EE 287 CIRCUIT THEORY

GIDEON ADOM-BAMFI

Course Content

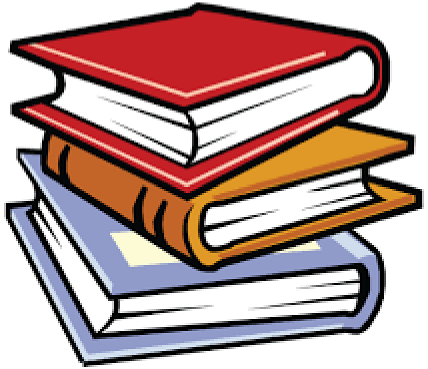
- Network Theorems in DC networks
- Network Theorems in AC networks
- RLC Circuits
- Resonant Circuits
- Two-port networks

Course Content

Grading System:

• Exams	70%
• Assignments	10%
• Mid-Semester Exams	15%
• Attendance	5 %

Software: Multisim



Recommended Textbooks

1. Engineering Circuit Analysis, William H. Hayt, and Jack E. Kemmerly, International Student Edition

2. 2. Shaum's Outline of Theory and Problems of Basic Circuit Analysis, John O'malley, Ph.D

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NETWORK THEOREMS IN DC NETWORKS

Electronic Components

Electronic components can be classified into:

1. Active Elements

These are elements that produce energy in the form of current or voltage.

Examples: Generators, batteries, Operational amplifiers, Transistors, Solar Cells

2. Passive Components

These are elements that use energy.

Examples: Resistors, Inductors, Capacitors

Types of Active Elements

1. Independent Voltage Source

It is a two-terminal element that maintains a specified voltage between its terminals regardless of the current through it.

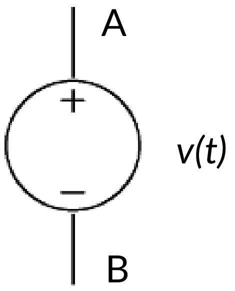


Figure 1: General Symbol

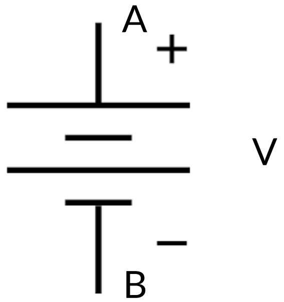


Figure 2: Constant Symbol

Figure 1 shows the symbol for time varying voltage (General symbol). The voltage v(t) is referenced positive at A.

Types of Active Elements

2. Independent Current Source

It is a two-terminal element that maintains a specified current regardless of the voltage across its terminals.

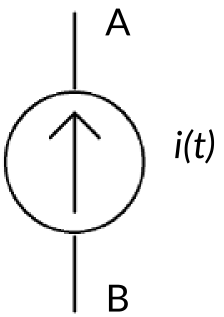


Figure 3: General Symbol

Figure 3 shows the general symbol, and the arrow indicates the direction of the current source when it is positive.

Types of Active Elements

3. Dependent or Controlled Voltage Source

A dependent or controlled voltage source is a voltage source whose terminal voltage depends on, or is controlled by a voltage or a current at a specified location in the circuit.

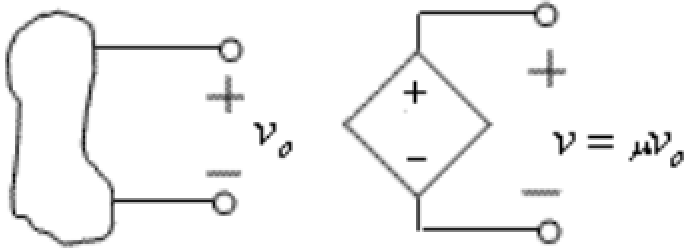


Figure 4 VCVS

The two types are:

- Voltage-Controlled Voltage Source (VCVS) controlled by a voltage.
- Current-Controlled Voltage Source (CCVS) controlled by a current.

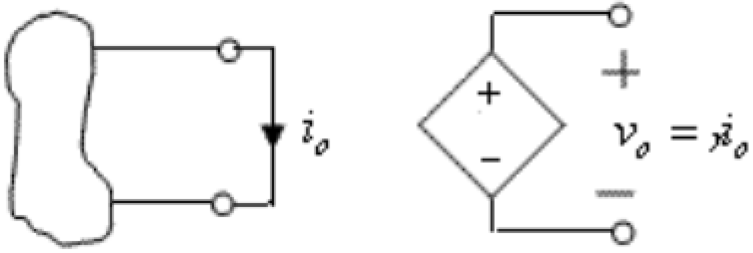


Figure 5 CCVS

Types of Active Elements

4. Dependent or Controlled Current Source

A dependent or controlled current source is a current source whose current depends on, or is controlled by a voltage or current at a specified location in the circuit.

The two types are:

- Voltage-Controlled Current Source (VCCS) controlled by voltage.

- Current-Controlled Current Source (CCCS) controlled by current.

The dependent sources are very important because they form part of the mathematical models used to describe the behaviour of many electronic circuit devices.

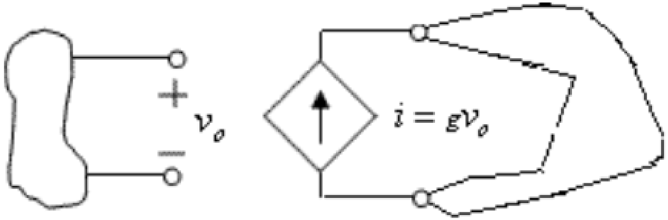


Figure 6 VCCS

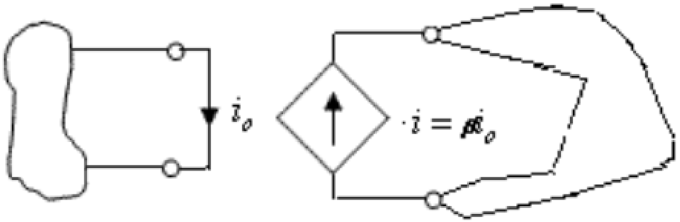
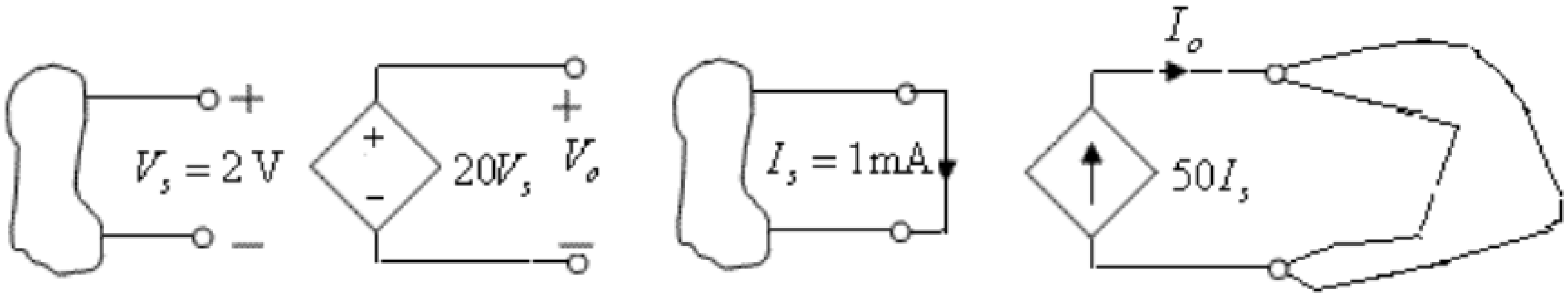


Figure 7 CCCS

Example 1

Find V_o and I_o.



(a) $V_o = 20V_s = 20{\times}2 = 40$ V

(b) $I_o = 50I_s = 50{\times}1 = 50$ mA

1. SUPERPOSITION THEOREM

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Superposition theorem states that the current through, or the voltage across an element in a linear network is equal to the algebraic sum of the currents or voltages produced by each source acting alone.

Steps to apply the Superposition Principle

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using some of the basic techniques. (Ohm's Law, Kirchhoff's Laws, Current Divider rule)

2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

$$\mathbf{V_o = V_{o1} + V_{o2} + V_{o3} \qquad or \qquad I_o = I_{o1} + I_{o2} + I_{o3}}$$

N.B. Superposition can be used for linear circuits containing dependent sources. However, it is not useful in this case because the dependent source is never made zero.

Two things we have to keep in mind:

1. We consider one independent source at a time while all other independent sources are turned off.

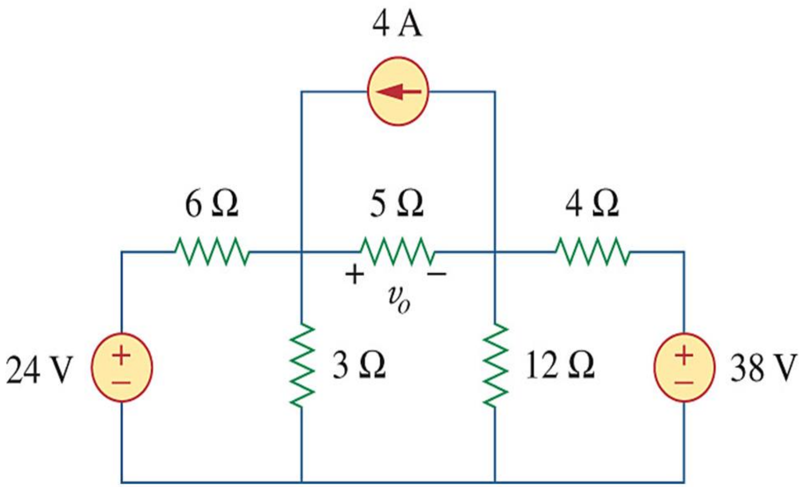
Turning off all other independent sources means:

- Independent voltage sources are replaced by 0 V - Short Circuit
- Independent current sources are replaced by 0 A - Open Circuit

2. Dependent sources are left intact because they are controlled by circuit variables.

Example 1

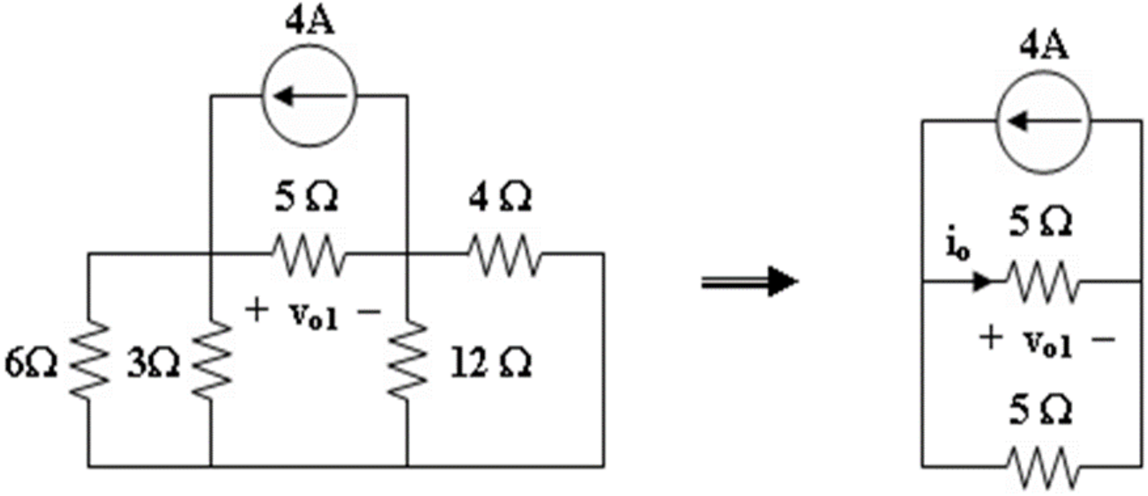
Determine V_o in the circuit below using the Superposition principle



Let $V_o = V_{o1} + V_{o2} + V_{o3}$,

where V_{o1}, V_{o2}, and V_{o3} are due to the 4-A, 24-V, and 38-V sources respectively

For V_{o1}, we keep the independent 4-A current source and turn off all other independent sources as shown in the circuit below



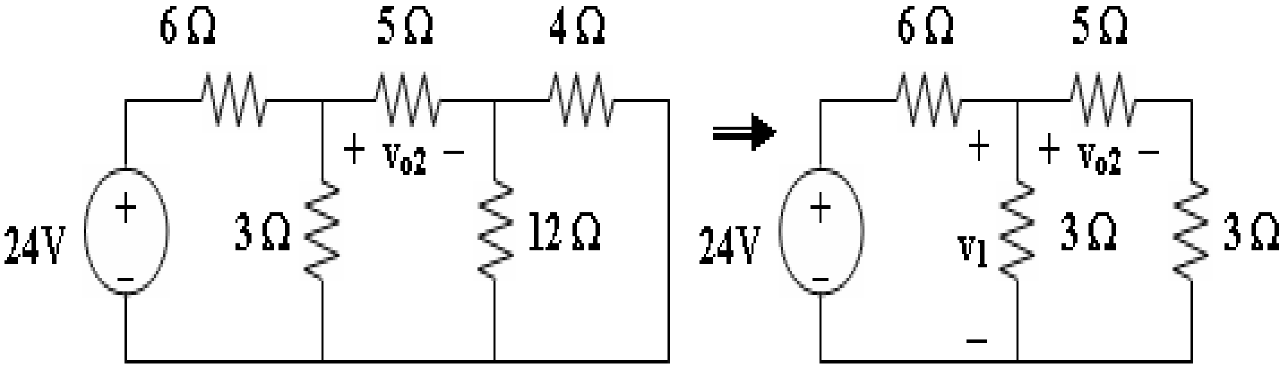
From the circuit above,

$6||3 = 2\ \Omega,\quad 4||12 = 3\ \Omega$

Hence, $I_o = \left(\frac{5}{5+5}\right) \times 4 = \frac{4}{2} = 2\ \text{A}$

$V_{o1} = 5I_o = 10\ \text{V}$

For V_{o2} we keep the independent 24 V voltage source and turn off all other independent sources as shown in the circuit below



From the circuit above,

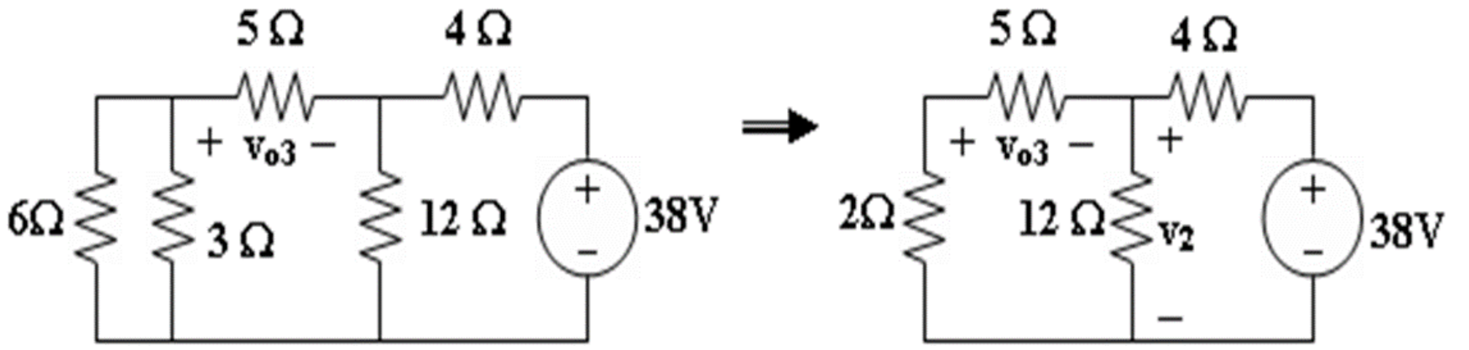
$$12||4 = 3$$

$$3||8 = \frac{24}{11}$$

$$V_1 = \left(\frac{\frac{24}{11}}{\frac{24}{11} + 6}\right) \times 24 = 6.4 \text{ V}$$

$$V_{o2} = \left(\frac{5}{5+3}\right) \times V_1 = \left(\frac{5}{8}\right) \times 6.4 = 4\ \text{V}$$

For V_{o3}, we keep the independent 38-V voltage source and turn off all other independent sources as shown in the circuit below



$6||3 = 2$

$7||12 = \left(\frac{84}{19}\right)\Omega$

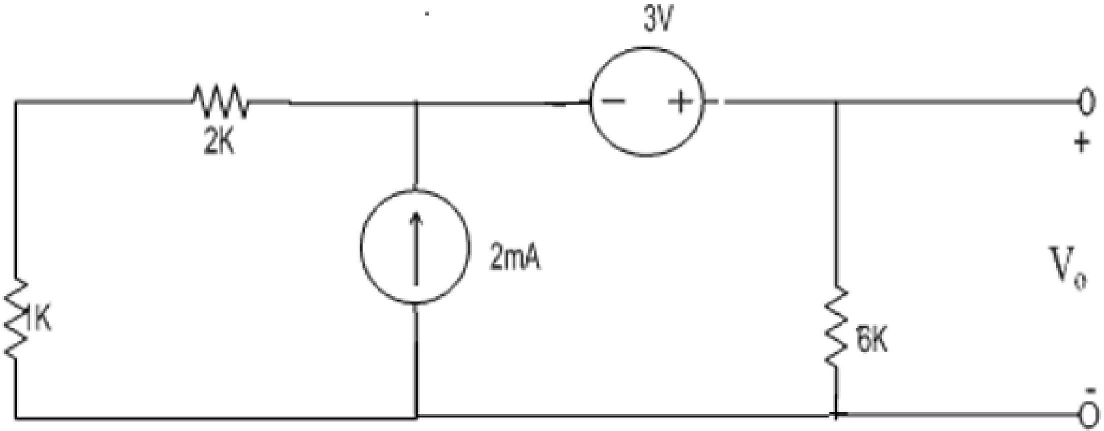
$$V_2 = \left(\frac{\frac{84}{19}}{\frac{84}{19}+4}\right) \times 38 = 19.95\ \mathrm{V}$$

$$V_{o3} = -\left(\frac{5}{2+5}\right) \times V_2 = -\left(\frac{5}{7}\right) \times 19.95 = -14.25\mathrm{V}$$

Therefore, $V_o = 10 + 4 - 14.25 = -0.25\ \mathrm{V}$

Example 2

Use superposition to find V_o in the circuit



Current source acting alone:

Current in the 6 k: using the current divider rule,

$$I'_o = \frac{(1+2)}{(1+2)+6} \times 2 = \frac{3\times 2}{9} = \frac{2}{3}\,\text{mA}$$

$$V'_o = \frac{2}{3}\,\text{mA} \times 6\text{k} = 4\text{ V}$$

Voltage Source acting alone:

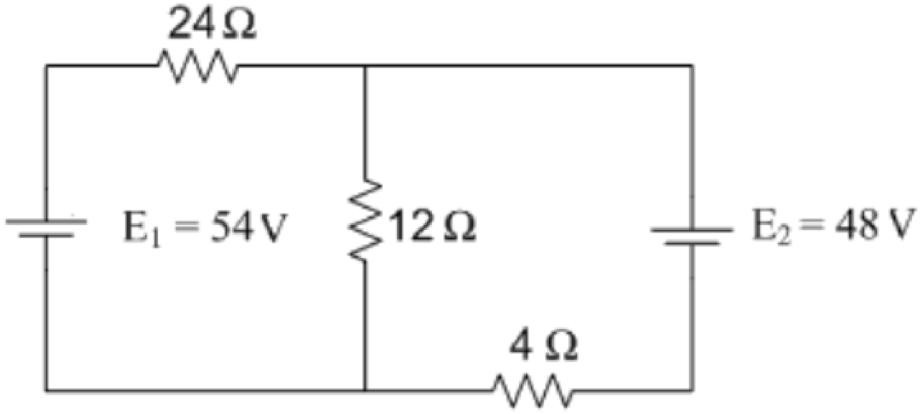
$$I_o'' = \frac{3}{1+2+6} = \frac{3}{9}\text{mA} \text{ and}$$

$$V_o'' = \frac{3}{1+2+6} = \frac{3}{9}\text{mA x } 6k = 2 \text{ V}$$

Therefore $Vo = V_o' + V_o'' = 4 + 2 = 6 \text{ V}$

Example 3

Using the Superposition principle, determine the current through the 4 Ω resistor



With the 54V source acting alone:

Total resistance $= 24 + 12//4 = 24 + \frac{12\times4}{16} = 27\ \Omega$

Total Current $= \frac{54}{27} = 2\ \text{A}$

Using current divider rule, current in the 4 Ω resistor:

$$I' = \frac{12}{12+4} \times 2 = \frac{12\times 2}{16} = 1.5 \text{ A}$$

With the 48 V source acting alone:

$$\text{Total resistance} = 4 + 24//12 = 4 + \frac{24 \times 12}{36} = 12\ \Omega$$

Total current = Current in the 4 Ω resistor,

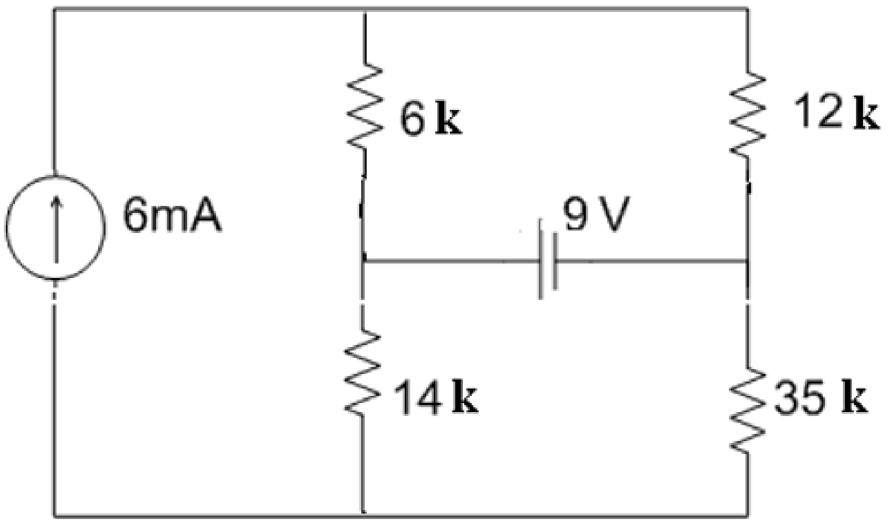
$$I'' = \frac{48}{12} = 4\ A$$

Since I' and I'' are not in the same direction through the 4 k resistor, the actual current

I = I' - I'' = 4 – 1.5 = 2.5 A (Actual current is in the direction of I'')

Example 4

Using the Superposition principle, find the current through the 12 k resistor



With the current source acting alone:

6 k and 12 k are in parallel

$$I' = \frac{6}{12+6} \times 6 = \frac{6 \times 6}{18} = 2 \text{ mA}$$

With the 9 V source acting alone:

$$I'' = \frac{9}{6+12} = \frac{9}{18} = 0.5 \text{ mA}$$

$$I = I' + I'' = 2 + 0.5 = 2.5 \text{ mA}$$

2. NODAL ANALYSIS

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The nodal analysis is a method used to solve circuits containing multiple nodes and loops.

This is based on Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL).

Definition of terms used

- **A branch:** It is a portion of a circuit containing a single element or more elements in series.

- **A node:** It is a point of connection of two or more branches or circuit elements.

Note that all the connecting wire in unbroken contact with the point is part of the node.

General Approach to Nodal Analysis

The Nodal Analysis is applied as follows:

1. Determine the number of nodes within the network, N.

2. Select a reference node and label the others as V_1, V_2, V_3, ...V_{N-1}

- **A reference node is often chosen to be the node to which the largest number of branches is connected.**

- In a practical electronic circuit, this usually corresponds to the chassis or ground line.

General Approach to Nodal Analysis

- In many cases such as in electric power systems, the chassis is shorted to the earth itself making its potential zero. For this reason, the reference node is frequently referred to as **ground or the ground node**.

3. Apply KCL at each node except the reference, assuming that all unknown currents leave the node for each application of KCL.

4. Solve the resulting equations of the nodal or node voltages.

The process is easy!

Independent Current Sources	Dependent Current Sources	Independent Voltage Sources	Dependent Voltage Sources

Independent Current Sources

These circuits contain only independent current sources.

Methods used:

- General Approach
- Matrix Form

Dependent Current Sources

These circuits contain dependent current sources.

Independent Voltage Sources

These circuits contain independent voltage sources.

Methods used:

- Supernode
- Convert voltage sources to current sources

Dependent Voltage Sources

These circuits contain dependent voltage sources.

Case 1

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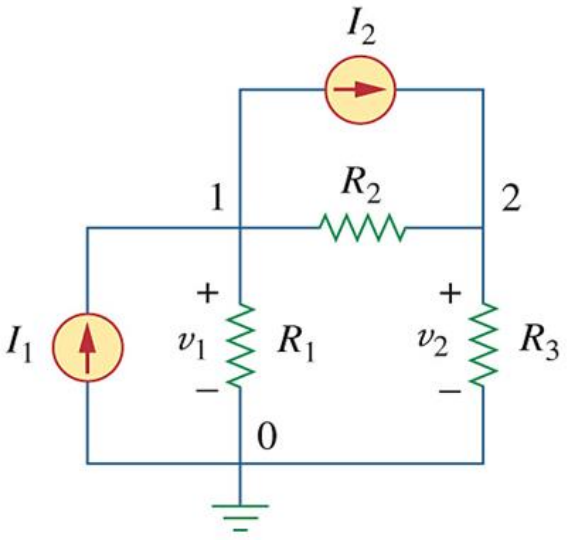
Circuits containing only Independent Current Sources.

General Approach

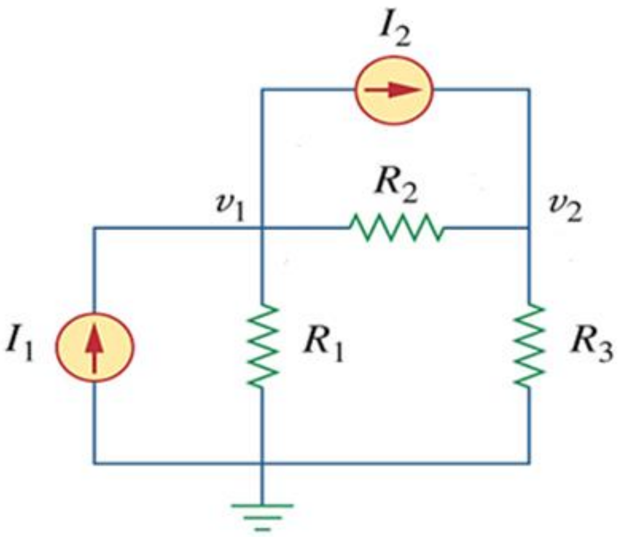
Nodal Analysis Procedure

Given a circuit with n nodes without voltage sources



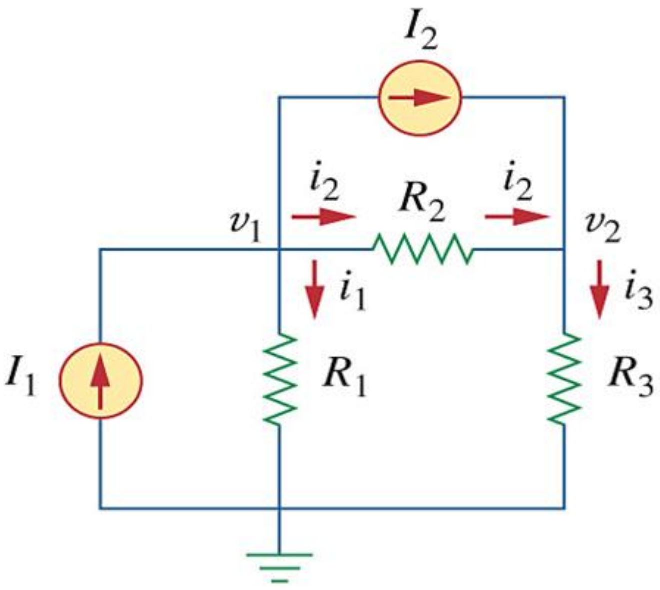
- The reference node is commonly called the ground.

- Assign voltage designations to the non-reference nodes.
- Nodes 1 and 2 are assigned voltages v_1 and v_2 respectively.



Apply KCL to each non-reference node in the circuit

- To be able to apply KCL, we need to know the **direction** of currents at each node
- Note that current flows from a **HIGHER** potential to a **LOWER** potential in a resistor



At node 1, applying KCL gives

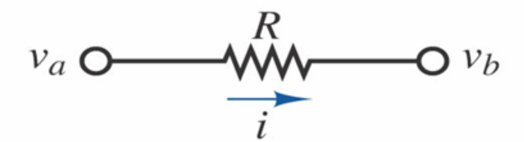
$$I_1 = I_2 + i_1 + i_2 \qquad (1)$$

At node 2,

$$I_2 + i_2 = i_3 \qquad (2)$$

In nodal analysis if

- We assign node voltages v_a and v_b, the branch current i flowing from a to b is then expressed as



$$i = \frac{v_a - v_b}{R}$$

We now apply Ohm's law to express the unknown current i_1, i_2 and i_3 in terms of node voltages.

$$i_1 = \frac{v_1 - 0}{R_1} \qquad i_2 = \frac{v_1 - v_2}{R_2} \qquad i_3 = \frac{v_3 - 0}{R_3} \qquad (3)$$

Substituting (3) in (1) and (2) yields

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

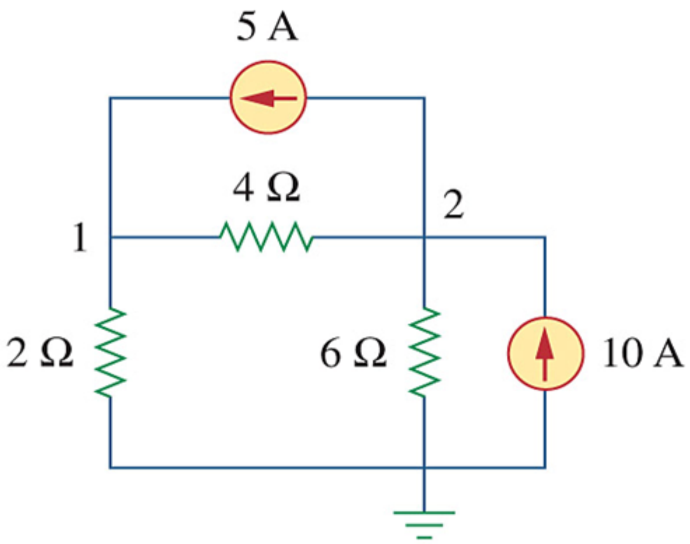
$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

Solve for the node voltages using the

- Substitution method
- Elimination method
- Cramer's rule

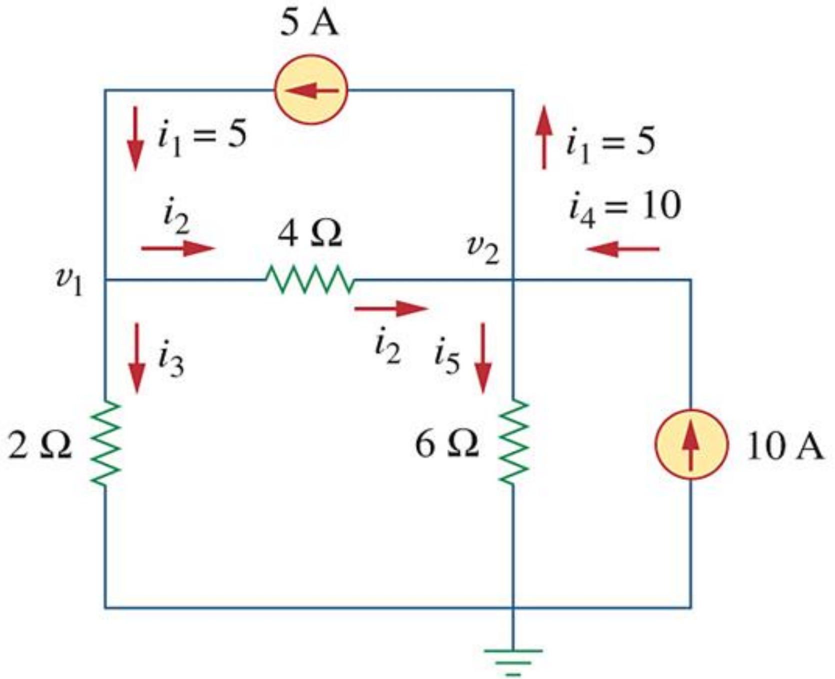
Example 1

Calculate the node voltages in the circuit shown below



Consider the circuit below prepared for nodal analysis:

- We assigned voltages v_1 and v_2 to nodes 1 and 2
- Current directions as shown on the circuit



Applying KCL at node 1 yields

$$i_1 = i_2 + i_3$$

Expressing the currents in terms of voltages we've

$$5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

Multiplying each term by4, we obtain

$$20 = v_1 - v_2 + 2v_1$$

or

$$3v_1 - v_2 = 20 \qquad (1)$$

Applying KCL at node 2 yields

$i_2 + i_4 = i_1 + i_5$

Expressing the currents in terms of voltages we've

$$\frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

Multiplying each term by 12 results in

$3v_1 - 3v_2 + 120 = 60 + 2v_2$

or

$$-3v_1 + 5v_2 = 60 \qquad (2)$$

Now we have two simultaneous equations, equation (1) and (2)

Method 1

Using the elimination technique, we add equations (1) and (2)

$4v_2 = 80, \qquad v_2 = 20$ V

Substituting $v_2 = 20$ in equation (1) gives

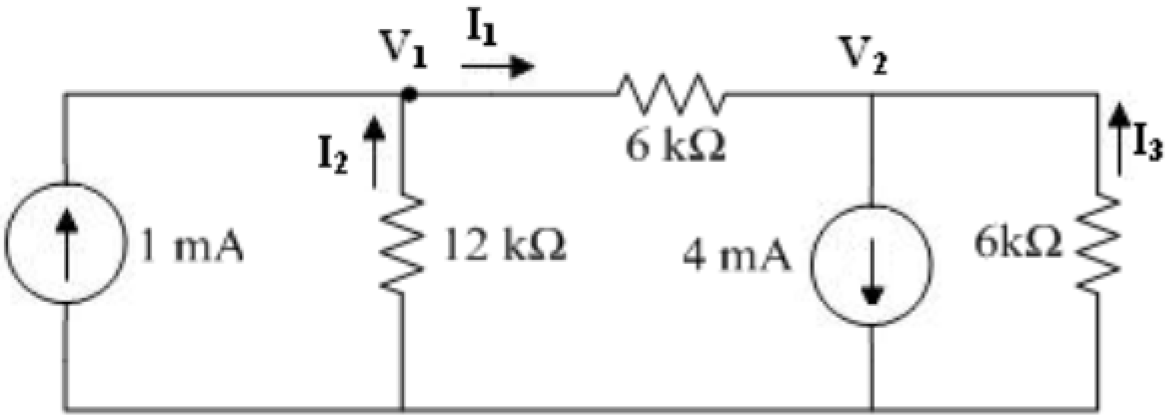
$3v_1 - 20 = 20$

$v_1 = \frac{40}{3}$

$v_1 = 13.33$ V

Example 2

Apply nodal analysis to the network



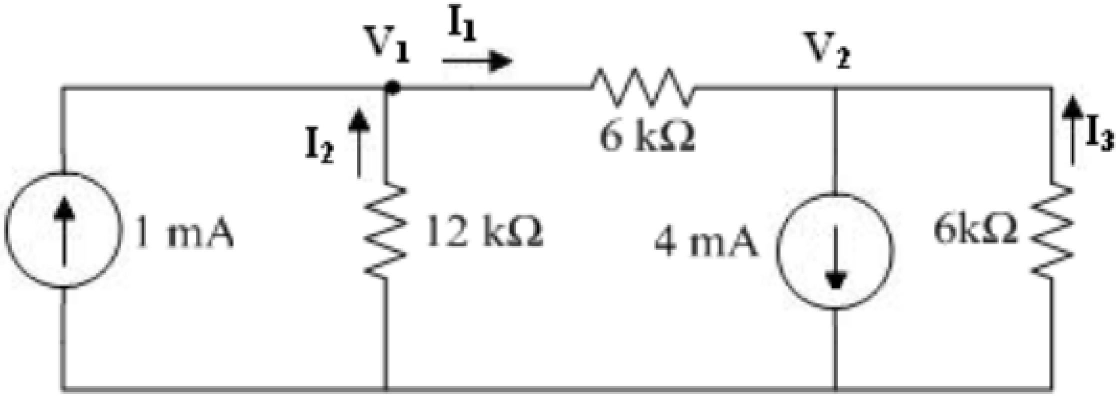
Apply KCL at node 1:

$$1 = \frac{V_1}{12} + \frac{V_1 - V_2}{6}$$

$$1 = \mathrm{V}_1 \times \left[\frac{1}{12} + \frac{1}{6}\right] - \frac{V_2}{6}$$

$$1 = \frac{V_1}{4} + \frac{V_2}{6} \qquad (1)$$

Apply KCL at node 2:



$$-4 = \frac{V_2}{6} + \frac{V_2 - V_1}{6}$$

$$-4 = -\frac{V_1}{6} + \mathrm{V}_2 \times \left[\frac{1}{6} + \frac{1}{6}\right]$$

$$-4 = -\frac{V_1}{6} + \frac{V_3}{3} \qquad (2)$$

$$(1) \times 2 + (2): -2 = \frac{V_1}{3},\ V_1 = -6\ \text{V}$$

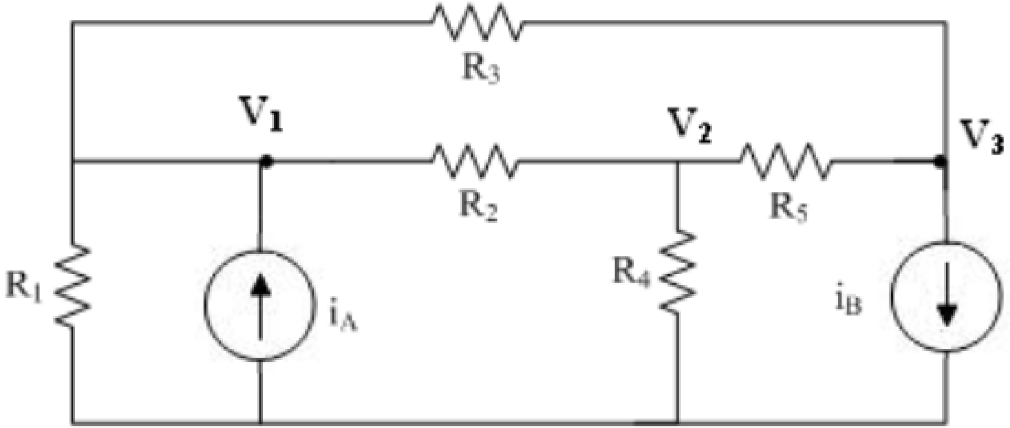
$$\text{From (1)}\ \frac{V_2}{6} = \frac{V_1}{4} - 1 = -\frac{6}{4} - 1 = -\frac{5}{2},\ V_2 = -\frac{5}{2} \times 6 = -15\ \text{V}$$

$$\text{Current}\ I_2 = \frac{0 - V_1}{12} = \frac{0-(-6)}{12} = \frac{6}{12} = 0.5\ \text{mA}$$

$$I_1 = 1 + I_2 = 1 + 0.5 = 1.5\ \text{mA or use}\ \frac{V_1 - V_2}{6} = \frac{9}{6} = 1.5\ \text{mA}$$

$$I_3 = 4 - I_1 = 4 - 1.5 = 2.5\ \text{mA or use}\ \frac{0 - V_2}{6} = \frac{15}{6} = 2.5\ \text{mA}$$

Example 3 – Matrix Form



$$\mathrm{i}_A = \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + \frac{v_1 - v_3}{R_3} = \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] v_1 - \frac{1}{R_2} v_2 - \frac{1}{R_3} v_3$$

$$0 = \frac{v_2}{R_4} + \frac{v_2 - v_1}{R_2} + \frac{v_2 - v_3}{R_5} = -\frac{1}{R_2} v_1 + \left[\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right] v_2 - \frac{1}{R_5} v_3$$

$$-\mathrm{i}_B = \frac{v_3 - v_2}{R_5} + \frac{v_3 - v_1}{R_3} = -\frac{1}{R_3} v_1 - \frac{1}{R_5} v_2 + \left[\frac{1}{R_3} + \frac{1}{R_5}\right] v_3$$

In matrix form, the equations are as follows:

$$\begin{bmatrix} \mathrm{i_A} \\ 0 \\ -\mathrm{i_B} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & -\frac{1}{R_3} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5} & -\frac{1}{R_5} \\ -\frac{1}{R_3} & -\frac{1}{R_5} & \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

The equation is in the form

$$[i] = [G][v]$$

The matrix G is called conductance or admittance matrix

Case 2

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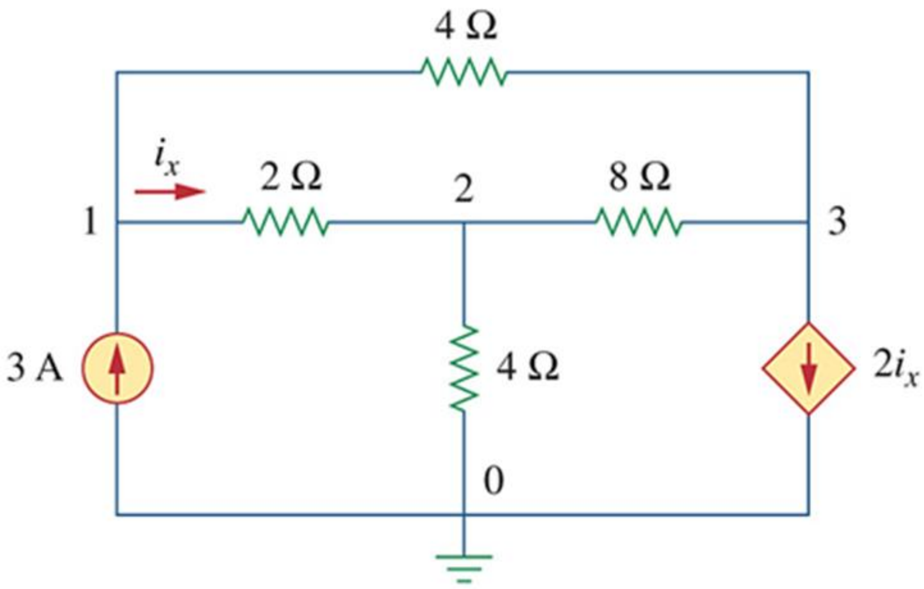
Circuits containing Dependent Current Sources.

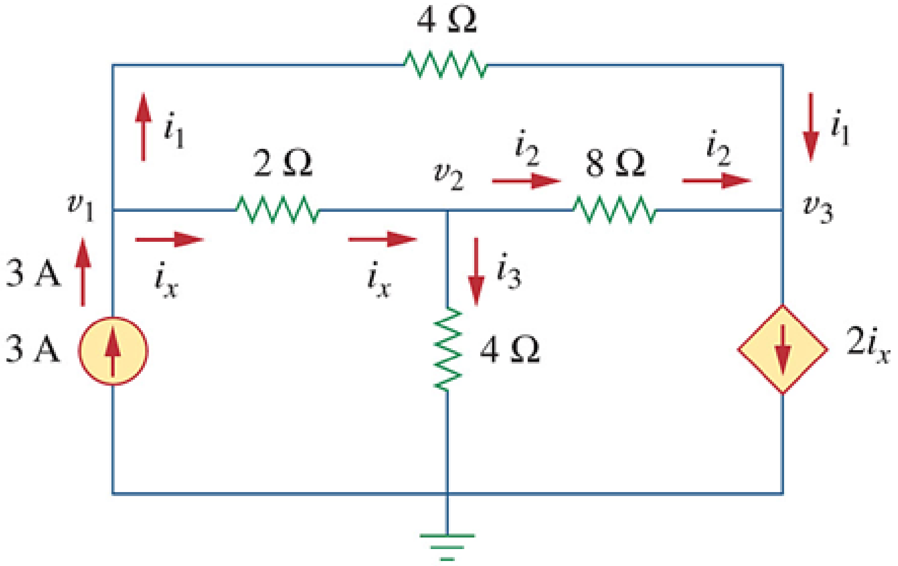
Example 1

Determine the voltages at the nodes of the circuit shown below



We assign

- voltages to the three nodes and
- direction of currents at the nodes as shown in the circuit below



At node 1

$3 = i_1 + i_x$, $$3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

Multiplying by 4 and rearranging terms, we get

$$3v_1 - 2v_2 - v_3 = 12 \qquad (1)$$

At node 2

$$i_x = i_2 + i_3, \quad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

Multiplying by 8 and rearranging terms, we get

$$-4v_1 + 7v_2 - v_3 = 0 \qquad (2)$$

At node 3

$$i_1 + i_2 = 2i_x$$

$$\frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{4}$$

Multiplying by 8, rearranging terms, and dividing by 3, we get:

$$2v_1 - 3v_2 + v_3 = 0 \qquad (3)$$

Solving equations (1), (2) and (3)

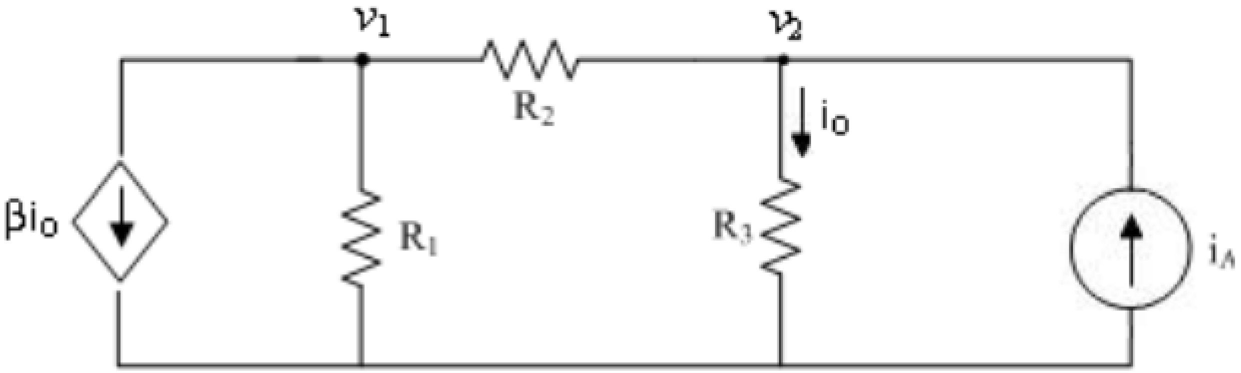
$v_1 = 4.8\ \text{V}$

$v_2 = 2.4\ \text{V}$

$v_3 = -\ 2.4\ \text{V}$

Example 2

Obtain the nodal equations for the network given below



Apply KCL at node 1:

$$-\beta i_o = \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} \quad \text{or} \quad -\beta\frac{v_2}{R_3} = \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

or

$$0 = \left[\frac{1}{R_1} + \frac{1}{R_2}\right] v_1 - \left[\frac{1}{R_2} - \frac{\beta}{R_3}\right] v_2$$

Apply KCL at node 2:

$$\mathrm{i_A} = \frac{v_2}{R_3} + \frac{v_2 - v_1}{R_2} = -\frac{1}{R_2} v_1 + \left[\frac{1}{R_2} + \frac{1}{R_3}\right] v_2$$

$$0 = \left[\frac{1}{R_1} + \frac{1}{R_2}\right] v_1 - \left[\frac{1}{R_2} - \frac{\beta}{R_3}\right] v_2$$

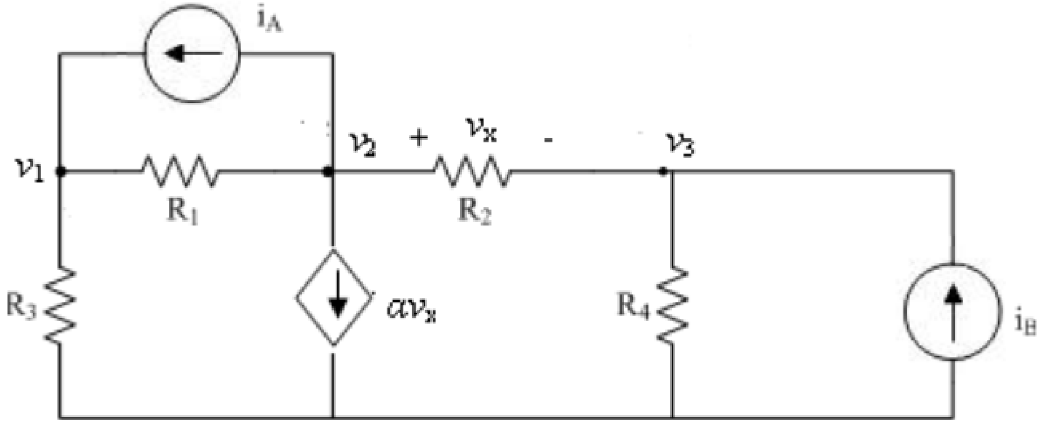
$$\mathrm{i_A} = -\frac{1}{R_2} v_1 + \left[\frac{1}{R_2} + \frac{1}{R_3}\right] v_2$$

In matrix form, we have:

$$\begin{bmatrix} 0 \\ i_A \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{R_2}\right) & -\left(\frac{1}{R_2} - \frac{\beta}{R_3}\right) \\ -\frac{1}{R_2} & \left(\frac{1}{R_2} + \frac{1}{R_3}\right) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Example 3

Obtain the nodal equations for the network given below



Apply KCL at node 1:

$$i_A = G_3 v_1 + G_1(v_1 - v_2) \; or \; iA = (G_1 + G_3)v_1 - G_1 v_2$$

Apply KCL at node 2:

$$-i_A - \alpha v_x = G_1(v_2 - v_1) + G_2(v_2 - v_3)$$

$$-i_A = G_1(v_2 - v_1) + G_2(v_2 - v_3) + \alpha(v_2 - v_3)$$

$$-i_A = -G_1 v_1 + (G_1 + G_2 + \alpha)\, v_2 - (G_2 + \alpha)\, v_3$$

Apply KCL at node 3:

$$i_B = G_2(v_3 - v_2) + G_4 v_3 = -G_2 v_2 + (G_2 + G_4) v_3$$

At node 1:

$$i_A = (G_1 + G_3) v_1 - G_1 v_2$$

At node 2:

$$-i_A = -G_1 v_1 + (G_1 + G_2 + \alpha) v_2 - (G_2 + \alpha) v_3$$

At node 3:

$\mathrm{i_B = -G_2v_2 + (G_2 + G_4)v_3}$

In matrix form, we have:

$$\begin{bmatrix} i_A \\ -i_A \\ i_B \end{bmatrix} = \begin{bmatrix} G_1 + G_3 & -G_1 & 0 \\ -G_1 & G_1 + G_2 + \alpha & G_2 + \alpha \\ 0 & -G_2 & G_2 + G_4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Case 3

“

Circuits containing only Independent Voltage Sources.

Circuits containing only Independent Voltage Sources

The presence of voltage sources **reduces the number of equations and the number of unknowns by one per voltage source**.

One of the following two approaches may be used:

- The concept of supernode.
- Converting voltage sources to current sources.

NB: Converting voltage sources to current sources is applicable when the independent voltage sources have resistances in series.

Method 1 - The Concept of Supernode

Voltage sources are enclosed in separate regions. Each closed region is called **a supernode**.

A supernode contains two nodes which may consist of

- a non-reference node and a reference node

  or

- two non–reference nodes.

The Concept of Supernode

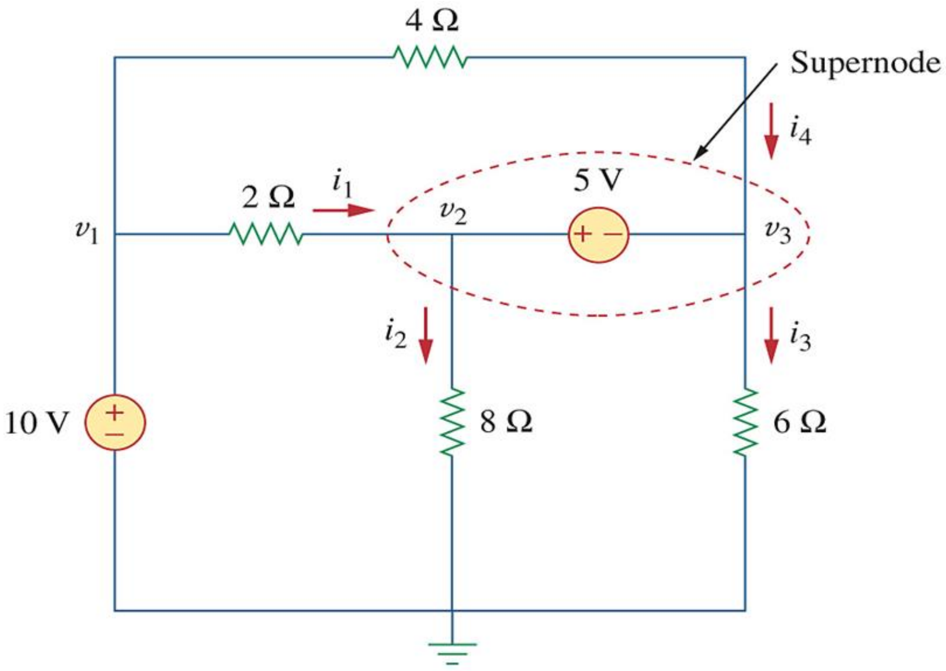
To obtain the nodal equations, we apply KCL to

- all supernodes not containing the reference node and
- to all other reference nodes.

KCL is applied to a supernode using the generalized form of KCL which states that “the algebraic sum of currents entering a closed region is zero.”

Example 1

Consider the circuit below



- The 10 V voltage source is connected between v_1 and the reference node.
- Thus v_1 is equal 10 V.
- The 5 V voltage source is between two non-reference nodes.
- v_2 and v_3 form a supernode

If the voltage source is connected between two non-reference nodes:

- the two non-reference nodes form a supernode.
- we apply both KCL and KVL to determine the node voltages.

In the circuit shown earlier

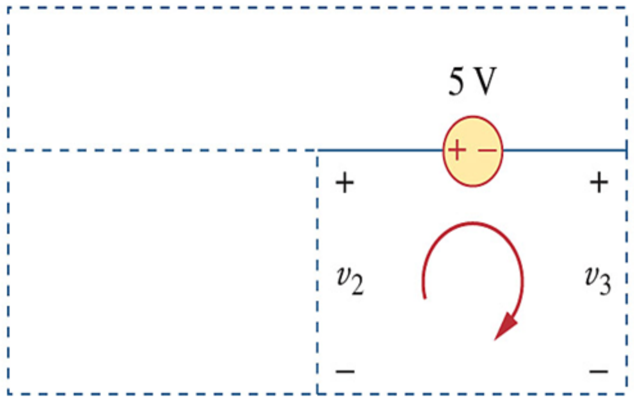
- Nodes 2 and 3 form a supernode
- KCL at the supernode is:

$$i_1 + i_4 = i_2 + i_3$$

or

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2}{8} + \frac{v_3}{6}$$

To apply KVL to the supernode we redraw the circuit as shown below:



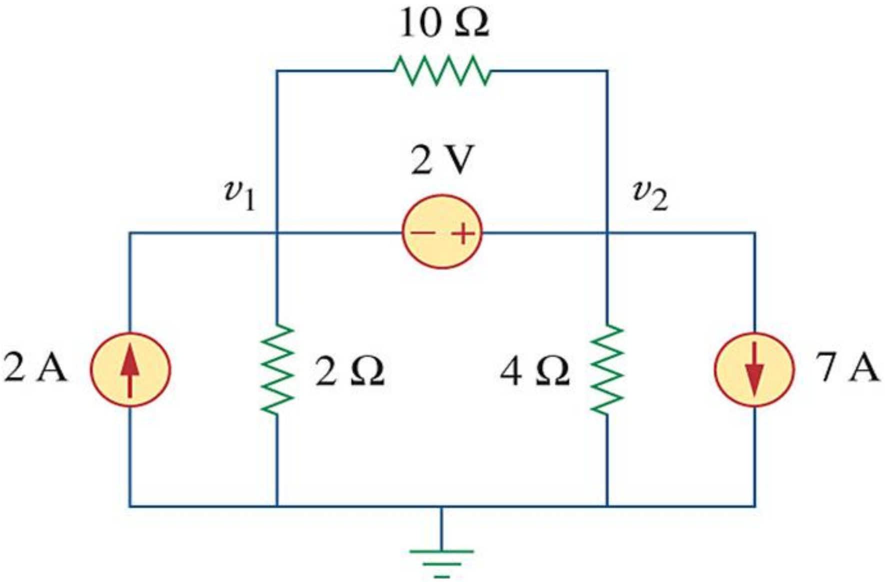
Going around the loop in the clockwise direction gives:

$$-v_2 + 5 + v_3 = 0$$

$$v_2 - v_3 = 5$$

Example 2

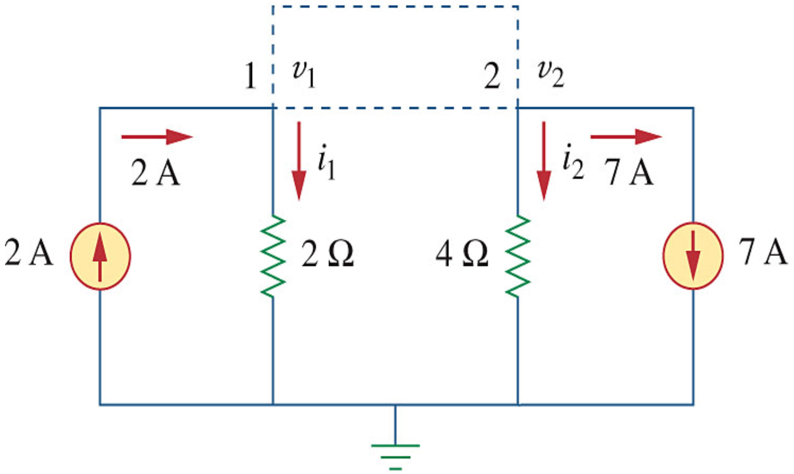
For the circuit shown below find the node voltages



- The supernode contains the 2 V source, nodes 1 and 2 and the 10-Ω resistor.

- Applying KCL to the supernode as shown in the circuit below gives:

$$2 = i_1 + i_2 + 7 \qquad (1)$$



Expressing i_1 and i_2 in terms of node voltages

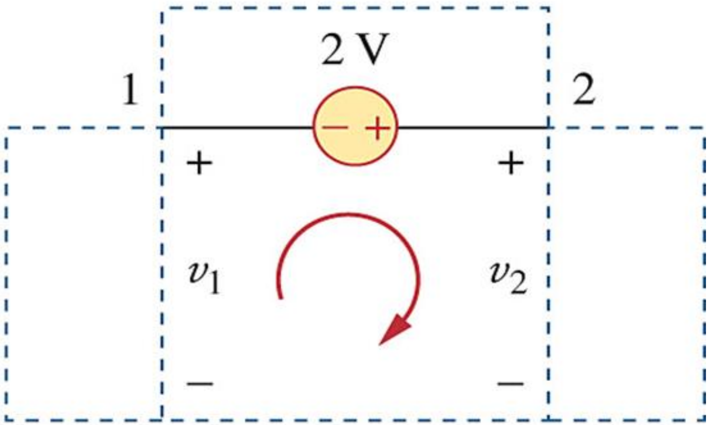
$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7$$

$$8 = 2v_1 + v_2 + 28$$

or

$$2v_1 + v_2 = -20 \qquad (2)$$

- To get the relationship between v_1 and v_2 ,we apply KVL to the circuit shown below:



- Going round the loop, we obtain:

$$-v_1 - 2 + v_2 = 0$$

$$v_2 = v_1 + 2 \qquad (3)$$

From equations (2) and (3), we write

$v_2 = v_1 + 2 = -20 - 2v_1$

or

$3v_1 = -22,$

$v_1 = -7.333\ \text{V}$

and

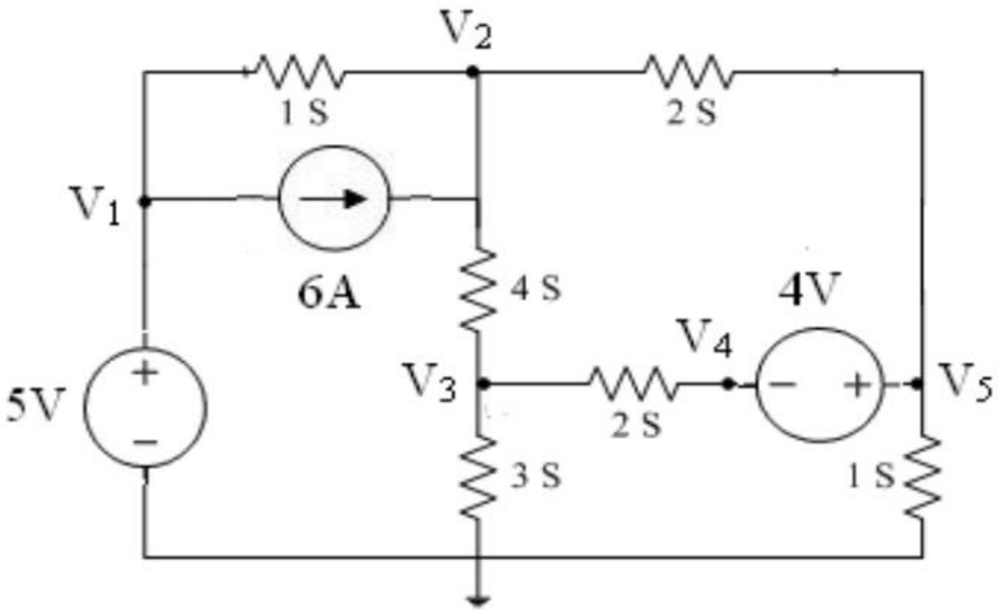
$v_2 = v_1 + 2 = -7.333 + 2$

$v_2 = -5.333\ \text{V}$

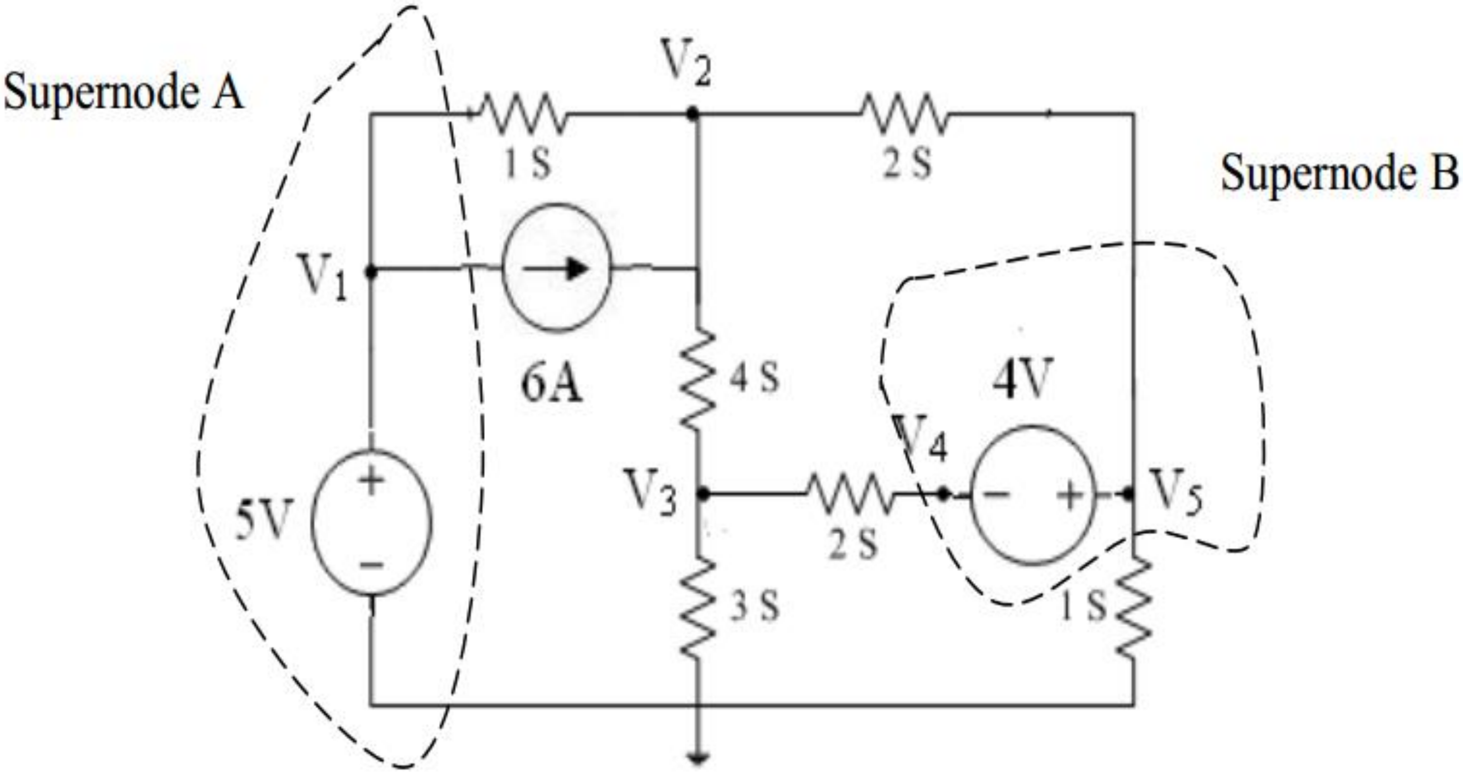
NB: Nodal analysis applies KCL to find unknown voltages in a given circuit

Example 3

Obtain the nodal equations for the network given below



Two supernodes A and B are shown



Voltage V_1 at node 1 in supernode A (containing the reference node) may be determined immediately as

$$V_1 = 5 + 0 = 5\ \text{V}$$

Voltage V_5 at node 5 in supernode B can be expressed in terms of V_4 at the other non-reference node 4 in the supernode as

$$V_5 = V_4 + 4$$

The number of nodal equations required $= 5 – 2 = 3$

Apply KCL to non-reference node 2:

$6 = 1(V_2 – 5) + 4(V_2 – V_3) + 2(V_2 – V_4 – 4)$

$19 = 7V_2 – 4V_3 – 2V_4$

Apply KCL to non-reference node 3:

$0 = 3V_3 + 4(V_3 – V_2) + 2(V_3 – V_4)$

$0 = –4V_2 + 9V_3 – 2V_4$

Apply KCL to supernode B:

$0 = 2(V_4 – V_3) + 1(V_4 + 4) + 2[(V_4 + 4) – V_2]$

$–12 = –2V_2 – 2V_3 + 5V_4$

At node 2:

$19 = 7V_2 - 4V_3 - 2V_4$

At node 3:

$0 = -4V_2 + 9V_3 - 2V_4$

At supernode B:

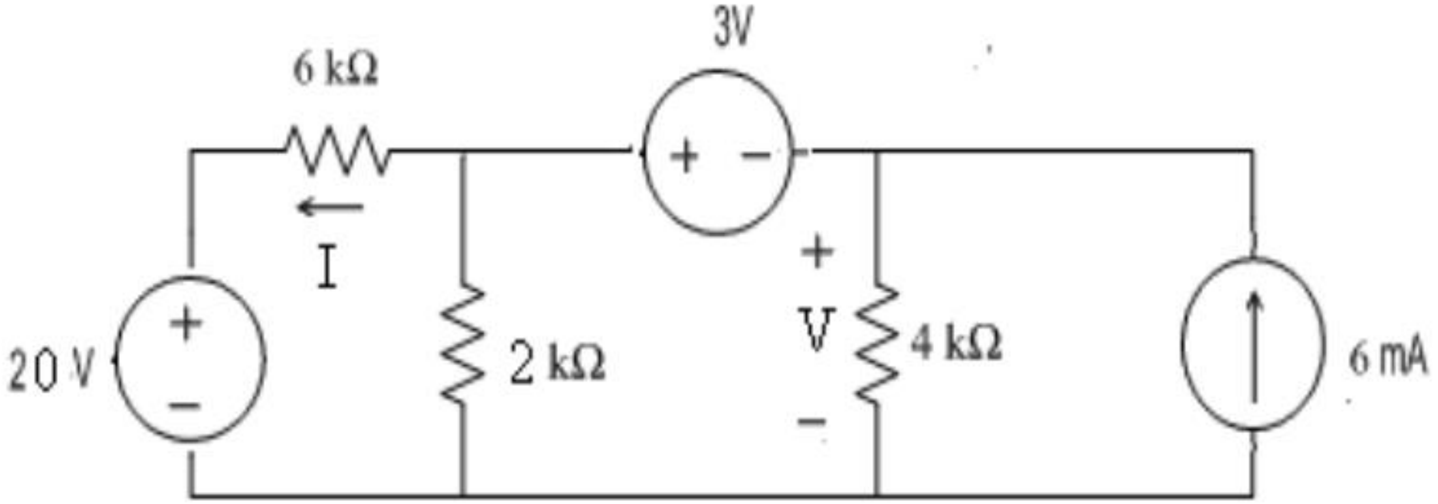
$-12 = -2V_2 - 2V_3 + 5V_4$

In matrix form, we have:

$$\begin{bmatrix} 19 \\ 0 \\ -12 \end{bmatrix} = \begin{bmatrix} 7 & -4 & -2 \\ -4 & 9 & -2 \\ -2 & -2 & 5 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

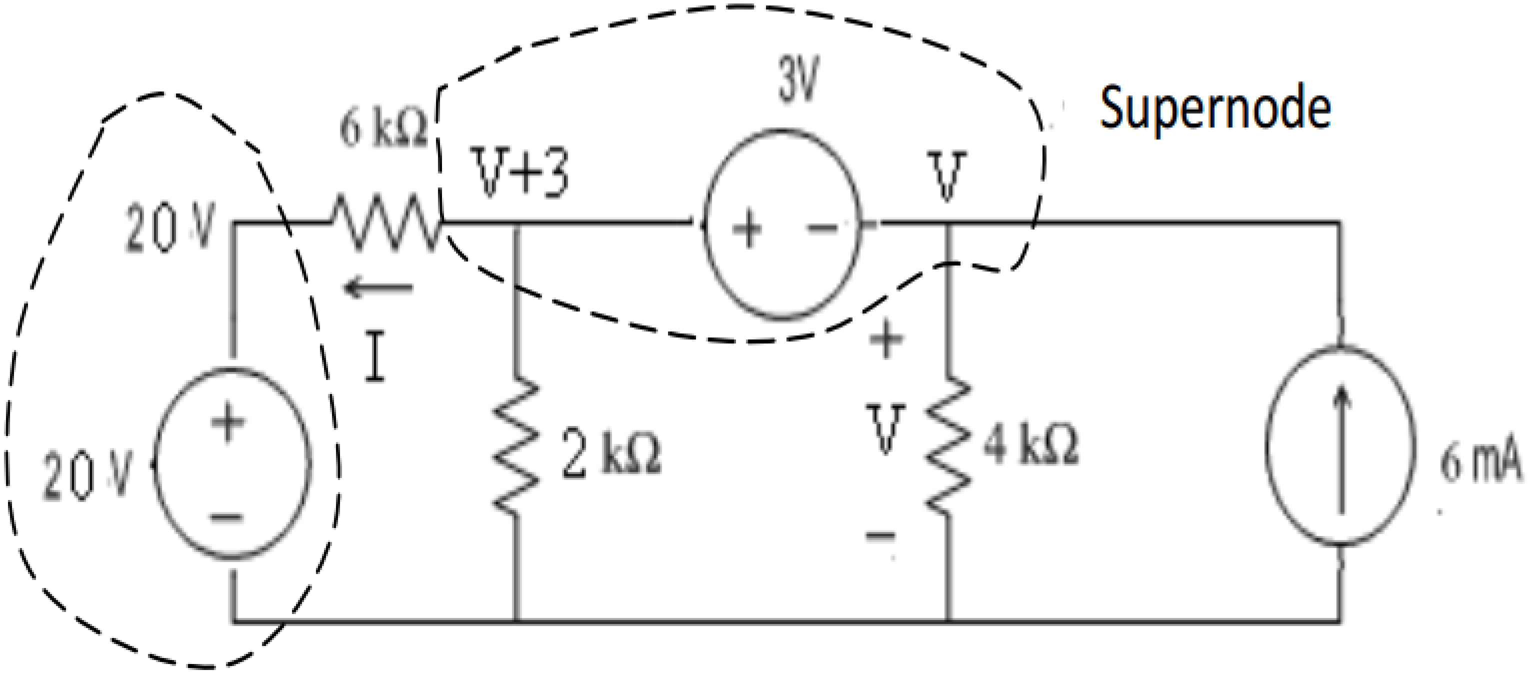
Example 4

Find V and I in the circuit using nodal analysis



Network has two supernodes and we must apply KCL to the supernode which does not contain reference node.

The supernodes are shown below



The voltage:

$$6 = \frac{V}{4} + \frac{V+3}{2} + \frac{(V+3) - 20}{6}$$

$6 \times 12 = 3V + 6(V + 3) + 2[(V + 3) – 20]$

$72 = 11V – 16$ or $11V = 88$

Therefore: $V = 8$ V

The current:

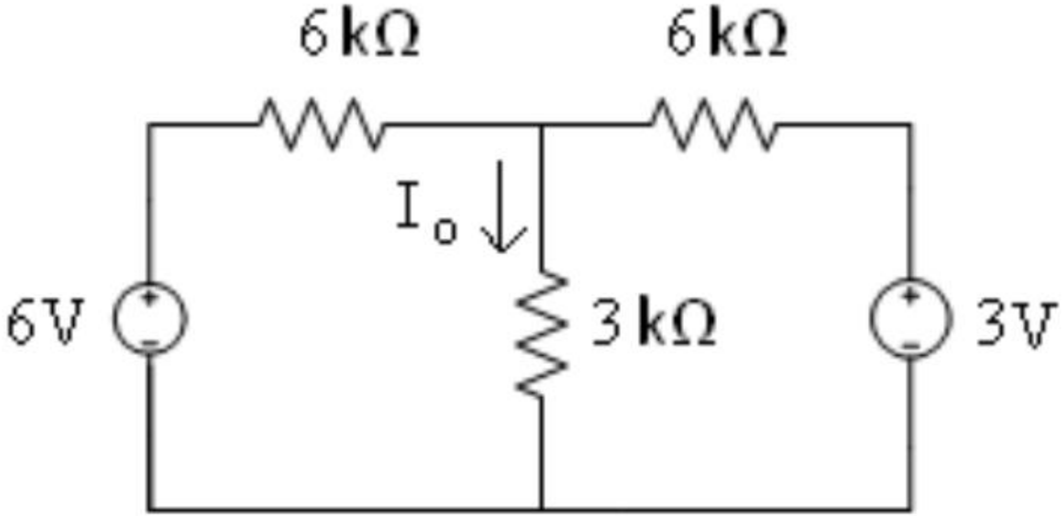
$$I = \frac{(V+3) - 20}{6} = \frac{(8 + 3) - 20}{6}$$

$$I = -\frac{9}{6}$$

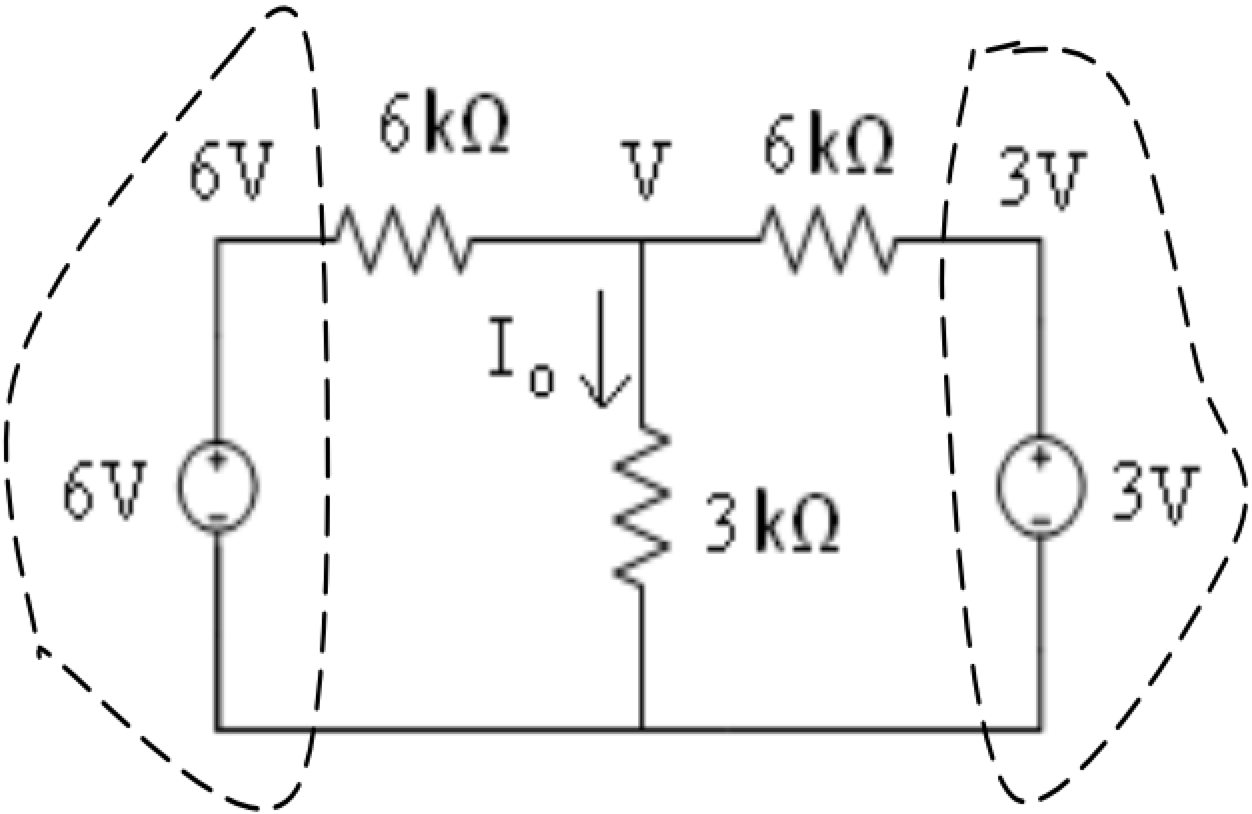
$I = 1.5$ mA

Example 5

Find the current I_o in the circuit below



Circuit has two supernodes all containing reference node and one non-reference node.



Applying KCL to the non-reference node, we obtain:

$$0 = \frac{V-6}{6} + \frac{V}{3} + \frac{V-3}{6}$$

$$0 = (V - 6) + 2V + V - 3$$

$$0 = 4V - 9$$

$$V = \frac{9}{4}\ V$$

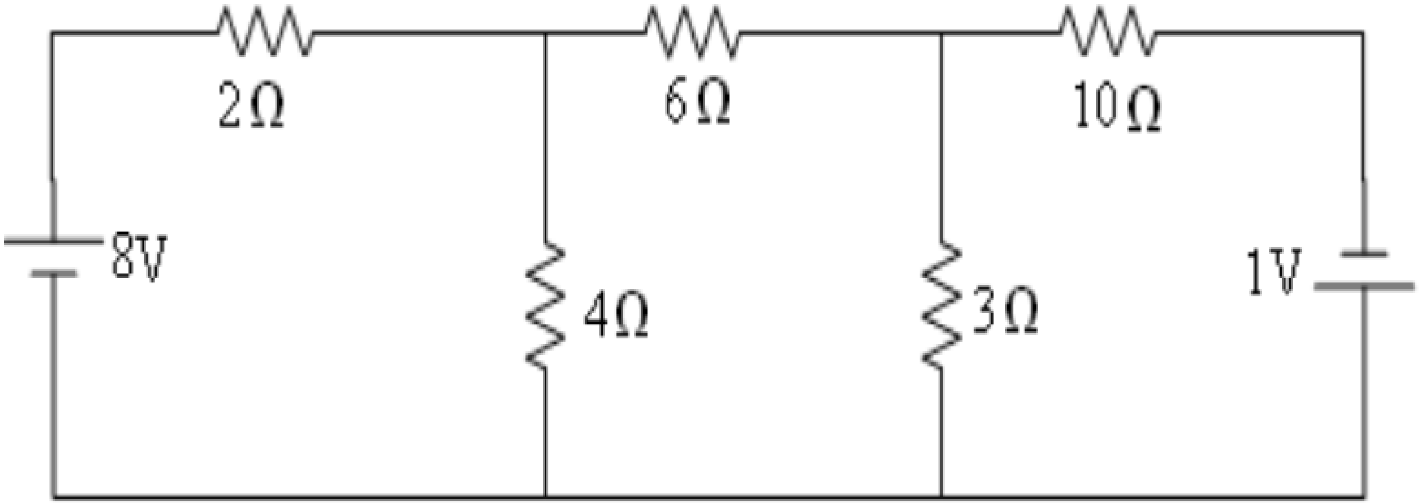
The current:

$$I_o = \frac{V}{3} = \frac{9}{4 \times 3} = \frac{3}{4}$$

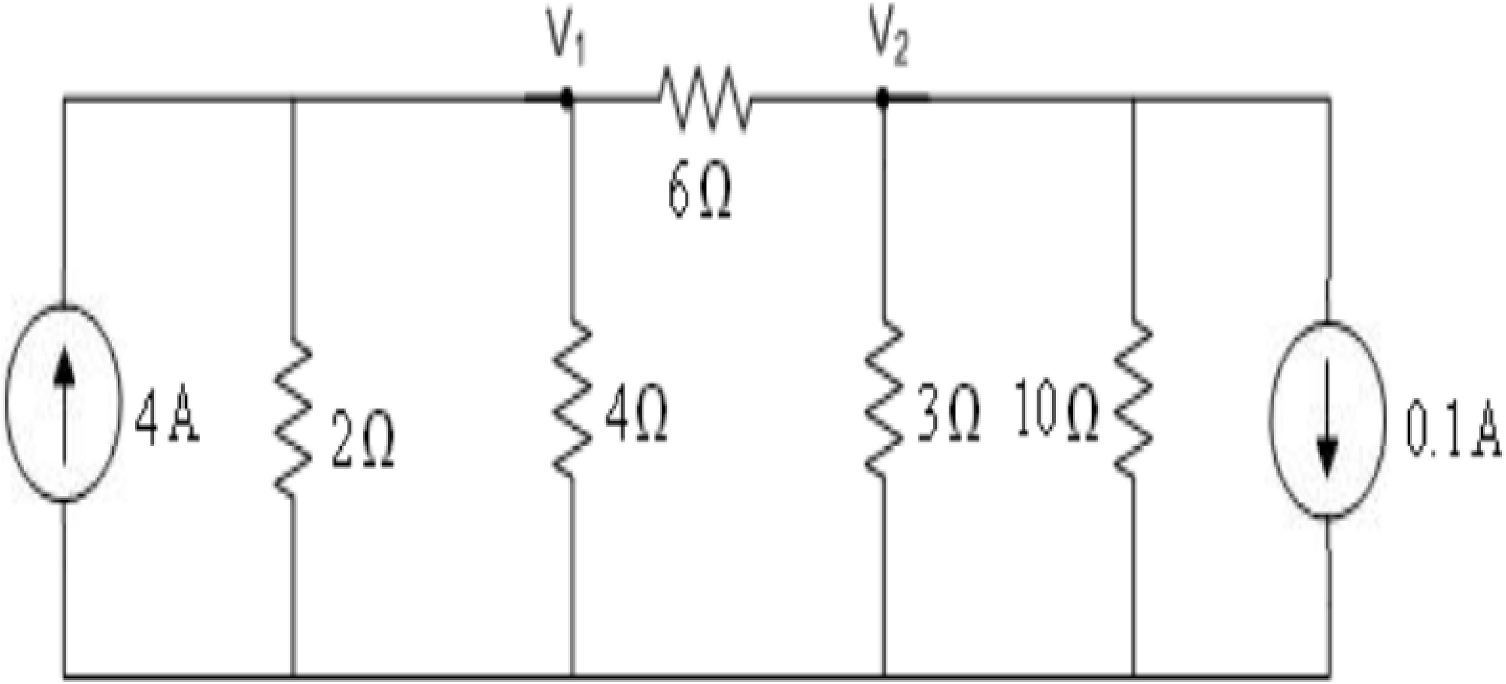
$$I_o = 0.75\ \text{mA}$$

Method 2 - Converting voltage sources to current sources

Find the voltage across the 3 Ω resistor by nodal analysis



Converting sources and choosing nodes, we obtain:



By inspection, we have

$$4 = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6}\right) V_1 - \frac{1}{6}\mathrm{V}_2$$

$$-0.1 = -\frac{1}{6}\mathrm{V}_1 + \left(\frac{1}{6} + \frac{1}{3} + \frac{1}{10}\right) V_2$$

or

$$4 = \frac{11}{12} V_1 - \frac{1}{6}\mathrm{V}_2$$

$$-0.1 = -\frac{1}{6}\mathrm{V}_1 + \frac{3}{5} V_2$$

Solving the two equations simultaneously, we obtain voltage across the 3 Ω resistor, $\mathrm{V}_2 = 1.101$ V

Case 4

“

Circuits containing Dependent Voltage Sources.

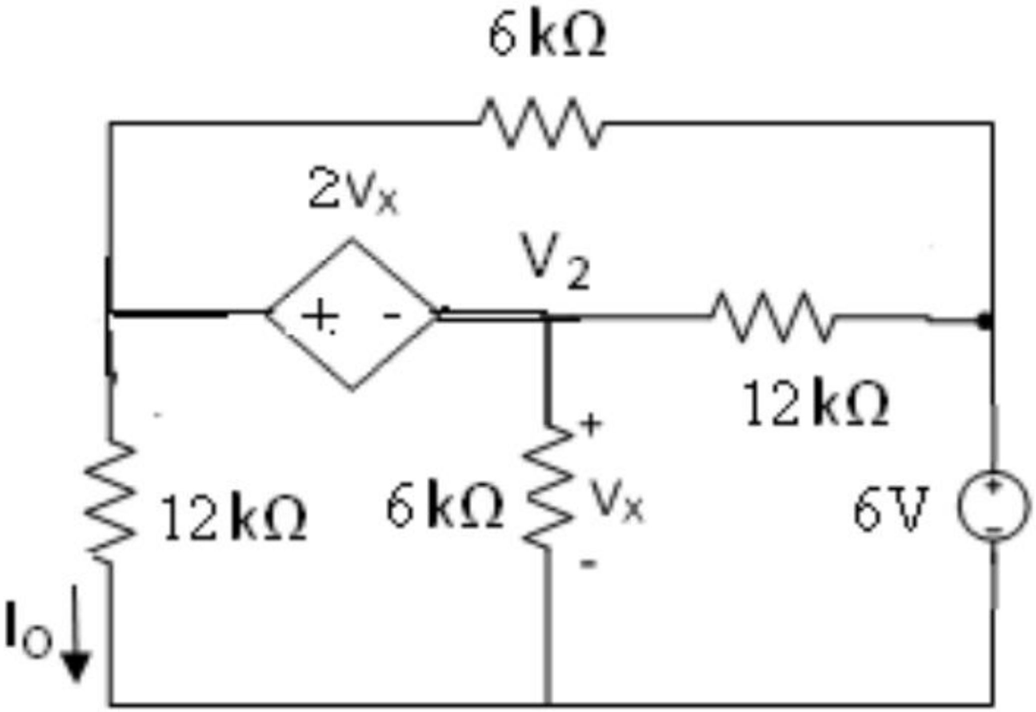
Circuits containing dependent voltage sources

- Treat them in the same manner as circuits containing independent voltage sources.

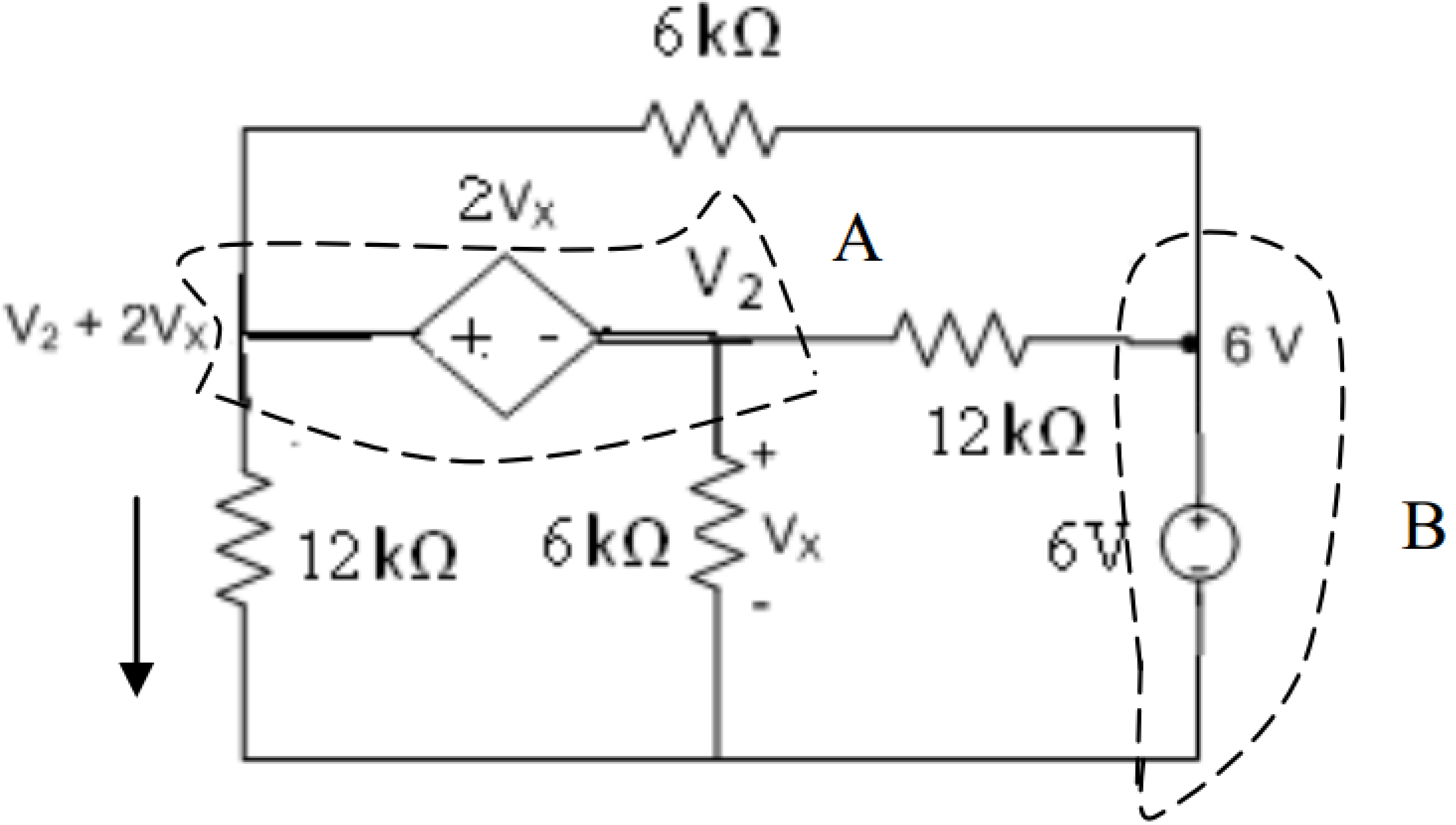
- When writing the circuit equations, first treat the dependent source as though it were an independent source and then write the controlling equations and substitute.

Example 1

Find the current Io in the network



The supernodes are shown below



Apply KCL to the supernode A:

$$0 = \frac{V_2 + 2Vx}{12} + \frac{(V_2 + 2Vx) - 6}{6} + \frac{V_2}{6} + \frac{V_2 - 6}{12}$$

$$0 = \left(\frac{1}{12} + \frac{1}{6} + \frac{1}{6} + \frac{1}{12}\right) V_2 + \left(\frac{2}{12} + \frac{2}{6}\right) V_x - 1 - \frac{6}{12}$$

$$0 = \frac{3}{6} V_2 + \frac{3}{6} V_x - \frac{18}{12}$$

Controlling equation: $V_x = V_2$

Substituting into the nodal equation, we obtain:

$$0 = \frac{3}{6}V_2 + \frac{3}{6}V_2 - \frac{18}{12} \ or \ \frac{18}{12} = \frac{6}{6}V_2$$

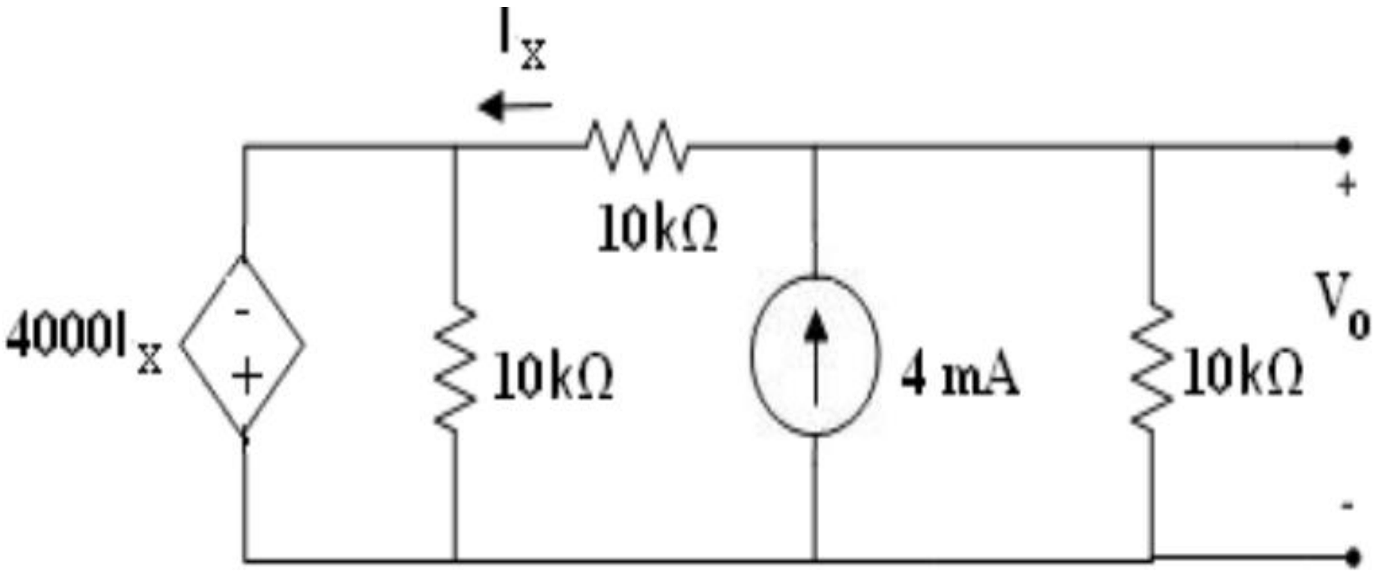
$V_2 = 1.5$ V

Current $I_o = \frac{V_2 + 2Vx}{12}$

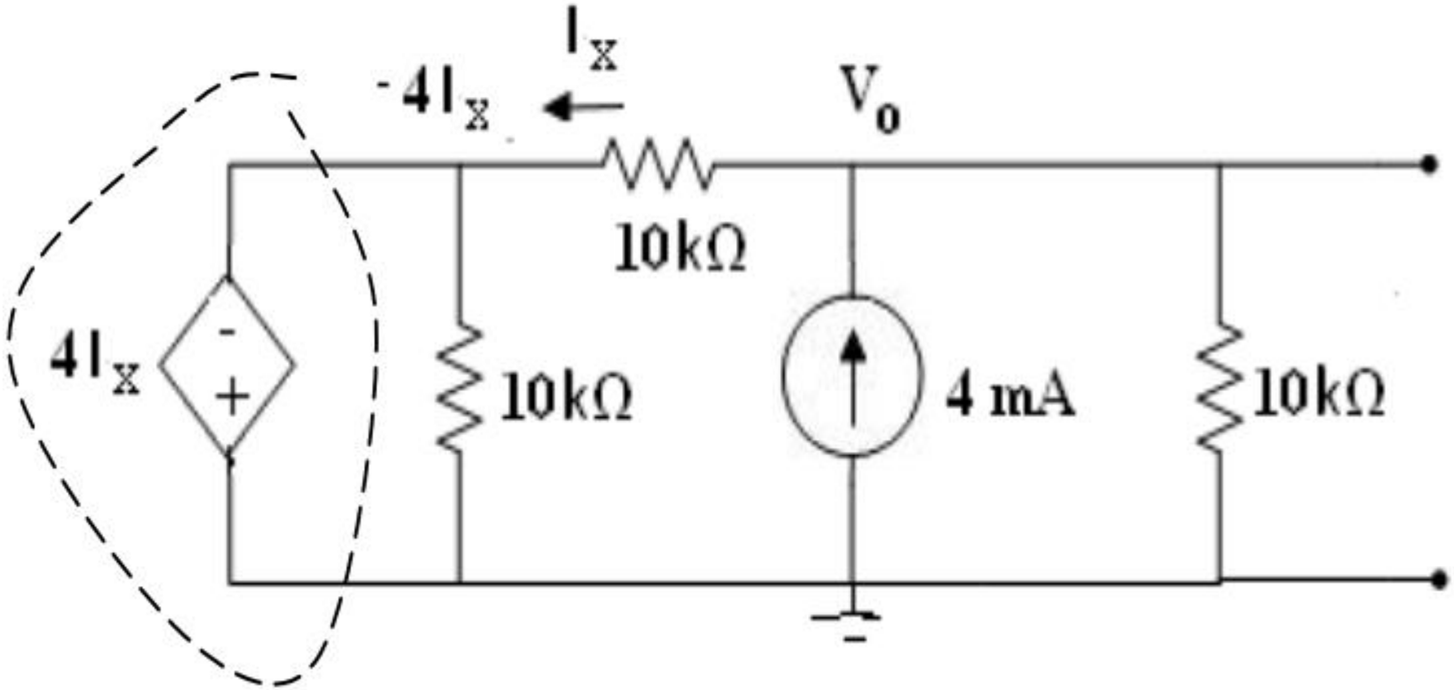
$$I_o = \frac{3V2}{12} = \frac{3}{12} \times \frac{3}{2} = \frac{3}{8}\text{mA}$$

Example 2

Find V_o in the circuit using nodal analysis



With currents in mA, the CCVS becomes $4I_x$



Applying KCL to the non-reference node V_o, we obtain:

$$4 = \frac{V_o}{10} + \frac{V_o - (-4\,I\,x\,)}{10} \quad \text{or}$$

$$4 = \frac{V_o}{5} + \frac{4Ix}{10} \;\; or\; 20 = V_o + 2I_x$$

$$V_o = 20 - 2I_x$$

The controlling equation is given by:

$$I_x = \frac{Vo - -4\,I\,x}{10} = \frac{V_o + 4Ix}{10}$$

$10I_x = V_o + 4I_x$ or $6I_x = V_o$ or $2I_x = \frac{V_o}{3}$

Substituting it in the nodal equation,

$V_o = 20 - 2I_x$

we obtain

$V_o = 20 - \frac{V_o}{3}$ *or* $V_o \times \frac{4}{3} = 20$

$V_o = 15V$

3. MESH ANALYSIS

"

Mesh analysis uses KVL to determine currents in the circuit.

Once the currents are known, Ohm’s law can be used to calculate voltages.

N independent equations are required if the circuit contains N meshes.

Mesh Analysis

- We assume that the circuits are planar. (Circuits that can be drawn on a plane surface with no wires crossing each other).

- In the case of non–planar circuits, we cannot define meshes and mesh analysis cannot be performed.

- Mesh analysis is thus not as general as nodal analysis, which has no topological restrictions.

Definition of terms used

- **A loop:** It is a closed path through a circuit in which no node is encountered more than once.

- **A mesh:** A mesh is a special kind of loop that does not contain any loop within it.

- We note that as we traverse the path of a mesh, we do not encircle any circuit element.

Definition of terms used

- **A planar circuit:** It is a circuit that can be drawn on a plane surface with no crossovers, i.e. **no element or connecting wire crosses another element or connecting wire.**

- In planar circuits meshes appear as windows.

- **A non–planar circuit:** It is a circuit that is not planar.

General Approach to Mesh Analysis

We follow the systematic approach given below:

- Place a loop current in the clockwise direction within each mesh or window of the network.
- Apply KVL around each mesh in the clockwise direction.
- Solve the resulting simultaneous equations for the loop currents.

Case 1

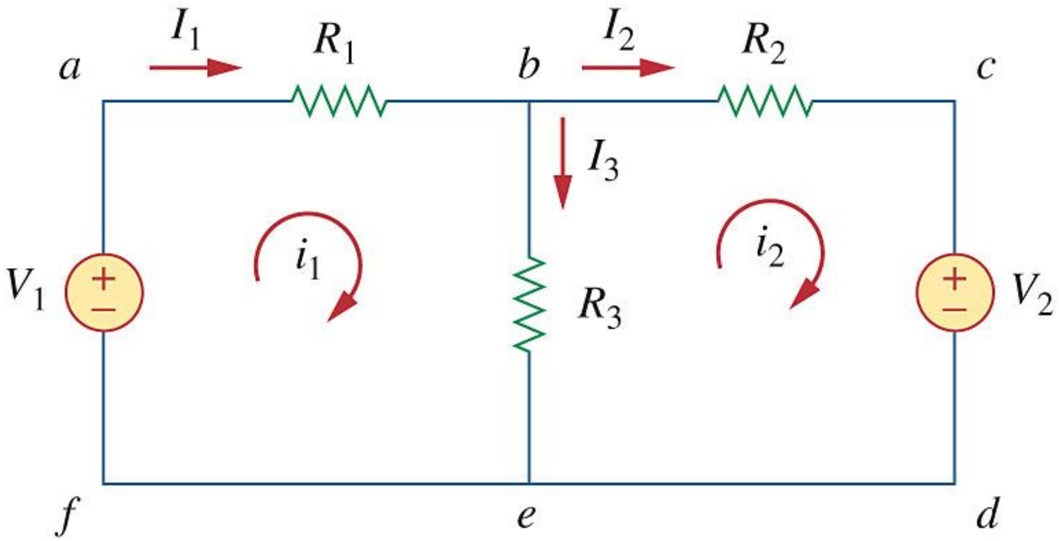
“

Circuits containing only Independent Voltage Sources.

The General Approach or Format Approach can be used

Independent Voltage Sources

To illustrate the steps, consider the circuit below



We use

- i to denote mesh current
- I to denote branch current

Step 1

Mesh currents i_1 and i_2 are assigned to meshes 1 and 2

Step 2

Apply KVL to each mesh

Mesh 1: $R_1 i_1 + R_3(i_1 - i_2) = V_1$

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1 \qquad (1)$$

Mesh 2: $R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$

$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2 \qquad (2)$$

Step 3

Solve for the mesh currents, putting equations (1) and (2) in matrix form yields:

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

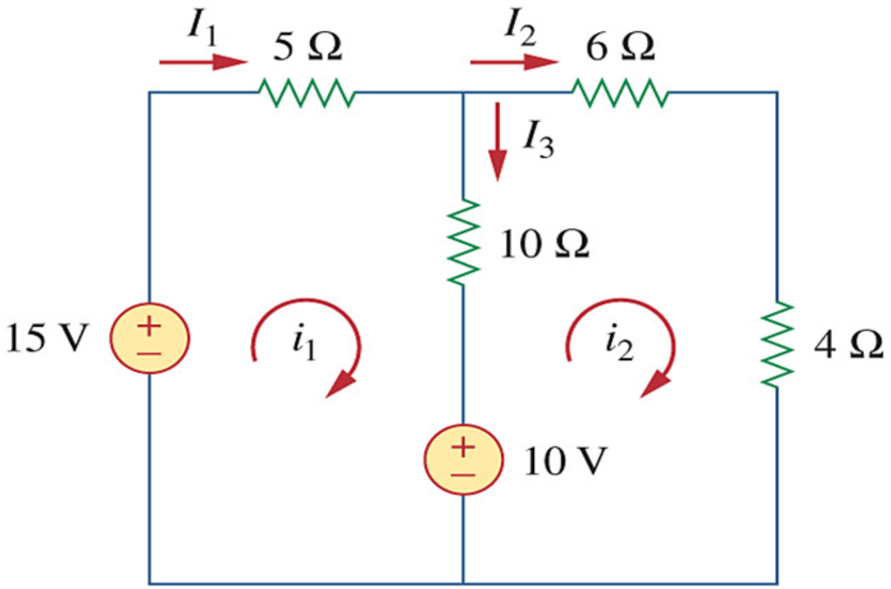
Note:

- In equation (1) that the coefficient of i_1 is the sum of the resistances in the first mesh, while

- The coefficient of i_2 is the negative of the resistance common to meshes 1 and 2.

Example 1

For the circuit below find the branch currents I_1, I_2 and I_3 using mesh analysis



We first obtain the mesh currents using KVL

Mesh 1:

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \qquad (1)$$

Mesh 2:

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 - 2i_2 = -1 \qquad (2)$$

Substituting (2) into (1)

$$6i_2 - 3 - 2i_2 = 1$$

$$i_2 = 1\ A$$

From (2):

$$i_1 = 2i_2 - 1 = 2 - 1 = 1\ A$$

Thus

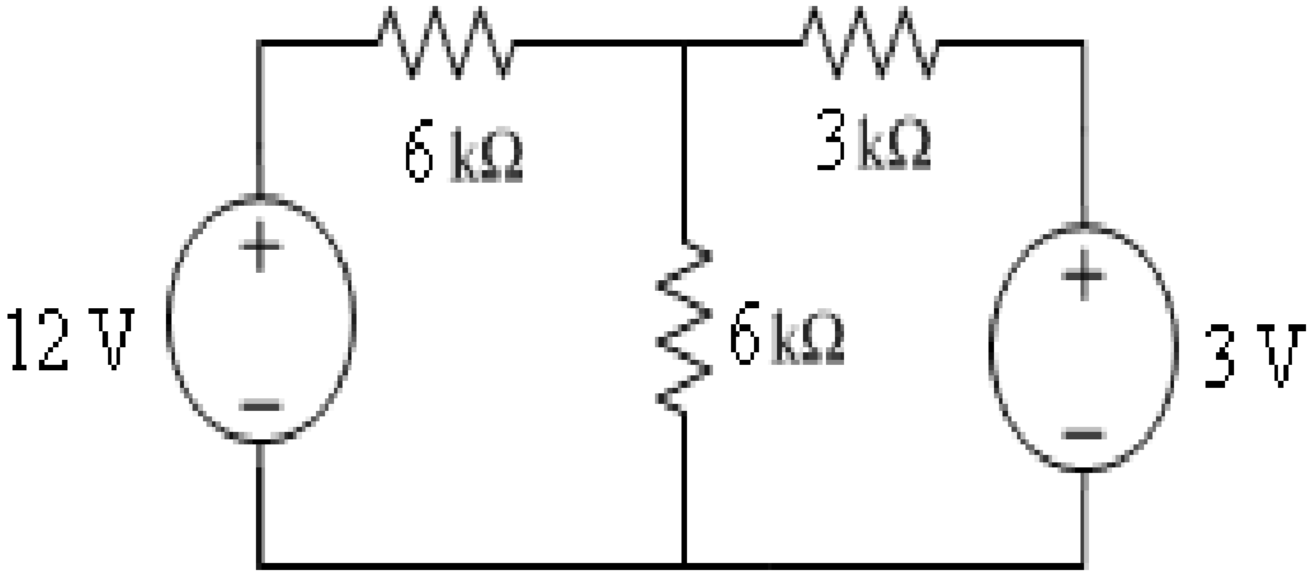
$$I_1 = i_1 = 1\ A$$

$$I_2 = i_2 = 1\ A$$

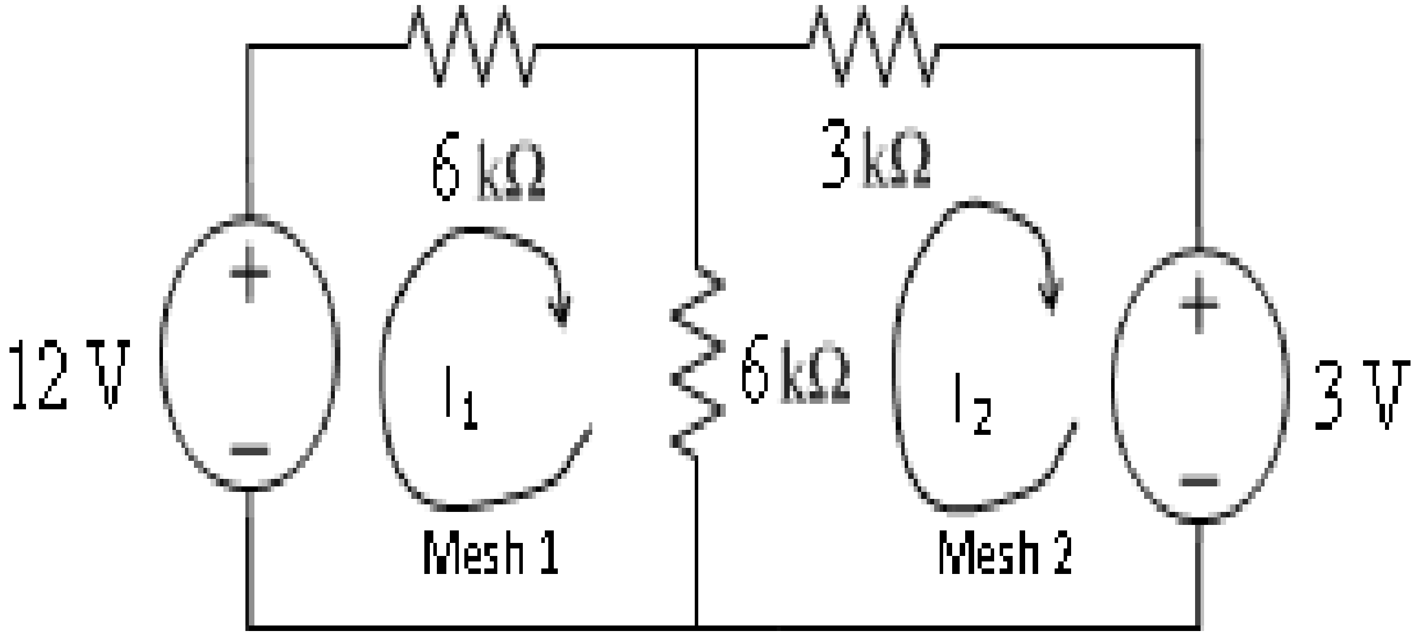
$$I_3 = i_1 - i_2 = 0$$

Example 2

Find the current through each branch of the network



The mesh currents are shown below:



Apply KVL to mesh 1:

$$12 = 6I_1 + 6(I_1 - I_2) = 12I_1 - 6I_2 \text{ or}$$

$$12 = 12I_1 - 6I_2 \qquad (1)$$

Apply KVL to mesh 2:

$$-3 = 3I_2 + 6(I_2 - I_1) \text{ or}$$

$$-3 = -6I_1 + 9I_2 \qquad (2)$$

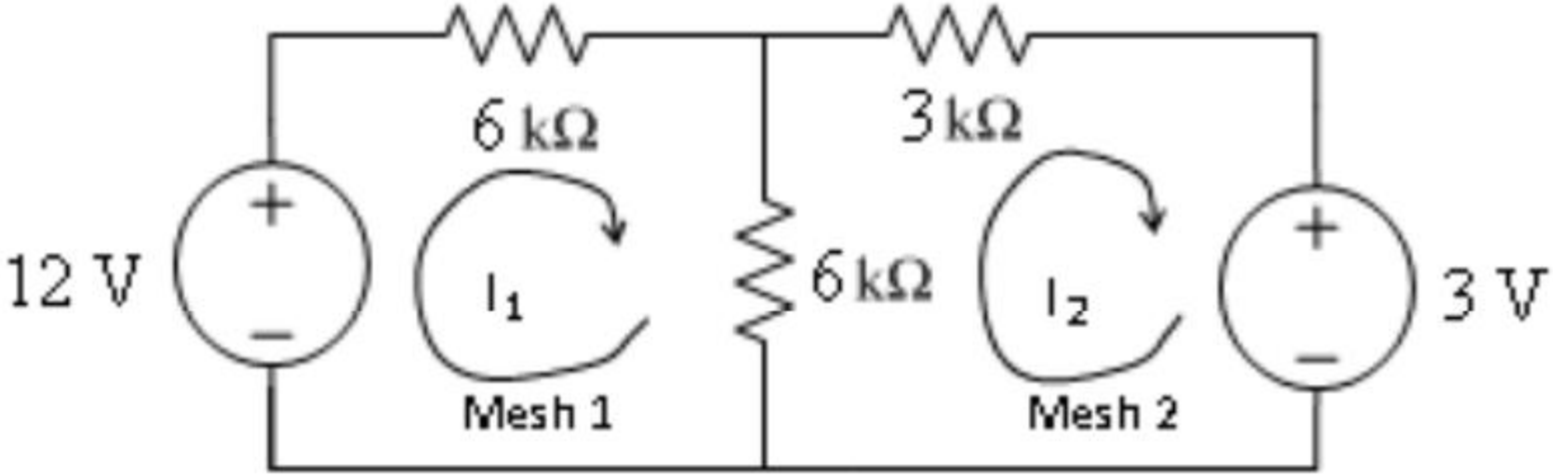
(1) + 2 × (2) gives:

$$6 = 12I_2$$

$$I_2 = \frac{6}{12} = 0.5 \text{ mA and from (2)}$$

$$6I_1 = 9I_2 + 3 = 9 \times 0.5 + 3$$

$$I_1 = 1.25 \text{ mA}$$



$I_1 = 1.25$ mA

$I_2 = 0.5$ mA

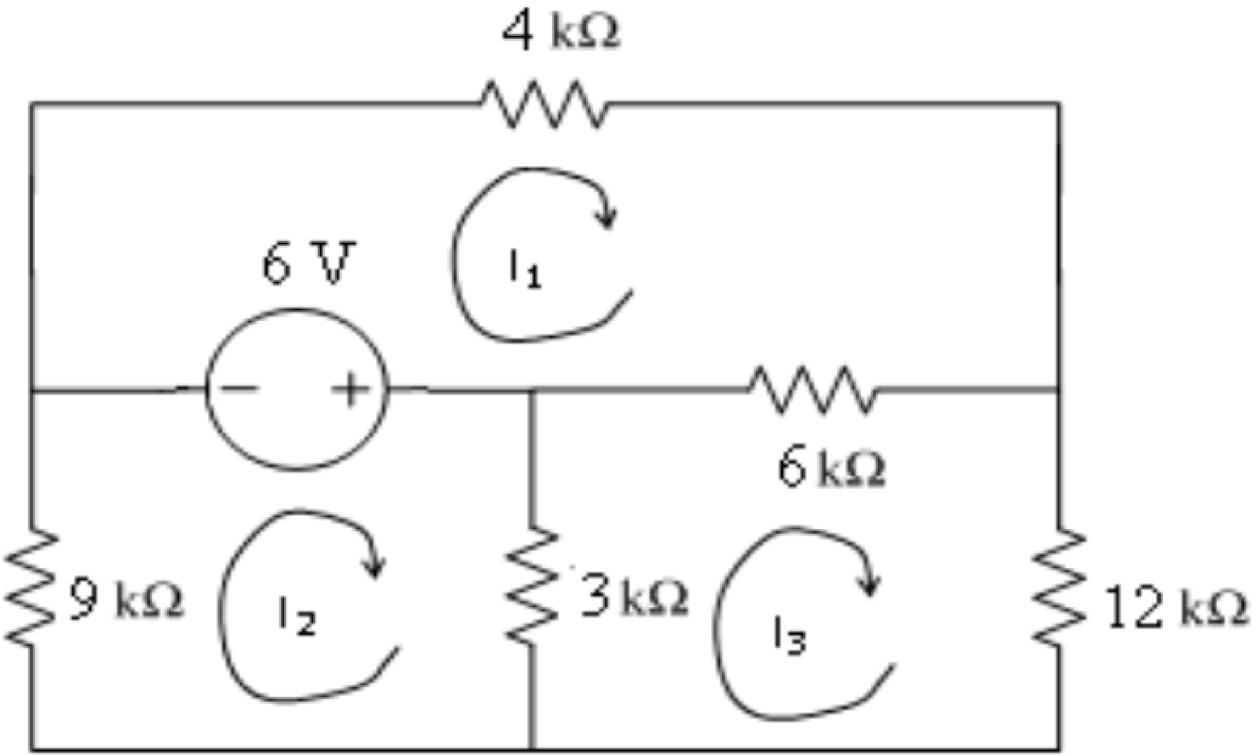
Current in the left outer branch = $I_1 = 1.25$ mA

Current in the middle branch = $I_1 – I_2 = 1.25 – 0.5 = 0.75$ mA

Current in the right outer branch = $I_2 = 0.5$ mA

Example 3

Find the current through each branch of the network



Loop 1

$-6 = 4I_1 + 6(I_1 - I_3)$ or

$-6 = 10I_1 - (0)I_2 - 6I_3$

Loop 2

$6 = 3(I_2 - I_3) + 9I_2$ or

$6 = (0)I_1 - 12I_2 - 3I_3$

Loop 3

$0 = 6(I_3 - I_1) + 12I_3 + 3(I_3 - I_2)$ or

$0 = -6I_1 - 3I_2 + 21I_3$

Therefore in matrix form:

$$\begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 & -6 \\ 0 & 12 & -3 \\ -6 & -3 & 21 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Case 2

“

Circuits containing Current Sources.

As in the case of nodal analysis with voltage sources, the presence of current sources reduces the number of unknowns in mesh analysis by one per current source.

Method 1

Converting current sources to voltage sources.

NB: The sources must have resistances connected in parallel

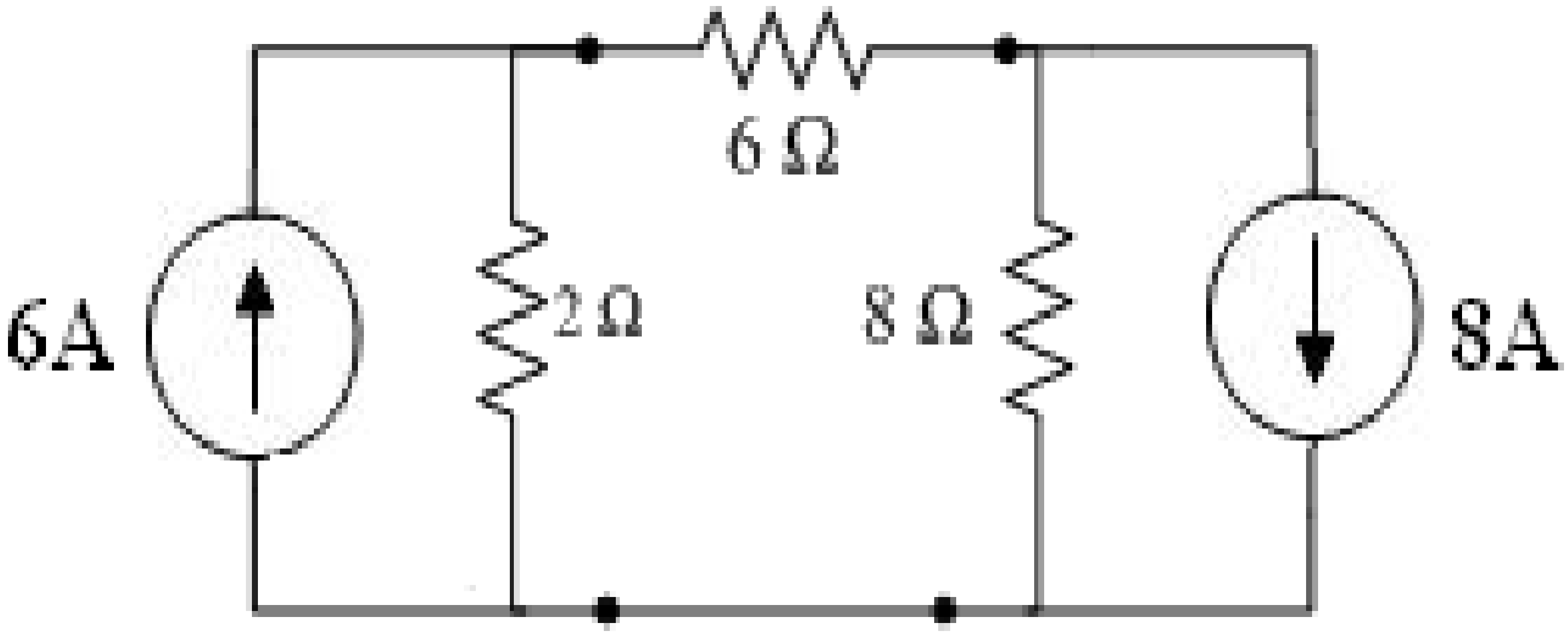
Method 2

Concept of Supermesh

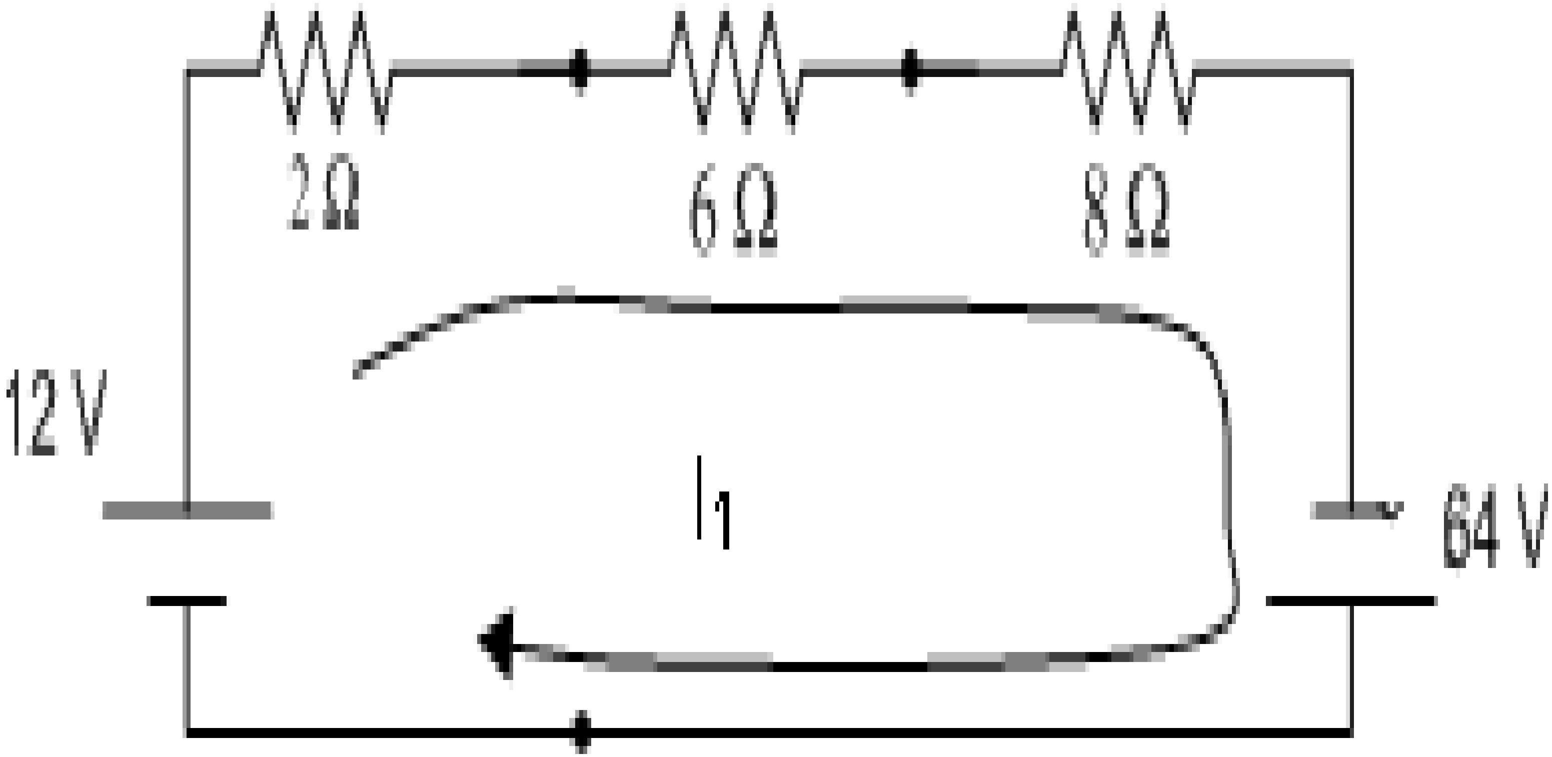
Converting current sources to voltage sources

Example 1

Use mesh analysis to determine the currents for the network

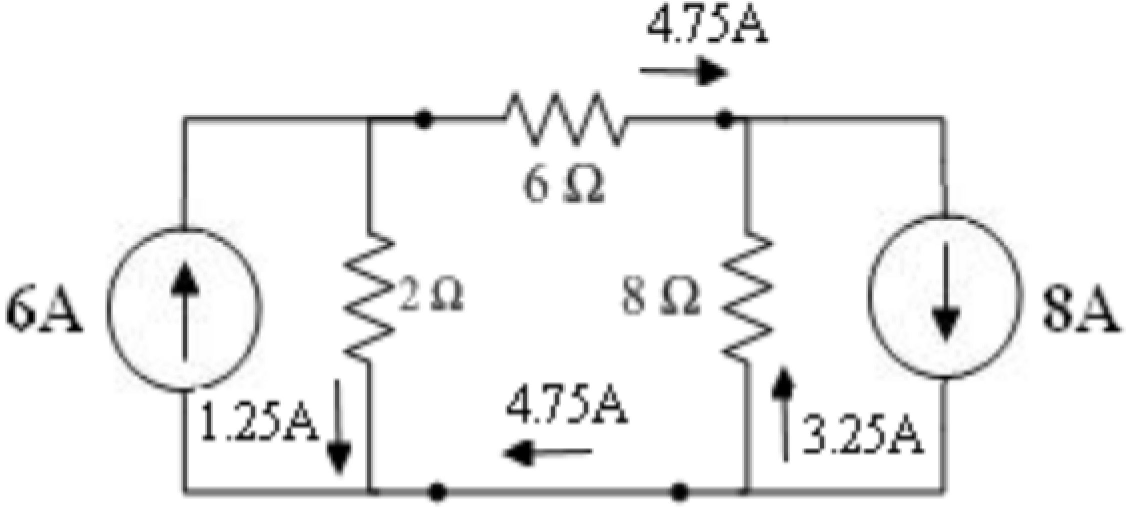


Convert current sources to voltage sources



$12 + 64 = (2 + 6 + 8)I_1$

$I_1 = \frac{76}{16} = 4.75 \text{ A}$



The currents in the various branches of the circuit:

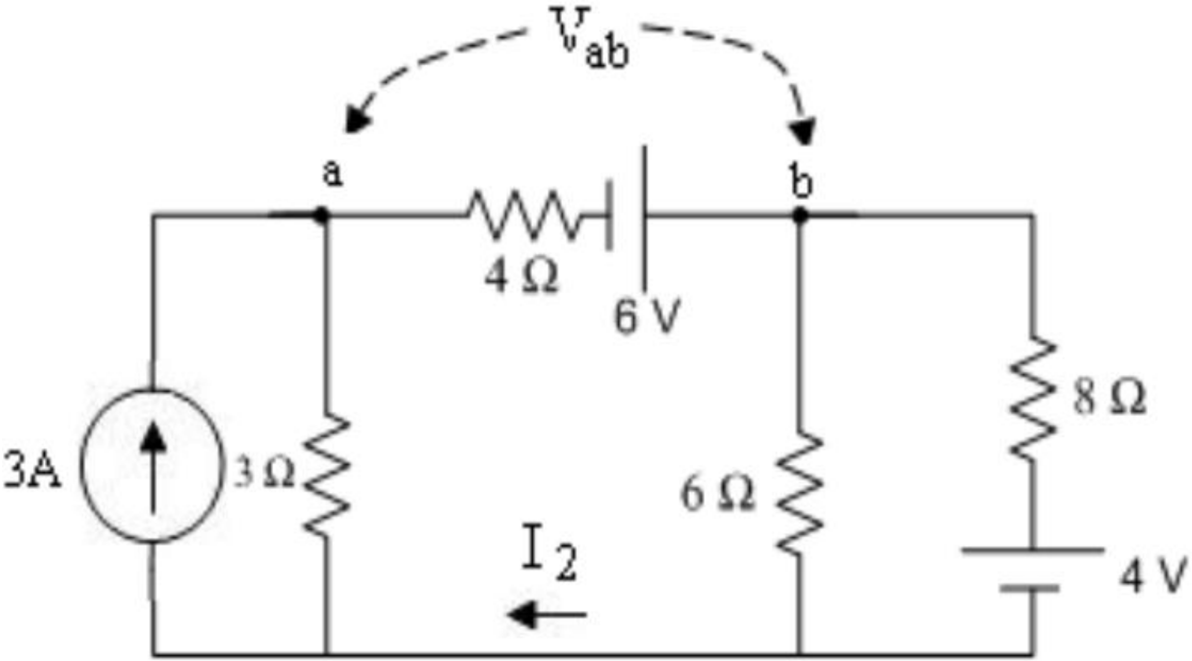
Current in 6 Ω resistor = 4.75 A

Current in 2 Ω resistor = 6 – 4.75 = 1.25 A

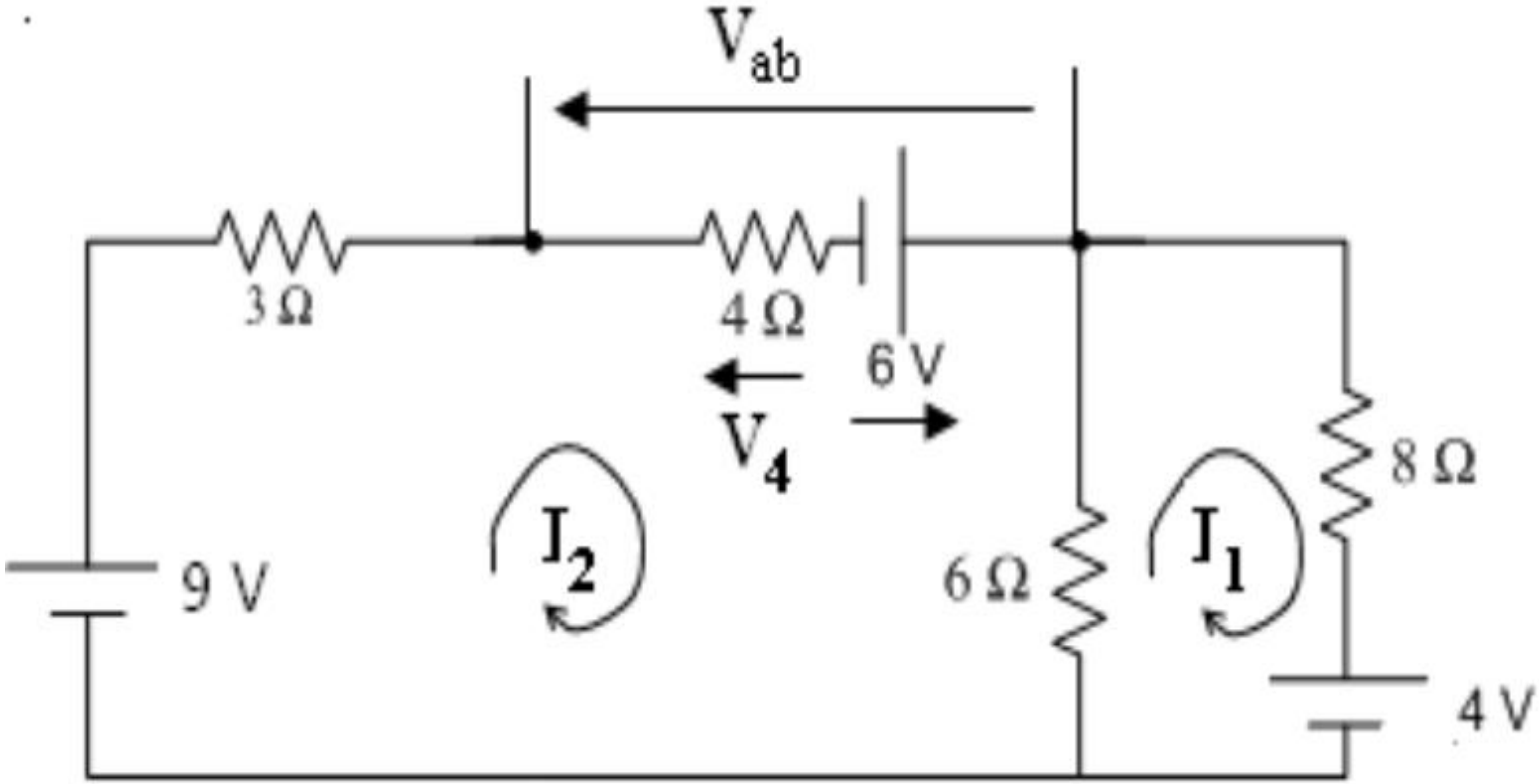
Current in 8 Ω resistor = 8 – 4.75 = 3.25 A

Example 2

For the network, determine the current I_2 using mesh analysis and then find the voltage V_{ab}



Convert current source to voltage sources:



By inspection:

$-4 = 14I_1 - 6I_2$

$15 = -6I_1 + 13I_2$

Solving the two simultaneous equations, we obtain:

$I_1 = 0.26$ A

$I_2 = 1.274$ A

$$
\begin{aligned}
\text{Voltage } V_{ab} &= V_4 - 6 \\
&= 4I_2 - 6 \\
&= 4 \times 1.274 - 6 \\
&= 0.904 \text{ V}
\end{aligned}
$$

The concept of supermesh

Scenario 1:

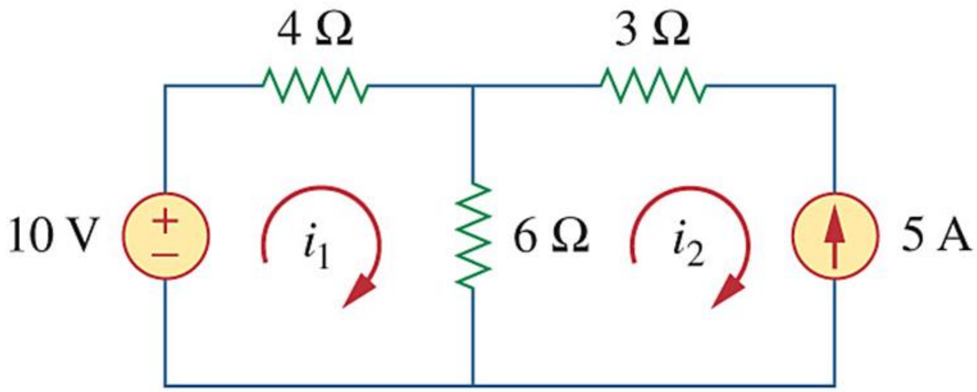
When a current source exists only in one mesh:

- set the mesh current to the value of the current source

$$i_2 = -5A$$

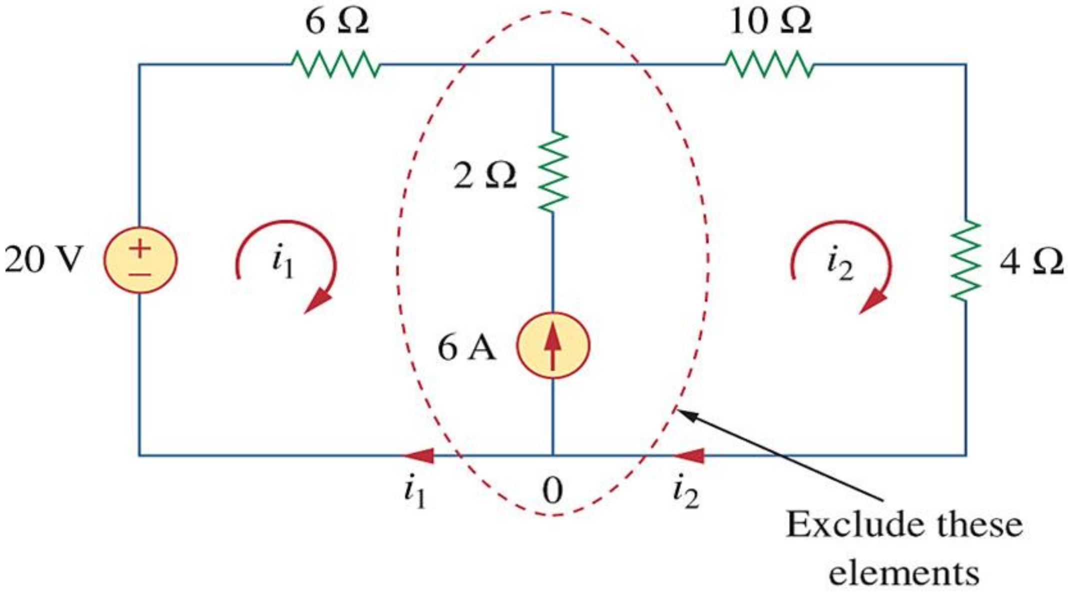
- The mesh equation for the other mesh is:

$$-10 + 4i_1 + 6(i_1 - i_2) = 0, \quad i_1 = -2\ A$$



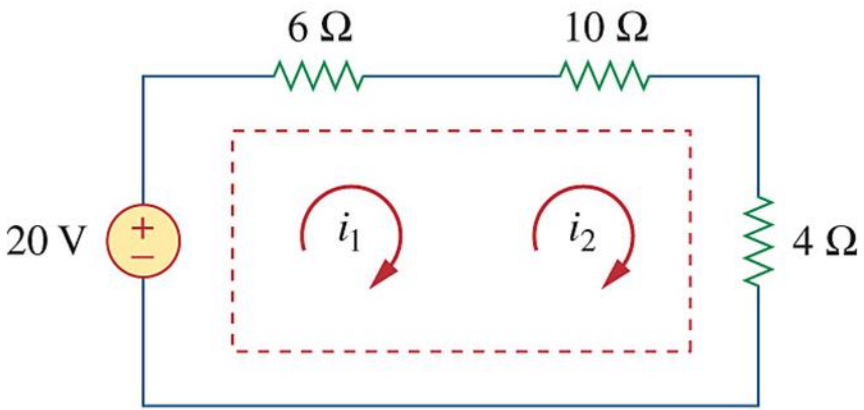
Scenario 2:

When a current source exists between two meshes



We create a supermesh by:

- excluding the current source and any element connected in series with it



- A supermesh results when two meshes have a (dependent or independent) current source in common

We create a supermesh as the periphery of the two meshes and treat it differently.

Applying KVL to the supermesh gives

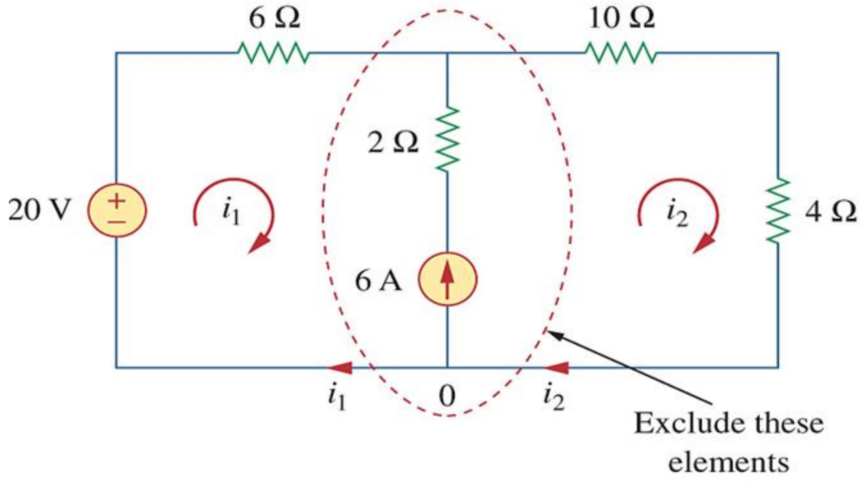
$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

or

$$6i_1 + 14i_2 = 20$$

We apply KCL to a node in the branch where the two meshes intersect.

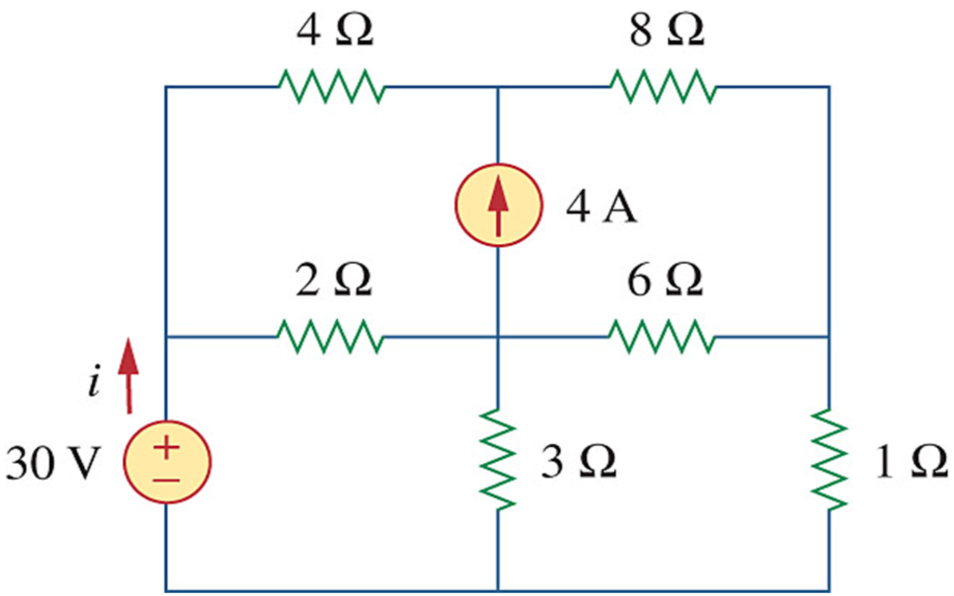
Applying KCL to node 0 and obtain:

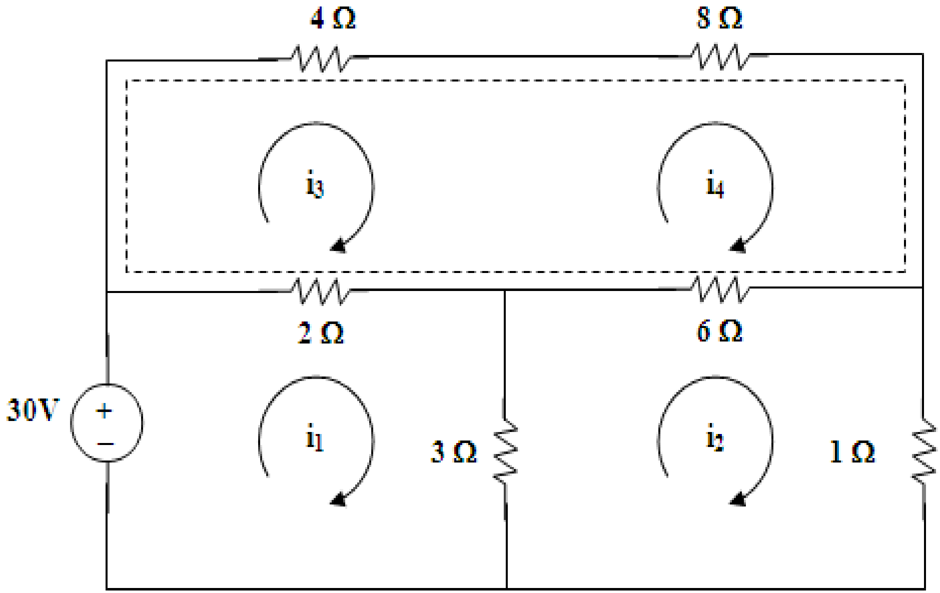


$$i_2 = i_1 + 6$$

Example 1

Find the current “I” in the circuit below





KVL for loop 1 gives:

$$-30 + 2(i_1 - i_3) + 3(i_1 - i_2) = 0$$

or

$$5i_1 - 3i_2 - 2i_3 = 30 \qquad (1)$$

KVL for loop 2 gives:

$$i_2 + 3(i_2 - i_1) + 6(i_2 - i_4) = 0$$

or

$$-3i_1 + 10i_2 - 6i_4 = 0 \qquad (2)$$

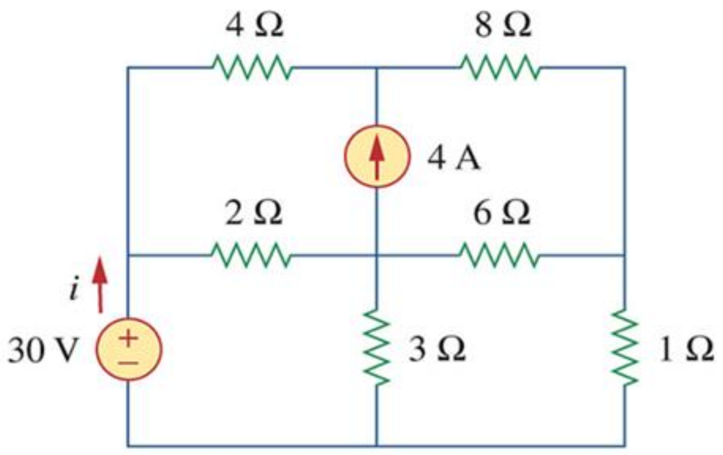
KVL for the supermesh yields:

$$2(i_3 - i_1) + 4i_3 + 8i_4 + 6(i_4 - i_2) = 0$$

or

$$-2i_1 - 6i_2 + 6i_3 + 14i_4 = 0 \qquad (3)$$

Applying KCL to a node in the branch where the two meshes intersect



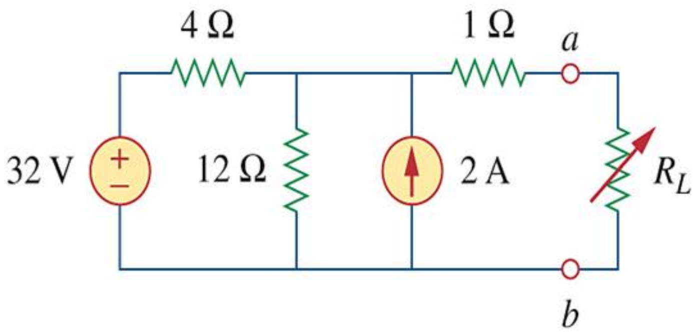
$$i_4 = i_3 + 4 \qquad (4)$$

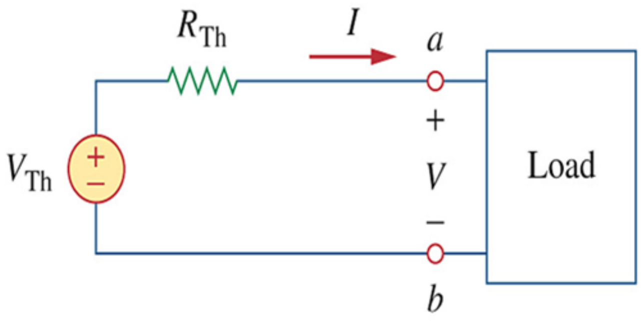
Solving (1) to (4) by elimination gives

$$i = i_1 = 8.561 \text{ A}$$

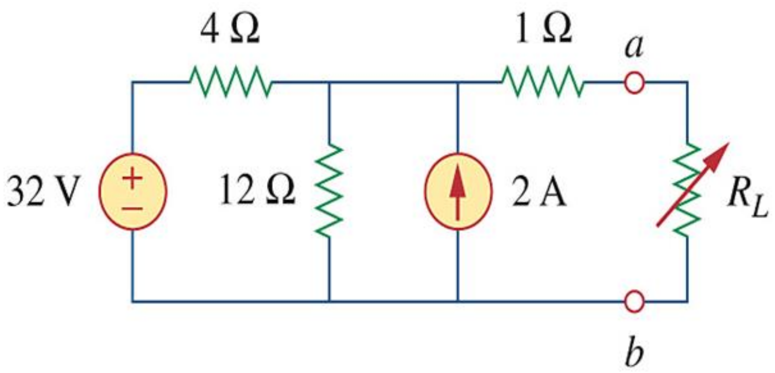
4. THEVENIN'S THEOREM

Thevenin's theorem states that any two-terminal linear dc network can be replaced by an equivalent circuit consisting of a voltage source, V_{TH} and a series resistance R_{TH}.



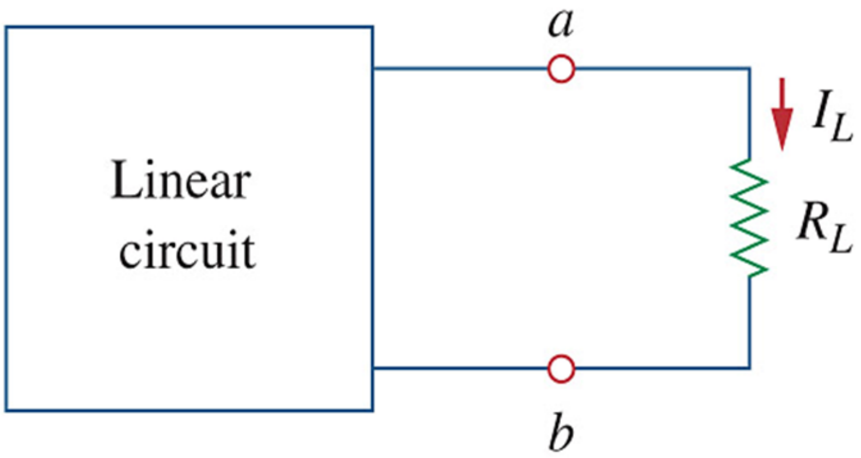


- V_{TH} is the open-circuit voltage at the terminals
- R_{TH} is the Thevenin (equivalent) resistance when all the independent sources are turned off.
- It often occurs in practice that a particular element in a circuit is variable (load) while other elements are fixed as shown in the circuit below



- Each time the variable element, R_L, changed, the entire circuit has to be analyzed again.

- Thevenin's theorem helps in the development of a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

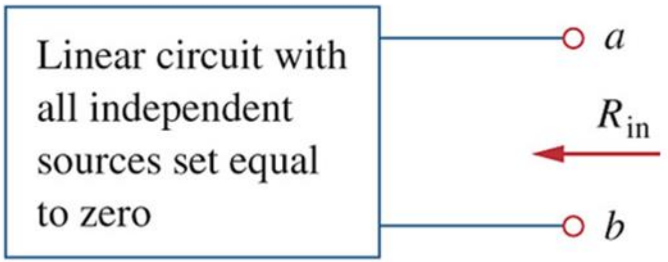


Thévenin Equivalent Resistance

Case 1

If the network has no dependent sources:

- Turn off all independent voltage and current sources.
- Compute the total resistance between the load terminal with the load removed.



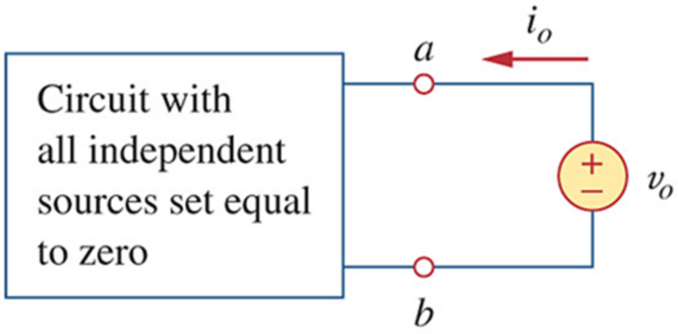
$$R_{Th} = R_{in}$$

Thévenin Equivalent Resistance

Case 2

If the network has dependent sources:

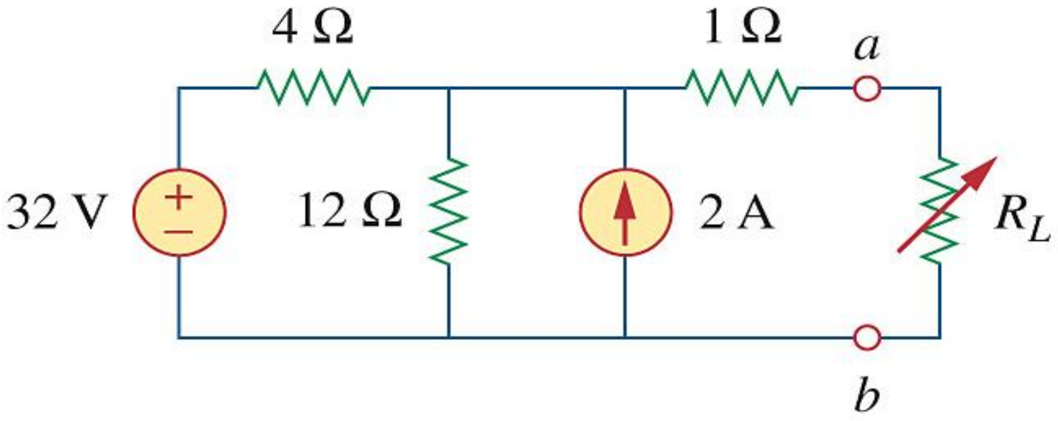
- Turn off all independent voltage and current sources.
- Dependent sources are not turned off.
- We apply a voltage source v_o (1V) at terminals a and b and determine the resulting current i_o



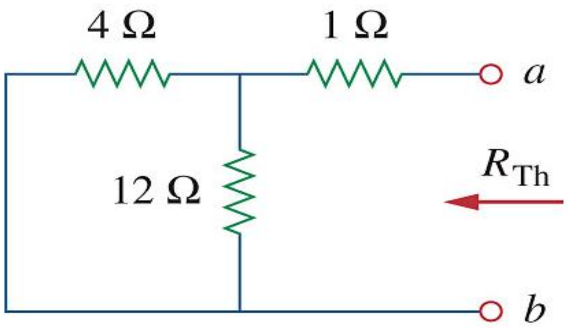
$$R_{\text{Th}} = \frac{v_o}{i_o}$$

Example 1

Compute the equivalent resistance the load RL ‘sees’ at port a-b for circuit the below



We find R_{TH} by short circuiting the 32-V voltage source and open circuiting the 2-A current source. The circuit becomes what is shown below:

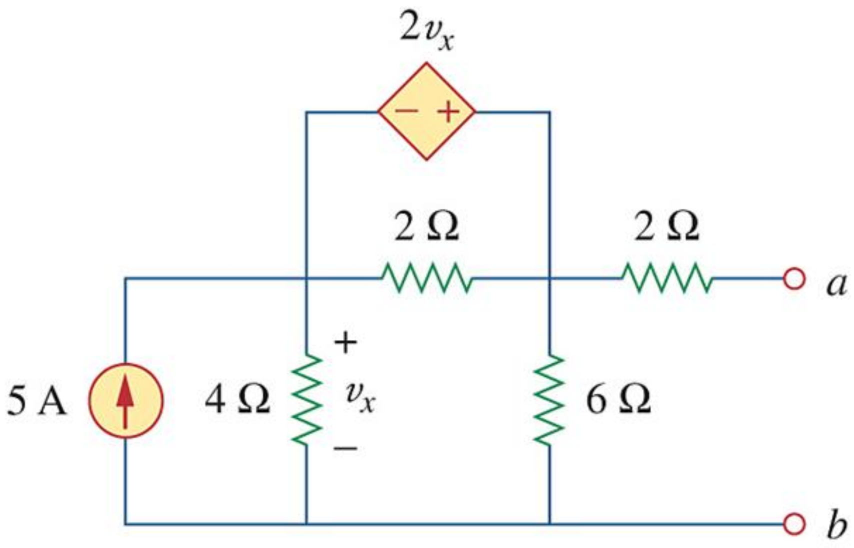


The Thevenin Equivalent Resistance is

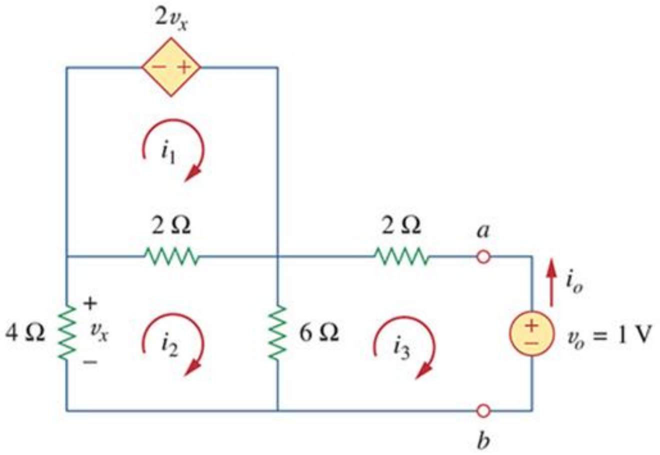
$$R_{TH} = (4||12) + 1 = \frac{4 \times 12}{16} + 1 = 4\,\Omega$$

Example 2

Compute the Thevenin equivalent resistance of circuit the below



- This circuit contains a dependent source:
- To find $R_{TH,}$ we set the independent source equal to zero but leave the dependent source alone.
- We excite the network with a voltage source v_o connected to the terminals as shown below:



- We set v_o = 1 V to ease calculation
- We find current i_o through the terminals and then obtain $R_{TH} = \frac{1}{i_o}$

Applying KVL to mesh 1 yields:

$$-2v_x + 2(i_1 - i_2) = 0$$

or

$$v_x = (i_1 - i_2)$$

But

$$V_x = -4i_2 = i_1 - i_2$$

Hence

$$i_1 = -3i_2 \qquad (1)$$

Applying KVL to meshes 2 and 3 produces

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0 \qquad (2)$$

and

$$6(i_3 - i_2) + 2i_3 + 1 = 0 \qquad (3)$$

Solving equations 1, 2 and 3 gives:

$$i_3 = -\frac{1}{6}\ \mathrm{A}$$

But

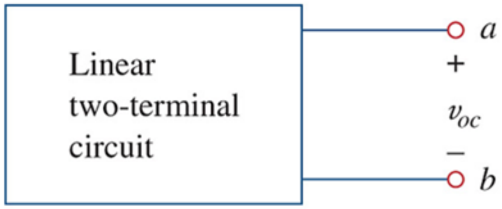
$$i_o = -i_3 = \frac{1}{6}$$

Hence

$$R_{TH} = \frac{1\ \mathrm{V}}{i_o} = 6\ \Omega$$

Computing the Thevenin Voltage

The Thevenin equivalent voltage is equal to the open-circuit voltage present at the load terminal



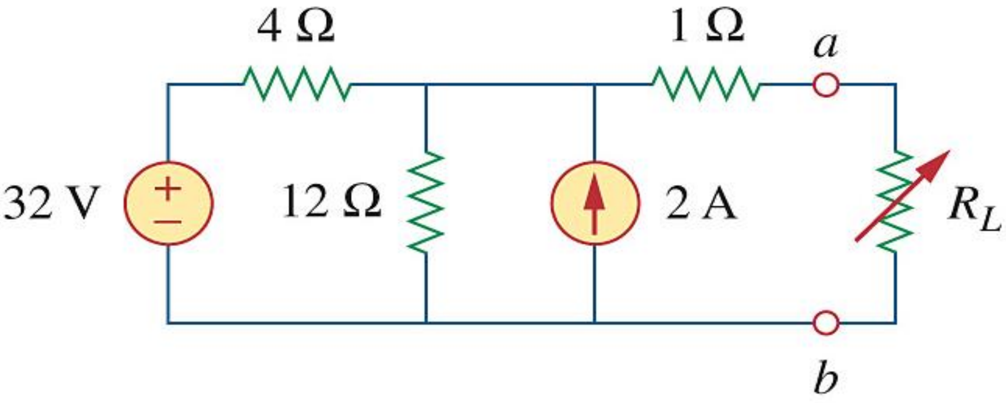
$V_{Th} = v_{oc}$

Procedure

- Remove the load, leaving the load terminals open-circuited
- Define the open-circuit voltage v_{oc} across the given load terminal
- Apply any preferred method (eg: node analysis) to solve for v_{oc}
- The Thevenin voltage is $v_{TH} = v_{oc}$

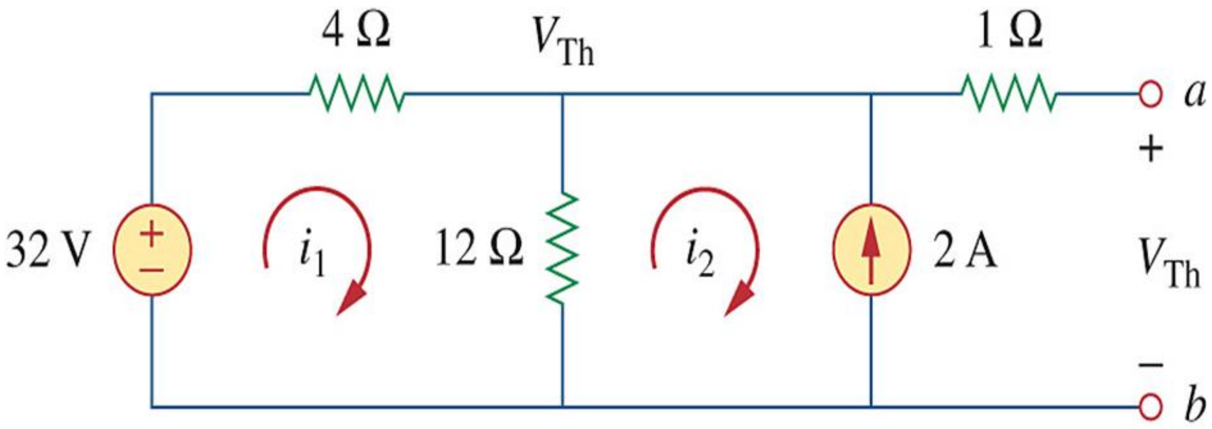
Example 3

Compute the Thevenin voltage in the circuit the below



Procedure

- Disconnect the load as shown below

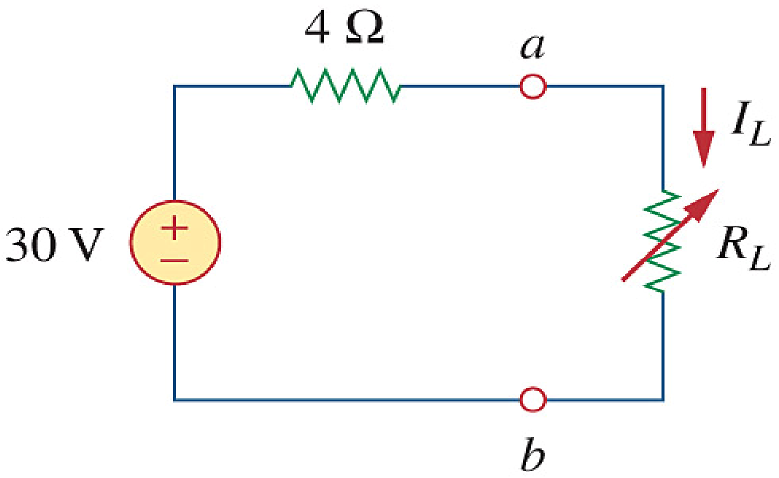


- Applying mesh analysis to the two loops we've

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2 \text{ A}$$

- Solving for i_1, we get $i_1 = 0.5$ A
- Thus, $V_{TH} = 12(i_1 - i_2) = 12(0.5 + 2) = 30$ V

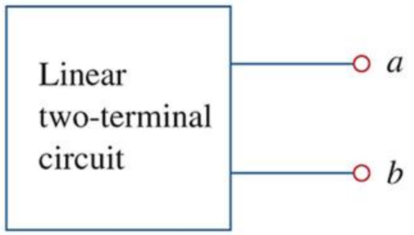
The Thevenin Equivalent circuit is as shown below:

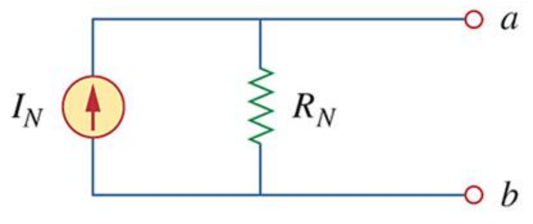


5. NORTON'S THEOREM

"

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source i_N in parallel with a resistor R_N





- Where i_N is the short-circuit current through the terminals

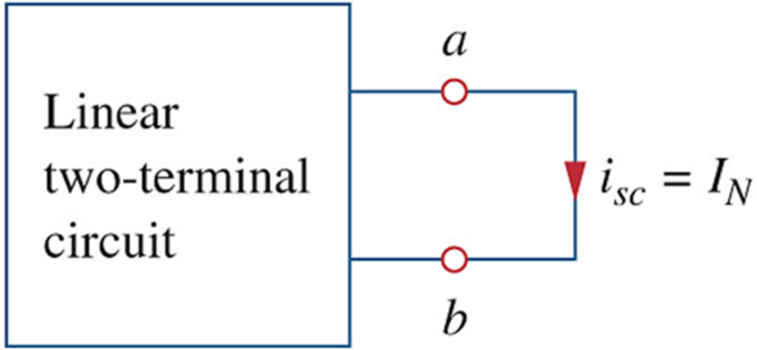
- R_N is the equivalent resistance at the terminals when the independent sources are turned off.

$$R_N = R_{TH}$$

Computing Norton Current

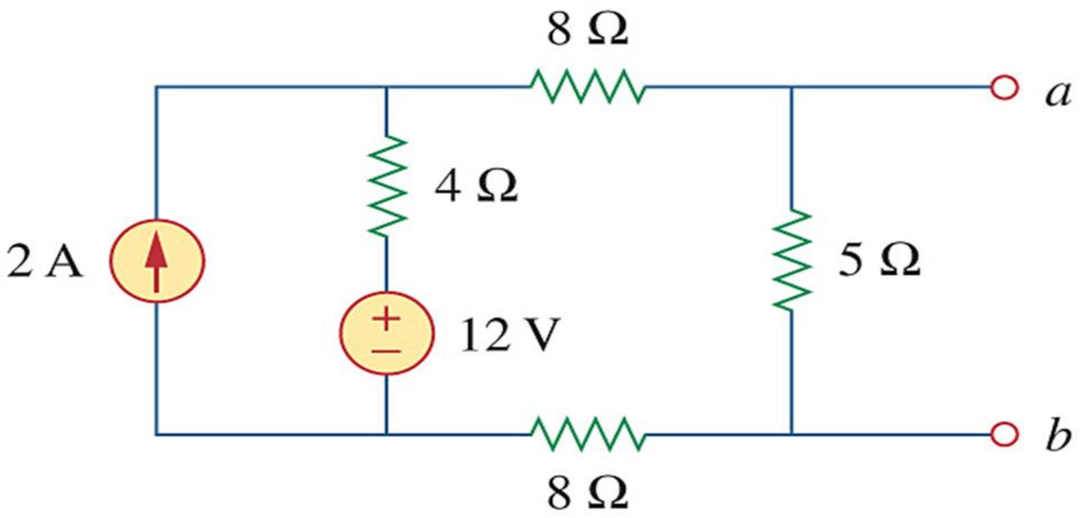
Procedure

- Replace the load with a short-circuit
- Define the short-circuit current i_{sc} to be the Norton equivalent current
- Apply any preferred method (eg: mesh analysis) to solve for i_{sc}
- The Norton current $i_N = i_{sc}$



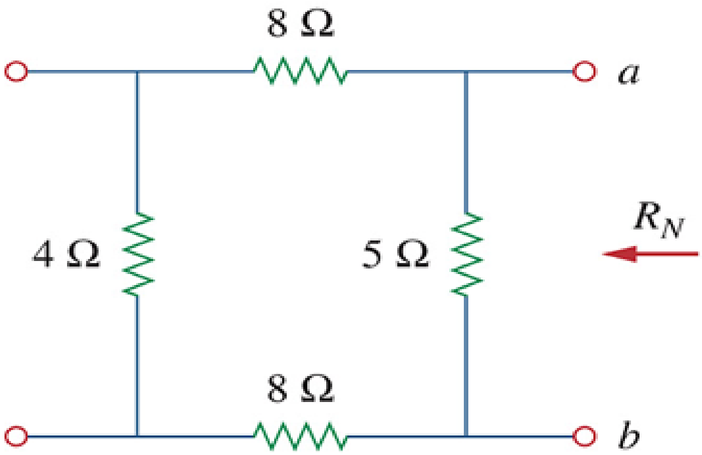
Example 1

- Find the Norton equivalent circuit of the circuit below at terminals a-b



We find

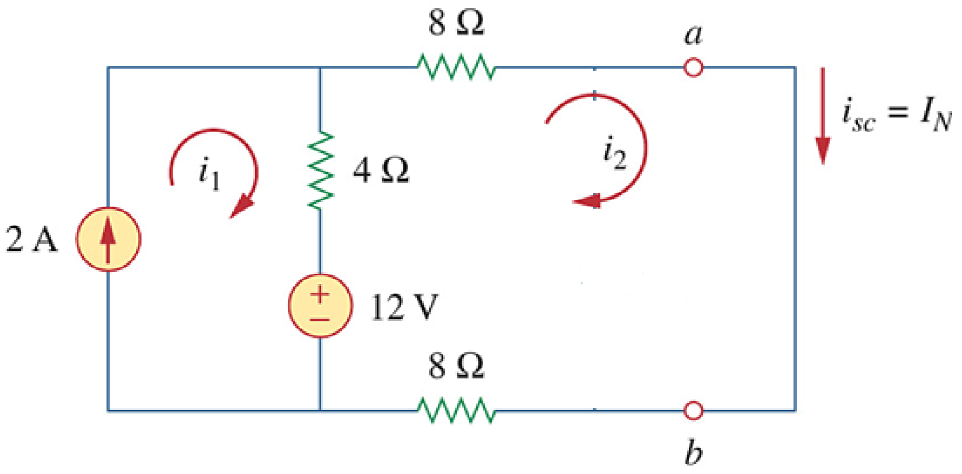
- R_N in the same way we find R_{TH} in the Thevenin equivalent circuit
- Set the independent sources equal to zero and this leads to



$$R_N = 5 \,||\, (8 + 4 + 8) = 5 \,||\, 20 = \frac{5 \times 20}{25} = 4\ \Omega$$

To find:

- I_N we short-circuit terminals a and b as shown below:



- We ignore the 5-Ω resistor because it has been short-circuited

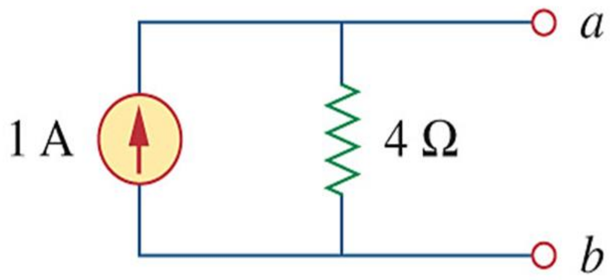
- Applying mesh analysis, we obtain:

  $i_1 = 2\text{ A}, \quad 20i_2 - 4i_1 - 12 = 0$

- From these equations, we obtain:

  $i_2 = 1\text{ A} = i_{sc} = I_N$

- Thus the Norton equivalent circuit is as shown below:



Thanks!

Any questions?