KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
SECOND YEAR
END-OF-SECOND-SEMESTER EXAMINATION, 2019/2020
BSc(Chemical, Petrochemical, Biomedical, Computer and Electrical) Engineering

---

MATH 252: CALCULUS OF SEVERAL VARIABLES
May, 2020

Index Number: 3562718

Faculty/Department: Electrical/Electronic Engineering

INSTRUCTIONS:

1. Answer **ALL** questions in section *A* and questions in SECTION B. Answer all the questions on the question paper.
2. Please make sure you have all **TWELVE**(**12**) pages of questions.
3. Write your **IndexNumber** boldly in the space provided on this front page and every other sheet.
4. Programmable and Graphing Calculators are **NOT ALLOWED**

Use the information below to answer questions 1 to 8. Given that $x = r\cos\phi$, $y = r\sin\phi$, $z = z$, $u(x, y, z)$, $\phi = \tan^{-1}\left(\frac{y}{x}\right)$ and $r = \sqrt{x^2 + y^2}$

1. Find $\dfrac{\partial u}{\partial y}$

   A. $\sin\phi\dfrac{\partial u}{\partial r} + \cos\phi\dfrac{\partial u}{\partial \phi}$

   B. $\sin\phi\dfrac{\partial u}{\partial r} + \dfrac{\cos\phi}{r}\dfrac{\partial u}{\partial \phi}$

   C. $\cos\phi\dfrac{\partial u}{\partial r} + \dfrac{\sin\phi}{r}\dfrac{\partial u}{\partial \phi}$

   D. $\dfrac{\sin\phi}{r}\dfrac{\partial u}{\partial r} + \dfrac{\cos\phi}{r}\dfrac{\partial u}{\partial \phi}$

2. Find $\dfrac{\partial u}{\partial y}$

   A. $\dfrac{-sin\phi}{r}$

   B. $\dfrac{\cos\phi}{r}$

   C. $\dfrac{\sin\phi}{r}$

   D. $\dfrac{-\cos\phi}{r}$

3. Simplify $\dfrac{\cos\phi}{r}\dfrac{\partial}{\partial\phi}\left(\sin\phi\dfrac{\partial u}{\partial r}\right)$

   A. $\dfrac{\cos\phi\sin\phi}{r}\dfrac{\partial^2 u}{\partial\phi\partial r}$

   B. $\dfrac{\cos^2\phi}{r}\dfrac{\partial u}{\partial r} + \dfrac{\cos\phi\sin\phi}{r^2}\dfrac{\partial^2 u}{\partial\phi\partial r}$

   C. $\dfrac{\cos^2\phi}{r}\dfrac{\partial u}{\partial r}$

   D. $\dfrac{\cos^2\phi}{r}\dfrac{\partial u}{\partial r} + \dfrac{\cos\phi\sin\phi}{r}\dfrac{\partial^2 u}{\partial\phi\partial r}$

4. Find $\dfrac{\partial \phi}{\partial y}$

   A. $\sin\phi$

   B. $\dfrac{-\sin\phi}{r}$

   C. $\dfrac{\cos\phi}{r}$

   D. $\cos\phi$

5. Find $\dfrac{\partial r}{\partial x}$

   A. $-r\cos\phi$

   B. $\sin\phi$

   C. $-r\sin\phi$

   D. $\cos\phi$

6. Find $\dfrac{\partial^2 u}{\partial x^2}$

   A. $\dfrac{\sin\phi\cos\phi}{r^2}\dfrac{\partial u}{\partial\phi}+\cos^2\phi\dfrac{\partial^2 u}{\partial r^2}+\dfrac{\sin^2\phi}{r^2}\dfrac{\partial^2 u}{\partial\phi^2}-\dfrac{\sin\phi\cos\phi}{r}\dfrac{\partial^2 u}{\partial\phi\partial u}+\dfrac{\cos\phi\sin\phi}{r}\dfrac{\partial^2 u}{\partial r\partial\phi}+\dfrac{\sin^2\phi}{r}\dfrac{\partial u}{\partial r}+\dfrac{\sin\phi\cos\phi}{r}\dfrac{\partial u}{\partial\phi}$

   B. $\dfrac{\sin^2\phi}{r^2}\dfrac{\partial^2 u}{\partial\phi^2}+\dfrac{\sin^2\phi}{r}\dfrac{\partial u}{\partial r}+\dfrac{\sin\phi\cos\phi}{r^2}\dfrac{\partial u}{\partial\phi}-\dfrac{\sin\phi\cos\phi}{r}\dfrac{\partial^2 u}{\partial\phi\partial r}-\dfrac{\cos\phi\sin\phi}{r}\dfrac{\partial^2}{\partial r\partial\phi}+\cos^2\phi\dfrac{\partial^2 u}{\partial r^2}+\dfrac{\cos\phi\sin\phi}{r^2}\dfrac{\partial u}{\partial\phi}$

   C. $\dfrac{-\cos\phi\sin\phi}{r}\dfrac{\partial^2 u}{\partial\phi^2}+\dfrac{\sin\phi\cos\phi}{r^2}\dfrac{\partial u}{\partial\phi}+\dfrac{\sin^2\phi}{r}\dfrac{\partial u}{\partial r}+\dfrac{\sin^2\phi}{r}\dfrac{\partial^2 u}{\partial\phi^2}-\dfrac{\sin\phi\cos\phi}{r}\dfrac{\partial^2 u}{\partial\phi\partial r}+\cos^2\phi\dfrac{\partial^2 u}{\partial r^2}+\dfrac{\cos\phi\sin\phi}{r^2}\dfrac{\partial u}{\partial\phi}$

   D. $\cos^2\phi\dfrac{\partial^2 u}{\partial r^2}+\dfrac{\sin^2\phi}{r}\dfrac{\partial u}{\partial r}-\dfrac{\sin\phi\cos\phi}{r}\dfrac{\partial u}{\partial r}+\dfrac{\cos\phi\sin\phi}{r^2}\dfrac{\partial u}{\partial\phi}-\dfrac{\cos\phi\sin\phi}{r}\dfrac{\partial^2 u}{\partial r\partial\phi}+\dfrac{\sin\phi\cos\phi}{r^2}\dfrac{\partial^2 u}{\partial r\partial\phi}+\dfrac{\sin^2\phi}{r^2}\dfrac{\partial^2 u}{\partial\phi^2}$

7. Simplify $(x^2+y^2)(\nabla.\nabla)$

A. $r^2\dfrac{\partial^2 u}{\partial z^2}+r^2\dfrac{\partial u}{\partial \phi}+\dfrac{\partial^2 u}{\partial r^2}+r\dfrac{\partial u}{\partial r}$

B. $\dfrac{\partial^2 u}{\partial \phi^2}+r^2\dfrac{\partial^2 u}{\partial r^2}-\dfrac{1}{r}\dfrac{\partial u}{\partial r}+r^2\dfrac{\partial^2 u}{\partial z^2}$

C. $r\dfrac{\partial u}{\partial r}+r^2\dfrac{\partial^2 u}{\partial z^2}+\dfrac{\partial^2 u}{\partial \phi^2}+r^2\dfrac{\partial^2 u}{\partial r^2}$

D. $r^2\dfrac{\partial^2 u}{\partial r^2}+\dfrac{\partial^2 u}{\partial \phi^2}+r\dfrac{\partial^2 u}{\partial r\partial \phi}+r^2\dfrac{\partial^2 u}{\partial z^2}$

8. Find $sin\phi\dfrac{\partial}{\partial r}\left(\sin\phi\dfrac{\partial u}{\partial r}+\dfrac{\cos\phi}{r}\dfrac{\partial u}{\partial \phi}\right)$

A. $\dfrac{-\sin\phi\cos\phi r^2}{r^2}\dfrac{\partial u}{\partial \phi}+\sin^2\phi\dfrac{\partial^2 u}{\partial r^2}+\dfrac{\sin\phi\cos\phi}{r}\dfrac{\partial^2 u}{\partial r\partial\phi}$

B. $\dfrac{-\sin\phi\cos\phi}{r^2}\dfrac{\partial^2 u}{\partial r\partial\phi}+\sin^2\phi\dfrac{\partial^2 u}{\partial r^2}$

C. $\sin^2\phi\dfrac{\partial^2 u}{\partial r^2}+\dfrac{\cos\phi\sin\phi}{r}\dfrac{\partial^2 u}{\partial r\partial\phi}$

D. $\sin^2\phi\dfrac{\partial^2 u}{\partial r^2}+\dfrac{\sin\phi\cos\phi}{r}\dfrac{\partial^2 u}{\partial r\partial\phi}+\dfrac{\sin\phi\cos\phi}{r^2}\dfrac{\partial u}{\partial\phi}$

Use this information to answer questions Q9 to Q12.
Given that $\dfrac{\partial u}{\partial t}=\lambda u-\omega v+\dfrac{\partial^2 u}{\partial x^2}$, $\dfrac{\partial v}{\partial t}=\omega u+\lambda v+\dfrac{\partial^2 v}{\partial x^2}$, $u=r\cos\phi$ and $v=r\sin\phi$

9. Find $\dfrac{\partial u}{\partial t}$, in terms of ϕ, r and t

A. $-r\sin\phi\dfrac{\partial\phi}{\partial t}-\cos\phi\dfrac{\partial r}{\partial t}$

B. $r\sin\phi\dfrac{\partial\phi}{\partial t}+\cos\phi\dfrac{\partial r}{\partial t}$

C. $r\sin\phi\dfrac{\partial\phi}{\partial t}+\cos\phi\dfrac{\partial r}{\partial t}$

D. $-r\sin\phi\dfrac{\partial\phi}{\partial t}+\cos\phi\dfrac{\partial r}{\partial t}$

10. Find $\dfrac{\partial^2 v}{\partial x^2}$

A. $-r\sin\phi\dfrac{\partial^2\phi}{\partial x^2}+\cos\phi\dfrac{\partial^2 r}{\partial x^2}$

B. $\cos\phi\dfrac{\partial^2\phi}{\partial x^2}+\sin\phi\dfrac{\partial^2 r}{\partial x^2}$

C. $r\cos\phi\dfrac{\partial^2\phi}{\partial x^2}+\sin\phi\dfrac{\partial^2 r}{\partial x^2}$

D. $-\cos\phi\dfrac{\partial^2 r}{\partial x^2}+r^2\sin\phi\dfrac{\partial^2\phi}{\partial x^2}$

11. $\dfrac{\partial r}{\partial t}$ is the same as ...

A. $r\lambda-\omega+\dfrac{\partial^2 r}{\partial x^2}$

B. $r\lambda+\phi\dfrac{\partial^2 r}{\partial x^2}$

C. $r\lambda+\omega+\dfrac{\partial^2\phi}{\partial x^2}+\dfrac{\partial^2 r}{\partial x^2}$

D. $r\lambda+\dfrac{\partial^2 r}{\partial x^2}$

12. Which of the following expression is equal to $\dfrac{\partial^2\phi}{\partial x^2}$

A. $r\omega-\dfrac{\partial\phi}{\partial t}$

B. $-\omega+\dfrac{\partial\phi}{\partial t}$

C. $-\omega+\dfrac{\partial r}{\partial t}+r\dfrac{\partial\phi}{\partial t}$

D. $\omega+\dfrac{1}{r}\dfrac{\partial\phi}{\partial t}$

13. Which of the following vector field is conservative?

A. $x^2y\underline{i}+(3x-yz)\underline{j}+z^3\underline{k}$

B. $\dfrac{-y}{x^2+y^2}\underline{i}+\dfrac{x}{x^2+y^2}\underline{j}+0\underline{k}$

C. $xy\underline{i}+z\underline{j}+z^2x\underline{k}$

D. $(x^3 - y)\underline{i} + y^5\underline{j} + e^z\underline{k}$

14. Which of the following statement is true?

A. The Stokes' theorem is a vector form of the Green's theorem

B. The divergence theorem is an extension of stokes' theorem

C. The Green's theorem relates the flow of fluid that surrounds a point and the fluid flow inside the point.

D. The curl of a fluid flow is a vector form of divergence of same fluid flow.

Use the information below to answer question 15 to 20
Given that $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, $r^2 = x^2 + y^2 + z^2$ and $u(x, y, z)$

15. Which of the following statement is equal to $\dfrac{\partial u}{\partial z}.\dfrac{\partial z}{\partial \theta}$

A. $-\dfrac{\partial u}{\partial y}.\dfrac{\partial y}{\partial \theta} - \dfrac{\partial u}{\partial \theta} + \dfrac{\partial u}{\partial r}.\dfrac{\partial r}{\partial \theta}$

B. $\dfrac{\partial u}{\partial \theta}.\dfrac{\partial \theta}{\partial x} - \dfrac{\partial u}{\partial y}.\dfrac{\partial y}{\partial \theta} - \dfrac{\partial u}{\partial x}.\dfrac{\partial x}{\partial \theta}$

C. $-\dfrac{\partial u}{\partial x}.\dfrac{\partial x}{\partial \theta} + \dfrac{\partial u}{\partial \theta} - \dfrac{\partial u}{\partial y}.\dfrac{\partial y}{\partial \theta}$

D. $\dfrac{\partial u}{\partial \theta} - \dfrac{\partial u}{\partial y}.\dfrac{\partial y}{\partial \theta} - \dfrac{\partial u}{\partial x}.\dfrac{\partial x}{\partial r}$

16. Find $\dfrac{\partial u}{\partial r}$

A. $\dfrac{\sin\phi}{\cos\phi}\dfrac{\partial u}{\partial x} + \dfrac{\sin\phi}{r\cos\phi}\dfrac{\partial u}{\partial \phi} + \cos\theta\dfrac{\partial u}{\partial z}$

B. $\cos\theta\dfrac{\partial u}{\partial z} + \dfrac{\sin\phi}{\cos\phi}\dfrac{\partial u}{\partial x} + \dfrac{\sin\phi}{\cos\phi}\dfrac{\partial u}{\partial \phi}$

C. $\dfrac{\sin\phi}{r\cos\phi}\dfrac{\partial u}{\partial \phi} - \dfrac{\sin\phi}{\cos\phi}\dfrac{\partial u}{\partial x} + \cos\phi\dfrac{\partial u}{\partial z}$

D. $\sin\theta\dfrac{\partial u}{\partial z} + r\cos\theta\dfrac{\partial u}{\partial x} + \dfrac{r\cos\phi}{\sin\phi}\dfrac{\partial u}{\partial \phi}$

17. Which of the following statement is equal to $\dfrac{\partial u}{\partial z}$

A. $\dfrac{\cos\theta\sin\phi}{r\sin^2\theta\cos\phi}\dfrac{\partial u}{\partial\phi}+\dfrac{\cos\phi}{\sin\theta\cos^2\theta}\dfrac{\partial u}{\partial x}-\dfrac{1}{\sin\theta}\dfrac{\partial u}{\partial\theta}$

B. $\dfrac{\sin\phi\cos\theta}{r\sin^2\theta\cos\phi}\dfrac{\partial u}{\partial\phi}+\dfrac{\cos\phi}{\sin\theta\cos\theta}\dfrac{\partial u}{\partial x}-\dfrac{1}{r\sin\theta}\dfrac{\partial u}{\partial\theta}$

C. $\dfrac{\sin\theta\cos\phi}{r\sin^2\theta\cos\phi}\dfrac{\partial u}{\partial\phi}+\dfrac{\cos\phi}{\sin\theta\cos\phi}\dfrac{\partial u}{\partial x}-\dfrac{1}{r\sin\theta}\dfrac{\partial u}{\partial\theta}$

D. $\dfrac{\cos\theta\sin\phi}{r\sin^2\theta\cos\phi}\dfrac{\partial u}{\partial\phi}+\dfrac{\cos\theta}{\sin\theta\cos\phi}\dfrac{\partial u}{\partial x}-\dfrac{1}{r\sin\theta}\dfrac{\partial u}{\partial\theta}$

18. Simplify $\dfrac{1}{r}\triangle u$

A. $\dfrac{1}{r^3}\left[\dfrac{\partial}{\partial r}\left(\dfrac{r^2}{\cos\theta}\dfrac{\partial u}{\partial r}\right)+\dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial\phi^2}\right]$

B. $\dfrac{1}{r^3}\left[\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial u}{\partial r}\right)+\dfrac{1}{\sin^2\theta}\dfrac{\partial^2 u}{\partial\phi^2}+\dfrac{1}{\sin\phi}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial u}{\partial\theta}\right)\right]$

C. $\dfrac{1}{r^3}\left[\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial u}{\partial r}\right)+\dfrac{1}{r\sin^2\theta}\dfrac{\partial^2 u}{\partial\phi^2}+\dfrac{1}{\sin\phi}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial u}{\partial\theta}\right)\right]$

D. $\dfrac{1}{r^3}\left[\dfrac{\partial}{\partial r}\left(\dfrac{1}{r}\dfrac{\partial u}{\partial r}\right)+\dfrac{1}{\sin\theta}\dfrac{\partial^2 u}{\partial\phi^2}+\dfrac{1}{\sin\phi}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial u}{\partial\theta}\right)\right]$

19. Find $\dfrac{\partial u}{\partial r}$

A. $\sin\theta\sin\phi\dfrac{\partial u}{\partial y}+\cos\theta\cos\phi\dfrac{\partial u}{\partial x}+\cos\theta\dfrac{\partial u}{\partial z}$

B. $\cos\theta\cos\phi\dfrac{\partial u}{\partial y}+\sin\theta\cos\phi\dfrac{\partial u}{\partial x}+\cos\theta\dfrac{\partial u}{\partial z}$

C. $\cos\theta\dfrac{\partial u}{\partial z}+\sin\theta\cos\phi\dfrac{\partial u}{\partial x}+\sin\theta\sin\phi\dfrac{\partial u}{\partial y}$

D. $\sin\phi\cos\theta\dfrac{\partial u}{\partial x}+\cos\theta\dfrac{\partial u}{\partial z}+\sin\theta\sin\phi\dfrac{\partial u}{\partial y}$

20. Find $\nabla X(\nabla u) + \triangle u$

A. $\dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \phi^2} + \dfrac{\partial^2 u}{\partial z^2} + \dfrac{\partial^2 u}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u}{\partial r}$

B. $\dfrac{1}{r}\dfrac{\partial u}{\partial r} + \sin\theta \dfrac{\partial^2 u}{\partial r \partial \phi} + \dfrac{\partial^2 u}{\partial z^2} + \dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \phi^2} + \dfrac{\partial^2 u}{\partial r^2}$

C. $\dfrac{\partial^2 u}{\partial z^2} + \dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \phi^2} + r^2\dfrac{\partial^2 u}{\partial r^2} + \dfrac{\partial u}{\partial r}$

D. $\dfrac{\cos\theta}{r}\dfrac{\partial u}{\partial r} + \dfrac{\partial^2 u}{\partial \phi^2} + \dfrac{1}{\sin\theta}\dfrac{\partial^2 u}{\partial z^2} + r\dfrac{\partial^2 u}{\partial r^2}$

[0]Benedict Barnes