Kwame Nkrumah University of Science and Technology,Kumasi

College of Engineering

Mid Semester Examination, 2015/2016

Math 251: Differential Equations Time:1hr15mins

Program: Index Number:

INSTRUCTIONS

Answer all questions, shade the correct answers to Section A on the scannable sheets and provide the answers to Section B in the spaces provided.

SECTION A

1. Which of the following is a solution of the differential equation $4y'' + 9y = 0$ Assume c_1 and c_2 are arbitrary constants

   A. $y = c_1e^t + c_2e^{-t}$ B. $y = c_1e^{\frac{3}{2}t} + c_2e^t$ C. $y = c_1\cos t + c_2\sin t$ D. $y = c_1\cos(\frac{3}{2}t) + c_2\sin(\frac{3}{2}t)$ E. $y = c_1\cos(\frac{2}{3}t) + c_2\sin(\frac{2}{3}t)$

2. Find the function $y(x)$ satisfying the differential equation and the initial condition $\frac{dy}{dx} = x\sqrt{x^2+9}; y(-4) = 0.$

   A. $y = \frac{1}{3}(x^2+9)^{\frac{3}{2}} + C$ B. $y = \frac{1}{3}[(x^2+9)^{\frac{3}{2}} - 125]$ C. $y = \frac{1}{3}[(x^2+9)^{\frac{3}{2}} + 125]$

   D. $y = \frac{1}{3}(x^2+9)^{\frac{3}{2}} - 125$ E. $y = \frac{1}{3}(x^2+9)^{\frac{3}{2}} + 125$

3. The inverse Laplace transform of $\frac{3s+1}{(s-1)(s^2+1)}$ is given as

   A. $e^t + 2\cos t + \sin t$ B. $2e^t + 2\cos t + \sin t$ C. $2e^t - 2\cos t + \sin t$ D. $2e^t + 2\cos t - \sin t$

4. Solve the given differential equation: $y' = 1 + x + y + xy$

   A. $y = x + \frac{x^2}{2} + c$ B. $\ln|1+y| = x + \frac{x^2}{2} + c$ C. $y = \ln|1+y|y + c$ D. $y = e^{3x}$

   E. $y = x - \frac{x^2}{2} + c$

5. Solve the differential equation $3xy' + y = 12x$

A. $y = 3x + c$ B. $y = 3x + Cx^{\frac{1}{3}}$ C. $y = 4x + c$ D. $y = 4x + Cx^{\frac{1}{3}}$
E. $y = 3x + Cx^{\frac{-1}{3}}$

6. Find the general solution to the DE $(3x^2y^3 + y^4)dx + (3x^3y^2 + y^4 + 4xy^3)dy = 0$

A. $5x^3y^3 + 5xy^4 + y^5 = c$ B. $y = 5x^3y^3 + 5xy^4 + y^5$ C. $10x^3y^3 + 10xy^4 + y^5 = c$
D. $y = 5x^3y^3 - 5xy^4 + y^5$

7. Find the Laplace transform of $\sin^2 3t$ is

A. $\frac{3}{s^2+36}$ B. $\frac{6}{s(s^2+36)}$ C. $\frac{18}{s(s^2+36)}$ D. $\frac{18}{(s^2+36)}$ E. $\frac{18}{s^2(s^2+36)}$

8. Let $y_1(t) = e^t \sin t$ and $y_2(t) = e^t \cos t$ be two solutions of a second order differential equations. Their Wronskian $W(t)$ is;

A. $-e^{2t}$ B. $2e^t \sin t$ C. e^{2t} D. $e^{2t} \cos t$ E. 1

9. The DE in which the conditions are specified at a single value of the independent variable say $x = x_0$ is called

A. Initial value problem B. Boundary value problem C. Initial condition
D. Boundary condition

10. Find the solution to the differential equation $\frac{dy}{dt} + \frac{1}{2}y = 2 + t$ that passes through the point (0,2).

A. $y = 2t + 2e^{\frac{-t}{2}}$ B. $y = 2t - 2e^{\frac{-t}{2}}$ C. $y = 2t + 2e^{\frac{t}{2}}$ D. $y = 2t - 2e^{\frac{t}{2}}$
E. None of the above

11. The integrating factor to the solution of $x' = x \tan t + \sin t$ is

A. $\sin t$ B. $\ln \cos t$ C. $\tan t$ D. $\cos t$ E. None of the above

12. The general solution to an nth order differential equation contains

A. At least n independent constants B. Exactly n independent constants C. At most n independent constants D. Exactly n dependent constants E. None of the above

13. The convolution of $f(t)$ and $g(t)$ is given by

A. $f*g)(t) = \int_0^t f(x)g(t-\tau)dx$ B. $(f*g)(t) = \int_0^t f(x)g(t-x)dx$ C. $f*g)(t) = \int_0^\infty f(x)g(t-x)dx$ D. $f*g)(t) = \int_0^t f(t)g(t-x)dx$ E. None of the above

14. $y = ce^{-\frac{1}{2}} + d$ is a solution of $\frac{dy}{dx} = \frac{y}{x^2}$ when

A. $c = 2, d = 0$ B. $c = 2, d = 2$ C. $c = 1, d = 1$ D. $c = 1, d = 0$
E. None of the above

15. What is the particular integral for $y'' + 3y' + 2y = cos^2 x$.

A. $y_p = \frac{3\sin 2x - \cos 2x}{20}$ B. $y_p = \frac{\sin 2x - 3\cos x - 1}{40}$ C. $y_p = \frac{3\sin 2x + 3\cos 2x + 1}{40}$ D. $y_p = \frac{3\sin 2x - \cos 2x + 10}{40}$ E. None of the above

16. Find the general solution of $x' = x\sin t + 2te^{-\cos t}$

A. $x(t) = t + Ce^{-\cos t}$ B. $x(t) = (t^2 + C)e^{-\cos t}$ C. $x(t) = (t + C)e^{\cos t}$
D. $x(t) = t^2 - Ce^{-\cos t}$

17. Determine the particular solution for $y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$

A. $\frac{-4}{3}x + \frac{23}{9} + \left(2x + \frac{4}{3}\right)e^{2x}$ B. $\frac{-4}{3}x + \frac{21}{9} + \left(2x - \frac{4}{3}\right)e^{2x}$ C. $\frac{4}{3}x + \frac{23}{9} + \left(2x + \frac{4}{3}\right)e^{2x}$
D. $\frac{-4}{3}x + \frac{23}{9} - \left(2x + \frac{4}{3}\right)e^{2x}$ E. None of the above

18. The velocity, given in meters per second, of a certain particle is given by the initial value problem , $\frac{dv}{dt} = 1000 - \frac{1}{40}v^2$, $v \geq 0$, $v(0) = 5$
Approximately how fast will the particle be moving after a very long time?

A. $25m/s$ B. $200m/s$ C. $40000m/s$ D. ∞ E. None of the above

19. What is the general solution to the differential equation
$cos(x+y) + 2x + (cos(x+y) + 4y)y' = 0$?

A. $cos(x+y) + x^2 = C$ B. $sin(x+y) + y^2 = C$ C. $sin(x+y) + 2x + 4y = C$
D. $sin(x+y) + x^2 + 2y^2 = C$ E. None of the above

20. Consider the initial value problem $t(t-2)(t-4)y' + y = e^{t^2}$, $y(3) = -6$
What is the largest interval in which a unique solution is guaranteed to exist?

A. $(4, \infty)$ B. $(-\infty, 0)$ C. $(2, 4)$ D. $(-\infty, \infty)$ E. None of the above

SECTION B

(a) Suppose $m_1 = 3, m_2 = -5$ and $m_3 = 1$ are roots of multiplicity one, two,and three, respectively, of an auxiliary equation. Write down the general solution of the corresponding homogeneous linear DE if it is

(i) an equation with constant coefficients.

(ii) a Cauchy - Euler equation.

(b) Consider the differential equation $ay'' + by' + cy = g(x)$, where a, b, and c are constant. Choose the input functions $g(x)$ for which the method of undetermined coefficients is applicable and the input functions for which the method of variation of parameters is applicable.

(i) $g(x) = e^x Inx$

(ii) $g(x) = x^3 cosx$

(iii) $g(x) = \frac{sinx}{e^x}$

(iv) $g(x) = 2x^{-2}e^x$

(v) $g(x) = sin^2 x$

(vi) $g(x) = \frac{e^x}{sinx}$