Question: Find the domain and range f(x, y, z) = ln(x+y) + xytan(z)

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Expert Answer

$f(x,y,z) = \ln(x+y) + xy\tan(z)$

For $f(x,y,z)$ to be defined

$x+y > 0$ and $z \neq \pm(2n+1)\frac{\pi}{2}$, $n = 0, 1, 2, 3, \ldots$

So,

Domain: $\left\{(x,y,z) \in \mathbb{R}^3 \mid x+y>0,\ z \neq \pm(2n+1)\frac{\pi}{2}\right\}$

For range

Consider the values from domain such that $y = 0$, $x > 0$

then $f = \ln(x+0) + x \times 0 \tan(z)$

$f = \ln x + 0$

$f = \ln x$, $x > 0$

$-\infty < \ln x < \infty$

$-\infty < f < \infty$

$\therefore$ Range of $f(x,y,z)$: $-\infty < f(x,y,z) < \infty$

Question: Evaluate Q Zdv : Where Q is enclosed by the paraboloid z = x^...

See this question in the app

Evaluate $\iiint$ Q Zdv : Where Q is enclosed by the paraboloid z = x^2 + y^2 and the plane z = 4.

dv is the Jacobian for either spherical or cylindrical coordinates.

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Expert Answer

Narendra Vaddella answered this
3,521 answers

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Using cylindrical co-ordinates

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

we have that

$$dV = dxdydz$$
$$= rdrd\theta dz$$

and $z = x^2 + y^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$$= r^2\left(\cos^2\theta + \sin^2\theta\right)$$
$$= r^2$$

Observe that $z = x^2 + y^2 = r^2$ is a paraboloid opens up.

Find the intersection point of this and the plane $z = 4$.

$$4 = r^2 \Rightarrow r = 2$$

Therefore,

$$\iiint_E zdV = \int_{\theta=0}^{2\pi}\int_{r=0}^{2}\int_{z=r^2}^{4} z\cdot rdrd\theta dz$$
$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{2}\left(\int_{z=r^2}^{4} zdz\right)rdrd\theta$$
$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{2}\left(\frac{1}{2}z^2\right)_{r^2}^{4} rdrd\theta$$
$$= \frac{1}{2}\int_{\theta=0}^{2\pi}\int_{r=0}^{2}\left(16 - r^4\right)rdrd\theta$$
$$= \frac{1}{2}\int_{\theta=0}^{2\pi}\left(\int_{r=0}^{2}\left(16r - r^5\right)dr\right)d\theta$$
$$= \frac{1}{2}\int_{\theta=0}^{2\pi}\left(8r^2 - \frac{1}{6}r^6\right)_0^2 d\theta$$
$$= \frac{32}{3}\cdot 2\pi$$
$$= \boxed{\frac{64\pi}{3}}$$

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