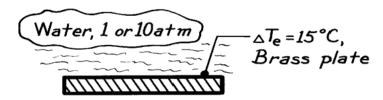
1. (**Problem 10.19 in the Book**) Estimate the power (W/m<sup>2</sup>) required to maintain a brass plate at  $\Delta T_e = 15$  ° C while boiling saturated water at 1 atm. What is the power requirement if the water is pressurized to 10 atm? At what fraction of the critical heat flux is the plate operating?

## **Schematic:**



**Assumptions:** (1) Nucleate pool boiling, (2)  $\Delta T_e = 15^{\circ}$ C for both pressure levels.

**Properties:** Table A-6, Saturated water, liquid (1 atm,  $T_{sat} = 100^{\circ}\text{C}$ ):  $\rho_l = 957.9 \text{ kg/m}^3$ ,  $c_{p,l} = 4217 \text{ J/kg·K}$ ,  $\mu_l = 279 \times 10^{-6} \text{ N·s/m}^2$ ,  $Pr_l = 1.76$ ,  $h_{fg} = 2257 \text{ kJ/kg}$ ,  $\sigma = 58.9 \times 10^{-3} \text{ N/m}$ ; Table A-6, Saturated water, vapor (1 atm):  $\rho_v = 0.596 \text{ kg/m}^3$ ; Table A-6, Saturated water, liquid (10 atm = 10.133 bar,  $T_{sat} = 453.4 \text{ K} = 180.4^{\circ}\text{C}$ ):  $\rho_l = 886.7 \text{ kg/m}^3$ ,  $c_{p,l} = 4410 \text{ J/kg·K}$ ,  $\mu_l = 149 \times 10^{-6} \text{ N·s/m}^2$ ,  $Pr_l = 0.98$ ,  $h_{fg} = 2012 \text{ kJ/kg}$ ,  $\sigma = 42.2 \times 10^{-3} \text{ N/m}$ ; Table A-6, Water, vapor (10.133 bar):  $\rho_v = 5.155 \text{ kg/m}^3$ .

**Analysis:** With  $\Delta T_e = 15$ °C, we expect nucleate pool boiling. The Rohsenow correlation with  $C_{s,f} = 0.006$  and n = 1.0 for the brass-water combination gives

$$\begin{split} q_s'' &= \mu_\ell h_{fg} \Bigg[ \frac{g \left( \rho_\ell - \rho_v \right)}{\sigma} \Bigg]^{1/2} \Bigg( \frac{c_{p,\ell} \Delta T_e}{C_{s,f} \; h_{fg} \; Pr_\ell^n} \Bigg)^3 \\ \textit{I atm:} \quad q_s'' &= 279 \times 10^{-6} \, \mathrm{N} \cdot \mathrm{s/m}^2 \times 2257 \times 10^3 \, \mathrm{J/kg} \Bigg[ \frac{9.8 \, \mathrm{m/s}^2 \left( 957.9 - 0.596 \right) \mathrm{kg/m}^3}{58.9 \times 10^{-3} \, \mathrm{N/m}} \Bigg]^{1/2} \times \\ & \left( \frac{4217 \; \mathrm{J/kg} \cdot \mathrm{K} \times 15 \mathrm{K}}{0.006 \times 2257 \times 10^3 \, \mathrm{J/kg} \times 1.76^1} \right)^3 = 4.70 \, \mathrm{MW/m}^2 \end{split}$$

$$10 \; atm: \qquad q_s'' &= 23.8 \, \mathrm{MW/m}^2 \end{split}$$

From Example 10.1,  $q''_{max}$  (1atm) = 1.26M W/m<sup>2</sup>. To find the critical heat flux at 10 atm, use the correlation of Eq. 10.6 with C = 0.149,

$$q''_{max} = 0.149 h_{fg} \rho_{v} \left[ \sigma g (\rho_{\ell} - \rho_{v}) / \rho_{v}^{2} \right]^{1/4}$$

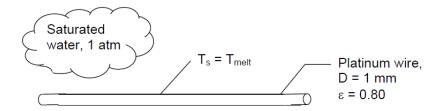
$$q_{max}''\left(10 \text{ atm}\right) = 0.149 \times 2012 \times 10^{3} \text{ J/kg} \times 5.155 \text{kg/m}^{3} \times \\ \left[\frac{42.2 \times 10^{-3} \text{ N/m} \times 9.8 \text{ m/s}^{2} \left(886.7 - 5.16\right) \text{kg/m}^{3}}{\left(5.155 \text{kg/m}^{3}\right)^{2}}\right]^{1/4} = 2.97 \text{MW/m}^{2}.$$

For both conditions, the Rohsenow correlation predicts a heat flux that exceeds the maximum heat flux,  $q''_{max}$ . We conclude that the boiling condition with  $\Delta T_e = 15$ °C for the brass-water combination is beyond the inflection point (P, see Fig. 10.4) where the boiling heat flux is no longer proportional to  $\Delta T_e^3$ .

$$q_{s}^{\prime\prime} \approx q_{max}^{\prime\prime} \left( 1 \text{ atm} \right) \leq 1.26 \, \text{MW} \, / \, m^{2} \qquad \qquad q_{s}^{\prime\prime} \approx q_{max}^{\prime\prime} \left( 10 \text{ atm} \right) \leq 2.97 \, \text{MW} \, / \, m^{2} \, . \label{eq:qs}$$

2. (**Problem 10.32 in the Book**) Consider a horizontal, D = 1-mm-diameter platinum wire suspended in saturated water at atmospheric pressure. The wire is heated by an electrical current. Determine the heat flux from the wire at the instant when the surface of the wire reaches its melting point. Determine the corresponding centerline temperature of the wire. Due to oxidation at very high temperature, the wire emissivity is  $\varepsilon = 0.80$  when it burns out. The water vapor properties at the film temperature of 1209 K are  $\rho_{\nu} = 0.189 \text{kg/m}^3$ ,  $c_{p,\nu} = 2404 \text{ J/kg} \cdot \text{K}$ ,  $v_{\nu} = 231 \times 10^{-6} \text{ m}^2/\text{s}$ ,  $k_{\nu} = 0.113 \text{W/m·K}$ .

## **Schematic:**



**Assumptions:** (1) Steady-state conditions, (2) Film pool boiling occurs.

**Properties:** *Table A.1*, Platinum:  $T_{melt} = 2045 \text{ K}$ ,  $k_p = 99.4 \text{ W/m·K}$ . *Table A.6*, saturated water, liquid ( $T_{sat} = 100^{\circ}$  C, 1 atm):  $\rho_l = 957.9 \text{ kg/m}^3$ ,  $h_{fg} = 2257 \text{ kJ/kg}$ ; Water vapor at film temperature ( $T_f = 1209 \text{ K}$ , 1 atm), given:  $\rho_v = 0.189 \text{ kg/m}^3$ ,  $c_{p,v} = 2404 \text{ J/kg·K}$ ,  $v_v = 231 \times 10^{-6} \text{ N·s/m}^2$ ,  $k_v = 0.113 \text{ W/m·K}$ .

**Analysis:** The heat flux is

$$q_{s}'' = \overline{h} \left( T_{s} - T_{sat} \right) = \overline{h} \Delta T_{e} \tag{1}$$

where  $\Delta T_e = (2045 - 373)K = 1672$  is indicative of film boiling. From Eq. 10.9,

$$\overline{h}^{4/3} = \overline{h}_{conv}^{4/3} + \overline{h}_{rad}\overline{h}^{1/3}$$

For  $\overline{h}_{conv}$  use Eq. 10.8 with C = 0.62 for a horizontal cylinder,

$$\overline{Nu}_{D} = \frac{\overline{h}_{conv}D}{k_{v}} = C \left[ \frac{g(\rho_{\ell} - \rho_{v})h'_{fg}D^{3}}{v_{v}k_{v}(T_{s} - T_{sat})} \right]^{1/4}$$

$$\frac{\overline{h}_{conv} \times 0.001 \text{m}}{0.113 \text{ W/m} \cdot \text{K}} = 0.62 \left[ \frac{9.8 \text{ m/s}^2 (957.9 - 0.189) \text{kg/m}^3 \times 5473 \times 10^3 \text{ J/kg} (0.001 \text{m})^3}{231 \times 10^{-6} \text{ m/s}^2 \times 0.113 \text{ W/m} \cdot \text{K} (2045 - 373) \text{ K}} \right]^{1/4}$$

$$\overline{h}_{conv} = 410 \text{ W/m}^2 \cdot \text{K}$$

where

 $h_{fg}' = h_{fg} + 0.8c_{p,v} \left( T_s - T_{sat} \right) = 2257 \, kJ/kg + 0.8 \times 2.404 \, k \, J/kg \cdot K \left( 2045 - 373 \right) K = 5473 \, kJ/kg. \quad To estimate the radiation coefficient, use Eq. 10.11,$ 

$$\overline{h}_{rad} = \frac{\epsilon \sigma \left( T_s^4 - T_{sat}^4 \right)}{T_s - T_{sat}} = \frac{0.80 \times 5.67 \times 10^{-8} \ \text{W/m}^2 \cdot \text{K}^4 \left( 2045^4 - 373^4 \right) \text{K}^4}{\left( 2045 - 373 \right) \text{K}} = 474 \, \text{W} \big/ \, \text{m}^2 \cdot \text{K} \, .$$

Then Eq. 10.9 becomes

$$\overline{h}^{4/3} = \left(410 \text{ W/m}^2 \cdot \text{K}\right)^{4/3} + \left(474 \text{ W/m}^2 \cdot \text{K}\right) \overline{h}^{1/3}$$

Solving iteratively, find  $\overline{h} = 802 \, W / m^2 \cdot K$ . Then, using Eq. (1), find

$$q_s'' = 802 \text{ W/m}^2 \cdot \text{K} (2045 - 373) \text{K} = 1.34 \text{ MW/m}^2$$

The volumetric heat generation rate due to the electrical current can be found from the energy balance,

$$q_s''\pi D = \dot{q}\pi D^2 / 4$$
  $\dot{q} = 4q'' / D = 4 \times 1.34 \text{ MW/m}^2 / 0.001 \text{ m} = 5.36 \times 10^9 \text{ W/m}^3$ 

From Eq. 3.59,

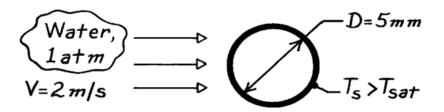
$$T_{c} = T(r = 0) = \frac{\dot{q}r_{0}^{2}}{4k} + T_{S}$$

$$= \frac{5.36 \times 10^{9} \text{ W/m}^{2} \times (0.0005 \text{ m})^{2}}{4 \times 99.4 \text{ W/m} \cdot \text{K}} + 2045 \text{ K} = 2048 \text{ K}$$

Comments: (1) The film boiling heat flux which causes the platinum wire to melt is not much greater than the critical heat flux. A system which was operating near the critical heat flux and underwent a small, unintentional increase in electrical power could cause destruction of the wire. (2) Radiation accounts for 60% of the heat flux from the wire at burnout. (3) Radial temperature differences in the wire are small because of the small radius and large thermal conductivity.

3. (**Problem 10.40 in the Book**) Saturated water at 1 atm and velocity 2 m/s flows over a cylindrical heating element of diameter 5 mm. What is the maximum heating rate (W/m) for nucleate boiling?

## **Schematic:**



Assumptions: Nucleate boiling in the presence of external forced convection.

**Properties:** *Table A-6*, Water (1 atm):  $T_{sat} = 100^{\circ}$  C,  $\rho_l = 957.9 \text{ kg/m}^3$ ,  $\rho_{\nu} = 0.5955 \text{ kg/m}^3$ ,  $h_{fg} = 2257 \text{ kJ/kg}$ ,  $\sigma_{loc} = 58.9 \times 10^{-3} \text{ N/m}$ .

**Analysis:** The Lienhard-Eichhorn correlation for forced convection with cross flow over a cylinder is appropriate for estimating  $q''_{max}$ . Assuming high-velocity region flow, Eq. 10.13 with Eq. 10.14 can be written as

$$q_{max}'' = \frac{\rho_{v} \, h_{fg} \, V}{\pi} \left[ \frac{1}{169} \left( \frac{\rho_{\ell}}{\rho_{v}} \right)^{3/4} + \frac{1}{19.2} \left( \frac{\rho_{\ell}}{\rho_{v}} \right)^{1/2} \left( \frac{\sigma}{\rho_{v} \, V^{2} D} \right)^{1/3} \right].$$

Substituting numerical values, find

$$q_{\text{max}}'' = \frac{1}{\pi} 0.5955 \text{kg/m}^3 \times 2257 \times 10^3 \,\text{J/kg} \times 2 \,\text{m/s} \left[ \frac{1}{169} \left( \frac{957.9}{0.5955} \right)^{3/4} + \frac{1}{19.2} \left( \frac{957.9}{0.5955} \right)^{1/2} \left( \frac{58.9 \times 10^{-3} \,\text{N/m}}{0.5955 \,\text{kg/m}^3 \left( 2 \,\text{m/s} \right)^2 0.005 \,\text{m}} \right)^{1/3} \right]$$

$$q''_{max} = 4.331 \text{ MW/m}^2.$$

The high-velocity region assumption is satisfied if

$$\frac{q_{max}''}{\rho_{v} \, h_{fg} \, v} \stackrel{?}{<} \frac{0.275}{\pi} \left( \frac{\rho_{\ell}}{\rho_{v}} \right)^{1/2} + 1$$

$$\frac{4.331\times10^{6} \text{ W/m}^{2}}{0.5955 \text{kg/m}^{3}\times2257\times10^{3} \text{ J/kg}\times2 \text{ m/s}} = 1.61 \frac{?}{<} \frac{0.275}{\pi} \left(\frac{957.9}{0.5955}\right)^{1/2} + 1 = 4.51.$$

The inequality is satisfied. Using the  $q''_{max}$  estimate, the maximum heating rate is

$$q'_{max} = q''_{max} \cdot \pi D = 4.331 MW / m^2 \times \pi (0.005 m) = 68.0 kW / m.$$

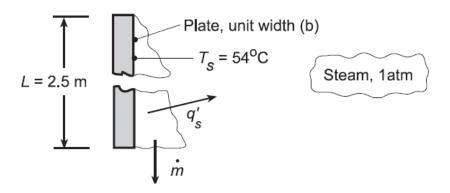
# Mech302-HEAT TRANSFER

# **HOMEWORK-10 Solutions**

**Comments:** Note that the effect of the forced convection is to increase the critical heat flux by 4.33/1.26 = 3.4 over the pool boiling case.

- 4. (**Problem 10.52 in the Book**) A vertical plate 2.5 m high, maintained at a uniform temperature of 54°C, is exposed to saturated steam at atmospheric pressure.
  - a) Estimate the condensation and heat transfer rates per unit width of the plate.
  - b) If the plate height were halved, would the flow regime stay the same or change?

## **Schematic:**



**Assumptions:** (1) Film condensation, (2) Negligible non-condensables in steam.

**Properties:** *Table A-6*, Water, vapor (1 atm):  $T_{sat} = 100^{\circ}$  C,  $h_{fg} = 2257$  kJ/kg; *Table A-6*, Water, liquid ( $T_f = (100 + 54)^{\circ}$  C/2 = 350 K):  $\rho_l = 973.7$  kg/m<sup>3</sup>,  $k_l = 0.668$  W/m·K,  $\mu_l = 365 \times 10^{-6}$  N·s/m<sup>2</sup>,  $c_{p,l} = 4195$  J/kg·K,  $Pr_l = 2.29$ ,  $v_l = \mu_l / \rho_l = 3.75 \times 10^{-7}$  m<sup>2</sup>/s.

#### **Analysis:**

(a) From Equation 10.27,  $h_{fg}' = h_{fg} + 0.68c_{p,\ell}(T_{sat} - T_s) = 2388$  kJ/kg. Then Eq. 10.42 yields,

$$P = \frac{k_{\ell}L(T_{\text{sat}} - T_{\text{s}})}{\mu_{\ell}h_{\text{fg}}^{\prime}(\nu_{\ell}^{-2}/g)^{1/3}} = \frac{0.668 \text{ W/m} \cdot K \times 2.5 \text{ m} \times (100 - 54)^{\circ}C}{365 \times 10^{-6} \text{ N} \cdot \text{s/m}^{2} \times 2388 \times 10^{3} \text{ J/kg} \times \left[ (3.75 \times 10^{-7} \text{ m}^{2}/\text{s})^{2}/9.8 \text{ m/s}^{2} \right]^{1/3}} = 3630 \times 10^{-6} \text{ N} \cdot \text{s/m}^{2} \times 2388 \times 10^{3} \text{ J/kg} \times \left[ (3.75 \times 10^{-7} \text{ m}^{2}/\text{s})^{2}/9.8 \text{ m/s}^{2} \right]^{1/3}}$$

Since P > 2530, the regime is turbulent and Eq. 10.45 yields

$$\begin{split} \overline{h}_{L} &= \frac{k_{\ell}}{\left(\nu_{\ell}^{2}/g\right)^{1/3}} \frac{1}{P} \bigg[ \left(0.024P - 53\right) P r_{\ell}^{1/2} + 89 \bigg]^{4/3} \\ &= \frac{0.668 \text{ W/m} \cdot \text{K}}{\bigg[ \left(3.75 \times 10^{-7} \text{ m}^{2}/\text{s}\right)^{2} / 9.8 \text{ m/s}^{2} \bigg]^{1/3}} \frac{1}{3630} \bigg[ \left(0.024 \times 3630 - 53\right) \times 2.29^{1/2} + 89 \bigg]^{4/3} = 5540 \text{ W/m}^{2} \cdot \text{K} \end{split}$$

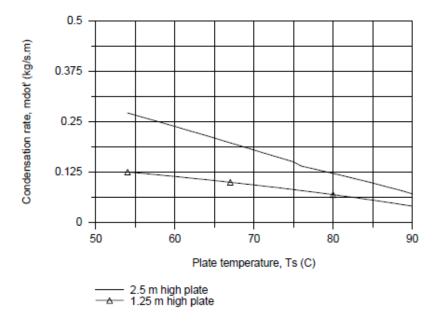
Then from Eqs. 10.33 and 10.34,

$$\begin{split} q' &= \overline{h}_L L (T_{sat} - T_s) = 5540 \text{ W/m}^2 \cdot \text{K} \times 2.5 \text{ m} \times (100 - 54)^\circ \text{C} = 637 \text{ kW/m} \\ \dot{m}' &= q' / h'_{fg} = 637 \times 10^3 \text{ W/m} / 2388 \times 10^3 \text{ J/kg} = 0.267 \text{ kg/m} \cdot \text{s} \end{split}$$

(b) If the length is halved, L = 1.25 m, then P will be halved, P = 1810.

Since 15.8 < P < 2530, the flow regime changes to wavy laminar flow.

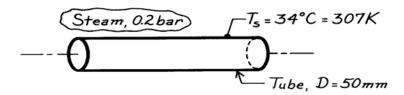
Eq. 10.44 then yields  $\overline{h}_L = 5190~W/m^2 \cdot K$  and we find  $q' = \overline{h}_L L(T_{sat} - T_s) = 299~kW/m$  and  $\dot{m}' = q'/h'_{fg} = 0.125~kg/m \cdot s$ . Note that the height was decreased by a factor of 2 while the rates decreased by a factor of 2.13.



The condensation rate decreases nearly linearly with increasing surface temperature. The inflection in the upper curve (L = 2.5 m) corresponds to the flow transition at P = 2530 between wavy-laminar and turbulent. For surface temperature lower than  $76^{\circ}$  C, the flow is turbulent over the 2.5 m plate. The flow over the 1.25 m plate is always in the wavy-laminar region. The fact that the 2.5 m plate experiences turbulent flow explains the height-rate relationship mentioned in the closing sentences of part (b).

5. (**Problem 10.58 in the Book**) A horizontal tube of 50-mm outer diameter, with a surface temperature of 34°C, is exposed to steam at 0.2 bar. Estimate the condensation rate and heat transfer rate per unit length of the tube.

### **Schematic:**



**Assumptions:** (1) Laminar film condensation, (2) Negligible non-condensibles in steam.

**Properties:** *Table A-6*, Saturated steam (0.2 bar):  $T_{sat} = 333 \text{K}$ ,  $\rho_v = 0.129 \text{ kg/m}^3$ ,  $h_{fg} = 2358 \text{ kJ/kg}$ ; *Table A-6*, Water, liquid ( $T_f = (T_s + T_{sat})/2 = 320 \text{K}$ ):  $\rho_1 = 989.1 \text{ kg/m}^3$ ,  $c_{p,l} = 4180 \text{ J/kg·K}$ ,  $\mu_1 = 577 \times 10^{-6} \text{ N·s/m}^2$ ,  $k_l = 0.640 \text{ W/m·K}$ .

### **Analysis:**

From Eqs. 10.33 and 10.34, the heat transfer and condensate rates per unit length of the tube are

$$q' = \overline{h}_D (\pi D) (T_{sat} - T_s) \qquad \qquad \dot{m}' = q' / h'_{fg}$$

where from Eq. 10.27 with  $\,\text{Ja}=c_{p,\ell}\left(T_{sat}-T_{s}\right)/\,h_{fg},$ 

$$\begin{split} h_{fg}' &= h_{fg} \left[ 1 + 0.68 \text{ Ja} \right] = 2358 \frac{kJ}{kg} \bigg[ 1 + 0.68 \times 4180 \, \text{J/kg} \cdot \text{K} \left( 333 - 307 \right) \text{K} \, / \, 2358 \times 10^3 \, \text{J/kg} \bigg] \\ &\quad h_{fg}' = 2432 \, \, k\text{J/kg}. \end{split}$$

For laminar film condensation, Eq. 10.45 is appropriate for estimating  $\overline{h}_D$  with C = 0.729,

$$\overline{h}_{D} = 0.729 \left[ \frac{g \, \rho_{\ell} \left( \rho_{\ell} - \rho_{v} \right) k_{\ell}^{3} \, h_{fg}'}{\mu_{\ell} \left( T_{sat} - T_{s} \right) D} \right]^{1/4}$$

$$\overline{h}_{D} = 0.729 \left[ \frac{9.8 \, \text{m/s}^2 \times 989.1 \, \text{kg/m}^3 \left(989.1 - 0.129\right) \text{kg/m}^3 \left(0.640 \, \text{W/m} \cdot \text{K}\right)^3 \times 2432 \times 10^3 \, \text{J/kg}}{577 \times 10^{-6} \, \text{N} \cdot \text{s/m}^2 \left(333 - 307\right) \, \text{K} \times 0.050 \text{m}} \right]^{1/4} + \frac{1}{1000 \times 1000} \left[ \frac{9.8 \, \text{m/s}^2 \times 989.1 \, \text{kg/m}^3 \left(989.1 - 0.129\right) \, \text{kg/m}^3 \left(0.640 \, \text{W/m} \cdot \text{K}\right)^3 \times 2432 \times 10^3 \, \text{J/kg}}{1000 \times 1000} \right]^{1/4} + \frac{1}{1000 \times 1000} \left[ \frac{9.8 \, \text{m/s}^2 \times 989.1 \, \text{kg/m}^3 \left(989.1 - 0.129\right) \, \text{kg/m}^3 \left(0.640 \, \text{W/m} \cdot \text{K}\right)^3 \times 2432 \times 10^3 \, \text{J/kg}}{1000 \times 1000} \right]^{1/4} + \frac{1}{1000 \times 1000} \left[ \frac{9.8 \, \text{m/s}^2 \times 989.1 \, \text{kg/m}^3 \left(989.1 - 0.129\right) \, \text{kg/m}^3 \left(0.640 \, \text{W/m} \cdot \text{K}\right)^3 \times 2432 \times 10^3 \, \text{J/kg}}{1000 \times 1000} \right]^{1/4} + \frac{1}{1000 \times 1000} \left[ \frac{9.8 \, \text{m/s}^2 \times 989.1 \, \text{kg/m}^3 \left(989.1 - 0.129\right) \, \text{kg/m}^3 \left(0.640 \, \text{W/m} \cdot \text{K}\right)^3 \times 2432 \times 10^3 \, \text{J/kg}}{1000 \times 1000} \right]^{1/4} + \frac{1}{1000 \times 1000} \left[ \frac{9.8 \, \text{m/s}^2 \times 989.1 \, \text{kg/m}^3 \left(989.1 - 0.129\right) \, \text{kg/m}^3 \left(0.640 \, \text{W/m} \cdot \text{K}\right)^3 \times 2432 \times 10^3 \, \text{J/kg}}{1000 \times 1000} \right]^{1/4} + \frac{1}{1000 \times 1000} \left[ \frac{9.8 \, \text{m/s}^2 \times 989.1 \, \text{kg/m}^3 \left(989.1 - 0.129\right) \, \text{kg/m}^3 \left(989.1 - 0.129$$

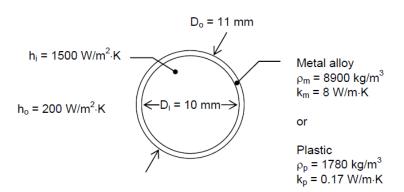
$$\overline{h}_D = 6926 \text{ W/m}^2 \cdot \text{K}.$$

Hence, the heat transfer and condensation rates are

$$q' = 6926 \text{ W/m}^2 \cdot \text{K} (\pi \times 0.050 \text{m}) (333 - 307) \text{K} = 28.3 \text{kW/m}$$
  
 $\dot{m}' = 28.3 \times 10^3 \text{ W/m} / 2432 \times 10^3 \text{ J/kg} = 1.16 \times 10^{-2} \text{kg/s} \cdot \text{m}.$ 

- 6. (**Problem 11.3 in the Book**) A shell-and-tube heat exchanger is to heat an acid liquid that flows in unfinned tubes of inside and outside diameters  $D_i = 10$  mm and  $D_o = 11$  mm, respectively. A hot gas flows on the shell side. To avoid corrosion of the tube material, the engineer may specify either N-Cr-Mo corrosion-resistant metal alloy ( $\rho_m = 8900 \text{kg/m}^3$ ,  $k_m = 8W/\text{m·K}$ ) or a polyvinylidene fluoride (PVDF) plastic ( $\rho_p = 1780 \text{kg/m}^3$ ,  $k_p = 0.17W/\text{m·K}$ ). The inner and outer heat transfer coefficients are  $h_i = 1500W/\text{m}^2 \cdot \text{K}$  and  $h_o = 20000W/\text{m}^2 \cdot \text{K}$ ,  $k_m = 8W/\text{m·K}$ , respectively.
- a) Determine the ratio of plastic to metal tube surface areas needed to transfer the same amount of heat.
- b) Determine the ration of plastic to metal mass associated with the two heat exchanger designs.
- c) The cost of the metal alloy per unit mass is three times that of the plastic. Determine which tube material should be specified on the basis of cost.

#### **Schematic:**



**Assumptions:** (1) Steady-state conditions, (2) Negligible fouling.

**Analysis:** (a) From Eq. 11.14, the heat transfer rates will be the same for the two wall materials when UA is the same for both. From Eq. 11.1, with no fouling or fins, and with the wall resistance given by Eq. 3.33,

$$\frac{1}{\text{UA}} = \left(\frac{1}{\text{h}_{i}\pi\text{D}_{i}} + \frac{\text{ln}(\text{D}_{o}/\text{D}_{i})}{2\pi\text{k}_{w}} + \frac{1}{\text{h}_{o}\pi\text{D}_{o}}\right)\frac{1}{\text{L}} = \left(\text{R}'_{conv,i} + \text{R}'_{w} + \text{R}'_{conv,o}\right)\frac{1}{\text{L}}$$
(1)

where

$$R_{conv,i}' = \frac{1}{h_i \pi D_i} = \frac{1}{1500 \text{ W/m}^2 \cdot \text{K} \times \pi \times 0.01 \text{ m}} = 0.0212 \text{ m} \cdot \text{K/W}$$

$$R'_{conv,o} = \frac{1}{h_o \pi D_o} = \frac{1}{200 \text{ W/m}^2 \cdot \text{K} \times \pi \times 0.011 \text{ m}} = 0.1447 \text{ m} \cdot \text{K/W}$$

and

$$R_{W}^{'} = \frac{\ln(D_{o}/D_{i})}{2\pi k_{W}} = \begin{cases} \frac{\ln(11/10)}{2\pi \times 8 \text{ W/m} \cdot \text{K}} = 0.0019 \text{ W/m} \cdot \text{K} & \text{metal alloy} \\ \frac{\ln(11/10)}{2\pi \times 0.17 \text{ W/m} \cdot \text{K}} = 0.0892 \text{ W/m} \cdot \text{K} & \text{plastic} \end{cases}$$

Thus, from Eq. (1),  $(UA)_m = (UA)_p$  implies the following ratio of areas,

$$\begin{split} \frac{A_p}{A_m} &= \frac{L_p}{L_m} = \frac{R_{conv,i} + R_{w,p} + R_{conv,o}}{R_{conv,i} + R_{w,m} + R_{conv,o}} \\ &= \frac{0.0212 \text{ m} \cdot \text{K/W} + 0.0892 \text{ m} \cdot \text{K/W} + 0.1447 \text{ m} \cdot \text{K/W}}{0.0212 \text{ m} \cdot \text{K/W} + 0.0019 \text{ m} \cdot \text{K/W} + 0.1447 \text{ m} \cdot \text{K/W}} \end{split}$$

$$\frac{A_p}{A_m} = 1.52$$

(b) The mass ratio is found as follows,

$$\frac{m_p}{m_m} = \frac{\rho_p A_p}{\rho_m A_m} = \frac{1780 \text{ kg/m}^3}{8900 \text{ kg/m}^3} 1.52 = 0.304$$

(c) The cost ratio is

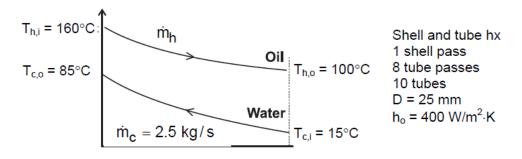
$$\frac{C_p}{C_m} = \frac{m_p}{3m_m} = \frac{1}{3}0.304 = 0.10$$

The plastic should be specified on the basis of cost.

COMMENTS: (1) Because of its lower thermal conductivity, the plastic heat exchanger wall requires 50% more surface area than the metal wall. Nonetheless, it is 70% lighter and 90% less expensive. (2) Plastic heat exchanger components must operate at temperatures below their glass transition point, which for PVDF is approximately 160° C. If the plastic heat exchanger is operated above the glass transition temperature, it will soften and lose all structural rigidity. (3) The cost-based selection of the material will change depending on the values of the inside and outside heat transfer coefficients. For example, as the inside and outside heat transfer coefficients approach infinity, the metal core should be selected on the basis of cost. For applications involving condensation or boiling, the heat transfer coefficients will depend strongly on the tube material, as discussed in Chapter 10.

7. (**Problem 11.22 in the Book**) A shell-and-tube heat exchanger must be designed to heat 2.5 kg/s of water from 15 to 85°C. The heating is to be accomplished by passing hot engine oil, which is available at  $160^{\circ}$ C, through the shell side of the exchanger. The oil is known to provide an average convection coefficient of  $h_o = 400$ W/m<sup>2</sup>·K on the outside of the tubes. Ten tubes pass the water through the shell. Each tube is thin walled, of diameter D = 25 mm, and makes eight passes through the shell. If the oil leaves the exchanger at  $100^{\circ}$ C, what is its flow rate? How long must the tube be to accomplish the desired heating?

#### **Schematic:**



**Assumptions:** (1) Negligible heat loss to the surroundings, (2) Constant properties, (3) Negligible tube wall thermal resistance and fouling effects, (4) Fully developed water flow in tubes.

## **Properties:**

$${\it Table~A.5}, \ unused \ engine \ oil: \ (\overline{T}_h = 130^{\circ} C): \ c_p = 2350 \ J/kg\cdot K. \ \ {\it Table~A.6}, \ water \ (\overline{T}_C = 50^{\circ} C): \ c_p = 4181 \ J/kg\cdot K, \ \mu = 548 \times 10^{-6} \ N\cdot s/m^2, \ k = 0.643 \ W/m\cdot K, \ Pr = 3.56.$$

**Analysis:** From the overall energy balance, Eq. 11.7b, the heat transfer required of the exchanger is

$$q = \dot{m}_c c_{p,c} (T_{c,o} - T_{c,i}) = 2.5 \ kg/s \times 4181 \ J/kg \cdot K(85-15)^{\circ}C = 7.317 \times 10^5 \ W$$
 Hence from Eq. 11.6b,

$$\dot{m}_{h} = \frac{q}{c_{p,h}(T_{h,i} - T_{h,o})} = \frac{7.317 \times 10^{5} \text{ W}}{2350 \text{ J/kg} \cdot \text{K}(160 - 100)^{\circ}\text{C}} = 5.19 \text{ kg/s}$$

The required tube length may be obtained using the  $\epsilon$ -NTU method. We first calculate the heat capacity rates,  $C_h = \dot{m}_h c_{p,h} = 12{,}195 \text{ W/K}, \ C_c = \dot{m}_c c_{p,c} = 10{,}453 \text{ W/K}.$  Thus,  $C_{min} = C_c$ , and  $C_r = C_{min}/C_{max} = 0.857$ . Then from Eq. 11.21,

$$\varepsilon = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{(85 - 15)^{\circ}C}{(160 - 15)^{\circ}C} = 0.483$$

Using Eqs. 11.30b,c for one shell pass and an even number of tube passes, we find

$$E = \frac{2/\epsilon - (1 + C_r)}{(1 + C_r^2)^{1/2}} = \frac{2/0.483 - (1 + 0.857)}{(1 + 0.857^2)^{1/2}} = 1.74$$

$$NTU = -(1 + C_r^2)^{-1/2} \ln \left( \frac{E - 1}{E + 1} \right) = -(1 + 0.857^2)^{-1/2} \ln \left( \frac{1.74 - 1}{1.74 + 1} \right) = 0.997$$

Thus UA = NTU×C $_{min}$  = 10,420 W/K. To find the required tube length, we must know the heat transfer coefficients for the water flow. We calculate the Reynolds number, with  $\dot{m}_1 = \dot{m}_c / N = 0.25$  kg/s defined as the water flow rate per tube, Eq. 8.6 yields

$$Re_{D} = \frac{4\dot{m}_{1}}{\pi D\mu_{c}} = \frac{4 \times 0.25 \text{ kg/s}}{\pi (0.025 \text{ m})548 \times 10^{-6} \text{ N} \cdot \text{s/m}^{2}} = 23,234$$

Hence the flow is turbulent, and from Eq. 8.60,

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023(23,234)^{4/5} (3.56)^{0.4} = 119$$

and

$$h_c = \frac{k_c}{D} Nu_D = \frac{0.643 \text{ W/m} \cdot \text{K}}{0.025 \text{ m}} 119 = 3060 \text{ W/m}^2 \cdot \text{K}$$

Hence  $U = [1/h_c + 1/h_h]^{-1} = 354 \text{ W/m}^2 \cdot \text{K}$  and we can find the required tube length from

$$L = \frac{UA}{UN\pi D} = \frac{10,420 \text{ W/K}}{354 \text{ W/m}^2 \cdot \text{K} \times 10 \times \pi \times 0.025 \text{ m}} = 37.5 \text{ m}$$

**Comments:** (1) With L/D = 1516, the assumption of fully developed conditions throughout the tube is justified. (2) With eight passes, the shell length is approximately L/8 = 4.7 m.