1.0 Theory of Hydraulic Machinery

1.1 Classification of Hydraulic Machines

Hydraulic energy is that possessed by a fluid and may be in the form of pressure, kinetic, potential and thermal energy. Mechanical energy is that associated with a moving or rotating part of machine transmitting power. Hydraulic machines transform hydraulic energy to mechanical energy or vice versa. They may be classified by direction of energy transfer and the mode of operation.

1.2 Direction of Energy Transfer

If they convert hydraulic energy to mechanical energy, they are referred to as Hydraulic Turbines or Motors. As they extract energy from the fluid, the energy of the fluid at the outlet of the machine is less than at its inlet. On other hand, if they convert mechanical energy to hydraulic energy, they are referred to as Hydraulic Pump**.** Since a pump adds energy to the fluid flowing through it, the energy is greater at the outlet of a pump than at its inlet.

Fluid is sometime used for smooth and gradual transfer of power from the shaft of a prime mover to the shaft of the associated driven machine in what is referred to as Hydraulic Transmission or Coupling**.** Mechanical energy is transformed into hydraulic in the first part of the coupling only to be changed back to mechanical in the other half of the coupling. A hydraulic transmission is thus a pump and turbine built in a single unit.

In Fig. 1a, is illustrated a hydroelectric power station. The hydraulic turbine is coupled to an electric generator so that the energy extracted from the fluid by the turbine powers the generator to produce electric power. In Fig. 1b, a pump is used to raise water from the lower reservoir (tailwater) into the upper basin (headwater).

1.3 Mode of Operation

Hydraulic machines may also be classified according their mode of operation as Positive Displacement Machines and Rotodynamic (or Turbo) Machines.

Positive displacement machines transfer energy by forcing fluid into or out of the chamber by changing the volume of the chamber. They are characterized by a movable member. When the volume of fluid increases, energy is transferred from the fluid to the mechanical system. Conversely, energy is transferred to the fluid when the volume decreases. Positive displacement pumps may be made from deformable materials such as rubber. In the animal heart, the muscle is the deformable material.

When the working member reciprocates, valves must be incorporated to ensure unidirectional energy flux; for a pump, the direction of energy flux is towards the fluid and for a motor, it is away from it. Gear pump using rotating working members do not require valves. Positive displacement machines essentially depend on hydrostatic principles (static forces) and dynamic effects are only incidental.

Rotodynamic machines depend essentially on dynamic forces and the exchange of energy occurs between a rotating rigid member (rotor or runner) and the fluid in motion; rotation of the rotor produces dynamic effects that either add energy to the fluid or extract energy from it.

1.4 Direction of Flow through Runner

Rotordynamic machines (turbine and pumps) are further classified by the direction of fluid flow in relation to the plane of impeller rotation as Radial, Axial and Mixed.

In Radial flow machines, the flow path through the runner is wholly or mainly in the radial direction; that is, in a plane perpendicular to the axis of rotation of the shaft. An example is the centrifugal pump. Further classification of radial flow machines is Inward Radial flow if the flow is from outwards to inwards radially, and Outward Radial flow if the flow is from inwards to outwards radially.

In Axial flow machines, the flow is wholly or mainly parallel to the axis of rotation (eg Kaplan turbine).

In Mixed flow (radial-axial) machine, the flow direction is predominantly radial at entry and axial at exit from the runner, eg. Francis turbine,

1.5 Additional Classification of Turbines:

i. Type of Energy at turbine inlet: Impulse turbine and Reaction turbine

In Impulse turbine, the energy available at the inlet is only kinetic energy. The fluid pressure as it flows over the vanes from the inlet to the outlet is atmospheric.

In Reaction turbines, the fluid possess both kinetic energy as well as pressure energy at the inlet of the turbine. As the fluid flows through the runner, the fluid is under pressure and the pressure energy continuously changes to kinetic energy. The runner is completely enclosed in an airtight casing and the runner is completely filled by the fluid.

ii. Head at inlet of turbine: <u>Low head</u>, <u>Medium head</u> and <u>High head</u> turbine.

iii. Specific Speed of turbine; <u>Low specific speed</u>, <u>Medium specific speed</u> and <u>High</u> <u>specific speed.</u>

2.0 Theory of Rotodynamic Machines

2.1 Euler's Equation

Shown in Fig. 2.1 is a sectional view of one dimensional (1D) fluid flow through a radial-flow turbine and a centrifugal pump. The assumption of 1D flow simplifies its analysis as it implies no variation of velocity and pressure across the blade passage (in other words, if at any point in the flow within the runner, a circle or arc is drawn with the same centre as the rotating wheel, the fluid velocity and pressure are assumed to be the same along that circle or arc). Real flow through a turbomachinery is rather three dimensional (3D) involving very complex velocity distribution that is dependent on many factors such as the shape, thickness and number of blades.

The following symbols are used throughout the analysis of the flow:

C: absolute velocity

C_u: whirl (tangential) component of C

C_F: radial (flow) component of C

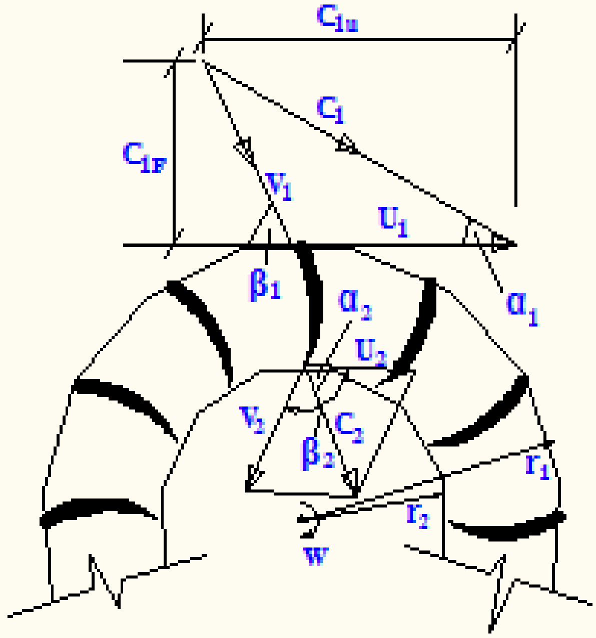
V: relative velocity

U: tangential velocity

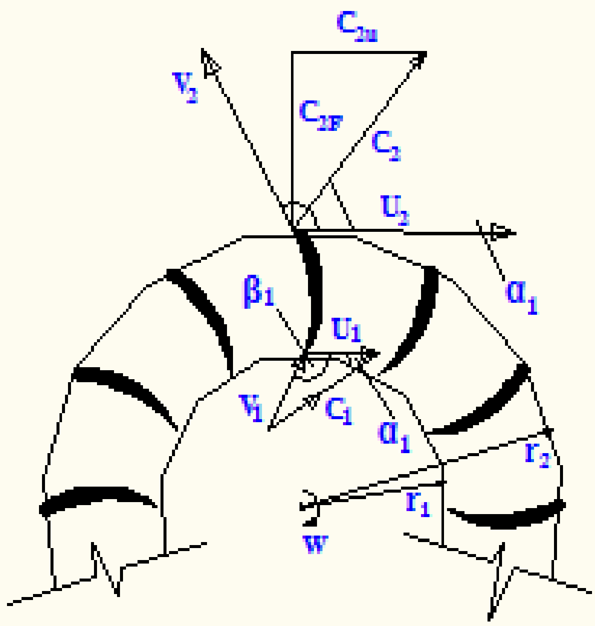
α: angle made by the C to the positive direction of U

β: angle made by V to the positive direction of U

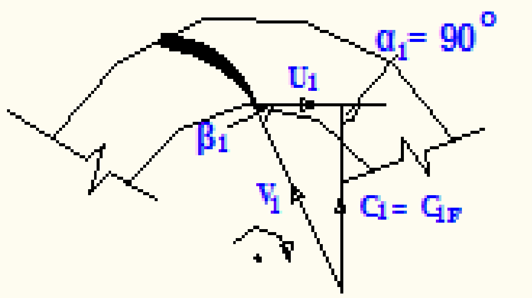
The angle, β, made by the V to the positive direction of U is equal to the blade angle, β', only for no-shock entry. For simplification, it is assumed β= β'. No-shock entry condition gives minimum entry loss and is a crucial design factor. Conditions at machine inlet and outlet are represented by subscript 1 and 2 respectively.



i. Radial Flow Turbine (inward Flow)



ii Centrifugal pump (outward flow)



ii Centrifugal pump with no prewhirl

Fig.2.1: One Dimensional Flow through a hydraulic Machine

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Newton's second law applied to angular motion states that the torque, T, is equal to the time rate of change of the angular momentum. Angular momentum (aka Moment of Momentum) is "mass x tangential velocity x radius". Thus, in Fig.2.1(i), the torque exerted by the fluid on the turbine rotor (which is equal and opposite to that exerted on the fluid by the turbine) is

$$T = \dot{m}\left(r_2 C_{2u} - r_1 C_{1u}\right) \qquad (2.1)$$

where

$\dot{m} = \rho\dot{Q}$: mass of fluid flowing per second

$\dot{m}C_{1u}r_1$: angular momentum entering the rotor per second

$\dot{m}C_{2u}r_2$: angular momentum leaving the rotor per second

Work done per second (Power) is

$$E = Tw = \dot{m}\left(U_2 C_{2u} - U_1 C_{1u}\right) \qquad (2.2)$$

Specific energy (energy per unit weight of flow)

$$h = \frac{E}{\dot{m}g} = \frac{1}{g}\left(U_2 C_{2u} - U_1 C_{1u}\right) \qquad (2.3)$$

Eq. 2.3 is known as the **Euler's Turbine Equation** and is applicable to all rotordynamic machines (turbines and pumps, axial or radial flow). For the turbine in which energy is extracted from the fluid, $U_1C_{1u} > U_2C_{2u}$ and for the pump, $U_1C_{1u} < U_2C_{2u}$ which indicates the reverse direction of energy transfer. The equation gives the maximum theoretical energy per unit weight (head, h) that can be extracted from or delivered to the fluid.

Head, h, of fluid represents the energy per unit weight of fluid.

The equation can be expressed in terms of absolute velocities C, U and V by inserting $C_{1u} = C_1 \cos\alpha_1$ and $C_{2u} = C_2 \cos\alpha_2$:

$$h = \frac{1}{g}\left(U_2 C_2 \cos\alpha_2 - U_1 C_1 \cos\alpha_1\right) \qquad (2.4a)$$

From the velocity triangle at inlet, we can write

$$V_1^2 = \left(C_1 \cos\alpha_1 - U_1\right)^2 + \left(C_1 \sin\alpha_1\right)^2$$

from which

$$U_1 C_1 \cos\alpha_1 = \frac{1}{2}\left(U_1{}^2 + C_1^2 - V_1^2\right) \qquad (2.4b)$$

Similarly,

$$U_2 C_2 \cos\alpha_2 = \frac{1}{2}\left(U_2{}^2 + C_2^2 - V_2^2\right) \qquad (2.4c)$$

Substitute (2.4b) & (2.4c) in (2.4a) gives

$$h = \frac{\left(C_2{}^2 - C_1^2\right)}{2g} + \frac{\left(U_2{}^2 - U_1^2\right)}{2g} + \frac{\left(V_1^2 - V_2^2\right)}{2g} \qquad (2.5)$$

Eq. 2.5 can also be derived by applying the Bernoulli's equation to the flow in the rotor blade. The specific energy of the fluid at rotor inlet (1) and outlet (2) is

$$h_1 = \frac{p_1}{\rho g} + \frac{C_1{}^2}{2g} + z_1, \qquad h_2 = \frac{p_2}{\rho g} + \frac{C_2{}^2}{2g} + z_2$$

and the change in fluid specific energy between inlet and outlet is thus

$$h = h_2 - h_1 = \frac{\left(C_2{}^2 - C_1^2\right)}{2g} + \frac{\left(p_2 - p_1\right)}{\rho g} + \left(z_2 - z_1\right) \qquad (2.6a)$$

For the relative flow in the rotating blades, the Bernoulli's energy conservation equation takes the form

$$\frac{p}{\rho g}+\frac{V^2}{2g}+z-\frac{U^2}{2g}=const. \qquad (2.6b)$$

Applied between the inlet and outlet:

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1-\frac{U_1^{\,2}}{2g}=\frac{p_2}{\rho g}+\frac{V_2^{\,2}}{2g}+z_2-\frac{U_2^{\,2}}{2g}$$

$$\frac{(p_2-p_1)}{\rho g}+(z_2-z_1)=\frac{\left(V_1^2-V_2^{\,2}\right)}{2g}+\frac{\left(U_2^{\,2}-U_1^{\,2}\right)}{2g} \qquad (2.6c)$$

Substituting (2.6c) in (2.6a) gives (2.5).

Eq. 2.5 comprise the following quantities:

$\frac{\left(C_2^{\,2}-C_1^{\,2}\right)}{2g}$: change in Kinetic Energy head of the fluid flowing through the rotor.

$\frac{\left(U_2^{\,2}-U_1^{\,2}\right)}{2g}$: change in Kinetic Energy head possessed by the fluid as a result of its rotation about the rotor axis. For a pump, it represents the work done by the centrifugal force on the fluid and is energy added to the fluid. For a turbine, it represents energy released by the fluid to the rotor.

$\frac{\left(V_1^2-V_2^{\,2}\right)}{2g}$: change in static head due to change in relative velocity from the inlet to the outlet. For a pump, the decrease in relative velocity is static regain.

2.2 Application of Euler's Equation

2.2.1 Centrifugal Machines

i. Pre-rotation of flow

Usually, the flow approaching the impeller channels of a centrifugal pump is set in rotation by the direct contact with the shaft and the hub of the impeller and partly by impulsive exchange between the fluid masses. This is referred to as "Prerotation" or "Prewhirl" of flow and is characterized by C_{1u} .

The theoretical head given by the Euler's equation

$$h = \frac{1}{g}\left(U_2 C_{2u} - U_1 C_{1u}\right) \tag{2.7a}$$

is only true if the prewhirl is caused by guide vanes or something else and not the impeller. If it is caused by the action of the impeller itself, it is an additional head (energy) input into the flow which can also expressed by the Euler's equation as

$$\Delta h = \frac{1}{g}\left(U_1 C_{1u}\right) \tag{2.7b}$$

The total theoretical head is then the sum of the head given by the prewhirl and the head given by the flow in the impeller blades:

$$h = \frac{1}{g}\left(U_2 C_{2u} - U_1 C_{1u}\right) + \frac{1}{g} U_1 C_{1u} = \frac{1}{g} U_2 C_{2u} \tag{2.7c}$$

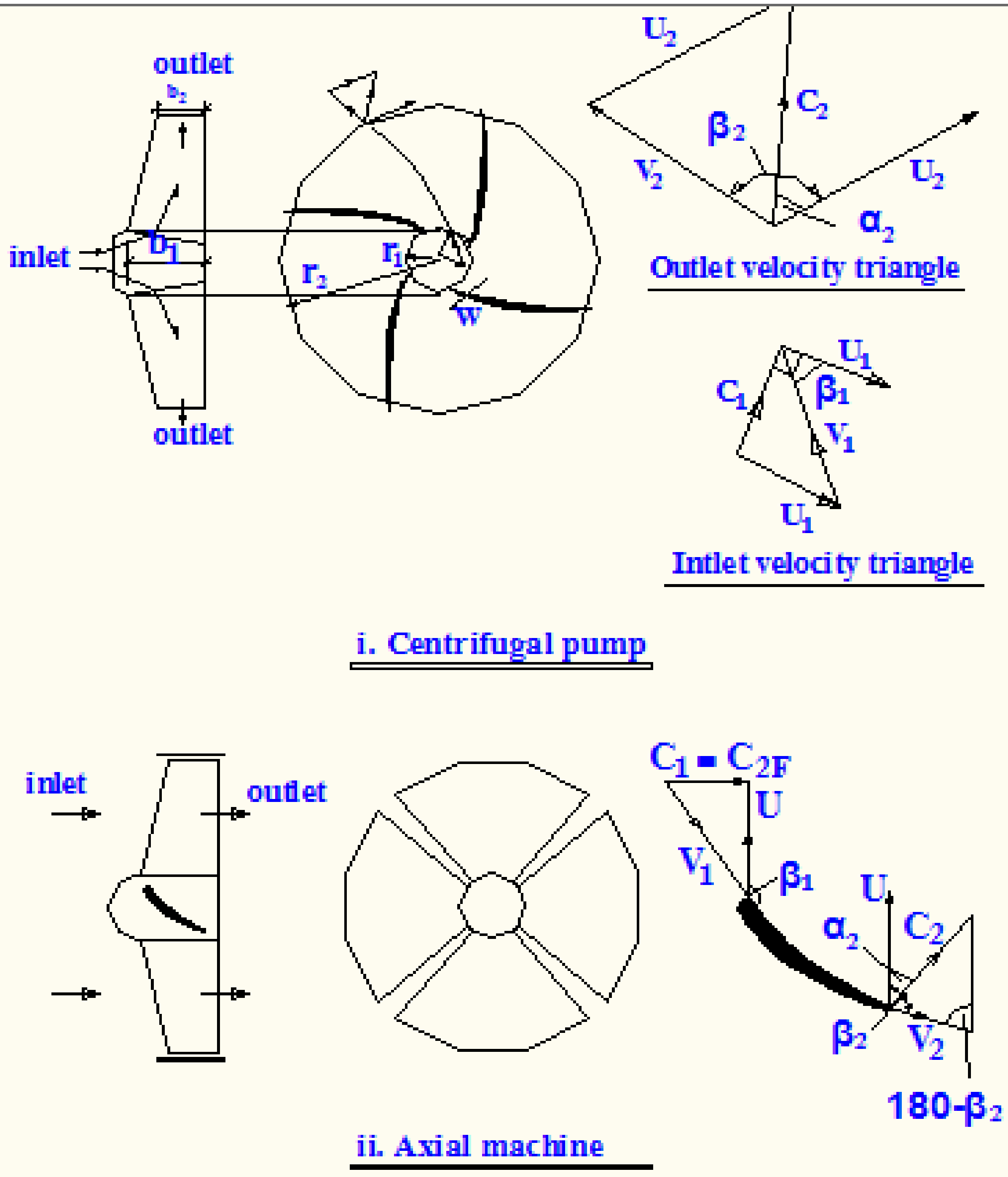

Fig.2.2. Velocity triangles for Centrifugal & Axial machines.

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Prewhirl caused by the impeller corresponds to assuming that the absolute velocity is radial at the entry ($\alpha_1 = 90^o$) hence $C_{1u}=0$ (no prewhirl at inlet).

ii. Energy Transferred

Continuity equation applied to the flow through the cylindrical surfaces at outlet and inlet gives:

$$\dot{m} = \rho_1 2\pi r_1 b_1 C_{1F} = \rho_2 2\pi r_2 b_2 C_{2F} \qquad (2.8a)$$

where b_1 and b_2 represent the width of the impeller blade at inlet and outlet respectively.

For incompressible flow, this simplifies to

$$r_1 b_1 C_{1F} = r_2 b_2 C_{2F} \qquad (2.8b)$$

Assuming that the absolute velocity at the inlet is radial, $C_1=C_{1F}$ can be calculated from Eq.2.8a knowing $\dot{m}$, ρ, r_1 and b_1. Knowing β_1 and ω, the velocity triangle at inlet is constructed.

At the outlet, the direction of V_2 can be drawn assuming that the fluid leaves the impeller tangentially and thus makes an angle β_2 with the whirl direction. The magnitude of C_{2F} is also calculated from Eq. 2.8 and drawn. A perpendicular is drawn from the tip C_{2F} to intersect V_2 and $U= \omega r_2$ drawn horizontally from the intersection. Vectorial addition of V_2 and U gives C_2. V_2. From the outlet triangle, we can write an expression for C_{2u}:

$$\cot(180-\beta_2)=\frac{U_2-C_{2u}}{C_{2F}} \quad \text{or} \quad C_{2u}=U_2-C_{2F}\cot(180-\beta_2) \tag{2.9a}$$

Substituting in Euler's equation:

$$h=\frac{U_2}{g}\left(U_2-C_{2F}\cot\{180-\beta_2\}\right) \tag{2.9b}$$

$$E=\dot{m}gh=\dot{m}U_2\left(U_2-C_{2F}\cot\{180-\beta_2\}\right) \tag{2.9c}$$

2.2.1 Axial Machines

In axial machine, changes in the fluid condition from inlet to outlet occur at the same radius, hence

$$U_1=U_2=U=\omega r \tag{2.10a}$$

Also, flow area at inlet and outlet are equal which gives

$$C_{1F}=C_{2F}=C_F \tag{2.10b}$$

The following assumption are used to determine the energy transferred:

i. No prewhirl at entry: $\alpha_1=90^o$, $C_{1u}=0$ and $C_1=C_{1F}=C_F$

ii. Relative velocity is tangential to blade at entry (no-shock, $\beta_1=\beta_1'$) and at exit.

From the outlet triangle, we get a similar expression as for centrifugal machines:

$$C_{2u} = U - C_F \cot(180 - \beta_2) \qquad (2.11a)$$

and

$$h = \frac{U}{g}\left(U - C_F \cot\{180 - \beta_2\}\right) \qquad (2.11b)$$

Eq.211b applies to any particular radius r and is not necessarily constant over the range R_i to R_o. Hence we cannot simply multiply it by the weight of flow ($\dot{m}g$) to get an expression for E as in Eq. 2.9c. For the value to remain constant at all radii, the increase of U with r must be counterbalanced by a corresponding decrease in $\left(C_F \cot\{180 - \beta_2\}\right)$. Since C_F is constant, this is achieved by twisting the blade (vary β_2) so that at any two radii, r_a & r_b, we get

$$U_a^{\ 2} - U_a C_F \cot\{180 - \beta_{2a}\} = U_b^{\ 2} - U_b C_F \cot\{180 - \beta_{2b}\} \qquad (2.12)$$

To write the expression for E, we apply Euler's equation to an element dr at radius r:

$$dE = (dW)\frac{U}{g}\left(U - C_F \cot\{180 - \beta_2\}\right)$$

where $dW = 2\pi\rho g C_F r dr$ and $U = \omega r$

This gives $E = 2\pi\rho\omega C_F \int_{R_i}^{R_o} r^2\left(\omega r - C_F \cot\{180 - \beta_2\}\right)dr$

which requires knowledge of the variation of β_2 with r before it can be integrated.

2.2.3 **Worked Example 1**

1. A jet of water is directed at the blades of a radial-flow turbine at an angle of 30^o to the tangent at the inlet and exits the blades at 120^o to the tangent at the outlet, both angles measured from the positive x-direction. The inner and outer blade radii are 300mm and 800mm respectively, blade width at inlet and outlet are 120mm and 320mm respectively and the flow rate is $2.5m^3/s$. Determine the torque required to hold the turbine stationary. [16.14kN-m]

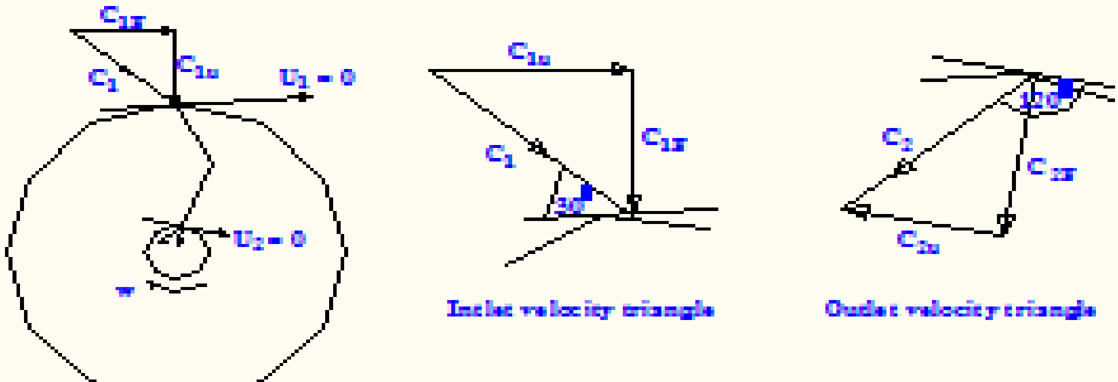
$$Q = 2\pi rbC_F \text{ or } C_F = Q/2\pi rb$$

At inlet, $C_{1F} = \dfrac{2.5}{2\pi \times 0.8 \times 0.12} = 4.14 \Rightarrow C_{1u} = C_{1F} \tan 60 = 7.17 m/s$

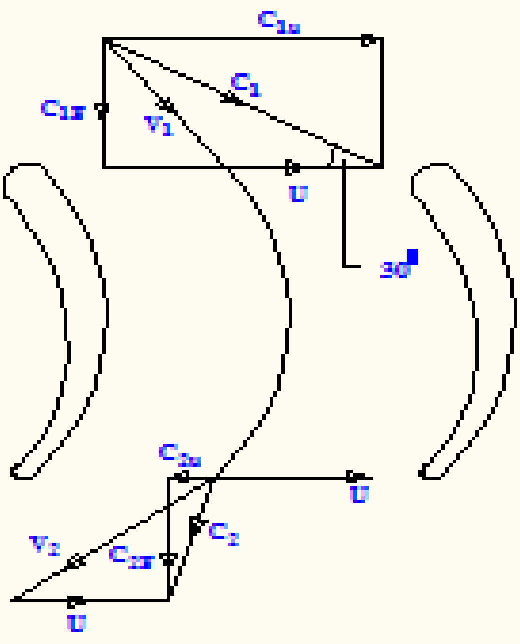
At outlet, $C_{2F} = \dfrac{2.5}{2\pi \times 0.3 \times 0.32} = 4.14 \Rightarrow C_{2u} = C_{2F} \tan 30 = 2.39 m/s$

$$T = m(C_{2u}r_2 - C_{1u}r_1) = 2.5 \times 10^3([-2.39 \times 0.3] - [7.17 \times 0.8]) = 16.14kN - m$$

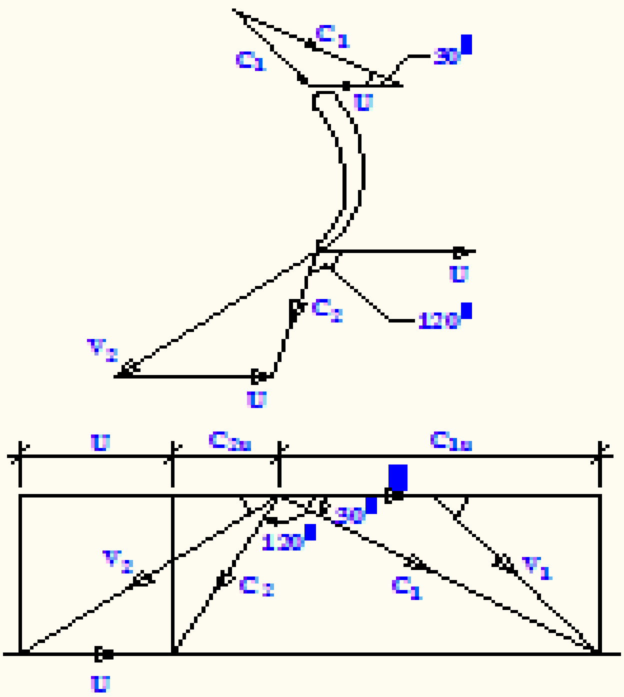
2. Water is directed at an axial flow turbine rotor with an absolute velocity of 20m/s directed at an angle of 30^o to the plane of rotation. The corresponding blade velocity is 12m/s. The water leaves the rotor blade such that the whirl component of the absolute velocity is 4m/s directed opposite to the direction of blade motion. If the stagnation pressure drop across the turbine is 3.5 bar, what is the efficiency of the turbine. Neglect change in elevation head. [73%]



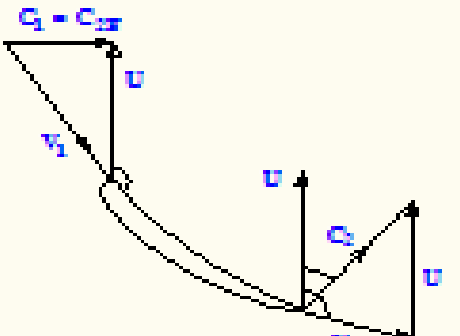
Worked Example 1



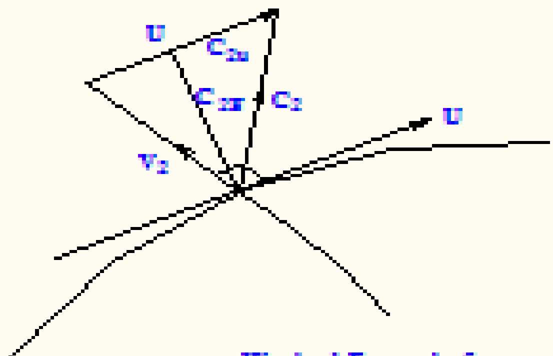
Worked Example 2



Worked Example 3



Worked Example 4



Worked Example 5

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$C_1 = 20m/s, \;\; \alpha_1 = 30^o, U_1 = U_2 = U = 12m/s,$

$$h = \frac{1}{g}\left(U_2 C_{2u} - U_1 C_{1u}\right) = \frac{U}{g}\left(C_{2u} - C_{1u}\right) = \frac{12}{g}\left(-[-4] - 20\cos 30\right) = -\frac{255.84}{g},$$

$$\Delta e_{fluid} = \left(\frac{p_2}{\rho g} + \frac{C_2^{\ 2}}{2g}\right) - \left(\frac{p_1}{\rho g} - \frac{p_1}{\rho g}\right) = \frac{-1}{\rho g}\left(\left[p_1 + \frac{\rho C_1^{\ 2}}{2}\right]\right) - \left[p_2 + \frac{\rho C_2^{\ 2}}{2}\right]$$

$$= \frac{-1}{10^3.g}\left(3.5x10^5\right) = \frac{-350}{g}$$

$$\eta = \frac{-255.84/g}{-350/g} = 73\%$$

3. The inlet nozzles of an axial flow turbine are inclined at an angle of 30° to the plane of rotation. The stream leaves the blades with an absolute velocity of 300 m/s at an angle of 120° to the direction of motion of the blades. Assuming equal inlet and outlet blade angles and zero axial thrust, calculate the blade angle and the output power per kg/s of fluid flow. (Hint: zero axial thrust implies C_{1F}=C_{2F})

$$C_F = C_2 \sin 60 = 300 \sin 60 = 260$$

$$C_{1u} = C_F \tan 60^o = 260 \times \sqrt{3} = 450$$

$$C_{2u} = 300\cos 60 = 150$$

$$\tan \beta_1 = \frac{C_F}{C_{1u} - U} = \frac{C_F}{C_{2u} + U}$$

$$150 + U = 450 - U \Rightarrow U = 150$$

$$\tan\beta_1 = \frac{260}{450-150} = \frac{260}{300}\text{, hence } \beta_1 = 41^o$$

Output power per unit weight of flow (mg) is $h_w = \frac{U}{g}(C_{2u} - C_{1u})$

Output power per unit mass flow rate:

$$h_m = U(C_{2u} - C_{1u}) = 150(-150-450) = 90 \text{ kW}$$

4. An axial flow fan has a hub diameter of 1.50 m and a tip diameter of 2.0 m It rotates at 18 rad/s and, when handling 5.0 m^3/s of air, develops a theoretical head equivalent to 17mm of water. Determine the blade outlet and inlet angles at the hub and at the tip. Assume that the velocity of flow is independent of radius and that the energy transfer per unit length of blade is constant. Take the density of air as 1.2 kg/m^3 and the density of water as 1000 kg/m^3.

$$C_F = \frac{Q}{A} = \frac{Q}{\pi\left(R_2{}^2 - R_1{}^2\right)} = \frac{5}{\pi(1-0.5625)} = 3.64 \text{ m/s}$$

Blade velocity at tip: $U_t = \omega R_2 = 18x1 = 18$ m/s

Blade velocity at hub: $U_h = \omega R_1 = 18x0.75 = 1.5$ m/s

For no-shock condition, inlet blade angle

at tip: $\beta_{1t} = 180 - \cot^{-1}\left(\frac{18}{3.64}\right) = 168.6^o$

at hub: $\beta_{1h} = 180 - \cot^{-1}\left(\frac{13.5}{3.64}\right) = 164.9^o$

Since the head generated by the tip and hub sections is the same, the outlet angles can be obtained by applying Euler’s Eq.2.11b to these sections:

$$h = \frac{U}{g}\left(U - C_F \cot\{180 - \beta_2\}\right)$$

where

h=17 mm of water = 0.017 m x 1000/1.2 =14.16 m of air

At the tip: $14.16 = \frac{18}{9.81}\left(18 - 3.64\cot\{180 - \beta_{2t}\}\right)$ which gives $\beta_{2t} = 160.5^o$

At the hub: $14.16 = \frac{13.5}{9.81}\left(13.5 - 3.64\cot\{180 - \beta_{2h}\}\right)$ which gives $\beta_{2h} = 131.4^o$

5. A centrifugal fan supplies air at a rate of 4.5m^3/s and total head of 100mm of water. The outer diameter of the impeller is 500mm and the outer width is 180mm. The blades are backward inclined and of negligible thickness. If the fan runs at 1800 rpm, and assuming that the conversion of velocity head to pressure head in the volute is counterbalanced by the friction losses there and in the runner, determine the blade angle at outlet. Assume zero whirl at inlet and take air density as 1.23kg/m^3.

$$Q = \pi \times d \times b \times C_{2F}$$

$$4.5 = \pi \times 0.5 \times 0.18 \times C_{2F} \Rightarrow C_{2F} = 47.12 m/s$$

$$U_2 = \omega r = \frac{1800}{60} \times 2\pi \times 0.25 = 47.12 m/s$$

$$h = \frac{U_2 C_{2U}}{g}$$

where h=0.1m of water corresponds to $h = \frac{1000}{1.23} \times 0.1 = 81.3$ m of air.

Thus:

$$81.3 = \frac{47.12 \times C_{2U}}{9.81} \Rightarrow C_{2U} = 16.9 m/s$$

$$\tan(180 - \beta_2) = \frac{C_{2F}}{U_2 - C_{2U}} = 27.8^o \Rightarrow \beta_2 = 152.2^o$$

2.3 Energy Equation for Gas Flow through Impeller

For flow of incompressible fluid through the impeller vanes, the thermodynamic state of flow is not changed; the temperature does not vary in the course of energy transfer. The energy transfer per unit weight of flow across the machine is given by the Bernoulli's equation as

$$h = \left(\frac{P_2}{\rho g} + \frac{C_2^{\,2}}{2g} + z_2\right) - \left(\frac{P_1}{\rho g} + \frac{C_1^{\,2}}{2g} + z_1\right) + \mathrm{h_L} \tag{2.13}$$

where "h" is positive for a pump and negative for a turbine and "$\mathrm{h_L}$" is the head loss in impeller channels.

Neglecting the elevation term and combining with Euler's turbine equation:

$$\frac{1}{\mathrm{g}}\left(\mathrm{U_2 C_{2u}} - \mathrm{U_1 C_{1u}}\right) = \frac{\left(\mathrm{P_2} - \mathrm{P_1}\right)}{\rho \mathrm{g}} + \frac{\mathrm{C_2^2} - \mathrm{C_1^2}}{2\mathrm{g}} + \mathrm{h_L} \tag{2.14}$$

Thus the energy added or extracted from the flow of incompressible fluid by the impeller vanes changes the stream pressure and kinetic energy and is partly expended in overcoming the resistance of the impeller channels.

For a compressible fluid (gas, steam, etc.), the thermodynamics state of the fluid will be changing as a result of energy transfer with the impeller vanes and the exchange of heat with the environment. The steady flow energy equation (from the First Law of Thermodynamics), which is applicable to all fluids (real or ideal, liquid, vapour or gas) is

$$q - w = (H_2 - H_1) + \frac{(C_2{}^2 - C_1{}^2)}{2} + g(z_2 - z_1) \qquad (2.15a)$$

Note that the quantities in Eq.2.15 represents energy per unit mass; "q" is the specific heat energy input to the flow, "w" is the specific work output by the flow and H is the specific enthalpy. Note that "H" is used here for <u>specific enthalpy</u> and not "h" which has already been used in the earlier section as <u>head</u>. Also, whilst "h" represents energy per unit weight, "w" is energy per unit mass; thus "w = hg"

From Eq.2.15a, the work input to the flow per unit mass is

$$-w = (H_2 - H_1) + \frac{(C_2{}^2 - C_1{}^2)}{2} + g(z_2 - z_1) - q \qquad (2.15b)$$

Euler's turbine equation on unit mass basis is

$$-w = gh = (U_2C_{2u} - U_1C_{1u}) \qquad (2.16)$$

Combining Eq.2.15b and 2.16 gives

$$(U_2C_{2u} - U_1C_{1u}) = (H_2 - H_1) + \frac{C_2^2 - C_1^2}{2} + g(z_2 - z_1) - q$$

Neglecting $g(z_2 - z_1)$ and writing for a gas $H = C_pT$ gives

$$(U_2C_{2u} - U_1C_{1u}) = C_p(T_2 - T_1) + \frac{(C_2 - C_1)}{2} - q \qquad (2.17)$$

Thus the mechanical energy transfer between the flow of compressible fluid and the impeller vanes changes the thermodynamic state of the gas and the kinetic energy and is partially expended as heat loss to the environment.

Stagnation or Total Enthalpy, H* , is defined as

$$\mathrm{H}^* = H + \frac{\mathrm{C}^2}{2} \qquad (2.18a)$$

Stagnation or Total Temperature, T*, is similarly defined by putting in Eq. 2.18a:

$$\mathrm{H} = c_p T \text{ and } \mathrm{H}^* = c_p T^*$$

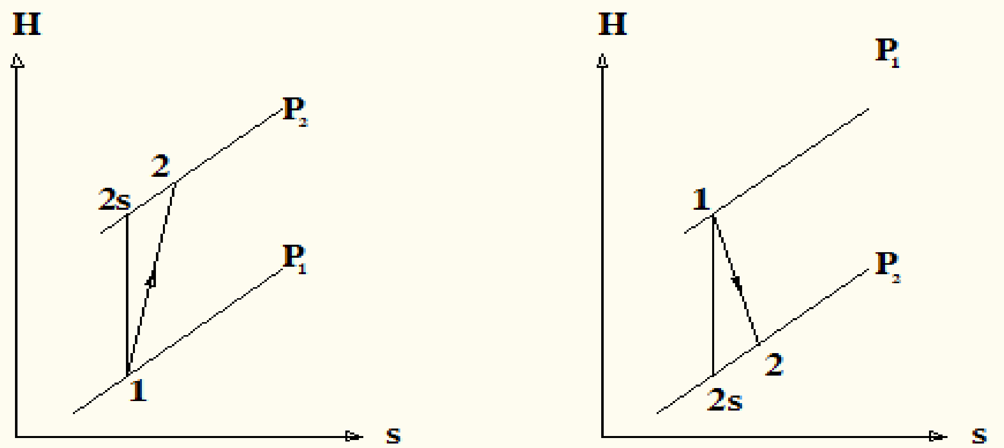
which gives

$$T^* = T + \frac{\mathrm{C}^2}{2\mathrm{c}_\mathrm{p}} \qquad (2.18b)$$

The flow in most turbomachinery is very nearly adiabatic ($q \approx 0$), hence Eq. 2.17 becomes

$$-w = gh = H^*{}_2 - H^*{}_1 = c_p\left(T^*{}_2 - T^*{}_1\right) \qquad (2.19)$$

The process occurring in the turbomachinery is illustrated on the " H-s" diagram below.



i. Compressor

ii. Turbine

Worked Example 2

A centrifugal compressor operating with a pressure ratio of 4 has an isentropic efficiency of 70% when running at 9,000 rpm. At the inlet, the gas temperature is 30°C and there is no pre-whirl. The gas exits the compressor with an absolute velocity inclined at an angle of 30° to the positive direction of blade motion. Find the absolute gas velocity at exit assuming rotor tip diameter of 600mm. Assume for the gas γ = 1.4 and c_p=1kJ/kg K.

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma}$$

$$T_{2s} = T_1 \times \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma} = (30+273)\times(4)^{1.4-1/1.4} = 450^o C$$

$$\frac{T_{2s}-T_1}{T_2-T_1} = 0.7$$

$$T_2 - T_1 = \frac{450-303}{0.7} = 210^o C$$

Energy per unit mass flow:

$$\mathrm{w} = U_2 C_{2u} - U_1 C_{1u} = c_p (T_2 - T_1)$$

$$\omega \times r_2 \times C_2 \times \cos 30^o - 0 = 210 \times 10^3$$

$$C_2 = 857 m/s$$

Note in the above solution that the pressure ratio of 4 refers to stagnation condition at suction and delivery (reservoir conditions) and thus T is stagnation value.

Assignment 1

(Deadline for submitting Assignment: 31st march 2011)

1. At the inlet of a rotodynamic machine, the blade speed at is 10m/s and the blade angle is 150^o measured from the positive direction of blade speed. At the outlet, the relative speed is 30m/s inclined at 120^o to the positive direction of blade speed. Assuming no prewhirl and no shock at entry, draw the velocity diagram on a sketch of blade section and show that the machine is a pump. Determine the energy exchange and the thrust on the machine per unit mass of fluid.

2. A centrifugal pump delivers $0.3m^3/s$ of water and 20m head at 1800rpm. The impeller is 300mm in diameter and 32mm wide at exit. Assuming there is no pre-whirl and the suction and delivery pipes have the same bore, calculate the following.

i. Blade outlet angle with respect to the positive direction of blade speed
ii. Torque required to rotate the pump.
iii. Change in fluid pressure across pump assuming negligible change in elevation between entry and exit of pump and negligible frictional losses.

3. The inlet nozzles of a radial-flow hydraulic turbine are inclined at an angle of 30^o to the tangent. The fluid exits the blades at 60^o to the tangent at the outlet, both angles measured from the positive x-direction. Assuming the outer and inner blade radii are 400mm and 150mm respectively, blade widths at inlet and outlet are 120mm and 150mm respectively, fluid flow rate is $2.5m^3/s$ and fluid density is $1000kg/m^3$, calculate the following:

i. Torque required to hold the turbine stationary.
ii. Speed of the turbine in revolutions per minute when allowed to rotate freely assuming head of water at turbine entry is 20m water and there are no frictional losses in the flow through the turbine.

4. Water enters an axial flow turbine rotor at an angle of 25^o to the plane of rotation and leaves the blades with an absolute velocity of 250 m/s at an angle of 150^o to the direction of motion of the blades. Assuming equal inlet and outlet blade angles and zero axial thrust, calculate the blade angle and the output power per kg/s of fluid flow. [Note that it is not assumed zero pre-whirl at entry]

2.4 Performance and Similarity Laws of Rotodynamic Machine

2.4.1 Performance Characteristics

In all hydraulic machines, the fluid quantities involved are the flow rate (Q) and the head (h). The associated mechanical quantities are the power (P), speed (N), size (D) and the efficiency (η).

For a pump, the output at a given speed is Q and h and for that matter, a plot of " h against Q" at constant speed forms the fundamental performance characteristic of a pump. To achieve this performance requires a power input which involves efficiency of energy input. Thus a complete set of performance characteristic of a rotodynamic pump is as illustrated in Fig.2.4.

The output of a turbine is the power developed at a given speed, thus the fundamental characteristic of a turbine consists of a plot of "P against N" at constant "h". The input is Q and hence Q and "η" are usually plotted against the speed.

The performance characteristic of a hydraulic machine is a graphical representation of the quantities that are relevant to its operation. The theoretical head of a rotodynamic machine is given by equations (2.9b) and (2.11b) as

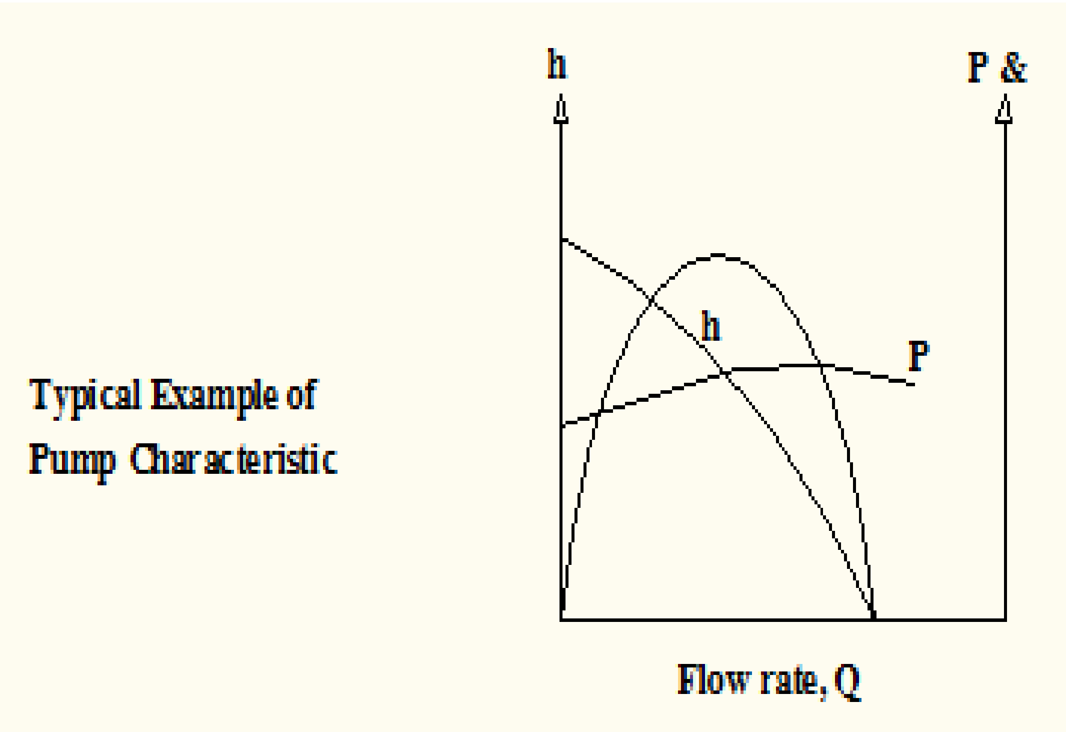
$$h = \frac{U_2}{g}\left(U_2 - C_{2F}\cot\{180 - \beta_2\}\right) = \frac{{U_2}^2}{g} - \frac{U_2}{A_2 g}Q\cot\left(180 - \beta_2\right)$$

where A_2 is impeller outlet area and $C_{2F} = Q/A_2$.

For a given impeller size and constant speed of rotation, U_2 and A_2 are constant, giving

$$h = K_1 - K_2 Q \cot(180 - \beta_2) \tag{2.20}$$

The constants in the equation are determined by experimental test. Note the importance of the blade outlet angle in the performance equation.



2.4.2 Losses and Efficiency

Several losses occur in the energy transfer process that occurs between a fluid and a hydraulic machine. First, the fluid flowing over the solid surfaces of the runner gives rise to frictional losses (boundary layer effects) and separation losses as the fluid changes direction. Additional losses can occur in the impeller as a result of secondary flows set up by pressure distribution in the impeller.

Secondly, as a result of leakage, the flow rate through the impeller is not the same as that going through the machine. In a turbine, some of the fluid supplied is discharged without striking the runners. In a pump, some fluid from the discharge end of the impeller (high pressure) can leak back to the inlet end (low pressure) and go through the impeller again; thus the impeller handles a greater volume of fluid than that discharged by the pump. Finally, there are mechanical losses occurring in bearings and seals (packing glands).

Following the above, the efficiency of a turbomachine can be broken down into three components; <u>hydraulic efficiency</u>, <u>volumetric efficiency</u> and <u>mechanical efficiency</u>.

<u>Turbine</u>

i. Volumetric Efficiency

$$\eta_v = \frac{Q - Q_L}{Q} \qquad (2.21a)$$

where Q is the net flow passing through the turbine and Q_L is the leakage loss.

ii. Hydraulic Efficiency

$$\eta_v = \frac{P_r}{P_a} = \frac{\rho g(Q-Q_L)\times(h-h_f)}{\rho g(Q-Q_L)h} = \frac{(h-h_f)}{h} \qquad (2.21b)$$

where

h :	net head available at the entrance to the turbine
h_f :	head loss due to fluid friction
$(h-h_f)$:	head utilized by the rotor,
$P_r = \rho g(Q-Q_L)(h-h_f)$:	real power delivered by the fluid to the rotor
$P_a = \rho g(Q-Q_L)h$:	available power in the fluid flowing through the rotor.
$P = \rho gQh$:	available power at the entrance to the turbine.

iii. Mechanical Efficiency

$$\eta_m = \frac{P_s}{P_r} = \frac{P_s}{\rho g(Q-Q_L)\times(h-h_f)} \qquad (2.21c)$$

where

P_s : power available at the shaft of the turbine

P_r : power delivered by the fluid to the turbine

iv. Overall Efficiency

$$\eta = \frac{P_s}{\rho gQh} = \frac{P_s}{P_r}\times\frac{P_r}{P_a}\times\frac{\rho g(Q-Q_L)h}{\rho gQh}$$

$$= \eta_v\eta_h\eta_m \qquad (2.21d)$$

Pump

i. Volumetric Efficiency

$$\eta_v = \frac{Q}{Q+Q_L} \tag{2.22a}$$

where Q_L represents the leakage loss from the high pressure side to the low pressure side and Q is the net flow passing through the pump. Thus "Q" is the actual quantity of liquid discharged per unit time from the pump whilst "$Q+Q_L$" is the fluid that is actually handled by the impeller. The loss "Q_L" is the leakage from the impeller that passes through the clearance between the impeller and the casing and finds its way back into the suction at the eye of the impeller.

ii. Hydraulic Efficiency

$$\eta_v = \frac{\rho g (Q+Q_L) h}{\rho g (Q+Q_L)(h+h_f)} = \frac{h}{(h+h_f)} \tag{2.22b}$$

where

h : net head delivered by the pump to the fluid

h_f : hydraulic loss

$(h+h_f)$: head transferred to the fluid from the rotor.

iii. Mechanical Efficiency

$$\eta_m = \frac{P_s - P_f}{P_s} = \frac{\rho g (Q+Q_L)(h+h_f)}{P_s} \tag{2.22c}$$

where P_s represents power at the shaft of the pump, P_f represents the power loss to mechanical friction and seals and "(P_s-P_f)" is the power delivered by the shaft to the rotor.

iv. Overall efficiency

$$\eta = \frac{\rho g Q h}{P_s}$$

$$= \frac{\rho g Q h}{\rho g (Q + Q_L) h} \times \frac{\rho g (Q + Q_L) h}{\rho g (Q + Q_L)(h + h_f)} \times \frac{\rho g (Q + Q_L)(h + h_f)}{P_s}$$

$$= \eta_v \eta_h \eta_m \tag{2.22d}$$

where "ρgQh" is the power delivered by the pump to the fluid and P_s represents the power at the shaft of the pump.

2.4.2 Similarity Laws

Similarity laws allow the performance of a prototype machine to be predicted from the test of a scaled model. In general, the laws of similarity (or similitude) enables us to test machines under practically convenient conditions (scaled model, speed, fluid, etc) and use the test result to predict the performance of machines belonging to the same family but having different size, operating at different speed and with different working fluid.

Similarity laws applied to turbomachines are based on the concept that two geometrically similar machines with similar velocity diagrams at entry to and exit from the rotating member are Homologous, meaning, their streamline pattern will be geometrically similar and bear resemblance to one another in behaviour.

Similarity laws are derived by dimensional analysis. The most significant variables and their respective dimensions that affect the operation of a turbomachine are the following:

Difference of head across machine	h	[L]
Power transferred between rotor and fluid	P	$[ML^{2}T^{-3}]$
Volumetric discharge	Q	$[L^{3}T^{-1}]$
Rotative speed	N	$[T^{-1}]$
Diameter of rotor	D	[M]
Density of fluid	ρ	$[ML^{-3}]$
Absolute viscosity of the fluid	μ	$[ML^{-1}T^{-1}]$
Bulk modulus of elasticity (= -v dp/dv)	K	$[ML^{-1}T^{-2}]$
Absolute roughness of machines internal passages	ε	[L].

To enable performance to be easily predicted under different gravitational forces, it is convenient to treat "gh" as the variable instead of "h". That way, a pump for example, will develop the same specific energy per unit mass (gh) on the earth as on the moon.

By the method of dimensional analysis, we write

$$\begin{aligned} gh &= \phi(Q, N, D, \rho, \mu, \varepsilon) \\ &= \sum k(Q^{a} N^{b} D^{c} \rho^{d} \mu^{e} K^{f} \varepsilon^{i}) \end{aligned} \qquad (2.23a)$$

where "k" is a dimensionless coefficient.

Thus

$$\left[L^2T^{-2}\right] \equiv \left[L^3T^{-1}\right]^a \left[T^{-1}\right]^b \left[L\right]^c \left[ML^{-3}\right]^d \left[ML^{-1}T^{-1}\right]^e \left[L\right]^i$$

Equating indices and solving gives;

$$d = -e - f,\;\; b = 2 - a - e - 2f,\;\; c = 2 - 3a - 2e - 2f - i$$

Substituting in Eq. 2.23;

$$gh = \sum kN^2D^2\left(\frac{Q}{ND^3}\right)^a\left(\frac{\mu}{ND^2\rho}\right)^e\left(\frac{K}{N^2D^2\rho}\right)^f\left(\frac{\varepsilon}{D}\right)^i$$

which gives the following functional relationship;

$$gh = N^2D^2\phi''\left[\left(\frac{Q}{ND^3}\right),\left(\frac{\mu}{ND^2\rho}\right),\left(\frac{K}{N^2D^2\rho}\right),\left(\frac{\varepsilon}{D}\right)\right]$$

$$\frac{gh}{N^2D^2} = \phi''\left[\left(\frac{Q}{ND^3}\right),\left(\frac{\mu}{ND^2\rho}\right),\left(\frac{K}{N^2D^2\rho}\right),\left(\frac{\varepsilon}{D}\right)\right] \qquad (2.23b)$$

The following dimensionless coefficients are defined in Eq.2.23b as follows:

Head Coefficient: $K_h = \dfrac{gh}{N^2D^2}$ (2.24a)

Discharge Coefficient $K_Q = \dfrac{Q}{ND^3}$ (2.24b)

Noting that $U = \omega R = \pi ND$, then $\dfrac{\mu}{ND^2\rho} \propto \dfrac{\mu}{UD\rho} \propto \dfrac{1}{\mathrm{Re}}$ (Re, Reynolds No.)

Similarly, $c^2 = K/\rho$ and $\frac{K}{N^2 D^2 \rho} \propto \frac{c^2}{U^2} \propto \frac{1}{Ma}$ (Ma, Mach No.)

Eq. 2.23b becomes

$$K_h = \phi''\left(K_Q,\ \text{Re},\ Ma,\ \varepsilon/D\right) \tag{2.25a}$$

where ε/D is the relative roughness of the machines internal passage

Similarly for P; $P = \phi(Q, N, D, \rho, \mu, \varepsilon)$

gives

$$P_h = \phi''\left(K_Q,\ \text{Re},\ Ma,\ \varepsilon/D\right) \tag{2.25b}$$

where the Power Coefficient: $P_h = \frac{P}{N^3 D^5 \rho}$ (2.25c)

The functional relationship between K_h & K_Q and K_P & K_Q is determined by experiment. For homologous machines operating under dynamically similar conditions (Re, Ma & ε/D), the respective coefficients have the same values at corresponding points of their characteristic.

Thus

$$K_Q = \frac{Q}{ND^3} \Rightarrow Q = K_Q N D^3, \tag{2.26a}$$

$$K_h = \frac{gh}{N^2 D^2} \Rightarrow gh = K_h N^2 D^2, \tag{2.26b}$$

$$K_P = \frac{P}{\rho N^3 D^5} \Rightarrow P = K_P N^3 D^5 \tag{2.26c}$$

The efficiency of homologous machines is also constant:

$$\eta = constant \qquad (2.26d)$$

For a pump, our primary interest is in its operation at a certain rotating speed, N, hence Eq.2.26 is suitable. For a turbine, the primary interest is in the its operation under a certain head, h, which is usually fixed and the relationship can be recast in terms “h”.

3.0 Hydraulic Turbines

3.1 Impulse and Reaction Turbines

A turbine may be classified as Impulse or Reaction turbine depending on the action of the water flowing through the turbine runners.

An impulse turbine is one in which the total head available is converted to kinetic energy in one or more stationary nozzles before entering the rotor. Thus, the energy available at the rotor inlet is only kinetic energy. The jet issuing from the nozzle(s) strike the rotor blades causing rotation and energy transfer. The following are some features of the impulse turbine:

i. there is no pressure drop in the flow over the vanes (the pressure everywhere is atmospheric)
ii. since the entering fluid energy is all kinetic, fluid absolute velocity and thereby its kinetic energy, will be less at turbine exit due to the energy transfer and losses.
iii. since there is no pressure variation, the fluid usually does not fill the entire space of the runner and is only in contact with part of the runner at all time.

The Pelton Wheel is the best known example of an Impulse turbine. It is named after Lester A. Pelton (1829 – 1908), an American engineer who contributed immensely to its development around 1880. There are other impulse turbines such as Turgo-impulse turbine and Girard turbine but the Pelton wheel is the only impulse turbine that is predominantly used.

In a reaction turbine, the head or pressure drop occurs partly in fixed guide vanes and in the runner. Thus, before entry into the runner, only a portion of the total available head is converted to kinetic energy implying a substantial part of the fluid energy remains in the form of pressure energy. The fluid is admitted around the entire circumference of the rotor and fills its passage entirely. In this way, all runner vanes are engaged in energy transfer at the same time. For this reason, the rotor is smaller than the impulse turbine of the same power.

An elementary example of a turbine with 100% reaction is the rotating lawn sprinkler. The Francis turbine is the best known example of a reaction turbine. The original design has seen some modifications but all these inward flow reaction turbines are referred to by the same name. The Kaplan turbine is an axial flow reaction turbine with adjustable blades.

3.2 Impulse Turbine - Pelton Wheel

The runner of the Pelton wheel consists of a circular wheel with a number of buckets evenly spaced round its periphery. The buckets have the shape of a double semi-ellipsoidal cups with each bucket divided into 2 symmetrical parts by a sharp edged ridge referred to as a splitter. The runner is encased in a casing to prevent splashing, of water and as a safeguard against accidents.

One or several nozzles are mounted such that each directs a jet along a tangent to the circle through the centres of the buckets called the pitch circle. The jet of water impinges on the splitter which splits is into 2 equal portions each of which flows through the smooth inner surface of the bucket and exits at its outer edge. By splitting the jet into 2 equal portions which flow in opposite direction, the axial thrust of the jets neutralize each other thereby eliminating axial thrust on the wheel shaft. Also at the lower tip of the bucket a notch is cut which prevents jet from striking the next bucket that follows the one the jet is striking.

3.3 Work Output and Efficiencies of Pelton Wheel

Now, $U_1 = U_2 = U$

Also at the inlet, C_1 and U are collinear and the velocity triangle reduces to a straight line. Thus,

$$V_1 = C_1 - U \text{ and } C_{1u} = C_1 \quad (3.1)$$

Due to inevitable frictional losses as the water flows over the inner faces of the bucket, V_2 is slightly less than V_1.

Thus, $$V_2 = kV_1 = k(C_1 - U) \quad (3.2)$$

where k is slightly less than unity in practice.

From the outlet velocity triangle,

$$C_{2u} = U - V_2 \cos(180 - \beta_2) = U - k(C_1 - U)\cos(180 - \beta_2) \quad (3.3)$$

By Euler's equation, the head (or energy/unit weight) exchange between the fluid and the turbine is

$$h = \frac{1}{g}(U_2 C_{2u} - U_1 C_{1u}) = \frac{U}{g}(C_{2u} - C_1)$$

Substituting for C_{2u} from Eq.3.3, this becomes

$$h = \frac{U}{g}(U - k[C_1 - U]\cos[180 - \beta_2] - C_1)$$

$$= \frac{-U}{g}([C_1 - U][1 + k\cos\{180 - \beta_2\}]) \quad (3.4)$$

i. <u>Hydraulic Efficiency</u>

Head (energy per unit weight) supplied to wheel in the form of kinetic energy is

$$h_a = C_1^2 / 2g \,. \quad (3.5)$$

Hence, the hydraulic efficiency, η_h, of the Pelton wheel is

$$\eta_h = \frac{h}{h_a} = \frac{-2U([C_1 - U][1 + k\cos\{180 - \beta_2\}])}{C_1^2} \quad (3.6)$$

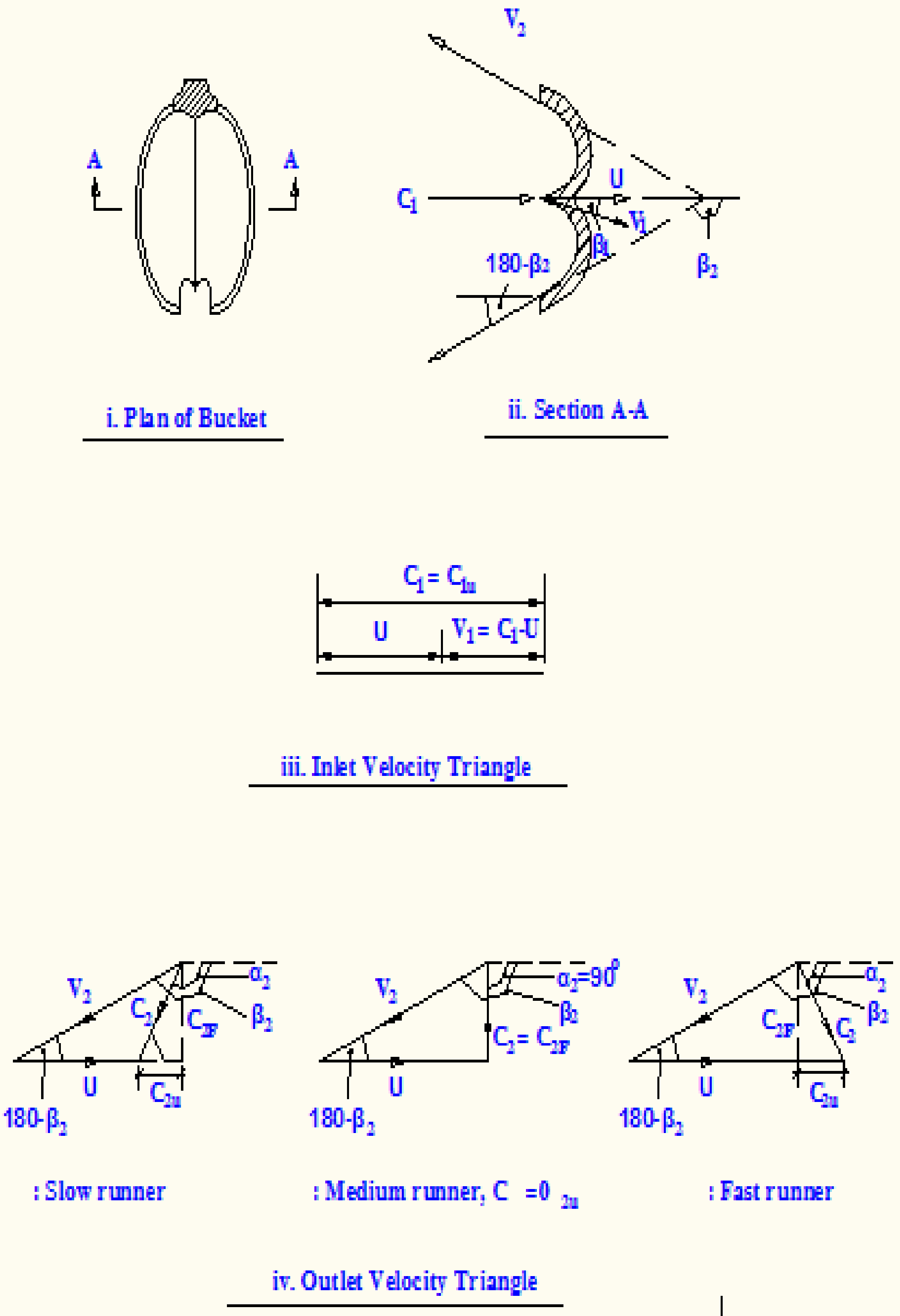


Fig. 3.1 Pelton Wheel Turbine: Bucket and Velocity Triangles

By E1. 2.6,

$$\eta_h = 0 \text{ for } U = 0 \text{ and } U = C_1$$

Thus, there exists a maximum hydraulic efficiency, $\eta_{h,max}$, between the 2 values of U.

To determine $\eta_{h,max}$ for a given jet velocity (C_1) and bucket tip angle β_2

$$\frac{d\,(\eta_h)}{dU} = \frac{2(1 + k\cos\{180 - \beta_2\})}{C_1^{\,2}}(C_1 - 2U) = 0$$

which gives $$U = \frac{C_1}{2} \qquad (3.7)$$

and $$\eta_{h,\max} = \frac{(1 + k\cos\{180 - \beta_2\})}{2} \qquad (3.8)$$

$\eta_{h,max}$ will be 100% if k = 1 and $\beta_2 = 180^o$ implying the flow is deflected through 180^o. The latter condition is not practicable because the deflected jet leaving the bucket will strike the next bucket following and thereby oppose the rotation.

Assuming no loss of energy as the water flows over the bucket (k=1), then the work output of the Pelton wheel may also be expressed as the difference in kinetic energies (or heads) between the incoming and leaving jets:

Work done per unit weight of flow, $$\Delta h = \frac{1}{2g}\left(C_1^{\,2} - C_2^{\,2}\right) \qquad (3.9)$$

and $$\eta_h = \frac{\left(C_1^{\,2} - C_2^{\,2}\right)/2g}{C_1^{\,2}/2g} = \frac{C_1^{\,2} - C_2}{C_1^{\,2}} \qquad (3.10)$$

Substituting the value of C_2 from the outlet velocity triangle and putting k=1 becomes exactly as Eq.3.6.

ii. <u>Spout Velocity, C_1</u>

Given a net head H at the entry to the jet, the theoretical jet velocity (also known as spout velocity) is

$$C_1 = \sqrt{2gH} \tag{3.11a}$$

which is obtained by applying Bernoulli's equation between the jet inlet and outlet. However, as a result of friction losses at the jet

$$C_1 = K_v\sqrt{2gH} \tag{3.11b}$$

where K_v is the Velocity Coefficient of the nozzle, with value ranging 0.97-0.99.

iii. Speed Ratio, K_v

It is also convenient to express U in terms of H by defining

$$U = K_u\sqrt{2gH} \tag{3.12}$$

where K_u is referred to as the Speed Ratio.

iv. Jet Diameter, d

The least diameter of the jet is found by

$$\frac{\pi d^2}{4}C_1 = \frac{\pi d^2}{4}K_v\sqrt{2gH} = Q$$

which gives

$$d = \left[\frac{4Q}{\pi K_v\sqrt{2gH}}\right]^{1/2} \tag{3.13}$$

Knowing the power, P, the flow rate may be determined by noting that

$$P = \eta_o(\rho QH)$$

where η_o is overall efficiency.

v. Pitch Diameter, D

The mean or pitch diameter D is found from

$$U = \frac{\pi DN}{60} = K_u\sqrt{2gH}$$

and

$$D_1 = \frac{60\left(K_u\sqrt{2gH}\right)}{\pi N} \tag{3.14}$$

3.4 Reaction Turbine - Francis Turbine

The Francis turbine was first developed as an inward radial flow reaction turbine by James B. Francis, an American engineer, after whom it is named. It was later modified to a mixed flow type whereby the water enters the runner radially at its outer periphery and exits at its centre.

From the penstock, the water enters the scroll casing (aka spiral scroll) which serves to provide an even distribution of water around the entire circumference of the turbine runner. To maintain the velocity of water constant throughout its path around the runner, the cross-sectional area of the casing is gradually reducing.

From the scroll casing, the water enters the speed ring which comprises of an upper and lower ring held together by the stay vanes. The stay vanes direct the water entering from the scroll casing to the guide vanes (aka wicket basket). Besides directing the flow onto the guide vanes, the speed ring also bears the weight of the column of water acting on the turbine and the electrical generator that it drives and transmits this load to the foundation. The number of stay vanes is usually half the number of guide vanes.

The guide vanes which are airfoil shaped regulate the volume and angle of water supplied to the runner. Regulation of the guide vanes is done by a series of levers and linkages which turn the vanes about their stems in unison. The regulation is done by means of a wheel (for small units) or automatically by a governor.

The runner comprises of series of curved vanes numbering 16 to 24. The vanes

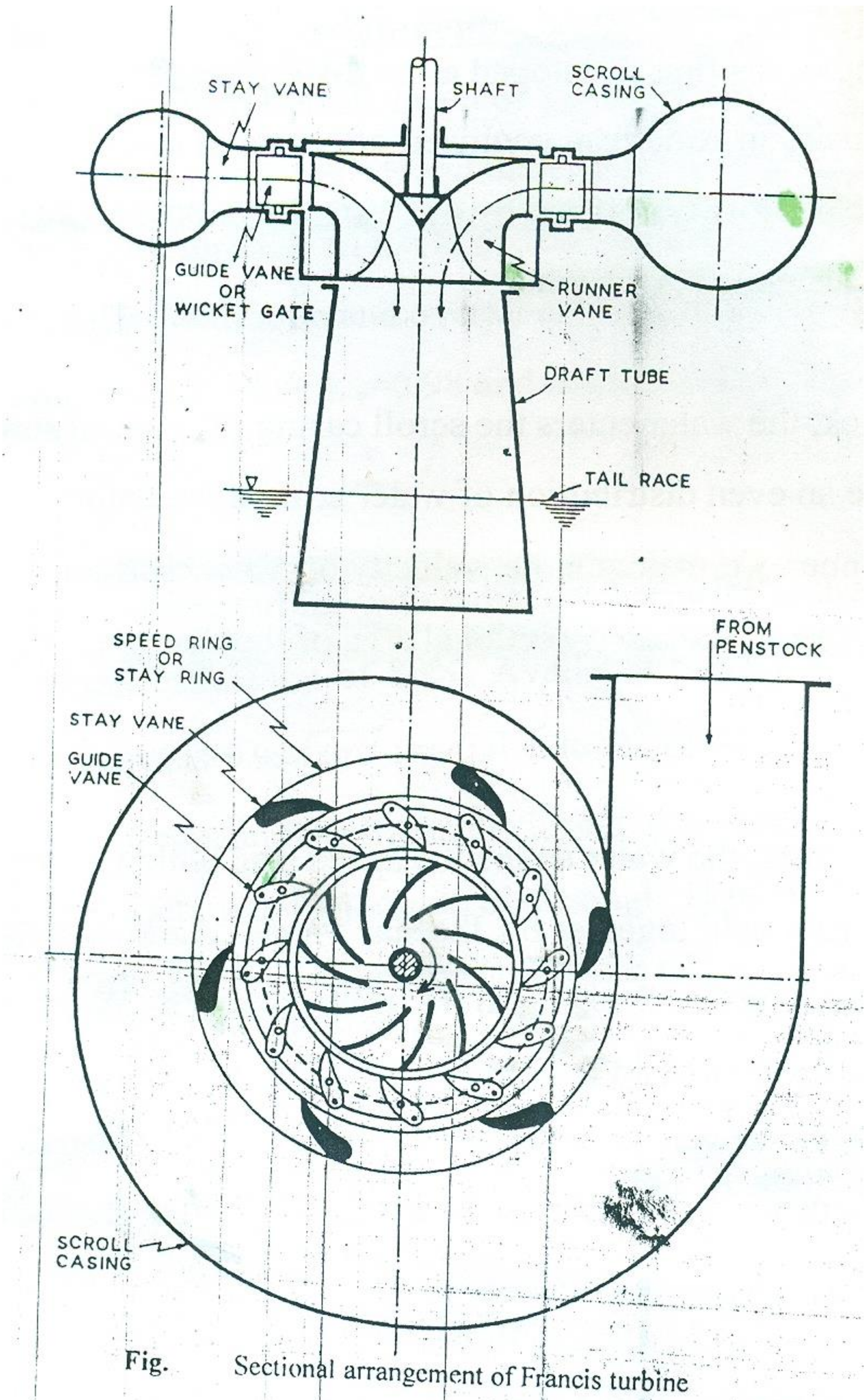


Fig. Sectional arrangement of Francis turbine are so shaped that the water enters the runner radially at the outer periphery and exits it axially at the inner periphery. The torque which rotates the runner comes from the change in the direction of flow of the water from radial to axial as it passes through the runner.

3.5 Work Output and Efficiencies of Francis Turbine

By Euler's equation, the head (or energy/unit weight) exchange between the fluid and the turbine is

$$h = \frac{1}{g}\left(U_2 C_{2u} - U_1 C_{1u}\right) \tag{3.15}$$

Recall in the derivation of the Euler's equation, "h" above will be the energy per unit weight or head input to the fluid and thus the energy delivered by the fluid to the turbine will be $\frac{1}{g}\left(U_1 C_{1u} - U_2 C_{2u}\right)$. Under specified conditions, the turbine power will be maximum when the whirl velocity is zero at the exit ($C_{2u}=0$).

If "H" is the input head, then the hydraulic efficiency is

$$\eta_h = \frac{1}{gH}\left(U_1 C_{1u} - U_2 C_{2u}\right) \tag{3.16}$$

The value of "η_h" for the Francis turbine usually runs between 85 to 95% and the overall efficiency between 80 to 90%.

3.6 Degree of Reaction, ∧

Degree of Reaction "∧" is defined as the ratio of the change in pressure head (energy transfer) occurring in the runner to the total energy transfer (hydraulic work done) on the runner.

$$\Lambda = \frac{\frac{P_1}{\rho g} - \frac{P_2}{\rho g}}{\left(U_1 C_{1u} - U_2 C_{2u}\right)/g} \tag{3.17}$$

A machine with any degree of reaction ($\rho \succ 0$) must have the rotor enclosed to prevent the fluid from expanding freely in all directions. When $\Lambda = 0$, there is no change of pressure (static) head in the flow through the rotor and such machines are Impulse machines.

3.7 Draft Tube

After passing through the runner, the water flows to the tail race through a Draft Tube which is a passage made of plate steel or concrete of increasing cross sectional are that connects the runner exit to the tail race. The draft tube must be airtight and its lower end must at all times be submerged below the level of water in the tail race (namely not exposed to the ambient pressure).

The function of the draft tube are

i. to establish a negative pressure or suction head at the runner exit thereby making it possible to install the turbine above the tail race level without loss of head.

ii. To convert a large fraction of the kinetic energy of the water leaving the runner into useful pressure energy (pressure recovery)

Shown in Fig.3.4 are different versions of draft tube. Types (a) and (b) are the most efficient but types (c) and (d) require less excavation. In type (a), the cone angle is kept at not more than 8^o to ensure that the water sticks to the inner surface; at higher cone angles, the water will not remain in contact with the inner surface thereby leading to formation of eddies which reduce the tube efficiency.

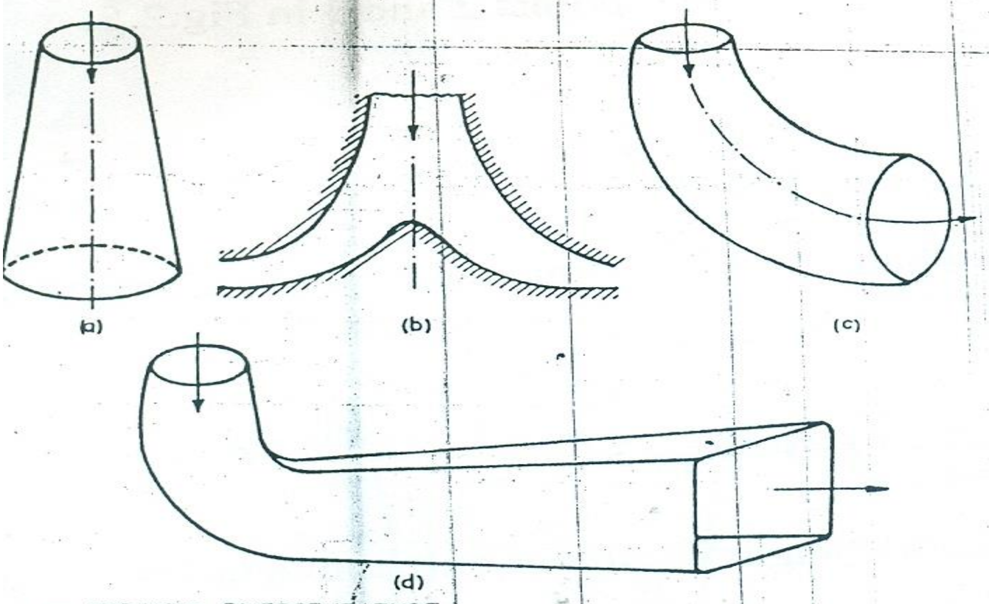


Fig. Different types of draft tubes

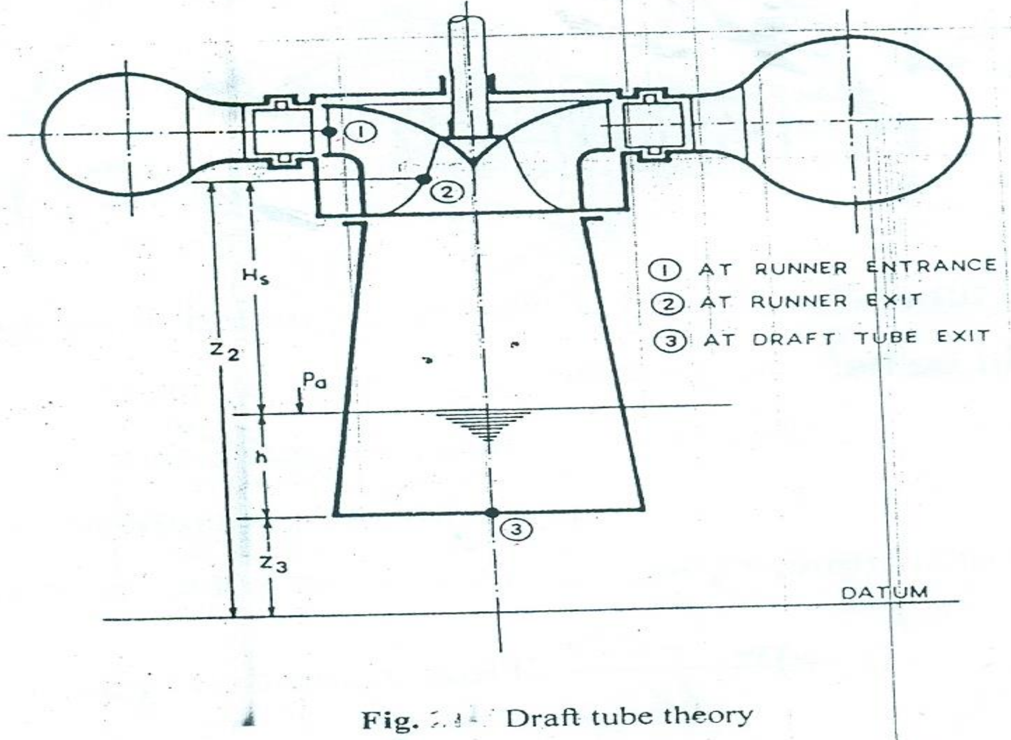


Fig. Draft tube theory

Applying Bernoulli's equation between points 2 and 3 in Fig.2.5,

$$\frac{p_2}{w}+\frac{C_2^{\ 2}}{2g}+z_2=\frac{p_3}{w}+\frac{C_3^{\ 2}}{2g}+z_3+h_f$$

where "h_f" is the loss of head in the draft tube.

Introducing $\frac{p_3}{w}=\frac{p_a}{w}+h$ where P_a is the ambient pressure and rearranging, the above equation becomes

$$\frac{p_2}{w}=\frac{p_a}{w}-\left[H_s+\frac{C_2^{\ 2}-C_1^{\ 2}}{2g}\right]+h_f \qquad (3.18)$$

where "$H_s = z_2 - z_3 - h$" is the height of runner exit above tail race level.
Eq. 3.18 shows that the pressure at the runner exit is suction pressure (below atmospheric). "H_2" is referred to as the Static Suction Head and $\left(C_2^{\ 2}-C_1^{\ 2}\right)/2g$ the Dynamic Suction Head. The head loss in the draft tube is usually expressed in terms of the change in flow velocity in the tube as

$$h_f=k\frac{\left(C_2^{\ 2}-C_3^{\ 2}\right)}{2g}$$

and Eq.3.18 becomes

$$\frac{p_2}{w}=\frac{p_a}{w}-\left[H_s+(1-k)\frac{C_2^{\ 2}-C_3^{\ 2}}{2g}\right] \qquad (33.19)$$

For cylindrical draft tube, $C_2 = C_3$ and if "h_f" is neglected, then the pressure at the runner exit would be below atmospheric pressure by an amount H_s which is the height of the runner exit above the tail race. Thus the turbine will not loose the head H_s because there will be equal reduction in pressure head at the runner exit. For a diverging draft tube, $C_3 \prec C_2$,and the lower pressure at the runner exit is further reduced by $h_f=(1-k)\frac{\left(C_2^{\ 2}-C_3^{\ 2}\right)}{2g}$. Thus in the divergent draft, a large portion of the kinetic head at the runner exit is converted to pressure head.

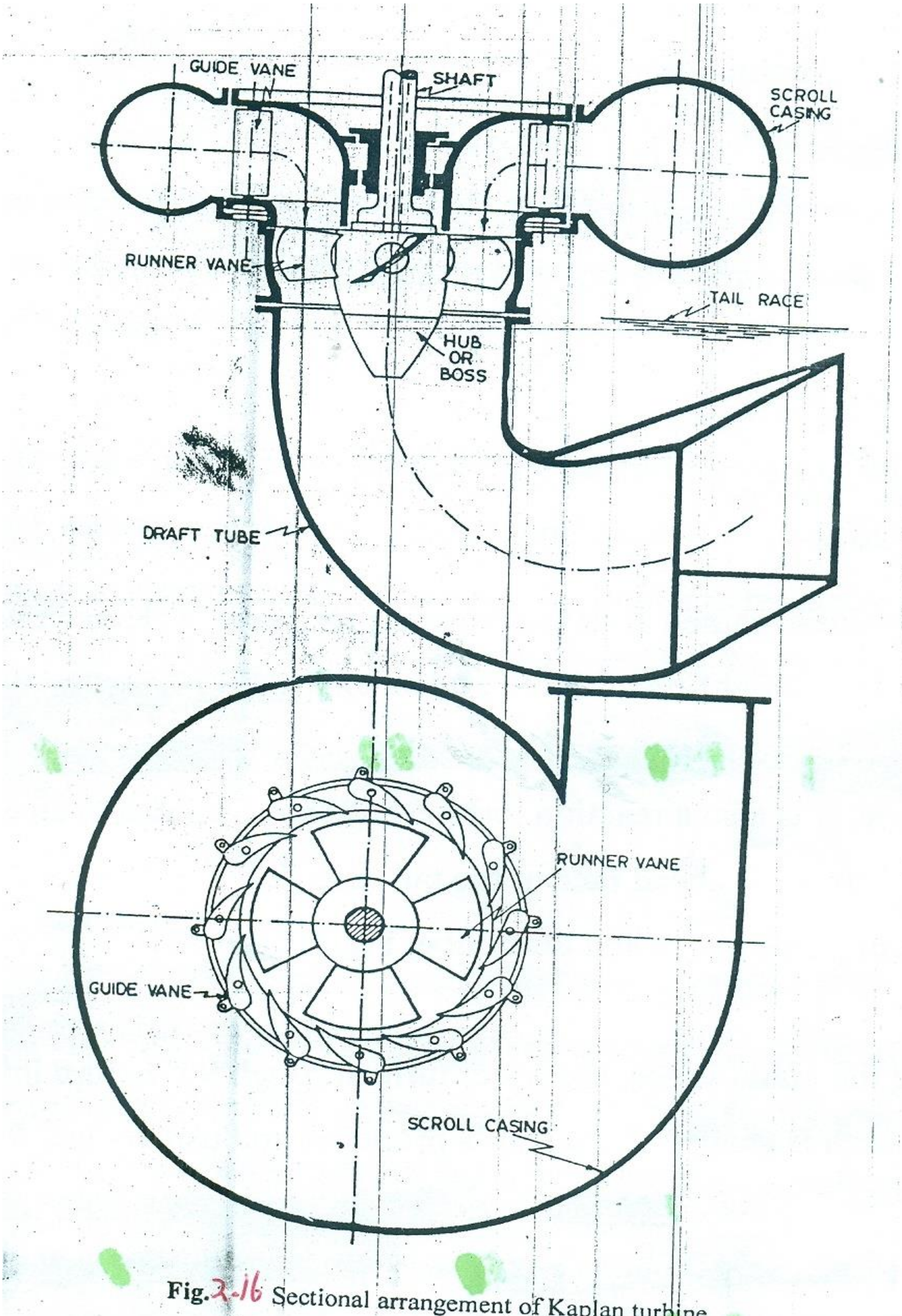


Fig. 2-16 Sectional arrangement of Kaplan turbine

Draft Tube Efficiency

The efficiency, η_d, of the draft tube is defined as the ratio of “Actual regain of pressure head” to the “Velocity head at entrance to Draft tube”:

Actual regain of pressure head: $\dfrac{\left(C_2^{\,2} - C_3^{\,2}\right)}{2g} - h_f = (1-k)\dfrac{C_2^{\,2} - C_3^{\,2}}{2g}$

Thus,
$$\eta_d = \frac{(1-k)\left\{C_2^{\,2} - C_3^{\,2}\right\}/2g}{\left(C_2^{\,2}/2g\right)}$$

$$= (1-k)\left\{1 - \left[\frac{C_3}{C_2}\right]^2\right\} \qquad (2.20)$$

3.8 Kaplan Turbine

The Kaplan turbine, named after the Austrian Engineer V. Kaplan who developed it, is an axial flow propeller turbine. It is well suited to very low heads and therefore requires a large flow volume to develop large power. Like the Francis turbine, it is also a reaction turbine and thus operates in an entirely closed conduit from the head race to the tail race. It has a scroll casing, stay ring, arrangement of guide vanes and draft tube, Fig. 3.16.

After exiting the guide vanes, the water turns through 90^o before impinging on the runner which is shaped like a ship’s propeller and usually has 4 or 6 (max. 8) blades attached to a hub. The runner blades can be automatically adjusted about their axis whilst in operation via a servomotor arrangement inside the hollow coupling of the turbine. Thus, regulation of both the guide vane angle and runner-blade angle enables the operation of the turbine to be varied to achieve high efficiency over a wide range of operating conditions such as at part load conditions due to the elimination of shock at entry. The runner blade angle of the Francis turbine is fixed and therefore shocks and eddy losses are inevitable.

3.9 Miscellaneous

i. Governor

Hydraulic turbines are today directly coupled to the electric generators that the drive. Electric generators operate at constant speed, N, according to the relation

$$f = \frac{pN}{60}$$

where "f" is the frequency of power generated and "p" is the number of pole pairs of the generator. The speed N also referred to as the synchronous speed must be held constant to ensure that the power is generated at constant "f", for example 50 Hz in Ghana.

As the load on the generator varies, the speed of the turbine runner will also vary if the input to the turbine remains the same. To maintain the speed of the runner constant, the volume of water running through the runner is regulated automatically by means of a governor. Illustrated in Fig. 3.7 is one type of governor referred to as Oil Pressure Governor, that is predominantly used in modern turbines.

ii. Runaway Speed

If the external load on a turbine drops to zero and at the same time the governing mechanism fails, the turbine runner will speed up to a maximum speed referred to as the Runaway Speed, N_{RS}. For safety reasons, all rotating components of the turbine must be designed to be able to withstand the runaway speed condition when it should occur. N_{RS} lies in the range 1.8 to 1.9 for the Pelton wheel, 2 to 2.2 for the Francis turbine and 2.5 to 3 for the Kaplan turbine.

Worked Examples 3

1. A Pelton wheel has a mean bucket speed of 10m/s with a jet of water flowing at the rate of 700 litres/s under a head of 30m. The bucket deflects the jet through an angle of 160^o. Calculate the power given by the water to the runner and the hydraulic efficiency of the turbine. Assume coefficient of velocity $K_v = 0.98$ and k=1.

   Solution

   $$U_1 = U_2 = U = 10m/s$$

   $$C_1 = K_v\sqrt{(2gh)} = 0.98 \times \sqrt{(2 \times 9.81 \times 30)} = 23.77m/s$$

   $$V = C_1 - U = 13.77m/s$$

   $$e = -h = \frac{U}{g}(C_1 - U)(1 - k\cos\{180 - \beta_2\}) = \frac{10}{9.81}(23.77 - 10)(1 - \cos 160) = 27.23m$$

   $$P = \rho Vg \times e = 1000 \times 0.7 \times 9.81 \times 27.23 = 187kW$$

   $$\eta = \frac{e}{C_1^2/2g} = \frac{27.23 \times 2 \times 9.81}{23.77^2} = 95\%$$

2. Given that the loss due to bucket friction and shock is $k_1(C_1 - U)^2/2g$ and that due to bearing friction and windage losses as $k_2U^2/2g$, where k_1 & k_2 are constants, show that the maximum efficiency of the Pelton wheel turbine occurs when the ratio of bucket velocity U to the jet velocity C_1 is given by

   $$\frac{U}{C_1} = \frac{1 - \cos\{180 - \beta_2\} + k_1}{2(1 - \cos\{180 - \beta_2\}) + k_1 + k_2}$$

   Solution

   Given the losses, the work output is the ideal with k=1 minus the losses:

   $$e = -h = \frac{U}{g}(C_1 - U)(1 - \cos\{180 - \beta_2\}) - k_1\frac{(C_1 - U)^2}{2g} - k_2\frac{U^2}{2g}$$

   Thus, the efficiency is:

$$\eta = \frac{e}{C_1^2/2g} = \frac{U}{g}(C_1 - U)[1-\cos\{180-\beta_2\}] - k_1\frac{(C_1-U)^2}{2g} - k_2\frac{U^2}{2g} \Big/ \frac{C_1^2}{2g}$$

$$= \frac{2U(C_1.-U)[1-\cos\{180-\beta_2\}] - k_1(C_1-U^2) - k_2U^2}{C_1^2}$$

For a fixed C_1, maximum η occurs when $\frac{d\eta}{dU} = 0$

$$\frac{2(C_1-2U)[1-\cos\{180-\beta_2\}] + 2k_1(C_1-U) - 2k_2U}{C_1^2} = 0$$

Solving,

$$\frac{U}{C_1} = \frac{1-\cos\{180-\beta_2\}+k_1}{2(1-\cos\{180-\beta_2\}) + k_1 + k_2}$$

3. A Pelton wheel has to be designed for the following data:

i.	Power to be developed	6,000 kW
ii.	Net available head	300 m
iii.	Speed	550 rpm
iv.	Ratio of jet diameter to wheel diameter	1/10
v.	Overall efficiency	85 %

Assuming $K_v = 0.98$ and $K_u = 0.46$, determine the number of jets, jet diameter, wheel diameter and the flow rate required.

Solution

$$C_1 = K_V\sqrt{2gH} = 0.98\sqrt{2\times 9.81\times 300} = 75.19\ m/s$$

$$U = K_u\sqrt{2gH} = 0.46\sqrt{2\times 9.81\times 300} = 35.29\ m/s$$

$$\eta = \frac{P}{wQH} = \frac{6000\times 10^3}{9810\times Q\times 300} \text{ or } Q = 2.399\ m^3/s$$

$$U = \frac{\pi DN}{60} = \frac{\pi\times D\times 550}{60} = 35.29\ m/s \text{ or } D = 1.225\ m$$

Thus, $\frac{d}{D} = \frac{1}{10}$ or $d = 0.10D = 122.5\ mm$

Jet area; $d = \frac{\pi}{4} \times (0.1225)^2 = 0.0118\ m^2$

Total jet area required; $\frac{Q}{C_1} = \frac{2.339}{75.19} = 0.319\ m^3$

No. of jets required: $\frac{0.0319}{0.0118} = 2.7 \approx 3$

Thus 3 jets are required each having a diameter $d = \left(\frac{0.0319}{3\{\pi/4\}} \right)^{1/2} = 116.4\ mm$

and $\frac{d}{D} = \frac{0.1164}{1.225} = \frac{1}{10.52}$

4. At the inlet to the runner of an inward flow reaction turbine, the whirl velocity is ($3.15\sqrt{H}$) m/s and the flow velocity is ($1.05\sqrt{H}$) m/s where H is the head in metres. At the exit the whirl velocity is ($0.22\sqrt{H}$) m/s in the same direction as at inlet and the flow velocity ($0.83\sqrt{H}$) m/s. The inner diameter of the runner is 0.6 times that of the outer. Assuming 80% hydraulic efficiency, compute the angles of the runner vanes at inlet and exit.

Solution

$$\eta_h = \frac{(C_{1u}U_1 - C_{2u}U_2)}{gH}; \qquad \frac{U_1}{D_1} = \frac{U_2}{D_2} \text{ or } U_2 = \frac{D_2}{D_1}U_1 = 0.6U_1$$

$$0.83 = \frac{(3.15\sqrt{H}U_1 - 0.22\sqrt{H} \times 0.6U_1)}{9.81H}$$

which gives $U_1 = 2.60\sqrt{H}$ and $U_2 = 0.6U_1 = 1.56\sqrt{H}$

From the inlet velocity triangle,

$$\tan\beta_1 = \frac{C_{1F}}{(U_1 - C_{1u})} = \frac{1.05\sqrt{H}}{(3.15\sqrt{H} - 2.60\sqrt{H})} = 1.9091 \text{ and } \beta_1 = 62^o\ 21'$$

From the outlet velocity triangle,

$$\tan(180 - \beta_2) = \frac{C_{1F}}{(U_1 - C_{1u})} = \frac{1.05\sqrt{H}}{(1.56\sqrt{H} - 0.22\sqrt{H})} = 0.6194 \text{ and } \beta_1 = 148^o\ 14'$$

5 In a Francis turbine, the flow velocity from inlet to exit od the turbine remains constant. Assuming the turbines discharges radially and neglecting the losses in the runner, show that the degree of reaction can be expressed as $\Lambda = \frac{1}{2} - \frac{1}{2}\left\{\frac{\cot\beta_1}{\cot\alpha_1 - \cot\beta_1}\right\}$

Solution

Applying Bernoulli's equation between the inlet and the exit of the runner and neglecting the difference in potential head, gives

$$\frac{p_1}{w} + \frac{C_1^{\ 2}}{2g} = \frac{p_2}{w} + \frac{C_2^{\ 2}}{2g} + h = \frac{p_2}{w} + \frac{C_2^{\ 2}}{2g} + \frac{C_{1u}U_1}{g}$$

where $h = \frac{C_{1u}U_1 - C_{2u}U_2}{g} = \frac{C_{1u}U_1}{g}$ for radial discharge

Drop in pressure head due to hydraulic work done in runner is given by

$$\frac{p_1}{w} - \frac{p_2}{w} = \frac{C_2^{\ 2}}{2g} - \frac{C_1^{\ 2}}{2g} + \frac{C_{1u}U_1}{g}$$

$$\Lambda = \left(\frac{p_1}{w} - \frac{p_2}{w}\right) \Big/ \frac{C_{1u}U_1}{g} = \left(\frac{C_2^{\ 2}}{2g} - \frac{C_1^{\ 2}}{2g} + \frac{C_{1u}U_1}{g}\right) \Big/ \frac{C_{1u}U_1}{g} = 1 + \frac{1}{2}\left[\frac{C_2^{\ 2} - C_1^{\ 2}}{C_{1u}U_1}\right]$$

For radial discharge, $C_2 = C_{2F} = C_{1F}$

Also, $C_{1u} = C_1 \cos\alpha_1$, $C_{1F} = C_1 \sin\alpha_1$,

$U_1 = C_{1u} - C_{1F}\cot\beta_1 = C_1[\cos\alpha_1 - \sin\alpha_1\cot\beta_1]$

$C_1 = C_{1F} = C_1 \sin\alpha_1$

Introducing these values into the expression for $\wedge$ above:

$$\Lambda = \frac{1}{2} - \frac{1}{2}\left[\frac{\cot\beta_1}{\cot\alpha_1 - \cot\beta_1}\right]$$

Assignment 2

(Deadline for submitting Assignment: …………..April 2011)

1. A Pelton wheel has a mean bucket speed of 12 m/s and is supplied with water at the rate of 750 litres per second under a head of 35 m. If the bucket deflects the jet through an angle of 160^o, find the power developed by the turbine and its hydraulic and overall efficiency assuming its mechanical efficiency is 80%, K_v=0.98 and neglect bucket friction.
[238.82 kW, 96.6%, 77.3%]

2. An inward flow reaction turbine with radial discharge is required to develop 150 kW at overall efficiency of 80%. Available head is 8 m; peripheral velocity of the wheel is $0.96\sqrt{2gH}$; radial velocity of the flow is $0.36\sqrt{2gH}$. Wheel speed is 150 rpm and the hydraulic losses in the turbine is 22% of the available energy. Determine (i). guide blade angle at inlet, (ii). wheel vane angle at inlet, (iii) wheel diameter and (iv) width of wheel at inlet.
[41^o 33', 147^o, 1.532 m, 110 mm]

4 A Pelton wheel is working under a gross head of 400m. The water is supplied through a penstock of diameter 1m and length 4km from the reservoir to the Pelton wheel. The coefficient of friction for the penstock is given as 0.008. The jet of water of diameter 150mm strikes the buckets of the wheel and gets deflected through an angle of 165^o. The relative velocity of the water at outlet is reduced by 15% due to friction between inside surface of the bucket and water. If the velocity of the buckets is 0.45 times the jet velocity at inlet and mechanical efficiency is 85%, determine the power given to the runner, shaft power, hydraulic and overall efficiencies.
[5,033 kW, 4,278.5 kW, 90%, 76.2%]