

**KWAME NKRUMAH UNIVERSITY OF SCIENCE AND
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ME 365: THERMODYNAMICS II

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PROF F. K. FORSON and DR. R. OPOKU---2021 Edition

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REVIEW OF FIRST AND SECOND LAWS OF THERMODYNAMICS

Introduction

In this unit, we review the first (1st) and second (2nd) laws of thermodynamics to provide a solid basis for applications to vapour power cycles, refrigeration cycles, and subsequently establish the importance of entropy as a consequence of the second (2nd) law. We also introduce the use of enthalpy-entropy chart in the analysis of steady flow devices such as turbines, compressors, and nozzles.



Learning Objectives

After studying this unit you should be able to:

1. Apply the first law of thermodynamics [i.e. Non flow Energy Equation (NFEE)] to closed systems and the Steady-Flow Energy Equation (SFEE) to open systems.
2. Establish the importance of entropy which is a consequence of the second law of thermodynamics.
3. Develop Tds relations for liquids, solids and gases and understand property diagrams involving Entropy.
4. Develop Entropy change relations for solids, liquids and gases as well as for pure substance.
5. Handle the use of enthalpy-entropy chart in the analysis of steady flow devices.
6. Understand the concept of “The Increase of entropy principle” and Entropy generation.

SESSION 1-1: FIRST AND SECOND LAWS OF THERMODYNAMICS

1-1.1 APPLICATION OF FIRST (1st) LAW OF THERMODYNAMICS TO CLOSED SYSTEMS (NON-FLOW)

When a closed system is taken through a Thermodynamic cycle, the net work done on the system is proportional to the net heat delivered to the surroundings. Because the 1st Law is associated with heat engine cycles, we restate it as follows:

When any closed system, as shown in Figure 1, is taken through a thermodynamic cycle, the net heat supplied from its surroundings plus the net work from the system must be zero. The 1st law is expressed by (European sign convention adopted in this presentation, please)

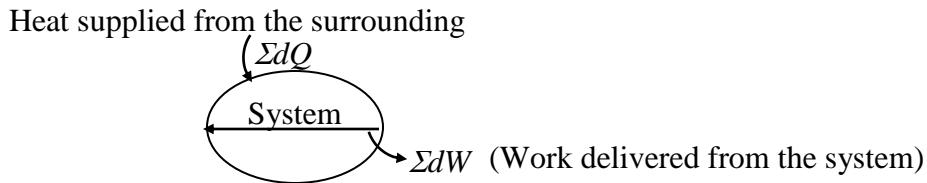


Figure 1 : closed system taken through a thermodynamic cycle

$$\text{Thus, } \boxed{\sum dQ + \sum dW = 0}$$

Thus, if $|Q_1|$ is the heat supplied, $|Q_2|$ the heat rejected, and $|W|$ the net work output by the system during the cycle, the First law becomes:

$$|Q_1| - |Q_2| = |W| \quad (1.1)$$

Where, $|Q_1| > |Q_2|$

A system operating in a cycle and producing a net quantity of work from a supply of heat is called a heat engine. The greater the proportion of the heat supplied converted into work, the better is the performance of the heat engine. Consequently, the cycle efficiency of a heat engine is defined as:

$$\eta = \frac{\text{Net work Output}}{\text{Heat Supplied}} = \frac{|W|}{|Q_1|} \quad (1.2)$$

From equations (1) and (2), the cycle efficiency is redefined as:

$$\eta = \frac{|Q_1| - |Q_2|}{|Q_1|} = 1 - \frac{|Q_2|}{|Q_1|} \quad (1.3a)$$

In general, for a closed system, for any process 1-2 we write,

$$Q_{12} \pm W_{12} = U_2 - U_1 \quad (1.3b)$$

$$\text{In differential form, } \delta Q_{12} \pm \delta W_{12} = dU \quad (1.3c)$$

The positive sign is used when the European sign convention is adapted with $W_{12} = - \int_1^2 P dV$

The negative sign is used when the American sign convention is adapted with $W_{12} = \int_1^2 P dV$

1-1.2 APPLICATION OF FIRST LAW OF THERMODYNAMICS TO OPEN (STEADY FLOW) SYSTEMS

The first law of thermodynamics applied to open (control volume) systems is given by the steady flow energy equation:

Generally, for a steady flow process 1-2,

$$\dot{Q}_{12} \pm \dot{W}_{12} = \dot{m} \left(h_2 - h_1 \right) + \frac{1}{2} \left(C_2^2 - C_1^2 \right) + g(z_2 - z_1) \quad (1.4)$$

In applying the first law, it is necessary to list all the kinds of energy involved in a flow process and adapt one of the sign conventions appropriately.

- i. Energy stored within the system. Internal energy, kinetic and potential energy and energy associated with other mechanical moving parts. In steady flow, this energy remains constant.
- ii. Simple energy transfer
Energy may leave or enter the system without being associated with flow. Shaft work \dot{W} , and heat transfer \dot{Q} .
- iii. Energy transfer due to flow
Fluid entering the system will possess internal energy, potential energy and kinetic energy transferred to the system as the fluid enters and similarly energy transfers from the system takes place as the fluid leaves. Pressure within the fluid is also another form.



Self Assessment 1-1

1. A thin walled and well-insulated cylinder device, **as shown in Figure Q1**, contains 5 L of saturated liquid water at a constant pressure of 175 kPa. Water is stirred by a paddle wheel while a current of 8 A flows through it for 45 min through a resistor placed in the water. If one-half of the liquid is evaporated during this constant pressure process and the paddle-wheel work amounts to 400 kJ, determine the voltage of the source, in Volts. Also, show the process on a *P-v* diagram with respect to saturation lines. **Suggested Answer: 224 V**
2. Consider a 1000-W iron **as shown in figure Q2**, whose base plate is made of 0.5-cm-thick aluminum alloy 2024-T6 ($\rho = 2770 \text{ kg/m}^3$ and $C_p = 875 \text{ J/kg} \cdot ^\circ\text{C}$), **as shown in Figure Q2**. The base plate has a surface area of 0.03 m^2 . Initially, the iron is in thermal equilibrium with the ambient air at 22°C . Assuming 85 percent of the heat generated in the resistance wires is transferred to the plate; determine the minimum time, in seconds, needed for the plate temperature to reach 140°C . **Suggested Answer: 50.5 s**
3. In **Figure Q3**, a piston–cylinder device contains helium gas ($R = 2.0769 \text{ kJ/kg K}$, $C_v = 3.1156 \text{ kJ/kg K}$) initially at 150 kPa, 20°C , and 0.5 m^3 . The helium is now compressed in a polytropic process ($PV^n = \text{constant}$) to 400 kPa and 140°C . Determine the heat loss or gain, in kJ, during this process. **Suggested Answer: 11.2 kJ loss**
4. A hand-held domestic hair-dryer, **such as the one shown in Figure Q4**, is basically a duct in which a few layers of electric resistors are placed. A small fan pulls the air in and forces it through the resistors where it is heated. Air ($R = 0.287 \text{ kJ/kg K}$; $C_p = 1.005 \text{ kJ/kg K}$) enters a 1200-W hair dryer at 100 kPa and 22°C and leaves at 47°C . The cross-sectional area of the hair dryer at the exit is 60 cm^2 . Neglecting the power consumed by the fan and the heat losses through the walls of the hair dryer, determine
 - (a) The volume flow rate of air at the inlet to the dryer, in m^3/s , and (b) the velocity of the air at the exit of the dryer, in m/s.

Suggested Answers: (a) **0.0404 m³/s** and (b) **7.31 m/s**

5. The schematic of a turbocharger of an internal combustion engine, **as depicted in Figure Q5**, consists of a turbine and a compressor. Hot exhaust gases flow through the turbine to produce work and the work output from the turbine is used as the work input to the compressor. The pressure of ambient air is increased as it flows through the compressor before it enters the engine cylinders. Thus, the purpose of a turbocharger is to increase the pressure of air so that more air gets into the cylinder. Consequently, more fuel can be burned and more power can be produced by the engine. In a turbocharger, exhaust gases enter the turbine at 400°C and 120 kPa at a rate of 0.02 kg/s and leave at 350°C . Air enters the compressor at 50°C and 100 kPa and leaves at 130 kPa at a rate of 0.018 kg/s. The compressor increases the air pressure with a side effect: It

also increases the air temperature, which increases the possibility of a gasoline engine to experience an engine knock. To avoid this, an after-cooler is placed after the compressor to cool the warm air by cold ambient air before it enters the engine cylinders. It is estimated that the after-cooler must decrease the air temperature below 80°C if knock is to be avoided. The cold ambient air enters the after-cooler at 30°C and leaves at 40°C . Disregarding any frictional losses in the turbine and the compressor and treating the exhaust gases as air, determine (a) the temperature of the air at the compressor outlet, $^{\circ}\text{C}$, and (b) the minimum volume flow rate of ambient air required to avoid knock, in m^3/s .

You may assume the following values:

$$C_{p\text{air}} = 1.005 \text{ kJ/(kg air K)} \quad C_{p\text{exhaust gas}} = 1.0063 \text{ kJ/(kg K)} \quad C_{p\text{warm air}} = 1.008 \text{ kJ/kg K}$$

Suggested Answers: (a) 108.6°C and (b) $0.0449 \text{ m}^3/\text{s}$

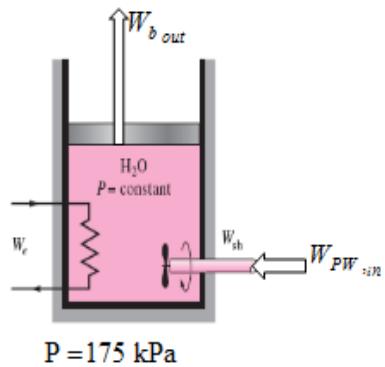


Figure Q1

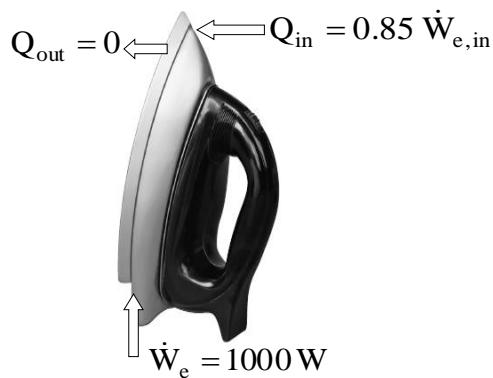


Figure Q2

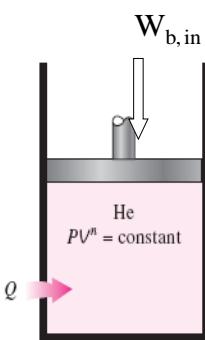


Figure Q3

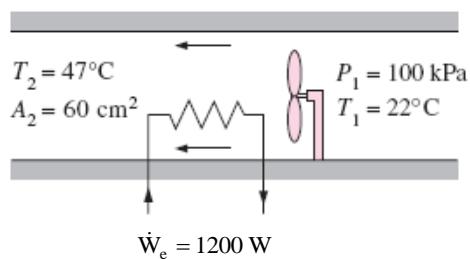


Figure Q4

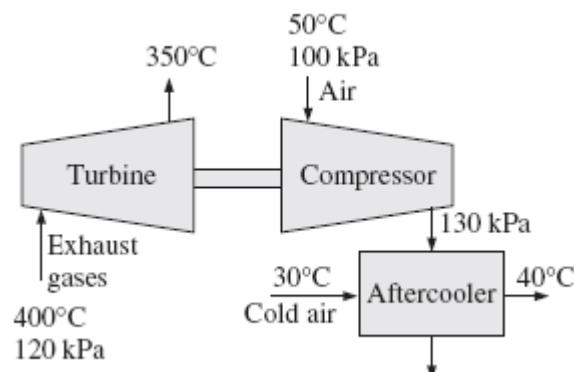


Figure Q5

SESSION 2-1: REVIEW OF THE SECOND (2ND) LAW OF THERMODYNAMICS AND ENTROPY

2-1.1 SECOND (2ND) LAW OF THERMODYNAMICS

Kelvin-Planck's statement of the Second Law (Heat Engine)

It is impossible to construct a system, which will operate in a cycle, extract heat from a reservoir, and do an equivalent amount of work on the surroundings. It follows from the Second Law that the net work output delivered must be less than the heat supplied for a cycle.

That is, $|W| < |Q_1|$

If $|W| < |Q_1|$, then it follows from Equation (1.1) that $|Q_2|$ must have some definite value and hence $\frac{|W|}{|Q_1|} < 1$. Thus, the 2nd law confirms the fact that the cyclic efficiency of a heat engine is less than 100%. It must be noted that $|Q_2| > 0$ constitutes a situation of thermal pollution (a problem for the world to solve which is of concern to many researchers).

Inference (Regarding means for transfer of heat)

If energy is to be supplied to a system in the form of heat, the system must be in contact with a reservoir at a temperature higher than that of the system at some point in the cycle. Similarly, if heat is to be rejected, the system must at some time be in contact with a reservoir of lower temperature than the system. The Second Law implies that if a system is to undergo a cycle and produce net work output, it must operate between at least two reservoirs operating at different temperatures. Note that all reversible engines operating between the same two reservoirs have the same efficiency. This leads to establishment of the thermodynamic scale

$$\frac{|Q_2|}{|Q_1|} = \frac{T_L}{T_H} \quad \text{Thus } \boxed{\eta_{\max} = 1 - \frac{T_L}{T_H}} \quad \text{since } \eta_{\max} < 1 \Rightarrow T_2 > 0$$

To maximise η_{\max} , the following two generic options are available:

- i) Reduce T_L ; and/or
- ii) Increase T_H

The second law places a limitation on the cycle efficiency. From the second law,

$$\begin{cases} \eta = \eta_{\max} \Rightarrow \text{reversible cycle} \\ \eta < \eta_{\max} \Rightarrow \text{irreversibilities present within the cycle} \\ \eta > \eta_{\max} \Rightarrow \text{impossibility situation} \end{cases}$$

2-1.1.1 Maximum Performance Measure for Power Cycles

The thermal efficiency of a system undergoing a reversible power cycle while operating between thermal reservoirs at absolute temperatures T_H and T_L such that $T_H > T_L$ is given by,

$$\eta_{\max} = 1 - \frac{T_L}{T_H}$$
 This is referred to as the Carnot efficiency. The value of the Carnot efficiency

increase as T_H increases and/or T_L decreases. Note that irreversibilities have the effect of decreasing work outputs and increasing work inputs.

The practical limitations on the maximum (thermal power cycle) efficiency is as follows:

T_L : T_L limited by environmental conditions- atmosphere, oceans or rivers with temperatures ranging between 25 and 30°C corresponding to saturation pressures of approximately 0.032 and 0.042 bar, respectively. T_L is also limited by the temperature difference required for heat transfer process. If the temperature differences for heat transfer narrows, the heat transfer would approach reversibility. The transfer of finite amount of energy between two bodies whose temperature difference is very small requires a considerable amount of time, a large heat transfer area, or both since $\dot{Q} = -KA \frac{dT}{dx}$ or $\dot{Q} = h_c A dT$.

Reasonable heat transfer can only be obtained with reasonable condenser size if a temperature difference of 10 - 15 K is provided. The possibility of increasing η_{\max} by reducing T_L below the environmental conditions is not practical, because maintaining T_L lower than the ambient temperature would require the deployment a refrigerator that would have to be supplied with input work to operate.

T_H : The efficiency increases rapidly as T_H increases, for a given value of T_L , T_H is limited by the strength of the materials to be used in making the parts of the hot reservoir. The metallurgical limit of heat resistant alloys is about 900 K for steam plants.

2-1.2 Entropy

2-1.2.1 What is Entropy?

Entropy can be viewed as a measure of *molecular disorder*, or *molecular randomness*. As a system becomes more disordered, the positions of the molecules become less predictable and the entropy increases. It is therefore obvious that the entropy of a substance is lowest in the solid phase in which the molecules of a substance continually oscillate about their equilibrium positions, but cannot move relative to each other thereby making it possible at any instant in time to predict the position of the molecules with good certainty, and highest in the gas phase in

which molecules move about at random, collide with each other, and change direction and therefore making it extremely difficult to predict accurately the microscopic state of a system at any instant.

Molecules in the gas phase possess a considerable amount of kinetic energy. This energy is however disorganised and as a result not capable of performing work. This explains why a paddle inserted in a gas would not rotate no matter how high the kinetic energy of the molecules of the gas is. It is even possible that the number of molecules trying to rotate the wheel in one direction at any instant is equal to the number of molecules that are trying to rotate it in the opposite direction, causing the wheel to remain stationary. In the case of a rotating shaft, the energy of the molecules is completely organised since the molecules of the shaft are rotating in the same direction together. This organised energy can readily be used to perform useful tasks such as raising a weight or generating electricity as in wind turbine application. Being an organised form of energy, work is free of disorder or randomness and thus free of entropy. *There is therefore no entropy transfer associated with energy transfer as work.* Hence, in the absence of friction, the process of raising a weight by a rotating shaft (or a flywheel) will not produce any entropy. Any process that does not produce net entropy is reversible, and thus the process described above can be reversed by lowering the weight. Therefore, energy is not degraded during the process, and no potential to do work is lost.

In the instance where a paddle wheel is operated in a gas, as shown in Figure 2a, the paddle wheel work is converted to the internal energy of the gas, as evidenced by a rise in gas temperature, creating a higher level of molecular chaos and disorder in the container. This process is quite different from the raising of a weight since the organised paddle-wheel energy is now converted back to a highly disordered form of energy, which cannot be converted back to the paddle wheel as the rotational kinetic energy. Only a portion of this energy can be converted to work by partially reorganising it through the use of a heat engine. Therefore, energy is degraded during this process and the ability to do work is reduced, and molecular disorder is produced, and associated with all this is an increase in entropy.

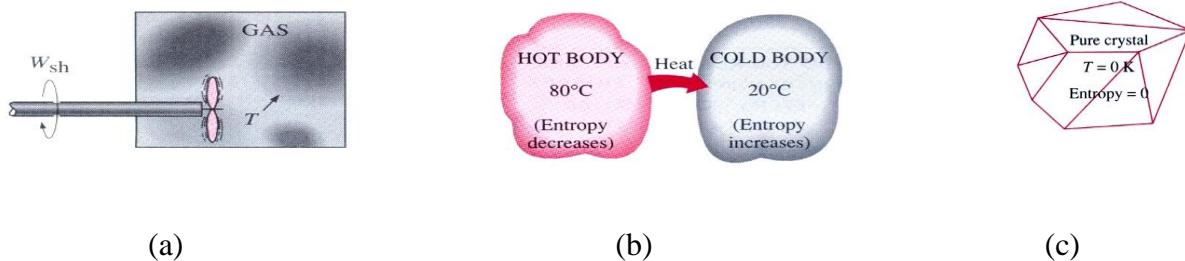


Figure 2: Entropy transfer and a decrease in the quality of energy



The **quantity** of energy is always preserved during an actual process (**the first law**)

But The **quality** is bound to decrease (**the second law**). This decrease in quality is always accompanied by an increase in entropy.

Heat is, in essence, a form of *disorganised energy*, and some disorganisation (entropy) will flow with heat (Figure 2b). As a result, the entropy and the level of molecular disorder or randomness of the hot body will decrease but the entropy and the level of molecular disorder of the cold body will increase. The second law requires that the increase in entropy of the cold body be greater than the decrease in entropy of the hot body, and thus the net entropy of the combined system (the cold body and the hot body) increases. That is, the combined system is at a state of greater disorder at the final state. Thus, we can conclude that processes can occur only in the direction of increased overall entropy or molecular disorder. That is, the entire universe is really getting more and more chaotic every day. From a statistical point of view, entropy is a measure of molecular randomness, that is, the uncertainty about the positions of molecules at any instant. Even in the solid phase, the molecules of a substance continually oscillate, creating an uncertainty about their position. The oscillations however, fade as the temperature is decreased, and the molecules become completely motionless at absolute zero. This represents a state of ultimate molecular order (and minimum energy). Therefore, *the entropy of a pure crystalline substance at absolute zero temperature is zero* since there is no uncertainty about the state of the molecules at that instant (Figure 2c). This statement is known as the third law of thermodynamics. The third law of thermodynamics provides an absolute reference point for the determination of entropy. The entropy determined relative to this point is called absolute entropy, and it is extremely useful in the thermodynamic analysis of chemical reactions. Notice that the entropy of a substance that is not pure crystalline (such as a solid solution) is not zero at absolute zero temperature. This is because more than one molecular configuration exists for such substances, which introduces some uncertainty about the microscopic state of the substance.

2-1.2.2 Entropy and Entropy Generation in Daily life

Entropy is a measure of disorder or disorganisation in a system. Likewise, entropy generation can be viewed as a measure of disorder or disorganisation generated during a process. Several ordinary events in daily life are relevant to the concept of entropy and entropy generation.

Efficient people lead low-entropy (highly-organised) lives. They have a place for everything (minimum uncertainty), and it takes minimum energy for them to locate something. Inefficient people, on the other hand, are disorganised and lead high-entropy lives. It takes them minutes (if not hours) to find something they need, and they are likely to create a bigger disorder in the process of searching since they will probably conduct the search in a disorganised manner. Note that people leading high-entropy lifestyles are always on the run, and never seem to catch up.

Some people seem to learn fast and remember well what they learn. This type of learning may be termed organised or low-entropy learning. These people make a conscientious effort to file the new information properly by relating it to their existing knowledge base and creating a solid information network in their minds. On the other hand, people who throw information into their minds as they study, with no effort to secure it, may think they are learning. They are bound to discover otherwise when they need to locate the information, for example during a test. But note that it is not easy to retrieve information from a database that is, in a sense, in the gas phase. Following from the above, students who have blackouts during tests should re-examine their study habits.

A library with good shelving and indexing system can be viewed as a low-entropy library because of the high level of organisation. Likewise, a library with a poor shelving and indexing system can be viewed as a high-entropy library because of the high level of disorganisation. A library with no indexing system is like no library, since a book is of no value if it cannot be found.

Two textbooks that seem to be identical because both cover basically the same topics and present the same information may actually be very different depending on how they cover the topics. After all, two seemingly identical cars are not so identical if one goes only half as many miles as the other one on the same amount of fuel. Likewise, two seemingly identical books are not so identical if it takes twice as long to learn a topic from one of them as it does from the other. Thus, comparisons made on the basis of the first law only may be highly misleading or deceptive.

From the military point of view, having a disorganised (high-entropy) army is like having no army at all. It is no coincidence that the command centres of any armed forces are among the primary targets during a war. Recall that in the Bible when Peter was imprisoned, four squads each consisting of four soldiers were commanded to guard him. That was a case of entropy minimisation in terms of command structure. One army that consists of ten divisions is ten times more powerful than ten armies each consisting of a single division. Likewise, one country that consists of ten states is more powerful than ten countries, each consisting of a single state. The United States would not be such a powerful country if there were fifty independent countries in its place instead of a single country with fifty states. The new European common market has the potential to be a new economic superpower. The old cliché “divide and conquer” can be rephrased as “increase the entropy and conquer.”



Following from the above presentation, I urge my students to do “low- entropy learning” or what I call “organised learning” based on conscientious effort to file related information.

2-1.3 Property Diagrams Involving Entropy

Property diagrams serve great visual aids in the thermodynamic analysis of processes. P - v and T - v diagrams have been extensively used in conjunction with the first law of thermodynamics. In the second law analysis, it is very helpful to plot the processes on diagrams for which one of the coordinates is entropy. The two diagrams commonly used in the second-law analysis are the *temperature-entropy* and the *enthalpy-entropy* diagrams. (See figure 6 for a typical h-s chart for steam)

2-1.3.1: The Temperature-Entropy (T-s) Diagram

Consider the defining equation of entropy given by one of the collaries of the second law of thermodynamics. It can be rearranged as:

$$\delta Q_{\text{int rev}} = T dS \quad (\text{kJ}) \quad (1.5)$$

As shown in Figure 3, $\delta Q_{\text{int rev}}$ corresponds to a differential area of a T - S diagram.

The total heat transfer during an internally reversible process is determined by integration to be:

$$Q_{\text{int rev}} = \int_1^2 T dS \quad (\text{kJ}) \quad (1.6)$$

which corresponds to the area under the process curve on a T - S diagram. Therefore, it can be concluded that *the area under the process curve on a T-S diagram represents heat transfer during an internally reversible process.*

This is somewhat analogous to reversible boundary work being represented by the area under the process curve on a P - V diagram. Note that the area under the process curve represents heat transfer for processes that are internally (or totally) reversible. *The area has no meaning for irreversible processes.*

Equations (1.5) and (1.6) can also be expressed on a unit-mass basis as:

$$\delta q_{\text{int rev}} = T ds \quad (\text{kJ/kg}) \quad (1.7)$$

and

$$q_{\text{int rev}} = \int_1^2 T ds \quad (\text{kJ/kg}) \quad (1.8)$$

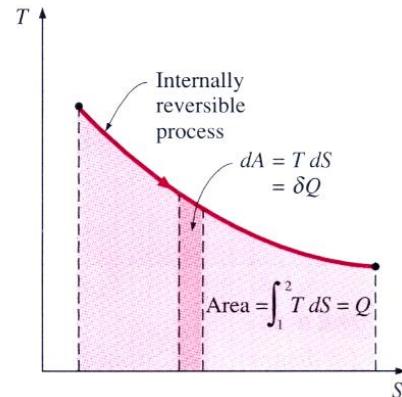


Figure 3: T-s diagram of a reversible process

To perform the integrations in Eqs 1.7 and 1.8, one needs to know the relationship between T and s during a process. One special case for which these integrations can be performed easily is the *internally reversible isothermal process*. It yields:

$$Q_{\text{int rev}} = T_0 \Delta S \quad (1.9)$$

or

$$q_{\text{int rev}} = T_0 \Delta s \quad (\text{kJ/kg}) \quad (1.10)$$

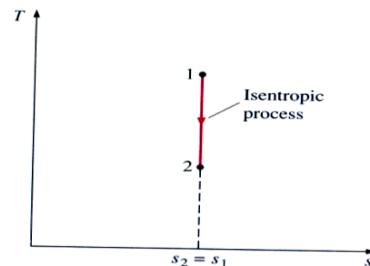


Figure 4: Isentropic Process

where T_0 is the constant temperature and ΔS is the entropy change of the system during the process.

In the relations above, T is the absolute temperature, which is always positive.

Therefore, heat transfer during internally reversible processes is positive when entropy increases and negative when entropy decreases. An isentropic process on a T - s diagram is easily recognised as a vertical-line segment. This is expected since an isentropic process involves no heat transfer, and therefore the area under the process path must be zero (Figure 4). The T - s diagrams serve as valuable tools for visualising the second-law aspects of processes and cycles, and thus they are frequently used in thermodynamics.

2-1.3.2 The Enthalpy-Entropy (h-s) Diagram

Another diagram commonly used in engineering is the enthalpy-entropy diagram, which is quite valuable in the analysis of steady-flow devices such as turbines, compressors, and nozzles. The coordinates of an h-s diagram represent two properties of major interest: enthalpy, which is a primary property in the first-law analysis of the steady-flow devices, and entropy, which is the property that accounts for irreversibilities during adiabatic processes. In analysing the steady flow of steam through an adiabatic turbine, for example, the vertical distance between the inlet and outlet states (Δh) is a measure of the work output of the turbine, and the horizontal distance (Δs) is a measure of the irreversibilities associated with the process (Figure 5a)

The h - s diagram is also called a **Mollier diagram** named after the German scientist R. Mollier (1863-1935). The general features of an h - s diagram are illustrated in Figure 5b. On an h - s diagram, the constant-temperature lines are straight in the saturated liquid-vapour mixture region. They become almost horizontal in the superheated vapour region, particularly at low pressures. This is not surprising since steam approaches ideal-gas behaviour as it moves away from the saturation region, and for ideal gases the enthalpy is a function of temperature only.

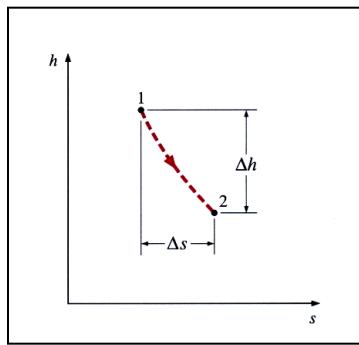


Figure 5a

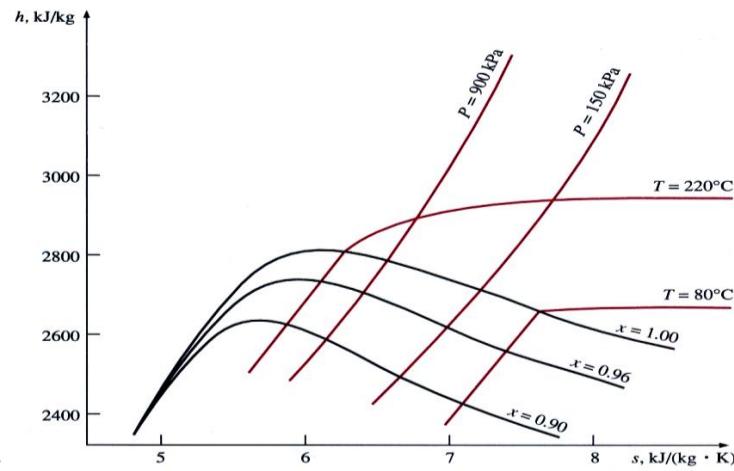


Figure 5b

Figure 5: Enthalpy-entropy (h-s) diagram

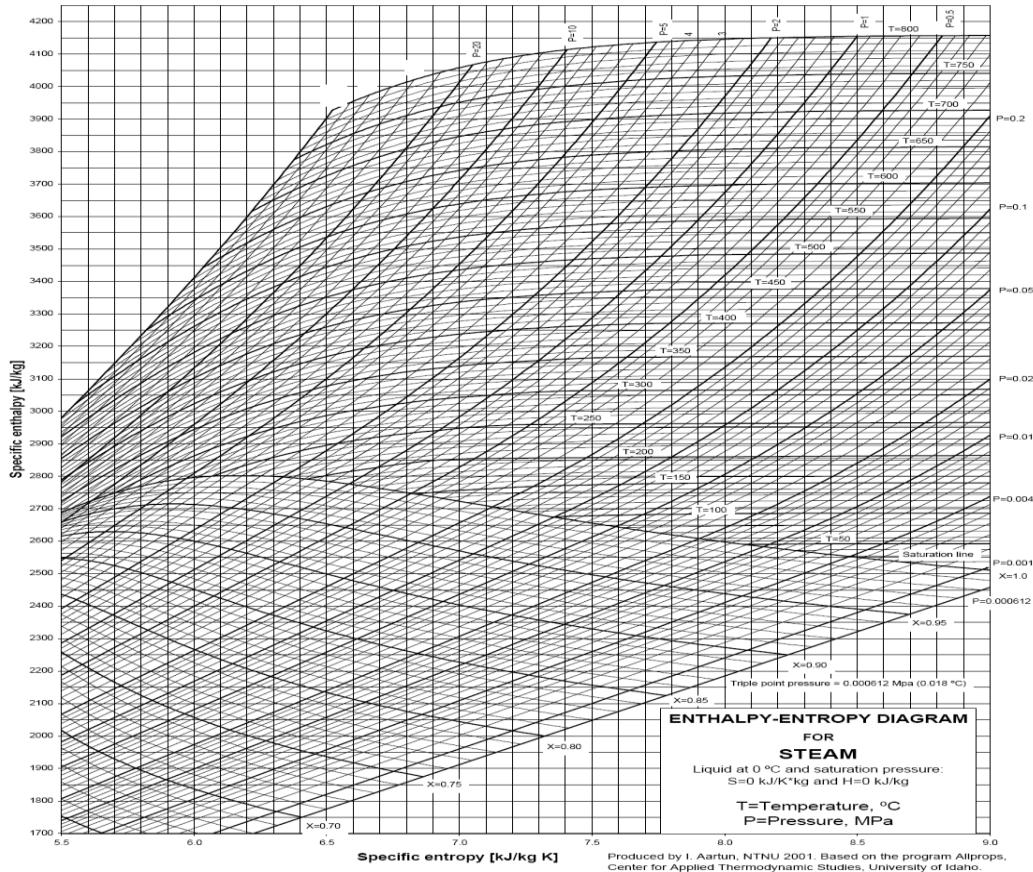


Figure 6: Enthalpy-Entropy (h-s) chart for steam

2-1.4 The T ds Relations

It has been shown that the quantity $(\delta Q/T)_{\text{int rev}}$ corresponds to a differential change in a property, called *entropy*. The entropy change for a process, then, was evaluated by integrating $\delta Q/T$ along some imaginary internally reversible path between the actual end states. For isothermal internally reversible processes, this integration is straightforward. But when the temperature varies during the process, there is the need for a relationship between δQ and T to perform this integration. Finding such relations is what is intended to be done in this section.

The differential form of the conservation of energy equation for a closed stationary system (a fixed mass) containing a simple compressible substance can be expressed for an internally reversible process as:

$$\delta Q_{\text{int rev}} + \delta W_{\text{int rev, out}} = dU \quad (1.11)$$

But

$$\delta Q_{\text{int rev}} = T dS \text{ and } \delta W_{\text{int rev, out}} = -P dV \text{ (European sign convention)}$$

Thus,

$$T dS = dU + P dV \quad (1.12)$$

or

$$T ds = du + P dv \quad (1.13)$$

per unit mass. Equation 1.13 is known as the first $T ds$, or *Gibbs, equation*. Notice that the only type of work interaction a simple compressible system may involve as it undergoes an internally reversible process is the quasi-equilibrium boundary work (PdV or $-PdV$ depending on which sign convention is adapted)

The second Tds equation is obtained by eliminating du from Equation 1.13 by using the definition of enthalpy ($h = u + Pv$):

$$\left. \begin{aligned} h &= u + Pv \rightarrow dh = du + Pdv + vdp \\ (\text{Eq. 1.13}) &\rightarrow T ds = du + Pdv \end{aligned} \right\} \quad T ds = dh - vdp \quad (1.14)$$

Relations for differential changes in entropy are obtained by solving for ds in Eqs. 1.13 and 1.14:

$$ds = \frac{du}{T} + \frac{Pdv}{T} \quad (1.15)$$

and

$$ds = \frac{dh}{T} - \frac{vdp}{T} \quad (1.16)$$

The entropy change during a process can be determined by integrating either of these equations between the initial and final states. To perform these integrations, however, there is the need for a defined relationship between du and dh and the temperature (such as $du = C_v dT$ and $dh = C_p dT$ for ideal gases) as well as the equation of state for the substance (such as the ideal-gas equation of state $Pv = RT$).

The $T ds$ relations for non simple systems, that is, systems that involve more than one mode of quasi-equilibrium work, can be obtained in a similar manner by including all the relevant quasi-equilibrium work modes.

2-1.4.1 Entropy Change of Liquids and Solids

Liquids and solids can be approximated as incompressible substances since their specific volumes remain nearly constant during a process. Thus, $dv \approx 0$ for liquids and solids, and Eq. 1.15 for this case reduces to

$$ds = \frac{du}{T} = \frac{C dT}{T} \quad (1.17)$$

since $C_p = C_v = C$ and $du = C dT$ for incompressible substances. Then the entropy change during a process is determined by integration to be:

$$\text{Liquids, solids: } s_2 - s_1 = \int_1^2 C(T) \frac{dT}{T} \cong C_{av} \ln \frac{T_2}{T_1} \quad [\text{kJ/(kg} \cdot \text{K)}] \quad (1.18)$$

where C_{av} is the *average* specific heat of the substance over the given temperature interval.

2-1.4.2 Entropy Change of Pure Substances

Entropy is a property, and thus the value of entropy of a system is fixed once the state of the system is specified. Specifying two intensive independent properties fixes the state of a simple compressible system, and thus the value of entropy, as well as the values of other properties at that state.

Using a suitable reference state, the entropies of substances are evaluated from measurable property data following rather involved computations, and the results are tabulated in the same manner as the other properties such as v , u , and h (Figure 7).

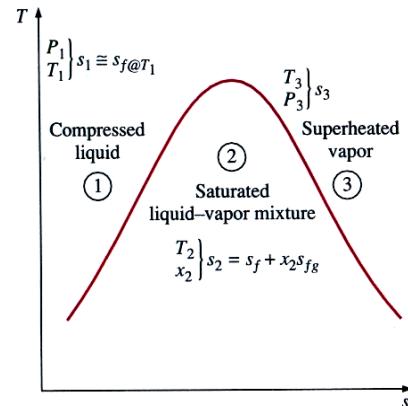


Figure 7: Phase description

The entropy values in the property tables are given relative to an arbitrary reference state. In steam tables, the entropy of saturated liquid s_f at 0.01 °C is assigned the value of zero. For refrigerant-134a, the zero value is assigned to saturated liquid at - 40 °C. The entropy values become negative at temperatures below the reference value.

The entropy change of a specified mass m (such as a closed system) during a process is simply:

$$\Delta S = m \Delta s = m(s_2 - s_1) \quad (\text{kJ/K}) \quad (1.19)$$

which is the difference between the entropy values at the final and initial states.

2-1.4.3 The Entropy Change of Ideal Gases

By substituting $du = C_v dT$ and $P = RT/v$ into Eq. 1.15 the differential entropy change of an ideal gas becomes:

$$ds = C_v \frac{dT}{T} + R \frac{dv}{v} \quad (1.20)$$

The entropy change for a process is obtained by integrating this relation between the end states:

$$s_2 - s_1 = \int_1^2 C_v(T) \frac{dT}{T} - R \ln \frac{v_2}{v_1} \quad (1.21)$$

A second relation for the entropy change of an ideal gas is obtained in a similar manner by substituting $dh = C_p dT$ and $v = RT/P$ into Eq. 1.16 and integrating. The result is:

$$s_2 - s_1 = \int_1^2 C_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1} \quad (1.22)$$

The specific heats of ideal gases, with the exception of monatomic gases, depend on temperature, and the integrals in Eqs. 1.21 and 1.22 cannot be performed unless the dependence of C_v and C_p on temperature is known. These integrations can be performed either by assuming constant specific heats or evaluating those integrals once and tabulating the results.

1. Constant Specific Heats: Approximate Analysis

For monatomic ideal gases such as helium, the specific heats are independent of temperature, and therefore the constant-specific-heat assumption introduces no error. For ideal gases whose specific heats vary almost linearly with temperature in the range of interest, the possible error is minimised by using specific-heat values evaluated at the average temperature (Figure 8). The results obtained in this way usually are sufficiently accurate for most ideal gases if the temperature range is not greater than a few hundred degrees. The entropy-change relations for ideal gases under the constant-specific-heat assumption are easily obtained by replacing $C_v(T)$ and $C_p(T)$ in Eqs. 1.21 and 1.22 by $C_{v,av}$ and $C_{p,av}$, respectively, and performing the integrations we obtain:

$$s_2 - s_1 = C_{v,av} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad [\text{kJ/(kg} \cdot \text{K})] \quad (1.23)$$

and

$$s_2 - s_1 = C_{p,av} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad [\text{kJ/(kg} \cdot \text{K})] \quad (1.24)$$

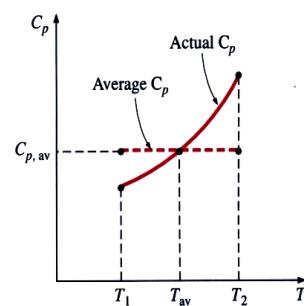


Figure 8: Specific heat- temperature diagram

Entropy changes can also be expressed on a unit-mole basis by multiplying these relations by molar mass:

$$\bar{s}_2 - \bar{s}_1 = \bar{C}_{v,av} \ln \frac{T_2}{T_1} + R_u \ln \frac{v_2}{v_1} \quad [\text{kJ/(kmol} \cdot \text{K})] \quad (1.25)$$

and $\bar{s}_2 - \bar{s}_1 = \bar{C}_{p,av} \ln \frac{T_2}{T_1} - R_u \ln \frac{P_2}{P_1} \quad [\text{kJ/(kmol} \cdot \text{K})]$ (1.26)

2-1.4.4 The Increase of Entropy Principle

Consider a cycle that is made up of two processes: process 1-2, which is arbitrary (reversible or irreversible), and process 2-1, which is internally reversible, as shown in Figure 9. From the Clausius inequality,

$$\oint \frac{\delta Q}{T} \leq 0$$

or

$$\int_1^2 \frac{\delta Q}{T} + \int_{2 \rightarrow 1}^1 \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \leq 0$$

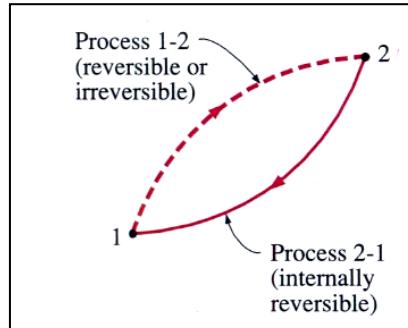


Figure 9: A cycle consisting of two processes

The second integral in the above relation is readily recognised as the entropy change $S_1 - S_2$. Therefore,

$$\int_1^2 \frac{\delta Q}{T} + S_1 - S_2 \leq 0$$

which can be arranged as,

$$S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T} \quad (1.27)$$

Equation 1.27 can be viewed as a mathematical statement of the second law of thermodynamics for a fixed mass. It can also be expressed in differential form as:

$$dS \geq \frac{\delta Q}{T} \quad (1.28)$$

where the equality hold for an internally reversible process and the inequality for an irreversible process. It may be concluded from these equations that *the entropy change of a closed system during an irreversible process is greater than the integral of $\delta Q/T$ evaluated for that process. In the limiting case of a reversible process, these two quantities become equal.* Note that T in the above relations is the *absolute temperature* at the *boundary* where the differential heat δQ is transferred between the system and the surroundings.

The quantity $\Delta S = S_2 - S_1$ represents the *entropy change* of the system. For a reversible process, it becomes equal to $\int_1^2 \delta Q/T$, which represents the *entropy transfer* with heat.

The inequality sign in the relations above is a constant reminder that the entropy change of a closed system during an irreversible process is always greater than the entropy transfer. That is, some entropy is generated or created during an irreversible process, and this generation is due entirely to the presence of irreversibilities. The entropy generated during a process is called **entropy generation** and is denoted by S_{gen} . Noting that the difference between the entropy change of a closed system and the entropy transfer is equal to entropy generation, Eq. 1.28 can be rewritten as equality as:

$$\Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}} \quad (1.29)$$



$$\Delta S_{\text{total}} = \sum_j \frac{Q_j}{T_j} + \sigma$$

Entropy production within the system itself as a result of internal irreversibility in the system itself and external irreversibility present within the portion of the surroundings included with the enlarged system

Entropy transfer as a result of heat transfer within the surroundings

$$\sigma \begin{cases} >0 & \text{irreversibility present within the system} \\ =0 & \text{no irreversibility present within the system} \end{cases}$$

Note that the entropy generation S_{gen} is always a *positive* quantity or zero. Its value depends on the process, and thus it is not a property of the system. Also, in the absence of any entropy transfer with heat, the entropy change of a system is equal to the entropy generation.

For an isolated system (or simply an adiabatic closed system), the heat transfer is zero, and Eq. 1.27 reduces to

$$\Delta S_{\text{isolated}} \geq 0 \quad (1.30)$$

This equation can be expressed as *the entropy change of an isolated system during a process always increases or, in the limiting case of a reversible process, remains constant*. In other words, it never decreases. This is known as the **increase of entropy principle**. Note that in the absence of any heat transfer, entropy change is due to irreversibilities only, and their effect is always to increase entropy.

Entropy is an extensive property, and thus the total entropy of a system is equal to the sum of the entropies of the parts of the system. A system and its surroundings can be viewed as the two subsystems of an isolated system, and the entropy change of this isolated system during a process is the sum of the entropy changes of the system and its surroundings, which is equal to the entropy generation since an isolated system involves no entropy transfer. That is,

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0 \quad (1.31)$$

Where, the equality holds for reversible processes and the inequality for irreversible ones.
Note that ΔS_{surr} refers to the change in the entropy of the surroundings as a result of the occurrence of the process under consideration.

Since no actual process is truly reversible, it can be concluded that some entropy is generated during a process, and therefore the entropy of the universe is continuously increasing. Entropy increase of the universe is a major concern not only to engineers but also to philosophers and theologians since entropy is viewed as a measure of the disorder (or “mixed-up-ness”) in the universe.

The increase of entropy principle does not imply that the entropy of a system cannot decrease. The entropy change of a system *can* be negative during a process (Figure 10), but entropy generation cannot. The increase of entropy principle can be summarised as follows:

$S_{\text{gen}} > 0$	Irreversible process
$S_{\text{gen}} = 0$	Reversible process
$S_{\text{gen}} < 0$	Impossible process

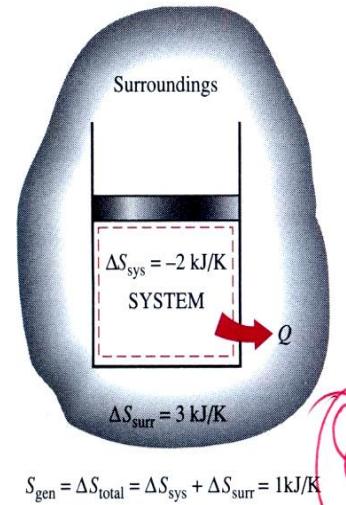


Figure 10: Entropy generation analysis

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} = 1 \text{ kJ/K}$$

The above relation serves as a criterion in determining whether a process is reversible, irreversible, or impossible.

Things in nature have a tendency to change until they attain a state of equilibrium. The increase of entropy principle dictates that the entropy of an isolated system will increase until the entropy of the system reaches a maximum value. At that point, the system is said to have reached an equilibrium state since the increase of entropy principle prohibits the system from undergoing a change of state that will result in a decrease in entropy.

Some Remarks about Entropy

In light of the preceding discussions, the following conclusions can be drawn:

1. Processes can occur in a certain direction only, not in any direction. A process must proceed in the direction that complies with the increase of entropy principle, that is $S_{\text{gen}} \geq 0$. A process that violates this principle is impossible. This principle often forces chemical reactions to come to a halt before reaching completion.
2. Entropy is a non conserved property, and there is no such thing as the conservation of entropy principle. Entropy is conserved during the idealised reversible processes only and increases during all actual processes. Therefore, the entropy of the universe is continuously increasing.
3. The performance of engineering systems is degraded by the presence of irreversibilities, and entropy generation is a measure of the magnitudes of the irreversibilities present during that process. The greater the extent of irreversibilities, the greater the entropy generation. Therefore, entropy generation can be used as a quantitative measure of irreversibilities associated with a process.

Example 1 Entropy Generation during Heat Transfer Processes

A heat source at 800 K loses 2000 kJ of heat through a partition to a sink at (a) 500 K and (b) 750 K. Determine which heat transfer process is more irreversible

Solution: A sketch of the reservoirs is shown in Figure 11. Both cases involve heat transfer through a finite temperature difference, and therefore both are irreversible. The magnitude of the irreversibility associated with each process can be determined by calculating the total entropy change for each case. The total entropy change for a heat transfer process involving two reservoirs (a source and a sink) is the sum of the entropy changes of each reservoir since the two reservoirs form an adiabatic system.

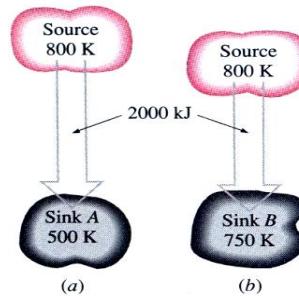


Figure 11: Schematic for example 1

Example 2

A frictionless cylinder-piston device contains saturated liquid-vapour mixture of water at 100 °C. During a constant-pressure process, 600 kJ of heat is transferred to the surrounding air at 20 °C. As a result, part of this water vapour condenses. Assuming that the water undergoes an internally reversible isothermal process, determine

- a) The entropy change of the water and
- b) The total entropy operation during the process by considering the extended system which includes the water, the piston-cylinder device and the region immediately outside the system that experiences a temperature change.

SOLUTION FOR EXAMPLE 2

$$a) \Delta S_{\text{system}} = \frac{Q}{T_{\text{system}}} = \frac{-600 \text{ kJ}}{(100 + 273) \text{ K}} = -1.61 \text{ kJ/K}$$

$$b) + \frac{Q_{\text{out}}}{T_0} + S_{\text{gen}} = \Delta S_{\text{System}}$$

$$\Delta S_{\text{gen}} = - \frac{Q_{\text{out}}}{T_0} + \Delta S_{\text{System}}$$

$$= \frac{600 \text{ kJ}}{(25 + 273) \text{ K}} - 1.61 \text{ kJ/K} = 2.01 \text{ kJ/K} - 1.61 \text{ kJ/K} = +0.40 \text{ kJ/K}$$

Comment: The entropy generation is due to irreversible heat transfer through a finite temperature difference.

It is justified to assume that $\Delta S_{\text{partition}} \approx 0$ since the entropy (as well as the energy) content of the partition essentially remained constant during this process.

SOLUTION FOR EXAMPLE 1

Noting that each reservoir undergoes an internally reversible isothermal process, the entropy change for each reservoir can be determined as:

(a) For the heat transfer process to a sink at 500 K.

$$\Delta S_{\text{source}} = \frac{Q_{\text{source}}}{T_{\text{source}}} = \frac{-2000 \text{ kJ}}{800 \text{ K}} = -2.5 \text{ kJ/K}$$

$$\Delta S_{\text{sink}} = \frac{Q_{\text{sink}}}{T_{\text{sink}}} = \frac{+2000 \text{ kJ}}{500 \text{ K}} = +4.0 \text{ kJ/K}$$

$$\text{and } S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{source}} + \Delta S_{\text{sink}} = (-2.5 + 4.0) \text{ kJ/K} = +1.5 \text{ kJ/K}$$

Therefore, 1.5 kJ/K of entropy is generated during this process. Noting that both reservoirs have undergone internally reversible processes, the entire entropy generation took place in the partition.

(b) Repeating the calculations in part (a) for a sink temperature of 750 K,

$$\Delta S_{\text{source}} = \frac{-2000 \text{ J}}{750 \text{ K}} = -2.5 \text{ kJ/K}$$

$$\Delta S_{\text{sink}} = \frac{+2000 \text{ J}}{750 \text{ K}} = +2.7 \text{ kJ/K}$$

$$\text{and } S_{\text{gen}} = \Delta S_{\text{total}} = (-2.5 + 2.7) \text{ kJ/K} = +0.2 \text{ kJ/K}$$

The total entropy change for the process in part (b) is smaller, and therefore it is less irreversible. This is expected since the process in (b) involves a smaller temperature difference and thus a smaller irreversibility. (a) is more irreversible and (b) is less irreversible.

Discussion: The irreversibilities associated with both processes could be eliminated by operating a Carnot heat engine between the source and the sink. For this case, it can be easily shown that $\Delta S_{\text{total}} = 0$.

2-1.4.5 Isentropic Processes

As previously mentioned, the entropy of a fixed mass can be changed by (1) heat transfer and (2) irreversibilities. It then follows that the entropy of a fixed mass will not change during a process that is internally reversible and adiabatic. A process during which the entropy remains constant is called an isentropic process. An isentropic process is characterised by:

$$\text{Isentropic process: } \Delta s = 0 \quad \text{or} \quad s_2 = s_1 \quad [\text{kJ/(kg} \cdot \text{K}]\text{}$$

That is, a substance will have the same entropy value at the end of the process as it does at the beginning if the process is carried out in an isentropic manner. Many engineering systems or devices such as pumps, turbines, nozzles, and diffusers are essentially adiabatic in their operation, and they perform best when the irreversibilities, such as the friction associated with the process are minimised. Therefore, an isentropic process can serve as an appropriate model for the actual processes. Also, isentropic processes enable the definition of efficiencies for processes to compare the actual performance of these devices to the performance under idealised conditions.

It should be recognised that a reversible adiabatic process is necessarily isentropic ($s_2 = s_1$), but an isentropic process is not necessarily reversible adiabatic. (the entropy increase of a substance during a process as a result of irreversibilities may be offset by a decrease in entropy as a result of heat losses, for example.) However, the term isentropic process is customarily used in thermodynamics to imply an internally reversible, adiabatic process.



Self Assessment 2-1

1. Steam at 0.05 bar, 100 °C is to be condensed completely by a reversible constant pressure process. Calculate the heat rejected per kilogram of steam, and the change of specific entropy. Sketch the process on a $T - s$ diagram and shade in the area which represents the heat flow.
2. 1 m³ of air is heated reversibly at constant pressure from 15 to 300 °C, and is then cooled reversibly at constant volume back to the initial temperature. The initial pressure is 1.03 bar. Calculate the net heat flow and the overall change of entropy, and sketch the processes on a $T - s$ diagram.
3. 1 kg of a fluid at 2.62 bar, - 3°C, is compressed according to a law $pv = \text{constant}$ to a pressure of 8.2 bar. Calculate the work input and the heat supplied (i) when the fluid is air, (ii) when the fluid is a refrigerant initially dry saturated with the properties given in the table below. Sketch each process on a $T - s$ diagram.

Table 1 : Table for answering self-assessment 2-1 Problem 3

Saturation values				
t_g (°C)	p_g (bar)	v_g (m³/kg)	h_f (kJ/kg)	h_g (kJ/kg)
-3.0	2.62	0.0757	96.07	292.94
32.3	8.20	0.0248	144.29	313.05

4. A certain perfect gas is compressed reversibly from 1 bar, 17°C to a pressure of 5 bar in a perfectly thermally insulated cylinder, the final temperature being 77°C. The work done on the gas during the compression is 45 kJ/kg. Calculate γ , c_v , R , and the molar mass of the gas.
 5. A system of 100 kg mass undergoes a process in which the specific entropy increases from 0.3 kJ/(kg.K) to 0.4 kJ/(kg. K). At the same time, the entropy of the surroundings of the system decreases from 80 kJ/K to 75 kJ/K. This process is:

(a) Reversible (b) irreversible (c) reversible and adiabatic (d) impossible

Hint: Determine the total change in entropy of the system and use the answer to select the correct option.

6. Air is flowing through a well-lagged duct. At section A, the pressure and temperature are 1.5 bar and 187 °C, respectively, and at some distance along the duct at a section B, the pressure and temperature are 1.3 bar and 160 °C. In what direction is the air flowing.

Hint: the total change in entropy of the system must be increasing in the direction of flow of the air since there are irreversibility's involved.

7. 300 kJ/s of heat is supplied at a constant fixed temperature of 290°C to a Heat engine. The heat rejection takes place at 8.5°C. The following results were obtained:
 (i) 215 kJ/s are rejected. (ii) 150 kJ/s are rejected. (iii) 75 kJ/s are rejected.

Classify which of the result report a reversible cycle or irreversible cycle or impossible Results.



Suggested Answers

- [2550 kJ/kg; 8.292 kJ/kg K]
- [101.5 kJ; 0.246 kJ/K]
- [88.41 kJ; - 88.41 kJ; 22.62 kJ; - 6.69 kJ]
- [1.132; 0.75 kJ/kg K; 0.099 kJ/kg K; 84 kg/kmol]

Table 2: Table for answering self-assessment 2-1 Problem 1

Saturation tables

Superheated values at P = 0.05 bar

P bar	t_s ° C	v_g m³/kg	h_f kJ/kg	h_g kJ/kg	h_{fg} kJ/kg
0.05	32.9	28.20	138	2561	2423

t ° C	s kJ/kg K	u kJ/kg	h kJ/kg	v_g m³/kg
100	8.768	2516	2688	34.42

POWER CYCLES

Introduction

In this unit, vapour and gas power cycles would be considered. We would investigate the various modifications of the basic Rankine vapour power cycles to increase the cycle thermal efficiency of steam power plants. The performance of gas power cycles for which the working fluid remains as a gas throughout the entire cycle would also be evaluated. Combined Power Cycles will be discussed briefly.



Learning Objectives

After studying this unit you should be able to:

1. Analyse vapour power cycles in which the working fluid is alternately vapourised and condensed.
2. Investigate ways to modify the basic Rankine vapour Power cycles to increase the cycle thermal efficiency of steam power plants.
3. Discuss other modifications for improving the efficiency of steam power plants.
4. Evaluate the performance of gas power cycles for which the working fluid remains as a gas throughout the entire cycle.
5. Analyse both closed and open gas power cycles.
6. Discuss ways of improving gas turbine cycle efficiency.
7. Analyse power cycles that consist of two separate cycles known as combined power cycles.

SESSION 1-2: VAPOUR POWER CYCLES

One method of producing mechanical power is the transfer of heat to a working fluid, which goes through a thermodynamic cycle, converts part of the heat to work and rejects heat to a sink. If the working fluid undergoes a phase change, the cycle is called a Vapour Power Cycle.

1-2.1 Characteristics of a vapour Power Cycle

- a) The working fluid is a condensable vapour, which is in the liquid phase during part of the cycle.
- b) The cycle consists of steady flow processes with each process carried out in a separate component specially designed for the purpose.
- c) Each component constitutes an open system and all the components are connected in series so that each fluid element passes through mechanical and thermodynamic states. In each component, matter flows into and out of a region of space as well as heat and work. All processes in power cycle are considered to be flow processes.

1-2.1.1 Decision Making in Design

Capacity: Total power output required

Energy Source: Chemical sources (coal, oil, natural gas, fuel cell and biomass); Nuclear energy; solar radiation etc.

Energy Sink: From the 2nd Law, we know that all heat engines must reject some heat. Also, the efficiency is higher if the sink temperature is low. Available natural heat sinks are the atmosphere, rivers, lakes, and oceans. At the moment, about up to approximately 60 % of heat absorbed is rejected. This is a serious ecological problem because of thermal pollution.

Working Fluid: Water because it is cheap and chemically stable.

Component selection: Design of hardware. Decide on component arrangement etc. and also work out the economic and technical feasibility of the project.

1-2.1.2 Criteria for comparing Cycles

The choice of a power plant is determined largely by:

Capital cost: This is determined by the size and complexity of the plant.

Operating cost: This is determined by the overall efficiency of the plant. In general, the efficiency can be improved by increasing the complexity of the plant. A compromise between low operating and capital costs is therefore required.

$$\eta_{\text{combustion}} = \frac{Q_i}{E_{\text{fuel}}}$$

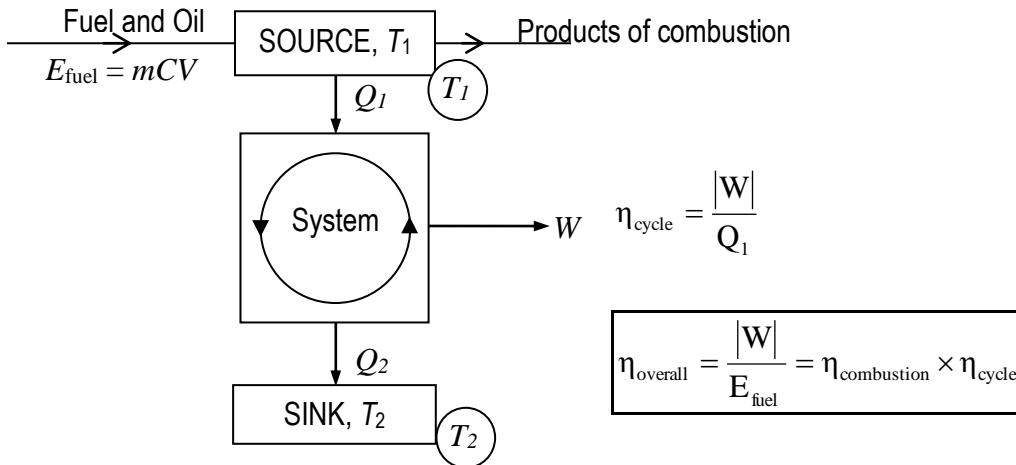


Figure 12: Vapour Power Cycle Analysis

The essential features of a vapour power cycle are as illustrated in Figure 12 above. The overall efficiency of a vapour power plant is suitably measured by the proportion of latent heat in the fuel, which is converted into useful mechanical work. The overall thermal efficiency can be expressed as a product of two efficiencies:

- Combustion efficiency:** This expresses the proportion of the internal energy or latent energy in the fuel transferred as heat to the working fluid.
- Cycle efficiency:** This expresses the proportion of heat energy transferred to the working fluid and which is subsequently converted into mechanical work.

The cycle efficiency is not unity for the following reasons:

- The 2nd law expresses the fact that even in the best power cycle (Carnot cycle); some form of heat must be rejected. The best power cycle is that in which all heat supplied is transferred while the working fluid is at a constant upper temperature T_H and all the heat rejected leaves while the working fluid is at a constant lower temperature T_L , and all the processes are reversible. The efficiency of such a cycle is given by the expression $\frac{T_H - T_L}{T_H}$ irrespective of the working fluid.
- From the practical point of view, all real processes are irreversible and irreversibility in cycles reduces the overall efficiency. The first law is still obeyed but the cycle efficiency $|W|/|Q_1|$ is reduced.

1-2.1.3 Ideal and Actual Cycle Efficiencies and Efficiency Ratio

If all the processes of a power cycle are assumed to be reversible, then the efficiency calculated is known as the ideal cycle efficiency. The ratio of the actual cycle efficiency to the ideal cycle efficiency is called the efficiency ratio.

$$\eta_{ratio} = \frac{\eta_{actual}}{\eta_{ideal}} \quad (2.1)$$

1-2.1.4 The Effect of Work Ratio

Table 1: Analysis of ideal and actual cycles

	Ideal		Actual $\eta_c = \eta_T = 0.90$	
	Cycle 1	Cycle 2	Cycle 1	Cycle 2
Q_{in}	120	120	120	120
$ W_t $	100	40	90	36
$ W_c $	61	1	67.8	1.1
$ W_{net} $	39	39	22.2	34.9
r_w	0.390	0.975	0.247	0.969
η_{cycle}	0.325	0.325	0.185	0.291

Some cycles are more sensitive to irreversibilities than others and a high ideal cycle efficiency only is not by itself a good indicator of whether or not the cycle will provide a power plant of high overall efficiency.

The work ratio, r_w , gives a direct indication of how sensitive a cycle is to any irreversibility introduced into it. It is defined as the ratio of the *net work output* $|W_{net}|$ to the *gross work output* $|W_t|$.

That is,

$$r_w = \frac{\text{Net work output}}{\text{Gross work output}} = \frac{|W_{net}|}{|W_t|} \quad (2.2)$$

Irreversibilities have the effect of decreasing the work outputs and increasing the work inputs and hence there is a decrease in the net work output. Summarising, we may say that a *high ideal cycle efficiency together with a high work ratio provide a reliable indication that the actual power plant will have a good overall efficiency*. A work ratio of unity means that the components producing work will be of least possible size for a given net power output. Work ratio is in itself not very informative and a more informative parameter will be defined subsequently.

1-2.1.5 Specific Steam Consumption

A more direct indication of the relative sizes of steam power plants is provided by the specific steam consumption (SSC). It is the mass flow of steam required per unit net power output. It is

usually expressed in kg/kWh and if $|W|$ is the magnitude of the net work output per unit mass of steam in kJ/kg,

$$\text{Then, } SSC = \frac{3600}{|W|} \text{ kg/kWh, if } |W| \text{ is in kJ/kg} \quad (2.3)$$

1-2.1.6 Process Efficiencies (η_t or η_c)

It is a measure of irreversibility and can be defined for a steady flow process as the ratio of isentropic to actual work or vice versa, depending on whether the process is a work producing process, or a work requiring process as follows,

$$\text{For a work producing device } \eta_t = \frac{\text{Actual work}}{\text{Isentropic work}} \quad \text{For a work requiring device } \eta_c = \frac{\text{Isentropic work}}{\text{Actual work}}$$

1-2.1.6.1 Turbine Process

$$\eta_{\text{turbine, isentropic}} = \frac{\text{work done during actual expansion}}{\text{work done during isentropic expansion}} = \frac{h_2 - h_1}{h_{2s} - h_1} \quad (2.4)$$

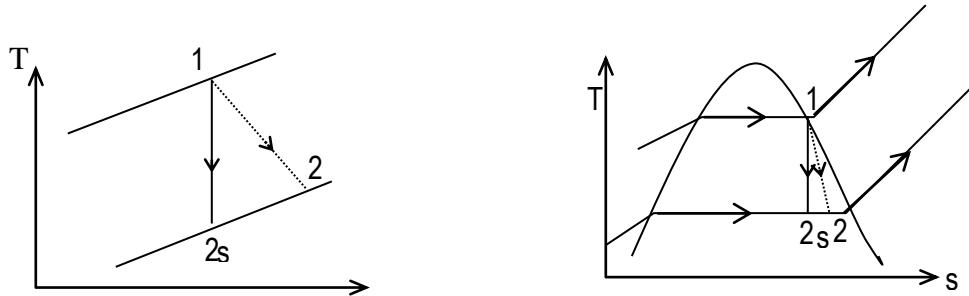


Figure 13: Isentropic and Actual processes of a turbine

Irreversibility within the turbine reduces the net power output of the plant

1-2.1.6.1 Compressor or Pump Process

$$\eta_{\text{compressor, isentropic}} = \frac{\text{work required during isentropic compression}}{\text{work required during actual compression}} = \frac{h_{2s} - h_1}{h_2 - h_1} \quad (2.5)$$

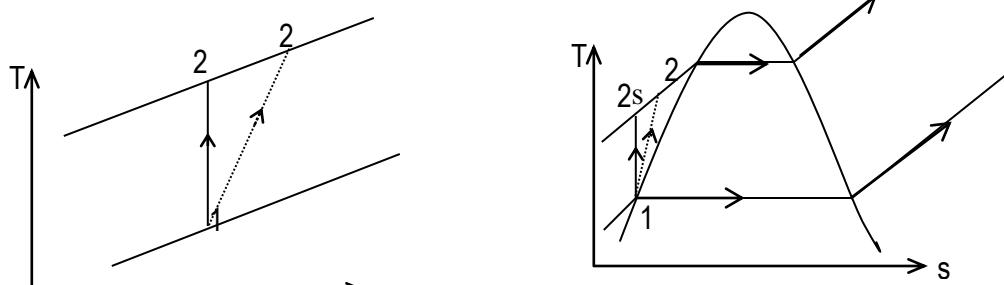


Figure 14: Isentropic and Actual processes of a compressor

1-2.2 CARNOT CYCLE

It consists of two reversible isothermal processes at T_H and T_L respectively, connected by two reversible adiabatic (isentropic) processes. When the working fluid is a condensable vapour the isothermal processes are obtained by heating and cooling at constant pressure while the fluid is a wet vapour.

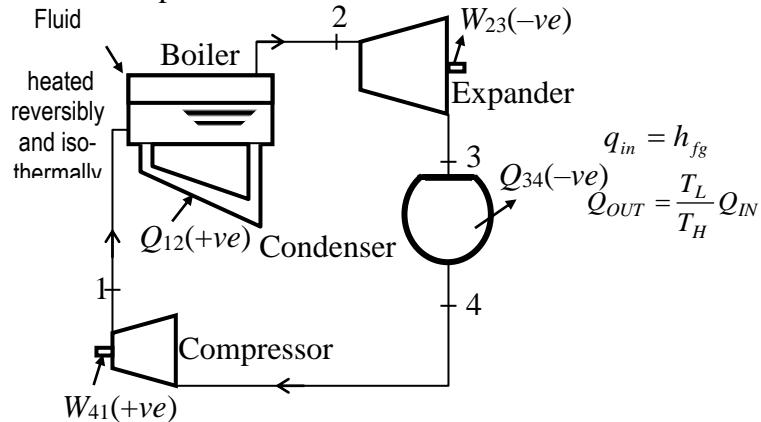


Figure 15: Flow diagram of the Carnot Cycle

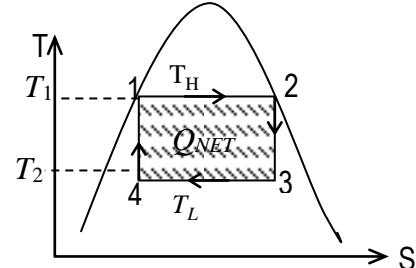


Figure 16: T-s diagram of the Carnot Cycle

Note For a Cycle, from 1st Law

$$Q_{NET} + W_{NET} = 0$$

$$Q_{NET} = Q_{IN} + Q_{OUT}$$

$$Q_{IN} = Q_{12} = T_1(S_2 - S_1)$$

$$Q_{OUT} = Q_{34} = -T_2(S_3 - S_4), \quad Q_{NET} = (T_H - T_L)(S_2 - S_1)$$

$$\boxed{\eta_{cycle} = \frac{|W_{NET}|}{Q_{IN}}} \quad (2.6)$$

1-2.2.1 Analysis of the processes of the Carnot Cycle

1-2: Saturated liquid in state 1 is evaporated in a boiler at constant pressure to form saturated steam in state 2.

Using the steady-state open flow energy equation,

$$Q_{12} \pm W_{12} = h_2 - h_1 + \Delta(KE) + \Delta(PE), \quad \text{but } \Delta(KE) = 0, \Delta(PE) = 0 \text{ and } W_{12} = 0$$

Hence,

$$\boxed{Q_{12} = h_2 - h_1}$$

$h_2 = h_g$ at the pressure of the boiler and
 $h_1 = h_f$ at the pressure of the boiler

(2.7)

2-3: Steam is expanded isentropically to state 3 while doing work in a turbine ($s_2 = s_3$). Note $s_2 = s_g$ and it is evaluated at the pressure of the boiler. Since the entropy at state 3 lies

between the s_f and s_g values corresponding to the condenser pressure, s_3 is evaluated as follows : $s_3 = s_{f,3} + x_3 s_{fg,3}$

where, x_3 is the dryness fraction of the working fluid at state 3. $s_{f,3}$ and $s_{fg,3}$ values are read at the pressure corresponding to the condenser pressure at state point 3.

Using the steady-state open flow energy equation, we obtain

$$\pm W_{23} = h_3 - h_2 \quad (2.8)$$

Activity: Explain concept of Dryness fraction, Quality and Moisture content to students.

Quality of steam decreases during the expansion process thus, the turbine handles steam with low quality (i.e. steam with high moisture content). The impingement of liquid droplets on the turbine blades causes erosion and is a major source of wear. $x \geq 90\%$ is desirable in practice.

$$x = \frac{m_g}{m_t} \quad \text{where } m_t = m_f + m_g$$

$$V = V_f + V_g = m_f v_f + m_g v_g$$

$$m_t v_{av} = m_f v_f + m_g v_g$$

$$\begin{aligned} v_{av} &= (1-x)v_f + xv_g \\ &= v_f + xv_{fg} \end{aligned}$$

3-4: After expansion the wet steam is then partially condensed at constant pressure and constant temperature while heat is rejected.

$$Q_{34} = h_4 - h_3 \quad (2.9)$$

$$s_1 = s_{f,4} + x_4 s_{fg,4}$$

Note: $s_1 = s_4$ and $s_{f,4}$, $s_{fg,4}$ are read at the pressure corresponding to the condenser at state point 4. Condensation is stopped at state 4.

4-1: Wet steam is compressed isentropically in a rotary or reciprocating compressor to state 1 that is the boiler pressure and temperature, the work required being:

$$\pm W_{41} = h_1 - h_4 \quad (2.10)$$

$s_1 = s_f$ and is read at the pressure corresponding to that of the boiler pressure.

$$\text{Thermal efficiency: } \eta_{th, carnot} = -\frac{W_{NET}}{Q_{IN}} = -\frac{W_{23} + W_{41}}{Q_{12}} = \frac{Q_{12} + Q_{34}}{Q_{12}} = 1 - \frac{T_L}{T_H} \quad (2.11)$$

The Carnot Cycle is **not** a realistic model for steam power plants because:

1. Limiting the heat transfer processes to two-phase systems to maintain isothermal conditions severely limits the maximum temperature that can be used in the cycle.
2. The turbine would have to handle steam with a high moisture content which causes erosion of turbine blades and
3. It is not practical to design a compressor that will handle a two-phase fluid.

1-2.3 THE BASIC RANKINE CYCLE [BASIC POWER CYCLE]

Although the Carnot cycle is the most efficient cycle, its work ratio is low. Further, there are practical difficulties in following it. Consider the Carnot cycle of Figure 16: at state 4, the steam is wet at T_L but it is difficult to stop the condensation process at the point 4 and then compress it to state 1. It is difficult to compress wet mixtures since the liquid tends to separate out from the vapour and the compressor would have to deal with a non-homogeneous mixture. It is practically convenient to allow the condensation process to proceed to completion, as in Figure 18. The working fluid is water at the new state point 4, and this can be conveniently pumped to boiler pressure as shown at state point 5. The pump has much smaller dimensions than it would have if it had to pump a wet vapour, the compression process is carried out more efficiently, and the equipment required is simpler and less expensive.



Thus, heat must be supplied to change the state from compressed water at 5 to saturated water at 1 and this is a constant pressure process, but it is not at constant temperature. If we let T_{eff} be the effective temperature at which heat is added or supplied at some constant pressure, then its value can be evaluated from the relation $Q_{52} = T_{eff}\Delta S$ where $\Delta S = S_2 - S_5$ and $S_5 = S_4$

With this new cycle, we realise that at state point 5; the water is not at the saturation temperature corresponding to the boiler pressure. . The effective temperature at which heat is added is lower than the corresponding Carnot temperature. Hence, the efficiency of the modified cycle is not as high as that of the Carnot cycle. But the net work output in the modified cycle is greater than that of the Carnot cycle. It follows that the SSC is less and the work ratio is greater.

Activity: Explain how T_{eff} is determined with an example in the lecture room to students.

This resulting ideal cycle, which is more suitable as a criterion for steam cycles rather than the Carnot cycle, is called the ideal *Basic Rankine cycle* or Basic Rankine Power Cycle.

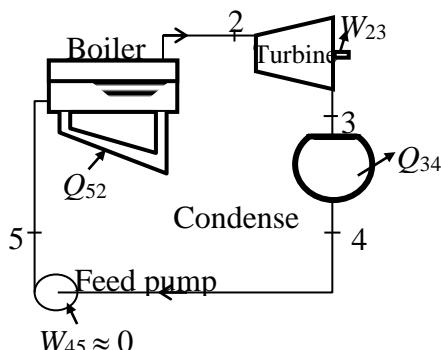


Figure 17: Flow diagram of the Basic Rankine Cycle

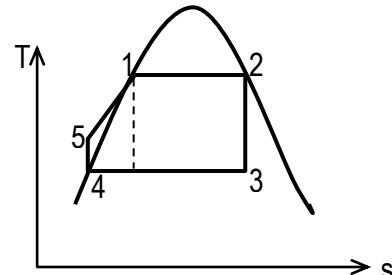


Figure 18: T-s diagram of the Basic Rankine Cycle

NB: The ideal basic Rankine Power Cycle consists of two constant pressure processes connected by two reversible adiabatic processes(i.e. isentropic processes).

Feed pump work:

For a reversible process: $dq = du - dw$ (1st Law) (a)

But $dw = -pdv$ and $h = u + pv$ (b)

$$dh = du + pdv + vdp \quad (c)$$

Substituting equations (b) and (c) into (a) we arrive at

$$dq = dh - vdp \quad (d)$$

For isentropic process, $dq = 0$, hence $dh = vdp$

$$\text{Thus } w_{45} = h_5 - h_4 = v_{4s}(P_5 - P_4) \quad (2.12)$$

where v_{4s} can be taken from tables for water at the pressure $P_4 = P_3$ (condenser pressure)

1-2.3.1 Effect of irreversibilities on the Basic ideal Rankine Cycle

In general, for a steady state, for a process say, 1-2, $\dot{m}(s_2 - s_1) - \frac{d\dot{Q}}{T} \geq 0$. But for an adiabatic process $dQ = 0$, hence $s_2 \geq s_1$ (increase of entropy principle). Thus for an irreversible process $s_2 > s_1$ and the irreversibility in process 1-2 is accounted for by introducing the process efficiency.

Consider the $T-s$ diagram of a Rankine cycle with superheat (which will be explained in a later section) shown in Figure 19. The actual compression and expansion processes are irreversible and are indicated by lines 1-2a and 3-4a, respectively.

$$\text{Turbine isentropic efficiency, } \eta_t = \frac{h_{4a} - h_3}{h_{4s} - h_3} \quad \text{and}$$

$$\text{Compression isentropic efficiency, } \eta_c = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

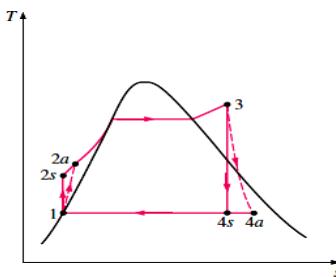


Figure 19: Effect of turbine and pump irreversibilities

1-2.3.2 Condenser Heat Load

This is the rate of heat removal from the condenser, per unit power output is given by the product of SSC and the heat removed in the condenser by the cooling water per unit mass of steam.

$$\text{Condenser heat load per kW of power output} = \text{SSC} \times |q_{out}| \quad (2.13)$$

1-2.3.3 Options for improving the efficiency of the Basic Rankine Cycle

The basic idea behind all the modifications to increase the thermal efficiency of a power cycle is the same: *Increase the average temperature at which heat is transferred to the working fluid in the boiler, or decrease the average temperature at which heat is rejected from the working fluid in the condenser.*

Three ways of increasing the efficiency of the Basic Rankine Cycle are discussed below:

(a) Lowering the condenser Pressure (Lowers $T_{low, av}$)

Steam exists as saturated mixture in the condenser at the saturation pressure corresponding to the pressure inside the condenser.

Therefore, lowering the condenser pressure lowers the temperature of the steam, and thus temperature at which heat is rejected.

The effect of lowering the condenser pressure is illustrated in Figure 20. For purposes of comparison, the inlet state temperature of the turbine is maintained the same. The net work increases. The heat input requirement increases (represented by the area under 2-2'), but this is small compared to the increase in the net work output (i.e. shaded area). Thus, *the overall effect of lowering the condenser pressure is an increase in the thermal efficiency of the cycle.*

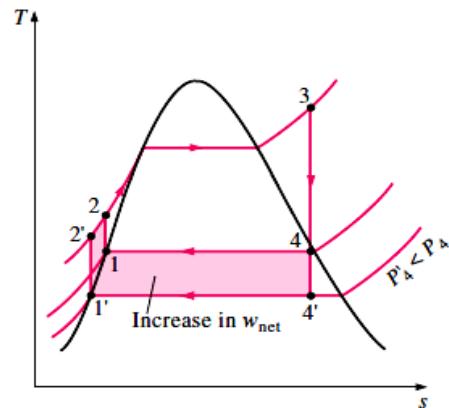


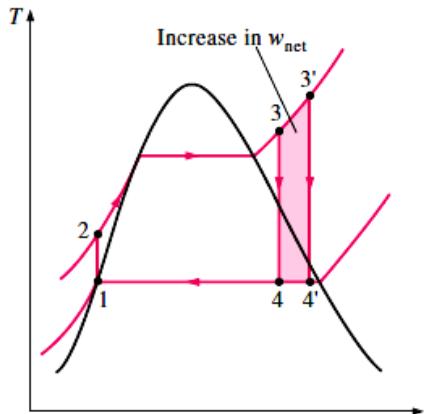
Figure 20: Effect of lowering condenser pressure on the Basic Rankine cycle performance

The condensers of steam power plants operate well below the atmospheric pressure. But there is a limit on the condenser pressure that can be used. It cannot be lower than the saturation pressure corresponding to the temperature of the cooling medium. For example, for a condenser that is cooled by nearby water at say 15 °C allowing for a temperature difference of 10 °C for effective heat transfer, the steam temperature in the condenser must be above 25 °C. From steam tables, the condenser pressure must be above 3.2 kPa (i.e. the corresponding pressure at 25 °C).

The side effects of lowering the condenser pressure option are:

- It creates the possibility of air leakage into the condenser
- It increases the moisture content of steam at the final stages of the turbine, which is undesirable in turbines because it decreases the turbine efficiency and results in corrosion of the turbine blades.

(b) Superheating the Steam to High Temperatures (increases $T_{high, av}$)



The average temperature at which heat is added to the steam can be increased without increasing the boiler pressure by superheating the steam to high temperatures. The effect of superheating on the performance of a vapour power cycle is illustrated in Figure 21.

We recognise an increase in the net work. The area under curve 3-3' represents the increase in heat input. Thus, superheating the steam increases both the net work and the heat input. The overall effect is an increase in cycle efficiency since the average temperature at which heat is added is increased.

Figure 21: Effect of superheating the steam to high temperatures

Other advantage of superheating:

- It decreases the moisture content of the steam at the turbine exit

Limitation of superheating:

- The temperature to which steam is superheated is limited by metallurgical limit of material used. $T = 620^{\circ}\text{C}$ is about the present optimum.

(c) Increasing Boiler Pressure (Increases $T_{high, av}$)

Another option of increasing the average temperature during the heat-addition process is to increase the operating pressure of the boiler, which automatically increases the temperature at which boiling takes places. This, in turn, raises the average temperature at which heat is added and thus raises the thermal efficiency.

The effect of increasing the boiler pressure is illustrated in Figure 22. We note that for a fixed turbine inlet temperature, the moisture content of steam at the turbine exit increases. This undesirable effect is corrected with reheating.

Today, modern power plants operate at supercritical pressures ($P > 22.1 \text{ MPa}$) and have thermal efficiencies of 40 % for fossil-fuel plants and 34 % for nuclear plants. The lower thermal efficiency values for nuclear plants are due to the fact that lower maximum temperatures values are used in those plants for safety reasons. This will lead to another improvement by reconsidering reheating cycles.

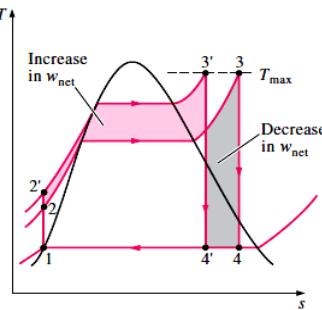


Figure 22: Effect of increasing the boiler pressure on the Basic Rankine Cycle performance

1-2.3.4 The Basic Rankine Cycle with Superheat

The metallurgical limit of the boiler material is not approached when the steam leaves the boiler in a saturated condition ($T_c \approx 374^\circ\text{C}$ but $T_{metallurgical} \approx 600^\circ\text{C}$). The quality of steam at turbine exit in the simple Rankine cycle is too low. The isentropic efficiency of turbines is affected by the wetness of steam. (Note that wet steam corrodes the turbine blades). In practice, we require a minimum quality of 0.9 at the turbine exit.

But by placing in the combustion gases a separate bank of tubes (the superheater) leading saturated steam away from the boiler, it is possible to raise the steam temperature without raising the boiler pressure. The resulting cycle with this modification is a *Basic Rankine cycle with superheat*, as shown on the T-s diagram in Figure 24.

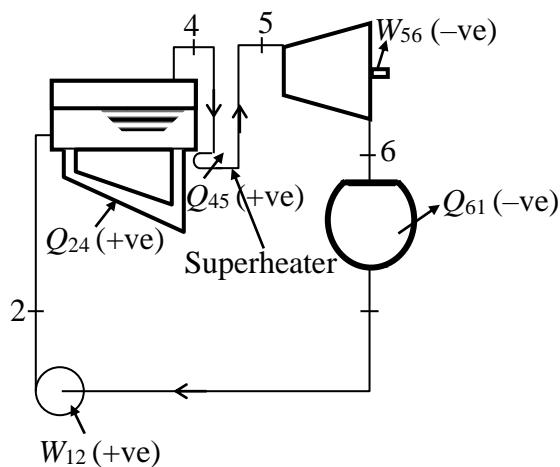


Figure 23: The Basic Rankine cycle with superheat

Thus, the net work output and heat input increase but overall effect is an increase in cycle efficiency.

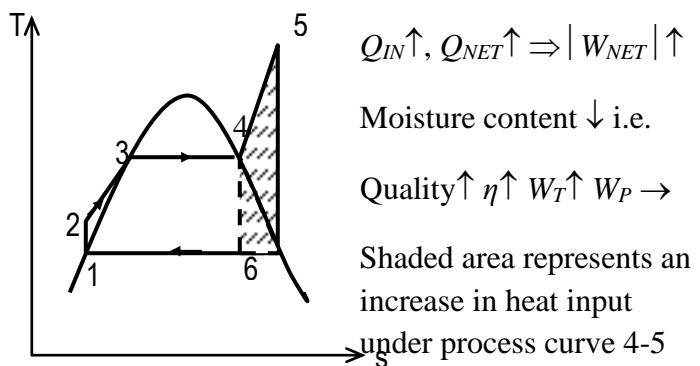


Figure 24: T-s diagram of the Basic Rankine cycle with superheat

The superheater is a heat exchanger in which heat is transferred to the saturated steam to increase its temperature. It may have its own source of heat or share with the boiler. If it is to share with the boiler, the bank is situated such that hot gases from the furnace heat it until the steam reaches the required temperature. Normally, they have smaller bores than the actual boiler tubes.

Comments:

- The effective temperature at which heat is added externally is increased. Hence efficiency of cycle increases. Unlike the efficiency of a simple Rankine cycle, the efficiency of the Rankine cycle with superheat increases continuously with pressure.
- The work ratio does not change since the work ratio in unsuperheated Rankine cycle is very near unity.
- The specific steam consumption is markedly reduced, the net work per unit mass of steam being much greater, so that the added complexity of the superheater is compensated by a reduction in the size of the other components.
- The dryness of the steam at the last stages of the steam turbine is increased. However, the required quality of steam at the turbine exit is not attained.

1-2.3.5 Reheat Cycle (Ideal Reheat Cycle)

It is desirable to increase the average temperature at which heat is supplied to the steam, and also to keep the steam as dry as possible at the lower pressure stages of the turbine. The wetness of steam at the turbine exit should be no greater than 10 %.

Higher boiler pressures are required for high efficiency, but expansion in one stage can result in exhaust steam which is wet. This condition is somehow improved by superheating but it is further improved by re-heating the steam, the expansion being carried out in two stages. With the reheat cycle, the expansion takes place in two turbines. The steam expands in the high-pressure turbine to some intermediate pressure, and is then passed back to yet another bank of tubes in the boiler where it is reheated at constant pressure, usually to the original superheat temperature. It then expands in the low pressure turbine to the condenser pressure. The T-s diagram for the Reheat cycle appears as in Figure 26.

Two options to increase the quality of steam at turbine exit:

- 1) Superheat steam to very high temperatures. OK, but not viable solely, since it will require raising the steam to metallurgical unsafe limits.
- 2) Expand steam in two stages, and reheat in between. This is a practical solution to reduce the wetness of steam at the turbine exit.

The larger the number of reheat, the larger the average temperature of the heat addition process. The use of more than two reheat stages is however not desirable or practical. Double reheating is only used on supercritical ($P > 22.1 \text{ MPa}$) power plants.

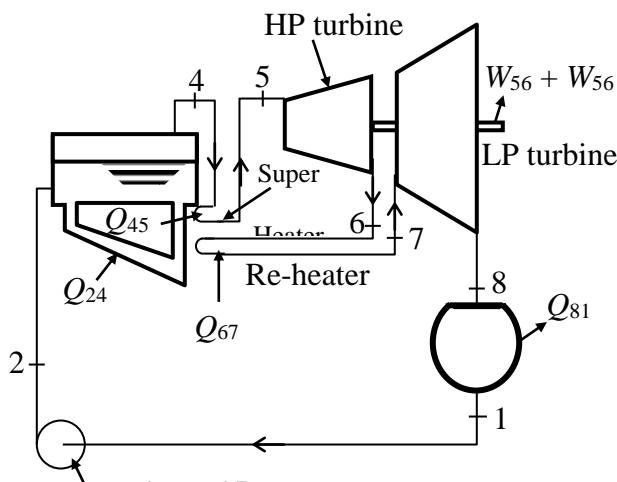


Figure 25: The Ideal Reheat cycle

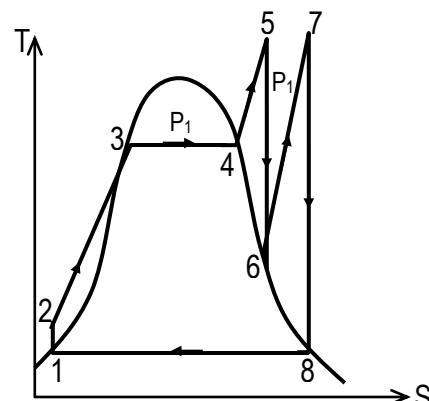


Figure 26: T-s diagram of the Ideal Reheat cycle

Processes 1-2, 2-4, 4-5 and 8-1 have already been described in previous Rankine Cycles shown.

5-6: Isentropic expansion in the high-pressure turbine to an intermediate pressure (P_i).

6-7: Heating at constant pressure (intermediate pressure, P_i) in the re-heater.

7-8: Isentropic expansion in the low pressure turbine.

Heat supplied = $Q_{245} + Q_{67}$

Gross work output = $W_{56} + W_{78}$

Also, $Q_{67} = h_7 - h_6$

Cycle efficiency,

$$\eta = \frac{W_{56} + W_{78} + W_{12}}{Q_{245} + Q_{67}} = \frac{(h_6 - h_5) + (h_8 - h_7) + (h_2 - h_1)}{(h_5 - h_2) + (h_7 - h_6)}$$

If the feed pump work is negligible, $h_2 \approx h_1$ and

$$\therefore Q_{245} = h_5 - h_1$$

Having the Rankine Cycle as the ideal cycle for vapour power cycles, the efficiency ratio of a power cycle is defined as:

$$\text{Efficiencyratio} = \frac{\text{cycleefficiency}}{\text{Rankine cycleefficiency}} \quad (2.14)$$

Boiler Efficiency:

The boiler efficiency is the heat supplied to the steam in the boiler expressed as a percentage of the chemical energy of the fuel which is available on combustion. That is;

$$\text{Boilerefficiency} = \frac{\dot{m}_s \times [h_5 - (\text{enthalpy of the feed water})]}{\dot{m}_f \times (\text{GCV or NCV})} \quad (2.15)$$

where h_5 is the enthalpy of the steam entering the turbine and \dot{m}_f the mass of fuel burned per unit time and \dot{m}_s the mass flow rate of steam. The GCV (Gross Calorific Value) and NCV (Net Calorific Value) are the higher and lower calorific values of the fuel. The size of a boiler, or its capacity, is quoted as the rate in kilogram per hour at which the steam is generated.

Comments:

Reheating reduces the steam consumption appreciably, because the area of the cycle on the T-s diagram, which equals the network done per unit mass of steam, increases. In high-pressure cycles, this implies a smaller boiler (expensive item in a high-pressure plant). This decrease in size goes with added complexity. Note that the effective average temperature at which the heat is added may be lower than that in the Rankine cycle if the intermediate pressure is too low. The use of re-heating gives drier steam at the turbine exit thus reducing blade erosion in the latter stages of the turbine. The cost of reheating, however, adds to the capital cost of the plant.

The sole purpose of reheating is to reduce the moisture content of steam at the turbine exit. Therefore, if we had materials that could withstand high temperatures there would be no need for reheating; and superheating alone would be an appropriate solution. In Reheating, the optimum reheating pressure is about $\frac{1}{4} \times P_{\text{maximum}}$ in the cycle. (i.e. if $P_{\text{max}} = 12 \text{ MPa}$, the optimum the reheating pressure must be 3 MPa)

1-2.4 REGENERATIVE CYCLE

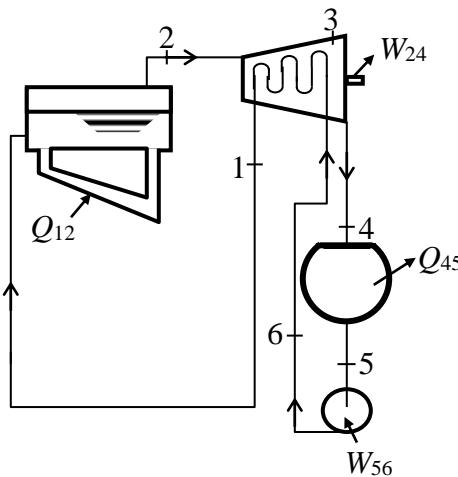


Figure 27: A hypothetical regenerative cycle

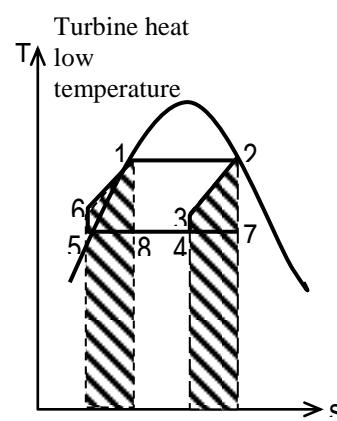


Figure 28: Corresponding T-s diagram

Heat is added while the working fluid is at a relatively low temperature, thereby lowering the temperature at which heat is added. The **Remedy is to:** Look at the ways of raising the temperature of the feed water entering the boiler.

Two immediate options available are:

- Compress feed water isentropically using system as in Carnot cycle.
- Transfer heat to feed water from the expanding steam in a counter flow heat exchanger built into the turbine (regeneration). This second option is impractical since steam at the turbine exit will be too wet i.e. below the practical 90% quality or higher values required of steam power plants in practice.

Practical means is to extract or “bleed” steam from the turbine at some points for feed-water heating. An open feed-water heater is basically a mixing chamber. Ideally, the mixture leaves the heater as a saturated liquid at the heater pressure.

- The efficiency of the un-superheated Rankine cycle (Figure 18) is less because the heat supplied is transferred while the temperature of the working fluid varies from T_5 to T_1 . If we could pre-heat the feed water such that it enters the boiler at the saturation temperature corresponding to the boiler pressure, then the efficiencies of the Rankine and Carnot Cycles would be identical.
- One method, which is theoretically possible, as indicated in Figure 28, is to pass the feed water through the turbine casing. If we arrange for counter flow, the heat transfer will be reversible. If the areas 23472 and 18561 are equal, then the Rankine cycle, 234561, and the Carnot cycle, 27812, are equivalent.

Impracticability of the above system:

- a) It is impossible to design a turbine, which would operate efficiently as both a turbine and a heat exchanger.
- b) The quality of steam at turbine exit or exhaust would be too low.

The commonly used method for increasing the thermal efficiency of vapour power plants is regenerative feed water heating or simply regeneration. The simplest approximation to a practical regenerative cycle is shown on the T-s diagram in Figure 29(b). When superheating is incorporated the T-s diagram is as shown in figure 29(c).

1-2.4.1 Regenerative Cycle Using One Open Feed Water Heater

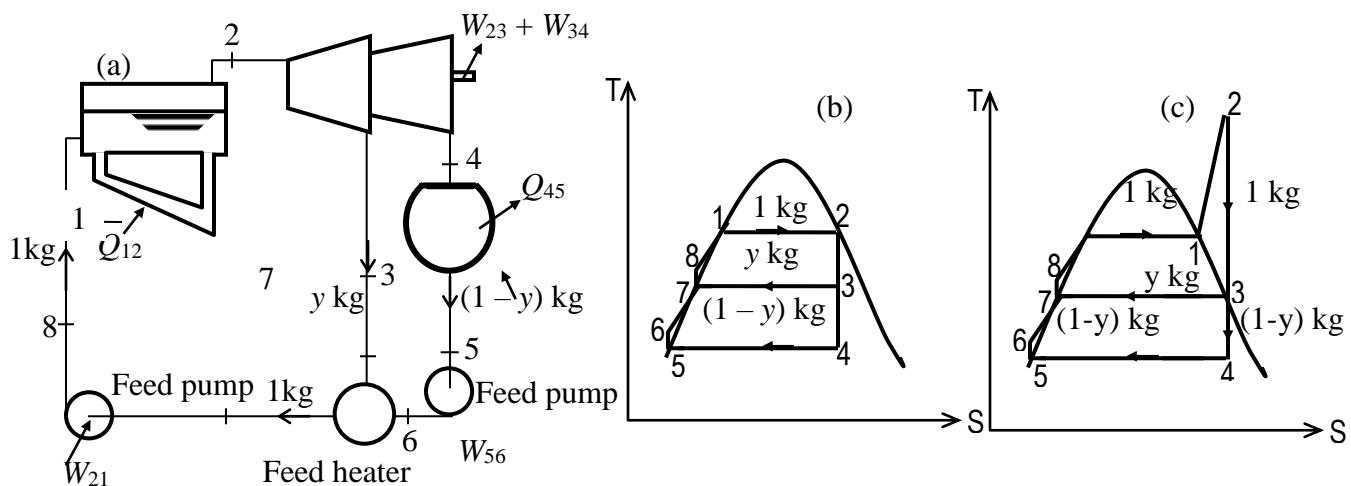


Figure 29: A simple regenerative cycle using one open feed water heater

Analysis

Consider 1 kg of steam through the plant, since it is more convenient to do so.

$$\text{Note: } y = \frac{\dot{m}_3}{\dot{m}_2}$$

2 - 3: 1 kg of steam expands to an intermediate pressure P_3 in a high-pressure turbine. A fraction, y , of the total flow is extracted or bled off at 3, into the open feed water heater operating at pressure, P_3 .

3: y kg extracted for feed heating.

3 - 4: $(1 - y)$ kg of steam continues expansion in the second-stage turbine to condenser pressure, P_4 .

4 - 5: $(1 - y)$ kg of steam, condensed to saturated liquid, state 5.

5 - 6: $(1 - y)$ kg liquid pumped by feed pump 1 to the extraction pressure, P_3 , and introduced into the feed water heater at state 6.

6 - 7: $(1 - y)$ kg in state 6, is reversibly and adiabatically, mixed with bled steam, y kg from state 3, to obtain saturated liquid at state 7.

7 - 8: Liquid (1 kg) at state 7 is pumped by pump 2 to the boiler pressure P_8 , and it enters the steam generator (boiler) at state 8.

8-1-2: working fluid is heated from state 8 to state 2 in the boiler

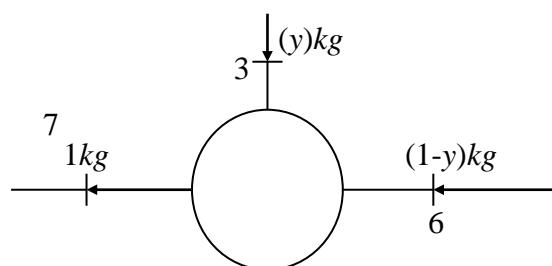
Notes:

- i) Average temperature at which heat is added is higher and therefore efficiency is higher
- ii) The mixing process is however irreversible. If the number of bled points is increased, this situation will improve.
- iii) For maximum cycle efficiency, T_3 should be approximately halfway between T_2 and T_4 . That is, $T_3 = \frac{T_2 + T_4}{2}$. Hence at this temperature the corresponding pressure P_7 is read, from saturation tables.

Calculation of the mass of steam bled off

Regeneration not only increases cycle efficiency but

- a) Provides a convenient measure of de-aerating the feed water to prevent corrosion in the boiler
- b) Helps control the large volume flow rate of steam at the final stages of the turbine (due to large specific volumes at low pressures)



FEED WATER HEATER

The feed water heater is a heat exchanger where heat is transferred from the steam to the feed water either by mixing the two fluid streams (open feed water heaters) or without mixing them (closed feed water heaters)

Figure 30: Energy balance on one open feed water heater

Note: \dot{m}_1 = total mass of steam flowing through the boiler in a unit time

Assume an adiabatic mixing process. From the 1st Law for an open system (with negligible $\Delta K.E$ and $\Delta P.E$) $Q + W = \sum h_{out} - \sum h_{in}$

$$0 = (1 \times h_7) - (y \times h_3) - (1 - y)h_6 \Rightarrow yh_3 + (1 - y)h_6 = h_7$$

$$y = \frac{h_7 - h_6}{h_3 - h_6} \quad (2.16)$$

Calculating work and heat transfers:

Computing turbine work output per unit mass of steam passing through the turbines

$$\frac{\dot{W}_t}{\dot{m}_1} = w_{23} + w_{34} = 1(h_3 - h_2) + (1 - y)(h_4 - h_3) \quad (2.17)$$

Total pump work per unit mass passing through the feed water

$$\frac{\dot{W}_p}{\dot{m}_1} = w_{56} + w_{78} = (1-y)(h_6 - h_5) + l(h_8 - h_7) \quad (2.18)$$

Energy added to working fluid passing through the boiler

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = q_{82} = h_2 - h_8 \quad (2.19)$$

If we employ a super-heater then the heat supplied in the boiler only (see Figure 39) is $Q_{81} = h_1 - h_8$ and heat supplied super-heater only is $Q_{12} = h_2 - h_1$

Energy rejected by heat transfer to the cooling water

$$\frac{\dot{Q}_{out}}{\dot{m}_1} = q_{45} = (1-y)(h_5 - h_4) \quad (2.20)$$

General comments:

- i) In practice, the best return for the additional capital cost is obtained with several feed water heaters. As many as 6 or 7 open feed water heaters have been used on large power plants.
- ii) The open feed water is not often used in large numbers because an equally large number of feed pumps are required. Instead the closed type is preferred. Open heaters for n bled points; will require $(n + 1)$ feed pumps.

1-2.4.2 Regenerative Cycle with One Closed Feed Water Heater

Closed feed water heaters are shell-and-tube-type recuperators in which the feed water temperature increases as the extracted steam condenses on the outside of the tubes carrying the feed water. Since the two streams do not mix, they can be at different pressures.

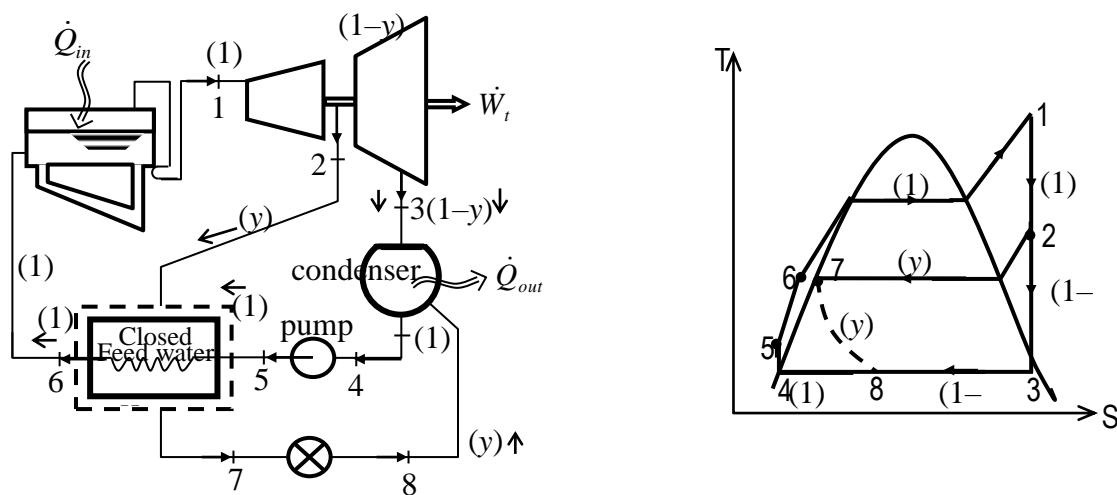


Figure 31: A simple regenerative cycle using one closed feed water heater

Consider 1 kg of steam in state 1 as shown in Fig. 31

- 1-2: Expansion through first-stage. A fraction of the flow, y kg, is bled off into the closed feed water heater where it condenses. Saturated liquid at the extraction pressure exits the feed water heater at state 7. The condensate is then trapped into the condenser, where it is re-united with the portion of the total flow passing through the second stage turbine.
- 2-3: $(1 - y)$ kg of steam goes to the low pressure turbine
- 3-4: Condensation of steam to saturated liquid at state 4
- 7-8: Expansion from state 7 to state 8 through the trap is irreversible, hence we use the dashed line.

The total flow exiting the condenser as saturated liquid at state 4 is pumped to the boiler pressure and enters the feed water heater at state 5. The temperature of the feed water is increased in passing through the feed water heater. The feed water then exits at state 6.

- 6 - 1: The working fluid is heated in boiler and superheater at constant pressure from state 6 to state 1.

Comments:

- i) Apply the conservation of mass and energy principles to the closed feed water heater. Assume no heat transfer between feed water heater and its surroundings, K.E and P.E effects neglected, we have

$$0 = yh_7 + (1 \times h_6) - yh_2 - (1 \times h_5) \Rightarrow y = \frac{h_6 - h_5}{h_2 - h_7} \quad (2.21)$$

- ii) Gross work output

$$\frac{\dot{W}_t}{\dot{m}_1} = w_{12} + w_{23} = (h_2 - h_1) + (1 - y)(h_3 - h_2) \quad (2.22)$$

- iii) Net pump work

$$\frac{\dot{W}_P}{\dot{m}_1} = w_{45} = v_{f_4}(P_5 - P_4) = 1 \times (h_5 - h_4) \quad (2.23)$$

- iv) Heat rejected in the condenser

$$\frac{\dot{Q}_{out}}{\dot{m}_1} = q_{34} = (1 \times h_4) - yh_8 - (1 - y)h_3 \quad (2.24)$$

- v) Heat supplied in the boiler and superheater

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = q_{61} = 1 \times (h_1 - h_6) \quad (2.25)$$

- vi) Cycle efficiency

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = -\frac{\dot{W}_{12} + \dot{W}_{23} + \dot{W}_{45}}{\dot{Q}_{61}} \quad (2.26)$$

1-2.4.5 Back-pressure and Extraction turbines

A turbine exhausting into a condenser at a relatively high pressure, in which the rejected heat is employed usefully, is called a back-pressure turbine as shown in Figure 32. The use of a back pressure turbine is practical if the power and heat requirements are fairly steady and well matched.

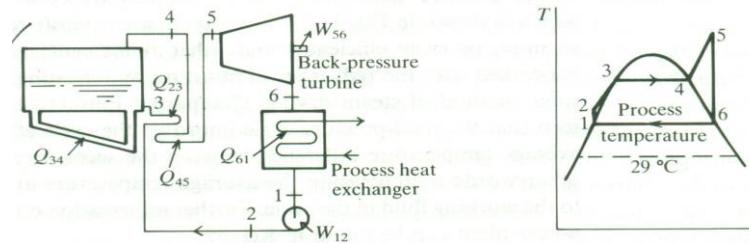


Figure 32: Back pressure turbine schematic with corresponding T-s diagram

When the power requirements are larger than those that can be obtained from a simple back-pressure turbine, an extraction or pass-out turbine can be used. In this alternative scheme, some of the steam is expanded to the lowest possible temperature of about 29 °C, the remainder being extracted from the turbine at the correct intermediate temperature (Figure 33). The extracted steam is condensed in the process heat exchanger and finally returned to the boiler. This scheme enables variations in power and heat requirements to be at least partly accommodated by controlling the amount of steam extracted.

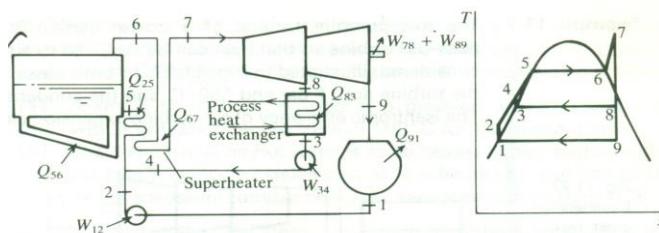


Figure 33: Extraction turbine schematic with corresponding T-s diagram



When the ratio of the power produced (P_E) to that of the heat required for process heating (P) is greater than 10, thus $\frac{P_E}{P} > 10$, then the back-pressure turbine is used. But when the ratio of the power produced (P_E) to that of the heat required for process heating (P) ranges between 4 and 10, thus $4 < \frac{P_E}{P} < 10$, then the extraction or pass-out turbine is used. When the ratio of the power produced (P_E) to that of the heat required for process heating (P) is less than 4, thus $\frac{P_E}{P} < 4$, use separate heating and power production scheme.

1-2.4.6 Cogeneration

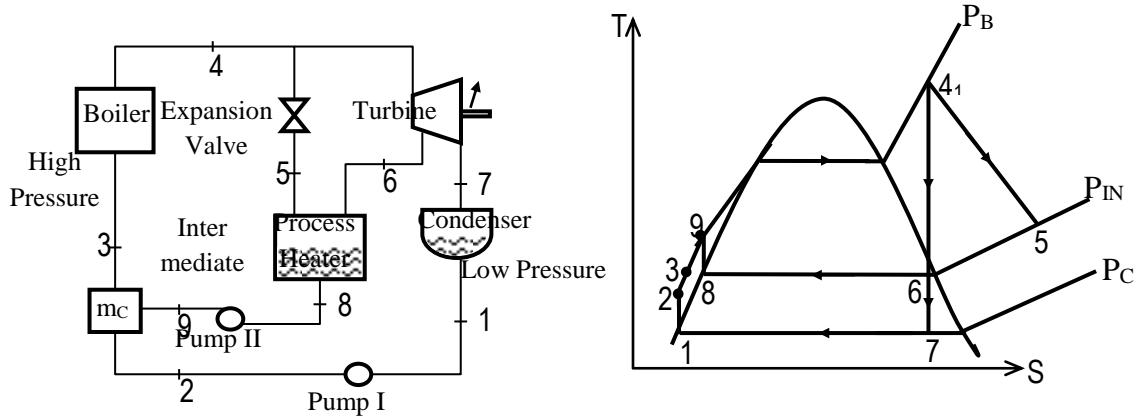


Figure 34: Schematic of a cogeneration plant with T-s diagram

In all cycles discussed so far, the sole purpose was to convert a portion of the heat transferred to the working fluid to work, which is the most valuable form of energy. The remaining portion of the heat is rejected to rivers, lakes, oceans or the atmosphere as waste heat. Wasting a large amount of heat is a price we have to pay to produce work. Many systems or devices, however, require energy input in the form of heat, called process heat. A plant that produces electricity while meeting the process-heat requirements of certain plants is called a co-generation plant.

Note: A Power plant as shown in figure 34 which is used for the production of more than one useful form of energy from the same energy source is called a co-generation plant. The utilisation factor is a measure of performance of co-generation plants and is defined in Equation (2.27) which takes account of the rate of process heat requirement and the rate steam turbine work output in relation to the rate of heat energy supplied to the plant.

$$\dot{Q}_{in} = \dot{m}_3(h_4 - h_3)$$

$$\dot{Q}_{out} = \dot{m}_7(h_1 - h_7)$$

$$\dot{W}_{pump} = \dot{m}_8(h_9 - h_8) + \dot{m}_1(h_2 - h_1)$$

$$\dot{W}_{turbine} = (\dot{m}_4 - \dot{m}_5)(h_6 - h_4) + \dot{m}_7(h_7 - h_6)$$

$$\text{Utilisation factor } \xi = \frac{|\dot{W}_{ST}| + |\dot{Q}_{process}|}{\dot{Q}_{IN}} \quad \xi = 1 - \frac{|\dot{Q}_{out}|}{\dot{Q}_{in}} \quad (2.27)$$

1-2.4.7 Further Considerations in Improving the Efficiency of a Steam Power Plant

Hitherto, considerations of efficiency have been based on optimising the use of the heat, which is actually supplied to the steam, and not the heat, which has been produced by the combustion of

the fuel in the boiler. The heat is transferred to the steam from gases, which are at a higher temperature (approx. 2000 K) than the steam, and the exhaust gases pass to the atmosphere at a high temperature.

Energy of the flue gases can be utilised to preheat the feed water before entry into the boiler. The external heat supplied is reduced thus increasing the thermal efficiency of the system.

The Economiser:

To utilise some of the energy in the flue gases, an *economiser* can be fitted (see Figure 35a). This is a heat exchanger placed in the flue gases, which extracts useful energy in the form of heat to preheat the feed water before entry to the boiler. (For the Carnot, ideal regenerative and complete feed heating cycles, no use can be made of an economiser since the feed water enters the boiler at the saturation temperature corresponding to the boiler pressure).

The air pre-heater:

To cool the flue gases even further and improve the plant efficiency, the air, which is required for the combustion of the fuel, can be pre-heated (see Figure 35b). For a given temperature of the combustion gases, the higher the initial temperature of the air then the less will be the energy input required, and hence less fuel will be used.

One practical consequence of cooling the combustion gases is that there is a pressure drop in the chimney and fans may be required to achieve the necessary forced draught for the flue gases.

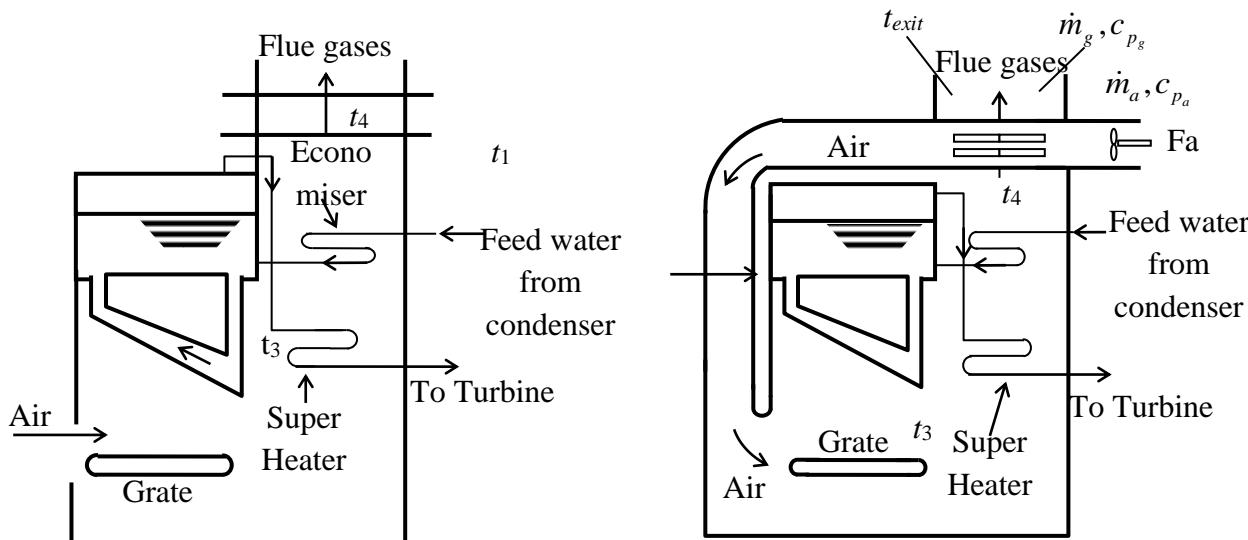


Figure 35: Diagrammatic sketch of a boiler plant with (a) an economizer and (b) an air preheater

Analysis of the Air pre-heater and economiser

Let us assume that the temperature of the feed water is raised from t_1 to t_2 in the economiser and that the flue gases are also cooled from t_3 to t_4 .

For air pre-heater:

$$\dot{m}_a \times c_{pa} \Delta t = c_{pg} \times \dot{m}_g (t_4 - t_{exit}) \quad (2.28)$$

Economiser and super-heater:

$$\dot{m}_g c_{pg} (t_3 - t_4) = \dot{m}_s (h_2 - h_1) + \dot{m}_s c_{ps} (t_{exit} - t_{entry}) \quad (2.29)$$

Where h_2 is evaluated at t_2 ; and t_{exit} and t_{entry} refer to temperature of the steam at exit and entry of the super-heater respectively. Subscript s refers to steam and subscript g refers to the flue gases.



Self Assessment 1-2

1. A steam plant operates on a simple ideal Rankine cycle between the pressure limits of 3 MPa and 50 kPa. The temperature of the steam at the turbine inlet is 400 °C, and the mass flow rate of steam through the cycle is 25 kg/s. Show the cycle on a T - s diagram with respect to saturation lines, and determine
 - (a) the thermal efficiency of the cycle and
 - (b) the net power output of the power plant
2. Consider a steam power plant that operates on a reheat Rankine cycle and has a net power output of 150 MW. Steam enters the high-pressure turbine at 10 MPa and 500 °C and the low-pressure turbine at 1 MPa and 500 °C. Steam leaves the condenser as a saturated liquid at a pressure of 10 kPa. The adiabatic efficiency of the turbine is 80 %, and that of the pump is 95%. Show the cycle on a T - s diagram with respect to saturation lines, and determine
 - (a) the quality (or temperature, if superheated) of steam at the turbine exit,
 - (b) the thermal efficiency of the cycle, and
 - (c) the mass flow rate of steam.
3. A steam power plant operates on an ideal reheat-regenerative Rankine cycle and has a net power output of 120 MW. Steam enters the high-pressure turbine at 10 MPa and 550 °C and leaves at 0.8 MPa. Some steam is extracted at this pressure to heat the feed-water in an open feed-water heater. The rest of the steam is reheated to 500 °C and is expanded in the low-pressure turbine to the condenser pressure of 10 kPa. Show the cycle on a T - s diagram with respect to saturation lines, and determine
 - (a) the mass flow rate of steam through the boiler and
 - (b) the thermal efficiency of the cycle.
4. How do the following quantities change when a simple ideal Rankine cycle is modified with reheating? Assume the mass flow rate is maintained the same.

Pump work input:	(a) increases	(b) decreases	(c) remains the same
Turbine work output:	(a) increases	(b) decreases	(c) remains the same
Heat supplied:	(a) increases	(b) decreases	(c) remains the same
Heat rejected:	(a) increases	(b) decreases	(c) remains the same
Moisture content at turbine exit:	(a) increases	(b) decreases	(c) remains the same



Suggested Answers

1. [28.4 %; 20.5 MW]
2. [87.5 °C; 34.1 %; 117.5 kg/s]
3. [81.9 kg/s; 44.4 %]
4. [(c); (a); (a); (a); (b)]

SESSION 2-2: GAS TURBINE CYCLES (BRAYTON CYCLE)

The gas turbine may have a closed or an open cycle as shown in Figures 36 and 37, respectively.

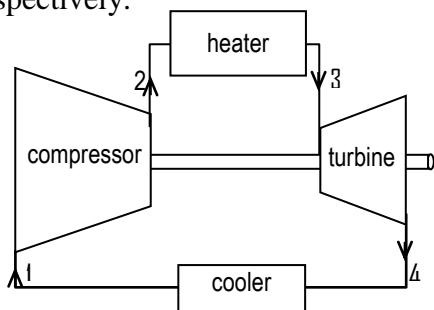


Figure 36: closed gas turbine cycle

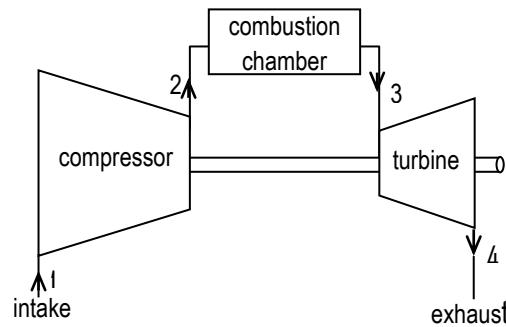


Figure 37: open gas turbine cycle

In both cases, heat is received and rejected at constant pressure, so that the theoretical cycle involves constant pressure heat addition and heat rejection processes. This is called the Joule or Brayton cycle.

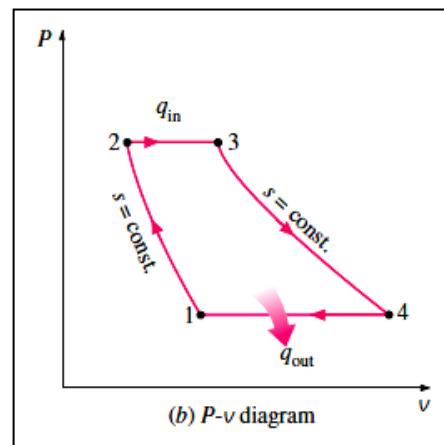
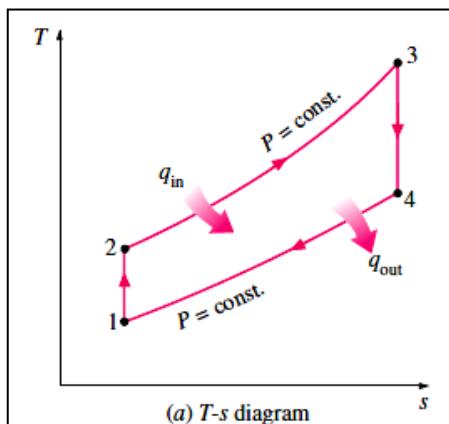


Figure 38: Ideal Brayton cycle with (a) T-s diagram and (b) P-v diagram

The heat transfers in the Brayton cycle as illustrated in Figures 38a and 38b are

$$\dot{Q}_{23} = \dot{m}c_p(T_3 - T_2) \quad \dot{Q}_{41} = \dot{m}c_p(T_1 - T_4) \quad (2.30)$$

Ideal cycle efficiency

$$\eta_{th} = \frac{|W|}{Q_{in}} = \frac{Q_{net}}{Q_{in}} = \frac{Q_{23} + Q_{41}}{Q_{23}} = 1 + \frac{Q_{41}}{Q_{23}} \quad (2.31)$$

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{T_1}{T_2} \left(\frac{\frac{T_4}{T_1} - 1}{\frac{T_3}{T_2} - 1} \right)$$

$$\text{Define } p_2/p_1 = r_p = p_3/p_4 \quad \text{Also } T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{1}{\gamma}} \quad \text{and} \quad T_4 = T_3 \left(\frac{p_4}{p_3} \right)^{\frac{1}{\gamma}}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{i.e.} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2} \quad \therefore \frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1$$

$$\boxed{\therefore \eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{r_p^{\frac{1}{\gamma}}} = 1 - r_p^{-\frac{1}{\gamma}}} \quad (2.32)$$

Work ratio

$$\boxed{r_w = \frac{W_{12} + W_{34}}{W_{34}} = \frac{h_2 - h_1 + h_4 - h_3}{h_4 - h_3} = 1 - \frac{T_1}{T_4} = 1 - \left(\frac{T_1}{T_3} \right) \left[r_p^{\frac{1}{\gamma}} \right]} \quad (2.33)$$

Comments

1. The inherent characteristics of high speed and high power output promote the use of gas turbines in a wide variety of applications (Aircraft engines).
2. They are used in stationary power generation.
3. Favourable power output-to-weight ratio makes it suited for transportation applications.
4. Lighter and more compact than vapour power plants (used for aircraft engine).
5. Increasing the pressure ratio increases the efficiency. However, for application in vehicles where engine weight must be kept small it is necessary to keep the compression ratio that yields the most net work per unit mass and NOT the pressure ratio for greatest efficiency. For this purpose $\frac{d(W/\dot{m})}{d(r_p)} = 0$.

Note:

$$\frac{|W|}{\dot{m}} = c_p [(T_4 - T_3) + (T_2 - T_1)] = c_p T_1 \left(\frac{T_4}{T_1} \frac{T_3}{T_2} - \frac{T_3}{T_1} + \frac{T_2}{T_1} - 1 \right)$$

$$c_p T_1 \left[\frac{T_3}{T_1} \left(\frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}} - \frac{T_3}{T_1} + \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$\frac{d(W/\dot{m})}{d(r_p)} = 0 \Rightarrow r_p = \left(\frac{T_3}{T_1} \right)^{\frac{\gamma}{2(\gamma-1)}}$$

Hence, for lightweight of the engine the optimal efficiency is given by

$$\eta_{th, optimal} = 1 - \left[\left(\frac{T_3}{T_1} \right)^{\frac{\gamma}{2(\gamma-1)}} \right]^{\frac{1-\gamma}{\gamma}} = 1 - \left(\frac{T_1}{T_3} \right)^{\frac{1}{2}} \quad (2.34)$$

There is a limit of about 1700 K imposed by metallurgical considerations on the maximum allowed temperature at the turbine inlet (T_3). If the gas turbine is operating in an environment with $T_1 = 298 K$, then the optimum efficiency for a maximum net work per unit mass is given as 58.1%.

Irreversible compression and expansion: The cycle shown by Fig. 39(a) assumes isentropic compression and expansion. Actual turbines cannot achieve this and the effect is shown by Fig. 2.21, from 1-2s-3-4s-1 to 1-2a-3-4a-1.

Isentropic efficiencies: The ratio of the actual work to the corresponding isentropic work for a turbine is called its *isentropic efficiency*. For the compressor, it is defined as the ratio of the isentropic work to the actual work.

Since both operate under conditions of steady flow and kinetic energies at entry and exit may usually be neglected, the work done or required is, in all cases, equal to the enthalpy change. For a gas, this is proportional to the temperature change, so that

$$\text{Compressor isentropic efficiency } \eta_c = (T_{2s} - T_1) / (T_{2a} - T_1) \quad (2.35)$$

$$\text{Turbine isentropic efficiency } \eta_t = (T_3 - T_{4a}) / (T_3 - T_{4s}) \quad (2.36)$$

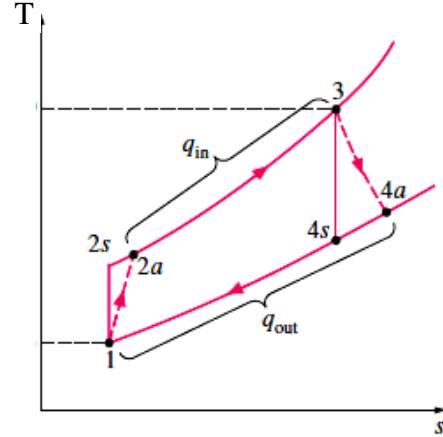


Figure 39: Irreversible compression and expansion

2-2.1 Air Standard Cycle for Jet Propulsion

In this cycle, the work done by the turbine is just sufficient to drive the compressor. The gases are expanded in the turbine to a pressure such that the turbine work output is equal to the compressor work input required. The exhaust pressure of the turbine will then be above that of the surroundings, and the gas can be expanded in a nozzle to the pressure of the surroundings.

Since the gases leave with a high velocity, the change in momentum the gas undergoes results in a thrust on the aircraft in which the engine is installed.

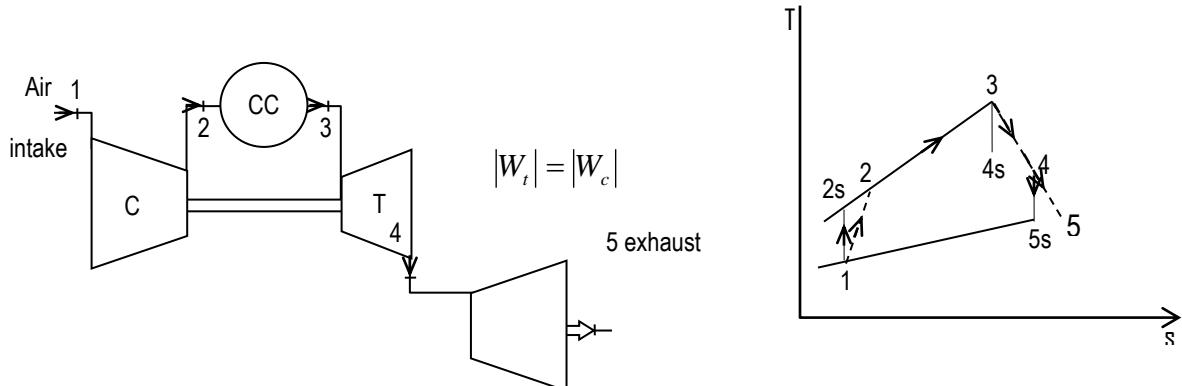


Figure 40: Simple jet engine cycle and corresponding T-s diagram

$$\text{From the first law, } h_4 = h_5 + V_5^2 / 2 \quad (2.37)$$

$$\text{But } dh = c_p dT \Rightarrow V_5^2 = 2c_p(T_4 - T_5) \quad (2.38)$$

The magnitude of the thrust developed is given by

$$F_N = \dot{m}_a(V_5 - V_a) \quad (2.39)$$

2-2.2 Improving the efficiency of the gas turbine cycle

a) Regeneration or the use of heat exchanger

In Figure 40, T_4 the exhaust temperature is greater than T_{2a} , the temperature after compression. Thus a heat exchanger could recover some of the heat rejected to the atmosphere, by passing the gases from the turbine through a heat exchanger, where heat transferred from the gases is used to heat the air leaving the compressor before it enters the combustion chamber, thus reducing the amount of heat to be supplied and increasing plant efficiency. **The effect of the use of a heat exchanger is to reduce the heat required from an external source.** The use of the heat exchanger is termed regeneration and the arrangement is shown in Figure 41.

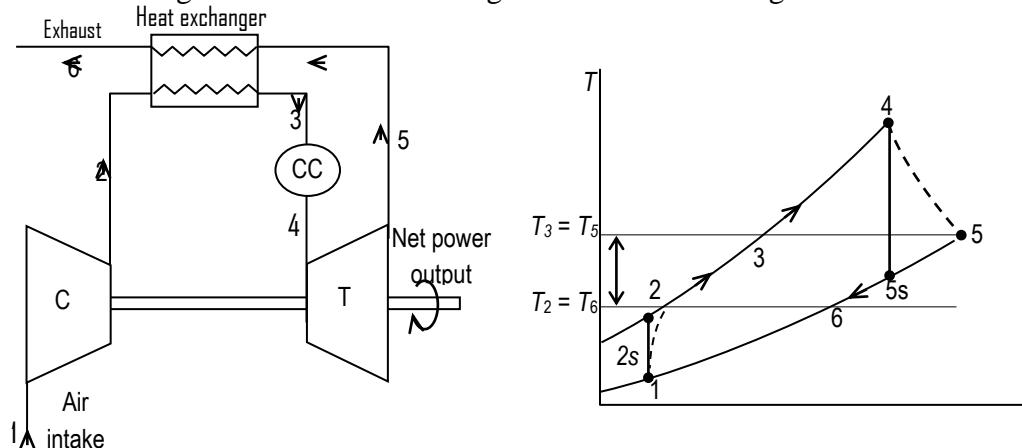


Figure 41: Gas turbine unit with heat exchanger and corresponding T-s diagram

The heat exchanger can only be used if there is a sufficiently large temperature difference between the gases leaving the turbine and the air leaving the compressor. ***This option is used to increase the efficiency of the gas-turbine but work ratio remains unaffected.***

b) *Inter-cooling and reheating*

Intercooling between compression stages is principally done to reduce the compressor work input.

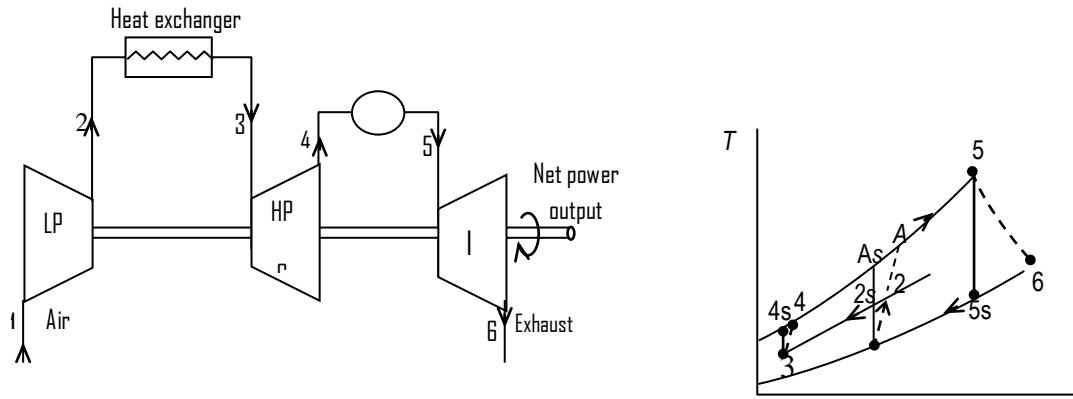


Figure 42: Gas turbine unit with inter-cooling and corresponding T-s diagram
Reheating between expansion stages is done to increase the turbine work output.

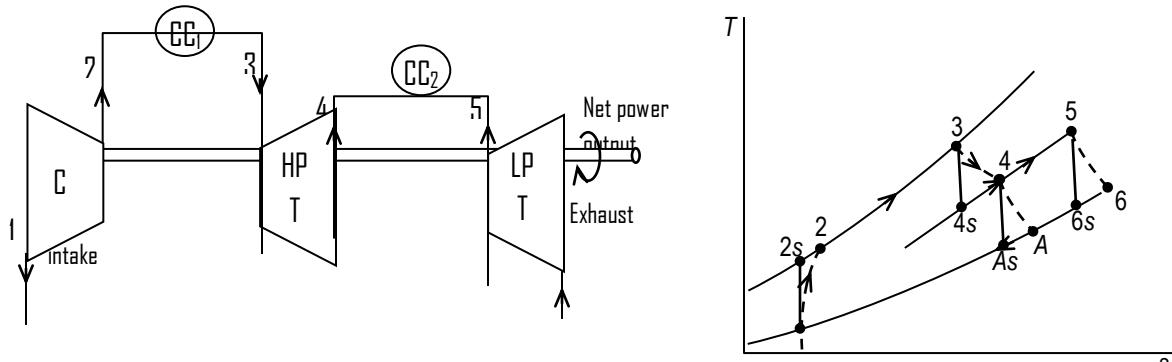


Figure 43: Gas turbine unit with reheating and corresponding T-s diagram

Intercooling however decreases the average temperature at which heat is added and reheating increases the average temperature at which heat is rejected.

Thus, in a gas turbine plant, inter-cooling and reheating will always decrease the thermal efficiency unless they are used in conjunction with regeneration.

NOTE: When an ideal Brayton Cycle is modified with several stages of compression and reheating with intercooling between stages in conjunction with regeneration of 100 % effectiveness, the cycle approaches the Brayton Cycle. ie $\eta_{th} = 1 - \frac{T_L}{T_H}$, irrespective of the pressure ratios in the cycle.

Example 2.1 Sample problem on the ideal Brayton cycle

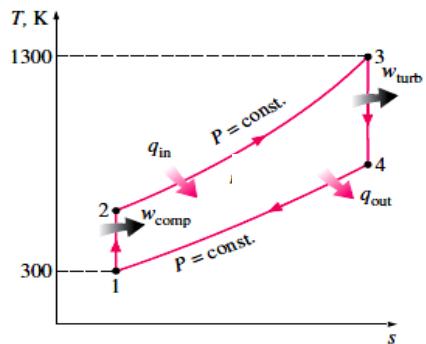
Consider a simple ideal Brayton cycle with air as the working fluid. The pressure ratio of the cycle is 6, and the minimum and maximum temperatures are 300 K and 1300 K, respectively. Now the pressure ratio is doubled without changing the minimum and maximum temperatures in the cycle. Determine the change in

- (a) The net work output per unit mass
- (b) The thermal efficiency of the cycle as a result of this modification

Use constant specific heats at room temperature

Solution:

Simple Ideal Brayton Cycle with air as the working fluid



Assumptions: (1) Air-standard assumptions applicable.
(2) Kinetic and potential energy changes are negligible.
(3) Air is an ideal gas with constant specific heats

Properties of air at room temperature are: $R = 0.287 \text{ kJ/kgK}$; $c_p = 1.005 \text{ kJ/kgK}$; $c_v = 0.718 \text{ kJ/kgK}$; $\gamma = 1.4$

Analysis: Processes 1-2 and 3-4 are isentropic processes

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = (300\text{K})(6)^{\frac{0.4}{1.4}} = 500.6\text{K}$$

also,

$$T_4 = T_3 \left(\frac{P_3}{P_4} \right)^{\frac{\gamma-1}{\gamma}} = (1300\text{K}) \left(\frac{1}{6} \right)^{\frac{0.4}{1.4}} = 779.1\text{K}$$

$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2) = 1.005\text{kJ/kgK}(1300 - 500.6)\text{K} = 803.4\text{kJ/kg}$$

$$|q_{out}| = h_4 - h_1 = c_p(T_4 - T_1) = 1.005\text{kJ/kgK}(779.1 - 300)\text{K} = 481.5\text{kJ/kg}$$

$$|w_{net}| = |q_{net}| = q_{in} - |q_{out}| = (803.4 - 481.5)\text{kJ/kg} = 321.9\text{kJ/kg}$$

$$\text{Thermal efficiency } \eta_{th} = \frac{|w_{net}|}{q_{in}} \times 100\% = \frac{321.9}{803.4} \times 100\% = 40.1\%$$

$$w_{comp} = h_2 - h_1 = c_p(T_2 - T_1) = 1.005\text{kJ/kgK}(500.6 - 300)\text{K} = 201.6\text{kJ/kg}$$

$$|w_{turb}| = h_3 - h_4 = c_p(T_3 - T_4) = 1.005\text{kJ/kgK}(1300 - 779.1)\text{K} = 523.5\text{kJ/kg}$$

$$\text{Back work ratio, } b_w = \frac{w_{comp}}{|w_{turb}|} = \frac{201.6}{523.5} = 0.385(38.5\%)$$

Considering the effect of doubling the pressure ratio for the cycle i.e. $r_p = 12$:

By similar analysis, T_2 and T_4 can be determined as 610.2 K and 639.2 K respectively.

$$\text{and } q_{in} = 1.005 \text{ kJ/kgK} (1300 - 610.2) \text{ K} = 693.2 \text{ kJ/kg}$$

$$|q_{out}| = 1.005 \text{ kJ/kgK} (639.2 - 300) \text{ K} = 340.9 \text{ kJ/kg}$$

$$|w_{net}| = (693.2 - 340.9) \text{ kJ/kgK} = 352.3 \text{ kJ/kg (increase)}$$

$$\eta_{th} = \frac{352.3}{693.2} \times 100\% = 50.8\% \text{ (increase)}$$

$$b_w = \frac{c_p (610.2 - 300)}{c_p (1300 - 639.2)} = 0.469(46.9\%) \text{ (increase)}$$

2-2.3 Combined Gas and Vapour Cycles

The continued quest for higher thermal efficiencies has resulted in rather innovative modifications to conventional power plants. The *binary vapour cycle* is one such modification. A more popular modification involves a gas power cycle topping a vapour power cycle, which is called the **combined gas-vapour cycle**, or just the **combined cycle**. The combined cycle of greatest interest is the gas-turbine (Brayton) cycle topping a steam-turbine (Rankine) cycle, which has a higher thermal efficiency than either of the cycles executed individually.

Gas-turbine cycles typically operate at considerably higher temperatures than steam cycles. The maximum fluid temperature at the turbine inlet is about 620 °C (1150 °F) for modern steam power plants, but over 1150 °C (2100 °F) for gas-turbine plants. It is over 1500 °C at the burner exit of turbojet engines. The use of higher temperatures in gas turbines is made possible by recent developments in cooling the turbine blades and coating the blades with high-temperature-resistant materials such as ceramics. Because of the higher average temperature at which heat is supplied, gas-turbine cycles have a greater potential for higher thermal efficiencies. However, the gas-turbine cycles have one inherent disadvantage: The gas leaves the gas turbine at very high temperatures (usually above 500 °C), which wipes out any potential gains in the thermal efficiency. The situation can be improved somewhat by using regeneration, but the improvement is limited. Consequently, the thermal efficiency of gas-turbine power plants, in general, is relatively low.

It makes engineering sense to take advantage of the very desirable characteristics of the gas-turbine cycle at high temperatures and to use the high temperature exhaust gases as the energy source for the bottoming cycle such as a steam power cycle. The result is a combined gas-steam cycle, as shown in Figure 44.

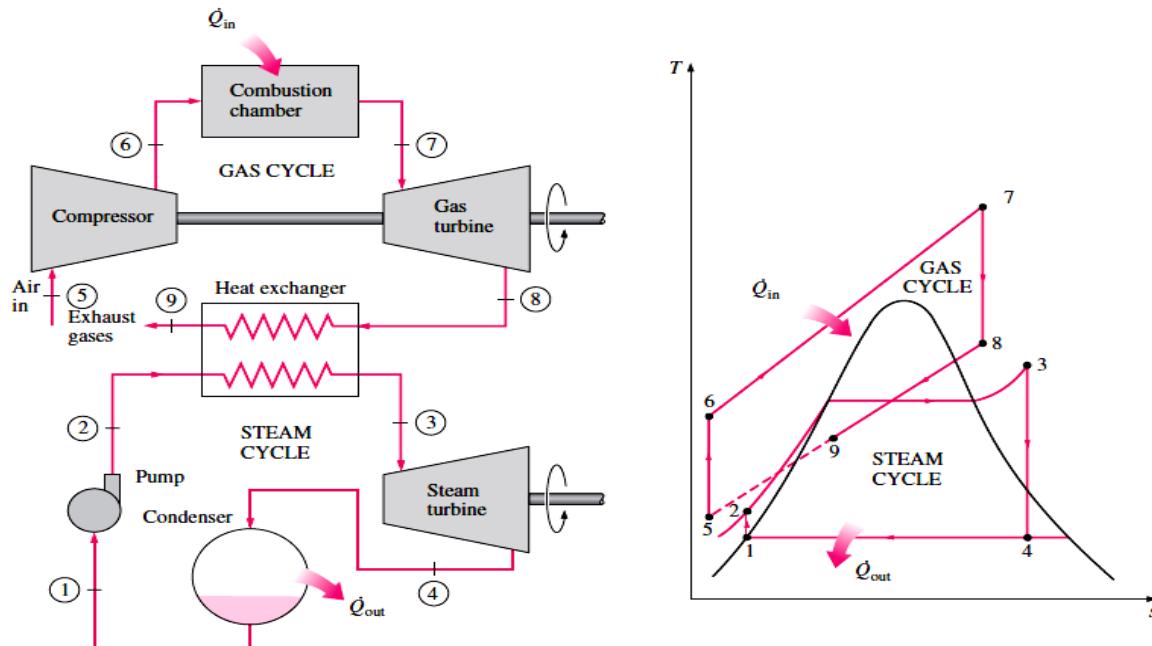


Figure 44: Combined gas-steam power plant with corresponding T-s diagram

In this cycle, energy is recovered from the exhaust gases by transferring it to the steam in a heat exchanger that serves as the boiler. In general, more than one gas turbine is needed to supply sufficient heat to the steam. Also, the steam cycle may involve regeneration as well as reheating. Energy for the reheating process can be supplied by burning some additional fuel in the oxygen-rich exhaust gases.

Recent developments in gas-turbine technology have made the combined gas-steam cycle economically very attractive. The combined cycle increases the efficiency without appreciably increasing the initial cost. Consequently, many new power plants operate on combined cycles, and many more existing steam- or gas-turbine plants are being converted to combined-cycle power plants. Thermal efficiencies well over 40 percent are reported as a result of conversion.

A 1090-MW Tohoku combined plant that was put in commercial operation in 1985 in Niigata, Japan, is reported to operate at a thermal efficiency of 44 percent. This plant has two 191-MW steam turbines and six 118-MW gas turbines. Hot combustion gases enter the gas turbines at 1154 °C, and steam enters the steam turbines at 500 °C. Steam is cooled in the condenser by cooling water at an average temperature of 15 °C. The compressors have a pressure ratio of 14, and the mass flow rate of air through the compressors is 443 kg/s.

A 1350-MW combined-cycle power plant built in Ambarli, Turkey, in 1988 by Siemens of Germany is the first commercially operating thermal plant in the world to attain an efficiency level as high as 52.5 percent at design operating conditions. This plant has six 150-MW gas turbines and three 173-MW steam turbines. Some recent combined-cycle power plants have achieved efficiencies above 60 percent.

(Source: Yunus A. Cengel, Michael A. Boles, *THERMODYNAMICS: An Engineering Approach* 5th Edition, page 583)

Example 2.2:

Show that the thermal efficiency of a combined gas-steam power plant η_{cc} can be expressed as:

$$\eta_{cc} = \eta_g + \eta_s - \eta_g \eta_s \quad (2.40)$$

Where $\eta_g = \frac{W_g}{Q_{in}}$ and $\eta_s = \frac{W_s}{Q_{g,out}}$ are the thermal efficiencies of the gas and steam cycles, respectively. Using this relation, determine the thermal efficiency of a combined power cycle that consists of a topping gas-turbine cycle with an efficiency of 40% and a bottoming steam-turbine cycle with an efficiency of 30%.

Solution:

Analysis:

$$\eta_{cc} = \frac{W_{total}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$\eta_g = \frac{W_g}{Q_{in}} = 1 - \frac{Q_{g,out}}{Q_{in}} \text{ and}$$

$$\eta_s = \frac{W_s}{Q_{g,out}} = 1 - \frac{Q_{out}}{Q_{g,out}}$$

where Q_{in} = heat supplied to the gas cycle

Q_{out} = heat rejected by the steam cycle

$Q_{g,out}$ = heat rejected from gas cycle and supplied to the steam cycle

$$\begin{aligned} \eta_g + \eta_s - \eta_g \eta_s &= \left(1 - \frac{Q_{g,out}}{Q_{in}}\right) + \left(1 - \frac{Q_{out}}{Q_{g,out}}\right) - \left(1 - \frac{Q_{g,out}}{Q_{in}}\right)\left(1 - \frac{Q_{out}}{Q_{g,out}}\right) \\ &= 1 - \frac{Q_{g,out}}{Q_{in}} + 1 - \frac{Q_{out}}{Q_{g,out}} - 1 + \frac{Q_{out}}{Q_{g,out}} + \frac{Q_{g,out}}{Q_{in}} - \frac{Q_{out}}{Q_{in}} \\ &= 1 - \frac{Q_{out}}{Q_{in}} = \eta_{cc} \end{aligned}$$

The proof is complete.

The proof assumes that the $Q_{g,out}$ for the topping gas-turbine cycle is exactly equal to the Q_{in} to the bottoming steam-turbine cycle such that the Q_{out} of the steam-turbine cycle is the net heat loss from the combined plant and Q_{in} into the topping gas-turbine cycle is the energy supply to the plant.

for $\eta_g = 0.40$ and $\eta_s = 0.30$

$$\eta_{cc} = \eta_g + \eta_s - \eta_g \eta_s = 0.40 + 0.30 - (0.40)(0.30) = 0.58$$

Example 2.3

It can be shown that the thermal efficiency of a combined gas-steam power plant η_{cc} can be expressed in terms of the thermal efficiencies of the gas- and steam-turbine cycles as

$$\eta_{cc} = \eta_g + \eta_s - \eta_g \eta_s$$

Prove that the value of η_{cc} is greater than either η_g or η_s . That is, the combined cycle is more efficient than either of the gas-turbine or steam turbine cycles alone.

Solution:

$$\begin{aligned}\text{Note: } \eta_g + \eta_s - \eta_g \eta_s &= \eta_g + \eta_s(1 - \eta_g) > \eta_g \quad \text{because } \eta_s(1 - \eta_g) \text{ is positive since } \eta_g < 1 \\ &= \eta_s + \eta_g(1 - \eta_s) > \eta_s \quad \text{because } \eta_g(1 - \eta_s) \text{ is positive since } \eta_s < 1\end{aligned}$$

Thus, we conclude that the combined gas-steam-turbine plant is more efficient than either the gas turbine or steam turbine cycles acting alone.

This relation proved is the ideal situation for a combined gas-steam power plant. The presence of irreversibilities in the cycle will make this ideal combined cycle efficiency η_{cc} unachievable. $\eta_{cc,actual}$ will be less than $\eta_{cc,ideal}$.

Let us now consider the analysis of some combined cycle worded problems with the ideal gas cycle as the topping cycle and the steam cycles as the bottoming cycles in all three cases.



Self Assessment 2-2

1. The gas turbine portion of a combined gas-steam power plant has a pressure ratio of 16. Air enters the compressor at 300 K at a rate of 14 kg/s and is heated to 1500 K in the combustion chamber. The combustion gases leaving the turbine are used to heat the steam to 400 °C at 10 MPa in a heat exchanger. The combustion gases leave the heat exchanger at 420 K. The steam leaving the turbine is condensed at 15 kPa. Assuming all the compression and expansion processes to be isentropic, determine:

- (a) The mass flow rate of the steam
- (b) The net power output
- (c) The thermal efficiency of the combined cycle.

For air, assume constant specific heats at room temperature.

Answers: (a) 1.275kg/s (b) 7818 kW (c) 66.3%

In questions 2 to 5 BELOW use the preamble to complete each of the statements that follow with either the *italicised* phrase: *decreases, increases or remains constant* to make the statement complete and true.

2. A simple ideal Brayton cycle with regeneration is modified to incorporate multistage compression with intercooling and multistage expansion with reheating without changing the pressure or temperature limits of the cycle. As a result of these two modifications:
 - a) The network output -----
 - b) The back work ratio-----
 - c) The thermal efficiency-----
 - d) The heat rejected-----
3. The single stage expansion process of an ideal Brayton Cycle without regeneration is replaced by a multistage expansion process with reheating between the same pressure limits. As a result of this modification:
 - a) The turbine work-----
 - b) The back work ratio-----
 - c) The thermal efficiency-----
4. The single stage compression process of an ideal Brayton cycle without regeneration is replaced by a multistage compression process with intercooling between the same pressure limits. As a result of this modification:
 - a) The compressor work-----
 - b) The back work ratio-----
 - c) The thermal efficiency-----
5. A simple ideal Brayton Cycle without regeneration is modified to incorporate multistage compression with intercooling and multistage expansion with reheating, without changing the pressure and temperature limits of the cycle. As a result of these modifications:
 - (a) The net work output-----
 - (b) The back work ratio-----
 - (c) The thermal efficiency-----
 - (d) The heat rejected -----.

REFRIGERATION CYCLES

Introduction

In the presentation of the contents of this unit, the description of refrigeration systems, thus systems that operate on reversed heat engines are introduced first. A number of modifications of the basic refrigeration cycle are considered and is followed by a discussion of household refrigeration systems and commercial refrigeration systems.



Learning Objectives

After studying this unit you should be able to:

1. Understand the concepts of refrigeration and heat pumps and their performance.
2. Analyse the ideal and actual mechanical vapour compression refrigeration cycles
3. Evaluate the performance of innovative vapour compression refrigeration cycles
4. Explain terms used in the study of refrigeration cycles.
5. Understand the concepts of absorption refrigeration systems.
6. Analyse household and commercial refrigeration systems
7. Introduce Gas Refrigeration System and discuss its performance.

SESSION 1-3: REFRIGERATION SYSTEMS

To lower the temperature of a body below that of its surroundings and then maintaining its temperature at this desired lower value, a continuous extraction of heat from the body is required. The lowering of the temperature of a body or space is accomplished by a mechanism called refrigeration. Thus, the purpose of a refrigerator is to transfer heat from a cold chamber which is at a temperature lower than that of its surroundings to a higher temperature region. A temperature gradient is thus established from the surroundings to the chamber and heat will flow naturally in this direction.

Every type of refrigeration system used for producing cold space must have the following four basic units:

- i) A Low temperature thermal sink from which the heat is extracted for cooling the space
- ii) Means of extracting the heat energy from the low temperature thermal sink and raising its temperature before delivering it to a heat receiver.
- iii) A receiver to which the heat is transferred from the high temperature, high pressure refrigerant.
- iv) A Means of reducing the pressure and temperature of the refrigerant before it returns to the sink.

In a power plant cycle, heat is received by the working fluid at a high temperature and rejected at a low temperature, while the fluid does a net amount of work. On the other hand, cycles can be conceived in which the reverse happens, heat supply being at a low temperature and rejected at a high temperature, while a net amount of work is done on the fluid. The latter are called heat pump or refrigeration cycles.

The term “heat pump” is usually applied to a machine whose principal purpose is to supply heat at an elevated temperature, and then the term “refrigeration” to one whose purpose is the extraction of heat from a cold space.

1-3.1 Reversed Heat Engine system operating on the Carnot cycle

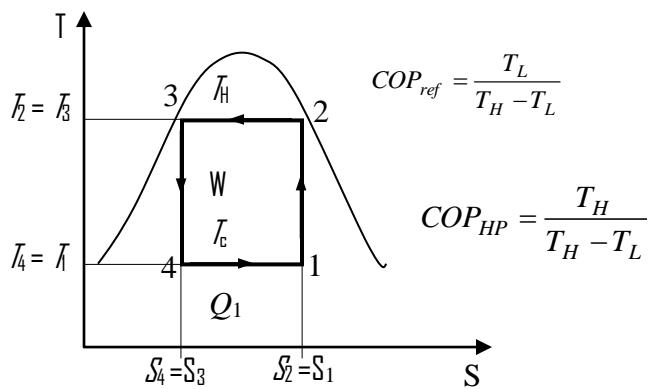


Figure 45: Layout of a revesed carnot heat engine with T-s diagram

The changes in the thermodynamic properties of the refrigerant throughout the cycle are indicated on the T-s diagram of Figure 45. The cycle events are as follows:

- 1-2: vapour at state 1 enters the compressor and is compressed isentropically from low pressure and temperature at state 1 to a higher pressure and temperature at state 2. The work input for this process is represented by W_{12} .
- 2-3: The vapour enters the condenser at state 2 and is condensed at constant pressure and temperature to state 3 where it is completely liquid. The heat rejected by the refrigerant is Q_{23}
- 3-4: The liquid expands isentropically to its original pressure at state 4. The work by the system being W_{34}
- 4-1: Evaporation at constant pressure and temperature to state 1. The main object is to extract as much heat during the evaporation process for a given expenditure of work.

Coefficient of performance

The criterion of performance of the cycle, expressed as the ratio output/input depends on what is regarded as the output. In a refrigerator, the object to is to extract the maximum amount of heat Q_{41} in the refrigerator for a net expenditure of work W . Therefore, the coefficient of performance of a refrigerator is defined by

$$C.O.P_{ref} = \frac{Q_{41}}{|W_{net}|} \quad (3.1)$$

In a heat pump, the object is to obtain maximum amount of heat Q_{23} from the condenser for the net expenditure of work W , and therefore, the coefficient of a heat pump is defined by

$$C.O.P_{hp} = -\frac{Q_{23}}{|W_{net}|} \quad (3.2)$$

The minus sign is introduced to make the *COP* a positive quantity since Q_{23} is negative.

The relationship between the two coefficients can be established by applying the first Law. Thus

$$Q_{41} + Q_{23} + W = 0 \quad \Rightarrow \quad \frac{Q_{41}}{|W_{net}|} + 1 = -\frac{Q_{23}}{|W_{net}|} \quad \text{and hence}$$

$$COP_{hp} = COP_{ref} + 1 \quad \Rightarrow \quad \begin{aligned} &\text{Heating effect of a heat pump is greater than} \\ &\text{the work input and this suggests that it would} \\ &\text{provide an effective heating.} \end{aligned} \quad (3.3)$$

The COP_{ref} may be greater than unity but the COP_{hp} is always greater than unity; hence, the reason why the term ‘efficiency’ is not employed.

$$COP_{ref} = \frac{Q_{41}}{|W_{net}|} = \frac{Q_{41}}{-Q_{41} - Q_{23}} = \frac{T_L(S_1 - S_4)}{-T_L(S_1 - S_4) - T_H(S_4 - S_1)} = \frac{T_L}{T_H - T_L} = \frac{1}{\frac{T_H}{T_L} - 1} \quad (3.4)$$

$$COP_{hp} = \frac{-Q_{23}}{W} = -\frac{Q_{23}}{-Q_{41} - Q_{23}} = -\frac{T_H(S_3 - S_2)}{-T_L(S_1 - S_4) - T_H(S_4 - S_1)} = \frac{T_H}{T_H - T_L} \quad (3.5)$$

Comments:

1. Equation (3.2) suggests that the COP_{hp} is the reciprocal of the efficiency of a power cycle.
2. The definitions, equations (3.4) and (3.5) show that both coefficients become smaller as the temperature difference of the cycle is increased.
3. Equation (3.3) indicates that ideally the heating effect of a heat pump is greater than the work input, and this suggest that it could serve as an effective heater. Since a heating effect can be obtained in a number of ways, which have a much lower capital cost than a heat pump system, a careful economic analysis is necessary before deciding whether a heat pump is financially viable.
4. A heat pump used for heating is much less effective when using the atmosphere as its low temperature heat source. The performance of a heat pump increases linearly with the temperature of the source of heat supply. It must be noted the heat loss from a building increases linearly as the ambient temperature increases. Hence a heat pump used for heating is more effective if the temperature of the low-temperature is a constant or approximately constant. This situation may require the low-temperature as ground water, or lake, or sea as the source.

1-3.2 Mechanical Vapour Compression Refrigeration Cycles

This is the most important system from the standpoint of commercial and domestic utility; it is the most practical form of refrigeration. The working fluid in this type of Refrigeration is a *refrigerant*. It readily vaporises and condenses or changes alternately between vapour and liquid phases without leaving the refrigerating plant and is compressed in the vapour state.

Practical consideration has led to several modifications to the reversed Carnot refrigeration cycle. These modifications are discussed in turn below:

1. Dispense with the turbine or the expansion cylinder and introduce a simple pressure reducing device - throttling valve or an expansion valve. Process 3-4 is now a constant enthalpy process. It is highly irreversible so that the whole cycle becomes irreversible. The capital cost of the plant is reduced. (The net-work required is now equal to the compressor work.) The heat transfer in the condenser is unaffected, but the heat extracted in the evaporator is diminished). Hence, the COP will be lower than the reversed Carnot cycle.

In addition, the flow rate of the working fluid is increased because the heat extracted per unit mass of the working fluid is reduced.

2. To make complete use of the specific enthalpy of vaporisation of the refrigerant in the evaporator it is desirable to continue the process until the vapour is dry saturated. In a

practical unit this process is extended to give the vapour a definite amount of superheat as it leaves the evaporator. This is really undesirable as the work to be done by the compressor is increased. It is, however, a practical necessity to allow the refrigerant to become superheated in this way in order to prevent the carry-over of liquid refrigerant into the compressor, where it tends to wash the lubricating oil. This is obviously undesirable if, the compressor is a reciprocating or the positive displacement rotary type with rubbing surfaces. Moreover, the oil might be carried to the evaporator where it can form a film over the tube surfaces and impair the heat transfer process. It is usual, therefore to transfer a slightly superheated vapour to the inlet of the compressor.

The condensed liquid is often sub-cooled before entering the throttle valve. Most of the heat transfer in the condenser must be rejected by virtue of an appreciable temperature difference, to keep the size of the condenser within reasonable limits. It is therefore possible to sub-cool the liquid after condensation to within a few degrees of the surrounding temperature. It must be noted that the temperature of the surrounding limits the amount of sub-cooling.

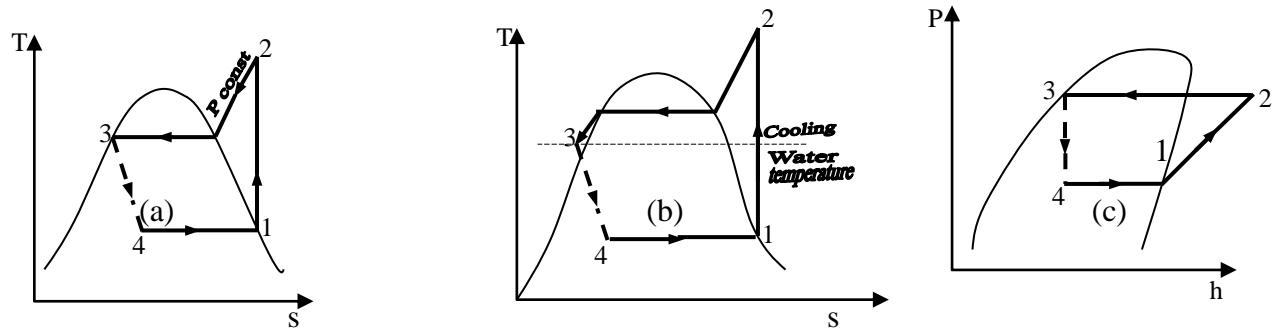


Figure 46: Modifications of a practical ideal refrigeration cycle with T-s and P-h diagrams showing superheating and sub cooling.

Figures 46a, b and c include all the modifications of this section, and this can be taken as showing the practical cycles.

The practical cycle consists of a number of flow processes and can be analysed by the application of the steady-flow energy equation,

$$h_1 + \frac{C_1^2}{2} + q + w = h_2 + \frac{C_2^2}{2} \quad (3.6)$$

If the changes in kinetic and potential energies are neglected, then

$$h_1 + q + w = h_2 \quad (3.7)$$

Applying this equation to each of the processes in Fig 46, we have for process 4-1

$$\begin{aligned} h_4 + q_{41} + 0 &= h_1 & \text{Therefore } q_{41} &= (h_1 - h_4) \\ \text{i.e.} & \quad \text{Refrigeration effect } q_{41} & &= (h_1 - h_4) \end{aligned} \quad (3.8)$$

$$\text{For process 1-2: } h_1 + 0 + w_{12} = h_2 \quad \text{therefore } w_{12} = h_2 - h_1$$

i.e. Work done on the refrigerant (Input work) $w_{12} = h_2 - h_1 \quad (3.9)$

Equation (3.9) applies to both reversible and irreversible compression between states 1 and 2, the only condition being that the process is adiabatic. If the process is reversible and adiabatic, then it is isentropic (i.e $S_1 = S_2$).

$$\text{For process 2-3: } h_2 + q_{23} + 0 = h_3 \quad \text{therefore } q_{23} = h_3 - h_2$$

i.e. Heat rejected by the refrigerant, $q_{23} = h_3 - h_2 \quad (3.10)$

1-3.2.1 Explanation of Terms

a. Refrigeration Load $\dot{Q}_{41} = \dot{m}(h_1 - h_4)$

Cooling capacity rate of removal of heat from the refrigerated space is often expressed in terms of refrigeration load.

The rate at which heat must be removed from the refrigerated space in order to produce and maintain the desired temperature condition is called the refrigeration load, the cooling load or the heat load. In most refrigeration applications, the total cooling load on the refrigerating equipment is the sum of the heat gains from several different sources as listed below:

- i) The heat transmitted by conduction through the insulated walls
- ii) The heat that must be removed from the warm air that enters through the opening and closing of the doors
- iii) The heat that must be removed from the refrigerated product to reduce the temperature of the product to the storage temperature
- iv) The heat given off by the people working in the space and by motors, lights, and other heat producing equipment operating in the space.

The America unit of refrigeration is called the ton and is defined as the rate of heat transfer of 200 Btu/min, based on the cooling rate required to produce 200 lb of ice at 32 °F from water at 32 °F in a time of 24 h. That is

$$1 \text{ ton of refrigeration} = 200 \text{ Btu/min} = 3.516 \text{ kW} = 211 \text{ kJ/min}$$

The Cooling load of a typical 200 m³ residence is the 3-ton (10-kW) range air –conditioner

b. Refrigerating Effect, (q_{41})

The refrigerating effect is the quantity of heat that each unit mass of refrigerant absorbs from the refrigerated space. When a liquid refrigerant vaporises as it flows through the evaporator it absorbs an amount of heat equal to that required to vaporise it, thus the refrigerating effect per unit mass of liquid refrigerant is potentially equal to its latent heat of vaporisation at the temperature of the refrigerant in the evaporator. This assumes that the entire mass of the liquid refrigerant will vaporise in the evaporator and produce useful cooling. However, in the actual cycle the temperature of the liquid entering the refrigerant control is always higher than the vaporising temperature in the evaporator and must first be cooled to the evaporator temperature

before the liquid can vaporise in the evaporator and absorb heat from the refrigerated space. For this reason, only part of each unit mass of the liquid refrigerant actually vaporises in the evaporator and produces useful cooling. Thus, the refrigerating effect per unit mass of liquid refrigerant is always less than the total latent heat of evaporation, h_{fg} .

c. Flow rate of refrigerant

The mass flow rate of the refrigerant is defined as the ratio of the refrigeration load to the refrigerating effect. That is

$$\dot{m} = \frac{\text{Refrigeration load}}{\text{Refrigerating effect}} \quad (3.11)$$

The volume flow rate of refrigerant drawn into the compressor is given by

$$\dot{V} = \dot{m}v \quad (3.12)$$

Where v is the specific volume of the refrigerant at entry to the compressor. If the compressor is the reciprocating type and has a volumetric efficiency η_v , then the swept volume is given by

$$V_s = \frac{\dot{V}}{fNn\eta_v} \quad \eta_v = \frac{\dot{V}}{nV_s fN} \quad (3.13)$$

Where n = the number of cylinders, N = the rational speed, for single-acting and double-acting machines $f = 1$ and 2 , respectively. For a reciprocating compressor, $V_s = \frac{\pi D^2 L}{4}$ where L is the stroke and D bore of the cylinder.

d. Refrigerating Agent

In any refrigerating process, the substance employed as the heat absorber or the cooling agent is called the refrigerant.

e. Cooking, Refrigeration, and Freezing of Foods: Their effect on micro-organisms

Micro-organisms are the prime cause for the spoilage of foods. Micro-organisms are bacteria, yeasts, moulds, and viruses. The undesirable changes caused by micro-organisms are off-flavours and colours, slime production, changes in texture and appearances, and the spoilage of foods. Cooking kills micro-organisms in foods, and prevents spoilage of foods. Refrigeration prevents or delays the spoilage of foods by reducing the rate of growth of micro-organisms. Freezing extends the storage life of foods for months by preventing the growth of micro-organisms.

Power requirement

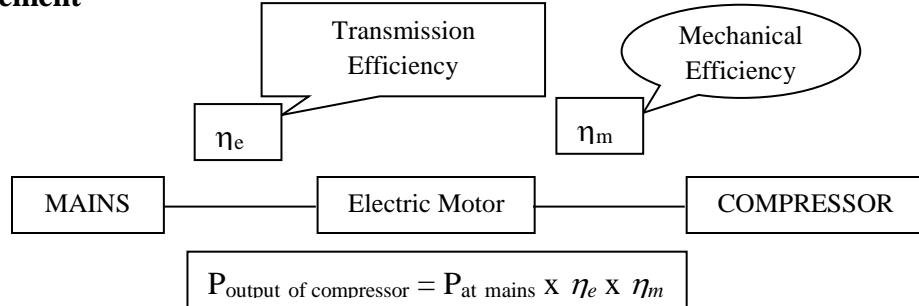


Figure 47: Analysing the Power output of a compressor

1-3.3 Innovative Vapour Refrigeration Systems

This section discusses the various modifications of the basic ideal mechanical vapour compression refrigeration systems.

1-3.3.1 The Use of Flash Chamber (Two-stage Compression with a Flash Chamber)

In practice, vaporisation does occur in the throttle valve resulting in the production of flash vapour, which merely passes through the evaporator without extracting any heat from the cold source (that is, the flash vapour makes no contribution to the refrigerating effect). However, work must be supplied to compress this vapour together with the ‘useful’ vapour through the full range of pressure. To address this problem, in a two-stage plant, the liquid is expanded to an intermediate pressure in the flash chamber, and the useless flash vapour is passed to the inlet of the second stage compressor. The liquid in the flash chamber is expanded in a second throttle valve to the final pressure and passes to the evaporator. By this arrangement less work is required, because the flash vapour removed at the intermediate pressure is only compressed over part of the pressure range of the cycle. For given amount of heat rejection, a reduction of the work required implies an increase in the heat extraction (First Law), and therefore the coefficient of performance is improved.

Figure 49(a) shows a diagrammatic arrangement of the plant, and Figure 49(b) shows the cycle plotted on a p-h diagram. Note that mass flow rate is not the same in all parts of the circuit. Let us consider 1 kg of refrigerant flowing through the condenser, and use the subscript I to denote the inter-stage and flash chamber conditions.

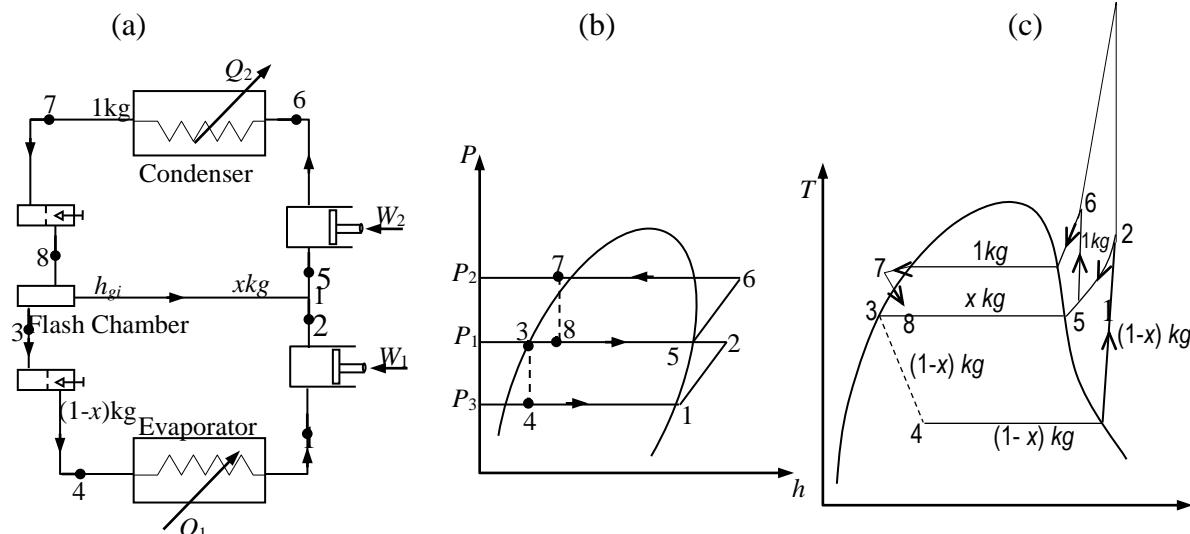


Figure 48: Two-stage compression refrigeration system with a flash chamber

At the flash chamber, x kg of dry saturated vapour at pressure p_i and enthalpy h_g is bled off to the inter-stage of the compressor. The remaining mass $(1-x)$ kg of liquid of enthalpy h_{f1} passes through the second throttle valve to the evaporator. In the first stage of compression $(1-x)$ kg of fluid is compressed from p_1 to p_i . At the inter-stage pressure, $(1-x)$ kg of fluid are mixed with x

kg of flash vapour of enthalpy, h_g . The resultant mixture at state 5 is compressed in the second stage compressor from p_i to p_6 at which it is delivered to the condenser. Note that, the actual amount of dry saturated vapour bled off is given by the dryness fraction at state 8.

The value of x is obtained by equating the enthalpy before throttling at 7 to the enthalpy after throttling at 8. That is,

$$h_{f_i} + xh_{fgi} = h_i \quad \text{or} \quad x = \frac{h_i - h_{f_i}}{h_{fgi}} \quad (3.14)$$

$$\text{The total work input} = w_{12} + w_{56} = (1-x)(h_2 - h_1) + (h_6 - h_5) \quad (3.15)$$

$$\text{Refrigerating effect, } q_{41} = (1-x)(h_1 - h_4) \quad (3.16)$$

$$\text{Heat rejected in the condenser, } q_{67} = h_7 - h_6 \quad (3.17)$$

$$\text{For the adiabatic mixing process: } 1(h_5) = xh_g + (1-x)h_2 \quad (3.18)$$

The condition at state 5 is determined from equation (3.18), if the intermediate pressure is known.

1-3.3.2 Compound Compression

Employing inter-cooling in multi-stage compression reduces power requirements. Inter-cooling can be economically achieved by allowing some of the refrigerant to physically bypass the evaporator and be used in flash type intercooler. The expansion valve employed can either be of the individual or the multi-expansion type. The two cases are illustrated below.

a) The arrangement for the use of individual expansion valves with compound compression and flash inter-cooling is shown in Figure 49 below:

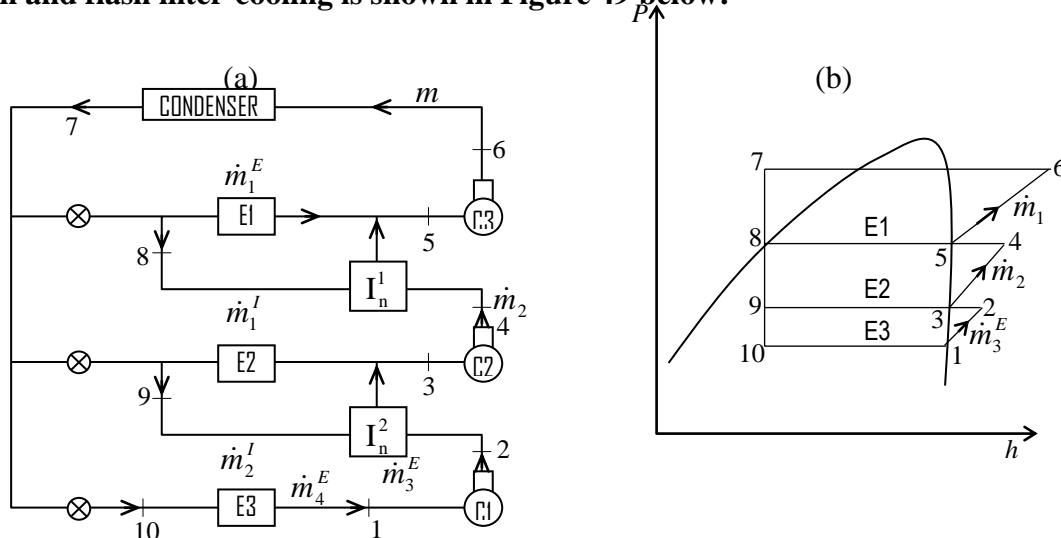


Figure 49: Compound compression with flash intercooling and corresponding P-h diagram

ANALYSIS:

For the analysis, you start with the low-pressure evaporator, i.e. E3

The low-pressure evaporator E3

Refrigerating effect, $q_{10,1} = h_1 - h_{10}$

Therefore the mass flow rate of refrigerant through E3 is given by

$$\dot{m}_3^E = \frac{\text{Refrigeration load}}{h_1 - h_{10}} \quad (3.19)$$

Power input to 1st stage compressor is given by

$$P_1 = \dot{m}_3^E (h_2 - h_1) \quad (3.20)$$

In the 2nd intercooler, the cooling effect required is given by

$$\dot{Q}_{23} = \dot{m}_3^E (h_3 - h_2) \quad (3.21)$$

Amount of refrigerant as at state 9 by-passed from the evaporator E2 to the intercooler to cool the suction of the 2nd stage compressor to dry saturated condition at state 3 is given by (It must be noted that the refrigerant bypassed is a refrigeration load to the intercooler).

$$\dot{m}_2^I = \frac{\dot{Q}_{23}}{h_3 - h_9} \quad (3.22)$$

The second evaporator, E2

Refrigerating effect of E2 is given by :

$$q_{93} = h_3 - h_9 \quad (3.23)$$

Refrigerant flow rate through E2 is given by:

$$\dot{m}_2^E = \frac{\text{Refrigeration load}}{\text{Refrigeration effect}} = \frac{\text{Refrigeration load}}{h_3 - h_9} \quad (3.24)$$

Therefore, the mass flow rate of refrigerant at entry (state 3) to the 2nd stage compressor is given by

$$\dot{m}_2 = \dot{m}_3^E + \dot{m}_2^I + \dot{m}_2^E \quad (3.25)$$

Power input for the 2nd stage compressor is given by $P_2 = \dot{m}_2 (h_4 - h_3)$ (3.26)

The high pressure evaporator E1

$$\text{Refrigerating effect } q_{85} = h_5 - h_8 \quad (3.27)$$

The refrigerant flow rate through the evaporator E1 is given by

$$\dot{m}_1^E = \frac{\text{Refrigeration load}}{h_5 - h_8} \quad (3.28)$$

The refrigerant flow rate through the 3rd stage intercooler is given by

$$\dot{m}_1^I = \frac{|\dot{Q}_{45}|}{h_5 - h_8} = \frac{|\dot{m}_2 (h_5 - h_4)|}{h_5 - h_8} \quad (3.29)$$

The refrigerant flow rate through the 3rd stage compressor is given by

$$m_1 = \dot{m}_1^I + \dot{m}_1^E + \dot{m}_2 \quad (3.30)$$

Power input for the 3rd stage compressor is given by

$$P_3 = \dot{m}_1(h_6 - h_5) \quad (3.31)$$

Hence, the total power input required from the compressor is:

$$P_{\text{input}} = P_1 + P_2 + P_3 \quad (3.32)$$

The heat rejected in the condenser is given by:

$$\dot{Q}_{67} = \dot{m}_1(h_7 - h_6) \quad (3.33)$$

b) Multiple expansion valves with compound compression and flash intercoolers

Multiple expansion valves with compound compression and flash intercoolers are connected as shown in Figure 50 below

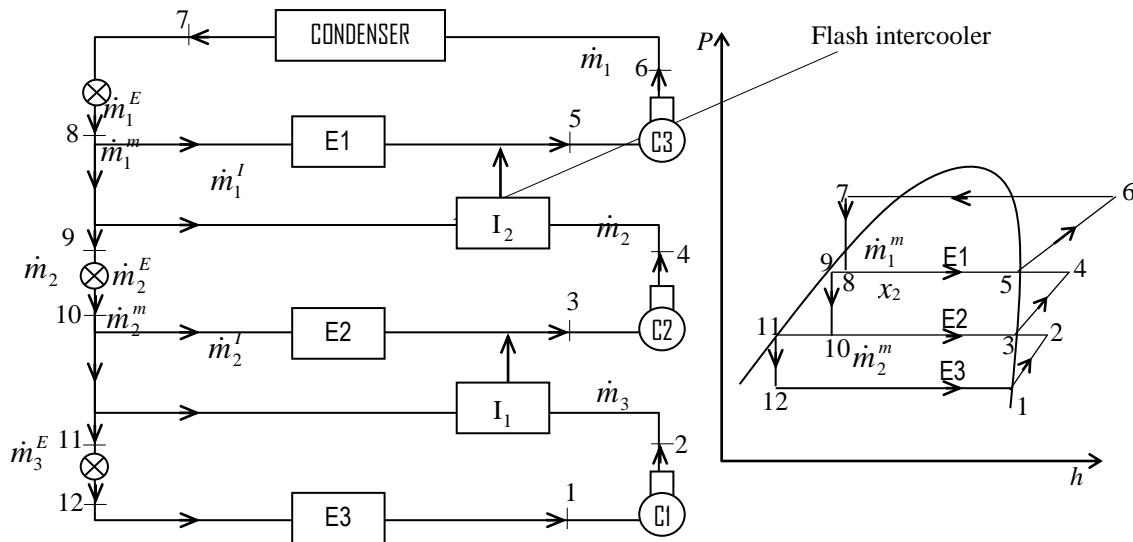


Figure 50: Multiple expansion valves with flash intercoolers and corresponding P-h diagram

Analysis is started with the low-pressure evaporator, E3

Evaporator E3

$$\text{Refrigerating effect } q_{12,1} = h_1 - h_{12} \quad (3.34)$$

The mass flow rate of refrigerant through E3 is given by

$$\dot{m}_3^E = \frac{\text{Refrigeration load}}{h_1 - h_{12}} \quad (3.35)$$

$$\text{The power input to the 1st compressor (C1)} \quad P_1 = \dot{m}_3^E (h_2 - h_1) \quad (3.36)$$

$$\text{In the 1st intercooler, the cooling effect required is given by: } Q_{23} = \dot{m}_3^E (h_3 - h_2) \quad (3.37)$$

The refrigerant flow rate bypassed from evaporator E2 at state 10 and which is required for intercooling in the 1st stage intercooler before entering the 2nd stage compressor as state 3 is given by

$$\dot{m}_2^1 = \frac{|Q_{23}|}{h_3 - h_{10}} \quad (3.38)$$

Evaporator E2

Refrigerating effect $q_{10,3} = h_3 - h_{10}$ (3.39)

The refrigerant flow rate through the 2nd stage evaporator is given by

$$\dot{m}_2^E = \frac{\text{Refrigeration load}}{h_1 - h_{10}} \quad (3.40)$$

Let the quality of mixture as it leaves the 2nd stage expansion valve be x_{10}

Additional vapour that flows through the evaporator E2 without any useful cooling is given by

$$\dot{m}_2^m = \frac{x_{10}}{1 - x_{10}} \dot{m}_3^E \quad (3.41)$$

Therefore, the total mass flow rate of refrigerant in the 2nd stage compressor is given by

$$\dot{m}_2 = \dot{m}_3^E + \dot{m}_2^I + \dot{m}_2^E + \dot{m}_2^m \quad (3.42)$$

Power required from the 2nd stage compressor is given by

$$P_2 = \dot{m}_3(h_4 - h_3) \quad (3.43)$$

Cooling effect required in the 2nd stage intercooler is given by

$$Q_{43} = \dot{m}_2(h_5 - h_4) \quad (3.44)$$

Refrigerant flow rate as at state 8 required for inter-cooling in the 2nd stage intercooler is given by

$$\dot{m}_1^I = \frac{|Q_{43}|}{h_5 - h_8} \quad (3.45)$$

Evaporator E1

Refrigerating effect $q_{85} = (h_5 - h_8)$ (3.46)

Refrigerating flow rate through E1 is given by $\dot{m}_1^E = \frac{\text{Refrigeration load}}{h_5 - h_8}$ (3.47)

Additional vapour flowing through the evaporator without any useful cooling is given by

$$\dot{m}_1^m = \frac{x_8}{1 - x_8} \dot{m}_2 \quad (3.48)$$

Total refrigerant flow rate through the 1st stage compressor is given by

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_1^I + \dot{m}_1^E + \dot{m}_1^m \quad (3.49)$$

Power required from the 3rd stage compressor is given by $P_1 = \dot{m}_1(h_6 - h_5)$ (3.50)

Total power required from the compressor is given by $P_{input} = P_1 + P_2 + P_3$ (3.51)

The heat rejected in the condenser is given by $\dot{Q}_{67} = \dot{m}_1(h_7 - h_6)$ (3.52)

1-3.4 Cascade Refrigeration systems

Refrigeration performed in stages, that is, two or more refrigeration cycles that operate in series.

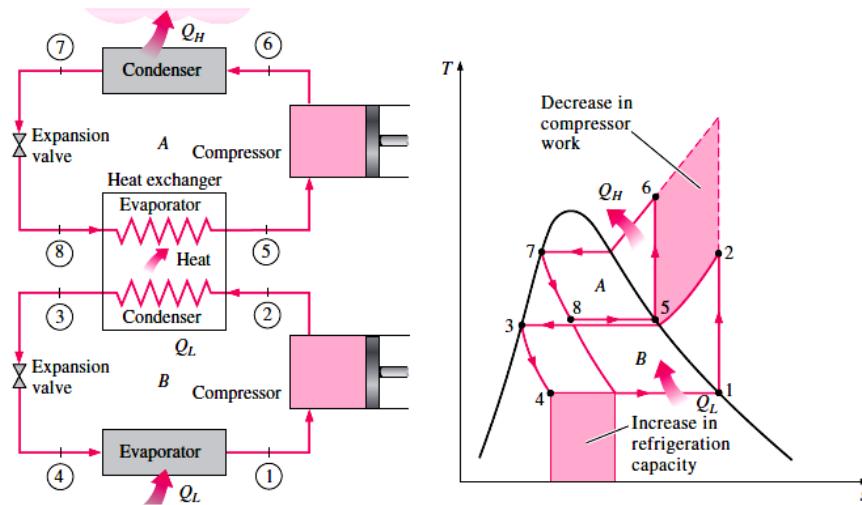


Figure 51: A two-stage cascade refrigeration system with corresponding T-s diagram

Note: There is decrease in compressor work

Amount of heat absorbed from refrigerated space increase

Hence cascading improves the *COP* of its refrigeration system.

1-3.4.1 Multi-purpose Refrigeration System with Single Compression

Providing two separate spaces for refrigeration and freezing. Not practical to maintain two spaces at same temperature since foods will get frozen. Ideal situation is to maintain freezer at about -18°C and fridge at about 3°C .

Option A

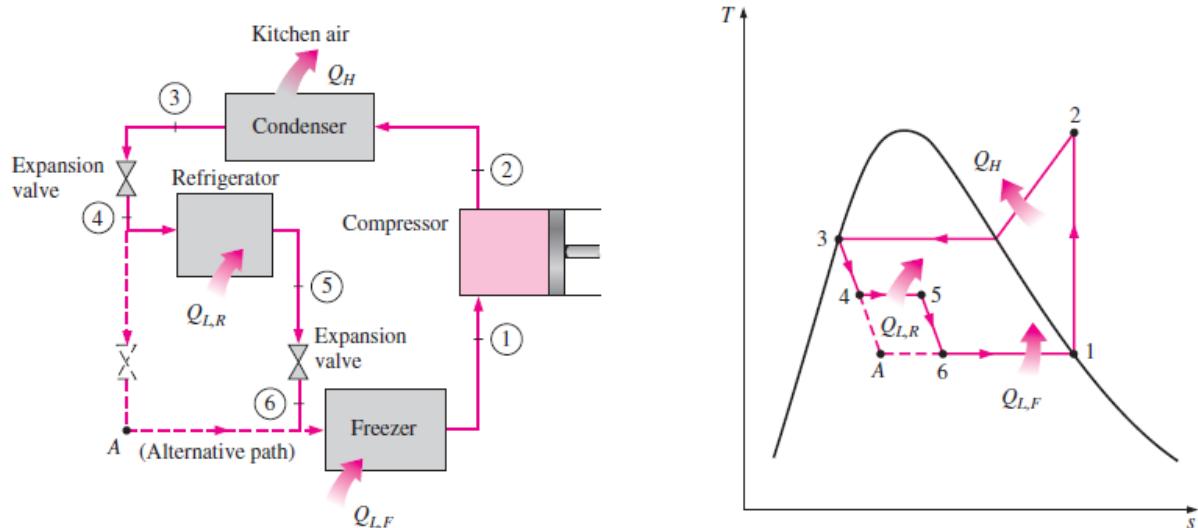


Figure 52: Refrigerator-freezer unit with one compressor and corresponding T-s diagram

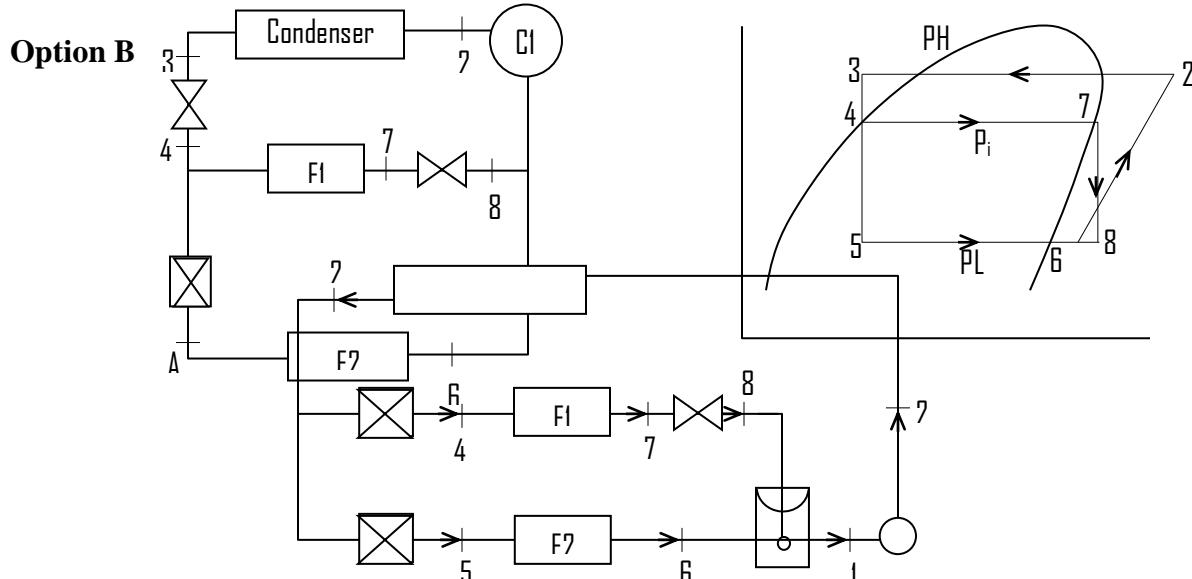


Figure 53: Schematic arrangements of Refrigerator-Freezer unit with one compressor



Self Assessment 1-3

1. The temperature in a refrigerator evaporator coil is -6°C and that in the condenser coil is 22°C . Assuming that the machine operates on the reversed Carnot cycle, calculate the COP_{ref} , the refrigerating effect per kilowatt of input work, and the heat rejected to the condenser. **Suggested Answers:** [9.54; 9.54 kW; 10.54 kW]

2. A refrigerator uses refrigerant-134a as the working fluid and operates on an ideal vapor-compression refrigeration cycle between 0.12 and 0.7 MPa. The mass flow rate of the refrigerant is 0.05 kg/s. Show the cycle on a T - s diagram with respect to saturation lines. Determine (a) the rate of heat removal from the refrigerated space and the power input to the compressor, (b) the rate of heat rejection to the environment, and (c) the coefficient of performance.

Suggested Answers: [(a) 7.41 kW, 1.83 kW, (b) 9.24 kW, (c) 4.05]

SESSION 2-3: REFRIGERANTS

From engineering standpoint the important parameters in the choice of a refrigerant are as follows:

- the saturation pressure at the desired low temperature should be above atmospheric, but not too high, otherwise leakage is harder to prevent and components must be designed to withstand the higher pressures; the pressure at the condenser must not be excessively high for the same reasons;
- the specific enthalpy of vaporisation at the low temperature should be as high as possible to give a reasonably low mass flow rate for a given refrigeration capacity;

- iii. the specific volume at compressor inlet should not be excessively high to avoid overly-large size compressors for a required mass flow rate;
- iv. the refrigerant should not react with the lubricating oil of the reciprocating compressor and should be miscible with the oil (miscibility is an advantage both from the heat transfer point of view and also a carrier of oil to moving parts); in a plant using an immiscible refrigerant with a reciprocating compressor an oil separator may have to be fitted after the compressor to prevent oil fouling the condenser and evaporator surfaces and hence reducing the heat transfer;
- v. the refrigerant should be non-toxic, non-corrosive, stable, non-flammable and non-explosive since safety requirements put limitations on the use of hazardous refrigerants.
- vi. the critical temperature should be well above the highest operating temperature in order to minimise the amount of superheat occurring during compression.

2-3.1 Types of Refrigerants

1. Ammonia, Advantages: Lowest ODP, higher COP, favourite and greater enthalpy of vaporisation. It has been reckoned to have zero Ozone depletion potential and Greenhouse potential. It is designated R717. Its toxicity is the main disadvantage, but secondary or even tertiary coolant systems can be employed to overcome this problem by confining the risk to a well-ventilated plant room. It is an immiscible refrigerant. It is used in industrial applications because of its high latent heat although it is toxic.

2. Halocarbon refrigerants

CFCs :Chlorofluorocarbons, CCl_3F (R11)trichlorofluoromethane;

CCl_2F_2 (R12)dichlorodifluoromethane; $\text{CCl}_2\text{FCClF}_2$ (R113)trichlorotrifluoromethane; R11 and R12 have now been phased out. Their Ozone Depletion Potential (ODP) are both 1.0, and their Green house potential are 1.0 and 3.10 respectively. R11, R12 and R113 are all miscible refrigerants. *HCFs*: CHClF_2 (R22), Chlorodifluoromethane CHCl_2CF_3 (R123) ,

Dichlorotrifluoroethane

Recent addition: CH_2FCF_3 (R134_a) Tetrafluoroethane is said to be the best replacement for the *CFCs*, and in the long term, the *HCFCs*. It contains no chlorine atoms and hence has a zero Ozone Depletion Potential (ODP). However, its greenhouse potential is 0.37.

- 1. *CO₂*. It is an immiscible refrigerant and it is not normally used for the following reasons:
 - a) It has a high vapour pressure at normal temperatures (need to design the cycle to withstand these high pressures)

- b) It has the volume and large plants are required and hence lead to increase in capital cost.
- c) It is colourless, therefore not easily detected where there is leakage
- d) It has a relatively low critical temperature and hence cannot be used in the tropics.
- e) CO_2 is the main greenhouse gas and its reintroduction into refrigeration is extremely unlikely.

2-3.1.1 Control of Refrigeration Capacity

- I. On-off control of the compressor with the on-off control metallic bellows, charged with a volatile liquid, is connected to a temperature sensor which is located in the evaporator coil. An increasing temperature at the evaporator causes an increase in the temperature and therefore the pressure in the bellows tend to increase. The bellows expand and the end of the bellows operates a switch, which closes the compressor motor circuit. A decreasing temperature in the evaporator produces the reverse effect.
- II. Regulating the flow rate by means of needle valve. By throttling the inlet to the compressor, the inlet pressure is reduced and hence the density of the incoming charge. The delivery pressure is reduced because of the lower inlet pressure. With constant-speed machine, control can be achieved by reducing the cooling refrigerant flow in the condenser. The temperature and pressure in the condenser increase and this reduces the effective capacity of the machine.

2-3.2 Vapour Absorption Refrigeration cycles

Vapour-absorption cycles have been in use for some time, but recently they have come of more interest because of their potential use as part of an energy-saving plant: also, they use more environmentally friendly refrigerants than current vapour-compression cycles.

A vapour-absorption system operates with a condenser, a throttle valve and an evaporator in the same way as a vapour-compression system. In these systems, the pump, and the absorber- generator combination, as shown in Fig. 54, replaces the compressor.

- After evaporation from state 1, the refrigerant is passed into a low-temperature absorbing medium. Solubility of the refrigerant is high and it gets dissolved in the absorbent. This process is accompanied by rejection of heat, Q_A to the surroundings at a temperature, T_a .
- The refrigerant-absorbent solution is then pumped to the condenser (higher) pressure and is heated in the generator, which is maintained at a constant temperature, T_g . Due to the reduced solubility of the refrigerant-absorbent solution at the higher pressure and temperature, refrigerant vapour is separated from the solution. The vapour passes to the condenser and the weakened refrigerant absorbent solution is throttled back to the absorber.
- A heat exchanger placed between the absorber and generator makes the system more efficient by transferring heat form the weak solution coming from the generator to the stronger solution pumped from the absorber gas, as shown in Figure 54.

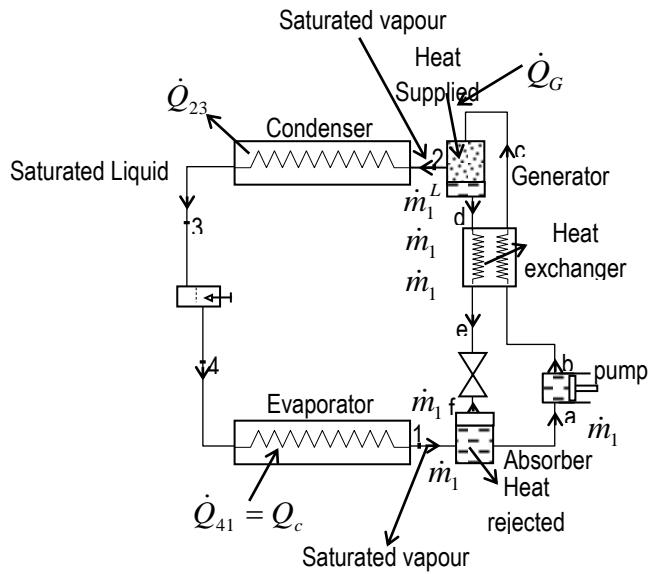


Figure 54: Diagrammatic arrangement of a Vapour-absorption refrigeration system

The work done in pumping the liquid solution is much less than that required for compressing vapour in the compressor of an equivalent vapour-compression-refrigeration cycle. The main energy input is the heat supplied in the generator. In addition, the absence of the compressor usually has the advantages of less vibration, noise and weight than with a vapour compression machine. Note that the main energy input into the system is the heat supplied in the generator; this may be supplied in several forms such as a fuel burning device, direct electrical heating, solar energy or waste heat.

Two refrigerant-absorbent combinations are already in use:

- NH_3/H_2O
- $H_2O/\text{Lithium Bromide}$

Analysis of the absorption cycle: assumes $H_2O/\text{lithium bromide}$ mixture

We note that if a solution of particular concentration is saturated at any pressure the temperature is fixed at that concentration.

Refrigerating effect $q_{41} = h_1 - h_4$

Hence, the mass flow rate of refrigerant $\dot{m} = \frac{\text{Refrigeration load}}{\text{Refrigerating effect}}$

Mass balance on the absorber is given by $\dot{m}_1 = \dot{m}_a - \dot{m}_f$ (3.53)

For the lithium bromide balance, $x_1 \dot{m}_1 = x_a \dot{m}_a - x_f \dot{m}_f$ (3.54)

where x is the concentration of the refrigerant absorbent solution

$$CH_2FC.F_3$$

Also,

$$\begin{aligned}\dot{m}_f &= \dot{m}_e = \dot{m}_d \\ \dot{m}_b &= \dot{m}_a = \dot{m}_c \\ \dot{m}_1 &= \dot{m}_2\end{aligned}\quad (3.55)$$

Applying an energy balance to the generator we have,

$$\dot{Q}_G + \dot{m}_c h_c = \dot{m}_2 h_2 + \dot{m}_d h_d \quad (3.56)$$

In the absence of a heat exchanger $h_a = h_c$, if the feed pump work is assumed negligible

$$\text{Heat rejected in the condenser is given by } \dot{Q}_{23} = \dot{m}_2 (h_3 - h_2) \quad (3.57)$$

Applying an energy balance to the complete system, neglecting pump work we obtain

$$\dot{Q}_G + \dot{Q}_C + \overbrace{\dot{Q}_{23} + \dot{Q}_{A_{bs}}}^{\dot{Q}_a} = 0 \quad (3.58)$$

The *COP* can be defined as the heat supplied in the generator divided into the refrigeration capacity, since the pump work is negligible. That is,

$$COP_{ref} = \frac{\dot{Q}_{41}}{\dot{Q}_G} \quad (3.59)$$

Note that $\begin{cases} h_1 = h_g & \text{at } p_s \text{ corresponding to temperature of the evaporator} \\ h_3 = h_{f3} & \text{at temperature of the condenser } (T_c) \\ h_2 = h_g & \text{at temperature of the generator } (T_g) \end{cases}$

Energy balance on the absorber is given by

$$\dot{Q}_{A_{bs}} + \dot{m}_a h_a = \dot{m}_f h_f + \dot{m}_1 h_1 \quad (3.60)$$

$$\text{Energy balance on the heat exchanger is given by: } m_d (h_d - h_e) = m_a (h_c - h_b) \quad (3.61)$$

For the throttling process $h_e = h_f$

Analysis of ideal absorption refrigeration cycle

Ideally (neglecting the quantity of energy transferred as work in the pump), energy is transferred in the form of heat at three levels:

- The atmospheric temperature, T_a at which heat is rejected in the condenser and absorber
- The temperature T_c at which heat is taken from the cold chamber
- The temperature T_g at which heat is received from the generator

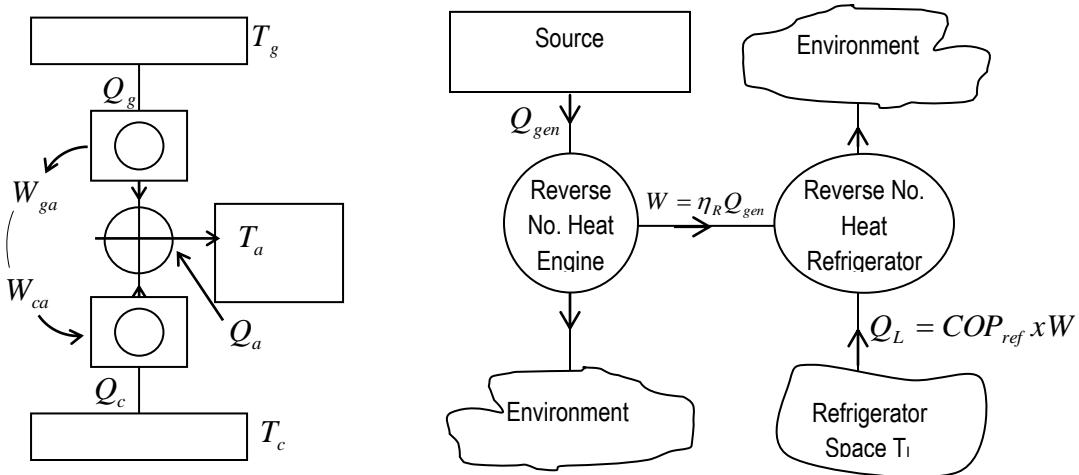


Figure 55: A ‘reversible equivalent’ of an Absorption refrigerator

An arrangement of a reversible machine performing a function equivalent to that of the absorption plant is shown in Figure 55.

First, a reversible heat engine receives a quantity of heat Q_g at T_g and rejects heat at T_a while producing a quantity of work W_{ga} with an efficiency

$$\frac{|W_{ga}|}{Q_g} = \frac{T_g - T_a}{T_g} \quad (3.62)$$

Secondly, a reversible refrigerator receives a quantity of heat Q_c at T_c and rejects heat at T_a while absorbing a quantity of work W_{ca} . The coefficient of performance of the refrigerator is

$$\frac{Q_c}{W_{ca}} = \frac{T_c}{T_a - T_c} \quad (3.63)$$

If $[W_{ga}]$ is made equal to W_{ca} , this plant will be equivalent to an absorption refrigerator for which the pump work is zero. The performance factor of the combined plant acting as refrigerator can be defined as Q_c/Q_g , which on combining equation (3.62) and (3.63) becomes

$$PF = \frac{Q_c}{Q_g} = \frac{T_c}{(T_a - T_c)} \times \frac{(T_g - T_a)}{T_g} = COP_{rev\ carnot\ cycle} \times \eta_{carnot\ cycle} \quad (3.64)$$

Equation (3.64) gives the theoretically possible maximum value of the **performance factor** (PF). The numerous thermodynamic irreversibilities occurring in practical cycle operation causes an appreciable decrease in the PF , and a practical upper limit of about 1.0 exists. To improve the cycle performance factor by reducing the heat required in the generator, a heat exchanger is provided to transfer heat from the hot solution returning to the absorber and the relatively cool solution going to the generator. Without the heat exchanger, the PF value further reduces.

This limiting practical value from Equation 3.64 is unity with normal operating temperatures, but the properties of refrigerant-absorbent solutions are such that the coefficient of performance is less than unity.

Applying the first and second laws of thermodynamics to the system as illustrated in Figure 56 equation (3.64) can also be derived as follows:

From the First law

$$Q_g + Q_c + Q_a = 0 \Rightarrow Q_a = Q_g + Q_c \quad (3.65)$$

From the Second law: $(\Delta S)_{\text{system}} + (\Delta S)_{\text{surrounding}} \geq 0$

$$\text{But } (\Delta S)_{\text{system}} = 0 \therefore (\Delta S)_{\text{surrounding}} \geq 0 = \frac{Q_g}{T_g} - \frac{Q_c}{T_c} - \frac{Q_a}{T_a} \geq 0 \quad (3.66)$$

For reversible system, $\Delta S = 0$ and therefore:

$$\frac{Q_g}{T_g} - \frac{Q_c}{T_c} + \frac{Q_g + Q_c}{T_a} = 0 \quad (3.67)$$

Multiplying Equation 3.67 through by $\frac{T_g + T_a}{Q_c}$ and simplifying the resulting expression we obtain:

$$\frac{Q_c}{Q_g} = \frac{(T_g - T_a)}{T_g} \times \frac{T_c}{(T_a - T_c)} \quad (3.68)$$

2-3.3 Household Refrigerators

Refrigerators to preserve perishable foods have long been one of the essential appliances in a household. A typical household refrigerator is actually a combination refrigerator-freezer since it has a freezer compartment to make ice and to store frozen food. Today's refrigerators use much less energy as a result of using smaller and higher-efficiency motors and compressors, better insulation materials, larger coil surface areas, and better door seals as shown in Figure 56.

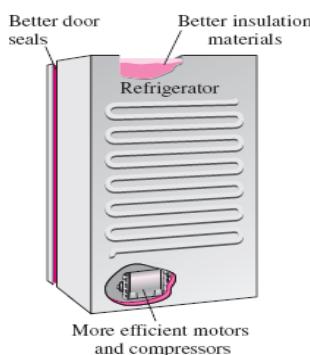


Figure 56: A typical household refrigerator in modern times

Replacing an old refrigerator with a new energy-efficient model will save over 1000 kWh of electricity per year. For the environment, this means a reduction of over 1 ton of CO₂, which causes global climate change, and over 10 kg of SO₂, which causes acid rain.

A household refrigerator is designed to maintain the freezer section at -18°C (0°F) and the refrigerator section at 3°C (37°F). Lower freezer temperatures increase energy consumption without improving the storage life of frozen foods significantly.



For specified external dimensions, a refrigerator is desired to have maximum food storage volume, minimum energy consumption, and the lowest possible cost to the consumer.

Household refrigerators consume from about 90 to 600 W of electrical energy when running and are designed to perform satisfactorily in environments at up to 43°C (110°F). Refrigerators run intermittently, as you may have noticed, running about 30 percent of the time under normal use in a house at 25°C (77°F). The total food storage volume has been increased over the years without an increase in the external dimensions by using thinner but more effective insulation and minimising the space occupied by the compressor and the condenser.

The size of the compressor and the other components of a refrigeration system are determined on the basis of the anticipated heat load (or refrigeration load), which is the rate of heat flow into the refrigerator. The heat load consists of the predictable part, such as heat transfer through the walls and door gaskets of the refrigerator, fan motors, and defrost heaters, and the unpredictable part, which depends on the user habits such as opening the door, making ice, and loading the refrigerator. The amount of energy consumed by the refrigerator can be minimised by practicing good conservation measures as discussed below.

1. Open the refrigerator door the fewest times possible for the shortest duration possible. Each time the refrigerator door is opened, the cool air inside is replaced by the warmer air outside, which needs to be cooled. Keeping the refrigerator or freezer full will save energy by reducing the amount of cold air that can escape each time the door is opened.
2. Cool the hot foods to room temperature first before putting them into the refrigerator. Moving a hot pan from the oven directly into the refrigerator not only wastes energy by making the refrigerator work longer, but it also causes the nearby perishable foods to spoil by creating a warm environment in its immediate surroundings.
3. Clean the condenser coils located behind or beneath the refrigerator. The dust and grime that collect on the coils act as insulation that slows down heat dissipation through them. Cleaning the coils a couple of times a year with a damp cloth or a vacuum cleaner will improve cooling ability of the refrigerator while cutting down the power consumption by a few percent.
4. Sometimes a fan is used to force-cool the condensers of large or built-in refrigerators, and the strong air motion keeps the coils clean.
5. Check the door gasket for air leaks. This can be done by placing a flashlight into the refrigerator, turning off the kitchen lights, and looking for light leaks. Heat transfer through the door gasket region accounts for almost one-third of the regular heat load of the refrigerators, and thus any defective door gaskets must be repaired immediately.

5. Avoid unnecessarily low temperature settings. The recommended temperatures for freezers and refrigerators are -18°C (0°F) and 3°C (37°F), respectively. Setting the freezer temperature below 18°C adds significantly to the energy consumption but does not add much to the storage life of frozen foods. Keeping temperatures 6°C (or 10°F) below recommended levels can increase the energy use by as much as 25 percent.

6. Avoid excessive ice build-up on the interior surfaces of the evaporator. The ice layer on the surface acts as insulation and slows down heat transfer from the freezer section to the refrigerant. The refrigerator should be defrosted by manually turning off the temperature control switch when the ice thickness exceeds a few millimetres.

7. Use the power-saver switch that controls the heating coils and prevents condensation on the outside surfaces in humid environments. About 10 percent of the total energy consumed by the refrigerator can be saved by turning this heater off and keeping it off unless there is visible condensation on the outer surfaces.

8. Do not block the air flow passages to and from the condenser coils of the refrigerator. The heat dissipated by the condenser to the air is carried away by air that enters through the bottom and sides of the refrigerator and leaves through the top. Any blockage of this natural convection air circulation path by large objects such as several cereal boxes on top of the refrigerator will degrade the performance of the condenser and thus the refrigerator (see Figure 57). These and other commonsense conservation measures will result in a reduction in the energy and maintenance costs of a refrigerator as well as an extended trouble-free life of the device.

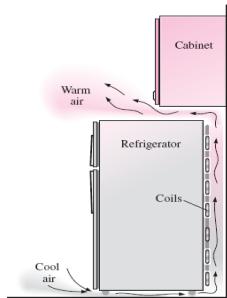


Figure 57: Air flow passages to and from condenser coils of a refrigerator

Worked Example 3.1

The interior lighting of refrigerators is provided by incandescent lamps whose switches are actuated by the opening of the refrigerator door. Consider a refrigerator whose 40-W light bulb remains on continuously as a result of a malfunction of the switch (Figure 58). If the refrigerator has a coefficient of performance of 1.3 and the cost of electricity is 8 cents per kWh, determine the increase in the energy consumption of the refrigerator and its cost per year if the switch is not fixed.

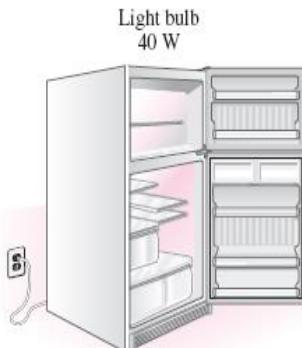


Figure 58: Schematic for worked example 3.1

Solution The light bulb of a refrigerator malfunctions and remains on. The increases in the electricity consumption and cost are to be determined.

Assumptions The life of the light bulb is more than 1 year.

Analysis The light bulb consumes 40 W of power when it is on, and thus adds 40 W to the heat load of the refrigerator. Noting that the COP of the refrigerator is 1.3, the power consumed by the refrigerator to remove the heat generated by the light bulb is

$$\dot{W}_{\text{refrig}} = \frac{Q_{\text{refrig}}}{\text{COP}_R} = \frac{40 \text{ W}}{1.3} = 30.8 \text{ W}$$

Therefore, the total additional power consumed by the refrigerator is

$$\dot{W}_{\text{total,additional}} = \dot{W}_{\text{light}} + \dot{W}_{\text{refrig}} = 40 + 30.8 = 70.8 \text{ W}$$

The total number of hours in a year is

$$\text{Annual hours} = (365 \text{ days/yr}) (24 \text{ h/day}) = 8760 \text{ h/yr}$$

Assuming the refrigerator is opened 20 times a day for an average of 30 s, the light would normally be on for

$$\begin{aligned} \text{Normal operating hours} &= (20 \text{ times/day}) (30 \text{ s/time}) (1 \text{ h}/3600 \text{ s}) (365 \text{ days/yr}) \\ &= 61 \text{ hr/yr} \end{aligned}$$

Then the additional hours the light remains on as a result of the malfunction becomes

$$\text{Additional operational hours} = \text{Annual hours} - \text{Normal operating hours}$$

$$= 8760 - 61 = 8699 \text{ h/yr}$$

Therefore, the additional electric power consumption and its cost per year are

$$\begin{aligned} \text{Additional power consumption} &= \dot{W}_{\text{total,additional}} \times (\text{Additional operating hours}) \\ &= (0.0708 \text{ kW}) (8699 \text{ h/yr}) = 616 \text{ kWh/yr} \end{aligned}$$

and

$$\begin{aligned} \text{Additional power cost} &= (\text{Additional power consumption}) (\text{unit cost}) \\ &= (616 \text{ kWh/yr}) (\$ 0.08/\text{kWh}) = \$ 49.3/\text{yr} \end{aligned}$$

Discussion Note that not repairing the switch will cost the homeowner about \$50 a year. This is alarming when we consider that at \$0.08/kWh, a typical refrigerator consumes about \$70 worth of electricity a year.

2-3.4 Commercial Refrigeration Systems

In commercial systems, the space to be cooled is not affected directly by the refrigerant. Rather, an auxiliary refrigerant is cooled and pumped around the area being cooled. This arrangement requires additional pumping work. The main purpose of introducing the auxiliary refrigerant is to avoid the dangers associated with refrigerant leakage since most of these systems use ammonia as refrigerant because of its high latent heat of evaporation. Brine circulating system is used as illustrated in Figure 59.

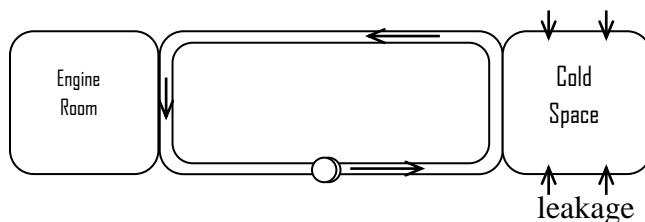


Figure 59: A typical arrangement of a commercial refrigerating plant

2-3.4.1 Compressor Types

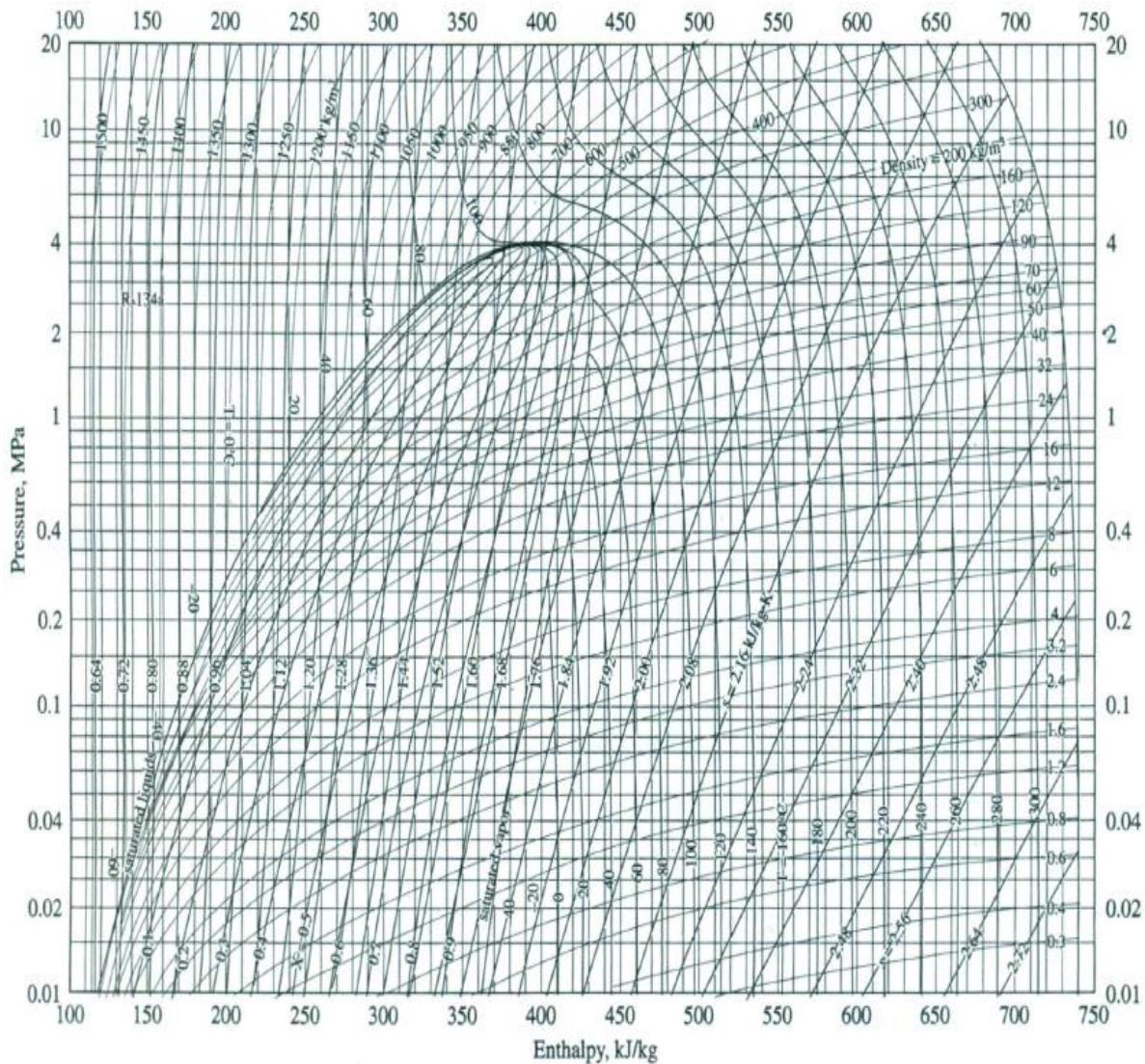
Compressors may be classified according to their housing. Thus, a compressor which is completely sealed in a straight housing with its motor is called a hermetic compressor. This type of compressor minimises gas leakage but cannot be field-serviced.

The semi-hermetic compressor combines the compressor and motor in one-housing, but its valve and other components can be serviced in the field.

Reciprocating compressors are in common use up to a power input of about 600 kW; at the lower end of the power scale (120 – 250 W) particularly for domestic applications the vane type compressors are used. Centrifugal compressors are used in the range from 300 kW up to 15 MW with screw-type compressors also used from 300 kW to about 3 MW. Centrifugal compressors are ideally suited for high volume flow machines; they run at speeds from 300 rev/min. up to about 20,000 rev/min.

Reciprocating machines run at much lower speeds, usually in the range from 200 to 600 rev/min. and increased capacity is obtained by using multi-cylinder machines. They are best suited for low specific volumes (high pressures) whereas centrifugal compressors are most suited for high specific volume (low pressures).

2-3.4.2 The Pressure – enthalpy (P-h) chart for R-134a



2.4 GAS REFRIGERATION CYCLES

As explained earlier, the Carnot Cycle (the standard of comparison for power cycles) and the reversed Carnot cycle (the standard of comparison for refrigeration cycles) are identical, except that the reversed Carnot cycle operates in the reverse direction. This suggests that the power cycles discussed in earlier chapters can be used as refrigeration cycles by simply reversing them. In fact, the vapour-compression refrigeration cycle is essentially a modified Rankine cycle operating in reverse. Another example is the reversed Stirling cycle, which is the cycle on which Stirling refrigerators operate. In this section, we discuss the *reversed Brayton cycle*, better known as the **gas refrigeration cycle**. In the *gas refrigeration cycle*, the refrigerant remains in the

gaseous phase throughout. Despite their relatively low COPs, the gas refrigeration cycles have two desirable characteristics: They involve simple, lighter components, which make them suitable for aircraft cooling, and they can incorporate regeneration, which makes them suitable for liquefaction of gases and cryogenic

applications. In an open-cycle aircraft cooling system, atmospheric air is compressed by a compressor, cooled by the surrounding air, and expanded in a turbine. The cool air leaving the turbine is then directly routed to the cabin.

Consider the gas refrigeration cycle shown in Fig. 11–18. The surroundings are at T_0 , and the refrigerated space is to be maintained at T_L . The gas is compressed during process 1-2. The high-pressure, high-temperature gas at state 2 is then cooled at constant pressure to T_0 ideally by rejecting heat to the surroundings. This is followed by an expansion process in a turbine, during which the gas temperature drops to T_4 . (Can we achieve the cooling effect by using a throttling valve instead of a turbine?) Finally, the cool gas absorbs heat from the refrigerated space until its temperature rises to T_1 . All the processes described are internally reversible, and the cycle executed is the *ideal* gas refrigeration cycle. In actual gas refrigeration cycles, the compression and expansion processes deviate from the isentropic ones and T_3 is higher than T_0 unless the heat exchanger employed is infinitely large. On a $T-s$ diagram, the area under process curve 4-1 represents the heat removed from the refrigerated space, and the enclosed area 1-2-3-4-1 represents the net work input. The ratio of these areas is the COP for the cycle, which may be expressed as

$$\text{COP}_{\text{ref}} = \frac{q_L}{w_{\text{net,in}}} = \frac{q_L}{w_{\text{comp}} - w_{\text{turb}}}$$

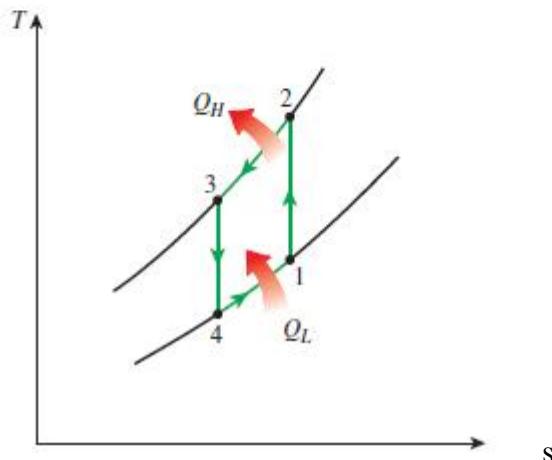
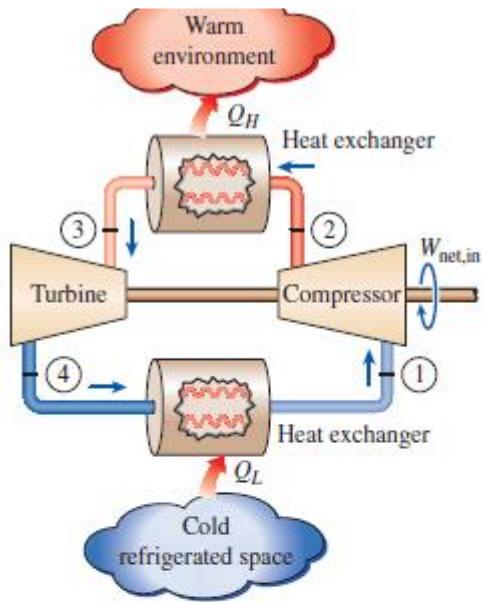
Where, $q_L = h_1 - h_4 = C_p \times (T_1 - T_4)$ $w_{\text{comp}} = C_p(T_2 - T_1)$ $w_{\text{turb}} = C_p(T_3 - T_4)$

$$T_2 = T_1 \times \left(\frac{p_2}{p_1} \right)^{(v-1)/v}, T_4 = T_3 \times \left(\frac{p_1}{p_2} \right)^{(v-1)/v}, q_H = C_p \times (T_2 - T_3)$$

$$w_{\text{net,in}} = C_p [(T_2 - T_1) - (T_3 - T_4)]$$

The properties of gases used in gas refrigeration systems are provided below:

Ideal-gas specific heats of various common gases					
(a) At 300 K					
Gas	Formula	Gas constant, R kJ/kg·K	c_p kJ/kg·K	c_v kJ/kg·K	k
Air	—	0.2870	1.005	0.718	1.400
Argon	Ar	0.2081	0.5203	0.3122	1.667
Butane	C ₄ H ₁₀	0.1433	1.7164	1.5734	1.091
Carbon dioxide	CO ₂	0.1889	0.846	0.657	1.289
Carbon monoxide	CO	0.2968	1.040	0.744	1.400
Ethane	C ₂ H ₆	0.2765	1.7662	1.4897	1.186
Ethylene	C ₂ H ₄	0.2964	1.5482	1.2518	1.237
Helium	He	2.0769	5.1926	3.1156	1.667
Hydrogen	H ₂	4.1240	14.307	10.183	1.405
Methane	CH ₄	0.5182	2.2537	1.7354	1.299
Neon	Ne	0.4119	1.0299	0.6179	1.667
Nitrogen	N ₂	0.2968	1.039	0.743	1.400
Octane	C ₈ H ₁₈	0.0729	1.7113	1.6385	1.044
Oxygen	O ₂	0.2598	0.918	0.658	1.395
Propane	C ₃ H ₈	0.1885	1.6794	1.4909	1.126
Steam	H ₂ O	0.4615	1.8723	1.4108	1.327



Worked Example on Gas Refrigeration

Consider an ideal gas refrigeration cycle using air as the working fluid. Air enters the compressor at 80 kPa and 25 °C and is compressed to 280 kPa. Air is then cooled to 35 °C before entering the turbine. For a mass flow rate of 0.2 kg/s, the net power input required is:

Solution:

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{(v-1)/v} = 298 \times \left(\frac{280}{80} \right)^{(1.4-1)/1.4} = 426.2 \text{ K} ,$$

$$T_4 = T_3 \left(\frac{P_1}{P_2} \right)^{(v-1)/v} = 308 \times \left(\frac{80}{280} \right)^{(1.4-1)/1.4} = 215 \text{ K}$$

$$\dot{W}_{\text{net,in}} = 0.2 \times 1.005 \times [(426.2 - 298) - (308 - 215)] = 7.08 \text{ kW}$$



Self Assessment 2-3

1. A refrigerator as shown in Figure Q1, uses refrigerant-134a as the working fluid and operates on the ideal vapor-compression refrigeration cycle. The refrigerant enters the evaporator at 120 kPa with a quality of 30 percent and leaves the compressor at 60°C. If the compressor consumes 450 W of power, determine (a) the mass flow rate of the refrigerant, (b) the condenser pressure, and (c) the COP of the refrigerator. **Suggested Answers:** (a) 0.00727 kg/s, (b) 672 kPa, (c) 2.43

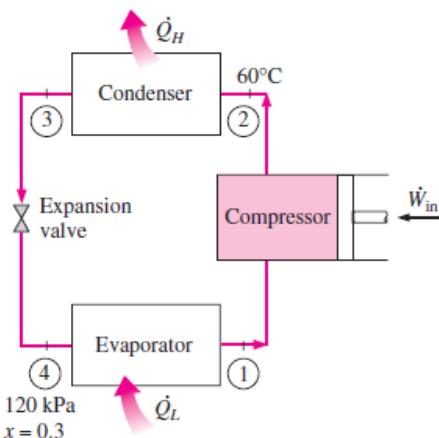


Figure Q1: Schematic of self assessment 2-3 Question 1

2. Heat is supplied to an absorption refrigeration system from a geothermal well at 130°C at a rate of 5×10^5 kJ/h. The environment is at 25°C, and the refrigerated space is maintained at -30 °C. Determine the maximum rate at which this system can remove heat from the refrigerated space. **Suggested Answers:** 5.75×10^5 kJ/h

3. A refrigerator, as shown in FIGURE P 3 with the corresponding T-s diagram, uses refrigerant R-134a as the working fluid and operates on the ideal vapour-compression refrigeration cycle except for the compression process. The refrigerant enters the evaporator at 120 kPa with a quality of 34.026 percent and leaves the compressor at 70 °C. If the compressor consumes 0.45 kW of power, Determine:
- The mass flow rate of the refrigerant, in kg/s, correct to 6 dp.
 - The condenser pressure, in kPa, and
 - The COP of the refrigerator, correct to 2dp.

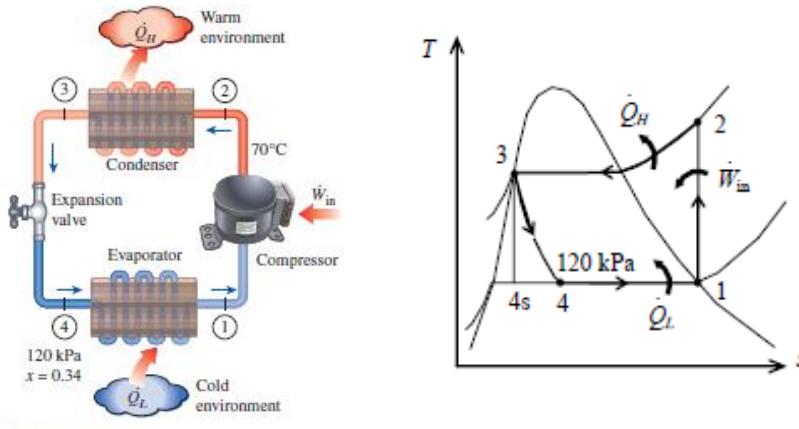


Figure P3

NON-REACTING IDEAL GAS MIXTURES AND PSYCHROMETRY

Introduction

In this unit, we develop rules for determining non reacting gas mixture properties from knowledge of mixture composition and the properties of individual constituents. Also the concept of psychrometry would be introduced and the use of psychrometric chart as a tool to determine the properties of atmospheric air would be discussed. Fundamental psychrometric processes would be introduced and discussed briefly.



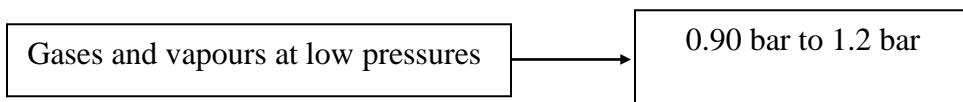
Learning Objectives

After studying this unit you should be able to:

1. Define the quantities used to describe the composition of a mixture, such as mass fractions, partial pressures etc.
2. Apply the rules for determining ideal gas properties.
3. Explain the fundamental psychrometric processes in isolation.
4. Apply the principles of conservation of mass and energy to various air – conditioning processes.
5. Analyse wet cooling towers with an example.

SESSION 1-4: IDEAL GAS MIXTURES

Any homogeneous mixture of gases can be regarded as a single substance if the constituents do not react chemically with one another and are in fixed proportions by mass. The properties of such a substance can be determined experimentally just as for a single substance and they can be tabulated or related in the same way. Air is regarded as a *simple substance since its constituents do not vary and its properties have been determined by direct measurement*. This chapter describes how this can be done for mixtures of perfect gases and vapours at low pressures. Thermodynamic analysis of processes which occur in an air conditioning system are subsequently provided as suitable examples of a system involving mixtures of gases and vapours at low pressures. The special nomenclature associated with the study of moist atmospheric mixtures of non-reacting gaseous mixtures and psychrometry is also introduced and a brief description of psychometric processes is then provided.



1-4.1 The Gas Laws

An ideal gas is an imaginary gas which obeys simple laws such as those of Boyle, Charles, Dalton, etc. These laws are based on the assumptions that the effects of intermolecular forces and volumes of the molecules of the gas are negligible owing to the fact that the average distance between the molecules is fairly large.

For processes involving high pressures or heterogeneous mixtures, these ideal gas laws break down and lose their accuracy and applicability. At low pressure or relatively high temperatures, many gases and homogeneous mixtures obey the ideal gas laws.

The main working fluid of the air conditioning system is a mixture of two gases, namely air and water vapour. Air conditioning processes involve a mixture of air and water vapour at relatively low pressures and temperatures, and therefore these mixtures of substances may be assumed to behave like ideal gases.

Boyle's Law

It states that for a given quantity of an ideal gas at a constant temperature, the volume is inversely proportional to the pressure.

$$\text{That is, } pV = \text{Constant} \quad \text{if } T = \text{constant} \quad (4.1)$$

Charles' Law

The law expresses the variation of the volume with temperature of an ideal gas undergoing a constant pressure process. It states that at constant pressure, the volume of an ideal gas is directly proportional to its absolute temperature.

$$\text{That is, } V = aT, \quad \text{if } p = \text{constant} \quad (4.2)$$

in which a is a constant

The General Gas Law

By application of the principle of joint variation, $V \propto \frac{T}{P}$

That is,

$$\frac{PV}{T} = R_g \quad (4.3)$$

Where R_g is the constant of proportionality and is a characteristic of every gas. The values of the constant for dry air and water are 0.286 kJ/kg K and 0.4619 kJ/kg K respectively.

Dalton's Law

This law is based on experiment and is found to be obeyed more accurately by gas mixtures at low pressures. It states that the pressure of a mixture of gases is equal to the sum of the partial

pressures of the constituents. ($P_T = \sum_{i=1}^n P_i$, where P_i is the partial pressure of the constituents)

The partial pressure of each constituent is that pressure which the gas would exert if it occupied alone that volume, occupied by the mixture at the same temperature.

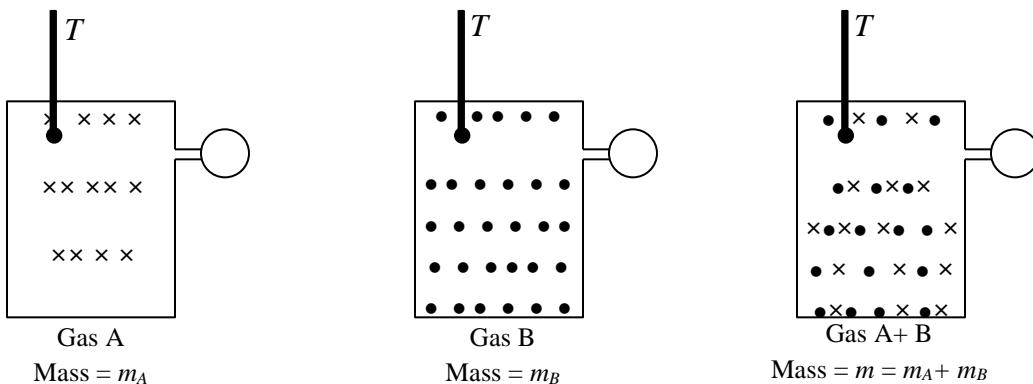


Figure 60: Illustrating the Partial Pressure of a gas

This is expressed diagrammatically in Figure 60. The gases A and B originally occupying volume V at temperature T , are mixed in the third vessel, which is of the same volume and is at the same temperature.

By the conservation of mass

$$m = m_A + m_B \quad (4.4)$$

By Dalton's Law

$$P = P_A + P_B \quad (4.5)$$

The law can be extended to a number of gases

$$\text{i.e } m = \sum m_i \quad (4.6)$$

where m_i is the mass of a constituent.

Similarly

$$P = \sum p_i \quad (4.7)$$

Where P_i is the pressure of a constituent.

Gibbs-Dalton Law

Dalton's law was reformulated by Gibbs to include a second statement on properties of mixtures. The combined statement, which is an empirical law, is stated as follows:

The internal energy, enthalpy and entropy of a gaseous mixture are respectively equal to the sums of the internal energies, enthalpies and entropies of the constituents.

Each constituent has internal energy, enthalpy and entropy, which it would have if it occupied alone that volume by the mixture at the same temperature of the mixture.

This law is valid for low pressures. This statement leads to the equation

$$mu = \sum m_i u_i \quad (4.8)$$

$$mh = \sum m_i h_i \quad (4.9)$$

$$ms = \sum m_i s_i \quad (4.10)$$

Amagat's Law or Leduc's Law

This is another empirical law and is stated as follows:

The volume of a mixture of gases is equal to the sum of the volumes of the individual constituents when it each exits alone at the pressure and temperature of the mixture.

If a constituent i is placed in an empty vessel having the volume of the mixture V , at the mixture temperature T , it will exert a partial pressure p_i . If the temperature is kept constant and the pressure is increased to the pressure of the mixture p , the volume reduces to V_i .

$$V_i = \frac{p_i}{p} V$$

(4.10a)

Summing up for all the constituents we obtain,

$$\sum V_i = \frac{V}{p} \sum p_i \quad (4.10b)$$

But from Dalton's law $\sum p_i = p$ and hence

$$V = \sum V_i \Big|_{p,T} \quad (4.11)$$

1-4.2 Mixtures of Perfect Gases

If a mixture occupies a volume V at a temperature T , the partial pressure p_i , of any constituent when it alone occupies the volume V at temperature T is given by

$$p_i V = n_i \tilde{R} T$$

p_i , is the partial pressure of the constituent, and n_i is the amount of substance of the constituent in the volume V . By writing an equation of the above from for each constituent and summing up, we have

$$V \sum p_i = \tilde{R}T \sum n_i$$

For a process which does not involve a chemical change, the number of molecules does not change and is given by

$$n = \sum n_i$$

Also, from Dalton's law we have

$$p = \sum p_i \Big|_{V,T}$$

Hence, for the mixture we can write

$$pV = n\tilde{R}T$$

But the molar mass of the mixture is

$$\tilde{m} = \frac{m}{n} \quad (4.12)$$

Where m is the mass of the mixture. We therefore have

$$pV = \frac{m}{\tilde{m}} \tilde{R}T = mRT$$

Note that \tilde{R} is the universal gas constant and $R = \tilde{R}/\tilde{m}$, is the specific gas constant for the mixture. Per unit mass of mixture this equation becomes.

$$pv = RT \quad (4.13)$$

An expression for the specific gas constant of a mixture in terms of its constituents can be found as follows:

$$R = \frac{\tilde{R}}{\tilde{m}} = \frac{n}{m} \tilde{R} \quad \rightarrow \quad R = \sum \frac{m_i}{m} R_i$$

Substituting $n = \sum n_i = \sum (m_i/\tilde{m}_i)$ and $\tilde{m}_i = (\tilde{R}/R_i)$, this becomes

$$R = \frac{\tilde{R}}{m} \sum m_i \frac{R_i}{\tilde{R}} = \sum \frac{m_i}{m} R_i \quad (4.14)$$

m_i/m is the mass fraction of the constituent i having the gas constant R_i

Partial Pressure

The partial pressure of the i th constituent may be obtained as follows,

$$p_i V = n_i \tilde{R}T \quad (4.15)$$

But for the mixture of gases, $pV = n\tilde{R}T$ (4.16)

Dividing equation (4.15) by (4.16) we have

$$\frac{p_i}{p} = \frac{n_i}{n} \Rightarrow p_i = \frac{n_i}{n} p \quad (4.17)$$

$\boxed{n_i/n}$ is called the mole fraction of the *i*th constituent of the mixture and P_i is the partial pressure of the *i*th constituent in the mixture.

From (4.17) and (4.10a)

$$\boxed{\frac{p_i}{p} = \frac{n_i}{n} = \frac{V_i}{V}} = y_i$$

Specific heat

$$dU = mc_v dT$$

From the Gibbs-Dalton Law

$$U = \sum U_i \Big|_{V,T}$$

In differential form, the equation therefore becomes

$$dU = m_i c_{vi} dT \therefore mc_v dT = \sum m_i c_{vi} dT$$

Hence the c_v for the mixture is given by; $c_p = \sum \frac{m_i}{m} c_{pi}$

$$c_v = \sum \frac{m_i}{m} c_{vi} \quad (4.18)$$

Another consequence of the equation of state of a perfect gas is that

$$c_p - c_v = R$$

The specific heat capacity at constant pressure of a mixture of perfect gases is therefore given by

$$c_p = \sum \frac{m_i}{m} R_i + \sum \frac{m_i}{m} c_{vi} = \sum \frac{m_i}{m} c_{pi} \quad (4.19)$$

The change in entropy from some reference state 0 to some other state is derived by compiling the equation of state and the first law of Thermodynamics and is given by

$$m(s - s_0) = \sum m_i c_{pi} \ln \frac{T}{T_o} - \sum m_i R_i \ln \frac{P_i}{P_{io}} = \sum m_i \left(c_{pi} \ln \frac{T}{T_o} - R_i \ln \frac{P_i}{P_{io}} \right) \quad (4.20)$$

1-4.3 Adiabatic Mixing of Perfect Gases

Consider two gases A and B separated from each other in a closed vessel by a thin diaphragm as shown in Figure 61. If the diaphragm is punctured or removed, then the gases mix as in Figure 62, and each constituent occupies the total volume of the mixture, behaving as if the other gas were not present. The process is equivalent to a free expansion of each gas, and the process is irreversible. Assuming that the vessel is thermally insulated, then the entropy of the system increases but the internal energy of the mixture remains constant. That is

Gas A	Gas B	Gas mixture
m_A	m_B	
n_A	n_B	
p_A	p_B	
T_A	T_B	
V_A	V_B	
		p, T
		$V = V_A + V_B$
		$m = m_A + m_B$
		$n = n_A + n_B$

Figure 61: Two gases separated by a partition

$$U_1 = n_A \tilde{c}_{vA} T_A + n_B \tilde{c}_{vB} T_B$$

and

$$U_2 = (n_A \tilde{c}_{vA} + n_B \tilde{c}_{vB}) T$$

Extending this result to any number of gases, then

$$U_1 = \sum n_i \tilde{c}_{vi} T_i \quad \text{and} \quad U_2 = T \sum n_i \tilde{c}_{vi}$$

Then

$$U_1 = U_2$$

And hence

$$T = \frac{\sum n_i \tilde{c}_{vi} T_i}{\sum n_i \tilde{c}_{vi}} \quad (4.21)$$

Another form of mixing is that which occurs when streams of fluid meet to form a common stream in a steady flow. The steady-flow energy equation can be applied to the mixing section, and kinetic and potential energies are normally assumed negligible. For such a situation, it can be shown that

$$T = \frac{\sum \dot{m}_i c_{pi} T_i}{\sum \dot{m}_i c_{pi}} \quad (4.22)$$

Since $\tilde{c}_p = \dot{m} c_p$ and $\tilde{m} = m/n$, then $n \tilde{c}_p = m c_p$

Relate this concept to what occurs in a mixing chamber. There is flow into the system and therefore the concept of enthalpy is used. Assume that in a quantity of atmospheric air the vapour pressure is 0.02337 bar at 30 °C and the total pressure is 1.013 bar.

Hence,

$$T = \frac{\sum (n_i \tilde{c}_{pi} T_i)}{\sum n_i \tilde{c}_{pi}} \quad (4.23)$$

1-4.4 Gas and Vapour Mixtures

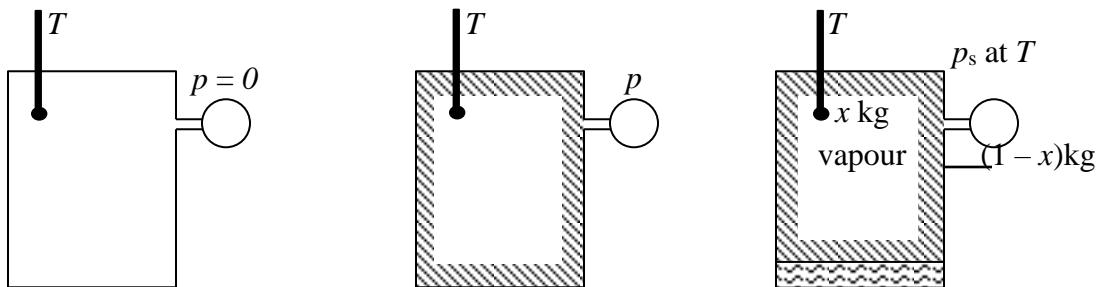


Figure 63: Analysis of gas and vapour mixture mixtures

Consider a vessel of fixed volume, which is maintained at a constant temperature as shown in Figure 64 (a). The vessel is evacuated and the absolute pressure is zero. In Figure 64 (b), a small quantity of water is introduced into the vessel and it evaporates to occupy the whole volume. For a small quantity of water introduced initially, the pressure in the vessel will be less than the saturation pressure corresponding to the temperature of the vessel. At this condition of pressure and temperature the vessel is occupied by superheated vapour.

As more water is introduced, the pressure increases and the water continues to evaporate until such a condition is reached that the volume can hold no more vapour. Any additional water introduced into the vessel after this state will not evaporate but will exist as water, the condition being as in Figure 63 (c). Per 1 kg of water introduced, the vessel contains x kg of dry saturated vapour and $(1-x)$ kg of water. The dryness fraction of the wet steam is therefore x .

During the process of evaporation, the temperature remains constant. However, if the temperature of the mixture is raised by the addition of heat, then more vapour will evaporate and the pressure in the vessel will increase. Eventually the vessel will contain superheated vapour as before, but at a higher pressure and temperature. The steady flow energy equation can be applied to this non-flow process.

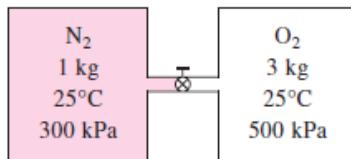
When a little water is sprayed into a vessel containing a gas mixture, then the vapour formed will exert the saturation pressure corresponding to the temperature of the vessel, and this, partial pressure of the vapour in the mixture. It must be noted that vapour is said to be saturated when it is in contact with its liquid. When a mixture contains a saturated vapour, then the partial pressure of the vapour can be found from tables at the temperature of the mixture.



Self Assessment 1-4

1. A gas mixture consists of 8 kmol of H₂ and 2 kmol of N₂. Determine the mass of each gas and the apparent gas constant of the mixture. **Suggested Answers:** 16 kg; 56 kg; 1.155 kJ/kg · K

2. A rigid tank that contains 1 kg of N₂ at 25°C and 300 kPa is connected to another rigid tank that contains 3 kg of O₂ at 25°C and 500 kPa. The valve connecting the two tanks is opened, and the two gases are allowed to mix. If the final mixture temperature is 25°C, determine the volume of each tank and the final mixture pressure. **Suggested Answers:** 0.295 m³, 0.465 m³, 422 kPa



3. Air has the following composition on a mole basis: 21 percent O₂, 78 percent N₂, and 1 percent Ar. Determine the gravimetric analysis of air and its molar mass.

Suggested Answers: 23.2 percent O₂, 75.4 percent N₂, 1.4 percent Ar, 28.96 kg/kmol

SESSION 2-4: AIR CONDITIONING APPLICATION

2-4.1 Psychrometry

The atmosphere is a mixture of air and superheated vapour. Knowledge of the properties of humid air is essential for the science of meteorology and for the design of air conditioning plants. The technical terms, which arise in psychrometry are explained below:

1. Humidity ratio (W)

It is the ratio of the mass of the water vapour to the mass of dry air in a given volume of mixture. This is

$$W = \frac{m_s}{m_a} = \frac{m_s/V}{m_a/V} = \frac{v_a}{v_s} \quad (4.24)$$

Where v_a and v_s are the specific volumes of the dry air and vapour in the mixture. Treating the water vapour as a perfect gas, and writing for the partial pressure of the vapour, we have

$$p_s V = m_s R_s T$$

Similarly for the dry air,

$$p_a V = m_a R_a T$$

It follows that

$$W = \frac{R_a p_s}{R_s p_a} \quad (4.25)$$

Now $R_a = 0.287 \text{ kJ/kg K}$ and $R_s = 8.3145/18.015 = 0.4615 \text{ kJ/kg K}$

Also $p_a = p - p_s$ where p is the barometric pressure. Therefore

$$W = 0.622 \frac{p_s}{p_a} = 0.622 \frac{p_s}{p - p_s} \quad (4.26)$$

2. Specific Humidity (ω)

It is the ratio of the mass of water vapour to the total mass of moist air in a given volume of the mixture

That is,

$$\omega = \frac{m_s}{(m_a + m_s)} \quad (4.27)$$

If the mass of water vapour is neglected compared to the mass of dry air then $[W = \omega]$. For the purpose of air conditioning, the simplified case is accepted.

3. Relative Humidity (ϕ)

It is the ratio of the actual mass of the water vapour in a given volume of air to that which it would contain if it were saturated at the same temperature i.e it is the ratio of the actual partial pressure p_s of the vapour to the partial pressure p_g of the vapour when the air is saturated at the same temperature.

Thus

$$\phi = \frac{p_s}{p_g} \quad (4.28)$$

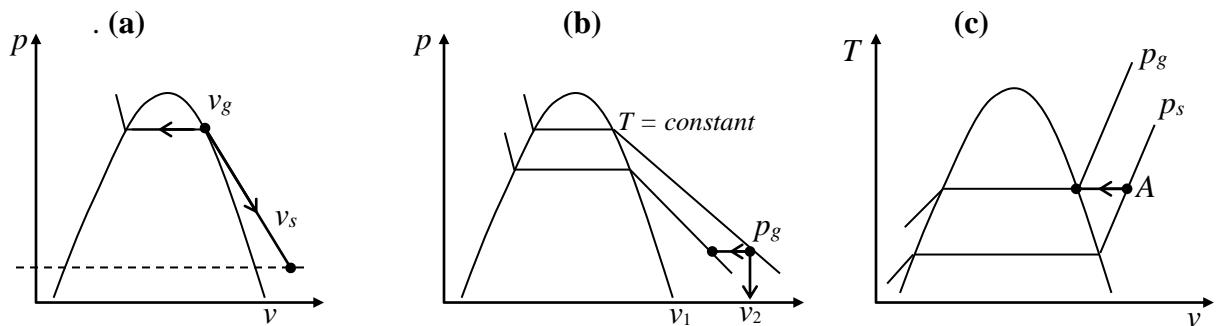


Figure 64: Property diagrams showing partial pressure and saturation vapour pressure

If the vapour is treated as a perfect gas then

$$\phi = \frac{m_s}{(m_s)_{sat}} = \frac{p_s}{p_g} \quad (4.29)$$

Dividing m_s and $(m_s)_{sat}$ by V , we have

$$\phi = \frac{m_s/V}{(m_s)_{sat}/V} = \frac{v_g}{v_s} \quad (4.30)$$

Where v_s is the actual specific volume of the vapour in the mixture, and v_g is the specific volume of the vapour when the air is saturated at the same temperature.

In air-conditioning, the concept of degree of saturation (μ) is used as an alternative to relative humidity. It is defined as:

$$\mu = \frac{\omega(p - p_g)}{0.622p_g} \quad (4.31)$$

2-4.2 Temperature and pressure parameters

2-4.2.1 Dew point

It is the temperature to which unsaturated air must be cooled at constant pressure for it to become saturated (or for condensation to begin). The cooling process undergone by the process is illustrated in Figure 65(c). The partial pressure of the vapour at the dew point is the saturation pressure p_{gd} corresponding to the dew-point temperature, and hence $p_s = p_{gd}$.

The relative humidity can be found from Equation 4.32

$$\phi = \frac{p_{gd}}{p_g} \quad (4.32)$$

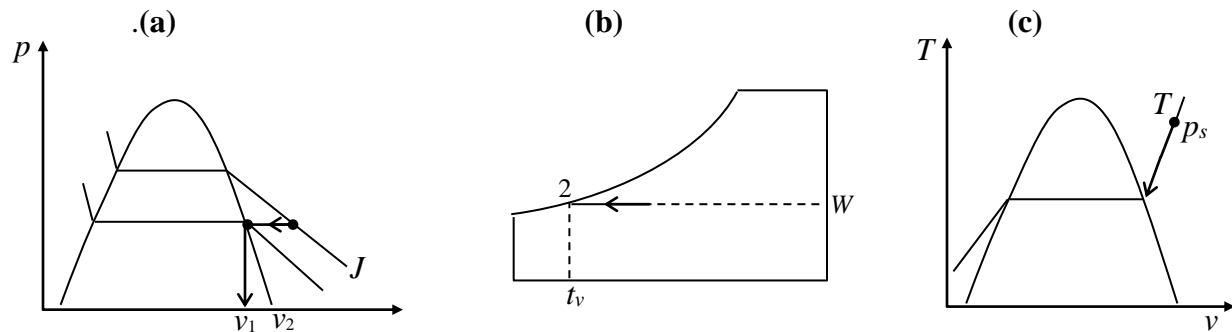


Figure 65: Property diagrams illustrating dew point temperature

2-4.2.2 Dry-bulb temperature, Wet-bulb temperature and Wet-bulb depression

If a stream of unsaturated air flows past a thermometer having a wetted sleeve of cotton or linen around the bulb, the temperature recorded will be less than the actual temperature of the air.

The temperature of the air that flows past the wetted sleeve falls because of evaporation from the wetted sleeve and subsequently heat is transferred from the air to the sleeve. A steady condition is then attained when the rate of heat transfer balances the loss of energy due to evaporation. The thermometer reading of this steady state is the **wet-bulb temperature**. The actual temperature of the air is referred to as the **dry-bulb temperature**.

When the air is unsaturated with vapour, the wet-bulb temperature lies between the dry-bulb temperature and the dew point. It must be noted that the lower the relative humidity of the air, the more rapid the evaporation from the wet-bulb and the larger the wet-bulb depression, i.e. the difference between the dry- and wet- bulb temperatures. When the air is saturated, the wet-bulb, dry-bulb and dew-point temperatures are identical.

2-4.2.3 Vapour pressure

This is the partial pressure of the water vapour in an air sample. If the air is saturated, the partial pressure is known as saturation pressure. The saturation pressure is the pressure at which a liquid vaporises or a vapour condenses. Note that for air water-vapour mixture, only the water vapour condenses or vaporises and so the saturation pressure strictly applies to the water portion. In air conditioning, we deal with problems in which the water vapour is in stable equilibrium with its own liquid in the presence of dry air. There are a number of empirical relations for evaluating the partial pressure of the water vapour but these would not be discussed at this level.

Humid specific volume

The specific volume of moist air is expressed in terms of dry air as

$$v = V_{\max}/m_a = R_a T / p_a = \frac{R_a T}{P_T} (1 + p_s/p_a) = \frac{R_a T}{P_T} [1 + 1.608W] \quad (4.33)$$

The total enthalpy of moist air

$$\begin{aligned} H_T &= H_a + H_s \\ m h_T &= m_a h_a + m_s h_s \end{aligned}$$

If we assume $m \approx m_a$ then, ie $m_s \ll m_a$

$$h = h_a + \omega h_s \quad (4.34)$$

h_a is the partial enthalpy of dry air and h_s is the partial enthalpy of the water vapour

But $h_s \approx h_g @ t$ and these values can be read from steam Tables or evaluated using Equation 4.35.

$$\begin{aligned} h_a &\approx c_{pa} t \\ h_s &= 2500.9 + 1.82t \end{aligned} \quad (4.35)$$

$$c_{pa} \approx 1.005 \text{ kJ/kg K}$$

Similarly, the humid specific heat at constant pressure is given by

$$c_p = c_{pa} + \omega c_{ps} \quad (4.36)$$

2-4.2.4 Psychrometric Chart

The psychrometric chart (Fig. 66) enables the enthalpy of humid atmospheric air, of any relative humidity, to be read off directly without resorting to rigorous and complex formulae. The psychrometric chart is drawn for fixed barometric pressure so that ω and t can be regarded as the only independent variables. The chart is used for normal temperatures and barometric pressure of 101.325 kPa but it can be used without loss of accuracy for pressures between 0.9 and 1.2 bar or 90 – 120 kPa.

The following are to be noted of the psychrometric chart:

- i) the enthalpy of the mixture h is expressed as kJ/kg dry air in the mixture. Equation 4.34 is used to find the value of h . The value ωh , depends on the specific humidity of the mixture and both the pressure and temperature of the air [see Equations (4.26), (4.34) and (4.35)]. Thus, Equation 4.26 indicates that when ω and p are known, the partial pressure of the vapour p_s is determined. If in addition the temperature t is known then h_s is fixed. The psychrometric chart is drawn for a fixed barometric pressure so ω and t can be regarded as the only independent variables. These form the y - and x -axes, respectively of the chart, and lines of constant enthalpy are plotted on the diagram.

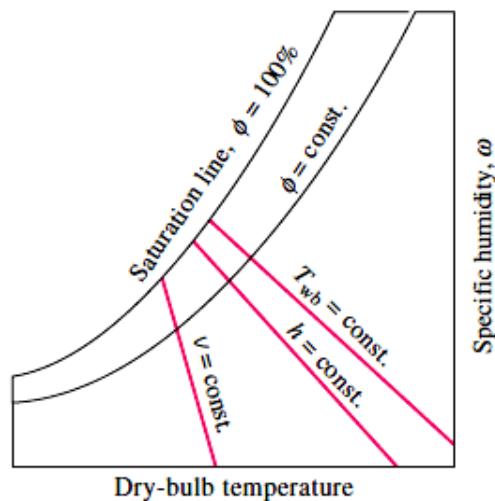


Figure 66: Schematic of a psychrometric chart

- ii) Equation 4.29 shows that for a given barometric pressure, the relative humidity ϕ is a function of ω_s , p_s and p_g . p_g is the saturation pressure corresponding to the dry-bulb temperature t , and we have already noted that p_s is a function of ω and t , and therefore the lines of constant relative humidity can be added to the diagram. Since relative humidity values are also given in relation to the dry- and wet-bulb temperatures, lines of constant wet-bulb temperature are also included. Lines of constant dew point are horizontal because they are also lines of constant specific humidity. The value of the wet-bulb temperature at any state along the saturation line ($\phi = 100\%$) is equal to the dry-bulb temperature.
- iii) Lines of constant specific volume v_a of dry air in the mixture are usually included. v_a is a function of the dry-bulb temperature, t and partial pressure, P_v of the mixture and therefore of the specific humidity [see Equation 4.33]. Therefore, for a given barometric pressure p , v_a is a function of t and ω .

2-4.3 Psychrometric Processes

The four basic essential air-conditioning processes are simple cooling (lowering the temperature while humidity ratio remains constant), simple heating (increasing the temperature while humidity ratio remains constant), humidifying (adding moisture), and dehumidifying (removing

moisture), these are illustrated by Figure 68, 67a, 67b, respectively. However, the section begins with the distinction between ‘latent’ and ‘sensible’ heat.

Subsequently, two other possible or realistic combinations of the four basic essential air-conditioning processes are introduced and explained.

Latent heat

Any situation in which evaporation of liquid takes place as is illustrated in Figure 67(a) without change in the dry-bulb temperature is accompanied by absorption of heat known as the latent heat of vaporisation. On the other hand, condensation of vapour without change in dry bulb temperature, as shown in Figure 67(b), involves removal of heat from the vapour phase.

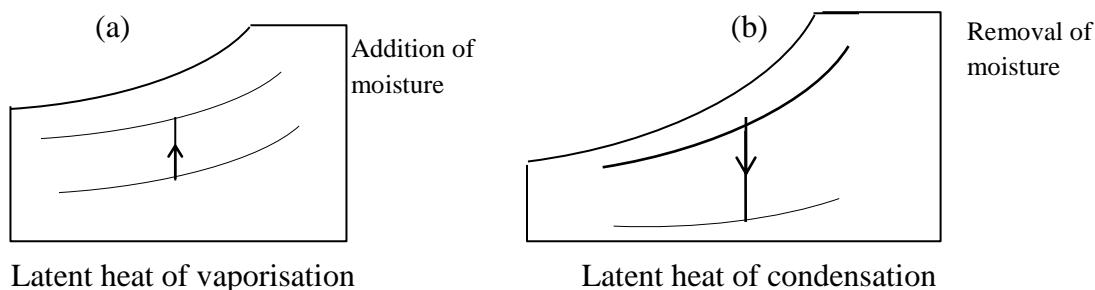


Figure 67:Latent heating process

Sensible heat

Sensible heat results from a process that raises the temperature of the system without changing its moisture content. Also, any cooling process in which the temperature drops without any effect on the moisture content is also accompanied by removal of sensible heat.

Sensible cooling or heating occurs when moist air flows over a cooling coil or heating coil, respectively. During the process, the air exchanges heat with a medium, while it flows through the coil or heater. For cooling there is the additional requirement that the coil size should be such as to prevent condensation throughout the process. Sensible cooling and heating processes are represented on psychrometric charts on Figure 68 and 69, respectively.

Simple Cooling

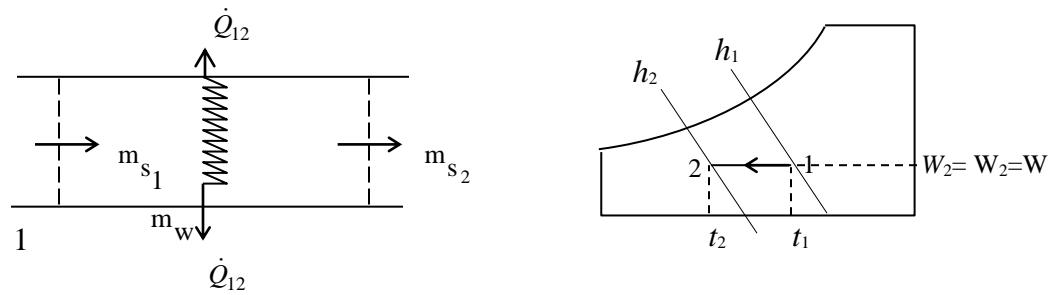


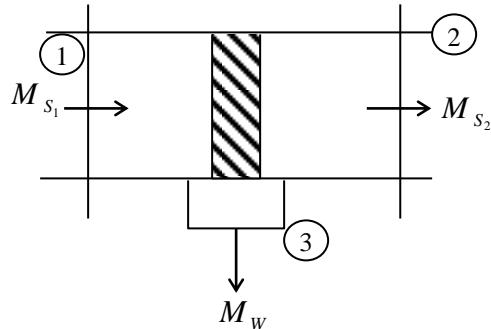
Figure 68: Simple cooling process

In sensible cooling, the mass of water condensed is given by:

$$\dot{m}_w = \dot{m}_{s1} - \dot{m}_{s2} = \dot{m}_a (\omega_1 - \omega_2)$$

And the heat transferred in the cooler is given also by:

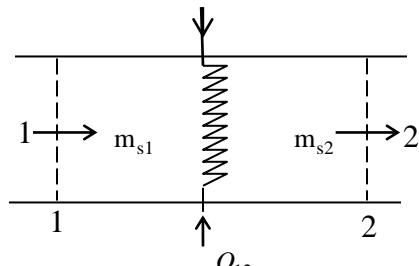
$$\dot{Q}_{12} = \dot{m}_a (h_1 - h_2) - \dot{m}_w h_{w3} \quad \text{Note: } h_{w3} = h_f @ t_3$$



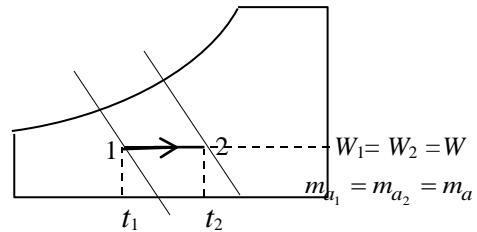
Simple Heating

For simple heating, the heat transferred is given by:

$$\dot{Q}_{12} = \dot{m}_a (h_2 - h_1)$$



(a)



(b)

Figure 69: Simple heating process

Cooling with Dehumidification

Cooling with dehumidification [Figure 70] is the simultaneous removal of heat (both sensible and latent) and moisture from moist air. It is achieved when air is cooled below the dew point.

Note that: Air is sometimes dehumidified to drop its relative humidity to more desirable levels.

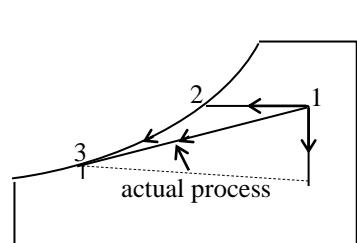


Figure 70: Cooling with dehumidification

Note: $\omega_3 < \omega_1$

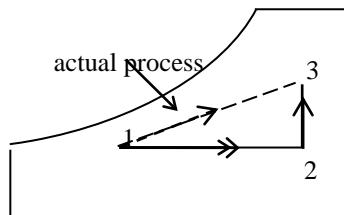


Figure 71: Heating with humidification

Applying the mass and energy balances on the cooling and dehumidification section gives:

$$\text{Dry air mass: } \dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$$

$$\text{Water mass balance: } \dot{m}_{a_1} \omega_1 = \dot{m}_{a_3} \omega_3 + \dot{m}_w \Rightarrow \dot{m}_w = \dot{m}_a (\omega_1 - \omega_3)$$

$$\text{Energy balance: } \sum_{\text{in}} \dot{m}h = \dot{Q}_{\text{out}} + \sum_{\text{out}} \dot{m}h \Rightarrow Q_{\text{out}} = \dot{m}_a (h_1 - h_2) - \dot{m}_w h_w$$

Heating and Humidifying

Air to be heated and humidified must first pass through a heating coil and then a spray chamber subsequently. Ideally, it is a process at constant humidity ratio followed by another at constant dry-bulb temperature during the passage of the air through the spray chamber [Figure 71].

Note that: Heated air is sometimes humidified to achieve a higher level of comfort, because very dry air causes dry skin, respiratory difficulties, and increased static electricity and hence the relevance of heating with humidification for comfort.

Evaporative Cooling

As water evaporates, h_{fg} is absorbed from the water body and the surrounding air. As a result both the water and air are cooled during the process.

This is the simultaneous removal of sensible heat from and addition of moisture to an air stream brought into contact with spray water whose temperature remains essentially constant at the wet bulb temperature.

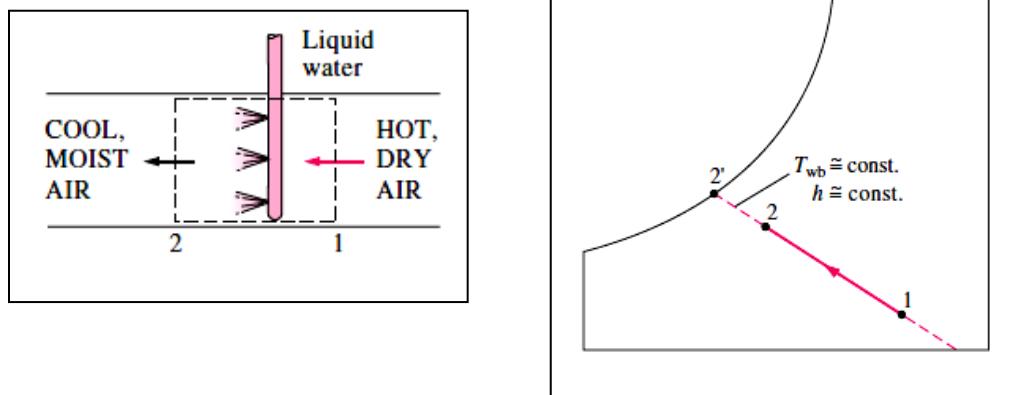
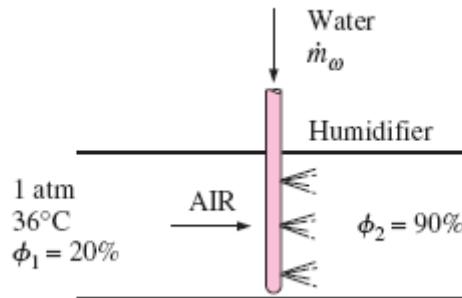


Figure 72: Evaporative Cooling

Note that: Heated air is sometimes humidified to achieve a higher level of comfort, because very dry air causes dry skin, respiratory difficulties, and increased static dectricity.

Worked Example 4.1 (Evaporative Cooling)

Air enters an evaporative cooler at 1 atm, 36°C, and 20 percent relative humidity at a rate of 3 m³/min, and it leaves with a relative humidity of 90 percent. Determine (a) the exit temperature of the air and (b) the required rate of water supply to the evaporative cooler.



Schematic for example 4.1

Assumptions:

1. The control volume shown in the accompanying figure operates at steady state
2. There is no heat and work transfer with the surroundings.
3. The water added to the evaporative cooler enters as a liquid and evaporates fully into the moist air.
4. The pressure remains constant throughout at 1 atm.

Solution:

From the psychrometric chart, for $T_{db_1} = 36^\circ\text{C}$ and $\phi_1 = 20\%$

$$T_{wb_1} = T_{wb_2} = 19.5^\circ\text{C}, \quad \omega_1 = 0.0075 \frac{\text{kg H}_2\text{O}}{\text{kg dry air}}, \quad h_1 = h_2 = 55.2 \frac{\text{kJ}}{\text{kg dry air}}, \quad v_1 = 0.886 \frac{\text{m}^3}{\text{kg}}$$

At state 2, for $T_{wb_1} = 19.5^\circ\text{C}$ and $\phi_2 = 90\%$

$$T_{db_2} = 21^\circ\text{C} \quad \text{and} \quad \omega_2 = 0.0138 \frac{\text{kg H}_2\text{O}}{\text{kg dry air}}$$

Hence the exit temperature of the air is 21 °C.

Applying mass balance to the water vapour;

$$\dot{m}_a \omega_1 + \dot{m}_w = \dot{m}_a \omega_2 \Rightarrow \dot{m}_w = \dot{m}_a (\omega_2 - \omega_1)$$

$$\dot{m}_a = \frac{\dot{V}_a}{v_1} = \frac{3}{0.886} \times \frac{\text{kg}}{\text{min}} = 3.386 \frac{\text{kg}}{\text{min}}$$

$$\text{Hence, } \dot{m}_w = 3.386 (0.0138 - 0.0075) = 0.0213 \frac{\text{kg}}{\text{min}}$$

Therefore, the required rate of water supplied is 0.0213 kg/min.

Example 4.2 (Heating and humidification)

Air at 1 atm, 15 °C and 60 % relative humidity is first heated to 20 °C in a heating section and then humidified by introducing water vapour. The air leaves the humidifying section at 25 °C and 65 % relative humidity. Determine

- a) the amount of steam added to the air, and
- b) the amount of heat transfer to the air in the heating section per unit mass of air.

SOLUTION:

$$\dot{m}_a = \dot{m}_{a_2} = \dot{m}_a$$

From the Psychrometric chart, at $T_l = 15^\circ\text{C}$ and $\phi_l = 60\%$

$$\omega_1 = 0.0064 \text{ kg H}_2\text{O/kg dry air} = \omega_2$$

$$h_1 = 31.1 \text{ kJ/kg dry air}$$

$$h_2 = 36.2 \text{ kJ/kg dry air (using the value of } \omega_2, T_2 = 20^\circ\text{C})$$

$$\omega_3 = 0.0129 \text{ kg H}_2\text{O/kg dry air (using the value of } T_3 = 25^\circ\text{C and } \phi_3 = 65\%)$$

$$\begin{aligned} \text{a) } \omega_3 &> \omega_2 \quad \Delta\omega = \omega_3 - \omega_2 = 0.0129 - 0.0064 \\ &= 0.0065 \text{ kg H}_2\text{O/kg dry air} \end{aligned}$$

$$\text{b) } \frac{\dot{Q}_{in}}{\dot{m}_a} = \dot{q}_{in} = h_2 - h_1 = 36.2 - 31.1 = 5.1 \text{ kJ/kg dry air}$$

Example 4.3 (cooling and dehumidification followed with heating)

An air conditioning system is to take in air at 1 atm, 34°C and 70% relative humidity and deliver it at 22°C and 50% relative humidity. The air flows first over the cooling coils, where it is cooled and dehumidified, and then over the resistance heating wires, where it is heated to the desired temperature. Assuming that the condensate is removed from the cooling section at 10°C . Determine:

- a) the temperature of air before it enters the heating section.
- b) the amount of heat removed in the cooling section and
- c) the amount of heat transferred in the heating section, both in kJ/kg dry air .

SOLUTION

From the Psychrometric chart, at $T_l = 15^\circ\text{C}$ and $\phi_l = 60\%$

$$h_1 = 95.2 \text{ kJ/kg dry air} = \omega_2$$

$$\omega_1 = 0.0238 \text{ kg H}_2\text{O/kg dry air and}$$

$$h_3 = 43.1 \text{ kJ/kg dry air}$$

$$\omega_3 = 0.0082 \text{ kg H}_2\text{O/kg dry air}$$

$$\begin{aligned} \text{b. } \dot{Q}_{out} &= \dot{m}_{a_1} h_1 - \left(\dot{m}_{a_1} h_2 + \dot{m}_w h_w \right) = \dot{m}_a (h_1 - h_2) - \dot{m}_w h_w \\ &= \dot{m}_a (95.2 - 31.8) - 42.02(0.0238 - 0.0082) \end{aligned}$$

$$\dot{Q}_{out} = 62.7 \text{ kJ/kg dry air}$$

$$\text{c. } q_{out} = h_3 - h_2 = 43.1 - 31.8 = 11.3 \text{ kJ/kg dry air}$$

Also,

$$h_{w_3} \approx h_f = 42.02 \text{ kJ/kg dry air}$$

@ 10 °C

But

$\omega_3 = \omega_2$ and $\phi_2 = 100\%$ and from the psychrometric chart

$T_2 = 11.1 \text{ }^{\circ}\text{C}$ and $h_2 = 31.8 \text{ kJ/kg dry air}$

2-4.4 Wet cooling towers

Power plants, large air-conditioning systems, and some industries generate large quantities of waste heat that is often rejected to cooling water from nearby lakes or rivers. In some cases, however, the cooling water supply is limited or thermal pollution is a serious concern. In such cases, the waste heat must be rejected to the atmosphere, with cooling water recirculating and serving as a transport medium for heat transfer between the source and the sink (the atmosphere). One way of achieving this is through the use of wet cooling towers.

A **wet cooling tower** is essentially a semi enclosed evaporative cooler. An induced-draft counter flow wet cooling tower is shown schematically in Figure 74.

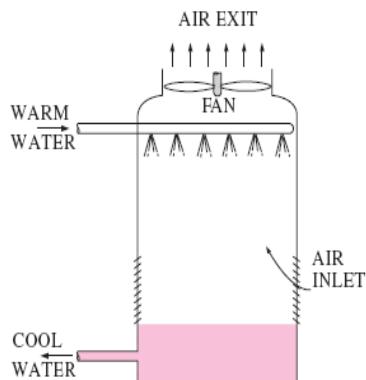


Figure 73: Induced-draft counter flow cooling tower

Air is drawn into the tower from the bottom and leaves through the top. Warm water from the condenser is pumped to the top of the tower and is sprayed into this airstream. The purpose of spraying is to expose a large surface area of water to the air. As the water droplets fall under the influence of gravity, a small fraction of water (usually a few percent) evaporates and cools the remaining water. The temperature and the moisture content of the air increase during this process. The cooled water collects at the bottom of the tower and is pumped back to the condenser to absorb additional waste heat. Makeup water must be added to the cycle to replace the water lost by evaporation and air draft. To minimize water carried away by the air, drift eliminators are installed in the wet cooling towers above the spray section.

The air circulation in the cooling tower described is provided by a fan, and therefore it is classified as a forced-draft cooling tower.

Another popular type of cooling tower is the **natural-draft cooling tower**, which looks like a large chimney and works like an ordinary chimney. The air in the tower has high water vapour content, and thus it is lighter than the outside air. Consequently, the light air in the tower rises, and the heavier outside air fills the vacant space, creating airflow from the bottom of the tower to the top. The flow rate of air is controlled by the conditions of the atmospheric air. Natural-draft cooling towers do not require any external power to induce the air, but they cost a lot more to build than forced-draft cooling towers. The natural-draft cooling towers are hyperbolic in profile, as shown in Figure 75, and some are over 100 m high. The hyperbolic profile is for greater structural strength, not for any thermodynamic reason.

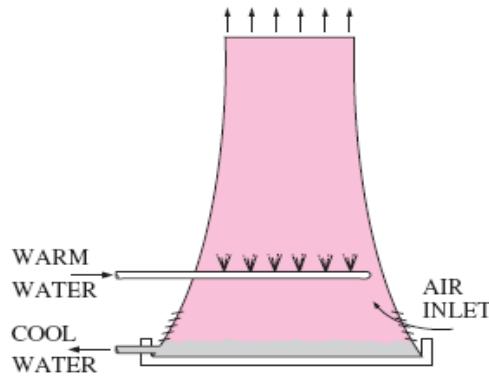


Figure 74: Natural-draft cooling tower

Worked Example 4.4: (Cooling of a Power Plant Warm Water by a Cooling Tower)

Warm water leaves the condenser of a power plant and enters a wet cooling tower at 35 °C at a rate of 100 kg/s. Water is cooled to 22 °C in the cooling tower by air that enters the tower at 1 atm, 20 °C, and 60 percent relative humidity and leaves saturated at 30°C. Neglecting the power input to the fan, determine (a) the volume flow rate of air into the cooling tower and (b) the mass flow rate of the required makeup water.

Solution Warm cooling water from a power plant is cooled in a wet cooling tower. The flow rates of makeup water and air are to be determined.

Assumptions 1 steady operating condition exists and thus the mass flow rate of dry air remains constant during the entire process. 2 Dry air and the water vapour are ideal gases. 3 The kinetic and potential energy changes are negligible. 4 The cooling tower is adiabatic.

Properties The enthalpy of saturated liquid water is 92.28 kJ/kg at 22°C and 146.64 kJ/kg at 35°C from steam tables. From the psychrometric chart,

$$h_1 = 42.2 \text{ kJ/kg dry air} \quad h_2 = 100 \text{ kJ/kg dry air}$$

$$\omega_1 = 0.0087 \text{ kg H}_2\text{O/kg dry air} \quad \omega_2 = 0.0273 \text{ kg H}_2\text{O/kg dry air}$$

$$v_1 = 0.842 \text{ m}^3/\text{kg dry air}$$

Analysis We take the entire *cooling tower* to be the system, which is shown schematically in Figure Q4.4. We note that the mass flow rate of liquid water decreases by an amount equal to the

amount of water that vaporizes in the tower during the cooling process. The water lost through evaporation must be made up later in the cycle to maintain steady operation.

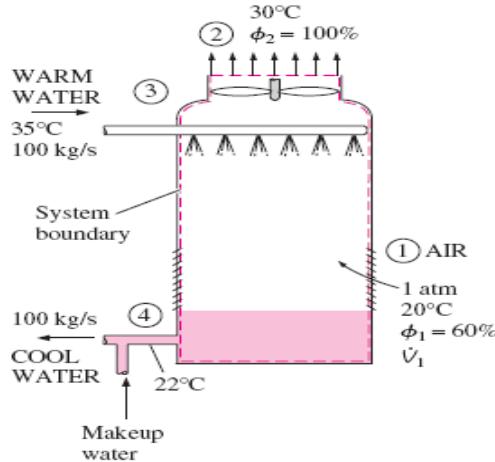


Figure Q4.4 Schematic for Example 4.4

Applying the mass and energy balances on the cooling tower gives

$$\text{Dry air mass balance: } \dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_{a_3}$$

$$\text{Water mass balance: } \dot{m}_3 + \dot{m}_{a_1} \omega_1 = \dot{m}_4 + \dot{m}_{a_2} \omega_2$$

$$\text{or } \dot{m}_3 - \dot{m}_4 = \dot{m}_a (\omega_2 - \omega_1) = \dot{m}_{\text{makeup}}$$

$$\text{Energy balance: } \sum_{\text{in}} \dot{m} h = \sum_{\text{out}} \dot{m} h \rightarrow \dot{m}_{a_1} h_1 + \dot{m}_3 h_3 = \dot{m}_{a_2} h_2 + \dot{m}_4 h_4$$

$$\text{Or } \dot{m}_3 h_3 = \dot{m}_a (h_2 - h_1) + (\dot{m}_3 - \dot{m}_{\text{makeup}}) h_4$$

Solving for \dot{m}_a gives

$$\dot{m}_a = \frac{\dot{m}_3 (h_3 - h_4)}{(h_2 - h_1) - (\omega_2 - \omega_1) h_4}$$

Substituting,

$$\dot{m}_a = \frac{(100 \text{ kg/s})[(146.64 - 92.28) \text{ kJ/kg}]}{[(100.0 - 42.2) \text{ kJ/kg}] - [(0.0273 - 0.0087)(9228) \text{ kJ/kg}]} = 96.9 \text{ kg/s}$$

Then the volume flow rate of air into the cooling tower becomes

$$\dot{V}_1 = \dot{m}_a v_1 = (96.9 \text{ kg/s})(0.842 \text{ m}^3/\text{kg}) = 81.6 \text{ m}^3/\text{s}$$

The mass flow rate of the required makeup water is determined from

$$\dot{m}_{\text{makeup}} = m_a (\omega_2 - \omega_1) = (96.9 \text{ kg/s})(0.0273 - 0.0087) = 1.80 \text{ kg/s}$$

Discussion Note that over 98 percent of the cooling water is saved and recirculated in this case.

2-4.5 Adiabatic Mixing of Airstreams

In many air-conditioning applications, there is the need to mix two airstreams. This is particularly manifested in production and process plants and hospitals which require that the conditioned air be mixed with a certain fraction of fresh air before it is routed into the living space (see Figure 76).

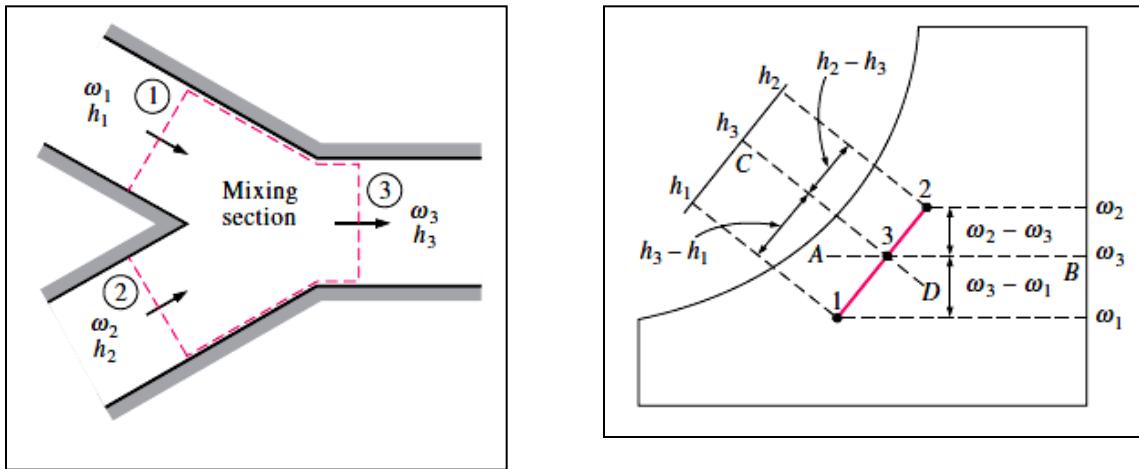


Figure 75: Adiabatic mixing of two air streams

The heat transfer with the surroundings is usually small, and thus the mixing processes can be assumed to be adiabatic. Mixing processes normally involve no work interactions, and the changes in kinetic and potential energies, if any, are negligible. Then the mass and energy balances for the adiabatic mixing of two airstreams reduces to

Mass of dry air :

$$\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3} \quad (4.37)$$

Mass of water vapour :

$$\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2} = \omega_3 \dot{m}_{a_3} \quad (4.38)$$

Energy :

$$\dot{m}_{a_1} h_1 + \dot{m}_{a_2} h_2 = \dot{m}_{a_3} h_3 \quad (4.39)$$

Eliminating \dot{m}_{a_3} from the relations above, we obtain:

$$\frac{\dot{m}_{a_1}}{\dot{m}_{a_2}} = \frac{\omega_2 - \omega_3}{\omega_3 - \omega_1} = \frac{h_2 - h_3}{h_3 - h_1} \quad (4.40)$$

When two airstreams at two different states (states 1 and 2) are mixed adiabatically, the state of the mixture (state 3) lies on a straight line connecting states 1 and 2 on the psychrometric chart, and the ratio of the distances 2-3 and 3-1 is equal to the ratio of mass flow rates \dot{m}_{a_1} and \dot{m}_{a_2} .

Worked Example 4.5: Adiabatic mixing of air streams

Saturated air leaving the cooling section of an air-conditioning system at 14°C at a rate of $50 \text{ m}^3/\text{min}$ is mixed adiabatically with the outside air at 32°C and 60 percent relative humidity at a rate of $20 \text{ m}^3/\text{min}$. Assuming that the mixing process occurs at a pressure of 1 atm, determine the specific humidity, the relative humidity, the dry-bulb temperature, and the volume flow rate of the mixture.

Assumptions: 1 steady operating condition exists. 2 Dry air and water vapor are ideal gases. 3 The kinetic and potential energy changes are negligible. 4 The mixing section is adiabatic.

Properties: The properties of each inlet stream are determined from the psychrometric chart to be

$$\begin{array}{ll} h_1 = 39.4 \text{ kJ/kg dry air} & h_2 = 79.0 \text{ kJ/kg dry air} \\ \omega_1 = 0.010 \text{ kg H}_2\text{O/kg dry air} & \text{And} \quad \omega_2 = 0.0182 \text{ kg H}_2\text{O/kg dry air} \\ v_1 = 0.826 \text{ m}^3/\text{kg dry air} & v_2 = 0.889 \text{ m}^3/\text{kg dry air} \end{array}$$

Analysis We take the mixing section of the streams as the system. The schematic of the system and the psychrometric chart of the process are shown in Figure Q 4.5.

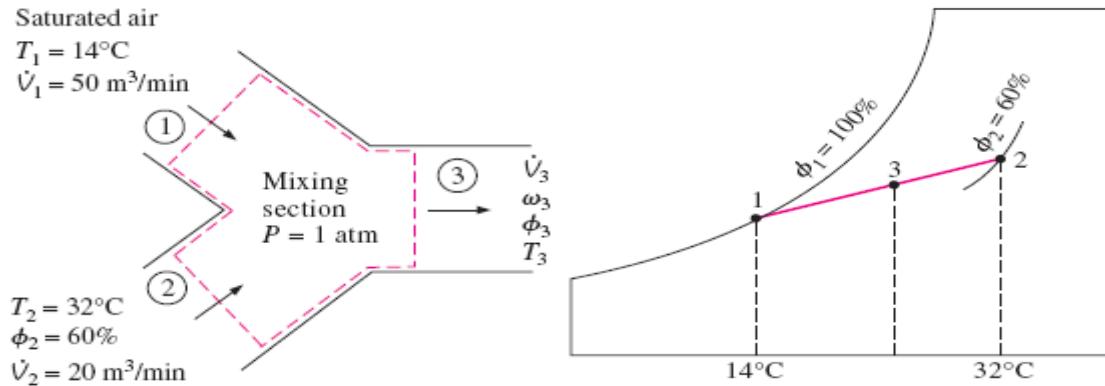


Figure Q 4.5 schematic for worked example 4.5

We note that this is a steady-flow mixing process. The mass flow rates of dry air in each stream are

$$\dot{m}_{a_1} = \frac{\dot{V}_1}{v_1} = \frac{50 \text{ m}^3/\text{min}}{0.826 \text{ m}^3/\text{kg dry air}} = 60.5 \text{ kg/min}$$

$$\dot{m}_{a_2} = \frac{\dot{V}_2}{v_2} = \frac{20 \text{ m}^3/\text{min}}{0.889 \text{ m}^3/\text{kg dry air}} = 22.5 \text{ kg/min}$$

From mass balance of dry air,

$$\dot{m}_{a_3} = \dot{m}_{a_1} + \dot{m}_{a_2} = (60.5 + 22.5) \text{ kg/min} = 83 \text{ kg/min}$$

The specific humidity and the enthalpy of the mixture can be determined from Eq. 4.40,

$$\frac{\dot{m}_{a_1}}{\dot{m}_{a_2}} = \frac{\omega_2 - \omega_3}{\omega_3 - \omega_1} = \frac{h_2 - h_3}{h_3 - h_1} \quad \text{hence} \quad \omega_3 = 0.0122 \text{ kg H}_2\text{O/kg dry air}$$

$$\frac{60.5}{22.5} = \frac{0.0182 - \omega_3}{\omega_3 - 0.010} = \frac{79.0 - h_3}{h_3 - 39.4} \quad h_3 = 50.1 \text{ kJ/kg dry air}$$

These two properties fix the state of the mixture. Other properties of the mixture are determined from the psychrometric chart:

$$T_3 = 19.0^\circ\text{C} \quad \phi_3 = 89\% \quad v_3 = 0.844 \text{ m}^3/\text{kg dry air}$$

Finally, the volume flow rate of the mixture is calculated as;

$$\dot{V}_3 = \dot{m}_{a_3} v_3 = (83 \text{ kg/min})(0.844 \text{ m}^3/\text{kg}) = 70.1 \text{ m}^3/\text{min}$$



Self Assessment 2-4

- An air-water vapour mixture enters a device with a pressure of 0.15 MPa, a temperature of 40 °C, and an unknown relative humidity. The mass of dry air entering per minute is 200 g. The mixture leaves the device at 30 °C, 0.15 MPa, and 80 percent relative humidity. Water is sprayed into the air and evaporated to do this cooling. If the water temperature is 30 °C and the device is adiabatic, how much water is required to run for 1 h?

Suggested Answer: [50 g/h]

- If the dew point temperature is 24 °C and the dry-bulb temperature is 30 °C, determine the wet-bulb temperature, $T_{\text{wet bulb}}$, the humidity ratio ω , the partial pressure of the vapour P_v , and the relative humidity ϕ . **Suggested Answers: [25.5 °C; 0.0189; 2.92kPa; 70%]**
- The air conditioner shown in Figure Q.3 operates at a pressure of 101.3 kPa.
 - Draw the entire process on a psychrometric chart.
 - Determine the mass flow rate of water required by the spray. **[0.3328 kg/min]**
 - Determine the rate of heat transfer. **[11.83 kW]**

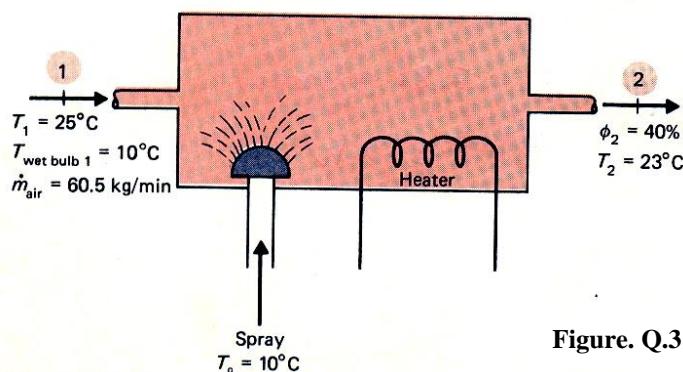


Figure. Q.3

4. The wet-bulb temperature is 25 °C, and the dry-bulb temperature is 32 °C. Using the psychrometric chart, determine ϕ , ω , P_v , and $T_{\text{dew point}}$.

Suggested Answers: [57%; 0.0171; 2.70 kPa; 22.5 °C]

5. Air enters a heating section at 95 kPa, 15 °C, and 30 % relative humidity at a rate of 6 m³/min, and it leaves at 25 °C. Determine:

- The rate of heat transfer in the heating section, and
- The relative humidity of the air at the exit

Suggested Answers: [(a) 69.39 kJ/min (c) 16.1 %]

6. Air enters a 40-cm-diameter cooling section at 1 atm, 32 °C, and 30 % relative humidity at 18 m/s. Heat is removed from the air at a rate of 1200 kJ/min. Determine:

- the exit temperature
- the exit relative humidity of the air, and
- the exit velocity

Suggested Answers: [(a) 24.4 °C (b) 46.6 % (c) 17.57 m/s]

7. Air enters a window air-conditioner at 1 atm, 32 °C and 30 % relative humidity at 8 m³/min, and it leaves as saturated air 12 °C. Part of the moisture in the air that condenses during the process is also removed at 12 °C. Determine:

- the rate of heat removal from the air
- the rate of moisture removal from the air

Suggested Answers: [(a) 0.112 kg/min (b) 462.4 kJ/min]

8. Air at 1 atm, 15 °C and 60 % relative humidity is first heated to 20 °C in a heating section and then humidified by introducing water vapour. The air leaves the humidifying section at 25 °C and 65 % relative humidity. Determine:

- The amount of water added to the air, and
- The amount of heat transfer to the air in the heat section.

Suggested Answers: [(a) 0.0065 kgH₂O/kg dry air (b) 5.1 kJ/kg dry air]

REACTING MIXTURES AND COMBUSTION

Introduction

In this unit we introduce reaction mixtures as a basis for introducing combustion of fuels. We also define the parameters used in combustion analysis such as air-fuel ratio, percentage theoretical air and dew point temperature as well as adiabatic flame temperature.



Learning Objectives

After studying this unit you should be able to:

1. Understand reaction mixtures as a basis for the combustion of pure hydrocarbon fuels.
2. Define the parameters used in combustion analysis, such as air-fuel ratio, percentage theoretical air and dew point temperature, etc.
3. Appreciate the meaning of adiabatic flame temperature.

SESSION 1-5: CHEMICAL REACTIONS

1-5.1 Combustion Process

The combustion process is a chemical reaction where fuel is oxidised and energy is released.

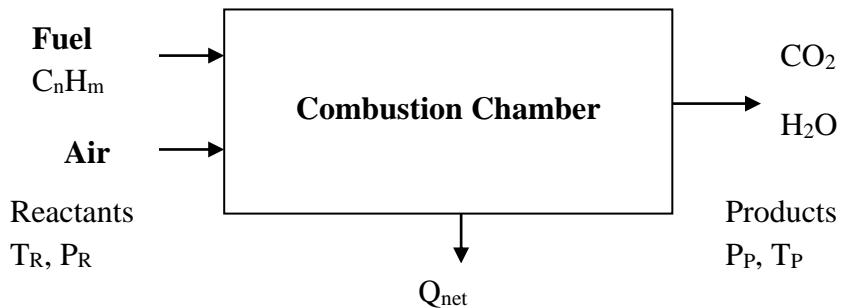


Figure 76: Combustion process

Fuels are usually composed of some compound or mixture containing carbon, C, and hydrogen, H₂.

Examples of hydrocarbon fuels are:

CH₄ Methane

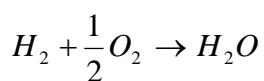
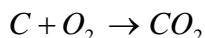
C₈H₁₈ Octane

Coal Mixture of C, H₂, S, O₂, N₂ and non-combustibles

Initially, we shall consider only those reactions that go to completion. The components prior to the reaction are called **Reactants** and the components after the reaction are called **Products**

Reactants → Products

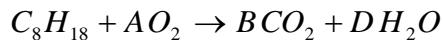
For example all carbon is burned to carbon dioxide and all hydrogen is converted into water.



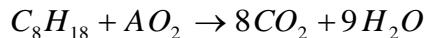
A complete combustion process is one where all carbon is burned to carbon dioxide (CO₂) and all hydrogen is converted into water (H₂O).

Example 5.1

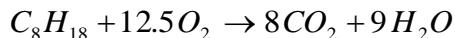
A complete combustion of octane in oxygen is represented by the balanced combustion equation. The balanced combustion equation is obtained by making sure we have the same number of atoms of each element on both sides of the equation. That is we make sure the mass is conserved.



Note we often can balance the C and H for complete combustion by inspection.



The amount of oxygen is found from the oxygen balance.



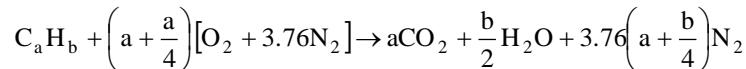
Note: Mole numbers are not conserved but we have conserved the mass on a total basis as well as a specie basis.

The complete combustion process is also called the **Stoichiometric combustion**, and all coefficients are called the **Stoichiometric coefficients**.

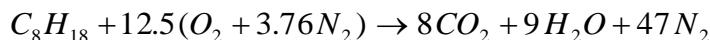
In most combustion processes, oxygen is supplied in the form of air rather than pure oxygen.

Air is assumed to be 21% oxygen and 79% nitrogen on a volume basis. For ideal gas mixtures, percent by volume is equal to percent by moles. Thus, for each mole of oxygen in air, there exists $79/21 = 3.76$ moles of nitrogen. Therefore, complete or theoretical combustion of octane with air can be written as:

Generally, the complete combustion equation of a pure hydrocarbon (C_aH_b) is given by:



Hence, the stoichiometric combustion Equation of iso-octane (C_8H_{18}) is written as:



1-5.2 Air-Fuel Ratio

Since the total moles of a mixture is equal to the sum of moles of each component, there are $12.5(1 + 3.76) = 59.5$ moles of air required for each mole of fuel for the complete combustion process.

Often complete combustion of the fuel will not occur unless there is an excess of air present greater than just the theoretical air required for complete combustion.

To determine the amount of excess air supplied for a combustion process, let us define the air-fuel ratio, AF which can be expressed on volume basis (usually for gaseous fuels) or mass basis (for solid and liquid fuels)

Knowing that the mole fraction is equivalent to the volume fraction, i.e. AF by volume, AF_v , is defined as:

$$AF_v = \frac{\text{kmol air}}{\text{kmol fuel}} \quad (5.1)$$

Thus for the above example, the theoretical air-fuel ratio by volume is:

$$AF_{th} = \frac{12.5(1+3.76)}{1} = 59.5 \text{ kmol air/kmol fuel}$$

Air-fuel ratio on a mass basis, AF_m , is defined as the ratio of the mass of air to the mass of fuel for a combustion process. That is,

$$AF_m = \frac{m_{\text{air}}}{m_{\text{fuel}}} \quad (5.2)$$

The mass m of a substance is related to the number of moles N through the relation $m = NM$ where M is the molar mass.

AF by mass, AF_m is related to AF_v by:

$$AF_m = AF_v \times \frac{\tilde{M}_{\text{air}}}{\tilde{M}_{\text{fuel}}} \quad (5.3)$$

where \tilde{M}_{air} is the molar mass of air and \tilde{M}_{fuel} is the molar mass of the fuel.

On a mass basis, the theoretical air-fuel ratio for the above example is therefore

$$AF_{th} = 59.5 \times \frac{28.97 \text{ kg/kmol}}{[8(12) + 18(1)] \text{ kg/kmol}} = 15.12 \text{ kg air/kg fuel}$$

The reciprocal of the air-fuel ratio (AF) is the fuel-air ratio (FA)

1-5.3 Percent Theoretical Air, Percentage Excess Air and Mixture Strength

In most cases, more than theoretical air is supplied to insure complete combustion and to reduce or eliminate carbon monoxide (CO) from the products of combustion. The amount of excess air is usually expressed as percent theoretical air and percent excess air.

$$\text{Percent theoretical air} = \frac{AF_{actual}}{AF_{th}} \times 100\% \quad (5.4)$$

$$\text{Percent excess air} = \frac{AF_{actual} - AF_{th}}{AF_{th}} \times 100\% \quad (5.5)$$

It can be shown that these results may be expressed in terms of the moles of oxygen only as:

$$\text{Percent theoretical air} = \frac{N(O_2)_{actual}}{N(O_2)_{th}} \times 100\% \quad (5.6)$$

percent excess air = % theoretical - 100 %

$$\text{Percent excess air} = \frac{N(O_2)_{actual} - N(O_2)_{th}}{N(O_2)_{th}} \times 100\% \quad (5.7)$$

The mixture strength or fuel/air equivalence ratio, ϕ , is defined as:

$$\phi = \frac{AF_{th}}{AF_{actual}} = \frac{FA_{actual}}{FA_{th}} \quad (5.8)$$

For fuel-lean mixtures (excess air, weak or lean mixture): $\phi < 1$

For stoichiometric mixtures: $\phi = 1$

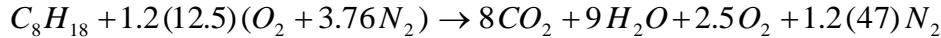
For fuel-rich mixtures (less air, rich mixture): $\phi > 1$

Example 5.2

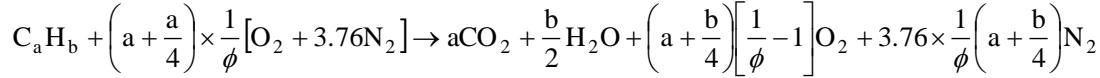
Write the combustion equation of octane with 120 % theoretical air (20 % excess air)



Note that (1)(12.5) O₂ is required for complete combustion to produce 8 kmol of carbon dioxide and 9 kmol of water; therefore, (0.2)(12.5) O₂ is found as excess oxygen in the products.



Generally, for a lean fuel the combustion equation for a pure hydrocarbon can be written as:



If $\phi=1$, there will be no excess oxygen in the products

If $\phi<1$, there will be excess oxygen in the product

If $\phi>1$, extra information is needed to able to write down the combustion equation completely.

1-5.4 Dew point temperature of combustion products

The dew point temperature for the product gases is the temperature at which the water in the product gases would begin to condense when the products are cooled at constant pressure. The dew point temperature is equal to the saturation temperature of the water at its partial pressure, P_v , in the products.

(5.9)

$T_{DP} = T_{sat}$ @ P_v of water in the products

where $P_v = y_v P_{products}$ (5.10)

$$\text{and } y_v = \text{mole fraction of water in the exhaust products} = \frac{N_{water}}{\sum_{products} N_e} \quad (5.11)$$

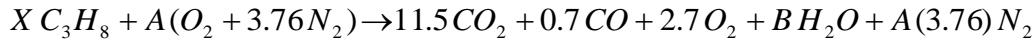
Example 5.3: Combustion equation when product gas analysis is known

Propane gas C_3H_8 is reacted with air such that the dry product gases are 11.5 % CO_2 , 2.7 % O_2 , and 0.7 % CO by volume. What percent theoretical air was supplied? What is the dew point temperature of the products if the total product pressure is 100 kPa?

Solution:

We assume 100 kmol of dry product gases; then, the percent by volume can be interpreted to be mole numbers. But, we do not know how much fuel and air were supplied or how much water was formed to get the 100 kmol of dry product gases.

We write the combustion equation generally as:



The unknown coefficients A, B, and X are found by conservation of mass for each species.

$$C : X(3) = 11.5(1) + 0.7(1) \quad X = 4.07$$

$$H : X(8) = B(2) \quad B = 16.28$$

$$O : A(2) = 11.5(2) + 0.7(1) + 2.7(2) + B(1) \quad A = 22.69$$

$$N_2 = A(3.76) = 85.31$$

The balanced equation is:



Second method to find A:

Assume the remainder of the 100 kmol of dry product gases is N_2 .

$$kmol N_2 = 100 - (11.5 + 0.7 + 2.7) = 85.1$$

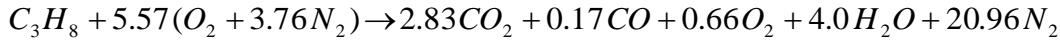
Then A is

$$A = \frac{85.1}{3.76} = 22.63 \quad (\text{fairly good check})$$

These two methods don't give the same results for A; but, they are close.

What would be the units on the coefficients in the balanced combustion equation?

Generally, we should write the combustion equation per kmol of fuel. To write the combustion equation per unit kmol of fuel divide by 4.07 i.e. X to obtain the following:



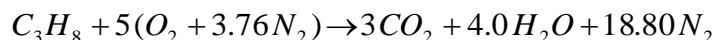
Actual mass of air, $m_{air} = N_{air} \times \tilde{M}_{air} = 5.57(1+3.76)\text{kmol} \times 28.97\text{kg/ kmol} = 768\text{kg}$

Mass of fuel, $m_{fuel} = N_{fuel} \times \tilde{M}_{fuel} = 1\text{kmol} \times [3(12)+8(1)]\text{kg/ kmol} = 44\text{ kg}$

The actual air-fuel ratio is:

$$AF_{actual} = \frac{768}{44} = 17.45\text{kg air/ kg fuel}$$

The theoretical combustion equation is:



Theoretical mass of air, $m_{air} = N_{air} \times \tilde{M}_{air} = 5(1+3.76)\text{kmol} \times 28.97\text{kg/ kmol} = 689\text{kg}$

The theoretical air-fuel ratio is:

$$AF_{th} = \frac{\text{Theoretical mass of air}}{\text{mass of fuel}} = \frac{689}{44} = 15.66\text{kg air/ kg fuel}$$

The percent theoretical air is:

$$\begin{aligned} \text{Percent theoretical air} &= \frac{AF_{actual}}{AF_{th}} \times 100\% \\ &= \frac{17.45}{15.66} \times 100\% = 111\% \end{aligned}$$

The percent excess air is:

$$\begin{aligned} \text{Percent excess air} &= \frac{AF_{actual} - AF_{th}}{AF_{th}} \times 100\% = \% \text{ theoretical air} - 100\% \\ &= \frac{17.45 - 15.66}{15.66} \times 100\% = 11\% \end{aligned}$$

Mixture strength:

$$\text{Mixture strength, } \phi = \frac{AF_{th}}{AF_{actual}} = \frac{15.66}{17.45} = 0.90 \quad (\phi < 1, \text{ lean mixture})$$

From actual combustion equation,

kmol of water, $N_v = 4\text{ kmol}$

Total kmol of exhaust products, $\sum_{products} N_e = (2.83 + 0.17 + 0.66 + 4 + 20.96)\text{kmol} = 28.62\text{kmol}$

$$\text{Mole fraction of water in the products, } y_v = \frac{N_v}{\sum_{\text{products}} N_e} = \frac{4}{28.62} = 0.1398$$

$$\begin{aligned}\text{Partial pressure of water in the products, } P_v &= y_v P_{\text{products}} = 0.1398(100\text{kPa}) \\ &= 13.98\text{kPa}\end{aligned}$$

Hence, the dew Point Temperature:

$$T_{DP} = T_{sat} @ 13.98\text{kPa} \text{ is read from steam Tables as :}$$

$$T_{dp} = 52.44^\circ\text{C}$$



Self Assessment 1-5

1. Propane (C_3H_8) is burned with 75 percent excess air during a combustion process. Assuming complete combustion, determine the air–fuel ratio. **Answer: 27.5 kg air/kg fuel**

2. A gaseous fuel with a volumetric analysis of 60 percent CH_4 , 30 percent H_2 , and 10 percent N_2 is burned to completion with 130 percent theoretical air. Determine (a) the air–fuel ratio and (b) the fraction of water vapor that would condense if the product gases were cooled to 20°C at 1 atm. **Answers: (a) 18.6 kg air/kg fuel, (b) 88 percent**

SESSION 2-5: ENTHALPY OF REACTING MIXTURES

2-5.1 Enthalpy of Formation, Enthalpy of Reaction and Enthalpy of Combustion

When a compound is formed from its elements (e.g. Methane, CH_4 , from C and H_2), heat transfer occurs. When heat is given off, the reaction is called *exothermic*. When heat is required, the reaction is called *endothermic*. Consider the following:

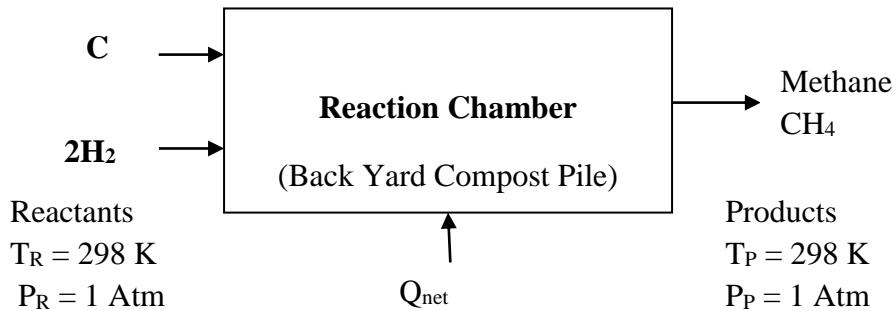
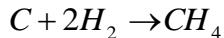


Figure 77: Energy balance on a reaction chamber



The conservation of energy for a steady-flow combustion process in Figure 78 is:

$$\begin{aligned}
E_{in} &= E_{out} \\
Q_{net} + H_{\text{reactants}} &= H_{\text{products}} \\
Q_{net} &= H_{\text{products}} - H_{\text{reactants}} \\
&= \sum_{\text{Products}} N_e \tilde{h}_e - \sum_{\text{Reactants}} N_i \tilde{h}_i \\
&= (\tilde{N}h)_{CH_4} - [(\tilde{N}h)_C + (\tilde{N}h)_{H_2}]
\end{aligned} \tag{5.12}$$

We need to establish a common reference state for the enthalpies of reacting components.

The enthalpy of the elements or their stable compounds is defined to be ZERO at 25 °C (298 K) and 1 atm (or 0.1 MPa).

$$\begin{aligned}
Q_{net} &= (1 \text{ kmol} \times \tilde{h}_{CH_4}) - [(1 \text{ kmol} \times \tilde{h}_C) + (2 \text{ kmol} \times \tilde{h}_{H_2})] \\
&= \tilde{h}_{CH_4} \text{ (kJ/kmol}_{CH_4}\text{)}
\end{aligned}$$

This heat transfer is called the **enthalpy of formation** for methane, \bar{h}_f^o . The superscript (o) implies the 1 atm pressure value and the subscript (f) implies 25 °C data. The enthalpy of formation can be viewed as the enthalpy of a substance at a specified state due to its chemical composition.

During the formation of methane from the elements at 298 K, 0.1 MPa, heat is given off (an exothermic reaction) such that

$$Q_{net} = \tilde{h}_{f CH_4}^o = -74,850 \text{ kJ/kmol}_{CH_4}$$

The heat transferred can also be referred to as the **enthalpy of reaction** h_R , which is defined as the difference between the enthalpy of the products at a specified state and the enthalpy of the reactants at the same state for a complete reaction. For combustion processes, the enthalpy of reaction is usually referred to as the **enthalpy of combustion** h_C , which represents the amount of heat released during a steady-flow combustion process when 1 kmol (or 1 kg) of fuel is burned completely at a specified temperature and pressure.

2-5.2 Adiabatic Flame Temperature

In the absence of any work interactions and any changes in kinetic or potential energies, the chemical energy released during a combustion process is lost as heat to the surroundings or is used internally to raise the temperature of the combustion products. The smaller the heat loss the larger the temperature rise. In the limiting case of no heat loss to the surroundings ($Q = 0$), the temperature of the products reaches a maximum, which is called the adiabatic flame or adiabatic combustion temperature of the reaction (Figure 78).

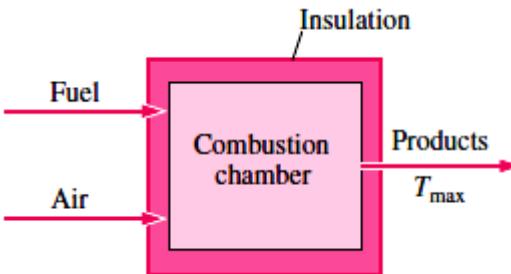


Figure 78: Adiabatic flame temperature



Self Assessment 2-5

- Benzene gas (C_6H_6) at $25^\circ C$ is burned during a steady-flow combustion process with 95 percent theoretical air that enters the combustion chamber at $25^\circ C$. All the hydrogen in the fuel burn to H_2O , but part of the carbon burns to CO. If the products leave at 1000 K , determine (a) the mole fraction of the CO in the products and (b) the heat transfer from the combustion chamber during this process. **Answers: (a) 2.1 percent, (b) 2,112,779 kJ/kmol C_6H_6**
- Acetylene gas (C_2H_2) at $25^\circ C$ is burned during a steady-flow combustion process with 30 percent excess air at $27^\circ C$. It is observed that 75,000 kJ of heat is being lost from the combustion chamber to the surroundings per kmol of acetylene. Assuming combustion is complete; determine the exit temperature of the product gases. **Answer: 2312 K**
- A cigarette lighter burns gaseous butane [$C_4H_{10}(g)$] with 200 percent theoretical air. The fuel and air are at $25^\circ C$, and the products of combustion are at $127^\circ C$ and $P = 100\text{ kPa}$.
 - How much excess air is used? **[100%]**
 - Work out the balanced actual combustion equation.
 - Find the air-fuel ratio on a mass basis. **[30.84 kg air/kg fuel]**
 - Find H_r . **[-126,150 kJ/kmol]**
 - Find H_p . **[-2,683,219 kJ/kmol]**
 - How much heat transfer is released during the combustion? **[2,557,069 kJ/kmol]**
 - Find the dew point temperature of the H_2O . **[40.9 °C]**
- Ethane gas ((C_2H_6)) at $25^\circ C$ is burned during a steady-flow combustion chamber at a rate of 5 kg/h with the stoichiometric amount of air, which is preheated to 500 K before entering the combustion chamber. An analysis of the combustion gases reveals that all the hydrogen in the fuels burns to H_2O but 95 % of the carbon in the fuel burns to CO_2 , the remaining 5 % forming, CO. If the products leave the combustion chamber at 800 K , determine the rate of heat transfer from the combustion chamber, in kJ/h. **Answer: 2100 kJ/h**

Note: $\tilde{M}_{butane} = 53.123\text{ kJ/k mol}$; $\tilde{M}_{air} = 28.97\text{ kJ/k mol}$ are to be used in problem 3 above.

Appendices

Appendix 1: Tutorial Problems Set 1

REVIEW PROBLEMS (TUTORIAL PROBLEMS SET 1)

Course Lecturer: Prof. F.K. Forson and Dr. R. Opoku

1. Complete the Table 2 below using steam tables. Insert a dash for irrelevant items, and interpolate where necessary.

Table 2: Incomplete table for Tutorial Problems Set 1 Q1

P (bar)	t (°C)	v (m ³ /kg)	x	Degree of superheat	h (kJ/kg)	u (kJ/kg)
	90	2.361				
20					2799	
5		0.3565				
	188					2400
34			0.9			
	81.3		0.85			
3	200					
15		0.152				
130					3335	
	250	1.601				
38.2			0.8			
	297		0.95			
2.3	300					
44	420					

2. Using the property values for refrigerant HFA 134a given in Table 3, calculate:
 - (i) The specific enthalpy and specific volume of HFA 134a at -8 °C, dryness fraction 0.85;

- (ii) The specific enthalpy of HFA 134a at 5.7024 bar, 35 °C.
(259.96 kJ/kg; 0.0775 m³/kg; 323.25 kJ/kg)

Table 3: Property values for refrigerant HFA 134a

Saturation values					Superheat values degree of superheat 20 K
<i>t_g</i> (°C)	<i>p_g</i> (bar)	<i>v_g</i> (m ³ /kg)	<i>h_f</i> (kJ/kg)	<i>h_g</i> (kJ/kg)	<i>h</i> (kJ/kg)
-10	2.0051	0.098	86.98	288.86	308.64
-5	2.4371	0.081	93.46	291.77	312.05
20	5.7024	0.036	126.92	306.22	328.93

3. Steam at 7 bar, dryness fraction 0.9, expands reversibly at constant pressure until the temperature is 200 °C. Calculate the work done and heat supplied per unit mass of steam during the process.
(-38.2 kJ/kg; 288.7 kJ/kg)

4. 0.05 kg of steam at 15 bar is contained in a rigid vessel of volume 0.0076 m³. What is the temperature of the steam? If the vessel is cooled, at what temperature will the steam be just dry saturated? Cooling is continued until the pressure in the vessel is 11 bar; calculate the final dryness fraction of the steam, and the heat rejected between the initial and the final states.

$$(250 \text{ °C}; 191.4 \text{ °C}; 0.857; 18.5 \text{ kJ})$$

5. A certain perfect gas is compressed reversibly from 1 bar, 17°C to a pressure of 5 bar in a perfectly thermally insulated cylinder, the final temperature being 77°C. The work done on the gas during the compression is 45 kJ/kg. Calculate γ , c_v , R , and the molar mass of the gas.

$$(1.132; 0.75 \text{ kJ/kg K}; 0.099 \text{ kJ/kg K}; 84 \text{ kg/kmol})$$

6. A refrigerant is dry saturated at 2 bar and is compressed reversibly in a cylinder according to a law $pv = \text{constant}$ to a pressure of 10 bar. Using the properties of refrigerant given in Table 4, calculate:

- (i) The final specific volume and temperature of the refrigerant

- (ii) The final specific volume and temperature when the working substance is air, compressed between the same pressures and from the same initial temperature.
(0.024 m³/kg, 24°C; 0.071 m³/kg, -25°C)

Table 4: Property values of refrigerant for Q6

t_g (°C)	p_g (bar)	v_g (m ³ /kg)
- 25	2	0.120
24	10	0.024

7. Steam at 0.05 bar, 100 °C is to be condensed completely by a reversible constant pressure process. Calculate the heat rejected per kilogram of steam, and the change of specific entropy. Sketch the process on a $T - s$ diagram and shade in the area which represents the heat flow.

(2550 kJ/kg; 8.292 kJ/kg K)

8. 0.05 kg of steam at 10 bar, dryness fraction 0.84, is heated reversibly in a rigid vessel until the pressure is 20 bar. Calculate the change of entropy and the heat supplied. Show the area which represents the heat supplied on a $T - s$ diagram.

(0.0704 kJ/ K; 36.85 kJ)

9. 1 m³ of air is heated reversibly at constant pressure from 15 to 300 °C, and is then cooled reversibly at constant volume back to the initial temperature. The initial pressure is 1.03 bar. Calculate the net heat flow and the overall change of entropy, and sketch the processes on a $T - s$ diagram.

(101.5 kJ; 0.246 kJ/K)

10. 1 kg of steam undergoes a reversible isothermal process from 20 bar and 250 °C to a pressure of 30 bar. Calculate the heat flow, stating whether it is supplied or rejected, and sketch the process on a $T - s$ diagram.

(-135 kJ; rejected)

11. Steam expands reversibly in a cylinder behind a piston from 6 bar dry saturated, to a pressure of 0.65 bar. Assuming that the cylinder is perfectly thermally insulated, calculate the work done during the expansion per kilogram of steam. Sketch the process on a $T - s$ diagram.

(323.8 kJ/kg)

12. 1 kg of a fluid at 30 bar, 300 °C, expands reversibly and isothermally to a pressure of 0.75 bar. Calculate the heat flow and the work done (i) when the fluid is air, (ii) when the fluid is steam. Sketch each process on a $T - s$ diagram.

(607 kJ; - 607 kJ; 1035 kJ; - 975 kJ)

13. 1 kg of a fluid at 2.62 bar, - 3°C, is compressed according to a law $pv = \text{constant}$ to a pressure of 8.2 bar. Calculate the work input and the heat supplied (i) when the fluid is air, (ii) when the fluid is a refrigerant initially dry saturated with the properties given in the table 5 below. Sketch each process on a $T - s$ diagram.

(88.41 kJ; - 88.41 kJ; 22.63 kJ; - 6.69 kJ)

Table 5: Property values of refrigerant for Q13

Saturation values				
t_g (°C)	p_g (bar)	v_g (m ³ /kg)	h_f (kJ/kg)	h_g (kJ/kg)
-3.0	2.62	0.0757	96.07	292.94
32.3	8.20	0.0248	144.29	313.05

14. At the start of the compression process in the reciprocating compressor of a refrigeration plant the refrigerant is at 1.5 bar, dry saturated. At the end of the compression process, which is according to a reversible polytropic law $pv^{1.2} = \text{constant}$, the pressure is 6.5 bar. Using the properties of refrigerant given in Table 6 and interpolating where necessary, calculate:

- (i) The change of specific entropy during the process;
- (ii) The degree of superheat of the refrigerant after compression.

(0.06 kJ/kg K; 35 K)

Table 6: Property values of refrigerant for Q14

Saturation values				Superheated values at 6.5 bar		
t_g (°C)	p_g (bar)	v_g (m ³ /kg)	s_g (kJ/kg K)	t (°C)	v (m ³ /kg)	s (kJ/kg K)
-20	1.5	0.109	1.12	50	0.030	1.15
25	6.5	0.027	1.11	70	0.034	1.21

15. 1 kg of air at 1.013 bar, 17°C, is compressed according to a law $pv^{1.3} = \text{constant}$, until the pressure is 5 bar. Calculate the change of entropy and sketch the process on a $T - s$ diagram, indicating the area which represents the heat flow.

(-0.0885 kJ)

16. 1 kg of air at 1.02 bar, 20 °C, undergoes a process in which the pressure is raised to 6.12 bar, and the volume becomes 0.25 m³. Calculate the change of entropy and mark the initial and final states on a $T - s$ diagram. **(0.083 kJ/ K)**

17. Steam at 15 bar is throttled to 1 bar and a temperature of 150 °C. Calculate the initial dryness fraction and the change of specific entropy. Sketch the process on a $T - s$ diagram and state the assumptions made in the throttling process.

(0.992; 1.202 kJ/kg K)

18. Two vessels, one exactly twice the volume of the other, are connected by a valve and immersed in a constant temperature bath of water. The smaller vessel contains hydrogen (molar mass 2 kg/kmol), and the other is completely evacuated. Calculate the change of entropy per kilogram of gas when the valve is opened and conditions are allowed to settle. Sketch the process on a $T - s$ diagram. Assume hydrogen to be a perfect gas. **(4.567 kJ/kg K)**

19. A turbine is supplied with steam at 40 bar, 400 °C, which expands through the turbine in steady flow to an exit pressure of 0.2 bar, and a dryness fraction of 0.93. The inlet velocity is negligible, but the steam leaves at high velocity through a duct of 0.14 m² cross-sectional area. If the mass flow is 3 kg/s, and the mechanical efficiency is 90%, calculate the power output of the turbine. Show that the process is irreversible and calculate the change of specific entropy. Heat losses from the turbine are negligible.

(2048 kW; 0.643 kJ/kg K)

20. In a centrifugal compressor, the air is compressed through a pressure ratio of 4 to 1, and the temperature of the air increases by a factor of 1.65. Show that the process is irreversible and calculate the change of entropy per kilogram of air. Assume that the process is adiabatic. Sketch the process on a $T - s$ diagram. **(0.105 kJ/kg K)**

21. In a gas turbine unit, the gases enter the turbine at 550 °C and 5 bar and leave at 1 bar. The process is approximately adiabatic, but the entropy changes by 0.174 kJ/kg K. Calculate the exit temperature of the gases. Assume the gases to act as a perfect gas, and take $\gamma = 1.333$ and $c_p = 1.11 \text{ kJ/kg K}$. Sketch the process on a $T - s$ diagram. **(370.9 °C)**

22. Steam enters a turbine at 70 bar, 500 °C and leaves at 2 bar in a dry saturated state. Calculate the isentropic efficiency and the effectiveness of the process. Neglect changes of kinetic and potential energy and assume that the process is adiabatic. The atmospheric temperature is 17 °C. **(84.4 %; 88 %)**

Appendix 2: Tutorial Problems Set 2

TUTORIAL PROBLEMS SET 2

(A) CARNOT VAPOUR CYCLE AND THE SIMPLE RANKINE CYCLE

1. (a) What four processes make up the simple Rankine cycle?
(b) Consider the simple ideal Rankine cycle with fixed turbine inlet conditions. What is the effect of lowering the condenser pressure on

Pump work input:	(a) increases	(b) decreases	(c) remains the same
Turbine work output:	(a) increases	(b) decreases	(c) remains the same
Heat supplied:	(a) increases	(b) decreases	(c) remains the same
Heat rejected:	(a) increases	(b) decreases	(c) remains the same
Cycle efficiency:	(a) increases	(b) decreases	(c) remains the same
Moisture content at turbine exit:	(a) increases	(b) decreases	(c) remains the same

2. (a) How do actual vapour cycles differ from idealised ones?
- (b) Consider a simple ideal Rankine cycle with fixed turbine inlet temperature and condenser pressure. What is the effect of increasing the boiler pressure on
- | | | | |
|--------------------------------------|---------------|---------------|----------------------|
| Pump work input: | (a) increases | (b) decreases | (c) remains the same |
| Turbine work output: | (a) increases | (b) decreases | (c) remains the same |
| Heat supplied: | (a) increases | (b) decreases | (c) remains the same |
| Heat rejected: | (a) increases | (b) decreases | (c) remains the same |
| Cycle efficiency: | (a) increases | (b) decreases | (c) remains the same |
| Moisture content at
turbine exit: | (a) increases | (b) decreases | (c) remains the same |
-
3. Consider a simple ideal Rankine cycle with fixed boiler and condenser pressures. What is the effect of superheating the steam to a higher temperature on
- | | | | |
|--------------------------------------|---------------|---------------|----------------------|
| Pump work input: | (a) increases | (b) decreases | (c) remains the same |
| Turbine work output: | (a) increases | (b) decreases | (c) remains the same |
| Heat supplied: | (a) increases | (b) decreases | (c) remains the same |
| Heat rejected: | (a) increases | (b) decreases | (c) remains the same |
| Cycle efficiency: | (a) increases | (b) decreases | (c) remains the same |
| Moisture content at
turbine exit: | (a) increases | (b) decreases | (c) remains the same |
-
4. A steam plant operates on a simple ideal Rankine cycle between the pressure limits of 3 MPa and 50 kPa. The temperature of the steam at the turbine inlet is 400 °C, and the mass flow rate of steam through the cycle is 25 kg/s. Show the cycle on a *T-s* diagram with respect to saturation lines, and determine
- (c) the thermal efficiency of the cycle and [28.4 %]
 (d) the net power output of the power plant [20.5 MW]
-
5. Consider a 300-MW steam power plant that operates on a simple ideal Rankine cycle. Steam enters the turbine at 10 MPa and 500 °C and is cooled in the condenser at a pressure of 10 kPa. Show the cycle on a *T-s* diagram with respect to saturation lines, and determine
- (a) the quality of steam at turbine exit, [0.793]
 (b) the thermal efficiency of the cycle, and [40.2 percent]
 (c) the mass flow rate of steam. [235.4 kg/s]

6. Repeat problem 5 assuming an adiabatic efficiency of 85 % for both the turbine and the pump.

[0.793, 34.1 percent, 277.8 kg/s]

7. Consider a coal-fired steam power plant that produces 300 MW of electric power. The power plant operates on a simple ideal Rankine cycle with turbine inlet conditions of 5 MPa and 450 °C and a condenser pressure of 25 kPa. The coal used has a heating value (energy released when the fuel is burned) of 29,300 kJ/kg. Assuming that 75 % of this energy is transferred to the steam in the boiler and that the electric generator has an efficiency of 96 %, determine

- (a) the overall plant efficiency (the ratio of the electric power output to the energy input as fuel) and
- (b) the required rate of coal supply.

[(a) 24.6 percent,(b) 41.62 kg/s]

8. Consider a steam power plant that operates on a simple ideal Rankine cycle and has a net power output of 30 MW. Steam enters the turbine at 7 MPa and 500 °C and is cooled in the condenser at a pressure of 10 kPa by running cooling water from a lake through the tubes of the condenser at a rate of 2000 kg/s. Show the cycle on a *T-s* diagram with respect to saturation lines, and determine

- (a) the thermal efficiency of the cycle, [38.9 percent]
- (b) the mass flow rate of steam, and [24.0 kg/s]
- (c) the temperature rise of the cooling water. [5.63 °C]

9. Repeat problem 8 assuming an adiabatic efficiency of 87 % for both the turbine and the pump.

(B) THE REHEAT RANKINE CYCLE, REGENERATIVE RANKINE CYCLE and COGENERATION

10. How do the following quantities change when a simple ideal Rankine cycle is modified with reheating? Assume the mass flow rate is maintained the same.

Pump work input:	(a) increases	(b) decreases	(c) remains the same
Turbine work output:	(a) increases	(b) decreases	(c) remains the same
Heat supplied:	(a) increases	(b) decreases	(c) remains the same
Heat rejected:	(a) increases	(b) decreases	(c) remains the same
Moisture content at turbine exit:	(a) increases	(b) decreases	(c) remains the same

11. How do the following quantities change when a simple ideal Rankine cycle is modified with regeneration? Assume the mass flow rate through the boiler is the same.
-
- | | | | |
|--------------------------------------|---------------|---------------|----------------------|
| Turbine work output: | (a) increases | (b) decreases | (c) remains the same |
| Heat supplied: | (a) increases | (b) decreases | (c) remains the same |
| Heat rejected: | (a) increases | (b) decreases | (c) remains the same |
| Moisture content at
turbine exit: | (a) increases | (b) decreases | (c) remains the same |
-
12. Consider a steam power plant that operates on a reheat Rankine cycle and has a net power output of 150 MW. Steam enters the high-pressure turbine at 10 MPa and 500 °C and the low-pressure turbine at 1 MPa and 500 °C. Steam leaves the condenser as a saturated liquid at a pressure of 10 kPa. The adiabatic efficiency of the turbine is 80 %, and that of the pump is 95%. Show the cycle on a *T-s* diagram with respect to saturation lines, and determine
- (d) the quality (or temperature, if superheated) of steam at the turbine exit, **[superheated, 87.5 °C]**
 - (e) the thermal efficiency of the cycle, and **[34.1 percent]**
 - (f) the mass flow rate of steam. **[117.5 kg/s]**
13. A steam power operates on an ideal reheat Rankine cycle between the pressure limits of 9 MPa and 10 kPa. The mass flow rate of steam through the cycle is 25 kg/s. Steam enters both stages of the turbine at 500 °C. If the moisture content of the steam at the exit of the low-pressure turbine is not to exceed 10 %, determine
- (a) the pressure at which reheating takes place, **[21.6 bar]**
 - (b) the total rate of heat input in the boiler, and **[91,075.3 kJ/s]**
 - (c) The thermal efficiency of the cycle. **[41 percent]**
- Also, show the cycle on a *T-s* diagram with respect to saturation lines.
14. Consider an ideal steam regenerative Rankine cycle with two feed-water heaters, one closed and one open, as illustrated in Figure Q14. Steam enters the turbine at 12.5 MPa and 550 °C and exhausts to the condenser at 10 kPa. Steam is extracted from the turbine at 0.8 MPa for the closed feed-water heater and at 0.3 MPa for the open one. The feed-water is heated to the condensation temperature of the extracted steam in the closed feed-water heater. The extracted steam leaves the closed feed-water heater as a saturated liquid, which is subsequently throttled to the open feed-water heater. Show the cycle on a *T-s* diagram with respect to saturation lines, and determine

- (a) the mass flow rate of steam through the boiler for a net power output of 250 MW and
(b) the thermal efficiency of the cycle.
[(a) 200.2 kg/s, (b) 45.3 percent]
15. A steam power plant operates on an ideal reheat-regenerative Rankine cycle and has a net power output of 120 MW. Steam enters the high-pressure turbine at 10 MPa and 550 °C and leaves at 0.8 MPa. Some steam is extracted at this pressure to heat the feed-water in an open feed-water heater. The rest of the steam is reheated to 500 °C and is expanded in the low-pressure turbine to the condenser pressure of 10 kPa. Show the cycle on a *T-s* diagram with respect to saturation lines, and determine
- (c) the mass flow rate of steam through the boiler and **[81.9 kg/s]**
(d) the thermal efficiency of the cycle. **[44.4 percent]**
16. Repeat problem 15, but replace the open feed-water heater with a closed feed-water heater, as illustrated in Figure Q16. Assume that the feed-water leaves the heater at the condensation temperature of the extracted steam and that the extracted steam leaves the heater as a saturated liquid and is pumped to the line carrying the feed-water.
- [(a) 81.9 kg/s (b) 44.4 percent]**
17. Steam enters the turbine of a cogeneration plant, as illustrated in Figure Q17, at 7 MPa and 500 °C. One-fourth of the steam is extracted from the turbine at 600 kPa for process heating. The remaining steam continues to expand to 10 kPa. The extracted steam is then condensed and mixed with feed-water at constant pressure and the mixture is pumped to the boiler pressure of 7 MPa. The mass flow rate of steam through the boiler is 30 kg/s. Disregarding any pressure drops and heat losses in the piping, and assuming the turbine and the pump to be isentropic, determine the power produced and the utilisation factor of the plant.
- [Power produced = 32,862 kW, Utilisation factor = 52.4 percent]**
18. Consider the cogeneration power plant modified with regeneration, as illustrated in Figure Q18. Steam enters the turbine at 6 MPa and 450 °C and expands to a pressure of 0.4 MPa. At this pressure, 60 percent of the steam is extracted from the turbine, and the remainder expands to 10 kPa. Part of the extracted steam is used to heat the feed-water in an open feed-water heater. The rest of the extracted steam is used for process heating and leaves the process heater as a saturated liquid at 0.4 MPa. It is subsequently mixed with the feed-water leaving the feed-water heater, and the mixture is pumped to the boiler pressure. Assuming the turbine and the pump to be isentropic, show the cycle on a *T-s* diagram with respect to saturation lines, and determine the mass flow rate of steam

through the boiler for a net power output of 15 MW. [Mass flow rate of steam = 17.7 kg/s]

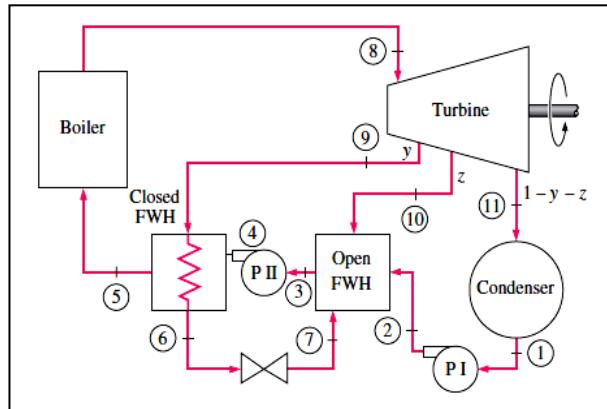


Figure Q14

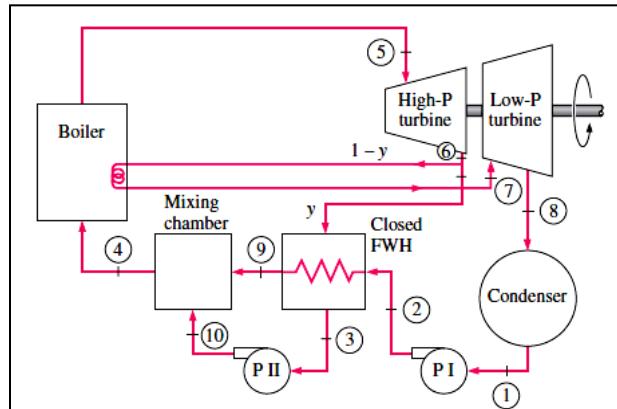


Figure Q16

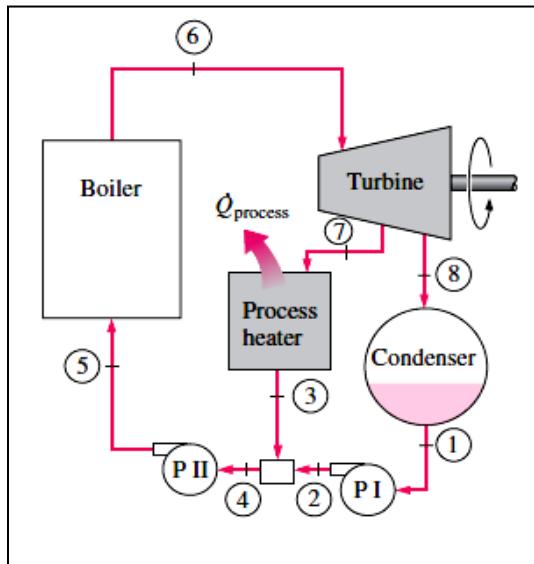


Figure Q17

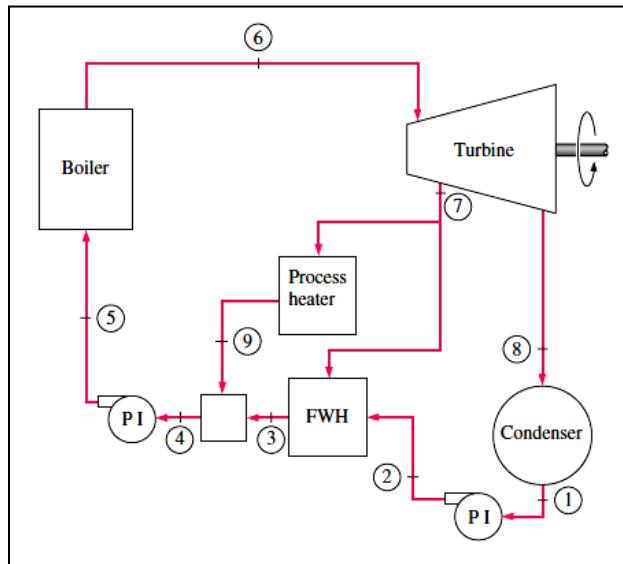


Figure Q18

19. The gas turbine portion of a combined gas-steam power plant has a pressure ratio of 16. Air enters the compressor at 300 K at a rate of 14 kg/s and is heated to 1500 K in the combustion chamber. The combustion gases leaving the turbine are used to heat the steam to 400 °C at 10 MPa in a heat exchanger. The combustion gases leave the heat exchanger at 420 K. The steam leaving the turbine is condensed at 15 kPa. Assuming all the compression and expansion processes to be isentropic, determine:
- The mass flow rate of the steam
 - The net power output
 - The thermal efficiency of the combined cycle.

For air, assume constant specific heats at room temperature.

Answers: (a) 1.275 kg/s

(b) 7818 kW

(c) 66.3%

20. Consider a combined gas-steam power plant that has a net power output of 450 MW. The pressure ratio of the gas-turbine cycle is 14. Air enters the compressor at 300 K and the turbine at 1400 K. The combustion gases leaving the gas turbine are used to heat the steam at 8 MPa to 400°C in a heat exchanger. The combustion gases leave the heat exchanger at 460K. An open feed water heater incorporated with the steam cycle operates at a pressure of 0.6 MPa. The condenser pressure is 20 kPa. Assuming all the compression and expansion processes to be isentropic, determine:
- (a) The mass flow rate ratio of air to steam
 - (b) The required rate of heat input in the combustion chamber
 - (c) The thermal efficiency of the combined cycle

21. Repeat problem 2.0 assuming adiabatic efficiencies of 100% for the pump, 82% for the compressor, and 86% for the gas and steam turbines.

Appendix 3: Tutorial Problems Set 3

ME 365, THERMODYNAMICS II, TUTORIAL PROBLEMS SET 3

Course Lecturer: Prof. F. K. Forson and Dr. R. Opoku

1. An air-water vapour mixture enters a device with a pressure of 0.15 MPa, a temperature of 40 °C, and an unknown relative humidity. The mass of dry air entering per minute is 200 g. The mixture leaves the device at 30 °C, 0.15 MPa, and 80 percent relative humidity. Water is sprayed into the air and evaporated to do this cooling. If the water temperature is 30 °C and the device is adiabatic, how much water is required to run for 1 h?

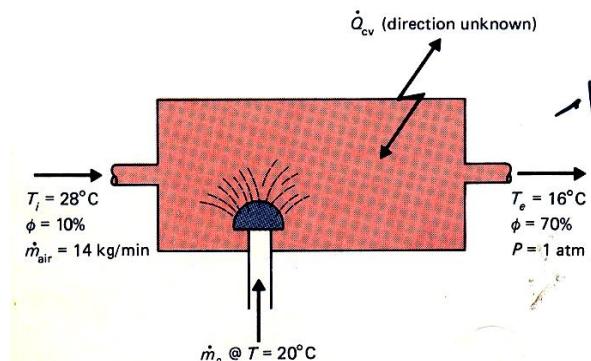
[50.5 g/h]

$$T_{\text{mix}} = 90^{\circ}\text{C}$$

$$P_{\text{mix}} = 400 \text{ kPa}$$

$$P_a = 380 \text{ kPa}$$

Find: $T_{\text{dew point}}$, ϕ , ω using
the psychrometric chart



[60.1 °C, 28.5 %, 0.0327]

Figure Q.5

3. Given an air-water vapour mixture at:

$$T_{\text{dry bulb}} = 25^{\circ}\text{C}$$

$$\phi = 70 \text{ percent}$$

Find : $T_{\text{wet bulb}}$, ω , $T_{\text{dew point}}$

using the psychrometric chart

[21 °C, 0.0138, 19 °C]

4. Given an air-water vapour mixture at:

$$T_{\text{dry bulb}} = 35^{\circ}\text{C}$$

$$\phi = 80 \text{ percent}$$

$$P = 1 \text{ atm}$$

Find : $T_{\text{wet bulb}}$, ω , P_v , $T_{\text{dew point}}$

[32 °C, 0.0289, 4.5 kPa, 31 °C]

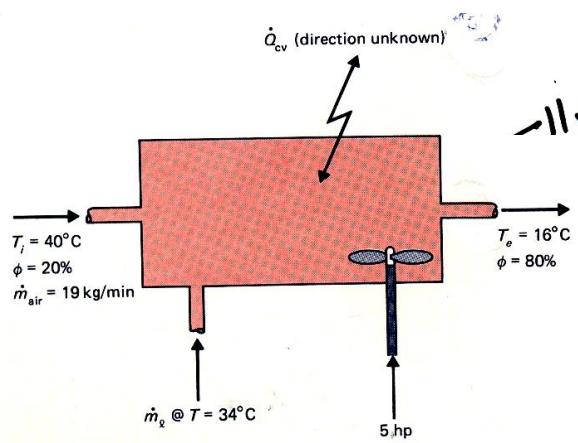


Figure Q.6

5. Given the air conditioner in Figure Q.5, determine \dot{Q}_{cv} .

[- 1.305 kW]

6. Given the device shown in Figure Q.6, determine \dot{Q}_{cv} . The pressure is 1 atm.

[- 11.66 kW]

7. The wet-bulb temperature is 25 °C, and the dry-bulb temperature is 32 °C. Using the psychrometric chart, determine ϕ , ω , P_v , and $T_{\text{dew point}}$.

[57%; 0.0171; 2.70 kPa; 22.5 °C]

8. If the dew point temperature is 24 °C and the dry-bulb temperature is 30 °C, determine the wet-bulb temperature, $T_{\text{wet bulb}}$, the humidity ratio ω , the partial pressure of the vapour P_v , and the relative humidity ϕ .

[25.5 °C; 0.0189; 2.92 kPa; 70%]

9. An air-vapour mixture in a steady state, steady flow process is cooled and dehumidified as shown in Figure Q.9. The process occurs at 0.1013 MPa. Do not use a psychrometric chart.

(a) Determine the rate of heat transfer (kW). [- 13.98 kW]

(b) Determine the mass flow rate of the water condensed (kg/s). [0.00302 kg/s]

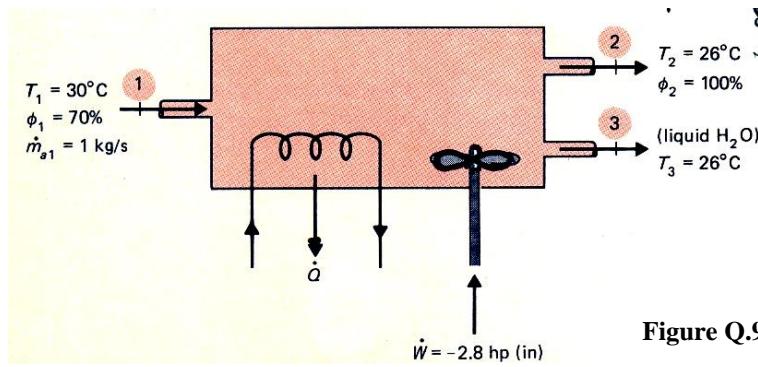


Figure Q.9

- 10.** Given the winter air conditioning system as shown in Figure Q.10. The pressure of this steady state, steady flow process is 101.3 kPa.
- Draw the entire process on a psychrometric chart.
 - Determine the mass flow rate of the water in the water sprays.
 - Determine the total flow rate of heat transfer to the system.
- [0.6237 kg/min; 43.9 kW]

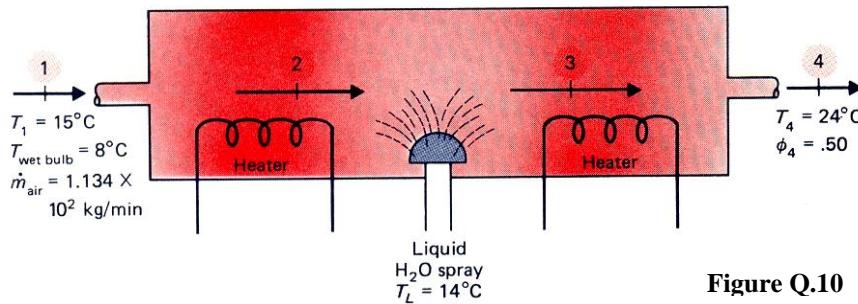


Figure Q.10

- 11.** A stream of air at 1 atm, 20 °C, and relative humidity of 80 percent has a mass flow rate of 0.05 kg/s of dry air. A second stream of air at 1 atm, 35 °C, and relative humidity of 40 percent is mixed adiabatically with the first stream to give a mixed stream temperature of 30 °C.
- What mass flow rate of the second stream is required?
 - What is the relative humidity of the mixed stream?
- 12.** In a certain northern city, a heating system is designed to heat outside air by injecting steam and by heating as shown in Figure Q.12.
- Determine the mass flow rate of steam required.
 - Determine the rate of heat transfer.
- [281 g/min; 7.56 kW]

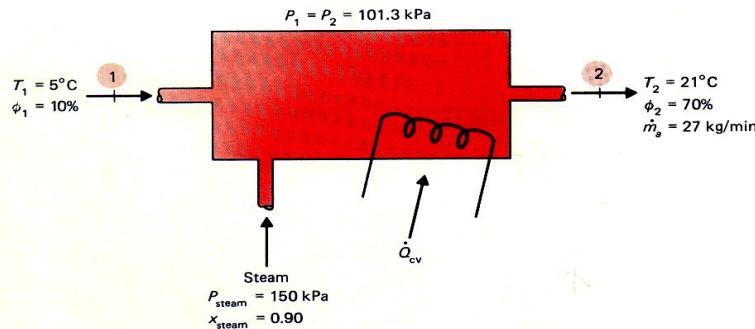


Figure Q.12

13. The air conditioner shown in Figure Q.13 operates at a pressure of 101.3 kPa.

- Draw the entire process on a psychrometric chart.
- Determine the mass flow rate of water required by the spray. [0.3328 kg/min]
- Determine the rate of heat transfer. [11.83 kW]

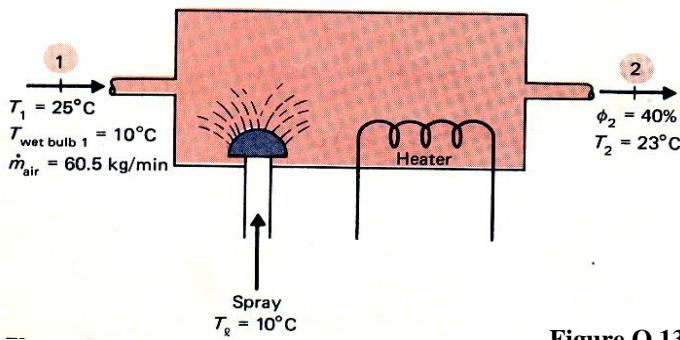


Figure Q.13

14. A room air conditioner treats the air as shown in Figure Q.14. Determine the mass of water removed per hour by the unit. [0.636 kg/h]

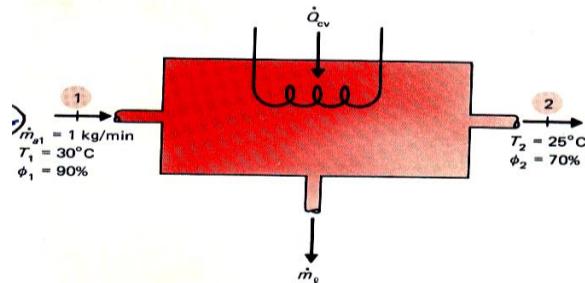


Figure Q.14

15. A residential air conditioner heats and humidifies outside air as shown in Figure Q.15.

- Draw the entire process on a psychrometric chart.
- Calculate the rate of heat transfer (kW). [25.0 kW]

(c) Calculate \dot{m}_l . [0.265 kg/min]

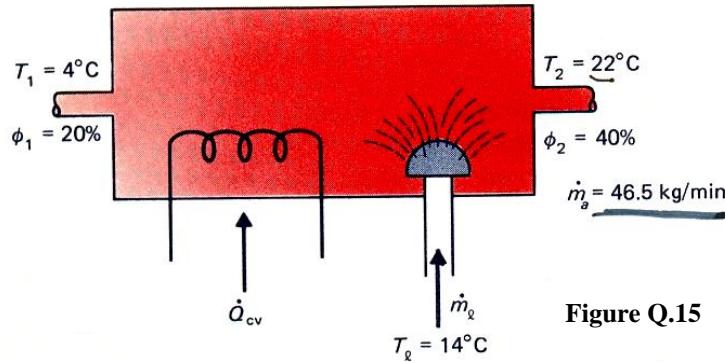


Figure Q.15

16. A central air conditioning system operates as shown in Figure Q.16.

- Draw the entire process on a psychrometric chart, as accurately as possible.
- Calculate the required cooling capacity of the cooling coils, (tons). [3.05 tons]
- Calculate \dot{Q}_{cv} for the heating coils in kW. [1.612 kW]
- Determine the amount of condensate produced in 1-h operation (kg). [10.84 kg/h]

Note: [1 ton = 3.516 kW]

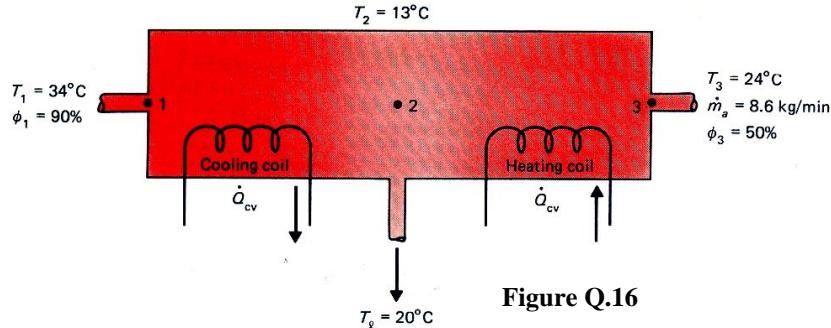


Figure Q.16

17. A large municipal power plant is cooled with an evaporative cooling tower as shown.

Find the required mass flow rate of dry air through the tower and the required flow rate of basin makeup water. [25974 kg/s; 475.2 kg/s]

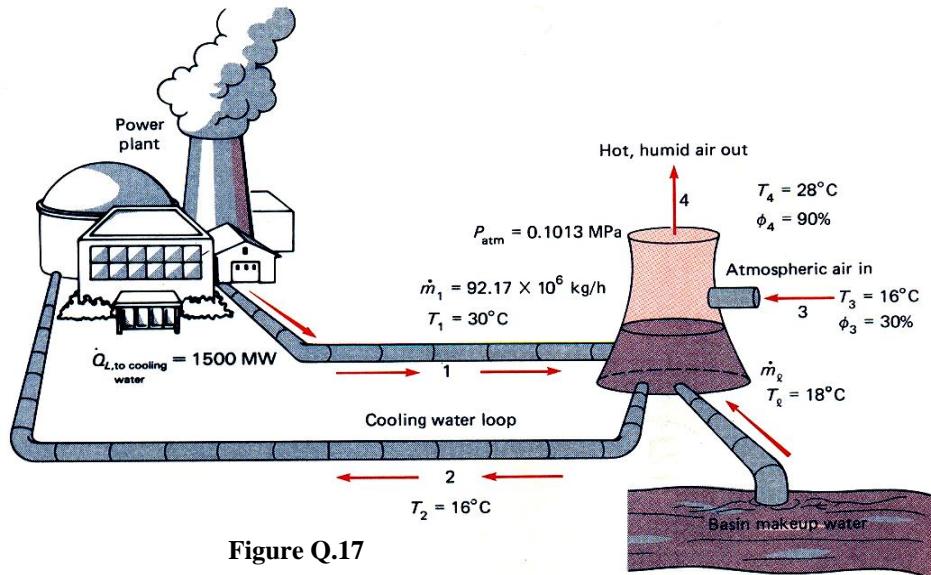


Figure Q.17

18. Some $100 \times 10^6 \text{ kg/h}$ of cooling water is cooled from 50 to 30 °C in a dry cooling tower. Atmospheric air enters the tower at 20 °C and leaves at 30 °C. Find the required flow rate of the air. [$2.31 \times 10^5 \text{ kg/s}$]
19. Some $150 \times 10^6 \text{ kg/h}$ of cooling water is cooled from 60 to 26 °C in a mechanical draft dry cooling tower (Figure Q.19). Atmospheric air enters the tower at 20 °C and leaves at 35 °C. The fan is driven by a 10,000-hp motor. Find the required flow rate of the air. [$3.94 \times 10^5 \text{ kg/s}$]

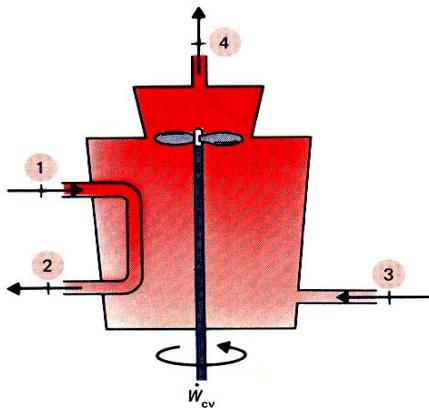


Figure Q.19

20. A power plant with a cooling lake operates as shown in Figure Q.20. The lake temperature seasonally varies between 2 and 18 °C. If the maximum temperature ecologically allowed at the exit is 30 °C, determine the seasonal range in \dot{m} of the cooling water. Note the follow: $\dot{Q}_H = 3000 \text{ MW}$, $\dot{Q}_{\text{stack}} = -500 \text{ MW}$, $\dot{W}_{\text{elect}} = 1000 \text{ MW}$.

$$[46.04 \times 10^6 \text{ to } 107.5 \times 10^6 \text{ kg/hr}]$$

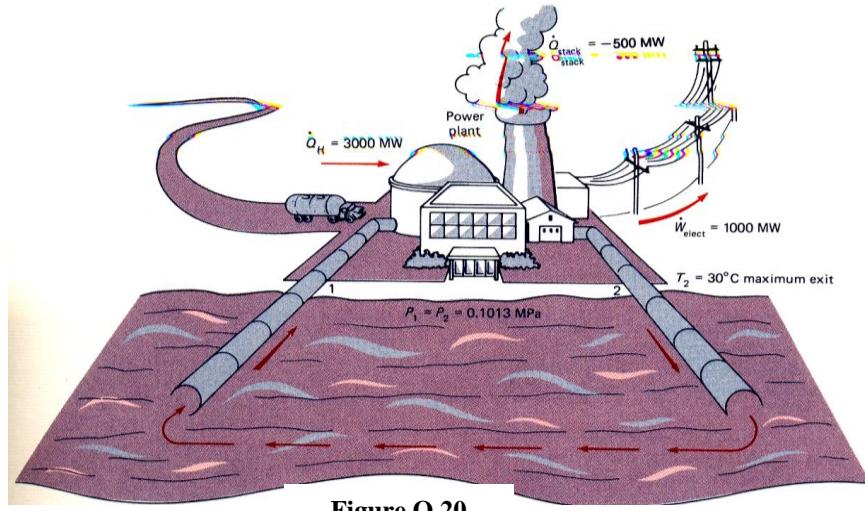


Figure Q.20

Note that:

$$\tilde{M}_{\text{air}} = 29 \text{ kg/mol}$$

$$\tilde{M}_{\text{C}_2\text{H}_4} = 28 \text{ kg/mol}$$

- 21.** An engine burns ethene (C_2H_4) with 200 percent theoretical air.
- How much excess air is used? [**100%**]
 - Workout the balanced combustion equation for 200 percent theoretical air.
 - Find the air-fuel ratio on a mass basis. [**29.49 kg air/kg fuel**]
- 22.** Combustion of a hydrocarbon of unknown composition resulted in the following equation (based on 100 mol of products):
- $$\text{C}_a\text{H}_b + d(\text{O}_2 + 3.76\text{N}_2) \rightarrow 8.0\text{CO}_2 + 0.9\text{CO} + 8.8\text{O}_2 + e\text{H}_2\text{O} + 82.3\text{N}_2$$
- Determine the composition of the fuel.
 - Determine the air-fuel ratio on a mass basis. [**24.08 kg air/kg fuel**]
 - Find the 100 percent theoretical air and determine the percentage of excess air supplied (you may neglect the CO contribution here). [**61.6%**]
- 23.** Determine the standardised molar enthalpy \bar{h} for 1 mol of H_2O at 1 atm and 800 K. [**-224,000 kJ/kmol**]
- 24.** In a rocket engine, complete combustion occurs when 2 mol of hydrogen combines with 1 mol of oxygen. The rocket carries 21 m^3 of O_2 at 100°C and 4 MPa. Determine the number of kilomoles of O_2 and the mass of H_2 required for complete combustion. [**27.087 kmol; 109.2 kg**]
- 25.** A new engine receives its energy by burning ethene (C_2H_4) with 150 percent theoretical air. The fuel and air are at 25°C . The products of combustion are at 627°C and $P = 100 \text{ kPa}$.

- (a) How much excess air is used? [50%]
 (b) Work out the balanced actual combustion equation.
 (c) Find the air-fuel ratio on a mass basis. [22.12 kg air/kg fuel]
 (d) Find H_p . [-834,200 kJ/kmol]
 (e) Find H_r . [52,320 kJ/kmol]
 (f) Find the heat transfer released during the combustion. [886,520 kJ/kmol]
 (g) Find the dew point temperature of the H₂O. [43.6 °C]
- 26.** A cigarette lighter burns gaseous butane [C₄H₁₀(g)] with 200 percent theoretical air. The fuel and air are at 25 °C, and the products of combustion are at 127 °C and $P = 100$ kPa.
 (h) How much excess air is used? [100%]
 (i) Work out the balanced actual combustion equation.
 (j) Find the air-fuel ratio on a mass basis. [30.84 kg air/kg fuel]
 (k) Find H_r . [-126,1500 kJ/kmol]
 (l) Find H_p . [-2,683,219 kJ/kmol]
 (m) How much heat transfer is released during the combustion? [2,557,069 kJ/kmol]
 (n) Find the dew point temperature of the H₂O. [40.9 °C]
- 27.** Determine the standard enthalpy of combustion of (a) H₂, (c) CO, and (c) C₂H₅OH.
 [-1.2 x 10⁵ kJ/kg; -1.01 x 10⁴ kJ/kg; -2.8 x 10⁵ kJ/kmol]
- 28.** Saturated steam leaves a boiler at $P = 200$ bars. Heat transfer occurs in a super-heater at constant pressure P until the temperature reaches 600 °C. The steam then enters a steam turbine, where it does work on the turbine, and leaves the turbine at a pressure of $P = 7.0$ bars. The turbine has an efficiency of 75 percent.
 (a) How much work is done on the turbine?
 (b) How much heat transfer occurs in the super-heater?
 (c) If liquid water enters the boiler at 165 °C and 200 bars, what is the Rankine cycle efficiency? (Neglect the pump work.)
 [-647.1 kJ/kg; 1123.9 kJ/kg; 22.9 °C]

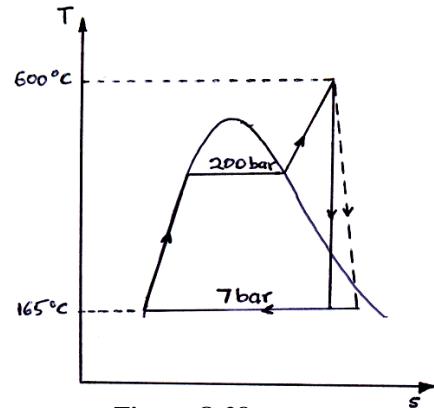


Figure Q.28

- 29.** A steam power plant operates with a boiler pressure of 1 MPa, and the steam leaving the boiler is superheated to 250 °C. The turbine used in the plant is a two-stage type. The first stage operates at an efficiency of 70 percent. Steam leaving the first stage exits at 600 kPa, is reheated to 250 °C, and is returned to the second (low-pressure) stage, which has an efficiency of 77 percent. The low-pressure stage exhausts to the condenser at a pressure of 8 kPa.
 (a) Sketch the equipment layout, and label the positions of states 1 through 7 on the layout to correspond with the state table.

- (b) Sketch the cycle on a T - s diagram, being as accurate as you can. Label the states to correspond to the state table given here.
(c) Fill in the state table (using steam tables).
(d) Fill in the energy table.

State Table

Location	State	$T, ^\circ\text{C}$	P, kPa	$h, \text{kJ/kg}$	$s, \text{kJ/(kg.K)}$	x
Pump inlet	1		8			0.0
Boiler inlet	2	45	1000			-
Super-heater inlet	3		1000			-
Super-heater outlet	4	250	1000			-
High-pressure turbine outlet	5		600			-
Low-pressure turbine inlet	6	250	600			-
Low-pressure turbine outlet	7		8			

Energy Table

$$q_{23}, \text{ kJ/kg} =$$

$$q_{34}, \text{ kJ/kg} =$$

$$q_{56}, \text{ kJ/kg} =$$

$$w_{45}, \text{ kJ/kg} =$$

$$w_{67}, \text{ kJ/kg} =$$

$$w_{12}, \text{ kJ/kg} =$$

$$\text{Cycle efficiency, \%} =$$

30. A Rankine cycle operates with an open feed-water heater. The boiler pressure is 6 MPa, and the super-heater provides a turbine inlet temperature of 400 °C. Bleed steam is provided from the turbine at 500 kPa.

- (a) What fraction of the boiler mass flow rate must be used in the bleed steam to provide a boiler feed-water inlet temperature of 150 °C? [0.817]
(b) What is the cycle efficiency? [40%]
(c) What is the cycle efficiency without regeneration? [37.5%]

31. Steam enters the high-pressure turbine in an ideal reheat cycle with a mass flow rate of 1000 g/s. Find w_t , w_p , and η_{th} .

$$\begin{aligned} T_3 &= 600 \text{ }^\circ\text{C} \\ P_3 &= 15.0 \text{ MPa} \\ P_4 &= 0.8 \text{ MPa} \\ T_5 &= 600 \text{ }^\circ\text{C} \\ P_6 &= 6 \text{ kPa} \end{aligned}$$

[2000 kW; 15.1 kW; 45.7%]

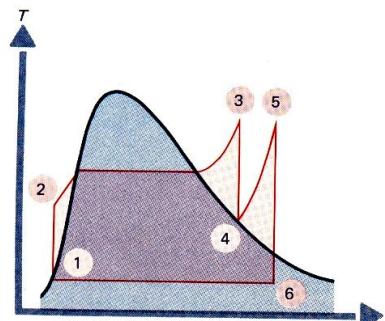


Figure Q.31

32. A steam power plant operates on the ideal Rankine cycle. The steam leaves the turbine at a pressure of 0.2 MPa and quality of 0.98. The turbine produces 1500 kW of power. If the mass flow rate is 1.5 kg/s, find the net power output and the cycle thermal efficiency.

[1492 MW; 33.4%]

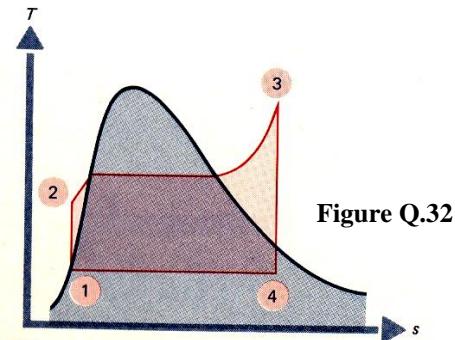


Figure Q.32

33. An ideal Rankine cycle operates with a high pressure of 6 MPa and a low pressure of 40 kPa. The quality of the steam leaving the turbine is 0.83. If $\dot{m} = 0.25 \text{ kg/s}$, find w_p , q_H , w_t , \dot{W}_{net} (hp), and q_L .

[6.12 kJ/kg; 2852.7 kJ/kg; 934.4 kJ/kg; 311.2 hp; 1922.4 kJ/kg]

34. Water enters the pump in an ideal Rankine cycle at $P = 0.1 \text{ MPa}$. The H₂O exits the pump at 3 MPa. The maximum temperature in the cycle is 450 °C, and $\dot{m} = 0.3 \text{ kg/s}$.

Find w_p , q_H , w_t , \dot{W}_{net} (hp), and q_L .

[3.03 kJ/kg; 2923.7 kJ/kg; 768.6 kJ/kg; 309.2 hp; 2155.1 kJ/kg]

35. An actual steam power plant with a two-stage turbine and an inter-stage heater operates as shown Figure Q.35. Determine the mass flow rate of steam, the net power output (kW), and the total heat transfer rate (kW) to the plant.

[0.6271 kg/s; 772.3 kW; 2273.3 kW]

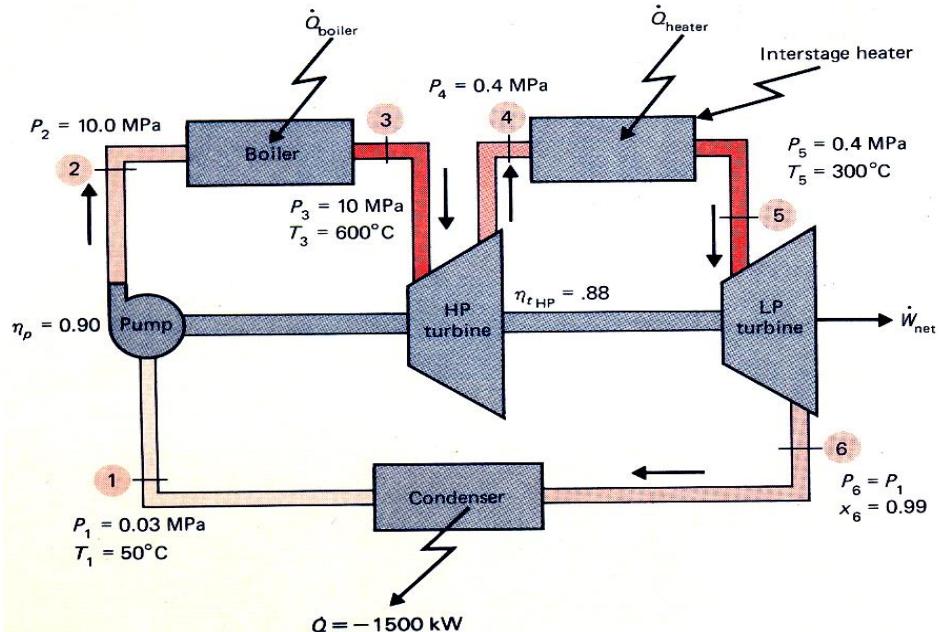


Figure Q35

- 36.** Determine the cycle thermal efficiency of a reheat cycle utilizing steam where the steam leaves the boiler and enters the turbine at 4 MPa and 400 °C. After expansion in the turbine to 400 kPa, the steam is reheated to 400 °C and then expanded in the low-pressure turbine to 10 kPa. The cycle is an ideal cycle. [35.9%]

- 37.** Consider the steam power plant shown in Figure Q.37. What is η_{th} for this plant? The pump and turbine are adiabatic. [16.7%]

State 1

$$P = 0.3 \text{ MPa}$$

$$x = 0$$

State 2

$$P = 2.0 \text{ MPa}$$

$$T = 140 \text{ }^{\circ}\text{C}$$

State 3

$$P = 2.0 \text{ MPa}$$

$$T = 350 \text{ }^{\circ}\text{C}$$

State 4

$$P = 0.3 \text{ MPa}$$

$$x = 98\%$$

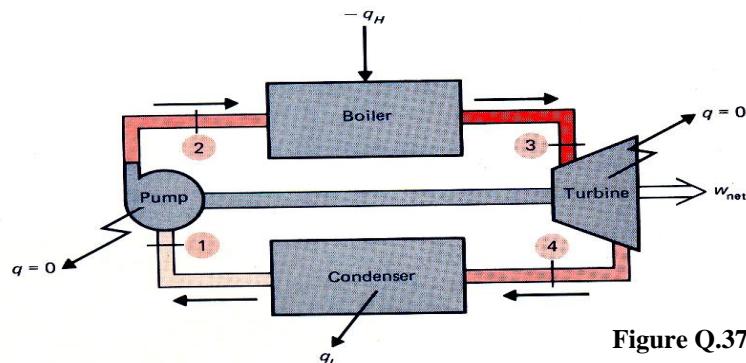


Figure Q.37

- 38.** The condenser of an ideal Rankine cycle power plant operates at 20 kPa, and steam exits the boiler at 525 °C and 2.0 MPa. Compare the thermal efficiency of this power plant to that of a Carnot engine operating between the same temperature extremes. Do not neglect pump work. [$\eta_R = 32\%$; $\eta_C = 58.3\%$; 82.2% more efficient]

- 39.** An actual steam power plant operates as shown in Figure Q.39. Find \dot{W}_{net} and η_{th} if the pump and turbine are adiabatic. [378 kW; 12.7%]

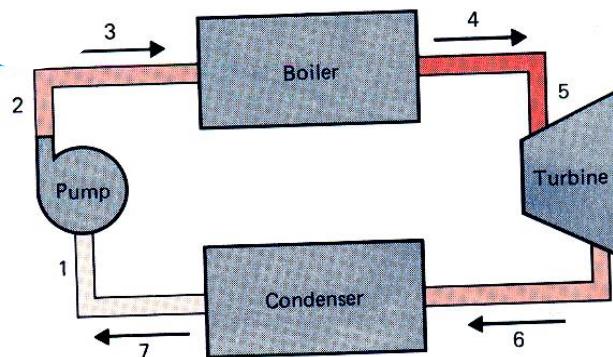


Figure Q.39

$$P_1 = 0.1013 \text{ MPa}$$

$$T_1 = 36^\circ\text{C}$$

$$P_2 = 2.08 \text{ MPa}$$

$$T_2 = 38^\circ\text{C}$$

$$P_3 = 2.05 \text{ MPa}$$

$$T_3 = 36^\circ\text{C}$$

$$P_4 = 2.0 \text{ MPa}$$

$$T_4 = 350^\circ\text{C}$$

$$P_5 = 1.5 \text{ MPa}$$

$$T_5 = 300^\circ\text{C}$$

$$P_6 = 0.15 \text{ MPa}$$

$$x_6 = 0.98$$

$$P_7 = 0.1013 \text{ MPa}$$

$$T_7 = 45^\circ\text{C}$$

$$\dot{m} = 1 \text{ kg/s}$$

40. A Rankine cycle steam power plant using geothermal energy as an energy source operates with steam leaving the steam generator at 2000 kPa and 400 °C. The condenser operates at 10 kPa. The turbine efficiency is 85 percent, and the feed pump efficiency is 100 percent. The power output of the turbine is 50 MW.

(a) Fill in all entries in a state table.

(b) What power input is required for the boiler (kW)? [**182 MW**]

41. In a “total energy system,” enough electricity is produced to satisfy the user’s electrical demand, and as much of the waste energy as possible is used to satisfy the user’s thermal demand (space heating, water heating, etc.). Considering the total energy system shown in Figure Q.41, find the rate of heat transfer to the river. [**70 MW**]

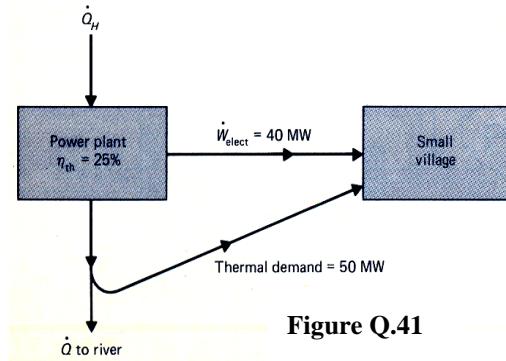


Figure Q.41

42. In the ideal vapour-compression refrigerator using refrigerant 12, the evaporator temperature is -20°C , and the inlet temperature to the condenser is 30°C . Sketch the cycle on a $P-h$ diagram and calculate

- (a) The work of the compressor.
(b) The heat transfer to the evaporator.
(c) The heat transfer from the condenser.
(d) The coefficient of performance.
(e) The degree of superheat of the refrigerant at inlet to the condenser.

43. An actual vapour-compression refrigerator, using refrigerant R-134a, has a 2-hp motor driving a compressor. The compressor efficiency is 70 percent, and it has an inlet pressure and temperature 0.05 MPa and -40°C while the exit pressure is 0.9 MPa. The condenser exit pressure and quality are 0.75 MPa and zero. The adiabatic expansion valve exit pressure is 0.06 MPa. Calculate the rejected heat transfer, the coefficient of performance, and the refrigeration capacity in tons and in Btu per hour.
(Hint: Assume that 1 hp = 746 W and that 1 ton of refrigeration = 3.516 kW = 200 Btu/min) [**217.5 kJ/kg; 7976 Btu/h or 0.663 tons**]

44. An ideal vapour-compression refrigeration system using refrigerant 12 operates with an evaporator temperature of -30°C and a condenser exit temperature of 50°C and requires a 100-hp motor to drive the compressor. Calculate the refrigerator’s capacity in tons.

(Hint: Assume that 1 hp = 746 W and that 1 ton of refrigeration = 3.516 kW = 200 Btu/min) [**57.1 tons**]

- 45.** An ideal steam power plant with reheat operates as shown in Figure Q.45. Determine the thermal efficiency and the mass flow rate. Do not neglect feed pump work. ($\dot{W}_p = 2.5 \text{ hp}$). [**0.311 kg/s; 44.4 %**]

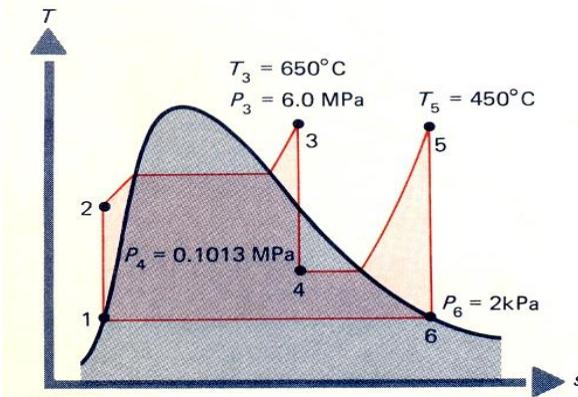
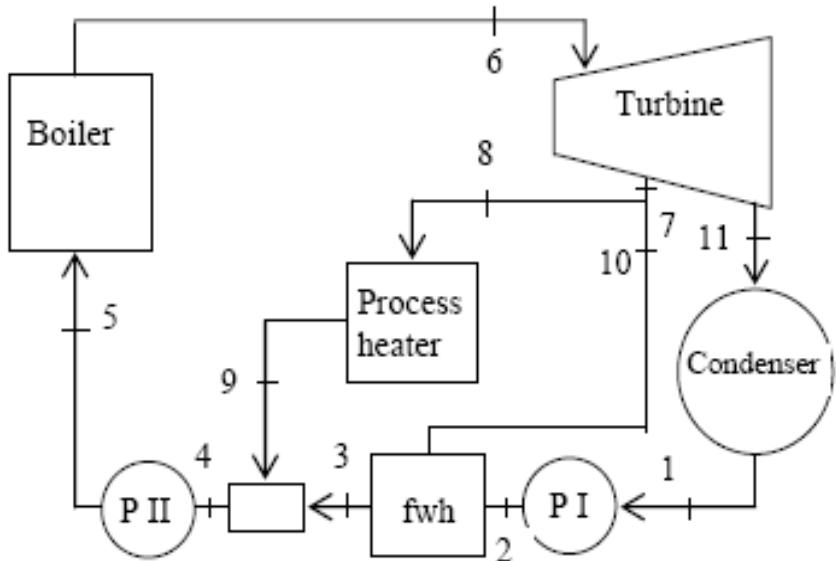


Figure Q.45

- 46.** Consider a **cogeneration power plant modified with regeneration**. Steam enters the turbine at 6 MPa and 450°C at a rate of 20 kg/s and expands to a pressure of 0.4 MPa. At this pressure, 60 percent of the steam is extracted from the turbine, and the remainder expands to a pressure of 10 kPa. Part of the extracted steam is used to heat feed-water in an open feed-water heater. The rest of the extracted steam is used for process heating and leaves the process heater as a saturated liquid at 0.4 MPa. It is subsequently mixed with the feed-water leaving the feed-water heater, and the mixture is pumped to the boiler pressure. The steam in the condenser is cooled and condensed by the cooling water from a nearby river, which enters the adiabatic condenser at a rate of 463 kg/s.



$$\begin{aligned}
 h_1 &= 191.81 \\
 h_2 &= 192.20 \\
 h_3 = h_4 = h_9 &= 604.66 \\
 h_5 &= 610.73 \\
 h_6 &= 3302.9 \\
 h_7 = h_8 = h_{10} &= 2665.6 \\
 h_{11} &= 2128.8
 \end{aligned}$$

The unit of each of the specific enthalpy (h_i) values provided is kJ/kg.

i. The total power output of the turbine is

- a. 17.0 MW
- b. 8.4 MW
- c. 12.2 MW
- d. 20.0 MW
- e. 3.4 MW

ii. The temperature rise of the cooling water from the river in the condenser is

- a. 8.0°C
- b. 5.2°C
- c. 9.6°C
- d. 12.9°C
- e. 16.2°C

iii. The mass flow rate of steam through the process heater is

- a. 1.6 kg/s
- b. 3.8 kg/s
- c. 5.2 kg/s
- d. 7.6 kg/s
- e. 10.4 kg/s

- iv. The rate of heat supply from the process heater per unit mass of steam passing *through it* is
- 246 kJ/kg
 - 893 kJ/kg
 - 1344 kJ/kg
 - 1891 kJ/kg
 - 2061 kJ/kg
- v. The rate of heat transfer to **the steam in the boiler is**
- 26.0 MJ/s
 - 53.8 MJ/s
 - 39.5 MJ/s
 - 62.8 MJ/s
 - 125.4 MJ/s

47. A gas turbine for an automobile is designed with a regenerator. Air enters the compressor of this engine at 100 kPa and 20°C. The compressor pressure ratio is 8; the maximum cycle temperature is 800°C; and the cold air stream leaves the regenerator 10°C cooler than the hot air stream at the inlet of the regenerator. Assuming both the compressor and turbine to be isentropic, determine the rates of heat addition and rejection for this cycle when it produces 150 kW. Use constant specific heats at room temperature.

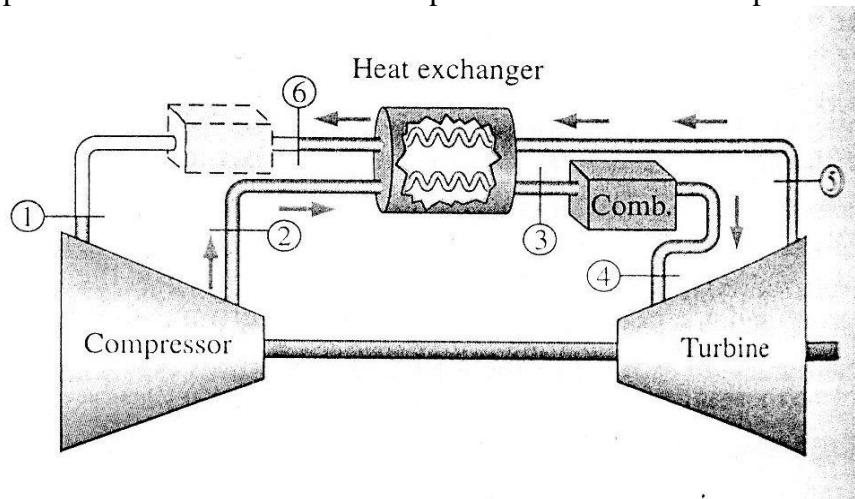


Figure Q47

48. Air enters a gas turbine with two stages of compression and two stages of expansion at 100 kPa and 17°C. This system uses a regenerator as well as reheating and intercooling. The pressure ratio across each compressor is 4; 300 kJ/kg of heat is added to the air in each combustion chamber; and the regenerator operates perfectly while increasing the temperature of the cold air by 20°C. Determine this system's thermal efficiency. Assume isentropic operations for the entire compressor and the turbine stages and use constant specific heats at room temperature.

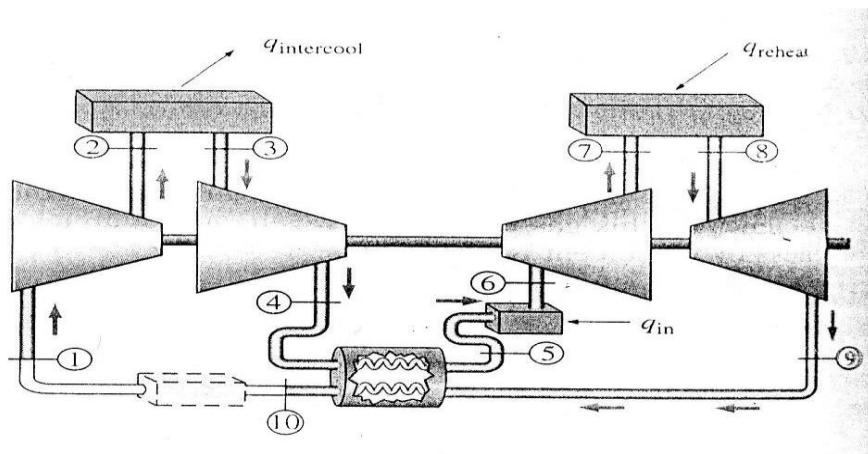


Figure Q48

49. A Brayton cycle with a pressure ratio of 15 operates with air entering the compressor at 70 kPa and 0°C, and the turbine at 600°C. Calculate the net specific work produced by this cycle treating the air as an ideal gas with (a) constant specific heat, and (b) variable specific heats.

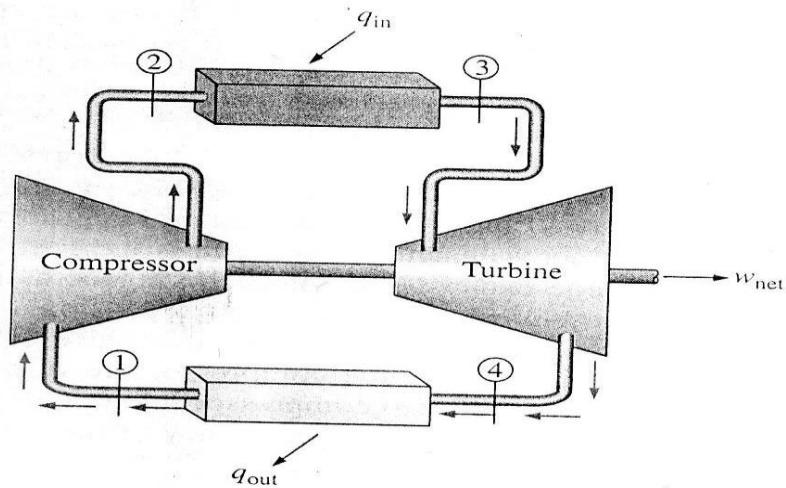


Figure Q49

50. A simple ideal Rankine cycle with water as the working fluid operates between the pressure limits of 3 MPa in the boiler and 30 kPa in the condenser and a turbine inlet temperature of 700°C. The boiler is sized to provide a steam flow of 50 kg/s. Determine the power produced by the turbine and consumed by the pump.

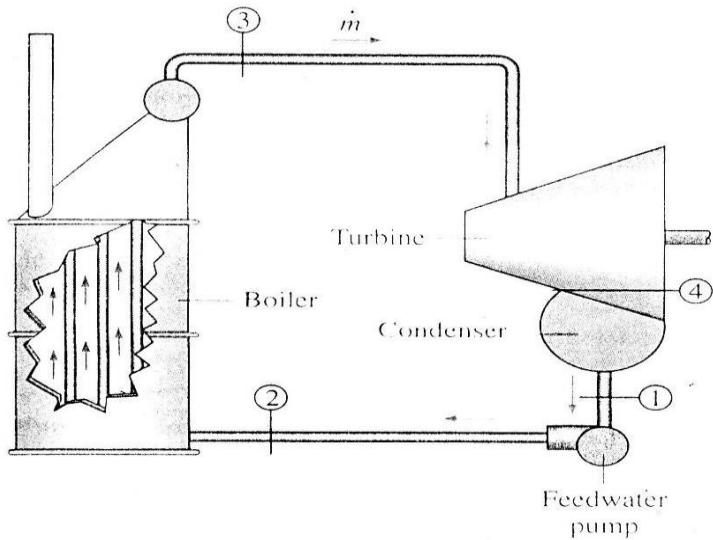


Figure Q50

51. Refrigerant -134a enters the condenser of a residential heat pump at 800 kPa and 55°C at a rate of 0.018 kg/s and leaves at 750 kPa sub cooled by 3°C. The refrigerant enters the compressor at 200 kPa superheated by 4°C. Determine
- The isentropic efficiency of the compressor
 - The rate of heat supplied to the heated room
 - The COP of the heat pump
 - The COP and the rate of heat supplied to the heated room if this heat pump is operated on the ideal vapour compression cycle between the pressure limits of 200 kPa and 800 kPa

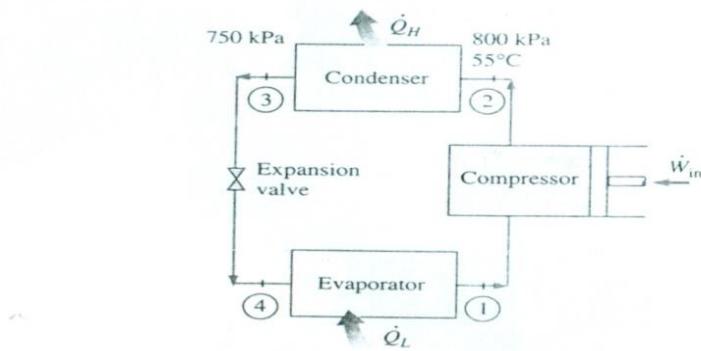


Figure Q51

52. A two-evaporator compression refrigeration system as shown in **Figure Q52** uses refrigerant- 134a as the working fluid. The system operates evaporator 1 at 0°C,

evaporator 2 at -26.4°C , and the condenser at 800 kPa. The refrigerant is circulated through the compressor at a rate of 0.1 kg/s and the low-temperature evaporator serves a cooling load of 8 kW. Determine the cooling rate of the high-temperature evaporator, the power required by the compressor, and the COP of the system. The refrigerant is saturated liquid at the exit of the condenser and saturated vapour at the exit of each evaporator, and the compressor is isentropic.

Answers: 6.58 kW, 4.50 kW, 3.24

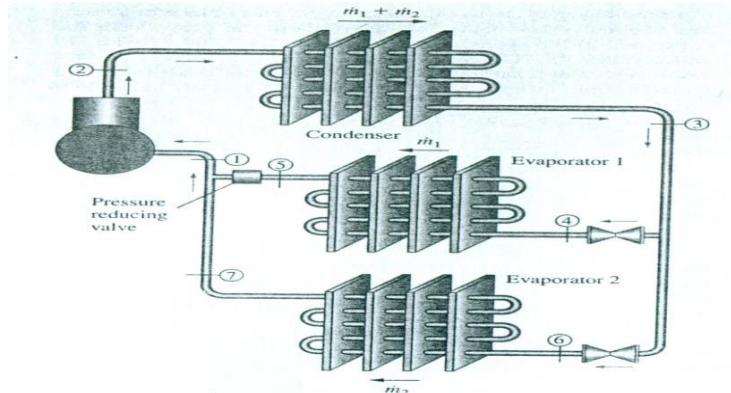


Figure Q52

53. A two-stage compression refrigeration system with an adiabatic liquid-vapour separation unit as shown in **figure Q53** uses refrigerant-134a as the working fluid. The system operates the evaporator at -40°C , the condenser at 800 kPa, and the separator at -10.1°C . This system is to serve a 30 kW cooling load. Determine the mass flow rate through each of the two compressors, the power used by the compressors, and the system's COP. The refrigerant is saturated liquid at the inlet of each expansion valve and saturated vapour at the inlet of each compressor, and the compressors are isentropic. **Answers: 0.160 kg/s, 0.230 kg/s, 10.9 kW, 3.24**

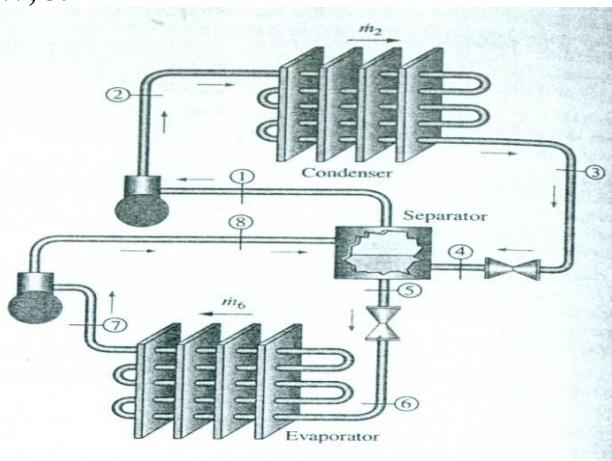


Figure Q53

54. A two-stage cascade refrigeration system is to provide cooling at -40°C while operating the high-temperature condenser at 1.6 MPa. Each stage operates on the ideal vapour compression refrigeration cycle. The upper vapour compression refrigeration system

(VCRS) uses water as its working fluid and operates its evaporator at 5°C. The lower cycle uses refrigerant -134a as its working fluid and operates its condenser at 400 kPa. This system produces a cooling effect of 20 kJ/s. Determine the mass flow rate of refrigerant -134a and water in their respective cycles, and the overall COP of this cascaded system.

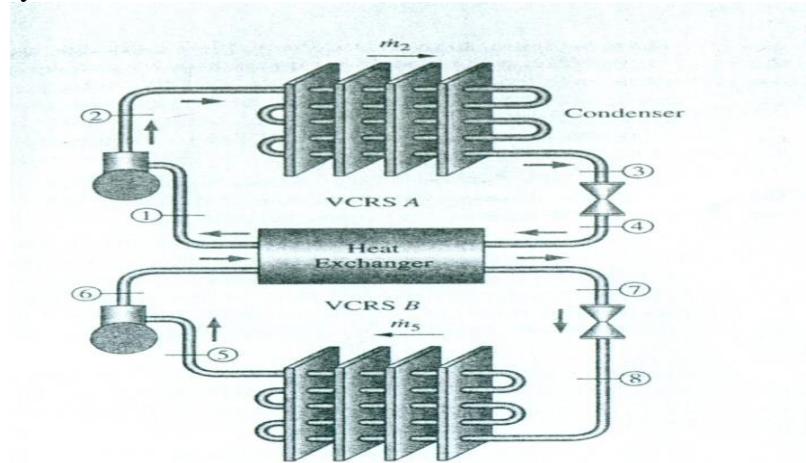


Figure Q54

55. A wet cooling tower is to cool 60 kg/s of water from 40 to 26°C. Atmospheric air enters the tower at 1 atm with dry- and wet-bulb temperatures of 22 and 16°C, respectively, and leaves at 34°C with a relative humidity of 90 percent. Using the psychrometric chart, determine (a) the volume flow rate of air into the cooling tower and (b) the mass flow rate of the required makeup water.

Answers: (a) 44.9 m³/s, (b) 1.16 kg/s

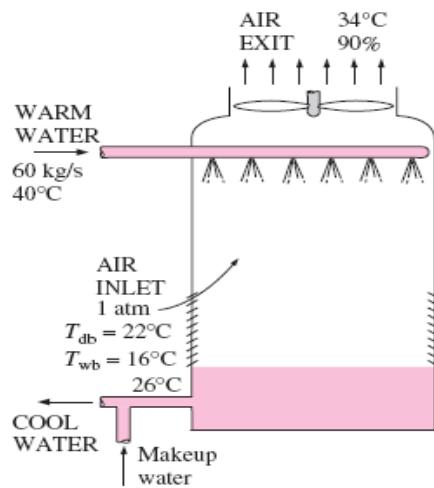


Figure Q55

- 56.** Air enters an evaporative cooler at 1 atm, 36 °C, and 20 percent relative humidity at a rate of 5 m³/min, and it leaves with a relative humidity of 90 percent. Determine (a) the exit temperature of the air and (b) the required rate of water supply to the evaporative cooler.

Answers: (a) $T_2 = 20.5 \text{ }^{\circ}\text{C}$ (b) 0.071 kg/min

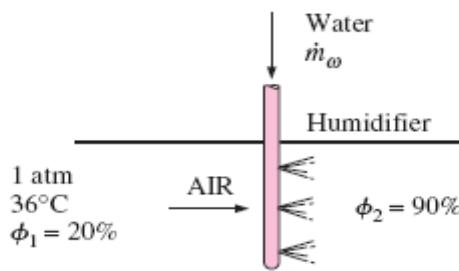


Figure Q56

- 57.** Air at 1 atm, 15°C, and 60 percent relative humidity is first heated to 30°C in a heating section and then passed through an evaporative cooler where its temperature drops to 25°C. Determine (a) the exit relative humidity and (b) the amount of water added to air, in kg H₂O/kg dry air.

Answers: (a) $\phi_3 = 42.3 \text{ \%}$ (b) $0.00201 \text{ kg H}_2\text{O/kg dry air}$

- 58.** Two airstreams are mixed steadily and adiabatically. The first stream enters at 32°C and 40 percent relative humidity at a rate of 20 m³/min, while the second stream enters at 12°C and 90 percent relative humidity at a rate of 25 m³/min. Assuming that the mixing process occurs at a pressure of 1 atm, determine the specific humidity, the relative humidity, the dry-bulb temperature, and the volume flow rate of the mixture. **Answers:** $0.0096 \text{ kg H}_2\text{O/kg dry air}$, 63.4 percent , $20.6 \text{ }^{\circ}\text{C}$, $45.0 \text{ m}^3/\text{min}$.

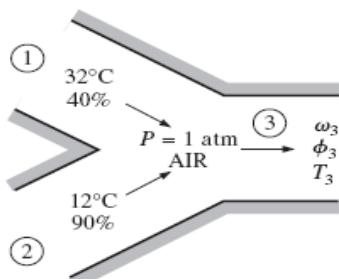


Figure Q58

- 59.** Air enters a natural draught cooling tower at 1.013 bar and 13°C and a relative humidity 50%. Water at 60 °C from turbine condensers is sprayed into the tower at the rate of 22.5 kg/s and leaves at 27 °C. The air leaves the tower at 38 °C, 1.013 bar and is saturated. Calculate:

- The air flow required in cubic metres per second; **Answer 21m³/s**
- The make-up water required in kilograms per second. **Answer 1 kg/s**

60. In a forced-draught cooling tower, hot water enters at a rate of 15 kg/s at 27 °C and leaves the tower at 21 °C. The ambient air drawn into the tower is at 1.01325 bar, 23 °C dry bulb and 17 °C wet bulb, and the air leaving the tower is saturated at 25 °C. The fan power input is 5 kW. Assuming that the specific enthalpy of superheated vapour is approximately equal to the specific enthalpy of saturated vapour at the same temperature, and that the pressure in the tower remains constant throughout, calculate:

- i. The required mass flow rate of air;
- ii. The required mass flow rate of make-up water to be added exterior to the tower.

Take the partial pressure of water vapour in air as

$$p = p_g - \left\{ 6.748 \times 10^{-4} (T_{db} - T_{wb}) \right\}$$

Where the pressures are in bar, dry bulb temperature, T_{db} , and the wet bulb temperature, T_{wb} , are in degrees centigrade. **Answers: 13.67 kg/s; 0.144 kg/s**