KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

CHEMICAL ENGINEERING DEPARTMENT

CHE 251: CHEMICAL PROCESS CALCULATIONS

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LECTURE 2: INTRODUCTION TO ENGINEERING CALCULATIONS

Learning Objectives

At the end of the lecture the student is expected to able to do the following:

- Add, subtract, multiply and divide units associated with numbers
- Convert one set of units in a function or equation into another equivalent set for mass, length, area, volume, time, energy and force
- Specify the basic and derived units in the SI and American engineering systems
- Explain the difference between weight and mass
- Define and know how to use the gravitational conversion factor g_c.

UNITS AND DIMENSIONS

Dimensions are our basic concept of measurement such as length, time, mass, temperature, and so on; **units** are means of expressing the dimensions, such as feet or centimetre for length, hours or seconds for time.

A measured or counted quantity has a numerical value (2.47) and a unit (whatever there are 2.47 of)It is essential in engineering calculations to write both the value and the unit of each quantity appearing in an equation: 2 meters, 1 second, 4.29 kilograms etc.

Units can be treated like algebraic variables when quantities are added, subtracted, multiplied, or divided. The numerical value of two quantities may be added or subtracted only if the units are the same.

Thus the operation 5 kilograms + 3 joules is meaningless because the dimensions of the two terms are different. The numerical operation 10 pounds + 5 grams can be performed (because the dimensions are the same, mass) only after the units are transformed to be the same, either pounds, or grams, or ounces and so on.

$$3\text{ cm} - 1\text{ cm} = 2\text{ cm} \qquad (3x - x = 2x)$$

but

$$3\text{ cm} - 1\text{ mm (or 1 s)} = ? \qquad (3x - y = ?)$$

On the other hand, *numerical values and their corresponding units may always be combined by multiplication or division.*

$$3\text{ N} \times 4\text{ m} = 12\text{ N}\cdot\text{m}$$

$$\frac{5.0\text{ km}}{2.0\text{ h}} = 2.5\text{ km/h}$$

$$7.0\ \frac{\text{km}}{\text{h}} \times 4\text{ h} = 28\text{ km}$$

$$3\text{ m} \times 4\text{ m} = 12\text{ m}^2$$

$$6\ \text{cm} \times 5\ \frac{\text{cm}}{\text{s}} = 30\ \text{cm}^2/\text{s}$$

$$\frac{6\ \text{g}}{2\ \text{g}} = 3 \qquad \text{(3 is a \textit{dimensionless} quantity)}$$

$$\left(5.0\ \frac{\text{kg}}{\text{s}}\right) \Big/ \left(0.20\ \frac{\text{kg}}{\text{m}^3}\right) = 25\ \text{m}^3/\text{s} \qquad \text{(Convince yourself)}$$

CONVERSION OF UNITS

A measured quantity can be expressed in terms of any units having the appropriate dimension. A particular velocity, for instance, may be expressed in ft/s, miles/h, cm/yr, or any other ratio of a length unit to a time unit. The numerical value of the velocity naturally depends on the units chosen.

The equivalence between two expressions of the same quantity may be defined in terms of a ratio:

$$\frac{1\ \text{cm}}{10\ \text{mm}} \qquad \text{(1 centimeter per 10 millimeters)}$$

$$\frac{10\ \text{mm}}{1\ \text{cm}} \qquad \text{(10 millimeters per centimeter)}$$

$$\left[\frac{10\ \text{mm}}{1\ \text{cm}}\right]^2 = \frac{100\ \text{mm}^2}{1\ \text{cm}^2}$$

The ratios of the form above are known as **conversion factors.**

To convert a quantity expressed in terms of one unit to its equivalent in terms of another unit, multiply the given quantity by the conversion factor (new unit/old unit). For example, to convert 36 mg to its equivalent in grams, write

$$(36\ \text{mg}) \times \left(\frac{1\ \text{g}}{1000\ \text{mg}}\right) = 0.036\ \text{g}$$

(Note how the old units cancel, leaving the desired unit.) An alternative way to write this equation is to use a vertical line instead of the multiplication symbol:

$$\begin{array}{c|c} 36\ \text{mg} & 1\ \text{g} \\ \hline & 1000\ \text{mg} \end{array} = 0.036\ \text{g}$$

Carrying along units in calculations of this type is the best way of avoiding the common mistake of multiplying when you mean to divide and vice versa. In the given example, the result is known to be correct because milligrams cancel leaving only grams on the left side, whereas

$$\frac{36\text{ mg} \mid 1000\text{ mg}}{\mid 1\text{ g}} = 36{,}000\text{ mg}^2/\text{g}$$

is clearly wrong. (More precisely, it is not what you intended to calculate.)

If you are given a quantity having a compound unit [e.g., miles/h, cal/(g·°C)], and you wish to convert it to its equivalent in terms of another set of units, set up a **dimensional equation**: write the given quantity and its units on the left, write the units of conversion factors that cancel the old units and replace them with the desired ones, fill in the values of the conversion factors, and carry out the indicated arithmetic to find the desired value.

Change 400 in.^{3}/day to cm^3/min.

Solution

$$\frac{400\text{ in.}^3}{\text{day}}\left(\frac{2.54\text{ cm}}{1\text{ in.}}\right)^3\frac{1\text{ day}}{24\text{ hr}}\,\frac{1\text{ hr}}{60\text{ min}} = 4.56\,\frac{\text{cm}^3}{\text{min}}$$

In this example note that not only are the numbers raised to a power, but the units also are raised to the same power.

Convert an acceleration of 1 cm/s^2 to its equivalent in km/yr^2.

$$\frac{1\text{ cm}}{\text{s}^2}\,\frac{3600^2\text{ s}^2}{1^2\text{ h}^2}\,\frac{24^2\text{ h}^2}{1^2\text{ day}^2}\,\frac{365^2\text{ day}^2}{1^2\text{ yr}^2}\,\frac{1\text{ m}}{10^2\text{ cm}}\,\frac{1\text{ km}}{10^3\text{ m}}$$

$$= \frac{(3600 \times 24 \times 365)^2}{10^2 \times 10^3}\,\frac{\text{km}}{\text{yr}^2} = \boxed{9.95 \times 10^9\text{ km/yr}^2}$$

A principle illustrated in this example is that raising a quantity (in particular, a conversion factor) to a power raises its units to the same power. The conversion factor for h^2/day^2 is therefore the square of the factor for h/day:

$$\left(\frac{24\text{ h}}{1\text{ day}}\right)^2 = 24^2\,\frac{\text{h}^2}{\text{day}^2}$$

SYSTEMS OF UNITS

A system of units has the following components:

1. ***Base units*** for mass, length, time, temperature, electrical current, and light intensity.
2. ***Multiple units,*** which are defined as multiples or fractions of base units such as minutes, hours, and milliseconds, all of which are defined in terms of the base unit of a second. Multiple units are defined for convenience rather than necessity: it is simply more convenient to refer to 3 yr than to 94,608,000 s.
3. ***Derived units,*** obtained in one of two ways:
   (a) By multiplying and dividing base or multiple units (cm^2, ft/min, $kg \cdot m/s^2$, etc.). Derived units of this type are referred to as **compound units.**
   (b) As defined equivalents of compound units (e.g., 1 erg $\equiv$ ($1 g \cdot cm/s^2$), 1 $lb_f \equiv 32.174\ lb_m \cdot ft/s^2$).

The "Système Internationale d'Unités," or **SI** for short, has gained widespread acceptance in the scientific and engineering community. Two of the base SI units—the ampere for electrical current and the candela for luminous intensity—will not concern us in this book. A third, the kelvin for temperature, will be discussed later. The others are the meter (m) for length, the kilogram (kg) for mass, and the second (s) for time.

Prefixes are used in SI to indicate powers of ten. The most common of these prefixes and their abbreviations are mega (M) for 10^6 (1 megawatt = 1 MW = 10^6 watts), kilo (k) for 10^3, centi (c) for 10^{-2}, milli (m) for 10^{-3}, micro (μ) for 10^{-6}, and nano (n) for 10^{-9}. The conversion factors between, say, centimeters and meters are therefore 10^{-2} m/cm and 10^2 cm/m. The principal SI units and prefixes are summarized in Table 2.3-1.

The **CGS system** is almost identical to SI, the principal difference being that grams (g) and centimeters (cm) are used instead of kilograms and meters as the base units of mass and length. The principal units of the CGS system are shown in Table 2.3-1.

The base units of the **American engineering system** are the foot (ft) for length, the pound-mass (lb_m) for mass, and the second (s) for time. This system has two principal difficulties. The first is the occurrence of conversion factors (such as 1 ft/12 in), which, unlike those in the metric systems, are not multiples of 10; the second, which has to do with the unit of force, is discussed in the next section.

Table 1: SI and CGS Systems

Base Units		
Quantity	Unit	Symbol
Length	meter (SI) centimeter (CGS)	m cm
Mass	kilogram (SI) gram (CGS)	kg g
Moles	gram-mole	mol or g-mole
Time	second	s
Temperature	kelvin	K
Electric current	ampere	A
Light intensity	candela	cd

Multiple Unit Preferences

tera (T) $= 10^{12}$	centi (c) $= 10^{-2}$
giga (G) $= 10^{9}$	milli (m) $= 10^{-3}$
mega (M) $= 10^{6}$	micro (μ) $= 10^{-6}$
kilo (k) $= 10^{3}$	nano (n) $= 10^{-9}$

Derived Units			
Quantity	Unit	Symbol	Equivalent in Terms of Base Units
Volume	liter	L	$0.001\ m^3$ $1000\ cm^3$
Force	newton (SI) dyne (CGS)	N	$1\ kg \cdot m/s^2$ $1\ g \cdot cm/s^2$
Pressure	pascal (SI)	Pa	$1\ N/m^2$
Energy, work	joule (SI) erg (CGS) gram-calorie	J cal	$1\ N \cdot m = 1\ kg \cdot m^2/s^2$ $1\ dyne \cdot cm = 1\ g \cdot cm^2/s^2$ $4.184\ J = 4.184\ kg \cdot m^2/s^2$
Power	watt	W	$1\ J/s = 1\ kg \cdot m^2/s^3$

1. What are the factors (numerical values and units) needed to convert
   (a) meters to millimeters?
   (b) nanoseconds to seconds?
   (c) square centimeters to square meters?
   (d) cubic feet to cubic meters (use the conversion factor table on the inside front cover)?
   (e) horsepower to British thermal units per second?
2. What is the derived SI unit for velocity? The velocity unit in the CGS system? In the American engineering system?

Conversion Between Systems of Units

Convert 23 $lb_m \cdot ft/min^2$ to its equivalent in $kg \cdot cm/s^2$.

As before, begin by writing the dimensional equation, fill in the units of conversion factors (new/old) and then the numerical values of these factors, and then do the arithmetic. The result is

$$\frac{23\ \text{lb}_m \cdot \text{ft}}{\text{min}^2}\left|\frac{0.453593\ \text{kg}}{1\ \text{lb}_m}\right|\frac{100\ \text{cm}}{3.281\ \text{ft}}\left|\frac{1^2\ \text{min}^2}{(60)^2\ \text{s}^2}\right. \quad \text{(Cancellation of units leaves kg·cm/s}^2\text{)}$$

$$= \frac{(23)(0.453593)(100)}{(3.281)(3600)}\frac{\text{kg}\cdot\text{cm}}{\text{s}^2} = \boxed{0.088\ \frac{\text{kg}\cdot\text{cm}}{\text{s}^2}}$$

FORCE AND WEIGHT

According to Newton's second law of motion, force is proportional to the product of mass and acceleration (length/time^2). *Natural force units* are, therefore, kg·m/s^2 (SI), g·cm/s^2 (CGS), and lb_m·ft/s^2 (American engineering). To avoid having to carry around these complex units in all calculations involving forces, *derived force units* have been defined in each system. In the metric systems, the derived force units (the newton in SI, the dyne in the CGS system) are defined to equal the natural units:

$$1\ \text{newton (N)} \equiv 1\ \text{kg}\cdot\text{m/s}^2$$

$$1\ \text{dyne} \equiv 1\ \text{g}\cdot\text{cm/s}^2$$

In the American engineering system, the derived force unit--called a pound-force (lb_f)-is defined as the product of a unit mass (1 lb_m) and the acceleration of gravity at sea level and 45° latitude, which is 32.174 ft/s^2:

$$1\ \text{lb}_f \equiv 32.174\ \text{lb}_m\cdot\text{ft/s}^2$$

The above equations define conversion factors between natural and derived force units.
For example: the force in newtons required to accelerate a mass of 4.00 kg at a rate of 9.00 rn/s^2 is

$$F = \frac{4.00\ \text{kg}}{}\left|\frac{9.00\ \text{m}}{\text{s}^2}\right|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2} = 36.0\ \text{N}$$

The force in lb_f required to accelerate a mass of 4.00 lb_m at a rate of 9.00 ft/s^2 is

$$F = \frac{4.00\ \text{lb}_m}{}\left|\frac{9.00\ \text{ft}}{\text{s}^2}\right|\frac{1\ \text{lb}_f}{32.174\ \text{lb}_m\cdot\text{ft/s}^2} = 1.12\ \text{lb}_f$$

The symbol g_c is sometimes used to denote the conversion factor from natural to derived force units: for example,

$$g_c = \frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}} = \frac{32.174\ \text{lb}_m\cdot\text{ft/s}^2}{1\ \text{lb}_f}$$

The weight of an object is the force exerted on the object by gravitational attraction. Suppose that an object of mass *m* is subjected to a gravitational force *W (W* is by definition the weight of the object) and that if this object were falling freely its acceleration would be *g*. The weight, mass, and free-fall acceleration of the object are related by ***W = mg***

The gravitational acceleration *(g)* varies directly with the mass of the attracting body (the earth, in most problems you will confront) and inversely with the square of the distance between the centers of mass of the attracting body and the object being attracted. The value of *g* at sea level and 45° latitude is given below in each system of units:

$$\boxed{\begin{aligned} g &= 9.8066\ \text{m/s}^2 \\ &= 980.66\ \text{cm/s}^2 \\ &= 32.174\ \text{ft/s}^2 \end{aligned}}$$

The acceleration of gravity does not vary much with position on the earth's surface and (within moderate limits) altitude and the values above may accordingly be used for most conversions between mass and weight.

Weight and Mass

Water has a density of 62.4 Ib_m/ft^3. How much does 2.000 ft^3 of water weigh (1) at sea level and 45° latitude and (2) in Denver, Colorado, where the altitude is 5374 ft and the gravitational acceleration is 32.139 ft/s^2?

The mass of the water is

$$M = \left(62.4\ \frac{\text{lb}_\text{m}}{\text{ft}^3}\right)(2\ \text{ft}^3) = 124.8\ \text{lb}_\text{m}$$

The weight of the water is

$$W = (124.8\ \text{lb}_\text{m})g\left(\frac{\text{ft}}{\text{s}^2}\right)\left(\frac{1\ \text{lb}_\text{f}}{32.174\ \text{lb}_\text{m}\cdot\text{ft/s}^2}\right)$$

1. At sea level, $g = 32.174\ \text{ft/s}^2$, so that $W = 124.8\ \text{lb}_\text{f}$.
2. In Denver, $g = 32.139\ \text{ft/s}^2$, and $W = 124.7\ \text{lb}_\text{f}$.

As this example illustrates, the error incurred by assuming that *9* = 32.174 ft/s^2 is normally quite small as long as you remain on the earth's surface. In a satellite or on another planet it would be a different story.