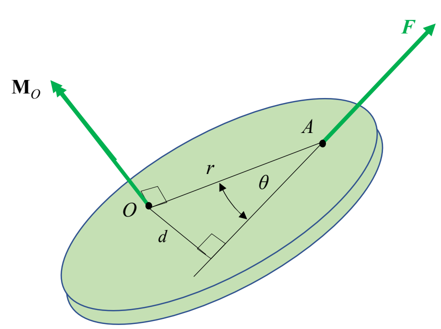
Moment of Forces

Scalar and vector approaches of calculating Moments
Principles of Moments
Equivalent Force-Couple Systems

Moment of a Force

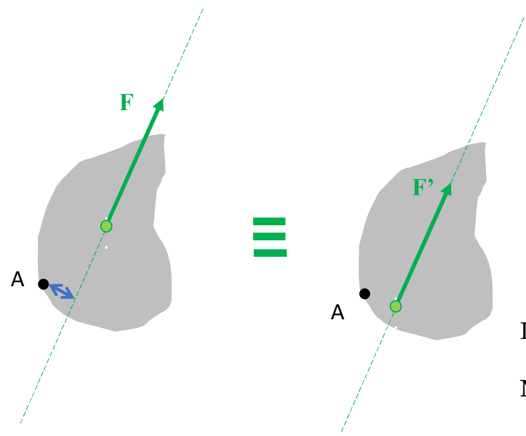
- Forces have the tendency to cause two types motions in rigid bodies; **translational** and **rotational** motions.
- The tendency of a force to rotate a body is referred to as moment.
- A moment may occur about a point; the ***Moment Centre***.



$$\vec{M}_o = \vec{r} \times \vec{F}$$
$$r\sin\theta = \mathrm{d}$$
$$M_o = (r\sin\theta)F = dF$$

Moment of a Force

- *d* must be perpendicular to the Force's line of action, so, the Force is treated as a sliding vector, due to the *Principle of Transmissibility* in rigid body mechanics.



If F and F' have the same magnitude,

Moment of F about A = Moment of F' about A

Moment of a Force

- **Scalar Approach**

$$M_o = d \times F$$

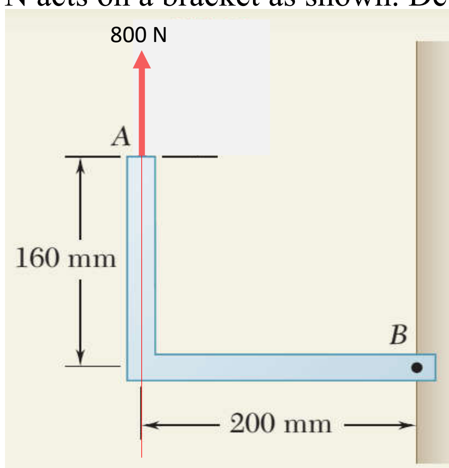
  - Only the magnitude of the moment is calculated using only the magnitudes of the force and the moment arm, *d,* defined as the perpendicular distance between the line of action of the force and the moment centre.
  - Often used when the moment, *d* can easily be determined. The sense of the moment is determined by inspection.

- **Vector Approach**
  - The position vector for the point of application of the force is multiplied by the components of the force to get the components of the Resultant moment.

Moment of a Force

Example

A force of 800 N acts on a bracket as shown. Determine the moment of the force about B.

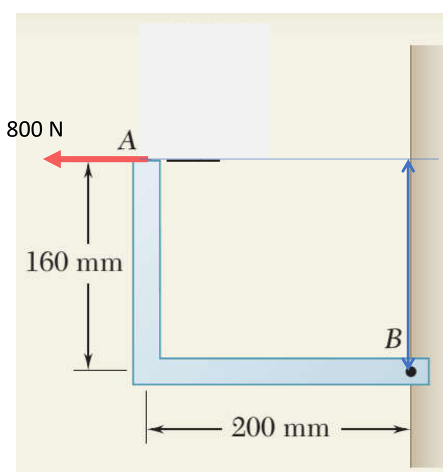


$$M_B = 0.2 \text{ m} \times 800$$
$$= 160\ Nm$$

Moment of a Force

Example

A force of 800 N acts on a bracket as shown. Determine the moment of the force about B.



$$M_B = 0.16m \times 800$$
$$= -128\ Nm$$

Moment of a Force

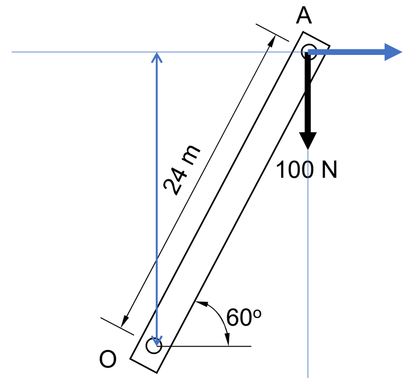
Example

A 100-N vertical force is applied to the end of a lever which is attached to a shaft at *O*.

Determine:

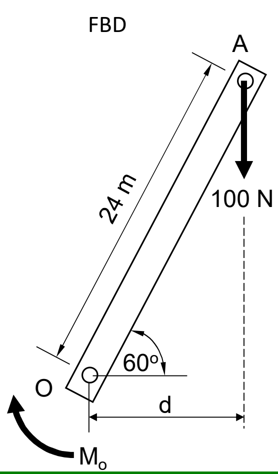
a) moment about *O* of the 100-N force,

b) horizontal force at *A* which creates the same moment,



Moment of a Force

Solution



Moment about *O* is equal to the product of the force and the perpendicular distance between the line of action of the force and *O*.

$$M_O = Fd$$

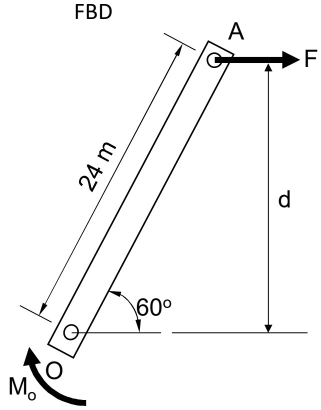
$$d = (24 \text{ m})\cos 60° = 12 \text{ m.}$$

$$M_O = (100 \text{ N})(12 \text{ m.}) =$$

Moment of a Force

Solution

FBD



Horizontal force at *A* that produces the same moment,

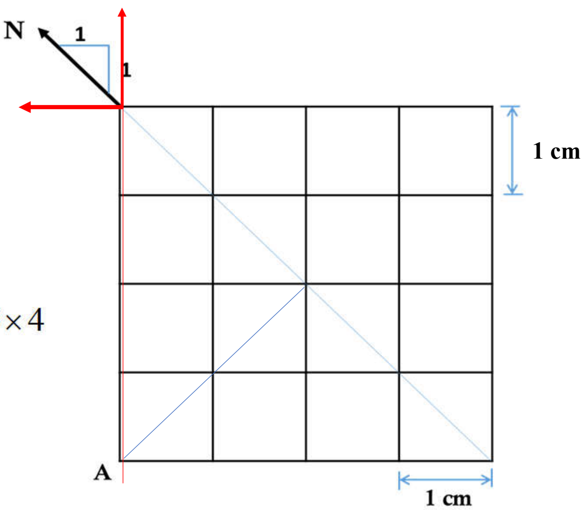
$$d = (24\text{ m})\sin 60° = 20.8\text{ m}$$
$$M_O = Fd$$
$$1200\text{ Nm.} = F(20.8\text{ m})$$
$$F = \frac{1200\text{ Nm}}{20.8\text{ m.}}$$
$$=$$

The Principle of Moments (Varignon's Theorem)

The moment of a force about a given point, is ***equal and equivalent*** to the sum of the moments of an equivalent system of forces in the same plane, about the same point.

The Principle of Moments (Varignon's Theorem)

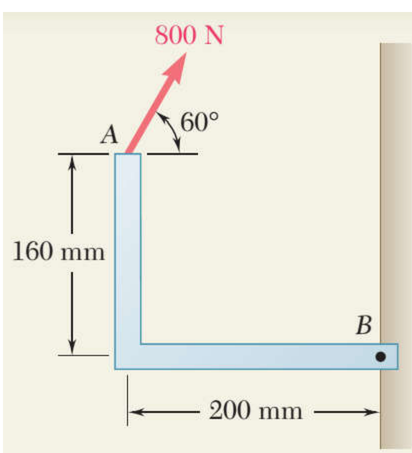
Determine the magnitude and sense of the moment due to the 100 N force shown below about A.

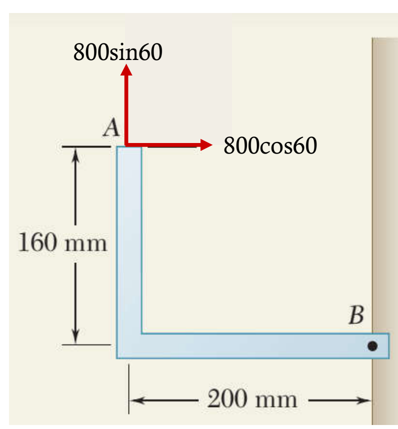


$$\circlearrowleft + \quad M_A = 100 \times \left(\frac{\sqrt{4^2 + 4^2}}{2} \right) = 100 \cos 45 \times 4$$

The Principle of Moments (Varignon's Theorem)

Determine the moment of the 800N force about B using the principle of moments,.





$$\circlearrowleft + \quad M_B = (0.16m \times 800 \cos 60) + (0.2m \times 800 \sin 60)$$
$$= 202.56\ Nm$$

Moment of a Force: Vector Formulation

- The position vector components for the point of application of the force (from the moment centre) are multiplied by the force vector components to get the of the Resultant moment components.

- Multiplication of the position and force vectors may be done in one of two ways;
  - ✓ Matrix approach
  - ✓ Expansion and simplification

Moment of a Force: Vector Formulation

- Vectors are expressed in components, arranged in a matrix form, and the determinant of the matrix taken.

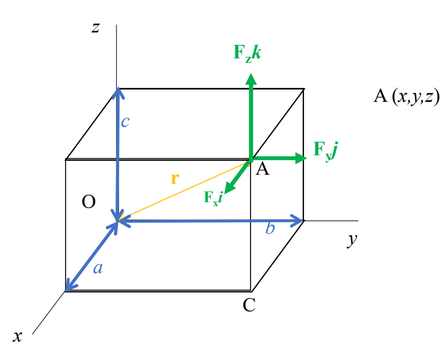
If $\vec{r} = a\vec{i} + b\vec{j} + c\vec{k}$ and $\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$

Expressing as a matrix,

$$\vec{M}_O = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ F_x & F_y & F_z \end{vmatrix}$$

Taking the determinant of the matrix,

$$\vec{M}_O = \left(bF_z - cF_y\right)\vec{i} - \left(aF_z - cF_x\right)\vec{j} + \left(aF_y - bF_x\right)\vec{k}$$
$$= M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$$



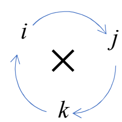
Moment of a Force: Vector Formulation

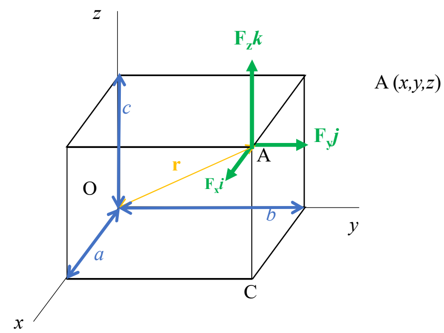
➢ Alternatively, a sort of expansion and simplification is done.

$$
\begin{aligned}
M_O &= \vec{r}\times\vec{F}\\
&= \left(a\vec{i}+b\vec{j}+c\vec{k}\right)\times\left(F_x\vec{i}+F_y\vec{j}+F_z\vec{k}\right)\\
&= \left[\left(a\vec{i}+b\vec{j}+c\vec{k}\right)\times F_x\vec{i}\right]+\left[\left(a\vec{i}+b\vec{j}+c\vec{k}\right)\times F_y\vec{j}\right]+\left[\left(a\vec{i}+b\vec{j}+c\vec{k}\right)\times F_z\vec{k}\right]
\end{aligned}
$$

But

$$
\begin{aligned}
&\vec{i}\times\vec{i}=0 \quad j\times i=-k \quad k\times i=j\\
&i\times j=k \quad j\times j=0 \quad k\times j=-i\\
&i\times k=-j \quad j\times k=i \quad k\times k=0
\end{aligned}
$$

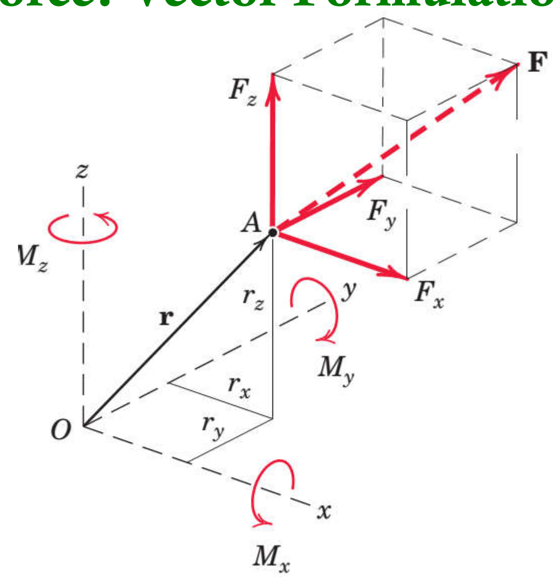




Therefore,

$$
\begin{aligned}
M_O &= -bF_x\vec{k}+cF_x\vec{j}+aF_y\vec{k}-cF_y\vec{i}-aF_z\vec{j}+bF_z\vec{i}\\
&= \left(bF_z-cF_y\right)\vec{i}-\left(aF_z-cF_x\right)\vec{j}+\left(aF_y-bF_x\right)\vec{k}\\
&= M_x\vec{i}+M_y\vec{j}+M_z\vec{k}
\end{aligned}
$$

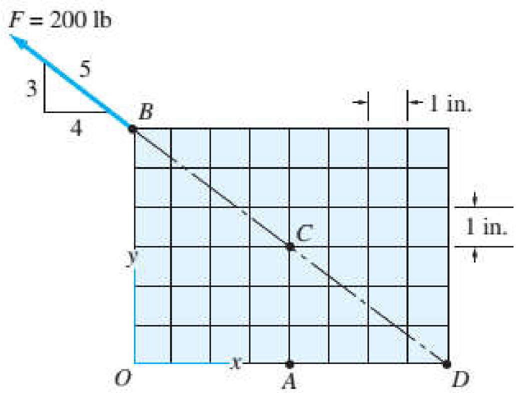
Moment of a Force: Vector Formulation



Directions of component moments

Moment of a Force: Vector Formulation

Determine the magnitude and sense of the moment due to the 100 N force shown below about A.



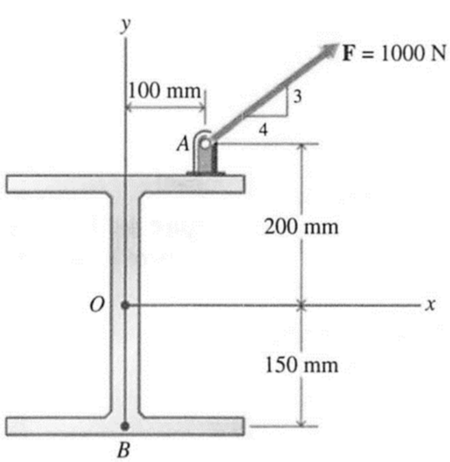
$$\vec{F} = -\left(\frac{4}{5}\right)200i + \left(\frac{3}{5}\right)200j$$
$$= -160i + 120j$$

$$\vec{r} = \vec{r}_{AB} = -4i + 6j$$

$$\vec{M}_A = \vec{r} \times \vec{F} = \vec{r}_{AB} \times \vec{F} = \begin{vmatrix} i & j & k \\ -4 & 6 & 0 \\ -160 & 120 & 0 \end{vmatrix}$$
$$= 480k \text{ lb.in}$$

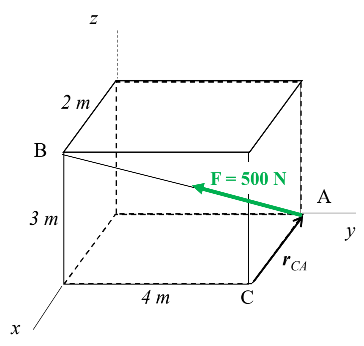
Moment of a Force: Vector Formulation

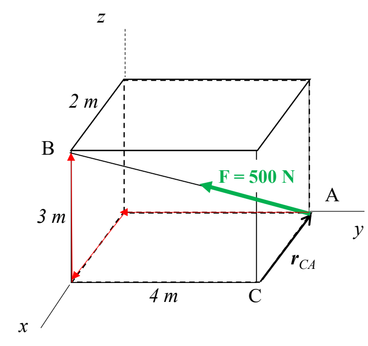
- A 1000-N force is applied to a beam cross section as show below. Determine the moment of the force about O, and the perpendicular distance from point B to the line of the action of the force.



Moment of a Force: Vector Formulation

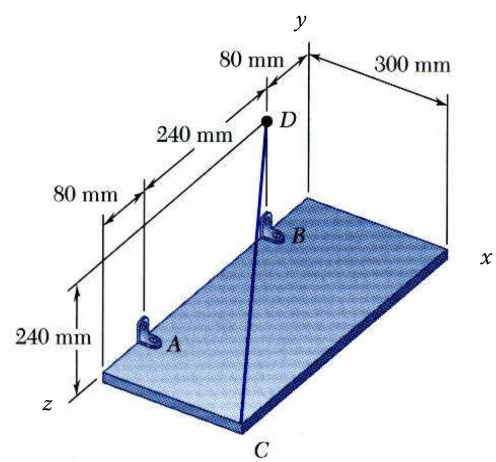
Determine the moment of the force *F,* about point *C*.



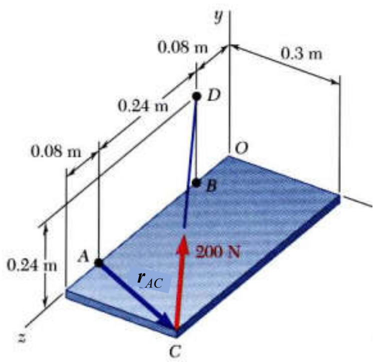


Moment of a Force: Vector Formulation

The rectangular plate is supported by the brackets at *A* and *B* and by a wire *CD*. Knowing that the tension in the wire is 200 N, determine the moment about *A* of the force exerted by the wire at *C*.



Moment of a Force: Vector Formulation



$$\vec{M}_A = \vec{r}_{AC} \times \vec{F}$$

$$\vec{r}_{AC} = (0.3\text{ m})\vec{i} + (0.08\text{ m})\vec{j}$$

$$\vec{F} = F\vec{\lambda} = (200\text{ N})\frac{-(0.3\text{ m})\vec{i} + (0.24\text{ m})\vec{j} - (0.32\text{ m})\vec{k}}{0.5\text{ m}}$$
$$= -(120\text{ N})\vec{i} + (96\text{ N})\vec{j} - (128\text{ N})\vec{k}$$

$$\vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.3 & 0 & 0.08 \\ -120 & 96 & -128 \end{vmatrix}$$

$$\vec{M}_A = -(7.68\text{ N}\cdot\text{m})\vec{i} + (28.8\text{ N}\cdot\text{m})\vec{j} + (28.8\text{ N}\cdot\text{m})\vec{k}$$