SPECIAL THEORY OF RELATIVITY

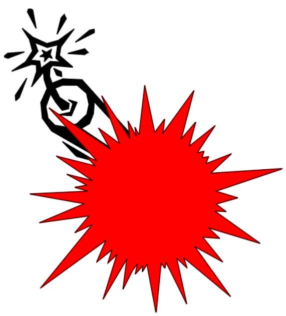
Newtonian (Classical) Relativity

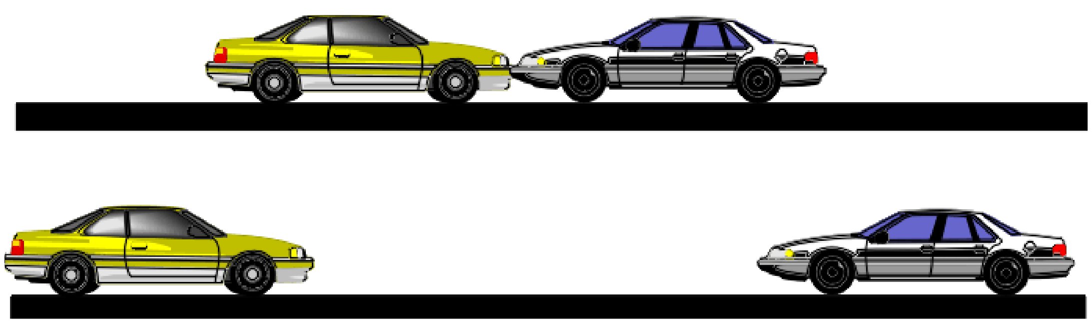
Assumption

- It is assumed that Newton's laws of motion must be measured with respect to (relative to) some reference frame.

Events and Coordinate

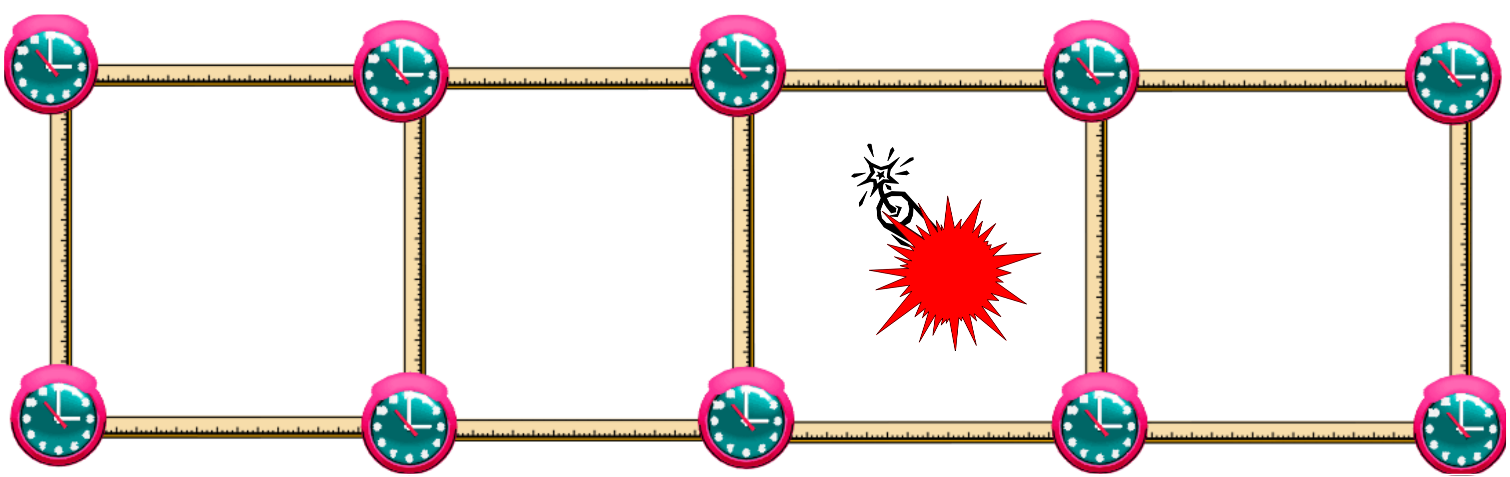
- In physics jargon, the word event has about the same meaning as it' s everyday usage.
- An event occurs at a specific location in space at a specific moment in time:





Reference Frames

- A reference frame is a means of describing the ***location*** of an event ***in space and time***.
- To construct a reference frame, lay out a bunch of ***rulers*** and synchronized ***clocks***
- You can then describe an event by ***where it occurs*** according to the ***rulers*** and ***when it occurs*** according to the ***clock***s.



Inertial Reference Frame

- A reference frame is called an **inertial frame** if Newton laws are valid in that frame.

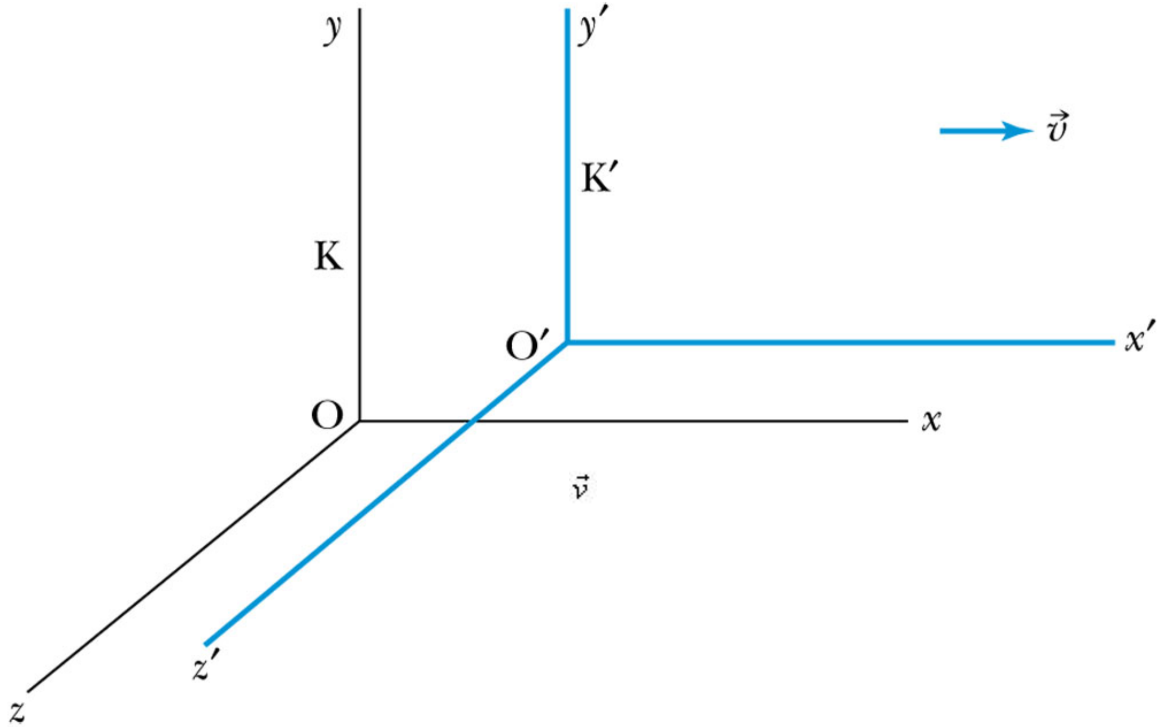
- Such a frame is established when a body, not subjected to net external forces, is observed to move in rectilinear motion at constant velocity.

Newtonian Principle of Relativity

- If Newton's laws are valid in one reference frame, then they are also valid in another reference frame moving at a uniform velocity relative to the first system.

- This is referred to as the **Newtonian principle of relativity** or **Galilean invariance.**

Inertial Frames K and K'

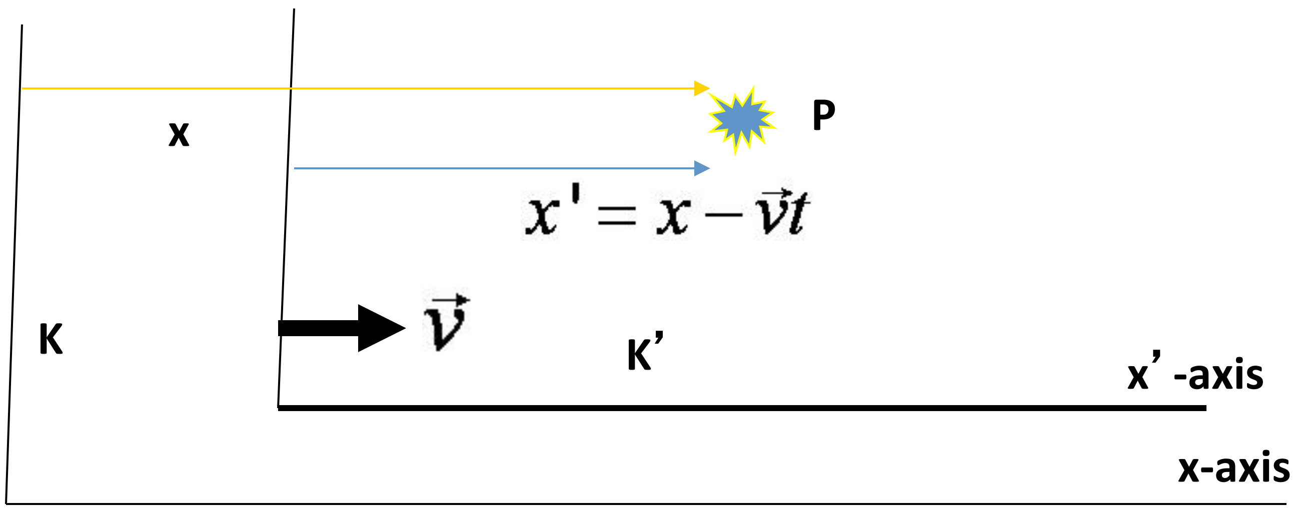


- K is at rest and K' is moving with velocity $\vec{v}$
- Axes are parallel
- K and K' are said to be *INERTIAL COORDINATE SYSTEMS*

The Galilean Transformation

For a point P

- In system K: P = (x, y, z, t)
- In system K' : P = (x' , y' , z' , t')



Conditions of the Galilean Transformation (Direct)

- Parallel axes
- K’ has a constant relative velocity in the *x*-direction with respect to K

$$x' = x - \vec{v}t$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

- **Time** (t) for all observers is a *Fundamental invariant*, i.e., the same for all inertial observers

The Inverse Relations

Step 1. Replace $\vec{v}$ with $-\vec{v}$

Step 2. Replace “primed” quantities with “unprimed” and “unprimed” with “primed.”

$$x = x' + \vec{v}t$$

$$y = y'$$

$$z = z'$$

$$t = t'$$

Galilean Velocity Transformations

Let the velocity component measured by observers O and O′ be (ux, uy, uz) and (ux′, uy′, uz′) respectively and using the function of a function relation,

$$\frac{dx'}{dt'} = \frac{dx'}{dt} \cdot \frac{dt}{dt'}$$

we have

$$u'_x = \frac{dx'}{dt'} = \frac{d}{dt}(x - vt) \cdot \frac{dt}{dt'} = \frac{dx}{dt} - v \qquad 1.9$$

but $$\frac{dx}{dt} = u_x \quad \Longrightarrow$$ (1.9) becomes

$$u'_x = u_x - v$$ **1.10** **Note this!!**

In addition

$$u'_y = u_y$$ 1.11

$$u'_z = u_z$$ 1.12

Galilean Acceleration Transformations

The acceleration transformations are obtained by either differentiating the distance relation twice or differentiating the velocity relation once. Thus if we let a_x denote the acceleration in the x direction then,

$$a_x = \frac{d^2 x}{dt^2} \qquad 1.13$$

or

$$a_x = \frac{du_x}{dt} \qquad 1.14$$

➔ $$a'_x = a_x \qquad a'_y = a_y \qquad a'_z = a_z \qquad 1.15$$

The Transition to Modern Relativity

- Although Newton's laws of motion had the same form under the Galilean transformation, Maxwell's equations did not.
- In 1905, Albert Einstein proposed a fundamental connection between space and time and that Newton's laws are only an approximation.

Lorentz Transformation

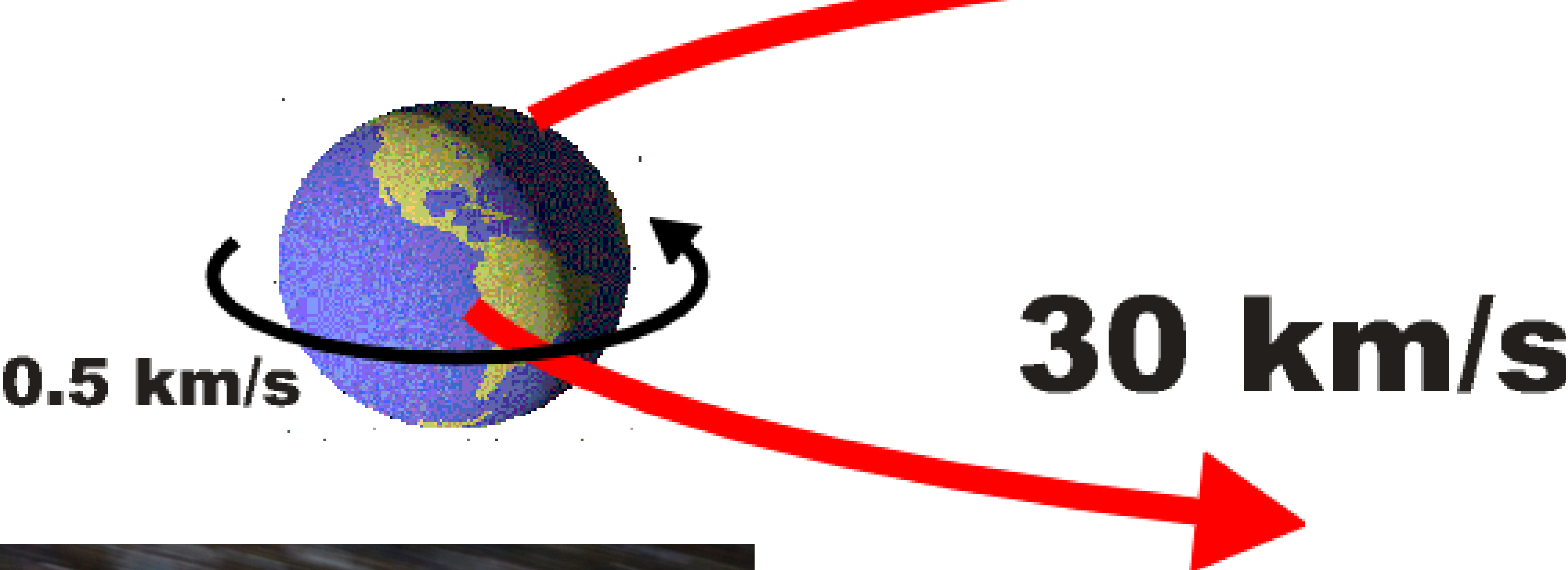
- As we shall see, space and time are not absolute as in Newtonian physics and everyday experience.
- The Mathematical relation between the description of two different observers is called the Lorentz transformation.
- Some phenomena which follow from the Lorentz transformation are:
  - Relativity of Simultaneous events
  - Time Dilation
  - Length Contraction

Reference Frames (cont.)

- What is the relation between the description of an event in a moving reference frame and a stationary one?
- To answer this question, we need to use the two principles of relativity

The First Principle of Relativity

- An inertial frame is one which moves through space at a constant velocity
- The first principle of relativity postulate is:
- *The Relativity Principl*
  - The laws of physics are identical in all inertial frames of reference frames.
- For example, if you are in a closed box moving through space at a constant velocity, there is no experiment you can do to determine how fast you are going
- In fact the idea of an observer being in motion with respect to space has no meaning.



Usain Bolt: *Only person to win back-to-back 100 and 200-meter Olympic gold medals. 9.58 100; 19.19 200; 14.35 150 — all world records. In 2013 Bolt became the only man to ever become three-time 200 World Champion. No one has ran faster and probably never will. EVER!*

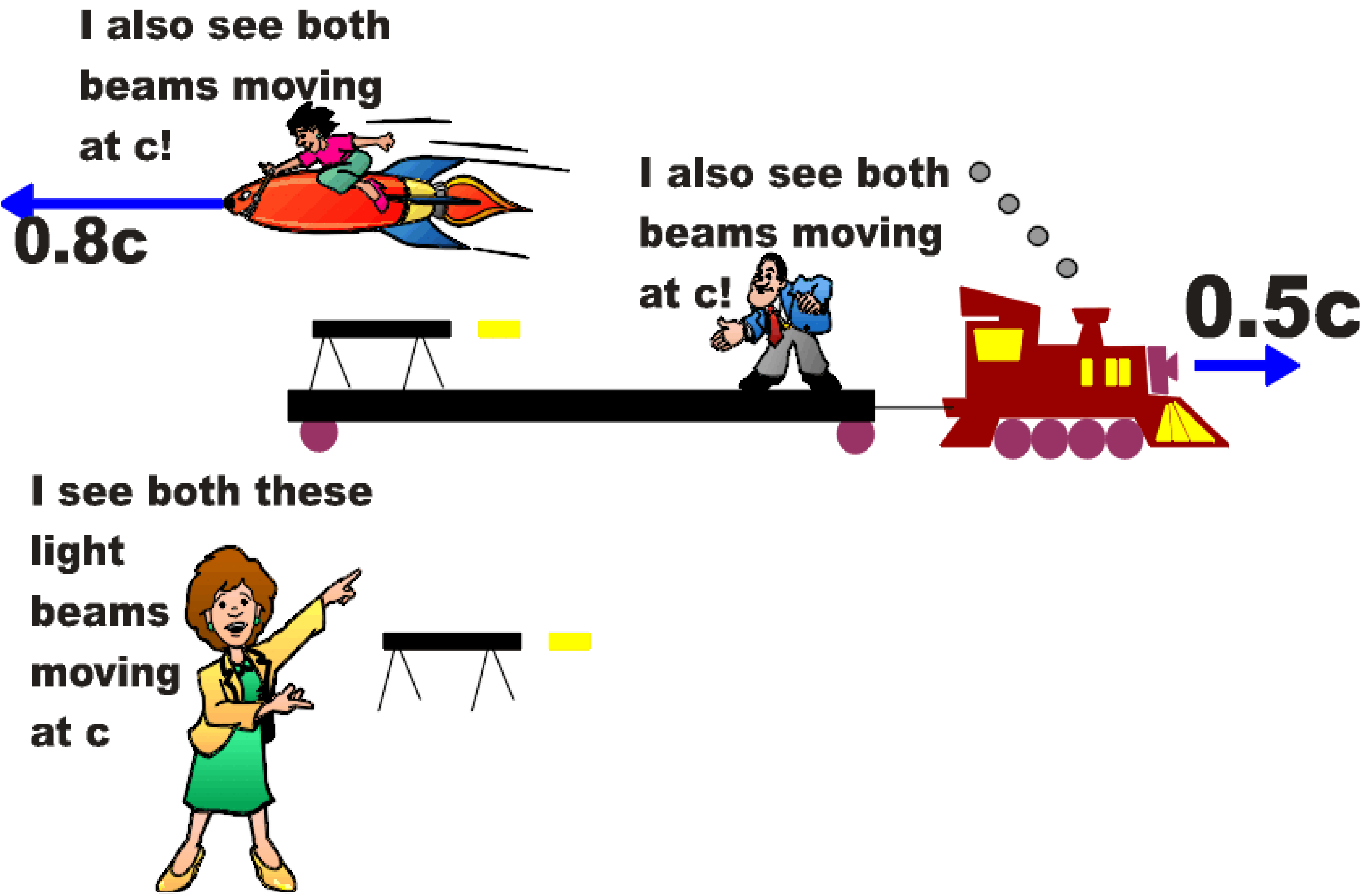
Source : http://charris.wordpress.com/the-list-top-10-fastest-human-beings/

The Second Principle of Relativity

- The second principle of relativity is a departure from Classical Physics:
  - *Constancy of the Speed of Light*
  - *Light propagates through empty space with a definite speed c independent of the speed of the source or observer.*
- This is what the Michelson Morley experiment shows.
- ***The speed of light is therefore very special***
- This principle is not obvious in everyday experience since things around us move much slower than c.
- *In fact, the effects of relativity only become apparent at high velocities*

With light, things look like this:

- A person on a cart moving at half the speed of light will see light moving at c.
- A person watching on the ground will see that same light moving at the same speed, whether the light came from a stationary or moving source


I also see both beams moving at c!
0.8c
I also see both beams moving at c!
0.5c
I see both these light beams moving at c

Welcome to The Strange World of Albert Einstein

- Some of the consequences of Special relativity are:
  - Events which are simultaneous to a stationary observer are not simultaneous to a moving observer.
  - Nothing can move faster than c, the speed of light in vacuum.
  - A stationary observer will see a moving clock running slow.
  - A moving object will be contracted along its direction of motion.
  - Mass can be shown to be a frozen form of energy according to the relation $E=mc^2$.

Inconsistency of the Galilean Transformation with Einstein's Postulates (The Lorentz Transformation)

- If light moves along the x-axis with a speed $u'_x = c$ then the Galilean transformation equations imply that the speed in the x′ frame would be $u_x = c + v$ instead of the expected value of $u_x = c$ which is consistent of Einstein's postulates and also with experiment.
- The classical transformation equation therefore will have to be modified to make consistent with Einstein's postulates.
- This is done by introducing a constant γ which is independent of the coordinates in the modified

equation. This gives the equations the form

$$x = \gamma(x' + vt') \qquad 1.16$$

γ depends on *v* or *c*.

Likewise,

$$x' = \gamma(x - vt) \qquad 1.17$$

Lorentz Transformation

Thought Experiment

Let us consider a propagating light source measured by two different observers from the x and x′ coordinate systems.

<u>Assumptions:</u>

$$t - t' = 0 \quad t' = 0$$

For the x-coordinate system, the equation for the propagating wavefront is given by

$$x = ct \qquad 1$$

Likewise for the x′- coordinate system,

$$x' = ct' \qquad 2$$

We earlier found that,

$$x = \gamma(x' + vt') \qquad 3$$

$$x' = \gamma(x - vt) \qquad 4$$

Substituting eqns 3 and 4 into 1 and 2 gives

$$ct = \gamma(ct' + vt') = \gamma(c + v)t' \qquad 5$$

and

$$ct' = \gamma(ct + vt) = \gamma(c - v)t \qquad 6$$

Eliminating t´/t from 5 and 5 and determining γ gives

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad 7$$

Some Time Dilation Factors

Velocity	Gamma
Space Shuttle 5000m/s	1.00000000036
Earth in Orbit 30000m/s	1.0000000047
0.01c	1.00005
0.1c	1.005
0.5c	1.15
0.8c	1.67
0.9c	2.29
0.95c	3.20
0.99c	7.09
0.999c	22.37
0.9999c	70.7

Substituting $x = \gamma(x' + vt')$ into $x' = \gamma(x - vt)$

gives

$$x' = \gamma[\gamma(x' + vt') - vt] \qquad 8$$

Solving 8 for t in terms of t´ and x´ gives
or alternative method

Set x=ct and substitution will do

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

The inverse relation is

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad 9$$

Time Dilation

Let us consider two events that occur at times t_1' and t_2' in the frame x' at a *common point* x_0'. The corresponding times recorded for the *same event* in the x frame will be t_1 and t_2.

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

Thus,

$$t_1 = \gamma\left(t_1' + \frac{vx_0'}{c^2}\right)$$

$$t_2 = \gamma\left(t_2' + \frac{vx_0'}{c^2}\right)$$

$$\Rightarrow\ t_2 - t_1 = \gamma\left(t_2' - t_1'\right)$$

$$\Delta t = \gamma\left(\Delta t_0\right)$$

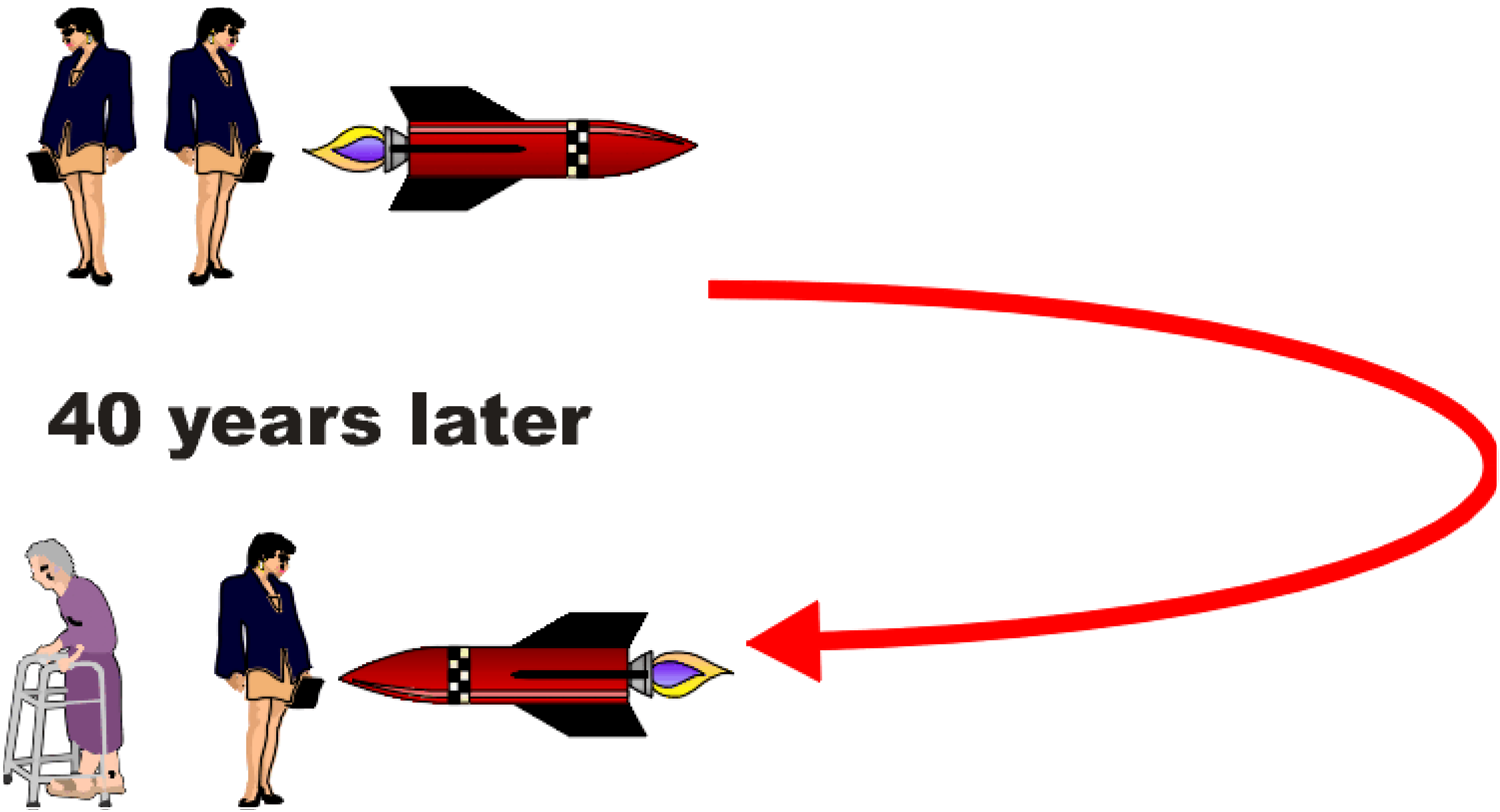
The time interval measured in any other reference frame is always found to be longer than the proper time. This time increase is called the time dilation.

Time Dilation (cont.)

- For example, suppose that a rocket ship is moving through space at a speed of 0.8c.
- According to an observer on earth 1.67 years pass for each year that passes for the rocket man, because for this velocity gamma=1.67
- But wait a second! According to the person on the rocket ship, the earth-man is moving at 0.8c. The rocket man will therefore observe the earth clock as running slow!
- Each sees the other's clock as running slow. HOW CAN THIS BE!!!!!

The Twin Paradox

- To bring this issue into focus, consider the following story:
  - Jane and Sally are identical twins. When they are both age 35, Sally travels in a rocket to a star 20 light years away at v=0.99c and then returns to Earth. The trip takes 40 years according to Jane and when Sally gets back, Jane has aged 40 years and is now 75 years old. Since gamma=7.09, Sally has aged only 5 years 8 months and is therefore only 40 years and 8 months old. Yet according to the above, when Sally was moving, she would see Jane's clock as running slow. How is this possible???


40 years later

Length Contraction

- The length of an object measured in a reference frame in which the object is at rest is called the proper length L_0.
- The length *L* measured in a reference in which the body is moving is always shorter than L_0.
- The *length decrease* between L_0 and L is called the **length contraction**.

Length Contraction

Now consider a rod *at rest* in a frame x' with one end at x_2' and the other at x_1' The proper length of the rod is

$$L_0 = x_2' - x_1'$$

In the frame *x* the *rod moves with a speed v* with a length

$$L = x_2 - x_1$$

Length Contraction

Now using $$x' = \gamma(x - vt)$$

$$x_2' = \gamma(x_2 - vt_2)$$

and

$$x_1' = \gamma(x_1 - vt_1)$$

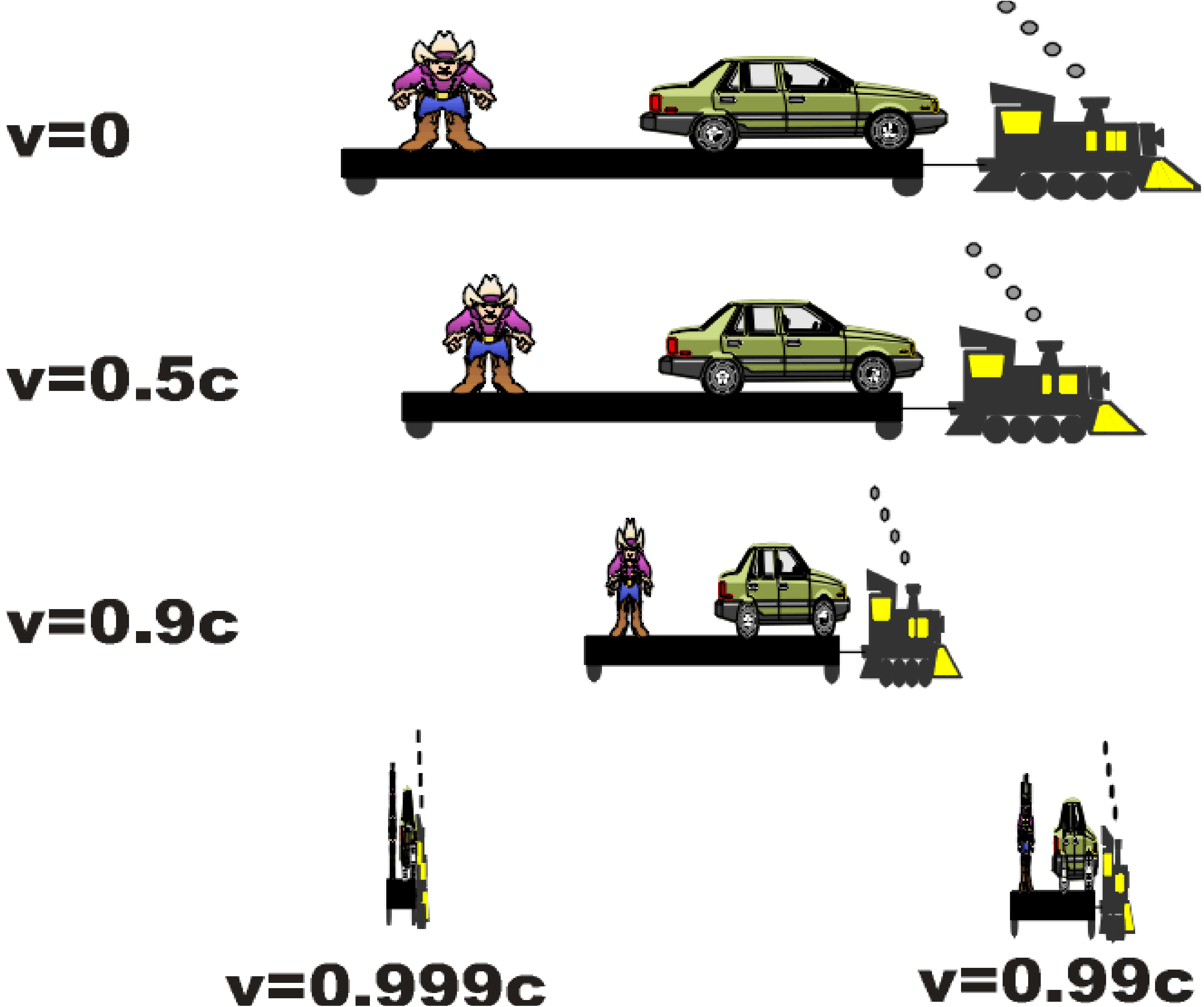
$$t_1 = t_2$$

´gives

$$x_2' - x_1' = \gamma(x_2 - x_1)$$

$$x_2 - x_1 = \frac{1}{\gamma}(x_2' - x_1')$$

$$L = \frac{1}{\gamma}L_0$$



v=0
v=0.5c
v=0.9c
v=0.999c
v=0.99c

WORKED EXAMPLES

A spaceship passes you at a speed of 0.750c. You measure its length to be 28.2m. How long would it be when at rest?

We measure the contracted length and find the rest length from

$$L = L_0\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}$$

$$28.2\,\mathrm{m} = L_0\left[1-(0.750)^2\right]^{\frac{1}{2}}$$

$L_0 = 42.6$ m

WORKED EXAMPLES

A certain type of elementary particle moves at a speed of 2.70 x 10^8 m/s. At this speed, the average lifetime is measured to be 4.76 x 10^{-6} s. What is the particle's lifetime at rest?

$$\Delta t = \frac{\Delta t_0}{\left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{1}{2}}} \qquad 4.76 \times 10^{-6}\ \text{s} = \frac{\Delta t_0}{\left\{1 - \left[\frac{(2.70 \times 10^8\ \text{m/s})}{(3.00 \times 10^8\ \text{m/s})}\right]^2\right\}^{\frac{1}{2}}}$$

$$\Delta t_0 = \boxed{2.07 \times 10^{-6}\ \text{s.}}$$

What Happens to Simultaneous Events?

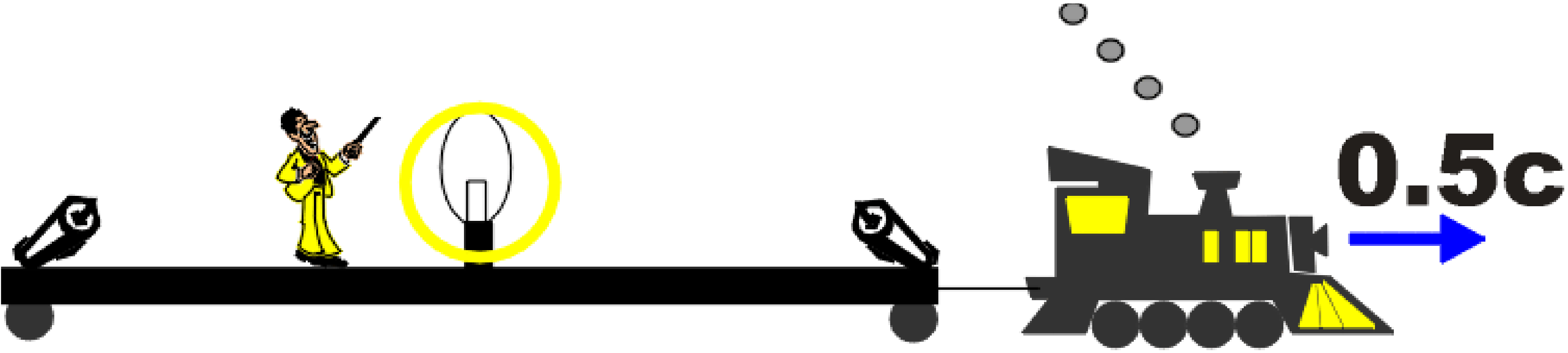
- Are events which are simultaneous to one observer also simultaneous to another observer?
- We can use the principles of relativity to answer this question.
- Imagine a train moving at half the speed of light…

Simultaneous Events

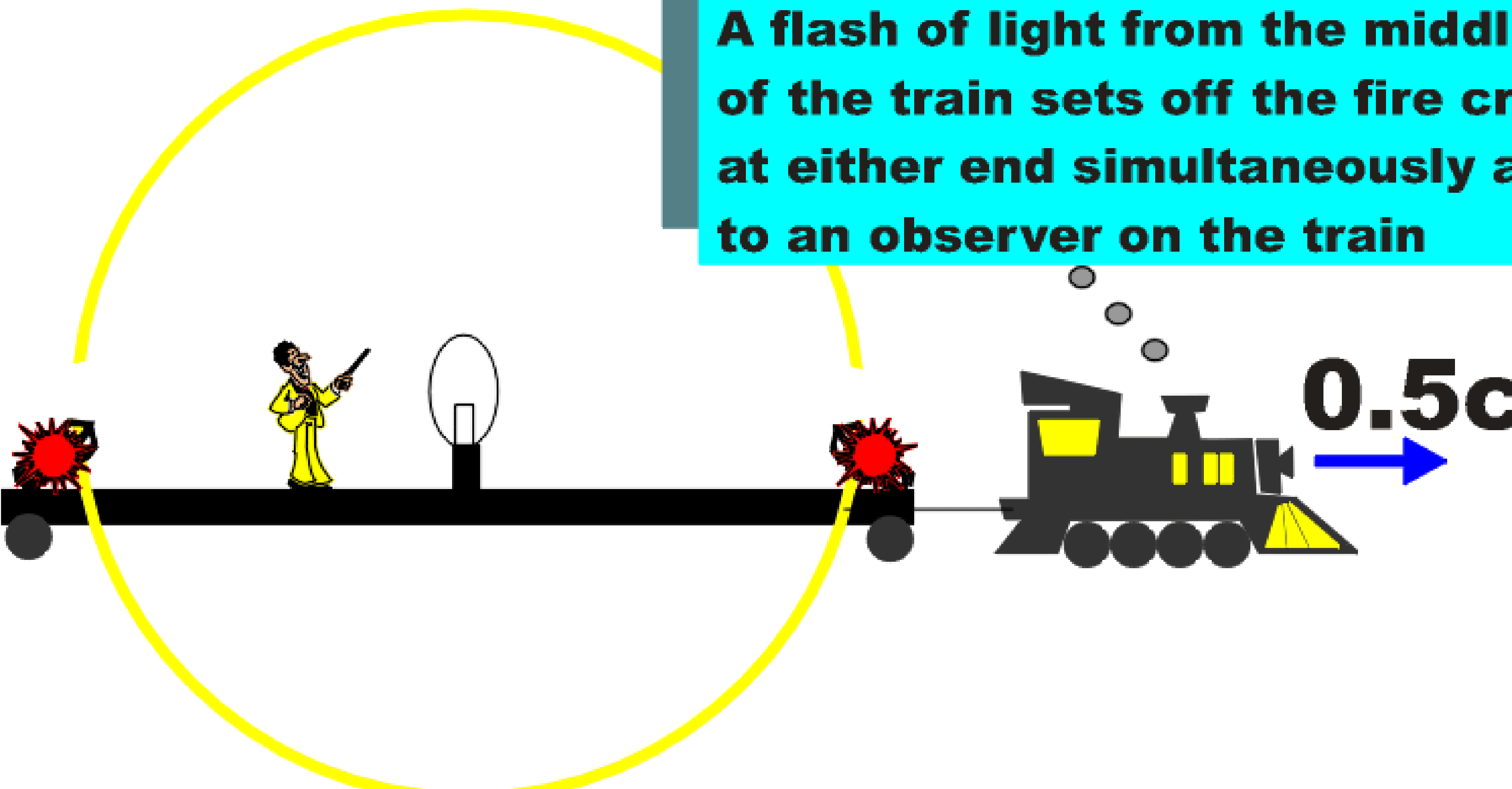
- Thus two events which are simultaneous to the observer on the train are not simultaneous to an observer on the ground
- The rearwards event happens first according to the stationary observer
- The stationary observer will therefore see a clock at the rear of the train ahead of the clock at the front of the train

View from the Train

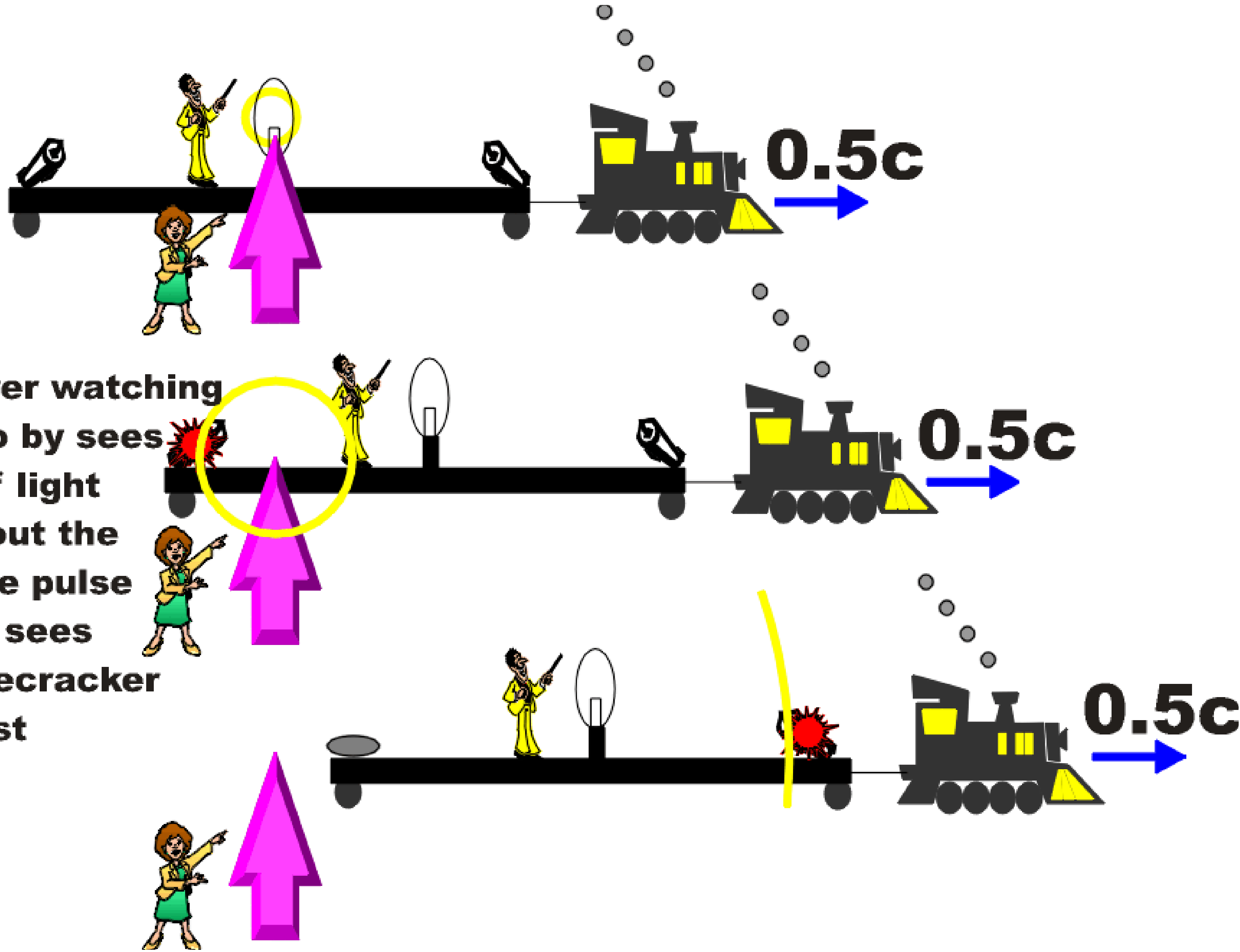
See next page



A flash of light from the middle of the train sets off the fire crackers at either end simultaneously according to an observer on the train

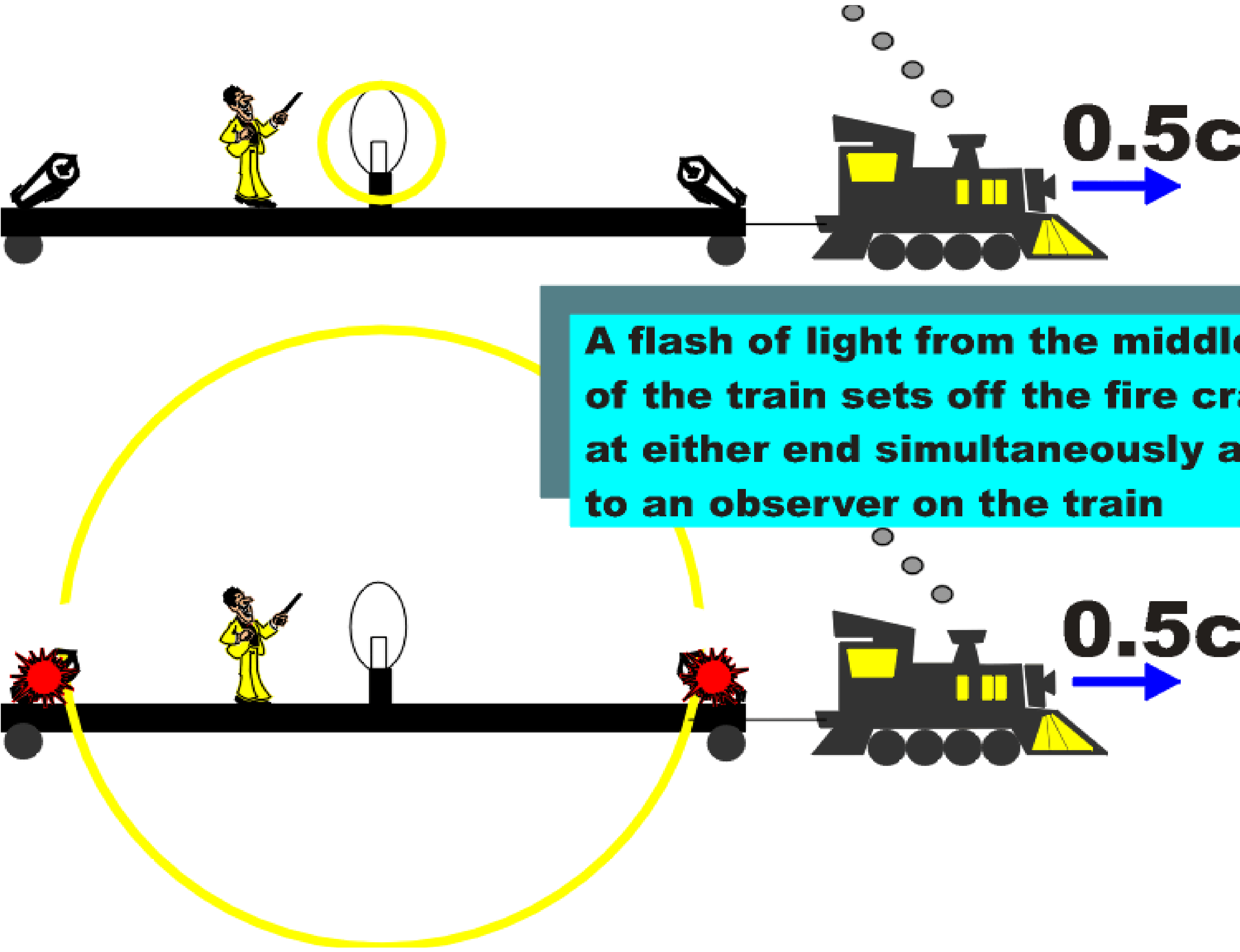


The View From The Ground



The observer watching the train go by sees the flash of light expand about the origin of the pulse and so she sees the rear firecracker explode first

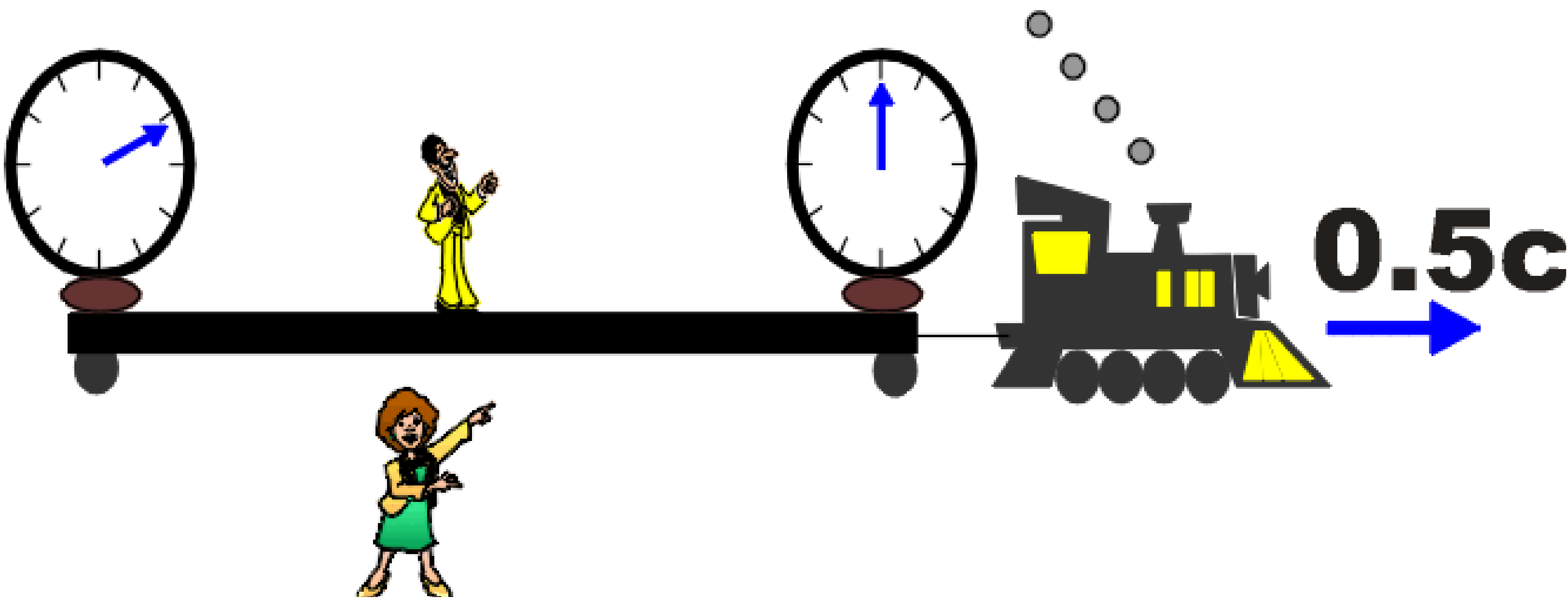
View from the Train



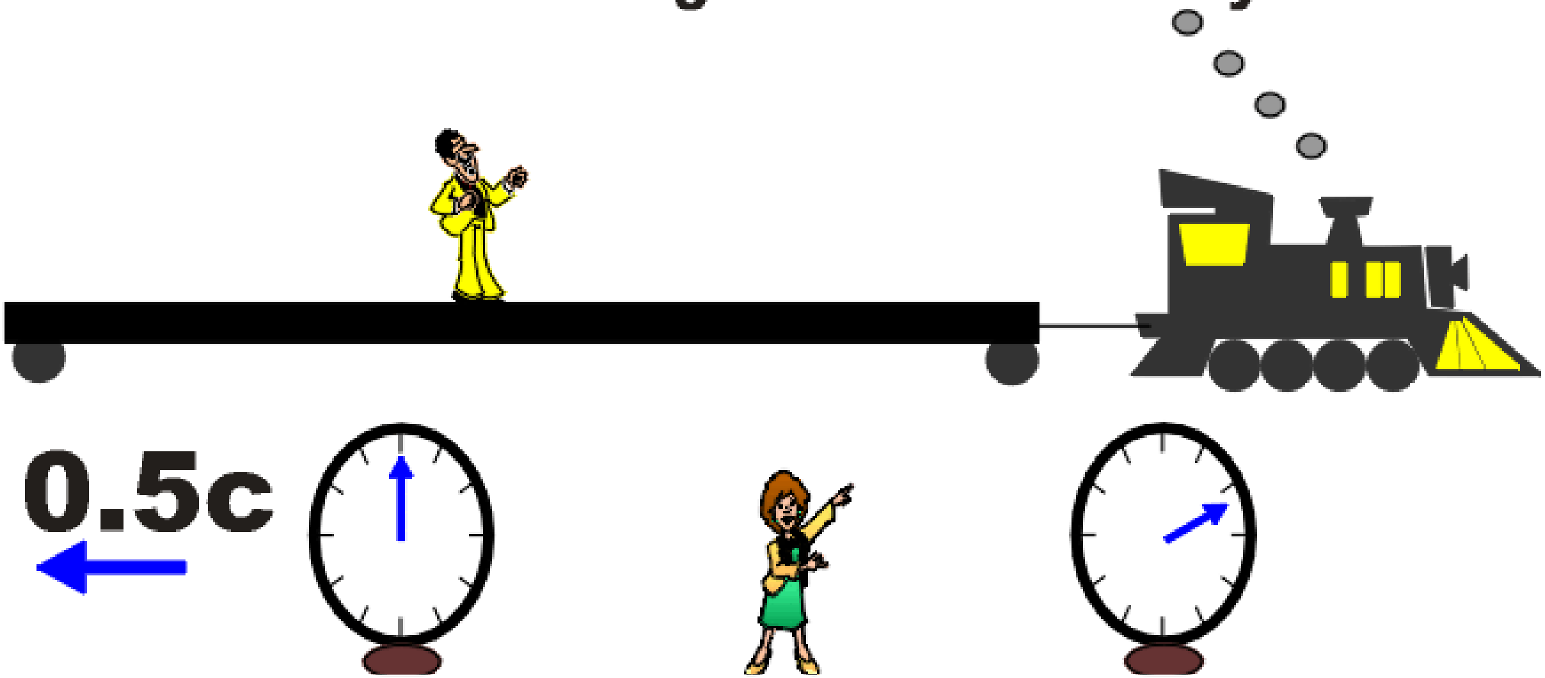
A flash of light from the middle of the train sets off the fire crackers at either end simultaneously according to an observer on the train

According to the person on the train, the two clocks are synchronized

According to the stationary observer, the rear clock runs ahead of the front clock



Of course it works the other way also. The observer on the train sees the stationary observer moving left at 0.5c. The person on the train sees the right hand stationary clock running ahead!



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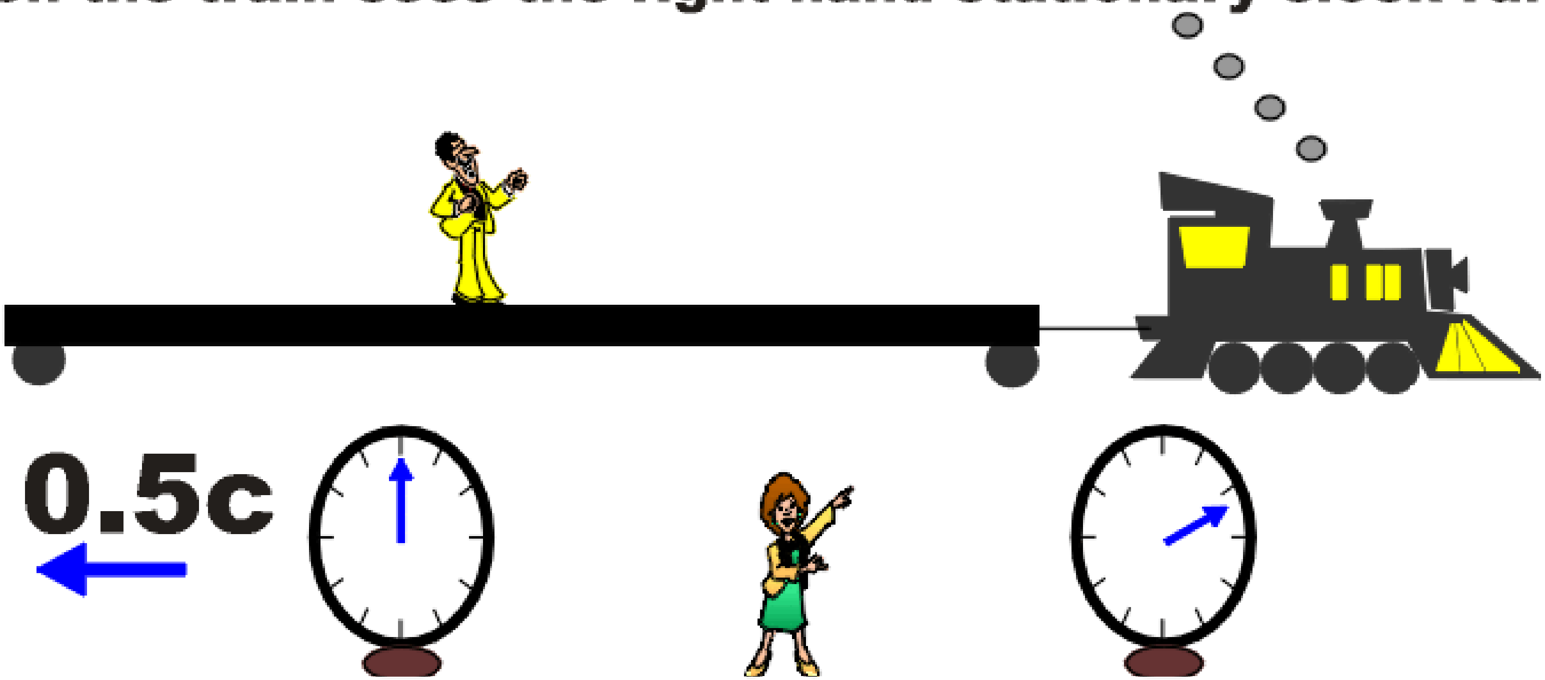

Critical Question

- Which of the following was a consequence of the Einstein Special Theory of Relativity?
  - A Events which are simultaneous to a stationary observer are simultaneous to a moving observer.
  - Nothing can move faster than c, the speed of light in vacuum.
  - A stationary observer will see a moving clock running at the same rate.
  - A moving object will be stretched along its direction of motion.
  - All of the above are true.

Tutorial Questions (Detailed Solutions at the end of the slides)

Question 1: **Lifetime of a moving muon.** (*a*) What will be the mean lifetime of a muon as measured in the laboratory if it is traveling at $v = 0.60c = 1.80 \times 10^8\ \mathrm{m/s}$ with respect to the laboratory? Its mean lifetime at rest is $2.20\ \mu\mathrm{s} = 2.20 \times 10^{-6}\ \mathrm{s}$. (*b*) How far does a muon travel in the laboratory, on average, before decaying?

An s: (a) 2.8×10^{-6} s (b) 500m

Question 2: **Painting's contraction.** A rectangular painting measures 1.00 m tall and 1.50 m wide. It is hung on the side wall of a spaceship which is moving past the Earth at a speed of $0.90c$. (*a*) What are the dimensions of the picture according to the captain of the spaceship? (*b*) What are the dimensions as seen by an observer on the Earth?

Ans: (a) Same dimension ie. 1.00m x 1.50m (b) 1.00m x 0.65m

Question 3: **A fantasy supertrain.** A very fast train with a proper length of 500 m is passing through a 200-m-long tunnel. The train's speed is so great that the train fits completely within the tunnel as seen by an observer at rest on the Earth (on the mountain above the tunnel); that is, the engine is just about to emerge from one end of the tunnel as the last car disappears into the other end. What is the train's speed?

Ans: 0.92c

Detailed Solutions to Tutorial Questions

Question 1: **Lifetime of a moving muon.** (*a*) What will be the mean lifetime of a muon as measured in the laboratory if it is traveling at $v = 0.60c = 1.80 \times 10^8 \text{ m/s}$ with respect to the laboratory? Its mean lifetime at rest is $2.20\ \mu\text{s} = 2.20 \times 10^{-6}\text{ s}$. (*b*) How far does a muon travel in the laboratory, on average, before decaying?

Solution

(*a*) $v = 0.60c$.

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2.20 \times 10^{-6}\text{ s}}{\sqrt{1 - \frac{0.36c^2}{c^2}}} = \frac{2.20 \times 10^{-6}\text{ s}}{\sqrt{0.64}} = 2.8 \times 10^{-6}\text{ s}.$$

(*b*) Relativity predicts that a muon would travel an average distance $d = v\,\Delta t = (0.60)(3.0 \times 10^8\text{ m/s})(2.8 \times 10^{-6}\text{ s}) = 500\text{ m}$, and this is the distance that is measured experimentally in the laboratory.

Question 2: **Painting's contraction.** A rectangular painting measures 1.00 m tall and 1.50 m wide. It is hung on the side wall of a spaceship which is moving past the Earth at a speed of $0.90c$. (*a*) What are the dimensions of the picture according to the captain of the spaceship? (*b*) What are the dimensions as seen by an observer on the Earth?

Solution: (*a*) The painting is at rest ($v = 0$) on the spaceship so it (as well as everything else in the spaceship) looks perfectly normal to everyone on the spaceship. The captain sees a 1.00-m by 1.50-m painting.

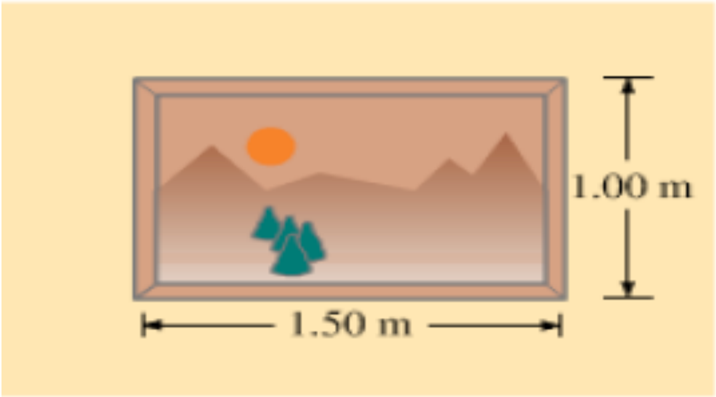

Fig.1

(b) Only the dimension in the direction of motion is shortened, so the height is unchanged at 1.00 m, fig.2 . The length, however, is contracted

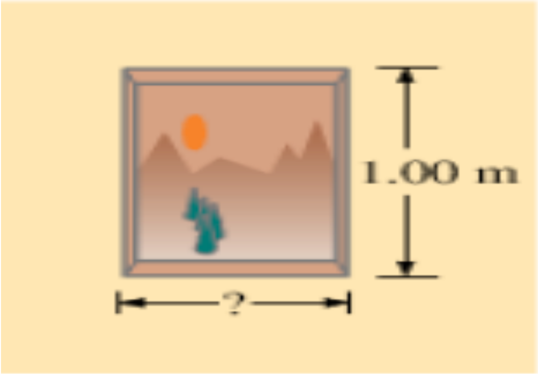

Fig.2

We apply the length contraction formula $L = L_0\sqrt{1 - \frac{v^2}{c^2}}$

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$= (1.50\ \text{m})\sqrt{1 - (0.90)^2} = 0.65\ \text{m}.$$

So the picture has dimensions 1.00 m × 0.65 m.

Question 3: **A fantasy supertrain.** A very fast train with a proper length of 500 m is passing through a 200-m-long tunnel. The train's speed is so great that the train fits completely within the tunnel as seen by an observer at rest on the Earth (on the mountain above the tunnel); that is, the engine is just about to emerge from one end of the tunnel as the last car disappears into the other end. What is the train's speed?

Solution: Since the train just fits inside the tunnel, its length measured by the person on the ground is 200 m. The length contraction formula can thus be used to solve for v.

Substituting $L = 200\text{ m}$ and $L_0 = 500\text{ m}$

$$200\text{ m} = 500\text{ m}\sqrt{1 - \frac{v^2}{c^2}};$$

dividing both sides by 500 m and squaring, we get

$$(0.40)^2 = 1 - \frac{v^2}{c^2}$$

or

$$\frac{v}{c} = \sqrt{1 - (0.40)^2}$$

$$v = 0.92c.$$