TABLE 2.1 Data for the mass and associated terminal velocities of a number of jumpers.

m, **kg**	83.6	60.2	72.1	91.1	92.9	65.3	80.9
v_t, **m/s**	53.4	48.5	50.9	55.7	54	47.7	51.1

Thus, if we measure the terminal velocity of a number of jumpers of known mass, this equation provides a means to estimate the drag coefficient. The data in Table 2.1 were collected for this purpose.

In this chapter, we will learn how MATLAB can be used to analyze such data. Beyond showing how MATLAB can be employed to compute quantities like drag coefficients, we will also illustrate how its graphical capabilities provide additional insight into such analyses.

2.1 THE MATLAB ENVIRONMENT

MATLAB is a computer program that provides the user with a convenient environment for performing many types of calculations. In particular, it provides a very nice tool to implement numerical methods.

The most common way to operate MATLAB is by entering commands one at a time in the command window. In this chapter, we use this interactive or *calculator mode* to introduce you to common operations such as performing calculations and creating plots. In Chap. 3, we show how such commands can be used to create MATLAB programs.

One further note. This chapter has been written as a hands-on exercise. That is, you should read it while sitting in front of your computer. The most efficient way to become proficient is to actually implement the commands on MATLAB as you proceed through the following material.

MATLAB uses three primary windows:

- Command window. Used to enter commands and data.
- Graphics window. Used to display plots and graphs.
- Edit window. Used to create and edit M-files.

In this chapter, we will make use of the command and graphics windows. In Chap. 3 we will use the edit window to create M-files.

After starting MATLAB, the command window will open with the command prompt being displayed

```
>>
```

The calculator mode of MATLAB operates in a sequential fashion as you type in commands line by line. For each command, you get a result. Thus, you can think of it as operating like a very fancy calculator. For example, if you type in

```
>> 55 - 16
```

MATLAB will display the result[1]

```
ans =
    39
```

[1] MATLAB skips a line between the label (`ans =`) and the number (`39`). Here, we omit such blank lines for conciseness. You can control whether blank lines are included with the `format compact` and `format loose` commands.

Notice that MATLAB has automatically assigned the answer to a variable, `ans`. Thus, you could now use `ans` in a subsequent calculation:

```
>> ans + 11
```

with the result

```
ans =
    50
```

MATLAB assigns the result to `ans` whenever you do not explicitly assign the calculation to a variable of your own choosing.

2.2 ASSIGNMENT

Assignment refers to assigning values to variable names. This results in the storage of the values in the memory location corresponding to the variable name.

2.2.1 Scalars

The assignment of values to scalar variables is similar to other computer languages. Try typing

```
>> a = 4
```

Note how the assignment echo prints to confirm what you have done:

```
a =
     4
```

Echo printing is a characteristic of MATLAB. It can be suppressed by terminating the command line with the semicolon (`;`) character. Try typing

```
>> A = 6;
```

You can type several commands on the same line by separating them with commas or semicolons. If you separate them with commas, they will be displayed, and if you use the semicolon, they will not. For example,

```
>> a = 4,A = 6;x = 1;

a =
     4
```

MATLAB treats names in a case-sensitive manner—that is, the variable `a` is not the same as `A`. To illustrate this, enter

```
>> a
```

and then enter

```
>> A
```

See how their values are distinct. They are distinct names.

We can assign complex values to variables, since MATLAB handles complex arithmetic automatically. The unit imaginary number $\sqrt{-1}$ is preassigned to the variable `i`. Consequently, a complex value can be assigned simply as in

```
>> x = 2+i*4

x =
   2.0000 + 4.0000i
```

It should be noted that MATLAB allows the symbol `j` to be used to represent the unit imaginary number for input. However, it always uses an `i` for display. For example,

```
>> x = 2+j*4

x =
   2.0000 + 4.0000i
```

There are several predefined variables, for example, `pi`.

```
>> pi

ans =
    3.1416
```

Notice how MATLAB displays four decimal places. If you desire additional precision, enter the following:

```
>> format long
```

Now when `pi` is entered the result is displayed to 15 significant figures:

```
>> pi

ans =
   3.14159265358979
```

To return to the four decimal version, type

```
>> format short
```

The following is a summary of the format commands you will employ routinely in engineering and scientific calculations. They all have the syntax: `format` *`type`*.

`type`	**Result**	**Example**
`short`	Scaled fixed-point format with 5 digits	`3.1416`
`long`	Scaled fixed-point format with 15 digits for double and 7 digits for single	`3.14159265358979`
`short e`	Floating-point format with 5 digits	`3.1416e+000`
`long e`	Floating-point format with 15 digits for double and 7 digits for single	`3.141592653589793e+000`
`short g`	Best of fixed- or floating-point format with 5 digits	`3.1416`
`long g`	Best of fixed- or floating-point format with 15 digits for double and 7 digits for single	`3.14159265358979`
`short eng`	Engineering format with at least 5 digits and a power that is a multiple of 3	`3.1416e+000`
`long eng`	Engineering format with exactly 16 significant digits and a power that is a multiple of 3	`3.14159265358979e+000`
`bank`	Fixed dollars and cents	`3.14`

2.2.2 Arrays, Vectors and Matrices

An array is a collection of values that are represented by a single variable name. One-dimensional arrays are called *vectors* and two-dimensional arrays are called *matrices*. The scalars used in Section 2.2.1 are actually matrices with one row and one column.

Brackets are used to enter arrays in the command mode. For example, a row vector can be assigned as follows:

```
>> a = [1 2 3 4 5]

a =
     1     2     3     4     5
```

Note that this assignment overrides the previous assignment of `a = 4`.

In practice, row vectors are rarely used to solve mathematical problems. When we speak of vectors, we usually refer to column vectors, which are more commonly used. A column vector can be entered in several ways. Try them.

```
>> b = [2;4;6;8;10]
```

or

```
>> b = [2
4
6
8
10]
```

or, by transposing a row vector with the `'` operator,

```
>> b = [2 4 6 8 10]'
```

The result in all three cases will be

```
b =
     2
     4
     6
     8
    10
```

A matrix of values can be assigned as follows:

```
>> A = [1 2 3; 4 5 6; 7 8 9]

A =
     1     2     3
     4     5     6
     7     8     9
```

In addition, the Enter key (carriage return) can be used to separate the rows. For example, in the following case, the Enter key would be struck after the 3, the 6 and the] to assign the matrix:

```
>> A = [1 2 3
        4 5 6
        7 8 9]
```

Finally, we could construct the same matrix by *concatenating* (i.e., joining) the vectors representing each column:

```
>> A = [[1 4 7]' [2 5 8]' [3 6 9]']
```

At any point in a session, a list of all current variables can be obtained by entering the `who` command:

```
>> who

Your variables are:
A    a    ans  b    x
```

or, with more detail, enter the `whos` command:

```
>> whos
  Name      Size               Bytes  Class

  A         3x3                   72  double array
  a         1x5                   40  double array
  ans       1x1                    8  double array
  b         5x1                   40  double array
  x         1x1                   16  double array (complex)

  Grand total is 21 elements using 176 bytes
```

Note that subscript notation can be used to access an individual element of an array. For example, the fourth element of the column vector `b` can be displayed as

```
>> b(4)

ans =
     8
```

For an array, `A(m,n)` selects the element in `m`th row and the `n`th column. For example,

```
>> A(2,3)

ans =
     6
```

There are several built-in functions that can be used to create matrices. For example, the `ones` and `zeros` functions create vectors or matrices filled with ones and zeros, respectively. Both have two arguments, the first for the number of rows and the second for the number of columns. For example, to create a 2×3 matrix of zeros:

```
>> E = zeros(2,3)

E =
     0     0     0
     0     0     0
```

Similarly, the `ones` function can be used to create a row vector of ones:

```
>> u = ones(1,3)

u =
     1     1     1
```

2.2.3 The Colon Operator

The colon operator is a powerful tool for creating and manipulating arrays. If a colon is used to separate two numbers, MATLAB generates the numbers between them using an increment of one:

```
>> t = 1:5

t =
     1     2     3     4     5
```

If colons are used to separate three numbers, MATLAB generates the numbers between the first and third numbers using an increment equal to the second number:

```
>> t = 1:0.5:3

t =
    1.0000    1.5000    2.0000    2.5000    3.0000
```

Note that negative increments can also be used

```
>> t = 10:-1:5

t =
    10     9     8     7     6     5
```

Aside from creating series of numbers, the colon can also be used as a wildcard to select the individual rows and columns of a matrix. When a colon is used in place of a specific subscript, the colon represents the entire row or column. For example, the second row of the matrix `A` can be selected as in

```
>> A(2,:)

ans =
      4     5     6
```

We can also use the colon notation to selectively extract a series of elements from within an array. For example, based on the previous definition of the vector `t`:

```
>> t(2:4)

ans =
      9     8     7
```

Thus, the second through the fourth elements are returned.

2.2.4 The `linspace` and `logspace` Functions

The `linspace` and `logspace` functions provide other handy tools to generate vectors of spaced points. The `linspace` function generates a row vector of equally spaced points. It has the form

```
linspace(x1, x2, n)
```

which generates *n* points between *x1* and *x2*. For example

```
>> linspace(0,1,6)

ans =
         0    0.2000    0.4000    0.6000    0.8000    1.0000
```

If the *n* is omitted, the function automatically generates 100 points.

The `logspace` function generates a row vector that is logarithmically equally spaced. It has the form

```
logspace(x1, x2, n)
```

which generates *n* logarithmically equally spaced points between decades 10^{x1} and 10^{x2}. For example,

```
>> logspace(-1,2,4)

ans =
    0.1000    1.0000   10.0000  100.0000
```

If *n* is omitted, it automatically generates 50 points.

2.2.5 Character Strings

Aside from numbers, *alphanumeric* information or *character strings* can be represented by enclosing the strings within single quotation marks. For example,

```
>> f = 'Miles ';
>> s = 'Davis';
```

Each character in a string is one element in an array. Thus, we can *concatenate* (i.e., paste together) strings as in

```
>> x = [f s]

x =
Miles Davis
```

Note that very long lines can be continued by placing an *ellipsis* (three consecutive periods) at the end of the line to be continued. For example, a row vector could be entered as

```
>> a = [1 2 3 4 5 ...
6 7 8]

a =
     1     2     3     4     5     6     7     8
```

However, you cannot use an ellipsis within single quotes to continue a string. To enter a string that extends beyond a single line, piece together shorter strings as in

```
>> quote = ['Any fool can make a rule,' ...
' and any fool will mind it']

quote =
Any fool can make a rule, and any fool will mind it
```

2.3 MATHEMATICAL OPERATIONS

Operations with scalar quantities are handled in a straightforward manner, similar to other computer languages. The common operators, in order of priority, are

^	Exponentiation
–	Negation
* /	Multiplication and division
\	Left division[2]
+ –	Addition and subtraction

These operators will work in calculator fashion. Try

```
>> 2*pi

ans =
    6.2832
```

Also, scalar real variables can be included:

```
>> y = pi/4;
>> y ^ 2.45

ans =
    0.5533
```

Results of calculations can be assigned to a variable, as in the next-to-last example, or simply displayed, as in the last example.

As with other computer calculation, the priority order can be overridden with parentheses. For example, because exponentiation has higher priority then negation, the following result would be obtained:

```
>> y = -4 ^ 2

y =
   -16
```

Thus, 4 is first squared and then negated. Parentheses can be used to override the priorities as in

```
>> y = (-4) ^ 2

y =
    16
```

[2] Left division applies to matrix algebra. It will be discussed in detail later in this book.

Calculations can also involve complex quantities. Here are some examples that use the values of `x` $(2 + 4i)$ and `y` (16) defined previously:

```
>> 3 * x

ans =
   6.0000 + 12.0000i

>> 1 / x

ans =
   0.1000 - 0.2000i

>> x ^ 2

ans =
 -12.0000 + 16.0000i

>> x + y

ans =
  18.0000 + 4.0000i
```

The real power of MATLAB is illustrated in its ability to carry out vector-matrix calculations. Although we will describe such calculations in detail in Chap. 8, it is worth introducing some examples here.

The *inner product* of two vectors (dot product) can be calculated using the `*` operator,

```
>> a * b

ans =
   110
```

and likewise, the *outer product*

```
>> b * a

ans =
      2     4     6     8    10
      4     8    12    16    20
      6    12    18    24    30
      8    16    24    32    40
     10    20    30    40    50
```

To further illustrate vector-matrix multiplication, first redefine `a` and `b`:

```
>> a = [1 2 3];
```

and

```
>> b = [4 5 6]';
```

Now, try

```
>> a * A

ans =
    30    36    42
```

or

```
>> A * b
ans =
    32
    77
   122
```

Matrices cannot be multiplied if the inner dimensions are unequal. Here is what happens when the dimensions are not those required by the operations. Try

```
>> A * a
```

MATLAB automatically displays the error message:

```
??? Error using ==> mtimes
Inner matrix dimensions must agree.
```

Matrix-matrix multiplication is carried out in likewise fashion:

```
>> A * A
ans =
    30    36    42
    66    81    96
   102   126   150
```

Mixed operations with scalars are also possible:

```
>> A/pi
ans =
    0.3183    0.6366    0.9549
    1.2732    1.5915    1.9099
    2.2282    2.5465    2.8648
```

We must always remember that MATLAB will apply the simple arithmetic operators in vector-matrix fashion if possible. At times, you will want to carry out calculations item by item in a matrix or vector. MATLAB provides for that too. For example,

```
>> A^2
ans =
    30    36    42
    66    81    96
   102   126   150
```

results in matrix multiplication of `A` with itself.

What if you want to square each element of `A`? That can be done with

```
>> A.^2
ans =
     1     4     9
    16    25    36
    49    64    81
```

The `.` preceding the `^` operator signifies that the operation is to be carried out element by element. The MATLAB manual calls these *array operations*. They are also often referred to as *element-by-element operations.*

MATLAB contains a helpful shortcut for performing calculations that you've already done. Press the up-arrow key. You should get back the last line you typed in.

```
>> A.^2
```

Pressing Enter will perform the calculation again. But you can also edit this line. For example, change it to the line below and then press Enter.

```
>> A.^3

ans =
     1     8    27
    64   125   216
   343   512   729
```

Using the up-arrow key, you can go back to any command that you entered. Press the up-arrow until you get back the line

```
>> b * a
```

Alternatively, you can type `b` and press the up-arrow once and it will automatically bring up the last command beginning with the letter `b`. The up-arrow shortcut is a quick way to fix errors without having to retype the entire line.

2.4 USE OF BUILT-IN FUNCTIONS

MATLAB and its Toolboxes have a rich collection of built-in functions. You can use online help to find out more about them. For example, if you want to learn about the `log` function, type in

```
>> help log

 LOG    Natural logarithm.
    LOG(X) is the natural logarithm of the elements of X.
    Complex results are produced if X is not positive.

    See also LOG2, LOG10, EXP, LOGM.
```

For a list of all the elementary functions, type

```
>> help elfun
```

One of their important properties of MATLAB's built-in functions is that they will operate directly on vector and matrix quantities. For example, try

```
>> log(A)

ans =
         0    0.6931    1.0986
    1.3863    1.6094    1.7918
    1.9459    2.0794    2.1972
```

and you will see that the natural logarithm function is applied in array style, element by element, to the matrix `A`. Most functions, such as `sqrt`, `abs`, `sin`, `acos`, `tanh`, and `exp`, operate in array fashion. Certain functions, such as exponential and square root, have matrix

definitions also. MATLAB will evaluate the matrix version when the letter `m` is appended to the function name. Try

```
>> sqrtm(A)
ans =
    0.4498 + 0.7623i   0.5526 + 0.2068i   0.6555 - 0.3487i
    1.0185 + 0.0842i   1.2515 + 0.0228i   1.4844 - 0.0385i
    1.5873 - 0.5940i   1.9503 - 0.1611i   2.3134 + 0.2717i
```

There are several functions for rounding. For example, suppose that we enter a vector:

```
>> E = [-1.6 -1.5 -1.4 1.4 1.5 1.6];
```

The `round` function rounds the elements of `E` to the nearest integers:

```
>> round(E)
ans =
    -2    -2    -1     1     2     2
```

The `ceil` (short for ceiling) function rounds to the nearest integers toward infinity:

```
>> ceil(E)
ans =
    -1    -1    -1     2     2     2
```

The `floor` function rounds down to the nearest integers toward minus infinity:

```
>> floor(E)
ans =
    -2    -2    -2     1     1     1
```

There are also functions that perform special actions on the elements of matrices and arrays. For example, the `sum` function returns the sum of the elements:

```
>> F = [3 5 4 6 1];
>> sum(F)
ans =
    19
```

In a similar way, it should be pretty obvious what's happening with the following commands:

```
>> min(F),max(F),mean(F),prod(F),sort(F)
ans =
    1
ans =
    6
ans =
    3.8000
ans =
    360
ans =
    1    3    4     5     6
```

A common use of functions is to evaluate a formula for a series of arguments. Recall that the velocity of a free-falling bungee jumper can be computed with [Eq. (1.9)]:

$$v = \sqrt{\frac{gm}{c_d}} \tanh\left(\sqrt{\frac{gc_d}{m}} t\right)$$

where v is velocity (m/s), g is the acceleration due to gravity (9.81 m/s^2), m is mass (kg), c_d is the drag coefficient (kg/m), and t is time (s).

Create a column vector `t` that contains values from 0 to 20 in steps of 2:

```
>> t = [0:2:20]'
t =
     0
     2
     4
     6
     8
    10
    12
    14
    16
    18
    20
```

Check the number of items in the `t` array with the `length` function:

```
>> length(t)
ans =
    11
```

Assign values to the parameters:

```
>> g = 9.81; m = 68.1; cd = 0.25;
```

MATLAB allows you to evaluate a formula such as $v = f(t)$, where the formula is computed for each value of the t array, and the result is assigned to a corresponding position in the v array. For our case,

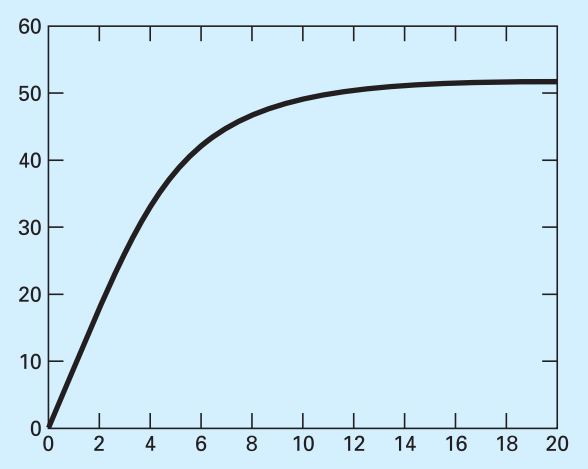
```
>> v = sqrt(g*m/cd)*tanh(sqrt(g*cd/m)*t)
v =
         0
   18.7292
   33.1118
   42.0762
   46.9575
   49.4214
   50.6175
   51.1871
   51.4560
   51.5823
   51.6416
```

2.5 GRAPHICS

MATLAB allows graphs to be created quickly and conveniently. For example, to create a graph of the `t` and `v` arrays from the data above, enter

```
>> plot(t, v)
```

The graph appears in the graphics window and can be printed or transferred via the clipboard to other programs.



You can customize the graph a bit with commands such as the following:

```
>> title('Plot of v versus t')
>> xlabel('Values of t')
>> ylabel('Values of v')
>> grid
```

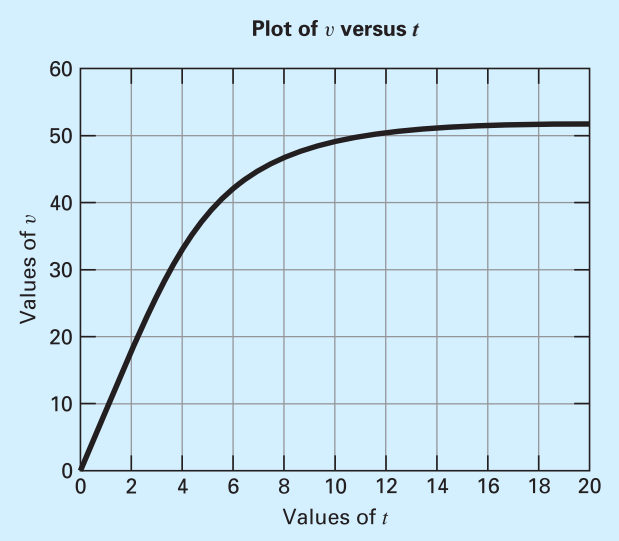


TABLE 2.2 Specifiers for colors, symbols, and line types.

Colors		Symbols		Line Types	
Blue	b	Point	.	Solid	-
Green	g	Circle	o	Dotted	:
Red	r	X-mark	x	Dashdot	-.
Cyan	c	Plus	+	Dashed	--
Magenta	m	Star	*		
Yellow	y	Square	s		
Black	k	Diamond	d		
White	w	Triangle(down)	v		
		Triangle(up)	^		
		Triangle(left)	<		
		Triangle(right)	>		
		Pentagram	p		
		Hexagram	h		

The `plot` command displays a solid thin blue line by default. If you want to plot each point with a symbol, you can include a specifier enclosed in single quotes in the `plot` function. Table 2.2 lists the available specifiers. For example, if you want to use open circles enter

```
>> plot(t, v, 'o')
```

You can also combine several specifiers. For example, if you want to use square green markers connected by green dashed lines, you could enter

```
>> plot(t, v, 's--g')
```

You can also control the line width as well as the marker's size and its edge and face (i.e., interior) colors. For example, the following command uses a heavier (2-point), dashed, cyan line to connect larger (10-point) diamond-shaped markers with black edges and magenta faces:

```
>> plot(x,y,'--dc','LineWidth',2,...
    'MarkerSize',10,...
    'MarkerEdgeColor','k',...
    'MarkerFaceColor','m')
```

Note that the default line width is 1 point. For the markers, the default size is 6 point with blue edge color and no face color.

MATLAB allows you to display more than one data set on the same plot. For example, an alternative way to connect each data marker with a straight line would be to type

```
>> plot(t, v, t, v, 'o')
```

It should be mentioned that, by default, previous plots are erased every time the `plot` command is implemented. The `hold on` command holds the current plot and all axis properties so that additional graphing commands can be added to the existing plot. The `hold off` command returns to the default mode. For example, if we had typed the following commands, the final plot would only display symbols:

```
>> plot(t, v)
>> plot(t, v, 'o')
```

In contrast, the following commands would result in both lines and symbols being displayed:

```
>> plot(t, v)
>> hold on
>> plot(t, v, 'o')
>> hold off
```

In addition to `hold`, another handy function is `subplot`, which allows you to split the graph window into subwindows or *panes*. It has the syntax

```
subplot(m, n, p)
```

This command breaks the graph window into an m-by-n matrix of small axes, and selects the p-th axes for the current plot.

We can demonstrate `subplot` by examining MATLAB's capability to generate three-dimensional plots. The simplest manifestation of this capability is the `plot3` command which has the syntax

```
plot3(x, y, z)
```

where x, y, and z are three vectors of the same length. The result is a line in three-dimensional space through the points whose coordinates are the elements of x, y, and z.

Plotting a helix provides a nice example to illustrate its utility. First, let's graph a circle with the two-dimensional `plot` function using the parametric representation: $x = \sin(t)$ and $y = \cos(t)$. We employ the `subplot` command so we can subsequently add the three-dimensional plot.

```
>> t = 0:pi/50:10*pi;
>> subplot(1,2,1);plot(sin(t),cos(t))
>> axis square
>> title('(a)')
```

As in Fig. 2.1*a*, the result is a circle. Note that the circle would have been distorted if we had not used the `axis square` command.

Now, let's add the helix to the graph's right pane. To do this, we again employ a parametric representation: $x = \sin(t)$, $y = \cos(t)$, and $z = t$

```
>> subplot(1,2,2);plot3(sin(t),cos(t),t);
>> title('(b)')
```

The result is shown in Fig. 2.1*b*. Can you visualize what's going on? As time evolves, the x and y coordinates sketch out the circumference of the circle in the x–y plane in the same fashion as the two-dimensional plot. However, simultaneously, the curve rises vertically as the z coordinate increases linearly with time. The net result is the characteristic spring or spiral staircase shape of the helix.

There are other features of graphics that are useful—for example, plotting objects instead of lines, families of curves plots, plotting on the complex plane, log-log or semilog plots, three-dimensional mesh plots, and contour plots. As described next, a variety of resources are available to learn about these as well as other MATLAB capabilities.

2.6 OTHER RESOURCES

The foregoing was designed to focus on those features of MATLAB that we will be using in the remainder of this book. As such, it is obviously not a comprehensive overview of all of MATLAB's capabilities. If you are interested in learning more, you should consult one

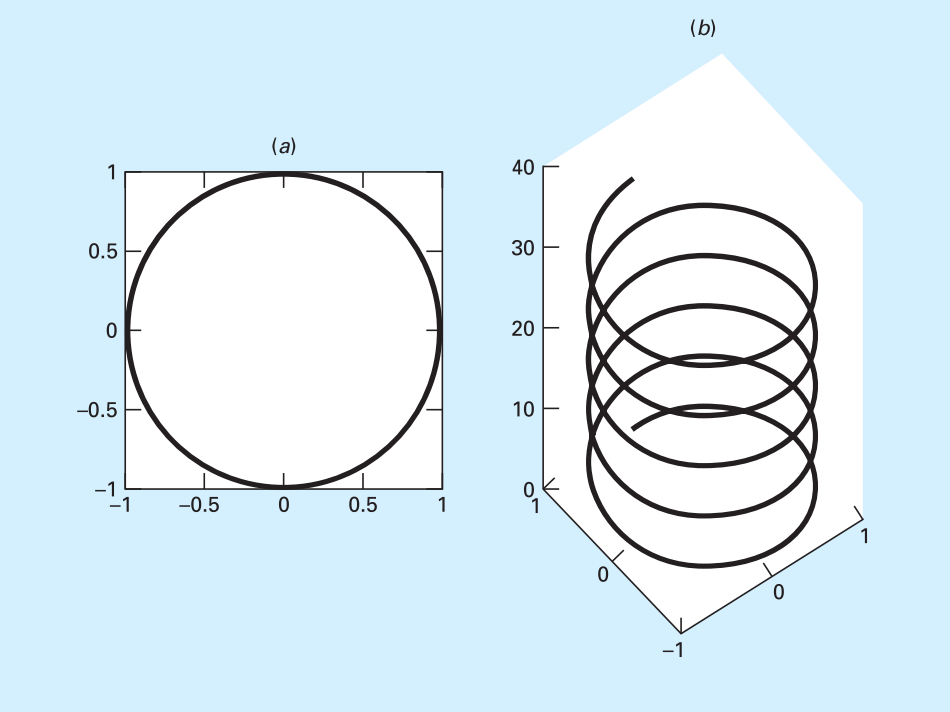


FIGURE 2.1
A two-pane plot of (*a*) a two-dimensional circle and (*b*) a three-dimensional helix.

of the excellent books devoted to MATLAB (e.g., Attaway, 2009; Palm, 2007; Hanselman and Littlefield, 2005; and Moore, 2008).

Further, the package itself includes an extensive Help facility that can be accessed by clicking on the Help menu in the command window. This will provide you with a number of different options for exploring and searching through MATLAB's Help material. In addition, it provides access to a number of instructive demos.

As described in this chapter, help is also available in interactive mode by typing the `help` command followed by the name of a command or function.

If you do not know the name, you can use the `lookfor` command to search the MATLAB Help files for occurrences of text. For example, suppose that you want to find all the commands and functions that relate to logarithms, you could enter

```
>> lookfor logarithm
```

and MATLAB will display all references that include the word `logarithm`.

Finally, you can obtain help from The MathWorks, Inc., website at www.mathworks.com. There you will find links to product information, newsgroups, books, and technical support as well as a variety of other useful resources.