Carnot Heat Engines

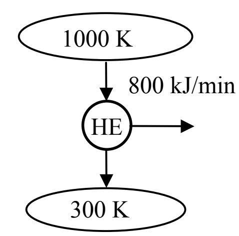
6-69C No.

6-70C The one that has a source temperature of 600°C. This is true because the higher the temperature at which heat is supplied to the working fluid of a heat engine, the higher the thermal efficiency.

6-71 The source and sink temperatures of a Carnot heat engine and the rate of heat supply are given. The thermal efficiency and the power output are to be determined.

Assumptions The Carnot heat engine operates steadily.

Analysis (a) The thermal efficiency of a Carnot heat engine depends on the source and the sink temperatures only, and is determined from



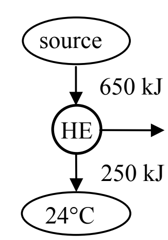
$$\eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300\text{ K}}{1000\text{ K}} = 0.70 \text{ or } \mathbf{70\%}$$

(b) The power output of this heat engine is determined from the definition of thermal efficiency,

$$\dot{W}_{\text{net,out}} = \eta_{\text{th}}\dot{Q}_H = (0.70)(800\text{ kJ/min}) = 560\text{ kJ/min} = \mathbf{9.33\text{ kW}}$$

6-72 The sink temperature of a Carnot heat engine and the rates of heat supply and heat rejection are given. The source temperature and the thermal efficiency of the engine are to be determined.

Assumptions The Carnot heat engine operates steadily.



Analysis (a) For reversible cyclic devices we have $\left(\frac{Q_H}{Q_L}\right)_{\text{rev}} = \left(\frac{T_H}{T_L}\right)$

Thus the temperature of the source T_H must be

$$T_H = \left(\frac{Q_H}{Q_L}\right)_{\text{rev}} T_L = \left(\frac{650\text{ kJ}}{250\text{ kJ}}\right)(297\text{ K}) = \mathbf{772.2\ \ K}$$

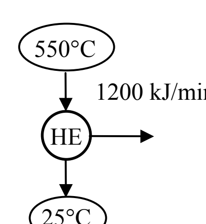
(*b*) The thermal efficiency of a Carnot heat engine depends on the source and the sink temperatures only, and is determined from

$$\eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{297\text{ K}}{772.2\text{ K}} = 0.615 \text{ or } \mathbf{61.5\%}$$

6-73 [*Also solved by EES on enclosed CD***]** The source and sink temperatures of a heat engine and the rate of heat supply are given. The maximum possible power output of this engine is to be determined.

Assumptions The heat engine operates steadily.

Analysis The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from



$$\eta_{\text{th,max}} = \eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{298\text{ K}}{823\text{ K}} = 0.638 \text{ or } 63.8\%$$

Then the maximum power output of this heat engine is determined from the definition of thermal efficiency to be

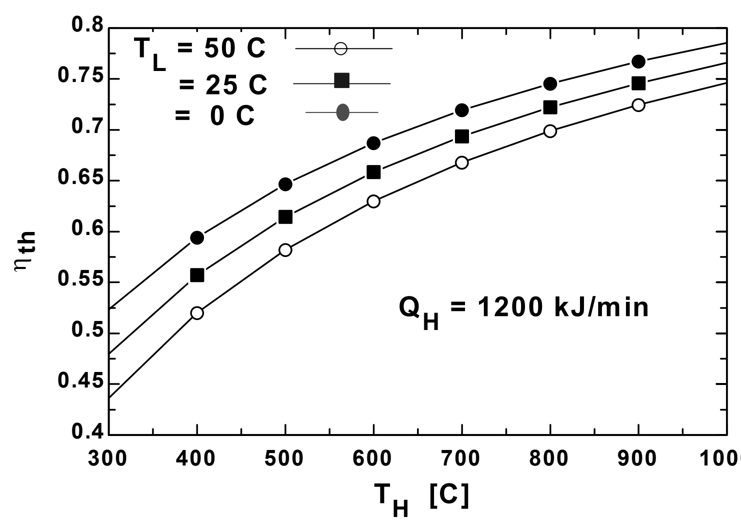
$$\dot{W}_{\text{net,out}} = \eta_{\text{th}}\dot{Q}_H = (0.638)(1200\text{ kJ/min}) = 765.6\text{ kJ/min} = \mathbf{12.8\text{ kW}}$$

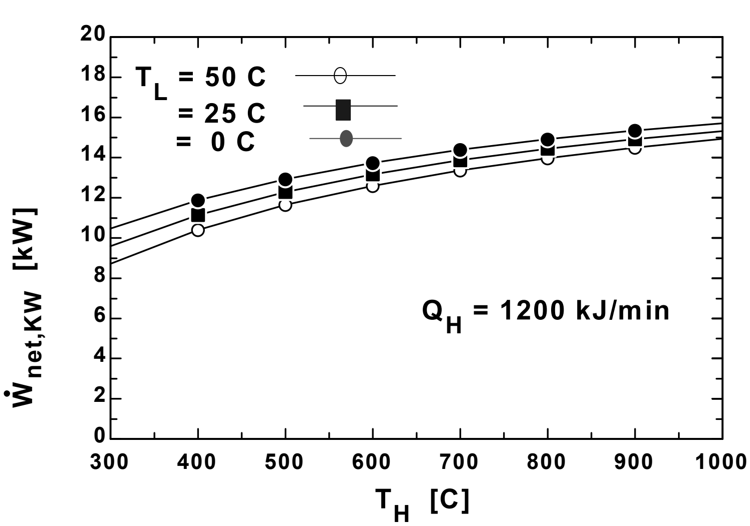
6-74 EES Problem 6-73 is reconsidered. The effects of the temperatures of the heat source and the heat sink on the power produced and the cycle thermal efficiency as the source temperature varies from 300°C to 1000°C and the sink temperature varies from 0°C to 50°C are to be studied. The power produced and the cycle efficiency against the source temperature for sink temperatures of 0°C, 25°C, and 50°C are to be plotted.

Analysis The problem is solved using EES, and the results are tabulated and plotted below.

```
"Input Data from the Diagram Window"
{T_H = 550 [C]
T_L = 25 [C]}
{Q_dot_H = 1200 [kJ/min]}

"First Law applied to the heat engine"

Q_dot_H - Q_dot_L- W_dot_net = 0
W_dot_net_KW=W_dot_net*convert(kJ/min,kW)

"Cycle Thermal Efficiency - Temperatures must be absolute"
eta_th = 1 - (T_L + 273)/(T_H + 273)

"Definition of cycle efficiency"
eta_th=W_dot_net / Q_dot_H
```

η_{th}	T_H [C]	W_{netkW} [kW]
0.52	300	10.47
0.59	400	11.89
0.65	500	12.94
0.69	600	13.75
0.72	700	14.39
0.75	800	14.91
0.77	900	15.35
0.79	1000	15.71





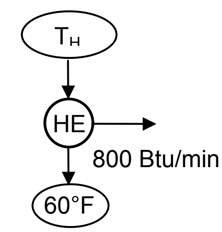
6-75E The sink temperature of a Carnot heat engine, the rate of heat rejection, and the thermal efficiency are given. The power output of the engine and the source temperature are to be determined.

Assumptions The Carnot heat engine operates steadily.

Analysis (a) The rate of heat input to this heat engine is determined from the definition of thermal efficiency,

$$\eta_{\text{th}} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} \longrightarrow 0.55 = 1 - \frac{800 \text{ Btu/min}}{\dot{Q}_H} \longrightarrow \dot{Q}_H = 1777.8 \text{ Btu/min}$$

Then the power output of this heat engine can be determined from



$$\dot{W}_{\text{net,out}} = \eta_{\text{th}} \dot{Q}_H = (0.55)(1777.8 \text{ Btu/min}) = 977.8 \text{ Btu/min} = \mathbf{23.1\ hp}$$

(*b*) For reversible cyclic devices we have $\left(\frac{\dot{Q}_H}{\dot{Q}_L}\right)_{\text{rev}} = \left(\frac{T_H}{T_L}\right)$

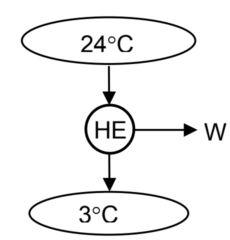
Thus the temperature of the source T_H must be

$$T_H = \left(\frac{\dot{Q}_H}{\dot{Q}_L}\right)_{\text{rev}} T_L = \left(\frac{1777.8 \text{ Btu/min}}{800 \text{ Btu/min}}\right)(520 \text{ R}) = \mathbf{1155.6\ R}$$

6-76 The source and sink temperatures of a OTEC (Ocean Thermal Energy Conversion) power plant are given. The maximum thermal efficiency is to be determined.

Assumptions The power plant operates steadily.

Analysis The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

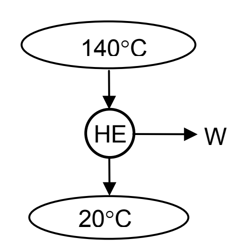


$$\eta_{\text{th,max}} = \eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{276 \text{ K}}{297 \text{ K}} = 0.071 \quad \text{or} \quad \mathbf{7.1\%}$$

6-77 The source and sink temperatures of a geothermal power plant are given. The maximum thermal efficiency is to be determined.

Assumptions The power plant operates steadily.

Analysis The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

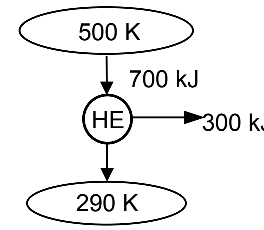


$$\eta_{\text{th,max}} = \eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{20 + 273 \text{ K}}{140 + 273 \text{ K}} = 0.291 \text{ or } \mathbf{29.1\%}$$

6-78 An inventor claims to have developed a heat engine. The inventor reports temperature, heat transfer, and work output measurements. The claim is to be evaluated.

Analysis The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{\text{th,max}} = \eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{290\text{ K}}{500\text{ K}} = 0.42 \text{ or } \mathbf{42\%}$$



The actual thermal efficiency of the heat engine in question is

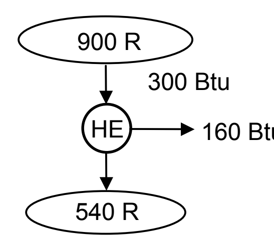
$$\eta_{\text{th}} = \frac{W_{net}}{Q_H} = \frac{300\text{ kJ}}{700\text{ kJ}} = 0.429 \text{ or } \mathbf{42.9\%}$$

which is greater than the maximum possible thermal efficiency. Therefore, this heat engine is a PMM2 and the claim is **false**.

6-79E An inventor claims to have developed a heat engine. The inventor reports temperature, heat transfer, and work output measurements. The claim is to be evaluated.

Analysis The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{\text{th,max}} = \eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{540\text{ R}}{900\text{ R}} = 0.40 \text{ or } \mathbf{40\%}$$



The actual thermal efficiency of the heat engine in question is

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_H} = \frac{160\text{ Btu}}{300\text{ Btu}} = 0.533 \text{ or } \mathbf{53.3\%}$$

which is greater than the maximum possible thermal efficiency. Therefore, this heat engine is a PMM2 and the claim is **false**.

6-80 A geothermal power plant uses geothermal liquid water at 160ºC at a specified rate as the heat source. The actual and maximum possible thermal efficiencies and the rate of heat rejected from this power plant are to be determined.

Assumptions **1** The power plant operates steadily. **2** The kinetic and potential energy changes are zero. **3** Steam properties are used for geothermal water.

Properties Using saturated liquid properties, the source and the sink state enthalpies of geothermal water are (Table A-4)

$$\left.\begin{array}{l} T_{\text{source}} = 160°\text{C} \\ x_{\text{source}} = 0 \end{array}\right\} h_{\text{source}} = 675.47 \text{ kJ/kg}$$

$$\left.\begin{array}{l} T_{\text{sink}} = 25°\text{C} \\ x_{\text{sink}} = 0 \end{array}\right\} h_{\text{sink}} = 104.83 \text{ kJ/kg}$$

Analysis (*a*) The rate of heat input to the plant may be taken as the enthalpy difference between the source and the sink for the power plant

$$\dot{Q}_{\text{in}} = \dot{m}_{\text{geo}}(h_{\text{source}} - h_{\text{sink}}) = (440 \text{ kg/s})(675.47 - 104.83) \text{ kJ/kg} = 251{,}083 \text{ kW}$$

The actual thermal efficiency is

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net,out}}}{\dot{Q}_{\text{in}}} = \frac{22 \text{ MW}}{251.083 \text{ MW}} = \mathbf{0.0876} = \mathbf{8.8\%}$$

(*b*) The maximum thermal efficiency is the thermal efficiency of a reversible heat engine operating between the source and sink temperatures

$$\eta_{\text{th,max}} = 1 - \frac{T_L}{T_H} = 1 - \frac{(25 + 273) \text{ K}}{(160 + 273) \text{ K}} = \mathbf{0.312} = \mathbf{31.2\%}$$

(*c*) Finally, the rate of heat rejection is

$$\dot{Q}_{\text{out}} = \dot{Q}_{\text{in}} - \dot{W}_{\text{net,out}} = 251.1 - 22 = \mathbf{229.1\,MW}$$

Carnot Refrigerators and Heat Pumps

6-81C By increasing T_L or by decreasing T_H.

6-82C It is the COP that a Carnot refrigerator would have, $\text{COP}_\text{R} = \dfrac{1}{T_H/T_L - 1}$.

6-83C No. At best (when everything is reversible), the increase in the work produced will be equal to the work consumed by the refrigerator. In reality, the work consumed by the refrigerator will always be greater than the additional work produced, resulting in a decrease in the thermal efficiency of the power plant.

6-84C No. At best (when everything is reversible), the increase in the work produced will be equal to the work consumed by the refrigerator. In reality, the work consumed by the refrigerator will always be greater than the additional work produced, resulting in a decrease in the thermal efficiency of the power plant.

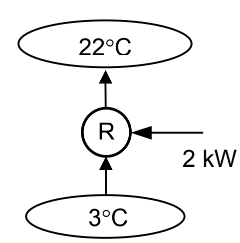
6-85C Bad idea. At best (when everything is reversible), the increase in the work produced will be equal to the work consumed by the heat pump. In reality, the work consumed by the heat pump will always be greater than the additional work produced, resulting in a decrease in the thermal efficiency of the power plant.

6-86 The refrigerated space and the environment temperatures of a Carnot refrigerator and the power consumption are given. The rate of heat removal from the refrigerated space is to be determined.

Assumptions The Carnot refrigerator operates steadily.

Analysis The coefficient of performance of a Carnot refrigerator depends on the temperature limits in the cycle only, and is determined from

$$\text{COP}_{\text{R,C}} = \frac{1}{(T_H/T_L)-1} = \frac{1}{(22+273\text{K})/(3+273\text{K})-1} = 14.5$$



The rate of heat removal from the refrigerated space is determined from the definition of the coefficient of performance of a refrigerator,

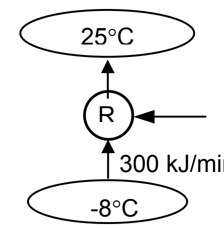
$$\dot{Q}_L = \text{COP}_\text{R}\dot{W}_{\text{net,in}} = (14.5)(2\text{ kW}) = 29.0\text{ kW} = \mathbf{1740\ \ kJ/min}$$

6-87 The refrigerated space and the environment temperatures for a refrigerator and the rate of heat removal from the refrigerated space are given. The minimum power input required is to be determined.

Assumptions The refrigerator operates steadily.

Analysis The power input to a refrigerator will be a minimum when the refrigerator operates in a reversible manner. The coefficient of performance of a reversible refrigerator depends on the temperature limits in the cycle only, and is determined from

$$COP_{\mathrm{R,rev}} = \frac{1}{(T_H/T_L)-1} = \frac{1}{(25+273\ \mathrm{K})/(-8+273\ \mathrm{K})-1} = 8.03$$



The power input to this refrigerator is determined from the definition of the coefficient of performance of a refrigerator,

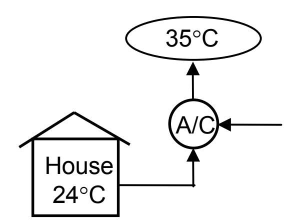
$$\dot{W}_{\mathrm{net,in,min}} = \frac{\dot{Q}_L}{COP_{\mathrm{R,max}}} = \frac{300\ \mathrm{kJ/min}}{8.03} = 37.36\ \mathrm{kJ/min} = \mathbf{0.623\ kW}$$

6-88 The cooled space and the outdoors temperatures for a Carnot air-conditioner and the rate of heat removal from the air-conditioned room are given. The power input required is to be determined.

Assumptions The air-conditioner operates steadily.

Analysis The COP of a Carnot air conditioner (or Carnot refrigerator) depends on the temperature limits in the cycle only, and is determined from

$$\mathrm{COP_{R,C}} = \frac{1}{(T_H/T_L)-1} = \frac{1}{(35+273\ \mathrm{K})/(24+273\ \mathrm{K})-1} = 27.0$$



The power input to this refrigerator is determined from the definition of the coefficient of performance of a refrigerator,

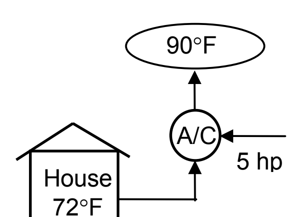
$$\dot{W}_{\mathrm{net,in}} = \frac{\dot{Q}_L}{\mathrm{COP_{R,max}}} = \frac{750\ \mathrm{kJ/min}}{27.0} = 27.8\ \mathrm{kJ/min} = \mathbf{0.463\ kW}$$

6-89E The cooled space and the outdoors temperatures for an air-conditioner and the power consumption are given. The maximum rate of heat removal from the air-conditioned space is to be determined.

Assumptions The air-conditioner operates steadily.

Analysis The rate of heat removal from a house will be a maximum when the air-conditioning system operates in a reversible manner. The coefficient of performance of a reversible air-conditioner (or refrigerator) depends on the temperature limits in the cycle only, and is determined from

$$COP_{\mathrm{R,rev}} = \frac{1}{(T_H/T_L)-1} = \frac{1}{(90+460\ \mathrm{R})/(72+460\ \mathrm{R})-1} = 29.6$$



The rate of heat removal from the house is determined from the definition of the coefficient of performance of a refrigerator,

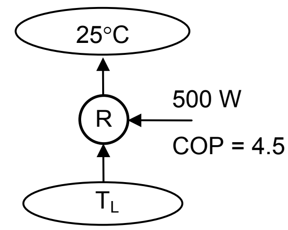
$$\dot{Q}_L = \mathrm{COP_R}\dot{W}_{\mathrm{net,in}} = (29.6)(5\ \mathrm{hp})\left(\frac{42.41\ \mathrm{Btu/min}}{1\ \mathrm{hp}}\right) = \mathbf{6277\ \ Btu/min}$$

6-90 The refrigerated space temperature, the COP, and the power input of a Carnot refrigerator are given. The rate of heat removal from the refrigerated space and its temperature are to be determined.

Assumptions The refrigerator operates steadily.

Analysis (*a*) The rate of heat removal from the refrigerated space is determined from the definition of the COP of a refrigerator,

$$\dot{Q}_L = \text{COP}_\text{R}\dot{W}_\text{net,in} = (4.5)(0.5\text{ kW}) = 2.25\text{ kW} = \mathbf{135\ kJ/min}$$



(*b*) The temperature of the refrigerated space T_L is determined from the coefficient of performance relation for a Carnot refrigerator,

$$\text{COP}_\text{R,rev} = \frac{1}{(T_H/T_L)-1} \longrightarrow 4.5 = \frac{1}{(25+273\text{ K})/T_L - 1}$$

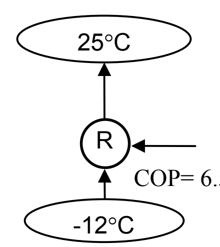
It yields

$$T_\text{L} = 243.8\text{ K} = \mathbf{-29.2°C}$$

6-91 An inventor claims to have developed a refrigerator. The inventor reports temperature and COP measurements. The claim is to be evaluated.

Analysis The highest coefficient of performance a refrigerator can have when removing heat from a cool medium at -12°C to a warmer medium at 25°C is

$$\text{COP}_\text{R,max} = \text{COP}_\text{R,rev} = \frac{1}{(T_H/T_L)-1} = \frac{1}{(25+273\text{ K})/(-12+273\text{ K})-1} = 7.1$$

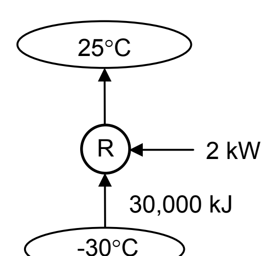


The COP claimed by the inventor is 6.5, which is below this maximum value, thus the claim is **reasonable**. However, it is not probable.

6-92 An experimentalist claims to have developed a refrigerator. The experimentalist reports temperature, heat transfer, and work input measurements. The claim is to be evaluated.

Analysis The highest coefficient of performance a refrigerator can have when removing heat from a cool medium at -30°C to a warmer medium at 25°C is

$$\text{COP}_\text{R,max} = \text{COP}_\text{R,rev} = \frac{1}{(T_H/T_L)-1} = \frac{1}{(25+273\text{ K})/(-30+273\text{ K})-1} = 4.42$$



The work consumed by the actual refrigerator during this experiment is

$$W_\text{net,in} = \dot{W}_\text{net,in}\Delta t = (2\text{ kJ/s})(20\times 60\text{ s}) = 2400\text{ kJ}$$

Then the coefficient of performance of this refrigerator becomes

$$\text{COP}_\text{R} = \frac{Q_L}{W_\text{net,in}} = \frac{30{,}000\text{kJ}}{2400\text{kJ}} = 12.5$$

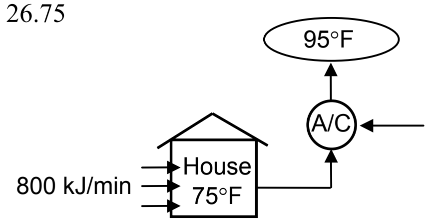
which is above the maximum value. Therefore, these measurements are **not reasonable**.

6-93E An air-conditioning system maintains a house at a specified temperature. The rate of heat gain of the house and the rate of internal heat generation are given. The maximum power input required is to be determined.

Assumptions The air-conditioner operates steadily.

Analysis The power input to an air-conditioning system will be a minimum when the air-conditioner operates in a reversible manner. The coefficient of performance of a reversible air-conditioner (or refrigerator) depends on the temperature limits in the cycle only, and is determined from

$$\mathrm{COP_{R,rev}} = \frac{1}{(T_H/T_L)-1} = \frac{1}{(95+460\ \mathrm{R})/(75+460\ \mathrm{R})-1} = 26.75$$



The cooling load of this air-conditioning system is the sum of the heat gain from the outside and the heat generated within the house,

$$\dot{Q}_L = 800+100 = 900\ \mathrm{Btu/min}$$

The power input to this refrigerator is determined from the definition of the coefficient of performance of a refrigerator,

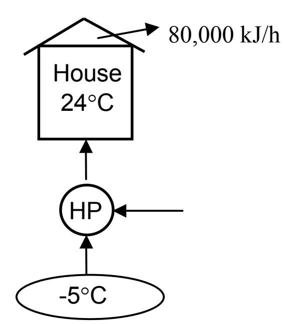
$$\dot{W}_{\mathrm{net,in,min}} = \frac{\dot{Q}_L}{\mathrm{COP_{R,max}}} = \frac{900\ \mathrm{Btu/min}}{26.75} = 33.6\ \mathrm{Btu/min} = \mathbf{0.79\ hp}$$

6-94 A heat pump maintains a house at a specified temperature. The rate of heat loss of the house is given. The minimum power input required is to be determined.

Assumptions The heat pump operates steadily.

Analysis The power input to a heat pump will be a minimum when the heat pump operates in a reversible manner. The COP of a reversible heat pump depends on the temperature limits in the cycle only, and is determined from

$$\mathrm{COP_{HP,rev}} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(-5+273\ \mathrm{K})/(24+273\ \mathrm{K})} = 10.2$$



The required power input to this reversible heat pump is determined from the definition of the coefficient of performance to be

$$\dot{W}_{\mathrm{net,in,min}} = \frac{\dot{Q}_H}{\mathrm{COP_{HP}}} = \frac{80{,}000\ \mathrm{kJ/h}}{10.2}\left(\frac{1\ \mathrm{h}}{3600\ \mathrm{s}}\right) = \mathbf{2.18\ \ kW}$$

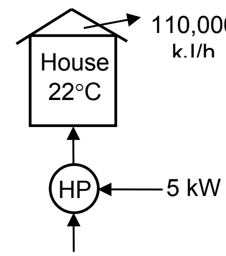
which is the *minimum* power input required.

6-95 A heat pump maintains a house at a specified temperature. The rate of heat loss of the house and the power consumption of the heat pump are given. It is to be determined if this heat pump can do the job.

Assumptions The heat pump operates steadily.

Analysis The power input to a heat pump will be a minimum when the heat pump operates in a reversible manner. The coefficient of performance of a reversible heat pump depends on the temperature limits in the cycle only, and is determined from

$$\text{COP}_{\text{HP,rev}} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(2+273\text{ K})/(22+273\text{ K})} = 14.75$$



The required power input to this reversible heat pump is determined from the definition of the coefficient of performance to be

$$\dot{W}_{\text{net,in,min}} = \frac{\dot{Q}_H}{\text{COP}_{\text{HP}}} = \frac{110{,}000\text{ kJ/h}}{14.75}\left(\frac{1\text{ h}}{3600\text{ s}}\right) = \mathbf{2.07\ kW}$$

This heat pump is **powerful enough** since 5 kW > 2.07 kW.

6-96 A heat pump that consumes 5-kW of power when operating maintains a house at a specified temperature. The house is losing heat in proportion to the temperature difference between the indoors and the outdoors. The lowest outdoor temperature for which this heat pump can do the job is to be determined.

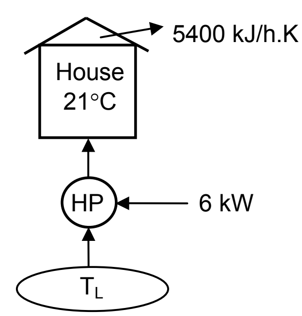
Assumptions The heat pump operates steadily.

Analysis Denoting the outdoor temperature by T_L, the heating load of this house can be expressed as

$$\dot{Q}_H = (5400\text{ kJ/h}\cdot\text{K})(294 - T_L) = (1.5\text{ kW/K})(294 - T_L)\text{K}$$

The coefficient of performance of a Carnot heat pump depends on the temperature limits in the cycle only, and can be expressed as

$$\text{COP}_{\text{HP}} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-T_L/(294\text{ K})}$$



or, as

$$\text{COP}_{\text{HP}} = \frac{\dot{Q}_H}{\dot{W}_{\text{net,in}}} = \frac{(1.5\text{ kW/K})(294 - T_L)\text{K}}{6\text{ kW}}$$

Equating the two relations above and solving for T_L, we obtain

$$T_L = 259.7\text{ K} = \mathbf{-13.3°C}$$

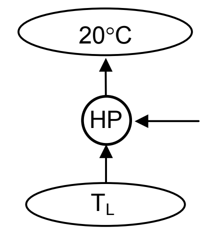
6-97 A heat pump maintains a house at a specified temperature in winter. The maximum COPs of the heat pump for different outdoor temperatures are to be determined.

Analysis The coefficient of performance of a heat pump will be a maximum when the heat pump operates in a reversible manner. The coefficient of performance of a reversible heat pump depends on the temperature limits in the cycle only, and is determined for all three cases above to be

$$COP_{HP,rev} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(10+273\text{K})/(20+273\text{K})} = \mathbf{29.3}$$

$$COP_{HP,rev} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(-5+273\text{K})/(20+273\text{K})} = \mathbf{11.7}$$

$$COP_{HP,rev} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(-30+273\text{K})/(20+273\text{K})} = \mathbf{5.86}$$



6-98E A heat pump maintains a house at a specified temperature. The rate of heat loss of the house is given. The minimum power inputs required for different source temperatures are to be determined.

Assumptions The heat pump operates steadily.

Analysis (*a*) The power input to a heat pump will be a minimum when the heat pump operates in a reversible manner. If the outdoor air at 25°F is used as the heat source, the COP of the heat pump and the required power input are determined to be

$$\text{COP}_{\text{HP,max}} = \text{COP}_{\text{HP,rev}} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(25+460\text{ R})/(78+460\text{ R})} = 10.15$$

and

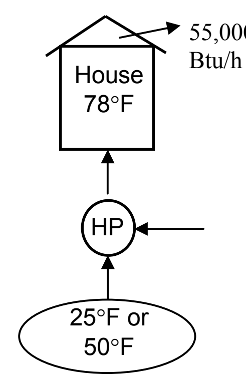
$$\dot{W}_{\text{net,in,min}} = \frac{\dot{Q}_H}{\text{COP}_{\text{HP,max}}} = \frac{55{,}000\text{ Btu/h}}{10.15}\left(\frac{1\text{ hp}}{2545\text{ Btu/h}}\right) = \mathbf{2.13\ hp}$$

(*b*) If the well-water at 50°F is used as the heat source, the COP of the heat pump and the required power input are determined to be

$$\text{COP}_{\text{HP,max}} = \text{COP}_{\text{HP,rev}} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(50+460\text{ R})/(78+460\text{ R})} = 19.2$$

and

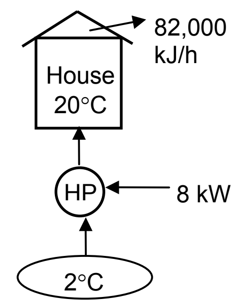
$$\dot{W}_{\text{net,in,min}} = \frac{\dot{Q}_H}{\text{COP}_{\text{HP,max}}} = \frac{55{,}000\text{ Btu/h}}{19.2}\left(\frac{1\text{ hp}}{2545\text{ Btu/h}}\right) = \mathbf{1.13\ hp}$$



6-99 A Carnot heat pump consumes 8-kW of power when operating, and maintains a house at a specified temperature. The average rate of heat loss of the house in a particular day is given. The actual running time of the heat pump that day, the heating cost, and the cost if resistance heating is used instead are to be determined.

Analysis (*a*) The coefficient of performance of this Carnot heat pump depends on the temperature limits in the cycle only, and is determined from

$$\mathrm{COP_{HP,rev}} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(2+273\ \mathrm{K})/(20+273\ \mathrm{K})} = 16.3$$



The amount of heat the house lost that day is

$$Q_H = \dot{Q}_H(1\ \mathrm{day}) = (82{,}000\ \mathrm{kJ/h})(24\ \mathrm{h}) = 1{,}968{,}000\ \mathrm{kJ}$$

Then the required work input to this Carnot heat pump is determined from the definition of the coefficient of performance to be

$$W_{\mathrm{net,in}} = \frac{Q_H}{\mathrm{COP_{HP}}} = \frac{1{,}968{,}000\ \mathrm{kJ}}{16.3} = 120{,}736\ \mathrm{kJ}$$

Thus the length of time the heat pump ran that day is

$$\Delta t = \frac{W_{\mathrm{net,in}}}{\dot{W}_{\mathrm{net,in}}} = \frac{120{,}736\ \mathrm{kJ}}{8\ \mathrm{kJ/s}} = 15{,}092\ \mathrm{s} = \mathbf{4.19\ h}$$

(*b*) The total heating cost that day is

$$\mathrm{Cost} = W \times \mathrm{price} = (\dot{W}_{\mathrm{net,in}} \times \Delta t)(\mathrm{price}) = (8\ \mathrm{kW})(4.19\ \mathrm{h})(0.085\ \$/\mathrm{kWh}) = \mathbf{\$2.85}$$

(*c*) If resistance heating were used, the entire heating load for that day would have to be met by electrical energy. Therefore, the heating system would consume 1,968,000 kJ of electricity that would cost

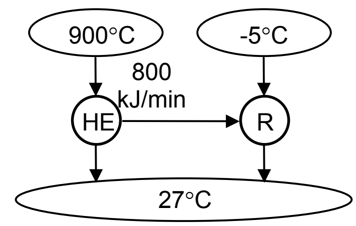
$$\mathrm{New\ Cost} = Q_H \times \mathrm{price} = (1{,}968{,}000\mathrm{kJ})\left(\frac{1\ \mathrm{kWh}}{3600\ \mathrm{kJ}}\right)(0.085\ \$/\mathrm{kWh}) = \mathbf{\$46.47}$$

6-100 A Carnot heat engine is used to drive a Carnot refrigerator. The maximum rate of heat removal from the refrigerated space and the total rate of heat rejection to the ambient air are to be determined.

Assumptions The heat engine and the refrigerator operate steadily.

Analysis (a) The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{\text{th,max}} = \eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300\text{ K}}{1173\text{ K}} = 0.744$$



Then the maximum power output of this heat engine is determined from the definition of thermal efficiency to be

$$\dot{W}_{\text{net,out}} = \eta_{\text{th}}\dot{Q}_H = (0.744)(800\text{ kJ/min}) = 595.2\text{ kJ/min}$$

which is also the power input to the refrigerator, $\dot{W}_{\text{net,in}}$.

The rate of heat removal from the refrigerated space will be a maximum if a Carnot refrigerator is used. The COP of the Carnot refrigerator is

$$\text{COP}_{\text{R,rev}} = \frac{1}{(T_H/T_L)-1} = \frac{1}{(27+273\text{ K})/(-5+273\text{ K})-1} = 8.37$$

Then the rate of heat removal from the refrigerated space becomes

$$\dot{Q}_{L,\text{R}} = (\text{COP}_{\text{R,rev}})(\dot{W}_{\text{net,in}}) = (8.37)(595.2\text{ kJ/min}) = \mathbf{4982\ kJ/min}$$

(b) The total rate of heat rejection to the ambient air is the sum of the heat rejected by the heat engine ($\dot{Q}_{L,\text{HE}}$) and the heat discarded by the refrigerator ($\dot{Q}_{H,\text{R}}$),

$$\dot{Q}_{L,\text{HE}} = \dot{Q}_{H,\text{HE}} - \dot{W}_{\text{net,out}} = 800 - 595.2 = 204.8\text{ kJ/min}$$

$$\dot{Q}_{H,\text{R}} = \dot{Q}_{L,\text{R}} + \dot{W}_{\text{net,in}} = 4982 + 595.2 = 5577.2\text{ kJ/min}$$

and

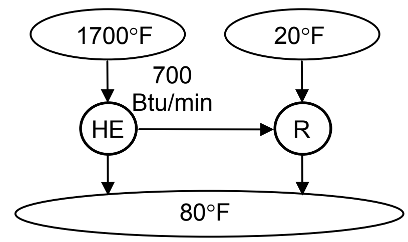
$$\dot{Q}_{\text{ambient}} = \dot{Q}_{L,\text{HE}} + \dot{Q}_{H,\text{R}} = 204.8 + 5577.2 = \mathbf{5782\ kJ/min}$$

6-101E A Carnot heat engine is used to drive a Carnot refrigerator. The maximum rate of heat removal from the refrigerated space and the total rate of heat rejection to the ambient air are to be determined.

Assumptions The heat engine and the refrigerator operate steadily.

Analysis (*a*) The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{\text{th,max}} = \eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{540\text{ R}}{2160\text{ R}} = 0.75$$



Then the maximum power output of this heat engine is determined from the definition of thermal efficiency to be

$$\dot{W}_{\text{net,out}} = \eta_{\text{th}}\dot{Q}_H = (0.75)(700\text{ Btu/min}) = 525\text{ Btu/min}$$

which is also the power input to the refrigerator, $\dot{W}_{\text{net,in}}$.

The rate of heat removal from the refrigerated space will be a maximum if a Carnot refrigerator is used. The COP of the Carnot refrigerator is

$$COP_{\text{R,rev}} = \frac{1}{(T_H/T_L)-1} = \frac{1}{(80+460\text{ R})/(20+460\text{ R})-1} = 8.0$$

Then the rate of heat removal from the refrigerated space becomes

$$\dot{Q}_{L,\text{R}} = (\text{COP}_{\text{R,rev}})(\dot{W}_{\text{net,in}}) = (8.0)(525\text{ Btu/min}) = \mathbf{4200\text{ Btu/min}}$$

(*b*) The total rate of heat rejection to the ambient air is the sum of the heat rejected by the heat engine ($\dot{Q}_{L,\text{HE}}$) and the heat discarded by the refrigerator ($\dot{Q}_{H,\text{R}}$),

$$\dot{Q}_{L,\text{HE}} = \dot{Q}_{H,\text{HE}} - \dot{W}_{\text{net,out}} = 700 - 525 = 175\text{ Btu/min}$$

$$\dot{Q}_{H,\text{R}} = \dot{Q}_{L,\text{R}} + \dot{W}_{\text{net,in}} = 4200 + 525 = 4725\text{ Btu/min}$$

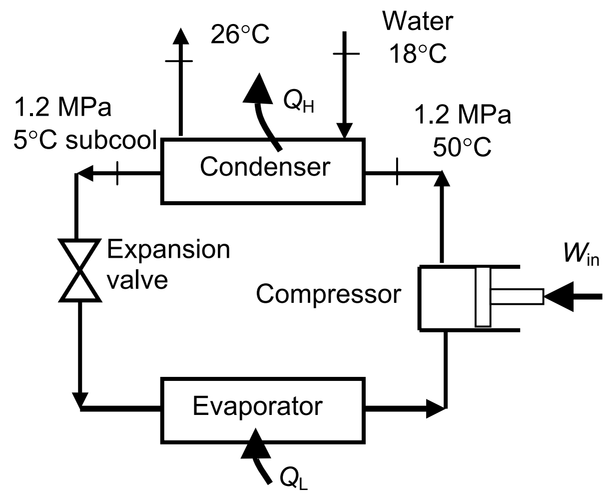
and

$$\dot{Q}_{\text{ambient}} = \dot{Q}_{L,\text{HE}} + \dot{Q}_{H,\text{R}} = 175 + 4725 = \mathbf{4900\ \ Btu/min}$$

6-102 A commercial refrigerator with R-134a as the working fluid is considered. The condenser inlet and exit states are specified. The mass flow rate of the refrigerant, the refrigeration load, the COP, and the minimum power input to the compressor are to be determined.

Assumptions **1** The refrigerator operates steadily. **2** The kinetic and potential energy changes are zero.

Properties The properties of R-134a and water are (Steam and R-134a tables)



$$\left.\begin{array}{l} P_1 = 1.2\text{ MPa} \\ T_1 = 50°\text{C} \end{array}\right\} h_1 = 278.27\text{ kJ/kg}$$

$$T_2 = T_{\text{sat@1.2 MPa}} + \Delta T_{\text{subcool}} = 46.3 - 5 = 41.3°\text{C}$$

$$\left.\begin{array}{l} P_2 = 1.2\text{ MPa} \\ T_2 = 41.3°\text{C} \end{array}\right\} h_2 = 110.17\text{ kJ/kg}$$

$$\left.\begin{array}{l} T_{w,1} = 18°\text{C} \\ x_{w,1} = 0 \end{array}\right\} h_{w,1} = 75.54\text{ kJ/kg}$$

$$\left.\begin{array}{l} T_{w,2} = 26°\text{C} \\ x_{w,2} = 0 \end{array}\right\} h_{w,2} = 109.01\text{ kJ/kg}$$

Analysis (*a*) The rate of heat transferred to the water is the energy change of the water from inlet to exit

$$\dot{Q}_H = \dot{m}_w (h_{w,2} - h_{w,1}) = (0.25\text{ kg/s})(109.01 - 75.54)\text{ kJ/kg} = 8.367\text{ kW}$$

The energy decrease of the refrigerant is equal to the energy increase of the water in the condenser. That is,

$$\dot{Q}_H = \dot{m}_R (h_1 - h_2) \longrightarrow \dot{m}_R = \frac{\dot{Q}_H}{h_1 - h_2} = \frac{8.367\text{ kW}}{(278.27 - 110.17)\text{ kJ/kg}} = \mathbf{0.0498\ kg/s}$$

(*b*) The refrigeration load is

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{\text{in}} = 8.37 - 3.30 = \mathbf{5.07\ kW}$$

(*c*) The COP of the refrigerator is determined from its definition,

$$\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = \frac{5.07\text{ kW}}{3.3\text{ kW}} = \mathbf{1.54}$$

(*d*) The COP of a reversible refrigerator operating between the same temperature limits is

$$\text{COP}_{\max} = \frac{1}{T_H / T_L - 1} = \frac{1}{(18 + 273)/(-35 + 273) - 1} = 4.49$$

Then, the minimum power input to the compressor for the same refrigeration load would be

$$\dot{W}_{\text{in,min}} = \frac{\dot{Q}_L}{\text{COP}_{\max}} = \frac{5.07\text{ kW}}{4.49} = \mathbf{1.13\ kW}$$

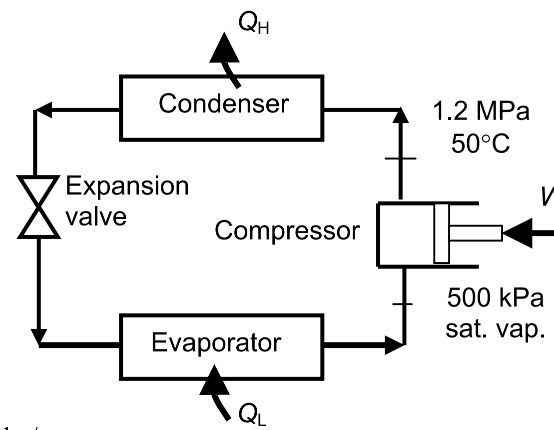
6-103 An air-conditioner with R-134a as the working fluid is considered. The compressor inlet and exit states are specified. The actual and maximum COPs and the minimum volume flow rate of the refrigerant at the compressor inlet are to be determined.

Assumptions **1** The air-conditioner operates steadily. **2** The kinetic and potential energy changes are zero.

Properties The properties of R-134a at the compressor inlet and exit states are (Tables A-11 through A-13)

$$\left.\begin{array}{l} P_1 = 500\text{ kPa} \\ x_1 = 1 \end{array}\right\} \begin{array}{l} h_1 = 259.30\text{ kJ/kg} \\ v_1 = 0.04112\text{ m}^3\text{/kg} \end{array}$$

$$\left.\begin{array}{l} P_2 = 1.2\text{ MPa} \\ T_2 = 50°\text{C} \end{array}\right\} h_2 = 278.27\text{ kJ/kg}$$



Analysis (*a*) The mass flow rate of the refrigerant and the power consumption of the compressor are

$$\dot{m}_R = \frac{\dot{V}_1}{v_1} = \frac{100\text{ L/min}\left(\frac{1\text{ m}^3}{1000\text{ L}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)}{0.04112\text{ m}^3\text{/kg}} = 0.04053\text{ kg/s}$$

$$\dot{W}_{\text{in}} = \dot{m}_R(h_2 - h_1) = (0.04053\text{ kg/s})(278.27 - 259.30)\text{ kJ/kg} = 0.7686\text{ kW}$$

The heat gains to the room must be rejected by the air-conditioner. That is,

$$\dot{Q}_L = \dot{Q}_{\text{heat}} + \dot{Q}_{\text{equipment}} = (250\text{ kJ/min})\left(\frac{1\text{ min}}{60\text{ s}}\right) + 0.9\text{ kW} = 5.067\text{ kW}$$

Then, the actual COP becomes

$$\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = \frac{5.067\text{ kW}}{0.7686\text{ kW}} = \mathbf{6.59}$$

(*b*) The COP of a reversible refrigerator operating between the same temperature limits is

$$\text{COP}_{\text{max}} \frac{1}{T_H / T_L - 1} = \frac{1}{(34 + 273)/(26 + 273) - 1} = \mathbf{37.4}$$

(*c*) The minimum power input to the compressor for the same refrigeration load would be

$$\dot{W}_{\text{in,min}} = \frac{\dot{Q}_L}{\text{COP}_{\text{max}}} = \frac{5.067\text{ kW}}{37.38} = 0.1356\text{ kW}$$

The minimum mass flow rate is

$$\dot{m}_{R,\text{min}} = \frac{\dot{W}_{\text{in,min}}}{h_2 - h_1} = \frac{0.1356\text{ kW}}{(278.27 - 259.30)\text{ kJ/kg}} = 0.007149\text{ kg/s}$$

Finally, the minimum volume flow rate at the compressor inlet is

$$\dot{V}_{\text{min},1} = \dot{m}_{R,\text{min}} v_1 = (0.007149\text{ kg/s})(0.04112\text{ m}^3\text{/kg}) = 0.000294\text{ m}^3\text{/s} = \mathbf{17.64\ L/min}$$