ME 261 Dynamics of Solid Mechanics
Introduction and Unit 1

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What is Mechanics?

- Dynamics is a branch of mechanics.
- It deals with the motions of bodies under action of unbalanced forces.
- Dynamics is further divided into
  - Kinematics
  - Kinetics.

COURSE OUTLINE

- Kinematics of Particles
- Kinetics of Particles: Equation of Motion
- Kinetics of Particles: Impulse and Momentum
- Kinetics of Particles: Energy
- Kinetics of Rigid Bodies: Equation of Motion
- Kinetics of Rigid Bodies: Impulse and Momentum
- Kinetics of Rigid Bodies: Energy
- Rotary Balancing

GRADING

- Continuous assessment: 30%
  - Assignments
  - Quizzes
  - Mid-Semester
  - Attendance
- End of semester examination: 70%
  - Section A: 7 questions, answer all
  - Section B: 3 questions, answer 2

RFERENCE BOOKS

- Robert L. Norton, Design of Machinery An introduction to Synthesis and Analysis of Mechanisms and Machines, 2nd Edition, McGraw-Hill. 1999
- A. G. Erdman and G. N. Sandor , Mechanism Design, Analysis and Synthesis, Vol. I 2nd Edition, Prentice Hall, 1991.
- Joseph E. Shigley and John J. Uicker, Jr, Theory of Machines and Mechanisms, McGraw-Hill, 1981.
- Charles E. Wilson and J. Peter Sadler, Kinematics and Dynamics of Machinery, Harper Collins College Publishers.
- R. S. Khurmi & J. K. Gupta, Theory of Machines, Eurasia Publishing House (PVT.) Ltd, 2005.
- R. C. Hibbeler, Principles of Dynamics, Pearson Prentice Hall
- Pytel Kiusalaas, Engineering Mechanics Dynamics, Thomson Learning
- F. P. Beer, E. R. Russell & W. E. Cluasen, Vector Mechanics for Engineers: Dynamics

Unit 1

KINEMATICS OF PARTICLES

Idealisations in Mechanics

- Time
- Length
- Mass
- Particles
- Rigid bodies
- Continuum
- Point force
- Deformable bodies

Definitions

- **Kinematics** is the study of motion of bodies without considering the manner in which the motion is produced
- **Kinetics**, on the other hand, deals with the relationships between the forces acting on a body
- **Plane motion is** a motion of a body confined to only one plane. The plane motion may be either rectilinear or curvilinear.

Definitions

- A particle moving along a straight line is said to be in **rectilinear motion**

- When a particle has **continuous or non-changing motion** then its position, velocity, and acceleration can be described by a single continuous mathematical function along the entire path. It may be conveniently represented by an equation.

- When a particle has **erratic or changing motion** then its position, velocity, and acceleration cannot be described by a single continuous mathematical function along the entire path.

Definitions

- **Curvilinear motion** is the motion along a curved path. Such a motion, when confined to one plane, is called plane curvilinear motion.

- When all the particles of a body travel in concentric circular paths of constant radii (about the axis of rotation perpendicular to the plane of motion) such as a pulley rotating about a fixed shaft or a shaft rotating about its own axis, then the motion is said to be a plane **rotational motion.**

Kinematics Quantities

- ***Time-*** *It is a measure of duration of an even or interval between two events.*
- ***Linear Displacement-***It may be defined as the distance moved by a body with respect to a certain fixed direction. The displacement may be along a straight or a curved path
- ***Linear Velocity-***It may be defined as the rate of change of linear displacement of a body with respect to the time. Mathematically, linear velocity, v = ds/dt.
- ***Linear Acceleration-***It may be defined as the rate of change of linear velocity of a body with respect to time. It is also a vector quantity. Mathematically, linear acceleration, a=dv/dt.

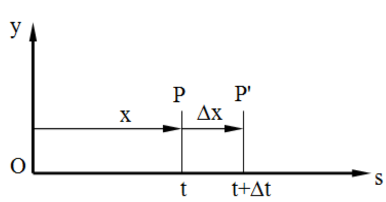
Rectilinear Motion

Consider a particle P moving along a straight-line path which, for convenience, is chosen to coincide with the x-axis

Velocity is defined as $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

Acceleration is defined as

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = v\frac{dv}{dx}$$



- SI unit of acceleration: m/s^2
- Acceleration is represented by an algebraic number which can be positive or negative.
- A positive value of acceleration a indicates that velocity is increasing, a negative value of a indicates that velocity decreasing.
- Negative acceleration is also called deceleration or retardation.

Example 1-1: Rectilinear Motion with variable Acceleration

A particle moving in a straight line assumes a position defined by the equation: $x = 6t^2 - t^3$
where t and x are expressed in seconds(s) and metres (m) respectively.
Determine the

(i) the velocity and acceleration of the particle at any time t.
(ii) the velocity and acceleration of the particle at time t= 2 s.
(iii) the instant the particle comes to rest.

Solution

(i) $\mathrm{v} = \frac{dx}{dt} = \frac{d}{dt}\left(6t^2 - t^3\right)$ $\quad \mathrm{v} = 12t - 3t^2$

$\mathrm{a} = \frac{dv}{dt} = \frac{d}{dt}\left(12t - 3t^2\right)$ $\quad \mathrm{a} = 12 - 6t$

$\mathrm{v}(\mathrm{t} = 2)) = 12t - 3t^2 = 12(2) - 3(2)^2$ v= 12 m/s

(ii) $\mathrm{a} = 12 - 6t = 12 - 6(2)$ a= 0

(iii) •At rest, v =0 $\quad \mathrm{v} = 12t - 3t^2 = 0$ $\quad$ t = 0 ad t = 4 s.

Example 1-2: Rectilinear Motion with variable Acceleration

The damping mechanism used to stop continues vibration of a vehicle's suspension consists essentially of a piston attached to the axle and moving in a cylinder filled with oil and attached to the frame. As the axle moves with an initial velocity v_0, the piston moves and oil is forced through orifices in the piston, causing the piston and the axle to decelerate at a rate proportional to their velocity; that is, $a = -kv$. Express (a) v in terms of time t, (b) piston displacement x in terms of t, (c) v in terms of x.

a)

$$a = \frac{dv}{dt} = -kv \qquad \frac{dv}{v} = -kdt$$

$$\int_{V_o}^{V} \frac{dv}{v} = \int_0^t -kdt \qquad \ln\left(\frac{v}{v_o}\right) = -kt$$

b)

$$v = \frac{dx}{dt} = v_o e^{-kt} \qquad \int_0^x dx = \int_0^t \left(v_o e^{-kt}\right) dt$$

$$x = -\frac{v_o}{k}\left[e^{-kt}\right]_0^t \qquad x = \frac{v_o}{k}\left(1 - e^{-kt}\right)$$

c)

$$v = v_o e^{-kt} \qquad a = v\frac{dv}{dx} = -kv$$

$$\int_{v_o}^{v} v\frac{dv}{v} = \int_0^x -kdx \qquad v = v_o - kx$$

Constant Acceleration

- Let a particle with initial velocity *u* move a distance *s* in time *t* to attain a final velocity *v*. Let the initial time be t=0. From

- Integrating and inserting the initial values, we have

$$a = \frac{dv}{dt}$$

$$\int_u^v dv = \int_0^t adt \Rightarrow v = u + at \ldots\ldots 1$$

Also from

$$v = \frac{dx}{dt} = u + at$$

$$\int_0^s dx = \int_0^t (u + at)dt \Rightarrow s = ut + \frac{1}{2}at^2$$

$$x = ut + \frac{1}{2}at^2 \ldots\ldots 2$$

Constant acceleration

Again, from

$$a = v\frac{dv}{dx}$$

$$\int_0^x adx = \int_u^v vdv \Rightarrow v^2 = u^2 + 2ax \ldots\ldots 3$$

Combining 1 and 3 gives

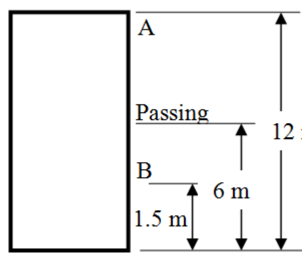
$$(v - u)(v + u) = 2ax$$

$$at(v + u) = 2ax$$

$$x = \frac{(v + u)}{2}t \ldots\ldots 4$$

Example 1-3: Rectilinear Motion with constant Acceleration

Ball A is released from rest at a height of 12 m and after 0.2 s ball B is thrown upward 1.5 m from the ground. The balls pass one another at a height of 6 m. Determine the speed at which ball B was thrown upward.



Solution

First we calculate the time required for ball A to drop 6 m

$$s_A = u_A t + \frac{1}{2} a_A t^2 \qquad -6 = (0)t + \frac{1}{2}(-9.81)t^2 \qquad t = 1.106\,\mathrm{s}$$

Ball B

It is throw upward from a height of 1.5 m above ground and travels (6-1.5) m to meet ball A.
It must reach a height of 6 m at time ball A reaches this height minus the elapse time of 0.2 s.

$$s_B = u_B t + \frac{1}{2} a_B t^2 \qquad (6-1.5) = u_B(1.106-0.2) + \frac{1}{2}(-9.81)(1.106-0.2)^2 \qquad u_B = 9.41\,\mathrm{m/s}$$

Example 1-4: Rectilinear Motion with constant Acceleration

When two cars A and B are next to one another, they are traveling in the same direction with speeds u_A and u_B respectively. If B maintains its constant speed, while A begins to decelerate at the rate a_A, determine the distance d between the cars at the instant A stops.

Solution

Motion of car A:

$$v_A = 0 = u_A + (-a_A)t \quad \Rightarrow t = \frac{u_A}{a_A} \quad s_A = u_A t + \frac{1}{2} a_A t^2 = u_A\left(\frac{u_A}{a_A}\right) + \frac{1}{2}(-a_A)\left(\frac{u_A}{a_A}\right)^2 \quad s_A = \frac{1}{2}\frac{u_A^{\ 2}}{a_A}$$

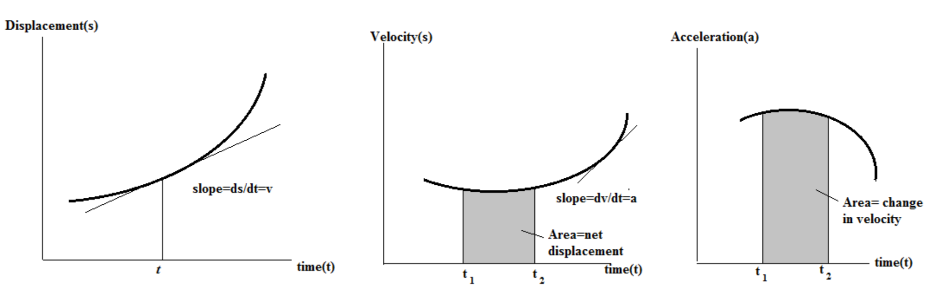
Motion of car B:

$$a_B = 0 \quad s_B = u_B t = u_B\left(\frac{u_A}{a_A}\right) \quad d = s_B - s_A = u_B\left(\frac{u_A}{a_A}\right) - \frac{1}{2}\frac{u_A^{\ 2}}{a_A} \quad d = \frac{(2u_B - u_A)u_A}{2a_A}$$

Variable Acceleration and Erratic Motion

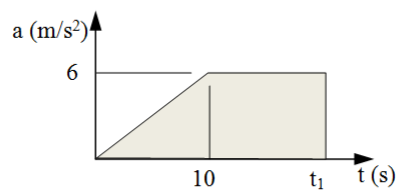
- When the acceleration is variable, no simple equations can be written from which the state of motion of the particle can be determined readily. However, if there is a simple mathematical expression for the displacement, velocity or the acceleration the other quantities can be determined by calculus.
- In problems where the motion is non-uniform and where no algebraic or trigonometric expressions are given for displacement, velocity or acceleration, graphical solutions are called for

Graphs of Displacement, Velocity and Acceleration vrs. time



Example 1-5: Erratic Motion

A car starting from rest moves along a straight track with acceleration as shown in Figure E1-5. Determine the time t_1 for the car to reach speed 50 m/s.



Solution

Velocity, v is given by area under the curve.

$$v = 50 = \frac{1}{2}(10)6 + (t - 10)6 \qquad t = 13.33\,\text{s}$$

Solution

Distance Travelled

Since v = f(t), the position as a function of time may be found by integrating v = ds/ dt with t = 0, s = 0.

$$s = \int v dt = \int (3t^2 - 6t + 10) dt \qquad s = t^3 - 3t^2 + 10t + A$$

Now, applying initial condition (at t=0, s=0); gives A=0

Therefore the distance covered at any given time is given by,

$$s = t^3 - 3t^2 + 10t$$

Acceleration: a=dv/dt, the acceleration as a function of time may be found by differentiating the velocity function with respect to time

And at t= 3s, the distance covered is,

$$s = (3)^3 - 3(3)^2 + 10(3) = 30\ m$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 6t + 10t) = 6t - 6$$

At time= 3 the acceleration is given by 6(3)-6= 12 m/s^2

Curvilinear Motion in Cartesian Coordinates

- Curvilinear motion is motion along a curved path.
- It consists of linear and rotational motion. For a curvilinear motion the velocity and acceleration of the particle can, in general, vary in both magnitude and direction. It can be shown that the velocity is always tangent to the path along which the particle moves.
- Curvilinear motion of particles can be described in terms of several different coordinate systems. Here, we will use Cartesian coordinates and show that curvilinear motion can be considered as the vector sum of three simultaneous rectilinear motions along the x, y and z coordinate axes.

Curvilinear Motion: Rectangular Components

The position vector of a particle can be written as

$$\boldsymbol{r}=x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k}$$

If the particle moves from an initial position $\boldsymbol{r}_i$ to a final position $\boldsymbol{r}_f$, the displacement of the particle during the time interval is $\Delta\boldsymbol{r}=\boldsymbol{r}_f-\boldsymbol{r}_i$

The velocity of the particle is

$$\boldsymbol{v}=\frac{d\boldsymbol{r}}{dt}=\dot{x}\boldsymbol{i}+\dot{y}\boldsymbol{j}+\dot{z} \qquad v=v_x i+v_y j+v_k k$$

At any instant the magnitude of v is defined as

$$v=\sqrt{v_x^{\,2}+v_y^{\,2}+v_z^{\,2}}$$

The acceleration is define as

$$\boldsymbol{a}=\frac{d^2\boldsymbol{r}}{dt^2}=\ddot{x}\,\boldsymbol{i}+\ddot{y}\boldsymbol{j}+\ddot{z}\boldsymbol{k} \qquad a=a_x i+a_y j+a_k k$$

At any instant the magnitude of a is defined as

$$a=\sqrt{a_x^{\,2}+a_y^{\,2}+a_z^{\,2}}$$

Example 1-8: Curvilinear Motion

The position vector of a particle is

$$\boldsymbol{r} = 3t^2\boldsymbol{i} + (t - 4t^2)\boldsymbol{j}$$

where ***r*** is measured in metres and the time is in seconds,

(a) Determine the displacement of the particle during the time interval t=1 s and t=3 s.

(b) Find the acceleration of the particle at time t = 3s.

Solution

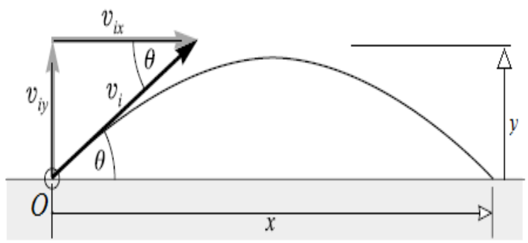
At t = 1 s, $\boldsymbol{r}_i = 3(1)^2\boldsymbol{i} + (1 - 4 \times 1^2)\boldsymbol{j} = 3\boldsymbol{i} - 3\boldsymbol{j}$

At t = 3 s, $\boldsymbol{r}_f = 3(3)^2\boldsymbol{i} + (3 - 4 \times 3^2)\boldsymbol{j} = 27\boldsymbol{i} - 33\boldsymbol{j}$

Therefore, the displacement during the interval is
$\Delta\boldsymbol{r} = \boldsymbol{r}_f - \boldsymbol{r}_i = (27\mathbf{i}-33\mathbf{j})-(3\mathbf{i}-3\mathbf{j}) = 24\mathbf{i}-30\mathbf{j}$
Differentiating the position vector, we have
$\boldsymbol{v} = 6t\,\mathbf{i} + (1 - 8t)\,\mathbf{j}$ and $\boldsymbol{a} = 6\boldsymbol{i} - 8\boldsymbol{j}$
Therefore, at t = 3 s,
$a = 6\mathbf{i} - 8\mathbf{j}$

Projectile Motion

- Motion of projectiles is a curvilinear motion.
- The free-flight motion of a projectile is often studied in terms of its rectangular components.
- Consider a projectile launched at point with an initial velocity of v_i, having components $(v_i)_x$ and $(v_i)_y$. When air resistance is neglected, the only force acting on the projectile is its weight, which causes the projectile to have a constant downward acceleration of approximately a= g = 9.81 m/s^2.



Projectile Motion

Horizontal Motion (to the right): $a = a_x = 0,\ s = x,\ u_x = v_i \cos\theta$

$$v = u + at; \qquad v = u = v_i \cos\theta$$

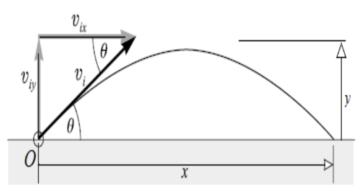
$$s = ut + \frac{1}{2}at^2 \qquad s = x = v_i t \cos\theta$$

Vertical Motion (upwards): $a_y = g = -9.81\ m/s^2, s = y, u_y = v_i \sin\theta$

$$v = u + at; \qquad v = v_i \cos\theta - gt$$

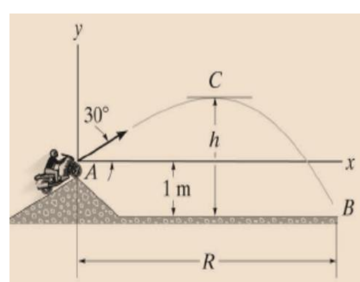
$$s = ut + \frac{1}{2}at^2 \qquad s = y = v_i t \sin\theta - \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2as \qquad v^2 = (v_i \sin\theta)^2$$



Example 1-9: Projectile

The track for a racing event was designed so that riders jump off the slope at 30°, from a height of 1 m. During a race it was observed that the rider remained in mid-air for 1.5s. Determine the speed at which he was traveling off the ramp, the horizontal distance he travels before striking the ground, and the maximum height he attains. Neglect the size of the bike and rider.



Solution

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Vertical Motion;
Initial velocity of rider= v_o
$u_y = v_o \sin 30°$; $a = -g = -9.81\ m/s^2$; $s = -1\ m$; $t = 1.5\ s$
Therefore using

$$s = ut + \frac{1}{2}at^2 \qquad -1 = 0.5v_o(1.5) - 0.5(9.81)(1.5)^2 \qquad \underline{\underline{v_o = 13.38\ m/s}}$$

Horizontal Motion;
$u_x = v_o \cos 30° = 11.59\ m/s$; $a = 0$; $s = R\ m$; $t = 1.5\ s$

$$using \quad v^2 = u^2 + 2as; \qquad 0^2 = 6.69^2 - 2(9.81)(h-1)$$

$\underline{h=3.28\ m}$

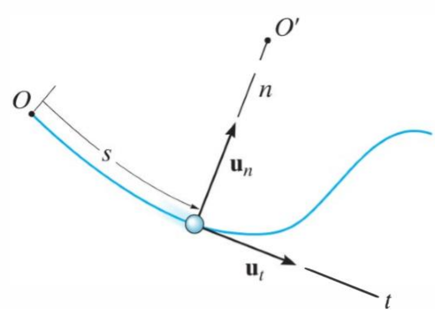
Maximum height obtained
@$v=0$, $s=h-1$; $u_y = v_o \sin 30° = 6.69\ m/s$; $a = -g = -9.81\ m/s^2$; $t=1.5\ s$

$$Again\ using \quad s = ut + \frac{1}{2}at^2; \qquad R = 11.59(1.5) = 17.38\ m$$

$\underline{R=17.4\ m}$

Curvilinear Motion: Normal and Tangential Components

When the path along which a particle travels is known, then it is often convenient to describe the motion using n and t coordinate axes which act normal and tangent to the path, respectively, and at the instant considered have their origin located at the particle.



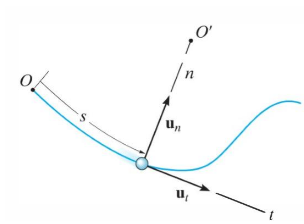
Curvilinear Motion: Normal and Tangential Components

Velocity

Since the particle movement, s is a function of time, the particle's velocity v has a direction that is always tangent to the path, and a magnitude determined by taking the time derivative of the path function s, i.e v = ds/dt

$v = Vu_t$ Where $V = \frac{ds}{dt}$

Hence the velocity has only tangential component.

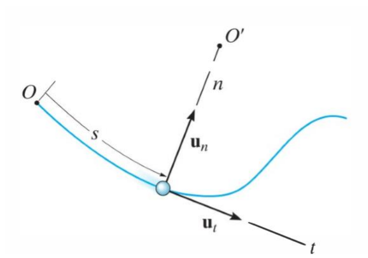


Curvilinear Motion: Normal and Tangential Components

Acceleration

The acceleration of the particle is the time rate of change of the velocity, given by, using the product rule

$$a = \frac{d}{dt}(v) = \frac{d}{dt}(Vu_t) \quad a = \dot{V}u_t + V\dot{u}_t \quad (1)$$



It can be shown that the derivative

$$\dot{u}_t = \dot{\theta}u_n = \frac{\dot{s}}{\rho}u_n = \frac{V}{\rho}u_n \quad (2)$$

$$\dot{u}_t = \dot{\theta}u_n = \frac{\dot{s}}{\rho}u_n = \frac{V}{\rho}u_n \quad (3)$$

Substituting (2) and (3) into (1) and simplifying, we have

$$a = \dot{V}u_t + \frac{V^2}{\rho}u_n$$

$$a = a_t u_t + a_n u_n$$

Magnitude is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}}$$

Example 1-10: Curvilinear Motion

A car travels around the horizontal circular track that has a radius of 95 m. If the car increases its speed at a constant rate of 2.3 m/s², starting from rest, determine the time needed for it to reach an acceleration of 2.8 m/s². Find the corresponding speed of the car at this instant.

Solution

$$a_t = 2.3\,\text{m/s}^2 \qquad v = u + a_t t = 0 + (2.3)\text{t} \qquad v = 2.3t$$

$$a_n = \frac{v^2}{\rho} = \frac{(2.3t)^2}{95} \qquad a_n = 0.05568t^2$$

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{(2.3t)^2 + \left(0.05568t^2\right)^2} = 2.8$$

$$(2.3t)^2 + \left(0.05568t^2\right)^2 = 2.8^2$$

Solving the above equation,

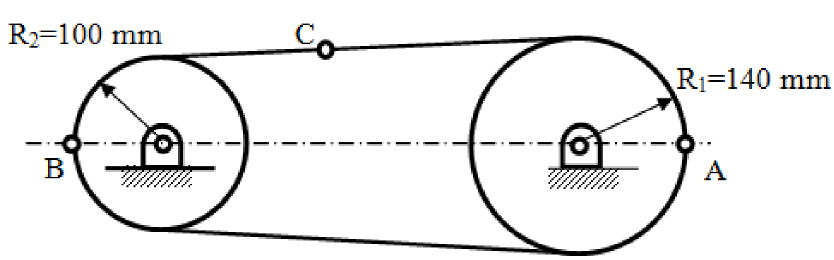
$$t^2 = -1706.6 \qquad t^2 = 0.5291$$

Disregarding the negative answer, we have

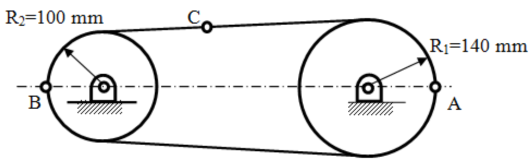
$$t = \sqrt{0.5291} \qquad t = 0.7274\,\text{s}$$

Example 1-11: Curvilinear Motion

The flexible belt shown runs around two pulleys of different radii. At the instant shown point C on the belt has a velocity of 4 m/s and acceleration of 40 m/s² in the direction in indicated in the figure. Calculate the magnitudes of the acceleration of points A and B on the belt at this instant.



Example 1-11: Curvilinear Motion



Solution

Assuming that the belt does not stretch, then

Every point of the belt has the same speed, $v_A=v_B=v_C = 4$ m/s

The rate of change of speed (dv/dt) of every point on the belt is the same, i.e. $(a_A)_t=(a_B)_t=(a_C)_t= 40$ m/s^2

$$(a_A)_n = \frac{v_A^2}{R_1} = \frac{(4)^2}{0.14} = 114.3\ \text{m/s}^2$$

$$a_A = \sqrt{(a_A)_n^2 + (a_A)_t^2} = \sqrt{114.3^2 + 60^2}$$

$$a_A = 129\ \text{m/s}^2$$

$$(a_B)_n = \frac{v_B^2}{R_2} = \frac{(4)^2}{0.1} = 160\ \text{m/s}^2$$

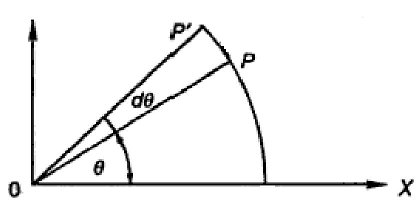
$$a_B = \sqrt{(a_B)_n^2 + (a_B)_t^2} = \sqrt{160^2 + 60^2}$$

$$a_B = 170.9\ \text{m/s}^2$$

Rotational Motion

Pure Rotational Motion

The figure below shows a line OP which rotates in a plane about point O. Let the point P move from position P to P' in a time interval Δt, the change in angular position (measured relative to the x-axis)



The instantaneous angular velocity,ω, is defined as the time rate of change of the angular position of the line OP, and is given by $\omega = \frac{d\theta}{dt} = \dot{\theta}$

Angular velocity is measured in radians per second.

The instantaneous angular acceleration is defined as the time rate of change of the angular velocity, and is given by

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

Rotational Motion

Let the line OP rotate through angle θ radians in time t seconds with initial and final velocities equal to ω_i and ω_f respectively. Then the following expressions can be derived:

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f{}^2 = \omega_i{}^2 + 2\alpha t$$

$$\theta = \omega t + \frac{1}{2}\alpha t^2$$

$$\theta = \left(\frac{\omega_i + \omega_f}{t}\right) t$$

Equations Governing Rectilinear and Rotational Motions

Parameter	Rectilinear (Straight-Line) Motion	Rotational (Angular) Motion
Time	t	t
Velocity	$v = \frac{ds}{dt}$	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$	$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
Displacement	s	θ
Constant Acceleration		
Velocity	$v = u + at$	$\omega_f = \omega_i + \alpha t$
Velocity	$v^2 = u^2 + 2as$	$\omega_f{}^2 = \omega_i{}^2 + 2\alpha\theta$
Displacement	$s = ut + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$
Displacement	$s = \left(\frac{u+v}{2}\right)t$	$\theta = \left(\frac{\omega_i + \omega_f}{2}\right)t$

Relationship between Linear and Rotational Motions

There is relationship between rotational motion and linear motion. Thus, a rotational motion may lead to linear motion, and vice versa. The relationships

s=rθ

v=rω

a=rα

Example 1-12: Rotational Motion

The rotor of a jet engine is rotating at 10 000 rpm when the fuel is shut off. The ensuing angular acceleration is α = -0.02ω, where ω is angular velocity in rad/s. Determine (a) the time it takes for the rotor to slow down to 1000 rpm, (b) the corresponding number of revolutions the rotor turns while decelerating to 1000 rpm.

$\omega = (2\pi/60) \times 10\,000 = 1000\,\pi/3$

(a) $\alpha = \dfrac{d\omega}{dt} = -0.02\omega$ $\quad \displaystyle\int_{1000\pi/3}^{100\pi/3} \frac{d\omega}{\omega} = \int_0^t -0.02dt$ $\quad \underline{\underline{t = 115\,\text{s}}}$

(b) $\alpha = \dfrac{d\omega}{dt} = \dfrac{d\omega}{d\theta}\dfrac{d\theta}{dt} = \omega\dfrac{d\omega}{d\theta} = -0.02\omega$ $\quad d\omega = -0.02d\theta$

$\displaystyle\int_{1000\pi/3}^{100\pi/3} d\omega = \int_0^\theta -0.02d\theta$ $\quad \theta = 15000\pi\ \text{rad}$ $\quad \underline{\underline{\theta = 7500\,\text{rev}}}$

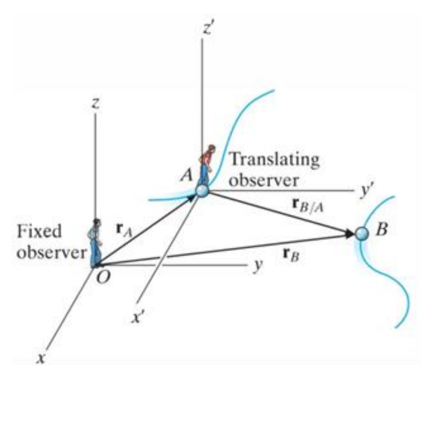
Relative Motion

- There are many motions where the path of motion for a particle is complicated.
- Such motions may be simplified by using two or more frames of reference.
- Such motions are considered as relative motion.
- Relative-motion analysis of particles using rotating frames of reference will be treated in under ***kinematics of rigid bodies***.

Relative Motion

Position

Consider particles A and B, which move along the arbitrary paths. The absolute position of each particle, r_A and r_B, is measured from the common origin O of the fixed x, y, z reference frame. Using vector addition, the position of B is related to position of A by the equation



$$r_B = r_A + r_{B/A}$$

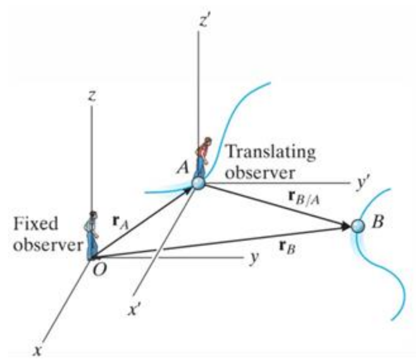
where $r_{B/A}$ is the position of B relative to A.

Relative Motion

Velocity

The velocity relation of points A and B is derived from the derivative of the position equation, which gives

$$v_B = v_A + v_{B/A}$$

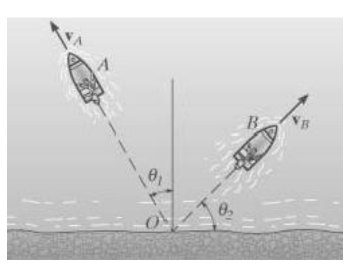


The acceleration relation of points A and B is derived from the derivative of the velocity equation , which gives

$$a_B = a_A + a_{B/A}$$

Example 1-13: Relative Motion

Two boats leave the shore at the same time and travel in the directions shown Figure E1-12 with given velocities. Using the data θ_1=45º, θ_2=30º, v_A=6 m/s, v_B=4.5 m/s. (a) Determine the speed of boat A with respect to boat B. (b) How long after leaving the shore will the boats be at a distance d = d=300 m, apart. Assume constant velocities.



Solution

Solution

(a) $v_A = V_A \begin{pmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{pmatrix} = 6 \begin{pmatrix} -\sin(45) \\ \cos(45) \end{pmatrix}$ $\qquad v_A = \begin{pmatrix} -4.2426 \\ 4.2426 \end{pmatrix}$

$v_B = V_B \begin{pmatrix} \cos(\theta_2) \\ \sin(\theta_2) \end{pmatrix} = 4.5 \begin{pmatrix} \cos(30) \\ \sin(30) \end{pmatrix}$ $\qquad v_B = \begin{pmatrix} 3.8971 \\ 2.25 \end{pmatrix}$

$v_{A/B} = v_A - a_B = \begin{pmatrix} -4.2426 \\ 4.2426 \end{pmatrix} - \begin{pmatrix} 3.8971 \\ 2.25 \end{pmatrix}$ $\qquad v_{A/B} = \begin{pmatrix} -8.1398 \\ 1.9926 \end{pmatrix}$

$|v_{A/B}| = \sqrt{(-8.1398)^2 + (1.9926)^2}$ $\qquad \underline{\underline{|v_{A/B}| = 8.38 \text{ m/s}}}$

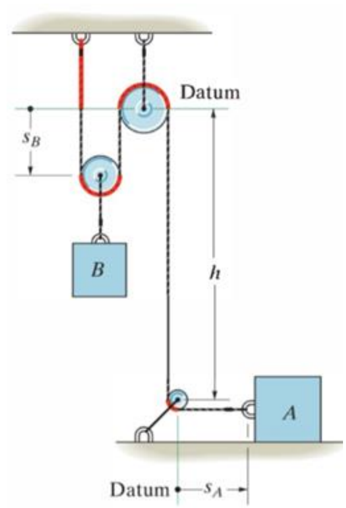
(b) $t = \dfrac{d}{|v_{A/B}|} = \dfrac{300}{8.38}$ $\qquad \underline{\underline{t = 35.8 \text{ s}}}$

Dependent Motion Analysis of Two Particles

- In some types of problems the motion of one particle will depend on the corresponding motion of another particle.

- This dependency commonly occurs if the particles, here represented by blocks, are interconnected by inextensible cords which are wrapped around pulleys.

Dependent Motion Analysis of Two Particles

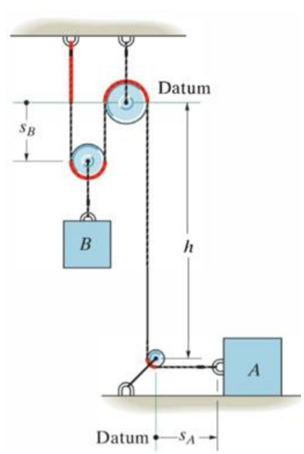


Consider the pulley system shown in Figure. The position of block A is specified by SA, and the position of the end of the cord from which block B is suspended is defined by SB.

We select coordinates such that:

a) their origin at fixed points or datums,
b) are measured in the direction of motion of each block,
c) are positive to the right for SA and positive downward for SB.

Dependent Motion Analysis of Two Particles



During the motion, the length of the darkened segments of the cord in the figure remains constant.

If L represents the total length of cord and LD represents each darkened segment, then the position coordinates can be related by the equation

$$2s_B + h + S_A + 4L_D = L$$

Taking time derivative of the above expression, realizing that LD, h and L remain constant, while S_A and S_B measure the segments of the cord that change in length. We have

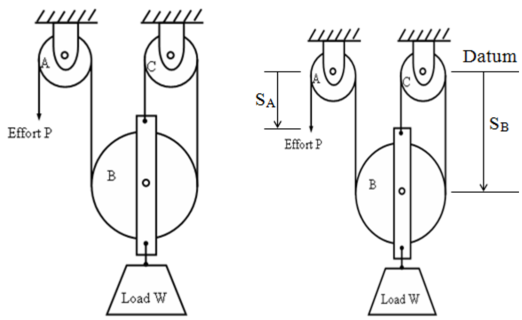
$$v_B = -\frac{v_A}{2}$$

In a similar manner, time differentiation of the velocities yields the relation between the accelerations, i.e.,

$$a_B = -\frac{a_A}{2}$$

Example 1-14: Dependent Motion

In the pulley system shown, the effort P is moving downward at 6 m/s, increasing at 3 m/s^2. Determine the speed and acceleration of the load W.



Solution

From the figure

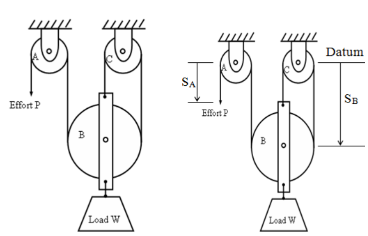
$$s_A + 3S_B = L$$

Taking time derivative of the above expression, realizing that L remains constant, we have

$$v_A + 3v_B = 0 \qquad v_B = -\frac{v_A}{3} = -\frac{(-6)}{3} \qquad \underline{\underline{v_B = 2\text{ m/s}\uparrow}}$$

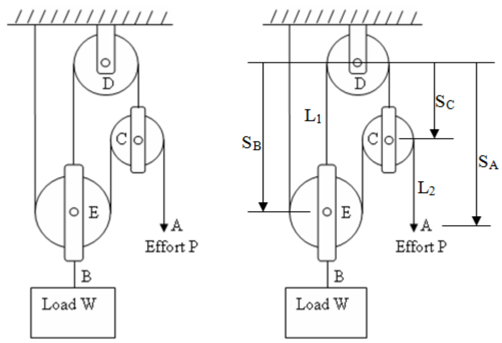
Taking time derivative of the velocity expression, we have

$$a_A + 3a_B = 0 \qquad a_B = -\frac{a_A}{3} = -\frac{(-3)}{3} \qquad \underline{\underline{a_B = 1\text{ m/s}^2\uparrow}}$$



Example 1-15: Dependent Motion

The figure shows a pulley system used to lift load. If the cable at A is pulled downward at a speed of 8 m/s increase at 2 m/s^2, find the speed and acceleration of the load at B.



Solution

$$s_B + S_C = L_1 \qquad (1)$$

$$s_B + (s_B - S_C) + (s_A - S_C) = L_2 \qquad \text{or} \qquad 2s_B + s_A - 2s_C = L_2 \quad (2)$$

Substituting for S_C in (2) using (1), and simplifying we have

$$s_A + 4s_B = L_2 + 2L_1$$

Taking time derivative of the above expression, realizing that both L_1 and L_2 remain constant, we have

$$v_A + 4v_B = 0 \qquad v_B = -\frac{v_A}{4} = -\frac{(-8)}{4} \qquad \underline{\underline{v_B = 2\ \text{m/s}\uparrow}}$$

Taking time derivative of the velocity expression, we have

$$a_A + 4a_B = 0 \qquad a_B = -\frac{a_A}{4} = -\frac{(-2)}{4} \qquad \underline{\underline{a_B = 0.5\ \text{m/s}^2\uparrow}}$$