ME 161/162 Basic Mechanics

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Unit 2

FORCES AND MOMENTS

Contents

Force Systems

- A *force* is any effect that may change the state of rest or motion of a body (Refer to Newton's 1st Law).

- A *force system* comprises of two or more forces acting on a body or a group of related bodies.

- Force systems are classified according to the arraignment of constituent forces.

- A body is in equilibrium when all the resultant forces acting on the body is zero.

- Note: No method exists for directly measuring a force. In mechanics, we use cause-and-effect relationship to measure/determine force e.g. spring scale uses deflection of spring to measure weight (force) and beam scale uses balancing of moment to measure weight (force).

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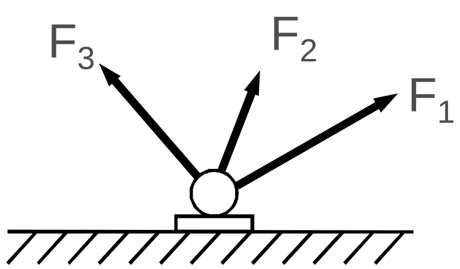
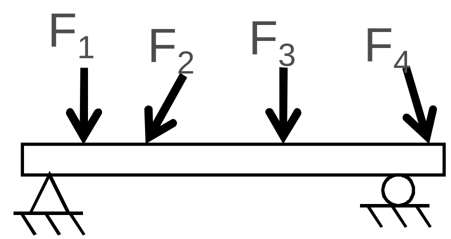
Types of Force System

A force system may be one-, two- (planar) or three-dimensional (spatial). A force system is said to be

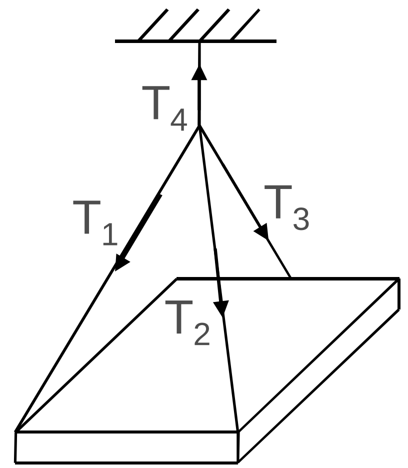
- *collinear* if the forces have a common line of action.
- *parallel* if the lines of action of the forces are parallel.
- *concurrent* if the lines of action of the forces intersect at a common point
- Coplanar when all the forces lie in the same plane.
- Spatial when all the forces do not lie in the same plane.



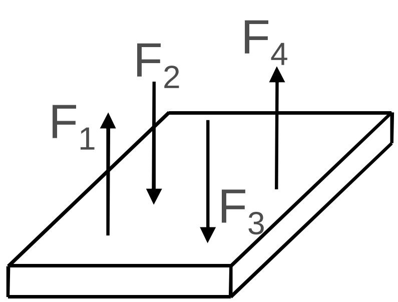
Concurrent Coplanar Force System



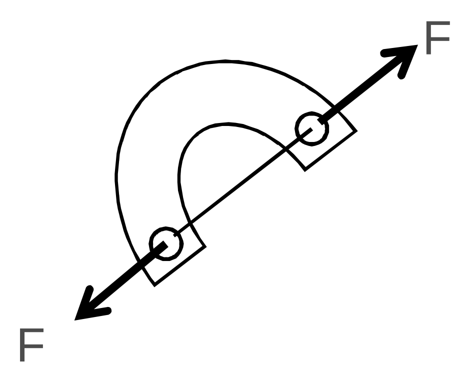
Non-Concurrent Coplanar Force System



Concurrent Spatial Force System

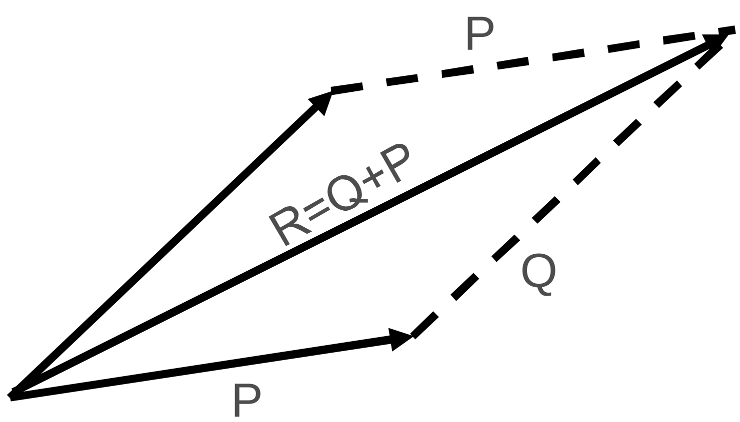


Parallel spatial Force System

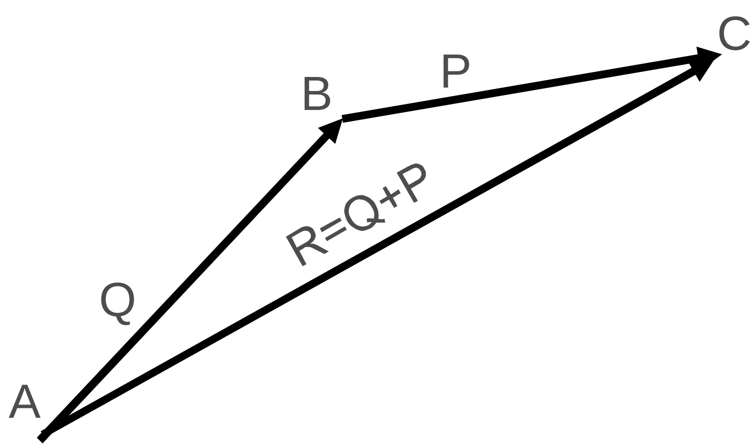


Collinear Force System

Addition of Vectors



▪Parallelogram Law



where A, B and C are angles

- Cosine Law (or Cosine rule)

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$

- Sine Law (or sin rule)

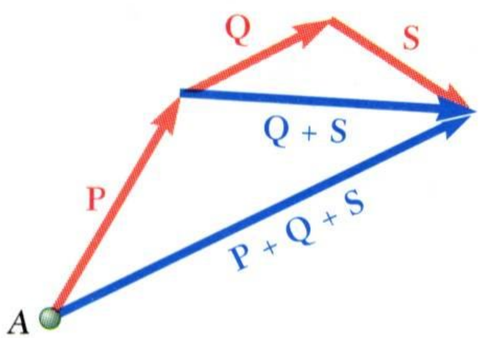
$$\frac{\sin A}{P} = \frac{\sin B}{R} = \frac{\sin C}{Q}$$

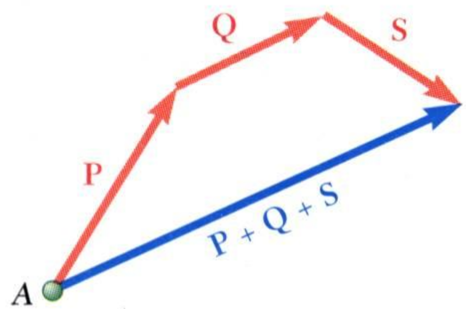
- Vector addition is commutative

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

- Note : P, Q and R are magnitudes of vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ respectively. A, B and C are interior angles of the vector triangle.

Addition of Vectors





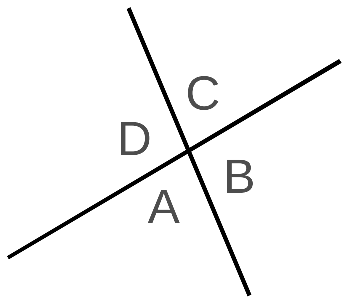
- Addition of three or more vectors through repeated application of the triangle rule

- The polygon rule for the addition of three or more vectors.

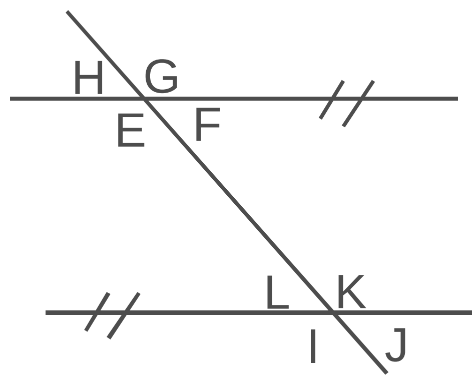
$$\vec{P}+\vec{Q}+\vec{S}=\left(\vec{P}+\vec{Q}\right)+\vec{S}=\vec{P}+\left(\vec{Q}+\vec{S}\right)$$

Brief Review of Geometry

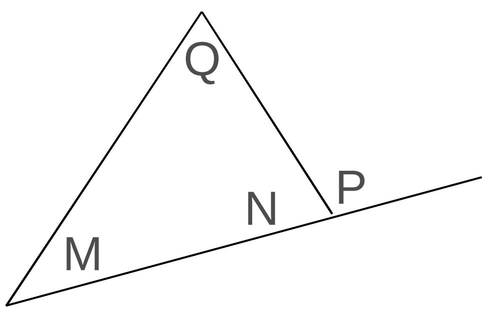
(a)



(b)



(c)



- From Diagram (a),

  $D + A = 180^{o}$

  $D + C = 180^{o}$

  $\therefore\ A = C$ and $B = D$

From Diagram (b),

$G = E=K=I$

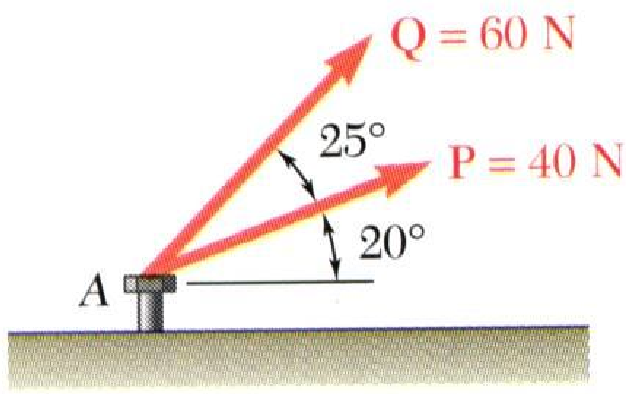
$H =F=L=J$

From Diagram (c),

$M + N + Q =180$

$N =180 - P$

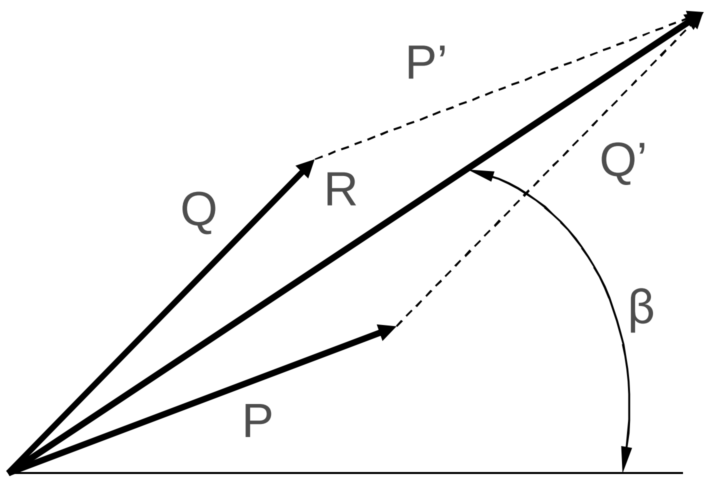
Example 2-1



Determine resultant of forces Q and P acting on the bolt shown above and angle the resultant makes with the horizontal axis.

Graphical Solution

Graphical solution - A half or full parallelogram with sides equal to **P** and **Q** is drawn to scale.

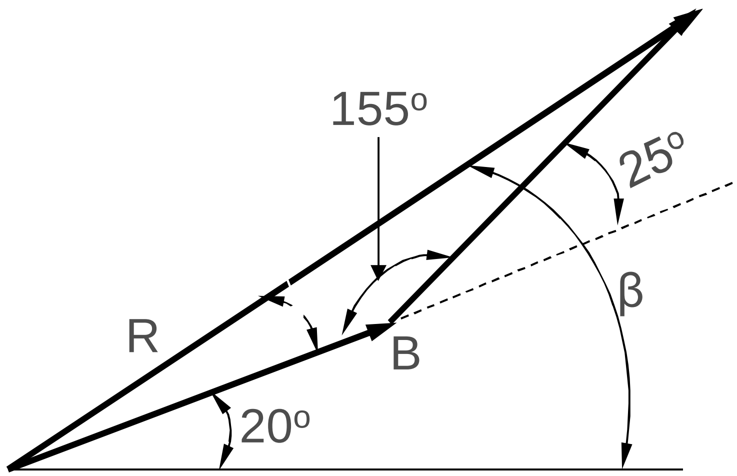


The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

R = 98 N $\beta = 35^{o}$

Example 2-1, Cont;

- Parallelogram solution



From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$

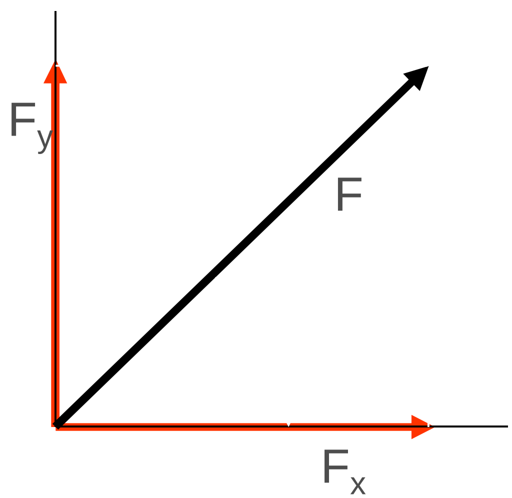
R = 97.73 N

From the Law of Sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$
$$\sin(\beta - 20^o) = \sin B \frac{Q}{R}$$
$$= \sin 155° \frac{60\text{N}}{97.73\text{N}}$$
$$\beta - 20^o = 15°$$
$$\beta = 35^o$$

Rectangular Components of Force

A force vector may be resolved into perpendicular components so that the resulting parallelogram is a rectangle. The resulting x and y components are referred to as *rectangular vector components* and



$$\vec{F} = \vec{F}_x + \vec{F}_y$$

Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

$$\vec{F} = \vec{F}_x i + \vec{F}_y j$$

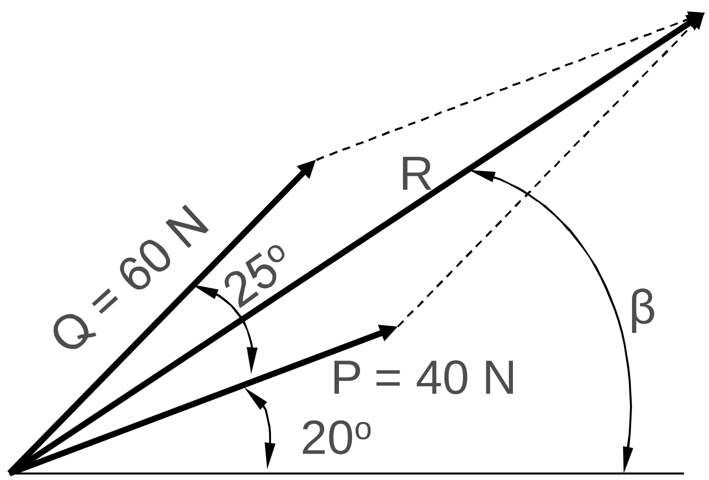
F_x and F_y are referred to as the *scalar components*

$$F = \sqrt{\left(F_x^{\ 2} + F_y^{\ 2}\right)}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Example 2-2

- Solve Example 3 using Rectangular Components solution



- $R_x = \Sigma F_x = P\cos 20^o + Q\cos (20^o + 25^o)$

  $= 40\cos 20^o + 60\cos 45^o$

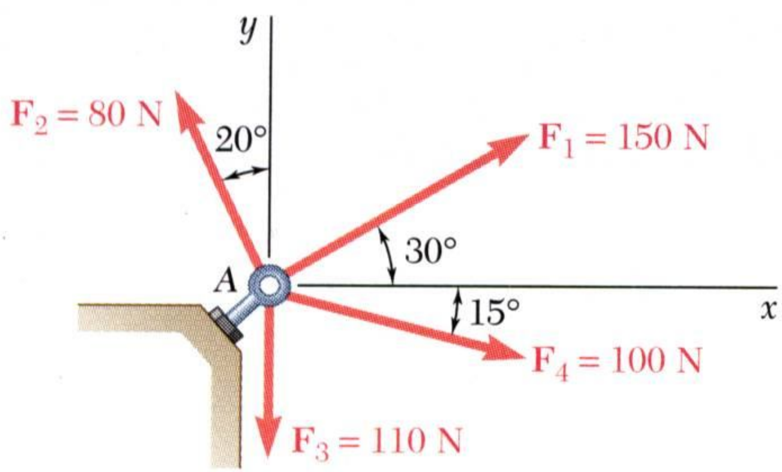
  $= 80.014$ N

- $R_y = \Sigma F_y = P\sin 20^o + Q\sin (20^o + 25^o)$

  $= 40\sin 20^o + 60\sin 45^o$

  $= 57.107$ N

Example 2-3

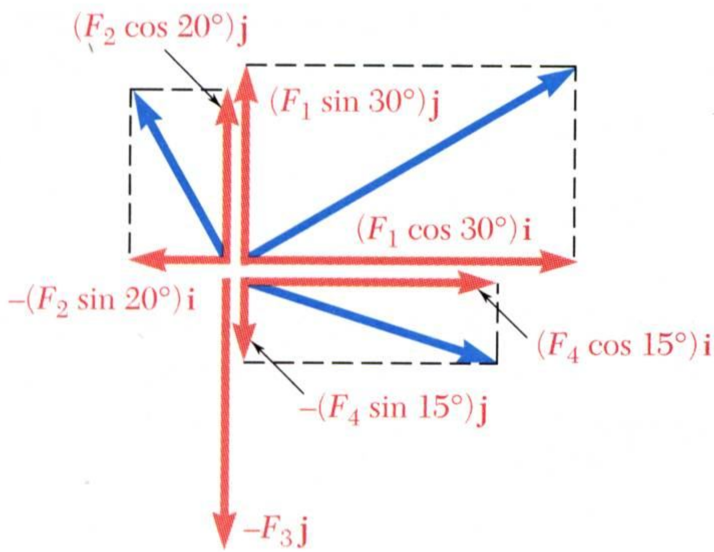


Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

SOLUTION:

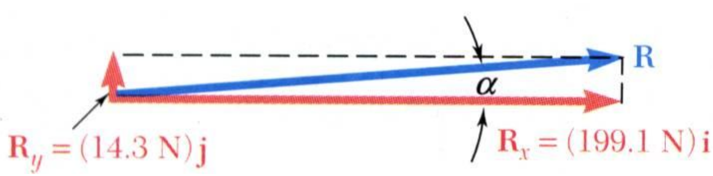
- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

Example 2-3, Cont;



- Resolve each force into rectangular components.

force	*mag*	$x-comp$	$y-comp$
$\vec{F}_1$	150	+129.9	+75.0
$\vec{F}_2$	80	−27.4	+75.2
$\vec{F}_3$	110	0	−110.0
$\vec{F}_4$	100	+96.6	−25.9
		$R_x = +199.1$	$R_y = +14.3$



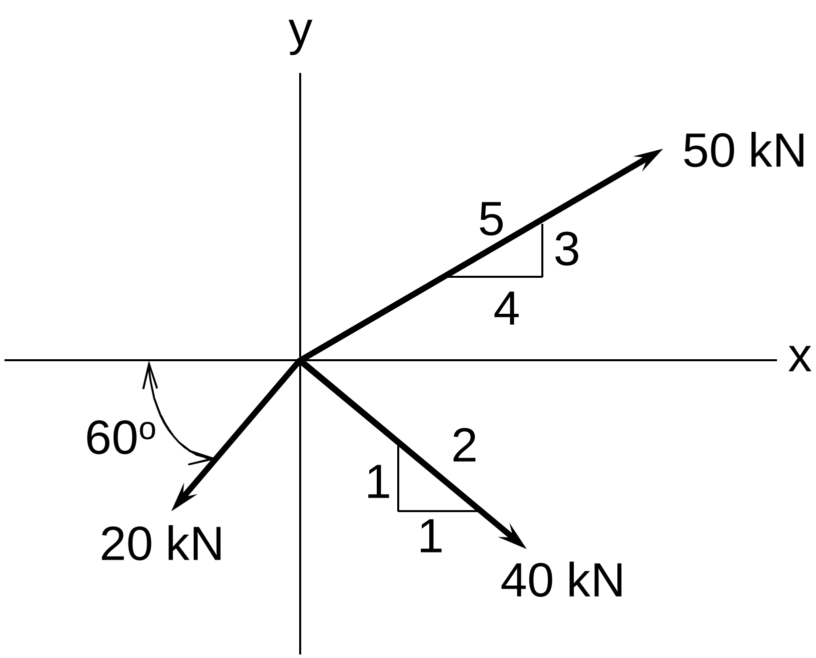
- Determine the components of the resultant by adding the corresponding force components.

$$R = \sqrt{199.1^2 + 14.3^2} \qquad \boxed{R = 199.6\mathrm{N}}$$

$$\tan\alpha = \frac{14.3\,\mathrm{N}}{199.1\,\mathrm{N}} \qquad \boxed{\alpha = 4.1°}$$

Example 2-4

- Determine the magnitude of the resultant force and its direction measured from the positive x axis.



Solution

$$R_x = \sum_{i=1}^{n} F_i Cos\theta_i$$

$$R_x = 50(4/5) - 20Cos60 + 40(1/\sqrt{2})$$

$$R_x = 58.284N$$

$$R_y = \sum_{i=1}^{n} F_i Sin\theta_i$$

$$R_y = 50(3/5) - 20Sin60 - 40(1/\sqrt{2})$$

$$R_y = -15.604N$$

$$R = \sqrt{\left(R_x{}^2 + R_y{}^2\right)}$$

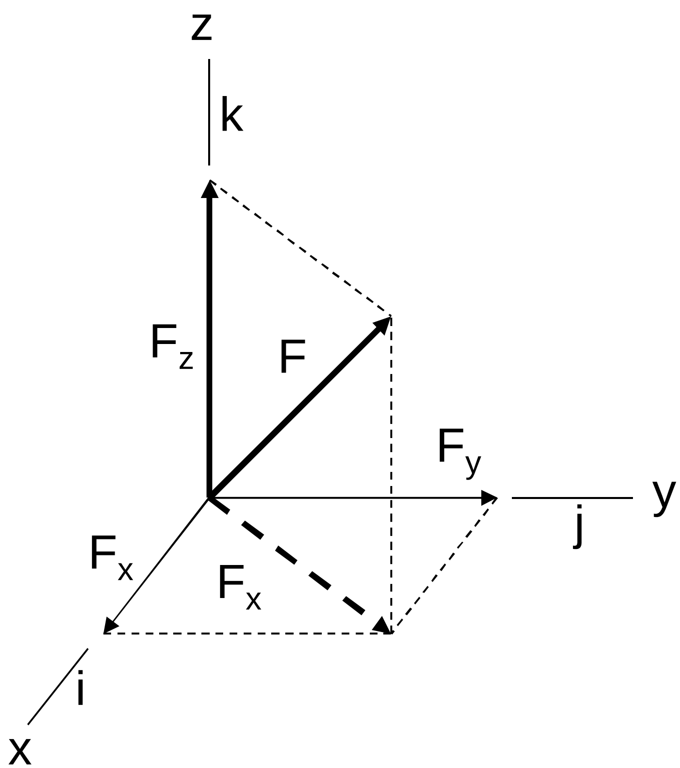
$$R = \sqrt{\left(58.284^2 + (-15.604)^2\right)}$$

$$R = 60.34N$$

$$\beta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$\beta = \tan^{-1}\left(\frac{-15.604}{58.284}\right) = -15^o \text{ or } 345^o$$

Cartesian Vectors



- In right-handed Cartesian coordinated system, the right thumb points in the positive z direction when the right hand figures are curled from positive x direction to positive y direction about the z axis.

- The three axes x, y and z are right angle to each other.

- The unit vectors I, j and k are the unit vectors along the x, y and z axes, respectively.

Vector Representation

- Vectors are represented by the magnitudes and directions of the three components using the unit vectors I, j and k. e.g.

$$F = F_x i + F_y j + F_z k$$

- The magnitude of the vector is given as

$$\left|F\right| = \sqrt{\left(F_x^2 + F_y^2 + F_z^2\right)}$$

- A unit vector U is vector with unit (1) magnitude.

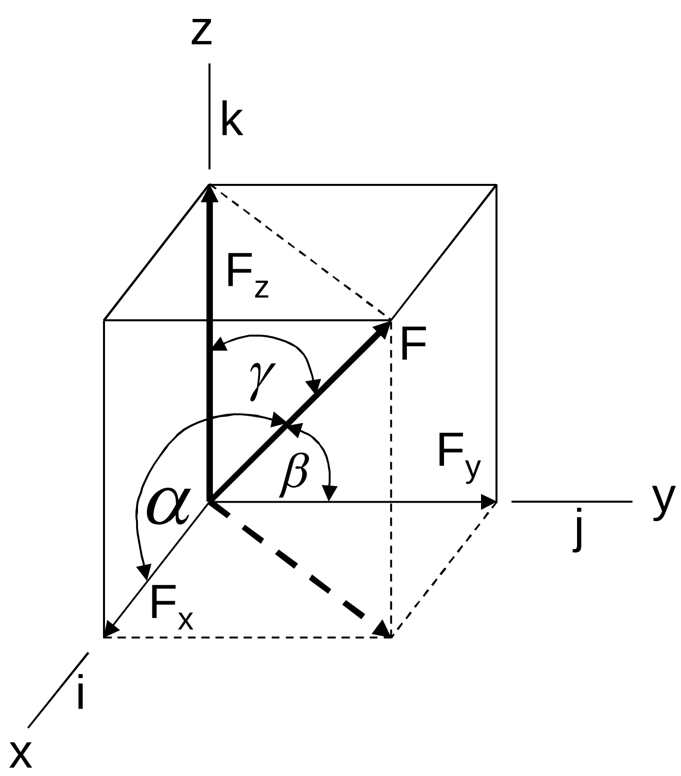
- If $\left|F\right| \neq 0$

$$U = \frac{1}{\left|F\right|}\left(F_x i + F_y j + F_z k\right)$$

$$U = \frac{F_x}{\left|F\right|} i + \frac{F_y}{\left|F\right|} j + \frac{F_z}{\left|F\right|} k$$

Direction Cosines

The direction cosines are the cosine of angles between the tail of a vector and the positive x, y and z axes.



Direction Cosines

$$\cos\alpha = \frac{F_x}{|F|} \qquad \cos\beta = \frac{F_y}{|F|} \qquad \cos\gamma = \frac{F_z}{|F|}$$

$$U_F = \cos\alpha i + \cos\beta j + \cos\gamma k$$

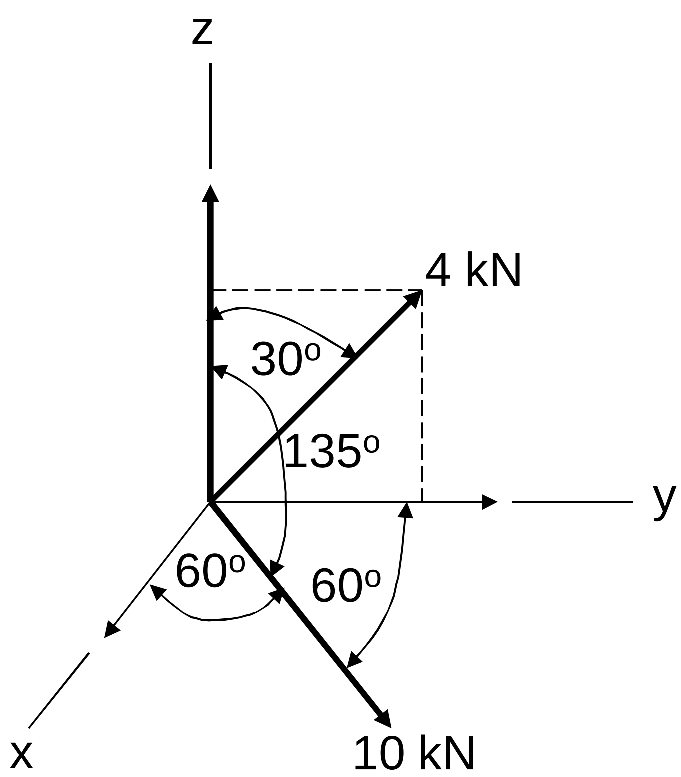
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

If a force of magnitude $|F|$ is directed along a line r, then the force vector is defined as

$$F = |F| \frac{r}{|r|}$$

Example 2-5

Determine the magnitude and coordinate direction angles of the resultant force.



Solution

$$F_1 = 4\left(0i + \sin 30^o\, j + \cos 30^o\, k\right)kN$$

$$F_1 = \left(2\,j + 3.464k\right)kN$$

$$F_2 = 10\left(\cos 60^o\, i + \cos 60^o\, j + \cos 135^o\, k\right)kN$$

$$F_1 = \left(5i + 5\,j - 7.0711k\right)kN$$

$$F = F_1 + F_2$$

$$F = \left(5i + 7\,j - 3.607k\right)kN$$

$$|F| = \sqrt{\left(5^2 + 7^2 + (-3.607)^2\right)} = 9.328$$

$$U_F = \frac{F}{|F|} = 0.536i + 0.7504\,j - 0.3867k$$

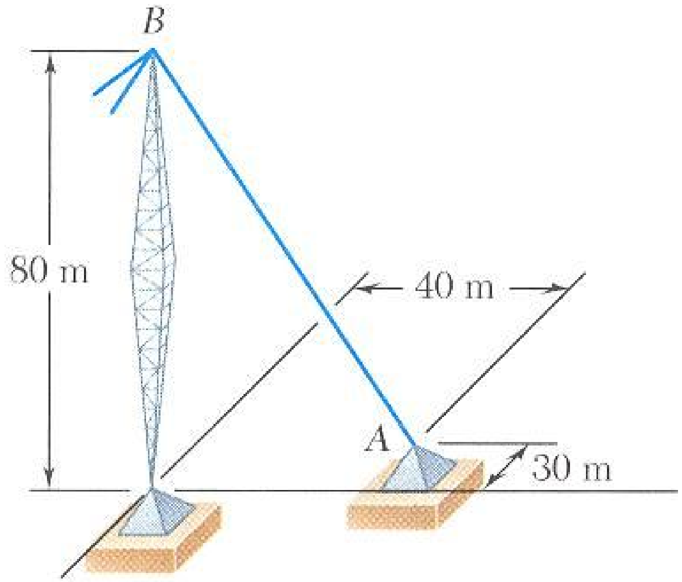
$$\text{Direction Cosine} = \cos^{-1}\left(U_F\right)$$

$$\alpha = 57.59^o$$

$$\beta = 41.37^o$$

$$\gamma = 112.70^o$$

Example 2-6



The tension in the guy wire is 2500 N. Determine:

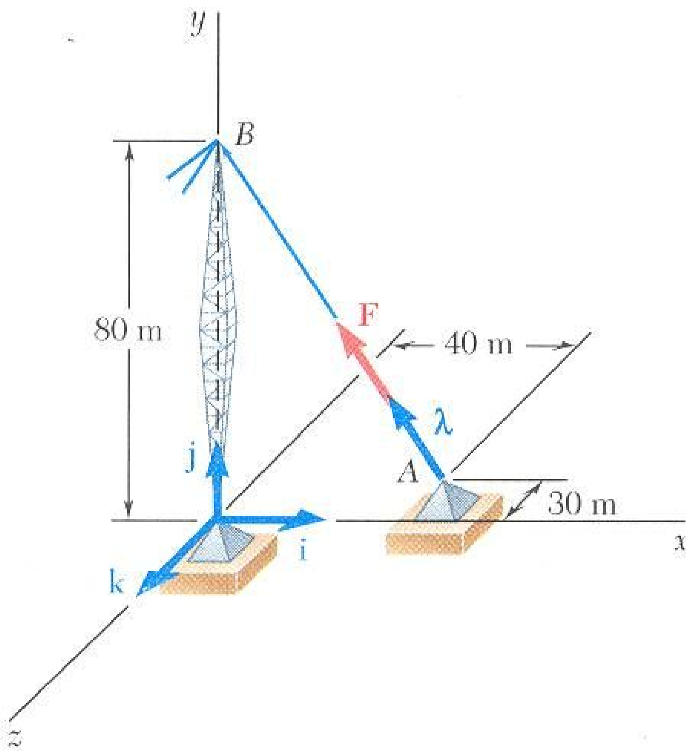
a) components F_x, F_y, F_z of the force acting on the bolt at *A*,

b) the angles θ_x, θ_y, θ_z defining the direction of the force

SOLUTION:

- Based on the relative locations of the points *A* and *B*, determine the unit vector pointing from *A* towards *B*.
- Apply the unit vector to determine the components of the force
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

Example 2-6



$$\overrightarrow{AB} = (-40\,\mathrm{m})\vec{i} + (80\,\mathrm{m})\vec{j} + (30\,\mathrm{m})\vec{k}$$

$$AB = \sqrt{(-40\,\mathrm{m})^2 + (80\,\mathrm{m})^2 + (30\,\mathrm{m})^2}$$

$$= 94.3\,\mathrm{m}$$

$$\vec{\lambda} = \left(\frac{-40}{94.3}\right)\vec{i} + \left(\frac{80}{94.3}\right)\vec{j} + \left(\frac{30}{94.3}\right)\vec{k}$$

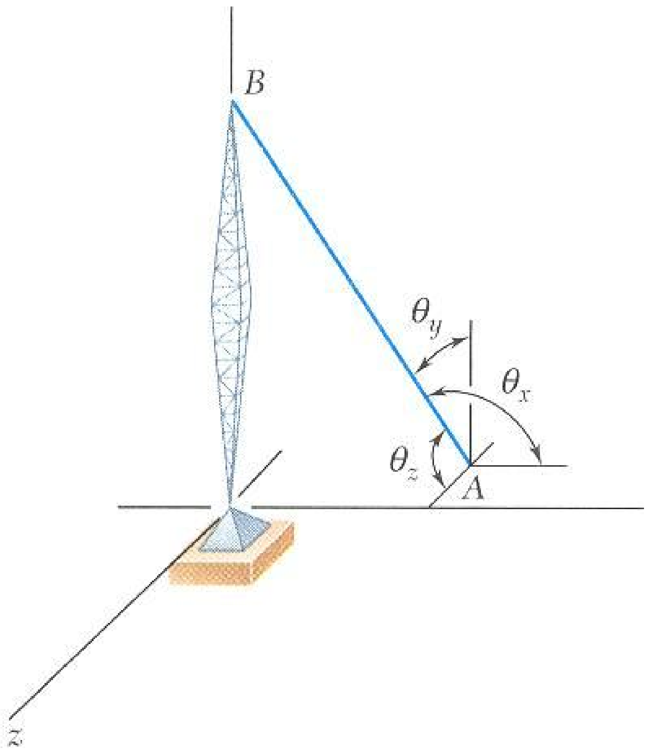
$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\vec{F} = F\vec{\lambda}$$

$$= (2500\,\mathrm{N})\left(-0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}\right)$$

$$= (-1060\,\mathrm{N})\vec{i} + (2120\,\mathrm{N})\vec{j} + (795\,\mathrm{N})\vec{k}$$

Example 2-6



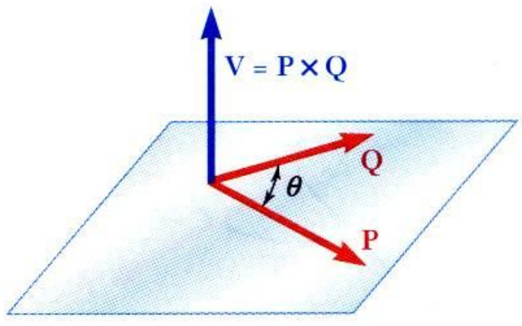
$$\vec{\lambda} = \cos\theta_x \vec{i} + \cos\theta_y \vec{j} + \cos\theta_z \vec{k}$$

$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\boxed{\begin{aligned} \theta_x &= 115.1^\circ \\ \theta_y &= 32.0^\circ \\ \theta_z &= 71.5^\circ \end{aligned}}$$

Product of Two Vectors

- Concept of the moment of a force about a point is more easily understood through applications of the *vector product* or *cross product*.
- Vector product of two vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ is defined as the vector $\boldsymbol{V}$ which satisfies the following conditions:
  1. Line of action of $\boldsymbol{V}$ is perpendicular to plane containing $\boldsymbol{P}$ and $\boldsymbol{Q}$.
  2. Magnitude of $\boldsymbol{V}$ is $V = PQ \sin\theta$
  3. Direction of $\boldsymbol{V}$ is obtained from the right-hand rule.



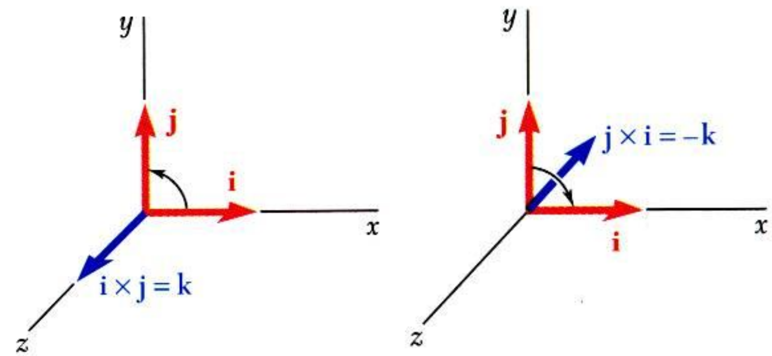
(a)

(b)

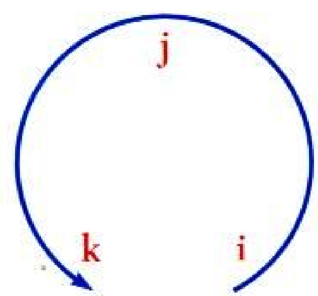
- Vector products:
  - are not commutative, $\boldsymbol{Q} \times \boldsymbol{P} = -(\boldsymbol{P} \times \boldsymbol{Q})$
  - are distributive, $\boldsymbol{P} \times (\boldsymbol{Q}_1 + \boldsymbol{Q}_2) = \boldsymbol{P} \times \boldsymbol{Q}_1 + \boldsymbol{P} \times \boldsymbol{Q}_2$
  - are not associative, $(\boldsymbol{P} \times \boldsymbol{Q}) \times \boldsymbol{S} \neq \boldsymbol{P} \times (\boldsymbol{Q} \times \boldsymbol{S})$

Vector Products

$$\vec{i}\times\vec{i}=0 \qquad \vec{j}\times\vec{i}=-\vec{k} \qquad \vec{k}\times\vec{i}=\vec{j}$$
$$\vec{i}\times\vec{j}=\vec{k} \qquad \vec{j}\times\vec{j}=0 \qquad \vec{k}\times\vec{j}=-\vec{i}$$
$$\vec{i}\times\vec{k}=-\vec{j} \qquad \vec{j}\times\vec{k}=\vec{i} \qquad \vec{k}\times\vec{k}=0$$



$$\vec{V}=\left(P_x\vec{i}+P_y\vec{j}+P_z\vec{k}\right)\times\left(Q_x\vec{i}+Q_y\vec{j}+Q_z\vec{k}\right)$$
$$=\left(P_yQ_z-P_zQ_y\right)\vec{i}+\left(P_zQ_x-P_xQ_z\right)\vec{j}$$
$$+\left(P_xQ_y-P_yQ_x\right)\vec{k}$$



$$=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ P_x & P_y & P_z\\ Q_x & Q_y & Q_z\end{vmatrix}$$

Moment of a Force

- The *moment* of $\boldsymbol{F}$ about O is defined as

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

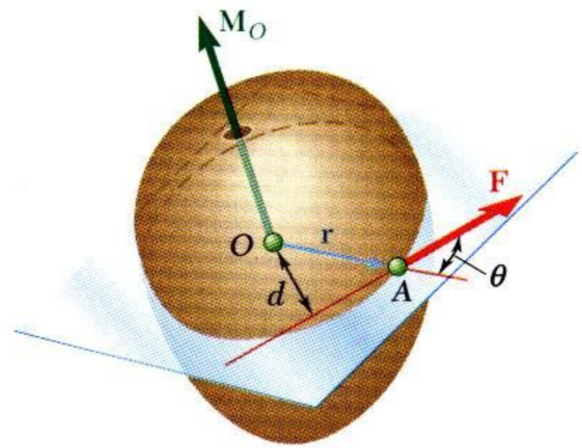
- The moment vector $\boldsymbol{M_O}$ is perpendicular to the plane containing O and the force $\boldsymbol{F}$.

- Magnitude of $\boldsymbol{M_O}$ measures the tendency of the force to cause rotation of the body about an axis along $\boldsymbol{M_O}$.

$$M_O = rF\sin\theta = Fd$$

The sense of the moment may be determined by the right-hand rule.

- Any force $\boldsymbol{F'}$ that has the same magnitude and direction as $\boldsymbol{F}$, is *equivalent* if it also has the same line of action and therefore, produces the same moment.

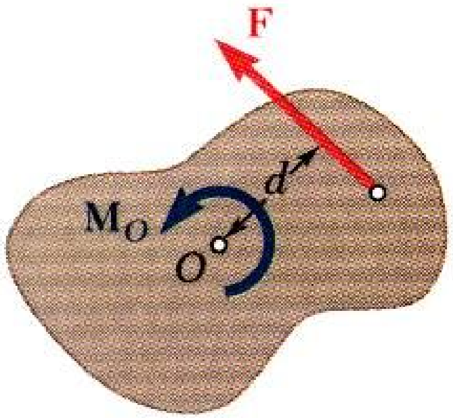


(a)

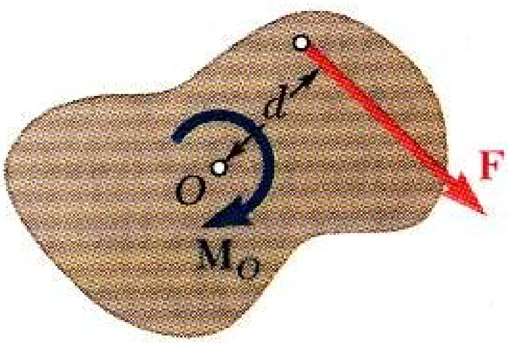
(b)

Moment of a Force

- *Two-dimensional structures* have length and breadth but negligible depth and are subjected to forces contained in the plane of the structure.
- The plane of the structure contains the point *O* and the force ***F***. $\boldsymbol{M_O}$, the moment of the force about *O* is perpendicular to the plane.
- If the force tends to rotate the structure clockwise, the sense of the moment vector is out of the plane of the structure and the magnitude of the moment is positive
- If the force tends to rotate the structure counterclockwise, the sense of the moment vector is into the plane of the structure and the magnitude of the moment is negative.



(a) $\mathbf{M}_O = + Fd$

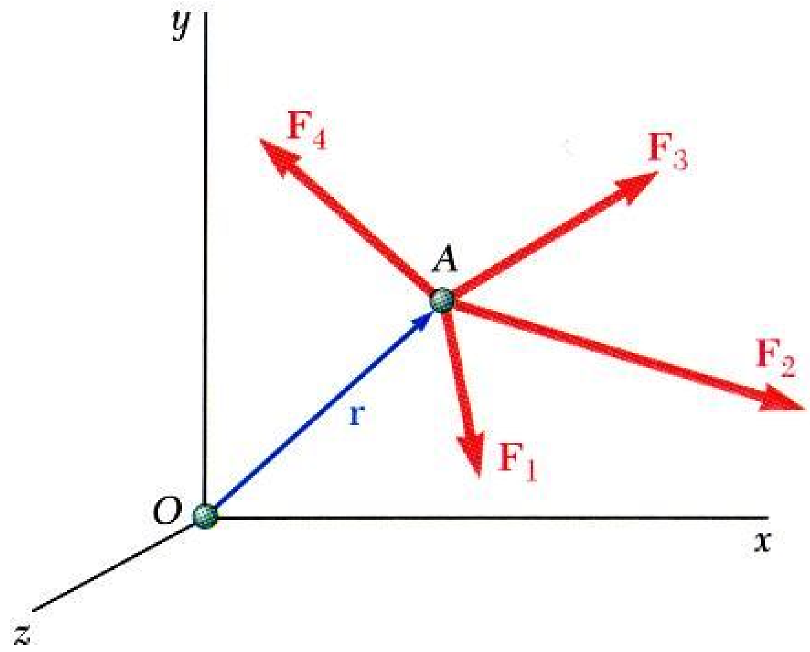


(b) $\mathbf{M}_O = - Fd$

Varignon's Theorem

- The moment about a give point *O* of the resultant of several concurrent forces is equal to the sum of the moments of the various moments about the same point *O*.

$$\vec{r} \times \left(\vec{F}_1 + \vec{F}_2 + \cdots\right) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \cdots$$



- Varignon's Theorem makes it possible to replace the direct determination of the moment of a force ***F*** by the moments of two or more component forces of ***F***.

Rectangular Components of Moment of a Force

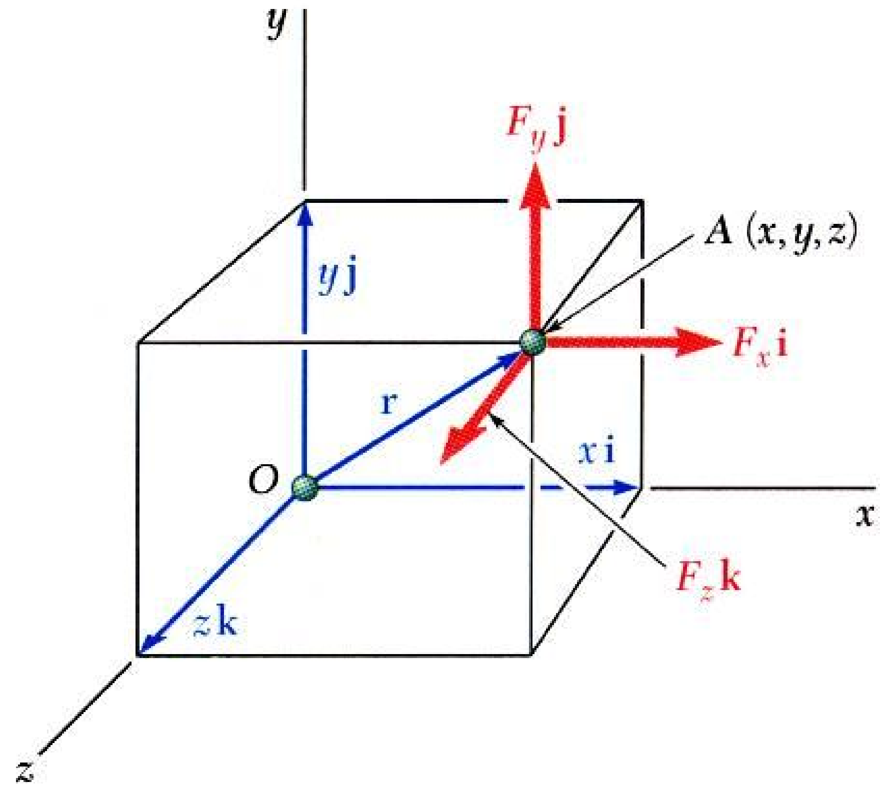
$$\vec{M}_O = \vec{r} \times \vec{F}, \qquad \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{M}_O = M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \left(yF_z - zF_y\right)\vec{i} + \left(zF_x - xF_z\right)\vec{j} + \left(xF_y - yF_x\right)\vec{k}$$



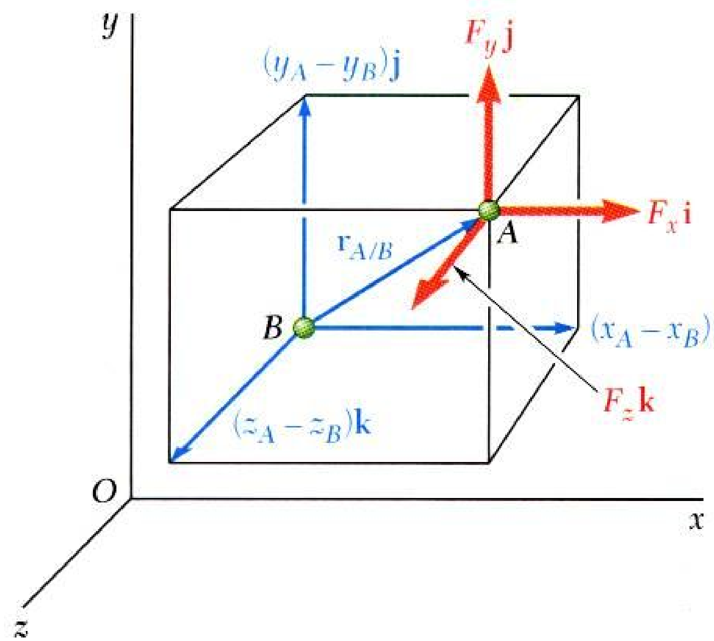
Rectangular Components of Moment of a Force

$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F}$$

$$\vec{r}_{A/B} = \vec{r}_A - \vec{r}_B$$

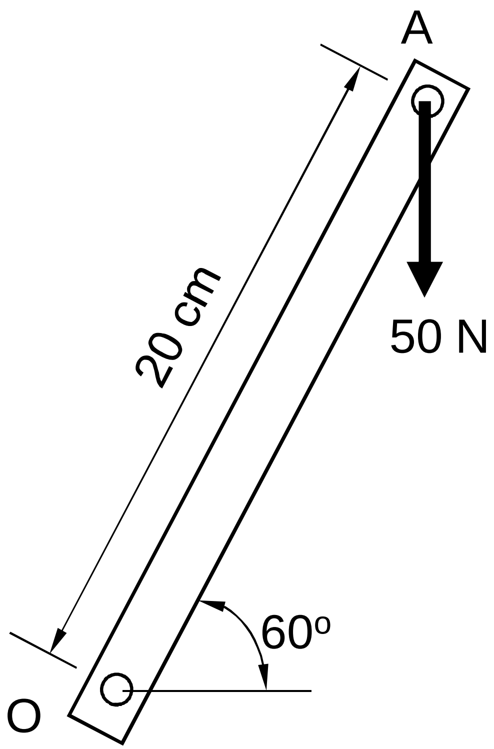
$$= (x_A - x_B)\vec{i} + (y_A - y_B)\vec{j} + (z_A - z_B)\vec{k}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$



$$\vec{M}_B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ (x_A - x_B) & (y_A - y_B) & (z_A - z_B) \\ F_x & F_y & F_z \end{vmatrix}$$

Example 2-7



50-N vertical force is applied to the end of a lever which is attached to a shaft at *O*.

Determine:

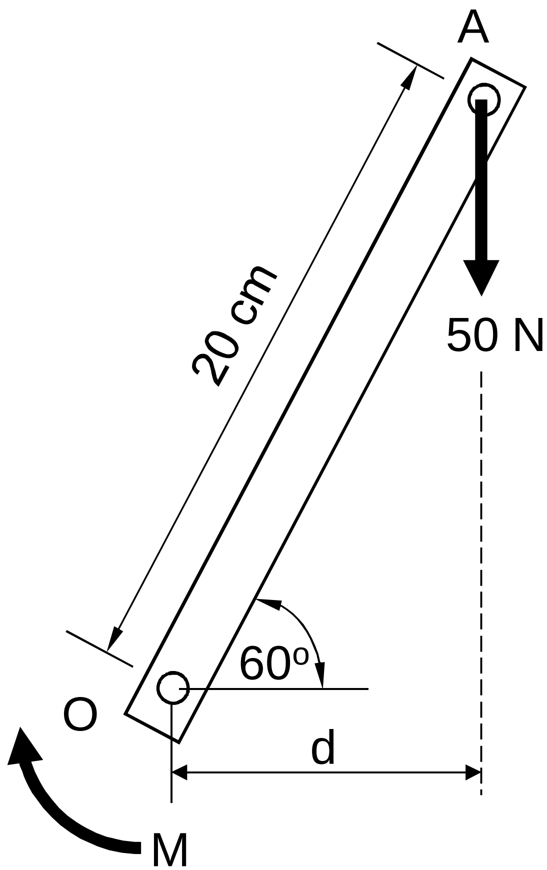
a) moment about *O*,

b) horizontal force at *A* which creates the same moment,

c) smallest force at A which produces the same moment,

d) location for a 125-N vertical force to produce the same moment,

Example 2-7



a) Moment about *O* is equal to the product of the force and the perpendicular distance between the line of action of the force and *O*. Since the force tends to rotate the lever clockwise, the moment vector is into the plane of the paper.

(a)

$$M_O = Fd$$

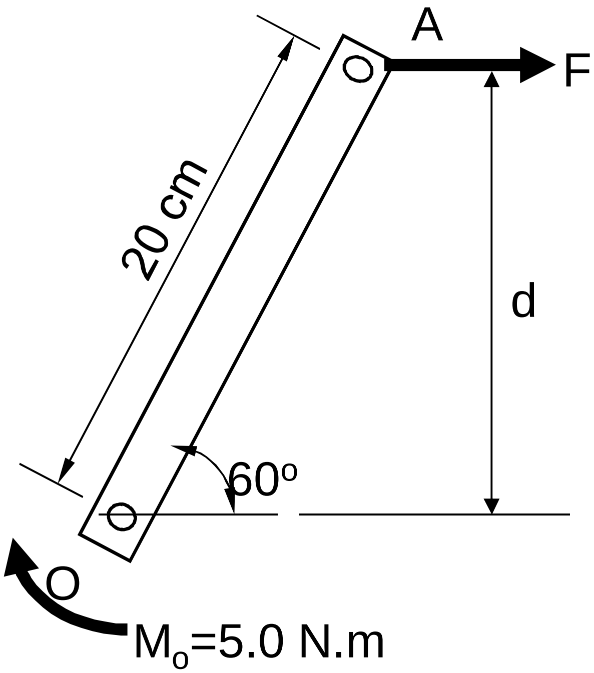
$$d = (0.20\,\text{m})\cos 60° = 0.10\,\text{m}.$$

$$M_O = (50\ )(0.10\ )$$

$$\boxed{M_O = 5.0\ N.\text{m}}$$

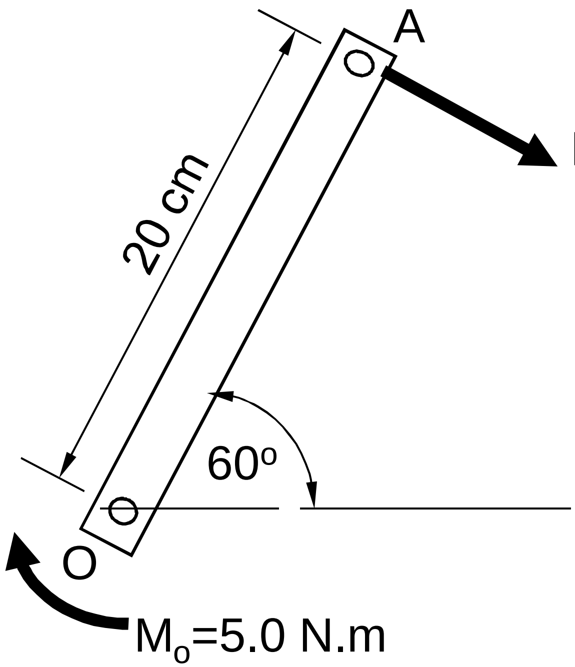
Example 2-7

(b) horizontal force at *A* which creates the same moment



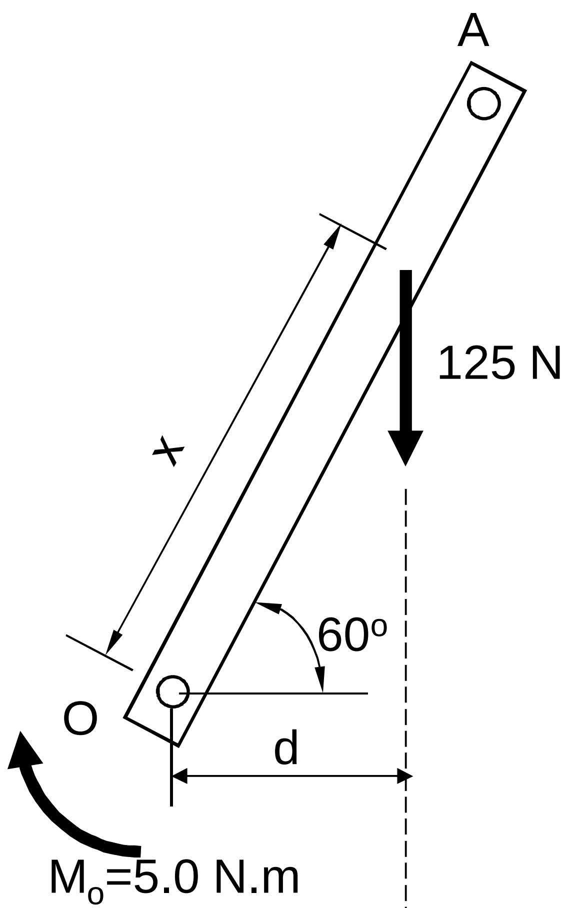
$$M_O = Fd$$
$$M_O = 5.0 N.m$$
$$d = (0.20\,\mathrm{m})\sin 60^\circ = 0.1732\,\mathrm{m}.$$
$$F = \frac{M_O}{d} = \frac{5.0}{0.1732}$$
$$\boxed{F = 28.9\ N.\mathrm{m}}$$

(c) smallest force at A which produc



$$M_O = Fd$$
$$M_O = 5.0 N.m$$
$$d = 0.20\,\mathrm{m}$$
$$F = \frac{M_O}{d} = \frac{5.0}{0.20}$$
$$\boxed{F = 25.0\ N.\mathrm{m}}$$

Example 2-7



(d) location for a 125-N vertical force that produce the same moment,

$$M_O = Fd$$

$$M_O = 5.0\,N.m$$

$$F = 125\,N$$

$$d = \frac{M_O}{F} = \frac{5.0}{125} = 0.04m$$

$$x\cos(60^o) = d = 0.04$$

$$x = \frac{d}{\cos(60^o)} = \frac{0.04}{\cos(60^o)} = 0.08m$$

$$\boxed{x = 8\text{ cm}}$$

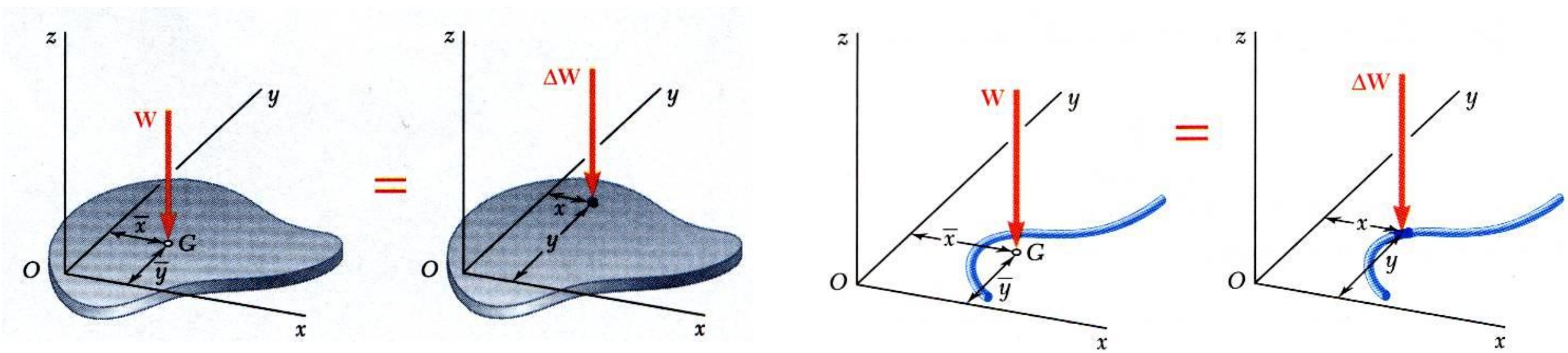
Centroids and Centres of Gravity

- The earth exerts a gravitational force on each of the particles forming a body. These forces can be replace by a single equivalent force equal to the weight of the body and applied at the *center of gravity* for the body.

- The *centroid of an area* is analogous to the center of gravity of a body. The concept of the *first moment of an area* is used to locate the centroid.

- Determination of the area of a *surface of revolution* and the volume of a *body of revolution* are accomplished with the *Theorems of Pappus-Guldinus*.

Center of Gravity of a 2D Body



$$\sum M_y \quad \bar{x}W = \sum x\Delta W$$ By Summation

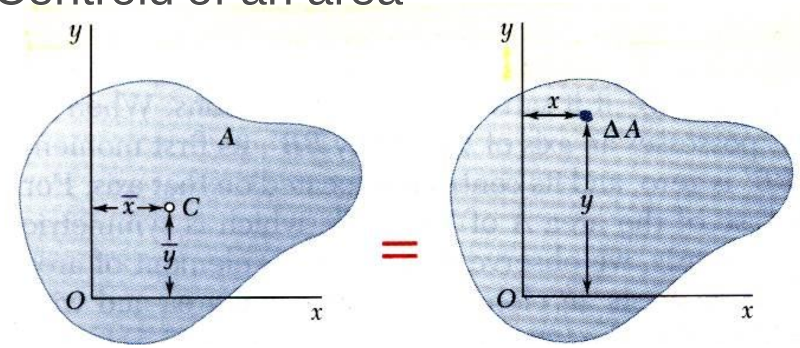
$$= \int x\,dW$$ By Integration

$$\sum M_y \quad \bar{y}W = \sum y\Delta W$$ By Summation

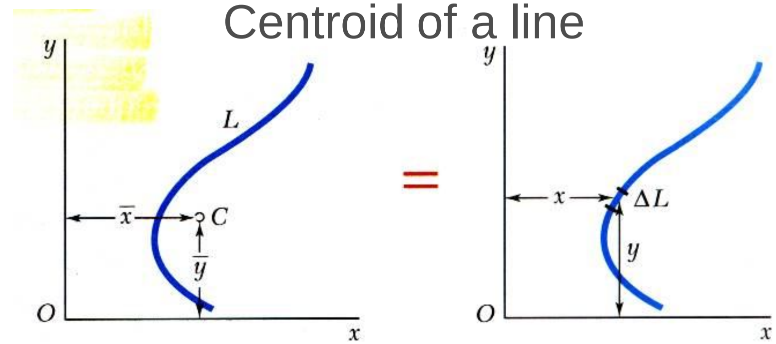
$$= \int y\,dW$$ By Integration

Centroids and First Moments of Areas and Lines

Centroid of an area



Centroid of a line



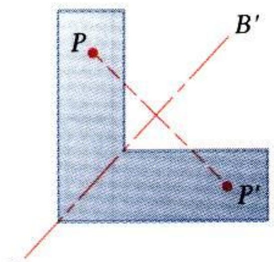
$$\bar{x}W = \int x\,dW$$
$$\bar{x}(\gamma At) = \int x(\gamma t)dA$$
$$\bar{x}A = \int x\,dA = Q_y$$
$$= \text{first moment with respect to } y$$
$$\bar{y}A = \int y\,dA = Q_x$$
$$= \text{first moment with respect to } x$$

$$\bar{x}W = \int x\,dW$$
$$\bar{x}(\gamma La) = \int x(\gamma a)dL$$
$$\bar{x}L = \int x\,dL$$
$$\bar{y}L = \int y\,dL$$

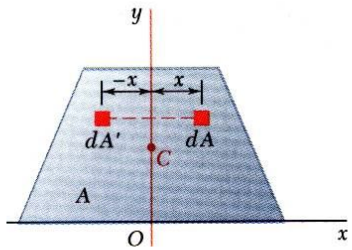
where γ is density and t is thickness

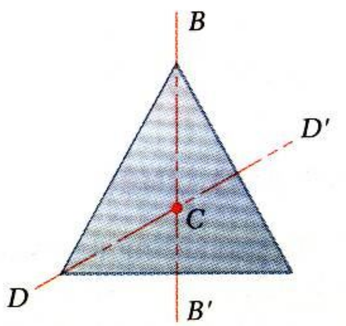
The centroid is located at $(\bar{x}, \bar{y})$

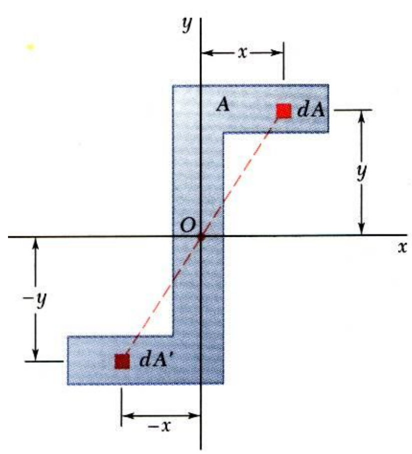
First Moments of Areas and Lines



(a)

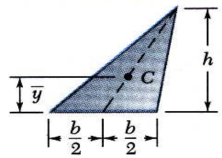
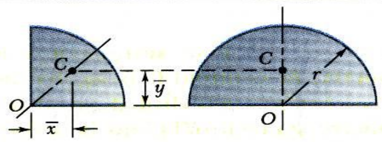
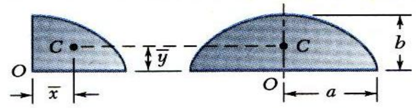
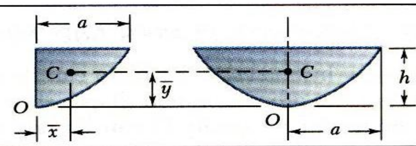
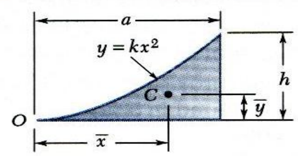
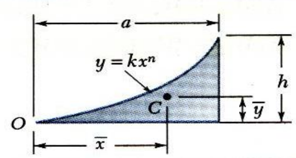
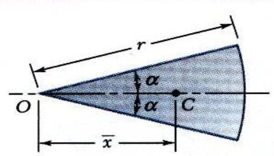




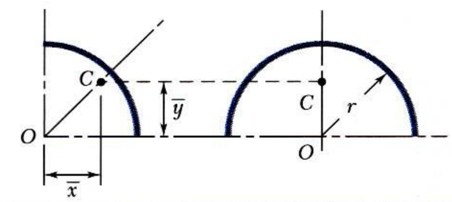
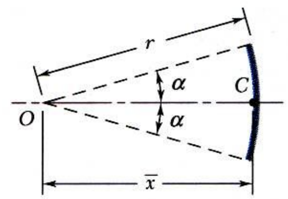


- An area is symmetric with respect to an axis *BB'* if for every point *P* there exists a point *P'* such that *PP'* is perpendicular to *BB'* and is divided into two equal parts by *BB'.*
- The first moment of an area with respect to a line of symmetry is zero.
- If an area possesses a line of symmetry, its centroid lies on that axis
- If an area possesses two lines of symmetry, its centroid lies at their intersection.
- An area is symmetric with respect to a center *O* if for every element *dA* at (*x*,*y*) there exists an area *dA'* of equal area at (*-x*,*-y*).
- The centroid of the area coincides with the center of symmetry.

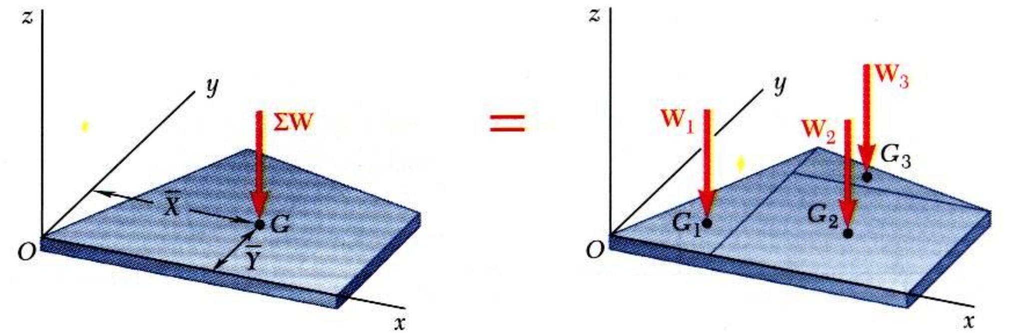
Centroids of Common Shapes of Areas

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

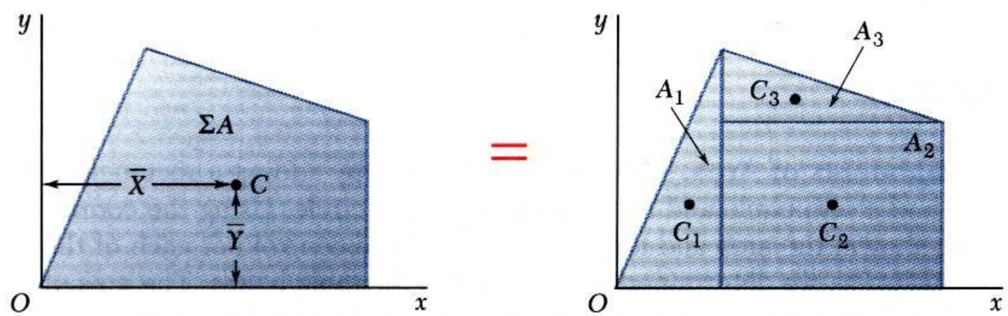
Centroids of Common Shapes of Lines

Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Composite Plates and Areas

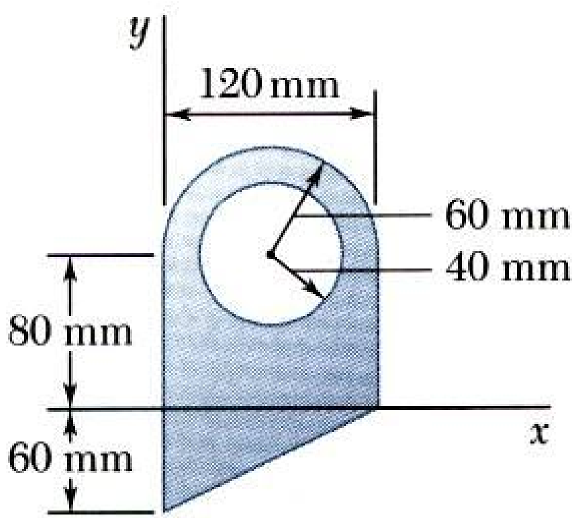


$$\bar{X}\sum W = \sum \bar{x} W$$
$$\bar{Y}\sum W = \sum \bar{y} W$$



$$\bar{X}\sum A = \sum \bar{x} A$$
$$\bar{Y}\sum A = \sum \bar{y} A$$

Example 2-8

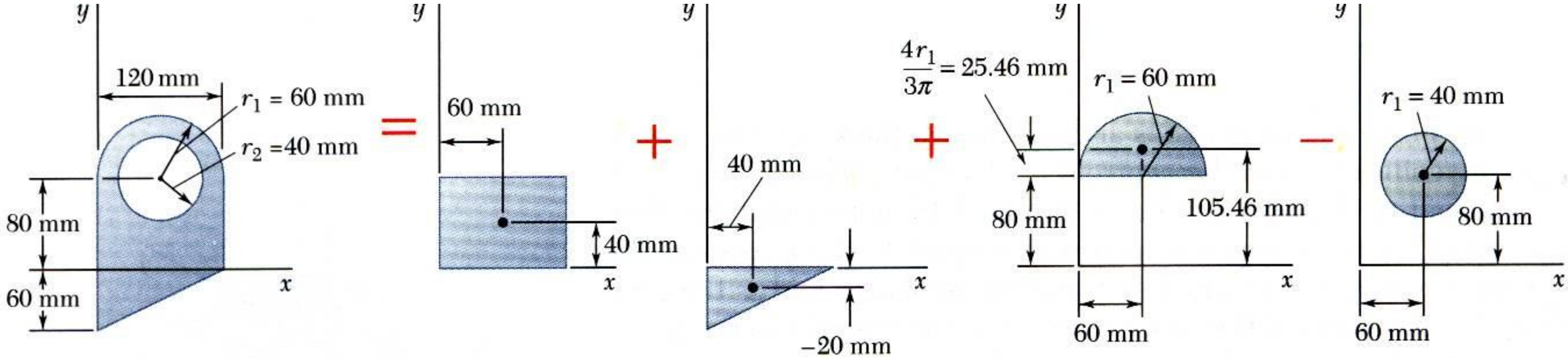


For the plane area shown, determine the first moments with respect to the *x* and *y* axes and the location of the centroid.

SOLUTION:

- Divide the area into a triangle, rectangle, and semicircle with a circular cutout.
- Calculate the first moments of each area with respect to the axes.
- Find the total area and first moments of the triangle, rectangle, and semicircle. Subtract the area and first moment of the circular cutout.
- Compute the coordinates of the area centroid by dividing the first moments by the total area.

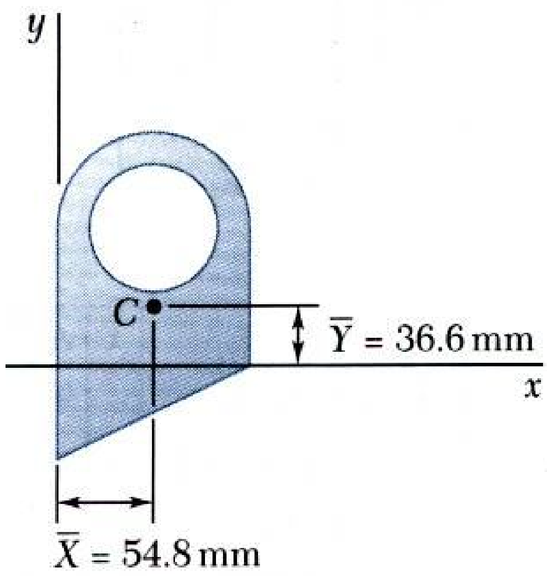
Example 2-8



Component	A, mm²	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm³	$\bar{y}A$, mm³
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

$$\boxed{\begin{aligned} Q_x &= +506.2 \times 10^3\,\text{mm}^3 \\ Q_y &= +757.7 \times 10^3\,\text{mm}^3 \end{aligned}}$$

Example 2-8



$$\bar{X} = \frac{\sum \bar{x} A}{\sum A} = \frac{+757.7 \times 10^3 \,\text{mm}^3}{13.828 \times 10^3 \,\text{mm}^2}$$

$$\boxed{\bar{X} = 54.8 \,\text{mm}}$$

$$\bar{Y} = \frac{\sum \bar{y} A}{\sum A} = \frac{+506.2 \times 10^3 \,\text{mm}^3}{13.828 \times 10^3 \,\text{mm}^2}$$

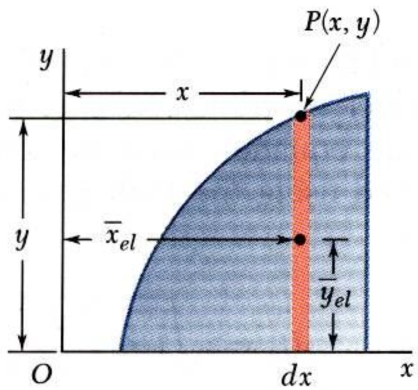
$$\boxed{\bar{Y} = 36.6 \,\text{mm}}$$

Determination of Centroids by Integration

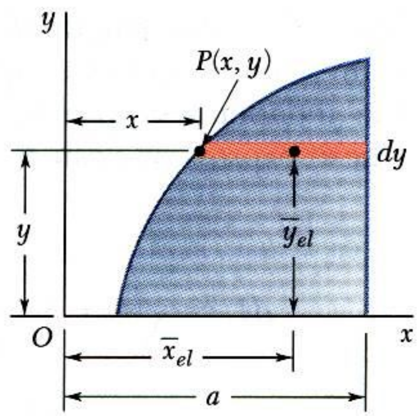
$$\bar{x}A = \int x\,dA = \iint x\,dxdy = \int \bar{x}_{el}\,dA$$

$$\bar{y}A = \int y\,dA = \iint y\,dxdy = \int \bar{y}_{el}\,dA$$

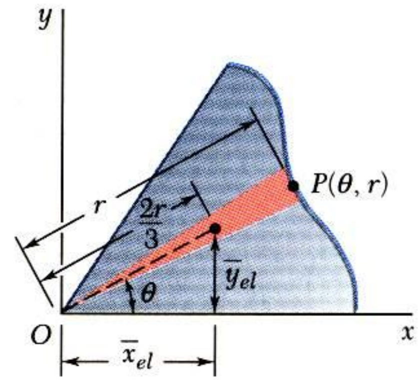
- Double integration to find the first moment may be avoided by defining *dA* as a thin rectangle or strip.



$$\bar{x}A = \int \bar{x}_{el}\,dA$$
$$= \int x\left(y\,dx\right)$$
$$\bar{y}A = \int \bar{y}_{el}\,dA$$
$$= \int \frac{y}{2}\left(y\,dx\right)$$

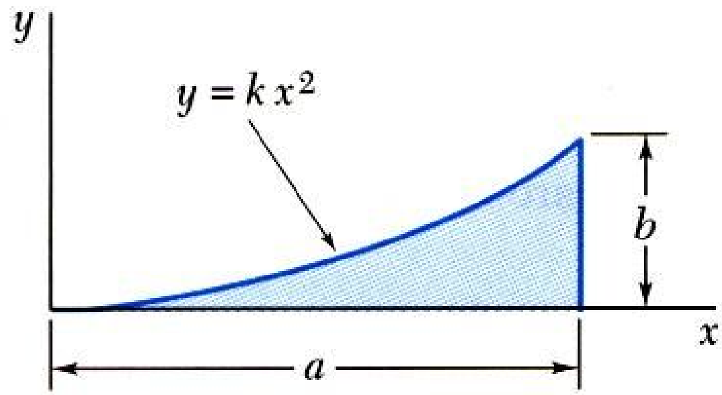


$$\bar{x}A = \int \bar{x}_{el}\,dA$$
$$= \int \frac{a+x}{2}\left[\left(a-x\right)dy\right]$$
$$\bar{y}A = \int \bar{y}_{el}\,dA$$
$$= \int y\left[\left(a-x\right)dy\right]$$



$$\bar{x}A = \int \bar{x}_{el}\,dA$$
$$= \int \frac{2r}{3}\cos\theta\left(\frac{1}{2}r^2 d\theta\right)$$
$$\bar{y}A = \int \bar{y}_{el}\,dA$$
$$= \int \frac{2r}{3}\sin\theta\left(\frac{1}{2}r^2 d\theta\right)$$

Example 2-9

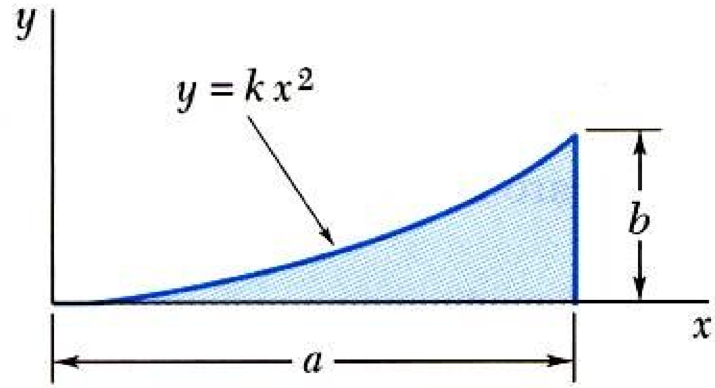


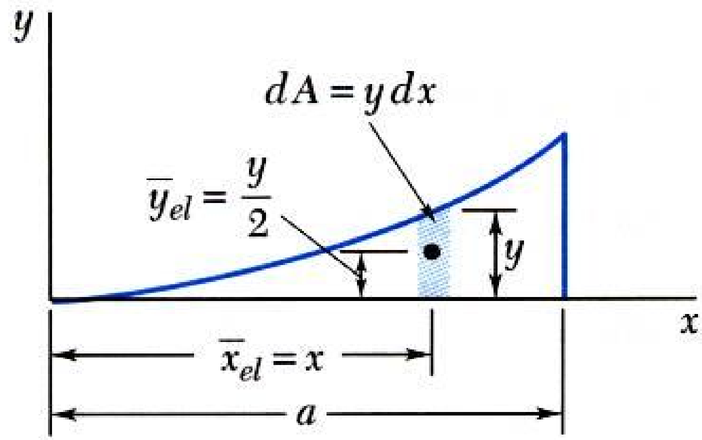
Determine by direct integration the location of the centroid of a parabolic spandrel.

SOLUTION:

- Determine the constant k.
- Evaluate the total area.
- Using either vertical or horizontal strips, perform a single integration to find the first moments.
- Evaluate the centroid coordinates.

Example 2-9





$$y = k\,x^2$$

$$b = k\,a^2 \;\Rightarrow k = \frac{b}{a^2}$$

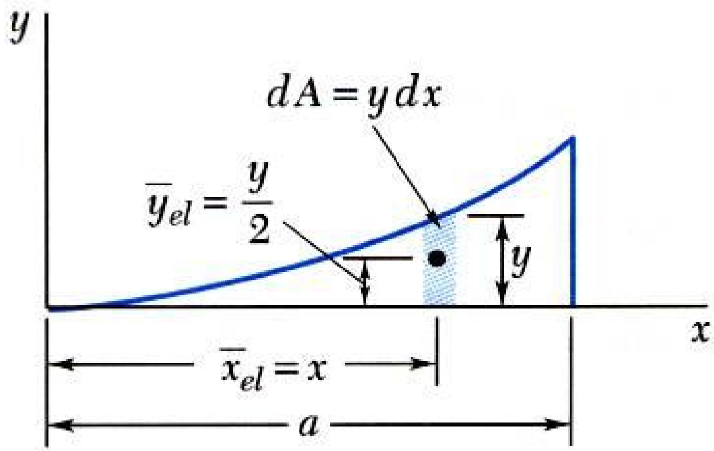
$$y = \frac{b}{a^2}x^2 \quad or \quad x = \frac{a}{b^{1/2}}y^{1/2}$$

$$A = \int dA$$

$$= \int y\,dx = \int_0^a \frac{b}{a^2}x^2 dx = \left[\frac{b}{a^2}\frac{x^3}{3}\right]_0^a$$

$$= \frac{ab}{3}$$

Example 2-9



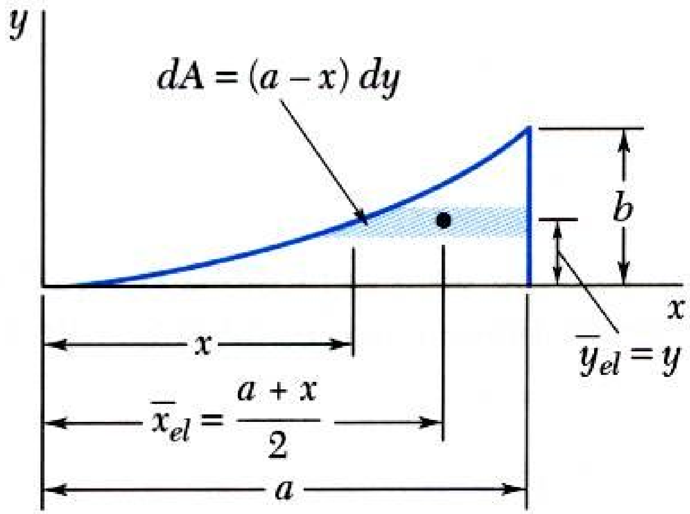
$$Q_y = \int \bar{x}_{el}\,dA = \int xy\,dx = \int_0^a x\left(\frac{b}{a^2}x^2\right)dx$$

$$= \left[\frac{b}{a^2}\frac{x^4}{4}\right]_0^a = \frac{a^2 b}{4}$$

$$Q_x = \int \bar{y}_{el}\,dA = \int \frac{y}{2}\,y\,dx = \int_0^a \frac{1}{2}\left(\frac{b}{a^2}x^2\right)^2 dx$$

$$= \left[\frac{b^2}{2a^4}\frac{x^5}{5}\right]_0^a = \frac{ab^2}{10}$$

Example 2-9



- Or, using horizontal strips, perform a single integration to find the first moments.

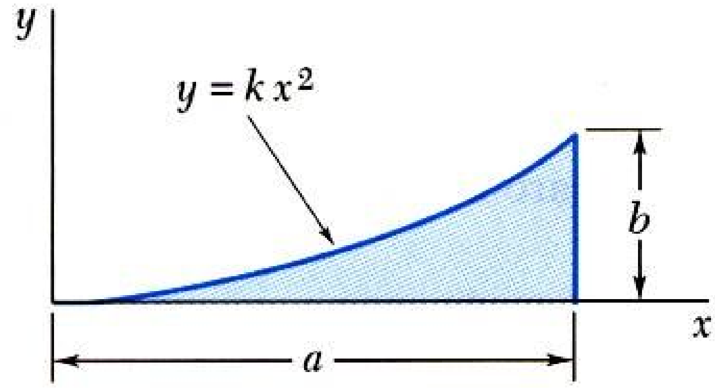
$$Q_y = \int \bar{x}_{el}\, dA = \int \frac{a+x}{2}(a-x)\,dy = \int_0^b \frac{a^2 - x^2}{2}\,dy$$

$$= \frac{1}{2}\int_0^b \left(a^2 - \frac{a^2}{b} y \right) dy = \frac{a^2 b}{4}$$

$$Q_x = \int \bar{y}_{el}\, dA = \int y(a-x)\,dy = \int y\left(a - \frac{a}{b^{1/2}} y^{1/2} \right) dy$$

$$= \int_0^b \left(ay - \frac{a}{b^{1/2}} y^{3/2} \right) dy = \frac{ab^2}{10}$$

Example 2-9



$$\bar{x}A = Q_y$$

$$\bar{x}\frac{ab}{3} = \frac{a^2 b}{4}$$

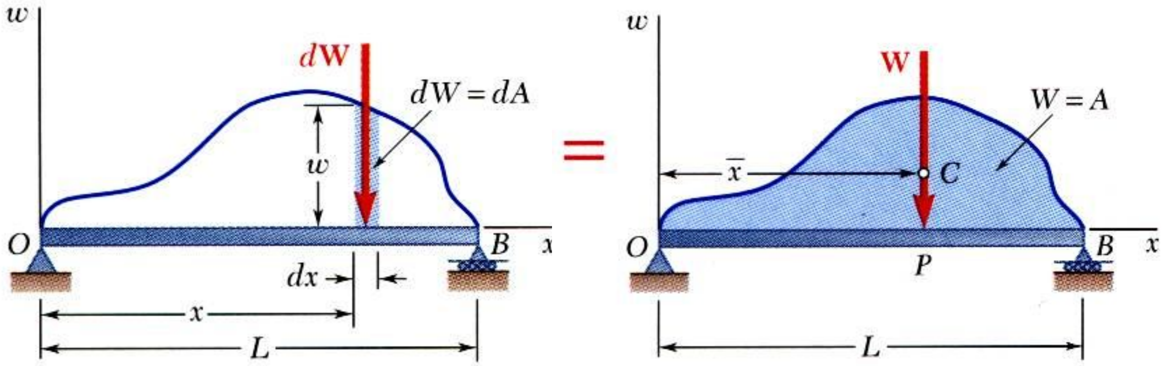
$$\boxed{\bar{x} = \frac{3}{4}a}$$

$$\bar{y}A = Q_x$$

$$\bar{y}\frac{ab}{3} = \frac{ab^2}{10}$$

$$\boxed{\bar{y} = \frac{3}{10}b}$$

Distributed Loads on Beams



$$W = \int_0^L w\,dx = \int dA = A$$

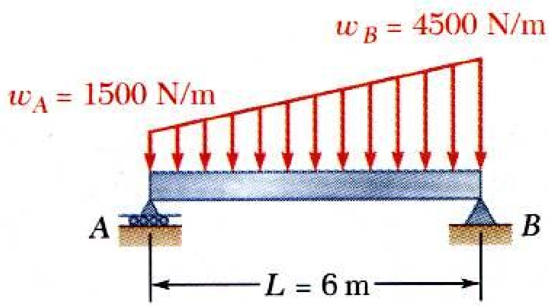
- A distributed load is represented by plotting the load per unit length, *w* (N/m) . The total load is equal to the area under the load curve.

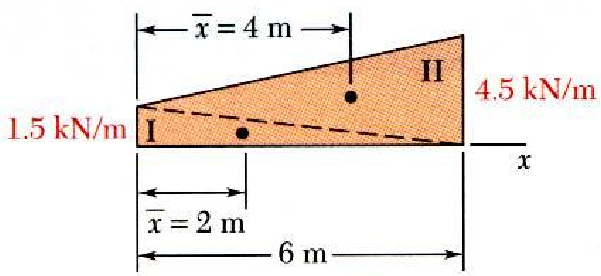
$$(OP)W = \int x\,dW$$

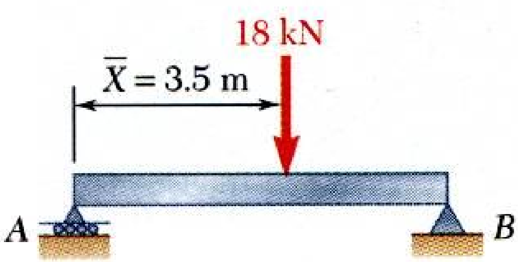
$$(OP)A = \int_0^L x\,dA = \bar{x}A$$

- A distributed load can be replace by a concentrated load with a magnitude equal to the area under the load curve and a line of action passing through the area centroid.

Example 2-10







SOLUTION:

- The magnitude of the concentrated load is equal to the total load or the area under the curve

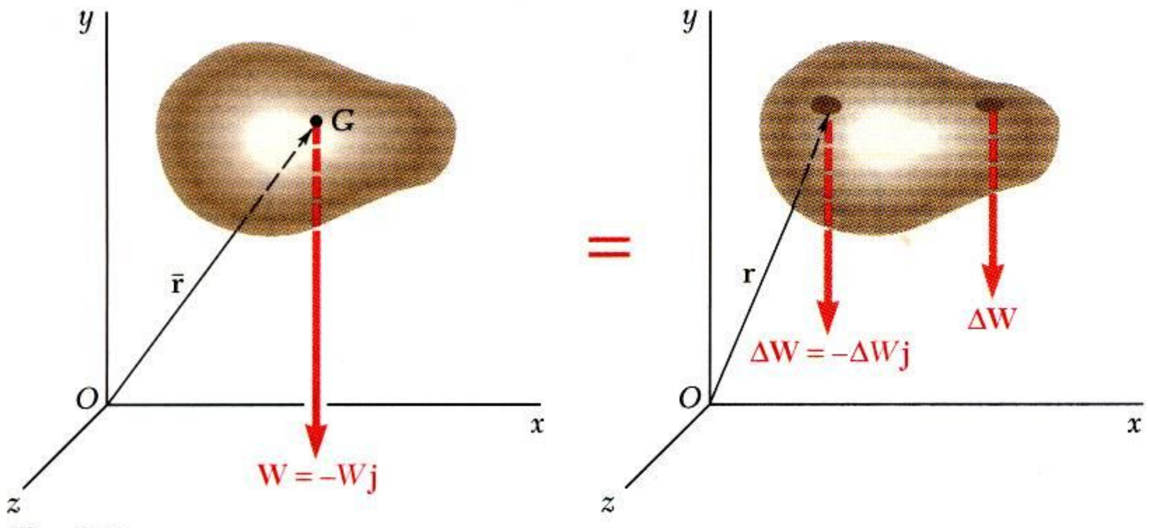
$$\boxed{F = 18.0\ \text{kN}}$$

- The line of action of the concentrated load passes through the centroid of the area under the curve.

$$\bar{X} = \frac{63\ \text{kN}\cdot\text{m}}{18\ \text{kN}} \qquad \boxed{\bar{X} = 3.5\ \text{m}}$$

Component	A, kN	$\bar{x}$, m	$\bar{x}A$, kN · m
Triangle I	4.5	2	9
Triangle II	13.5	4	54
	$\Sigma A = 18.0$		$\Sigma \bar{x}A = 63$

Center of Gravity of a 3D Body: Centroid of a Volume



- Center of gravity *G*

$$-W\vec{j} = \sum\left(-\Delta W\vec{j}\right)$$

$$\vec{r}_G \times \left(-W\vec{j}\right) = \sum\left[\vec{r} \times \left(-\Delta W\vec{j}\right)\right]$$

$$\vec{r}_G W \times \left(-\vec{j}\right) = \left(\sum \vec{r}\Delta W\right) \times \left(-\vec{j}\right)$$

$$W = \int dW \qquad \vec{r}_G W = \int \vec{r}\, dW$$

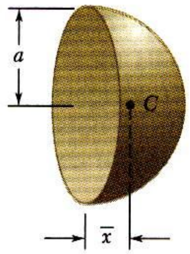
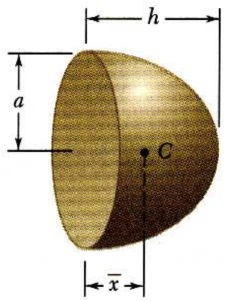
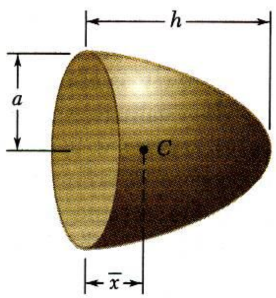
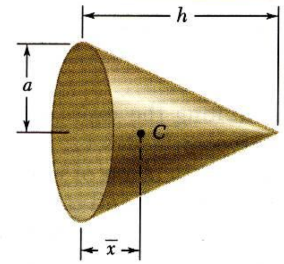
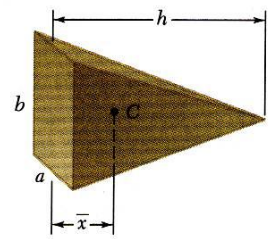
- Results are independent of body orientation,

$$\bar{x}W = \int x\,dW \quad \bar{y}W = \int y\,dW \quad \bar{z}W = \int z\,dW$$

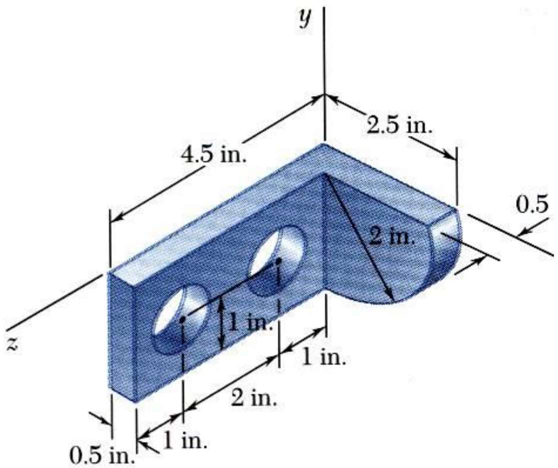
$$W = \gamma V \text{ and } dW = \gamma\, dV$$

$$\bar{x}V = \int x\,dV \quad \bar{y}V = \int y\,dV \quad \bar{z}V = \int z\,dV$$

Centroids of Common 3D Shapes

Shape		$\bar{x}$	Volume
Hemisphere		$\frac{3a}{8}$	$\frac{2}{3}\pi a^3$
Semiellipsoid of revolution		$\frac{3h}{8}$	$\frac{2}{3}\pi a^2 h$
Paraboloid of revolution		$\frac{h}{3}$	$\frac{1}{2}\pi a^2 h$
Cone		$\frac{h}{4}$	$\frac{1}{3}\pi a^2 h$
Pyramid		$\frac{h}{4}$	$\frac{1}{3}abh$

Composite 3D Bodies

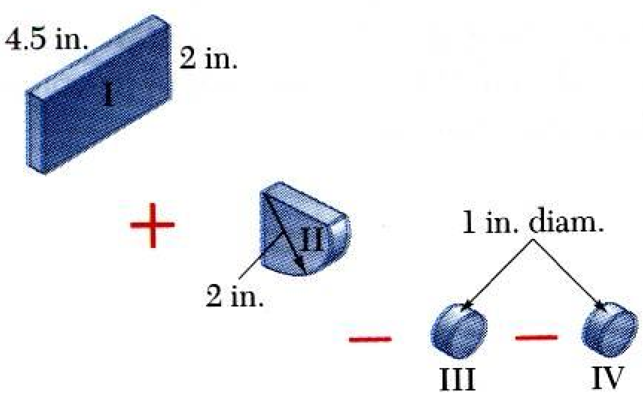


- Moment of the total weight concentrated at the center of gravity G is equal to the sum of the moments of the weights of the component parts.

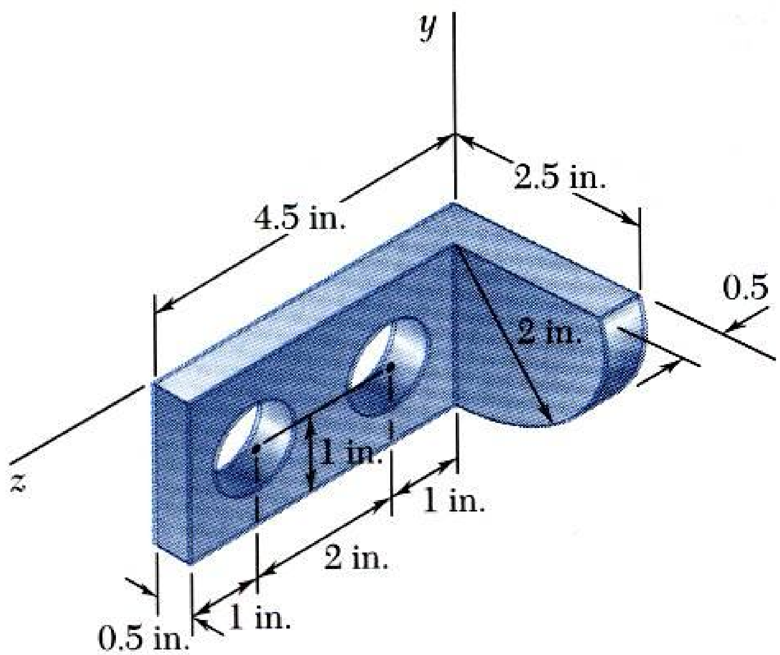
$$\bar{X}\sum W = \sum \bar{x}W \quad \bar{Y}\sum W = \sum \bar{y}W \quad \bar{Z}\sum W = \sum \bar{z}W$$

- For homogeneous bodies,

$$\bar{X}\sum V = \sum \bar{x}V \quad \bar{Y}\sum V = \sum \bar{y}V \quad \bar{Z}\sum V = \sum \bar{z}V$$



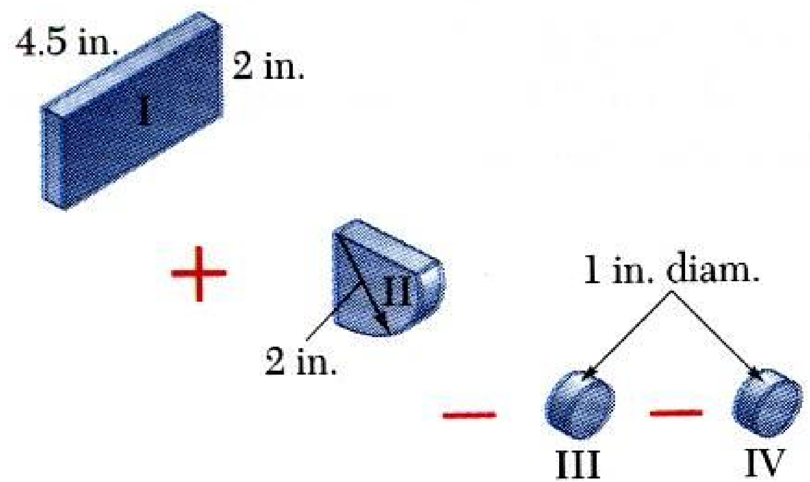
Example 2-11



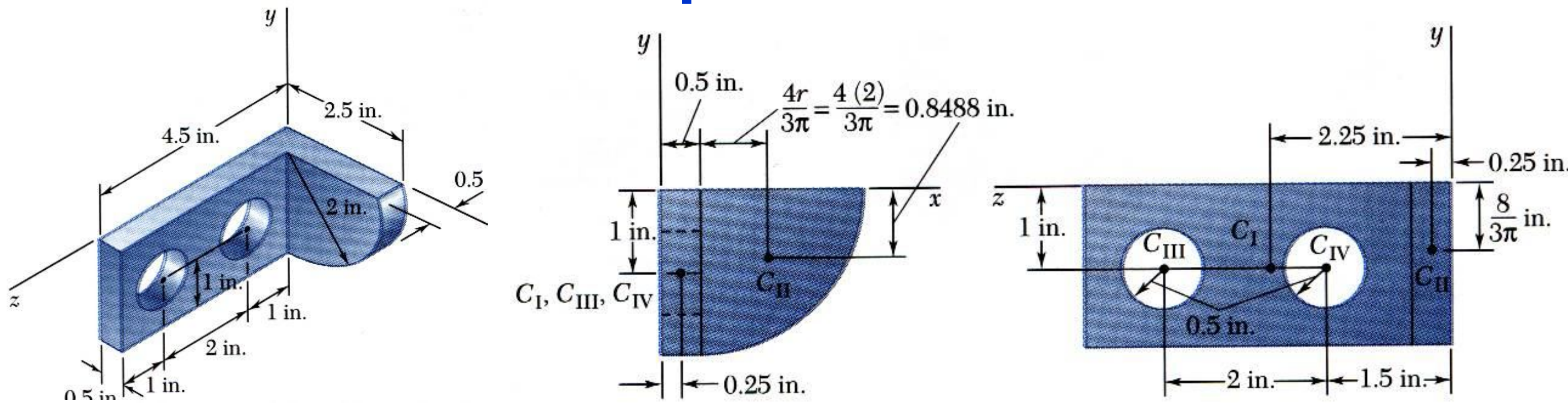
Locate the center of gravity of the steel machine element. The diameter of each hole is 1 in.

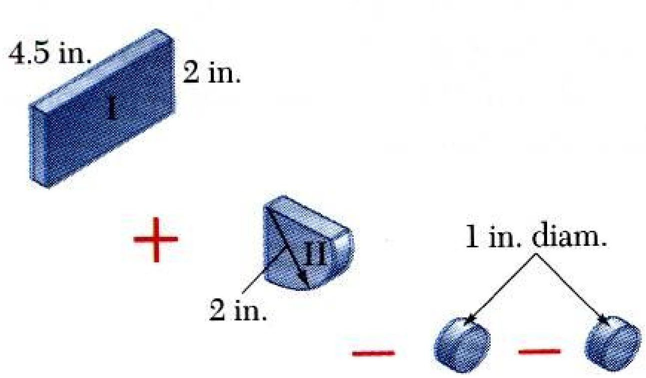
SOLUTION:

- Form the machine element from a rectangular parallelepiped and a quarter cylinder and then subtracting two 1-in. diameter cylinders.



Example 2-11

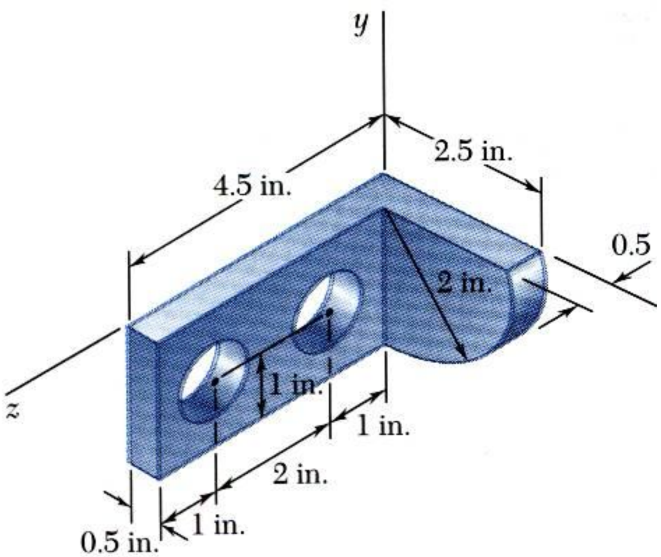




	V, in³	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{x}V$, in⁴	$\bar{y}V$, in⁴	$\bar{z}V$, in⁴
I	$(4.5)(2)(0.5) = 4.5$	0.25	−1	2.25	1.125	−4.5	10.125
II	$\frac{1}{4}\pi(2)^2(0.5) = 1.571$	1.3488	−0.8488	0.25	2.119	−1.333	0.393
III	$-\pi(0.5)^2(0.5) = -0.3927$	0.25	−1	3.5	−0.098	0.393	−1.374
IV	$-\pi(0.5)^2(0.5) = -0.3927$	0.25	−1	1.5	−0.098	0.393	−0.589
	$\Sigma V = 5.286$				$\Sigma\bar{x}V = 3.048$	$\Sigma\bar{y}V = -5.047$	$\Sigma\bar{z}V = 8.555$

Example 2-11

	V, in^3	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{x}V$, in^4	$\bar{y}V$, in^4	$\bar{z}V$, in^4
I	$(4.5)(2)(0.5) = 4.5$	0.25	−1	2.25	1.125	−4.5	10.125
II	$\frac{1}{4}\pi(2)^2(0.5) = 1.571$	1.3488	−0.8488	0.25	2.119	−1.333	0.393
III	$-\pi(0.5)^2(0.5) = -0.3927$	0.25	−1	3.5	−0.098	0.393	−1.374
IV	$-\pi(0.5)^2(0.5) = -0.3927$	0.25	−1	1.5	−0.098	0.393	−0.589
	$\Sigma V = 5.286$				$\Sigma\bar{x}V = 3.048$	$\Sigma\bar{y}V = -5.047$	$\Sigma\bar{z}V = 8.555$



$$\bar{X} = \sum \bar{x}V / \sum V = \left(3.08\,\text{in}^4\right) / \left(5.286\,\text{in}^3\right)$$

$$\boxed{\bar{X} = 0.577\,\text{in.}}$$

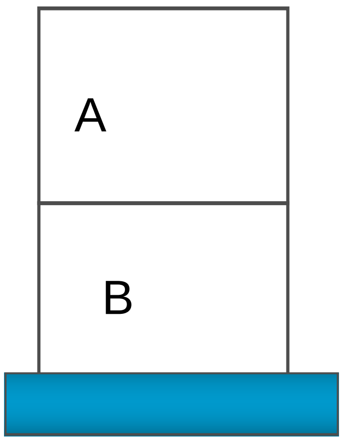
$$\bar{Y} = \sum \bar{y}V / \sum V = \left(-5.047\,\text{in}^4\right) / \left(5.286\,\text{in}^3\right)$$

$$\boxed{\bar{Y} = 0.577\,\text{in.}}$$

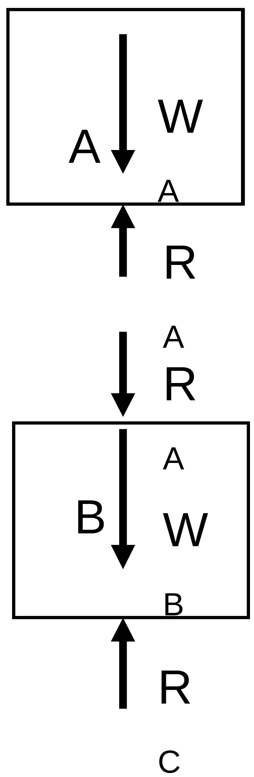
$$\bar{Z} = \sum \bar{z}V / \sum V = \left(1.618\,\text{in}^4\right) / \left(5.286\,\text{in}^3\right)$$

$$\boxed{\bar{Z} = 0.577\,\text{in.}}$$

Free-Body Diagram



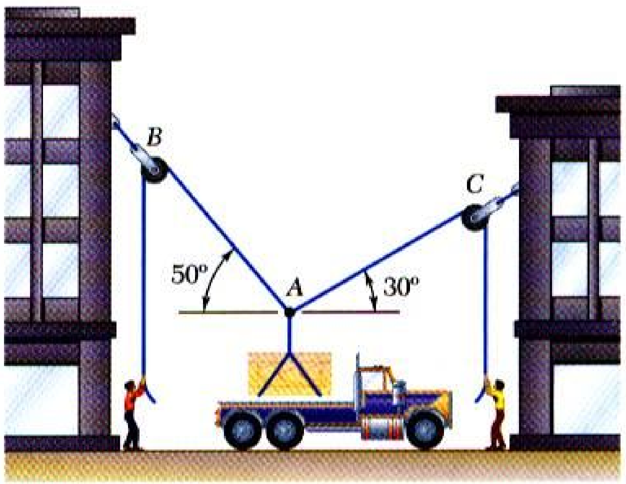
Two blocks of wood

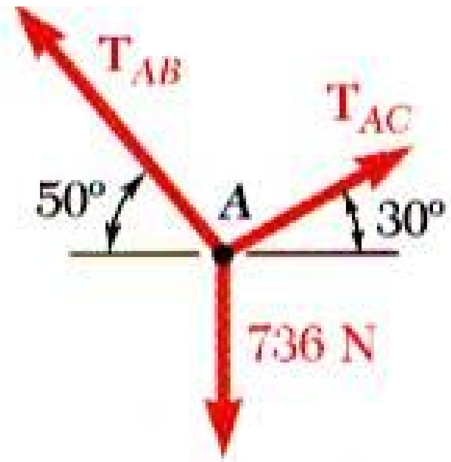


Steps for Drawing a Free-body Diagram

- Decide which body (or group of bodies) is under consideration , imagine it to be isolated from all other bodies, and sketch the outlined shape of the body.
- Indicate by means of arrows all external forces and moments acting on the body. This should include (a) the weight of the body, (b) all external forces (c) reactions at supports and other contacts with other bodies.
- For each unknown force, indicate its point of application and assumed a direction, if it is unknown.
- Include the dimensions and angles needed for computing moments of forces and resolving forces.
- The weight of a body always acts vertically downward through the centre of gravity of the body.
- Forces acting at joints should considered as internal forces and should not be shown if the joint is not separated. Once the joint is separated, the forces acting at the joints must consider as external forces and indicated on the free-body diagram

Free-Body Diagrams

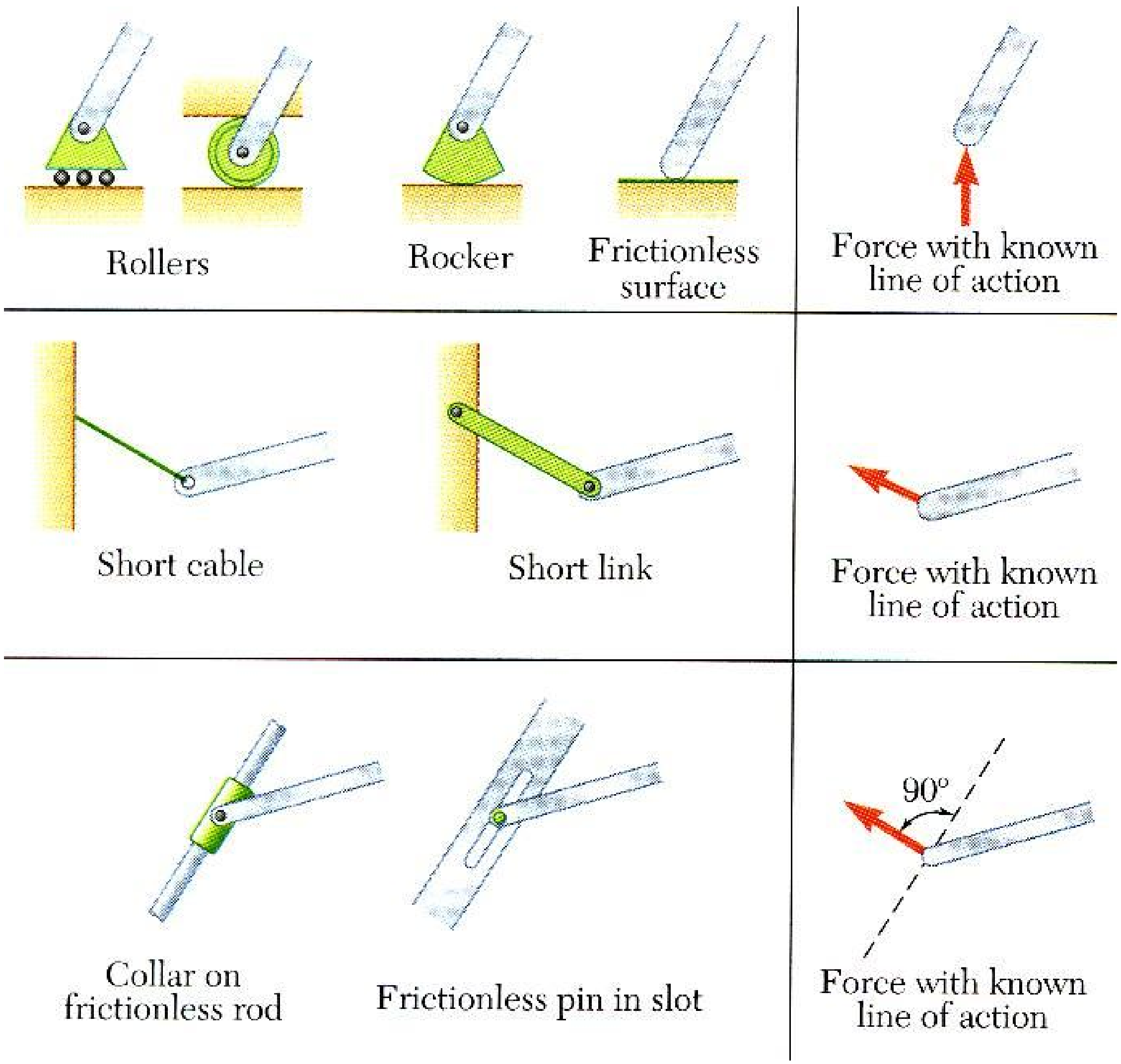




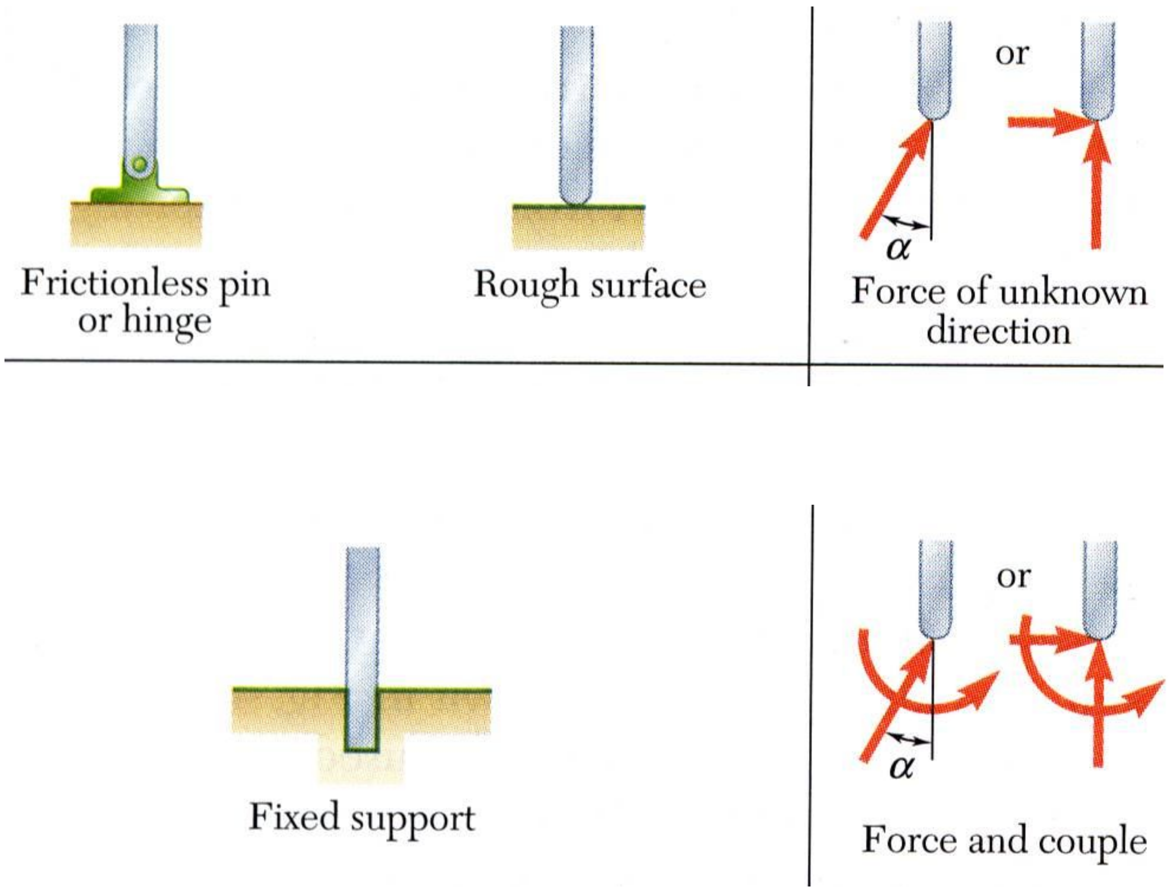
Space Diagram: A sketch showing the physical conditions of the problem.

Free-Body Diagram: A sketch showing only the forces on the selected particle

Reactions at Supports and Connections for a 2D Structure



Reactions at Supports and Connections for a 2D Structure



Equilibrium Conditions for a Particle

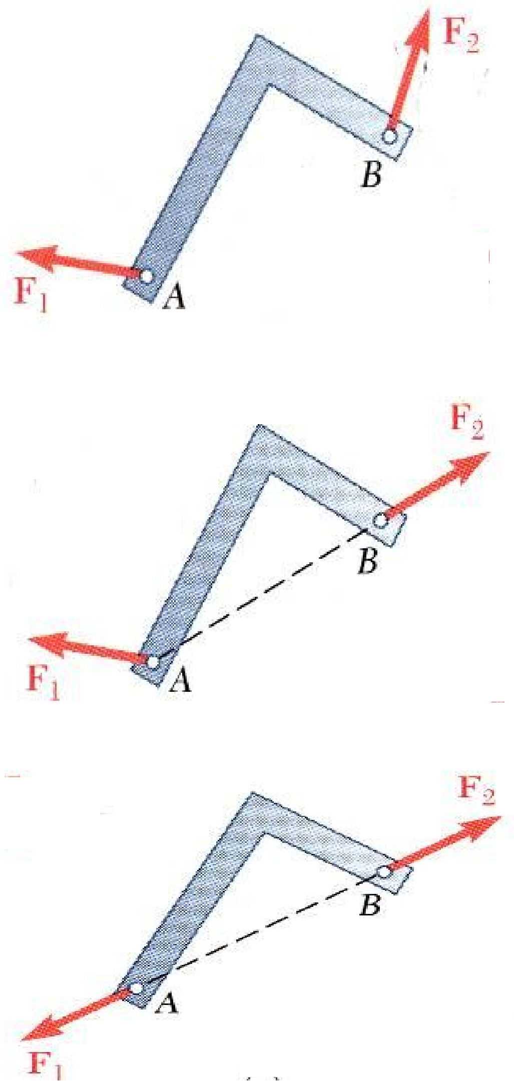
A particle is in equilibrium if

- it is at rest relative to an initial reference frame
- the body moves with constant velocity along a straight line relative to an initial frame

The conditions are :

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

Equilibrium of a Two-Force Body

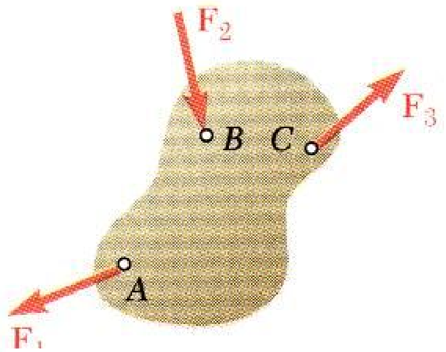


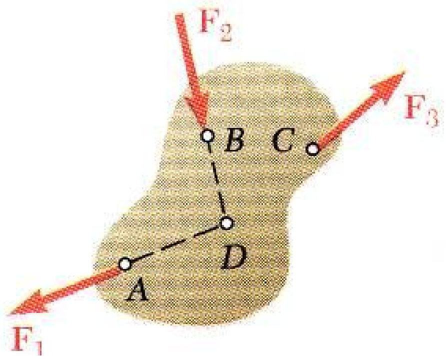
- For static equilibrium, the sum of moments about *A* must be zero. The moment of $\boldsymbol{F_2}$ must be zero. It follows that the line of action of $\boldsymbol{F_2}$ must pass through *A*.

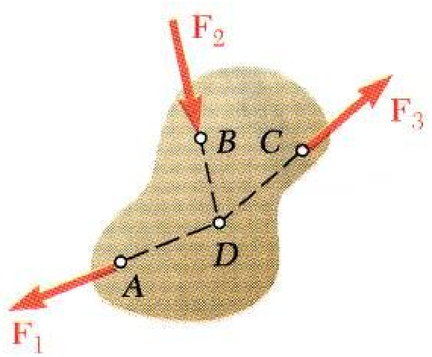
  - Similarly, the line of action of $\boldsymbol{F_1}$ must pass through *B* for the sum of moments about *B* to be zero.

  - Requiring that the sum of forces in any direction be zero leads to the conclusion that $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ must have equal magnitude but opposite sense.

Equilibrium of a Three-Force Body

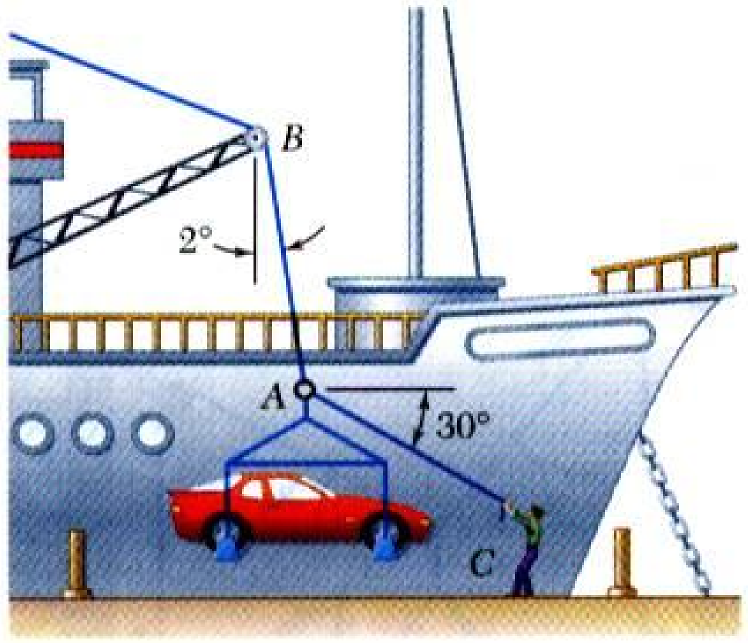






- Consider a rigid body subjected to forces acting at only 3 points.
- Assuming that their lines of action intersect, the moment of ***F***$_1$ and ***F***$_2$ about the point of intersection represented by *D* is zero.
- Since the rigid body is in equilibrium, the sum of the moments of ***F***$_1$, ***F***$_2$, and ***F***$_3$ about any axis must be zero. It follows that the moment of ***F***$_3$ about *D* must be zero as well and that the line of action of ***F***$_3$ must pass through *D*.
- The lines of action of the three forces must be concurrent or parallel.

Example 2-7

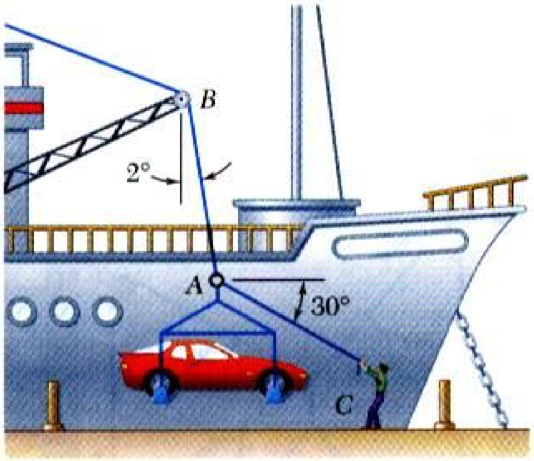


In a ship-unloading operation, a 3500-kg automobile is supported by a cable. A rope is tied to the cable and pulled to center the automobile over its intended position. What is the tension in the rope?

SOLUTION:

- Construct a free-body diagram for the particle at the junction of the rope and cable.
- Apply the conditions for equilibrium by creating a closed polygon from the forces applied to the particle.
- Apply trigonometric relations to determine the unknown force magnitudes.

Example 2-7

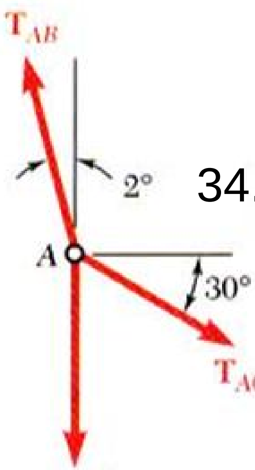


SOLUTION:

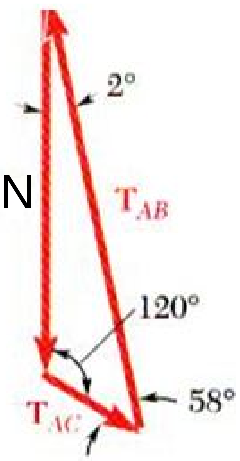
- Construct a free-body diagram for the particle at *A*.

- Apply the conditions for equilibrium.

- Solve for the unknown force magnitudes.



34.3 kN



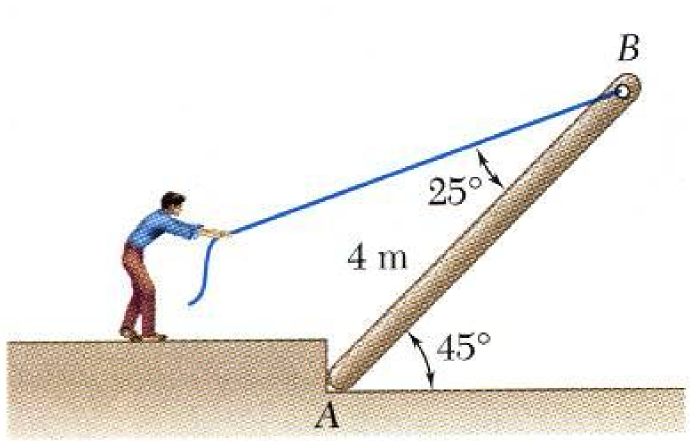
3500 x 9.81

$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 2°} = \frac{34.3x1000}{\sin 58°}$$

$$T_{AB} = 35.1\,\text{kN}$$

$$\boxed{T_{AC} = 1.41\,\text{kN}}$$

Example 2-8



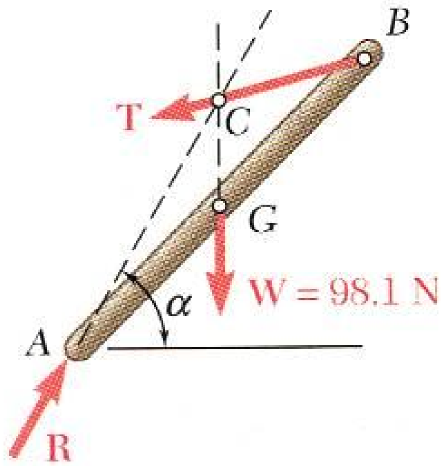
A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

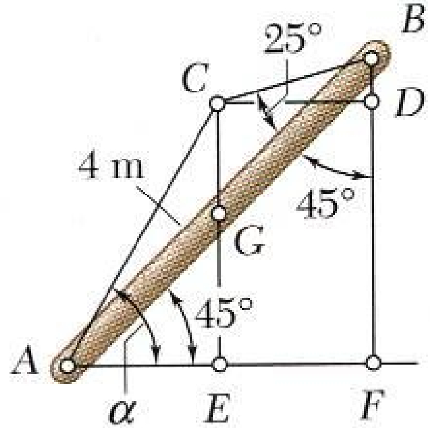
Find the tension in the rope and the reaction at *A*.

SOLUTION:

- Create a free-body diagram of the joist. Note that the joist is a 3 force body acted upon by the rope, its weight, and the reaction at *A*.
- The three forces must be concurrent for static equilibrium. Therefore, the reaction ***R*** must pass through the intersection of the lines of action of the weight and rope forces. Determine the direction of the reaction force ***R***.
- Utilize a force triangle to determine the magnitude of the reaction force ***R***.

Example 2-8





- Create a free-body diagram of the joist.
- Determine the direction of the reaction force ***R***.

$$AF = AB\cos 45 = (4\,\text{m})\cos 45 = 2.828\,\text{m}$$

$$CD = AE = \tfrac{1}{2}AF = 1.414\,\text{m}$$

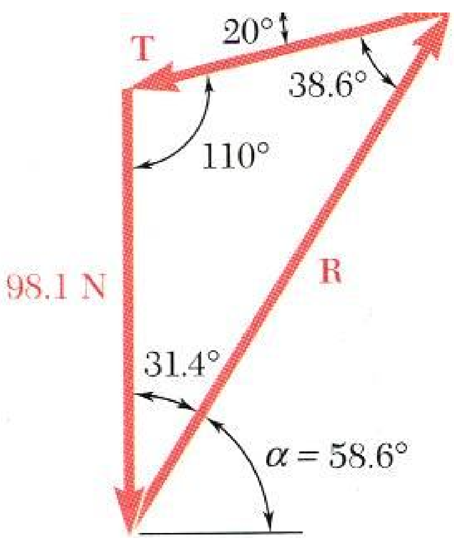
$$BD = CD\cot(45+20) = (1.414\,\text{m})\tan 20 = 0.515\,\text{m}$$

$$CE = BF - BD = (2.828 - 0.515)\,\text{m} = 2.313\,\text{m}$$

$$\tan\alpha = \frac{CE}{AE} = \frac{2.313}{1.414} = 1.636$$

$$\boxed{\alpha = 58.6^\circ}$$

Example 2-8



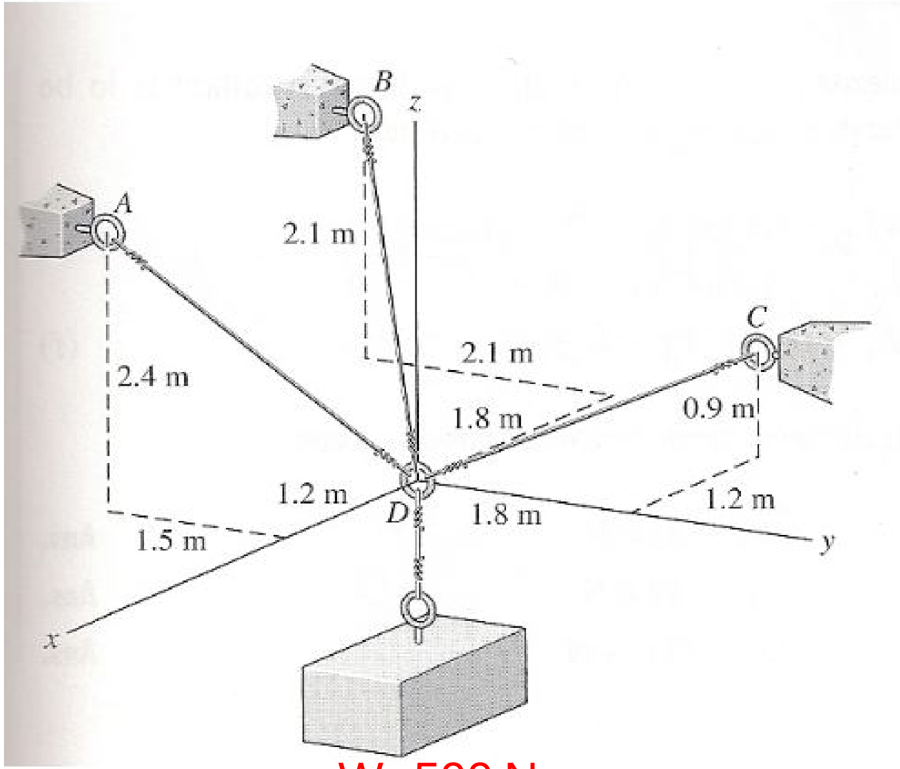
- Determine the magnitude of the reaction force ***R***.

$$\frac{T}{\sin 31.4^\circ} = \frac{R}{\sin 110^\circ} = \frac{98.1\,\mathrm{N}}{\sin 38.6^\circ}$$

$$\boxed{\begin{aligned} T &= 81.9\,\mathrm{N} \\ R &= 147.8\,\mathrm{N} \end{aligned}}$$

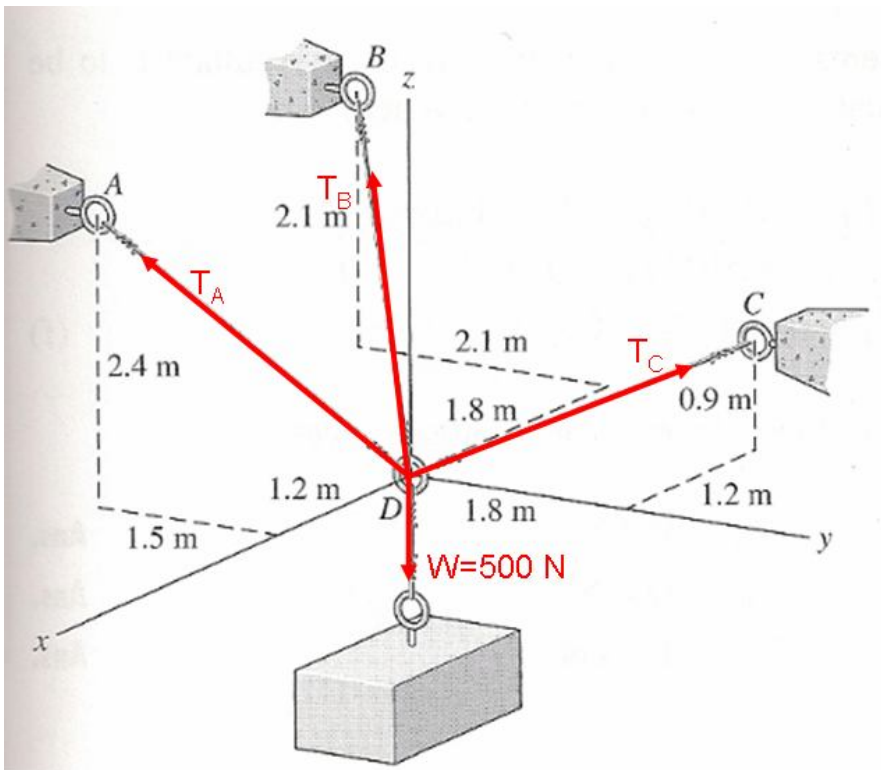
Example 2-9

A 500-N block is supported by a system of cables as shown in . Determine the tension in the cables A, B and C.



W=500 N

Example 2-9

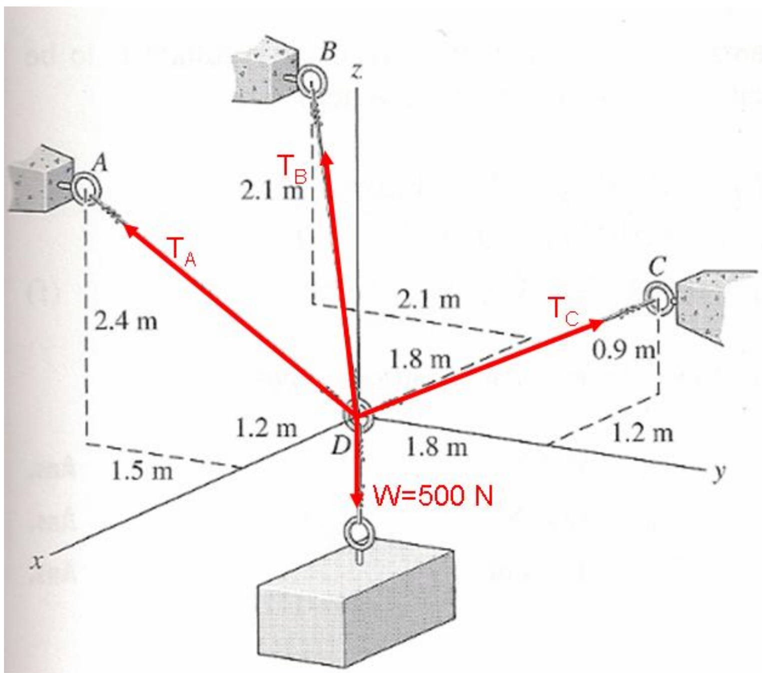


$$r_A = (1.2-0)i + (-1.5-0)j + (2.4-0)k$$
$$r_A = 1.2i - 1.5j + 2.4k$$
$$|r_A| = \sqrt{\left(1.2^2 + (-1.5)^2 + 2.4^2\right)}$$
$$|r_A| = 2.773$$
$$\hat{r}_A = \frac{r_A}{|r_A|} = 0.4327i - 0.5409j + 0.8655k$$

$$r_B = -1.8i - 2.1j + 2.1k$$
$$|r_B| = \sqrt{\left((-1.8)^2 + (-2.1)^2 + 2.1^2\right)}$$
$$|r_B| = 3.4727$$
$$\hat{r}_B = \frac{r_B}{|r_B|} = -0.5183i - 0.6047j - 0.6047k$$

$$r_C = -1.2i + 1.8j + 0.9k$$
$$|r_C| = \sqrt{\left((-1.2)^2 + 1.8^2 + 0.9^2\right)}$$
$$|r_C| = 2.3431$$
$$\hat{r}_C = \frac{r_C}{|r_C|} = -0.5121i + 0.7682j + 0.3841k$$

Example 2-9



$$\sum F = 0$$

$$T_A(0.4327i - 0.5409j + 0.8655k) +$$
$$T_B(-0.5183i - 0.6047j - 0.6047k) +$$
$$T_C(-0.5121i + 0.7682j + 0.3841k) - 500k = 0$$

$$0.4327T_A - 0.5183T_B - 0.5121T_C = 0 \quad (1)$$

$$-0.5409T_A - 0.6047T_B + 0.7682T_C = 0 \quad (2)$$

$$0.8655T_A - 0.6047T_B + 0.3841T_C - 500 = 0 \quad (3)$$

Equilibrium of a Rigid Body in 3D

- Six scalar equations are required to express the conditions for the equilibrium of a rigid body in the general three dimensional case.

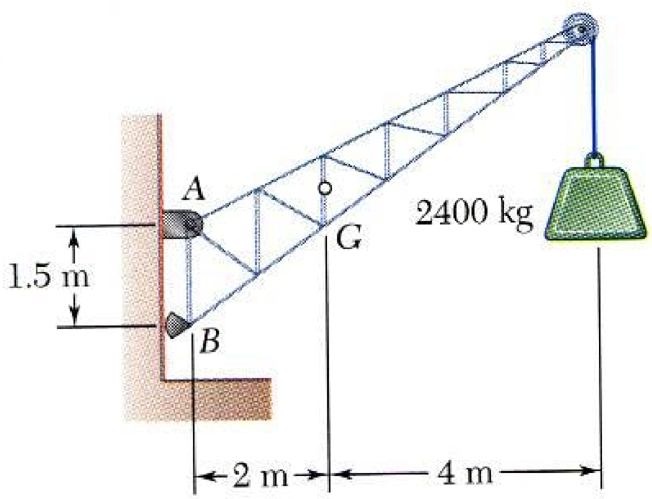
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

- These equations can be solved for no more than 6 unknowns which generally represent reactions at supports or connections.

- The scalar equations are conveniently obtained by applying the vector forms of the conditions for equilibrium,

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F}\right) = 0$$

Example 2-11



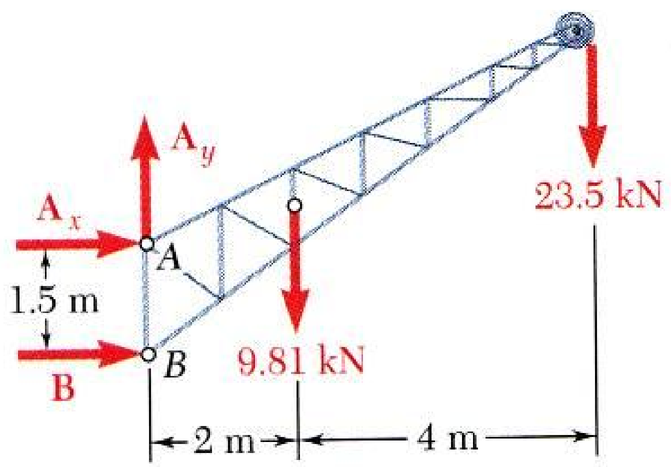
A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*.

Determine the components of the

SOLUTION:

- Create a free-body diagram for the crane.
- Determine *B* by solving the equation for the sum of the moments of all forces about *A*. Note there will be no contribution from the unknown reactions at *A*.
- Determine the reactions at *A* by solving the equations for the sum of all horizontal force components and all vertical force components.
- Check the values obtained for the reactions by verifying that the sum of the moments about *B* of all forces is zero.

Example 2-11



- Create the free-body diagram.

- Determine B by solving the equation for the sum of the moments of all forces about A.

$$\sum M_A = 0: \quad +B(1.5\text{m}) - 9.81\,\text{kN}(2\text{m}) - 23.5\,\text{kN}(6\text{m}) = 0$$

$$\boxed{B = +107.1\,\text{kN}}$$

- Determine the reactions at A by solving the equations for the sum of all horizontal forces and all vertical forces.

$$\sum F_x = 0: \quad A_x + B = 0$$

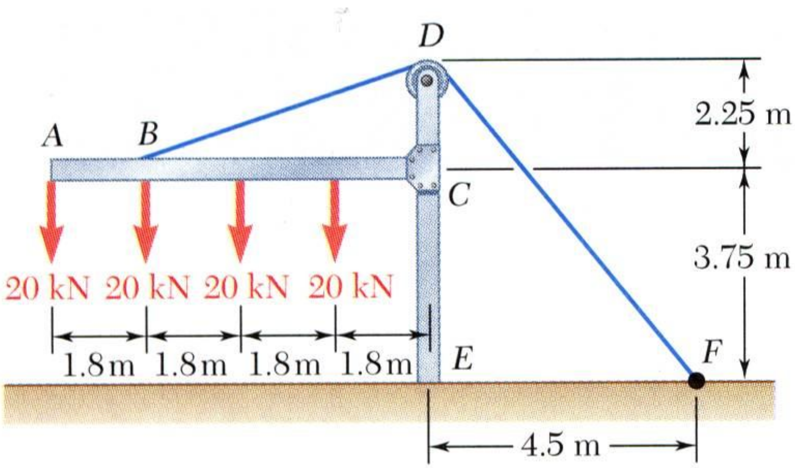
$$\boxed{A_x = -107.1\,\text{kN}}$$

$$\sum F_y = 0: \quad A_y - 9.81\,\text{kN} - 23.5\,\text{kN} = 0$$

$$\boxed{A_y = +33.3\,\text{kN}}$$

- Check the values obtained.

Example 2-12



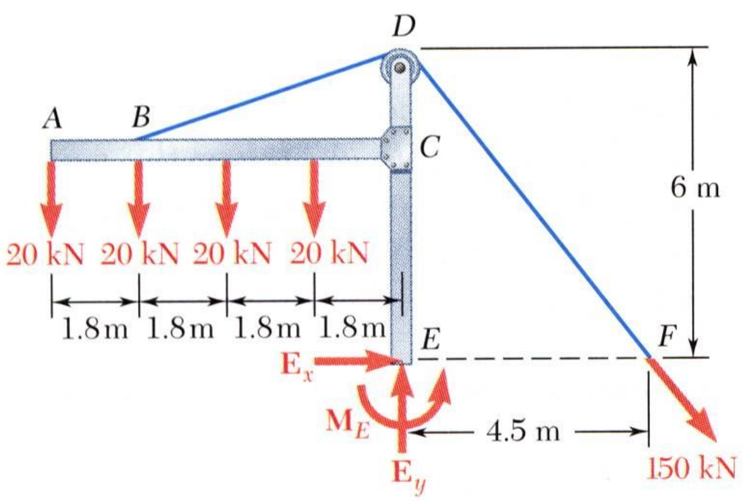
The frame supports part of the roof of a small building. The tension in the cable is 150 kN.

Determine the reaction at the fixed end *E*.

SOLUTION:

- Create a free-body diagram for the frame and cable.
- Solve 3 equilibrium equations for the reaction force components and couple at *E*.

Example 2-12



- Free-body diagram for the frame and cable.

- Solve 3 equilibrium equations for the reaction force components and couple.

$$\sum F_x = 0: \quad E_x + \frac{4.5}{7.5}(150\,\mathrm{kN}) = 0$$

$$\boxed{E_x = -90.0\,\mathrm{kN}}$$

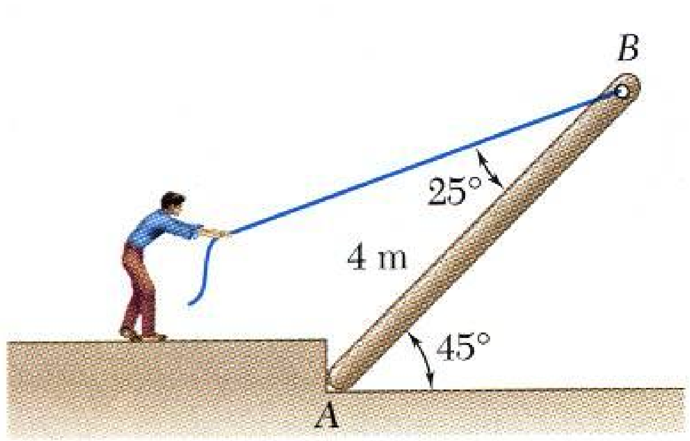
$$\sum F_y = 0: \quad E_y - 4(20\,\mathrm{kN}) - \frac{6}{7.5}(150\,\mathrm{kN}) = 0$$

$$\boxed{E_y = +200\,\mathrm{kN}}$$

$$\sum M_E = 0: \quad +20\,\mathrm{kN}(7.2\,\mathrm{m}) + 20\,\mathrm{kN}(5.4\,\mathrm{m}) + 20\,\mathrm{kN}(3.6\,\mathrm{m}) + 20\,\mathrm{kN}(1.8\,\mathrm{m}) - \frac{6}{7.5}(150\,\mathrm{kN})4.5\,\mathrm{m} + M_E = 0$$

$$\boxed{M_E = 180.0\,\mathrm{kN}\cdot\mathrm{m}}$$

Example 2-13

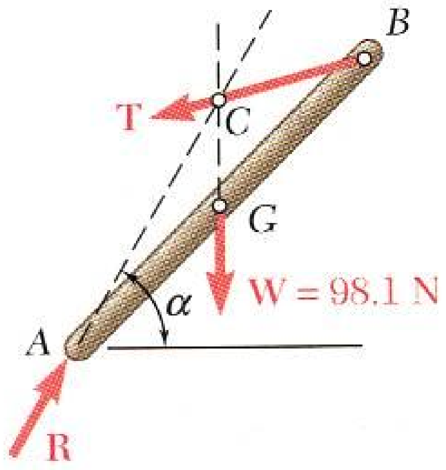


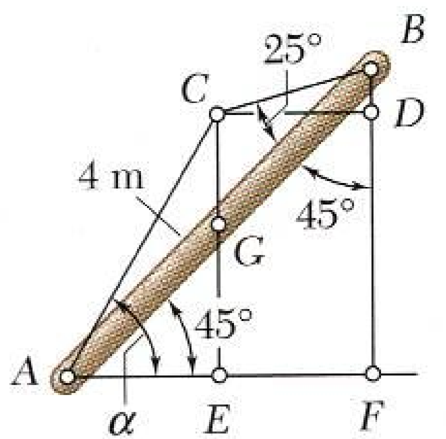
Solve Example 2-8 by summing moment.

SOLUTION:

- Create a free-body diagram of the joist. Note that the joist is a 3 force body acted upon by the rope, its weight, and the reaction at *A*.
- The three forces must be concurrent for static equilibrium. Therefore, the reaction ***R*** must pass through the intersection of the lines of action of the weight and rope forces. Determine the direction of the reaction force ***R***.
- Utilize a force triangle to determine the magnitude of the reaction force ***R***.

Example 2-13





- Create a free-body diagram of the joist.
- Determine the tension in the rope by taking moment about A

$$T\cos 20.L\sin 45 - T\sin 20.L\cos 45 - 98.1\text{x}\tfrac{1}{2}L\cos 45 = 0$$

$$\boxed{T = 82.1\,\text{N}}$$

- Solve equilibrium equations for the reaction force components

$$\sum F_x = 0: \quad \text{R}\cos\alpha - (82.1)\cos 20 = 0$$

$$\text{R}\cos\alpha = 77.15\,\text{N} \qquad (1)$$

$$\sum F_y = 0: \quad \text{R}\sin\alpha - (82.1)\sin 20 - 98.1 = 0$$

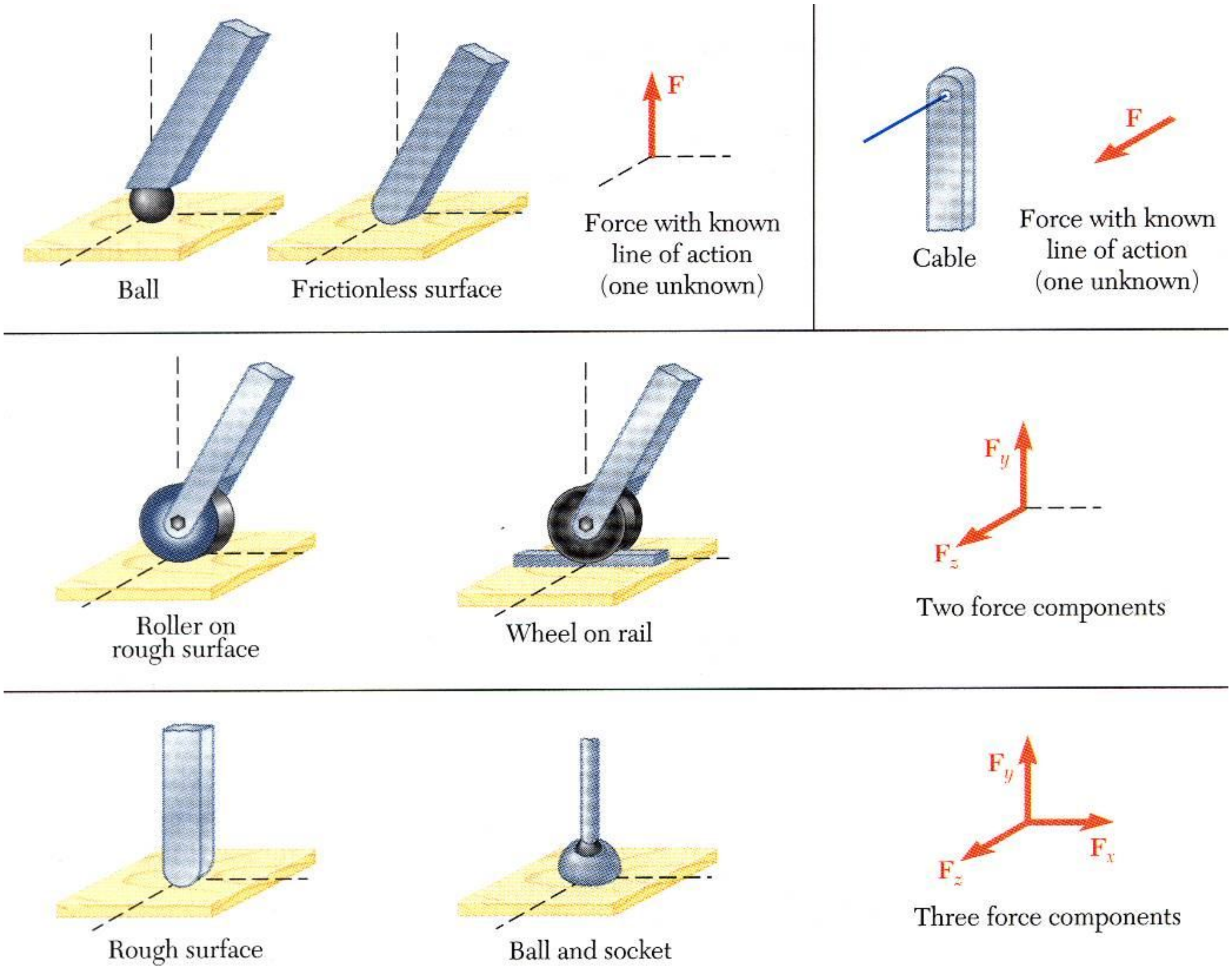
$$\text{R}\sin\alpha = 126.2\,\text{N} \qquad (2)$$

- Solving (1) and (2)

$$\boxed{R = 147.9\,\text{N}}$$

$$\alpha = 58.6^\circ$$

Reactions at Supports and Connections for a 3D Structure



Reactions at Supports and Connections for a 3D Structure