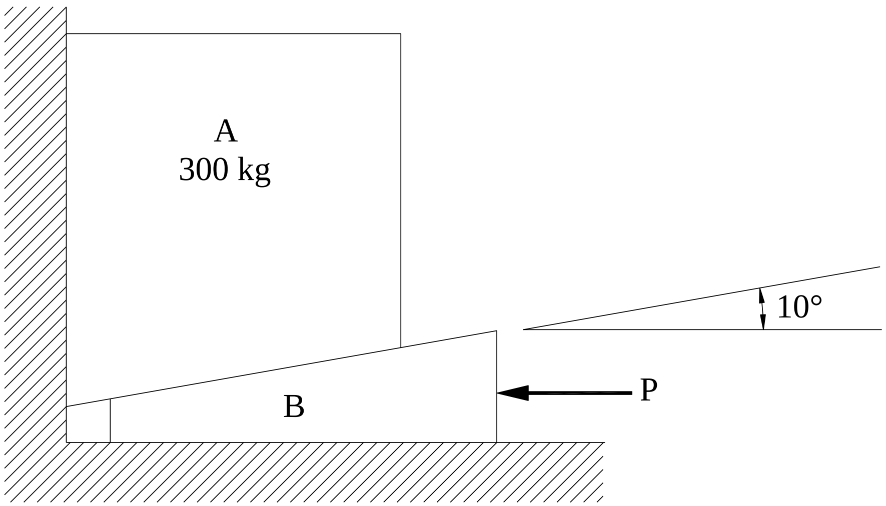
7.2 Wedges

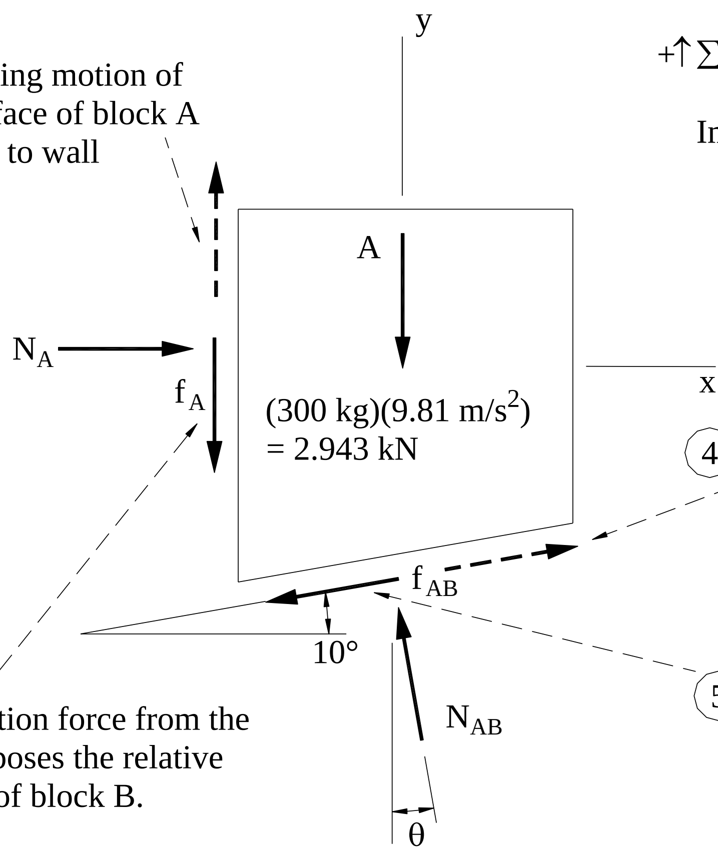
1. If the coefficient of static friction equals 0.3 for all surfaces of contact, determine the smallest value of force P necessary to raise the block A. Neglect the weight of the wedge B.



7.2 Wedges Example 1, page 2 of 4

1 Free-body diagram of block A

2 Impending motion of left surface of block A relative to wall



3 The friction force from the wall opposes the relative motion of block B.

4 Impending motion of lower surface of block A relative to block B (This is the motion that an observer sitting on block B would see as he observes block A move past.)

5 The friction force from block B opposes the relative motion of block A.

6 Equations of equilibrium for block A

$$\xrightarrow{+} \sum F_x = 0: \; N_A - f_{AB} \cos 10° - N_{AB} \sin \theta = 0 \qquad (1)$$

$$+\uparrow \sum F_y = 0: \; -f_A - f_{AB} \sin 10° + N_{AB} \cos \theta - 2.943 \text{ kN} = 0 \qquad (2)$$

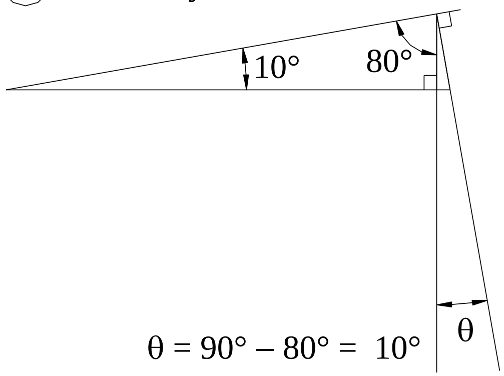
Impending motion so,

$$f_A = f_{A\text{-max}} \equiv \mu N_A = 0.3 N_A \qquad (3)$$

$$f_{AB} = f_{AB\text{-max}} \equiv \mu N_{AB} = 0.3 N_{AB} \qquad (4)$$

7.2 Wedges Example 1, page 3 of 4

7 Geometry



8 Using θ = 10° in Eqs. 1- 4 and solving simultaneously gives

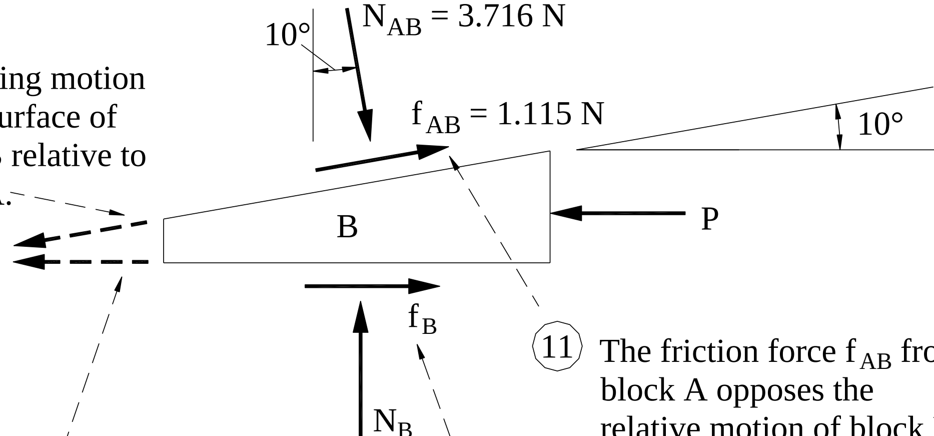
$f_A = 0.523$ N

$N_A = 1.743$ N

$f_{AB} = 1.115$ N

$N_{AB} = 3.716$ N

9 Free-body diagram of wedge B



10 Impending motion of top surface of block B relative to block A.

11 The friction force f_{AB} from block A opposes the relative motion of block B.

12 Impending motion of lower surface of block B relative to floor

13 The friction force f_B from the floor opposes the relative motion of block B.

14 Equations of equilibrium for block B

$$\overset{+}{\rightarrow} \sum F_x = 0: \; f_B + (1.115 \text{ N}) \cos 10° + (3.716 \text{ N}) \sin 10° - P = 0 \qquad (5)$$

$$+\uparrow \sum F_y = 0: \; N_B + (1.115 \text{ N}) \sin 10° - (3.716 \text{ N}) \cos 10° = 0 \qquad (6)$$

7.2 Wedges Example 1, page 4 of 4

15 Slip impends between block B and the floor, so

$$f_B = f_{B\text{-max}} \equiv \mu N_B = 0.3N_B \qquad (7)$$

Solving Eqs. 5, 6, and 7 simultaneously gives

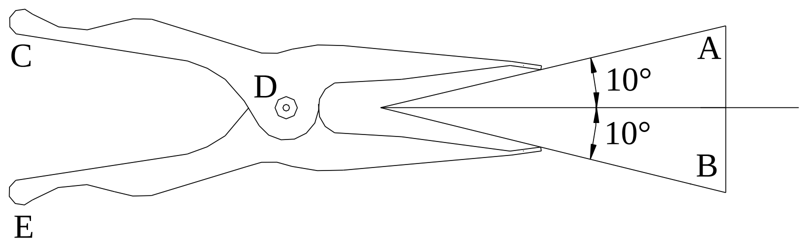
$f_B = 1.04$ N

$N_B = 3.47$ N

$P = 2.78$ N ←Ans.

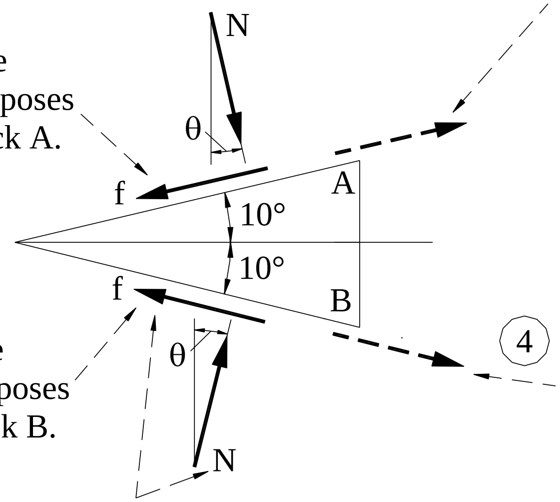
7.2 Wedges Example 2, page 1 of 3

2. Wedges A and B are to be glued together. Determine the minimum coefficient of static friction μ required, if clamp CDE is to be able to hold the wedges in the position shown, while the glue dries.



7.2 Wedges Example 2, page 2 of 3

1 Free-body diagram of wedges



2 Impending motion of upper surface of wedge A relative to the upper jaw of the clamp (The wedge is just about to slip out from the jaws of the clamp.)

3 The friction force from the upper jaw of the clamp opposes the relative motion of block A.

4 Impending motion of lower surface of wedge B relative to the lower jaw of the clamp.

5 The friction force from the lower jaw of the clamp opposes the relative motion of block B.

6 Because of symmetry, the normal force N and friction force f acting on the bottom of the wedge are given the same variable names as those on the top.

7 Equations of equilibrium

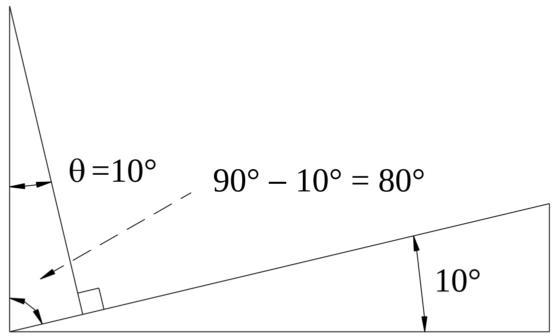
$$\overset{+}{\rightarrow}\sum F_x = 0: \quad 2N \sin\theta - 2f \cos 10° = 0 \qquad (1)$$

$$+\uparrow\sum F_y = 0: \quad N \cos\theta - N \cos\theta - f \sin 10° + f \sin 10° = 0 \qquad (2)$$

7.2 Wedges Example 2, page 3 of 3

8 Since we have already used symmetry in labeling the forces on the free-body diagram, the ΣF_y equilibrium equation degenerates to 0 = 0 and gives us no new information.

9 Geometry



10 Slip impends between both jaws of the clamp and the wedge, and thus

$$f = f_{max} \equiv \mu N \qquad (3)$$

11 Eqs. 1 and 3 constitute two equations in three unknowns (f, N, and μ), so it appears that we can't solve the problem. But we have not been asked to find f and N, only μ. Thus solve Eq. 1 with θ = 10° for the *ratio* f/N:

$$\frac{f}{N} = \tan 10° \qquad (4)$$

Also solve Eq. 3 for f/N:

$$\frac{f}{N} = \mu \qquad (5)$$

Eqs. 4 and 5 imply that

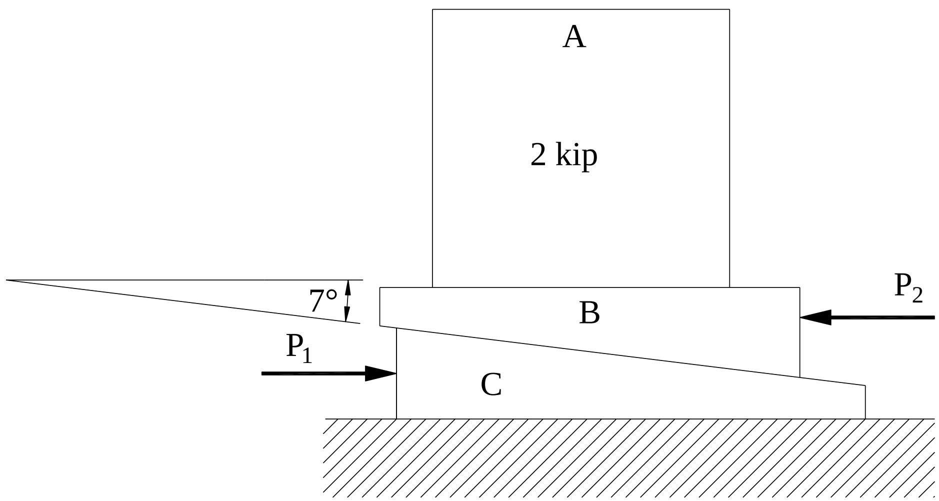
$$\mu = \tan 10°$$

$$= 0.176 \qquad \leftarrow \text{Ans.}$$

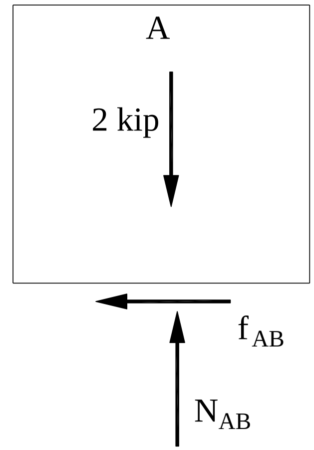
This result is independent of the clamp force N. That is, no matter how strong the clamp spring is, if the coefficient of friction is less than 0.176, then the clamp will slip.

7.2 Wedges Example 3, page 1 of 3

3. Determine the smallest values of forces P_1 and P_2 required to raise block A while preventing A from moving horizontally. The coefficient of static friction for all surfaces of contact is 0.3, and the weight of wedges B and C is negligible compared to the weight of block A.



(1) Free-body diagram of block A



(2) Equations of equilibrium for block A

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $-f_{AB} = 0$ (1)

Therefore, $f_{AB} = 0$

$+\uparrow\Sigma F_y = 0$: $N_{AB} - 2\text{ kip} = 0$ (2)

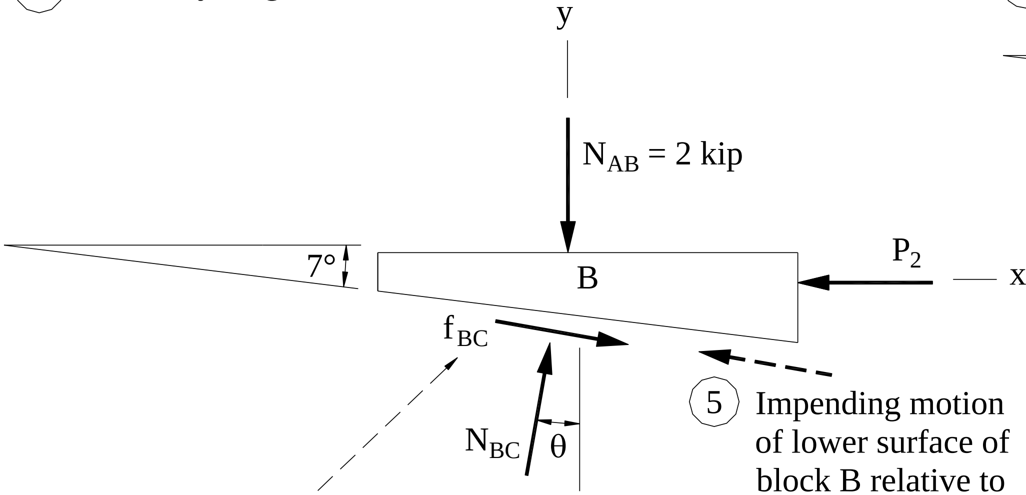
Solving gives

$N_{AB} = 2\text{ kip}$

(3) The friction force f_{AB} has to be zero, since we know that block A is not to move horizontally and no other horizontal force acts. In fact, we could have just shown $f_{AB} = 0$ on the free body initially.

7.2 Wedges Example 3, page 2 of 3

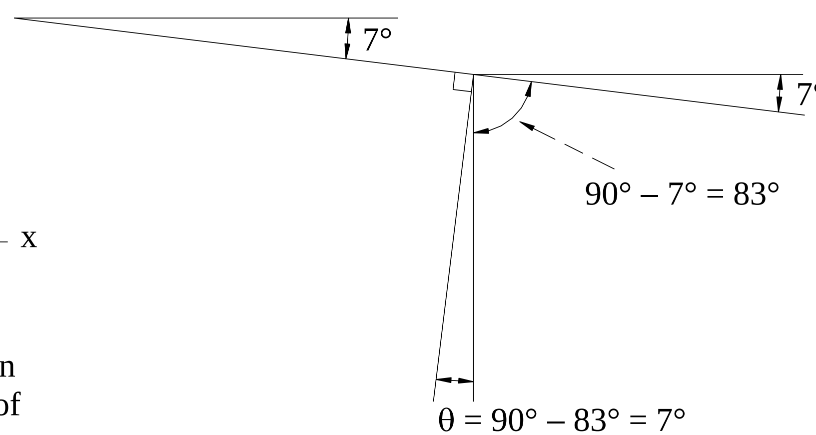
4 Free-body diagram of block B



5 Impending motion of lower surface of block B relative to block C.

6 The friction force from block C opposes the relative motion of block B.

9 Geometry



7 Equations of equilibrium for block B

$$\overset{+}{\rightarrow} \sum F_x = 0: \quad f_{BC} \cos 7° + N_{BC} \sin \theta - P_2 = 0 \qquad (3)$$

$$+\uparrow \sum F_y = 0: \quad -f_{BC} \sin 7° + N_{BC} \cos \theta - 2 \text{ kip} = 0 \qquad (4)$$

8 Slip impends between blocks B and C, so

$$f_{BC} = f_{BC\text{-max}} \equiv \mu N_{BC} = 0.3 N_{BC} \qquad (5)$$

10 Solving Eqs. 3, 4, and 5 with θ = 7° gives

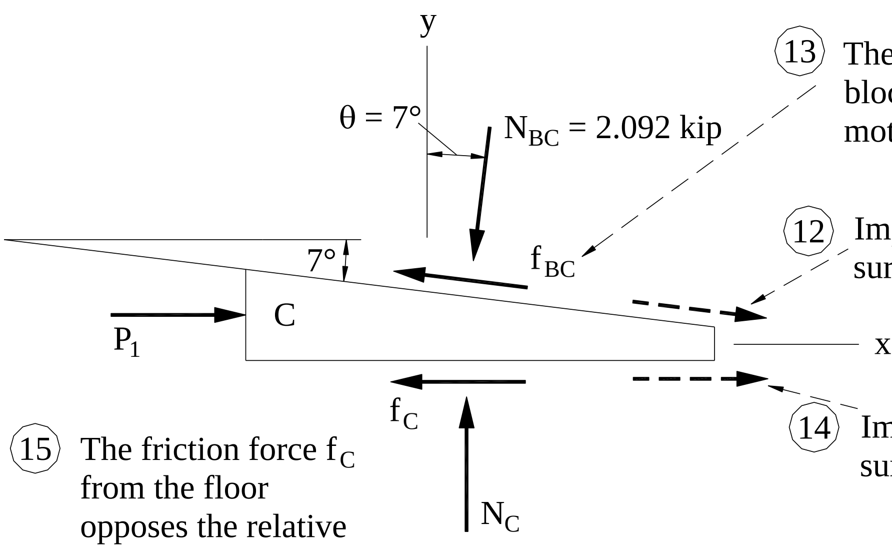
N_{BC} = 2.092 kip

f_{BC} = 0.628 kip = 628 lb

P_2 = 0.878 kip = 878 lb ←Ans.

7.2 Wedges Example 3, page 3 of 3

(11) Free-body diagram of block C



(13) The friction force f_{BC} from block B opposes the relative motion of block C.

(12) Impending motion of upper surface of C relative to B

(14) Impending motion of lower surface of C relative to the floor

(15) The friction force f_C from the floor opposes the relative motion of block B.

(16) Equilibrium equations for block C

$$\overset{+}{\rightarrow} \Sigma F_x = 0:\ -f_{BC} \cos 7° - (2.092 \text{ kip}) \sin 7° + P_1 - f_C = 0 \qquad (6)$$

$$+\uparrow \Sigma F_y = 0:\ f_{BC} \sin 7° - (2.092 \text{ kip}) \cos 7° + N_C = 0 \qquad (7)$$

Slip impends between block C and the floor, so

$$f_C = f_{C\text{-max}} \equiv \mu N_C = 0.3 N_C \qquad (8)$$

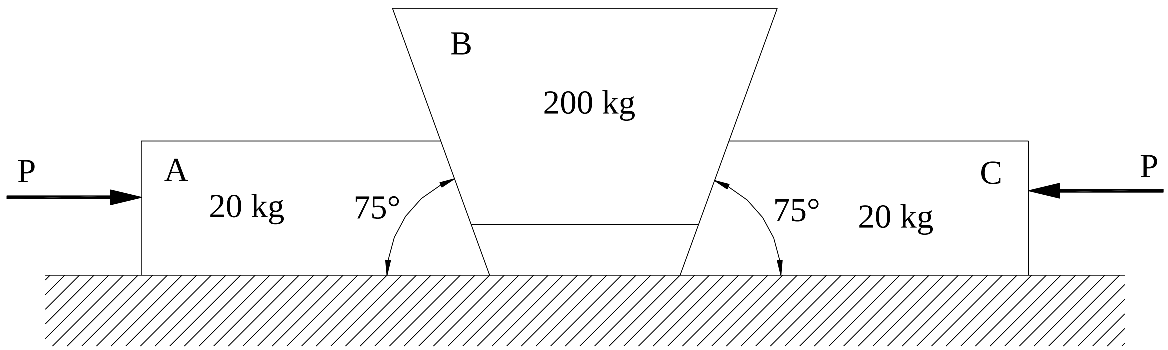
(17) Solving Eqs. 6, 7, and 8 simultaneously gives

$$f_C = 0.6 \text{ kip} = 600 \text{ lb}$$

$$N_C = 2 \text{ kip}$$

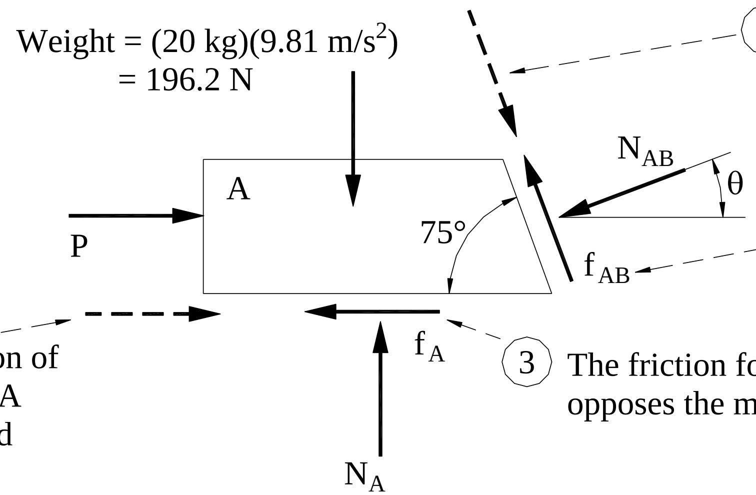
$$P_1 = 1.478 \text{ kip} \qquad \leftarrow\text{Ans.}$$

4. If the coefficient of static friction for all surfaces of contact is 0.25, determine the smallest value of the forces P that will move wedge B upward.



7.2 Wedges Example 4, page 2 of 4

1 Free-body diagram of block A



4 Impending motion of right-side of block A relative to block B (An observer on block B would see block A move down.)

5 The friction force from block B opposes the relative motion of block A.

2 Impending motion of bottom of block A relative to ground

3 The friction force from the floor opposes the motion of block A.

6 Equations of equilibrium

$\overset{+}{\rightarrow} \sum F_x = 0$: $P - f_A - f_{AB} \cos 75° - N_{AB} \cos \theta = 0$ (1)

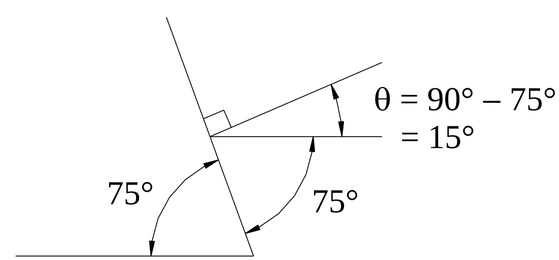
$+\uparrow \sum F_y = 0$: $N_A - 196.2\text{ N} + f_{AB} \sin 75° - N_{AB} \sin \theta = 0$ (2)

Slip impends so,

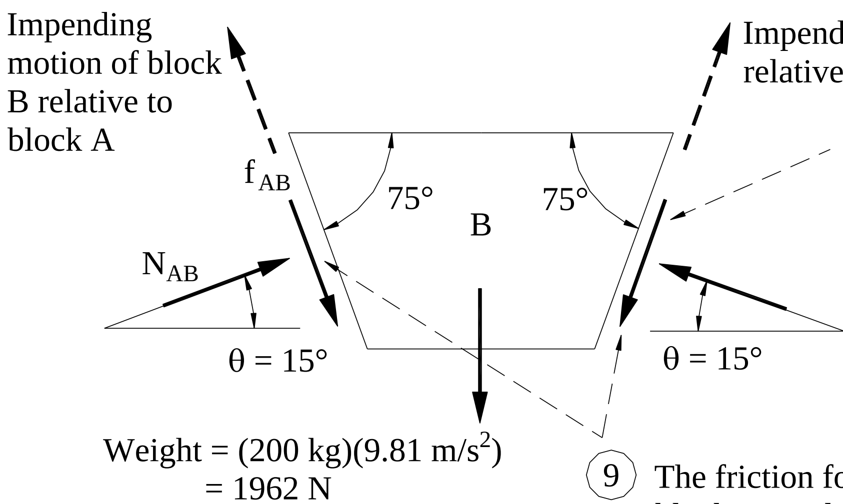
$f_A = f_{A\text{-}max} \equiv \mu N_A = 0.25N_A$ (3)

$f_{AB} = f_{AB\text{-}max} \equiv \mu N_{AB} = 0.25N_{AB}$ (4)

7 Geometry



8 Free-body diagram of block B



Impending motion of block B relative to block C

$f_{BC} = f_{AB}$ by symmetry (You can show this by summing moments about the point where the lines of action of N_{AB}, N_{BC}, and the weight intersect)

$N_{BC} = N_{AB}$ by symmetry (You can show this by summing moments about the point where the lines of action of f_{AB}, f_{BC}, and the weight intersect.)

9 The friction forces from blocks A and C oppose impending upward relative motion of block B.

10 Equations of equilibrium

$$\overset{+}{\rightarrow}\sum F_x = 0: \; N_{AB}\cos 15° - N_{AB}\cos 15° + f_{AB}\cos 75° - f_{AB}\cos 75° = 0 \qquad (5)$$

(Note that this equation reduces to 0 = 0. This happens because we have assumed symmetry to conclude that $f_{BC} = f_{AB}$ and $N_{BC} = N_{AB}$.)

$$+\uparrow\sum F_y = 0: \; N_{AB}\sin 15° + N_{AB}\sin 15° - f_{AB}\sin 75° - f_{AB}\sin 75° - 1962\text{ N} = 0 \qquad (6)$$

7.2 Wedges Example 4, page 4 of 4

11 Solving Eqs. 1– 4 and 6 simultaneously, with $\theta = 75°$, gives

$f_A = 294$ N = 0.294 kN

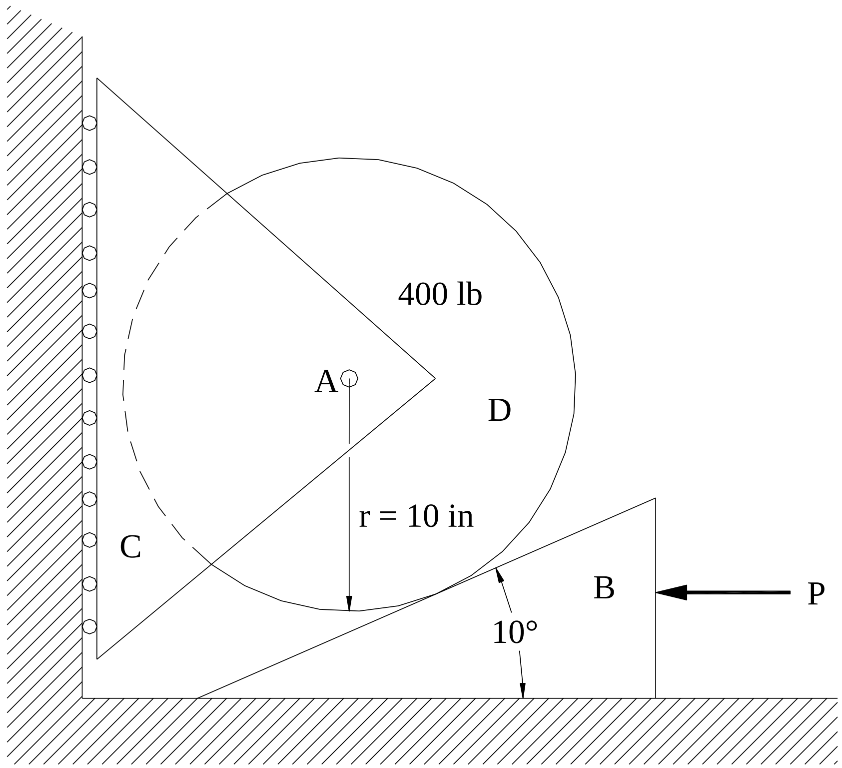
$N_A = 1\,180$ N = 1.18 kN

$f_{AB} = 14\,150$ N = 14.15 kN

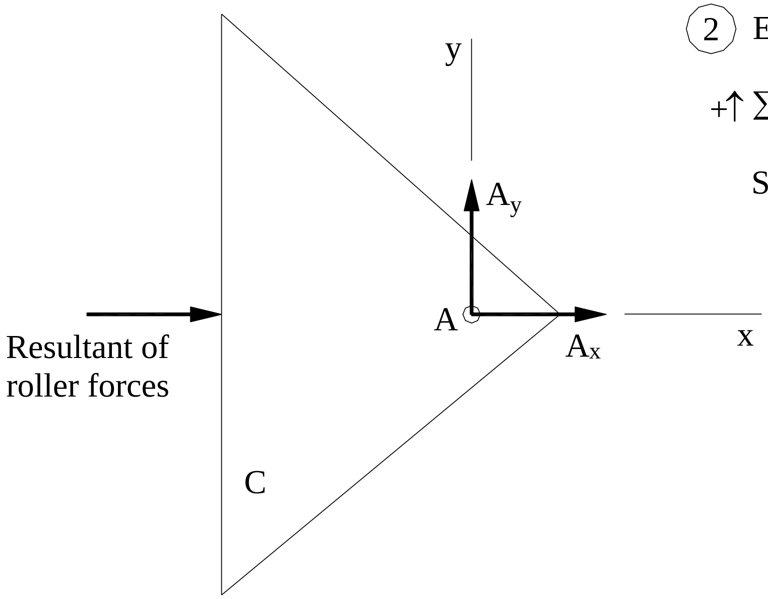
$N_{AB} = 56\,600$ N = 56.6 kN

$P = 58\,600$ N = 58.6 kN ←Ans.

5. The cylinder D, which is connected by a pin at A to the triangular plate C, is being raised by the wedge B. Neglecting the weight of the wedge and the plate, determine the minimum force P necessary to raise the cylinder if the coefficient of static friction is 0.3 for the surfaces of contact of the wedge.



1 Free-body diagram of triangular plate

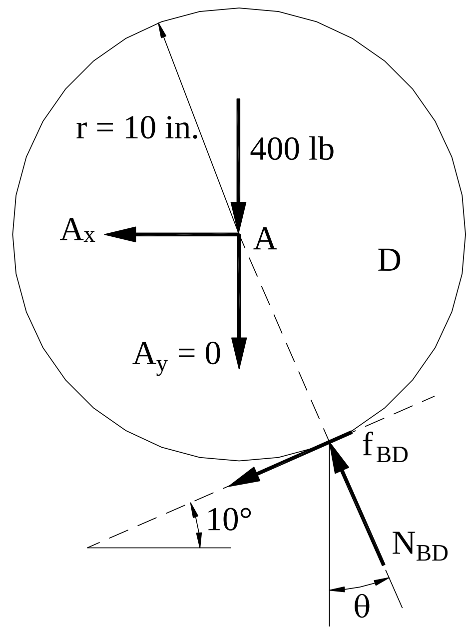


2 Equilibrium equation for triangular plate

$+\uparrow \sum F_y = 0: \quad A_y = 0$ (1)

So, no vertical force is transmitted by pin A.

3 Free-body diagram of cylinder D



4 Equilibrium equations for cylinder D.

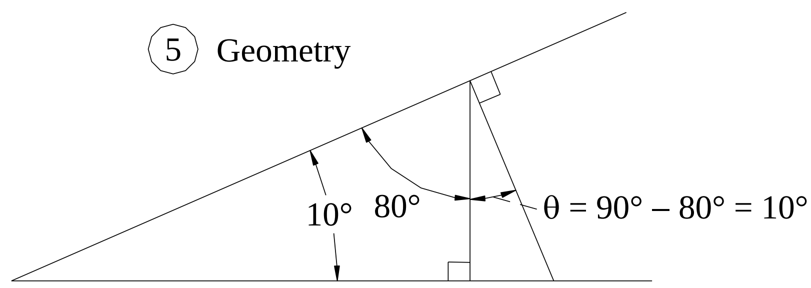
The equation for the sum of forces in the x-direction has not been included because including it would introduce an additional unknown, A_x, which we have not been asked to determine.

$$+\uparrow \Sigma F_y = 0: \ -f_{BD} \sin\theta + N_{BD}\cos\theta - 400 \text{ lb} = 0 \qquad (2)$$

$$\circlearrowleft + \Sigma M_A = 0: \ -f_{BD}(10 \text{ in.}) = 0 \qquad (3)$$

Therefore $f_{BD} = 0$, that is, no friction force acts on the cylinder.

5 Geometry

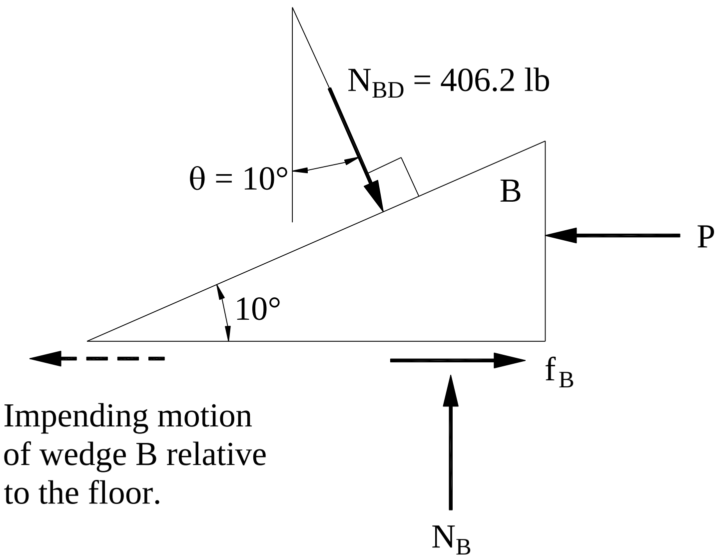


6 Solving Eq. 2 with θ = 10° gives

$$N_{BD} = 406.2 \text{ lb}$$

7.2 Wedges Example 5, page 4 of 4

7 Free-body diagram of wedge B



8 Impending motion of wedge B relative to the floor.

9 Equilibrium equations for wedge B

$$\overset{+}{\rightarrow}\Sigma F_x = 0:\ f_B + (406.2\text{ lb})\sin 10^\circ - P = 0 \qquad (4)$$

$$+\uparrow\Sigma F_y = 0:\ N_B - (406.2\text{ lb})\cos 10^\circ = 0 \qquad (5)$$

Slip impends, so

$$f_B = f_{B\text{-max}} \equiv \mu N_B = 0.3 N_B \qquad (6)$$

Solving Eqs. 4, 5, and 6 simultaneously gives

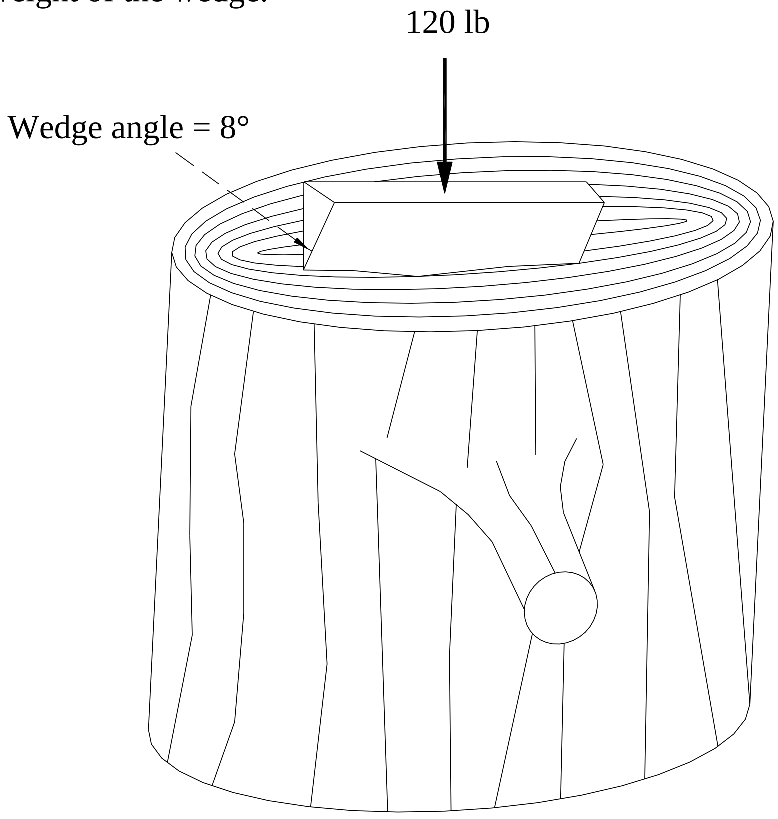
$f_B = 120$ lb

$N_B = 400$ lb

$P = 190.5$ lb ←Ans.

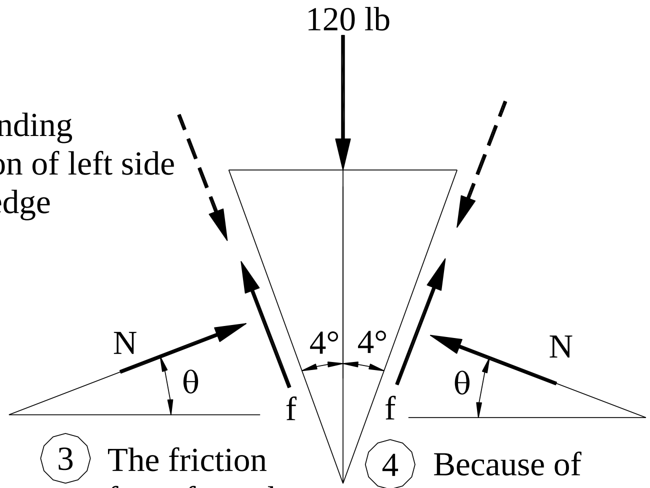
7.2 Wedges Example 6, page 1 of 3

6. To split the log shown, a 120-lb force is applied to the top of the wedge, which causes the wedge to be about to slip farther into the log. Determine the friction and normal forces acting on the sides of the wedge, if the coefficient of static friction is 0.6. Also determine if the wedge will pop out of the log if the force is removed. Neglect the weight of the wedge.



7.2 Wedges Example 6, page 2 of 3

1 Free-body diagram of wedge



2 Impending motion of left side of wedge

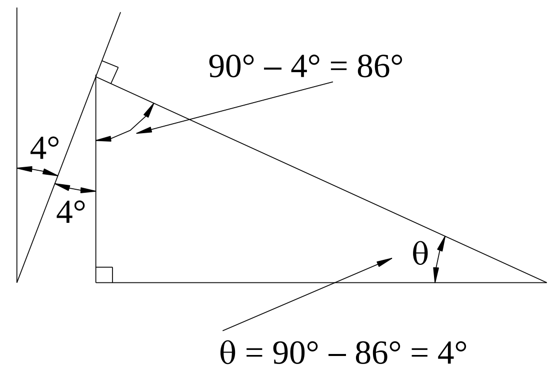
3 The friction force from the wood opposes the motion of the wedge.

4 Because of symmetry, the same variables, N and f, are used on the right side of the wedge.

5 Equilibrium equation for the wedge

$$+\uparrow \sum F_y = 0: \; 2f \cos 4° + 2N \sin \theta - 120 \text{ lb} = 0 \qquad (1)$$

6 Geometry



7 Slip impends, so

$$f = f_{max} \equiv \mu N = 0.6N \qquad (2)$$

Solving Eqs. 1 and 2 simultaneously gives

$f = 53.9$ lb ←Ans.

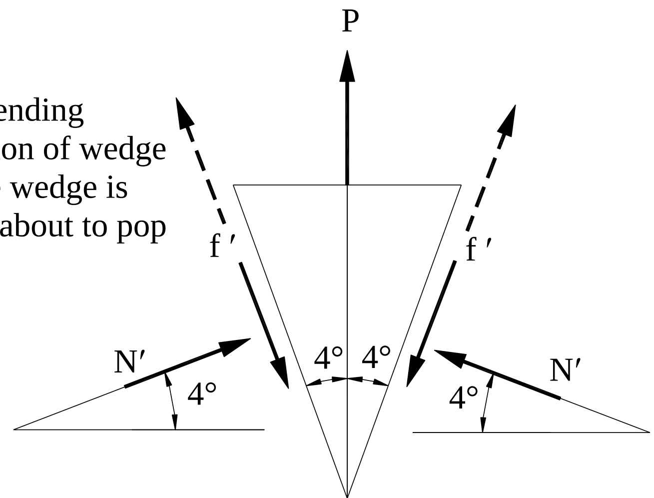
$N = 89.8$ lb ←Ans.

7.2 Wedges Example 6, page 3 of 3

8 Second part of the problem (Determine if the wedge will pop out, if no force acts down on the top). To determine if the wedge will pop out, let's first determine what force would be needed to *pull* the wedge out.

9 Free-body diagram of wedge based on assumption that a force P is applied to pull the wedge out.

10 Impending motion of wedge (The wedge is just about to pop out.)



11 The friction force from the wood opposes the motion of the wedge (The force tries to keep the wedge in the stump.)

12 Equilibrium equation for the wedge

$$+\uparrow \sum F_y = 0: \; -2f' \cos 4° + 2N' \sin 4° + P = 0 \qquad (3)$$

Slip impends, so

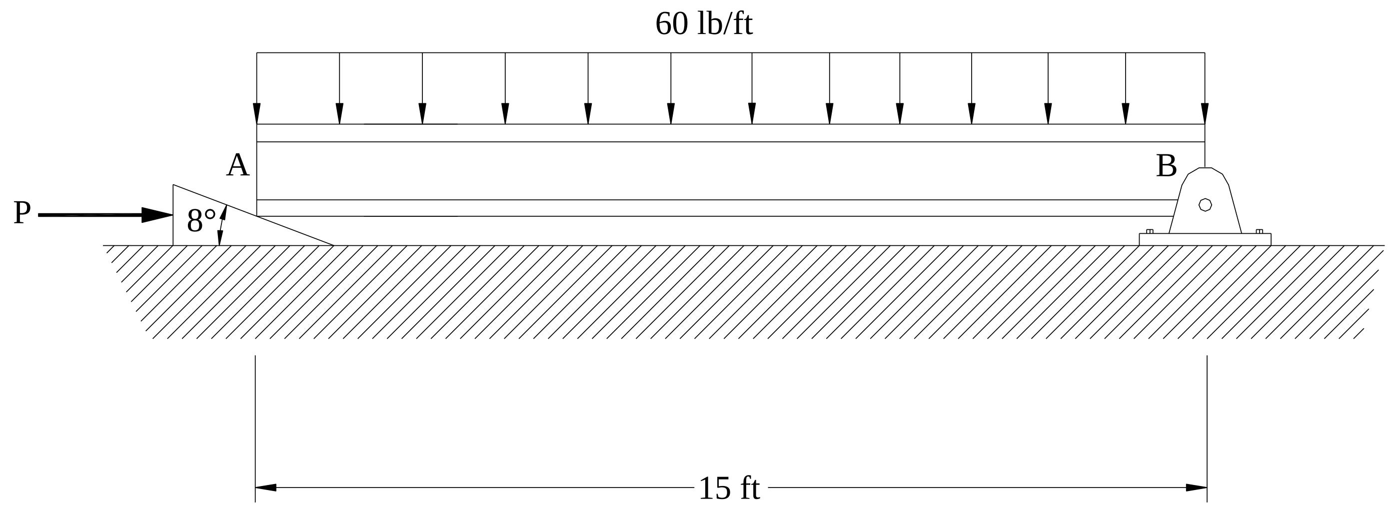
$$f' = f'_{max} \equiv \mu N' = 0.6N' \qquad (4)$$

Substituting the expression for f ' of Eq. 4 into Eq. 3 and solving for P gives

$$P = 2(0.6 \cos 4° - \sin 4°)N' = 1.058\ N' \qquad (5)$$

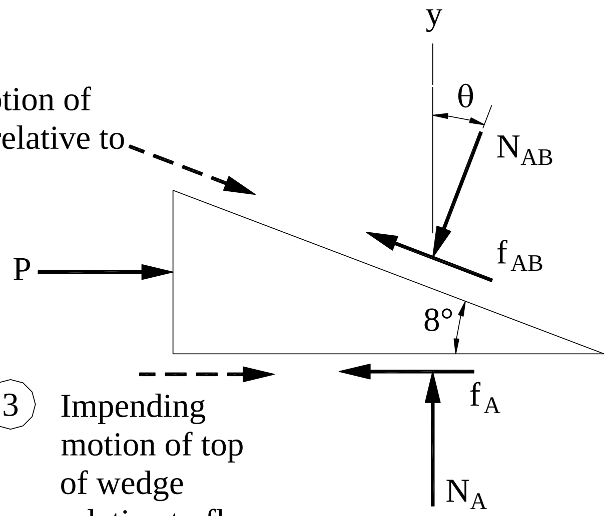
Since the normal force N' always points towards the wedge, it is always positive. Eq. 5 thus implies that the wedge will pop out *only* if an upward force greater than or equal to 1.058 times the normal force is applied. For any smaller value of P, the wedge will remain in place. Thus in particular for the special case of P = 0 (no vertical force applied), the wedge will remain in place.

7. The end A of the beam needs to be raised slightly to make it level. If the coefficient of static friction of the contact surfaces of the wedge is 0.3, determine the smallest value of the horizontal force P that will raise end A. The weight and size of the wedge are negligible. Also, if the force P is removed, determine if the wedge will remain in place, that is, is the wedge self-locking?



1 Free-body diagram of wedge

2 Impending motion of top of wedge relative to beam



3 Impending motion of top of wedge relative to floor

4 Equilibrium equations for wedge

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - f_A - f_{AB}\cos 8° - N_{AB}\sin\theta = 0 \qquad (1)$$

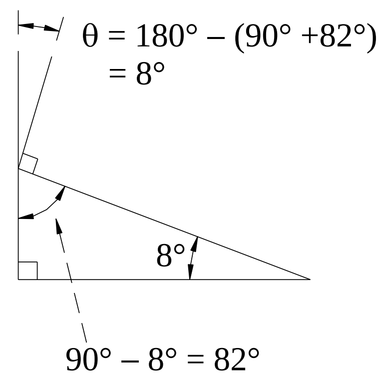
$$+\uparrow \Sigma F_y = 0: \quad N_A + f_{AB}\sin 8° - N_{AB}\cos\theta = 0 \qquad (2)$$

Slip impends so,

$$f_A = f_{A\text{-max}} \equiv \mu N_A = 0.3N_A \qquad (3)$$

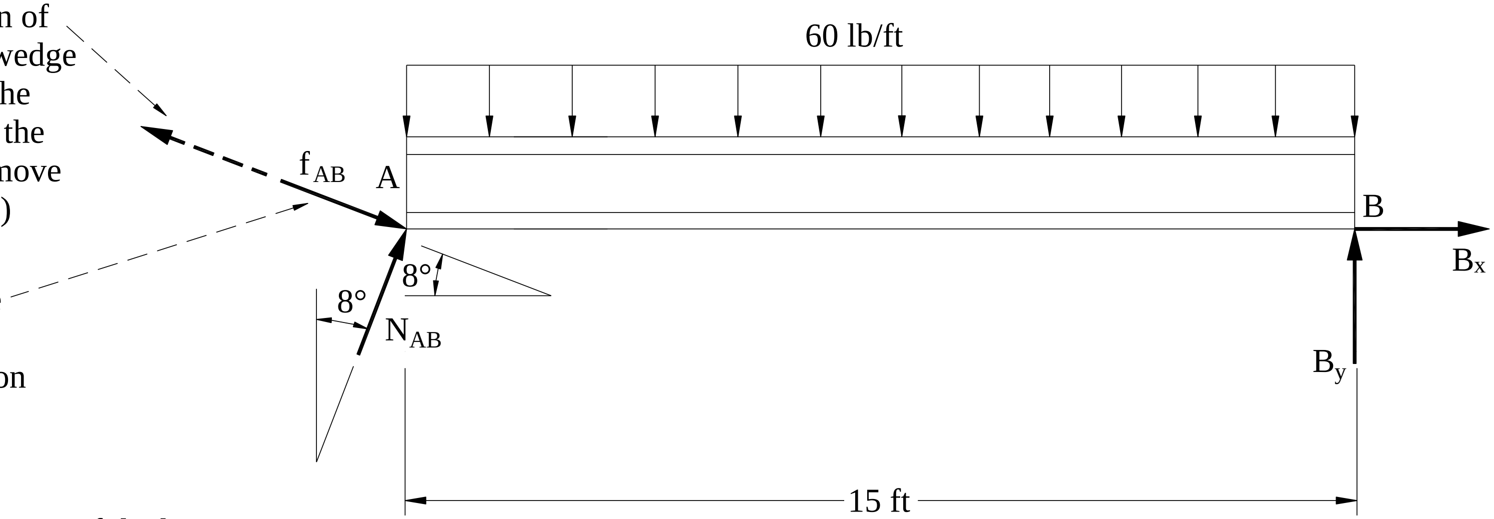
$$f_{AB} = f_{AB\text{-max}} \equiv \mu N_{AB} = 0.3N_{AB} \qquad (4)$$

5 Geometry



6 Impending motion of beam relative to wedge (An observer on the wedge would see the end of the beam move up and to the left.)

7 The friction force from the wedge opposes the motion of the beam.



8 Equilibrium equation of the beam

$$\circlearrowleft + \Sigma M_B = 0: \ (60 \text{ lb/ft})(15 \text{ ft})\left(\frac{15 \text{ ft}}{2}\right) - N_{AB} \cos 8° \, (15 \text{ ft}) + f_{AB} \sin 8° \, (15 \text{ ft}) = 0 \qquad (5)$$

Solving Eqs. 1-5 simultaneously gives

$f_A = 135$ lb

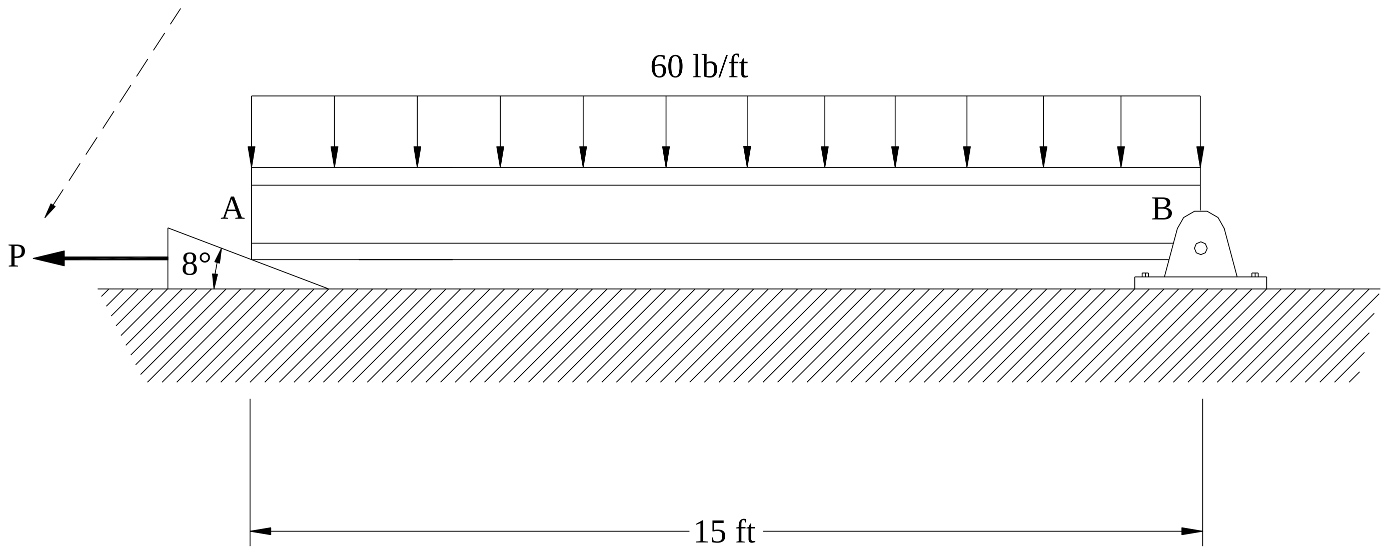
$N_A = 450$ lb

$f_{AB} = 142$ lb

$N_{AB} = 474$ lb

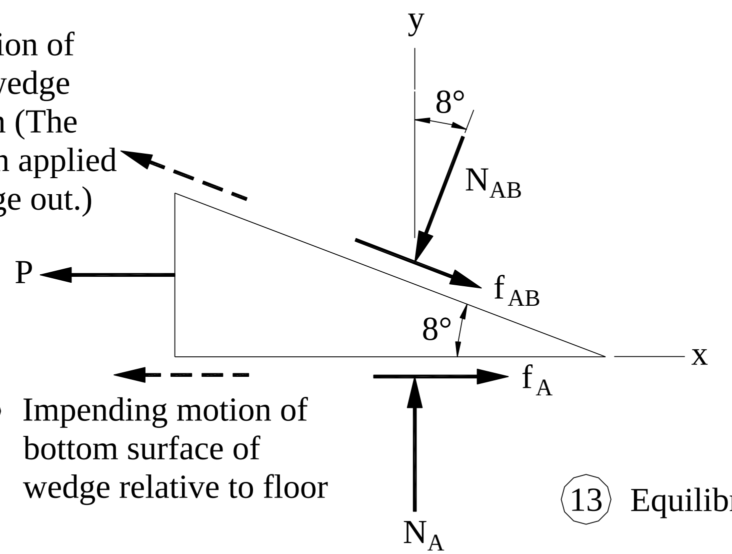
$P = 342$ lb ←Ans.

9 Second part of the problem: Determine if the wedge is self-locking. To do so, reverse the direction of force P and calculate the value of P necessary to cause impending motion of the wedge *to the left*.



10 Free-body diagram of wedge

11 Impending motion of top surface of wedge relative to beam (The force P has been applied to pull the wedge out.)



12 Impending motion of bottom surface of wedge relative to floor

13 Equilibrium equations for wedge

$$\overset{+}{\rightarrow}\Sigma F_x = 0: \quad -P + f_A + f_{AB}\cos 8° - N_{AB}\sin 8° = 0 \qquad (1)$$

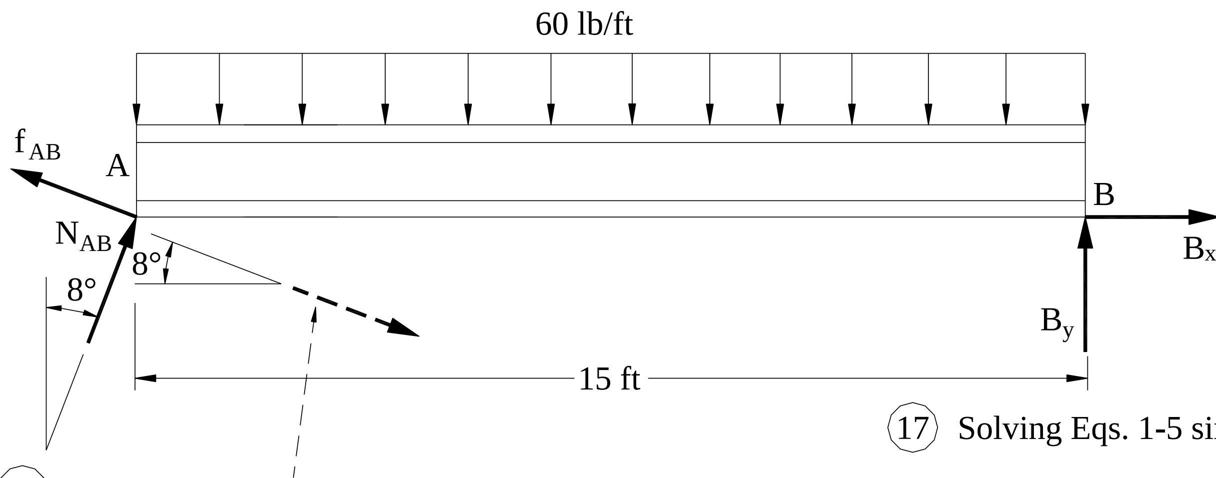
$$+\uparrow\Sigma F_y = 0: \quad N_A - f_{AB}\sin 8° - N_{AB}\cos 8° = 0 \qquad (2)$$

Slip impends so,

$$f_A = f_{A\text{-max}} \equiv \mu N_A = 0.3N_A \qquad (3)$$

$$f_{AB} = f_{B\text{-max}} \equiv \mu N_{AB} = 0.3N_{AB} \qquad (4)$$

(14) Free-body diagram of beam



(15) Impending motion of beam relative to wedge (an observer on the wedge sees the beam move down and to the right)

(16) Equilibrium equation of the beam

$$\circlearrowleft\!+\sum M_B = 0\!:\ (60\ \text{lb/ft})(15\ \text{ft})\left(\frac{15\ \text{ft}}{2}\right) - N_{AB}\cos 8^\circ\,(15\ \text{ft})$$

$$- f_{AB}\sin 8^\circ\,(15\ \text{ft}) = 0 \qquad (5)$$

(17) Solving Eqs. 1-5 simultaneously gives

$f_A = 135$ lb

$N_A = 450$ lb

$f_{AB} = 131$ lb

$N_{AB} = 436$ lb,

$P = 204$ lb.

A force P of at least 204 lb is necessary to pull out the wedge; any thing less than 204 lb is not enough—the wedge will remain in place. In particular, when *no* force is applied (P = 0), the wedge remains in place. Thus it is self-locking.