Course 157

Note on solutions to Example sheet 1

1) Absolute error $= \left|A - \tilde{A}\right|$ a) 0.012 b) 0.0002 c) 2×10^9

Relative error $= \left|A - \tilde{A}\right| / |A|$ a) ≈ 0.0012 b) ≈ 0.0423 c) ≈ 0.003

2) $a + b := 0.469 \times 10^{-2}$ $|RE| \approx 0.15 \times 10^{-5}$

$a - b := 0.473 \times 10^{-2}$ $|RE| \approx 0.15 \times 10^{-5}$

$a \times b := -0.871 \times 10^{-7}$ $|RE| \approx 0.35 \times 10^{-10}$

$a/b := -0.255 \times 10^{3}$ $|RE| \approx -0.4054$

note the symbol := is used to denote *computed as.*

3) Analysis of the truncation error as in the notes for $e^{0.1}$ does not work here. Why? To get the truncation error compare the computed result with the exact answer using MATLAB.

4) For very small h there is a cancellation between $\{\sin(x+h) - \sin(x)\}$ which makes the estimate of derivative poor. For even smaller h, the computed value$x + h$ is stored as x and the estimate of the derivative is zero.

5) $a = 2$

$x_0 = \frac{a}{2} = 1$

$x_1 = \frac{1}{2}\left(1 + \frac{2}{1}\right) = \frac{3}{2} = 1.5$ error $= 0.8 \times 10^{-1}$

$x_2 = \frac{1}{2}\left(\frac{3}{2} + \frac{4}{3}\right) = \frac{17}{12} = 1.416666666667$ error $= 0.2 \times 10^{-2}$

$x_3 = \frac{577}{408} = 1.414215686275$ error $= 0.2 \times 10^{-5}$

$x_4 = \frac{665857}{470832} = 1.414213562375$ error $= 0.1 \times 10^{-11}$

Note: the error at step $i+1$ approximately satisfies $\varepsilon_{i+1} = O\left(\varepsilon_i^2\right)$

6) 111 (since $7 = 4 + 2 + 1 = 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$)

11011

0.001 (since $0.125 = 2^{-3} = 0 \times 2^0 + 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$)

1110.11

$0.01001100110011\cdots$

7) (0.111000, +11) (Note: the exponent is also in binary form i.e $3 \to 11$)

(0.110110, +101)

(0.100000, −10)

(0.111011, +100)

(0.100110, −1)