EE 262 – SYNCHRONOUS MACHINES

COURSE CONTENT:

Basic Theory. Construction, Phasor Diagram and Equivalent Circuit of Non-Salient Pole Machine. Machine Characteristics. Two-Axis Theory. Phasor Diagram of Salient-Pole Machine. Assessment of Reactance. Determination of Voltage Regulation.

Parallel Operation: Synchronizing, Effects of Changing Excitation and Mechanical Torque, Load Sharing of Two Machines, Hunting, Performance Equations, Circle Diagrams

V-Curve of Synchronous Motor. Starting of Synchronous Motor and Its Industrial Control Circuit. Synchronous Induction Motor. Single-Phase Synchronous Generator.

EE 262 – SYNCHRONOUS MACHINES

1. THREE-PHASE SYNCHRONOUS MACHINES

1.1 Introduction

Synchronous machines are a type of rotating machines where electromechanical energy conversion takes place when the change in flux is associated with mechanical motion. Generally in rotating machines, voltages are induced in windings or group of coils either by

- rotating the windings mechanically through a magnetic field or
- mechanically rotating a magnetic field past the windings or
- designing the magnetic circuit so that the reluctance varies with rotation of the rotor.

By any of these methods, the flux linking a specific coil is changed cyclically, and an induced voltage $e = \frac{d}{dt}(N\phi)$ is generated in accordance with Lenz's Law of electromagnetic induction.

The group of such coils so interconnected that their generated or induced voltages all make a positive contribution to the desired result is called an ***armature winding***. The coils are wound on iron cores so as to concentrate the flux and thus ensure that the flux path through them is as effective as possible.

Because the armature iron is subjected to a varying magnetic field, eddy currents will be induced in it. To minimize the eddy-current loss, the armature iron core is built up of thin laminations. The magnetic circuit is completed through the iron of the other machine member, and ***exciting coils***, or ***field windings***, are placed on that member to act as the primary sources of flux. Permanent magnets may be used in small machines.

1.2 Definition of Synchronous Machines

A *synchronous machine is an AC machine whose mechanical speed (or rotor speed) under steady-state conditions is proportional to the frequency of the voltage and current in its armature*. The frequency in cycles per second (hertz or Hz) is the same as the speed of the rotor in revolutions per second, i.e., the electrical frequency is *synchronized* with the mechanical speed, and this is the reason for the designation *synchronous machine*.

At synchronous speed, the *rotating* magnetic field created by the armature current travels at the same speed as the field created by the field current, and a steady torque results from the interaction of the two magnetic fields.

For a P-pole machine, the synchronous speed N_s (in rev/min.) is related to the frequency f of the armature induced voltages and currents and the number of poles P by the equation $N_s = \frac{120f}{P}$.

The number of poles is chosen according to the desired speed. The speed is in turn determined by the driven equipment or prime mover attached to the shaft of the synchronous machine. Thus a 2-pole machine must revolve at 3000 rpm to produce a 50-Hz voltage. But a great many synchronous machines have more than 2 poles. It is worth noting that essentially, electric motors and generators are the less expensive the faster they run.

With a few exceptions, synchronous machines are 3-phase machines because of the advantages of 3-phase systems for generation, transmission and heavy-power utilization. For the production of a set of three voltages phase-displaced by 120 electrical degrees in time, it is obvious that a minimum of 3 coils displaced 120 electrical degrees in space must be used.

1.3 Constructional Features of Synchronous Machines

Unlike *induction* machines in which there are alternating currents in both stator and rotor windings, ***synchronous machines have alternating currents (AC) in the stator windings and DC currents in the rotor windings***. The 3-phase synchronous machine has the following essential constructional parts:

- *stator* comprising
    - stator frame or yoke
    - armature coils
- *rotor* comprising
    - rotor core
    - rotor or field windings
- *slip-rings or collector rings*
- *brushes and bearings*

Of these parts, the stator frame or yoke, the pole-cores, the rotor core and air gap between the poles and rotor core, form the magnetic circuit, whereas the rest form the electrical circuit.

1.4 The Stator:

The stator unit is the stationary part of the machine and consists of the stator frame or yoke and the armature coils. The stator core contains the set of slots that carry 3-phase winding, and is ***laminated*** to minimize loss due to hysteresis and eddy-currents. The laminations are insulated from each other and have spaces between them for allowing cooling air to pass through. The windings have hollow passages through which cooling water is circulated.

The *stator frame or yoke* serves the purposes of

- providing mechanical support for the poles
- protection for the machine and
- carries the armature flux produced by the poles.

In small machines, where weight is of little importance and cheapness is the main consideration, the yoke is made of cast iron. But for large machines, cast steel or rolled steel is usually employed to fabricate the stator frame.

1.5 The Rotor:

The rotor, which is the rotating part of the machine, is located on a shaft running on bearings, and is free to rotate between magnetic poles. The *rotor core* is cylindrical or drum-shaped, and is built of steel laminations with *slots* to house the *field windings*. Besides housing the field windings in *slots* and causing them to rotate to cut the magnetic flux of the magnetic fields, the rotor core also provides a magnetic path of low reluctance to the flux from the poles.

The rotor or field windings are insulated from each other and placed in *slots* which are lined with tough insulating material. The rotor carries the dc windings or field windings. These field windings are excited by direct current conducted to them by means of carbon *brushes* bearing on *slip rings* or *collector rings*.

1.6 Damper Windings on Rotor

In addition to the DC winding, the rotor carries the so-called *damper windings* (also called *squirrel-cage winding*). In the salient-pole (engine-driven) machines, the damper winding is embedded in the pole-shoes or pole faces and connected (short-circuited) at their end with brass or heavy copper rings. The damper windings are not usually required in cylindrical-rotor machines driven by reciprocating steam engines, water wheels or steam turbines.

The *damper winding* serves the following functions:

- produces forces which dampen the oscillation of the rotor, thereby reducing *hunting* (momentary speed fluctuations)
- helps to start synchronous motors
- maintains balanced 3-phase voltage under unbalanced load conditions.
- improves parallel operation of salient-pole generators driven by internal-combustion engines.

1.7 The Slip-Rings (or Collector Rings):

The *slip-ring* is made of copper segments, and has the same functions in the motor as in a generator. Its purpose is to facilitate the collection of current from the DC excitation source to the field windings.

1.8 The Brushes and Bearings:

The purpose of brushes is to carry current from the external circuit to the commutator. They are usually made of blocks of carbon or graphite, and are rectangular in shape. The brushes should slide freely in their holder so as to follow any irregularity in the commutator. Because of their reliability, ball-bearings are frequently employed, though for heavy duties, roller-bearings are preferable.
The ball and rollers are lubricated by hard oil for quieter operation and for reducing the wear of the bearings.

1.9 Types of Synchronous Machines

Synchronous machines may be classified broadly under:

- Synchronous Generators (or Alternators)
- Synchronous Motors (synchronous motors operating with load attached)
- Synchronous Condensers (synchronous motors operating on no-load)

2. SYNCHRONOUS GENERATORS

They are the main primary source of electrical energy we consume. They convert mechanical energy into electrical energy, in powers ranging up to 1500MW. In general, the greater the generated power, the higher the voltage rating. However the rated generated voltage seldom exceeds 25 kV because the increased slot insulation takes up valuable space at the expense of copper conductors. *The stator winding (i.e., the armature winding which collects the generated voltage) is always star-connected and the neutral is connected to ground*. The reasons for this arrangement are:

1. The highest effective voltage between a stator conductor and the grounded stator core is only $1/\sqrt{3}$, that is, 57.7% of the line voltage. We can therefore reduce the amount of insulation in the slots, which in turn enables us to increase the cross-section of the conductor. A larger conductor permits us to increase the current and hence the power output of the machine.
2. When a generator is under load, the voltage per phase becomes distorted and the waveform is no longer sinusoidal. This distortion is mainly due to undesired third harmonic voltages. With star connection the harmonics do not appear between the lines. With delta connection, the harmonic voltages add up and produce large circulating current in the delta connected winding which causes additional I^2R losses.
3. The neutral is available for protective gear.

2.1 Types of Synchronous Generators

Synchronous generators are classified according to the type of rotors they use, which in turn depends on the speed of the prime mover. The two types of rotors used in generators are:

- Salient-Pole Rotor Type
- Cylindrical-Rotor (or Non-salient Pole) Type

The constructional reasons for some synchronous generators having salient-pole rotor structures and other having cylindrical rotors can be appreciated with the aid of the synchronous speed equation $N_s = \dfrac{120f}{P}$.

2.1.1 Salient-Pole Generators – (Low-speed Generators)

The power system in most countries, including Ghana, operates at a constant system frequency f of 50 Hz. And so for *low-and medium-speed (engine-driven) rotors*, a relatively large number of poles are required to produce the desired frequency.

A *salient- or projecting pole* construction is thus characteristic of ***hydroelectric generators***, because hydraulic and engine-driven turbines operate at relatively *low speeds*, with running speed between 50 and 300 rpm for hydraulic turbines and about 200 rpm for internal-combustion engines.

The salient-pole construction with *concentrated* windings is thus best adapted mechanically to multi-polar slow- and medium-speed hydroelectric generators, hydraulic and engine-driven turbines and some synchronous motors. Such generators are characterized by their *large diameters* and short axial lengths. The poles and pole-shoes (which cover 2/3 of pole pitch) are laminated to reduce heating due to eddy currents. The field coils for small machines are wound with round wire, while rectangular copper strips wound on edge are used for large machines.

2.1.2 Cylindrical-Rotor Generators – (High-speed Generators)

When the steam engine or gas turbine is operated at a high speed, it has a high efficiency, and for this reason, it is often used to drive generators at high speeds. But for high speeds, it is difficult to build a rotating field with projecting poles strong enough to withstand the centrifugal force. Projecting poles also cause excessive wind losses and make the generator noisy.

To overcome these undesirable features, generators intended for *high-speed steam-turbine or gas-turbine* drive have their field structure made *non-salient or cylindrical* in form and *small in diameter*, with *distributed field windings* placed in slots and arranged so as to produce an approximately sinusoidal 2- or 4-pole field.

Such high-speed turbine-driven generators are also called ***turbo generators***, and are commonly 2- or 4-pole cylindrical- rotor machines running at speeds of 3000 rpm or 1500 rpm. In turbo generators, considerable heat is liberated in small spaces, and this heat must be carried away by air currents forced through passages in the heated parts. Hence forced ventilation is required in turbo generators. The turbo generator must be enclosed to control the direction of the air currents, as well as to reduce noise.

2.2 Principle of Operation of Synchronous Generator

Rotating machines (be they AC or DC) operate on the same fundamental principles of electromagnetic induction. It may be recalled that they consist of an *armature winding* (i.e., a group of coils so interconnected that their generated or induced voltages all make a positive contribution to the desired result) and a *field winding or exciting coils* (to act as the primary sources of flux).

The field winding (or rotor winding) is excited or energized by the so-called ***exciter system*** supplying direct current from a DC source that needs not exceed 250 volts. In most cases, the necessary exciting (or magnetising) current is obtained from a small

DC shunt generator mounted on the shaft of the synchronous machine itself. Because the field magnets are rotating, the direct current is supplied through two slip rings. As the exciting DC voltage is relatively small, the slip rings and brush gear are of light construction. Recently, brushless excitation systems have been developed in which a 3-phase AC exciter and a group of rectifiers supply DC to the machine.

When the rotor is made to rotate by the turbine of the prime mover, the stator or armature conductors (being stationary) are cut by the rotated DC magnetic flux from the field coils on the rotor. Hence they have induced emf produced in them.

Because the magnetic poles are alternately N and S, they induce an emf and hence current in the armature conductors, which first flow in one direction and then in the other. Hence an *alternating emf* is induced in the stator windings *whose frequency depends on the number of* N and S *poles moving past a conductor in one second and whose direction is given by Fleming's Right-Hand Rule*.

2.3 Advantages of Stationary Armature and Revolving Field System

In DC generators, a commutator assembly (consisting of *split-rings*, brushes, etc) is used to change the alternating emf induced in the coils to a unidirectional emf for the external circuit. Since the armature of a synchronous machine rather utilizes ***slip rings*** to supply alternating current to the external load and therefore has no need of a commutator, the armature needs not be the rotating member. *In fact, it is more desirable to have a stationary armature with the field poles rotating inside it, as this structure has several advantages*.

The **advantages** of a having stationary armature and rotating field system include the following:

1. For polyphase power, a rotating armature would require three or more slip rings to deliver power to an external load. These slip rings, being exposed, are difficult to insulate, particularly for the high AC voltages (30 kV or more) synchronous generators are required to supply.
2. Because of the difficulty of insulation of the slip rings on a rotating armature, *arc-overs and short circuits* are apt to occur.
3. *No slip rings are required in a stationary armature*. The output current can be led directly from fixed terminals on the stator (or the armature windings) to the load circuit, without having to pass it through brush-contacts. With the doing away with slip rings, power and frictional losses due to contact resistance with slips are thus avoided.
4. It is much *easier to properly insulate a stationary armature* (where winding space is less concentrated) for high voltage than it is to insulate a high-voltage rotating winding. The heavy insulation must have adequate mechanical strength to withstand the mechanical forces due to centrifugal force.

3. PARAMETERS AND OPERATING CHARACTERISTICS OF SYNCHRONOUS GENERATORS

3.1 Rotating Magnetic Fields Polyphase AC Machines (e.g. Synchronous Generators

To understand the theory of polyphase a-c machines, it is necessary to study the nature of the magnetic field produced by a polyphase winding. In particular, we shall consider the mmf patterns of a 3-phase winding such as those found on the stator of 3-phase induction and synchronous machines.

In a 3-phase machine, the windings of the individual phases are displaced from each other by 120 electrical degrees in space around the airgap circumference. When the 3-phase windings are excited by a 3-phase power source, alternating currents flow in the windings. Under balanced 3-phase conditions, the instantaneous currents are given as:

$$\begin{aligned} i_a &= I_{\max} \cos\omega t \\ i_b &= I_{\max} \cos(\omega t - 120^0) \\ i_c &= I_{\max} \cos(\omega t - 240^0) \end{aligned} \qquad (1)$$

where $I_{\max}$ is the maximum value of the current and the time origin is arbitrarily taken as the instant when the phase-a current is a positive maximum. See Fig below

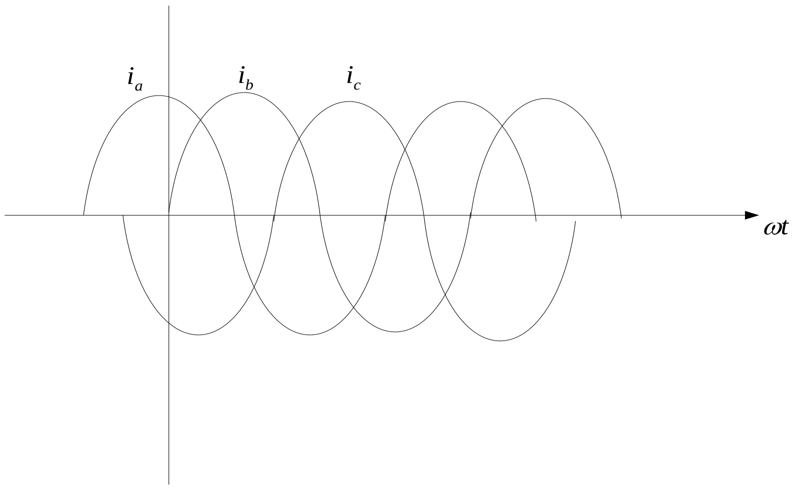


Fig: Instantaneous 3-Phase Currents

The corresponding component mmf waves will also vary sinusoidally with time. Each component is a stationary, pulsating sinusoidal distribution of mmf around the airgap with *its peak located along the magnetic axis of its phase and its amplitude proportional to the instantaneous phase current*. In other words, *a standing space wave is varying sinusoidally with time*.

Each component can be represented by an oscillating space vector drawn along the magnetic axis of its phase, with length proportional to the instantaneous phase current. The resultant mmf is the sum of the components from all three phases.

3.2 Analytical Analysis of Rotating Mmf Wave

To study the resultant field analytically, let the origin for angle θ around the airgap periphery be placed at the axis of phase a. At any time *t*, all 3 phases contribute to the airgap mmf at any point θ. See Fig below.

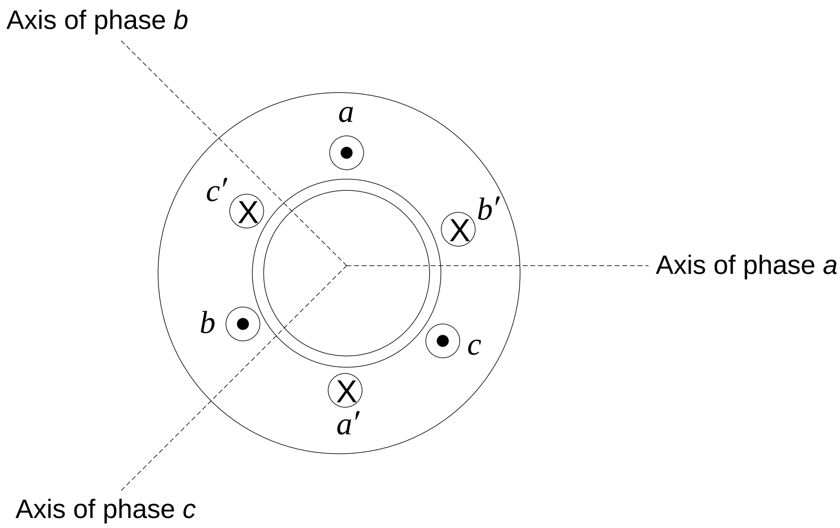


Fig: Simplified 2-Pole 3-Phase Winding

The *mmf* contributions from the 3 phases are:

$$\begin{aligned} F_a &= F_{apeak}\cos\theta \\ F_b &= F_{bpeak}\cos(\theta - 120^0) \\ F_c &= F_{cpeak}\cos(\theta - 240^0) \end{aligned} \qquad (2)$$

The 120 electrical degrees displacements appear because the machine is so wound that the axes of the 3-phases are 120 electrical degrees apart in space.

The ***resultant mmf*** at any point θ is then

$$F(\theta) = F_{apeak}\cos\theta + F_{bpeak}\cos(\theta - 120^0) + F_{cpeak}\cos(\theta - 240^0) \qquad (3)$$

But the mmf amplitudes vary with time in accordance with the current variations. Thus, with the time origin arbitrarily taken at the instant when the phase-a current is a positive maximum, the mmf amplitudes can be written as function of time *t* as:

$$\begin{aligned} F_{apeak} &= F_{a\max}\cos\omega t \\ F_{bpeak} &= F_{b\max}\cos(\omega t - 120^0) \\ F_{cpeak} &= F_{c\max}\cos(\omega t - 240^0) \end{aligned} \qquad (4)$$

The quantities $F_{a\max}$, $F_{b\max}$ and $F_{c\max}$ are respectively the time-maximum values of the amplitudes F_{apeak}, F_{bpeak} and F_{cpeak}. The 120^0 displacements appear here because the three currents are 120^0 phase-displaced in time. Under balanced conditions, the currents in the 3-phases are also balanced and therefore of equal amplitude. The three amplitudes $F_{a\max}$, $F_{b\max}$ and $F_{c\max}$ are also then equal, and the symbol $F_{\max}$ may be used for all three.

Equation (3) therefore becomes

$$F(\theta,t) = \{F_{\max}\cos\theta\cos\omega t\} + \{F_{\max}\cos(\theta-120^0)\cos(\omega t-120^0)\} + \{F_{\max}\cos(\theta-240^0)\cos(\omega t-240^0)\} \tag{5}$$

Each of the three components on the right-hand side of Eqn (5) is a *pulsating* standing wave. ***In each term, the trigonometric function of θ defines the space distribution as a stationary sinusoid, and the trigonometric function of t indicates that the amplitudes pulsate with time***.

The first of the terms expresses the phase-a component; the second and third terms express, respectively, the phase-b and phase-c components.

By use of the trigonometric transformation

$$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta) \tag{6}$$

each of the components in Eqn (5) can be expressed as cosine functions of sum and difference angles. Thus using the trigonometric transformation, Eqn (5) is transformed into

$$\begin{aligned} F(\theta,t) &= \left\{\frac{1}{2}F_{\max}\cos(\theta-\omega t) + \frac{1}{2}F_{\max}\cos(\theta+\omega t)\right\} \\ &+ \left\{\frac{1}{2}F_{\max}\cos(\theta-\omega t) + \frac{1}{2}F_{\max}\cos(\theta+\omega t-240^0)\right\} \\ &+ \left\{\frac{1}{2}F_{\max}\cos(\theta-\omega t) + \frac{1}{2}F_{\max}\cos(\theta+\omega t-480^0)\right\} \end{aligned} \tag{7}$$

Now the three cosine terms involving the angles $(\theta+\omega t)$, $(\theta+\omega t-240^0)$, and $(\theta+\omega t-480^0)$ are equal sinusoid displaced in phase by 120^0. Note that a lag angle of 480^0 is equivalent to a lag angle of $(480^0 - 360^0) = 120^0$. Their sum is therefore zero, and Eqn (13) reduces further to:

$$F(\theta,t) = 1.5F_{\max}\cos(\theta-\omega t) \tag{8}$$

The ***resultant*** mmf wave described by Eqn (8) is a sinusoidal function of the space angle θ, having a *constant amplitude* of $1.5F_{\max}$ and ***rotating*** at angular velocity ω or a space-phase angle ωt which is a linear function of time.

In general, it may be shown that a rotating field of constant amplitude will be produced by a $q-phase$ winding excited by balanced $q-phase$ currents when the respective phases are wound $2\pi/q$ electrical radians apart in space. The constant amplitude will be $q/2$ times the maximum contribution of one phase, and the speed will be $\omega = 2\pi f$ electrical radians per second.

3.3 Generated Voltage in *Rotating* AC Machines (e.g. Alternators)

The study of voltages induced in any of the armature windings of rotating machines resolves into study of the voltage induced in a single coil followed by addition of the individual coil voltages in the manner dictated by the specific interconnection of the coils forming the complete winding. The general nature of induced voltage has already been discussed. The focus in this section is the determination of the voltage magnitude by Faraday's Law.

Consider a single N-turn *full-pitch* coil, that is a coil that spans 180 electrical degrees or complete pole pitch. For simplicity, consider a 2-pole cylindrical rotor machine. The field winding on the rotor is excited by a d-c source, and assumed to produce a sinusoidal space wave of flux density $B = \dfrac{\Phi}{A}$ at the stator surface. The rotor is spinning at constant angular velocity ω.

When the rotor poles are in line with the magnetic axis of the stator (armature) coil, the flux linkage with the stator coil is $N\Phi$, where Φ is the airgap flux per pole. For the assumed sinusoidal flux-density wave,

$$B = B_{\max}\cos\theta \tag{9}$$

where $B_{\max}$ is the peak value at the rotor pole center and θ is measured in electrical radians from the rotor pole axis. The airgap flux per pole is the integral of the flux density over the pole area.

As the rotor turns, the flux linkage varies as the cosine of the angle α between the magnetic axes of the stator coil and rotor. By Faraday's Law, the voltage induced in the stator coil is

$$e = -\frac{d\lambda}{dt} = \omega N\Phi_{\max}\sin\omega t - N\frac{d\Phi_{\max}}{dt}\cos\omega t \tag{10}$$

The minus sign associated with Faraday's Law in Eqn (10) implies that while the flux linking the coil is decreasing, an emf will be induced in it in a direction to try to produce a current which would tend to prevent the flux linking it from decreasing.

The first term $\omega N\Phi_{\max}\sin\omega t$ is called the ***rotational or speed voltage*** due to the relative motion of field and coil. The second term $N\dfrac{d\Phi_{\max}}{dt}\cos\omega t$ is the ***transformer voltage***, and is only present when the amplitude of the flux density wave changes with time.

In the normal steady state operation of most rotating machines, the amplitude of the airgap flux wave is constant, and the induced voltage is simply the rotational voltage.

$$e = \omega N\Phi_{\max}\sin\omega t \tag{11}$$

In normal steady-state operation of AC machines, we are usually interested in the rms values of voltages and currents rather than their instantaneous values. And hence from Eqn (11), the maximum value of the induced instantaneous voltage is:

$$E_{\max} = \omega N\Phi_{\max} = 2\pi f N\Phi_{\max} \tag{12}$$

Its rms value is given as

$$E_{rms} = \frac{E_{\max}}{\sqrt{2}} = 4.44\, fN\Phi_{\max} \tag{13}$$

These equations are identical in form to the corresponding emf equations for a transformer. *Therefore relative motion of a coil (rotor field coil is this case) and a constant-amplitude spatial flux density wave (like the DC flux from the field windings on the rotor) produces the same voltage effect as does a time-varying flux in association with stationary coils in a transformer. Rotation, in effect, introduces the time element and **transforms a space** distribution of flux density into a **time variation** of voltage*.

For a distributed winding, the induced emf in Eqn (13) must be modified by the winding factor k_w.

$$E_{rms} = 4.44 k_w fN\Phi \tag{14}$$

where N is the total series turns per phase.

3.4 Armature Reaction Flux in Synchronous Generators

When the DC field winding on the rotor is energized, voltage is induced in the armature windings on the stator of the alternator. Under ***no-load*** condition, no current will flow through the armature windings. In that situation, the flux in the airgap is uniformly distributed and due only to that of the field windings. However, **under loading situation** when a synchronous generator supplies electrical power to a load, armature current flows. The armature current creates a component flux wave in the airgap which rotates at synchronous speed, as shown in Section 1.2.2. This armature flux reacts with the flux created by the field, and an electromagnetic torque results from the tendency of the two magnetic fields to align themselves.

In a generator, this torque ***opposes*** rotation, and a counter mechanical torque must be applied from the prime mover in order to sustain rotation. The electromagnetic torque is the mechanism through which greater electrical power output calls for greater mechanical power input.

3.5 Distorting Effect of Armature Reaction Flux

The effect of the armature flux is to ***distort*** the main flux distribution in the airgap due to the field flux. The effect on the flux magnitude is important because both the generated voltage and torque per unit of armature current are influenced thereby.

The armature mmf F_a combines with (or superimposes on) the main flux in the airgap due to the field mmf F_f to give a resultant mmf F_r that is distorted. This net mmf distribution due to the combined magnetizing action of the field and armature fluxes magnetizes the machine to a resultant gap flux per pole Φ_r, which in turn generates a voltage $E_g = 4.44 k_w fN\Phi_r$ in the stator or armature winding. In the absence of ***magnetic saturation***, F_a and F_f can be considered to produce separate gap fluxes Φ_a and Φ_f, which superpose to give Φ_r as resultant. The two flux components can then be considered to induce separate emfs E_a and E_f in the stator windings having the phasor sum E_g.

The net flux distribution in the airgap of an alternator depends on the amount of stator current and on the phase relation existing between the current and voltage; that is, the power factor of the load.

3.6 Armature Reaction Reactance in Synchronous Generators

The net effect of the armature current is to distort or twist the main flux from the field windings and hence the generated voltage in the stator (armature windings). This effect is known as ***armature reaction***. The mmf wave created by the armature current is called *armature-reaction mmf*.

For salient-pole machines, the effect of the armature mmf is to be seen as creating flux sweeping across the pole-faces. Thus its path in the pole shoes crosses the path of the main-field flux. For this reason, armature reaction of this type is called ***cross-magnetizing armature reaction***. It evidently causes a decrease in the resultant airgap flux density under one half of the pole and an increase under the other half.

And so when the field winding is excited and the armature is connected to supply load, the resultant airgap flux distribution is the superposition of the flux distribution from the field windings and that from the armature windings. Because of saturation of iron, the flux density is decreased by a greater amount under one pole tip than it is increased under the other.

Accordingly, the resultant flux per pole is lower than would be produced by the field winding alone, a consequence known as the ***demagnetizing effect of cross-magnetizing armature reaction***. Since the demagnetizing effect of cross-magnetizing armature reaction is caused by saturation, its magnitude is a non-linear function of both the field current and the armature current.

3.6.1 Counteracting Armature Reaction

The effect of cross-magnetizing armature reaction may be limited in the design and construction of the machine. The mmf of the main field should exert predominating control on the airgap flux, so that the condition of weak field mmf and strong armature mmf may be avoided through the following measures:

1. ***Increasing the reluctance of the cross-flux path*** – essentially the armature teeth, pole shoes and the airgap by increasing the degree of saturation in the teeth and pole faces, by avoiding too small an airgap and by using chamfered pole face which increases the airgap at the pole tips.

2. The best but also the most expensive curative measure is compensate the armature mmf by means of a winding embedded in the pole faces called ***compensating or pole-face winding***. The compensating winding is connected in *series* with the armature and in arranged in such a way as to supply a magnetizing action that is ***equal and opposite*** to that of the armature coils at all loads, thereby neutralizing the cross-magnetizing effect of the armature ampere-turns.

Furthermore, the addition of the compensating winding improves the speed of response, because it reduces the armature circuit time-constant. The main disadvantage of commutating or pole-face winding is their expense. They are used therefore in machines designed for heavy loads or rapidly changing loads.

3.6.2 Armature Reaction Reactance

As explained, the magnetomotive force (mmf) produced by the armature currents in the armature winding when the alternator is loaded is called *armature reaction*. The load current, flowing through the armature windings, builds up *local* flux which on cutting the winding generates a *counter (or reactance) emf*. This effect gives the armature a *reactance* that is numerically equal to $2\pi fL$, where L is the leakage inductance of the armature winding (similar to the leakage inductance in transformer windings).

This leakage reactance arising from the armature reaction is called ***armature reaction reactance***, since the *flux which causes it is around the armature turns only and does not affect the field directly*. This armature reaction flux is proportional to the armature currents, since the magnetic paths it covers is not normally saturated.

3.7 Phasor and Equivalent Circuit Diagrams of The *Cylindrical-Rotor* (Non-Salient Pole) Synchronous Generators

An elementary picture of how a synchronous generator works has already been given in the previous sections. In this section, analytical methods of examining the study-state performance of polyphase synchronous generator will be considered. Initial consideration will be given to non-salient pole (cylindrical-rotor) synchronous generators, with the effects of salient poles considered thereafter.

3.7.1 Phasor Diagrams of *Cylindrical-Rotor* Synchronous Generators

It will be recalled from Eqn (8) that balanced polyphase in a symmetrical polyphase winding create an mmf wave whose space-fundamental component rotates at synchronous speed. Recall also that the mmf wave is directly *opposite* say, arbitrarily chosen phase a, at the instant when the phase a current has its maximum.

When a synchronous generator is loaded, current flows in both armature and field windings, and they create mmf waves in the respective windings. The mmf wave created by the armature current is commonly called the *armature-reaction mmf*. The resultant magnetic field Φ_r in the machine is the sum of the two components produced by the field current Φ_f and the armature reaction Φ_{ar}. Because the fields are sinusoid, and sinusoids can conveniently be added by phasor methods, the airgap flux and mmf conditions in a synchronous machine can thus be represented by phasor diagrams.

The Fig below shows the space-phasor diagram for two situations of the armature current in phase with and lagging the excitation voltage in a synchronous generator.

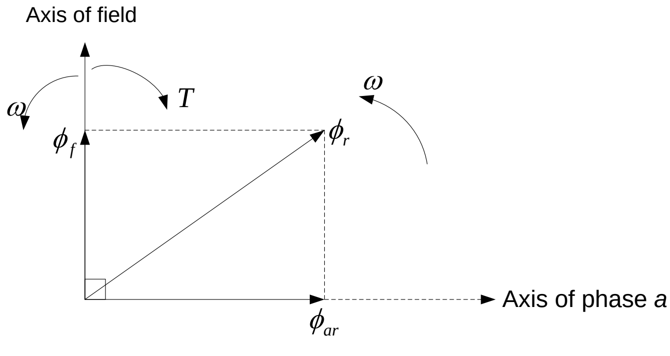


Fig (a)

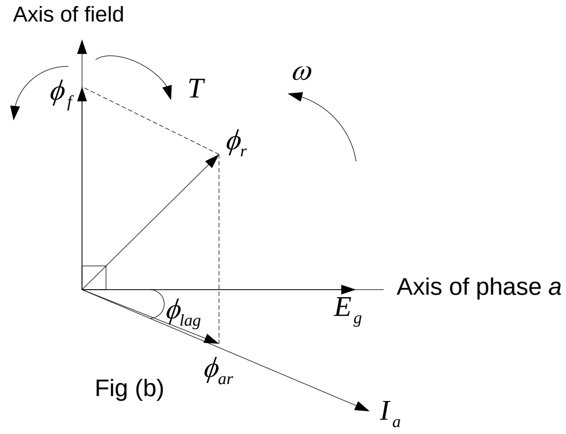


Fig (b)

*Fig: Phasor Diagram of Fluxes in Cylindrical-Rotor **Synchronous Generator***
*Armature current in phase with excitation (**unity** power factor)*
*Armature current lagging the excitation (**lagging** power factor)*

3.7.2 Equivalent Circuit of Cylindrical-Rotor Synchronous Generator

A very useful and simple equivalent representing the steady-sate behaviour of a cylindrical-rotor synchronous machine under balanced, polyphase conditions is obtained if the effect of the armature reaction flux is represented by an inductive reactance. For the start, let us consider an *unsaturated cylindrical-rotor* machine. Although neglect of magnetic saturation may appear to be a drastic simplification, it can be shown that the results obtained can be modified so as to take saturation into account.

As stated already, the resultant airgap flux in the machine can be considered as the phasor sum of the component fluxes created by the field and armature-reaction mmfs. From the viewpoint of the armature windings, these fluxes manifest themselves as generated emfs. The resultant airgap voltage E_r can then be considered as the ***phasor sum*** of the excitation or generated voltage E_g generated by the field flux and the voltage E_{ar} generated by the armature-reaction flux.

The component emfs E_g and E_{ar} are proportional to the field and armature currents respectively, and *each lags the flux which generates it by 90⁰*. The armature-reaction flux Φ_{ar} is in phase with the armature current I_a, and consequently the armature-reaction emf E_{ar} lags the armature current by 90^0. See Fig below.

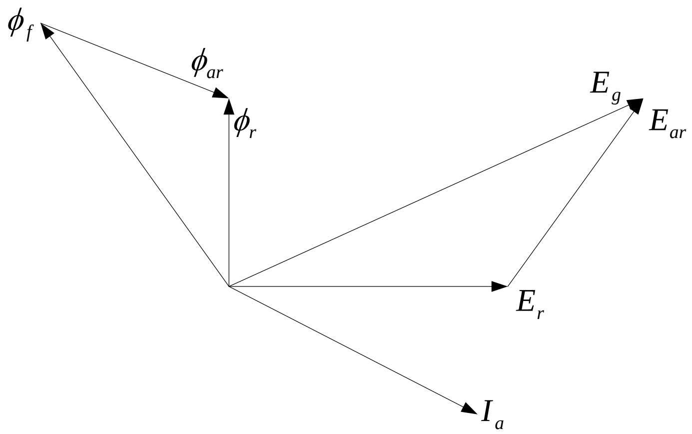


Fig: Phasor Diagram of Component Fluxes and Corresponding Induced Voltages

The effect of armature reaction is simply that of an inductive reactance X_{ar} (the *armature-reaction or magnetizing reactance*) accounting for the component voltage generated by the space-fundamental flux created by armature reaction. Part of the excitation or generated voltage E_g is used to overcome the voltage E_{ar} generated by the armature-reaction flux and the rest appears as the resultant airgap voltage E_r. Thus,

$$\begin{aligned} E_g &= E_r + E_{ar} \\ &= E_r + jI_a X_{ar} \end{aligned} \tag{15}$$

The resultant airgap voltage E_r differs from the terminal voltage V by the armature resistance R_a and leakage-reactance X_a drops. See Fig (a) below. The armature leakage reactance accounts for the voltages induced by the component fluxes which are not included in the airgap voltage E_r.

These fluxes include not only leakage across the armature slots and around the coil ends, but also those associated with the space-harmonic field effects created by the departure from a sinusoid necessarily present in actual armature mmf wave.

Finally, the equivalent circuit for an unsaturated non-salient-pole (cylindrical-rotor) machine under balanced polyphase conditions reduces to the form shown in Fig (b) below, in which the machine is represented on a per-phase basis by its excitation voltage E_g in series with a simple impedance.

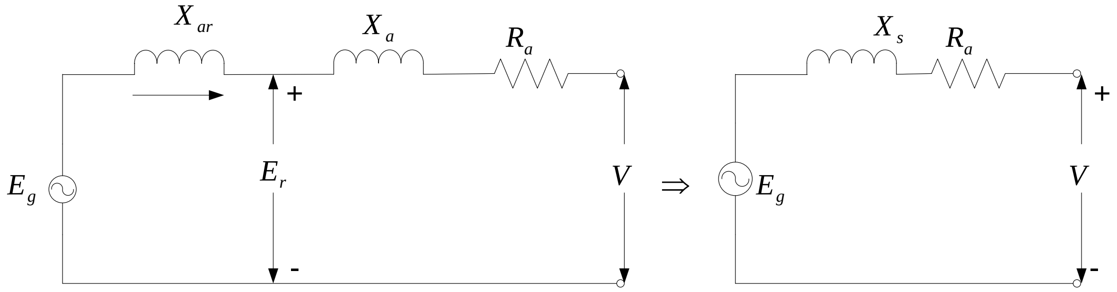


Fig: Equivalent Circuits of ***Cylindrical-Rotor*** *(Non-Salient Pole) Synchronous Machines*

3.7.3 Synchronous Reactance

This simple impedance is called *synchronous impedance*. Its reactance X_s is called the ***synchronous reactance***. In terms of the magnetizing and leakage reactances,

$$X_s = X_{ar} + X_a$$

$$\Rightarrow \quad Z_s = R_a + jX_s = R_a + j(X_a + X_{ar}) \qquad (16)$$

The synchronous reactance takes into account all the flux produced by balanced polyphase armature currents, while the excitation or generated voltage takes into account the flux produced by the field current. In an unsaturated cylindrical-rotor (non-salient pole) machine at constant frequency, the synchronous reactance is a constant. Furthermore, the ***excitation voltage is proportional to the field current*** and ***equals the voltage which would appear at the terminals if the armature were open-circuited, the speed and field current being held constant***.

The value of X_s is 10 to 100 times greater than R_a. Therefore we can neglect the resistance, unless we are interested in efficiency or heating.

4. STEADY-STATE PERFORMANCE CHARACTERISTICS OF THE SYNCHRONOUS MACHINE

Two basic sets of characteristic curves for a synchronous machine are involved in the inclusion of saturation effects and in the determination of the appropriate machine constants. The tests performed to obtain these characteristics are the ***open-circuit and short-circuit tests***. These tests yield the open-circuit and short-circuit characteristics, which are necessary for determining the performance of synchronous machines.

Except for a few remarks on the degree of validity of certain assumptions, the discussions apply to both non-salient-pole (cylindrical-rotor) and salient-pole machines.

4.1 Open-Circuit Characteristics

Like the magnetization curve for a d-c machine, the *open-circuit characteristic* of a synchronous machine is a curve of the armature terminal voltage on open circuit (or no-load) as a function of the field excitation, when the machine is running at synchronous speed. Essentially, the open-circuit characteristic represents the relation between the space-fundamental component of the airgap flux and the mmf on the magnetic circuit when the field winding constitutes the only mmf source.

The open-circuit characteristic is determined ***experimentally*** by driving the machine mechanically at rated synchronous speed as an *unloaded* generator, that is, with armature terminals on open circuit, and measuring the open-circuit terminal voltages for a series of field current values. The OCC is shown in the Fig below.

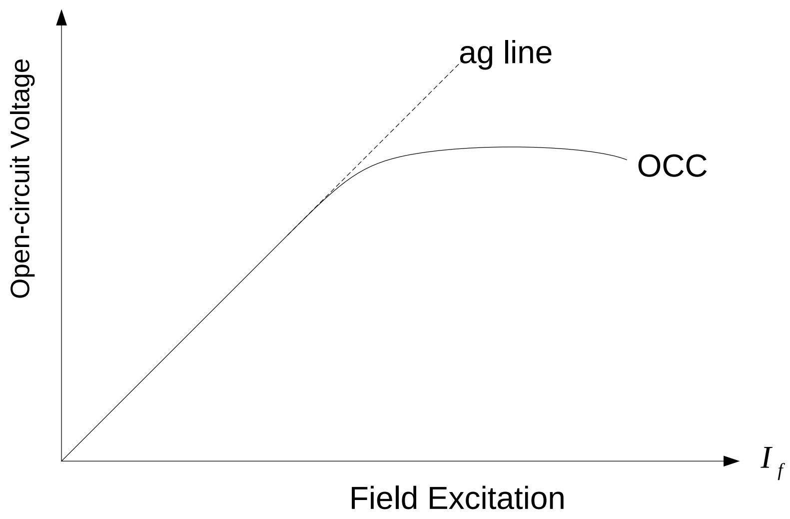


Fig: Open-Circuit Characteristic (OCC) of a Synchronous Machine

Since the generated voltage is proportional to the flux and therefore the flux density, the open-circuit characteristic (OCC) is, to some scale, just like the $B-H$ magnetization or hysteresis curve. The OCC may be plotted in per-unit terms, where unity voltage is the rated voltage and unity field current is the excitation corresponding to rated voltage on the airgap line.

The straight line tangent to the lower portion of the magnetization curve is called the *airgap line*. It indicates very closely the mmf required to overcome the reluctance of the airgap. *If it were not for the effects of saturation, the airgap line and OCC would coincide, so that the departure of the curve from the airgap line is an indication of the degree of saturation present*. In a normal machine, the ratio at rated voltage of the total mmf to that required by the airgap alone usually is between 1.10 and 1.25.

4.2 Short-Circuit Characteristics

The *short-circuit characteristic* (SCC) is obtained ***experimentally*** by short-circuiting the armature (i.e. stator) windings of a synchronous machine being driven as a generator at synchronous speed through low-resistance ammeters. The excitation or field current is gradually adjusted from its zero value until the armature current has reached a maximum safe value, about 1.5 to 2 times rated or full-load current.

Corresponding values of short-circuit armature current and field current are measured, and the SCC is drawn as shown in Fig below.

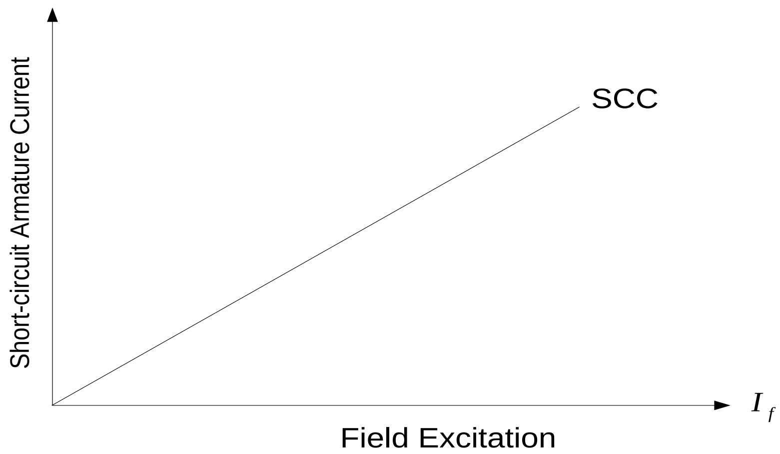


Fig: Short-Circuit Characteristic (SCC) of a Synchronous Machine

The SCC curve is ***linear*** because the armature reaction mmf F_{ar} is almost as large as the field excitation, so that the resultant excitation and flux are small and therefore the iron is not saturated.

The phasor relation between the excitation (or generated) voltage E_g and the steady-state armature current I_a under polyphase *short-circuit conditions* is:

$$\begin{aligned} E_g &= I_a(R_a + jX_s) \\ &= I_{sc} Z_s \end{aligned} \tag{17}$$

Because the resistance R_a is much smaller than the synchronous reactance X_s, the armature current I_a lags the excitation voltage E_g by very nearly 90^0. Consequently, the armature-reaction mmf wave is very nearly in line with the axis of the field poles and in opposition to the field mmf.

The resultant mmf, obtained from the phasor sum of the field and armature-reaction mmfs, creates the resultant airgap flux wave which generates the airgap voltage E_r equal to the voltage consumed in armature resistance R_a and leakage reactance X_a. As an equation,

$$E_r = I_a(R_a + jX_a) \tag{18}$$

In most synchronous machines, the armature resistance is negligible, and the leakage reactance is about 0.15 p.u., that is, at rated armature current, the leakage-reactance voltage drop $I_a X_a$ is about 0.15 p.u. And so from Eqn (18), the airgap voltage E_r at rated armature current on short-circuit is about 0.15 p.u., that is to say, the resultant airgap flux is only about 0.15 of its normal-voltage value. Consequently, the machine is operating in an *unsaturated* condition. ***The short-circuit armature current therefore is directly proportional to the field current over the range from zero to well above rated armature current.***

From Eqn (17), for the same field current,

$$Z_s = \left.\frac{E_g}{I_{sc}}\right|_{\text{for same field current } I_f} \tag{19}$$

This impedance is called ***unsaturated synchronous impedance***. At low excitations, the OCC is linear and the unsaturated synchronous impedance is constant. But at high excitations, the OCC is nonlinear and the unsaturated synchronous impedance is not constant. The unsaturated synchronous impedance can be found from the open-circuit and short-circuit data. See Fig below.

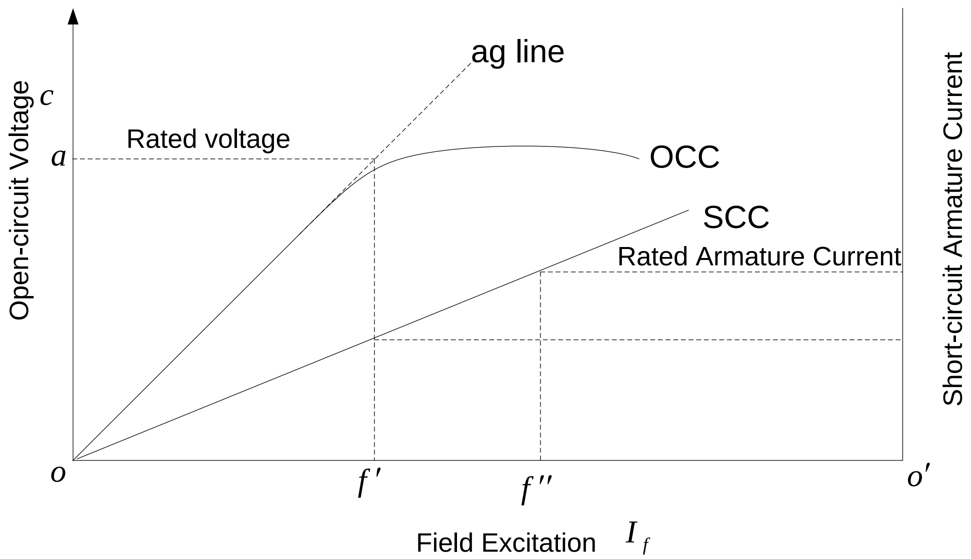


Fig: Typical Open-Circuit and Short-Circuit Characteristics of Synchronous Machine

4.3 Determination of the armature resistance R_a

The armature resistance R_a per phase can be measured directly by voltmeter and ammeter method or by using Wheatstone bridge. However, under working conditions, the ***effective*** value of R_a is increased due to skin effect. The value of R_a so obtained by direct measurement is increased by 60 % or so to allow for this effect. Generally, an *effective value of 1.6 times the DC value* R_{dc} *is taken*.

Example 1

The following test results were obtained from a 3-phase, 6000 kVA, 66 kV star-connected, 2-pole, 50 Hz turbo alternator. With a field current of 125 A, the open circuit voltage is 8000 V at rated speed. With the same excitation and rated speed, the short-circuit current was 800 A. If at the rated full-load, the resistance drop is 3%, determine the following:

i. synchronous impedance Z_s
ii. armature resistance R_a
iii. synchronous reactance X_s

Solution 1

i. The synchronous impedance is calculated using the equation

$$Z_s = \left.\frac{E_{oc}}{I_{sc}}\right|_{\text{for same field current } I_f} = \frac{8000/\sqrt{3}}{800} = \underline{\underline{5.77\,\Omega}}$$

ii. The voltage per phase is $E_{oc} = 8000/\sqrt{3} = \underline{3810V}$

Full load current, $I = \dfrac{S_{3ph}}{V_L\sqrt{3}} = \dfrac{6000}{6.6\times\sqrt{3}} = \underline{525\,A}$

Resistance drop, $IR_a = 3\% \times E_{oc} = 0.03\times 3810 = \underline{114.3V}$

Hence $R_a = 114.3/525 = \underline{\underline{0.218\,\Omega}}$

iii. Synchronous reactance , $X_s = \sqrt{{Z_s}^2 - {R_a}^2} = \sqrt{5.77^2 - 0.218^2} = \underline{\underline{5.74\,\Omega}}$

Example 2

A three-phase 50 Hz star-connected 2000 kVA, 2300 V alternator gives a short-circuit current of 600 A for a certain field excitation. With the same excitation, the open circuit voltage was 900 V. The resistance measured between a pair of terminal was 0.12 Ω. Determine the following:

i. synchronous impedance Z_s
ii. armature resistance R_a
iii. synchronous reactance X_s

Solution 2

i. The synchronous impedance is calculated using the equation

$$Z_s = \left.\frac{E_{oc}}{I_{sc}}\right|_{\text{for same field current } I_f} = \frac{900/\sqrt{3}}{600} = \underline{\underline{0.866\Omega}}$$

ii. Resistance between the pair of terminals is 0.12 Ω. It is the resistance between two phases *connected in series*. Thus measured resistance per phase 0.12/2 = 0.06 Ω.

Hence ***effective*** armature resistance, $R_a = 1.6 \times R_{measured} = 1.6 \times 0.06 = \underline{\underline{0.096\Omega}}$

iii. Synchronous reactance , $X_s = \sqrt{{Z_s}^2 - {R_a}^2} = \sqrt{0.866^2 - 0.096^2} = \underline{\underline{0.86\Omega}}$

4.4 Output Power Delivered By *Cylindrical-Rotor* Synchronous Generator

Consider a simple circuit of a cylindrical-rotor synchronous generator of excitation voltage E_g supplying power to a bus of voltage V_{bus} through an impedance Z such that current I flows. The phasor diagram is also shown below.

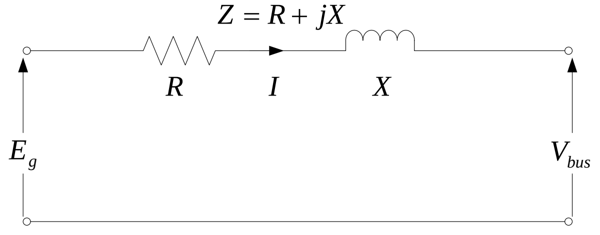


Fig (a)

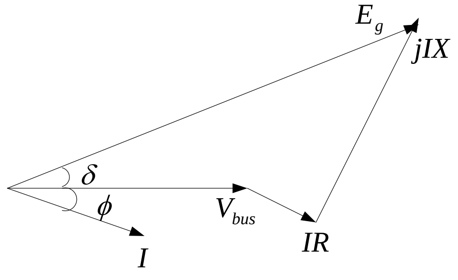


Fig (b)

Fig: (a) Circuit Diagram (b) Phasor Diagram

The power P_2 delivered to the load or bus end V_{bus} is

$$P_2 = V_{bus} I \cos\phi \tag{20}$$

where ϕ is the phase angle of the current I with respect to V_{bus}. The phasor current is

$$I = \frac{E_g - V_{bus}}{Z} \tag{21}$$

If the phasor voltages and impedance are expressed in polar forms,

$$I = \frac{E_g\angle\delta - V_{bus}\angle 0}{Z\angle\phi_z} \quad = \quad \frac{E_g}{Z}\angle(\delta - \phi_z) \; - \; \frac{V_{bus}}{Z}\angle(-\phi_z) \tag{22}$$

where δ is the phase angle (called *power angle*) by which E_g leads V_{bus}, Z is the magnitude of the impedance.

The ***real part*** of the phasor Eqn (22) is the component of I in phase with V_{bus}.

Hence

$$\begin{aligned} I\cos\phi &= \mathrm{Re}\,al\ part\left[\frac{E_g}{Z}\angle(\delta - \phi_z) \; - \; \frac{V_{bus}}{Z}\angle(-\phi_z)\right] \\ &= \frac{E_g}{Z}\cos(\delta - \phi_z) - \frac{V_{bus}}{Z}\cos(-\phi_z) \end{aligned} \tag{23}$$

We note that $\cos(-\phi_z) = \cos(\phi_z) = R/Z$. Substituting Eqn (23) into (20), we obtain,

$$\begin{aligned} P_2 &= V_{bus} I \cos\phi \\ &= V_{bus} \cdot \left\{\frac{E_g}{Z}\cos(\delta - \phi_z) - \frac{V_{bus}}{Z}\cos(-\phi_z)\right\} \\ &= \frac{E_g V_{bus}}{Z}\cos(\delta - \phi_z) - \frac{V_{bus}{}^2 R}{Z^2} \\ &= \frac{E_g V_{bus}}{Z}\sin(\delta + \alpha_z) - \frac{V_{bus}{}^2 R}{Z^2} \end{aligned} \tag{24}$$

where $\alpha_z = \tan^{-1}\dfrac{R}{X} = 90^0 - \phi_z$ (25)

and is usually a small angle.

Similarly, the power P_1 at the source end E_g of the impedance can be expressed as

$$P_1 = \frac{E_g V_{bus}}{Z}\sin(\delta - \alpha_z) + \frac{{V_{bus}}^2 R}{Z^2} \tag{26}$$

If, as is frequently the case, the ***resistance is assumed negligible***, then

$$P_1 = P_2 = P_{real} = \frac{E_g V_{bus}}{X}\sin\delta \tag{27}$$

4.5 Maximum Power Output From Cylindrical Rotor Alternator

There is a maximum output that an alternator is capable of delivering for given values of terminal voltage, frequency and excitation. For a cylindrical rotor, if the resistance is negligible (in which case the IR_a drop is neglected), and the synchronous reactance X_s and voltages V and E_g are constant (of course E_g is fixed by excitation), then the maximum power per phase occurs at $\delta = 90^0$.

Then from Eqns (26) and (27), the maximum power per phase is given as:

$$\begin{aligned} P_{\max} &= \frac{E_g V}{X_s} && \text{if } R_a \text{ is neglected} \\ &= \frac{V}{Z_s}\left(E_g - V\cos\alpha\right) && \text{if } R_a \text{ is considered} \\ \text{where } \cos\alpha &= \frac{R_a}{Z_s} \end{aligned} \tag{28}$$

Example 3

A 3-phase 11 kV 5 MVA star-connected alternator has a synchronous impedance of $(1 + j12)\ \Omega$ per phase. Its excitation is such that the generated line emf is 12 kV. If the alternator is connected to an infinite busbar, determine the maximum output at the given excitation.

Solution 3

The generated phase voltage is $E_g = \frac{12000}{\sqrt{3}} = \underline{6928V}$

The terminal voltage per phase is $V = \frac{11000}{\sqrt{3}} = \underline{6351V}$

Since the armature resistance is NOT negligible, we need to calculate the internal angle $\cos\alpha = \frac{R_a}{Z_s} = \frac{1}{\sqrt{1^2 + 12^2}} = 0.083$

Hence the maximum power output per phase is

$$
\begin{aligned}
P_{\max} &= \frac{V}{Z_s}\left(E_g - V\cos\alpha\right) \\
&= \frac{6351}{\sqrt{1^2 + 12^2}}\left(6928 - 6351 \times 0.083\right) \\
&= 3375.88\,kW
\end{aligned}
$$

$$\Rightarrow Total\ P_{\max} = 3 \times 3375.88 = \underline{\underline{10127.64\,kW}}$$

4.6 Phasor and Equivalent Diagrams of *Salient-Pole Rotor* Synchronous Generator (*Two-Axis or Two-Reactance Theory to Account for Saliency*)

The effect of salient poles is taken into account in the so-called ***two-axis or two-reactance theory***, by ***resolving the armature current or the sinusoidal armature flux into two perpendicular components***, one in time quadrature (say Φ_{aq}) and the other in time phase (say Φ_{ad}) with ***excitation voltage E_g taken as the reference or d-axis***. Each of the fluxes Φ_{ad} and Φ_{aq} may then be thought of as inducing its own voltage E_d and E_q respectively.

4.6.1 Phasor Diagram of *Salient-Pole* Synchronous Generator

This diagram is drawn for an unsaturated salient-pole generator operating at a lagging power factor.

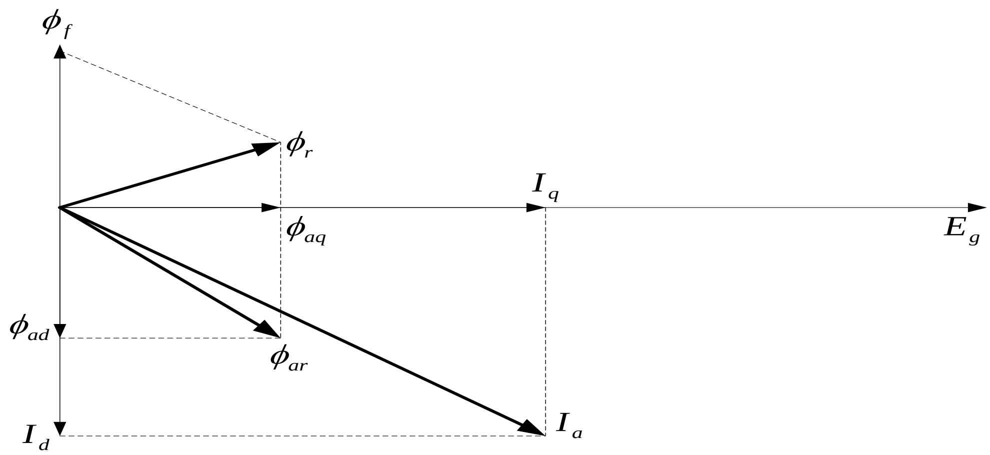


*Fig: Phasor Diagram of Salient-Pole Synchronous Generator (**Lagging pf**)*

The d-component I_d of the armature current, in time quadrature with the excitation voltage, produces its component fundamental armature-reaction flux Φ_{ard} along the axes of the field poles. On the other hand, the q-component I_q of the armature current, in phase with the excitation voltage, produces its component fundamental armature-reaction flux Φ_{arq} in space quadrature with the field poles.

A *direct-axis* quantity is one whose magnetic effect is centered on the axes of the field. Direct-axis mmfs act on the main magnetic circuit.

A *quadrature-axis* quantity is one whose magnetic effect is centered on the interpolar space. For an unsaturated machine, the armature-reaction flux Φ_{ar} is thus the phasor sum of the components Φ_{ard} and Φ_{arq}.

The *resultant flux* Φ_r is then the phasor sum of the armature-reaction flux Φ_{ar} and the main field flux Φ_f.

Thus

$$\Phi_r = \Phi_{ar} + \Phi_f \quad , \qquad \text{where} \quad \Phi_{ar} = \Phi_{ard} + \Phi_{arq}$$
$$= \Phi_{ard} + \Phi_{arq} + \Phi_f$$

4.6.2 Equivalent Circuit of *Salient-Pole* Synchronous Machines

With each of the armature component currents I_d and I_q, there is associated a component synchronous reactance voltage drop $jI_d X_d$ and $jIqX_q$ respectively, where the reactances X_d and X_q are, respectively, called the ***direct- and quadrature axis synchronous rectances***.

Principally, the synchronous reactance accounts for the inductive effects of all the fundamental-frequency-generating fluxes created by the armature currents, including both armature leakage and armature-reaction fluxes. Thus the inductive effects of the direct- and quadrature axis armature-reaction flux waves Φ_{ard} and Φ_{arq} can be accounted for by the equivalent d-axis and q-axis magnetizing reactances X_{ard} and X_{arq} respectively, similar to the magnetizing armature-reaction reactance X_{ar} of the non-salient-pole *(cylindrical-rotor)* theory.

Expressed mathematically, the direct- and quadrature-axis synchronous reactances X_d and X_q are given as

$$\begin{aligned} X_d &= X_a + X_{ard} \\ X_q &= X_a + X_{arq} \end{aligned} \qquad (29)$$

where

X_{ard} = d-axis armature-reaction reactance (due to I_d or Φ_{ard})
X_{arq} = q-axis armature-reaction reactance (due to I_q or Φ_{arq})
X_a = armature leakage reactance and is assumed to be the same for both d-axis and q-axis currents.
X_d = d-axis synchronous reactance
X_q = q-axis synchronous reactance

Note: *The synchronous reactance expression of Eqn (29), deduced from the salient-pole two-reactance theory, compares with that of Eqn (16), deduced from the cylindrical-rotor theory*.

The generated voltage E_g equals the terminal voltage V on no-load. If the armature resistance is introduced, then the excitation voltage must equal the terminal voltage V plus the armature resistance drop $I_a R_a$ and the component synchronous-reactance drops due to the d- and q-axis armature currents. Thus

$$E_g = V + I_a R_a + jI_d X_d + jI_q X_q \tag{30}$$

See Fig below for the phasor representation of Eqn (30)

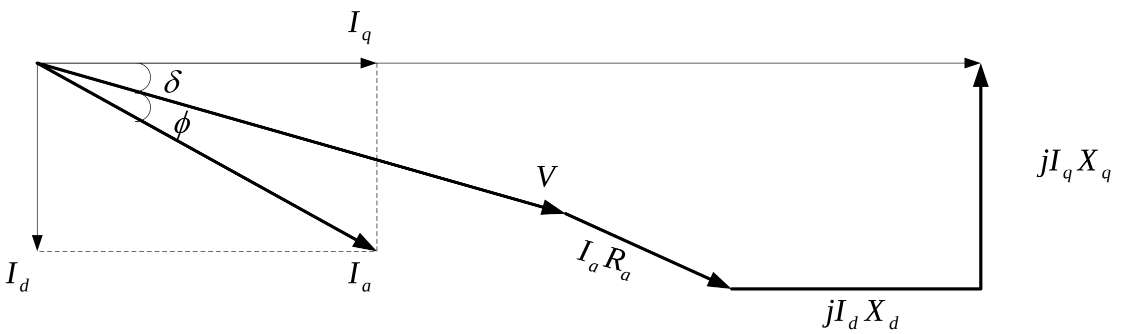


Fig: Phasor Diagram of Salient-Pole Synchronous Generator

The q-axis synchronous reactance X_q is less than the d-axis synchronous reluctance X_d because of the greater reluctance of the airgap in the quadrature axis. Usually, X_q is between 0.6 X_d and 0.7 X_d.

Note that a small salient-pole effect is present in turbo-alternators, even though they are cylindrical-rotor machines, because of the effect of the rotor slots on the quadrature-axis reluctance.

4.6.3 Relations Among Component Voltages In Phasor Diagram For *Salient-Pole* Synchronous Generators Operating at *Lagging* Power factor

In using or drawing the phasor diagram of the salient-pole synchronous generator in the Fig above, the armature current must be resolved into its d-axis and q-axis components. *This resolution assumes that the phase angle ($\phi + \delta$) of the armature current* ***with respect to*** *the excitation voltage is known*. Often, however, the phase angle (power-factor angle) ϕ at the machine terminals is explicitly known, whilst the power angle δ (i.e. angle between the excitation voltage E_g and terminal voltage V) is not known and must be calculated.

The phasor diagram above is repeated by the solid-line phasors in the Fig below.

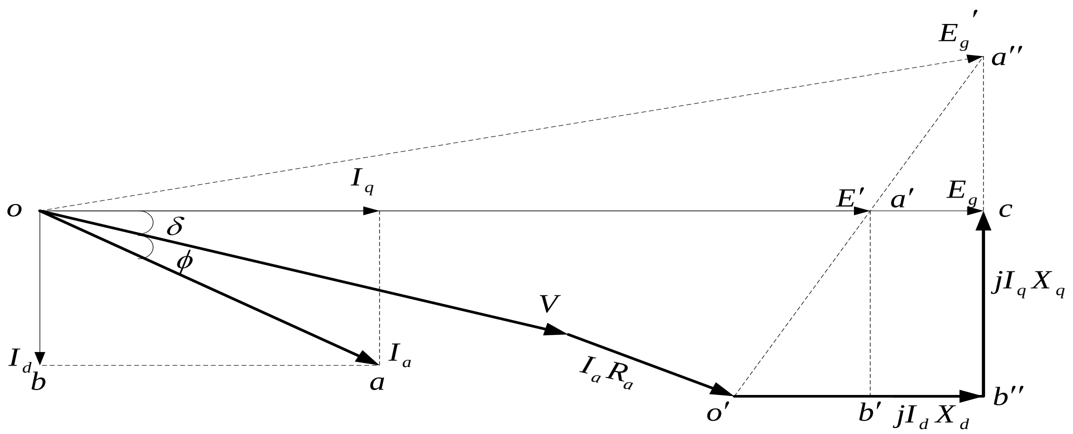


Fig: Relations Among Component Voltages in Phasor Diagram of Salient-Pole Synchronous Generator Operating at Lagging Power Factor

The study of the diagram shows that the dashed phasor $o'a'$, perpendicular to the armature current I_a, equals jI_aX_q. This result follows geometrically from the fact that triangles $o'a'b'$ and oab are *similar, because their corresponding sides are perpendicular*.

Thus using the ***properties*** of ***similar triangles***,

$$\frac{o'a'}{oa} = \frac{b'a'}{ba}$$
$$\Rightarrow o'a' = \frac{b'a'}{ba}oa \quad = \frac{jI_qX_q}{I_q}I_a \quad = jI_aX_q \tag{31}$$

Similarly, other relations are obtained.

$o'a' = jI_aX_q$	$o'b' = jI_dX_q$	$a'c = jI_d(X_d - X_q)$
$b'a' = b''c = jI_qX_q$	$o'b'' = jI_dX_d$	$o'a'' = jI_aX_d$

The phasor sum $V + I_a R_a + jI_a X_q$ then locates the angular position of the excitation voltage E_g and therefore the d- and q-axes. Physically, this must be so, because all the field excitation in a normal machine is in the direct axis.

One use of these relations in determining the excitation requirements for specified operating conditions at the terminals of a salient-pole machine is illustrated in the following example.

Example 4:

The reactances X_d and X_q of a salient-pole synchronous generator are 1.00 and 0.60 p.u., respectively. The armature resistance is negligible. Compute the excitation voltage when the generator delivers rated kVA at 0.80 pf lagging current, and rated terminal voltage.

Solution 4:

The power factor angle ϕ is known but the power angle δ is unknown. And so first of all, the phase ($\phi + \delta$) of the excitation voltage E_g must be found, so that the armature current I_a can be resolved into its d-axis and q-axis components.

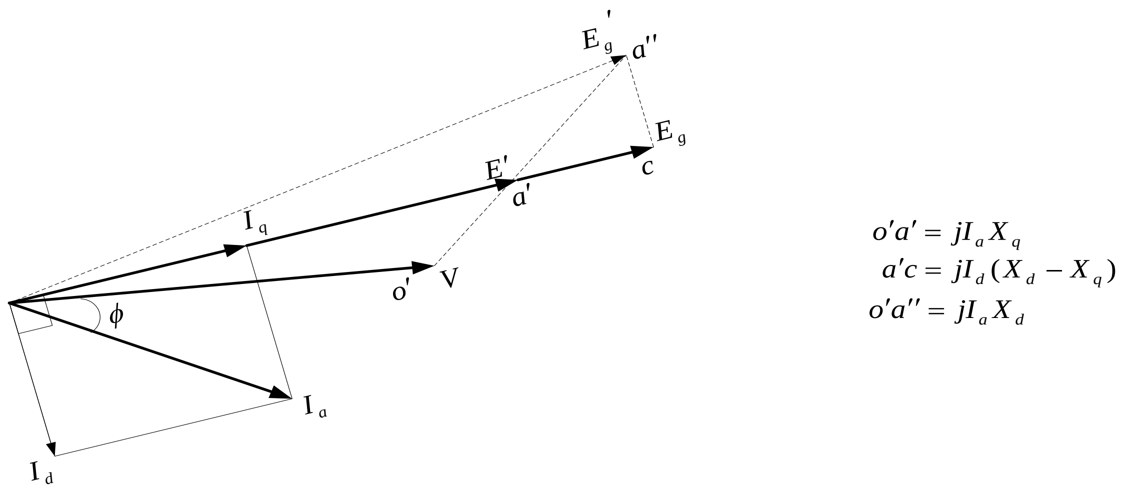


Fig: Salient-Pole Generator Phasor Diagram for Example 1

The armature current is given as:

$$I_a = 0.80 - j0.60 = 1.00\angle -36.9^0\ p.u. \qquad (32)$$

With the terminal voltage as the reference phasor, $V = 1\angle 0^0 = 1 + j0$, and the phasor sum E' in the phasor diagram is:

$$E' = V + jI_a X_q$$
$$= (1 + j0) + j(0.80 - j0.60)(0.60)$$
$$= 1.36 + j0.48$$
$$= 1.44\angle 19.4^0 \ p.u.$$

The power angle δ = 19.4^0 and so the phase angle between the excitation voltage E_g and the armature current I_a is $(\phi + \delta)$ = (36.9^0 + 19.4^0) = 56.3^0.

The armature current can now be resolved into its d- and q-axis components. Their *magnitudes* are

$$|I_d| = |I_a|\sin(\phi + \delta) = 1.00\sin 56.3^0 = 0.832$$
$$|I_q| = |I_a|\cos(\phi + \delta) = 1.00\cos 56.3^0 = 0.555 \qquad (33)$$

As *phasors*,

$$I_d = |I_d|\angle(-90^0 + 19.4^0) = 0.832\angle -70.6^0$$
$$I_q = |I_q|\angle(19.4^0) = 0.555\angle 19.4^0 \qquad (34)$$

We can now find the excitation voltage E_g by simply adding ***numerically*** the length $a'c = I_d(X_d - X_q)$ to the magnitude of E'.

Thus the ***magnitude*** of the excitation voltage is the ***algebraic sum***

$$E_g = E' + I_d(X_d - X_q)$$
$$= 1.44 + (0.832)(1.00 - 0.60) \qquad (35)$$
$$= 1.77 \, p.u.$$

As a *phasor*, $E_g = 1.77\angle 19.4^0 \ p.u.$

4.6.4 Output Power Delivered by *Salient-Pole* Synchronous Generators

The discussion will be limited to a simple system shown in the schematic diagram below comprising a salient-pole machine SM connected to an ***infinite bus*** (e.g. a large power system network) of voltage V_{net} through a series impedance (e.g. a transmission line) of reactance X_{ser} per phase. Resistance is neglected.

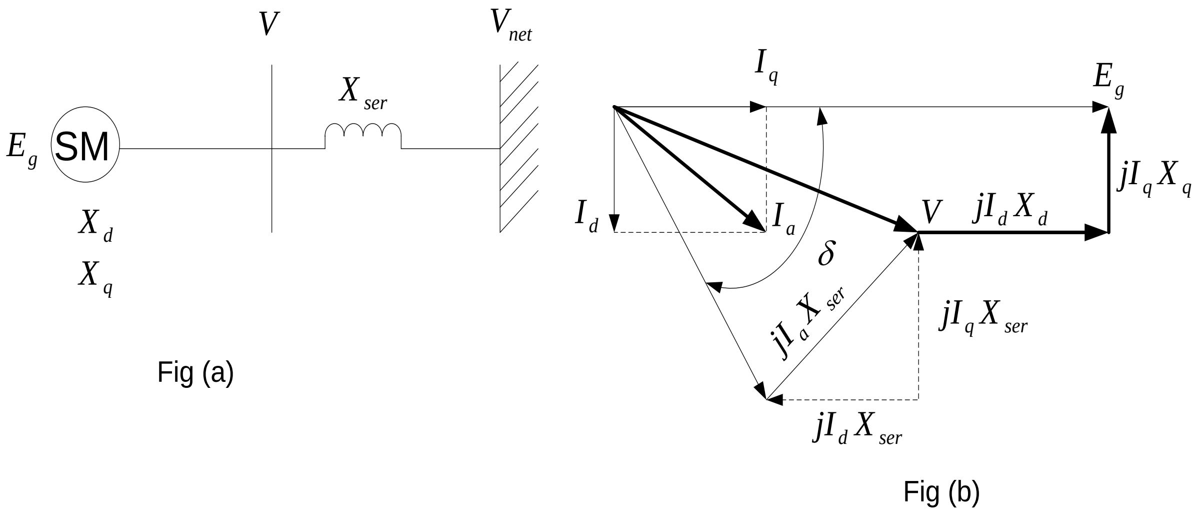


Fig: Salient-Pole Synchronous Machine and Series Impedance (a) Single-Line Diagram (b) Phasor Diagram

The dashed phasors show the external reactance drop resolved into components due to I_d and I_q. The effect of the external impedance is merely to add its reactance to the reactances of the machine; i.e., the *total* values of reactance interposed between the excitation voltage E_g and the infinite bus voltage V_{net} are

$$X_{dT} = X_d + X_{ser} \tag{36}$$
$$X_{qT} = X_q + X_{ser} \tag{37}$$

If the infinite bus voltage V_{net} is resolved into components $V_{net}\sin\delta$ and $V_{net}\cos\delta$ ***in phase*** with I_d and I_q respectively, then the active power P delivered to the bus per phase is

$$P = I_d V_{net}\sin\delta + I_q V_{net}\cos\delta \tag{38}$$

Also, from the Fig (b) above,

$$E_g - V_{net}\cos\delta = I_d X_{ser} + I_d X_d = I_d(X_{ser} + X_d)$$
$$= I_d X_{dT}$$

Hence $I_d = \dfrac{E_g - V_{net}\cos\delta}{X_{dT}}$ (39)

Similarly,

$$V_{net}\sin\delta = I_q X_q + I_q X_{ser} = I_q(X_{ser} + X_d)$$
$$= I_q X_{qT}$$

Hence $I_q = \dfrac{V_{net}\sin\delta}{X_{qT}}$ (40)

Substituting Eqns (39) and (40) into Eqn (38) gives

$$P = \frac{E_g V_{net}}{X_{dT}} \sin\delta + V_{net}^{\ 2} \frac{X_{dT} - X_{qT}}{2X_{dT} X_{qT}} \sin 2\delta \qquad (41)$$

NB: *Eqn (41) applies to all possible combinations of a synchronous machine and an external system, so long as the resistance is negligible***.**

This power-angle characteristic is shown in the Fig below.

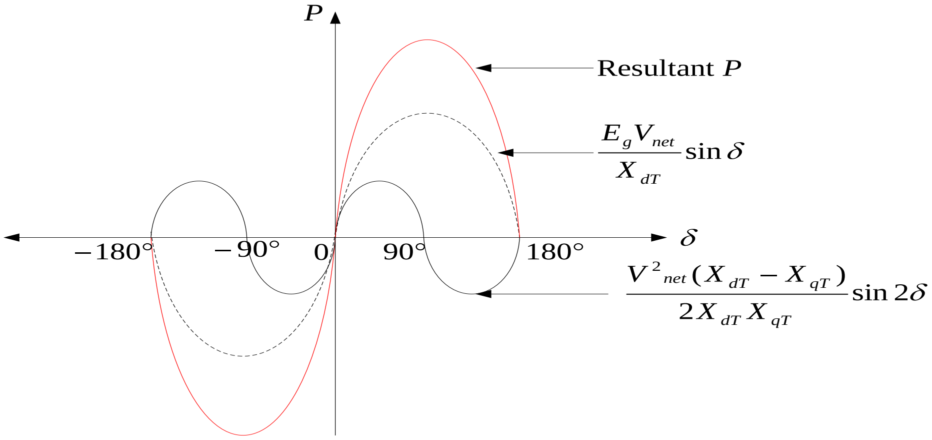


Fig: Power-Angle Characteristic of a ***Salient-Pole*** *Synchronous Machine Showing Fundamental Component Due to Field Excitation and Second Harmonic Component Due to Reluctance Torque*

The first term of Eqn (41) is the same as the expression obtained for a cylindrical-rotor machine. This term is merely an extension of the concept to include the effects of series reactance. The second term introduces the effects of salient poles. It represents the fact that the airgap flux wave creates torque tending to align the field poles in the position of minimum reluctance. This term is the power corresponding to the ***reluctance torque***.

Note that the reluctance is independent of field excitation. Note also that if $X_d = X_q$ *as in a* ***uniform-airgap*** *machine, there is no preferential direction of magnetization, the reluctance torque is zero, and Eqn (41) reduces to the power-angle equation for a cylindrical rotor machine whose synchronous reactance is* X_d.

Because of the reluctance torque, a salient-pole machine is stiffer than one with a cylindrical rotor, i.e., for equal voltages and equal values of X_d, a salient-pole machine develops a given torque at a smaller value of δ, and the maximum torque which can be developed is somewhat greater.

For a normally excited machine, the effect of salient poles usually amounts to a few percent. Only at small excitations does the reluctance torque become important. ***Except at small excitations or when exceptionally accurate results are required, a salient-pole machine usually can be treated by simple cylindrical-rotor theory***.

Example 5:

A 1500 kVA, star-connected, 2300 V, three-phase salient pole synchronous generator has reactances X_d =1.95 ohms and X_q =1.40 ohms per phase. All losses may be neglected. Find the excitation voltage for operation at rated kVA and power factor of 0.85 lagging.

Solution 5:

The terminal ***phase*** voltage is given as

$$V = \frac{2300}{\sqrt{3}} = 1328\ V$$

The rated armature current *per phase* is given as

$$I_a = \frac{S_{ph}}{V} = \frac{(1500 \times 10^3)/3}{1328} = 377\ A$$

Using the terminal voltage as reference

$$V = 1328\angle 0^0 = (1328 + j0)\,V$$

The ***per phase*** armature current is then

$$\begin{aligned} I_a &= 377\angle(-\cos^{-1} 0.85^0) \\ &= 377\angle -31.8^0 \\ &= (320 - j199)\ A \end{aligned}$$

The phasor sum E' is given as

$$\begin{aligned} E' &= V + jI_a X_q \\ &= (1328 + j0) + j(320 - j199)(1.40) \\ &= 1607 + j448 \\ &= 1668\angle 15.6^0\ V \end{aligned}$$

Hence the power angle δ = 15.6^0, and so the phase angle φ between the excitation voltage E_g and the per phase armature current I_a is

φ = ($\phi + \delta$), when ϕ is lagging, = (15.6^0 + 31.8^0) = 47.4^0, The armature current can now be resolved into its d- and q-axis components. Their magnitudes are

$$\left|I_d\right| = \left|I_a\right| \sin(\phi + \delta) = 377 \sin 47.4^0 = 278\,A$$
$$\left|I_q\right| = \left|I_a\right| \cos(\phi + \delta) = 377 \cos 47.4^0 = 255\,A$$

As phasors, the d- and q-axis components of the armature currents are

$$I_d = \left|I_d\right| \angle(-90^0 + 15.6^0) = 278\angle -74.4^0$$
$$I_q = \left|I_q\right| \angle(15.6^0) = 255\angle 15.6^0$$

We can now find the excitation voltage E_g by simply adding ***numerically*** $I_d(X_d - X_q)$ to the magnitude of E'. Thus the ***magnitude*** of the excitation voltage is the ***algebraic sum***

$$\begin{aligned} E_g &= E' + I_d(X_d - X_q) \\ &= 1668 + (278)(1.95 - 1.40) \\ &= 1821 V \end{aligned}$$

As a *phasor*, $E_g = \underline{\underline{1821\angle 15.6^0 V}}$ (with reference to the terminal voltage).

Example 6:

A 36 MVA, 21 kV, 1800 rev/min alternator has a synchronous reactance of 5 ohm per phase. If the excitation voltage is 12 kV (line-to-neutral) and the output voltage is 17.3 kV (line-to-line), calculate the power delivered by the machine when the torque angle is 30°.

Solution 6:

Power delivered per phase is

$$P = \frac{VE_g}{X_s} \sin\delta = \frac{12\,(17.3/\sqrt{3})}{5} \sin 30^o = 12 \text{ MW}$$

Therefore the ***total power*** delivered by all the three phases = 3 x 12 = <u>36 MW</u>

4.7 Voltage Regulation and Load Excitation

It is a known fact that with a change in load, there is a change in terminal voltage of an alternator. The ***magnitude of this change depends not only on the load but also on the load power factor***.

The *voltage regulation* of a synchronous generator is defined as the *ratio of the change (actually a rise) in voltage when full load at rated voltage and a given power factor is removed from the machine, to the rated terminal voltage, the field excitation and speed remaining constant*.

Thus if V is the rated voltage, and the terminal voltage becomes E_0 when full-load is thrown off, then the per-unit voltage regulation is given as

$$VR = \frac{E_0 - V}{V} \tag{42}$$

Notes on Eqn (42).

i. ($E_0 - V$) is the *arithmetic differenc*e and not the vector difference.
ii. For a *leading* load power factor, the terminal voltage will fall on removing the full-load. Hence regulation is *negative* in that case.
iii. The rise in voltage when full-load is thrown off is not the same as the fall in voltage when full-load is applied.

4.7.1 Determination of Voltage Regulation

The voltage regulation of a generator depends on the armature resistance and synchronous reactance and power factor.

<u>Small Machines</u>:

In the case of small machines, the voltage regulation is found by *direct loading*. The ***procedure*** is as follows: The alternator is driven at synchronous speed and the terminal voltage is adjusted to its rated value V. The load is varied until the wattmeter and ammeter (connected for the purpose) indicate the rated values at desired power factor. Then the entire load is thrown off while the speed and field excitation are kept constant. The open-circuit or no-load voltage E_0 is read. The voltage regulation can thus be determined from the formula $VR = \frac{E_0 - V}{V}$.

Large Machines:

For large machines, the cost of finding the voltage regulation by direct loading is prohibitive. Hence other *indirect methods* are used as described below. It must be pointed out that all these methods differ chiefly in the way the no-load voltage E_0 is found in each case.

The indirect methods employed for determining the voltage regulation and load excitation of large machines include the following:

i. Synchronous Impedance or EMF Method (due to Behn Eschenberg)
ii. Ampere-Turn or MMF Method (due to Rothert)
iii. Zero Power Factor or Potier Method (as name implies, due to Potier)

All these methods require the determination of

a. armature (or stator) resistance R_a
b. open-circuit/no-load characteristic (OCC) and
c. short-circuit characteristic SCC.

The value of the armature resistance per phase can be measured directly by the voltmeter and ammeter method or by using the Wheatstone Bridge method. However under working conditions, the effective value of R_a is increased due to the so-called ***skin effect***, and so the value of R_a so obtained is increased by 60% to account for the skin effect. Generally, a value 1.6 times the d-c value is taken.

4.7.2 Synchronous Impedance or EMF Method:

The general data required in this method are the OCC, SCC and the synchronous impedance which is determined in the following procedure.

I. The O/C test is performed by running the machine on no-load and noting the values of induced voltage and filed excitation. The OCC is then plotted from data obtained. It is just like the B-H curve.

II. Similarly, the SCC is drawn from the data given by the S/C test. It is a straight line passing through the origin.

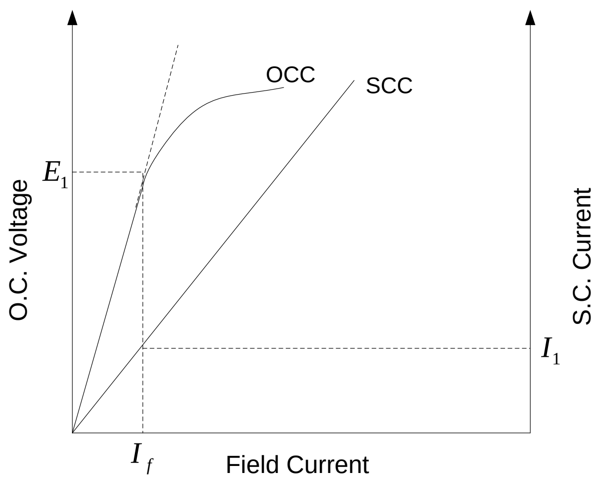


Fig (a)

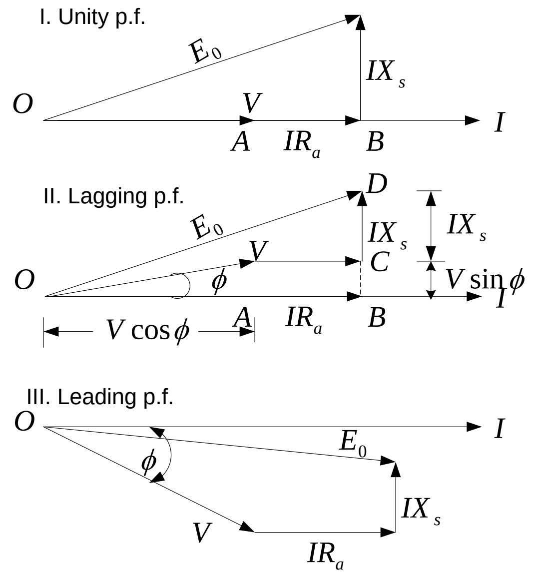


Fig (b)

Fig: Synchronous Impedance or EMF Method of Determining Voltage Regulation

Both these curves are drawn on a *common field-current base* as shown below in Fig (a) and its vector diagram in Fig (b).

III. Consider a field current I_f. The O/C voltage corresponding to this field current is, say, E_1, and I_1 is the corresponding short-circuit current obtained on the SCC. When the winding is shorted-circuited, the terminal voltage is zero. Hence it may be assumed that the whole of this voltage E_1 is used to circulate the armature short-circuit current I_1 against the synchronous impedance Z_s.

Thus $E_1 = I_1 Z_s \quad \Rightarrow \quad Z_s = \dfrac{E_1(open-circuit)}{I_1(short-circuit)}$ (43)

IV. Since the armature resistance R_a can be found as described earlier, the synchronous reactance is obtained as $X_s = \sqrt{(Z_s^2 - R_a^2)}$

V. Knowing R_a and X_s, the vector diagram of Fig (b) can be drawn for any load and any power factor.

From it, the **no-load generated voltage** is determined as

$$E_0 = \sqrt{\left(OB^2 + BD^2\right)} = \sqrt{\left\{(OA + AB)^2 + (BC + CD)^2\right\}}$$

or

$$E_0 = \sqrt{\left(V\cos\phi + IR_a\right)^2 + \left(V\sin\phi \pm IX_s\right)^2} \qquad (44)$$

$+$ sign : lagging pf

$-$ sign : leading pf

$\phi = 0$: unity pf

Hence the voltage regulation is obtained as $VR = \frac{E_0 - V}{V}$. The internal voltage E_g behind synchronous impedance is taken as the no-load voltage. The method is referred to as the ***saturated synchronous impedance method***. Practical experience has shown and confirmed that reasonable good results are obtained by this method.

From the vector diagram above, it is clear that for the same field excitation, the terminal voltage is decreased from its no-load value E_0 to V (for a lagging power factor). This is because of the following:

1. drop in voltage, IR_a, due to armature resistance
2. drop in voltage, IX_a, due to armature leakage reactance
3. drop in voltage, IX_{ar} , due to armature reaction

The drop in voltage due to armature reaction has been accounted for by assuming the presence of the fictitious reactance X_{ar} in the armature winding. The value of the armature reaction reactance X_{ar} is such that IX_{ar} represents the voltage drop due to armature reaction. Hence the vector difference between the no-load voltage E_0 and terminal voltage V is equal to the drop IZ_s across the synchronous impedance Z_s. Note that IZ_s represents the total drop in the alternator under load.

Example 7

A $3300V$, $100kVA$ 3-phase star-connected synchronous generator has an effective armature resistance of $0.5\Omega/ph$. A field current of $5A$ was necessary to produce a rated current on short-circuit and a voltage of $1000V$ (line) on open-circuit. Determine the full-load voltage regulation for the following types of loads:

a) unity power factor
b) 0.80 leading power factor
c) 0.71 lagging power factor

Solution 7

For a star-connected winding, the rated phase current equals the line current.

$$I = I_L = \frac{S}{\sqrt{3} \cdot V_L} = \frac{100 \times 10^3}{\sqrt{3} \times 3300} = \underline{\underline{17.5A}}$$

The synchronous impedance is given as

$$Z_s = \left. \frac{E_{oc}}{I_{sc}} \right|_{same I_f} = \frac{1000/\sqrt{3}}{17.5} = \underline{\underline{33\Omega}}$$

The synchronous reactance is then calculated as

$$X_s = \sqrt{Z_s^{\,2} - R_a^{\,2}} = \sqrt{(33)^2 - (0.5)^2} = \underline{\underline{32.99\,\Omega}}$$

The voltage regulation is $VR = \dfrac{E-V}{V}$, where V is the terminal voltage and E is the generated voltage, and is given as $E = \sqrt{(V\cos\varphi + IR_a)^2 + (V\sin\varphi \pm IX_s)^2}$.

The terminal voltage per phase is

$$V = V_L/\sqrt{3} = 3300/\sqrt{3} = \underline{\underline{1905V}}$$

a) **Unity power factor** $\cos\varphi = 1.0$, $\sin\varphi = 0.0$

The generated voltage is

$$\begin{aligned} E &= \sqrt{(V + IR_a)^2 + (IX_s)^2} \\ &= \sqrt{[(1905) + (17.5\times 0.5)]^2 + [(17.5\times 32.99)]^2} \\ &= \underline{1999V} \end{aligned}$$

Hence the percentage voltage regulation is

$$VR = \left(\frac{E-V}{V}\right)\times 100 = \left(\frac{1999-1905}{1905}\right)\times 100 = \underline{\underline{4.93\%}}$$

b) **0.8 leading power factor** $\cos\varphi = 0.8$, $\sin\varphi = 0.6$

The generated voltage is

$$\begin{aligned} E &= \sqrt{(V\cos\varphi + IR_a)^2 + (V\sin\varphi - IX_s)^2} \\ &= \sqrt{[(1905\times 0.8) + (17.5\times 0.5)]^2 + [(1905\times 0.6) - (17.5\times 32.99)]^2} \\ &= \underline{1634V} \end{aligned}$$

Hence the percentage voltage regulation is

$$VR = \left(\frac{E-V}{V}\right)\times 100 = \left(\frac{1634-1905}{1905}\right)\times 100 = \underline{\underline{-14.24\%}}$$

b) **0.71 lagging power factor** $\cos\varphi = 0.71, \sin\varphi = 0.71$

The generated voltage is

$$E = \sqrt{(V\cos\varphi + IR_a)^2 + (V\sin\varphi + IX_s)^2}$$
$$= \sqrt{[(1905\times 0.71)+(17.5\times 0.5)]^2 + [(1905\times 0.71)+(17.5\times 32.99)]^2}$$
$$= \underline{2362V}$$

Hence the percentage voltage regulation is

$$VR = \left(\frac{E-V}{V}\right)\times 100 = \left(\frac{2362-1905}{1905}\right)\times 100 = \underline{\underline{23.97\,\%}}$$

In the Table below, find a **summary** of the values for the generated voltage, terminal voltage and voltage regulation for the different loads and power factors.

	Resistive load (unity pf)	Capacitive load (leading pf)		Inductive load (lagging pf)	
		0.60	0.80	0.71	0.80
Generated voltage E (volts)	1999	1491	1634	2362	2304
Voltage regulation VR (%)	+4.93	-21.74	-14.24	+23.97	+20.94

Observations:

1. The *magnitude* of the generated voltage depends on the *load type* and its power factor.
2. Much *more voltages* would have to be generated *for inductive loads than for capacitive loads*. The reason is that, *inductive loads are basically reactive power absorbers, whilst capacitive loads are reactive power producers*.
3. For a *particular inductive load*, the better the power factor, the less voltage would have to be generated, and vice versa. For example, at 0.8 pf lagging, the generated voltage is 2304 V, compared to that of 2362 V for 0.71 pf lagging.
4. For a *particular capacitive load*, the better the power factor, the more voltage is generated, and vice versa. For example, at 0.8 pf leading, the generated voltage is 1634 V, compared to that of 1491 V for 0.60 pf leading.
5. The voltage regulation is *negative* for *capacitive loads*. This means that for capacitive loads, the terminal voltage is more than the generated voltage. *Capacitors therefore enhance terminal voltage*.
6. The voltage regulation is *positive* for *inductive loads*. This means that for inductive loads, the terminal voltage is less than the generated voltage. *Inductive loads therefore tend to reduce the terminal voltage*.

Example 8

A 600 V 60 kVA single-phase alternator has an effective resistance of 0.2 Ω. A field current of 10 A produces an armature current of 210 A on short-circuit and a emf of 480 V on open-circuit. Calculate

(a) synchronous impedance and reactance
(b) full-load voltage regulation with 0.8 pf lagging

Solution 8

(a) The synchronous impedance is given as $Z_s = \left.\frac{E_{oc}}{I_{sc}}\right|_{\text{for same field current I}_f} = \frac{480}{210} = \underline{2.28\,\Omega}$.

Hence the reactance is calculated as $X_s = \sqrt{{Z_s}^2 - {R_a}^2} = \sqrt{(2.28)^2 - (0.2)^2} = \underline{2.27\,\Omega}$

(b) full-load voltage regulation with 0.8 pf *lagging*; $\cos\phi = 0.8 \;\Rightarrow \sin\phi = 0.6$

For the *single-phase* alternator, the *full-load* phase-current $I = \frac{S}{V} = \frac{60\times 1000}{600} = \underline{100\,A}$

The no-load voltage for a lagging load is given as

$$\begin{aligned} E_0 &= \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_s)^2} \\ &= \sqrt{(600\times 0.8 + 100\times 0.2)^2 + (600\times 0.6 + 100\times 2.27)^2} \\ &= \underline{771V} \end{aligned}$$

The percentage voltage regulation is

$$VR = \left(\frac{E_0 - V}{V}\right)\times 100 = \left(\frac{771-600}{600}\right)\times 100 = \underline{28.5\,\%}$$

Example 9

A 3-phase star-connected alternator is rated at 1500 kVA 12000 V. The armature effective resistance and synchronous reactance are respectively 2 Ω and 35 Ω per phase. Calculate the percentage regulation for a load of 1200 kW at power factors of

(a) 0.8 lagging
(b) 0.8 leading

Solution 9

It must be noted that the given voltage is a ***line*** value, but we need the phase voltage for our calculation of the no-load terminal voltage.

For a *star-connected* winding, the phase voltage is given as

$$V = \frac{V_L}{\sqrt{3}} = \frac{12{,}000}{\sqrt{3}} = \underline{6928V}$$

The *load current* ***drawn by*** the 1200 kW 0.8 power factor lagging load is

$$I = \frac{P_{3\phi}}{V_L \times \sqrt{3} \times \cos\phi} = \frac{1200\times 10^3}{12{,}000 \times \sqrt{3} \times 0.8} = \underline{72.2\ A}$$

(a) Voltage regulation with 0.8 pf ***lagging***; $\cos\phi = 0.8 \ \Rightarrow \sin\phi = 0.6$

The no-load voltage for a lagging load is given as

$$\begin{aligned} E_0 &= \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_s)^2} \\ &= \sqrt{(6928\times 0.8 + 72.2\times 2)^2 + (6928\times 0.6 + 72.2\times 35)^2} \\ &= \underline{8775.7\,V} \end{aligned}$$

The corresponding percentage voltage regulation is

$$VR = \left(\frac{E_0 - V}{V}\right)\times 100 = \left(\frac{8775. - 6928}{6928}\right)\times 100 = \underline{26.67\,\%}$$

(b) Voltage regulation with 0.8 pf ***leading***; $\cos\phi = 0.8 \ \Rightarrow \sin\phi = 0.6$

Note that for a leading load, the ***sign*** before the IX_s drop in the no-load voltage equation is ***negative***. Moreover, the voltage regulation is expected to be negative.

The *load current* ***supplied by*** the 1200 kW 0.8 power factor leading load is

$$I = \frac{P_{3\phi}}{V_L \times \sqrt{3} \times \cos\phi} = \frac{1200\times 10^3}{12{,}000 \times \sqrt{3} \times 0.8} = \underline{72.2\ A}$$

The no-load voltage for a leading load is given as

$$\begin{aligned}E_0 &= \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi - IX_s)^2} \\ &= \sqrt{(6928\times 0.8 + 72.2\times 2)^2 + (6928\times 0.6 - 72.2\times 35)^2} \\ &= \underline{5915.7\,V}\end{aligned}$$

The percentage voltage regulation is

$$VR = \left(\frac{E_0 - V}{V}\right)\times 100 = \left(\frac{5915.7 - 6928}{6928}\right)\times 100 = \underline{-17.11\%}$$

NOTE: The ***full-load current*** can be obtained ***from the alternator parameters*** as $I_{fl} = \dfrac{S}{V_L\sqrt{3}} = \dfrac{1500\times 10^3}{12{,}000\times\sqrt{3}} = \underline{72.2\,A}$. Since the load current (for both lagging and leading loads) is the same as the full-load current, the calculated voltage regulation is then the ***full-load voltage regulation***.

4.8 Steady-State *Operating Characteristics* of Synchronous Generators

The principal steady-state operating characteristics are the ***interrelations*** among terminal voltage, field current, armature current and power factor and efficiency. A selection of performance curves, which are of importance in practical applications of the machine, will be considered.

These operating characteristics can be obtained qualitatively from the equivalent circuit based on a constant synchronous impedance. The main characteristics are

1. Terminal Voltage Characteristics ($V - I_a$ *curves for a given power factor*)
2. Compounding Curves ($I_f - I_a$ *curves at rated terminal voltage* V)
3. Reactive-Active Power Capability Curves ($Q - P$ *curves for a given power factor*)

4.8.1 Terminal Voltage Characteristics ($V - I_a$ *curves For a Given Power Factor*)

The ***generator voltage characteristic*** shows the ***variation of terminal voltage with load (or armature) current for a given power factor*** when the generator is driven at constant speed and excited with constant current. Typical voltage characteristics of an alternator are shown below.

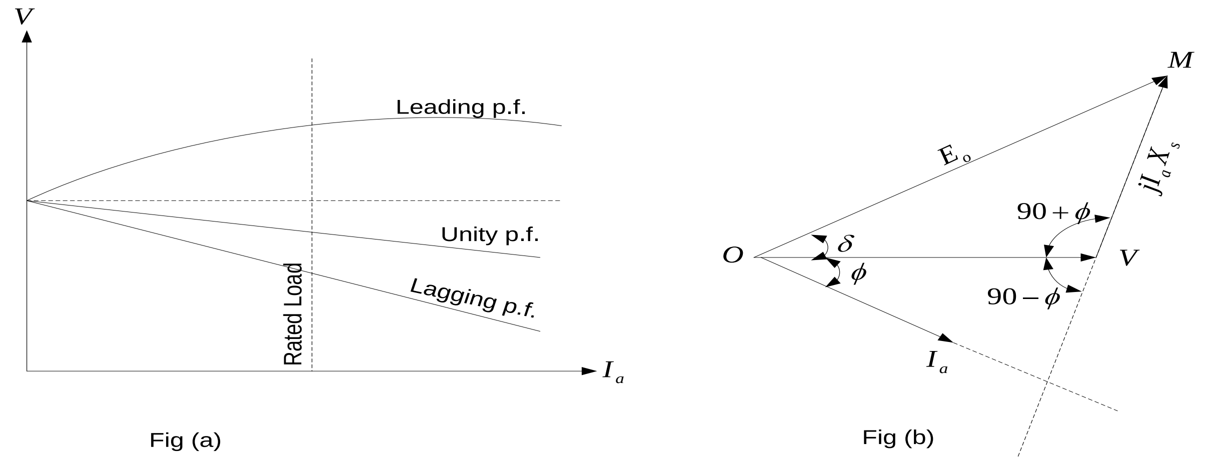


Fig (a) Terminal Voltage Characteristics
(b) Phasor Diagram for ***Negligible*** *Armature Resistance*

Neglecting the stator resistance the phasor diagram in Fig (b) is obtained. The power factor is lagging.

Referring to Fig (b), the angle $\angle OVM = 90 + \phi$, where $\cos\phi$ is the power factor.

Using the **cosine rule**, we obtain from $OM^2 = OV^2 + VM^2 - 2 \times OV \times VM \cos\angle OVM$, the equation

$$\begin{aligned} E_0^2 &= V^2 + I^2 X_s^2 - 2 \times V \times IX_s \cos(90+\phi) \\ &= V^2 + I^2 X_s^2 + 2VIX_s \sin\phi \end{aligned} \qquad (45)$$

Dividing Eqn (45) through by E_0^2 yields

$$1 = \left(\frac{V}{E_0}\right)^2 + I^2\left(\frac{X_s}{E_0}\right)^2 + 2.\left(\frac{V}{E_0}\right).I.\left(\frac{X_s}{E_0}\right).\sin\phi$$

or $\qquad (46)$

$$1 = \left(\frac{V}{E_0}\right)^2 + \left(\frac{I}{I_{sc}}\right)^2 + 2.\left(\frac{V}{E_0}\right).\left(\frac{I}{I_{sc}}\right).\sin\phi$$

where $I_{sc} = \dfrac{E_0}{X_s}$ *(for negligible armature resistance)* is the armature current when the machine terminals are short-circuited. This may be about 150% of normal current in large machines.

Consider the following cases:

Case I: Unity Power Factor (i.e. ϕ = 0)

For unity power factor, ϕ = 0 and $\sin\phi$ is zero. Hence Eqn (46) reduces to

$$1 = \left(\frac{V}{E_0}\right)^2 + \left(\frac{I}{I_{sc}}\right)^2 \qquad (47)$$

Eqn (47) is the equation of a ***circle***, if V is expressed as a percentage (or fraction) of E_0 and I as a percentage of (or fraction) of I_{sc}.

Case II: Zero Power Factor *Lagging* (i.e. ϕ = 90º lagging)

For zero lagging power factor, ϕ = 90^0 and sin ϕ = 1. Hence Eqn (46) reduces to

$$\begin{aligned} 1 &= \left(\frac{V}{E_0}\right)^2 + \left(\frac{I}{I_{sc}}\right)^2 + 2\times\left(\frac{V}{E_0}\right)\times\left(\frac{I}{I_{sc}}\right). \\ &= \left[\left(\frac{V}{E_0}\right)+\left(\frac{I}{I_{sc}}\right)\right]^2 \\ &or \\ 1 &= \left(\frac{V}{E_0}\right)+\left(\frac{I}{I_{sc}}\right) \end{aligned} \qquad (48)$$

$$\Rightarrow \left(\frac{V}{E_0}\right) = 1 - \left(\frac{I}{I_{sc}}\right)$$

Drawing (V/E_0) against (I/I_{sc}) makes Eqn (48) a ***straight line*** with negative unity gradient and cutting the (V/E_0) and (I/I_{sc}) axes at points (0, 1) and (1, 0) respectively.

Case III: Zero Power Factor *Leading* (i.e. ϕ = - 90º leading)

Similarly for zero leading power factor, ϕ = -90^0 and sin ϕ = -1. Hence Eqn (47) reduces to

$$1 = \left(\frac{V}{E_0}\right) - \left(\frac{I}{I_{sc}}\right) \qquad (49)$$

Eqn (49) is another ***straight line*** with positive unity gradient and cutting the (V/E_0) and (I/I_{sc}) axes at the points (0, 1) and (1, 0) respectively, and for which by reason of the direct magnetization effect of leading currents, the voltage increases with load.

4.8.1.1 Observations About Terminal Voltage Characteristics

The following observations can be made from the *terminal voltage characteristics***:**

1. For unity and lagging power factors the voltage always drops with increase of load.
2. At a certain leading power factor the full load regulation is zero, i.e., the terminal voltage is the same for both full- and no-load conditions.
3. At lower leading power factors the voltage rises with increase of load and the voltage regulation is negative.

4.8.2 Compounding Curves ($I_f - I_a$ *Curves at Rated Terminal Voltage* V)

Consider a synchronous generator delivering power at constant frequency to a load whose power factor is constant. The curve showing the ***field current required to maintain rated terminal voltage*** as the constant-power-factor-load is varied is known as a ***compounding curve***.

The Eqn (46) derived above, ie,

$$E_g^2 = V^2 + I^2 X_s^2 + 2 \times V \times IX_s \sin\phi \qquad (50)$$

can be used to obtain the field excitation I_f necessary to maintain constant output voltage for all loads, because the excitation voltage E_g is proportional to the excitation when synchronous reactance is constant.

Three compounding curves (or E_g-current for constant output voltage) at various constant power factors are shown in the Fig below.

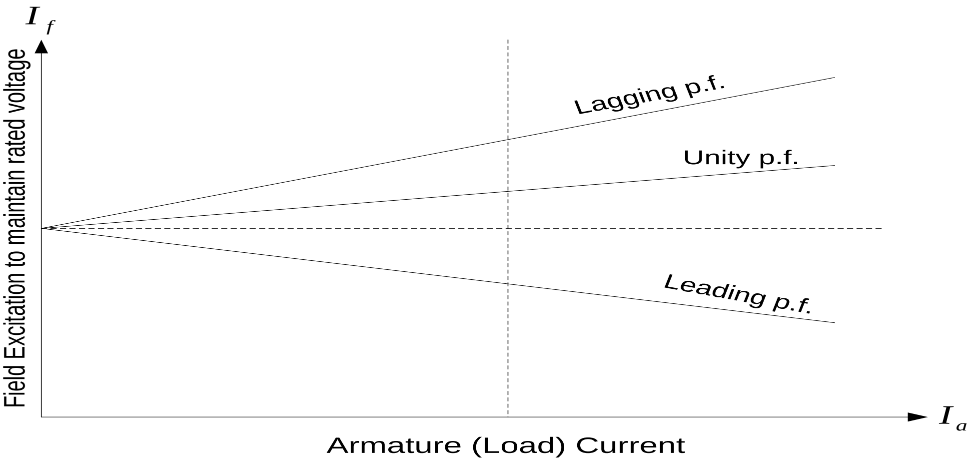


Fig: Generator Compounding Curves

4.8.2.1 Observations About The Compounding Curves:

The following observations can be made of the *compounding curves* above.

1. If the field current is held constant while the load varies, the terminal voltage will vary.
2. All unity- and lagging-pf loads will require an increase of excitation with increase of load current
3. Low leading-pf loads will require a decrease of excitation with increase of load current.

4.8.3 Reactive-Active Power Capability Curves ($Q-P$ *curves For a Given pf*)

Synchronous generators are usually rated in terms of the maximum kVA load at a specific voltage and power factor (often 0.8, 0.85, or 0.9), which they can carry without overheating. The *active power output of the generator is limited to a value within the* kVA *rating by the capability of its prime mover*.

By virtue of its voltage-regulating system, the machine normally operates at a constant voltage whose value is within ± 5% of rated voltage. When the active-power loading and voltage are fixed, the allowable reactive-power loading is limited by either armature or field heating.

A typical set of active-reactive power capability curves are shown below.

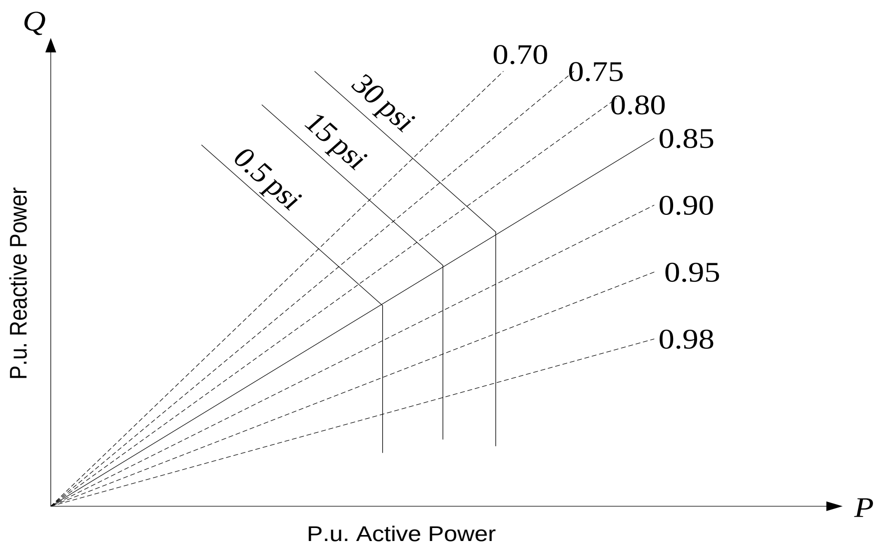


Fig: Reactive-Active Power Capability Curves of a Turbogenerator

These curves give the maximum reactive-power loadings corresponding to various active power loadings with operation at rated voltage. ***Armature heating is the limiting factor in the region from unity to rated power factor (say 0.85). For lower power factors, field heating is the limiting factor***.

Such set of curves form a valuable guide in planning and operating the power system of which the generator is a part. Also shown in the Fig above is the effect of increased pressure of the coolant (e.g. hydrogen) on the allowable machine loadings.

5. PARALLELING OF GENERATORS

Synchronous generators can readily be operated in parallel. In fact, modern alternating current power systems usually consist of several generating units connected in parallel to a common bus line, interconnected by hundreds of kilometers of transmission lines and supplying electrical energy to load centers scattered over hundreds of thousands of square kilometers. These systems must be designed in such a way that synchronism will be maintained following disturbances on the system.

The ***principal reasons*** for interconnected systems are ***reliability***, ***continuity of service*** and ***economy*** in plant investment and operating costs. The generators may be divided into two groups as follows:

Group 1: It will consist of one or more generators, which are continuously adjusted to maintain the magnitude and frequency of the system voltage.

Group 2: It will consist of all other generators, which are used to regulate the active and reactive power requirements on the system. Generators of this group are considered to be connected to an *infinite bus* as are all the loads, including synchronous motors. As mentioned already, the power (both active and reactive) contributed by a generator connected to an infinite bus is determined by its excitation and the mechanical power to the prime mover.

5.1 Reasons for Paralleling Generators

Two or more generators may be put into parallel operation for the following ***reasons***:

1. Local or regional power demand may *exceed* the power of a *single* available generator.
2. Parallel generators allow one or more units to be *shut down for scheduled or emergency maintenance* without having to interrupt power supply to the load.
3. Generators operate at *reduced efficiency at light or part load*. And so shutting down one or more generators allows the remaining load to be supplied by fewer machines that are efficiently loaded.
4. *Load growth* can be handled by added machines without disturbing the original installation.
5. Available machine prime movers and generators can be *matched for economic or optimal utilization* in terms of economy and flexibility of use.

5.2 Requirements for Generator Paralleling

Before paralleling two or more generators, some conditions have to be fulfilled. These include the following:

1. The *terminal voltages* must be the *same* at the *paralleling or interconnection or tie point* or junction, even though not the same at the generators.
2. The *phase sequences or rotations* for three-phase generators must be the *same* at the paralleling point.
3. The line *frequencies* must be *identical or approximately equal* at the paralleling point.

In the vast majority of cases, this means the same frequency at the generator, because frequency changing is not economical. Mixed frequencies must be paralleled through some frequency conversion means for compatibility at the point of interconnection.

4. With reference to the load, the voltage of the incoming generator must be *in phase* with that of the running machine. It will stay in phase under normal conditions after paralleling.
5. The incoming generator must generate a *voltage wave* of approximately the *same shape* as that of the running generator. The *prime movers* of the generators must have relatively *similar drooping speed-load characteristics*. This is to prevent a machine with a rising speed load characteristic from taking more of the load until it fails from overload.

NB: *Violations of the above-mentioned requirements for paralleling would result in circulating currents between the machines, resulting in serious and even disastrous consequences.* The series of operations required to bring about conditions 1, 2 and 3 is called ***synchronization***.

5.3 Synchronization

It may be logically assumed that a generator will be placed in parallel with one or more generators only when additional load requirement necessitates it. Those generators already in the system and carrying load are called *running machines*, while that which is to be placed in the system is known as the *incoming machine*.

At the time of synchronization, ***all*** the conditions for paralleling two or more generators must be met, namely, the

2. effective *terminal voltage* of the incoming generator *must be exactly equal to that of the others, or of the busbar connecting them*.
3. *frequencies should be the same*, although it is more desirable that the frequencies at the instant of paralleling be almost, but not quite, identical.
4. *phase sequence or rotation* of the running and incoming generators *must be the same*.
5. individual phase voltages which are to be connected to each other must be in exact phase opposition. This is the same as saying that the terminals of DC generators must be connected positive to positive (+ to +) and negative to negative (- to -).

The requirements of 1, 2 and 3 above are satisfied as follows:

- The incoming generator *terminal voltage is made equal to the bus line voltage* to which the running machine(s) have already been connected, *by manipulating the DC field excitation*. The effective terminal voltage magnitude can be checked by means of voltmeters.
- The frequency, $f = \dfrac{N_s \cdot P}{120}$, which depends directly on the speed, is adjusted by changing the speed of the prime mover (driving machine).
- A *synchronizing device or equipment*, called ***synchroscope***, is used in modern day times to satisfy the condition of *equal phase sequence or rotation*.

5.4 Synchroscope Method for Synchronization

In practice, in large central power station installations, generators are synchronized by means of an indicating instrument called the *synchroscope*. A synchroscope has a rotating hand and a dial labeled with slow and fast direction arrows to show the incoming machine speed relation.

The pointer rotates clockwise if the incoming machine is fast and anticlockwise if it is slow. In addition, it has a pointer that continually indicates the phase angle between the two source voltages.

The phase angles are not shown but the dial has a zero marker to indicate when the voltages are in-phase. See Fig below.

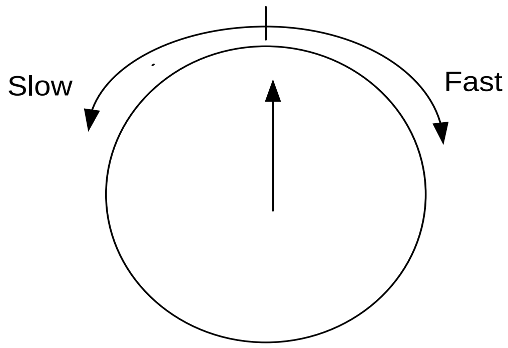


Fig: Synchroscope

It has two independent circuits, one being connected to the incoming generator and the other to the bus line. The magnetic fields set up by these two circuits cause the hand to rotate. During synchronization, as the incoming rotational speed nears synchronism, the speed of the synchroscope hand drops enough to become visible. *The hand speed is proportional to the difference in speed*.

When the hand comes to a standstill at the mark on the dial indicating synchronism, the synchronizing switch connecting the incoming generator to the bus line may be closed, after making the final check on the magnitude of the voltages.

In modern generating stations, synchronization is automated.

5.5 Load Sharing or Allocation

Any two or more generators that have been synchronized together can be made to ***share*** *the active-power and reactive-power load by appropriate adjustments of the prime-mover throttles and field rheostats respectively*. In discussing load distribution among two paralleled generators, we shall be looking at the

1. effect of change in field excitation
2. effect of change in mechanical driving torque of the prime mover

5.5.1 Effect of Change in Excitation – Reactive Load Sharing

5.5.1.1 Concept of Synchronising Current

And if *two generators operating in parallel supply equal current and have the same power factor,* and the *field excitation of* ***only one*** *of them is changed (say increased)*, a resultant emf $E_r = E_2 - E_1$ will exist around the local circuit.

See Fig (b) below.

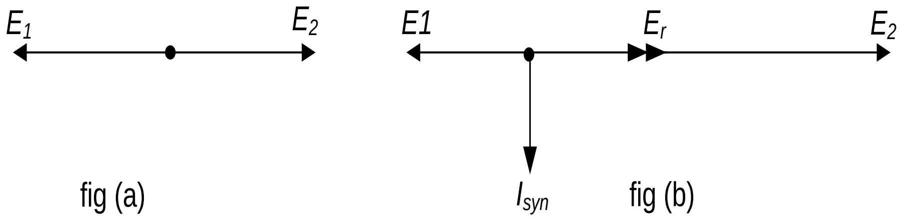


Fig: Concept of Synchronizing Current

This resultant emf will cause the flow of a circulating current called *synchronizing current* I_{syn} between the two generators, which is limited by the impedance of the two machines. Because the *resistance of the generator windings is negligible* in comparison to their reactance, the circulating synchronizing current I_{syn} will lag behind the resultant emf E_r or E_2 by ***nearly*** 90^0, but lead E_1 by 90^0, as evident in Fig (b).

$$I_{syn} = \frac{E_r}{Z} \cong \frac{E_r}{X_s} \text{ (lagging behind } E_r \text{ by } 90^0). \qquad (51)$$

It is thus seen that the circulating synchronizing current flows at zero *lagging* power factor through Generator 2 and at zero *leading* power factor through Generator 1. Consequently, the ***circulating synchronizing current*** I_{syn} ***produces two effects simultaneously:***

- It produces *demagnetizing effect* (because of its lagging) on Generator 2 and decreases the flux in Generator 2, resulting thereby in a reduction in E_2.
- But it produces a *magnetizing effect* (because of its leading) on Generator 1 and increases the flux in Generator 1, resulting thereby in an increase in E_1.
- However, the active power supplied by each of the generators is not materially affected.

Thus if an attempt is made to increase the field excitation of one generator, a reactive circulating current is established that tends to keep the field strengths of the two generators the same. The increase in induced voltage in Generator 2 is only slight in comparison to the change in its field current.

E_2 is greater than E_1 only by the amount of the impedance drop ($I_{syn}Z$) produced by the synchronizing current I_{syn} circulating through both machines.

5.5.1.2 Effect of Change in Field Excitation for Paralleled Generators

Consider two already synchronized generators each supplying current I_a to an external load. If the field excitation of one generator is changed, a reactive circulating current (synchronizing current) results, that changes the reactive kVA-output and hence the power factor at which each of the generator is producing. See Fig below.

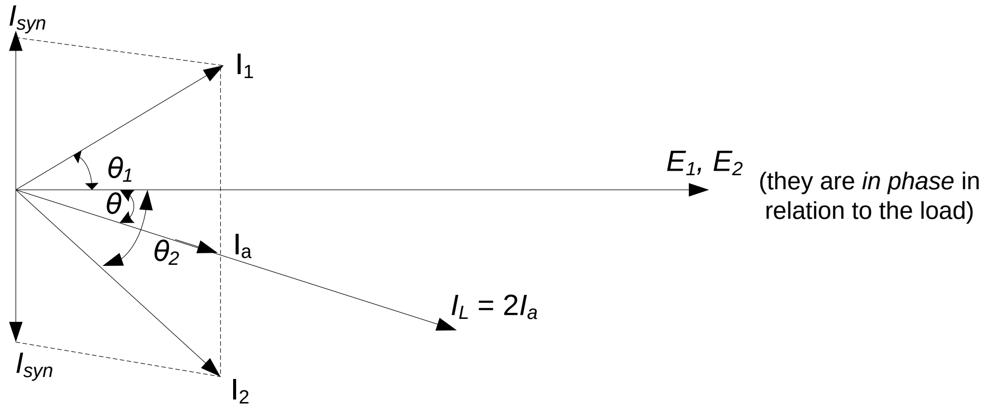


Fig: Effect of Change in Excitation for Two Paralleled Generators

The vectors E_2 and E_1 are shown ***in-phase because they are represented relative to the load***. The synchronous I_{syn} current circulating through Generator 1 is leading E_1, and that through Generator 2 is lagging E_2. *The total current carried by Generator 1 is I_1 (the vector sum of the leading I_{syn} and I_a) and that by Generator 2 is I_2 (the vector sum of the lagging I_{syn} and I_a) and that by the load is $I_L = 2I_a$.*

And so if *two generators in parallel supply* ***equal current*** *and have the* ***same power factor,*** and the *field excitation of one of them is changed*, a reactive circulating current (synchronizing current) is established. This synchronizing current may *lead* the induced emf of one machine and increase the total current in that machine, and *lag* the induced emf of the other machine and so decrease the total current in the other machine. However, the power supplied by each of the generators is not materially affected.

The following points must be noted:

1. If *excitation is changed, only the power factor* at which the load is delivered *by the respective generators* is changed.
2. By keeping the input of the prime mover of a generator constant, any change in field excitation merely changes the reactive-kVA component of the output and the terminal voltage, but not the active-kW output.
3. A generator cannot therefore be made to take an active load merely by increasing its field excitation.

The change in excitation is achieved by the ***exciter system***, and is thus effected to ***achieve reactive power control*** and ***hence voltage regulation***.

5.5.2 Effect of Change in Prime Mover Input – Active Load Sharing

The following points are worth noting:

1. The load taken up by a generator directly depends on its driving mechanical torque
2. Generators in parallel tend to remain synchronized. If the speed of one is increased, it immediately supplies more load and slows down, and the other supplies less load and speeds up, so that the speeds of the alternators again become equal.
3. By increasing the input to its prime mover, a generator can be made to take a greater share of the load, though at different power factor.

Operators in power plants have control of the steam or fuel or water supply to the prime mover, and so they can shift the real or active load from one generator to another as desired.

The exact sharing of ***active power*** between synchronous generators is determined by their speed-load or frequency-load characteristics, which take the form shown in Fig below.

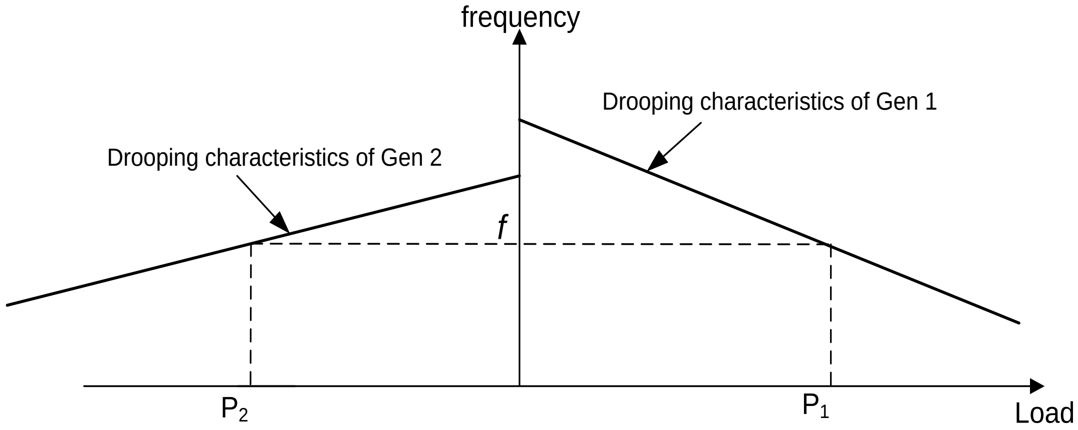


Fig: Drooping Frequency-Load Characteristics

For steady state operation, the frequencies of the two machines must be equal. Hence

$$\omega = \omega_{01} - P_1 \tan\delta_1 = \omega_{02} - P_2 \tan\delta_2 \qquad (52)$$

The slope, $\tan\delta$, is termed the ***drooping*** of the characteristics.

Changing the speed-load characteristic changes the load sharing, and this involves an alteration to the governor setting. The speed regulation is so adjusted that changes in frequency are small (of the order of 5 % from no load to full load). Unless the speed–load characteristics are identical and drooping in nature, the machines can never share the total load in accordance with their ratings.

Example 10:

Two generators rated 200 MW and 400 MW are operating in parallel. The droop characteristics of their governors are 4 % and 5 % respectively from no load to full load. Assuming that the generators are operating at 50 Hz at no load, how would a load of 600 MW be shared between them? What will be the system frequency at this load?

Solution 10:

Let x_1 and x_2 be the loads contributed by Generator 1 and Generator 2 ***expressed as fraction of their rated powers***. Then from Eqn (52),

$$\omega_1 = \omega_{01} - P_1 \tan\delta_1 = 1.0 - 0.04x_1 \qquad \text{for Generator 1}$$

$$\omega_2 = \omega_{02} - P_2 \tan\delta_2 = 1.0 - 0.05x_2 \qquad \text{for Generator 2}$$

Since the generators are in parallel, they will operate at the same (angular) frequency $\omega_1 = \omega_2 = \omega$**.** Therefore

$$\omega_1 = \omega_2 \Rightarrow 1.0 - 0.04x_1 = 1.0 - 0.05x_2$$

or $$\frac{x_1}{x_2} = \frac{0.05}{0.04} = \frac{5}{4} \qquad \Rightarrow x_1 = \frac{5}{4}x_2 \ .$$

Now to share a total load of 600 MW, we also have

$$x_1P_1 + x_2P_2 = 600 \qquad \Rightarrow 200x_1 + 400x_2 = 600 \qquad \Rightarrow x_1 + 2x_2 = 3$$

Solving for x_2 by substituting the value of x_1, we obtain

$$\left(\frac{5}{4}x_2\right) + 2x_2 = 3 \quad \Rightarrow x_2 = \frac{12}{13}$$

Hence $x_1 = \frac{5}{4}x_2 \qquad \Rightarrow x_1 = \frac{15}{13}$

Thus the share of the load supplied by Generator 2 is

$$P_{2L} = x_2 \times P_2 = \frac{12}{13} \times 400 = \underline{369\,MW}$$

And the share of the load supplied by Generator 1 is

$$P_{1L} = x_1 \times P_1 = \frac{15}{13} \times 200 = \underline{231\,MW}$$

The system frequency in per unit is

$$\omega = 1.0 - 0.05x_2 = 1.0 - 0.05 \times \frac{12}{13} = 1 - 0.046 = \underline{0.954\ p.u\ Hz}$$

Hence the ***actual*** system frequency $f = 0.954 \times 50 = \underline{\underline{47.70\,Hz}}$

Note that with this load sharing, the Generator 1 will be overloaded by almost 15 %.

Example 11:

Two 750 kW alternators operate in parallel. The speed regulation of one set is 100 % to 103 % from full-load to no-load and that of the other 100 % to 104 %. How will the two alternators share a load of 1000 kW and at what load will one machine cease to supply any portion of the load?

Solution 11:

$\omega_1 = \omega_{01} - P_1 \tan\delta_1 = 1.04 - 0.04x_1$ for Generator 1

$\omega_2 = \omega_{02} - P_2 \tan\delta_2 = 1.03 - 0.03x_2$ for Generator 2

Since the generators are connected in parallel, they will operate at the same frequency, that is, $\omega_1 = \omega_2 = \omega$. Therefore

$$\omega_1 = \omega_2 \Rightarrow 1.04 - 0.04x_1 = 1.03 - 0.03x_2 \quad \Rightarrow x_1 = 0.25 + 0.75x_2$$

For the two generators of the same 750 kW rating to share the load of 1000 kW,

$$P_1x_1 + P_2x_2 = P_L \Rightarrow 750x_1 + 750x_2 = 1000 \quad \Rightarrow x_1 + x_2 = \frac{4}{3}$$

Substituting x_1 to solve for x_2, we obtain $(0.25 + 0.75x_2) + x_2 = \frac{4}{3} \quad \Rightarrow x_2 = \underline{0.619}$

Thus the load contributed by Generator 2 is $P_{2L} = x_2P_2 = 0.619 \times 750 = \underline{464\,kW}$

And the load contributed by Generator 1 is $P_{1L} = P_L - P_{2L} = 1000 - 464 = \underline{536\,kW}$

Generator 2 will cease to contribute when the speed is **1.03 *p.u***. At that speed, power supplied by Generator 1, that is x_1, is determined from the relation:

$$\omega_1 = 1.04 - 0.04x_1 = 1.03\,\text{p.u.} \quad \Rightarrow x_1 = 0.25\ p.u.$$

$$\text{or } P_{1L} = x_1 \cdot P_1 = 0.25 \times 750 = \underline{187.5\,kW}$$

5.5.3 Synchronizing Power and Torque – Effect of Increasing the Driving Mechanical Torque

Should the ***torque driving incoming alternator 2 be increased***, by adjusting the throttle opening of the prime mover to admit more steam or water or fuel as the case may be, *the alternator 2 would increase in speed* ***for only a small fraction of a revolution until its induced voltage*** E_2 ***has pulled slightly ahead in phase relation*** α.

See Fig below.

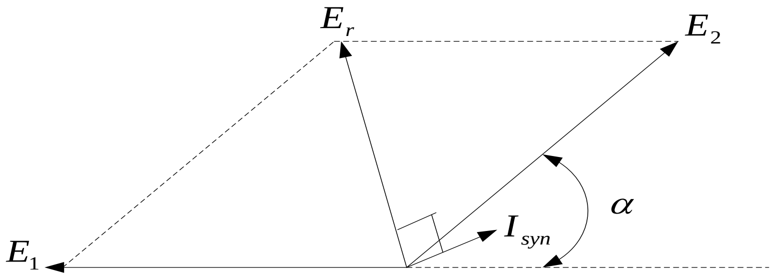


Fig: Effect of Changing (Increasing) The Mechanical Torque

In that case, the load on incoming Alternator 2 will increase to such an extent that will cause this alternator to slow down to its original speed. The induced emf of alternator 2 has now advanced in phase position by α degrees from its original position. ***The two induced voltages*** E_1 ***and*** E_2 ***are no longer in exact phase opposition to neutralize each other, and a resultant induced voltage*** E_r ***exists to circulate a synchronizing current*** I_{syn} ***through the two alternators***.

Since the resistance of the two alternators is small compared to their reactance, the circulating synchronizing current I_{syn} will lag the resultant induced voltage E_r by ***nearly*** 90 degrees. ***The synchronizing current is seen to be <u>nearly in phase</u> <u>with</u>*** E_2 ***and <u>nearly 180 degrees out of phase with</u>*** E_1.

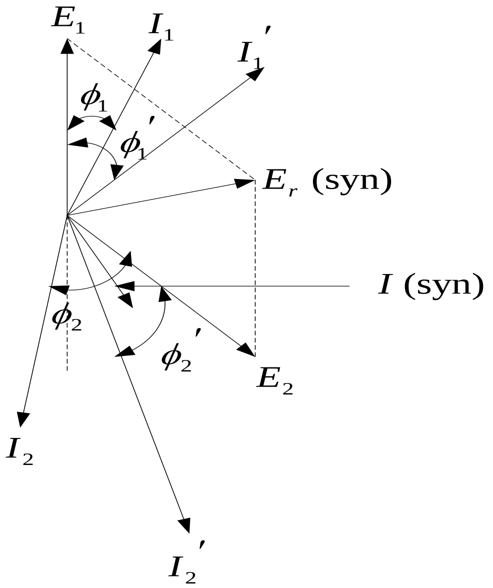
Consequently, I_{syn} will be nearly in phase (have a magnetizing effect) with the load current carried by Alternator 2 and nearly 180 degrees out of phase (have a demagnetizing effect) with the load current carried by Alternator 1. Hence Alternator 2 will carry a high current and Alternator 1 will carry a low current.

The above statement shows that if two alternators in parallel supply *equal currents* to a load, and the driving torque of one machine is increased, the power supplied by this machine is increased and the power supplied by the other is decreased.

5.5.3.1 Effect of *Increasing* Prime Mover Power Input to One of Two Alternators Supplying Currents and Different Power Factors

Consider two paralleled alternators whose excitations and power inputs into their prime movers have been adjusted to supply currents of I_1 and I_2 at respective (different) power factors $\cos\phi_1$ and $\cos\phi_2$.

Now *if the power input to the prime mover of machine 2 is increased*, then its emf vector will swing ahead by a certain angle as shown in the Fig below.



$$\overrightarrow{I_2}' = \overrightarrow{I_2} + \overrightarrow{I_{syn}}$$
$$\overrightarrow{I_1}' = \overrightarrow{I_1} + \overrightarrow{I_{syn}}$$

Fig: Effect of Increasing Power Input to Prime Movers of Two Machines Delivering Different Currents at Different Power Factors

Now a resultant synchronizing voltage E_r or E_{syn} is produced which, acting on the local circuit, sets flowing the synchronizing current I_{syn} which lags almost by 90^0 behind E_r.The new armature current I_2' of second machine is the vector sum of I_2 and I_{syn}.

It will be noted that I_2' ***is greater than*** I_2 *and is less lagging than it*. Hence the Alternator 2 takes up increased share of the load. This synchronizing current is taken up by machine 1. *Its armature current also changes from* I_1 *to* I_1' *.It may be noted that* I_1' ***is lesser and is at a greater angle from*** E_1. Hence Alternator 1 is taking up a *lesser share of the load*.

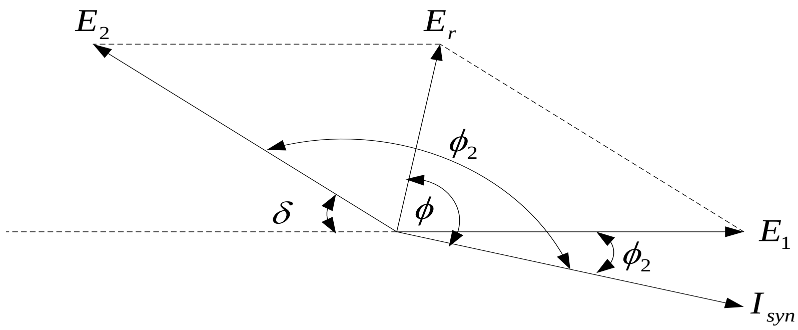
Thus it is found that:

- The load taken up by an alternator directly depends on its driving mechanical torque (or angular advance of the rotor)
- By increasing the input to its prime mover, an alternator can be made to take a greater share of the load, though at different power factor.

5.5.4 Synchronizing Power

Suppose that two machines are driven at the same speed and that their emfs are adjusted to equality, i.e, $E_1 = E_2 = E$.

If additional power input or torque is supplied to say machine 2 causing it to accelerate, its induced voltage will advance by a small angle δ ahead that of machine 1, and machine 2 will take a greater part of the load as depicted in Fig below.



$E_1 = E_2 = E$.

Let $(180-\delta)$ = angular phase difference between the induced emfs E_1 and E_2.
E = ***per phase*** emf of each of the machines.

The resultant emf is given as

$$E_r = 2E\cos\left(\frac{180^0-\delta}{2}\right) = 2E\cos\left(90-\frac{\delta}{2}\right) = 2E\sin\frac{\delta}{2}$$

$$\delta \text{ is very small, and so for small } \delta, \quad \sin\frac{\delta}{2} \cong \frac{\delta}{2}. \tag{53}$$

Thus

$$E_r \cong 2E\cdot\frac{\delta}{2} = E\delta$$

The synchronizing current is

$$I_{syn} = \frac{E_r}{Z} \cong \frac{E\cdot\delta}{Z} \tag{54}$$

lagging behind the resultant emf E_r by θ, where $\theta = \tan^{-1}\frac{X_s}{R}$, where Z is the combined impedance *per phase* of the two alternators, *or one of the alternator,* ***only if it is connected to an infinite bus***.

If resistance is small as compared to synchronous reactance of the alternator, then synchronizing current is:

$$I_{syn} = \frac{E_r}{Z} \cong \frac{E_r}{X_s} \cong \frac{E\cdot\delta}{X_s} \text{ (lagging behind } E_r \text{ by } 90^0) \tag{55}$$

The Fig. above shows this synchronizing current to be *nearly in phase* with E_1 and anti-phase with E_2. Consequently machine 2 produces a power $P_{syn} \cong E_2 I_{syn}$ as a generator, and supplies it (I^2R losses excepted) to machine 1 as synchronous motor.

Power supplied by Alternator 1 is: $E_1 I_{syn} \cos\phi_1$

Power supplied by (accelerated) Alternator 2 is: $E_2 I_{syn} \cos\phi_2$

The ***power supplied by the accelerated alternator*** is called ***synchronizing power***, and is given by the expression P_{syn} as

$$P_{syn} = E_2 I_{syn} \cos\phi_2 = E \cdot \frac{E \cdot \delta}{X_s} \cdot \cos\phi_2$$

$$\text{Since } \phi_2 \text{ is close to } 180 \text{ degrees}, \cos\phi_2 \cong -1. \text{ Hence} \tag{56}$$

$$P_{syn} = \frac{\delta \cdot E^2}{X_s}$$

The ***total*** synchronizing power for ***3-phases*** is

$$P_{syn(3\phi)} = \frac{3\delta \cdot E^2}{X_s} \tag{57}$$

The synchronizing power tends to retard the faster Machine 2 and accelerates the slower Machine 1, pulling the two back into step.

We note that:

1. The synchronizing power is produced because the armature impedance is highly inductive
2. The synchronizing power is produced even when their emfs are not equal.
3. ***Alternators in parallel tend to remain synchronized. If the speed of one is increased, it immediately supplies more load and slows down, and the other supplies less load and speeds up, so that the speeds of the alternators again become equal.***

Operators in power plants have control of the steam or fuel or water supply to the prime mover, and so they can shift the load from one alternator to another as desired.

5.5.5 Synchronizing Torque

As explained already, a change of ***load angle*** occurs whenever there is unbalance between the applied mechanical torque and the electromagnetic torque. This in turn can be caused by:

- a change to the mechanical input from the prime mover or
- a change of armature current due to circuit conditions or
- a change of excitation.

For a given excitation of a non-salient-pole machine, a change from load angle δ to $\delta+\Delta\delta$ results in additional power

$$\Delta P=\frac{\partial P}{\partial\delta}\Delta\delta=\frac{VE_g}{X_s}\cos\delta\,\Delta\delta \tag{58}$$

where ΔP is the synchronizing power. The corresponding synchronizing torque is $\Delta P/\omega_s$ .This torque acts against the angular disturbance to return the rotor axis towards its balance conditions. Thus there is inherent tendency of synchronous machine to remain in synchronism with the supply.

The synchronizing torque is maximum for no load ($\delta=0$) and reduces as the machine is more heavily loaded. For a load angle $\delta=\pi/2$ it is zero. $P_{syn}=\dfrac{\partial P}{\partial\delta}$ is the synchronizing power/radian and T_{syn} the torque/radian.

*More practically both P_{syn} and T_{syn} are expressed in terms of power or **torque per mechanical degree of displacement** by **multiplying the** P_{syn} **expression by** $\frac{P}{2}\left(\frac{\pi}{180^o}\right)$.*

If T_{syn} is the synchronizing torque in $N-m$, the total synchronizing power is related to the synchronizing torque as:

$$P_{syn(3\phi)}=\frac{T_{syn}\cdot 2\pi N_s}{60}=\frac{3\delta\cdot E^2}{X_s}$$

$$\Rightarrow T_{syn}=\frac{P_{syn(3\phi)}}{\omega_s}=\frac{P_{syn(3\phi)}}{\left(2\pi N_s/60\right)} \tag{59}$$

$$=\frac{3\delta E^2\cdot 60}{X_s\cdot 2\pi N_s}$$

Example 12:

A 2000-kVA, 3-phase, 8-pole star-connected synchronous generator runs on 6000-V, 50-Hz infinite bus bars. Find the synchronizing power and torque per mechanical degree of displacement (a) for no load with excitation adjusted to give 6000 V on open circuit (b) for full load at a power factor of 0.8 lagging. The synchronous reactance of the machine is 21.6 ohms.

Solution 12:

(a) **No-load** conditions:

The synchronising power and torque *per mechanical degree of displacement* on ***no-load*** are given as:

$$P_{syn} = \frac{\delta \cdot E^2}{X_s} = \frac{V^2}{X_s} \times \frac{P}{2}\left(\frac{\pi}{180}\right) = \frac{6000^2}{21.60} \times \frac{8}{2}\left(\frac{\pi}{180}\right) = \underline{\underline{116\,kW}}$$

$$T_{syn} = \frac{P_{syn}}{\omega_s} = \frac{P_{syn}}{2\pi(f/p)} = \frac{116 \times 10^3}{2\pi \times (50/4)} = \underline{\underline{1470\,Nm}}$$

(b) **Full-load** conditions

The full-load current is

$$\begin{aligned} I_L &= \frac{S_{3\phi}}{V_L\sqrt{3}} = \frac{2000 \times 10^3}{6000\sqrt{3}} = 192.45\,A \\ &= I_\phi (star-connected) \\ &= I_a \end{aligned}$$

The generated voltage is

$$\begin{aligned} E_g &= V + jI_a X_s = \frac{6000}{\sqrt{3}} + j21.6 \times 192.45\angle -\cos^{-1}(0.8^o) \\ &= 3464.10 + 4156.92\angle 53.1^o = 5960 + j3324 \\ &= 6824\angle 29.15^o \text{ V (per phase)} \\ &= \underline{11820\text{V (line value)}} \end{aligned}$$

The synchronizing power is

$$P_{syn} = \frac{VE_g}{X_s}\cos\delta \quad \text{per electrical degree of displacement}$$

$$= \frac{VE_g}{X_s}\cos\delta \times \frac{P}{2}\left(\frac{\pi}{180}\right) \text{ per mechanical degree of displacement}$$

$$= \frac{6000\times 11820}{21.60}\cos 29.15^o \times \frac{8}{2}\left(\frac{\pi}{180}\right)$$

$$= \underline{\underline{200\,kW}}$$

The synchronizing torque is thus

$$T_{syn} = \frac{P_{syn}}{\omega_s} = \frac{P_{syn}}{2\pi(f/p)} = \frac{200\times 10^3}{2\pi(50/4)} = \underline{\underline{2550\,N-m}}$$

Note that the synchronizing torque is greater in (b) than in (a) because of the increased value of E_g

Example 13

A 3 MVA 6-pole alternator runs at 1000 rpm in parallel with other machines on 3.3 kV busbars. The synchronous reactance is 20 %. Calculate the synchronising power per one mechanical degree of displacement and the corresponding synchronizing torque.

Solution 13

Generated phase voltage $E = \frac{V_L}{\sqrt{3}} = \frac{3.3\times 10^3}{\sqrt{3}} = \underline{1905V}$

Full-load current $I = \frac{S}{V_L\sqrt{3}} = \frac{3\times 10^6}{3.3\times 10^3\times\sqrt{3}} = \underline{524.9A}$

Synchronous impedance $Z_s = \frac{E}{I} = \frac{1905}{524.9} = \underline{3.63\Omega}$

Synchronous reactance $X_s = 0.2Z_s = 0.2\times 3.63 = \underline{0.726\Omega}$

Recall the relationship between mechanical degree of displacement α and electrical degree of displacement δ given as $\delta_{elec} = \frac{P}{2}\alpha_{mech}$, where P is the number of poles

The synchronising power and torque *per mechanical degree of displacement* on ***no-load*** are given as:

$$
\begin{aligned}
P_{3\phi,syn} &= \frac{3\alpha E^2}{X_s} \; per\; mechanical \deg ree\; of\; displacement \\
&= \frac{3\delta E^2}{X_s} \; per\; \text{electrical} \deg ree\; of\; displacement \\
&= \frac{3E^2}{X_s} \times \frac{P}{2}\left(\frac{\pi}{180}\right) \times 1^0 = \frac{3 \times (1905)^2}{0.726} \times \frac{6}{2}\left(\frac{\pi}{180}\right) \times 1 \\
&= \underline{\underline{785.19\,kW}}
\end{aligned}
$$

$$
\begin{aligned}
T_{syn} &= \frac{P_{3\phi,syn}}{\omega_s} = \frac{P_{3\phi,syn}}{2\pi(f/p)} \\
&= \frac{P_{3\phi,syn}}{2\pi(N_s/60)} \\
&= \frac{785.19 \times 10^3 \times 60}{2\pi \times 1000} \\
&= \underline{\underline{7498\,Nm}}
\end{aligned}
$$

Example 14

A 2-pole 50 Hz 3-phase turbo-alternator is excited to generate the busbar voltage of 11 kV on no-load. The machine is star-connected and the short-circuit current for this excitation is 1000 A. Calculate the synchronising power per degree of mechanical displacement of the rotor and the corresponding synchronizing torque.

Solution 14

The generated voltage per phase $E = \frac{V_L}{\sqrt{3}} = \frac{11000 \times 10^3}{\sqrt{3}} = \underline{6351V}$

Synchronous speed $N_s = \frac{120f}{P} = \frac{120 \times 50}{2} = \underline{3000\,rpm}$

For negligible resistance, synchronous reactance $X_s \cong Z_s = \frac{E_{oc}}{I_{sc}} = \frac{6351}{1000} = \underline{6.35\,\Omega}$

Again recall the relationship between mechanical degree of displacement α and electrical degree of displacement δ given as $\delta_{elec} = \frac{P}{2}\alpha_{mech}$, where P is the number of poles

The synchronising power and torque *per mechanical degree of displacement* on ***no-load*** are given as:

$$
\begin{aligned}
P_{3\phi,syn} &= \frac{3\alpha E^2}{X_s} \; per\ mechanical\ \deg ree\ of\ displacement \\
&= \frac{3\delta E^2}{X_s} \; per\ \mathrm{electrical}\ \deg ree\ of\ displacement \\
&= \frac{3E^2}{X_s} \times \frac{P}{2}\left(\frac{\pi}{180}\right) \times 1^0 = \frac{3 \times (6351)^2}{6.35} \times \frac{2}{2}\left(\frac{\pi}{180}\right) \times 1 \\
&= \underline{\underline{332.6\, kW}}
\end{aligned}
$$

$$
\begin{aligned}
T_{syn} &= \frac{P_{3\phi,syn}}{\omega_s} = \frac{P_{3\phi,syn}}{2\pi(f/p)} \\
&= \frac{P_{3\phi,syn}}{2\pi(N_s/60)} \\
&= \frac{332.6 \times 10^3 \times 60}{2\pi \times 3000} \\
&= \underline{\underline{1058.67\, Nm}}
\end{aligned}
$$

5.6 Relationship Between Mechanical (Rotor) Angular Displacement α_{mech} and Electrical Displacement δ.

Since one cycle of voltage (360 electrical degrees of the voltage wave) is generated every time a pair of poles passes a coil, we must distinguish between electrical degrees used to express voltage and current, and the mechanical degrees used to express position of the rotor. In a two-pole machine, electrical and mechanical degrees are equal. In an four-pole machine, two cycles or 720 electrical degrees are produced per revolution of 360 mechanical degrees.

The number of electrical degrees equals $P/2$ times the number of mechanical degrees in any machine, where P is the number of poles. Accordingly

$$\delta_{elec} = \frac{P}{2}\alpha_{mech} \tag{60}$$

5.7 Hunting of Alternators

When two alternators are operating in parallel, *any instantaneous reduction in the angular velocity of one machine*, due to variations in the load, causes:

- A change in load division between them and
- A synchronizing current to circulate

The *circulating current acts as an additional load on one machine and lightens the load on the other machine*. This *retards the former* and *permits the latter to accelerate* until the two are once more in the proper relative phase positions where no circulating currents flows, if the excitations have been equal.

The change to correct the phase position cannot be accomplished without some *overshooting* on the part of the rotors, accompanied by retardation, with a repetition of the entire cycle. *This action of the alternators in experiencing momentary speed fluctuations is termed* ***hunting***, their action being exactly equivalent to those of synchronous motors under similar conditions. The period of the swing agrees with the natural oscillating period of the rotor as a torsional pendulum.

Below are some relevant points to note about hunting.

1. Hunting generally occurs in alternators driven by reciprocating engines, because the driving torque of reciprocating engines is not uniform in a revolution of the flywheel.
2. The torque output will *pulsate* if the prime mover of one of the alternators is a reciprocating engine. If this pulsation has a forced frequency within 20% of that of the natural oscillating frequency of the alternator rotor, the oscillation following any load change will be *cumulative.* The successive oscillations increase in magnitude, causing the alternator to be thrown out of synchronism in a short time, or the loads at certain instances may be sufficient to open the circuit breakers.
3. Turbo-generators seldom hunt, because the prime mover supplies a uniform driving torque.
4. It is customary to *specify*, for alternators to be operated in parallel, the *allowable torque-angle variation*.
5. *Machines driven by internal combustion engines must have* ***large flywheels*** *or* ***heavy damping windings*** *to prevent excessive oscillations.*

5.7.1 Reducing Hunting in Alternators

The methods generally used to *reduce hunting* are:

1. Dampening of the oscillations by use of a *heavy squirrel-cage dampening winding* placed on the field poles
2. Changing the *natural* period of vibration of the machine by changing the flywheel (a *heavier flywheel* usually gives a more dampening effect).
3. Dampening the governor, if the oscillations are started by the action of the governor.

6.0 THREE-PHASE SYNCHRONOUS MOTORS:

6.1 Introduction

The *synchronous motor* is one type of 3-phase AC *motor which operates at a constant speed form no-load to full-load*. This is due to the fact that because the source frequency is fixed, the motor speed $N_s = \frac{120f}{P}$ also stays constant irrespective of the load or source voltage. The speed can be changed by changing the frequency only. *It runs either at synchronous speed or not at all*. The synchronous motor has many industrial applications because of its *fixed* speed from no-load to full-load, its high efficiency and low initial cost.

Synchronous motors cannot be used where there are sudden applications of heavy loads, because such loads will pull the rotor out of step with the rotating magnetic field. And so they are generally used for driving loads requiring constant-speed operation and infrequent starting and stopping such as d-c generators, blowers and compressors.

Synchronous motors are identical in construction to three-phase alternators, in that *it has a revolving field which must be separately excited from a DC source. Synchronous motors are used not so much because they are run at constant speed, but because they possess other unique electrical properties*. *For example, they can improve the power factor of a system or plant while carrying its load*.

By varying the DC field excitation current, synchronous motors can be made to supply either leading or lagging reactive power, thereby varying the power factor of the three-phase synchronous motor over a wide range of lagging and leading values. They can go to full rating in providing leading VAR (over-excited operation) and approximately 50% to 80% of rating in absorbing lagging VAR (under-excited operation).

The reactive power taken up by synchronous motors depends on two factors, namely, the DC field excitation and mechanical load delivered by the machine. Maximum leading power is taken by synchronous motors with maximum field excitation and at zero loads.

They may be built with either cylindrical or salient-pole rotors but the salient-pole design is more common. And so *synchronous motors are generally of the salient-pole type, whilst alternators are either of the turbo- or salient-pole type*. Most synchronous motors are rated between 15 kW and 15 MW and turn at speeds ranging from125 rev/min to 150 rev/min. Consequently these machines are mainly used in heavy industries.

A synchronous motor per se has no net starting torque, and special means must be provided for bringing it up to synchronous speed by induction-motor action, as will be described in a later section.

6.2 Constructional Features of Three-Phase Synchronous Motors

A three-phase synchronous motor consists of the following essential parts:

- *Laminated stator core* with three-phase stator (*armature*) windings
- *Rotating DC field* windings on the rotor, complete with slips rings
- Brushes and brush holders
- Two-end shields to house the bearings that support the shaft.

The stator core and its AC *stator windings* of a synchronous motor are similar to that of a 3-phase *squirrel-cage* induction motor or a wound-rotor induction motor. The leads for the stator windings terminate in a terminal box usually mounted on the side of the motor frame.

The *rotor* is generally *of salient-pole design*. *The number of rotor field poles must equal the number of stator field poles*. In order to *eliminate hunting* and to *develop the necessary starting torque* when AC voltage is applied to the stator,the rotor poles contain *damper windings of solid copper bars* which are short-circuited at each end.

The *field circuit* leads are brought out to two slip rings mounted on the rotor shaft. Carbon brushes mounted in brush holders make contact with the two slip rings. The terminals of the field circuit are brought out from the brush holders to a second terminal box mounted on the motor frame.

6.3 Principle of Operation of Synchronous Motors

In a 3-phase synchronous motor, a polyphase current is supplied to the stator winding, and it produces a ***revolving magnetic field or flux*** traveling at synchronous speed $\omega_s = \frac{2}{P}\omega$ or $N_s = \frac{120f}{P}$, as in an induction motor. A direct current is supplied to the rotor winding, and it produces a ***fixed polarity*** at each pole of the rotor.

See Fig below.

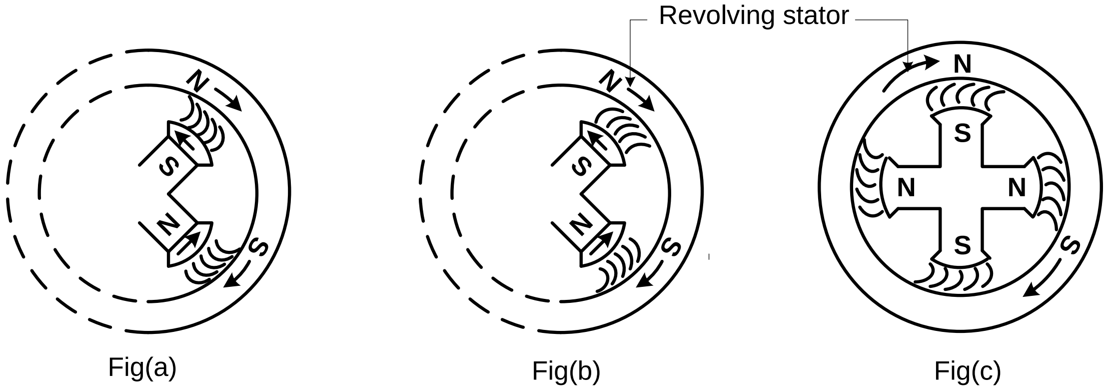


Fig: Principle of Operation of Locking Synchronous Motors Into Synchronism With The Revolving Stator Magnetic Field

6.3.1 Why Synchronous Motors Are Not Self-Starting

Suppose that the stator and rotor winding of the synchronous motor are ***energized at the same time***. *As the poles of the revolving stator magnetic field approach the fixed magnetic rotor poles of opposite polarity, they are attracted to the* ***unlike*** *poles as in Fig (a) above.* The attractive force tends to turn the rotor in the direction opposite to that of the revolving stator field. Just as the rotor gets started in this direction, the revolving stator field poles are leaving the rotor poles as in Fig (b) above, and this tends to pull the rotor poles in the opposite direction.

Hence the revolving stator magnetic field tends to pull the rotor poles first inn one direction and then in the other direction, and a result, **the starting torque is zero**. The ***synchronous motor is thus inherently not self-starting***, unless some starting mechanisms or techniques are employed.

6.3.2 Starting of Synchronous Motors – Use of *Damper Windings* as Squirrel-Cage Induction Motor

As explained in the preceding Section 8.3.1, it is practically impossible to start a synchronous motor with its d.c field energized, as the net starting torque is zero. A *squirrel-cage winding* is generally placed on the rotor poles of a synchronous motor to make the machine self-starting. ***To start the motor, the rotor winding is first left de-energized and a polyphase voltage is supplied to the stator windings***. A revolving stator magnetic field is consequently produced. This revolving stator magnetic flux cuts across the wound-rotor or squirrel-cage winding of the rotor, and induces alternating voltages which will then cause currents to flow in the squirrel-cage rotor conductors.

The resultant magnetic field produced by the induced currents in the **squirrel-cage windings** embedded in the rotor field poles **reacts** with the stator field in such a manner as to produce an electromagnetic torque that brings about the rotation of the motor, the direction of which is determined by Fleming’s Left Hand Rule. However, the *initial speed of the synchronous motor is slightly lower than the synchronous speed of the armature (stator) revolving field*. The rotor of the typical synchronous motor accelerates to about 95% to 97% of the synchronous speed.

After the motor comes up to speed close to the synchronous speed, the rotor winding is then energized with DC current from a DC supply source*. The DC current produces magnetic poles of fixed polarity at each pole of the rotor core.* Now opposite polarity poles on the stator will attract. *By this magnetic attraction, the rotor field poles are locked into synchronism with the unlike poles of the stator field, and the rotor then rotates at the same synchronous speed as that of the stator (armature) field*. See Fig (c) above.

The synchronous motor is thus inherently not self-starting, and some means must be employed to run it up to synchronous or near synchronous speed, before being synchronized to the supply.

It has been made obvious that for the production of a ***steady unidirectional*** torque, the rotating fields of stator and rotor must be traveling at the same speed, and therefore the rotor must turn at precisely the synchronous speed for the flux distribution to remain unaltered so that rotation can be maintained.

A synchronous motor connected to a constant-frequency source therefore operates at a constant steady-state speed regardless of load.

When load is applied in a synchronous motor, the rotor poles are pulled behind the stator poles through a small angle (about 20^0 in a fully loaded, 2-pole machine). Then the magnetic coupling keeps the rotor turning at exactly the same speed as that of the revolving stator field.

6.3.3 Necessary Precaution When Starting Synchronous Motor – Use of *External Starting Resistance*

Suppose that a synchronous motor is started with its rotor winding open. At the instant that its stator is energized, a rapidly revolving stator magnetic field is established, which sweeps past the squirrel-cage rotor winding at a rapid rate, since it is at standstill.

*This winding has a large number of turns and therefore an **extremely high voltage is induced in the many turns of the field winding, which will appear between its terminals**.*

A high-peak terminal voltage also appears when the rotor circuit of a machine in operation is opened. These high voltages will puncture the insulation, unless the winding is highly insulated.

In practice, to contain this phenomenon, the DC *field winding is short-circuited during the starting period through a very low resistance* connected to the switch of the rotor circuit as shown below.

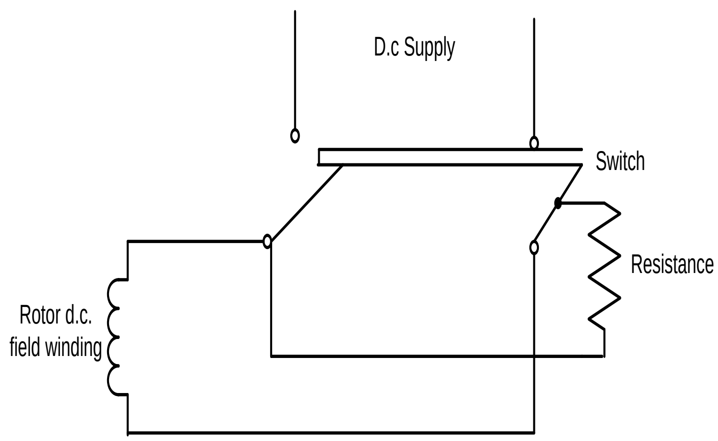


Fig: Precautionary Measure Taken During The Starting of Synchronous Motor

During the starting period, the switch is in the open position, in which case the resistance is connected across the field terminals. This allows alternating current to flow in the field winding. *Since the impedance of the field winding is high, compared to that of the external resistor, a high-voltage drop is produced in the winding, reducing the terminal voltage to a safe value*.

The voltage induced when the field switch is open is reduced by the back emf set up in the winding, as is evident from Lenz's law. This, together with the IX drop in the winding, gives a low terminal voltage.

In a synchronous motor, the rotor current is approximately in phase opposition to the induced or generated voltage. The effect of *armature reaction* is to increase the flux in one half of each pole and to decrease it in the trailing half. Consequently, the flux is distorted in the direction of rotation, and the lines of flux in the air gap are skewed in such a direction as to exert a rotational torque on the rotor.

As the rotor nears synchronous speed, DC current is applied to the rotor field coils, and the rotor is accelerated until it rotates synchronously with the stator revolving field.

6.4 Synchronous Condensers:

Synchronous condensors are actually synchronous motors that are operating on *no-load (without a connected mechanical load)* for the purpose of power factor correction or improving the voltage regulation of a transmission line. It is the only motor that can work at leading power factor (through over-excitation) and at the same time supply mechanical power. They are connected at load points in ac power networks to absorb or deliver reactive power in order to stabilize the voltage. Both rotor constructional forms are used but salient-pole design with four or six poles is usual.

6.4.1 Advantages of Synchronous Condensers over Capacitors

The principal *advantages of the synchronous condensers* are:

1. The ease with which the power factor can be controlled by either producing or absorbing reactive power. An over-excited synchronous motor having a leading power factor can be operated in parallel with induction motors having lagging power factor, thereby improving the overall power factor of the supply system.
2. Their performance can be regulated continuously and smoothly over a wide range.
3. They are more practical than capacitors, because such motors may be used to deliver a mechanical load in addition to operating with a leading power factor.
4. The speed of the synchronous motor is constant and independent of load. This characteristic is mainly of use when the motor is required to drive another alternator to generate a supply at a different frequency, as in frequency-changers.

6.4.2 Disadvantages of Synchronous Condensers compared with Capacitors

The main *disadvantages of synchronous condensers* are:

1. Some arrangement must be provided for starting and synchronizing the motor
2. A DC supply is necessary for the rotor excitation. This is usually provided by a small DC shunt generator carried on an induction motor
3. Synchronous motors contribute to the current flowing into a short-circuit fault on the system, and may call for additional expenditure in switchgear.
4. They are more expensive that static capacitors, and their use is justified only for voltage regulation of *high-voltage* transmission systems.

6.5 Effect of Loading on the Synchronous Motor

When the mechanical load on a DC or AC motor is increased, the speed N falls. Consequently, in accordance with the relation

$$E_b = E_g = \frac{\Phi ZNP}{60b} \qquad (81)$$

where Φ = flux per pole (Wb)
Z = total number of armature conductors
= no. of slots x no. of conductors per slot
P = no. of poles
b = no. of parallel paths in armature
= $2m$ for wave winding
= mP for lap winding
m = multiplicity (i.e., be it simplex, duplex, triplex, etc) of the winding
N = armature speed in revolutions per minute (*r.p.m*)

This fall in speed in turn decreases the back or counter emf E_b (equal to the generated voltage E_g), so that the source is able to supply more current (for a DC motor $I = \frac{V - E_b}{R_a}$) to meet the increased load demands. ***However, this action <u>cannot take place in the synchronous motor</u>, because the rotor must turn at synchronous speed at all loads***.

The load on the synchronous motor rather has effect on the relative position of the stator and rotor poles, i.e., on the angle between the rotating stator flux and the rotor poles.

6.5.1 Torque Angle at No-Load

The Fig below shows the relative position of the stator pole and the corresponding rotor pole of the synchronous motor at ***no-load*** as well as the corresponding no-load condition vector diagrams.

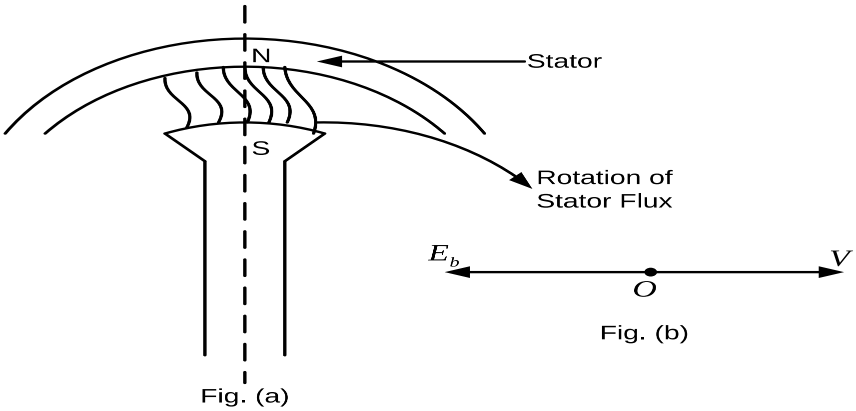


Fig (a) Torque Angle at No-Load (b) No-Load Condition Vector Diagram

At ***no-load***, the stator and rotor ***pole centres are directly in line*** with each other as seen in Fig (a). When the three-phase synchronous motor is properly synchronized to the supply and carrying no-load, the induced voltage is practically 180 degrees out of phase with the applied voltage. It is seen that $V = E_g = E_b$, and hence their vector sum is zero, and so is the stator (armature) current negligibly small. The motor intake is thus zero, as there is no load to be met by it. In other words, the motor just ***floats***.

6.5.2 Torque Angle at Rated Load

The Fig below shows the relative position of the stator pole and the corresponding rotor pole of the synchronous motor at ***rated load*** and the corresponding no-load condition vector diagrams.

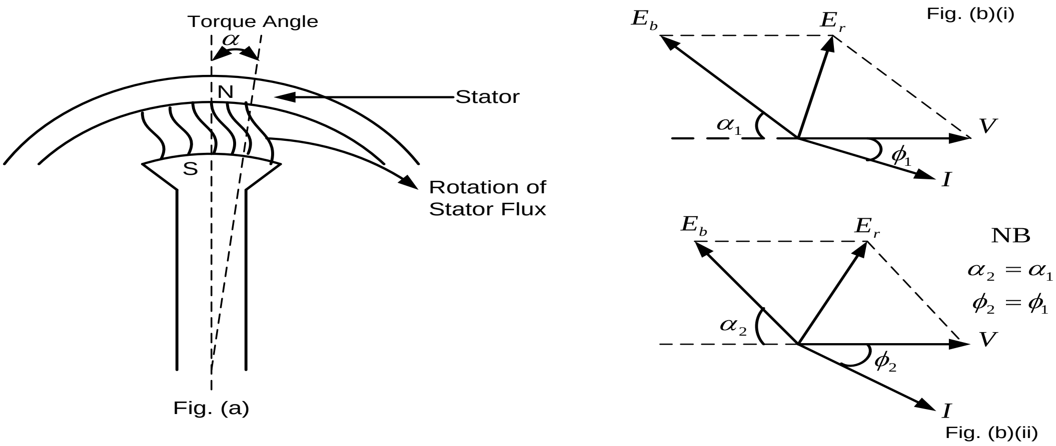


Fig (a) Torque Angle at Rated Load (b) Rated Load Condition Vector Diagrams

When the motor is loaded, it should be borne in mind that there should be no change in the overall speed as the rotor must continue to rotate at synchronous speed. Practically, however, when the motor is loaded, it *slows down momentarily* to adjust itself to the change in load condition, and thereby the rotor pole falls back a little more relative to the stator pole, as shown in Fig (a) above.

And so ***the loading causes the poles to be shifted α degrees behind their no-load positions***, as indicated in Fig (a) above, ***and consequently the induced or generated stator voltage E_g occurs α degrees later***.

NB: The emf induced in the stator coil depends on the position of the rotor with respect to the stator. The induced emf is maximum if the coil sides are opposite the pole centers, and minimum when its sides are midway between pole tips. *The angular displacement between the centres of the stator and rotor field poles is called the torque angle α.*

In a DC *motor*, the armature current I is determined by dividing the difference between the terminal voltage V and the generated voltage or back emf E_b by the armature resistance, that is, $I = \dfrac{V - E_b}{R_a}$. Similarly, in a *synchronous motor*, the stator current is determined by dividing the voltage-vector resultant E_r between V and E_b by the stator impedance, or $I = \dfrac{E_r}{Z_a} = \dfrac{E_r}{R_a + jX_a}$. Since the stator reactance is large compared with its resistance, the current $I \approx \dfrac{E_r}{jX_a}$ lags the resultant voltage E_r by nearly 90 degrees, and hence the current *I* lags the applied voltage V by ϕ degrees. See Fig (b) (i). In this case, the synchronous motor is operating with a lagging power factor.

If the load is increased, the rotor poles are pulled further behind the stator poles, and this causes E_b to lag further or α to increase. ***Hence the torque angle increases with the increase in load***. Due to the increase in load or torque angle, the resultant voltage E_r across the armature (or stator) circuit increases, and therefore, stator current I drawn from the supply mains increases while lagging the applied voltage *V* through a greater angle.

Thus a synchronous motor is able to supply increased mechanical load, not by reduction in speed, but by shift in relative positions of the rotor and revolving magnetic field (or stator flux). From Fig (b) above, it is obvious that ***for increasing load with a constant value of generated or back emf E_b, the phase angle ϕ increases in lagging direction.***

If the angle between stator and rotor pole centres becomes too large, due to heavy overloading, then the rotor will pull out of step (or out of synchronism) with the rotating stator field and operate as an induction motor with the aid of the amortisseur (squirrel-cage like) winding, and a heavy current will flow through the armature.

This causes the circuit breakers to open and the synchronous motor will come to a standstill. The maximum value of a torque which a synchronous motor can develop without pulling out of synchronism is called the ***pull-out torque***. In most synchronous motors, the pull-out torque is 150 to 200% of rated torque output. Most synchronous motors are rated larger than 100 hp and are used for many industrial applications requiring constant-speed drives.

6.6 Operating Characteristics of Synchronous Motors

Electrical utility companies usually charge industries for operating at low power factors below a specified level, say, 0.9. It is therefore desirable for industries to invest in power-factor improvement devices to improve or "correct" their power factor to avoid such charges and to make more economical use of electrical energy.

Utility companies also attempt to correct the power factor of their systems. A certain quantity of inductance is present in most of the power distribution system, including the generator windings, the transformer windings, and the power lines. To counteract the inductive effects and increase the power factor, two methods can be used. These are:

1. Power-factor corrective capacitors
2. Three-phase synchronous motors

When used only as a power-factor correcting device, in which case no-load is connected to the shaft of the three-phase synchronous motor, it is called a *synchronous condenser* or *capacitor*.

To understand how a three-phase synchronous motor machine operates as a power factor-corrective, we would consider the so-called *V-curves* (or *compounding curves*) of the synchronous motor.

6.6.1 *V-Curves* or *Compounding Curves* of The Synchronous Motor (Effect of Rotor Field Excitation on Armature Current and Power Factor *For Constant Mechanical Load*)

Synchronous motors operate at a constant speed. Thus a variation in rotor dc excitation current has no effect on the speed. *The power factor at which a synchronous motor operates, and hence its armature current, can be controlled by adjusting its DC field excitation.*

The curve showing the relation between armature current and the dc field current at a constant terminal voltage with a *constant shaft load* is known as a ***V-curve*** because of its ***characteristic V-like shape***.

Three operational conditions may exist, depending on the amount of excitation applied to the rotor. These conditions are:

1. Under-excitation – operation at lagging power factor (inductive effect)
2. Normal excitation – operation at unity power factor
3. Over-excitation – operation at leading power factor (capacitive effect)

Case 1: Under-Excitation $E_b < V$ (*Lagging* Power factor)

Suppose the *field excitation is decreased*. Since the speed is constant, the induced voltage $E_g = E_b$ *decreases*. See Fig (a) below.

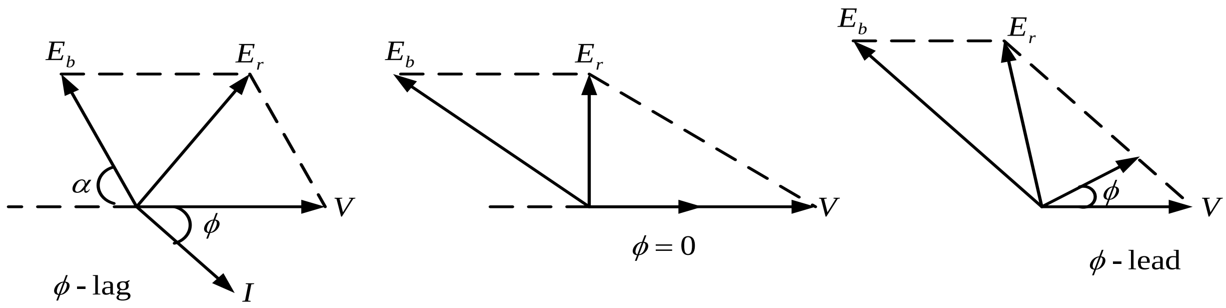


Fig (a) Fig (b) Fig (c)

Fig: Effect of Varying DC Excitation on Armature Current for Constant Shaft Load

(a) Under-Excitation (b) Normal Excitation (c) Over-Excitation

With the reduction in induced voltage (back emf), the resultant voltage E_r becomes greater. As a result, the current becomes greater and ***lags*** the applied voltage at a greater amount.

Case 2: Normal Excitation $E_b = V$ (*Unity* Power factor)

Suppose that the *field excitation is increased* (thus increasing the induced voltage) until the current is in phase with the applied voltage, making the power factor of the synchronous motor unity. For a given load at unity power factor, the resultant voltage E_r and therefore the stator current are minima. See Fig (b) above.

Case 3: Over-Excitation $E_b > V$ (*Leading* Power factor)

If the field excitation is still further increased, the induced voltage is increased, the current increases and ***leads*** the applied voltage. See Fig (c) above.

Hence for a given load, the power factor is governed by the dc field excitation. A weak field excitation produces a lagging current and a strong field produces a leading current. The current is minimum at unity power factor and increases as the power factor becomes poor, either leading or lagging.

A family of V-curves showing the variation of power factor and stator current at no-load and at full-load with a varying field excitation is shown below.

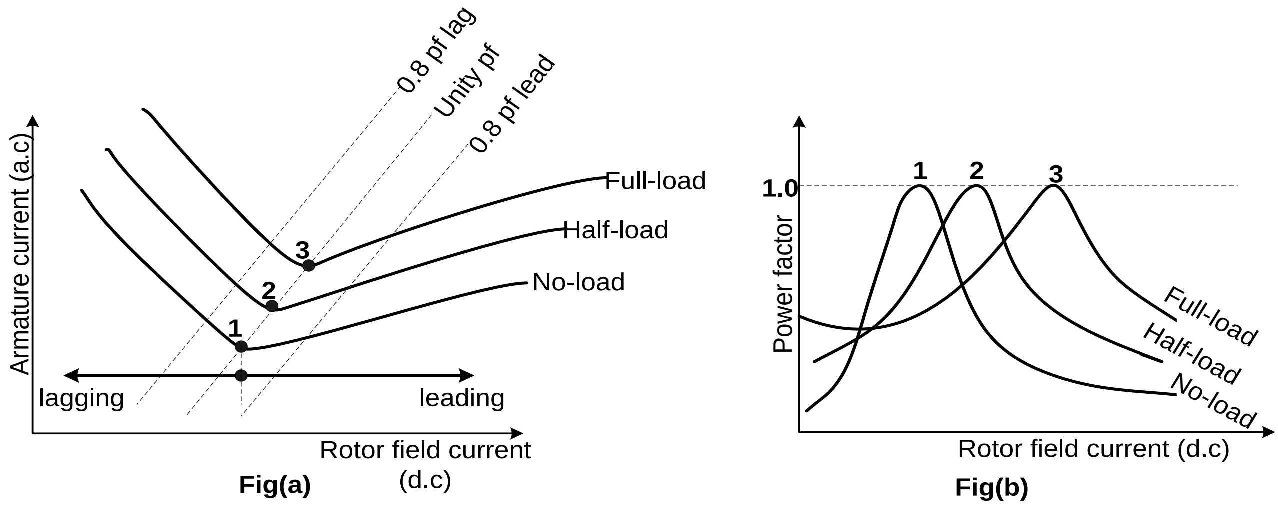


Fig: Characteristic V-Curves of a Three-Phase Synchronous Motor
(a) Stator Current versus Rotor Current at a Particular Load
(b) Power Factor versus Rotor Current at a Particular Load

Observations about V-curves of Synchronous Motor

The following points are worth noting about the V-curves:

1. Note the variation of stator current drawn by the synchronous motor as rotor dc excitation current varies. By applying a given ***constant load*** to the shaft of a synchronous motor and varying the field current from under-excitation through normal excitation to over-excitation, and recording the armature current at each step, the V-curves are obtained.
2. The situations shown on the graph of Fig (a) above indicate no-load, half-load and full-load conditions with power factors equal to unity, 0.8 leading and 0.8 lagging.
3. The graph of Fig (b) shows variation of power factor with changes in rotor dc excitation under three different load conditions (no-load, half-load and full-load).
4. Observe that for a given load or output power, the stator current is minimum when the power factor equals unity.
5. The dashed lines are loci of constant power factor (0.8 leading, unity and 0.8 lagging).
6. Points to the right of the unity power factor compounding curve correspond to over-excitation and leading current input
7. Points to the left of the unity power factor compounding curve correspond to under-excitation and lagging current input
8. The synchronous motor compounding curves, as in Fig (a) above, are very similar to the generator compounding curve. (Note the interchange of armature current and field current axes when comparing the two similar curves). In fact, if it were not for the small effects of armature resistance, the motor and generator compounding curves would be identical, except that the lagging- and leading- power-factor curves would be interchanged.

6.7 Circuit and Phasor Diagrams of Synchronous Motors

Assume the synchronous motor has been synchronized with and taking its supply from a constant-voltage constant-frequency source commonly referred to as an *infinite bus*. Depending upon the excitation, different phasor diagrams may be obtained.

6.7.1 Circuit and Phasor Diagram for Over-Excitation and Leading Pf

For a DC field over-excitation in *leading power factor* at rated load, the equivalent circuit and the corresponding phasor diagram of a synchronous motor are as given in the Fig below.

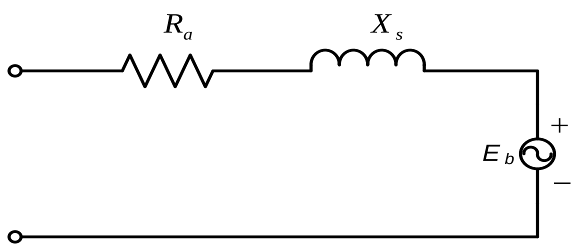


Fig (a)

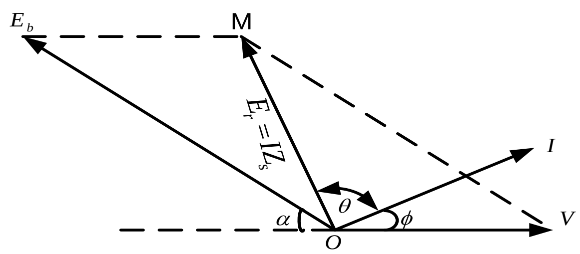


Fig (b)

Fig (a) Equivalent Circuit (b) Phasor Diagram of Synchronous Motor For Over-Excitation and Leading Power Factor ϕ

OV	= supply voltage per phase
I	= leading armature current
VM	= back emf at a torque (or load) angle α
OM	= impedance drop per phase
	= resultant voltage $E_r = I \cdot Z_s \approx I \cdot X_s$ (if R_a is negligible).
E_b	= generated emf per phase

NB: The current $I = \dfrac{E_r}{Z_s}$ *leads* the supply voltage V by the leading power factor angle ϕ but *lags* the behind the resultant voltage E_r by an angle θ (internal angle)

$$\theta = \tan^{-1}\left(\frac{X_s}{R_a}\right) \tag{82}$$

6.7.2 Generated emf per phase in Synchronous Motor

In general, the generated emf *per phase* for a given power factor is:

$$\begin{aligned} E_b &= \sqrt{V^2 + E_r^2 - 2 \times V \times E_r \cos(\theta \pm \phi)} \\ &= \sqrt{V^2 + (IZ_s)^2 - 2 \times V \times IZ_s \cos(\theta \pm \phi)} \end{aligned}$$

$$\begin{aligned} &+\text{sign} &&: \text{leading pf} \\ &-\text{sign} &&: \text{lagging pf} \\ &\phi = 0 &&: \text{unity pf} \end{aligned} \qquad (83)$$

6.7.3 Mechanical Power per Phase Developed By Synchronous Motor

The mechanical power developed *per phase* in the rotor is

$$P_{mech} = \frac{E_b V}{Z_s}\cos(\theta - \alpha) - \frac{E_b^2}{Z_s}\cos\theta \qquad (84)$$

This is the expression for the mechanical power developed in terms of the load angle α and the ***internal angle*** θ of the motor for a constant voltage V and induced voltage E_b (or excitation because E_b depends on excitation only).

6.7.3.1 Maximum Mechanical Power Developed in Synch. Motor

The condition for maximum power developed can be found by differentiating the power expression of Eqn (83) with respect to the load angle, and then equating it to zero.

$$\frac{dP_{mech}}{d\alpha} = -\frac{E_b V}{Z_s}\sin(\theta - \alpha) = 0$$

$$\Rightarrow \sin(\theta - \alpha) = 0 \qquad (85)$$

$$\Rightarrow \theta = \alpha$$

The value of maximum power is then obtained for $\theta = \alpha$ as

$$\begin{aligned} P_{mech\max} &= \frac{E_b V}{Z_s} - \frac{E_b^{\,2}}{Z_s}\cos\alpha \\ &= \frac{E_b V}{Z_s} - \frac{E_b^{\,2}}{Z_s}\cos\theta \qquad (86) \\ &= \frac{E_b}{Z_s}\left(V - E_b\cos\theta\right) \end{aligned}$$

Notes on maximum power developed:

1. The *maximum power and hence torque* (speed is constant at synchronous value) *depends on* V *and* E_b*, i.e., excitation*.
2. The maximum value of θ and hence α is 90^0. For all values of V and E_b, this limiting value of α is the same, but maximum torque will be proportional to the maximum power developed as given in Eqn (86).

The graph of the mechanical power *per phase* developed in the rotor of the synchronous motor is plotted against the coupling angle α.

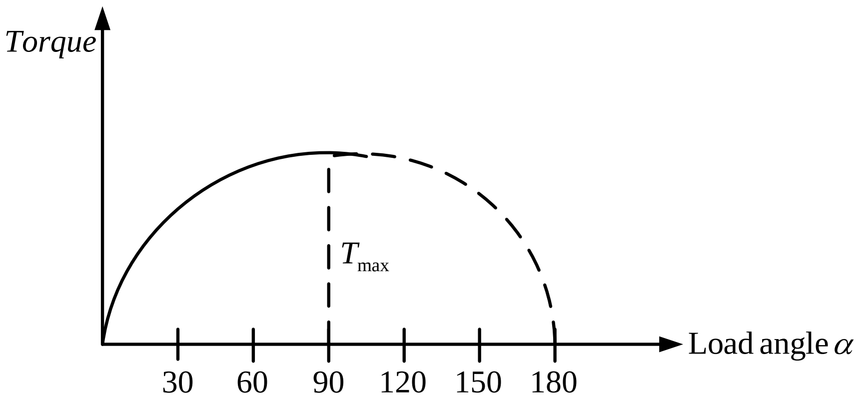


Fig: Mechanical Torque of Synchronous Motor as Function of Load Angle α

6.7.3.2 Mechanical Power Developed in Rotor of Synchronous Motor for Negligible Armature Resistance $R_a \cong 0$:

If armature resistance R_a is neglected, then

$$Z_s \approx X_s \quad \text{and} \quad \theta = \tan^{-1}\left[\frac{X_s}{(R_a = 0)}\right] = 90^0 \qquad (87)$$

Hence for negligible armature resistance $R_a \cong 0$, the mechanical power expression of Eqn (83) reduces to

$$\begin{aligned} P_{mech} &= \frac{E_b V}{X_s}\cos(90^0 - \alpha) \\ &= \frac{E_b V}{X_s}\sin\alpha \quad (\text{for negligible armature resistance}) \end{aligned} \qquad (88)$$

Eqn (88) is the value of the mechanical power developed in terms of α, the basic variable of a synchronous machine.

Maximum Power from Synchronous Motor

Generally, the maximum torque from a synchronous motor is

$$P_{mech\,\max} = \frac{E_b V}{X_s} \qquad \text{armature resistance neglected}$$

$$= \frac{E_b}{Z_s}\left(V - E_b \cos\theta\right) \qquad \text{armature resistance considered} \tag{89}$$

where $\cos\theta = \frac{R_a}{Z_s}$

This corresponds to the *"pull-out"* torque

$$T_{\max} = \frac{P_{mech\max}}{\omega_s} = \frac{P_{mech\max}}{2\pi(f/p)} \tag{90}$$

where f is the frequency and p is the *pole pairs*.

6.7.4 Circuit and Phasor Diagram for Normal Excitation and Unity Pf

Assuming that the dc field excitation is held constant at a value which results in *unity power factor* at rated load, the equivalent circuit and the corresponding phasor diagram of a synchronous motor are as given in the Fig below.

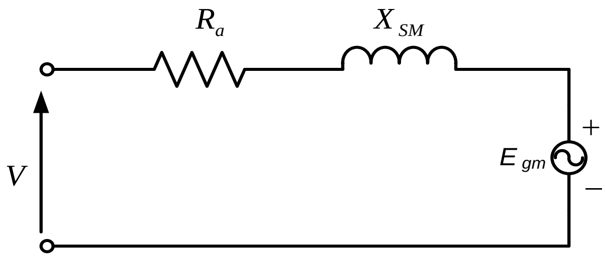


Fig (a)

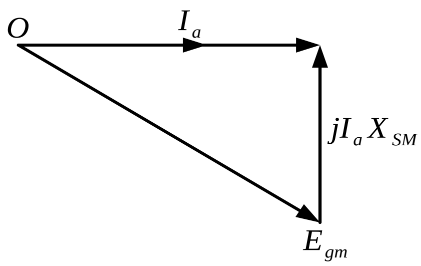


Fig (b)

Fig (a) Equivalent Circuit (b) Phasor Diagram of Synchronous Motor at Normal Excitation and Unity Power Factor

From the phasor diagram at full-load,

$$E_b = \sqrt{V^2 + (IZ_s)^2} \tag{91}$$

Example 15:

A 2000 hp 1.0 power factor 3-phase star-connected 2300 V 30 pole 60 Hz synchronous motor has a synchronous reactance of 1.95 ohms per phase. For the purposes of this problem, all losses may be neglected.

(a) Compute the maximum torque which this motor can deliver if it is supplied with power from a constant-voltage constant-frequency source (called *infinite bus*), and if its field excitation is constant at a value which results in 1.0 power factor at rated load.

(b) Instead of the infinite bus of part (a), suppose that the motor were supplied with power from a 3-phase star-connected 2300 V 1750 kVA 2-pole 3600 rpm turbine generator whose synchronous reactance is 2.65 ohms per phase. The generator is driven at rated speed, and the field excitations of generator and motor are adjusted so that the motor runs at 1.0 power factor and rated terminal voltage at full load. The field excitations of both machines are then held constant, and the mechanical load on the synchronous motor is gradually increased. Compute the maximum motor torque under these conditions. Also compute the terminal voltage when the motor is delivering its maximum torque.

Solution 15:

Although this machine is undoubtedly of the salient pole type, the problem shall be solved by the simple cylindrical-rotor theory. The solution accordingly neglects reluctance torque. The machine would actually develop a maximum torque somewhat greater than the computed value.

The equivalent circuit and phasor diagram at full-load are shown below, where E_{gm} is the excitation voltage of the motor and X_{sm} is its synchronous reactance.

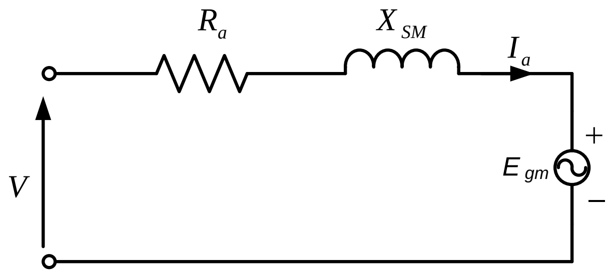


Fig (a)

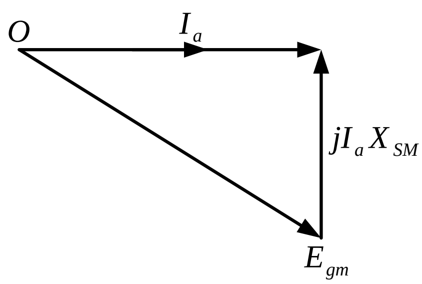


Fig (b)

Fig: Equivalent Circuit and Phasor Diagram for Example7

From the motor rating with *losses neglected*,

$$\text{Rated kVA, } S_n = \frac{P}{\cos\phi} = \frac{2000\times 0.746}{1.0} = 1492\,kVA\ (3-\text{phase}) = \underline{497\,kVA\ (\text{per phase})}$$

$$\text{Rated voltage per phase, } V_\phi = \frac{V_L}{\sqrt{3}} = \frac{2300}{1.732} = \underline{1330V}$$

$$\text{Rated current } I_a = \frac{S_\phi}{V_\phi} = \frac{49700}{1330} = \underline{374\,A} \text{ (star-connected)}$$

From the phasor diagram, at full-load and neglecting losses ($R_a \cong 0$)

$$E_{gm} = \sqrt{V^2 + (I_a X_{sm})^2} = \sqrt{1330^2 + (374 \times 1.95)^2} = \underline{1515V}$$

With an *infinite bus* as power source and the field excitation being constant, then V and E_{gm} are also constant.

The maximum power is given as

$$P_{\max} = \frac{VE}{X} = \frac{VE_{gm}}{X_{sm}} = \frac{1330 \times 1515}{1.95} = 1030kW \text{ per phase} = \underline{3090kW} \text{ (3-phase)}$$

With 30 poles (or 15 pole pairs) at 60 Hz frequency, the maximum torque is given as

$$T_{\max} = \frac{P_{\max}}{\omega_s} = \frac{P_{\max}}{2\pi(f/p)} = \frac{3090}{2\pi \times (60/15)} = \underline{123kN-m}$$

When the power source is the turbine generator, the equivalent circuit and phasor diagram are that shown below, where E_{gg} is the excitation voltage of the generator and X_{sg} is its synchronous reactance

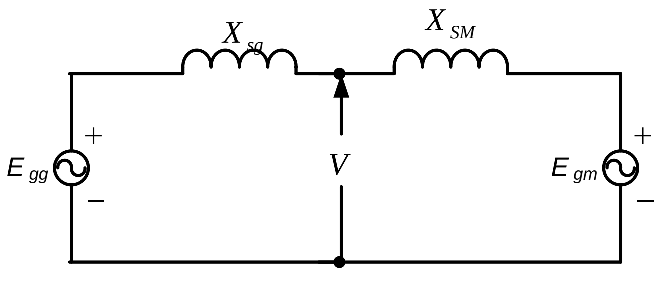


Fig (a)

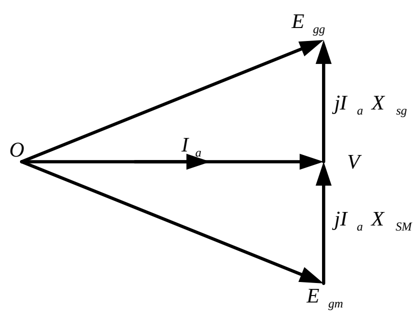


Fig (b)

Fig: Equivalent Circuit And Phasor Diagram With Turbine Generator as Power Source

As before, V = 1330 V at full-load, and E_{gm} = 1515 V. From the phasor diagram at full-load and neglecting losses ($R_a \cong 0$)

$$E_{gg} = \sqrt{V^2 + (I_a X_{sg})^2} = \sqrt{1330^2 + (374 \times 2.65)^2} = \underline{1655V}$$

Since the field excitations and speeds of both machines are constant, E_{gg} and E_{gm} are constant. The maximum power in this instance is given as

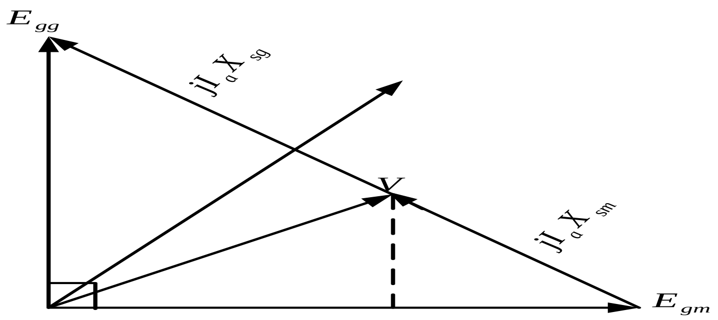
$$P_{\max} = \frac{VE}{X} = \frac{E_{gg} \cdot E_{gm}}{X_{sg} + X_{sm}} = \frac{1655 \times 1515}{(1.95 + 2.65)} = 545\,kW \text{ per phase} = \underline{1635\,kW} \text{ (3-phase)}$$

The maximum torque in this instance is then given as

$$T_{\max} = \frac{P_{\max}}{\omega_s} = \frac{P_{\max}}{2\pi(f/p)} = \frac{1635}{2\pi \times (60/15)} = \underline{65\,kN - m}$$

NOTE: Synchronism would be lost if a load torque greater than this *"pull-out"* torque were applied to the motor shaft. In that case, the motor would stall, the generator would tens to speed, and so the circuit would be interrupted by circuit breaker action.

With fixed excitations, *maximum power occurs when* E_{gg} *leads* E_{gm} *by* 90^0 as shown in the phasor diagram below.



From the phasor diagram,

$$\left[I_a(X_{sg} + X_{sm})\right]^2 = E_{gg}^2 + E_{gm}^2 \qquad \Rightarrow I_a = \frac{\sqrt{E_{gg}^2 + E_{gm}^2}}{(X_{sg} + X_{sm})} = \frac{\sqrt{1655^2 + 1515^2}}{(1.95 + 2.65)} = \underline{488\,A}$$

$$\cos\alpha = \frac{E_{gm}}{I_a(X_{sg} + X_{sm})} = \frac{1515}{488(2.65 + 1.95)} = \underline{0.767}$$

$$\sin\alpha = \frac{E_{gg}}{I_a(X_{sg} + X_{sm})} = \frac{1655}{488(2.65 + 1.95)} = \underline{0.739}$$

The phasor equation for the terminal voltage is

$$\begin{aligned} V &= E_{gm} - I_a X_{sm}\cos\alpha + jI_a X_{sm}\sin\alpha \\ &= 1515 - (488 \times 1.95 \times 0.676) + j(488 \times 1.95 \times 0.739) \\ &= 872 + j703 \\ &= 1120V \text{ per phase} \\ &= \underline{1940V} \text{ (line-to-line)} \end{aligned}$$

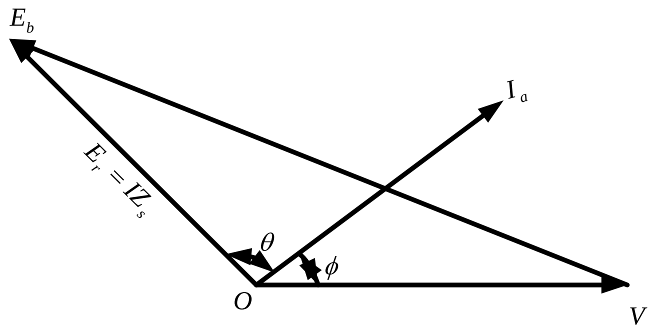
NB: When the source is the turbine generator, as in part (b), the effect of its impedance causes the terminal voltage to decrease with increasing load, thereby reducing the maximum power from 3090 kW in part (a) to 1635 kW in part (b).

Example 16:

A 2200 V, 3-phase star-connected synchronous motor has a resistance of 0.22 Ω/phase and a synchronous reactance of 2.4 Ω/phase. The motor is operating at 0.6 power factor leading with a line current of 180 A. Determine the value of the generated emf per phase.

Solution 16:

The phasor diagram is as shown below.



The rated voltage per phase is $V_\phi = \frac{V_L}{\sqrt{3}} = \frac{2200}{1.732} = \underline{1270V}$

The impedance drop per phase is given as:

$$E_r = I \cdot Z_s = I\sqrt{R_a{}^2 + X_s{}^2} = 180\sqrt{(0.22)^2 + (2.4)^2} = 433.8V$$

The internal angle is given as

$$\theta = \tan^{-1}\frac{X_s}{R_a} = \tan^{-1}\frac{2.4}{0.22} \quad \Rightarrow \theta = \underline{84.8^0}$$

The power factor (*leading)* is $\phi = \cos^{-1} 0.6 = \underline{53.13^0 (lead)}$

The generated emf per phase for *leading* power factor is

$$\begin{aligned} E_b &= \sqrt{V^2 + E_r{}^2 - 2 \cdot V \cdot E_r \cos(\theta + \phi)} \\ &= \sqrt{(1270)^2 + (433.8)^2 - 2 \times 1270 \times 433.8 \times \cos(84.8^0 + 53.13^0)} \\ &= \underline{1618.3V \text{ per phase}} \end{aligned}$$

Hence synchronous emf (line) is $\sqrt{3} \times E_b = \sqrt{3} \times 1618.3 = \underline{2803V}$

Example 17:

A 11 kV, 3-phase star-connected synchronous motor draws a current of 45 A, and has effective resistance and synchronous reactance per phase of 0.9 Ω and 28 Ω respectively. Calculate the power supplied to the motor and the induced emf when the motor is operating at a power factor of: (a) 0.8 lagging (b) 0.8 leading

Solution 17:

The phasor diagrams for lagging and leading power factors are as shown in the Figs (a) and (b) below.

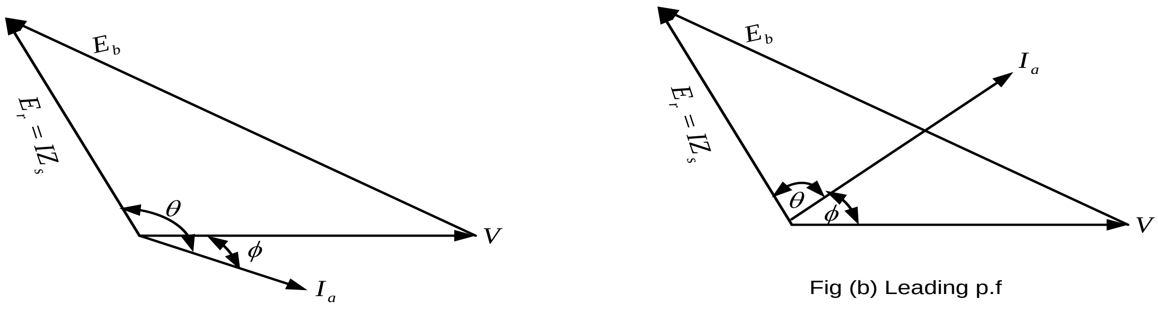


Fig (a) Lagging p.f

Fig (b) Leading p.f

Fig: Phasor Diagrams for Lagging and Leading Power Factors

The supply voltage per phase is $V_\phi = \dfrac{V_L}{\sqrt{3}} = \dfrac{11{,}000}{1.732} = \underline{6351V}$

The impedance drop per phase is given as:

$$E_r = I \cdot Z_s = I\sqrt{R_a{}^2 + X_s{}^2} = 45\sqrt{(0.9)^2 + (28)^2} = \underline{1260V}$$

The *internal* angle is given as

$$\theta = \tan^{-1}\frac{X_s}{R_a} = \tan^{-1}\frac{28}{0.9} \quad \Rightarrow \theta = \underline{88.1^0}$$

(a) For 0.8 power factor ***lagging***:

The power supplied to the motor is

$$P_{in} = \sqrt{3}E_L I_L \cos\phi = \sqrt{3}\times 11{,}000\times 45\times 0.8 = \underline{685.9\,kW}$$

The generated (induced) emf per phase is

$$\begin{aligned} E_b &= \sqrt{V^2 + E_r^{\ 2} - 2\cdot V\cdot E_r\cos(\theta-\phi)} \\ &= \sqrt{(6351)^2 + (1260)^2 - 2\times 6351\times 1260\times\cos(88.1^0 - 36.9^0)} \\ &= \underline{5647.5\,V \text{ per phase}} \end{aligned}$$

Hence induced synchronous emf (line) is $\sqrt{3}\times E_b = \sqrt{3}\times 5647.5 = \underline{9781.7\,V}$

(b) For 0.8 power factor ***leading***:

The power supplied to the motor is

$$\begin{aligned} P_{in} &= \sqrt{3}E_L I_L \cos\phi \\ &= \sqrt{3}\times 11{,}000\times 45\times 0.8 \\ &= \underline{685.9\,kW} \end{aligned}$$

The generated (induced) emf per phase is

$$\begin{aligned} E_b &= \sqrt{V^2 + E_r^{\ 2} - 2\cdot V\cdot E_r\cos(\theta+\phi)} \\ &= \sqrt{(6351)^2 + (1260)^2 - 2\times 6351\times 1260\times\cos(88.1^0 + 36.9^0)} \\ &= \underline{7148.6\,V \text{ per phase}} \end{aligned}$$

Hence induced synchronous emf (line) is $\sqrt{3}\times E_b = \sqrt{3}\times 7148.6 = \underline{12381.4\,V}$

Example 18

A 2000 V 3-phase 4-pole star-connected synchronous motor runs at 1500 rpm. The excitation is constant and corresponds to an open-circuit voltage of 2000 V. The resistance is negligible compared to the synchronous reactance of 3.5 Ω per phase. For an armature current of 200 A, determine

(a) Power factor
(b) Power input
(c) Torque developed

Solution 18

Supply voltage is $V = \dfrac{2000}{\sqrt{3}} = \underline{1155V}$

Induced (back) emf per phase $E_b = \dfrac{2000}{\sqrt{3}} = \underline{1155V}$

Since *resistance is negligible*, synchronous impedance $Z_s = X_s = \underline{3.5\,\Omega}$

Internal angle $\theta = \tan^{-1}\left(\dfrac{X_s}{R_a}\right) = \tan^{-1}\left(\dfrac{3.5}{0}\right) = \underline{90^0}$

Impedance drop $E_r = IZ_s = 200 \times 3.5 = \underline{700V}$

Assuming the armature current *lagging* behind the supply voltage, we have

$$
\begin{aligned}
&E_b = \sqrt{V^2 + E_r^{\;2} - 2 \cdot V \cdot E_r \cos(\theta - \phi)} \\
&\Rightarrow (1155)^2 = (1155)^2 + (700)^2 - 2 \times 1155 \times 700 \cos(90 - \phi) \\
&\Rightarrow \sin\phi = \frac{700^2}{2 \times 1155 \times 700} \\
&\Rightarrow \phi = 17.6^0
\end{aligned}
$$

(a) Power factor $\cos\phi = \cos 17.6^0 = \underline{\underline{0.9532}}$

(b) Power input is $P_{in} = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 2000 \times 200 \times 0.9532 = \underline{\underline{660.396\,kW}}$

(c) Torque developed $T = \dfrac{P_{in} - losses}{2\pi N_s / 60} = \dfrac{(660396 - 0)}{2\pi \times 1500/60} = 4204.2\ Nm$

Example 19

A 6600 V 3-phase star-connected synchronous motor draws a full-load current of 80 A at 0.8 p.f. leading. The synchronous impedance per phase is $(2.2 + j22)\ \Omega$. If the stray losses of the machine are 3200 W, find:

(a) Induced emf
(b) Output power
(c) Efficiency of the machine

Solution 19

Supply voltage is $V = \frac{6600}{\sqrt{3}} = \underline{3810V}$

Internal angle $\theta = \tan^{-1}\left(\frac{X_s}{R_a}\right) = \tan^{-1}\left(\frac{22}{02.2}\right) = \underline{84.3^0}$

Impedance drop $E_r = IZ_s = I\sqrt{R_a{}^2 + X_s{}^2} = 80\sqrt{(2.2)^2 + (22)^2} = \underline{1768.8V}$

(a) Induced emf per phase for *leading* current is

$$\begin{aligned} E_b &= \sqrt{V^2 + E_r{}^2 - 2 \cdot V \cdot E_r \cos(\theta + \phi)} \\ &= \sqrt{(3810)^2 + (1768.8)^2 - 2 \times 3810 \times 1768.8 \times \cos(84.3^0 + 36.9^0)} \\ &= \underline{4962.5V \text{ per phase}} \end{aligned}$$

Hence induced line emf is $\sqrt{3} \times E_b = \sqrt{3} \times 4962.5 = \underline{\underline{8595V}}$

(b) Total input power $P_{in} = \sqrt{3} E_L I_L \cos\phi = \sqrt{3} \times 6600 \times 80 \times 0.8 = \underline{731618W}$

Total copper losses $P_{Cu} = 3I^2 R_a = 3 \times (80)^2 \times 2.2 = \underline{42240W}$

Total stray losses $P_{stray} = \underline{3200W}$

Hence power output of the machine is

$$\begin{aligned} P_{out} &= \text{power input - copper losses- stray losses} \\ &= 731618 - 42240 - 3200 \\ &= \underline{\underline{686178W}} \end{aligned}$$

(c) Efficiency $\eta = \left(\frac{output}{input}\right) \times 100 = \left(\frac{686178}{731618}\right) \times 100 = \underline{\underline{93.79\,\%}}$

Exercises on Maximum Power Output

Exercise 1

A 3-phase 11 kV 10 MW star-connected synchronous generator has a synchronous impedance of $(0.6 + j10)\ \Omega$ per phase. If the excitation is such that the open-circuit voltage is 12 kV, determine:

(a) Maximum output of the generator
(b) Current and p.f. at the maximum output

Hint: Assume negligible armature resistance

[Ans: 13,200 kW ; 939.8 A ; 0.737 lead]

Exercise 2

A 3-phase 11 kV 5 MVA star-connected alternator has a synchronous impedance of $(1 + j10)\ \Omega$ per phase. Its excitation is such that the generated line emf is 14 kV. If the alternator is connected to infinite busbars, determine maximum output at the given excitation

[Ans: 14133 kW]

Exercise 3

A 3-phase 11 kV 10 MW star-connected synchronous alternator has a synchronous impedance of $(0.8 + j8)\ \Omega$ per phase. If its excitation is such that the open-circuit voltage is 14 kV, determine:

(a) Maximum output of the generator
(b) Current and p.f. at the maximum output

[Ans: 19.25 MW ; 1287 A ; 0.78 lead]

Exercises on Voltage Regulation

Exercise 4

A 3300 V 3-phase star-connected alternator has a full-load current of 100 A. On short-circuit, a field current of 5 A was necessary to produce full-load current. The emf on open-circuit for the same excitation was 900 V. The armature effective resistance per phase was $0.8\,\Omega$. Calculate the full-load percentage regulation for a load of

(a) 0.8 lagging [Ans: 36.18 %]
(b) 0.8 leading [Ans: -14.80 %]

[Hint: You need to calculate the synchronous impedance and reactance]

Exercise 5

A 3-phase star-connected alternator is rated at 1600 kVA13500 V. The armature effective resistance and synchronous reactance are 1.5Ω and 30 Ω respectively per phase. Calculate the percentage regulation for a load of 1280 kW at power factors of

(a) 0.8 lagging [Ans: 18.59 %]
(b) 0.8 leading [Ans: -11.98 %]

Exercise on Parallel Operation of Alternators

Exercise 6

Two exactly similar turbo-alternators are rated 20 MW each and run in parallel. The speed-load characteristics of the driving turbines are such that the frequency of Alternator 1 drops uniformly 50 Hz on no-load to 48 Hz on full-load and that of Alternator 2 from 50 Hz to 8.5 Hz. How will the two machines share a load of 30 MW?

[Ans: 12.8572 MW ; 17.1428 MW]

Exercises on Synchronizing Power and Torque

Exercise 7

A 4 MVA 10 kV 1500 rpm 50 Hz alternator runs in parallel with other machines The synchronous reactance is 25 %. Find for no-load situation the synchronising power per unit mechanical angle of phase displacement as well as the corresponding synchronizing torque if the mechanical displacement is 0.7 degree.

[Ans: 558.6 kW ; 2489.3 Nm]

Exercise 8

A 4500 kVA 50 Hz 3-phase star-connected synchronous generator having a synchronous reactance of 0.3 p.u is running at 1500 rpm and is excited to give 11000 V. If the rotor deviates slightly from its equilibrium position, what is the synchronizing torque in N-m per degree of mechanical displacement?

Hint: The synchronous reactance has been given in per unit, and there is the need to determine the actual ohmic value. Taking the generated phase voltage E as the reference, the actual synchronous reactance drop is calculated as $IX_s = 0.3 \times E$. In this way, the synchronous reactance can be calculated.

[Ans: 524.06 kW ; 3336 N-m]

Exercises on Synchronous Motors

Exercise 9

A 2300 V 3-phase star-connected synchronous motor has a resistance of 0.2 Ω per phase and a synchronous reactance of 2.2 Ω per phase. The motor is operating at 0.5 p.f. leading with a line current of 200 A. Determine the value of the generated emf per phase. [Ans: 1708 V]

Exercise 10

A 6600 V 3-phase star-connected draws a full-load current of 70 A at 0.8 p.f. leading. Find the emf induced, output power and efficiency of the machine if the stray losses total 30 kW, and the armature resistance is 2 Ω per phase and the synchronous reactance is 20 Ω per phase

[Ans: 8130 V (line) ; 580.6 kW ; 90.72 %]

Exercise 11

The input to a 11,000 V 3-phase star-connected synchronous motor is 50 A. The effective resistance and synchronous reactance per phase are 0.95 Ω and 29 Ω respectively. Calculate the power supplied to the motor and the induced emf for a power factor of

(a) 0.8 lagging
(b) 0.8 leading

[Ans: (a) 9635 V ; 762.1 kW ; (b) 12610 V ; 762.1 kW]

Exercise 12

A 2500 V 3-phase 4-pole star-connected synchronous motor runs at 1500 rpm. The excitation is constant and corresponds to an open-circuit voltage of 2500 V. The resistance is negligible in comparison with the synchronous reactance of 3 Ω per phase. Determine the power input, power factor, and torque developed for an armature current of 250 A.

[Ans: 1045 kW ; 0.9656 lag ; 6655 Nm]

Exercises

1. A 6 pole round rotor 3 phase star connected synchronous machine has the following test results:
Open circuit test: 4000 V line to line at 1000 rev/min
50 A rotor current
Short circuit test: 300 A at 500 rev/min
50 A rotor current
Neglect the stator resistance and core losses, and assuming a linear open circuit characteristic, calculate:
(a) the machine synchronous reactance at 50 Hz,
(b) the rotor current required for the machine to operate as a motor at 0.8 power factor leading from a supply of 3.3 kV line to line with an output power of 1000 kW,
(c) the rotor current required for the machine to operate as a generator on an infinite bus of 3.3 kV line to line when delievering 1500 kVA at 0.8 power factor lagging,
(d) the load angle for (b) and (c), and
Sketch the phasor diagram for (b) and (c).
Answer: 7.7 W, 69.54 A, 76 A, 24.8_o, 27.4_o
2. For a 3 phase star connected 2500 kVA 6600 V synchronous generator operating at full load, calculate
(a) the percent voltage regulation at a power factor of 0.8 lagging,
(b) the percent voltage regulation at a power factor of 0.8 leading.
The synchronous reactance and the armature resistance are 10.4 Ω and 0.071 Ω respectively.
Answer: 44%, -20%
3. Determine the rotor speed in rev/min of the following 3 phase synchronous machines:
(a) f = 60 Hz, number of poles = 6,
(b) f = 50 Hz, number of poles = 12, and
(c) f = 400 Hz, number of poles = 4.
Answer: 1200 rev/min, 500 rev/min, 12000 rev/min
4. A Y connected 3 phase 50 Hz 8 pole synchronous alternator has a induced voltage of 4400 V between the lines when the rotor field current is 10 A. If this alternator is to generate 60 Hz voltage, compute the new synchronous speed and induced voltage for the same rotor current of 10 A.
Answer: E_{new}*=5280 V,* $n_{syn-new}$*=900 rev/min*
5. A 3 phase Y connected 6 pole alternator is rated at 10 kVA 220 V at 60 Hz. Synchronous reactance X_s=3 Ω. The no load line to neutral terminal voltage at 1000 rev/min follows the magnetization curve shown below. Determine
(a) the rated speed in rev/min,
(b) the field current required for full load operation at 0.8 power factor lagging.
E (V) 11 38 70 102 131 156 178 193 206 215 221 224
I_f (A) 0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2
Synchronous Machines
31
Answer: n_{syn}*=1200 rev/min,* I_f*=1.0 A*
6. The alternator of question 3 s rated at 10 kVA 220 V 26.2 A at 1200 rev/min. Determine the torque angle and the field current for unity power factor operation as a motor at rated load.
Answer: torque angle=32_o*,* I_f *= 0.75 A*
7. A 3 phase induction furnace draws 7.5 kVA at 0.6 power factor lagging. A 10 kVA synchronous motor is available. If the overall power factor of the combination is to be unity, determine the mechanical load which can be carried by the motor.
Answer: 8 kW
8. The following data are taken from the open circuit and short circuit characteristics of a 45 kVA 3 phase Y connected 220 V (line to line) 6 pole 60 Hz synchronous machine:
from the open circuit characteristic: Line to line voltage = 220 V
Field current = 2.84 A
from the short circuit characteristic: Armature current (A) 118 152
Field current (A) 2.20 2.84
from the air gap line: Field current = 2.20 A
Line to line voltage = 202 V

Calculate the unsaturated value of the synchronous reactance, and its saturated value at the rated voltage. Express the synchronous reactance in ohms per phase and also in per unit on the machine rating as a base.
Answer: 0.987 W per phase, 0.92 per unit, 0.836 W per phase, 0.775 per unit
9. From the phasor diagram of a synchronous machine with constant synchronous reactance X_s operating a constant terminal voltage V_t and constant excitation voltage E_f, show that the locus of the tip of the armature current phasor is a circle. On a phasor diagram with terminal voltage chosen as the reference phasor indicate the position of the center of this circle and its radius. Express the coordinates of the center and the radius of the circle in terms of V_t, E_f, and X_s.