: 0.00445 Ω.: 0.00870 Ω]
(4) A 150-kVA, 2,400/240-V, 50-Hz single-phase transformer has the following resistances and reactances: R_1 =0.225 Ω X_{l1} = 0.525 Ω, R_2 = 0.00220 Ω, X_{l2} = 0.00445 Ω, R_m = 10 k Ω and X_m = 1.5 k Ω. The transformer is supplying full-rated load at 0.85 lagging power factor and rated secondary terminal voltage. Calculate (a) I_2 (b) I_p (c) I_m (d) I_o (e) I_1 (f) V_1. Use the exact equivalent circuit referred to the primary side. [625$\angle$-31.79° A: 0.24$\angle$0.39°: 1.62$\angle$-89.61° A: 1.64$\angle$-81.22°: 75.95$\angle$-22.10°: 2,457.62$\angle$1.10° V]
(5) The results of open- and short-circuit tests carried out on a 230/115-V, 60-Hz single – phase transformer are

Test type	Primary	Secondary
Open circuit	Open	115 V, 6.5 A, 192 W
Short circuit	17.5 V, 43.5 A, 234 W	Short-circuited

Calculate the parameter of the approximate equivalent circuit [R_{e1} = 0.124 Ω, X_{e1} = 0.37 Ω]
(6) The results of open- and short-circuit tests on a 100 kVA, 11,000/2,200-V, and 50-Hz single –phase transformer are

Test type	Primary	Secondary
Open circuit	Open	2,200 V, 1.5 A, 800 W
Short circuit	600 V, 10.0 A, 1,000 W	Short-circuited

Determine (a) R_{e1} and X_{e1} (b) R_{e2} and X_{e2} (c) the percentage regulation at 0.75 power factor leading, unity power factor and 0.85 power factor lagging [12 Ω: 58.79 Ω: 0.48 Ω: 2.35 Ω: -2.46 %: 1.11 %: 3.64 %]

(7) From the data of Exercise 6 calculate the transformer efficiency at 0.8 power factor lagging for (a) 50 % (b) 100 % of rated full load. [97.44 %: 97.8 %]

(8) A 100-kVA, 11,000/220-V, 50-Hz transformer has a core loss P_o of 800 W and R_{e2} of 0.48 Ω. Calculate the secondary current for maximum efficiency. [40.82 A]

(9) A 10-kVA 2,400/240-V 50-Hz single-phase transformer is reconnected to step down a voltage from 2,640 to 2,400 V. Calculate (a) the kVA rating as an autotransformer [110 kVA]

(10) A 75-kVA 230-V three-phase load is supplied from a 6,600-V three-phase supply using a star-star-connected transformer bank. What are the voltage, current and kVA ratings of the single-phase transformers? [3,810 V: 6.56 A: 25 kVA]

(11) Three single-phase transformers are connected in delta-delta, and are used to step down a line voltage of 110 kV to 66 kV to supply an industrial plant drawing 50 MW at a 0.80 power factor. Calculate (a) the high-voltage-side line current (b) the low-voltage-side line current (c) the primary phase currents (d) the secondary phase currents [328.04 A: 546.73 A: 189.39 A: 315.65 A]

Exercises Four

(1) Explain why an induction motor cannot develop torque when running at synchronous speed.

(2) The rotor of an induction motor should never be locked while full voltage is being applied to the stator. Explain

(3) Why does the rotor of an induction motor turn slower than the revolving field?

(4) Give two advantages of a slip-ring motor over a squirrel-cage motor.

(5) How can we change the direction of rotation of a 3-phase induction motor?

(6) We can bring an induction motor to a quick stop either by plugging it or by exciting the stator from a d.c. source. Which method produces the least amount of heat in the motor? Explain.

(7) An induction motor has four poles and is connected to a 50 Hz supply. If the machine runs on full load at 2 per cent slip, determine the running speed and the frequency of the rotor current. [1470 rev/min: 1 Hz]

(8) A three-phase induction motor, at standstill, has a rotor voltage of 100 V between the slip rings when they are open-circuited. The rotor winding is star-connected and has a leakage reactance of 1 ohm per phase at standstill and a resitance of 0.2 ohms per phase. Calculate (a) the rotor current when the slip is 4 per cent and the rings are short circuited (b) the slip and the rotor current when the motor is developing maximum torque. Assume the flux remains constant. [11.33 A: 0.2, 40.8 A]

(9) If the star-connected rotor winding of a three-phase induction motor has resistance of 0.01 ohm per phase and a standstill leakage reactance of 0.08 ohms per phase, what must be the value of the resistance per phase of a starter to give the maximum starting torque? What is the percentage slip when the starting resistance has been reduced to 0.02 ohms per phase, if the motor is still exerting its maximum torque? [0.07 Ω, 37.5 %]

(10) A three-phase, 50 Hz, six-pole induction motor has a slip of 0.04 per unit when the output power is 20 kW. The frictional loss is 250 W. Calculate (a) the rotor speed (b) the rotor copper loss. [960 rev/min, 855 W]

(11) In a certain eight-pole, 50 Hz machine, the rotor reactance per phase is 0.04 ohms and the maximum torque occurs at a speed of 645 rev/min. Assuming that the air-gap flux is constant at all loads, determine the percentage of maximum torque (a) at starting (b) when the slip is 3 per cent. 27.5 %, 41 %]

(12) A three-phase, 50 Hz induction motor has four poles and runs at a speed of 1440 rev/min when the total torque developed by the rotor is 70 Nm. Calculate (a) the total input to the rotor (b) the rotor copper loss. [11 kW, 440 W]

Exercises Five

(1) A variable-reluctance stepper motor has 36 rotor teeth and 4 stator phases. Determine its step angle. [2.5^o]

(2) What is the main use of stepper motors?

(3) What is the difference between a reluctance and permanent magnet reluctance stepper motor?

(4) Describe the construction of hybrid stepper motor.

(5) For what reasons and applications would you select a hysteresis motor?

(6) Why is the speed drop of a universal motor operating on ac greater than might be expected?