INSTITUTE OF DISTANCE LEARNING

KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI, GHANA

ME 355
STRENGTH OF MATERIALS II

UNIT 6

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Introduction

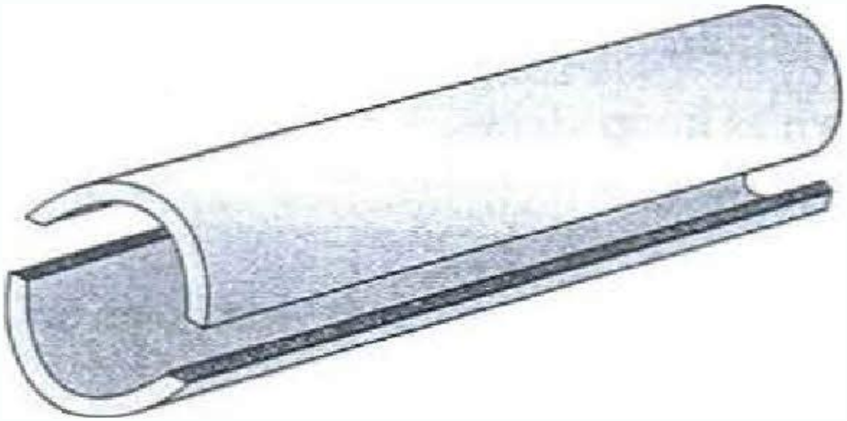
- ***THIN SHELLS***

Learning Objectives

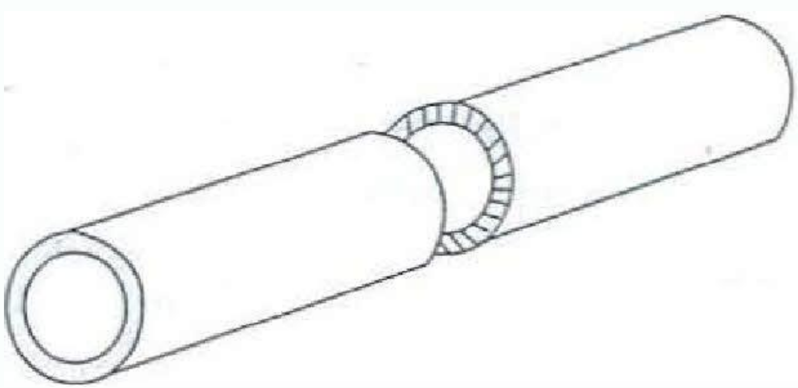
After reading this unit you should be able to:

- Derive the equations for axial and tangential stresses in cylindrical and spherical vessels
- Compute the a axial and tangential stresses using the derived equations
- Derive the equations for axial and tangential strains in cylindrical and spherical vessels
- Compute the a axial and tangential strains using the derived equations

Failure of a Thin Cylindrical Shell due to an Internal Pressure



(a) Split into two troughs



(b) Split into two cylinders

Stresses in a Thin Cylindrical Shell

i. Circumferential stress and

ii. Longitudinal stress

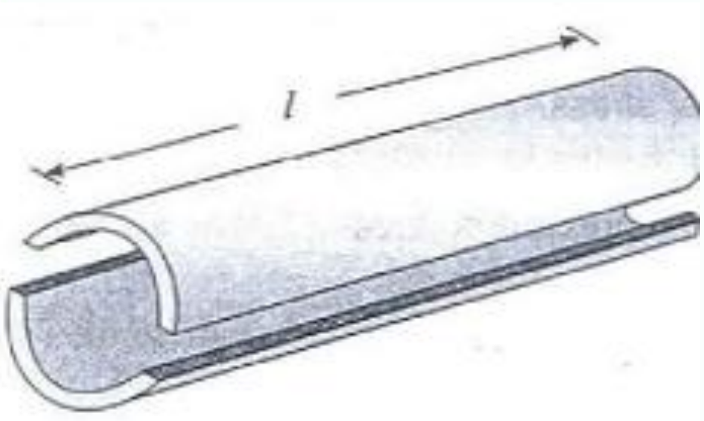
Circumferential Stress

Let l Length of the shell,

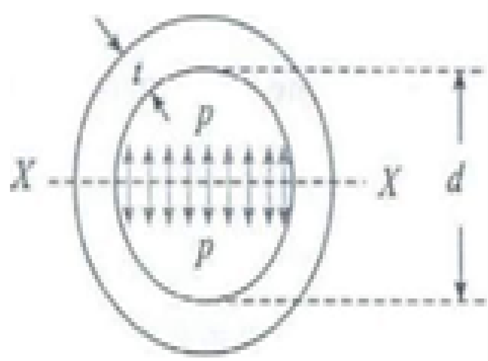
d Diameter of the shell,

t Thickness of the shell

p Intensity of internal pressure



(a)



(b)

Total pressure along the diameter

$$P == pxdxl$$

Circumferential stress

$$\sigma_c = \frac{Total\ pressure}{\text{Re}\,sisting\ section} = \frac{pdl}{2tl} = \frac{pd}{2t}$$

If η is the efficiency of the riveted joints of the shell, then stress

$$\sigma_c = \frac{pd}{2t\eta}$$

Longitudinal Stress

- Let

p Intensity of internal pressure,

l Length of the shell,

d Diameter of the shell and

t Thickness of the shell.

Total pressure along its length

$$P == px\frac{\pi}{4}(d)^2$$

Longitudinal stress

$$\sigma_l = \frac{Total\ \ pressure}{\text{Re}\,sisting\ \ section} = \frac{px\frac{\pi}{4}(d)^2}{\pi dt} = \frac{pd}{4t}$$

If η is the efficiency of the riveted joint of the shell, then the stress

$$\sigma_l = \frac{pd}{4t\eta}$$

Example 6-1:

A stream boiler of 800 mm diameter is made up of 10 mm thick plates. If the boiler is subjected to an internal pressure of 2.5 MPa, find the circumferential and longitudinal stresses induced in the boiler plates.

Solution

Given: Diameter of boiler (d) = 800 mm; Thickness of plates (t) = 10 mm and internal pressure (p) = 2.5 MPa = 2.5 N/mm^2

Circumferential stress induced in the boiler plates

$$\sigma_c = \frac{pd}{2t} = \frac{2.5x800}{2x10} = 100\ N/mm^2$$

Longitudinal stress induced in the boiler plates

$$\sigma_l = \frac{pd}{4t} = \frac{2.5x800}{4x10} = 50\ N/mm^2$$

Example 6-2:

- *A cylindrical shell of 1.3 m diameter is made up of 18 mm thick plates. Find the circumferential and longitudinal stress in the plates. if the boiler is subjected to an internal pressure of 2.4 MPa. Take efficiency of the joints as 70%.*

Solution

- Given: Diameter of shell (d) = 1.3 m = 1.3 x 10^3 mm;. Thickness of plates (t) = 18 -mm; Internal pressure (p) = 2.4MPa = 2.4 N/mm^2 and efficiency (η) = 70% = 0.7

Circumferential stress

$$\sigma_c = \frac{pd}{2t\eta} = \frac{2.4x\left(1.3x10^3\right)}{2x10x0.7} = 124\ N/mm^2$$

Longitudinal stress

$$\sigma_l = \frac{pd}{4t\eta} = \frac{2.4x\left(1.3x10^3\right)}{4x10x0.7} = 62\ N/mm^2$$

Example 6-.3:

- *A gas cylinder of internal diameter 40 mm is 5 mm thick, if the tensile stress in the material* is *not to exceed 30 MPa, find the maximum pressure which can be allowed in the cylinder.*

Solution

Given: Diameter of cylinder (d) = 40 mm ; Thickness of plates (t) = 5 mm and tensile stress (σ_c = 30 MPa = 30 N/mm^2)

Since σ_c. is double σ_l, therefore in order to find the maximum pressure the given stress should be taken as circumferential stress

$$30 = \frac{pd}{2t} = \frac{px40}{2x5} = 4p \Rightarrow p = \frac{30}{4} = 7.50\ N/mm^2$$

Design of Thin Cylindrical Shells

- Given length *(l)*, diameter *(d)*, intensity of maximum internal pressure *(p)* and circumference stress *(σ_c)*,

$$t = \frac{pd}{2\sigma_c}$$

Example 6-4:

- A thin cylindrical shell of4oo mm diameter is to be designed for an internal pressure of 2.4 MPa. Find the suitable thickness of the shell, if the allowable circumferential stress is 50 MPa.

Solution

Given: Diameter of shell (d) = 400 mm; Internal pressure (p)) = 2.4 MPa = 2.4 N/mm$^{2'}$ and circumferential stress (σ_c) = 50 MPa = 50 N/mm^2

We know that thickness of the shell

$$t = \frac{pd}{2\sigma_c} = \frac{2.4x400}{2x50} = 9.6\ mm$$

Example 6-5:

- *A cylindrical shell of 500 mm diameter is required to withstand an. internal pressure of 4 MPa. Find the minimum thickness of the shell, if maximum tensile strength in the plate material is 400 MPa and efficiency of the joints is 65%. Take factor of safety as 5.*

Solution

Given: Diameter of shell (d) = 500 mm; Internal pressure (p) = 4 MPa = 4 N/mm^2; Tensile strength = 400 MPa = 400 N/mm^2; Efficiency (η) = 65% = 0.65 and factor of safety = 5

Allowable tensile stress *(i.e.,* circumferential stress),

$$\sigma_c = \frac{Tensile \quad strength}{Factor \quad of \quad safety} = \frac{400}{5} = 80\ N/mm^2$$

The minimum thickness of shell,

$$t = \frac{pd}{2\sigma_c} = \frac{4x500}{2x80x0.65} = 19.2\ mm$$

Change in Dimensions due to Internal Pressure

- Let
  - *l* Length of the shell,
  - *d* Diameter of (he shell.
  - *t* Thickness of the shell and
  - *p* Intensity of the internal pressure
  - *δd* Change in diameter of the shell.
  - *δl* Change in the length of the shell and
  - *1/m* Poisson's ratio.

Changes in diameter

$$\delta d = \varepsilon_1 . d = \frac{pd}{2tE}\left(1 - \frac{1}{2m}\right) . xd = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

Change in length

$$\delta l = \varepsilon_2 . l = \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right) . xl = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

Example 6-6:

- *A cylindrical thin drum 800 mm in diameter and 4 m long is made of 1.0 mm thick plates. If the drum is subjected to an internal pressure of 2.5 MPa, determine its changes in diameter and length Take E as 200 GPa and Poisson's ratio as 0.25.*

Solution

Given: Diameter of drum (d) = 800 mm . Length of drum (l) = 4 m = 4 x 10^3 mm; 'Thickness of plates (t) = 1O mm; Internal pressure (p) = 2.5 MPa = 2.5 N/mm^2. Modulus of elasticity (E) = 200 GPa = 200 x 10^3 N/mm^2 and Poisson's ratio (1/m) = 0.25

Change in diameter $\delta d = \dfrac{pd^2}{2tE}\left(1-\dfrac{1}{2m}\right) = \dfrac{2.5x(800)^2}{2x10x(200x10^3)}\left(1-\dfrac{0.25}{2}\right) = 0.35\,mm$

Change in length $\delta l = \dfrac{pdl}{2tE}\left(\dfrac{1}{2}-\dfrac{1}{m}\right) = \dfrac{2.5x800x(4x10^3)}{2x10x(200x10^3)}\left(\dfrac{1}{2}-0.25\right) = 0.5\,mm$

Change in Volume due to an Internal Pressure

- Let

  l Original length,

  d Original diameter,

  δl Change in length due to pressure and

  δd Change in diameter due to pressure.

Original volume
$$V = \frac{\pi}{4} x d^2 x l$$

Final volume
$$V_1 = \left[\frac{\pi}{4} (d + \delta d)^2 x (l + \delta l) \right]$$

Change in volume
$$\frac{\delta V}{V} == \frac{\delta l}{l} + \frac{2\delta d}{d} = \varepsilon_l + 2\varepsilon_c \Rightarrow \delta V = V(\varepsilon_l + 2\varepsilon_c)$$

Example .6-7:

- *A cylindrical vessel 2 m. long and 500 mm in diameter with 10 mm thick plates is subjected to an internal pressure of 3 MPa. Calculate the change in volume of the vessel. Take E 200 GPa and. Poisson's ratio = 0.3 for the vessel material.*

Solution

Given: Length of vessel (l) = 2 m = 2 x 10^3 mm; Diameter of vessel (d) = 500 mm: Thickness of plates (t) = 10 mm; Internal pressure (p) := 3 MPa := 3 N/mm^2; Modulus of elasticity (E) = 200 GPa = 200 x 10^3 N/mm^2 and Poisson's ratio (1/m) = 0.3.

Circumferential strain,

$$\varepsilon_c = \frac{pd}{2tE}\left(1-\frac{1}{2m}\right) = \frac{3x500}{2x10x\left(200x10^3\right)}\left(1-\frac{0.3}{2}\right) = 0.32x10^{-3}$$

- Longitudinal strain,

$$\delta l = \frac{pd}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right) = \frac{3x500}{2x10x\left(200x10^{3}\right)}\left(\frac{1}{2}-0.3\right) = 0.075x10^{-3}$$

- Original volume of the vessel,

- The change in volume,

$$V = \frac{\pi}{4}xd^{2}xl = \frac{\pi}{4}x(500)^{2}x\left(2x10^{3}\right) = 392.7x10^{6}\ mm^{3}$$

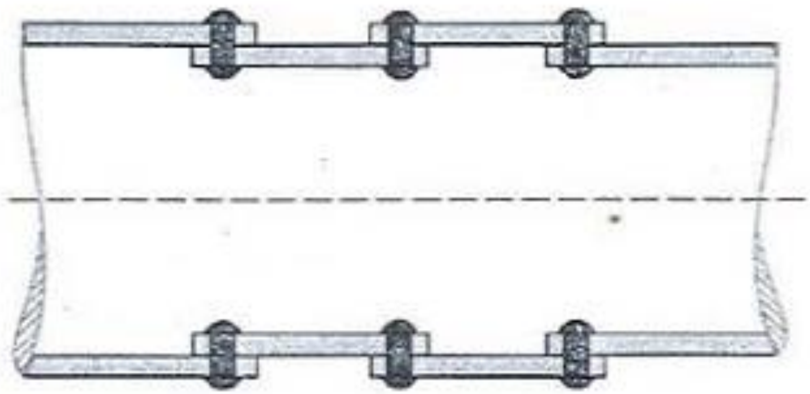
$$\delta V = V\left(\varepsilon_{l} + 2\varepsilon_{c}\right)$$

$$= \left(392.7x10^{6}\right)\left[\left(0.32x10^{-3}\right) + 2\left(0.075x10^{-3}\right)\right] = 185x10^{3}\ mm^{3}$$

Application of Thin Cylindrical Shells

Riveted Cylindrical Shells

(a) Joining by Butt Joint



(b) Joining by Lap Joint

The circumferential stress

$$\sigma_c = \frac{pd}{2t\eta}$$

The longitudinal stress

$$\sigma_l = \frac{pd}{4t\eta}$$

Example 6-8:

- *A boiler of shell of 2 m diameter is made up of mild steel plate of 20 mm thick: The efficiency of the longitudinal and circumferential joints is 70% and 60% respectively. Determine the safe pressure in the boiler; if the permissible tensile stress in the plate section through the rivets is 100 MPa. Also, determine the circumferential stress in the plate and longitudinal stress through the rivets.*

Solution

Given: Diameter of boiler (d) = 2 m = 2 x 10^3 mm; Thickness (t) = 20 mm: Longitudinal efficiency (η_l) = 70% = 0.7; Circumferential efficiency (η_c) = 60% = 0.6 and permissible stress (σ) = 100 MPa = 100 N/mm^2

Safe stress in the boiler

$$\sigma_c = \frac{pd}{2t\eta_c} = \frac{1.4x\left(2x10^3\right)}{2x20x0.6} = 116.7\ N/mm^2$$

Circumferential stress in the rivets

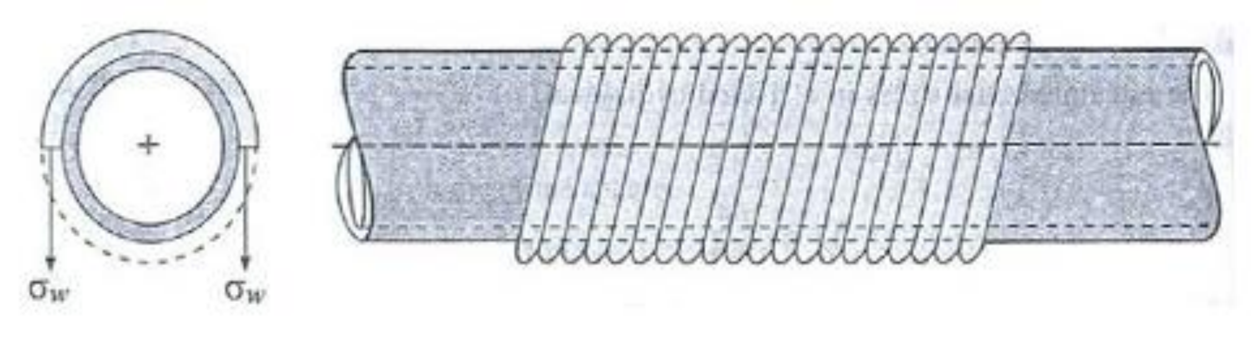
$$100 = \frac{pd}{4t\eta_l} = \frac{px\left(1.3x10^3\right)}{4x10x0.7} = \frac{500p}{7}$$

$$\Rightarrow p = \frac{100x7}{500} = 1.4 \ N/mm^2$$

Longitudinal stress in the rivets

$$\sigma_l = \frac{pd}{4t\eta_l} = \frac{1.4x\left(2x10^3\right)}{4x20x0.7} = 50 \ N/mm^2$$

Wire-Bound Thin Cylindrical Shells



Example 6-9:

A cast iron pipe of 300 mm internal diameter and 12 mm. thick is wound closely with a single layer of circular steel wire of 5 mm diameter under a tension of 60 MPa. Find the initial compressive stress in the pipe section. Also find the stresses set up in the pipe and steel wire when water under a pressure of 4 MPa is admitted into the pipe. Take E for cast iron and steel as 100 GPa and 200 GPa respectively. Poisson's ratio = 0.3.

Solution

- Given: Diameter of pipe (d) = 300 mm; Pipe Thickness (t) = 1.2 mm; Diameter of wire = 5 mm; Tension in wire = 60 MPa = 60 N/mm^2; Pressure of water (p) = 4 MPa = 4 N/mm^2 Modulus of elasticity for cast iron (E_C) = 100 GPa = 100 x 10^3 N/mm^2; Modulus of elasticity for steel (E_s) = 200 GPa = 200 x 10^3 N/mm^2; and Poisson's ratio (1/m) = 0.3

Number of wire sections for 1 mm pipe length

$$n_w = \frac{2}{Diameter\ of\ wire} = \frac{2}{5} = 0.4$$

The initial compressive force

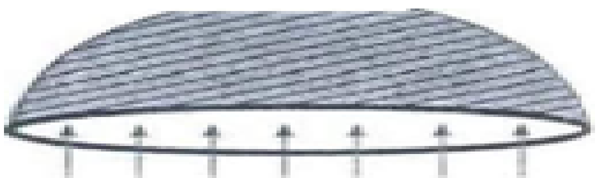
$$F_{cw} = n_w x A_w x \sigma_w = 0.4x\left[\frac{\pi}{4}x(5)^2\right]x60 = 471.3\ N$$

Therefore, initial compressive stress

$$\sigma_c = \frac{F_{cw}}{2t} = \frac{471.3}{2x12} = 19.6\ N/mm^2$$

THIN SPHERICAL SHELLS

Stresses in Thin Spherical Shells



Spherical shell

Let p Intensity of internal pressure,

d Diameter of the shell

t Thickness of the shell,

P Intensity of internal pressure x Area

$$P = \text{Intensity of internal pressure x Area} = px\frac{\pi}{4}d^2$$

and the stress in the material, $\sigma = \dfrac{Total\ pressure}{\text{Re}\,sisting\ \ section} = \dfrac{px\frac{\pi}{4}d^2}{\pi dt} = \dfrac{pd}{4t}$

This is a tensile stress across the x-x. It is also known as hoop stress.

NOTE. If η is the efficiency of the riveted joints of the shell, then stress, $\sigma = \dfrac{pd}{4t\eta}$

Example 6-10:

- *A spherical gas vessel of 1.2 m diameter is subjected to a pressure of 1.8 MPa. Determine the stress induced in the vessel plate, if its thickness is 5 mm.*

Solution

Given: Diameter of vessel (d) = 1.2 m = 1.2×10^3 mm; Internal pressure (p) = 1.8; MPa = 1.8 N/mm^2 and thickness of plates (t) = 5 mm.

We know that stress in the vessel plates, $\sigma = \dfrac{pd}{4t} = \dfrac{1.8x\left(12x10^3\right)}{4x5} = 108\ N/mm^2$

Example 11:

- *A spherical vessel of 2 m diameter is subjected to an internal pressure of 2 MPa. Find the minimum thickness of the plates required. if the maximum stress is not to exceed 100 MPa. Take efficiency of the joint as 80%.*

Solution

Given: Diameter of vessel (d) = 2 m = 2 x 10^3 mm; Internal pressure (p) = 2 MPa = 2 Nmm^2; maximum stress (σ) = 100 MPa = 100 /mm^2 and efficiency of joint (η) = 80% = 0.8.

Let t Minimum thickness of the plates in mm.

We know that the stress in the plate (σ), $100 = \frac{pd}{4t\eta} = \frac{2x12x10^3}{4tx0.8} = \frac{1250}{t} \Rightarrow t = \frac{1250}{100} = 12.5\ mm$

Change in Diameter and Volume due to an Internal Pressure

- Consider a thin spherical shell subjected to an internal pressure.
- Let

  d Diameter of the shell,

  P Intensity of internal pressure and

  t Thickness of the shell.

We have already discussed in the last session that the stress in a spherical shell, $\sigma = \frac{pd}{4t}$

and the strain in any one direction, $\varepsilon = \frac{\sigma}{E} - \frac{\sigma}{mE} = \frac{pd}{4tE} - \frac{pd}{4tEm} = \frac{pd}{4tE}\left(1 - \frac{1}{m}\right)$

Therefore, change in diameter, $\delta d = \varepsilon.d = \frac{pd}{4tE}\left(1 - \frac{1}{m}\right)xd = \frac{pd^2}{4tE}\left(1 - \frac{1}{m}\right)$

We also know that original volume of the sphere, $V = \frac{\pi}{6}xd^3$

and final volume due to pressure, $V + \delta V = \frac{\pi}{6}x(d + \delta d)^3$

Therefore, volumetric strain, $\frac{\delta V}{V} = \frac{(V + \delta V) - V}{V} = \frac{\left[\frac{\pi}{6}x(d + \delta d)^3\right] - \left(\frac{\pi}{6}x(d)^3\right)}{\frac{\pi}{6}x(d)^3}$

Neglecting second and higher powers of δd, $\frac{\delta V}{V} = \frac{d^3 + (3d^2.\delta d) - d^3}{d^3} = \frac{3.\delta d}{d} = 3\varepsilon$

and $\delta V = 3\varepsilon.V = 3x\frac{pd}{4tE}\left(1 - \frac{1}{m}\right)x\left(\frac{\pi}{6}xd^3\right) = \frac{\pi pd^4}{8tE}\left(1 - \frac{1}{m}\right)$

Example 12.

- A spherical shell of 2 m diameter is made up of 10 mm thick plates. Calculate the change in diameter and volume of the shell, when it is subjected to an internal pressure of 1.6 MPa. Take E = 200 GPa and 1/m = 0.3.

Solution

Given: Diameter of shell (d) = 2 m = 2 x 10^3 mm; Thickness of plates (t) = 10 mm; Internal pressure (P) = 1.6 MPa = 1.6 N/mm^2; Modulus of elasticity (E) = 200 GPa = 200 x 10 N/mm^2 and Poisson's ratio (1/m) = 0.3. Change in diameter

Change in diameter

We know that change in diameter. $\delta d = \frac{pd^2}{4tE}\left(1-\frac{1}{m}\right) = \frac{1.6x\left(2x10^3\right)^2}{4x10x\left(200x10^3\right)}\left(1-0.3\right) = 0.56\ mm$

Change in volume

We also know that change in volume,

$$\delta V = \frac{\pi pd^4}{8tE}\left(1-\frac{1}{m}\right) = \frac{\pi x1.6x\left(2x10^3\right)^4}{8x10x\left(200x10^3\right)}\left(1-0.3\right) = 3.52x10^6\ mm^3$$

Bulk Modulus K

- If a hydrostatic pressure *p* (is equal in all direction) acting on a body of initial volume V ,
- Causes a reduction in volume equal to δV,
- Then the bulk modulus K is defined as the ratio between the fluid pressure and volumetric strain.

$$K = \frac{-p}{\delta V / V}$$

Example 6-13:

A boiler drum consists of a cylindrical portion 2m long, 1m diameter, and 25mm thick is closed by a hemispherical ends. In a hydrostatic test to 10N/mm^2, how much additional water will be pumped in, after initial filling at atmospheric pressure?

Assume the circumferential strain at the junction of cylinder hemisphere is the same for both for the drum material. E==207,000 N/mm^2; v==0.3, for water K==2100 N/mm^2

Solution

For cylinder, the hoop stress is

$$\sigma_h = \frac{pr}{t} = \frac{(10)(500)}{25} = 200\, N/mm^2$$

Longitudinal stress is

$$\sigma_L = \frac{pr}{2t} = \frac{(10)(500)}{2(25)} = 100\, N/mm^2$$

Hoop strain is

$$\varepsilon_h = \frac{1}{E}(\sigma_h - \upsilon\sigma_L) = \frac{1}{E}(200 - 0.3(100)) = \frac{170}{E}$$ Longitudinal strain is

$$\varepsilon_L = \frac{1}{E}(\sigma_L - \upsilon\sigma_h) = \frac{1}{E}(100 - 0.3(200)) = \frac{40}{E}$$ Increase in volume:

$$\Delta V = (2\varepsilon_h + \varepsilon_L)V = \left[2\left(\frac{170}{E}\right) + \frac{40}{E}\right]V = 380\frac{V}{E}$$ But $V = \frac{\pi d^2 L}{4} = \frac{\pi}{4}(1m)^2(2m) = 1.57x10^9\, mm^3$

Therefore $\Delta V_1 = 380\frac{V}{E} = 380\left(\frac{1{,}570{,}000{,}000}{207{,}000}\right) = 2.88x10^6\, mm^3$ For the two hemispherical ends,

Hoop strain: $\varepsilon = \varepsilon_h$

Increase in volume:

$$\Delta V = (2\varepsilon + \varepsilon)V = \left[2\left(\frac{170}{E}\right) + \frac{170}{E}\right]V = 510\frac{V}{E} \text{ But } V = \frac{\pi d^2 L}{4} = \frac{\pi}{6}(1m)^3 = 0.5x10^9 mm^3$$

Therefore

$$\Delta V_2 = 510\,\frac{V}{E} = 510\left(\frac{530{,}000{,}000}{207{,}000}\right) = 1.3x10^6 mm^3$$

Decrease in volume of water due to pressure

$$\Delta V_3 = \frac{p}{K}V = \frac{10}{2100}\left[\frac{(\pi)(1000)^2(2000)}{4} + \frac{\pi(1000)^3}{6}\right]$$

$$= 10x10^6 mm^3$$

Total additional of water required

$$\Delta V_T = \Delta V_1 + \Delta V_2 + \Delta V_3 = 14.2x10^6 mm^3$$

Sample Questions

Problem 3: *A cylindrical vessel whose ends are closed by means of rigid flange of steel plate 3 mm thick. The internal length and diameter of vessel are 50 cm and 25 cm. Determine the longitudinal and circumferential stresses in the cylindrical shell due to an internal pressure of 3 N/mm^2. Also calculate increase in length, diameter and volume of the vessel. Take: $E = 200$ GPa and $1/m = 0{\cdot}3$.*

Problem 6: *A boiler drum consists of a cylindrical portion 4 m long, 1·5 m in diameter and 2.25 cm thick. It is closed by hemispherical ends. In a hydraulic test to 6 MPa, how much water will be pumped in after initial filling at atmospheric pressure? The circumferential strain at the junction of the cylinder and hemisphere may be assumed as same for both. Take = 200 GPa, K (for water) = 2·13 GPa , and $1/m = 0{\cdot}3$*

Problem 8*: A gun metal tube of 100 mm bore, wall thickness 2·5 mm is closely wound externally by a steel wire 1 mm diameter. Determine the tension under which the wire must be wound on the tube, if an internal radial pressure of 3 MPa is required before the tube is subjected to the tensile stress in the circumferential direction. Take E (For gun metal) = 102 GPa, E (For steel) = 210 GPa and $1/m = 0{\cdot}35$ m*