CHAPTER 2

STRUTS AND COLUMNS

Chapter Outline

(A) EULER CRIPPING LOAD FORMULA

1. Definition of Strut
2. The Euler's Formula

(B) EMPIRICAL METHODS

1. Rankine Formula
2. Perry-Robertson's Formula

EULER CRIPPLING LOAD FORMULA

Definition of Strut

- A structural member, subjected to an axial compressive force, is called a strut.

- A strut may be
  - horizontal,
  - inclined or
  - even vertical.

- A vertical strut, used in buildings or frames is called a *column*.

- A strut is subjected to some compressive force, then the compressive stress induced,

$$\sigma = \frac{p}{A}$$

Euler’s Formula

Assumptions in the Euler's Column Theory

The following simplifying assumptions are made in the Euler's column theory:

- Initially the column is perfectly straight and the load applied is truly axial.
- The cross-section of the column is uniform throughout its length.
- The column material is perfectly elastic, homogeneous and isotropic and thus obeys Hooke’s law.
- The length of column is very large as compared to its cross-sectional dimensions.
- The shortening of column, due to direct compression (being very small) is neglected.
- The failure of column occurs due to buckling alone.

Euler's Formula

Types of End Conditions of Columns

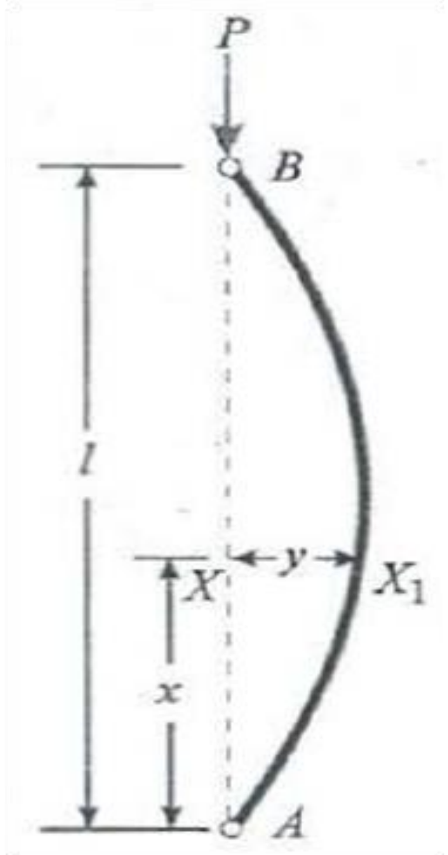
- In actual practice, there are a number of end conditions, for columns.
- But, we shall study the Euler's column theory based on the following four types of end conditions:

1. Both ends hinged
2. Both ends fixed
3. One end is fixed and the other hinged, and
4. One end is fixed and the other free.

- Now we shall discuss the value of critical load for all the above mentioned types of end conditions one after the other.

Euler's Formula

***Case 1:* Both Ends Hinged**



Now consider any section $X\text{-}X_1$, at a distance x from A. Let P denote the critical load on the column, and y, the deflection of the column at section $X\text{-}X_1$

Moment due to the critical load P is given by

$$M = -Py$$

Differential Equation

$$\frac{d^2y}{dx^2} + \alpha^2 y = 0$$

where

$$\alpha^2 = P/EI$$

Euler's Formula

Solution $$y = A\sin\alpha x + B\cos\alpha x$$

Boundary Conditions

At x = 0; y = 0; $$\therefore B = 0$$

At x==*l*; y==0; $$\therefore A\sin\alpha l = 0$$

Since $A \neq 0$, then $\sin\alpha l = 0$, therefore $\alpha l = \pi$

$$\alpha^2 = \pi^2 / l^2 = P/EI$$

Hence, the Euler load $P_e = \pi^2 EI / l^2$

Example 2.1

A straight bar of alloy, 1 m long and 12·5 mm by 4·8 mm in section, is mounted in a strut-testing machine and loaded axially until it buckles. Assuming the Euler formula to apply, estimate the maximum central deflection before the material attains its yield point of 280 N/mm^2. E = 72,000 N/mm^2.

Solution

There will be no deflection at all until the Euler load is reached, i.e.

$$load = \left(\frac{\pi}{l}\right)^2 EI = \left(\frac{\pi}{1000}\right)(72000)\left[\frac{(12.5)(4.8^3)}{12}\right] = 82N$$

Maximum bending moment

$$P\delta = 82\delta$$

Maximum bending stress $\sigma_m = \dfrac{My}{I_x}$

Maximum stress is the sum of direct and bending stresses at the centre

$$280 = \frac{82}{(12.5)(4.8)} + \frac{82\delta(6)}{(12.5)(4.8^2)} = 1.37 + 1.71\delta$$

$$\Rightarrow \delta = 163mm$$

Example 2.2

A steel rod 5 m long and of 40 mm diameter is used as a column, with one end fixed and the other free. Determine the crippling load by Euler's formula. Take E as 200 GPa

Solution

Given : Length (l) = 5 m = 5 x 10^3 mm; Diameter of column (d) = 40 mm and modulus of elasticity (E) = 200 GPa = 200 x 10^3 N/mm^2

Moment of inertia of the column section $$I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(40)^4 = 40{,}000\pi \ mm^4$$

Euler's crippling load $$P_E = \frac{\pi^2 EI}{4l^2} = \frac{\pi^2 (200 \times 10^3)(40000\pi)}{4 \times (5000)^2} = 2480 \ N$$

Example 2.3

A hollow alloy tube 4 m long with external and internal diameters of 40 mm and 25 mm respectively was found to extend 4.8 mm under a tensile load of 60 kN. Find the buckling load for the tube with both ends pinned. Also find the safe load on the tube, taking a factor of safety as 5.

Solution

Given: Length l = 4 m ; External diameter of column (D) = 40 mm; Internal diameter of column (d) = 25 mm ; Extension (δl) = 4.8 mm ; Tensile load = 60 kN = 60 x 10^3 N and factor of safety = 5.

Area of the tube $$A = \frac{\pi}{4}\left(D^2 - d^2\right) = \frac{\pi}{4}\left(40^2 - 25^2\right) = 765.8\, mm^2$$

Moment of inertia of the tube $$I = \frac{\pi}{64}\left(D^4 - d^4\right) = \frac{\pi}{4}\left(40^4 - 25^4\right) = 106{,}500\, mm^4$$

Example 2.3 (continued)

Strain in the alloy tube $\varepsilon = \dfrac{\delta l}{l} = \dfrac{4.8}{4000} = 0.0012$

The modulus of elasticity for the alloy $E = \dfrac{P}{A\varepsilon} = \dfrac{60000}{(765.8)(0.0012)} = 65{,}290\ N/mm^2$

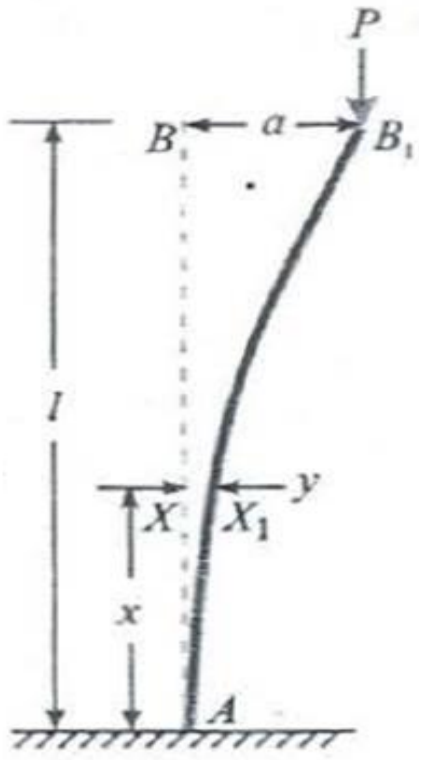
Euler's buckling load $P_E = \pi^2 EI \big/ L_e^2 = \dfrac{\pi^2 (65290)(106500)}{(4000)^2} = 4290\ N$

Safe load for the tube

$$Safe\, load = \frac{Buckling\, load}{Factor\ of\ safety} = \frac{4290\ N}{5} = 858\ N$$

Euler's Formula

Case 2: One End Fixed; Other Free



Now consider any section *X*, at a distance *x* from *A*.

Let *P* be the critical load on the column, and

y the deflection of the column at X-X_1

Moment due to the critical load *P*,

$$M = P(a - y) = -P(y - a)$$

Differential Equation

$$\frac{d^2 y}{dx^2} + \alpha^2 y = \alpha^2 a$$

Euler’s Formula

Solution $$y = A\sin\alpha x + B\cos\alpha x + a$$

Boundary Condition

$$x = 0; \quad y = 0; \quad B + a = 0 \Rightarrow B = -a$$

$$\frac{dy}{dx} = A\alpha\cos\alpha x - B\alpha\sin\alpha x$$

$$x = 0; \quad \frac{dy}{dx} = 0; \quad \therefore A\alpha = 0$$

$$\alpha \neq 0; \quad A = 0$$

Therefore

$$y = -a\cos\alpha x + a$$

Boundary Condition

$$x = l; \quad y = a$$

$$\therefore a = a - a\cos\alpha l$$

$$\Rightarrow 1 = 1 - \cos\alpha l$$

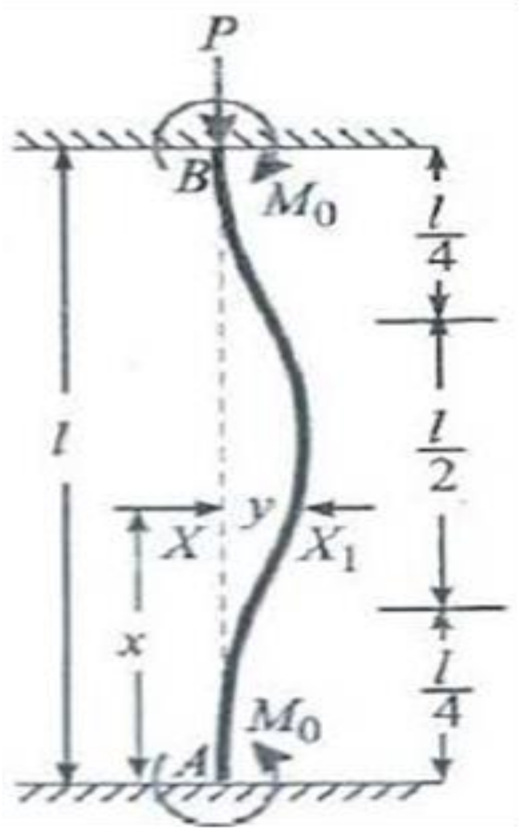
$$\therefore \alpha l = \pi/2$$

Hence, the Euler load

$$P_e = \pi^2 EI \big/ 4l^2$$

Euler's Formula

Case 3: Both Ends Fixed



Moment due to the critical load *P*

$$M = -Py + M_0$$

Differential Equation

$$\frac{d^2y}{dx^2} + \alpha^2 y = M_0/(EI)$$

Euler's Formula

Solution

$$y = A\sin\alpha x + B\cos\alpha x + M_0 / EI\alpha^2$$

Boundary Condition

$x = 0; \quad y = 0;$

$\therefore B = -M_0 / EI\alpha^2 = -M_0 / P$

$$x = 0; \quad \frac{dy}{dx} = 0; \quad \therefore A\alpha = 0$$

$$\alpha \neq 0; \quad A = 0$$

Therefore $\quad y = \left(\dfrac{M_0}{P}\right)(1 - \cos\alpha x)$

Boundary Condition

$$x = l; \quad y = 0; \quad \therefore \cos\alpha l = 1$$

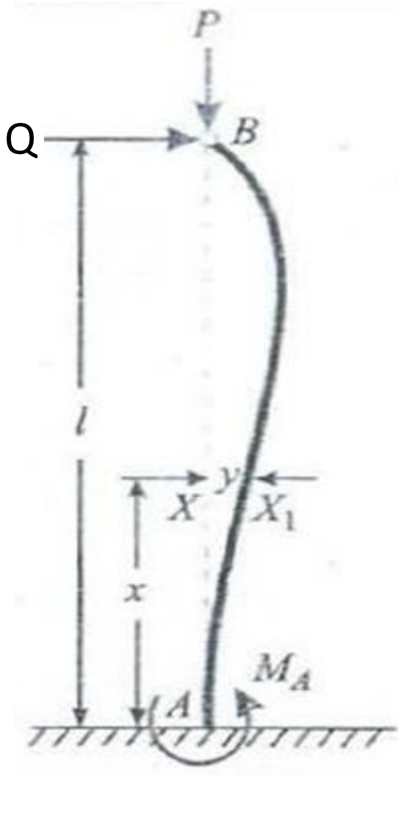
$$\alpha l = 2\pi$$

Hence, the Euler load

$$P_e = 4\pi^2 EI \Big/ l^2$$

Euler's Formula

Case 4: One End Fixed; Other Hinged



Now consider any section X-X_1, at a distance x from A.

Let P denote the axial load on the column

Q the lateral force required hold end B in position, and

y the deflection of the column at section X-X_1

Moment due to the critical load P, $M = -Py + Q(l - x)$

Differential Equation $$\frac{d^2 y}{dx^2} + \alpha^2 y = \frac{Q}{EI}(l - x)$$

Solution $$\therefore y = A\sin\alpha x + B\cos\alpha x + \frac{Q}{P}(l - x)$$

Euler's Formula

Boundary Condition $x = 0; \quad y = 0; \quad \therefore B = -\dfrac{Ql}{P}$

Therefore $x = l; \quad y = 0; \quad \dfrac{dy}{dx} = 0 \quad \therefore \tan \alpha l = \alpha l = 4.493$

$$\Rightarrow \alpha = 4.493/l$$

$$\alpha^2 = \frac{P}{EI} \Rightarrow P = \alpha^2 EI = \frac{2.047\pi^2 EI}{L^2}$$

Hence, the Euler load $P_e = 2.07\pi^2 EI \big/ l^2$

Euler’s Formula

Euler's Formula and Equivalent length of a Column

General equation for Euler 's formula

$$P_E = \pi^2 EI \big/ L_e^2$$

where L_e is the equivalent or effective length of column.

Table 1: The equivalent lengths (L) for the given end conditions

S.No.	End conditions	Relation between equivalent length (L_e) and actual length (l)	Crippling load (P)
1.	Both ends hinged	$L_e = l$	$P = \frac{\pi^2 EI}{(l)^2} = \frac{\pi^2 EI}{l^2}$
2.	One end fixed and the other free	$L_e = 2\,l$	$P = \frac{\pi^2 EI}{(2l)^2} = \frac{\pi^2 EI}{4l^2}$
3.	Both ends fixed	$L_e = \frac{l}{2}$	$P = \frac{\pi^2 EI}{\left(\frac{l}{2}\right)^2} = \frac{4\pi^2 EI}{l^2}$
4.	One end fixed and the other hinged	$L_e = \frac{l}{\sqrt{2}}$	$P = \frac{\pi^2 EI}{\left(\frac{l}{\sqrt{2}}\right)^2} = \frac{2\pi^2 EI}{l^2}$

Sample Problems on Euler’s Formula

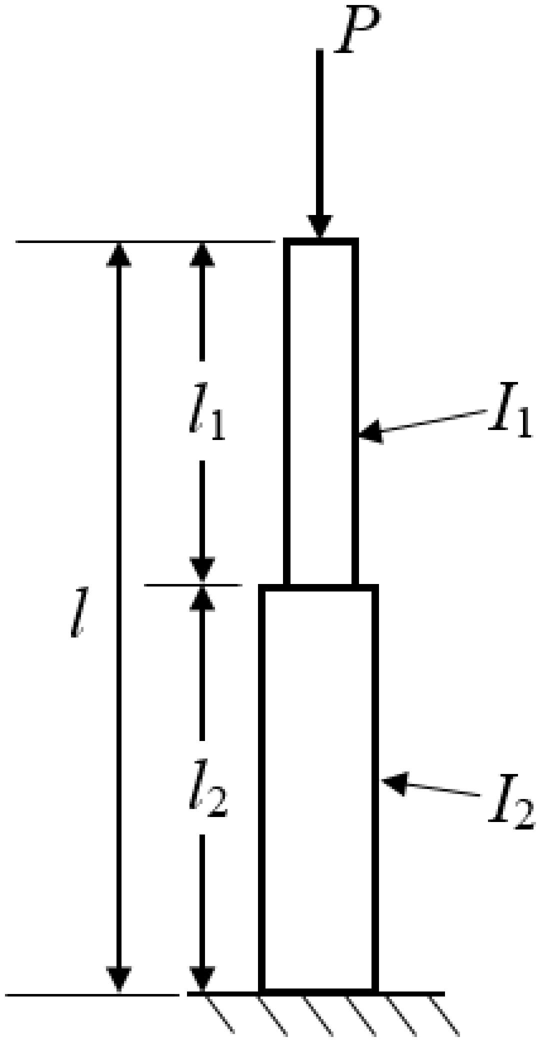
1. A solid rectangular bar 60 mm by 45 mm is used as a strut. Determine the Euler crippling load for the following end conditions. Take E = 200 GPa.

(a) Both ends of the strut are hinged
(b) One end fixed and the other end is free
(c) One end is fixed and the end is hinged
(d) Both ends of the strut are fixed

2. A simply supported beam of length 5 m is subjected to a central point load of magnitude 200 kN. Under the action of the load, the beam experiences a deflection of 20 mm at the center. Determine the Euler crippling load when the beam is used as a column with one end fixed and the other end hinged.

3. A steel bar of rectangular cross section 25 mm by 50 mm with hinged ends is axially compressed. Determine the minimum length at which the Euler crippling formula applies if E = 200 GPa. Also, determine the magnitude of the critical stress if the length of the bar is 6m.

Further Examples on Euler's Formula



4. The compound column fixed at one end and free at the other consists of two prismatic bars with moments of inertia I_1 and I_2. Show that

$$\alpha_2 tan\alpha_1 l_1 = \alpha_1 cot\alpha_2 l_2$$

where $\alpha_1^2 = P/(EI_1)$ and $\alpha_2^2 = P/(EI_2)$

Euler’s Formula

Slenderness Ratio

Euler’s formula for the crippling load

$$P_E = \pi^2 EI \Big/ L_e^2 \quad \ldots(i)$$

Let $I = Ak^2$

$$P_E = \pi^2 E\left(Ak^2\right) \Big/ L_e^2 = \pi^2 EA \Big/ \left(L_e/k\right)^2$$

Slenderness Ratio

$$L_e \big/ k$$

Euler’s Formula

Limitation of Euler's Formula

- Euler’s formula for the crippling load $P_E = \pi^2 EA \big/ (L_e/k)^2$
- Euler's crippling stress

$$\sigma_E = \frac{P}{A} = \pi^2 E \big/ (L_e/k)^2$$

- Now let us consider a mild steel column having a crushing stress of 320 MPa or 320 N/mm^2 and Young's modulus of 200 GPa or 200 x I 0^3 N/mm^2.

- Thus, if the slenderness ratio is less than 80 the Euler's formula is not valid for a mild steel column

$$320 = \pi^2 E \big/ (L_e/k)^2 = \pi^2 \left(200x10^3\right) \big/ (L_e/k)^2 \Rightarrow \frac{L_e}{k} = 78.5 \approx 80$$

EMPIRICAL METHODS

Empirical Formulae for Columns

In this session, we shall study the other methods used to derive the critical load of a strut:

- Case 1: Rankine formula
- Case 2: Perry-Robertson formula
- Case 3: Johnson's formula

Case 1: Rankine Formula

For very long struts the failure will occur through buckling as in Euler load

$$P_e = \frac{\pi^2 EI}{L_e^2}$$

For a very short columns failure is by crushing (or yielding)

$$P_c = A.\sigma_c = area \times crushing\ stress$$

Rankine load for the failure of any length of strut

$$\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_e}$$

For a very short column, P_e is large

$$\frac{1}{P_e} \approx small$$

$$\frac{1}{P_R} \cong \frac{1}{P_c} \quad \therefore P_R \cong P_c$$

For a very long column P_e is small

$$\frac{1}{P_e} \approx \text{large}$$

$$\frac{1}{P_R} \cong \frac{1}{P_e} \quad \therefore P_R \cong P_e$$

Case 1: Rankine Formula

Rewriting
$$\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_e} = \frac{P_e + P_c}{P_c P_e}$$

Thus
$$P_R = \frac{P_c P_e}{P_e + P_c} = \frac{P_c}{1 + P_c / P_e} = \frac{A\sigma_c}{1 + a\left(L_e / k\right)^2}$$

Where

- ❑ P_c Crushing load of the column material
- ❑ σ_c Crushing stress of the column material
- ❑ A Cross-sectional area of the column
- ❑ a Rankine's constant
- ❑ L_e Equivalent length of the column, and
- ❑ K Least radius of gyration

Case 2: Rankine Formula

The following table gives the values of crushing stress (σ_c) and Rankine's constant (a) for various materials:

S.No.	Material	σ_C in MPa	$a = \frac{\sigma_C}{\pi^2 E}$
1.	Mild Steel	320	$\frac{1}{7500}$
2.	Cast Iron	550	$\frac{1}{1600}$
3.	Wrought Iron	250	$\frac{1}{9000}$
4.	Timber	40	$\frac{1}{750}$

Note : The above values arc only for a column with both ends hinged. For other end conditions, the equivalent length should be used.

Example 2.4

Find the Euler's crippling load for a hollow cylindrical steel column of 38 mm external diameter and 2.5 mm thick. Take length of the column as 2.3 m and hinged at its both ends. Take E = 205 GPa. Also determine crippling load by Rankine's formula using constants as 335 MPa and 1/7500

Solution

Give: External diameter (D) = 38 mm; Thickness = 2.5 mm or inner diameter (d) = 38- (2 x 2.5) = 33 mm ; Length of the column (l) = 2.3 m = 2.3 x 10^3 mm; Yield stress (σ_c) = 335 MPa = 335 N/mm^2 and Rankine's constant (a) =1/7500

For both ends hinged, effective length of the column, $Le = l$ = 2.3 x 10^3 mm

Moment of inertia of the column section

$$I_{XX} = \frac{\pi}{64}\left(D^4 - d^4\right) = \frac{\pi}{64}\left[(38)^4 - (33)^4\right] = 14.05x10^3 \pi\ mm^4$$

Example 2.4 (continued)

Area of the column section

$$A = \frac{\pi}{4}\left(D^2 - d^2\right) = \frac{\pi}{4}\left[(38)^2 - (33)^2\right] = 88.75\pi\ mm^2$$

The least radius of gyration

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{14.05x10^3\pi}{88.75\pi}} = 12.6\ mm$$

Euler's crippling load

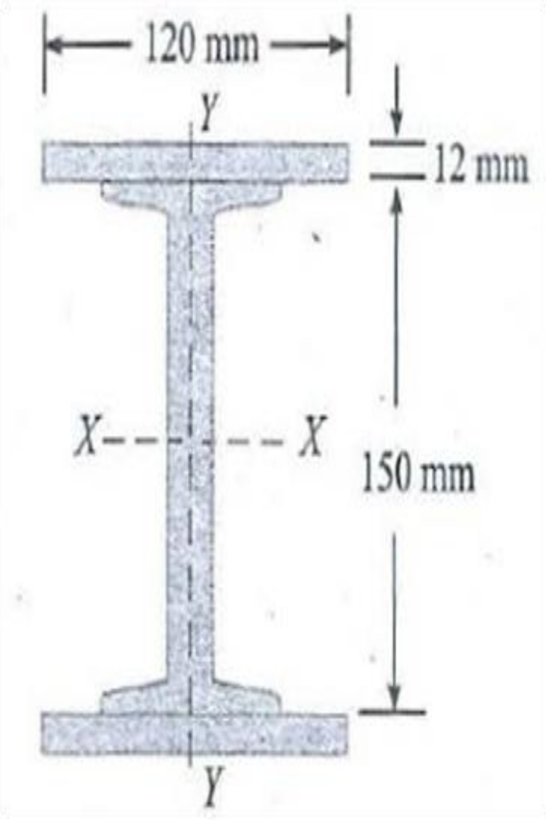
$$P_E = \pi^2 EI \Big/ L_e^2 = \frac{\pi^2\left(205x10^3\right)\left(14.05x10^3\pi\right)}{(2300)^2} = 16{,}880\ N$$

Rankine's crippling load

$$P_R == \frac{A\sigma_c}{1 + a\left(L_e/k\right)^2} = \frac{(88.75\pi)(335)}{1 + \left(\frac{1}{7500}\right)\left(\frac{2300}{12.6}\right)^2} = 17{,}160\ N$$

Example 2.5

Fig. 27 shows a built-up column consisting of 150 mm x 100 mm R. S. J. with 120 mm x 12 mm plate riveted to each flange. Calculate the safe load, the column can carry, if it is 4 m long having one end fixed and the other hinged with a factor of safety 3.5. Take the properties of the joist as Area= 2167 mm^2 I_{XX} = 8.391 x 10^6 mm^4, I_{YY} = 0.948 x 10^6 mm^4. Assume the yield stress as 315 MPa and Rankine's constant (a) = 1/7500



Solution

Given: Length of the column (l)= 4 m = 4 x 10^3 mm; Factor of safety = 3.5; Yield stress (σ_c) = 315.MPa = 315 N/mm^2; Area of joist= 2167 mm^2; Moment of inertia, about *X*-*X* axis (I_{XX}) = 8.391 x 10^6 mm^4; Moment of inertia about *Y*-*Y* axis (I_{YY}) = 0.948 x 10^6 mm^4 and Rankine's constant *(a)* = 1/7500

Example 2.5 (continued)

Area of the column section, A = 2167 + (2 x 120 x 12) = 5047 mm^2

Moment of inertia of the column section

$$I_{XX} = \left(83.91x10^6\right) + 2\left[\frac{(120)(12)^3}{12} - (120)(12)(81)^2\right] = 27.32x10^6 \ mm^4$$

$$I_{YY} = \left(0.948x10^6\right) + 2\left[\frac{(12)(120)^3}{12}\right] = 4.404x10^6 \ mm^4$$

The least of two, I_{YY} = 4.404 x 10^6 mm^4

For fixed at one end and hinged at the other,

$$L_e = \frac{l}{\sqrt{2}} = \frac{4000}{\sqrt{2}} = 2.83x10^3 \ mm$$

The least radius of gyration

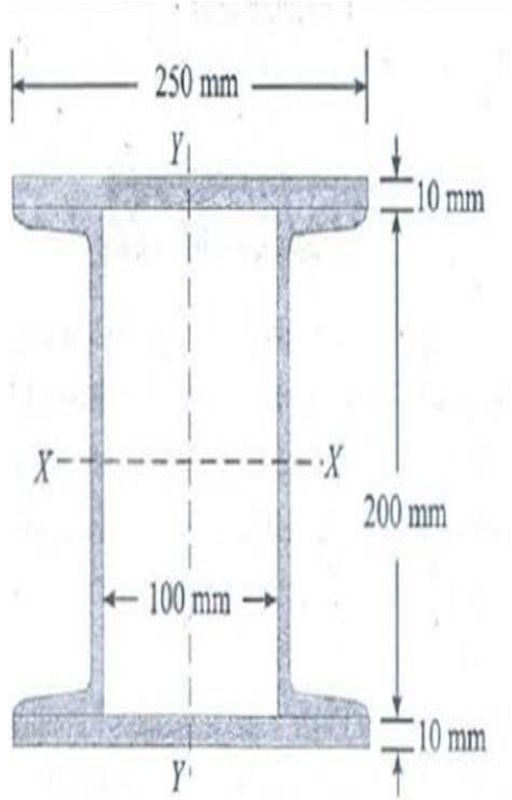
$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{4.404x10^6}{5047}} = 29.5 \, mm$$

Rankine's crippling load

$$P_R == \frac{A\sigma_c}{1 + a\left(L_e/k\right)^2} = \frac{(5047)(315)}{1 + \left(\frac{1}{7500}\right)\left(\frac{2830}{29.5}\right)^2} = 714 \, kN$$

Example 2.6

A column is made up of two channels. ISJC 200 and two 250 mm x 10 mm flange plates as shown in Fig.28. Determine by Rankine's formula the safe load, the column of 6 m length, with both ends fixed, can carry with a factor of safety 4. The properties of one channel are Area = I 777 mm^2, I_{XX} = 11.612 X 106 mm^4 and I_{YY} = 0.842 x 10^6 mm^4. Distance of centroid from back to web=. 19.7 mm. Take (σ_c) = 320 MPa and (a)= 1/7500



Solution

Given : Length of the column (l)= 6 m = 6 X 10^3 mm ; Factor of safety= 4 ; Area of channel = 1777 mm^2; Moment of inertia about X-X axis (I_{XX}) = 11.612 x 10^6 mm^4; Moment of inertia about Y-Y axis (I_{YY}) = 0.842 x 10^6 mm^4; Distance of centroid from the back of web= 19.7 mm; Crushing stress (σ_c) = 320 MPa = 320 N/mm^2 and Rankine's constant (a)= 1/7500

Example 2-7 (continued)

Area of the column section, A = 2 [1777 + (250 x 10)] = 8554 mm^2

Moment of inertia of the column section,

$$I_{XX} = \left(2x11.612x10^6\right) + 2\left[\frac{(250)(10)^3}{12} - (250)(10)(105)^2\right] = 78.391x10^6\ mm^4$$

$$I_{YY} = 2\left[\frac{(10)(250)^3}{12} + \left(0.846x10^6\right) + 1777x(50+19.7)^2\right] = 44.992x10^6\ mm^4$$

The least of two, I_{YY} = 44.992 x 10^6 mm^4

For fixed at both ends, $L_e = \frac{l}{2} = \frac{6000}{2} = 3x10^3\ mm$

The least radius of gyration $k = \sqrt{\frac{I}{A}} = \sqrt{\frac{44.992x10^6}{8554}} = 72.5\,mm$

Example 2.6 (continued)

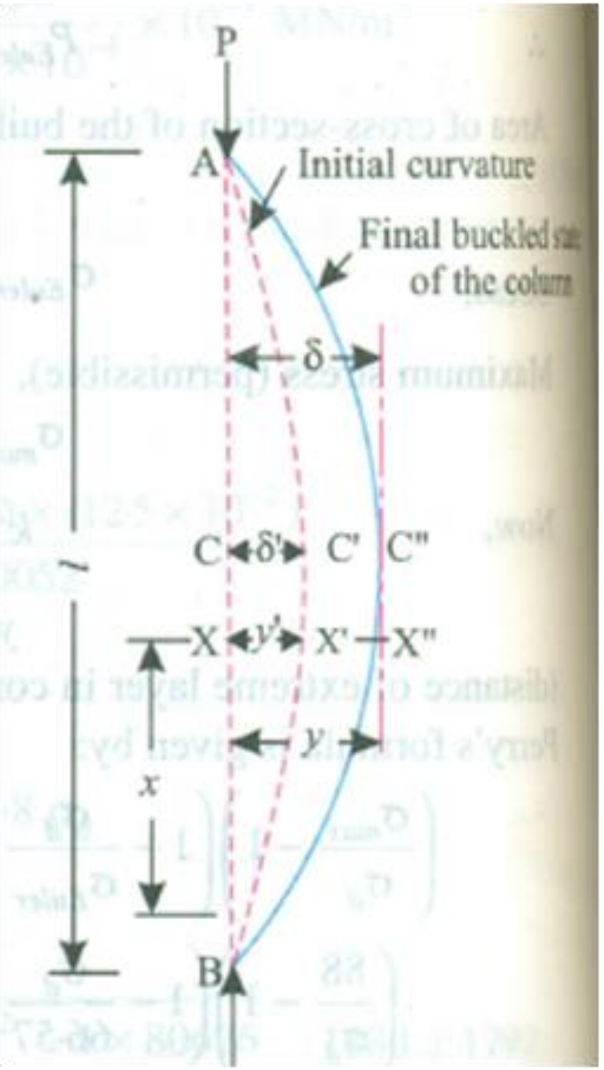
Rankine's crippling load

$$P_R == \frac{A\sigma_c}{1+a\left(L_e/k\right)^2} = \frac{(8554)(320)}{1+\left(\frac{1}{7500}\right)\left(\frac{3000}{72.5}\right)^2} = 2228.5\,kN$$

Safe load on the column

$$Safe\,load = \frac{Crippling\,load}{Factor\ of\ \ safety} = \frac{2228.5}{4} = 557.1\,kN$$

Case 2: Perry-Robertson's Formula



Initial deflection at a distance x from the end B $$y' = \delta'.\sin\frac{\pi x}{l}$$

$$\frac{dy'}{dx} = \frac{\pi\delta'}{l}.\cos\frac{\pi x}{l}$$

$$\frac{d^2y'}{dx^2} = -\frac{\pi^2\delta'}{l^2}.\sin\frac{\pi x}{l}$$

The deflection at x changes from y' to y $$\therefore\ EI\frac{d^2(y-y')}{dx^2} = -Py$$

$$\frac{d^2(y-y')}{dx^2} = -\frac{Py}{EI}$$

$$\Rightarrow \frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{d^2y'}{dx^2} = -\frac{\pi^2}{l^2}\delta'.\sin\frac{\pi x}{l}$$

Case 2: Perry-Robertson's Formula

Solution $y = C\delta' \sin \frac{\pi x}{l}$

$$\frac{dy}{dx} = \frac{\pi}{l} C\delta' \cos \frac{\pi x}{l}$$

$$\frac{d^2 y}{dx^2} = -\left(\frac{\pi}{l}\right)^2 C\delta' \sin \frac{\pi x}{l}$$

Inserting the values of y and dy/dx

$$C == \frac{P_E}{P_E - P}$$

Hence the equation to the deflected form of the column

$$y = \frac{P_E}{P_E - P} \delta'.\sin \frac{\pi x}{l}$$

The deflection will be maximum at the mid-point

$$y = \delta \Rightarrow \delta = \frac{P_E}{P_E - P} \delta'$$

Maximum bending moment

$$M = P\delta = \frac{P.P_E}{P_E - P} \delta'$$

Maximum compressive stress

$$\left[\frac{\sigma_{\max}}{\sigma_d} - 1\right]\left[1 - \frac{\sigma_d}{\sigma_E}\right] = \frac{\delta' y_c}{k^2}$$

Example 2.7

A steel strut has an outside diameter of 180mm and inside diameter of 120mm and is 6m long. It is hinged at both ends and is initially bent. Assuming the centre line of the strut as sinusoidal with maximum deviation of 9mm, determine the maximum stress developed due to an axial load of 150kN.take E=208 Gpa

Solution

Given: Outside diameter of the strut, (D) = 180 mm; Inside diameter of the strut, (d) = 120 mm; Length of the strut, (*l*) = 6 m =6 x 10^3 mm; Maximum deviation at the centre, (δ') = 9 mm; Young's modulus, (E) = 208 GPa = 208 x 10^3 N/mm^2; Axial load, (P) = 150 kN = 150 x 10^3 N

Area of cross-section

$$A = \frac{\pi}{4}\left(D^2 - d^2\right) = \frac{\pi}{4}\left(180^2 - 120^2\right) = 14.14x10^3 \ mm^2$$

Moment of inertia

$$I = \frac{\pi}{64}\left(D^4 - d^4\right) = \frac{\pi}{64}\left(180^4 - 120^4\right) = 41.35x10^6 \ mm^4$$

Example 2.7 (continued)

Radius of gyration

$$k^2 = \frac{I}{A} = \frac{41.35x10^6 \ mm^4}{14.14x10^3 \ mm^2} = 2.924x10^3 \ mm^2$$

Euler load, for pinned at both ends, $L_e = l = 6x10^3$ mm

$$P_E = \left(\frac{\pi}{L_e}\right)^2 EI = \left(\frac{\pi}{6x10^3}\right)^2 \left(208x10^3\right)\left(41.35x10^6\right) = 2.36x10^6 \ N$$

Euler Stress

$$\sigma_E = \frac{P_E}{A} = \frac{2.36x10^6}{14.14x10^3} = 166.75 \ N/mm^2$$

Direct stress

$$\sigma_d = \frac{P}{A} = \frac{150x10^3}{14.14x10^3} = 10.6 \ N/mm^2$$

Example 2.7 (continued)

Distance of the extreme layer in compression from the neutral axis

$$y_c = \frac{D}{2} = \frac{180}{2} = 90\ mm$$

We know that

$$\left[\frac{\sigma_{\max}}{\sigma_d} - 1\right]\left[1 - \frac{\sigma_d}{\sigma_E}\right] = \frac{\delta' y_c}{k^2} \Rightarrow \left[\frac{\sigma_{\max}}{10.6} - 1\right]\left[1 - \frac{10.6}{166.75}\right] = \frac{9x90}{2.924\,x10^3}$$

Therefore

$$\left[\frac{\sigma_{\max}}{10.6} - 1\right] = \frac{0.277}{0.936} = 0.296$$

$$\Rightarrow \sigma_{\max} = 10.6x\left(1 + 0.296\right) = 13.74\ N/mm^2$$