Second Law – Sample Problems

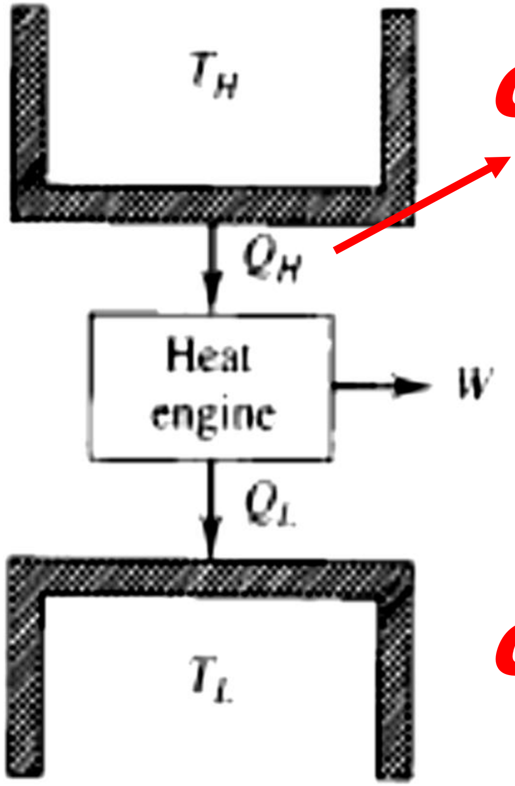
D. A. Quansah, PhD

Heat Engines – Worked Example

A gasoline truck engine takes in 10,000 J of heat and delivers 2000 J of mechanical work per cycle. The heat is obtained by burning gasoline with heat of combustion of 5.0×10^4 J/g.

a) What is the thermal efficiency of this engine?
b) How much heat is discarded in each cycle?
c) If the engine goes through 25 cycles per second, what is its power output in watts?
d) How much gasoline is burned in each cycle?
e) How much gasoline is burned per second?

Heat Engines – Worked Example



Q_H=10,000 J

2,000 J

Q_L=?

a) What is the thermal efficiency of this engine?

$$\eta_{thermal} = \frac{\text{Work Output}}{\text{Required Heat Input}}$$

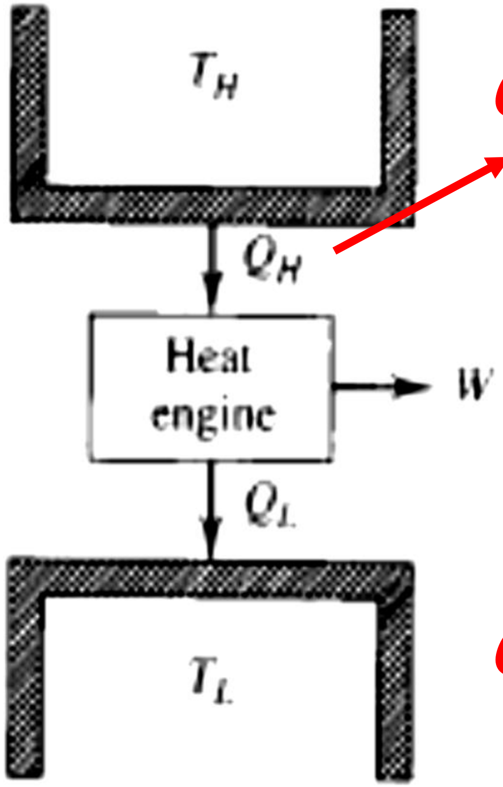
$$= \frac{2000\ \text{J}}{10000\ \text{J}} \qquad = 20\%$$

b) How much heat is discarded in each cycle?

$Q_H - Q_L = W \Rightarrow Q_L = Q_H - W$

$Q_L = 10000\ J – 2000\ J \qquad = 8000\ J$

Heat Engines – Worked Example



Q_H=10,000 J

2,000 J

Q_L=?

c) If the engine goes through 25 cycles per second, what is its power output in watts?

Power Output in watt $\left(\mathbf{W\ or\ \frac{J}{s}}\right)$

$$= \left(\frac{2000\ \mathrm{J}}{cycle}\right) \times \left(\frac{25\ cycles}{s}\right) = 50000\ W\ = 50\ \mathrm{kW}$$

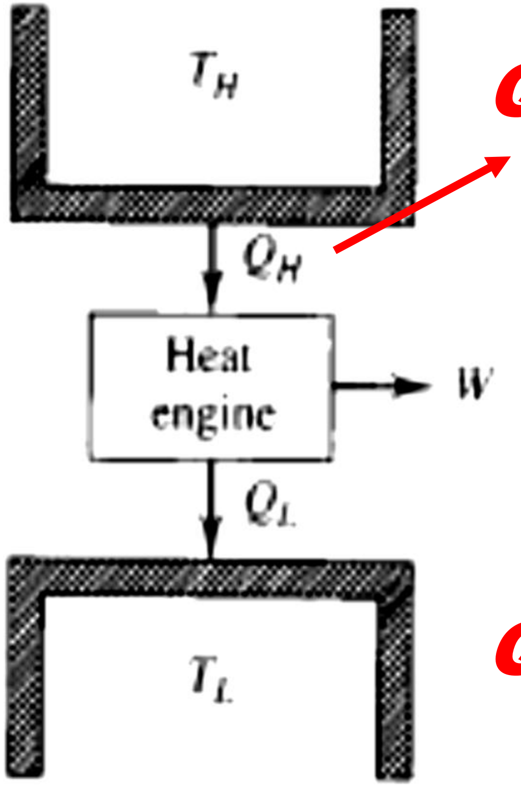
d) How much gasoline is burned in each cycle?

Since 10,000 J of heat (i.e. Q_H) is required per cycle, gasoline burned (m) will be:

$$\mathrm{m} = \frac{10000\ \mathrm{J}}{5 \times 10^{4}\ \mathrm{J/g}}$$

=0.20 g

Heat Engines – Worked Example



Q_H=10,000 J

2,000 J

Q_L=?

e) How much gasoline is burned per second?

$$\left(\frac{0.2\ g}{cycle}\right)\times\left(\frac{25\ cycles}{s}\right)=5.0\ {}^{g}\!/_{s}$$

from previous section

$$={}^{18\ kg}\!/_{h}$$

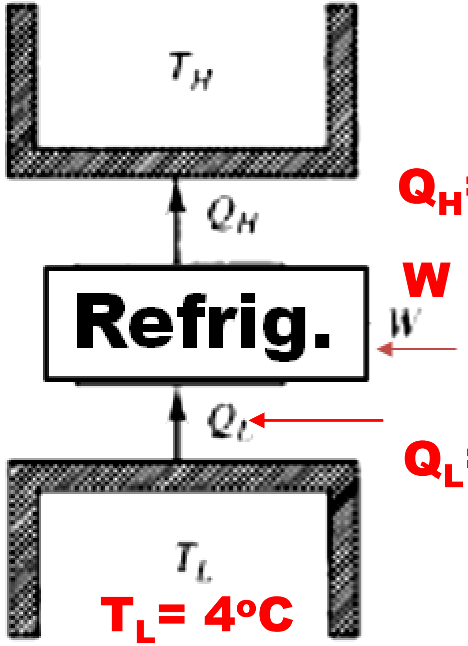
If the density of gasoline is taken as 700 kg/m^3 and the truck is assumed to be travelling at 90 km/h. What is the *mileage* of the the truck (km covered per litre of fuel consumed)?

Refrigerators – Worked Example

The food compartment of a refrigerator is maintained at 4 ^{o}C by removing heat from it at a rate of 360 kJ/min. If the required power input to the refrigerator is 2 kW, determine:

(a) the coefficient of performance of the refrigerator and

(b) the rate of heat rejection to the room that houses the refrigerator.



(a) the coefficient of performance of the refrigerator

$$COP_R = \frac{Desired\ Output}{Required\ Input} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{360\ \mathrm{kJ/min}}{2\ kW} \times \frac{1\ \mathrm{kW}}{60 kJ/min}$$

$$\boxed{\mathbf{COP_R = 3}}$$

(b) the rate of heat rejection to the room that houses the refrigerator.

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{in}$$

$$= 360\frac{\mathrm{kJ}}{\mathrm{min}} + 2\ \mathrm{kW}\left(\frac{60\ \mathrm{kJ/kW}}{1\ \mathrm{kW}}\right) \quad \mathbf{= 480\ kJ/min}$$

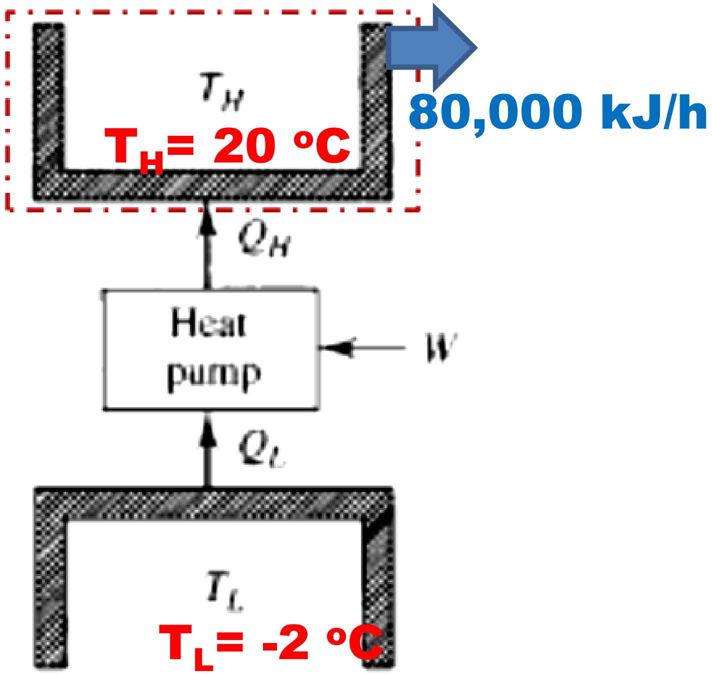
$$= xx\ \mathrm{kW}$$

Heat Pumps – Worked Example

A heat pump is used to meet the heating requirements of a house and maintain it at 20 °C. On a day when the outdoor air temperature drops to -2 °C, the house is estimated to lose heat at a rate of 80,000 kJ/h. If the heat pump has a COP of 2.5, determine:

(a) the power consumed by the heat pump

(b) the rate at which heat is absorbed from the cold outdoor air.



(a) the power consumed by the heat pump

$$\mathrm{COP_H} = \frac{\text{Desired Output}}{\text{Required Input}} = \frac{\dot{Q}_H}{\dot{W}_{in}}$$

$$\dot{W}_{in} = \frac{\dot{Q}_H}{COP_H} = \frac{80{,}000\ \mathrm{kJ/h}}{2.5} = 32{,}000\ \mathrm{kJ/h} = 8.9\ \mathrm{kW}$$

(b) the rate at which heat is absorbed from the cold outdoor air.

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{in} = 80{,}000\ \mathrm{kJ/h} - 32{,}000\ \mathrm{kJ/h}$$

$$= 48{,}000\ \mathrm{kJ/h}$$

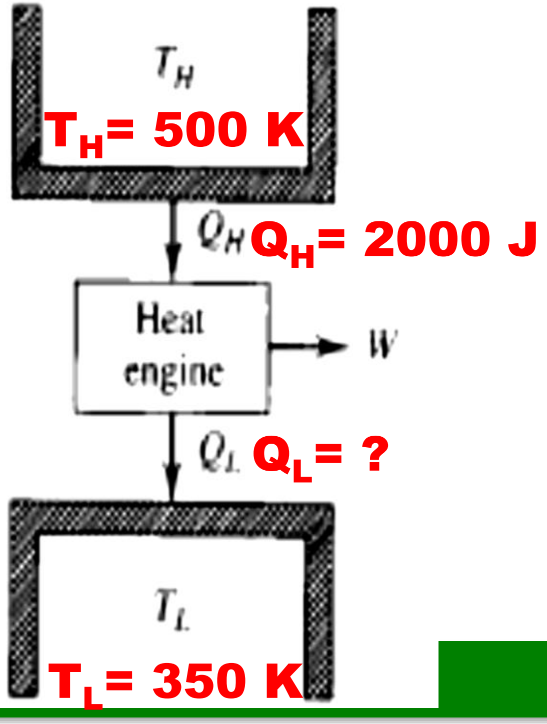
Carnot Heat Engine– Worked Example

A carnot engine takes 2000 J of heat from a reservoir at 500 K, does some work and discards some heat to a reservoir at 350 K.

(a) How much heat is discarded?

(b) How much work does it do?, and

(c) What is its efficiency?



$$\eta_{th} = \frac{W}{Q_H}$$

for a cyclic process

$$\eta_{th} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

for a Carnot Engine

$$\eta_{th} = 1 - \frac{T_L}{T_H}$$

from the above, we may therefore write:

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$$

$$Q_L = \left(\frac{T_L}{T_H}\right) Q_H = \left(\frac{350\ \mathrm{K}}{500\ \mathrm{K}}\right) 200\mathrm{J} \quad \mathbf{= 1400\ J}$$

Work done: $W = Q_H - Q_L \quad = 2000\ \mathrm{J} - 1400\ \mathrm{J}$ $\mathbf{= 600\ J}$

Efficiency: $\eta_{th} = 1 - \frac{T_L}{T_H} \quad = 1 - \frac{350\ \mathrm{K}}{500\ \mathrm{K}}$ $\mathbf{= 30\%}$

Entropy Changes– Worked Example

- Enthropy is a quantitative measure of randomness.
- Consider an infinitesimal isothermal expansion by an ideal gas. An amount of heat ***dQ*** is added and the gas expands by a small amount **dV** such that the gas **Temperature** is kept constant.
- Recall: internal energy remains constant, since it depends only on temperature.
- From the first law, one may write:

$$dQ = dW \quad = pdV \quad = \frac{nRT}{V}dV \Rightarrow \frac{dV}{V} = \frac{dQ}{nRT}$$

- The gas is obviously more disordered after expansion than before, i.e. increased randomness due to volume for mobility.
- The fractional change in volume $\frac{dV}{V}$ is a measure of randomness and is proportional to $\frac{dQ}{T}$.
- The symbol **S** is introduced for entropy of the system. The infinitesimal entropy change ***ds*** for an infinitesimal reversible process at temperature ***T*** is given as:

$$dS = \frac{dQ}{T}$$

Entropy Changes– Worked Example

If an amount of heat **Q** is added during a reversible isothermal process at absolute temperature **T**, the total entropy change ***ΔS*** is given by:

$$\Delta S = S_2 - S_1 = \frac{\mathrm{Q}}{\mathrm{T}} \qquad \text{(reversible isothermal process)}$$

Entropy has unit of J/K.

$$\Delta S = \int_1^2 \frac{\mathrm{Q}}{\mathrm{T}} \qquad \text{(for any reversible process)}$$

Where 1 and 2 represent initial and final states of the system.
Note that entropy is a point function.

Entropy Changes– Worked Example

For a reversible cyclic process: $\int \frac{dQ}{T} = 0$

For a irreversible cyclic process: $\int \frac{dQ}{T} < 0$

Recall: the Clausius inequality: $\oint \frac{dQ}{T} \leq 0$

Consider a reversible Carnot heat engine, the entropy change for the cycle may written as:

$$\frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0$$

For an irreversible cycle, less work will be extracted and $Q_{L\_irr} > Q_L$.

Consequently:

$$\frac{Q_H}{T_H} - \frac{Q_{L\_irr}}{T_L} < 0$$

Hence the Clausius Inequality!

Entropy Changes– Worked Example

What is the change of entropy of 1 kg of ice that is melted reveribly at 0 ^{o}C and converted to water at 0 ^{o}C? The heat of fusion of water is 3.34×10^5 J/kg.

Notes:

- Melting occurs at constant temperature, T = 0 ^{o}C = 273 K
- The heat added can be computed as:

$$Q = mL_f = 1\text{kg} * 3.34 \times 10^5 \text{ J/kg} = 3.34 \times 10^4 \text{ J}$$

$$\Delta S = \frac{Q}{T} = \frac{3.34 \times 10^5 \text{J}}{273 \text{ K}} = 1.22 \times 10^3 \text{J/K}$$

Entropy Changes– Worked Example

One kilogram of water at 0 ^oC is heated to 100 ^oC. Compute its change in entropy. Take specific heat to be 4190 J/kg.K and assume it to be constant over the given temperature range.

$$\Delta S = \int_1^2 \frac{\mathrm{Q}}{\mathrm{T}} \qquad = \int_1^2 mc\frac{dT}{T} \qquad = mc\, ln\left(\frac{T_2}{T_1}\right)$$

$$\Delta S = (1\ \mathrm{kg})(4190\ \mathrm{J/kg})\left(ln\left(\frac{373\ \mathrm{K}}{273\ \mathrm{K}}\right)\right)$$

$$\Delta S = 1.31 \times 10^3 \mathrm{J/K}$$