

## Chapter 7: Exergy: A Measure of Work Potential

The energy content of the universe is constant, just as its mass content is. Yet at times of crisis we are bombarded with speeches and articles on how to “conserve” energy. As engineers, we know that energy is already conserved. What is not conserved is *exergy*, which is the useful work potential of the energy. Once the exergy is wasted, it can never be recovered. When we use energy (to heat our homes, for example), we are not destroying any energy; we are merely converting it to a less useful form, a form of less exergy.

### Exergy and the Dead State

The useful work potential of a system at the specified state is called *exergy*. Exergy is a property and is associated with the state of the system and the environment. A system that is in equilibrium with its surroundings has zero exergy and is said to be at the *dead state*. The exergy of the thermal energy of thermal reservoirs is equivalent to the work output of a Carnot heat engine operating between the reservoir and the environment.

### Reversible Work

*Reversible work*  $W_{\text{rev}}$  is defined as the maximum amount of useful work that can be produced (or the minimum work that needs to be supplied) as a system undergoes a process between the specified initial and final states. This is the useful work output (or input) obtained when the process between the initial and final states is executed in a totally reversible manner.

### Irreversibility

The difference between the reversible work  $W_{\text{rev}}$  and the useful work  $W_u$  is due to the irreversibilities present during the process and is called the *irreversibility*  $I$ . It is equivalent to the *exergy destroyed* and is expressed as

$$I = X_{\text{destroyed}} = T_o S_{\text{gen}} = W_{\text{rev}, \text{out}} - W_{u, \text{out}} = W_{u, \text{in}} - W_{\text{rev}, \text{in}}$$

where  $S_{\text{gen}}$  is the entropy generated during the process. For a totally reversible process, the useful and reversible work terms are identical and thus irreversibility is zero.

Exergy destroyed represents the lost work potential and is also called the *wasted work* or *lost work*.

### **Second-Law Efficiency**

The *second-law efficiency* is a measure of the performance of a device relative to the performance under reversible conditions for the same end states and is given by

$$\eta_{II} = \frac{\eta_{th}}{\eta_{th, rev}} = \frac{W_u}{W_{rev}}$$

for heat engines and other work-producing devices and

$$\eta_{II} = \frac{COP}{COP_{rev}} = \frac{W_{rev}}{W_u}$$

for refrigerators, heat pumps, and other work-consuming devices.

In general, the second-law efficiency is expressed as

$$\eta_{II} = \frac{\text{Exergy recovered}}{\text{Exergy supplied}} = 1 - \frac{\text{Exergy destroyed}}{\text{Exergy supplied}}$$

### **Exergy Forms**

Now let's determine the exergy of various forms of energy.

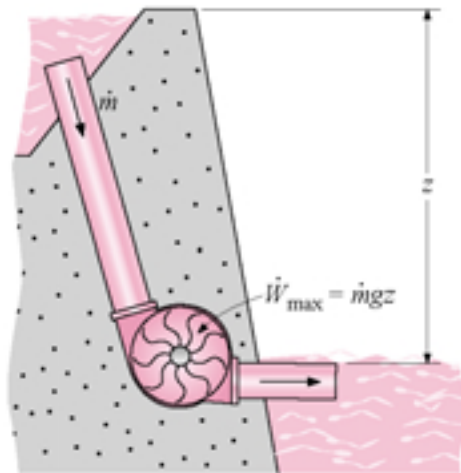
### Exergy of kinetic energy

Kinetic energy is a form of mechanical energy and can be converted directly into work. Kinetic energy itself is the work potential or exergy of kinetic energy independent of the temperature and pressure of the environment.

$$\text{Exergy of kinetic energy: } x_{ke} = ke = \frac{\vec{V}^2}{2}$$

### Exergy of potential energy

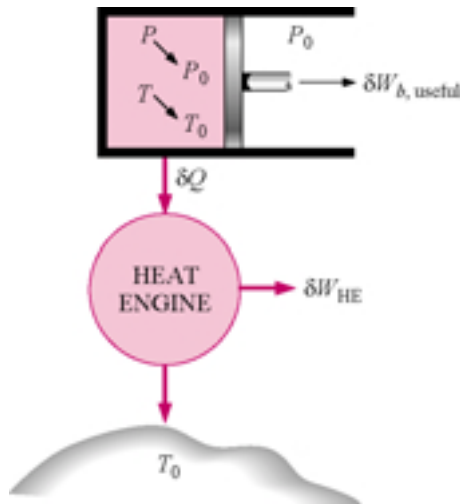
Potential energy is a form of mechanical energy and can be converted directly into work. Potential energy itself is the work potential or exergy of potential energy independent of the temperature and pressure of the environment.



$$\text{Exergy of potential energy: } x_{pe} = pe = gz$$

### Exergy of internal energy

Consider heat transferred to or from a closed system whenever there is a temperature difference across the system boundary. The exergy for internal energy may be determined by considering how much of this heat transfer is converted to work entirely. Let's look at the following figure.



Taking the heat transfer to be from the system to its surroundings, the conservation of energy is

$$\delta E_{in} - \delta E_{out} = dE_{system}$$

$$0 - \delta Q - \delta W = dU$$

The work is the boundary work and can be written as

$$\delta W = P dV = (P - P_0) dV + P_0 dV$$

$$= \delta W_{b, useful} + P_0 dV$$

Any useful work delivered by a piston-cylinder device is due to the pressure above the atmospheric level.

To assure the reversibility of the process, the heat transfer occurs through a reversible heat engine.

$$\delta W_{HE} = \eta_{th} \delta Q = (1 - \frac{T_o}{T}) \delta Q = \delta Q - T_o \frac{\delta Q}{T}$$

$$dS = \frac{\delta Q_{net}}{T} = \frac{-\delta Q}{T}$$

$$\delta W_{HE} = \delta Q + T_o dS$$

$$\delta Q = \delta W_{HE} - T_o dS$$

$$-(\delta W_{HE} - T_o dS) - (\delta W_{b, useful} + P_o dV) = dU$$

$$\begin{aligned} \delta W_{total\ useful} &= \delta W_{b, useful} + \delta W_{HE} \\ &= -dU - P_o dV + T_o dS \end{aligned}$$

Integrating from the given state (no subscript) to the dead state (0 subscript), we have

$$\begin{aligned} W_{total\ useful} &= -(U_0 - U) - P_o (V_0 - V) + T_o (S_0 - S) \\ &= (U - U_0) + P_o (V - V_0) - T_o (S - S_0) \end{aligned}$$

The total useful work due to a change in internal energy is the exergy of internal energy. Per unit mass this is

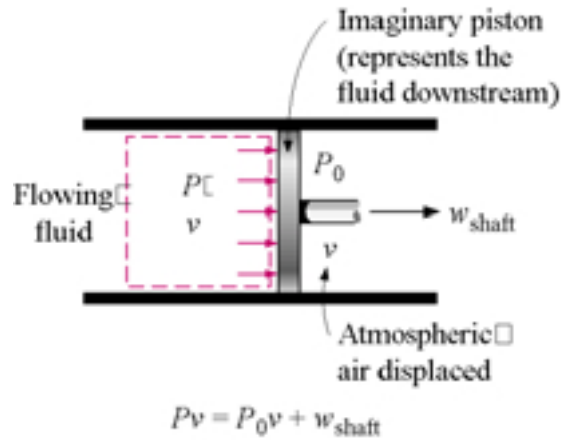
*Exergy of internal energy:*

$$x_u = (u - u_o) + P_o (v - v_o) - T_o (s - s_o)$$

## Exergy of flow work

The energy needed to force mass to flow into or out of a control volume is the flow work per unit mass given by (see Chapter 4)

$$w_{flow} = Pv$$



The exergy of flow work is the excess of flow work done against atmospheric air at  $P_0$  to displace it by volume  $v$ . According to the above figure, the useful work potential due to flow work is

$$w_{flow, useful} = Pv - P_0v$$

Thus, the exergy of flow work is

Exergy of flow work:  $x_{pv} = Pv - P_0v = (P - P_0)v$

## Exergy of enthalpy energy

Since enthalpy is the sum of internal energy and the flow work, the exergy of enthalpy is the sum of the exergies of internal energy and flow work

$$\begin{aligned}x_h &= x_u + x_{pv} \\&= (u - u_o) + P_o(v - v_o) - T_o(s - s_o) + (P - P_o)v \\&= (u - u_o) + P(v - v_o) - T_o(s - s_o) \\&= (h - h_o) - T_o(s - s_o)\end{aligned}$$

*Exergy of enthalpy:*  $x_h = (h - h_o) - T_o(s - s_o)$

The exergy of enthalpy can be negative if the pressure is lower than atmospheric pressure.

## Exergy Change of a System (Closed System)

The exergy of a fixed mass (nonflow exergy) is expressed as the sum of the exergies of its internal energy, kinetic energy, and potential energy.

$$x_{nonflow} = x_u + x_{ke} + x_{pe}$$

This is called the nonflow (or closed system) exergy  $\phi$  and is given on a unit mass basis as

*Nonflow exergy:*

$$\begin{aligned}\phi &= (u - u_o) + P_o(v - v_o) - T_o(s - s_o) + \frac{\vec{V}^2}{2} + gz \\&= (e - e_o) + P_o(v - v_o) - T_o(s - s_o)\end{aligned}$$

The *exergy change* of a fixed mass as it undergoes a process from state 1 to state 2 is given by

$$\begin{aligned}\Delta X &= X_2 - X_1 = m(\phi_2 - \phi_1) \\ &= (E_2 - E_1) + P_o(V_2 - V_1) - T_o(S_2 - S_1) \\ &= (U_2 - U_1) + P_o(V_2 - V_1) - T_o(S_2 - S_1) + m\frac{\vec{V}_2^2 - \vec{V}_1^2}{2} + mg(z_2 - z_1)\end{aligned}$$

### Exergy Change of a Flow Stream (Open System)

The exergy of a flow stream is expressed as the sum of the exergies of its enthalpy energy, kinetic energy, and potential energy.

$$x_{flow} = x_h + x_{ke} + x_{pe}$$

This is called the flow (or open system) exergy  $\psi$  and is given on a unit mass basis as

*Flow exergy:*

$$\psi = (h - h_o) - T_o(s - s_o) + \frac{\vec{V}^2}{2} + gz$$

The *exergy change* of a fluid stream as it undergoes a process from state 1 to state 2 is given by

$$\Delta\psi = \psi_2 - \psi_1 = (h_2 - h_1) - T_o(s_2 - s_1) + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} + g(z_2 - z_1)$$



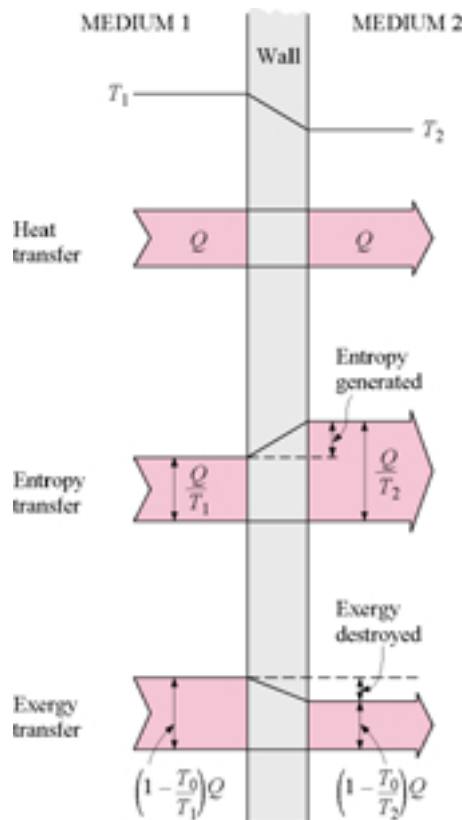
## Exergy Transfer by Heat, Work, and Mass

Exergy can be transferred by heat, work, and mass flow, and exergy transfer accompanied by heat, work, and mass transfer are given by the following.

### Exergy transfer by heat transfer

By the second law we know that only a portion of heat transfer at a temperature above the environment temperature can be converted into work. The maximum useful work is produced from it by passing this heat transfer through a reversible heat engine. The exergy transfer by heat is

*Exergy transfer by heat:* 
$$X_{heat} = \left(1 - \frac{T_o}{T}\right) Q$$



Note in the above figure that entropy generation is always by exergy destruction and that heat transfer  $Q$  at a location at temperature  $T$  is always accompanied by entropy transfer in the amount of  $Q/T$  and exergy transfer in the amount of  $(1-T_o/T)Q$ . Note that exergy transfer by heat is zero for adiabatic systems.

### Exergy transfer by work

Exergy is the useful work potential, and the exergy transfer by work can simply be expressed as

$$\text{Exergy transfer by work: } X_{\text{work}} = \begin{cases} W - W_{\text{surr}} & \text{(for boundary work)} \\ W & \text{(for other forms of work)} \end{cases}$$

where  $W_{\text{surr}} = P_o(V_2 - V_1)$ ,  $P_o$  is atmospheric pressure, and  $V_1$  and  $V_2$  are the initial and final volumes of the system. The exergy transfer for shaft work and electrical work is equal to the work  $W$  itself.

### Exergy transfer by mass

Mass flow is a mechanism to transport exergy, entropy, and energy into or out of a system. As mass in the amount  $m$  enters or leaves a system the exergy transfer is given by

$$\text{Exergy transfer by mass: } X_{\text{mass}} = m\psi$$

### The Decrease of Exergy Principle and Exergy Destruction

The exergy of an isolated system during a process always decreases or, in the limiting case of a reversible process, remains constant. This is known as the *decrease of exergy principle* and is expressed as

$$\Delta X_{\text{isolated}} = (X_2 - X_1)_{\text{isolated}} \leq 0$$

## Exergy Destruction

Irreversibilities such as friction, mixing, chemical reactions, heat transfer through finite temperature difference, unrestrained expansion, non-quasi-equilibrium compression, or expansion always generate entropy, and anything that generates entropy always destroys exergy. The exergy destroyed is proportional to the entropy generated as expressed as

$$X_{destroyed} = T_o S_{gen}$$

The decrease of exergy principle does not imply that the exergy of a system cannot increase. The exergy change of a system can be positive or negative during a process, but exergy destroyed cannot be negative. The decrease of exergy principle can be summarized as follows:

$$X_{destroyed} \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases}$$

## Exergy Balances

Exergy balance for *any system* undergoing *any process* can be expressed as

*General:*

$$\underbrace{X_{in} - X_{out}}_{\text{Net exergy transfer by heat, work, and mass}} - \underbrace{X_{destroyed}}_{\text{Exergy destruction}} = \underbrace{\Delta X_{system}}_{\text{Change in exergy}}$$

General, rate form:

$$\underbrace{\dot{X}_{in} - \dot{X}_{out}}_{\text{Rate of net exergy transfer by heat, work, and mass}} - \underbrace{\dot{X}_{destroyed}}_{\text{Rate of exergy destruction}} = \underbrace{\Delta \dot{X}_{system}}_{\text{Rate of change of exergy}}$$

General, unit-mass basis:

$$(x_{in} - x_{out}) - x_{destroyed} = \Delta x_{system}$$

where

$$\begin{aligned}\dot{X}_{heat} &= \left(1 - \frac{T_o}{T}\right) \dot{Q} \\ \dot{X}_{work} &= \dot{W}_{useful} \\ \dot{X}_{mass} &= \dot{m} \psi \\ \Delta \dot{X}_{system} &= dX_{system} / dt\end{aligned}$$

For a *reversible process*, the exergy destruction term,  $X_{destroyed}$ , is zero.

Considering the system to be a general control volume and taking the positive direction of heat transfer to be to the system and the positive direction of work transfer to be from the system, the general exergy balance relations can be expressed more explicitly as

$$\sum \left(1 - \frac{T_o}{T_k}\right) \dot{Q}_k - [\dot{W} - P_o(V_2 - V_1)] + \sum \dot{m}_i \psi_i - \sum \dot{m}_e \psi_e - \dot{X}_{destroyed} = X_2 - X_1$$

$$\sum \left(1 - \frac{T_o}{T_k}\right) \dot{Q}_k - \left[\dot{W} - P_o \frac{dV_{CV}}{dt}\right] + \sum \dot{m}_i \psi_i - \sum \dot{m}_e \psi_e - \dot{X}_{destroyed} = \frac{dX_{CV}}{dt}$$

where the subscripts are  $i$  = inlet,  $e$  = exit, 1 = initial state, and 2 = final state of the system. For closed systems, no mass crosses the boundaries and we omit the terms containing the sum over the inlets and exits.

### Example 7-1

Oxygen gas is compressed in a piston-cylinder device from an initial state of  $0.8 \text{ m}^3/\text{kg}$  and  $25^\circ\text{C}$  to a final state of  $0.1 \text{ m}^3/\text{kg}$  and  $287^\circ\text{C}$ . Determine the reversible work input and the increase in the exergy of the oxygen during this process. Assume the surroundings to be at  $25^\circ\text{C}$  and  $100 \text{ kPa}$ .

We assume that oxygen is an ideal gas with constant specific heats. From Table A-2,  $R = 0.2598 \text{ kJ/kg}\cdot\text{K}$ . The specific heat is determined at the average temperature

$$\begin{aligned} T_{av} &= \frac{T_1 + T_2}{2} = \frac{(25 + 287)^\circ\text{C}}{2} = 156^\circ\text{C} \\ &= (156 + 273)\text{K} = 429\text{K} \end{aligned}$$

Table A-2(b) gives  $C_{v, \text{ave}} = 0.690 \text{ kJ/kg}\cdot\text{K}$ .

The entropy change of oxygen is

$$\begin{aligned} s_2 - s_1 &= C_{v, \text{ave}} \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right) \\ &= 0.690 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \ln\left(\frac{(287 + 273)\text{K}}{(25 + 273)\text{K}}\right) + 0.2598 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \ln\left(\frac{0.1 \frac{\text{m}^3}{\text{kg}}}{0.8 \frac{\text{m}^3}{\text{kg}}}\right) \\ &= -0.105 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \end{aligned}$$

We calculate the reversible work input, which represents the minimum work input  $W_{\text{rev, in}}$  in this case, from the exergy balance by setting the exergy destruction equal to zero.

$$\underbrace{X_{in} - X_{out}}_{\text{Net exergy transfer by heat, work, and mass}} - \underbrace{X_{destroyed}^0}_{\text{Exergy destruction}} = \underbrace{\Delta X_{system}}_{\text{Change in exergy}}$$

$$W_{rev,in} = X_2 - X_1$$

Therefore, the change in exergy and the reversible work are identical in this case. Substituting the closed system exergy relation, the reversible work input during this process is determined to be

$$\begin{aligned} w_{rev,in} &= \phi_2 - \phi_1 \\ &= (u_2 - u_1) + P_o(v_2 - v_1) - T_o(s_2 - s_1) \\ &= C_{v,ave}(T_2 - T_1) + P_o(v_2 - v_1) - T_o(s_2 - s_1) \\ &= 0.690 \frac{kJ}{kg \cdot K} (287 - 25) K + 100 kPa (0.1 - 0.8) \frac{m^3}{kg} \frac{kJ}{m^3 kPa} \\ &\quad - (25 + 273) K (-0.105 \frac{kJ}{kg \cdot K}) \\ &= 142.1 \frac{kJ}{kg} \end{aligned}$$

The increase in exergy of the oxygen is

$$x_2 - x_1 = \phi_2 - \phi_1 = w_{rev,in} = 142.1 \frac{kJ}{kg}$$

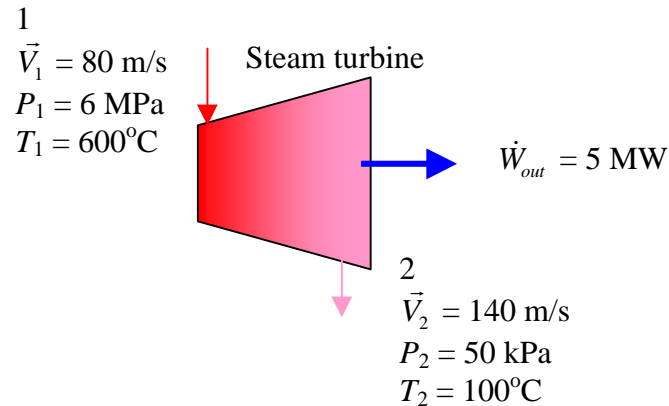
### Example 7-2

Steam enters an adiabatic turbine at 6 MPa, 600°C, and 80 m/s and leaves at 50 kPa, 100°C, and 140 m/s. The surroundings to the turbine are at 25°C.

If the power output of the turbine is 5MW, determine

- the power potential of the steam at its inlet conditions, in MW.
- the reversible power, in MW.
- the second law efficiency.

We assume steady-flow and neglect changes in potential energy.



The mass flow rate of the steam is determined from the steady-flow energy equation applied to the actual process,

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}}_{\text{Rate of change of energy}} \quad 0 \text{ (steady)}$$

$$\dot{m}_1 \left( h_1 + \frac{\vec{V}_1^2}{2} \right) - \dot{m}_2 \left( h_2 + \frac{\vec{V}_2^2}{2} \right) - \dot{W}_{out} = 0$$

Conservation of mass for the steady flow gives

$$\underbrace{\dot{m}_{in} - \dot{m}_{out}}_{\text{Rate of net mass transfer}} = \underbrace{\Delta \dot{m}_{system}}_{\text{Rate of change of mass}}$$

$$\dot{m}_1 - \dot{m}_2 = 0$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

The work done by the turbine and the mass flow rate are

$$\dot{W}_{out} = \dot{m} \left[ (h_1 - h_2) + \left( \frac{\vec{V}_1^2}{2} - \frac{\vec{V}_2^2}{2} \right) \right]$$

$$\dot{m} = \frac{\dot{W}_{out}}{(h_1 - h_2) - \Delta ke}$$

where

$$\begin{aligned} \Delta ke &= \left( \frac{\vec{V}_2^2}{2} - \frac{\vec{V}_1^2}{2} \right) \\ &= \frac{(140 \text{ m/s})^2 - (80 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \\ &= 6.6 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

From the steam tables



$$\left. \begin{array}{l} P_1 = 6 \text{ MPa} \\ T_1 = 600^\circ \text{ C} \end{array} \right\} \begin{cases} h_1 = 3658.4 \frac{\text{kJ}}{\text{kg}} \\ s_1 = 7.1677 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \end{cases}$$

$$\left. \begin{array}{l} P_2 = 50 \text{ kPa} \\ T_2 = 100^\circ \text{ C} \end{array} \right\} \begin{cases} h_2 = 2682.5 \frac{\text{kJ}}{\text{kg}} \\ s_2 = 7.6947 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \end{cases}$$

$$\left. \begin{array}{l} P_0 = 100 \text{ kPa} \\ T_0 = 25^\circ \text{ C} \end{array} \right\} \begin{cases} h_0 \cong h_{f @ 25^\circ \text{ C}} = 104.89 \frac{\text{kJ}}{\text{kg}} \\ s_0 \cong s_{f @ 25^\circ \text{ C}} = 0.3674 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \end{cases}$$

$$\begin{aligned} \dot{m} &= \frac{\dot{W}_{out}}{(h_1 - h_2) - \Delta ke} \\ &= \frac{5 \text{ MW}}{(3658.4 - 2682.5 - 6.6) \frac{\text{kJ}}{\text{kg}}} \left( \frac{1000 \text{ kJ} / \text{s}}{\text{MW}} \right) \\ &= 5.16 \frac{\text{kg}}{\text{s}} \end{aligned}$$

The power potential of the steam at the inlet conditions is equivalent to its exergy at the inlet state. Recall that we neglect the potential energy of the flow.

$$\dot{\Psi}_1 = \dot{m} \psi_1 = \dot{m} \left( (h_1 - h_o) - T_o (s_1 - s_o) + \frac{\vec{V}_1^2}{2} + g \overset{\nearrow 0}{z_1} \right)$$

$$\begin{aligned}
\dot{\Psi}_1 &= 5.16 \frac{kg}{s} \left( (3658.4 - 104.89) \frac{kJ}{kg} - (298 K)(7.1677 - 0.3674) \frac{kJ}{kg \cdot K} \right) \\
&\quad + \frac{(80 m/s)^2}{2} \left( \frac{kJ/kg}{1000 m^2/s^2} \right) \\
&= 5.16 \frac{kg}{s} \left( 1530.2 \frac{kJ}{kg} \right) \left( \frac{MW}{1000 kJ/s} \right) \\
&= 7.90 MW
\end{aligned}$$

The power output of the turbine if there are no irreversibilities is the reversible power and is determined from the rate form of the exergy balance applied on the turbine and setting the exergy destruction term equal to zero.

$$\underbrace{\dot{X}_{in} - \dot{X}_{out}}_{\text{Rate of net exergy transfer by heat, work, and mass}} - \underbrace{\dot{X}_{destroyed}^0}_{\text{Rate of exergy destruction}} = \underbrace{\Delta \dot{X}_{system}^0}_{\text{Rate of change of exergy}} \quad 0 \text{ (steady)}$$

$$\dot{X}_{in} = \dot{X}_{out}$$

$$\dot{m}\psi_1 = \dot{W}_{rev, out} + \dot{m}\psi_2$$

$$\begin{aligned}
\dot{W}_{rev, out} &= \dot{m}(\psi_2 - \psi_1) \\
&= \dot{m}[(h_1 - h_2) - T_0(s_1 - s_2) - \Delta ke - \Delta pe] \quad 0
\end{aligned}$$

$$\begin{aligned}
 \dot{W}_{rev, out} &= 5.16 \frac{kg}{s} \left( (3658.4 - 2682.5) \frac{kJ}{kg} - (298 K)(7.1677 - 7.6947) \frac{kJ}{kg \cdot K} \right) \\
 &= 5.16 \frac{kg}{s} \left( 1126.0 \frac{kJ}{kg} \right) \left( \frac{MW}{1000 kJ / s} \right) \\
 &= 5.81 MW
 \end{aligned}$$

The second-law efficiency is determined from

$$\eta_{II} = \frac{\dot{W}}{\dot{W}_{rev}} = \frac{5 MW}{5.81 MW} = 86.1\%$$