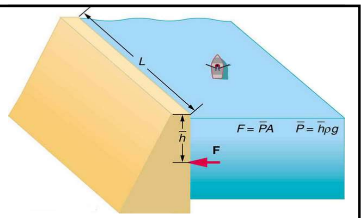

ME 251 Introduction to Fluid Mechanics

Pressure in Fluid

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1

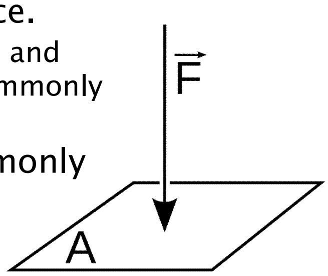
Areas to Cover

- Pressure in Fluid
  - Pressure variation in a fluid at rest
  - Absolute, gauge, atmospheric and vacuum pressures
  - Measurement of pressure: Manometers, mechanical gauges

2

PRESSURE

- Pressure is a normal force per unit area.
- Unit is Newton per square meter (N/m^2), or Pascal (Pa).
- The unit Pascal is too small for pressures encountered in practice.
  - Multiples: kilopascal (1 kPa $=10^3$ Pa) and megapascal (1 MPa $= 10^6$ Pa) are commonly used.
- Three other pressure units commonly used in practice, are
  - bar,
  - standard atmosphere, and
  - kilogram-force per square centimeter



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PRESSURE

- English system:
  - the pressure unit is pound-force per square inch (lbf/in^2,or psi),
  - and 1 atm = 14.696 psi.
  - The pressure units kgf/cm^2 and lbf/in^2 are also denoted by kg/cm^2 and lb/in^2,respectively,
  - It can be shown that 1 kgf/cm^2 =14.223 psi.

$$1 \text{ bar} = 10^5 \text{ Pa} = 0.1 \text{ MPa} = 100 \text{ kPa}$$

$$1 \text{ atm} = 101{,}325 \text{ Pa} = 101.325 \text{ kPa} = 1.01325 \text{ bars}$$

$$\begin{aligned} 1 \text{ kgf/cm}^2 &= 9.807 \text{ N/cm}^2 = 9.807 \times 10^4 \text{ N/m}^2 = 9.807 \times 10^4 \text{ Pa} \\ &= 0.9807 \text{ bar} \\ &= 0.9679 \text{ atm} \end{aligned}$$

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PRESSURE

- The actual pressure at a given position is **absolute pressure**, measured relative to absolute vacuum (i.e., absolute zero pressure).
- Most pressure measuring devices are calibrated to read zero in the atmosphere
- Measured the difference between the absolute pressure and the local atmospheric pressure called the **gauge pressure.**
- Pressures below atmospheric pressure are **vacuum pressures** and are measured by vacuum gauges that indicate the difference between the atmospheric pressure and the absolute pressure.

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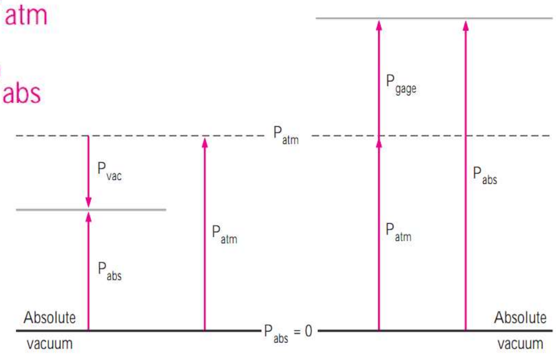
PRESSURE

- Absolute, gauge, and vacuum pressures are all positive quantities and are related to each other by

$$P_{gage} = P_{abs} - P_{atm}$$

$$P_{vac} = P_{atm} - P_{abs}$$



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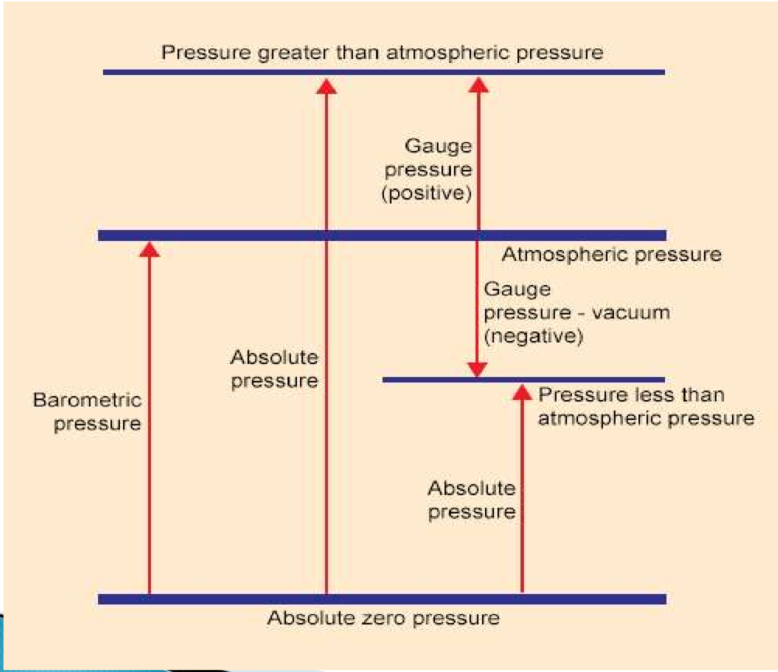

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A vacuum gauge connected to a chamber reads 5.8 psi at a location where the atmospheric pressure is 14.5 psi. Determine the absolute pressure in the chamber.

SOLUTION The gage pressure of a vacuum chamber is given. The absolute pressure in the chamber is to be determined.

Analysis The absolute pressure is easily determined from Eq. 3–2 to be

$$P_{abs} = P_{atm} - P_{vac} = 14.5 - 5.8 = 8.7 \text{ psi}$$

Discussion Note that the *local* value of the atmospheric pressure is used when determining the absolute pressure.

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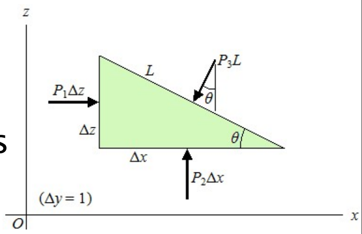
Pressure at a Point

- The mean pressures at the three surfaces are P_1, P_2,and P_3
- The force acting on a surface is the product of mean pressure and the Area



$$\sum F_x = ma_x = 0: \qquad P_1\,\Delta z - P_3 l \sin\theta = 0$$

$$\sum F_z = ma_z = 0: \qquad P_2\,\Delta x - P_3 l \cos\theta - \frac{1}{2}\rho g\,\Delta x\,\Delta z = 0$$

$$P_1 - P_3 = 0$$

$$P_2 - P_3 - \frac{1}{2}\rho g\,\Delta z = 0$$

$$P_1 = P_2 = P_3 = P$$

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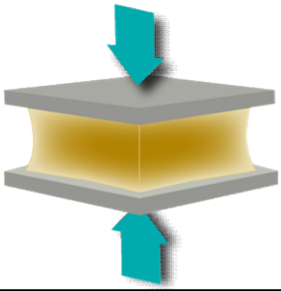
Pressure at a Point

- Pressure is the compressive force per unit area, and it gives the impression of being a vector.
- Pressure at any point in a fluid is the same in all directions.
- That is, it has magnitude but not a specific direction,
  - a scalar quantity.



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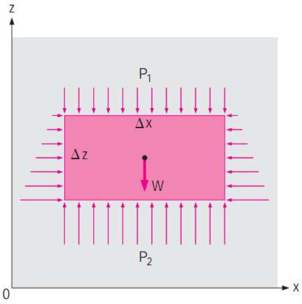
Variation of Pressure with Depth

- consider a rectangular fluid element of height Δz, length Δx, and unit width in equilibrium,
- Density of the fluid ρ is constant
- A force balance in the vertical z-direction gives



$$\sum F_z = ma_z = 0: \qquad P_2\,\Delta x - P_1\,\Delta x - \rho g\,\Delta x\,\Delta z = 0$$

$$\Delta P = P_2 - P_1 = \rho g\,\Delta z = \gamma_s\,\Delta z$$

The pressure difference between two points in a constant density fluid is proportional to the vertical distance Δz between the points and the density ρ of the fluid.
Pressure in a fluid increases linearly with depth.

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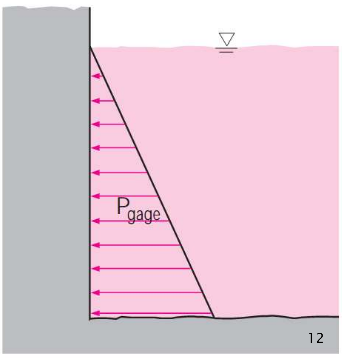
Variation of Pressure with Depth

- For fluids whose density changes significantly with elevation, a relation for the variation of pressure with elevation can be obtained by dividing by ΔxΔz, and taking the limit as Δz→0.

$$P_2\,\Delta x - P_1\,\Delta x - \rho g\,\Delta x\,\Delta z = 0$$

$$\frac{dP}{dz} = -\rho g$$



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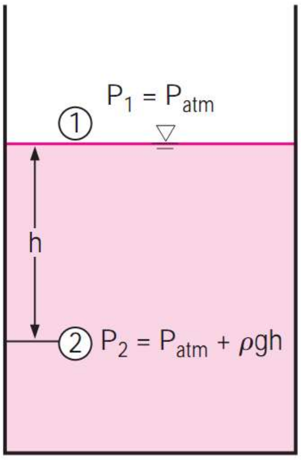
Variation of Pressure with Depth

- The pressure difference between points 1 and 2 can be determined by integrating

$$\frac{dP}{dz} = -\rho g$$

$$\Delta P = P_2 - P_1 = -\int_1^2 \rho g \, dz$$



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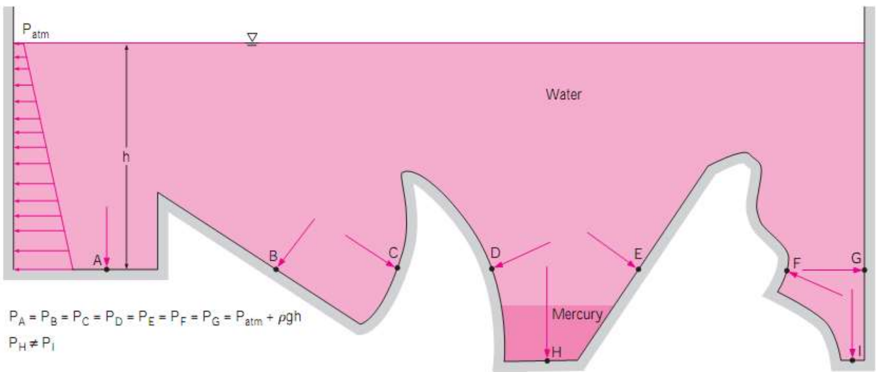
Pressure in a fluid at rest

- Independent of the shape or cross section of the container.
- Changes with the vertical distance, but remains constant in Horizontal direction.



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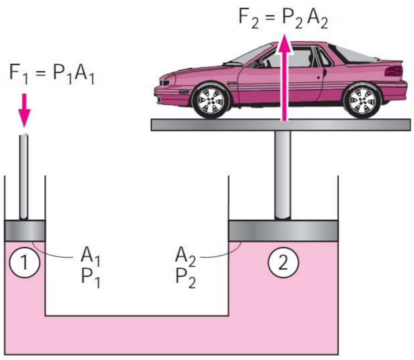
Pascal's law

- The pressure applied to a confined fluid transmit the pressure throughout by the same amount.
- A consequence of the pressure in a fluid remaining constant in the horizontal direction,



$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

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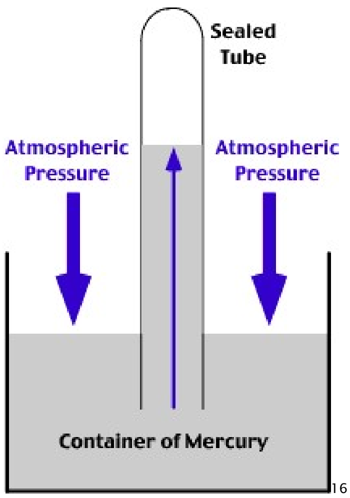
THE BAROMETER

- A device for measuring atmospheric pressure.
- It is Inverted mercury-filled tube in a mercury container that is open to the atmosphere.



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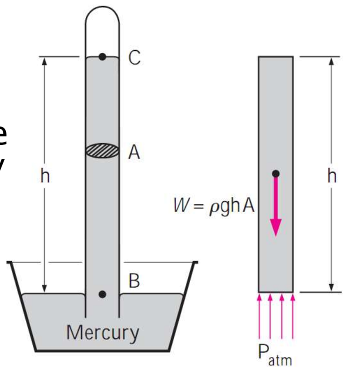
THE BAROMETER

- The pressure at point B is equal to the atmospheric pressure.
- Pressure at C can be taken to be zero since there is only mercury vapor above point C and the pressure is very low relative to P_{atm} and can be neglected to an excellent approximation.
- Writing a force balance in the vertical direction gives



$$P_{atm} = \rho g h$$

Atmospheric pressure = Barometric pressure

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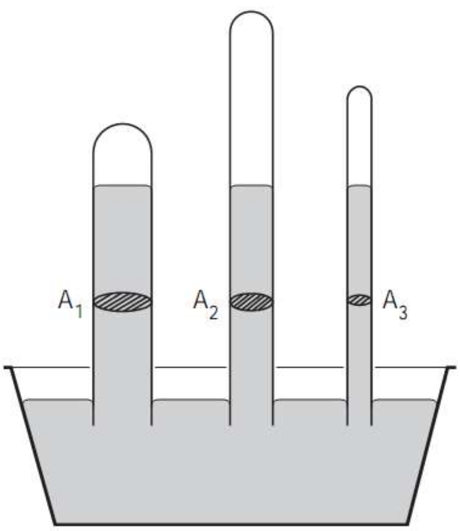
THE BAROMETER

- Effect of cross-section is negligible



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Atmospheric Pressure Unit

- A frequently used pressure unit is the standard atmosphere.
  - the pressure produced by a column of mercury 760 mm in height at 0°C (ρHg =13,595 kg/m^3) under standard gravitational acceleration (g " 9.807 m/s^2).
- If water instead of mercury were used to measure the standard atmospheric pressure, a water column of about 10.3 m would be needed.
- Pressure is sometimes expressed (especially by weather forecasters) in terms of the height of the mercury column.
  - The standard atmospheric pressure, for example, is 760 mmHg (29.92 inHg) at 0°C.
  - The unit mmHg is also called the torr in honor of Torricelli.
  - Therefore, 1 atm = 760 torr and 1 torr = 133.3 Pa

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Determine the atmospheric pressure at a location where the barometric reading is 740 mmHg and the gravitational acceleration is g = 9.81 m/s^2.
Assume the temperature of mercury to be 10°C, at which its density is13,570 kg/m^3.

SOLUTION The barometric reading at a location in height of mercury column is given. The atmospheric pressure is to be determined.
Assumptions The temperature of mercury is assumed to be 10°C.
Properties The density of mercury is given to be 13,570 kg/m^3.
Analysis From Eq. 3–15, the atmospheric pressure is determined to be

$$\begin{aligned} P_{atm} &= \rho g h \\ &= (13{,}570\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.74\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= 98.5\ \text{kPa} \end{aligned}$$

Discussion Note that density changes with temperature, and thus this effect should be considered in calculations.

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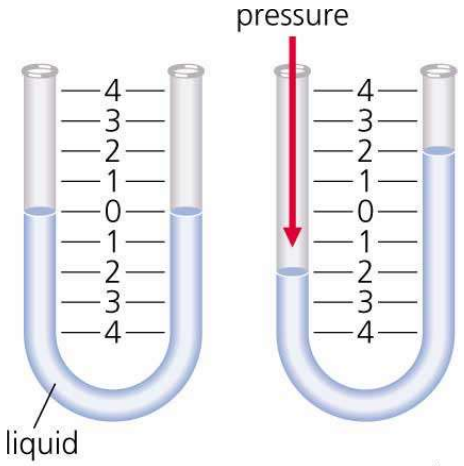
THE MANOMETER

- A manometer is a glass or plastic U-tube containing one or more fluids such as mercury, water, alcohol, or oil.
- commonly used to measure small and moderate pressure differences.



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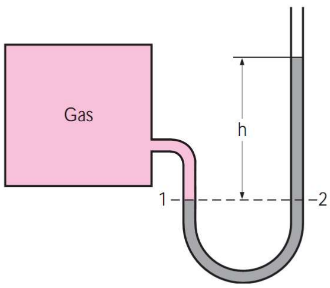
THE MANOMETER

- Since the gravitational effects of gases are negligible, the pressure anywhere in the tank and at position 1 has the same value.
- Since pressure in a fluid does not vary in the horizontal direction within a fluid, the pressure at point 2 is the same as the pressure at point 1, $P_2 = P_1$.
- The differential fluid column of height **h** is in static equilibrium, and it is open to the atmosphere.
- Pressure at point 2 is determined directly as



$$P_2 = P_{atm} + \rho g h$$

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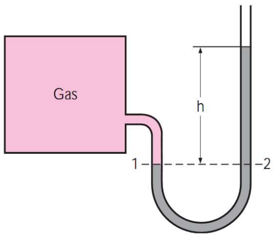
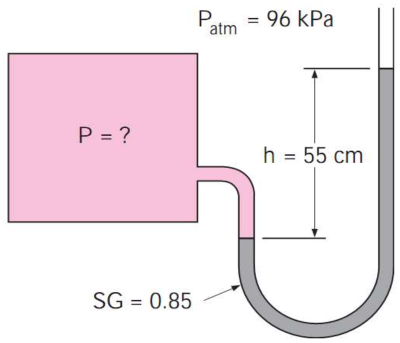
THE MANOMETER

- The cross-sectional area of the tube has no effect on the differential height **h**, and thus the pressure exerted by the fluid.
- However, the diameter of the tube should be large enough (more than a few millimeters) to ensure that the surface tension effect and thus the capillary rise is negligible.



A manometer is used to measure the pressure in a tank. The fluid used has a specific gravity of 0.85, and the manometer column height is 55 cm, as shown. If the local atmospheric pressure is 96 kPa, determine the absolute pressure within the tank.



SOLUTION The reading of a manometer attached to a tank and the atmospheric pressure are given. The absolute pressure in the tank is to be determined.

Assumptions The fluid in the tank is a gas whose density is much lower than the density of manometer fluid.

Properties The specific gravity of the manometer fluid is given to be 0.85. We take the standard density of water to be 1000 kg/m³.

Analysis The density of the fluid is obtained by multiplying its specific gravity by the density of water, which is taken to be 1000 kg/m³:

$$\rho = SG\,(\rho_{H_2O}) = (0.85)(1000\ \text{kg/m}^3) = 850\ \text{kg/m}^3$$

Then from Eq. 3–12,

$$\begin{aligned} P &= P_{atm} + \rho g h \\ &= 96\ \text{kPa} + (850\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.55\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= 100.6\ \text{kPa} \end{aligned}$$

Discussion Note that the gage pressure in the tank is 4.6 kPa.

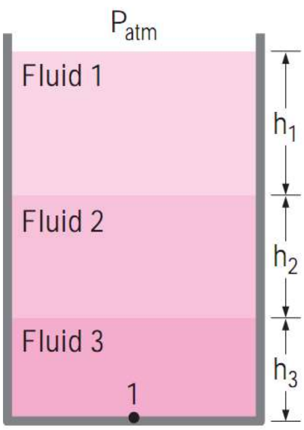
Multifluid System

(1) the pressure change across a fluid column of height h is

$\Delta P = \rho g h$,

(2) pressure increases downward in a given fluid and decreases upward ($P_{bottom} > P_{top}$), and

(3) two points at the same elevation in a continuous fluid at rest are at the same pressure.

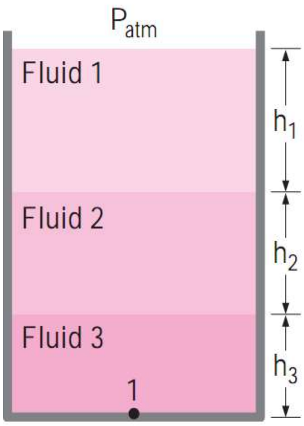


Multifluid System

- The pressure at any point can be determined by starting with a point of known pressure and adding or subtracting

$$\rho g h$$

terms as we advance toward the point of interest.



$$P_{atm} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 = P_1$$

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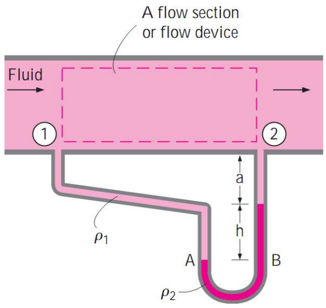
Pressure Difference

- The working fluid can be either a gas or a liquid with density, ρ_1.
- The density of the manometer fluid is ρ_2,
- The differential fluid height is h.
- Starting at point 1 with P_1, moving along the tube by adding or subtracting the $\rho g h$ terms until we reach point 2, and setting the result equal to P_2
- Pressure difference $P_1 - P_2$ is obtained



$$P_1 + \rho_1 g(a + h) - \rho_2 g h - \rho_1 g a = P_2$$

$$P_1 - P_2 = (\rho_2 - \rho_1) g h$$

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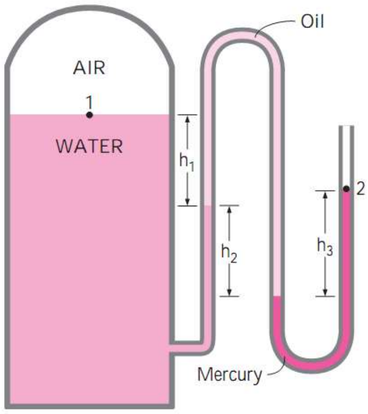
The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in Fig. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if $h_1 = 0.1$ m, $h_2 = 0.2$ m, and $h_3 = 0.35$ m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.



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SOLUTION The pressure in a pressurized water tank is measured by a multi-fluid manometer. The air pressure in the tank is to be determined.

Assumption The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus we can determine the pressure at the air–water interface.

Properties The densities of water, oil, and mercury are given to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.

Analysis Starting with the pressure at point 1 at the air–water interface, moving along the tube by adding or subtracting the ρgh terms until we reach point 2, and setting the result equal to P_{atm} since the tube is open to the atmosphere gives

$$P_1 + \rho_{water} g h_1 + \rho_{oil} g h_2 - \rho_{mercury} g h_3 = P_{atm}$$

Solving for P_1 and substituting,

$$\begin{aligned} P_1 &= P_{atm} - \rho_{water} g h_1 - \rho_{oil} g h_2 + \rho_{mercury} g h_3 \\ &= P_{atm} + g(\rho_{mercury} h_3 - \rho_{water} h_1 - \rho_{oil} h_2) \\ &= 85.6 \text{ kPa} + (9.81 \text{ m/s}^2)[(13{,}600 \text{ kg/m}^3)(0.35 \text{ m}) - (1000 \text{ kg/m}^3)(0.1 \text{ m}) \\ &\quad - (850 \text{ kg/m}^3)(0.2 \text{ m})]\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) \\ &= 130 \text{ kPa} \end{aligned}$$

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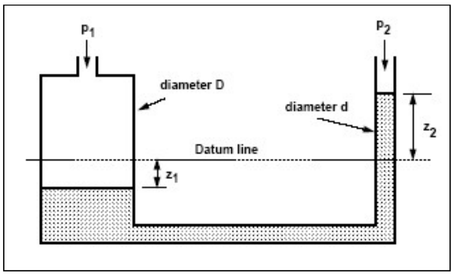
Magnification

The volume that move from large side (left) = the volume that rise in right tube



volume of liquid moved from the left side to the right
$= z_2 \times (\pi d^2 / 4)$

The fall in level of the left side is

$$z_1 = \frac{\text{Volume moved}}{\text{Area of left side}}$$

$$= \frac{z_2\left(\pi d^2 / 4\right)}{\pi D^2 / 4}$$

$$= z_2\left(\frac{d}{D}\right)^2$$

Putting this in the equation,

$$p_1 - p_2 = \rho g\left[z_2 + z_2\left(\frac{d}{D}\right)^2\right]$$

$$= \rho g z_2\left[1 + \left(\frac{d}{D}\right)^2\right]$$

If D >> d then $(d/D)^2$ is very small so

$$p_1 - p_2 = \rho g z_2$$

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The volume that move from x-x to z-z datum = the volume that rise in right tube above x-x datum

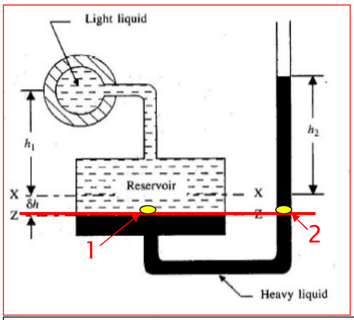
$$P1 = P2$$

$$P + \rho_1 g(h_1 + \delta h) = \rho_2 g(h_2 + \delta h)$$

$$P + \rho_1 g(h_1 + \frac{a}{A}h_2) = \rho_2 g(h_2 + \frac{a}{A}h_2)$$

$\frac{\text{a}}{\text{A}}$ is very small

$$\therefore P = \rho_2 g(h_2) - \rho_1 g(h_1)$$



$$A \times \delta h = a \times h_2 \quad \text{or} \quad \delta h = \frac{a \times h_2}{A}$$

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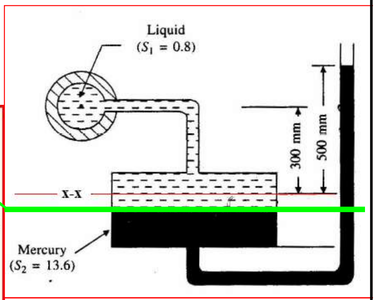
The manometer shown is connected to a pipe containing a liquid of specific gravity of 0.8. the ratio of area of the reservoir to that of the limb is 100. find the pressure in the pipe

Solution



$$P1 = P2$$

$$P + 800 \times 9.81 \times (0.3 + \delta h) = 13600 \times 9.81 \times (0.5 + \delta h)$$

$$P + 800 \times 9.81(0.3 + \frac{1}{100}0.5) = 13600 \times 9.81(0.5 + \frac{1}{100}0.5)$$

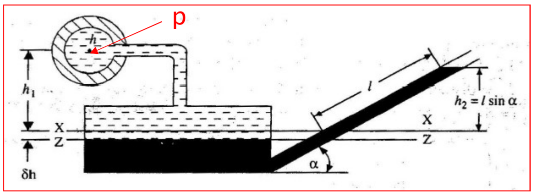
$$P + 2393.6 = 67375.1$$

$$\mathrm{P} = 64981.5$$

$$h = \frac{P}{\rho_1 g} = \frac{64981.5}{800 \times 9.81} = (8.28\ m) \text{ of liquid}$$

Inclined single column manometers

- This type of manometer is useful for measurement of small pressures and is more sensitive than vertical tube type.



$$P1 = P2$$

$$P + \rho_1 g(h_1 + \delta h) = \rho_2 g(h_2 + \delta h)$$

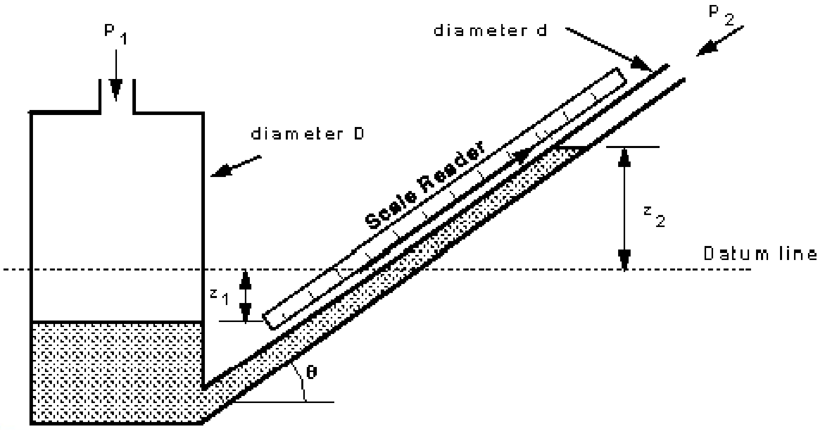
$$P + \rho_1 g(h_1 + \frac{a}{A}h_2) = \rho_2 g(h_2 + \frac{a}{A}h_2)$$

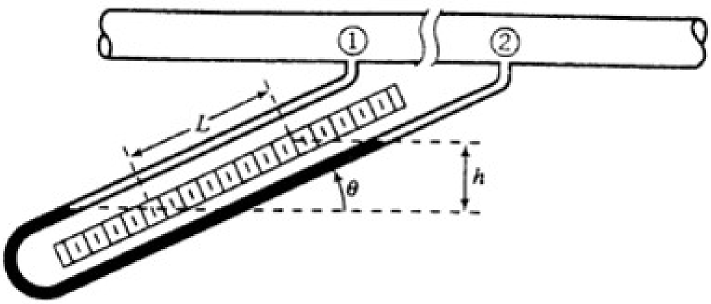
$$\frac{\mathrm{a}}{\mathrm{A}} \text{ is very small}$$

$$\therefore P = \rho_2 g(h_2) - \rho_1 g(h_1)$$

$$P = \rho_2 g(l \sin \alpha) - \rho_1 g(h_1)$$

$$h = \frac{P}{\rho_1 g} = \frac{\rho_2}{\rho_1} l \sin \alpha - h_1$$





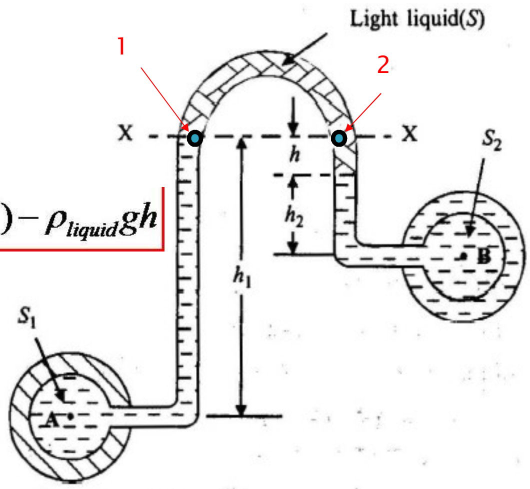
$$P_1 = (\rho_2 - \rho_1)gh + P_2$$
$$P_1 - P_2 = (\rho_2 - \rho_1)gh$$
But $h = L\sin\theta$
$$P_1 - P_2 = (\rho_2 - \rho_1)g\, L\sin\theta$$

Inverted U-tube Differential manometers

At x-x

$P_1 = P_2$

$$P_A - \rho_A g(h_1) = P_B - \rho_B g(h_2) - \rho_{liquid} g h$$



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An inverted manometer having an oil of specific gravity 0.8 connected to two different pipes carrying water under pressure. Determine the pressure in the pipe B the pressure in the pipe A is 2.0 meters of water

At datum x-x

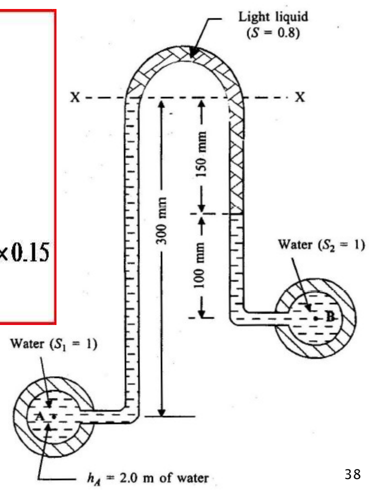
$P1 = P2$

$P_A = 2 \times 1000 \times 9.81 = 19620\ \text{N/m}^2$

$$P_A - \rho_A g(h_1) = P_B - \rho_B g(h_2) - \rho_{liquid} g h$$

$$19620 - 1000 \times 9.81 \times 0.3 = P_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.15$$

$$P_B = 18835\ \text{N/m}^2 = 18.8\ \text{kN/m}^2$$



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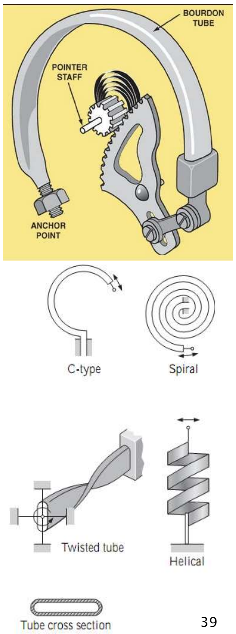
Other Pressure Measuring Devices

- Bourdon tube is a mechanical pressure measurement device.
- Consists of a hollow metal tube bent like a hook whose end is closed and connected to a dial indicator needle.
- When the tube is open to the atmosphere, the tube is undeflected, and the needle on the dial at this state is calibrated to read zero (gauge pressure).
- When the fluid inside the tube is pressurized, the tube stretches and moves the needle in proportion to the pressure applied.



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Pressure Transducers

- Pressure transducers use various techniques to convert the pressure effect to an electrical effect such as a change in voltage, resistance, or capacitance.
- They are smaller and faster, and can be more sensitive, reliable, and precise than their mechanical counterparts.
- Pressures from less than a millionth of 1 atm to several thousands of atm.

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Pressure Transducers

- A wide variety available to measure gauge, absolute, and differential pressures in a wide range of applications.
  - Gauge pressure transducers use the atmospheric pressure as a reference
  - The absolute pressure transducers are calibrated to have a zero signal output at full vacuum.
  - Differential pressure transducers measure the pressure difference.

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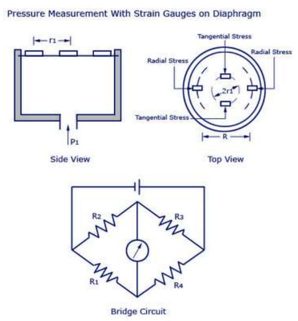
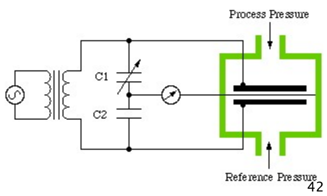
Strain-gauge Pressure Transducers

- Worked by having a diaphragm deflect between two chambers open to the pressure inputs.
- As the diaphragm stretches in response to a change in pressure difference across it, the strain gauge stretches and a Wheatstone bridge circuit amplifies the output.
- A capacitance transducer works similarly, but capacitance change is measured instead of resistance change as the diaphragm stretches.





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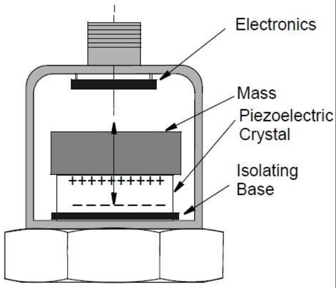
Piezoelectric Transducers

- Piezoelectric transducers = solid-state pressure transducers.
- Work on the principle that an electric potential is generated in a crystalline substance when it is subjected to mechanical pressure.
- Very suitable for high-pressure applications.
- Generally not as sensitive as the diaphragm-type transducers.



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