CHAPTER 4

BENDING OF CURVED BARS

Introduction

Recall, for straight beams, the bending moment is related to the bending stress and radius of curvature by

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

The above equation can be applied with sufficient accuracy to beams with **small initial curvature** (i.e., beams whose radius of curvature *R* is large compared to the cross-section dimensions). For elements with **large initial curvature**, such as rings, crane hooks, chain links, etc., the simple bending equation cannot be used.

small *R*

Introduction

Assumptions Used in Deriving the Bending Stress Equations

1. The bar material is stressed within the elastic limit, and thus obeys Hooke's law.

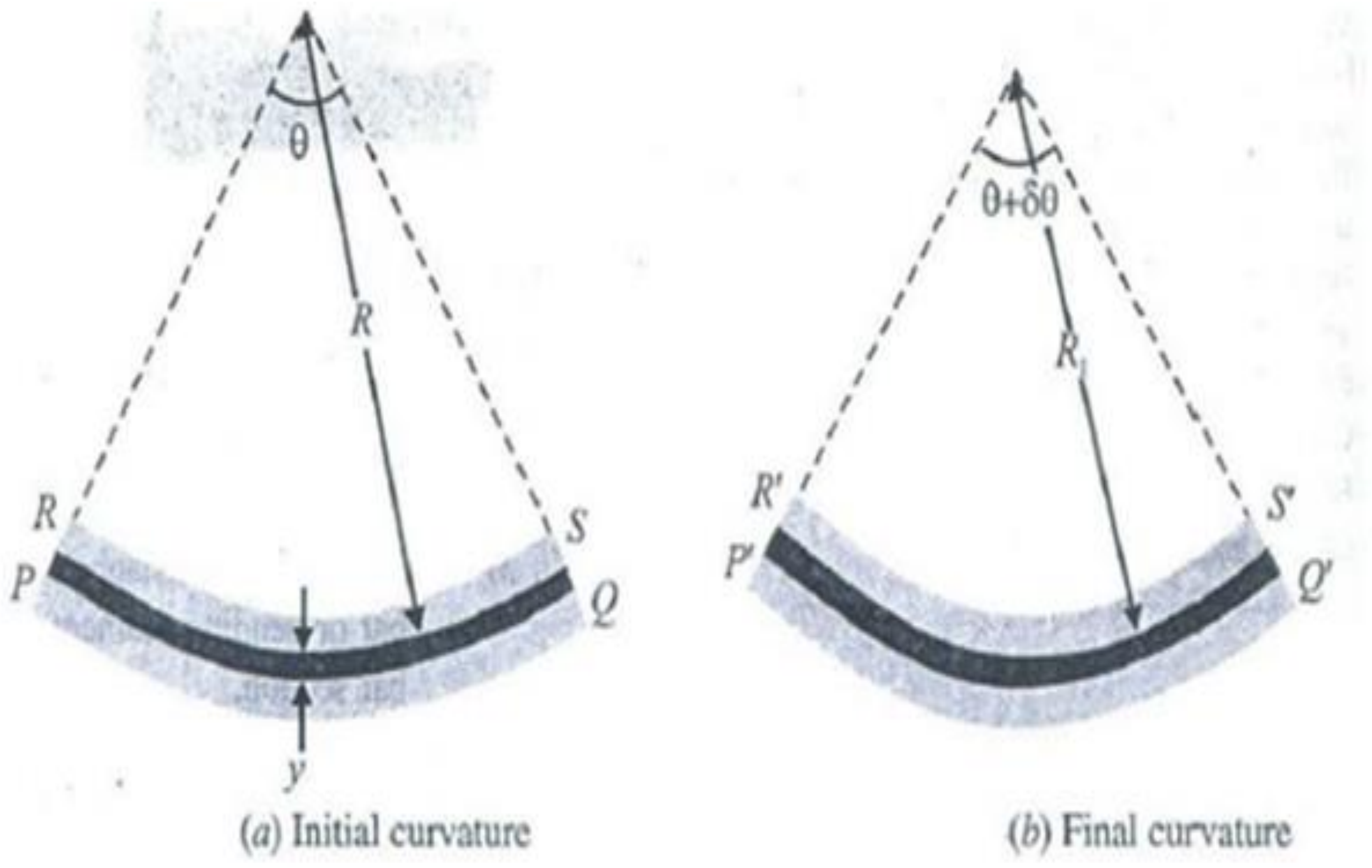
2. The transverse sections, which were plane before bending, remain plane after bending.

3. The longitudinal fibres of the bar, parallel to the central axis, exert no pressure on each other.

4. The transverse cross-section has at least one axis of symmetry, and the bending moment lies on this plane.

5. The value of E (*i.e.*, modulus of elasticity) is the same in tension and compression.

Bars with Small Initial Curvature



Bars with Small Initial Curvature

Let the bar be given more curvature after the application of the end moments as shown in Fig. (b) above.

Let

R denote the initial radius of curvature of the bar

R_1 the Final radius of curvature

θ the initial angle subtended at the centre of curvature, and

$(\theta + \delta\theta)$ the final angle subtended at the centre of curvature.

The change in length

$$\delta l = P'Q' - PQ$$

Bars with Small Initial Curvature

Therefore, strain

$$\varepsilon = \frac{\delta l}{PQ} = \frac{P'Q' - PQ}{PQ} \;..........(i)$$

$$\varepsilon = \frac{(R_1 + y)(\theta + \delta\theta) - (R + y)\theta}{(R + y)\theta}$$

$$\varepsilon = \frac{R_1(\theta + \delta\theta) + y.\delta\theta - R\theta}{(R + y)\theta} \;....(ii)$$

From the geometry of the bar,

$$RS = R\theta$$

$$R'S' = R_1(\theta + \delta\theta)$$

But $RS = R'S'$

$$\Rightarrow R\theta = R_1(\theta + \delta\theta) = R_1\theta + R_1\delta\theta$$

$$\Rightarrow R\theta - R_1\theta = R_1\delta\theta$$

$$\Rightarrow (R - R_1)\theta = R_1\delta\theta$$

Hence $\dfrac{\delta\theta}{\theta} = \left(\dfrac{R - R_1}{R_1}\right)$

Bars with Small Initial Curvature

Substituting $R_1(\theta+\delta\theta) = R\theta$ in (ii)

$$\varepsilon = \frac{R\theta + y\delta\theta - R\theta}{(R+y)\theta} = \frac{y\delta\theta}{(R+y)\theta}$$

$$= \frac{y}{(R+y)}\left(\frac{\delta\theta}{\theta}\right)$$

$$\therefore \ \ \varepsilon = \left(\frac{y}{R+y}\right)\left(\frac{R-R_1}{R}\right)$$

Since $y << R$,

Then, $(R + \mathrm{y}) = R$

Therefore $$\varepsilon = y\left(\frac{1}{R_1} - \frac{1}{R}\right)$$

$$\sigma = E\varepsilon = Ey\left(\frac{1}{R_1} - \frac{1}{R}\right)$$

Example 1

A steel bar 50 mm in diameter, is formed into a circular arc of 4 m radius and supports an angle of 90^o. A couple is applied at each end of the bar, which changes the slope to 95^o at one end relative to the other. Calculate the maximum bending stress due to the couple. Take E as 200 GPa.

Solution

Given: Diameter of bar (d) = 50 mm; Radius of arc (R) = 4 m = 4000 mm; Initial angle subtended at the centre (θ) = 90°; Final angle subtended at the centre ($\theta + \delta\theta$) = 95° and modulus of elasticity (E) = 200 GPa = 200 x 10^3 N/mm^2

$$\delta\theta = 95^o - 90^0 = 5^o$$

$$\frac{\delta\theta}{\theta} = \frac{(R - R_1)}{R_1} \Rightarrow \frac{5}{90} = \frac{(4000 - R_1)}{R_1}$$

$$5R_1 = 360000 - 90R_1$$

$$\therefore R_1 = \frac{360000}{95} = 3789\ mm$$

Example 1 (continued)

$$\boxed{\sigma = E\varepsilon = Ey\left(\frac{1}{R_1} - \frac{1}{R}\right)}$$

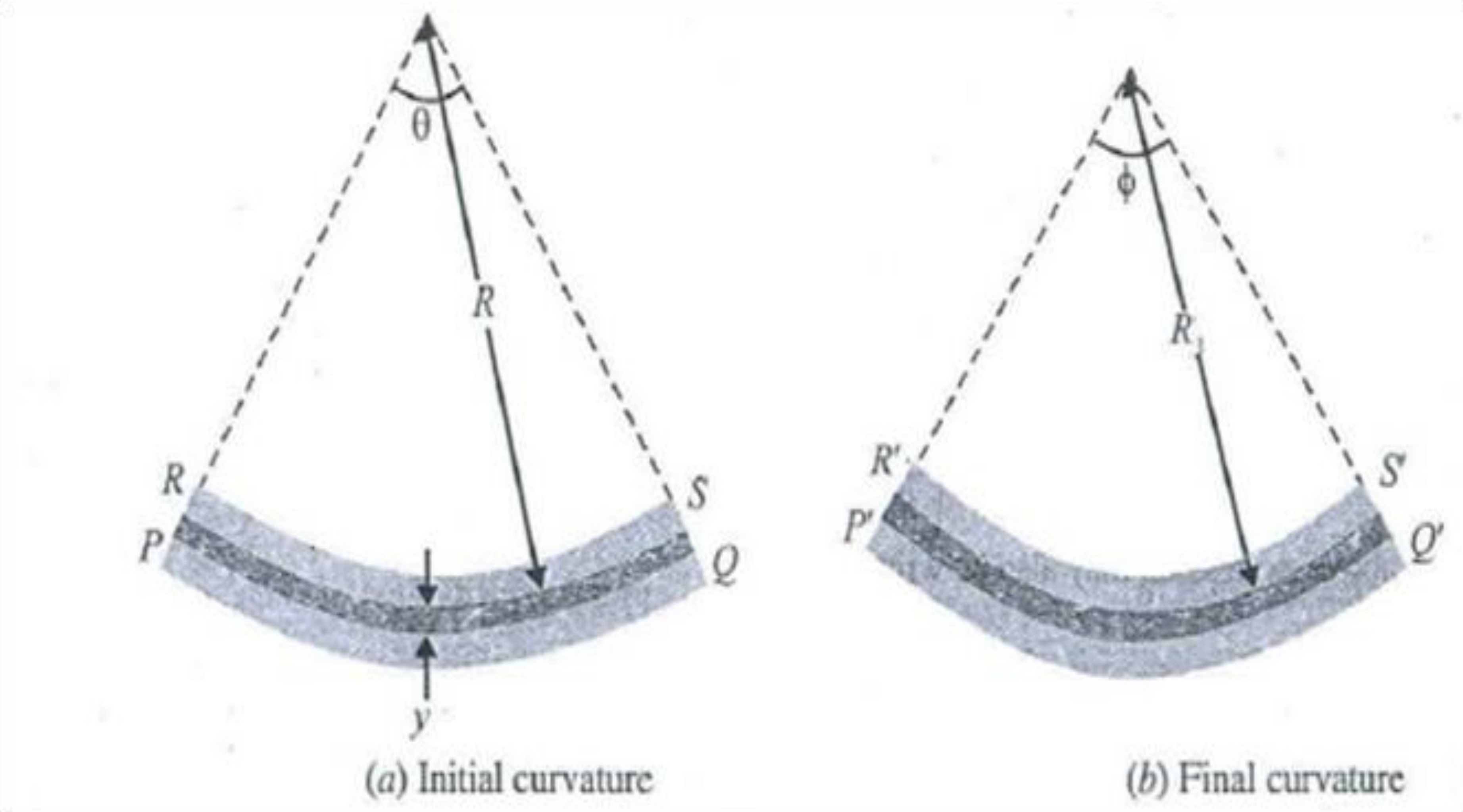
Distance between centre line of bar and extreme fibre

$$y = \frac{d}{2} = \frac{50}{2} = 25\,mm$$

The maximum bending stress due to the couple

$$\sigma = E\varepsilon = Ey\left(\frac{1}{R_1} - \frac{1}{R}\right) = \left(200\times 10^3\right)(25)\left[\frac{1}{3789} - \frac{1}{4000}\right] = 69.6\ \mathrm{N/mm^2}$$

Bars with a Large Initial Curvature



Bars with a Large Initial Curvature

Again, let

R denote the initial radius of curvature of the bar
R_1 the final radius of curvature
θ the initial angle subtended at the centre by the bar,
ϕ final angle subtended at the centre of the bar
σ_0 bending stress in the centroidal fibre R'S'
σ bending stress in the fibre P'Q' and
dA area of fibre *P'Q'*.

Now consider a layer *PQ*, which has been bent up to P'*Q'* after bending.

Let *y* be the distance of the layer *PQ* from *RS*, the neutral axis of the bar.

We know that increase in the length of the bar.

Bars with a Large Initial Curvature

We know that the increase in the length of the bar at the centroidal axis, $\delta l = R'S' - RS$
Strain,

$$\varepsilon_0 = \frac{\delta l}{PQ} = \frac{R'S' - RS}{RS} = \frac{R'S'}{RS} - 1 \qquad \varepsilon_0 + 1 = \frac{R'S'}{RS} = \frac{R_1\varphi}{R\theta} \quad \text{(i)}$$

and increase in the length of the bar at a distance y from the centroidal axis is $\delta l = P'Q' - PQ$

Strain is

$$\varepsilon = \frac{\delta l}{PQ} = \frac{P'Q' - PQ}{PQ} = \frac{P'Q'}{PQ} - 1 \qquad \varepsilon + 1 = \frac{P'Q'}{PQ} = \frac{(R_1+y)\varphi}{(R+1)\theta} \quad \text{(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\varepsilon+1}{\varepsilon_0+1} = \frac{\frac{(R_1+y)\varphi}{(R+y)\theta}}{\frac{R_1\varphi}{R\theta}} = \frac{\frac{R_1+y}{R_1}}{\frac{R+y}{R}} \qquad \varepsilon = \varepsilon_0 + \frac{(\varepsilon_0+1)y\left(\frac{1}{R_1}-\frac{1}{R}\right)}{1+\frac{y}{R}} \text{ (iv)}$$

Bars with a Large Initial Curvature

From equation (iv) $\varepsilon = \varepsilon_0 + (\varepsilon_0 + 1)y\left(\frac{1}{R_1} - \frac{1}{R}\right)/(1 + y/R)$

The bending stress in the fibre P'*Q'*,

$$\sigma = E.\varepsilon = E\left[\varepsilon_0 + \frac{(\varepsilon_0+1)y\left(\frac{1}{R_1}-\frac{1}{R}\right)}{1+\frac{y}{R}}\right] \qquad \text{(v)}$$

Eqns. (iv) and (v) imply that for bars of large initial curvature, stress and strain are no longer proportional to *y*. That is, σ ≠ 0 on the centroidal axis.

The force in an element of area *dA* at a distance *y* of from the centroidal axis,

$$dP = \sigma dA = E\left|\varepsilon_0 + \frac{(\varepsilon_0 + 1)y\left(\frac{1}{R_1} - \frac{1}{R}\right)}{1 + \frac{y}{R}}\right| dA$$

The total normal force

$$P = E\varepsilon_0 A + E(\varepsilon_0 + 1)\left(\frac{1}{R_1} - \frac{1}{R}\right)\int \frac{y}{1+\left(y/R\right)} dA$$

Bars with a Large Initial Curvature

Since the beam is in equilibrium, therefore the total normal force on the cross-section is zero.

$$P = E\varepsilon_0 A + E(\varepsilon_0 + 1)\left(\frac{1}{R_1} - \frac{1}{R}\right)\int \frac{y}{1+\left(y/R\right)} dA = 0..(vi)$$

Let us find out the value of separately

$$\int \frac{y}{1+\left(y/R\right)} dA = \int y.dA - \int \frac{y^2}{R+y} dA$$

Since $\int y.dA$ being first moment of area about the neutral axis is zero, hence

$$\int \frac{y}{1+(y/R)} dA = -\int \frac{y^2}{R+y} dA = -\frac{Ah^2}{R}.. \text{ (vii)}$$

h^2 is the constant of the section; *h* is called the link radius.

Bars with a Large Initial Curvature

Therefore,

$$E\varepsilon_0 A + E(\varepsilon_0+1)\left(\frac{1}{R_1}-\frac{1}{R}\right)\left(-\frac{Ah^2}{R}\right)=0$$

$$\varepsilon_0 = (\varepsilon_0+1)\left(\frac{1}{R_1}-\frac{1}{R}\right)\left(\frac{h^2}{R}\right) \ldots (viii)$$

The total moment of the section

$$M = \int y.\sigma.dA = \int y.E.\varepsilon.dA$$

$$\varepsilon_0 = \frac{M}{EAR}$$

$$\sigma = \frac{M}{AR}\left[1+\frac{R^2 y}{h^2(R+y)}\right]$$

The bending stress

$$\sigma == \left[\frac{M}{EAR}+\frac{MRy}{EAh^2(R+y)}\right]$$

Link Radius for Standard Sections

From Eq. (vii)

$$\int \frac{y^2}{R+y} dA = \frac{Ah^2}{R}$$

Simplifying

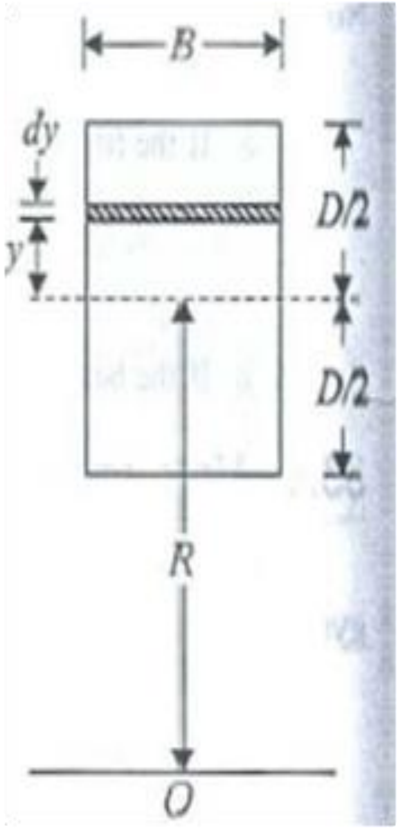
$$h^2 = \frac{R}{A} \int \frac{y^2}{R+y} dA$$

Hence

$$h^2 = \frac{R^3}{A} \left(\int \frac{dA}{R+y} \right) - R^2$$

Link Radius for Standard Sections

Value of Link Radius for a Rectangular Section



Therefore area of the strip, $dA = Bdy..(i)$

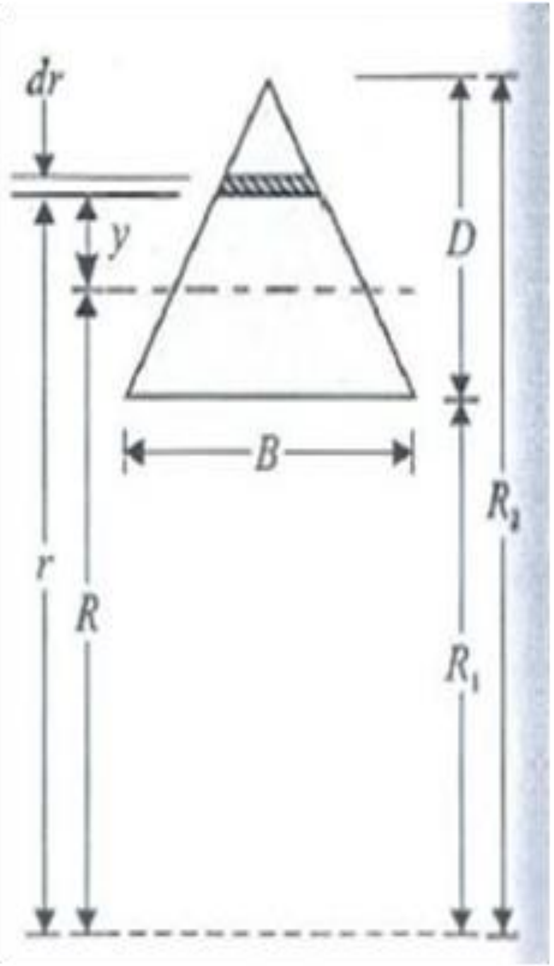
We know that $$h^2 = \frac{R^3}{A}\left(\int \frac{dA}{R+y}\right) - R^2$$

Substituting the value of dA and simplifying,

$$h^2 = \frac{R^3}{BD}\int_{-\frac{D}{2}}^{\frac{D}{2}} \frac{Bdy}{R+y} - R^2 = \frac{R^3}{D}\ln\left(\frac{2R+D}{2R-D}\right) - R^2$$

Link Radius for Standard Sections

Value of link Radius for Triangular Section



From geometry, the width of the bar $b = \frac{B}{D}(R_2 - r)$

Area of strip $dA = \frac{B}{D}(R_2 - r)dr$

We know that $h^2 = \frac{R^3}{A}\left(\int \frac{dA}{R+y}\right) - R^2$

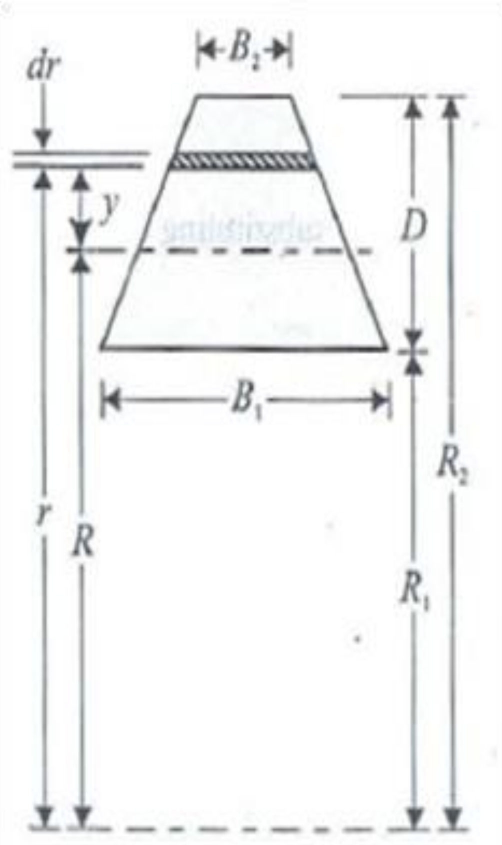
But $R + y = r$

Substituting the value of dA and integrating, $h^2 = \frac{R^3}{A}\int_{R_1}^{R_2} \frac{\frac{B}{D}(R_2 - r)dr}{R+y}$

Hence, $h^2 == \frac{R^3}{A} \times \frac{B}{D}\left[R_2 \log\frac{R_2}{R_1} - D\right] - R^2$

Link Radius for Standard Sections

Value of link Radius for a Trapezoidal Section



From geometry, the width of the bar $b = B_2 + \left(\frac{B_1 - B_2}{D}\right)(R_2 - r)$

Area of strip
$$dA = b.dr = \left[B_2 + \left(\frac{B_1 - B_2}{D}\right)(R_2 - r)\right]dr$$

We know that
$$h^2 = \frac{R^3}{A}\left(\int \frac{dA}{R+y}\right) - R^2$$

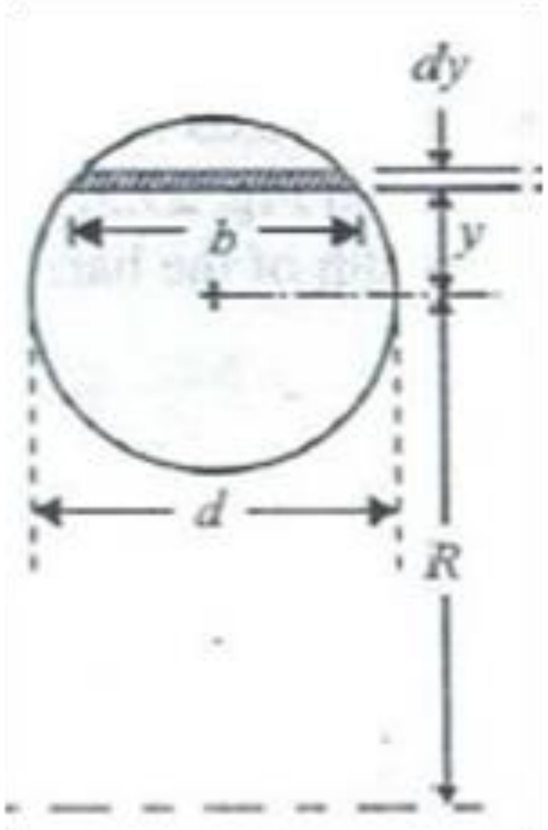
But $R + y = r$

Therefore
$$h^2 = B_2 \int_{R_1}^{R_2} \frac{dr}{r} + \left(\frac{B_1 - B_2}{D}\right) R_2 \int_{R_1}^{R_2} \frac{dr}{r} - \left(\frac{B_1 - B_2}{D}\right) \int_{R_1}^{R_2} \frac{rdr}{r} - R^2$$

Hence
$$h^2 = \frac{R^3}{A}\left\{\left(\log \frac{R_2}{R_1}\right)\left[B_2 + \frac{(B_1 - B_2)R_2}{D}\right] - (B_1 - B_2)\right\} - R^2$$

Link Radius for Standard Sections

Value of link Radius for a Circular Section



From geometry, the width of the bar $b = 2\left[\sqrt{\left(\frac{d}{2}\right)^2 - y^2}\right] = 2\times\sqrt{\left(\frac{d^2}{4} - y^2\right)}$

Area of strip $dA = bdy = 2\times\sqrt{\left(\frac{d^2}{4} - y^2\right)}dy$

We know that

$$h^2 = \frac{R^3}{A}\left(\int\frac{dA}{R+y}\right) - R^2$$

Substituting the value of dA and integrating,

$$h^2 = \frac{R^3}{\frac{\pi}{4}d^2}\int_{-\frac{D}{2}}^{\frac{D}{2}}\frac{2\times\sqrt{\left(\frac{d^2}{4} - y^2\right)}}{R+y}dy - R^2 = \frac{d^2}{16} + \frac{1}{8}\times\frac{d^4}{16R^2}$$

Example 2

A curved bar of square section, 3-cm sides and radius of curvature 4½ cm is initially unstressed. If a bending moment of 300 Nm is applied to the tending to straighten it, find the stresses at the inner and outer face

Solution

Given: Beam width (B) = 30 mm; Beam depth (*D*) = 30 mm; Radius of beam *(R)* = 45 mm and bending moment *(M)* = 3x10^5 N-mm.

Area, *A* = 30 x 30 = 900 mm^2

Distance between centre Line and extreme fibre $y = \frac{D}{2} = \frac{30}{2} = 15\ \mathrm{mm}$

Link radius $h^2 = \frac{R^3}{D}\log\left(\frac{2R+D}{2R-D}\right) - R^2 = \left(\frac{45^3}{30}\right)\log\left[\frac{2(45)+30}{2(45)-30}\right] - 45^2 = 80.43\ \mathrm{mm}$

Example 2 (continued)

At the bottom $y = +15\,mm$

Maximum stress at bottom surface

$$\sigma = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right] = \frac{3\times10^5}{(900)(45)}\left[1 + \frac{(45^2)(15)}{(80.43)(45+15)}\right]$$

$$\sigma = 7.407[1 + 6.294] = 54.03\ \mathrm{N/mm^2}$$

At the top $y = -15\,mm$

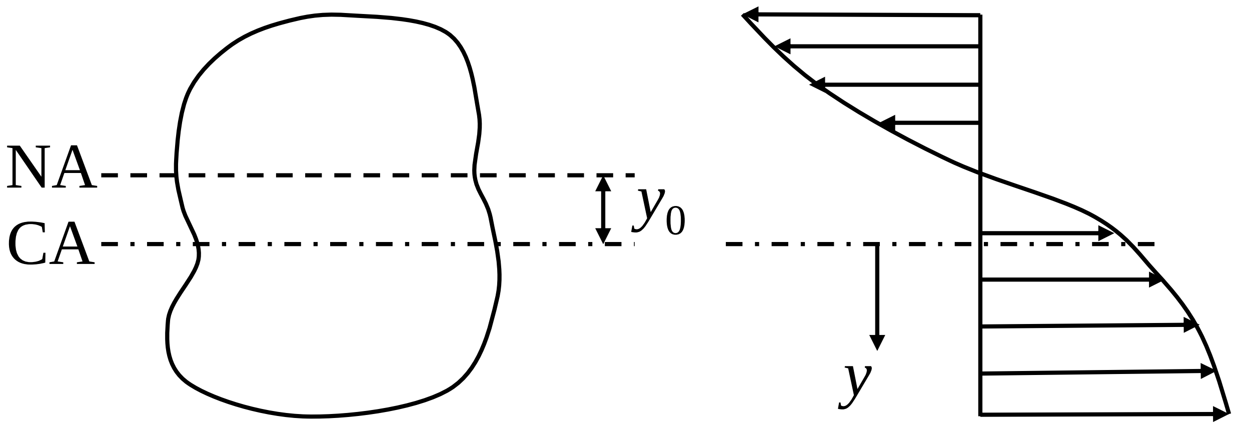
Maximum stress at top surface

$$\sigma = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right] = \frac{3x10^5}{(900)(45)}\left[1 + \frac{(45^2)(-15)}{(80.43)(45+(-15))}\right]$$

$$\sigma = 7.407[1 - 12.589] = -85.84\ \mathrm{N/mm^2}$$

Location of the Neutral Axis

It was mentioned that due to the large initial curvature, stress is no longer proportional to distance from the neutral axis. The direct effect is that neutral axis no longer coincides with the centroidal axis of the bar cross-section.



Let y_0 denote the distance between the neutral and centroidal axes. Recall, the stress σ over the cross-section is given by

$$\sigma = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right]$$

On the neutral axis, $\sigma = 0$ and $y = -y_0$

$$\Longrightarrow \quad 0 = \frac{M}{AR}\left[1 + \frac{R^2(-y_0)}{h^2(R-y_0)}\right]$$

which can be simplified to obtain

$$y_0 = \frac{h^2 R}{(R^2 + h^2)}$$

Example 3

Determine the location of the neutral axis and the intensity of maximum stresses when a curved beam of rectangular section 20 m wide and 40 m deep is subjected to a pure bending moment of magnitude 600 Nm. The beam is curved in a plane parallel to its depth and the mean radius of curvature is 50 mm.

Solution

Given: Beam width = 20 mm; Beam depth (D) = 40 mm; Radius of beam (R) = 50 mm and bending moment (M) = 6 x 10^5 N-mm.

Area, $A = 40 \text{ x } 20 = 800 \text{ mm}^2$

Link radius, $$h^2 = \frac{R^3}{D}\log\left(\frac{2R+D}{2R-D}\right) - R^2 = \left(\frac{50^3}{40}\right)\log\left[\frac{2(50)+40}{2(50)-40}\right] - 50^2 = 147.806\, mm$$

Location of neutral axis, $$y_0 = \frac{h^2 R}{(R^2+h^2)} = \frac{147.806(50)}{50^2+147.806} = 2.79 \text{ mm}$$

Example 3 (continued)

Distance between centre line and extreme fibre, $y = \dfrac{D}{2} = \dfrac{40}{2} = 20\ mm$

Maximum stress at bottom surface

At the bottom $y = +20\ mm$

The stress is given by

$$\sigma = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right] = \frac{6\times10^5}{(800)(50)}\left[1 + \frac{(50^2)(20)}{(147.806)(50+20)}\right] = 87.5\ \mathrm{N/mm^2}$$

Maximum stress at top surface

At the top $y = -20\ mm$

The stress is given by $\sigma = \dfrac{M}{AR}\left[1 + \dfrac{R^2 y}{h^2(R+y)}\right] = \dfrac{6\times10^5}{(800)(50)}\left[1 + \dfrac{(50^2)(-20)}{(147.806)(50+(-20))}\right] = -154.14\ \mathrm{N/mm^2}$

Example 4

A beam of circular section of diameter 20 mm has its centre line curved to a radius of 50 mm. Find the intensity of maximum stresses in the beam, when subjected to a moment of 5 kN-mm.

Solution

Given: Diameter of section (d) = 20 mm; Radius of curvature (R) = 50 mm and moment (M) = 5 kN-mm = 5 x 10^3 N-mm

Area $$A = \frac{\pi}{4} x d^2 = \frac{\pi}{4} x 20^2 = 100\pi \ mm^2$$

The distance between centre line and extreme fibre $$y = \frac{d}{2} = \frac{20}{2} = 10 \ mm$$

Link radius

$$h^2 = \frac{d^2}{16} + \frac{1}{8} x \frac{d^4}{16R^2} = \frac{20^2}{16} + \frac{1}{8} x \frac{20^4}{16x50^2} = 25.05 \, mm$$

Example 4 (continued)

At the bottom $y = +10\ mm$

Maximum stress at bottom surface

$$\sigma = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right] = \frac{5x10^5}{(100\pi)(50)}\left[1 + \frac{(50^2)(10)}{(25.05)(50+10)}\right] = 5.61\ N/mm^2$$

At the top $y = -10\ mm$

Maximum stress at top surface

$$\sigma = \frac{M}{AR}\left[1 - \frac{R^2 y}{h^2(R+y)}\right] = \frac{5x10^5}{(100\pi)(50)}\left[1 + \frac{(50^2)(10)}{(25.05)(50+(-10))}\right] = -4.98\ N/mm^2$$

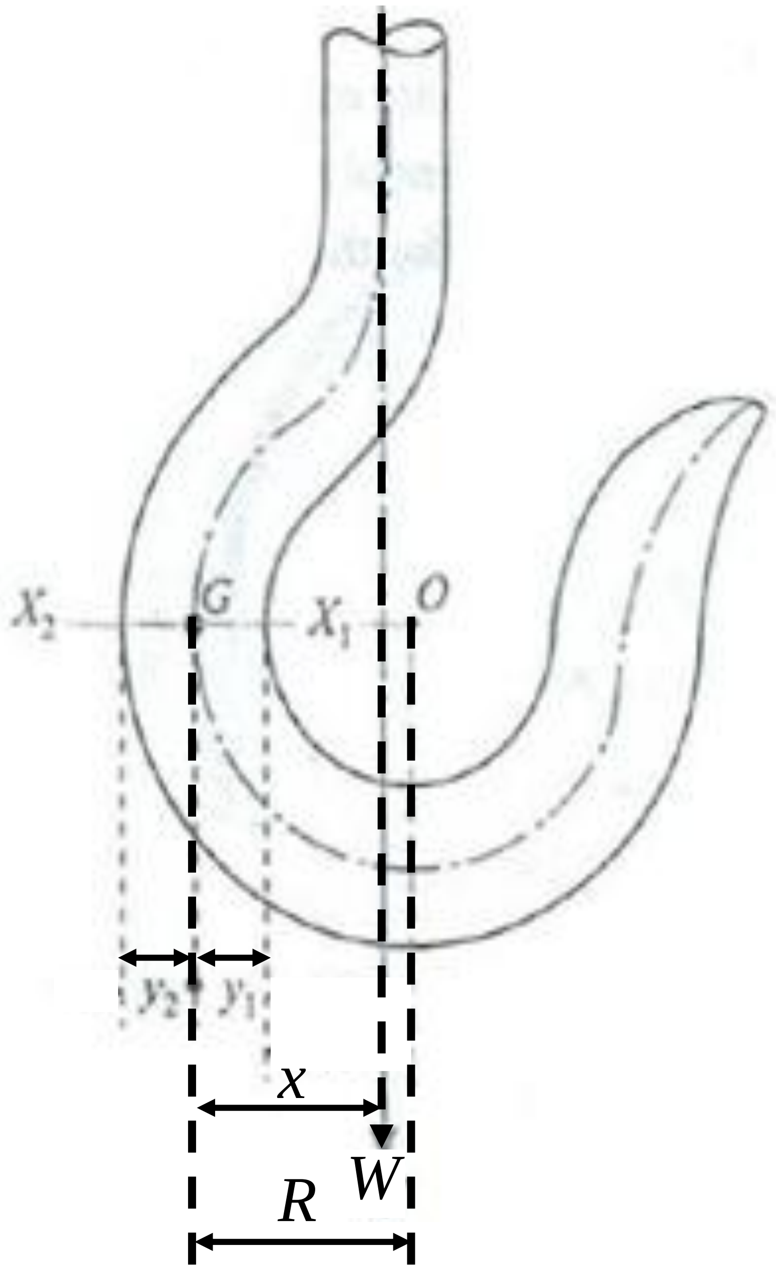
Application of Bars with Large Initial Curvature

We have already discussed and derived the relations for finding the bending stress in bars with large initial curvature.

The results may be applied in finding the stresses in

1. Crane Hooks,
2. Rings and
3. Chain links

when subjected to a load.



Crane Hook

Let W denote the load supported by the hook, x the distance between the line of action of W and the centroidal axis, and R the radius of curvature of the hook

Let bending moment caused by the load W is given by

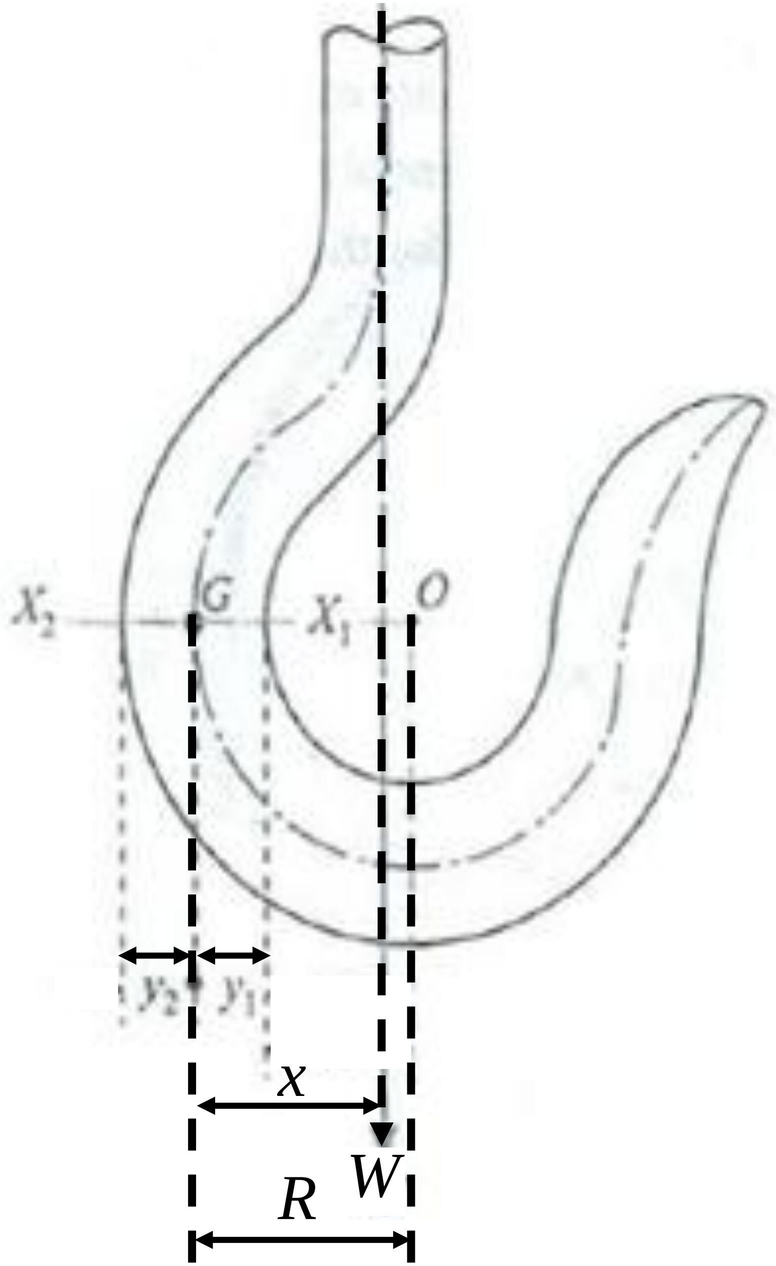
$$M = -Wx$$

The total stress, σ at section X_1X_2 is the sum of direct stress, σ_D and bending stress, σ_B due to M

$$\sigma = \sigma_D + \sigma_B$$

$$\sigma_D = \frac{W}{A} \quad \text{and} \quad \sigma_B = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right]$$

Crane Hook



Substituting $M = -Wx$

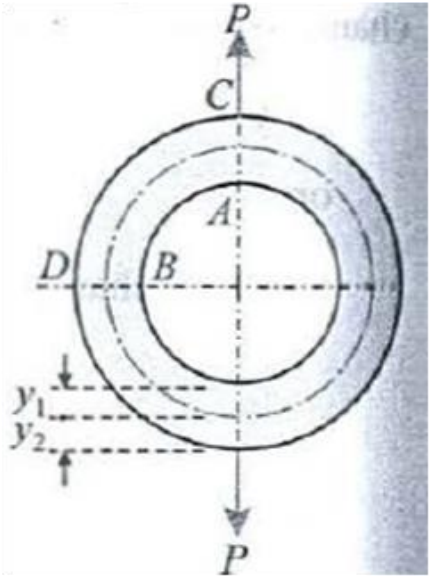
$$\sigma = \frac{W}{A} - \frac{Wx}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right]$$

Maximum tensile stress $$\sigma = \frac{W}{A} - \frac{Wx}{AR}\left[1 + \frac{R^2 y_1}{h^2(R+y_1)}\right]$$

Maximum compressive stress $$\sigma = \frac{W}{A} - \frac{Wx}{AR}\left[1 + \frac{R^2 y_2}{h^2(R+y_2)}\right]$$

Application of Bars with Large Initial Curvature

Rings



Stress at the bottom (point A)

$$\sigma = \frac{P}{\pi A}\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R - y_1}\right]$$

Stress at the inner (point B)

$$\sigma = \frac{P}{2A} - \frac{0.182P}{A}\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R - y_1}\right]$$

Stress at the top (point C)

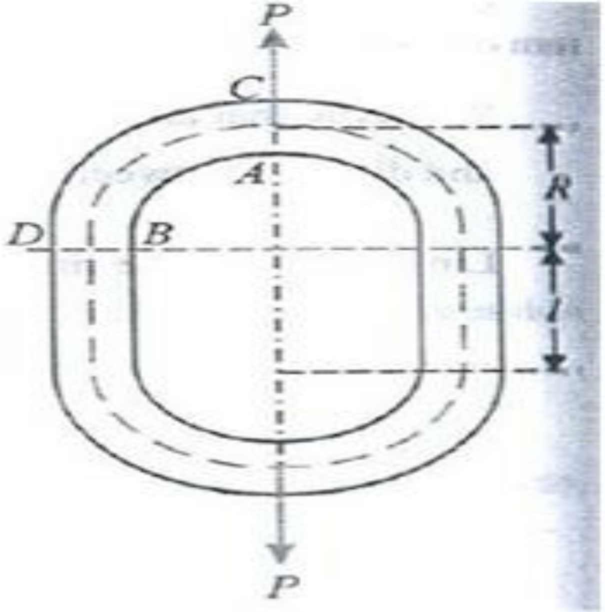
$$\sigma = \frac{P}{\pi A}\left[1 + \frac{R^2}{h^2} x \frac{y_2}{R + y_2}\right]$$

Stress at the outer (point D)

$$\sigma = \frac{P}{2A} - \frac{0.182P}{A}\left[1 + \frac{R^2}{h^2} x \frac{y_2}{R + y_2}\right]$$

Application of Bars with Large Initial Curvature

Chain Links



Stress at the bottom (point A)

$$\sigma_A = \frac{P}{2A}\left(\frac{l+2R}{l+\pi R}\right)\left[1-\frac{R^2}{h^2}x\frac{y_1}{R-y_1}\right]$$

Stress at the inner (point B)

$$\sigma = \frac{P}{2A} - \frac{PR}{2A}\left(\frac{\pi-2}{l+\pi R}\right)\left[1-\frac{R^2}{h^2}x\frac{y_1}{R-y_1}\right]$$

Stress at the top (point C)

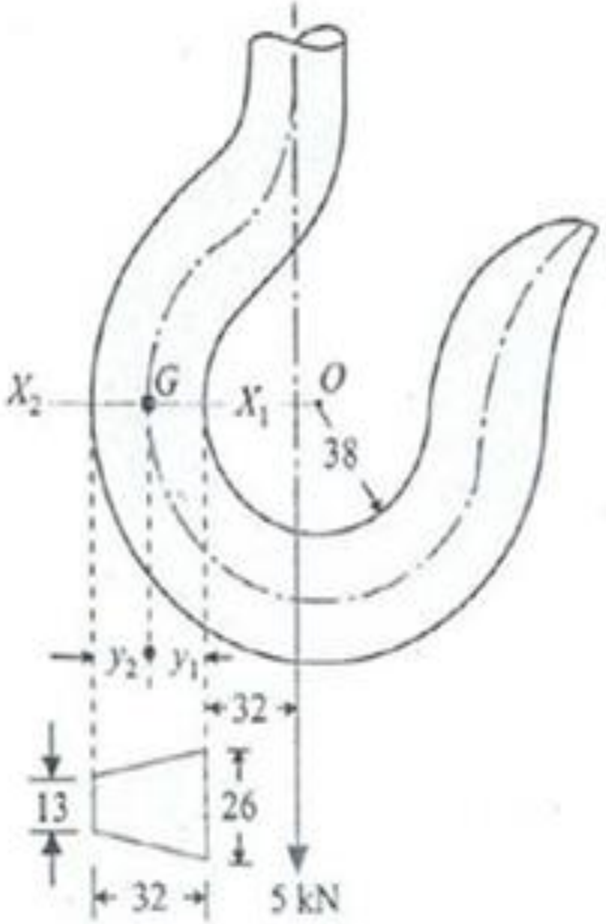
$$\sigma = \frac{P}{2A}\left(\frac{l+2R}{l+\pi R}\right)\left[1+\frac{R^2}{h^2}x\frac{y_2}{R+y_2}\right]$$

Stress at the outer (point D)

$$\sigma = \frac{P}{2A} - \frac{PR}{2A}\left(\frac{\pi-2}{l+\pi R}\right)\left[1+\frac{R^2}{h^2}x\frac{y_2}{R+y_2}\right]$$

Example 5

A crane hook carries a load of 5 kN the line of load being at a horizontal distance of 32 mm from the inside edge of a horizontal section through the centre of curvature; and the centre of curvature being 38 mm from the same edge. The horizontal section is a trapezium whose parallel sides are 13 mm and 26 mm and height is 32 mm. Determine the greatest tensile and compressive stresses in the hook as shown in Fig.



Solution

Given: Load (W) = 5 kN = 5 x 10^3 N; Distance between the centre line and inner edge *(x)* = 32 mm; Distance between centre of curvature and inner edge = 38 mm; Outer width (B_2) = 13 mm; Inner width (B_1) = 26 mm and depth *(D)* = 32 mm.

Example 5 (continued)

Distance between centre line and extreme fibres:

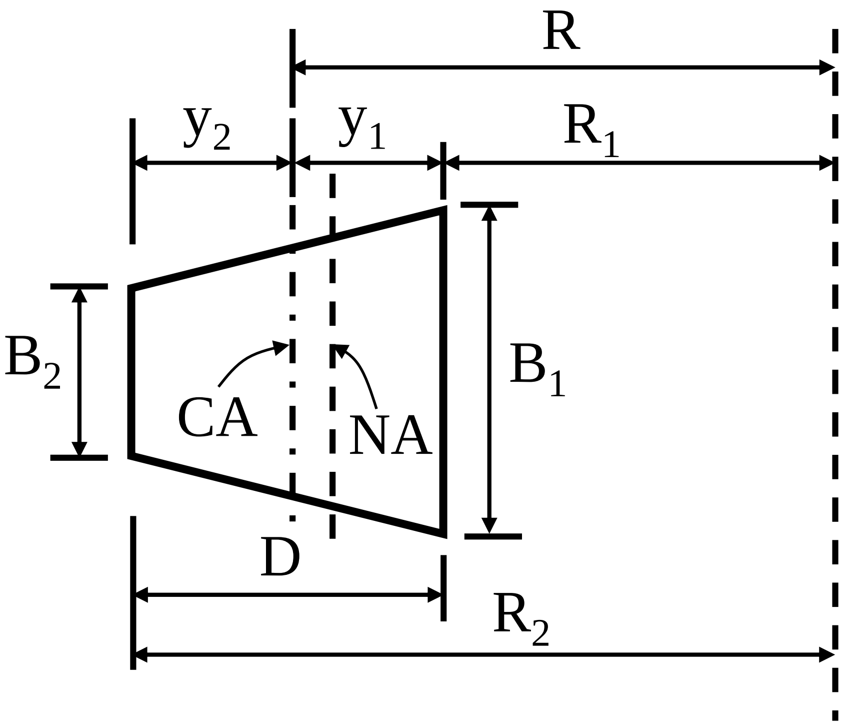
$$y_1 = \frac{B_1 + 2B_2}{B_1 + B_2}\left(\frac{D}{3}\right) = \left(\frac{26 + 2\times 13}{26+13}\right)\left(\frac{32}{3}\right) = 14.2\ mm$$

and

$$y_2 = D - y_1 = 32 - 14.2 = 17.8\ mm$$

Area

$$A = \frac{D}{2}\left(B_1 + B_2\right) = \frac{32}{2}\left(26+13\right) = 624\ mm^2$$



Radius of inner edge,

R_1 = 38 mm

Radius of outer edge,

R_2 = 38 + 32 = 70 mm

Radius of central line,

R = 38 + 14.2 = 52.2 mm

Example 5 (continued)

Link radius

$$h^2 = \frac{R^3}{A}\left\{\log\frac{R_2}{R_1}\left[B_2 + \frac{(B_1 - B_2)R_2}{D}\right] - (B_1 - B_2)\right\} - R^2$$

$$\Rightarrow h^2 = \frac{52.22^3}{624}\left\{\left(\log\frac{70}{38}\right)\left[13 + \frac{(26-13)(70)}{32}\right] - (26-13)\right\} - 52.22^2 = 82.9$$

At the outer edge $y = +17.8\ mm$

Maximum stress at outside edge

$$\sigma = \frac{W}{A}\left\{1 - \frac{x}{R}\left[1 + \frac{R^2 y_2}{h^2(R + y_2)}\right]\right\}$$

$$= \frac{5\times10^3}{624}\left\{1 - \frac{38}{52.22}\left[1 + \frac{(52.22^2)(17.8)}{(82.9)(52.22+17.8)}\right]\right\} = -46.58\ \mathrm{N/mm^2}$$

At the inner edg $y = -14.2\ mm$

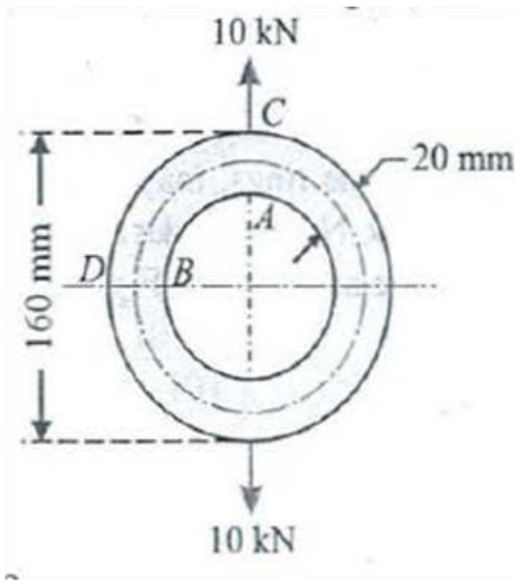
Maximum stress at inside edge

$$\sigma = \frac{W}{A}\left\{1 - \frac{x}{R}\left[1 + \frac{R^2 y_1}{h^2(R + y_1)}\right]\right\}$$

$$= \frac{5\times10^3}{624}\left\{1 - \frac{38}{52.22}\left[1 - \frac{(52.22^2)(14.2)}{(82.9)(52.22-14.2)}\right]\right\} = 73.82\ \mathrm{N/mm^2}$$

Example 6

A close circular ring made up of 20 mm diameter steel bar is subjected to a pull of 10 kN, whose line of action passes through the centre of the ring. Find the maximum value of tensile and compressive stresses in the ring, if the mean diameter of the ring is 160 mm as shown in Fig.



Solution

Given: Diameter of steel bar *(d)*= 20 mm; Pull *(P)* = 10 kN = .10^4 N and diameter of the ring *(D)*= 160 mm or radius of ring *(R)* = 80 mm.

Area, $$A = \frac{\pi}{4} x d^2 = \frac{\pi}{4} x 20^2 = 100\pi \ mm^2$$

The distance between centre line of the ring and extreme fibre

$$y = y_1 = y_2 = 10\, mm$$

Example 6 (continued)

Link radius $$h^2 = \frac{d^2}{16} + \frac{1}{8} x \frac{d^4}{16R^2} = \frac{20^2}{16} + \frac{1}{8} x \frac{20^4}{16x80^2} = 25.5$$

Stress at the bottom (point A)

$$\sigma = \frac{P}{\pi A}\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R - y_1}\right]$$

$$= \frac{10^4}{\pi(100\pi)}\left[1 - \frac{80^2}{25.2} x \frac{10}{80 - 10}\right] = -357.83\ N/mm^2$$

Stress at the inner (point B)

$$\sigma = \frac{P}{2A} - \frac{0.182P}{A}\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R - y_1}\right]$$

$$= \frac{10^4}{2x100\pi} - \frac{0.182x10^4}{100\pi}\left[1 - \frac{80^2}{25.2} x \frac{10}{80 - 10}\right] = 220.3\ N/mm^2$$

Example 6 (continued)

Stress at the top (point C)

$$\sigma = \frac{P}{\pi A}\left[1 + \frac{R^2}{h^2} x \frac{y_2}{R + y_2}\right]$$

$$= \frac{10^4}{\pi(100\pi)}\left[1 + \frac{80^2}{25.2} x \frac{10}{80 + 10}\right] = 296\ N/mm^2$$

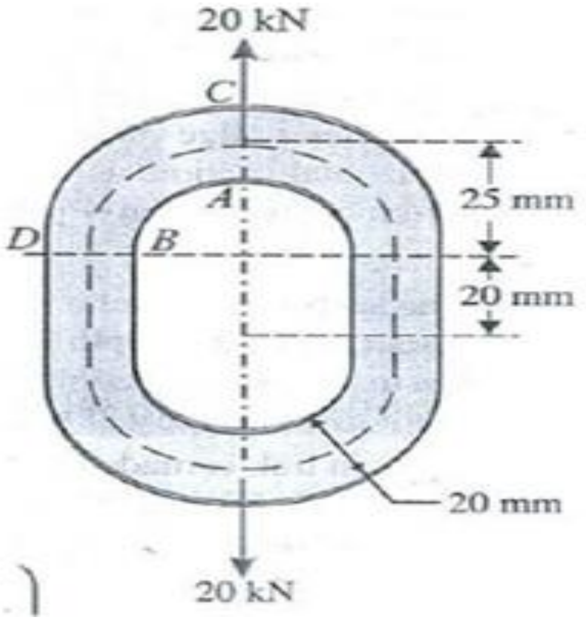
Stress at the outer (point D)

$$\sigma = \frac{P}{2A} - \frac{0.182P}{A}\left[1 + \frac{R^2}{h^2} x \frac{y_2}{R + y_2}\right]$$

$$= \frac{10^4}{2x100\pi} - \frac{0.182x10^4}{100\pi}\left[1 + \frac{80^2}{25.2} x \frac{10}{80 + 10}\right] = -153.4\ N/mm^2$$

Example 7

A chain link is made of 20 mm diameter round steel with mean radius of circular ends 25 mm, the length of straight portion being 20 mm. Determine the values of maximum tensile and compressive stresses, when the link is subjected to a pull of 20 kN at its ends as shown in Fig.



Solution

Given: Diameter of steel bar *(d)* = 20 mm; Radius of link *(R)* = 25 mm; Length of straight portion (l) = 20 mm and pull *(P)* = 20 kN = 2 x 10^4 N

Area $$A = \frac{\pi}{4} x d^2 = \frac{\pi}{4} x 20^2 = 100\pi \ mm^2$$

The distance between centre line of the ring and extreme fibre,

$$y = y_1 = y_2 = 10 \, mm$$

Example 7 (continued)

Link radius

$$h^2 = \frac{d^2}{16} + \frac{1}{8} \times \frac{d^4}{16R^2} = \frac{20^2}{16} + \frac{1}{8} \times \frac{20^4}{16x25^2} = 27$$

Stress at the bottom (point A)

$$\sigma_A = \frac{P}{2A}\left(\frac{l+2R}{l+\pi R}\right)\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R-y_1}\right]$$

$$= \frac{2x10^3}{2x100\pi}\left(\frac{20+2x25}{20+\pi x25}\right)\left[1 - \frac{25^2}{27} x \frac{10}{(25-10)}\right] = -326.2\ N/mm^2$$

Stress at the inner (point B)

$$\sigma_B = \frac{P}{2A} - \frac{PR}{2A}\left(\frac{\pi-2}{l+\pi R}\right)\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R-y_1}\right]$$

$$= \frac{2x10^3}{2x100\pi} - \frac{(2x10^3)x25}{2x100\pi}\left(\frac{\pi-2}{20+\pi x25}\right)\left[1 - \frac{25^2}{27} x \frac{10}{(25-10)}\right] = 164.8\ N/mm^2$$

Example 7 (continued)

Stress at the top (point C)

$$\sigma = \frac{P}{2A}\left(\frac{l+2R}{l+\pi R}\right)\left[1+\frac{R^2}{h^2}x\frac{y_2}{R+y_2}\right]$$

$$= \frac{2x10^3}{2x100\pi}\left(\frac{20+2x25}{20+\pi x25}\right)\left[1+\frac{25^2}{27}x\frac{10}{25+10}\right] = 172.2\ N/mm^2$$

Stress at the outer (point D)

$$\sigma = \frac{P}{2A}-\frac{PR}{2A}\left(\frac{\pi-2}{l+\pi R}\right)\left[1+\frac{R^2}{h^2}x\frac{y_2}{R+y_2}\right]$$

$$= \frac{2x10^3}{2x100\pi}-\frac{\left(2x10^3\right)x25}{2A}\left(\frac{\pi-2}{20+\pi R}\right)\left[1+\frac{25^2}{27}x\frac{10}{(25+10)}\right] = -38.3\ N/mm^2$$

Thus the maximum tensile stress will occur at C equal to 172.2 N/mm^2 and maximum compressive will occur at A equal to 326.3 N/mm^2.