Static Equilibrium Of Particles And Rigid Bodies

Static Equilibrium
Procedure for analyzing static equilibrium problems
Free Body Diagrams

Static Equilibrium

- A particle or body is said to be in equilibrium if the resultant force and moment acting on it is zero. In other words, the sum of forces (or moments) **must** be equal to zero.

- For particles;

$$\vec{R} = \sum F = 0$$
$$\Rightarrow \sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

- For bodies;

$$\vec{R} = \sum F = 0$$
$$\Rightarrow \sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$
$$\vec{M} = \sum M = 0$$
$$\Rightarrow \sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

Solving Static Equilibrium Problems

- Involves three main steps;
  - ✓Sketch a free body diagram for the problem
  - ✓Sum up forces and moments to obtain the **equations of equilibrium** for the problem.
  - ✓Solve the equations and interpret your results.

Sketching Free Body Diagrams

- Select the extent of the body that is of interest, detach it from the ground and all other bodies and supports, and (basically) sketch the outline of the "free-body".

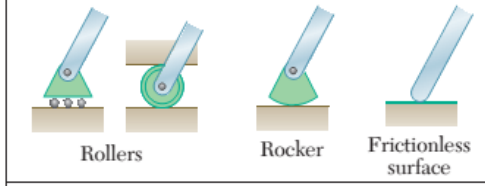
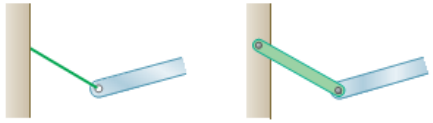
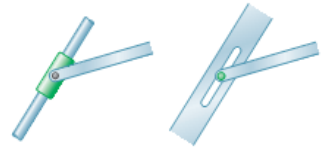
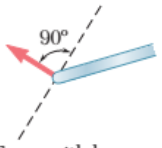
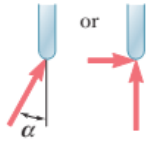
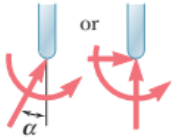
- Indicate force reactions which the ground and other supports exert on the "free-body".

- Indicate external forces and moments, including the rigid body weight where it cannot be ignored at their points of application.

- Include the required dimensions to compute the moments of the forces where necessary.

Free Body Diagrams: Support Reactions

➢Reactions at Supports and Connections for Two-Dimensional Structures

Support or Connection	Reaction	Number of Unknowns
 Rollers; Rocker; Frictionless surface	 Force with known line of action	1
 Short cable; Short link	 Force with known line of action	1
 Collar on frictionless rod; Frictionless pin in slot	 Force with known line of action	1
 Frictionless pin or hinge; Rough surface	 Force of unknown direction	2
 Fixed support	 Force and couple	3

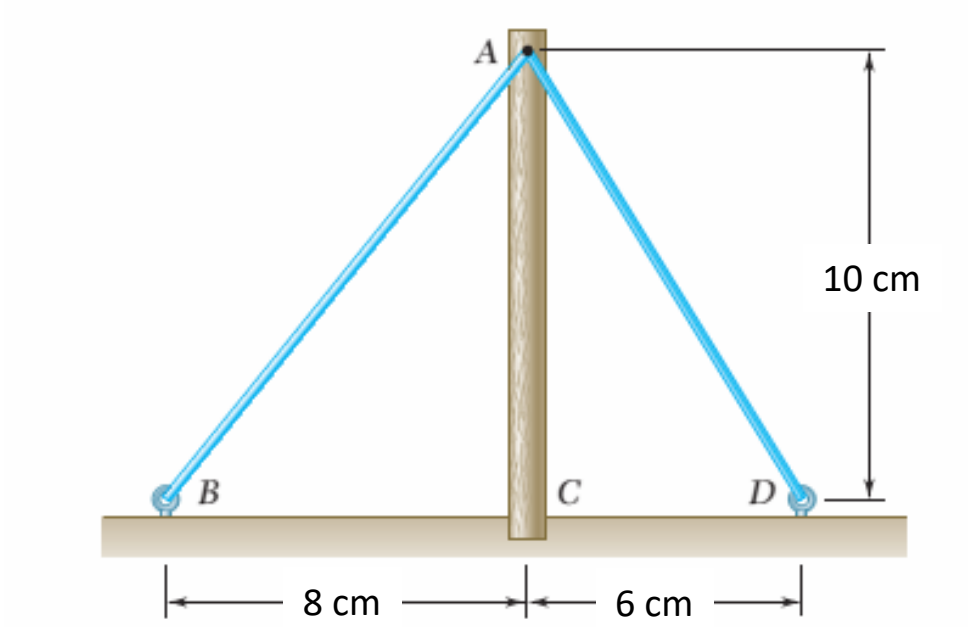
- Reactions equivalent to a force with known line of action.
- Reactions equivalent to a force of unknown direction and magnitude.
- Reactions equivalent to a force of unknown direction and magnitude and a couple of unknown magnitude

Source: Vector Mechanics for Engineers, Beer *et al.*

Free Body Diagrams

Example

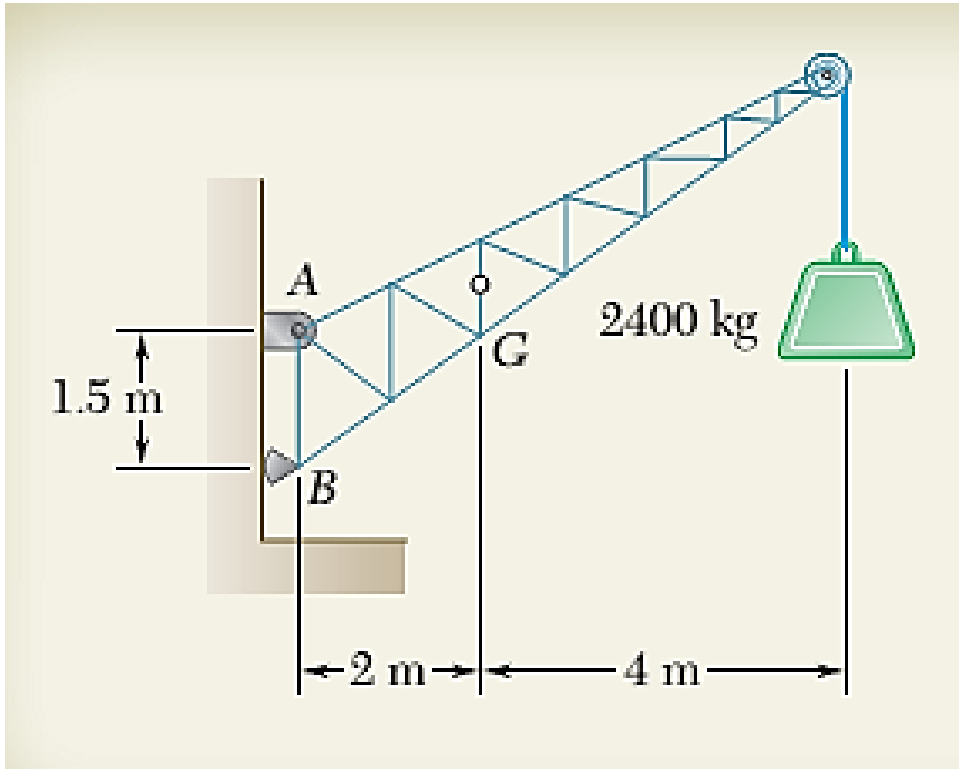
Cables AB and AD help support pole AC. Knowing that the tension is 120 N in AB and 40 N in AD, sketch the free body diagram for the pole.

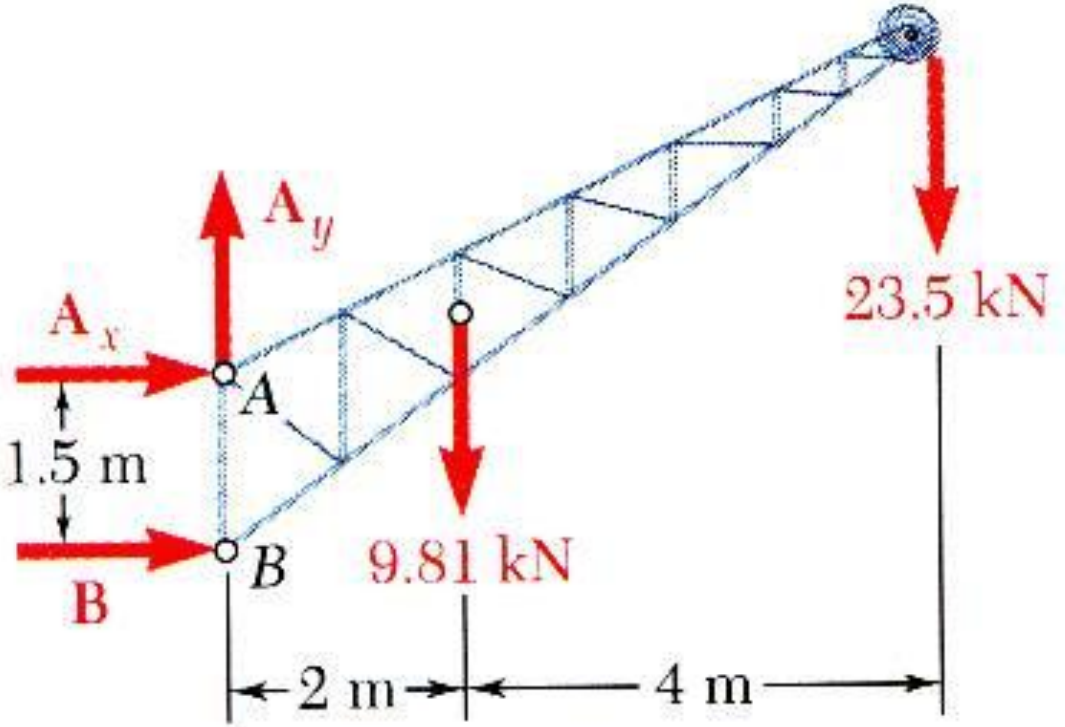


Sketching Free Body Diagrams

➢Example

A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G. Sketch the free body diagram for the crane.

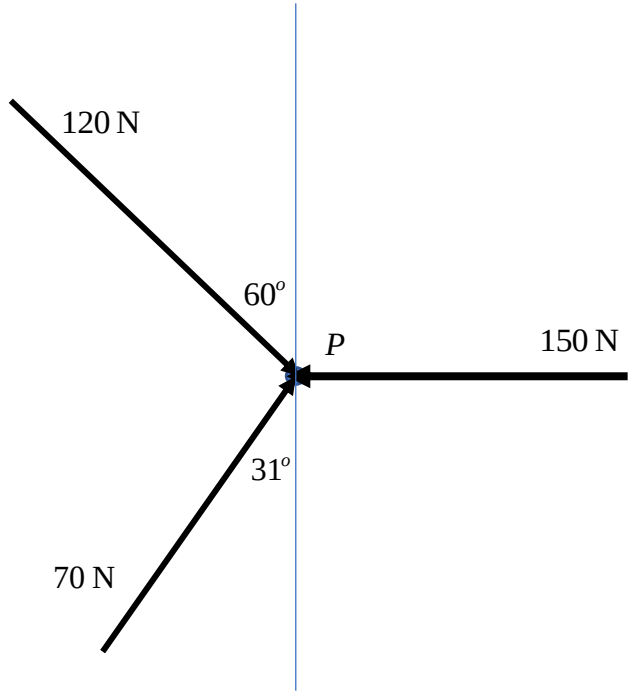




Static Equilibrium - Particles

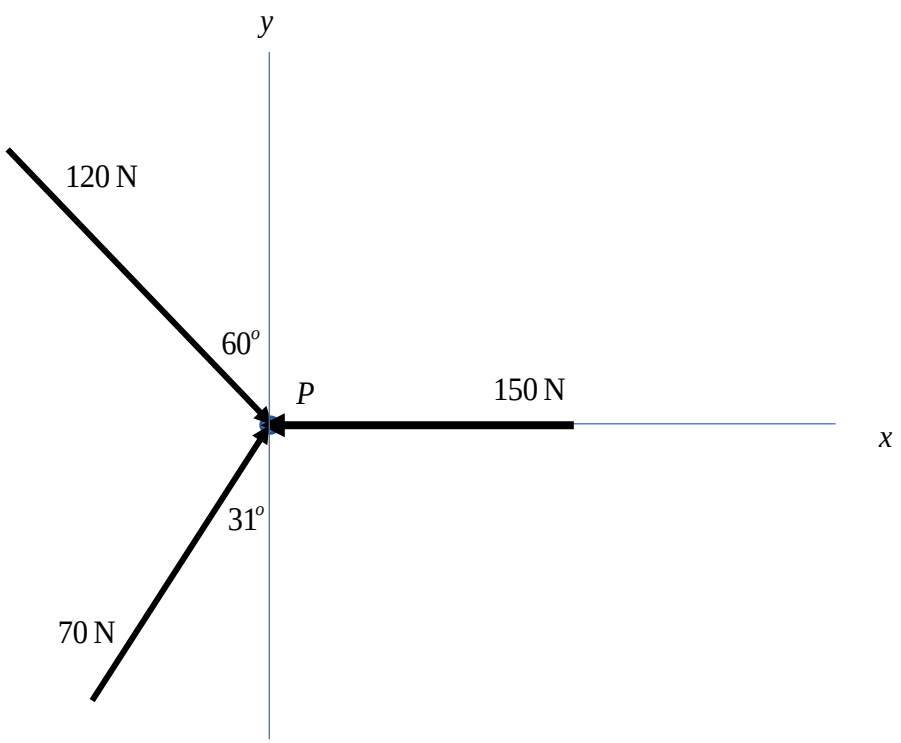
Example

Determine if the particle P is in equilibrium under the influence of the forces shown.



Static Equilibrium - Particles

➢ Solution



Equations of Equilibrium

$$\overset{+}{\rightarrow}\sum F_x = 120\sin 60^o + 70\sin 31^o - 150 = -11.02 \text{ N}$$

$$+\uparrow \sum F_y = -120\cos 60^o + 70\cos 31^o = -0.0017 \text{ N} = 0.00 \text{ N}$$

For equilibrium, $\sum F_x = 0 = \sum F_y$

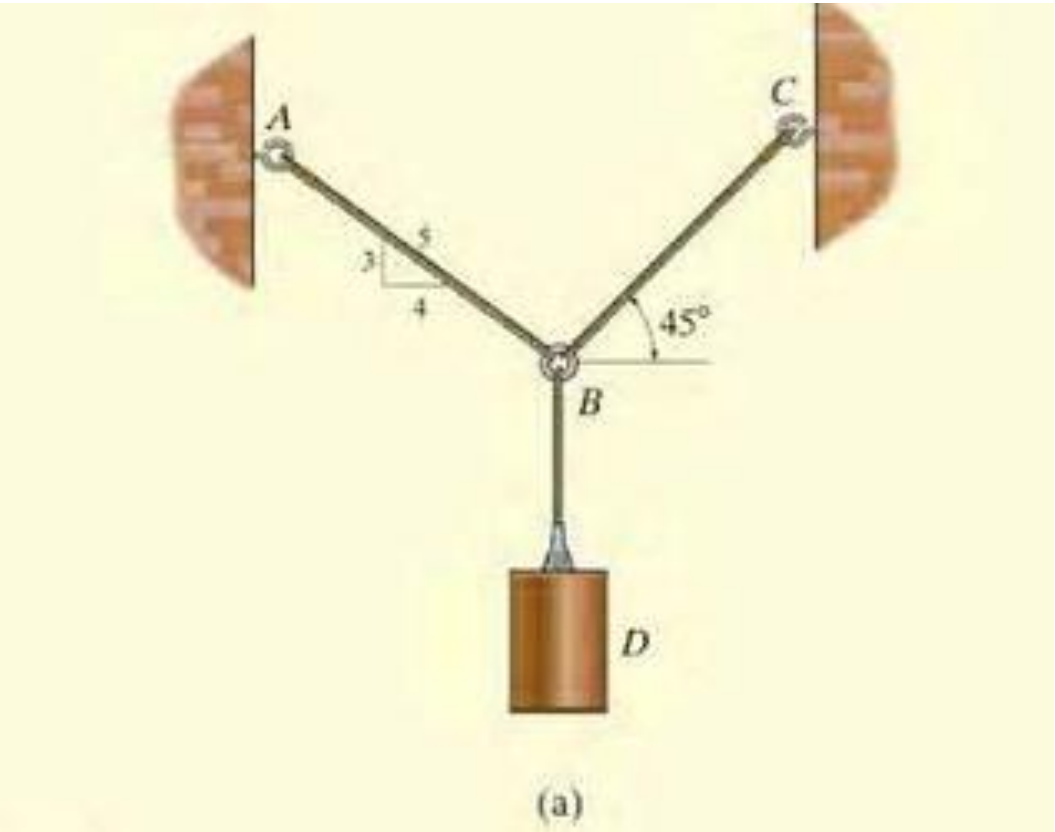
But $\sum F_x \neq 0$

Hence, P is not in equilibrium

Static Equilibrium - Particles

➢ Example

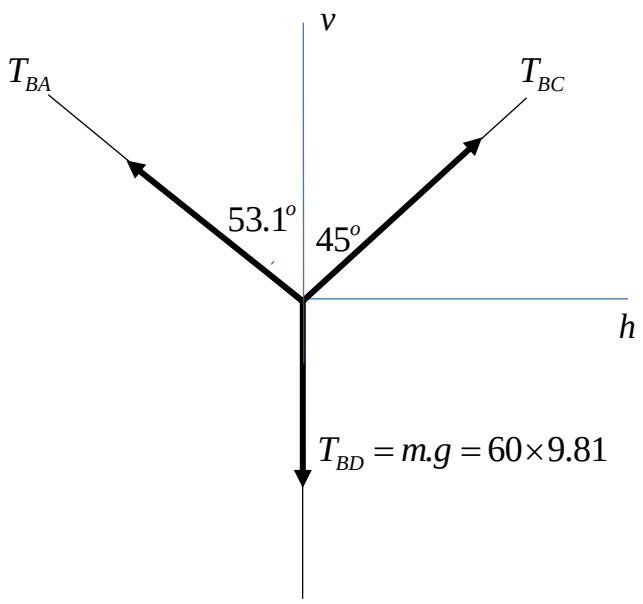
Determine the tensions required in cables BC and BA to keep the ring at B in equilibrium.



Static Equilibrium - Particles

➢ Solution

FBD



Equations of Equilibrium

$$\overset{+}{\rightarrow} \sum F_h = 0 : T_{BC} \sin 45^o - T_{BA} \sin 53.1^o = 0 \quad \text{--- (1)}$$

$$+\uparrow \sum F_v = 0 : T_{BC} \cos 45^o + T_{BA} \cos 53.1^o - T_{BD} = 0$$

$$= T_{BC} \cos 45^o + T_{BA} \cos 53.1^o = 588.6 \text{ N} \quad \text{--- (2)}$$

Solving (1) and (2) simultaneously,

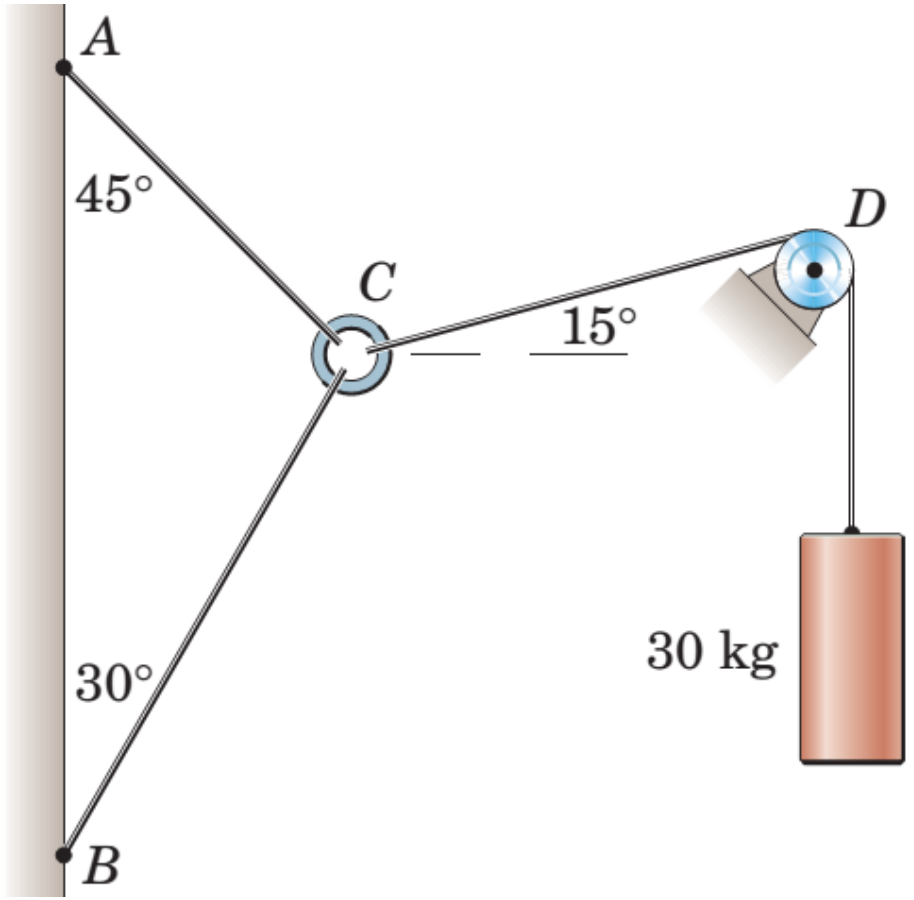
$T_{BC} = 475.41$ N

$T_{BA} = 420.43$ N

Static Equilibrium - Particles

Example

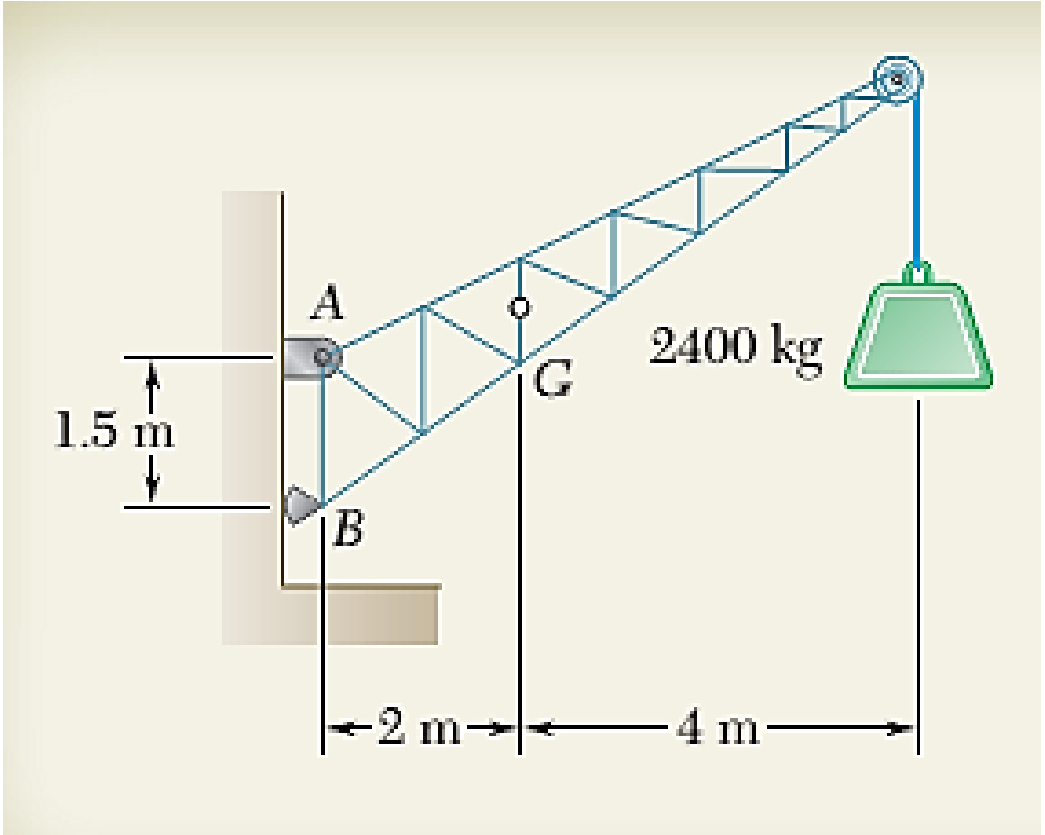
Three cables are joined at the junction ring, *C*. Determine the magnitudes of the tensions in cables *AC* and *BC* on the ring.



Static Equilibrium – Rigid Bodies

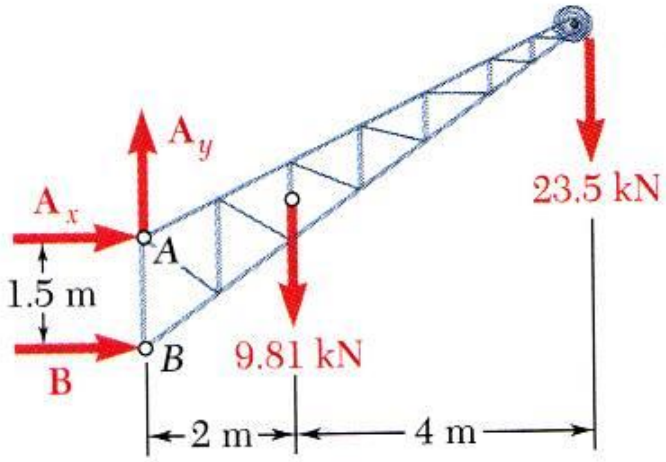
➢Example

A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G. Determine the components of the reactions at A and B.



Static Equilibrium – Rigid Bodies

➢Solution



At A,

$$\overset{+}{\rightarrow}\sum F_x = 0: \quad A_x + B = 0$$

$$+\uparrow \sum F_y = 0: \quad A_y - 9.81\,\text{kN} - 23.5\,\text{kN} = 0$$

$$A_y = +33.3\,\text{kN}$$

Taking moments about A,

$$\circlearrowleft + \sum M_A = 0: \quad +B(1.5\text{m}) - 9.81\,\text{kN}(2\text{m}) - 23.5\,\text{kN}(6\text{m}) = 0$$

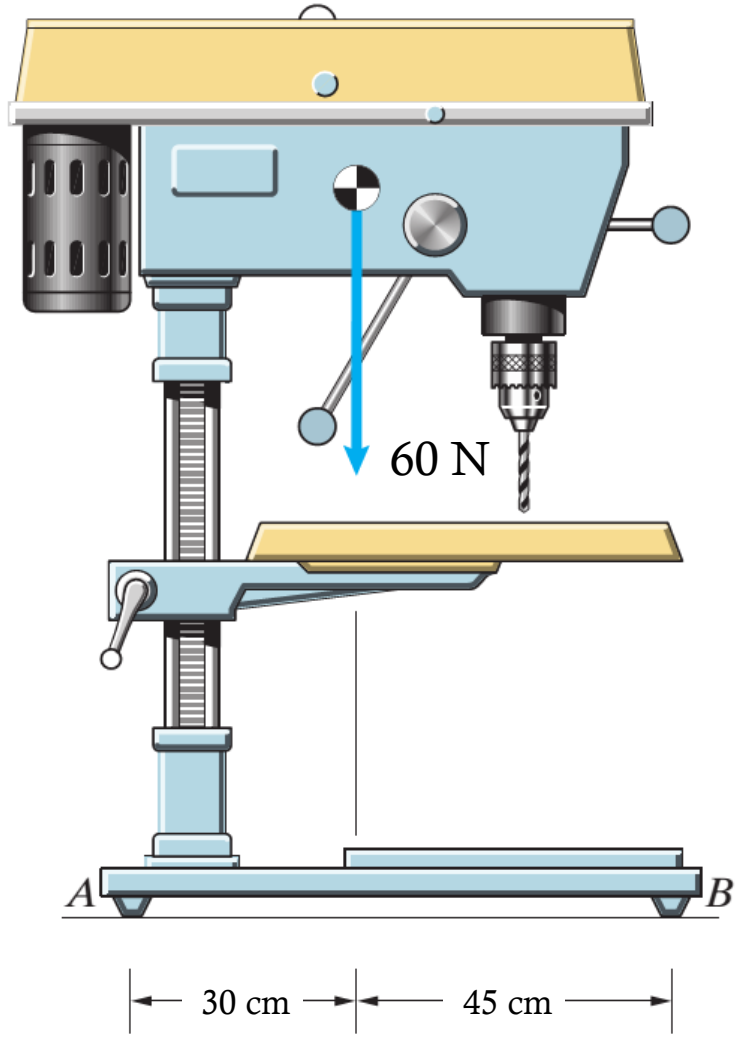
$$B = +107.1\,\text{kN}$$

$$\therefore A_x = -107.1\,\text{kN}$$

Static Equilibrium – Rigid Bodies

➢Example

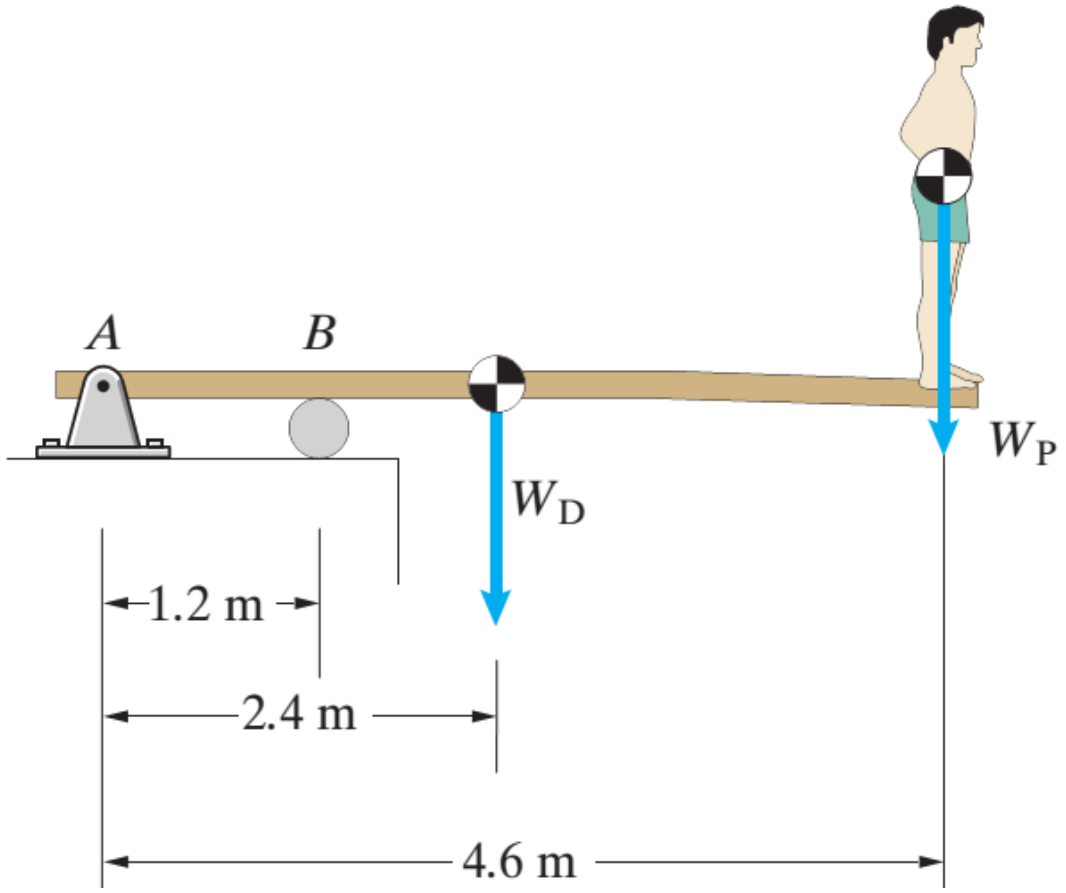
Determine the reactions at A and B. Assume that the surfaces at A and B are frictionless.



Static Equilibrium – Rigid Bodies

The masses of the man and the diving board are 54 kg and 36 kg, respectively. Assume that they are in equilibrium.

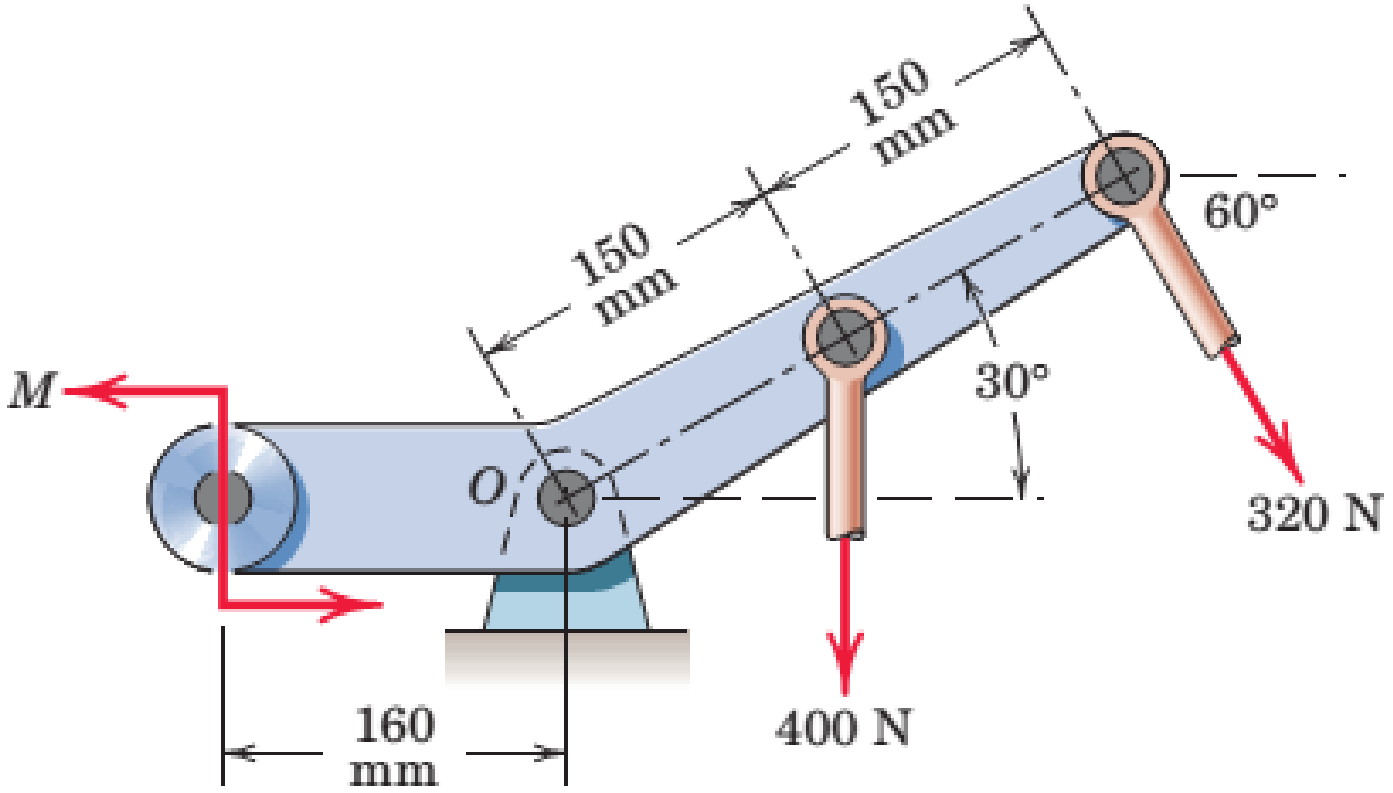
(a) Sketch the free-body diagram of the diving board.
(b) Determine the reactions at the supports A and B.



Static Equilibrium – Rigid Bodies

Example

Neglecting weight, determine *M* if the link is in equilibrium.



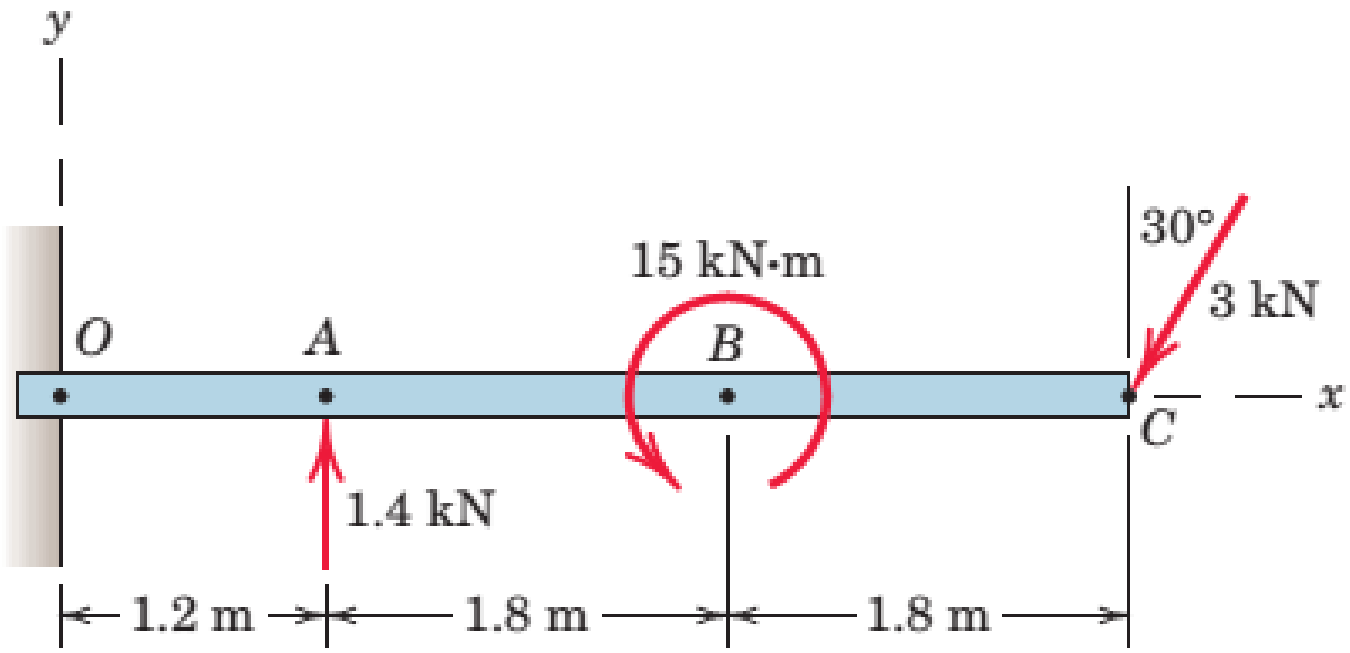
Ans:

M = 147.96 Nm

Static Equilibrium – Rigid Bodies

Example

The 500 kg uniform beam is subjected to the three external loads shown. Compute the reactions at the support point *O*.



Ans:

F_x = 1.5 kN, F_y = -1.198 kN, M_O = 0.85 kNm (clockwise)

Static Equilibrium – Rigid Bodies

Example

A 150 N crate rests on the 100 N pickup tailgate as shown. Calculate the tension T in each of the two restraining cables, one of which is shown. The centres of gravity are at G_1 and G_2. Assume the crate is located midway between the two cables.