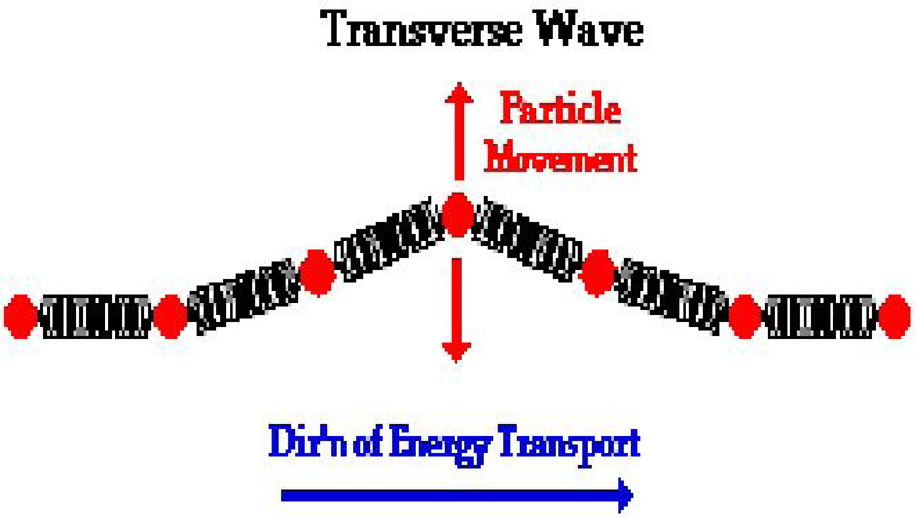
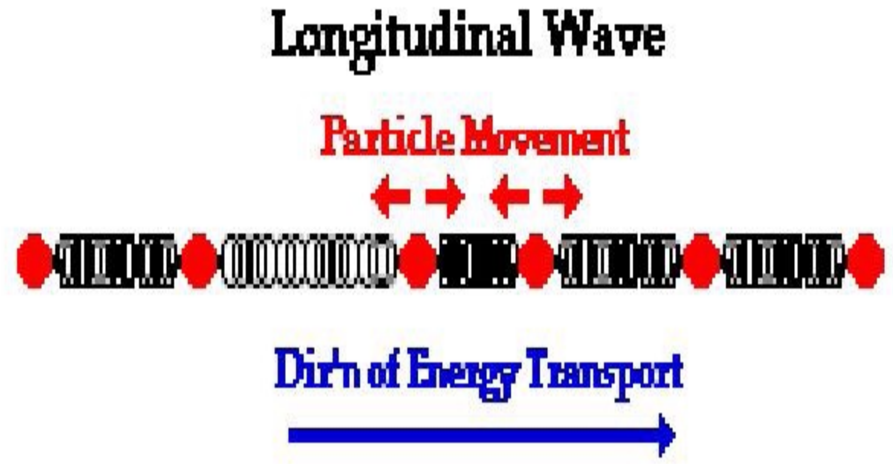
GENERAL PROPERTIES OF OCILLATIONS AND WAVES

Categories of Waves

Transverse Waves



Longitudinal Waves



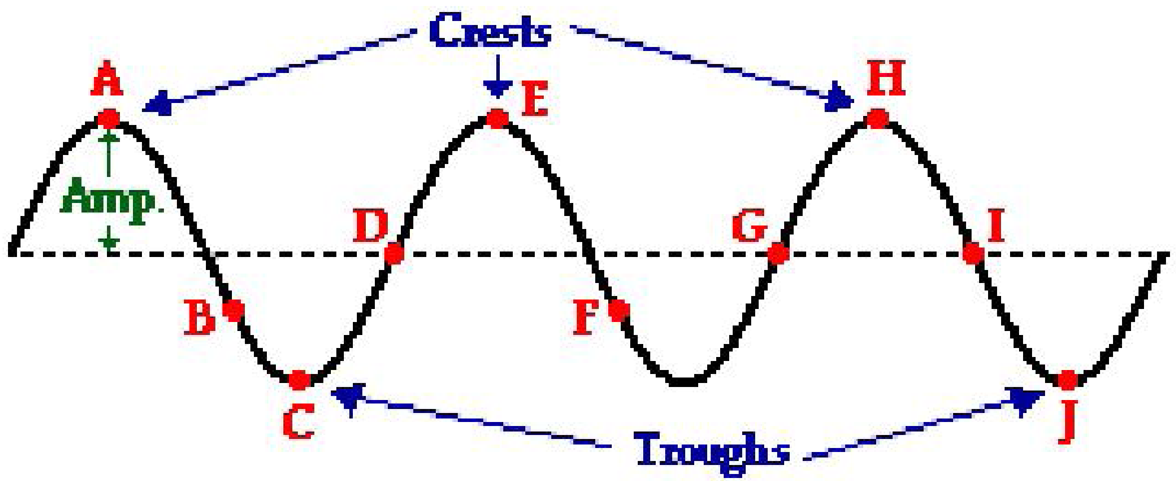
Categories of Waves

Surface Waves

Electromagnetic Waves

Mechanical Waves

Description of a Wave



Crest, Trough, Wavelength, Amplitude, Frequency, Period, Intensity, Phase Angle, Phase Velocity

General Wave Equation

$$\frac{\partial^2 \psi}{\partial t^2} = v \frac{\partial^2 \psi}{\partial x^2}$$

$\psi(r,t)$ is the wave amplitude, v and the characteristic phase velocity.

Equation of a travelling wave

$$\psi(x,t) = A\cos(kx - \omega t - \phi)$$

Superposition of Waves

- The principle of superposition may be applied to waves whenever two (or more) waves travelling through the same medium at the same time.

- The waves pass through each other without being disturbed. The net displacement of the medium at any point in space or time is simply the sum of the individual wave displacements.

Two sine waves travelling in the same direction: Constructive and Destructive Interference

Consider two similar travelling waves in the same direction. Principle of superposition gives

$$y(x,t) = A\sin(kx-\omega t) + A\sin(kx-\omega t+\phi)$$

and using the trigonometrically identity

$$\sin(A) + \sin(B) = 2\cos\left(\frac{B-A}{2}\right)\sin\left(\frac{A+B}{2}\right)$$

gives

$$2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

- The superposition of two or more waves is called **interference**

Constructive interference:

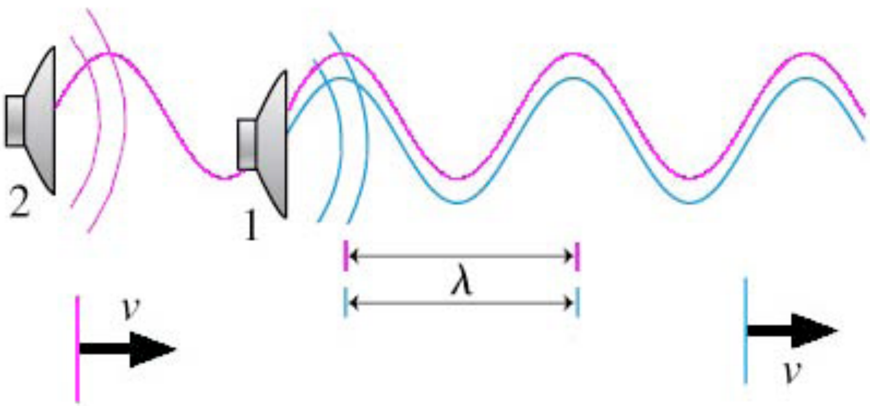
These two waves are in phase.

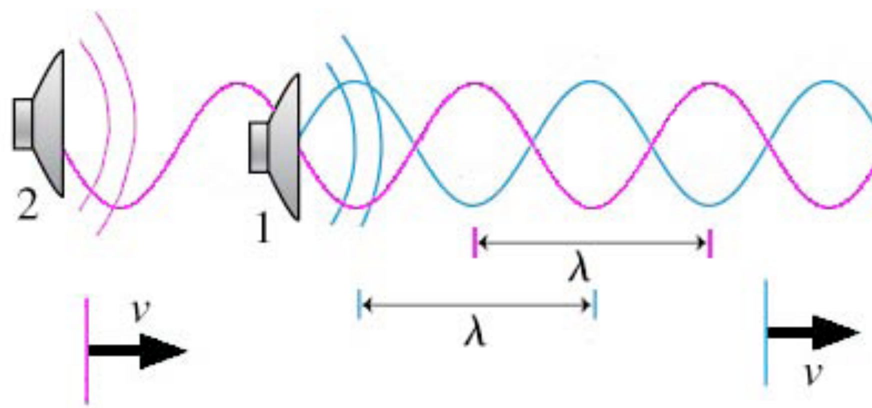
Their crests are aligned.

Destructive interference:

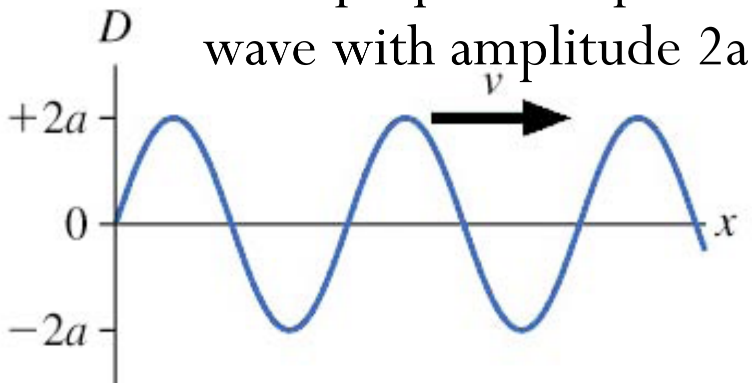
These two waves are out of phase.

The crests of one are aligned with the troughs of the other.

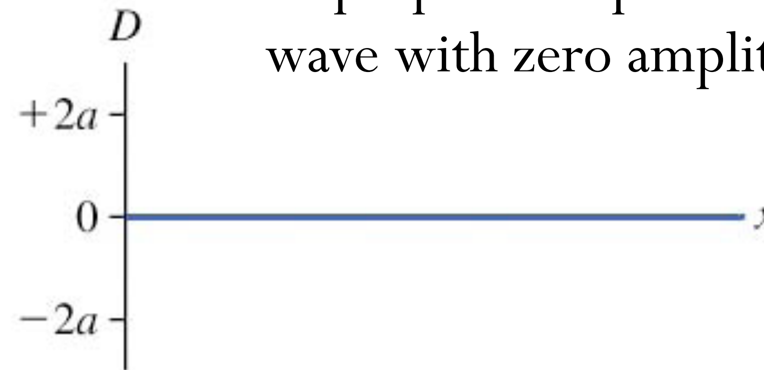




Their superposition produces a wave with amplitude 2a



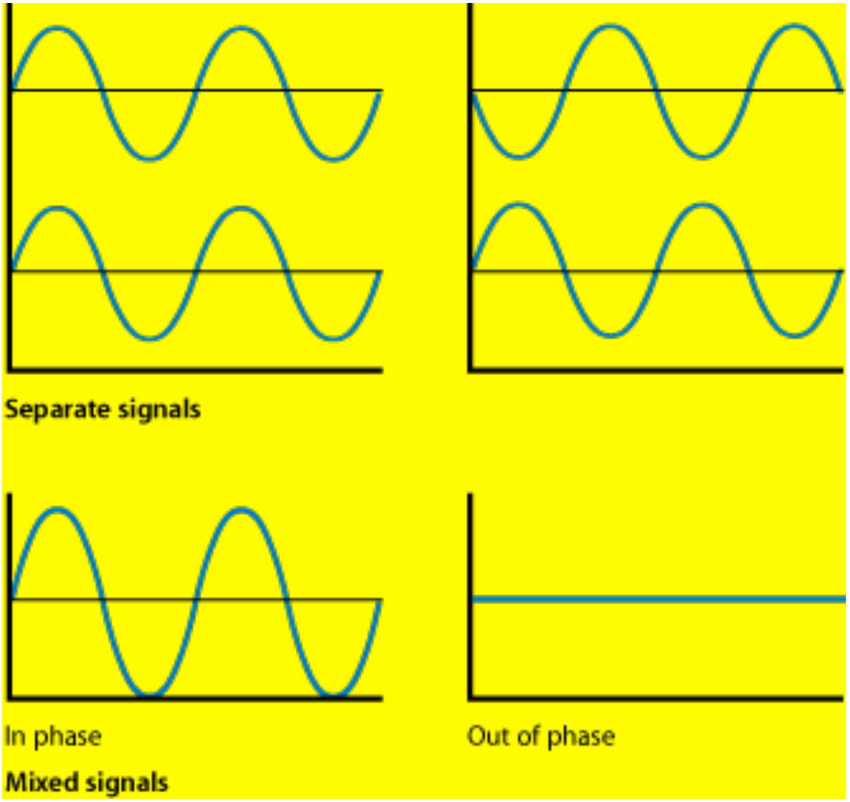
Their superposition produces a wave with zero amplitude

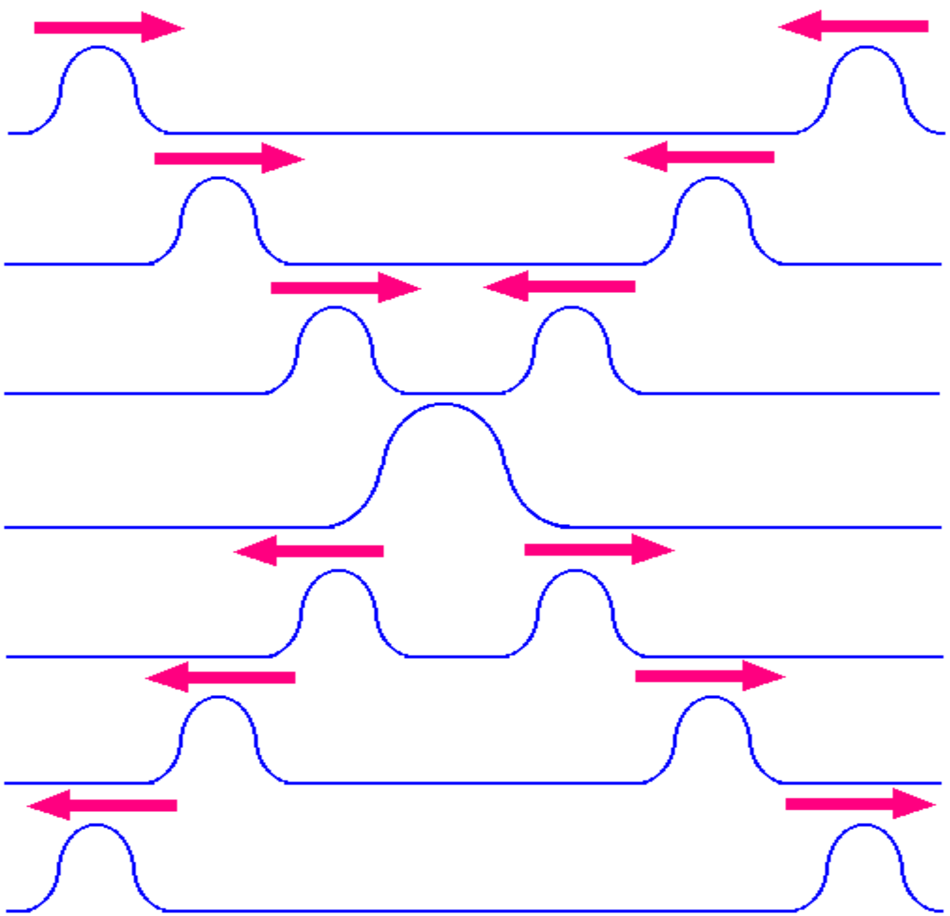


Phase – The term phase is used to describe the relative position of the wave crests as they meet.

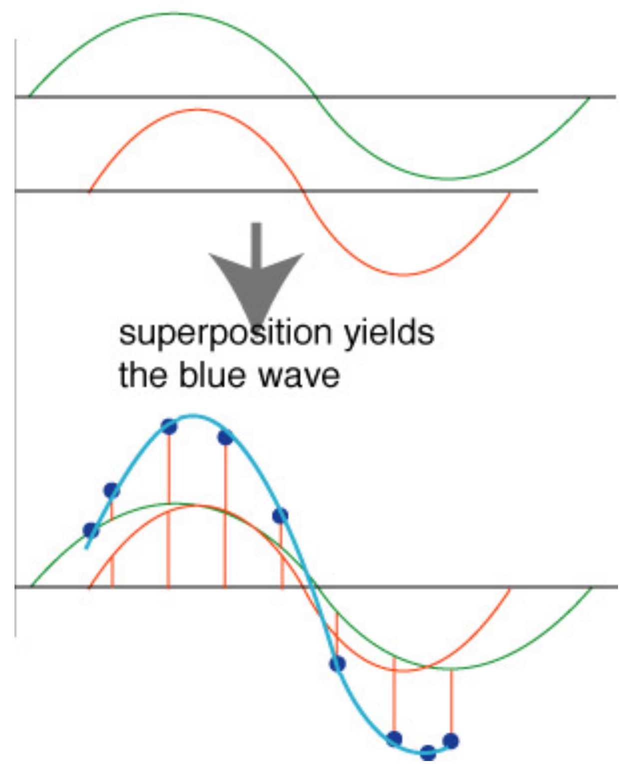
a. In phase – results in an amplitude that is the sum of both crests.

b. Out of phase – occurs when the crests do not meet together and the waves partially or completely cancel each other. This complete cancellation occurs when a trough of one wave meets the crest of another.





Superposition of Two Waves



Two sine waves travelling in opposite directions create a standing wave

Consider two oppositelly travelling waves

$$\psi(x,t) = A\sin(kx - \omega t)$$

$$\psi(x,t) = A\sin(kx + \omega t)$$

Adding these gives

$$\psi(x,t) = A[\sin(kx - \omega t) + \sin(kx + \omega t)]$$

$$\psi(x,t) = A[\sin kx\cos\omega t - \cos kx\sin\omega t + \sin kx\cos\omega t + \cos kx\sin\omega t]$$

$$\psi(x,t) = A[\sin kx\cos\omega t + \sin kx\cos\omega t]$$

$$\psi(x,t) = 2A\sin kx\cos\omega t$$

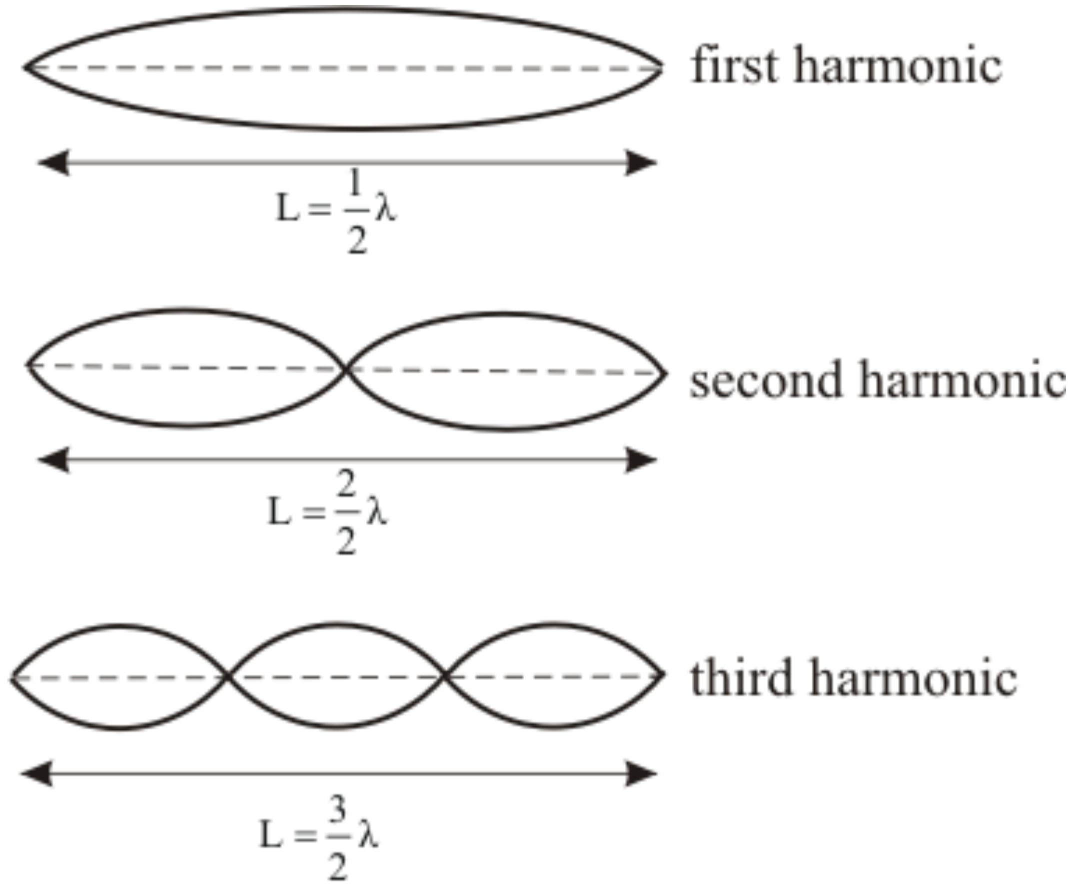
Formation of Standing waves

A standing wave pattern is a vibrational pattern created within a medium when the vibrational frequency of the source causes reflected waves from one end of the medium to interfere with incident waves from the source.

This interference occurs in such a manner that specific points along the medium appear to be standing still.

A careful study of the standing wave patterns reveal a clear mathematical relationship between the wavelength of the wave which produces the pattern and the length of the medium in which the pattern is displayed.

Formation of Standing Waves



For the nth harmonic $L = \dfrac{n}{2}\lambda$

Two sine (or cosine) waves with different frequencies: Beats

Consider two sine waves

$$y(x,t) = A\sin(k_1 x - \omega_1 t)$$

$$y(x,t) = A\sin(k_2 x - \omega_2 t)$$

Application of Principle of Superposition gives

$$y(x,t) = A\sin(k_1 x - \omega_1 t) + A\sin(k_2 x - \omega_2 t)$$

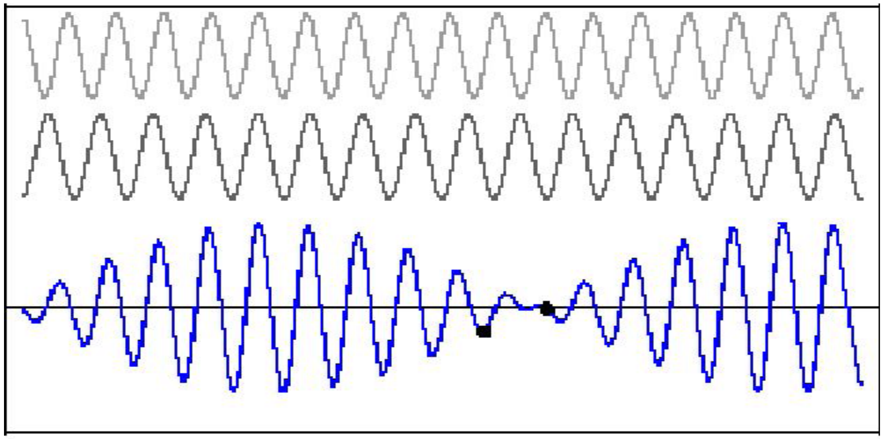
$$= \quad 2A\cos\left[\frac{(k_1 - k_2)}{2}x - \frac{(\omega_1 - \omega_2)}{2}t\right]\sin\left[\frac{k_1 + k_2}{2}x - \frac{(\omega_1 + \omega_2)}{2}t\right]$$

Beat Frequency

$$f = 1/2\left(f_1 - f_2\right)$$

Beat

$$f_{beat} = \left(f_1 - f_2\right)$$



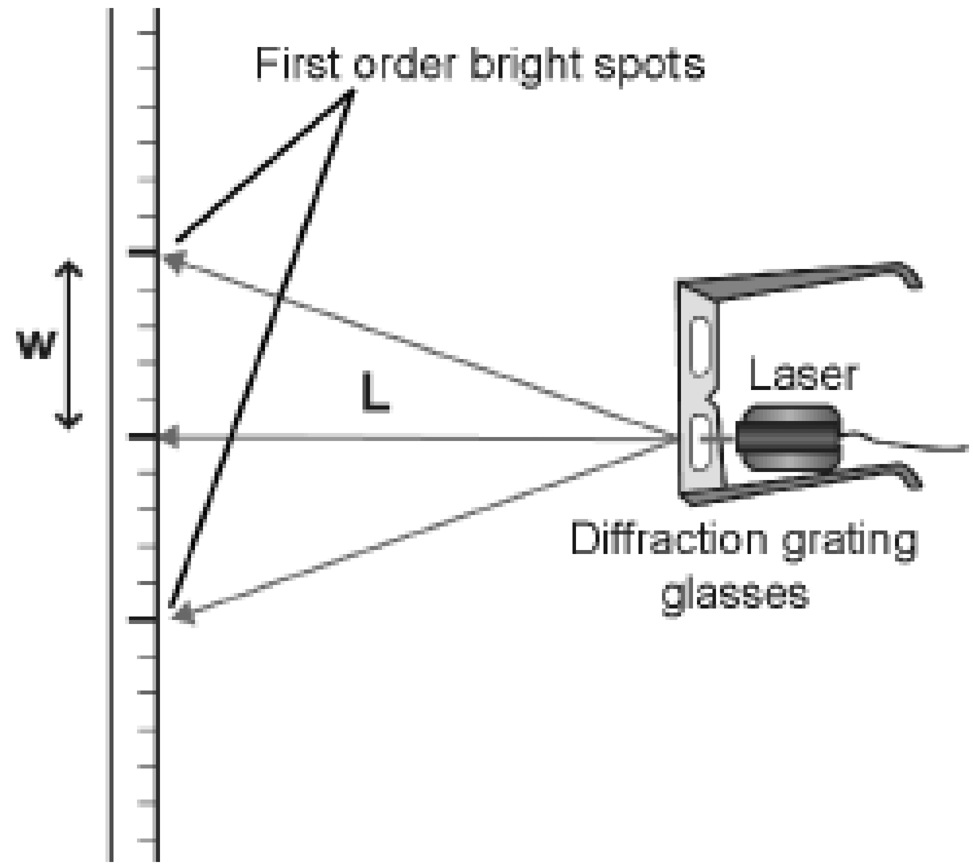
- The **group velocity** of a wave is the velocity with which the overall shape of the wave's amplitudes —known as the *modulation* or *envelope* of the wave —propagates through space.
- The group velocity v_g is defined by the equation

$$v_g = \frac{\partial \omega}{\partial k}$$

Interference, Diffraction, and Polarization

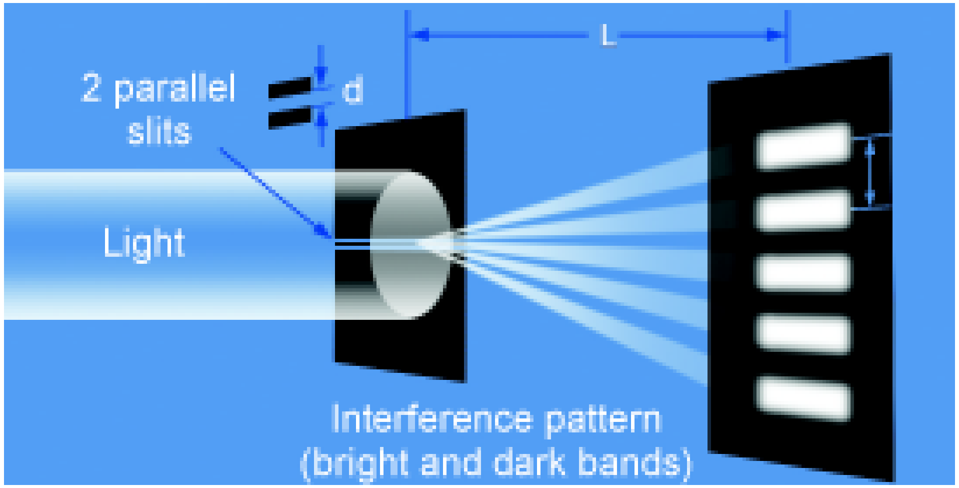
Key Question:

What are some ways light behaves like a wave?



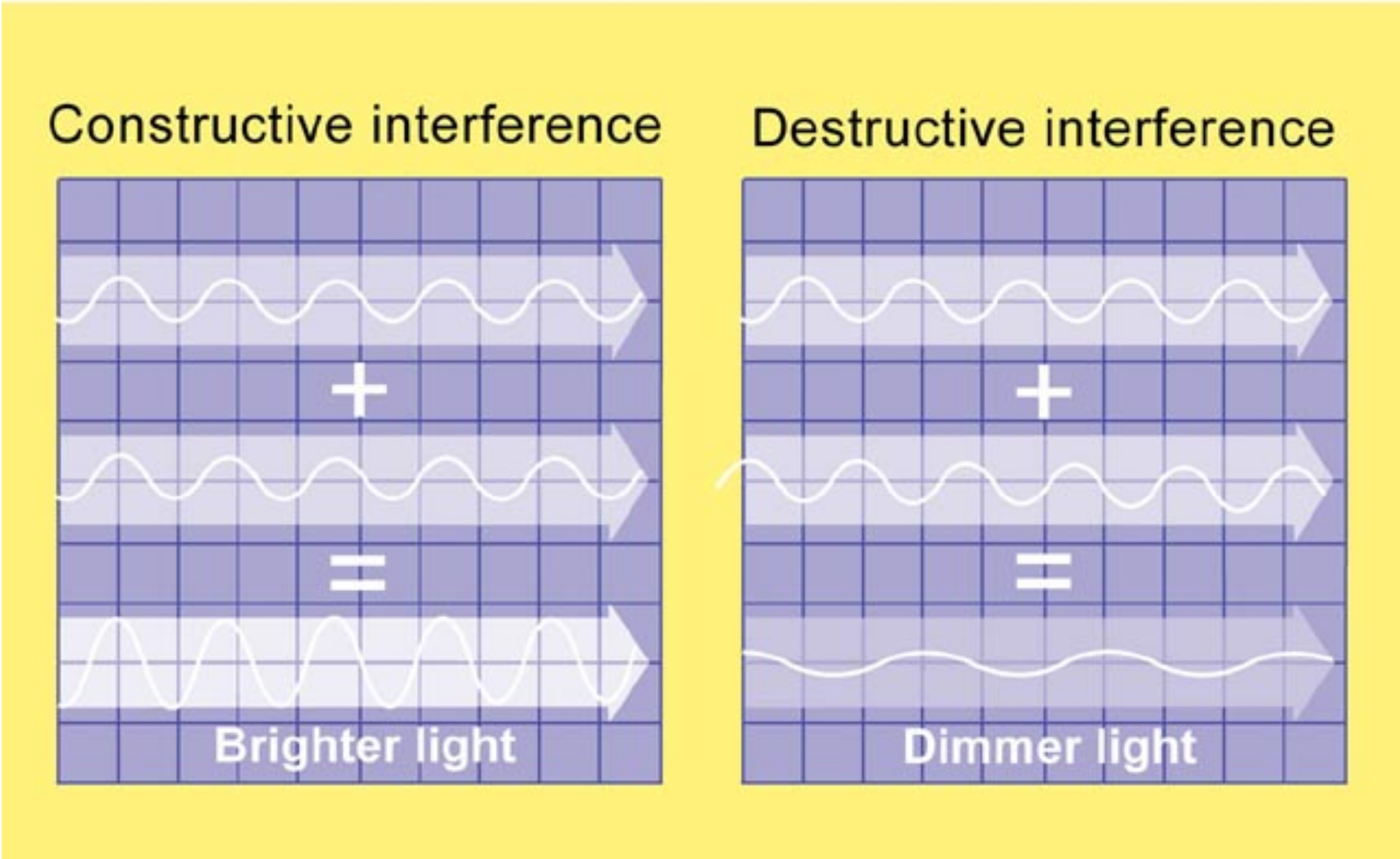
Interference, Diffraction, and Polarization

- In 1807, Thomas Young (1773-1829) did the most convincing experiment demonstrating that light is a wave.
- A beam of light fell on a pair of parallel, very thin slits in a piece of metal.
- After passing through the slits, the light fell on a screen.

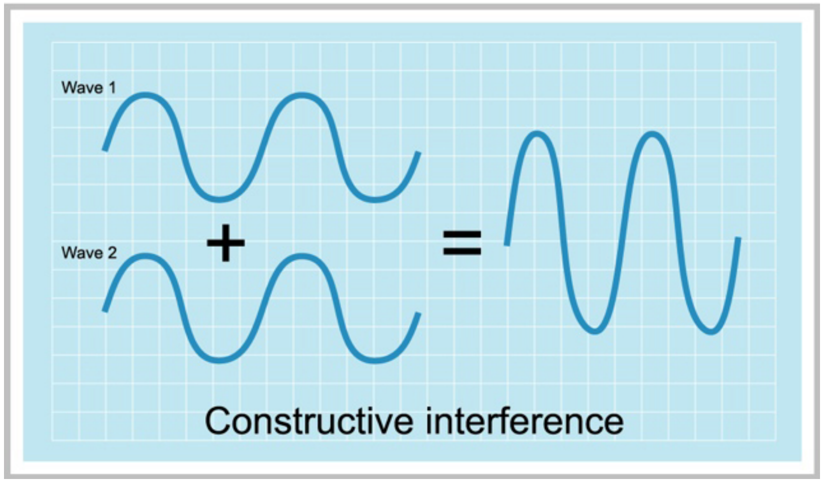


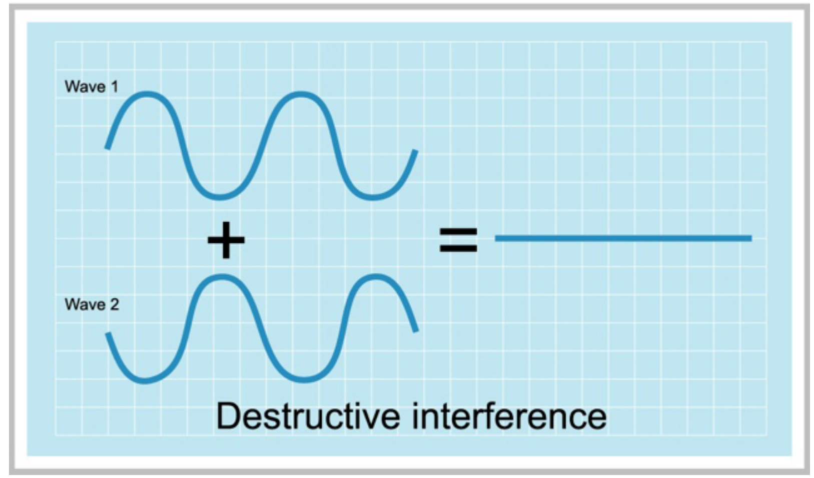
- A pattern of alternating bright and dark bands formed is called an interference pattern.

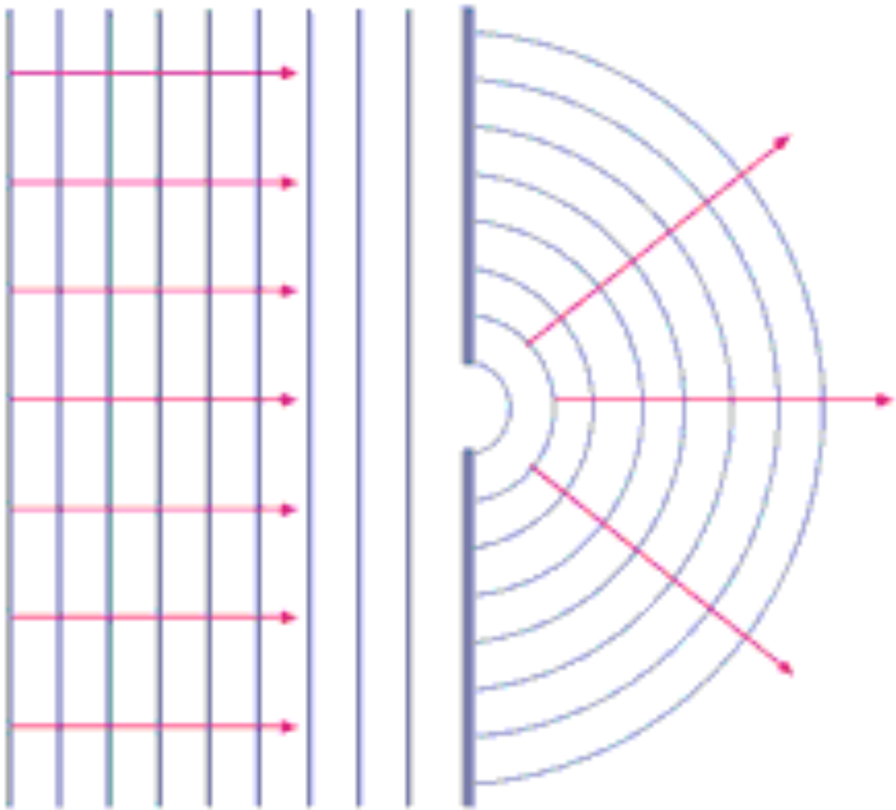
The Interference of Light Waves



Interference





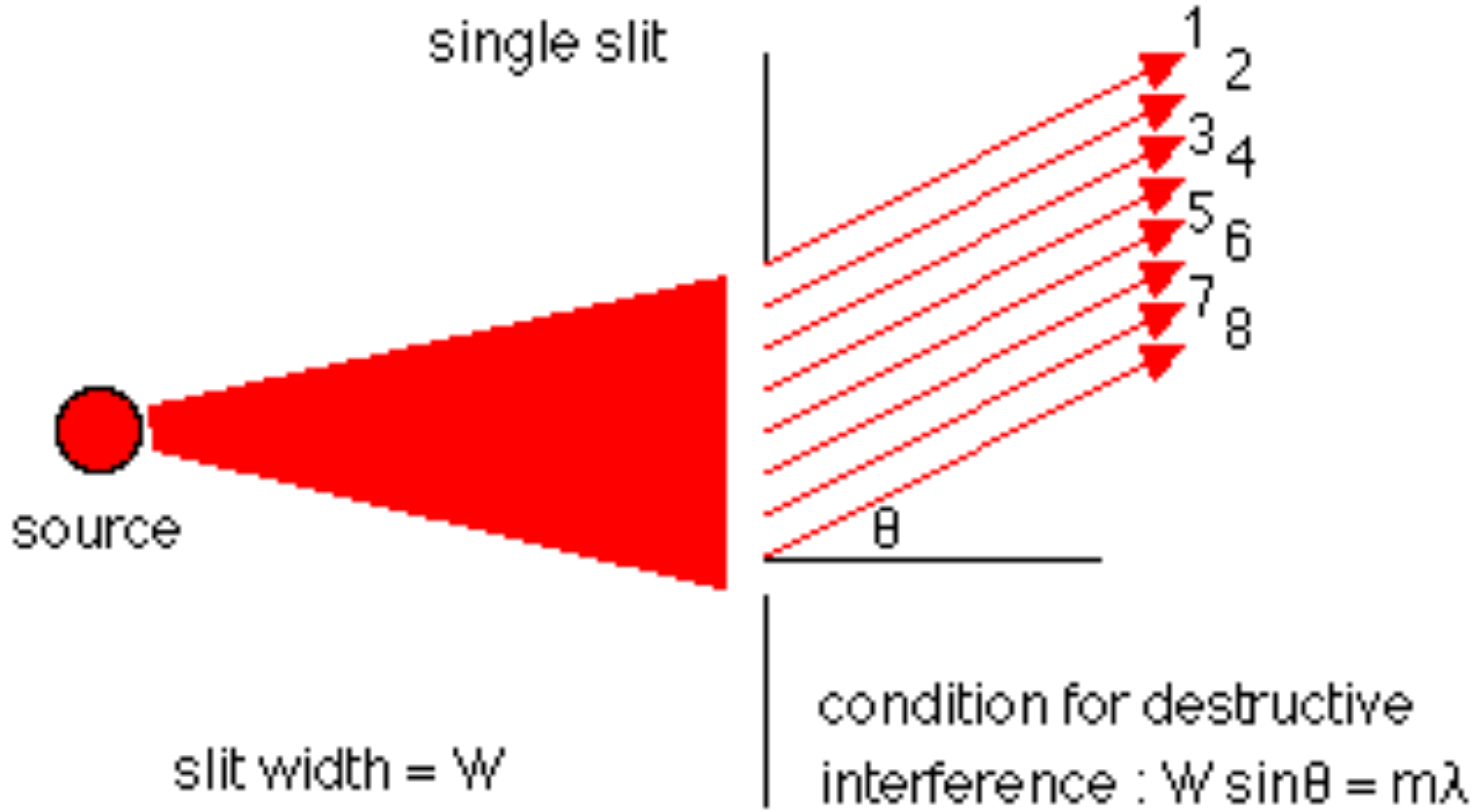


One hole

diffraction

- Diffraction is the bending of waves that occurs when a wave passes through a narrow opening

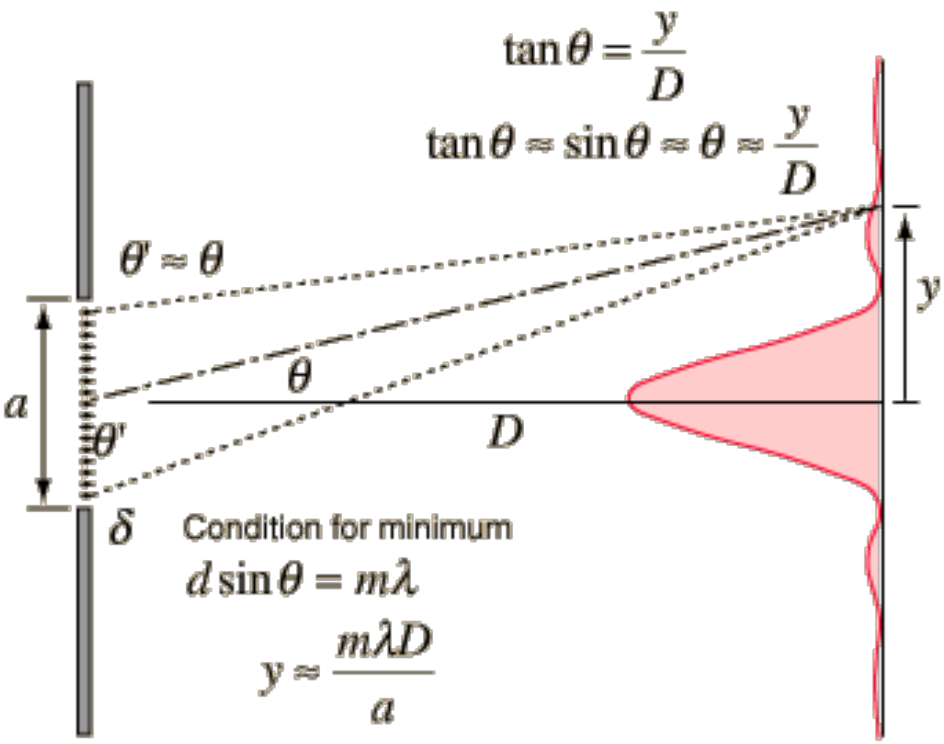
- If a wave falls on a barrier that has an opening of dimension similar to the wavelength, the wave will flair into the region beyond

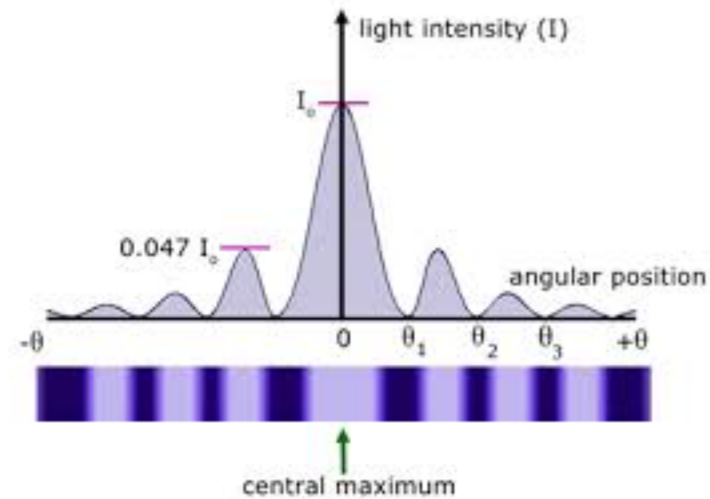


Dark fringes occur at :

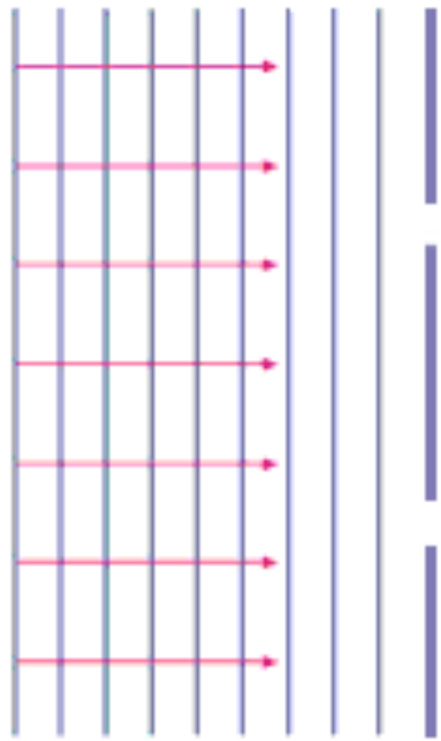
$$w \sin \theta = m\lambda$$

Diffraction by a single slit

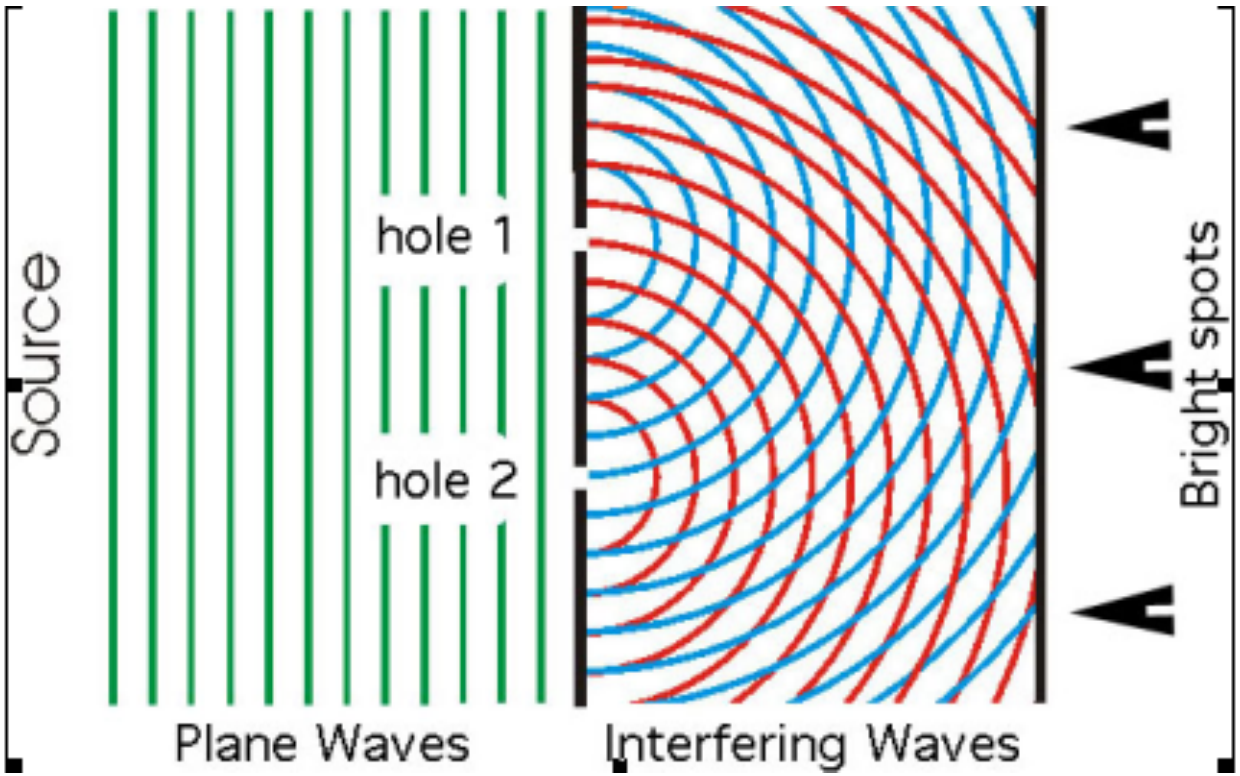




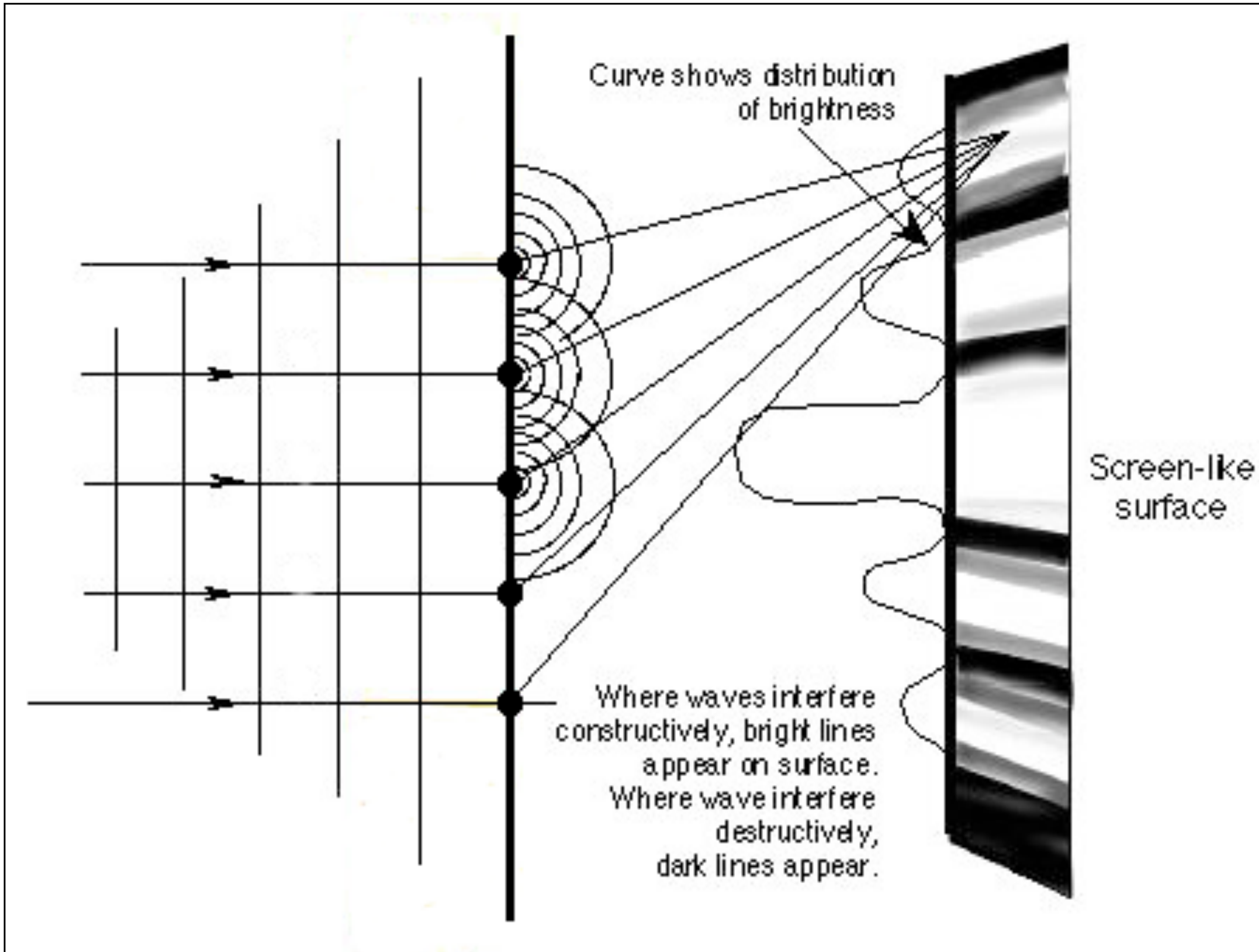
Dark fringes
Bright fringes

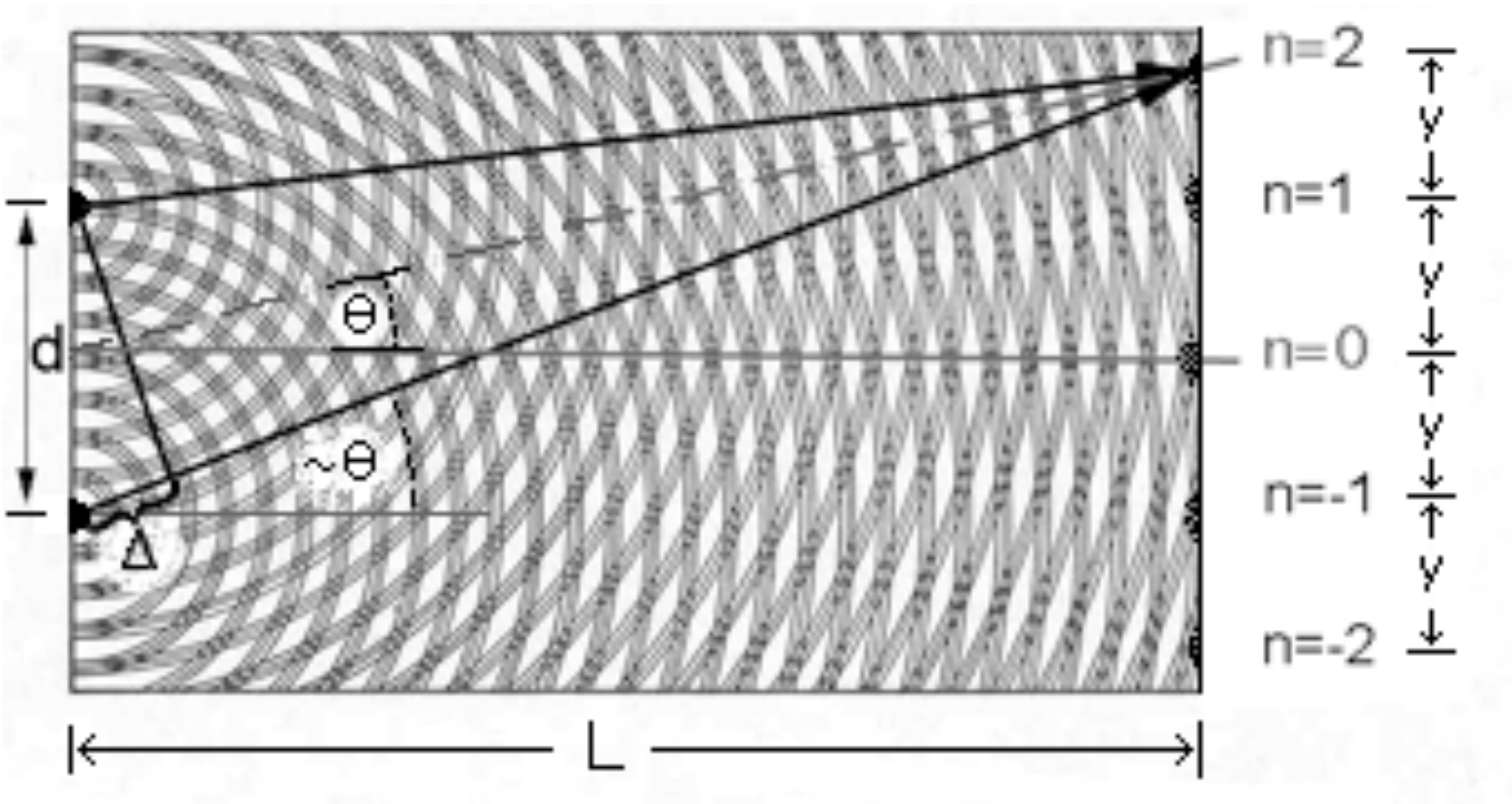


Predict pattern for two holes

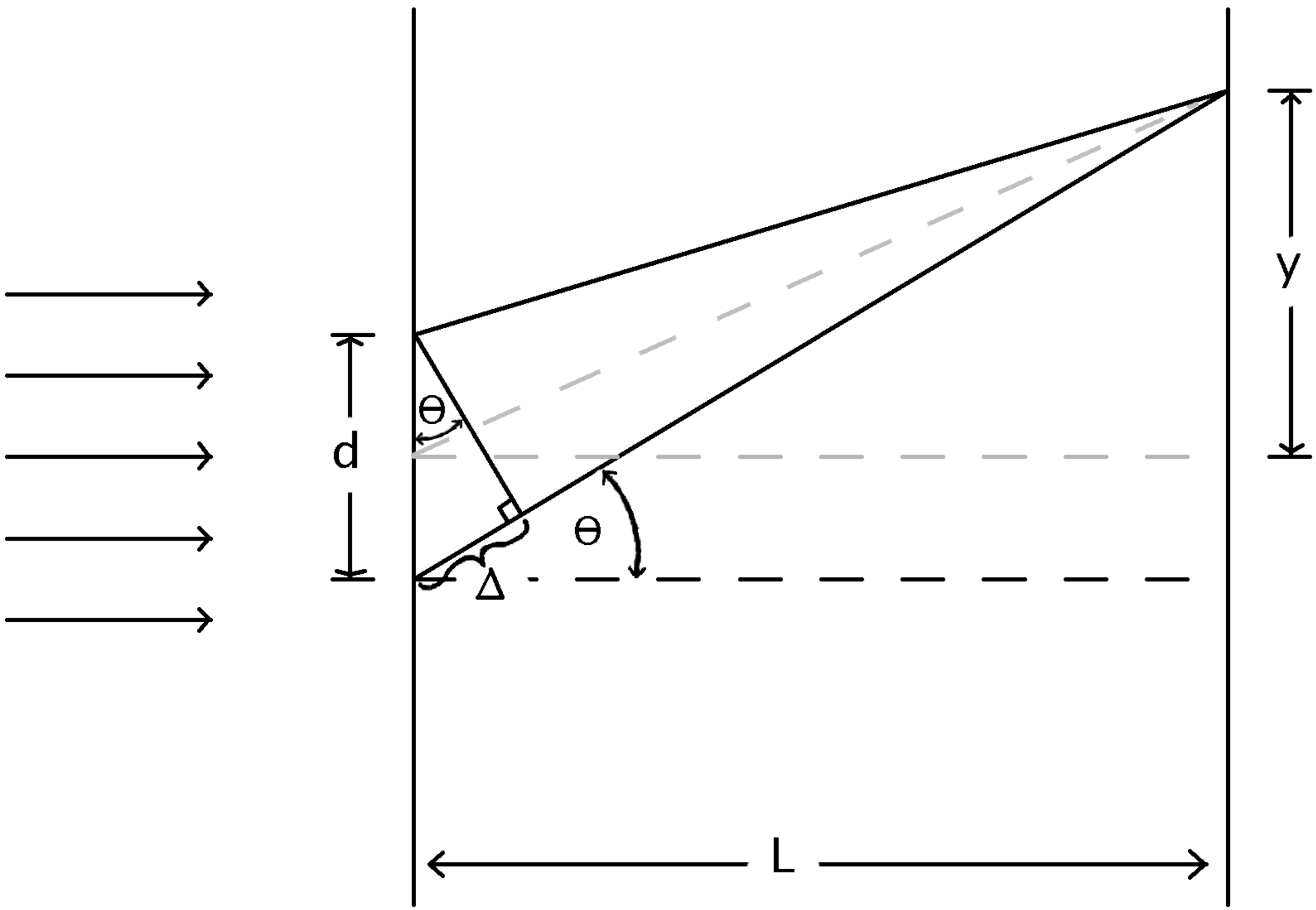


Source
hole 1
hole 2
Plane Waves
Interfering Waves
Bright spots


Curve shows distribution
of brightness
Screen-like
surface
Where waves interfere
constructively, bright lines
appear on surface.
Where wave interfere
destructively,
dark lines appear.



n=2
n=1
n=0
n=-1
n=-2
y
y
y
y
d
Θ
~Θ
Δ
L


θ
d
θ
Δ
y
L

Condition for bright fringes:

Δ= nλ = d sin q where n = 0, 1, 2, 3 ...

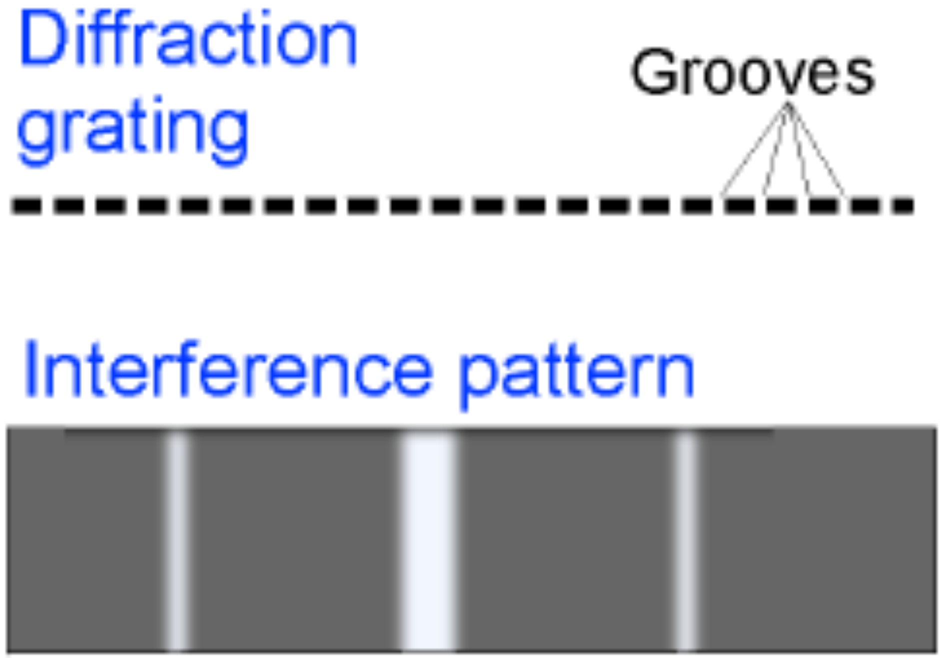
Condition for dark fringes:

Δ = ½λ, 3/2 λ, 5/2λ or $$\Delta = 1/2(2n+1)\lambda$$

where n = 0, 1, 2, 3

Diffraction gratings

A diffraction grating is a precise array of tiny engraved lines, each of which allows light through.
The spectrum produced is a mixture of many different wavelengths of light.



BASICS OF DIFFRACTION

- Single slit interference

P– 1st maximum
Q– 1st secondary maximum
$\theta = n\lambda/d$

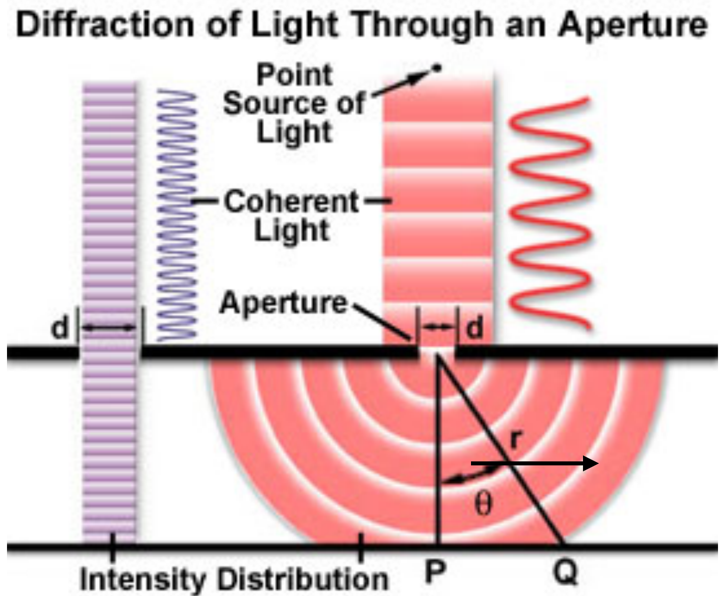


Figure 3

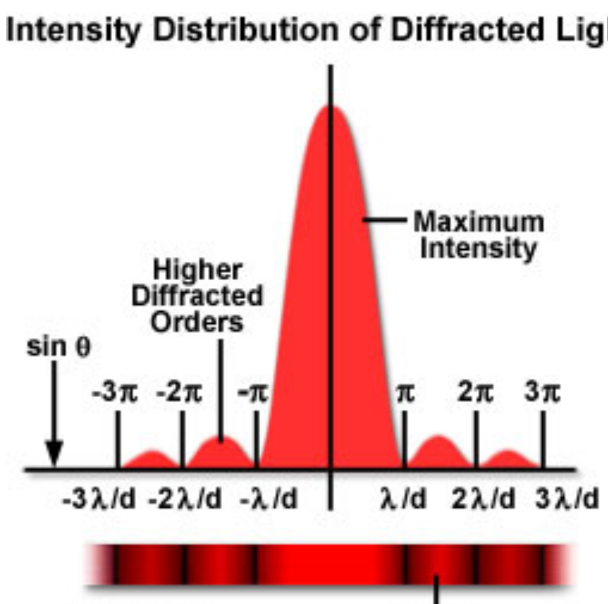


Figure 4

Intensity of the beam is governed by

$$I = I_0 \{ \sin\beta / \beta \}^2$$

Where $\beta = (\pi / \lambda)\, d \sin\theta$

DOPPLER EFFECT

Doppler Effect is the apparent change in the observed frequency of a wave, as of sound or light, occurring when the source and observer are in motion relative to each other, with the frequency increasing when the source and observer approach each other and decreasing when they move apart.

Source moving away from stationary observer

$$f = \left(\frac{v}{v + v_s} \right) f_o$$

Observer hears sound of longer wavelength, lower frequency and lower pitch

DOPPLER EFFECT

Source moving towards from stationary observer

$$f = \left(\frac{v}{v - v_s} \right) f_o$$

Observer hears sound of shorter wavelength, higher frequency and higher pitch

DOPPLER EFFECT

Observer moving towards from stationary source

$$f = \left(\frac{v + v_r}{v} \right) f_o$$

Observer hears sound of shorter wavelength, higher frequency and higher pitch

DOPPLER EFFECT

Observer moving away from stationary source

$$f = \left(\frac{v - v_r}{v} \right) f_o$$

Observer hears sound of longer wavelength, lower frequency and lower pitch

Doppler Effect in Light

Redshift

Occurs whenever a light source moves away from the observer, corresponding to the Doppler shift that changes the perceived frequency of sound waves. Observer sees a longer wavelength

$$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}$$

$z > 0$ for redshift

$z < 0$ for blueshift

Bel and Decibel

Bel

It is a measurement unit used when comparing two sound intensities. Whenever the intensity of sound is increased by a factor of 10, the increase in intensity is said to be 1 bel. The dynamic range of audibility of the human ear is 12 bels or 120 decibels

Decibel (dB)

The sound intensity I may be expressed in decibels above the standard threshold of hearing Io . The expression is

$$1 \ \mathrm{dB} = 10 \ \log \frac{I}{I_0}$$