ANALYSIS OF STRUCTURES

Analysis of Planar Trusses using the Method Of Joints.
Analysis of Planar Trusses using the Method Of Sections.
Analysis of Frames.

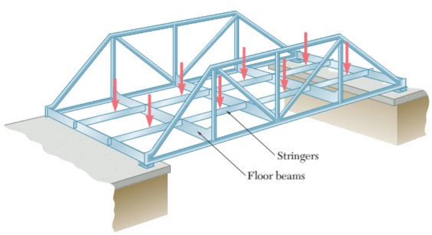
ANALYSIS OF STRUCTURES

Trusses

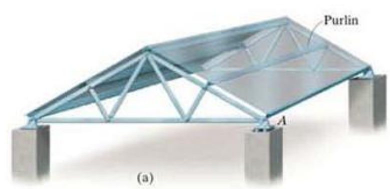
- Trusses may be classified as simple or compound.
- A truss is therefore often considered simple if its design or construction is such that each time we add 2 new members, they are joined such the number of joints in the truss increases by 1.
- A Truss is designed such that its members are two are two-force members (i.e. they are either in tension or compression).
- In designing a truss, it is desirable to know the force each individual member must sustain.

ANALYSIS OF STRUCTURES
Trusses

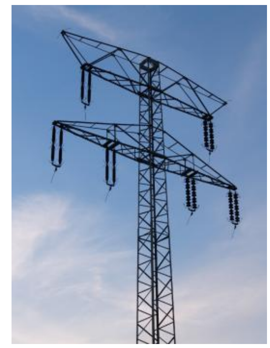
➢Common applications are roofs, bridges and power pylons.



Bridge Truss
Source: Mechanics for Engineers by Beet *et al*



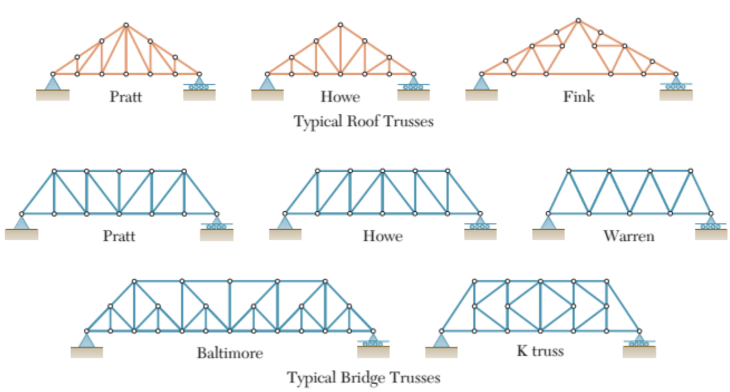
Roof Supporting Truss
Source: Engineering Mechanics Statics by Hiebbler



Power Pylon
Source: http://upload.wikimedia.org/wikipedia/commons/7/7e/Electricity_pylon_power_outage.jpg

ANALYSIS OF STRUCTURES
Planar Trusses

➢Trusses may be Planar or Space Trusses.



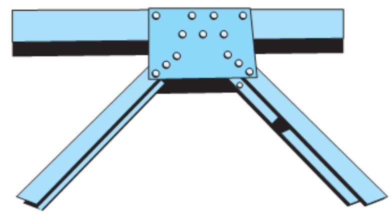
Some Planar Trusses
Source: Beet *et al*

Space Truss
Source
*http://www.picstopin.com/1000/truss/http:%7C%7Cwww*timplex*com%7Ctrusstypess*png/*

ANALYSIS OF STRUCTURES
Analysis of Planar Trusses

- Assumptions for analysis:
  - The weights of the slender bars/members are negligible.
  - Forces act at the ends of the members such that they are in either tension or compression.
  - All joints are pins.

- There are two approaches to analysis:
  - The method of Joints
  - The method of Sections.



Truss members welded or riveted to a gusset plate
Source: Engineering Mechanics – Statics by Pytel

ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Joints

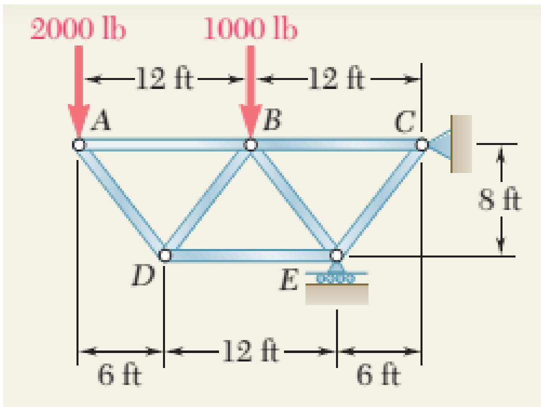
- This involves an equilibrium analysis on each joint to determine the forces being exerted on the end of each member at that joint.
- It is based on the assumption that if the whole truss is in equilibrium, each of its members or joints are also in equilibrium.

- Analysis is done in two main steps:
  - Determine the reactions at the support reactions using the FBD of the entire truss.
  - Conduct an equilibrium analysis at each joint/pin to determine the forces in each member.

ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Joints

➢Example

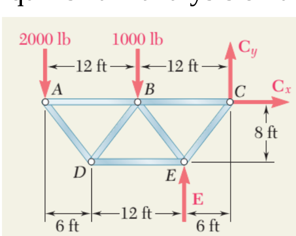
Determine the force in each member of the truss shown below using the method of joints.



ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Joints

➢Solution

➢Equilibrium analysis on the entire truss to determine the support reactions



For equilibrium,

$$+\rightarrow \sum F_x = 0 : C_x = 0$$

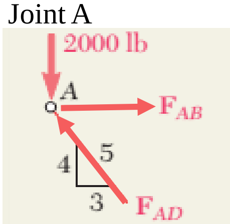
$$+\uparrow \sum F_y = 0 : C_y + E = 3000\text{ lb}$$

$$\circlearrowleft\!+ \sum M_C = 0 : (6\text{ ft})E - (2000\text{ lb})(24\text{ ft}) - (1000\text{ lb})(12\text{ ft}) = 0$$

$$E = 10000\text{ lb}\uparrow$$

➢Equilibrium analysis on the individual joints

Joint A



For equilibrium,

$$+\rightarrow \sum F_x = 0 : F_{AB} - \frac{3}{5}F_{AD} = 0 \qquad ----(1)$$

$$+\uparrow \sum F_y = 0 : \frac{4}{5}F_{AD} - 2000\text{ lb} = 0 \qquad ----(2)$$

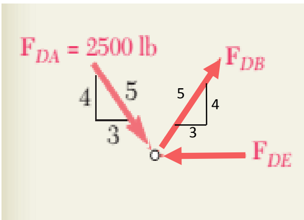
Solving (1) and (2) simultaneously,

$F_{AD} = 2500\text{ lb} \qquad F_{AB} = 1500\text{ lb}$

ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Joints

- Solution

Joint D



For equilibrium,

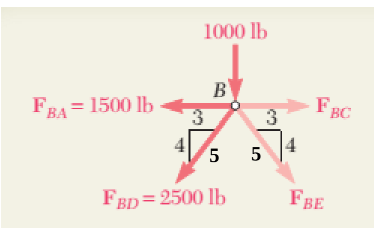
$$+\rightarrow\sum F_x = 0: \frac{3}{5}(2500\text{ lb}) - F_{DE} + \frac{3}{5}F_{DB} = 0 \qquad \text{----}(1)$$

$$+\uparrow\sum F_y = 0: \frac{4}{5}F_{DB} - \frac{4}{5}(2000\text{ lb}) = 0 \qquad \text{----}(2)$$

Solving (1) and (2) simultaneously,

$F_{DB} = 2500\text{ lb} \qquad F_{DE} = 3000\text{ lb}$

Joint B



For equilibrium,

$$+\rightarrow\sum F_x = 0: -\frac{3}{5}(2500\text{ lb}) - 1500\text{ lb} + F_{BC} + \frac{3}{5}F_{BE} = 0 \qquad \text{----}(1)$$

$$+\uparrow\sum F_y = 0: -1000\text{ lb} - \frac{4}{5}(2000\text{ lb}) - \frac{4}{5}F_{BE} = 0 \qquad \text{----}(2)$$

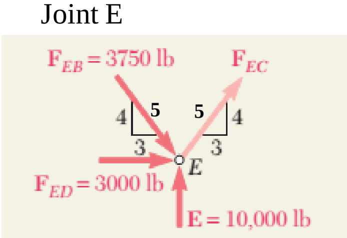
Solving (1) and (2) simultaneously,

$F_{BE} = 3750\text{ lb} \qquad F_{BC} = 5250\text{ lb}$

ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Joints

- Solution

Joint E



For equilibrium,

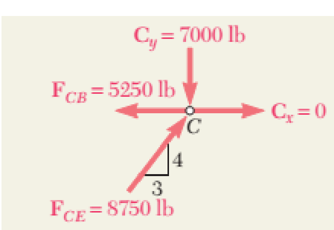
$$+\rightarrow\sum F_x = 0: \frac{3}{5}(3750\text{ lb}) + 3000\text{ lb} + \frac{3}{5}F_{EC} = 0 \qquad \text{----}(1)$$

$$+\uparrow\sum F_y = 0: -\frac{4}{5}(3750\text{ lb}) + \frac{4}{5}F_{EC} + 10000\text{ lb} = 0 \qquad \text{----}(2)$$

Solving (1) and (2) simultaneously,

$F_{DB} = 2500\text{ lb} \qquad F_{DE} = 3000\text{ lb}$

Joint C



For equilibrium,

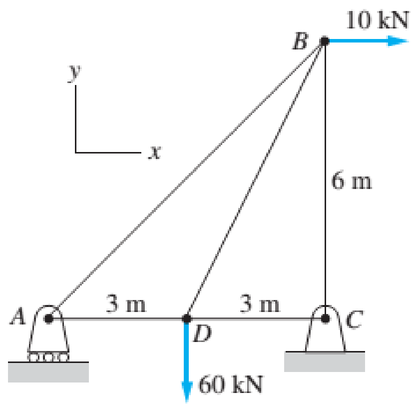
$$+\rightarrow\sum F_x = 0: \frac{3}{5}(8750\text{ lb}) - 5250\text{ lb} + 0 = 0$$

$$+\uparrow\sum F_y = 0: -7000\text{ lb} - \frac{4}{5}(8750\text{ lb}) = 0$$

ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Joints

➢Example

Determine the force in each member of the truss shown below using the method of joints. Indicate whether each member is in tension or compression



$P_{AB} = -28.3\text{ kN (Compression)}$

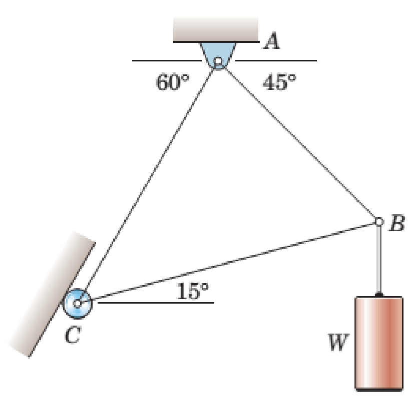
$P_{AD} = 20\text{ kN (Tension)}$

$P_{BC} = -40\text{ kN (Compression)}$

ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Joints

➢Solution

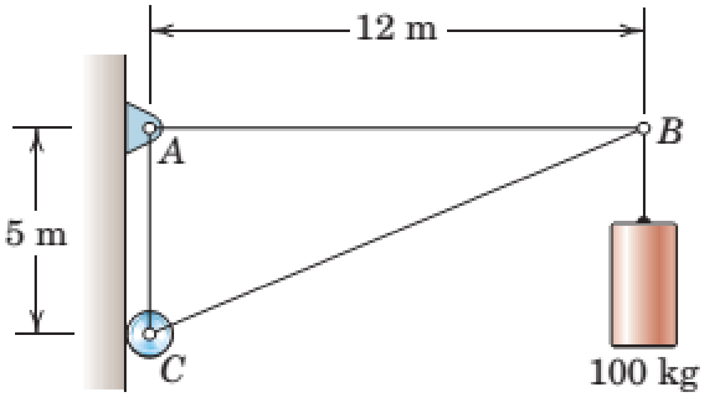
Determine the force in each of the members of the loaded truss



ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Joints

➢Solution

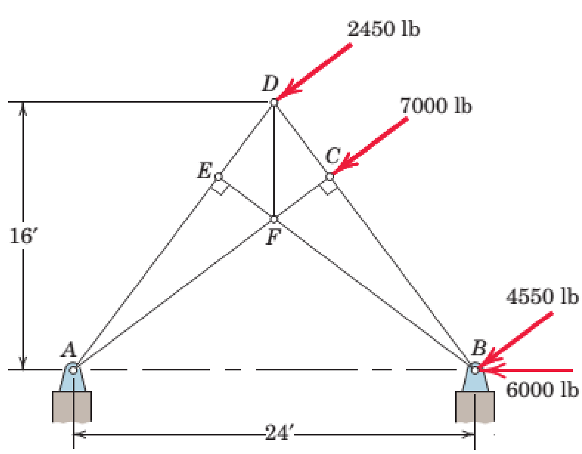
Determine the force in each of the members of the loaded truss



ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Joints

➢Solution

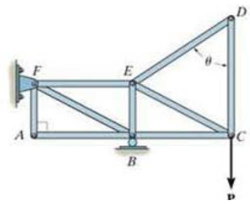
Determine the force in each of the members of the loaded truss

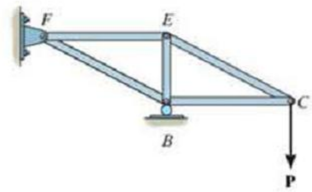


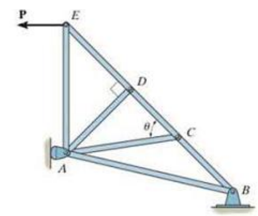
ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Joints

Zero Force Members

- It is possible during that some members of a Truss do not experience any external forces. Such members are referred to as ***Zero Force Members.***
- Identification of such members can significantly make analysis of planar trusses using the method of joints easier.
- Identification is normally by inspection.
- Two rules of thumb for identifying such members are;
  1. *if three members form a truss joint for which two of the members are collinear (have the same line of action), the third member may be a zero force member provided no external force or support reaction is applied to the joint.*
  2. *if a joint is formed by only two members and the joint is not subjected to any external load or support reactions, then the two members may be zero force members.*







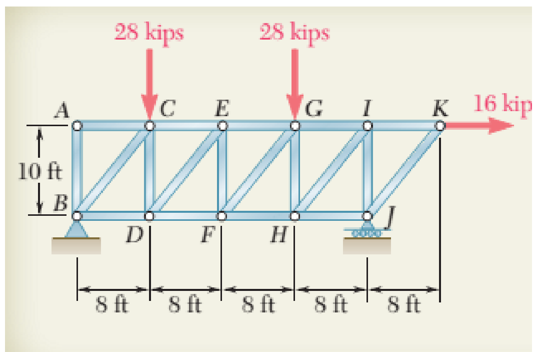
ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Sections

- This method is useful if it is desired to find the forces in only few members.
- It is based on the assumption that if the whole truss is in equilibrium, any section of it must also be in equilibrium.
- Analysis in the following steps:
  - Determine the support reactions using the FBD of the entire truss.
  - Draw a line which divides the truss into two separate portions but does not intersect more than three members. One of the three intersected members must be the member of interest.
  - Use one of the two portions as a free body diagram and determine the force due to the member of interest.

ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Sections

➢Example

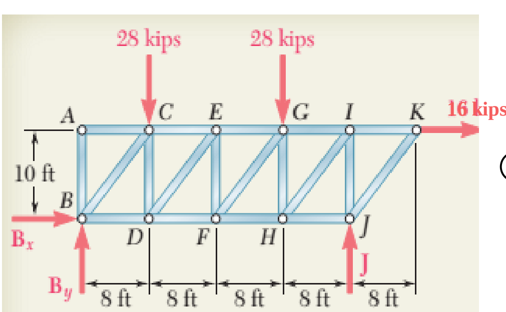
Determine the force in members *EF* and *GI* of the truss shown



ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Sections

➢Example Solution

Equilibrium analysis on the whole truss to determine support reactions.



For equilibrium,

$$+\rightarrow \sum F_x = 0: B_x + 16\text{ kips} = 0$$

$$+\uparrow \sum F_y = 0: B_y + J - 28\text{ kips} - 28\text{ kips} = 0$$

$$\circlearrowleft + \sum M_B = 0: 28\text{ kips}(8\text{ ft}) + 28\text{ kips}(24\text{ ft}) + 16\text{ kips}(10\text{ ft}) - J(32\text{ ft}) = 0$$

$$J = 33\text{ kips}$$

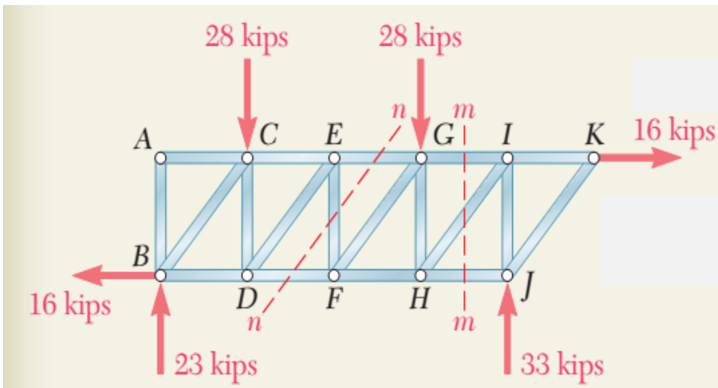
$$B_x = -16\text{ kips}$$

$$B_y = 23\text{ kips}$$

ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Sections

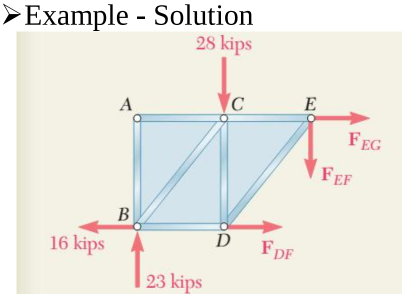
➢Example - Solution

Dividing the truss into two with line *nn* (to solve for member *EF*), then with *mm* (to solve for *GI*)



ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Sections

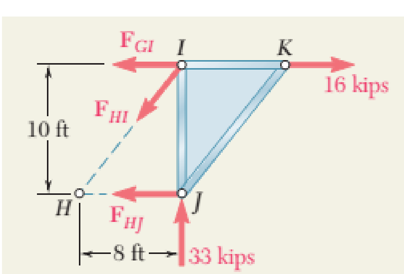
➢Example - Solution



Performing an equilibrium analysis on the left side of *nn* to obtain *EF*,

$$+\uparrow\sum F_y = 0: 23\text{ kips} - 28\text{ kips} - F_{EF} = 0$$

$$F_{EF} = -5\text{ kips}$$



Performing an equilibrium analysis on the right side of *mm* to obtain *GI*

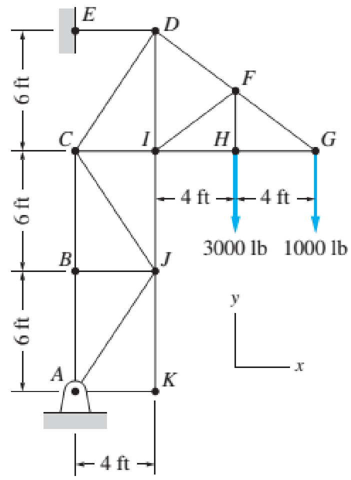
$$\circlearrowleft +\sum M_H = 0: -F_{GI}(10\text{ ft}) - 33\text{ kips}(8\text{ ft}) + 16\text{ kips}(10\text{ ft}) = 0$$

$$F_{GI} = -10.4\text{ kips}$$

ANALYSIS OF STRUCTURES
Analysis of Planar Trusses - The Method of Sections

➢Example

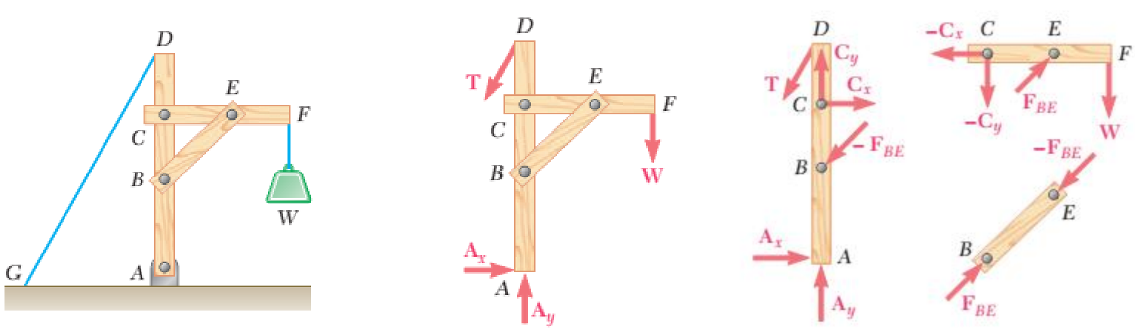
Determine the forces in members *FI* and *JC* of the truss shown. Indicate tension or compression.



ANALYSIS OF STRUCTURES
Frames

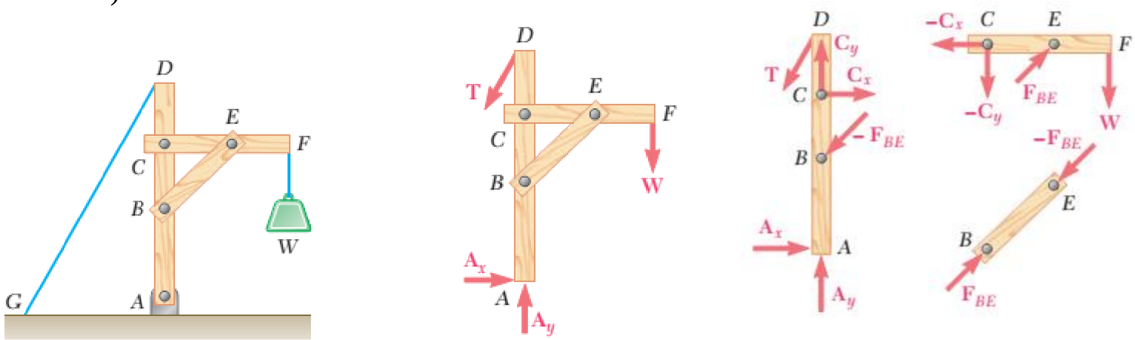
➢Frames and Machines are structures in which at least one member is a multiforce member (acted upon by three or more forces).

➢While frames are designed to support forces, machines transmit and modify forces.



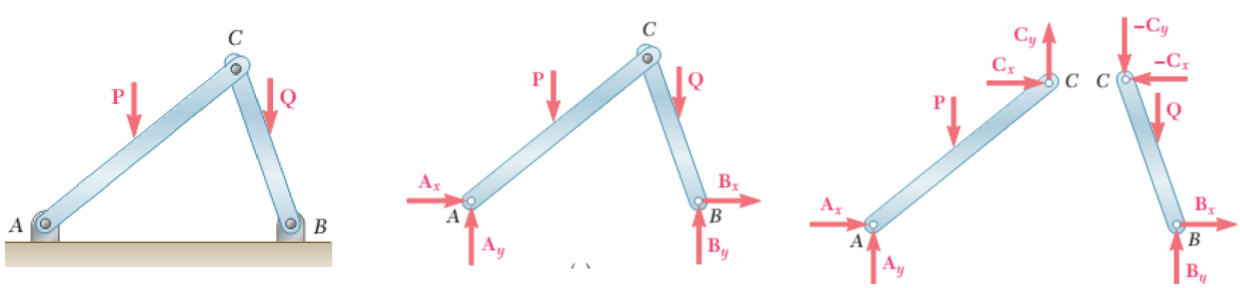
ANALYSIS OF STRUCTURES
Analysis of Frames

- Procedure is similar to what is used in trusses.
- An equilibrium analysis is first carried out on the entire frame.
- An equilibrium is then carried out on the individual members of the frame to determine all forces acting on each of them (It is sometimes easier if this is done first on the two force, then three force members).



ANALYSIS OF STRUCTURES
Analysis of Frames

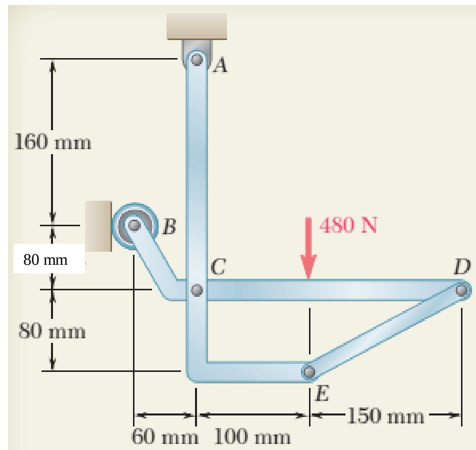
- During analysis of non-rigid frames, the individual members of frame are considered as rigid members.



ANALYSIS OF STRUCTURES
Analysis of Frames

➢Example

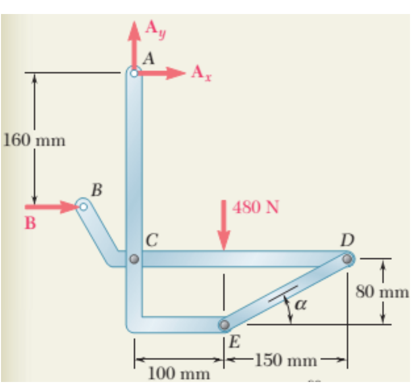
In the frame shown, members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*. Is link *DE* tension or compression?



ANALYSIS OF STRUCTURES
Analysis of Frames

➢Example – Solution

Equilibrium analysis on the entire structure



For equilibrium,

$$+\rightarrow \sum F_x = 0 : B + A_x = 0$$

$$+\uparrow \sum F_y = 0 : A_y = 480 \text{ N}$$

$$\circlearrowleft + \sum M_A = 0 : -B(160 \text{ mm}) + 480 \text{ N}(100 \text{ mm}) = 0$$

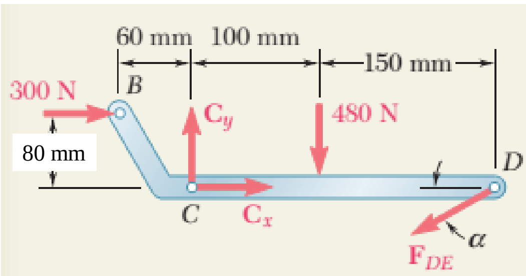
$$B = 300 \text{ N}$$

So,

$$A_x = -300 \text{ N}$$

ANALYSIS OF STRUCTURES
Analysis of Frames

Example – Solution (Equilibrium analysis on individual links)



For equilibrium,

$$+\rightarrow \sum F_x = 0: 300\text{ N} + C_x - F_{DEx} = 0$$

$$+\uparrow \sum F_y = 0: C_y - 480\text{ N} - F_{DEy} = 0$$

$$+ \sum M_C = 0: 300\text{ N}(80\text{ mm}) + 480\text{ N}(100\text{ mm}) + F_{DEy}(250\text{ mm}) = 0$$

$$F_{DEy} = -264\text{ N} \qquad C_y = 216\text{ N}$$

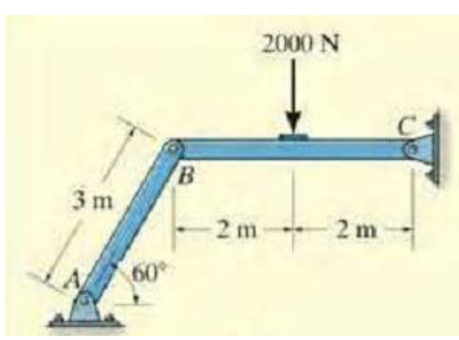
$$\alpha = \tan^{-1}\left(\frac{80}{150}\right)$$

$$F_{DE} = -561\text{ N} \qquad F_{DEx} = -495\text{ N} \qquad C_x = -795\text{ N}$$

ANALYSIS OF STRUCTURES
Analysis of Frames

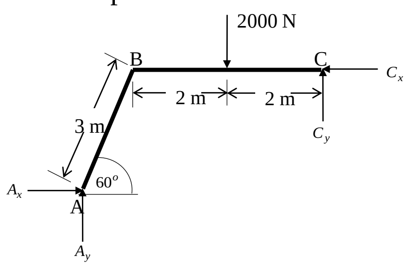
➢Example

Determine the horizontal and vertical components of the force which the pin at *C* exerts on member *BC* of the frame shown below. Also determine the forces acting on both members at *B*



ANALYSIS OF STRUCTURES
Analysis of Frames

➢Example - Solution



For equilibrium,

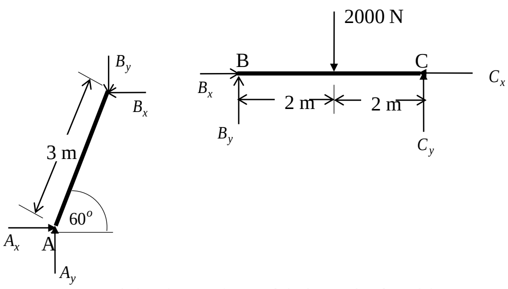
$$+\rightarrow \sum F_x = 0: A_x - C_x = 0$$

$$+\uparrow \sum F_y = 0: A_y + C_y - 2000\ \text{N} = 0$$

$$\circlearrowleft + \sum M_C = 0: -A_y(4 + 3\cos 60^o) + A_x(3\sin 60^o) + 2000\ \text{N}(2\ \text{m}) = 0$$

$$\circlearrowleft + \sum M_A = 0: -C_y(4 + 3\cos 60^o)\text{m} + C_x(3\sin 60^o)\text{m} + 2000\ \text{N}(2 + 3\cos 60^o)\text{m}$$

$$A_x = C_x = 577\ \text{N} \qquad A_y = C_y = 1000\ \text{N}$$



Taking member BC, for equilibrium,

$$+\rightarrow \sum F_x = 0: B_x - C_x = 0$$

$$+\uparrow \sum F_y = 0: B_y + C_y - 2000\ \text{N} = 0$$

$$B_x = 577\ \text{N}$$

$$B_y = 1000\ \text{N}$$

For member AB,

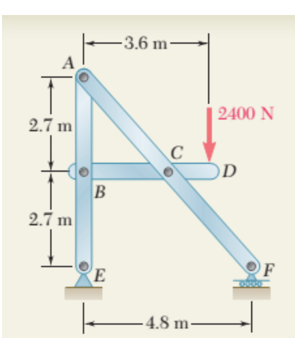
$$B_x = 577\ \text{N} \leftarrow$$

$$B_y = 1000\ \text{N} \downarrow$$

ANALYSIS OF STRUCTURES
Analysis of Frames

➢Example

➢Determine the components of the forces acting on each member of the frame shown.



$$F = 1800\ \text{N}$$

$$E_y = 600\ \text{N}$$

$$E_x = 0\ \text{N}$$

$$A_x = 0\ \text{N}$$

$$A_y = 1800\ \text{N}$$

$$B_x = 0\ \text{N}$$

$$B_y = 1200\ \text{N}$$

$$C_x = 0\ \text{N}$$

$$C_y = 1000\ \text{N}$$