

# ME 164 - STATICS OF SOLID MECHANICS / ME 162 - BASIC MECHANICS

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### LECTURE 2

### Static Equilibrium of Particles and Rigid Bodies Analysis of Planar Trusses and Frames

### Static Equilibrium Of Particles And Rigid Bodies

Static Equilibrium
Procedure for analyzing static equilibrium problems
Free Body Diagrams

### Static Equilibrium

A particle or body is said to be in equilibrium if the resultant force and moment acting on it is zero. In other words, the sum of forces (or moments) **must** be equal to zero.

For particles; 
$$\vec{R} = \sum F = 0$$
  
 $\Rightarrow \sum F_x = 0$   $\sum F_y = 0$   $\sum F_z = 0$ 

For bodies; 
$$\vec{R} = \sum F = 0$$

$$\Rightarrow \sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\vec{M} = \sum M = 0$$

$$\Rightarrow \sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

# Solving Static Equilibrium Problems

➤Involves three main steps;

✓ Sketch a free body diagram for the problem

✓ Sum up forces and moments to obtain the **equations of equilibrium** for the problem.

✓ Solve the equations and interpret your results.

### Sketching Free Body Diagrams

Select the extent of the body that is of interest, detach it from the ground and all other bodies and supports, and (basically) sketch the outline of the "free-body".

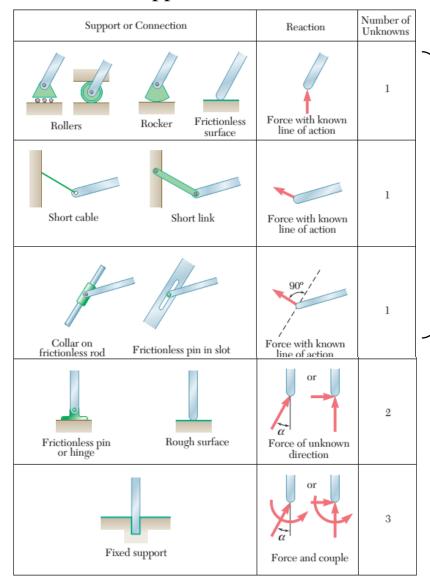
Indicate force reactions which the ground and other supports exert on the "free-body".

Indicate external forces and moments, including the rigid body weight where it cannot be ignored at their points of application.

Include the required dimensions to compute the moments of the forces where necessary.

### Free Body Diagrams: Support Reactions

➤ Reactions at Supports and Connections for Two-Dimensional Structures



• Reactions equivalent to a force with known line of action.

Reactions equivalent to a force of unknown direction and magnitude.

Reactions equivalent to a force of unknown direction and magnitude and a couple of unknown magnitude

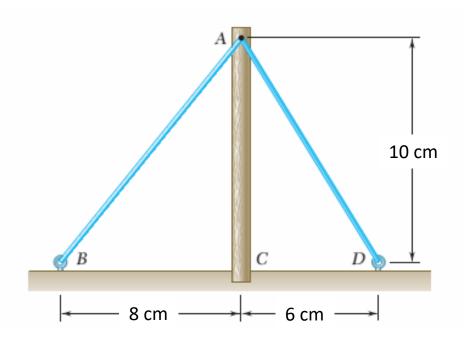
Source:



### Free Body Diagrams

#### Example

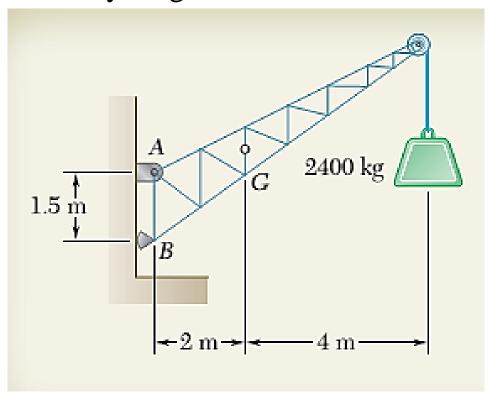
Cables AB and AD help support pole AC. Knowing that the tension is 120 N in AB and 40 N in AD, sketch the free body diagram for the pole.

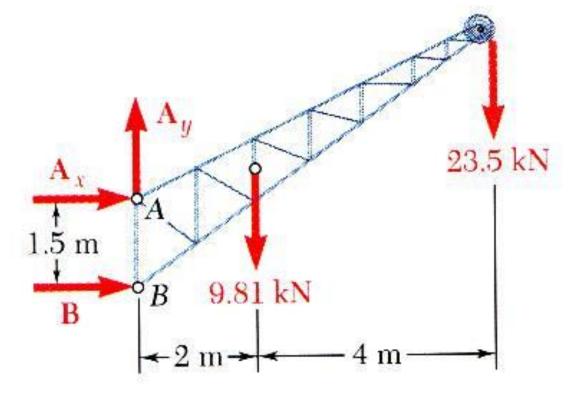


### Sketching Free Body Diagrams

### **Example**

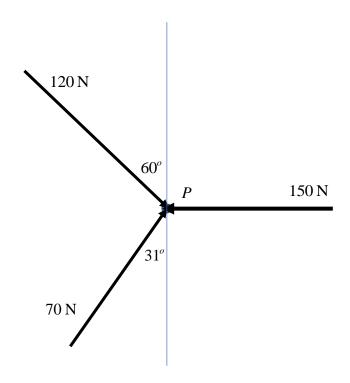
A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G. Sketch the free body diagram for the crane.



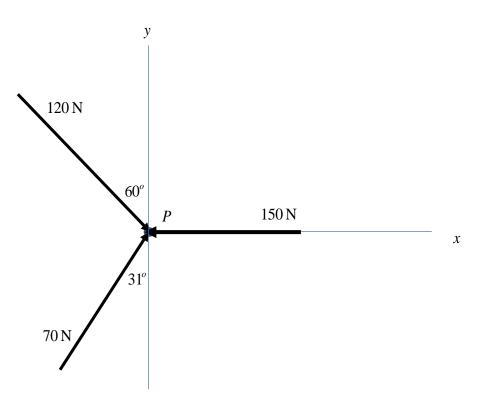


### Example

Determine if the particle P is in equilibrium under the influence of the forces shown.



#### **>** Solution



#### Equations of Equilibrium

For equilibrium, 
$$\sum F_x = 0 = \sum F_y$$

But 
$$\sum F_x \neq 0$$

Hence, P is not in equilibrium

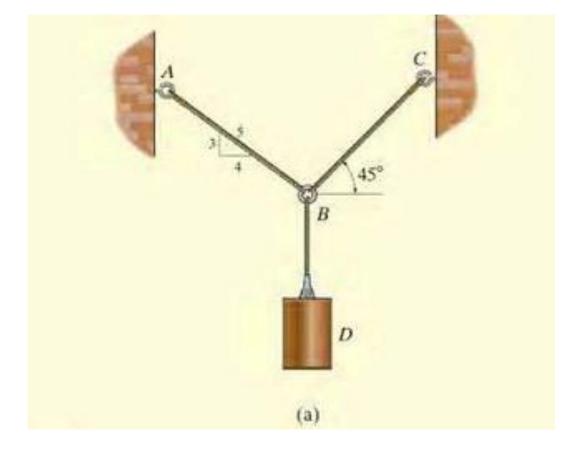




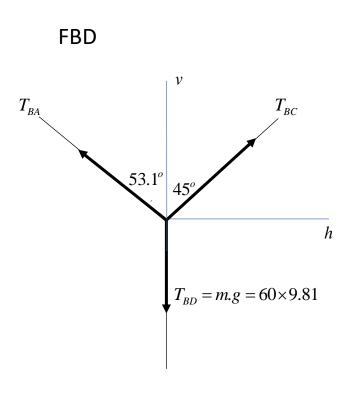
**Example** 

Determine the tensions required in cables BC and BA to keep the ring at B in

equilibrium.



#### **>** Solution



#### Equations of Equilibrium

$$\xrightarrow{+} \sum F_h = 0: T_{BC} \sin 45^o - T_{BA} \sin 53.1^o = 0 \qquad ---- (1)$$

$$+ \uparrow \sum F_v = 0: T_{BC} \cos 45^o + T_{BA} \cos 53.1^o - T_{BD} = 0$$

$$= T_{BC} \cos 45^o + T_{BA} \cos 53.1^o = 588.6 \text{ N} \qquad ---- (2)$$

Solving (1) and (2) simultaneously,

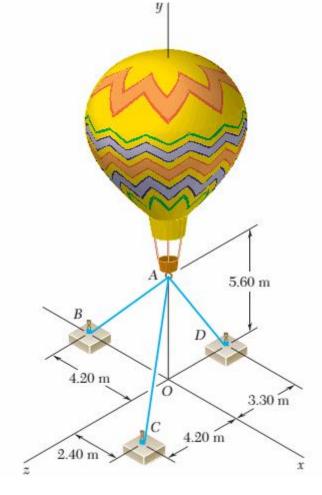
$$T_{BC} = 475.41 \,\mathrm{N}$$

$$T_{BA} = 420.43 \text{ N}$$

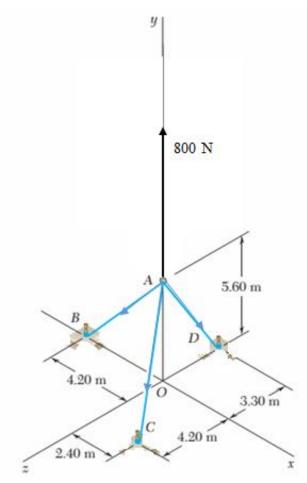
### Example

# Static Equilibrium - Particles

Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800-N vertical force on the ring at A, determine the tension in each cable assuming equilibrium.



#### Solution



**Equations of Equilibrium** 

$$\vec{T}_{AB} = T_{AB} \frac{-4.2\vec{i} - 5.6\vec{j}}{\sqrt{4.2^2 + 5.6^2}} = -0.6T_{AB}\vec{i} - 0.8T_{AB}\vec{j}$$

$$\vec{T}_{AC} = T_{AC} \frac{2.4\vec{i} - 5.6\vec{j} + 4.2\vec{k}}{\sqrt{2.4^2 + 5.6^2 + 4.2^2}} = 0.37T_{AC}\vec{i} - 0.87T_{AC}\vec{j} + 0.65T_{AC}\vec{k}$$

$$\vec{T}_{AD} = T_{AD} \frac{-5.6\vec{j} - 3.3\vec{k}}{\sqrt{5.6^2 + 3.3^2}} = -0.86T_{AD}\vec{j} - 0.51T_{AD}\vec{k}$$

$$F_{Av} = 800 \text{ N}\vec{j}$$

$$\sum F_x = -0.6T_{AB} + 0.37T_{AC} = 0 --- (1)$$

$$\sum F_{y} = -0.8T_{AB} - 0.87T_{AC} - 0.86T_{AD} + 800 = 0 \qquad --- (2)$$

$$\sum F_z = 0.65T_{AC} - 0.51T_{AD} = 0 --- (3)$$

Solving (1), (2) and (3) simultaneously,

$$T_{AB} = 200.6 \text{ N}$$

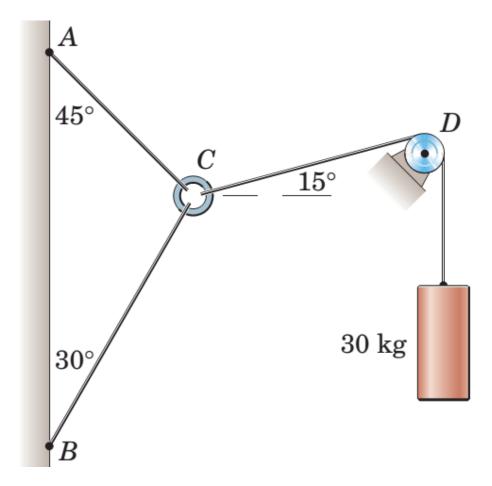
$$T_{AC} = 325.3 \text{ N}$$

$$T_{AD} = 414.6 \text{ N}$$

### Example

Three cables are joined at the junction ring, C. Determine the magnitudes of the tensions in

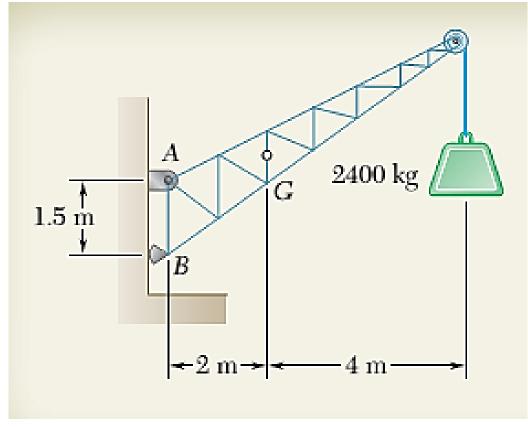
cables AC and BC on the ring.



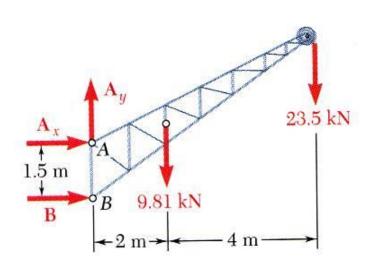
### **Example**

A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G. Determine

the components of the reactions at A and B.



#### **Solution**



At A,

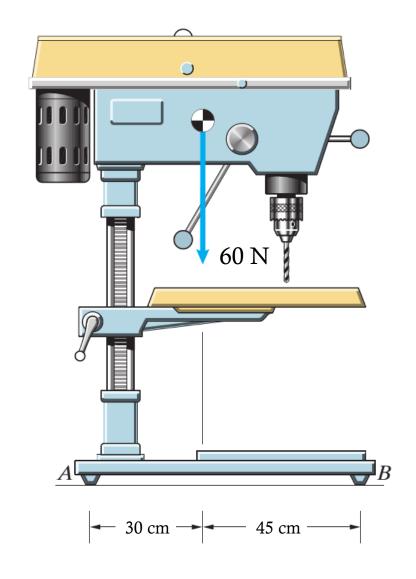
Taking moments about A,

$$\sum M_A = 0: +B(1.5\text{m}) - 9.81 \text{ kN}(2\text{m}) - 23.5 \text{ kN}(6\text{m}) = 0$$
$$B = +107.1 \text{ kN}$$

$$\therefore A_r = -107.1$$
kN

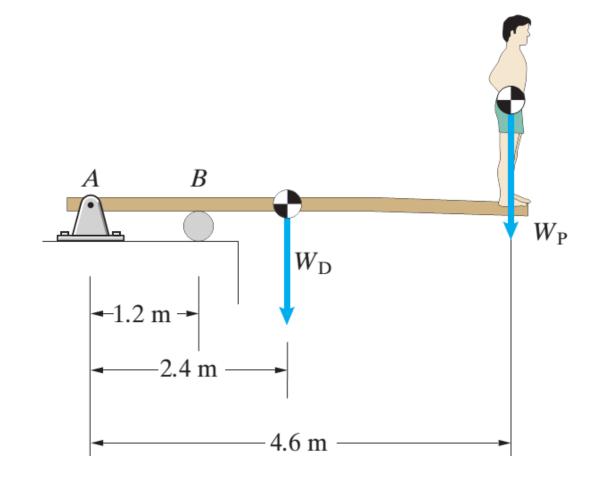
### **≻**Example

Determine the reactions at A and B. Assume that the surfaces at A and B are frictionless.



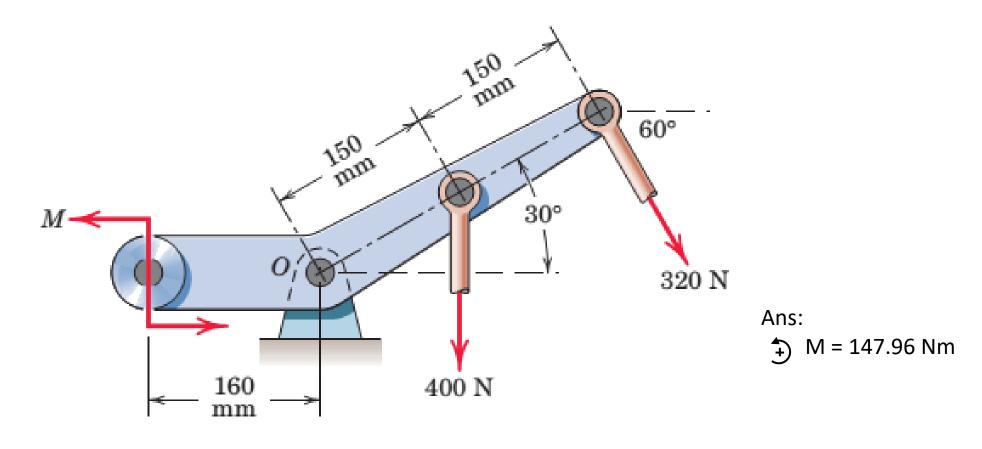
The masses of the man and the diving board are 54 kg and 36 kg, respectively. Assume that they are in equilibrium.

- (a) Sketch the free-body diagram of the diving board.
- (b) Determine the reactions at the supports A and B.



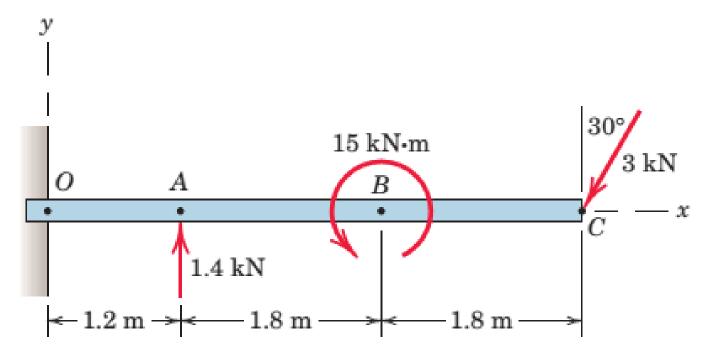
### Example

Neglecting weight, determine *M* if the link is in equilibrium.



### Example

The 500 kg uniform beam is subjected to the three external loads shown. Compute the reactions at the support point *O*.

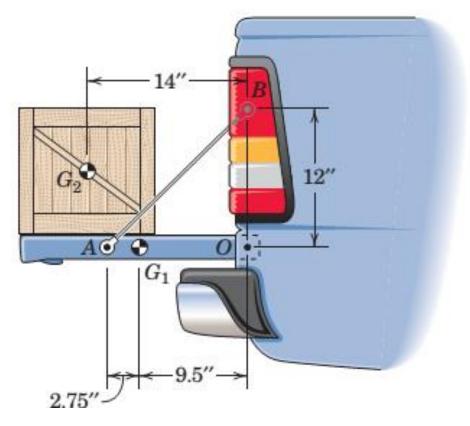


Ans:

 $F_x = 1.5 \text{ kN}, F_v = -1.198 \text{ kN}, M_O = 0.85 \text{ kNm (clockwise)}$ 

#### Example

A 150 N crate rests on the 100 N pickup tailgate as shown. Calculate the tension T in each of the two restraining cables, one of which is shown. The centres of gravity are at  $G_1$  and  $G_2$ . Assume the crate is located midway between the two cables.







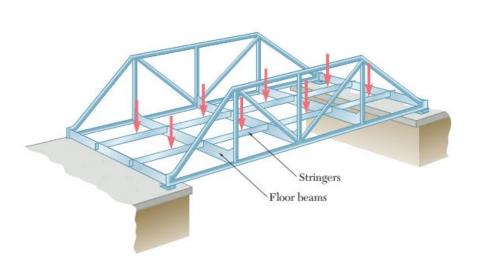
# Static Equilibrium Applications:

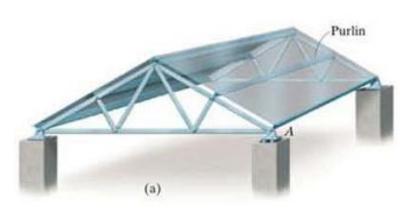
**Analysis of Planar Trusses and Frames** 

- Engineering structures basically, are designed to support or transfer forces safely. They normally comprise of a number of structural elements of members connected to form a main structure.
- Trusses are designed to transmit forces over long distances and comprise straight, slender bars that are joined to often form pattern of triangles.

- The loads on trusses are always applied at the joints, while in frames, loads may not necessarily be applied at the joints. Also, members of trusses are normally two force members, whereas in frames, at least one member is a sustains more than two forces.
- In designing a structure, it is desirable to know the force each individual member must sustain.

➤ Common applications are roofs, bridges and power pylons.







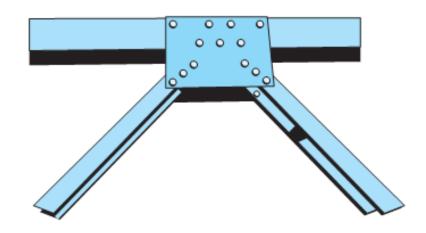
Bridge Truss Source: Mechanics for Engineers by Beet *et al* 

Roof Supporting Truss
Source: Engineering Mechanics Statics by Hiebbler

Power Pylon Source: http://upload.wikimedia.org/wikipedia/ commons/7/7e/Electricity\_pylon\_power\_outage .jpg

### >Assumptions:

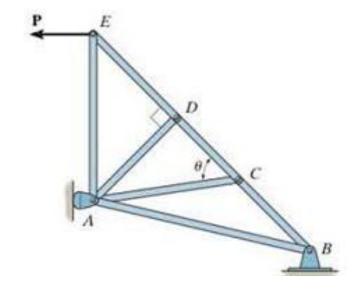
- The weights of the slender bars/members are negligible.
- Forces act at the ends of the members such that they are in either tension or compression.
- All joints are pins.

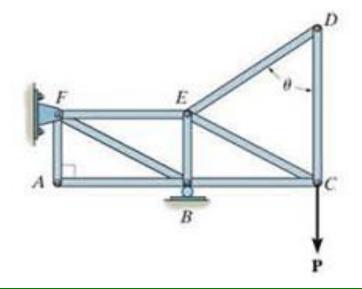


Truss members welded or riveted to a gusset plate Source: Engineering Mechanics – Statics by Pytel

### Analysis of Planar Trusses – Zero Force Members

- It is possible that some members of a Truss do not experience any external forces. Such members are referred to as *Zero Force Members*.
- ➤ Identifying of such members can significantly make analysis of planar trusses easier.
- > Two rules of thumb for identifying such members are;
  - 1. if three members form a truss joint and two of the members are collinear, the third member may be a zero force member provided no external force or support reaction is applied to that joint (See top right Figure).
  - 2. if a joint is formed by only two members and there is no external load or support reactions at the joint, then the two members may be zero force members (Bottom right Figure).





# Analysis of Planar Trusses – The Method of Joints

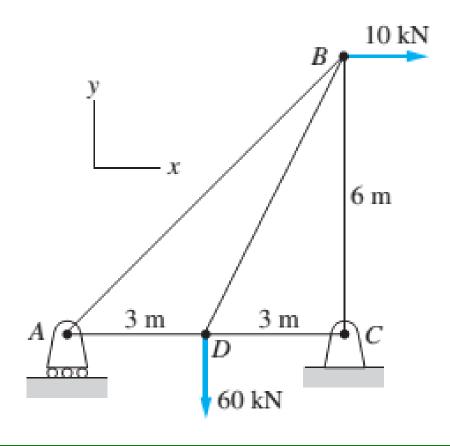
This involves an equilibrium analysis on each joint to determine the forces being exerted on the end of each member at that joint.

➤It is based on the assumption that if the whole truss is in equilibrium, each of its members or joints are also in equilibrium.

- Analysis is done in two main steps:
  - Determine all support reactions on the entire truss.
  - Conduct an equilibrium analysis at each joint/pin to determine the forces in each member.

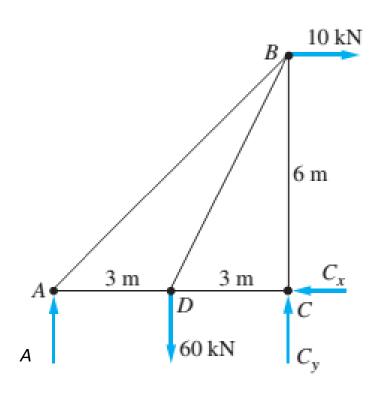
### Example

Determine the force in each member of the truss shown below using the method of joints. Indicate whether each member is in tension or compression



### Example

Equilibrium analysis of the whole structure to determine support reactions



For equilibrium,

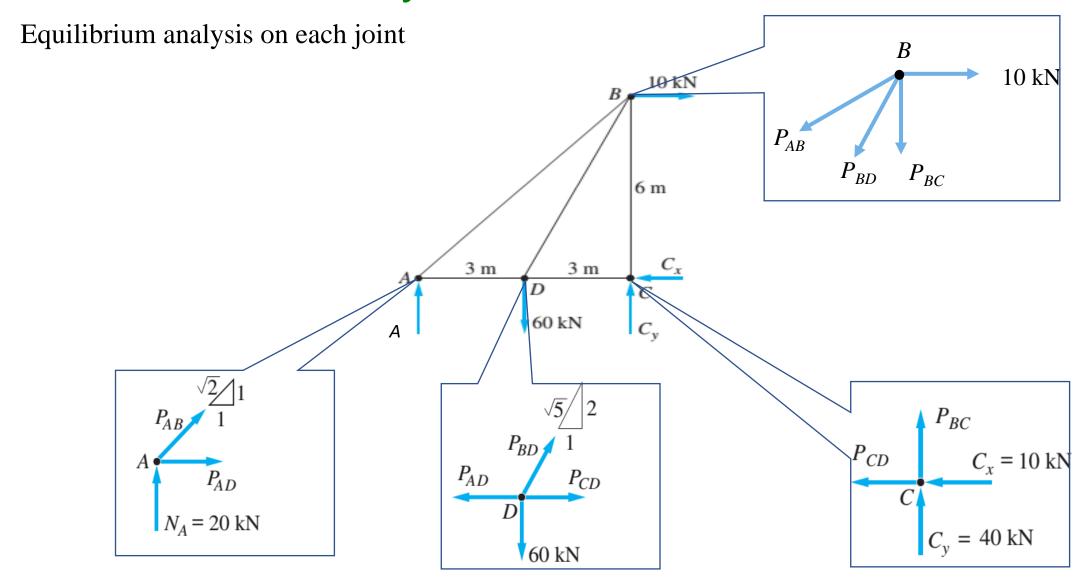
$$+ \rightarrow \sum F_x = 0 : -C_x + 10 \text{ kN} = 0$$

$$+ \uparrow \sum F_y = 0 : C_y + A - 60 \text{ kN} = 0$$

$$(\pm) \sum M_C = 0 : A(6 \text{ m}) + (10 \text{ kN})(6 \text{ m}) - (60 \text{ kN})(3 \text{ m}) = 0$$

$$A = 20 \text{ kN}$$

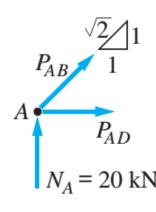
$$\therefore C_v = 40 \,\mathrm{kN}$$



#### **>**Solution

Equilibrium analysis on the joints

Joint A



For equilibrium,

$$P_{AB} = 0 \qquad + \sum F_{x} = 0 : P_{AD} + \frac{1}{\sqrt{2}} P_{AB} = 0 \qquad ----- (1)$$

$$+ \sum F_{y} = 0 : 20 \text{ kN} + \frac{1}{\sqrt{2}} P_{AB} = 0 \qquad ----- (2)$$

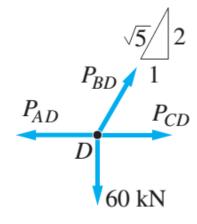
$$N_{A} = 20 \text{ kN}$$

$$P_{AB} = 0 \qquad ----- (2)$$

$$+ \uparrow \sum F_y = 0:20 \text{ kN} + \frac{1}{\sqrt{2}} P_{AB} = 0$$
 -----(2)

$$P_{AB} = -28.3 \text{ kN (Compression)}$$
  $P_{AD} = 20 \text{ kN (Tension)}$ 

Joint D



For equilibrium,

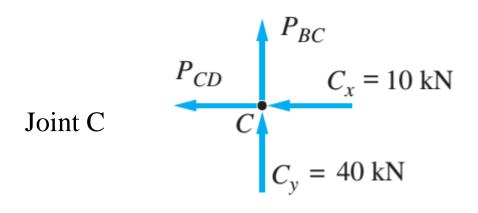
$$+ \rightarrow \sum F_x = 0 : -P_{AD} + \frac{1}{\sqrt{5}} P_{BD} + P_{CD} = 0$$
 ---- (1)

$$+ \uparrow \sum F_y = 0 : -60 \text{ kN} + \frac{2}{\sqrt{5}} P_{BD} = 0$$
 -----(2)

$$P_{BD} = 67.1 \,\text{kN} \text{ (Tension)}$$
  $P_{CD} = -10 \,\text{kN} \text{ (Compression)}$ 

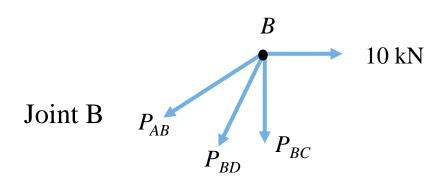
#### **>**Solution

Equilibrium analysis on the joints



For equilibrium,  

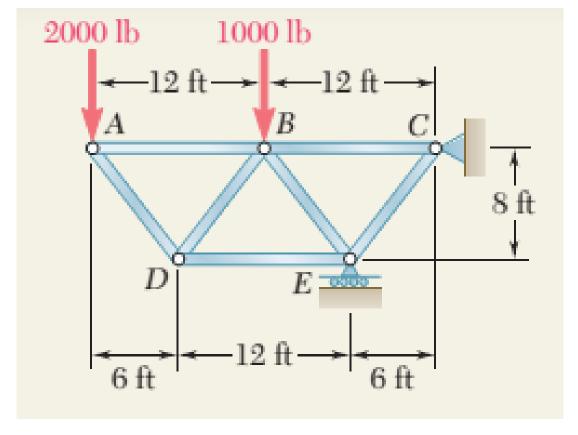
$$C_x = 10 \text{ kN}$$
  $+ \sum F_x = 0 : -P_{CD} - 10 \text{ kN} = 0$  ----- (1)  
 $C_y = 40 \text{ kN}$   $+ \sum F_y = 0 : 40 \text{ kN} + P_{BC} = 0$  ----- (2)  
 $C_y = 40 \text{ kN}$  (Compression)



### **Example**

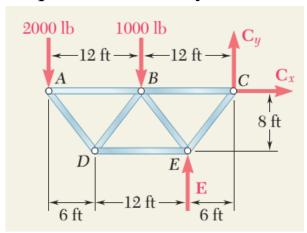
Determine the force in each member of the truss shown below using the method of

joints.



#### **>** Solution

Equilibrium analysis on the entire truss to determine the support reactions



For equilibrium,

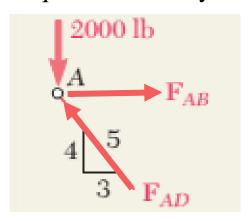
$$+ \rightarrow \sum F_x = 0 : C_x = 0$$

$$+ \uparrow \sum F_y = 0 : C_y + E = 3000 \text{ lb}$$

$$\sum M_C = 0 : (6 \text{ ft})E - (2000 \text{ lb})(24 \text{ ft}) - (1000 \text{ lb})(12 \text{ ft}) = 0$$

$$+ \uparrow E = 10000 \text{ lb} \uparrow$$

> Equilibrium analysis on the individual joints



For equilibrium,

$$+ \rightarrow \sum F_x = 0: F_{AB} - \frac{3}{5}F_{AD} = 0$$
 ----(1)

$$+ \uparrow \sum F_y = 0 : \frac{4}{5} F_{AD} - 2000 \,\text{lb} = 0$$
 ---- (2)

Solving (1) and (2) simultaneously,

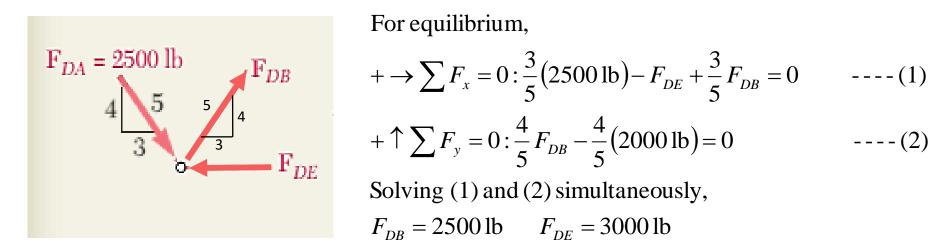
$$F_{AD} = 2500 \,\text{lb}$$
  $F_{AB} = 1500 \,\text{lb}$ 



# **Analysis of Planar Trusses - The Method of Joints**



Joint D



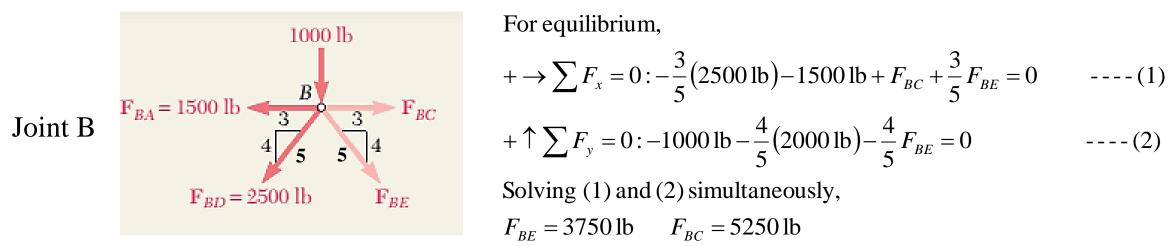
For equilibrium,

$$+ \rightarrow \sum F_x = 0: \frac{3}{5} (2500 \,\text{lb}) - F_{DE} + \frac{3}{5} F_{DB} = 0$$
 ---- (1)

$$+ \uparrow \sum F_y = 0 : \frac{4}{5} F_{DB} - \frac{4}{5} (2000 \,\text{lb}) = 0$$
 ---- (2)

Solving (1) and (2) simultaneously,

$$F_{DB} = 2500 \,\text{lb}$$
  $F_{DE} = 3000 \,\text{lb}$ 



For equilibrium,

$$+ \rightarrow \sum F_x = 0 : -\frac{3}{5} (2500 \,\text{lb}) - 1500 \,\text{lb} + F_{BC} + \frac{3}{5} F_{BE} = 0$$
 ---- (1)

$$+ \uparrow \sum F_y = 0 : -1000 \,\text{lb} - \frac{4}{5} (2000 \,\text{lb}) - \frac{4}{5} F_{BE} = 0 \qquad ---- (2)$$

Solving (1) and (2) simultaneously,

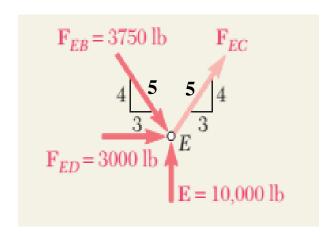
$$F_{BE} = 3750 \,\text{lb}$$
  $F_{BC} = 5250 \,\text{lb}$ 



# **Analysis of Planar Trusses - The Method of Joints**



Joint E



For equilibrium,

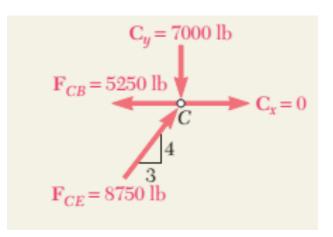
$$+ \rightarrow \sum F_x = 0: \frac{3}{5} (3750 \,\text{lb}) + 3000 \,\text{lb} + \frac{3}{5} F_{EC} = 0$$
 ---- (1)

$$+ \uparrow \sum F_y = 0 : -\frac{4}{5} (3750 \,\text{lb}) + \frac{4}{5} F_{EC} + 10000 \,\text{lb} = 0$$
 ---- (2)

Solving (1) and (2) simultaneously,

$$F_{DB} = 2500 \,\text{lb}$$
  $F_{DE} = 3000 \,\text{lb}$ 

Joint C



For equilibrium,

$$+ \rightarrow \sum F_x = 0: \frac{3}{5} (8750 \text{ lb}) - 5250 \text{ lb} + 0 = 0$$

$$+ \uparrow \sum F_y = 0 : -7000 \,\text{lb} - \frac{4}{5} (8750 \,\text{lb}) = 0$$

#### Analysis of Planar Trusses – The Method of Sections

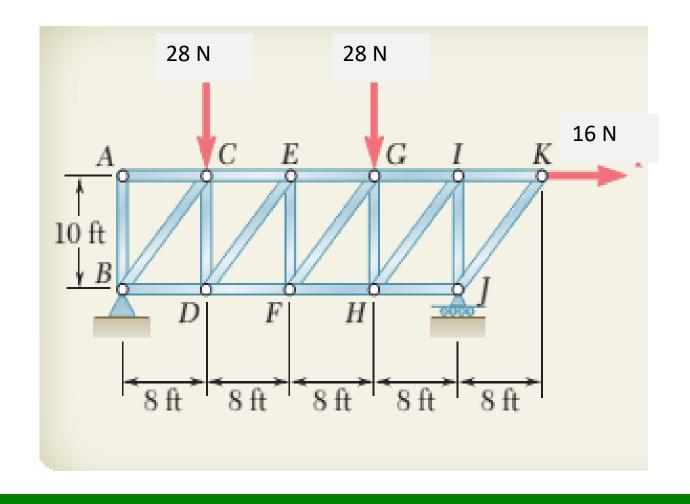
This method is useful if it is desired to find the forces in only a few members.

➤It is based on the assumption that if the whole truss is in equilibrium, any section of it must also be in equilibrium.

- ➤ Analysis in the following steps:
  - ➤ Perform an equilibrium analysis to determine all support reactions on the truss.
  - Draw a line which divides the truss into two separate portions, intersecting the member of interest in the process.
  - Perform an equilibrium analysis on one of the sections to determine the force on the member of interest.

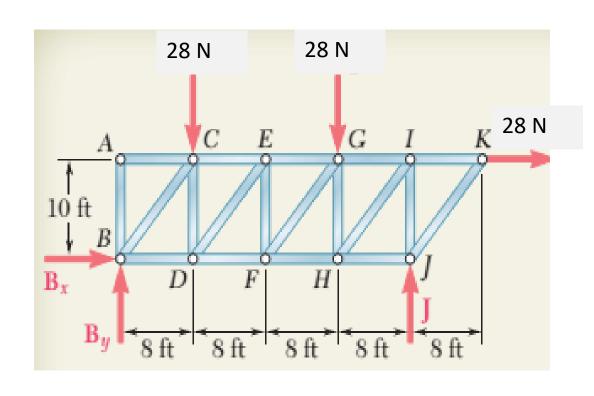
#### **Example**

Determine the force in members *EF* and *GI* of the truss shown



#### **Example Solution**

Equilibrium analysis on the whole truss to determine support reactions.



For equilibrium,

+ → 
$$\sum F_x = 0$$
:  $B_x + 16$  N = 0  
+  $\uparrow \sum F_y = 0$ :  $B_y + J - 28$  N - 28 N = 0

$$\underbrace{+} \sum M_B = 0:28 \text{ N}(8 \text{ ft}) + 28 \text{ N}(24 \text{ ft}) + 16 \text{ N}(10 \text{ ft}) - J(32 \text{ ft}) = 0$$

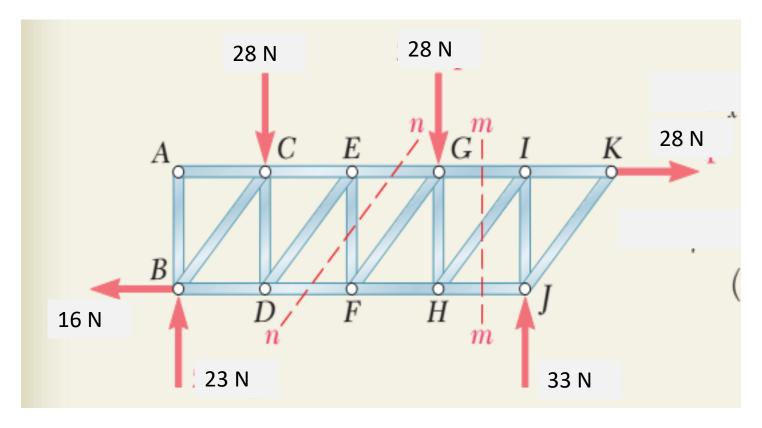
$$J = 33 \, \text{N}$$

$$B_{x} = -16 \text{ N}$$

$$B_{v} = 23 \, \text{N}$$

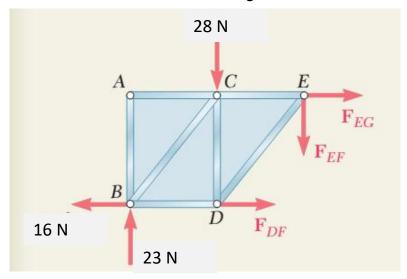
Example - Solution

Dividing the truss into two with line nn (to solve for member EF), then with mm (to solve for GI)

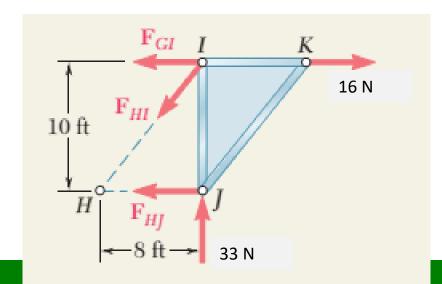


#### Example - Solamalysis of Planar Trusses - The Method of Sections





Performing an equilibrium analysis on the left side of nn to obtain EF,  $+ \uparrow \sum F_y = 0:23 \text{ N} - 28 \text{ N} - F_{EF} = 0$   $F_{EF} = -5 \text{ N}$ 



Performing an equilibrium analysis on the right side of mm to obtain GI,

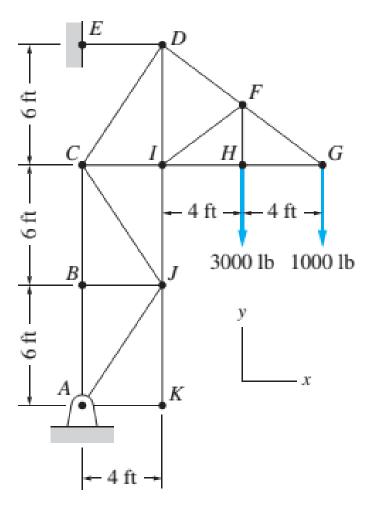
$$\underbrace{+ \sum_{H} M_{H}} = 0: -F_{GI}(10 \text{ ft}) - 33 \text{ N}(8 \text{ ft}) + 16 \text{ N}(10 \text{ ft}) = 0$$

$$F_{GI} = -10.4 \text{ N}$$

**≻**Example

Determine the forces in members FI and JC of the truss shown. Indicate tension or

compression.

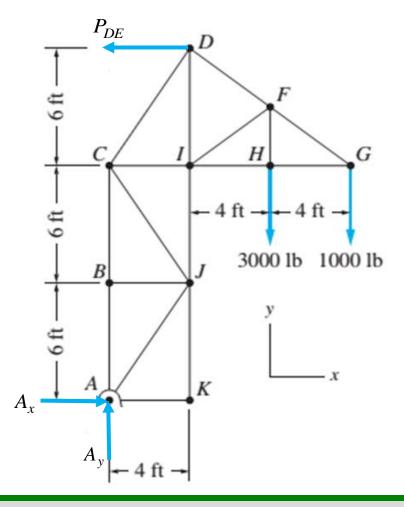






## Example - Solution Analysis of Planar Trusses - The Method of Sections

Equilibrium analysis on the whole truss to determine support reactions gives.



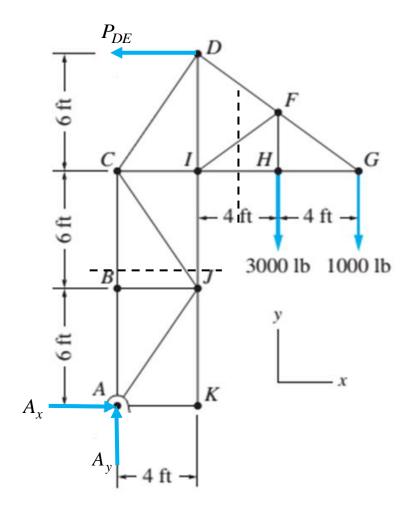
$$P_{DE} = 2000 \, \text{lb}$$

$$A_x = 20001b$$

$$A_{y} = 4000 \, \text{lb}$$

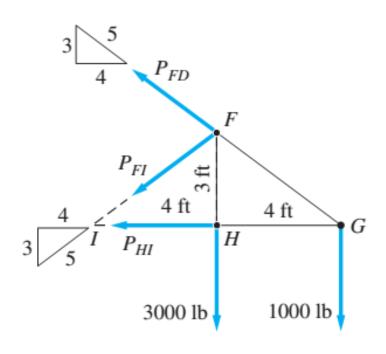
#### **≻**Example - Solution

The truss is divided into 2 tie lines so FI and JC can be determined.



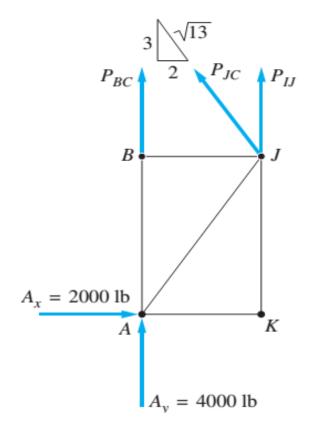
#### **Example - Solution**

For FI



$$P_{FI} = -2500 \, \text{lb}$$



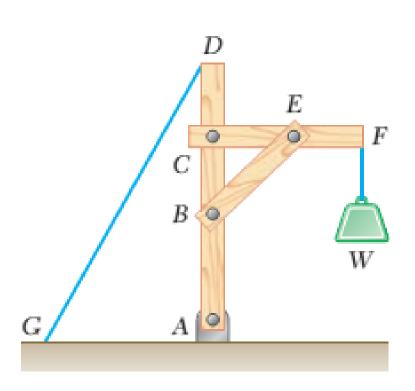


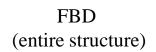
$$P_{JC} = -36101b$$

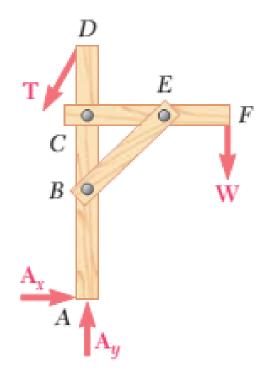
Frames and Machines are structures in which at least one member is acted upon by at least three or more forces member.

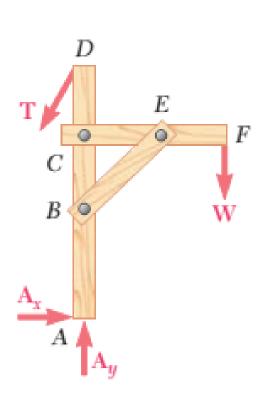
➤In addition, forces can act on any point of a frame member (not just the ends)

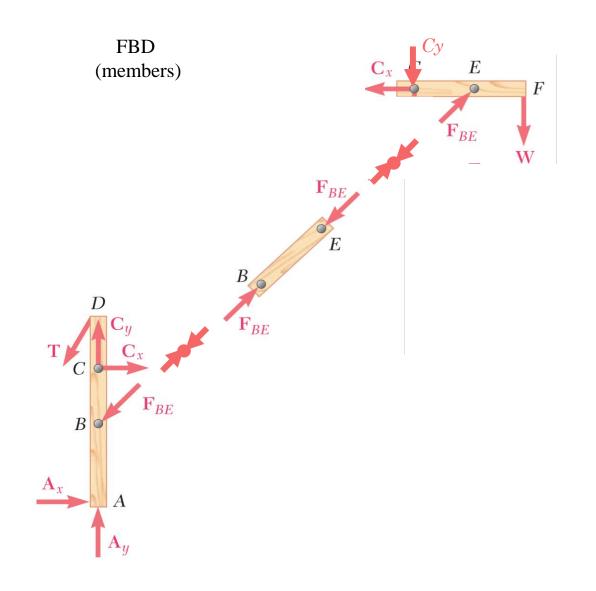
- Analysis procedure is similar to what is used in trusses;
  - An equilibrium analysis is first carried out on the entire structure.
  - An equilibrium analysis is then carried out on the individual members of the frame to determine all forces acting on each of them (It is sometimes easier if this is done first on the two force, then three force members).





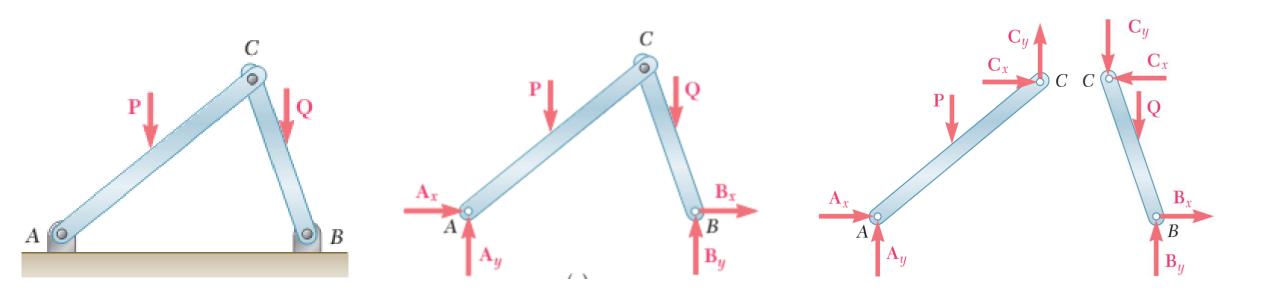






#### **Analysis of Frames**

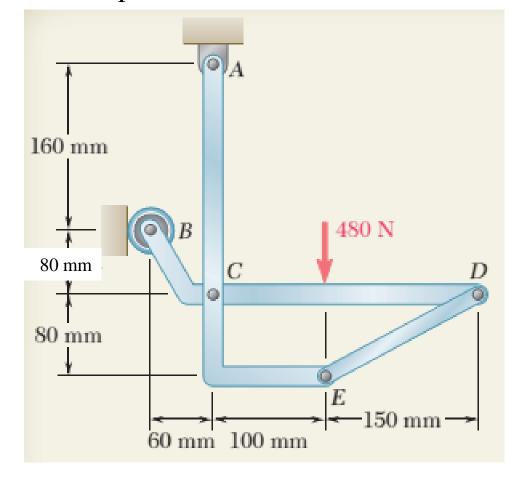
During analysis of non-rigid frames, the individual members of frame are considered as rigid members.



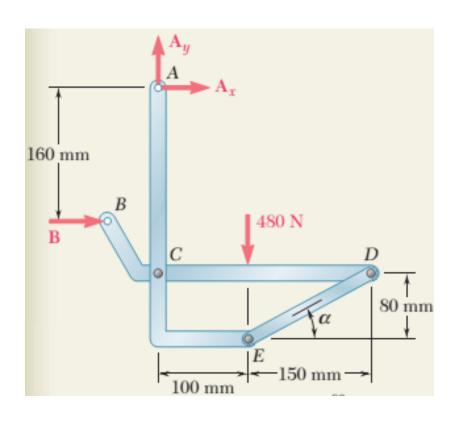
#### **Example**

In the frame shown, members ACE and BCD are connected by a pin at C and by the link DE. For the loading shown, determine the force in link DE and the components of the force exerted at C on member

BCD. Is link DE tension or compression?



Equilibrium analysis on the entire structure



For equilibrium,

$$+ \rightarrow \sum F_x = 0 : B + A_x = 0$$

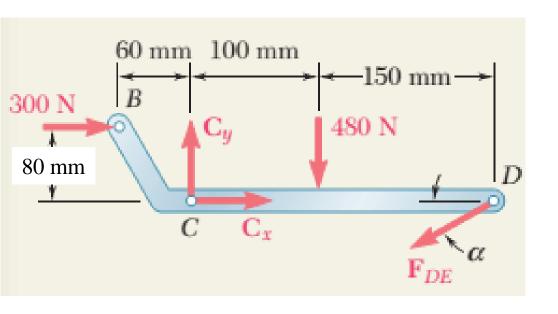
$$+ \uparrow \sum F_y = 0 : A_y = 480 \text{ N}$$

$$(\pm) \sum M_A = 0 : -B(160 \text{ mm}) + 480 \text{ N}(100 \text{ mm}) = 0$$

$$B = 300 \text{ N}$$

$$A_x = -300 \text{ N}$$

Example – Solution (Equilibrium analysis on individual links)



For equilibrium,

$$+ \rightarrow \sum_{x} F_x = 0:300 \text{ N} + C_x - F_{DEx} = 0$$

$$+ \uparrow \sum F_y = 0 : C_y - 480 \text{ N} - F_{DEy} = 0$$

$$\sum M_C = 0:300 \text{ N}(80 \text{ mm}) + 480 \text{ N}(100 \text{ mm}) + F_{DEy}(250 \text{ mm}) = 0$$

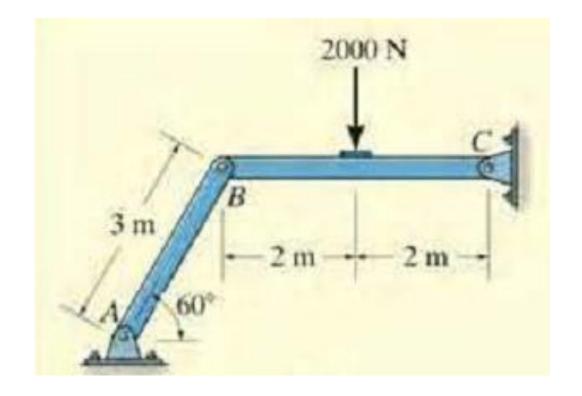
$$F_{DEy} = -288 \text{ N}$$
  $C_y = -192 \text{ N}$ 

$$\alpha = \tan^{-1} \left( \frac{80}{150} \right)$$

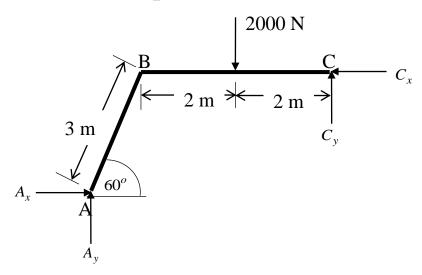
$$F_{DE} = -612 \text{ N}$$
  $F_{DEx} = -\text{ N}$   $C_x = -\text{ N}$ 

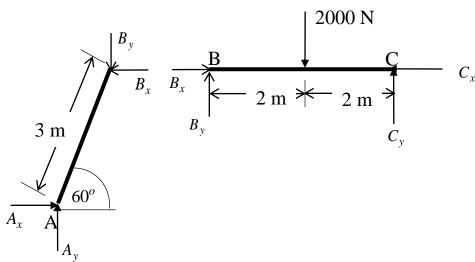
#### **Example**

Determine the horizontal and vertical components of the force which the pin at C exerts on member BC of the frame shown below. Also determine the forces acting on both members at B



#### **≻**Example - Solution





For equilibrium,

$$+ \rightarrow \sum F_x = 0$$
:  $A_x - C_x = 0$ 

$$+ \uparrow \sum F_y = 0 : A_y + C_y - 2000 \text{ N} = 0$$

$$\sum M_C = 0: -A_y (4 + 3\cos 60^\circ) + A_x (3\sin 60^\circ) + 2000 \text{ N}(2\text{ m}) = 0$$

$$\sum M_A = 0: -C_y (4 + 3\cos 60^\circ) \text{m} + C_x (3\sin 60^\circ) \text{m} + 2000 \text{ N} (2 + 3\cos 60^\circ) \text{m} = 0$$

$$A_x = C_x = 577 \text{ N}$$
  $A_y = C_y = 1000 \text{ N}$ 

Taking member BC, for equilibrium,

$$+ \rightarrow \sum F_x = 0 : B_x - C_x = 0$$

$$+ \uparrow \sum F_y = 0 : B_y + C_y - 2000 \text{ N} = 0$$

$$B_x = 577 \text{ N}$$

$$B_{y} = 1000 \,\text{N}$$

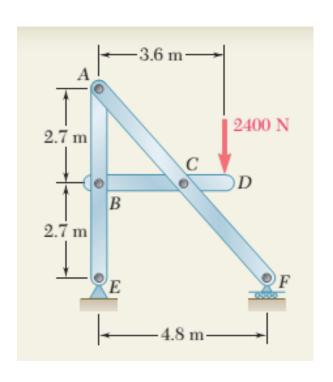
For member AB,

$$B_x = 577 \text{ N} \leftarrow$$

$$B_{\rm v} = 1000 \,\mathrm{N} \downarrow$$

#### **Example**

Determine the components of the forces acting on each member of the frame shown.



$$F = 1800 \,\mathrm{N}$$

$$E_y = 600 \,\mathrm{N}$$

$$E_x = 0 \text{ N}$$

$$A_x = 0 \text{ N}$$

$$A_y = 1800 \,\mathrm{N}$$

$$B_x = 0 \text{ N}$$

$$B_{y} = 1200 \,\mathrm{N}$$

$$C_x = 0 \text{ N}$$

$$C_{v} = 1000 \,\mathrm{N}$$