18.06 Professor A.J. de Jong Exam 2 April 9, 2003

Your name is: ____________________

Please circle your recitation:

Important: Briefly explain all of your answers.

1 (29 pts.)

(a) Compute the determinant of the following matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 & 2 \end{pmatrix}$$

Mention the method used for each step in the calculation.

(b) Give a basis for each of the four fundamental subspaces associated to the following matrix

$$\begin{pmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

2 (29 pts.)

(a) Apply the Gram-schmidt algorithm to the columns of the matrix A below. (Use the order in which they occur in the matrix!) Use this to write $A = QR$, where Q is a matrix with orthonormal columns, and R is upper triangular.

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ -1 & -1 \end{pmatrix}.$$

(b) Compute the matrix of the projection onto the column space of A. What is the distance of the point $(1, 1, 1, 0)$ to this column space?

3 (14 pts.) Show that the following determinant is zero for any values of a, b and c:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix}$$

4 (28 pts.) Let A be the matrix

$$\begin{pmatrix} 7 & 5 \\ 3 & -7 \end{pmatrix}.$$

(a) Find matrices S and Λ such that A has a factorization of the form

$$A = S\Lambda S^{-1},$$

where S is invertible and Λ is diagonal: $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2)$.

(b) Find a matrix B such that $B^3 = A$. (Hint: First find such a matrix for Λ. Then use the formula above.)