INSTITUTE OF
DISTANCE LEARNING
KWAME NKRUMAH UNIVERSITY OF SCIENCE
AND TECHNOLOGY, KUMASI, GHANA

ME 362 VIBRATIONS I

Lecture 1

Introduction

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REFERENCE BOOKS

- Mechanics of Machines, Advanced Theory and Examples, John Hannah, R. C. Stephens.
- Mechanics of Machines, Elementary Theory and Examples, John Hannah, R. C. Stephens.
- Theory of Machines, R.S. Khurmi and J.K. Gupta, 2004
- Vibration with Control, Daniel J. Inman, Wiley, 2006.
- Vibrations in Rotating Machinery, Professional Engineering Publishers, Wiley, 2000
- Vibrations in Rotating Machinery, IMechE, Wiley, 2004.
- VECTOR MECHANICS FOR ENGINEERS (Statics and Dynamics), Ferdinand P. Beer, E. Russell Johnston, Jr., David F. Mazurek, Phillip J. Cornwell, Elliot R. Eisenberg

Grading:

- Continuous Assessment 30%
- Final Exam 70%

Prerequisites: A previous course in:

ME 161-Basic Engineering Mechanics

ME 262/264/260-Theory of Machines

Homework: About 4-7 problems will be assigned after every meeting and these will be collected for grading. Collaboration is encouraged, but independent work is required and essential in order to perform well on the exams.

COURSE OUTLINE

- Unit 1: Introduction to Vibration, its Modelling, and Related Concepts
- Unit 2: Response to Excitations
- Unit 3: Multiple-Degree-of-Freedom Systems
- Unit 4: Torsional Vibration Systems
- Unit 5: Design for Vibration Suppression
- Unit 6: Vibration Testing and Measurement

UNIT 1

Introduction

Mechanical vibration

- In ME 261/260/361 we studied kinematics and kinetics of rigid bodies separately
- For flexible(non rigid) members the kinematic and kinetic analysis cannot be decoupled
- Flexible members that have mass oscillates when subjected to a force

Mechanical vibration cont'd…

- Mechanical vibration is repetitive motion of a particle or system of bodies relative to a stationary reference frame or equilibrium position.
- Vibrations occur everywhere in our lives
  - ➢ heart rate
  - ➢ music(sound)
  - ➢ earthquakes
  - ➢ Machines and structures etc.
- The performances of many engineering systems are limited by the vibrational properties of the systems.
- Vibration can be harmful and in such cases it must be avoided.
- However, it can be very useful and desired, or may increase the performance of an engineering system.

Mechanical vibration cont'd…

- Present-day machines and structures often contain high-energy sources which create intense vibration excitation problems,
- and modern construction methods result in systems with low mass and low inherent damping.
- Therefore careful design and analysis is necessary to avoid resonance or an undesirable dynamic performance.

Uses of vibration analysis

- Preventive maintenance
- Incoming inspection
- Quality control
- Engineering and research
- Plant engineering
- Field service and machinery start-up
- Sales and customer satisfaction
- Noise control
- Auscultation

Some definitions

- **Vibration** is a motion repeated at some intervals. It is expressed in terms of frequency.
- **Cycle** is a single complete repetition of a vibratory motion.
- **Period** is the time required to complete one cycle
- **Frequency** is the number of cycles completed in a given interval of time, usually one seco but occasionally, one minute.
- **Amplitude** is the maximum displacement of a vibrating object from its initial position.
- **Torque** is an action tending to produce rotation of an object.
- **Torsional Vibration** is the twisting and untwisting of a shaft resulting from the perio application of torque.
- **Damping** is a reduction in the amplitude of an oscillation as a result of energy dissipation fr a system to overcome friction or other resistive forces.

Classification of Vibration

- Free and Forced Vibration
- Undamped and Damped Vibration
- Linear and Nonlinear Vibration
- Deterministic and Random Vibration

Dynamic analysis

Dynamic analysis can be carried out most conveniently by adopting the following three-stage approach:

I. MODEL
II. FORMULATE EQUATIONS OF MOTION
III. EXCITATION(RESPONSE)

MECHANICAL SYSTEM COMPONENTS

A mechanical system comprises inertia components, stiffness components, and damping components.

- The inertia components have kinetic energy when the system is in motion. The kinetic energy of a rigid body undergoing planar motion is

$$K.E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

MECHANICAL SYSTEM COMPONENTS

- A linear stiffness component (a linear spring) has a force displacement relation of the form $F=kx$
- where F is applied force and x is the component's change in length from its unstreched length.
- The stiffness k has dimensions of force per length.

MECHANICAL SYSTEM COMPONENTS

- A *dashpot(damper)* is a mechanical device that adds viscous damping to a mechanical system. A linear viscous damping component has a force-velocity relation of the form; $F=cv$
- where c, is the damping coefficient of dimensions mass per time.

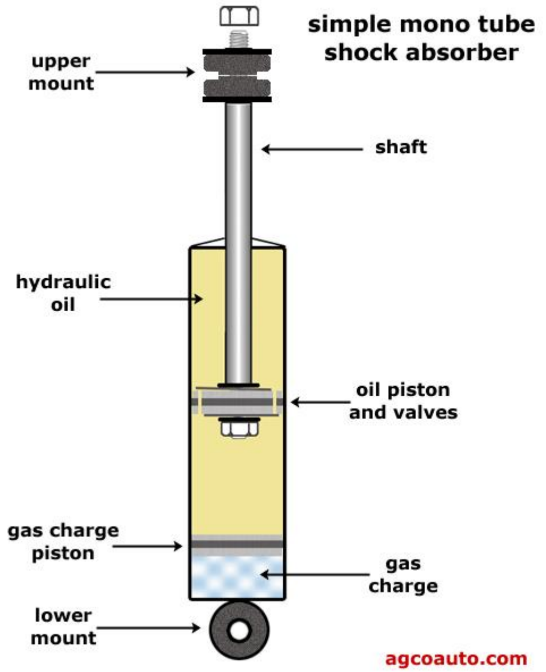


MECHANICAL SYSTEM COMPONENTS

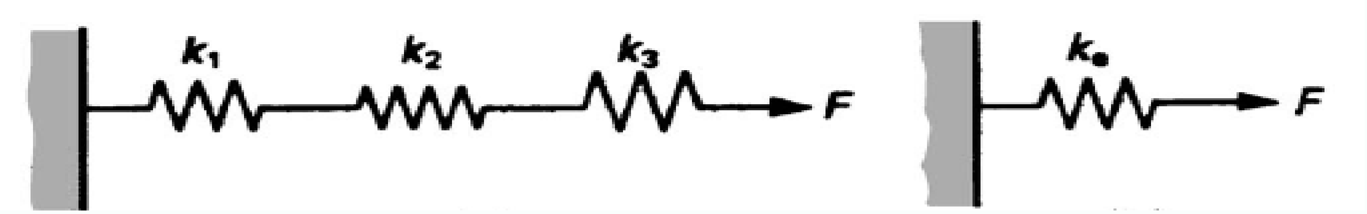
For a damper

- Mechanical resistance that produces force proportional to the velocity is called **viscous friction**.
- If the fluid flow rate is small, the flow is **laminar** and the force is proportional to the velocity.
- If the fluid flow rate is large, the flow is **turbulent** and the force is proportional to the square of the velocity.
- The seal produces a constant friction force that is independent of velocity (Coulomb friction)
- Resistance due to both Coulomb friction and viscous friction is given by

$$F = F_c + cv$$



Springs connected in series

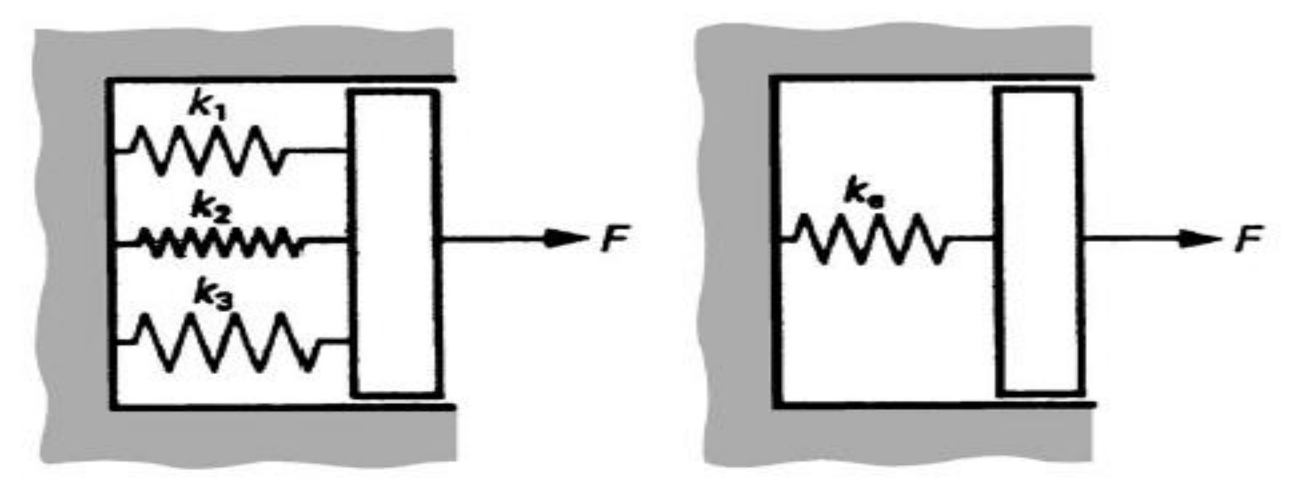


$$\delta = \delta_1 + \delta_2 + \delta_3$$

$$\frac{F}{k} = \frac{F}{k_1} + \frac{F}{k_2} + \frac{F}{k_3};$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

Springs connected in parallel

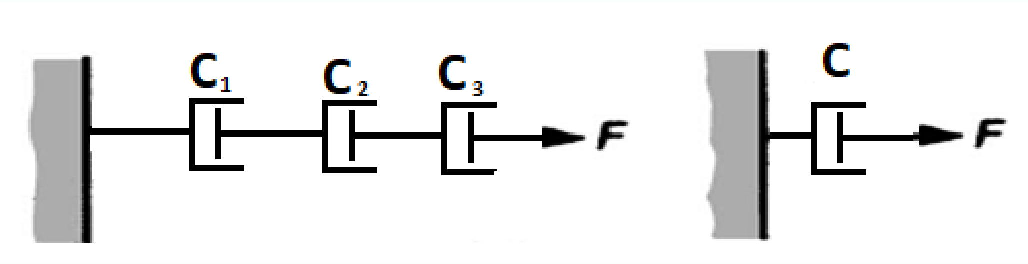


$$F = F_1 + F_2 + F_3$$

$$k\delta = k_1\delta + k_2\delta + k_3\delta$$

$$k = k_1 + k_2 + k_3$$

Dampers connected in Series

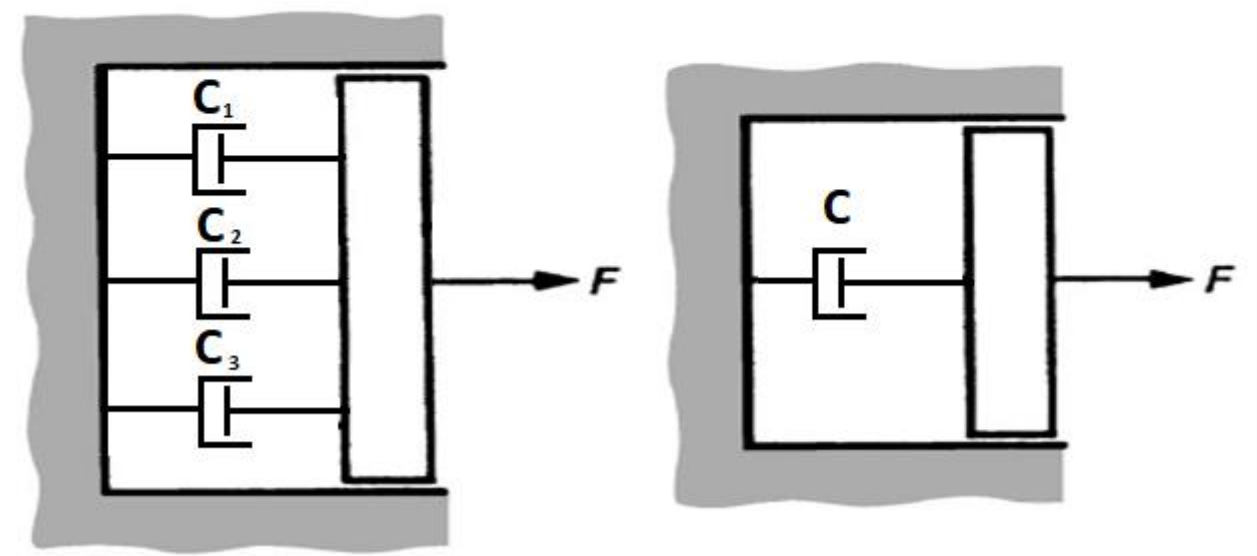


$$v = v_1 + v_2 + v_3$$

$$\frac{F}{c} = \frac{F}{c_1} + \frac{F}{c_2} + \frac{F}{c_3};$$

$$\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}$$

Dampers connected in parallel

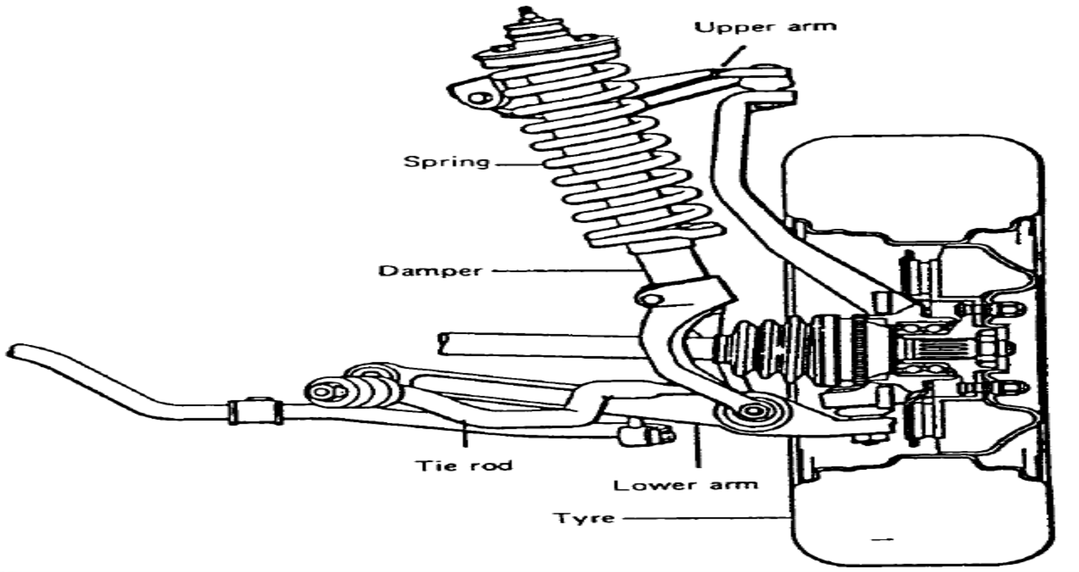


$$F = F_1 + F_2 + F_3$$

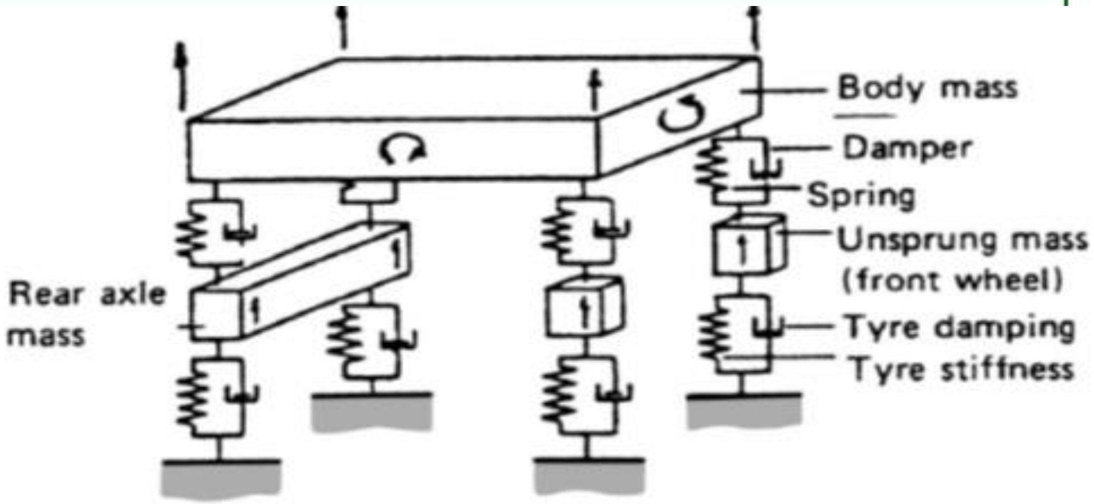
$$cv = c_1 v + c_2 v + c_3 v$$

$$c = c_1 + c_2 + c_3$$

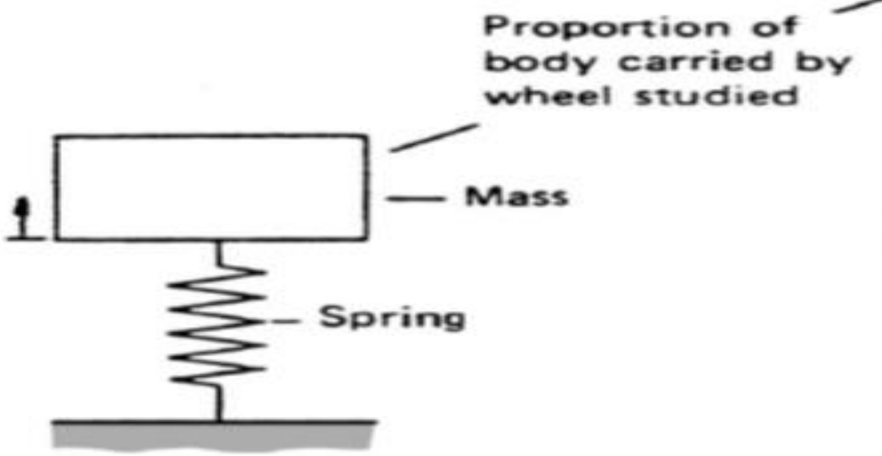
Modeling



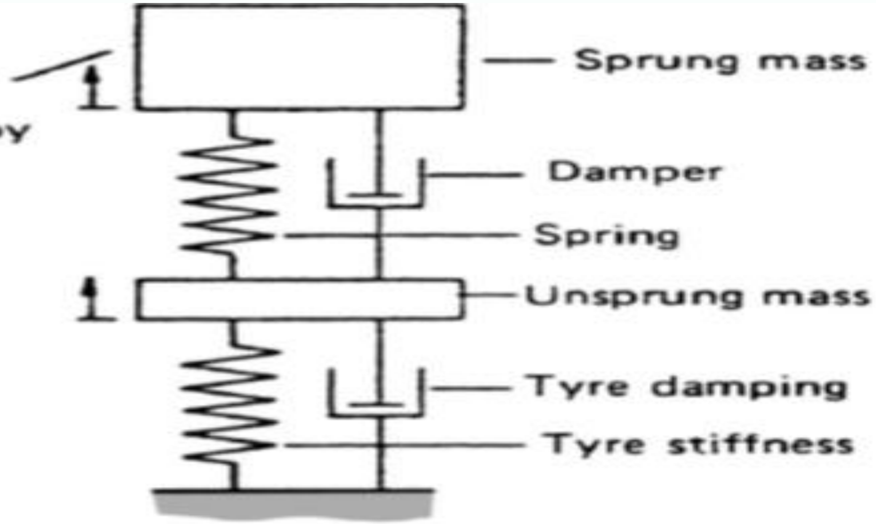
Rover 800 front suspension. (By courtesy of Rover Group.)



Motion in a vertical direction, roll, and pitch can be analysed

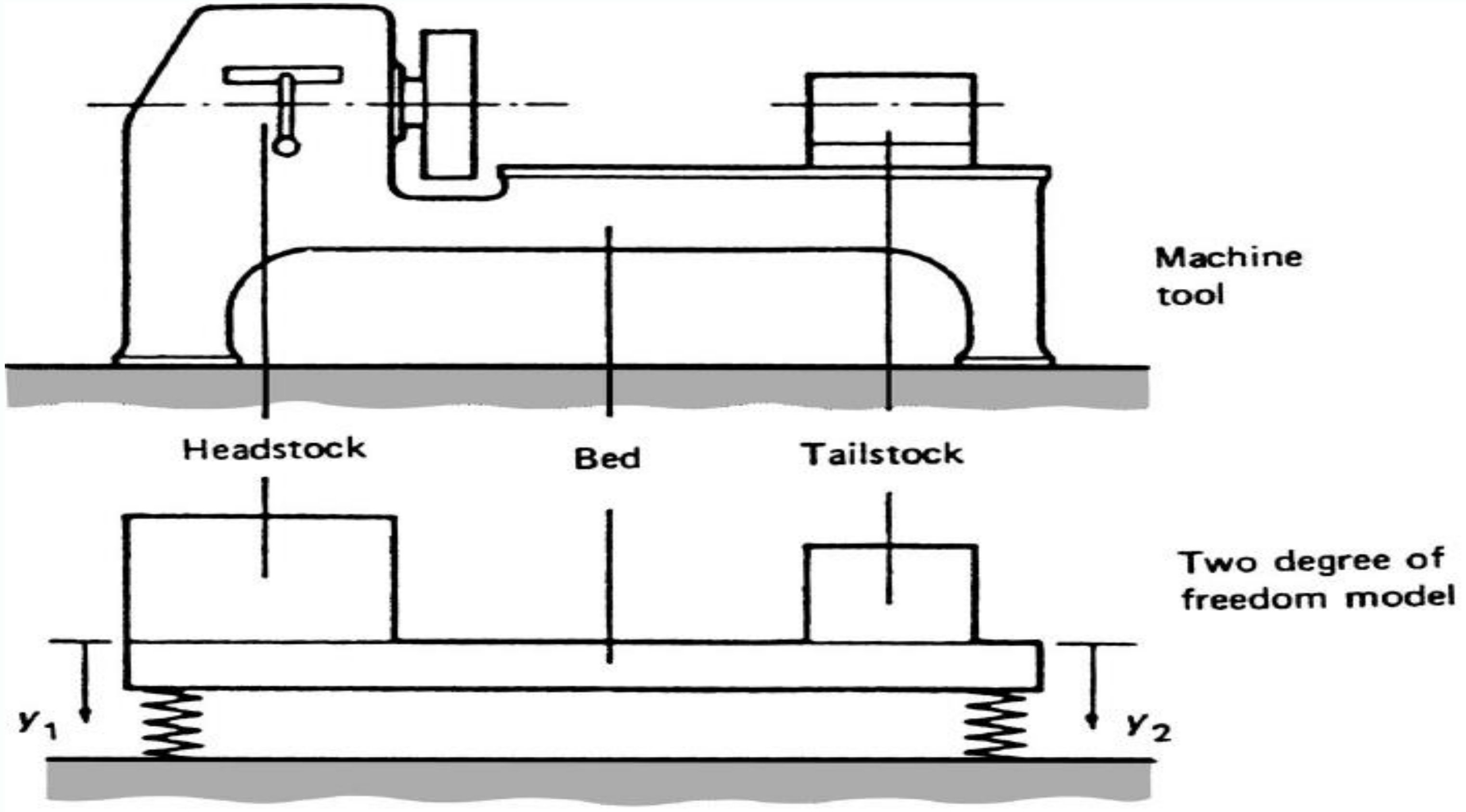


Simplest model – motion in a vertical direction only can be analysed.



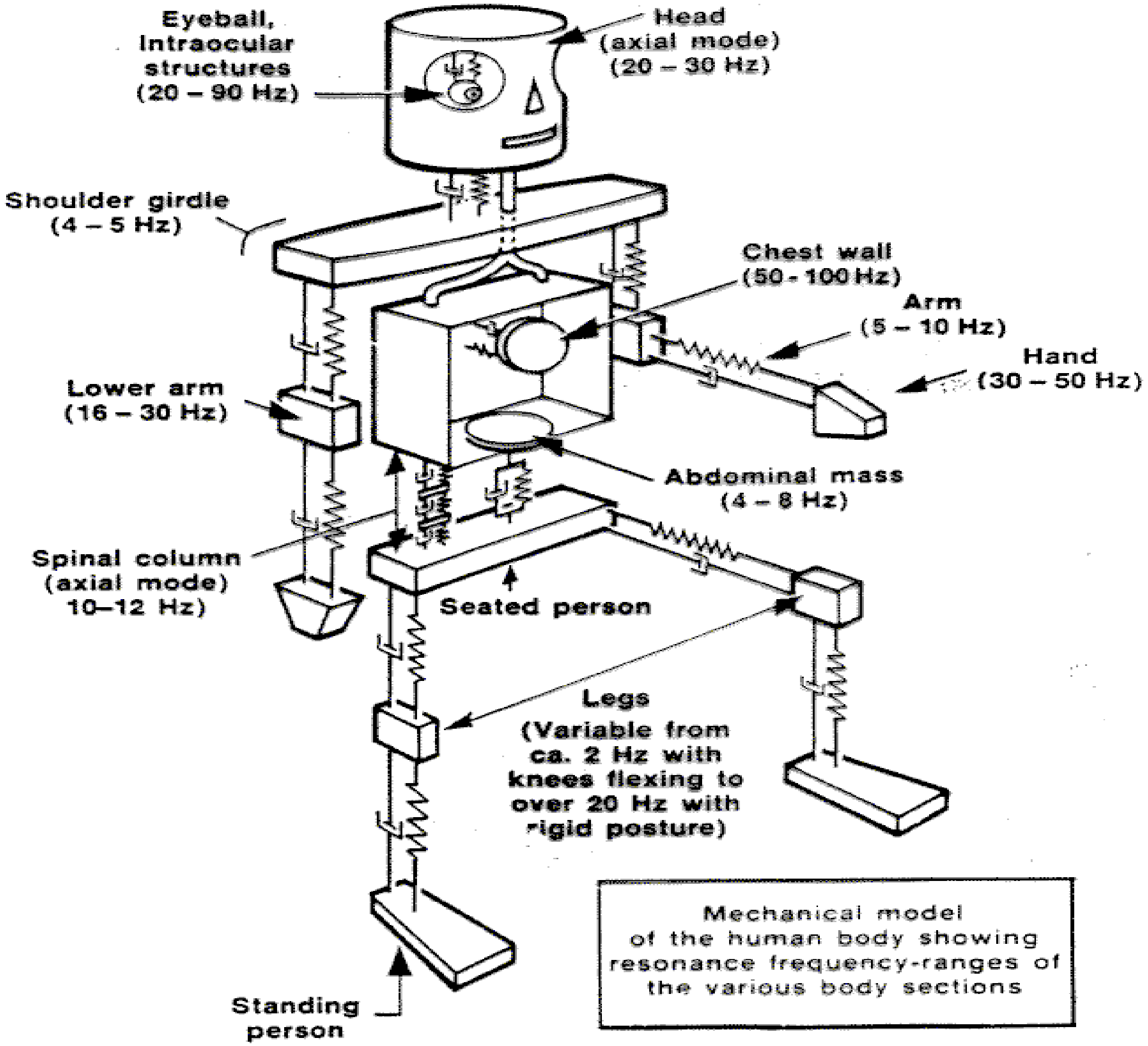
Motion in a vertical direction only can be analysed.

Modeling

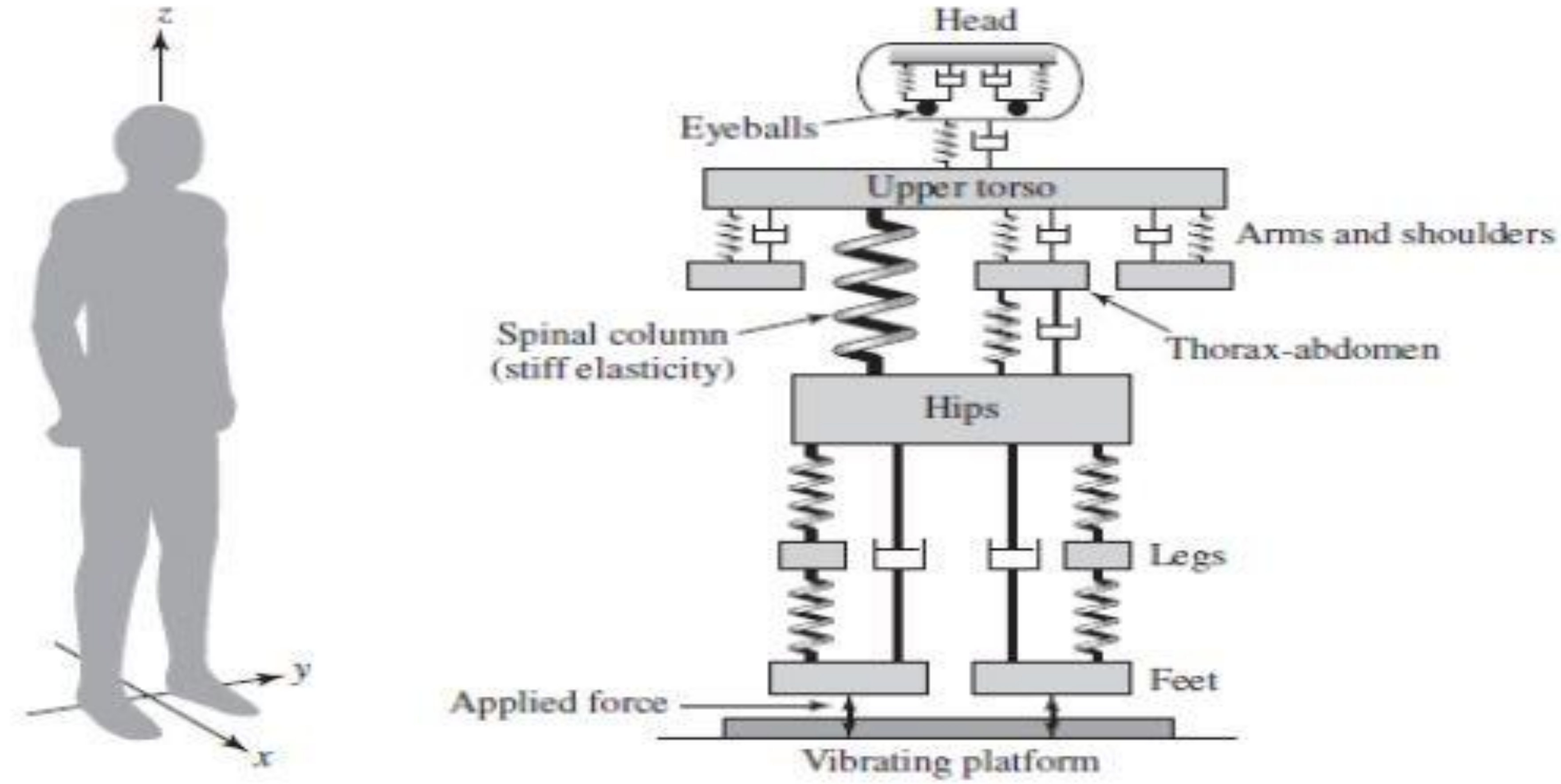


Machine tool vibration analysis model.

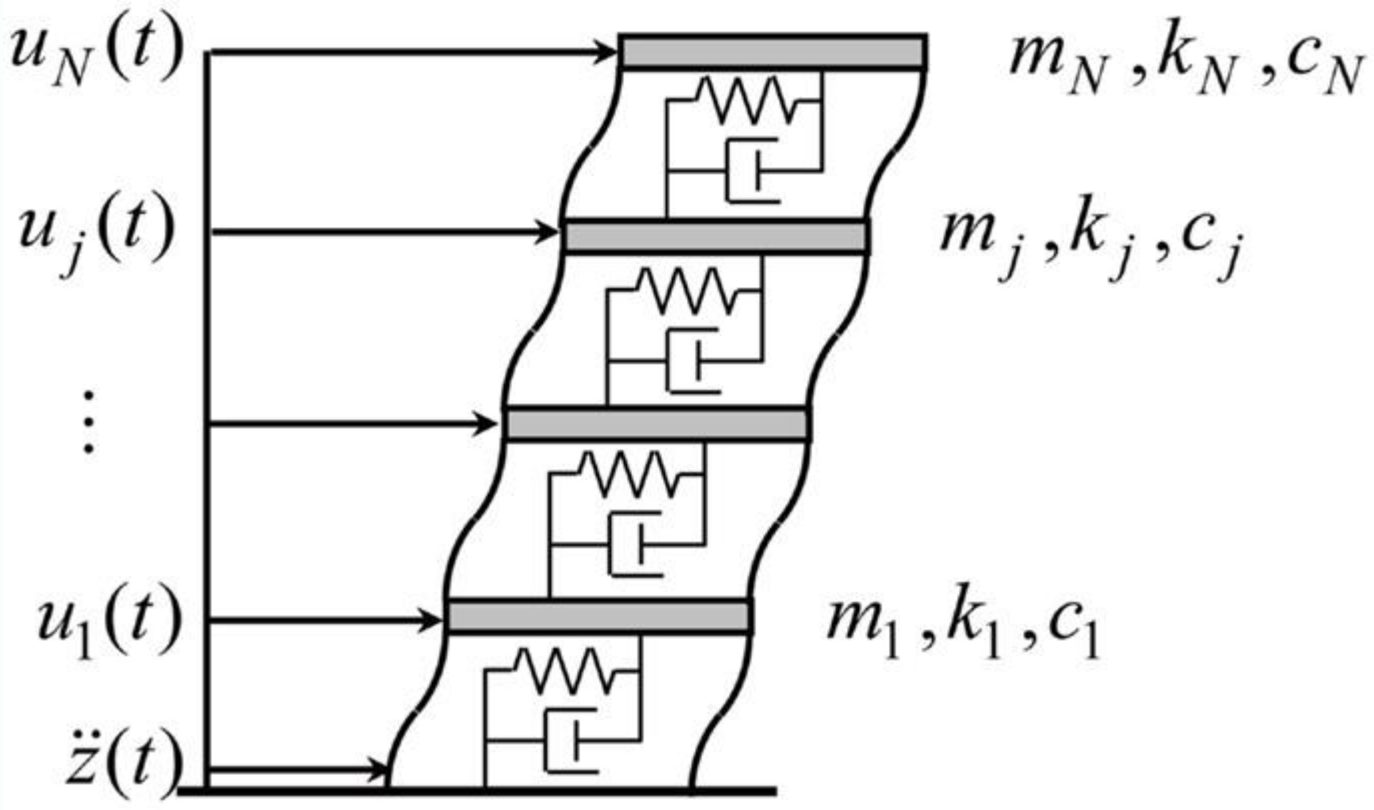
Human body resonance frequencies



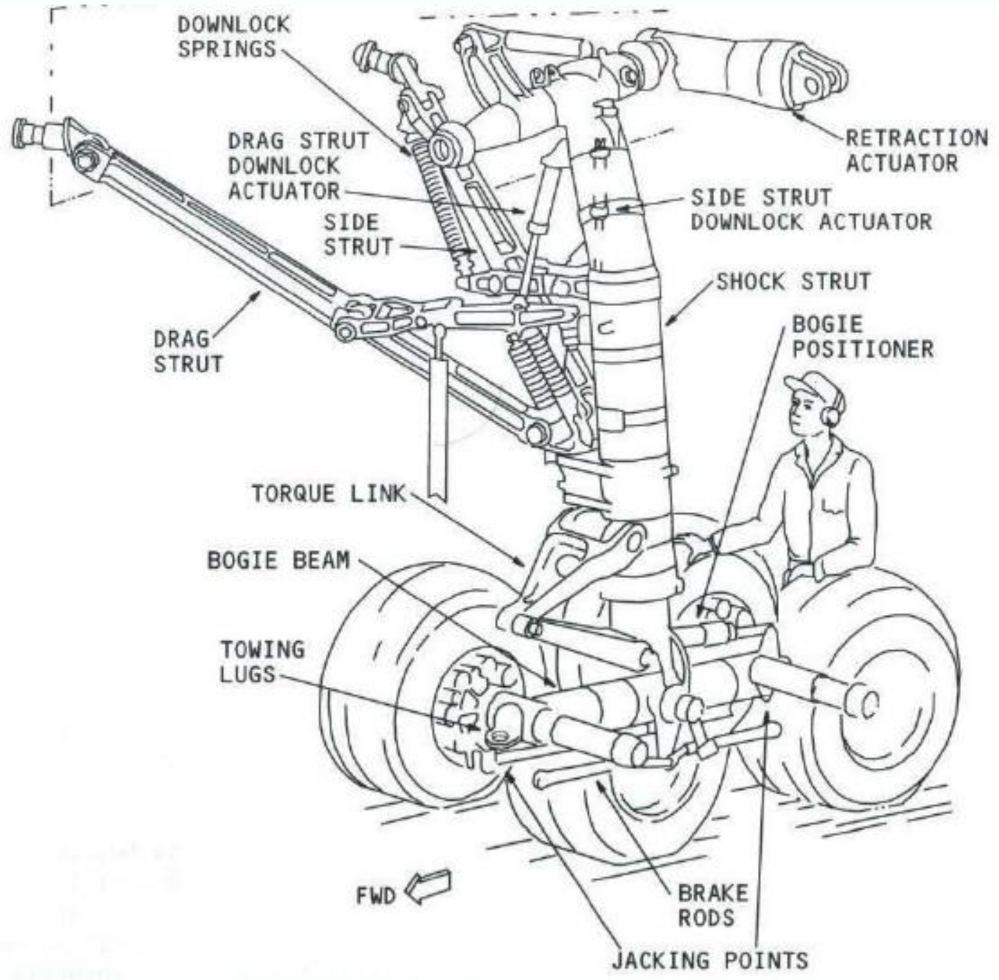
Human Model

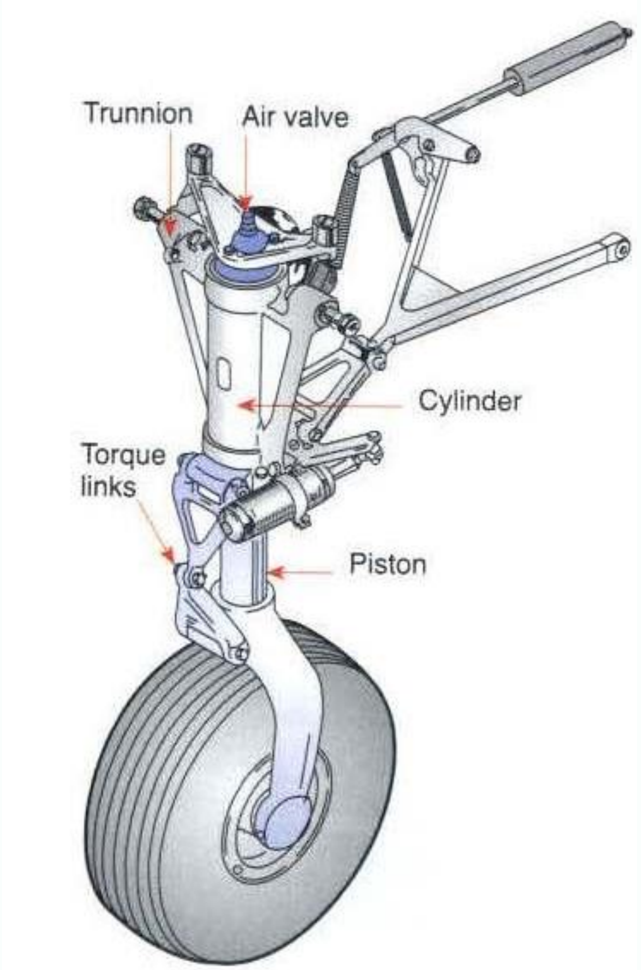

z
y
x
Head
Eyeballs
Upper torso
Arms and shoulders
Spinal column
(stiff elasticity)
Thorax-abdomen
Hips
Legs
Feet
Applied force
Vibrating platform

Multistory Building Model

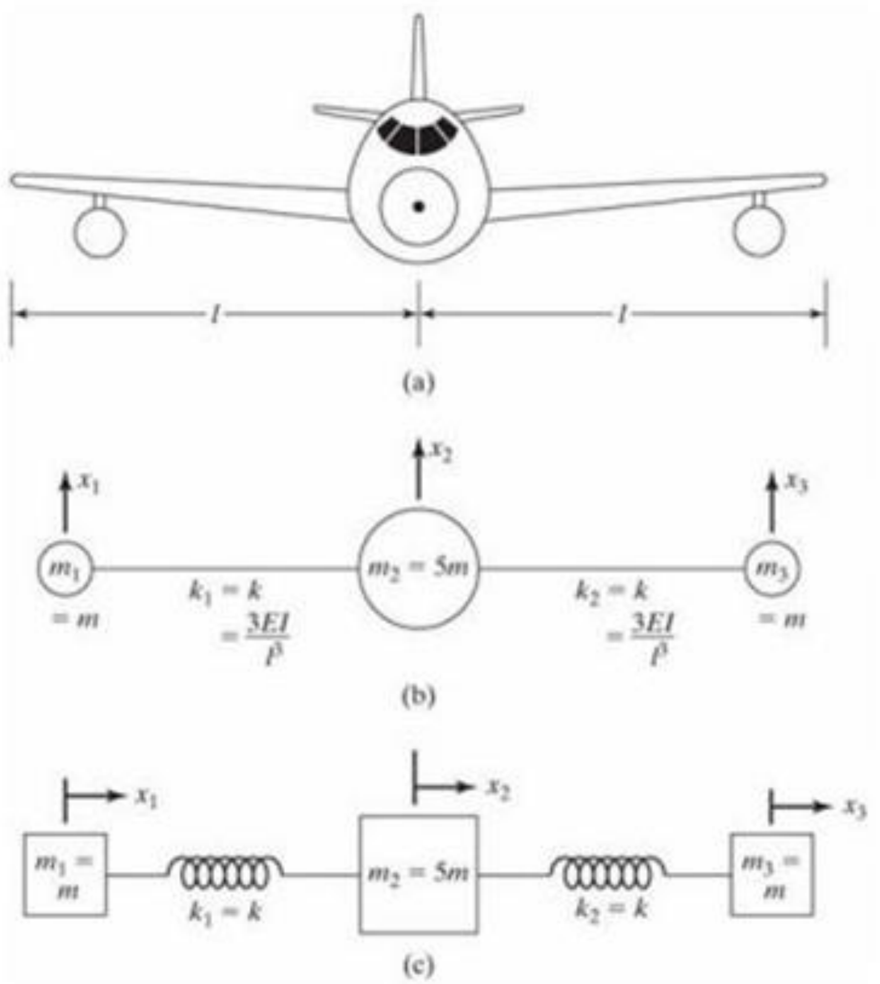


Main Landing Gear





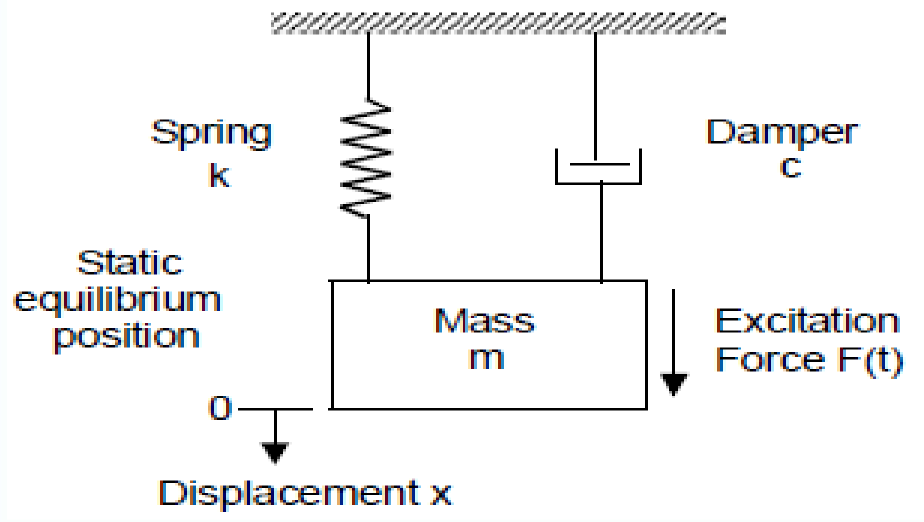
Airplane Model



Compactors

Sand sieving machine

EQUIVALENT SYSTEMS ANALYSIS



The kinetic energy of a linear system can be written in the form

$$K.E = \frac{1}{2}mv^2$$

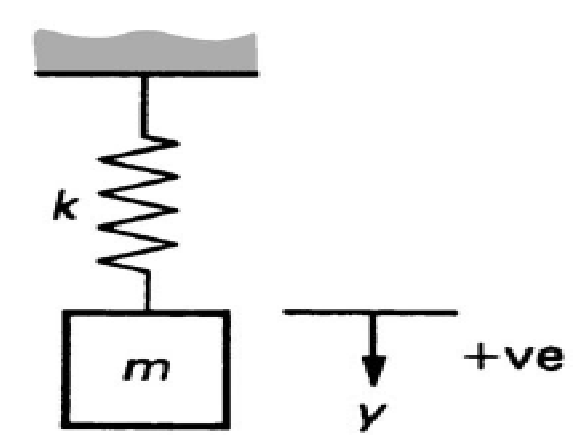
The potential energy of a linear system can be written in the form

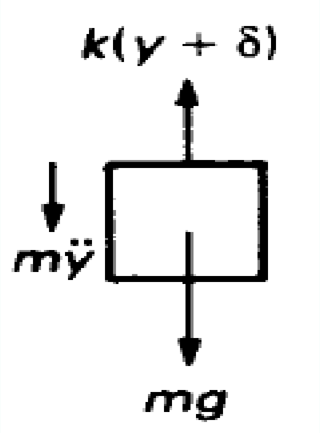
$$P.E = \int kx\,dx = \frac{1}{2}kx^2$$

The work done by the viscous damping force in a linear system between two arbitrary locations x_1, and x_2, can be written as

$$W = \int_{x_1}^{x_2} -cv\,dx$$

FREE UNDAMPED VIBRATION–S.H.M





$$mg = k\delta$$

$$m\ddot{y} = -k(y + \delta) + mg;$$

$$\ddot{y} + \left(\frac{k}{m}\right)y = 0;$$

Energy Method

- For undamped free vibration the total energy in the vibrating system is constant throughout the cycle. Therefore the maximum potential energy *V*, is equal to the maximum kinetic energy *T*, although these maxima occur at different times during the cycle of vibration.

$T + V =$ constant, and thus $\frac{d}{dt}(T + V) = 0.$

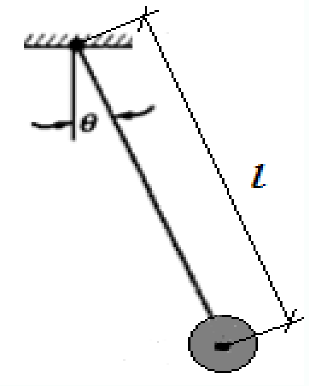
$$T = \frac{1}{2}m\dot{y}^2; \; V = \frac{1}{2}ky^2$$

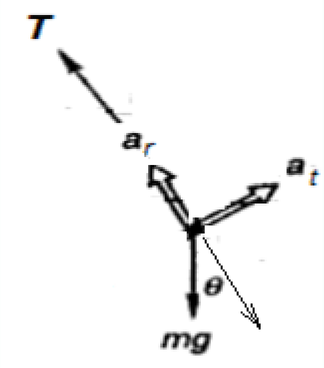
$$\frac{d}{dt}\left(\frac{1}{2}m\dot{y}^2 + \frac{1}{2}ky^2\right) = 0$$

$$m\ddot{y} + ky = 0$$

The Simple Pendulum

Newtonian Approach





$$\sum F_t = ma_t$$

$$a_t = \alpha l = l\ddot{\theta};\ ma_t = -mgsin\theta$$

$$ma_t = -mgsin\theta$$

$$\ddot{\theta} + mg\theta = 0$$

$$\ddot{\theta} + \omega^2\theta = 0;\ \ since\ \omega^2 = g/l$$

Energy Method

$$P.E = V = mgl(1 - \cos\theta)$$

$$K.E = T = \frac{1}{2}I\dot{\theta}^2$$

$$\frac{d}{dt}(V + T) = 0$$

$$\frac{d}{dt}\left(mgl(1 - \cos\theta) + \frac{1}{2}I\dot{\theta}^2\right) = 0$$

$$I\ddot{\theta} + mgl\theta = 0$$

$$\ddot{\theta} + \omega^2\theta = 0;\ \ since\ I = ml^2$$

$$where\ \ \omega^2 = \frac{g}{l}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

Spring–mass system in which the mass of the spring matters

The kinetic energy of the mass *M shown in* the Figure is as before

$$\frac{1}{2}M(\omega_n X)^2$$

The mass of the element δl is $m\frac{\delta l}{L}$, *so* its kinetic energy at the mid position of the element is $\frac{1}{2}\left(m\frac{\delta l}{L}\right)\left(\omega_n \frac{l}{L}X\right)^2$, and of the whole spring is therefore

$$\frac{1}{2}m\omega_n{}^2X^2\frac{1}{L^3}\int_0^L l^2 dl = \frac{1}{2}\frac{m}{3}\omega_n{}^2X^2$$

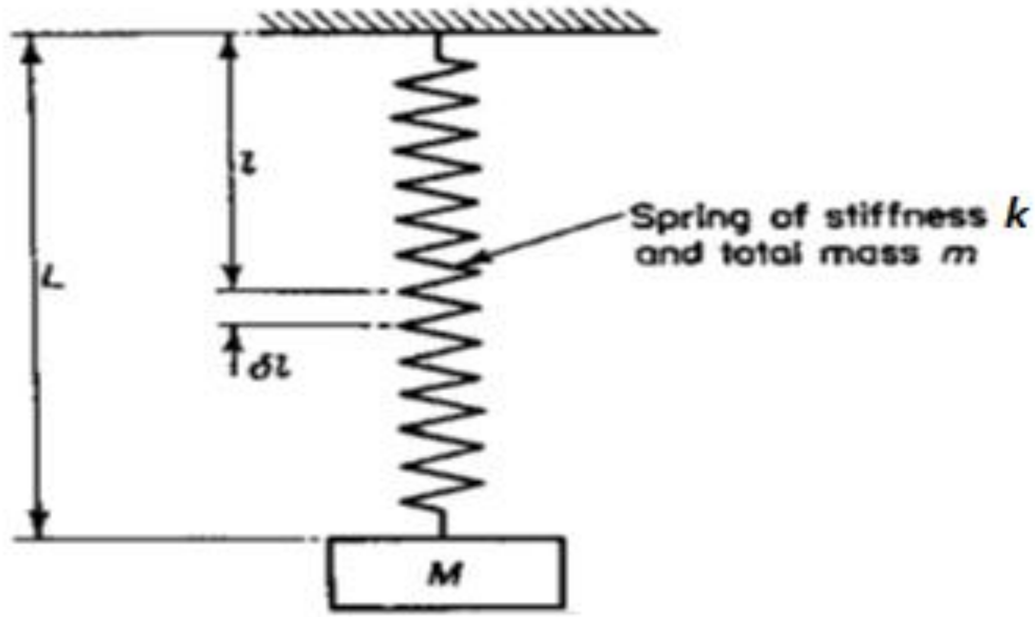
So the total kinetic energy -in the system is

$$\frac{1}{2}\left(M+\frac{m}{3}\right)\omega_n{}^2X^2$$

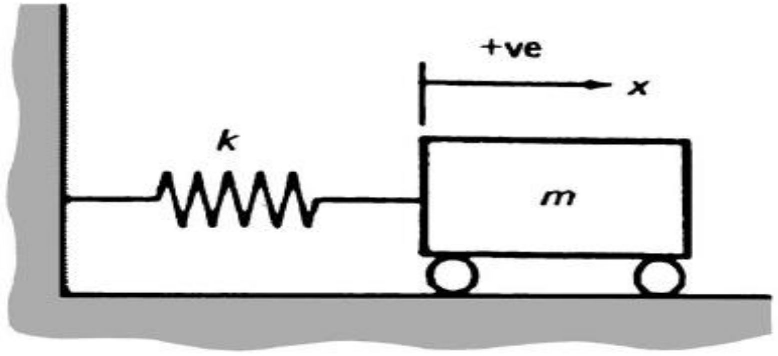
and equating this to the strain energy we get

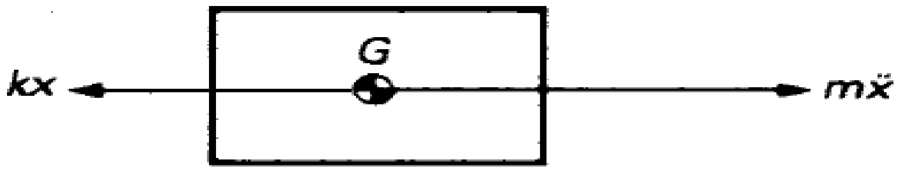
$$\omega_n = \sqrt{\frac{k}{M+\frac{1}{3}m}}\ rad/s$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{M+\frac{1}{3}m}}\ Hz$$



FREE UNDAMPED VIBRATION





$$m\ddot{x} + kx = 0; \quad \ddot{x} + \left(\frac{k}{m}\right)x = 0;$$

$$\ddot{x} + {\omega_n}^2 x = 0; \, {\omega_n}^2 = \frac{k}{m}; f = \frac{\omega_n}{2\pi};$$

$$T = 1/f$$

$$x = A\sin\omega_n t + B\cos\omega_n t$$

or

$$x = C\sin(\omega_n t + \emptyset)$$

Assumed initial conditions

$$x(0) = x_o; \;\; \dot{x}(0) = v_o$$

$$@\, t = 0; x = x_o$$

$$x_o = B$$

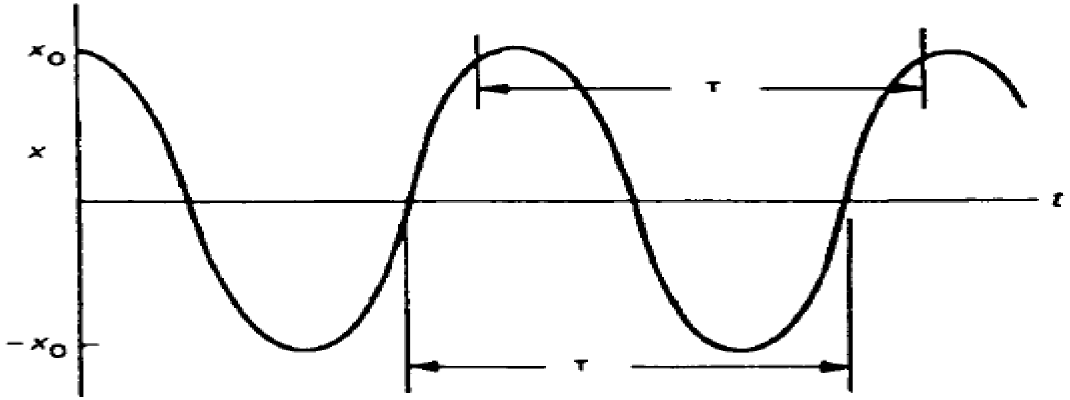
$$@\, t = 0; \dot{x} = v_o$$

$$\dot{x} = A\omega\cos\omega_n t - B\omega\sin\omega_n t$$

$$v_o = A\omega_n$$

$$A = v_o/\omega_n$$

$$x = (v_o/\omega_n)\sin\omega_n t + x_o\cos\omega_n t$$



Example 1–1

Consider a small spring about 30 mm long, welded to a stationary table (ground) so that it is fixed at the point of contact, with a 12-mm bolt welded to the other end which is free to move. The mass of this system is about 49.2 g. The spring stiffness is measured to be *k* = 857.8 N/m. Calculate the natural frequency and period. Also determine the maximum amplitude of the response if the spring is initially deflected 10 mm.

Solution The natural frequency is

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{857.8\ N/m}{49.2\ x\ 10^{-3}\ kg}} = 132\ \text{rad/s}$$

In hertz, this becomes

$$f_n = \frac{\omega_n}{2\pi} = 21\ \text{Hz}$$

The period is $T = \frac{2\pi}{\omega_n} = \frac{1}{f_n} = 0.0476\ \text{s}$

Solution

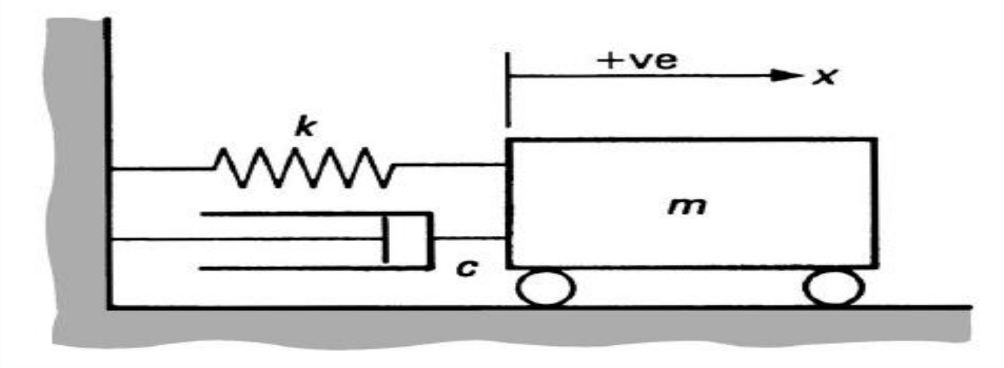
The maximum value of the displacement response,, corresponds to the value of the constant *A.* Assuming that no initial velocity is given to the spring (v_o = 0),

$$x(t)_{max} = A = \frac{\sqrt{\omega_n^2 x_o^2 + v_o^2}}{\omega_n} = x_o = 10 \text{ mm}$$

The maximum value of the velocity response is $\omega A \ or \ \omega x_o$ = 1320 mm/s and the acceleration response has maximum value $\omega_n^2 A$ = $\omega_n^2 x_o$ = 174.24 X 10^3 mm/s^2. Since v_o= 0, the phase is ϕ = tan^{-1} $(\omega_n x_o/0)$ = π/2, or 90^o. Hence, in this case, the response is

$$x(t) = 10 \sin\left(132t + \frac{\pi}{2}\right) = 10 \cos\ (132t) \text{ mm}$$

FREE DAMPED VIBRATION



$$m\ddot{x} + c\dot{x} + kx = 0$$

$$x = X_o e^{\lambda t}$$

$$\dot{x} = \lambda X_o e^{\lambda t}; \quad \ddot{x} = \lambda^2 X_o e^{\lambda t}$$

$$m\lambda^2 + c\lambda + k = 0$$

$$\lambda = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

$$d = c^2 - 4mk$$

$d = 0$; *Critically damped system*

$d > 0$; Overdamped system

$d < 0$; Less damped – underdamped System

$$d = c^2 - 4mk = 0$$

Critical damping; $c_c = 2\sqrt{mk} = 2m\omega_n$

Damping ratio, $\xi = \frac{c}{c_c} = \frac{c}{2m\omega_n}$

$\Rightarrow \xi = 1$; *Critically damped system*

$\xi > 1$; Overdamped system

$\xi < 1$; Less damped – underdamped System

$$\lambda = \frac{-2\xi m\omega_n \pm \sqrt{(2\xi m\omega_n)^2 - (2m\omega_n)^2}}{2m}$$

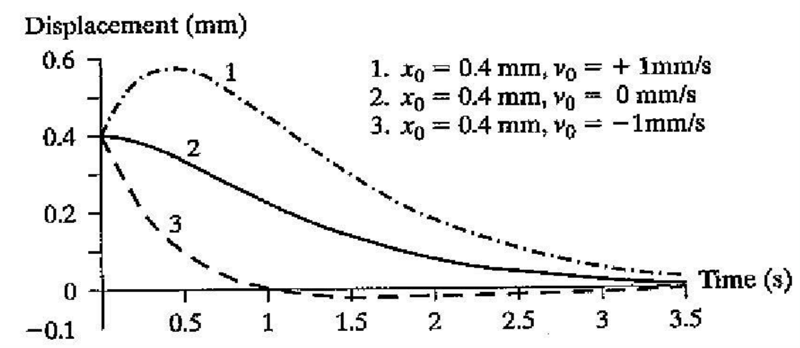
$$\lambda = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

FREE DAMPED VIBRATION

- $\xi = 1; \textit{Critically damped system}$

$$\lambda_1 = \lambda_2 = -\omega_n$$

$$x(t) = (A + Bt)e^{-\omega_n t}$$

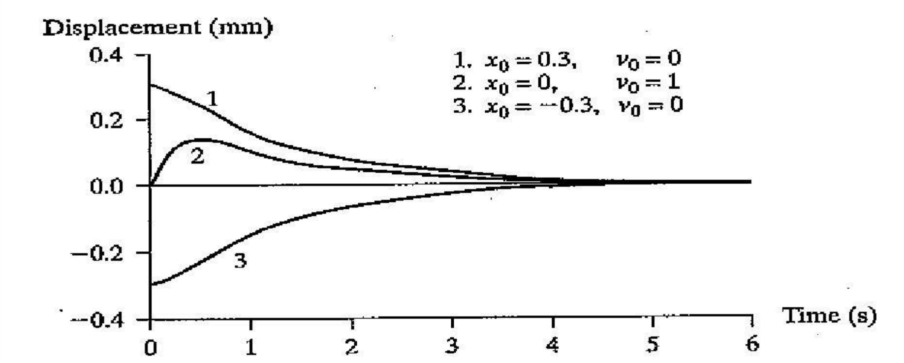


- $\xi > 1;$ Overdamped system

$$\lambda_1 = -\xi\omega_n - \omega_n\sqrt{\xi^2 - 1}$$

$$\lambda_2 = -\xi\omega_n + \omega_n\sqrt{\xi^2 - 1}$$

$$x(t) = e^{-\xi\omega_n t}(Ae^{\lambda_1 t} + Be^{\lambda_2 t})$$



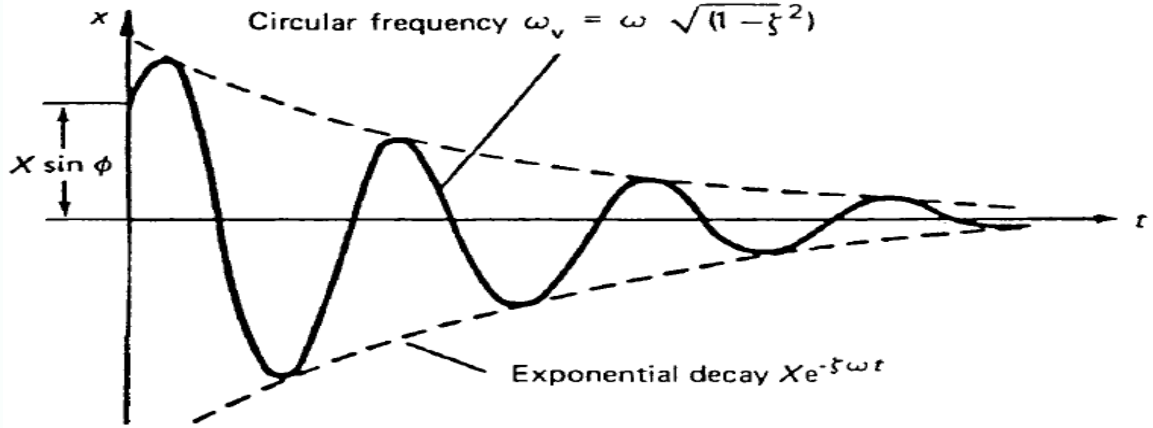
- $\xi < 1;$ Less damped – underdamped System

$$\lambda_1 = -\xi\omega_n - \omega_n j\sqrt{1 - \xi^2}$$

$$\lambda_2 = -\xi\omega_n + \omega_n j\sqrt{1 - \xi^2}$$

$$x(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$$

$\omega_d = \omega_n\sqrt{1 - \xi^2}$; Damped Natural Frequency



Example 1–2

The damping rate of the spring is measured to be 0.11 kg/s. Calculate the damping ratio and determine if the free motion of the spring-bolt system(Example 1-1) is over-damped, under-damped, or critically damped. *m* = 49.2 g and *k* = 857.8 N/m

Solution.

Using the definition of the critical damping coefficient of equation

$$c_{cr} = 2\sqrt{km} = 2\sqrt{(2857.8\ \mathrm{N/m})(49.2 \times 10^{-3}\mathrm{kg}}$$

= 12.993 kg/s

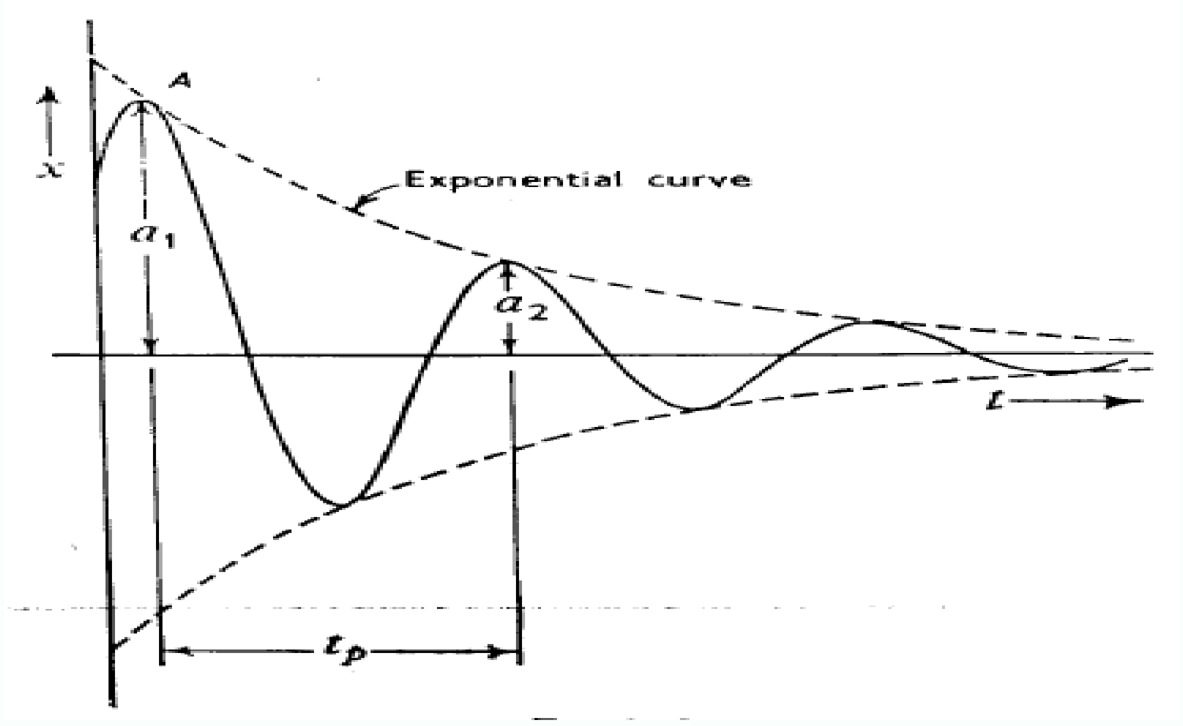
If *c* is measured to be 0.11 kg/s, the critical damping ratio becomes

$$\xi = \frac{c}{c_{cr}} = \frac{0.11\ (\mathrm{kg/s})}{12.993\ (\mathrm{kg/s})} = 0.0085$$

or 0.85% damping. Since ξ is less than 1, the system is under-damped. The motion re-sulting from giving the spring-bolt system a small displacement will be oscillatory.

Logarithmic decrement

- A convenient way of determining the damping in a system is to measure the rate of decay of oscillation.



$$a_1 = Ae^{-\xi\omega_n t}\sin(\omega_v t + \phi)$$

$$a_2 = Ae^{-2\xi\omega_n t}\sin(2\omega_v t + \phi)$$

$$a_r = Ae^{-r\xi\omega_n t}\sin(r\omega_v t + \phi)$$

$$t = t_p = \frac{2\pi}{\omega_d}; r = 1, 2, 3 \ldots$$

$$\sin(r\omega_d t + \phi) = \sin(\omega_d t + \phi)$$

$$\frac{a_1}{a_r} = e^{\xi\omega_n t(r-1)}$$

$$ln\left(\frac{a_1}{a_r}\right) = \xi\omega_n t(r-1) = \frac{2\pi\xi(r-1)}{\sqrt{1-\xi^2}}$$

Example 1–3

A static deflection test is performed on a less damped system with a mass of 2 kg and the stiffness is determined to be 1.5 X 10^3 N/m. The displacements at t_1 and t_2 are measured to be 9 and 1 mm, respectively. Calculate the damping coefficient.

Solution

From the definition of the logarithmic decrement

$$\delta = \ln\left[\frac{x(t_1)}{x(t_2)}\right] = \ln\left[\frac{9\text{mm}}{1\text{mm}}\right] = 2.1972$$

$$\xi = \frac{2.1972^2}{4\pi^2 + 2.1972^2} = 0.109\text{or } 10.9\%$$

Also

$$c_{cr} = 2\sqrt{km} = 2\sqrt{(1.5\times 10^3\text{N/m})(2\text{ kg})} = 1.095\times 10^2\text{ kg/s}$$

and from equation (1.23) the damping coefficient becomes

$$c = c_{cr}\xi = (1.095\times 10^2)(0.109) = 11.94\text{ kg/s}$$

Stability Considerations

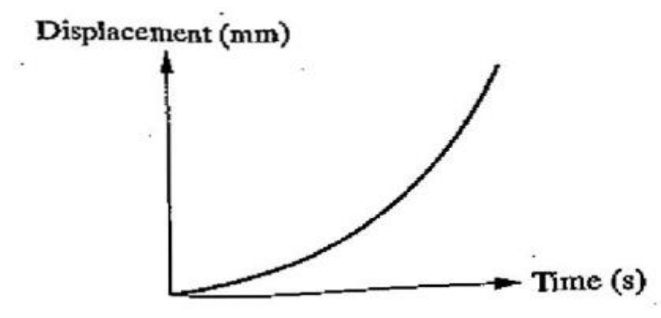
- In cases where the vibration equation takes the form

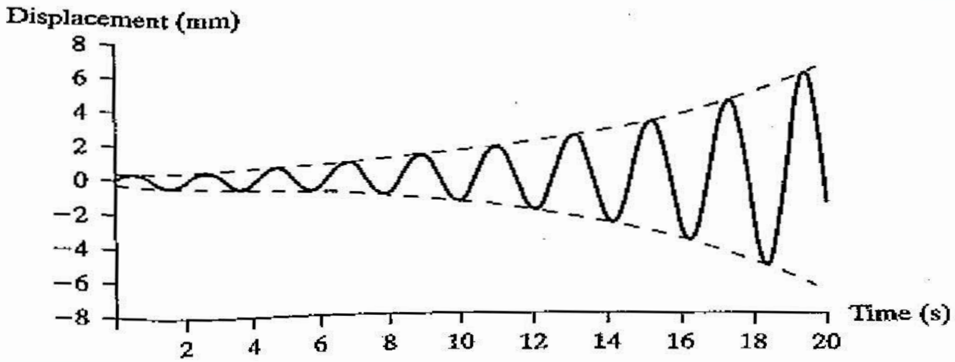
$$m\ddot{x} - kx = 0$$

The solution takes the form

$$x(t) = A \sinh \omega_n t + B \cosh \omega_n t$$

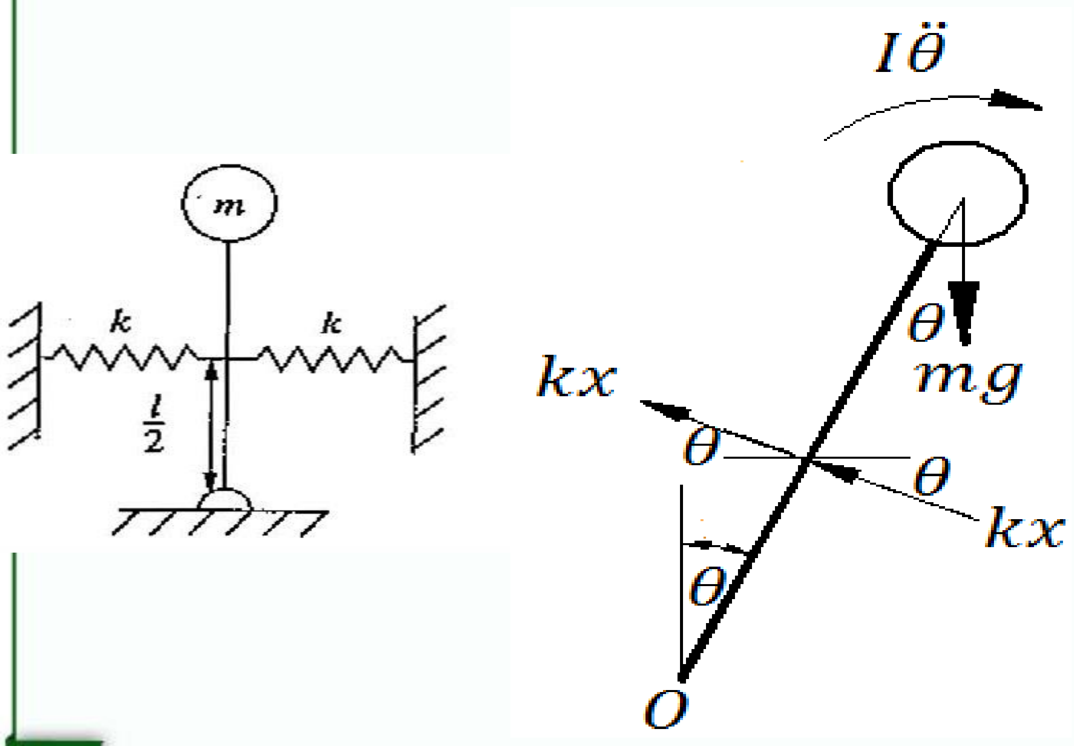
which increases without bound as *t* does. In this case $|x(t)|$ no longer remains finite and such solutions are called *divergent* or *unstable*.





Example

Consider the inverted pendulum connected to two equal springs, shown. Assume that the springs are undeflected when in the vertical position and that the mass *m* of the ball at the end of the pendulum rod is substantially larger than the mass of the rod itself, so that the rod is considered to be massless. If the rod is of length l and the springs are attached at the point $l/2$, the equation of motion becomes



$$ml^2\ddot{\theta} + \left(\frac{kl^2}{2}\sin\theta\right)\cos\theta - mgl\sin\theta = 0$$

For $\theta <$ about $\pi/20$, $\sin\theta \cong \theta$ and $\cos\theta \cong 1$. $ml^2\ddot{\theta} + \left(\frac{kl^2}{2}\theta\right) - mgl\theta = 0$

which upon rearranging becomes

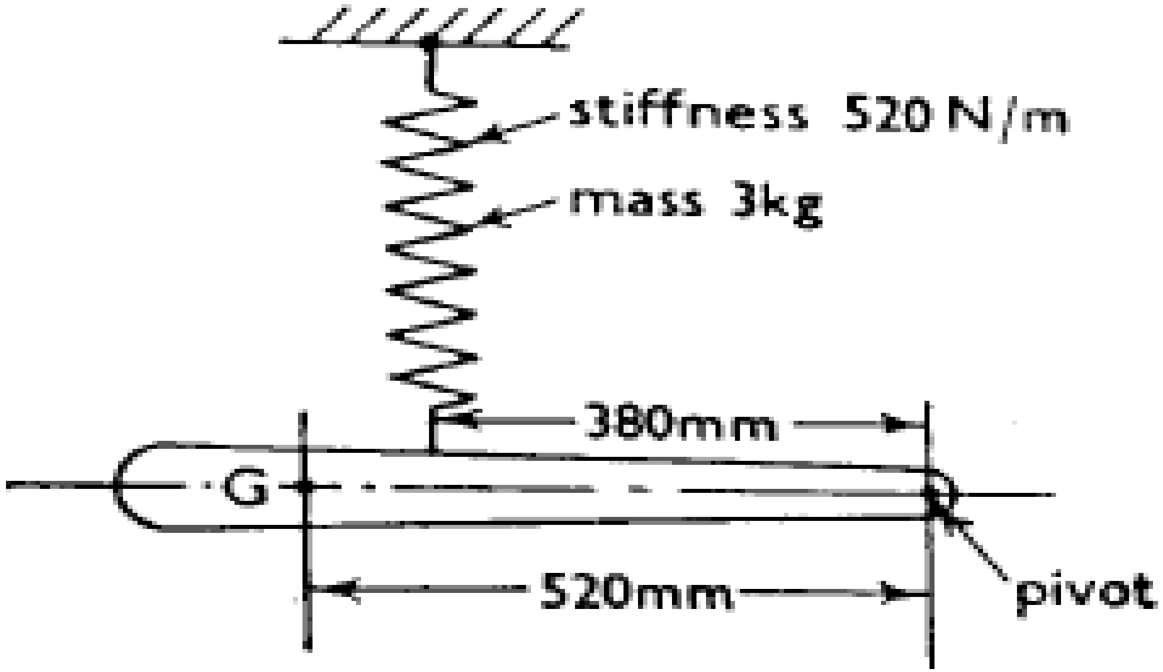
$$2ml\ddot{\theta}(t) + (kl - 2mg)\theta(t) = 0$$

where θ is now restricted to be small (smaller than $\pi/20$). If *k*, *l*, and *m* are all such that the coefficient of θ, called the effective stiffness, is negative, that is, if

$$kl - 2mg < 0; \text{system unstable}$$

ASSIGNMENT 1

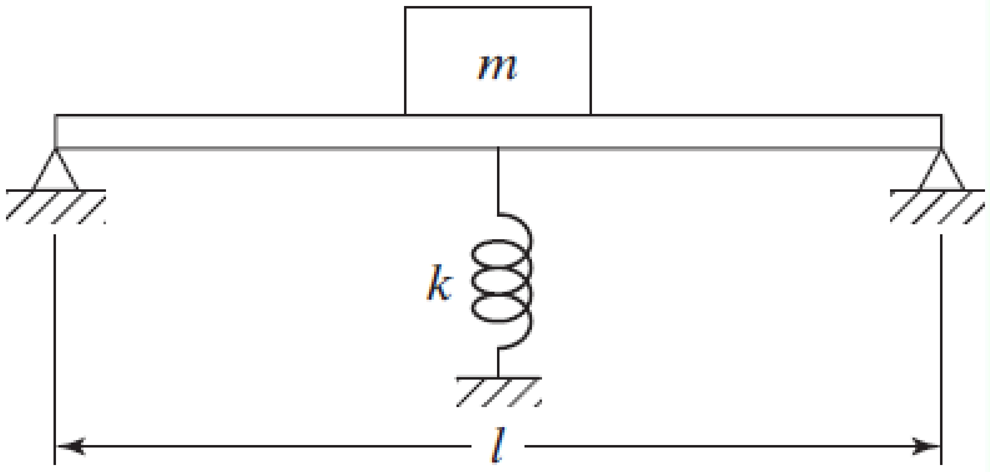
The bar shown has a mass of 15 kg and the radius of gyration about the c.g. is 0.26 m. Find the natural period of oscillation.



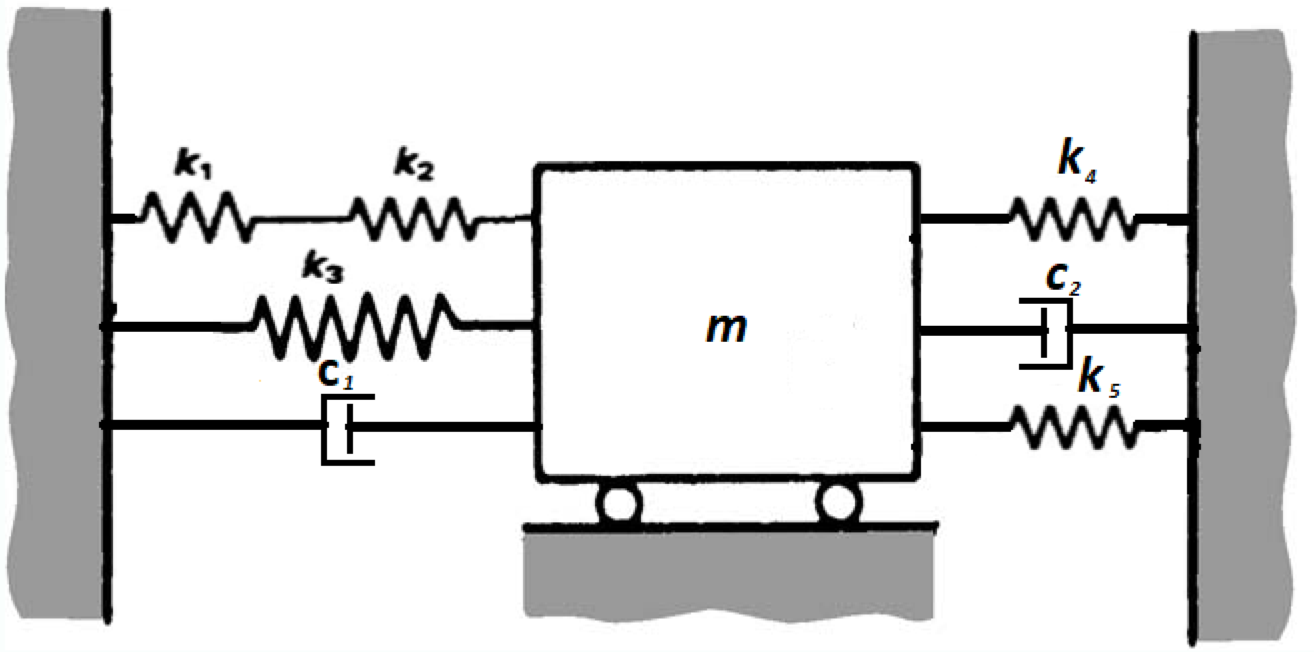
ASSIGNMENT 2

A machine of mass $m = 500\ kg$ is mounted on a simply supported steel beam of length $l = 2\ m$ having a rectangular cross section $(depth = 0.1\ m,\ width = 1.2\ m)$ and Young's modulus $E = 2.06 \times 10^{11}\ N/m^2$. To reduce the vertical deflection of the beam, a spring of stiffness k is attached at mid-span, as shown. Determine the value of k needed to reduce the deflection of the beam by 25 percent of its original value.

Assume that the density of steel is 7850 kg/m^3.



ASSIGNMENT 3



$$m = 10\,kg$$
$$k_1 = 2\,kN/m$$
$$k_2 = 3\,kN/m$$
$$k_3 = 4\,kN/m$$
$$k_4 = 10\,kN/m$$
$$k_5 = 1\,kN/m$$
$$c_1 = 1\,kg/s$$
$$c_2 = 5\,kg/s$$

Calculate the Natural frequency and the response of the system shown.

$$x(0) = 5\,mm \;\; and \;\; \dot{x} = -2\,mm/s$$

ASSIGNMENT 4

In a vibration suspension system, a mass of 25 kg is suspended from a helical spring of stiffness 15 kN/m, the motion being controlled by a dash-pot such that the amplitude of the vibration decreases to one-fifth of Its original value after two complete vibrations.

Find (a) the value of the damping force,

(b) the frequency of the vibration.

ASSIGNMENT 5

Part of a structure is modelled by a thin rigid rod of mass ***m*** pivoted at the lower end, and held in the vertical position by two springs, each of stiffness ***k,*** as shown. Write down the equation of motion and comment on its stability