

## ASSIGNMENT ONE

1. The bar shown in Figure 1 has a mass of 25 kg and the radius of gyration about the c.g. is 0.235 m. Find the natural period of oscillation.

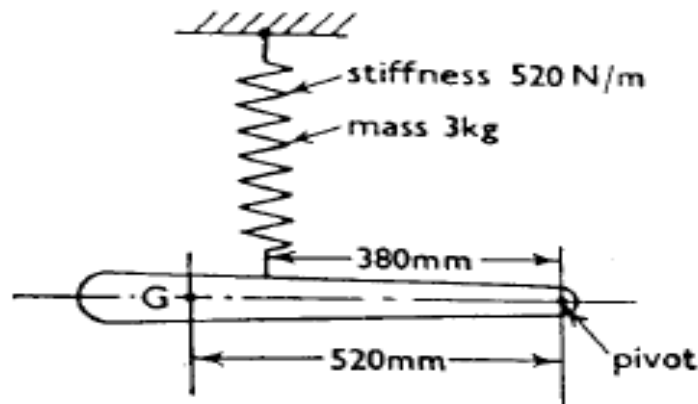


Figure 1

2. In a vibration suspension system, a mass of 25 kg is suspended from a helical spring of stiffness 15 kN/m, the motion being controlled by a dash-pot such that the amplitude of the vibration decreases to one-fifth of its original value after two complete vibrations. Calculate the value of the damping coefficient and the frequency of vibration.
3. Consider the system shown in Figure 2. Write down the equation of motion and comment on its stability.

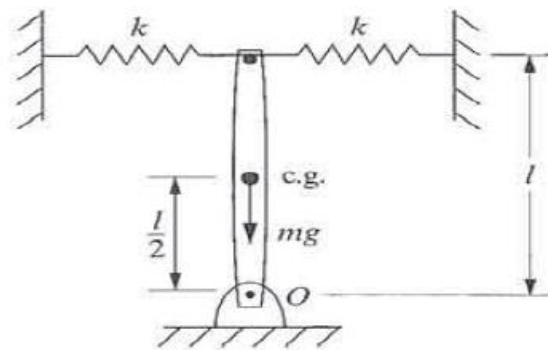


Figure 2

## ASSIGNMENT TWO

1. A spring-mass –damper system has mass of 150 kg, stiffness of 1500 N/m and damping coefficient of 200 kg/s.

- Calculate the undamped natural frequency
- Calculate the damping ratio
- Calculate the damped natural frequency
- Is the system overdamped, underdamped or critically damped?
- Does the solution oscillate?

The system above is given an initial velocity of 10 mm/s and an initial displacement of -5 mm.

- Calculate the form of the response and plot it as long as it takes to die out. How long does it take to die out?

2. Figure 2 shows a simple model of a vehicle which is moving with a constant speed  $80 \text{ km/h}$  over a surface with a sinusoidal profile. The mass of the vehicle is  $9 \text{ tons}$  and the stiffness of its suspension springs is  $100 \text{ kN/m}$ . The wavelength of the surface profile is  $0.8 \text{ m}$  and the amplitude of its undulations is  $15 \text{ mm}$ . Obtain an equation for the steady state vertical displacement of the vehicle and plot the response. (Use MS Excel or MATLAB for the plot).

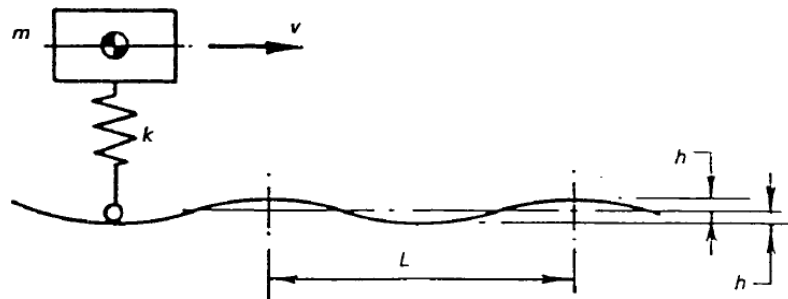


Figure 2

3. A fan is mounted on a spring and viscous damper in parallel so that only linear motion in the vertical direction occurs.

Briefly derive an expression for the force transmitted to the ground through the spring and damper if the fan generates a harmonic disturbing force in the vertical direction. Given that the fan has a mass of 40 kg and a rotating unbalance of 0.01 kg m, determine the spring stiffness required for 10% force transmission. Take the damping ratio of the system to be 0.2 and the fan speed as 1480 rev/min.

When running under these conditions what effect on the transmission would removal of the damping element have?

### ASSIGNMENT THREE

- Figure 3 shows a trolley of mass  $M$ , which runs on a frictionless horizontal plane. At  $O$  the trolley carries a simple pendulum of length  $l$  with a body of mass  $m$  at its end.

Two equal springs, each of stiffness  $k$ , are attached to the trolley and to the fixed walls. By using the independent co-ordinates  $x$  and  $\varphi$  as shown in the figure, determine, for the small free oscillations, the equations of motion.

$$(M + m)\ddot{x} + 2kx + ml\ddot{\varphi} = 0$$

$$\ddot{x} + l\ddot{\varphi} + g\varphi = 0$$

If  $M=100 \text{ kg}$ ,  $m=10 \text{ kg}$ ,  $k= 2 \text{ kN/m}$ ,  $l= 2 \text{ m}$ . Determine the natural frequencies and mode shapes of the system.

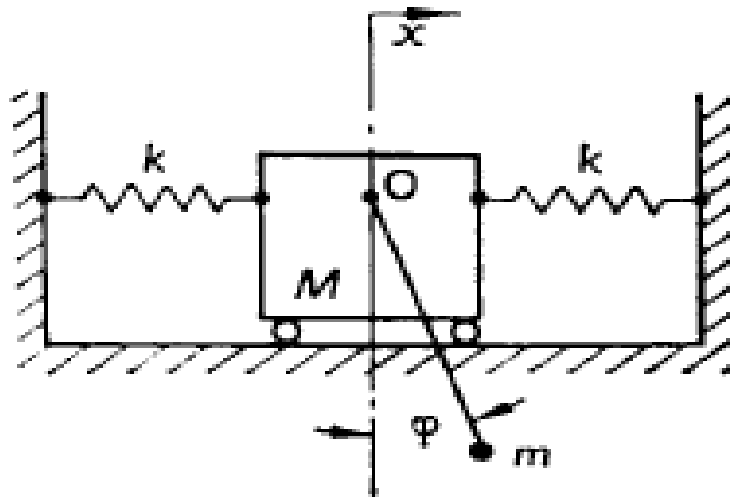


Figure 3

- Figure 4 shows a system consisting of two bodies and three springs with a harmonic force  $F(t) = 15 \cos 3t \text{ kN}$ , applied to one mass. Determine the natural frequencies of this system and calculate its steady-state response. Obtain the solution in the physical coordinate system  $x(t)$ . Work from first principles using matrix methods. Take  $m_1 = 7 \text{ kg}$ ,  $m_2 = 17 \text{ kg}$ ,  $k_1 = 30 \text{ kN/m}$ ,  $k_2 = 20 \text{ kN/m}$ , and  $k_3 = 16 \text{ kN/m}$ . Assume initial conditions  $x = \begin{Bmatrix} 0 \\ 2 \end{Bmatrix} \text{ mm}$  and  $\dot{x} = \begin{Bmatrix} 1 \\ 5 \end{Bmatrix} \text{ mm/s}$

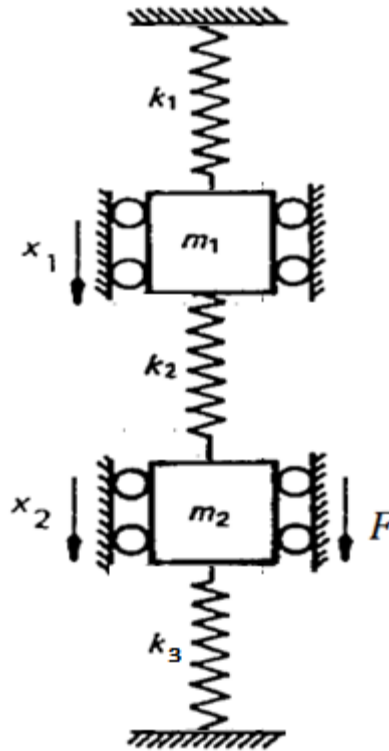


Figure 4

3. A slightly more sophisticated model of a vehicle suspension system is given in Figure 5. Write the equations of motion in matrix form. Calculate the natural frequencies for  $k_1 = 10^3 \text{ N/m}$ ,  $k_2 = 10^4 \text{ N/m}$ ,  $m_2 = 50 \text{ kg}$ , and  $m_1 = 2000 \text{ kg}$ . Add modal damping to this model of  $\xi_1 = 0.01$  and  $\xi_2 = 0.2$  and calculate the response of the body  $x_1(t)$  to a harmonic input at the second mass of  $10 \sin 3t \text{ N}$ .

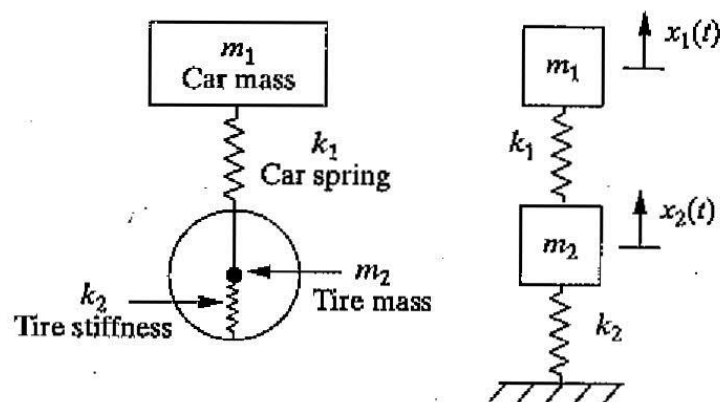


Figure 5: Two-degree-of-freedom model of a vehicle suspension System.