UNIT 2
SELF ASSESSMENT 1-2 (Pages 38-40)

Unit 2, Problem 1

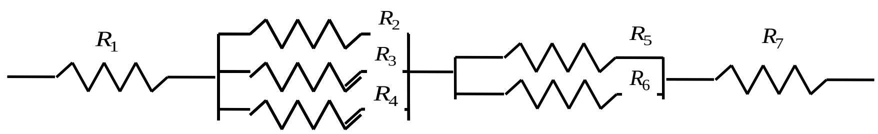
$$R_{th} = \frac{1}{h_i A} + \frac{L_{glass}}{k_{glass}} + \frac{L_{stagnat\,air}}{k_{air} A} + \frac{1}{h_o A} = \frac{1}{10x2.4} + \frac{0.003}{0.78x2.4} + \frac{0.012}{0.026x2.4} + \frac{0.003}{0.78x2.4} + \frac{1}{25x\,2.4} = 0.25385$$

$$\dot{Q} = \frac{\Delta T}{R_{th}} = \frac{(T_{\infty 1} - T_{\infty o})}{R_{th}} = \frac{(T_{\infty 1} - T_{s1})}{R_{fi}} = \frac{(24-(-5))}{0.25385} = 114\,W \Rightarrow T_{si} = T_{\infty 1} - 114xR_{fi} = 24 - 4..75000038 = 19.2\,^0C.$$

Unit 2, Problem 2.

Properties: The thermal conductivities of various materials used are given to be $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, and $k_E = 35$ W/m·°C.

Analysis: The representative surface area is $A = 0.12 \times 1 = 0.12\ m^2$ and the equivalent electrical network is as shown below:



(*a*) The thermal resistance network and the individual thermal resistances are

$$R_1 = R_A = \left(\frac{L}{kA}\right)_A = \frac{0.01\,m}{(2\ W/m.°C)(0.12\,m^2)} = 0.04167\ °C/W$$

$$R_2 = R_4 = R_C = \left(\frac{L}{kA}\right)_C = \frac{0.05\,m}{(20\ W/m.°C)(0.04\,m^2)} = 0.0625\ °C/W$$

$$R_3 = R_B = \left(\frac{L}{kA}\right)_B = \frac{0.05\,m}{(8\ W/m.°C)(0.04\,m^2)} = 0.1563\ °C/W$$

$$R_5 = R_D = \left(\frac{L}{kA}\right)_D = \frac{0.1\,m}{(15\ W/m.^oC)(0.06\,m^2)} = 0.1111\ °C/W$$

$$R_6 = R_E = \left(\frac{L}{kA}\right)_E = \frac{0.1\,m}{(35\ W/m.°C)(0.06\,m^2)} = 0.0476\ ^oC/W$$

$$R_7 = R_F = \left(\frac{L}{kA}\right)_F = \frac{0.06\,m}{(2\ W/m.°C)(0.12\,m^2)} = 0.25\ °C/W$$

$$\frac{1}{R_{mid,1}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{0.0625} + \frac{1}{0.1563} + \frac{1}{0.0625} \longrightarrow R_{mid,1} = 0.026\ °C/W$$

$$\frac{1}{R_{mid,2}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{0.1111} + \frac{1}{0.0467} \longrightarrow R_{mid,2} = 0.033\ °C/W$$

$$R_{total} = R_1 + R_{mid,1} + R_{mid,2} + R_7 + R_8 = 0.04167 + 0.026 + 0.033 + 0.25$$
$$= 0.35067\ °C/W$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(300-100)°C}{0.35067\ °C/W} = 570.34\ W\ \ \text{(for a 0.12 m} \times \text{1 m section)}$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (570.34\ W)\frac{(5\ m)(8\ m)}{0.12\ m^2} = \mathbf{1.90 \times 10^5\ W}$$

(*b*) The total thermal resistance between left surface and the point where the sections B, D, and E meet is

$$R_{total} = R_1 + R_{mid,1} = 0.04167 + 0.026 = 0.06767\ °C/W$$

Then the temperature at the point where The sections B, D, and E meet becomes

$$\dot{Q} = \frac{T_1 - T}{R_{total}} \longrightarrow T = T_1 - \dot{Q}R_{total} = 300°C - (57.34\ W)(0.06767\ °C/W) = \mathbf{261°C}$$

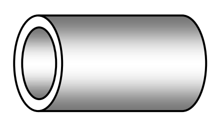
(*c*) The temperature drop across the section F can be determined from

$$\dot{Q} = \frac{\Delta T}{R_F} \longrightarrow \Delta T = \dot{Q}R_F = (570.34W)(0.25°C/W) = \mathbf{142.6°C}$$

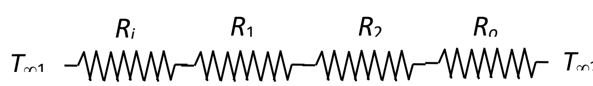
Unit 2, Problem 3.

Properties: The thermal conductivities are given to be k = 15 W/m·°C for steel and k = 0.038 W/m·°C for glass wool insulation

Analysis: The inner and the outer surface areas of the insulated pipe per unit length are



$$A_i = \pi D_i L = \pi(0.05\ m)(1\ m) = 0.157\ m^2$$
$$A_o = \pi D_o L = \pi(0.055 + 0.06\ m)(1\ m) = 0.361\ m^2$$



The individual thermal resistances are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(80\ W/m^2.°C)(0.157\ m^2)} = \frac{0.08\ °C/W}{L}$$

$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(15\ W/m.°C)(1\ m)} = \frac{0.00101\ °C/W}{L}$$

$$R_2 = R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.038\ W/m.°C)(1\ m)} = \frac{3.089\ °C/W}{L}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(15\ W/m^2.°C)(0.361\ m^2)} = \frac{0.1847\ °C/W}{L}$$

$$R_{total} = R_i + R_1 + R_2 + R_o = \frac{0.08}{L} + \frac{0.00101}{L} + \frac{3.089}{L} + \frac{0.1847}{L} = \frac{3.355\ °C/W}{L}$$

Then the steady rate of heat loss from the steam per metre. pipe length becomes

$$\frac{\dot{Q}}{L} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(320-5)°C}{3.355\ °C/W} = 93.9\ W/m$$

The temperature drops across the pipe and the insulation are

$$\Delta T_{pipe} = \dot{Q}R_{pipe} = (93.9\ W)(0.00101\ °C/W) = \mathbf{0.095°C}$$

$$\Delta T_{insulation} = \dot{Q}R_{insulation} = (93.9\ W)(3.089\ °C/W) = \mathbf{290°C}$$

Unit 2 Problem 4

For the furnace wall, we determine the overall heat transfer coefficient based on a unit area as follows

$$\frac{1}{U} = \frac{1}{h_o} + \frac{L_{fb}}{k_{fb}} + \frac{L_{ib}}{k_{ib}} + \frac{L_{bb}}{k_{bb}} = \frac{1}{10} + \frac{0.25}{1.4} + \frac{0.125}{0.2} + \frac{0.25}{0.7} = 1.2606\,W/m^2$$

Therefore per unit area is

$$q = U\,\Delta T = U(T_{inside} - T_{outside}) = \frac{1}{1.2606}(600-20) = 460\,W/m^2$$

The outside temperature of the wall, t, 10(t- 20) = 460, hence t = 66 ^{0}C

Unit 2 Problem 5

$$R_{th} = \frac{1}{h} + \frac{0.00008}{1.05x10^{-3}} + \frac{0.025}{50x10^{-3}} = 1.444 m^2 K/kW$$

$$\dot{q} = \frac{\Delta T}{R_{th}} = \frac{350-95}{1.444} = 176.6\,kW/m^2$$

Unit 2 Problem 6

The new effective resistance will be 1.444 +35 m^2/kW = 36.444 m^2/kW

And q = (350-95)/36.444 = 7 kW/m^2

Unit 2 Problem 7

For the window the thermal resistance is given by

$$R_{th} = \frac{1}{h_i A} + \frac{1}{h_o A} + \frac{L_{tg}}{k_{tg} A} = \frac{1}{8.5x2.16} + \frac{1}{31x2.16} + \frac{0.0015}{0.76x2.16} = 0.070314\,K/W$$

For a temperature change of 1K the heat flow through the window will be $1/0.070314 = 14.22$ W

Total resistance of the external wall is $R_{total} = \frac{1}{8.5x\,7.84} + \frac{1}{31x7.84} + \frac{0.25}{0.45x7.84} + \frac{0.01}{0.14x7.84} + \frac{0.005}{0.86x7.84} + \frac{0.16}{7.84}$

$R_{total} = 0.12360\,K/W$

For a temperature change of 1 K, the heat flow through the external only will be $1/0.12360 = 8.09$ W

The proportion of heat through the window is given by $\frac{14.22}{(14.22+8.09)}x100\% = 63.8\%$

Unit 2 Problem 8

The thermal resistance of the pipe wall is given by

$$R_{th} = \frac{\ln(31/25)}{2\pi\pi x4} = 0.000713\,\mathrm{m^2K/W}$$

Thermal resistance of both fluid films is given by

$$R_{film} = \frac{1}{2800x2\pi 80.025} + \frac{1}{17\ x2\pi x2\pi x0.031} = 0.304276\,\mathrm{m^2K/W}$$

Total thermal resistance of the system is $0.000713 + 0.304276 = 0.305\,\mathrm{m^2K/W}$

Heat loss per unit length of the pipe $= \dfrac{\Delta T}{R_{total}} = \dfrac{(80-15)}{0.305} = 213\mathrm{W/m} = 0.213\mathrm{kW/m}$

Unit 2 Problem 9

$$\frac{\ln(43/31)}{2\pi\pi\, 0.03} = 1.736\,\mathrm{m^2K/W}$$

Thermal resistance of felt = therefore new total resistance $= 0.305 + 1.736 = 2.041\,\mathrm{m^2K/W}$

Percentage reduction in heat loss $= \dfrac{[213 - (80 - 15)/2.041]}{213} x100\% = 85.1\%$

Unit 2 Problem 10

For the inner layer, the thermal resistance, $R_1 = \ln(115/75)/\{2x3.142x0.09x1\} = 0.7559\ m^2K/W$

For the outer layer the thermal resistance $R_2 = \ln(140/115)/\{2x3.142x0.06x\ 1\} = 0.5218\ m^2K/W$

For the outer surface, the film resistance $R_{fo} = 1/\{17x\ 2x3.142x\ 0.14x\ 1\} = 0.0668\ m^2K/W$

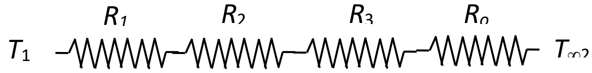
The total thermal resistance = 0.7559 +0.5218 +0.0669 = 1.3446 m^2 K/W

Therefore the heat lost per unit length = $\dfrac{(230-20)}{1.3446} = 156\mathrm{W/m}$ or $0.156\,\mathrm{kW/m}$

The outside surface temperature = 20 + (156x 0.0669) = 30.5 ^{0}C

Unit 2, Problem P11.

The same assumptions hold as in **P3** above. The individual thermal resistances are;



$$R_1 = R_{steel} = \frac{r_2 - r_1}{4\pi \cdot k_1 r_1 r_2} = \frac{0.52 - 0.5}{4\pi \times 48 \times 0.52 \times 0.5} = 0.000128\,°C/W$$

$$R_2 = R_{vermiculite} = \frac{r_3 - r_2}{4\pi \cdot k_2 r_2 r_3} = \frac{0.545 - 0.52}{4\pi \times 0.047 \times 0.545 \times 0.52} = 0.1494\,°C/W$$

$$R_3 = R_{asbestos} = \frac{r_4 - r_3}{4\pi \cdot k_3 r_4 r_3} = \frac{0.555 - 0.545}{4\pi \times 0.21 \times 0.545 \times 0.555} = 0.01253\,°C/W$$

$$R_o = R_{conv} = \frac{1}{h_o A_o} = \frac{1}{20 \times 4\pi \times (0.555)^2} = 0.01292\,°C/W$$

$$R_{total} = R_1 + R_2 + R_3 + R_o = 0.000128 + 0.1494 + 0.01253 + 0.01292 = 0.17498\,°C/W$$

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} = \frac{500 - 20}{0.17498} = 2743\text{ W} = 2.743\text{ kW}$$

Unit 2 Problem 12

$$m = \sqrt{\left(100x\ 2\pi\pi v 0.01/2x\ \pi x\ 0.01^2\right)} = 28.284$$

then $mL = 28.284x\, 0.1 = 2.8284$

and, tanh mL = 0.993

Also $h/mk = 1 = 00/(2.8264x\ 24) = 0.141$

$$\dot{Q} = \sqrt{hPkA}\,(\theta_b)\tanh mL\left[\frac{1 + \dfrac{h}{mk\tanh mL}}{1 + \dfrac{h}{mk}\tanh mL}\right]$$

$$\dot{Q} = \sqrt{100x\ 2\pi\pi\ 0.01x24x\pi.\ 0.01^2}\ (250)\,x0.993\left[\frac{1 + \dfrac{0.141}{0.993}}{1 + 0.141x0.993}\right] = 54.134\text{ W}$$

For the base per pitch, $\dot{Q} = 100x\ 250x[0.03^2 - (\pi\pi x 0.0^2)] = 14.645\text{ W}$

Heat loss per $1\,m^2$ of studded surface $(54.134 + 14.645)/(0.03^2) = 76421\text{ W/m}^2 = 76.42\text{ kW/m}^2$

Unit 2 Problem 13

i.) $m = \sqrt{2x40/(26x0.003)} = 32.026$

and $mL = 32.026x\,0.03 = 0.9608$

therefore fin efficiency = $\tanh mL/mL = \tanh(0.9608)/0.9608 = 77.5\%$

ii.) pitch of fins $= 100/12.5 = 8\,mm = 0.008\,m$

therefore base surface area per fin $= (0.008 - 0.003)x\,1\,m^2 = 0.005\,m^2$

neglecting the tip of the fin the surface area per metre length along each fin $= 2x0.03 = 0.06\,m^2$

heat lost per fin per metre length of the primary surface $= 40\ x\ 280(0.005 + 0.775x\ 0.06) = 576.8\ W$

Also number of fins in 1 metre square area $= 12.5\ x\,10 = 125$

heat lost for 1 mete square $= 576.8\ x125 = 72100\ W = 72.1\,kW$

iii.) $\dfrac{\Delta t_{tip}}{\Delta t_{base}} = \dfrac{1}{\cosh mL} = \dfrac{1}{\cosh(0.9608)} = 0.6675$

i.e. temperature at the tip $= 20 + (300 - 20)x\ 0.6675 = 206.9\ {}^0C$

Unit 2 Problem 14

$$q = \sqrt{hPkA}\theta_0 = \left[\frac{(3.5)\pi(0.025)(372)\pi(0.025)^2}{4}\right]^{1/2}(90-40) = 11.2\ W$$

Unit 2 Problem 15

$$q = \sqrt{hPkA}\theta_0 = \left[\frac{(20)\pi(0.0005)(372)\pi(0.0005)^2}{4}\right]^{1/2}(120-20) = 0.152\ W$$

Unit 2 Problem 16

$t = 1.0$ mm $\quad L = 2.0$ cm $\quad r_1 = 1.0$ cm $\quad h = 150 \quad k = 204$

$L_c = 2.05$ cm $\quad r_{2c} = 3.05$ cm $\quad \dfrac{r_{2c}}{r_1} = 3.05 \quad T_0 = 150$

$T_\infty = 20$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0205)^{3/2}\left[\frac{150}{(204)(0.001)(0.0205)}\right]^{1/2} = 0.556$$

$\eta_f = 0.75$

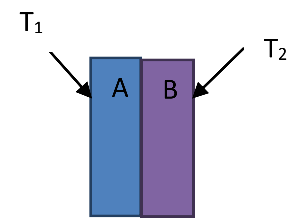
$q = (0.75)(150)(2)\pi(0.0305^2 - 0.01^2)(150-20) = 76.3\ W$

UNIT 2 MCQs SOLUTION

1. $\Delta T = \frac{275 \times 0.15}{-1.1} = 37.5°C$

⇨ **Temperature drop is 37.5°C**

2. $\Delta T = 18°C$



$$R = \frac{L_A}{K_A} + \frac{L_B}{K_B} = \frac{0.08}{0.8} + \frac{0.05}{0.2} = 0.1 + 0.25 = \mathbf{0.35}\ \mathbf{m^2K/W}$$

$$\frac{\dot{Q}}{A_X} = \frac{\Delta T}{R_{total}} = \frac{18\ K}{0.35\ m^2K/W} = 51.4\ W/m^2$$

3. $r_{cr} = \frac{2\times k}{h} = \frac{2\times 0.15}{7.5} = 0.04\ m$

$thickness = r_{cr} - r_1 = 0.04 - 0.015 = 0.025\ m = \mathbf{2.5\ cm}$

4. $A_X = 1.5 \times 2 = 3m^2$

$$\dot{Q} = \frac{\Delta T}{\Sigma R} \Sigma R = R_{glass} + R_{air}$$

$$\Sigma R = \frac{3 \times 0.005}{0.8 \times 3} + \frac{2 \times 0.01}{0.025 \times 3} = 0.27292\ k/W$$

$$\dot{Q}_X = \frac{10\ k}{0.27292\ k/W} = 36.64\ W \cong \mathbf{37\ W}$$

5. $\dot{Q}_X = n_f \times hA_s(T_s - T_{in})$

$$\dot{Q}_X = 0.75 \times 30 \times 100 \times \pi \times 0.01 \times 0.1 \times (80 - 20)$$

$$\dot{Q}_X = 424.17 \cong \mathbf{424\ W}$$

6. $\varepsilon = \frac{A_{fin}}{A_C}\eta_f = \frac{\pi\times 0.006\times 0.03\times 4}{\pi\times(0.006)^2} \times 0.7 = \mathbf{14}$

7. $\varepsilon_{fin} = \frac{A_{fin}}{A_C}\eta_f = \frac{4\times 0.002\times 0.03}{0.002\times 0.002} \times 0.7 = \mathbf{39}$

8. $V_d = \frac{\pi D^3}{6} \qquad m = \rho V_d = 1141 \times 3.142 \times \frac{6^3}{6} = 129{,}060.792$

$$\frac{\dot{Q}}{A_X} = \frac{mc_p\Delta T}{time} = \frac{129{,}060.792 \times 1.71 \times 10^3 \times 1}{144 \times 3600} = 425.752 \cong \mathbf{426W}$$

9. $A_X = 4 \times 2.5m^2 = 10m^2 R_{wall} = \frac{\delta}{kA_X}$

$$\Rightarrow k = \frac{\delta}{R_{wall}A_X} = \frac{0.20}{0.0125 \times 10} = \mathbf{1.6}\ \mathbf{W/m°C}$$

10. $\frac{k_A}{k_B} = 4 \qquad \frac{\delta_A}{\delta_B} = 2 \qquad \dot{Q}_A = -k_A\frac{A_X\Delta T}{\delta_A}\ \dot{Q}_B = -k_B\frac{A_X\Delta T}{\delta_B}$

$$\frac{\dot{Q}_A}{\dot{Q}_A} = \frac{\delta_B}{\delta_A} \times \frac{k_A}{k_B} = \frac{1}{2} \times 4 = \mathbf{2}$$