Fall or Decay of Current in a R-L Circuit:

- Let us consider the circuit shown in fig. 3.6(a).
- In this circuit, the switch 'S' is closed sufficiently long duration in position '1'.
- This means that the current through the inductor reaches its steady-state value and it acts, as a short circuit i.e. the voltage across the inductor is nearly equal to zero
- If the switch 'S' is opened at time **'t'=0** and kept in position '2' for ***t* > 0** as shown in Fig. 3.6(b), this situation is referred to as source free circuit.

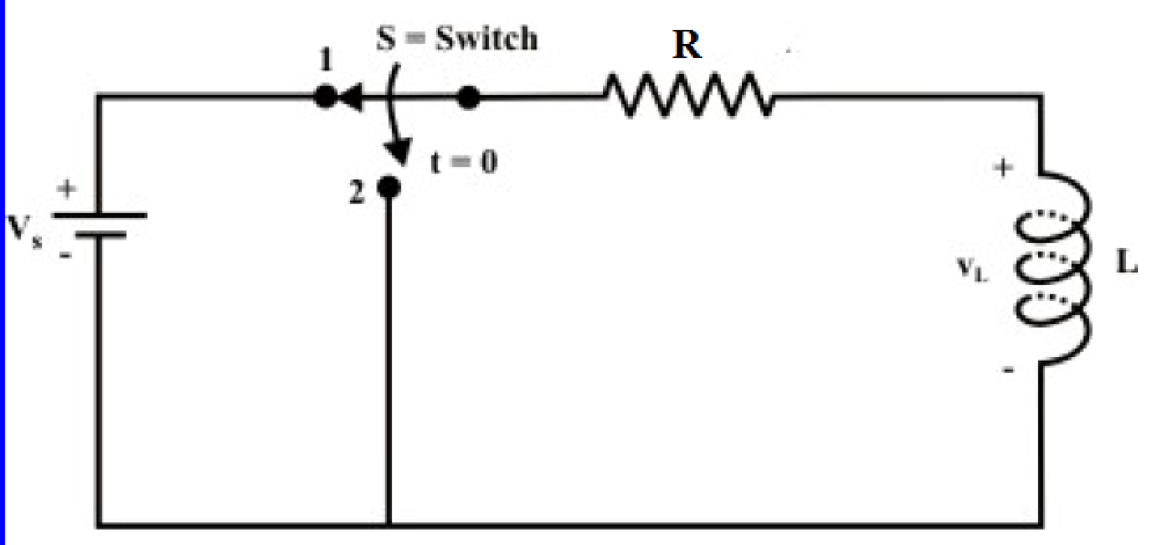


Fig. 3.6 (a)

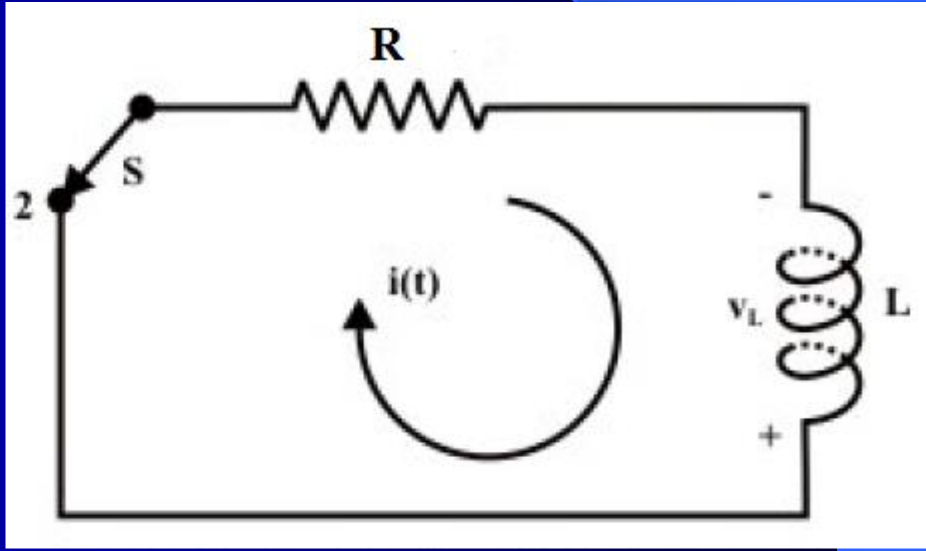


Fig. 3.6 (b): Decay of current in R-L circuit

- Since the current through an inductor cannot change instantaneously, the current through the inductor just before ($i(0^-)$) and after ($i(0^+)$) opening the switch ‘S’ must be same.
- Because there is no source to sustain the current flow in inductor, the magnetic field in inductor starts to collapse and this, in turn, will induce a voltage across the inductor.
- The polarity of this induced voltage across the inductor is just in reverse direction compared to the situation that occurred during the growth of current in inductor (i.e. when the switch ‘S’ is kept in position ‘1’).
- This is illustrated in fig. 3.6(b), where the voltage induced in inductor is positive at the bottom of the inductor terminal and negative at the top. This implies that the current through inductor will still flow in the same direction, but with a magnitude decaying toward zero.

- Applying KVL around the closed circuit in Fig. 3.6(b), we obtain

$$L\frac{di(t)}{dt} + Ri(t) = 0 \qquad 3.15$$

- The solution of the homogenous (input free) first-order differential equation above is given by

$$i(t) = i_n(t) = Ae^{at}$$

- Solving the differential equation (3.15)
- Rearrange the equation into a form that is easier to integrate

$$\frac{di(t)}{dt} = \frac{R}{L}i(t) \qquad (3.16)$$

- Divide by the i(t) and integrate.

$$\int \frac{di(t)}{dt}\frac{1}{i(t)}dt = -\frac{R}{L}\int dt \qquad (3.17)$$

- The integral becomes,

$$In\big(i(t)\big) = -\frac{t}{L/R} + D \qquad (3.18)$$

where D is the constant of integration

- Remove the natural log and solve for the inductor current

$$i(t) = e^{D} e^{-\frac{t}{L/R}} \qquad (3.19)$$

- Let $e^{D} = A$, *a constant*
- Hence

$$i(t) = A e^{-\frac{t}{L/R}} \qquad (3.20)$$

- The constant A is revealed at time t=0 when the switch 'S' is just closed in position 2. From equation (3.20)

$$i(0) = A \qquad (3.21)$$

- Now current before the switch is closed in position 2 $i(0^{-})$ is $\frac{V_S}{R}$, hence $i(0) = \frac{V_S}{R}$
- It means the constant A is,

$$\frac{V_S}{R} = A$$

Therefore the expression of the current in a source-free R-L circuit is

$$i(t) = \frac{V_S}{R} e^{-\frac{t}{\tau}} \qquad where \quad \tau = \frac{L}{R} \qquad (3.22)$$

- A sketch of i(t) for $t \geq 0$ is shown in fig.3.7. Here, transient has ended and steady state has been reached when both current in inductor i(t) and voltage across the inductor including its internal resistance are zero.

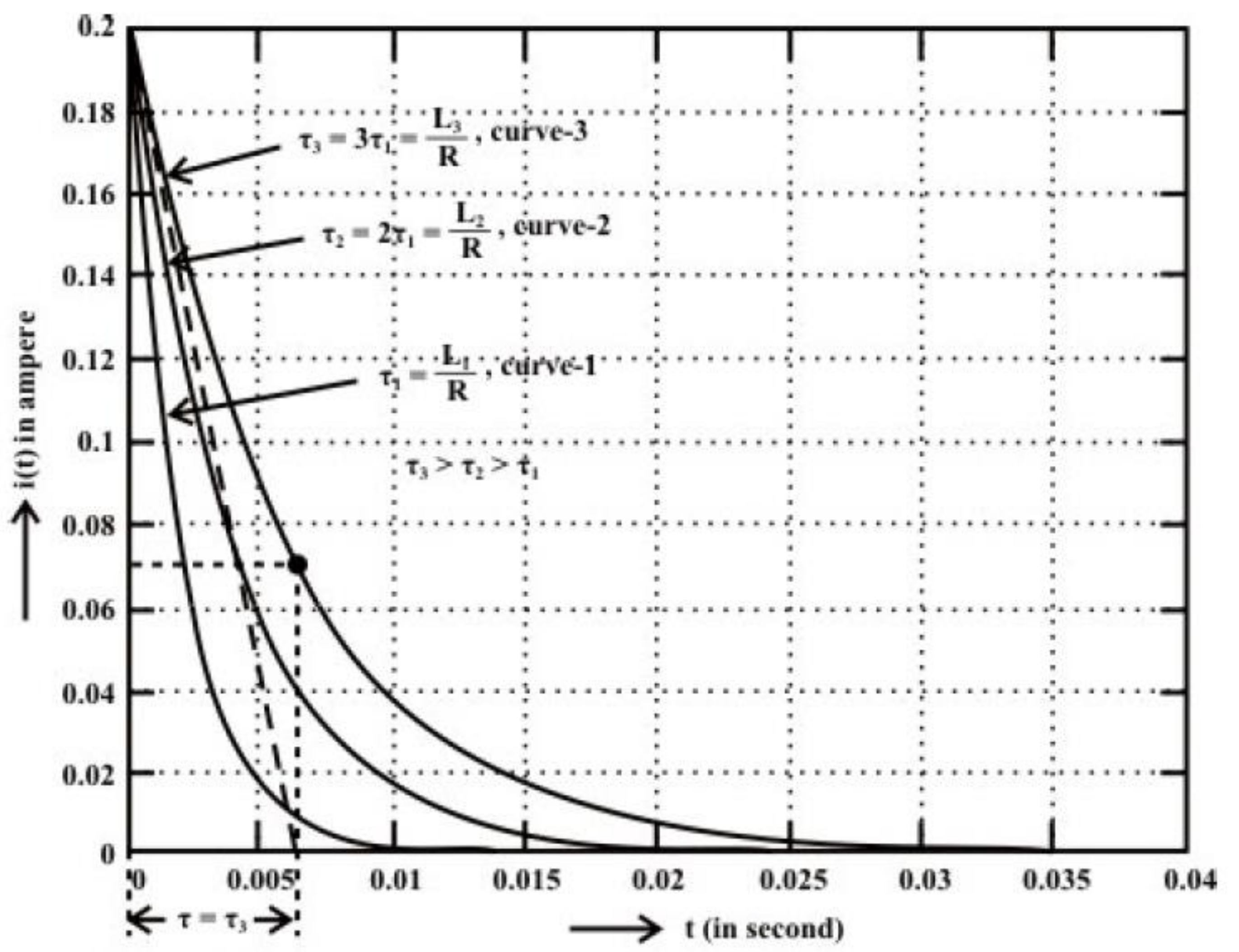


Fig. 3.7: Fall of current in R-L circuit (assumed initial current through inductor is I)

- **Time Constant (τ) for exponential decay response:**
- For the source free circuit, it is the time τ by which the current falls to 36.8 percent of its initial value.
- The initial condition in this case (see fig. 3.6(b)) is considered to be the value of inductor's current at the moment the switch is opened and kept in position '2'.
- Mathematically, τ is computed as

$$i(t) = 0.368\, x\, \frac{V_S}{R} = \frac{V_S}{R} e^{-\frac{R}{L}t}$$

$$\Rightarrow t = \tau = \frac{L}{R}$$

- **Example 3.1:** Fig. 3.8 shows the plot of current i(t) through a series R-L circuit when a constant forcing function of magnitude $V_S = 50\,V$ is applied to it. Calculate the values of resistance R and inductance L.

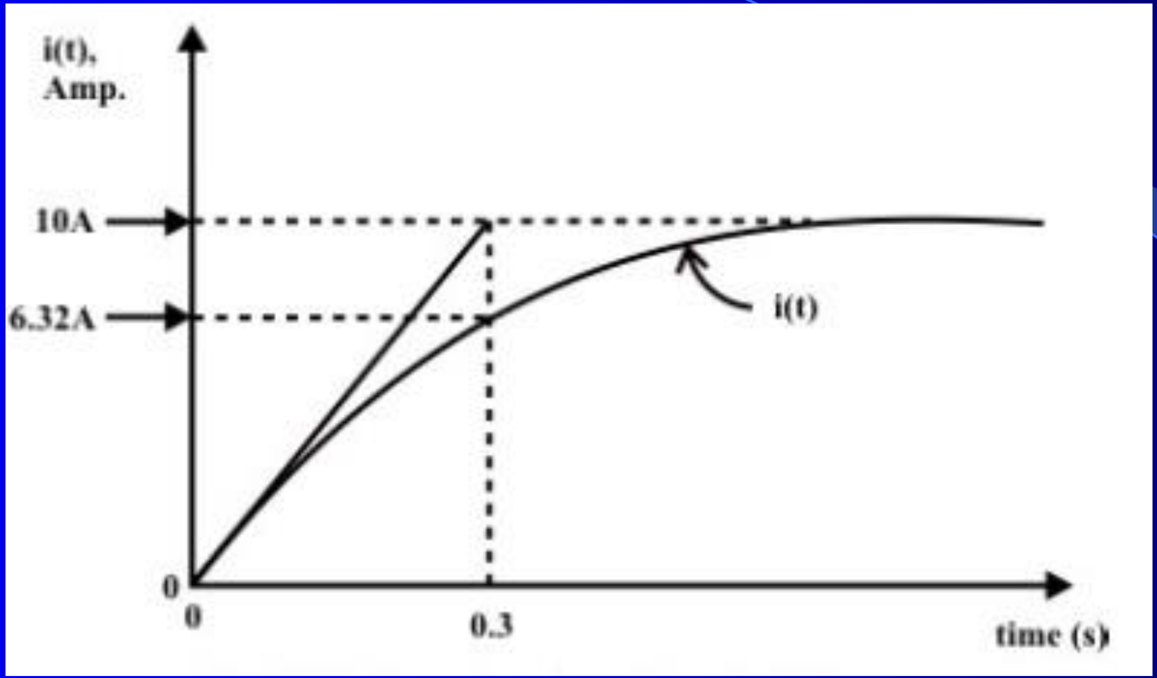


Fig. 3.8: Current –time characteristic

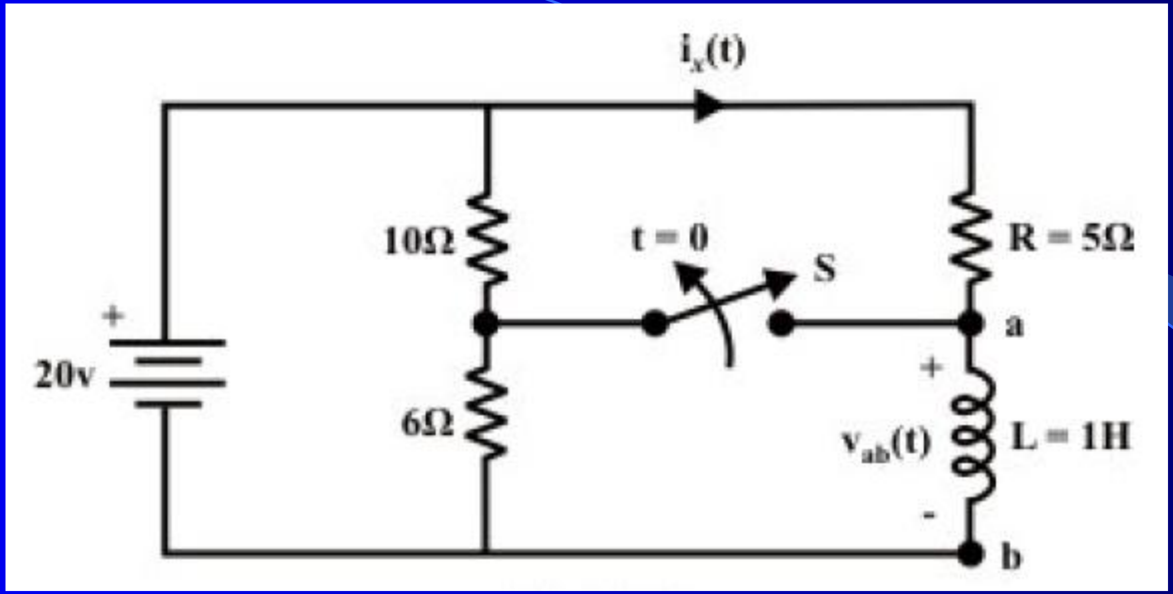
- **Solution:** From Fig.3.8 one can easily see that the steady state current flowing through the circuit is $10A$ and the time constant of the circuit $\tau = 0.3\,sec$. The following relationships can be written as

$$i_{steady\ state} = \frac{V_S}{R} \;=>\; 10 = \frac{50}{R} \quad =>\; R = 5\,\Omega$$

- And

$$\tau = \frac{L}{R} \Rightarrow 0.3 = \frac{L}{5} \Rightarrow L = 1.5\ H$$

- **Example 3.2:** For the circuit shown in Fig. 3.9, the switch ‘S’ has been closed for a long time and then opens at t=0.



- Find

(i) $v_{ab}(0^-)$ (ii) $i_x(0^-)$, $i_L(0^-)$ (iii) $i_x(0^+)$ (iv) $v_{ab}(0^+)$ (v) $i_x(t \to \infty)$ (vi) $v_{ab}(t \to \infty$ (vii) $i_x(t)$ for t > 0

Find

(i) $v_{ab}(0^-)$ (ii) $i_x(0^-)$, $i_L(0^-)$ (iii) $i_x(0^+)$ (iv) $v_{ab}(0^+)$ (v) $i_x(t \to \infty)$ (vi) $v_{ab}(t \to \infty)$

Solution:

When the switch *S* was in closed position for a long time, the circuit reached in steady state condition i.e. the current through inductor is constant and hence, the voltage across the inductor terminals *a and b* is zero or in other words, inductor acts as short circuit i.e.,

(i) $\boldsymbol{v_{ab}(0^-) = 0\ \mathrm{V}}$.

It can be seen that no current is flowing through resistor 6Ω. The following are the currents through different branches just before the switch 'S' is opened i.e., at $t = 0^-$.

(ii) $\boldsymbol{i_x(0^-) = \frac{20}{5} = 4A}$ **and the current through the 10Ω resistor,** $\boldsymbol{i_{10\Omega}(0^-) = \frac{20}{10} = 2A}$**.** The algebraic sum of these two currents is flowing through the inductor i.e.,

$\boldsymbol{i_L(0^-) = 2 + 4 = 6A}$

- *When the switch 'S' is in open position*
- The current through inductor at time $t = 0^+$ is same as that of current $i_L(0^-)$, since inductor cannot change its current instantaneously. Therefore the current $i_x(0^+)$ is given by

(iii) $\boldsymbol{i_x(0^+) = i_L(0^+) = 6A}$

(iv)Applying KVL around the closed loop at $t = 0^+$ we get,

$$20 - i_x(0^+) \text{ x } R = v_{ab}(0^+) \Rightarrow 20 - 6 \text{ x } 5 = v_{ab}(0^+) \;\Rightarrow\; v_{ab}(0^+) = -10V$$

The negative sign indicates that inductor terminal 'b' as +ve terminal and it acts as a source of energy.

(v and vi)At steady state condition ($t \to \infty$) the current through inductor is constant and hence inductor acts as a short circuit. This establishes the following relations:

$$v_{ba}(t = \infty) = 0V \text{ and } i_x(t = \infty) = \frac{20}{5} = 4A$$