DEPARTMENT OF COMPUTER ENGINEERING

KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI, GHANA

COE 271
Semiconductor Devices

Lecture 8

Operational Amplifiers OPAMP

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"Learn from yesterday, live for today, hope for tomorrow. The important thing is to not stop questioning."

Albert Einstein

Overview

- Operational Amplifier
  - General Considerations
  - Op-Amp-Based Circuits
  - Linear Functions
  - Nonlinear Functions
  - Op-Amp Nonidealities
  - Design Examples

Overview

- Linear Functions
  - Noninverting Amplifier
  - Inverting Amplifier
  - Integrator /Differentiator

Overview

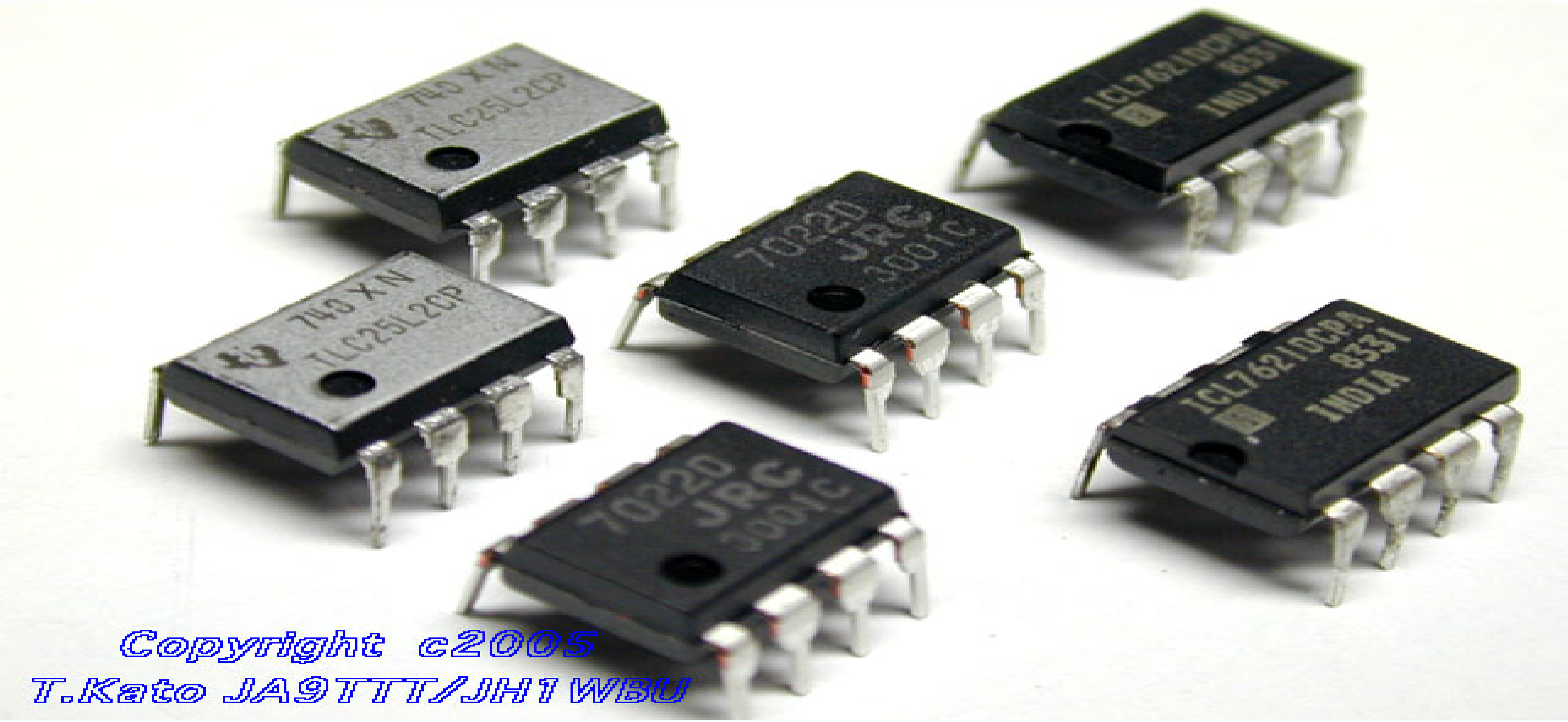
Nonlinear Functions

- Precision Rectifier
- Logarithmic Amplifier
- Square Root Circuit

Overview

- Op-Amp Nonidealities
  - DC Offsets
  - Input Bias
  - Speed Limitations
  - Finite Input and Output Impedances

Introduction to OP



Introduction to OP

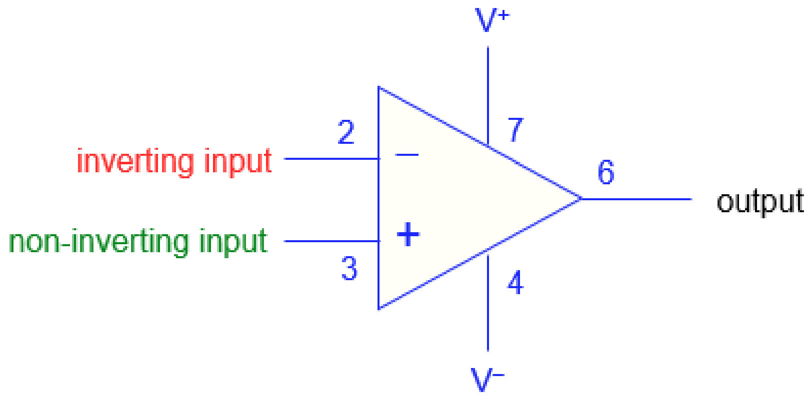
OP (-also called amplifiers or buffers in general)

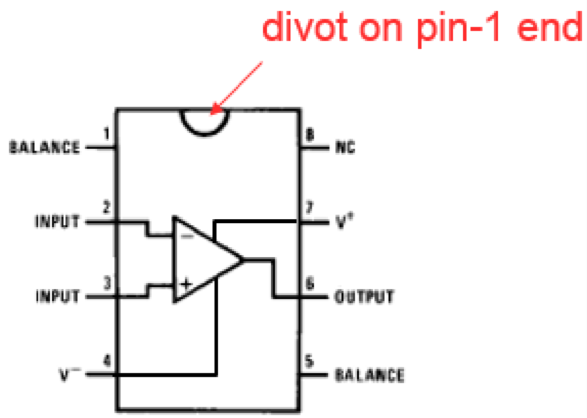
Op is drawn as a triangle in a schematic

Op has two inputs (inverting and non-inverting)

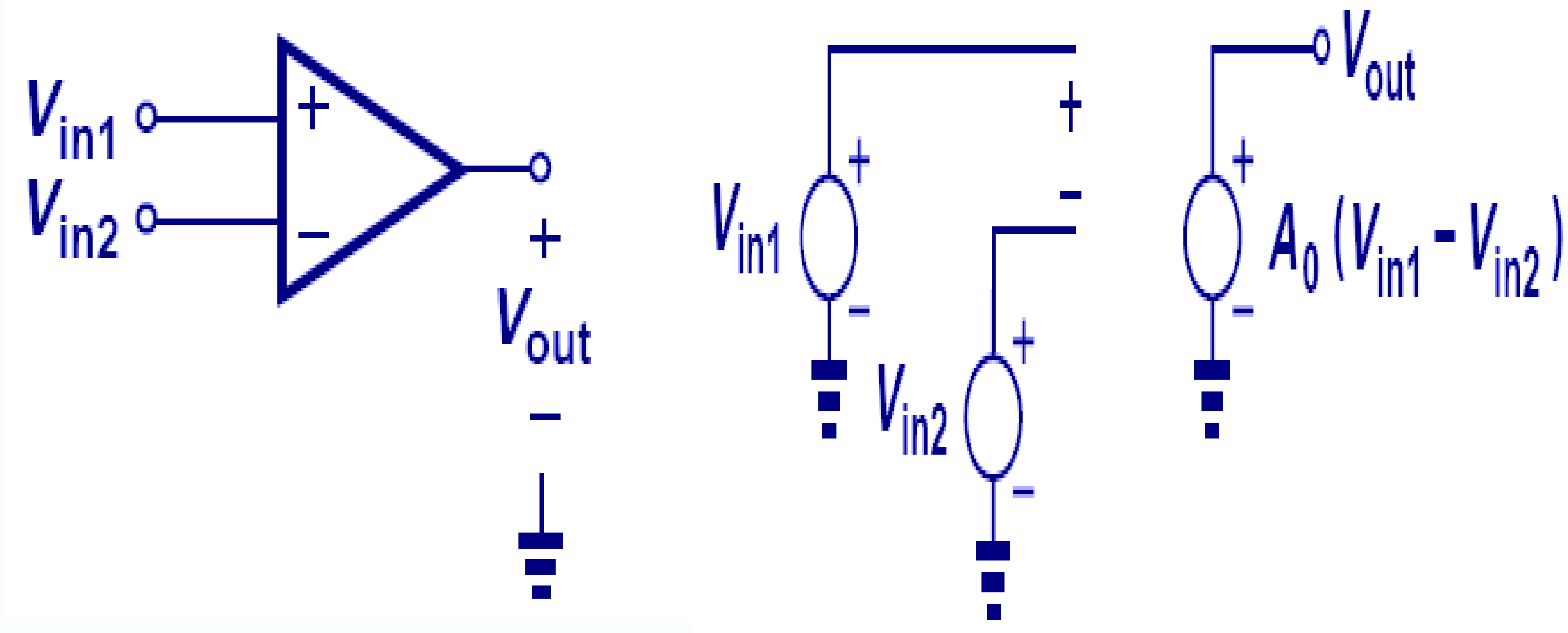
Op has one output

Op also has power connections (NB: no explicit ground)





Basic Operational Amplifier

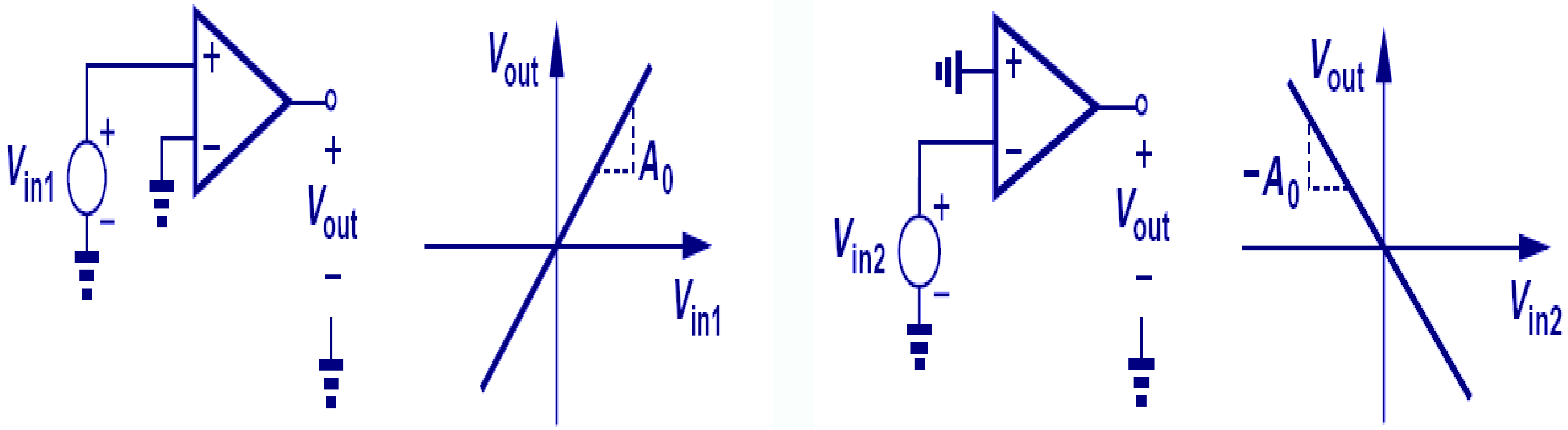


Internal Op-amplifier formula

$$V_{out} = A_0\left(V_{in1} - V_{in2}\right)$$

- Op amp is a circuit that has two inputs and one output.
- It amplifies the difference between the two inputs.

Inverting and Non-inverting Op Amp



- If the negative input is grounded, the gain is positive.

  If the positive input is grounded, the gain is negative.

Ideal Operational Amplifier

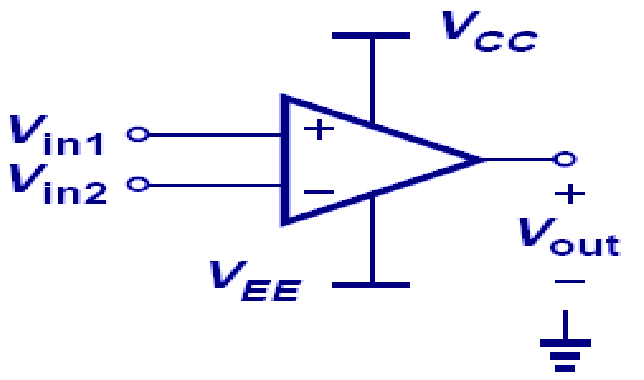
- Infinite gain
- (- a voltage difference at the two inputs is magnified infinitely)

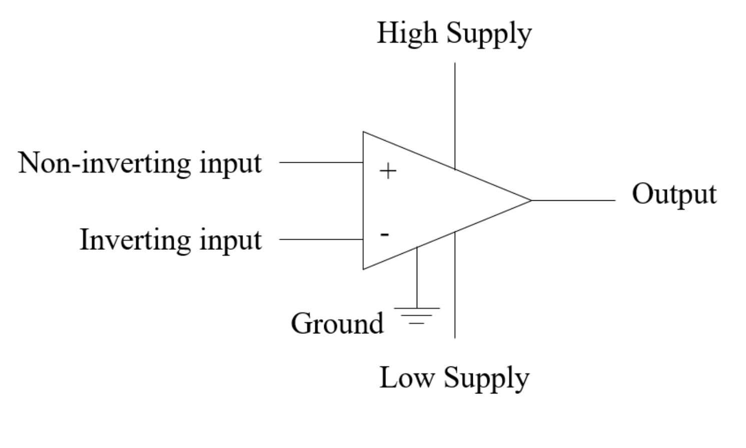
- (-meaning that the difference between + and – terminal is amplified by say 200000)

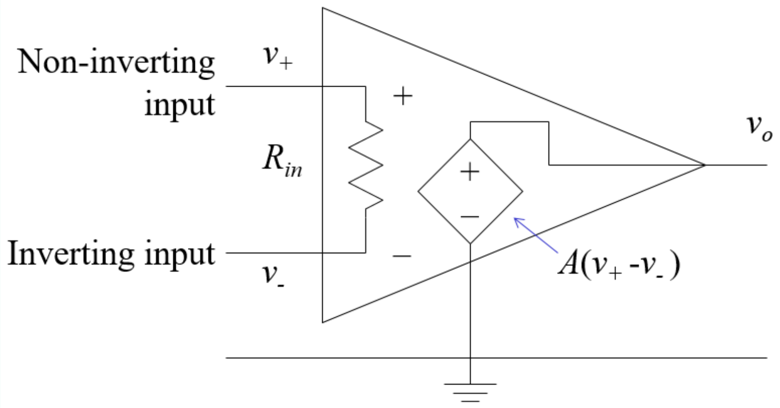
Ideal Operational Amplifier

- Infinite input impedance
- (-no current flows into the inputs)
- (-impedance is about $10^{12}\Omega$ for FET)
- Zero output impedance
- (-rock-solid independent of load)
- Infinite speed
- (-limited to few MHz range)

Op Amp with Supply Rails

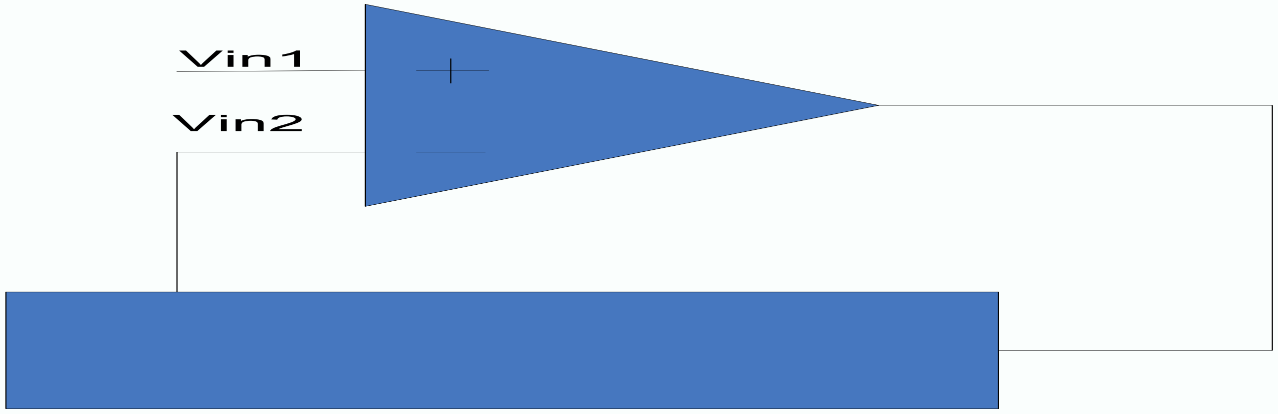


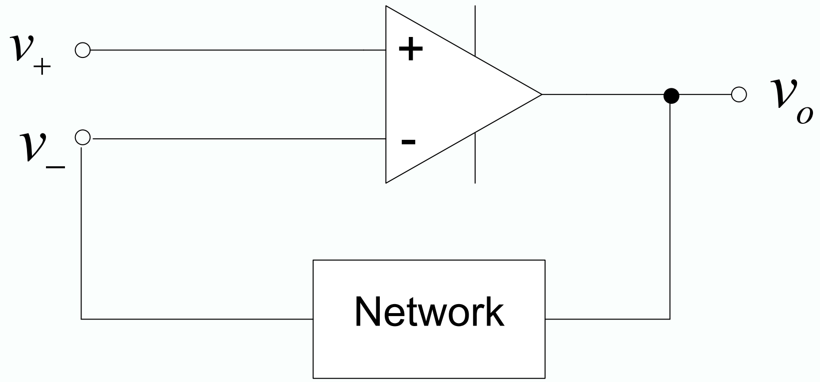




- To explicitly show the supply voltages, V_{CC} and V_{EE} are shown.
- In some cases, V_{EE} is zero.

Virtual Short (Ideal Op with Neg. FB)





- Due to infinite gain of op amp, the circuit forces V_{in2} to be close to V_{in1}, thus creating a virtual short.

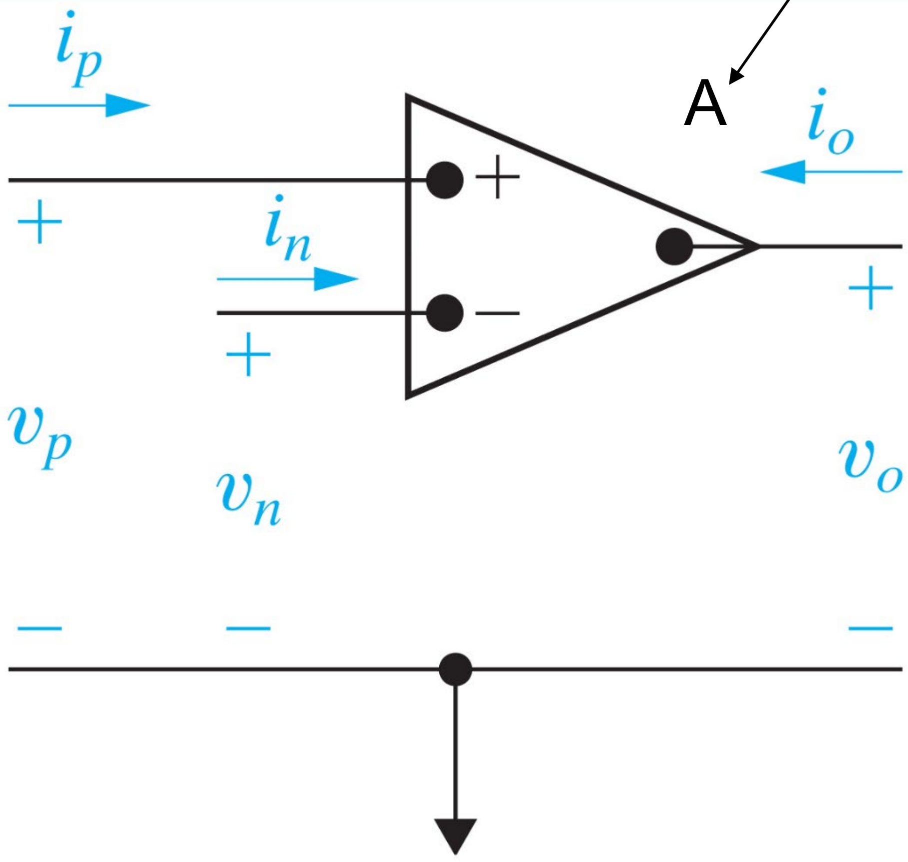
The negative feedback forces the "virtual short" condition to occur

Golden Rules of Op Amps:

1. The output attempts to do whatever is necessary to make the voltage difference between the inputs zero.
2. The inputs draw no current.

Ideal OP Analysis (Open Loop)

A = "open-loop" gain



$$v_o = A(v_p - v_n)$$

$$R_{in} \to \infty$$

$$A \to \infty$$

$$v_p = v_n$$

$$i_p = i_n = 0$$

Consequences of the Ideal

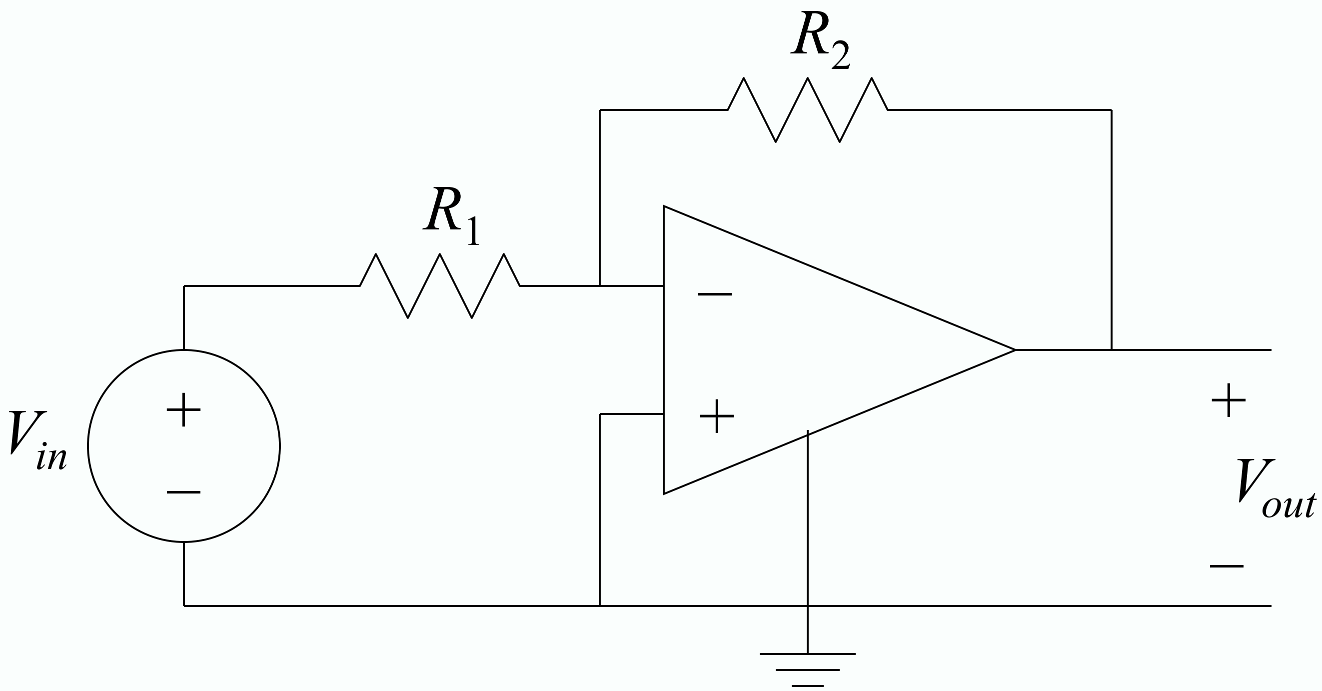
- Infinite input resistance means the current into the inverting input is zero:

$$i_- = 0$$

- Infinite gain means the difference between v_+ and v_- is zero:

$$v_+ - v_- = 0$$

Feedback Analysis

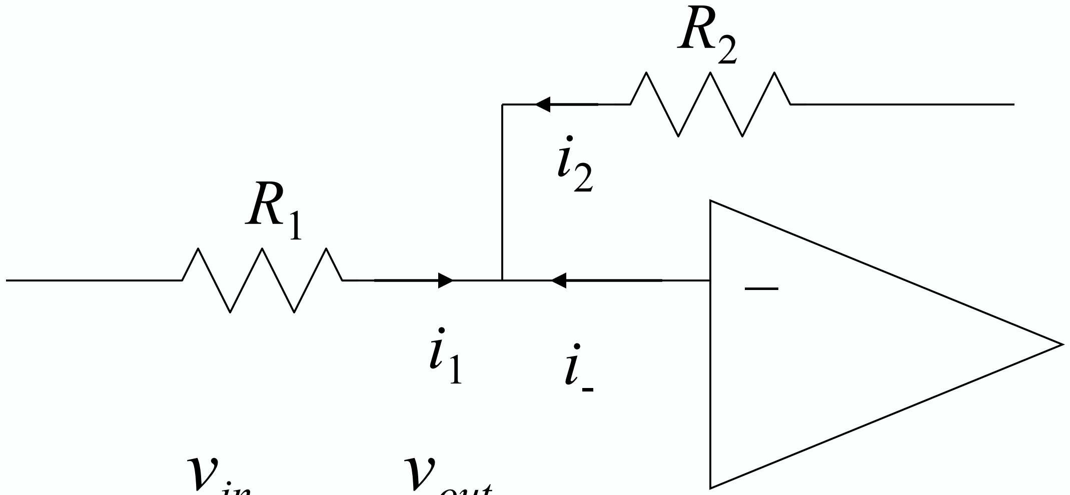


Solving the Amplifier Circuit

Apply KCL at the inverting input:

$$i_1 + i_2 + i_- = 0$$

$$i_- = 0$$



$$i_1 = \frac{v_{in} - v_-}{R_1} = \frac{v_{in}}{R_1}$$

$$i_2 = \frac{v_{out} - v_-}{R_2} = \frac{v_{out}}{R_2}$$

Solve for v_{out}

$$\frac{v_{in}}{R_1} = -\frac{v_{out}}{R_2}$$

Amplifier gain: $$\frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1}$$

Recap

- The ideal op-amp model leads to the following conditions:

$$i_- = 0 = i_+$$

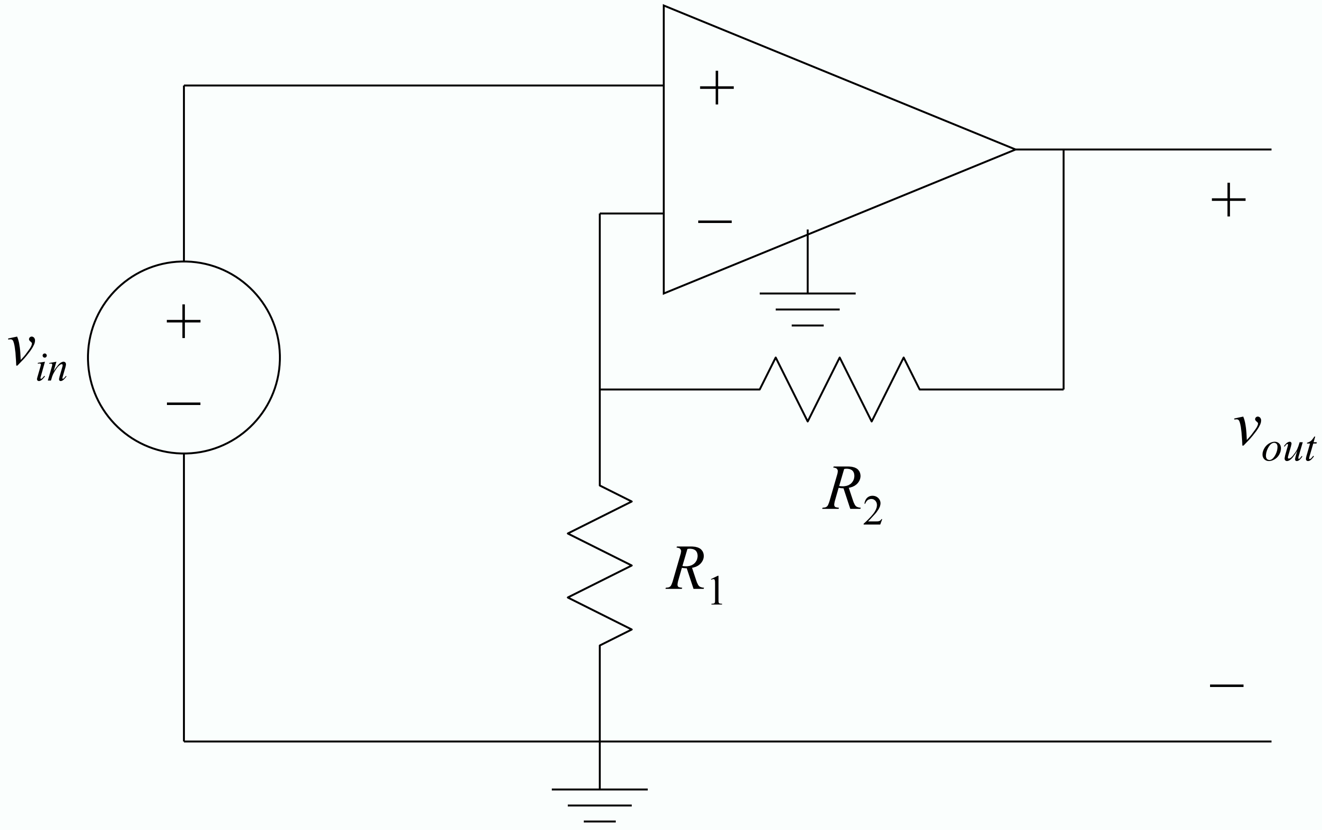
$$v_+ = v_-$$

- These conditions are used, along with KCL and other analysis techniques, to solve for the output voltage in terms of the input(s).

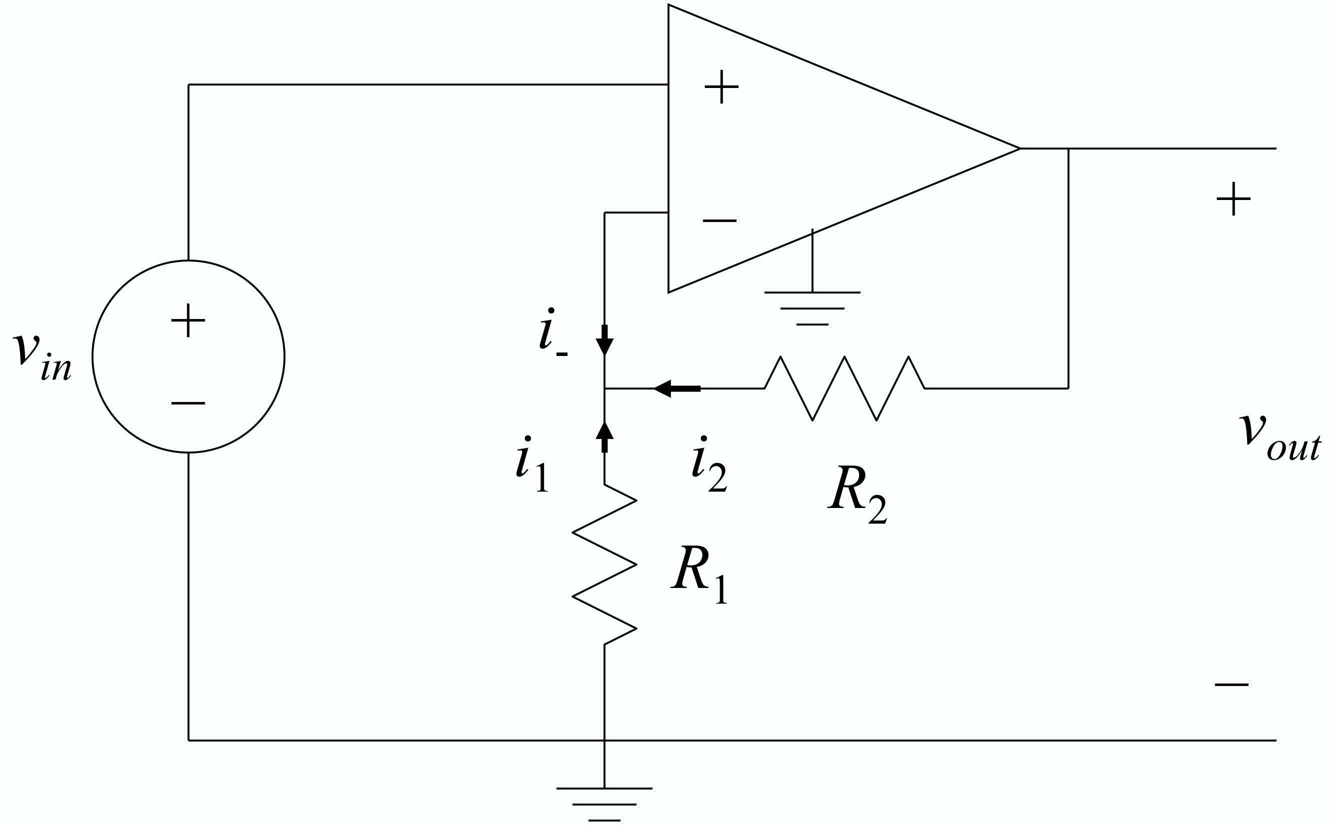
Review

- To solve an op-amp circuit, we usually apply KCL at one or both of the inputs.
- We then invoke the consequences of the ideal model.
  - The op amp will provide whatever output voltage is necessary to make both input voltages equal.
- We solve for the op-amp output voltage.

The Non-Inverting Amplifier



KCL at the Inverting Input



$$i_- = 0$$

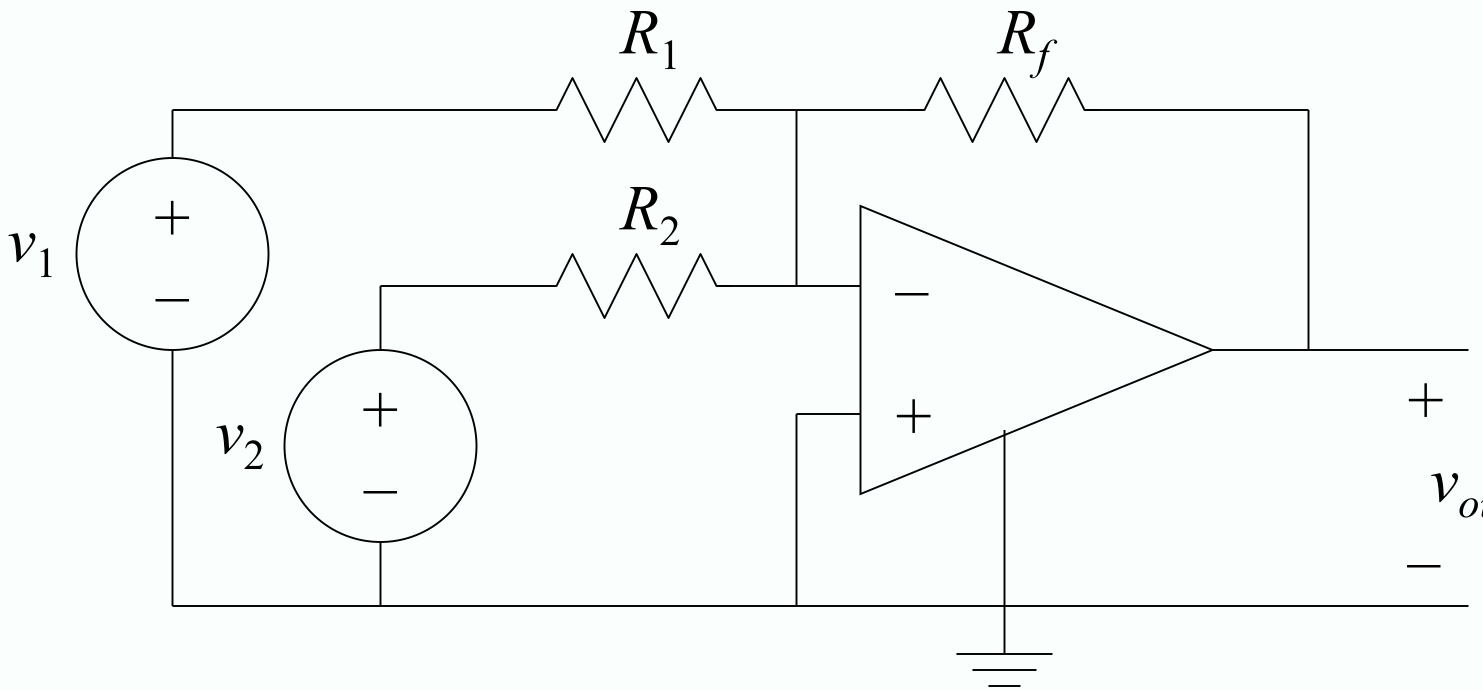
$$i_1 = \frac{-v_-}{R_1} = \frac{-v_{in}}{R_1}$$

$$i_2 = \frac{v_{out} - v_-}{R_2} = \frac{v_{out} - v_{in}}{R_2}$$

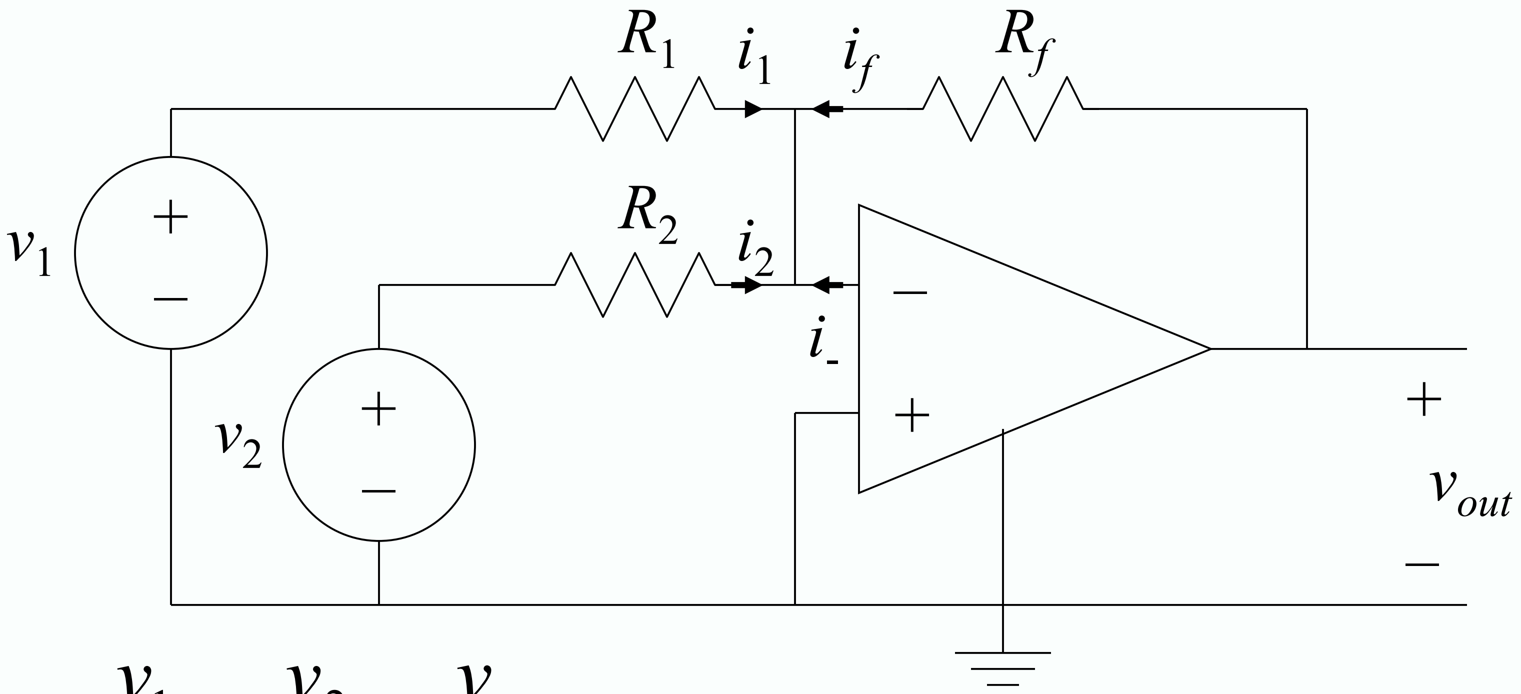
$$\frac{-v_{in}}{R_1} + \frac{v_{out} - v_{in}}{R_2} = 0$$

$$v_{out} = v_{in}\left(1 + \frac{R_2}{R_1}\right)$$

A Mixer Circuit



KCL at the Inverting Input



$$i_1 = \frac{v_1 - v_-}{R_1} = \frac{v_1}{R_1}$$

$$i_2 = \frac{v_2 - v_-}{R_2} = \frac{v_2}{R_2}$$

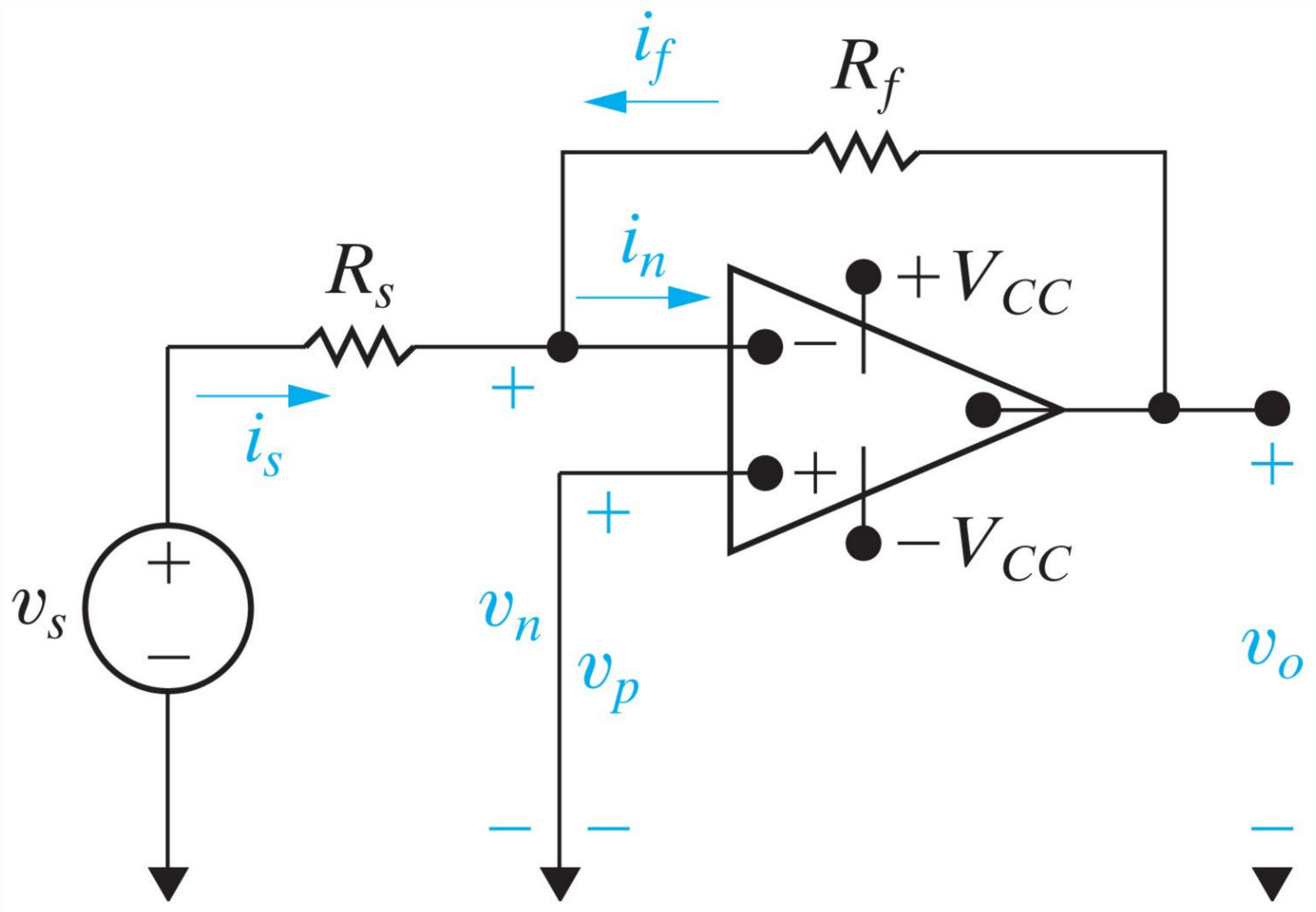
$$i_- = 0$$

$$i_f = \frac{v_{out} - v_-}{R_f} = \frac{v_{out}}{R_f}$$

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_{out}}{R_f} = 0$$

$$v_{out} = -\frac{R_f}{R_1} v_1 - \frac{R_f}{R_2} v_2$$

Inverting Amplifier Analysis



Analysis Using the Ideal OP AMP

“Virtual” ground

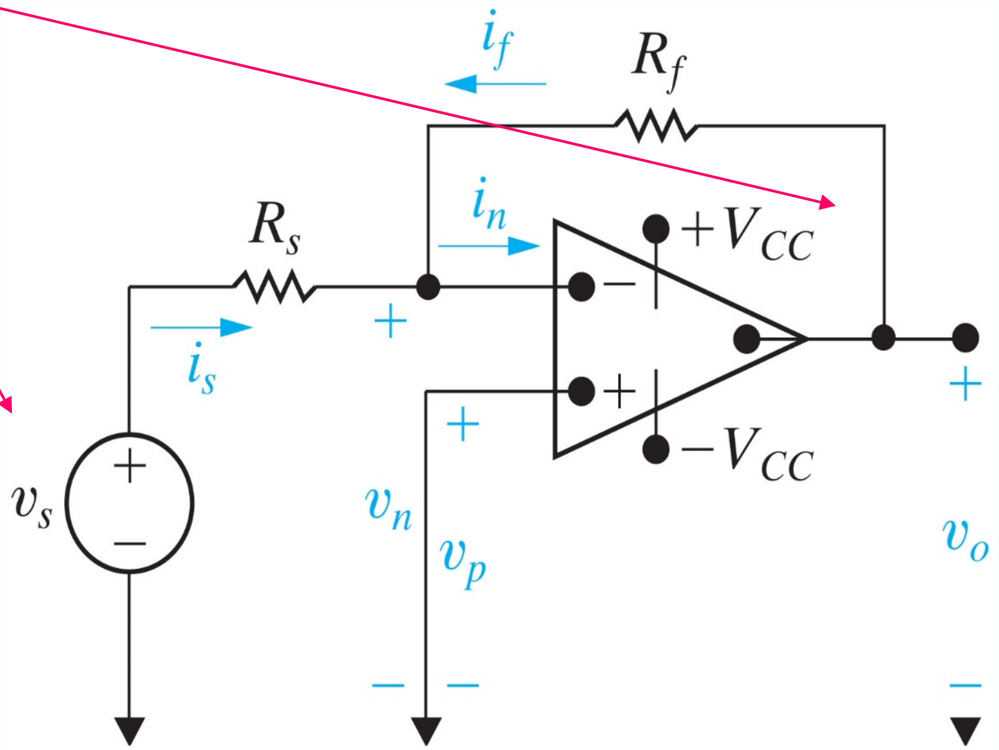


Figure: 05-09

$$i_s + i_f = i_n$$

$$v_n = v_p = 0$$

$$i_s = \frac{v_s}{R_s}$$

$$i_f = \frac{v_o}{R_f}$$

Inverting Amplifier Analysis

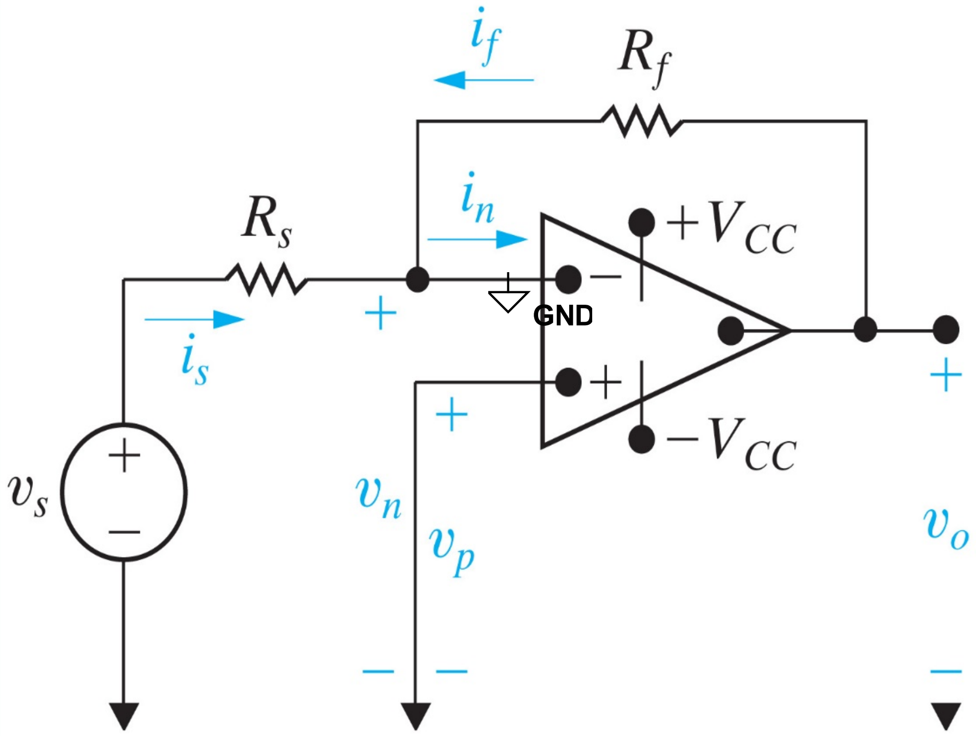


Figure: 05-09

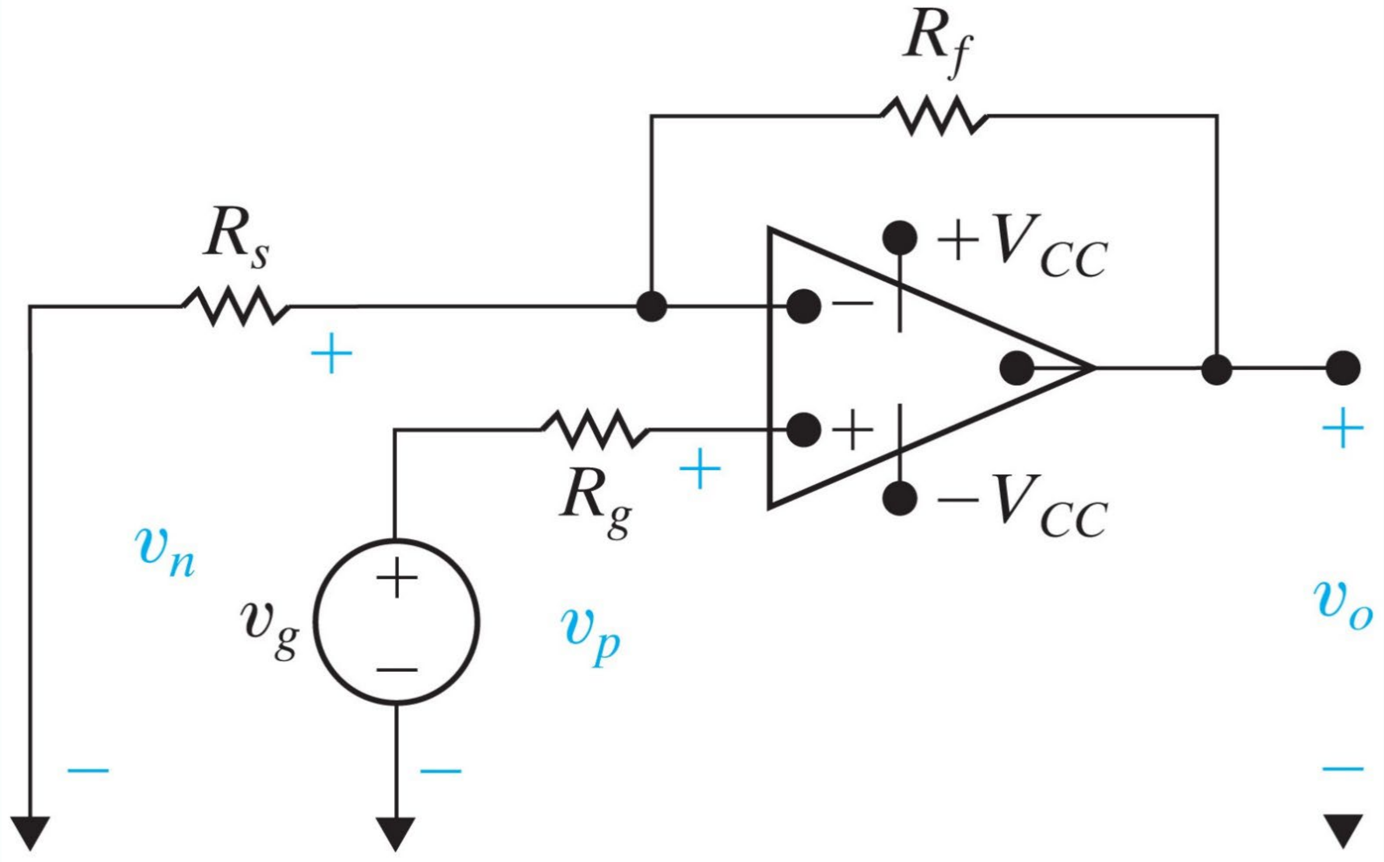
$$i_n = 0$$

$$i_f = -i_s$$

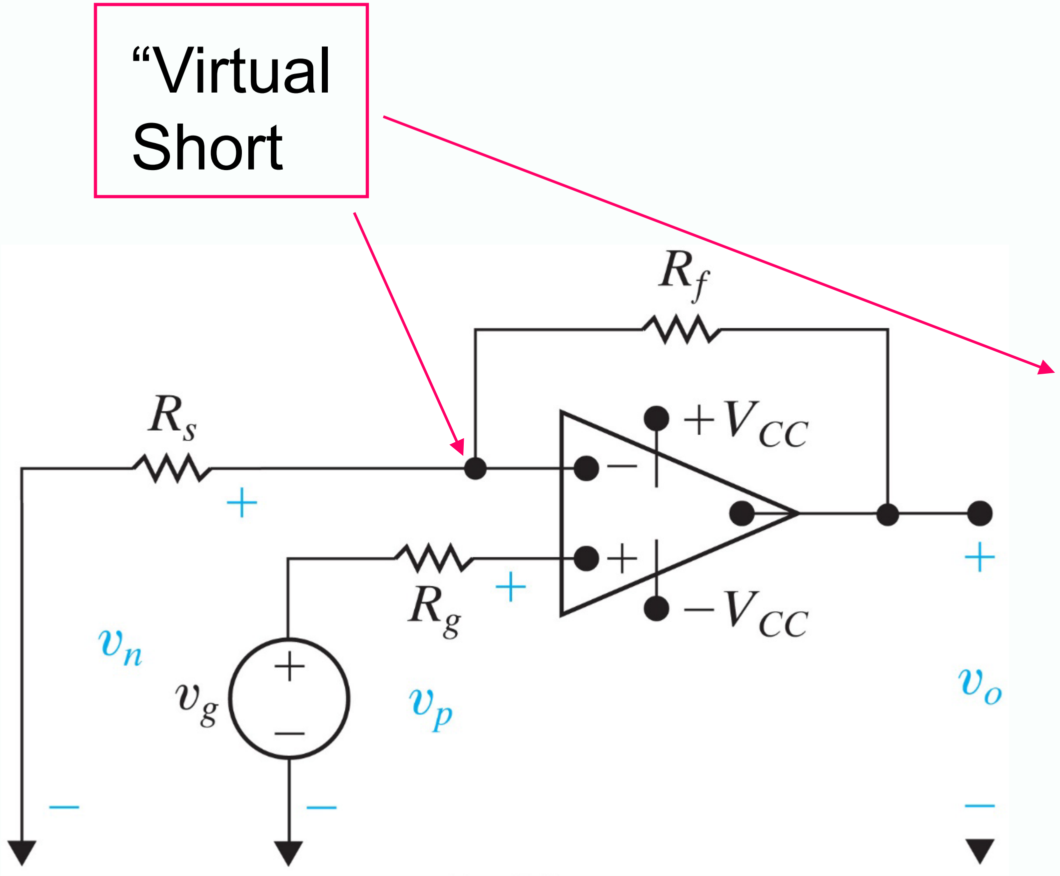
$$\frac{v_o}{R_f} = -\frac{v_s}{R_s}$$

$$\boxed{v_o = -\frac{R_f}{R_s} v_s}$$

Non-Inverting Amplifier Analysis



Analysis Using the Ideal OP AMP



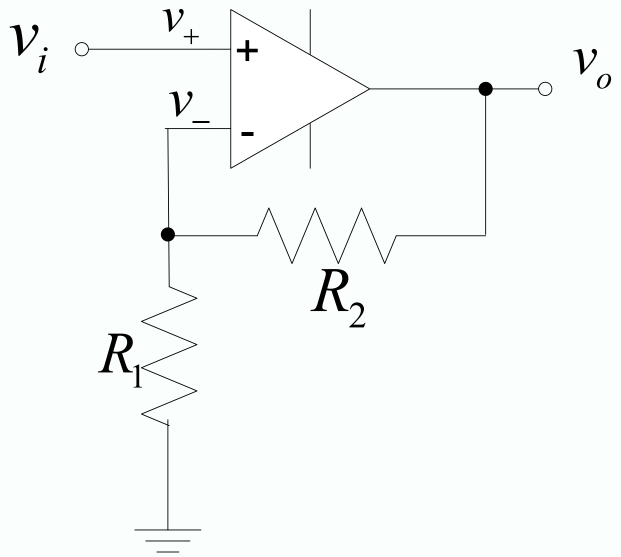
$$v_p = v_g$$

$$v_n = v_p = v_g = v_o \frac{R_s}{R_s + R_f}$$

$$v_o = \frac{R_s + R_f}{R_s} v_g$$

$$\boxed{v_o = \left(1 + \frac{R_f}{R_s}\right) v_g}$$

Non-Inverting Amplifier



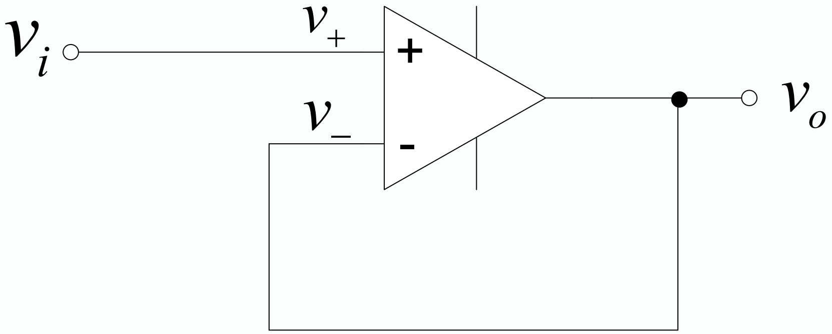
Closed-loop voltage gain

$$A_F = \frac{v_o}{v_i}$$

$$v_i = v_+ = v_- = \frac{R_1}{R_1 + R_2} v_o$$

$$A_F = \frac{v_o}{v_i} = 1 + \frac{R_2}{R_1}$$

Unity-Gain Buffer



Closed-loop voltage gain

$$A_F = \frac{v_o}{v_i}$$

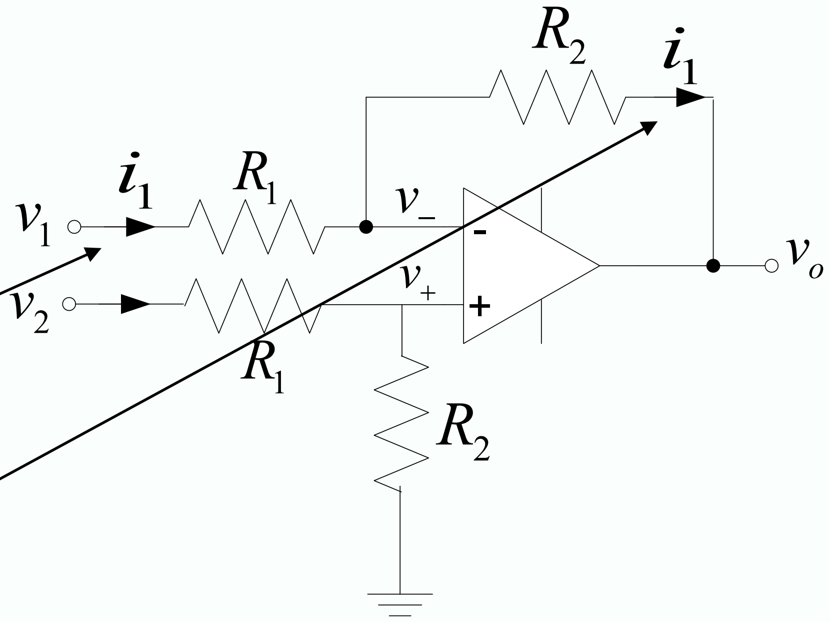
$$v_i = v_+ = v_- = v_o$$

$$A_F = \frac{v_o}{v_i} = 1$$

Used as a "line driver" that transforms a high input impedance (resistance) to a low output impedance. Can provide substantial current gain.

Differential Amplifier

Current into op amp is zero



$$v_- = v_+$$

$$i_1 = \frac{v_1 - v_-}{R_1}$$

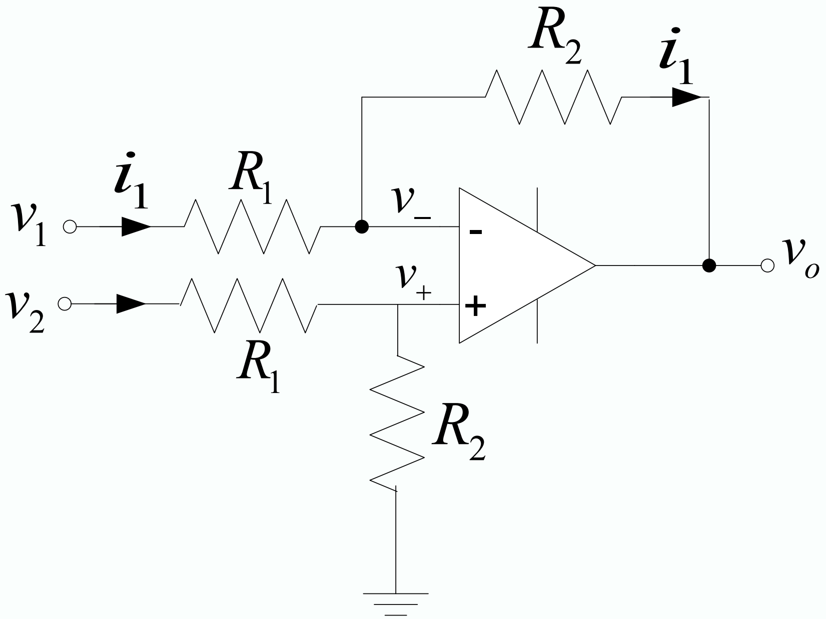
$$i_1 = \frac{v_- - v_0}{R_2}$$

$$v_+ = \frac{R_2}{R_1 + R_2} v_2$$

$$\frac{v_1 - v_+}{R_1} = \frac{v_+ - v_0}{R_2}$$

$$\frac{v_1 - \frac{R_2}{R_1 + R_2} v_2}{R_1} = \frac{\frac{R_2}{R_1 + R_2} v_2 - v_0}{R_2}$$

Differential Amplifier



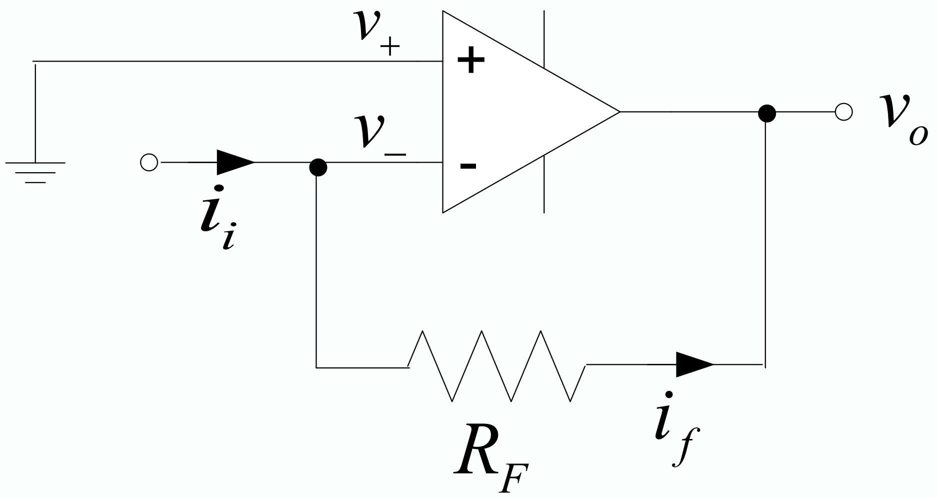
$$\frac{v_1 - \dfrac{R_2}{R_1 + R_2} v_2}{R_1} = \frac{\dfrac{R_2}{R_1 + R_2} v_2 - v_0}{R_2}$$

$$v_0 = -\frac{R_2}{R_1} v_1 + \frac{R_2}{R_1 + R_2} v_2 + \frac{R_2^2}{R_1 \left(R_1 + R_2\right)} v_2$$

$$v_0 = -\frac{R_2}{R_1} v_1 + \frac{R_2}{R_1 + R_2} \left(1 + \frac{R_2}{R_1}\right) v_2$$

$$v_0 = \frac{R_2}{R_1} \left(v_2 - v_1\right)$$

Current-to-Voltage Converter



$$i_i = i_f$$

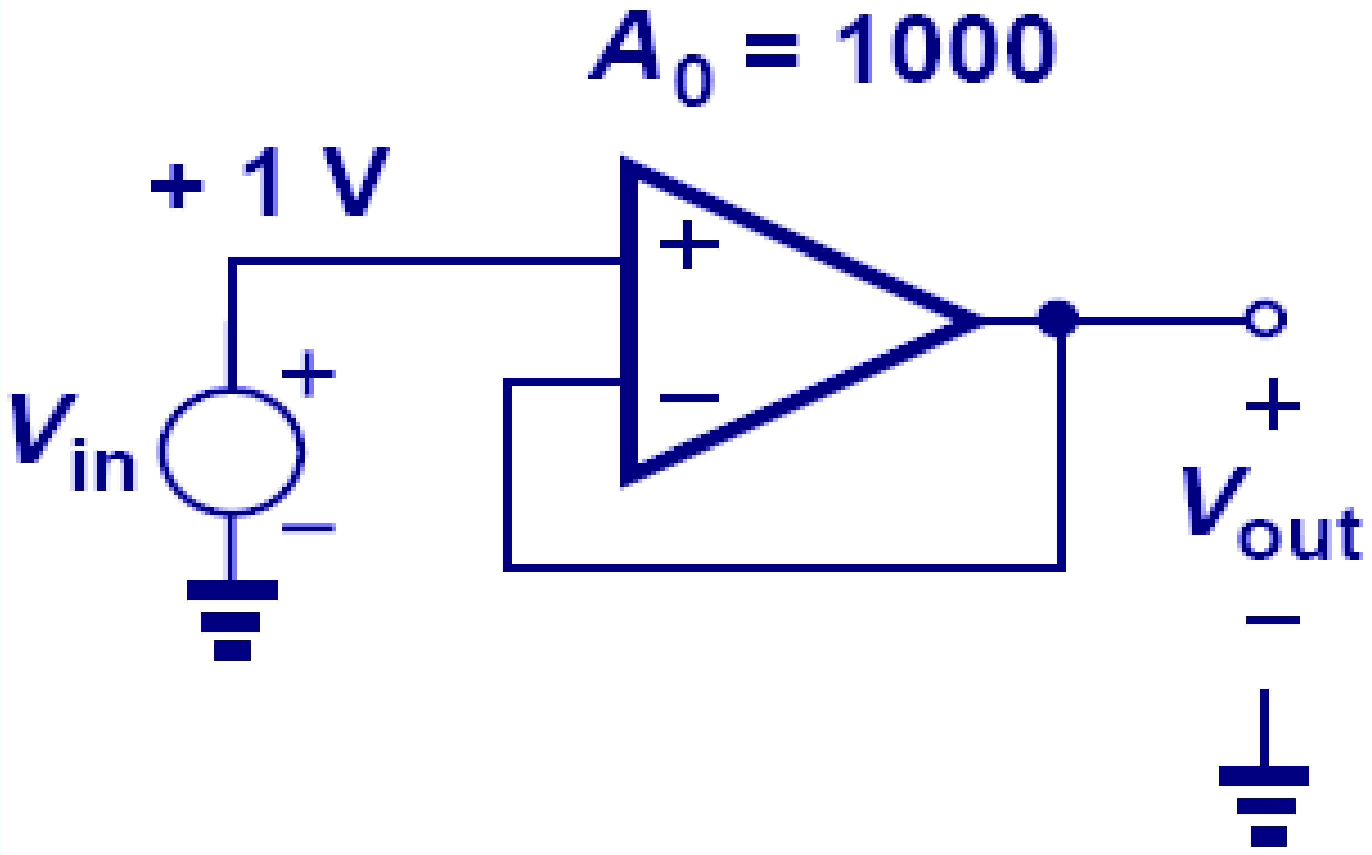
$$v_- = v_+ = 0$$

$$0 - v_0 = i_f R_F$$

$$v_0 = -i_i R_F$$

$$\text{Transresistance } = \Delta v_0 / \Delta i_i = -R_F$$

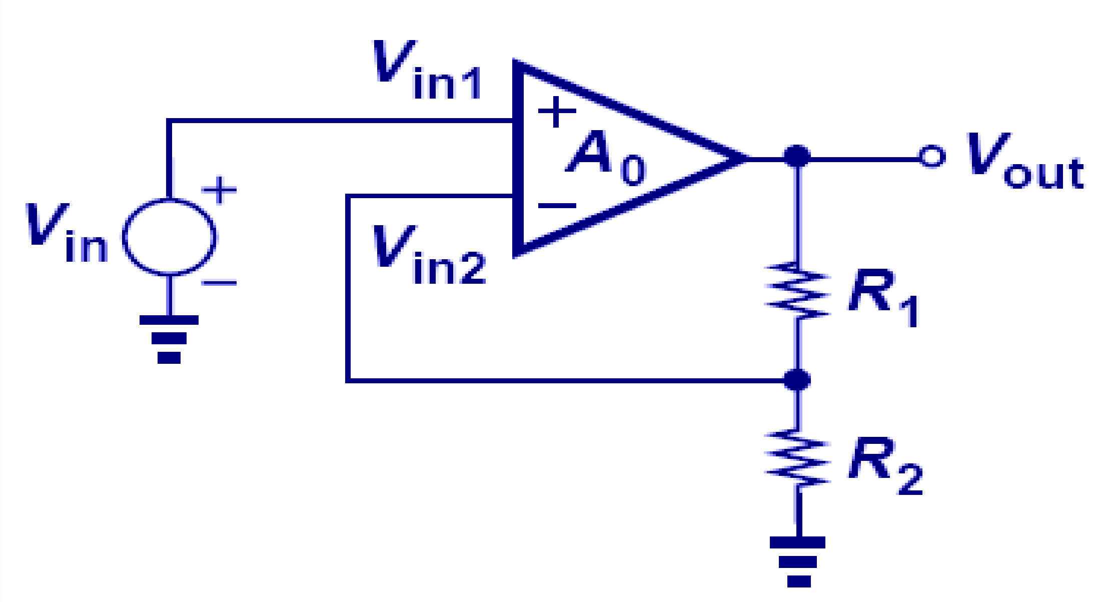
Unity Gain Amplifier



$$V_{out} = A_0(V_{in} - V_{out})$$

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + A_0}$$

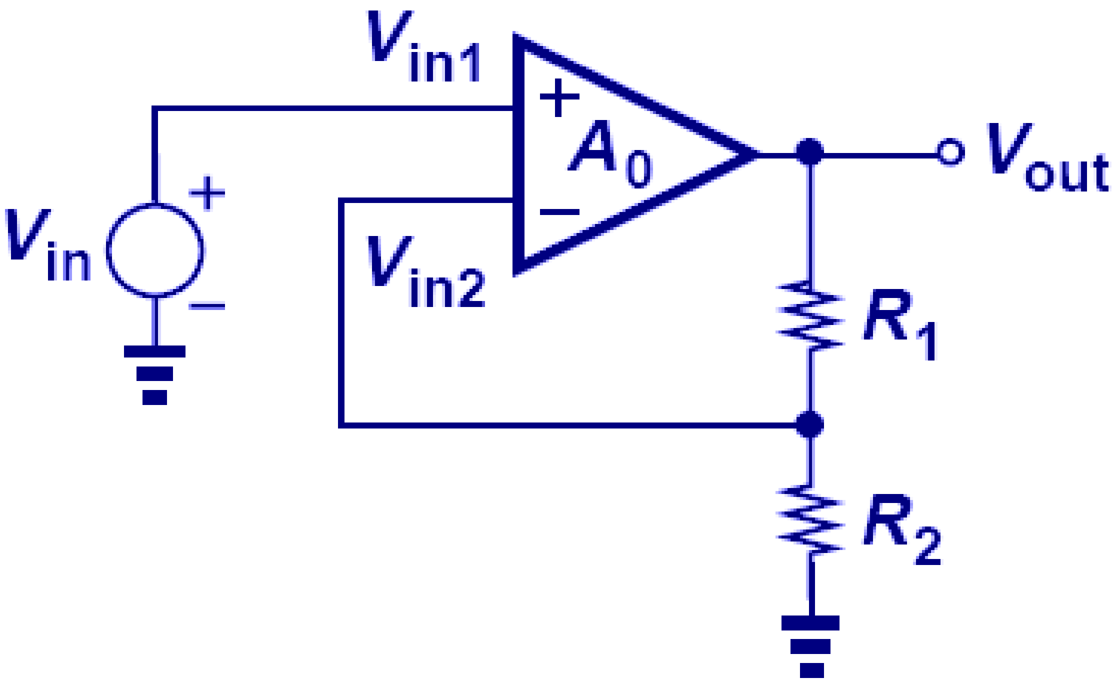
Noninverting Amplifier (Infinite A_0)



$$\frac{V_{out}}{V_{in}} = 1 + \frac{R_1}{R_2}$$

- A noninverting amplifier returns a fraction of output signal thru a resistor divider to the negative input.
- With a high A_o, V_{out}/V_{in} depends only on ratio of resistors, which is very precise.

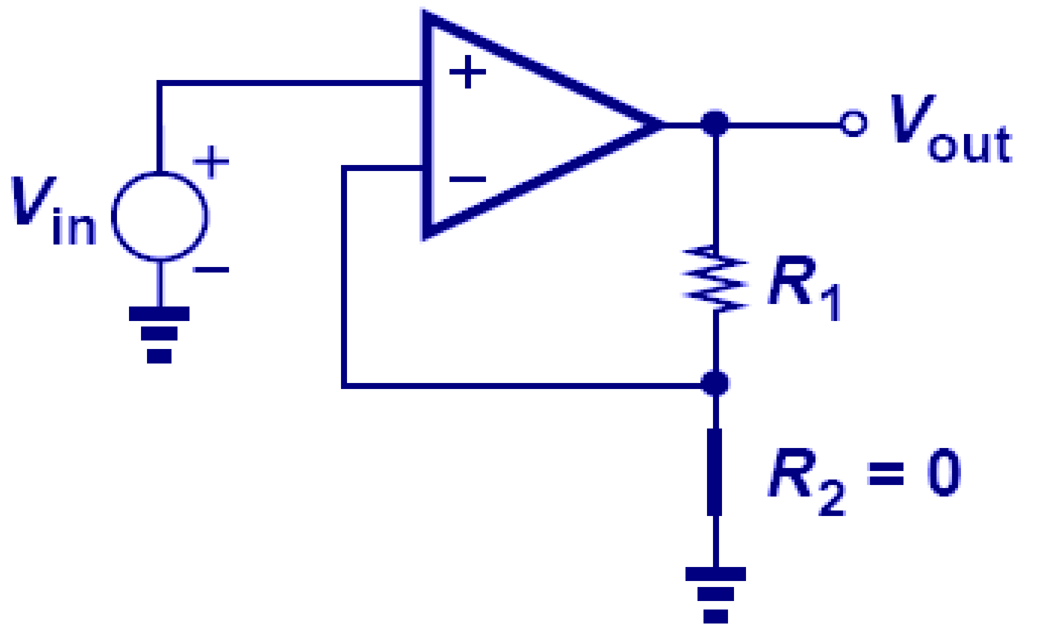
Noninverting Amplifier (Finite A_0)

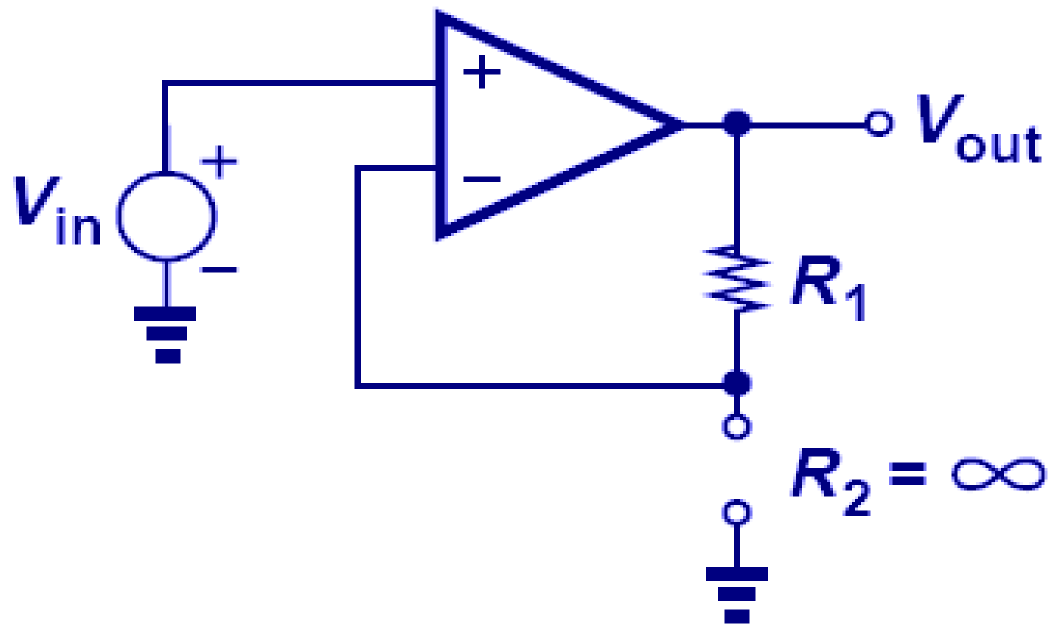


$$\frac{V_{out}}{V_{in}} \approx \left(1+\frac{R_1}{R_2}\right)\left[1-\left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0}\right]$$

- The error term indicates that the larger the closed-loop gain, the less accurate the circuit becomes.

Extreme Cases of R_2 (Infinite A_0)

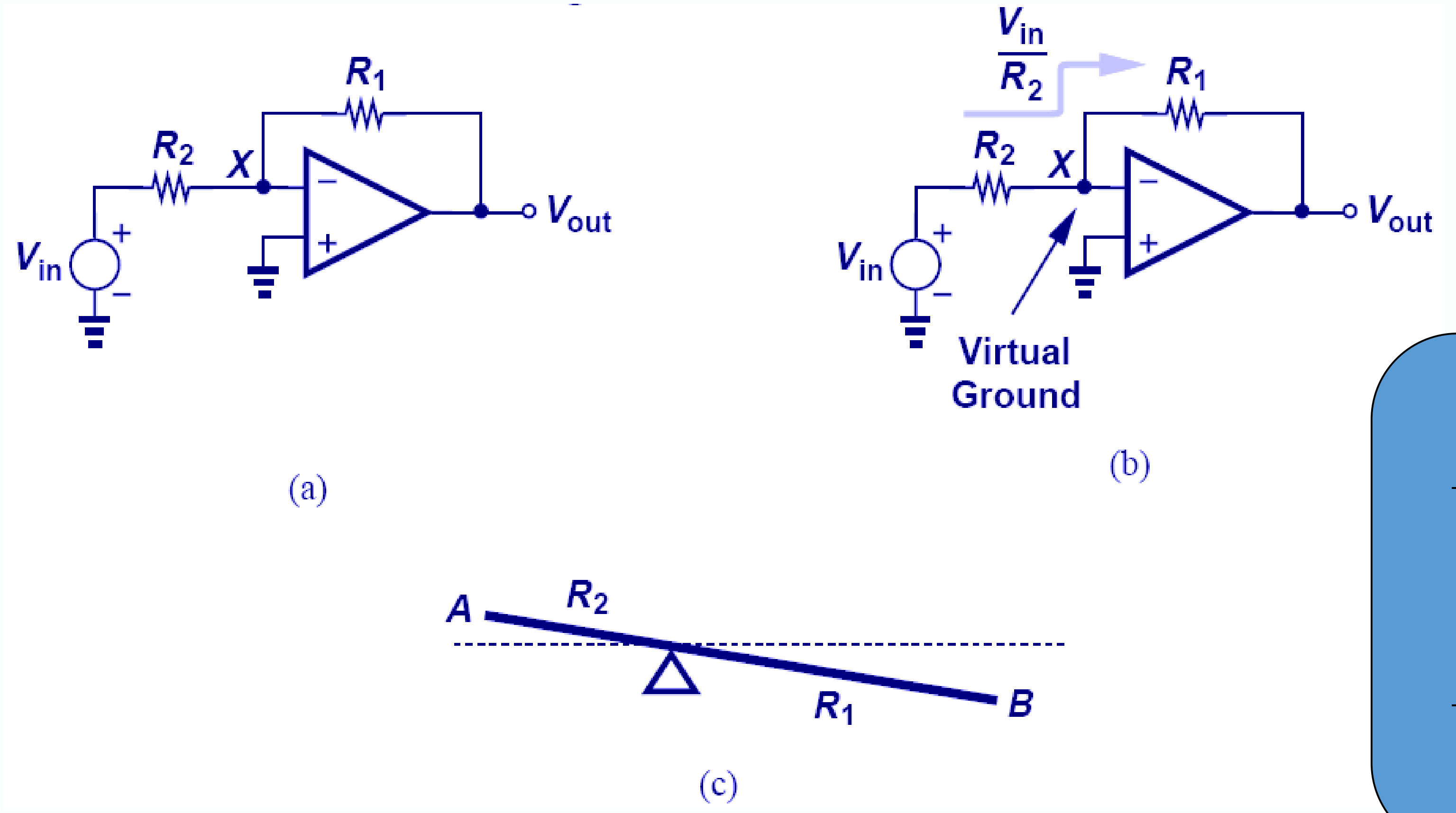




- If R_2 is zero, the loop is open and V_{out}/V_{in} is equal to the intrinsic gain of the op amp.

If R_2 is infinite, the circuit becomes a unity-gain amplifier and V_{out}/V_{in} becomes equal to one.

Inverting Amplifier

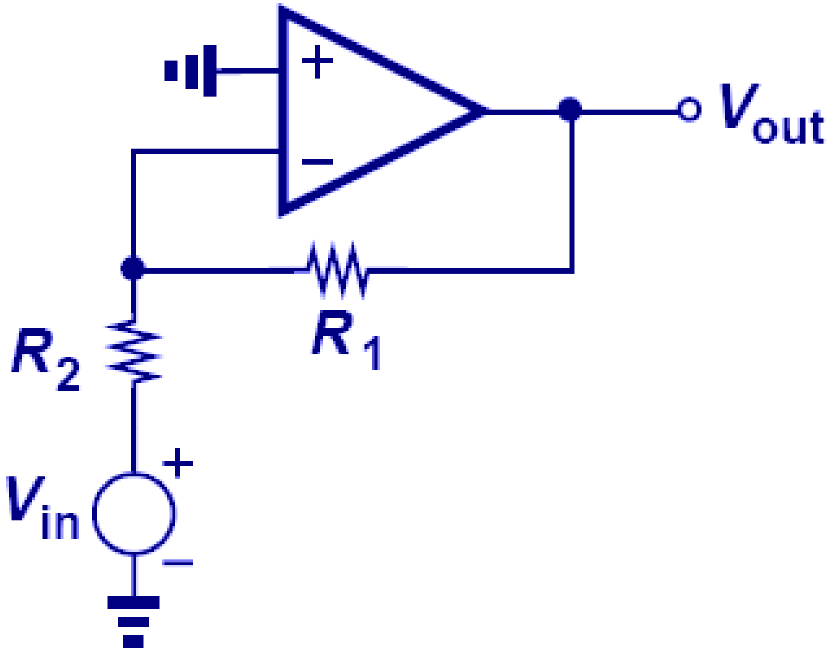


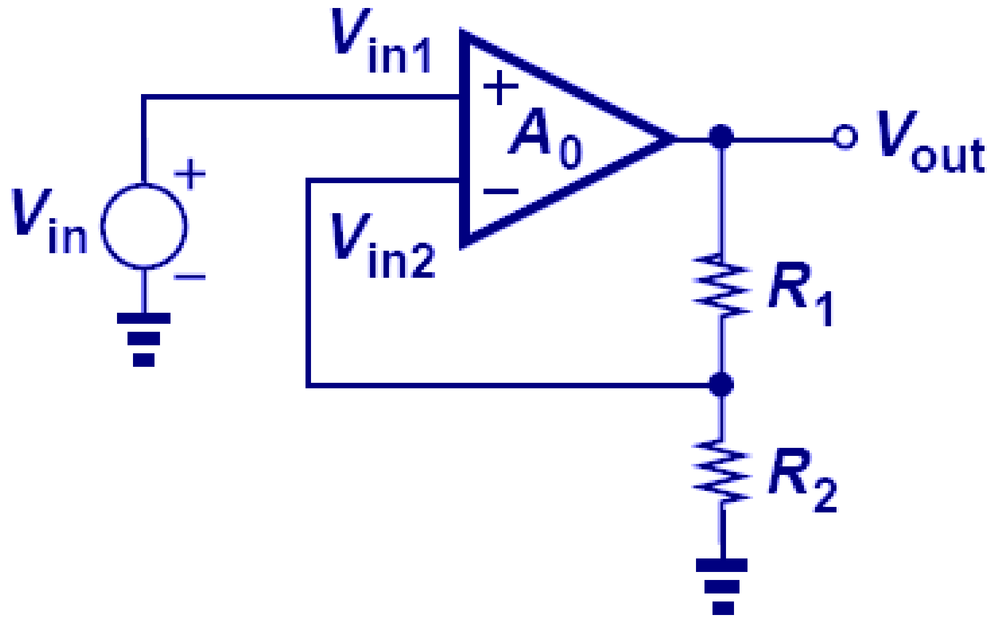
$$\frac{0 - V_{out}}{R_1} = \frac{V_{in}}{R_2}$$

$$\frac{V_{out}}{V_{in}} = \frac{-R_1}{R_2}$$

- Infinite A_0 forces the negative input to be a virtual ground.

Another View of Inverting Amplifier

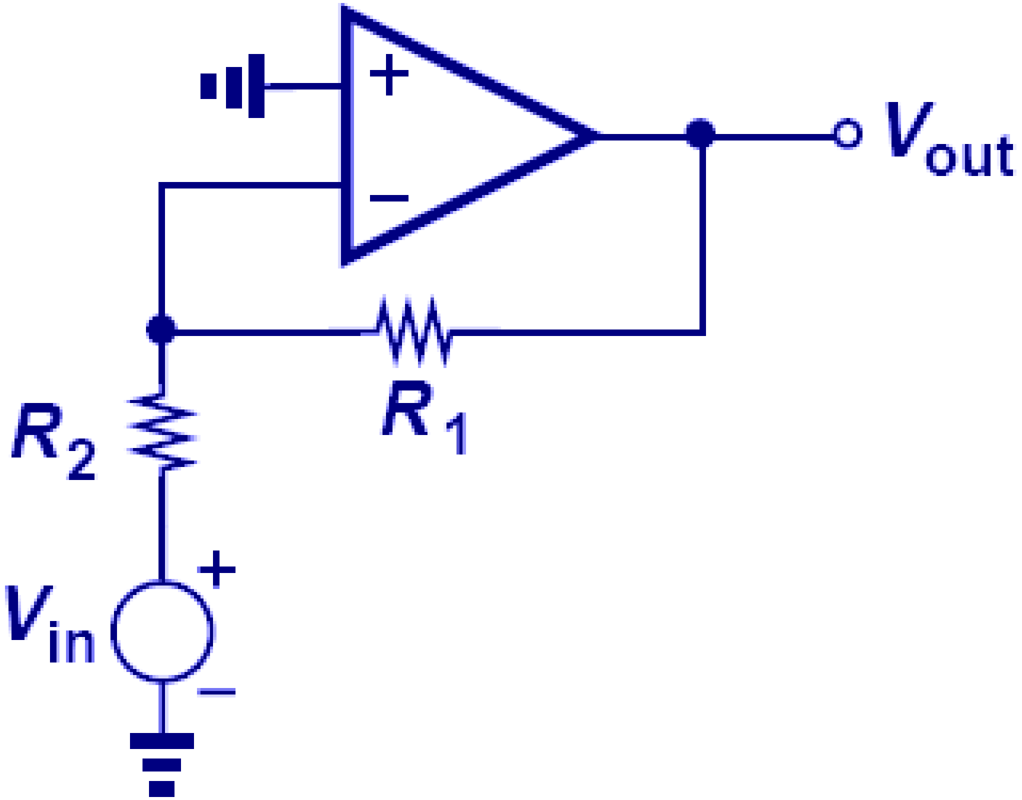




Inverting

Noninverting

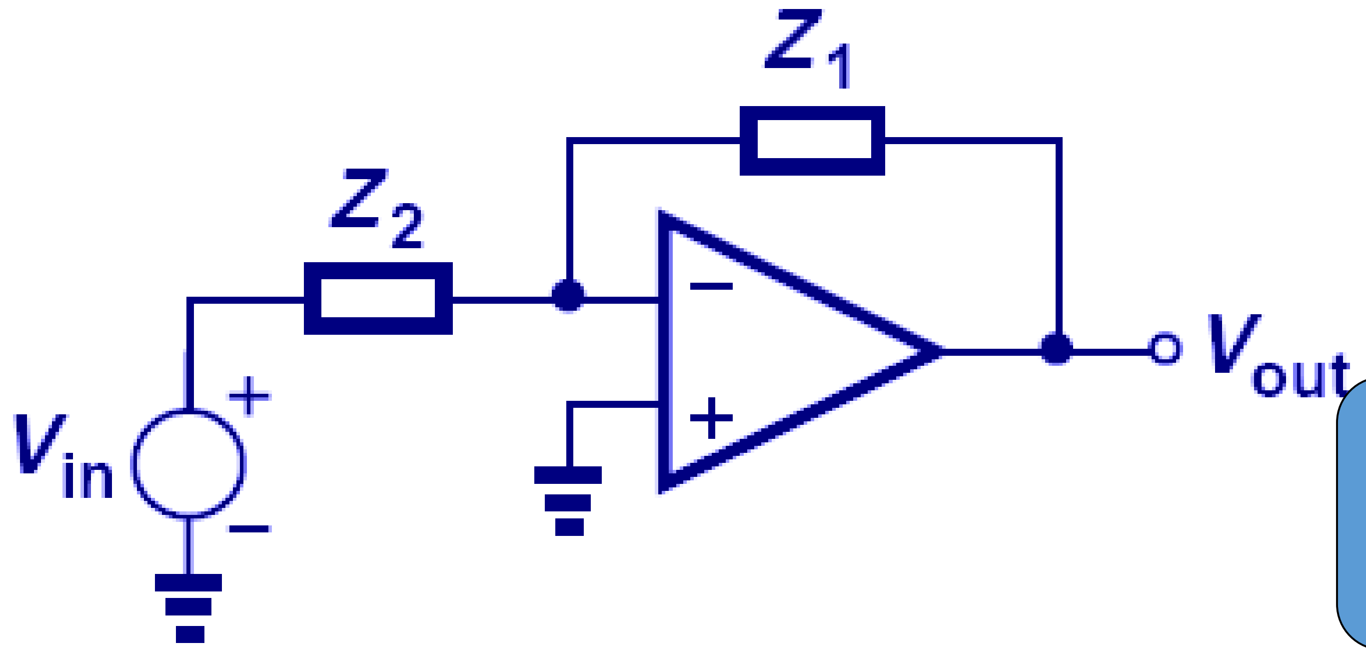
Gain Error Due to Finite A_0



$$\frac{V_{out}}{V_{in}} \approx -\frac{R_1}{R_2}\left[1-\frac{1}{A_0}\left(1+\frac{R_1}{R_2}\right)\right]$$

The larger the closed loop gain, the more inaccurate the circuit is.

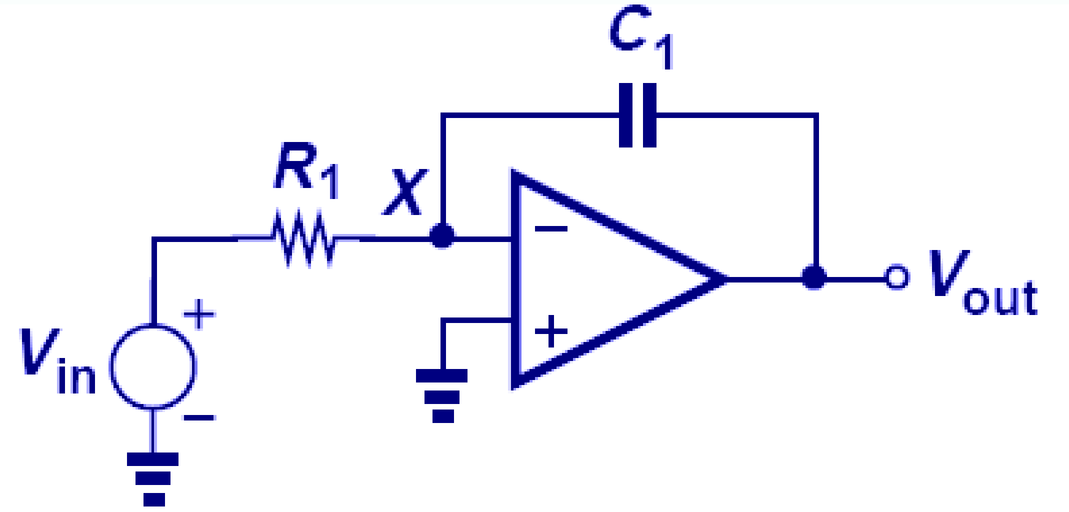
Complex Impedances Around the Op Amp

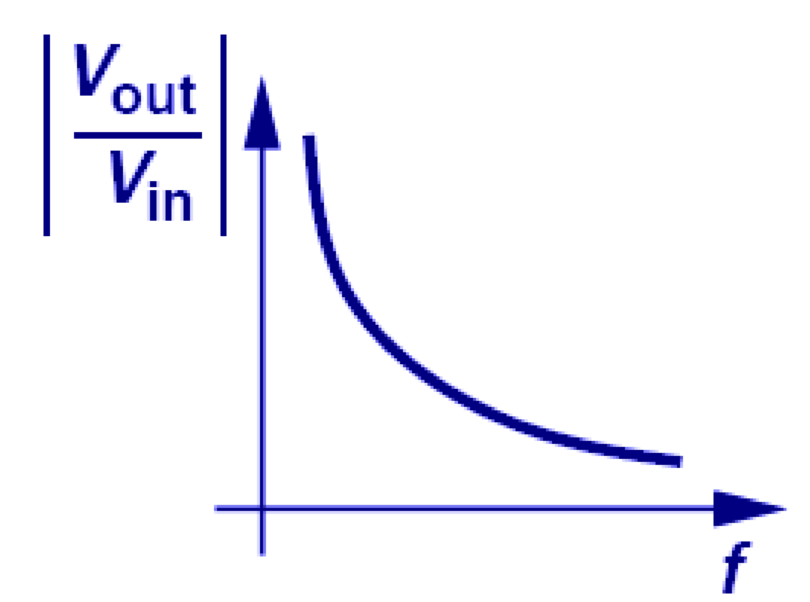


$$\frac{V_{out}}{V_{in}} \approx -\frac{Z_1}{Z_2}$$

- The closed-loop gain is still equal to the ratio of two impedances.

Integrator

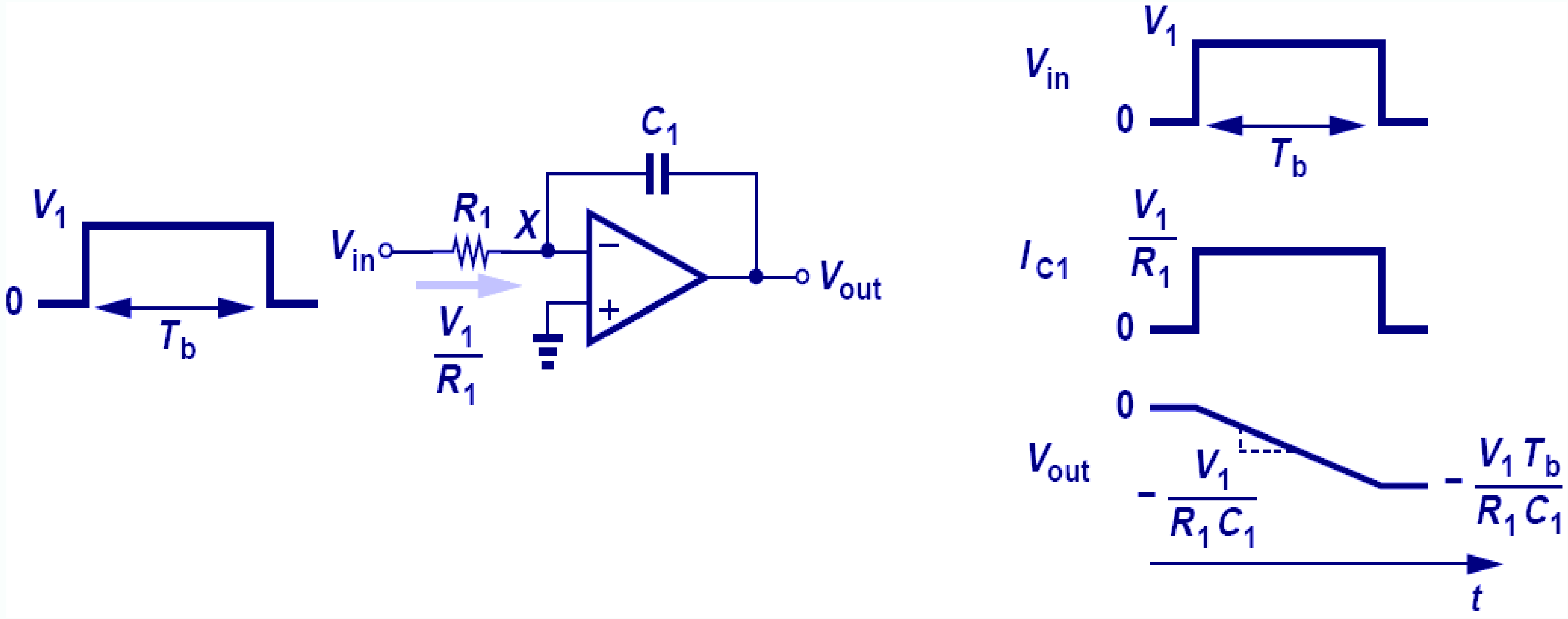




$$\frac{V_{out}}{V_{in}} = -\frac{1}{R_1 C_1 s}$$

$$V_{out} = -\frac{1}{R_1 C_1}\int V_{in}\,dt$$

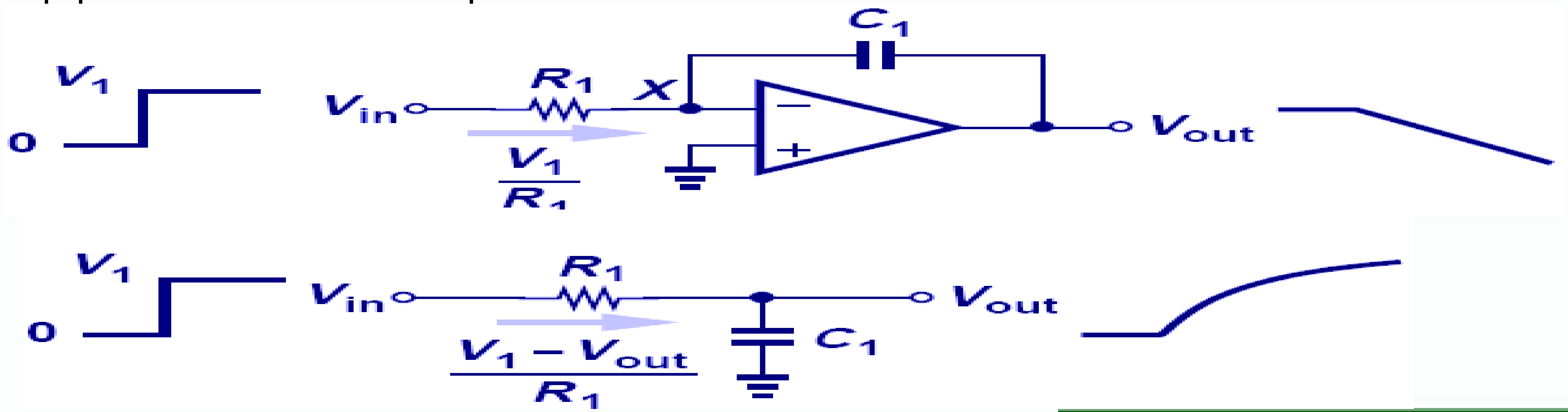
Integrator with Pulse Input



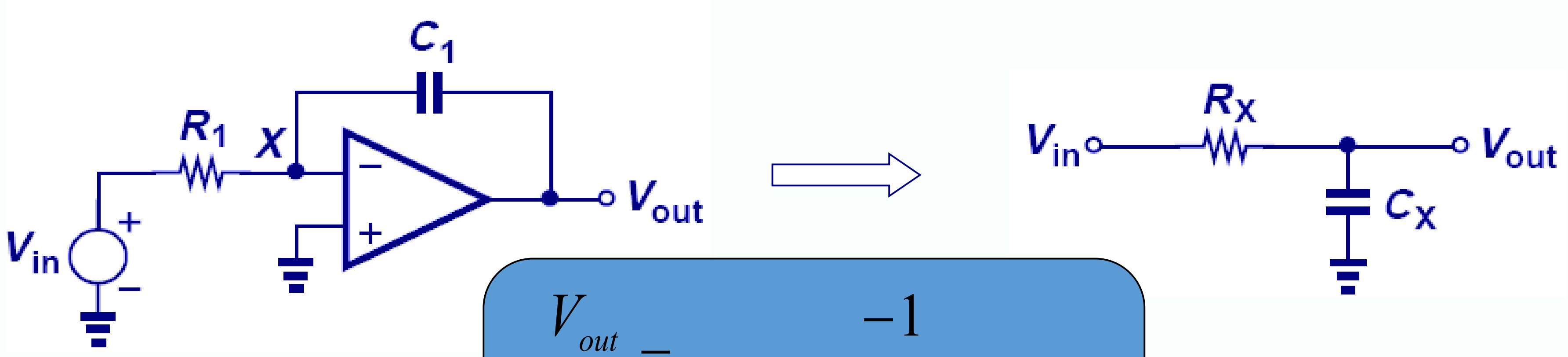
$$V_{out} = -\frac{1}{R_1C_1}\int V_{in}dt = -\frac{V_1}{R_1C_1}t \quad 0 < t < T_b$$

Comparison of Integrator and RC Lowpass Filter

- The RC low-pass filter is actually a "passive" approximation to an integrator.
- With the RC time constant large enough, the RC filter output approaches a ramp.



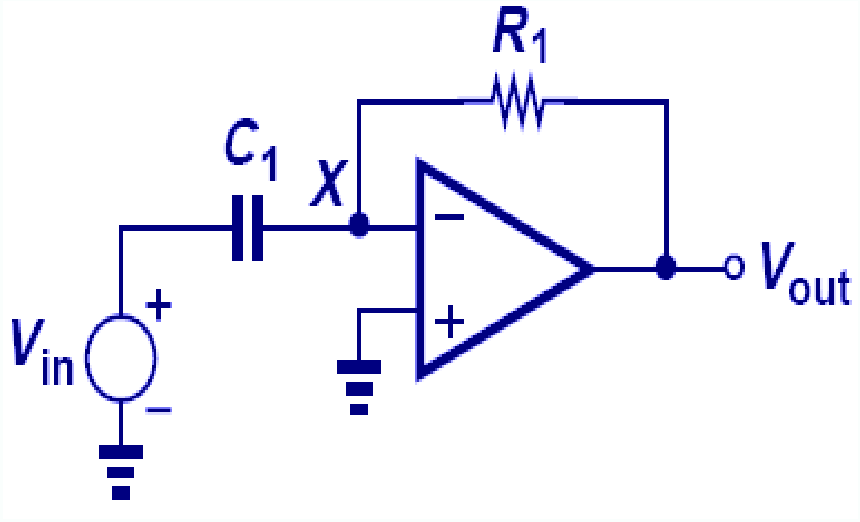
Lossy Integrator

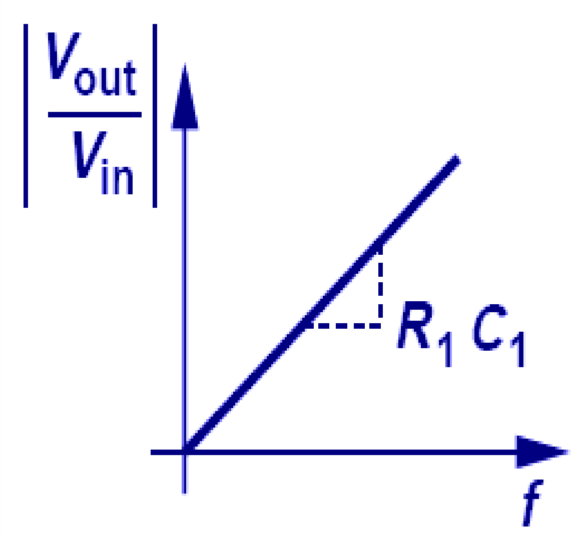


$$\frac{V_{out}}{V_{in}} = \frac{-1}{\frac{1}{A_0} + \left(1 + \frac{1}{A_0}\right) R_1 C_1 s}$$

- When finite op amp gain is considered, the integrator becomes lossy as the pole moves from the origin to $-1/[(1+A_0)R_1C_1]$.
- It can be approximated as an RC circuit with C boosted by a factor of A_0+1.

Differentiator

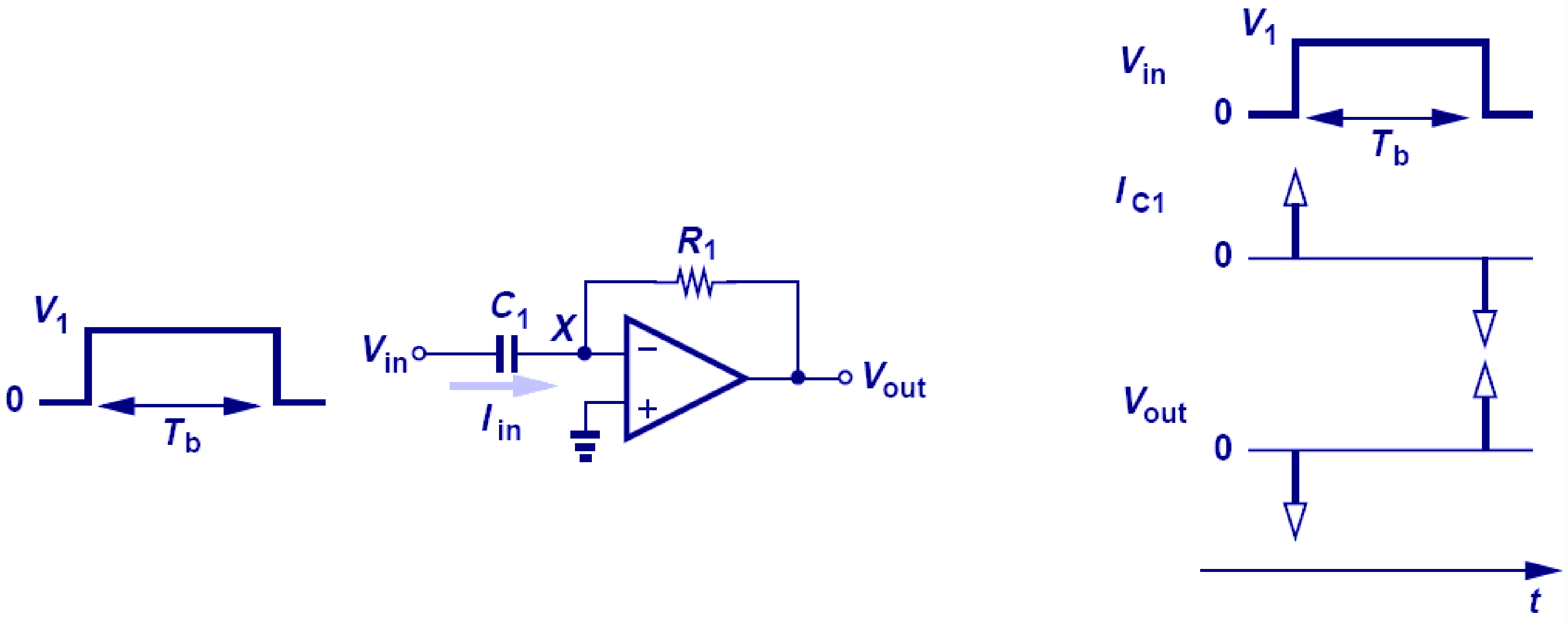




$$V_{out} = -R_1 C_1 \frac{dV_{in}}{dt}$$

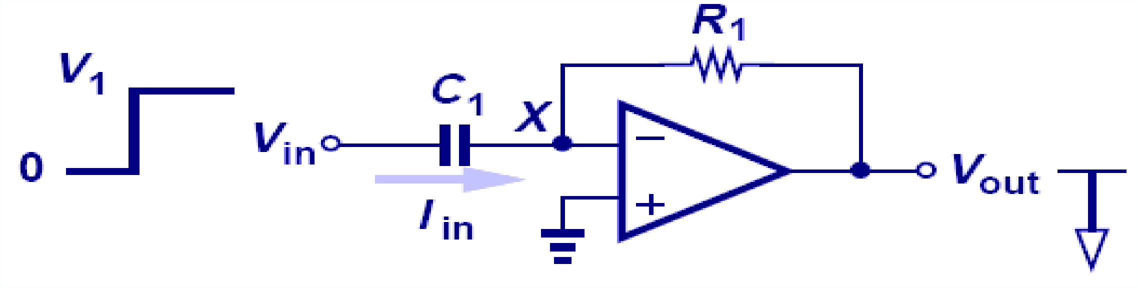
$$\frac{V_{out}}{V_{in}} = -\frac{R_1}{\frac{1}{C_1 s}} = -R_1 C_1 s$$

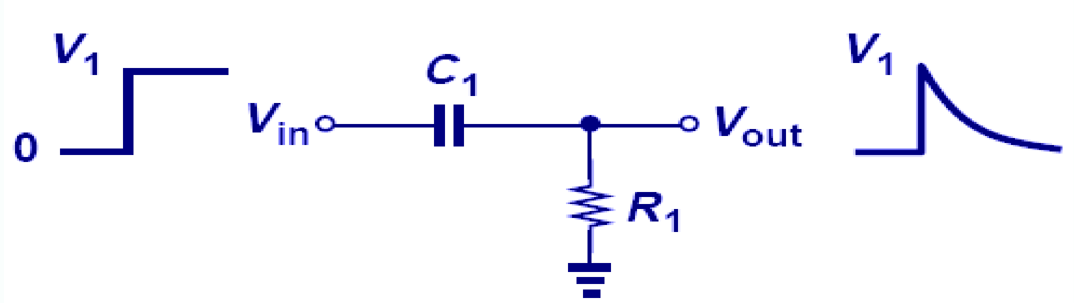
Differentiator with Pulse Input



$$V_{out} = \mp R_1 C_1 V_1 \delta(t)$$

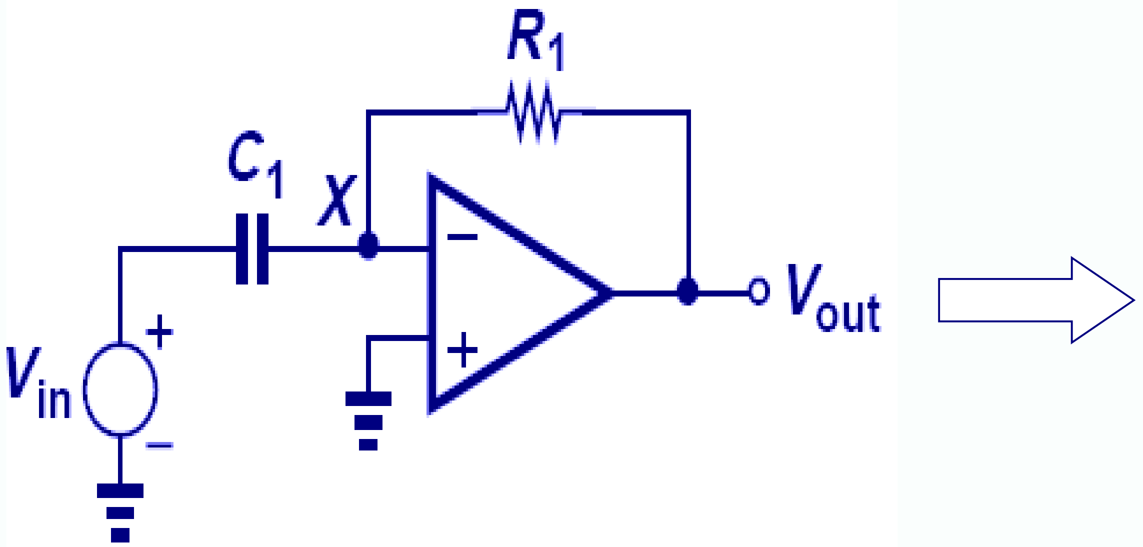
Comparison of Differentiator and High-Pass Filter

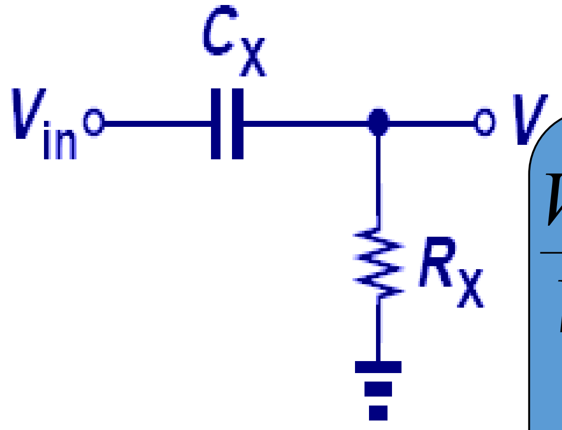




- The RC high-pass filter is actually a passive approximation to the differentiator.
- When the RC time constant is small enough, the RC filter approximates a differentiator.

Lossy Differentiator

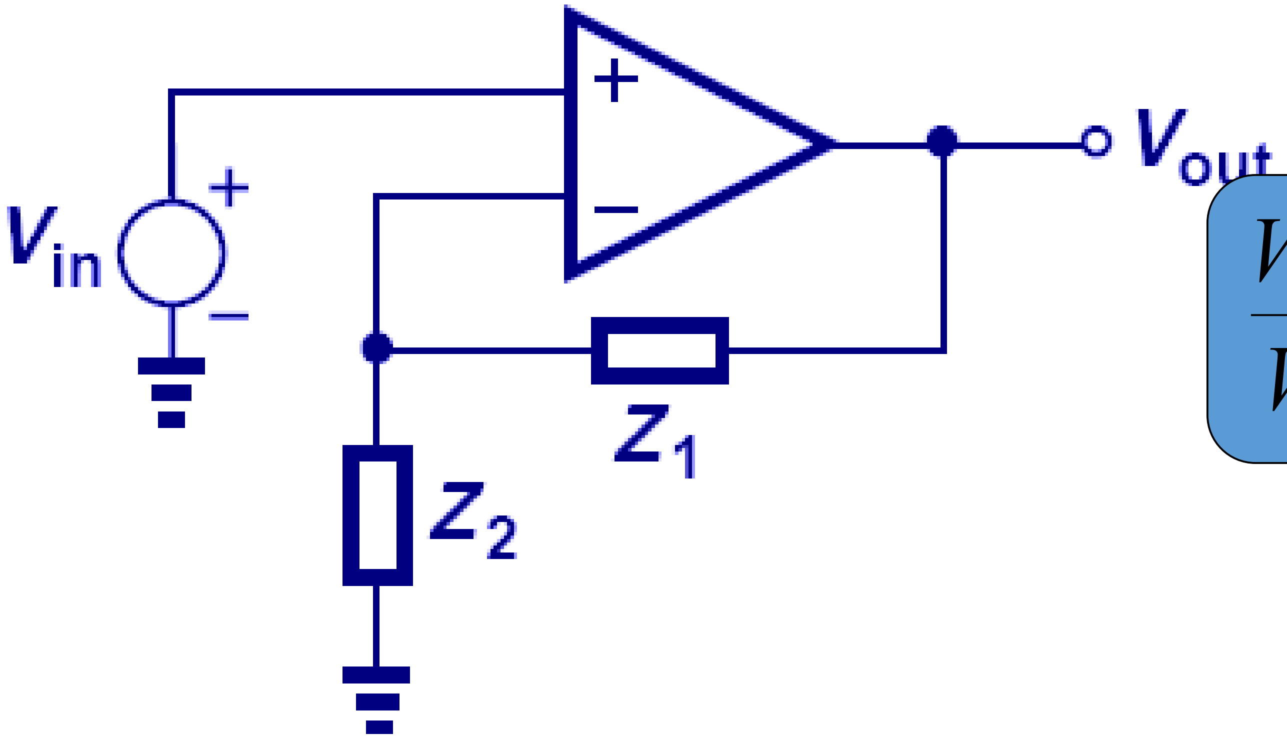




$$\frac{V_{out}}{V_{in}} = \frac{-R_1C_1s}{1+\frac{1}{A_0}+\frac{R_1C_1s}{A_0}}$$

- When finite op amp gain is considered, the differentiator becomes lossy as the zero moves from the origin to $-(A_0+1)/R_1C_1$.
- It can be approximated as an RC circuit with R reduced by a factor of (A_0+1).

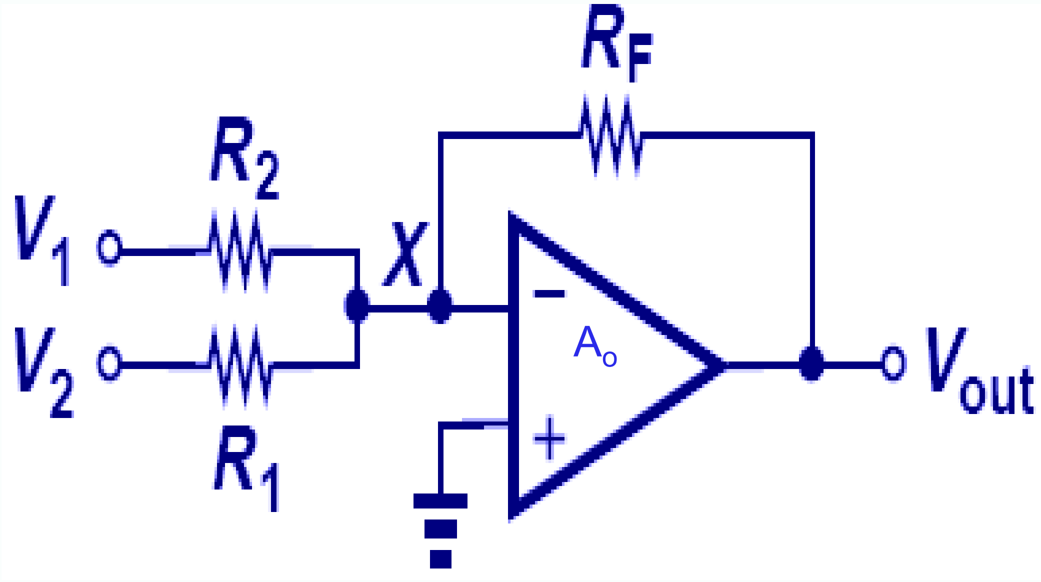
Op Amp with General Impedances



$$\frac{V_{out}}{V_{in}} = 1 + \frac{Z_1}{Z_2}$$

- This circuit cannot operate as ideal integrator or differentiator.

Voltage Adder



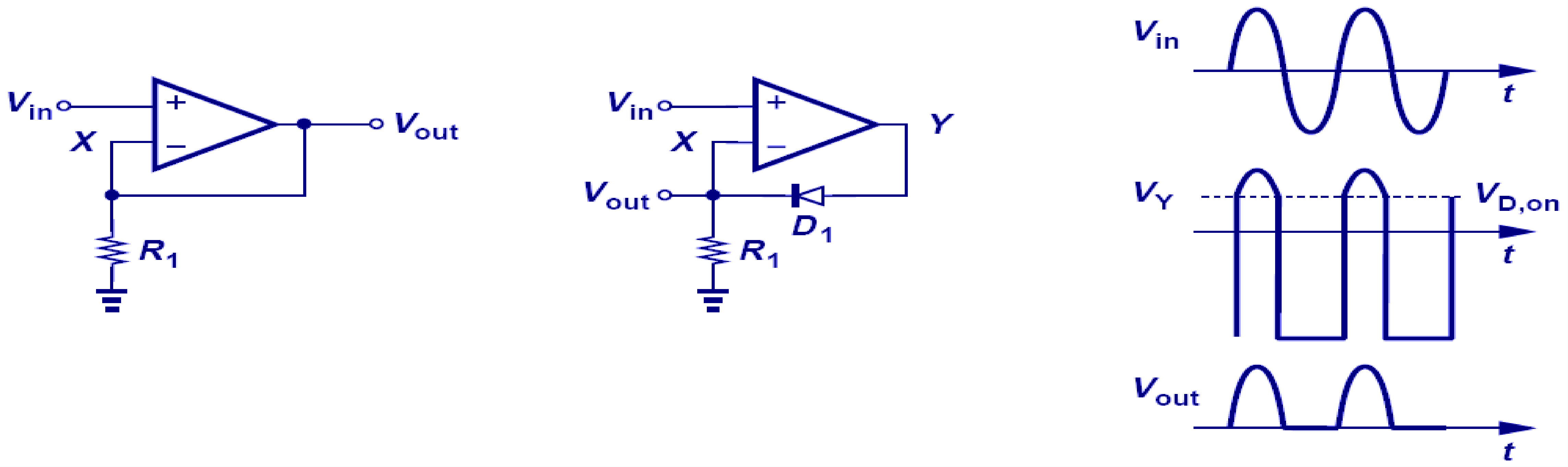
$$V_{out} = -R_F\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

$$V_{out} = \frac{-R_F}{R}(V_1 + V_2)$$

If $R_1 = R_2 = R$

- If A_o is infinite, X is pinned at ground, currents proportional to V_1 and V_2 will flow to X and then across R_F to produce an output proportional to the sum of two voltages.

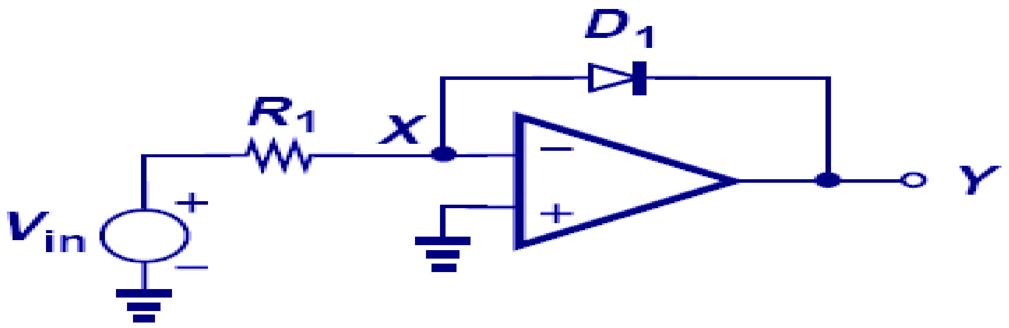
Precision Rectifier

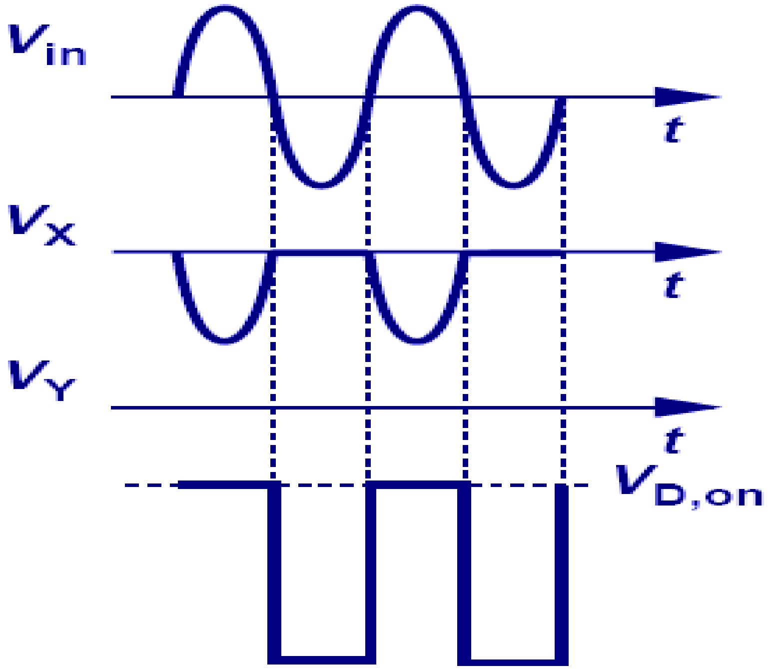


- When V_{in} is positive, the circuit in b) behaves like that in a), so the output follows input.
- When V_{in} is negative, the diode opens, and the output drops to zero. Thus performing rectification.

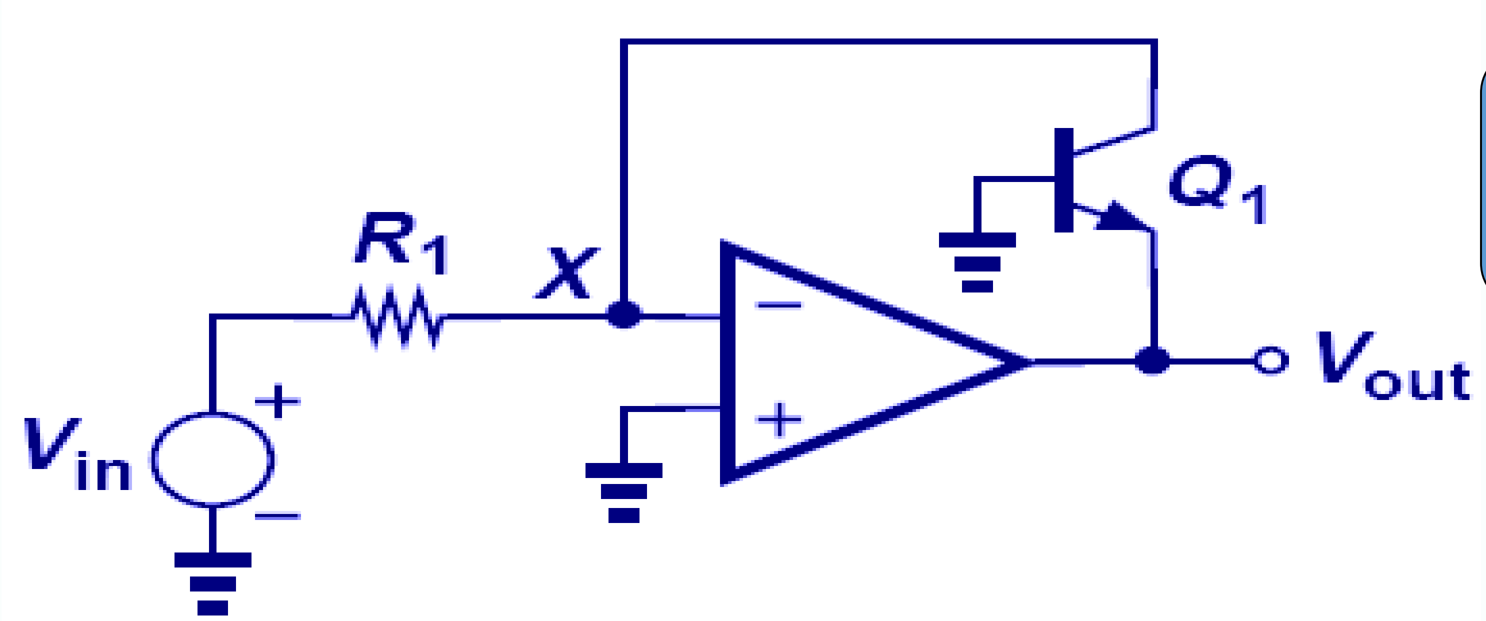
Inverting Precision Rectifier

- When V_{in} is positive, the diode is on, V_y is pinned around $V_{D,on}$, and V_x at virtual ground.
- When V_{in} is negative, the diode is off, V_y goes extremely negative, and V_x becomes equal to V_{in}.





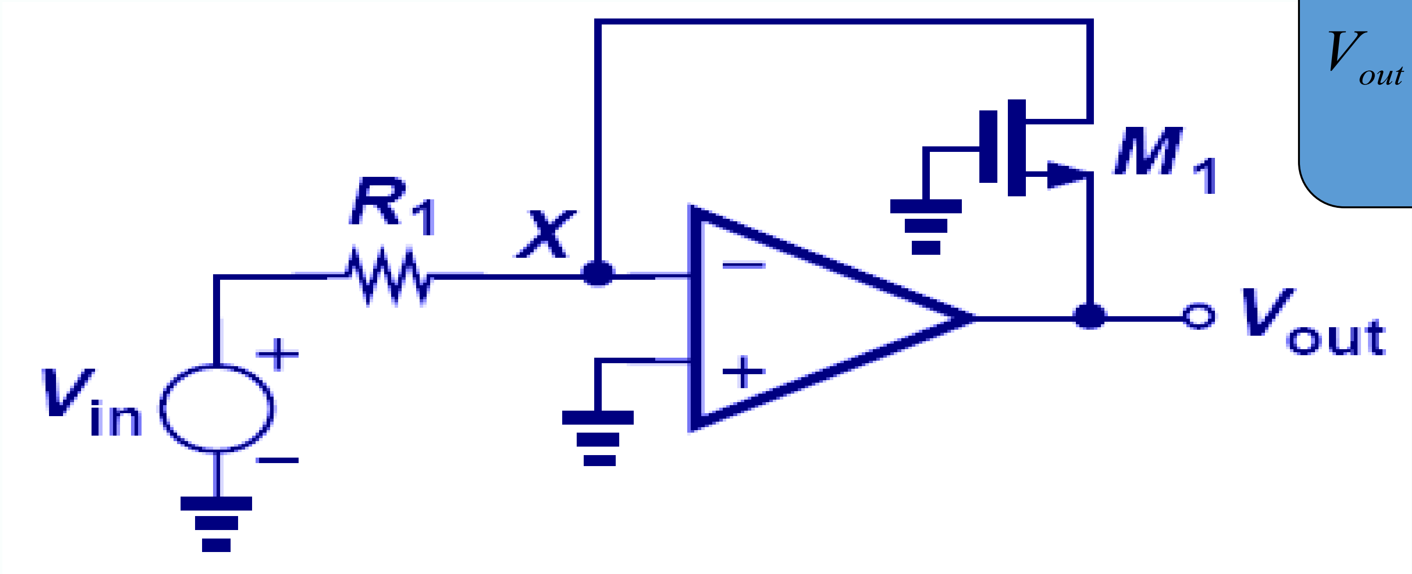
Logarithmic Amplifier



$$V_{out} = -V_T \ln \frac{V_{in}}{R_1 I_S}$$

- By inserting a bipolar transistor in the loop, an amplifier with logarithmic characteristic can be constructed.
- This is because the current to voltage conversion of a bipolar transistor is a natural logarithm.

Square-Root Amplifier



$$V_{out} = -\sqrt{\frac{2V_{in}}{\mu_n C_{ox} \frac{W}{L} R_1}} - V_{TH}$$

- By replacing the bipolar transistor with a MOSFET, an amplifier with a square-root characteristic can be built.
- This is because the current to voltage conversion of a MOSFET is square-root.

Thank You

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