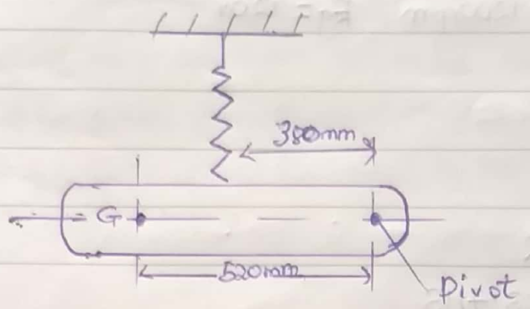
9371517

Mechanical Engineering 3

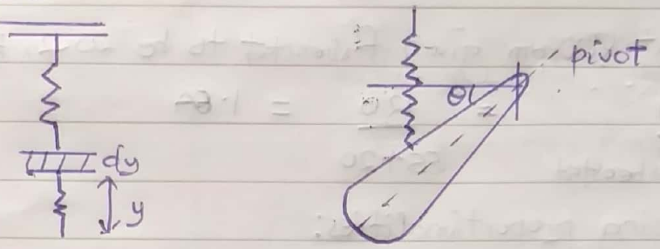
ME 362 VIBRATIONS ASSIGNMENT 1

1. Mass = 25kg, Radius of gyration = 0.235m, period of osillation, T = ?



Let the system displaced by θ about the pivot and deflection of spring be x

$\therefore x = 380\theta = 0.38\theta$



Let spring of length ℓ and mass per unit length, m

Let a strip dy at distance y from the base.

Then, dm (strip mass) $= m dy$

Let force and velocity v

velocity of strip $= \frac{v}{\ell} y$

$\therefore$ kinetic energy $= \int_0^{\ell} \frac{1}{2} m' dy \left(\frac{v}{\ell} y\right)^2$

$$k = \frac{1}{2} \frac{m' v^2}{\ell^2} \times \frac{1}{3} [\ell^3 - 0]$$

$$k = \frac{1}{2} m' \ell v^2 \times \frac{1}{3}$$

But $m'\ell = ms$

$$k = \frac{1}{2} \left(\frac{ms}{3} \times v^2\right)$$

Applying the Energy Theorem,

Total energy

$$T = \frac{1}{2} \frac{ms}{3} v^2 + \frac{1}{2} I\dot{\theta}^2 + \frac{1}{2} kx^2$$

where $\frac{1}{2} kx^2$ is the strain energy of the spring.

$\frac{1}{2} I\dot{\theta}^2$ is the rotational kinetic energy of the bar about O.

$I = mk^2 + m \times 0.52^2$

$v = \dot{x} = 0.38\dot{\theta}$

$T = \frac{1}{2}\frac{m_S}{3} \times (0.38\dot{\theta})^2 + \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}k(0.38\theta)^2$

Differentiating about θ and putting $dT/d\theta = 0$ yields:

$\frac{dT}{d\theta} = \frac{1}{2}\frac{m_S}{3} \times 0.38^2 \times 2\dot{\theta} \times \ddot{\theta} + \frac{1}{2}I \times 2\dot{\theta} \times \ddot{\theta} + \frac{1}{2}k \times 0.38^2 \times \theta \times 2\dot{\theta} -$

$\dot{\theta}\left[\left(\frac{m_S}{3} \times 0.38^2 + I\right)\ddot{\theta} + k \times 0.38^2\theta\right] = 0$

$\therefore \dot{\theta} \neq 0$

$\Rightarrow \left(\frac{m_S}{3} \times 0.38^2 + I\right)\ddot{\theta} + k \times 0.38^2\theta = 0$

$I = 25 \times 0.235^2 + 25 \times 0.52^2$

$I = \frac{521}{64}$ kg-m^2

$\left(\frac{3}{3} \times 0.38^2 + \frac{521}{64}\right)\ddot{\theta} + 520 \times 0.38^2\theta = 0$

$\ddot{\theta} + 9.063098784 = 0$

Compare the above with $\ddot{\theta} + w^2\theta = 0$

$w^2 = 9.063098784$

$w = 3.010498096$ rad/s

$\therefore \frac{2\pi}{T} = 3.010498096$

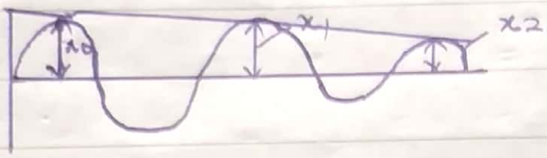
Period, $T = 2.087091606s$

2. Mass = 25kg Spring stiffness = 15kN/m

Let x_0 be the initial amplitude

let x_1 be final amplitude after two consecutive vibrations

Let x_2 be $\frac{1}{5}(x_1)$



$$x_2 = \left(\frac{1}{5}\right)x_1$$

$$\therefore \frac{x_1}{x_2} = 5$$

Therefore, the logarithmic decreament

$$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right) = \ln\left(\frac{x_1}{x_2}\right)$$

$$\delta = \ln(5) = 1.609$$

Now from the formula

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$1.60 = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

Continuation

$$(1.60)^2 = \frac{4\pi^2\xi^2}{1-\xi^2}$$

$$2.59(1-\xi^2) = 4\pi^2\xi^2$$

$$2.57 - 2.5\xi^2 = 39.4\xi^2$$

$$41.99\xi^2 = 2.59$$

$$\xi = 0.2483$$

$\therefore$ the damping ratio, ξ is 0.2483

Calculating the damping co-efficient yields:

$$\xi = \frac{c}{c_c}$$

where c = actual damping coefficient

c_c = critical damping coefficient

$$\text{But } C_c = 2\sqrt{km} = 2\sqrt{15\times10^3\times25}$$

$$C_c = 1224.7\ Ns/m$$

$$\xi = \frac{c}{C_c}$$

$$0.2483 = \frac{c}{1224.7}$$

$$c = 0.2483\times1224.7$$

$$c = 304.1\ Ns/m$$

9371517
Mechanical Engineering 3
ME 362

2. calculations of frequency of Vibration

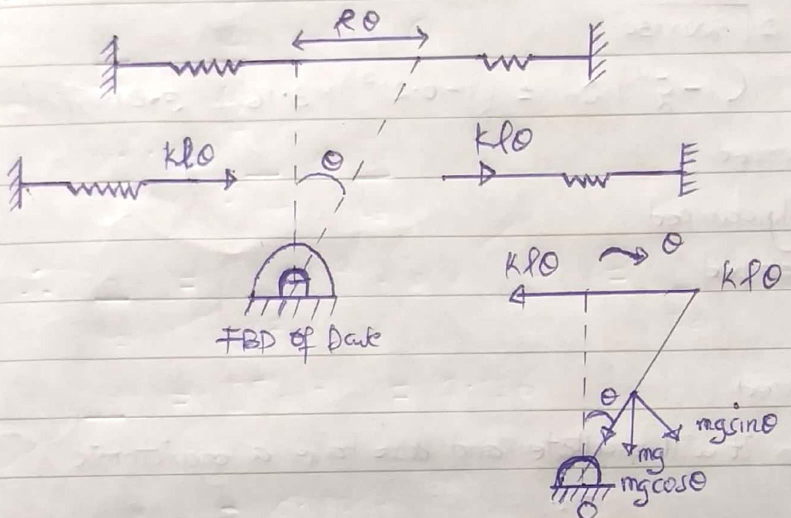
$$w_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{15\times10^3}{25}}$$

$w_n = 24.5$ rad/s

$$f_n = \frac{w_n}{2\pi} = \frac{24.5}{2\times3.14}$$

$\therefore f_n = 3.9$ Hz

3.



Writing torque equation about O yields;

$$-mg\sin\theta\times\frac{l}{2} + 2kl^2\theta = I\ddot{\theta} = \frac{-ml^2\ddot{\theta}}{3}$$

$$\frac{ml^2\ddot{\theta}}{3} - mg\sin\theta\frac{l}{2} + 2kl^2\theta = 0 \quad \text{since } \theta \text{ is small, } \sin\theta = \theta$$

$$\frac{ml^2\ddot{\theta}}{3} + \left(-mg\frac{l}{2} + 2kl^2\right)\theta = 0$$

$$\frac{ml\ddot{\theta}}{3} + \left(\frac{mg}{2} + 2kl\right)\theta = 0$$

$$w_n = \sqrt{\frac{-mg/2 + 2kl}{ml/3}}$$

$$w_n = \sqrt{\frac{\frac{4kl-mg}{2}}{ml/3}}$$

$$w_n = \sqrt{\frac{3(4kl-mg)}{2ml}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{3}{2}\left(\frac{4kl-mg}{ml}\right)}$$

The system is stable and will vibrate with the above frequency

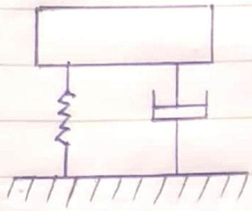
9371517

Mechanical Engineering 3

ME 362 VIBRATIONS

ASSIGNMENT 2

1. Mass = 150kg stiffness of spring, $k = 1500 N/m$, damping coefficient, $c = 200 kg/s$



i. Undamped natural frequency, $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1500}{150}} = 3.16 rad/s$

ii. Damping ratio, $\xi = \frac{c}{2\sqrt{km}} = \frac{200}{2\sqrt{1500 \times 150}} = 0.21$

iii. Damped natural frequency, $\omega_d = (1-\xi^2)\omega_n = (1-0.21^2) \times 3.16 = 3.019 rad/s$

iv. w.k.T if $\xi > 1 \Rightarrow$ overdamped

$\xi = 1 \Rightarrow$ critically damped

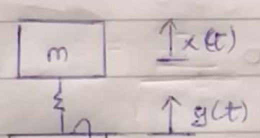
$\xi < 1 \Rightarrow$ under damped

$\xi = 0.21 < 1$

$\therefore$ the system is under damped

v. YES, for an under damped system, it will oscillate and also have a logarithmic decrement in every oscillation.

2.



Mass = 900kg $k = 100 kN/m$ $v = 80 kmph$

$\lambda = 0.8m$ $Y = 15mm = 0.015m$

Equation of motion:

$m\ddot{x} + k(x-y) = 0$

$m\ddot{x} + kx = ky$ — ①

But $y = Y\sin\omega t$

$\omega = 2\pi f = 2\pi\left(\frac{v \times 1000}{3600}\right)\frac{1}{\lambda}$

$\omega = \frac{2\pi \times 80 \times 1000}{3600} \times \frac{1}{8} = 174.533 rad/s$

$\therefore y = 0.015\sin(174.533t)$ — ②

Equation ① the becomes

$m\ddot{x} + kx = kY\sin\omega t$ — ③

The steady state response is given by particular solution of Equation ③

9371517

ME362 Assignment 2

2. Let $x_p(t) = A\cos\omega t + B\sin\omega t$ — (4)

$\therefore \dot{x}(t) = -A\omega\sin\omega t + B\omega\cos\omega t$

$\ddot{x}(t) = -A\omega^2\cos\omega t - B\omega^2\sin\omega t$

Substituting or putting in Eqn (3) yields:

$-m\omega^2(A\cos\omega t + B\sin\omega t) + k(A\cos\omega t + B\sin\omega t) = kY\sin\omega t$

$\cos\omega t(-m\omega^2 A + kA) + \sin\omega t(-m\omega^2 B + kB) = kY\sin\omega t$

Now, comparing coefficients:

$-m\omega^2 A + kA = 0 \Rightarrow A = 0$

$-m\omega^2 B + kB = kY \Rightarrow B = \dfrac{kY}{k - m\omega^2}$

from Eqn (4),

$x_p(t) = x(t) = \dfrac{kY}{k - m\omega^2}\sin\omega t$

Substituting values gives:

$x(t) = \dfrac{100\times10^2\times0.015}{100\times10^3 - 9000\times174.533^2}\sin(174.533t)$

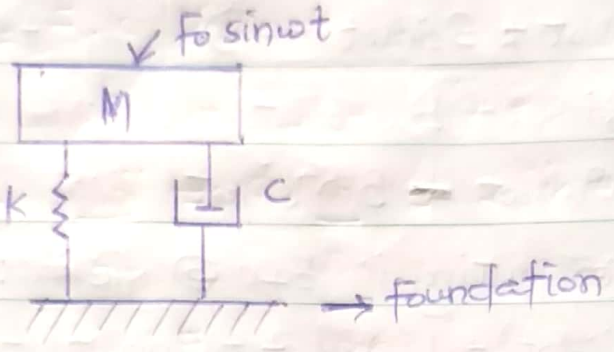
$x(t) = -5.473\times10^{-6}\sin(174.533t)$ — (5)

9371517

ME 362

ASSIGNMENT 2

3.



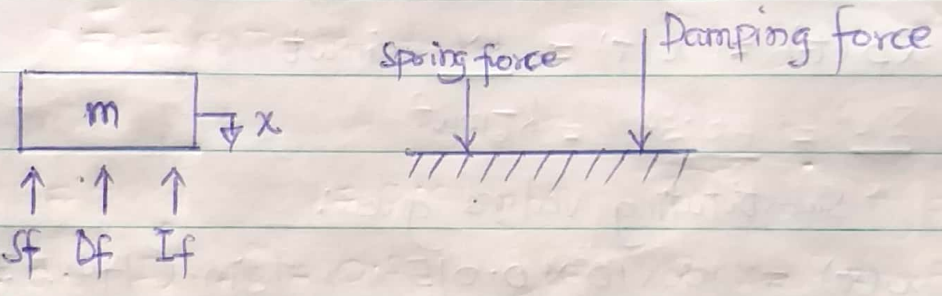
Total forces in the system are:

i F_0

ii Damper force $= C\omega x$

iii Inertia force $= m\omega^2 x$

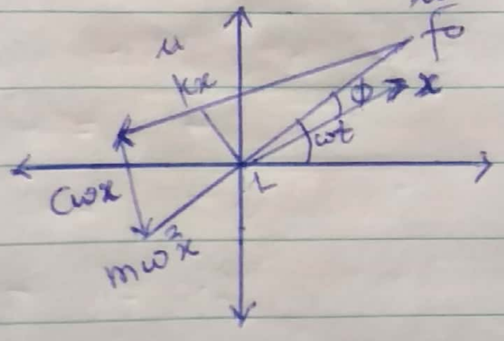
iv. Spring force $= kx$



Let the force transmitted to the ground through spring and damper be F_T (Force Transmissibility)

$$T_r = \frac{\text{force transmitted to the foundation}}{\text{force impulsed upon the system}}$$

Free Body Diagram



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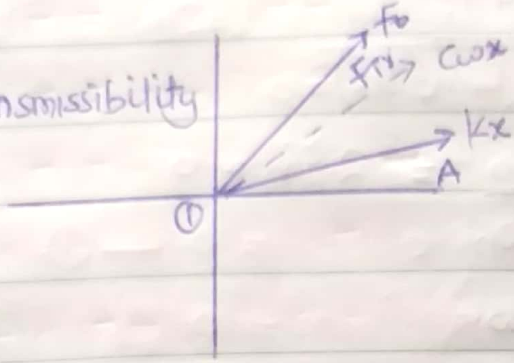
8 Phasor Diagram

Force transmissibility



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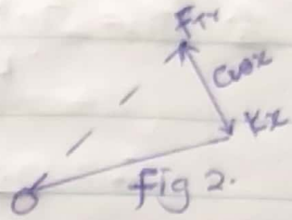
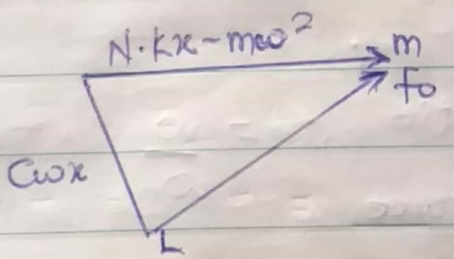


Fig 2.

From fig. 2

$$F_{Tr} = \sqrt{(c\omega x)^2 + (kx)^2}$$

$$F_{Tr} = x\sqrt{(c\omega)^2 + k^2} \quad \text{— (1)}$$



$$F_o = \sqrt{(kx - m\omega^2 x)^2 + (c\omega x)^2}$$

$$f_o = \sqrt{x^2 (K - m\omega^2)^2 + (c\omega)^2}$$

$$F_o = x\sqrt{(K - m\omega^2) + (c\omega)^2}$$

$$\Rightarrow x = \frac{f_o}{\sqrt{(K - m\omega^2)^2 + (c\omega)^2}} \quad \text{— (2)}$$

Putting the value of x in Eqn (1) gives:

$$F_{Tr} = \frac{f_o\sqrt{(c\omega)^2 + k^2}}{\sqrt{(K - m\omega^2)^2 + (c\omega)^2}} \quad \text{— (2)}$$

$$F_{Tr} = \frac{F_o \cdot K\sqrt{\left(\frac{c\omega}{K}\right)^2 + 1}}{K^2\left(1 - \frac{m\omega^2}{K}\right)^2 + (c\omega)^2}$$

9371517

ME362 Assignment 2

3. $F_{Tr} = \frac{F_0 \cdot K\sqrt{(cw/K)^2+1}}{K\sqrt{(1-mw^2/K)+(cw/K)^2}}$

$\therefore \frac{cw}{K} = \frac{2\xi w}{w_n} = 2\xi r$

$r = w/w_n$

$$\frac{F_{Tr}}{F_0} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}} \quad — ②$$

The expression about is for force transmitted to foundation

where :

ξ = damping ratio , $r = \frac{w}{w_n}$

+ F_0 = external force

Mass = 40kg, $m \cdot r = 0.01$ kg·m, Transmissibility = 10%

$F_T = 0.1 \times F_0$, $\xi = 0.2$, $N = 1480$ rev/min $= w = \frac{2\pi \times 1480}{60}$

$w = 155$ rad/s

Total rotating unbalance

$F_0 = m \cdot r \cdot w^2$

$F_0 = (0.01) \times (155)^2 = 240N$

force transmitted $= \frac{10}{100} F_0$

$F_T = 24N$

From Eqn ②, the force transmitted to foundation is

$F_T = \sqrt{1+(2\xi r)^2}$

$24 = \sqrt{1+(2(\xi r))^2}$

But $\xi = 0.2$

9371517
ME 362 Assignment 2

3. $r = 59.94 = \frac{w}{w_n}$, $w = 155$ rad/s

$w_n = \sqrt{\frac{k}{m}} = \frac{w}{r} = \frac{155}{59.94} = 2.58$

$2.58^2 = \frac{k}{40}$

$k = 267.4$ N/m

∴ the stiffness of the spring is 267.4 N/m

Removing the damping element from the system makes the damping ratio = 0. So we have to take (r^2-1) instead of $(1-r^2)$

$\frac{F_T}{F_0} = \xi = 0.1 = \frac{1}{(1-r^2)}$ from eqn ①

$(1-r^2) = 10$

$r^2 = -9$

Taking (r^2-1),

$0.1 = \frac{1}{r^2-1}$

$r^2-1 = 10$

$r = \sqrt{11} = 3.32$

∴ $w/w_n = 3.32$

In conclusion, removing the damping element will make the frequency of the system greater than the natural frequency.

9371517

Mechanical Engineering 3

ME 362 VIBRATIONS

ASSIGNMENT 3

1 $M = 100kg$, $m = 10kg$; $K = 2kN/m$, $l = 2m$

$$(M+m)\ddot{x} + 2kx + ml\ddot{\phi} = 0$$

$$(100+10)\ddot{x} + 2(2)x + (10\times2)\ddot{\phi} = 0$$

$$110\ddot{x} + 20\ddot{\phi} + 4x = 0 \quad — ①$$

$$\ddot{x} + l\ddot{\phi} + g\phi = 0$$

$$\ddot{x} + 2\ddot{\phi} + 9.8\phi = 0 \quad — ②$$

Assuming the above solution in second order is:

$x = X\cos(\omega_n t + \theta)$, $\ddot{x} = -\omega_n^2 X\cos(\omega_n t + \theta)$

$\phi = \Theta\cos(\omega_n t + \theta)$, $\ddot{\phi} = -\omega_n^2 \Theta\cos(\omega_n t + \theta)$

where θ is the phase angle.

Now, substituting or putting the values in the equation:

$$-110\omega_n^2 x\cos(\omega_n t + \theta) - 20\omega_n^2\Theta\cos(\omega_n t + \theta) + 4x\cos(\omega_n t + \theta) = 0$$

$$[(-110\omega_n^2 + 4)x - 20\omega_n^2 t]\cos(\omega_n t + \theta) = 0 \quad — ③$$

$$-\omega_n^2 x\cos(\omega_n t + \theta) - 2\omega_n^2\Theta\cos(\omega_n t + \theta) + 9.81\Theta\cos(\omega_n t + \theta) = 0$$

$$[-\omega_n^2 x + (-2\omega_n^2 + 9.81)\Theta]\cos(\omega_n t + \theta) = 0$$

multiplying the above by 20 gives:

$$[-20\omega_n^2 x + (-40\omega_n^2 + 196)\Theta]\cos(\omega_n t + \theta) = 0 \quad — ④$$

Using eqns ③ & ④, the natural frequency is calculated by

$$\det\begin{vmatrix} -110\omega_n^2 t + 4 & -20\omega_n^2 \\ -20\omega_n^2 & -40\omega_n^2 + 196 \end{vmatrix} = 0$$

$$(-110\omega_n^2 + 4)(-40\omega_n^2 + 196) - (20\omega_n^2)^2 = 0$$

$$4000\omega_n^4 - 21640\omega_n^2 + 784 = 0 \quad — ⑤$$

From eqn ⑤,

$$\omega_n^2 = 0.036475,\ 5.373525$$

9371517

ME 362

ASSIGNMENT 3

1. $\omega_1 = 0.19$ rad/s, $\omega_2 = 2.318$ rad/s

For the mode shapes,

$$x_1 = \frac{\Theta^{(1)}}{x^{(1)}} = \frac{-110\omega_1^2+4}{20\omega_1^2} = \frac{20\omega_1^2}{40\omega_1^2+196}$$

$$x_1 = \frac{-110(0.191)^2+4}{20\times(0.191)^2} = -0.0168$$

$$x_2 = \frac{\Theta^{(2)}}{x^{(2)}} = \frac{-110\omega_2^2+4}{20\omega_2^2} = \frac{-110(2.318)^2+4}{20(2.318)^2} = -5.46278$$

$$x^{(1)} = \begin{Bmatrix} x_1^{(1)} \\ \Theta^{(1)} \end{Bmatrix} = \begin{Bmatrix} x_1^{(1)} \\ \Theta x_1 x^{(1)} \end{Bmatrix} = \begin{Bmatrix} x^{(1)} \\ -0.0168x^{(1)} \end{Bmatrix}$$

$$x^{(2)} = \begin{Bmatrix} x^{(2)} \\ \Theta^{(2)} \end{Bmatrix} = \begin{Bmatrix} x^{(2)} \\ x_2 x^{(2)} \end{Bmatrix} = \begin{Bmatrix} x^{(2)} \\ -5.46278x^{(2)} \end{Bmatrix}$$

$$x^{(1)}t = \begin{Bmatrix} x^{(1)} \\ \phi^{(1)} \end{Bmatrix} = \begin{Bmatrix} x^{(1)}\cos(\omega_1 t+\phi_1) \\ -0.0168x^{(1)}\cos(\omega_1 t+\phi_1) \end{Bmatrix} = \text{first mode}$$

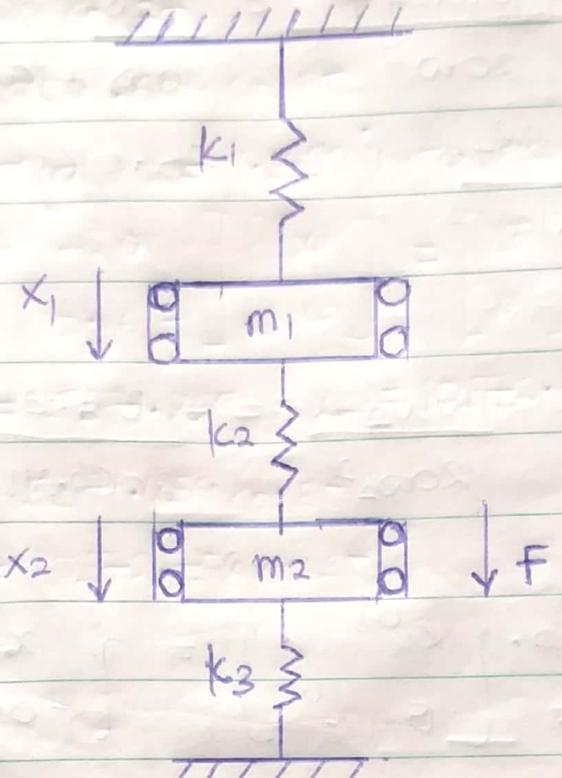
$$x^{(2)}t = \begin{Bmatrix} x^{(2)} \\ \phi^{(2)} \end{Bmatrix} = \begin{Bmatrix} x^{(2)}\cos(\omega_2 t+\phi_2) \\ -5.46278x^{(2)}\cos(\omega_2 t+\phi_2) \end{Bmatrix} = \text{second mode}$$

93715IT

ME 362 VIBRATIONS

ASSIGNMENT 3

2. $f(t) = 15\cos 3t$ kN, $m_1 = 7$kg, $m_2 = 17$kg, $k_1 = 30$kN/m, $k_2 = 20$kN/m, $k_3 = 16$kN/m



Applying Newton's Law of motion,

$$m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = F(t) \quad — ①$$

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) + k_3x_2 = 0 \quad — ②$$

Writing Eqns ① and ② in matrix form gives

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} (k_1+k_2) & -k_2 \\ -k_2 & (k_2+k_3) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix}$$

Calculating the undamped system solution of the system:

$x_1(t) = x_1 \cos\omega t$

$= x_1 \cos 3t$

$\ddot{x}_1(t) = -\omega^2 x_1 \cos 3\omega t$

$= -\omega^2 x_1 \cos 3t$

9371517

ME 362 Assignment 3

2. $$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} -\omega^2 x_1 \\ -\omega^2 x_2 \end{bmatrix}\cos 3t + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\cos 3t = \begin{bmatrix} 15 \\ 0 \end{bmatrix}\cos 3t$$

$$\begin{bmatrix} (k_1+k_2)-\omega^2 m_1 & -k_2 \\ -k_2 & (k_2+k_3)-\omega^2 m_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix}$$

Putting the values in the matrix, let $\omega^2 = \lambda$

$$\begin{bmatrix} 50-7\lambda & -20 \\ -20 & 36-17\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix} \longrightarrow (3)$$

Solving for the determinant of eqn (3) gives:

$$\begin{bmatrix} 50-7\lambda & -20 \\ -20 & 36-17\lambda \end{bmatrix} = 0$$

$$(50-7\lambda)(36-17\lambda) - 400 = 0$$

$$119\lambda^2 - 1102\lambda + 1400 = 0$$

$$\lambda_1 = 7.7406, \quad \lambda_2 = 1.5199$$

For the Natural frequency (ω),

$$\omega_1 = \sqrt{\lambda_1}$$
$$\omega_1 = \sqrt{7.7406}$$
$$\omega_1 = 2.78 \text{ rad/s}$$
$$\omega_2 = \sqrt{\lambda_2}$$
$$= \sqrt{1.5199}$$
$$\omega_2 = 1.23 \text{ rad/s}$$

Substituting $\lambda_1 = 7.7406$ into eqn (3)

$$\begin{bmatrix} -4.1842 & -20 \\ -20 & -95.5902 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix}$$

$$\therefore x_1 = -44361.426, \quad x_2 = 9281.604$$

9371517

ME 362 VIBRATIONS

ASSIGNMENT 3

2. $x_1(t) = X_1 \cos 3t$

$= -44361.426 \cos 3t$

$x_2(t) = X_2 \cos 3t$

$= 9281.604 \cos 3t$

Putting $\lambda_2 = 1.5199$ into eqn ②

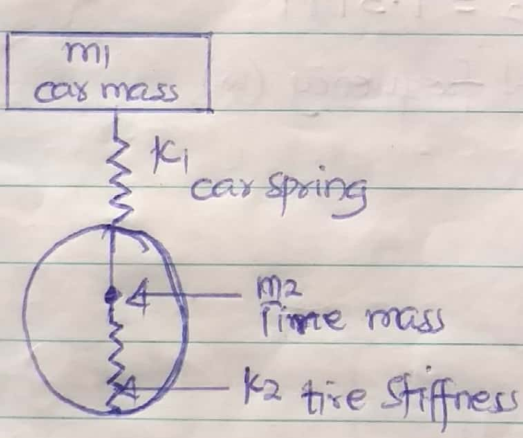
$$\begin{bmatrix} 39.3607 & -20 \\ -20 & 10.1617 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix}$$

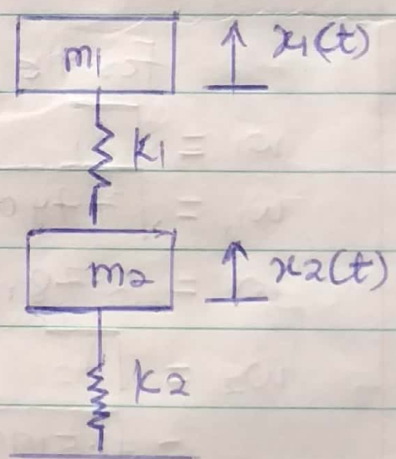
$x_1 = 5371.859$, $x_2 = 10572.758$

$x_1(t) = 5371.859 \cos 3t$

$x_2(t) = 10572.758 \cos 3t$

3.





$m_1 = 200kg$, $m_2 = 50kg$, $k_1 = 1000N/m$, $k_2 = 10000N/m$

Equations of motion for the system:

$m_1\ddot{x}_1 + k_1x_1 - k_1x_2 = 0$

$m_2\ddot{x}_1 - k_1x_1 + (k_1+k_2)x_2 = 0$

Putting the values into the equations of motion yields:

$200\ddot{x}_1 + 1000x_1 - 1000x_2 = 0$

$50\ddot{x}_2 - 1000x_1 + (1000 + 10000)x_2 = 0$

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ME 362 VIBRATIONS

ASSIGNMENT 3

3. Rewriting the above equations in matrix form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \ddot{x} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1+k_2 \end{bmatrix} x = 0$$

$$\begin{bmatrix} 2000 & 0 \\ 0 & 50 \end{bmatrix} \ddot{x} + \begin{bmatrix} 1000 & -1000 \\ -1000 & 11000 \end{bmatrix} x = 0$$

For the natural frequency,

$$\det(-\omega^2 M + K) = 0$$

$$\begin{vmatrix} -m_1\omega^2 + k_1 & -k_1 \\ -k_1 & -m_2\omega^2 + (k_1+k_2) \end{vmatrix} = 0$$

Substituting the values into the natural frequency,

$$\begin{vmatrix} -2000\omega^2 + 1000 & -1000 \\ -1000 & -50\omega^2 + 11000 \end{vmatrix} = 0$$

$$100000\omega^4 - 2.205 \times 10^7 \omega^2 + 10^7 = 0$$

$\therefore \omega_1^2 = 0.454, \quad \omega_2^2 = 220.046$

Therefore the natural frequency turns to be,

$\omega_1^2 = 0.454$

$\omega_1 = \sqrt{0.454}$

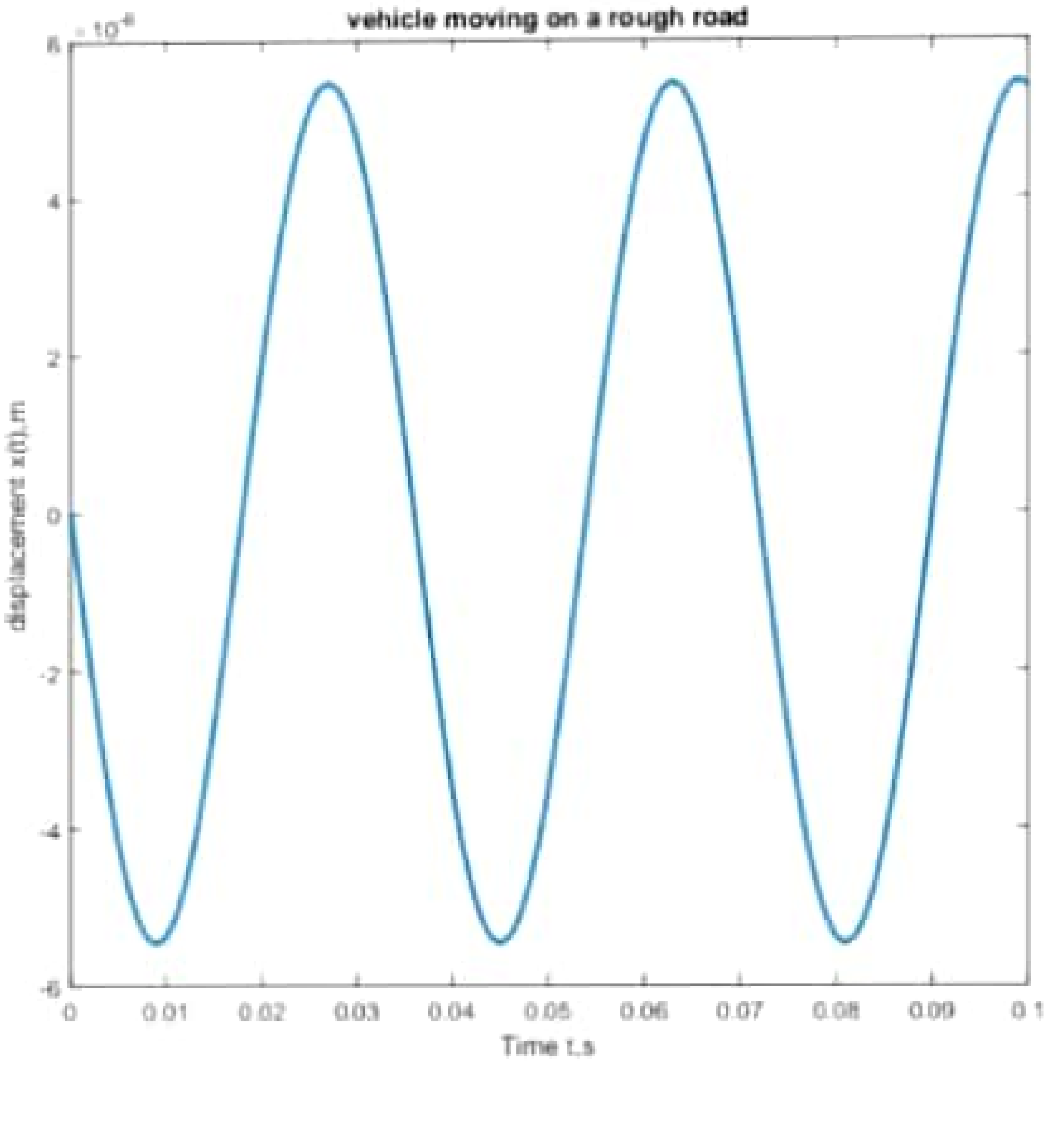
$\omega_1 = 0.674 \text{ rad/s}$

$\omega_2^2 = 220.046$

$\omega_2 = \sqrt{220.046}$

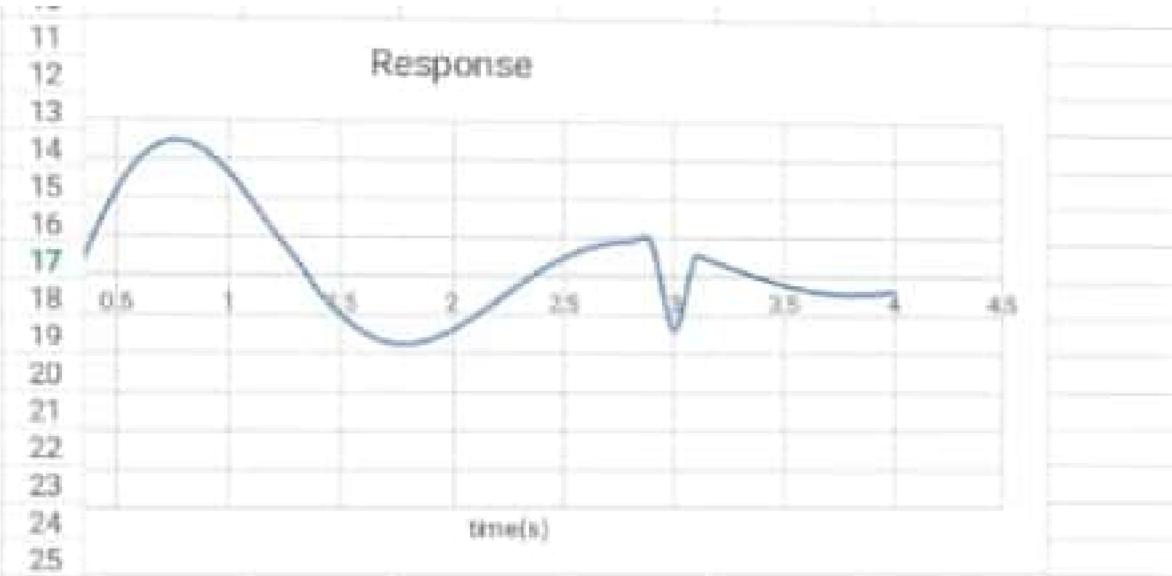
$\omega_2 = 14.834 \text{ rad/s}$

$\therefore$ the values of the natural frequencies are $\omega_1 = 0.674$ rad/s and $\omega_2 = 14.834$ rad/s



vehicle moving on a rough road
6
4
2
0
-2
-4
-6
displacement x(t),m
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Time t,s

t/s	x(t)/m
0	-0.005000027
0.1	-0.003555737
0.2	-0.001961684
0.3	-0.000384571
0.4	0.001031289
0.5	0.002174579
0.6	0.002973191
0.7	0.003396022
0.8	0.003450743
0.9	0.003178246
1	0.002644695
1.1	0.00193223
1.2	0.001129343
1.3	0.000418432
1.4	-0.000414645
1.5	-0.001020683
1.6	-0.00145627
1.7	-0.001702252
1.8	-0.001759468
1.9	-0.001646181
2	-0.001394202
2.1	-0.001044247
2.2	-0.000641042
2.3	-0.000228674
2.4	0.000153426
2.5	0.000473563
2.6	0.000709752
2.7	0.000850547
2.8	0.00089479
2.9	0.000850431
3	-0.001394202
3.1	0.000561561
3.2	0.000359697
3.3	0.000149647
3.4	-4.80647E-05
3.5	-0.000216623
3.6	-0.000344018
3.7	-0.000423563
3.8	-0.000453849
3.9	-0.000438214
4	-0.000383846


Response
time(s)