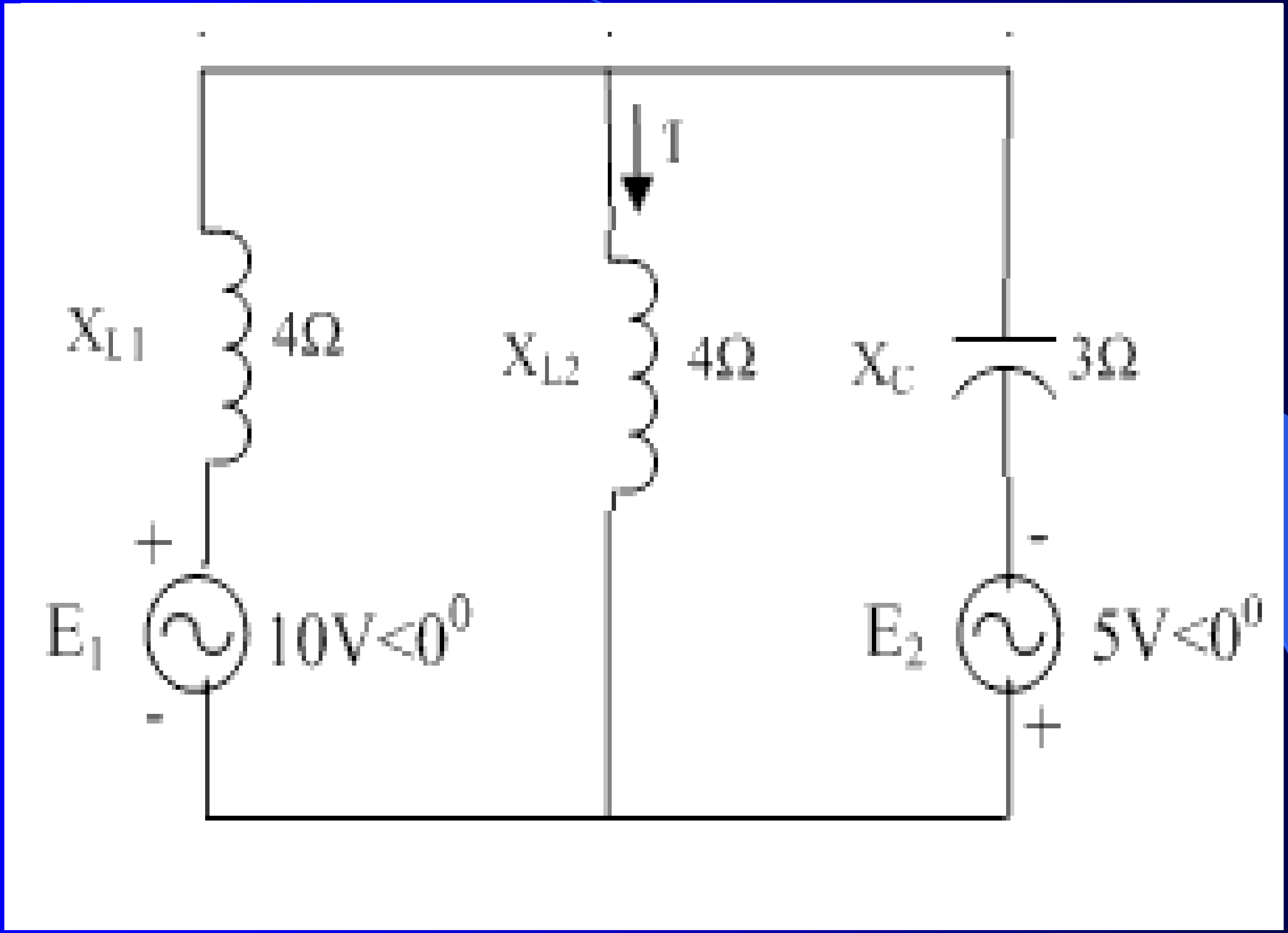
Application of Network Theorems to AC Networks

- **Introduction**
  - When voltages, currents and impedances are treated as complex numbers or phasors, the solution of ac circuits becomes the same as that of dc circuits.
  - In this chapter, we will be working with impedances and phasors instead of just resistors and real numbers.
- Superposition
  - No change

Example 1: Find the current *I* by the superposition theorem



Solution:

<u>Voltage source E_1 acting alone:</u>

Effective impedance of parallel branch,

$$X_{L2}//X_C = \frac{(j4)(-j3)}{j4 - j3} = -j12$$

Total impedance = j4 – j12

$= -j8\ \Omega = 8\ \Omega\angle -90^0$

Total current = $10\angle 0^0 / 8\angle -90^0$

$= 1.25\ A\angle 90^0$ and

I^1 = voltage across parallel branch / X_{L2}

$= (12\angle -90^0 \times 1.25\angle 90^0) / 4\angle 90^0$

$= 3.75A\angle -90$

Voltage source E_2 acting alone:

Impedance of parallel branch, $X_{L1}//X_{L2}$ = j4//j4 = j2

Total Impedance = j2 – j3 = -j1 = $1\angle -90^0$

Total Current = 5 / $1\angle -90^0$ = $5A\angle 90^0$

I^{11} = $5\angle 90^0$ / 2 = $2.5A\angle 90^0$ because $X_{L1} = X_{L2}$

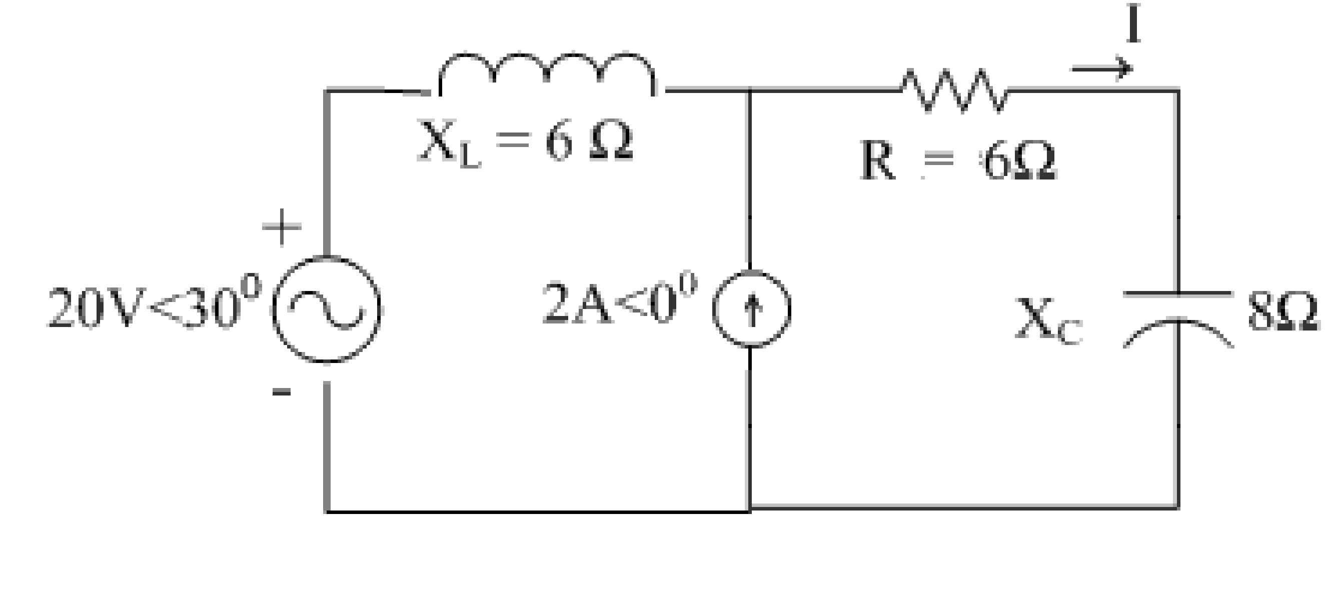
Actual Current, $I = I^1 - I^{11}$

$= 3.75A\angle -90 - 2.5A\angle 90^0$

$= -j3.75 - j2.5$

$= -j6.25 = 6.25A\angle -90^0$

Example 2:Using superposition, find the current I



With the current source acting alone,

the current through the resistor by current divider rule is

$$I^1 = \frac{j6}{6 + j(6 - 8)} \times 2 = \frac{j12}{6 - j2}$$

$$= \frac{12\angle 90^0}{6.32\angle -18.43^0} = 1.9\angle 108.43^0$$

With the voltage source acting alone,

$$I^{11} = \frac{20\angle 30^0}{6 + j(6 - 8)} = \frac{20\angle 30^0}{6 - j2}$$

$$= \frac{20\angle 30^0}{6.32\angle -18.43^0} = 3.16\angle 48.43^0$$

Actual Current,

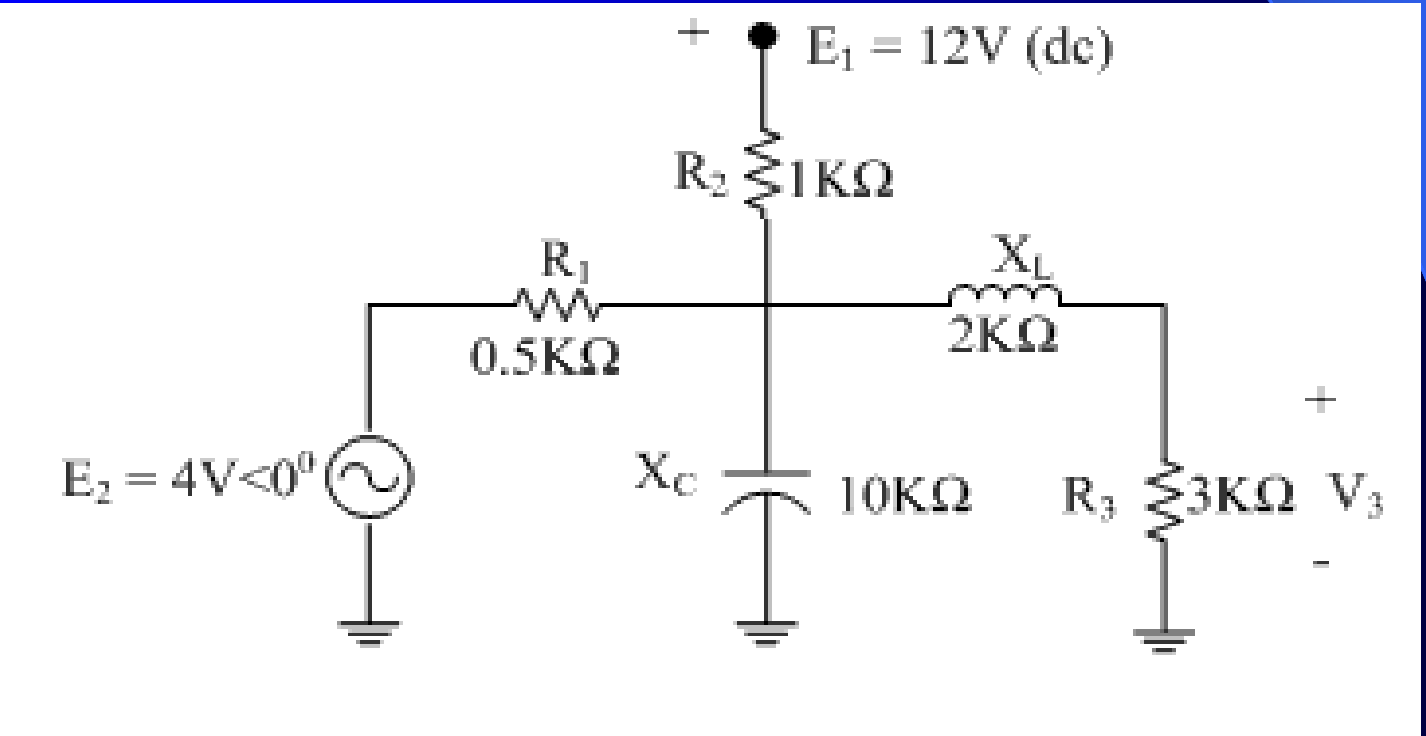
$$I = I^1 + I^{11} = 1.9\angle 108.43^0 + 3.16\angle 48.43^0$$

$$= (-0.60 + j1.80) + (2.10 + j2.36)$$

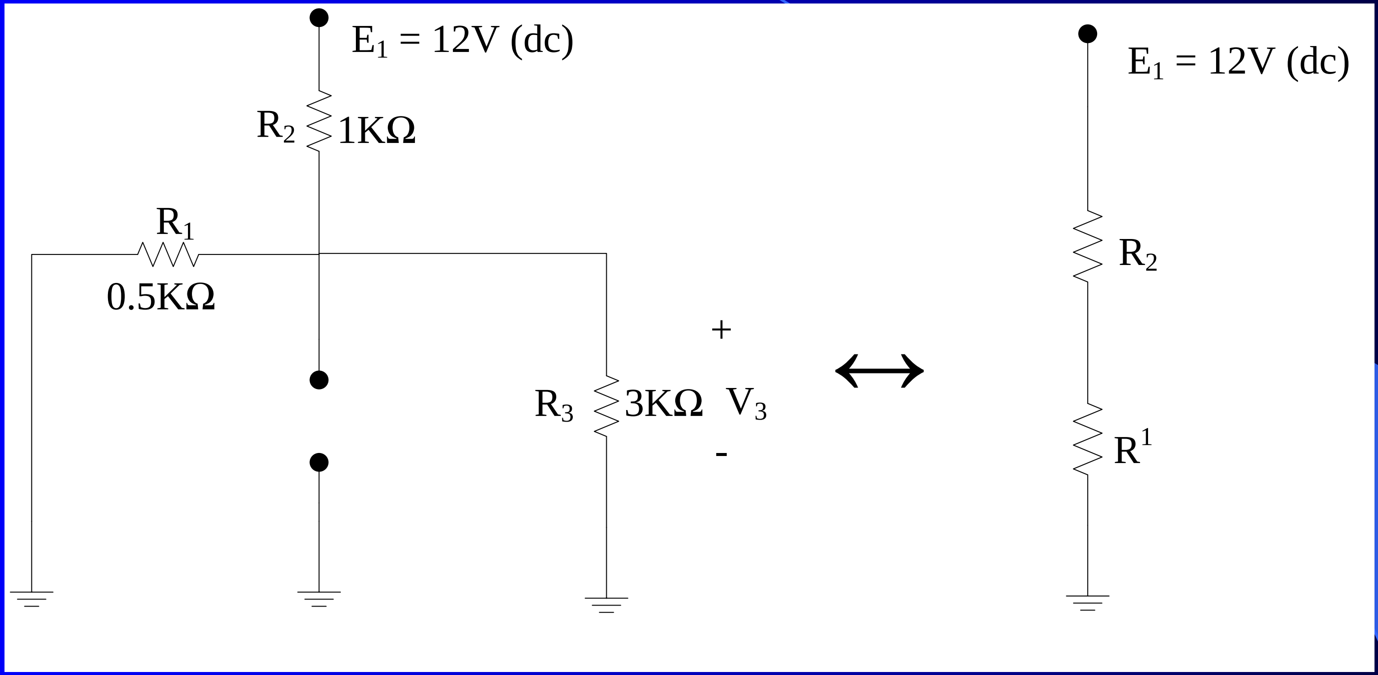
$$= 1.50 + j4.16 = 4.42\angle 70.2^0$$

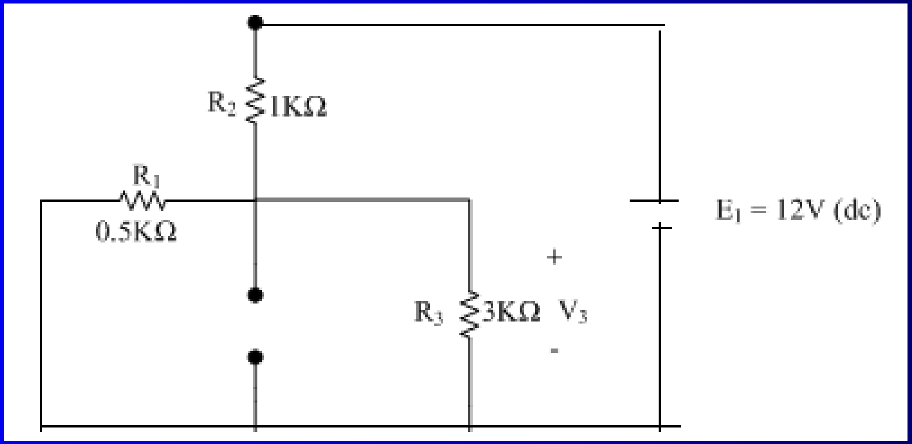
- NOTE: One of the most frequent applications of the superposition theorem is to electronic systems in which the dc and ac analysis are treated separately and the total solution is the sum of the two.

- Example 3: For the network below, determine the sinusoidal expression for the voltage v_3 using superposition.



- For the dc analysis, in steady state, the capacitor is open-circuited and the inductor short-circuited.
- The result is the network below.





Resistance of parallel branch is given by

$$R^1 = \frac{(0.5 \times 3)}{(0.5 + 3)} = 0.429\ \text{k}\Omega$$

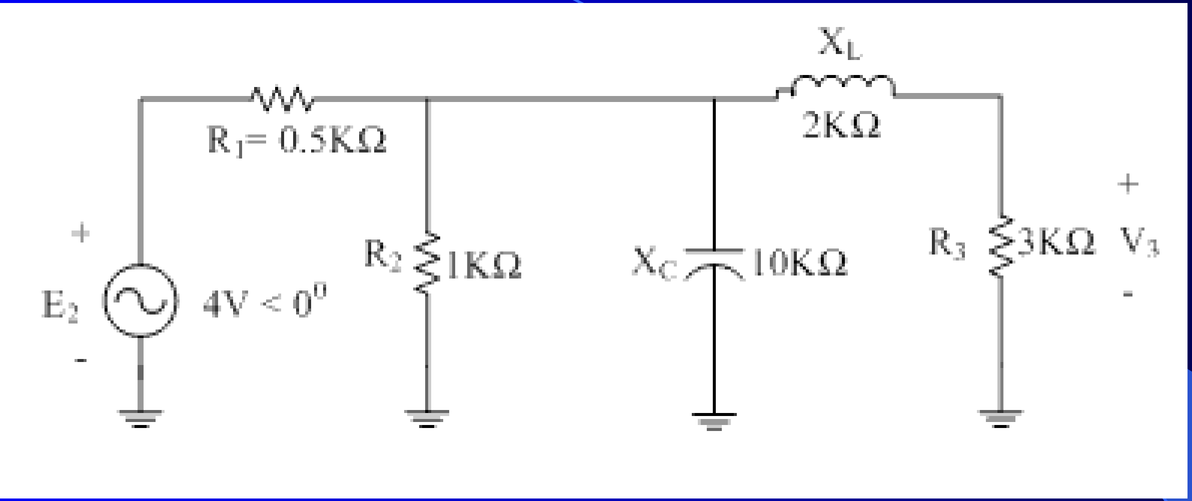
and the voltage across R_3

= voltage across the parallel branch is

$$V^1 = \frac{R^1}{R^1 + R_2} \times E_1 = \frac{0.429}{(0.429 + 1)} \times 12$$

$$= \frac{5.148}{1.429} = 3.6\ \text{V dc}$$

- For the ac source acting alone, the network is redrawn as shown below:



The impedance of the parallel branch is given by

$$\frac{1}{Z_P} = \frac{1}{R_2} + \frac{1}{-jX_C} + \frac{1}{R_3 + jX_L} = \frac{1}{1} + \frac{1}{-\text{j}10} + \frac{1}{3 + j2}$$

$$= 1 + j0.1 + \frac{1}{3.61\angle 33.69^0}$$

$$= 1 + \text{j}0.1 + 0.231 - \text{j}0.154 = 1.231 - \text{j}0.054 = 1.231\angle - 2.51^0$$

$$Z_P = \frac{1}{1.231\angle{-2.51^0}} = 0.812 + j0.036$$

$$\text{Total impedance } Z_T = Z_1 + Z_p = 0.5 + 0.812 + j0.036$$
$$= 1.312 + j0.036$$
$$= 1.312\angle -1.57^0$$

$$\text{Total or source current } = \frac{4\angle 0^0}{1.312\angle -1.57^0} = 3.05\text{mA}\angle -1.57^0$$

$$\text{Current in } R_3 = \frac{\text{voltage across parallel branch}}{R_3 + jX_L}$$

$$= \left(\frac{1}{1.23\angle -2.51^0} \times 3.05\angle -1.57^0\right) \times \frac{1}{8.61\angle -33.69^0}$$
$$= 0.686 \text{ mA}\angle -32.75^0$$

$$V_3 = \text{voltage across } R_3 = 0.686 \text{ mA}\angle -32.75^0 \times 3\angle 0^0$$
$$= 2.06V\angle -32.75^0$$

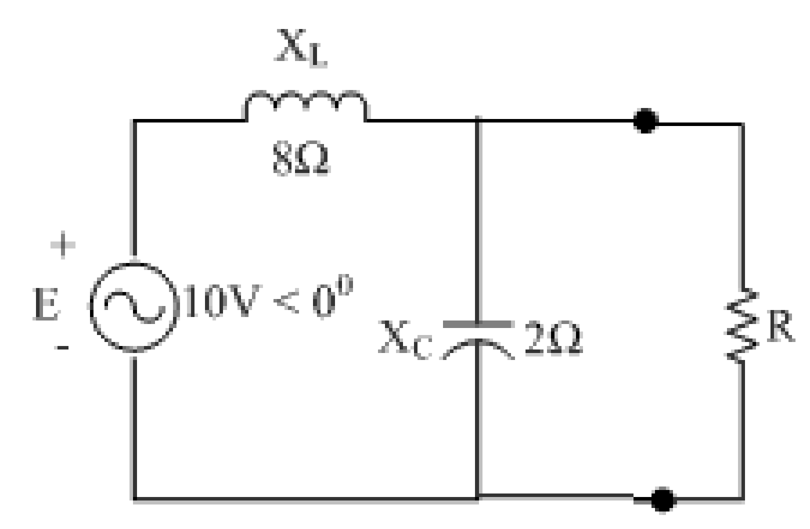
The total solution $V_3 = V_3\,(\text{dc}) + V_3\,(\text{ac}) = 3.6\text{ V} + 2.06\text{ V}\angle -32.75^0$

From which $v_3(t) = 3.6 + 2.06\sqrt{2}\sin(\omega t - 32.75°)$

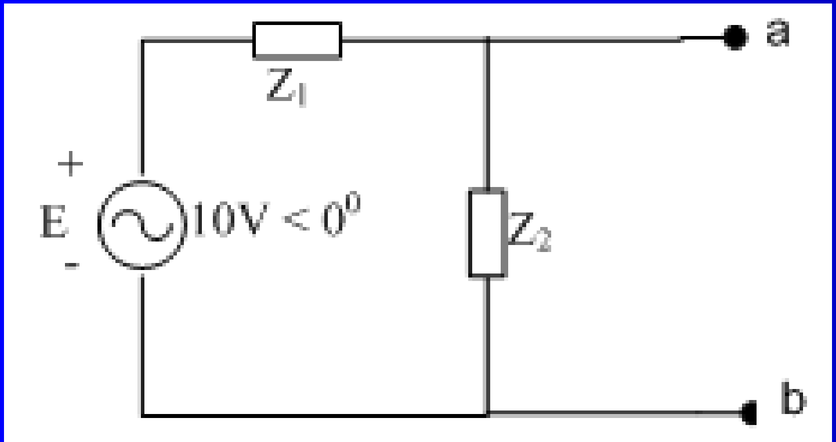
$$= v_3(t) = 3.6 + 2.91\sin(\omega t - 32.75°) \text{ volts}$$

- **Thevenin'sTheorem**
  - The only change is replacement of the term resistance with impedance.
  - Unlike the superposition, it is applicable to only one frequency since reactance is frequency dependent.
  - We note that if the frequencies are more than one and the impedances are frequency dependent (XL's or XC's are present) then we cannot have a single Thevenin equivalent circuit.

Example 4: Find the Thevenin equivalent circuit for the network external to *R*



Solution: With *R* disconnected, the circuit becomes



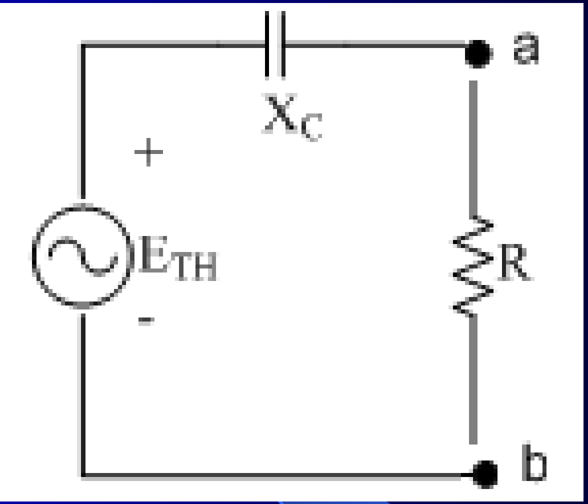
where $Z_1 = j8 \; and \; Z_2 = -j2$

By voltage divider rule,

$$E_{TH} = V_{ab} = \frac{Z_2}{Z_1 + Z_2} \times E$$

$$= -j2 \times 10\angle 0^0 = \frac{-j20}{j6} \text{V}$$

$$= 3.33 \text{ V}\angle -180^0$$



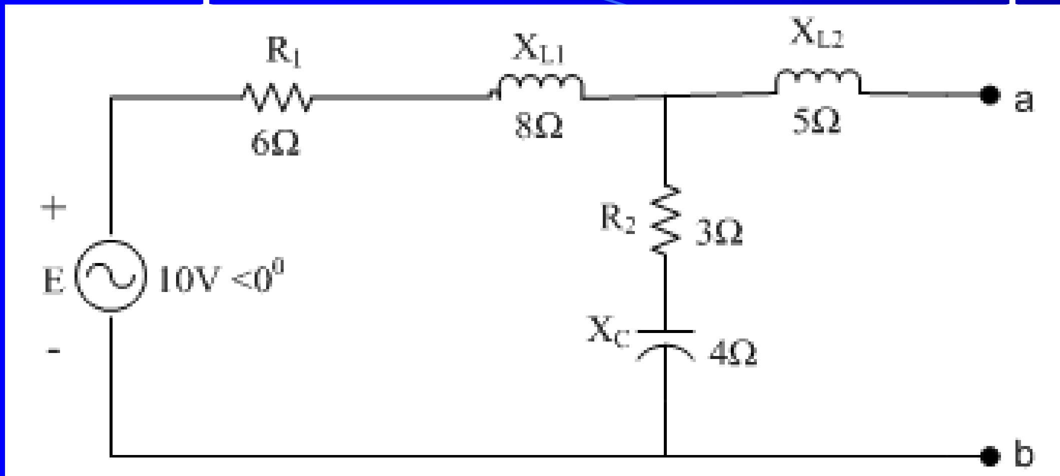
$$E_{TH} = 3.33 \text{ V}\angle -180^0$$

$$X_C = Z_{TH} = -j2.67\ \Omega$$

$$Z_{TH} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(j8)(-j2)}{(j8 - j2)} = \frac{16}{j6}$$

$$= \frac{16}{6\angle 90^0} = 2.67\Omega\angle -90^0$$

Example 5: Find the Thevenin equivalent circuit seen at a-b



Solution

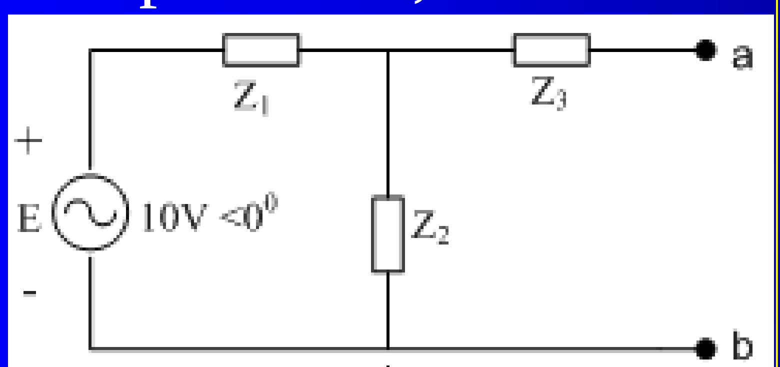
Let

$$Z_1 = R_1 + jX_{L1} = 6 + j8 = 10\,\Omega\angle 53.13^0$$

$$Z_2 = R_2 - jX_C = 3 - j4 = 5\,\Omega\angle - 53.13^0$$

$$Z_3 = jX_{L2} = j5$$

With these impedances, the circuit becomes



$$Z_{TH} = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= j5 + \frac{10\angle 53.13^0 \times 5\angle - 53.13^0}{(6 + j8) + (3 - j4)}$$

$$= j5 + \frac{50\angle 0^0}{9 + j4} = \frac{50\angle 0^0}{9.85\angle 23.96^0} + j5$$

$$= 4.64 + j2.94 = 5.49\Omega\angle 32.36^0$$

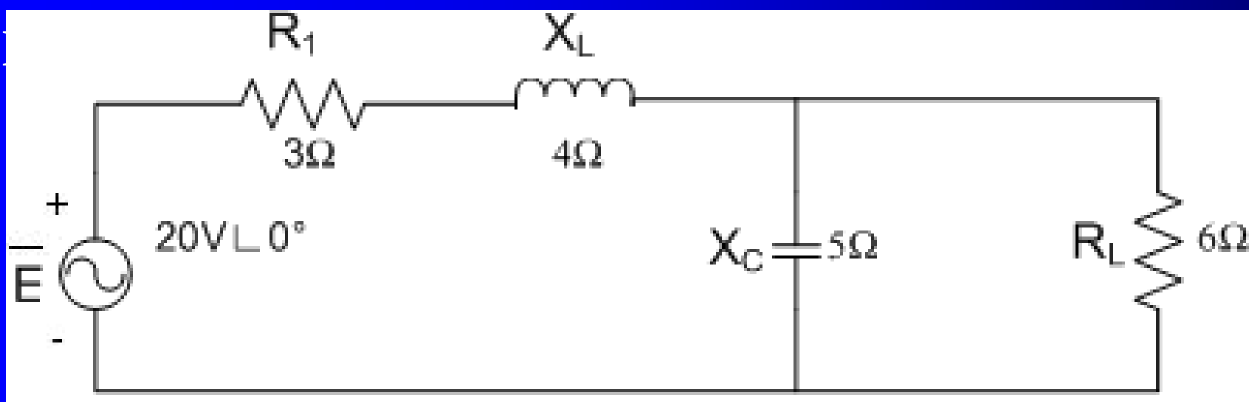
$$E_{TH} = \frac{Z_2}{Z_1 + Z_2} \times E$$

$$= \frac{5\angle - 53.13^0 \times 10\angle 0^0}{9.85\angle 23.96^0}$$

$$= 5.08\angle{-77.09^0}$$

Norton's Theorem

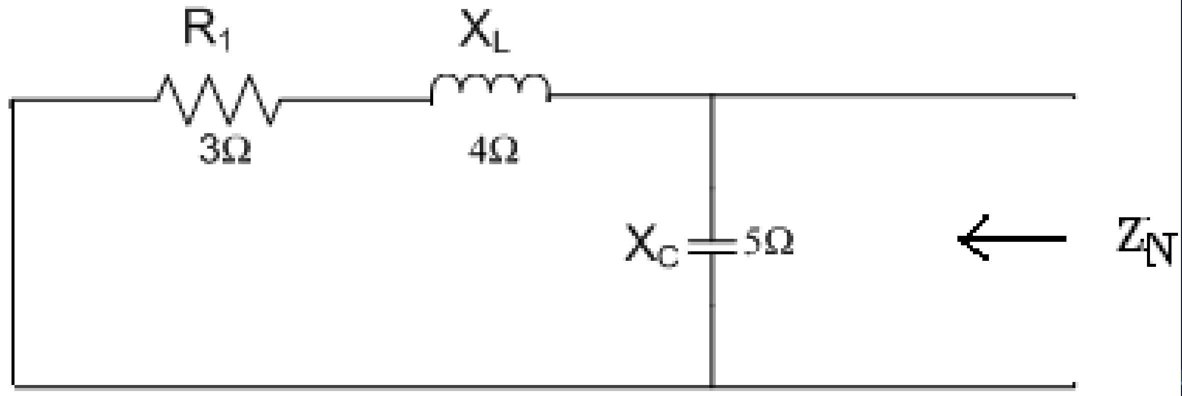
- Here too, resistance is replaced by impedance and
- It is applicable to only one frequency since reactance is frequency dependent.
- Example 9: Determine the Norton equivalent circuit for the



Solution:

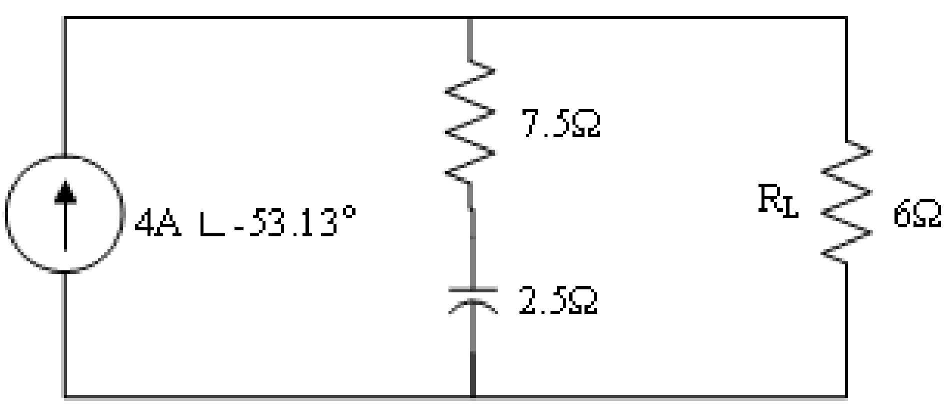
<u>Calculate Norton's current:</u> Since X_C is short-circuited

$$\bar{I}_{\mathrm{N}} = \frac{\bar{E}}{R_{\mathrm{I}} + jX_{\mathrm{L}}} = \frac{20\angle 0°}{3 + j4} = \frac{20\angle 0°}{5\angle 53.13°} = 4\mathrm{A}\angle -53.13°$$

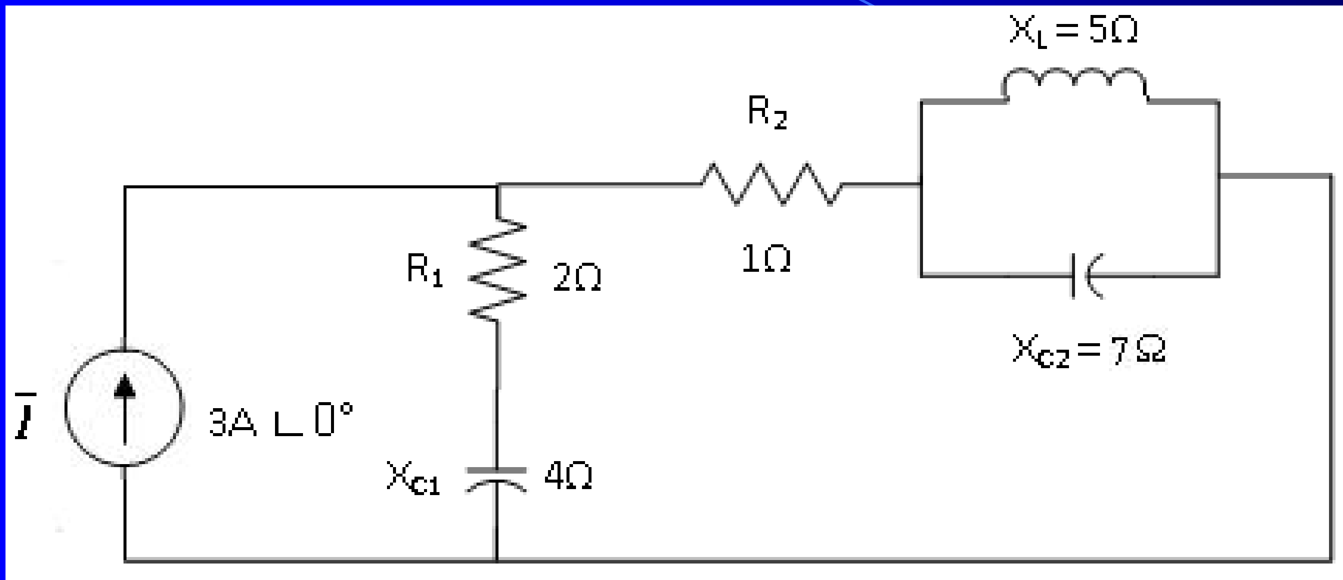


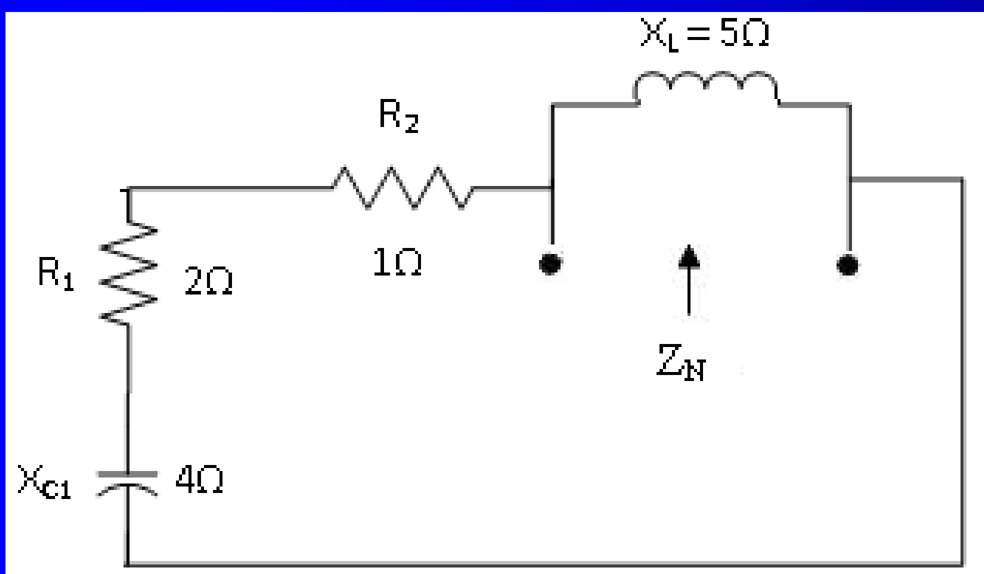
Calculate Z_N:

$$Z_N = \frac{(R_1 + jX_L)(-jX_L)}{R_1 + J(X_L - X_C)} = 7.91\Omega \angle\text{-}18.44° = 7.50 - j2.50\ \Omega$$



- Example 10: Find the Norton equivalent circuit for the network external to the 7-Ω capacitive reactance.





Calculate Z_N:

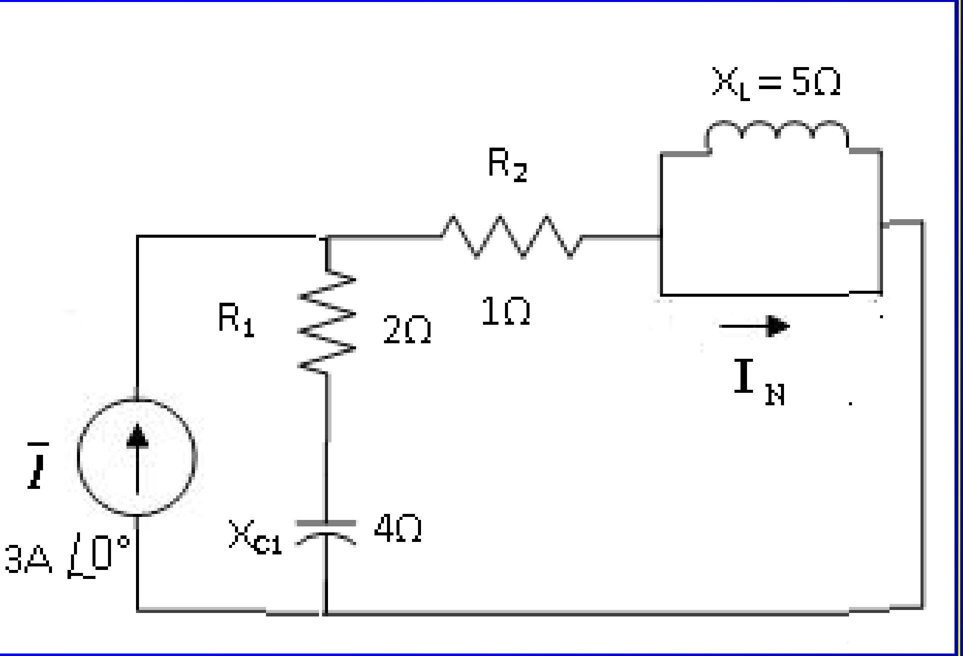
$$Z_N = jX_L // (R_1 + R_2 - jX_C)$$

$$= \frac{(j5)(3 - j4)}{3 + j1} = 7.91\Omega \angle 18.44^\circ$$

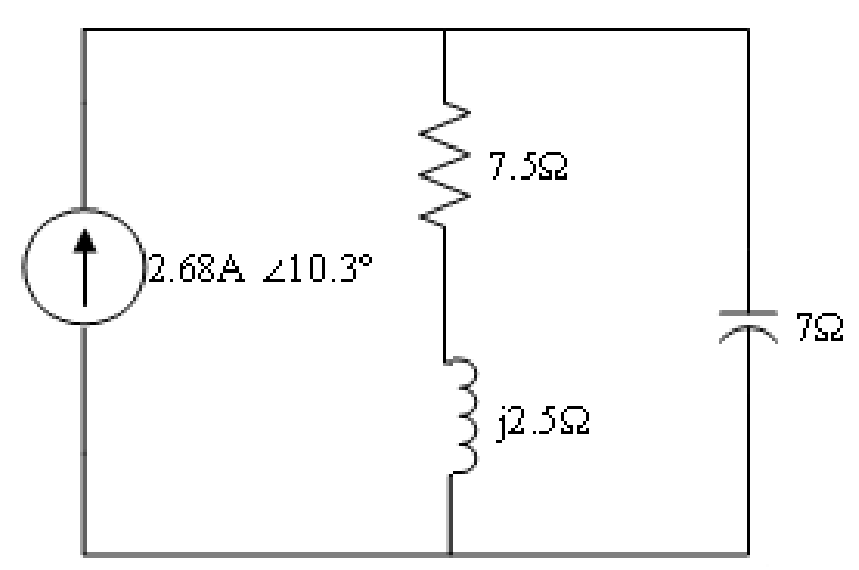
$$= 7.50 + j2.50\ \Omega$$

Calculate short-circuit current:

X_L is short–circuited and the parallel branch consists of $(R_1 + jX_{C1})$ // R_2.



Equivalent circuit:



Using the current divider rule, the Norton's current = current through R_2 is given by

$$\bar{I}_N = \left[\frac{R_1 - jX_{C1}}{R_1 + R_2 - jX_{C1}}\right] \times \bar{I}$$

$$= 2.68\text{A} \angle 10.3°$$