ANSAH OFEI KWADWO

I/N: 3548318 REF N^{O}: 20589490

<u>MATH 252: CALCULUS WITH SEVERAL VARIABLES</u>

<u>Question 1</u>

a) Suppose that $f(x,t) = 2e^{-t/6}$ is the monthly rate of change of the price per barrel of oil. If *x* is the number of millions of barrels and *t* is the number of months since January, 2016, find

$$\int_0^{10}\int_0^{4} 2\boldsymbol{e}^{\frac{-t}{6}}dt\,dx$$

Interpret your answer.

b) Find the sum of monthly rate of change of price per barrel of oil from January, 2016 to March, 2016 if the quantum of oil increases from 2 million barrels to 8 million barrels.

Solution

a) *x* is the number of millions of barrels
t is the number of months from January

$$\int_0^{10}\int_0^{4} 2\boldsymbol{e}^{\frac{-t}{6}}dt\,dx$$

$$\int_0^{10}\left(\int_0^{4} 2\boldsymbol{e}^{\frac{-t}{6}}dt\right)dx$$

$$\int_0^{10}\left[-12e^{-t/6}\right]_0^4 dx$$

$$\int_0^{10}\left(-12\left(\boldsymbol{e}^{\frac{-4}{6}} - e^0\right)\right)dx$$

$$\int_0^{10}\left(-12\left(\boldsymbol{e}^{\frac{-2}{3}} - 1\right)\right)dx$$

$$\int_0^{10} 5.8389945716089\,dx$$

$$[5.83899457160892x]_0^{10}$$

58.389945716089 or 58.39

This means the sum of the monthly rate of change of the price per barrel of oil from January 2016 to May 2016 (4 months ahead) with an increase in number of barrels from 0 to 10 million barrels is <u>58.39 price/barrel</u>.

b) <u>Initial values</u>
x => 2
t => 0

<u>Final values</u>
x => 8
t => 2

The equation: $\int\int 2\boldsymbol{e}^{-\frac{t}{6}}\,dt\,dx$ then becomes:

$$\int_2^8\int_0^2 2\boldsymbol{e}^{\frac{-t}{6}}dt\,dx$$

$$\int_2^8\left(\int_0^2 2\boldsymbol{e}^{\frac{-t}{6}}dt\right)dx$$

$$\int_2^8\left[-12e^{-t/6}\right]_0^2 dx$$

$$\int_2^8 -12\left(\boldsymbol{e}^{\frac{-2}{6}}-\boldsymbol{e}^0\right)dx$$

$$\int_2^8 -12\left(\frac{1}{\sqrt[3]{e}}-1\right)dx$$

$$\int_2^8 3.4016242731145\,dx$$

$$[3.4016242731145x]_2^8$$

$$(3.4016242731145\times 8)-(3.4016242731145\times 2)$$

$$20.409745638687\ or\ 20.41$$

This makes the sum of the monthly change in price per barrel of oil is <u>20.41 price/barrel</u>.

Question 2

If R is the total resistance of the two resistors, connected in parallel, with resistance R_1 and R_2 ,then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If the resistances are measured in ohms as $R_1 = 10\Omega$ and $R_2 = 20\Omega$ with an error 0.2 in each case, calculate the maximum error in R.

Solution

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

In order to obtain the maximum error in R, the differential of R must be determined.

$$dR = \frac{\partial R}{\partial R_1} dR_1 + \frac{\partial R}{\partial R_2} dR_2$$

$$R = R_1 R_2 (R_1 + R_2)^{-1}$$

For $\frac{\partial R}{\partial R_1}$:

$$\frac{\partial R}{\partial R_1} = R_2 (R_1 + R_2)^{-1} + (-1)(1)(R_1 R_2)(R_1 + R_2)^{-2}$$

$$\frac{\partial R}{\partial R_1} = \frac{R_2}{(R_1 + R_2)} - \frac{R_1 R_2}{(R_1 + R_2)^2}$$

$$\frac{\partial R}{\partial R_1} = \frac{{R_2}^2}{(R_1 + R_2)^2}$$

For $\frac{\partial R}{\partial R_2}$:

$$\frac{\partial R}{\partial R_2} = R_1(R_1+R_2)^{-1} + (-1)(1)(R_1R_2)(R_1+R_2)^{-2}$$

$$\frac{\partial R}{\partial R_2} = \frac{R_1}{(R_1+R_2)} - \frac{R_1R_2}{(R_1+R_2)^2}$$

$$\frac{\partial R}{\partial R_2} = \frac{{R_1}^2}{(R_1+R_2)^2}$$

Taking $dR_1 = \Delta R_1 = 0.2$ and $R_1 = 10\Omega$ and $dR_2 = \Delta R_2 = 0.2$ and $R_2 = 20\Omega$

$$dR = \left(\frac{{R_2}^2}{(R_1+R_2)^2}\right) dR_1 + \left(\frac{{R_1}^2}{(R_1+R_2)^2}\right) dR_2 \text{ becomes}$$

$$dR = \left(\frac{20^2}{(10+20)^2}\right) 0.2 + \left(\frac{10^2}{(10+20)^2}\right) 0.2$$

$$dR = \frac{1}{9}$$

$$dR = 0.1111111111111$$

Hence the maximum error in R is 0.111

Question 3

Explain the following theorems in your own words and state two applications of each theorem in your field of study.

a) Divergence theorem
b) Stokes' theorem

Solution

a) Divergence theorem: This theorem states that the triple (volume) integral of the divergence of a vector field ($\underset{F}{\rightarrow}$) is the same as the closed surface integral of the vector field enclosed or bounded by a surface (s). Mathematically:

$$\iiint_V \nabla \vec{F}\, dV = \iint_s \vec{F}\, ds$$

Applications

- The divergence theorem can be used to derive Gauss' law, a fundamental law in electrostatics.
- In electromagnetism, there are continuity equations that describe the conservation of mass, momentum, energy, probability, or other quantities. The divergence theorem states that any such continuity equation can be written in a differential form (in terms of a divergence) and an integral form (in terms of a flux).
- Used to derive Gauss' law for magnetization

b) Stokes Theorem: It states that a surface integral of the curl of a vector field is equal to a line integral of the vector field around the boundary of the surface. Mathematically

$$\iint \nabla \times \vec{F}\, ds = \oint_c \vec{F}\, ds$$

Applications

In the theory of electromagnetism, the classical Stokes Theorem moves between the differential and integral forms of two of Maxwell's four equations which are below

- Used in derivation of Faraday's law
- Used in derivation of Ampere's law

daed_Ragen