1. In the design of beams, it is necessary to quantify the deflection, y, to ensure that it is
(a) greater than the minimum allowable deflection
(b) less than one-half the maximum allowable deflection
(c) greater than the maximum allowable deflection
(d) less than the maximum allowable deflection

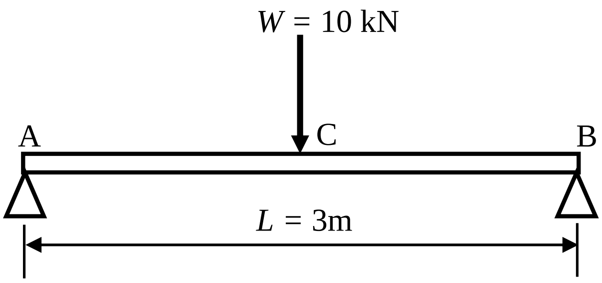
An initially straight prismatic beam is subjected to a bending moment of magnitude M. If the radius of curvature of the beam after deformation is R, use this information to answer questions 2 to 4.

2. Which of the following is the correct relation for the longitudinal bending stress in the beam?
(a) $\sigma = E/(Ry)$
(b) $\sigma = ER/y$
(c) $\sigma = Ry/E$
(d) $\sigma = yE/R$

3. The magnitude of the bending moment acting on the beam is given by
(a) $MI = ER$
(b) $MR = EI$
(c) $ME = IR$
(d) $My = R/EI$

4. The maximum bending stress in the beam is given by
(a) $\sigma_{max} = MR/(y_{max}/I)$
(b) $\sigma_{max} = MI/(y_{max})$
(c) $\sigma_{max} = M/(I/y_{max})$
(d) $\sigma_{max} = M/(E/y_{max})$

The diagram below shows a simply supported beam of length L = 3 m that is subjected to a central load W = 10 kN. If for the beam, the second moment of area, I = 12 × 10^6 mm^4 and Young's modulus of elasticity, E = 200 GPa, use this information to answer questions 5 to 11.



5. The correct algebraic expression for the bending moment at a distance x from end B is
(a) 10000x Nm
(b) 4000x Nm
(c) 5000x Nm
(d) 2000x Nm

6. The correct algebraic expression for the equation of the deflection curve is
(a) $y = Wx^3/(16EI) - WL^2x/(12EI)$
(b) $y = Wx^3/(12EI) - WL^2x/(18EI)$
(c) $y = Wx^3/(12EI) - WL^2x/(16EI)$
(d) $y = Wx^2/(4EI) - WL^2/(16EI)$

7. Calculate the maximum slope of the deflected beam.
(a) 0.0023 rad
(b) -0.0023 rad
(c) 0.023 rad
(d) -0.023 rad

8. Calculate the maximum deflection of the beam.
(a) – 2.34 mm
(b) – 5.32 mm
(c) – 10.34 mm
(d) – 20.34 mm

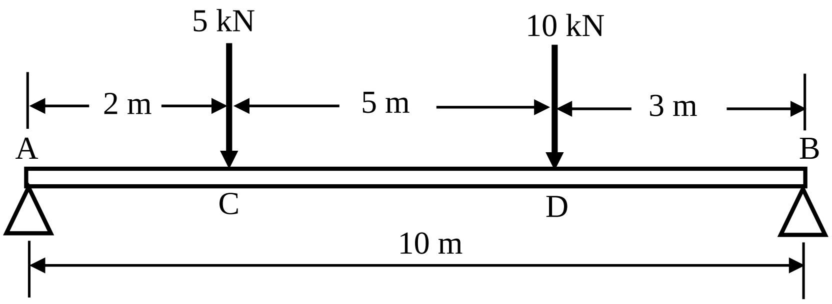
9. Calculate the radius of curvature of the beam at *x* = 1.5 m.
(a) 1.6×10^3 mm
(b) 1.6×10^5 mm
(c) 3.2×10^3 mm
(d) 3.2×10^5 mm

10. What is the maximum bending stress in the beam?
(a) – 7.50 MPa
(b) – 1.46 MPa
(c) – 10.10 MPa
(d) – 3.92 MPa

11. What is the moment of resistance of the deflected beam?
(a) 10.53 kNm
(b) 2.83 kNm
(c) 7.49 kNm
(d) 5.50 kNm

12. A beam 2.5 m long, simply supported at its ends, carries a point load *W* at its centre. If the maximum slope of the beam is 0.025 radians, find the magnitude of the deflection at the centre of the beam.
(a) 12.65 mm
(b) 4.52 mm
(c) 20.83 mm
(d) 10.23 mm

The figure below shows a horizontal steel girder AB of length 10 m having a uniform cross-section with $I = 160 \times 10^6\ mm^4$ and Young's modulus, $E = 200$ GPa. It is simply supported at its ends and carries two concentrated loads at C and D.



Use this information to answer questions 13 to 17.

13. Calculate the magnitude of the reaction at A.
(a) 7 kN
(b) 15 kN
(c) 20 kN
(d) 8 kN

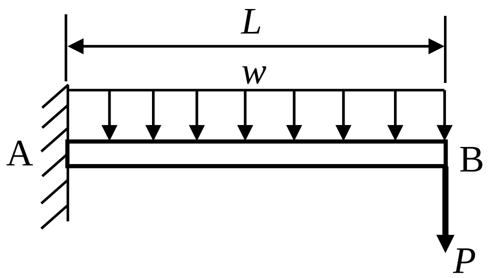
14. Calculate the magnitude of the reaction at B.
(a) 7 kN
(b) 15 kN
(c) 20 kN
(d) 8 kN

15. Using Macaulay's method, which of the following expressions represents the bending moment on the beam?
(a) $7000x - 5000[x - 2000] + 10000[x - 7000]$
(b) $7000x - 5000[x - 2000] - 10000[x - 7000]$
(c) $5000x - 7000[x - 2000] - 10000[x - 7000]$
(d) $7000x - 5000[x - 2000] - 8000[x - 7000]$

16. Using Macaulay's method calculate the deflection under the load at C.
(a) – 8.55 mm
(b) – 5.95 mm
(c) – 10.34 mm
(d) – 4.05 mm

17. Using Macaulay's method calculate the deflection under the load at D.
(a) – 8.55 mm
(b) – 5.95 mm
(c) – 10.34 mm
(d) – 4.05 mm

The cantilever beam AB shown below supports a uniformly distributed load w and a concentrated load P at B. Using this as a preamble, answer questions 18 to 20. Take $L = 2$ m, $w = 4$ kN/m, $P = 1.7$ kN and $EI = 2.4 \times 10^6$ Nm^2.



18. Determine the algebraic expression for the bending moment at a distance x from the free end B.
(a) $M = Px - wx^2/2$
(b) $M = -Px + wx^2/2$
(c) $M = -Px - wx^2$
(d) $M = -Px - wx^2/2$

19. Determine the algebraic expression for the strain energy in the cantilever beam.
(a) $U = (1/EI)(P^2L^3/2 - PwL^4/3 - w^2L^5/16)$
(b) $U = (1/EI)(P^2L^3/2 + PwL^4/3 + w^2L^5/16)$
(c) $U = (1/EI)(P^2L^3/6 + PwL^4/8 + w^2L^5/40)$
(d) $U = (1/EI)(P^2L^3/6 - PwL^4/8 - w^2L^5/40)$

20. Using Castigliano's theorem, calculate the deflection of the end B of the beam.
(a) 14.4 mm
(b) 8.52 mm
(c) 2.97 mm
(d) 5.22 mm

21. A steel rod 8 m long and 50 mm in diameter is used as a column, with one end fixed and the other end free. Determine the Euler crippling load for the column. Take $E = 200$ GPa.
(a) 10.65 kN
(b) 8.78 kN
(c) 2.37 kN
(d) 15.88 kN

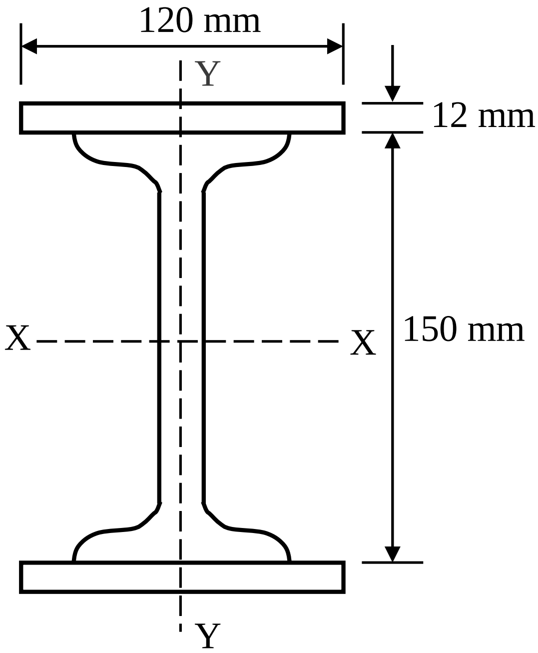
22. The Rankine formula for a strut of given length is given by $1/P_R = (1/P_C) + (1/P_E)$, where P_C and P_E are the crushing and Euler loads, respectively. Which of the following statements is/are true?

I. For a very short column, $1/P_C$ is negligibly small
II. For a very short column, $1/P_E$ is large
III. For a very long column, $1/P_C$ is large
IV. For a very long column, $1/P_E$ is large

(a) II only

(b) I and III only
(c) II and IV only
(d) IV only

The built-up column shown in the diagram below consists of a 150 mm × 100 mm rolled steel joist (R.S.J) with a 120 mm × 12 mm plate riveted to each flange. Use this information to answer questions 23 to 25. Take the properties of the joist as Area (A) = 2167 mm^2, I_{XX} = 8.391 × 10^6 mm^4, I_{YY} = 0.948 × 10^6 mm^4 and assume the yield stress and Rankine constant are 315 MPa and 1/7500, respectively.



23. Determine the moment of inertia of the column about the X-X axis.
(a) 4.404 × 10^6 mm^4
(b) 5.464 × 10^7 mm^4
(c) 2.732 × 10^7 mm^4
(d) 2.732 × 10^6 mm^4

24. Determine the moment of inertia of the column about the Y-Y axis.
(a) 4.404 × 10^6 mm^4
(b) 5.464 × 10^7 mm^4
(c) 2.732 × 10^7 mm^4
(d) 2.732 × 10^6 mm^4

25. Calculate the Rankine crippling load of the column, if it is 6.5 m long having one end fixed and the other hinged.
(a) 576 kN
(b) 376 kN
(c) 147 kN
(d) 212 kN

GFK Tay

USEFUL FORMULAE

1. Differential equation for the deflection curve of a beam

$$EI\frac{d^2y}{dx^2} = M(x)$$

2. Strain energy in a beam of length, L.

$$U = \int_0^L \left(\frac{M^2}{2EI}\right) dx$$

3. Euler crippling load for strut of length l and equivalent length L_e with generic end conditions

$$P_E = \frac{\pi^2 EI}{{L_e}^2}$$

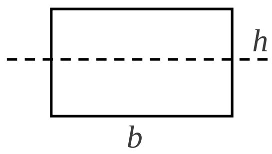
End Conditions	Equivalent Length (L_e)
Both ends hinged	l
One end fixed; the other free	$2l$
Both ends fixed	$l/2$
One end fixed; the other hinged	$l/\sqrt{2}$

4. Rankine crippling load

$$P_R = \frac{A\sigma_C}{1 + a(L_e/k)^2}$$

5. Second moment of area for a rectangular section

$$I = \frac{1}{12}bh^3$$



6. Second moment of area for a circular section

$$I = \frac{\pi d^4}{64}$$