6 CONVECTION HEAT TRANSFER IN CLOSED CONDUIT

The most important convective heat-transfer process industrially is that of cooling or heating a fluid flowing inside a closed conduit or pipe. The relationship between temperature profile and heat transfer at a bounding surface is defined in terms of the heat transfer coefficient introduced in Newton's law of cooling. The heat transfer coefficient depends on the temperature difference selected. For flow in a conduit, there are several temperature differences of importance, and each has an associated heat transfer coefficient.

Although the solution for the differential energy balance in the laminar flow regime is available, engineers prefer to work with heat transfer coefficients. As a result, it is customary to express heat transfer performance, even for laminar flow, in terms of heat transfer coefficient.

The heat transfer coefficients for laminar flow have a strong dependence on position. This is not usually the case for heat transfer with turbulent flow.

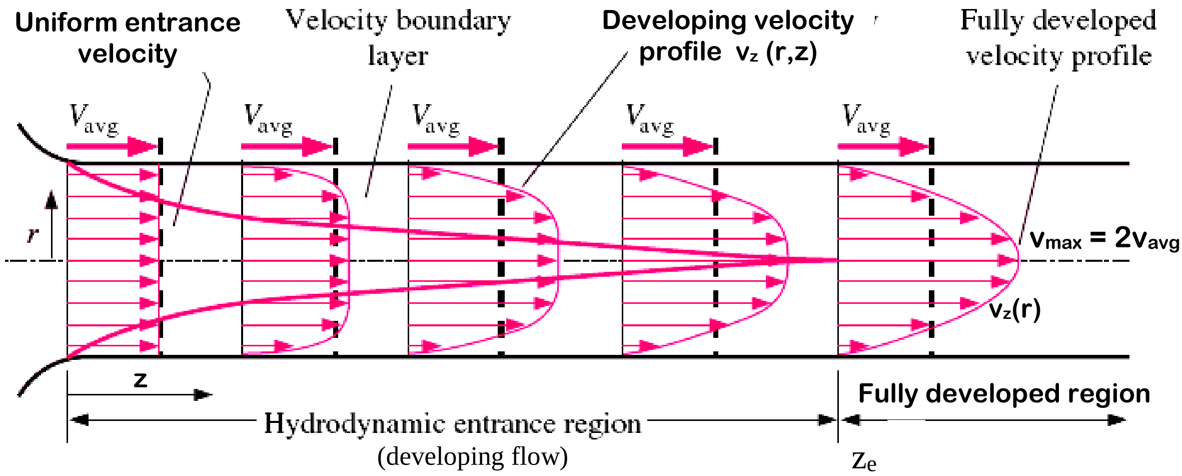
There are 2 objectives:

(1) To relate the heat transferred to the temperature change experienced by the fluid,

(2) To relate the heat transferred to the fluid to the difference in temperature between the wall and the fluid

6.1 Laminar flow, entrance region

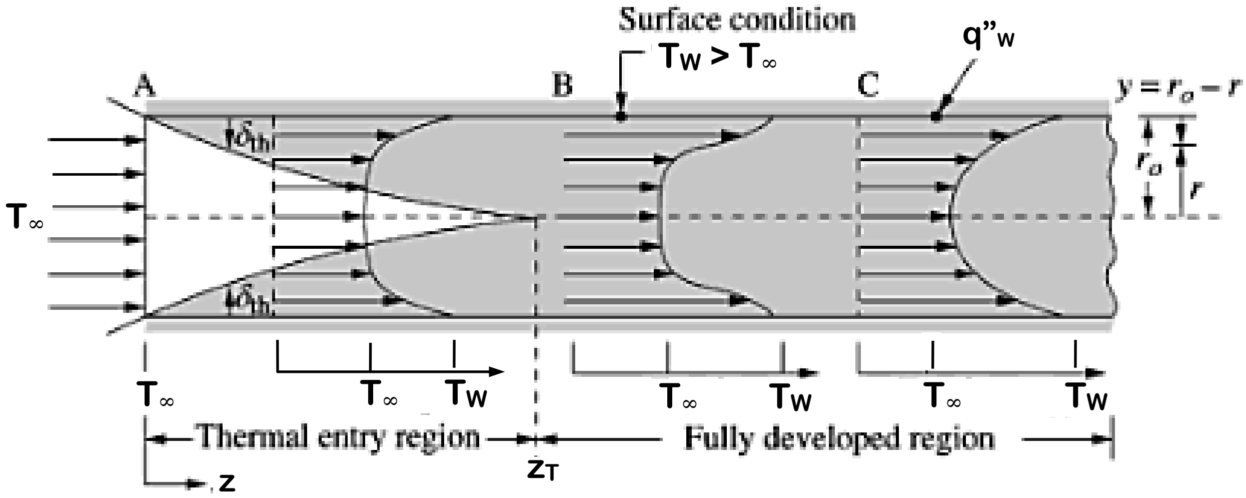
Consider a reservoir from which fluid flows under laminar conditions into a circular duct. The duct entrance is well rounded. The mouth of the tube is located at $z = 0$.



Development of the hydrodynamic boundary layer in a pipe

A fluid usually enters a pipe with uniform velocity (v_{avg}) and temperature (T_∞) profiles. If the pipe wall is heated or cooled, then both thermal and hydrodynamic boundary layers begin developing at the pipe inlet. The boundary layers coat the inner surface of the tube. As the boundary layers grow in thickness, eventually they fill the tube. Beyond this location, the velocity profile or the temperature distribution no longer changes, the flow becomes hydrodynamically and/or thermally fully developed.

As the fluid moves along the pipe, the velocity profile changes from being a constant at inlet (v_{avg}) to **fully developed** downstream. A fully developed velocity profile does not change any longer in the flow direction. The axial distance required for the velocity profile to become fully developed is called the **hydrodynamic entrance length** z_e. At this distance the boundary layer fills the pipe cross-section, that is $\delta = R$ at $z = z_e$.



Development of thermal boundary layer in pipe and temperature profiles for (B) constant T_W, (C) constant q″

When the fluid enters the pipe, convective heat transfer will occur, a thermal boundary layer will start to develop, and the fluid's uniform T_∞ temperature will increase. In the thermal entrance region (A), the temperature of the central portion of the flow outside the thermal boundary layer δ_T remains unchanged, but in the boundary layer the temperature increases sharply to the pipe surface temperature T_W. The thermal boundary layer builds up till it fills the pipe cross-section. After this point, fully developed flow condition has been reached.

In the fully developed region, the fluid at the centerline begins to heat up as well.

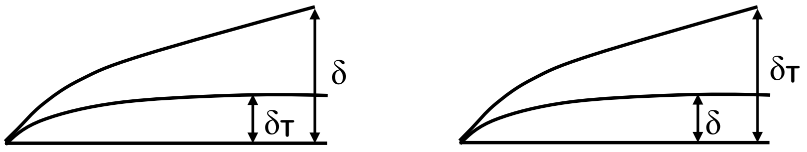
The **thermal entry length** z_T is where $\delta_T = R$.

$$\frac{z_e}{D} \cong 0.05\,\text{Re} \qquad \text{where} \qquad \text{Re} = \frac{\rho v_{avg} D}{\mu} < 2100 \text{ (laminar flow)}$$

$$\frac{z_T}{D} \cong 0.05\,\text{Re Pr}$$

v_{avg} = average flow velocity
z_e = hydrodynamic entrance length
z_T = thermal entry length

For gases (Pr < 1), therefore, the hydrodynamic boundary layer develops more slowly than the thermal boundary layer ($z_e > z_T$). For liquids (Pr > 1), the flow develops hydrodynamically more rapidly than it does thermally. For substances with high Prandtl numbers, such as oils (Pr > 100), the flow develops hydrodynamically so much more quickly that a fully developed velocity profile can be reasonably assumed throughout.



Pr >> 1
(e.g. oils)

Pr << 1
(e.g. liquid metals)

A fully developed velocity profile is mathematically defined as:

$$\frac{d\,(v_z / v_{z\,\max})}{dz} = 0 \quad \Rightarrow \quad v_z = v_z(r) \quad \text{it is independent of z.}$$

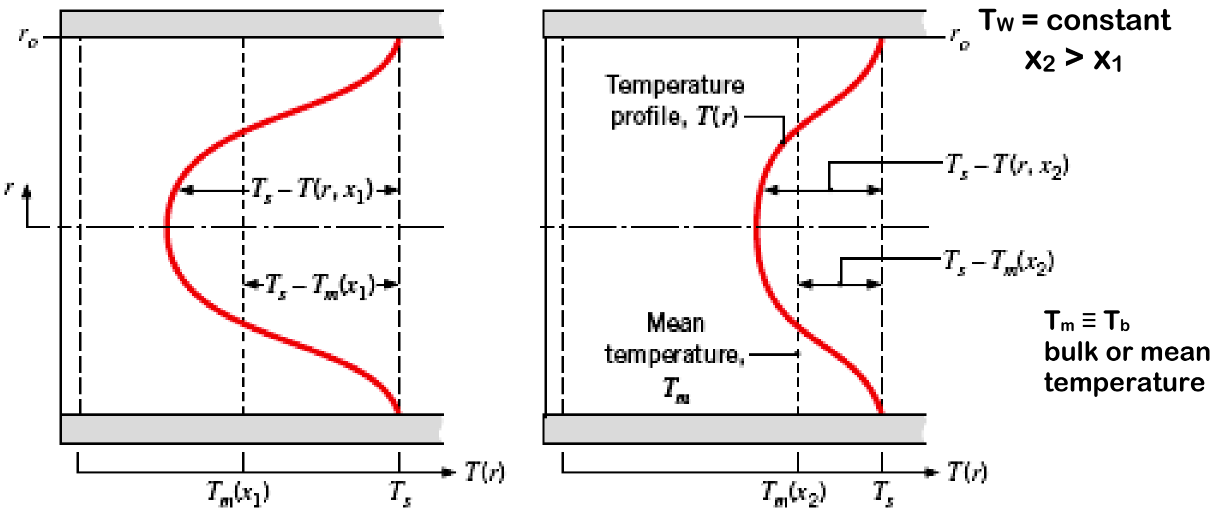
A fully developed temperature profile is one in which

$$\frac{d\,[\,(T_W - T)/(T_W - T_b)\,]}{dz} = 0$$

where T_W = wall temperature
T_b = bulk fluid temperature or mean fluid temperature

These equations are valid at each radial location in the cross section.

Unlike the velocity profile, the temperature profile can be different at different cross sections of the tube in the developed region, and it usually is. However, above temperature ratio (dimensionless temperature profile) remains unchanged when T_W or q″ is constant.



Temperature profiles for thermally fully developed flow for constant temperature heating. The relative shape of the temperature profiles is the same.

The bulk temperature for a circular conduit (ρ, c_p = constant) is:

$$T_b(z) = \frac{(v_z T)_{avg}}{v_{z,avg}} = \frac{\iint v_z T \, dA}{\iint v_z \, dA} = \frac{\int_0^{2\pi}\int_0^R v_z(r)\; T(r,z)\; r\, dr\, d\Theta}{\int_0^{2\pi}\int_0^R v_z(r)\, r\, dr\, d\Theta}$$

The local heat transfer coefficient h_z is: $q = h_z A_S (T_W - T_b)$

$T_W - T_b$ is the temperature difference between the wall and the fluid. It is an important quantity. Both T_W and T_b can vary in the z-direction.

If we integrate the local coefficient h_z over a definite area (such as the surface area A_S), the integrated result would be $\bar{h}$ average heat transfer coefficient:

$$\bar{h} = \frac{1}{A_s} \int_{A_s} h_z \, dA_s$$

The solution in the entrance region

Nusselt number, thus h heat transfer coefficient is much higher in the entrance region.

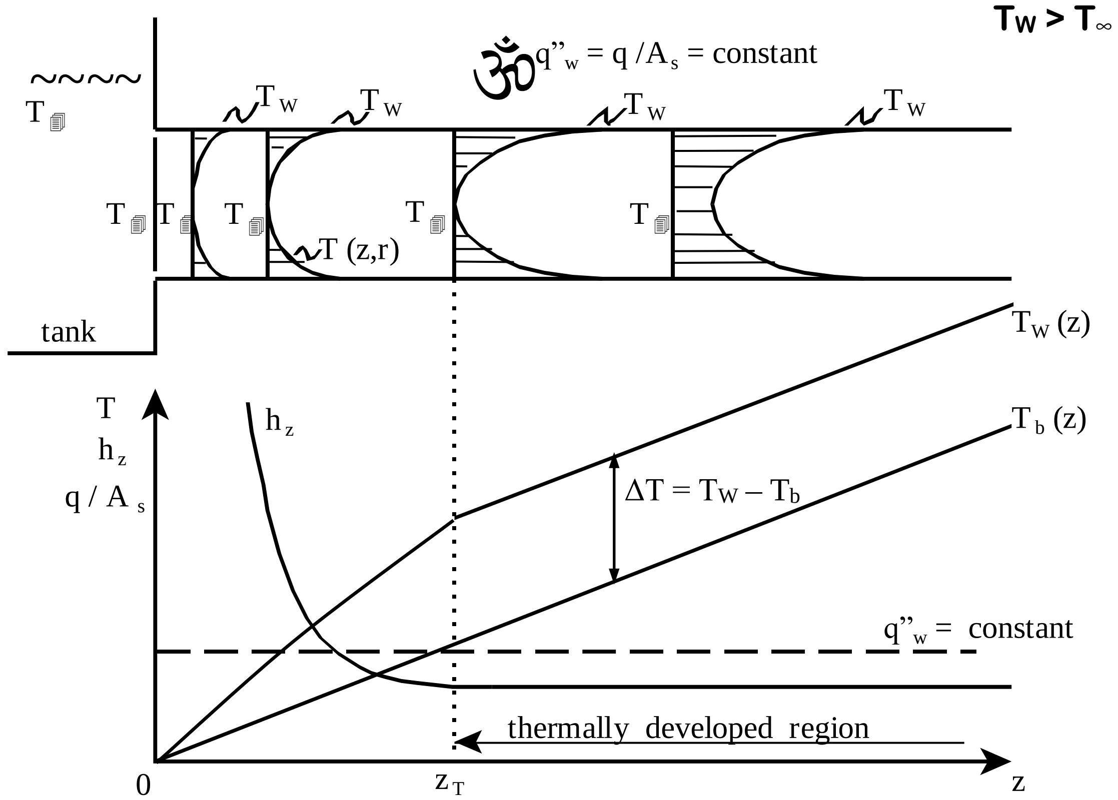
The Leveque solution (1928) for the Nusselt number in the entrance region for laminar flow and constant wall temperature

$$Nu = 1.077\,(\mathrm{Re\,Pr})^{1/3}\left(\frac{D}{z}\right)^{1/3} \qquad \text{for} \qquad 100 < \mathrm{Re\,Pr}\,\frac{\mathrm{D}}{\mathrm{z}} < 5000$$

6.2 Heat transfer in laminar flow — Constant heat flux at the wall

The heat flux at the wall $q''_w = q/A_S$ = constant. Experimentally this can be achieved by passing electric current through a metal pipe or by wrapping the duct with a material through which an electric current is passed. The resulting temperature-profile variation with axial distance is illustrated in the next figure.

The velocity profile and the temperature profile both begin changing at the tube entrance. The fluid temperature at the inlet is T_∞, the tank-fluid temperature and varies within the tube. At any axial location z, T varies from T_W, the wall temperature, to some value at the centerline. Just as the velocity profile eventually becomes fully developed, so does the temperature profile. The temperature profile reaches a nonchanging shape, but with the constant heat input that exists, $T - T_\infty$ increases with increasing axial distance z.



The figure shows also the variation of the wall temperature and the bulk temperature with increasing z. The bulk temperature increases linearly with distance z. The wall temperature changes over the thermal entry length z_T. Beyond z_T, the wall- and bulk-temperature lines are parallel.

The temperature difference between the wall and the fluid ($T_W - T_b$) is used in Newton's law of cooling to define a local heat transfer coefficient:

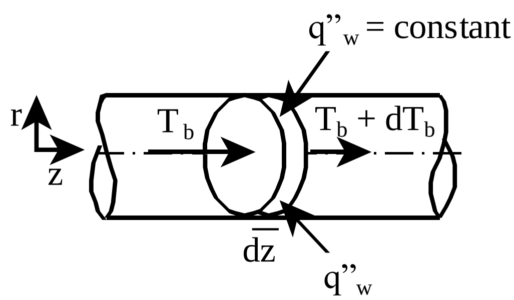
$$q'' = h_z \underbrace{(T_W - T_b)}$$

In the entrance region: T_W and T_b vary with z $\Rightarrow$ h_z also varies with z
In the thermally developed region: $T_W - T_b$ = constant $\Rightarrow$ h_z = constant

and $T_W - T_b = \Delta T = \dfrac{q''}{h_z}$

Note: h_z is infinite at the entrance of the tube.

The relationship between T_b and q''



$P = 2\pi R$ = perimeter of tube
$A = \pi R^2$ = flow cross-section

Perform a heat balance for the element:

$$q''_w = \frac{dq}{dA_S} = \frac{dq}{P\,dz} \quad \Rightarrow \quad dq = q''_w\,P\,dz$$

The heat added increases the enthalpy of the fluid:

$$dq = \dot{m}\,dh = \dot{m}c_p\,dT_b = \rho v_{avg} A c_p\,dT_b$$

$$\left.\begin{matrix} dq = q''_w\,P\,dz \\ dq = \rho v_{avg} A c_p\,dT_b \end{matrix}\right\} =$$

$$2\pi R\,q''_w\,dz = \rho v_{avg}\,\pi R^2 c_p\,dT_b$$

$$\Downarrow$$

$$\frac{dT_b}{dz} = \frac{2\,q''_w\,\alpha}{v_{avg}\,k\,R} = \text{constant} \qquad \text{where} \qquad \alpha = \frac{k}{\rho c_p}$$

The slope of the bulk temperature line $T_b(z)$ is constant $\Rightarrow$ T_b varies linearly with z, this result is independent whether or not fully developed conditions exist.

Integrating from T_{bi} to T_{bo} and from 0 to L yields:

$$\underline{\underline{T_{bo} - T_{bi} = \frac{2\,q''\alpha\,L}{v_{avg}\,k\,R}}} \qquad \text{applies to laminar or turbulent flow regime}$$

The bulk temperature T_b

$$\underline{\underline{T_W - T_b = \frac{11}{48}\,\frac{q''_w\,D}{k}}}$$

The heat transfer coefficient

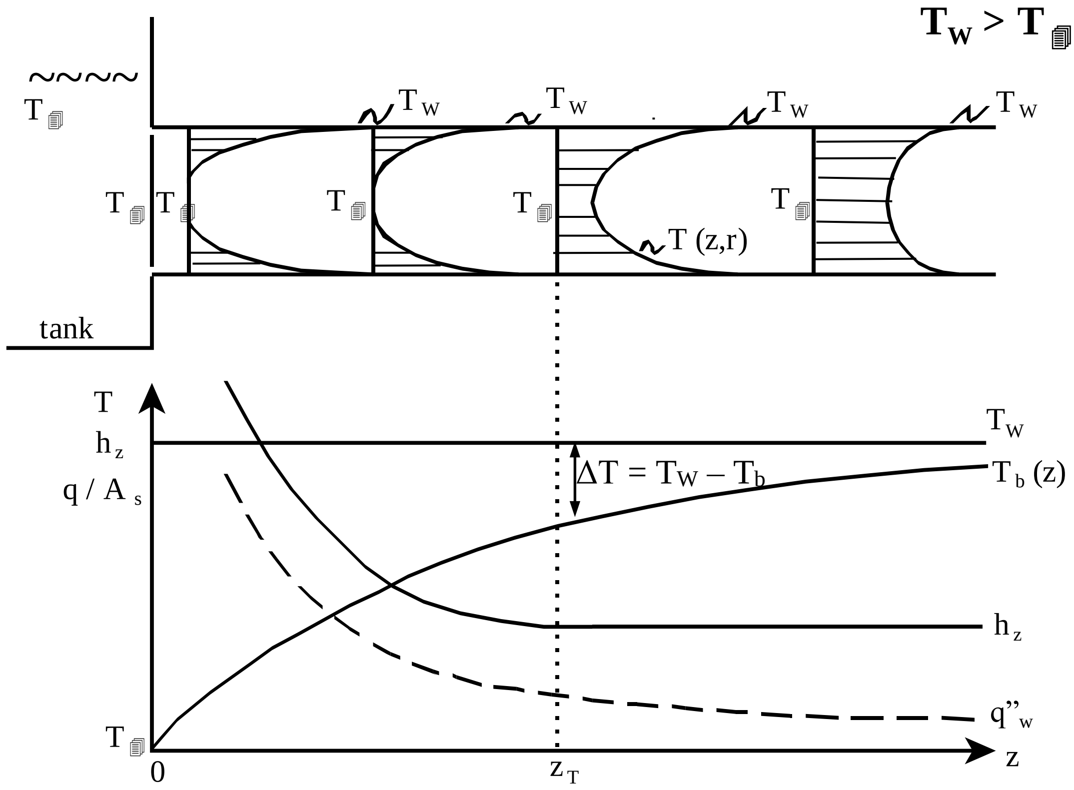
Introducing above result into Newton's law of cooling [$q''_w = h_z\,(T_W - T_b)$] results

$$\underline{\underline{Nu = \frac{h_z\,D}{k} = \frac{48}{11} = 4.364}}$$

constant wall heat flux
hydrodynamically and thermally developed
laminar flow

6.3 Heat transfer in laminar flow — Constant wall temperature

A fluid at temperature T_∞ flows from a reservoir through a pipe of constant wall temperature T_W ($>T_\infty$). The resulting variation of temperature profile with axial distance is illustrated in the figure below.



Experimentally constant wall temperature can be achieved by allowing a fluid to condense on the outside surface of the tube. Condensation is an isothermal process for pure substances.

- T_b changes from inlet and approaches the constant T_W with increasing z.
- T_b does not increase linearly.
- q'' heat flux decreases with increasing z.
- $\Delta T = (T_W - T_b)$ difference decreases with z, but so does the heat flux q''. The combined effect is that the local heat transfer coefficient h_z becomes constant in the thermally developed region. [$q''_w = h_z (T_W - T_b)$]
- ΔT decays exponentially.

The heat transfer rate

$$q = 2\pi R L \bar{h}_L \frac{(T_W - T_{bo}) - (T_W - T_{bi})}{\ln \dfrac{T_W - T_{bo}}{T_W - T_{bi}}}$$

2πRL = A_S the surface area of a circular duct

Introducing the **logarithmic temperature difference** ΔT_{lm}:

$$\boxed{\Delta T_{lm} \equiv \frac{(T_W - T_{bo}) - (T_W - T_{bi})}{\ln \dfrac{T_W - T_{bo}}{T_W - T_{bi}}}}$$

$$\boxed{q = \bar{h}_L A_S \Delta T_{lm}}$$

for laminar flow, T_W = constant

Note:

– the integrated coefficient $\bar{h}_L$ **is defined in terms of** $\mathbf{\Delta T_{lm}}$
– the **local coefficient** $\mathbf{h_z}$ **is defined in terms of** $\mathbf{(T_W - T_b)}$
– ΔT_{lm} is an exact representation of the changing average temperature difference between the fluid and the surface.
– $\mathbf{\Delta T_{lm}}$ **should always be used when determining the convection heat transfer in a tube whose surface is maintained at a constant temperature** $\mathbf{T_W}$**.**

Above equations and results do not require that fully developed conditions exist.

The heat transfer coefficient

$$Nu = \frac{h_z D}{k} = 3.658$$

constant wall temperature
hydrodynamically and thermally developed
laminar flow

Graphical representation of results

The Nusselt number based on a local coefficient h_z is a significant parameter. From dimensional analysis it can be seen that

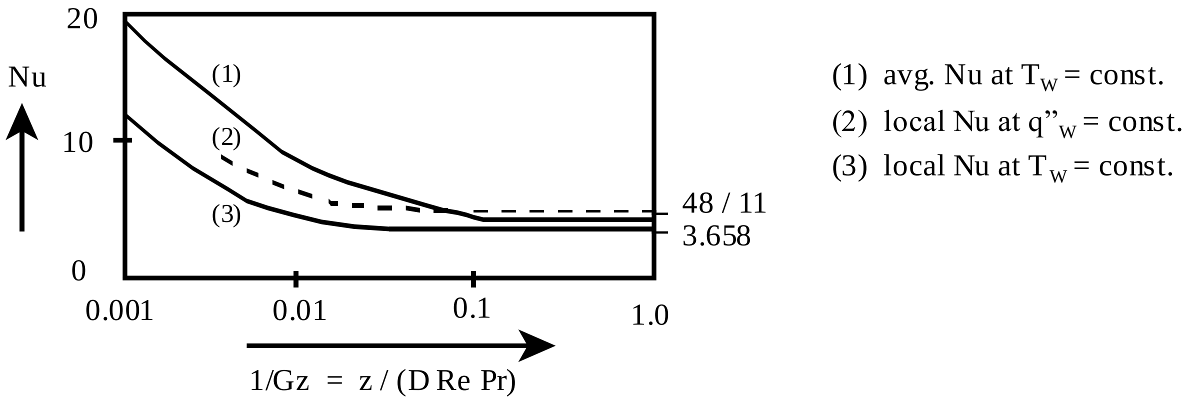
$h_z = h_z (v_{avg}, \rho, \mu, D, c_p, k, z)$

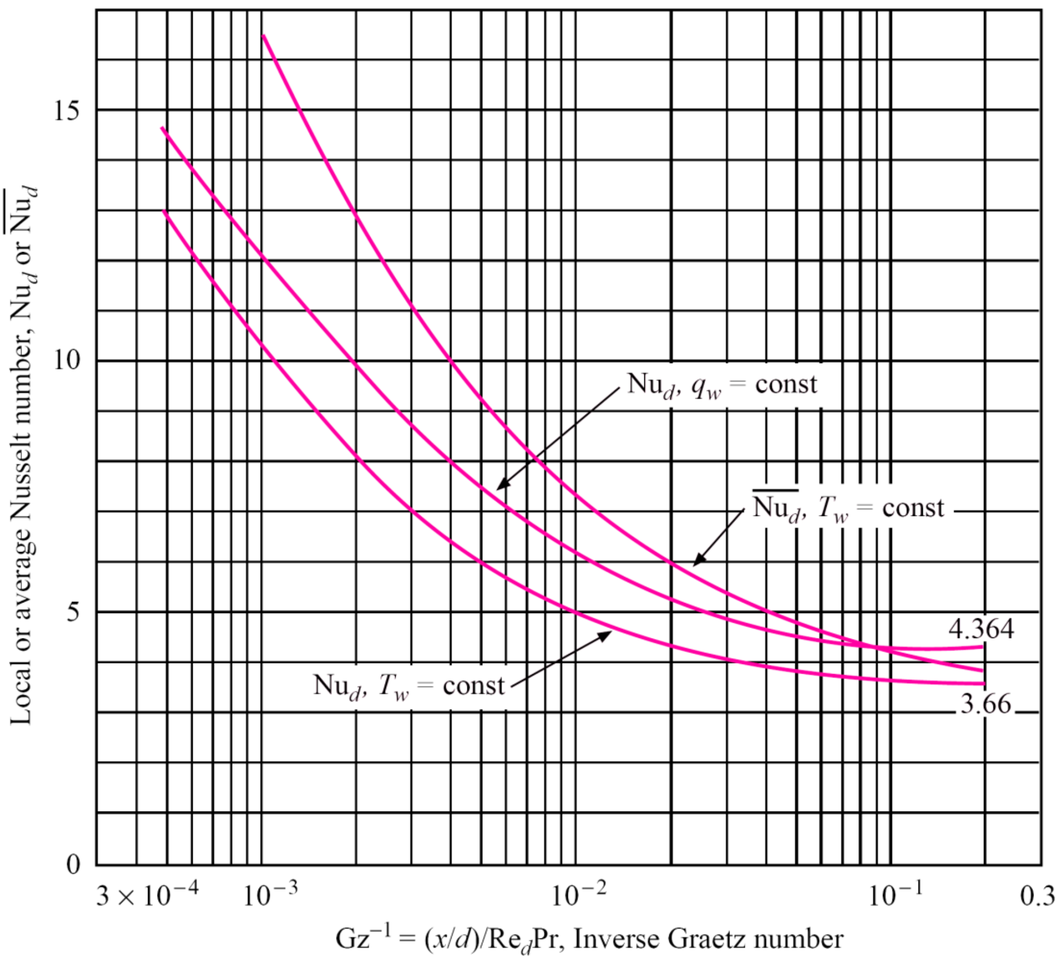
$$\Rightarrow \quad \frac{h_z D}{k} = h_z \left(Re, Pr, \frac{z}{D}\right) \qquad \text{or} \qquad Nu = Nu \left(Re, Pr, \frac{z}{D}\right)$$

Defining Grätz (Graetz) number as:

$$Gz \equiv Re\, Pr \frac{D}{z} \quad \text{so} \quad Nu = Nu(Gz) \quad \text{or} \quad Nu = Nu(1/Gz)$$

The following graph summarizes the results obtained from the solution of the differential equation of energy. The temperature profile is thermally developed when $1/Gz \geq 0.05$.





Local and average Nusselt numbers for circular tube thermal entrance regions in fully developed laminar flow.

6.4 The combined-entry-length problem for laminar flow

When the laminar flow is not developed hydrodynamically or thermally, the case is referred to as the combined-entry-length problem. For this case, the heat transfer in the entrance region is more sensitive to the Prandtl number.

Graphical representation of the problem (as graphs of Nu vs. 1/Gz) is available in literature for constant wall temperature and for constant wall heat flux as well. It is often impractical to have graphs from which to obtain Nusselt number.

For the combined problem of developing velocity and temperature profiles, the **Sieder-Tate equation** is useful:

$$\overline{Nu} = \frac{\bar{h}_L\, D}{k} = 1.86 \left(\mathrm{Re}\, \mathrm{Pr}\, \frac{D}{L} \right)^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14}$$

where μ_w = dynamic viscosity of fluid at T_W

$\bar{h}_L$ = average heat-transfer coefficient over the heat-transfer length

Fluid properties (k, ρ, c_p, μ) are evaluated at the average of the bulk temperature:

$$\boxed{T_{avg} = \frac{T_{bo} + T_{bi}}{2}}$$

It applies to steady laminar flow of Newtonian fluid in a tube having constant wall temperature under the following conditions:

T_W = constant (in condensers when heating a fluid with condensing vapor)
0.48 < Pr < 16 700 (for gases, organic liquids, water solutions −but not water)
$0.044 < (\mu / \mu_w) < 9.75$
Re < 2100 (laminar flow)
Re Pr (D/L) > 10
Deviation: ± 12%

In laminar flow, the average coefficient $\bar{h}_L$ depends strongly on the heated length. The average temperature drop (ΔT_{avg}) between the bulk fluid and the wall is used to calculate the heat-transfer rate:

$$q = \bar{h}\, A \Delta T_{avg} = \bar{h}\, A\, \frac{(T_W - T_{bi}) + (T_W - T_{bo})}{2}$$

6.5 Heat transfer in turbulent flow in tubes

In turbulent flow, the hydrodynamic and thermal entry lengths are of the same size and are independent of the Prandtl number. The entry length is much shorter than in laminar flow and its dependence of the Reynold number is weak. The entrance effect becomes insignificant beyond a tube length of 10 diameters:

$$z_e \approx z_T \leq 10\ D$$

The tubes used in practice in forced convection are usually several times longer than the length of the entrance region, and thus we assume fully developed flow for the entire length of the tube.

Heat transfer coefficients are higher with turbulent flow than with laminar flow.

The Nusselt numbers for T_W = constant or q''_w = constant are identical in the fully developed regions, and nearly identical in the entrance regions.

It is not possible to obtain a closed-form solution for the velocity profile in turbulent flow. Dimensional analysis predicts that the Nusselt number depends on Reynolds and Prandtl numbers. Combining with experimental results, the following is found to apply:

Sieder-Tate equation (1936)

$$Nu = \frac{h_L D}{k} = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

Conditions:

T_W = constant or q''_w = constant
Re > 10 000 (turbulent flow)
$0.7 \leq \mathrm{Pr} \leq 16\ 700$
$L/D \geq 60$ thermally and hydrodynamically developed flow
Deviation: +15% to −10%

h_L is the heat-transfer coefficient based on the log mean driving force ΔT_{lm}:

$$q = h_L\ A\ \mathbf{\Delta T_{lm}}$$

This correlation is applicable for fluids whose viscosity changes greatly for small temperature change. It is not recommended for water.

The use of this equation may be trial and error, since value of h_L must be known to evaluate T_W, and hence μ_w, at the wall temperature.

Since the rate of heat transfer is greater in turbulent flow than in laminar flow, most equipment are operated in the turbulent range.

In place of this type of correlation, another is often used, which consists of using a dimensionless group j called the **Colburn factor**:

$$j = \left(\frac{h}{c_p G}\right)\left(\frac{c_p \mu}{k}\right)^{2/3}$$

where $G = \rho v$ = mass velocity or mass flux, kg/m^2s
h/c_pG = St = Stanton number
$c_p\mu/k$ = Pr = Prandtl number

$$\left.\begin{aligned} \Rightarrow \quad & j = St\, Pr^{2/3} \\ \text{Colburn showed that} \quad & j = \frac{f_F}{2} \end{aligned}\right\} =$$

By this, Colburn related the Fanning friction factor to the heat transfer coefficient.
The dimensionless group j is often plotted against the Re number.

Dittus-Boelter equation:
It is an alternative expression that is useful when the wall temperature is unknown.

$$\underline{\underline{Nu = 0.023\, Re^{0.8}\, Pr^{n}}}$$

where $n = 0.4$ for heating $(T_W > T_b)$
$n = 0.3$ for cooling $(T_W < T_b)$

Conditions:
T_W = constant or q''_w = constant
$0.7 \leq Pr \leq 160$
Re > 10 000 (maybe Re > 8 000) turbulent flow
$L / D \geq 60$
Deviation: ± 20%

This correlation is applicable in case of moderate temperature differences between the wall and the fluid. This equation should be used for water.

The transition region extending from Re = 2100 to 10000 is not well understood and is usually avoided in design if possible.

These correlations for the turbulent regime predict the local Nusselt number. The average Nusselt number based on a heat transfer coefficient for the entire tube is not available.

When dealing with non-circular conduits, the hydraulic diameter or equivalent diameter is used in the heat transfer correlations whenever D diameter appears.

$$\boxed{D_e = 4 \cdot r_H = 4\, \frac{\text{cross sectional area}}{\text{wetted perimeter}}}$$

Summary of heat transfer correlations in forced convection

Geometry	Equation	Restrictions
Tube flow	$Nu_d = 0.023\ Re_d^{0.8} Pr^n$	Fully developed turbulent flow, $n = 0.4$ for heating, $n = 0.3$ for cooling, $0.6 < Pr < 100$, $2500 < Re_d < 1.25 \times 10^5$
Tube flow	$Nu_d = 0.0214(Re_d^{0.8} - 100)Pr^{0.4}$	$0.5 < Pr < 1.5$, $10^4 < Re_d < 5 \times 10^6$
	$Nu_d = 0.012(Re_d^{0.87} - 280)Pr^{0.4}$	$1.5 < Pr < 500$, $3000 < Re_d < 10^6$
Tube flow	$Nu_d = 0.027\ Re_d^{0.8} Pr^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14}$	Fully developed turbulent flow
Tube flow, entrance region	$Nu_d = 0.036\ Re_d^{0.8} Pr^{1/3} \left(\frac{d}{L}\right)^{0.055}$ See also Figures 6-5 and 6-6	Turbulent flow $10 < \frac{L}{d} < 400$
Tube flow	Petukov relation	Fully developed turbulent flow, $0.5 < Pr < 2000$, $10^4 < Re_d < 5 \times 10^6$, $0 < \frac{\mu_b}{\mu_w} < 40$
Tube flow	$Nu_d = 3.66 + \frac{0.0668(d/L)\ Re_d Pr}{1 + 0.04[(d/L)\ Re_d Pr]^{2/3}}$	Laminar, $T_w = \text{const.}$
Tube flow	$Nu_d = 1.86(Re_d Pr)^{1/3} \left(\frac{d}{L}\right)^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14}$	Fully developed laminar flow, $T_w = \text{const.}$ $Re_d Pr \frac{d}{L} > 10$
Rough tubes	$St_b Pr_f^{2/3} = \frac{f}{8}$	Fully developed turbulent flow
Noncircular ducts	Reynolds number evaluated on basis of hydraulic diameter $D_H = \frac{4A}{P}$ A = flow cross-section area, P = wetted perimeter	Same as particular equation for tube flow

Approximate magnitude of some heat-transfer coefficients

Mechanism	**h, W/m^2K**
Condensing steam	5500-30 000
Condensing organics	1000-3000
Boiling liquids	1500-30 000
Moving water	250-17 000
Moving hydrocarbons	50-1700
Still air	5-25
Moving air	10-55

7 HEAT EXCHANGERS

A heat exchanger is a device designed for exchanging heat between 2 fluids.

7.1 Classification of heat exchangers

Heat exchangers can be classified in a number of ways, depending on their construction or how the fluid move relative to each other through the device.

(1) By flow arrangement:
- parallel flow — the two fluid streams travel in the same direction along the exchanger,
- counterflow (or countercurrent flow) — the two fluid streams travel in the opposite direction,
- crossflow — the two fluid paths cross each other at right angles.

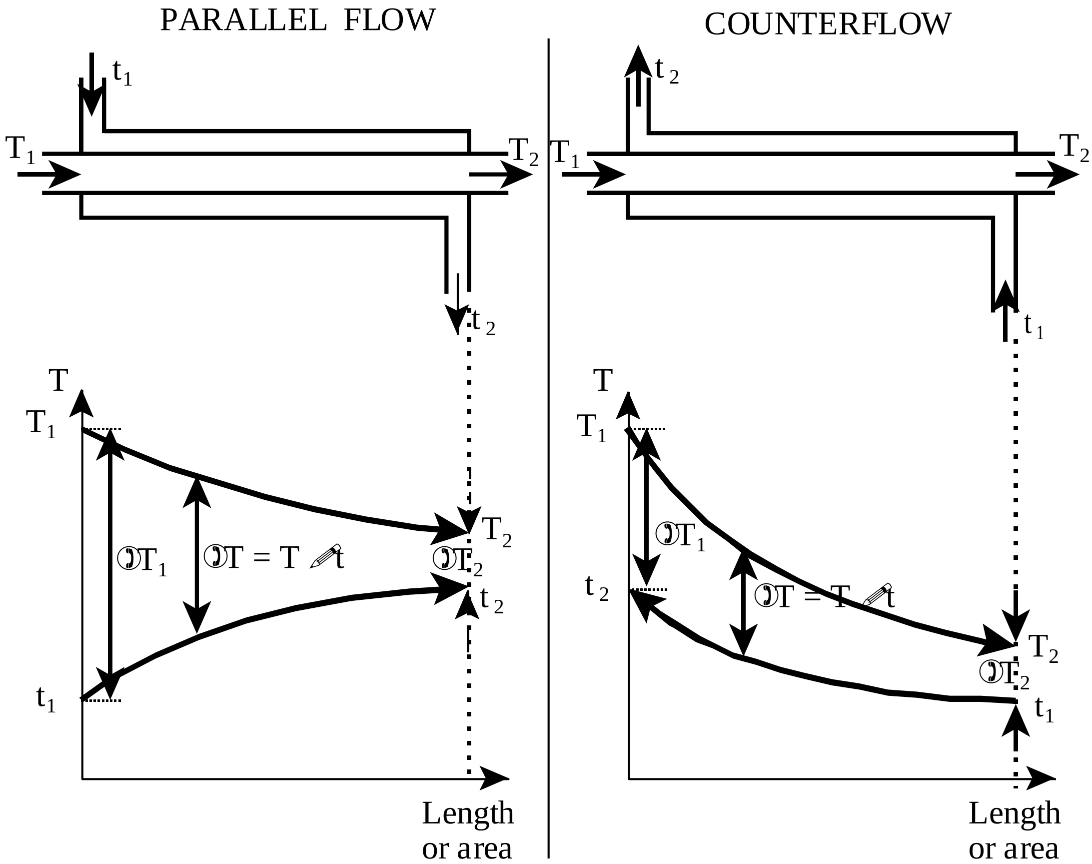
(2) By construction:
- double-pipe heat exchanger — It consists of two concentric pipes. One fluid flows through the inner pipe, the other flows through the annulus.
- shell-and-tube heat exchanger —It consists of a huge outer cylinder (shell) within which are contained many tubes (tube bundle). One fluid flows through the shell, the other flows through the tubes.
- others (pipecoils, plate-type heat exchanger, extended surface heat exchangers)

(3) By the number of passes:
- single pass
- multi pass

The **objective** is to predict:
- the amount of heat transferred,
- the outlet temperature of one or both fluid streams,
- the required heat transfer area, depending on what is known.

It is usually assumed, that all the heat lost by the hot fluid is transferred to the cold fluid (that is there are no heat losses).

7.2 Temperature distribution



$\Delta T_1 = T_1 - t_1$
$\Delta T_2 = T_2 - t_2$

$\Delta T_1 = T_1 - t_2$
$\Delta T_2 = T_2 - t_1$

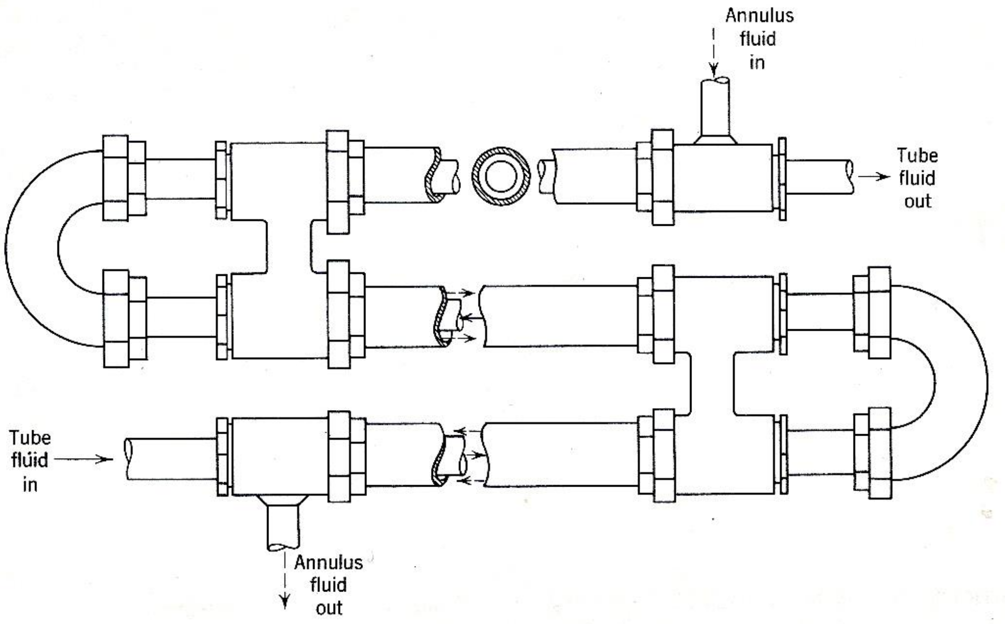
where ΔT_1 and ΔT_2 = terminal temperature differences
T = hot fluid
t = cold fluid
1 = inlet
2 = outlet

In counterflow arrangement t_2 (cold fluid outlet temperature) can be greater than T_2 (hot fluid outlet temperature). In parallel flow, however, the upper limit of t_2 is T_2 .

7.3 Double-pipe heat exchanger

One fluid flows through the center pipe while another flows in the annulus. Such an exchanger may consist of several passes arranged in a vertical stack. The two lengths of inner pipe are connected by a return bend, which is usually exposed and does not provide effective heat transfer surface. Two legs make a hairpin.

Usually common copper water tubing is used to construct the exchanger, although pipe and special tubing can also be used.



Effective lengths:
- 12 ft (3.65 m)
- 15 ft (4.57 m)
- 20 ft (6.1 m)

Standard sizes of double-pipe exchangers

Outer pipe	**Inner pipe**
2 in (50.8 mm)	1¼ in (31.75 mm)
2½ in (63.5 mm)	1¼ in (31.75 mm)
3 in (76.2 mm)	2 in (50.8 mm)
4 in (101.6 mm)	3 in (76.2 mm)

Application of double-pipe exchangers:
- It is useful, when not more, than 9-14 m^2 of heat transfer surface is required.
- It is preferred for small capacity, high pressure, and countercurrent operation.
- For low flow rates,
- If the heat duty (q) is less than 500 kW,
- If for the shell-and-tube heat exchanger the number of tubes are less than about 30 (¾ in = 19 mm OD tubes) and the diameter of the shell is less than 200 mm.

Under these conditions, a shell-and-tube exchanger is uneconomic; a large number of small diameter shells in series is required to provide adequate velocities and near-countercurrent flow.

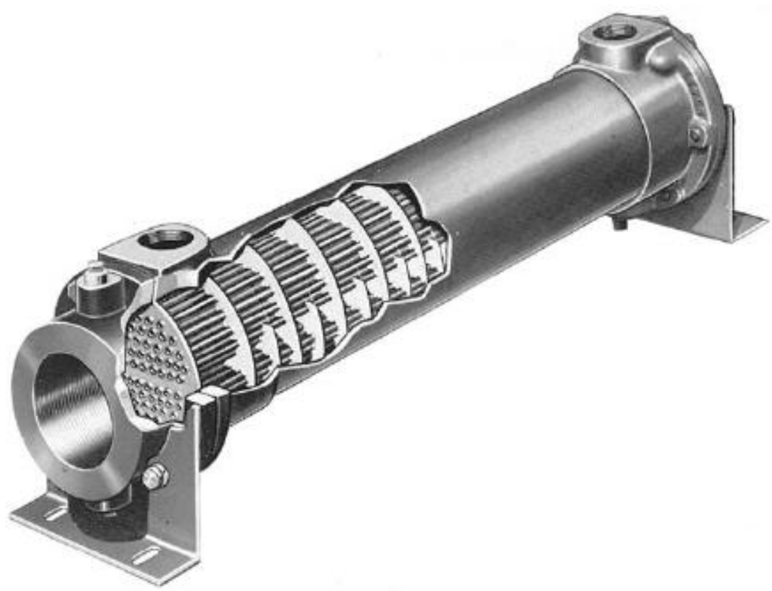
Disadvantages:
- Each double-pipe exchanger introduces no fewer than 14 points at which leakage might occur.
- Considerable time and expense required for dismantling and periodically cleaning it compared with other types of equipment.
- When a large number are connected, they require considerable space.

7.4 Shell-and-tube heat exchanger

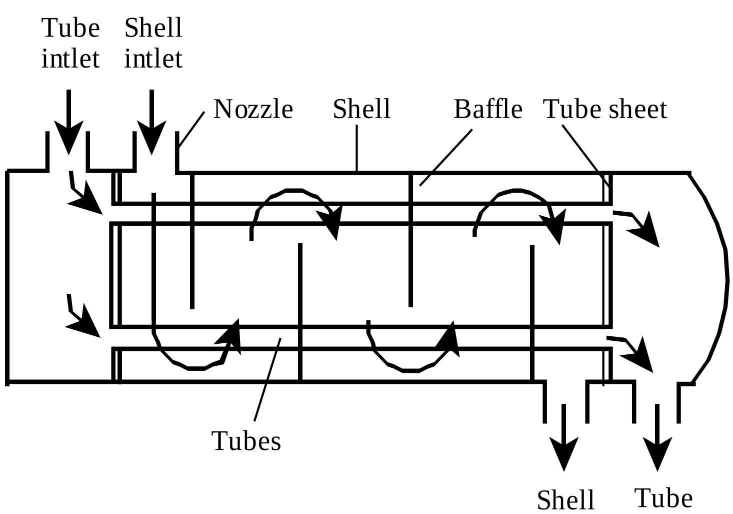
Where a high flow rate is involved, the number of double-pipe heat exchangers required becomes large (high floor space requirement, high capital cost), so an alternative apparatus, a shell-and-tube heat exchanger can be used. Shell-and-tube heat exchangers become the industry standard because of several advantages:

- high flow rates
- compactness (large ratio of heat transfer area to volume and weight)
- good design methods exist
- ease of construction in a wide range of sizes
- ease of maintenance
- high heat transfer coefficients
- can handle either high or low pressures and temperatures

The Tubular Exchanger Manufacturer's Association (TEMA), as an association of heat exchanger equipment fabricators, has developed a standard for basic construction.



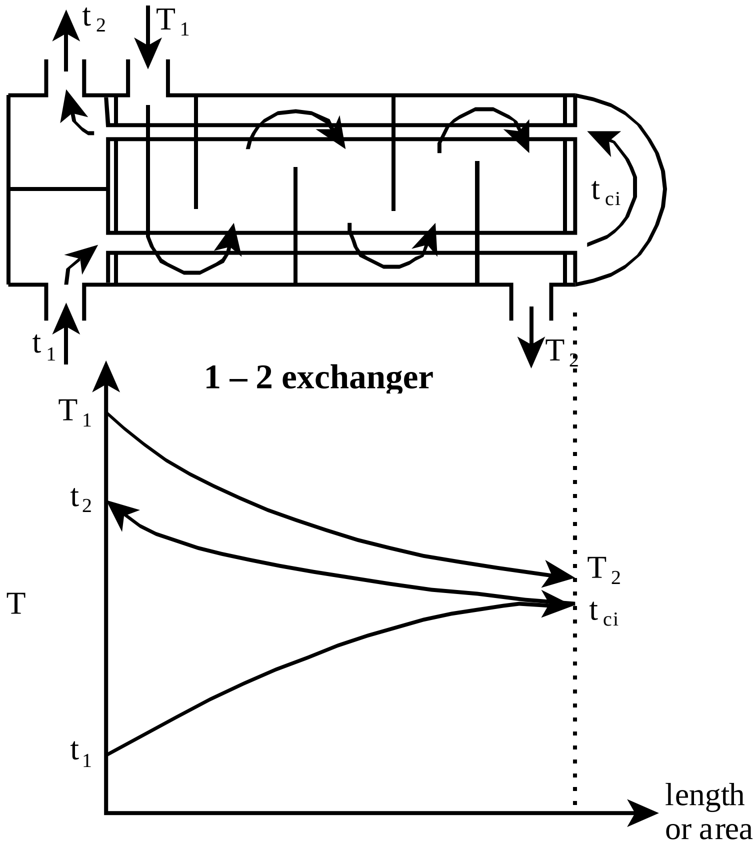
Shell-and-tube heat exchanger with 1 tube pass



Single pass 1 – 1 exchanger

The number of tubes in the tube bundle ranges from about 20 to over 1000. The tubes are attached to perforated flat plates, termed **tube sheets**, at each end. The tubes pass through a number of flat plates, called **baffles,** along their length, which serve to support them and direct the fluid flow in the shell in such a way that heat transfer is enhanced. The assembly of tubes and baffles, called the

tube bundle, is held together by a system of **tie rods**, and **spacer tubes**.



The simplest multi-pass shell-and-tube exchanger is the 1-2 type. This configuration has higher velocities and shorter tubes. The higher velocity increases the heat transfer coefficient.

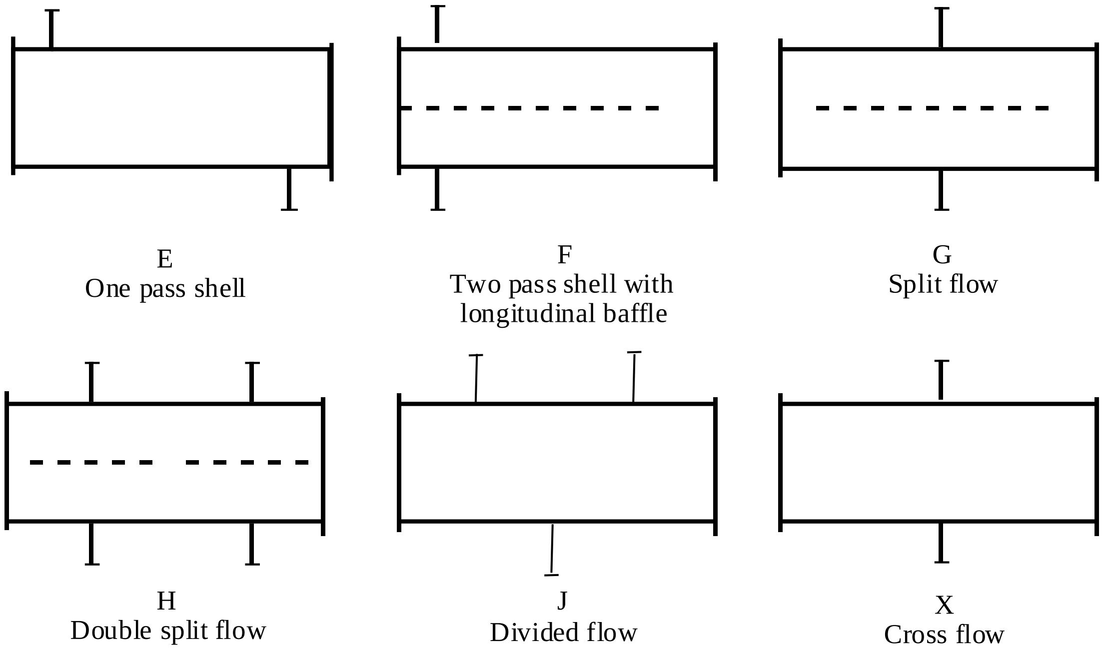
It has an important limitation: because of the parallel flow pass, the exchanger is unable to bring the exit temperature of the cold fluid very near to the entrance temperature of the hot fluid.

Further increasing the number of passes (e.g. 1-4, 1-6, 2-4, 2-6, 2-8 etc.), the fluid velocities could be further increased.

Shell

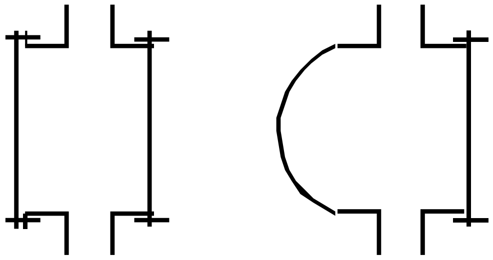
The type of shell often is established before thermal rating of the unit takes place.

The TEMA standard type of designated system:



Shell types

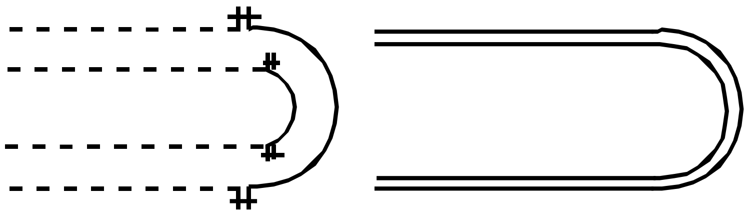
Shell inside diameters are standardized. Up to 23 in (60 cm) diameters are fixed. Sizes above 60 cm are specified to the nearest inch not exceeding 60 in (152.4 cm). Shells are constructed of rolled plates. Pressures are not exceeding 200 bar.



A
Channel and
removable cover

B
Bonnet
(integral cover)

Front and stationary head types



Floating head

U-tube bundle

Rear end head types

L	Fixed tube sheet like "A" stationary head
M	Fixed tube sheet like "B" stationary head
P	Outside packed floating head
S	Floating head with backing device
T	Pull through floating head
U	U-tube bundle
W	Externally sealed floating tube sheet

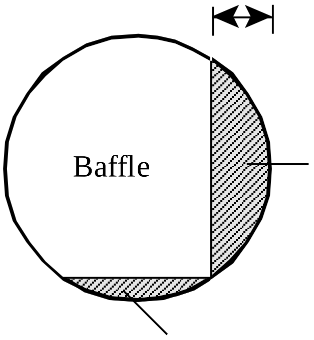
Advantages of fixed tube sheet exchanger:

- There are no internal joints, thus eliminating a potential source of leakage of one fluid into another
- The peripheral tubes may be placed close to the inside of the shell, due to the absence of an internal joint, so that more tubes can be accommodated in a shell of given diameter for the fixed tube sheet type than for any other type.
- In addition to the fact that there are no flanges on the shell side makes it desirable for high-pressure and/or lethal services.
- U-tube has no internal joints either, but there is a practical limit to the bend of the internal tubes; the number of tubes accommodated within a given shell diameter is slightly less, than that for the fixed tube sheet type.

Fixed tube sheet may be used with maximum 120°C temperature difference between the 2 fluids; floating head may be used above 180°C temperature difference.

Shell side velocities: 0.6 – 1.5 m/s for water and water-like materials.

Baffles

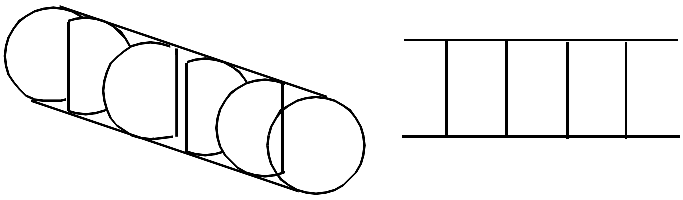


Baffles are very important part of the performance of a heat exchanger.

Shell side baffles
Shell-side baffles have their thermal significance in that the shell-side fluid is made to flow to and fro across the bundle from one end of the exchanger to the other. There are only a few popular and practical arrangements.

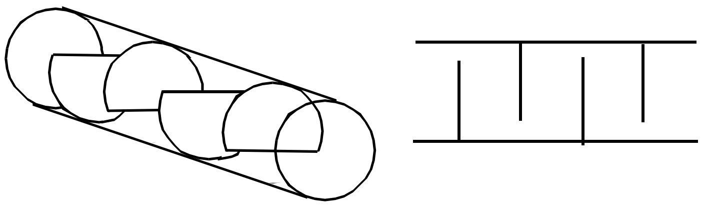
The cut-out portion represents the free flow area for the shell side fluid. It is usually 25% of the inside diameter of the shell.

Baffles can be arranged to direct the shell side fluid in "side-to-side" or "up-and-down" flow.



Standard segmental baffle
designed for side to side flow (vertival cut)

This type of baffle is used for:
- single phase clean operation,
- vaporizing, and
- single phase fouling operation,
- condensing.



Standard segmental baffle
designed for up and down flow (horizontal cut)

This type of baffle is used for:
- single phase clean operation (horizontal cut is more common), and
- vaporizing.

The baffles serve as tube support as well. The baffles are supported by 1 or more guide rods, which are fastened between the tube sheets.

Baffles in a small shell-and-tube heat exchanger, only 6 of the 37 tubes are in position.

Baffle thickness: it depends on the shell diameter and unsupported tube length, but it is usually between $^1/_8$ - $^3/_4$ in (3.2-19 mm).
Baffle clearance: the holes for the tubes are drilled 0.4-1 mm larger than the tube D_o outside diameter. The clearance between shell and baffles are usually 3-12 mm.
Baffle spacing: The baffle spacing varies between $^1/_5$ - 1 times the inside diameter of the shell. The distance between baffles should not be less than $^1/_5$ the diameter of the shell or 2 in (50.8 mm), whichever is the greater.

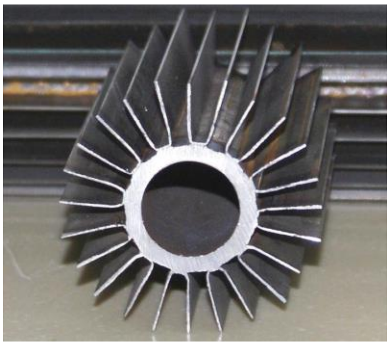
Tube side baffles
They are built into the head and return ends of an exchanger to direct the fluid through the tubes at the proper relative position in the bundle for good heat transfer. The arrangement may take any of several designs depending on the number of tube side passes required. The number of tubes per pass is usually arranged about equal.

Tubes
There are 2 basic types of tubes:
- plain
- finned (external or internal)

Plane tubes are used in the usual heat exchange applications. Plain tube mechanical data and dimensions are given in tables. The usual tube sizes for most exchangers are ¾ in and 1 in OD (19.05 and 25.4 mm). Tubes of 1¼ in and 1½ in OD and sometimes larger are used in boilers, evaporators, and reboilers. They are available in all common metals. Standard lengths of tubes for heat exchanger construction are 8, 12, 16, 20, and 24 ft (2.438, 3.658, 4.877, 6.096, and 7.325 m) for both straight and U-tubes, but other lengths might be used.

Tube side velocities: 0.9 – 2.4 m/s for water and similar liquids.

For a given heat exchange area the cheapest exchanger is one, which has a small shell diameter and a long tube length.

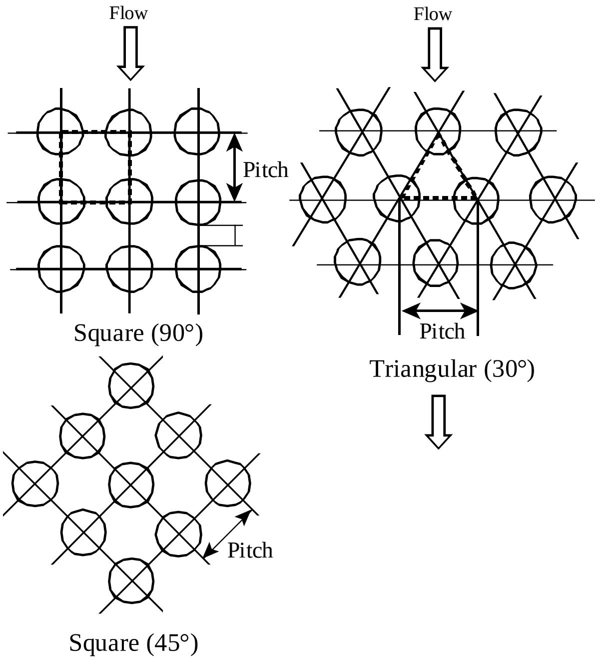


Finned tubes

Stegosaurus

Tube pitch

The tubes are arranged on triangular (most popular) or square orientations. The **tube pitch** gives the distance between the tube centers. Unless the shell side fluid tends to foul badly, triangular pitch is used, because about 15% more tube can be packed into a shell of given diameter and pitch than in square pitch.



Tubes in triangular pitch cannot be cleaned by running a brush between the rods because no space exists for cleaning lanes. So it is suitable for clean services or fouling services handled by chemical treatment (not mechanical).

Square pitch allows mechanical cleaning of the outside of the tubes. Also square pitch gives a lower shell-side pressure drop than triangular pitch.

The 90° square pitch is used in turbulent flow, the 45° square pitch is used in laminar flow.

Fluid allocation

In principle, either stream entering a shell-and tube heat exchanger may be put on either tube-side or shell-side. However, there are four considerations that influence the choice to result in the most economical exchanger.

(1) **High pressure:** If one of the streams is at high pressure, it should be put through the tubes. In this case, only the tube-side fittings needs to withstand the high pressure, the shell may be made of lighter metal.

(2) **Corrosion:** Since most corrosion resistant alloys are more expensive than the ordinary materials of construction, the corrosive fluid should be placed in the tubes.

(3) **Fouling:** Certain streams foul so badly that the entire design is dominated by features, which seek (a) to minimize fouling (e.g. high velocity, avoidance of dead or eddy flow regions) (b) to facilitate cleaning (fouling fluid on tube-side, wide pitch and rotated square layout if shell-side fluid is fouling) or (c) to extend operational life by multiple units.

(4) **Low heat transfer coefficient:** If one stream has an inherent low heat transfer coefficient (such as low pressure gases or viscous liquids), this stream should be put on shell-side so that extended surface may be used to reduce the total cost of the exchanger.

7.5 Energy balance

The first step in design is to set up material and energy balances over the heat exchanger. The exchange of heat between the 2 fluids is assumed to be complete (100%); that is the heat transfer to or from the ambient is usually neglected (negligible in comparison with the heat transfer in the exchanger).

$$q = \dot{m}\,(h_{out} - h_{in})$$

where

q = rate of heat transfer into stream; also called heat load or heat duty, W
h = enthalpy per unit mass of stream, J/kg
$\dot{m}$ = mass flow rate of stream, kg/s

Applying it for both the hot and cold streams:

$$\left.\begin{aligned} q &= \dot{m}_h\,(h_{h,out} - h_{h,in}) \\ q &= \dot{m}_c\,(h_{c,out} - h_{c,in}) \end{aligned}\right\} \qquad q_c = -\,q_h$$

$$\dot{m}_h\,(h_{h,out} - h_{h,in}) = \dot{m}_c\,(h_{c,out} - h_{c,in})$$

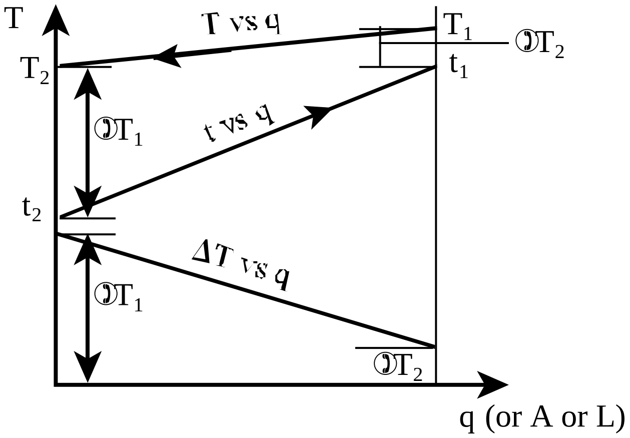
This is the overall enthalpy balance

If constant specific heats (c_p) are assumed:

$$q = \dot{m}_h\,c_{p,h}\,(T_{h,in} - T_{h,out}) = \dot{m}_c\,c_{p,c}\,(t_{c,in} - t_{c,out})$$

$$\boxed{q = \dot{m}_h\,c_{p,h}\,(T_1 - T_2) = \dot{m}_c\,c_{p,c}\,(t_2 - t_1)}$$

7.6 Logarithmic mean temperature difference (LMTD)



Let's consider counterflow.

T, t = average temperatures of hot and cold fluid respectively, °C

$\Delta T = T - t$ = overall local temperature difference, °C

It is reasonable to expect the heat flux to be proportional to a driving force (ΔT). The two fluid stream temperatures T and t vary with distance z within the exchanger. Therefore the driving force for heat transfer $\Delta T = T - t$ also varies throughout the exchanger. Consequently, it is necessary to start with a differential equation.

$dq = U\,\Delta T\,dA$ where $\Delta T = T - t$,°C
U = overall heat transfer coefficient, W/m^{2}K

To integrate this equation, we have to assume that:

- U is constant,
- $c_{p,hot}$ and $c_{p,cold}$ are constant,
- heat exchange with the ambient is negligible (that is there are no heat losses),
- the flow is steady.

Since T and t vary linearly with q (assumption 2 and 4), therefore ΔT does likewise, and $d(\Delta T)/dq$, the slope of the graph is constant. Therefore

$$\frac{d\,(\Delta T)}{dq} = \frac{\Delta T_2 - \Delta T_1}{q_T}$$

where q_T = rate of heat transfer in entire exchanger
ΔT_1, ΔT_2 = terminal temperature differences or approach

$$\frac{d\,(\Delta T)}{U\,\Delta T\,dA} = \frac{\Delta T_2 - \Delta T_1}{q_T}$$

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d\,(\Delta T)}{\Delta T} = \frac{U\,(\Delta T_2 - \Delta T_1)}{q_T} \int_0^{A_T} dA$$

$$\ln\frac{\Delta T_2}{\Delta T_1} = \frac{U\,(\Delta T_2 - \Delta T_1)}{q_T}\,A_T$$

$$q_T = U\,A_T\,\frac{\Delta T_2 - \Delta T_1}{\ln\frac{\Delta T_2}{\Delta T_1}} = U\,A_T\,\Delta T_{lm}$$

$$\boxed{\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln\frac{\Delta T_2}{\Delta T_1}}}$$ logarithmic mean temperature difference (LMTD)

Therefore, the heat transferred for the entire exchanger is: $\boxed{q = U\,A\,\Delta T_{lm}}$

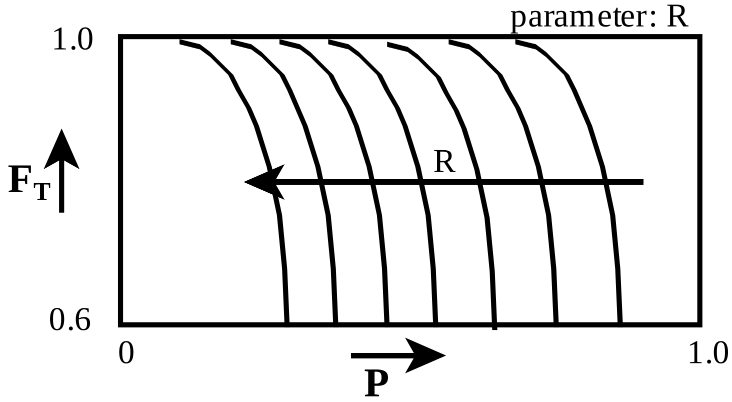
This last equation is true not only for counterflow but parallel flow as well.

7.7 Temperature correction factor F_T

In multipass shell-and-tube exchangers the flow pattern is complex (parallel, countercurrent and crossflow are all present). A 1-2 shell-and-tube exchanger has 1 parallel pass and 1 counterflow pass. Hence, ΔT_{lm} , which applies to either parallel or counterflow but not to a mixture of both types, cannot be used to calculate the mean temperature drop without a correction. It is customary to define a temperature correction factor F_T , which is so determined that when it is multiplied by the LMTD (ΔT_{lm}) for counterflow, the product is the true average temperature drop:

$$\boxed{\text{true } \Delta T = F_T\,\Delta T_{lm}}$$

In the literature correction factor (F_T) charts are available for different exchangers.



Each curved line in the chart corresponds to a constant value of the dimensionless ratio R:

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{C_c}{C_h} \qquad \text{heat capacity ratio}$$

The abscissas are values of the dimensionless ratio P defined as:

$$P = \frac{t_2 - t_1}{T_1 - t_1} \qquad \text{thermal effectiveness}$$

$t_2 - t_1$ = the actual temperature rise of the cold fluid
$T_1 - t_1$ = the maximum possible temperature rise if the warm end approach were zero ($T_1 - t_2 = 0$) based on counterflow

The temperature correction factor expresses the efficiency of an exchanger:

$$\eta = \frac{\text{temperature difference in the exchanger}}{\text{temperature difference for counterflow}} = F_T$$

In above definition the counterflow exchanger is assigned an efficiency of 100%.

Temperature correction values less than 0.75 are generally unacceptable because the exchanger configuration chosen is inefficient. To raise the F_T factor the exchanger configuration or temperature levels, or both must be changed.

7.8 Overall heat transfer coefficient

The heat transferred for the entire exchanger is:

$q = U\,A\,\Delta T_{lm}$ where ΔT_{lm} = LMTD, °C
A = heat transfer area. It is customary to use the outside surface area A_o of the inner tube (or tubes), m^2
U = overall heat transfer coefficient, $W/m^2\,K$

The resistances to heat transfer are:

$$R_{Ti} = \frac{1}{h_i\,A_i}, \qquad R_{To} = \frac{1}{h_o\,A_o}, \qquad R_W = \frac{\ln(R_o/R_i)}{2\pi k L} = \frac{\ln(D_o/D_i)}{2\pi k L}$$

where
R_{Ti} = film resistance for the inside tube-surface area
R_{To} = film resistance for the outside tube-surface area
R_W = tube-wall conduction resistance
R_o = outside radius of tube
R_i = inside radius of tube
h_i = film transfer coefficient based on the inside surface area $A_i = \pi D_i L$
h_o = film transfer coefficient based on the outside surface area $A_o = \pi D_o L$

It is necessary to specify the area. If A is taken as the outside tube area A_o, U becomes a coefficient based on that area (U_o). If the inside area A_i is chosen, the coefficient is also based on that area (U_i).

Note:
$$U_o = \frac{1}{\dfrac{1}{h_o} + \dfrac{D_o \ln(D_o/D_i)}{2k} + \dfrac{1}{h_i}\dfrac{D_o}{D_i}}$$

When not stated, the heat transfer area in conventional shell-and-tube heat exchangers is considered as A_o outside tube surface area.

Typical values of overall heat-transfer coefficients in shell-and-tube heat exchangers

	U, W/m²K
Water to water	850-1700
Water to brine	570-1150
Water to condensing steam	570-1150
Water to petrol or kerosene	300-1000
Water to oil, gas oil or vegetable oil	100-350
Gas oil to gas oil	110-285
Steam to boiling water	1450-2250
Light organics to light organics	230-425
Heavy organics to heavy organics	55-230
Feedwater heaters	1000-8500
Steam to light fuel oil	200-400
Steam to heavy fuel oil	50-200
Gas to gas	10-40
Steam condenser	1000-6000

7.9 Fouling factors

In actual service, heat-transfer surfaces do not remain clean. Scale, dirt and other solids deposit on one or both sides of the tubes provide additional resistances to heat flow and reduce the overall heat transfer coefficient. In petroleum processes coke and other substances can deposit. Corrosion products may form on the surfaces. Biological growth (e.g. algae) can occur with cooling water and in the biological industries. Particles held in suspension in the flow stream will deposit on the surface in low velocity regions. Chemical reaction in the flow may take place on the heat transfer surface producing an adhering solid product of reaction. Dissolved materials can crystallize out under certain circumstances from the fluid on the warmer surfaces.

To avoid or reduce these fouling resistances chemical inhibitors are often added to minimize corrosion, salt deposition, and algae growth. Water velocities above 1 m/s are generally used to reduce fouling. Large temperature differences should also be avoided to reduce salt deposition on surfaces.

The effect of fouling is taken care of in design by adding the fouling resistances R_{di} and R_{do} to the total resistance. As a rule, they are applied without correcting for inside diameter to outside diameter, since they are not known to any great degree of accuracy. So the overall heat transfer coefficient becomes:

$$\boxed{U_o = \frac{1}{\dfrac{1}{h_o} + R_{do} + \dfrac{D_o \ln(D_o / D_i)}{2k} + R_{di} + \dfrac{1}{h_i}\dfrac{D_o}{D_i}}}$$

R_{di} = fouling resistance or fouling factor for the inside tube surface, $m^2 K/W$
R_{do} = fouling resistance or fouling factor for the outside tube surface, $m^2 K/W$

The fouling resistances are usually measured for various combinations of fluids and metals. Typical fouling factors recommended for use in designing heat exchangers are available in many references.

Cross section of fouled tubes

Cost of fouling

The cost of fouling is estimated to be above $6 billion/year for the petroleum refining industry alone. It is made up of different cost elements:

- Additional capital cost: additional surface area is needed to compensate for reduction in heat transfer due to fouling. An increase in exchanger size increases the installation cost as well. A larger exchanger has increased pressure loss, therefore larger pumps and motors are needed.
- Energy cost: increased pumping power is needed to meet increased pressure loss.
- Maintenance cost: Dismantling, cleaning, and reassembling, cost of chemicals and additives to reduce fouling.
- Shutdown cost: is the value of lost production when the exchanger is shut down for cleaning.

Control of fouling

Once the combination of mechanisms of fouling problem is recognized, fouling rate may be substantially reduced. Corrosion fouling can be reduced by selecting corrosion resistant materials for the equipment. Biofouling can be reduced by the use of Cu alloys and/or chemical additives to the fluid stream to control the growth and reproduction of the organisms. Particulate fouling can be reduced by ensuring sufficient flow velocities (e.g. tubeside water velocity should be minimum 0.9-1.0 m/s) and reducing stagnant areas.

Approximate fouling factors for heat exchanger tubes

Material	Fouling factor R_d, m^2K/W
Distilled water	0.000 1
Sea water	0.000 2 - 0.000 4
Treated boiler feed water	0.000 1 - 0.000 2
Clean river or lake water	0.000 2 - 0.000 6
Steam (oil free)	0.000 1
Steam (oil-bearing)	0.000 3
Organic vapours	0.000 1
Engine oil	0.000 2
Most industrial liquids	0.000 2
Most refinery liquids	0.000 2 - 0.000 9
Gases	0.000 2 - 0.000 4
Vaporizing liquids	0.000 35
Air	0.000 4
Vegetable oils and gas oils	0.000 5
Flue gases	0.001 - 0.002
Diesel engine exhaust	0.002

Removal of fouling deposits

Chemical removal of fouling can be achieved in some cases by weak acid, special solvent, and so on. Other deposits adhere weakly and can be washed off by periodic operation at very high velocities or by flushing with a high-velocity steam or water jet or using a sand-water slurry. These methods may be applied to both the shell side and the tube side without pulling the bundle. Many fouling deposits however must be removed by mechanical cleaning such as rodding, brushing, or scraping the surface.

Fluid allocation

- Fixed tube sheet: The inside of the tubes may be cleaned mechanically after removing the channel covers or complete channels, but because the tube bundle cannot be removed, cleaning of the outside of the tubes can only be achieved by chemical means. Therefore, this type is limited to applications where the shell-side fluid is virtually non-fouling; fouling fluids must be routed through the tubes.
- U-tube: The tube bundle can be withdrawn, therefore the outside of the tubes can be cleaned mechanically, but the inside of the tubes is usually cleaned by chemical methods.
- Corrosive fluids must always be routed through the tubes.
- The higher temperature fluid should be inside the tubes to minimize heat loss.
- If the lower flow rate fluid is allocated to the shell side, a better heat transfer performance is achieved.
- Higher heat transfer rates are ordinarily obtained by placing a viscous fluid on the shell-side.

7.10 Viscosity correction factor ϕ_v

The properties of fluid (especially viscosity) are temperature dependent. Since a temperature field (in which the temperature varies from point to point) exist in a flowing stream undergoing heat transfer, a problem appears in the choice of temperature at which the properties should be evaluated.

One of the methods is then to evaluate the properties at the average fluid temperature and then a correction factor (Φ_v) is specified.
The heat transfer coefficient h is given by

$h = h'\Phi_v$ where h' = heat transfer coefficient calculated for $\Phi_v = 1$

$$\Phi_v = \left(\frac{\mu}{\mu_w}\right)^n = \text{viscosity correction factor}$$

Liquids being heated $\mu_w < \mu \Rightarrow \Phi_v > 1$
being cooled $\mu_w > \mu \Rightarrow \Phi_v < 1$
For gases the inequalities are reversed.

n = 0.14 usually
μ = dynamic viscosity at mean fluid temperature, Pa · s
μ_w = dynamic viscosity at the wall temperature, Pa · s

Φ_v cannot be calculated directly because it is related to the T_W surface temperature, which in turn depends on h heat transfer coefficient. To avoid tedious trial and error solution, the surface temperature may be calculated directly from the following equation:

$$T_W = T_{avg} - U_o\,(T_{avg} - t_{avg})\sum R_T$$

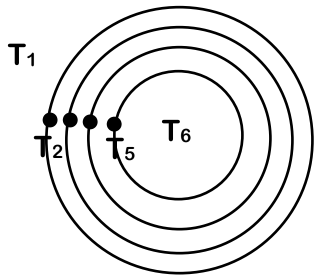
where $\sum R_T$ = sum of appropriate thermal resistances

This T_W temperature value is used to calculate Φ_v viscosity correction factor and the corrected heat transfer coefficient h.

Trial-and-error calculation

Assume T_W, get μ_w at T_W, then substitute into the heat balance equation:

$$h_o\left(\frac{\mu}{\mu_w}\right)^n (T_5 - T_6) = \frac{T_1 - T_5}{\sum R_{1-5}}$$



or directly from $T_W = T_1 - U\,\Delta T\,\Sigma R_{1-W}$

If it does not balance, assume a new T_W. When there is a significant viscosity gradient, Φ_v viscosity correction factor is no longer unity.

7.11 Heat transfer correlations for the shell-side

The mechanism of heat flow in forced convection outside tubes differs from that of flow inside tubes, because of differences in fluid flow mechanism.

Inside tube: No form drag exists except perhaps for a short distance at the entrance end, and all friction is **wall friction**. Therefore, there is no variation in the local heat transfer coefficient at different points in a given circumference and a close analogy exists between friction and heat transfer. Also, a sharp distinction exists between laminar and turbulent flow, which calls for different treatment of heat-transfer relations for the 2 flow regime.

Outside tube: In flow of fluids across a cylindrical shape, boundary-layer separation occurs, and a wake develops that causes **form friction**. No sharp distinction is found between laminar and turbulent flow, and a common correlation can be used for both low and high Re numbers. Also, the local heat transfer coefficient varies from point to point around the circumference. In practice, the variation in the local heat transfer coefficient is often of no importance, and average values based on the entire circumference are used.

The variables affecting the heat transfer coefficient h are the same that in case of heat flow to fluids inside tubes: D_o, c_p, μ, k, and G (mass velocity of the fluid approaching the tube).

There are many correlations and nomograms for shell-side heat transfer coefficient calculation in literature.

8 CONDENSATION HEAT TRANSFER

Condensation is a convection process associated with a phase change of fluid. Condensation occurs when a saturated vapour comes into contact with a solid surface whose surface temperature T_W is below the saturation temperature T_{sat}, to form a liquid. Processes of heat transfer accompanied by phase change are more complex than simple heat exchange between fluids.

The condensing vapour may consist of:
– a single substance (e.g. steam),
– a mixture of condensable and non-condensable substances (e.g. steam and air),
– a mixture of 2 or more condensable vapours.

The condensing temperature of a **single pure substance** depends only on the pressure; therefore, the process is isothermal. **Mixed vapours** condensing at constant pressure condense over a temperature range and yield a condensate of variable composition until the entire vapour stream is condensed.

There are 2 types of condensation:
– dropwise condensation,
– film-type condensation.

Dropwise condensation

The condensate begins to form at microscopic nucleation sites (e.g. tiny pits, scratches, dust specks, ...). The drops grow and coalesce with their neighbours to form visible fine drops. The fine drops, in turn, coalesce into small streams, which flow down the surface under the force of gravity, sweep away condensate, and clear the surface for more droplets. During dropwise condensation, large portion of the surface area is covered with an extremely thin film of liquid of negligible thermal resistance; consequently, the heat transfer coefficient at these bare areas is very high.

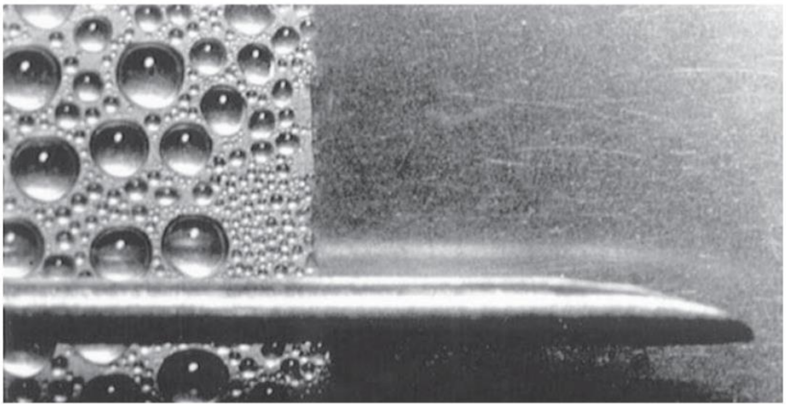
The average coefficient for dropwise condensation may be 5-10 times that for film-type condensation and can be as high as 110 000 W/m^2K.

Dropwise condensation occurs when the condensate does not wet the surface (e.g. ethylene glycol, glycerine, nitrobenzene, isoheptane, steam, etc.). This type of condensation is so unstable and is so difficult to maintain (because the surfaces become wetted after prolonged exposure to a condensing vapour) that the method is not common.

For normal design purposes, film-type condensation is assumed.

Film-type condensation

When the condensate wets the surface, it forms a continuous liquid film through which heat must be transferred. The liquid film flows over the surface by the action of gravity. It is this film of liquid between the surface and the vapour that forms the main resistance to heat transfer. The thickness of the film increases rapidly in the firs few cm and then more and more slowly.



(a) (b)

Condensation on a vertical surface (a) Dropwise (b) Film

Condensing film coefficients are much greater than those in forced convection and are of the order of magnitude of several thousand of $W/m^2 K$.

The film may flow in the laminar or in the turbulent regime, depending upon:

- the rate of condensation,
- fluid properties,
- length of path of the growing condensate film, and
- geometry.

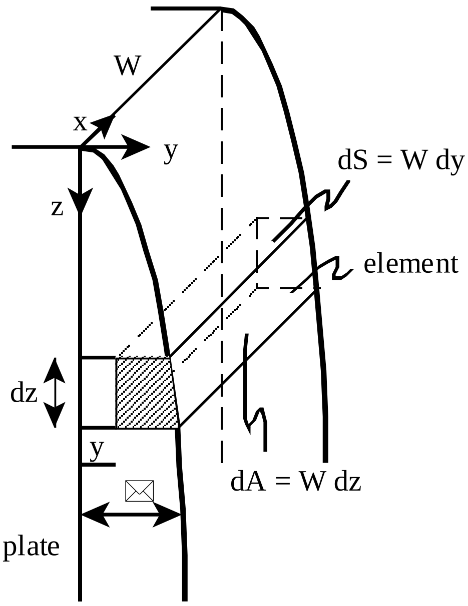
The usual design for condensers employs tubes, with the condensate formed inside or outside of the tubes. **Pure vapours** are usually condensed on the **outside** of tubes (**shell side**).

The same basic design equations used for heat exchangers are valid, but the nature of the coefficient correlations is somewhat different, depending upon whether the condensation process takes place on vertical or horizontal tubes. Friction losses in a condenser are normally small, so that condensation is a constant pressure process.

The basic equations for the rate of heat transfer in film-type condensation were first developed by Nusselt (1916).

8.1 Vertical surfaces — Laminar flow

On a vertical flat surface of width W a vapour is condensing in a film-type manner. The film is 2-dimensional and has a thickness of δ at any z location [$\delta = \delta(z)$].



Nusselt's assumptions:

(1) Pure vapor is at its T_{sat} saturation temperature.
(2) The condensate film flows in laminar flow.
(3) Heat is transferred through the film solely by conduction ($\Rightarrow h_z = k/\delta$).
(4) The temperature distribution through the film is linear.
(5) The temperature of the condensing surface T_W is constant.
(6) The physical properties of the condensate are constant and evaluated at a mean film temperature T_f :

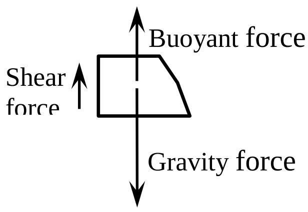
$$T_f = \frac{T_{sat} + T_W}{2}$$

(7) Negligible vapour shear exists at the interface.

The velocity distribution in the falling film

Force balance:

gravity force = buoyancy + friction force



$$\rho g(\delta - y) W dz = \rho_v g(\delta - y) W dz + \tau_{yz} W dz$$

$(\delta - y)$ Wdz = volume of the element (control volume)
ρ = liquid density
ρ_v = vapour density
$\tau_{yz} = \mu \, (dv_z/dy)$ (in $-z$ direction, laminar flow)

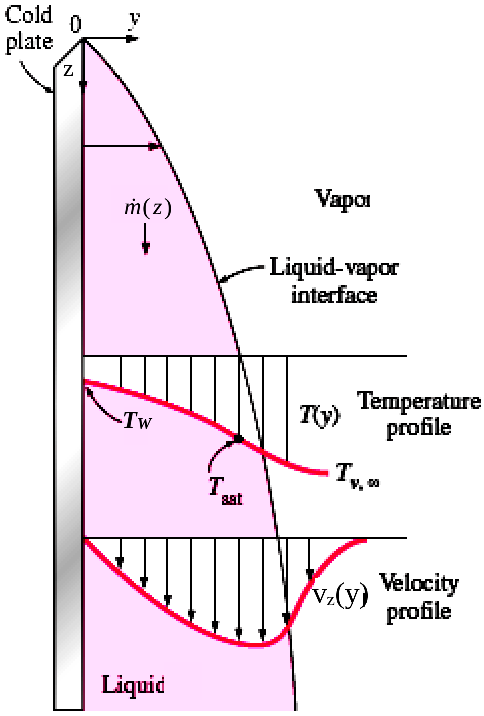
$$(\rho - \rho_v) g \, (\delta - y) = \mu \frac{dv_z}{dy}$$

integrating

$$v_z = \frac{g}{\mu}(\rho - \rho_v)\left(\delta y - \frac{y^2}{2}\right) + C_1$$

B.C.: at $y = 0$ $v_z = 0$ $\Rightarrow$ $C_1 = 0$

$$v_z = \frac{g}{\mu}(\rho-\rho_v)\,\delta^2\left(\frac{y}{\delta} - \frac{1}{2}\frac{y^2}{\delta^2}\right) \qquad \text{velocity profile}$$



Mass flow rate of condensate

The differential mass flow rate through the element is $\rho v_z\,dS$. (dS=Wdy or dS=dydx)

The total mass flow rate at any point z is:

$$\dot{m} = \iint \rho v_z dS = \int_0^W\!\!\int_0^\delta \rho v_z dy\,dx = W\int_0^\delta \rho\,\frac{g}{\mu}(\rho-\rho_v)\left(\delta y - \frac{1}{2}y^2\right)dy =$$

$$= \frac{\rho(\rho-\rho_v)g\,W}{\mu}\left(\delta\frac{y^2}{2} - \frac{y^3}{6}\right)\Bigg|_0^\delta = \frac{W\rho(\rho-\rho_v)g\,\delta^3}{3\mu}$$

The mass flow rate is traditionally expressed as mass flow rate per unit width of fall:

$$\boxed{\Gamma \equiv \frac{\dot{m}}{W}}$$ **liquid loading** or condensate loading, kg/ms

$$\frac{\dot{m}}{W} = \Gamma = \frac{\delta^3\rho(\rho-\rho_v)g}{3\mu} \quad\Rightarrow\quad \delta = \left(\frac{3\mu\Gamma}{\rho^2 g}\right)^{1/3} \qquad \text{for } \rho >> \rho_v$$

and $$\underline{\underline{\Gamma}} = \frac{\dot{m}}{W} = \frac{m/t}{W} = \frac{\rho\,(W\,L\,\delta)}{W\,t} = \underline{\underline{\rho\,\delta\,v_{avg}}}$$

Rate of heat transfer

The heat flow at the wall is: $$q_y = -k\,dA\,\frac{\partial T}{\partial y}\bigg|_{y=0}$$

Assuming a linear temperature profile: $$\frac{\partial T}{\partial y} = \frac{\Delta T}{\delta} \Rightarrow q_y = -k\,W\,dz\,\frac{T_W - T_{sat}}{\delta}$$

In the same dz distance, the increase in mass flow rate from condensation is $d\dot{m}$, or dΓ for unit width:

$$\frac{d\dot{m}}{W} = d\Gamma = d\left[\frac{\delta^3\rho(\rho-\rho_v)g}{3\mu}\right] = \frac{\cancel{3}\delta^2\rho\,(\rho-\rho_v)g}{\cancel{3}\mu}\,d\delta$$

Making a heat balance for dz distance:

heat added by condensation = heat removed at the wall by conduction

$$d\dot{m}\,\Delta H_v = k\,W dz\,\frac{T_{sat} - T_W}{\delta}$$

the **temperature drop across the condensate film**:

$$\boxed{\Delta T_o = T_{sat} - T_W}$$

ΔH_v = latent heat of vaporization, kJ/kg

$$\frac{d\dot{m}}{W}\,\Delta H_v = k\,dz\,\frac{\Delta T_o}{\delta}$$

substituting for $d\dot{m}/W$

$$\frac{\delta^2 g\,(\rho-\rho_v)g\,\Delta H_v}{\mu}\,d\delta = k\,\frac{\Delta T_o}{\delta}\,dz$$

$$\delta^3 d\delta = \frac{k\,\mu\,\Delta T_o}{\rho(\rho-\rho_v)g\,\Delta H_v}\,dz$$

integrating

$$\frac{\delta^4}{4} = \frac{k\,\mu\,\Delta T_o}{\rho(\rho-\rho_v)g\,\Delta H_v}\,z + C_2$$

$\Big\downarrow$ B.C.: at $z = 0$ $\delta = 0 \Rightarrow C_2 = 0$

$$\delta(z) = \left(\frac{4k\,\mu\,\Delta T_o}{\rho(\rho - \rho_v)g\,\Delta H_v}\, z \right)^{1/4} \qquad \Rightarrow \quad \delta \sim z^{1/4}$$

At the wall, using the local heat transfer coefficient h_z, a heat balance for the element:

heat flow through the wall by conduction = heat taken away by convection

$$h_z\, dA\Delta T_o = k\, dA\Delta T_o / \delta$$

$$\Downarrow$$

$$\boxed{h_z = \frac{k}{\delta}}$$

$$h_z = \frac{k}{\delta} = k\left(\frac{\rho(\rho - \rho_v)g\,\Delta H_v}{4k\,\mu\,\Delta T_o\, z} \right)^{1/4}$$

By integrating the local heat transfer coefficient h_z over the entire surface we obtain the average coefficient $\bar{h}_z$.

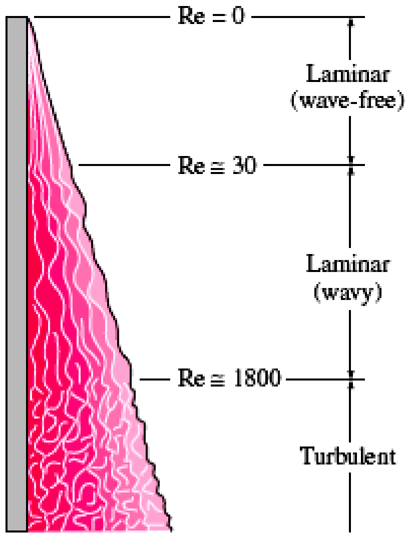
$$\bar{h} = \frac{1}{LW}\int_0^W\!\!\int_0^L h_z\, dz\, dx = \frac{1}{L}\left(\frac{k^3\rho(\rho - \rho_v)g\,\Delta H_v}{4\,\mu\,\Delta T_o} \right)^{1/4} \int_0^L \left(\frac{1}{z} \right)^{1/4} dz$$

$$\bar{h} = \frac{4}{3}\left(\frac{k^3\rho(\rho - \rho_v)g\,\Delta H_v}{\mu\,\Delta T_o\, L} \right)^{1/4} = \frac{4}{3}\, h_z\big|_L = 0.943\left(\frac{k^3\rho(\rho - \rho_v)g\,\Delta H_v}{\mu\,\Delta T_o\, L} \right)^{1/4}$$

$$\Rightarrow \qquad \bar{h} = 0.943\left(\frac{k^3\rho(\rho - \rho_v)g\,\Delta H_v}{\mu\,\Delta T_o\, L} \right)^{1/4} \quad \text{Nusselt's equation for vertical surfaces}$$

Although Nusselt's analysis was carried out for a vertical plane surface, the result is applicable to condensation ***inside*** or ***outside vertical tubes***, because the condensate film thickness δ is typically 2 or 3 order of magnitude smaller than the typical tube diameters.

Nusselt's equation gives lower $\bar{h}$ values than the experimental values. This implies that the assumption of laminar flow is not quite right. For laminar flow $Re_\delta < 1800$. At values as low as $Re_\delta = 30 - 40$, ripples begin to appear in the film surface because of vapour shear.



Flow regimes during film condensation on a vertical plate

Because of these turbulent effects and the effect of vapour shear McAdams recommended 20% increase of the $\bar{h}$ value (0.943x1.2=1.13). Calculation of $\bar{h}$ from this equation is direct, if L is known.

McAdams further recommended a different formula for mean film temperature:

$$\underline{\underline{T_f}} = T_{sat} - \frac{3\,(T_{sat} - T_W)}{4} = \underline{\underline{T_{sat} - \frac{3}{4}\,\Delta T_o}}$$

When L is known:

$$\boxed{\Gamma \equiv \frac{\dot{m}}{P} = \frac{\dot{m}}{\pi D}}$$

where P = wetted perimeter or drainage premier
P = πD for vertical pipe

and $q = \dot{m}\,\Delta H_v = \bar{h}\,\pi D L\,\Delta T_o \quad \Rightarrow \quad \frac{\Delta H_v}{L\,\Delta T_o} = \frac{\bar{h}\,\pi D}{\dot{m}} = \frac{\bar{h}}{\Gamma}$

$$\underline{\underline{\bar{h} = 1.13\left(\frac{k^3 \rho(\rho - \rho_v) g\,\Delta H_v}{\mu\,\Delta T_o\,L}\right)^{1/4} = 1.18\left(\frac{k^3 \rho^2 g}{\mu \Gamma}\right)^{1/3}}}$$

McAdams equation laminar flow vertical tube

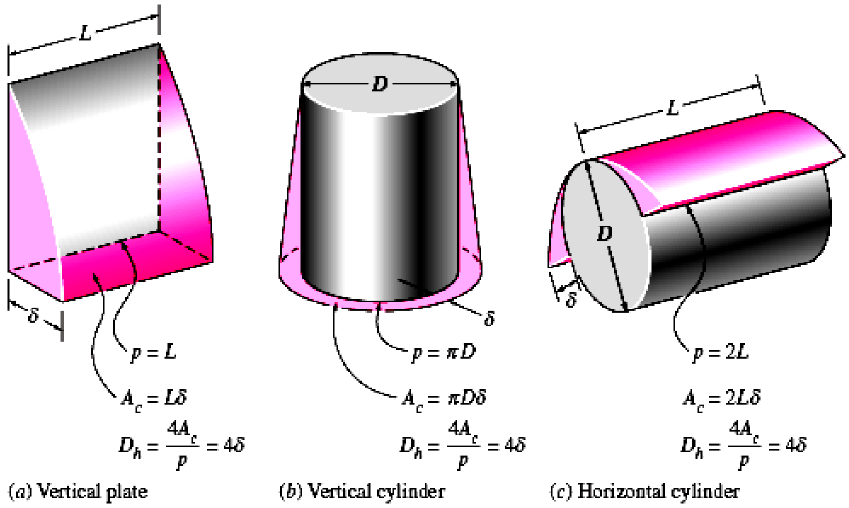
For a single tube: $\Gamma = \dfrac{\dot{m}}{\pi D}$, for a tube bundle: $\Gamma = \dfrac{\dot{m}/N_t}{\pi D}$

Either part of the equation may be used, depending upon the information at hand.

The criterion for determining whether the film is laminar or turbulent is the Reynolds number:

$$\boxed{Re = \frac{\rho v D_e}{\mu} = \frac{\rho v\,4\delta}{\mu} = \frac{4\Gamma}{\mu}}$$

for laminar flow $Re < 1800$
for turbulent flow $Re > 1800$



The wetted perimeter (P), the condensation cross-sectional area (A_c), and the hydraulic diameter or equivalent diameter (D_h) for some common geometries

8.2 Vertical surfaces — Turbulent flow

From experimental results developed by Kirkbride (1934):

$$\bar{h} = 0.0077\left(\frac{k^3\rho^2 g}{\mu^2}\right)^{1/3}\left(\frac{4\Gamma}{\mu}\right)^{0.4}$$

turbulent flow (Re > 1800)
vertical surface

Mixtures of vapours and non-condensing gases are usually cooled and condensed **inside vertical tubes**, so that the inert gas is continually swept away from the heat transfer surface by the incoming stream.

8.3 Horizontal surfaces

Outside tubes

The condensate film begins growing from the top of the tube with zero thickness and increases in thickness as it flows around to the bottom and then drips off.

Nusselt's equation for horizontal surfaces and single horizontal tube:

$$\bar{h} = 0.725\left(\frac{k^3\rho(\rho-\rho_v)g\,\Delta H_v}{\mu\,\Delta T_o\,D_o}\right)^{1/4} = 0.954\left(\frac{k^3\rho^2 g}{\mu\Gamma}\right)^{1/3}$$

outside
horizontal surface

where $\boxed{\Gamma = \frac{\dot{m}}{L}}$

Horizontal tube bundle

The liquid condensate from the top tube drips onto the one below therefore, the above equation for horizontal surface is slightly modified:

$$\bar{h} \;=\; 0.725\left(\frac{k^3\rho^2\, g\;\Delta H_v}{\mu\,\Delta T_o\; D_o\, N}\right)^{1/4}$$

where N = number of tubes in a vertical row (stack) [Geankoplis; Bejan]

N_t = total number of tubes in the tube bank [Janna]

$N^{2/3}$ = ⅔ power is because the condensate splashes away from each individual tube as it falls cumulatively from tube to tube instead of dripping completely to the tube below [McCabe; Kern]

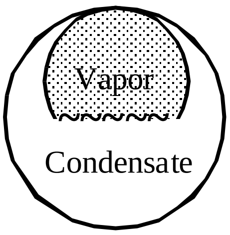
On horizontal tubes, the flow of condensate film is always laminar because of the short path length.

In general, the coefficient of a film condensing on a horizontal tube is considerably larger than on a vertical tube under otherwise similar conditions unless the tubes are very short or there are very many horizontal tubes in each stack.

$$\boxed{\bar{h}_{horizontal} \;>\; \bar{h}_{vertical}}$$

Inside tubes

During the condensation process, a mixture of vapour and liquid travels through the tube. At high flow rates (turbulent flow) the liquid (in the form of droplets) and the vapour travel together as a uniform mixture in the core of the tube, with a liquid film condensing along the tube surface.



For low flow rates, however, vapour condenses on the tube surface and collects in the lower portion of the tube under the action of gravity.

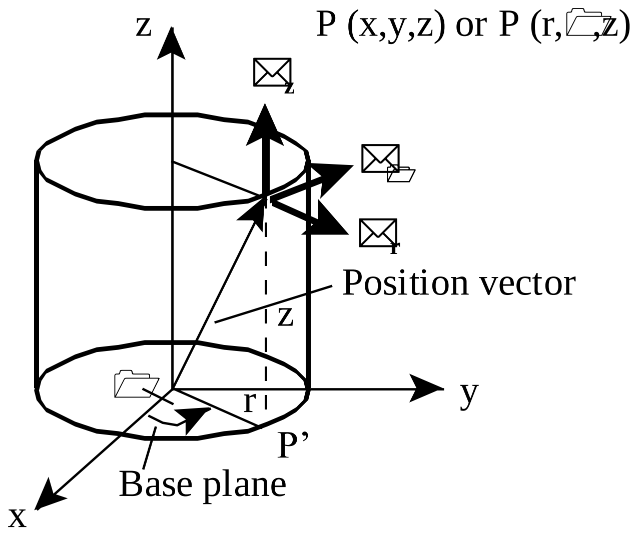
For low vapour velocities $\mathrm{Re} \;=\; \dfrac{\rho_v\, v_v\, D_i}{\mu_v} \;<\; 35\,000$ where Re is evaluated at inlet conditions to the tube.

Chato equation:

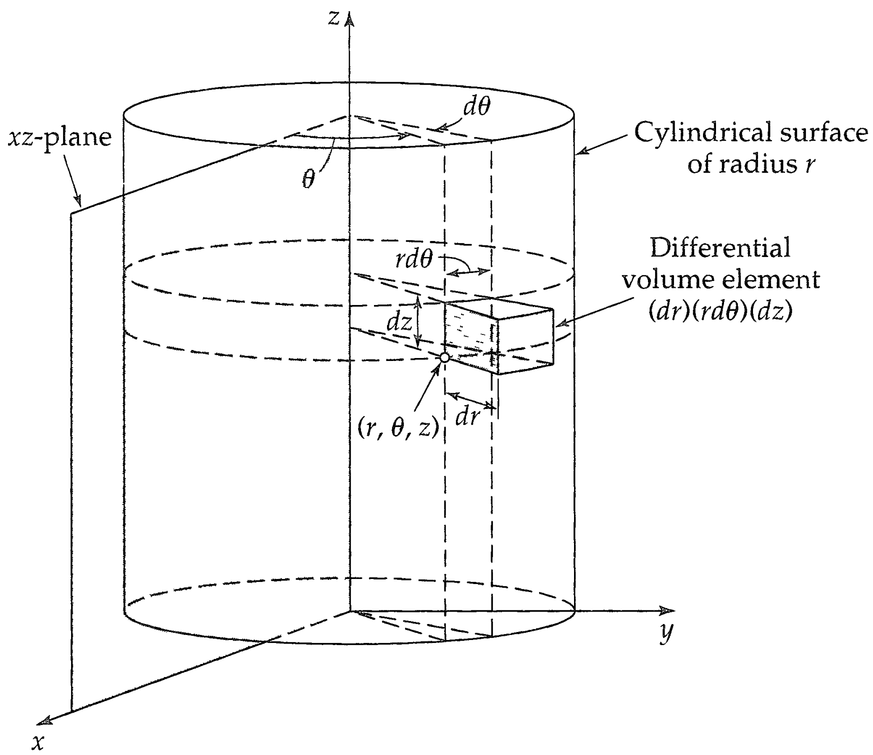
$$\bar{h} \;=\; 0.555\left(\frac{k^3\rho^2\, g\;\Delta H_v'}{\mu\,\Delta T_o\; D_i}\right)^{1/4} \qquad \text{where} \qquad \Delta H_v' = \Delta H_v \;+\; \frac{3}{8}c_p\;\Delta T_o$$

APPENDIX

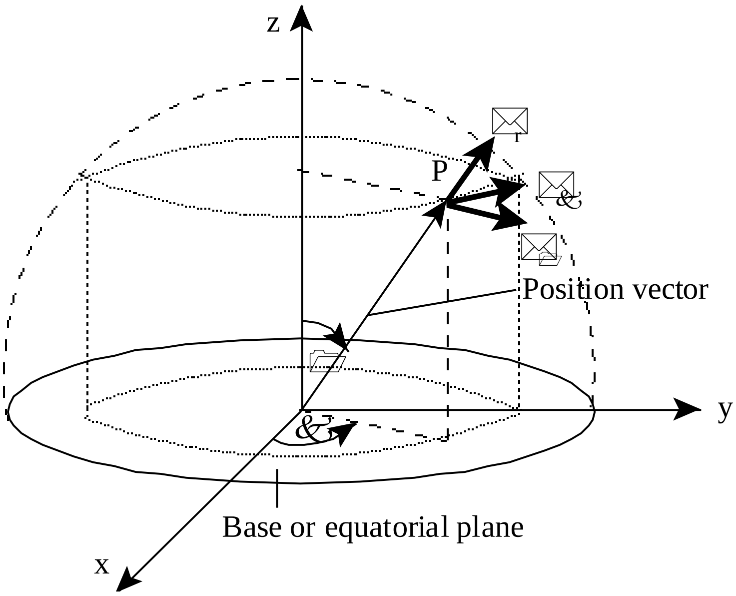
Cylindrical and spherical coordinate systems



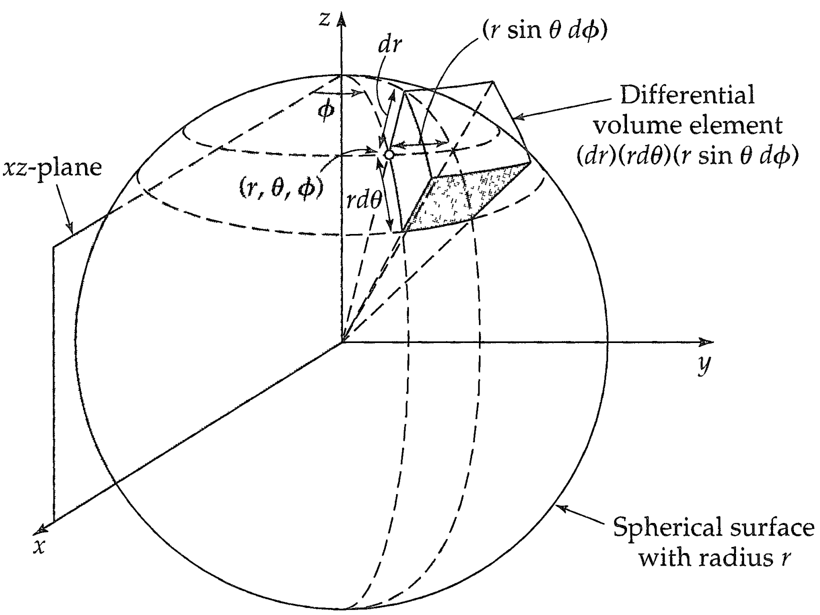
Unit vectors and cylindrical coordinates of point P



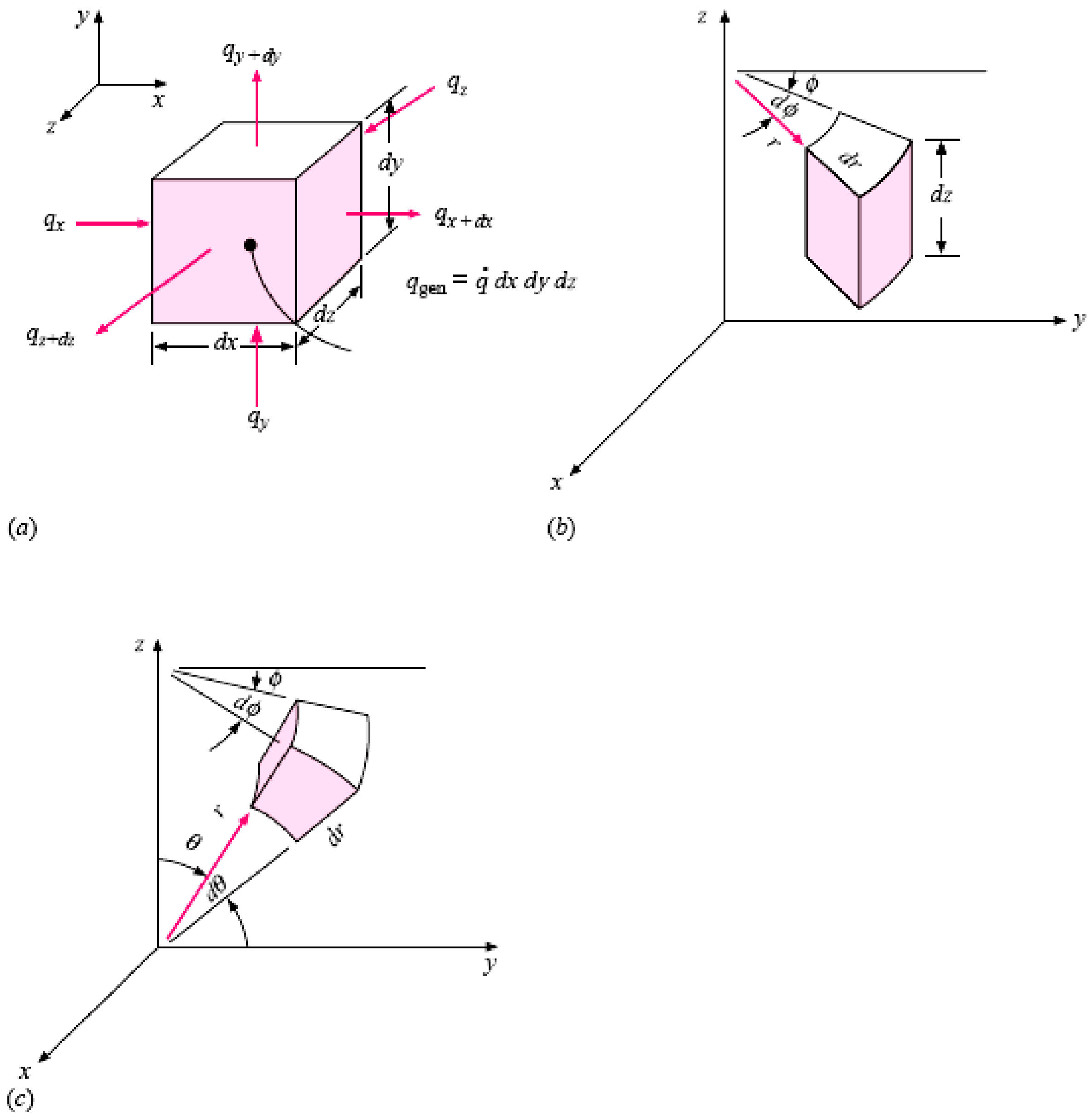
Differential volume element	dV = r dr dΘ dz	
Differential line elements	dr, r dΘ, and dz	
Differential surface elements	(r dΘ)(dz)	perpendicular to the r direction
	(dz)(dr)	perpendicular to the Θ direction
	(dr)(r dΘ)	perpendicular to the z direction



Unit vectors and spherical coordinates of point P

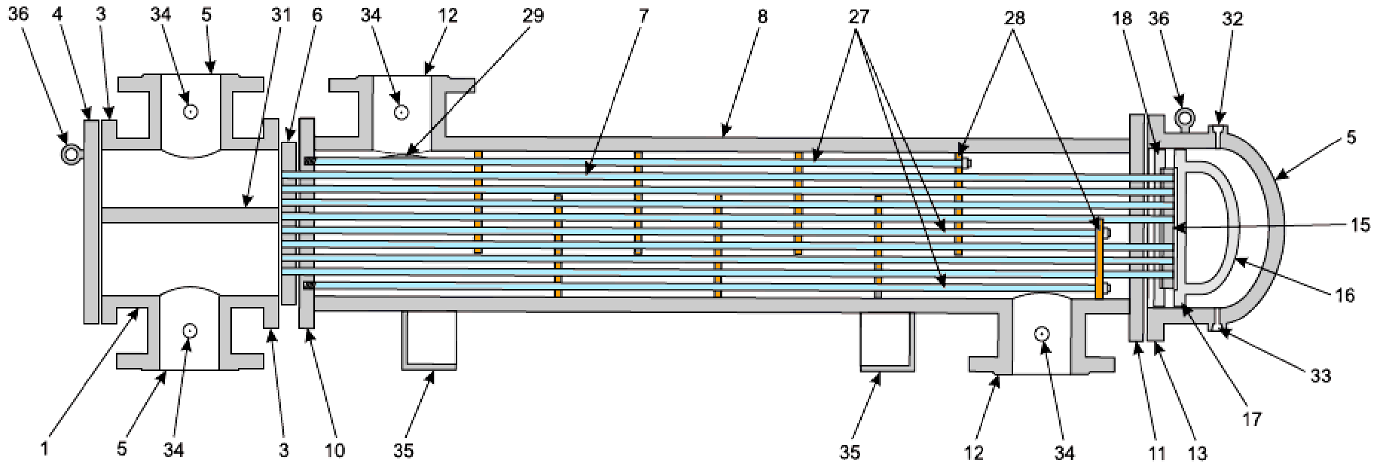


Differential volume element:	$dV = r^2 \sin\Theta\, dr\, d\Theta\, d\Phi$
Differential line elements:	dr, $r\, d\Theta$, and $r \sin\Theta\, d\Phi$
Differential surface elements:	
$(r\, d\Theta)(r \sin\Theta\, d\Phi)$	perpendicular to the r direction
$(r \sin\Theta\, d\Phi)(dr)$	perpendicular to the Θ direction
$(dr)(r\, d\Theta)$	perpendicular to the Φ direction



Elemental volumes
(a) rectangular coordinates; (b) cylindrical coordinates; (c) spherical coordinates

Heat exchangers



Floating head heat exchanger

1. Stationary Head-Channel
2. Stationary Head-Bonnet
3. Stationary Head Flange - Channel or Bonnet
4. Channel Cover
5. Stationary Head Nozzle
6. Stationary Tubesheet
7. Tubes
8. Shell
9. Shell Cover
10. Shell Flange - Stationary Head End
11. Shell Flange - Rear Head End
12. Shell Nozzle
13. Shell Cover Flange
14. Expansion Joint
15. Floating Tubesheet
16. Floating Head Cover
17. Floating Head Cover Flange
18. Floating Head Backing Device
19. Split Shear Ring
20. Slip-On Backing Ring
21. Floating Head Cover – External
22. Floating Tubesheet Skirt
23. Packing Box
24. Packing
25. Packing Gland
26. Lantern Ring
27. Tierods and Spacers
28. Transverse Baffles or Support Pates
29. Impingement Plate
30. Longitudnal Baffle
31. Pass Partition
32. Vent Connection
33. Drain Connection
34. Instrument Connection
35. Support Saddle
36. Lifting Lug
37. Support Bracket
38. Weir
39. Liquid Level Connection

Tube sheet

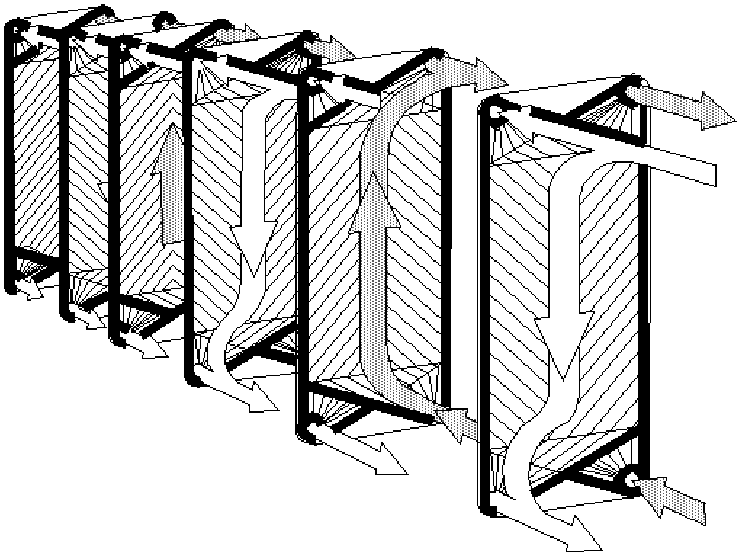


Plate and frame heat exchanger **Plate and frame heat exchanger flow pattern**

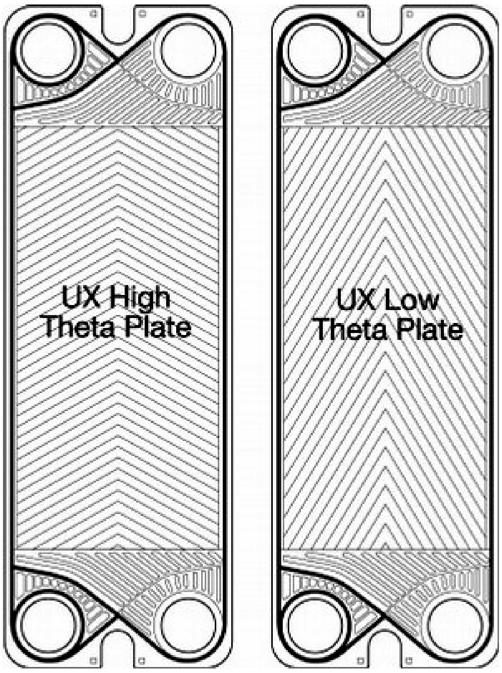


Plate and frame heat exchanger plates

Herringbone or chevron plates offer a variety of thermal lengths. They are used in a multitude of applications and are generally more efficient with relatively clean solutions due to greater induced turbulence within the flow stream.

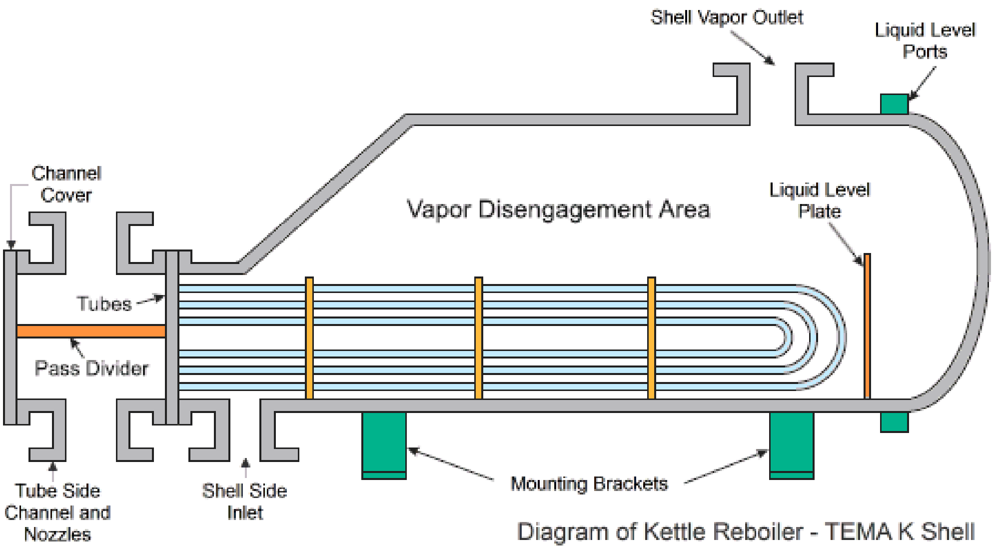
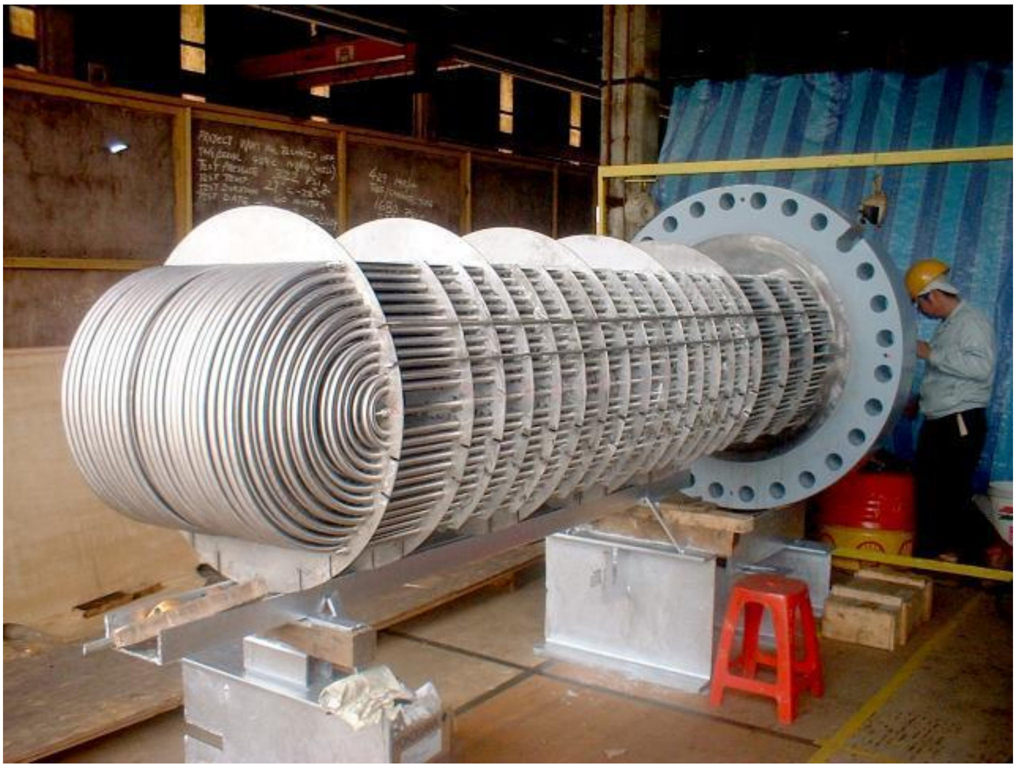


Diagram of Kettle Reboiler - TEMA K Shell



U-tube bundle

QUANTITY	SI unit	CONVERSION
Density (ρ)	kg/m^3	**$1 g/cm^3 = 1000 kg/m^3$** $1 lb/ft^3 = 16.018 kg/m^3$
Energy, Work	kJ	1 Btu = 1.05506 kJ 1 kcal = 4.1868 kJ 1 Btu = 252.16 cal 1 kWh = 3.6 MJ
Force (F)	N	$1 lb_f = 4.4482 N$ $1 N = 10^5 dyn$
Heat flux (q'')	W/m^2	$1 Btu/ft^2 h = 3.154 W/m^2$
Heat rate (q)	W / J/s	1 Btu/h = 0.29307 W
Heat transfer coefficient (h,U)	$W/m^2 K$	$1 Btu/ft^2 h°F = 5.6783 W/m^2 K$
Latent heat (ΔH_v)	kJ/kg	1 Btu/lb = 2.326 kJ/kg
Length (L)	m	**1 m = 100 cm = 1000 mm** 1 ft = 0.3048 m 1 in = 2.54 cm
Mass	kg	1 lb = 0.4536 kg
Power	W	1 hp = 0.7457 kW
Pressure (p)	Pa / N/m^2 (bar)	**$1 bar = 10^5 Pa$** **1 atm = 101.3 kPa** **1 atm = 760 mmHg** 1 atm = 14.7 psia 1 psia = 6894.76 Pa
Specific heat (c_p)	kJ/kgK	1 Btu/lb°F = 4.1868 kJ/kgK 1 Btu/lb°F = 1 kcal/kg°C
Temperature (T)	K (°C)	**T(K) = t(°C) + 273.15** **ΔT(°C) = ΔT(K)** = 1.8ΔT(°F) X(°F) = 5/(X!32)/9 (°C) 1K = 1.8°R T(°C) = (1/1.8)(°R! 492)
Thermal conductivity (k)	W/mK	1 Btu/ft h°F = 1.7307 W/mK
Viscosity Dynamic (μ) Kinematic (ν)	Pa≅s = kg/ms m^2/s	**1 cP = $10^{!3}$ Pa≅s** 1 cSt = $10^{!6}$ m^2/s
Volume (V)	m^3	**$1 m^3 = 1000$ P** 1 gal (US) = 3.7854 l 1 gal (GB) = 4.5435 l $1 ft^3 = 28.3168$ l