8 THE OVERALL MOMENTUM BALANCE

The overall momentum balance is useful for the analysis of the forces involved in fluid systems.

The quantity characterizing the motion of a body is its momentum, which is defined as m**v**, kgm/s. It is a vector quantity. We will consider only linear momentum (angular momentum is not considered). The momentum per unit volume is ρ**v**.

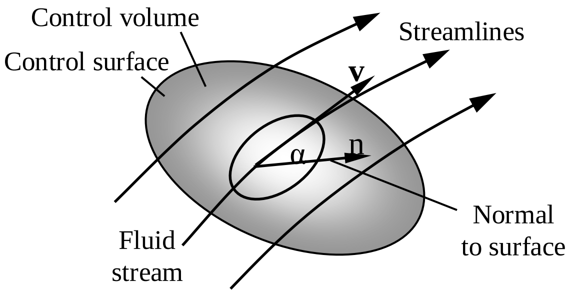
Newton's 2 nd law states that the (time) rate of change of the momentum of the body (system) is equal to the sum of all forces acting on the body (system) and takes place in the direction of the net force.

$$\boxed{\sum \bar{F} = \frac{D(m\bar{v})}{Dt}}$$ where **F** = force, N

8.1 The overall momentum balance equation

An overall momentum balance, similar to the overall mass balance can be written for the control volume:

$$\left\{\begin{matrix}\text{Sum of.}\\ \text{forces acting}\\ \text{on C.V.}\end{matrix}\right\} = \underbrace{\left\{\begin{matrix}\text{Rate of mom.}\\ \textbf{out}\\ \text{of C.V.}\end{matrix}\right\} - \left\{\begin{matrix}\text{Rate of mom.}\\ \textbf{in}\\ \text{into C.V.}\end{matrix}\right\}}_{\text{net rate of momentum efflux from C.V.}} + \left\{\begin{matrix}\text{Rate of mom.}\\ \textbf{accumulation}\\ \text{in C.V.}\end{matrix}\right\}$$



The fluid flow is through a differential area dA on a control surface.

dA cosα is the area projected in a direction normal to the net velocity vector **v**.

Rate of momentum accumulation: $$\frac{\partial(m\bar{v})}{\partial t} = \frac{\partial}{\partial t}\iiint_{C.V.} \rho\bar{v}\, dV$$

Rate of momentum **efflux** for a small element of area dA on the control surface

v dm = **v** (ρv)(dA cosα) = ρ **v** (**v** · **n**) dA

Integrating over the entire control surface A:

$$\left\{\begin{matrix}\text{net mom. efflux}\\ \text{from C.V.}\end{matrix}\right\} = \iint_A \bar{v}\,(\rho v)\cos\alpha\, dA$$

The overall linear momentum balance for a control volume becomes:

$$\boxed{\frac{D(m\bar{v})}{Dt} = \sum \bar{F} = \iint\limits_{A} \bar{v}\,(\rho v)\cos\alpha\, dA + \frac{\partial}{\partial t}\iiint\limits_{C.V.} \rho \bar{v}\, dV}$$

This vector equation is recognized as an application of Reynolds' transport equation.

The components of the vector equation:

$$\sum F_x = \iint\limits_{A} v_x\ \rho v \cos\alpha\ dA + \frac{\partial}{\partial t}\iiint\limits_{V} \rho v_x\ dV$$

$$\sum F_y = \iint\limits_{A} v_y\ \rho v \cos\alpha\ dA + \frac{\partial}{\partial t}\iiint\limits_{V} \rho v_y\ dV$$

$$\sum F_z = \iint\limits_{A} v_z\ \rho v \cos\alpha\ dA + \frac{\partial}{\partial t}\iiint\limits_{V} \rho v_z\ dV$$

F = the force exerted by the surrounding **on** the control volume fluid

3F_x is composed of several forces. These are:

(1) **Body force** (F_{xg}): is the x-directed force caused by gravity acting on the total mass in the C.V. $F_{xg} = m\ g_x$ $F_{xg} = 0$ if the x-direction is horizontal.

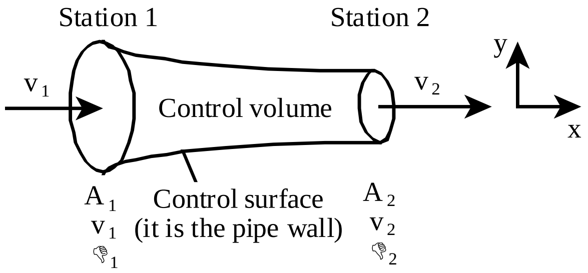
(2) **Pressure force** (F_{xp}): is the x-directed force caused by the pressure acting on the surface of the C.V. Where the control surface cuts through the fluid, the pressure is taken to be directed **inward perpendicular** to the surface; the appropriate component must be considered.
If part of the control surface is solid and this wall is included inside the control surface, there is a contribution to F_{xp} from the pressure on the outside of the wall, which is typically atmospheric pressure.

(3) **Drag, friction, or shear force** (F_{xs}): is the integrated x-directed drag, friction, or shear force. It is exerted on the fluid by a solid wall when the control surface cuts between the fluid and the solid wall. This force is parallel to the fluid flow, the appropriate component must be considered for areas not directed along the x axis. In many cases it is negligible compared to the other forces.

(4) **Solid surface force** (R_x): is the x component of the resultant of the forces acting on the C.V. at places where the control surface cuts through a solid. This occurs typically when the control volume includes a section of pipe and the fluid it contains.

$$3F_x = F_{xg} + F_{xp} + F_{xs} + R_x$$

8.2 Overall momentum balance in one direction



Flow through a horizontal nozzle
(in x- direction only)

Assumptions:

- steady-state $\Rightarrow$ no momentum accumulation
- flow only in x-direction $\Rightarrow$ $v_x = v$
- ρ = constant over the area of integration
- all the flow inward is normal to A_1 and outward normal to A_2

$$\sum F_x = F_{xg} + F_{xp} + F_{xs} + R_x = \iint_A v_x \ \rho v_x \ \cos\alpha \ dA$$

The right hand side can be expanded as:

$$\iint_A v_x \ \rho v_x \ cos\,\alpha \ dA = \iint_{A_2} v_{x2} \ \rho_2 \, v_{x2} \, cos\,\alpha_2 \ dA + \iint_{A_1} v_{x1} \ \rho_1 \, v_{x1} \, cos\,\alpha_1 \ dA$$

at station 1 $\alpha = 180°$ $\Rightarrow$ $\cos\alpha_1 = -1$
at station 2 $\alpha = 0°$ $\Rightarrow$ $\cos\alpha_2 = +1$

$$\dot{m} = \rho v_{avg} A \quad \Rightarrow \quad \rho = \frac{\dot{m}}{v_{avg} A}$$

$$= \frac{\dot{m}}{v_{x2,avg}} \frac{1}{A_2} \iint_{A_2} v_{x2}^2 \ dA - \frac{\dot{m}}{v_{x1,avg}} \frac{1}{A_1} \iint_{A_1} v_{x1}^2 \ dA = \ \dot{m} \frac{(v_{x2}^2)_{avg}}{v_{x2,avg}} - \dot{m} \frac{(v_{x1}^2)_{avg}}{v_{x1,avg}} =$$

where if velocity is not constant and varies across the surface:

$$(v_x^2)_{avg} \equiv \frac{1}{A} \iint_A v_x^2 \ dA \qquad \text{average of squared velocity}$$

$$\frac{(v_x^2)_{avg}}{v_{x,avg}} \quad \text{is replaced by} \quad \frac{v_{x,avg}}{\beta} \quad \Rightarrow \quad \boxed{\beta \equiv \frac{(v_{avg})^2}{(v^2)_{avg}}}$$

β = momentum velocity correction factor
β = 0.95 – 0.99 for turbulent flow.
In most turbulent flow applications β is taken to be 1
$\beta = {}^3/_4$ for laminar flow

If β is known, the momentum flow rate is $\dfrac{\dot{m} v_{avg}}{\beta}$

$$= \dot{m}\,\frac{(v_{x2})_{avg}}{\beta} - \dot{m}\,\frac{(v_{x1})_{avg}}{\beta} = \dot{m}v_2 - \dot{m}v_1$$

$F_{xg} = 0$ (since gravity is acting only in y direction)
$\beta = 1$
$F_{xs} = 0$
$F_{xp} = p_1A_1 - p_2A_2$

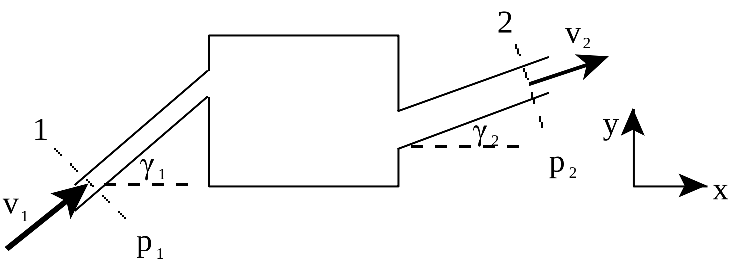
$$\boxed{p_1A_1 - p_2A_2 + R_x = \dot{m}v_2 - \dot{m}v_1}$$

$R_x = \dot{m}v_2 - \dot{m}v_1 + p_2A_2 - p_1A_1$ where R_x = the force exerted **by the solid on the fluid**. The force of the fluid on the solid wall is $-R_x$.

If the conduit axis is curved, the analogous balance of y-momentum must be solved as well in order to complete the solution.

8.3 Overall momentum balance in two directions

Consider a flow system with fluid entering at point 1 and leaving at point 2.



Assumptions:

- Steady-state flow of fluid
- $F_{xs} = 0$ (frictional force is negligible)
- $\beta = 1$ (turbulent flow)

$$F_{xg} + F_{xp} + R_x = \iint_A v_x \rho v \cos\alpha \, dA$$

$F_{xg} = 0$
$F_{xp} = p_1A_1 \cos\gamma_1 - p_2A_2 \cos\gamma_2$
$\iint_A v_x \rho v \cos\alpha \cos\gamma \, dA = \dot{m}v_2 \cos\gamma_2 - \dot{m}v_1 \cos\gamma_1$

$$\boxed{F_{xg} + p_1A_1 \cos\gamma_1 - p_2A_2 \cos\gamma_2 + R_x = \dot{m}v_2 \cos\gamma_2 - \dot{m}v_1 \cos\gamma_1}$$

$$R_x = \dot{m}v_2 \cos\gamma_2 - \dot{m}v_1 \cos\gamma_1 + p_2A_2 \cos\gamma_2 - p_1A_1 \cos\gamma_1$$

The overall momentum balance for the y direction:

$F_{yg} = -\,mg$ the negative sign indicates that it acts in the −y direction
cosγ is replaced by sinγ

therefore $$R_y = \dot{m}v_2 \sin\gamma_2 - \dot{m}v_1 \sin\gamma_1 + p_2A_2 \sin\gamma_2 - p_1A_1 \sin\gamma_1 + mg$$

Failing to keep a consistent sign convention is the most common source of problems when applying the momentum balance equation.

If you work in gauge pressures, then you only need to show pressure forces on those parts of the boundary of the control volume where the pressure is different from atmospheric.

9 VELOCITY DISTRIBUTION IN LAMINAR FLOW; SHELL MOMENTUM BALANCE

In engineering problems we need to know the maximum velocity, the average velocity or the shear stress at the surface. These quantities can be obtained easily once the velocity profiles are known.

9.1 Shell momentum balances

For steady-state flow, the momentum balance is:

$$\left\{\begin{array}{c}\text{Rate of}\\ \text{momentum}\\ \textbf{in}\end{array}\right\} - \left\{\begin{array}{c}\text{Rate of}\\ \text{momentum}\\ \textbf{out}\end{array}\right\} + \left\{\begin{array}{c}\text{Sum of forces}\\ \text{acting on}\\ \text{C.V.}\end{array}\right\} = 0$$

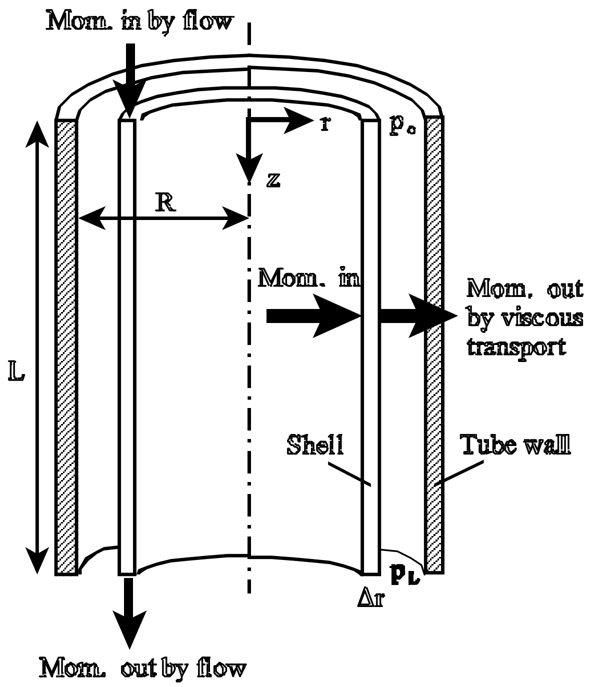
Momentum can flow in to the control volume (shell) by
(1) momentum transport according to Newton’s law of viscosity — (**conductive transport**)
(2) virtue of the over-all fluid motion — (**convective transport**).

The forces acting on the control volume (represent external momentum source):
(1) pressure forces — acting on surfaces,
(2) gravity forces — acting on the volume as a whole.

Generally the procedure:
(1) Set up a momentum balance of the above form for a shell of finite thickness.
(2) Let the shell thickness approach zero.
(3) Obtain a differential equation for momentum flux distribution.
(4) Integrate the differential equation and apply the boundary conditions to get the **momentum flux distribution**.
(5) Insert Newton’s law of viscosity for the momentum flux to obtain a differential equation for the velocity distribution.
(6) Integrate the differential equation and apply the boundary conditions to get the **velocity distribution**.
(7) Calculate the average velocity, the maximum velocity, the volumetric flow rate, the pressure drop, forces on the boundaries, ...

The integration constants are evaluated by the use of boundary conditions (B.C.). The most used boundary conditions are:
(1) At solid-fluid interfaces the fluid velocity equals the velocity with which the surface itself is moving (**no slip** condition).
(2) At gas-liquid interfaces the momentum flux (and therefore the velocity gradient) in the liquid phase is assumed to be zero (it is very nearly zero).
(3) At liquid-liquid interfaces the momentum flux is perpendicular to the interface; and the velocity is continuous across the interface.

9.2 Flow through a circular tube



L = tube length, shell length
R = tube radius
Δr = shell thickness over which momentum balance is made
p_o , p_L = static pressures

We use cylindrical coordinates.

Assumptions:

- The flow is laminar (Re < 2100)
- Incompressible fluid (ρ = constant)
- Steady-state flow
- Newtonian fluid (μ = constant)
- Fully developed flow (end effects are neglected) ⇒ the pressure gradient is constant ⇒ $dp/dz = \Delta p/L = (p_o - p_L)/L$
  An entrance length of 0.035 D is required for building up the velocity profile.
- The fluid behaves as a continuum.
  It is valid except for very narrow capillary tubes or very dilute gases.
- There is NO SLIP at the wall.

The components of the momentum balance:

Rate of momentum in (by conduction)
across cylindrical surface at r $\quad 2\pi rL\tau_{rz}|_r$

Rate of momentum out
across cylindrical surface r + Δr $\quad 2\pi rL\tau_{rz}|_{r+\Delta r}$

Rate of z-momentum in (by convection)
across annular surface at z = 0 $\quad 2\pi r\Delta r\, v_z\ \rho v_z|_{z=0}$

Rate of z-momentum out
across annular surface at z = L $\quad 2\pi r\Delta r\, v_z\ \rho v_z|_{z=L}$

Gravity force acting on
cylindrical shell $\quad 2\pi r\Delta rL\, \rho g$

Pressure force acting on
annular surface at z = 0 $2\pi r \Delta r\, p_o$

Pressure force acting on
annular surface at z = L !$2\pi r \Delta r\, p_L$

The momentum balance:

Since the flow is fully developed, v_z is the same at z = 0 and z = L therefore the convective momentum terms cancel one another.

$2\pi r L\, (\tau_{rz}|_{r+\Delta r} - \tau_{rz}|_r) = 2\pi r \Delta r L\, \rho g + 2\pi r \Delta r\, (p_o - p_L)$

Divide by $2\pi L \Delta r$ and take limits as Δr approaches zero

$$\frac{d}{dr}(r\tau_{rz}) = \left(\frac{p_o - p_L}{L} + \rho g\right) r$$

Introducing $\mathcal{P} = p - \rho g z$

p = static pressure

$\rho g z$ = pressure resulted from gravity force

$\mathcal{P}$ = it represents the combined effect of static pressure and gravitational force

then $p_o = \mathcal{P}_o$

$p_L = \mathcal{P}_L + \rho g L$

$$\frac{p_o - p_L}{L} + \rho g = \frac{(\mathcal{P}_o - \mathcal{P}_L)}{L} \quad \text{or} \quad \Delta\mathcal{P} = \Delta p + \rho g L$$

$$\frac{d}{dr}(r\tau_{rz}) = \frac{(\mathcal{P}_o - \mathcal{P}_L)}{L} r$$

$$\tau_{rz} = \frac{(\mathcal{P}_o - \mathcal{P}_L)}{2L} r + \frac{C_1}{r}$$

B.C. at r = 0 τ_{rz} is not infinite $\Rightarrow$ $C_1 = 0$

$$\tau_{rz} = \frac{(\mathcal{P}_o - \mathcal{P}_L)}{2L} r \quad \text{or} \quad \tau_{rz} = \frac{\Delta\mathcal{P}}{2L} r$$

=============

This is the momentum flux distribution.

It is a linear function of radius r.

The maximum value occurs at the wall (r = R): $\tau = \tau_w = \dfrac{\Delta\mathcal{P}}{2L} R$

Substituting Newton's law of viscosity: $\tau_{rz} = -\mu \dfrac{dv_z}{dr}$

$$\frac{dv_z}{dr} = -\frac{(\mathcal{P}_o - \mathcal{P}_L)}{2\mu L} r$$

$$v_z = -\frac{(\mathcal{P}_o - \mathcal{P}_L)}{4\mu L} r^2 + C_2$$ The velocity distribution is **parabolic**

B.C. at r = R $\quad v_z = 0 \quad \Rightarrow \quad C_2 = \dfrac{(\mathcal{P}_o - \mathcal{P}_L)}{4\mu L} R^2$

$$v_z = \frac{(\mathcal{P}_o - \mathcal{P}_L) R^2}{4\mu L}\left[1 - \left(\frac{r}{R}\right)^2\right]$$ The velocity distribution

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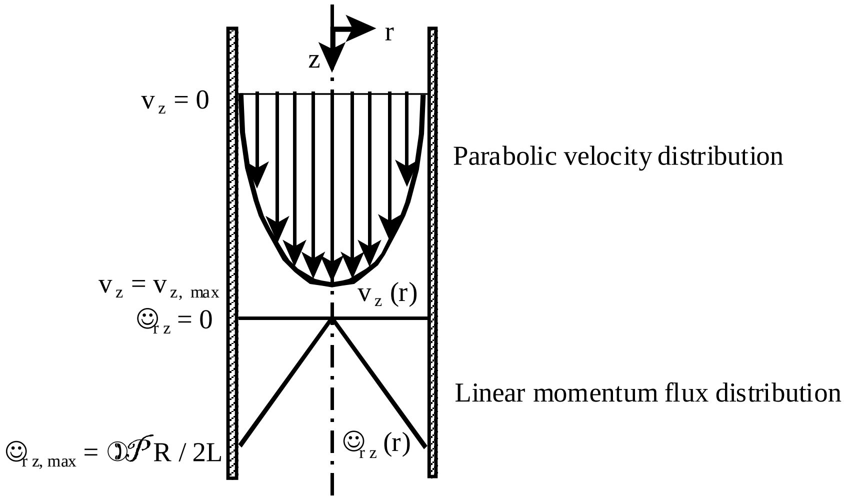
$\Rightarrow \quad v_z - \Delta\mathcal{P}$



Knowledge of the complete velocity distribution is not usually needed in engineering problems. Rather, we need to know the maximum velocity, the average velocity, or the shear stress at a surface. These quantities can be obtained easily once the velocity profiles are known.

The maximum velocity ($v_{z,max}$)

$$v_{z,max} = \frac{(\mathcal{P}_o - \mathcal{P}_L) R^2}{4\mu L} = \frac{\Delta\mathcal{P}}{4\mu L} R^2$$ It occurs at r = 0 (at the center line)

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The average velocity ($v_{z,\,avg}$)

$$v_{z,avg} = \frac{1}{A}\iint_A v_z\, dA = \frac{1}{R^2\pi}\int_0^{2\pi}\int_0^R v_z\, r\, dr\, d\Theta = \frac{2\pi}{R^2\pi}\int_0^R \frac{\Delta\mathcal{P}}{4\mu L}(R^2 - r^2) r\, dr =$$

$$= \frac{\Delta\mathcal{P}}{2R^2\mu L}\left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{\Delta\mathcal{P}}{2R^2\mu L} R^4\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$\boxed{v_{z,avg} = \frac{\Delta\mathcal{P}\, R^2}{8\mu L}}$$ **Hagen-Poiseuille equation**

Note: $\boxed{v_{z,avg} = \frac{1}{2} v_{z,max}}$

The volumetric flow rate

$$\dot{V} = A v_{z,avg} = R^2 \pi v_{z,avg} = \frac{\Delta \mathcal{P} R^4 \pi}{8 \mu L} \quad \Rightarrow \quad \Delta \mathcal{P} – \dot{V}$$

One of the use of the Hagen-Poiseuille equation is in the experimental measurement of viscosity. By measuring $\Delta\mathcal{P}$ (or Δp) pressure drop and $\dot{V}$ volumetric flow rate through a tube of known length L and diameter D, μ can be calculated by

$$\mu = \frac{\Delta p \, D^4 \pi}{128 \, \dot{V} L}$$

The force of the fluid on the surface (F_z)

The z-component of the force **F** of the fluid on the wetted surface of the pipe is:

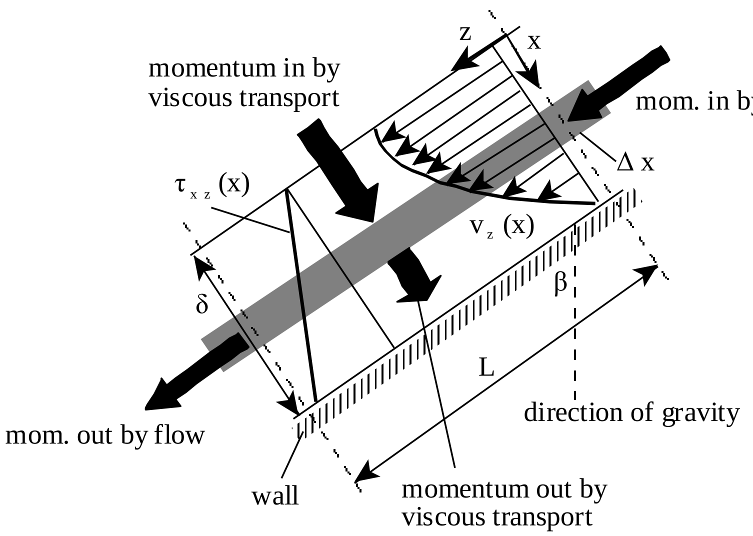
$$F_z = \iint_A \tau \, dA = \int_0^L \int_0^{2\pi} \tau_{rz}\big|_{r=R} \underbrace{r \, d\Theta \, dz}_{\text{dA}} = 2\pi R L \, \frac{\Delta\mathcal{P}}{2L} \, R =$$

$$= \pi R^2 \, \Delta\mathcal{P} = \pi R^2 \, (\mathcal{P}_o \, ! \, \mathcal{P}_L) = \pi R^2 (\Delta p + \rho g L) = \pi R^2 \, (p_o \, ! \, p_L) + \pi R^2 \, \rho g L$$

It says that the net force acting on the cylinder of fluid by virtue of the pressure difference Δp and gravitational acceleration is just counterbalanced by the viscous force F_z, which tends to resist the fluid motion.

9.3 Flow of a falling film

Let’s consider the flow of a fluid along an inclined flat surface (examples: evaporators, gas absorption equipment, wetted-wall towers, ...)



ρ = constant
μ = constant
δ = constant

The region L is sufficiently far from the ends of the wall that the entrance and exit disturbances are not included in L. Therefore v_z velocity component does not depend on z (the velocity distribution is **fully developed**).
Set up z-momentum balance over a control volume (shell) of thickness Δx, bounded by the planes z = 0 and z = L, and extending a distance W in the y-direction.

The components of the momentum balance:

Rate of momentum in (by conduction)
across surface LW at x $\quad LW\,\tau_{xz}\,|_{x}$

Rate of momentum out
across surface LW at x + Δx $\quad LW\,\tau_{xz}\,|_{x+\Delta x}$

Rate of z-momentum in (by convection)
across surface WΔx at z = 0 $\quad W\Delta x\, v_z\, \rho v_z\,|_{z=0}$

Rate of z-momentum out
across surface WΔx at z = L $\quad W\Delta x\, v_z\, \rho v_z\,|_{z=L}$

Gravity force acting on fluid $\quad LW\Delta x\,\rho\, g\, \cos\beta$

The momentum balance:

$$LW\,\tau_{xz}\Big|_{x} - LW\,\tau_{xz}\Big|_{x+\Delta x} + \cancel{W\Delta x\,\rho v_z^2\Big|_{z=0}} - \cancel{W\Delta x\,\rho v_z^2\Big|_{z=L}} + LW\Delta x\,\rho g\cos\beta = 0$$

v_z is the same at z = 0 and z = L (fully developed flow) therefore the 3[rd] and 4[th] terms cancel one another.

Divide by LWΔx and take the limits as Δx approaches zero

$$\lim_{\Delta x\to 0}\frac{\tau_{xz}\big|_{x+\Delta x}-\tau_{xz}\big|_{x}}{\Delta x} = \rho g\cos\beta$$

$$\frac{d}{dx}\tau_{xz} = \rho g\cos\beta$$

This is the differential equation for the momentum flux

$\int$

$$\tau_{xz} = \rho g x\cos\beta + C_1$$

The momentum flux distribution is a linear function

Determination of C_1 with the help of B.C. at the liquid-gas interface:
B.C.: at $x = 0$ $\quad \tau_{xz} = 0 \quad \Rightarrow \quad C_1 = 0$

$$\tau_{xz} = \rho g x\cos\beta$$
==========

This is the momentum flux distribution

If the fluid is Newtonian, then $\qquad \tau_{xz} = -\mu\,\dfrac{dv_z}{dx}$

$$\frac{dv_z}{dx} = -\frac{\rho g \cos\beta}{\mu}\, x$$

$$\Big|\int$$

$$v_z = -\frac{\rho g \cos\beta}{2\mu}\, x^2 + C_2$$

The velocity distribution is parabolic

Determination of C_2 with the help of B.C. at the solid-liquid interface:
B.C.: at $x = \delta$ $v_z = 0$ (no slip)

$$v_z = \frac{\rho\, g \cos\beta}{2\mu}\,(\delta^2 - x^2)$$

$$v_z = \frac{\rho g \delta^2 \cos\beta}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]$$

This is the velocity distribution

====================

The maximum velocity ($v_{z,max}$)

$$v_z = \frac{\rho g \delta^2 \cos\beta}{2\mu}$$

The maximum velocity is the velocity at x = 0

===========

The average velocity ($v_{z,\, avg}$)

The average velocity over the cross-section of the film:

$$v_{z,avg} = \frac{1}{A}\iint_A v_z\, dA = \frac{1}{W\delta}\int_0^W\!\int_0^\delta v_z\, dx\, dy = \frac{W}{W\delta}\int_0^\delta v_z\, dx = \frac{1}{\delta}\int_0^\delta \frac{\rho g \cos\beta}{2\mu}(\delta^2 - x^2)\, dx =$$

$$= \frac{1}{\delta}\,\frac{\rho g \cos\beta}{2\mu}\left[\delta^2 x - \frac{x^3}{3}\right]_0^\delta = \frac{1}{\delta}\,\frac{\rho g \cos\beta}{2\mu}\left(\delta^3 - \frac{\delta^3}{3}\right) =$$

$$v_{z,avg} = \frac{\rho g \delta^2 \cos\beta}{3\mu}$$

==============

Combining $v_{z,\,max}$ and $v_{z,\,avg}$ we obtain: $v_{z,\,avg} = b\; v_{z,\,max}$

============

The volumetric flow rate ($\dot{V}$)

$$\dot{V} = A v_{z,avg} = W \delta v_{z,avg} = \frac{\rho g W \delta^3 \cos\beta}{3\mu}$$

The film thickness (δ)

It may be given in terms of
- ! average velocity,
- ! volumetric flow rate, m^3/s
- ! mass flow rate, kg/s
- ! liquid loading, kg/ms

Liquid loading (**Γ**) is defined as the mass flow rate per unit width of wall:

$$\boxed{\Gamma = \frac{\dot{m}}{W}} \qquad \Gamma = \frac{\dot{m}}{W} = \frac{\rho V}{tW} = \frac{\rho W L \delta}{tW} = \rho \delta v_{avg}$$

The force of the fluid on the surface (F_z)

The z-component of the force **F** of the fluid on the surface is:

$$F_z = \iint_A \tau \, dA = \int_0^L \int_0^W \tau_{xz}\Big|_{x=\delta} dydz = \int_0^L \int_0^W -\mu \frac{dv_z}{dx}\Big|_{x=\delta} dy\,dz =$$

$$-\mu \frac{d}{dx} \frac{\rho g \cos\beta}{2\mu} (\delta^2 - x^2)\Big|_{x=\delta} = -\mu \frac{d}{dx} \frac{\rho g \cos\beta}{2\mu} (0 - 2x)\Big|_{x=\delta} = \rho g \delta \cos\beta$$

$\Rightarrow$ $F_z = \rho g \delta L W \cos\beta$ This is just the z-component of the weight of the entire fluid in the film.

The Reynolds number for falling film

$$\boxed{Re = \frac{\rho v_{avg} D_e}{\mu}} \qquad \boxed{D_e = 4 r_H} \qquad \boxed{r_H = \frac{\text{cross sectional area of flow}}{\text{wetted perimeter}}}$$

where D_e = the equivalent diameter (or characteristic length)
r_H = the hydraulic radius

For falling film: $r_H = \delta W / W = \delta$ and

$$Re = \frac{\rho v_{avg} D_e}{\mu} = \frac{\rho v_{avg} 4 r_H}{\mu} = \frac{4 \rho v_{avg} \delta}{\mu} = \frac{4\Gamma}{\mu}$$

$$\boxed{\mathrm{Re} = \frac{4\Gamma}{\mu}}$$

For the flow of a falling film:

laminar flow with straight streamlines	Re < 25
laminar flow with rippling	25 < Re < 1200
turbulent flow	Re > 1200

Above analytical results are valid only when the film is falling in laminar flow with straight streamlines.

10 THE DIFFERENTIAL MOMENTUM BALANCE; EQUATION OF MOTION

It is not necessary to formulate new balances for each new flow problem. It is often easier to start with the differential equations of the conservation of mass (equation of continuity) and the conservation of momentum (equation of motion) in general form. Then these equations are simplified discarding unneeded terms for each particular problem — based on assumptions.

10.1 Stresses on a fluid element

The force per unit area is known as stress and is defined as:

$$\text{stress} = \lim_{\Delta A \to \delta A} \frac{\Delta F}{\Delta A}$$

where δA = the smallest area for which statistical averages are meaningful.

Each of the 6 faces of the cube will be subjected to mechanical stresses arising from the adjacent fluid, and each of these can in turn be resolved into 3 components, parallel to the 3 axes.

When the force acts perpendicularly to the surface, the stress is known as **normal** stress, and when it acts parallel to the surface, it is known as **shear** stress. In an x-y plane the shear stress τ_{yx} is defined as:

$$\tau_{yx} = \frac{\partial F_x}{\partial A_y}$$

The shear stress (τ) is a tensor quantity. It has 2 indexes.

1 st subscript: indicates the face on which the shear stress is acting by giving the axis perpendicular to the face (e.g. τ_{yx}: the momentum is transported in y-direction).

2 nd subscript: gives the direction in which the shear stress acts (e.g. τ_{yx}: the shear stress is exerted in x-direction).

The components of the shear stress tensor: $$\bar{\tau} = \begin{vmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{vmatrix}$$

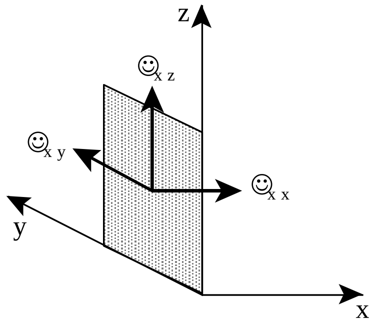
The stresses with the mixed subscript (e.g. τ_{yx}) are shear stresses tending to deform the element and change the angles between the faces. The stresses with repeated subscript (e.g. τ_{xx}) are normal stresses, which are closely related to the hydrostatic pressure. Normal stresses are considered positive (+) for tension. The normal stresses are related to the pressure by the following arbitrary definition:

$$-p = \frac{1}{3}(\tau_{xx} + \tau_{yy} + \tau_{zz})$$

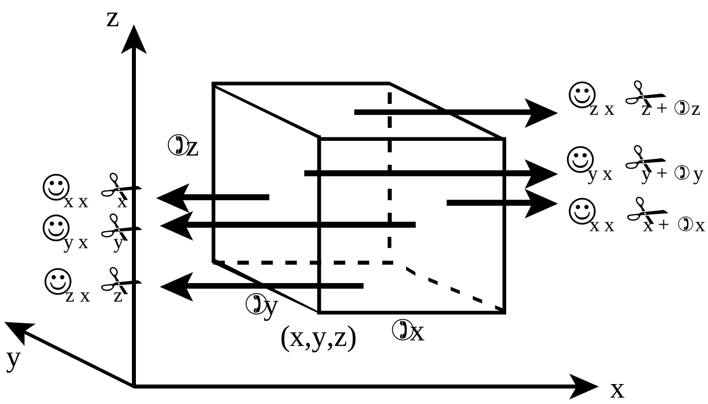
Since the normal stresses are positive (+) for tension, and the pressure is positive (+) for compression, the minus sign is necessary.

$\tau_{xz} = \tau_{zx}$, $\tau_{yz} = \tau_{zy}$, ... it is a symmetric tensor.

3 stress components on a single face of fluid element:



Stress components acting in the x-direction on a fluid element:



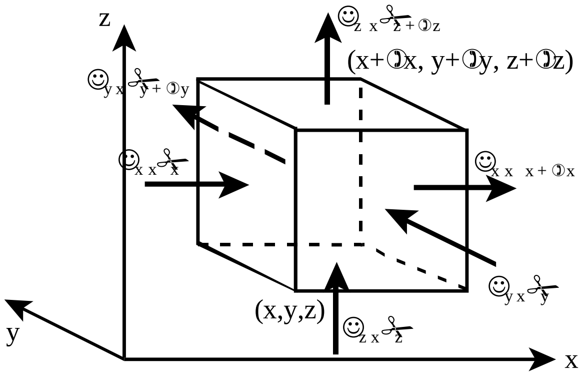
10.2 The equation of motion

The equation of motion is a differential momentum balance over a stationary volume element ΔxΔyΔz for unsteady-state laminar flow.

$$\left\{\begin{matrix}\text{Rate of mom.}\\ \textbf{accumulation}\\ \text{in C.V.}\end{matrix}\right\} = \left\{\begin{matrix}\text{Rate of mom.}\\ \textbf{in}\\ \text{into C.V.}\end{matrix}\right\} - \left\{\begin{matrix}\text{Rate of mom.}\\ \textbf{out}\\ \text{of C.V.}\end{matrix}\right\} + \left\{\begin{matrix}\text{Sum of}\\ \text{forces acting}\\ \text{on C.V.}\end{matrix}\right\}$$

The fluid can flow through all six faces of the volume element in any arbitrary direction. The momentum balance equation is a vector equation with components in each of the coordinate directions x, y, and z. Arrows indicate the direction in which the x-component of momentum is transported through the surfaces. They are not the stresses but the directions in which momentum is transported!

We begin by considering the x-component of each term in the momentum balance equation; y- and z-components may be handled analogously.



x-component of momentum:

(1) Convective flow (by virtue of the bulk fluid flow)

Rate of x-momentum in
through the face at x $\qquad \dot{m}\, v_x = \dot{V}\rho\, v_x = A v_x \rho\, v_x = \Delta y\, \Delta z\, v_x \rho\, v_x \quad \Rightarrow$

$$\rho v_x v_x \big|_x \, \Delta y\, \Delta z$$

Rate of x-momentum out
through the face at x + Δx $\qquad \rho v_x v_x \big|_{x+\Delta x} \, \Delta y\, \Delta z$

Rate of x-momentum in
through the face at y $\qquad \dot{m} v_x = \dot{V} \rho\, v_x = A v_y \rho\, v_x = \Delta x\, \Delta y\, v_y \rho\, v_x \quad \Rightarrow$

$$\rho v_y v_x \big|_y \, \Delta x\, \Delta z$$

and so on

The net convective x-momentum flow into the volume element:

$$\Delta y\, \Delta z\, (\rho v_x v_x\big|_x - \rho v_x v_x\big|_{x+\Delta x}) + \Delta x\, \Delta z\, (\rho v_y v_x\big|_y - \rho v_y v_x\big|_{y+\Delta y}) + \Delta x\, \Delta y\, (\rho v_z v_x\big|_z - \rho v_z v_x\big|_{z+\Delta z})$$

(2) Conductive flow (by molecular transport)

Rate of x-momentum in
through the face at x $\qquad \tau_{xx}\big|_x \, \Delta y \Delta z$

Rate of x-momentum out
through the face at x +Δx $\qquad \tau_{xx}\big|_{x+\Delta x} \, \Delta y \Delta z$

Rate of x-momentum in
through the face at y $\qquad \tau_{yx}\big|_y \, \Delta x \Delta z$

and so on

The net conductive x-momentum flow into the volume element:

$$\Delta y\Delta z\ (\tau_{xx}\big|_{x} - \tau_{xx}\big|_{x+\Delta x}) + \Delta x\Delta z\ (\tau_{yx}\big|_{y} - \tau_{yx}\big|_{y+\Delta y}) + \Delta x\Delta y\ (\tau_{zx}\big|_{z} - \tau_{zx}\big|_{z+\Delta z})$$

(3) Sum of forces

The only important forces are
- ! those arising from the fluid pressure p
- ! gravitational force

The resultant forces in the x-direction:

$$\Delta y\Delta z\ (p|_{x} - p|_{x+\Delta x}) + \rho g_x\ \Delta x\Delta y\Delta z$$

(4) The rate of accumulation of x-momentum within the volume element:

$$\Delta x\Delta y\Delta z\ \frac{\partial \rho v_x}{\partial t}$$

Substitute (1), (2), (3), and (4) into the momentum balance equation, then divide by $\Delta x\Delta y\Delta z$ and take the limits as Δx, Δy, and Δz approach zero.

The x-component of the equation of motion:

$$\frac{\partial \rho v_x}{\partial t} = -\left(\frac{\partial}{\partial x}\rho v_x v_x + \frac{\partial}{\partial y}\rho v_y v_x + \frac{\partial}{\partial z}\rho v_z v_x\right) - \left(\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

The y-component of the equation of motion:

$$\frac{\partial \rho v_y}{\partial t} = -\left(\frac{\partial}{\partial x}\rho v_x v_y + \frac{\partial}{\partial y}\rho v_y v_y + \frac{\partial}{\partial z}\rho v_z v_y\right) - \left(\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right) - \frac{\partial p}{\partial y} + \rho g_y$$

The z-component of the equation of motion:

$$\frac{\partial \rho v_z}{\partial t} = -\left(\frac{\partial}{\partial x}\rho v_x v_z + \frac{\partial}{\partial y}\rho v_y v_z + \frac{\partial}{\partial z}\rho v_z v_z\right) - \left(\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}\right) - \frac{\partial p}{\partial z} + \rho g_z$$

We can combine these 3 equations to a single vector equation:

$$\frac{\partial}{\partial t}\rho\bar{v} = -\nabla\cdot\rho\bar{v}\bar{v} - \nabla\bar{\tau} - \nabla p + \rho\bar{g} \qquad \text{The equation of motion}$$

ρ**v** : mass velocity (or momentum per unit volume) vector, its components are ρv_x , ρv_y , ρv_z
g : gravity acceleration, its components are g_x , g_y , g_z
Λp : pressure gradient vector, its components are Mp/Mx , Mp/My , Mp/Mz
τ : shear stress tensor, its 9 components are τ_{xx} , τ_{xy} , τ_{xz} , τ_{yx} , ...
ρ**vv** : convective momentum flux tensor (dyadic product), its 9 components are

$$\rho\bar{v}\bar{v} = \begin{vmatrix} \rho v_x v_x & \rho v_x v_y & \rho v_x v_z \\ \rho v_y v_x & \rho v_y v_y & \rho v_y v_z \\ \rho v_z v_x & \rho v_z v_y & \rho v_z v_z \end{vmatrix}$$

$\left.\begin{array}{l}\Lambda \cdot \rho\mathbf{vv} \\ \Lambda \cdot \tau\end{array}\right\}$ are not simple divergences, because $\rho\mathbf{vv}$ and $\boldsymbol{\tau}$ are tensors

1 st term: rate of increase of momentum per unit volume at a fixed point
2 nd term: rate of momentum gain by convection per unit volume
3 rd term: rate of momentum gain by conduction (viscous transport) per unit volume
4 th term: pressure force acting on element per unit volume
5 th term: gravitational force on element per unit volume

The equation of motion can be rearranged with the help of the equation of continuity to give:

$$\rho \frac{D\bar{v}}{Dt} = -\nabla \cdot \bar{\tau} - \nabla p + \rho\bar{g}$$

In this form the equation of motion states that a small volume element moving with a fluid is accelerated because of the forces acting upon it. This statement is Newton's 2 nd law of motion. Therefore the momentum balance is completely equivalent to Newton's 2 nd law of motion. All these forms of the equation of motion are valid for any continuous medium.

The shear stress components for Newtonian fluids have been related to the velocity gradients and the fluid viscosity. These relations are available in the literature. E.g.:

$$\tau_{xx} = -2\,\mu \frac{\partial v_x}{\partial x} + \frac{2}{3}\,\mu\,\nabla \cdot \bar{v}$$

Substituting these shear stress expressions into the x-, y-, and z-components of the equation of motion we get the general equations of motion for a Newtonian fluid with varying ρ and μ. But these equations in their complete forms are seldom used. Usually restricted forms of the equations of motion are used.

10.3 Navier-Stokes equation

To use the differential momentum balance equation it is necessary to replace the normal-stress and shear-stress terms with terms involving measurable properties, such as viscosities. It can only be done by introducing severe restrictive assumptions such as:

- Laminar flow
- Incompressible fluid (ρ = constant) $\Rightarrow$ the equation of continuity becomes $\Lambda \cdot \mathbf{v} = 0$
- Newtonian fluid (μ = constant)

For ρ = constant the equation of motion becomes:

$$\rho \frac{D\bar{v}}{Dt} = \mu\,\nabla^2\bar{v} - \nabla p + \rho\bar{g}$$

The x-component:

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

The Navier-Stokes equations are the differential form of Newton's 2 nd law of motion.

THE EQUATION OF MOTION IN RECTANGULAR COORDINATES (x, y, z)

In terms of velocity gradients for a Newtonian fluid with constant ρ and μ:

x-component
$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x$$

y-component
$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) + \rho g_y$$

z-component
$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \rho g_z$$

THE EQUATION OF MOTION IN CYLINDRICAL COORDINATES (r, θ, z)

In terms of velocity gradients for a Newtonian fluid with constant ρ and μ:

r-component[a]
$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho g_r$$

θ-component[b]
$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta$$

z-component
$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

THE EQUATION OF MOTION IN SPHERICAL COORDINATES (r, θ, ϕ)

In terms of velocity gradients for a Newtonian fluid with constant ρ and μ:[a]

r-component

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right)$$
$$= -\frac{\partial p}{\partial r} + \mu\left(\nabla^2 v_r - \frac{2}{r^2}v_r - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} - \frac{2}{r^2}v_\theta \cot\theta - \frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right) + \rho g_r$$

θ-component

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2 \cot\theta}{r}\right)$$
$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left(\nabla^2 v_\theta + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\phi}{\partial \phi}\right) + \rho g_\theta$$

φ-component

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r}{r} + \frac{v_\theta v_\phi}{r}\cot\theta\right)$$
$$= -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi} + \mu\left(\nabla^2 v_\phi - \frac{v_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\theta}{\partial \phi}\right) + \rho g_\phi$$

---

[a] In these equations

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2}{\partial \phi^2}\right)$$

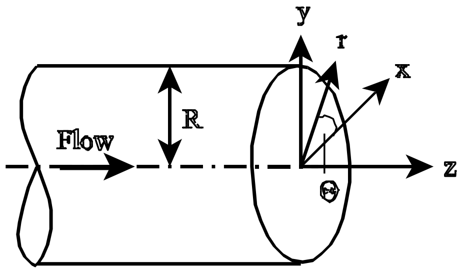
Use of the equations of change to set up steady flow problems

The procedure when applying the Navier-Stokes equations:

(1) Read the problem carefully, identify the given quantities and change them to the appropriate units.
(2) Choose a coordinate system describing the geometry of the physical situation.
(3) Sketch.
(4) Analyze the problem by making assumptions, or restrictions, that may apply.
(5) Select the appropriate Navier-Stokes equation and based on the assumptions eliminate terms found to be inappropriate.
(6) Solve the resulting differential equations by integration and using the appropriate B.C.
(7) Comment on the results.

10.4 Flow through a circular pipe

We solved the problem earlier by setting up a shell momentum balance and solved for the velocity distribution. Now let us use the equations of change after simplification. Cylindrical coordinates are the most appropriate for this problem.



Assumptions:

- laminar flow
- steady-state
- μ, ρ = constant (Newtonian fluid)
- fully developed flow (v_z does not depend on z)
- flow is only in z-direction ($v_r = v_\Theta = 0$)
- v_z is not a function of Θ because of cylindrical symmetry

The z-component of the equation of motion:

$$\rho v_z \frac{\partial v_z}{\partial z} = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{\partial^2 v_z}{\partial z^2}\right]$$

The equation of continuity: $\dfrac{\partial v_z}{\partial z} = 0 \quad \Rightarrow \quad \dfrac{\partial^2 v_z}{\partial z^2} = 0$

So the equation of motion reduces to: $\dfrac{dp}{dz} = \mu \dfrac{1}{r}\dfrac{d}{dr}\left(r\dfrac{dv_z}{dr}\right)$

The L.H.S. is a function of z only. The R.H.S. is a function of r only. This is possible only when each side is equal to the same constant.

Determining this constant:

$$\frac{dp}{dz} = \text{constant}$$

$$dp = \text{constant}\ dz$$

Integrate with the limits of
at $z = 0$ $\quad p = p_o$
at $z = \Delta z$ $\quad p = p_L$

$$[p]_{p_o}^{p_L} = \text{constant}\ [z]_0^{\Delta z}$$

$$\text{constant} = \frac{p_L - p_o}{\Delta z} = -\frac{p_o - p_L}{\Delta z} = -\frac{\Delta p}{\Delta z}$$

Thus dp/dz = constant = $-\Delta p/\Delta z$ where $\mathbf{\Delta p = p_o - p_L}$ **pressure drop** (not as in B.E.)

The equation of motion becomes:

$$-\frac{\Delta p}{\Delta z} = \mu \frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)$$

$$\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = -\frac{\Delta p}{\mu\,\Delta z} r$$

$$\Big|\ \int$$

$$r\frac{dv_z}{dr} = -\frac{\Delta p}{\mu\,\Delta z}\frac{r^2}{2} + C_1$$

B.C.1: at $r = 0$ v_z is maximum $\Rightarrow$ $dv_z/dr = 0$ Ψ $C_1 = 0$

$$\frac{dv_z}{dr} = -\frac{\Delta p}{\mu\,\Delta z}\frac{r}{2}$$

$$\Big|\ \int$$

$$v_z = -\frac{\Delta p}{2\mu\,\Delta z}\frac{r^2}{2} + C_2$$

B.C.2: at $r = R$ $v_z = 0$ (no slip at the wall)

$$\Rightarrow \quad C_2 = +\frac{\Delta p}{4\mu\Delta z}R^2$$

$$v_z = \frac{\Delta p}{4\mu\,\Delta z}(R^2 - r^2) \quad \text{or} \quad v_z = \frac{\Delta p R^2}{4\mu\,\Delta z}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

The same result as earlier

11 FLUID FRICTION

Fluid friction falls into 2 broad category: – skin friction
– form friction

Skin friction is generated in laminar flow in unseparated boundary layers.
Form friction: the presence of fittings and changes in flow direction will cause boundary layer separation. When boundary layer separates and forms wakes, additional energy dissipation appears within the wake and this type of friction is called form friction.

The friction loss h_f includes both type of frictional loss.

11.1 Friction factor for laminar flow in pipes

Note:– for mechanical energy balance in the Bernoulli equation $\Delta p = p_2 ! p_1$, but usually
$p_1 > p_2 \Rightarrow p_2 ! p_1 = \Delta p < 0$

– commonly **Δp** is used for **pressure drop**, that is $\mathbf{\Delta p = p_1 ! p_2}$ and this terminology is used here!

For laminar flow the only type of friction is the skin friction between the wall and the fluid stream. So the friction loss from the Bernoulli equation is:

$$\boxed{h_f = \frac{\Delta p}{\rho}}$$

Δp can be determined from the Hagen-Poiseuille equation (**for laminar flow only!**):

$$v_{avg} = \frac{\Delta p R^2}{8\mu L} = \frac{\Delta p D^2}{32\mu L}$$

The total transport of momentum to the wall (τ_w) is equal the rate of transport of momentum from the fluid by molecular transport:

Shear force = Pressure force

$\tau_w A = \Delta p S$ where $A = \pi DL$ = surface area of pipe
$S = \pi D^2/4$ = cross-sectional area for fluid flow

$$\Rightarrow \quad \tau_w = \frac{\Delta p D}{4L} = \frac{8\mu v_{avg}}{D}$$ by expressing Δp from the Hagen-Poiseuille equation

Therefore $$h_f = \frac{\Delta p}{\rho} = \frac{4\,\tau_w L}{\rho D} = \frac{32\,\mu L v_{avg}}{\rho D^2}$$

The friction factor, f , is another parameter used to characterize the friction losses.

The definition of friction factor:

$$\boxed{F_{drag} = A\,\frac{E_{kin}}{V}\,f}$$

where F_{drag} = drag force. It points in the same direction as the average velocity. Its magnitude may be arbitrarily expressed as the product of A, E_{kin}/V, and f,
A = characteristic area,
E_{kin}/V = characteristic kinetic energy per unit volume,
f = friction factor, dimensionless.

For any flow system f is not defined until A and E_{kin}/V is specified. Rearranging the defining equation of f gives for the friction factor:

$$f = \frac{F_{drag}/A}{E_{kin}/V} = \frac{\tau_w}{E_{kin}/V} = \frac{\textit{total momentum transfer}}{\textit{momentum transfer by turbulence}}$$

f is a relatively simple function of Re and the system shape: f = f (Re, L /D)

For flow in conduits:

A = 2πRL wetted surface

$$\frac{E_{kin}}{V} = \frac{{}^1/_2\, mv^2}{V} = \frac{1}{2}\rho v_{avg}^2$$

$\Rightarrow$ $F_{drag} = (2\pi RL)(\tfrac{1}{2}\rho v^2)\, f_F$

Generally the quantity measured is not F_{drag} the drag force, but $\Delta p = p_1 - p_2$ pressure drop.

$$\left.\begin{aligned} F_{drag} &= \Delta p R^2 \pi \\ \text{and} \quad F_{drag} &= (2\pi RL)(\tfrac{1}{2}\rho v^2)\, f_F \end{aligned}\right\} \quad \boxed{f_F = \frac{1}{4}\frac{D}{L}\frac{\Delta p}{{}^1/_2\,\rho v^2}}$$

where f_F = Fanning friction factor

$\boxed{f_D = 4 f_F}$ the **Darcy friction factor**

The relationship between skin-friction parameters (h_{fs}, Δp_s, τ_w, and f_F):

$$h_{fs} = \frac{\Delta p}{\rho} = \frac{4\tau_w L}{\rho D} = 4 f_F \frac{L}{D}\frac{v^2}{2}$$

$\boxed{h_f = f_D \frac{L}{D}\frac{v^2}{2}}$ This is the **Darcy-Weisbach equation**

The Darcy-Weisbach equation is valid for both laminar and turbulent flow.

The head loss in meter is h_f/g.

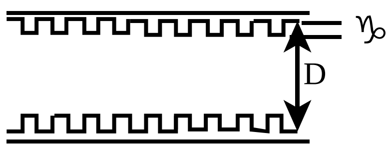
The friction factor for laminar flow in pipes:

$$f_F = \frac{\tau_w}{E_{kin}/V} = \frac{\tau_w}{{}^1/_2\ \rho v^2} = \frac{8\mu v}{{}^1/_2\ \rho v^2 D} = \frac{16}{\mathrm{Re}} \qquad \text{for Re} < 2100 \qquad \Rightarrow \qquad \boxed{f_D = \frac{64}{\mathrm{Re}}}$$

11.2 Friction factor for turbulent flow

In turbulent flow the friction factor also depends on Re, but the Hagen-Poiseuille equation does not hold and it is not possible to predict it theoretically as it was done for laminar flow. The friction factor must be determined experimentally, and it not only depends on Reynolds number but also on the surface roughness of the pipe: f = f (Re, γ/D). In laminar flow the roughness has no effect.

Moody (1944) presented a chart to predict the friction factor, hence the frictional pressure drop Δp_{fr} of round pipes. He based his chart on Nikuradse's data and on all the other available data on flow in pipes. Moody also suggested the working values for the absolute roughness shown in the table below.



γ = absolute roughness (the average height of roughness), m

The estimation of the roughness of the surface of the pipe normally presents considerable difficulty. At high Re, the effect of pipe roughness is considerable.

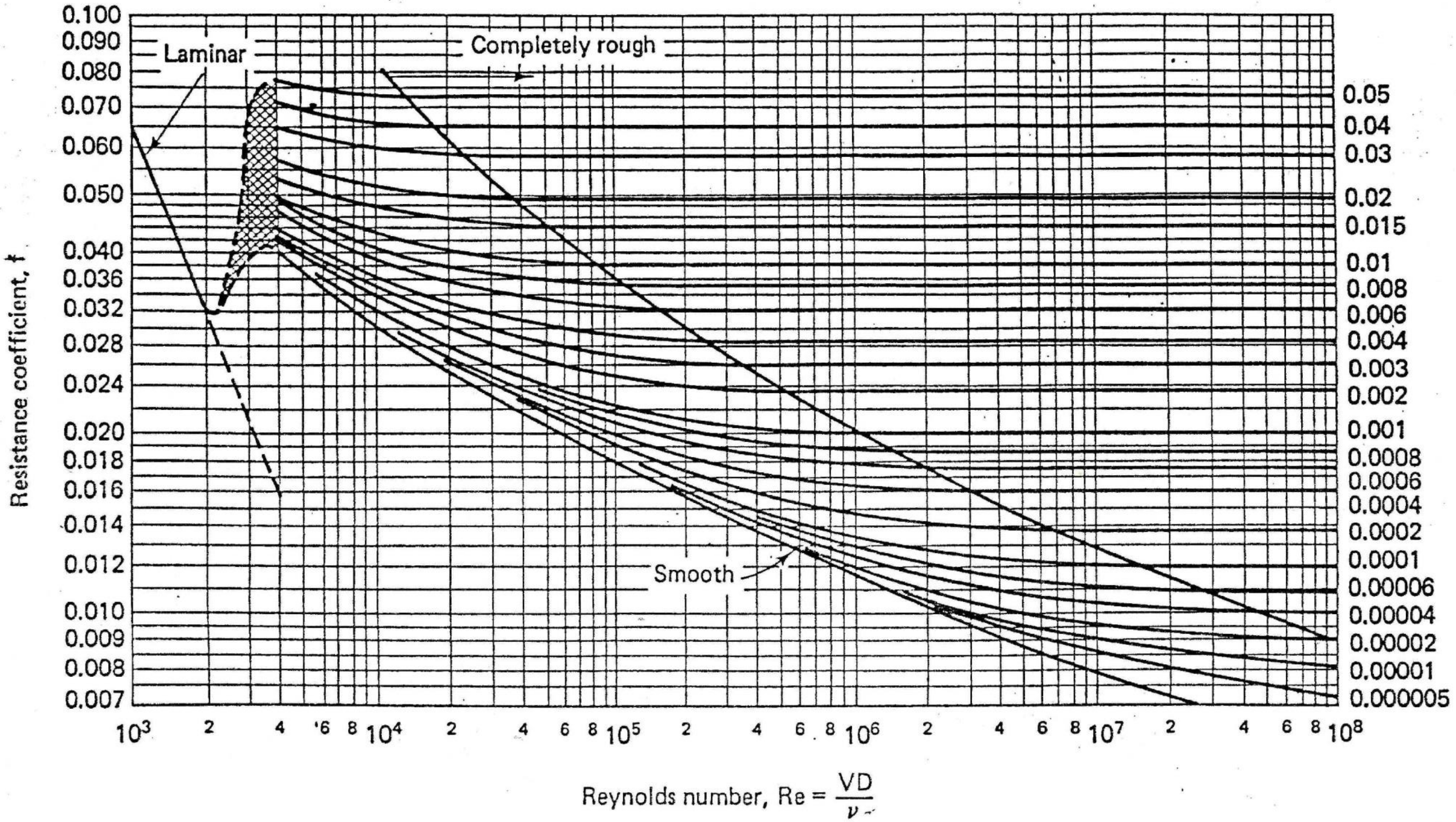
Absolute roughness of some materials

	γ, mm
Drawn tubing (brass, lead, glass, etc.)	0.0015
Commercial steel and wrought iron	0.046
Galvanized iron	0.15
Cast iron	0.26
Concrete	0.3 - 3.0
Steel	
commercial, new	0.002
stainless steel, new	0.002
sheet metal, new	0.05
rusted	2.0
riveted	3.0

Where pipes have become corroded, the values of the roughness will commonly be increased up to tenfold.

The Moody diagram is a log-log plot of f_D vs. Re.

for Re < 2100 $f_D = 64/\mathrm{Re}$ a straight line
for Re > 4000 (turbulent flow) — the lower line represents the friction factor for **hydraulically smooth** pipes. The other lines have a parameter γ/D relative roughness.



Moody resistance diagram for Darcy–Weisbach equation.

In turbulent flow higher pressure drops are required for a given flow rate for a rough pipe (f_D is higher).

For flow through channels of noncircular cross section we use the equations for circular pipes, but instead of the diameter D the equivalent diameter $D_{ekv.}$ (= $4r_H$) is used. This applies only to turbulent flow (but not for laminar flow).

Empirical relations are developed to calculate Re:

for Re < 20 000 $$f_D = \frac{0.316}{\mathrm{Re}^{0.25}}$$ by Blasius

for Re > 20 000 $$f_D = \frac{0.183}{Re^{0.2}}$$

for 3000 < Re < 300 000 $$f_D = 0.0056 + \frac{0.5}{Re^{0.32}}$$

The turbulent and transition region curves ($4x10^3 < \mathrm{Re} < 10^7$), can be represented with very good accuracy ($\pm$5%) by:

$$f_F = 0.001375\left[1 + \left(20000\,\frac{\varepsilon}{D} + \frac{10^6}{\mathrm{Re}}\right)^{1/3}\right]$$

The Haaland equation can also be used with accuracy of $\pm$ 1.5%

for $4x10^4 \leq \mathrm{Re} \leq 10^8$ $$\frac{1}{f_D} = -1.8\log\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon}{3.7D}\right)^{10/9}\right]$$

For computer use, many other algebraic expressions are also available instead of the Moody diagram.

For practical installations it must be remembered that the frictional losses cannot be estimated with better than $\pm$10% accuracy, because the exact values of the roughness are seldom known to better than that accuracy and because the roughness of pipes will change with use as they corrode or collect deposits. The pumping unit must therefore always have ample excess capacity.

The equations for turbulent flow in pipes hold for incompressible liquids. They also hold for a gas if the density (or pressure) changes less than 10%. Then an average density should be used and the errors involved will be less than the uncertainty limits in the friction factor. In this case the average density, ρ_{avg}, is the density at the average pressure $p_{avg} = (p_1 + p_2)/2$.

$$Re = \frac{\rho v D}{\mu} = \frac{GD}{\mu}$$ where G = ρv = mass velocity = constant, independent of the density and the velocity variation for the gas

and $\quad p_1 - p_2 = \Delta p_{fr} = f_D \dfrac{L}{D}\dfrac{v^2}{2} = f_D \dfrac{L}{D}\dfrac{G^2}{2\rho_{avg}}$

or $\quad p_1^2 - p_2^2 = f_D \dfrac{L}{D}\dfrac{G^2 RT}{M} \quad$ since $\quad \rho_1 = p_1 \dfrac{M}{RT}$

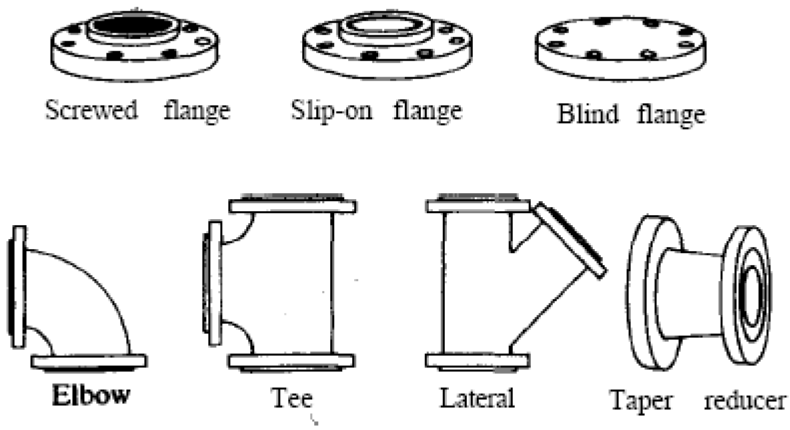
11.3 Friction losses for fittings

Fittings, valves, transition in pipe size disturb the normal flow lines and cause additional friction losses. Whenever the velocity of a fluid is changed, either in direction or magnitude, **form friction** is generated in addition to **skin friction**.

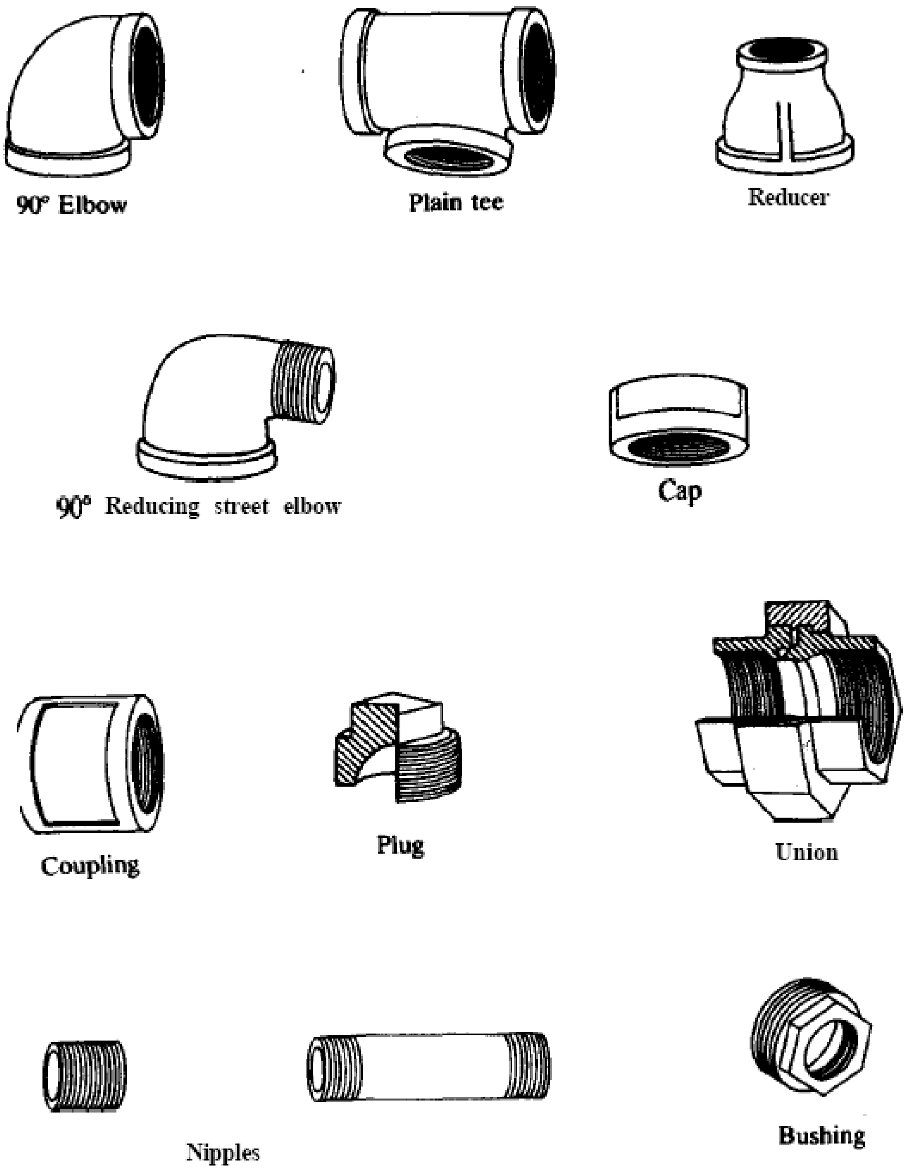
These losses are generally expressed as:

$h_f = K\dfrac{v^2}{2}$, J/kg where K = loss coefficient, dimensionless

The various loss coefficients are in some cases determined theoretically, but the majorities are entirely empirical. These losses are called **minor losses**, but in many cases they become more significant than pipe wall friction (**major losses**).



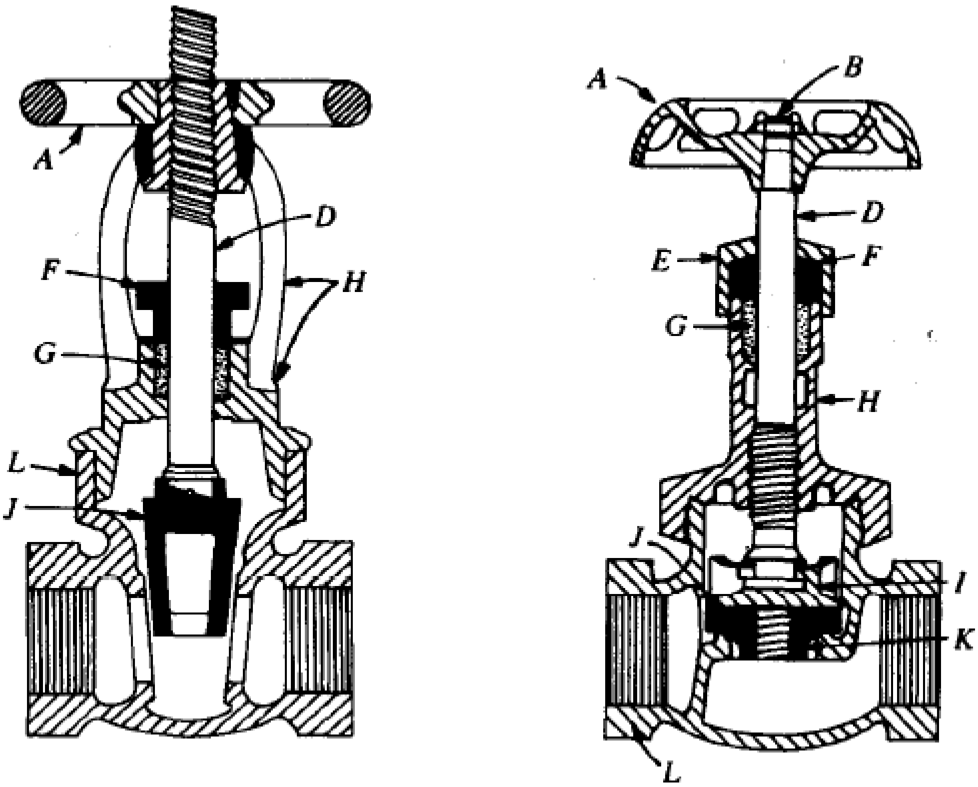
Typical flanged pipe fittings



Typical screwed pipe fittings

Gate valve

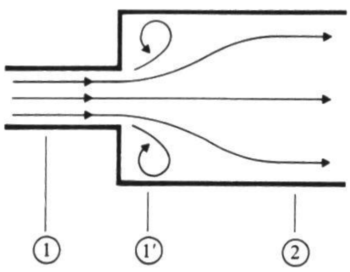
Globe valve



Sectional view of gate valve (left) and globe valve (right)

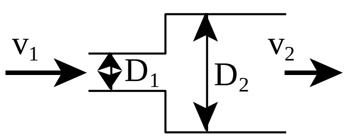
A-wheel; B-wheel nut; D-spindle; E-packing nut; F-gland; G-packing; H-bonnet; I-disk holder; J-disk; K-disk nut; L,-body.

Friction loss in sudden enlargement of cross-section



If the diameter of the pipe suddenly increases, the effective area available for flow will gradually increase from that of the smaller pipe to that of the larger one and the velocity of the flow will progressively decrease. Thus, fluid with a relatively high velocity will be injected into relatively slow moving fluid; turbulence will be set up and much of the excess kinetic energy will be converted to internal energy (heat) and therefore wasted. The larger the change of diameter, the greater the pressure losses. At position 1' turbulent eddies occur which give rise to the local friction loss.

If the change of cross-section is gradual, the kinetic energy can be recovered as pressure energy. For circular pipe the optimum angle for a tapering enlarging section is 7° and for a rectangular duct is about 11°.

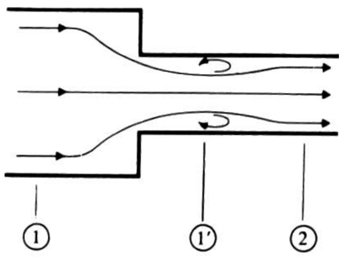


Applying the momentum balance and the Bernoulli equation we get:

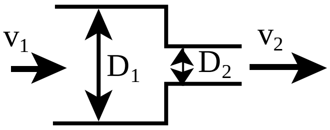
$$h_f = \frac{v_1^2}{2\alpha}\left[1-\frac{D_1^2}{D_2^2}\right] \Rightarrow K_e = \left(1-\frac{D_1^2}{D_2^2}\right)^2$$

When a pipe expands in to a large tank $A_1 << A_2$, i.e. $A_1/A_2 = D_1/D_2 = 0$ so $K_e = 1$. That is, the head loss is equal to the velocity head just before the expansion into the tank.

Friction loss in sudden contraction of cross-section



In a sudden contraction, flow contracts from point 1 to point 1', forming a vena contracta. From experiment it has been shown that this contraction is about 40% (i.e. $A_{1'} = 0.6\ A_2$).



$$h_f = 0.55\,\frac{v_1^2}{2\alpha}\left[1-\frac{D_1^2}{D_2^2}\right] \Rightarrow K_e = 0.55\left(1-\frac{D_1^2}{D_2^2}\right)^2$$

K_f for fittings are found in handbooks.

An equivalent method determining the head loss in fittings is to introduce an **equivalent length,** L_{eq}, such that $h_f = f_D\,(L_{eq}/D)\,(v^2/2)$ where L_{eq} is the length of pipe that produces a friction loss equivalent to the friction loss in a particular fitting. Therefore, $K_f = f_D\,L_{eq}/D$. We can add the equivalent length to the actual length of pipe to find an adjusted length, which gives practically the same friction effect as does the actual pipe including fittings.

$$\boxed{h_f = \sum\left(f_D\,\frac{L}{D}\,\frac{v^2}{2} + K_e\,\frac{v_1^2}{2} + K_c\,\frac{v_2^2}{2} + K_f\,\frac{v_1^2}{2}\right)}$$

Examples of loss coefficients and equivalent lengths

Fitting	K_f	L/D
Globe valve, wide open	6.0	300
half open	9.5	475
Angle valve, wide open	2.0	100
Gate valve, wide open	0.17	9
half open	4.5	225
Check valve, ball type	70.0	3500
swing type	2.0	100
Elbow, 90°	0.75	35
Elbow, 45°	0.35	17
Tee	1	50
Coupling	0.04	2
Union	0.04	2

These procedures give only a fair estimate of the pressure drop, not as reliable an estimate we can make for flow in a straight pipe. It is suggested, that the equivalent length method matches experimental results better when $Re < 10^5$ and the K method matches experimental results better when $Re > 10^5$.

There are no similar correlations for laminar flow.

11.4 Pipes and tubes

Fluids are usually transported in pipe or tubing. They are available in varying sizes, wall thickness, and material of construction (e.g. metals, alloys, ceramics, glass, and various plastics). There is no clear-cut distinction between the terms pipes and tubing. Generally pipes have thicker walls as compared to tubes, have larger diameters and come in moderate lengths of 6-12 m. Pipe walls are usually slightly rough; tubing has very smooth walls.

Pipes are commonly joined by welding, by using pipe threads, or by using a mechanical coupling. If frequent disconnection will be required, gasketed pipe flanges or union fittings provide better reliability than threads. Large above ground pipes typically use flanged joints. Pieces of tubing are connected by compression fittings, flare fittings, or soldered fittings.

Pipes and tubing are specified in terms of their diameter and their wall thickness.

Pipes

Nominal Pipe Size (NPS) is a North American set of standard sizes for pipes. The pipes are specified according to their **nominal size** based on inches. The nominal size of a pipe is related to it's **inside diameter**. The wall thickness is indicated by the **schedule number**. Usually the higher the schedule number, the thicker the wall. With steel pipe, the nominal diameter range from $\subset$ to 30 in (3 - 760 mm).

All pipes of a given nominal size, regardless of the wall thickness, have the same outside diameter, to ensure the interchangeability of fittings. Pipes of other materials are also made with the same outside diameters as steel pipe.

The wall thickness of pipes are designed according to allowable stress and pressure to prevent bursting. With pipes less than 8 in (200 mm) only schedule numbers 40, 80, 120, and 160 are common. With other alloys the wall thickness may be greater or less than that of steel pipes, depending on the strength of the alloy.

In Europe, pressure piping uses the same pipe IDs and wall thicknesses as the Nominal Pipe Size, but labels them with a metric Diameter Nominal (DN) instead of the imperial NPS. For NPS larger than 14, the DN is equal to the NPS multiplied by 25 (**not** 25.4). They are often called ISO pipes.

Pipes are either supported from below or hung from above. These devices are called supports. The inside of pipes can be cleaned with the tube cleaning process, if they are contaminated with debris or fouling. This depends on the process that the pipe will be used for and the cleanliness needed for the process. In some cases the pipes are cleaned using a "pig" or displacement device; alternately they may be chemically flushed. In some cases, the lines are blown clean with compressed air or nitrogen.

Tubing

The size of tubing is indicated by the **outside diameter**. The nominal value is the actual outer diameter. Inside diameter depends on the thickness of the tube. The wall thickness is ordinarily given by the **BWG** (Birmingham Wire Gauge) number, which ranges from 24 (very light) to 7 (very heavy). They are frequently made from non-ferrous metals such as Cu, brass, Al, and they are widely used in heat exchangers.

The optimum size of pipe for a specific situation depends on the relative cost of investment, power, maintenance, and fittings. In small installations rules of thumb are sufficient.

Representative ranges of velocity in pipes for ordinary practice

Fluid	Type of flow	Velocity, m/s
Thin liquid	Gravity flow	0.15 – 0.30
	Pump inlet	0.3 – 0.9
	Pump discharge	1.2 – 3
	Process line	1.2 – 1.4
Viscous liquid	Pump inlet	0.06 – 0.15
	Pump discharge	0.15 – 0.6
Steam		9 – 15
Air or gas		9 – 30

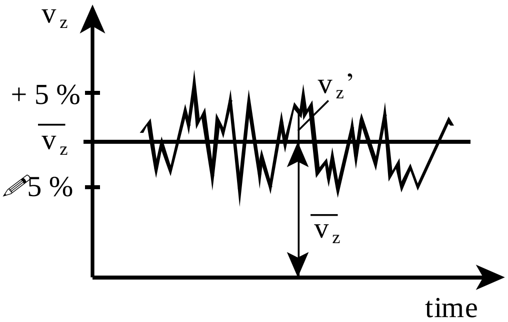

The West African Gas Pipeline

12 TURBULENT FLOW

The detailed behaviour of turbulent flows is so complex that in spite of considerable research effort over the past hundred years we do not now possess a comprehensive theory of turbulence or a simple conceptual model of how it works in detail. We can write suitable equations for turbulent flow, but the computation overwhelms the largest computers. Most of what we know consists of qualitative observations, measurements of various properties of turbulent flow, and some definitions and correlations of these measurements.

12.1 Velocity

All 3 components of the actual velocity vector vary rapidly in magnitude and direction also at a point, the instantaneous pressure at the same point fluctuates rapidly and simultaneously with the fluctuation of velocity. At first sight turbulence seems to be chaotic, structureless, and randomized, but this is not quite so. The figure below shows the turbulent velocity fluctuation at a point in time.



v_z = instantaneous velocity
$\bar{v}_z$ = mean or average velocity
v_z' = fluctuating velocity component

$$\boxed{v_z = \bar{v}_z + v_z'}$$

Throughout the turbulence literature (and this chapter) a bar over a symbol indicates the time average value of that quantity.

$$\boxed{\bar{v}_z \equiv \frac{1}{t_o}\int_t^{t+t_o} v_z \, dt}$$

This is the definition of mean or time average velocity.
t_o = time period of the order of a few second

The time averages of the fluctuating components of velocity and pressure are 0.

$$\boxed{\bar{v}_z' \equiv \frac{1}{t_o}\int_t^{t+t_o} v_z' \, dt = 0}$$

The fluctuations in turbulent flow in pipes are mostly so fast that ordinary fluid-flow measuring devices do not detect them at all; those devices record only values associated with $\bar{v}_z$.

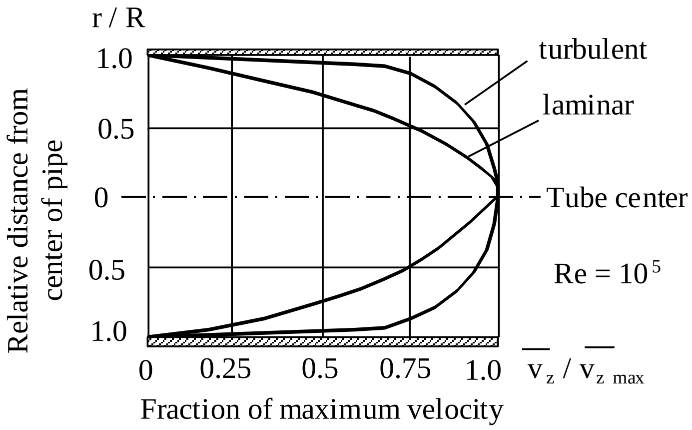
In laminar flow there are no eddies; the deviating velocities and pressure fluctuation do not exist; the total velocity in the direction of flow v_z (instantaneous velocity) is constant and equal to $\bar{v}_z$ (mean velocity). If the mean velocity is constant with time, then this is included in our definition of steady flow.

The ultimate description we would like to have of turbulent flow would be an explicit expression for $\bar{v}_x, \bar{v}_y, \bar{v}_z$, and v_x, v_y, and v_z as a function of time and position. Then we could predict the average and the fluctuating velocities at any point and any time.

The equation of continuity and motion do apply to turbulent flow, but we cannot solve them. If they could be solved, would give the instantaneous values of the velocity and pressure.

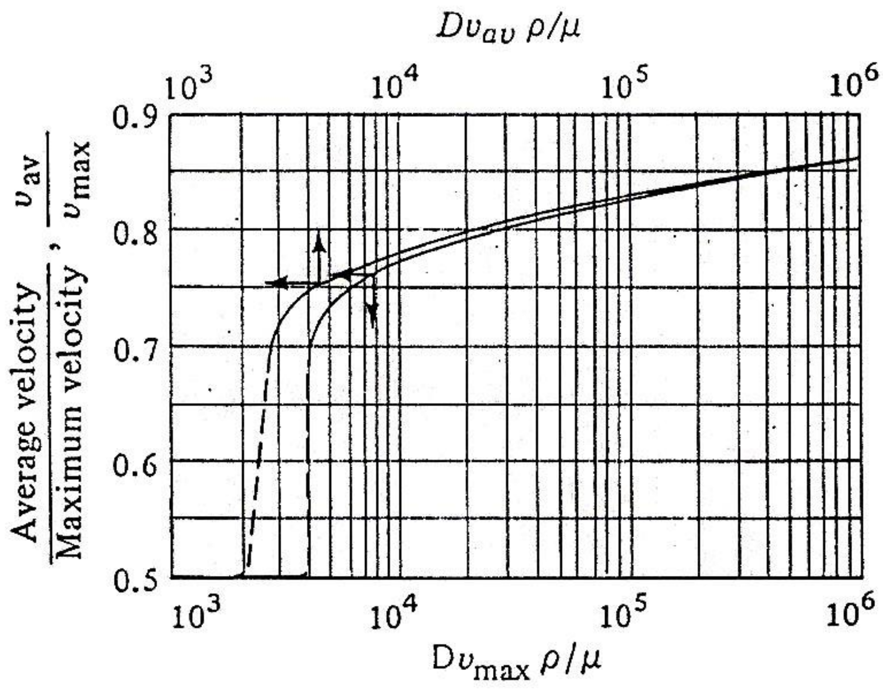
12.2 Velocity distribution in pipe

The qualitative comparison of laminar and turbulent velocity distributions in tubes is shown in the figure below.



The turbulent curve is somewhat flattened, $v_{avg} / v_{max} \cong 0.8$ and $\Delta p \sim v^2$
(for laminar flow $v_{avg} / v_{max} \cong 0.5$ and $\Delta p \sim v$).

The v_{avg}/v_{max} ratio as a function of Reynolds number can be read from the following chart:



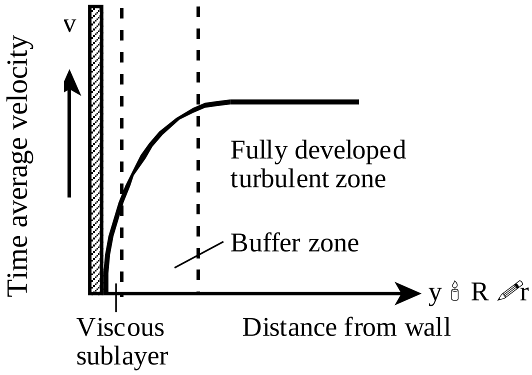
As the Reynolds number is increased, the velocity profile approaches closer and closer to **plug flow**.

There are 3 arbitrary zones in the tube:

(1) At the center of the tube the velocity fluctuations are almost completely random — **fully developed turbulence**. Purely viscous effects are negligible in it.

(2) At the wall the fluctuations in the axial direction are greater than those in the radial direction. All fluctuations approach zero at the wall itself — **viscous sublayer**. Newton's law of viscosity is used to describe the flow.

(3) **Buffer zone** (or transition layer): laminar and turbulent effects are both important. This is a relatively thin layer.



The change between the zones is continuous. This figure shows the velocity distribution for turbulent flow near the wall.

An important experimental result is the **power-law** relation for the turbulent velocity profile formulated by Prandtl. It is valid over smooth surfaces.

$$\boxed{\bar{v}_z = \bar{v}_{z,max}\left(\frac{R-r}{R}\right)^{\frac{1}{n}}}$$

where n = 6 – 10

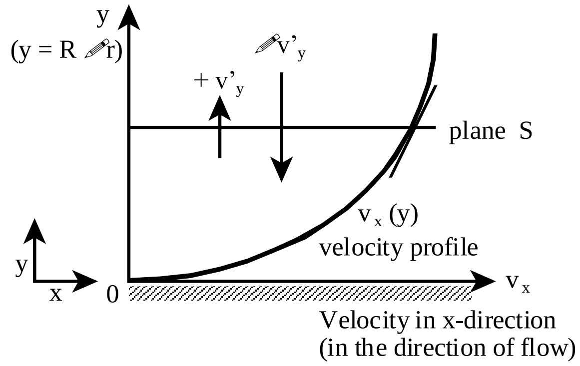
Re	**n**
4.0×10^3	6.0
2.3×10^4	6.6
$\mathbf{1.1 \times 10^5}$	**7.0**
1.1×10^6	8.8
2.0×10^6	10
3.2×10^6	10

Prandtl selected $^1/_7$ as the best average, so above equation is called $^1/_7$ th law as well. It is widely used because it is simple and gives useful results.

12.3 Reynolds stresses

In turbulent flow there is no net motion perpendicular to the tube axis, but there exist an intense, local, oscillating motion perpendicular to the tube axis. The transfer of fluid perpendicular to the net axial motion causes an increase in shear stress over the value given for laminar flow. This extra stress is called **Reynolds stress** after Reynolds, who first explained it. The most dramatic effect of these stresses is the large increase of frictional losses in turbulent flow. The other dramatic effect is the change in the shape of the velocity profile from laminar to turbulent flow.

Shear forces much larger than those occurring in laminar flow exist in turbulent flow wherever there is a velocity gradient across a shear plane. In laminar flow the components of τ shear stress are expressed according to Newton's law of viscosity. The turbulent or Reynolds stresses $\tau^{(t)}$ are handled empirically.



Consider a fluid in turbulent flow moving in x-direction. Plane S is parallel to the flow. The instantaneous velocity in plane S is v_x. The velocity gradient is positive (v_x increases with y).

The mechanism of turbulent shear depends upon the deviating velocities v'_y. An eddy moving toward the wall has a negative value of fluctuating velocity $!v'_y$, and its movement represents a mass flow rate $\rho(!v'_y)$ into the fluid below plane S. The velocity of the eddy in the x-direction is $v_x = \bar{v}_x + v'_x$. The rate of momentum transport per unit area is $\rho(!v'_y)v'_x$, if the eddy crossing is slowed down to the mean velocity $\bar{v}_x$. This momentum flux, after time-averaging for all eddies, is the turbulent shear stress or Reynolds stress:

$$\bar{\tau}_{yx}^{(t)} = \rho\, \bar{v}'_y\, \bar{v}'_x$$

$$\bar{\tau}_{yx}^{(t)} = -\,\mu^{(t)}\, \frac{d\,\bar{v}_x}{dy}$$

where $\mu^{(t)}$ = eddy viscosity; it was introduced by Boussinesq (1877).

Although the eddy viscosity $\mu^{(t)}$ is analogous to the viscosity $\mu^{(P)}$, there is a basic difference between the 2 quantities:

$\mu^{(P)}$: the viscosity is a true property of the fluid and is the macroscopic result of averaging motions and momentums of myriads of molecules.

$\mu^{(t)}$: the eddy viscosity is not just a property of the fluid but depends on the fluid velocity and the geometry of the system.

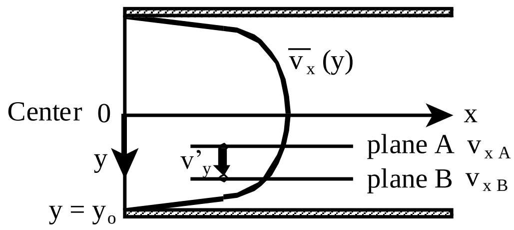
- It is a function of all factors that influences the detailed patterns of turbulence and the deviating velocities.
- It is especially sensitive to location in the turbulent field.
- It is determined by experiments on the flow itself (with great difficulty).

The total shear stress in the turbulent fluid is the sum of viscous stress and the turbulent stress:

$$\bar{\tau}_{yx}^{(t)} = -\,(\mu^{(t)} + \mu^{(\ell)})\, \frac{d\,\bar{v}_x}{dy}$$

There are semiempirical expressions for the Reynolds stresses in literature. One of them uses a quantity called Prandtl mixing length.

12.4 Prandtl mixing length (1925)



$$v_{xA} = \bar{v}_{xA} + v'_{xA}$$

$$v_{xB} = \bar{v}_{xb} + v'_{xB}$$

$v_{xA} > v_{xB}$

Turbulent velocity distribution in a slit

Consider a small pocket or "lump" of fluid, which is displaced from plane A to B in the y-direction with velocity v'_y. In reality, the lump of fluid will gradually lose its identity, but in definition of the mixing length, it is assumed to retain its identity until it has traveled a distance P defined as Prandtl mixing length.

In plane B, the lump of fluid will differ in mean velocity from that in plane A by $\bar{v}_{xB} - \bar{v}_{xA}$.
Since the pocket of liquid is assumed to retain its original velocity, $\bar{v}_{xB} - \bar{v}_{xA} = -v'_x$

For small distance involved: $\dfrac{d\bar{v}_x}{dy} = \dfrac{\bar{v}_{xB} - \bar{v}_{xA}}{\ell}$

$\Rightarrow \quad v'_x = -\ell \dfrac{d\bar{v}_x}{dy}$ and in general $|\bar{v}'_x| = \ell \left| \dfrac{d\bar{v}_x}{dy} \right|$

Prandtl also assumed that $|\bar{v}'_y| \approx |\bar{v}'_x| \quad \Rightarrow \quad \bar{v}'_x \bar{v}'_y = \ell^2 \left| \dfrac{d\bar{v}_x}{dy} \right|^2$

Since the sign of $\bar{v}'_x \bar{v}'_y$ depends on the sign of $d\bar{v}_x / dy$, we can write this as

$$\bar{v}'_x \bar{v}'_y = -\ell^2 \left| \frac{d\bar{v}_x}{dy} \right| \frac{d\bar{v}_x}{dy}$$

Then $\quad \tau_{yx}^{(t)} = \rho \bar{v}'_x \bar{v}'_y = -\rho \ell^2 \left| \dfrac{d\bar{v}_x}{dy} \right| \dfrac{d\bar{v}_x}{dy}$

and $\quad \mu^{(t)} = \rho \ell^2 \left| \dfrac{d\bar{v}_x}{dy} \right|$

Although it may appear that we have just replaced one empirical, noncomputable quantity ($\mu^{(t)}$) with another, but P is easier to estimate, than $\mu^{(t)}$.

- P cannot be greater than the dimensions of the channel,
- it should approach zero near the wall.

P = 0.4 y assumption gives a very good velocity distribution for turbulent flow (where y is the distance from the wall).

12.5 The universal velocity distribution

We divide the fluid flow in the pipe into

(1) a laminar sublayer ($\tau_{rz} = \tau_{rz}^{(P)}$)
(2) a buffer zone ($\tau_{rz} = \tau_{rz}^{(P)} + \tau_{rz}^{(t)}$), and
(3) a central core ($\tau_{rz} = \tau_{rz}^{(t)}$).

Let $y = R - r$ be the distance from the wall
v represents $\bar{v}$
τ represents $\bar{\tau}_{rz}^{(t)}$ or $\tau^{(\ell)}$
τ_w = shear stress at the wall.

Prandtl mixing length relation is used for the turbulent shear stress expression.

$$\left.\begin{aligned} &\text{We saw} \quad \tau = \frac{\Delta p}{2L} r \\ &\text{and} \quad \tau_w = \frac{\Delta p}{2L} R \end{aligned}\right\} \Rightarrow \quad \tau = \tau_w \frac{r}{R} = \tau_w \left(1 - \frac{y}{R}\right)$$

(1) Viscous sublayer

Assume: $\tau = \tau_w$ that is neglect any variation in τ in the viscous sublayer, since it is very thin.

It means $\tau = \tau_w \underbrace{\left(1 - \frac{y}{R}\right)}_{=1}$ Prandtl mathematical simplification physically indefensible but simplifies the mathematics and the result differs very little from the correct solution.

$$\Rightarrow \quad \tau = \tau_w = \mu \frac{dv}{dy} = \text{constant}$$

On integration, $\tau_w y = \mu v$

$$\Rightarrow \quad v = \frac{\tau_w}{\mu} y$$

This is a **linear velocity distribution**. It is in contrast with the parabolic velocity profile in the entire tube when there is laminar flow.

Define $v^* \equiv \sqrt{\dfrac{\tau_w}{\rho}}$ **friction velocity**

(The friction velocity is not a physical velocity, which could be measured at some point in the flow, but a combination of terms that has the dimensions of a velocity.)

$v^+ \equiv \dfrac{v}{v^*}$ dimensionless velocity

$y^+ \equiv \dfrac{y v^* \rho}{\mu}$ dimensionless distance from the wall

$$\frac{v}{v^*} = \frac{\tau_w y}{\mu v^*} = \frac{\tau_w y}{\mu \sqrt{\dfrac{\tau_w}{\rho}}} = \frac{y\sqrt{\tau_w \rho}}{\mu}$$

$\Rightarrow \quad \boxed{v^+ = y^+}$ for $0 < y^+ < 5$

(3) Turbulent core

Assumptions:

- $\tau = \tau_w$ = constant (as for viscous sublayer)
- $\ell = \kappa_1 y$ where κ_1 is a constant (This equation is not correct far from the wall.)
- $\tau = \tau^{(t)}$ (neglect any viscous shear stress)

$$\bar{\tau}_{yx}^{(t)} = \rho \ell^2 \left(\frac{d\bar{v}_x}{dy} \right)^2 = \rho \kappa_1^2 y^2 \left(\frac{d\bar{v}_x}{dy} \right)^2 \quad \Rightarrow \quad \tau_w = \rho \kappa_1^2 y^2 \left(\frac{d\bar{v}_x}{dy} \right)^2$$

$$\underbrace{\frac{\tau_w}{\rho}}_{= v^{*2}} = \rho \kappa_1^2 y^2 \left(\frac{d\bar{v}_x}{dy} \right)^2$$

$$v^* = \kappa_1 y \frac{dv}{dy}$$

$$v^* \frac{dy}{y} = \kappa_1 \, dv$$

Integrate from the outer edge of the buffer zone $y = y_1$ to any position y

$$\frac{1}{\kappa_1} v^* \ln \frac{y}{y_1} = v - v_1$$

$$v^+ - v_1^+ = \frac{1}{\kappa_1} \ln \frac{y^+}{y_1^+} \qquad \text{for} \qquad y^+ > y_1^+$$

$$v^+ = \frac{1}{\kappa_1} \ln y^+ + \underbrace{(v_1^+ - \ln y_1^+)}_{\text{constant;}}$$

This is a **logarithmic velocity distribution**

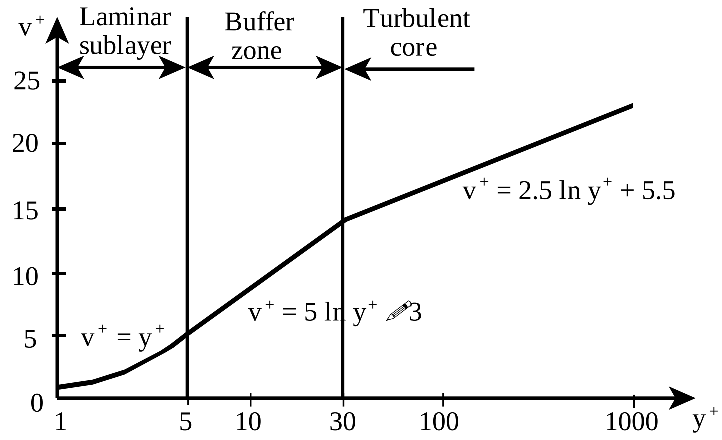
von Kármán has found $\kappa_1 = 0.4$

$\boxed{v^+ = 2.5 \ln y^+ + 5.5}$ for $y^+ > 30$

(2) Buffer zone

An empirical equation of the above form describes the velocity distribution in the buffer zone:

$\boxed{v^+ = 5.0 \ln y^+ - 3.05}$ for $5 < y^+ < 30$



The universal velocity profile for flow in a smooth circular tube

Although these velocity distribution equations are in general found to be adequate, there is apparently no layer near the wall where the flow is steady - laminar; eddies enter the region below $y^+ = 5$. In this region viscous effects are predominant, but the flow is not really laminar.

13 DIMENSIONAL ANALYSIS

In majority of fluid-mechanics and heat transfer problems, turbulent flow conditions exist. The equations of motion and energy cannot be solved explicitly for turbulent flow problems, unless some significant and limiting assumptions are made. However, we can use the differential equations to find out which dimensionless number can be used in correlating experimental data for a given physical situation — the method is called dimensional analysis.

13.1 Buckingham method

In many cases we are not even able to formulate a differential equation which clearly applies. Then Buckingham method is required. Buckingham theorem states that the functional relationship among q quantities in terms of u fundamental units (or dimensions) may be written as (q – u) independent dimensionless groups called π's.

There are 5 fundamental quantities which serve as a measure of physical systems:

mass	M	
length	L	
time	t	
temperature	T	
electric charge	Q	
(heat	H)	are not essential but can be added
(force	F)	

Example:
An incompressible fluid is flowing inside a circular pipe of inside diameter D.

– The significant variables are: $\Delta p, v, D, L, \mu, \rho$ $\Rightarrow$ $q = 6$ the total number of variables
– The fundamental dimensions are: M, L, t $\Rightarrow$ $u = 3$
– The number of dimensionless groups or π's is $q - u = 6 - 3 = 3$

$$\text{Thus } \pi_1 = f(\pi_2, \pi_3)$$

– Select a core group of u (or 3) variables which will appear in each π group and among them contain all the fundamental dimensions. Also, no 2 of the variables selected for the core can have the same dimensions. In choosing the core, the variable whose effect one desires to isolate is often excluded (Δp).

This leaves us with the variables: v, D, μ, ρ (L and D have the same dimension)

We select the core variables to be: D, v and ρ (common to all 3 groups)

The dimensionless groups are then:

$$\pi_1 = D^a\, v^b\, \rho^c\, \Delta p^1$$
$$\pi_2 = D^d\, v^e\, \rho^f\, L^1$$
$$\pi_3 = D^g\, v^h\, \rho^i\, \mu^1$$

To be dimensionless, the variables must be raised to certain exponents a, b, c, ...

– Evaluate the exponents
Write the equations for π_1, π_2, π_3 dimensionally:

for π_1 $$M^o L^o t^o = 1 = L^a \left(\frac{L}{t}\right)^b \left(\frac{M}{L^3}\right)^c \frac{M}{Lt^2}$$

equate the exponents of M, L, and t:

$$\left.\begin{array}{ll} \text{(L)} & 0 = a + b - 3c - 1 \\ \text{(M)} & 0 = c + 1 \\ \text{(t)} & 0 = -b - 2 \end{array}\right\} \quad \begin{array}{l} a = 0 \\ b = -2 \\ c = -1 \end{array}$$

$$\Rightarrow \quad \pi_1 = \frac{\Delta p}{v^2 \rho} = Eu \qquad \text{Euler number}$$

for π_2 $$M^o L^o t^o = 1 = L^d \left(\frac{L}{t}\right)^e \left(\frac{M}{L^3}\right)^f L^1$$

$$\left.\begin{array}{ll} \text{(L)} & 0 = d + e - 3f + 1 \\ \text{(M)} & 0 = f \\ \text{(t)} & 0 = -e \end{array}\right\} \quad \begin{array}{l} d = -1 \\ e = 0 \\ f = 0 \end{array}$$

$$\Rightarrow \quad \pi_2 = \frac{L}{D}$$

for π_3 $$M^o L^o t^o = 1 = L^g \left(\frac{L}{t}\right)^h \left(\frac{M}{L^3}\right)^i \frac{M}{Lt}$$

$$\left.\begin{array}{ll} \text{(L)} & 0 = g + e - 3i - 1 \\ \text{(M)} & 0 = i + 1 \\ \text{(t)} & 0 = -h - 1 \end{array}\right\} \quad \begin{array}{l} g = -1 \\ h = -1 \\ f = -1 \end{array}$$

$$\Rightarrow \quad \pi_3 = \frac{\mu}{Dv\rho} \quad \Rightarrow \quad \pi_3 = \frac{Dv\rho}{\mu} = Re$$

Since $\pi_1 = f(\pi_2, \pi_3)$ therefore $$\frac{\Delta p}{\rho v^2} = f\left(\frac{L}{D}, \frac{\rho v D}{\mu}\right)$$

It is permissible to manipulate this equation algebraically and obtain another dependence. This type of analysis is useful in empirical correlation of data. However, it does not tell the importance of each dimensionless group, nor does it select the variables to be used.

14 PUMPS

Fluids are moved through pipe, equipment, or the ambient atmosphere by pumps, fans, blowers, and compressors. They increase the mechanical energy of the fluid. The energy increase may be used to increase the velocity, the pressure, or the elevation of the fluid. The energy required by the pump will depend on the height through which the fluid is raised, the pressure required on delivery, the length and diameter of the pipe, the flow rate, together with the physical properties of the fluid, particularly its viscosity and density.

14.1 Classification of pumps

The liquids used in chemical industries differ considerably in physical and chemical properties, and it has been necessary to develop a wide variety of pumps.

(1) Positive displacement pumps (apply direct pressure to the fluid)
Positive displacement (P.D.) pumps allow the fluid to flow into some enclosed cavity from a low pressure source, trapping the fluid, and then forcing it out into a high-pressure receiver by decreasing the volume of the cavity.

Before 150 years ago all pumps and compressors were P.D. pumps. They are practically constant volumetric flow rate devices (at fixed drive motor speed). The volume of liquid delivered increases directly with speed of the motor and is not appreciably influenced by the pressure. They can generate high pressures.

Several types have been developed:
- Reciprocating pumps (the force applied to the fluid by a piston acting in a cylinder)
  Types: membrane pumps, diaphragm pumps, direct acting pumps, piston pumps...
- Rotating pumps (the force is applied to the fluid by rotating pressure members)
  Types: gear pumps, sliding vane pumps, peristaltic pumps, screw pumps...

(2) Centrifugal pumps (use torque to generate rotation)
Starting about 1850, industrial countries learned to build high-speed rotating machines. Before that time no such devices existed. Rotating high-speed pumps have almost completely displaced P.D. devices for high flow rates, and for most medium-pressure, medium-flow-rate pumping operations. They are simpler, smaller, cheaper, and more robust than the P.D. pumps they replace. They are the most common type of pump in chemical engineering processes.

They can be classified
- by the number of stages: single stage
  multistage
- by the impeller type: radial flow
  axial flow
  mixed flow
- by the casing type: volute
  diffuser
- by the position of shaft: horizontal
  vertical

– by the suction: single suction
double suction

Fans, blowers and compressors are also centrifugal devices for handling gases.

Fans discharge large volume of gas (usually air) into open spaces or large ducts. They accept gases at near atmospheric pressure and raise the pressure by –3%.
Blowers raise the pressure to an intermediate level, usually to less than 3 atm (that is develop a maximum pressure of –2 atm), but more than accomplished by fans.
Compressors are machines that raise the pressure above the levels for which fans are used. They include blowers.

In pumps and fans the density of the fluid does not change appreciably — in discussing them incompressible-flow theory is adequate. In blowers and compressors compressible-flow theory is required.

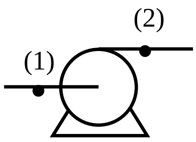
14.2 Pump characteristics

***Pump capacity* (Q)**
Pump capacity, or flow rate, is the liquid volume flowing through the pump per unit time:

$$\boxed{Q = \frac{V}{t}\,, \quad m^3/s}$$

Pumping head (H)
Equivalent terms for pumping head: developed head, pump head, pump total head. It represents the increase in useful energy content of the pumped liquid. At the pump itself, the liquid enters the suction connection and leaves the discharge connection. A Bernoulli equation can be written:

Total suction head (1): $H_{sc} \equiv \dfrac{p_{sc}}{\rho g} + z_{sc} + \dfrac{v_{sc}^2}{2g}\,, \quad m$



Total discharge head (2): $H_{dc} \equiv \dfrac{p_{dc}}{\rho g} + z_{dc} + \dfrac{v_{dc}^2}{2g}\,, \quad m$

$$\boxed{H = H_{dc} - H_{sc} \equiv \Delta z + \frac{p_{dc} - p_{sc}}{\rho g} + \frac{v_{dc}^2 - v_{sc}^2}{2g}\,, \quad m}$$

where $\Delta z = z_{dc}$! z_{sc} = the height difference of the center line of the discharge and suction pipe, m (usually negligible)

The head is a measurement of the height of a liquid column that the pump could create from the kinetic energy imparted to the liquid. (If a pipe were shooting a jet of water straight up into the air, the height the water goes up would be the head.)

The main reason for using head instead of pressure to measure the energy the pump develops that the pressure from a pump will change if the density of the liquid changes, but the head will not change.

Pump power (P)

The useful power (P_{useful}) is the hydraulic power transmitted to the fluid:

$$\boxed{P_{useful} = \dot{m}gH = \Delta p\dot{V} = \rho QgH, \quad W}$$

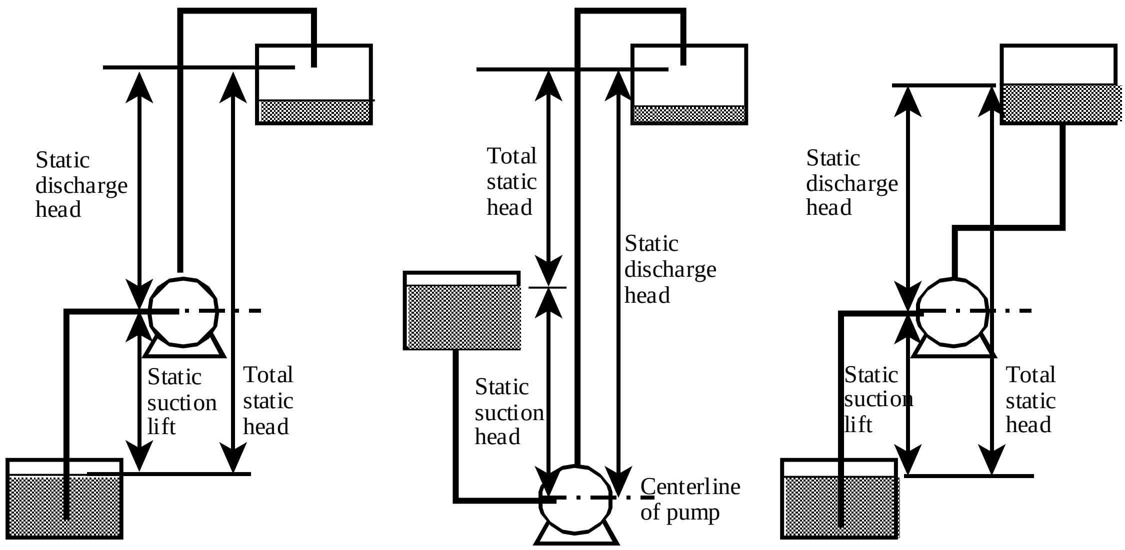
The **power requirement** (P) is the power supplied to the pump from an external source. It is also called **power rating**.

Pump efficiency (η)

$$\boxed{\eta = \frac{P_{useful}}{P} = \frac{\rho gQH}{P} \quad \Rightarrow \quad P = \frac{P_{useful}}{\eta}}$$

$\eta = 0.65 - 0.90$. It represents the overall efficiency of the pump.
Generally, the larger the pump the higher the attainable efficiency.

14.3 Head terms used in pumping



(a) (b) (c)

Static suction lift: it is the vertical distance, in m, from the liquid supply level to the pump center line, the pump being **above** the supply level.

Static suction head: it is the vertical distance, in m, between the liquid supply level and the pump centerline, where the pump is **below** the supply level.

Suction lift: it is a negative suction head.

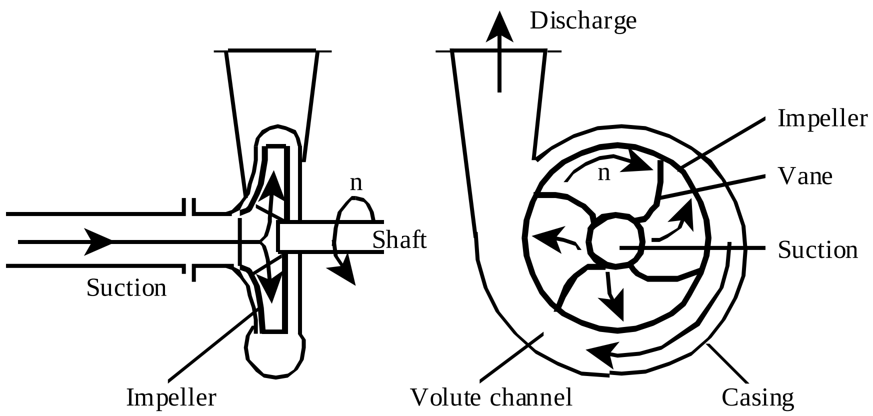
Static discharge head: it is the vertical distance, in m, from the pump centerline to the point of free delivery of the liquid.

Total static head: it is the vertical distance between the supply level and the discharge level of the liquid.

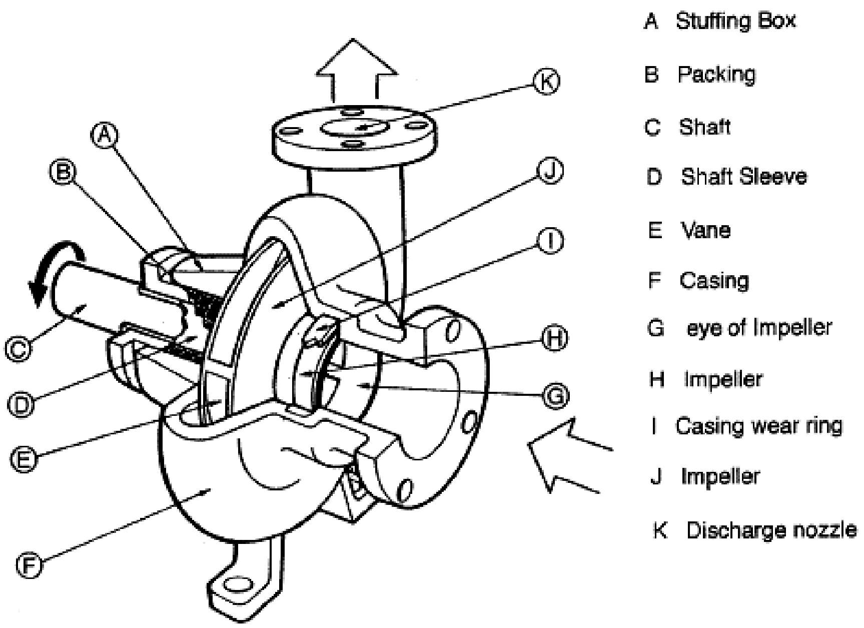
14.4 Centrifugal pumps

Centrifugal pumps constitute the most common type of pumping machinery in ordinary plant practice. They can pump liquids with very wide ranging properties and suspensions with a high solid content and may be constructed from a very wide range of corrosion resistant materials.

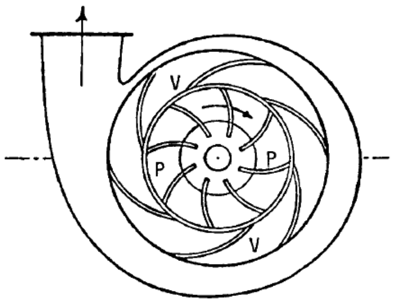
Working principle and common types of centrifugal pumps
The liquid enters through a suction connection concentric with the axis of the high speed rotary element called **impeller** (rotor, runner). The impeller carries radial **vanes**. Liquid flows outward between the vanes and leaves the impeller at a considerably greater velocity than at the entrance to the impeller. The liquid leaving the impeller is collected in the **casing** and leaves the pump through a tangential discharge connection. In the casing the velocity of the liquid is gradually decreasing because of its increasing cross section. According to Bernoulli equation the kinetic energy is converted into pressure energy. The shaft is driven by a direct-connected motor at a constant speed (commonly $n = 1750\ \mathrm{min}^{-1}$). At the entrance to the impeller the pressure is less so suction is generated and the pump continuously carries the liquid.



Volute pump



Parts of a centrifugal pump



Diffuser pump: vanes V are fixed, impeller P rotate

Centrifugal pumps are **not self-priming**. At start, the vacuum produced by the impeller is not sufficient (because of the big gap between the impeller and the casing) to suck in the liquid to be carried. Priming with the liquid to be pumped is required before start.

Volute type: The impeller discharges into a gradually expanding spiral passage in which velocity head is partially converted to pressure head at the outlet.

Single suction: The liquid enters on one side at the edge of the impeller.

Multistage: The maximum head that is generated by a single impeller is limited by the peripheral speed reasonably attainable to –20-30 m. When a greater head is needed, 2 or more impellers can be mounted in series on a single shaft to obtain a multistage pump.

Impeller types

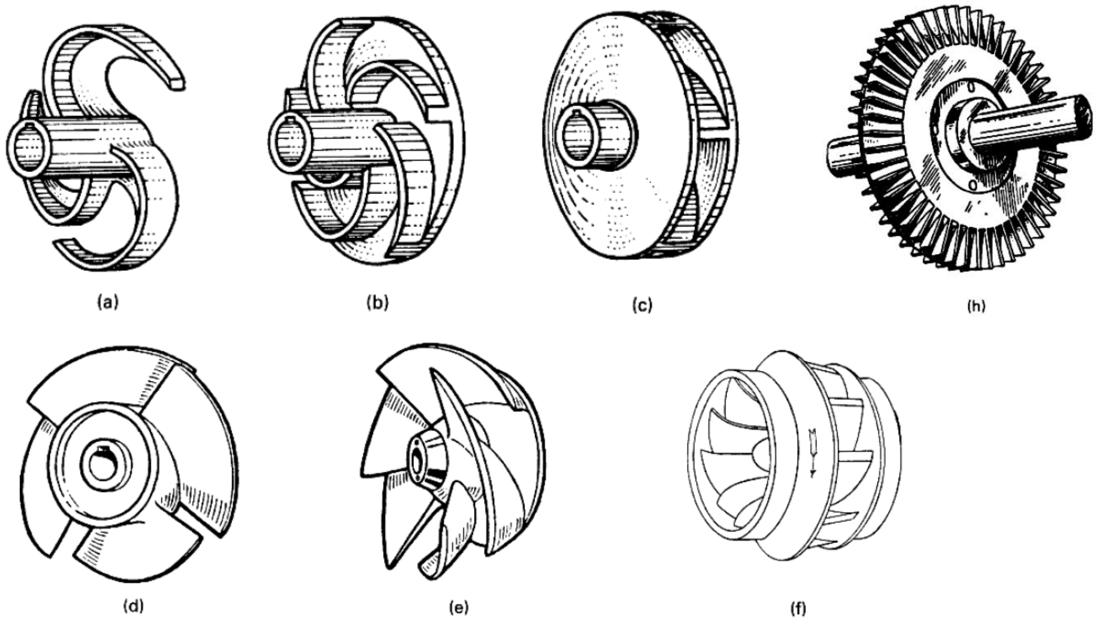
Open impeller: It consists of vanes attached to a shaft without any form of supporting sidewall and is suited to handling slurries without clogging.

Semienclosed impeller: It has a complete shroud on one side. It is nonclogging, used primarily in small size pumps.

Closed impeller: It has shrouds on both sides of the vanes from the eye to the periphery. It is used for clear liquids.

Axial flow: The flow develops by axial thrust of a propeller blade. It is limited to heads under 15 m or so.

Mixed flow: It develops head by combined centrifugal action and propeller action in the axial direction. It is suited to high flow rates at moderate heads.



Some types of impellers for centrifugal pumps. (a) Open impeller. (b) Semi open impeller. (c) Shrouded impeller. (d) Axial flow (propeller) type. (e) Combined axial and radial flow, open type. (f) Shrouded mixed-flow

A centrifugal pump

14.5 Cavitation

Bubbles will form in a liquid when the local pressure falls below the vapor pressure of the liquid. **Cavitation** is the local boiling of the liquid which occurs where its pressure is at or below the vapor pressure (and this may happen where the local velocity is high, as follows from Bernoulli equation). If the suction pressure is only slightly greater than the vapour pressure, some liquid may flash to vapour inside the pump and cavitation will occur.

When cavitation first begins, only very small bubbles are formed but there may have serious effects. Thus, when a bubble is formed and later collapses, a very high transient pressure is generated (800 MPa ≈ 8000 atm) where the fluid elements meet. If this collapse occurs on the surface of an immersed body, it will receive a blow and in persistent cavitation such blows will be applied with great frequency (many thousands time a second) and for long periods. This hammering of the surface is the cause of the pitting on parts of hydraulic machinery. Parts of the surface may even be torn completely away. When cavitation occurs in a pump it may sound as though gravel were passing through the machine. The noise is caused by the collapse of the vapor bubbles.

Cavitation greatly reduces the pump capacity and causes severe erosion. Since no liquid pump can pump only vapour, liquid flow can even stop because of cavitation. The satisfactory operation of a pump requires that vaporization of the liquid being pumped does not occur at any condition of operation.

Net Positive Suction Head (NPSH)

As the liquid passes from the pump suction to the eye of the impeller, the velocity increases and the pressure decreases. There are also pressure losses due to shock and turbulence as the liquid strikes the impeller. The centrifugal force of the impeller vanes further increases the velocity and decreases the pressure of the liquid.

To avoid cavitation, the suction head (H_{sc}) must exceed the vapour pressure by a certain value called NPSH: **NPSH = H_{sc} – vapour pressure head**

For any pump, the manufacturers specify the minimum value of the NPSH required, which must exist at the suction point of the pump. The required NPSH is the amount by which the pressure at the suction point of the pump, expressed as a head of the liquid to be pumped, must exceed the vapor pressure of the liquid. For any installation, the available NPSH must be calculated.

NPSH =	Pressure head at the source
	+ static suction head (negative for suction lift)
	− friction head in the suction line
	− vapor pressure of the liquid

Available NPSH: It is the pressure head available to force a given flow (m^3/s) through the suction piping into the pump. This is a function of the system. It is easily calculated with above formula.

Required NPSH: this is a function of pump design and it varies with capacity and speed of any given pump. It is supplied by the manufacturer based on tests.
Common range is 1.5 - 6 m.

Condition of cavitation free operation: **available NPSH > required NPSH**

The available NPSH should always be calculated if

- the pump is installed above the liquid level
- the pump draws from a tank under vacuum
- the liquid has a high vapour pressure
- the suction line is unusually long

Causes of cavitation

- Clogged suction pipe
- Suction line too long
- Suction line diameter too small
- Suction lift too high
- Valve on suction line only partially open

Cavitation may be minimized by paying attention to the design of pump installation on the suction side.

Remedies

- Remove debris from suction line
- Move pump closer to source tank/sump — reduces h_f
- Increase suction line diameter — reduces h_f
- Decrease suction lift requirement
- Install larger pump running slower which will decrease the NPSH required by the pump
- Increase discharge pressure
- Fully open suction line valve — reduces h_f

14.6 Similitude

Whenever it is necessary to perform tests on a model to obtain information that cannot be obtained by analytical means alone, the rules of similitude must be applied. Similitude is the theory and art of predicting prototype performance from model observations.

Model Study: Present engineering practice makes use of model tests more frequently than most people realize. Information derived from these model studies often indicates potential problems that can be corrected before prototype is built, thereby saving considerable time and expense in development of the prototype (e.g. airplanes, cars, ships, reactors, pumps etc.)

Geometric similarity refers to linear dimensions. Two vessels of different sizes are geometrically similar if the ratios of the corresponding dimensions on the two scales are the same.

Kinematic similarity refers to motion and requires geometric similarity and the same ratio of velocities for the corresponding positions in the vessels.

Dynamic similarity concerns forces and requires all force ratios for corresponding positions to be equal in kinematically similar vessels.

The requirement for similitude of flow between model and prototype is that the significant dimensionless parameters must be equal for model and prototype.

Operation similarity conditions for pumps

Manufacturers usually build centrifugal pumps in series of geometrically similar types. The pump operating conditions can be set by 2 parameters: pump capacity Q and rotational speed n. Operation of a pump is similar to another pump if the conditions of geometrical, kinematic, and dynamic similitude are satisfied.

For geometrically similar pumps, operating under similar conditions, the **similarity relations** are:

$$\frac{Q_2}{Q_1} = \frac{n_2}{n_1}\left(\frac{D_2}{D_1}\right)^3 \quad \Rightarrow \quad \frac{Q}{nD^3} = \text{constant} = \textit{capacity coefficient } c_Q$$

$$\frac{H_2}{H_1} = \left(\frac{n_2 D_2}{n_1 D_1}\right)^2 \quad \Rightarrow \quad \frac{gH}{n^2 D^2} = \text{constant} = \textit{head coefficient } c_H$$

$$\frac{P_2}{P_1} = \frac{\rho_2}{\rho_1}\left(\frac{n_2}{n_1}\right)^3\left(\frac{D_2}{D_1}\right)^5 \quad \Rightarrow \quad \frac{P}{\rho n^3 D^5} = \text{constant} = \textit{power coefficient } c_P$$

For a given pump:

$$\frac{Q_2}{Q_1} = \frac{n_2}{n_1} \quad \Rightarrow \quad \frac{Q}{n} = \text{constant}$$

$$\frac{H_2}{H_1} = \left(\frac{n_2}{n_1}\right)^2 \quad \Rightarrow \quad \frac{H}{n^2} = \text{constant}$$

$$\frac{P_2}{P_1} = \left(\frac{n_2}{n_1}\right)^3 \quad \Rightarrow \quad \frac{P}{n^3} = \text{constant}$$

With these relationships it is possible to calculate the pump characteristics from one speed to the other.

Specific speed (n_s)

It is another constant characteristic of geometrically similar pumps. It is used to describe the geometry (shape) of a pump impeller. It is frequently used to classify centrifugal pumps.

$$n_s = \frac{nQ^{0.5}}{(gH)^{0.75}}$$

It is a dimensionless parameter; its value does not depend on the units used. However, dimensional formulae are used in practice. The numerical value of the specific speed indicates the type of pump.

Europe

Specific speed is the speed required to discharge water at a unit head of 1 m when the pump is driven at the unit power of 1 brake hp (horse power) under best efficiency conditions with an impeller geometrically similar to the one under consideration. This definition involves the obsolete unit of power, n_s is however universally accepted and commonly used.
(1 brake hp = 0.7457 kW or 1 kW = 1.36 break hp)

An equivalent definition: Specific speed is the speed required to produce 1 m head at 75 P/s = 0.075 m^3/s capacity of water.

$$n_s = 3.65 \frac{n\sqrt{Q}}{H^{3/4}}$$

where n = rotational speed, 1/min
Q = pump capacity, m^3/s
H = pump head, m

Imperial units (US)

Specific speed is the speed required to produce 1 ft head with 1 break hp.

$$n_s = \frac{n\sqrt{Q}}{H^{3/4}}$$

where n = rotational speed, rpm (1/min)
Q = pump capacity, US gal/min
(1 US = 3.78 ℓ)
H = pump head, ft

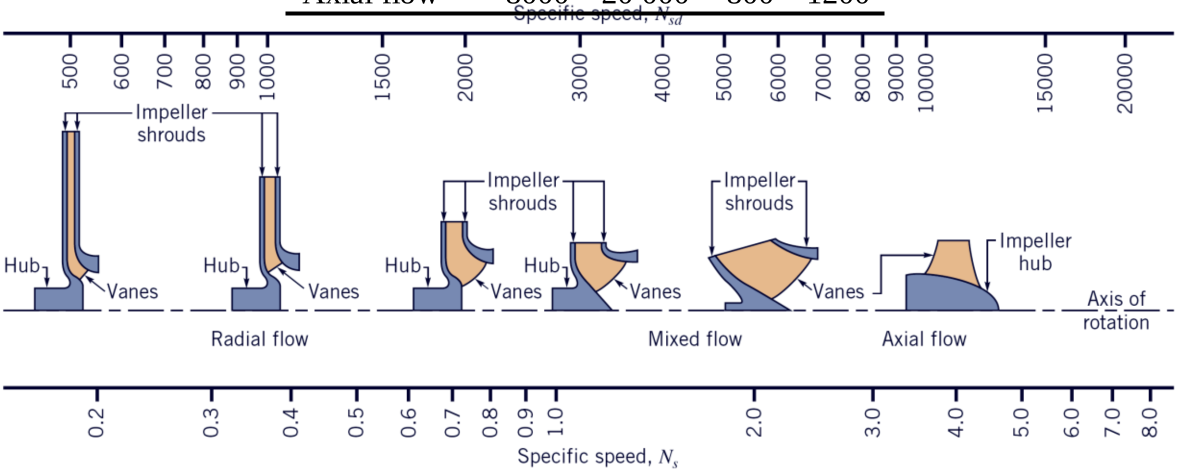
The specific speed determines the general shape or class of the impeller. As the specific speed increases, the ratio of the impeller outlet diameter, D_2, to the inlet or eye diameter, D_1, decreases. Radial flow impellers develop head principally through centrifugal force. Pumps of higher specific speeds develop head partly by centrifugal force and partly by axial force. A higher specific speed indicates a pump design with head generation more by axial forces and less by centrifugal forces.

In general, the efficiency of the pump increases as the specific speed increases.

Typical values for specific speed:

Pump	**USA**	**Europe**
Radial flow	500 – 4000	40 – 320

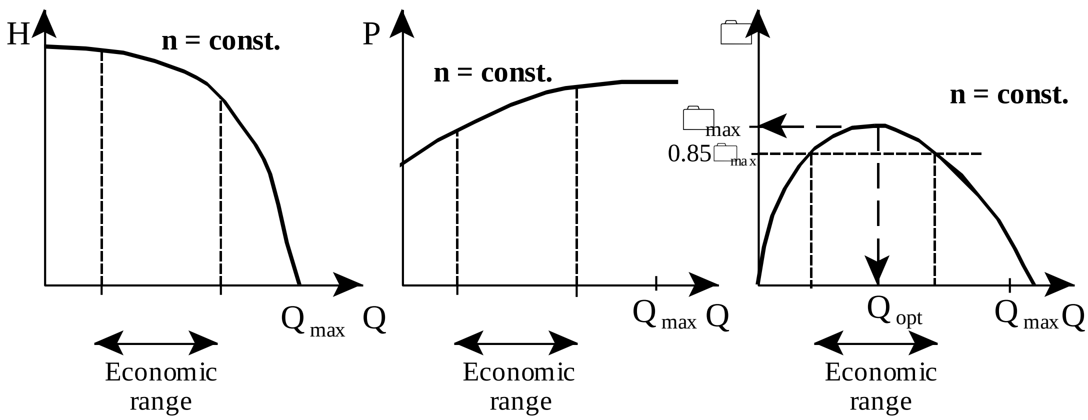
Mixed flow	4000 – 8000	320 – 800
Axial flow	8000 – 20 000	800 – 1200



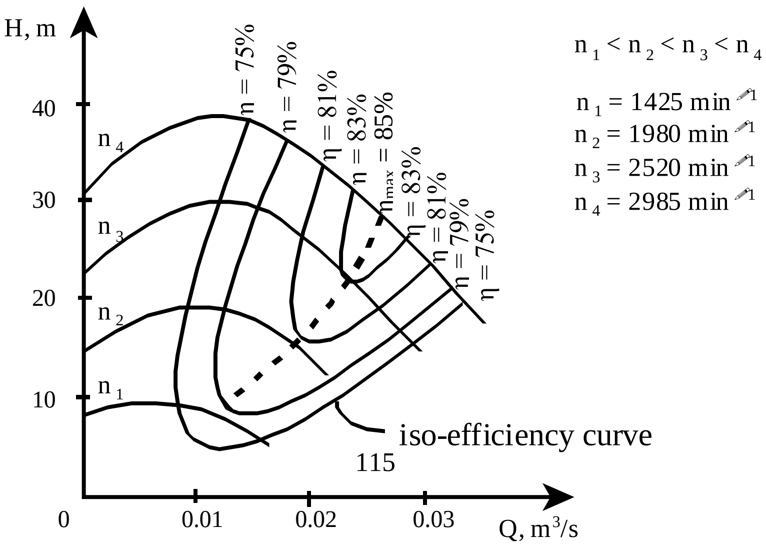
Specific speed of impellers (top scale - USA, bottom scale - dimensionless)

Characteristic curves

The characteristic curves of a centrifugal pump are the graphical representation of relationships of H(Q), P(Q), and η(Q) **at n = constant**.



Universal characteristics: The characteristic curves are drawn for variable speed or for several impeller diameters or for several blade angles and the iso-efficiency curves (η/η_{max}) appear on the same diagram. **Iso-efficiency curve** joins the coordinates of the points of equal efficiency.



When similitude is perfect, the iso-efficiency curves are parabolic. The same centrifugal pump can operate at various flow rates (Q), pumping heads (H) and rotation speeds (n).

Advantages of centrifugal pumps:
- They give steady delivery,
- They can handle slurries,
- They can have high capacities (Q) but at small head (H),
- They operate at high speed so they can be driven directly by electric motors,
- They are available in a large variety of materials,
- They are simple in construction therefore they are inexpensive,
- They take up little floor space and don’t need heavy foundation,
- They have low maintenance cost,
- They cannot be overloaded.

Disadvantages of centrifugal pumps:
- Single stage pumps cannot develop high pressure except at very high speed (10 000 min^{-1}),
- Multistage pumps for high pressures are expensive, particularly in corrosion-resistant materials,
- The developed head (H) is greatly dependent on the delivery (Q),
- Efficiencies drop off rapidly at flow rates much different from those at peak efficiency,
- Their performance (η) drops off rapidly with increasing viscosity,
- Their deliveries (Q) drop when the system resistance increases,
- They are not self-priming.

SI UNITS AND CONVERSION FACTORS

QUANTITY	SI unit	CONVERSION
Density (ρ)	kg/m^3	**1 g/cm^3 = 1000 kg/m^3** 1 lb/ft^3 = 16.018 kg/m^3
Energy, Work	kJ	1 Btu = 1.05506 kJ 1 kcal = 4.1868 kJ 1 Btu = 252.16 cal 1 kWh = 3.6 MJ
Force (F)	N	1 lb_f = 4.4482 N 1 N = 10^5 dyn
Length (L)	m	**1 m = 1000 mm** **1 m = 100 cm** 1 ft = 0.3048 m 1 in = 2.54 cm
Mass	kg	**1 t = 1000 kg** **1 kg = 1000 g** 1 lb = 0.4536 kg
Power	W	1 hp = 0.7457 kW
Pressure (p)	Pa ≡ N/m^2 (bar)	**1 bar = 10^5 Pa** **1 atm = 101.32 kPa** **1 atm = 760 mmHg** 1 atm = 14.7 psia 1 psia = 6894.76 Pa
Temperature (T)	K (°C)	**T(K) = t(°C) + 273.15** **ΔT(°C) = ΔT(K)** = 1.8 ΔT(°F) X(°F) = 5/(X!32)/9 (°C)
Viscosity Dynamic (μ) Kinematic (ν)	Pa≅s = kg/ms m^2/s	**1 cP = 10!3 Pa≅s** 1 cSt = 10!6 m^2/s
Volume (V)	m^3	**1 m^3 = 1000 P** 1 gal (US) = 3.7854 ℓ 1 gal (GB) = 4.5435 ℓ 1 ft^3 = 28.3168 ℓ

Constants

g = 9.81 m/s^2

R =8.314 J/mol K = 8314 J/kmol K = 8314 Pa m^3/kmol K = 0.082 ℓ atm/mol K