

# ME 251 Introduction to Fluid Mechanics

## Properties

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## Course Objectives

- ▶ Understand the Characteristics/Properties of fluid
- ▶ Have good knowledge of fluid static
  - Static Pressure in Fluid
  - Manometers
  - Hydrostatic forces
  - Buoyancy and flotation
- ▶ Appreciate the importance of dimensions and dimensional analysis in problem solving.

## Areas to Cover

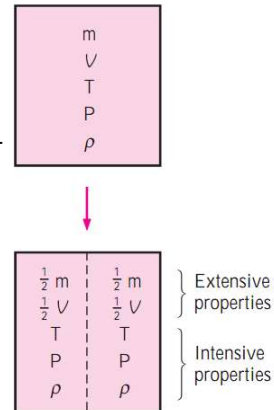
- ▶ Characteristics/Properties of Fluid
  - Properties of fluid (Density, mass density, viscosity).
  - Thermodynamic properties (compressibility, surface tension and capillarity).

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## Properties of Fluid

## Intensive and Extensive properties

- ▶ **Intensive** properties are **independent** of the mass of a system.
  - temperature,
  - pressure,
  - density.
- ▶ **Extensive** properties are those whose values depend on the size—or extent—of the system.
  - Total mass,
  - total volume  $V$ ,
  - total momentum
- ▶ Extensive properties per unit mass are called **specific** properties.
  - specific volume ( $v = V/m$ ) and
  - specific total energy ( $e = E/m$ ).



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## Continuum

- ▶ Matter is made up of atoms that are widely spaced in the gas phase. Yet it is very convenient to disregard the atomic nature of a substance and view it as **a continuous, homogeneous matter** with no holes, that is, **a continuum**.
- ▶ The continuum idealization allows us to treat properties as point functions and to assume that the properties vary continually in space with no jump discontinuities.
- ▶ This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules.
- ▶ The continuum idealization is implicit in many statements we make, such as “the density of water in a glass is the same at any point.”

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## DENSITY AND SPECIFIC GRAVITY

- ▶ **Density** is defined as mass per unit volume.

$$\text{Density} = M/V$$

$$\rho = \frac{m}{V} \quad (\text{kg/m}^3)$$

- ▶ The reciprocal of density is the **specific volume**  $v$ , (volume per unit mass).

$$v = V/m = 1/\rho$$

- ▶ For a differential volume element of mass  $dm$  and volume  $dV$ , density can be expressed as

$$\rho = dm/dV$$

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## DENSITY AND SPECIFIC GRAVITY

- ▶ The density of a substance, in general, depends on temperature and pressure.
- ▶ The density of most gases is proportional to pressure and inversely proportional to temperature.

$$\rho = kP$$

$$\rho = k \frac{1}{T}$$

$$\Rightarrow \rho = k \frac{P}{T}$$

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## DENSITY AND SPECIFIC GRAVITY

- ▶ Liquids and solids are essentially **incompressible** substances, and the variation of their density with pressure is usually negligible.
  - At 20°C, for example, the density of water changes from 998 kg/m<sup>3</sup> at 1 atm to 1003 kg/m<sup>3</sup> at 100 atm, a change of just 0.5 percent.
- ▶ The density of liquids and solids depends more strongly on temperature.
  - At 1 atm, for example, the density of water changes from 998 kg/m<sup>3</sup> at 20°C to 975 kg/m<sup>3</sup> at 75°C, a change of 2.3 percent, which can still be neglected in many engineering analyses.

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## DENSITY AND SPECIFIC GRAVITY

- ▶ When a density of a substance is given relative to the density of a well-known substance it is called **specific gravity**, or **relative density**,
- ▶ It the ratio of the density of a substance to the density of some standard substance at a specified temperature.
- ▶ Usually water is used at 4°C, for which  $\rho_{\text{H}_2\text{O}} = 1000 \text{ kg/m}^3$ ).

$$\text{SG} = \frac{\rho}{\rho_{\text{H}_2\text{O}}}$$

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## DENSITY AND SPECIFIC GRAVITY

- ▶ The weight of a unit volume of a substance is called specific weight

$$\gamma_s = \rho g \quad (\text{N/m}^3)$$

- where  $g$  is the gravitational acceleration.

Specific gravities of some substances at 0°C

Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl alcohol	0.79
Mercury	13.6
Wood	0.3–0.9
Gold	19.2
Bones	1.7–2.0
Ice	0.92
Air (at 1 atm)	0.0013

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## Density of Ideal Gases

- ▶ An equation that relates the pressure, temperature, and density (or specific volume) of a substance is called an **equation of state**.
- ▶ The simplest and best-known equation of state for substances in the gas phase is the **ideal-gas equation** of state, expressed as

$$Pv = RT \quad \text{or} \quad P = \rho RT$$

- where  $P$  is the absolute pressure,
- $v$  is the specific volume,
- $T$  is the thermodynamic (absolute) temperature,
- $\rho$  is the density, and
- $R$  is the gas constant.

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## Density of Ideal Gases

$$Pv = RT \quad \text{or} \quad P = \rho RT$$

- ▶ The gas constant  $R$  is different for each gas and is determined from

$$R = R_u / M,$$

- where  $R_u$  is the universal gas constant and  $R_u = 8.314 \text{ kJ/kmol} \cdot \text{K} = 1.986 \text{ Btu/lbmol} \cdot \text{R}$ , and
- $M$  is the molar mass (also called molecular weight) of the gas.

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## Thermodynamic Temperature Scale

- ▶ The thermodynamic temperature scale in the SI is the Kelvin scale, and the unit is the **kelvin**, K.
- ▶ In the English system, it is the Rankine scale, and the unit is the **Rankine**, R.
- ▶ Various temperature scales are related by

$$T(\text{K}) = T(^{\circ}\text{C}) + 273.15$$

$$T(\text{R}) = T(^{\circ}\text{F}) + 459.67$$

- ▶ It is common practice to round the constants 273.15 and 459.67 to 273 and 460, respectively.

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Determine the density, specific gravity and mass of the air in a room whose dimensions are 4m X 5m X 6m at 100kPa and 25°C

**Solution** The density, specific gravity, and mass of the air in a room are to be determined.

**Assumptions** At specified conditions, air can be treated as an ideal gas.

**Properties** The gas constant of air is  $R = 0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$ .

**Analysis** The density of air is determined from the ideal-gas relation  $P = \rho RT$  to be

$$\rho = \frac{P}{RT} = \frac{100 \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(25 + 273) \text{ K}} = 1.17 \text{ kg/m}^3$$

Then the specific gravity of air becomes

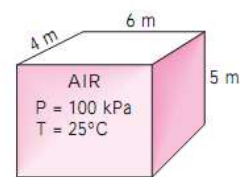
$$\text{SG} = \frac{\rho}{\rho_{\text{H}_2\text{O}}} = \frac{1.17 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = 0.00117$$

Finally, the volume and the mass of air in the room are

$$V = (4 \text{ m})(5 \text{ m})(6 \text{ m}) = 120 \text{ m}^3$$

$$m = \rho V = (1.17 \text{ kg/m}^3)(120 \text{ m}^3) = 140 \text{ kg}$$

**Discussion** Note that we converted the temperature to the unit K from °C before using it in the ideal-gas relation.



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## VAPOUR PRESSURE

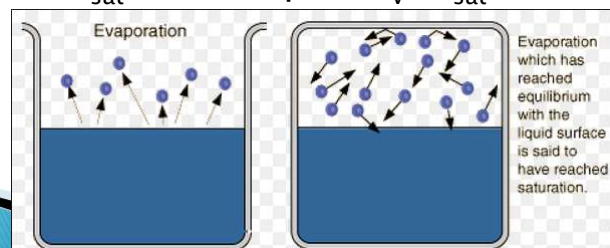
- ▶ At a given pressure, the temperature at which a pure substance changes phase is called the **Saturation Temperature,  $T_{\text{sat}}$**
- ▶ At a given temperature, the pressure at which a pure substance changes phase is called the **Saturation Pressure,  $P_{\text{sat}}$** 
  - At an absolute pressure of 1 standard atmosphere (1 atm or 101.325 kPa), for example, the saturation temperature of water is 100°C.
  - Conversely, at a temperature of 100°C, the saturation pressure of water is 1 atm.

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## VAPOUR PRESSURE

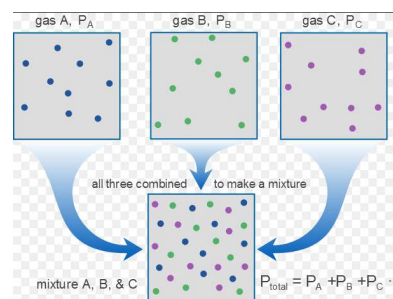
- ▶ The **Vapour Pressure**  $P_v$  of a pure substance is the pressure exerted by its vapour in **phase equilibrium** with its liquid at a given temperature.
- ▶  $P_v$  is a property of the pure substance, and turns out to be identical to the saturation pressure  $P_{sat}$  of the liquid ( $P_v = P_{sat}$ ).



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## VAPOUR PRESSURE

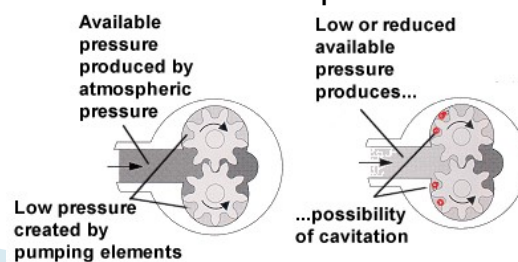
- ▶ **Partial pressure** is the pressure of a gas or vapour in a mixture with other gases.
- ▶ The partial pressure of a vapour must be less than or equal to the vapour pressure if there is no liquid present.
- ▶ When both vapour and liquid are present and the system is in phase equilibrium, the partial pressure of the vapour must equal the vapour pressure, and the system is said to be saturated.



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## CAVITATION

- ▶ It is possible that the liquid pressure in liquid-flow systems dropped below the vapour pressure at some locations
- ▶ the result is unplanned vaporization.
- ▶ The vapour bubbles collapse as they are swept away from the low-pressure regions, generating highly destructive, extremely high-pressure waves.
- ▶ the formation of bubbles and their collapse is called **CAVITATION**



## CAVITATION

- ▶ The phenomenon, is a common cause for drop in performance and even the erosion of impeller blades
- ▶ Cavitation is an important consideration in the design of hydraulic turbines and pumps and must be avoided (or at least minimized) in flow systems.



In a water distribution system, the temperature of water is observed to be as high as 30°C. Determine the minimum pressure allowed in the system to avoid cavitation.

**SOLUTION** The minimum pressure in a water distribution system to avoid cavitation is to be determined.

**Properties** The vapor pressure of water at 30°C is 4.25 kPa.

**Analysis** To avoid cavitation, the pressure anywhere in the flow should not be allowed to drop below the vapor (or saturation) pressure at the given temperature. That is,

$$P_{\min} = P_{\text{sat}@30^\circ\text{C}} = 4.25 \text{ kPa}$$

Therefore, the pressure should be maintained above 4.25 kPa everywhere in the flow.

**Discussion** Note that the vapor pressure increases with increasing temperature, and thus the risk of cavitation is greater at higher fluid temperatures.

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Properties of saturated water

Temp. <i>T</i> , °C	Saturation Pressure <i>P</i> <sub>sat</sub> , kPa	Density <i>ρ</i> , kg/m <sup>3</sup>		Enthalpy of Vaporization <i>h</i> <sub>fg</sub> , kJ/kg	Specific Heat <i>c</i> <sub>p</sub> , J/kg · K		Thermal Conductivity <i>k</i> , W/m · K		Dynamic Viscosity <i>μ</i> , kg/m · s		Prandtl Number <i>Pr</i>		Volume Expansion Coefficient <i>β</i> , 1/K
		Liquid	Vapor		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	
0.01	0.6113	999.8	0.0048	2501	4217	1854	0.561	0.0171	1.792 × 10 <sup>-3</sup>	0.922 × 10 <sup>-5</sup>	13.5	1.00	-0.068 × 10 <sup>-3</sup>
5	0.8721	999.9	0.0068	2490	4205	1857	0.571	0.0173	1.519 × 10 <sup>-3</sup>	0.934 × 10 <sup>-5</sup>	11.2	1.00	0.015 × 10 <sup>-3</sup>
10	1.2276	999.7	0.0094	2478	4194	1862	0.580	0.0176	1.307 × 10 <sup>-3</sup>	0.946 × 10 <sup>-5</sup>	9.45	1.00	0.733 × 10 <sup>-3</sup>
15	1.7051	999.1	0.0128	2466	4186	1863	0.589	0.0179	1.138 × 10 <sup>-3</sup>	0.959 × 10 <sup>-5</sup>	8.09	1.00	0.138 × 10 <sup>-3</sup>
20	2.339	998.0	0.0173	2454	4182	1867	0.598	0.0182	1.002 × 10 <sup>-3</sup>	0.973 × 10 <sup>-5</sup>	7.01	1.00	0.195 × 10 <sup>-3</sup>
25	3.169	997.0	0.0231	2442	4180	1870	0.607	0.0186	0.891 × 10 <sup>-3</sup>	0.987 × 10 <sup>-5</sup>	6.14	1.00	0.247 × 10 <sup>-3</sup>
30	4.246	996.0	0.0304	2431	4178	1875	0.615	0.0189	0.798 × 10 <sup>-3</sup>	1.001 × 10 <sup>-5</sup>	5.42	1.00	0.294 × 10 <sup>-3</sup>
35	5.628	994.0	0.0397	2419	4178	1880	0.623	0.0192	0.720 × 10 <sup>-3</sup>	1.016 × 10 <sup>-5</sup>	4.83	1.00	0.337 × 10 <sup>-3</sup>
40	7.384	992.1	0.0512	2407	4179	1885	0.631	0.0196	0.653 × 10 <sup>-3</sup>	1.031 × 10 <sup>-5</sup>	4.32	1.00	0.377 × 10 <sup>-3</sup>
45	9.593	990.1	0.0655	2395	4180	1892	0.637	0.0200	0.596 × 10 <sup>-3</sup>	1.046 × 10 <sup>-5</sup>	3.91	1.00	0.415 × 10 <sup>-3</sup>
50	12.35	988.1	0.0831	2383	4181	1900	0.644	0.0204	0.547 × 10 <sup>-3</sup>	1.062 × 10 <sup>-5</sup>	3.55	1.00	0.451 × 10 <sup>-3</sup>
55	15.76	985.2	0.1045	2371	4183	1908	0.649	0.0208	0.504 × 10 <sup>-3</sup>	1.077 × 10 <sup>-5</sup>	3.25	1.00	0.484 × 10 <sup>-3</sup>
60	19.94	983.3	0.1304	2359	4185	1916	0.654	0.0212	0.467 × 10 <sup>-3</sup>	1.093 × 10 <sup>-5</sup>	2.99	1.00	0.517 × 10 <sup>-3</sup>
65	25.03	980.4	0.1614	2346	4187	1926	0.659	0.0216	0.433 × 10 <sup>-3</sup>	1.110 × 10 <sup>-5</sup>	2.75	1.00	0.548 × 10 <sup>-3</sup>
70	31.19	977.5	0.1983	2334	4190	1936	0.663	0.0221	0.404 × 10 <sup>-3</sup>	1.126 × 10 <sup>-5</sup>	2.55	1.00	0.578 × 10 <sup>-3</sup>
75	38.58	974.7	0.2421	2321	4193	1948	0.667	0.0225	0.378 × 10 <sup>-3</sup>	1.142 × 10 <sup>-5</sup>	2.38	1.00	0.607 × 10 <sup>-3</sup>
80	47.39	971.8	0.2935	2309	4197	1962	0.670	0.0230	0.355 × 10 <sup>-3</sup>	1.159 × 10 <sup>-5</sup>	2.22	1.00	0.653 × 10 <sup>-3</sup>
85	57.83	968.1	0.3536	2296	4201	1977	0.673	0.0235	0.333 × 10 <sup>-3</sup>	1.176 × 10 <sup>-5</sup>	2.08	1.00	0.670 × 10 <sup>-3</sup>
90	70.14	965.3	0.4235	2283	4206	1993	0.675	0.0240	0.315 × 10 <sup>-3</sup>	1.193 × 10 <sup>-5</sup>	1.96	1.00	0.702 × 10 <sup>-3</sup>
95	84.55	961.5	0.5045	2270	4212	2010	0.677	0.0246	0.297 × 10 <sup>-3</sup>	1.210 × 10 <sup>-5</sup>	1.85	1.00	0.716 × 10 <sup>-3</sup>
100	101.33	957.9	0.5978	2257	4217	2029	0.679	0.0251	0.282 × 10 <sup>-3</sup>	1.227 × 10 <sup>-5</sup>	1.75	1.00	0.750 × 10 <sup>-3</sup>
110	143.27	950.6	0.8263	2230	4229	2071	0.682	0.0262	0.255 × 10 <sup>-3</sup>	1.261 × 10 <sup>-5</sup>	1.58	1.00	0.798 × 10 <sup>-3</sup>
120	198.53	943.4	1.121	2203	4244	2120	0.683	0.0275	0.232 × 10 <sup>-3</sup>	1.296 × 10 <sup>-5</sup>	1.44	1.00	0.858 × 10 <sup>-3</sup>
130	270.1	934.6	1.496	2174	4263	2177	0.684	0.0288	0.213 × 10 <sup>-3</sup>	1.330 × 10 <sup>-5</sup>	1.33	1.01	0.913 × 10 <sup>-3</sup>
140	361.3	921.7	1.965	2145	4286	2244	0.683	0.0301	0.197 × 10 <sup>-3</sup>	1.365 × 10 <sup>-5</sup>	1.24	1.02	0.970 × 10 <sup>-3</sup>
150	475.8	916.6	2.546	2114	4311	2314	0.682	0.0316	0.183 × 10 <sup>-3</sup>	1.399 × 10 <sup>-5</sup>	1.16	1.02	1.025 × 10 <sup>-3</sup>
160	617.8	907.4	3.256	2083	4340	2420	0.680	0.0331	0.170 × 10 <sup>-3</sup>	1.434 × 10 <sup>-5</sup>	1.09	1.05	1.145 × 10 <sup>-3</sup>
170	791.7	897.7	4.119	2050	4370	2490	0.677	0.0347	0.160 × 10 <sup>-3</sup>	1.468 × 10 <sup>-5</sup>	1.03	1.05	1.178 × 10 <sup>-3</sup>
180	1,002.1	887.3	5.153	2015	4410	2590	0.673	0.0364	0.150 × 10 <sup>-3</sup>	1.502 × 10 <sup>-5</sup>	0.983	1.07	1.210 × 10 <sup>-3</sup>
190	1,254.4	876.4	6.388	1979	4460	2710	0.669	0.0382	0.142 × 10 <sup>-3</sup>	1.537 × 10 <sup>-5</sup>	0.947	1.09	1.280 × 10 <sup>-3</sup>
200	1,553.8	864.3	7.852	1941	4500	2840	0.663	0.0401	0.134 × 10 <sup>-3</sup>	1.571 × 10 <sup>-5</sup>	0.910	1.11	1.350 × 10 <sup>-3</sup>
220	2,318	840.3	11.60	1859	4610	3110	0.650	0.0442	0.122 × 10 <sup>-3</sup>	1.641 × 10 <sup>-5</sup>	0.865	1.15	1.520 × 10 <sup>-3</sup>
240	3,344	813.7	16.73	1767	4760	3520	0.632	0.0487	0.111 × 10 <sup>-3</sup>	1.712 × 10 <sup>-5</sup>	0.836	1.24	1.720 × 10 <sup>-3</sup>
260	4,688	783.7	23.69	1663	4970	4070	0.609	0.0540	0.102 × 10 <sup>-3</sup>	1.788 × 10 <sup>-5</sup>	0.832	1.35	2.000 × 10 <sup>-3</sup>

## ENERGY AND SPECIFIC HEATS

- ▶ Energy can exist in numerous forms such as
  - thermal,
  - mechanical,
  - kinetic,
  - potential,
  - electrical,
  - magnetic,
  - chemical, and
  - Nuclear.
- ▶ the sum constitutes the total energy E (or e on a unit mass basis) of a system
- ▶ The forms of energy related to the molecular structure of a system and the degree of the molecular activity are referred to as the microscopic energy.

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## ENERGY AND SPECIFIC HEATS

- ▶ The sum of all microscopic forms of energy is **internal energy** of a system, denoted by U (or u on a unit mass basis).
- ▶ The energy that a system possesses as a result of its motion relative to some reference frame is **kinetic energy** per unit mass is expressed as

$$ke = V^2/2$$

- where V denotes the velocity of the system relative to some fixed reference frame.

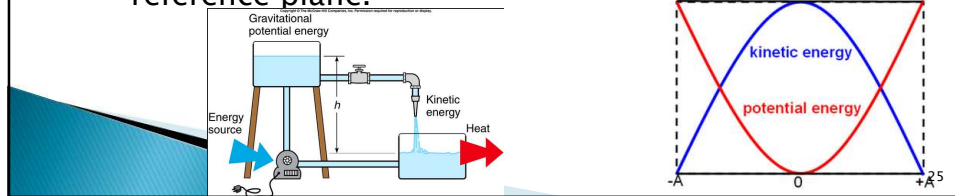
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## ENERGY AND SPECIFIC HEATS

- ▶ The energy that a system possesses as a result of its elevation in a gravitational field is called **potential energy** and is expressed on a per-unit mass basis as

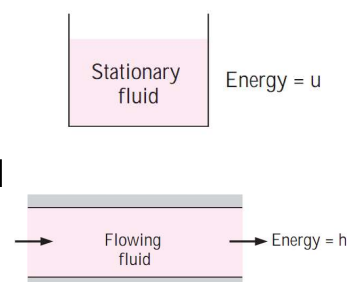
$$pe = g z$$

- where  $g$  is the gravitational acceleration and
- $z$  is the elevation of the center of gravity of a system relative to some arbitrarily selected reference plane.



## ENERGY AND SPECIFIC HEATS

- ▶ **Thermal energy** is the sensible and latent forms of internal energy.
- ▶ The **internal energy**,  $u$ , represents the microscopic energy of a non-flowing fluid per unit mass,
- ▶ **Enthalpy**,  $h$ , represents the microscopic energy of a flowing fluid per unit mass



$$h = u + Pv = u + \frac{P}{\rho}$$

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## ENERGY AND SPECIFIC HEATS

- ▶ In the absence of such effects as magnetic, electric, and surface tension, a system is a **simple compressible** system.
- ▶ The total energy of a simple compressible system consists of three parts:
  - internal,
  - kinetic, and
  - potential energies
 expressed, on a unit-mass basis, as

$$e = u + ke + pe.$$

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## ENERGY AND SPECIFIC HEATS

- ▶ The fluid entering or leaving a control volume possesses an additional form of energy—the **flow energy**  $P/\rho$ .
- ▶ the total energy of a flowing fluid on a unit-mass basis becomes

$$\text{total energy} = \text{flow energy } P/\rho + e$$

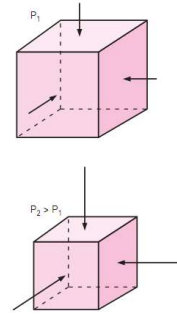
$$e_{\text{flowing}} = P/\rho + e = h + ke + pe = h + \frac{V^2}{2} + gz \quad (\text{kJ/kg})$$

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## COEFFICIENT OF COMPRESSIBILITY

- ▶ The volume (or density) of a fluid changes with a change in its temperature or pressure.
- ▶ the coefficient of compressibility,  $\kappa$ , (bulk modulus of compressibility or bulk modulus of elasticity) represents the change in pressure corresponding to a fractional change in volume or density of the fluid while the temperature remains constant.
- ▶ A large value of  $\kappa$  indicates that a large change in pressure is needed to cause a small fractional change in volume.
  - A fluid with a large  $\kappa$  is essentially incompressible.



$$\kappa = -v \left( \frac{\partial P}{\partial v} \right)_T = \rho \left( \frac{\partial P}{\partial \rho} \right)_T \quad (\text{Pa}) \quad \kappa \cong - \frac{\Delta P}{\Delta v/v} \cong \frac{\Delta P}{\Delta \rho/\rho} \quad (T = \text{constant})$$

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## COEFFICIENT OF COMPRESSIBILITY

- ▶ For an ideal gas,

$$P = \rho RT \text{ and } (\partial P / \partial \rho)_T = RT = P / \rho,$$

$$\kappa_{\text{ideal gas}} = P \quad (\text{Pa})$$

- ▶ Coefficient of compressibility of an ideal gas is equal to its absolute pressure,
  - the coefficient of compressibility of the gas increases with increasing pressure.

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## COEFFICIENT OF COMPRESSIBILITY

- ▶ Substituting  $\kappa = P$  into the definition of the coefficient of compressibility

$$\kappa \cong -\frac{\Delta P}{\Delta v/v} \cong \frac{\Delta P}{\Delta \rho/\rho} \quad \frac{\Delta \rho}{\rho} = \frac{\Delta P}{P} \quad (T = \text{constant})$$

- ▶ the percent increase of density of an ideal gas during isothermal compression is equal to the percent increase in pressure.
- ▶ For air at 1 atm pressure,  $\kappa = P = 1 \text{ atm}$ .

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## Isothermal Compressibility

- ▶ The inverse of the coefficient of compressibility is called the **isothermal compressibility,  $\alpha$** .
- ▶ The isothermal compressibility of a fluid represents the fractional change in volume or density corresponding to a unit change in pressure.

$$\alpha = \frac{1}{\kappa} = -\frac{1}{v} \left( \frac{\partial v}{\partial P} \right)_T = \frac{1}{\rho} \left( \frac{\partial \rho}{\partial P} \right)_T \quad (1/\text{Pa})$$

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## Coefficient of Volume Expansion

- ▶ The density of a fluid, in general, depends more strongly on temperature than it does on pressure,
- ▶ The variation of density with temperature is responsible for numerous natural phenomena such as
  - winds,
  - currents in oceans,
  - rise of plumes in chimneys,
  - the operation of hot-air balloons,
  - heat transfer by natural convection, and
  - rise of hot air (thus the phrase “heat rises”).

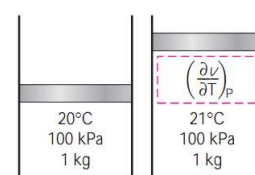
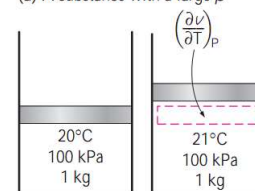
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## Coefficient of Volume Expansion

- ▶ The coefficient of volume expansion (or volume expansivity),  $\beta$  is a measure of the change in volume of a substance with temperature at constant pressure.

$$\beta = \frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_P = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_P \quad (1/K)$$

$$\beta \approx \frac{\Delta v/v}{\Delta T} = -\frac{\Delta \rho/\rho}{\Delta T} \quad (\text{at constant } P)$$

(a) A substance with a large  $\beta$ (b) A substance with a small  $\beta$ 

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## Coefficient of Volume Expansion

- ▶ A large value of  $\beta$  for a fluid means a large change in density with temperature.
- ▶ The product  $\beta\Delta T$  represents the fraction of volume change of a fluid that corresponds to a temperature change of  $\Delta T$  at constant pressure.
- ▶ It can be shown easily that the volume expansion coefficient of an ideal gas ( $P=\rho RT$ ) at a temperature  $T$  is equivalent to the inverse of the temperature

$$\beta_{\text{ideal gas}} = \frac{1}{T} \quad (1/K)$$

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## Pressure and Temperature Effect

- ▶ For a combined effects of pressure and temperature changes on the volume
- ▶ The specific volume can be a function of  $T$  and  $P$ .  
i.e.  $v = v(T, P)$
- ▶ Differentiating and using the definitions of the compression and expansion coefficients  $\alpha$  and  $\beta$  give

$$dv = \left(\frac{\partial v}{\partial T}\right)_P dT + \left(\frac{\partial v}{\partial P}\right)_T dP = (\beta dT - \alpha dP)v$$

- ▶ Fractional change in volume (or density) can be expressed approximately as

$$\frac{\Delta v}{v} = -\frac{\Delta \rho}{\rho} \cong \beta \Delta T - \alpha \Delta P$$

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Consider water initially at 20°C and 1 atm.  
 Determine the final density of water  
 (a) if it is heated to 50°C at a constant pressure of 1 atm, and (b) if it is compressed to 100-atm pressure at a constant temperature of 20°C. Take the isothermal compressibility of water to be  $\alpha = 4.80 \times 10^{-5} \text{ atm}^{-1}$ .

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**SOLUTION** Water at a given temperature and pressure is considered. The densities of water after it is heated and after it is compressed are to be determined.

**Assumptions** 1 The coefficient of volume expansion and the isothermal compressibility of water are constant in the given temperature range. 2 An approximate analysis is performed by replacing differential changes in quantities by finite changes.

**Properties** The density of water at 20°C and 1 atm pressure is  $\rho_1 = 998.0 \text{ kg/m}^3$ . The coefficient of volume expansion at the average temperature of  $(20 + 50)/2 = 35^\circ\text{C}$  is  $\beta = 0.337 \times 10^{-3} \text{ K}^{-1}$ . The isothermal compressibility of water is given to be  $\alpha = 4.80 \times 10^{-5} \text{ atm}^{-1}$ .

**Analysis** When differential quantities are replaced by differences and the properties  $\alpha$  and  $\beta$  are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as (Eq. 2-23)

$$\Delta\rho = \alpha\rho\Delta P - \beta\rho\Delta T$$

(a) The change in density due to the change of temperature from 20°C to 50°C at constant pressure is

$$\begin{aligned}\Delta\rho &= -\beta\rho\Delta T = -(0.337 \times 10^{-3} \text{ K}^{-1})(998 \text{ kg/m}^3)(50 - 20) \text{ K} \\ &= -10.0 \text{ kg/m}^3\end{aligned}$$

Noting that  $\Delta\rho = \rho_2 - \rho_1$ , the density of water at 50°C and 1 atm is

$$\rho_2 = \rho_1 + \Delta\rho = 998.0 + (-10.0) = 988.0 \text{ kg/m}^3$$

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which is almost identical to the listed value of  $988.1 \text{ kg/m}^3$  at  $50^\circ\text{C}$  in Table A-3. This is mostly due to  $\beta$  varying with temperature almost linearly, as shown in Fig. 2-10.

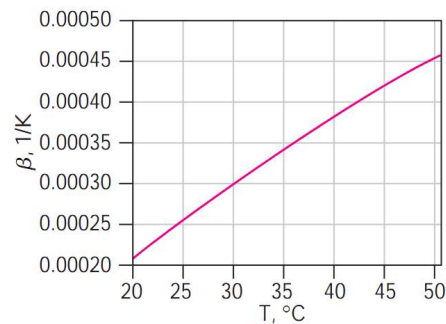
(b) The change in density due to a change of pressure from 1 atm to 100 atm at constant temperature is

$$\Delta\rho = \alpha\rho\Delta P = (4.80 \times 10^{-5} \text{ atm}^{-1})(998 \text{ kg/m}^3)(100 - 1) \text{ atm} = 4.7 \text{ kg/m}^3$$

Then the density of water at 100 atm and  $20^\circ\text{C}$  becomes

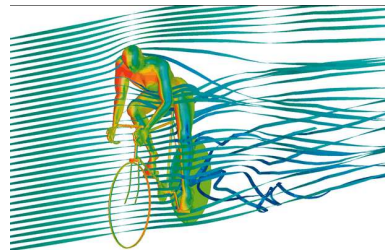
$$\rho_2 = \rho_1 + \Delta\rho = 998.0 + 4.7 = 1002.7 \text{ kg/m}^3$$

**Discussion** Note that the density of water decreases while being heated and increases while being compressed, as expected. This problem can be solved more accurately using differential analysis when functional forms of properties are available.



## VISCOSITY

- ▶ Viscosity is a property that represents the **internal resistance** of a fluid to motion or the “fluidity,”.
- ▶ The force a flowing fluid exerts on a body in the flow direction is called the drag force, and the magnitude of this force depends, in part, on viscosity.

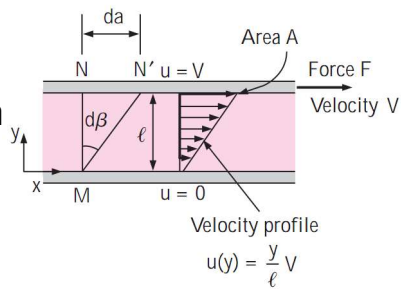


## VISCOSITY

- ▶ The fluid velocity between the plates varies linearly between 0 and  $V$ ,
  - The velocity profile and the velocity gradient are

$$u(y) = \frac{y}{\ell} V \quad \text{and} \quad \frac{du}{dy} = \frac{V}{\ell}$$

For a differential time interval  $dt$ , the sides of fluid particles along a vertical line  $MN$  rotate through a differential angle  $d\beta$  while the upper plate moves a differential distance  $da = Vdt$ .

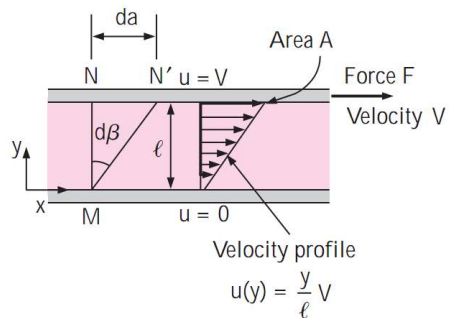


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## VISCOSITY

- ▶ The angular displacement or deformation (or shear strain) can be expressed as

$$d\beta \approx \tan \beta = \frac{da}{\ell} = \frac{V dt}{\ell} = \frac{du}{dy} dt$$



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## VISCOSITY

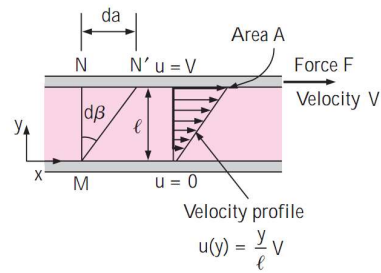
- ▶ The rate of deformation of a fluid element is equivalent to the velocity gradient  $du/dy$ .

$$\frac{d\beta}{dt} = \frac{du}{dy}$$

- ▶ Further, it can be verified experimentally that for most fluids the rate of deformation (and thus the velocity gradient) is directly proportional to the shear stress.

$$\tau \propto \frac{d\beta}{dt}$$

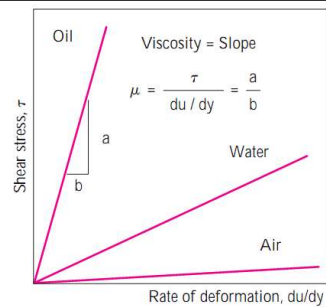
$$\text{or } \tau \propto \frac{du}{dy}$$



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## VISCOSITY

- ▶ **Newtonian fluids** – Fluids for which the rate of deformation is proportional to the shear stress.
  - water,
  - air,
  - gasoline, and
  - oils.
- ▶ **non-Newtonian fluids**
  - Blood and liquid plastics



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## VISCOSITY

- ▶ In one-dimensional shear flow of Newtonian fluids, shear stress can be expressed by the linear relationship

$$\tau = \mu \frac{du}{dy} \quad (\text{N/m}^2)$$

- ▶ The constant of proportionality  $\mu$  is coefficient of viscosity or the dynamic (or absolute) viscosity of the fluid,
- ▶ unit is  $\text{kg/m} \cdot \text{s}$ , or equivalently,  $\text{N} \cdot \text{s}/\text{m}^2$  (or  $\text{Pa} \cdot \text{s}$ ).
- ▶ A common viscosity unit is poise, which is equivalent to  $0.1 \text{ Pa} \cdot \text{s}$  (or centipoise, which is one-hundredth of a poise).

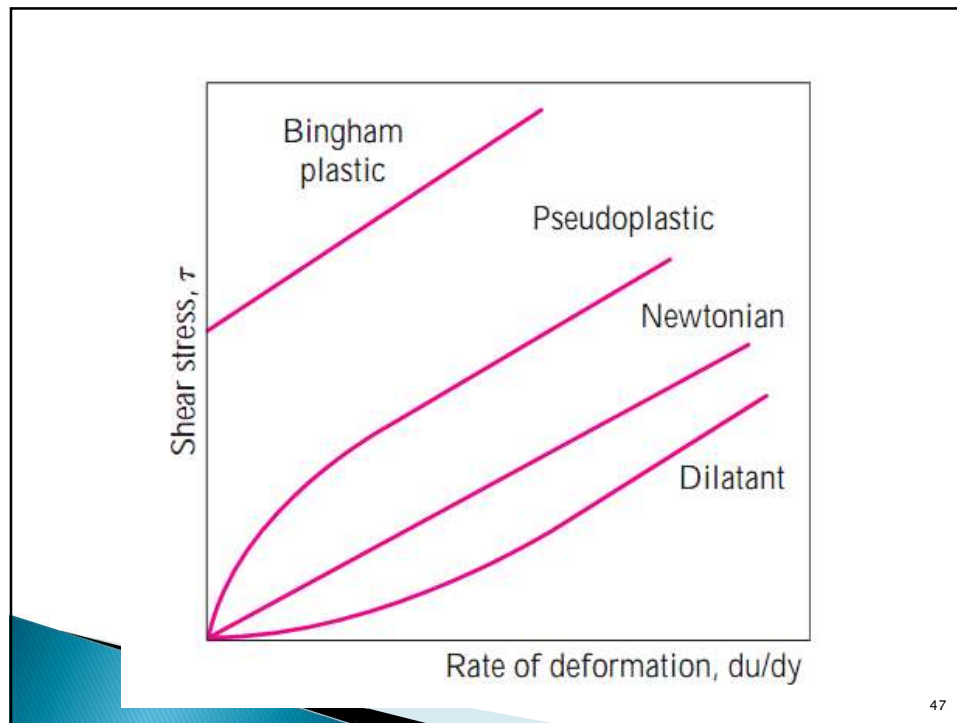
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## Viscosity

- ▶ Fluids for which the apparent viscosity increases with the rate of deformation (such as solutions with suspended starch or sand) are referred to as **dilatant** or **shear thickening** fluids, and
- ▶ those that exhibit the opposite behavior (the fluid becoming less viscous as it is sheared harder, such as some paints, polymer solutions, and fluids with suspended particles) are referred to as **pseudoplastic** or **shear thinning** fluids.
- ▶ Some materials such as toothpaste can resist a finite shear stress and thus behave as a solid, but deform continuously when the shear stress exceeds the yield stress and thus behave as a fluid. Such materials are referred to as **Bingham plastics**.

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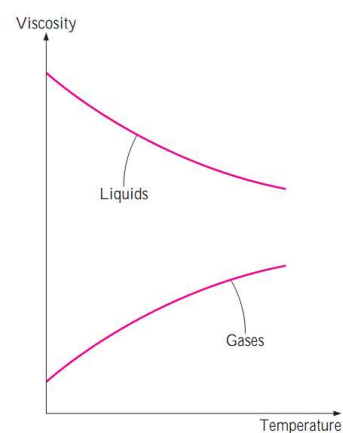




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## Viscosity

- ▶ **Kinematic viscosity,  $\nu$ ,** is the ratio of dynamic viscosity to density,
 
$$\nu = \mu/\rho.$$
  - units of kinematic viscosity are  $\text{m}^2/\text{s}$  and
  - stoke (1 stoke =  $1 \text{ cm}^2/\text{s} = 0.0001 \text{ m}^2/\text{s}$ ).
- ▶ The viscosity of liquids decreases with temperature,
- ▶ The viscosity of gases increases with temperature



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## Viscosity of Gas

- ▶ The kinetic theory of gases predicts the viscosity of gases to be proportional to the square root of temperature.  $\mu_{\text{gas}} \propto \sqrt{T}$
- ▶ The viscosity of gases is expressed as a function of temperature by the Sutherland correlation (from The U.S. Standard Atmosphere) as 
$$\mu = \frac{aT^{1/2}}{1 + b/T}$$

- where T is absolute temperature
- a and b are experimentally determined constants.

$$a = 1.458 \times 10^{-6} \text{ kg/(m} \cdot \text{s} \cdot \text{K}^{1/2}) \text{ and } b = 110.4 \text{ K}$$

## Viscosity of Liquid

- ▶ For liquids, the viscosity is approximated as

$$\mu = a10^{b/(T-c)}$$

- Where T is absolute temperature and
- a, b, and c are experimentally determined constants.

## Viscosity of Liquid

- ▶ Considering a fluid layer of thickness  $l$  within a small gap between two concentric cylinders.
- ▶ The gap between the cylinders can be modeled as two parallel flat plates separated by a fluid.
- ▶ Torque is  $T = FR$  (force times the moment arm, which is the radius  $R$  of the inner cylinder in this case),
- ▶ the tangential velocity is  $V = \omega R$  (angular velocity times the radius), and
- ▶ The wetted surface area of the inner cylinder is  $A = 2\pi RL$ 
  - disregarding the shear stress acting on the two ends of the inner cylinder, torque can be expressed as

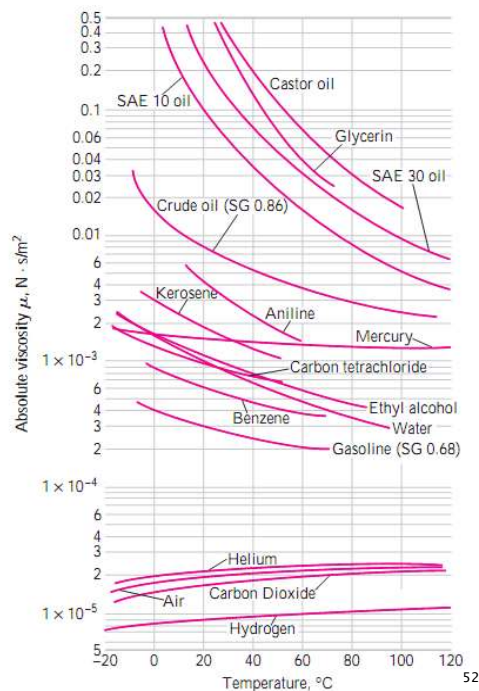
$$T = FR = \mu \frac{2\pi R^3 \omega L}{l} = \mu \frac{4\pi^2 R^3 n L}{l}$$

where  $L$  is the length of the cylinder and  $n$  is the number of revolutions per unit time, which is usually expressed in rpm (revolutions per minute).

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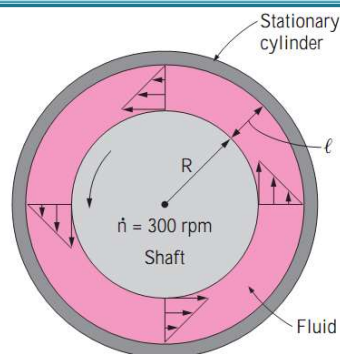
Dynamic viscosities of some fluids  
at 1 atm and 20°C (unless  
otherwise stated)

Fluid	Dynamic Viscosity $\mu$ , kg/m · s
Glycerin:	
-20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.00040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088



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The viscosity of a fluid is to be measured by a viscometer constructed of two 40-cm-long concentric cylinders. The outer diameter of the inner cylinder is 12 cm, and the gap between the two cylinders is 0.15 cm. The inner cylinder is rotated at 300 rpm, and the torque is measured to be 1.8 Nm. Determine the viscosity of the fluid.



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**SOLUTION** The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

**Assumptions** 1 The inner cylinder is completely submerged in oil. 2 The viscous effects on the two ends of the inner cylinder are negligible.

**Analysis** The velocity profile is linear only when the curvature effects are negligible, and the profile can be approximated as being linear in this case since  $\ell/R \ll 1$ . Solving Eq. 2-34 for viscosity and substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{T\ell}{4\pi^2 R^3 \dot{n} L} = \frac{(1.8 \text{ N} \cdot \text{m})(0.0015 \text{ m})}{4\pi^2 (0.06 \text{ m})^3 (300/60 \text{ 1/s})(0.4 \text{ m})} = 0.158 \text{ N} \cdot \text{s/m}^2$$

**Discussion** Viscosity is a strong function of temperature, and a viscosity value without a corresponding temperature is of little value. Therefore, the temperature of the fluid should have also been measured during this experiment, and reported with this calculation.

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# Viscosity

## Question:

- ▶ The velocity distribution of a viscous liquid (dynamic viscosity  $\mu = 0.75 \text{ Ns/m}^2$ ) flowing over a fixed plate is given by  $u = 0.64y - y^2$  ( $u$  is velocity in m/s and  $y$  is the distance from the plate in m). What are the shear stresses at the plate surface and at  $y=0.34\text{m}$ ?

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# SURFACE TENSION

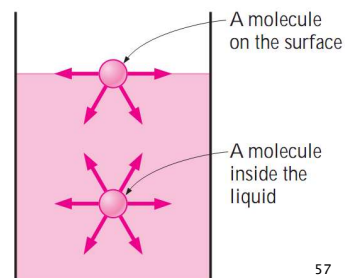
- ▶ Some observations:
  - a drop of blood forms a hump on a horizontal glass;
  - a drop of mercury forms a near-perfect sphere and can be rolled just like a steel ball over a smooth surface;
  - water droplets from rain or dew hang from branches or leaves of trees;
  - a liquid fuel injected into an engine forms a mist of spherical droplets;
  - water dripping from a leaky faucet falls as spherical droplets;
  - a soap bubble released into the air forms a spherical shape; and
- ▶ In these and other observances, liquid droplets behave like small spherical balloons filled with the liquid, and the surface of the liquid acts like a stretched elastic membrane under tension.



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## SURFACE TENSION

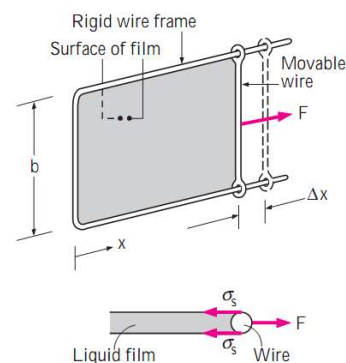
- ▶ The pulling force that causes this tension acts parallel to the surface and is due to the attractive forces between the molecules of the liquid.
- ▶ The magnitude of this force per unit length is **surface tension**  $\sigma_s$  (N/m).
- ▶ This effect is also a surface energy (Nm/m<sup>2</sup> or J/m<sup>2</sup>).
- ▶  $\sigma_s$  represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount.



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## SURFACE TENSION

- ▶ Consider a liquid film (such as the film of a soap bubble) suspended on a U-shaped wire frame with a movable side. Normally, the liquid film tends to pull the movable wire inward in order to minimize its surface area.
- ▶ A force  $F$  needs to be applied on the movable wire in the opposite direction to balance this pulling effect. The length along which the tension acts is  $2b$ .
- ▶ Then a force balance on the movable wire gives  $F = 2b\sigma_s$ ,
- ▶ and thus the surface tension can be expressed as



$$\sigma_s = \frac{F}{2b}$$

$$W = \text{Force} \times \text{Distance} = F \Delta x = 2b\sigma_s \Delta x = \sigma_s \Delta A$$

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## SURFACE TENSION

Surface tension of some fluids in air at 1 atm and 20°C (unless otherwise stated)

- ▶ The surface tension varies greatly from substance to substance, and with temperature for a given substance.
- ▶ The surface tension of a substance can be changed considerably by impurities.
- ▶ chemicals, called surfactants, can be added to a liquid to decrease its surface tension.
  - soaps and detergents lower the surface tension of water and enable it to penetrate through the small openings between fibers for more effective washing.
- ▶ It is important to specify the adjacent liquid or gas when specifying surface tension.

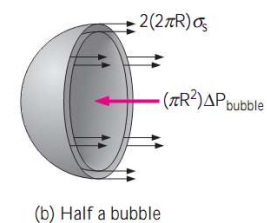
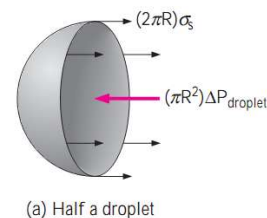
Fluid	Surface Tension $\sigma_s$ , N/m*
Water:	
0°C	0.076
20°C	0.073
100°C	0.059
300°C	0.014
Glycerin	0.063
SAE 30 oil	0.035
Mercury	0.440
Ethyl alcohol	0.023
Blood, 37°C	0.058
Gasoline	0.022
Ammonia	0.021
Soap solution	0.025
Kerosene	0.028

\* Multiply by 0.06852 to convert to lbf/ft.

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## SURFACE TENSION

- ▶ A curved interface indicates a pressure difference (or “pressure jump”) across the interface with pressure being higher on the concave side.
- ▶ The excess pressure  $\Delta P$  inside a droplet or bubble above the atmospheric pressure, for example, can be determined by considering the free-body diagram of half a droplet or bubble.
- ▶ Noting that surface tension acts along the circumference and the pressure acts on the area, horizontal force balances for the droplet and the bubble give



$$\text{Droplet: } (2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{droplet}} \rightarrow \Delta P_{\text{droplet}} = P_i - P_o = \frac{2\sigma_s}{R}$$

$$\text{Bubble: } 2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{bubble}} \rightarrow \Delta P_{\text{bubble}} = P_i - P_o = \frac{4\sigma_s}{R}$$

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## Capillary Effect

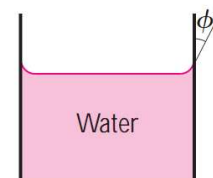
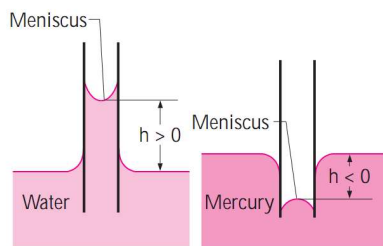
- ▶ capillary effect is the rise or fall of a liquid in a small-diameter tube inserted into the liquid.
- ▶ The rise of kerosene through a cotton wick inserted into the reservoir of a kerosene lamp is due to capillary effect.
- ▶ The capillary effect is also partially responsible for the rise of water to the top of tall trees.



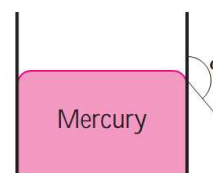
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## Capillary Effect

- ▶ The strength of the capillary effect is quantified by the contact (or wetting) angle  $\phi$ , defined as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact.
- ▶ The surface tension force acts along this tangent line toward the solid surface. A liquid is said to wet the surface when  $\phi < 90^\circ$  and not to wet the surface when  $\phi > 90^\circ$ .



(a) Wetting fluid

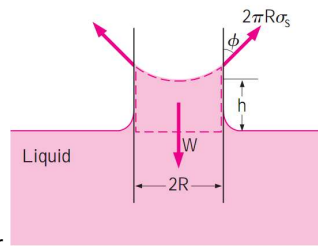


(b) Nonwetting fluid

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## Capillary Effect

- ▶ The magnitude of the capillary rise in a circular tube can be determined from a force balance on the cylindrical liquid column of height,  $h$ , in the tube.
- ▶ The bottom of the liquid column is at the same level as the free surface of the reservoir, and thus the pressure there must be atmospheric pressure.
- ▶ This balances the atmospheric pressure acting at the top surface, and thus these two effects cancel each other. The weight of the liquid column is approximately



$$W = mg = \rho Vg = \rho g(\pi R^2 h)$$

$$W = F_{\text{surface}} \rightarrow \rho g(\pi R^2 h) = 2\pi R \sigma_s \cos \phi$$

$$h = \frac{2\sigma_s}{\rho g R} \cos \phi \quad (R = \text{constant})$$

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A 0.6 mm diameter glass tube is inserted into water at 20°C in a cup.  
Determine the capillary rise of water in the tube.

**SOLUTION** The rise of water in a slender tube as a result of the capillary effect is to be determined.

**Assumptions** 1 There are no impurities in the water and no contamination on the surfaces of the glass tube. 2 The experiment is conducted in atmospheric air.

**Properties** The surface tension of water at 20°C is 0.073 N/m (Table 2–3). The contact angle of water with glass is 0° (from preceding text). We take the density of liquid water to be 1000 kg/m³.

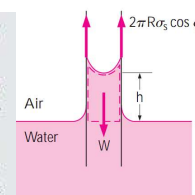
**Analysis** The capillary rise is determined directly from Eq. 2–15 by substituting the given values, yielding

$$h = \frac{2\sigma_s}{\rho g R} \cos \phi = \frac{2(0.073 \text{ N/m})}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \times 10^{-3} \text{ m})} (\cos 0^\circ) \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right)$$

$$= 0.050 \text{ m} = \mathbf{5.0 \text{ cm}}$$

Therefore, water rises in the tube 5 cm above the liquid level in the cup.

**Discussion** Note that if the tube diameter were 1 cm, the capillary rise would be 0.3 mm, which is hardly noticeable to the eye. Actually, the capillary rise in a large-diameter tube occurs only at the rim. The center does not rise at all. Therefore, the capillary effect can be ignored for large-diameter tubes.



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