Assignment 1 - Solution

Q.1. The CDF is defined by $F_X(x) = P(X \le x)$. Thus,

$$F_X(x) = \begin{cases} 0 & x < 3 \\ P_X(3) = 0.3 & 3 \le x < 5 \\ P_X(3) + P_X(5) = 0.5 & 5 \le x < 8 \\ P_X(3) + P_X(5) + P_X(8) = 0.8 & 8 \le x < 10 \\ 1 & x \geq 10 \end{cases}$$

Q.2

First find the density function of X: $f(y) = \frac{d}{dy}(F(y))$

$$\Rightarrow f(y) = \begin{cases} \frac{1}{2} & \text{if } y = 1 \\ y-1 & \text{if } 1 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

Hence;

$$E(y) = \frac{1}{2} + \int_1^2 y(y-1)\,dy$$

$$= \frac{1}{2} + \left[\frac{1}{3}y^3 - \frac{1}{2}y^2\right]_1^2$$

$$= \frac{1}{2} + \frac{8}{3} - \frac{4}{2} - \frac{1}{3} + \frac{1}{2}$$

$$= \frac{7}{3} - 1 = \frac{4}{3}$$

Also; $E(y^2) = \frac{1}{2} + \int_1^2 y^2(y-1)\,dy$

$$= \frac{1}{2} + \left[\frac{1}{4}y^4 - \frac{1}{3}y^3\right]\Big|_1^2$$

$$= \frac{1}{2} + \frac{16}{4} - \frac{8}{3} - \frac{1}{4} + \frac{1}{3}$$

$$= \frac{23}{12}$$

$\therefore Var(Y) = E(Y^2) - E(Y)^2 = \frac{23}{12} - \left(\frac{4}{3}\right)^2$

$$= \frac{5}{36}$$

Q.3

Let D_1 = First Delay
D_2 = Second Delay
D_3 = Third Delay

$\therefore P(D_1) = 6/10$

$P(D_2 | D_1) = 2/10$

$P(D_3 | D_1 \cap D_2) = 1/10$

$$P(D_2 | D_1) = \frac{P(D_1 \cap D_2)}{P(D_1)}$$

$$\Rightarrow P(D_1 \cap D_2) = 2/10 \times 6/10$$

$$= 3/25$$

Also; $P(D_3 | D_1 \cap D_2) = \frac{P(D_1 \cap D_2 \cap D_3)}{P(D_1 \cap D_3)}$

$$= 1/10 \times 3/25$$

$$= 3/250 = 0.012$$

$\therefore$ Probability that there is delay in all the 3 transfers is 0.012

Q.4: [BONUS]

Given $Var(2A - B) = 6$ and $Var(A + 2B) = 9$

$\Rightarrow Var(2A - B) = 6$

$= 4Var(A) + var(B) = 6$ — (1)

$var(A) + 4var(B) = 9$ — (2)

Solve simultaneously to get:

a) $var(A) = 1$ and $var(B) = 2$

b) $Var(2A + 3B) = 4Var(A) + 9var(B)$

$= 4(1) + 9(2)$

$= 22$