KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

INSTITUTE OF DISTANCE LEARNING

(BSC MECHANICAL ENGINEERING, TOP-UP, YEAR 3)

ME 366: HEAT TRANSFER

Credit: 3

F. K. FORSON

Publisher Information

For any information contact:

Dean
Institute of Distance Learning
New Library Building
Kwame Nkrumah University of Science and Technology
Kumasi, Ghana

Phone: +233-03220-60013
+233-03220-61287
+233-03220-60023

Fax: +233-03220-60014

E-mail: idldean@kvcit.org
idloffice@kvcit.org
kvcit@idl-knust.edu.gh
kvcitavu@yahoo.ca

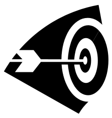
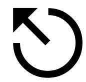
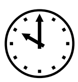
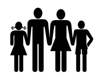
Web: www.idl-knust.edu.gh
www.kvcit.org

ISBN:

Editors:

Publisher notes to the Learners:

1. Icons: - the following icons have been used to give readers a quick access to where similar information may be found in the text of this course material. Writer may use them as and when necessary in their writing. Facilitator and learners should take note of them.

Icon #1	Icon #2	Icon #3	Icon #4	Icon #5
				
Learning Objective	Activity	Assignments	Information	Summary
Icon #6	Icon #7	Icon #8	Icon #9	Icon #10
				
Time For Activity	Self Assessment	Group Discussion	Read	New Terms
Icon #11	Icon #12	Icon #13	Icon #14	Icon #15
				
Well Done	Note/Learning Tip	Pause	Question	Online

2. Guidelines for making use of learning support (virtual classroom, etc.)

This course material is also available online at the virtual classroom (v-classroom) Learning Management System. You may access it at www.kvcit.org

Course Writer

F. K. FORSON received his BSc. and MSc. degrees in Mechanical Engineering in 1983 and 1994, respectively, specialising in Thermo-fluids and Energy Systems Engineering from the University of Science and Technology (UST), now Kwame Nkrumah University of Science and Technology (KNUST), Kumasi-Ghana. He received his PhD degree in 1999 from De MontFort University, Leicester, United Kingdom.

He worked with the Department of Mechanical Engineering of UST from November 01, 1982 to September 30, 1984 as a National Service Person. He then joined the department as an academic staff in the capacity as an Assistant Lecturer in October 01, 1984.

Prof. F. K. Forson has served KNUST in a couple of academic and other responsibilities as listed below:

- Feb. 01, 2005 to September 30, 2006; Head, Department of Mechanical Engineering
- Oct. 01, 2006 to Sept. 30, 2007; Dean, Faculty of Mechanical and Agricultural Engineering
- Jan. 01, 2006 to Dec. 31, 2007; University Council Member as Convocation Non-Professorial Representative

He has published several Technical papers in the area of Thermo-fluids and Energy Systems in international peer reviewed journals like *Ambient Energy*, *Energy Conversion and Management*, *Renewable Energy* and *Solar Energy*.

F. K. Forson has lectured in the following courses over the period of his service with the University: Applied Thermodynamics, Heat Transfer, Internal Combustion Engines, Fluid Mechanics, Theory of Machines, and Strength of Materials at the under-graduate level; and in Advanced Thermodynamics, Heat and Mass Transfer, and Engineering Research Methods at the post-graduate level. He has a wide range of experience in teaching of thermodynamics at undergraduate and graduate level.

He is presently, an Associate Professor and the Head of Department of Mechanical Engineering, KNUST, Kumasi - Ghana.

Course developer(s):

- Prof. Francis Kofi Forson; BSc (Mech. Eng), MSc (Mech. Eng), PhD; Lecturer and Head of Department; Department of Mechanical Engineering; Kwame Nkrumah University of Science and Technology, Kumasi.
- Mr. Amoabeng Owura Kofi; BSc (Mech. Eng); Graduate Student and Demonstrator; Department of Mechanical Engineering; Kwame Nkrumah University of Science and Technology, Kumasi.

Acknowledgement

Firstly, the constructive comments and suggestions of Prof. Frederick Ohene Akuffo and Dr. Kwame Owusu-Achaw who have used the earlier versions in teaching regular students of the third year Mechanical Engineering Programme is acknowledged.

Secondly, the efforts of Mr. Michael Benissan Gbikpi and Mr. Amoabeng Owura Kofi who tirelessly word processed and formatted the document in its present form is acknowledged.

The additions and suggested ordering of the material by Mr. Charles Kofi Kafui Sekyere, who has also used the material for teaching in the past three years cannot go without acknowledgement.

All who have contributed in diverse forms materially and spiritually are also acknowledged.

Course Introduction

ME 366: HEAT TRANSFER is a course in BSc. Mechanical Engineering HND Top-Up year 3 taken in the second semester. It is a theory course and has 3 credits.

COURSE OVERVIEW

This module is a one-semester course in heat transfer designed for undergraduate engineering students. The text covers the basic principles of heat transfer with a broad range of engineering applications. The course module is used to extend thermodynamic analysis through the study of the modes of heat transfer and through the development of relations to calculate heat transfer rates.

COURSE OBJECTIVES

On completion of the course, you should be able to:

1. Appreciate the physical mechanisms that underlie heat transfer processes and its relevance to industrial and environmental problems.
2. Calculate heat transfer rates.
3. Apply heat transfer principles in combination with thermodynamics to design simple devices such as solar collectors, solar chimney based on sound appreciation of the physics involved.

COURSE OUTLINE

The course is divided into six units. Each Unit is broken into Sessions, each of which will address one or more of the course objectives.

- Unit 1: Introduction and importance of heat transfer
- Unit 2: Conduction heat transfer
- Unit 3: Radiation heat transfer phenomena
- Unit 4: View factors and Radiation Exchange between surfaces
- Unit 5: Convection heat transfer
- Unit 6: Application of heat transfer mechanisms to heat exchanger design

COURSE STUDY GUIDE

This provides a monthly/weekly schedule of progress of your learning.

Meeting #	Unit/Session	FFFS/Practical/Exam/Quiz
1	Unit 1/ Sessions 1,2	FFFS/ Assignment 1,2 & 3 given out to be submitted two weeks after the 6th FFFS
2	Unit 2/ Session1, 2	FFFS/ Quiz 1
3	Unit 3/ Session 1, 2	FFFS
4	Unit 4/ Sessions 1,2	FFFS
5	Unit 5/ Sessions 1,2	FFFS/ Quiz 2
6	Unit 6/ Sessions 1,2	FFFS/Exam/; Take-Home Mid Semester Exams and Quizzes to be submitted by 5pm on the 2nd Friday following this last meeting.

COURSE POLICY:

COURSE: ME 366 HEAT TRANSFER
COURSE LECTURER: PROF. F. K. FORSON
COURSE OUTLINE AND POLICY, 2011

1. COURSE OUTLINE

Conduction, Convection, and Radiation Heat transfer. Applications to heat exchangers and solar collector design.

2. REFERENCES/READING LIST /RECOMMENDED TEXTBOOKS

(a) Fundamentals of Heat and Mass Transfer, F.P. Incropera and D. P. DeWitt, 4th Edition, 1996.

(b) Heat Transfer: A Practical Approach, Yunus A. Cengel, McGraw Hill Inc., 1998.

(c) Engineering Thermodynamics: Work and Heat Transfer, 4th Edition, Longman Group Ltd., 1992 Chapters 21-24

(d) Applied Thermodynamics for Engineering Technologists, 5th Edition, Longman Scientific and Technical, 1993, Chapter 16.

(e) Heat Transfer, J. P. Holman, 10th Edition, 2010.

3. ASSIGNMENTS (160, POINTS)

(a) You will be responsible for doing the background reading corresponding to a particular LECTURE before the scheduled lecture for that period.
(b) Assignments are due on or before the specified last dates for submission. Assignments SHALL be graded and returned to you before the final examination in the course.
(c) THREE (3) SETS OF **HOME-WORK** PROBLEMS - shall be assigned to you in the semester. : All assignments shall be considered as assigned on the date of receipt of your course handout. These homework problems shall be assigned from the set of tutorial problems in the handout as detailed in the handout.

1. ***Assignment 1 AREA OF COVERAGE PAGE 1 TO 12 OF LECTURE NOTES: Tutorial PROBLEMS Questions 44, 45, 47, 48 [40 points]***
2. ***Assignment 2 AREA OF COVERAGE PAGE 1 TO 41 OF LECTURE NOTES) : Tutorial PROBLEMS Questions 1,6, 16 25,37 [40 points]***
3. ***Assignment 3 AREA OF COVERAGE PAGE 1 TO 93 OF LECTURE NOTES : Tutorial PROBLEMS: Questions 7, 10, 40, 43 [40 points]***

For regular students

(d) Last Dates for submission of **Assignments 1, 2, and 3 shall be February 11, 2011; March 11, 2011; and April 08, 2011, respectively**. All assignments shall be submitted in class.

Distance learners:

(a) You will be responsible for doing the reading corresponding to a particular period before the scheduled lecture.
(b) Assignments are due on or before the specified last dates for submission. Assignments will be graded and returned to you.

THREE (3) SETS OF **HOME-WORK** PROBLEMS - will be assigned to you over the semester on the first day of lectures. All assignments shall be considered as assigned on the date of receipt of your course handout. These homework problems will be assigned from the set of tutorial problems in the handout as detailed in the handout. All the assignments must be submitted by 5pm on the 2nd Friday following the last meeting.

4. Quizzes (40 points):

Regular Students:

TWO (2) **open-book** quizzes of duration of 30 minutes will be administered in the semester, in order to highlight key points and test the understanding and promote individual participation in lectures. Each of these may involve material from reading to homework assigned for the lecture or material developed during the lecture. These will be graded and returned to you in lecture. Towards this end please, remember to bring a calculator to each lecture .

The following shall constitute the tentative periods for the quizzes**: Quiz 1 will be administered in the month of February; Quiz 2 will be administered in the month of April.**

A. QUIZ 1 [20 points]: AREA OF COVERAGE: PAGES 1 TO 12 OF LECTURE NOTES (HAND-OUT) FOR ME 366.

B. QUIZ 2 [20 points]: AREA OF COVERAGE: PAGES 1 TO THE END OF LECTURE NOTES (HAND-OUT) FOR ME 366.

<u>Distance learners:</u>

TWO (2) **take-home** quizzes will be administered in the semester, in order to highlight key points and test the understanding and promote individual participation in lectures. Quiz 1 will be given out at meeting #2 and quiz 2 will be given out during meeting #5. Both Quizzes will be submitted by 5pm on the 2^{nd} Friday following the last meeting.

5. Mid Semester Examination (100 points)

<u>Regular Students:</u>

An **open-book** mid-semester examination will be conducted in the semester. The duration of the mid-semester will be one hour. You will be required to answer all questions in the one hour. No make-up exams will be given *except* in the event of excusable absences due to illness or serious emergency. **<u>The mid –semester examination will consist of two sections; Section A will comprise twenty multiple choice questions and in section B you will choose two out of four analysis-type problems and answer them in addition to the section A compulsory questions</u>**.

<u>Distance learners:</u>

A **take-home** mid-semester examination will be conducted in the semester. The Question paper will be given out during the 5^{th} meeting. The scripts for the mid-semester Examination will be submitted by 5pm on the 2^{nd} Friday following the last meeting.

6. Attendance (60 points)

Classroom attendance is expected except in cases of illness, emergencies, or special circumstances. You will be held individually responsible for any material that is discussed in lecture, whether treated in the notes or not. There will be no opportunity for make-up of missed lectures. You will be required to sign the attendance register.

7. Participation

You are encouraged to ask questions during lectures or face-to-face sessions regarding aspects of reading, homework or lecture material that is unclear to you. You may be called upon to answer questions during lectures, comment on problem solutions, and/or lead discussions related to the lecture material**. Remember to bring a calculator to each lecture.**

8. Final Examination

A final **closed-book** examination given at the end of the course will cover all materials (main and supplementary materials) assigned or provided through the course. It will be closed book examination lasting three (3) hours. **All questions in section A of the final examination will be compulsory**. **In section B, you will be required to answer three out of five problems**.

9. Grading

The following gives the weighting of the various items to be used in the determination of grades for the course: Homework Problems: 13.33 %; Quizzes 3.34 %; Mid-semester 8.33 %; Attendance at lectures 5 %; Final Examination 70 %.

10. Unacceptable conduct

Communication between students or learners in solving assignment problems is encouraged. However, each student or learner is expected to do his own work in satisfying the homework problem requirements, and failure to do so will result in a zero grade for the assignment. Rude behaviour during lectures or face-to-face sessions is unacceptable and will not be tolerated. Indecent and provocative dressings are considered to be not morally right and will not be tolerated in class. All mobile or cell phones are to be switched off or put on vibration mode during my lectures or face-to-face sessions. In case you need to attend to a an urgent call, all you need to do is to raise an arm and walk out quietly to a distance not within the reach of the lecture hall or meeting place and attend to the call making sure that we shall not over hear you. When you finish with your call, with an arm up, you can quietly return to your seat making sure that you not distract the attention of the class or make yourself the centre of attraction for the rest of the lecture period or the face-to-face session.

11. Office hours for personal consultation on academic issues

For regular students

I will be available to attend to students on individual basis between the hours of 2 pm and 5 pm on all Tuesdays in my personal office

For Distance learners

You can reach me for consultation via 0322064340 (personal office phone) between the hours of 2 pm and 5 pm or via fkforson@yahoo.co.uk between the hours of 6 pm and 8 pm on Tuesdays.

USERS' NOTE:

Numerous textbooks have been consulted and materials from them have been cautiously adapted and assembled to achieve the pertinent information needed for the intended target group of this very course – Heat transfer. It is hoped that users of this revised handout will find it a delightful and useful complement to other textbooks obtainable on the subject. Users of this handout are advised and encouraged to go through all the worked examples included in this edition and also try and solve all the tutorial set problems. The tutorial set problems have been carefully selected from a myriad of intriguing problems available in textbooks to address most of the practical areas of application that students who have subscribed to this course have shown interest in the past years.

RESOURCES/WEBSITES: www.mhhe.com , www.mitcourseware.com

ME 366 HEAT TRANSFER LECTURING ACTIVITIES FOR THE 2010/2011 ACADEMIC YEAR

REGULAR STUDENTS

Week	Period	Activity	Feed back
1	Jan 24 – Jan 28		
2	**Jan 31 – Feb 04**	**Assignment 1 assigned**	
3	Feb 07 – Feb 11	Submission of Assignment 1	
4	**Feb 14 – Feb 18**	**QUIZ 1 administered**	Assignment 1 scripts returned
5	Feb 21 – Feb 25		Quiz 1 scripts returned
6	**Feb 28 – Mar 04**	**Assignment 2 assigned**	
7	Mar 07 – Mar 11	Submission of Assignment 2	
8	Mar 14 – Mar 18		Assignment 2 scripts returned
9	**Mar 21 – Mar 25**	**Mid- Semester Exam**	
10	**Mar 28 – April 01**	**Assignment 3 assigned**	Mid- Semester scores released and problems discussed
11	April 04 – April 08	Submission of Assignment 3	
12	April 11 – April 15	**QUIZ 2 administered**	Assignment 3 scripts returned
13	**April 18 – April 22**	**LECTURES END**	Quiz 2 scripts returned
14	**April 25 – April 29**	**End of Semester Exams**	
15	**May 02 – May 06**	**End of Semester Exams**	
16	**May 09 – May 13**	**VACATION**	

DISTANCE LEARNERS

Meeting #	Unit/Session	FFFS/Practical/Exam/Quiz
1	Unit 1/ Sessions 1,2	FFFS/ Assignment 1,2 & 3 given out to be submitted two weeks after the 6th FFFS
2	Unit 2/ Session1, 2	FFFS/ Quiz 1
3	Unit 3/ Session 1, 2	FFFS
4	Unit 4/ Sessions 1,2	FFFS
5	Unit 5/ Sessions 1,2	FFFS/ Quiz 2
6	Unit 6/ Sessions 1,2	FFFS/Exam/; Take-Home Mid Semester Exams to be submitted by 5pm on the 2nd Friday following this last meeting.

Table of Contents

List of Tables

List of Figures

List of Appendices

Unit 1

INTRODUCTION AND IMPORTANCE OF HEAT TRANSFER

Introduction

In this unit, we define heat transfer and show the relationship between thermodynamics and heat transfer as well as demonstrate the importance of heat transfer. We also define the three modes of heat transfer namely conduction, convection and radiation and explain the mechanisms of heat transfer for each of the three modes. Rate equations for each of the three modes of heat transfer for one dimensional steady temperature distribution are defined with their application through worked examples also demonstrated. The importance of the conservation of energy requirement for analysing unsteady and steady- state heat transfer problems are also discussed.

Learning Objectives

After studying this unit, you should be able to:

1. Define heat transfer.
2. Show the relationship between thermodynamics and heat transfer.
3. Demonstrate the importance of heat transfer.
4. Define the three modes of heat transfer- conduction, convection and radiation.
5. Explain the mechanism of heat transfer for each heat transfer mode.
6. Define the rate equations for the three modes of heat transfer for one dimensional steady temperature distribution and demonstrate their application.
7. Demonstrate the importance of the conservation of energy requirement for analyzing unsteady and steady-state heat transfer problems.

Unit content

SESSION 1-1: INTRODUCTORY REMARKS AND IMPORTANCE OF HEAT TRANSFER

In the science of thermodynamics, which deals with energy in its various forms and with its transformation from one form to another, two particularly important transient forms are defined, work and heat. These energies are termed transient since, by definition, they exist only when there is an exchange of energy between two systems or between a system and its surroundings. When the exchange takes place without the transfer of mass from the system and not by means of temperature difference, the energy is said to have been transferred through the performance of work. If, on the other hand, the exchange is due to a temperature difference, the energy is said to have been transferred by the flow of heat. It must be noted that the existence of temperature difference is the distinguishing feature of the energy form known as heat.

The subject of heat transfer deals with the analysis of heat flows in systems, which generally arise due to temperature difference. The subject is essential for the economic design of a large range of engineering equipment of practical interest; for example, in the design of heat exchangers such as boilers, condensers, radiators, etc. Heat transfer analysis is essential for the proper sizing of such equipment. In heating and air-conditioning applications for buildings, a proper heat transfer analysis is necessary to estimate the amount of insulation needed to prevent excessive heat losses or gains. Other application areas include solar energy technology, aerospace technology, and nuclear technology among others.

> Heat transfer (or heat) is energy in transit due to a temperature difference

1-1.1 FUNDAMENTAL LAWS AND CONCEPTS

As shown in Figure 1, we refer to different types of heat transfer processes as *modes*. Traditionally, three modes of heat transfer are recognised, namely: conduction, convection, and radiation. It is certainly a rare instance when one encounters a problem of practical interest, which does not involve at least two, and sometimes all three, of these modes occurring simultaneously.

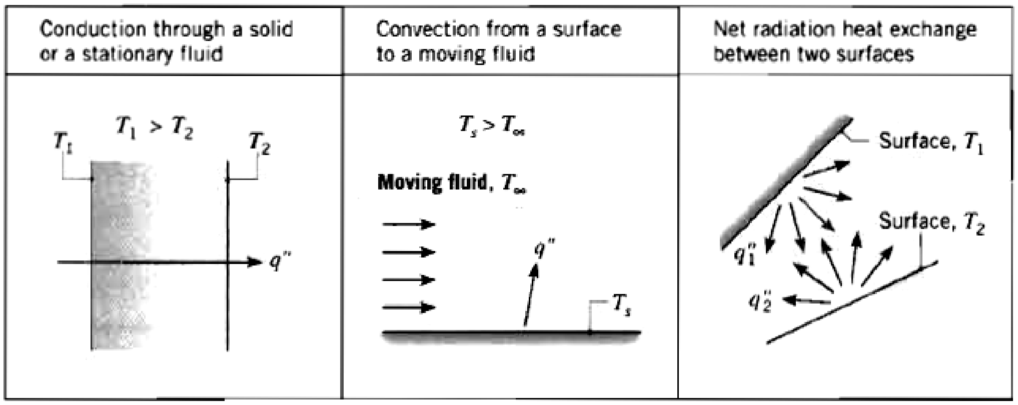


Figure 1: Conduction, convection and radiation heat transfer modes

1-1.1.1 CONDUCTION HEAT TRANSFER

Heat conduction is that mode of energy transfer between solids brought into direct contact having different temperatures or within solids as a result of temperature difference or with liquids or gases as a result of temperature difference without an appreciable movement of matter. This takes places as a result of kinetic motion or direct impact of molecules, as in the case of fluid at rest, and by the drift of electrons as in the case of metals. In a solid, which is a good electric conductor a large number of free electrons move about in the lattice; hence materials that are good electric conductors are generally good heat conductors (e.g. copper, silver etc)

1-1.1.1a Fourier's law of heat conduction

The basic law of heat conduction is the Fourier's law. It states that the rate of flow of heat through a single homogenous solid is directly proportional to the area A of the section at right angles to the direction of heat flow, and to the change of temperature with respect to the length of the path of heat flow, dT/dx. This is an empirical law based on observation.

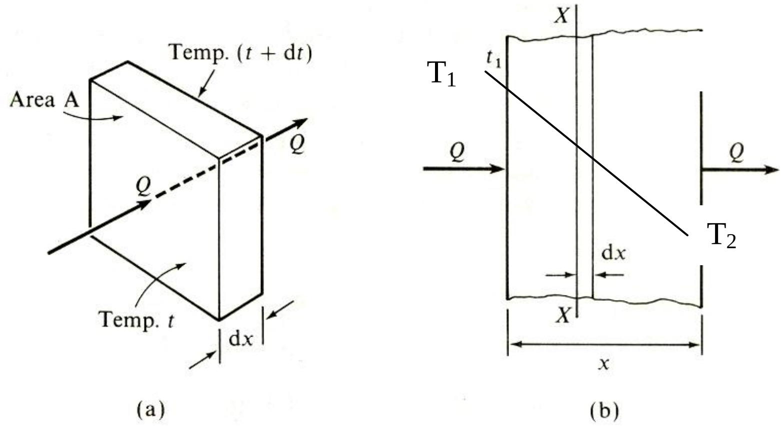


Figure 2: Heat flow through a thin slab of material

Consider the infinitesimal body with thickness dx in the solid body with one face at a temperature T_1 and the other at a temperature T_2 as indicated (Figure 2a). Then, applying the Fourier's law we have for the rate of heat flow in the positive direction.

Rate of heat flow, $\dot{Q} \;\alpha\; A \dfrac{dT}{dx}$ or $\dot{Q} = -\lambda\; A \dfrac{dT}{dx}$ or $\dot{q} = -\lambda \dfrac{(T_2 - T_1)}{L}$ (1.1)

The rate of heat flow in the direction of x is taken as positive hence the negative sign in equation (1.1) since dT is always negative, i.e. the driving potential is negative in the direction of the heat flow. The term λ is called the thermal conductivity of the material. The units of λ are usually written as W/m K or kW/mK.

Consider the transfer of heat through a slab of material as shown in Figure 2(b). At section X-X, using equation (1.1)

$$\dot{Q} = -A_x \lambda \frac{dT}{dx} \quad \text{or} \quad \dot{Q}\; dx = -\lambda A_x dT$$

Integrating

$$\int_0^x \dot{Q}\, dx = -\int_{T_1}^{T_2} \lambda \; AdT \;\; \text{or} \;\; \dot{Q}x = -A\int_{T_1}^{T_2} \lambda dT$$

The equation can be solved when the variation of thermal conductivity, λ, with temperature, T, is known. By considering λ as a constant we obtain

$$\dot{Q} = -\lambda\, A\frac{(T_2 - T_1)}{x} = \lambda\, A\frac{(T_1 - T_2)}{x} \qquad (1.2)$$

The thermal conductivity λ varies greatly between materials. The thermal conductivity of materials varies over a wide range as indicated in Table 1.

For many materials, the thermal conductivity, λ is a linear function of temperature Thus,

$$\lambda(T) = \lambda_0(1 + \beta T)$$

In such cases, the integration of equation (1.1) gives

$$\dot{Q} = \frac{\lambda_0 A}{x}\left[(T_1 - T_2) + \frac{\beta}{2}\left(T_1^{\,2} - T_2^{\,2}\right)\right] = \frac{\lambda_{av} A}{x}(T_1 - T_2) \qquad (1.3)$$

Where λ_{av} the value of which is λ evaluated at the average temperature $(T_1 + T_2)/2$

Table 1: The range of thermal conductivity of various materials at room temperature

Material	λ, W/m K
Nonmetallic crystals (quartz, Beryllium oxide, Silicon carbide, Diamond)	10 – 2300
Pure metals (Manganese, Iron, Copper, Silver)	10 – 420
Metals Alloys (Nichrome, Steel, Bronze, Aluminium alloys	8 – 170
Non - Metallic solids (rubber, food, rock, oxides)	0.15 – 70
Liquids (Oils, Water, Mercury)	0.12 – 8.5
Insulating Materials (foams, wood, fibres)	0.04 – 0.7
Gases at atmospheric pressure (CO_2, Air, He, H_2)	0.02 – 0.20

Note that for gases; $\lambda \; \alpha \sqrt{\frac{T}{\breve{m}}}$, where T is the absolute temperature of the gas and $\breve{m}$ is its corresponding molar mass.

Table 2: Thermal conductivity of some materials

Substance	λ, W/m K
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminium	237
Commercial bronze	52
Iron	80.2
Mild steel	48.5
Lead	34.6
Concrete	0.85 – 1.4
Building brick	0.35 – 0.7
Wood (oak)	0.15 – 0.2
Rubber	0.15
Helium (g)	0.152
Cork board	0.043
Air (g)	0.026

Thermal conductivity often varies with temperature. For most pure metals, the thermal conductivity decreases with temperature, while for gases and insulating materials it increases with temperature. It follows from equation (1.1) that materials with high thermal conductivities are good conductors and those with low thermal conductivities are good insulators. Conduction of heat occurs most readily in pure metals, less so in alloys, and much less readily in non-metals. Gases and liquids are good insulators, but unless a completely stagnant layer of fluid is obtained, heat is transferred by convection currents.

1-1.1.2 CONVECTION HEAT TRANSFER

Convection is the mode of heat transfer, which takes place between a solid surface and an adjacent fluid that is in motion as long as the solid and the fluid are at different temperatures and it involves the combined effects of conduction and fluid motion. The faster the fluid motion, the greater the convection heat-transfer. In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction. If the bulk fluid motion is artificially induced, say with a fan or a pump, the convection is called forced convection. On the other hand, if buoyancy forces generated because of density differences resulting from temperature difference cause the fluid motion it is called free or natural convection.

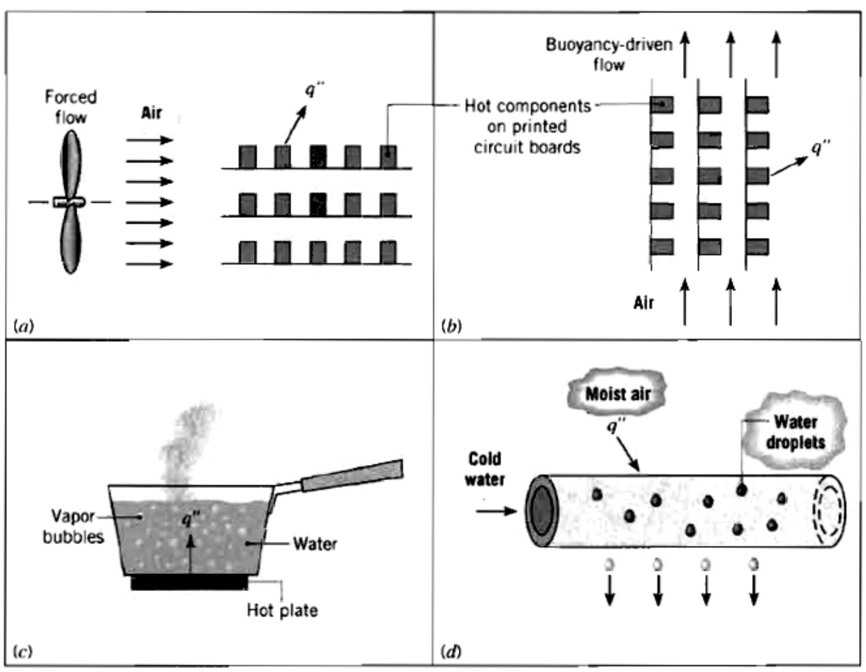


Figure 3: Convection heat transfer processes: a) Forced convection b) Natural convection c) Boiling d) Condensation

1-1.1.2a Newton's Law of Cooling:

Newton's law of cooling states that the heat transfer from a solid surface of Area A, at a temperature, T_w, to a fluid of temperature, *T*, is given by

$$\dot{Q} = \alpha_c A(T_w - T) \quad (1.4)$$

Where α_c is called the heat transfer coefficient. The units of α are seen to be W/m^2 K or kW/m^{2}K. The heat transfer coefficient is an experimentally determined parameter whose value depends on all the variables influencing convection such as surface geometry, the nature of fluid motion, the properties of the fluid, and the bulk fluid velocity. Equation (1.4) does not include the heat loss from the surface by radiation.

We have described the convection heat transfer mode as energy transfer occurring within a fluid due to the combined effects of conduction and bulk fluid motion. Typically, the energy that is being transferred is the *sensible* or internal thermal energy of the fluid. However, there are convection processes for which there is, in addition, *latent heat exchange*. This latent heat exchange is generally associated with a phase change between the liquid and vapour states of the fluid. Two special cases of interest are *boiling* and *condensation*. Regardless of the nature of the convection heat transfer process, the appropriate rate equation is of the form of equation (1.4).

The convection transfer coefficient varies greatly for different types of f1ow. Typical values of the convection heat transfer coefficient are provided in Table 3.

Table 3: Typical values of the convection heat transfer coefficient

Process	*h*, W/m^2 K
Free convection	
Gases	2 – 25
Liquids	10 – 1,000
Forced convection	
Gases	25 – 250
Liquids	50 – 20,000
Boiling water	2,500 – 25,000
Condensation steam	5,000 – 100,000

Source: Fundamentals of Heat and Mass Transfer by Incropera and DeWitt (1996)

1-1.1.3 RADIATION HEAT TRANSFER

Radiation is the energy emitted by matter in the form of electromagnetic waves as a result of the changes in the electronic configurations of the atoms or molecules. Radiation is the mode of heat transfer by means of electromagnetic waves between two surfaces or two bodies separated by space, which does not have any medium what so ever. Energy from the sun, for example, reaches the earth by radiation. It must be noted that heat transfer by radiation is a significant mechanism of energy transport. Radiation heat transfer occurs more efficiently in a vacuum.

1-1.1.3a Stefan-Boltzmann Law

Consider radiation transfer processes for the surface of Figure 4(a)

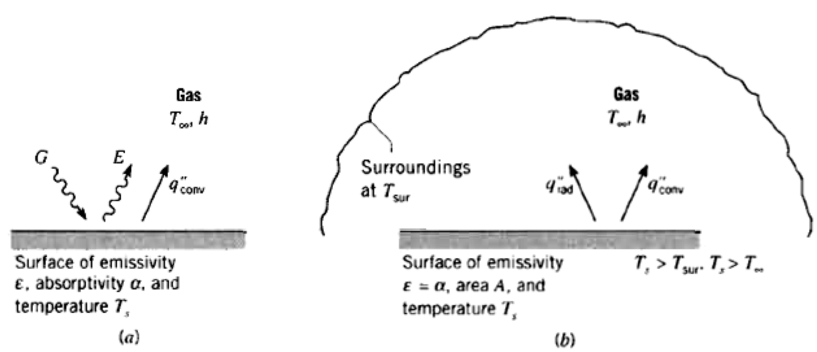


Figure 4: Radiation exchange: a) at a surface and b) between a surface and large surroundings

Radiation that is emitted by a surface originates from the thermal energy of the matter bounded by the surface, and the rate at which energy is released per unit area (W/m^2) is termed the surface *emissive power*, E_b. There is an upper limit to the emissive power, which is prescribed by the *Stefan-Boltzmann law*:

$$E_b = \sigma T_s^4 \quad (1.5a)$$

Where T_s is the absolute temperature of the surface and σ is the Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W/m^2 K^4. A surface that emits radiation according this law is called an *ideal radiation* or *blackbody*.

The heat flux emitted by a real surface is less than that of a blackbody at the same temperature and is given by:

$$E = \varepsilon\sigma T_s^4 \quad (1.5b)$$

where ε is a radiative property of the surface termed *emissivity*. With values in the range $0 \le \varepsilon \le 1$, this property provides a measure of how efficiently a surface emits energy relative to a blackbody. It depends strongly on the surface material and finish. It must be noted that a smooth surface reflects radiation specularly, while a rough surface reflects radiation diffusely. Hence, rough surfaces are much betters absorbers of radiation than smooth surfaces.

Radiation may also be incident on a surface from its surroundings. The radiation may originate from a special source, such as the sun, or from other surfaces to which the surface of interest is exposed. Irrespective of the source(s), we designate the rate at which radiation is incident on a unit area of the surface as the *irradiation*, G. (Figure 4a)

A portion, or all, of the irradiation may be *absorbed* by the surface, thereby increasing the thermal energy of the material The rate at which radiant energy is absorbed per unit area may be evaluated from knowledge of a surface radiative property termed the *absorptivity* α. That is,

$$G_{abs} = \alpha G$$

where $0 \leq \alpha \leq 1$. If $\alpha \leq 1$ and the surface is *opaque*, a portion of the irradiation is *reflected*. If the surface is semitransparent, portions of the irradiation may also be *transmitted*.

A special case that occurs frequently involves radiation exchange between a small surface at T_s and a much larger, isothermal surface that completely surrounds the smaller one. The *surroundings* could, for example, be the walls of a room or a furnace whose temperature T_{sur} differs from that of an enclosed surface and the irradiation from the surroundings may be approximated by the emission from a blackbody at T_{sur}, in which case $G = \sigma T_{sur}{}^4$. If the surface is assumed to be that for which $\alpha = \varepsilon$ (a grey body), the net rate radiation heat transfer *from* the surface, expressed per unit area of the surface is

$$\dot{q} = \frac{\dot{Q}}{A} = \varepsilon E_b - \alpha G = \varepsilon\sigma\left(T_s^4 - T_{sur}{}^4\right) \qquad (1.6)$$

i.e. when a surface of emissivity ε and surface area A at an absolute temperature T_s is completely enclosed by a much larger (or black) surface at an absolute temperature T_{sur} separated by a gas (such as air) that does not interfere with radiation, the net rate of radiation between these two surfaces is given by equation 1.6. Equation (1.6) provides the difference between thermal energy that is released due to radiation emission and that which is gained due to radiation absorption.

There are many applications for which it is convenient to express the net radiation heat exchange in the form.

$$\dot{Q} = \alpha_r A(T_s - T_{sur}) \qquad (1.7)$$

Where, from Equation (1.7), the radiation heat transfer coefficient α_r is:

$$\alpha_r = \varepsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2) \qquad (1.8)$$

Here, we have modelled the radiation mode in a manner similar to convection. In this sense, we have linearised the radiation rate equation. Its importance is in solar energy problems for purposes of modelling.

Notes:

1. The rate of heat transfer per unit area normal to the direction of heat transfer is called heat flux, and the average heat flux on a surface is expressed.

$$\dot{q} = \frac{\dot{Q}}{A} \tag{1.9}$$

where A is the heat transfer area normal to the direction of heat transfer. The unit of heat flux in SI units is W/m^2.

2. Application to solar air-heaters and solar collectors. (Energy balance analysis for the different components, steady-state solution and transient analysis.)

Self Assessment 1-1

1. The inner and outer surfaces of a 0.5-cm-thick 2-m x 2-m window glass in winter are 10°C and 3°C, respectively. If the thermal conductivity of the glass is 0.78 W/m·°C, determine the amount of heat loss, in kJ, through the glass over a period of 5 hours. What would your answer be if the glass were 1 cm thick? **Suggested Answers: 78,624 kJ, 39,312 kJ**

2. Consider a person standing in a room at 23°C. Determine the total rate of heat transfer from this person if the exposed surface area and the skin temperature of the person are 1.7 m^2 and 32°C, respectively, and the convection heat transfer coefficient is 5 W/m^2·°C. Take the emissivity of the skin and the clothes to be 0.9, and assume the temperature of the inner surfaces of the room to be the same as the air temperature.
Suggested Answer: 161 W

3. Hot air at 80°C is blown over a 2-m x 4-m flat surface at 30°C. If the average convection heat transfer coefficient is 55 W/m^2 · °C, determine the rate of heat transfer from the air to the plate, in kW. **Suggested Answer: 22 kW**

SESSION 2-1: THE CONSERVATION OF ENERGY REQUIREMENT

2-1.1 THE FIRST LAW OF THERMODYNAMICS

For many heat transfer problems, the first law of thermodynamics (the law of conservation of - energy) provides a useful tool. In applying the first law, we need to identify the control volume, a region of space bounded by a control surface through which energy and matter may pass. Once the control volume is identified, an appropriate time basis must be specified. One option involves formulating the law on a *rate basis*. That is, at any instant, there must be a balance between all *energy rates*. Alternatively, the first law must also be satisfied over any time interval. For such an interval, there must be a balance between the *amounts of all energy changes*.

According to the time basis, the first law formulations that we are suited for heat transfer analysis may be stated as follows.

At an Instant (t)

The rate at which thermal and mechanical energy enters a control volume, plus the rate at which thermal energy is generated within the control volume, minus the rate at which thermal and mechanical energy leaves the control volume must equal the rate of increase of energy stored within the control volume.

Over an Instant (Δt)

The amount of thermal and mechanical energy that enters a control volume, plus the amount of thermal energy that is generated within the control volume, minus the amount of thermal and mechanical energy that leaves the control volume must equal the increase in the amount of energy stored in the control volume.

A general form of the conservation requirement may then be expressed on a rate basis as

$$\boxed{\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \frac{dE_{st}}{dt} = \dot{E}_{st}} \qquad (1.10)$$

Equation (1.10) may be applied at any instant of time. The alternative form that applies for a time interval Δt is obtained by integrating equation (1.10) over time:

$$\boxed{E_{in} + E_g - E_{out} = \Delta E_{st}} \tag{1.11}$$

The inflow and outflow terms are *surface phenomena*. That is, they are associated exclusively with processes occurring at the control surface and are proportional to the surface area. A common situation involves energy inflow and outflow due to heat transfer by conduction, convection, and/or radiation.

The *energy generation term* is associated with conversion from some other energy form to thermal energy and it is a *volumetric phenomenon*. That is, it occurs within the control volume and is proportional to the magnitude of this volume. (e.g. conversion from electrical energy that occurs due resistance heating when an electric current is passed through a conductor).

Energy storage is also a volumetric phenomenon, and changes within the control volume may be due to changes in the internal, kinetic, and/or potential energies of its contents.

We will frequently have occasion to apply the conservation of energy requirement at the surface of a medium. In this special case, the control surface includes no mass or volume. Equation 1.10, are no longer relevant and it is only necessary to deal with surface phenomena. For this case, the conservation requirement becomes

$$\boxed{\dot{E}_{in} - \dot{E}_{out} = 0} \tag{1.12}$$

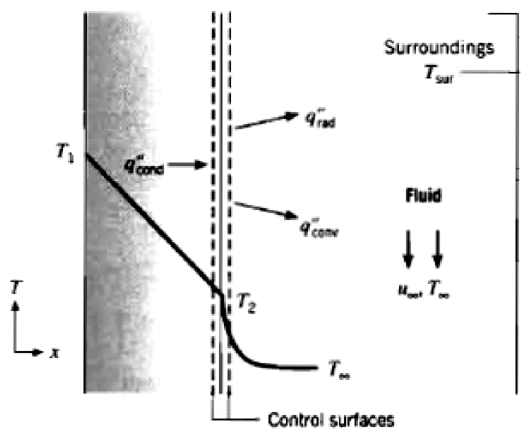
Even though thermal energy generation may be occurring in the medium, the process would not affect the energy balance at the control surface. Moreover, this conservation requirement holds for both *steady state* and *transient* conditions.

Worked Example 1.1

The hot combustion gases of a furnace are separated from the ambient air and its surroundings, which are at 25°C, by a brick wall 0.15 m thick. The brick has a thermal conductivity of 1.2 W/m K and a surface emissivity of 0.8. Under steady state conditions, an outer surface temperature of 100°C is measured. Free convection heat transfer to the air adjoining the surface is characterised by a convection coefficient of α = 20 W/m^2 K. What is the brick inner surface temperature?

Assumptions

1. Steady-state conditions
2. One-dimensional heat transfer by conduction across the walls.
3. Radiation exchange between the outer surface of the wall and the surroundings is between a small surface and a large enclosure.



The inside surface temperature may be obtained by performing an energy balance at the outer surface.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

Hence, on a unit area basis, $\dot{q}_{cond} - \dot{q}_{conv} - \dot{q}_{rad} = 0$

$$\lambda \frac{T_1 - T_2}{L} = \alpha\left(T_2 - T_\infty\right) + \varepsilon\sigma\left(T_2^4 - T_{sur}{}^4\right)$$

Substituting the appropriate numerical values, we find

$$1.2 \times \frac{(T_1 - 373)}{0.15} = 20(373 - 298) + 0.8(5.67 \times 10^{-8})(373^4 - 298^4)$$

$$1.2 \times \frac{(T_1 - 373)}{0.15} = 1500\,\mathrm{W/m^2} + 520\,\mathrm{W/m^2}$$

Solving for T_1

$$T_1 = 373 \;+\; \frac{0.15}{1.2}(2020) = 625\,\mathrm{K} = 352^0\,\mathrm{C}$$

Comments

1. Note that the contribution of radiation to heat transfer from outer surface is significant. The relative contribution would diminish, however, with increasing α_c and/or decreasing T_2.

2. When using energy balances involving radiation exchange and other modes, it is good practice to express all temperatures in Kelvin units. This practice is necessary when the unknown temperature appears in the radiation term and in one or more of the other terms

---

Worked Example 1.2

An insulated steam pipe passes through a room in which the air and walls are at 25^0C. The outside diameter of the pipe is 70 mm, and its surface temperature and emissivity are 200^0C and 0.8, respectively. What are the surface emissive power and irradiation? If the coefficient associated with free convection heat transfer from the surface to the air is 15 $\mathrm{W/m^2\,K}$, what is the rate of heat loss from the surface per unit length of the pipe.

Assumptions

a. Steady state conditions

b. Radiation exchange between the pipe and the room is between a small surface in a much larger enclosure.

c. The surface emissivity and absorptivity are equal. (Grey body)

The surface emissive power may be evaluated as

$$E = \varepsilon\sigma T_s^4 = 0.8(5.67 \times 10^{-8}) \times 473^4 = 2270\,\mathrm{W/m^2}$$

The irradiation is given by

$$G = \sigma T_{sur}{}^4 = 5.67 \times 10^{-8} \times 298^4 = 447\,\mathrm{W/m^2}$$

Heat loss from the pipe is by convection to the room air and by radiation exchange with the walls. Hence, with $A = \pi DL$

$$\dot{Q} = h(\pi DL)(T_s - T_\infty) + \varepsilon(\pi DL)\sigma\left(T_s^4 - T_{sur}{}^4\right)$$

The heat loss per unit length is given by

$$\dot{q} = \frac{\dot{Q}}{L} = 15 \times (\pi \times 0.07)(473 - 298) + 0.8(\pi \times 0.07) \times 5.67 \times 10^{-8}\left(473^4 - 298^4\right) = 998\text{W/m}$$

Comments

a. Note that the temperature may be expressed in units of ^{0}C or K when evaluating the temperature difference for a convection (or conduction) heat transfer rate. However, temperature must be expressed in Kelvin when evaluating a radiation transfer rate.

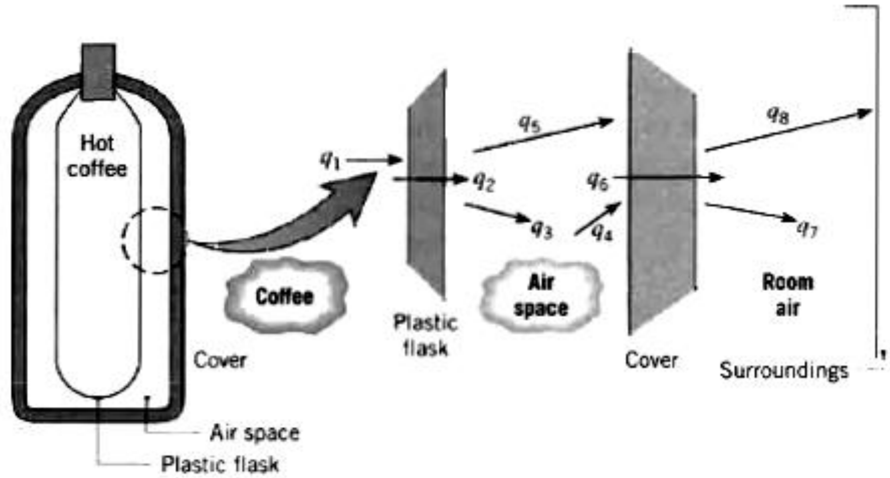
b. The radiation and convection heat transfer rates are comparable because T_s is large compared to T_{sur} and the coefficient associated with free convection is small.

Review Problem: Formulate a practical problem on the Application of energy balance on control surface to solar air-heater analysis for students in class.

Q.1 A closed container filled with hot coffee is in a room whose air and walls are at a fixed temperature. Identify all heat transfer processes, (q_1 to q_8) as shown in the figure below, that contribute to cooling of the coffee. Comment on features that would contribute to a superior container design.

Q.2 A heat rate of 3 kW is conducted through a section of an insulating material of cross-sectional area 10m^2 and thickness 2.5 cm. If the inner (hot) surface temperature is 415^{0}C and the thermal conductivity of the material is 0.2 W/m K, what is the outer surface temperature? **Suggested Answer: 377.5 ^{0}C**

Solution to **Q.1**



q_1 free convection from the coffee to the flask
q_2 conduction through the flask
q_3 free convection from the flask to the air
q_4 free convection from the air to the cover
q_5 net radiation exchange between the outer surface of the flask and the inner surface of the cover
q_6 conduction through the cover
q_7 free convection from the cover to the room air
q_8 net radiation exchange between the outer surface of the cover and the surroundings

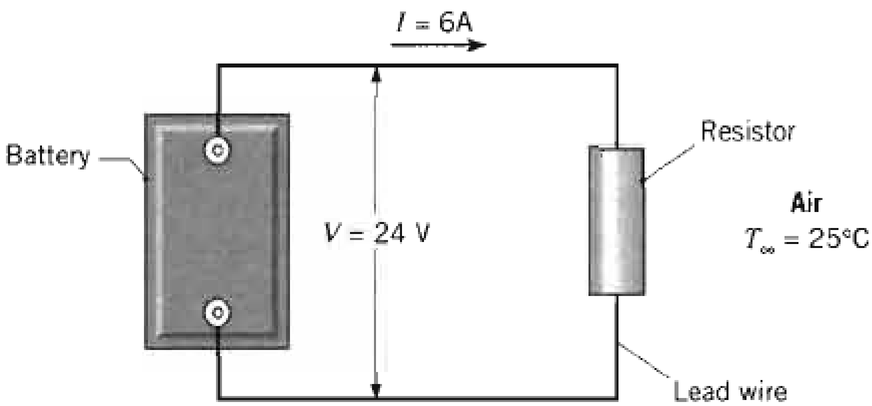
Improving container design

1. Use of aluminized (low emissivity) surfaces for the flask and cover to reduce net radiation, and
2. Evacuating the air space or using a filler material to retard free convection.

Individual/Group Discussion Problems: Tutorial Problems Questions 6, 44, 45, 47, and 48

Self Assessment 2-1

1. An electrical resistor is connected to a battery, as shown schematically. After a brief transient, the resistor assumes a nearly uniform, steady-state temperature of 95 °C, while the battery and lead wires remain at the ambient temperature of 25 °C.



Neglecting the electrical resistance of the lead wires,

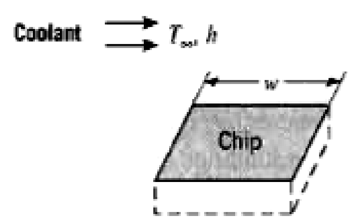
(a) Consider the resistor as the system about which a control surface is placed and equation 1.10 is applied. Determine the corresponding values of $\dot{E}_{in}(W)$, $\dot{E}_g(W)$, $\dot{E}_{out}(W)$ and $\dot{E}_{st}(W)$. If a control surface is placed about the entire system, what are the values of $\dot{E}_{in}$, $\dot{E}_g$, $\dot{E}_{out}$ and $\dot{E}_{st}$?

(b) If electrical energy is dissipated uniformly within the resistor, which is a cylinder of diameter $D = 60\ mm$ and length $L = 250\ mm$, what is the volumetric heat generation rate, $q\left(W/m^2\right)$?

(c) Neglecting radiation from the resistor, what is the convection heat transfer?

2. (a) A square isothermal chip is of width *w* = 5mm on a side and is mounted in a substrate such that its side and back surfaces are well insulated, while the front surface is exposed to the flow of a coolant at $T_\infty = 15^oC$. From reliability considerations, the chip temperature must not exceed $T = 85^oC$.

If the coolant is air and the corresponding convection coefficient is $h = 200\ W/m^2K$, what is the maximum allowable chip power? If the coolant is a dielectric liquid for which $h = 3000\ W/m^2K$, what is the maximum allowable power?



(b) A spherical interplanetary probe of 0.5-m diameter contains electronics that dissipate 150W. If the probe surface has an emissivity of 0.8 and the probe does not receive radiation from other surfaces, as, for example, from the sun, what is its surface temperature?

Suggested Answers

1. **[For the resistor**, $\dot{E}_{in} = \dot{E}_{st} = 0, E_{out} = E_g = 144W$ and **For the Battery-Resistor Combination**, $\dot{E}_{in} = \dot{E}_g = 0, E_{out} = 144W, E_{st} = -144W$ (b) $\dot{q} = 2.04x10^5 W/m^3$ (c) $h = 39W/m^2K$ **]**

2. **[** (a) 0.35W, 5.25W and (b) 254.7K **]**

***Learning Track Activities** ensure you understand all the terms listed below and can work heat transfer problems by applying the three modes of heat transfer as well as the conservation of energy requirement.*

Key terms/ New Words in Unit

i. Heat transfer
ii. Transient
iii. Work energy
iv. Heat energy
v. Temperature difference
vi. Conduction
vii. Convection
viii. Radiation
ix. Fourier's law
x. Newton's Law of cooling
xi. Stefan Boltzmann Law
xii. Forced convection
xiii. Natural convection
xiv. Heat transfer rate
xv. Heat flux
xvi. Thermal conductivity
xvii. Emissivity
xviii. Absorptivity
xix. Opaque surface
xx. Grey surface
xxi. Irradiation
xxii. Convection heat transfer coefficient
xxiii. Radiation heat transfer coefficient
xxiv. Boiling
xxv. Condensation
xxvi. Emissive power
xxvii. Black body
xxviii. Transmisivity

xxix. Reflectivity
xxx. Surface phenomena
xxxi. Volumetric phenomenon
xxxii. Energy generation
xxxiii. Energy storage
xxxiv. Buoyancy forces
xxxv. insolation

Discussion Question

1. On a hot summer day, a student turns his fan on when he leaves his room in the morning. When he returns in the evening, will his room be warmer or cooler than the neighboring rooms? Why? Assume all the doors and windows are kept closed.

2. Consider two identical rooms, one with a refrigerator in it and the other without one. If all the doors and windows are closed, will the room that contains the refrigerator be cooler or warmer than the other room? Why?

3. Consider heat loss through the two walls of a house on a winter night. The walls are identical, except that one of them has a tightly fit glass window. Through which wall will the house lose more heat? Explain.

Review Question: Consider a person standing in a room maintained at 20°C at all times. The inner surfaces of the walls, floors, and ceiling of the house are observed to be at an average temperature of 12°C in winter and 23°C in summer. Determine the rates of radiation heat transfer between this person and the surrounding surfaces in both summer and winter if the exposed surface area, emissivity, and the average outer surface temperature of the person are 1.6 m^2, 0.95, and 32°C, respectively.

Web Activity: www.mhhe.com/cengel ,www.mitcourseware.com

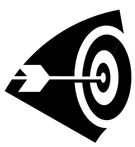
Reading: Read chapter 1, Heat Transfer- A practical Approach by Yunus. A. Cengel, 2^{nd} Edition, McGraw Hill Inc.

Unit 2

CONDUCTION HEAT TRANSFER

Introduction

In this unit, steady state heat conduction problems involving one-dimensional temperature distribution are treated. The concept of thermal resistance for cases with one-dimensional steady state temperature distribution is introduced and the use of equivalent thermal circuits for analysing problems with multiple layers of different materials. The application of heat transfer to extended surfaces is also discussed. A general heat conduction equation in a three dimensional approach is also derived.



Learning Objectives

After studying this unit you should be able to:

1. Solve steady- state conduction problems involving one dimensional temperature distribution.
2. Understand the concept of thermal resistance for cases with one dimensional steady temperature distribution.
3. Demonstrate the use of equivalent thermal circuits for analyzing problems with multiple layers of different materials.
4. Analytically determine heat transfer rates in extended surfaces.
5. Understand the concept of contact thermal resistance in heat transfer analysis.
6. Examine the effect of internal heat generation on temperature distribution and heat transfer.
7. Solve transient problems with uniform temperature distribution.

Unit content

SESSION 1-2: DEFINITION OF CONDUCTION HEAT TRANSFER

Heat conduction is that mode of energy transfer between solids brought into direct contact having different temperatures or within solids as a result of temperature difference or with liquids or gases as a result of temperature difference without an appreciable movement of matter. This takes places as a result of kinetic motion or direct impact of molecules, as in the case of fluid at rest, and by the drift of electrons as in the case of metals. In a solid, which is a good electric conductor a large number of free electrons move about in the lattice; hence materials that are good electric conductors are generally good heat conductors (e.g. copper, silver etc).

1-2.1 one dimensional steady state conduction

In this section, we treat situations for which heat is transferred by diffusion under *one-dimensional, steady state* conditions. The term one-dimensional refers to the fact that only one co-ordinate is needed to describe the spatial variation of the dependent variables. Hence, in a one-dimensional system, temperature gradients exist along only a single space coordinate direction, and heat transfer occurs exclusively in that direction. The system is characterised by steady state conditions if the temperature at each point is independent of time.

We begin our consideration of one-dimensional, steady state conduction by discussing heat transfer with no internal generation in common geometries. The concept of thermal resistance (analogous to electrical resistance) is introduced as an aid to solving conduction heat transfer problems. The effect of internal heat generation on the temperature distribution and heat rate is then treated. The general heat conduction equation is introduced. Finally, conduction analysis is used to describe the performance of extended surfaces or fins, wherein the role of convection at the external boundary must be considered.

1-2.1a Direction of heat flow in response to a temperature gradient

Figure 5 illustrates the direction of heat flow, q_x'', in a one-dimensional coordinate system when the gradient, *dT/dx*, is positive or negative. Note that conduction heat transfer is determined using Fourier's Law

$\dot{Q} = -\lambda A \dfrac{dT}{dx}$ or in terms of the heat flux $q_x'' = \dfrac{\dot{Q}}{A} = -\lambda \dfrac{dT}{dx}$

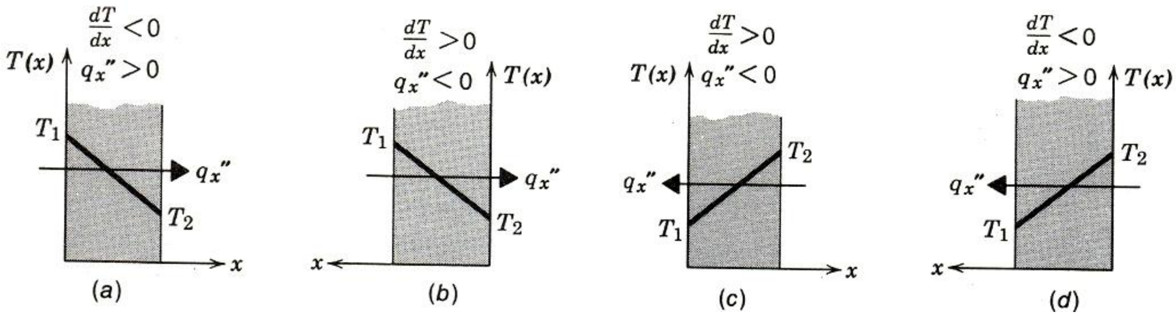


Figure 5: The relationship between coordinate system, heat flow direction, and temperature gradient in a one dimensional plane wall of constant cross-section and thermal conductivity

1-2.1b Thermal diffusivity

The product ρc_p called heat capacity is frequently used in heat transfer analysis and it represents the heat storage capacity of the material per unit volume. In heat transfer analysis, the ratio of the thermal conductivity to the heat capacity is an important property termed the *thermal diffusivity* α_d, which has units of m^2/s:

$$\alpha_d = \frac{Heat\,conducted}{Heat\,stored} = \frac{\lambda}{\rho c_p}(m^2/s)$$

The thermal conductivity λ represents how well a material conducts heat, and the heat capacity ρc_p represents how much energy a material stores per unit volume. The larger the thermal diffusivity, the faster the propagation of heat into the medium. A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat will be conducted further. Thermal diffusivity ranges from $\alpha_d = 0.14 \times 10^{-6}\,\mathrm{m}^2$/s for water to $149 \times 10^{-6}\,\mathrm{m}^2$/s for silver. The thermal diffusivities of beef and water are the same, since meat as well as fresh vegetables and fruits are mostly water, and thus they possess the thermal properties of water.

1-2.1.1 Plane wall

For one-dimensional heat conduction in a plane wall, temperature is a function of the x coordinate only and heat is transferred exclusively in this direction. In Figure 6a, a plane wall separates two fluids of different temperatures. Heat transfer occurs by convection from the hot fluid at $T_{\infty,1}$ to one surface; of the wall at $T_{s,1}$, by conduction through the wall, and by convection from the outer surface of the wall at $T_{s,2}$ to the cold fluid at $T_{\infty,2}$.

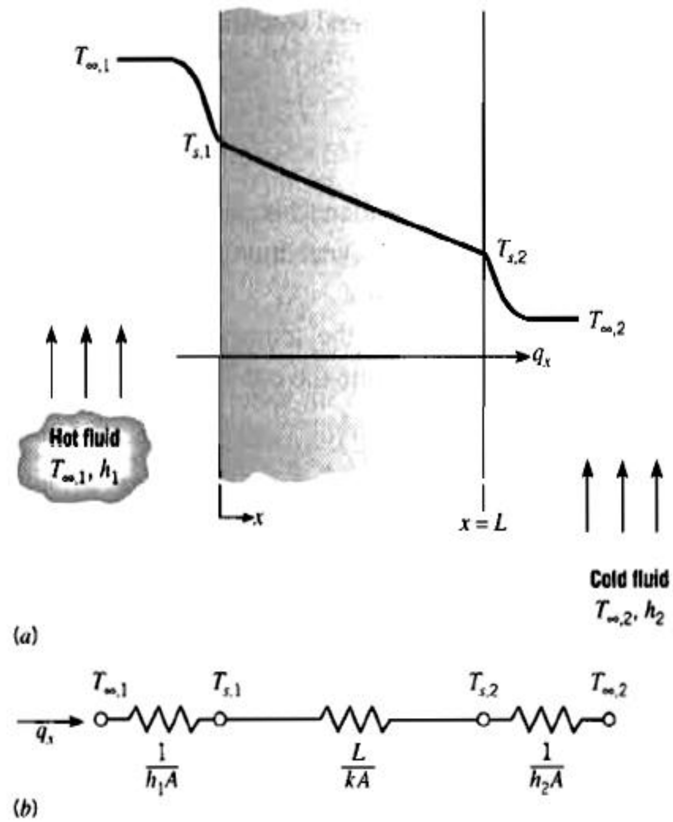


Figure 6: Heat Transfer through a plane wall (a) Temperature distribution (b) Equivalent thermal circuit

Heat equation for one dimensional heat flow

The temperature distribution in the wall can be determined by solving the heat equation with the proper boundary conditions imposed. The heat equation for a plane wall of constant cross-section is:

$$\frac{d}{dx}\left(\lambda \frac{dT}{dx}\right) = 0 \qquad (2.1)$$

If the thermal conductivity, λ, of the wall material is assumed constant then,

$$T(x) = C_1 x + C_2 \tag{2.2}$$

To find values of the constants we apply the boundary conditions of the first kind

$x = 0$ and $x = L$, in which case

$$T(0) = T_{s,1} \quad \text{and} \quad T(L) = T_{s,2}$$

Applying the conditions to the general solution, we have:

$$T(x) = \left(T_{s,2} - T_{s,1}\right)\frac{x}{L} + T_{s,1} \tag{2.3}$$

From equation (2.3), it is evident that *for one-dimensional, steady state heat conduction in a plane wall with no heat generation and constant thermal conductivity, the temperature varies linearly with x.*

Now that we have the temperature distribution, we can use the Fourier's law to determine the conduction heat transfer rate. That is,

$$\dot{Q}_x = -A_x \lambda \frac{dT}{dx} = \frac{\lambda A}{L}\left(T_{s,1} - T_{s,2}\right) \tag{2.4}$$

Notes: a. A_x is the area of the wall normal to the direction of heat transfer and, for the plane wall, it is a constant independent of x.

b. The heat flux, $\dot{q}_x$ is obtained by dividing $\dot{Q}_x$ by A_x

1-2.1.1.1 Thermal Resistance

Just as an electrical resistance is associated with the conduction of electricity, a thermal resistance is associated with the conduction of heat. Defining resistance as the ratio of a driving potential to the corresponding transfer rate, it follows from Equation 2.4 that the *thermal resistance* for *conduction* is,

i.e. $\mathrm{R_t} = \dfrac{\text{potential difference}}{\text{corresponding transfer rate}}$,

Hence,

$$R_{t,cond} = \frac{T_{s,1} - T_{s,2}}{\dot{Q}_x} = \frac{L}{\lambda A_x} \tag{2.5}$$

Similarly, for electrical conduction in the same system, Ohm's law provides an electrical resistance of the form

$$R_e = \frac{E_1 - E_2}{I} = \frac{L}{kA} \quad \text{where } k \text{ is the electrical conductivity} \tag{2.6}$$

Comparing Equations 2.5 & 2.6, the heat flow rate $\dot{Q}_x$ is analogous to electric current I, while the temperature difference ($T_{s,1} - T_{s,2}$) is analogous to the potential difference $E_1 - E_2$. We shall now develop expressions for computing the thermal resistance for convection and radiation.

A thermal resistance may also be associated with heat transfer by convection at a surface. From Newton's law of cooling,

$$\dot{Q}_x = \alpha A (T_s - T_\infty) \tag{2.7}$$

The *thermal resistance* for *convection* is then

$$R_{t,conv} = \frac{T_s - T_\infty}{\dot{Q}_x} = \frac{1}{\alpha\, A} \tag{2.8}$$

The *equivalent thermal circuit* for the plane wall with convection surface conditions is shown in Figure 6b. The heat transfer rate may be determined from separate considerations of each element in the network. Since $\dot{Q}_x$ is constant throughout the network, it follows that

$$\dot{Q}_x = \frac{T_{\infty,1} - T_{s,1}}{1/\alpha_1 A} = \frac{T_{s,1} - T_{s,2}}{L/\lambda A} = \frac{T_{s,2} - T_{\infty,2}}{1/\alpha_2 A} \tag{2.9}$$

In terms of the *overall temperature difference*, $T_{\infty,1} - T_{\infty,2}$, and the *total thermal resistance*, R_{tot}, the heat transfer rate may also be expressed as

$$\dot{Q}_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}} \tag{2.10}$$

Because the conduction and convection resistances are in series and may be summed, it follows that

$$R_{tot} = \frac{1}{\alpha_1 A} + \frac{L}{\lambda A} + \frac{1}{\alpha_2 A} \tag{2.11}$$

Another resistance may be pertinent if a surface is separated from *large surroundings* by a gas. In particular, the radiation exchange between the surface and its surroundings may be required, and the rate is determined from equation (1.7). lt follows that a thermal resistance for radiation may be defined as

$$R_{t,rad} = \frac{T_s - T_{sur}}{\dot{Q}_{rad}} = \frac{1}{\alpha_r A} \qquad (2.12)$$

Convection and radiation resistances act in parallel and they may be combined to obtain a single, effective surface resistance.

1-2.1.1.2 The Composite Wall

Equivalent thermal circuits may also be used for more complex systems, such as composite walls. Such walls may involve any number of series and parallel thermal resistances due to layers of different materials. Consider the series composite wall of Figure 7. The one-dimensional heat transfer rate for this system may be expressed as

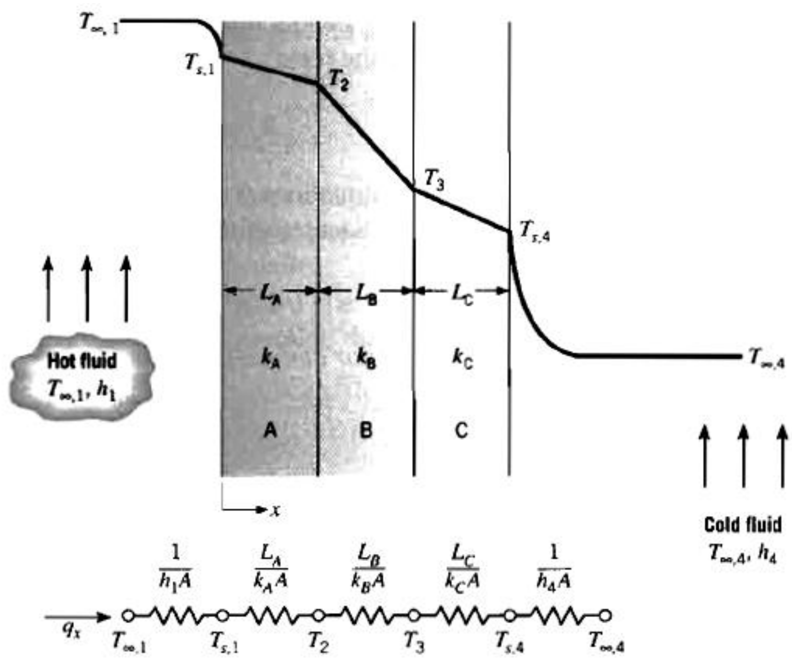


Figure 7: Equivalent thermal circuit for a series composite wall

$$\dot{Q} = \frac{T_{\infty,1} - T_{\infty,4}}{\sum R_t} \qquad (2.13)$$

Where $T_{\infty,1} - T_{\infty,4}$ is the overall temperature difference and the summation includes all thermal resistance.

Hence, $\dot{Q} = \dfrac{T_{\infty,1} - T_{\infty,4}}{\left[(1/\alpha_1 A) + (L_A/\lambda_A A) + (L_B/\lambda_B A) + (L_C/\lambda_C A) + (1/\alpha_4 A)\right]}$ (2.14)

Alternatively, the heat transfer rate can be related to the temperature difference and resistance associated with each element. For example,

$$\dot{Q} = \frac{T_{\infty,1} - T_{s,1}}{(1/\alpha_1 A)} = \frac{T_{s,1} - T_2}{(L_A/\lambda_A A)} = \frac{T_2 - T_3}{(L_B/\lambda_B A)} = \ldots \tag{2.15}$$

With composite system it is convenient to work with an overall heat transfer coefficient, U, which is defined by an expression analogous to Newton's law of cooling.

Accordingly,

$$\dot{Q} = U\ A\ \Delta T \tag{2.16}$$

where ΔT is the overall temperature difference. The overall heat transfer coefficient is related to the total thermal resistance, and from Equations (2.13) and (2.16) we see that $UA = 1/R_{tot}$. Hence, for the composite wall of Figure 7,

$$U = \frac{1}{R_{tot}A} = \frac{1}{\left[(1/\alpha_1) + (L_A/\lambda_A) + (L_B/\lambda_B) + (L_C/\lambda_C) + (1/\alpha_4)\right]} \tag{2.17}$$

In general, we may write

$$R_{tot} = \sum R_t = \frac{\Delta T}{\dot{Q}} = \frac{1}{UA} \tag{2.18}$$

Composite walls may also be characterised by series-parallel configurations, such as that shown in Figure 8.

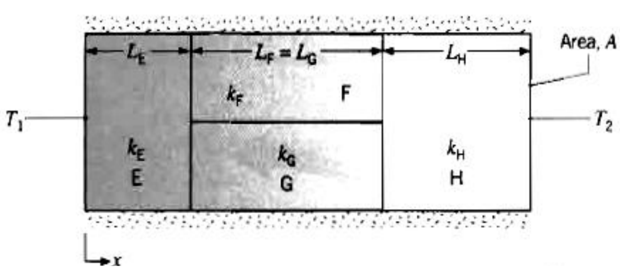


Figure 8: Series-parallel composite wall

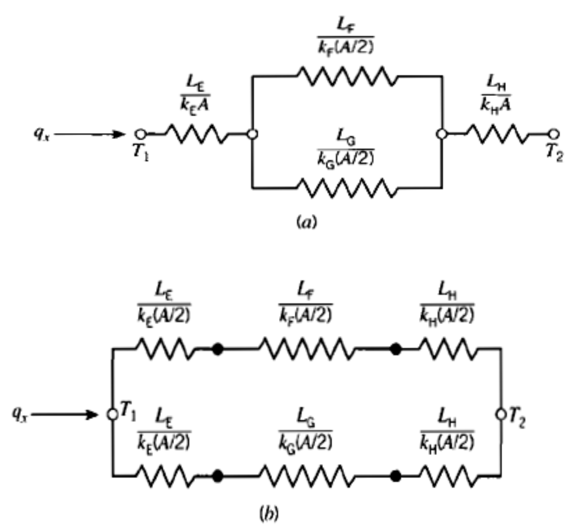


Figure 9: Two possible thermal circuits

For case (a), it is presumed that surfaces normal to the x direction are isothermal; while for case (b), it is assumed that surfaces parallel to the x direction are adiabatic. Both cases are based on the assumption of one-dimensional heat flow.

1-2.1.1.3 Contact Resistance

It is important to recognise that in composite systems, the temperature drop across the interface between materials may be appreciable.

This temperature change may be attributed to what is known as the thermal contact resistance, $R_{t,c}$. The effect is shown in Figure 10, and for a unit area of the surface, the resistance is defined as

$$R_{t,c}'' = \frac{T_A - T_B}{\dot{q}} \qquad (2.19a)$$

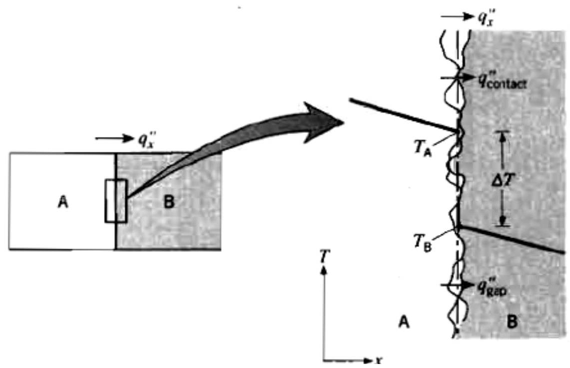


Figure 10: Effect of contact resistance

The heat transfer through the interface is the sum of the heat transfers through the solid contact spots and the gaps in the non-contact areas and can be expressed as:

$$\dot{Q} = \dot{Q}_{\text{contact}} + \dot{Q}_{\text{gap}} = \alpha_c \, A \Delta T_{\text{interface}}$$

where α_c is the thermal contact conductance and is related to the thermal contact resistance by

$$R_{t,c}'' = \frac{1}{\alpha_c} = \frac{\Delta T_{\text{interface}}}{\dot{Q}/A} \qquad (2.19b)$$

Contact spots are interspersed with gaps that are, in most instances, air filled. Heat transfer is therefore due to conduction across the actual contact area and due to convection and/or radiation across the gaps.

The value of thermal contact resistance depends on the *surface roughness* and the *material properties* as well as the *temperature* and *pressure* at the interface and type of fluid trapped at the interface. Thermal contact resistance is observed to *decrease* with *decreasing surface roughness* and *increasing interface pressure*. Thermal contact resistance can be minimised by applying thermal conducting liquid called *thermal grease* such as silicon oil.

1-2.2 Radial Systems

Cylindrical and spherical systems often experience temperature gradients in the radial direction only and therefore may be treated as one-dimensional. Under steady state conditions with no heat generation, such systems may be analysed by using the standard method, which begins with the appropriate form of heat equation, or the alternative method, which begins with the appropriate form of Fourier's law.

1-2.2.1The cylinder

A common example is the hollow cylinder whose inner and outer surfaces are exposed to fluids at different temperatures (Figure 11). For steady state conditions with no heat generation, the appropriate form of the one-dimensional heat conduction equation in cylindrical coordinates is:

$$\frac{1}{r}\frac{d}{dr}\left(\lambda r\frac{dT}{dr}\right)=0 \qquad (2.20)$$

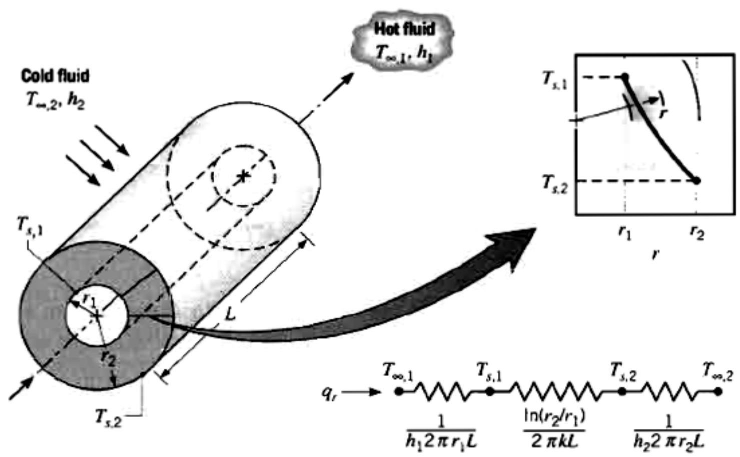


Figure 11: Hollow cylinder with convective surface conditions

From the Fourier's law, the rate at which energy is conducted across any cylindrical surface in the solid may be expressed as:

$$\dot{Q}_r = -\lambda A_r \frac{dT}{dr} = -\lambda(2\pi rL)\frac{dT}{dr} \qquad (2.21)$$

where $A_r = 2\pi rL$ is the area normal to the direction of heat transfer.

We determine the temperature distribution in the cylinder by solving Equation 2.20 and applying appropriate boundary conditions. Assuming the value of λ to be constant, Equation 2.20 may be integrated twice to obtain the general solution

$$T(r) = C_1 \text{In}(r) + C_2 \tag{2.22}$$

To evaluate the constants we introduce the boundary conditions:

$T(r_1) = T_{s,1}$ and $T(r_2) = T_{s,2}$

Solving for the constants and substituting in the general solution, we then obtain

$$T(r) = \frac{T_{s,1} - T_{s,2}}{\text{In}(r_1 / r_2)} \text{In}\left(\frac{r}{r_2}\right) + T_{s,2} \tag{2.23}$$

Thus, the temperature distribution associated with radial conduction through a cylindrical wall is logarithmic, not linear, as it is for the plane wall.

If the temperature distribution, Equation 2.23, is now used with Fourier's law, Equation 2.21, we obtain the following for the heat transfer rate:

$$\dot{Q}_r = \frac{2\pi L\lambda(T_{s,1} - T_{s,2})}{\text{In}(r_2 / r_1)} \tag{2.24}$$

From this result, it is evident that for radial conduction in a cylindrical wall, the thermal resistance is of the form:

$$R_{t,cond} = \frac{\text{In}(r_2 / r_1)}{2\pi L\lambda} \tag{2.25}$$

The alternative method of obtaining Equation (2.25) is to integrate Equation 2.21

Consider now the composite system of Figure 12, neglecting interfacial contact resistance the heat transfer rate is expressed as

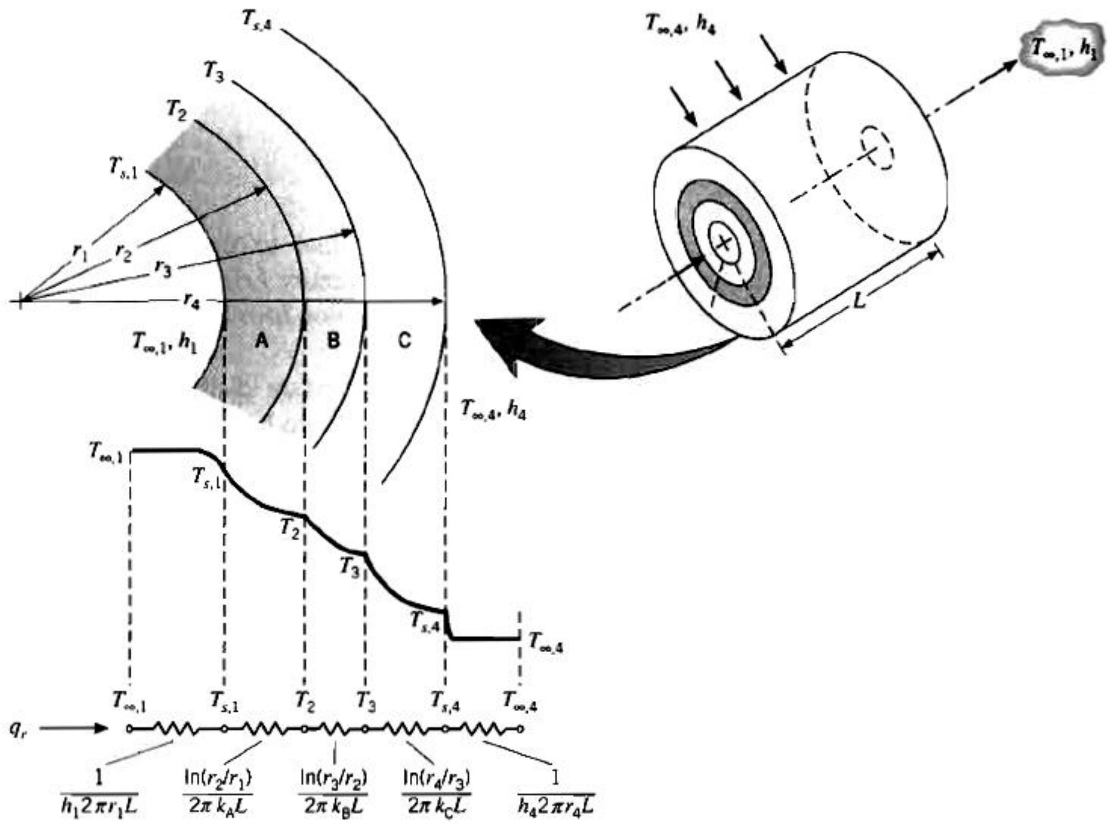


Figure 12: Temperature distribution for a composite cylindrical wall

$$\dot{Q}_r = \frac{T_{\infty,1} - T_{\infty,4}}{\left[\frac{1}{2\pi r_1 L\alpha_1} + \frac{\text{In}(r_2/r_1)}{2\pi\lambda_A L} + \frac{\text{In}(r_3/r_2)}{2\pi\lambda_B L} + \frac{\text{In}(r_4/r_3)}{2\pi\lambda_C L} + \frac{1}{2\pi r_4 L\alpha_4}\right]} \tag{2.26}$$

The above result is expressed in terms of the overall heat transfer coefficient as

$$\dot{Q}_r = \frac{T_{\infty,1} - T_{\infty,4}}{R_{tot}} = UA\left(T_{\infty,1} - T_{\infty,4}\right) = U_1 A_1 (\Delta T) = U_2 A_2 (\Delta T) = ... \tag{2.27}$$

Evaluating U based as inside area, A_1

If U is defined in terms of the inside area, $A_1 = 2\pi r_1 L$, Equations 2.26 and 2.27 may be equated to yield

$$U_1 = \frac{1}{\frac{1}{\alpha_1} + \frac{r_1}{\lambda_A}\text{In}\frac{r_2}{r_1} + \frac{r_1}{\lambda_B}\text{In}\frac{r_3}{r_2} + \frac{r_1}{\lambda_C}\text{In}\frac{r_4}{r_3} + \frac{r_1}{r_4}\frac{1}{\alpha_4}} \tag{2.28}$$

This definition is arbitrary, and the overall coefficient may also be defined in terms of A_4 or any of the intermediate area. Note that

$$U_1A_1 = U_2A_2 = U_3A_3 = U_4A_4 = \frac{1}{\sum R_t} \qquad (2.29)$$

and the specific forms of U_2, U_3 and U_4 may be inferred from Equations 2.27 and 2.28

1-2.2.2 Spherical Shells

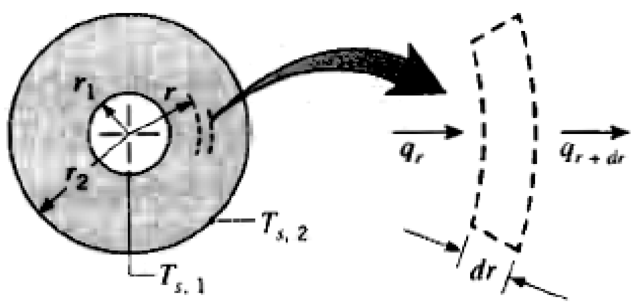


Figure 13: Hollow spherical shell

For the differential control volume, energy conservation requires that $\dot{q}_r = \dot{q}_{r+dr}$ .For steady-state, one-dimensional conditions with no heat generation, the appropriate form of the heat conduction equation in spherical coordinates is:

$$\frac{1}{r^2}\frac{d}{dr}(\lambda r^2 \frac{dT}{dr}) = 0 \qquad (2.30a)$$

And from Fourier's Law

$$\dot{Q}_r = -\lambda A_r \frac{dT}{dr} = -\lambda(4\pi r^2)\frac{dT}{dr} \qquad (2.30b)$$

Where $A_r = 4\pi r^2$ is the area normal to the direction of heat transfer. For constant $\dot{Q}_r$ and also constant λ, equation 2.30b may be integrated with the appropriate limits to yield

$$\dot{Q}_r = \frac{4\pi\lambda(T_{s,1} - T_{s,2})}{(1/r_1) - (1/r_2)} = \frac{(T_{s,1} - T_{s,2})}{R_{t,cond}} \qquad (2.31)$$

where $R_{t,cond} = \frac{1}{4\pi\lambda}(\frac{1}{r_1} - \frac{1}{r_2}) = \frac{r_2 - r_1}{4\pi\lambda r_2 r_1}$

r_1 is the inner radius and r_2 is the outer radius

1-2.3 Critical Radius of Insulation

We know that adding more insulation to a plane wall always decreases heat transfer. The thicker the insulation, the lower the heat transfer rate. This is expected, since the heat transfer area *A* is constant, and adding insulation always increases the thermal resistance of the wall without affecting the convection resistance.

Adding insulation to a cylindrical pipe or a spherical shell, however, is a different matter. The additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection. The heat transfer from the pipe may increase or decrease, depending on which effect dominates.

Consider a cylindrical pipe of outer radius r_1 whose outer surface temperature T_1 is maintained constant. The pipe is now insulated with a material whose thermal conductivity is λ and outer radius is r_2, Heat is lost from the pipe to the surrounding medium at temperature T_∞, with a convection heat transfer coefficient α. The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\dfrac{\text{In}(r_2 / r_1)}{2\pi L\lambda} + \dfrac{1}{\alpha(2\pi r_2 L)}} \tag{2.32}$$

The variation of $\dot{Q}$ with the outer radius of insulation r_2 is plotted in Figure 15. The value of r_2 at which $\dot{Q}$ reaches a maximum is determined from the requirement that $d\dot{Q}/dr_2$ is zero. Performing the differentiation and solving for r_2 yields the critical radius of insulation for a cylindrical body to be $r_{cr,cylinder} = \dfrac{\lambda}{\alpha}$.

To determine whether the foregoing result maximises or minimises the total resistance, the second derivative must be evaluated. A positive value of the second derivative for all possible values of r_2 will show that $r_{cr,cylinder} = \dfrac{\lambda}{\alpha}$ is the insulation radius for which the total resistance is a minimum, and *not* a maximum. Hence, an optimum insulation thickness *does not exist*.

Note that the critical radius of insulation depends on the thermal conductivity of the insulation λ and the external convection heat transfer coefficient α. The rate of heat transfer from the cylinder increases with the addition of insulation for $r_2 < r_{cr}$. It reaches a maximum when $r_2 = r_{cr}$, and starts to decrease for $r_2 > r_{cr}$. Thus, insulating the pipe may actually increase the rate of heat transfer from the pipe instead of decreasing it when $r_2 < r_{cr}$.

The value of the critical radius r_{cr} will be the largest when λ is large and α is small. Noting that the lowest value of α encountered in practice is about 2W/(m^2 K) for the case of natural convection gases, and that the thermal conductivity of common insulating materials is about 0.05 W/(m K), the largest value of the critical radius we are likely to encounter is

$$R_{cr,\max} = \frac{\lambda_{\max \text{insulation}}}{\alpha_{\min}} = \frac{0.05}{2} = 0.025\text{m} = 2.5\text{cm} = 25\text{mm}\,.$$

This value would be even smaller when the radiation effects are considered. The critical radius would be much less in forced convection. The radius or electric wires may be smaller than the critical radius; therefore the plastic electrical insulation may actually enhance the heat transfer from electric wires and thus keep their steady operating temperatures at a lower value and thus safer levels.

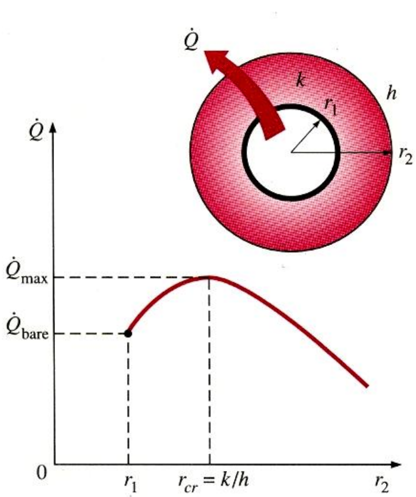


Figure 15: Variation of $\dot{Q}$ with the outer radius of Insulation, r_2

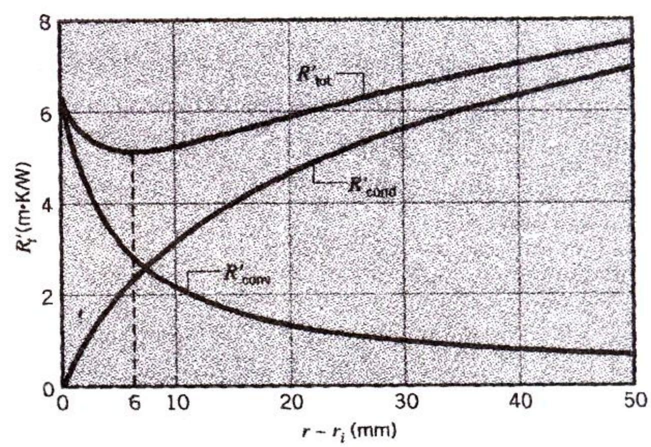


Figure 14: Variation of R_{th} with the outer radius of Insulation

The discussion above can be repeated for a sphere, and it can be shown in a similar manner that the critical radius of insulation for a spherical shell is

$$r_{cr\,sphere} = \frac{2\lambda}{\alpha}$$

1-2.4 Conduction with thermal energy generation

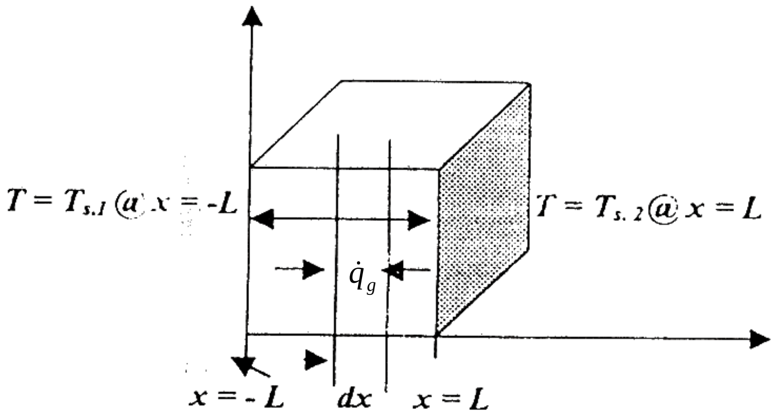


Figure 16: Control volume of a slab with thermal energy generation

Considering a slab of width $2L$ and an infinitely large length as shown in Figure 16, Let $\dot{q}_g$ = rate at which heat is generated per unit volume.

Considering the infinitesimal volume of length dx, Energy balance yield:

$$\dot{Q}_x + \dot{q}_g A dx = \dot{Q}_{x+dx} \tag{2.33}$$

Recall that, by the Taylor's series expansion,

$$f(x+dx) = f(x) + \frac{\partial f}{\partial x} dx + ---$$

$$= f(x) + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n f}{\partial x^n} dx^n$$

Since f(x) is dependent on position only, we can use total derivative

$$f(x+dx) = f(x) + \frac{df}{dx} dx + \ldots$$

$$\dot{Q}_{x+dx} = \dot{Q}_x + \frac{d\dot{Q}_x}{dx} dx + \ldots$$

$$\dot{Q}_{x+dx} - \dot{Q}_x = \frac{d\dot{Q}_x}{dx} dx + \ldots$$

According to Fourier's Law,

$$\dot{Q}_x = -A_x \lambda \frac{dT}{dx}$$

$$\therefore \dot{Q}_{x+dx} - \dot{Q}_x = \frac{d}{dx}\left[-A_x \lambda \frac{dT}{dx}\right] dx = -A_x \lambda \frac{d^2T}{dx^2} dx \qquad \text{if } A_x, \lambda = \text{constant}$$

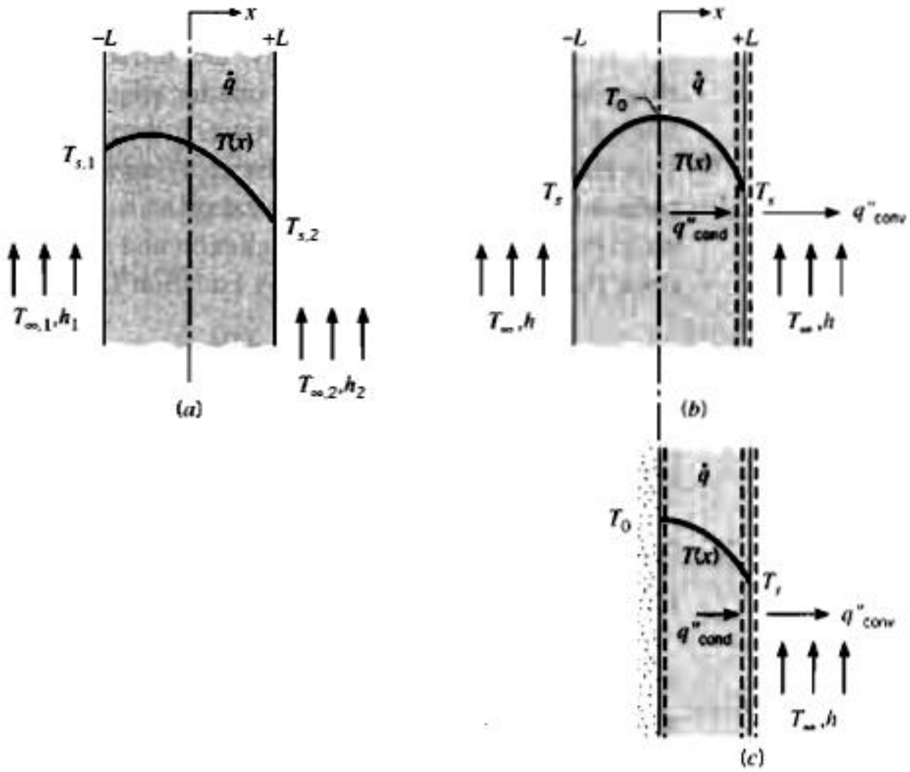


Figure 17: Conduction in a plane wall with heat generation

Substitute in Equation 2.33 to get

$$\dot{q}_g A_x dx = -A_x \lambda \frac{d^2T}{dx^2} dx$$

$$\frac{d^2T}{dx^2} = -\frac{\dot{q}_g}{\lambda} \qquad \textit{Second order differential equation}$$

Boundary condition is $T = T_{s,1}$ at $x = -L$ and $T = T_{s,2}$ at $x = L$

Integrating the differential equation we have

$$\frac{dT}{dx} = -\frac{\dot{q}_g}{\lambda} x + c_1$$

Integrating again we have

$$T(x) = -\frac{\dot{q}_g}{2\lambda} x^2 + C_1 x + C_2$$

Applying boundary conditions $T(-L) = T_{s,1}$ and $T(L) = T_{s,2}$

The constants of integration may be evaluated and are of the form

$$C_1 = \frac{T_{s,2} - T_{s,1}}{2L} \text{ and } C_2 = \frac{\dot{q}_g}{2\lambda} L^2 + \frac{T_{s,1} + T_{s,2}}{2}$$

$$T(x)=\frac{\dot{q}_g L^2}{2\lambda}(1-\frac{x^2}{L^2})+\frac{T_{s,1}-T_{s,2}}{2}\frac{x}{L}+\frac{T_{s,1}+T_{s,2}}{2} \qquad (2.36)$$

Note, however; that with generation the heat flux is no longer independent of *x*.

The preceding simplifies when both are maintained at a common temperature, such that $T_{s1} = T_{s2} = T_s$. The temperature distribution is then *symmetrical* about the mid-plane as in Figure 17b, as is given by

$$T(x)=\frac{\dot{q}_g L^2}{2\lambda}\left(1-\frac{x^2}{L^2}\right)+T_s \qquad (2.37)$$

The maximum temperature exists at the mid-plane

$$T(0)=T_o=\frac{\dot{q}L^2}{2\lambda}+T_s \qquad (2.38)$$

in which case the temperature distribution may be expressed as

$$\frac{T(x)-T_0}{T_s-T_0}=\left(\frac{x}{L}\right)^2 \qquad (2.39)$$

It is important to note that at the plane of symmetry in Figure 17b, the temperature gradient is zero, $(dT/dx)_{x=0}=0$. Accordingly, there is no heat transfer across this plane, and it may be represented by an adiabatic surface. One implication of this result is that equation 2.37 also applies to plane walls that are perfectly insulated on one side $(x=0)$ and maintained at a fixed temperature T_s on the other side $(x=L)$.

Note: A material having a thermal heat generation cannot be represented by a thermal circuit.

In figure 17c, we can write the energy balance equation at $x=L$ as,

$$-\lambda\left.\frac{dT}{dx}\right|_{x=L}=\alpha(T_s-T_\infty) \qquad (2.40)$$

Assignment:

Derive the above relation for a radial system

Note that:

$$\dot{q}_r+\dot{q}_g A_r dr=\dot{q}_{r+dr} \qquad q_r+q_g A_r dr=q$$

$$\dot{q}_{r+dr}=\dot{q}_r+\frac{d\dot{q}_r}{dr}dr$$

$$\dot{q}_{r+dr}-\dot{q}_r=\frac{d}{dr}\left[-A_r\lambda\frac{dT}{dr}\right]dr$$

Boundary conditions $T(r_o)=T_s$, and $\left.\frac{dT}{dr}\right|_{r,0}=0$

Self Assessment 1-2

1. Consider a 5-m-high, 8-m-long, and 0.22-m-thick wall whose representative cross-section is as given in Figure Q1. The thermal conductivities of various materials used, in W/m °C, are $\lambda_A = \lambda_F = 2$, $\lambda_B = 8$, $\lambda_C = 20$, $\lambda_D = 15$, and $\lambda_E = 35$. The left and right surfaces of the wall are maintained at uniform temperatures of 300°C and 100°C, respectively. Assuming heat transfer through the wall to be one-dimensional, determine (a) the rate of heat transfer through the wall; (b) the temperature at the point where the sections B, D, and E meet; and (c) the temperature drop across the section F. Disregard any contact resistances at the interfaces.

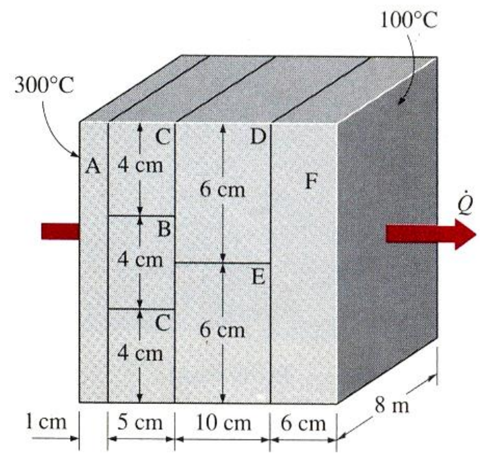


Figure Q1

2. Steam at 320°C flows in a stainless steel pipe (λ = 15 W/m °C) whose inner and outer diameter are 5 cm and 5.5 cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation (λ = 0.038 W/m °C). Heat is lost to the surroundings at 5°C by natural convection and radiation, with a combined natural convection and radiation heat transfer coefficient of 15 W/m^2 °C. Taking the heat transfer coefficient inside the pipe to be 80 W/m^2 °C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

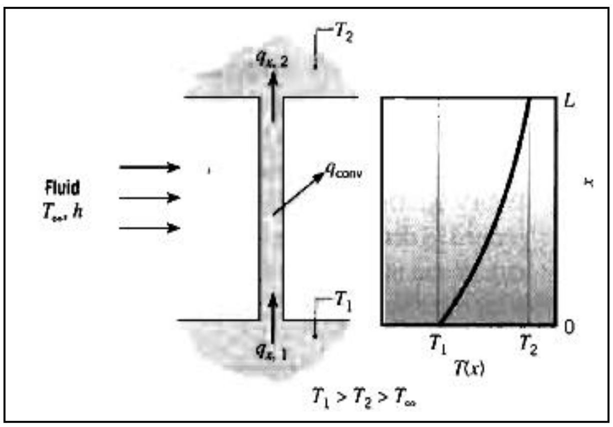
3. A spherical pressure vessel of 1 m inside diameter is made of 20 mm steel plate. The vessel is lagged with a 25 mm thickness of vermiculite held in position by 10 mm thick asbestos. The heat transfer coefficient for the outside surface is 20 W/m^2 K, and the thermal conductivities of steel, vermiculite, and asbestos are 48, 0.047, and 0.21 W/m K, respectively. Neglecting radiation, calculate the rate of heat loss from the sphere when the inside surface is at 500 ^{0}C, and the room temperature is 20 ^{0}C.

Suggested Answers

1. **[189 kW; 258 °C, 141 °C]**
2. **[94 W; 0.095 °C; 290.5 °C]**
3. **[2.744 kW]**

SESSION 2-2: HEAT TRANSFER FROM EXTENDED SURFACES (CONDUCTION-CONVECTION SYSTEMS)

The term *extended surface* is commonly used in reference to a solid that experiences energy transfer by conduction within its boundaries, as well as energy transfer by convection (and/or radiation) between its boundaries and the surroundings. Such a system is schematically shown in Figure 18. Although there are many different situations that involve combined conduction-convection effects, the most frequent application is one in which an extended surface is used specifically to *enhance* the heat transfer rate between a solid and an adjoining fluid. Such an extended surface is termed a *fin*.



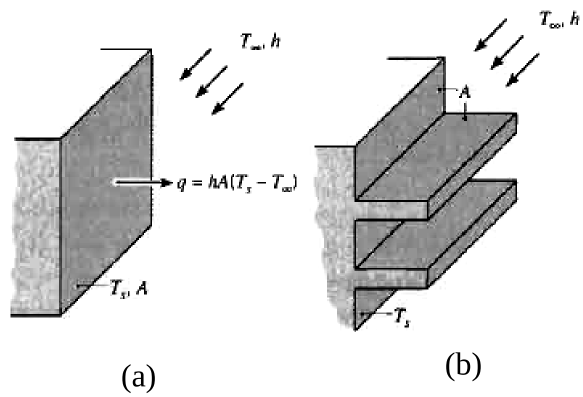


Figure 19: Combined conduction-convection in a structural member

Figure 18: Use of fins to enhance heat transfer

Consider the plane wall of Figure 19a. If T_s is fixed, there are two ways in which the heat transfer rate may be increased. The convection heat transfer coefficient α could be increased by increasing the fluid velocity, and/or the fluid temperature T_∞ could be reduced. However, many situations would be encountered in which increasing α to the maximum possible value is either insufficient to obtain the desired heat transfer rate or the associated costs are prohibitive. The second option of reducing T_∞ is impractical. There exists a third option. That is, increasing the surface area across which the convection occurs may increase the heat transfer rate. Employing *fins* that *extend* from the wall into the surrounding fluid may do this.

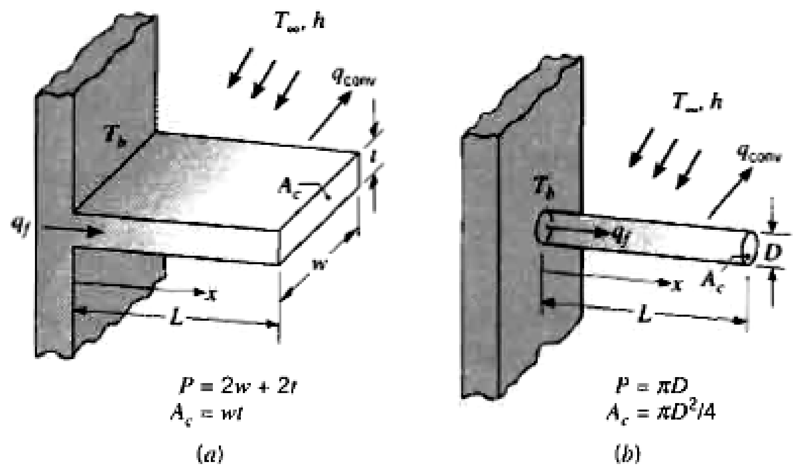


Figure 20: Straight fins of uniform cross section

Applying the conservation energy requirement to a differential element of Figure 20, we obtain heat transfer by conduction into section at x = Heat conduction away from section at $x + dx$ plus heat transfer by convection away from the section

$$\Rightarrow \dot{Q}_x = \dot{Q}_{(x+dx)} + d\dot{Q}_{conv} \tag{2.41}$$

From Fourier's law, we know that $\dot{Q}_x = -\lambda A_c \dfrac{dT}{dx}$

$$\dot{Q}_{x+dx} = \dot{Q}_x + \frac{d\dot{Q}_x}{dx} dx \tag{2.42}$$

It follows that

$$\dot{Q}_{x+dx} = -\lambda A_c \frac{dT}{dx} - \lambda \frac{d}{dx}\left(A_c \frac{dT}{dx} \right) dx \tag{2.43}$$

The convection heat transfer rate may be expressed as

$$d\dot{Q}_{conv} = \alpha dA_s (T - T_\infty) \tag{2.44}$$

where dA_s, is the surface area of the differential element. Substituting the foregoing rates into the energy balance, Equation 2.41, we obtain,

$$\frac{d}{dx}\left(A_c \frac{dT}{dx} \right) - \frac{\alpha}{\lambda} \frac{dA_s}{dx} (T - T_\infty) = 0$$

or (2.45)

$$\frac{d^2T}{dx^2} + \left(\frac{1}{Ac} \frac{dA_c}{dx} \right) \frac{dT}{dx} - \left(\frac{1}{A_c} \frac{\alpha}{\lambda} \frac{dA_s}{dx} \right) (T - T_\infty) = 0$$

Equation 2.45 provides a general form of the energy equation for one-dimensional conditions in an extended surface.

2-2.1 Fins of uniform cross-sectional area

We consider straight rectangular and pin fins of uniform cross section (Figure 20). Each fin is attached to a base surface of temperature $T(0) = T_b$ and extends into a fluid of temperature T_∞ For the prescribed fins, A_c is a constant and $A_s = Px$. A_s is the surface area measured from the base to x and P is the fin perimeter. Accordingly, with $dA_c / dx = 0$, and $dA_s / dx = P$, Equation 2.45 reduces to

$$\frac{d^2T}{dx^2} - \frac{\alpha P}{\lambda A}(T - T_\infty) = 0 \tag{2.46}$$

In simplifying the above relation, we transform the dependent variable by defining an *excess temperature* θ as

$$\theta(x) = T(x) - T_\infty \tag{2.47}$$

where, T_∞ is a constant, $d\theta / dx = dT / dx$. Substituting Equation 2.47 into Equation 2.46, we obtain

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \tag{2.48}$$

where

$$m^2 = \frac{\alpha P}{\lambda A_c} \tag{2.49}$$

Equation 2.48 is a linear, homogeneous, second-order differential equation with constant coefficients. Its general solution is of the form

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \tag{2.49a}$$

We seek to equation 2.49a for the following three boundary conditions:

Boundary conditions

a. The fin is infinitely long (Case 1)
b. Insulated at the end (Case 2)
c. Convection at the end (Case 3)

Case 1: (Infinitely long fin, L = x = ∞)

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$\theta(0) = T_b - T_\infty = \theta_b$

$\Rightarrow T_b - T_\infty = C_1 e^0 + C_2 e^0$

$T_b - T_\infty = C_1 + C_2 = \theta_b$

$\theta = 0$ @ $x = \infty$

$\Rightarrow C_1 e^\infty + C_2 e^{-\infty} = 0$. Note that this condition will be satisfied by e^{-mx} but not by the other prospective solution function e^{mx} since it turns to infinity as x gets larger. Therefore, a general solution of this will consist of a constant multiple of e^{-mx}. $\Rightarrow C_1 = 0$ and $\therefore C_2 = \theta_b$

The general solution is thus $\theta(x) = \theta_b e^{-mx}$ *or* $\dfrac{\theta(x)}{\theta_b} = e^{-mx}$ (2.51a)

$$\boxed{T(x) - T_\infty = (T_b - T_\infty)e^{-mx} = e^{-x\sqrt{\alpha P/\lambda A_c}}(T_b - T_\infty)} \tag{2.51b}$$

The steady rate of heat transfer from the entire fin can be determined from Fourier's law of heat conduction as

$$\boxed{\dot{Q}_{\text{long fin}} = -\lambda A_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{\alpha P \lambda A_c}\,(T_b - T_\infty)} \tag{2.52}$$

Case 2: Negligible heat loss from fin tip (Insulated fin tip, ($\dot{Q}_{fin\ tip} = 0$)

$$\theta(0) = T_b - T_\infty\ ; \qquad \left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

$$C_1 + C_2 = T_b - T_\infty$$

and $C_1 e^{mL} - C_2 e^{-mL} = 0$

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

The rate of heat transfer from the fin can be determined again from Fourier's law of heat conduction

$$\boxed{\dot{Q}_{\text{insulatedtip}} = -\lambda A_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{\alpha P \lambda A_c}\,(T_b - T_\infty)\tanh mL} \tag{2.54}$$

Comments:

1. Since there is no heat loss from the tip of an infinitely long rod, an estimate of the validity of this approximation may be made by comparing Equations 2.52 and 2.54. To a satisfactory approximation, the expressions provide equivalent results if $\tanh mL \geq 0.99$ or $mL \geq 2.65$. Hence a rod may be assumed to be infinitely long if:

$$\boxed{L \geq L_\infty \equiv \frac{2.65}{m} = 2.65\left(\frac{\alpha \lambda A_c}{\lambda P}\right)^{1/2}}$$

2. Fin heat transfer rate may accurately be predicted from the infinite fin approximation if $ml \geq 2.65$. However, if the infinite fin approximation is to accurately predict the temperature distribution *T(x)*, a larger value of *mL* would be required. This value may be inferred from Equation 2.51a and the requirement that the tip temperature be very close to the fluid temperature. Hence, if it is required that $\theta(L)/\theta_b = e^{-mL} < 0.01$, then it follows that $mL > 4.6$.

Case 3: Convection at the end (or combined convection and radiation)

$$T - T_\infty = T_b - T_\infty \Rightarrow \theta(0) = T_b - T_\infty$$

$$\dot{q}_{cond} = -A\lambda \frac{dT}{dx}\, at = x = L$$

$$\dot{q}_{conv} = A\alpha(T - T_\infty)at = x = L$$

$$-A\lambda \frac{dT}{dx}\bigg|_{x=L} = A\alpha(T(L) - T_\infty)$$

$$C_1 + C_2 = T_b - T_\infty$$

and

$$\alpha\left(C_1 e^{mL} + C_2 e^{-mL}\right) = \lambda m\left(C_2 e^{-mL} - C_1 e^{mL}\right)$$

Solving for C_1 and C_2 we obtain,

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (\alpha/m\lambda)\sinh m(L-x)}{\cosh mL + (a/m\lambda)\sinh mL} \tag{2.55}$$

The rate of heat transfer from the fin base is given by the Fourier's law as

$$\boxed{\dot{Q}_{\text{convectionfrom fin tip}} = \sqrt{\alpha P \lambda A_c}\,\frac{\sinh mL + (\alpha/m\lambda)\cosh mL}{\cosh mL + (\alpha/m\lambda)\sinh mL}(T_b - T_\infty)} \tag{2.56}$$

2-2.1.1 Fin Effectiveness

Fins are used to increase the heat transfer from a surface by increasing the effective surface area. However, a fin itself represents a conduction resistance to heat transfer from the original surface. For this reason, there is no assurance that the heat transfer rate will be increased through the use of fins. An assessment of this matter may be made by evaluating the fin effectiveness ε_f. It is defined as the ratio of the fin heat transfer rate to the heat transfer rate that would exist without the fin.

$\varepsilon_{fin} = \dfrac{\dot{Q}_{fin}}{\alpha A_{c,b}(T_b - T_\infty)}$, where $A_{c,b}$ is the fin cross-sectional area at the base of the material

In a rational design, ε_{fin} should be large but the use of fins is justified if $\varepsilon_{fin} \geq 2$

The rate of heat transfer from a sufficiently long fin of uniform cross section under steady conditions is given by Eq. 2.52. The effectiveness of such a long fin is determined to be

$$\varepsilon_{longfin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{nofin}} = \frac{\sqrt{\alpha P \lambda A_c}(T_b - T_\infty)}{\alpha A_b (T_b - T_\infty)} = \sqrt{\frac{\lambda P}{\alpha A_c}} \quad \text{since } A_c = A_b \text{ in this case.}$$

Comments on fin effectiveness:

1. Fin effectiveness is enhanced by the choice of a material of high thermal conductivity. Aluminium alloys are mostly common. Copper and iron are other options. However, although copper is superior from the stand-point of thermal conductivity, aluminium alloys are the more common choice because of additional benefits related to lower cost and weight.

2. Fin effectiveness is also enhanced by increasing the ratio of the perimeter to the cross-sectional area. For this reason the use of thin plate and slender pin, but closely spaced fins, is preferred, with the proviso that the fin gap not be reduced to a value for which flow between the fins is severely impeded, thereby reducing the convection coefficient.

3. The use of fins is most effective in applications involving a low convection heat transfer coefficient. Thus, the use of fins is more easily justified when the medium is a gas instead of a liquid and the heat transfer is by natural convection instead of by forced convection (Table 1)

2-2.1.2 Fin efficiency

It is defined as the ratio of the heat transferred from the fin surface to that which would be transferred if the whole of the fin surface were at the temperature of the base surface

The fin efficiency is defined as

$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin\ \max}} = \frac{\text{Actual heat transfer rate from the fin surface}}{\text{Ideal heat transfer from the fin if the entire fin surface were at the base temperature}}$$

$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\alpha A_{fin}\left(T_b - T_\infty\right)}$$

A_{fin} is the total surface area of the fin

The fin efficiency and fin effectiveness are related to each other by

$$\boxed{\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\dot{Q}_{fin}}{\alpha A_b (T_b - T_\infty)} = \frac{\eta_{fin}\alpha A_{fin}(T_b - T_\infty)}{\alpha A_b (T_b - T_\infty)} = \frac{A_{fin}}{A_b}\eta_{fin}}$$

Note: charts and formulae for determining efficiency of some common fin shapes are presented in Figures 21 and 22 as well as in Table 4

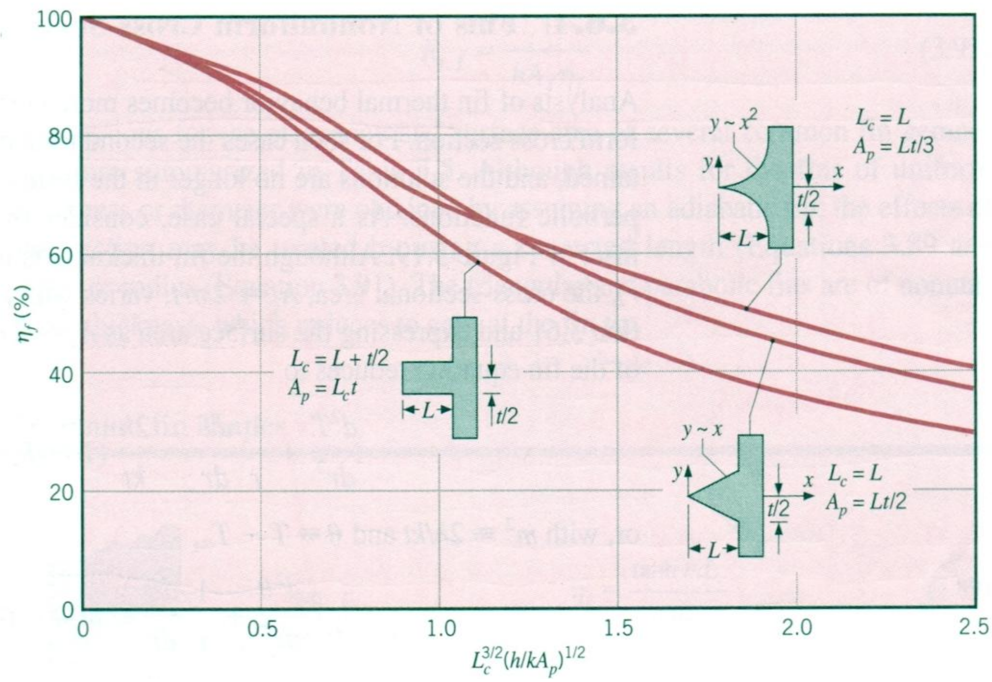


Figure 21: Efficiency of straight fins (rectangular, triangular and parabolic profiles)

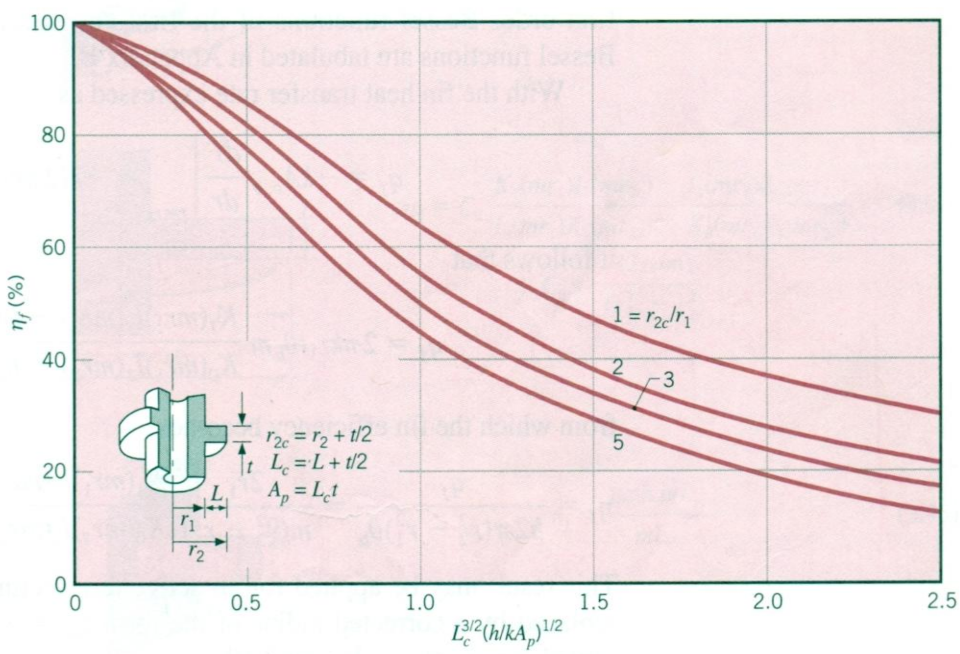
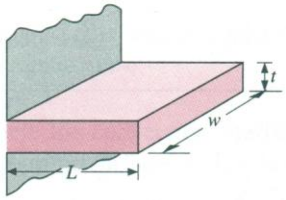
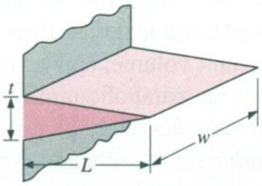
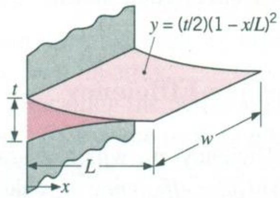
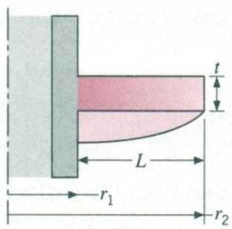
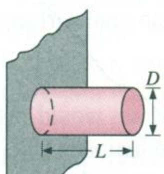
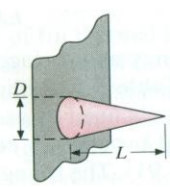
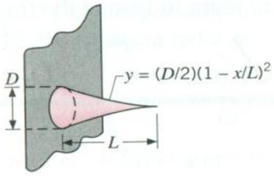


Figure 22: Efficiency of annular fins of rectangular profile.

Table 4: Efficiency of some common fin shapes

Straight Fins		
Rectangular[a] $A_f = 2wL_c$ $L_c = L + (t/2)$		$\eta_f = \dfrac{\tanh mL_c}{mL_c}$
Triangular[a] $A_f = 2w[L^2 + (t/2)^2]^{1/2}$		$\eta_f = \dfrac{1}{mL}\dfrac{I_1(2mL)}{I_0(2mL)}$
Parabolic[a] $A_f = w[C_1L + (L^2/t)\ln(t/L + C_1)]$ $C_1 = [1 + (t/L)^2]^{1/2}$		$\eta_f = \dfrac{2}{[4(mL)^2 + 1]^{1/2} + 1}$
Circular Fin		
Rectangular[a] $A_f = 2\pi(r_{2c}^2 - r_1^2)$ $r_{2c} = r_2 + (t/2)$		$\eta_f = C_2\dfrac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$ $C_2 = \dfrac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$
Pin Fins		
Rectangular[b] $A_f = \pi DL_c$ $L_c = L + (D/4)$		$\eta_f = \dfrac{\tanh mL_c}{mL_c}$
Triangular[b] $A_f = \dfrac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$		$\eta_f = \dfrac{2}{mL}\dfrac{I_2(2mL)}{I_1(2mL)}$
Parabolic[b] $A_f = \dfrac{\pi L^3}{8D}\{C_3C_4 - \dfrac{L}{2D}\ln[(2DC_4/L) + C_3]\}$ $C_3 = 1 + 2(D/L)^2$ $C_4 = [1 + (D/L)^2]^{1/2}$		$\eta_f = \dfrac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$

[a] $m = (2h/kt)^{1/2}$.
[b] $m = (4h/kD)^{1/2}$.

Worked Example 2.1

Aluminium fins 1.5 cm wide and 1.0 mm thick are placed on 2.5 cm diameter tube to dissipate the heat. The tube surface temperature is 170 °C, and the ambient fluid temperature is 25 °C. Calculate the heat loss per fin for $h = 130\ \mathrm{W/(m^2.°C)}$. Assume $\lambda = 200\ \mathrm{W/(m.°C)}$ for aluminium.

SOLUTION

We compute the heat transfer by using the fin efficiency curves and formulae in Figures 21 and 22 respectively. The parameters needed are

$$L_c = L + t/2 = 1.5 + 0.05 = 1.55\ \mathrm{cm}$$

$$r_1 = \frac{D_1}{2} = \frac{2.5}{2} = 1.25\ \mathrm{cm},\ r_{2c} = r_1 + L_c = 1.25 + 1.55 = 2.80\ \mathrm{cm}$$

$$\frac{r_{2c}}{r_1} = \frac{2.80}{1.25} = 2.24$$

$$A_m = t\left(r_{2c} - r_1\right) = 0.001(2.8 - 1.25)\left(10^{-2}\right) = 1.55 \times 10^{-5}\ \mathrm{m^2}$$

$$L_c^{3/2}\left(\frac{h}{\lambda A_m}\right)^{\frac{1}{2}} = (0.0155)^{3/2}\left[\frac{130}{200 \times 1.55 \times 10^{-5}}\right]^{\frac{1}{2}} = 0.396$$

From figure 21, the efficiency of the fins, η_f = 82%.

The heat transferred if the entire fin were at the base temperature with both sides of the fin exchanging heat is given by

$$\dot{Q}_{max} = hA_{fin}\left(T_b - T_\infty\right) = 2\pi\left(r_{2c}^2 - r_1^2\right)h\left(T_b - T_\infty\right) = 2\pi\left(2.8^2 - 1.25^2\right) \times 10^{-4} \times 130(170 - 25) = 74.35\mathrm{W}$$

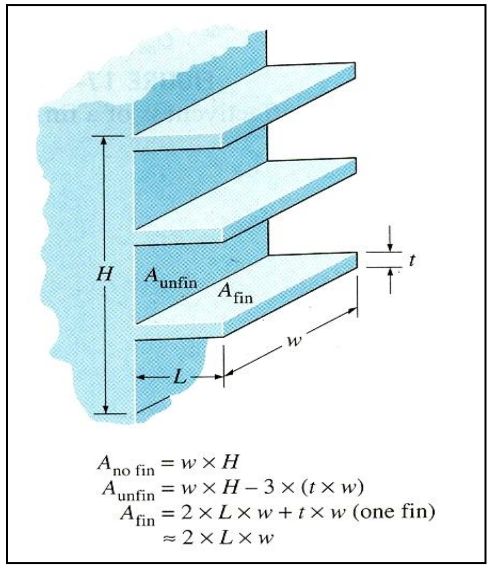
Hence the actual heat transfer from the fin surface is

$$\dot{Q}_{fin} = \dot{Q}_{fin,max} \times \eta_{fin} = 74.35 \times 0.82 = 60.97\ \mathrm{W}$$

2-2.1.3 Overall effectiveness

Total heat transfer from a surface will consider the unfinned portion of the surface as well as the fins

$$\begin{aligned}\dot{Q}_{total\,fin} &= \dot{Q}_{unfin} + \dot{Q}_{fin}\\ &= \alpha A_{unfin}(T_b - T_\infty) + \eta_{fin}\alpha A_{fin}(T_b - T_\infty)\\ &= \alpha(A_{unfin} + \eta_{fin}A_{fin})(T_b - T_\infty)\end{aligned}$$



Overall effectiveness is defined as

$$\varepsilon_{fin,overall} = \frac{\dot{Q}_{total,\,fin}}{\dot{Q}_{total,no\,fin}} = \frac{\alpha(A_{unfin} + \eta_{fin}A_{fin})(T_b - T_\infty)}{\alpha A_{no\,fin}(T_b - T_\infty)}$$

where $A_{no\,fin}$ is the area of the surface when there are no fins, A_{fin} is the total surface area of all the fins on the surface, and A_{unfin} is the area of the unfinned portion of the surface.
The overall effectiveness is a better measure of the performance of a finned surface than the effectiveness of the individual fins.

2-2.1.4 Overall Surface Efficiency

The overall surface efficiency η_o characterises an array of fins and the base surface to which they are attached. It is defined as

$$\eta_o = \frac{\dot{Q}_{total}}{\dot{Q}_{\max}} = \frac{\dot{Q}_t}{\alpha A_t \theta_b}$$

where $\dot{Q}_t$ is the total heat rate from the surface area A_t associated with both the fins and the exposed portion of the base (often termed the prime surface).

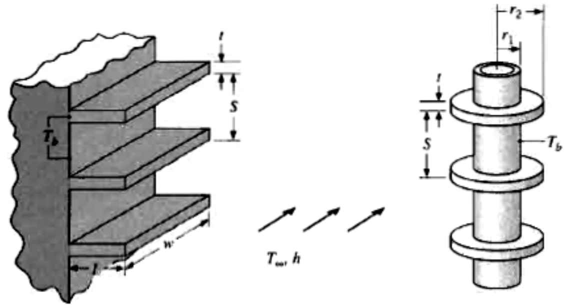


Figure 23: Representative fin arrays (a) Rectangular fins (b) Annular fins

For N fins in the array, each of surface area A_f, and the area of the prime surface is designated as A_b, the total surface area is:

$$A_t = NA_f + A_b$$

The maximum possible heat rate would result if the entire fin surface, as well as the exposed base, were maintained at T_b.
The total rate of heat transfer by convection from the fins and the prime (unfinned) surface may be expressed as

$$\dot{Q}_t = N\eta_f \alpha A_f \theta_b + \alpha A_b \theta_b$$

where η_f is the efficiency of a single fin. It follows that

$$\dot{Q}_t = \alpha\left[N\eta_f A_f + (A_t - NA_f)\right]\theta_b = \alpha A_t\left[1 - \frac{NA_f}{A_t}(1-\eta_f)\right]\theta_b$$

Thus, $\boxed{\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)}$

2-2.1.5 Pitch

The pitch, S, of an array of regularly spaced fins (Figure 23) is defined as the distance from a point on a fin to the corresponding point on an adjacent fin along a vertical or horizontal line between the two fins. It can be obtained by dividing a given distance by the number of fins within that distance. The pitch is the sum of the fin thickness and the inter-fin spacing and is the unit that is repeated throughout the fin array.

$$Pitch, S = \frac{\text{Distance}}{\text{Number of fins within the distance}} = t + s$$

where t is the fin thickness and
s is the interfin space

Heat transfer within a pitch can be considered; with the total heat transfer for the entire finned surface obtained by multiplying the computed result for the pitch by the total number of fins present.

Comments (for Practical fin design):

1. Fin length such that $mL = 1$ is an ideal fin length, which is a compromise between heat transfer, performance and fin size. For design purposes, a good guide is to assume that fins should not be used unless $\alpha b/\lambda < 0.2$, where b is the characteristic thickness of the fin.
2. Fins should also have a high thermal conductivity and hence would not normally be used on non-metallic surface.

2-2.2 Three Dimensional Heat Conduction

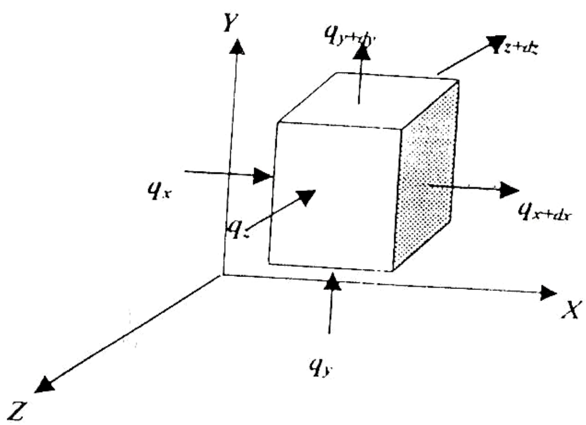


Figure 24: Illustration of heat flow in three dimensional Cartesian coordinate system

Unsteady or transient state considered

(Heat conducted into the volume + Heat generated within the volume) = (Heat conducted away from the volume + change in internal energy)

For unsteady state, change in internal energy is not zero and temperature is a function of time

- Heat conducted into the volume is given by Q and during time dt , total energy is $Q_x + Q_y + Q_z$
- If $\dot{q}_g$ is the rate of generation of heat energy per unit volume

  $\therefore$ Total energy generated is given by $\dot{q}_g\, dxdydz$
- Heat conducted away from the volume in $\left(\dot{Q}_{x+dx} + \dot{Q}_{y+dy} + \dot{Q}_{y+dz}\right)dt$
- Change in internal energy $mcdT = c\rho dT\left(dxdydz\right)$

Where $m = \rho dv = \rho dxdydz$

$$\therefore \left(\dot{Q}_x + \dot{Q}_y + \dot{Q}_z\right)dt + \dot{q}_g\, dxdydzdt = \left(\dot{Q}_{x+dx} + \dot{Q}_{y+dy} + \dot{Q}_{z+dz}\right)dt + c\rho dT\left(dxdydz\right)$$

Now

$$\begin{aligned}\left(\dot{Q}_x - \dot{Q}_{x+dx}\right)dt &= -\frac{\partial}{\partial x}\left[\dot{Q}_x\right]dxdt \\ &= -\frac{\partial}{\partial x}\left[-A\lambda\frac{\partial T}{\partial x}\right]dxdt\end{aligned} \qquad (2.58)$$

but A is the cross sectional area in the x-direction
$\therefore A = dydz$

Replacing A in Eq 2.58

$$\therefore -\frac{\partial}{\partial x}\left[-\lambda dydz\frac{\partial T}{\partial x}\right]dxdt = \frac{\partial}{\partial x}\left[\lambda\frac{\partial T}{\partial x}\right]dxdydzdt$$

Similarly

$$\left(\dot{q}_y - \dot{q}_{y+dy}\right)dt = \frac{\partial}{\partial y}\left[\lambda\frac{\partial T}{\partial y}\right]dxdydzdt$$

Etc

Finally we have:

$$\left[\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)\right]dvdt + \dot{q}_g\, dvdt = c\rho dTdv$$

Dividing by $dvdt$

$$\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right) + \dot{q}_g = \frac{\lambda}{\alpha_d}\frac{\partial T}{\partial t} \qquad (2.59)$$

Thus Equation 2.59 is the general conduction equation. Assuming λ is constant, we obtain:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}_g}{\lambda} = \frac{1}{\alpha_d}\frac{\partial T}{\partial t} \quad \text{(Fourier-Biot Equation)} \qquad (2.60)$$

Where $\alpha_d = \dfrac{\lambda}{c\rho}$ (Thermal diffusivity)

For no internal heat generation, $\dot{q}_g = 0$

$$\frac{\partial^2 T}{\partial \chi^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha_d}\frac{\partial T}{\partial t} \text{ (Fourier Equation)} \tag{2.61}$$

For steady state but $\dot{q}_g \neq 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}_g}{\lambda} = 0 \quad \text{(Poisson Equation)} \tag{2.62}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \text{ (Laplace Equation)} \ (\mathit{for}\ \dot{q}_g = 0, \mathit{at\ steady\ state}) \tag{2.63}$$

Equations using cylindrical or spherical coordinates may be derived in a similar way, or obtained from Equation 2.60 by transforming the coordinates. For one-dimensional problems (e.g. an infinitely long cylinder or a sphere), it is simpler to derive the equations directly.

(a) Infinite slab (see Figure 2). From equation 2.60 we have:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}_g}{\lambda} = \frac{1}{\alpha_d}\frac{\partial T}{\partial t} \tag{2.64}$$

(b) Infinitely long cylinder (see Figure 25). Applying an energy balance to an element of thickness, dr, we have:

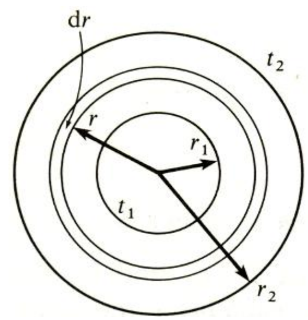


Figure 25: Cross-section through a cylinder

$$\dot{\mathrm{e}}_{\mathrm{in}} + \dot{\mathrm{e}}_{\mathrm{gen}} - \dot{\mathrm{e}}_{\mathrm{out}} = \dot{\mathrm{e}}_{\mathrm{st}}$$

$$\dot{q}_r + \dot{q}_g(2\pi r L\mathrm{d}r) - \dot{q}_{r+dr} = \rho c 2\pi r L \mathrm{d}r \frac{\partial T}{\partial t}$$

$$-\frac{\partial \dot{q}_r}{\partial r}\mathrm{d}r + \dot{q}_g(2\pi r L\mathrm{d}r) = \rho c 2\pi r L \mathrm{d}r \frac{\partial T}{\partial t}$$

$$-\frac{\partial}{\partial r}(-\lambda 2\pi r L\frac{\partial T}{\partial r})\mathrm{d}r + \dot{q}_g(2\pi r L\mathrm{d}r) = \rho c 2\pi r L \mathrm{d}r \frac{\partial T}{\partial t}$$

dividing through by $2\pi L\mathrm{d}r$

$$\frac{\partial}{\partial r}(\lambda r\frac{\partial T}{\partial r}) + \dot{q}_g r = \rho c r \frac{\partial T}{\partial t}$$

for λ = constant

$$(\lambda r\frac{\partial^2 T}{\partial r^2} + \lambda\frac{\partial T}{\partial r}) + \dot{q}_g r = \rho c r \frac{\partial T}{\partial t}$$

i.e.

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\dot{q}_g}{\lambda} = \frac{1}{\alpha_d}\frac{\partial T}{\partial t} \tag{2.65}$$

(c) Sphere (see Figure 26). applying an energy balance similar to the process followed in (b) above:

$$-\frac{\partial \dot{q}_r}{\partial r}\mathrm{d}r + \dot{q}_g(4\pi r^2\mathrm{d}r) = \rho c 4\pi r^2 \mathrm{d}r\frac{\partial T}{\partial t}$$

therefore

$$-\frac{\partial}{\partial r}(-\lambda 4\pi r^2\frac{\partial T}{\partial r})\mathrm{d}r + \dot{q}_g(4\pi r^2\mathrm{d}r) = \rho c 4\pi r^2 \mathrm{d}r\frac{\partial T}{\partial t}$$

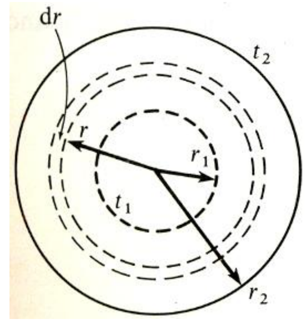


Figure 26: Cross section through a hollow sphere

i.e.

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{\dot{q}_g}{\lambda} = \frac{1}{\alpha_d}\frac{\partial T}{\partial t} \qquad (2.66)$$

For steady-state cases, the right-hand side of equations 2.64, 2.65 and 2.66 becomes zero, and the equations become ordinary differential equations.

Worked Example 2.2

A hollow cylindrical copper conductor of 30 mm outside diameter and 14 mm inside diameter has a current density of 40 A/mm^2. The external surface is covered with a uniform layer of insulation of thickness 10 mm, and the ambient temperature is 10^{o}C. Neglecting axial conduction and assuming that the temperature of the insulation must not exceed 135^{o}C at any point, calculate:

(i) The heat required to be removed per unit time by forced cooling from the inside of the conductor;

(ii) The temperature at the inside surface of the conductor.

Data Thermal conductivity of copper = 380 W/mK; thermal conductivity of insulating material = 0.3 W/mK; heat transfer coefficient at outside surface = 40 W/m^{2}K; electrical resistivity of copper = $2\times10^{-5}\,\Omega\mathrm{mm}$.

Solution From Equation 2.65, for the steady state:

$$\frac{\mathrm{d}^2 T}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}T}{\mathrm{d}r} + \frac{\dot{q}_g}{\lambda} = 0$$

or

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}T}{\mathrm{d}r}\right) = -\frac{\dot{q}_g}{\lambda}$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}T}{\mathrm{d}r}\right) = -\frac{\dot{q}_g}{\lambda}r$$

Hence integrating

$$r\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{\dot{q}_g r^2}{2\lambda} + C_1$$

therefore

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{\dot{q}_g r}{2\lambda} + \frac{C_1}{r} \qquad \text{(a)}$$

Integrating

$$T = -\frac{\dot{q}_g r^2}{4\lambda} + C_1 \mathrm{In}\, r + C_2 \qquad \text{(b)}$$

where C_1 and C_2 are integration constants.

The heat generated per unit volume due to the current flowing is given by:

$$\dot{q}_g = \frac{I^2 R}{AL}$$

where I is the current, R the electrical resistance, A the cross-sectional area, and L the length.

Also, current density, $J = I/A$ and resistance, $R = sL/A$

where s is the electrical resistivity of the conductor material,

i.e. $$\dot{q}_g = \frac{J^2 A^2 sL}{AL \cdot A} = J^2 s \qquad (2.67)$$

$$\dot{q}_g = 40^2 \times 2 \times 10^{-5}\ \mathrm{W/\ mm^3} = 32 \times 10^6\ \mathrm{W/m^3}$$

(i) The maximum temperature of the insulation (t_i = 135°C) occurs at the interface between the insulation and the copper tube. Hence for the insulation, heat transfer to the outside:

$$\dot{Q}_0 = \frac{(t_i - t)}{R_{\mathrm{ins}}} = \frac{2\pi \times 0.3(135 - t)}{\mathrm{In}(50/30)}$$

where t is the temperature of the outside surface of the insulation,

i.e. $$135 - t = 0.271\dot{Q}_0 \qquad \text{(c)}$$

For the heat transferred from the outside surface of the insulation by convection,

$$\dot{Q}_0 = \alpha A(t - t_\infty)$$

$$\dot{Q}_0 = 40 \times 2\pi \times 0.025(t - 10)$$

Therefore,

$$t - 10 = 0.159\dot{Q}_0 \qquad \text{(d)}$$

Adding equations (c) and (d)

$$135 - 10 = 0.43\dot{Q}_0 \qquad \text{therefore } \dot{Q}_0 = 290.7\,\mathrm{W}$$

$$\text{Total heat generated internally} = \dot{q}_g \times \text{volume}$$

$$= 32 \times 10^6 \times \frac{\pi}{4}(0.03^2 - 0.014^2)$$

$$= 17\,693.5\,\text{W}$$

Hence,

$$\text{Heat removed from inside of conductor} = (17\,693.5 - 290.7)\,\text{W}$$

$$= 17.4\,\text{kW}$$

(ii) Two boundary conditions are required to find the constants C_1 and C_2 and hence to obtain the solution of Equation (b).

At the inside of the conductor:

$$\text{Heat supplied to conductor} = -\lambda A\left(\frac{dt}{dr}\right)_{r=0.007} = -17\,400\,\text{W}$$

Therefore,

$$\left(\frac{dt}{dr}\right)_{r=0.007} = \frac{17\,400}{380 \times \pi \times 0.014} = 1041.1\,\text{K/m}$$

Substituting in equation (a)

$$1041.1 = -\left\{\frac{32 \times 10^6 \times 0.007}{2 \times 380}\right\} + \frac{C_1}{0.007}$$

Therefore,

$C_1 = 9.351$

At the outside surface of the conductor, $t = 135^{o}$C, hence in equation (b)

$$135 = -\left\{\frac{32 \times 10^6 \times 0.015^2}{4 \times 380}\right\} + 9.351\,\text{In}(0.015) + C_2$$

Therefore,

$C_2 = 179$

Therefore, the complete solution for the temperature distribution in the conductor is:

$$t = -\left(\frac{32 \times 10^6}{4 \times 380}\right) r^2 + (9.351\,\text{In}\,r) + 179$$

Hence, at the inside surface, when $r = 0.007$

$$t = -\left\{\frac{32 \times 10^6 \times 0.007^2}{4 \times 380}\right\} + (9.351\ \text{In}\,0.007) + 179$$

$$= 131.6^0 C$$

2-2.3 Unsteady Heat flow (cooling of a billet)

In heat transfer, bodies are observed to behave like a "lump" whose interior remains essentially uniform at all times during a heat transfer process. The initial temperature of the body in a given environment (α, T_∞) is (T_i, t = 0) and the temperature of such bodies may be taken to be a function of time only, i.e T (t). Heat transfer analysis that utilizes this idealization is known as lumped capacitance method of analysis and it provides great simplification to certain classes of heat transfer without much loss of accuracy from exact solutions. The following questions can be answered with this analysis

1. The temperature of the body after a time t.
2. The time taken by the body to attain or reach a defined temperature in a given environment.
3. The rate of heat transfer between the body and the environment at any time t as determined by Newton's law of cooling as $\dot{Q}(t) = \alpha A\left(T(t) - T_\infty\right)$
4. The total amount of heat transfer between the body and the environment over the time interval (t = 0 to t) is $\dot{Q} = \dot{m}\,C_p\left(T(t) - T_i\right)$, simply obtained from the rate of change of internal energy of the body.
5. The upper limit of heat transfer from the body as obtained when the body reaches the temperature of the environment i.e. $\dot{Q}_{max} = \dot{m}\,C_p\left(T_\infty - T_i\right)$

We develop the criteria for analysis by doing the following:

(a) Approximating the shape of the body as being (i) a slab (ii) a cylinder and (iii) a sphere

(b) Determining the characteristic length Lc of the body where $L_c = \left(V/A_s \right)$

(c) Determining the governing parameter for lumped capacitance analysis called the Biot Number (B_i)

$$B_i = \frac{\text{Conduction resistance of body}}{\text{Convection resistance from the body in that environment}} = \frac{\left(L_c/\lambda A \right)}{\left(1/\alpha A \right)} = \frac{\alpha L_c}{\lambda}$$

Criteria applicable with about 5% loss of accuracy when $B_i < 0.1$. So first step is to evaluate B_i and decide whether to use the method. In doing so we define the following;

$$L_c = \frac{\text{Volume of body}}{\text{surface area of body}}$$

For a slab, $L_c = t/2$

For a cylinder, $L_c = r/2 = d/4$

For a sphere, $L_c = r/3 = d/6$

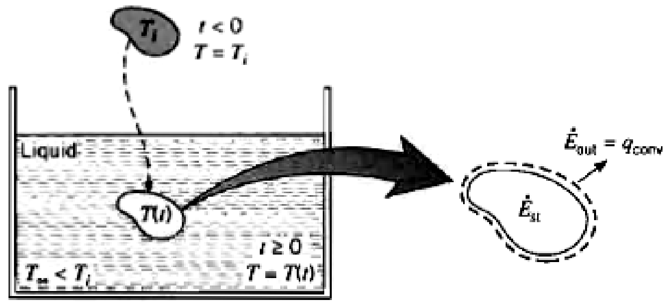


Figure 27: Cooling of a hot metal

Consider a hot metal forging (Figure 27) that is initially at a uniform temperature T and is quenched by immersing it in a liquid of lower temperature T_∞. If the quenching is said to begin at time t = 0, the temperature of the solid will decrease for time t > 0, until it eventually reaches T_∞. This reduction is due to convection heat transfer at the solid-liquid interface. The essence of the lumped capacitance method is the assumption that the temperature of the solid is *spatially uniform* at any instant during the transient period. This assumption implies that temperature gradients within the solid are negligible.

From Fourier's law, heat conduction in the absence of temperature gradient implies the existence of infinite thermal conductivity. Such a condition is impossible. A better approximation to the real situation is to assume that the resistance to conduction within the solid is small compared with the resistance to heat transfer between the solid and its surroundings.

In neglecting temperature gradients within the solid, we can no longer consider the problem from within the framework of the heat equation. Instead, the transient temperature response is determined by formulating an overall energy balance on the solid. This balance must relate the rate of loss at the surface to the rate of change of the internal energy. Applying Equation 1.10 to the control volume to have

$$-\dot{E}_{out} = \dot{E}_{stored}$$

or

$$-\alpha_c A(T - T_\infty) = \rho V c \frac{dT}{dt} \qquad (2.68)$$

$$R_c \text{(Convection Resistance)} = \frac{1}{A\alpha_c}$$

$$R_k \text{(Conduction Resistance)} = \frac{L_c}{A\lambda}$$

The Biot Number is the ratio

$\frac{R_k}{R_c} = \frac{\alpha_c L_c}{\lambda}$ which is a dimensionless parameter

Where L_c is the characteristic length and defined as ($L_c = V/A_s$). V is the volume and A_s is the total surface area.

If the Biot Number ≤ 0.1 it implies that λ is so high that the conduction resistance is very low hence the temperature is uniform. In this case, $T = T(t)$

Since internal energy changes as temperature changes, the internal energy is expressed as $mcdT = \rho VcdT$

$$\frac{dT}{dt} = -\frac{A_s \alpha_c}{\rho c V}(T - T_\infty) \tag{2.69}$$

$$\theta \equiv T - T_\infty$$

$$\frac{d\theta}{dt} = -\frac{A_s \alpha_c}{\rho c V}\theta$$

Initial conditions $T = T_i$ @ $t = 0$

Separating variables

$$\frac{d\theta}{\theta} = -\frac{A_s \alpha_c}{\rho c V}dt$$

$$let\ \ b = \frac{A_s \alpha_c}{\rho c V}$$

But θ = $T - T_\infty$

Boundary conditions: $T = T_i$ @ $t = 0$

$$\int \frac{d\theta}{\theta} = -b\int dt$$

$$\Rightarrow \text{In}\theta = -bt + C$$

$$\therefore \text{In}(T - T_\infty) = -bt + \text{In}(T_i - T_\infty)$$

$$\text{In}\frac{T - T_\infty}{T_i - T_\infty} = -bt \tag{2.70}$$

Equation 2.70 may be used to determine the time required for the solid to reach some temperature T.

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-bt} \tag{2.71}$$

Equation 2.71 may be used to compute the temperature reached by the solid at some time *t*. The foregoing results indicate that the difference between the solid and fluid temperature must decay exponentially to zero as *t* approaches infinity.

Worked Example 2.3

A steel ball $\left[C_p = 0.46\ \mathrm{kJ}/(\mathrm{kg.°C}),\ \lambda = 35\ \mathrm{W}/(\mathrm{m.°C})\right]$ 5cm in diameter and initially at a uniform temperature of 450 °C is suddenly placed in a controlled environment in which the temperature is maintained at 100 °C. The convection heat transfer coefficient is 10 W/ (m^2. °C). Calculate the time required for the ball to attain a temperature of 150 °C.

SOLUTION
Assume that the lumped capacitance analysis is applicable because of the low value of α and high value of λ.

We determine the Biot number = $\dfrac{\alpha L_c}{\lambda} = \dfrac{\alpha(V/A)}{\lambda} = \dfrac{10\left(0.05/6\right)}{35} = 0.00238 < 0.1$

Since Bi is < 0.1, the condition for applicability of the lumped-capacitance analysis is met.
From the equation;

$$\frac{T(t) - T_\infty}{T_o - T_\infty} = e^{-bt} \quad \text{where } b = \frac{\alpha}{\rho C_p L_c} = \frac{10}{7800 \times 460 \times (0.05/6)} = 3.344 \times 10^{-4}\,s^{-1}$$

$$\frac{150 - 100}{450 - 100} = e^{-3.344 \times 10^{-4} t} \quad \text{solving for t gives } t = 5819s = 1.62h$$

Thus it will take the ball 1hour and 62 minutes to attain a temperature of 150 °C

Worked Example 2.4

A person is found dead at 5 PM in a room whose temperature is 20°C. The temperature of the body is measured to be 25°C when found, and the heat transfer coefficient is estimated to be h = 8 W/m^2 · °C. Modeling the body of the person as a 30-cm-diameter, 1.70-m-long cylinder; estimate the time of death of that person.

SOLUTION A body is found while still warm. The time of death is to be estimated.

Assumptions 1 The body can be modeled as a 30-cm-diameter, 1.70-m-long cylinder. **2** The thermal properties of the body and the heat transfer coefficient are constant. **3** The radiation effects are negligible. **4** The person was healthy (!) when he or she died with a body temperature of 37°C.

Properties The average human body is 72 percent water by mass, and thus we can assume the body to have the properties of water at the average temperature of (37 + 25)/2 = 31°C; $k = 0.617$ W/m · °C, $\rho = 996$ kg/m^3, and $C_p = 4178$ J/kg · °C .

Analysis The characteristic length of the body is

$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(0.15 \text{ m})^2(1.7 \text{ m})}{2\pi(0.15 \text{ m})(1.7 \text{ m}) + 2\pi(0.15 \text{ m})^2} = 0.0689 \text{ m}$$

Then the Biot number becomes

$$\text{Bi} = \frac{hL_c}{k} = \frac{(8 \text{ W/m}^2 \cdot °\text{C})(0.0689 \text{ m})}{0.617 \text{ W/m} \cdot °\text{C}} = 0.89 > 0.1$$

Therefore, lumped system analysis is *not* applicable. However, we can still use it to get a "rough" estimate of the time of death. The exponent b in this case is

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{8 \text{ W/m}^2 \cdot °\text{C}}{(996 \text{ kg/m}^3)(4178 \text{ J/kg} \cdot °\text{C})(0.0689 \text{ m})}$$
$$= 2.79 \times 10^{-5} \text{ s}^{-1}$$

We now substitute these values into Eq. 2.71,

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25 - 20}{37 - 20} = e^{-(2.79 \times 10^{-5} \text{ s}^{-1})t}$$

Which yields; t = 43,600 s = (12.2h)

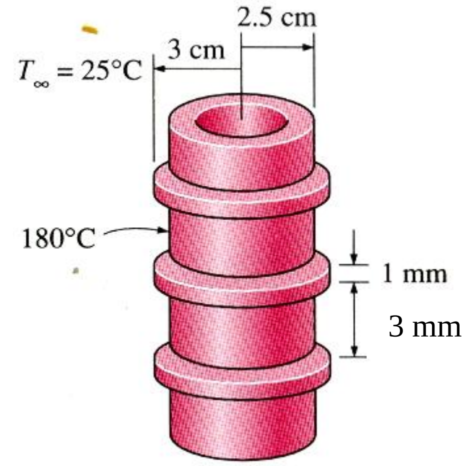
Therefore, as a rough estimate, the person died about 12 h before the body was found, and thus the time of death is 5 AM. A simple analysis yields vital information which may require carbon dating to establish the same results.

Individual/Group Discussion Problems: Tutorial Problems Questions 1, 2, 3, 4, 5, 6, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 37, 38.

Self Assessment 2-2

1. Steam in a heating system flows through tubes whose outer diameter is 5 cm and whose walls are maintained at a temperature of 180^{o}C. Circular aluminium alloy 2024-T6 fins (λ = 186 W/m ^{o}C) of outer diameter 6 cm and constant thickness 1 mm are attached to the tube. The space between the fins is 3 mm, and thus there are 250 fins per meter length of the tube. Heat is transferred to the surrounding air at T_∞ = 25^{o}C, with a heat transfer coefficient of 40 W/m^2 ^{o}C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins. Assume $\eta_f = 98\%$

   Suggested Answer: 2666 W



2. The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1.2-mm-diameter sphere. The properties of the junction are λ = 35 W/m · °C, ρ = 8500 kg/m^3, and C_p = 320 J/kg · °C, and the heat transfer coefficient between the junction and the gas is h = 65 W/m^2 · °C.
Determine how long it will take for the thermocouple to read 99 percent of the initial temperature difference. **Suggested Answer: 38.5 s**

Learning Track Activities ***ensure you understand key terms listed below***

Key terms/ New Words in Unit

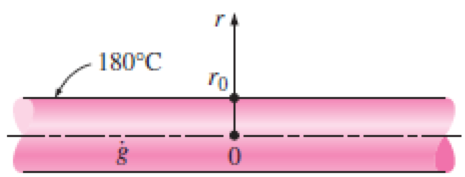
i. Thermal diffusivity
ii. Heat storage capacity
iii. Thermal conductivity
iv. Thermal resistance
v. Overall heat transfer coefficient
vi. Thermal contact resistance
vii. Plane walls
viii. Composite walls
ix. Thermal circuit analysis
x. Biot number
xi. Critical radius of Insulation
xii. Thermal energy generation
xiii. Fin efficiency
xiv. Fin effectiveness
xv. Pitch of a fin

Discussion Question: Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

Review Question: A long homogeneous resistance wire of radius r_0 = 5 mm is being used to heat the air in a room by the passage of electric current. Heat is generated in the wire uniformly at a rate of $\dot{g} = 5 \times 10^7$ W/m^3 as a result of resistance heating. If the temperature of the outer surface of the wire remains at 180°C, determine the temperature at r =2 mm after steady operation conditions are reached. Take the thermal conductivity of the wire to be λ = 8 W/m · °C.**[Suggested *Answer:* 212.8 °C]**



Web Activity: www.mitcourseware.com, www.mhhe.com

Reading: Read chapter 2 of Heat transfer: A practical approach, Yunus .A. Cengel, McGraw Hill Inc, 1998.

Unit 3

RADIATION HEAT TRANSFER PHENOMENA

Introduction

In the presentation of the contents of this unit, we distinguish between radiation and diffusion; define emissive power, black body, and monochromatic emissive power as well as other terms that are specific to radiative heat transfer. We shall also discuss wavelength dependent properties as well as radiative properties in greater detail.

Learning Objectives

After studying this unit you should be able to:

1. Distinguish between radiation and diffusion.
2. Define emissive power, black body, monochromatic emissive power and other terms that are specific to radiation heat transfer.
3. Understand wavelength dependent properties and define total hemispherical emissivity and absorptivity of surfaces.
4. Define radiative properties; emissivity, absorptivity, reflectivity and transmissivity of surfaces.

Unit content

Session 1-3: Introduction to Radiation Heat Transfer
- 1-3.1: Thermal Radiation
- 1-3.2: Black body Radiation

Session 2-3: Radiation Properties
- 2-3.1: Emissivity
- 2-3.2: Absorptivity, Reflectivity and Transmissivity
- 2-3.3: Kirchhoff's Law
- 2-3.4: The Atmosphere and Solar Radiation

SESSION 1-3: INTRODUCTION TO RADIATION HEAT TRANSFER

Radiation is a very important mode of heat transfer, and may be defined as the transfer of energy between bodies, which may be separated by a vacuum, or an intervening medium such as a fluid. Radiation is propagated by electromagnetic waves, which arises as a result of accelerated changes or changing electric currents giving rise to electric and magnetic fields. Electromagnetic waves transport energy just like other waves and all electromagnetic waves travel at the speed of light. Electromagnetic waves are characterised by their frequency *v* and wavelength λ, These two properties in a medium are related by

$$\lambda = \frac{c}{v} \tag{3.1}$$

Where c is the speed of light in the medium. For propagation in a vacuum, $c = c_o = 2.998 \times 10^8\ m/\mathrm{sec}$. The speed of light in a medium is related to the speed of light in a vacuum by $c = c_o$ / n, where n is the index of refraction of that medium. For air and most gases, $n = 1$, while for water and glass, $n \cong 1.5$. Wavelength is normally measured in micrometre, (μm) where $\mu m = 10^{-6}$ m. Unlike the wavelength and the speed of propagation, the frequency of an electromagnetic wave depends only on the source and is independent of the medium through which the wave travels. The *frequency* (the number of oscillations per second) of an electromagnetic wave can range from a few cycles to millions of cycles and even higher per second, depending on the source.

In radiation studies, it has proven useful to view electromagnetic radiation as the propagation of a collection of discrete packets (or quantum) of energy called **photons** or **quanta**, as proposed by Max Planck in 1990 in conjunction with his quantum theory.

In this view, each photon of frequency *v* is considered to have an energy of value given by:

$$e = hv = \frac{hc}{\lambda} \tag{3.2}$$

where $h = 6.625 \times 10^{-34}$ J s is called the *Planck's constant*. Since h and c are constants, it follows that the energy of a photon is inversely proportional to its wavelength. Therefore, shorter-wavelength radiation possesses larger photon energies, and is highly destructive. For this reason shorter wavelength such as gamma rays and X-rays are to be avoided.

1-3.1 Thermal Radiation

Although all electromagnetic waves have the same general features, waves of different wavelength differ significantly in their behaviour. The electromagnetic radiation encountered in practice covers a wide range of wavelengths, varying from less than $10^{-10}\ \mu m$ for cosmic rays to more than $10^{10}\ \mu m$ for electrical power waves. The electromagnetic spectrum also includes gamma rays, X-rays, ultraviolet radiation, visible light, infrared radiation, thermal radiation, microwaves, and radio waves, as shown in Figure 28.

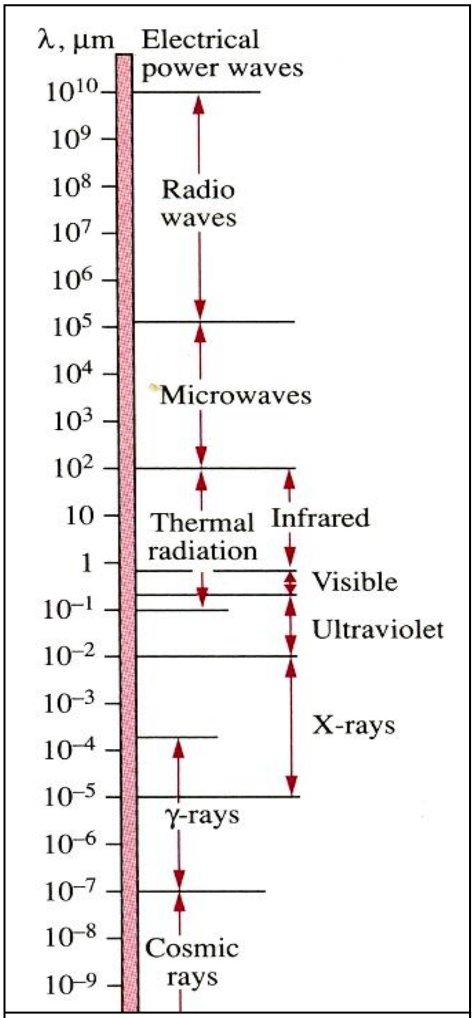


Figure 28: Electromagnetic spectrum

The type of electromagnetic radiation that is pertinent to heat transfer is **thermal radiation** emitted as a result of vibrational and translational motions of the molecules, atoms, electrons of a substance. Temperature is a measure of the strength of these activities at the microscopic level, and the rate of thermal radiation emission increases with increasing temperature. All matter whose temperature is above absolute zero continuously emits thermal radiation. That is, everything around our friends and us such as walls and furniture constantly emits (and absorbs) radiation. Thermal radiation is also defined as the portion of the electromagnetic spectrum that extends from about 0.1 to 100 μm, since the radiation emitted by bodies because of their temperature falls almost entirely into this wavelength range. Thus, thermal radiation includes the entire visible and infrared (IR) radiation as well as a portion of the ultraviolet (UV) radiation.

What we call light is simply the visible portion of the electromagnetic spectrum that lies between 0.40 and $0.76\ \mu m$. Light or the visible spectrum consists of narrow bands of colour from violet to red, as shown in Table 3.1. The colour of a surface depends on its ability to *reflect* certain wavelengths. For example, a surface that reflects all of the light appears *white*, while a surface that absorbs the entire light incident on it appears black.

Table 5: The wavelength ranges of different colours

Colour	Wavelength band, μm
Violet	0.40 – 0.44
Blue	0.44 – 0.49
Green	0.49 – 0.54
Yellow	0.54 – 0.60
Orange	0.60 – 0.63
Red	0.63 – 0.76

A body that emits some radiation in the visible range is called a light source. The sun is obviously our primary light source. The electromagnetic radiation emitted by the sun is known as solar radiation, and nearly all of it falls into the wavelength band of 0.3 - 3 μm. Almost half of solar radiation is light, with the remaining being ultraviolet (0.01 – 0.40 μm) and infrared (0.76 - 100 μm).

Bodies at room temperature emit radiation in the infrared range, which extends from 0.76 to 100 μm. Bodies start emitting noticeable visible radiation at temperatures above 800K. The tungsten filament of a light bulb must be heated above 2000 K before it can emit any significant amount of radiation in the visible range.

The ultraviolet radiation occupies the low-wavelength end of the thermal radiation spectrum and lies between 0.01 and 0.40 μm. About 12% of solar radiation is in the ultraviolet range, and it would be devastating if it were to reach the surface of the earth. Ultraviolet rays can kill microorganisms and cause serious damage to humans and other living organisms. Fortunately, the zone (O_3) layer in the atmosphere acts as a protective blanket, and absorbs most of this ultraviolet radiation. The ultraviolet rays that remain in sunlight are still sufficient to cause serious sunburn during prolonged exposure. Recent discoveries of 'holes' in the ozone layer has prompted the international community to ban the use of chemicals that destroy ozone, such as the widely used refrigerant Freon – 12, in order to save the earth. Ultraviolet radiation is also produced artificially in fluorescent lamps for use in medicine as bacteria killer.

Microwave ovens utilise electromagnetic radiation in the microwave region of the spectrum generated by microwave tubes called magnetrons. Microwaves in the range of 10^2 – 10^5 μm are very suitable for use in cooking since they are *reflected* by metals, *transmitted* by glass and plastics, and *absorbed* by food (especially water) molecules. Thus, the electric energy converted to radiation in a microwave oven eventually becomes part of the internal energy of the food. The fast and efficient cooking of microwave ovens has made them some of the essential appliances in modern kitchens.

Radars and cordless telephones also use electromagnetic radiation in the microwave region. For radio and TV the wavelength is between 1 and 1000 μm.

In heat transfer studies; we are interested in the energy emitted by bodies because of their temperature only. Therefore, we shall limit ourselves to *thermal radiation* or simply *radiation*. Though radiation is a **volumetric phenomenon**, for opaque (non-transparent) solids such as metals, wood and rocks, radiation is considered to be a **surface phenomenon**. This is because the radiation emitted by the interior regions can never reach the surface, and the radiation incident on such bodies is usually absorbed within a few microns from the surface.

1-3.2 Black body Radiation

Anybody at temperature above absolute zero emits radiation in all directions (i.e. diffusely) over a wide range of wavelengths. The amount of energy emitted from a surface at a given wavelength depends on the material of the body and the condition of its surface as well as the surface temperatures. In view of this, different bodies may emit different amount of radiation per unit surface area, even when they are at the same temperature. An idealized body, called a *blackbody*, serves as a standard against which the radiative properties of real surfaces may be compared.

A **blackbody** is defined as a body, which emits the maximum amount of radiation that can be emitted from a surface at any temperature. A blackbody also absorbs all incident radiation, regardless of wavelength and direction It is therefore a perfect emitter and a perfect absorber of radiation. In addition, a blackbody emits radiation energy uniformly in all directions. As a result, a blackbody is often called a **diffuse** emitter. The terms diffuse means “independent of direction”.

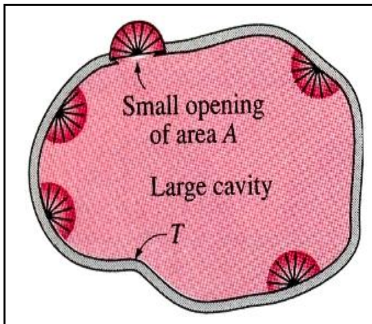


Figure 29: A large cavity with a small opening

The total radiant energy emitted by a blackbody per unit time per unit surface area is given by

$$E_b = \sigma T^4 \qquad (3.3)$$

Where, $\sigma = 5.67 \times 10^{-8} W/m^2K^4$ is the Stefan-Boltzmann constant and T is the absolute temperature of the surface in Kelvin. The quantity E_b is called the blackbody emissive power. Incidentally, surfaces coated with mat-black paint approach idealized blackbody behaviour. Another type of body that closely resembles a blackbody is a large cavity with a small opening (Figure 29).

In this case, radiation coming in through the opening of area *A* will undergo multiple reflections, and thus it will have several chances to be absorbed by the interior surfaces of the cavity before any part of it can possibly escape. If the surface of the cavity is isothermal at temperature *T*, the radiation emitted by the interior surfaces will stream through the opening after undergoing multiple reflections, and thus it will have a diffuse nature. Therefore, the cavity will act as a perfect absorber and perfect emitter, and the opening will resemble a blackbody of surface area *A* at temperature *T*, regardless of the actual radiative properties of the cavity.

The Stefan-Boltzmann law gives the total blackbody emissive power, which is the sum of the radiation emitted over all wavelengths. The **spectral (monochromatic) blackbody emissive power**, which is *the amount of radiation energy emitted by a blackbody at an absolute temperature T per unit time, per unit surface area, and per unit wavelength about the wavelength* λ is sometimes desired, We are also often interested in the amount of radiation emitted over some wavelength band. For example, an incandescent light bulb is judged on the basis of the radiation it emits in the visible range, than the radiation it emits at all wavelengths.

1-3.2.1 Monochromatic emissive power

This is the rate at which energy is emitted by a body per unit surface area and per unit wavelength about the wavelength λ. This depends of the temperature and the surface characteristics. It is often necessary to know the monochromatic or spectral blackbody emissive power. Monochromatic emissive power, $E_{b\lambda}$ which is defined as the amount of radiant energy emitted by a blackbody at an absolute temperature *T* per unit time, per unit surface area, per unit wavelength λ. For example, we are more interested in the amount of radiation that an incandescent light bulb emits in the visible wavelength spectrum than we are in the total amount of radiation that the light bulb emits.

The monochromatic (spectral) blackbody emissive $E_{b\lambda}$ (W/m^2.μm) is expressed by the Planck's distribution law according to the equation.

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp(C_2/\lambda T) - 1\right]} \qquad (W/m^2.\mu m) \text{------------- Planck's Law} \qquad (3.4)$$

where $C_1 = 2\pi hc_0{}^2 = 3.742\times10^8 W\mu m^4/m^2; C_2 = hc_o/k = 1.439\times10^4\,\mu mK$

Also *T* is the absolute temperature of the surface, λ is the wavelength of the radiation emitted, and k = 1.3805 x 10^{-23} J/K is called the *Boltzmann's constant*. The term spectrum indicates dependence on wavelength.

The radiant energy emitted by a blackbody per unit area over a wavelength band from $\lambda = 0$ to λ is determined b integrating the equation for $E_{b\lambda}$ namely

$$E_{b.0-\lambda}(T) = \int_0^{\lambda} E_{b\lambda}(T)d\lambda \qquad (3.5)$$

Because of the nature of the expression for $E_{b\lambda}$ the above integration does not have a closed form solution. To simplify the handling of the integration, we define a dimensionless quantity f_λ called the **blackbody radiation function** as

$$f_\lambda(T) = \frac{\int_0^{\lambda} E_{b\lambda}(T)d\lambda}{E_b} = \frac{\int_0^{\lambda} E_{b\lambda}(T)d\lambda}{\sigma T^4} \qquad (3.6)$$

The function f_λ represents the fraction of radiation emitted from a blackbody at temperature *T* in the wavelength band $\lambda = 0$ to λ. The values of f_λ are tabulated extensively in textbooks as a function of λT, and selected values are given in Table 6 below.

Table 6: Blackbody radiation function, f_λ

$\lambda T(\mu m K)$	f_λ	$\lambda T(\mu m K)$	f_λ
200	0.000000	4200	0.516014
400	0.000000	4400	0.548796
800	0.000016	4800	0.607559
1000	0.000321	5000	0.633747
1400	0.007790	5400	0.680360
1800	0.039341	5800	0.720158
2000	0.066728	6000	0.737818
2600	0.183120	7000	0.808109
2800	0.227897	8000	0.856288
3000	0.273232	9000	0.890029
3400	0.361735	10000	0.914199
4000	0.480877	100000	0.999905

Tables of the blackbody radiation function are useful in determining, for example, the fraction of radiant energy emitted by a blackbody at temperature *T* over a finite wavelength band from $\lambda = \lambda_1$ to $\lambda = \lambda_2$ from the expression

$$f_{\lambda_1-\lambda_2}(T) = f_{\lambda_2}(T) - f_{\lambda_1}(T) \qquad (3.7)$$

where $f_{\lambda_1}(T)$ and $f_{\lambda_2}(T)$ are the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, respectively.

The variation of the blackbody emissive power with wavelength is plotted in Figure 3.3 for selected temperatures. The following observations can be made from this figure:

1. The emitted radiation is a continuous function of wavelength. At any specified temperature, it increases with wavelength, reaches a peak, and then decreases with increasing wavelength.
2. At any given wavelength, the black body emissive power increases with increasing temperature.
3. As temperature increases, the curves get steeper and shift to the left to the shorter-wavelength region. Consequently, a larger fraction of the radiation is emitted at shorter *wavelengths* at higher temperatures.

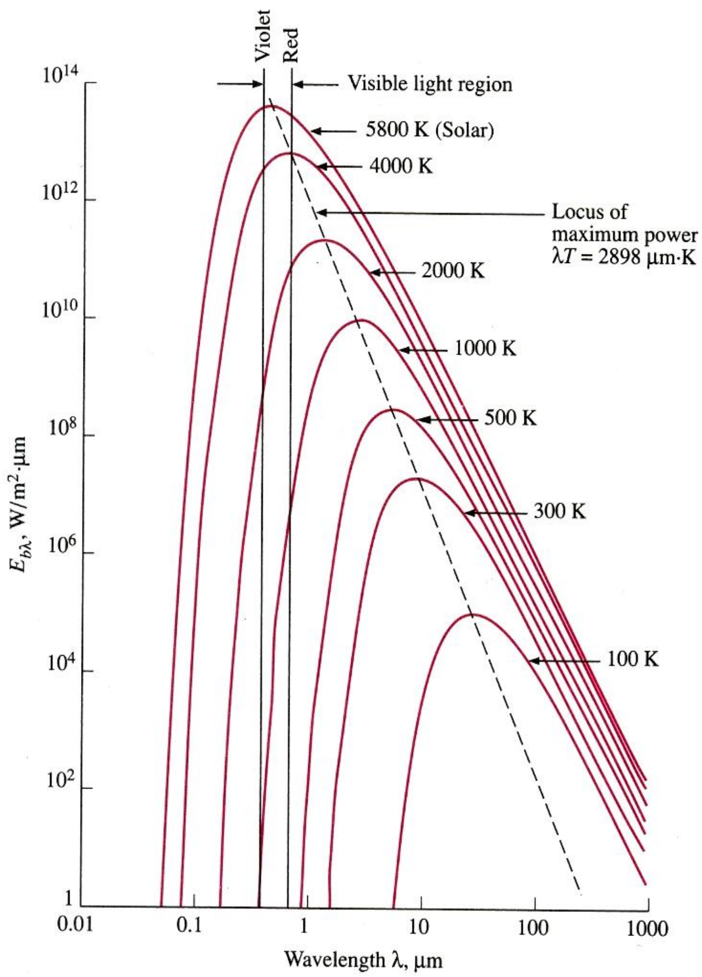


Figure 30: Variation of spectral black body emissive power with wavelength

4. The radiation emitted by the sun which is considered to be a blackbody at 5762 *K* (or roughly 5800 K), reaches its peak in the visible region of the spectrum. Therefore, the sun is in tune with our eyes. On the other hand surfaces at $T \leq 800$ K emit entirely in the infrared region and thus are not visible to the eye unless they reflect light coming from other surfaces.

5. As temperature increases, the peak shifts towards shorter wave1engths. The wavelength at which the peak occurs for a specified temperature is given by the **Wien's displacement** law.

A plot of Wien's displacement law, which is the locus of the peaks of the radiation emission curves, is given in Figure 30 as the broken line.

From the graph, the peaks or the maximum heights of all the curves lie in a pattern. It has been shown that

$$\lambda_{\max} T = 2897.8 \mu m K \tag{3.8}$$

Thus for a black body, knowing the temperature, one can determine the maximum wavelength of it. For example the sun, $T = 5800$ *K*.

$$\therefore \lambda_{\max} = \frac{2897.8 \mu m K}{5800 K} = 0.5 \mu m$$

which near the middle of the visible range. Thus the maximum intensity of the sun's radiation is over a small band around the visible range of the spectrum.

Again for a black body

$$E_b = \int_0^\infty E_{b\lambda} d\lambda = \sigma T^4 \tag{3.9}$$

This gives total emissive power over the entire spectrum, where $\sigma = 5.67 \times 10^{-8} \ W/m^2 . \ K^4$ known as the Stefan-Boltzmann constant. For any body, σT^4 is the maximum radiant energy that it is capable of emitting.

The Stefan-Boltzmann law $E_b = \sigma T^4$ gives the total radiation emitted by a blackbody at all wavelengths from $\lambda = 0$ to $\lambda = \infty$.

Worked Example 3.1

The temperature of the filament of an incandescent light bulb is 2500 K. Assuming the filament to be a blackbody, determine the fraction of the radiant energy emitted by the filament, which falls within the visible range. Also determine the wavelength at which the emission of radiation from the filament peaks.

Solution

The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.4\mu m$ to $\lambda_2 = 0.76\mu m$. Noting that T = 2500 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from the table above to be

$$\lambda_1 T = (0.40\mu m)\ (2500K) = 1000\mu mK \Rightarrow f_{\lambda 1} = 0.000321$$

$$\lambda_2 T = (0.76\mu m)\ (2500K) = 1900\mu mK \Rightarrow f_{\lambda 2} \cong 0.053035$$

This implies that 0.03% of the radiation is emitted at wavelengths < 0.4 μm and 5.3% at wavelengths less than 0.76 μm. Thus, the fraction of radiation emitted between these two wavelengths is $f_{\lambda 1-\lambda 2} = f_{\lambda 2} - f_{\lambda 1} = 0.053035 - 0.00321 = 0.0527135$.

Thus, only about 5% of the radiation emitted by the filament of the light bulb falls in the visible range. The remaining 95% of the radiation appears in the infrared region in the form of radiant heat. Hence, it is very inefficient to use incandescent lights for lighting and explains why fluorescent tubes are preferable.

The wavelength at which the emission of radiation from the filament peaks is obtained from the expression $\lambda_{\max} T = 2897.8\mu mK \Rightarrow \lambda_{\max} = \dfrac{2897.8}{2500} = 1.16\mu m$.

It follows that the radiation from the filament peaks in the infrared region.

Self Assessment 1-3

1. Consider a 20 cm x 20 cm x 20 cm cubical body at 1000 K suspended in air. Assuming the body closely approximates a blackbody, determine
   a. The rate at which the cube emits radiation energy, in W and
   b. The spectral black body emissive power at a wavelength of 4 μm.

Suggested Answer: [13,600 W; 10,300 kW/m^2]

2. A 3 mm thick glass window transmits 90 percent of the radiation between $\lambda = 0.3$ and 3.0 μm and is essentially opaque for radiation at other wavelengths. Determine the rate of radiation transmitted through a 2 m x 2 m glass window from blackbody sources at (a) 5800 K and (b) 1000 K.
Suggested Answer: [218,400 kW; 55.8 kW]

SESSION 2-3: RADIATION PROPERTIES

Most materials encountered in practice, such as metal, wood and bricks are opaque to thermal radiation, and radiation is considered to be surface phenomena for such materials. That is thermal radiation is emitted or absorbed within the first few microns of the surface, and we speak of radiation properties of surfaces for opaque materials.

Some other materials, such as glass and water, allow visible radiation to penetrate to considerable depths before any significant absorption takes places. Such materials are called semitransparent. On the other hand, both glass and water, though transparent to visible radiation, are practically opaque to infrared radiation. Therefore, materials can exhibit different behaviour at different wavelengths, and the dependence on wavelength is an important consideration in the study of radiation properties.

The radiation properties of real surfaces are often described in terms of those of the blackbody, which is a perfect emitter and absorber of radiation. The most important radiation characteristics are: emissivity, absorptivity, reflectivity and transmissivity.

2-3.1 Emissivity

The emissivity of a surface is defined as the ratio of the radiation emitted by the surface to the radiation emitted by a blackbody at the same temperature. The emissivity of a surface is denoted by ε, where $0 < \varepsilon < 1$.

The emissivity of a real surface generally varies with temperature, wavelength and direction. Therefore, different emissivities can be defined for a surface, depending on the effects considered for example; the emissivity of a surface at a specified wavelength is called the monochromatic or spectral emissivity, ε_λ. Similarly, the emissivity in a specified direction is called directional emisivity, ε_θ, where θ is the angle between the direction of radiation and the normal of the surface. The emissivity of a surface averaged over all directions is called hemispherical emissivity, while the emissivity averaged over all wavelengths and is called the total emissivity. Thus, total hemispherical emissivity of a surface is simply the average emissivity over all directions and wavelengths and can be expressed as

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{E(T)}{\sigma T^4} \quad (3.10)$$

Where *E(T)* is the total emissive power of the real surface. From the above equation, the total emissive power of a real surface can be written as

$$E(T) = \varepsilon(T)\sigma T^4 \quad (3.11)$$

In the same way, monochromatic or spectral emissivity is defined as

$$\varepsilon_\lambda(T) = \frac{E_\lambda(T)}{E_{b\lambda}(T)} \quad (3.12)$$

Where $E_\lambda(T)$ is the spectral emissive power of the real surface.

In order to reduce the complexities introduced into the analysis of radiation by its dependence on wavelength and directions, the grey and diffuse approximations have been developed. A surface is said to be diffuse if its properties are independent of direction, and grey if the properties are independent of wavelength. Therefore, the emissivity of a grey, diffuse surface is simply the total hemisphere emissivity of that surface which is independent of direction and wavelength.

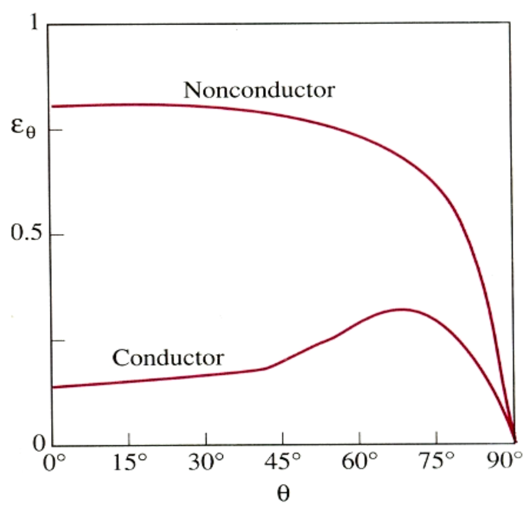
The results may be summarized as follows:

Real surface: $\varepsilon_\theta \neq$ Constant, $\varepsilon_\lambda \neq$ Constant

Diffuse surface: $\varepsilon_\theta =$ Constant

Grey surface: $\varepsilon_\lambda =$ Constant

Diffuse, grey surface: $\varepsilon = \varepsilon_\lambda = \varepsilon_\theta =$ Constant



Although real surfaces do not emit radiation in a completely diffuse manner as a blackbody does, they usually approximate it as shown in the Figure 31.

For conductors such as metals, ε_θ remains nearly constant for about $\theta < 40^0$; while for nonconductors such as plastic, it is nearly constant for $\theta < 70^0$, For this reason, the directional emissivity of a surface in the normal direction $(\theta = 0)$ is representative of the hemispherical emissivity of the surface.

Figure 31: Typical variation of emissivity with direction for electrical conductors

A grey surface should emit as much radiation as the real surface it represents at the same temperature. For this to be realistic, the areas under the emission curves of the real and grey surfaces must be equal. That is,

$$\varepsilon(T)\sigma T^4 = \int_0^\infty \varepsilon_\lambda(T) E_{b\lambda}(T)\,d\lambda \tag{3.13}$$

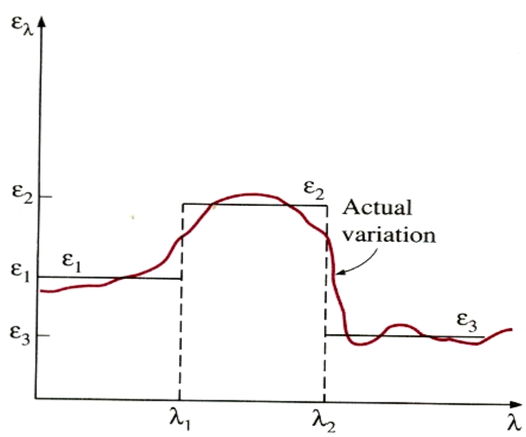
The average emissivity can then be given as

$$\varepsilon(T) = \frac{\int_0^\infty \varepsilon_\lambda(T) E_{b\lambda}(T)\,d\lambda}{\sigma T^4} \tag{3.14}$$

The integration is usually a complicated function; hence the integration has to be performed numerically. Alternatively, by dividing the spectrum into a sufficient number of wavelength bands and assuming the emissivity to remain constant over each band we can perform the integration quite easily. This is equivalent to expressing the function $\varepsilon(T)$ as a step function. This simplification offers equal convenience for little sacrifice of accuracy, since it allows us to transform the integration into a summation in terms of blackbody emission functions.

As an example, consider the emissivity function below (Figure 32.).

This function can be approximated reasonably well by a step function of the form



$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = \text{constant}, 0 \le \lambda < \lambda_1 \\ \varepsilon_2 = \text{constant}, \lambda_1 \le \lambda < \lambda_2 \\ \varepsilon_3 = \text{constant}, \lambda_2 \le \lambda < \infty \end{cases}$$

Figure 32: Approximating the actual variation of emissivity with wavelength by a step function

Then the average emissivity can be determined by breaking the integral into three parts and utilizing the definition of the blackbody radiation function as

$$\varepsilon(T)=\frac{\varepsilon_1\int_0^{\lambda 1}E_{b\lambda}(T)d\lambda}{\sigma T^4}+\frac{\varepsilon_2\int_{\lambda 1}^{\lambda 2}E_{b\lambda}(T)d\lambda}{\sigma T^4}+\frac{\varepsilon_3\int_{\lambda 2}^{\infty}E_{b\lambda}(T)d\lambda}{\sigma T^4}$$

$$=\varepsilon_1 f_{0-\lambda_1}(T)+\varepsilon_2 f_{\lambda_1-\lambda_2}(T)+\varepsilon_3 f_{\lambda_2-\infty}(T) \tag{3.15}$$

Worked Example 3.2

The monochromatic emissivity function of an opaque surface at 800 K is approximated as

$$\varepsilon_\lambda=\begin{cases}\varepsilon_1=0.3, & 0\le\lambda<3\mu m\\ \varepsilon_2=0.8, & 3\mu m\le\lambda<7\mu m\\ \varepsilon_3=0{,}1 & 7\mu m\le\lambda<\infty\end{cases}$$

Determine the average emissivity of the surface and its emissive power.

Solution

The variation of the emissivity of the surface with wavelength is given as a step function. Therefore, the average emissivity of the surface can be determined by breaking the integral into three parts as shown in the above equation, namely

$$\varepsilon(T)=\frac{\varepsilon_1\int_0^{\lambda 1}E_{b\lambda}(T)d\lambda}{\sigma T^4}+\frac{\varepsilon_2\int_{\lambda 1}^{\lambda 2}E_{b\lambda}(T)d\lambda}{\sigma T^4}+\frac{\varepsilon_3\int_{\lambda 2}^{\infty}E_{b\lambda}(T)d\lambda}{\sigma T^4}$$
$$=\varepsilon_1 f_{0-\lambda_1}(T)+\varepsilon_2 f_{\lambda_1-\lambda_2}(T)+\varepsilon_3 f_{\lambda 2-\infty}(T)$$
$$=\varepsilon_1 f_{\lambda_1}+\varepsilon_2\left(f_{\lambda_2}-f_{\lambda_1}\right)+\varepsilon_3\left(1-f_{\lambda_2}\right)$$

where $f_{\lambda 1}$ and $f_{\lambda 2}$ are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined to be

$$\lambda_1 T=(3\mu m)(800K)=2400\mu m.K\rightarrow f_{\lambda 1}=0.140256$$
$$\lambda_2 T=(7\mu m)(800K)=5600\mu m.K\rightarrow f_{\lambda 2}=0.701046$$

Substituting, we have

$$\varepsilon=0.3(0.140256)+0.8(0.701046-0.140256)+0.1(1-0.701046)=0.521$$

Thus, the surface will emit as much radiation energy at 800 K as a grey surface having a constant emmisity $\varepsilon=0.521$. The emissive power of the surface is

$$E=\varepsilon\sigma T^4=0.521\left(5.67\times10^{-8}\right)(800K)^4=12{,}100W/m^2$$

2-3.2 Absorptivity, Reflectivity and Transmisivity

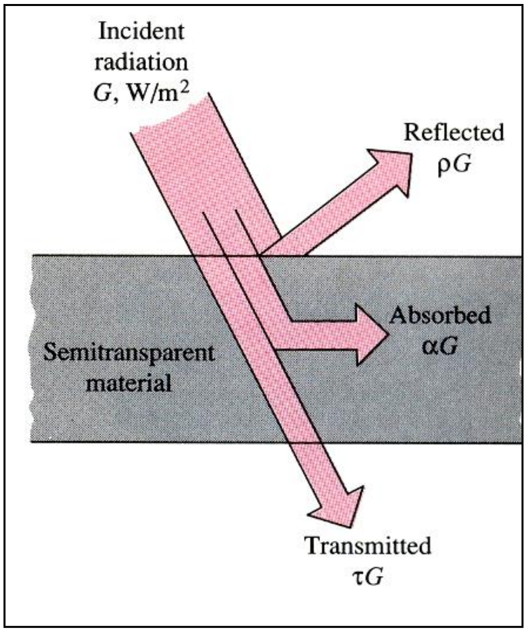


Figure 33: Radiative properties of a surface

When radiation strikes a surface, part of it is absorbed, part of it is reflected, and the remaining part, if any, is transmitted, as illustrated in Figure 33.

The fraction of incident radiation (or irradiation) absorbed by a surface is called the absorptivity, α; the fraction reflected by the surface is called the reflectivity, ρ; and the fraction transmitted is called the transmissivity, τ

Thus,

$$\alpha = \frac{absorbed\ \ radiation}{incident\ \ radiation} = \frac{G_{abs}}{G}, 0 \le \alpha \le 1 \qquad (3.16)$$

$$\rho = \frac{reflected\ \ radiation}{incident\ \ radiation} = \frac{G_{ref}}{G}, 0 \le \rho \le 1 \qquad (3.17)$$

$$\tau = \frac{transmitted\ \ radiation}{incident\ \ radiation} = \frac{G_{tr}}{G}, 0 \le \tau \le 1 \qquad (3.18)$$

where G is the radiation energy incident on the surface, and G_{abs}, G_{ref}, G_{tr}, are the absorbed, reflected and transmitted portions of it, respectively, The first law of thermodynamics requires that the sum of the absorbed, reflected and transmitted radiation be equal to the incident radiation.

Thus

$$G = G_{abs} + G_{ref} + G_{tr} \qquad (3.19)$$

Consequently, dividing through by G we have

$$\alpha + \rho + \tau = 1 \qquad (3.20)$$

For opaque surfaces, $\tau = 0$, and thus

$$\alpha + \rho = 1 \qquad (3.21)$$

This is an important property relation, since it allows us to determine both the absorptivity and reflectivity of an opaque surface by measuring either of these properties.

Like emissivity, the properties α , ρ and τ can also be defined for a specific wavelength or direction. For example, the spectral or monochromatic absorptivity, reflectivity and transmissivity of a surface are defined in a similar manner as

$$\alpha_\lambda = \frac{G_{\lambda,abs}}{G_\lambda}, \ \rho_\lambda = \frac{G_{\lambda,ref}}{G_\lambda}, \ \tau_\lambda = \frac{G_{\lambda,tr}}{G_\lambda} \tag{3.22}$$

where G_λ is the radiant energy incident at the wavelength λ, and $G_{\lambda,abs}$, $G_{\lambda,ref}$, $G_{\lambda,tr}$ are the absorbed, reflected and transmitted portions of it respectively. Similar definitions can be given for directional properties in direction θ by replacing all occurrences of the subscript λ by θ.

The average absorptivity, reflectivity and transmissivity of a surface can also be defined in terms of their spectral counterparts, namely

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}, \quad \rho = \frac{\int_0^\infty \rho_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}, \quad \tau = \frac{\int_0^\infty \tau_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} \tag{3.23}$$

The reflectivity differs from the other properties in the sense that it is bi-directional in nature. That is, the value of the reflectivity of a surface depends not only on the direction of the incident radiation but also the direction of reflection.

In general, the radiant beam incident on a real surface in a given direction will be reflected irregularly as shown in figure (34a) below.

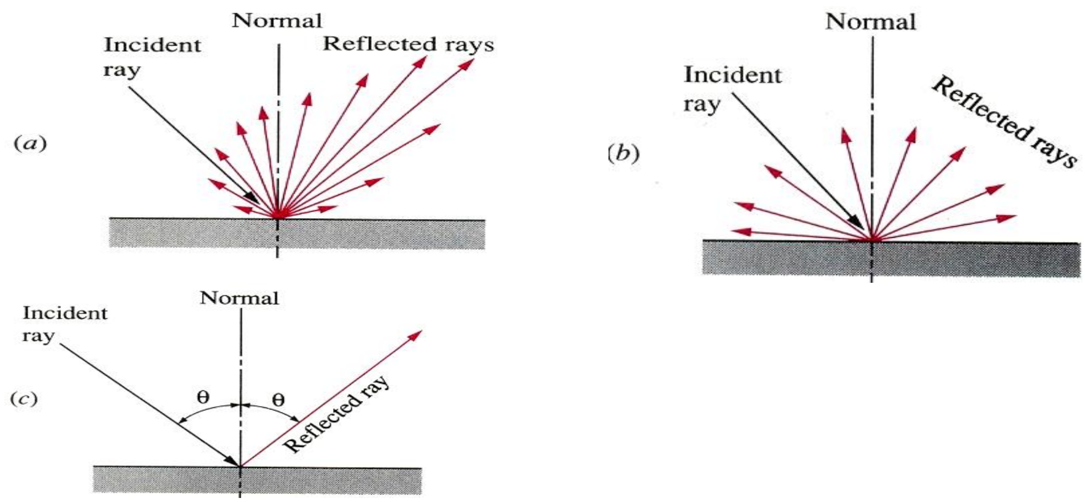


Figure 34: Different types of reflection from a surface

The computation of radiation for an irregular reflection would require an extensive data set and would involve rather complicated analysis. For simplicity, surfaces are assumed to reflect in a perfectly specular or diffuse manner.

In specular (or mirror like) reflection, the angle of reflection equals the angle of incidence of the radiation beam (Fig 34c). In diffuse ref1ection, radiation is reflected equally in all directions, as shown in figure (34b) above. Reflection from smooth and polished surfaces approximates specular reflection, while reflection from rough surfaces approximates diffuse reflection. In radiation analysis, a surface is said to be smooth if the height of the surface roughness is much smaller than the wavelength of the incident radiation.

Definition of some terms:

The emissive Power (E): Total amount of radiation emitted by a body per unit time and unit area

Black Body: An ideal body which absorbs all radiations incident on it and reflects or transmits none. Or it is a radiator, which emits at any specified temperature, the maximum possible amount of thermal radiation at all wavelengths.

Intensity of radiation (I): The radiant energy propagated in space in a particular direction per unit solid angle and per unit of area as projected on a plane perpendicular to the direction of propagation. Note that for a diffuse surface it is constant and does not vary with the emission angle.

Solid Angle: The area subtended on a sphere of unit radius; or for a sphere of radius r, it is defined as the intercepted area divided by r^2.

Radiosity (J): The rate at which radiation leaves a surface per unit area.

Irradiation (G): The rate of which radiation is incident on a unit surface area.

2-3.3 Kirchhoff's Law

Consider a small body of surface area A, emissivity ε, and absorptivity α, at temperature T contained in a large isothermal enclosure at the same temperature, as shown in Figure 35. We note that a large isothermal enclosure forms a blackbody cavity regardless of the radiative properties of the enclosure surface, and the body in the enclosure is too small to interfere with the blackbody nature of the cavity.

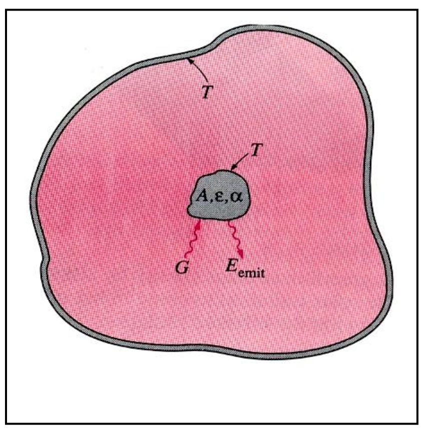


Figure 35: A small body in a large isothermal enclosure

Consequently, the radiation incident on any part of the small body is equal to the radiation emitted by a blackbody at temperature *T*. Thus, $G = E_b(T) = \sigma T^4$, and the radiation absorbed by the small body per unit area is:

$$G_{abs} = \alpha G = \alpha\sigma T^4 \tag{3.24}$$

The radiation emitted by the small body is:

$$E_{emit} = \varepsilon\sigma T^4 \tag{3.25}$$

Since the radiation emitted by the small body must be equal to the radiation absorbed by it, we have:

$$A\varepsilon\sigma T^4 = A\alpha\sigma T^4 \tag{3.26}$$

Thus,

$$\varepsilon(T) = \alpha(T) \tag{3.27}$$

Kirchhoff's law states that the total hemispherical emissivity of a surface at temperature T is equal to its total hemispherical absorptivity for radiation coming from a blackbody at the *same* temperature as the surface.

Similar considerations may be used to derive the spectral form of Kirchhoff's law, as

$$\varepsilon_\lambda(T) = \alpha_\lambda(T) \tag{3.28}$$

Similarly,

$$\varepsilon_{\lambda,\theta}(T) = \alpha_{\lambda,\theta}(T) \tag{3.29}$$

This implies that the emissivity of a surface at a specified wavelength, direction and temperature is always equal to its absorptivity at the same wavelength, direction and temperature.

These relations

$$\begin{aligned} \varepsilon &= \alpha \\ \rho &= 1 - \alpha \end{aligned} \tag{3.30}$$

enable us to determine all three properties of an opaque surface from a knowledge of only one property.

In using the above relations, care should be exercised when there is considerable difference between the surface temperature and the temperature of the source of incident radiation.

For bodies in thermal equilibrium, the ratio of the emissive power to that of its absorptivity is the same for all the bodies. That is:

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3} = \frac{E_n}{\alpha_n} \tag{3.31}$$

If Body is black then, $\frac{E_{b1}}{1} = \frac{E_2}{\alpha_2}$, $\therefore E_2 = \alpha E_{b1}$. Comparing this to $E = \varepsilon E_b$

For grey bodies, $\alpha = \varepsilon$

Grey Body

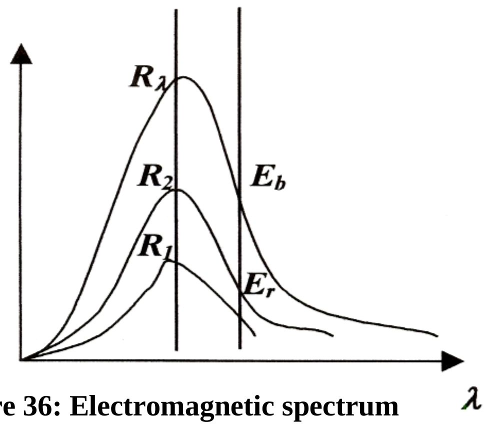


Figure 36: Electromagnetic spectrum as a function of wavelength

A grey body is one whose emissivity is constant through the entire spectrum or is not a function of wavelength.

$$\varepsilon_{\lambda_1} = \varepsilon_{\lambda_2} = \varepsilon_{\lambda_3} = \varepsilon_{\lambda_5} = \varepsilon$$

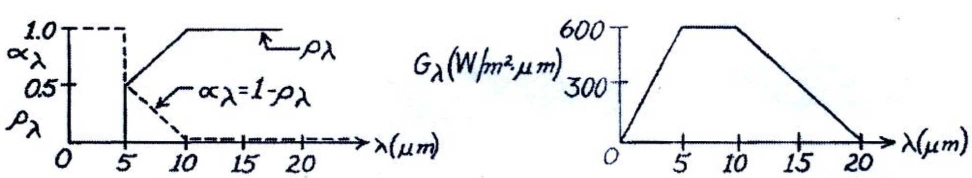
Monochromatic emissivity

$$\varepsilon_\lambda = \frac{E_{r\lambda}}{E_{b\lambda}} \tag{3.32}$$

Worked Example 3.3: An opaque surface with prescribed spectral hemispherical reflectivity distribution is subjected to a prescribed spectral irradiation.

Find: (a) The spectral, hemispherical absorptivity, (b) Total irradiation, (c) The absorbed radiant flux, and (d) Total, hemispherical absorptivity

Schematic:



Assumption: Surface is opaque

Analysis: (a) The spectral, hemispherical absorptivity, α_λ, for an opaque surface is given by:

$$\alpha_\lambda = 1 - \rho_\lambda$$

which is shown as a dashed line on the ρ_λ distribution axes.

(b) The total irradiation, G, can be obtained by integrating by parts

$$G = \int_0^{\infty} G_\lambda d\lambda = \int_0^{5\mu m} G_\lambda d\lambda + \int_{5\mu m}^{10\mu m} G_\lambda d\lambda + \int_{10\mu m}^{20\mu m} G_\lambda d\lambda$$

$$G = \frac{1}{2} \times 600 \frac{W}{m^2.\mu m}(5-0)\mu m + 600 \frac{W}{m^2.\mu m}(10-5)\mu m + \frac{1}{2} \times 600 \frac{W}{m^2.\mu m} \times (20-10)\mu m$$

$$G = 7500 W / m^2$$

(c). The absorbed irradiation is obtained by

$$G_{abs} = \int_0^{\infty} \alpha_\lambda G_\lambda d\lambda = \alpha_1 \int_0^{5\mu m} G_\lambda d\lambda + G_{\lambda,2} \int_{5\mu m}^{10\mu m} \alpha_\lambda d\lambda + \alpha_3 \int_{10\mu m}^{20\mu m} G_\lambda d\lambda$$

Noting that $\alpha_1 = 1.0$ for $\lambda = 0 \rightarrow 5$ μm; $G_{\lambda,2} = 600$W/m^2.μm for $\lambda = 5 \rightarrow 10$ μm and $\alpha_3 = 0$ for $\lambda > 10$ μm, find that

$$G_{abs} = 1.0(0.5 \times 600 W / m^2 \mu m)(5-0)\mu m + 600 W / m^2 \mu m (0.5 \times 0.5)(10-5)\mu m + 0$$

$$G_{abs} = 2250 W / m^2$$

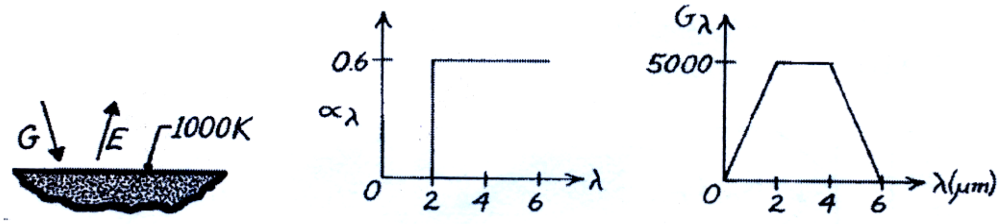
(d) The total hemispherical absorptivity is defined as the fraction of the total irradiation that is absorbed. From Equation 3.16

$$\alpha = \frac{G_{abs}}{G} = \frac{2250 W / m^2}{7500 W / m^2} = 0.30$$

Comment: Recognise that the total hemispherical absorptivity, α = 0.3, is for the given spectral irradiation. For a different G_λ, one would then expect a different value for α.

Worked Example 3.4: Given the spectral distribution of the absorptivity and irradiation of a surface at 1000K find (a) Total, hemispherical absorptivity, (b) Total, hemispherical emissivity, (c) Net radiant flux to the surface

Schematic:



Assumption: $\alpha_\lambda = \varepsilon_\lambda$

Analysis:

(a) From Equation 3.23

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} = \frac{\int_0^{2\,\mu\mathrm{m}} \alpha_\lambda G_\lambda d\lambda + \int_2^{4\,\mu m} \alpha_\lambda G_\lambda d\lambda + \int_4^{6\,\mu m} \alpha_\lambda G_\lambda d\lambda}{\int_0^{2\,\mu m} G_\lambda d\lambda + \int_2^{4\,\mu m} G_\lambda d\lambda + \int_4^{6\,\mu m} G_\lambda d\lambda}$$

$$\alpha = \frac{0 \times 1/2(2-0)5000 + 0.6(4-2)5000 + 0.6 \times 1/2(6-4)5000}{1/2(2-0)5000 + (4-2)(5000) + 1/2(6-2)5000}$$

$$\alpha = \frac{9000}{20{,}000} = 0.45$$

(b) From Equation 3.15,

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b} d\lambda}{E_b} = \frac{0\int_0^{2\,\mu m} E_{\lambda,b} d\lambda}{E_b} + \frac{0.6\int_2^\infty E_{\lambda,b} d\lambda}{E_b}$$

$$\varepsilon = 0.6F_{(2\,\mu m \to \infty)} = 0.6\left[1 - F_{(0\to 2\,\mu m)}\right]$$

From Table 3.2, with $\lambda \mathrm{T} = 2\mu\mathrm{m} \times 1000\mathrm{K} = 2000\mu\mathrm{mK}$. find $F_{(0\to 2\,\mu m)} = 0.0667$

Hence

$$\varepsilon = 0.6[1 - 0.0667] = 0.56$$

(c) The net radiant heat flux *to* the surface is

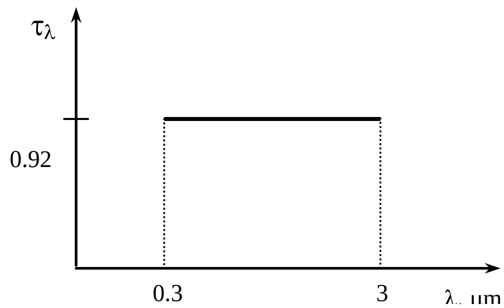
$$q''_{\mathrm{rad,\,net}} = \alpha G - E = \alpha G - \varepsilon\sigma T^4$$

$$q''_{\mathrm{rad,\,net}} = 0.45\,(20{,}000\,\mathrm{W/m^2}) - 0.56 \times 5.67 \times 10^{-8}\ \mathrm{W/m^2.K^4} \times (1000\,\mathrm{K})^4$$

$$q''_{\mathrm{rad,\,net}} = (9000 - 31{,}751)\,\mathrm{W/m^2} = -22{,}751\ \mathrm{W/m^2}$$

Worked Example 3.5

The variation of the spectral transmissivity of a 0.6 cm thick glass window is as shown in the figure below. Determine the average transmissivity of this window for solar radiation (T = 5800 K) and radiation coming from surfaces at room temperature (T = 300K). Also determine the amount of solar radiation transmitted through the window for incident solar radiation of 650W/m^2.



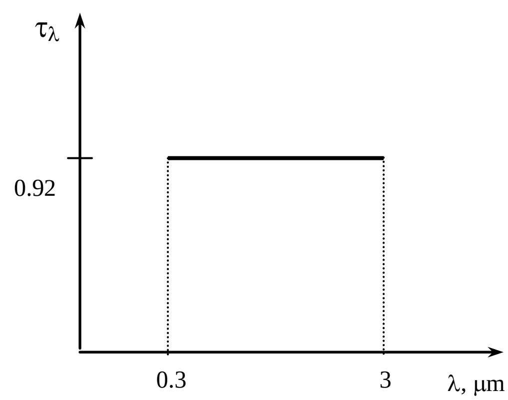
Solution

The variation of transmissivity of a glass is given. The average transmissivity of the glass at two temperatures and the amount of solar radiation transmitted through the glass are to be determined.

Analysis for T=5800 K:

$$\lambda_1 T_1 = (0.3\ \mu\text{m})(5800\ \text{K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.035$$

$$\lambda_2 T_1 = (3\ \mu\text{m})(5800\ \text{K}) = 17{,}400\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.977$$



The average transmissivity of this surface is

$$\tau(T) = \tau_1 (f_{\lambda_2} - f_{\lambda_1}) = (0.9)(0.977 - 0.035) = \mathbf{0.848}$$

For T=300 K:

$$\lambda_1 T_2 = (0.3\ \mu\text{m})(300\ \text{K}) = 90\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0$$

$$\lambda_2 T_2 = (3\ \mu\text{m})(300\ \text{K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.0001685$$

Then,

$$\tau(T) = \tau_1 (f_{\lambda_2} - f_{\lambda_1}) = (0.9)(0.0001685 - 0.0) = \mathbf{0.00015 \approx 0}$$

The amount of solar radiation transmitted through this glass is

$$G_{\text{tr}} = \tau G_{\text{incident}} = 0.848(650\ \text{W/m}^2) = \mathbf{551\ W/m^2}$$

2-3.4 The Atmosphere and Solar Radiation

The sun is our primary source of energy. The energy coming off the sun, called solar energy, reaches us in the form electromagnetic waves after experiencing considerable interactions with the atmosphere. The radiation energy emitted by the constituents of the atmosphere form the atmospheric radiation.

The sun is a nearly spherical body that has a diameter, D = 1.39 x 10^9 m and a mass of 2 x 10^{30} kg and is located at a mean distance, L = 1.50 x 10^{11} m from the earth (Figure 37).

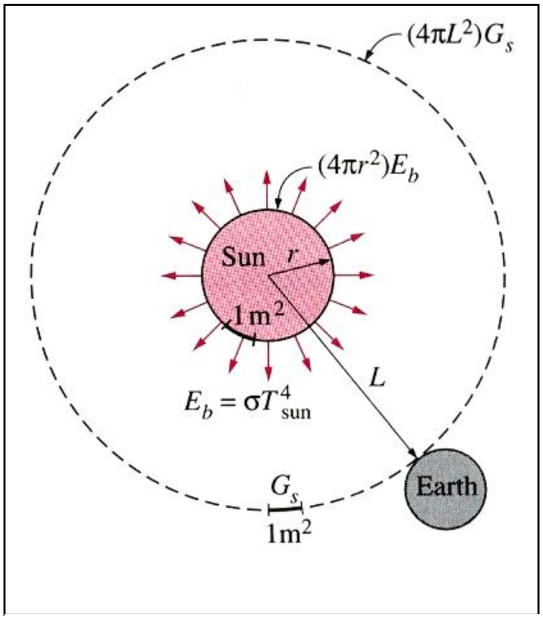


Figure 37: Solar radiation reaching the earth's atmosphere

It emits radiation continuously at a rate of 3.8 x 10^{26} W. Less than a billionth of this energy strikes the earth, which is sufficient to warm the earth and to maintain life through the photosynthesis process. The rate at which solar energy is incident on a surface normal to the sun's rays at the outer edge of the atmosphere when the earth is at its mean distance from the sun is called the solar constant. Its value is given by $G_s = 1353 \text{ W/m}^2$

The measured value of the solar constant can be used to estimate the effective surface temperature of the sun from the requirement that

$$\left(4\pi L_1^2\right)G_s = \left(4\pi r^2\right)\sigma T_{sun}^{\ 4} \qquad (3.33)$$

The gas molecules and the suspended particles in the atmosphere emit radiation as well as absorb it. Although this emission is far from resembling the distribution of radiation from a blackbody, it is found convenient in radiation calculations to treat the atmosphere as a blackbody at some lower fictitious temperature that emits an equivalent amount of radiation energy. This fictitious temperature is called the **effective sky temperature** T_{sky}. Then, the radiation from the atmosphere to the earth's surface is expressed as

$$G_{sky} = \sigma T_{sky}^{\ 4} \qquad (3.34)$$

The value of the sky temperature depends on the atmospheric conditions. It ranges from about 230 K for cold, clear-sky conditions to about 285 K for warm, cloudy-sky conditions. The sky radiation absorbed by a surface can be expressed as:

$$E_{sky,absorbed} = \alpha G_{sky} = \alpha\sigma T_{sky}^4 = \varepsilon\sigma T_{sky}^4 \qquad (3.35)$$

The net rate of radiation heat transfer to a surface exposed to solar radiation and atmospheric radiation is determined from an energy balance

$$\dot{q}_{\text{net,rad}} = \sum E_{\text{absorbed}} - \sum E_{\text{emitted}}$$

$$\begin{aligned}\dot{q}_{\text{net, rad}} &= E_{\text{solar,absorbed}} + E_{\text{sky,absorbed}} + E_{\text{emitted}} \\ &= \alpha_s G_s + \varepsilon\sigma T_{\text{sky}}^4 - \varepsilon\sigma T_s^4 \\ &= \alpha_s G_{\text{solar}} + \varepsilon\sigma (T_{\text{sky}}^4 - T_s^4)\end{aligned} \tag{3.36}$$

where T_s, is the temperature of the surface in *K* and ε is its emissivity at room temperature. A positive result for $\dot{q}_{\text{net,rad}}$ indicates a radiation heat gain by the surface and a negative result indicates a heat loss.

The absorption and emission of radiation by the elementary gases such H_2, O_2, and N_2 at moderate temperatures are negligible, and a medium filled with these gases can be treated as a vacuum in radiation analysis. The absorption and emission of radiation by gases with larger molecules such as H_2O and CO_2, however, can be significant and may be considered. Radiation properties of surfaces are quite different for the incident and emitted radiation, and the surfaces cannot be assumed to be grey.

Surfaces that are intended to collect solar energy, such as the absorber surfaces of solar collectors, are desired to have high absorptivity and low emissivity values to maximise the absorption of solar radiation and minimise the emission of radiation. On the other hand, surfaces that are intended to remain cool under the sun, such as the outer surfaces of fuel tanks and refrigerator trucks, are designed to have just the opposite properties. Surfaces are kept cool by simply painting them white $\left(\alpha_s = 0.14, \quad \varepsilon = 0.93\right)$. Materials such as glass and water are semitransparent to short wavelength (0.3μm to 3μm) radiation but opaque to longer wavelength (heat) radiation. This property makes it desirable in using glass as the cover plate material for solar collector application.

Self Assessment 2-3

1. The spectral emissivity function of an opaque surface at 1000 K is approximated as

$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = 0.4, & 0 \le \lambda < 2\ \mu\text{m} \\ \varepsilon_2 = 0.7, & 2\ \mu\text{m} \le \lambda < 6\ \mu\text{m} \\ \varepsilon_3 = 0.3, & 6\ \mu\text{m} \le \lambda < \infty \end{cases}$$

Determine the average emissivity of the surface and the rate of radiation emission from the surface, in W/m^2. **Suggested Answers: 0.575, 32.6 kW/m^2**

2. The variation of the spectral absorptivity of a surface is as given in Figure Q2. Determine the average absorptivity and reflectivity of the surface for radiation that originates from a source at T =2500 K. Also, determine the average emissivity of this surface at 3000 K.

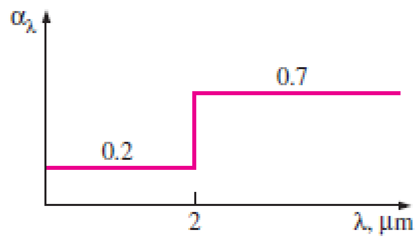


Figure Q2

Suggested Answer: [0.38; 0.62; 0.33]

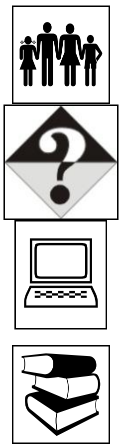
3. The emissivity of a surface coated with aluminum oxide can be approximated to be 0.2 for radiation at wavelengths less than 5 μm and 0.9 for radiation at wavelengths greater than 5 μm. Determine the average emissivity of this surface at (*a*) 5800 K and (*b*) 300 K. What can you say about the absorptivity of this surface for radiation coming from sources at 5800 K and 300 K? **Suggested Answers: [0.203; 0.89]**

Learning Track Activities ensure you understand all the terms listed below

Key terms/ New Words in Unit

i. Electromagnetic waves
ii. Frequency
iii. Wavelength
iv. Photons
v. Thermal radiation
vi. Electromagnetic spectrum
vii. Blackbody radiation
viii. Spectral blackbody emissive power
ix. Wien's displacement law

x. Blackbody radiation function
xi. Emissivity
xii. Reflectivity
xiii. Transmisivity
xiv. Absorptivity
xv. Kirchhoff's law
xvi. Intensity of Radiation
xvii. Solid Angle
xviii. Specular reflection
xix. Diffuse reflection
xx. Effective sky temperature
xxi. Solar constant



Review Question:

The spectral transmisivity of a 3-mm-thick regular glass can be expressed as

$$\tau_1 = 0 \quad \text{for} \quad \lambda < 0.35\ \mu\text{m}$$
$$\tau_2 = 0.85 \quad \text{for} \quad 0.35 < \lambda < 2.5\ \mu\text{m}$$
$$\tau_3 = 0 \quad \text{for} \quad \lambda > 2.5\ \mu\text{m}$$

Determine the transmisivity of this glass for solar radiation. What is the transmisivity of this glass for light?

Discussion Question:

1. Consider two identical bodies, one at 1000 K and the other at 1500 K. Which body emits more radiation in the shorter-wavelength region? Which body emits more radiation at a wavelength of 20 μm?

2. For a surface, how is irradiation defined? For diffusely incident radiation, how is irradiation on a surface related to the intensity of incident radiation?

Web Activity: www.mhhe.com/cengel, www.mitcourseware.com

Reading: Read chapter 12, Fundamentals of heat and mass transfer, F.P.Incropera and D.P.Dewit, 4th Edition.

Unit 4

VIEW FACTORS AND RADIATION EXCHANGE BETWEEN SURFACES

Introduction

In this unit, we shall explain how view factors are computed and evaluate view factors for different configurations and understand view factor algebra. Problems involving radiative heat transfer among grey, opaque surfaces separated by a medium that does not participate in radiative heat transfer would be considered. Lastly, we shall apply the radiosity method for the solution of radiant heat transfer among diffuse, grey surfaces in an enclosure.

Learning Objectives

After reading this unit you should be able to:

1. Explain how view factors are computed.
2. Evaluate view factors for different configurations and understand view factor algebra.
3. Solve problems involving radiative heat transfer among grey, opaque surfaces separated by a medium that does not participate in radiation heat transfer.
4. Apply the radiosity method for the solution of radiant heat transfer among diffuse, grey surfaces in an enclosure.

Unit content

Session 1-4: The View Factor

1-4.1: Determination of view factors
1-4.2: View Factor relations and charts
1-4.3: View factors between infinitely long surfaces

Session 2-4: Radiation Exchange between surfaces

2-4.1: The Concept of Radiosity
2-4.2: Radiation heat transfer in two surface enclosures
2-4.3: Radiation shields
2-4.4: Radiation Exchange in three or more surface enclosures

SESSION 1-4: THE VIEW FACTOR

We have so far considered the radiation properties of surfaces and the radiation interactions of a single surface. We now consider *radiation heat transfer* between two or more surfaces. Radiation exchange between two surfaces depend not only on their radiation properties and temperatures, but also on their relative orientation (see Figure 38).

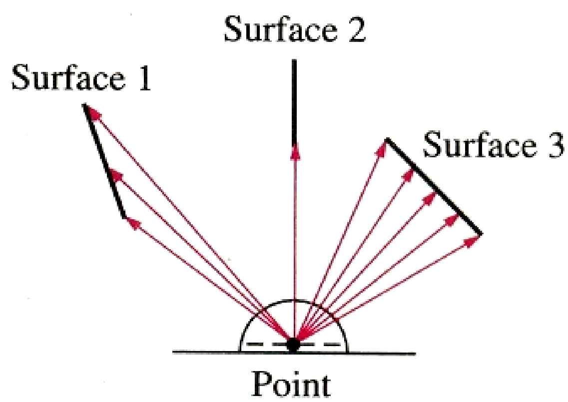


Figure 38: Effect of orientation on radiation interchange

The parameter *view factor* is used to account for the effects of orientation on radiation exchange. It is purely a geometric quantity and is not affected by the surface properties or temperature.

The view factor $F_{i,j}$ from a surface *i* to a surface *j* is defined by:

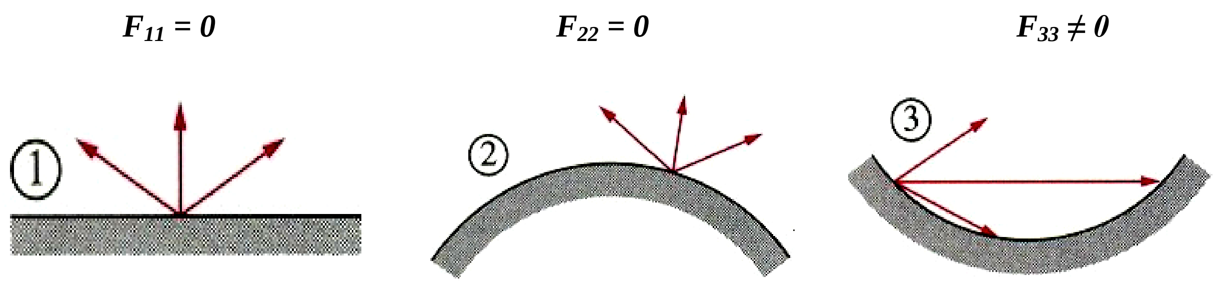
$F_{i,j}$ = the fraction of the radiation leaving surface *i* that strikes *j* directly.

Thus, the view factor F_{12} represent the fraction of the radiation leaving surface 1 that strikes surface 2 directly, while F_{21} represents the fraction of the radiation leaving surface 2 that strikes surface 1 directly. Note that the radiation that strikes a surface after being reflected by other surfaces is not considered in the evaluation of view factors.

The fraction of radiation leaving a surface that strikes itself can also be accounted for using the view factor concept. In that case, $j = i$, and we can write

F_{ii} = the fraction of radiation leaving the surface *i* that strikes itself directly.

For plane and convex surfaces, F_{ii} = 0, but for a concave surface, $F_{ii} \neq 0$, as illustrated in the Figure 39



(a) Plane surface **(b) Convex surface** **(c) Concave surface**

Figure 39: View factor from a surface to itself

The view factor F_{ij} may take a minimum value of 0 and a maximum value of 1. If F_{ij} = 0, it implies that no radiation leaving surface *i* strikes surface *j* directly. On the other hand, if F_{ij} = 1, it implies that all the radiation leaving surface *i* is intercepted directly by surface *j*. A typical example of this is in the case of two concentric spheres, where the entire radiation leaving the smaller surface (surface 1) is intercepted by the large (surface 2), as shown in Figure 40.

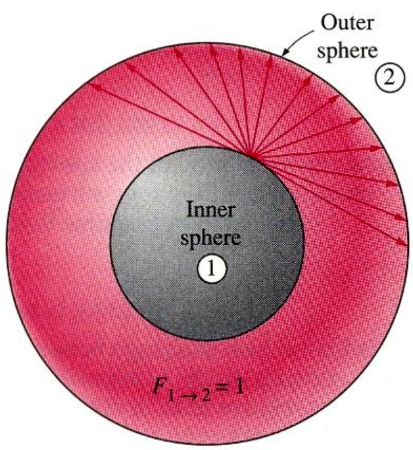


Figure 40: View factor for two concentric spheres

1-4.1 Determination of view factors

The view factor F_{12} between two surfaces A_1 and A_2 can be determined in a systematic manner by first expressing the view factor between two elemental areas d_{A1} and d_{A2} in terms of the spatial variables and then performing the necessary integration.

Consider for example, the two surfaces A_1 and A_2 shown in Figure 41. Let *r* be the distance between the two surfaces.

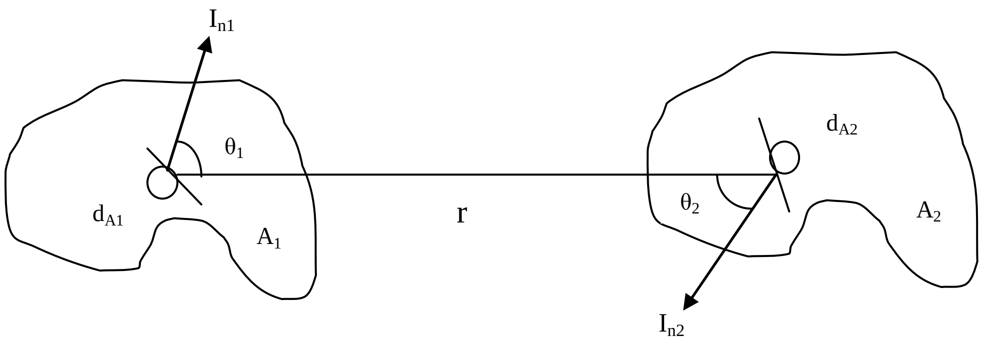


Figure 41: Determination of view factor

θ represents the polar angle between the normal to a surface and the line joining the two areas

The view factor is determined by:

$$F_{12} = \frac{1}{A_1} \iint_A \frac{\cos\theta_1 \cos\theta_2 dA_1 dA_2}{\pi r^2} \tag{4.1}$$

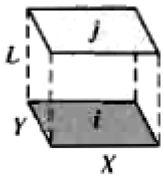
View factors for selected geometries are given in Tables 7 and 8 in analytical form, and in Figures 42 to 45 in the graphical form.

Table 7: View factor expressions for some common geometries of finite size (3D)

Geometry	Relation

Aligned Parallel Rectangles

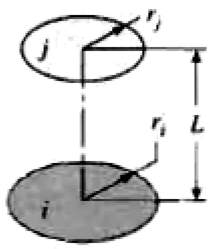
(Figure 13.4)



$\bar{X} = X/L, \bar{Y} = Y/L$

$$F_{ij} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$$

Coaxial Parallel Disks
(Figure 13.5)

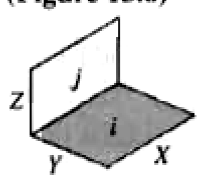


$R_i = r_i/L, R_j = r_j/L$

$$S = 1 + \frac{1+R_j^2}{R_i^2}$$

$$F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$$

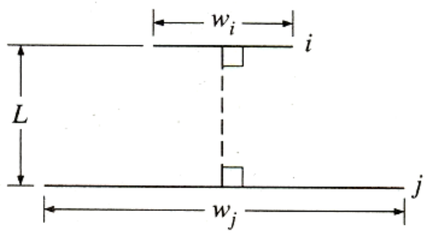
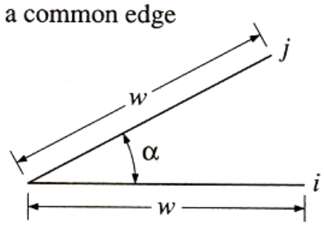
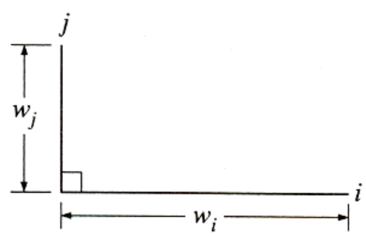
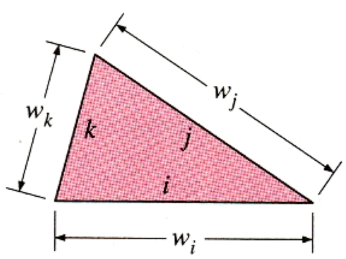
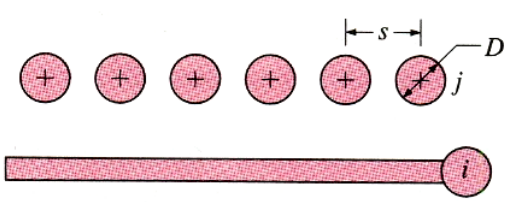
Perpendicular Rectangles with a Common Edge
(Figure 13.6)



$H = Z/X, W = Y/X$

$$F_{ij} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$$

Table 8: View factor expressions for some infinitely long (2D) geometries

Geometry	Relation
Parallel plates with midlines connected by perpendicular 	$W_i = w_i/L,\ W_j = w_j/L$ $F_{i\to j} = \dfrac{[(W_i + W_j)^2 + 4]^{1/2} - (W_j - W_i)^2 + 4]^{1/2}}{2W_i}$
Inclined parallel plates of equal width and with a common edge 	$F_{i\to j} = 1 - \sin\frac{1}{2}\alpha$
Perpendicular plates with a common edge 	$F_{i\to j} = \frac{1}{2}\left\{1 + \frac{w_j}{w_i} - \left[1 + \left(\frac{w_j}{w_i}\right)^2\right]^{1/2}\right\}$
Three-sided enclosure 	$F_{i\to j} = \dfrac{w_i + w_j - w_k}{2w_i}$
Infinite plane and row of cylinders 	$F_{i\to j} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \frac{D}{s}\tan^{-1}\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}$

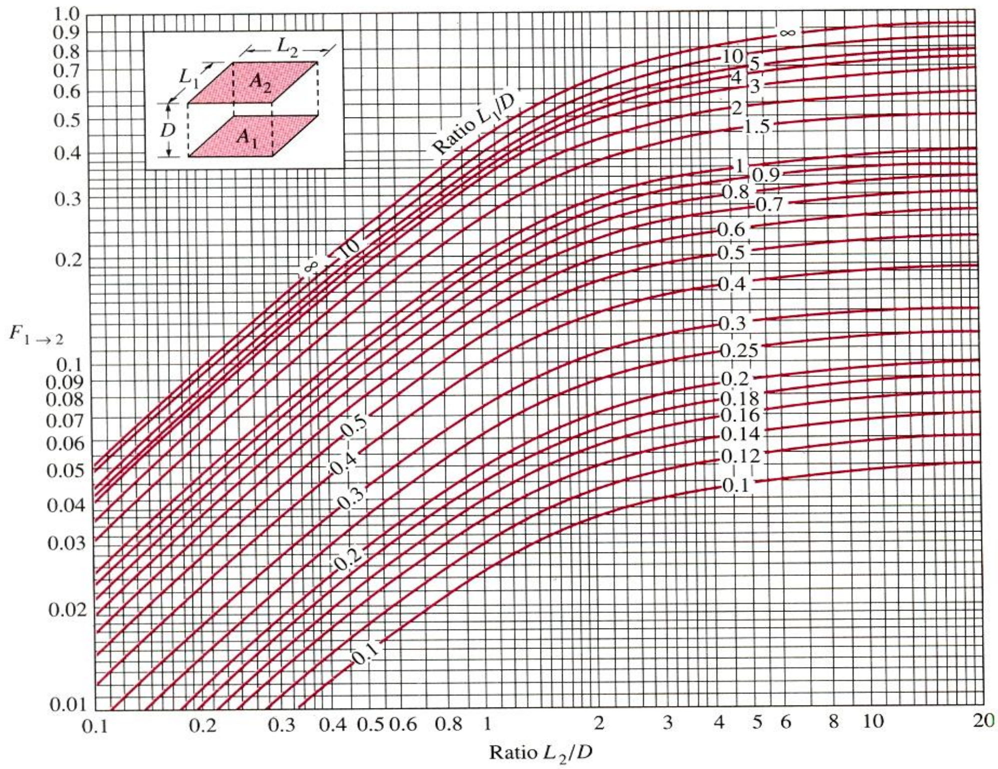


Figure 42: View factor between two aligned parallel rectangles of equal size

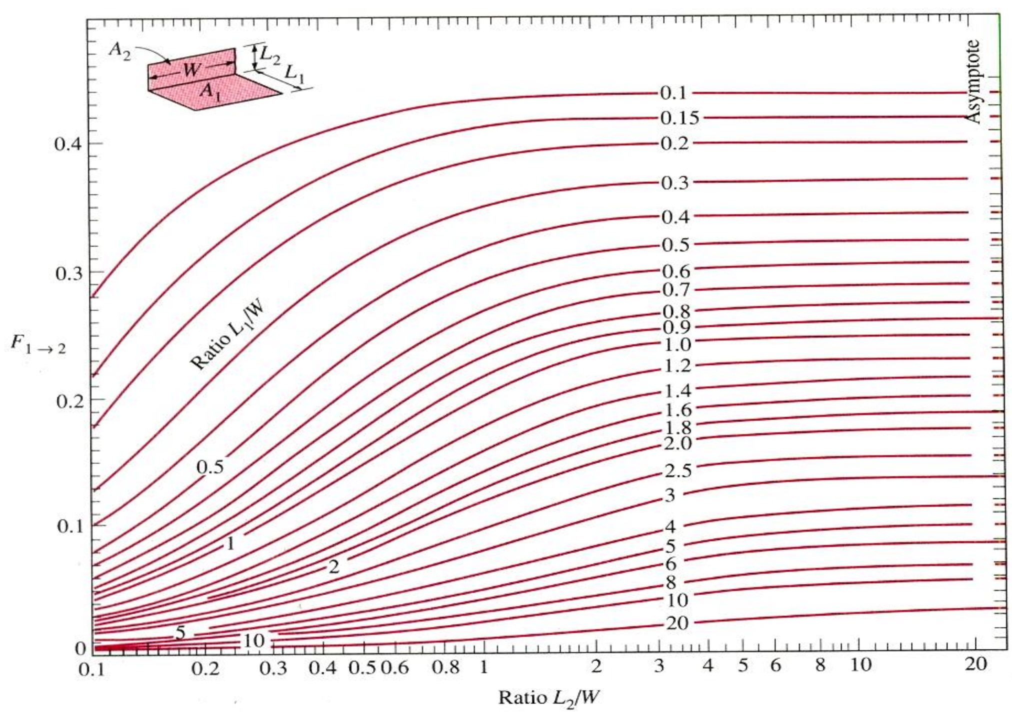


Figure 43: View factor between two perpendicular surfaces with a common edge

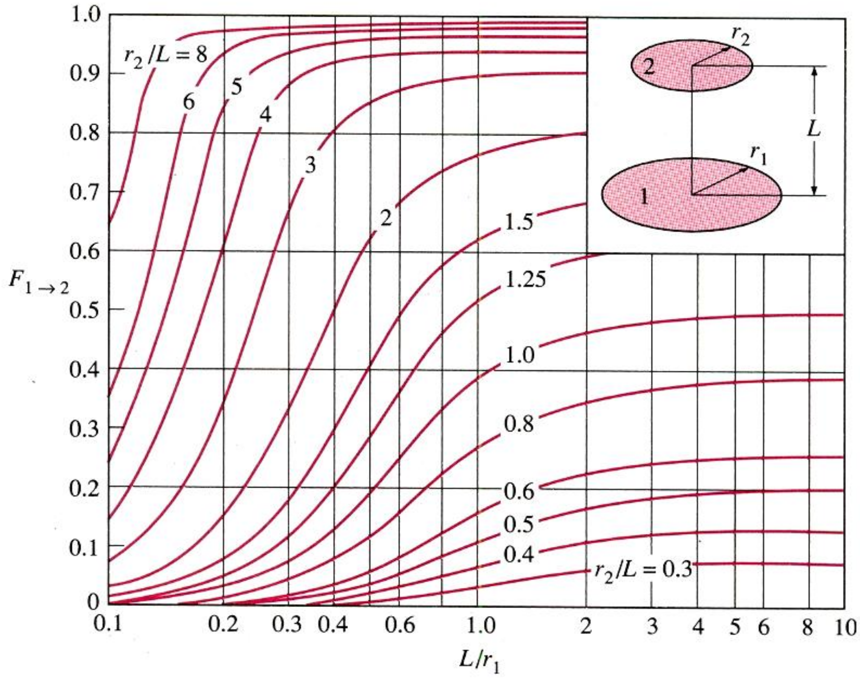
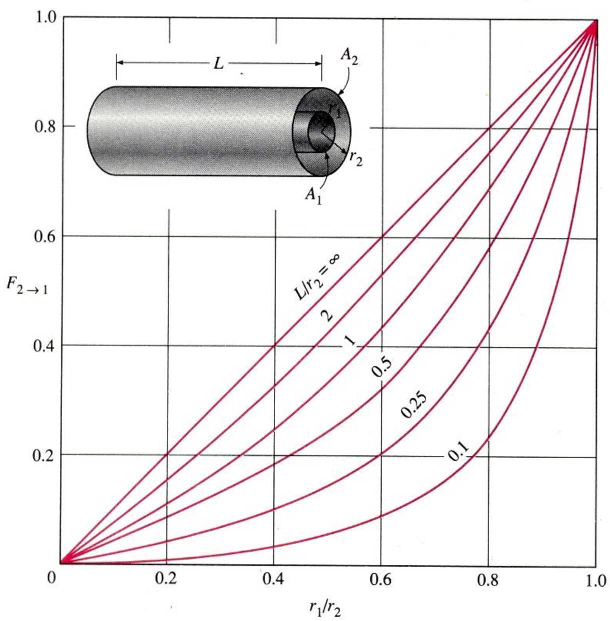


Figure 44: View factor between two coaxial parallel disks



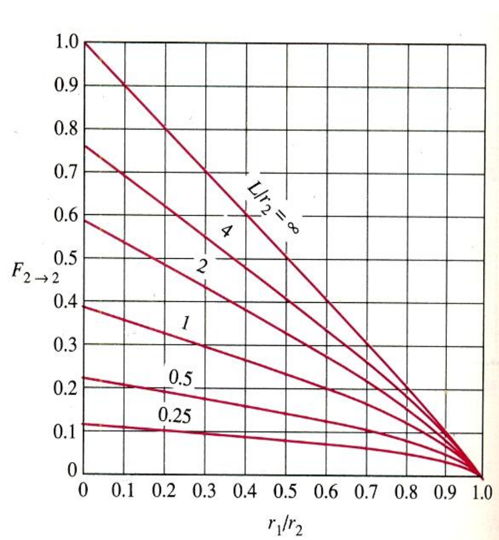


Figure 45: View factors for two concentric cylinders of finite length: (a) outer cylinder to inner cylinder; (b) outer cylinder to itself

1-4.2 View Factor Relations and Charts

Radiation analysis of an enclosure consisting of N number of surfaces required the evaluation of N^2 view factors and this evaluation can be time consuming. Fortunately, once a sufficient number of view factors are available, the remaining ones can be calculated using some fundamental relations presented below

1-4.2.1The Reciprocity Rule

The view factors F_{ij} and F_{ji} are *not* equal to each other unless the areas of the two surfaces are equal. That is,

$$F_{ji} = F_{ij} \qquad \text{when} \qquad A_i = A_j$$
$$F_{ji} \neq F_{ij} \qquad \text{when} \qquad A_i \neq A_j$$

Using the radiation intensity concept and going through some manipulations it can be shown that a pair of view factors F_{ij} and F_{ji} are related to each other by

$$A_i F_{ij} = A_j F_{ji} \tag{4.2}$$

This relation is known as the **reciprocity rule**, and it facilitates the determination of the counterpart of a view factor from knowledge of the view factor itself and the areas of the two surfaces.

1-4.2.2 The Summation Rule

This rule states that the sum of the view factors from surface *i* of an enclosure to all the surfaces of the enclosure, including to itself, must be equal to unity. (Figure 46)

This may be written mathematically as

$$\sum_{j=1}^{N} F_{ij} = 1 \tag{4.3}$$

where N is the number of surfaces of the enclosure.

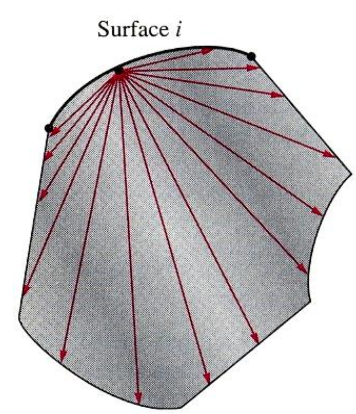


Figure 46: Demonstrating summation rule

The summation rule can be applied to each surface of an enclosure by varying *i* from 1 to N. Thus the summation rule applied to each of the N surfaces of an enclosure gives N relations for the determination of the view factors. Also, the reciprocity rule gives $\frac{1}{2}N(N-1)$ additional relations. Consequently, the total number of view factors that need to be evaluated directly for an N – surface enclosure becomes

$$N^2 - \left[N + \frac{1}{2} N(N-1) \right] = \frac{1}{2} N(N-1) \tag{4.4}$$

For example, for a six-surface enclosure, we need to determine only $\frac{1}{2} \times 6(6-1) = 15$ of the $6^2 = 36$ view factors directly. The remaining 21 view factors can be determined from the 21 equations that are obtained by applying the reciprocity and summation rules.

Example 3.5: Determine the view factors associated with an enclosure formed buy two concentric spheres as shown in Figure 47.

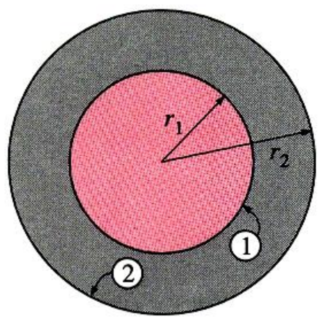


Figure 47: Geometry considered in example 3.5

The outer surface of the smaller sphere (surface 1) and inner surface of the larger sphere (surface 2) form a two-surface enclosure. Thus, N = 2; implying that the enclosure involves $N^2 = 2^2 = 4$ view factors. These view factors are: F_{11}, F_{12}, F_{21}, F_{22}. In this two-surface enclosure, we need to determine only

$$\frac{1}{2} N(N-1) = \frac{1}{2} \times 2(2-1) = 1$$

view factors directly**.** The remaining three view factors can be determined by the application of the summation and reciprocity rules. But it turns out that we can determine not only one but two view factors directly in this case by a simple inspection.

$F_{11} = 0$, since no radiation leaving surface 1 strikes itself and $F_{12} = 1$, since all radiation leaving surface 1 strikes surface 2. The view factor F_{21} is determined by applying the reciprocity rule to surfaces 1 and 2.

$$A_1 F_{12} = A_2 F_{21}$$

which yields,

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{4\pi r_1^2}{4\pi r_2^2} \times 1 = \left(\frac{r_1}{r_2}\right)^2$$

Finally, the view factor F_{22} is determined by applying the summation rule to surface 2:

$$F_{21} + F_{22} = 1$$

and thus,

$$F_{22} = 1 - F_{21} = 1 - \left(\frac{r_1}{r_2}\right)^2$$

Note that if $r_2 >> r_1$, F_{22} approaches 1.

This is expected since the fraction of radiation leaving the outer sphere that is intercepted by the inner sphere will be negligible in that case. Also, note that the two spheres considered above do not need to be concentric. However, the radiation analysis will be most accurate for the case of concentric spheres, since the radiation is most likely to be uniform on the surfaces in that case.

1-4.2.3 The Superposition Rule

The view factor from a surface *i* to a surface *j* is equal to the sum of the view factors from the surface *i* to the parts of surface *j*. The reverse is not true. That is, the view factor from a surface *j* to the surface *i* is not equal to the sum of the view factors from the parts of surface *j* to surface *i*.

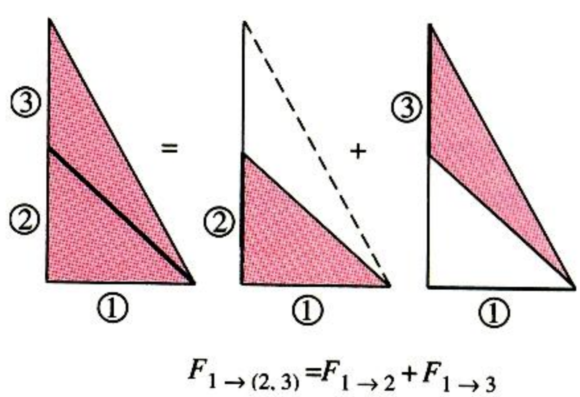


Figure 48: Demonstrating superposition rule

For the geometry shown in Figure 48, which is infinitely long in the direction perpendicular to the plane of the paper, the radiation that leaves surface 1 and strikes surfaces 2 and 3 is equal to the sum of the radiation that strikes surfaces 2 and 3. The view factor from surface 1 to the combined surfaces 2 and 3 can be expressed as:

$$F_{1\to(2,3)} = F_{12} + F_{13} \tag{4.5}$$

The view factor F_{13} cannot be determined directly but can be obtained from equation 4.5 by evaluating $F_{1\to(2,3)}$ and F_{12} using view factor expressions and charts.

To obtain a relation for the view factor $F_{(2,3)\to1}$, multiply equation 4.5 by A_1:

$$A_1F_{1\to(2,3)} = A_1F_{12} + A_1F_{13} \tag{4.5}$$

and apply the reciprocity rule to each term to get

$$(A_2 + A_3)F_{(2,3)\to1} = A_2F_{21} + A_3F_{31} \tag{4.6}$$

or

$$F_{(2,3)\to1} = \frac{A_2F_{21} + A_3F_{31}}{(A_2 + A_3)} \tag{4.7}$$

Areas that are expressed as the sum of more than two parts can be handled in a similar manner.

Example 3.6: Fraction of radiation leaving an opening

Determine the fraction of the radiation leaving the base of the cylindrical enclosure shown in Figure 49 that escapes through a coaxial ring opening at its top surface. The radius and the length of the enclosure are r_1 = 10 cm and L = 10 cm, while the inner and outer radii of the ring are r_2 = 5 cm and r_3 = 8 cm, respectively.

Solution: The fraction of radiation leaving the base of a cylindrical enclosure through a coaxial ring opening at its top surface is to be determined.

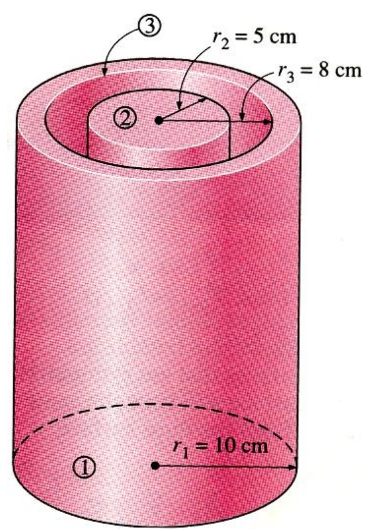


Figure 49: Schematic diagram for worked example 3.6

Assumption: The base surface is a diffuse emitter and reflector.

Analysis: View factor $F_{1\to \text{ring}}$ is in fact what is to be determined. This view factor cannot however be directly evaluated since no analytical expression nor chart is available for the determination of view factors between a circular area and a coaxial ring. However, view factors between two coaxial parallel disks can be determined using the chart in Figure 44 and a ring can always be expressed in terms of disks.

Using the superposition rule, the view factor from surface 1 to 3 can be expressed as

$$F_{13} = F_{12} + F_{1\to \text{ring}}$$

Since surface 3 is the sum of surface 2 and the ring area. The view factors F_{12} and F_{13} are determined from the chart in Figure 44 as follows:

$$\frac{L}{r_1} = \frac{10\,\text{cm}}{10\,\text{cm}} = 1 \quad \text{and} \quad \frac{r_2}{L} = \frac{5\,\text{cm}}{10\,\text{cm}} = 0.5 \xrightarrow{\text{(Figure 44)}} F_{12} = 0.11$$

$$\frac{L}{r_1} = \frac{10\,\text{cm}}{10\,\text{cm}} = 1 \quad \text{and} \quad \frac{r_3}{L} = \frac{8\,\text{cm}}{10\,\text{cm}} = 0.8 \xrightarrow{\text{(Figure 44)}} F_{13} = 0.28$$

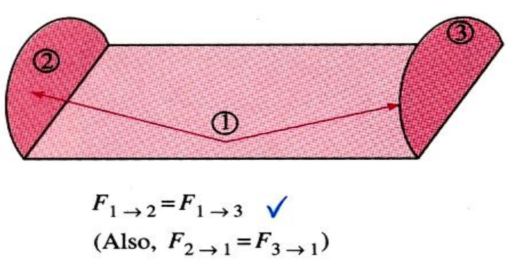
Therefore,

$$F_{1\to \text{ring}} = F_{13} - F_{12} = 0.28 - 0.11 = 0.17$$

Note that F_{12} and F_{13} represent the fraction of radiation leaving the base that strikes the circular surfaces 2 and 3, respectively, and their difference gives the fraction that strikes the ring area.

1-4.2.4 The Symmetry Rule

Two or more surfaces that possess symmetry about a third surface will have identical view factors from that surface (Figure 50)



The symmetry rule can also be expressed as follows: *if the surfaces j and k are symmetric about the surface i then* $F_{ij} = F_{ik}$. Using the reciprocity rule, it can be shown that the relation $F_{ji} = F_{ki}$ is also true in this case.

Figure 50: Demonstrating symmetry rule

Example 3.7: View factors associated with a Tetragon

Determine the view factors from the base of the pyramid shown in Figure 51 to each of its four side surfaces. The base of the pyramid is a square, and its side surfaces are isosceles triangles.

Solution: The view factors from the base of a tetragon to each of its four side surfaces for the case of a square base are to be determined.

Assumption: The surfaces are diffuse emitters and reflectors

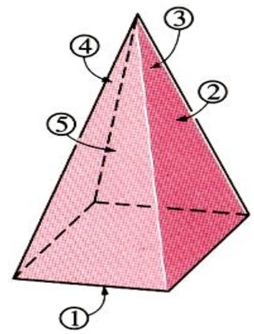


Figure 51: Schematic for worked Example 3.7

Analysis: The base of the pyramid and its four side surfaces form a five-surface enclosure. The four side surfaces are symmetric about the base surface. From the symmetry rule,

$$F_{12} = F_{13} = F_{14} = F_{15}$$

Also, applying the summation rule to surface 1

$$\sum_{j=1}^{5} F_{1j} = F_{11} + F_{12} + F_{13} + F_{14} + F_{15} = 1$$

However, $F_{11} = 0$, since the base is a flat surface. Then the two relations above yield

$$F_{12} = F_{13} = F_{14} = F_{15} = 0.25$$

Each of the four side surfaces of the pyramid receive one-fourth of the entire radiation leaving the base surface, as expected. Note that the presence of symmetry greatly simplified the determination of the view factors.

Example 3.8: View factors associated with a long triangular duct

Determine the view factor from any one side to any other side of the infinitely long triangular duct whose cross-section is given in Figure 52.

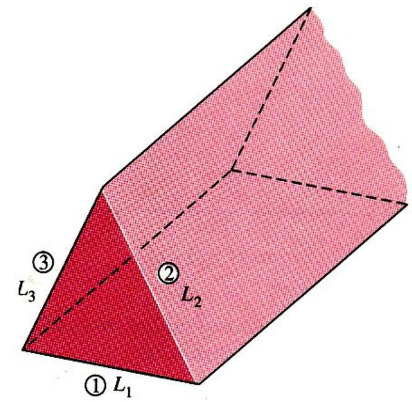


Figure 52: Schematic diagram for Worked example 3.8

Assumption: The surfaces are diffuse emitters and reflectors.

Analysis: The widths of the sides of the triangular cross-section of the duct are L_1, L_2, and L_3, and the surface areas corresponding to them are A_1, A_2, and A_3, respectively. Since the duct is infinitely long, the fraction of radiation leaving any surface that escapes through the ends of the duct is negligible. Therefore, the infinitely long duct can be considered to be a three-sided enclosure, N = 3.

This enclosure involves $N^2 = 3^2 = 9$ view factors, and there is the need to determine

$$\frac{1}{2}N(N-1) = \frac{1}{2}\times 3(3-1) = 3$$

of these view factors directly.

All 3 can be determined by inspection as $F_{11} = F_{22} = F_{33} = 0$ since all three surfaces are flat. The remaining six view factors can be determined by the application of the summation and reciprocity rules.

Applying the summation rule to each of the three surfaces gives:

$$F_{11} + F_{12} + F_{13} = 1$$
$$F_{21} + F_{22} + F_{23} = 1$$
$$F_{31} + F_{32} + F_{33} = 1$$

Knowing that $F_{11} = F_{22} = F_{33} = 0$ and multiplying the first equation by A_1, the second by A_2, and the third by A_3 gives

$$
\begin{aligned}
A_1F_{12} + A_1F_{13} &= A_1 \\
A_2F_{21} + A_2F_{23} &= A_2 \\
A_3F_{31} + A_3F_{32} &= A_3
\end{aligned}
$$

Finally, applying the three reciprocity rules $A_1F_{12} = A_2F_{21}$, $A_1F_{13} = A_3F_{31}$, and $A_2F_{23} = A_3F_{32}$ gives

$$
\begin{aligned}
A_1F_{12} + A_1F_{13} &= A_1 \\
A_1F_{12} + A_2F_{23} &= A_2 \\
A_1F_{13} + A_2F_{23} &= A_3
\end{aligned}
$$

This is a set of three algebraic equations with three unknowns, which can be solved to obtain

$$
\begin{aligned}
F_{12} &= \frac{A_1 + A_2 - A_3}{2A_1} = \frac{L_1 + L_2 - L_3}{2L_1} \\
F_{13} &= \frac{A_1 + A_3 - A_2}{2A_1} = \frac{L_1 + L_3 - L_2}{2L_1} \\
F_{23} &= \frac{A_2 + A_3 - A_1}{2A_2} = \frac{L_2 + L_3 - L_1}{2L_2}
\end{aligned}
\tag{4.8}
$$

Note that the areas of the side surfaces have been replaced by their corresponding widths since $A = Ls$ and the length s can be factored out and cancelled.

The result obtained can be generalised as follows: *the view factor from a surface of a very long triangular duct to another surface is equal to the sum of the widths of these two surfaces minus the width of the third surface, divided by twice the width of the first surface.*

1-4.3 View Factors between Infinitely long surfaces: The Crossed-Strings Method

Many practical problems involve geometries of constant cross-section such as channels and ducts that are very long in one direction relative to the other directions. Such geometries can conveniently be considered to be two-dimensional, since any radiation interaction through their end surfaces will be negligible. They can therefore be modeled as being infinitely long and the view factor between their surfaces can be determined by the cross-strings method developed by H.C. Hottel. The surfaces of the geometry do not need to be flat; they can be convex, concave, or any irregular shape.

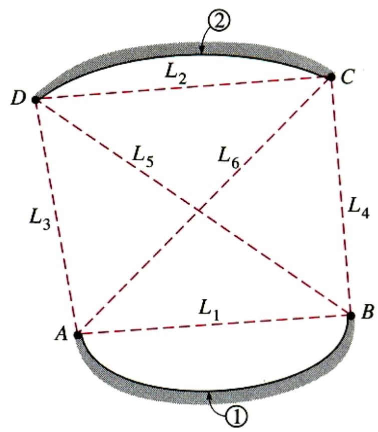


Figure 53: Determination of the view factor F_{12} by the application of the crossed-strings method

To determine the view factor F_{12}, between surfaces 1 and 2 there is first the need to identify the endpoints of the surfaces (the points A, B, C and D) and connect them to each other with tightly stretched strings, which are indicated by dashed lines. F_{12} can be expressed in terms of the lengths of these stretched strings, which are straight lines as

$$F_{12} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1} \tag{4.9}$$

Hottel's cross-string method can be expressed verbally as

$$F_{ij} = \frac{\sum (\text{Crossed strings}) - \sum (\text{Uncrossed strings})}{2 \times (\text{String on surface } i)} \tag{4.10}$$

The crossed-strings method is applicable even when the surfaces considered share a common edge, as in a triangle. In such cases, the common edge can be treated as an imaginary string of zero length. The method can also be applied to surfaces that are partially blocked by other surfaces by allowing the strings to bend around the blocking surfaces.

Example 3.9: The Crossed-Strings Method for View Factors

Two infinitely long parallel plates of widths a = 12 cm and b = 5 cm are located a distance c = 6 cm apart, as shown in Figure 54. (a) Determine the view factor F_{12} from surface 1 to surface 2 using the crossed-strings method. (b) Derive the crossed-strings formula by forming triangles on the given geometry and using equation 4.8 for view factors between the sides of triangles.

Solution. The view factors between two infinitely long parallel plates are to be determined using the crossed-strings method, and the formula for the view factor is to be derived.

Assumption: The surfaces are diffuse emitters and reflectors

Analysis: (a) First the endpoints of both surfaces are labeled and straight dashed lines are drawn between the points as shown in the figure. The crossed and uncrossed strings are identified after which the crossed-strings method is applied to determine the view factor F_{12}.

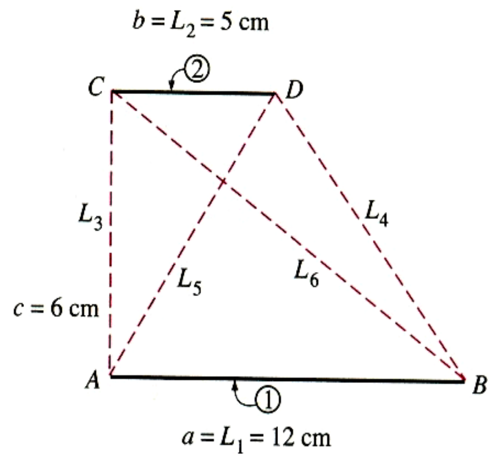


Figure 54: Schematic for worked example 3.9

$$F_{12} = \frac{\sum(\text{Crossed strings}) - \sum(\text{Uncrossed strings})}{2\times(\text{String on surface 1})} = \frac{(L_5+L_6)-(L_3+L_4)}{2L_1}$$

where

$$L_1 = a = 12\,\text{cm} \qquad L_4 = \sqrt{7^2+6^2} = 9.22\,\text{cm}$$
$$L_2 = b = 5\,\text{cm} \qquad L_5 = \sqrt{5^2+6^2} = 7.81\,\text{cm}$$
$$L_3 = c = 6\,\text{cm} \qquad L_6 = \sqrt{12^2+6^2} = 13.42\,\text{cm}$$

Substituting,

$$F_{12} = \frac{[(7.81+13.42)-(6+9.22)]\text{cm}}{2\times 12\,\text{cm}} = 0.250$$

(b) The geometry is infinitely long in the direction perpendicular to the plane of the paper, and thus the two plates (surfaces 1 and 2) and the two openings (imaginary surfaces 3 and 4) form a four-surface enclosure. Applying the summation rule to surface 1 yields:

$$F_{11} + F_{12} + F_{13} + F_{14} = 1$$

But $F_{11} = 0$ since it is a flat surface. Therefore,

$$F_{12} = 1 - F_{13} - F_{14}$$

Where the view factors F_{13} and F_{14} can be determined by considering the triangles ABC and ABD, respectively, and applying Equation 4.8 for view factors between the sides of triangles:

$$F_{13} = \frac{L_1 + L_3 - L_6}{2L_1}, \qquad F_{14} = \frac{L_1 + L_4 - L_5}{2L_1}$$

Substituting,

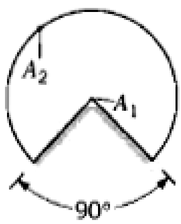
$$\begin{aligned} F_{12} &= 1 - \frac{L_1 + L_3 - L_6}{2L_1} - \frac{L_1 + L_4 - L_5}{2L_1} \\ &= \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1} \end{aligned}$$

This is also some kind of a proof of the crossed-strings method for the case of two infinitely long plain parallel surfaces.

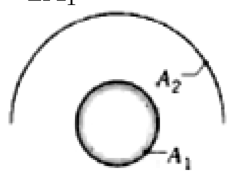
More Examples on the Determination of View Factors

Example 3.10 Determine F_{12} and F_{21} for the following configurations using the reciprocity theorem and other basic shape factor relations. Do not use tables or charts.

(a) Long duct

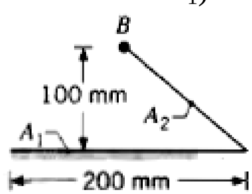


(b) Small sphere of area A_1 under a concentric hemisphere of area $A_2 = 2A_1$

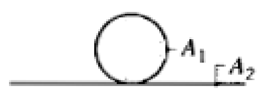


(c) Long duct. What is F_{22} for this case?

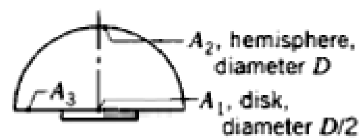
(d) Long inclined plates (point B is directly above the center of A_1)



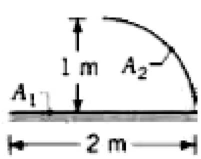
(e) Sphere lying on infinite plane



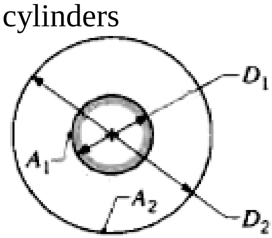
(f) Hemisphere-disk arrangement



(g) Long, open channel



(h) Long concentric cylinders

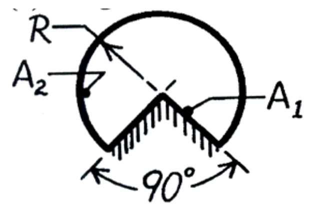


Solution:

Assumption: Surfaces are diffuse

Analysis: No tables or charts are to be used. This approach involves use of the reciprocity relation and summation rule. Note that reciprocity applies to two surfaces; summation applies to an enclosure. Some shape factors will be identified by inspection. L is the length normal to the page.

(a) Long duct (L) By inspection, $F_{12} = 1.0$

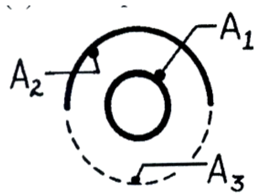


By reciprocity,

$A_2F_{21} = A_1F_{12}$

$$F_{21} = \frac{A_1F_{12}}{A_2} = \frac{2RL}{(3/4)\cdot 2\pi RL}\times 1.0 = \frac{4}{3\pi} = 0.424$$

(b) Small sphere, A_1 under concentric hemisphere, A_2, where $A_2 = 2A_1$

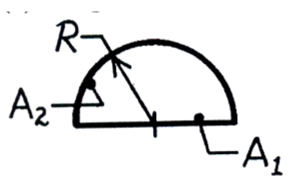


Summation rule: $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$

By reciprocity, $F_{21} = \dfrac{A_1F_{12}}{A_2} = \dfrac{A_1}{2A_1}\times 0.5 = 0.25$

(c) Long duct (L)



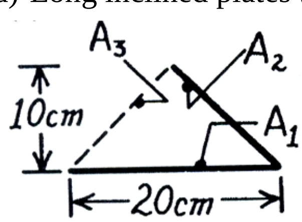
By inspection, $F_{12} = 1.0$

By reciprocity, $F_{21} = \dfrac{A_1F_{12}}{A_2} = \dfrac{2RL}{\pi RL}\times 1.0 = \dfrac{2}{\pi} = 0.637$

Summation rule, $F_{21} + F_{22} = 1$

$F_{22} = 1 - F_{21} = 1 - 0.64 = 0.36$

(d) Long inclined plates (L)



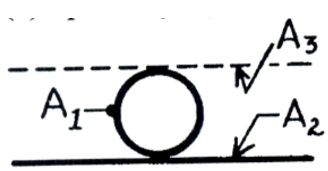
Summation rule, $F_{11} + F_{12} + F_{13} = 1$

A_2 and A_3 are symmetrical about A1, $\Rightarrow F_{12} = F_{13}$

Since $F_{11} = 0$, $F_{12} = F_{13} = 0.5$

By reciprocity, $F_{21} = \dfrac{A_1F_{12}}{A_2} = \dfrac{20L}{10(\sqrt{2})L}\times 0.5 = 0.707$

(e) Sphere lying on an infinite plane

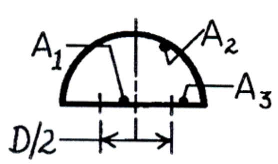


Summation rule, $F_{11} + F_{12} + F_{13} = 1$

But $F_{11} = 0$ and $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.5$

By reciprocity, $F_{21} = \dfrac{A_1F_{12}}{A_2} \to 0 \quad \text{since } A_2 \to \infty$

(f) Hemisphere over a disc of diameter D/2; find also F_{22} and F_{23}



By inspection, $F_{12} = 1.0$

Summation rule for surface A_3 is written as

$$F_{31} + F_{32} + F_{33} = 1. \quad \text{Hence, } F_{32} = 1.0$$

By reciprocity,

$$A_2 F_{23} = A_3 F_{32}$$

$$F_{23} = \frac{A_3 F_{32}}{A_2}$$

$$F_{23} = \left\{ \left[\frac{\pi D^2}{4} - \frac{\pi (D/2)^2}{4} \right] / \frac{\pi D^2}{2} \right\} \times 1.0 = 0.375$$

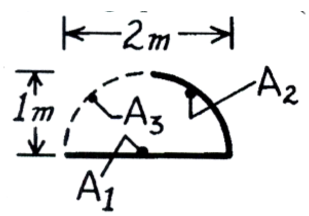
By reciprocity, $F_{21} = \dfrac{A_1 F_{12}}{A_2} = \left\{ \dfrac{\pi}{4} \left[\dfrac{D}{2} \right]^2 / \dfrac{\pi D^2}{2} \right\} \times 1.0 = 0.125$

Summation rule for A_2, $\quad F_{21} + F_{22} + F_{23} = 1$

$$F_{22} = 1 - F_{21} - F_{23} = 1 - 0.125 - 0.375 = 0.5$$

Note that by inspection, it can be deduced that $F_{22} = 0.5$

(g) Long open channel (L)



Summation rule of A_1: $\quad F_{11} + F_{12} + F_{13} = 1$

But $F_{11} = 0$ and $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$

By reciprocity,

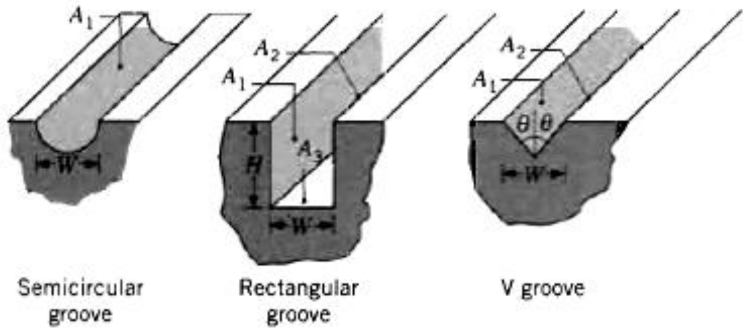
$$F_{21} = \frac{A_1 F_{12}}{A_2} = \frac{2 \times L}{(\frac{2\pi 1}{4} \times L)} \times 0.5 = \frac{4}{\pi} \times 0.50 = 0.637$$

Comments:

(1) Note that the summation rule is applied to an enclosure. To complete the enclosure, it was necessary in several cases to define a third surface which was shown by dashed lines.

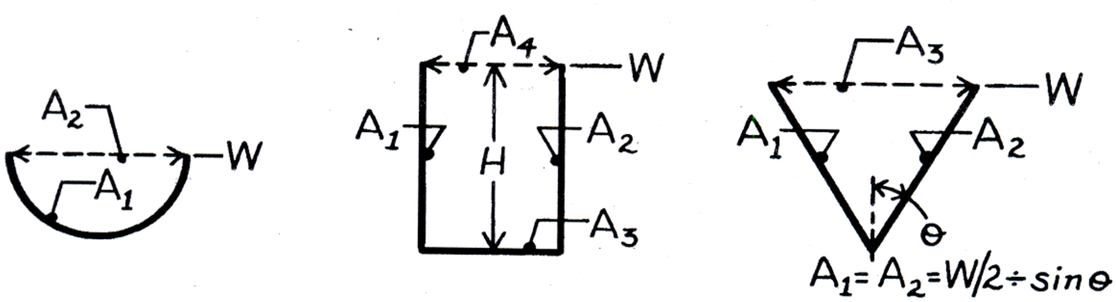
(2) Recognise that the solutions follow a systematic procedure; in many instances it is possible to deduce a shape factor by inspection.

Example 3.11: Consider the following grooves, each of width W, that have been machined from a solid block of material.



(a) For each case, obtain an expression for the view factor of the groove with respect to the surroundings outside the groove.
(b) For the V groove, obtain an expression for the view factor F_{12} where A_1 and A_2 are opposite surfaces.
(c) If $H = 2W$ in the rectangular groove, what is the view factor F_{12}?

Solution:



Assumptions: (1) Diffuse surfaces, (2) Negligible end effects, "long grooves".
Analysis:

(a) Consider a unit length of each groove and represent the surroundings by a hypothetical surface (dashed line).

Semi-Circular Groove:

$$F_{21} = 1; \qquad F_{12} = \frac{A_2}{A_1} F_{21} = \frac{W}{(\pi\pi W/2} \times 1 = \frac{2}{\pi}$$

Rectangular Groove:

By reciprocity, $(A_1 + A_2 + A_3)F_{(1,2,3)4} = A_4 F_{4(1,2,3)}$

But, $F_{4(1,2,3)} = 1$

$$F_{(1,2,3)4} = \frac{A_4}{A_1 + A_2 + A_3} F_{4(1,2,3)} = \frac{W}{H + W + H} \times 1 = \frac{W}{W + 2H}$$

V Groove:

By reciprocity, $(A_1 + A_2) F_{(1,2)3} = A_3 F_{3(1,2)}$

But, $F_{3(1,2)} = 1$

$$F_{(1,2)3} = \frac{A_3}{A_1 + A_2} F_{3(1,2)} = \frac{W}{\frac{W/2}{\sin\theta} + \frac{W/2}{\sin\theta}} = \frac{W}{\frac{W}{\sin\theta}} = \sin\theta$$

(b) Summation rule: $F_{11} + F_{12} + F_{13} = 1$

But $F_{11} = 0$, Hence, $F_{12} = 1 - F_{13}$

Also, $A_1 F_{13} = A_3 F_{31}$, by reciprocity

Hence, $F_{13} = \frac{A_3}{A_1} F_{31}$

$\therefore F_{12} = 1 - \frac{A_3}{A_1} F_{31}$ but $F_{31} = \frac{1}{2}$ from symmetry

Hence, $F_{12} = 1 - \frac{W}{(W/2)/\sin\theta} \times \frac{1}{2} = 1 - \sin\theta$

(c). From Fig 3.15, with $L_2/D = H/W = 2$ and $L_1/D \to \infty$, $F_{12} \approx 0.62$

Comments: (1) Note that for the V groove, $F_{13} = F_{23} = F_{(1,2)3} = \sin\theta$

(2) In part (c), Fig 3.15 could also be used with $L_1/D = 2$ and $L_2/D = \infty$. However, obtaining the limit of F_{ij} as $L_2/D \to \infty$ from the figure is somewhat uncertain.

Self Assessment 1-4

1. Consider an enclosure consisting of 12 surfaces. How many view factors does this geometry involve? How many of these view factors can be determined by the application of the reciprocity and the summation rules? **Suggested Answers: 144, 78**

2. Consider a hemispherical furnace with a flat circular base of diameter *D*. Determine the view factor from the dome of this furnace to its base. **Suggested Answer: 0.5**

SESSION 2-4: RADIATION EXCHANGE BETWEEN SURFACES

2-4a Black Surface

Consider two black surfaces of arbitrary shape maintained at uniform and constant temperatures T_1 and T_2 as shown in Figure 55.

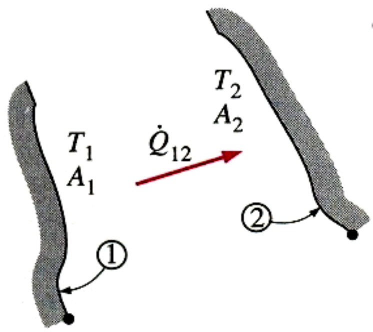


Figure 55: Radiation Exchange between two black surfaces

The net rate of heat transfer by radiation from 1 to 2 may be determined by noting that for a black surface, any radiation striking it is completely absorbed. Using the concept of view factors already introduced, we can write the expression:

$$\begin{Bmatrix} \text{Net rate of} \\ \text{radiative heat} \\ \text{transfer from} \\ \text{1 to 2} \end{Bmatrix} = \begin{Bmatrix} \text{Radaition leaving} \\ \text{surface 1 that} \\ \text{strikes surface 2} \end{Bmatrix} - \begin{Bmatrix} \text{Radiation leaving} \\ \text{surface 2 that} \\ \text{strikes surface 1} \end{Bmatrix}$$

Mathematically, this is written as

$$\dot{Q}_{12} = A_1 F_{12} E_{b1} - A_2 F_{21} E_{b2} \tag{4.11}$$

Now, we recall that

$$E_b = \sigma T^4 \tag{4.12}$$

$$A_1 F_{12} = A_2 F_{21} \tag{4.13}$$

Substituting, equation (4.12 & 4.13 into 4.11), gives

$$\dot{Q}_{12} = A_1 F_{12} \sigma \left(T_1^4 - T_2^4 \right) \tag{4.14}$$

If $\dot{Q}_{12}$ is found to be negative, it implies that net radiation heat transfer is from surface 2 to surface 1

For an enclosure consisting of N black surfaces maintained at fixed uniform temperature, Equation 4.11 above can be generalised to give the *net* radiation leaving any given surface *i*. The resulting expression is:

$$\dot{Q}_i = \sum_{j=1}^{N} Q_{ij} = \sum_{j=1}^{N} A_i F_{ij} \sigma (T_i^4 - T_j^4) \qquad (4.15)$$

Example 3.12

The surfaces of the cubical furnace shown in Figure 56 have dimensions 5 m x 5 m x 5 m. The bottom, top and side surfaces are maintained at uniform fixed temperatures of 800 K, 1500 K and 500 K respectively. Determine

(a) The net rate of radiation heat exchange from the bottom to the side surfaces.

(b) The net rate of radiation heat exchange from the bottom to the top surface.

(c) The overall net rate of radiation heat transfer from the bottom surface

Assume that the view factor $F_{12} = 0.2$

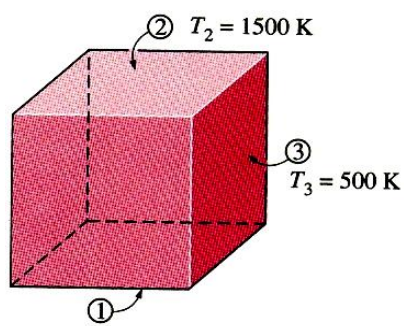


Figure 56: Heat exchange in a cubical furnace of black surfaces

Solution

Although the cube has six surfaces, we notice that all the sides are identical in dimensions and temperature, and could therefore be treated as one. Consequently, there are only 3 different surfaces. The surfaces are identified as 1, 2 and 3 as shown. The problem then reduces to determining $\dot{Q}_{12}$, $\dot{Q}_{13}$ and $\dot{Q}_1$.

(a) The net rate of radiation heat exchange between the bottom and side surfaces is given by

$$\dot{Q}_{13} = A_1 F_{13} \sigma \left(T_1^4 - T_3^4\right)$$

To determine F_{13}, we note that $F_{11} = 0$.

Using the summation rule, we have

$$F_{11} + F_{12} + F_{13} = 1$$

$$\Rightarrow \quad F_{13} = 1 - F_{11} - F_{12} = 1 - 0 - 0.2 = 0.8$$

Substituting, it follows that

$$\begin{aligned} \dot{Q}_{13} &= 25 \times 0.8 \times \left(5.67 \times 10^{-8}\right) \times \left(800^4 - 500^4\right) \\ &= 393{,}611 \text{ W} \end{aligned}$$

(b) The rate of radiation heat exchange from surface 1 to surface 2, $\dot{Q}_{12}$ is obtained in a similar way,

Namely,

$$\begin{aligned} \dot{Q}_{12} &= A_1 F_{12} \sigma \left(T_1^4 - T_2^4\right) \\ &= 25 \times 0.2 \times \left(5.67 \times 10^{-8}\right) \times \left(800^4 - 1500^4\right) \\ &= -1{,}319{,}097 \text{ W} \end{aligned}$$

The negative sign indicates that net radiation heat transfer is from surface 2 to surface 1

(c) The overall net radiative heat transfer from surface 1 is obtained by summation; namely,

$$\begin{aligned} \dot{Q}_1 &= \sum_{j=1}^{3} \dot{Q}_{ij} = \dot{Q}_{11} + \dot{Q}_{12} + \dot{Q}_{13} \\ &= 0 + (-1{,}319{,}097) + (393{,}611) \\ &= -925{,}486 \text{ W} \end{aligned}$$

Thus, the bottom surface is ***gaining*** net radiation at a rate of about 925 kW.

2-4b Grey Surfaces

In the previous section, we considered radiation interchange between black surfaces. The resulting expressions were relatively easy to develop and apply, mainly because a black surface does not reflect or transmit any radiation incident on it, but rather absorbs ***all***.

In real life, many surfaces are encountered which cannot be considered as black body surfaces, since they may allow reflections and transmission to occur. The detailed analysis of radiation, which incorporates reflection and transmission, is rather complicated. We shall, therefore, restrict our consideration to surfaces, which can be considered as ***opaque, diffuse*** and ***grey***. That is, the surfaces are non-transparent, they are diffuse emitters and diffuse reflectors, and their radiation properties are independent of wavelength. In addition, each surface will be assumed to be isothermal, and the radiation leaving and arriving at each surface are uniform over the surface.

2-4.1 The Concept of Radiosity

In order to simplify the analysis of radiation interchange between non-black surfaces, we introduce the concept of radiosity. The radiosity of a surface, **J**, is defined as the ***total radiation energy leaving the surface per unit time and per unit area***. This is illustrated in Figure 57.

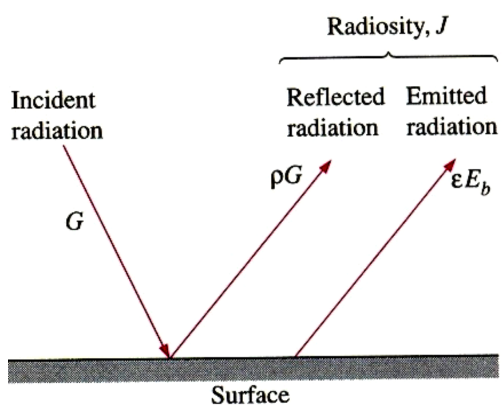


Figure 57: Illustrating radiation leaving a surface

For a surface *i* that is grey and ***opaque***, if the incident radiation intensity is ***G***, then the radiosity ***J*** may be written as

$$J_i = \begin{Bmatrix} \text{radiation emitted} \\ \text{by surface i} \end{Bmatrix} + \begin{Bmatrix} \text{radiation reflected} \\ \text{by surface i} \end{Bmatrix}$$

For an ***opaque*** grey surface, we note that

$$\varepsilon_i = \alpha_i \quad \text{and} \quad \alpha_i + \rho_i = 1 \Rightarrow \rho_i = 1 - \varepsilon_i$$

The expression for radiosity then becomes

$$J_i = \varepsilon_i E_{bi} + (1-\varepsilon_i)G_i \tag{4.16}$$

For a black body, we not that $\varepsilon_i = 1$, hence

$$J_i(blackbody) = E_{bi} = \sigma T_1^4$$

Thus, the radiosity of a black body is equal to its emissive power.

The characteristic of a grey body is such that its surface has an "internal resistance".
The net rate at which radiation leaves the area *dA*:

$$d\dot{Q}_i = dA_i(J_i - G_i) \tag{4.17}$$

From Equation 4.16

$$G_i = \frac{J_i - \varepsilon_i E_{bi}}{1-\varepsilon_i} = \frac{J_i - \varepsilon_i E_{bi}}{\rho_i} \tag{4.18}$$

Substituting Equation 4.18 into 4.17

$$\frac{d\dot{Q}_i}{dA_i} = J_i - \frac{J_i - \varepsilon_i E_{bi}}{\rho_i}$$

$$= \frac{1}{\rho_i}\left[\varepsilon_i E_{bi} - J_i(1-\rho_i)\right] \qquad \text{but } 1 - \rho_i = \varepsilon_i$$

$$= \frac{\varepsilon_i}{\rho_i}(E_{bi} - J_i)$$

$$\therefore \dot{Q}_{\text{net}} = \frac{A_i \varepsilon_i}{\rho_i}(E_{bi} - J_i) \tag{4.19}$$

In an electrical analogy to Ohm's Law, this equation can be arranged as:

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i} \tag{4.20}$$

where

$$R_i = \frac{\rho_i}{A_i \varepsilon_i} \tag{4.21}$$

is the **surface resistance** to radiation. The quantity $E_{bi} - J_i$ corresponds to potential difference and the net rate of heat transfer corresponds to current in the electrical analogy.

2-4.1.1 Net Radiation Heat Transfer between Any Two Surfaces

Consider two diffuse, grey, and opaque surfaces of arbitrary shape maintained at uniform temperatures, as shown in Figure 58. Recognizing that the radiosity J represents the rate of radiation leaving a surface per unit surface area and that the view factor F_{ij} represents the fraction of radiation leaving surface i that strikes surface j, the *net* rate of radiation heat transfer from surface i to surface j can be expressed as

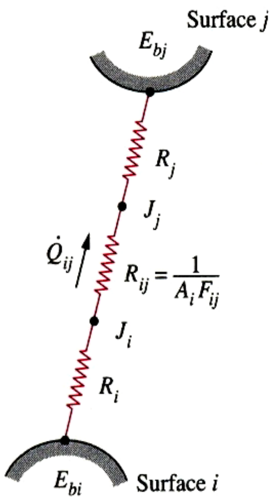


Figure 58: Two diffuse, grey and opaque surfaces

$$\dot{Q}_{ij} = \begin{pmatrix} \text{Radiation leaving} \\ \text{the entire surface i} \\ \text{that stikes surface j} \end{pmatrix} - \begin{pmatrix} \text{Radiation leaving} \\ \text{the entire surface j} \\ \text{that strikes surface i} \end{pmatrix} \tag{4.22}$$

$$= A_i J_i F_{ij} - A_j J_j F_{ji}$$

From the reciprocity relation $A_i F_{ij} - A_j F_{ji}$

$$\therefore \dot{Q}_{ij} = A_i F_{ij} (J_i - J_j) \tag{4.23}$$

In analogy to Ohm's Law, this equation can be arranged as

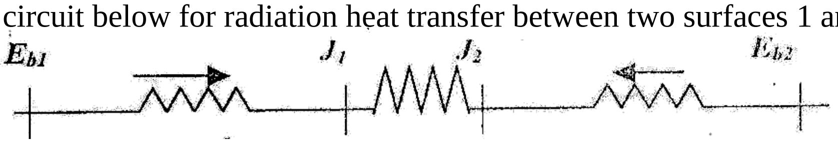
$$\dot{Q}_{ij} = \frac{J_i - J_j}{R_{ij}} \tag{4.24}$$

where

$$R_{ij} = \frac{1}{A_i F_{ij}} \tag{4.25}$$

is the **space resistance** to radiation. The quantity $J_i - J_j$ corresponds to a *potential difference*, and the net rate of heat transfer between two surfaces corresponds to *current* in the electrical analogy.

Consider the circuit below for radiation heat transfer between two surfaces 1 and 2



$$R = \frac{\rho_1}{A_1 \varepsilon_1} \qquad \frac{1}{A_1 F_{12}} or \frac{1}{A_2 F_{12}} \qquad R_2 = \frac{\rho_2}{A_2 \varepsilon_2}$$

Figure 59: Thermal Circuit Analysis

From the above analysis, before the radiation leaves the body there seems to be an internal resistance

$$\frac{\rho_n}{A_n \varepsilon_n} \quad n = 1,2...$$

$$\dot{Q}_{12} = \frac{E_{b_1} - E_{b_2}}{\sum R} \equiv A_1 F_{12} (E_{b1} - E_{b2})$$

where $$\sum R = \frac{\rho_1}{A_1 \varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{\rho_2}{A_2 \varepsilon_2} \tag{4.26}$$

For three cases considered

- Infinitely long parallel plates
- Infinitely long concentric cylinders
- Sphere within sphere

$$\dot{Q}_{12} = \frac{E_{b_1} - E_{b_2}}{R_{th}}$$

For infinitely long parallel plates,

$$\therefore \dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R} = A_1 F_{12} \sigma (T_1^4 - T_2^4) = \frac{A\sigma (T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 \right)} \tag{4.27}$$

For a small body in a large enclosure (Table 8)

$$\sum R = \frac{\rho_1}{\varepsilon_1 A_1} + \frac{\rho_2}{\varepsilon_2 A_2} + \frac{1}{A_1 F_{12}} \qquad \text{but } F_{12} = 1$$

$$= \frac{1}{A_1} \left[\frac{\rho_1}{\varepsilon_2} + \frac{A_1}{A_2} \left(\frac{\rho_1}{\varepsilon_2} \right) + 1 \right] \qquad \frac{A_1}{A_2} \cong 0$$

$$\sum R = \frac{1}{A_1} \left[\frac{1 - \varepsilon_1}{\varepsilon_1} + 1 \right] = \frac{1}{A_1} \left(\frac{1}{\varepsilon_1} \right)$$

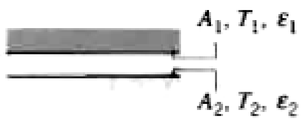
$$\Rightarrow \left[A_1 F_{12} = A_1 \varepsilon_1 \right]$$

Thus, for a small body in a large enclosure the radiation exchange is

$$\dot{Q}_{12} = \varepsilon_1 A_1 \left(E_{b_1} - E_{b_2} \right) \tag{4.28}$$

Table 3.5 Radiation exchange for common configurations

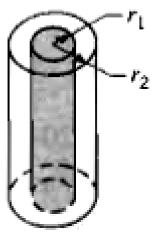
Large (Infinite) Parallel Planes



$$A_1 = A_2 = \qquad F_{12} = 1$$

$$\dot{Q}_{12}\ {}_{2} = \frac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

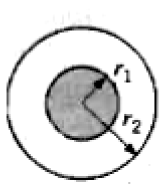
Long (Infinite) Concentric Cylinders



$$\frac{A_1}{A_2} = \frac{r_1}{r_2} \qquad F_{12} = 1$$

$$\dot{Q}_{12}\ {}_{2} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

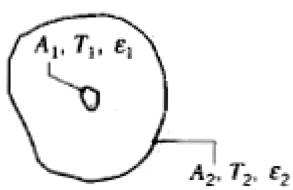
Concentric Spheres



$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} \qquad F_{12} = 1$$

$$\dot{Q}_{12}\ {}_{2} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2}$$

Small Convex Object in a Large Cavity



$$\frac{A_1}{A_2} \approx 0 \qquad F_{12} = 1$$

$$\dot{Q}_{12}\ {}_{2} = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4)$$

2-4.2 Radiation Heat Transfer in two surface Enclosures

Consider an enclosure consisting of two opaque surfaces at specified temperatures T_1 and T_2 as shown in Figure 60. Surfaces 1 and 2 have emissivities ε_1 and ε_2 and surface areas A_1 and A_2, There are only two surfaces in the enclosure and we can write $\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$.

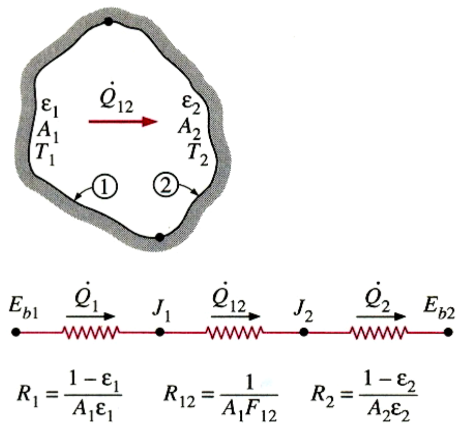


Figure 60: Heat Transfer between two surface enclosures

That is, the net rate of radiation transfer from surface 1 to surface 2 must equal the net rate of radiation transfer *from* surface 1 and the net rate of radiation transfer *to* surface 2. From Figure 60.

$$\dot{Q}_{12} = \frac{E_{b_1} - E_{b_2}}{R_1 + R_{12} + R_2} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_1\varepsilon_2}} \tag{4.29}$$

This result is applicable to any two grey, diffuse, opaque surfaces that form an enclosure. The view factor depends on the geometry and must be determined in advance.

2-4.3 Radiation Shields

Radiation heat transfer between two surfaces can be reduced greatly by inserting a thin, high reflective (low emissivity) sheet of material between the two surfaces. Such high reflective thin plates or shells are called radiation shields. Radiation shields are also used in temperature measurements of fluids to reduce the error caused by the radiation effect when the temperature sensor is exposed to surfaces that are much hotter or colder than the fluid itself. The role of the radiation shield is to reduce the rate of radiation heat transfer by placing additional resistance in the path of radiation heat flow. The lower the emissivity of the shield, the higher the resistance.

For two parallel plates with no shield and maintained at temperatures T_1 and T_2 with emissivities ε_1 and ε_2 , respectively, the rate of heat transfer is given by

$$\dot{Q}_{12,noshield} = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \tag{4.30}$$

With one shield placed in between the two parallel plates (Figure 61), the radiation heat transfer from surface 1 to surface 2 is given by

$$\dot{Q}_{12}, \text{one shield} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{13}} + \frac{1-\varepsilon_{3,1}}{A_1\varepsilon_{3,1}} + \frac{1-\varepsilon_{3,2}}{A_3\varepsilon_{3,2}} + \frac{1}{A_3F_{32}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}} \tag{4.31}$$

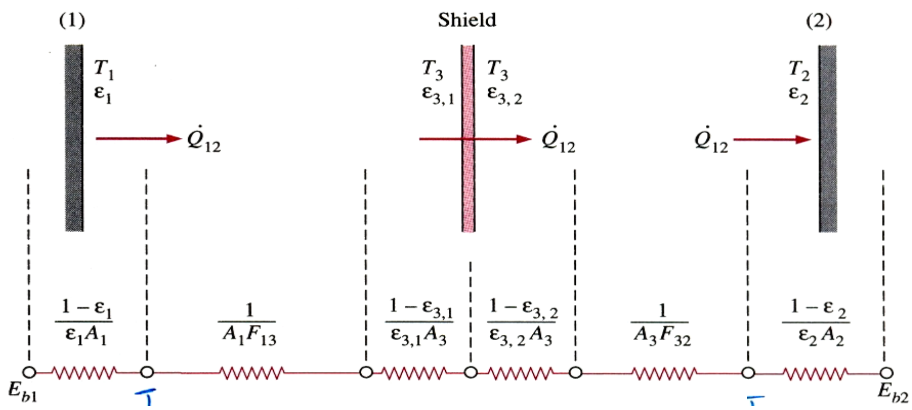


Figure 61: Heat transfer between two surfaces with a radiation shield

Noting that $F_{13} = F_{23} = 1$, $A_1 = A_2 = A_3$ for parallel plates, the equation reduces to:

$$\dot{Q}_{12} = \frac{1}{1+1}\left[\frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}\right]$$

which is basically ½ of the value for radiation heat transfer between the two parallel plates without a shield. In general, for N radiation shields inserted in between the two plates, the net radiation heat transfer from surface 1 to surface 2 is given by

$$\dot{Q}_{12,\,\text{N shields}} = \left[1/(N+1)\right]\ \dot{Q}_{12,\ \text{noshield}}. \tag{4.32}$$

When a thermometer or any other temperature measuring device such as a thermocouple is placed in a medium (see Figure 62), heat transfer takes place between the sensor of the thermometer and the medium by convection until the sensor reaches the temperature of the medium. But the when the sensor is surrounded by surfaces that are at a different temperature from the fluid, radiation exchange will take place between the sensor and the surrounding surfaces. When the heat transfers by convection and radiation balance each other, the sensor will indicate a temperature that falls between the fluid and the surface temperatures.

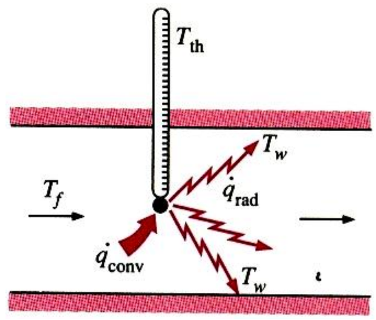


Figure 62: Thermocouple placed in a medium

When equilibrium is established the reading of the sensor will stabilise when heat gain by convection as measured by the sensor, equals the heat loss by radiation. Thus, on a unit area basis, we have

$$\alpha_c(T_f - T_{th}) = \varepsilon_{th}\sigma\left(T_{th}^{\ 4} - T_w^4\right)$$

$$\therefore T_f = T_{th} + \frac{\varepsilon_{th}\sigma\left(T_{th}^{\ 4} - T_w^4\right)}{\alpha_c} \tag{4.33}$$

where

T_f = actual temperature of the fluid, K

T_{th} = temperature value measured by the thermometer, K

T_w = temperature of the surrounding surfaces, K

α_c = convection heat transfer coefficient, W/m^2 K

ε = emissivity of the sensor of the thermometer

Example 3.13: Radiation effect on Temperature Measurements

A thermocouple used to measure the temperature of hot air flowing in a duct whose walls are maintained at T_w = 400 K shows a temperature reading of T_{th} = 650 K (Figure 63). Assuming the emissivity of the thermocouple junction to be ε = 0.6 and the convection heat transfer coefficient to be α_c = 80 W/m^2°C, determine the actual temperature of the air.

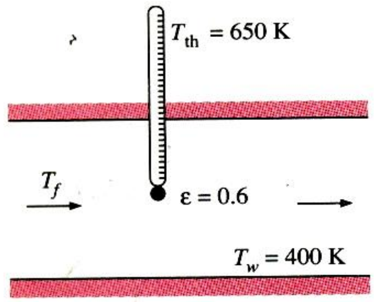


Figure 63: Radiation effect of temperature measurement

Solution:

Assumptions: The surfaces are opaque, diffuse and grey

Analysis: The walls of the duct are at a much lower temperature as compared to the air in it. As a result of radiation effect, the thermocouple will show a reading lower than the actual air temperature. The actual air temperature is determined using Equation 4.33 to be

$$T_f = T_{th} + \frac{\varepsilon\sigma\left(T_{th}^{\ 4} - T_w^4\right)}{\alpha_c}$$

$$= (650\,\mathrm{K}) + \frac{0.6 \times (5.67 \times 10^{-8}\ \mathrm{W/m^2K^4})\left[(650\,\mathrm{K})^4 - (400\,\mathrm{K})^4\right]}{80\ \mathrm{W/m^{2\,o}C}}$$

$$= 715.0\,\mathrm{K}$$

Radiation effect causes a difference of 65 °C (65 K) in temperature reading in this case.

2-4.4 Radiation Exchange in Three or More Zone Enclosures

The radiation network approach described for radiation exchange between two surfaces can readily be generalised to determine the radiation exchange among the surfaces of a three or more zone enclosure by utilising the resistances defined by Equations 4.21 and 4.25. That is the radiation resistance at surface A_i is given by:

$$R_i = \frac{\rho_i}{A_i\,\varepsilon_i} = \frac{1-\varepsilon_i}{A_i\,\varepsilon_i} \tag{4.34}$$

And the radiation resistance across the fictitious surfaces above A_i and A_j, is given by:

$$R_{ij} = \frac{1}{A_i F_{ij}} \tag{4.35}$$

To illustrate the application, consider a three-zone enclosure as shown in Figure 64. Zones 1,2 and 3 have surface areas A_1, A_2, and A_3 respectively. Figure 65 shows the corresponding radiation network.

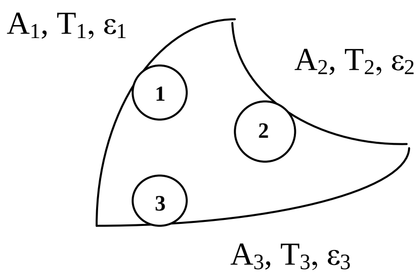


Figure 64: Illustration of a three surface enclosure

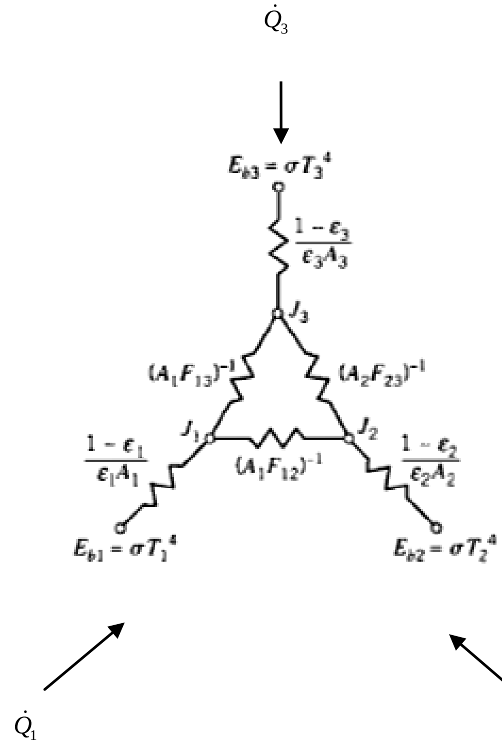


Figure 65: Radiation network analysis of a three surface enclosure

To solve the problem, the algebraic sum of the currents at the nodes J_1, J_2 and J_3 is set equal to zero. By this, the sum of the currents arriving at a junction must be equal to the sum of the currents leaving the junction. This leads to three algebraic equations for the determination of three unknown radiosities J_1, J_2, and J_3. Knowing the radiosities, the heat flow rates $\dot{Q}_1$, $\dot{Q}_2$, and $\dot{Q}_3$ at each zone are determined by the application of Ohm's Law as:

$$
\begin{aligned}
\dot{Q}_1 &= \frac{\sigma T_1^4 - J_1}{R_1} \qquad \text{where } R_1 = \frac{1-\varepsilon_1}{A_1\varepsilon_1} \\
\dot{Q}_2 &= \frac{\sigma T_{21}^4 - J_2}{R_2} \qquad \text{where } R_2 = \frac{1-\varepsilon_2}{A_2\varepsilon_2} \\
\dot{Q}_3 &= \frac{\sigma T_3^4 - J_3}{R_3} \qquad \text{where } R_3 = \frac{1-\varepsilon_3}{A_3\varepsilon_3}
\end{aligned}
\tag{4.36}
$$

The equations above are not applicable to black surfaces that have no surface resistance since $\varepsilon = 1$. A more generally applicable expression for the net radiative heat transfer from surface *i* to other surfaces in the enclosure is given by:

$$\dot{Q}_i = \sum_{j=1}^{N} A_i F_{ij} (J_i - J_j) \qquad (4.37)$$

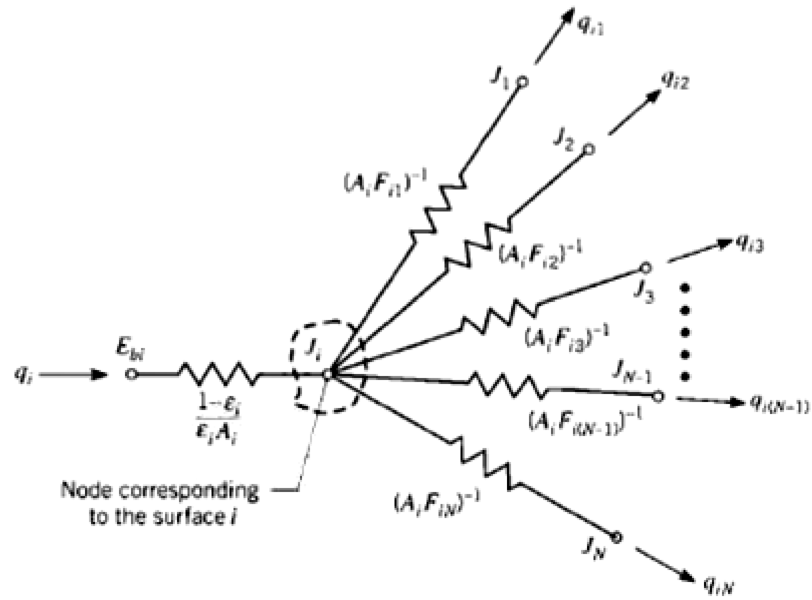


Figure 66: Network representation of radiation exchange between surface *i* and the remaining surfaces of an enclosure

Consider Figure 66 (in which $\dot{Q}$ is represented simply as *q*), the radiation balance for the radiosity node associated with surface *i* can be expressed as:

$$\frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{(A_i F_{ij})^{-1}} \qquad (4.38)$$

The rate of radiation transfer (current flow) to *i* through its surface resistance must equal the net rate of radiation transfer (current flows) from *i* to all other surfaces through the corresponding geometrical resistances.

2-4.4.1 The Reradiating Surface

The assumption of a *reradiating surface* is common to many industrial applications. This idealised surface is characterised by zero net radiation transfer ($q_i = 0$). It is closely approached by real surfaces that are well insulated on one side and for which convection effects may be neglected on the opposite (radiating side). With $q_i = 0$, if follows from Equations 3.51 and 3.54 that $G_i = J_i = E_{bi}$. Hence, if the radiosity of a reradiating surface is known, its temperature is readily determined. In an enclosure, the equilibrium temperature of a reradiating surface is determined by its interaction with the other surfaces, and it is *independent of the emissivity of the reradiating surface*.

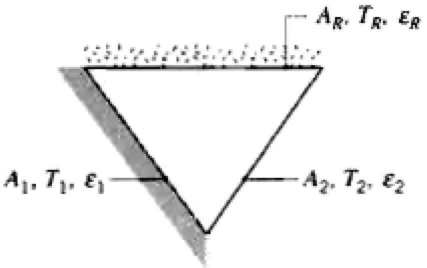


Figure 68: A three surface enclosure with one surface reradiating

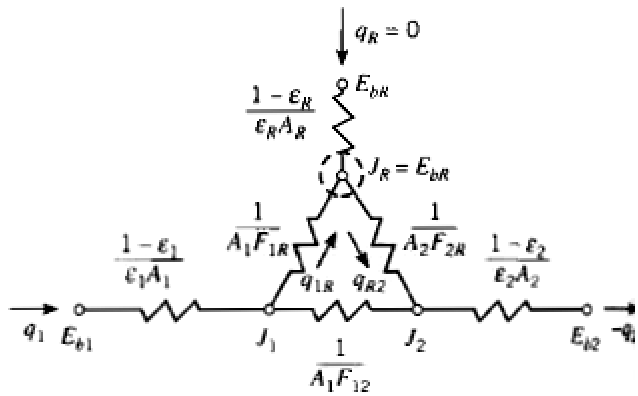


Figure 67: Corresponding Radiation network analysis

A three-surface enclosure, for which the third surface, surface R, is reradiating, is shown in Figure 67, and the corresponding network is shown in Figure 68. Note that $\dot{Q}$ is represented simply as q. Surface R is presumed to be well insulated and convection effects are assumed to be negligible. Hence, with $q_R = 0$, the net radiation *transfer* from surface 1 must equal the net radiation transfer to surface 2. The network is a simple series-parallel arrangement, and from its analysis it is readily shown that:

$$q_1 = -q_2 = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12} + \left[(1/A_1 F_{1R}) + (1/A_2 F_{2R})\right]^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}} \tag{4.39}$$

Knowing that $q_1 = -q_2$, Equation 3.54 may be applied to surfaces 1 and 2 to determine their radiosities J_1 and J_2. Knowing J_1 and J_2, and the geometrical resistances, the radiosity of the reradiating surface J_R may be determined from the radiation balance:

$$\frac{J_1 - J_R}{(1/A_1F_{1R})} = \frac{J_R - J_2}{(1/A_2F_{2R})} \tag{4.40}$$

The temperature of the reradiating surface may then be determined from the requirement that $\sigma T_R^4 = J_R$.

Worked Example 3.14

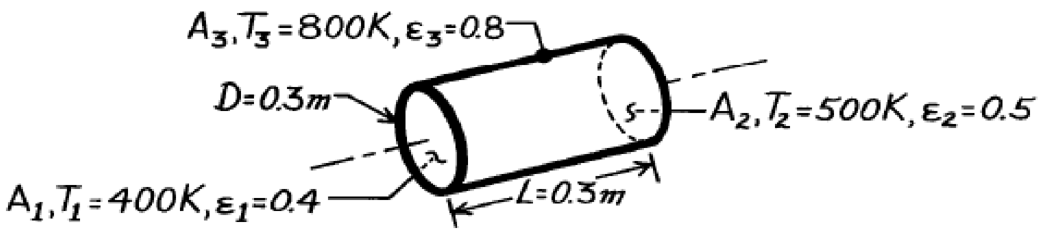
Consider a circular furnace that is 0.3 m long and 0.3 m in diameter. The two ends have diffuse, grey surfaces that are maintained at 400 and 500 K with emissivities of 0.4 and 0.5, respectively. The lateral surface is also diffuse and grey with an emissivity of 0.8 and a temperature of 800 K. Determine the net radiation heat transfer from each of the surfaces.

SOLUTION

KNOWN: Circular furnace with prescribed temperatures and emissivities of the lateral and end surfaces.

FIND: Net radiative heat transfer from each surface.

SCHEMATIC:



ASSUMPTIONS: (1) Surfaces are isothermal and diffuse-gray.

ANALYSIS: To calculate the net radiation heat transfer from each surface, we need to determine its radiosity. First, evaluate terms which will be required. The radiation network is the same as in Figure 65.

$E_{b1} = \sigma T_1^4 = 1452\ \mathrm{W/m^2}$ $\qquad A_1 = A_2 = \pi D^2/4 = 0.07069\ \mathrm{m^2}$ $\qquad F_{12} = F_{21} = 0.17$

$E_{b2} = \sigma T_2^4 = 3544\ \mathrm{W/m^2}$ $\qquad A_3 = \pi DL = 0.2827\ \mathrm{m^2}$ $\qquad F_{23} = F_{13} = 0.83$

$E_{b3} = \sigma T_3^4 = 23{,}224\ \mathrm{W/m^2}$

The view factor F_{12} results from Figure 44 with $L/r_1 = 2$ and $r_2/L = 0.5$. The radiation balances using Eq. 3.72, omitting units for convenience, are:

$$A_1: \quad \frac{1452 - J_1}{\dfrac{(1-0.4)}{0.4 \times 0.07069}} = 0.07069 \times 0.17 (J_1 - J_2) + 0.07069 \times 0.83 (J_1 - J_3)$$

$$-2.500 J_1 + 0.2550 J_2 + 1.2450 J_3 = -1452 \qquad (1)$$

$$A_2: \quad \frac{3544 - J_2}{\dfrac{(1-0.5)}{0.5 \times 0.07069}} = 0.07069 \times 0.17 (J_2 - J_1) + 0.07069 \times 0.83 (J_2 - J_3)$$

$$-0.1700 J_1 - 2.0000 J_2 + 0.8300 J_3 = -3544 \qquad (2)$$

$$A_3: \quad \frac{23,224 - J_3}{\dfrac{(1-0.8)}{0.8 \times 0.2827}} = 0.07069 \times 0.83 (J_3 - J_1) + 0.07069 \times 0.83 (J_3 - J_2)$$

$$0.05189 J_1 + 0.05189 J_2 - 1.1037 J_3 = -23,224 \qquad (3)$$

Solving Eqs. (1) – (3) simultaneously, find

$$J_1 = 12,877 \text{ W/m}^2 \qquad J_2 = 12,086 \text{ W/m}^2 \qquad J_3 = 22,216 \text{ W/m}^2.$$

Using Eq. 3.71, the net radiation heat transfer for each surface follows:

$$q_i = \sum_{j=1}^{N} A_i F_{ij} (J_i - J_j)$$

$$A_1: \; q_1 = 0.07069 \times 0.17 (12,877 - 12,086) \text{ W} + 0.07069 \times 0.83 (12,877 - 22,216) \text{ W} = -538 \text{ W}$$

$$A_2: \; q_2 = 0.07069 \times 0.17 (12,086 - 12,877) \text{ W} + 0.07069 \times 0.83 (12,086 - 22,216) \text{ W} = -603 \text{ W}$$

$$A_3: \; q_3 = 0.07069 \times 0.83 (22,216 - 12,877) \text{ W} + 0.07069 \times 0.83 (22,216 - 12,086) \text{ W} = 1141 \text{ W}$$

COMMENTS: Note that $\sum q_i = 0$. Also, note that $J_2 < J_1$ despite the fact that $T_2 > T_1$; note the role emissivity plays in explaining this.

Individual/Group Discussion Problems: Tutorial Problems Questions 7, 8, 9, 10, 40, 41, 42, 43.

Self Assessment 2-4

1. Two large surfaces are maintained at 500 K and 1000 K respectively. Determine the rate of radiant heat exchange between the plates per unit area.

i. If they are black bodies
ii. If they are grey bodies with $\varepsilon = 0.8$

2.A shield having emissivity of 0.5 i.e. $\varepsilon = 0.5$ is placed between the bodies at the above temperature. Determine the rate of radiant heat flow per unit area between

i. The black bodies
ii. The grey bodies with $\varepsilon = 0.8$

3. Determine the temperature of the shield of the two cases in (**2**) above.

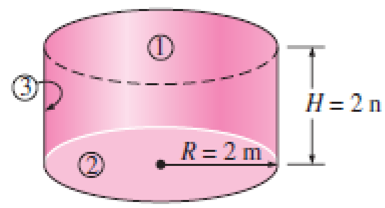
Suggested Answers;	**(1) i. 53,156.3 W/m^2**	**ii. 35, 437. 5 W/m^2**
	(2) i. 13, 289. 1 W/m^2	**ii. 9,665 W/m^2**
	(3) i. 853.7 K	**ii. 817.2 K**

***Learning Track Activities ensure** you understand the terms listed below for radiation heat transfer between surfaces*

Key terms/ New Words in Unit

i. Radiosity
ii. Surface resistance
iii. Space resistance
iv. Reradiating surface
v. Radiation shield
vi. Grey surface
vii. Black surface
viii. View factor
ix. Opaque surface
x. Diffuse surface
xi. Summation rule
xii. Reciprocity rule
xiii. Superposition rule
xiv. Symmetry rule

Review Question: A furnace is of cylindrical shape with $R = H = 2$ m. The base, top, and side surfaces of the furnace are all black and are maintained at uniform temperatures of 500, 700, and 1200 K, respectively. Determine the net rate of radiation heat transfer to or from the top surface during steady operation.



Discussion Question: 1. What does the view factor represent? When is the view factor from a surface to itself not zero?

2. How can you determine the view factor F_{12} when the view factor F_{21} and the surface areas are available?

3. What are the summation rule and the superposition rule for view factors?

4. What is the crossed-strings method? For what kind of geometries is the crossed-strings method applicable?

Web Activity: www.mhhe.com , www.mitcourseware.com

Reading: Read chapter 12, Heat transfer, A practical approach by Yunus. A. Cengel, 2nd Edition.

Unit 5

CONVECTION HEAT TRANSFER

Introduction

In this unit we shall distinguish between natural and forced convection heat flows and also determine the convective heat transfer coefficient in forced and natural convection for both internal and external flows.

Learning Objectives

After reading this unit you should be able to:

1. Distinguish between natural and forced convection heat flows.
2. Determine the convective heat transfer coefficient in forced and natural convection for external and internal flows.
3. Work with convective heat transfer coefficient in heat exchanger problem analysis.

Unit content

Session 1-5: Introduction to Convection Heat Transfer

1-5.1 Forced Convection
1-5.2 Dimensional Analysis
1-5.3 Reynolds Analogy
1-5.4 The Heat Transfer Coefficient

Session 2-5: Natural Convection

2-5.1: Natural Convection Correlations
2-5.2: Natural Convection Inside Enclosures
2-5.3: Natural Convection from finned Surfaces
2-5.4: Combined Natural and Forced Convection

SESSION 1-5: INTRODUCTION TO CONVECTION HEAT TRANSFER

The study of heat transfer by convection is concerned with the calculation of rates of heat exchange between fluids and solid boundaries. The solution of the differential equations that govern the transfer of heat by convection usually presents considerable mathematical difficulties, and exact solutions can rarely be obtained. This chapter is confined to a description of methods giving approximate solutions to some simple but important cases of steady heat flow.

1t is known from experience that the main resistance to heat flow from a wall to a fluid, is in a comparatively thin boundary layer adjacent to the wall. The first method of calculating rates of heat transfer makes use of the hydrodynamic concept of a *velocity boundary layer* and the analogous thermal concept of a *temperature boundary layer*. The object of this procedure is to find the value of the temperature gradient $\left(dT/dy\right)\big|_0$ in the fluid in the immediate vicinity of the wall. It is assumed that the fluid at the wall adheres to it, and therefore the heat flow at the wall is by conduction and not by convection, i.e.

$$\dot{Q} = \lambda A \left(\frac{dT}{dy}\right)_{y=0} \tag{5.1a}$$

The second method is like the first in that it studies the behaviour of the boundary layer. It makes use of the similarity, first pointed out by Reynolds, between the mechanism of fluid friction in the boundary layer (i.e. the transfer of fluid momentum to the wall) and the transfer of heat by convection. When this analogy is valid, rates of heat transfer can be predicted from the measurement of the shear stress between a fluid and a wall.

The third method is concerned with model testing, used to obtain information in a form, which is applicable to similar systems of any scale. It is based on the principle of dynamic similarity, and employs the method of dimensional analysis for the correlation of empirical data. It was pointed out in chapter 1 that it is customary in engineering to express the rate of heat transfer between a fluid and a wall by a relation of the form $\dot{Q} = \alpha A\left(T_s - T_w\right)$

where α is a *convection* or *film* heat transfer coefficient. It must be remembered that α is not a physical constant of the fluid. α is a function of all the parameters that affect the heat flow, such as viscosity and velocity, and it may even depend on A and $\left(T_s - T_w\right)$ because $\dot{Q}$ does not necessarily vary with these variables. The results of the three methods of approaches outlined are usually expressed in terms of the coefficient α, and the problem of finding $\dot{Q}$ is simply transferred to the evaluation of α.

In this course, we shall principally employ method 3 to establish the needed relations for the analysis of convection heat transfer problems.

1-5.1 Forced Convection

The study of forced convection is concerned with the transfer of heat between a moving fluid and a solid surface. In order to apply Newton's law of cooling, given by equation (1.4), it is necessary to find a value for the heat transfer coefficient, α. The value of the convection heat transfer coefficient is given by λ/δ, where λ is the thermal conductivity of the fluid and δ is the thickness of the fluid film on the surface. The problem is then to find a value for δ in terms of the fluid properties and the fluid velocity; δ depends on the type of fluid flow across the surface and is governed by the Reynolds number.

The Reynolds number is dimensionless group given by $\mathrm{Re} = \dfrac{\rho u L}{\mu} = \dfrac{uL}{\nu}$

where ρ is the fluid density, u the fluid mean velocity, L the characteristic linear dimension, μ the dynamic viscosity of the f1uid, and ν the kinematic viscosity of the fluid.

1-5.1.1 The Reynolds number

The Reynolds number represents the ratio of the *inertia forces* to *viscous forces* in the fluid. At *large* Reynolds numbers, the inertia forces, which are proportional to the density and the velocity of the fluid, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuation of the fluid. At *small* Reynolds numbers, however, the viscous forces are large enough to overcome the inertia forces to keep the fluid "in line." Thus, the flow is *turbulent* in the first case and *laminar* in the second. The Reynolds number at which the flow becomes turbulent is called the *critical Reynolds number*. This value is different for different geometries. For flow over a flat plate, transition from laminar to turbulent occurs at the critical Reynolds number $\mathrm{Re}_{\text{critical, flat plate}} \approx 5 \times 10^5$.

This generally accepted value of the critical Reynolds number, for a flat plate, may vary somewhat depending on the surface roughness, the turbulence level, and the variation of pressure along the surface.

The various kinds of forced convection, such as flow in a tube, flows across a tube, flows across a flat plate, etc. can be solved mathematically when certain assumptions are made with regard to the boundary conditions. It is exceedingly difficult to obtain an exact mathematical solution to such problems, particularly in the case of turbulent flow, but approximate solutions can be obtained by making suitable assumptions.

It is not within the scope of this course to approach the subject of forced convection fundamentally. However, many of the results used in heat transfer are derived from experiment, and in fact for many problems no mathematical solution is available and empirical values are essential. These empirical values can be generalised using dimensional analysis, which will now be considered.

1-5.2 Dimensional Analysis

In order to apply dimensional analysis, it is necessary to know from experience all the variables upon which the desired function depends. Since the results must apply to geometrically similar bodies therefore one of the variables must always be a characteristic linear dimension.

Consider the dimensional analysis for forced convection, assuming that the effects of free convection, due to differences in density, may be neglected. It is found that the heat transfer coefficient, α, depends on the fluid viscosity, μ, the f1uid density, ρ, the thermal conductivity of the fluid, λ, the specific heat capacity of the fluid, *c*, the temperature difference between the surface and the fluid Δt, and the fluid velocity, *u*. Therefore we have

$$\alpha = f(\mu, \rho, \lambda, c, \Delta t, u, l) \qquad (5.1b)$$

where l is a characteristic dimension of length

Equation (5.1b) can be written as follows:

$$\alpha = A\mu^{a_1}\rho^{b_1}\lambda^{c_1}c^{d_1}\Delta t^{e_1}u^{f_1}l^{g_1} + B\mu^{a_2}\rho^{b_2}\lambda^{c_2}c^{d_2}\Delta t^{e_2}u^{f_2}l^{g_2} + \text{etc.} \qquad (5.2)$$

where *A* and *B* are constants, and a_1, b_1, c_1, etc. are arbitrary indices.

Each term on the right-hand side of the equation must have the same dimensions as the dimensions of α. Considering the first term only, we can write

Dimensions of α = dimensions of ($\mu^{a_1}\rho^{b_1}\lambda^{c_1}c^{d_1}\Delta t^{e_1}u^{f_1}l^{g_1}$)

Each of the properties in the equation can be expressed in terms of five fundamental dimensions; these are mass, *M*, length, *L*, time, *T*, temperature, *t*, and heat *H*.

For α the units are $\dfrac{W}{m^2K}, i.e. \dfrac{H}{L^2Tt}$

For μ the units are $\dfrac{kg}{ms}, i.e. \dfrac{M}{LT}$

For λ the units are $\frac{W}{mK}, i.e. \frac{H}{LTt}$

For ρ the units are $\frac{kg}{m^3}, i.e. \frac{M}{L^3}$

For c the units are $\frac{kJ}{kgK}, i.e. \frac{H}{Mt}$

For Δt the units are K, i.e. t

For u the units are $\frac{m}{s}, i.e. \frac{L}{T}$

For l the units are m, i.e. L

Hence, substituting

$$\frac{H}{L^2Tt} = \left(\frac{M}{LT}\right)^a \times \left(\frac{M}{L^3}\right)^b \times \left(\frac{H}{LTt}\right)^c \times \left(\frac{H}{Mt}\right)^d \times (t)^e \times \left(\frac{L}{T}\right)^f \times (L)^g$$

$$\text{i.e. } \frac{H}{L^2Tt} = (M)^{a+b-d} \times (L)^{f+g-a-3b-c} \times (T)^{-a-c-f} \times (t)^{e-c-d} \times (H)^{c+d}$$

For dimensions of each side of the equation to be the same, the power to which each fundamental dimension is raised must be the same on both sides of the equation, Therefore, equating indices we have

For H: $1 = c + d$

For L: $-2 = f + g - a - 3b - c$

For T: $-1 = -a - c - f$

For t: $-1 = e - c - d$

For M: $0 = a + b - d$

We have five equations and seven unknowns, therefore a solution can only be obtained in terms of two of the indices. It is most useful to express a, b, c, e, and g in terms of d and f Then it can be shown that

$$a = (d - f); \qquad b = f; \qquad c = (1 - d); \qquad e = 0; \qquad a = (f - 1)$$

Substituting these values in equation (5.2), we have

$$\alpha = A\mu^{(d_1 - f_1)}\rho^{f_1}\lambda^{(1-d_1)}c^{d_1}\Delta t^0 \mu^{f_1} l^{(f_1-1)}$$

$$+ B\mu^{(d_2 - f_2)}\rho^{f_2}\lambda^{(1-d_2)}c^{d_2}\Delta t^0 \mu^{f_2} l^{(f_2-1)} + etc.$$

i.e. $$\alpha = A\frac{\lambda}{l}\left(\frac{c\mu}{\lambda}\right)^{d_1}\left(\frac{\rho ul}{\mu}\right)^{f_1} + B\frac{\lambda}{l}\left(\frac{c\mu}{\lambda}\right)^{d_2}\left(\frac{\rho ul}{\mu}\right)^{f_2} + etc.$$

Hence, it can be seen that

$$\frac{\alpha l}{\lambda} = KF\left\{\left(\frac{c\mu}{\lambda}\right),\left(\frac{\rho ul}{\mu}\right)\right\}$$

where *K* is a constant and F is some function.

The dimensionless group, $\alpha l / \lambda$, is called the Nusselt number, *Nu*; the dimensionless group, $c\mu/\lambda$, is called the Prandtl number, *Pr*; and the dimensionless group, $\rho ul / \mu$ is the Reynolds number, *Re*,

i.e. $$Nu = KF\{(\mathrm{Pr}),(\mathrm{Re})\} \qquad Nu = c\left(\mathrm{Pr}^n \, \mathrm{Re}^m\right) \tag{5.3}$$

Experiments can be performed in order to evaluate K, and to determine the nature of the function F. When evaluating Nu, Pr and Re it is customary to take the fluid properties at a suitable mean temperature, since the properties vary with temperature. For cases in which the temperature of the bulk of the fluid is not different from the temperature of the solid surface, then fluid properties are evaluated at the mean bulk temperature of the fluid. When the temperature difference is large, using a mean bulk temperature may cause errors, and a mean film temperature is sometimes used, defined by $T_f = (T_b + T_w)/2$ where T_b is the mean bulk temperature and T_w the surface temperature. When using an empirical equation it is essential to know at what temperature the experimenter has evaluated the properties. It should also be noted that the Prandtl number $\mathrm{Pr} = \mu c / \lambda$, consists entirely of fluid properties and therefore is itself a property, Values of the Prandtl number are tabulated in tables.

Since heat is a form of energy it can be seen that there is no need to choose heat as one of the fundamental dimensions, Instead, we can determine the dimension of heat as follows

Force x distance = mass x acceleration x distance = $M\dfrac{L}{T^2}L = \dfrac{ML^2}{T^2}$ and the units of heat are replaced by ML^2/T^2 whenever they occur, then four dimensionless groups are obtained.

i.e., $$Nu = KF\left\{(\Pr),(\mathrm{Re}),\left(\frac{u^2}{c\Delta t}\right)\right\} \tag{5.4}$$

Now, if the group $\left(u^2/c\Delta t\right)$ is divided by, $(\gamma - 1)$, which is a constant for any one gas, and if the absolute bulk temperature of the gas, T, replaces Δt then we have

$$\frac{u^2}{cT(\gamma-1)} = \frac{u^2}{\gamma RT} = \frac{u^2}{a^2} = (Ma)^2 \tag{5.5}$$

where a is the velocity of sound in the gas and Ma, the Mach number. The influence of the Mach-number on heat transfer is negligible for most problems. For high-speed flows, the influence of the Mach number is an important parameter.

1-5.3 Reynolds Analogy

Reynolds postulated that the heat transfer from a solid surface is similar to the transfer of fluid momentum from the surface, and hence that it is possible to express the heat transfer in terms of the frictional resistance to the flow.

Consider turbulent flow. It can be assumed that particles of mass, m, transport heat and momentum to and from the surface, moving perpendicular to the surface. Then, on the average

Heat transfer per unit area, $$\dot{q} = \dot{m}c\Delta t \tag{5.6}$$

Where, c is the specific heat capacity of the fluid and Δt the temperature difference between the surface and the bulk of the fluid. Also, the rate of change of momentum across the stream is given by

$$\dot{m}(u - u_w) = \dot{m}u \tag{5.7}$$

Where, u is the velocity of the bulk of the fluid and u_w the fluid velocity at the surface.

Force per unit area $$\tau_w = \dot{m}u \tag{5.8}$$

Where, τ_w is the shear stress in the fluid at the wall.

Combining the equations for the heat flow and the momentum transfer, then

$$\frac{\dot{q}}{c\Delta t} = \frac{\tau_w}{u} \Rightarrow \dot{q} = \frac{\tau_w c\Delta t}{u} \tag{5.9}$$

For a turbulent flow, in practice there is always a thin layer of f1uid on the surface in which viscous effects predominate. This film is known as the laminar sub-layer. In this layer heat is transferred purely by conduction. Therefore, from the Fourier's law, for a unit area,

$$\dot{q} = -\lambda \left(\frac{dT}{dy} \right) \Bigg|_{y=0} \tag{5.10}$$

Where, y is the distance from the surface perpendicular to the surface and λ is the thermal conductivity of the f1uid.

Also, for viscous flow,

Shear stress = dynamic viscosity x velocity gradient; hence, the shear stress at the wall is given:

$$\tau_w = \mu \frac{du}{dy} \Bigg|_{y=0} \tag{5.11}$$

Now, since the laminar sub-layer is very thin it may be assumed that the temperature and velocity vary linearly with the distance from the wall

i.e., $$\dot{q} = -\frac{\lambda \Delta t}{\delta_b} \text{ and } \tau_w = \frac{\mu u}{\delta_b} \tag{5.12}$$

where, δ_b is the thickness of the laminar sub-layer.

Then, eliminating δ_b, and neglecting the minus sign, we have:

$$\frac{\dot{q}}{\lambda \Delta t} = \frac{\tau_w}{\mu u} \quad \text{i.e.} \quad \dot{q} = \frac{\tau_w \lambda \Delta t}{\mu u} \tag{5.13}$$

It can be seen that this equation is identical with equation (5.9), when $c = \lambda / \mu$

That is, when $\frac{c\mu}{\lambda} = 1$ or Pr = 1.

Therefore, for fluids whose Prandtl number is approximately unity the simple Reynolds analogy can be applied, since the heat transferred across the laminar sub-layer can be considered in a similar way to the heat transferred from the sub-layer to the bulk of the fluid. For most gases, dry vapours, and superheated vapour Pr lies between about 0.65 and 1.2.

For a unit surface area, $\dot{q} = \alpha \Delta t$, therefore substituting in equation (5.9), we have $\dfrac{\alpha}{c} = \dfrac{\tau_w}{u}$

Dividing through by ρu where ρ is the mean density of the fluid, we have $\dfrac{\alpha}{\rho c u} = \dfrac{\tau_w}{\rho u^2}$

Both sides of this equation are dimensionless. The term on the left-hand side is called the Stanton number, St.

A dimensionless friction factor, f, is defined by $f = \dfrac{\tau_w}{\rho u^2 / 2}$

Therefore, we have for the Reynolds analogy $\mathrm{St} = \dfrac{f}{2}$, which can be rewritten in terms of Nu, Re and Pr as,

$$\mathrm{St} = \frac{\alpha}{\rho c u} = \frac{\alpha l}{\lambda} \times \frac{\mu}{\rho u l} \times \frac{\lambda}{C\mu} = \frac{\mathrm{Nu}}{\mathrm{Re}\,\mathrm{Pr}} \qquad \Rightarrow \left[f/2 = \frac{\mathrm{Nu}}{\mathrm{Re}\,\mathrm{Pr}} \right] \tag{5.14}$$

For turbulent flow in a pipe, a simple measurement of the pressure drop gives *f* and then using Equation (5.9) or (5.10), the approximate heat flow can be found.

For flow in a pipe of diameter, d, the resistance to flow over unit length is given by

$$\text{Resistance} = \tau_w \pi d = \Delta p \frac{\pi}{4} d^2 \qquad \Rightarrow \tau_w = \frac{\Delta p d}{4} \tag{5.15}$$

where Δp is the pressure drop in unit length.

An important factor in heat exchanger design is the pumping power required. The pumping power is the rate at which work is done in overcoming the frictional resistance, i.e. For flow in a pipe,

$$\text{Pumping power per unit length, } \dot{W} = \tau_w \pi d u \tag{5.16}$$

Also, from equation (5.9), Heat flow per unit area is $\dot{q} = \dfrac{\tau_w c \Delta t}{u}$

That is, Heat flow per unit length, $\dot{Q} = \dfrac{\tau_w c \Delta t \pi d}{u}$

Then, the ratio of the pumping power required to the rate of heat flow can be expressed as:

$$\frac{\dot{W}}{\dot{Q}} = \frac{\tau_w \pi d u^2}{\tau_w c \Delta t \pi d} = \frac{u^2}{c \Delta t} \tag{5.17}$$

Equation (5.17) suggests that the power required for a given heat transfer rate can be reduced by decreasing the velocity of flow, *u*. However, a reduction in the fluid velocity means that the required surface area must be increased, and hence a compromise must be made.

Example 5.1 Heating of air in a tube

Air is heated by passing it through a 25 mm bore copper tube which is maintained at 280 ^oC. The air enters at 15 ^oC and leaves at 270 ^oC at a mean velocity of 30 m/s. Using the Reynolds analogy, calculate the length of the tube and the pumping power required. For turbulent flow in a tube take $f = 0.0791/(\mathrm{Re})^{1/4}$, and all properties at mean film temperature. Take the mean temperature difference as $\Delta \bar{t}_{\mathrm{In}} = (\Delta t_1 - \Delta t_2)/\mathrm{In}(\Delta t_1/\Delta t_2)$.

Solution

The mean film temperature can be found as:

$$\mathrm{t_f} = \frac{\mathrm{t_b} + \mathrm{t_w}}{2} = \frac{1}{2}\left(\frac{15+270}{2} + 280\right) = \frac{142.5+280}{2} 211.25^0\,\mathrm{C}$$

From Table 10 at t_f = 211.25^{o}C = 484.4 K, the properties of air can be found as:

λ = 3.938 x10^{-5} kW/m K; Pr = 0.681; ν = 3.591 x 10^{-5} m^2/s; ρ = 0.73 kg/m^3; c_p = 1.027 kJ/kgK

Then,

$$\mathrm{Re} = \frac{ud}{\nu} = \frac{30 \times 0.025}{3.591 \times 10^{-5}} = 20900$$

Therefore,

$$f = \frac{0.0791}{(20900)^{1/4}} = \frac{0.0791}{12.02} = 0.00658$$

From Equation 5.14,

$$\mathrm{St} = \frac{\mathrm{Nu}}{\mathrm{RePr}} = f/2 = \frac{0.00658}{2} = 0.00329$$

That is, $\mathrm{Nu} = \mathrm{St} \cdot \mathrm{Re} \cdot \mathrm{Pr} = 0.00329 \times 20900 \times 0.681 = 46.8$

Therefore, $\dfrac{\alpha d}{\lambda} = 46.8$

That is, $\alpha = \dfrac{46.8 \times 3.938 \times 10^{-5}}{0.025} = 0.0737 \text{ kW/m}^2\text{ K}$

$$\text{Mass flow of air} = \rho A u = 0.73 \times \frac{\pi}{4} \times 0.025^2 \times 30 = 0.01075 \text{ kg/s}$$

Hence,

$$\begin{aligned}\text{Heat received by the air} &= \dot{m}c(t_{a,2} - t_{a,1}) \\ &= 0.01075 \times 1.027 \times (270 - 15) \\ &= 2.815 \text{ kW}\end{aligned}$$

Also, $\dot{Q} = \alpha A \Delta t = 2.815 \text{ kW}$

and $\Delta t = \Delta \bar{t}_{\ln} = \dfrac{\Delta t_1 - \Delta t_2}{\ln(\Delta t_1 / \Delta t_2)} = \dfrac{(280 - 15) - (280 - 270)}{\ln\{(280 - 15)/(280 - 270)\}} = 77.8 \text{ K}$

Then, $\dot{Q} = 2.815 = 0.0737 \times A \times 77.8$

Therefore, $A = \dfrac{2.815}{0.0737 \times 77.8} = 0.49 \text{ m}^2$

Thus,

$$\text{Tube length} = \frac{A}{\pi d} = \frac{0.49}{\pi \times 0.025} = 6.24 \text{ m}$$

From Equation 5.17,

$$\frac{\dot{W}}{\dot{Q}} = \frac{u^2}{c \Delta t}$$

Therefore,

$$\dot{W} = \frac{\dot{Q} u^2}{c \Delta t} = \frac{2.815 \times 30^2}{1.027 \times 77.8} = 31.7 \text{ W}$$

That is, Pumping power = 31.7 W

1-5.4 The Heat Transfer Coefficient

Flow across cylinders and spheres, in general, involve flow separation which is difficult to handle analytically. Therefore such flows must be studied experimentally. Numerous investigators have studied such flows and several empirical correlations have been developed for the heat transfer coefficient. Churchill and Bernstein presented a relation for the determination of the average Nusselt number for flow over a *cylinder*:

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{\alpha D}{\lambda} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{28{,}200}\right)^{5/8}\right]^{4/5} \tag{5.18}$$

The fluid properties are evaluated at the *film temperature* $T_f = \frac{1}{2}(T_\infty + T_s)$,which is the average of the free-stream and surface temperatures. For flow over a *sphere*, Whitaker proposed the following comprehensive correlation:

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{\alpha D}{\lambda} = 2 + \left[0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}\right]\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \tag{5.19}$$

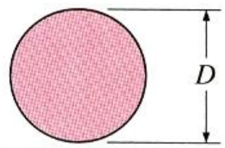
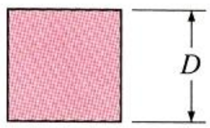
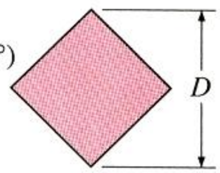
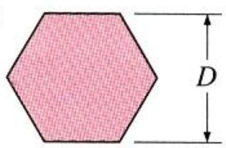
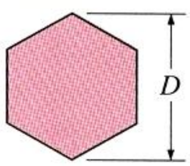
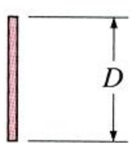
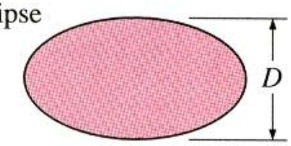
which is valid for 3.5 ≤ Re ≤ 80,000 and 0.7 ≤ Pr ≤ 380. The fluid properties in this case are evaluated at the free-stream temperature T_∞, except for μ_s, which is evaluated at the surface temperature T_s. Although the two relations above are considered to be quite accurate, the results obtained from them can be off by as much as 30 percent.

The average Nusselt number for flow across cylinders can be expressed compactly as

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{\alpha D}{\lambda} = C\,\mathrm{Re}^m\,\mathrm{Pr}^{\mathrm{n}} \tag{5.20}$$

Where, $n = \frac{1}{3}$ and the experimentally determined constants *C* and *m* are given in Table 9 for circular as well as various noncircular cylinders. The characteristic length D for use in the calculation of the Reynolds and Nusselt numbers for different geometries is as indicated on the figure. Note that all fluid properties are evaluated at the *film temperature* $T_f = \frac{1}{2}(T_\infty + T_s)$. The relations for cylinders as presented are for *single* cylinders or cylinders oriented such that the flow over them is not affected by the presence of others. Also they are applicable to *smooth* surfaces only. *Surface roughness* and *free-stream turbulence* may affect the drag and heat transfer coefficients significantly.

Table 9: Empirical correlations for the average Nusselt number for forced convection over circular and non-circular cylinders in cross flow (from Zhukauskas and Jacob)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\,Pr^{1/3}$ $Nu = 0.911Re^{0.385}\,Pr^{1/3}$ $Nu = 0.683Re^{0.466}\,Pr^{1/3}$ $Nu = 0.193Re^{0.618}\,Pr^{1/3}$ $Nu = 0.027Re^{0.805}\,Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\,Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\,Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\,Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\,Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\,Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\,Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\,Pr^{1/3}$

Example 5.2 Heat Loss from a Steam Pipe in Windy Air

A long 10-cm diameter steam pipe whose external surface temperature is 110 ^oC passes through some open area that is not protected against the winds. Determine the rate of heat loss from the pipe per unit of its length when the air is at 1 atm pressure and 4 ^oC and the wind is blowing across the pipe at a velocity of 8 m/s.

Solution: A steam pipe is exposed to windy air. The rate of heat loss form the steam is to be determined.

Assumptions: (1) Steady operating conditions exist. (2) Radiation effects are negligible. (3) Air is an ideal gas

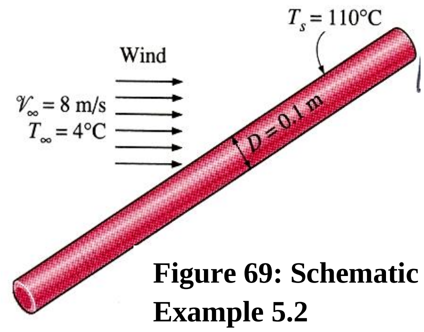


Figure 69: Schematic for Worked Example 5.2

Properties: The properties of air at the film temperature of $T_f = (T_\infty + T_s)/2 =$

$(110+4)/2 = 57^0\text{C} = 330\,\text{K}$ and 1 atm pressure from property tables (Table 10) are:

$\lambda = 0.0283$ W/m^{o}C Pr = 0.708 $\nu = 1.86 \times 10^{-5}$m^2/s

Analysis: This is an *external flow* problem, since we are interested in the heat transfer from the pipe to the air that is flowing outside the pipe. The Reynolds number of the flow is

$$\text{Re} = \frac{V_\infty D}{\nu} = \frac{(8\,\text{m/s})(0.1\text{m})}{1.86\times10^{-5}\,\text{m}^2/\text{s}} = 43{,}011$$

Then the Nusselt number in this case can be determined from

$$\text{Nu}_{\text{cyl}} = \frac{\alpha D}{\lambda} = 0.3 + \frac{0.62\,\text{Re}^{1/2}\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{28{,}200}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62\,(43{,}011)^{1/2}(0.708)^{1/3}}{\left[1+(0.4/0.708)^{2/3}\right]^{1/4}}\left[1+\left(\frac{43{,}011}{28{,}200}\right)^{5/8}\right]^{4/5}$$

$$=196.3$$

and

$$\alpha = \frac{\lambda}{D}\text{Nu} = \frac{0.0283\,\text{W/m}^{\text{o}}\text{C}}{0.1\text{m}}(196.3) = 55.6\,\text{W/m}^{2\,\text{o}}\text{C}$$

Then the rate of heat transfer from the pipe per unit of its length becomes

$$\mathrm{A} = pL = \pi DL = \pi(0.1\mathrm{m})(1\mathrm{m}) = 0.314\,\mathrm{m}^2$$
$$\dot{Q} = \alpha A(T_s - T_\infty) = (55.6\ \mathrm{W/m^2\,^oC})(0.314\,\mathrm{m})(110 - 4)^o\mathrm{C} = 1851\,\mathrm{W}$$

The rate of heat loss from the entire pipe can be obtained by multiplying the value above by the length of the pipe in m.

Discussion: The simpler Nusselt number relation in Table 9 in this case would give Nu = 129, which is 34 percent lower than the value obtained above using Equation 5.18

Table 10: Properties of gases at 1 atm pressure

Temperature, T K	Density, ρ kg/m³	Specific heat, C_p J/kg · °C	Thermal conductivity, k W/m · °C	Thermal diffusivity, α m²/s	Dynamic viscosity, μ kg/m · s	Kinematic viscosity, ν m²/s	Prandtl number, Pr
Air							
200	1.766	1003	0.0181	1.02×10^{-5}	1.34×10^{-5}	0.76×10^{-5}	0.740
250	1.413	1003	0.0223	1.57×10^{-5}	1.61×10^{-5}	1.14×10^{-5}	0.724
280	1.271	1004	0.0246	1.95×10^{-5}	1.75×10^{-5}	1.40×10^{-5}	0.717
290	1.224	1005	0.0253	2.08×10^{-5}	1.80×10^{-5}	1.48×10^{-5}	0.714
298	1.186	1005	0.0259	2.18×10^{-5}	1.84×10^{-5}	1.55×10^{-5}	0.712
300	1.177	1005	0.0261	2.21×10^{-5}	1.85×10^{-5}	1.57×10^{-5}	0.712
310	1.143	1006	0.0268	2.35×10^{-5}	1.90×10^{-5}	1.67×10^{-5}	0.711
320	1.110	1006	0.0275	2.49×10^{-5}	1.94×10^{-5}	1.77×10^{-5}	0.710
330	1.076	1007	0.0283	2.64×10^{-5}	1.99×10^{-5}	1.86×10^{-5}	0.708
340	1.043	1007	0.0290	2.78×10^{-5}	2.03×10^{-5}	1.96×10^{-5}	0.707
350	1.009	1008	0.0297	2.92×10^{-5}	2.08×10^{-5}	2.06×10^{-5}	0.706
400	0.883	1013	0.0331	3.70×10^{-5}	2.29×10^{-5}	2.60×10^{-5}	0.703
450	0.785	1020	0.0363	4.54×10^{-5}	2.49×10^{-5}	3.18×10^{-5}	0.700
500	0.706	1029	0.0395	5.44×10^{-5}	2.68×10^{-5}	3.80×10^{-5}	0.699
550	0.642	1039	0.0426	6.39×10^{-5}	2.86×10^{-5}	4.45×10^{-5}	0.698
600	0.589	1051	0.0456	7.37×10^{-5}	3.03×10^{-5}	5.15×10^{-5}	0.698
700	0.504	1075	0.0513	9.46×10^{-5}	3.35×10^{-5}	6.64×10^{-5}	0.702
800	0.441	1099	0.0569	11.7×10^{-5}	3.64×10^{-5}	8.25×10^{-5}	0.704
900	0.392	1120	0.0625	14.2×10^{-5}	3.92×10^{-5}	9.99×10^{-5}	0.705
1000	0.353	1141	0.0672	16.7×10^{-5}	4.18×10^{-5}	11.8×10^{-5}	0.709
1200	0.294	1175	0.0759	22.2×10^{-5}	4.65×10^{-5}	15.8×10^{-5}	0.720
1400	0.252	1201	0.0835	27.6×10^{-5}	5.09×10^{-5}	20.2×10^{-5}	0.732
1600	0.221	1240	0.0904	33.0×10^{-5}	5.49×10^{-5}	24.9×10^{-5}	0.753
1800	0.196	1276	0.0970	38.3×10^{-5}	5.87×10^{-5}	29.9×10^{-5}	0.772
2000	0.177	1327	0.1032	44.1×10^{-5}	6.23×10^{-5}	35.3×10^{-5}	0.801
Ammonia (NH_3)							
200	1.038	2199	0.0153	0.67×10^{-5}	6.89×10^{-6}	0.66×10^{-5}	0.990
250	0.831	2248	0.0197	1.05×10^{-5}	8.53×10^{-6}	1.03×10^{-5}	0.973
300	0.692	2298	0.0246	1.55×10^{-5}	10.27×10^{-6}	1.48×10^{-5}	0.969
350	0.593	2349	0.0302	2.17×10^{-5}	12.06×10^{-6}	2.03×10^{-5}	0.938
400	0.519	2402	0.0364	2.92×10^{-5}	13.90×10^{-6}	2.68×10^{-5}	0.917
450	0.461	2455	0.0433	3.82×10^{-5}	15.76×10^{-6}	3.42×10^{-5}	0.894
500	0.415	2507	0.0506	4.86×10^{-5}	17.63×10^{-6}	4.25×10^{-5}	0.873
550	0.378	2559	0.0580	6.00×10^{-5}	19.5×10^{-6}	5.16×10^{-5}	0.860
600	0.346	2611	0.0656	7.26×10^{-5}	21.4×10^{-6}	6.18×10^{-5}	0.852
700	0.297	2710	0.0811	10.1×10^{-5}	25.1×10^{-6}	8.45×10^{-5}	0.839
800	0.260	2810	0.0977	13.4×10^{-5}	28.8×10^{-6}	11.1×10^{-5}	0.828
Argon							
200	2.435	523.6	0.0124	0.98×10^{-5}	1.60×10^{-5}	0.66×10^{-5}	0.674
250	1.948	522.2	0.0152	1.49×10^{-5}	1.95×10^{-5}	1.00×10^{-5}	0.672
300	1.623	521.6	0.0177	2.09×10^{-5}	2.27×10^{-5}	1.40×10^{-5}	0.669
350	1.392	521.2	0.0201	2.78×10^{-5}	2.57×10^{-5}	1.85×10^{-5}	0.666
400	1.218	521.0	0.0223	3.52×10^{-5}	2.85×10^{-5}	2.34×10^{-5}	0.665
450	1.082	520.9	0.0244	4.33×10^{-5}	3.12×10^{-5}	2.88×10^{-5}	0.665

Source: Fundamentals of thermal fluid sciences by Y.A.Cengel & R. Turner, Mc Graw Hill

Table 10: Properties of gases at 1 atm pressure continued.

Temperature, T K	Density, ρ kg/m^3	Specific heat, C_p J/kg · °C	Thermal conductivity, k W/m · °C	Thermal diffusivity, α m^2/s	Dynamic viscosity, μ kg/m · s	Kinematic viscosity, ν m^2/s	Prandtl number, Pr
500	0.974	520.8	0.0264	5.20×10^{-5}	3.37×10^{-5}	3.45×10^{-5}	0.664
550	0.886	520.7	0.0283	6.14×10^{-5}	3.60×10^{-5}	4.07×10^{-5}	0.662
600	0.812	520.6	0.0301	7.12×10^{-5}	3.83×10^{-5}	4.72×10^{-5}	0.662
700	0.696	520.6	0.0336	9.28×10^{-5}	4.25×10^{-5}	6.11×10^{-5}	0.658
800	0.609	520.5	0.0369	11.6×10^{-5}	4.64×10^{-5}	7.62×10^{-5}	0.655
900	0.541	520.5	0.0398	14.1×10^{-5}	5.01×10^{-5}	9.26×10^{-5}	0.654
1000	0.487	520.5	0.0427	16.8×10^{-5}	5.35×10^{-5}	11.0×10^{-5}	0.652
1200	0.406	520.5	0.0481	22.8×10^{-5}	5.99×10^{-5}	14.8×10^{-5}	0.648
1400	0.348	520.4	0.0535	29.6×10^{-5}	6.56×10^{-5}	18.9×10^{-5}	0.638
			Carbon dioxide (CO_2)				
200	2.683	759	0.0095	0.47×10^{-5}	1.02×10^{-5}	0.38×10^{-5}	0.814
250	2.146	806	0.0129	0.75×10^{-5}	1.26×10^{-5}	0.59×10^{-5}	0.790
300	1.789	852	0.0166	1.09×10^{-5}	1.50×10^{-5}	0.84×10^{-5}	0.768
350	1.533	897	0.0205	1.49×10^{-5}	1.73×10^{-5}	1.13×10^{-5}	0.755
400	1.341	939	0.0244	1.94×10^{-5}	1.94×10^{-5}	1.45×10^{-5}	0.747
450	1.192	979	0.0283	2.43×10^{-5}	2.15×10^{-5}	1.80×10^{-5}	0.743
500	1.073	1017	0.0323	2.96×10^{-5}	2.35×10^{-5}	2.19×10^{-5}	0.740
550	0.976	1049	0.0363	3.55×10^{-5}	2.54×10^{-5}	2.60×10^{-5}	0.734
600	0.894	1077	0.0403	4.18×10^{-5}	2.72×10^{-5}	3.04×10^{-5}	0.727
700	0.767	1126	0.0487	5.64×10^{-5}	3.06×10^{-5}	3.99×10^{-5}	0.708
800	0.671	1169	0.0560	7.14×10^{-5}	3.39×10^{-5}	5.05×10^{-5}	0.708
900	0.596	1205	0.0621	8.65×10^{-5}	3.69×10^{-5}	6.19×10^{-5}	0.716
1000	0.537	1235	0.0680	10.25×10^{-5}	3.97×10^{-5}	7.40×10^{-5}	0.721
1200	0.447	1283	0.0780	13.6×10^{-5}	4.49×10^{-5}	10.04×10^{-5}	0.739
1400	0.383	1315	0.0867	17.2×10^{-5}	4.97×10^{-5}	13.0×10^{-5}	0.754
			Carbon monoxide (CO)				
200	1.708	1045	0.0175	0.98×10^{-5}	1.27×10^{-5}	0.75×10^{-5}	0.763
250	1.366	1048	0.0214	1.50×10^{-5}	1.54×10^{-5}	1.13×10^{-5}	0.753
300	1.138	1051	0.0252	2.11×10^{-5}	1.78×10^{-5}	1.56×10^{-5}	0.743
350	0.976	1056	0.0288	2.80×10^{-5}	2.01×10^{-5}	2.05×10^{-5}	0.735
400	0.854	1060	0.0323	3.57×10^{-5}	2.21×10^{-5}	2.59×10^{-5}	0.727
450	0.759	1065	0.0355	4.39×10^{-5}	2.41×10^{-5}	3.18×10^{-5}	0.723
500	0.683	1071	0.0386	5.28×10^{-5}	2.60×10^{-5}	3.80×10^{-5}	0.720
550	0.621	1077	0.0416	6.22×10^{-5}	2.77×10^{-5}	4.46×10^{-5}	0.717
600	0.569	1084	0.0444	7.20×10^{-5}	2.94×10^{-5}	5.17×10^{-5}	0.718
700	0.488	1099	0.0497	9.27×10^{-5}	3.25×10^{-5}	6.66×10^{-5}	0.718
800	0.427	1114	0.0549	11.5×10^{-5}	3.54×10^{-5}	8.29×10^{-5}	0.718
900	0.379	1128	0.0596	13.9×10^{-5}	3.81×10^{-5}	10.04×10^{-5}	0.721
1000	0.342	1142	0.0644	16.5×10^{-5}	4.06×10^{-5}	11.9×10^{-5}	0.720
1100	0.310	1155	0.0692	19.3×10^{-5}	4.30×10^{-5}	13.9×10^{-5}	0.718
1200	0.285	1168	0.0738	22.2×10^{-5}	4.53×10^{-5}	15.9×10^{-5}	0.717
			Helium				
200	0.2440	5197	0.115	0.91×10^{-4}	1.50×10^{-5}	0.61×10^{-4}	0.676
250	0.1952	5197	0.134	1.54×10^{-4}	1.75×10^{-5}	0.90×10^{-4}	0.680

Source: Fundamentals of thermal fluid sciences by Y.A.Cengel & R. Turner, Mc Graw Hill

Table 10: Properties of gases at 1 atm pressure continued.

Temperature, T K	Density, ρ kg/m^3	Specific heat, C_p J/kg · °C	Thermal conductivity, k W/m · °C	Thermal diffusivity, α m^2/s	Dynamic viscosity, μ kg/m · s	Kinematic viscosity, ν m^2/s	Prandtl number, Pr
300	0.1627	5197	0.150	1.77×10^{-4}	1.99×10^{-5}	1.22×10^{-4}	0.690
350	0.1394	5197	0.165	2.28×10^{-4}	2.21×10^{-5}	1.59×10^{-4}	0.698
400	0.1220	5197	0.180	2.83×10^{-4}	2.43×10^{-5}	1.99×10^{-4}	0.703
450	0.1085	5197	0.195	3.45×10^{-4}	2.63×10^{-5}	2.43×10^{-4}	0.702
500	0.0976	5197	0.211	4.17×10^{-4}	2.83×10^{-5}	2.90×10^{-4}	0.695
550	0.0887	5197	0.229	4.97×10^{-4}	3.02×10^{-5}	3.40×10^{-4}	0.684
600	0.0813	5197	0.247	5.84×10^{-4}	3.20×10^{-5}	3.93×10^{-4}	0.673
700	0.0697	5197	0.278	7.67×10^{-4}	3.55×10^{-5}	5.09×10^{-4}	0.663
800	0.0610	5197	0.307	9.68×10^{-4}	3.88×10^{-5}	6.37×10^{-4}	0.657
900	0.0542	5197	0.335	11.9×10^{-4}	4.20×10^{-5}	7.75×10^{-4}	0.652
1000	0.0488	5197	0.363	14.3×10^{-4}	4.50×10^{-5}	9.23×10^{-4}	0.645
1200	0.0407	5197	0.416	19.7×10^{-4}	5.08×10^{-5}	12.5×10^{-4}	0.635
1400	0.0349	5197	0.469	25.9×10^{-4}	5.61×10^{-5}	16.1×10^{-4}	0.622
1600	0.0305	5197	0.521	32.9×10^{-4}	6.10×10^{-5}	20.0×10^{-4}	0.608
1800	0.0271	5197	0.570	40.4×10^{-4}	6.57×10^{-5}	24.2×10^{-4}	0.599
2000	0.0244	5197	0.620	48.9×10^{-4}	7.00×10^{-5}	28.7×10^{-4}	0.587
			Hydrogen				
200	0.1299	13,540	0.128	0.77×10^{-4}	0.68×10^{-5}	0.55×10^{-4}	0.717
250	0.0983	14,070	0.156	1.13×10^{-4}	0.79×10^{-5}	0.80×10^{-4}	0.713
300	0.0819	14,320	0.182	1.55×10^{-4}	0.89×10^{-5}	1.09×10^{-4}	0.705
350	0.0702	14,420	0.203	2.01×10^{-4}	0.99×10^{-5}	1.42×10^{-4}	0.705
400	0.0614	14,480	0.221	2.49×10^{-4}	1.09×10^{-5}	1.78×10^{-4}	0.714
450	0.0546	14,500	0.239	3.02×10^{-4}	1.18×10^{-5}	2.17×10^{-4}	0.719
500	0.0492	14,510	0.256	3.59×10^{-4}	1.27×10^{-5}	2.59×10^{-4}	0.721
550	0.0447	14,520	0.274	4.22×10^{-4}	1.36×10^{-5}	3.04×10^{-4}	0.722
600	0.0410	14,540	0.291	4.89×10^{-4}	1.45×10^{-5}	3.54×10^{-4}	0.724
700	0.0351	14,610	0.325	6.34×10^{-4}	1.61×10^{-5}	4.59×10^{-4}	0.724
800	0.0307	14,710	0.360	7.97×10^{-4}	1.77×10^{-5}	5.76×10^{-4}	0.723
900	0.0273	14,840	0.394	10.8×10^{-4}	1.92×10^{-5}	7.03×10^{-4}	0.723
1000	0.0246	14,990	0.428	11.6×10^{-4}	2.07×10^{-5}	8.42×10^{-4}	0.724
1200	0.0205	15,370	0.495	15.7×10^{-4}	2.36×10^{-5}	11.5×10^{-4}	0.733
			Nitrogen				
200	1.708	1043	0.0183	1.02×10^{-5}	1.29×10^{-5}	0.75×10^{-5}	0.734
250	1.367	1042	0.0222	1.56×10^{-5}	1.55×10^{-5}	1.13×10^{-5}	0.725
300	1.139	1040	0.0260	2.19×10^{-5}	1.79×10^{-5}	1.57×10^{-5}	0.715
350	0.967	1041	0.0294	2.92×10^{-5}	2.01×10^{-5}	2.08×10^{-5}	0.711
400	0.854	1045	0.0325	3.64×10^{-5}	2.21×10^{-5}	2.59×10^{-5}	0.710
450	0.759	1050	0.0356	4.47×10^{-5}	2.41×10^{-5}	3.17×10^{-5}	0.709
500	0.683	1057	0.0387	5.36×10^{-5}	2.59×10^{-5}	3.79×10^{-5}	0.708
550	0.621	1065	0.0414	6.26×10^{-5}	2.76×10^{-5}	4.45×10^{-5}	0.711
600	0.569	1075	0.0441	7.20×10^{-5}	2.93×10^{-5}	5.14×10^{-5}	0.713
700	0.488	1098	0.0493	9.20×10^{-5}	3.24×10^{-5}	6.63×10^{-5}	0.720
800	0.427	1122	0.0541	11.3×10^{-5}	3.52×10^{-5}	8.24×10^{-5}	0.730
900	0.380	1146	0.0587	13.5×10^{-5}	3.79×10^{-5}	9.97×10^{-5}	0.739
1000	0.342	1168	0.0631	15.8×10^{-5}	4.04×10^{-5}	11.8×10^{-5}	0.747

Source: Fundamentals of thermal fluid sciences by Y.A.Cengel & R. Turner, Mc Graw Hill

Table 10: Properties of gases at 1 atm pressure continued.

Temperature, T K	Density, ρ kg/m^3	Specific heat, C_p J/kg · °C	Thermal conductivity, k W/m · °C	Thermal diffusivity, α m^2/s	Dynamic viscosity, μ kg/m · s	Kinematic viscosity, ν m^2/s	Prandtl number, Pr
1200	0.285	1205	0.0713	20.8×10^{-5}	4.50×10^{-5}	15.8×10^{-5}	0.761
1400	0.244	1233	0.0797	26.5×10^{-5}	4.92×10^{-5}	20.2×10^{-5}	0.761
Oxygen							
200	1.951	906	0.0182	1.03×10^{-5}	1.47×10^{-5}	0.75×10^{-5}	0.728
250	1.561	914	0.0225	1.58×10^{-5}	1.78×10^{-5}	1.14×10^{-5}	0.721
300	1.301	920	0.0267	2.23×10^{-5}	2.07×10^{-5}	1.59×10^{-5}	0.711
350	1.115	929	0.0306	2.95×10^{-5}	2.34×10^{-5}	2.10×10^{-5}	0.710
400	0.976	942	0.0342	3.72×10^{-5}	2.59×10^{-5}	2.65×10^{-5}	0.713
450	0.867	956	0.0377	4.55×10^{-5}	2.83×10^{-5}	3.26×10^{-5}	0.717
500	0.780	971	0.0412	5.44×10^{-5}	3.05×10^{-5}	3.91×10^{-5}	0.720
550	0.709	987	0.0447	6.38×10^{-5}	3.27×10^{-5}	4.61×10^{-5}	0.722
600	0.650	1003	0.0480	7.36×10^{-5}	3.47×10^{-5}	5.34×10^{-5}	0.725
700	0.557	1032	0.0544	9.46×10^{-5}	3.85×10^{-5}	6.91×10^{-5}	0.730
800	0.488	1054	0.0603	11.7×10^{-5}	4.21×10^{-5}	8.63×10^{-5}	0.736
900	0.434	1074	0.0661	14.2×10^{-5}	4.54×10^{-5}	10.5×10^{-5}	0.738
1000	0.390	1091	0.0717	16.8×10^{-5}	4.85×10^{-5}	12.4×10^{-5}	0.738
1200	0.325	1116	0.0821	22.6×10^{-5}	5.42×10^{-5}	16.7×10^{-5}	0.737
1400	0.278	1136	0.0921	29.1×10^{-5}	5.95×10^{-5}	21.3×10^{-5}	0.734
Water vapor (steam)							
300	0.0253*	2041	0.0181	35.1×10^{-5}*	0.91×10^{-5}	36.1×10^{-5}*	1.03
350	0.258*	2037	0.0222	4.22×10^{-5}*	1.12×10^{-5}	4.33×10^{-5}*	1.02
400	0.555	2000	0.0264	2.38×10^{-5}	1.32×10^{-5}	2.38×10^{-5}	1.00
450	0.491	1968	0.0307	3.17×10^{-5}	1.52×10^{-5}	3.10×10^{-5}	0.98
500	0.441	1977	0.0357	4.09×10^{-5}	1.73×10^{-5}	3.92×10^{-5}	0.96
550	0.401	1994	0.0411	5.15×10^{-5}	1.93×10^{-5}	4.82×10^{-5}	0.94
600	0.367	2022	0.0464	6.25×10^{-5}	2.13×10^{-5}	5.82×10^{-5}	0.93
700	0.314	2083	0.0572	8.74×10^{-5}	2.54×10^{-5}	8.09×10^{-5}	0.93
800	0.275	2148	0.0686	11.6×10^{-5}	2.95×10^{-5}	10.7×10^{-5}	0.92
900	0.244	2217	0.078	14.4×10^{-5}	3.36×10^{-5}	13.7×10^{-5}	0.95
1000	0.220	2288	0.087	17.3×10^{-5}	3.76×10^{-5}	17.1×10^{-5}	0.99

Source: Fundamentals of thermal fluid sciences by Y.A.Cengel & R. Turner, Mc Graw Hill

Self Assessment 1-5

1 Water at 10°C (ρ = 999.7 kg/m^3 and μ = 1.307 x 10^{-3} kg/m.s) is flowing in a 0.20-cm-diameter 15-m-long pipe steadily at an average velocity of 1.2 m/s. Determine (*a*) the pressure drop and (*b*) the pumping power requirement to overcome this pressure drop.

Suggested Answers: (a) 188 kPa, (b) 0.71 W

2. Air enters a 7-m-long section of a rectangular duct of cross section 15 cm x 20 cm at 50°C at an average velocity of 7 m/s. If the walls of the duct are maintained at 10°C, determine (*a*) The outlet temperature of the air, (*b*) the rate of heat transfer from the air, and (*c*) the fan power needed to overcome the pressure losses in this section of the duct.

Suggested Answers: (a) 32.8°C, (b) 3674 W, (c) 4.2 W

SESSION 2-5: NATURAL CONVECTION

Heat transfer by free convection is due to differences in density in the fluid causing a natural circulation, and hence a transfer of heat. For the majority of problems in which a fluid flows across a surface, the superimposed effect of natural convection is small enough to be neglected. When there is no forced velocity of the fluid then the heat is transferred entirely by natural convection (when radiation is negligible), The heat transfer in this case depends on the coefficient of cubical expansion, β, which is given by:

$$\rho_1 = \rho_2(1+\beta\Delta t) \qquad \text{or} \qquad (\rho_1 - \rho_2) = \rho_2\beta\Delta t \tag{5.21}$$

where Δt is the temperature difference between the two parts of the fluid of density ρ_1 and ρ_2. It can be shown that the volume expansion coefficient β of an ideal gas at an absolute temperature T is equivalent to the inverse of the temperature:

$$\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\mathrm{K}) \tag{5.22}$$

The upthrust per unit volume of fluid is: $(\rho_1 - \rho_2)g$, and the velocity of the convection current is dependent on the upthrust. That is the convection currents depends on; $(\rho_1 - \rho_2)g = \rho_2\beta\Delta t g$.

The heat transfer also depends on the fluid viscosity, thermal conductivity of the fluid, and a characteristic dimension of length. Since the coefficient of cubical expansion and the local acceleration due to gravity, do not have a separate effect on the heat transfer, then only the product, βg , needs to be considered.

From the procedure of dimensional analysis, we obtain:

$$\mathrm{Nu} = KF\{(\mathrm{Pr}),(\mathrm{Gr})\} \tag{5.23}$$

where

$$\mathrm{Gr} = \frac{\beta g \rho^2 \delta^3 \Delta t}{\mu^2} = \frac{\beta g \delta^3 \Delta t}{\nu^2} \tag{5.24}$$

Gr is the Grashof number, δ is the characteristic length of the geometry and ν is the kinematic viscosity of the fluid. In many cases of natural convection it is possible to use an approximate equation to evaluate the heat transfer coefficient.

2-5a The Grashof Number

The Grashof number represents the ratio of the buoyancy force to the viscous force acting on a fluid. The role played by the *Reynolds number* in forced convection is played by the *Grashof number* in natural convection. As such, the Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection. For vertical plates, for example, the critical Grashof number is observed to be about 10^9. Therefore, the flow regime on a vertical plate becomes turbulent at Grashof numbers greater than 10^9.

The heat transfer rate in natural convection from a solid surface to the surrounding fluid is expressed by Newton's Law of cooling as $\dot{Q}_{conv} = \alpha A(T_s - T_\infty)$

where A is the heat transfer surface area and α is the average heat transfer coefficient on the surface.

2-5.1 Natural Convection correlations

Although the mechanism of natural convection is well understood, the complexities of fluid motion make it very difficult to obtain simple analytical relations for heat transfer by solving the governing equations of motion and energy.

The simple empirical correlations for the average *Nusselt number* Nu in natural convection are of the form

$$\mathrm{Nu} = \frac{\alpha\delta}{\lambda} = C(\mathrm{Gr}\,\mathrm{Pr})^n = C\,\mathrm{Ra}^n \tag{5.25}$$

where Ra is the Rayleigh number, which is the product of the Grashof and Prandtl numbers:

$$\mathrm{Ra} = \mathrm{Gr}\,\mathrm{Pr} = \frac{\mathrm{g}\beta(T_s - T_\infty)\delta^3}{\nu^2}\mathrm{Pr} \tag{5.26}$$

The values of the constants C and *n* depend on the *geometry* of the surface and the *flow regime*, which is characterised by the range of the Rayleigh number. The value of *n* is usually $\frac{1}{4}$ for laminar flow and $\frac{1}{3}$ for turbulent flow. The value of the constant *C* is normally less < 1.

Simple relations for the average Nusselt number for various geometries are given in Table 10, together with sketches of the geometries. Also given in this table are the characteristic lengths of the geometries and the ranges of the Rayleigh number in which the relation is applicable. All fluid properties are to be evaluated at the film temperature $T_f = \frac{1}{2}(T_\infty + T_s)$.

Example 5.3   Heat Loss from Hot Water Pipes

A 6-m-long section of an 8-cm-diameter horizontal hot water pipe shown in Figure 70 passes through a large room whose temperature is 18°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection.

Solution: A horizontal hot water pipe passes through a large room. The rate of heat loss from the pipe by natural convection is to be determined.

Assumptions: (1) Steady operating conditions exist. (2) Air is an ideal gas. (3) The local atmospheric pressure is 1 atm.

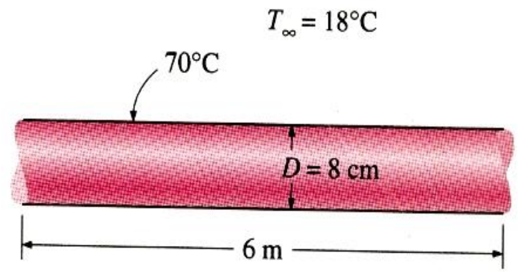


Figure 70: Schematic for worked example 5.3

Properties: The properties of air at the film temperature of $T_f = \frac{1}{2}(T_\infty + T_s) = (70+18)/2 =$ 44°C = 317 K and 1 atm pressure are:

$\lambda = 0.0273$ W/m°C   Pr = 0.710 $\nu = 1.74 \times 10^{-5}$ m²/s $\beta = 1/T_f = 1/317$ K $= 0.00315$ K^{-1}

Analysis: The characteristic length in this case is the outer diameter of the pipe, $\delta = D = 0.08$m Then, the Rayleigh number becomes:

$$\mathrm{Ra} = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2}\mathrm{Pr}$$

$$= \frac{(9.81\,\mathrm{m/s^2})(0.00315\,\mathrm{K^{-1}})[(70-18)\,\mathrm{K}](0.08\,\mathrm{m})^3}{(1.74\times10^{-5}\ \mathrm{m^2/s})^2}(0.710) = 1.930\times10^6$$

Then, the natural convection Nusselt number in this case can be determined from the appropriate equation from Table 11 to be:

$$\mathrm{Nu} = \left\{0.6 + \frac{0.387\,\mathrm{Ra}^{1/6}}{\left[1+(0.559/\mathrm{Pr})^{9/16}\right]^{8/27}}\right\}^2$$

$$= \left\{0.6 + \frac{0.387\,(1.930\times10^6)^{1/6}}{\left[1+(0.559/0.710)^{9/16}\right]^{8/27}}\right\}^2 = 17.2$$

Then,

$$\alpha = \frac{\lambda}{D}\,\mathrm{Nu} = \frac{0.0273\ \mathrm{W/m\,^oC}}{0.08\mathrm{m}}(17.2) = 5.9\ \mathrm{W/m^{2\ o}C}$$
$$\mathrm{A} = \pi \mathrm{DL} = \pi(0.08\mathrm{m})(6\mathrm{m}) = 1.51\,\mathrm{m}^2$$

and

$$\dot{Q} = \alpha A(T_s - T_\infty) = (5.9\ \mathrm{W/m^{2\ o}C})(1.51\,\mathrm{m}^2)(70-18)^0\mathrm{C} = 463\,\mathrm{W}$$

Therefore, the pipe will lose heat to the air in the room at a rate of 463 W by natural convection.

Discussion: The pipe will lose heat to the surroundings by radiation as well as by natural convection. A radiation analysis should normally accompany a natural convection analysis unless the emissivity of the surface is low.

Example 5.4 Heat Loss from a Wall

A wall 0.6 m high by 3 m wide is maintained at 79 ^{o}C in an atmosphere at 15 ^{o}C. Neglecting end effects and radiation, calculate the rate of heat loss by natural convection. For natural convection from a vertical flat surface take, at any distance, *x*:

$$\mathrm{Nu_x} = 0.509(\mathrm{Pr})^{1/2}(\mathrm{Pr}+0.952)^{-1/4}(\mathrm{Gr_x})^{1/4} \qquad \text{for Gr} > 10^{10}$$
$$\mathrm{Nu_x} = 0.679(\mathrm{Pr})^{1/2}(\mathrm{Pr}+0.952)^{-1/4}(\mathrm{Gr_x})^{1/4} \qquad \text{for Gr} < 10^{10}$$

Where all properties are at the mean film temperature, and $\beta = 1/T$, where T is the absolute temperature of the bulk of the air.

Solution The rate of heat loss from a wall by natural convection is to be determined given specified conditions and expressions for the local Nusselt number.

Assumptions: (1) Steady operating conditions exist. (2) Air is an ideal gas. (3) The local atmospheric pressure is 1 atm.

Properties: The properties of air at the film temperature of $T_f = \frac{1}{2}(T_\infty + T_w) = (15+79)/2 =$

47^{o}C = 320 K and 1 atm pressure are:

$$\lambda = 0.02778\ \mathrm{W/m^oC} \qquad \mathrm{Pr} = 0.7022 \qquad \nu = 1.759 \times 10^{-5}\ \mathrm{m^2/s}$$

But $\beta = 1/T_\infty = 1/288\ \mathrm{K} = 0.00347\ \mathrm{K}^{-1}$,where T_∞ is the bulk air temperature.

Analysis: The characteristic length in this case is the outer height of the wall, δ = H = 0.6 m.

Then, the Grashof number becomes:

$$\mathrm{Gr} = \frac{g\beta(T_w - T_\infty)\delta^3}{\nu^2}$$
$$= \frac{(9.81\,\mathrm{m/s^2})(0.00347\,\mathrm{K^{-1}})\left[(79-15)\,\mathrm{K}\right](0.6\,\mathrm{m})^3}{(1.759\times10^{-5}\,\mathrm{m^2/s})^2} = 1.521\times10^9$$

$\mathrm{Gr} < 10^{10}$ hence,

$$\mathrm{Nu_x} = 0.679(0.7022)^{1/2}(0.7022 + 0.952)^{-1/4}(1.521\times10^9)^{1/4}$$
$$= 99.08$$

Therefore, the heat transfer coefficient α is:

$$\alpha = \frac{\lambda}{\delta}\mathrm{Nu} = \frac{0.02778\,\mathrm{W/m\,K}}{0.6\mathrm{m}}(99.08) = 4.59\,\mathrm{W/m^2\,K}$$

Heat loss by convection can be determined from Newton's Law of Cooling as:

$$\dot{Q} = \alpha A(T_w - T_\infty)$$
$$= 4.59\,\mathrm{W/m^2K} \times (0.6\mathrm{m}\times3\mathrm{m})(79-15)\mathrm{K}$$
$$= 528.8\,\mathrm{W}$$

2-5.2 Natural Convection inside Enclosures

The Rayleigh number for an enclosure is determined from

$$\mathrm{Ra} = \frac{g\beta(T_1 - T_2)\delta^3}{\nu^2}\mathrm{Pr} \tag{5.27}$$

Where, the characteristic length δ is the distance between the hot and cold surfaces, and T_1 and T_2 are the temperatures of the hot and cold surfaces, respectively. All fluid properties are to be evaluated at the average fluid temperature $T_{av} = \frac{1}{2}(T_1 + T_2)$

In an experimental study using air, Holland *et al*., (1976) give the relationship between the Nusselt Number for tilt angles from 0 to 75^0C, for natural convection between parallel plates, as:

$\mathrm{Nu} = 1 + 1.44\left[1 - \frac{1708(\sin 1.8\beta)^{1.6}}{\mathrm{Ra}\cos\beta}\right]\left[1 - \frac{1708}{\mathrm{Ra}\cos\beta}\right]^+ + \left[\left(\frac{\mathrm{Ra}\cos\beta}{5830}\right)^{1/3} - 1\right]^+$ where the meaning of the + exponent is that only positive values of the terms in the square brackets are used (i.e., use zero, if the term is negative)

Table 11: Empirical correlations for average Nusselt number for natural convection over surfaces

Empirical correlations for the average Nusselt number for natural convection over surfaces

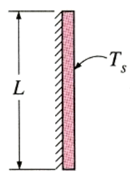
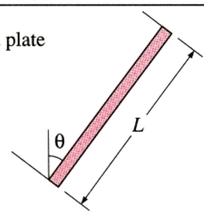
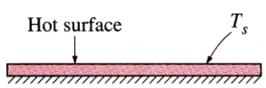
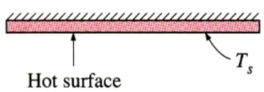
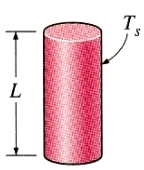
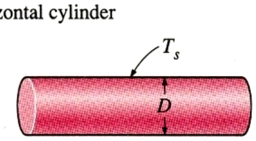
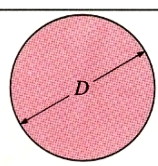
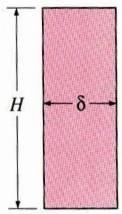
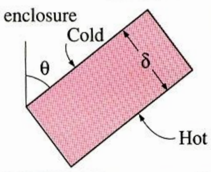
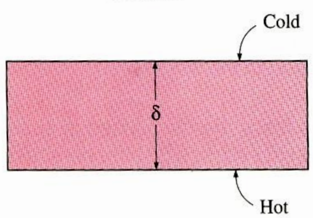
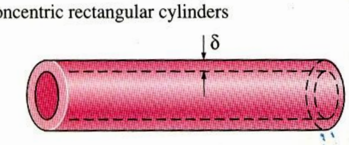
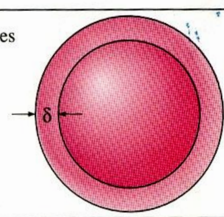
Geometry	Characteristic length δ	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}^{1/4}$ $\text{Nu} = 0.1\text{Ra}^{1/3}$ $\text{Nu} = \left\{0.825 + \dfrac{0.387\text{Ra}^{1/6}}{\left(1 + (0.492/\text{Pr})^{9/16}\right)^{8/27}}\right\}^2$ (complex but more accurate)
Inclined plate 	L		Use vertical plate equations as a first degree of approximation. Replace g by $g\cos\theta$ for $\text{Ra} < 10^9$
Horizontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate) 	A/p	10^4–10^7 10^7–10^{11}	$\text{Nu} = 0.54\text{Ra}^{1/4}$ $\text{Nu} = 0.15\text{Ra}^{1/3}$
(*b*) Lower surface of a hot plate (or upper surface of a cold plate) 		10^5–10^{11}	$\text{Nu} = 0.27\text{Ra}^{1/4}$
vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \dfrac{35L}{\text{Gr}^{1/4}}$
Horizontal cylinder 	D	10^{-5}–10^{12}	$\text{Nu} = \left\{0.6 + \dfrac{0.387\text{Ra}^{1/6}}{\left(1 + (0.559/\text{Pr})^{9/16}\right)^{8/27}}\right\}^2$
Sphere 	$\frac{1}{2}\pi D$	$\text{Ra} \leq 10^{11}$ $(\text{Pr} \geq 0.7)$	$\text{Nu} = 2 + \dfrac{0.589\text{Ra}^{1/4}}{\left(1 + (0.469/\text{Pr})^{9/16}\right)^{4/9}}$

Table 12: Empirical correlations for the average Nusselt number for natural convection in enclosures (the characteristic length δ is as indicated on the respective diagram)

Geometry	Fluid	H/δ	Range of Pr	Range of Ra	Nusselt number
Vertical rectangular enclosure (or vertical cylindrical enclosure) 	Gas or liquid	—	—	Ra < 2000	Nu = 1 (7-22)
	Gas	11–42	0.5–2	2×10^3–2×10^5	$Nu = 0.197 Ra^{1/4} \left(\frac{H}{\delta}\right)^{-1/9}$ (7-23)
	Gas	11–42	0.5–2	2×10^5–10^7	$Nu = 0.073 Ra^{1/3} \left(\frac{H}{\delta}\right)^{-1/9}$ (7-24)
	Liquid	10–40	1–20,000	10^4–10^7	$Nu = 0.42 Pr^{0.012} Ra^{1/4} \left(\frac{H}{\delta}\right)^{-0.3}$ (7-25)
	Liquid	1–40	1–20	10^6–10^9	$Nu = 0.046 Ra^{1/3}$ (7-26)
Inclined rectangular enclosure 					Use the correlations for vertical enclosures as a first-degree approximation for $\theta \le 20°$ by replacing g in the Ra relation by $g \cos\theta$
Horizontal rectangular enclosure (hot surface at the top)	Gas or liquid	—	—	—	Nu = 1 (7-27)
Horizontal rectangular enclosure (hot surface at the bottom) 	Gas or liquid	—	—	Ra < 1700	Nu = 1 (7-28)
	Gas	—	0.5–2	1.7×10^3–7×10^3	$Nu = 0.059 Ra^{0.4}$ (7-29)
	Gas	—	0.5–2	7×10^3–3.2×10^5	$Nu = 0.212 Ra^{1/4}$ (7-30)
	Gas	—	0.5–2	$Ra > 3.2 \times 10^5$	$Nu = 0.061 Ra^{1/3}$ (7-31)
	Liquid	—	1–5000	1.7×10^3–6×10^3	$Nu = 0.012 Ra^{0.6}$ (7-32)
	Liquid	—	1–5000	6×10^3–3.7×10^4	$Nu = 0.375 Ra^{0.2}$ (7-33)
	Liquid	—	1–20	3.7×10^4–10^8	$Nu = 0.13 Ra^{0.3}$ (7-34)
	Liquid	—	1–20	$Ra > 10^8$	$Nu = 0.057 Ra^{1/3}$ (7-35)
Concentric rectangular cylinders 	Gas or liquid	—	1–5000	6.3×10^3–10^6	$Nu = 0.11 Ra^{0.29}$ (7-36)
	Gas or liquid	—	1–5000	10^6–10^8	$Nu = 0.40 Ra^{0.20}$ (7-37)
Concentric spheres 	Gas or liquid	—	0.7–4000	10^2–10^9	$Nu = 0.228 Ra^{0.226}$ (7-38)

Simple empirical correlations for the Nusselt number for various enclosures are given in Table 12. Once the Nusselt number is available, the heat transfer coefficient and the rate of heat transfer through the enclosure can be determined from:

$$\alpha = \frac{\lambda}{\delta}\text{Nu} \tag{5.28}$$

and

$$\dot{Q} = \alpha A(T_1 - T_2) = \lambda \,\text{Nu}\, A\frac{T_1 - T_2}{\delta} \tag{5.29}$$

Where,

$$A = \begin{cases} HL & \text{rectangular enclosures} \\ \dfrac{\pi L(D_2 - D_1)}{\text{In}\,(D_2 / D_1)} & \text{concentric cylinders} \\ \pi D_1 D_2 & \text{concentric spheres} \end{cases} \tag{5.30}$$

Example 5.5 Heat Loss through a Double-Pane Window

The vertical 0.8-m-high, 2-m-wide double pane window shown in Figure 71 consists of two sheets of glass separated by a 2-cm air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 12°C and 2°C, determine the rate of heat transfer through the window.

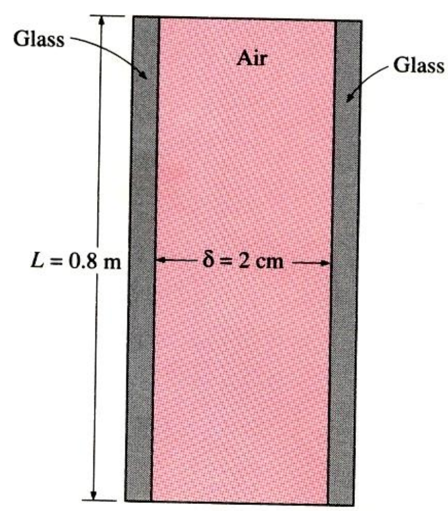


Figure 71: Schematic for example 5.5

Solution: Two glasses of a double-pane window are maintained at specified temperatures. The rate of heat transfer through the window is to be determined.

Assumptions: (1) Steady operating conditions exist. (2) Air is an ideal gas

Properties: The properties of air at the average temperature of $T_{av} = \frac{1}{2}(T_1 + T_2) = \frac{1}{2}(12 + 2) =$

7°C = 280 K and 1 atm pressure are:

$\lambda = 0.0246$ W/m°C; Pr = 0.717; $\nu = 1.40 \times 10^{-5}$ m²/s; $\beta = 1/T_{av} = 1/280$ K $= 0.00357$ K^{-1}

Analysis: We have a rectangular enclosure filled with air. The characteristic length in this case is the distance between the two glasses, $\delta = 0.02$m. Then, the Rayleigh number becomes:

$$\mathrm{Ra} = \frac{g\beta(T_1 - T_2)\delta^3}{\nu^2}\mathrm{Pr}$$
$$= \frac{(9.81\,\mathrm{m/s^2})(0.00357\,\mathrm{K^{-1}})[(12-2)\,\mathrm{K}](0.02\,\mathrm{m})^3}{(1.40\times10^{-5}\,\mathrm{m^2/s})^2}(0.717) = 1.024\times10^4$$

Then, the natural convection Nusselt number in this case can be determined from Table 12 to be

$$\mathrm{Nu} = 0.197\,\mathrm{Ra}^{1/4}\left(\frac{H}{\delta}\right)^{-1.9} = 0.197\,(1.024\times10^4)^{1/4}\left(\frac{0.8\,\mathrm{m}}{0.02\,\mathrm{m}}\right)^{-1.9} = 1.32$$

Then,

$$A = H \text{ x } L = (0.8\text{ m})(2\text{ m}) = 1.6\text{ m}^2$$

and

$$\dot{Q} = \lambda\,\mathrm{Nu}\,A\frac{T_1 - T_2}{\delta}$$
$$= (0.0246\,\mathrm{W/m^oC})(1.32)(1.6\,\mathrm{m^2})\frac{(12-2)^0\mathrm{C}}{0.02\,\mathrm{m}} = 25.9\,\mathrm{W}$$

Therefore, heat will be lost through the window at a rate of 25.9 W

2-5.3 Natural Convection from Finned Surfaces

In the selection of a heat sink for a particular application, there is always the question of whether to select one with *closely packed* or *widely spaced* fins for a given base area. A heat sink with closely packed fins will have greater surface area for heat transfer but a smaller heat transfer coefficient because of the extra resistance the additional fins will introduce to fluid flow through the interfin passages. A heat sink with widely spaced fins, on the other hand, will have a higher heat transfer coefficient by a smaller surface area. Then, there must be an *optimum spacing* that maximises the natural convection heat transfer from the heat sink for a given base area *WL*, where *W* and *L* are the width and height of the base of the sink, respectively, as shown in Figure 72.

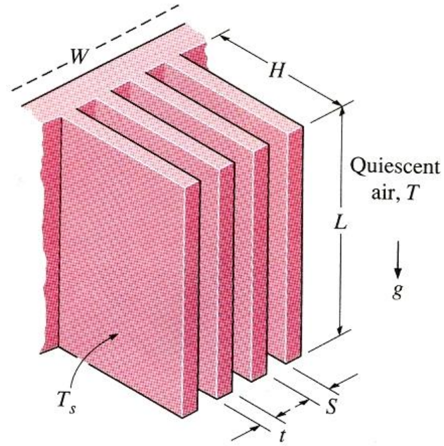


Figure 72: Various dimensions of a finned surface oriented vertically

When the fins are essentially isothermal and the fin thickness t is small relative to the fin spacing S, the optimum fin spacing for a vertical heat sink is determined by Bar-Cohen and Rohsenow to be

$$S_{\text{opt}} = 2.714 \frac{L}{\text{Ra}^{1/4}} \tag{5.31}$$

Where, the fin length L in the vertical direction is taken to be the characteristic length in the evaluation of the Rayleigh number. The heat transfer coefficient for the optimum spacing case was determined to be

$$\alpha = 1.31 \frac{\lambda}{S_{\text{opt}}} \tag{5.32}$$

Then, the rate of heat transfer by natural convection from the fins can be determined from

$$\dot{Q} = \alpha(2nLH)(T_s - T_\infty) \tag{5.33}$$

Where, $n = W/(S+t) \approx W/S$ is the number of fins on the heat sink and T_s is the surface temperature of the fins.

Example 5.6 Heat Loss from the Ducts of a Heating system

A 12-cm-wide and 18-cm-high vertical hot surface in 25 ^{o}C air is to be cooled by a heat sink with equally spaced fins of rectangular profile (Figure 73). The fins are 0.1 cm thick and 18 cm long in the vertical direction and have a height of 2.4 cm from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 80 ^{o}C.

Solution: A heat sink with equally spaced rectangular fins is to be used to cool a hot surface. The optimum fin spacing and the rate of heat transfer from the heat sink are to be determined.

Assumptions: (1) Steady operating conditions exist. (2) Air is an ideal gas. (3) The atmospheric pressure at that location is 1 atm. (4) The thickness *t* of the fins is very small relative to the fin spacing *S* so that Equations 5.31 and 5.32 for optimum fin spacing are applicable.

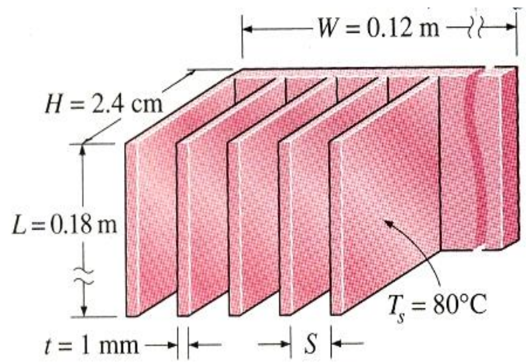


Figure 73: Schematic for worked example 5.6

Properties: The properties of air at the film temperature of $T_f = \frac{1}{2}(T_s + T_\infty) = \frac{1}{2}(80 + 25) =$ 52.5^{o}C or 325.5 K and 1 atm pressure are:

λ = 0.0279 W/m^{o}C; Pr = 0.709; ν = 1.82 x 10^{-5}m^{2}/s; $\beta = 1/T_f$ = 1/325.5 K = 0.003072 K^{-1}

Analysis: The characteristic length in this case is the length of the fins in the vertical direction, which is given to be L = 0.18 m. Then, the Rayleigh number becomes

$$\mathrm{Ra} = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2}\mathrm{Pr}$$
$$= \frac{(9.81\,\mathrm{m/s^2})(0.003072\,\mathrm{K^{-1}})[(80-25)\,\mathrm{K}](0.18\,\mathrm{m})^3}{(1.82\times10^{-5}\,\mathrm{m^2/s})^2}(0.709) = 2.067\times10^7$$

The optimum fin spacing is determined from Equation 5.31 to be

$$S_{\mathrm{opt}} = 2.714\frac{L}{\mathrm{Ra}^{1/4}} = 2.714\frac{0.18\,\mathrm{m}}{(2.067\times10^7)^{1/4}} = 0.0072\,\mathrm{m} = 7.2\,\mathrm{mm}$$

which is about 7 times the thickness of the fins. Therefore, the assumption of negligible fin thickness in this case is acceptable for practical purposes. The number of fins and the heat transfer coefficient for this optimum fin spacing case are:

$$n = \frac{W}{S+t} = \frac{0.12\,\text{m}}{(0.0072+0.001)\,\text{m}} \approx 15\,\text{fins}$$

$$\alpha = 1.31\frac{\lambda}{\text{S}_{\text{opt}}} = 1.31\frac{0.0279\,\text{W/m}^{\text{o}}\text{C}}{0.0072\text{m}} = 5.08\,\text{W/m}^{2\,\text{o}}\text{C}$$

Then, the rate of natural convection heat transfer from the fins becomes:

$$\begin{aligned}\dot{Q} &= \alpha(2nLH)(T_s - T_\infty)\\ &= (5.08\,\text{W/m}^{2\,\text{o}}\text{C})\left[2\times 15\times(0.18\,\text{m})(0.024\,\text{m})\right](80-25)^{\text{o}}\text{C} = 36.2\,\text{W}\end{aligned}$$

Therefore, the heat sink can dissipate heat by natural convection at a rate of 36.2 W from the fins.

2-5.4 Combined Natural and Forced Convection

The presence of a temperature gradient in a fluid in a gravity field always gives rise to natural convection currents, and thus heat transfer by natural convection. Therefore, forced convection is always accompanied by natural convection. Natural convection can however be ignored when high fluid velocities are involved.

For a given fluid, it is observed that the parameter Gr/Re^2 represents the importance of natural convection relative to forced convection. This is not surprising since the convection heat transfer coefficient is a strong function of the Reynolds number, Re, in forced convection and the Grashof number, Gr, in natural convection.

Natural convection is negligible when $Gr/Re^2 < 0.1$, forced convection is negligible when $Gr/Re^2 > 10$, and neither is negligible when $0.1 < Gr/Re^2 < 10$. Therefore, both natural and forced convection must be considered in heat transfer calculations when the Gr and Re^2 are of the same order of magnitude (one is within a factor of 10 times the other). Forced convection is small relative to natural convection only in the rare case of extremely low forced flow velocities.

Natural convection may *help* or *hurt* forced convection heat transfer, depending on the relative directions of *buoyancy-induced* and the *forced convection* motions (Figure 74)

1. In *assisting flow*, the buoyant motion is in the *same* direction as the forced motion. Therefore, natural convection assists forced convection and *enhances* heat transfer. An example is upward forced flow over a hot surface.

2. In *opposing flow*, the buoyant motion is in the *opposite* direction to the forced motion. Therefore, natural convection resists forced convection and *decreases* heat transfer. An example is upward forced flow over a cold surface.

3. In *transverse flow*, the buoyant motion is *perpendicular* to the forced motion. Transverse flow enhances fluid mixing and thus *enhances* heat transfer. An example is horizontal flow over a hot or cold cylinder or sphere.

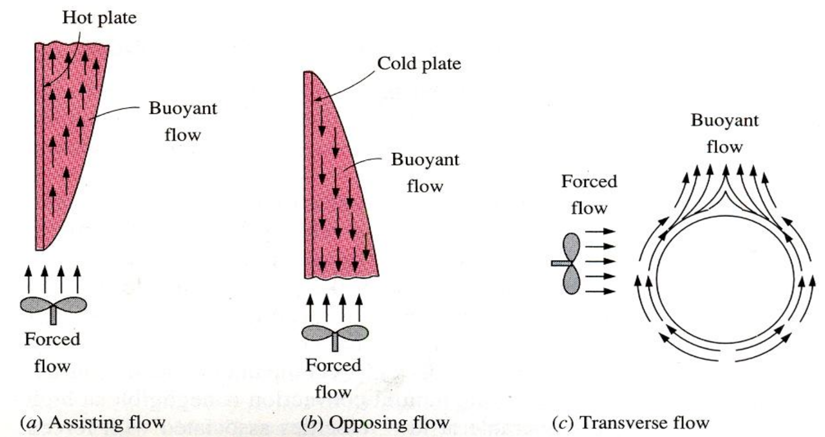


Figure 74: Buoyancy induced and forced convection motions

Combined natural and forced convection heat transfer is not obtained merely by adding the contributions of natural and forced convection in assisting flows and subtracting them in opposing flows. A review of experimental data suggests a correlation of the form:

$$\mathrm{Nu}_{\mathrm{combined}} = (\mathrm{Nu}^n_{\mathrm{forced}} \pm \mathrm{Nu}^n_{\mathrm{natural}})^{1/n} \qquad (5.34)$$

where $\mathrm{Nu}_{\mathrm{forced}}$ and $\mathrm{Nu}_{\mathrm{natural}}$ are determined from the correlations of pure forced and pure natural convection, respectively. The plus sign is for assisting and transverse flows and the minus sign is for opposing flows. The value of the exponent n varies between 3 and 4, depending on the geometry involved. It is observed that n = 3 correlate experimental data for vertical surfaces as well. Larger values of n are better suited for horizontal surfaces.

The choice of whether to utilise *natural* or *forced* convection in the cooling of equipment depends on the maximum allowable operating temperature. From Newton's Law of cooling

$$\dot{Q} = \alpha A(T_s - T_\infty)$$

it can be noted that for a fixed value of power dissipation and surface area, α and T_s are *inversely proportional*. Therefore, the device will operate at a *higher* temperature when the α is low (typical of natural convection) and at a *lower* temperature when α is high (typical of forced convection).

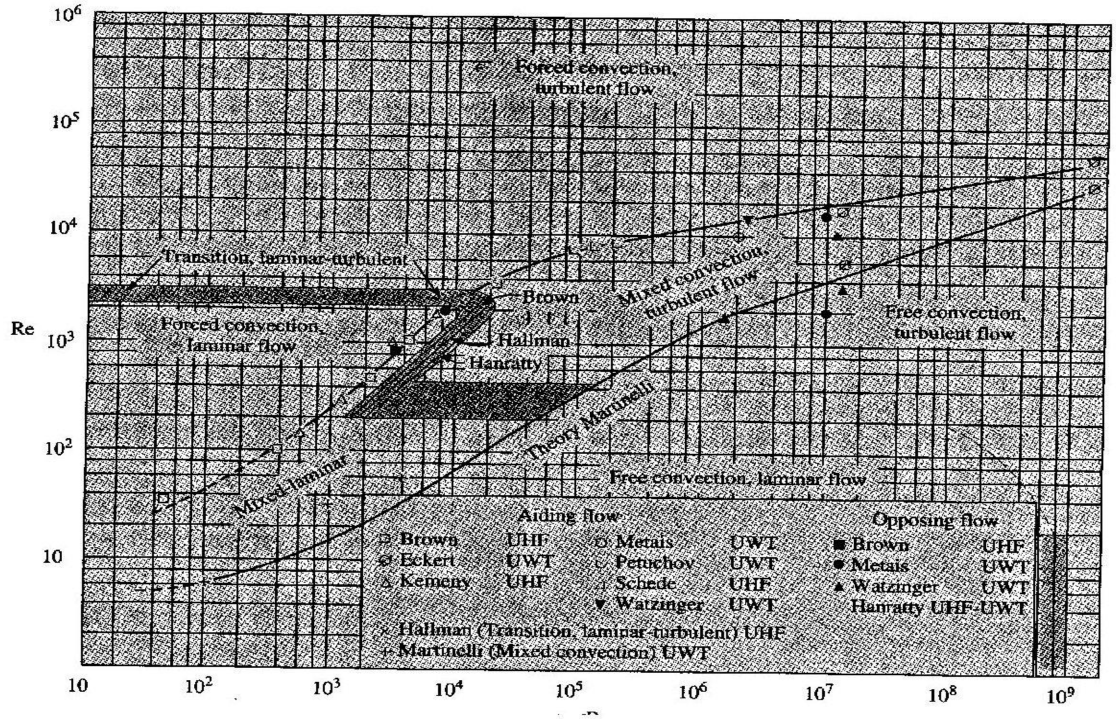
Natural convection is the preferred mode of heat transfer since no blowers or pumps are needed and thus all the problems associated with these, such as noise, vibration, power consumption, and malfunctioning, are avoided. Natural convection is adequate for cooling low-power-output devices, especially when they are attached to extended surfaces such as heat sinks. High-power-output devices however will require the use of pumps or blowers to keep the operating temperature within limits. For very-high-power-output devices, even forced convection may not be sufficient to limit the operating temperature and boiling and condensation may be employed to take advantage of the very high heat transfer coefficients associated with phase change processes.

A summary of combined free and forced convection effects in tubes has been given by Metais and Eckert (1964) and figure 75 represents the regimes for combined convection in vertical tubes while the regimes for combined convection for flow in horizontal tubes is presented in figure 76. The applicable range of figures 75 and 76 is for $10^{-2} < P_r\left(\frac{d}{L}\right) < 1$. An example to illustrate the use of figures 75 and 76 in solving combined free and forced convection problem is illustrated in example 5.7.

Note that Brown and Gauvin (1965) have developed a better correlation for the mixed convection laminar flow region which is preferred over the relation by Oliver provided in figure 76. The Brown and Gauvin (1965) correlation is

$$N_u = 1.75\left(\frac{\mu_b}{\mu_w}\right)^{0.14}\left[G_Z + 0.012\left(G_Z G_r^{\frac{1}{3}}\right)^{\frac{4}{3}}\right]^{\frac{1}{3}} \qquad (5.35)$$

Where μ_b is evaluated at the bulk temperature and μ_w is evaluated at the surface temperature of the tube wall. Note that the Graetz number is defined as $G_Z = R_e P_r\left(\frac{d}{L}\right)$ (5.36)



$$\text{Re.Pr}\frac{d}{L}$$

Figure 75: Regimes of free, forced and mixed convection for flow through vertical tubes according to Metais and Eckert

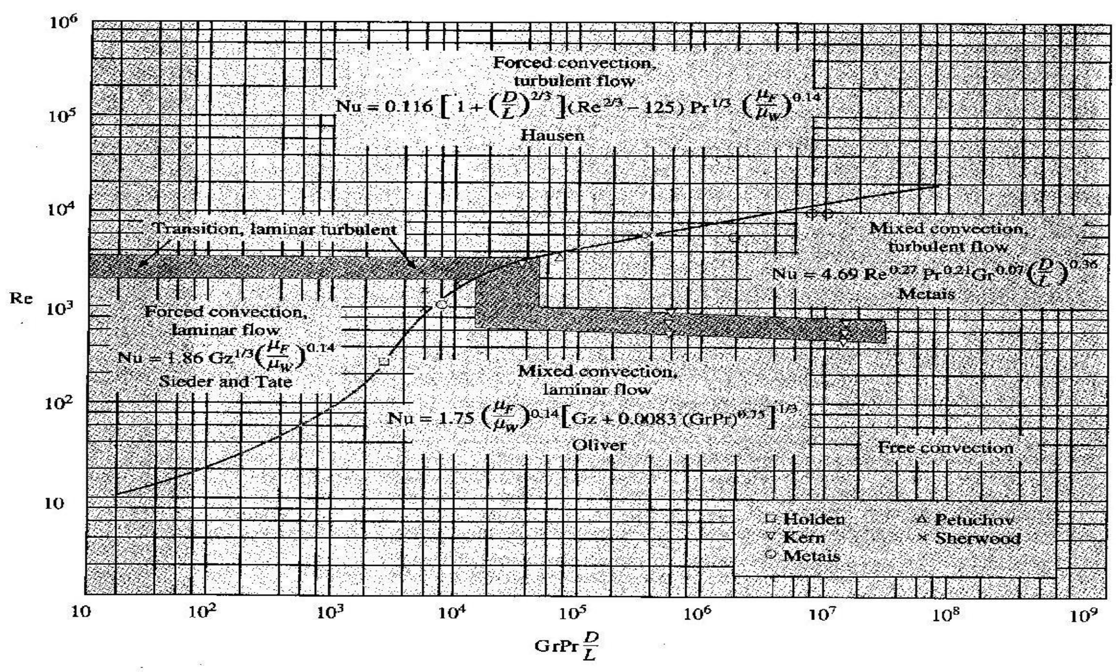


Figure 76: Regimes of free forced and mixed convection for flow through Horizonal tubes according to Metais and Eckert.

Worked example 5.7

Air at 1 atm and 27 °C is forced through a horizontal 25mm diameter tube at an average velocity of 30 cm/s. The tube wall is maintained at a constant temperature of 140 °C. Calculate the heat transfer coefficient for this situation if the tube is 0.4 m long.

Solution

For this calculation we evaluate properties at the film temperature:

$$T_f = \frac{140+27}{2} = 83.5\,°C = 356.5\,K$$

$$\rho_f = \frac{P}{RT} = \frac{1.0132\times10^5}{(287)(356.5)} = 0.99\ \text{kg/m}^3, \beta = \frac{1}{T_f} = 2.805\times10^{-3}\,K^{-1}, \mu_f = 2.102\times10^{-5}\ \text{kg/m.s}$$

$$\mu_w = 2.337\times10^{-5}\ \text{kg/m.s},\ \lambda_f = 0.0305\ \text{W/m.°C},\ P_r = 0.685$$

Let us take the bulk temperature to be 27 °C for evaluating μ_b; then $\mu_b = 1.8462\times10^{-5}$ kg/m.s

The significant parameters are calculated as

$$Re_f = \frac{\rho u d}{\mu} = \frac{(0.99)(0.3)(0.025)}{2.102\times10^{-5}} = 3.53$$

$$Gr = \frac{\rho^2 g\beta(T_w - T_b)d^3}{\mu^2} = \frac{(0.99)^2(9.8)(2.805\times10^{-3})(140-27)(0.025)^3}{(2.102\times10^{-5})^2} = 1.077\times10^5$$

$$GrPr\frac{d}{L} = (1.077\times10^5)(0.695)\frac{0.025}{0.4} = 4677$$

According to figure 76, the mixed convection flow regime is encountered. Thus we must use equation 5.35. The Graetz number is calculated as

$$G_Z = Re.Pr\frac{d}{L} = \frac{(353)(0.695)(0.025)}{0.4} = 15.33$$

The numerical calculation for equation 5.35 becomes

$$Nu = 1.75\left(\frac{1.8462}{2.337}\right)^{0.14}\left\{15.33 + (0.012)\left[(15.33)(1.077\times10^5)^{1/3}\right]^{4/3}\right\}^{\frac{1}{3}} = 7.70$$

The average heat transfer coefficient is then calculated as

$$\bar{h} = \frac{\lambda}{d} Nu = \frac{(0.0305)(7.70)}{0.025} = 9.40 \ W/m^2.°C$$

It is interesting to compare this value with that which would be obtained for strictly laminar forced convection. The Sieder – Tate relation applies, so that

$$Nu = 1.86 (Re.Pr)^{1/3} \left(\frac{\mu_f}{\mu_w}\right)^{0.14} \left(\frac{d}{L}\right)^{1/3} = 1.86 Gz^{1/3} \left(\frac{\mu_f}{\mu_w}\right)^{0.14} = (1.86)(15.33)^{1/3} \left(\frac{2.102}{2.337}\right)^{0.14} = 4.55$$

And $\bar{h} = \frac{\lambda}{d} Nu = \frac{(0.0305)(4.55)}{0.025} = 5.55 \ W/m^2.°C$

Thus there would be an error of – 41 percent if the calculation were made strictly on the basis of laminar forced convection.

Individual/ Group Discussion Problems: Tutorial Problems Questions 11, 12, 26, 27, 28, 29, 30, 31.

Self Assessment 2-5

1. A 30-cm x 30-cm circuit board that contains 121 square chips on one side is to be cooled by combined natural convection and radiation by mounting it on a vertical surface in a room at 25°C. Each chip dissipates 0.05 W of power, and the emissivity of the chip surfaces is 0.7. Assuming the heat transfer from the back side of the circuit board to be negligible, and the temperature of the surrounding surfaces to be the same as the air temperature of the room, determine the surface temperature of the chips. **Suggested Answer: 33.4°C**

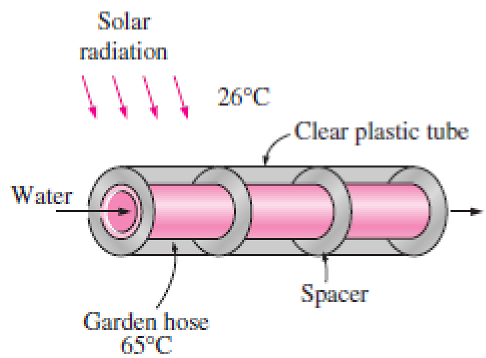
2. Consider a wall-mounted power transistor that dissipates 0.18 W of power in an environment at 35°C. The transistor is 0.45 cm long and has a diameter of 0.4 cm. The emissivity of the outer surface of the transistor is 0.1, and the average temperature of the surrounding surfaces is 25°C. Disregarding any heat transfer from the base surface, determine the surface temperature of the transistor. Use air properties at 100°C. **Suggested Answer: 183°C**

Learning Track Activities *ensure you understand the terms listed below for convection heat transfer*

Key terms/ New Words in Unit

i. Forced Convection
ii. Newton's law of cooling
iii. Reynolds number
iv. Laminar flow
v. Turbulent flow
vi. Prandtl number
vii. Nusselt number
viii. Mean bulk temperature
ix. Mean film temperature
x. Reynolds analogy
xi. Stanton number
xii. Heat transfer coefficient
xiii. Natural convection
xiv. Grashof number
xv. Rayleigh number
xvi. Optimum fin spacing

Review Question: A simple solar collector is built by placing a 5-cmdiameter clear plastic tube around a garden hose whose outer diameter is 1.6 cm. The hose is painted black to maximize solar absorption, and some plastic rings are used to keep the spacing between the hose and the clear plastic cover constant. During a clear day, the temperature of the hose is measured to be 65°C, while the ambient air temperature is 26°C. Determine the rate of heat loss from the water in the hose per meter of its length by natural convection. Also, discuss how the performance of this solar collector can be improved. **Suggested Answer: 8.2 W**



Discussion Question: 1. Consider a fluid whose volume does not change with temperature at constant pressure. What can you say about natural convection heat transfer in this medium?

2. Physically, what does the Grashof number represent? How does the Grashof number differ from the Reynolds number?

Web Activity: www.mhhe.com , www.mitcourseware.com

Reading: Read chapter 9, Heat transfer, a practical approach by Yunus. A. Cengel, 2^{nd} Edition.

Unit 6

HEAT EXCHANGERS

Introduction

In this unit, we shall classify the different types of heat exchangers and study the importance of the overall heat transfer coefficient in heat transfer rate analysis. We shall also analyse heat exchanger problems using the logarithmic mean temperature difference and the effectiveness - NTU methods as well as the performance of heat exchangers under designed and off- design conditions.

Learning Objectives

After reading this unit you should be able to:

1. Classify the different types of heat exchangers
2. Understand the importance of the overall heat transfer coefficient in heat transfer rate analysis.
3. Analyse heat exchanger problems using the logarithmic mean temperature difference (LMTD) and the effectiveness- NTU methods
4. Analyse the performance of heat exchangers under designed and off-design conditions.

Unit content

Session 1-6: Introduction to heat exchangers

1-6.1Types of heat exchangers
1-6.2Overall heat transfer coefficient
1-6.3Analysis of heat exchangers

Session 2-6: Logarithmic Mean Temperature Difference and Effectiveness-NTU methods

2-6.1Logarithmic mean temperature difference (LMTD)
2-6.2Effectiveness – NTU method

SESSION 1-6: INTRODUCTION TO HEAT EXCHANGERS

Heat exchangers are devices that allow the exchange of heat between two fluids that are at different temperatures without allowing them to mix with each other. Heat exchangers are manufactured in a variety of types, the simplest being the *double-pipe* heat exchanger (see Figure 75)

Heat transfer in a heat exchanger usually involves *convection* in each fluid and *conduction* through the wall separating the two fluids. In the analysis of heat exchangers, it is convenient to work with an *overall heat transfer coefficient U* that accounts for the contribution of all these effects on heat transfer. In the analysis of heat exchangers, it is usually convenient to work with the *logarithmic mean temperature difference* LMTD, which is an equivalent mean temperature difference between the two fluids for the entire heat exchanger. However, it must be stated that the LMTD method is best suited for determining the size of a heat exchanger when the inlet and the outlet temperatures are known. The second method used for heat exchanger analysis is the *effectiveness-NTU* method. This latter method is best suited to predict the outlet temperatures of the hot and cold fluid streams in a specified heat exchanger.

Heat exchangers are manufactured in a variety of types; and thus we start this chapter with the classification of heat exchangers. We then discuss the determination of the overall heat transfer coefficient in heat exchangers, and the logarithmic mean temperature difference LMTD for some configurations. We shall then introduce the *correction factor F* to account for the deviation of the mean temperature difference from the LMTD in complex configuration. Next we discuss the effectiveness-NTU method, which enables us to analyse heat exchangers when the outlet temperatures of the fluids are not known.

1-6.1 Types of Heat Exchangers

Different innovative heat exchanger designs have emerged in an attempt to match heat transfer hardware to the heat transfer requirements. The simplest type of heat exchanger consists of two concentric pipes of different diameters, as shown in Figure 75, called the double-pipe heat exchanger. One fluid in a double-pipe heat exchanger flows through the smaller pipe while the other flows through the annular space between the two pipes. Two types of flow arrangement are possible in a double-pipe heat exchanger: In **parallel flow**, both the hot and cold fluids enter the heat exchanger at the same end and move in the *same* direction. In **counter flow**, on the other hand, the hot and cold fluids enter the heat exchanger at opposite ends and flow in *opposite* directions.

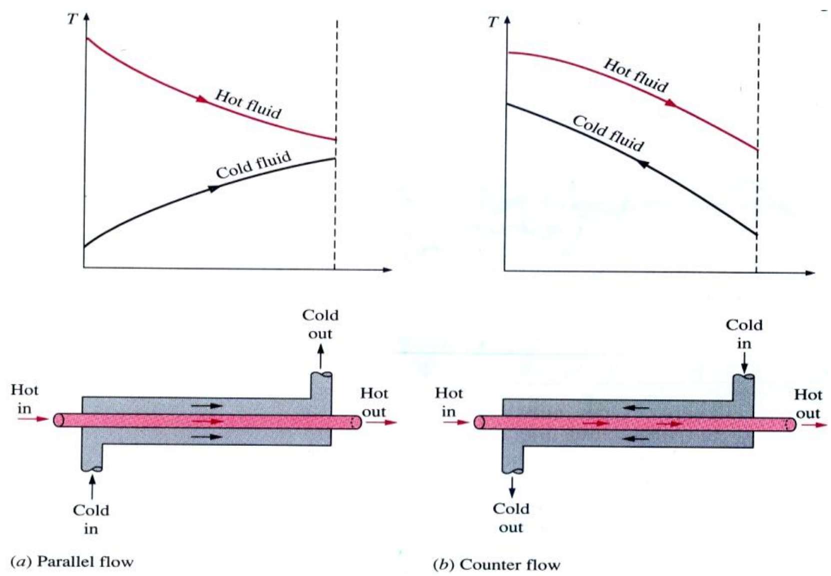


Figure 77: Different flow regimes and associated temperature profiles in a double-pipe heat exchanger

Another type of heat exchanger, which is specifically to realise a large heat transfer surface area per unit volume, is the **compact** heat exchanger. The ratio of the heat transfer surface area of a heat exchanger to its volume is called the *area density* β. A heat exchanger with $\beta > 700m^2/m^3$ is classified as being compact. Examples are the car radiator $\left(\beta = 1000m^2/m^3\right)$, human lung $\left(\beta = 20{,}000m^2/m^3\right)$. These heat exchangers enable us to achieve high heat transfer rates between two fluids in a small volume, and are used in applications with strict limitations on the weight and volume of heat exchangers. The large surface in compact heat exchangers is obtained by attaching fins to the walls separating the two fluids. They are normally used in gas-to-gas and gas-to-fluid heat exchangers to counteract the low heat transfer coefficient associated with gas flow with increased surface area.

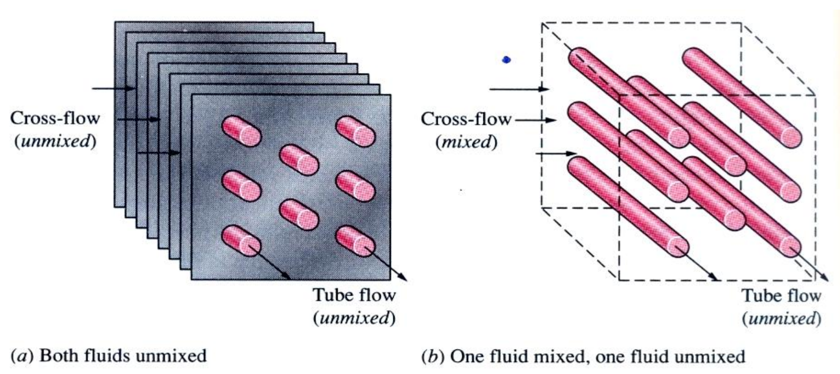


Figure 78: Examples of different flow configurations in cross-flow heat exchangers

In compact heat exchangers, the two fluids usually move perpendicular to each other, and such flow configuration is called **cross-flow** (see Figure 76). The cross flow is further classified as **mixed** or **unmixed flow**. In the unmixed cases the plate fins force the fluid to flow through a particular inter-fin spacing and prevent it from moving in the transverse direction (e.g. car radiator). In the mixed case fluid is free to move in the transverse direction (e.g. flow of fluid through tubes with a second fluid flowing freely around the tubes, see Figure 76).

The most common type of heat exchanger in industrial applications is the **shell-and-tube** heat exchanger, shown in Figure 77. Shell-and-tube heat exchangers contain a large number of tubes packed in a shell with their axes parallel to that of the shell. Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the tubes through the shell. *Baffles* are commonly placed in the shell to force the shell-side fluid to flow across the shell to enhance heat transfer and to ensure uniform spacing between tubes. Such heat exchangers have *headers* (large flow areas at the ends where the tube-side fluid accumulates before entering the tubes and after leaving them) at both ends of the shell. Shell-and-tube heat exchangers are not suitable for use in automobile, aircraft, and marine applications because of their relatively large size and weight.

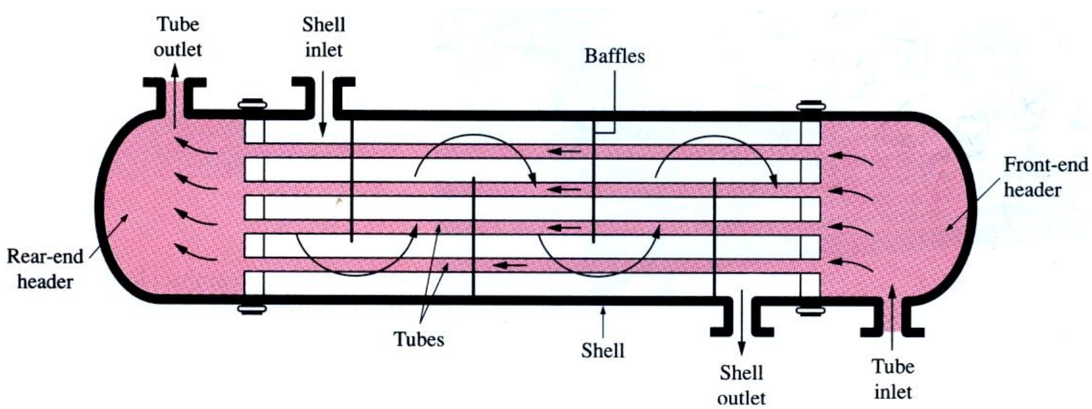


Figure 79: The schematic of a shell-and-tube heat exchanger (one-shell pass and one-tube pass)

Shell-and-tube heat exchangers are further classified according to the number of shell and tube passes involved. Heat exchangers in which all the tubes make one U-turn in the shell, for example, are called *one-shell pass* and *two-tube passes* heat exchangers (see Figure 78(a)). Likewise, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a two-shell pass and four-tube pass heat exchanger (see Figure 78(b)).

(*b*) Two-shell passes and four-tube passes

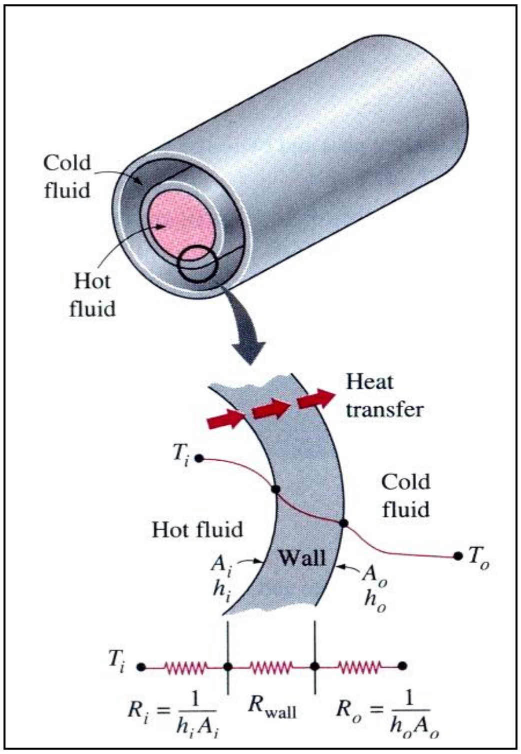
(*a*) One-shell pass and two-tube passes

Figure 80: Multiple shell and tube passes

Another type of heat exchanger that involves alternate passage of the hot and cold fluid streams through the same flow area is the regenerative heat exchanger. The *static-type* regenerative heat exchanger is basically a porous mass that has a heat storage capacity, such as a ceramic wire mesh. Hot and cold f1uids flow through this porous mass alternatively. Heat is transferred from the hot fluid to the matrix of the regenerator during the flow of the hot fluid, and from the matrix to the cold fluid during the flow of the cold fluid. Thus, the matrix serves as a temporary heat storage medium. The *dynamic-type* regenerator involves a rotating drum and continuous flow of the hot fluid and cold f1uid through different portions of the drum so that any portion of the drum passes periodically through the hot stream, storing heat, and then through the cold stream, *rejecting* this *stored* heat. Again the drum serves as the medium to transport the heat from the hot to the cold fluid stream.

Heat exchangers are often given specific names to reflect the specific application for which they are used. For example, a *condenser* is a heat exchanger in which one of the fluids gives up heat and condenses as it flows through the heat exchanger. A *boiler* is another heat exchanger in which one of the fluids absorbs heat and vaporises. A space radiator is a heat exchanger that transfers heat from the hot fluid to the surrounding space by radiation.

1-6.2 Overall Heat Transfer Coefficient



A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by convection, through the wall by conduction, and from the wall to the cold fluid again by convection. Any radiation effects may usually be included in the convection transfer coefficients. The thermal resistance network associated with this heat transfer process involves two convection and one conduction resistances, as shown in Figure 79.

$$R_w = \frac{\text{In}(r_o / r_i)}{2\pi\lambda L} \tag{6.1}$$

where λ is the thermal conductivity of the wall material and L is the length of the tube. The total resistance becomes:

$$R = \frac{1}{h_i A_i} + \frac{\text{In}(r_o / r_i)}{2\pi\lambda L} + \frac{1}{h_0 A_o} \tag{6.2}$$

Figure 81: Thermal resistance network of a double shell heat exchanger

A_i is the area of the inner surface of the wall that separates the two fluids, and A_o is the area of the outer surface of the wall. In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold fluid into a single resistance R, and to express the rate of the heat transfer between the two fluids as:

$$\dot{Q} = \frac{\Delta T}{R} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T \tag{6.3}$$

where U is the overall heat transfer coefficient, whose units is W/m^2 K. Cancelling Δt, Equation 6.3 reduces to:

$$\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + \frac{\text{In}(r_o / r_i)}{2\pi\lambda L} + \frac{1}{h_o A_o} \tag{6.4}$$

Note that $U_i A_i = U_o A_o$ but $U_i \neq U_o$ unless $A_i = A_o$. Therefore, the overall heat transfer coefficient U is meaningless unless the area on which it is based is specified. This is especially the case when one side of the tube wall is finned and the other side is not, since the surface area of the finned side is several times that of the unfinned side.

When the wall thickness of the tube is small and the thermal conductivity of the tube material is high the thermal resistance of the tube is negligible ($R_{wall} \approx 0$) and $U_i \approx U_o$ and $\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$.

When the tube is finned on one side to enhance heat transfer, the total heat transfer surface area on the finned side becomes,

$$A = A_{total} = A_{fin} + A_{unfinned} \text{ (For isothermal fins)}$$

For non isothermal fins $A = \eta_{fin} A_{fin} + A_{unfinned}$

where η_{fin} is the fin efficiency.

1-6.2.1Fouling factor

Table 13: Representative fouling factors for various fluids in Heat exchanger devices

Representative fouling factors (thermal resistance due to fouling for a unit surface area)

Fluid	R_f, $m^2 \cdot °C/W$
Distilled water, sea water, river water, boiler feedwater:	
Below 50°C	0.0001
Above 50°C	0.0002
Fuel oil	0.0009
Steam (oil-free)	0.0001
Refrigerants (liquid)	0.0002
Refrigerants (vapor)	0.0004
Alcohol vapors	0.0001
Air	0.0004

Source: Tubular Exchange Manufacturers Association.

The performance of heat exchangers usually deteriorates with time as a result of accumulation of *deposits* on heat transfer surfaces. The layer of deposits represents *additional resistance* to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease. The net effect of these accumulations on heat transfer is represented by a **fouling factor**, R_f, which is a measure of the thermal resistance introduced by the fouling.

One type of fouling is the precipitation of solid deposits in a fluid on the heat transfer surfaces. Hard water is the cause of this. The scales of such deposit come off by scratching, and the surfaces can be cleaned of such deposits by chemical treatment. Another form of fouling, which is common in the chemical industry, is *corrosion* and other *chemical fouling*.

The fouling factor depends on the operating temperature and the velocity of the fluids, as well as the length of service. Fouling *increases* with *increasing temperature* and *decreasing velocity*. To account for the effects of fouling, the fouling resistance concept is introduced. In Equation 6.2, introducing the fouling factors at the inner at outer surfaces we have:

$$R = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\text{In}(r_o / r_i)}{2\pi\lambda L} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o} \qquad (6.5)$$

1-6.3 Analysis of Heat Exchangers

In the following sections, we shall discuss the two methods used in the analysis of heat exchangers. Some general considerations are presented first.

Heat exchangers operate for long periods of time with no change in their operating conditions. Therefore, they are modelled as *steady-flow* devices, As such, the mass flow rate of each fluid remains constant, and the fluid properties such as temperature and velocity at any inlet or outlet remain the same. Also, the fluid streams experience little or no change in their velocities and elevations, and thus the kinetic and potential energy changes are negligible. The specific heat of the fluid changes with temperature but, in a general temperature range, it can be treated as a constant at some value with very little loss of accuracy. Axial heat conduction along the tube is usually insignificant and can be neglected.

Under these assumptions, the first law of thermodynamics requires that the rate of heat transfer from the hot fluid be equal to the rate of heat transfer to the cold one, That is,

$$\dot{Q} = \dot{m}_c C_{pc}\left(T_{c,out} - T_{c,in}\right) = \dot{m}_h C_{ph}\left(T_{h,in} - T_{h,out}\right) \tag{6.6}$$

where the subscripts *h* and *c* refer to hot and cold fluids respectively. In heat exchanger analysis, it is convenient to combine the product of the mass flow rate and the specific heat of a fluid into a single quantity. This quantity is called the heat capacity rate and is defined as $C = \dot{m}C_p$.

The heat capacity rate of a fluid stream represents the rate of heat transfer needed to change the temperature of the fluid stream by 1^{o} C as it flows through the heat exchanger. The fluid with a large heat capacity rate will experience a small temperature change, and the fluid with a small heat capacity rate will experience a large temperature change.

Two special types of heat exchangers commonly used in practice are condensers and boilers. One of the fluids in a condenser or a boiler undergoes a phase-change process, and the rate of heat transfer is expressed as

$$\dot{Q} = \dot{m}h_{fg} \tag{6.7}$$

where $\dot{m}$ is the rate of evaporation or condensation of the fluid and h_{fg} is the enthalpy of evaporation of the fluid at the specified temperature or pressure. The heat capacity rate of a fluid during a phase-change process must approach infinity since the temperature change is practically zero (see Figure 80).

That is, $C = \dot{m}C_p \to \infty$ when $\Delta T \to 0$, so that the heat transfer rate $\dot{Q} = \dot{m}C_p\Delta T$ is a finite quantity. Therefore, in heat exchanger analysis, a condensing or boiling fluid is modelled as a fluid whose heat capacity rate is infinity.

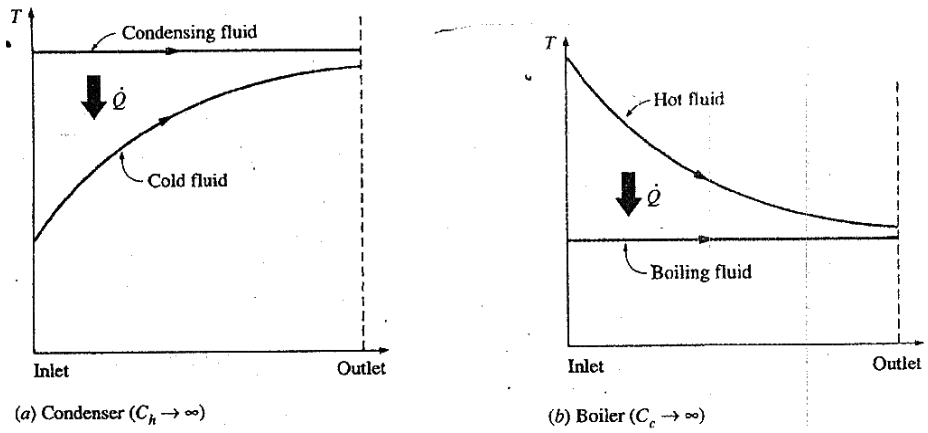


Figure 82: Variation of fluid temperatures in a heat exchanger when one all' the fluid condenses or boils

The rate of heat transfer in a heat exchanger can also be expressed in an analogous manner to Newton's law of cooling as

$$\dot{Q} = UA\Delta T_m \qquad (6.8)$$

where U is the overall heat transfer coefficient, A is the heat transfer area, and ΔT_m is an appropriate average temperature difference between the two fluids assumed constant along the entire length of the exchanger. The appropriate form of the mean temperature difference between the fluids is logarithmic in nature, and its determination is presented in the next section.

Self Assessment 1-6

1. A long thin-walled double-pipe heat exchanger with tube and shell diameters of 1.0 cm and 2.5 cm, respectively, is used to condense refrigerant 134a at 20 °C by water. The refrigerant flows through the tube, with a convection heat transfer coefficient of h_i = 5000 W/m^2. °C. Water flows through the shell at a rate of 0.3 kg/s. Determine the overall heat transfer coefficient of this heat exchanger.

   Suggested Answer: 2100W/m^2. °C

SESSION 2-6: LOGARITHMIC MEAN TEMPERATURE DIFFERENCE AND THE EFFECTIVENESS- NTU METHOD

2-6.1 The Logarithmic Mean Temperature Difference (LMTD)

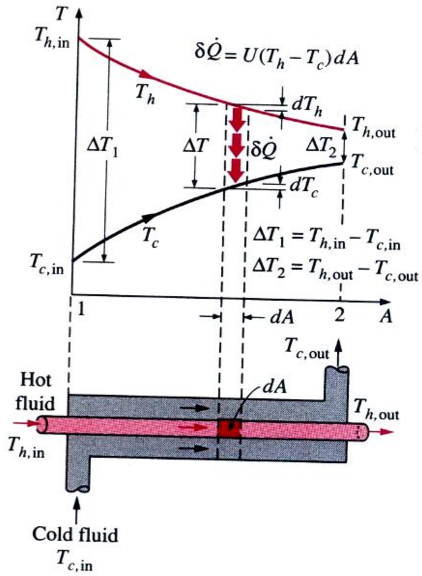


Figure 83: Parallel flow heat exchanger

2-6.1.1 Parallel-flow heat exchanger

In order to develop a relation for the equivalent average temperature difference between the fluids, consider the *parallel-flow double-pipe* heat exchanger, as shown in Figure 81. The temperature difference between the hot and the cold fluids is large at inlet of the heat exchanger but decreases exponentially toward the outlet. The temperature of the hot fluid decrease, and that of the cold fluid increases along the heat exchanger, but the temperature of the cold fluid can never exceed that of the hot fluid no matter how long the heat exchanger is.

An energy balance on each fluid in a differential section of the heat exchanger can be expressed as:

$$\delta\dot{Q} = -\dot{m}_h C_{ph} dT_h \qquad \text{and} \qquad \delta\dot{Q} = \dot{m}_c C_{pc} dT_c \tag{6.9}$$

Solving the equation (6.9) above for dT_h and dT_c and taking their difference we get

$$dT_h - dT_c = d(T_h - T_c) = -\delta\dot{Q}\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \tag{6.10}$$

The rate of heat transfer in the differential section of the heat exchanger can also be expressed as:

$$\delta\dot{Q} = U(T_h - T_c)dA \tag{6.11}$$

Substituting this into Equation 6.10 and rearranging gives:

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -UdA\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \tag{6.12}$$

Integrating from the inlet of the heat exchanger to its outlet, we obtain:

$$\text{In}\frac{T_{h,out}-T_{c,out}}{T_{h,in}-T_{c,in}} = -UA\left(\frac{1}{\dot{m}_h C_{ph}}+\frac{1}{\dot{m}_c C_{pc}}\right) \tag{6.13}$$

Finally, solving Equation 6.6 for $\dot{m}_c C_{pc}$ and $\dot{m}_h C_{ph}$ and substituting into Equation 6.13 gives, after some rearrangement,

$$\dot{Q} = UA\Delta T_{lm} \tag{6.14}$$

where

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{In(\Delta T_1 / \Delta T_2)} \tag{6.15}$$

is the logarithm mean temperature difference, which is the suitable form of the average temperature difference for use in the analysis of heat exchangers. Here, ΔT_1 and ΔT_2 represent the temperature difference between the two fluids at the two ends (inlet and outlet) of the heat exchanger and are defined as shown in Figure 82(a) It makes no difference which end is designated as the inlet or the outlet.

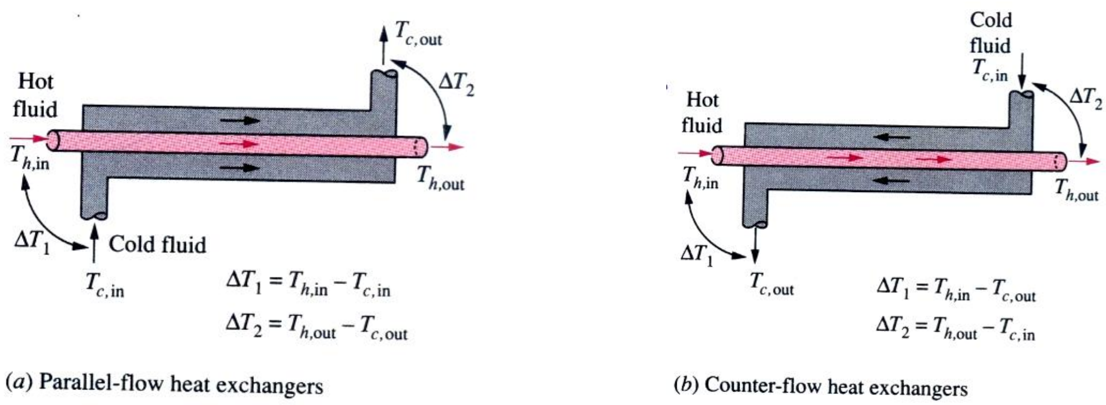


Figure 84: The temperature difference expressions in the parallel-flow and counter-flow heat exchangers

1-6.1.2 Counter-flow heat exchanger

The variation of fluid temperatures in a counter-flow double-pipe heat exchanger is presented in Figure 83. Note that the hot and cold fluids enter the heat exchanger from opposite ends, and outlet temperature of the *cold fluid* in this case may exceed the outlet temperature of the *hot fluid*. In the limiting case, the cold fluid will be heated to the inlet temperature of the hot fluid. However, the outlet temperature of the cold fluid never exceeds the inlet temperature of the hot fluid.

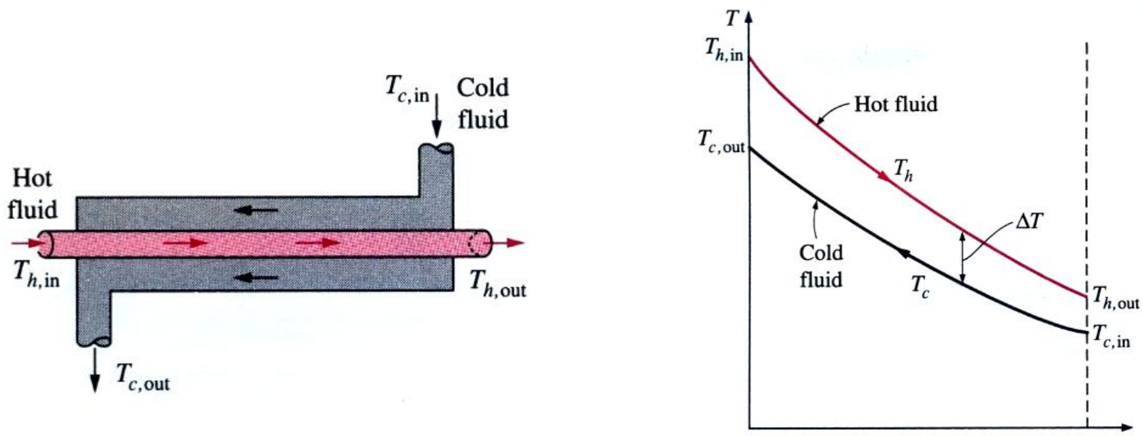


Figure 85: The variation of fluid temperature in a counter-flow double-pipe heat exchanger

$$\delta\dot{Q} = -\dot{m}_h C_{ph} dT_h \qquad \text{and} \qquad \delta\dot{Q} = -\dot{m}_c C_{pc} dT_c \tag{6.16}$$

Solving the equation (5.16) above for dT_h and dT_c and taking their difference we get:

$$dT_h - dT_c = d(T_h - T_c) = -\delta\dot{Q}\left(\frac{1}{\dot{m}_h C_{ph}} - \frac{1}{\dot{m}_c C_{pc}}\right) \tag{6.17}$$

The rate of heat transfer in the differential section of the heat exchanger can also be expressed as:

$$\delta\dot{Q} = U(T_h - T_c)dA \tag{6.18}$$

Substituting this into Equation 6.17 and rearranging gives:

$$\frac{d(T_h - T_c)}{T_h - T_c} = -UdA\left(\frac{1}{\dot{m}_c C_{ph}} - \frac{1}{\dot{m}_c C_{pc}}\right) \tag{6.19}$$

Integrating from the inlet of the heat exchanger to its outlet, we obtain:

$$\text{In}\frac{T_{h,out}-T_{c,out}}{T_{h,in}-T_{c,in}}=-UA\left(\frac{1}{\dot{m}_h C_{ph}}-\frac{1}{\dot{m}_c C_{pc}}\right) \tag{6.20}$$

Finally, solving Equation 6.6 for $\dot{m}_c C_{pc}$ and $\dot{m}_h C_{ph}$ and substituting into Equation 6.20 gives, after some rearrangement,

$$\dot{Q}=UA\Delta T_{lm} \tag{6.21}$$

where

$$\Delta T_{lm}=\frac{\Delta T_1-\Delta T_2}{\text{In}\left(\Delta T_1/\Delta T_2\right)} \tag{6.22}$$

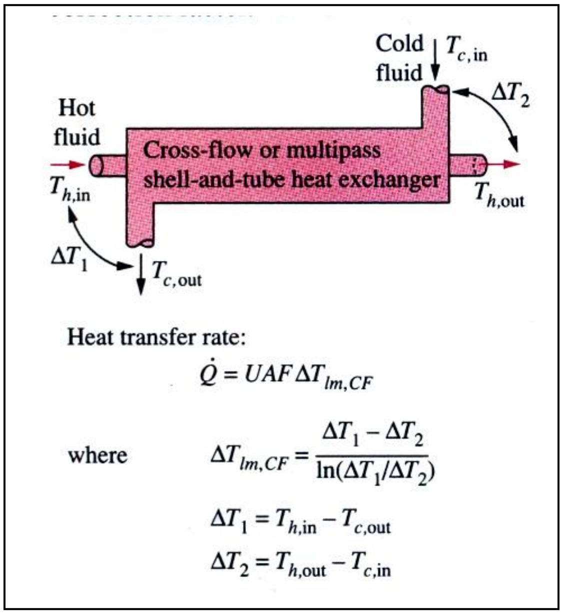
is the logarithmic mean temperature difference, which is the suitable form of the average temperature difference for use in the analysis of heat exchangers. Here, ΔT_1 and ΔT_2 represent the temperature differences between the two fluids at the two ends (inlet and outlet) of the heat exchanger but this time, they are expressed as shown in Figure 82(b).

For specified inlet and outlet temperatures, the log mean temperature difference for a counter-flow heat exchanger is always greater than that for a parallel-flow heat exchanger. Thus, a smaller surface area (and thus a smaller heat exchanger) is needed to achieve a specified heat transfer rate in the counter-flow heat exchanger.

In counter-flow heat exchanger, the temperature difference between the hot and the cold fluids will remain constant along the heat exchanger when the heat capacity rates of the two fluids are equal (that is, ΔT = constant when C_h = C_c). Then, we have ΔT_1 = ΔT_2 and the log mean temperature difference relation above gives $\Delta T_{lm}=\frac{0}{0}$, which is indeterminate. In this case, we have $\Delta T_{lm}=\Delta T_1=\Delta T_2$.

A condenser or boiler can be considered to be either a parallel- or counter-flow heat exchanger since both approaches give the same result.

2-6.1.3Multi-pass and cross-flow heat exchangers: Use of a correction factor



In such cases, it is convenient to relate the equivalent temperature difference to the log mean temperature difference relation for the counter-flow case as:

$$\Delta T_{lm} = F\Delta T_{lm,CF} \qquad (6.23)$$

where F is the correction factor, which depends on the geometry of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams. The correction factor is less than unity for a cross-flow and multi-pass shell-and-tube heat exchanger. Thus, the correction factor is a measure of deviation of the log mean temperature from the corresponding values of the counter-flow case.

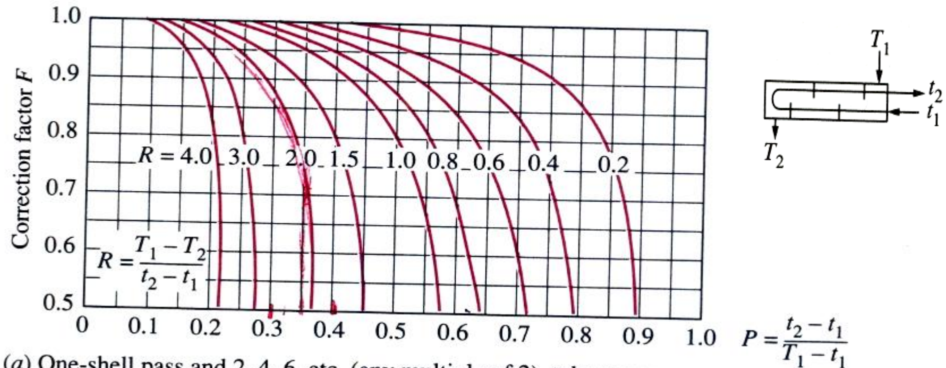
Figure 86: Determination of heat transfer rates for cross-flow and multi-flow heat exchangers

The **correction factor** F for common cross-flow and shell-and-tube heat exchanger configurations is given in Figure 85 versus two temperature ratios defined as:

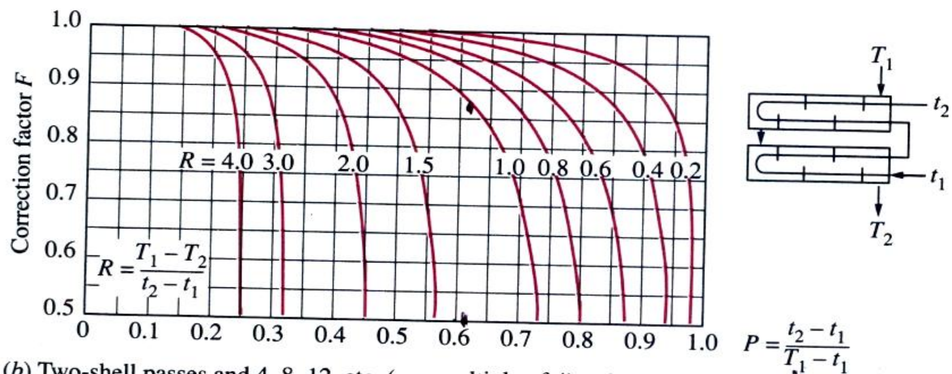
$$P = \frac{t_2 - t_1}{T_1 - t_1} \quad \text{and} \quad R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}C_p)_{\text{tubeside}}}{(\dot{m}C_p)_{\text{shellside}}} \qquad (6.24)$$

where the subscripts 1 and 2 represent the inlet and outlet, respectively. Note that for the shell-and-tube heat exchanger, T and t represent the *shell*- and *tube*-side temperatures, respectively, as shown in the correction factor charts. The determination of the correction factor F requires the availability of the inlet and outlet temperatures for both the cold and hot fluids.

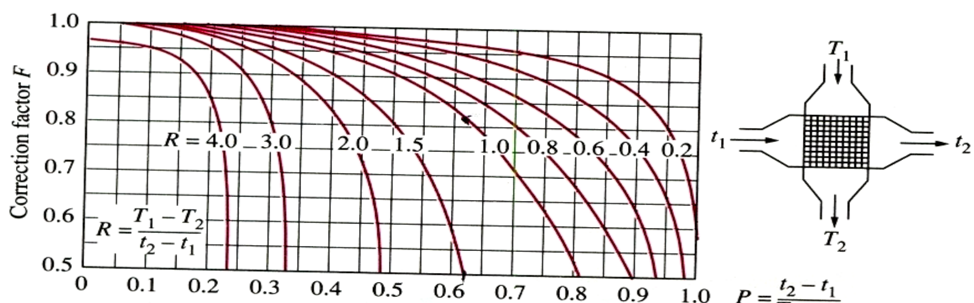
Note that the value of P ranges from 0 to 1. The value of R ranges from 0 (for phase change condition on shell side) to infinity (phase change on the tube side). The correction factor for a condenser or boiler is unity regardless of the configuration of the heat exchanger.



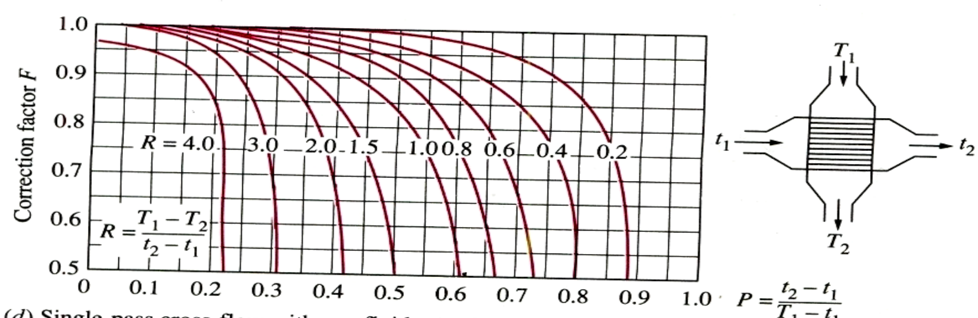
(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes



(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes



(*c*) Single-pass cross-flow with both fluids *unmixed*



(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

Figure 87: Correction factor *F* charts for common shell-and-tube and cross-flow heat exchangers.

Example 6.1 Heating Water in a Counter-Flow Heat Exchanger

A counter-flow double-pipe heat exchanger is to heat water from 20 ^oC to 80 ^oC at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal available at 160^oC at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m^2.^oC, determine the length of the heat exchanger required to achieve the desired heating.

Solution: Water is heated in a counter-flow double-pipe heat exchanger by geothermal water. The required length of the heat exchanger is to be determined.

Assumptions: (1) Steady operating conditions exist. (2) The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. (3) Changes in the kinetic and potential energies of the fluid streams are negligible. (4) There is no fouling. (5) Fluid properties are constant.

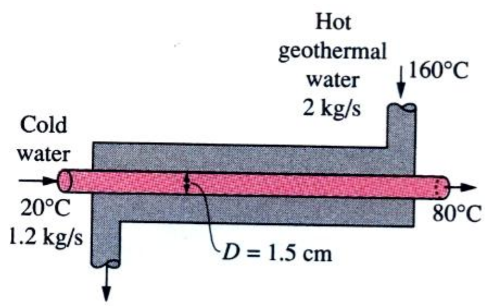


Figure 88: Schematic for example 6.1

Properties: Specific heats of water and geothermal fluid are 4.18 and 4.31 kJ/kg.^oC, respectively.

Analysis: The schematic of the heat exchanger is given in Figure 86. The rate of heat transfer in the heat exchanger can be determined from:

$$\dot{Q} = \left[\dot{m}C_p(T_{0ut} - T_{in})\right]_{\text{water}} = (1.2\,\text{kg/s})(4.18\,\text{kJ/kg}^o\text{C})(80-20)^o\text{C} = 301.0\,\text{kW}$$

Noting that all of this heat is supplied by the geothermal water, the outlet temperature of the geothermal water is determined to be:

$$\dot{Q} = \left[\dot{m}C_p(T_{in} - T_{out})\right]_{\text{geothermal}} \quad \rightarrow \quad T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}C_p}$$

$$= 160^o\text{C} - \frac{301.0\,\text{kW}}{(2\,\text{kg/s})(4.31\,\text{kJ/kg}\cdot^o\text{C})} = 125.1^0\,\text{C}$$

Knowing the inlet and outlet temperatures of both fluids, the logarithmic mean temperature difference for this counter-flow heat exchanger becomes:

$$\Delta T_1 = T_{h,in} - T_{c,out} = (160-80)^o\text{C} = 80^o\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = (125.1-20)^o\text{C} = 105.1^o\text{C}$$

And

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\text{In}\,(\Delta T_1 / \Delta T_2)} = \frac{(80 - 105.1)^{\text{o}}\,\text{C}}{\text{In}\,(80/105.1)} = 92^{\text{o}}\,\text{C}$$

Then, the surface area of the heat exchanger is determined to be:

$$\dot{Q} = UA\Delta T_{\text{lm}} \quad \rightarrow \quad A = \frac{\dot{Q}}{U\Delta T_{\text{lm}}} = \frac{301{,}000\,\text{W}}{(640\,\text{W/m}^2\cdot{}^{\text{o}}\,\text{C})(92^{\text{o}}\,\text{C})} = 5.11\,\text{m}^2$$

To provide this much heat transfer surface area, the length of the tube must be:

$$A = \pi DL \quad \rightarrow \quad L = \frac{A}{\pi D} = \frac{5.11\text{m}^2}{\pi(0.015\text{m})} = 108.4\,\text{m}$$

Discussion: The inner tube of this counter-flow heat exchanger (and thus the heat exchanger itself) needs to be over 100 m long to achieve the desired heat transfer, which is impractical. In cases like this, there is the need to use a plate heat exchanger or a multi pass shell-and-tube heat exchanger with multiple passes of tube bundles.

Example 6.2 Heating of Glycerin in a Multipass Heat Exchanger

A 2-shell passes and 4-tube passes heat exchanger is used to heat glycerin from 20 $^{\text{o}}$C to 50$^{\text{o}}$C by hot water, which enters the thin-walled 2-cm-diameter tubes at 80 $^{\text{o}}$C and leaves at 40 $^{\text{o}}$C (Figure 87). The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 W/m^2.$^{\text{o}}$C on the glycerin (shell) side and 160 W/m^2.$^{\text{o}}$C on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (a) before any fouling occurs and (b) after fouling with a fouling factor or 0.0006 m^2.$^{\text{o}}$C/W occurs on the outer surfaces of the tubes.

Solution: Glycerin is heated in a 2-shell passes and 4-tube passes heat exchanger by hot water. The rate of heat transfer for the cases on no fouling and fouling are to be determined.

Assumptions: (1) Steady operating conditions exist. (2) The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. (3) Changes in the kinetic and potential energies of the fluid streams are negligible. (4) Heat transfer coefficients and fouling factors are constant and uniform. (5) The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

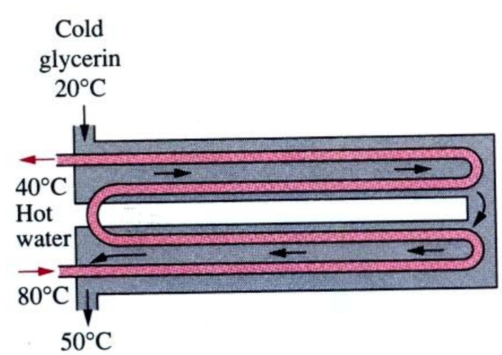


Figure 89: Schematic for example 6.2

Analysis: The tubes are said to be thin-walled, and thus it is reasonable to assume the inner surface area of the tubes to be equal to the outer surface area. Then the heat transfer surface area of this heat exchanger becomes:

$$A = \pi DL = \pi(0.02\,\text{m})(60\,\text{m}) = 3.77\,\text{m}^2$$

The rate of heat transfer in this heat exchanger can be determined from:

$$\dot{Q} = UAF\Delta T_{\text{lm},CF}$$

Where F is the correction factor and $\Delta T_{\text{lm},CF}$ is the log mean temperature difference for the counter-flow arrangement. These two quantities are determined from:

$$\Delta T_1 = T_{h,in} - T_{c,out} = (80-50)^\circ\text{C} = 30^\circ\text{C}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = (40-20)^\circ\text{C} = 20^\circ\text{C}$$

$$\Delta T_{\text{lm},CF} = \frac{\Delta T_1 - \Delta T_2}{\text{In}(\Delta T_1/\Delta T_2)} = \frac{(30-20)^\circ\text{C}}{\text{In}(30/20)} = 24.7^\circ\text{C}$$

and

$$\left.\begin{aligned} \text{P} &= \frac{\text{t}_2 - \text{t}_1}{\text{T}_1 - \text{t}_1} = \frac{40-80}{20-80} = 0.67 \\ \text{R} &= \frac{\text{T}_1 - \text{T}_2}{\text{t}_2 - \text{t}_1} = \frac{20-50}{40-80} = 0.75 \end{aligned}\right\}\text{F} = 0.90 \qquad \text{(Figure 87b)}$$

(a) In the case of no fouling, the overall heat transfer coefficient U is determined from:

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{160\,\text{W/m}^2\cdot^\circ\text{C}} + \dfrac{1}{25\,\text{W/m}^2\cdot^\circ\text{C}}} = 21.6\,\text{W/m}^2\cdot^\circ\text{C}$$

Then the rate of heat transfer becomes:

$$\dot{Q} = UAF\Delta T_{\text{lm},CF} = (21.6\,\text{W/m}^2\cdot^\circ\text{C})(3.77\,\text{m}^2)(0.90)(24.7^\circ\text{C}) = 1810\,\text{W}$$

(b) When there is fouling on one of the surfaces, the overall heat transfer coefficient U is determined from:

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o} + R_f} = \frac{1}{\dfrac{1}{160\,\text{W/m}^2\cdot^\circ\text{C}} + \dfrac{1}{25\,\text{W/m}^2\cdot^\circ\text{C}} + 0.0006\,\text{m}^2\cdot^\circ\text{C/W}} = 21.3\,\text{W/m}^2\cdot^\circ\text{C}$$

The rate of heat transfer in this case becomes:

$$\dot{Q} = UAF\Delta T_{\mathrm{lm},CF} = (21.3\,\mathrm{W/m^2\cdot{}^\circ C})(3.77\,\mathrm{m^2})(0.90)(24.7^\circ\mathrm{C}) = 1785\,\mathrm{W}$$

Discussion: Note that the rate of heat transfer decreases as a result of fouling, as expected. The decrease is not dramatic, however, because of the relatively low convection heat transfer coefficients involved.

2-6.2 The Effectiveness- NTU Method

The log mean temperature difference (LMTD) method discussed in the previous section is easy to use in heat exchanger analysis when the inlet and the outlet temperatures of the hot and cold fluids are known or can be determined from an energy balance. Once the LMTD, the mass flow rates, and the overall heat transfer coefficient are available, the heat transfer surface area of the heat exchanger can be determined from Equation 6.21.

Therefore, the LMTD method is very suitable for determining the size of a heat exchanger to realise prescribed outlet temperatures when the mass flow rates and the inlet and outlet temperatures of the hot and cold fluids are specified. With the LMTD method, the task is to select a heat exchanger that will meet the prescribed heat transfer requirements. The procedure to follow in the selection process is as follows:

- Select the type of heat exchanger suitable for the application.
- Determine any unknown inlet and outlet temperature and the heat transfer rates using an energy balance.
- Calculate the log mean temperature difference and the correction factor, if necessary.
- Obtain (select or calculate) the value of the overall heat transfer coefficient U.
- Calculate the heat transfer surface area A.

The task is completed by selecting a heat exchanger that has a heat transfer surface area equal to or larger than A.

A second kind of problem encountered in heat exchanger analysis is the determination of *heat transfer rate* and the *outlet temperatures* of the hot and cold fluids for prescribed fluid mass flow rates and inlet temperatures when the *type* and *size* of the heat exchanger are specified. The LMTD method can be used but it involves tedious iterations. The effectiveness-NTU method is employed in this case. The new method is based on a dimensionless parameter called the **heat transfer effectiveness** ε, defined as:

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}} \tag{6.25}$$

The actual heat transfer rate in a heat exchanger can be determined from an energy balance on the hot or cold fluid streams and can be expressed as,

$$\dot{Q} = C_c\left(T_{c,out} - T_{c,in}\right) = C_h\left(T_{h,in} - T_{h,out}\right) \tag{6.26}$$

where all symbols are as previously defined.

The *maximum temperature difference* in a heat exchanger is the difference between the *inlet* temperatures of the hot and cold fluids. That is, $\Delta T_{\max} = T_{h,in} - T_{c,in}$. It must be noted that the heat transfer in a heat exchanger will reach its maximum value when (1) the cold fluid is heated to the inlet temperature of the hot fluid or (2) the hot fluid is cooled to the inlet temperature of the cold fluid.

These limiting conditions will not be attained unless the heat capacity rates of the two fluids are identical (i.e. $C_c = C_h$). When $C_c \neq C_h$, the fluid with the smaller heat capacity rate will experience the maximum temperature, at which heat transfer will come to a halt. Therefore, the maximum possible heat transfer rate in a heat exchanger is (see Figure 90)

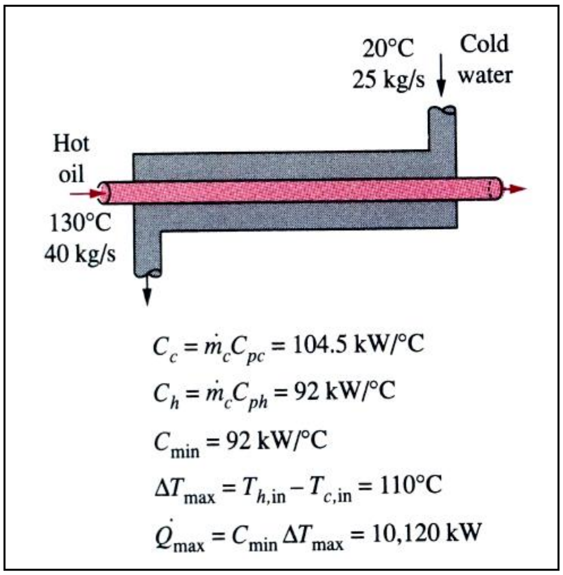


Figure 90: Counter flow heat exchanger

$$\dot{Q}_{\max} = C_{\min}\left(T_{h,in} - T_{c,in}\right) \tag{6.27}$$

where C_{min} is the smaller of $C_h = \dot{m}_h C_{ph}$ and $C_c = \dot{m}_c C_{pc}$

Hence, to determine $\dot{Q}_{\max}$ requires knowledge of the inlet temperature of the hot and cold fluids and their mass flow rates. Thus, once the effectiveness is known the actual heat transfer rate can be evaluated from the relation:

$$\dot{Q} = \varepsilon\dot{Q}_{\max} = \varepsilon C_{\min}\left(T_{h,in} - T_{c,in}\right) \tag{6.28}$$

In this way, the heat transfer can be determined without knowing the outlet temperatures of the fluids. The effectiveness depends on the geometry of the heat exchanger as well as the flow arrangement.

2-6.2.1 Parallel flow in double-pipe heat exchanger

Rearranging equation (6.13), developed for a parallel-flow heat exchanger, we obtain:

$$\text{In}\left(\frac{T_{h,out}-T_{c,out}}{T_{h,in}-T_{c,in}}\right)=-\frac{UA}{C_c}\left(1+\frac{C_c}{C_h}\right) \tag{6.29}$$

Also, solving Equation 6.6 for $T_{h,out}$ gives:

$$T_{h,out}=T_{h,in}-\frac{C_c}{C_h}\left(T_{c,out}-T_{c,in}\right) \tag{6.30}$$

Substituting this into equation (6.29), after adding and subtracting $T_{c,in}$ gives:

$$\text{In}\,\frac{T_{h,in}-T_{c,in}+T_{c,in}-T_{c,out}-\frac{C_c}{C_h}\left(T_{c,out}-T_{c,in}\right)}{T_{h,in}-T_{c,in}}=-\frac{UA}{C_c}\left(1+\frac{C_c}{C_h}\right) \tag{6.31}$$

which simplifies to:

$$\text{In}\left[1-\left(1+\frac{C_c}{C_h}\right)\frac{T_{c,out}-T_{c,in}}{T_{h,in}-T_{c,in}}\right]=-\frac{UA}{C_c}\left(1+\frac{C_c}{C_h}\right) \tag{6.32}$$

But,

$$\varepsilon=\frac{\dot{Q}}{\dot{Q}_{\max}}=\frac{C_c\left(T_{c,out}-T_{c,in}\right)}{C_{\min}\left(T_{h,in}-T_{c,in}\right)}\Rightarrow\frac{\left(T_{c,out}-T_{c,in}\right)}{\left(T_{h,in}-T_{c,in}\right)}=\varepsilon\frac{C_{\min}}{C_c} \tag{5.33}$$

Substituting this result into Equation 6.32, and solving for ε gives the following relation for the effectiveness of a *parallel-flow* heat exchanger:

$$\varepsilon_{parallel-flow}=\frac{1-\exp\left[-\frac{UA}{C_c}\left(1+\frac{C_c}{C_h}\right)\right]}{\left(1+\frac{C_c}{C_h}\right)\frac{C_{\min}}{C_c}} \tag{6.34}$$

Taking either C_c or C_h to be $C_{\min}$ (both approaches the same result), the relation can be expressed more conveniently as:

$$\varepsilon_{parallel-flow} = \frac{1 - \exp\left[-\frac{UA}{C_{\min}}\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]}{\left(1 + \frac{C_{\min}}{C_{\max}}\right)} = \frac{1 - \exp\left[-\mathrm{NTU}(1 + C)\right]}{1 + C} \qquad (6.35)$$

Effectiveness relations involve the dimensionless group $UA/C_{\min}$. This quantity is called **the number of transfer units (NTU)** and is expressed as:

$$\boxed{\mathrm{NTU} = \frac{UA}{C_{\min}} = \frac{UA}{(mC_p)_{\min}}} \qquad (6.36)$$

Note that the NTU is proportional to A. For specified values of *U* and $C_{\min}$, the NTU is *a measure of the heat transfer surface area A*. Thus, the larger the NTU, the larger the heat exchanger. The effectiveness is a function of the number of transfer units (NTU) and the capacity ratio C (i.e. $C_{\min}$ / $C_{\max}$). The effectiveness of some common types of heat exchangers are plotted in Figure 91. Effectiveness relations have also been developed for a large number of heat exchangers, and the results are given in Table 14. Table 15 gives NTU directly when the effectiveness is known.

Table 14: Effectiveness relations for heat exchangers

Effectiveness relations for heat exchangers: NTU $= UA/C_{min}$ and $C = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$

Heat exchanger type	Effectiveness relation
1 *Double pipe:*	
Parallel-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 + C)]}{1 + C}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 - C)]}{1 - C\exp[-\text{NTU}(1 - C)]}$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1 + C + \sqrt{1 + C^2}\dfrac{1 + \exp[-\text{NTU}\sqrt{1 + C^2}]}{1 - C\exp[-\text{NTU}\sqrt{1 + C^2}]}\right\}^{-1}$
3 *Cross-flow (single-pass)*	
Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{C}[\exp(-C\,\text{NTU}^{0.78}) - 1]\right\}$
C_{max} mixed, C_{min} unmixed	$\varepsilon = \dfrac{1}{C}(1 - \exp\{1 - C[1 - \exp(-\text{NTU})]\})$
C_{min} mixed, C_{max} unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{C}[1 - \exp(-C\,\text{NTU})]\right\}$
4 *All heat exchangers with C = 0*	$\varepsilon = 1 - \exp(-\text{NTU})$

Source: Kays and London, Ref. 7.

Table 15: NTU relations for heat exchangers

NTU relations for heat exchangers NTU $= UA/C_{min}$ and $C = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$

Heat exchanger type	NTU relation
1 *Double-pipe:*	
Parallel-flow	$\text{NTU} = -\dfrac{\ln[1 - \varepsilon(1 + C)]}{1 + C}$
Counter-flow	$\text{NTU} = \dfrac{1}{C - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon C - 1}\right)$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$\text{NTU} = -\dfrac{1}{\sqrt{1 + C^2}}\ln\left(\dfrac{2/\varepsilon - 1 - C - \sqrt{1 + C^2}}{2/\varepsilon - 1 - C + \sqrt{1 + C^2}}\right)$
3 *Cross-flow (single-pass)*	
C_{max} mixed, C_{min} unmixed	$\text{NTU} = -\ln\left[1 + \dfrac{\ln(1 - \varepsilon C)}{C}\right]$
C_{min} mixed, C_{max} unmixed	$\text{NTU} = -\dfrac{\ln[C\ln(1 - \varepsilon) + 1]}{C}$
4 *All heat exchangers with C = 0*	$\text{NTU} = -\ln(1 - \varepsilon)$

Source: Kays and London, Ref. 7.

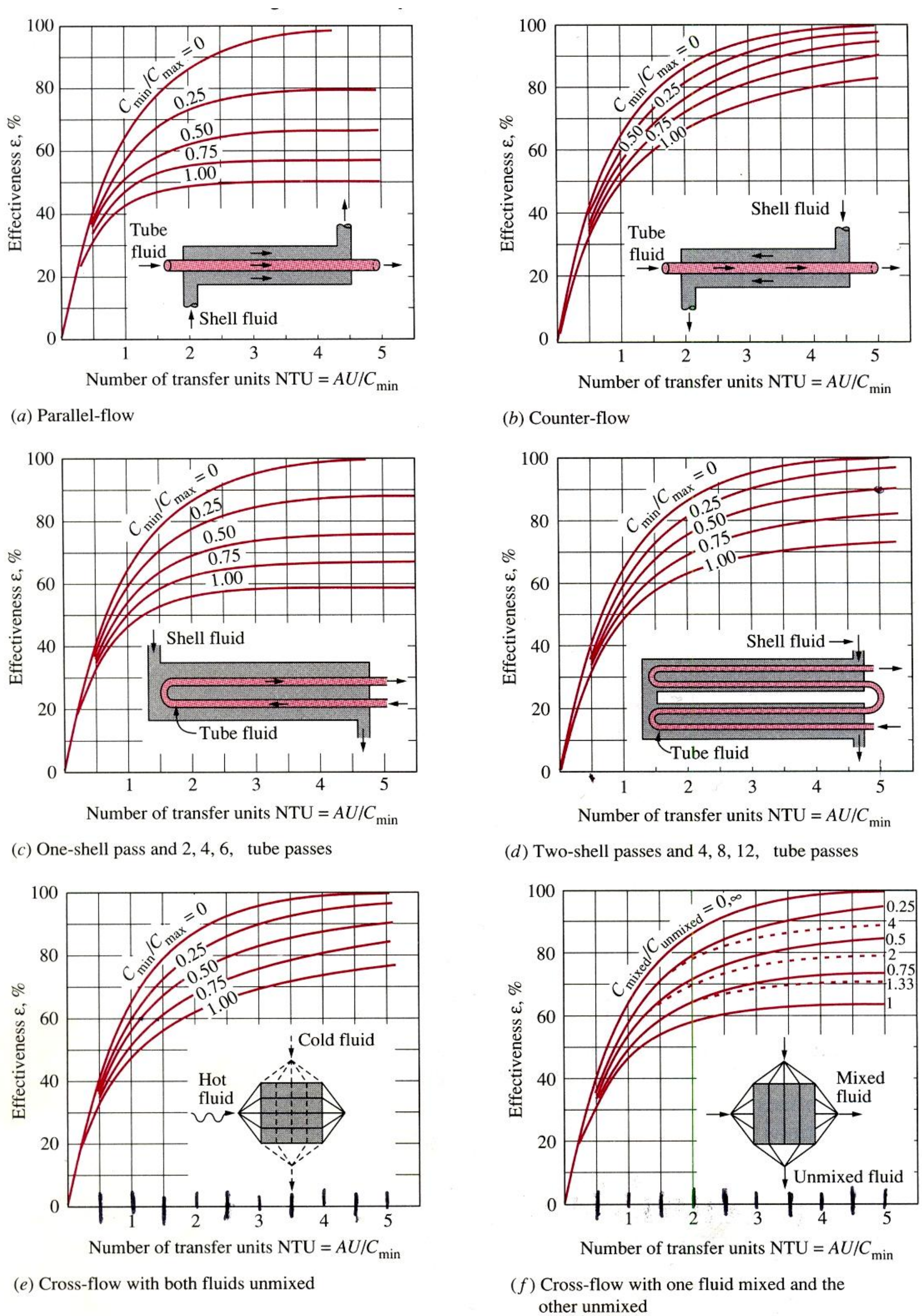


Figure 91: Effectiveness of heat exchangers

Example 6.3 Cooling Hot Oil by Water in a Multi-pass Heat Exchanger

Hot oil is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is 310 W/m^2.^{o}C. Water flows through the tubes at a rate of 0.2 kg/s, and the oil through the shell at a rate of 0.3 kg/s. The water and the oil enter at temperatures of 20 ^{o}C and 150 ^{o}C, respectively. Specific heats of water and oil are to be assumed as 4.18 and 2.13 kJ/kg.^{o}C, respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.

Solution: Hot oil is to be cooled by water in a heat exchanger. The mass flow rates and the inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined.

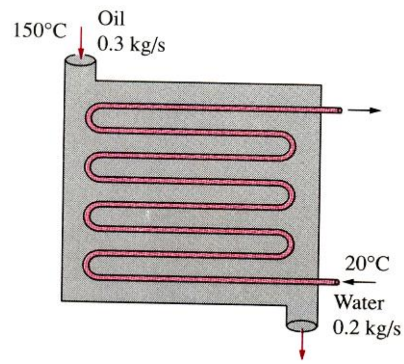


Figure 92: Schematic for example 6.3

Assumptions: (1) Steady operating conditions exist. (2) The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. (3) Changes in the kinetic and potential energies of the fluid streams are negligible. (4) The thickness of the tube is negligible since it is thin-walled. (5) The overall heat transfer coefficient is constant and uniform.

Analysis: The schematic of the heat exchanger is given in Figure 92. The outlet temperatures are not specified, and they cannot be determined from an energy balance. The use of the LMTD method in this case will involve tedious iterations, and thus the ε-NTU method is indicated. The first step in the ε-NTU method is to determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h C_{ph} = (0.3\text{kg/s})(2.13\text{kJ/kg}\cdot^o\text{C}) = 0.639\,\text{kW/}^o\text{C}$$

$$C_c = \dot{m}_c C_{pc} = (0.2\text{kg/s})(4.18\text{kJ/kg}\cdot^o\text{C}) = 0.836\,\text{kW/}^o\text{C}$$

Therefore,

$$C_{\min} = C_h = 0.639\text{kW/}^o\text{C}$$

and

$$C = \frac{C_{\min}}{C_{\max}} = \frac{0.639}{0.836} = 0.764$$

Then the maximum heat transfer rate is determined from Equation 6.27 to be:

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,in} - T_{c,in}) \\ &= (0.639\,\text{kW}/^{\text{o}}\text{C})(150-20)^{\text{o}}\text{C} = 83.1\,\text{kW}\end{aligned}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 83.1 kW. The heat transfer surface area is:

$$A = n(\pi DL) = 8\pi(0.014\text{m})(5\text{m}) = 1.76\text{m}^2$$

Then the NTU of this heat exchanger becomes:

$$\text{NTU} = \frac{UA}{C_{\min}} = \frac{(310\,\text{W/m}^2\cdot{}^{\text{o}}\text{C})(1.76\,\text{m}^2)}{639\,\text{W}/^{0}\text{C}} = 0.854$$

The effectiveness of this heat exchanger corresponding to C = 0.764 and NTU = 0.854 is determined from Figure 91 (c) to be:

$$\varepsilon = 0.49$$

Alternatively, the effectiveness could be determined more accurately from the third relation in Table 14 but with more labour. Then, the actual rate of heat transfer becomes:

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.49)(83.1\,\text{kW}) = 40.7\,\text{kW}$$

Finally, the outlet temperatures of the cold and the hot fluid streams are determined to be:

$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) \quad \rightarrow \quad T_{c,out} = T_{c,in} + \frac{\dot{Q}}{C_c}$$

$$= 20\ ^{\text{o}}\text{C} + \frac{40.7\,\text{kW}}{0.836\,\text{kW}/^{\text{o}}\text{C}} = 68.7\ ^{0}\text{C}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \quad \rightarrow \quad T_{h,out} = T_{h,in} - \frac{\dot{Q}}{C_h}$$

$$= 150\ ^{\text{o}}\text{C} - \frac{40.7\,\text{kW}}{0.639\,\text{kW}/^{\text{o}}\text{C}} = 86.3\ ^{0}\text{C}$$

Therefore, the temperature of the cooling water will rise from 20 $^{\text{o}}$C to 68.7 $^{\text{o}}$C as it cools the hot oil from 150 $^{\text{o}}$C to 86.3 $^{\text{o}}$C in this heat exchanger.

Example 6.4 Use of a two-pass shell-and-tube heat exchanger to condense a chemical

A two-pass shell-and-tube heat exchanger is used to condense a chemical on the shell side at a rate of 50 kg/s at a saturation temperature of 80 ^{o}C. The chemical enters as a dry saturated vapour and is not under-cooled during the process. Water at 10 ^{o}C and a mass flow rate of 100 kg/s is available as coolant; the velocity of the water is to be approximately 1.5 m/s. Using the data below and taking a nominal tube diameter of 25 mm, neglecting tube wall thickness, calculate:

i. The number of tubes required;
ii. The tube length;
iii. The number of transfer units;
iv. The effectiveness of the heat exchanger.

Data: Specific enthalpy of vaporisation of chemical, 417.8 kJ/kg; heat transfer coefficient for shell side, 10 W/m^{2}K; fouling factor for shell side, 0.1 m^{2}K/kW; fouling factor for tube side, 0.2 m^{2}K/kW.

For turbulent flow in a pipe, take

$$\mathrm{Nu} = 0.023(\mathrm{Re})^{0.8}(\mathrm{Pr})^{0.4}$$

with all properties at the mean bulk temperature.

Solution: Water flows in the tubes and the chemical flow in the shell. Water is the cold fluid while the chemical is the hot fluid.

Assumptions: (1) Steady operating conditions exist. (2) The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. (3) Changes in the kinetic and potential energies of the fluid streams are negligible. (4) The thickness of the tube is negligible since it is thin-walled. (5) The overall heat transfer coefficient is constant and uniform.

Heat transferred is calculated as:

$$\dot{Q} = \dot{m}_{chem} h_{fg,chem} = (50\mathrm{kg/s})(417.8\mathrm{kJ/kg}) = 20{,}890\,\mathrm{kW}$$

Assuming the specific heat capacity of water as 4.2 kJ/kg K, the exit temperature of water in the tube can be determined from:

$$\dot{Q} = \dot{m}_w C_{pw}(T_{c,out} - T_{c,in}) \quad \rightarrow \quad T_{c,out} = T_{c,in} + \frac{\dot{Q}}{\dot{m}_w C_{pw}}$$

$$= 10^{\circ}\mathrm{C} + \frac{20{,}890\,\mathrm{kW}}{(100\mathrm{kg/s} \times 4.2\,\mathrm{kJ/kg}\cdot\mathrm{K})}$$

$$= 60^{\circ}\mathrm{C}$$

(Note: C_p of water remains fairly constant at 4.2 kJ/kgK from 0.01°C to about 120°C, i.e. until water starts to boil)

Mean bulk temperature of water in the tube can then be obtained as:

$$T_b = \frac{T_{c,out} + T_{c,in}}{2} = \frac{10^\circ\mathrm{C} + 60^\circ\mathrm{C}}{2} = 35^\circ\mathrm{C}$$

From page 10 of the steam tables, properties of liquid water at 35°C can be read as:

$v_\mathrm{f} = 0.001006\mathrm{m}^3/\mathrm{kg}$, $\mu_\mathrm{f} = 718 \times 10^{-6}$ kg/ms, $\lambda_\mathrm{f} = 625 \times 10^{-6}$ kW/mK, $\mathrm{Pr}_\mathrm{f} = 4.80$

i. Mass flow rate of water = (number of tubes) x (mass flow rate per tube)

$$\dot{m}_w = n_p \times \rho A V = \frac{n_p\, \rho \pi d^2 V}{4} = \frac{n_p\, \pi d^2 V}{4 v_\mathrm{f}}$$

and

$$n_p = \frac{4 v_\mathrm{f}\, \dot{m}_w}{\pi d^2 V} \tag{5.37}$$

where n_p is the number of tubes per pass, v_f is the specific volume of water at the mean bulk temperature, d is the tube diameter and V the velocity of water.

From the relation above,

$$n_p = \frac{4 v_\mathrm{f}\, \dot{m}_w}{\pi d^2 V} = \frac{4(0.001006\mathrm{m}^3/\mathrm{kg})(100\mathrm{kg/s})}{\pi(0.025\mathrm{m})^2(1.5\mathrm{m/s})} = 136.6 \approx 137$$

For the two tube passes in the shell [see Figure 88(a)], total number of tubes = 2 x 137 = 274 tubes.

ii. From Equation 6.37, n is inversely proportional to V. Hence, for 137 tubes, the actual velocity of water can be obtained as:

$$\text{Actual water velocity} = 1.5\mathrm{m/s} \times \frac{136.6}{137} = 1.496\mathrm{m/s}$$

$$\text{Then, } \mathrm{Re} = \frac{\rho V d}{\mu} = \frac{Vd}{v_\mathrm{f}\,\mu} = \frac{(1.496\mathrm{m/s})(0.025\mathrm{m})}{(0.001006\mathrm{m}^3/\mathrm{kg})(718 \times 10^{-6}\,\mathrm{kg/ms})} = 51{,}778.5$$

The value of the Re exceeds the critical value of 2300 for flow through a tube; hence, the flow is turbulent.

The Nusselt number can then be obtained by:

$$\begin{aligned}\text{Nu} &= 0.023(\text{Re})^{0.8}(\text{Pr})^{0.4} \\ &= 0.023(51778.5)^{0.8}(4.8)^{0.4} \\ &= 254.4\end{aligned}$$

The heat transfer coefficient within the tube h_t can be determined as:

$$\text{Nu} = \frac{h_t d}{\lambda} \quad \rightarrow \quad h_t = \frac{\lambda}{d}\text{Nu} = \frac{625 \times 10^{-6}\,\text{kW/mK}}{0.025\text{m}} \times 254.4 = 6.36\ \text{kW/m}^2\text{K}$$

For $A_i \approx A_o$, the overall heat transfer coefficient will be:

$$\begin{aligned}\frac{1}{U} &= \frac{1}{h_s} + \frac{1}{h_t} + R_{f,s} + R_{f,t} \\ &= \frac{1}{10\,\text{kW/m}^2\text{K}} + \frac{1}{6.36\,\text{kW/m}^2\text{K}} + 0.1\,\text{m}^2\text{K/W} + 0.2\,\text{m}^2\text{K/W} \\ &= 0.5572\ \text{m}^2\text{K/W}\end{aligned}$$

$$\therefore U = 1.7947\ \text{W/ m}^2\text{K}$$

The rate of heat transfer in this heat exchanger can also be expressed as:

$$\dot{Q} = UAF\Delta T_{\text{lm},CF}$$

Where F is the correction factor and $\Delta T_{\text{lm},CF}$ is the log mean temperature difference for the counter-flow arrangement. These two quantities are determined from:

$$\begin{aligned}\Delta T_1 &= T_{h,in} - T_{c,out} = (80-60)^{\circ}\text{C} = 20^{\circ}\text{C} \\ \Delta T_2 &= T_{h,out} - T_{c,in} = (80-10)^{\circ}\text{C} = 70^{\circ}\text{C}\end{aligned}$$

$$\Delta T_{\text{lm},CF} = \frac{\Delta T_1 - \Delta T_2}{\text{In}\,(\Delta T_1 / \Delta T_2)} = \frac{(20-70)}{\text{In}\,(20/70)} = 39.91\text{K}$$

The correction factor for a condenser or boiler is unity regardless of the configuration of the heat exchanger; hence, F = 1.

Area required can then be computed as:

$$A = \frac{\dot{Q}}{UF\Delta T_{\mathrm{lm},CF}} = \frac{20890\,\mathrm{kW}}{(1.7947\,\mathrm{W/m^2K})(1)(39.91\,\mathrm{K})} = 291.65\,\mathrm{m}^2$$

Tube length to provide the required area:

$$A = \mathrm{pn_p} \times \pi\mathrm{dL} \qquad \rightarrow \quad \mathrm{L} = \frac{\mathrm{A}}{\mathrm{pn_p}\ \pi\mathrm{d}} \qquad \text{where p is the number of tube passes}$$

$$\text{and } \mathrm{n_p} \text{ is the number of tubes per pass}$$

$$= \frac{291.65\,\mathrm{m}^2}{2 \times 137 \times \pi \times 0.025\,\mathrm{m}}$$

$$= 13.55\,\mathrm{m}$$

iii. Number of Transfer Units:

$$\mathrm{NTU} = \frac{AU}{C_{\min}}$$

The chemical will have an infinite heat capacity since it is undergoing a phase change. Hence, $C_{\min}$ will simply be the product of the mass flow rate of water and the specific heat capacity of water.

$$\mathrm{NTU} = \frac{AU}{C_{\min}} = \frac{291.65\,\mathrm{m}^2 \times 1.7947\ \mathrm{W/m^2K}}{(100\ \mathrm{kg/s} \times 4.2\ \mathrm{kJ/kgK})} = 1.246$$

iv. Effectiveness (ε) of the heat exchanger:

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\dot{Q}}{C_{\min}(T_{h,in} - T_{c,in})}$$

$$= \frac{20890\,\mathrm{kW}}{100\ \mathrm{kg/s} \times 4.2\ \mathrm{kJ/kgK} \times (80 - 10)\,\mathrm{K}} = 0.711$$

$$= 71.1\%$$

Example 6.5 Single-pass multiple tubes counter-flow shell-and-tube heat exchanger

An oil cooler consists of a single-pass, counter-flow shell-and-tube heat exchanger with 300 tubes of internal diameter 7.3 mm and length 8 m. The oil flows in the tube side entering at a mass flow rate of 8 kg/s at a temperature of 70^{o}C. Cooling water in the shell side enters at a mass flow rate of 12 kg/s at a temperature of 15^{o}C. Using the data below, calculate:

i. the number of transfer units;
ii. the effectiveness of the heat exchanger;
iii. the outlet temperature of the oil.

Data: Shell side heat transfer coefficient, 1000 W/m^{2}K; heat transfer coefficient for the tube side given by $\mathrm{Nu} = 0.023(\mathrm{Re})^{0.8}(\mathrm{Pr})^{0.4}$ with properties as follows: specific heat capacity of oil, 3.42 kJ/kgK; density of oil, 900 kg/m^3; dynamic viscosity of oil, 1.5 x 10^{-3} kg/ms; thermal conductivity of oil, 0.15 W/mK.

Solution: Two fluids exchange heat. Water flows in the shell and oil flows in the tubes of a counter-flow shell-and-tube heat exchanger.

Assumptions: (1) Steady operating conditions exist. (2) The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. (3) Changes in the kinetic and potential energies of the fluid streams are negligible. (4) The thickness of the tube is negligible since it is thin-walled. (5) The overall heat transfer coefficient is constant and uniform.

Analysis:

Considering a single tube, $\mathrm{Re} = \dfrac{\rho V d}{\mu}$

But $\dot{m}_{oil} = \rho A V = \rho V \dfrac{\pi d^2}{4}$

Expressing V in terms of $\dot{m}_{oil}$, ρ and d, and substituting in the expression for Re:

$$\mathrm{Re} = \frac{4\dot{m}_{oil}}{\pi d \mu}$$

Where $\dot{m}_{oil}$ is the mass flow rate of oil per tube. (Note: there are 300 tubes)

$$\therefore \mathrm{Re} = \frac{4 \times 8\,\mathrm{kg/s}}{300 \times \pi \times 0.0073\mathrm{m} \times (1.5 \times 10^{-3}\,\mathrm{kg/ms})} = 3100.7$$

which falls in the turbulent flow regime for flow through a tube.

To determine the Prandtl number:

$$\begin{aligned}\text{Pr} &= \frac{C_{p,oil}\mu_{oil}}{\lambda_{oil}} \\ &= \frac{(3420\,\text{J/kgK})(1.5\times10^{-3}\,\text{kg/ms})}{0.15\,\text{W/mK}} \\ &= 34.2\end{aligned}$$

The Nusselt number can be obtained using the relation:

$$\begin{aligned}\text{Nu} &= 0.023(\text{Re})^{0.8}(\text{Pr})^{0.4} \\ &= 0.023(3100)^{0.8}(34.2)^{0.4} \\ &= 58.67\end{aligned}$$

But $\text{Nu} = \dfrac{h_t d}{\lambda} \rightarrow h_t = \dfrac{\lambda}{d}\text{Nu} = \dfrac{0.15\,\text{W/mK}}{0.0073\text{m}}\times 58.67 = 1205.5\,\text{W/m}^2\text{K}$

where h_t represents the heat transfer coefficient on the tube side.

Ignoring the thickness of the tube and fouling effects, the overall heat transfer coefficient is computed by:

$$\begin{aligned}\frac{1}{\text{U}} &= \frac{1}{\text{h}_\text{s}} + \frac{1}{\text{h}_\text{t}} \\ &= \frac{1}{1000\,\text{W/m}^2\text{K}} + \frac{1}{1205.5\,\text{W/m}^2\text{K}} \\ &= 0.0018295\,\text{m}^2\text{K/W} \\ \therefore \text{U} &= 546.6\,\text{W/m}^2\text{K}\end{aligned}$$

There is the need to determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h C_{ph} = (8\text{kg/s})(3.42\text{kJ/kg}\cdot\text{K}) = 27.36\,\text{kW/K}$$
$$C_c = \dot{m}_c C_{pc} = (12\text{kg/s})(4.20\,\text{kJ/kg}\cdot\text{K}) = 50.4\,\text{kW/K}$$

Therefore,

$$C_{\min} = C_h = 27.36\,\text{kW/K}$$

and

$$C = \frac{C_{\min}}{C_{\max}} = \frac{27.36}{50.4} = 0.543$$

Then, the maximum heat transfer rate is determined from Equation 6.27 to be:

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,in} - T_{c,in}) \\ &= (27.36\,\text{kW/K})(70-15)\text{K} = 1504.8\,\text{kW}\end{aligned}$$

The heat transfer surface area is:

$$A = n(\pi DL) = 300\pi(0.0073\text{m})(8\text{m}) = 55.04\text{m}^2$$

Then, the NTU of this heat exchanger becomes:

$$\text{NTU} = \frac{UA}{C_{\min}} = \frac{(546.6\,\text{W/m}^2 \cdot K)(55.04\,\text{m}^2)}{27360\,\text{W/K}} = 1.1$$

The effectiveness of this heat exchanger corresponding to C = 0.543 and NTU = 1.1 is determined from Figure 91 (b) to be:

$$\varepsilon = 0.59 \ (59\ \%)$$

Alternatively, the effectiveness could be determined more accurately from the second relation in Table 14 as 0.588 (58.8 %). Then the actual rate of heat transfer becomes:

$$\dot{Q} = \varepsilon\dot{Q}_{\max} = (0.588)(1504.8\,\text{kW}) = 884.82\,\text{kW}$$

Finally, the outlet temperature of the oil which is the hot fluid stream is determined to be:

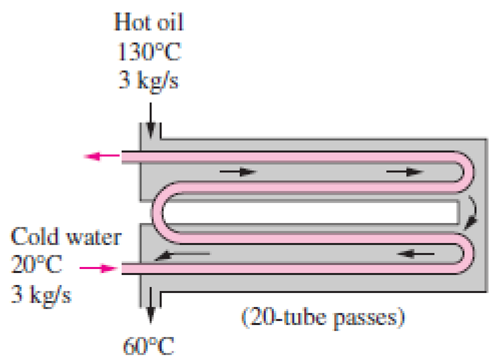
$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \quad \rightarrow \quad T_{h,out} = T_{h,in} - \frac{\dot{Q}}{C_h}$$

$$= 70\ ^{\circ}\text{C} - \frac{884.82\,\text{kW}}{27.36\,\text{kW/K}} = 37.7\ ^{0}\text{C}$$

Individual/Group Discussion Problems: Tutorial Problems Questions 13, 14, 15, 32, 33, 34, 35, 36.

Self Assessment 2-6

1. Cold water (C_p = 4180 J/kg · °C) enters the tubes of a heat exchanger with 2-shell passes and 20–tube passes at 20°C at a rate of 3 kg/s, while hot oil (C_p =2200 J/kg · °C) enters the shell at 130°C at the same mass flow rate and leaves at 60°C. If the overall heat transfer coefficient based on the outer surface of the tube is 300 W/m^2 · °C, determine (*a*) the rate of heat transfer and (*b*) the heat transfer surface area on the outer side of the tube.

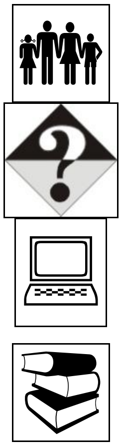


Suggested Answers: (a) 462 kW, (b) 29.2 m^2

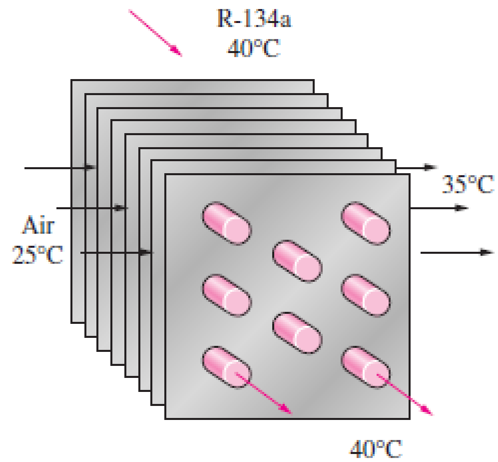
Learning Track Activities ensure you understand all the terms listed below

☑ Key terms/ New Words in Unit

i. parallel flow heat exchanger
ii. counter flow heat exchanger
iii. log mean temperature difference
iv. heat exchanger effectiveness
v. overall heat transfer coefficient
vi. fouling factor
vii. compact heat exchanger
viii. shell and tube heat exchanger
ix. cross flow heat exchanger
x. Number of transfer units
xi. Capacity ratio
xii. Heat capacity rate



Review Question: The condenser of a room air conditioner is designed to reject heat at a rate of 15,000 kJ/h from Refrigerant-134a as the refrigerant is condensed at a temperature of 40°C. Air (*Cp* =1005 J/kg · °C) flows across the finned condenser coils, entering at 25°C and leaving at 35°C. If the overall heat transfer coefficient based on the refrigerant side is 150 W/m2 · °C, determine the heat transfer area on the refrigerant side.



Suggested Answer: 3.05 m^2

Discussion Question: There are two heat exchangers that can meet the heat transfer requirements of a facility. Both have the same pumping power requirements, the same useful life, and the same price tag. But one is heavier and larger in size. Under what conditions would you choose the smaller one?

- **Web Activity:** www.mitcourseware.com , www.mhhe.com

- **Reading:** Read chapter 13, Heat Transfer, A practical approach by Yunus. A. Cengel, 2nd Edition

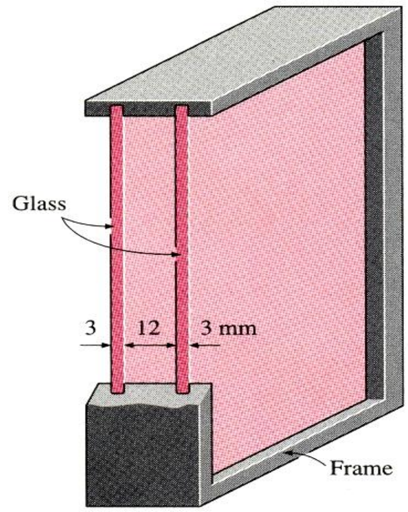
Appendices

Appendix 1: Tutorial set problems

ME 366: HEAT TRANSFER TUTORIAL PROBLEMS

---

1. Consider a 1.2-m-high and 2-m-wide double pane window consisting of two 3-mm-thick layers of glass (λ = 0.78 W/m ^{o}C) separated by a 12-mm-wide stagnant air space (λ = 0.026 W/m ^{o}C). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 24^{o}C while the temperature of the outdoors is –5^{o}C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be α_1 = 10 W/m^2 ^{o}C and α_2 = 25 W/m^2 ^{o}C, and disregard any heat transfer by radiation.

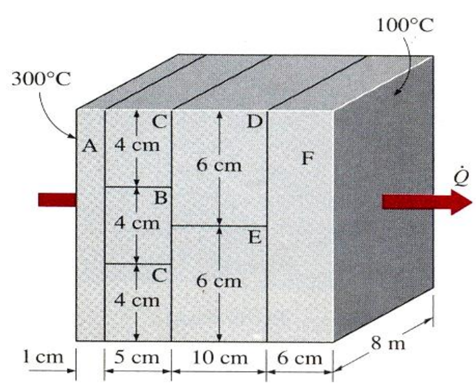


[114 W; 19.2 °C]

2. Two 5-cm-diameter, 15-cm-long aluminum bars (λ = 176 W/m ^{o}C) with ground surfaces are pressed against each other with a pressure of 20 atm. The bars are enclosed in an insulation sleeve and, thus, heat transfer from the lateral surfaces is negligible. If the top and bottom surfaces of the two bar system are maintained at temperatures of 150^{o}C and 20^{o}C, respectively, determine (a) the rate of heat transfer along the cylinders under steady conditions and (b) the temperature drop at the interface. Assume that the thermal contact conductance h_c = 11, 400 W/(m^{2}. ^{0}C)

[(a) 142.4 W, (b) 6.4°C]

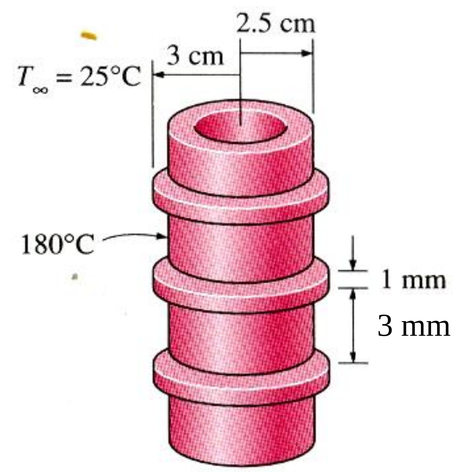
3. Consider a 5-m-high, 8-m-long, and 0.22-m-thick wall whose representative cross-section is as given in the figure. The thermal conductivities of various materials used, in W/m ^{o}C, are $\lambda_A = \lambda_F$ = 2, λ_B = 8, λ_C = 20, λ_D = 15, and λ_E = 35. The left and right surfaces of the wall are maintained at uniform temperatures of 300^{o}C and 100^{o}C, respectively. Assuming heat transfer through the wall to be one-dimensional, determine (a) the rate of heat



transfer through the wall; (b) the temperature at the point where the sections B, D, and E meet;and (c) the temperature drop across the section F. Disregard any contact resistances at the interfaces.
[189 kW; 258 °C ;141 °C]

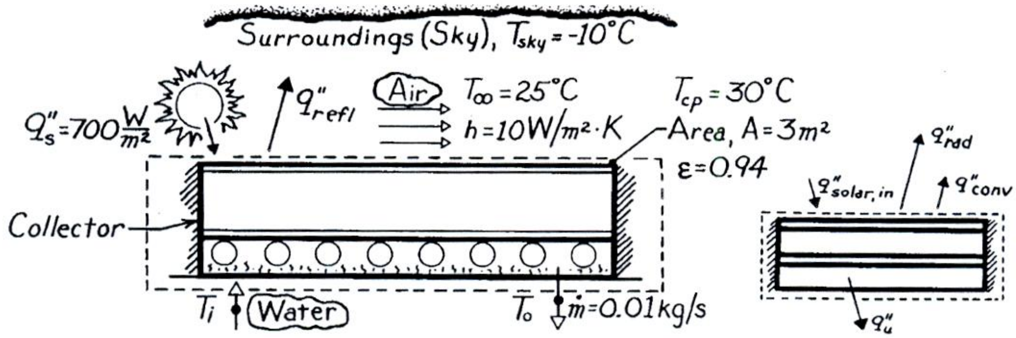
4. Steam at 320^{o}C flows in a stainless steel pipe (λ = 15 W/m ^{o}C) whose inner and outer diameter are 5 cm and 5.5 cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation (λ = 0.038 W/m ^{o}C). Heat is lost to the surroundings at 5^{o}C by natural convection and radiation, with a combined natural convection and radiation heat transfer coefficient of 15 W/m^2 ^{o}C. Taking the heat transfer coefficient inside the pipe to be 80 W/m^2 ^{o}C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.
[94 W; 0.095 °C; 290 °C]

5. Steam in a heating system flows through tubes whose outer diameter is 5 cm and whose walls are maintained at a temperature of 180^{o}C. Circular aluminum alloy 2024-T6 fins (λ = 186 W/m ^{o}C) of outer diameter 6 cm and constant thickness 1 mm are attached to the tube. The space between the fins is 3 mm, and thus there are 250 fins per meter length of the tube. Heat is transferred to the surrounding air at T_∞ = 25^{o}C, with a heat transfer coefficient of 40 W/m^2 ^{o}C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins. Assume $\eta_f = 98\%$ **[2670 W]**



6. A solar flux of 700 W/m^2 is incident on a flat-plate solar collector used to heat water. The area of the collector is 3 m^2, and 90% of the solar radiation passes through the cover glass and is absorbed by the absorber plate. The remaining 10% is reflected away from the collector. Water flows through the tube passages on the back side of the absorber plate and is heated from an inlet temperature T_i to an outlet temperature T_o. The cover glass, operating at a temperature of 30^{o}C, has an emissivity of 0.94 and experiences radiation exchange with the sky at -10^{o}C. The convection coefficient between the cover glass and the ambient air at 25^{o}C is 10 W/m^{2}K.

Schematic:



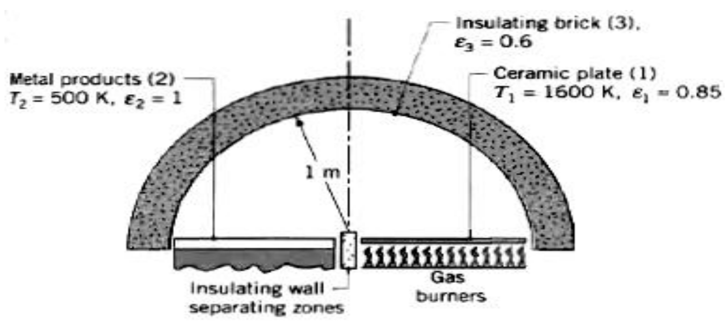
(a) Perform an overall energy balance on the collector to obtain an expression for the rate at which useful heat is collected per unit area of the collector, q_u''. Determine the value of q_u''.

(b) Calculate the temperature rise of the water, $T_o - T_i$, if the flow rate is 0.01 kg/s. Assume the specific heat of the water to be 4179 J/kgK.

(c) The collector efficiency η is defined as the ratio of the useful heat collected to the rate at which solar energy is incident on the collector. What is the value of η?

[385 W/m^2; 27 °C; 55.1 %]

7. A long, hemi cylindrical (1-m radius) shaped furnace used to heat treat sheet metal products is comprised of three zones. The heating zone (1) is constructed from a ceramic plate of emissivity 0.85 and is operated at 1600 K by gas burners. The load zone (2) consists of sheet metal products, assumed to be black surfaces, that are maintained at 500 K. The refractory zone (3) is fabricated from insulating bricks having an emissivity of 0.6. Assume steady-state conditions, diffuse, grey surfaces, and negligible convection.

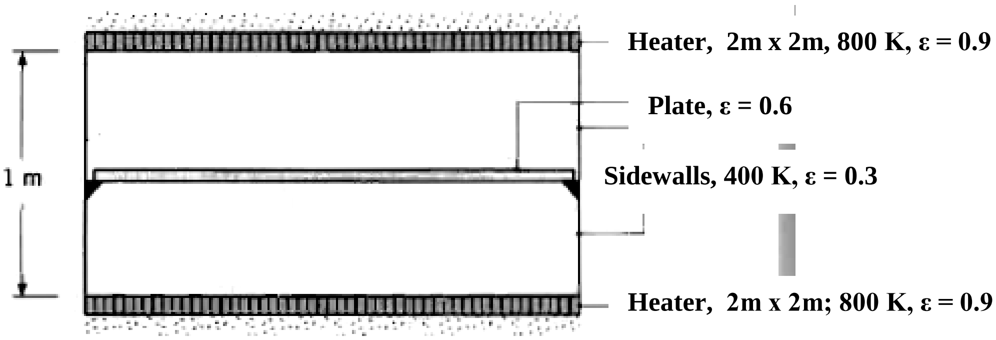


(a) What is the heat rate per unit length of the furnace (normal to the page) that must be supplied by the gas burners for the prescribed conditions?

(b) What is the temperature of the insulating brick surface for the prescribed conditions?

[169.1 kW/m; 1320 K]

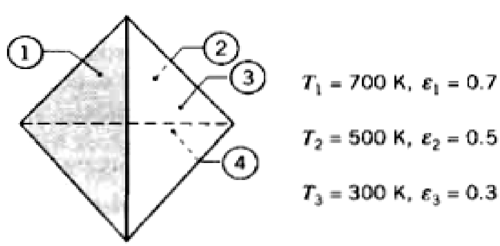
8. An electric furnace consisting of two heater sections, top and bottom, is used to heat treat a coating that is applied to both surfaces of a thin metal plate inserted midway between the heaters.



The heaters and plate are 2 m by 2 m on a side, and each heater is separated from the plate by a distance of 0.5 m. Each heater is well insulated on its back side and has an emissivity of 0.9 at its exposed surface. The plate and the sidewalls have emissivities of 0.6 and 0.3, respectively. Sketch the equivalent radiation network for the system and label all pertinent resistances and potentials. For the prescribed conditions, obtain the required electrical power and the plate temperature.
[43.8 kW; 764 K]

9. Consider a circular furnace that is 0.3 m long and 0.3 m in diameter. The two ends have diffuse, grey surfaces that are maintained at 400 and 500 K with emissivities of 0.4 and 0.5, respectively. The lateral surface is also diffuse and grey with an emissivity of 0.8 and a temperature of 800 K. Determine the net radiation heat transfer from each of the surfaces.
[-538 W; -603 W; 1141 W]

10. Consider the diffuse, gray, four-surface enclosure with all sides equal as shown. The temperatures of three surfaces are specified, while the fourth surface is well insulated and can be treated as a reradiating surface. Determine the temperature of the reradiating surface (4).



[610.9 K]

11. (a) Air at 15°C and 1 bar is to be heated to 285°C while flowing at 34.2 m^3/h through a 25 mm diameter tube which is maintained at 455°C. Assuming that the simple Reynolds analogy is valid, taking $f = 0.0791/(\mathrm{Re})^{1/4}$, and all properties at the mean bulk temperature, calculate the length of the tube required. **[1.88 mm]**

    (b) Air flows through a 20 mm diameter tube 2 m long with a mean velocity of 40 m/s. the tube wall temperature is 150°C and the air temperature increases from 15 to 100°C. Using the simple Reynolds analogy with all properties at the mean bulk temperature, estimate the pressure loss in millimetres of water in the tube due to friction, and the pumping power required. Take the mean air pressure as 1 atm.
    [173 mm of water; 21.3 W]

12. Air at a temperature of 15°C is blown across a flat plate with a surface temperature of 550°C at a mean velocity of 6 m/s. Neglecting radiation, calculate the rate of heat transfer per meter width from both sides of the plate over the first 150 mm of the plate. For heat transfer from a flat plate with a large temperature difference between the plate and the fluid, the local Nusselt number is given by:

$$\mathrm{Nu} = 0.332(\mathrm{Pr})^{1/3}(\mathrm{Re})^{1/2}(T_w / T_s)^{0.117}$$

    where all properties are at the mean film temperature, Re is based on the distance from the leading edge of the plate, and T_w and T_s are the absolute temperatures of the plate and the free stream of the air. **[4.39 kW]**

13. An exhaust pipe of 75 mm outside diameter is cooled by surrounding it by an annular space containing water. The exhaust gas enters the exhaust pipe at 350°C, and the water enters from the mains at 10°C. The heat transfer coefficients for the gases and water may be taken as 0.3 and 1.5 kW/m^2K, and the pipe thickness may be taken to be negligible. The gases are required to be cooled to 100°C and the mean specific heat capacity at constant pressure is 1.13 kJ/kgK. The gas flow rate is 200 kg/h and the water flow rate is 1400 kg/h. Taking the specific heat of water as 4.19 kJ/kgK, calculate:
    i. the required pipe length for parallel flow; **[1.48 m]**
    ii. the required pipe length for counter flow. **[1.44 m]**

14. In a chemical plant a solution of density 1100 kg/m^3 and specific heat capacity 4.6 kJ/kgK is to be heated from 65°C to 100°C; the required flow rate of the solution is 11.8 kg/s. It is desired to use a tubular heat exchanger, the solution flowing at about 1.2 m/s in 25 mm bore iron tubes, and being heated by wet steam at 115°C. The length of the tubes must not exceed 3.5 m.

Taking the inside and outside heat transfer coefficients as 5 and 10 kW/m^2K, and neglecting the thermal resistance of the tube wall, estimate the number of tubes and the number of tube passes required. **[18; 4]**

15. An oil engine develops 300 kW and the specific fuel consumption is 0.21 kg/kWh. The exhaust from the engine is used in a tubular water heater, flowing through 25 mm diameter tubes, entering with a velocity of 12 m/s, at 340^oC and leaving at 90^oC. The water enters the heater at 10^oC and leaves at 90^oC, flowing in counter-flow to the hot gases. The air-fuel ratio of the engine is 20, and the exhaust pressure is 1.01 bar. The overall heat transfer coefficient of the heat exchanger when designed is found to be 56 W/m^2K, but after running for some time a fouling factor of 0.5 m^2K/kW must be assumed. Taking the specific heat capacity and the gas constant for the gases as 1.11 kJ/kg K and 0.29 kJ/kg K, and the specific heat capacity for the water as 4.19 kJ/kg K, calculate:
    - i. The mass flow rate of water; **[1096 kg/h]**
    - ii. The number of tubes required; **[110]**
    - iii. The required tube length. **[1.457 m]**

16. A furnace wall consists of 250mm firebrick, 125 mm insulting brick, and 250 mm building brick. The inside of wall is at a temperature of 600 ^0C and the atmospheric temperature is 20 ^0C. The heat transfer coefficient for the outside surface is 10 W/m^2 K, and the thermal conductivities of the firebrick, insulating brick, and building brick are 1.4, 0.2, and 0.7 W/m K, respectively. Neglecting radiation, calculate the rate of heat loss per unit wall surface area and the temperature of the outside wall surface of the furnace. **[460 W/m^2; 66 °C]**

17. An electric hot-plate is maintained at a temperature of 350 ^0C and is used to keep a solution just boiling at 95 ^0C. The solution is contained in an enamelled cast -iron vessel of wall thickness 25 mm and enamel thickness 0.8 mm. The heat transfer coefficient for the boiling solution is 5.5 kW/m^2 K, and the thermal conductivities of cast-iron and enamel are 50 and 1.05 W/m K, respectively. Calculate the resistance to the heat transfer for unit area, and the rate of heat transfer per unit area.
    [1.444 m^2K/kW; 176.6 kW]

18. In problem 17, recalculate the rate of heat transfer per unit area if the base of the cast-iron vessel is not perfectly flat, and the resistance of the resultant air film is 35 m^2 K/ kW.
    [7 kW/m^2]

19. The wall of a house consists of two, 125 mm thick brick walls with an inner cavity. The inside wall has a 10 mm coating of plaster, and there is a cement rendering of 5 mm on the outside wall. In one room of the house, the external wall is 4 m by 2.5 m, and contains a window of 1.8 m by 1.2 m of 1.5 mm thick glass. The heat transfer coefficients for the inside and outside surfaces of the wall and window are 8.5 and 31 W/m^2 K, respectively. The thermal conductivities of brick, plaster, cement, and glass are 0.43, 0.14, 0.86, and 0.76W/m K, respectively. Assuming that the resistance of the air cavity is 0.16 m^2 K/ W, neglecting radiation, calculate the proportion of the total heat transfer which is due to the heat loss through the window. **[63.8%]**

20. Water at 80 ^{0}C flows through a 50 mm bore steel pipe of 6 mm thickness, and the atmospheric temperature is 15 ^{0}C. The thermal conductivity of the steel is 48 W/m K and the inside and outside heat transfer coefficients are 2800 and 17 W/m^2 K, respectively. Neglecting radiation, calculate the rate of heat loss per unit length of pipe. **[0.213 kW/m]**

21. Calculate the percentage reduction in heat loss for the pipe in problem 20 when a layer of hair felt 12 mm thick, of thermal conductivity 0.03 W/m K, is wrapped round the outside surface. Assume that the heat transfer coefficient for the outside surface remains unchanged. **[85.1 %]**

22. A steam main of 150 mm outside diameter containing wet steam at 28 bar is insulated with an inner layer of diatomaceous earth, 40 mm thick, and an outer layer of 85% magnesia, 25 mm thick. The inside surface of the pipe is at the steam temperature, and the heat transfer coefficient for the outside surface of the lagging is 17 W/m^2 K. The thermal conductivities of diatomaceous earth and 85 % magnesia are 0.09, and 0.06 W/m K, respectively. Neglecting radiation and thermal resistance of the pipe wall, calculate the rat of heat loss per unit length of the pipe and the temperature of the outside surface of the lagging, when the room temperature is 20^{0}C. **[156 W/m; 30.5 °C]**

23. A spherical pressure vessel of 1 m inside diameter is made of 20 mm steel plate. The vessel is lagged with a 25 mm thickness of vermiculite held in position by 10 mm thick asbestos. The heat transfer coefficient for the outside surface is 20 W/m^2 K, and the thermal conductivities of steel, vermiculite, and asbestos are 48, 0.047, and 0.21 W/m K, respectively. Neglecting radiation, calculate the rate of heat loss from the sphere when the inside surface is at 500 ^{0}C, and the room temperature is 20 ^{0}C. **[2.744 kW]**

24. A solid copper conductor of 13 mm diameter carries a current density of 5 A /mm^2. The conductor is electrically insulated with a thickness of rubber insulation such that the wire temperature is kept to the minimum possible. Assuming that the surrounding air is at 30 ^{0}C, calculate,:

(i) The thickness of insulation; **[3.5 mm]**
(ii) The wire temperature at the axis; **[105.6 °C]**
(iii) The temperature of the outside of the insulation; **[82.8 °C]**
(iv) The wire temperature at the axis with the insulation removed and the new steady state reached. **[111.3 °C]**
Give a physical explanation why any larger or smaller thickness of insulation will lead to a higher wire temperature.

Data Heat transfer coefficient for outside surface of rubber or copper (assumed constant), 20 W/m^2 K; thermal conductivities of copper and rubber, 380 and 0.2 W/m K; electrical resistivity of copper 2x 10^{-5} Ω mm

25. A thick fin of rectangular cross-section, 1 m x 1 m, projects from a flat surface at 200 ^{0}C into a fluid at 20 ^{0}C. Using the data below, estimate the temperature distribution in the steady state assuming two-dimensional conduction , and hence calculate the rate of heat loss from the fin surface per unit length.

Data: Thermal conductivity of fin material, 25 W/ (m. K); Heat transfer coefficient for all parts of the fin surface, 10 W/ (m^2. K) **[2.5 kW/m]**

26. In an engine oil cooler, the engine oil enters 10 mm diameter tubes, each of length 1.2 m at 160 ^{0}C and is cooled to 40 ^{0} C. The mean velocity of the engine oil in the tubes is 1.5 m/s. The tube wall is maintained at a constant temperature of 20 ^{0} C by a coolant.

(a) Calculate the mean bulk temperature of the flow

(b) Calculate the Reynolds number based on the mean bulk properties

(c) Calculate the Prandtl number based on the mean bulk properties

(d) Allowing for entry length effect, calculate the mean Nusselt number and hence the mean convection heat transfer coefficient for the flow inside the tube.

<u>Hints:</u>

(a) Determine whether the flow inside the tube is Laminar or Turbulent and use the appropriate mean Nusselt expression.

(b) For turbulent flow in a tube of diameter d, over a length L, for oil being cooled, the mean Nusselt number is given by:

$$Nu = 0.026(d/L)\left(\frac{\mu}{\mu_w}\right)^{0.17} (\mathrm{Re})^{0.8} (\mathrm{Pr})^{0.4}, \qquad \mathrm{Re_d} > 2300$$

(c) For laminar flow in a tube of diameter d, over a length L, for oil being cooled, the mean Nusselt number is given by $\quad \mathrm{Nu} = 3.65, \quad \mathrm{Re_d} < 2300$

Evaluate all properties, in both expressions, at mean bulk temperature and take the viscosity of the engine oil at the tube surface, $\mu_w = 0.8\,\text{kg/ms}$. Use the properties of the engine oil provided in **Table 16 below:**

Table 16: Information for Problem 26

$t/^0C$	$\rho/(kg/m^3)$	$\upsilon/(cSt)$	$\lambda/(W/m\,K)$	$c_p/(kJ/kg\,K)$
40	878	251.0	0.144	1.96
100	839	20.4	0.137	2.22
160	806	5.7	0.131	2.48

1centistoke (cSt) = 10^{-6} m^2/s

27. A wall 0.60 m high by 3.0 m wide is maintained at 79 ^{0}C in an atmosphere at 15 ^{0}C. Neglecting end effects and radiation, calculate the rate of heat loss by natural convection. For natural convection from a vertical surface take, at any distance x :

$$Nu_x = 0.509(Pr)^{1/3}(Pr + 0.952)^{-1/4}(Gr_x)^{1/4},$$

Where all properties are at the mean film temperature, and $\beta = 1/T$, where T is the absolute temperature of the bulk of the air. **[528 W]**

28. A pipe containing dry saturated steam at 177 ^{0}C is 150 mm bore and has a 50 mm thickness of 85 % magnesia covering. The steam velocity is 6.0 m/s and the heat transfer coefficient may be found from $Nu = 0.023(Re)^{0.8}(Pr)^{0.4}$ where all properties are at mean bulk temperature. The atmospheric temperature is 17 ^{0}C and the heat transfer coefficient from a horizontal cylinder is given approximately by $\alpha = 1.32\,(\Delta t/d)^{1/4}$ where α is in W/ m^2 K, Δt is in K and d in m.
The pipe wall is 7 mm thick and the thermal conductivity of the pipe metal is 50 W/m K ; the thermal conductivity of 85 % magnesia insulation is 0.06 W/m K. Neglecting radiation, taking arithmetic mean areas for the pipe wall and lagging, and using a trial-and-error method, calculate:
(i) The temperature of the outside surface of the lagging; **[46.3 °C]**
(ii) The rate of heat loss from the pipe per unit length. **[104 W/m]**

29. In an air cooler, the air is blown across a bank of tubes at the rate of 240 kg/h at a velocity of 24 m/s, the air entering at 97 ^{0}C and leaving at 27 ^{0}C. The cooling water enters the tubes at 10 ^{0}C and leaves at 20 ^{0}C, at a mean velocity of 0.6 m/s. The tubes are 6 mm diameter and the wall thickness may be neglected. The heat transfer coefficient from the air to the tubes may be calculated from $Nu = 0.33\,(Re)^{0.6}(Pr)^{0.33}$ with properties at the mean bulk temperature.

The heat transfer coefficient from the water to the tubes is given by

$St = \dfrac{f/2}{1 + (\mathrm{Pr})^{-1/6}\,(\mathrm{Re})^{-1/8}\,(\mathrm{Pr} - 1)}$ Where $f = (0.0791/\mathrm{Re})^{1//4}$ and properties are at the mean bulk temperature. Assuming that the tubes are arranged in six passes, and that the logarithmic mean temperature difference for counter –flow can be assumed, calculate:

(i) The number of tubes required in each pass; **[7]**

(ii) The necessary tube length. **[0.528 m]**

30. A double pipe heat exchanger has an effectiveness of 0.5 when the flow is counter-current and the thermal capacity of one fluid is twice that of the other fluid. Calculate the effectiveness of the heat exchanger if the direction of flow of one of the fluids is reversed with the same mass flow rates as before. **[0.469]**

31. 500 kg/h of oil at 120^0C is to be cooled in the annulus of a double pipe counter-flow heat exchanger by water which enters the inside pipe at 10^0C. The inner pipe has an inside diameter of 25 mm and a wall thickness of 2 mm, and the inside diameter of the outer pipe is 50 mm; the effective length is 12 m. Using the data below, calculate the exit temperature of the oil.

Data: **Oil** take Nu = 30, based on an equivalent diameter d_e given by
d_e = 4x (flow area)/(heat transfer area per unit length);
Specific heat capacity, 2.31 kJ/kg K; thermal conductivity, 0.135 W/m K;
Fouling factor 0.001 m^2 $K^/$W in the annulus
Water assume the simple Reynolds analogy holds true, taking the velocity as 1m/s and the friction factor, f, as 0.0002; specific heat capacity, 4.18 kJ/kg K; density, 1000 kg/m^3; fouling factor, 0.0002 m^2 K/W.
Neglect thermal resistance of the pipe wall. **[98.8 °C]**

32. Circular cross-section studs of radius 10 mm, fixed length 100 mm, thermal conductivity 24 W/m K are attached to flat surface with their axes perpendicular to the surface on a square pitch of 30 mm. The primary surface is at 300 ^{0}C. A fluid at 50 ^{0}C is forced across the surface such that the mean heat transfer coefficient is 100 W/m^2 K. Calculate the rate of heat loss per unit area of the studded surface. Assume that the heat transfer coefficient is the same for the primary surface and for the rod surfaces. **[77.66 kW/m²]**

33. A flat surface at temperature of 300 ^{0}C has a rectangular section cooling fins perpendicular to the surface projecting into a fluid at 20 ^{0}C. There are 12.5 fins per 100 mm and the fins have a thickness of 3 mm and a length of 30 mm. The thermal conductivity of the fin material is 26 W/m K and the heat transfer coefficient for all surfaces may be taken as 40 W/m^2 K. Neglecting the heat loss at the tip of each fin, calculate:

(i) The fin efficiency; **[77.5 %]**

(ii) The rate of the heat loss from the unit area of the flat surface; **[72.1 kW/m^2]**
(iii) The temperature at the tip of each fin; **[206.9 °C]**

34. A cylindrical storage tank, 1 m diameter by 1.2 m long has an outside surface temperature of 60 ^{0}C, and an emissivity of 0.9. Calculate the rate of heat loss by radiation when the tank is in a large room, the walls of which are at 15 ^{0}C. Calculate also the reduction in the rate of heat loss by radiation if the tank is painted with aluminium paint of emissivity 0.4. Assume that the tank is a grey body.
[1474 W; 819 W]

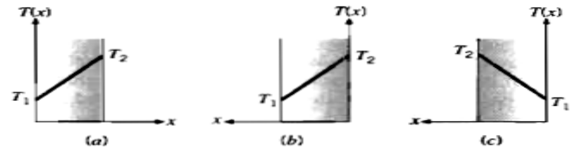
35. A copper pipe at 260 ^{0}C is in a large room at 15 ^{0}C. Calculate the rate of heat loss per unit area of pipe surface by radiation, taking the emissivity of copper as 0.61 at 260 ^{0}C, and as 0.56 at 15 ^{0}C. Assume that the absorptivity of a surface depends only on the temperature of the source of radiation. **[2571.5 W/m^2]**

36. Calculate the rate of heat transfer per unit area by radiation between two brick walls a short distance apart, when the temperatures of the surfaces are 30 ^{0}C and 15 ^{0}C. The emissivity of the brick may be taken as 0.93, and the surface may be assumed to be grey. **[76.2 kW/m^2]**

37. A thermos flask consists of an inner cylindrical vessel of 60 mm outside diameter and an outer cylindrical vessel of 65 mm inside diameter. Both surfaces are polished silver, emissivity 0.02. Calculate the rate of heat loss per millimetre length of the flask when it contains boiling water and the temperature of the outside surface is 17 ^{0}C. Neglect the thermal resistance of the metal walls of the flask. (NB Polished surfaces reflect specularly and hence in this case the surfaces act as large parallel planes.) **[0.00133 W]**

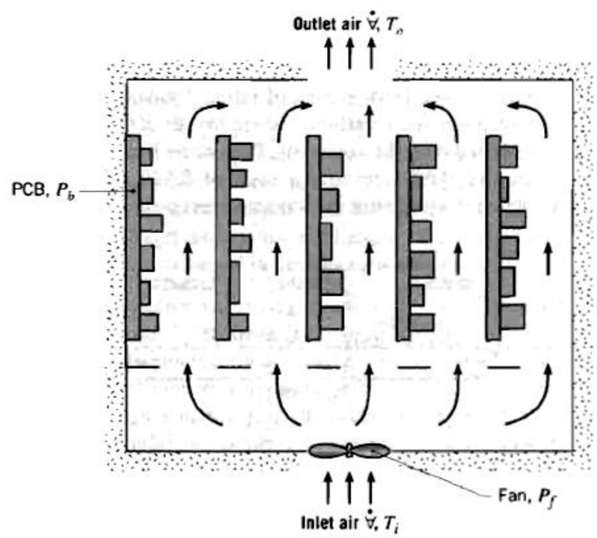
38. In a muffle furnace, the floor 4.5 m by 4.5 m, is constructed of refractory material (emissivity = 0.7). Two rows of oxidised steel tubes are placed 3 m above and parallel to the floor but for the purpose of analysis these can be replaced by 4.5 m by 4.5 m plane surfaces having emissivity of 0.9. The average temperature of the floor and tubes are 900 and 270 ^{0}C, respectively. Taking the geometric factor for radiation from floor to tubes as 0.32, calculate:
(i) The net rate of heat transfer to the tubes;
(ii) The mean temperature of the refractory walls of the furnace, assuming that these are well insulated.
[1009 kW; 687 °C]

39. Consider a plane wall 100 mm thick and of thermal conductivity 100 W/ (m.K). Steady-state conditions are known to exist with T_1 = 400 K and T_2 = 600 K. Determine the heat flux q_x'' and the temperature gradient dT/dx for the coordinate systems shown.



[-200 kW/m^2, 2000 K/m; 200 kW/m^2, -2000 K/m; -200 kW/m^2, 2000 K/m]

40. A computer consists of an array of five printed circuit boards (PCBs), each dissipating P_b = 20W of power. Cooling of the electronic components on a board is provided by the forced flow of air, equally distributed in passages formed by adjoining boards, and the convection coefficient associated with heat transfer from the components to the air is approximately h = 200 W/m^{2}K. Air enters the computer console at a temperature of T_i = 20^{o}C, and flow is driven by a fan whose power consumption is P_f = 25 W.



(a) If the temperature rise of the air flow, (T_o – T_i), is not to exceed 15^{o}C, what is the minimum allowable volumetric flow rate $\dot{\forall}$ of the air? The density and specific heat of the air may be approximated as ρ = 1.161 kg/m^3 and c_p = 1007 J/kgK, respectively.
[0.428 m^3/min]

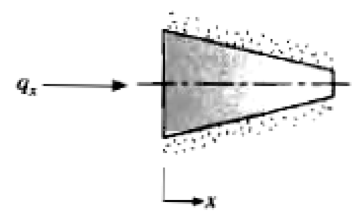
(b) The component that is most susceptible to thermal failure dissipates 1 W/cm^2 of surface area. To minimize the potential for thermal failure, where should the component be installed on a PCB? What is its surface temperature at this location? **[70 °C]**

41. Microwave radiation is known to be transmitted by plastics, glass, and ceramics, but to be absorbed by materials having polar molecules such as water. Water molecules exposed to

microwave radiation align and reverse alignment with the microwave radiation at frequencies up to 10^9 s^{-1}, causing heat to be generated. Contrast cooking in a microwave oven with cooking in a conventional radiant or convection oven. In each case what is the physical mechanism responsible for heating the food? Which oven has the greater energy utilization factor? Why? Microwave heating is being considered for drying clothes. How would operation of a microwave clothes dryer differ from a conventional dryer? Which is likely to have the greater energy utilization efficiency and why?

42. (a) Assume steady-state, one-dimensional heat conduction through the symmetric shape shown.

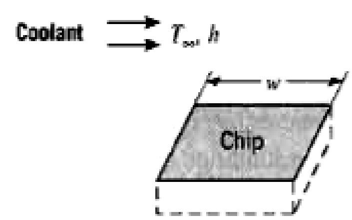
Assuming that there is no internal heat generation, derive an expression for the thermal conductivity $k(x)$ for these conditions: $A(x) = (1 - x)$, $T(x) = 300(1 - 2x - x^3)$ and $q = 6000$W, where A is in square meters, T in kelvins, and x in meters.



(b) To determine the effect of the temperature dependence of the thermal conductivity on the temperature distribution in a solid, consider a material for which this dependence may be expressed as $k = k_o + aT$, where k_o is a positive constant and a is a coefficient that may be positive or negative. Sketch the steady-state temperature distribution associated with heat transfer in a plane wall for three cases corresponding to $a > 0$, $a = 0$, and $a < 0$.

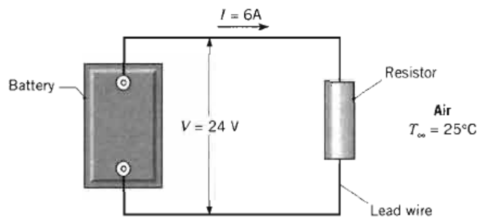
43. (a) A square isothermal chip is of width w = 5mm on a side and is mounted in a substrate such that its side and back surfaces are well insulated, while the front surface is exposed to the flow of a coolant at T_∞ = 15°C. From reliability considerations, the chip temperature must not exceed T = 85°C.

If the coolant is air and the corresponding convection coefficient is h = 200 W/m^{2}K, what is the maximum allowable chip power? If the coolant is a dielectric liquid for which h = 3000 W/m^{2}K, what is the maximum allowable power? **[0.35W; 5.25W]**



(b) A spherical interplanetary probe of 0.5-m diameter contains electronics that dissipate 150W. If the probe surface has an emissivity of 0.8 and the probe does not receive radiation from other surfaces, as, for example, from the sun, what is its surface temperature? **[254.7 K]**

44. An electrical resistor is connected to a battery, as shown schematically. After a brief transient, the resistor assumes a nearly uniform, steady-state temperature of 95 °C, while the battery and lead wires remain at the ambient temperature of 25 °C.



Neglecting the electrical resistance of the lead wires,

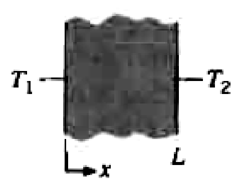
(a) Consider the resistor as the system about which a control surface is placed and equation 1.10 is applied. Determine the corresponding values of $\dot{E}_{in}(W)$, $\dot{E}_g(W)$, $\dot{E}_{out}(W)$ and $\dot{E}_{st}(W)$. If a control surface is placed about the entire system, what are the values of $\dot{E}_{in}$, $\dot{E}_g$, $\dot{E}_{out}$ and $\dot{E}_{st}$?

(b) If electrical energy is dissipated uniformly within the resistor, which is a cylinder of diameter $D = 60\,\text{mm}$ and length $L = 250\,\text{mm}$, what is the volumetric heat generation rate, $q\left(W/m^2\right)$?

(c) Neglecting radiation from the resistor, what is the convection heat transfer coefficient?

44. By considering the steady-state conditions for one-dimensional conduction in a slab as shown in Figure Q44, having thermal conductivity $\lambda = 50\,\text{W/m K}$ and thickness L = 0.25 m with no internal heat generation, determine the unknown quantities and complete the Table below for each case.

Figure Q44



Fill in the missing quantities

T_1/ (^{0}C)	T_2/ (^{0}C)	[dT/dx]/ (K/m)	$\dot{q}$ /(W/m^2)
65	- 35		
- 20	- 5		
90		140	
	45	- 90	
	15	120	

45. To warm up some milk for a baby, a mother pours milk into a thin-walled glass whose diameter is 6 cm. The height of the milk in the glass is 7 cm. She then places the glass into a large pan filled with hot water at 60°C. The milk is stirred constantly, so that its temperature is uniform at all times. If the heat transfer coefficient between the water and the glass is 120 $W/m^2 \cdot °C$, determine how long it will take for the milk to warm up from 3°C to 38°C. Take the properties of the milk to be the same as those of water. Can the milk in this case be treated as a lumped system? Why? **[5.8 min]**

46. Stainless steel ball bearings (ρ = 8085 kg/m^3, k = 15.1 W/m · °C, Cp = 0.480 kJ/kg · °C, and $\alpha = 3.91 \times 10^{-6}$ m^2/s) having a diameter of 1.2 cm are to be quenched in water. The balls leave the oven at a uniform temperature of 900°C and are exposed to air at 30°C for a while before they are dropped into the water. If the temperature of the balls is not to fall below 850°C prior to quenching and the heat transfer coefficient in the air is 125 $W/m^2 \cdot °C$, determine how long they can stand in the air before being dropped into the water. **[3.7 s]**

47. The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1.2-mm-diameter sphere. The properties of the junction are k = 35 W/m · °C, ρ = 8500 kg/m^3, and Cp = 320 J/kg · °C, and the heat transfer coefficient between the junction and the gas is h = 65 $W/m^2 \cdot °C$. Determine how long it will take for the thermocouple to read 99 percent of the initial temperature difference. **[38.5 s]**

Learner Feedback Form/[insert course code]

detach and return to IDL, KNUST

Dear Learner,
While studying the units in the course, you may have found certain portions of the text difficult to comprehend. We wish to know your difficulties and suggestions, in order to improve the course. Therefore, we request you to fill out and send the following questionnaire, which pertains to this course. If you find the space provided insufficient, kindly use a separate sheet.

1. How many hours did you need for studying the units

Unit no.	1	2	3	4	5	6
No. of hours						

2. Please give your reactions to the following items based on your reading of the course

Items	**Excellent**	**Very good**	**Good**	**Poor**	*Give specific examples, if poor*
Presentation quality	☐	☐	☐	☐	
Language and style	☐	☐	☐	☐	
Illustrations used (diagrams, tables, etc.)	☐	☐	☐	☐	
Conceptual clarity	☐	☐	☐	☐	
Self assessment	☐	☐	☐	☐	
Feedback to SA	☐	☐	☐	☐	

3. Any other comments

Unit 1 ……………………………………………………………………………………..

Unit 2 ……………………………………………………………………………………..

Unit 3 ……………………………………………………………………………………..

Unit 4 ……………………………………………………………………………………..

Unit 5 ……………………………………………………………………………………..

Unit 6 ……………………………………………………………………………………..

(May use reverse side of page if necessary)