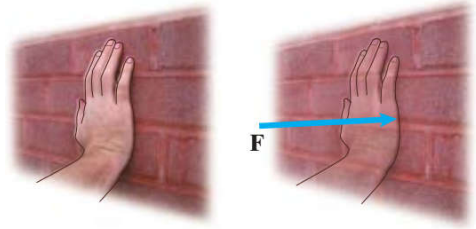
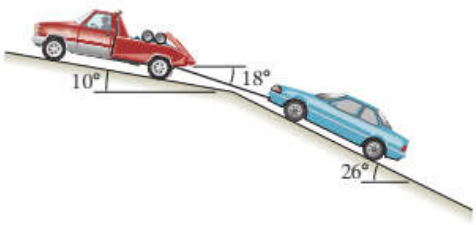
FORCES

Characteristics and Resultants of Forces

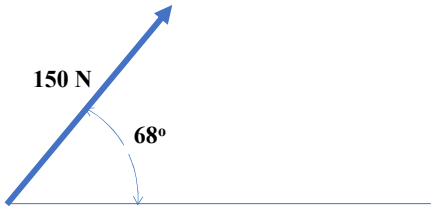
Forces

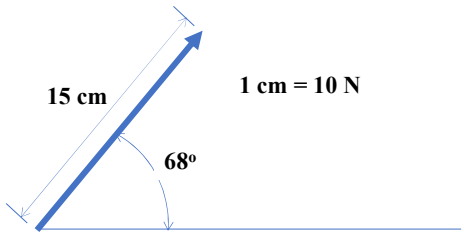




Forces: Some Characteristics

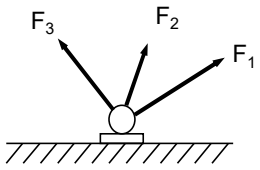
- Forces are vector quantities.



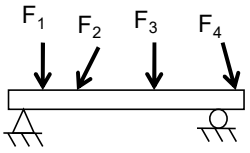


- Forces considered equal if they have the same magnitude and direction.
- Forces are equivalent if they produce the same resultant effect on a rigid body.

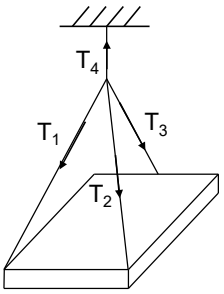
Systems of Forces



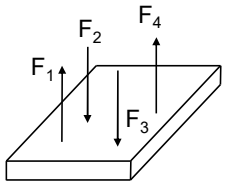
Concurrent Coplanar Force System



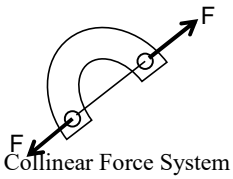
Non-Concurrent
Coplanar Force System



Concurrent Spatial Force System



Parallel spatial Force System



Collinear Force System

A system of forces can be replaced with an equivalent system of forces or a Resultant Force (one equivalent force).

How do we find the Resultant of a System of Forces?

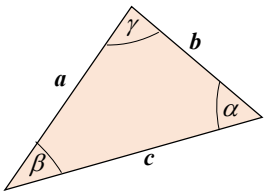
The Resultant of a Systems of Forces

There are several approaches:

- Graphical approach –Polygon rules of vector addition.
- Force Triangle with Sine and Cosine rules.
- Summation of Rectangular/Perpendicular Force components

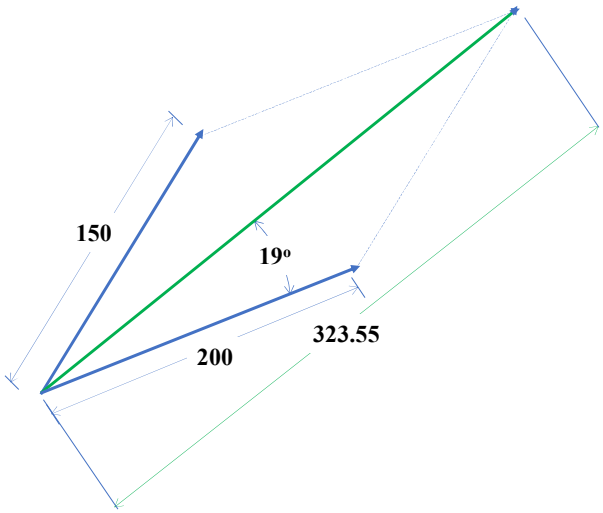
Resultants of Forces: Force Triangle Approach

- The sine and cosine laws:

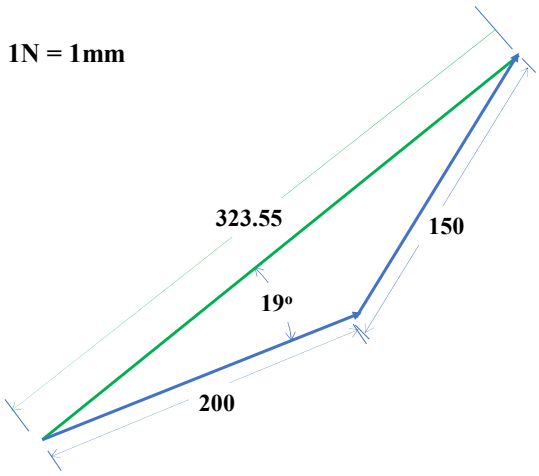


Law of Sines	$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
Law of Cosines	$a^2 = b^2 + c^2 - 2bc\cos\alpha$ $b^2 = c^2 + a^2 - 2ca\cos\beta$ $c^2 = a^2 + b^2 - 2ab\cos\gamma$

Resultants of Forces: Graphical Approach



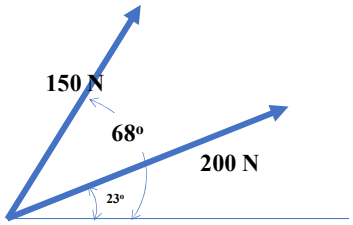
1N = 1mm

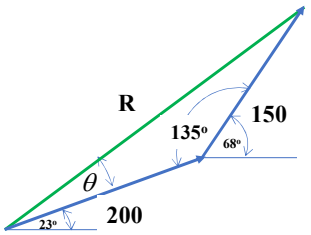


Triangle Law

Resultants of Forces: Force Triangle Approach

Example





From the Cosine Rule,

$R^2 = 200^2 + 150^2 - 2(200)(150)\cos 135^\circ$

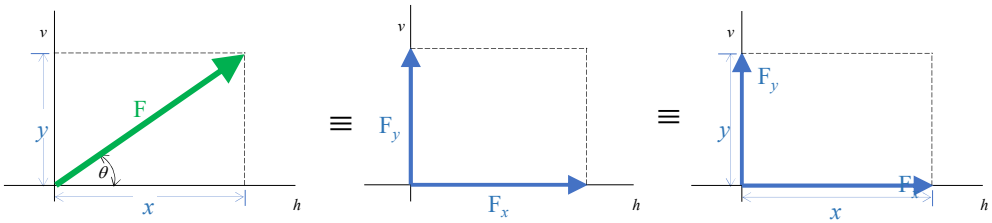
$R = 323.9$ N.

From the Sine Law

$\frac{323.9 \text{ N}}{\sin 135^\circ} = \frac{150 \text{ N}}{\sin \theta^\circ}, \quad \theta = 19.11^\circ$

Resultants of Forces: Force Components Approach

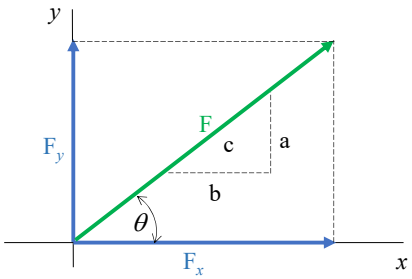
- This approach requires the forces to be decomposed/resolved into Rectangular/Cartesian/perpendicular components (along principal directions).



$\vec{F} = \vec{F}_x + \vec{F}_y = F_x\vec{i} + F_y\vec{j}$

- Like components (along the same direction) are then summed to get the resultant components.
- Magnitude and direction of the resultant force can be obtained through appropriate Trigonometry techniques.

Resultants of Forces: Resolve Forces into Components



$$\vec{F} = \vec{F}_x + \vec{F}_y$$
$$= F\vec{i} + F\vec{j}$$

Pythagoras Theorem

$$\vec{F} = \vec{F}_x + \vec{F}_y$$
$$\vec{F} = F\cos\theta + F\sin\theta$$

Note:

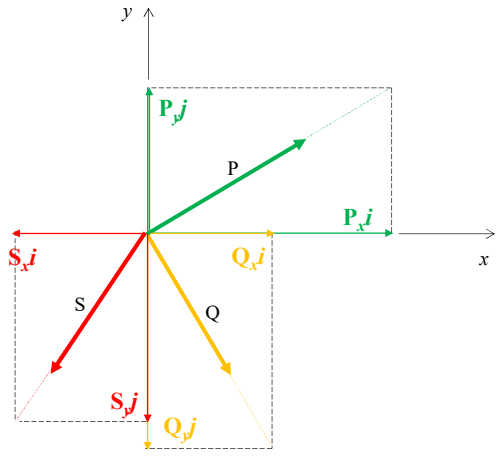
$$\frac{a}{\sqrt{b^2+a^2}} = \cos\theta, \qquad \frac{b}{\sqrt{b^2+a^2}} = \sin\theta$$

Directional Vectors

$$\vec{F} = F\lambda = F\left(\frac{b\vec{i} + a\vec{j}}{\sqrt{b^2+a^2}}\right)$$
$$= F\left(\frac{b\vec{i}}{\sqrt{b^2+a^2}}\right) + F\left(\frac{a\vec{j}}{\sqrt{b^2+a^2}}\right)$$
$$= F\vec{i} + F\vec{j}$$

Resultants of Forces: Resolve Forces into Components

- Sum like components to get the components of the resultant.



$$\vec{R} = \sum F_x + \sum F_y$$
$$\vec{R} = (P_x + Q_x + S_x)\vec{i} + (P_y + Q_y + S_y)\vec{j}$$

The magnitude of the Resultant Force is given by;

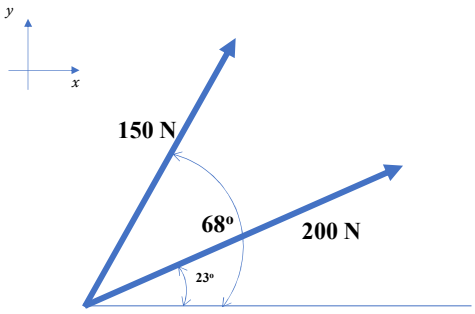
$$R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}$$

And the direction;

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right)$$

Resultants of Forces

Example



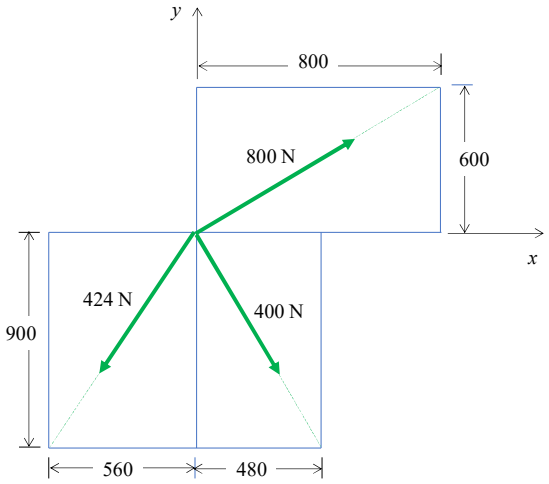
Force	F_x	F_y
200 N	200 cos 23 N	200 sin 23 N
150 N	150 cos 68 N	150 sin 68 N
Σ	240.292 N	217.224 N

$$F = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}$$
$$= \sqrt{240.292^2 + 217.224^2}$$
$$= 323.924 \text{ N}$$

Resultants of Forces

Example

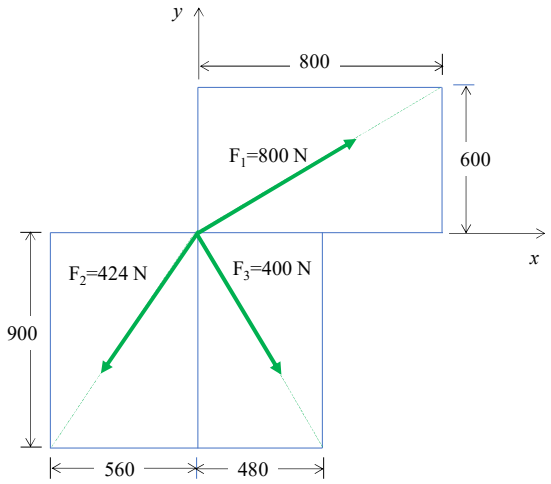
Find the resultant of the forces shown



Dimensions are in mm

Resultants of Forces

Example - Solution



$$\vec{F} = F\lambda = F\left(\frac{x\vec{i} + y\vec{j}}{\sqrt{x^2 + y^2}}\right)$$

$$\vec{F}_1 = 800\left(\frac{800\vec{i} + 600\vec{j}}{\sqrt{800^2 + 600^2}}\right) = 800.\frac{800\vec{i}}{1000} + 800.\frac{600\vec{j}}{1000} = 640\vec{i} + 480\vec{j}$$

$$\vec{F}_2 = 424\left(\frac{-560\vec{i} - 900\vec{j}}{\sqrt{560^2 + 900^2}}\right) = -224\vec{i} - 360\vec{j}$$

$$\vec{F}_3 = 400\left(\frac{480\vec{i} - 900\vec{j}}{\sqrt{480^2 + 900^2}}\right) =$$

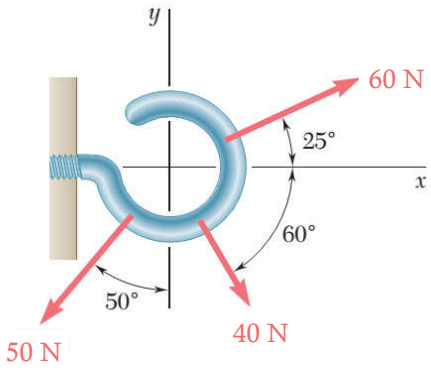
Resultant, $\vec{F} = \sum F\vec{i} + \sum F\vec{j} =$

The magnitude of the Resultant, $F = \sqrt{\left(\sum F\vec{i}\right)^2 + \left(\sum F\vec{j}\right)^2}$

Resultants of Forces

Example

Find the resultant of the forces shown.



$$F = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}$$

$$+\rightarrow \sum F_x = 60\cos 25 + 40\cos 60 - 50\sin 50 = 36.076 \text{ N}$$

$$+\uparrow \sum F_y = 60\sin 25 - 40\sin 60 - 50\cos 50 = -41.423 \text{ N}$$

OR

Force	F_x(+→)	F_y(+↑)
60 N	60 cos 25	60 sin 25
40 N	40 cos 60	- 40 sin 60
50 N	- 50 cos 40	- 50 cos 50
Σ	36.076 N	- 41.523 N

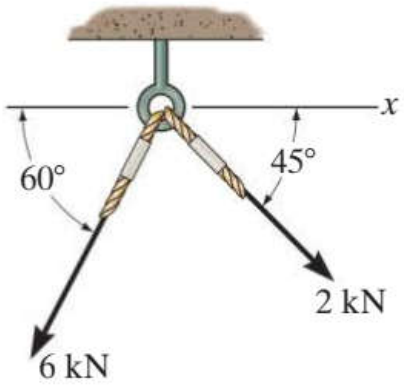
$$F = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}$$

$$F = \sqrt{36.076^2 + (-41.423)^2} = 54.930 \text{ N}$$

Resultants of Forces

Example

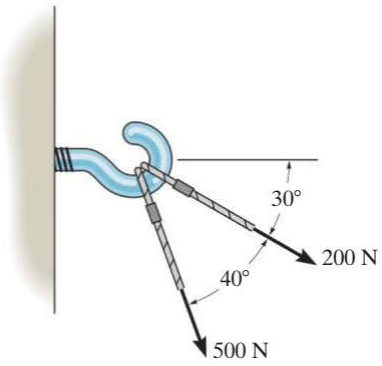
Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x-axis.



Resultants of Forces

Example

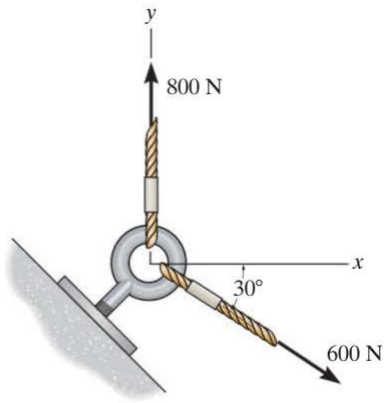
Determine the magnitude of the resultant force acting om the hook.



Resultants of Forces

Example

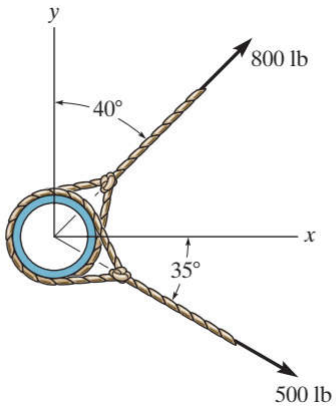
Determine the magnitude of the resultant force acting on the screw eye and its direction measured counter-clockwise from the x-axis.



Resultants of Forces

Example

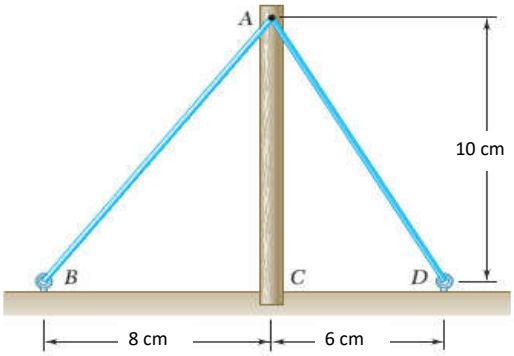
Determine the magnitude of the resultant force acting on the screw eye and its direction measured counter-clockwise from the x-axis.



Resultants of Forces

Example

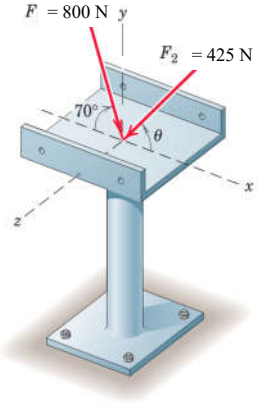
Cables AB and AD help support pole AC. Knowing that the tension is 120 N in AB and 40 N in AD, determine the magnitude of the resultant of the forces exerted by the cables at A.



Resultants of Forces

Example

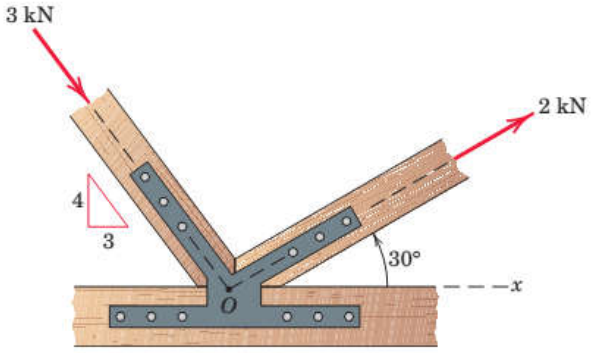
Two forces on the same plane are applied to the construction bracket as shown. Determine the angle Θ which makes the resultant of the two forces vertical. Determine the magnitude R of the resultant.



Resultants of Forces

Example

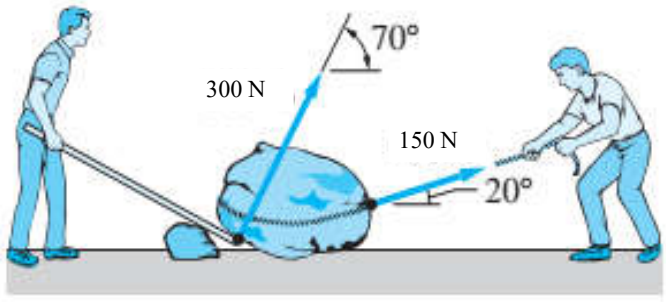
The two structural members, one of which is in tension and other in compression, exert the indicated forces on joint *O*. If we were to replace to two members with an equivalent one at the same point, determine the its angle of inclination with respect to the positive x axis. Also determine the magnitude of the force that will be acting through it.



Resultants of Forces

Example

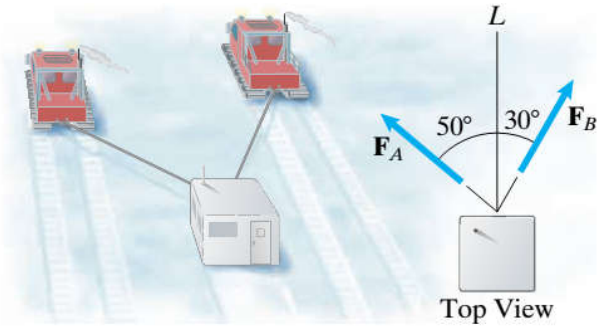
Two men are trying to roll the boulder by applying the forces as shown. Determine the magnitude and direction of the force that is equivalent to the forces the two men are applying.



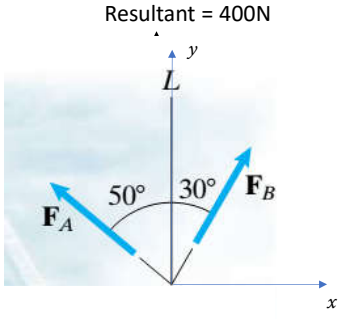
Resultants of Forces

Example

Two snow carts tow an emergency shelter. The towing cables are horizontal. The total force $F_A + F_B$ on the shelter is parallel to the direction L and its magnitude is 400 N. Determine the magnitudes of F_A and F_B



Resultants of Forces



$$\overset{+}{\rightarrow}\sum F_x = F_B \sin 30^o - F_A \sin 50^o$$

$$+\uparrow \sum F_y = F_B \cos 30^o - F_A \cos 50^o$$

Resultant force is vertical. This means;

$R_x = 0$

$R_y = 400$ N

And

$R_x = \Sigma F_x$

$R_y = \Sigma F_y$

So,

$$F_B \sin 30^o - F_A \sin 50^o = 0$$

$$F_B \cos 30^o - F_A \cos 50^o = 400$$

Solve simltaneously to get;

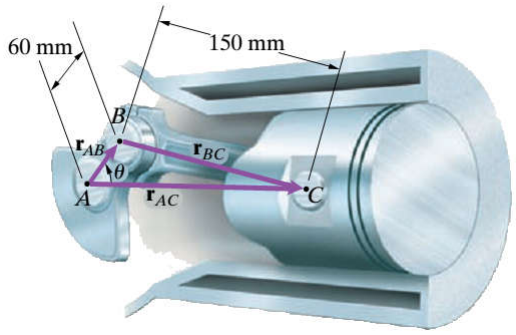
$F_A = 203\ N$

$F_B = 311\ N$

Resultants of Forces

Example

The angle $\Theta = 50°$. determine the length of the line representing vector r_{AC}. (Hint: all three lines lie in the same plane)

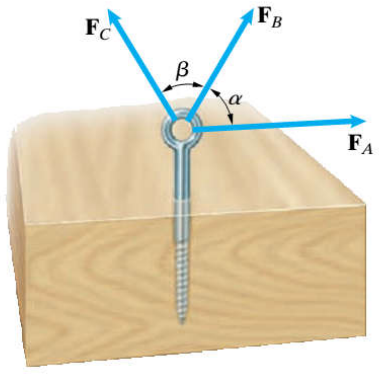


r_{AC} = 181 mm

Resultants of Forces

Example

The forces F_A = 40 N, F_B = 50 N, and F_C = 40 N act on the screw pin as illustrated in the Figure. α = 50° and β = 80°. Determine the magnitude of the resultant of the three forces on the eye of the screw pin, assuming they are coplanar.



R = 83 N