Chapter 22

D.C. machines

At the end of this chapter you should be able to:

- distinguish between the function of a motor and a generator
- describe the action of a commutator
- describe the construction of a d.c. machine
- distinguish between wave and lap windings
- understand shunt, series and compound windings of d.c. machines
- understand armature reaction
- calculate generated e.m.f. in an armature winding using $E = 2p\Phi nZ/c$
- describe types of d.c. generator and their characteristics
- calculate generated e.m.f. for a generator using $E = V + I_a R_a$
- state typical applications of d.c. generators
- list d.c. machine losses and calculate efficiency
- calculate back e.m.f. for a d.c. motor using $E = V - I_a R_a$
- calculate the torque of a d.c. motor using $T = EI_a/2\pi n$ and $T = p\Phi ZI_a/\pi c$
- describe types of d.c. motor and their characteristics
- state typical applications of d.c. motors
- describe a d.c. motor starter
- describe methods of speed control of d.c. motors
- list types of enclosure for d.c. motors

22.1 Introduction

When the input to an electrical machine is electrical energy, (seen as applying a voltage to the electrical terminals of the machine), and the output is mechanical energy, (seen as a rotating shaft), the machine is called an electric **motor**. Thus an electric motor converts electrical energy into mechanical energy.

The principle of operation of a motor is explained in Section 8.4, page 91. When the input to an electrical machine is mechanical energy, (seen as, say, a diesel motor, coupled to the machine by a shaft), and the output is electrical energy, (seen as a voltage appearing at the electrical terminals of the machine), the machine is called a **generator**. Thus, a generator converts mechanical energy to electrical energy.

The principle of operation of a generator is explained in Section 9.2, page 98.

22.2 The action of a commutator

In an electric motor, conductors rotate in a uniform magnetic field. A single-loop conductor mounted between permanent magnets is shown in Fig. 22.1. A voltage is applied at points A and B in Fig. 22.1(a)

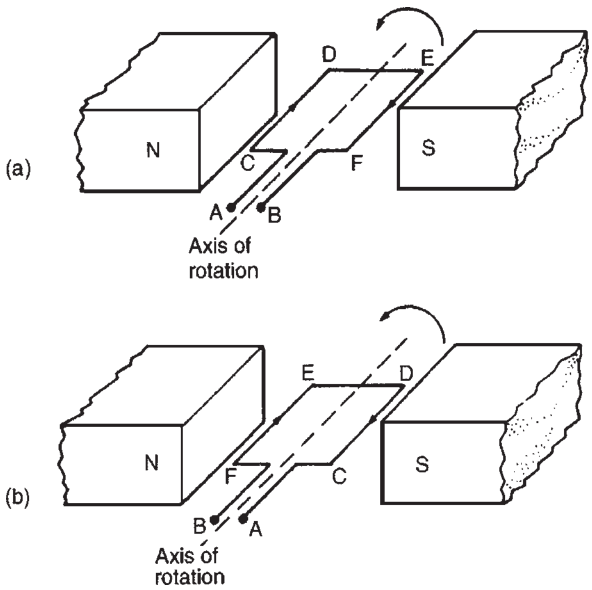


Figure 22.1

A force, F, acts on the loop due to the interaction of the magnetic field of the permanent magnets and the magnetic field created by the current flowing in the loop. This force is proportional to the flux density, B, the current flowing, I, and the effective length of the conductor, l, i.e. $F = BIl$. The force is made up of two parts, one acting vertically downwards due to the current flowing from C to D and the other acting vertically upwards due to the current flowing from E to F (from Fleming's left hand rule). If the loop is free to rotate, then when it has rotated through 180°, the conductors are as shown in Fig. 22.1(b) For rotation to continue in the same direction, it is necessary for the current flow to be as shown in Fig. 22.1(b), i.e. from D to C and from F to E. This apparent reversal in the direction of current flow is achieved by a process called **commutation**. With reference to Fig. 22.2(a), when a direct voltage is applied at A and B, then as the single-loop conductor rotates, current flow will always be away from the commutator for the part of the conductor adjacent to the N-pole and towards the commutator for the part of the conductor adjacent to the S-pole. Thus the forces act to give continuous rotation in an anti-clockwise direction. The arrangement shown in Fig. 22.2(a) is called a 'two-segment' commutator and the voltage is applied to the rotating segments by stationary **brushes**, (usually carbon blocks), which slide on the commutator material, (usually copper), when rotation takes place.

In practice, there are many conductors on the rotating part of a d.c. machine and these are attached to many commutator segments. A schematic diagram of a multi-segment commutator is shown in Fig. 22.2(b).

Poor commutation results in sparking at the trailing edge of the brushes. This can be improved by using **interpoles** (situated between each pair of main poles), high resistance brushes, or using brushes spanning several commutator segments.

22.3 D.C. machine construction

The basic parts of any d.c. machine are shown in Fig. 22.3, and comprise:

(a) a stationary part called the **stator** having,

(i) a steel ring called the **yoke**, to which are attached

(ii) the magnetic **poles**, around which are the

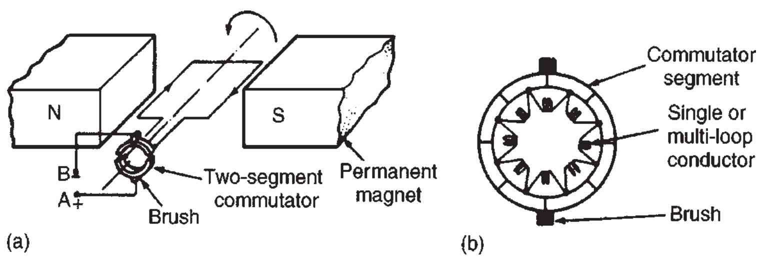


Figure 22.2

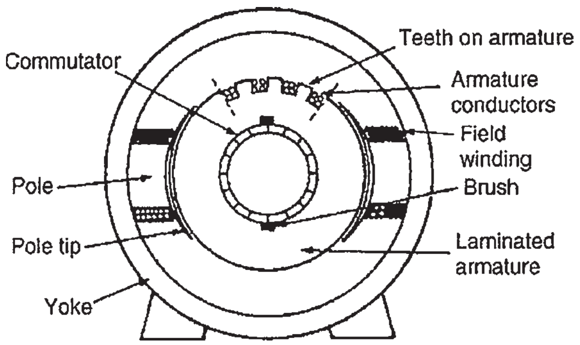


Figure 22.3

(iii) **field windings**, i.e. many turns of a conductor wound round the pole core; current passing through this conductor creates an electromagnet, (rather than the permanent magnets shown in Figs. 22.1 and 22.2),

(b) a rotating part called the **armature** mounted in bearings housed in the stator and having,

(iv) a laminated cylinder of iron or steel called the **core**, on which teeth are cut to house the
(v) **armature winding**, i.e. a single or multi-loop conductor system, and
(vi) the **commutator**, (see Section 22.2)

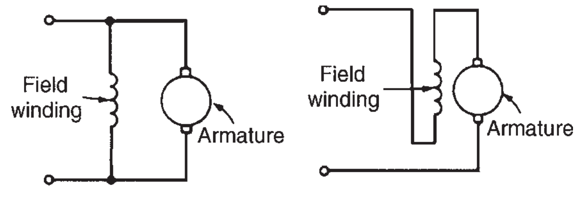
Armature windings can be divided into two groups, depending on how the wires are joined to the commutator. These are called **wave windings** and **lap windings**.

(a) In **wave windings** there are two paths in parallel irrespective of the number of poles, each path supplying half the total current output. Wave wound generators produce high voltage, low current outputs.

(b) In **lap windings** there are as many paths in parallel as the machine has poles. The total current output divides equally between them. Lap wound generators produce high current, low voltage output.

22.4 Shunt, series and compound windings

When the field winding of a d.c. machine is connected in parallel with the armature, as shown in Fig. 22.4(a), the machine is said to be **shunt** wound. If the field winding is connected in series with the armature, as shown in Fig. 22.4(b), then the machine is said to be **series** wound. A **compound** wound machine has a combination of series and shunt windings.



(a) Shunt-wound machine (b) Series-wound machine

Figure 22.4

Depending on whether the electrical machine is series wound, shunt wound or compound wound, it behaves differently when a load is applied. The behaviour of a d.c. machine under various conditions is shown by means of graphs, called characteristic curves or just **characteristics**. The characteristics shown in the following sections are theoretical, since they neglect the effects of armature reaction.

Armature reaction is the effect that the magnetic field produced by the armature current has on the magnetic field produced by the field system. In a generator, armature reaction results in a reduced output voltage, and in a motor, armature reaction results in increased speed.

A way of overcoming the effect of armature reaction is to fit compensating windings, located in slots in the pole face.

22.5 E.m.f. generated in an armature winding

Let Z = number of armature conductors,
Φ = useful flux per pole, in webers,
p = number of **pairs** of poles
and n = armature speed in rev/s

The e.m.f. generated by the armature is equal to the e.m.f. generated by one of the parallel paths. Each conductor passes $2p$ poles per revolution and thus cuts $2p\Phi$ webers of magnetic flux per revolution. Hence flux cut by one conductor per second $= 2p\Phi n$ Wb and so the average e.m.f. E generated per conductor is given by:

$$E2p\Phi n \text{ volts}$$

(since 1 volt = 1 Weber per second)
Let c = number of parallel paths through the winding between positive and negative brushes

$c = 2$ **for a wave winding**

$c = 2p$ **for a lap winding**

The number of conductors in series in each path $= Z/c$

The total e.m.f. between

$$\text{brushes} = (\text{average e.m.f./conductor})\ (\text{number of conductors in series per path}) = 2p\Phi nZ/c$$

i.e. **generated e.m.f.** $\boldsymbol{E = \dfrac{2p\Phi nZ}{c}}$ **volts** (1)

Since Z, p and c are constant for a given machine, then $E \propto \Phi n$. However $2\pi n$ is the angular velocity ω in radians per second, hence the generated e.m.f. is proportional to Φ and ω,

i.e. **generated e.m.f.** $\boldsymbol{E \propto \Phi\omega}$ (2)

Problem 1. An 8-pole, wave-connected armature has 600 conductors and is driven at 625 rev/min. If the flux per pole is 20 mWb, determine the generated e.m.f.

$Z = 600$, $c = 2$ (for a wave winding), $p = 4$ pairs, $n = 625/60$ rev/s and $\Phi = 20 \times 10^{-3}$ Wb.

Generated e.m.f.

$$\boldsymbol{E} = \frac{2p\Phi nZ}{c} = \frac{2(4)(20 \times 10^{-3})\left(\frac{625}{60}\right)(600)}{2} = \textbf{500 volts}$$

Problem 2. A 4-pole generator has a lap-wound armature with 50 slots with 16 conductors per slot. The useful flux per pole is 30 mWb. Determine the speed at which the machine must be driven to generate an e.m.f. of 240 V.

$E = 240$ V, $c = 2p$ (for a lap winding), $Z = 50 \times 16 = 800$ and $\Phi = 30 \times 10^{-3}$ Wb.

Generated e.m.f.

$$\boldsymbol{E} = \frac{2p\Phi nZ}{c} = \frac{2p\Phi nZ}{2p} = \Phi nZ$$

Rearranging gives, speed,

$$n = \frac{E}{\Phi Z} = \frac{240}{(30 \times 10^{-3})(800)} = \textbf{10 rev/s or 600 rev/min}$$

Problem 3. An 8-pole, lap-wound armature has 1200 conductors and a flux per pole of 0.03 Wb. Determine the e.m.f. generated when running at 500 rev/min.

Generated e.m.f.,

$$\boldsymbol{E} = \frac{2p\Phi nZ}{c} = \frac{2p\Phi nZ}{2p} \text{ for a lap-wound machine,}$$

i.e. $\boldsymbol{E} = \Phi nZ$

$$= (0.03)\left(\frac{500}{60}\right)(1200) = \textbf{300 volts}$$

Problem 4. Determine the generated e.m.f. in Problem 3 if the armature is wave-wound.

Generated e.m.f.

$$\boldsymbol{E} = \frac{2p\Phi nZ}{c} = \frac{2p\Phi nZ}{2} \quad (\text{since } c = 2 \text{ for wave-wound})$$

$$= p\Phi nZ = (4)(\Phi nZ)$$

$$= (4)(300) \text{ from Problem 3}$$

$$= \textbf{1200 volts}$$

Problem 5. A d.c. shunt-wound generator running at constant speed generates a voltage of 150 V at a certain value of field current. Determine the change in the generated voltage when the field current is reduced by 20 per cent, assuming the flux is proportional to the field current.

The generated e.m.f. E of a generator is proportional to $\Phi\omega$, i.e. is proportional to Φn, where Φ is the flux and n is the speed of rotation. It follows that $E = k\Phi n$, where k is a constant.

At speed n_1 and flux Φ_1, $E_1 = k\Phi_1 n_1$

At speed n_2 and flux Φ_2, $E_2 = k\Phi_2 n_2$

Thus, by division:

$$\frac{E_1}{E_2} = \frac{k\Phi_1 n_1}{k\Phi_2 n_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2}$$

The initial conditions are $E_1 = 150\,V$, $\Phi = \Phi_1$ and $n = n_1$. When the flux is reduced by 20 per cent, the new value of flux is 80/100 or 0.8 of the initial value, i.e. $\Phi_2 = 0.8\Phi_1$. Since the generator is running at constant speed, $n_2 = n_1$

Thus $\quad \dfrac{E_1}{E_2} = \dfrac{\Phi_1 n_1}{\Phi_2 n_2} = \dfrac{\Phi_1 n_1}{0.8\Phi_1 n_2} = \dfrac{1}{0.8}$

that is, $\quad E_2 = 150 \times 0.8 = 120\,V$

Thus, a reduction of 20 per cent in the value of the flux **reduces the generated voltage to 120 V** at constant speed.

Problem 6. A d.c. generator running at 30 rev/s generates an e.m.f. of 200 V. Determine the percentage increase in the flux per pole required to generate 250 V at 20 rev/s.

From Equation (2), generated e.m.f., $E \propto \Phi\omega$ and since $\omega = 2\pi n$, $E \propto \Phi n$

$$\text{Let } E_1 = 200\,V,\ n_1 = 30\,\text{rev/s}$$

and flux per pole at this speed be Φ_1

$$\text{Let } E_2 = 250\,V,\ n_2 = 20\,\text{rev/s}$$

and flux per pole at this speed be Φ_2

Since $\quad E \propto \Phi n$ then $\dfrac{E_1}{E_2} = \dfrac{\Phi_1 n_1}{\Phi_2 n_2}$

Hence $\quad \dfrac{200}{250} = \dfrac{\Phi_1(30)}{\Phi_2(20)}$

from which, $\quad \Phi_2 = \dfrac{\Phi_1(30)(250)}{(20)(200)} = 1.875\Phi_1$

Hence **the increase in flux per pole needs to be 87.5 per cent**

Now try the following exercise

Exercise 130 Further problems on generator e.m.f.

1. A 4-pole, wave-connected armature of a d.c. machine has 750 conductors and is driven at 720 rev/min. If the useful flux per pole is 15 mWb, determine the generated e.m.f. [270 volts]

2. A 6-pole generator has a lap-wound armature with 40 slots with 20 conductors per slot. The flux per pole is 25 mWb. Calculate the speed at which the machine must be driven to generate an e.m.f. of 300 V [15 rev/s or 900 rev/min]

3. A 4-pole armature of a d.c. machine has 1000 conductors and a flux per pole of 20 mWb. Determine the e.m.f. generated when running at 600 rev/min when the armature is (a) wave-wound (b) lap-wound. [(a) 400 volts (b) 200 volts]

4. A d.c. generator running at 25 rev/s generates an e.m.f. of 150 V. Determine the percentage increase in the flux per pole required to generate 180 V at 20 rev/s [50%]

22.6 D.C. generators

D.C. generators are classified according to the method of their field excitation. These groupings are:

(i) **Separately-excited generators**, where the field winding is connected to a source of supply other than the armature of its own machine.

(ii) **Self-excited generators**, where the field winding receives its supply from the armature of its own machine, and which are sub-divided into (a) shunt, (b) series, and (c) compound wound generators.

22.7 Types of d.c. generator and their characteristics

(a) Separately-excited generator

A typical separately-excited generator circuit is shown in Fig. 22.5.

When a load is connected across the armature terminals, a load current I_a will flow. The terminal voltage V will fall from its open-circuit e.m.f. E due to a volt drop caused by current flowing through the armature resistance, shown as R_a

i.e. **terminal voltage, $V = E - I_a R_a$**

or **generated e.m.f., $E = V + I_a R_a$** (3)

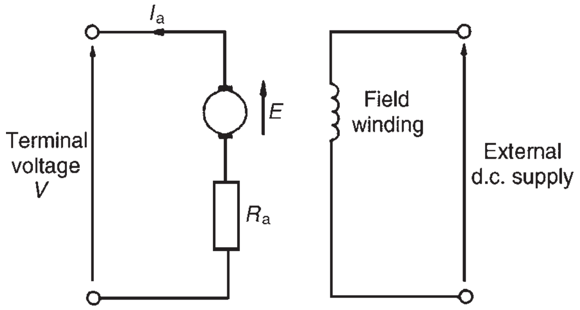


Figure 22.5

Problem 7. Determine the terminal voltage of a generator which develops an e.m.f. of 200 V and has an armature current of 30 A on load. Assume the armature resistance is 0.30 Ω.

With reference to Fig. 22.5, terminal voltage,

$$V = E - I_a R_a$$
$$= 200 - (30)(0.30)$$
$$= 200 - 9$$
$$= \textbf{191 volts}$$

Problem 8. A generator is connected to a 60 Ω load and a current of 8 A flows. If the armature resistance is 1 Ω determine (a) the terminal voltage, and (b) the generated e.m.f.

(a) Terminal voltage, $V = I_a R_L = (8)(60) = \textbf{480 volts}$

(b) Generated e.m.f.,

$$E = V + I_a R_a \quad \text{from Equation (3)}$$
$$= 480 + (8)(1) = 480 + 8 = \textbf{488 volts}$$

Problem 9. A separately-excited generator develops a no-load e.m.f. of 150 V at an armature speed of 20 rev/s and a flux per pole of 0.10 Wb. Determine the generated e.m.f. when (a) the speed increases to 25 rev/s and the pole flux remains unchanged, (b) the speed remains at 20 rev/s and the pole flux is decreased to 0.08 Wb, and (c) the speed increases to 24 rev/s and the pole flux is decreased to 0.07 Wb.

(a) From Section 22.5, generated e.m.f. $E \propto \Phi n$

from which, $\dfrac{E_1}{E_2} = \dfrac{\Phi_1 N_1}{\Phi_2 N_2}$

Hence $\dfrac{150}{E_2} = \dfrac{(0.10)(20)}{(0.1)(25)}$

from which, $E_2 = \dfrac{(150)(0.10)(25)}{(0.10)(20)}$

$= \textbf{187.5 volts}$

(b) $\dfrac{150}{E_3} = \dfrac{(0.10)(20)}{(0.08)(20)}$

from which, e.m.f., $\boldsymbol{E_3} = \dfrac{(150)(0.08)(20)}{(0.10)(20)}$

$= \textbf{120 volts}$

(c) $\dfrac{150}{E_4} = \dfrac{(0.10)(20)}{(0.07)(24)}$

from which, e.m.f., $\boldsymbol{E_4} = \dfrac{(150)(0.07)(24)}{(0.10)(20)}$

$= \textbf{126 volts}$

Characteristics

The two principal generator characteristics are the generated voltage/field current characteristics, called the **open-circuit characteristic** and the terminal voltage/load current characteristic, called the **load characteristic**. A typical separately-excited generator **open-circuit characteristic** is shown in Fig. 22.6(a) and a typical **load characteristic** is shown in Fig. 22.6(b).

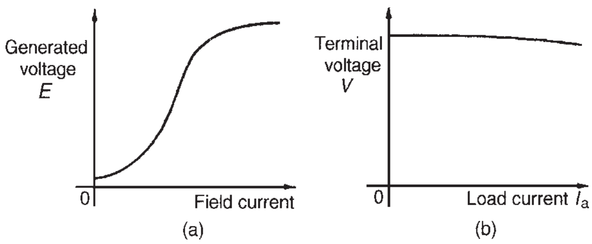


Figure 22.6

A separately-excited generator is used only in special cases, such as when a wide variation in terminal p.d. is required, or when exact control of the field current is necessary. Its disadvantage lies in requiring a separate source of direct current.

(b) Shunt wound generator

In a shunt wound generator the field winding is connected in parallel with the armature as shown in

Fig. 22.7. The field winding has a relatively high resistance and therefore the current carried is only a fraction of the armature current.

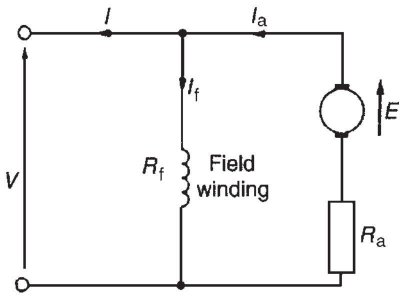


Figure 22.7

For the circuit shown in Fig. 22.7

terminal voltage, $V = E - I_aR_a$

or **generated e.m.f., $E = V + I_aR_a$**

$I_a = I_f + I$ from Kirchhoff's current law, where I_a = armature current, I_f = field current ($= V/R_f$) and I = load current.

Problem 10. A shunt generator supplies a 20 kW load at 200 V through cables of resistance, $R = 100\,\text{m}\Omega$. If the field winding resistance, $R_f = 50\,\Omega$ and the armature resistance, $R_a = 40\,\text{m}\Omega$, determine (a) the terminal voltage, and (b) the e.m.f. generated in the armature.

(a) The circuit is as shown in Fig. 22.8

Load current, $I = \dfrac{20\,000 \text{ watts}}{200 \text{ volts}} = 100\,\text{A}$

Volt drop in the cables to the load $= IR = (100)\,(100 \times 10^{-3}) = 10\,\text{V}$.

Hence **terminal voltage, $V = 200 + 10 = 210$ volts**.

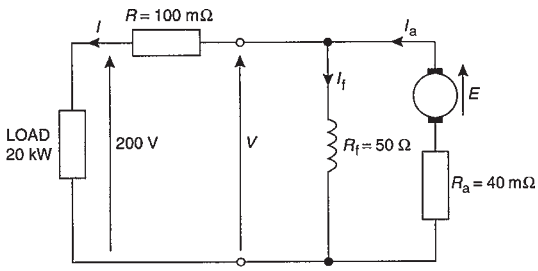


Figure 22.8

(b) Armature current $I_a = I_f + I$

Field current, $I_f = \dfrac{V}{R_f} = \dfrac{210}{50} = 4.2\,\text{A}$

Hence $I_a = I_f + I = 4.2 + 100 = 104.2\,\text{A}$

Generated e.m.f. $E = V + I_aR_a$

$= 210 + (104.2)(40 \times 10^{-3})$

$= 210 + 4.168$

$=$ **214.17 volts**

Characteristics

The generated e.m.f., E, is proportional to $\Phi\omega$, (see Section 22.5), hence at constant speed, since $\omega = 2\pi n$, $E \propto \Phi$. Also the flux Φ is proportional to field current I_f until magnetic saturation of the iron circuit of the generator occurs. Hence the open circuit characteristic is as shown in Fig. 22.9(a).

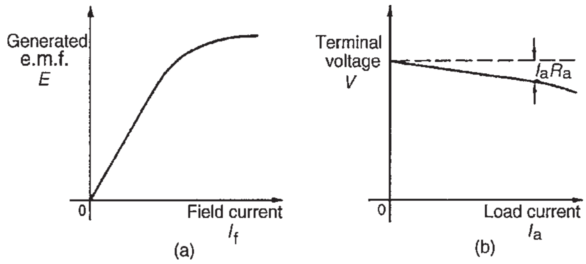


Figure 22.9

As the load current on a generator having constant field current and running at constant speed increases, the value of armature current increases, hence the armature volt drop, I_aR_a increases. The generated voltage E is larger than the terminal voltage V and the voltage equation for the armature circuit is $V = E - I_aR_a$. Since E is constant, V decreases with increasing load. The load characteristic is as shown in Fig. 22.9(b). In practice, the fall in voltage is about 10 per cent between no-load and full-load for many d.c. shunt-wound generators.

The shunt-wound generator is the type most used in practice, but the load current must be limited to a value that is well below the maximum value. This then avoids excessive variation of the terminal voltage. Typical applications are with battery charging and motor car generators.

(c) Series-wound generator

In the series-wound generator the field winding is connected in series with the armature as shown in Fig. 22.10.

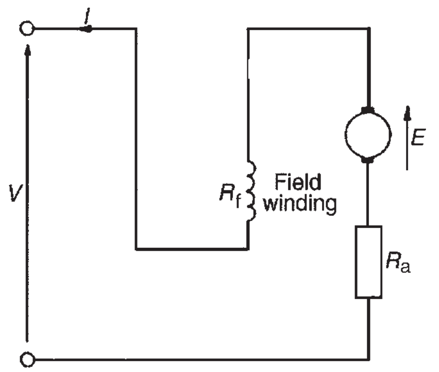


Figure 22.10

Characteristic

The load characteristic is the terminal voltage/current characteristic. The generated e.m.f. E, is proportional to $\Phi\omega$ and at constant speed $\omega(=2\pi n)$ is a constant. Thus E is proportional to Φ. For values of current below magnetic saturation of the yoke, poles, air gaps and armature core, the flux Φ is proportional to the current, hence $E \propto I$. For values of current above those required for magnetic saturation, the generated e.m.f. is approximately constant. The values of field resistance and armature resistance in a series wound machine are small, hence the terminal voltage V is very nearly equal to E. A typical load characteristic for a series generator is shown in Fig. 22.11.

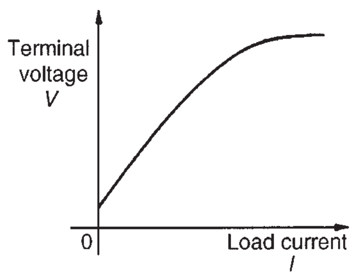


Figure 22.11

In a series-wound generator, the field winding is in series with the armature and it is not possible to have a value of field current when the terminals are open circuited, thus it is not possible to obtain an open-circuit characteristic.

Series-wound generators are rarely used in practise, but can be used as a ‘booster’ on d.c. transmission lines.

(d) Compound-wound generator

In the compound-wound generator two methods of connection are used, both having a mixture of shunt and series windings, designed to combine the advantages of each. Fig. 22.12(a) shows what is termed a **long-shunt** compound generator, and Fig. 22.12(b) shows a **short-shunt** compound generator. The latter is the most generally used form of d.c. generator.

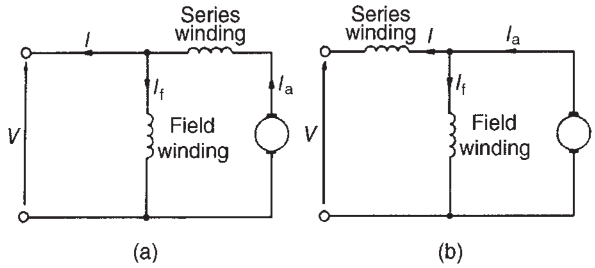


Figure 22.12

Problem 11. A short-shunt compound generator supplies 80 A at 200 V. If the field resistance, $R_f = 40\ \Omega$, the series resistance, $R_{Se} = 0.02\ \Omega$ and the armature resistance, $R_a = 0.04\ \Omega$, determine the e.m.f. generated.

The circuit is shown in Fig. 22.13.

Volt drop in series winding $= IR_{Se} = (80)(0.02)$
$= 1.6\,\text{V}$.

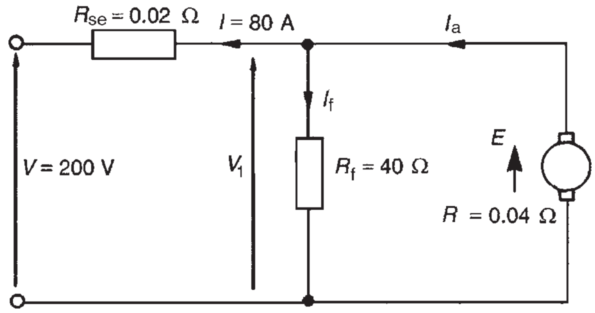


Figure 22.13

P.d. across the field winding = p.d. across armature $= V_1 = 200 + 1.6 = 201.6\,\text{V}$

$$\text{Field current } I_f = \frac{V_1}{R_f} = \frac{201.6}{40} = 5.04\,\text{A}$$

Armature current, $I_a = I + I_f = 80 + 5.04 = 85.04\,\text{A}$

$$\textbf{Generated e.m.f., } E = V_1 + I_a R_a$$
$$= 201.6 + (85.04)(0.04)$$
$$= 201.6 + 3.4016$$
$$= \mathbf{205\ volts}$$

Characteristics

In cumulative-compound machines the magnetic flux produced by the series and shunt fields are additive. Included in this group are **over-compounded**, **level-compounded** and **under-compounded machines** — the degree of compounding obtained depending on the number of turns of wire on the series winding.

A large number of series winding turns results in an over-compounded characteristic, as shown in Fig. 22.14, in which the full-load terminal voltage exceeds the no-load voltage. A level-compound machine gives a full-load terminal voltage which is equal to the no-load voltage, as shown in Fig. 22.14.

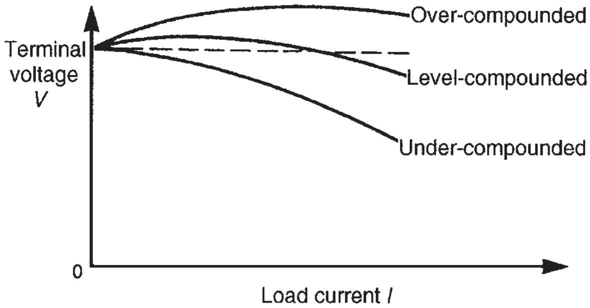


Figure 22.14

An under-compounded machine gives a full-load terminal voltage which is less than the no-load voltage, as shown in Fig. 22.14. However even this latter characteristic is a little better than that for a shunt generator alone. Compound-wound generators are used in electric arc welding, with lighting sets and with marine equipment.

Now try the following exercise

Exercise 131 Further problems on the d.c. generator

1. Determine the terminal voltage of a generator which develops an e.m.f. of 240 V and has an armature current of 50 A on load. Assume the armature resistance is 40 mΩ [238 volts]

2. A generator is connected to a 50 Ω load and a current of 10 A flows. If the armature resistance is 0.5 Ω, determine (a) the terminal voltage, and (b) the generated e.m.f. [(a) 500 volts (b) 505 volts]

3. A separately excited generator develops a no-load e.m.f. of 180 V at an armature speed of 15 rev/s and a flux per pole of 0.20 Wb. Calculate the generated e.m.f. when:
   (a) the speed increases to 20 rev/s and the flux per pole remains unchanged
   (b) the speed remains at 15 rev/s and the pole flux is decreased to 0.125 Wb
   (c) the speed increases to 25 rev/s and the pole flux is decreased to 0.18 Wb
   [(a) 240 volts (b) 112.5 volts (c) 270 volts]

4. A shunt generator supplies a 50 kW load at 400 V through cables of resistance 0.2 Ω. If the field winding resistance is 50 Ω and the armature resistance is 0.05 Ω, determine (a) the terminal voltage, (b) the e.m.f. generated in the armature [(a) 425 volts (b) 431.68 volts]

5. A short-shunt compound generator supplies 50 A at 300 V. If the field resistance is 30 Ω, the series resistance 0.03 Ω and the armature resistance 0.05 Ω, determine the e.m.f. generated [304.5 volts]

6. A d.c. generator has a generated e.m.f. of 210 V when running at 700 rev/min and the flux per pole is 120 mWb. Determine the generated e.m.f.
   (a) at 1050 rev/min, assuming the flux remains constant,
   (b) if the flux is reduced by one-sixth at constant speed, and
   (c) at a speed of 1155 rev/min and a flux of 132 mWb
   [(a) 315 V (b) 175 V (c) 381.2 V]

7. A 250 V d.c. shunt-wound generator has an armature resistance of 0.1 Ω. Determine the generated e.m.f. when the generator is supplying 50 kW, neglecting the field current of the generator. [270 V]

22.8 D.C. machine losses

As stated in Section 22.1, a generator is a machine for converting mechanical energy into electrical energy and a motor is a machine for converting electrical energy into mechanical energy. When such conversions take place, certain losses occur which are dissipated in the form of heat.

The principal **losses of machines** are:

(i) **Copper loss**, due to I^2R heat losses in the armature and field windings.

(ii) **Iron (or core) loss**, due to hysteresis and eddy-current losses in the armature. This loss can be reduced by constructing the armature of silicon steel laminations having a high resistivity and low hysteresis loss. At constant speed, the iron loss is assumed constant.

(iii) **Friction and windage losses**, due to bearing and brush contact friction and losses due to air resistance against moving parts (called windage). At constant speed, these losses are assumed to be constant.

(iv) **Brush contact loss** between the brushes and commutator. This loss is approximately proportional to the load current.

The total losses of a machine can be quite significant and operating efficiencies of between 80 per cent and 90 per cent are common.

22.9 Efficiency of a d.c. generator

The efficiency of an electrical machine is the ratio of the output power to the input power and is usually expressed as a percentage. The Greek letter, 'η' (eta) is used to signify efficiency and since the units are, power/power, then efficiency has no units. Thus

$$\textbf{efficiency, } \eta = \left(\frac{\textbf{output power}}{\textbf{input power}}\right) \times 100\%$$

If the total resistance of the armature circuit (including brush contact resistance) is R_a, then **the total loss in the armature circuit is $I_a^2 R_a$**

If the terminal voltage is V and the current in the shunt circuit is I_f, then **the loss in the shunt circuit is $I_f V$**

If the sum of the iron, friction and windage losses is C then **the total losses is given by: $I_a^2 R_a + I_f V + C$** ($I_a^2 R_a + I_f V$ is, in fact, the 'copper loss').

If the output current is I, then **the output power is VI**. Total input power $= VI + I_a^2 R_a + I_f V + C$. Hence

$$\textbf{efficiency, } \eta = \frac{\textbf{output}}{\textbf{input}}, \text{ i.e.}$$

$$\eta = \left(\frac{VI}{VI + I_a^2 R_a + I_f V + C}\right) \times 100\% \qquad (4)$$

The **efficiency of a generator is a maximum** when the load is such that:

$$I_a^2 R_a = VI_f + C$$

i.e. when the variable loss = the constant loss

Problem 12. A 10 kW shunt generator having an armature circuit resistance of 0.75 Ω and a field resistance of 125 Ω, generates a terminal voltage of 250 V at full load. Determine the efficiency of the generator at full load, assuming the iron, friction and windage losses amount to 600 W.

The circuit is shown in Fig. 22.15

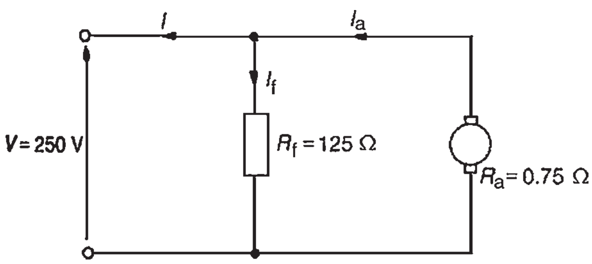


Figure 22.15

Output power $= 10\,000\text{ W} = VI$ from which, load current $I = 10\,000/V = 10\,000/250 = 40\text{ A}$.
Field current, $I_f = V/R_f = 250/125 = 2\text{ A}$.
Armature current, $I_a = I_f + I = 2 + 40 = 42\text{ A}$

$$\text{Efficiency, } \eta = \left(\frac{VI}{VI + I_a^2 R + I_f V + C}\right) \times 100\%$$

$$= \left(\frac{10\,000}{10\,000 + (42)^2(0.75) + (2)(250) + 600}\right) \times 100\%$$

$$= \left(\frac{10\,000}{12\,423}\right) \times 100\%$$

$$= \mathbf{80.50\%}$$

Now try the following exercise

Exercise 132 A further problem on the efficiency of a d.c. generator

1. A 15 kW shunt generator having an armature circuit resistance of 0.4 Ω and a field resistance of 100 Ω, generates a terminal voltage of 240 V at full load. Determine the efficiency of the generator at full load, assuming the iron, friction and windage losses amount to 1 kW [82.14%]

22.10 D.C. motors

The construction of a d.c. motor is the same as a d.c. generator. The only difference is that in a generator the generated e.m.f. is greater than the terminal voltage, whereas in a motor the generated e.m.f. is less than the terminal voltage.

D.C. motors are often used in power stations to drive emergency stand-by pump systems which come into operation to protect essential equipment and plant should the normal a.c. supplies or pumps fail.

Back e.m.f.

When a d.c. motor rotates, an e.m.f. is induced in the armature conductors. By Lenz's law this induced e.m.f. E opposes the supply voltage V and is called a **back e.m.f.**, and the supply voltage, V is given by:

$$\boldsymbol{V = E + I_aR_a} \quad \text{or} \quad \boldsymbol{E = V - I_aR_a} \qquad (5)$$

Problem 13. A d.c. motor operates from a 240 V supply. The armature resistance is 0.2 Ω. Determine the back e.m.f. when the armature current is 50 A.

For a motor, $V = E + I_aR_a$ hence back e.m.f.,

$$E = V - I_aR_a$$
$$= 240 - (50)(0.2)$$
$$= 240 - 10 = \mathbf{230\ volts}$$

Problem 14. The armature of a d.c. machine has a resistance of 0.25 Ω and is connected to a 300 V supply. Calculate the e.m.f. generated when it is running: (a) as a generator giving 100 A , and (b) as a motor taking 80 A.

(a) As a generator, generated e.m.f.,

$$E = V + I_aR_a, \text{ from Equation (3),}$$
$$= 300 + (100)(0.25)$$
$$= 300 + 25$$
$$= \mathbf{325 volts}$$

(b) As a motor, generated e.m.f. (or back e.m.f.),

$$E = V - I_aR_a, \text{ from Equation (5),}$$
$$= 300 - (80)(0.25)$$
$$= \mathbf{280\ volts}$$

Now try the following exercise

Exercise 133 Further problems on back e.m.f.

1. A d.c. motor operates from a 350 V supply. If the armature resistance is 0.4 Ω determine the back e.m.f. when the armature current is 60 A [326 volts]

2. The armature of a d.c. machine has a resistance of 0.5 Ω and is connected to a 200 V supply. Calculate the e.m.f. generated when it is running (a) as a motor taking 50 A, and (b) as a generator giving 70 A [(a) 175 volts (b) 235 volts]

3. Determine the generated e.m.f. of a d.c. machine if the armature resistance is 0.1 Ω and it (a) is running as a motor connected to a 230 V supply, the armature current being 60 A, and (b) is running as a generator with a terminal voltage of 230 V, the armature current being 80 A [(a) 224 V (b) 238 V]

22.11 Torque of a d.c. motor

From Equation (5), for a d.c. motor, the supply voltage V is given by

$$V = E + I_a R_a$$

Multiplying each term by current I_a gives:

$$VI_a = EI_a + I_a^2 R_a$$

The term $\boldsymbol{VI_a}$ is the **total electrical power supplied to the armature**, the term $\boldsymbol{I_a^2 R_a}$ is the **loss due to armature resistance**, and the term $\boldsymbol{EI_a}$ is the **mechanical power developed by the armature**. If T is the torque, in newton metres, then the mechanical power developed is given by $T\omega$ watts (see '*Science for Engineering*')

Hence $\quad T\omega = 2\pi nT = EI_a$

from which,

$$\textbf{torque } \boldsymbol{T = \frac{EI_a}{2\pi n}} \textbf{ newton metres} \qquad (6)$$

From Section 22.5, Equation (1), the e.m.f. E generated is given by

$$E = \frac{2p\Phi nZ}{c}$$

Hence $\quad 2\pi nT = EI_a = \left(\dfrac{2p\Phi nZ}{c}\right) I_a$

Hence torque $\quad T = \dfrac{\left(\dfrac{2p\Phi nZ}{c}\right)}{2\pi n} I_a$

i.e. $\quad \boldsymbol{T = \dfrac{p\Phi Z I_a}{\pi c}}$ **newton metres** $\qquad (7)$

For a given machine, Z, c and p are fixed values

Hence $\quad$ torque, $\boldsymbol{T \propto \Phi I_a}$ $\qquad (8)$

Problem 15. An 8-pole d.c. motor has a wave-wound armature with 900 conductors. The useful flux per pole is 25 mWb. Determine the torque exerted when a current of 30 A flows in each armature conductor.

$p = 4$, $c = 2$ for a wave winding,
$\Phi = 25 \times 10^{-3}$ Wb, $Z = 900$ and $I_a = 30$ A.

From Equation (7),

$$\textbf{torque, } \boldsymbol{T} = \frac{p\Phi Z I_a}{\pi c} = \frac{(4)(25 \times 10^{-3})(900)(30)}{\pi(2)} = \textbf{429.7 Nm}$$

Problem 16. Determine the torque developed by a 350 V d.c. motor having an armature resistance of 0.5 Ω and running at 15 rev/s. The armature current is 60 A.

$V = 350$ V, $R_a = 0.5\,\Omega$, $n = 15$ rev/s and $I_a = 60$ A.
Back e.m.f. $E = V - I_a R_a = 350 - (60)(0.5) = 320$ V.
From Equation (6),

$$\textbf{torque, } \boldsymbol{T} = \frac{EI_a}{2\pi n} = \frac{(320)(60)}{2\pi(15)} = \textbf{203.7 Nm}$$

Problem 17. A six-pole lap-wound motor is connected to a 250 V d.c. supply. The armature has 500 conductors and a resistance of 1 Ω. The flux per pole is 20 mWb. Calculate (a) the speed and (b) the torque developed when the armature current is 40 A.

$V = 250$ V, $Z = 500$, $R_a = 1\,\Omega$, $\Phi = 20 \times 10^{-3}$ Wb, $I_a = 40$ A and $c = 2p$ for a lap winding

(a) Back e.m.f. $E = V - I_a R_a = 250 - (40)(1) = 210$ V

$$\text{E.m.f. } E = \frac{2p\Phi nZ}{c}$$

$$\text{i.e. } 210 = \frac{2p(20 \times 10^{-3})n(500)}{2p} = 10n$$

$$\text{Hence } \textbf{speed } \boldsymbol{n} = \frac{210}{10} = \textbf{21 rev/s} \text{ or } (21 \times 60) = \textbf{1260 rev/min}$$

(b) $\quad \textbf{Torque } \boldsymbol{T} = \dfrac{EI_a}{2\pi n} = \dfrac{(210)(40)}{2\pi(21)} = \textbf{63.66 Nm}$

Problem 18. The shaft torque of a diesel motor driving a 100 V d.c. shunt-wound generator is

25 Nm. The armature current of the generator is 16 A at this value of torque. If the shunt field regulator is adjusted so that the flux is reduced by 15 per cent, the torque increases to 35 Nm. Determine the armature current at this new value of torque.

From Equation (8), the shaft torque T of a generator is proportional to ΦI_a, where Φ is the flux and I_a is the armature current, or, $T = k\Phi I_a$, where k is a constant.

The torque at flux Φ_1 and armature current I_{a1} is $T_1 = k\Phi_1 I_{a1}$ Similarly, $T_2 = k\Phi_2 I_{a2}$

By division $\quad \dfrac{T_1}{T_2} = \dfrac{k\Phi_1 I_{a1}}{k\Phi_2 I_{a2}} = \dfrac{\Phi_1 I_{a1}}{\Phi_2 I_{a2}}$

Hence $\quad \dfrac{25}{35} = \dfrac{\Phi_1 \times 16}{0.85\Phi_1 \times I_{a2}}$

i.e. $\quad I_{a2} = \dfrac{16 \times 35}{0.85 \times 25} = 26.35\,\text{A}$

That is, **the armature current at the new value of torque is 26.35 A**

Problem 19. A 100 V d.c. generator supplies a current of 15 A when running at 1500 rev/min. If the torque on the shaft driving the generator is 12 Nm, determine (a) the efficiency of the generator and (b) the power loss in the generator.

(a) From Section 22.9, the efficiency of a generator = output power/input power × 100 per cent. The output power is the electrical output, i.e. VI watts. The input power to a generator is the mechanical power in the shaft driving the generator, i.e. $T\omega$ or $T(2\pi n)$ watts, where T is the torque in Nm and n is speed of rotation in rev/s. Hence, for a generator,

$$\text{efficiency, } \eta = \frac{VI}{T(2\pi n)} \times 100\%$$

$$= \frac{(100)(15)(100)}{(12)(2\pi)\left(\dfrac{1500}{60}\right)}$$

i.e. **efficiency = 79.6%**

(b) The input power = output power + losses

Hence, $T(2\pi n) = VI + \text{losses}$

i.e. losses $= T(2\pi n) - VI$

$$= \left[(12)(2\pi)\left(\frac{1500}{60}\right)\right] - [(100)(15)]$$

i.e. **power loss** = 1885 − 1500 = **385 W**

Now try the following exercise

Exercise 134 Further problems on losses, efficiency, and torque

1. The shaft torque required to drive a d.c. generator is 18.7 Nm when it is running at 1250 rev/min. If its efficiency is 87 per cent under these conditions and the armature current is 17.3 A, determine the voltage at the terminals of the generator [123.1 V]

2. A 220 V, d.c. generator supplies a load of 37.5 A and runs at 1550 rev/min. Determine the shaft torque of the diesel motor driving the generator, if the generator efficiency is 78 per cent [65.2 Nm]

3. A 4-pole d.c. motor has a wave-wound armature with 800 conductors. The useful flux per pole is 20 mWb. Calculate the torque exerted when a current of 40 A flows in each armature conductor [203.7 Nm]

4. Calculate the torque developed by a 240 V d.c. motor whose armature current is 50 A, armature resistance is 0.6 Ω and is running at 10 rev/s [167.1 Nm]

5. An 8-pole lap-wound d.c. motor has a 200 V supply. The armature has 800 conductors and a resistance of 0.8 Ω. If the useful flux per pole is 40 mWb and the armature current is 30 A, calculate (a) the speed and (b) the torque developed
[(a) 5.5 rev/s or 330 rev/min (b) 152.8 Nm]

6. A 150 V d.c. generator supplies a current of 25 A when running at 1200 rev/min. If

the torque on the shaft driving the generator is 35.8 Nm, determine (a) the efficiency of the generator, and (b) the power loss in the generator [(a) 83.4 per cent (b) 748.8 W]

22.12 Types of d.c. motor and their characteristics

(a) Shunt wound motor

In the shunt wound motor the field winding is in parallel with the armature across the supply as shown in Fig. 22.16.

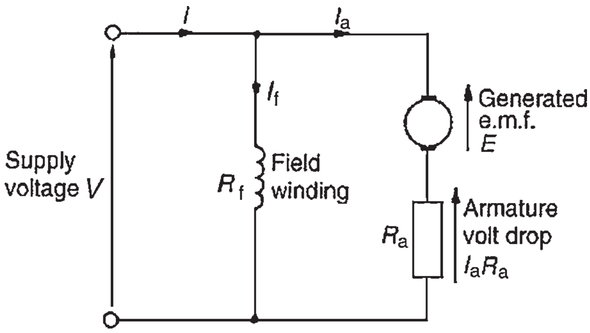


Figure 22.16

For the circuit shown in Fig. 22.16,

$$\textbf{Supply voltage, } V = E + I_a R_a$$

$$\textbf{or generated e.m.f., } E = V - I_a R_a$$

$$\textbf{Supply current, } I = I_a + I_f$$

from Kirchhoff's current law

Problem 20. A 240 V shunt motor takes a total current of 30 A. If the field winding resistance $R_f = 150\,\Omega$ and the armature resistance $R_a = 0.4\,\Omega$ determine (a) the current in the armature, and (b) the back e.m.f.

(a) Field current $I_f = \dfrac{V}{R_f} = \dfrac{240}{150} = 1.6\,\text{A}$

Supply current $I = I_a + I_f$

Hence armature current, $I_a = I - I_f = 30 - 1.6 = \mathbf{28.4\,A}$

(b) Back e.m.f.

$E = V - I_a R_a = 240 - (28.4)(0.4) = \mathbf{228.64\ volts}$

Characteristics

The two principal characteristics are the torque/armature current and speed/armature current relationships. From these, the torque/speed relationship can be derived.

(i) The theoretical torque/armature current characteristic can be derived from the expression $T \propto \Phi I_a$, (see Section 22.11). For a shunt-wound motor, the field winding is connected in parallel with the armature circuit and thus the applied voltage gives a constant field current, i.e. a shunt-wound motor is a constant flux machine. Since Φ is constant, it follows that $T \propto I_a$, and the characteristic is as shown in Fig. 22.17

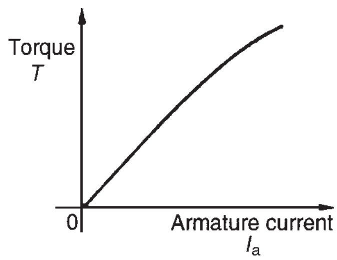


Figure 22.17

(ii) The armature circuit of a d.c. motor has resistance due to the armature winding and brushes, R_a ohms, and when armature current I_a is flowing through it, there is a voltage drop of $I_a R_a$ volts. In Fig. 22.16 the armature resistance is shown as a separate resistor in the armature circuit to help understanding. Also, even though the machine is a motor, because conductors are rotating in a magnetic field, a voltage, $E \propto \Phi\omega$, is generated by the armature conductors. From Equation (5), $V = E + I_a R_a$ or $E = V - I_a R_a$ However, from Section 22.5, $E \propto \Phi n$, hence $n \propto E/\Phi$ i.e.

$$\textbf{speed of rotation, } n \propto \frac{E}{\Phi} \propto \frac{V - I_a R_a}{\Phi} \qquad (9)$$

For a shunt motor, V, Φ and R_a are constants, hence as armature current I_a increases, I_aR_a increases and $V - I_aR_a$ decreases, and the speed is proportional to a quantity which is decreasing and is as shown in Fig. 22.18 As the load on the shaft of the motor increases, I_a increases and the speed drops slightly. In practice, the speed falls by about 10 per cent between no-load and full-load on many d.c. shunt-wound motors. Due to this relatively small drop in speed, the d.c. shunt-wound motor is taken as basically being a constant-speed machine and may be used for driving lathes, lines of shafts, fans, conveyor belts, pumps, compressors, drilling machines and so on.

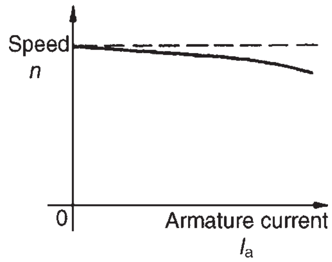


Figure 22.18

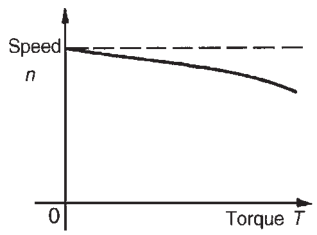


Figure 22.19

(iii) Since torque is proportional to armature current, (see (i) above), the theoretical speed/torque characteristic is as shown in Fig. 22.19.

Problem 21. A 200 V, d.c. shunt-wound motor has an armature resistance of 0.4 Ω and at a certain load has an armature current of 30 A and runs at 1350 rev/min. If the load on the shaft of the motor is increased so that the armature current increases to 45 A, determine the speed of the motor, assuming the flux remains constant.

The relationship $E \propto \Phi n$ applies to both generators and motors. For a motor, $E = V - I_aR_a$, (see equation (5))

Hence $\quad E_1 = 200 - 30 \times 0.4 = 188\,\text{V}$

and $\quad E_2 = 200 - 45 \times 0.4 = 182\,\text{V}$

The relationship

$$\frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2}$$

applies to both generators and motors. Since the flux is constant, $\Phi_1 = \Phi_2$. Hence

$$\frac{188}{182} = \frac{\Phi_1 \times \left(\frac{1350}{60}\right)}{\Phi_1 \times n_2}$$

i.e. $$n_2 = \frac{22.5 \times 182}{188} = 21.78\,\text{rev/s}$$

Thus **the speed of the motor when the armature current is 45 A** is 21.78 × 60 rev/min i.e. **1307 rev/min**.

Problem 22. A 220 V, d.c. shunt-wound motor runs at 800 rev/min and the armature current is 30 A. The armature circuit resistance is 0.4 Ω. Determine (a) the maximum value of armature current if the flux is suddenly reduced by 10 per cent and (b) the steady state value of the armature current at the new value of flux, assuming the shaft torque of the motor remains constant.

(a) For a d.c. shunt-wound motor, $E = V - I_aR_a$. Hence initial generated e.m.f., $E_1 = 220 = 30 \times 0.4 = 208\,\text{V}$. The generated e.m.f. is also such that $E \propto \Phi n$, so at the instant the flux is reduced, the speed has not had time to change, and $E = 208 \times 90/100 - 187.2\,\text{V}$ Hence, the voltage drop due to the armature resistance is 220 − 187.2 i.e. 32.8 V. The **instantaneous value of the current** $= 32.8/0.4 = $ **82 A**. This increase in current is about three times the initial value and causes an increase in torque, ($T \propto \Phi I_a$). The motor accelerates because of the larger torque value until steady state conditions are reached.

(b) $T \propto \Phi I_a$ and, since the torque is constant, $\Phi_1 I_{a1} = \Phi_2 I_{a2}$. The flux Φ is reduced by 10 per cent, hence $\Phi_2 = 0.9\Phi_1$. Thus, $\Phi_1 \times 30 = 0.9\Phi_1 \times I_{a2}$ i.e. the steady state value of armature current, $I_{a2} = 30/0.9 = $ **33.33 A**.

(b) Series-wound motor

In the series-wound motor the field winding is in series with the armature across the supply as shown in Fig. 22.20.

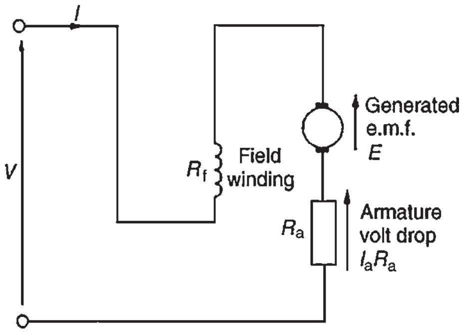


Figure 22.20

For the series motor shown in Fig. 22.20,

Supply voltage $V = E + I(R_a + R_f)$

or generated e.m.f. $E = V - I(R_a + R_f)$

Characteristics

In a series motor, the armature current flows in the field winding and is equal to the supply current, I.

(i) **The torque/current characteristic**
It is shown in Section 22.11 that torque $T \propto \Phi I_a$. Since the armature and field currents are the same current, I, in a series machine, then $T \propto \Phi I$ over a limited range, before magnetic saturation of the magnetic circuit of the motor is reached, (i.e. the linear portion of the B–H curve for the yoke, poles, air gap, brushes and armature in series). Thus $\Phi \propto I$ and $T \propto I^2$. After magnetic saturation, Φ almost becomes a constant and $T \propto I$. Thus the theoretical torque/current characteristic is as shown in Fig. 22.21.

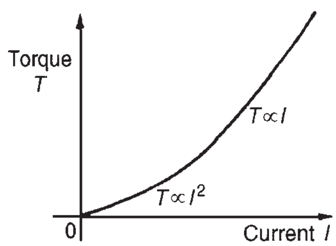


Figure 22.21

(ii) **The speed/current characteristic**
It is shown in equation (9) that

$$n \propto \frac{V - I_aR_a}{\Phi}$$

In a series motor, $I_a = I$ and below the magnetic saturation level, $\Phi \propto I$. Thus $n \propto (V - IR)/I$ where R is the combined resistance of the series field and armature circuit. Since IR is small compared with V, then an approximate relationship for the speed is $n \propto V/I \propto 1/I$ since V is constant. Hence the theoretical speed/current characteristic is as shown in Fig. 22.22. The high speed at small values of current indicate that this type of motor must not be run on very light loads and invariably, such motors are permanently coupled to their loads.

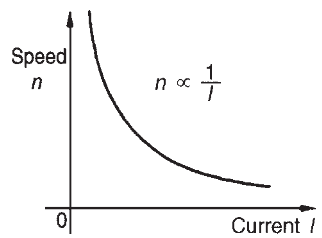


Figure 22.22

(iii) The theoretical **speed/torque characteristic** may be derived from (i) and (ii) above by obtaining the torque and speed for various values of current and plotting the co-ordinates on the speed/torque characteristics. A typical speed/torque characteristic is shown in Fig. 22.23.

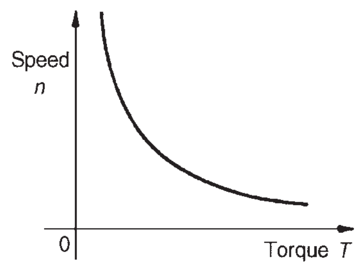


Figure 22.23

A d.c. series motor takes a large current on starting and the characteristic shown in Fig. 22.21 shows that the series-wound motor has a large torque when the current is large. Hence these motors are used for traction (such as trains, milk delivery vehicles, etc.), driving fans and for cranes and hoists, where a large initial torque is required.

Problem 23. A series motor has an armature resistance of 0.2 Ω and a series field resistance of 0.3 Ω. It is connected to a 240 V supply and at a particular load runs at 24 rev/s when drawing 15 A from the supply. (a) Determine the generated e.m.f. at this load (b) Calculate the speed of the motor when the load is changed such that the current is increased to 30 A. Assume that this causes a doubling of the flux.

(a) With reference to Fig. 22.20, generated e.m.f., E_1 at initial load, is given by

$$\begin{aligned} E_1 &= V - I_a(R_a + R_f) \\ &= 240 - (15)(0.2 + 0.3) \\ &= 240 - 7.5 = \mathbf{232.5\ volts} \end{aligned}$$

(b) When the current is increased to 30 A, the generated e.m.f. is given by:

$$\begin{aligned} E_2 &= V - I_2(R_a + R_f) \\ &= 240 - (30)(0.2 + 0.3) \\ &= 240 - 15 = 225\ \text{volts} \end{aligned}$$

Now e.m.f. $E \propto \boldsymbol{\Phi n}$ thus

$$\frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2}$$

i.e. $\dfrac{232.5}{22.5} = \dfrac{\Phi_1(24)}{(2\Phi_1)n_2}$ since $\Phi_2 = 2\Phi_1$

Hence

$$\textbf{speed of motor, } n_2 = \frac{(24)(225)}{(232.5)(2)} = \mathbf{11.6\ rev/s}$$

As the current has been increased from 15 A to 30 A, the speed has decreased from 24 rev/s to 11.6 rev/s. Its speed/current characteristic is similar to Fig. 22.22.

(c) Compound wound motor

There are two types of compound wound motor:

(i) **Cumulative compound**, in which the series winding is so connected that the field due to it assists that due to the shunt winding.

(ii) **Differential compound**, in which the series winding is so connected that the field due to it opposes that due to the shunt winding.

Figure 22.24(a) shows a **long-shunt** compound motor and Fig. 22.24(b) a **short-shunt** compound motor.

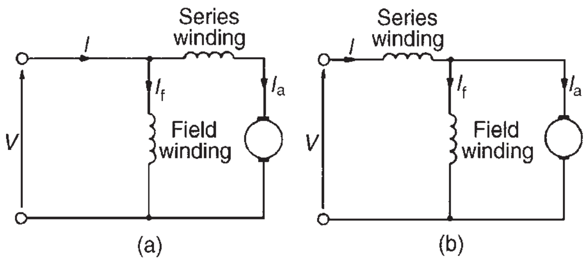


Figure 22.24

Characteristics

A compound-wound motor has both a series and a shunt field winding, (i.e. one winding in series and one in parallel with the armature), and is usually wound to have a characteristic similar in shape to a series wound motor (see Figs. 22.21–22.23). A limited amount of shunt winding is present to restrict the no-load speed to a safe value. However, by varying the number of turns on the series and shunt windings and the directions of the magnetic fields produced by these windings (assisting or opposing), families of characteristics may be obtained to suit almost all applications. Generally, compound-wound motors are used for heavy duties, particularly in applications where sudden heavy load may occur such as for driving plunger pumps, presses, geared lifts, conveyors, hoists and so on.

Typical compound motor torque and speed characteristics are shown in Fig. 22.25.

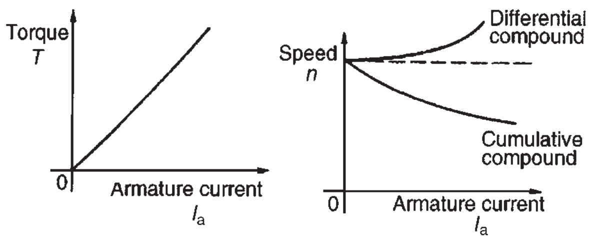


Figure 22.25

Section 3

22.13 The efficiency of a d.c. motor

It was stated in Section 22.9, that the efficiency of a d.c. machine is given by:

$$\text{efficiency, } \eta = \frac{\text{output power}}{\text{input power}} \times 100\%$$

Also, the total losses $= I_a^2 R_a + I_f V + C$ (for a shunt motor) where C is the sum of the iron, friction and windage losses.

For a motor,

$$\text{the input power} = VI$$

$$\text{and the output power} = VI - \text{losses}$$

$$= VI - I_a^2 R_a - I_f V - C$$

Hence **efficiency**,

$$\boldsymbol{\eta = \left(\frac{VI - I_a^2 R_a - I_f V - C}{VI}\right) \times 100\%} \qquad (10)$$

The **efficiency of a motor is a maximum** when the load is such that:

$$\boldsymbol{I_a^2 R_a = I_f V + C}$$

Problem 24. A 320 V shunt motor takes a total current of 80 A and runs at 1000 rev/min. If the iron, friction and windage losses amount to 1.5 kW, the shunt field resistance is 40 Ω and the armature resistance is 0.2 Ω, determine the overall efficiency of the motor.

The circuit is shown in Fig. 22.26.

Field current, $I_f = V/R_f = 320/40 = 8$ A.

Armature current $I_a = I - I_f = 80 - 8 = 72$ A.

C = iron, friction and windage losses = 1500 W.

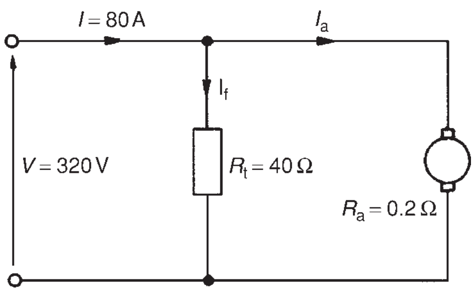


Figure 22.26

Efficiency,

$$\eta = \left(\frac{VI - I_a^2 R_a - I_f V - C}{VI}\right) \times 100\%$$

$$= \left(\frac{(320)(80) - (72)^2(0.2) - (8)(320) - 1500}{(320)(80)}\right) \times 100\%$$

$$= \left(\frac{25\,600 - 1036.8 - 2560 - 1500}{25\,600}\right) \times 100\%$$

$$= \left(\frac{20\,503.2}{25\,600}\right) \times 100\%$$

$$= \mathbf{80.1\%}$$

Problem 25. A 250 V series motor draws a current of 40 A. The armature resistance is 0.15 Ω and the field resistance is 0.05 Ω. Determine the maximum efficiency of the motor.

The circuit is as shown in Fig. 22.27

From equation (10), efficiency,

$$\eta = \left(\frac{VI - I_a^2 R_a - I_f V - C}{VI}\right) \times 100\%$$

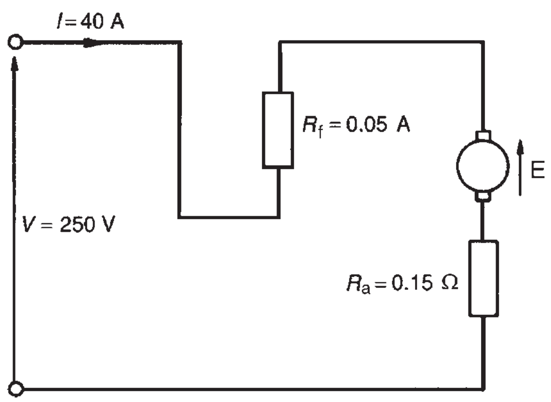


Figure 22.27

However for a series motor, $I_f = 0$ and the $I_a^2 R_a$ loss needs to be $I^2(R_a + R_f)$ Hence efficiency,

$$\eta = \left(\frac{VI - I^2(R_a + R_f) - C}{VI}\right) \times 100\%$$

For maximum efficiency $I^2(R_a + R_f) = C$. Hence efficiency,

$$\eta = \left(\frac{VI - 2I^2(R_a + R_f)}{VI}\right) \times 100\%$$

$$= \left(\frac{(250)(40) - 2(40)^2(0.15 + 0.05)}{(250)(40)}\right) \times 100\%$$

$$= \left(\frac{10\,000 - 640}{10\,000}\right) \times 100\%$$

$$= \left(\frac{9360}{10\,000}\right) \times 100\% = \mathbf{93.6\%}$$

Problem 26. A 200 V d.c. motor develops a shaft torque of 15 Nm at 1200 rev/min. If the efficiency is 80 per cent, determine the current supplied to the motor.

The efficiency of a motor $= \dfrac{\text{output power}}{\text{input power}} \times 100\%$.

The output power of a motor is the power available to do work at its shaft and is given by $T\omega$ or $T(2\pi n)$ watts, where T is the torque in Nm and n is the speed of rotation in rev/s. The input power is the electrical power in watts supplied to the motor, i.e. VI watts.

Thus for a motor,

efficiency, $$\eta = \frac{T(2\pi n)}{VI} \times 100\%$$

i.e. $$80 = \left[\frac{(15)(2\pi n)\left(\frac{1200}{60}\right)}{(200)(I)}\right] \times 100$$

Thus the current supplied,

$$I = \frac{(15)(2\pi)(20)(100)}{(200)(80)}$$

$$= \mathbf{11.8\,A}$$

Problem 27. A d.c. series motor drives a load at 30 rev/s and takes a current of 10 A when the supply voltage is 400 V. If the total resistance of the motor is 2 Ω and the iron, friction and windage losses amount to 300 W, determine the efficiency of the motor.

Efficiency,

$$\eta = \left(\frac{VI - I^2R - C}{VI}\right) \times 100\%$$

$$= \left(\frac{(400)(10) - (10)^2(2) - 300}{(400)(10)}\right) \times 100\%$$

$$= \left(\frac{4000 - 200 - 300}{4000}\right) \times 100\%$$

$$= \left(\frac{3500}{4000}\right) \times 100\% = \mathbf{87.5\%}$$

Now try the following exercise

Exercise 135 Further problems on d.c. motors

1. A 240 V shunt motor takes a total current of 80 A. If the field winding resistance is 120 Ω and the armature resistance is 0.4 Ω, determine (a) the current in the armature, and (b) the back e.m.f. [(a) 78 A (b) 208.8 V]

2. A d.c. motor has a speed of 900 rev/min when connected to a 460 V supply. Find the approximate value of the speed of the motor when connected to a 200 V supply, assuming the flux decreases by 30 per cent and neglecting the armature volt drop. [559 rev/min]

3. A series motor having a series field resistance of 0.25 Ω and an armature resistance of 0.15 Ω, is connected to a 220 V supply and at a particular load runs at 20 rev/s when drawing 20 A from the supply. Calculate the e.m.f. generated at this load. Determine also the speed of the motor when the load is changed such that the current increases to 25 A. Assume the flux increases by 25 per cent [212 V, 15.85 rev/s]

4. A 500 V shunt motor takes a total current of 100 A and runs at 1200 rev/min. If the shunt field resistance is 50 Ω, the armature resistance is 0.25 Ω and the iron, friction and windage losses amount to 2 kW, determine the overall efficiency of the motor. [81.95 per cent]

5. A 250 V, series-wound motor is running at 500 rev/min and its shaft torque is 130 Nm. If

its efficiency at this load is 88 per cent, find the current taken from the supply. [30.94 A]

6. In a test on a d.c. motor, the following data was obtained. Supply voltage: 500 V, current taken from the supply: 42.4 A, speed: 850 rev/min, shaft torque: 187 Nm. Determine the efficiency of the motor correct to the nearest 0.5 per cent. [78.5 per cent]

7. A 300 V series motor draws a current of 50 A. The field resistance is 40 mΩ and the armature resistance is 0.2 Ω. Determine the maximum efficiency of the motor. [92 per cent]

8. A series motor drives a load at 1500 rev/min and takes a current of 20 A when the supply voltage is 250 V. If the total resistance of the motor is 1.5 Ω and the iron, friction and windage losses amount to 400 W, determine the efficiency of the motor. [80 per cent]

9. A series-wound motor is connected to a d.c. supply and develops full-load torque when the current is 30 A and speed is 1000 rev/min. If the flux per pole is proportional to the current flowing, find the current and speed at half full-load torque, when connected to the same supply. [21.2 A, 1415 rev/min]

22.14 D.C. motor starter

If a d.c. motor whose armature is stationary is switched directly to its supply voltage, it is likely that the fuses protecting the motor will burn out. This is because the armature resistance is small, frequently being less than one ohm. Thus, additional resistance must be added to the armature circuit at the instant of closing the switch to start the motor.

As the speed of the motor increases, the armature conductors are cutting flux and a generated voltage, acting in opposition to the applied voltage, is produced, which limits the flow of armature current. Thus the value of the additional armature resistance can then be reduced.

When at normal running speed, the generated e.m.f. is such that no additional resistance is required in the armature circuit. To achieve this varying resistance in the armature circuit on starting, a d.c. motor starter is used, as shown in Fig. 22.28.

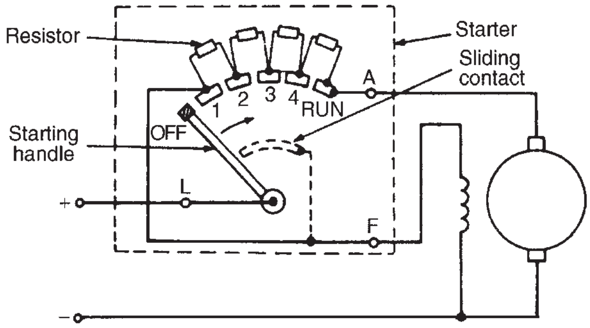


Figure 22.28

The starting handle is moved **slowly** in a clockwise direction to start the motor. For a shunt-wound motor, the field winding is connected to stud 1 or to L via a sliding contact on the starting handle, to give maximum field current, hence maximum flux, hence maximum torque on starting, since $T \propto \Phi I_a$. A similar arrangement without the field connection is used for series motors.

22.15 Speed control of d.c. motors

Shunt-wound motors

The speed of a shunt-wound d.c. motor, n, is proportional to

$$\frac{V - I_a R_a}{\Phi}$$

(see equation (9)). The speed is varied either by varying the value of flux, Φ, or by varying the value of R_a. The former is achieved by using a variable resistor in series with the field winding, as shown in Fig. 22.29(a) and such a resistor is called the **shunt field regulator**.

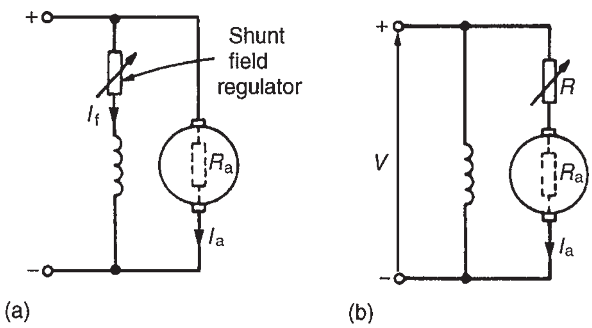


Figure 22.29

As the value of resistance of the shunt field regulator is increased, the value of the field current, I_f, is decreased.

This results in a decrease in the value of flux, Φ, and hence an increase in the speed, since $n \propto 1/\Phi$. Thus only speeds **above** that given without a shunt field regulator can be obtained by this method. Speeds **below** those given by

$$\frac{V - I_a R_a}{\Phi}$$

are obtained by increasing the resistance in the armature circuit, as shown in Fig. 22.29(b), where

$$n \propto \frac{V - I_a(R_a + R)}{\Phi}$$

Since resistor R is in series with the armature, it carries the full armature current and results in a large power loss in large motors where a considerable speed reduction is required for long periods.

These methods of speed control are demonstrated in the following worked problem.

Problem 28. A 500 V shunt motor runs at its normal speed of 10 rev/s when the armature current is 120 A. The armature resistance is 0.2 Ω.
(a) Determine the speed when the current is 60 A and a resistance of 0.5 Ω is connected in series with the armature, the shunt field remaining constant
(b) Determine the speed when the current is 60 A and the shunt field is reduced to 80 per cent of its normal value by increasing resistance in the field circuit.

(a) With reference to Fig. 22.29(b), back e.m.f. at 120 A, $E_1 = V - I_a R_a = 500 - (120)(0.2)$
$= 500 - 24 = 476$ volts.

When $I_a = 60$ A,

$$E_2 = 500 - (60)(0.2 + 0.5)$$
$$= 500 - (60)(0.7)$$
$$= 500 - 42 = 458 \text{ volts}$$

Now $\frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2}$

i.e. $\frac{476}{458} = \frac{\Phi_1(10)}{\Phi_1 n_2}$ since $\Phi_2 = \Phi_1$

from which,

$$\textbf{speed } \boldsymbol{n_2} = \frac{(10)(458)}{476} = \mathbf{9.62\ rev/s}$$

(b) Back e.m.f. when $I_a = 60$ A,

$$E_3 = 500 - (60)(0.2)$$
$$= 500 - 12 = 488 \text{ volts}$$

Now $\frac{E_1}{E_3} = \frac{\Phi_1 n_1}{\Phi_3 n_3}$

i.e. $\frac{476}{488} = \frac{\Phi_1(10)}{0.8\Phi_1 n_3}$ since $\Phi_3 = 0.8\,\Phi_1$

from which,

$$\textbf{speed } \boldsymbol{n_3} = \frac{(10)(488)}{(0.8)(476)} = \mathbf{12.82\ rev/s}$$

Series-wound motors

The speed control of series-wound motors is achieved using either (a) field resistance, or (b) armature resistance techniques.

(a) The speed of a d.c. series-wound motor is given by:

$$n = k\left(\frac{V - IR}{\Phi}\right)$$

where k is a constant, V is the terminal voltage, R is the combined resistance of the armature and series field and Φ is the flux. Thus, a reduction in flux results in an increase in speed. This is achieved by putting a variable resistance in parallel with the field winding and reducing the field current, and hence flux, for a given value of supply current. A circuit diagram of this arrangement is shown in Fig. 22.30(a). A variable resistor connected in parallel with the series-wound field to control speed is called a **diverter**. Speeds above those given with no diverter are obtained by this method. Problem 29 below demonstrates this method.

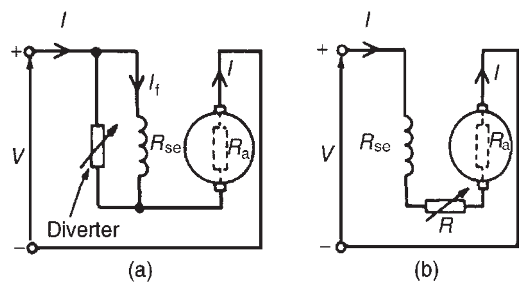


Figure 22.30

Section 3

(b) Speeds below normal are obtained by connecting a variable resistor in series with the field winding and armature circuit, as shown in Fig. 22.30(b). This effectively increases the value of R in the equation

$$n = k\left(\frac{V - IR}{\Phi}\right)$$

and thus reduces the speed. Since the additional resistor carries the full supply current, a large power loss is associated with large motors in which a considerable speed reduction is required for long periods. This method is demonstrated in problem 30.

Problem 29. On full-load a 300 V series motor takes 90 A and runs at 15 rev/s. The armature resistance is 0.1 Ω and the series winding resistance is 50 mΩ. Determine the speed when developing full load torque but with a 0.2 Ω diverter in parallel with the field winding. (Assume that the flux is proportional to the field current).

At 300 V, e.m.f.

$$\begin{aligned} E_1 &= V - IR = V - I(R_a + R_{se}) \\ &= 300 - (90)(0.1 + 0.05) \\ &= 300 - (90)(0.15) \\ &= 300 - 13.5 = 286.5 \text{ volts} \end{aligned}$$

With the 0.2 Ω diverter in parallel with R_{se} (see Fig. 22.30(a)), the equivalent resistance,

$$R = \frac{(0.2)(0.05)}{0.2 + 0.05} = \frac{(0.2)(0.05)}{0.25} = 0.04\,\Omega$$

By current division, current

$$I_1 \text{ (in Fig. 22.30(a))} = \left(\frac{0.2}{0.2 + 0.05}\right) I = 0.8\,I$$

Torque, $T \propto I_a \Phi$ and for full load torque, $I_{a1}\Phi_1 = I_{a2}\Phi_2$. Since flux is proportional to field current $\Phi_1 \propto I_{a1}$ and $\Phi_2 \propto 0.8\,I_{a2}$ then $(90)(90) = (I_{a2})(0.8\,I_{a2})$

from which, $$I_{a2}^2 = \frac{90^2}{0.8}$$

and $$I_{a2} = \frac{90}{\sqrt{0.8}} = 100.62\text{ A}$$

Hence e.m.f. $$\begin{aligned} E_2 &= V - I_{a2}(R_a + R) \\ &= 300 - (100.62)(0.1 + 0.04) \\ &= 300 - (100.62)(0.14) \\ &= 300 - 14.087 = 285.9 \text{ volts} \end{aligned}$$

Now e.m.f., $E \propto \Phi n$, from which,

$$\frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2} = \frac{I_{a1} n_1}{0.8 I_{a2} n_2}$$

Hence $$\frac{286.5}{285.9} = \frac{(90)(15)}{(0.8)(100.62)n_2}$$

and **new speed,** $$\boldsymbol{n_2} = \frac{(285.9)(90)(15)}{(286.5)(0.8)(100.62)} = \mathbf{16.74\ rev/s}$$

Thus the speed of the motor has increased from 15 rev/s (i.e. 900 rev/min) to 16.74 rev/s (i.e. 1004 rev/min) by inserting a 0.2 Ω diverter resistance in parallel with the series winding.

Problem 30. A series motor runs at 800 rev/min when the voltage is 400 V and the current is 25 A. The armature resistance is 0.4 Ω and the series field resistance is 0.2 Ω. Determine the resistance to be connected in series to reduce the speed to 600 rev/min with the same current.

With reference to Fig. 22.30(b), at 800 rev/min,

e.m.f., $$\begin{aligned} E_1 &= V - I(R_a + R_{se}) \\ &= 400 - (25)(0.4 + 0.2) \\ &= 400 - (25)(0.6) \\ &= 400 - 15 = 385 \text{ volts} \end{aligned}$$

At 600 rev/min, since the current is unchanged, the flux is unchanged.

Thus $E \propto \Phi n$ or $E \propto n$ and

$$\frac{E_1}{E_2} = \frac{n_1}{n_2}$$

Hence $$\frac{385}{E_2} = \frac{800}{600}$$

from which, $$E_2 = \frac{(385)(600)}{800} = 288.75 \text{ volts}$$

and $E_2 = V - I(R_a + R_{se} + R)$

Hence $288.75 = 400 - 25(0.4 + 0.2 + R)$

Rearranging gives:

$$0.6 + R = \frac{400 - 288.75}{25} = 4.45$$

from which, extra series resistance, $R = 4.45 - 0.6$ i.e. $\mathbf{R = 3.85\,\Omega}$.

Thus the addition of a series resistance of 3.85 Ω has reduced the speed from 800 rev/min to 600 rev/min.

Now try the following exercise

Exercise 136 Further problems on the speed control of d.c. motors

1. A 350 V shunt motor runs at its normal speed of 12 rev/s when the armature current is 90 A. The resistance of the armature is 0.3 Ω.
   (a) Find the speed when the current is 45 A and a resistance of 0.4 Ω is connected in series with the armature, the shunt field remaining constant
   (b) Find the speed when the current is 45 A and the shunt field is reduced to 75 per cent of its normal value by increasing resistance in the field circuit.
   [(a) 11.83 rev/s (b) 16.67 rev/s]

2. A series motor runs at 900 rev/min when the voltage is 420 V and the current is 40 A. The armature resistance is 0.3 Ω and the series field resistance is 0.2 Ω. Calculate the resistance to be connected in series to reduce the speed to 720 rev/min with the same current. [2 Ω]

3. A 320 V series motor takes 80 A and runs at 1080 rev/min at full load. The armature resistance is 0.2 Ω and the series winding resistance is 0.05 Ω. Assuming the flux is proportional to the field current, calculate the speed when developing full-load torque, but with a 0.15 Ω diverter in parallel with the field winding. [1239 rev/min]

22.16 Motor cooling

Motors are often classified according to the type of enclosure used, the type depending on the conditions under which the motor is used and the degree of ventilation required.

The most common type of protection is the **screen-protected type**, where ventilation is achieved by fitting a fan internally, with the openings at the end of the motor fitted with wire mesh.

A **drip-proof type** is similar to the screen-protected type but has a cover over the screen to prevent drips of water entering the machine.

A **flame-proof type** is usually cooled by the conduction of heat through the motor casing.

With a **pipe-ventilated type**, air is piped into the motor from a dust-free area, and an internally fitted fan ensures the circulation of this cool air.

Now try the following exercises

Exercise 137 Short answer questions on d.c. machines

1. A ...... converts mechanical energy into electrical energy
2. A ...... converts electrical energy into mechanical energy
3. What does ‘commutation’ achieve?
4. Poor commutation may cause sparking. How can this be improved?
5. State any five basic parts of a d.c. machine
6. State the two groups armature windings can be divided into
7. What is armature reaction? How can it be overcome?
8. The e.m.f. generated in an armature winding is given by $E = 2p\Phi nZ/c$ volts. State what p, Φ, n, Z and c represent.
9. In a series-wound d.c. machine, the field winding is in ...... with the armature circuit
10. In a d.c. generator, the relationship between the generated voltage, terminal voltage, current and armature resistance is given by $E = \ldots\ldots$
11. A d.c. machine has its field winding in parallel with the armatures circuit. It is called a ...... wound machine

12. Sketch a typical open-circuit characteristic for (a) a separately excited generator (b) a shunt generator (c) a series generator
13. Sketch a typical load characteristic for (a) a separately excited generator (b) a shunt generator
14. State one application for (a) a shunt generator (b) a series generator (c) a compound generator
15. State the principle losses in d.c. machines
16. The efficiency of a d.c. machine is given by the ratio (......) per cent
17. The equation relating the generated e.m.f., E, terminal voltage, armature current and armature resistance for a d.c. motor is $E =$ ......
18. The torque T of a d.c. motor is given by $T = p\Phi Z I_a / \pi c$ newton metres. State what p, Φ, Z, I and c represent
19. Complete the following. In a d.c. machine
    (a) generated e.m.f. $\propto$ ...... $\times$ ......
    (b) torque $\propto$ ...... $\times$ ......
20. Sketch typical characteristics of torque/armature current for
    (a) a shunt motor
    (b) a series motor
    (c) a compound motor
21. Sketch typical speed/torque characteristics for a shunt and series motor
22. State two applications for each of the following motors:
    (a) shunt (b) series (c) compound
    In questions 23 to 26, an electrical machine runs at n rev/s, has a shaft torque of T, and takes a current of I from a supply voltage V
23. The power input to a generator is ...... watts
24. The power input to a motor is ...... watts
25. The power output from a generator is ...... watts
26. The power output from a motor is ...... watts
27. The generated e.m.f. of a d.c machine is proportional to ...... volts
28. The torque produced by a d.c. motor is proportional to ...... Nm
29. A starter is necessary for a d.c. motor because the generated e.m.f. is ...... at low speeds
30. The speed of a d.c. shunt-wound motor will ...... if the value of resistance of the shunt field regulator is increased
31. The speed of a d.c. motor will ...... if the value of resistance in the armature circuit is increased
32. The value of the speed of a d.c. shunt-wound motor ...... as the value of the armature current increases
33. At a large value of torque, the speed of a d.c. series-wound motor is ......
34. At a large value of field current, the generated e.m.f. of a d.c. shunt-wound generator is approximately ......
35. In a series-wound generator, the terminal voltage increases as the load current ......
36. One type of d.c. motor uses resistance in series with the field winding to obtain speed variations and another type uses resistance in parallel with the field winding for the same purpose. Explain briefly why these two distinct methods are used and why the field current plays a significant part in controlling the speed of a d.c. motor.
37. Name three types of motor enclosure

Exercise 138 Multi-choice questions on d.c. machines

(Answers on page 399)

1. Which of the following statements is false?
    (a) A d.c. motor converts electrical energy to mechanical energy
    (b) The efficiency of a d.c. motor is the ratio input power to output power
    (c) A d.c. generator converts mechanical power to electrical power
    (d) The efficiency of a d.c. generator is the ratio output power to input power

A shunt-wound d.c. machine is running at n rev/s and has a shaft torque of T Nm. The supply current is IA when connected to d.c. bus-bars of voltage V volts. The armature resistance of the machine is R_a ohms, the armature current is I_aA and the generated voltage is E volts. Use this data to find the formulae of the quantities stated in questions 2 to 9, selecting the correct answer from the following list:

(a) $V - I_aR_a$ (b) $E + I_aR_a$
(c) VI (d) $E - I_aR_a$
(e) $T(2\pi n)$ (f) $V + I_aR_a$

2. The input power when running as a generator
3. The output power when running as a motor
4. The input power when running as a motor
5. The output power when running as a generator
6. The generated voltage when running as a motor
7. The terminal voltage when running as a generator
8. The generated voltage when running as a generator
9. The terminal voltage when running as a motor
10. Which of the following statements is false?
    (a) A commutator is necessary as part of a d.c. motor to keep the armature rotating in the same direction
    (b) A commutator is necessary as part of a d.c. generator to produce unidirectional voltage at the terminals of the generator
    (c) The field winding of a d.c. machine is housed in slots on the armature
    (d) The brushes of a d.c. machine are usually made of carbon and do not rotate with the armature
11. If the speed of a d.c. machine is doubled and the flux remains constant, the generated e.m.f. (a) remains the same (b) is doubled (c) is halved
12. If the flux per pole of a shunt-wound d.c. generator is increased, and all other variables are kept the same, the speed (a) decreases (b) stays the same (c) increases
13. If the flux per pole of a shunt-wound d.c. generator is halved, the generated e.m.f. at constant speed (a) is doubled (b) is halved (c) remains the same
14. In a series-wound generator running at constant speed, as the load current increases, the terminal voltage
    (a) increases (b) decreases
    (c) stays the same
15. Which of the following statements is false for a series-wound d.c. motor?
    (a) The speed decreases with increase of resistance in the armature circuit
    (b) The speed increases as the flux decreases
    (c) The speed can be controlled by a diverter
    (d) The speed can be controlled by a shunt field regulator
16. Which of the following statements is false?
    (a) A series-wound motor has a large starting torque
    (b) A shunt-wound motor must be permanently connected to its load
    (c) The speed of a series-wound motor drops considerably when load is applied
    (d) A shunt-wound motor is essentially a constant-speed machine
17. The speed of a d.c. motor may be increased by
    (a) increasing the armature current
    (b) decreasing the field current
    (c) decreasing the applied voltage
    (d) increasing the field current
18. The armature resistance of a d.c. motor is $0.5\,\Omega$, the supply voltage is 200 V and the back e.m.f. is 196 V at full speed. The armature current is:
    (a) 4 A (b) 8 A
    (c) 400 A (d) 392 A
19. In d.c. generators iron losses are made up of:
    (a) hysteresis and friction losses
    (b) hysteresis, eddy current and brush contact losses
    (c) hysteresis and eddy current losses

(d) hysteresis, eddy current and copper losses

20. The effect of inserting a resistance in series with the field winding of a shunt motor is to:
    (a) increase the magnetic field
    (b) increase the speed of the motor
    (c) decrease the armature current
    (d) reduce the speed of the motor

21. The supply voltage to a d.c. motor is 240 V. If the back e.m.f. is 230 V and the armature resistance is 0.25 Ω, the armature current is:
    (a) 10 A (b) 40 A
    (c) 960 A (d) 920 A

22. With a d.c. motor, the starter resistor:
    (a) limits the armature current to a safe starting value
    (b) controls the speed of the machine
    (c) prevents the field current flowing through and damaging the armature
    (d) limits the field current to a safe starting value

23. From Fig. 22.31, the expected characteristic for a shunt-wound d.c. generator is:
    (a) P (b) Q
    (c) R (d) S

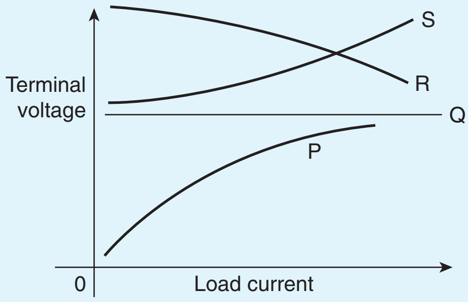


Figure 22.31

24. A commutator is a device fitted to a generator. Its function is:
    (a) to prevent sparking when the load changes
    (b) to convert the a.c. generated into a d.c. output
    (c) to convey the current to and from the windings
    (d) to generate a direct current

Chapter 23

Three-phase induction motors

At the end of this chapter you should be able to:

- appreciate the merits of three-phase induction motors
- understand how a rotating magnetic field is produced
- state the synchronous speed, $n_s = (f/p)$ and use in calculations
- describe the principle of operation of a three-phase induction motor
- distinguish between squirrel-cage and wound-rotor types of motor
- understand how a torque is produced causing rotor movement
- understand and calculate slip
- derive expressions for rotor e.m.f., frequency, resistance, reactance, impedance, current and copper loss, and use them in calculations
- state the losses in an induction motor and calculate efficiency
- derive the torque equation for an induction motor, state the condition for maximum torque, and use in calculations
- describe torque-speed and torque-slip characteristics for an induction motor
- state and describe methods of starting induction motors
- state advantages of cage rotor and wound rotor types of induction motor
- describe the double cage induction motor
- state typical applications of three-phase induction motors

23.1 Introduction

In d.c. motors, introduced in Chapter 22, conductors on a rotating armature pass through a stationary magnetic field. In a **three-phase induction motor**, the magnetic field rotates and this has the advantage that no external electrical connections to the rotor need be made. Its name is derived from the fact that the current in the rotor is **induced** by the magnetic field instead of being supplied through electrical connections to the supply. The result is a motor which: (i) is cheap and robust, (ii) is explosion proof, due to the absence of a commutator or slip-rings and brushes with their associated sparking, (iii) requires little or no skilled maintenance, and (iv) has self-starting properties when switched to a supply with no additional expenditure on auxiliary equipment. The principal disadvantage of a three-phase induction motor is that its speed cannot be readily adjusted.

23.2 Production of a rotating magnetic field

When a three-phase supply is connected to symmetrical three-phase windings, the currents flowing in the windings produce a magnetic field. This magnetic field is constant in magnitude and rotates at constant speed as shown below, and is called the **synchronous speed**.

With reference to Fig. 23.1, the windings are represented by three single-loop conductors, one for each phase, marked R_SR_F, Y_SY_F and B_SB_F, the S and F signifying start and finish. In practice, each phase winding comprises many turns and is distributed around the stator; the single-loop approach is for clarity only.

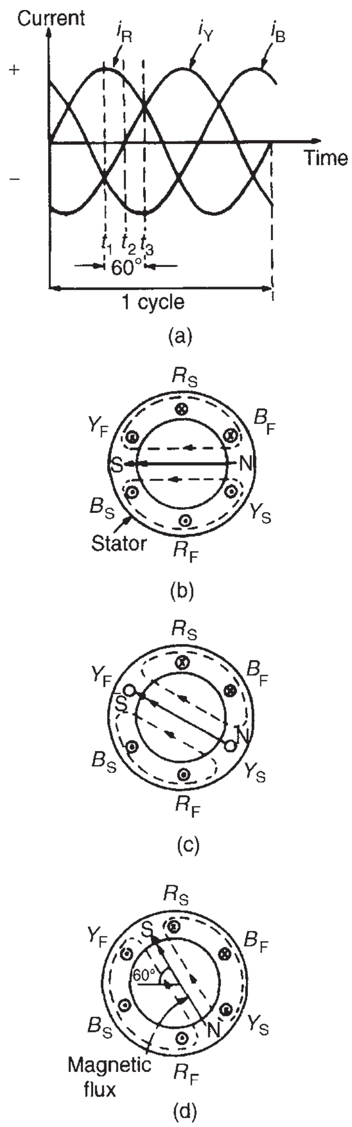


Figure 23.1

When the stator windings are connected to a three-phase supply, the current flowing in each winding varies with time and is as shown in Fig. 23.1(a). If the value of current in a winding is positive, the assumption is made that it flows from start to finish of the winding, i.e. if it is the red phase, current flows from R_S to R_F, i.e. away from the viewer in R_S and towards the viewer in R_F. When the value of current is negative, the assumption is made that it flows from finish to start, i.e. towards the viewer in an 'S' winding and away from the viewer in an 'F' winding. At time, say t_1, shown in Fig. 23.1(a), the current flowing in the red phase is a maximum positive value. At the same time t_1, the currents flowing in the yellow and blue phases are both 0.5 times the maximum value and are negative.

The current distribution in the stator windings is therefore as shown in Fig. 23.1(b), in which current flows away from the viewer, (shown as ⊗) in R_S since it is positive, but towards the viewer (shown as ⊙) in Y_S and B_S, since these are negative. The resulting magnetic field is as shown, due to the 'solenoid' action and application of the corkscrew rule.

A short time later at time t_2, the current flowing in the red phase has fallen to about 0.87 times its maximum value and is positive, the current in the yellow phase is zero and the current in the blue phase is about 0.87 times its maximum value and is negative. Hence the currents and resultant magnetic field are as shown in Fig. 23.1(c). At time t_3, the currents in the red and yellow phases are 0.5 of their maximum values and the current in the blue phase is a maximum negative value. The currents and resultant magnetic field are as shown in Fig. 23.1(d).

Similar diagrams to Fig. 23.1(b), (c) and (d) can be produced for all time values and these would show that the magnetic field travels through one revolution for each cycle of the supply voltage applied to the stator windings.

By considering the flux values rather than the current values, it is shown below that the rotating magnetic field has a constant value of flux. The three coils shown in Fig. 23.2(a), are connected in star to a three-phase supply. Let the positive directions of the fluxes produced by currents flowing in the coils, be ϕ_A, ϕ_B and ϕ_C respectively. The directions of ϕ_A, ϕ_B and ϕ_C do not alter, but their magnitudes are proportional to the currents flowing in the coils at any particular time. At time t_1, shown in Fig. 23.2(b), the currents flowing in the coils are:

i_B, a maximum positive value, i.e. the flux is towards point P; i_A and i_C, half the maximum value and negative, i.e. the flux is away from point P.

These currents give rise to the magnetic fluxes ϕ_A, ϕ_B and ϕ_C, whose magnitudes and directions are as shown

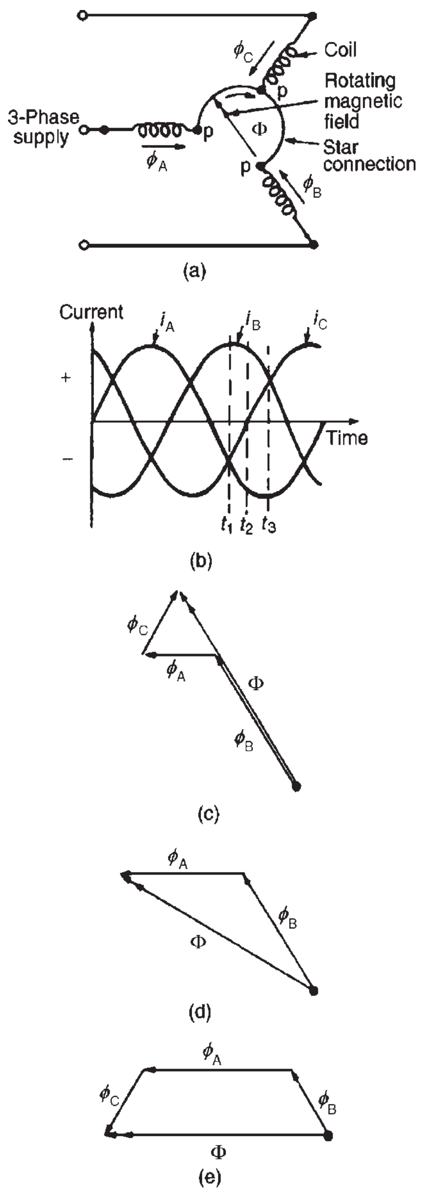


Figure 23.2

in Fig. 23.2(c). The resultant flux is the phasor sum of ϕ_A, ϕ_B and ϕ_C, shown as Φ in Fig. 23.2(c). At time t_2, the currents flowing are:

i_B, 0.866 × maximum positive value, i_C, zero, and i_A, 0.866 × maximum negative value.

The magnetic fluxes and the resultant magnetic flux are as shown in Fig. 23.2(d).

At time t_3,

i_B is 0.5 × maximum value and is positive

i_A is a maximum negative value, and

i_C is 0.5 × maximum value and is positive.

The magnetic fluxes and the resultant magnetic flux are as shown in Fig. 23.2(e)

Inspection of Fig. 23.2(c), (d) and (e) shows that the magnitude of the resultant magnetic flux, Φ, in each case is constant and is $1\frac{1}{2}$ × the maximum value of ϕ_A, ϕ_B or ϕ_C, but that its direction is changing. The process of determining the resultant flux may be repeated for all values of time and shows that the magnitude of the resultant flux is constant for all values of time and also that it rotates at constant speed, making one revolution for each cycle of the supply voltage.

23.3 Synchronous speed

The rotating magnetic field produced by three-phase windings could have been produced by rotating a permanent magnet's north and south pole at synchronous speed, (shown as N and S at the ends of the flux phasors in Fig. 23.1(b), (c) and (d)). For this reason, it is called a 2-pole system and an induction motor using three phase windings only is called a 2-pole induction motor. If six windings displaced from one another by 60° are used, as shown in Fig. 23.3(a), by drawing the current and resultant magnetic field diagrams at various time values, it may be shown that one cycle of the supply current to

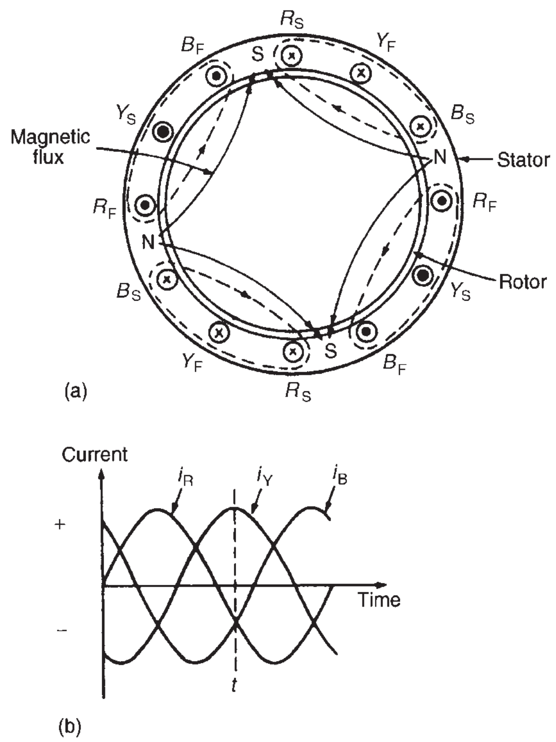


Figure 23.3

the stator windings causes the magnetic field to move through half a revolution. The current distribution in the stator windings are shown in Fig. 23.3(a), for the time t shown in Fig. 23.3(b).

It can be seen that for six windings on the stator, the magnetic flux produced is the same as that produced by rotating two permanent magnet north poles and two permanent magnet south poles at synchronous speed. This is called a 4-pole system and an induction motor using six phase windings is called a 4-pole induction motor. By increasing the number of phase windings the number of poles can be increased to any even number.

In general, if f is the frequency of the currents in the stator windings and the stator is wound to be equivalent to p **pairs** of poles, the speed of revolution of the rotating magnetic field, i.e. the synchronous speed, n_s is given by:

$$\boldsymbol{n_s = \frac{f}{p}\ \text{rev/s}}$$

Problem 1. A three-phase two-pole induction motor is connected to a 50 Hz supply. Determine the synchronous speed of the motor in rev/min.

From above, $n_s = (f/p)$ rev/s, where n_s is the synchronous speed, f is the frequency in hertz of the supply to the stator and p is the number of **pairs** of poles. Since the motor is connected to a 50 hertz supply, $f = 50$.

The motor has a two-pole system, hence p, the number of pairs of poles, is 1. Thus, synchronous speed, $\boldsymbol{n_s} = (50/1) = 50$ rev/s $= 50 \times 60$ rev/min $=$ **3000 rev/min**.

Problem 2. A stator winding supplied from a three-phase 60 Hz system is required to produce a magnetic flux rotating at 900 rev/min. Determine the number of poles.

Synchronous speed,

$$n_s = 900\ \text{rev/min} = \frac{900}{60}\ \text{rev/s} = 15\ \text{rev/s}$$

Since

$$n_s = \left(\frac{f}{p}\right) \text{ then } p = \left(\frac{f}{n_s}\right) = \left(\frac{60}{15}\right) = 4$$

Hence **the number of pole pairs is 4** and thus **the number of poles is 8**

Problem 3. A three-phase 2-pole motor is to have a synchronous speed of 6000 rev/min. Calculate the frequency of the supply voltage.

Since $n_s = \left(\frac{f}{p}\right)$ then

$$\textbf{frequency, } \boldsymbol{f} = (n_s)(p) = \left(\frac{6000}{60}\right)\left(\frac{2}{2}\right) = \textbf{100 Hz}$$

Now try the following exercise

Exercise 139 Further problems on synchronous speed

1. The synchronous speed of a 3-phase, 4-pole induction motor is 60 rev/s. Determine the frequency of the supply to the stator windings. [120 Hz]
2. The synchronous speed of a 3-phase induction motor is 25 rev/s and the frequency of the supply to the stator is 50 Hz. Calculate the equivalent number of pairs of poles of the motor. [2]
3. A 6-pole, 3-phase induction motor is connected to a 300 Hz supply. Determine the speed of rotation of the magnetic field produced by the stator. [100 rev/s]

23.4 Construction of a three-phase induction motor

The stator of a three-phase induction motor is the stationary part corresponding to the yoke of a d.c. machine. It is wound to give a 2-pole, 4-pole, 6-pole, ... rotating magnetic field, depending on the rotor speed required. The rotor, corresponding to the armature of a d.c. machine, is built up of laminated iron, to reduce eddy currents.

In the type most widely used, known as a **squirrel-cage rotor**, copper or aluminium bars are placed in slots cut in the laminated iron, the ends of the bars being welded or brazed into a heavy conducting ring, (see Fig. 23.4(a)). A cross-sectional view of a three-phase induction motor is shown in Fig. 23.4(b).

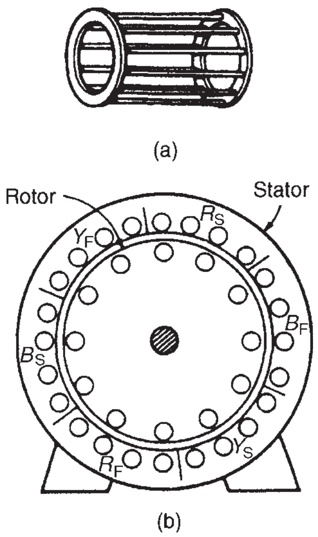


Figure 23.4

The conductors are placed in slots in the laminated iron rotor core. If the slots are skewed, better starting and quieter running is achieved. This type of rotor has no external connections which means that slip rings and brushes are not needed. The squirrel-cage motor is cheap, reliable and efficient. Another type of rotor is the **wound rotor**. With this type there are phase windings in slots, similar to those in the stator. The windings may be connected in star or delta and the connections made to three slip rings. The slip rings are used to add external resistance to the rotor circuit, particularly for starting (see Section 23.13), but for normal running the slip rings are short-circuited.

The principle of operation is the same for both the squirrel cage and the wound rotor machines.

23.5 Principle of operation of a three-phase induction motor

When a three-phase supply is connected to the stator windings, a rotating magnetic field is produced. As the magnetic flux cuts a bar on the rotor, an e.m.f. is induced in it and since it is joined, via the end conducting rings, to another bar one pole pitch away, a current flows in the bars. The magnetic field associated with this current flowing in the bars interacts with the rotating magnetic field and a force is produced, tending to turn the rotor in the same direction as the rotating magnetic field, (see Fig. 23.5). Similar forces are applied to all the conductors on the rotor, so that a torque is produced causing the rotor to rotate.

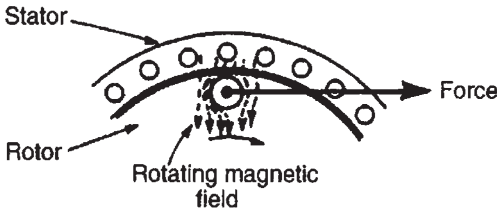


Figure 23.5

23.6 Slip

The force exerted by the rotor bars causes the rotor to turn in the direction of the rotating magnetic field. As the rotor speed increases, the rate at which the rotating magnetic field cuts the rotor bars is less and the frequency of the induced e.m.f.'s in the rotor bars is less. If the rotor runs at the same speed as the rotating magnetic field, no e.m.f.'s are induced in the rotor, hence there is no force on them and no torque on the rotor. Thus the rotor slows down. For this reason the rotor can never run at synchronous speed.

When there is no load on the rotor, the resistive forces due to windage and bearing friction are small and the rotor runs very nearly at synchronous speed. As the rotor is loaded, the speed falls and this causes an increase in the frequency of the induced e.m.f.'s in the rotor bars and hence the rotor current, force and torque increase. The difference between the rotor speed, n_r, and the synchronous speed, n_s, is called the **slip speed**, i.e.

$$\textbf{slip speed} = n_s - n_r \ \textbf{rev/s}$$

The ratio $(n_s - n_r)/n_s$ is called the **fractional slip** or just the **slip**, s, and is usually expressed as a percentage. Thus

$$\textbf{slip}, s = \left(\frac{n_s - n_r}{n_s}\right) \times 100\%$$

Typical values of slip between no load and full load are about 4 to 5 per cent for small motors and 1.5 to 2 per cent for large motors.

Problem 4. The stator of a 3-phase, 4-pole induction motor is connected to a 50 Hz supply. The rotor runs at 1455 rev/min at full load. Determine (a) the synchronous speed and (b) the slip at full load.

(a) The number of pairs of poles, $p = (4/2) = 2$. The supply frequency $f = 50$ Hz. The **synchronous speed**, $n_s = (f/p) = (50/2) = $ **25 rev/s**.

(b) The rotor speed, $n_r = (1455/60) = 24.25$ rev/s.

$$\textbf{Slip, s} = \left(\frac{n_s - n_r}{n_s}\right) \times 100\%$$
$$= \left(\frac{25 - 24.25}{25}\right) \times 100\%$$
$$= \mathbf{3\%}$$

Problem 5. A 3-phase, 60 Hz induction motor has 2 poles. If the slip is 2 per cent at a certain load, determine (a) the synchronous speed, (b) the speed of the rotor, and (c) the frequency of the induced e.m.f.'s in the rotor.

(a) $f = 60$ Hz and $p = (2/2) = 1$. Hence **synchronous speed**, $\boldsymbol{n_s} = (f/p) = (60/1) =$ **60 rev/s** or 60×60 = **3600 rev/min**.

(b) Since slip,

$$s = \left(\frac{n_s - n_r}{n_s}\right) \times 100\%$$
$$2 = \left(\frac{60 - n_r}{60}\right) \times 100$$

Hence

$$\frac{2 \times 60}{100} = 60 - n_r$$

i.e.

$$n_r = 60 - \frac{2 \times 60}{100} = 58.8 \text{ rev/s}$$

i.e. the rotor runs at $58.8 \times 60 =$ **3528 rev/min**

(c) Since the synchronous speed is 60 rev/s and that of the rotor is 58.8 rev/s, the rotating magnetic field cuts the rotor bars at $(60 - 58.8) = 1.2$ rev/s.

Thus the frequency of the e.m.f.'s induced in the rotor bars, is $\boldsymbol{f = n_s p = (1.2)(\tfrac{2}{2}) = 1.2}$ **Hz**.

Problem 6. A three-phase induction motor is supplied from a 50 Hz supply and runs at 1200 rev/min when the slip is 4 per cent. Determine the synchronous speed.

$$\text{Slip, } s = \left(\frac{n_s - n_r}{n_s}\right) \times 100\%$$

Rotor speed, $n_r = (1200/60) = 20$ rev/s and $s = 4$.

Hence

$$4 = \left(\frac{n_s - 20}{n_s}\right) \times 100\% \text{ or } 0.04 = \frac{n_s - 20}{n_s}$$

from which, $n_s(0.04) = n_s - 20$ and
$20 = n_s - 0.04\, n_s = n_s(1 - 0.04)$.

Hence **synchronous speed**,

$$\boldsymbol{n_s} = \frac{20}{1 - 0.04} = 20.8\dot{3} \text{ rev/s}$$
$$= (20.8\dot{3} \times 60) \text{ rev/min}$$
$$= \mathbf{1250 \text{ rev/min}}$$

Now try the following exercise

Exercise 140 Further problems on slip

1. A 6-pole, 3-phase induction motor runs at 970 rev/min at a certain load. If the stator is connected to a 50 Hz supply, find the percentage slip at this load. [3%]

2. A 3-phase, 50 Hz induction motor has 8 poles. If the full load slip is 2.5 per cent, determine
   (a) the synchronous speed,
   (b) the rotor speed, and
   (c) the frequency of the rotor e.m.f.'s
   [(a) 750 rev/min (b) 731 rev/min (c) 1.25 Hz]

3. A three-phase induction motor is supplied from a 60 Hz supply and runs at 1710 rev/min when the slip is 5 per cent. Determine the synchronous speed. [1800 rev/min]

4. A 4-pole, 3-phase, 50 Hz induction motor runs at 1440 rev/min at full load. Calculate
   (a) the synchronous speed,
   (b) the slip and
   (c) the frequency of the rotor induced e.m.f.'s
   [(a) 1500 rev/min (b) 4% (c) 2 Hz]

23.7 Rotor e.m.f. and frequency

Rotor e.m.f.

When an induction motor is stationary, the stator and rotor windings form the equivalent of a transformer as shown in Fig. 23.6

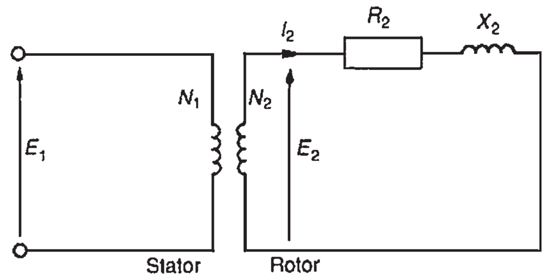


Figure 23.6

The rotor e.m.f. at standstill is given by

$$E_2 = \left(\frac{N_2}{N_1}\right) E_1 \qquad (1)$$

where E_1 is the supply voltage per phase to the stator.

When an induction motor is running, the induced e.m.f. in the rotor is less since the relative movement between conductors and the rotating field is less. The induced e.m.f. is proportional to this movement, hence it must be proportional to the slip, s. Hence **when running**, rotor e.m.f. per phase $= E_r = sE_2$

$$\text{i.e. rotor e.m.f. per phase } = s\left(\frac{N_2}{N_1}\right) E_1 \qquad (2)$$

Rotor frequency

The rotor e.m.f. is induced by an alternating flux and the rate at which the flux passes the conductors is the slip speed. Thus the frequency of the rotor e.m.f. is given by:

$$f_r = (n_s - n_r)p = \left(\frac{n_s - n_r}{n_s}\right)(n_s p)$$

However $(n_s - n_r)/n_s$ is the slip s and $(n_s p)$ is the supply frequency f, hence

$$f_r = sf \qquad (3)$$

Problem 7. The frequency of the supply to the stator of an 8-pole induction motor is 50 Hz and the rotor frequency is 3 Hz. Determine (a) the slip, and (b) the rotor speed.

(a) From Equation (3), $f_r = sf$. Hence $3 = (s)(50)$ from which,

$$\textbf{slip, s} = \frac{3}{50} = \mathbf{0.06} \text{ or } \mathbf{6\%}$$

(b) Synchronous speed, $n_s = f/p = 50/4 = 12.5$ rev/s or $(12.5 \times 60) = 750$ rev/min

$$\text{Slip, } s = \left(\frac{n_s - n_r}{n_s}\right)$$

$$\text{hence} \qquad 0.06 = \left(\frac{12.5 - n_r}{12.5}\right)$$

$$(0.06)(12.5) = 12.5 - n_r$$

and **rotor speed**,

$$\mathbf{n_r} = 12.5 - (0.06)(12.5)$$
$$= \mathbf{11.75 \text{ rev/s}} \text{ or } \mathbf{705 \text{ rev/min}}$$

Now try the following exercise

Exercise 141 Further problems on rotor frequency

1. A 12-pole, 3-phase, 50 Hz induction motor runs at 475 rev/min. Determine
   (a) the slip speed,
   (b) the percentage slip and
   (c) the frequency of rotor currents
   [(a) 25 rev/min (b) 5% (c) 2.5 Hz]

2. The frequency of the supply to the stator of a 6-pole induction motor is 50 Hz and the rotor frequency is 2 Hz. Determine
   (a) the slip, and
   (b) the rotor speed, in rev/min
   [(a) 0.04 or 4% (b) 960 rev/min]

23.8 Rotor impedance and current

Rotor resistance

The rotor resistance R_2 is unaffected by frequency or slip, and hence remains constant.

Rotor reactance

Rotor reactance varies with the frequency of the rotor current. At standstill, reactance per phase, $X_2 = 2\pi f L$. When running, reactance per phase,

$$X_r = 2\pi f_r L$$
$$= 2\pi(sf)L \quad \text{from equation (3)}$$
$$= s(2\pi f L)$$

$$\text{i.e.} \qquad \mathbf{X_r = sX_2} \qquad (4)$$

Section 3

Figure 23.7 represents the rotor circuit when running.

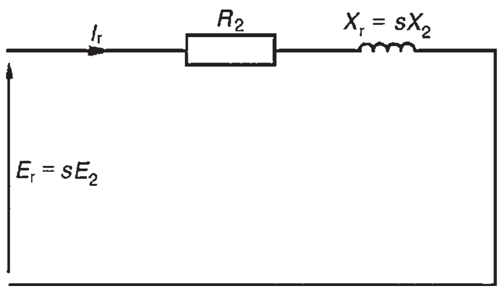


Figure 23.7

Rotor impedance

Rotor impedance per phase,

$$Z_r = \sqrt{R_2^2 + (sX_2)^2} \quad (5)$$

At standstill, slip $s = 1$, then

$$Z_2 = \sqrt{R_2^2 + X_2^2} \quad (6)$$

Rotor current

From Fig. 23.6 and 23.7, **at standstill, starting current,**

$$I_2 = \frac{E_2}{Z_2} = \frac{\left(\frac{N_2}{N_1}\right)E_1}{\sqrt{R_2^2 + X_2^2}} \quad (7)$$

and when running, current,

$$I_r = \frac{E_r}{Z_r} = \frac{s\left(\frac{N_2}{N_1}\right)E_1}{\sqrt{R_2^2 + (sX_2)^2}} \quad (8)$$

23.9 Rotor copper loss

Power $P = 2\pi nT$, where T is the torque in newton metres, hence torque $T = (P/2\pi n)$. If P_2 is the power input to the rotor from the rotating field, and P_m is the mechanical power output (including friction losses)

then $$T = \frac{P_2}{2\pi n_s} = \frac{P_m}{2\pi n_r}$$

from which, $$\frac{P_2}{n_s} = \frac{P_m}{n_r} \quad \text{or} \quad \frac{P_m}{P_2} = \frac{n_r}{n_s}$$

Hence $$1 - \frac{P_m}{P_2} = 1 - \frac{n_r}{n_s}$$

$$\frac{P_2 - P_m}{P_2} = \frac{n_s - n_r}{n_s} = s$$

$P_2 - P_m$ is the electrical or copper loss in the rotor, i.e. $P_2 - P_m = I_r^2 R_2$. Hence

$$\textbf{slip, } s = \frac{\textbf{rotor copper loss}}{\textbf{rotor input}} = \frac{I_r^2 R_2}{P_2} \quad (9)$$

or power input to the rotor,

$$P_2 = \frac{I_r^2 R_2}{s} \quad (10)$$

23.10 Induction motor losses and efficiency

Figure 23.8 summarises losses in induction motors. Motor efficiency,

$$\eta = \frac{\textbf{output power}}{\textbf{input power}} = \frac{P_m}{P_1} \times 100\%$$

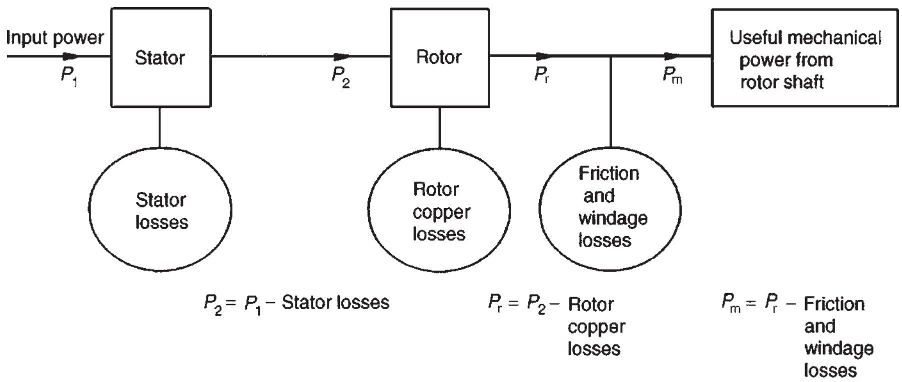


Figure 23.8

Problem 8. The power supplied to a three-phase induction motor is 32 kW and the stator losses are 1200 W. If the slip is 5 per cent, determine (a) the rotor copper loss, (b) the total mechanical power developed by the rotor, (c) the output power of the motor if friction and windage losses are 750 W, and (d) the efficiency of the motor, neglecting rotor iron loss.

(a) Input power to rotor = stator input power − stator losses

= 32 kW − 1.2 kW

= 30.8 kW

From Equation (9),

$$\text{slip} = \frac{\text{rotor copper loss}}{\text{rotor input}}$$

i.e.

$$\frac{5}{100} = \frac{\text{rotor copper loss}}{30.8}$$

from which, **rotor copper loss** = (0.05)(30.8) = **1.54 kW**

(b) Total mechanical power developed by the rotor

= rotor input power − rotor losses
= 30.8 − 1.54 = **29.26 kW**

(c) Output power of motor

= power developed by the rotor − friction and windage losses

= 29.26 − 0.75 = **28.51 kW**

(d) Efficiency of induction motor,

$$\eta = \left(\frac{\text{output power}}{\text{input power}}\right) \times 100\%$$

$$= \left(\frac{28.51}{32}\right) \times 100\%$$

$$= \mathbf{89.10\%}$$

Problem 9. The speed of the induction motor of Problem 8 is reduced to 35 per cent of its synchronous speed by using external rotor resistance. If the torque and stator losses are unchanged, determine (a) the rotor copper loss, and (b) the efficiency of the motor.

(a) Slip, $s = \left(\frac{n_s - n_r}{n_s}\right) \times 100\%$

$$= \left(\frac{n_s - 0.35n_s}{n_s}\right) \times 100\%$$

$$= (0.65)(100) = 65\%$$

Input power to rotor = 30.8 kW (from Problem 8)

Since $s = \frac{\text{rotor copper loss}}{\text{rotor input}}$

then **rotor copper loss** = (s)(rotor input)

$$= \left(\frac{65}{100}\right)(30.8)$$

$$= \mathbf{20.02\ kW}$$

(b) Power developed by rotor

= input power to rotor − rotor copper loss

= 30.8 − 20.02 = 10.78 kW

Output power of motor

= power developed by rotor − friction and windage losses

= 10.78 − 0.75 = 10.03 kW

Efficiency,

$$\eta = \left(\frac{\text{output power}}{\text{input power}}\right) \times 100\%$$

$$= \left(\frac{10.03}{32}\right) \times 100\%$$

$$= \mathbf{31.34\%}$$

Now try the following exercise

Exercise 142 Further problems on losses and efficiency

1. The power supplied to a three-phase induction motor is 50 kW and the stator losses are 2 kW. If the slip is 4 per cent, determine
   (a) the rotor copper loss,
   (b) the total mechanical power developed by the rotor,

(c) the output power of the motor if friction and windage losses are 1 kW, and
(d) the efficiency of the motor, neglecting rotor iron losses.
[(a) 1.92 kW (b) 46.08 kW (c) 45.08 kW (d) 90.16%]

2. By using external rotor resistance, the speed of the induction motor in Problem 1 is reduced to 40 per cent of its synchronous speed. If the torque and stator losses are unchanged, calculate
(a) the rotor copper loss, and
(b) the efficiency of the motor.
[(a) 28.80 kW (b) 36.40%]

23.11 Torque equation for an induction motor

Torque

$$T = \frac{P_2}{2\pi n_s} = \left(\frac{1}{2\pi n_s}\right)\left(\frac{I_r^2 R_2}{s}\right)$$

(from Equation (10))

From Equation (8),
$$I_r = \frac{s\left(\frac{N_2}{N_1}\right)E_1}{\sqrt{R_2^2 + (sX_2)^2}}$$

Hence torque per phase,

$$T = \left(\frac{1}{2\pi n_s}\right)\left(\frac{s^2\left(\frac{N_2}{N_1}\right)^2 E_1^2}{R_2^2 + (sX_2)^2}\right)\left(\frac{R_2}{s}\right)$$

i.e.

$$T = \left(\frac{1}{2\pi n_s}\right)\left(\frac{s\left(\frac{N_2}{N_2}\right)^2 E_1^2 R_2}{R_2^2 + (sX_2)^2}\right)$$

If there are m phases then torque,

$$T = \left(\frac{m}{2\pi n_s}\right)\left(\frac{s\left(\frac{N_2}{N_1}\right)^2 E_1^2 R_2}{R_2^2 + (sX_2)^2}\right)$$

i.e.

$$\boldsymbol{T = \left(\frac{m\left(\frac{N_2}{N_1}\right)^2}{2\pi n_s}\right)\left(\frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2}\right)} \qquad (11)$$

$$= k\left(\frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2}\right)$$

where k is a constant for a particular machine, i.e.

$$\textbf{torque, } \boldsymbol{T \propto \left(\frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2}\right)} \qquad (12)$$

Under normal conditions, the supply voltage is usually constant, hence Equation (12) becomes:

$$T \propto \frac{sR_2}{R_2^2 + (sX_2)^2}$$
$$\propto \frac{R_2}{\frac{R_2^2}{s} + sX_2^2}$$

The torque will be a maximum when the denominator is a minimum and this occurs when

$$\frac{R_2^2}{s} = sX_2^2$$

i.e. when

$$s = \frac{R_2}{X_2} \quad \text{or} \quad R_2 = sX_2 = X_r$$

from Equation (4). Thus **maximum torque** occurs when rotor resistance and rotor reactance are equal, i.e. when $\boldsymbol{R_2 = X_r}$

Problems 10 to 13 following illustrate some of the characteristics of three-phase induction motors.

Problem 10. A 415 V, three-phase, 50 Hz, 4 pole, star-connected induction motor runs at 24 rev/s on full load. The rotor resistance and reactance per phase are 0.35 Ω and 3.5 Ω respectively, and the effective rotor-stator turns ratio is 0.85:1. Calculate (a) the synchronous speed, (b) the slip, (c) the full load torque, (d) the power output if mechanical losses amount to 770 W, (e) the maximum torque, (f) the speed at which maximum torque occurs, and (g) the starting torque.

(a) Synchronous speed, $n_s = (f/p) = (50/2) =$ **25 rev/s** or $(25 \times 60) =$ **1500 rev/min**

(b) Slip, $s = \left(\frac{n_s - n_r}{n_s}\right) = \frac{25 - 24}{25} = \mathbf{0.04}$ or **4%**

(c) Phase voltage,

$$E_1 = \frac{415}{\sqrt{3}} = 239.6 \text{ volts}$$

Full load torque,

$$T = \left(\frac{m\left(\frac{N_2}{N_1}\right)^2}{2\pi n_s}\right)\left(\frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2}\right)$$

from Equation (11)

$$= \left(\frac{3(0.85)^2}{2\pi(25)}\right)\left(\frac{(0.04)(239.6)^2(0.35)}{(0.35)^2 + (0.04 \times 3.5)^2}\right)$$

$$= (0.01380)\left(\frac{803.71}{0.1421}\right)$$

$$= \mathbf{78.05\ Nm}$$

(d) Output power, including friction losses,

$$P_m = 2\pi n_r T$$
$$= 2\pi(24)(78.05)$$
$$= 11\,770 \text{ watts}$$

Hence, **power output** $= P_m -$ mechanical losses

$$= 11\,770 - 770$$
$$= 11\,000\text{ W}$$
$$= \mathbf{11\ kW}$$

(e) Maximum torque occurs when $R_2 = X_r = 0.35\,\Omega$

Slip, $$s = \frac{R_2}{X_2} = \frac{0.35}{3.5} = 0.1$$

Hence **maximum torque,**

$$\mathbf{T_m} = (0.01380)\left(\frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2}\right) \text{ from part (c)}$$

$$= (0.01380)\left(\frac{0.1(239.6)^2 0.35}{0.35^2 + 0.35^2}\right)$$

$$= (0.01380)\left(\frac{2009.29}{0.245}\right) = \mathbf{113.18\ Nm}$$

(f) For maximum torque, slip $s = 0.1$

Slip, $$s = \left(\frac{n_s - n_r}{n_s}\right)$$

i.e.

$$0.1 = \left(\frac{25 - n_s}{25}\right)$$

Hence $(0.1)(25) = 25 - n_r$ and
$n_r = 25 - (0.1)(25)$

Thus speed at which maximum torque occurs, $n_r = 25 - 2.5 =$ **22.5 rev/s** or **1350 rev/min**

(g) At the start, i.e. at standstill, slip $s = 1$. Hence,

$$\text{starting torque} = \left(\frac{m\left(\frac{N_2}{N_1}\right)^2}{2\pi n_s}\right)\left(\frac{E_1^2 R_2}{R_2^2 + X_2^2}\right)$$

from Equation (11) with $s = 1$

$$= (0.01380)\left(\frac{(239.6)^2 0.35}{0.35^2 + 3.5^2}\right)$$

$$= (0.01380)\left(\frac{20\,092.86}{12.3725}\right)$$

i.e. **starting torque = 22.41 Nm**

(Note that the full load torque (from part (c)) is 78.05 Nm but the starting torque is only 22.41 Nm)

Problem 11. Determine for the induction motor in Problem 10 at full load, (a) the rotor current, (b) the rotor copper loss, and (c) the starting current.

(a) From Equation (8), **rotor current**,

$$\mathbf{I_r} = \frac{s\left(\dfrac{N_2}{N_1}\right)E_1}{\sqrt{R_2^2 + (sX_2)^2}} = \frac{(0.04)(0.85)(239.6)}{\sqrt{0.35^2 + (0.04 \times 3.5)^2}} = \frac{8.1464}{0.37696} = \mathbf{21.61\,A}$$

(b) Rotor copper

$$\text{loss per phase} = I_r^2 R_2 = (21.61)^2(0.35) = 163.45\,\text{W}$$

Total copper loss (for 3 phases)

$$= 3 \times 163.45 = \mathbf{490.35\,W}$$

(c) From Equation (7), starting current,

$$I_2 = \frac{\left(\dfrac{N_2}{N_1}\right)E_1}{\sqrt{R_2^2 + X_2^2}} = \frac{(0.85)(239.5)}{\sqrt{0.35^2 + 3.5^2}} = \mathbf{57.90\,A}$$

(Note that the starting current of 57.90 A is considerably higher than the full load current of 21.61 A)

Problem 12. For the induction motor in Problems 10 and 11, if the stator losses are 650 W, determine (a) the power input at full load, (b) the efficiency of the motor at full load and (c) the current taken from the supply at full load, if the motor runs at a power factor of 0.87 lagging.

(a) Output power $P_m = 11.770$ kW from part (d), Problem 10. Rotor copper loss = 490.35 W = 0.49035 kW from part (b), Problem 11.
Stator input power,

$$P_1 = P_m + \text{rotor copper loss} + \text{rotor stator loss}$$
$$= 11.770 + 0.49035 + 0.650$$
$$= \mathbf{12.91\,kW}$$

(b) Net power output = 11 kW from part (d), Problem 10. Hence efficiency,

$$\eta = \frac{\text{output}}{\text{input}} \times 100\% = \left(\frac{11}{12.91}\right) \times 100\% = \mathbf{85.21\%}$$

(c) Power input, $P_1 = \sqrt{3}\,V_L I_L \cos\phi$ (see Chapter 20) and $\cos\phi = \text{p.f.} = 0.87$ hence, **supply current**,

$$\boldsymbol{I_L} = \frac{P_1}{\sqrt{3}\,V_L \cos\phi} = \frac{12.91 \times 1000}{\sqrt{3}(415)0.87} = \mathbf{20.64\,A}$$

Problem 13. For the induction motor of Problems 10 to 12, determine the resistance of the rotor winding required for maximum starting torque.

From Equation (4), rotor reactance $X_r = sX_2$. At the moment of starting, slip, $s = 1$. Maximum torque occurs when rotor reactance equals rotor resistance hence for **maximum torque**,
$\boldsymbol{R_2} = X_r = sX_2 = X_2 = \mathbf{3.5\,\Omega}$.

Thus if the induction motor was a wound rotor type with slip rings then an external star-connected resistance of $(3.5 - 0.35)\,\Omega = 3.15\,\Omega$ per phase could be added to the rotor resistance to give maximum torque at starting (see Section 23.13).

Now try the following exercise

Exercise 143 Further problems on the torque equation

1. A 400 V, three-phase, 50 Hz, 2-pole, star-connected induction motor runs at 48.5 rev/s on full load. The rotor resistance and reactance per phase are 0.4 Ω and 4.0 Ω respectively, and the effective rotor-stator turns ratio is 0.8:1. Calculate
   (a) the synchronous speed,
   (b) the slip,
   (c) the full load torque,
   (d) the power output if mechanical losses amount to 500 W,
   (e) the maximum torque,
   (f) the speed at which maximum torque occurs, and
   (g) the starting torque.
   [(a) 50 rev/s or 3000 rev/min (b) 0.03 or 3% (c) 22.43 Nm (d) 6.34 kW (e) 40.74 Nm (f) 45 rev/s or 2700 rev/min (g) 8.07 Nm]

2. For the induction motor in Problem 1, calculate at full load
   (a) the rotor current,
   (b) the rotor copper loss, and
   (c) the starting current.
   [(a) 13.27 A (b) 211.3 W (c) 45.96 A]

3. If the stator losses for the induction motor in Problem 1 are 525 W, calculate at full load
   (a) the power input,
   (b) the efficiency of the motor and
   (c) the current taken from the supply if the motor runs at a power factor of 0.84
   [(a) 7.57 kW (b) 83.75% (c) 13.0 A]

4. For the induction motor in Problem 1, determine the resistance of the rotor winding required for maximum starting torque
   [4.0 Ω]

23.12 Induction motor torque-speed characteristics

From Problem 10, parts (c) and (g), it is seen that the normal starting torque may be less than the full load torque. Also, from Problem 10, parts (e) and (f), it is seen that the speed at which maximum torque occurs is determined by the value of the rotor resistance. At synchronous speed, slip $s = 0$ and torque is zero. From these observations, the torque-speed and torque-slip characteristics of an induction motor are as shown in Fig. 23.9.

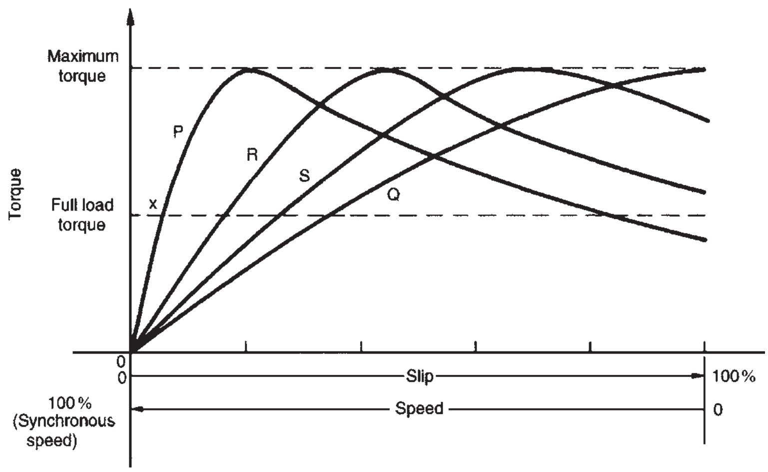


Figure 23.9

The rotor resistance of an induction motor is usually small compared with its reactance (for example, $R_2 = 0.35\ \Omega$ and $X_2 = 3.5\ \Omega$ in the above Problems), so that maximum torque occurs at a high speed, typically about 80 per cent of synchronous speed.

Curve P in Fig. 23.9 is a typical characteristic for an induction motor. The curve P cuts the full-load torque line at point X, showing that at full load the slip is about 4–5 per cent. The normal operating conditions are between 0 and X, thus it can be seen that for normal operation the speed variation with load is quite small — the induction motor is an almost constant-speed machine. Redrawing the speed-torque characteristic between 0 and X gives the characteristic shown in Fig. 23.10, which is similar to a d.c. shunt motor as shown in Chapter 22.

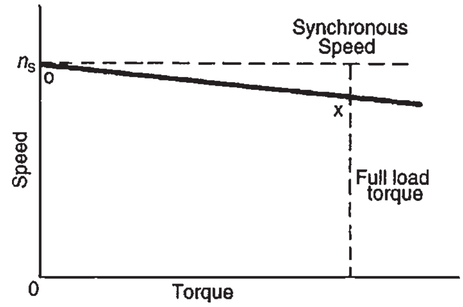


Figure 23.10

If maximum torque is required at starting then a high resistance rotor is necessary, which gives characteristic Q in Fig. 23.9. However, as can be seen, the motor has a full load slip of over 30 per cent, which results in a drop in efficiency. Also such a motor has a large speed variation with variations of load. Curves R and S of Fig. 23.9 are characteristics for values of rotor resistance's between those of P and Q. Better starting torque than for curve P is obtained, but with lower efficiency and with speed variations under operating conditions.

A **squirrel-cage induction motor** would normally follow characteristic P. This type of machine is highly efficient and about constant-speed under normal running conditions. However it has a poor starting torque and must be started off-load or very lightly loaded (see Section 23.13 below). Also, on starting, the current can be four or five times the normal full load current, due to the motor acting like a transformer with secondary short circuited. In Problem 11, for example, the current at starting was nearly three times the full load current.

A **wound-rotor induction motor** would follow characteristic P when the slip-rings are short-circuited, which is the normal running condition. However, the slip-rings allow for the addition of resistance to the rotor circuit externally and, as a result, for starting, the motor can have a characteristic similar to curve Q in Fig. 23.9 and the high starting current experienced by the cage induction motor can be overcome.

In general, for three-phase induction motors, the power factor is usually between about 0.8 and 0.9 lagging, and the full load efficiency is usually about 80–90 per cent.

From Equation (12), it is seen that torque is proportional to the square of the supply voltage. Any voltage variations therefore would seriously affect the induction motor performance.

23.13 Starting methods for induction motors

Squirrel-cage rotor

(i) **Direct-on-line starting**
With this method, starting current is high and may cause interference with supplies to other consumers.

(ii) **Auto transformer starting**
With this method, an auto transformer is used to reduce the stator voltage, E_1, and thus the starting current (see Equation (7)). However, the starting torque is seriously reduced (see Equation (12)), so the voltage is reduced only sufficiently to give the required reduction of the starting current. A typical arrangement is shown in Fig. 23.11. A double-throw switch connects the auto transformer in circuit for starting, and when the motor is up to speed the switch is moved to the run position which connects the supply directly to the motor.

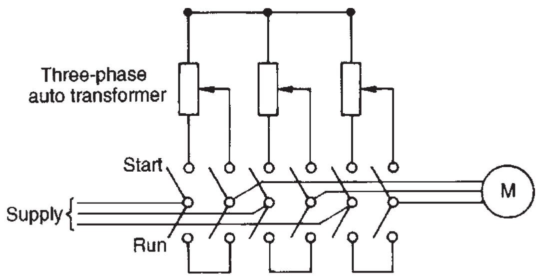


Figure 23.11

(iii) **Star-delta starting**
With this method, for starting, the connections to the stator phase winding are star-connected, so that the voltage across each phase winding is $(1/\sqrt{3})$ (i.e. 0.577) of the line voltage. For running, the windings are switched to delta-connection. A typical arrangement is shown in Fig. 23.12 This method of starting is less expensive than by auto transformer.

Wound rotor

When starting on load is necessary, a wound rotor induction motor must be used. This is because maximum torque at starting can be obtained by adding external resistance to the rotor circuit via slip rings, (see Problem 13). A face-plate type starter is used, and as the resistance is gradually reduced, the machine characteristics at each stage will be similar to Q, S, R and P of Fig. 23.13. At each resistance step, the motor operation will transfer from one characteristic to the next so that the overall starting characteristic will be as shown by the bold line in Fig. 23.13. For very large induction motors, very gradual and smooth starting is achieved by a liquid type resistance.

23.14 Advantages of squirrel-cage induction motors

The advantages of squirrel-cage motors compared with the wound rotor type are that they:

(i) are cheaper and more robust

(ii) have slightly higher efficiency and power factor

(iii) are explosion-proof, since the risk of sparking is eliminated by the absence of slip rings and brushes.

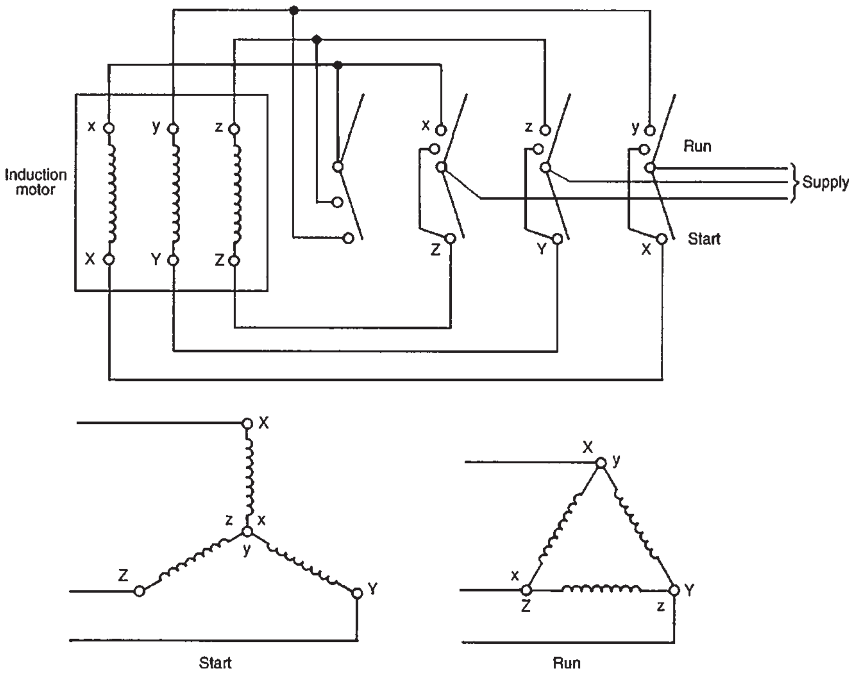


Figure 23.12

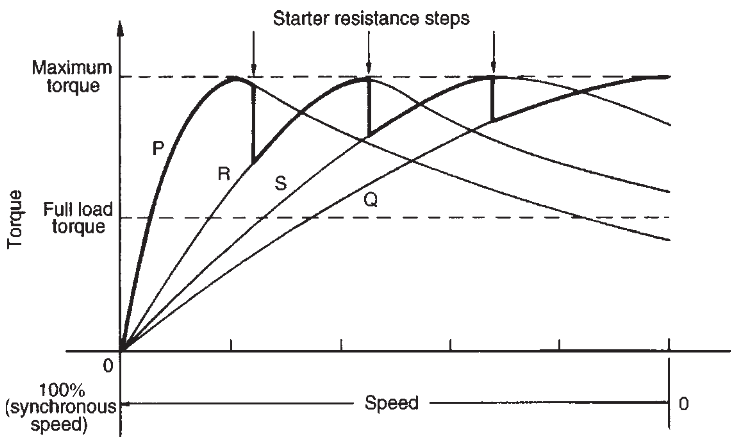


Figure 23.13

23.15 Advantages of wound rotor induction motors

The advantages of the wound rotor motor compared with the cage type are that they:

(i) have a much higher starting torque

(ii) have a much lower starting current

(iii) have a means of varying speed by use of external rotor resistance.

23.16 Double cage induction motor

The advantages of squirrel-cage and wound rotor induction motors are combined in the double cage induction motor. This type of induction motor is specially constructed with the rotor having two cages, one inside the other. The outer cage has high resistance conductors so that maximum torque is achieved at or near starting. The inner cage has normal low resistance copper conductors but high reactance since it is embedded deep in the iron core. The torque-speed characteristic of the inner cage is that of a normal induction motor, as shown in Fig. 23.14. At starting, the outer cage produces the torque, but when running the inner cage produces the torque. The combined characteristic of inner and outer cages is shown in Fig. 23.14 The double cage induction motor is highly efficient when running.

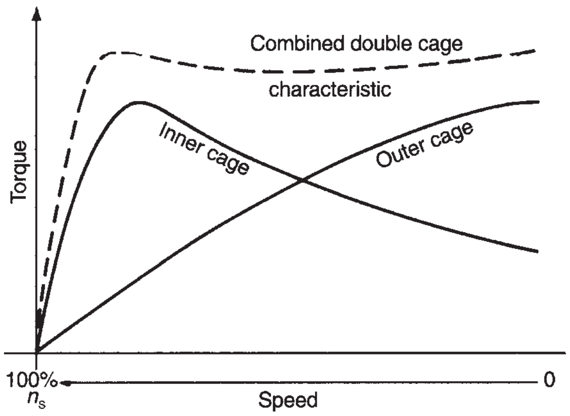


Figure 23.14

23.17 Uses of three-phase induction motors

Three-phase induction motors are widely used in industry and constitute almost all industrial drives where a nearly constant speed is required, from small workshops to the largest industrial enterprises.

Typical applications are with machine tools, pumps and mill motors. The squirrel cage rotor type is the most widely used of all a.c. motors.

Now try the following exercises

Exercise 144 Short answer questions on three-phase induction motors

1. Name three advantages that a three-phase induction motor has when compared with a d.c. motor
2. Name the principal disadvantage of a three-phase induction motor when compared with a d.c. motor
3. Explain briefly, with the aid of sketches, the principle of operation of a 3-phase induction motor
4. Explain briefly how slip-frequency currents are set up in the rotor bars of a 3-phase induction motor and why this frequency varies with load
5. Explain briefly why a 3-phase induction motor develops no torque when running at synchronous speed. Define the slip of an induction motor and explain why its value depends on the load on the rotor.
6. Write down the two properties of the magnetic field produced by the stator of a three-phase induction motor
7. The speed at which the magnetic field of a three-phase induction motor rotates is called the . . . . . . speed
8. The synchronous speed of a three-phase induction motor is . . . . . . proportional to supply frequency
9. The synchronous speed of a three-phase induction motor is . . . . . . proportional to the number of pairs of poles
10. The type of rotor most widely used in a three-phase induction motor is called a . . . . . .
11. The slip of a three-phase induction motor is given by: $s = \dfrac{\dots\dots}{\dots} \times 100\%$
12. A typical value for the slip of a small three-phase induction motor is . . . %
13. As the load on the rotor of a three-phase induction motor increases, the slip . . . . . .
14. $\dfrac{\text{Rotor copper loss}}{\text{Rotor input power}} = \dots\dots$
15. State the losses in an induction motor
16. Maximum torque occurs when . . . . . . = ...
17. Sketch a typical speed-torque characteristic for an induction motor
18. State two methods of starting squirrel-cage induction motors
19. Which type of induction motor is used when starting on-load is necessary?
20. Describe briefly a double cage induction motor

21. State two advantages of cage rotor machines compared with wound rotor machines
22. State two advantages of wound rotor machines compared with cage rotor machines
23. Name any three applications of three-phase induction motors

Exercise 145 Multi-choice questions on three-phase induction motors

(Answers on page 399)

1. Which of the following statements about a three-phase squirrel-cage induction motor is false?
   (a) It has no external electrical connections to its rotor
   (b) A three-phase supply is connected to its stator
   (c) A magnetic flux which alternates is produced
   (d) It is cheap, robust and requires little or no skilled maintenance

2. Which of the following statements about a three-phase induction motor is false?
   (a) The speed of rotation of the magnetic field is called the synchronous speed
   (b) A three-phase supply connected to the rotor produces a rotating magnetic field
   (c) The rotating magnetic field has a constant speed and constant magnitude
   (d) It is essentially a constant speed type machine

3. Which of the following statements is false when referring to a three-phase induction motor?
   (a) The synchronous speed is half the supply frequency when it has four poles
   (b) In a 2-pole machine, the synchronous speed is equal to the supply frequency
   (c) If the number of poles is increased, the synchronous speed is reduced
   (d) The synchronous speed is inversely proportional to the number of poles

4. A 4-pole three-phase induction motor has a synchronous speed of 25 rev/s. The frequency of the supply to the stator is:
   (a) 50 Hz (b) 100 Hz
   (c) 25 Hz (d) 12.5 Hz

   Questions 5 and 6 refer to a three-phase induction motor. Which statements are false?

5. (a) The slip speed is the synchronous speed minus the rotor speed
   (b) As the rotor is loaded, the slip decreases
   (c) The frequency of induced rotor e.m.f.'s increases with load on the rotor
   (d) The torque on the rotor is due to the interaction of magnetic fields

6. (a) If the rotor is running at synchronous speed, there is no torque on the rotor
   (b) If the number of poles on the stator is doubled, the synchronous speed is halved
   (c) At no-load, the rotor speed is very nearly equal to the synchronous speed
   (d) The direction of rotation of the rotor is opposite to the direction of rotation of the magnetic field to give maximum current induced in the rotor bars

   A three-phase, 4-pole, 50 Hz induction motor runs at 1440 rev/min. In questions 7 to 10, determine the correct answers for the quantities stated, selecting your answer from the list given below:
   (a) 12.5 rev/s (b) 25 rev/s (c) 1 rev/s
   (d) 50 rev/s (e) 1% (f) 4%
   (g) 50% (h) 4 Hz (i) 50 Hz
   (j) 2 Hz

7. The synchronous speed
8. The slip speed
9. The percentage slip
10. The frequency of induced e.m.f.'s in the rotor
11. The slip speed of an induction motor may be defined as the:
    (a) number of pairs of poles ÷ frequency
    (b) rotor speed − synchronous speed
    (c) rotor speed + synchronous speed
    (d) synchronous speed − rotor speed

12. The slip speed of an induction motor depends upon:
    (a) armature current (b) supply voltage
    (c) mechanical load (d) eddy currents

13. The starting torque of a simple squirrel-cage motor is:
    (a) low
    (b) increases as rotor current rises
    (c) decreases as rotor current rises
    (d) high

14. The slip speed of an induction motor:
    (a) is zero until the rotor moves and then rises slightly
    (b) is 100 per cent until the rotor moves and then decreases slightly
    (c) is 100 per cent until the rotor moves and then falls to a low value
    (d) is zero until the rotor moves and then rises to 100 per cent

15. A four-pole induction motor when supplied from a 50 Hz supply experiences a 5 per cent slip. The rotor speed will be:
    (a) 25 rev/s (b) 23.75 rev/s
    (c) 26.25 rev/s (d) 11.875 rev/s

16. A stator winding of an induction motor supplied from a three-phase, 60 Hz system is required to produce a magnetic flux rotating at 900 rev/min. The number of poles is:
    (a) 2 (b) 8
    (c) 6 (d) 4

17. The stator of a three-phase, 2-pole induction motor is connected to a 50 Hz supply. The rotor runs at 2880 rev/min at full load. The slip is:
    (a) 4.17% (b) 92%
    (c) 4% (d) 96%

18. An 8-pole induction motor, when fed from a 60 Hz supply, experiences a 5 per cent slip. The rotor speed is:
    (a) 427.5 rev/min (b) 855 rev/min
    (c) 900 rev/min (d) 945 rev/min

Revision Test 7

This revision test covers the material contained in Chapters 22 and 23. *The marks for each question are shown in brackets at the end of each question.*

1. A 6-pole armature has 1000 conductors and a flux per pole of 40 mWb. Determine the e.m.f. generated when running at 600 rev/min when (a) lap wound (b) wave wound. (6)

2. The armature of a d.c. machine has a resistance of 0.3 Ω and is connected to a 200 V supply. Calculate the e.m.f. generated when it is running (a) as a generator giving 80 A (b) as a motor taking 80 A. (4)

3. A 15 kW shunt generator having an armature circuit resistance of 1 Ω and a field resistance of 160 Ω generates a terminal voltage of 240 V at full-load. Determine the efficiency of the generator at full-load assuming the iron, friction and windage losses amount to 544 W. (6)

4. A 4-pole d.c. motor has a wave-wound armature with 1000 conductors. The useful flux per pole is 40 mWb. Calculate the torque exerted when a current of 25 A flows in each armature conductor. (4)

5. A 400 V shunt motor runs at its normal speed of 20 rev/s when the armature current is 100 A. The armature resistance is 0.25 Ω. Calculate the speed, in rev/min when the current is 50 A and a resistance of 0.40 Ω is connected in series with the armature, the shunt field remaining constant. (7)

6. The stator of a three-phase, 6-pole induction motor is connected to a 60 Hz supply. The rotor runs at 1155 rev/min at full load. Determine (a) the synchronous speed, and (b) the slip at full load. (6)

7. The power supplied to a three-phase induction motor is 40 kW and the stator losses are 2 kW. If the slip is 4 per cent determine (a) the rotor copper loss, (b) the total mechanical power developed by the rotor, (c) the output power of the motor if frictional and windage losses are 1.48 kW, and (d) the efficiency of the motor, neglecting rotor iron loss. (9)

8. A 400 V, three-phase, 100 Hz, 8-pole induction motor runs at 24.25 rev/s on full load. The rotor resistance and reactance per phase are 0.2 Ω and 2 Ω respectively and the effective rotor-stator turns ratio is 0.80:1. Calculate (a) the synchronous speed, (b) the slip, and (c) the full load torque. (8)

Formulae for electrical power technology

Three-phase systems:

Star $I_L = I_p \quad V_L = \sqrt{3}\, V_p$

Delta $V_L = V_p \quad I_L = \sqrt{3}\, I_p$

$P = \sqrt{3}\, V_L I_L \cos\phi$ or $P = 3I_p^2 R_p$

Two-wattmeter method

$P = P_1 + P_2 \quad \tan\phi = \sqrt{3}\dfrac{(P_1 - P_2)}{(P_1 + P_2)}$

Transformers:

$\dfrac{V_1}{V_2} = \dfrac{N_1}{N_2} = \dfrac{I_2}{I_1} \quad I_0 = \sqrt{(I_M^2 + I_C^2)}$

$I_M = I_0 \sin\phi_0 \quad I_c = I_0 \cos\phi_0$

$E = 4.44 f \Phi_m N$

$\text{Regulation} = \left(\dfrac{E_2 - E_1}{E_2}\right) \times 100\%$

Equivalent circuit: $R_e = R_1 + R_2\left(\dfrac{V_1}{V_2}\right)^2$

$X_e = X_1 + X_2\left(\dfrac{V_1}{V_2}\right)^2 \quad Z_e = \sqrt{(R_e^2 + X_e^2)}$

Efficiency, $\eta = 1 - \dfrac{\text{losses}}{\text{input power}}$

Output power $= V_2 I_2 \cos\phi_2$

Total loss $=$ copper loss $+$ iron loss

Input power $=$ output power $+$ losses

Resistance matching: $R_1 = \left(\dfrac{N_1}{N_2}\right)^2 R_L$

D.C. Machines:

Generated e.m.f. $\text{E} = \dfrac{2p\Phi nZ}{c} \propto \Phi\omega$

($c = 2$ for wave winding, $c = 2p$ for lap winding)

Generator: $E = V + I_a R_a$

Efficiency, $\eta = \left(\dfrac{VI}{VI + I_a^2 R_a + I_f V + C}\right) \times 100\%$

Motor: $E = V - I_a R_a$

Efficiency, $\eta = \left(\dfrac{VI - I_a^2 R_a - I_f V - C}{VI}\right) \times 100\%$

$\text{Torque} = \dfrac{EI_a}{2\pi n} = \dfrac{p\Phi Z I_a}{\pi c} \propto I_a \Phi$

Three-phase induction motors:

$n_S = \dfrac{f}{p} \quad s = \left(\dfrac{n_s - n_r}{n_s}\right) \times 100$

$f_r = sf \quad X_r = sX_2$

$I_r = \dfrac{E_r}{Z_r} = \dfrac{s\left(\dfrac{N_2}{N_1}\right)E_1}{\sqrt{[R_2^2 + (sX_2)^2]}} \quad s = \dfrac{I_r^2 R_2}{P_2}$

Efficiency,

$$\eta = \frac{P_m}{P_1} = \frac{\text{input} - \text{stator loss} - \text{rotor copper loss} - \text{friction \& windage loss}}{\text{input power}}$$

Torque,

$$T = \left(\frac{m\left(\dfrac{N_2}{N_1}\right)^2}{2\pi n_s}\right)\left(\frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2}\right) \propto \frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2}$$

Answers to multiple choice questions

Chapter 1. Exercise 4 (page 7)

1 (c)	4 (a)	7 (b)	9 (d)	11 (b)
2 (d)	5 (c)	8 (c)	10 (a)	12 (d)
3 (c)	6 (b)			

Chapter 2. Exercise 10 (page 19)

1 (b)	4 (b)	7 (b)	10 (c)	12 (d)
2 (b)	5 (d)	8 (c)	11 (c)	13 (a)
3 (c)	6 (d)	9 (b)		

Chapter 3. Exercise 15 (page 27)

1 (c)	3 (b)	5 (d)	7 (b)	9 (d)
2 (d)	4 (d)	6 (c)	8 (c)	

Chapter 4. Exercise 18 (page 38)

1 (d)	4 (c)	7 (d)	10 (d)	12 (a)
2 (a)	5 (b)	8 (b)	11 (c)	13 (c)
3 (b)	6 (d)	9 (c)		

Chapter 5. Exercise 24 (page 53)

1 (a)	4 (c)	6 (b)	8 (b)	10 (d)
2 (c)	5 (a)	7 (d)	9 (c)	11 (d)
3 (c)				

Chapter 6. Exercise 31 (page 70)

1 (b)	4 (c)	6 (b)	8 (a)	10 (c)
2 (a)	5 (a)	7 (b)	9 (c)	11 (d)
3 (b)				

Chapter 7. Exercise 37 (page 82)

1 (d)	5 (c)	8 (c)	11 (a) and (d),	12 (a)
2 (b)	6 (d)	9 (c)	(b) and (f),	13 (a)
3 (b)	7 (a)	10 (c)	(c) and (e)	
4 (c)				

Chapter 8. Exercise 41 (page 94)

1 (d)	3 (d)	5 (b)	7 (d)	9 (a)
2 (c)	4 (a)	6 (c)	8 (a)	10 (b)

Chapter 9. Exercise 49 (page 108)

1 (c)	4 (b)	7 (c)	9 (c)	11 (a)
2 (b)	5 (c)	8 (d)	10 (a)	12 (b)
3 (c)	6 (a)			

Chapter 10. Exercise 59 (page 137)

1 (d)	7 (c)	13 (b)	19 (d)
2 (a) or (c)	8 (a)	14 (p)	20 (a)
3 (b)	9 (i)	15 (d)	21 (d)
4 (b)	10 (j)	16 (o)	22 (c)
5 (c)	11 (g)	17 (n)	23 (a)
6 (f)	12 (c)	18 (b)	

Chapter 11. Exercise 63 (page 152)

1 (c)	3 (d)	5 (b)	7 (c)	9 (a)
2 (a)	4 (c)	6 (b)	8 (d)	10 (b)

Chapter 12. Exercise 67 (page 172)

1 (b)	5 (a)	9 (b)	13 (b)	17 (c)
2 (b)	6 (d)	10 (c)	14 (b)	18 (b)
3 (c)	7 (b)	11 (a)	15 (b)	19 (a)
4 (a)	8 (d)	12 (b)	16 (b)	20 (b)

Chapter 13. Exercise 75 (page 202)

1 (d)	5 (a)	8 (a)	11 (b)	14 (b)
2 (c)	6 (d)	9 (c)	12 (d)	15 (c)
3 (b)	7 (c)	10 (c)	13 (d)	16 (a)
4 (c)				

Chapter 14. Exercise 81 (page 219)

1 (c)	4 (a)	7 (b)	9 (b)	11 (b)
2 (d)	5 (d)	8 (c)	10 (c)	12 (d)
3 (d)	6 (c)			

Chapter 15. Exercise 89 (page 241)

1 (c)	5 (a)	9 (d)	13 (b)	17 (c)
2 (a)	6 (b)	10 (d)	14 (c)	18 (a)
3 (b)	7 (a)	11 (b)	15 (b)	19 (d)
4 (b)	8 (d)	12 (c)	16 (b)	

Chapter 16. Exercise 97 (page 258)

1 (d)	5 (h)	9 (a)	12 (d)
2 (g)	6 (b)	10 (d), (g), (i) and (l)	13 (c)
3 (i)	7 (k)	11 (b)	14 (b)
4 (s)	8 (l)		

Chapter 17. Exercise 102 (page 270)

1 (d)	4 (c)	7 (b)	9(d)	11 (d)
2 (b)	5 (c)	8 (a)	10 (b)	12 (c)
3 (a)	6 (a)			

Chapter 18. Exercise 106 (page 287)

1 (c)	5 (g)	9 (a)	13 (c)	16 (c)
2 (b)	6 (e)	10 (d)	14 (j)	17 (a)
3 (b)	7 (l)	11 (g)	15 (h)	18 (a)
4 (g)	8 (c)	12 (b)		

Chapter 19. Exercise 110 (page 303)

1 (c)	3 (b)	5 (a)	7 (d)	9 (c)
2 (b)	4 (d)	6 (b)	8 (a)	10 (c)

Chapter 20. Exercise 116 (page 325)

1 (g)	5 (f)	9 (l)	12 (j)	15 (c)
2 (c)	6 (a)	10 (d)	13 (d)	16 (b)
3 (a)	7 (g)	11 (f)	14 (b)	17 (c)
4 (a)	8 (l)			

Chapter 21. Exercise 129 (page 349)

1 (a)	5 (c)	9 (b)	13 (h)	17 (c)
2 (d)	6 (a)	10 (g)	14 (k)	18 (b) and (c)
3 (a)	7 (b)	11 (d)	15 (j)	19 (c)
4 (b)	8 (a)	12 (a)	16 (f)	20 (b)

Chapter 22. Exercise 138 (page 375)

1 (b)	6 (a)	11 (b)	16 (b)	21 (b)
2 (e)	7 (d)	12 (a)	17 (b)	22 (a)
3 (e)	8 (f)	13 (b)	18 (b)	23 (c)
4 (c)	9 (b)	14 (a)	19 (c)	24 (d)
5 (c)	10 (c)	15 (d)	20 (b)	

Chapter 23. Exercise 145 (page 394)

1 (c)	5 (b)	9 (f)	13 (a)	16 (b)
2 (b)	6 (d)	10 (j)	14 (c)	17 (c)
3 (d)	7 (b)	11 (d)	15 (b)	18 (b)
4 (a)	8 (c)	12 (c)		

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Index