Lecture 4

Fatigue Failures Resulting from Varying Loading

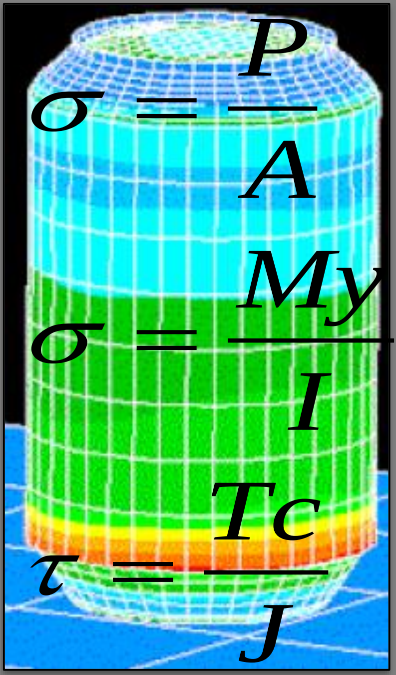
Ing. Dr. P. Y. Andoh, PhD, MGhIE

Mechanical Engineering Dept. Kwame Nkrumah University of Science and Technology, Kumasi

Phone: 024 2235674; 020 0960067

Whatsapp: 026 1099725

Email: andohp_2@yahoo.com; pyandoh.coe@knust.edu.gh



$$\sigma = \frac{P}{A}$$

$$\sigma = \frac{My}{I}$$

$$\tau = \frac{Tc}{J}$$

ME 274: DESIGN PROJECT II

Lecture Outline

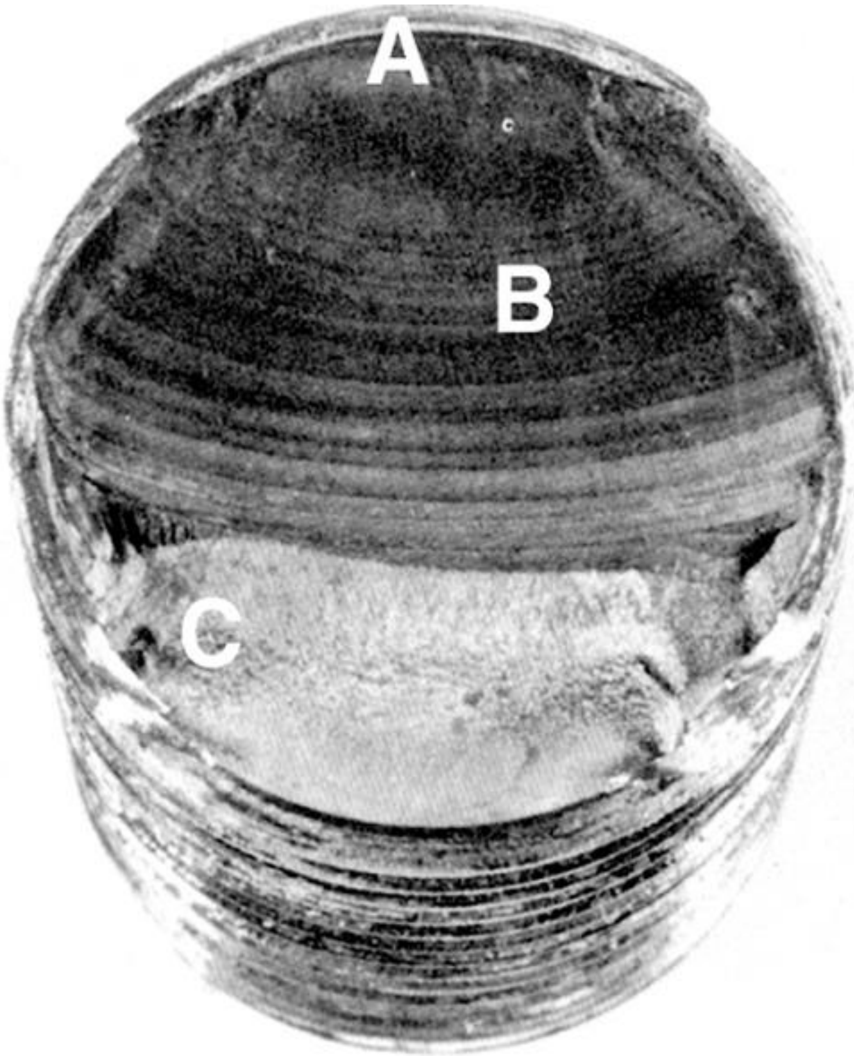
- Introduction to Fatigue in Metals
- Stages of Fatigue Failure
- Schematics of Fatigue Fracture Surfaces
- Fatigue Fracture Examples
- Fatigue-Life Methods
- The Endurance Limit
- Stress Concentration and Notch Sensitivity
- Characterizing Fluctuating Stresses
- Fatigue Loading
- Failure Criteria for Ductile Materials
- Fatigue Criteria for Brittle Materials

Introduction to Fatigue in Metals

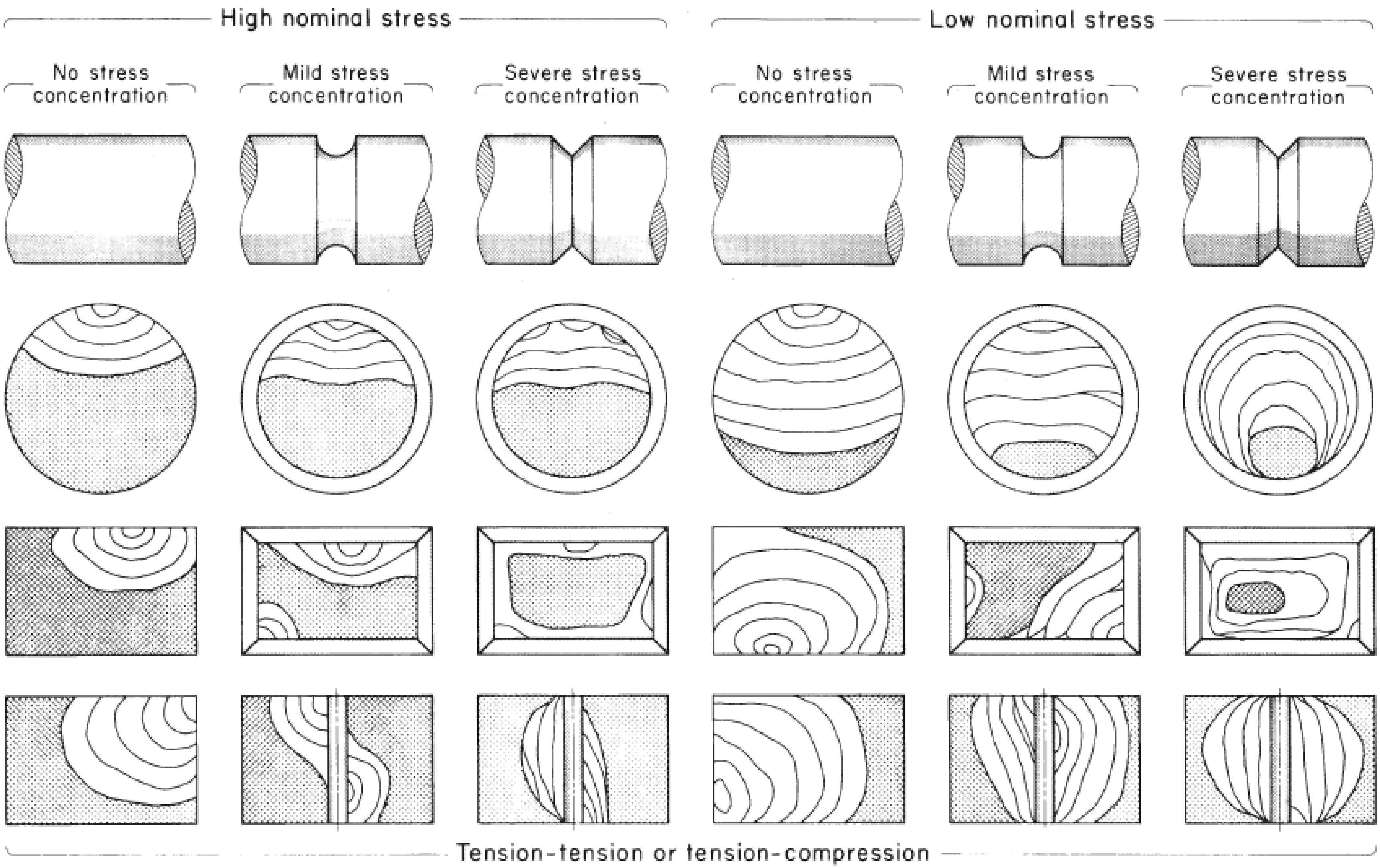
- Loading produces stresses that are variable, repeated, alternating, or fluctuating
- Maximum stresses well below yield strength
- Failure occurs after many stress cycles
- Failure is by sudden ultimate fracture
- No visible warning in advance of failure

Stages of Fatigue Failure

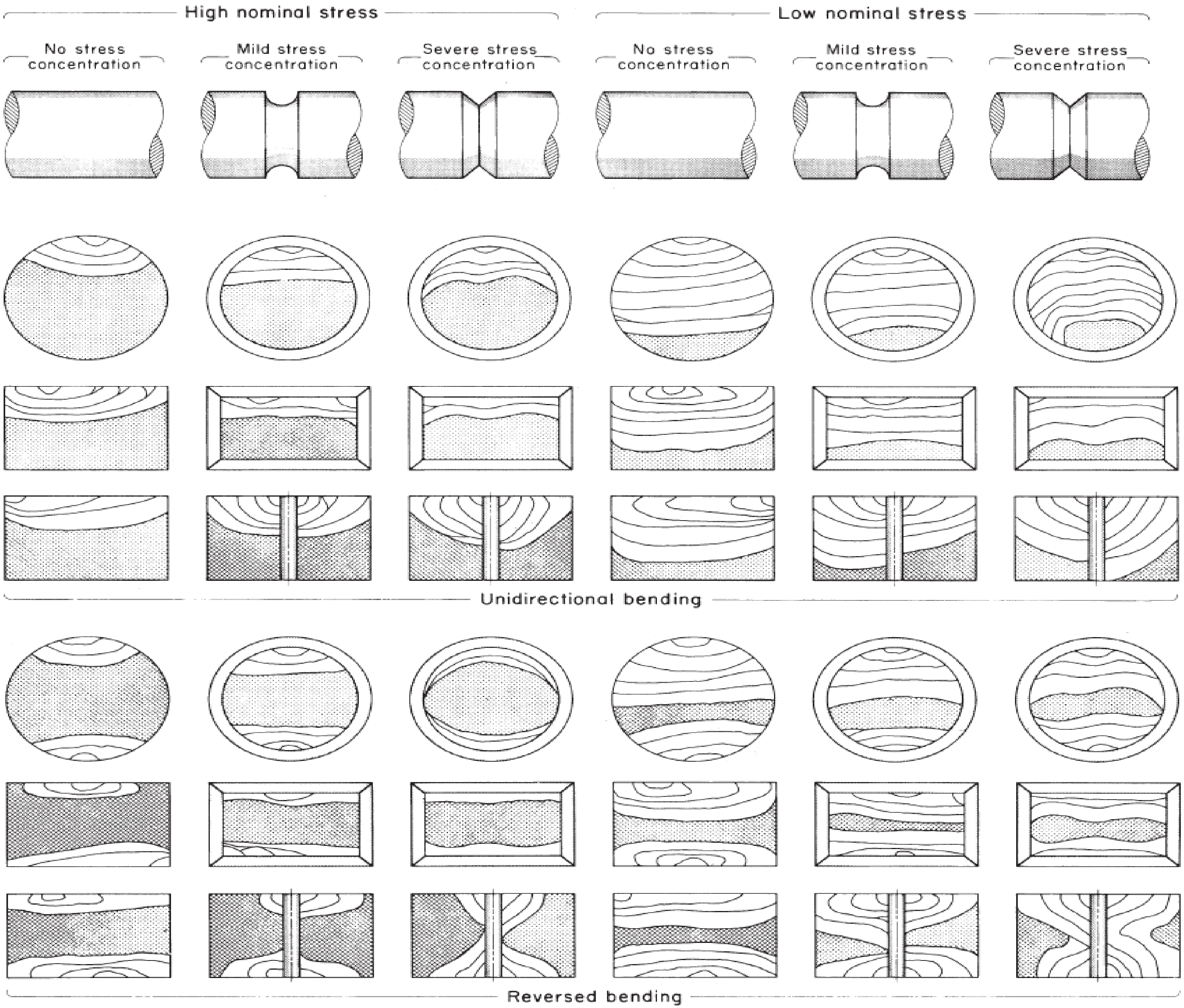
- *Stage I* – Initiation of micro-crack due to cyclic plastic deformation
- *Stage II* – Progresses to macro-crack that repeatedly opens and closes, creating bands called *beach marks*
- *Stage III* – Crack has propagated far enough that remaining material is insufficient to carry the load, and fails by simple ultimate failure



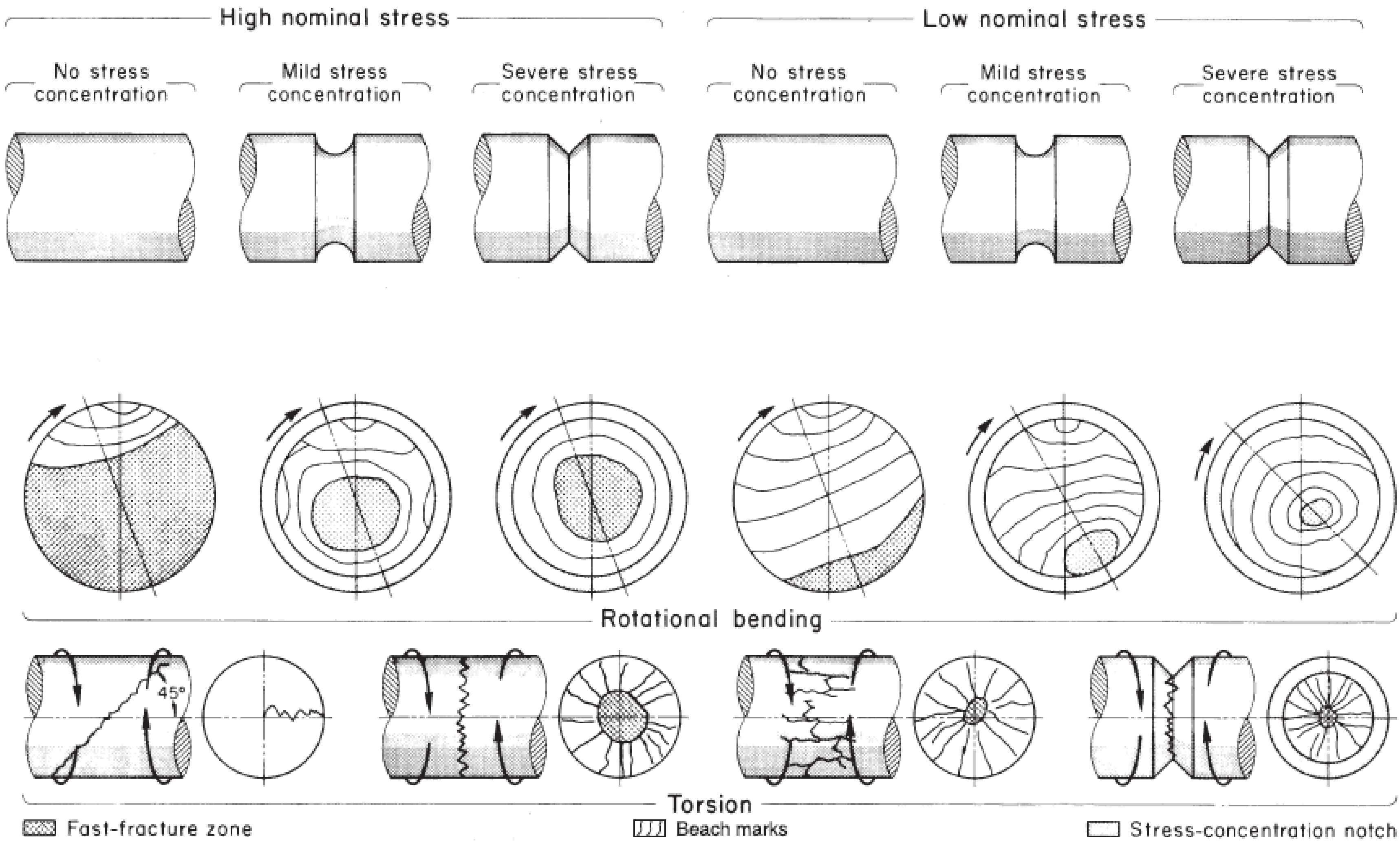
Schematics of Fatigue Fracture Surfaces



Schematics of Fatigue Fracture Surfaces



Schematics of Fatigue Fracture Surfaces

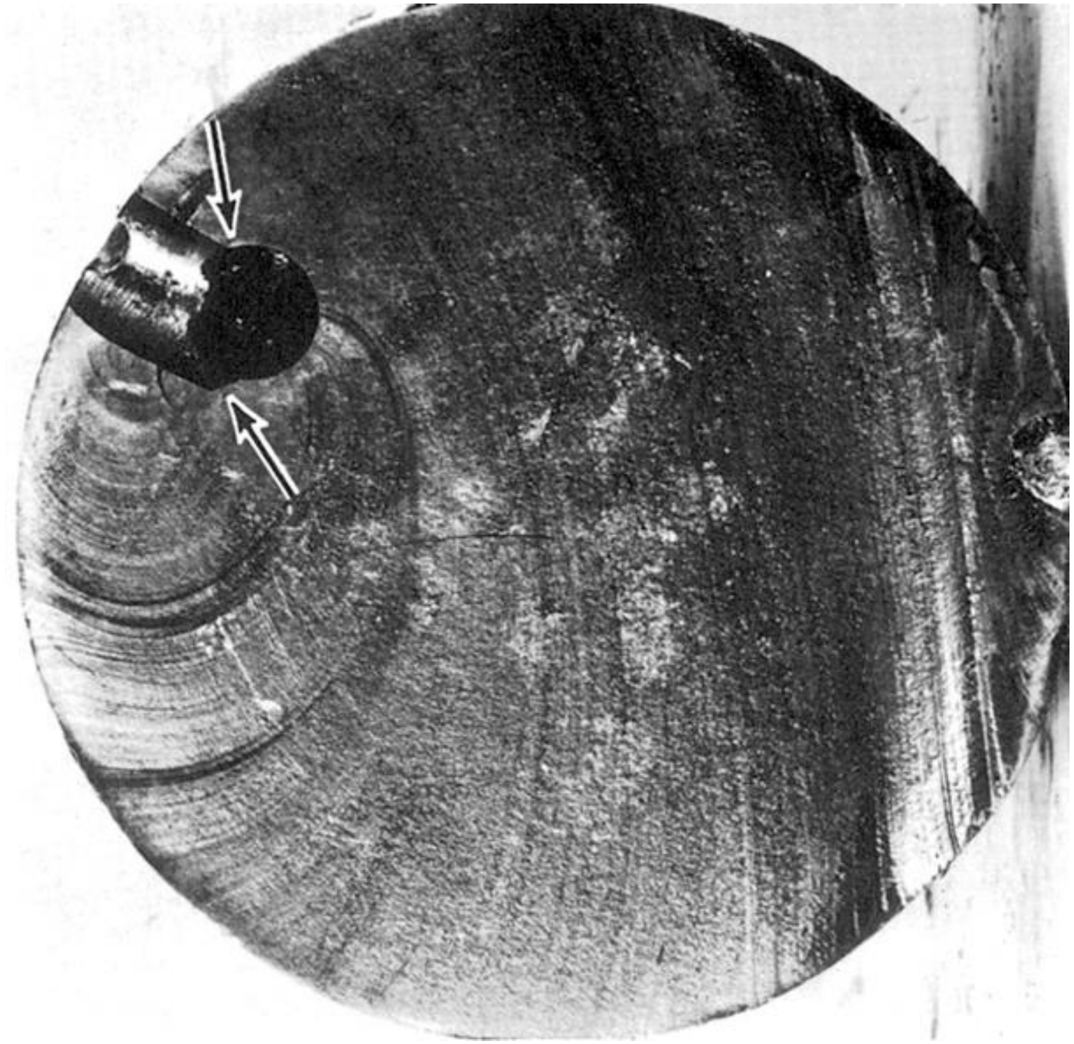


Fatigue Fracture Examples

- AISI 4320 drive shaft
- B– crack initiation at stress concentration in keyway
- C– Final brittle failure

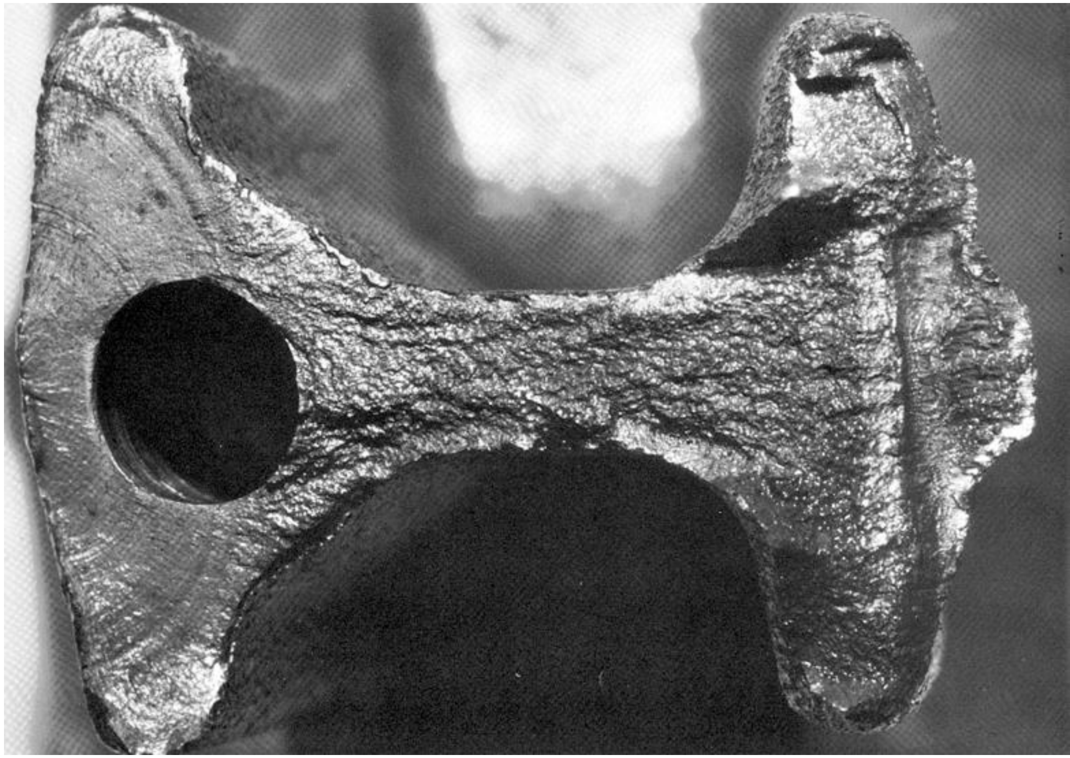
Fatigue Fracture Examples

- Fatigue failure initiating at mismatched grease holes
- Sharp corners (at arrows) provided stress concentrations



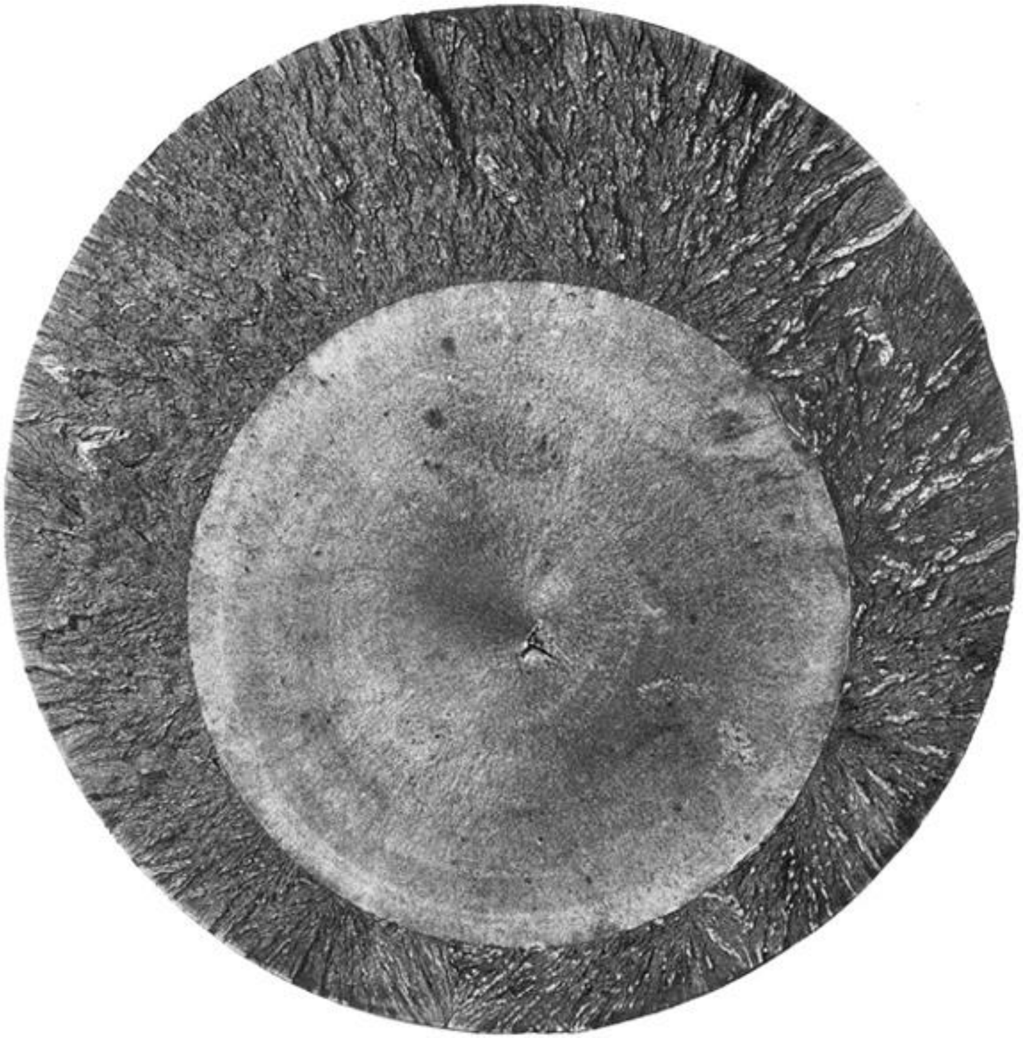
Fatigue Fracture Examples

- Fatigue failure of forged connecting rod
- Crack initiated at flash line of the forging at the left edge of picture
- Beach marks show crack propagation halfway around the hole before ultimate fracture



Fatigue Fracture Examples

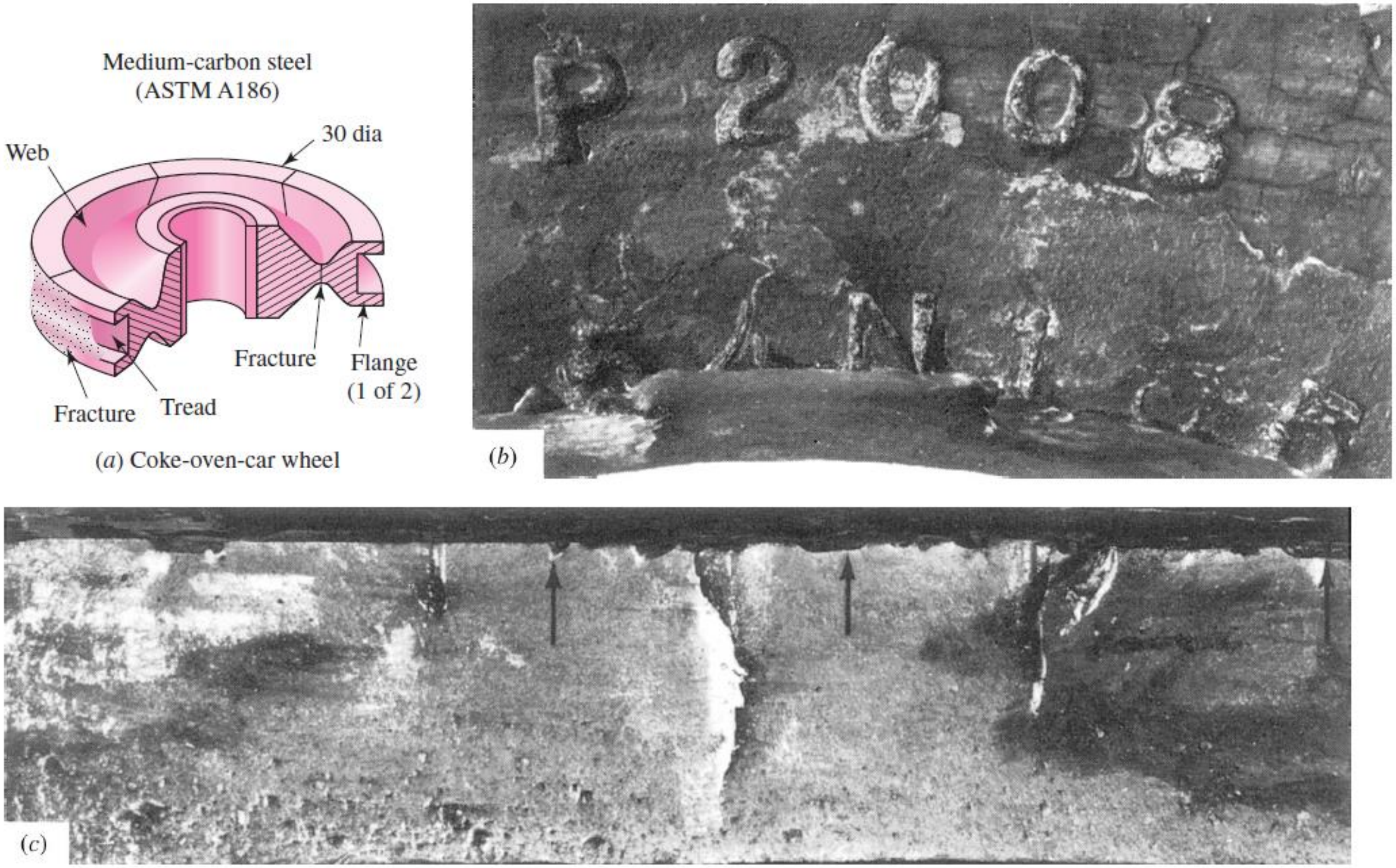
- Fatigue failure of a 200-mm diameter piston rod of an alloy steel steam hammer
- Loaded axially
- Crack initiated at a forging flake internal to the part
- Internal crack grew outward symmetrically



Fatigue Fracture Examples

- Double-flange trailer wheel

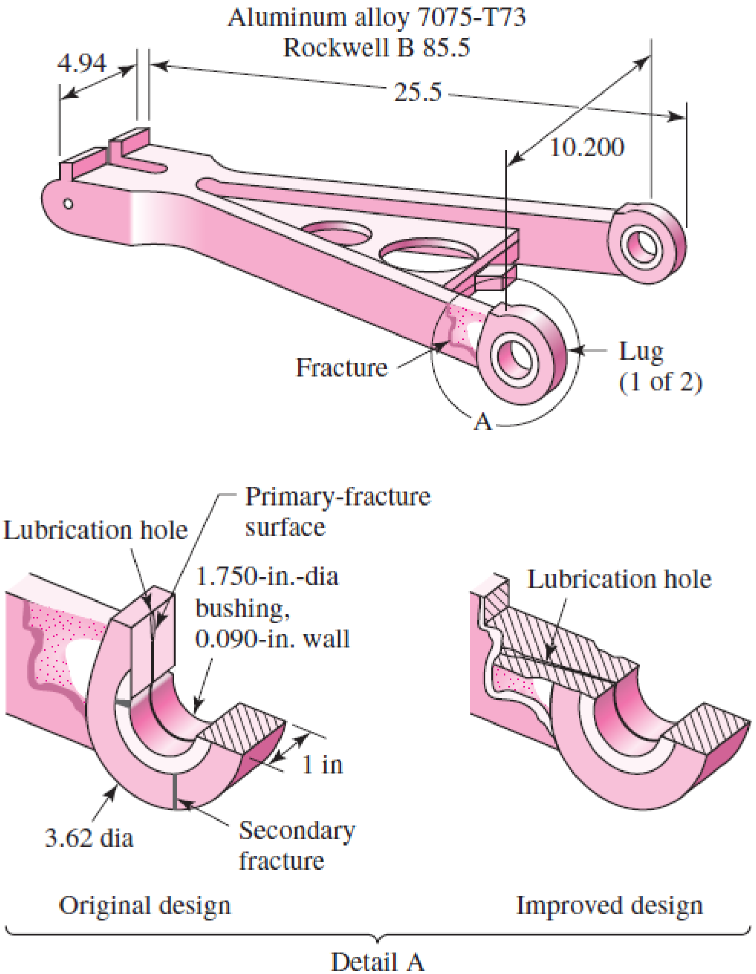
- Cracks initiated at stamp marks



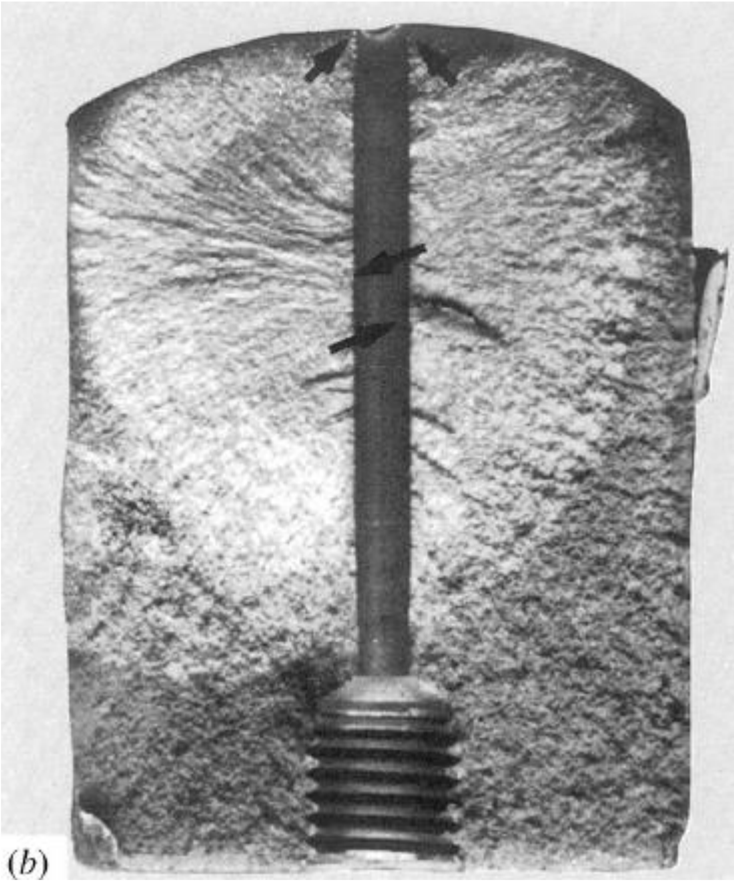
(*a*) Coke-oven-car wheel (*b*) (*c*)

Fatigue Fracture Examples

- Aluminum allow landing-gear torque-arm assembly redesign to eliminate fatigue fracture at lubrication hole



(*a*)


(*b*)

Fatigue-Life Methods

- Three major fatigue life models

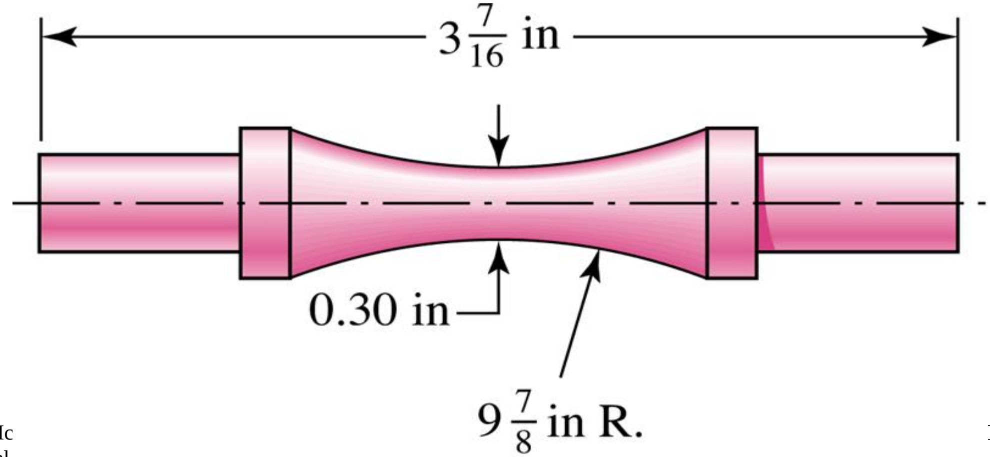
- Methods predict life in number of cycles to failure, *N*, for a specific level of loading

- Stress-life method
  - ✓Least accurate, particularly for low cycle applications
  - ✓Most traditional, easiest to implement

- Strain-life method
  - ✓Detailed analysis of plastic deformation at localized regions
  - ✓Several idealizations are compounded, leading to uncertainties in results

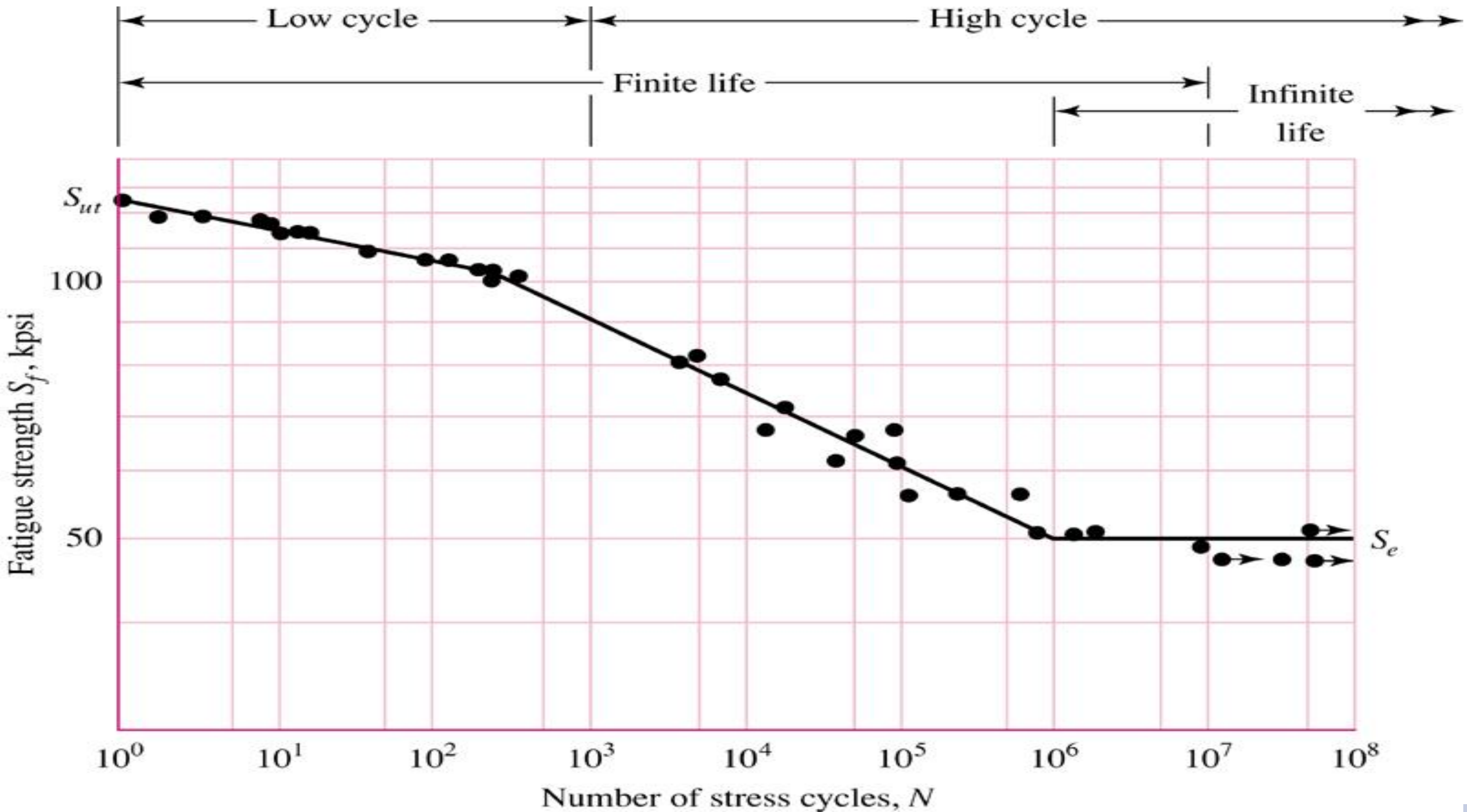
Stress-Life Method

- Test specimens are subjected to repeated stress while counting cycles to failure
- Most common test machine is R. R. Moore high-speed rotating-beam machine
- Subjects specimen to pure bending with no transverse shear
- As specimen rotates, stress fluctuates between equal magnitudes of tension and compression, known as *completely reversed* stress cycling
- Specimen is carefully machined and polished



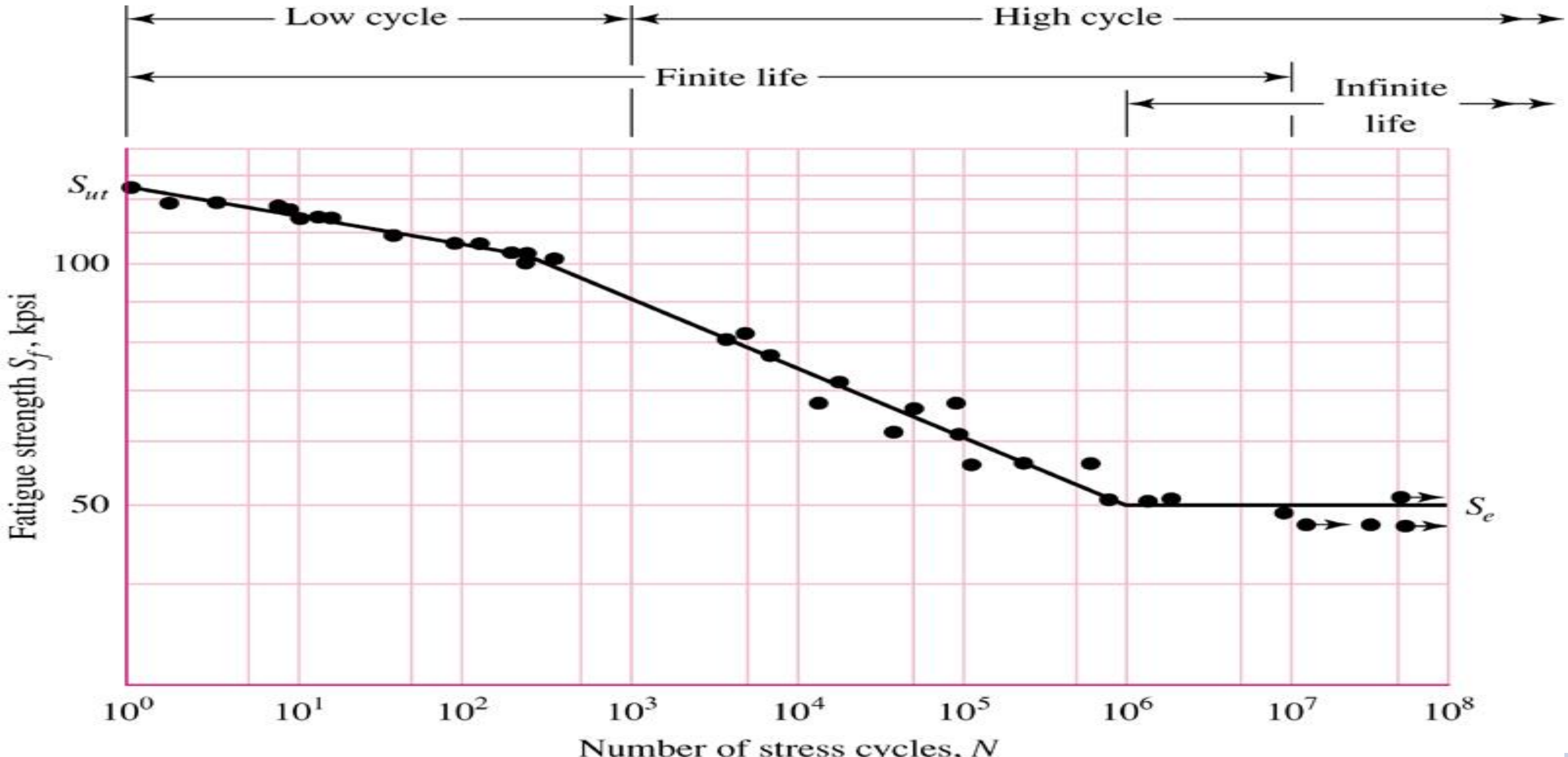
S-*N* Diagram

- Number of cycles to failure at varying stress levels is plotted on log-log scale
- For steels, a knee occurs near 10^6 cycles
- Strength corresponding to the knee is called *endurance limit* S_e



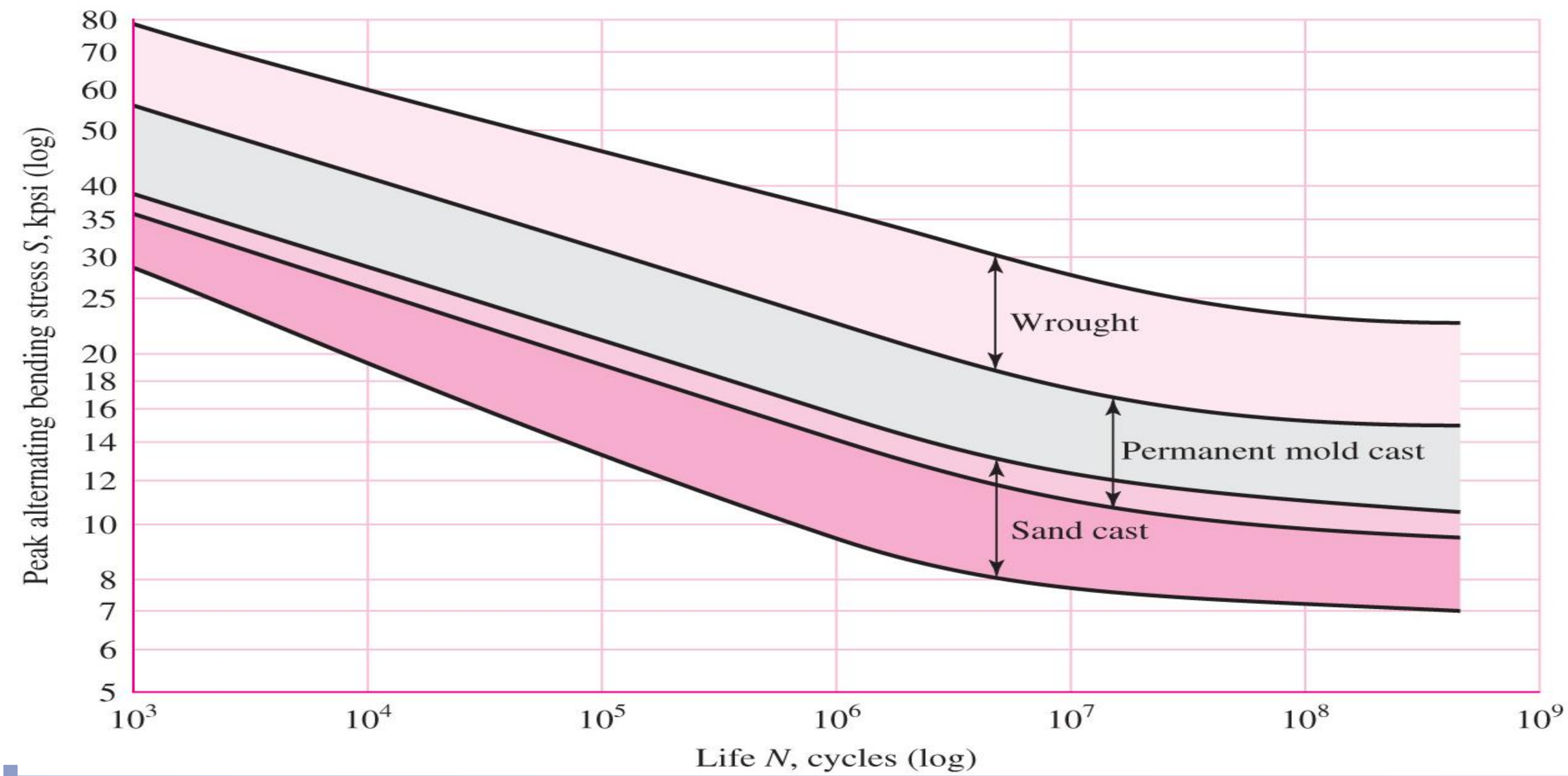
S-*N* Diagram for Steel

- Stress levels below S_e predict infinite life
- Between 10^3 and 10^6 cycles, finite life is predicted
- Below 10^3 cycles is known as *low cycle*, and is often considered quasi-static. Yielding usually occurs before fatigue in this zone.



S-N Diagram for Nonferrous Metals

- Nonferrous metals often do not have an endurance limit.
- Fatigue strength S_f is reported at a specific number of cycles
- Figure shows typical S-N diagram for aluminums

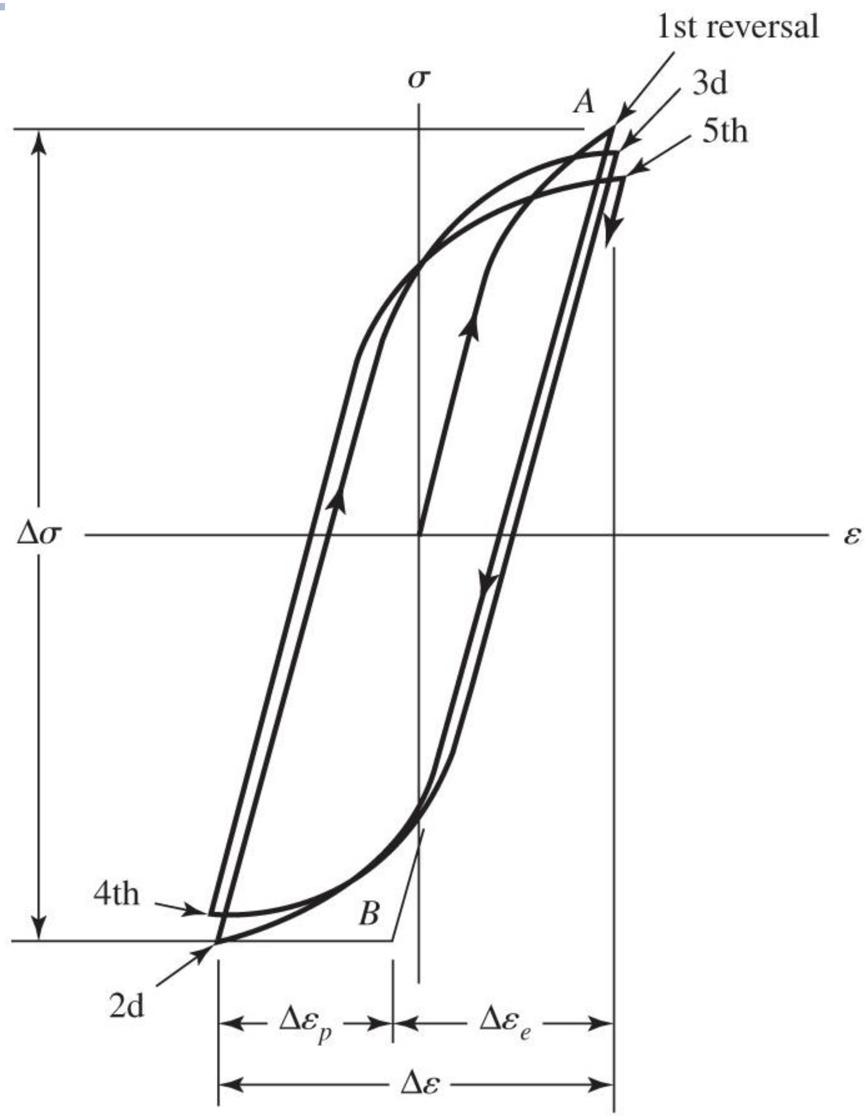


Strain-Life Method

- Method uses detailed analysis of plastic deformation at localized regions
- Compounding of several idealizations leads to significant uncertainties in numerical results
- Useful for explaining nature of fatigue

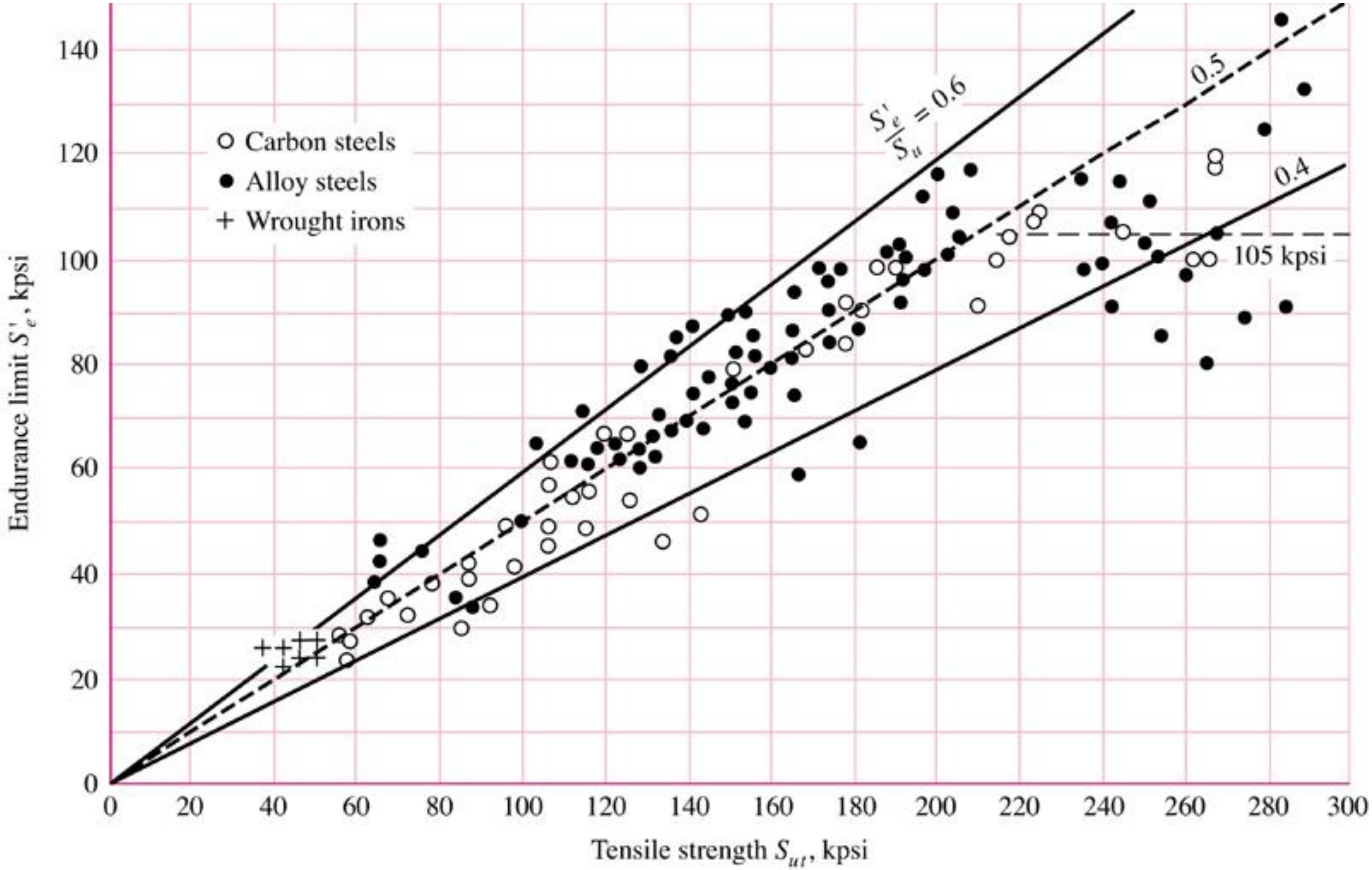
Strain-Life Method

- Fatigue failure almost always begins at a local discontinuity
- When stress at discontinuity exceeds elastic limit, plastic strain occurs
- Cyclic plastic strain can change elastic limit, leading to fatigue
- Fig. shows true stress-true strain hysteresis loops of the first five stress reversals



The Endurance Limit

- The endurance limit for steels has been experimentally found to be related to the ultimate strength



The Endurance Limit for Steel ($S_e' = S_{10^6}$)

- Simplified estimate of endurance limit for steels for the rotating-beam specimen, S'_e

US Units For $S_{ut} \le 200$ Ksi

$$\boxed{S_e' = 0.5 S_{ut} \text{ Ksi}}$$

For $S_{ut} > 200$ Ksi

$$\boxed{S_e' = 100 \text{ Ksi}}$$

SI Units For $S_{ut} \le 1400$ MPa

$$\boxed{S_e' = 0.5 S_{ut} \text{ MPa}}$$

For $S_{ut} > 1400$ MPa

$$\boxed{S_e' = 700 \text{ MPa}}$$

Note: For Steel, $S_{ut} = 0.5 H_B$ (Ksi) $= 3.41 H_B$ (MPa)

The Endurance Limit for Wrought Al-Alloys ($S_e' = S_{5x10^8}$)

- Simplified estimate of endurance limit for Wrought Al-Alloy for the rotating-beam specimen, S'_e

US Units

For $S_{ut} \leq 47$ Ksi

$$\boxed{S_e' = 0.4 S_{ut} \text{ Ksi}}$$

For $S_{ut} > 47$ Ksi

$$\boxed{S_e' = 19 \text{ Ksi}}$$

SI Units

For $S_{ut} \leq 325$ MPa

$$\boxed{S_e' = 0.4 S_{ut} \text{ MPa}}$$

For $S_{ut} > 325$ MPa

$$\boxed{S_e' = 130 \text{ MPa}}$$

Endurance Limit Modifying Factors

- Endurance limit S'_e is for carefully prepared and tested specimen
- If warranted, S_e is obtained from testing of actual parts

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

k_a = surface condition modification factor

k_b = size modification factor

k_c = load modification factor

k_d = temperature modification factor

k_e = reliability factor[13]

k_f = miscellaneous-effects modification factor

S'_e = rotary-beam test specimen endurance limit

S_e = endurance limit at the critical location of a machine part in the geometry and condition of use

Surface Factor k_a

- Stresses tend to be high at the surface

- Surface finish has an impact on initiation of cracks at localized stress concentrations

- Surface factor is a function of ultimate strength. Higher strengths are more sensitive to rough surfaces.

$$k_a = a S_{ut}^b$$

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

Size Factor k_b

- Larger parts have greater surface area at high stress levels
- Likelihood of crack initiation is higher
- Size factor is obtained from experimental data with wide scatter
- For bending and torsion loads, the trend of the size factor data is given by

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \leq d \leq 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \leq 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \leq d \leq 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \leq 254 \text{ mm} \end{cases} \qquad \textbf{Eq. i}$$

- Applies only for round, rotating diameter
- For axial load, there is no size effect, so $k_b = 1$

Size Factor k_b

- For parts that are not round and rotating, an equivalent round rotating diameter is obtained.
- Equate the volume of material stressed at and above 95% of the maximum stress to the same volume in the rotating-beam specimen.
- Lengths cancel, so equate the areas.
- For a rotating round section, the 95% stress area is the area of a ring,

$$A_{0.95\sigma} = \frac{\pi}{4}[d^2 - (0.95d)^2] = 0.0766d^2 \quad \textbf{Eq. ii}$$

- Equate 95% stress area for other conditions to Eq.ii and solve for d as the equivalent round rotating diameter

Size Factor k_b

- For non-rotating round,

$$A_{0.95\sigma} = 0.01046d^2 \qquad \textbf{Eq. iii}$$

- Equating to Eq. ii and solving for equivalent diameter,

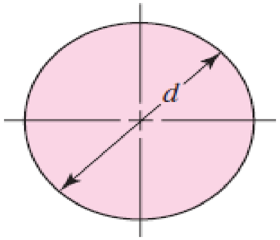
$$d_e = 0.370d \qquad \textbf{Eq. iv}$$

- Similarly, for rectangular section $h \times b$, $A_{95_\sigma} = 0.05\,hb$. Equating to Eq. ii

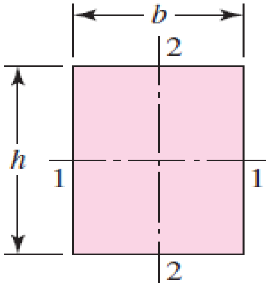
$$d_e = 0.808(hb)^{1/2} \qquad \textbf{Eq. v}$$

- Other common cross sections are given in Table

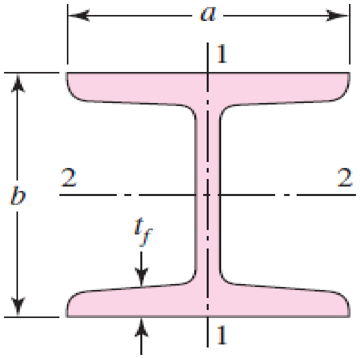
Size Factor k_b



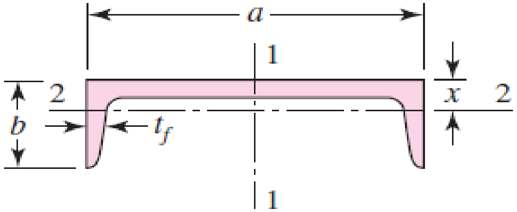
$$A_{0.95\sigma} = 0.01046d^2$$
$$d_e = 0.370d$$



$$A_{0.95\sigma} = 0.05hb$$
$$d_e = 0.808\sqrt{hb}$$



$$A_{0.95\sigma} = \begin{cases} 0.10at_f & & \text{axis 1-1} \\ 0.05ba & t_f > 0.025a & \text{axis 2-2} \end{cases}$$



$$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b - x) & \text{axis 2-2} \end{cases}$$

Loading Factor k_c

- Accounts for changes in endurance limit for different types of fatigue loading.

- Only to be used for single load types. Use Combination Loading method when more than one load type is present.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases}$$

Temperature Factor k_d

- Endurance limit appears to maintain same relation to ultimate strength for elevated temperatures as at room temperature

- This relation is summarized in Table

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

Temperature Factor k_d

- If ultimate strength is known for operating temperature, then just use that strength. Let $k_d = 1$ and proceed as usual.

- If ultimate strength is known only at room temperature, then use Table to estimate ultimate strength at operating temperature. With that strength, let $k_d = 1$ and proceed as usual.

-

- Alternatively, use ultimate strength at room temperature and apply temperature factor from Table to the endurance limit.

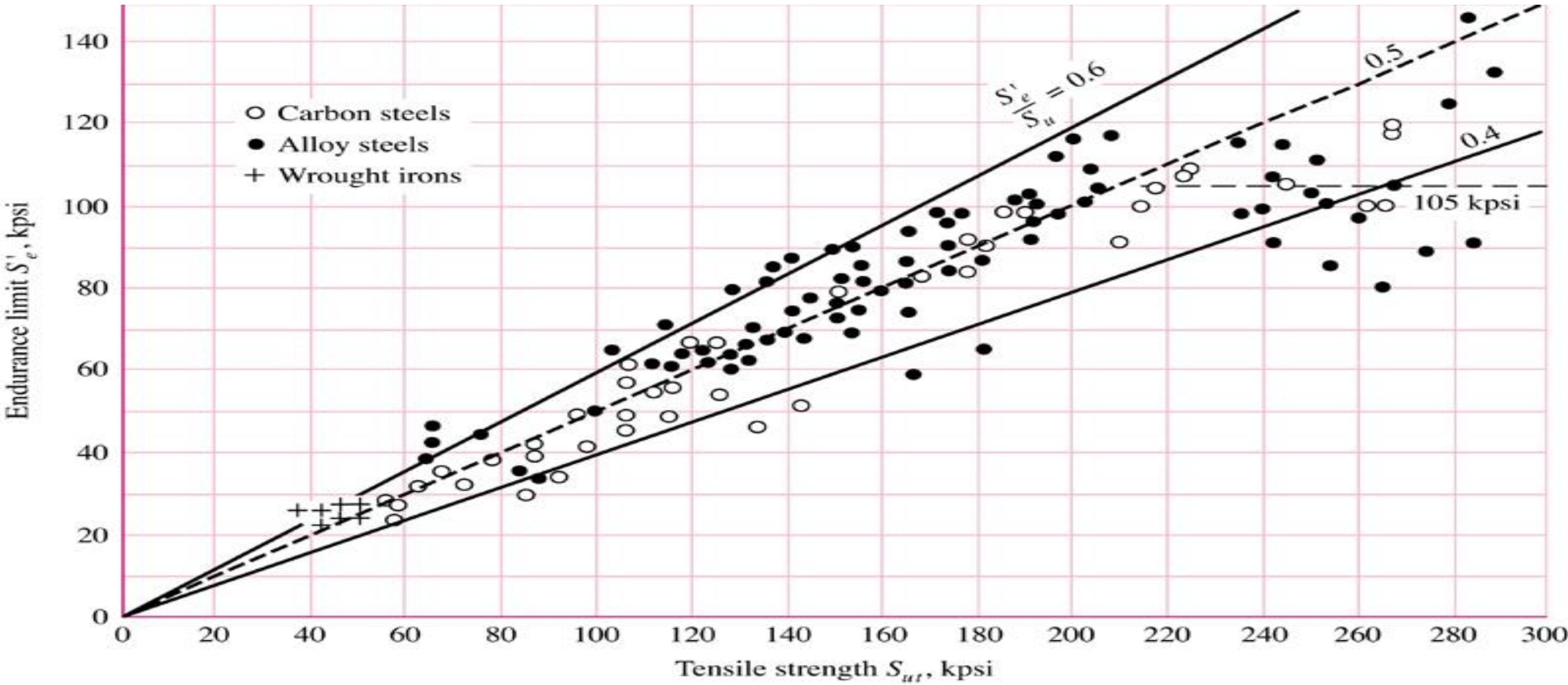
$$k_d = \frac{S_T}{S_{RT}} \quad \textbf{Eq. i}$$

- A fourth-order polynomial curve fit of the underlying data of Table can be used in place of the table, if desired.

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4 \quad \textbf{Eq. ii}$$

Reliability Factor k_e

- From Fig., $S'_e = 0.5\, S_{ut}$ is typical of the data and represents 50% reliability.
- Reliability factor adjusts to other reliabilities.
- *Only* adjusts Fig. assumption. *Does not* imply overall reliability.



Reliability Factor k_e

- Simply obtain k_e for desired reliability from Table.

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Miscellaneous-Effects Factor k_f

- Reminder to consider other possible factors.
  - Residual stresses
  - Directional characteristics from cold working
  - Case hardening
  - Corrosion
  - Surface conditioning, e.g. electrolytic plating and metal spraying
  - Cyclic Frequency
  - Frettage Corrosion
- Limited data is available.
- May require research or testing.

Endurance Strength at 10^3 Cycles

- The endurance strength at 10^3 is given by

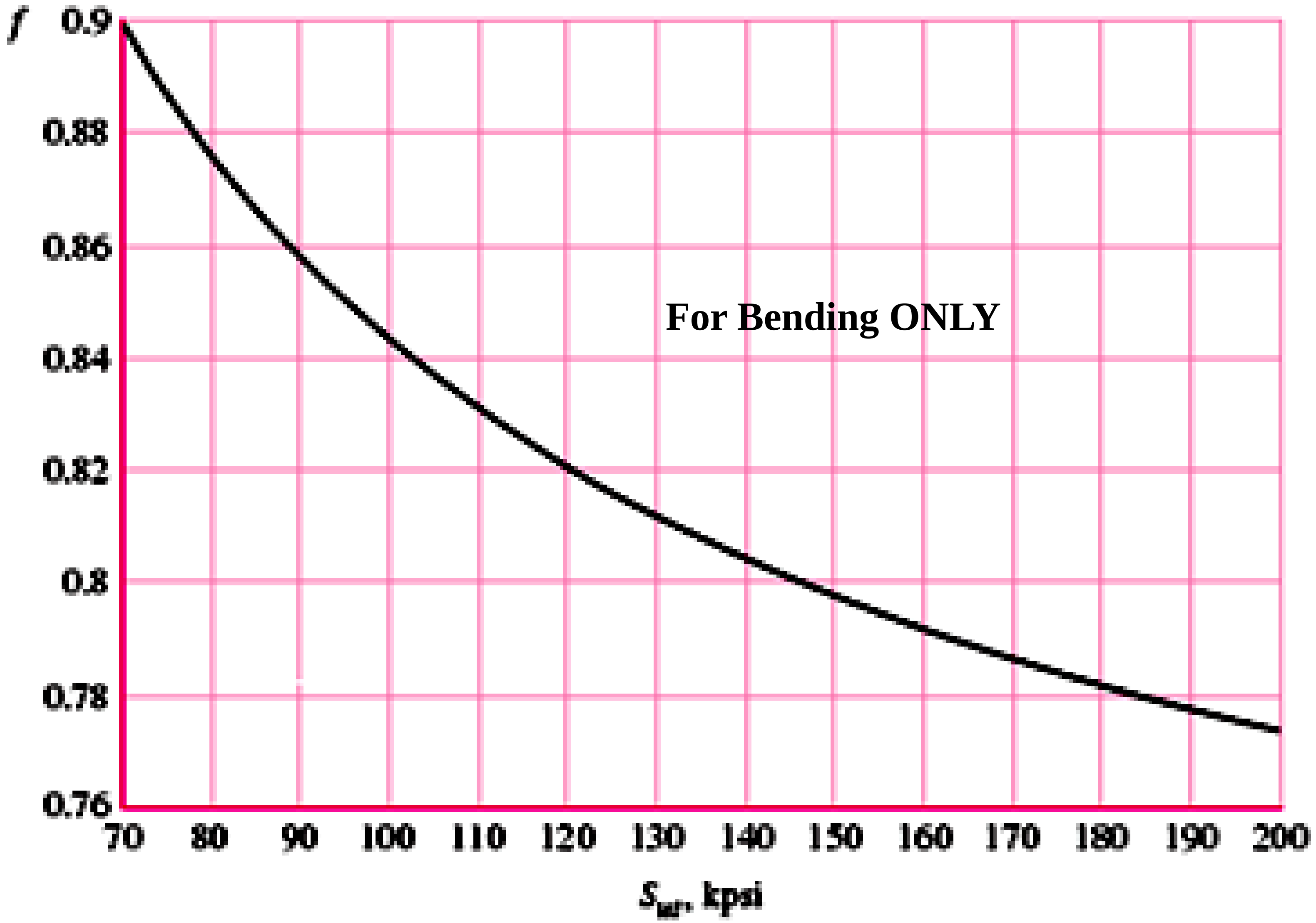
$$\boxed{S_{10^3} = f\, S_{ut}}$$

- Where f is

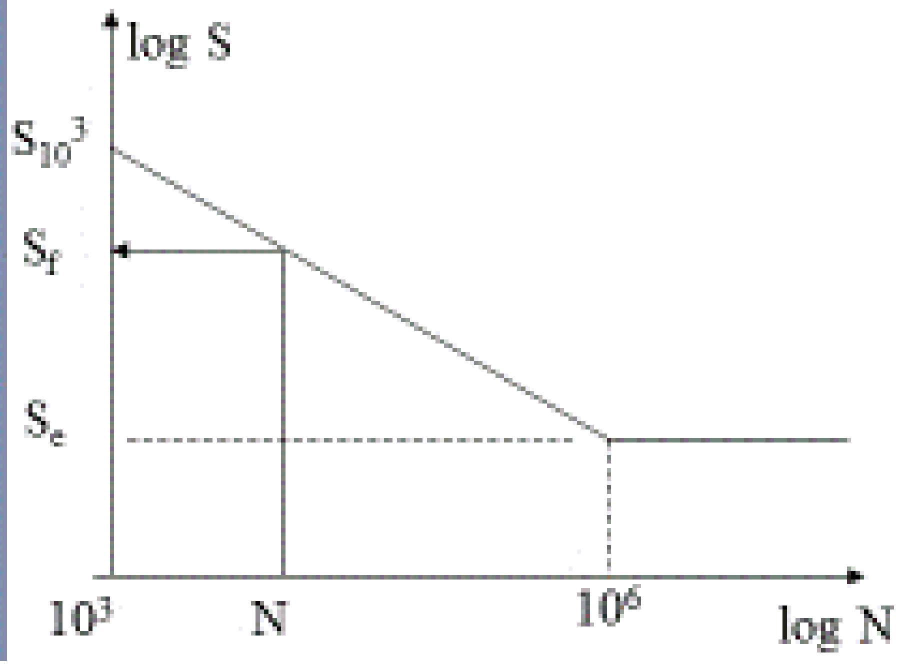
$$f = \begin{cases} 0.9 \text{ for Bending} \\ 0.75 \text{ for Axial/Direct Load} \\ 0.72 \text{ for Torsion (Steels)} \\ 0.63 \text{ for Torsion (All other ductile materials)} \end{cases}$$

- See Fig. to find the value of "f" (fatigue strength fraction)

Endurance Strength at 10^3 Cycles



Endurance Strength at N Cycles Life



1. Find S_{10^3}
2. Find S_e
3. Make S-N Plot
4. Use extrapolation to find S_f
5. Results of Interpolation

$$S_f = S_N$$

It can be shown that:

$$\boxed{S_f = aN^b}$$

Where:

$$a = (S_{10^3})^2 / S_e$$

$$b = (-1/3) \log [S_{10^3} / S_e]$$

Example 4–1

A steel has a minimum ultimate strength of 520 MPa and a machined surface. Estimate k_a.

Solution From Table 6–2, $a = 4.51$ and $b = -0.265$. Then, from Eq. (6–19)

Answer $$k_a = 4.51(520)^{-0.265} = 0.860$$

Example 4–2

A steel shaft loaded in bending is 32 mm in diameter, abutting a filleted shoulder 38 mm in diameter. The shaft material has a mean ultimate tensile strength of 690 MPa. Estimate the Marin size factor k_b if the shaft is used in

(*a*) A rotating mode.

(*b*) A nonrotating mode.

Solution (*a*) From Eq. (6–20)

Answer

$$k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{32}{7.62}\right)^{-0.107} = 0.858$$

(*b*) From Table 6–3,

$$d_e = 0.37d = 0.37(32) = 11.84 \text{ mm}$$

From Eq. (6–20),

Answer

$$k_b = \left(\frac{11.84}{7.62}\right)^{-0.107} = 0.954$$

Example 4–3

A 1035 steel has a tensile strength of 70 kpsi and is to be used for a part that sees 450°F in service. Estimate the Marin temperature modification factor and $(S_e)_{450^\circ}$ if

(*a*) The room-temperature endurance limit by test is $(S'_e)_{70^\circ} = 39.0$ kpsi.

(*b*) Only the tensile strength at room temperature is known.

Solution (*a*) First, from Eq. (6–27),

$$k_d = 0.975 + 0.432(10^{-3})(450) - 0.115(10^{-5})(450^2) + 0.104(10^{-8})(450^3) - 0.595(10^{-12})(450^4) = 1.007$$

Thus,

Answer $$(S_e)_{450^\circ} = k_d(S'_e)_{70^\circ} = 1.007(39.0) = 39.3 \text{ kpsi}$$

Example 4–3 (continued)

(*b*) Interpolating from Table 6–4 gives

$$(S_T/S_{RT})_{450^\circ} = 1.018 + (0.995 - 1.018)\frac{450 - 400}{500 - 400} = 1.007$$

Thus, the tensile strength at 450°F is estimated as

$$(S_{ut})_{450^\circ} = (S_T/S_{RT})_{450^\circ}\,(S_{ut})_{70^\circ} = 1.007(70) = 70.5\text{ kpsi}$$

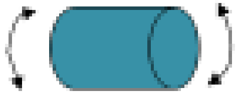
From Eq. (6–8) then,

Answer $$(S_e)_{450^\circ} = 0.5\,(S_{ut})_{450^\circ} = 0.5(70.5) = 35.2\text{ kpsi}$$

Part *a* gives the better estimate due to actual testing of the particular material.

Example 4–4

Estimate the design endurance limit at 50% reliability for the 50-mm diameter machined bar loaded as shown. The material is AISI 1050 CD Steel. The part is non-rotating will be used at room temperature.



Material:	S_y =580 MPa S_{ut} = 690 MPa
Specimen Endurance Limit,	$S_e' = 0.5S_{ut} = 0.5\,(690) = 345$ MPa
Design Endurance Limit,	$S_e = k_a k_b k_c k_d k_e S_e' = k_a k_b k_c k_d k_e\,(345)$
Surface factor (machined),	$k_a = aS_{ut}^{b}$ **= 4.51 (690)$^{-0.265}$ = 0.798**
Size factor (non-rotating),	$d = 50$mm
	$d_e = 0.370d = 18.5$ (Table)
Bending	$k_b = 1.24 d_e^{-0.107} = 0.907$
Load Factor (Bending),	$k_c = 1.0$
Temperature (Room),	$k_d = 1.0$
Reliability (50%),	$k_e = 1.0$

Finally, $S_e = (0.798)(0.907)(1.0)(1.0)(1.0)(0.5\text{x}690)$

$= 250$ MPa

Example 4–5

A machine part is made from AISI 1035 Steel. It has ground finish and the load is variable and torsional. The material is heat-treated to a tensile strength of 710 MPa. The part diameter is 32 mm and will be rotating during service. It will be used in a machine at temperature of 300 F. Estimate: (a) the endurance limit at 95% reliability and (b) endurance strength at 10^4 cycles and show it on S-N Plot

S_{ut} = 710 MPa **$S_e' = 0.5S_{ut}$ = 355 MPa**

A.

Design Endurance Limit, $S_e = k_a k_b k_c k_d k_e S_e'$

Surface factor (ground), $k_a = aS_{ut}^{\mathbf{b}} = \mathbf{1.58\ (710)^{-0.085} = 0.904}$

Size factor (rotating), $d = 32\text{mm}$

Torsion $k_b = 1.24d^{\mathbf{-0.107}} = 0.856$

Load Factor (Torsion), $k_c = 0.59$

Temperature (300F), $k_d = 1.024$

Reliability (95%), $k_e = 0.868$

Finally,

$S_e = (0.904)(0.856)(0.59)(1.024)(0.868)(355)$

= 144 MPa

Example 4-5 (Cont'd)

B. <u>To find</u> S_{10^4}

$S_f = S_{10^4} = aN^b = a(10^4)^b$

Where $a = (S_{10^3})^2 / S_e$

$S_{10^3} = f\, S_{ut} = 0.72\, S_{ut} = 511$ MPa

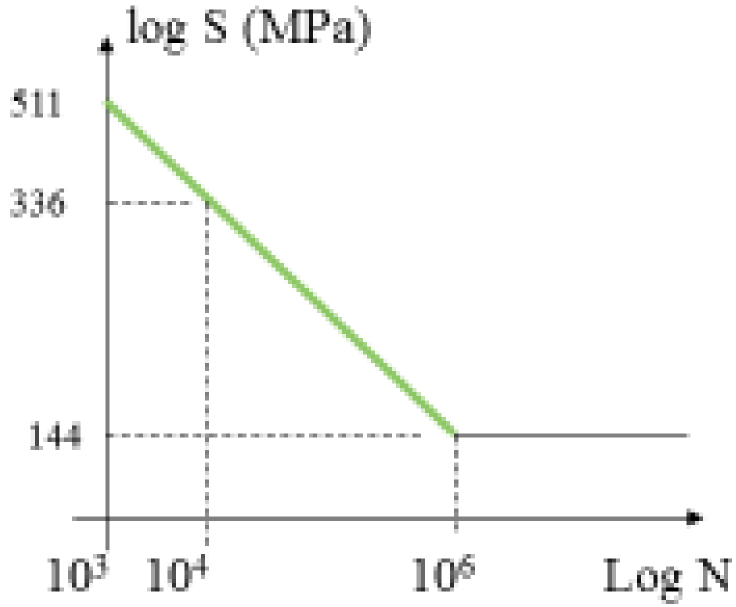
$a = (511)^2 / S_e = 1813$ MPa

$b = (-1/3) \log [S_{10^3} / S_e]$

$= (-1/3) \log [511/ 144]$

$= -0.183$

Thus $S_{10^4} = 1813(10^4)^{-0.183}$ $\underline{= 336 \text{ MPa}}$



Stress Concentration and Notch Sensitivity

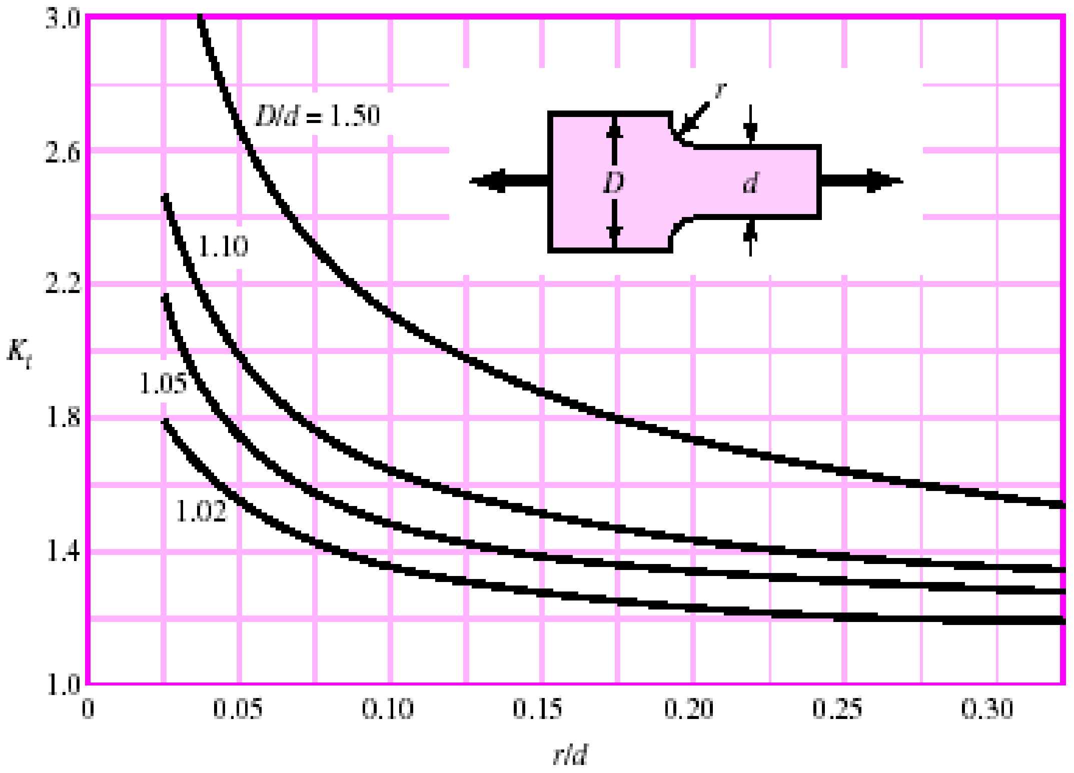
- For dynamic loading, stress concentration effects must be applied.
- Obtain K_t as usual (e.g. Appendix A–15)
- For fatigue, some materials are not fully sensitive to K_t so a reduced value can be used.
- Define K_f as the *fatigue stress-concentration factor*.
- Define q as *notch sensitivity*, ranging from 0 (not sensitive) to 1 (fully sensitive).

$$q = \frac{K_f - 1}{K_t - 1} \qquad \text{or} \qquad q_{\text{shear}} = \frac{K_{fs} - 1}{K_{ts} - 1}$$

- For $q = 0$, $K_f = 1$
- For $q = 1$, $K_f = K_t$

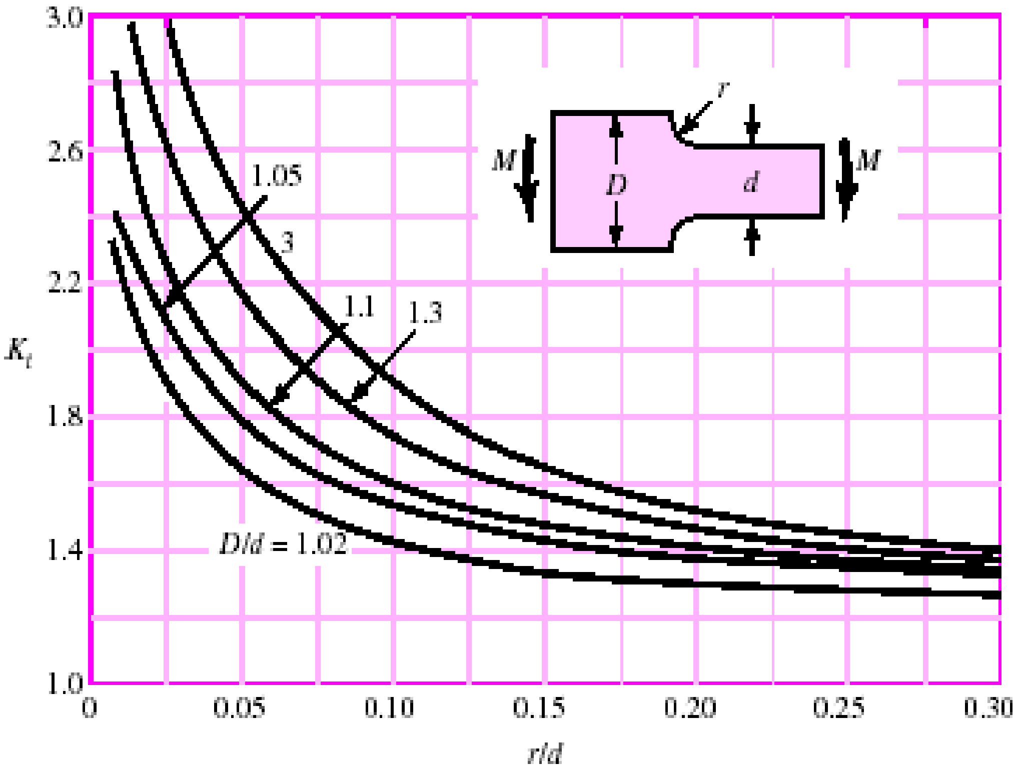
Notch Sensitivity

- Theoretical Stress Concentration Factor, K_t for Axial Loading



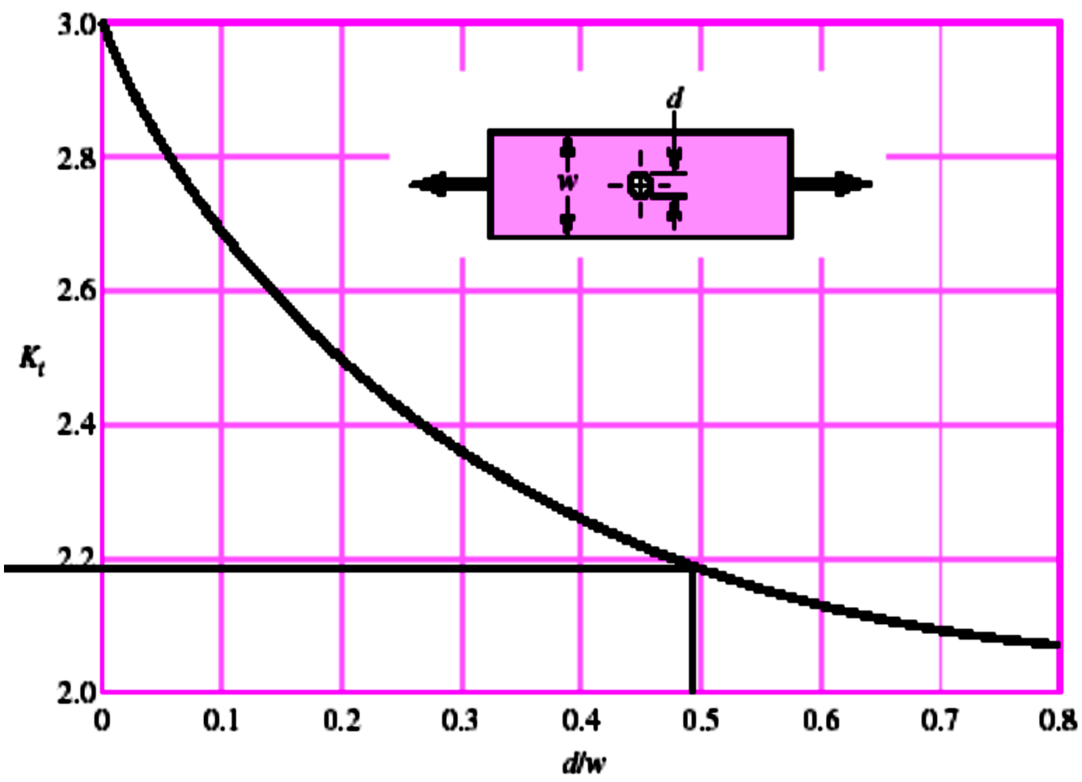
Notch Sensitivity

➢ Theoretical Stress Concentration Factor, K_t for Bending Loading



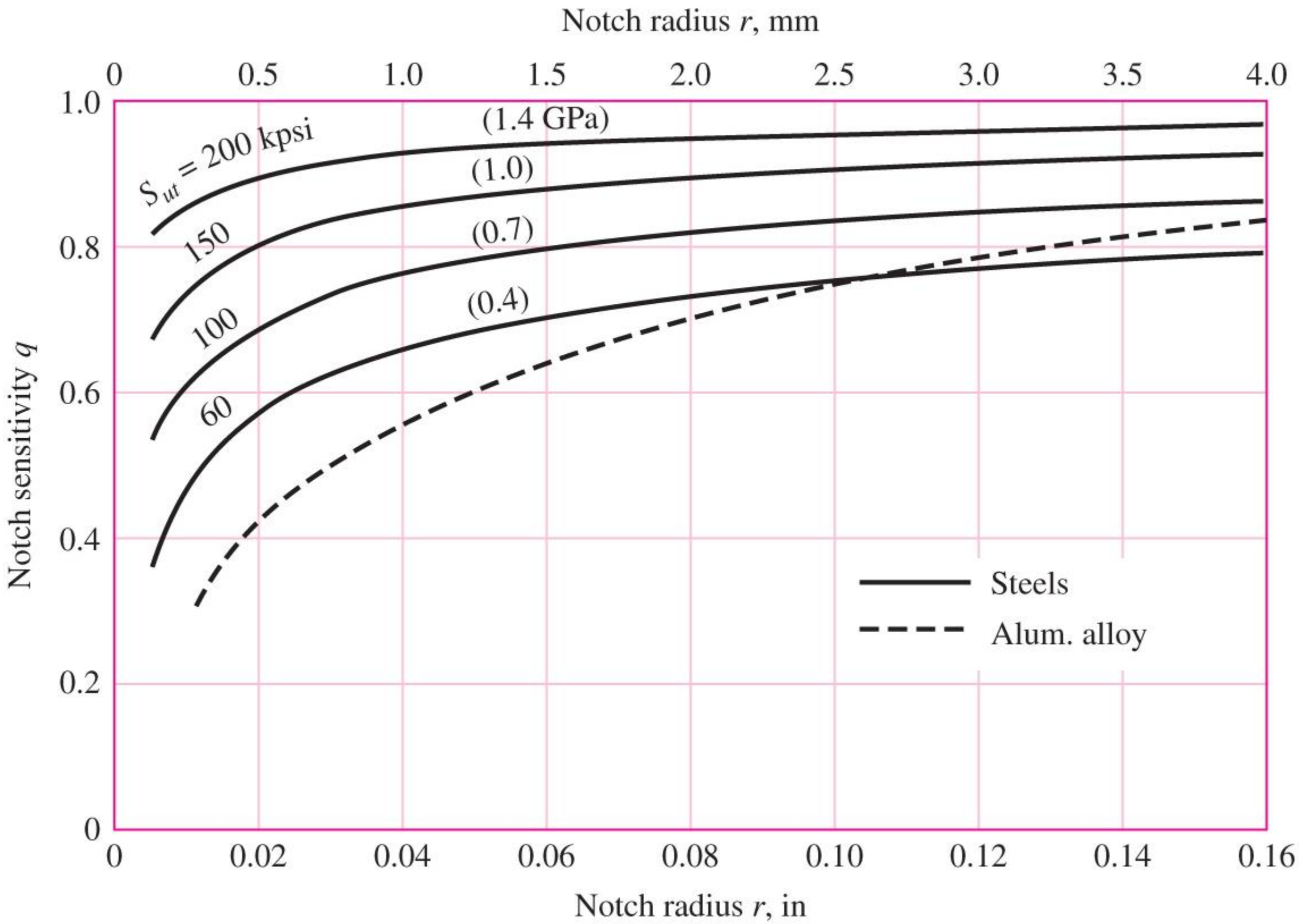
Notch Sensitivity

- Theoretical Stress Concentration Factor, K_t for Axial Loading



Notch Sensitivity

- Obtain q for bending or axial loading from Fig. Then get K_f from Eq. (6–32): $K_f = 1 + q(K_t - 1)$



Notch Sensitivity

- Obtain q_s for torsional loading from Fig. 6–21.
- Then get K_{fs} from Eq. (6–32): $K_{fs} = 1 + q_s(K_{ts} - 1)$

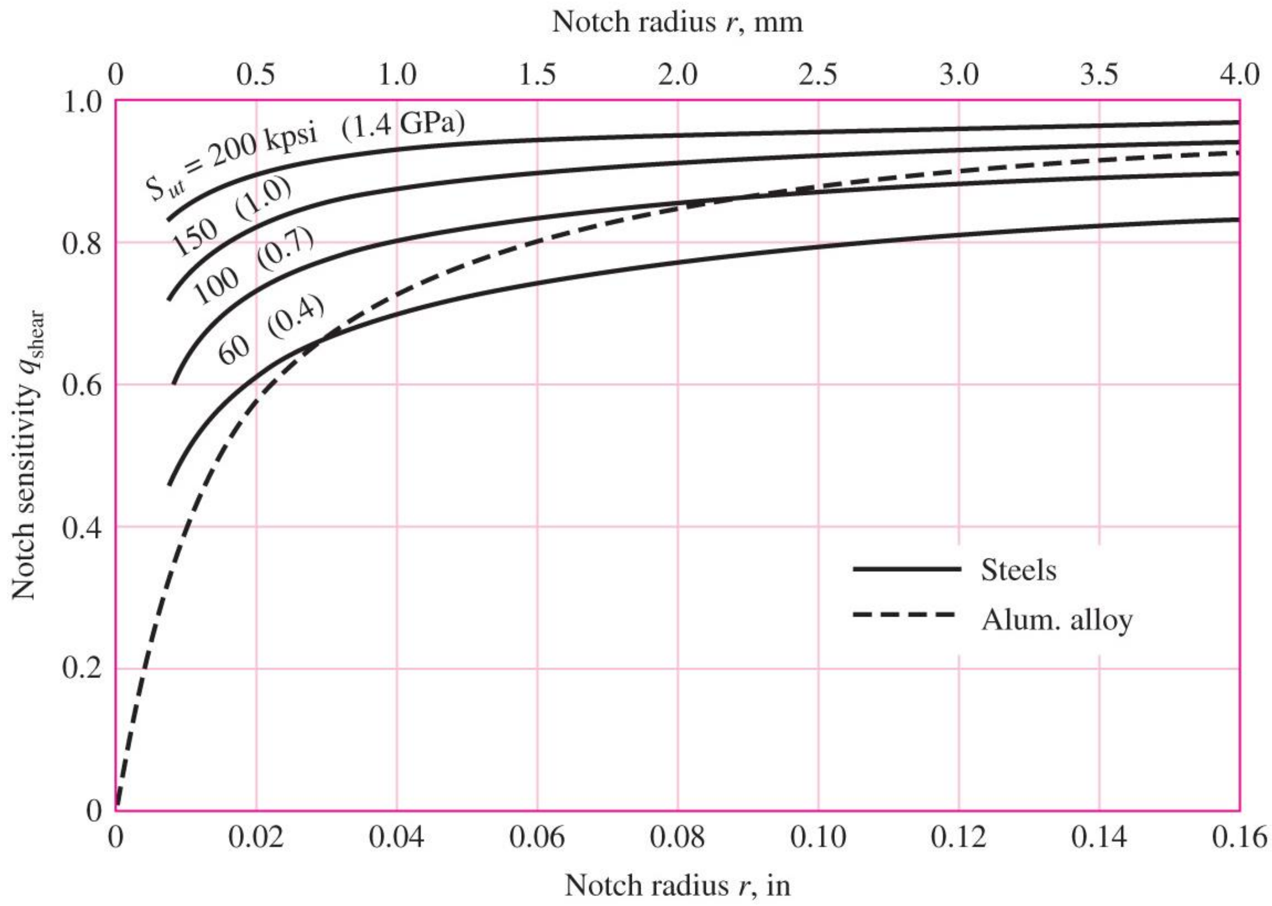


Fig. 6–21

Notch Sensitivity

- Alternatively, can use curve fit equations to get notch sensitivity, or go directly to K_f .

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}}$$

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}}$$

- For steels, with S_{ut} in kpsi

- Bending or axial:

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$$

- Torsion:

$$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$$

Notch Sensitivity for Cast Irons

- Cast irons are already full of discontinuities, which are included in the strengths.
- Additional notches do not add much additional harm.
- Recommended to use $q = 0.2$ for cast irons.

Application of Fatigue Stress Concentration Factor

- Use K_f as a multiplier to increase the nominal stress.
- Some designers (and previous editions of textbook) sometimes applied $1/K_f$ as a Marin factor to reduce S_e.
- For infinite life, either method is equivalent, since

$$n_f = \frac{S_e}{K_f \sigma} = \frac{\left(1/K_f\right) S_e}{\sigma}$$

- For finite life, increasing stress is more conservative. Decreasing S_e applies more to high cycle than low cycle.

Example 4–6

A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f using:
(*a*) Figure 6–20.
(*b*) Equations (6–33) and (6–35).

Solution

From Fig. A–15–9, using $D/d = 38/32 = 1.1875$, $r/d = 3/32 = 0.093\,75$, we read the graph to find $K_t = 1.65$.
(*a*) From Fig. 6–20, for $S_{ut} = 690$ MPa and $r = 3$ mm, $q = 0.84$. Thus, from Eq. (6–32)

$$K_f = 1 + q(K_t - 1) = 1 + 0.84(1.65 - 1) = 1.55 \qquad \text{Answer}$$

(*b*) From Eq. (6–35*a*) with $S_{ut} = 690\text{ MPa} = 100\text{ kpsi}$, $\sqrt{a} = 0.0622\sqrt{\text{in}} = 0.313\sqrt{\text{mm}}$. Substituting this into Eq. (6–33) with $r = 3$ mm gives

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} = 1 + \frac{1.65 - 1}{1 + \dfrac{0.313}{\sqrt{3}}} = 1.55 \qquad \text{Answer}$$

Example 4–7

For the step-shaft of Ex. 6–6, it is determined that the fully corrected endurance limit is $S_e = 280$ MPa. Consider the shaft undergoes a fully reversing nominal stress in the fillet of $(\sigma_{rev})_{nom} = 260$ MPa. Estimate the number of cycles to failure.

Solution

From Ex. 6–6, $K_f = 1.55$, and the ultimate strength is $S_{ut} = 690$ MPa $= 100$ kpsi. The maximum reversing stress is

$$(\sigma_{rev})_{max} = K_f(\sigma_{rev})_{nom} = 1.55(260) = 403 \text{ MPa}$$

From Fig. 6–18, $f = 0.845$. From Eqs. (6–14), (6–15), and (6–16)

$$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.845(690)]^2}{280} = 1214 \text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{f\,S_{ut}}{S_e} = -\frac{1}{3}\log\left[\frac{0.845(690)}{280}\right] = -0.1062$$

$$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b} = \left(\frac{403}{1214}\right)^{1/-0.1062} = 32.3(10^3) \text{ cycles}$$ Answer

Example 4–8

A 1015 hot-rolled steel bar has been machined to a diameter of 1 in. It is to be placed in reversed axial loading for 70 000 cycles to failure in an operating environment of 550°F. Using ASTM minimum properties, and a reliability of 99 percent, estimate the endurance limit and fatigue strength at 70 000 cycles.

Solution

From Table A–20, $S_{ut} = 50$ kpsi at 70°F. Since the rotating-beam specimen endurance limit is not known at room temperature, we determine the ultimate strength at the elevated temperature first, using Table 6–4. From Table 6–4,

$$\left(\frac{S_T}{S_{RT}}\right)_{550^\circ} = \frac{0.995 + 0.963}{2} = 0.979$$

The ultimate strength at 550°F is then

$$(S_{ut})_{550^\circ} = (S_T/S_{RT})_{550^\circ}\,(S_{ut})_{70^\circ} = 0.979(50) = 49.0 \text{ kpsi}$$

The rotating-beam specimen endurance limit at 550°F is then estimated from Eq. (6–8) as

$$S_e' = 0.5(49) = 24.5 \text{ kpsi}$$

Example 4–8 (continued)

Next, we determine the Marin factors. For the machined surface, Eq. (6–19) with Table 6–2 gives

$$k_a = aS_{ut}^b = 2.70(49^{-0.265}) = 0.963$$

For axial loading, from Eq. (6–21), the size factor $k_b = 1$, and from Eq. (6–26) the loading factor is $k_c = 0.85$. The temperature factor $k_d = 1$, since we accounted for the temperature in modifying the ultimate strength and consequently the endurance limit. For 99 percent reliability, from Table 6–5, $k_e = 0.814$. Finally, since no other conditions were given, the miscellaneous factor is $k_f = 1$. The endurance limit for the part is estimated by Eq. (6–18) as

$$\begin{aligned} S_e &= k_a k_b k_c k_d k_e k_f S'_e \\ &= 0.963(1)(0.85)(1)(0.814)(1)24.5 = 16.3 \text{ kpsi} \end{aligned}$$

Answer

Example 4–8 (continued)

For the fatigue strength at 70 000 cycles we need to construct the S-N equation. From p. 293, since $S_{ut} = 49 < 70$ kpsi, then $f = 0.9$. From Eq. (6–14)

$$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.9(49)]^2}{16.3} = 119.3 \text{ kpsi}$$

and Eq. (6–15)

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{0.9(49)}{16.3}\right] = -0.1441$$

Finally, for the fatigue strength at 70 000 cycles, Eq. (6–13) gives

$$S_f = a\,N^b = 119.3(70\,000)^{-0.1441} = 23.9 \text{ kpsi} \qquad \text{Answer}$$

Example 4–9

Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM “minimum” strengths, estimate the life of the part.

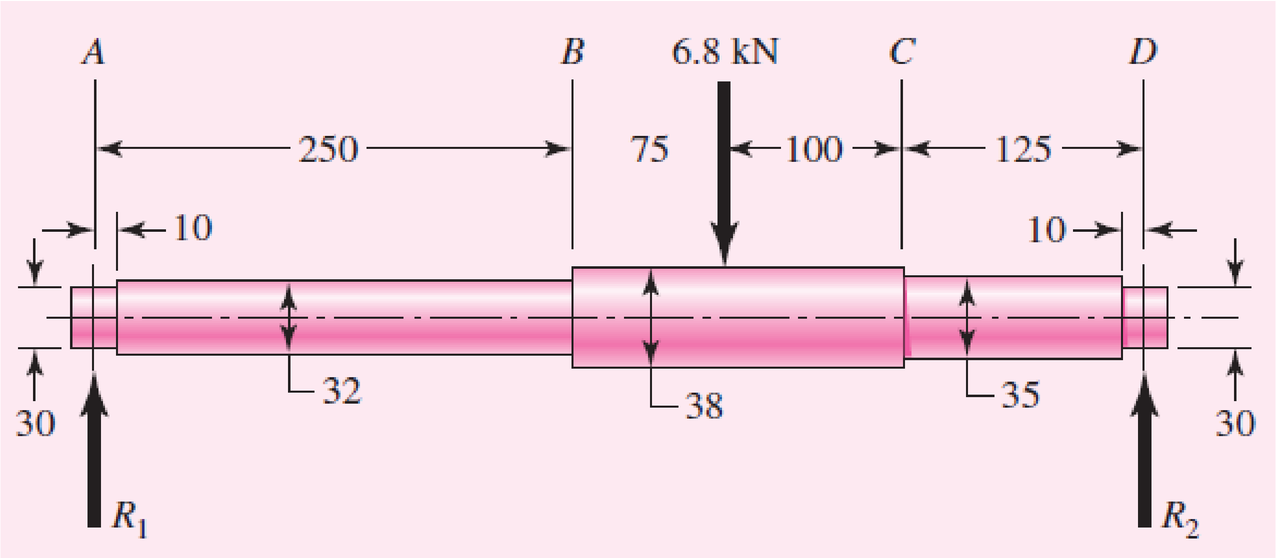
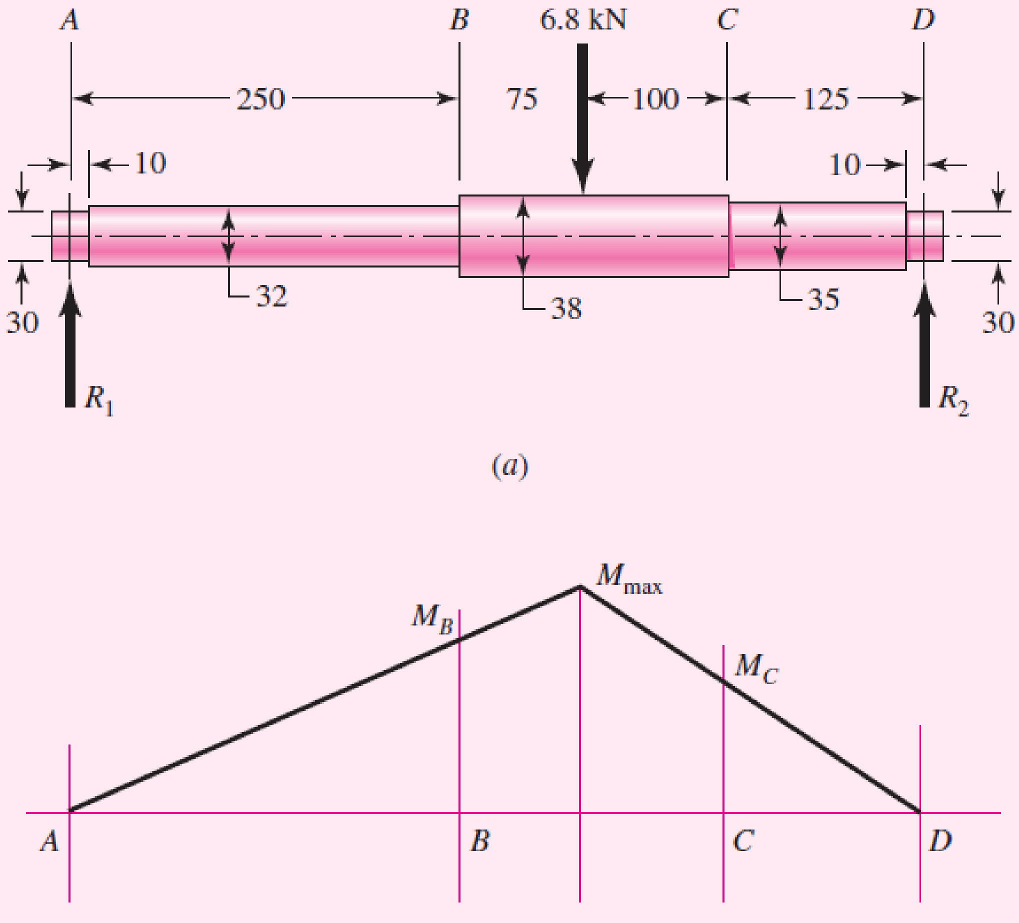


Fig. a

Example 4–9 (continued)

From Fig. 6–22*b* we learn that failure will probably occur at *B* rather than at *C* or at the point of maximum moment. Point *B* has a smaller cross section, a higher bending moment, and a higher stress-concentration factor than *C*, and the location of maximum moment has a larger size and no stress-concentration factor.



(*a*)

(*b*)

Example 4–9 (continued)

We shall solve the problem by first estimating the strength at point B, since the strength will be different elsewhere, and comparing this strength with the stress at the same point.

From Table A–20 we find $S_{ut} = 690$ MPa and $S_y = 580$ MPa. The endurance limit S'_e is estimated as

$$S'_e = 0.5(690) = 345 \text{ MPa}$$

From Eq. (6–19) and Table 6–2,

$$k_a = 4.51(690)^{-0.265} = 0.798$$

From Eq. (6–20),

$$k_b = (32/7.62)^{-0.107} = 0.858$$

Since $k_c = k_d = k_e = k_f = 1$,

$$S_e = 0.798(0.858)345 = 236 \text{ MPa}$$

Example 4–9 (continued)

To find the geometric stress-concentration factor K_t we enter Fig. A–15–9 with $D/d = 38/32 = 1.1875$ and $r/d = 3/32 = 0.093\,75$ and read $K_t \doteq 1.65$. Substituting $S_{ut} = 690/6.89 = 100$ kpsi into Eq. (6–35*a*) yields $\sqrt{a} = 0.0622\sqrt{\text{in}} = 0.313\sqrt{\text{mm}}$. Substituting this into Eq. (6–33) gives

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} = 1 + \frac{1.65 - 1}{1 + 0.313/\sqrt{3}} = 1.55$$

The next step is to estimate the bending stress at point B. The bending moment is

$$M_B = R_1 x = \frac{225F}{550}250 = \frac{225(6.8)}{550}250 = 695.5 \text{ N} \cdot \text{m}$$

Just to the left of B the section modulus is $I/c = \pi d^3/32 = \pi 32^3/32 = 3.217\ (10^3)\ \text{mm}^3$. The reversing bending stress is, assuming infinite life,

$$\sigma_{\text{rev}} = K_f \frac{M_B}{I/c} = 1.55 \frac{695.5}{3.217}(10)^{-6} = 335.1(10^6)\ \text{Pa} = 335.1\ \text{MPa}$$

This stress is greater than S_e and less than S_y. This means we have both finite life and no yielding on the first cycle.

Example 4–9 (continued)

For finite life, we will need to use Eq. (6–16). The ultimate strength, $S_{ut} = 690$ MPa = 100 kpsi. From Fig. 6–18, $f = 0.844$. From Eq. (6–14)

$$a = \frac{(f\ S_{ut})^2}{S_e} = \frac{[0.844(690)]^2}{236} = 1437 \text{ MPa}$$

and from Eq. (6–15)

$$b = -\frac{1}{3}\log\left(\frac{f\ S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{0.844(690)}{236}\right] = -0.1308$$

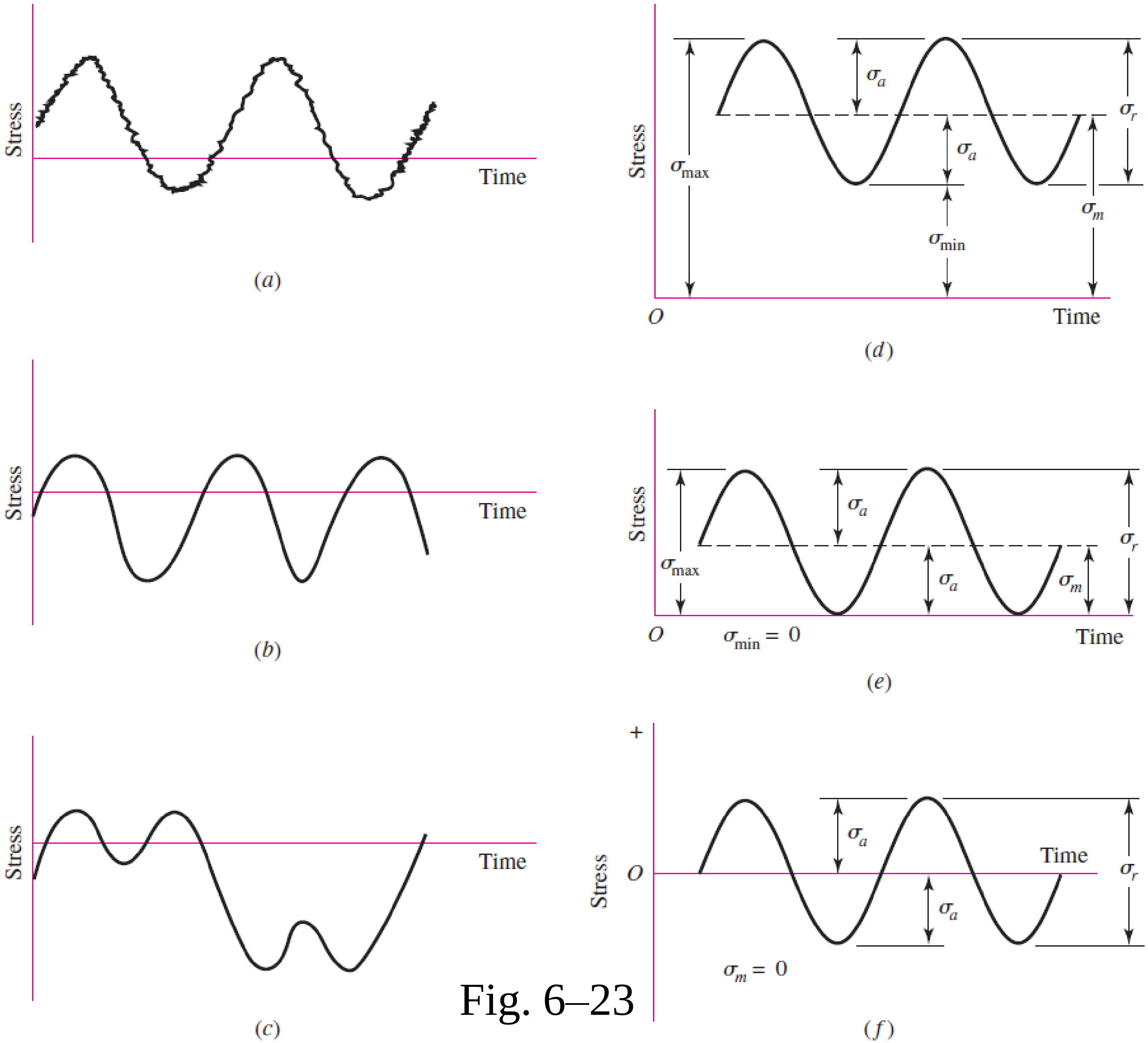
From Eq. (6–16),

$$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{335.1}{1437}\right)^{-1/0.1308} = 68(10^3) \text{ cycles}$$ Answer

Characterizing Fluctuating Stresses

- The *S*-*N* diagram is applicable for *completely reversed* stresses
- Other fluctuating stresses exist
- Sinusoidal loading patterns are common, but not necessary
- Vary the σ_m and σ_a to learn about the fatigue resistance under fluctuating loading
- Three common methods of plotting results follow.

Characterizing Fluctuating Stresses



General Fluctuating

Repeated

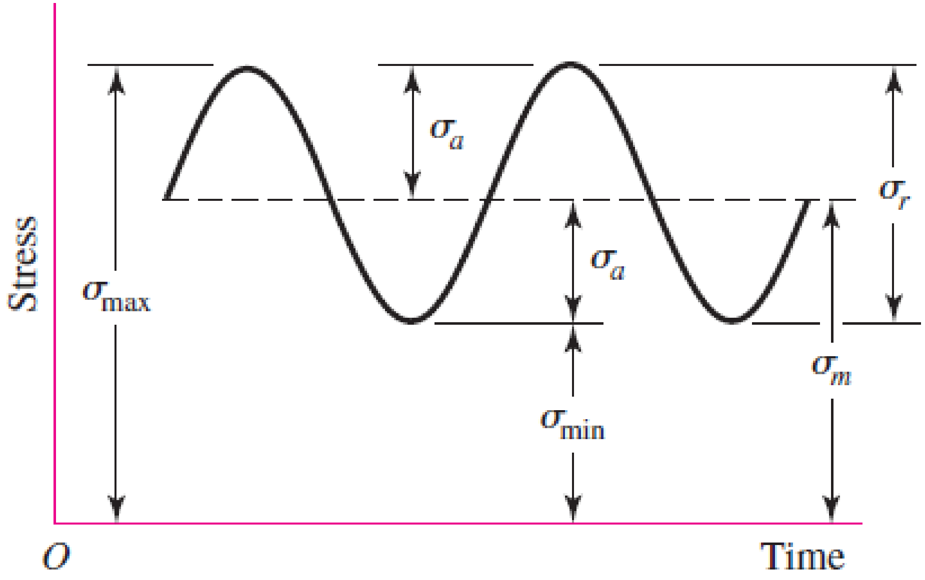
Completely Reversed

Fig. 6–23

Characterizing Fluctuating Stresses

- Fluctuating stresses can often be characterized simply by the minimum and maximum stresses, $\sigma_{\min}$ and $\sigma_{\max}$

- Define σ_m as *midrange* steady component of stress (sometimes called *mean* stress) and σ_a as amplitude of *alternating* component of stress

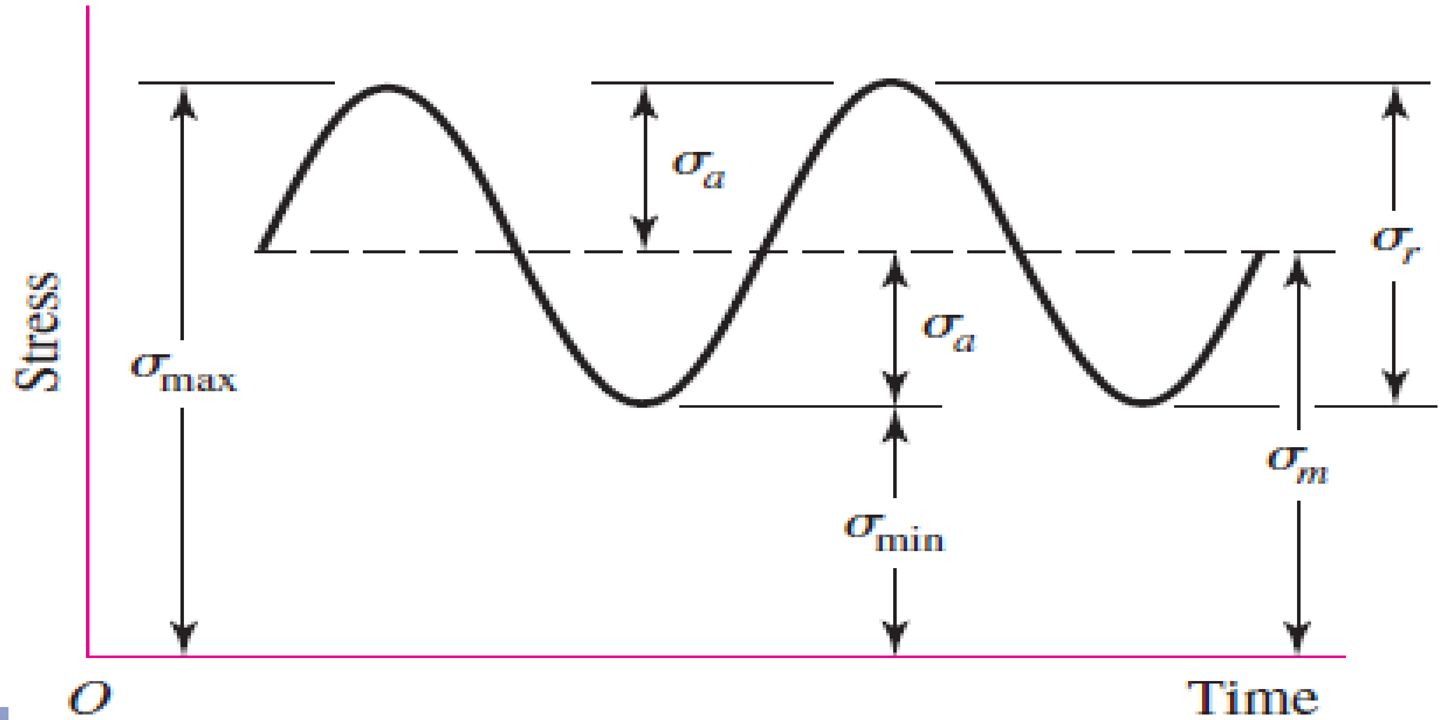


$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

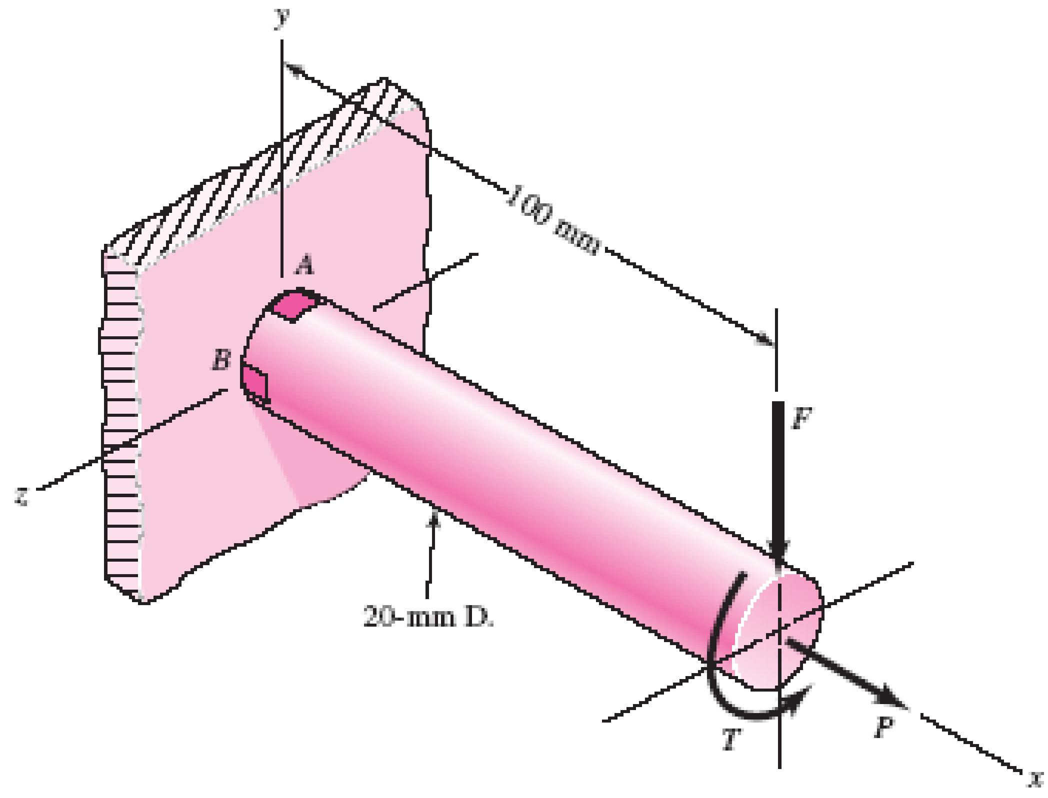
$$\sigma_a = \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right|$$

Characterizing Fluctuating Stresses

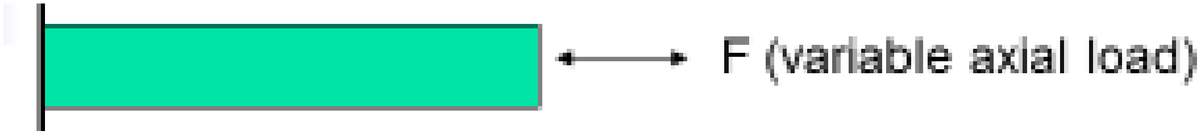
- Other useful definitions
  - *Stress ratio* $$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$
  - *Amplitude ratio* $$A = \frac{\sigma_a}{\sigma_m}$$



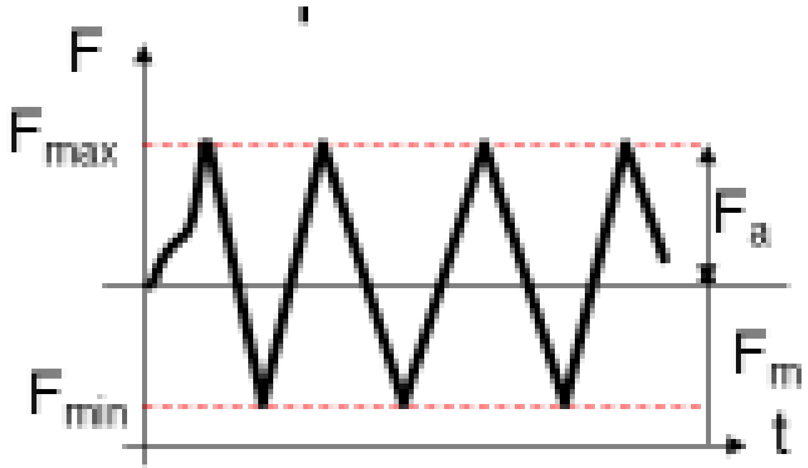
Fatigue Loading



Axial Fatigue Load & Stresses



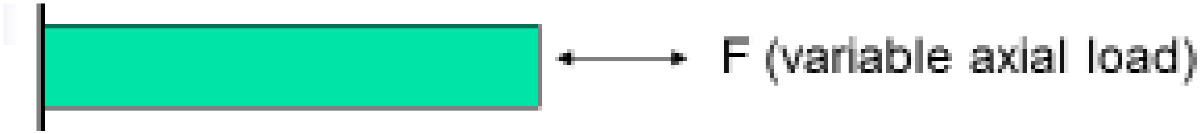
Fluctuating Load/Stress: **$F \sim (F_{max}, F_{min})$**



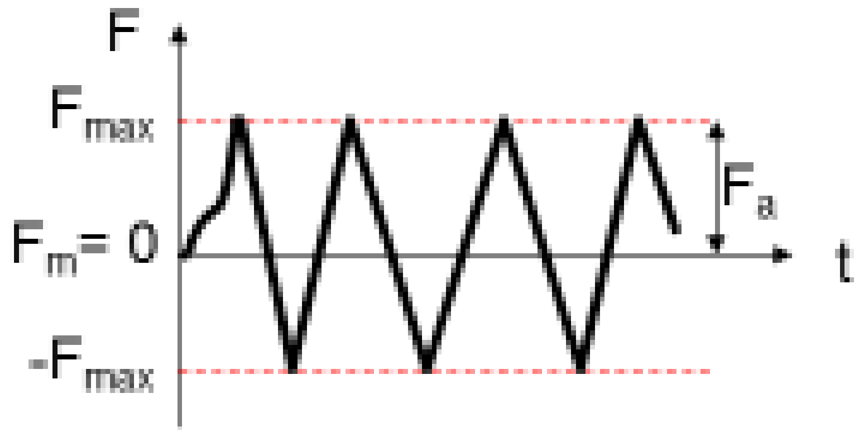
$F_m = \frac{1}{2}(F_{max} + F_{min})$ $\quad\quad$ $F_a = \frac{1}{2}(F_{max} - F_{min})$

$\sigma_m = F_m/A$ $\quad\quad$ $\sigma_a = F_a/A$

Axial Fatigue Load & Stresses



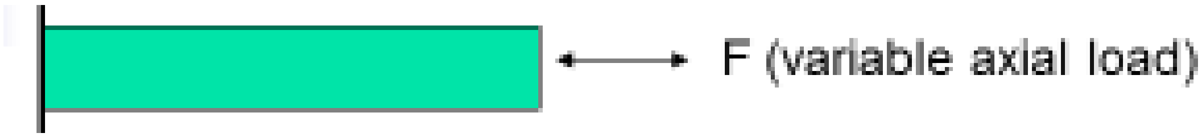
Reversed Load/Stress: $F \sim (F_{max}, -F_{max})$



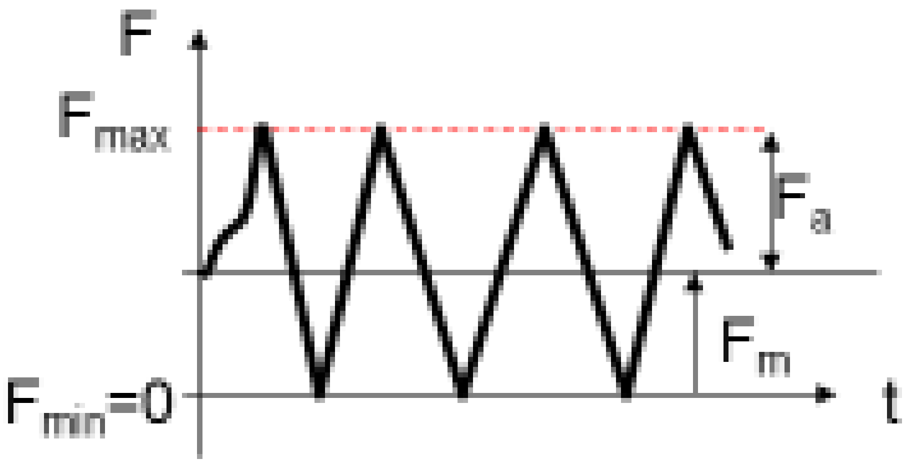
$F_m = ½(F_{max} + F_{min}) = 0$ $\qquad$ $F_a = ½(F_{max} - F_{min}) = F_{max}$

$\sigma_m = F_m/A = 0$ $\qquad$ $\sigma_a = F_a/A = F_m/A$

Axial Fatigue Load & Stresses



Repeated Load/Stress: F ~ (F_{max}, 0)



$F_m = ½(F_{max} + F_{min}) = ½\, F_{max}$ $\qquad$ $F_a = ½(F_{max} - F_{min}) = ½\, F_{max}$

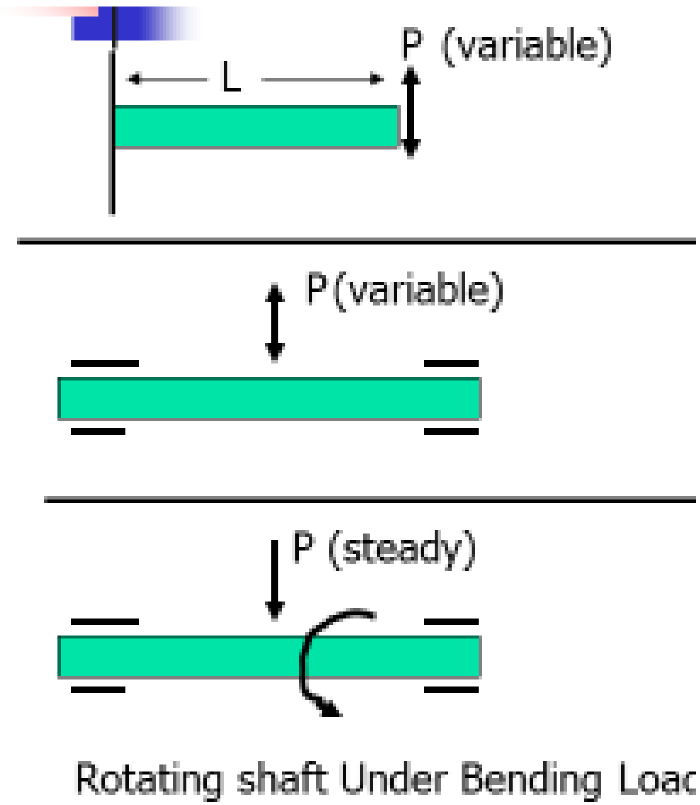
$\sigma_m = F_m/A$ $\qquad$ $\sigma_a = F_a/A$

NOTE:

$F_m = F_a = ½\, F_{max}$

$\sigma_m = \sigma_a = F_m/A$

Bending Fatigue Load & Stresses



$P_m = ½(P_{max} + P_{min})$ $P_a = ½(P_{max} - P_{min})$

$M_m = P_m L$ $\sigma_m = M_m/Z$

$M_a = P_a L$ $\sigma_a = M_a/Z$

Draw SFD and BMD using P_m --> M_m
Draw SFD and BMD using P_a --> M_a

Then $\sigma_m = M_m/Z$
$\sigma_a = M_a/Z$

Reversed Bending
$\sigma_m = 0$
$\sigma_a = M_a/Z = M/Z$

Torsion Fatigue Load & Stresses

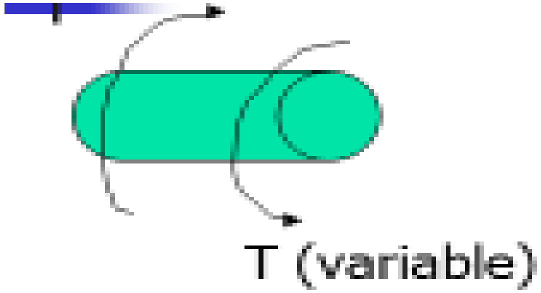
Find T_m and T_a

$T_m = ½(T_{max} + T_{min})$ $T_a = ½(T_{max} - T_{min})$

AND

$\tau_m = T_m/Z_p$

$\tau_a = T_a/Z_p$



Combined Biaxial Fluctuating Stresses

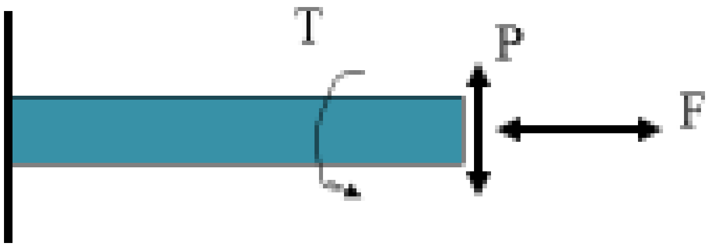
1. Find: (P_m, P_a) and (F_m, F_a)

$$P_m = ½(P_{max} + P_{min}) \qquad P_a = ½(P_{max} - P_{min})$$
$$F_m = ½(F_{max} + F_{min}) \qquad F_a = ½(F_{max} - F_{min})$$

Calculate:
$$M_m = P_m L \quad \text{and} \quad M_a = P_a L$$
$$\sigma_m = F_m/A + M_m/Z$$
$$\sigma_a = F_a/A + M_a/Z$$

2. Find: (T_m, T_a)

$$T_m = ½(T_{max} + T_{min}) \qquad T_a = ½(T_{max} - T_{min})$$



Calculate:
$$\tau_m = T_m/Z_p$$
$$\tau_a = T_a/Z_p$$

3. Calculate equivalent von-Mises Normal Stresses:

$$\sigma_m' = (\sigma_m^2 + 3\tau_m^2)^{½} \qquad \sigma_a' = (\sigma_a^2 + 3\tau_a^2)^{½}$$

Application of K_f for Fluctuating Stresses

- For fluctuating loads at points with stress concentration, the best approach is to design to avoid all localized plastic strain.
- In this case, K_f should be applied to both alternating and midrange stress components.
- When localized strain does occur, some methods (e.g. *nominal mean stress* method and *residual stress* method) recommend only applying K_f to the alternating stress.
- The *Dowling method* recommends applying K_f to the alternating stress and K_{fm} to the mid-range stress, where K_{fm} is

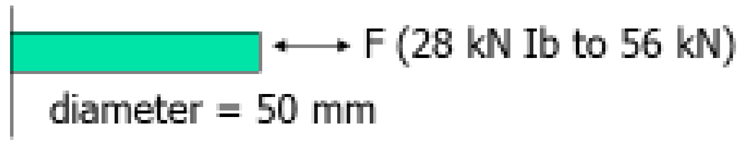
$$K_{fm} = K_f \qquad K_f|\sigma_{\max,o}| < S_y$$

$$K_{fm} = \frac{S_y - K_f\sigma_{ao}}{|\sigma_{mo}|} \qquad K_f|\sigma_{\max,o}| > S_y$$

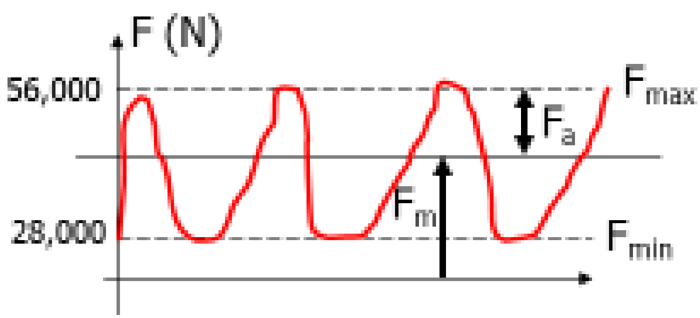
$$K_{fm} = 0 \qquad K_f|\sigma_{\max,o} - \sigma_{\min,o}| > 2S_y$$

Example 4-10

For the situation shown, find the mean and the alternating stresses in the member:



Solution



$F_m = ½(F_{max} + F_{min})$ $F_a = ½(F_{max} - F_{min})$

$F_m = ½(56,000 + 28,000) = 42,000$ N

$F_a = ½(56,000 – 28,000) = 14,000$ N

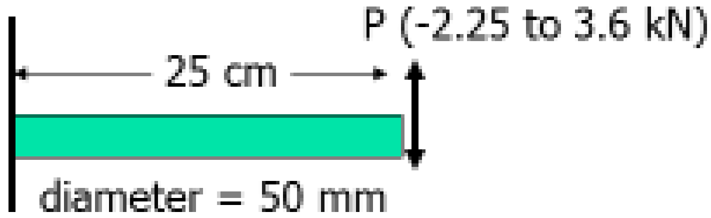
Direct Axial Load

$\sigma_m = 4F_m/\pi d^2$
$= 4\,(42,000)/\pi(50)^2$
$= 21$ N/mm² (MPa)

$\sigma_a = 4F_a/\pi d^2$
$= 4\,(14,000)/\pi(50)^2$
$= 7$ N/mm² (MPa)

Example 4-11

For the situation shown, find the mean and the alternating stresses in the member:



Bending Stress

$M_m = 250P_m \quad M_a = 10P_a$

$\sigma_m = 32M_m/\pi d^3$

$= 32\,(250P_m)\,/\pi(50)^3$

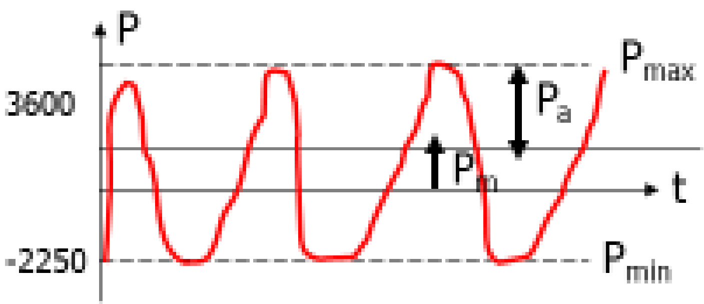
$= 14$ N/mm^2 (MPa)

$\sigma_a = 32M_a/\pi d^3$

$= 32\,(250P_a)\,/\pi(50)^3$

$= 60$ N/mm^2 (MPa)

Solution



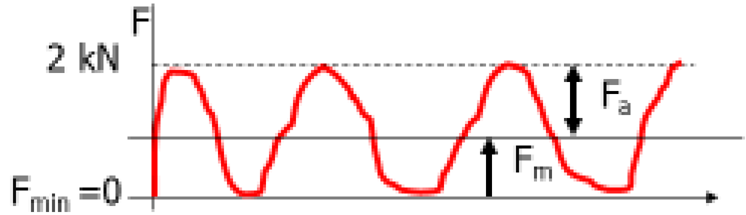
$F_a = \frac{1}{2}(P_{max} - P_{min}) \quad P_a = \frac{1}{2}(P_{max} - P_{min})$

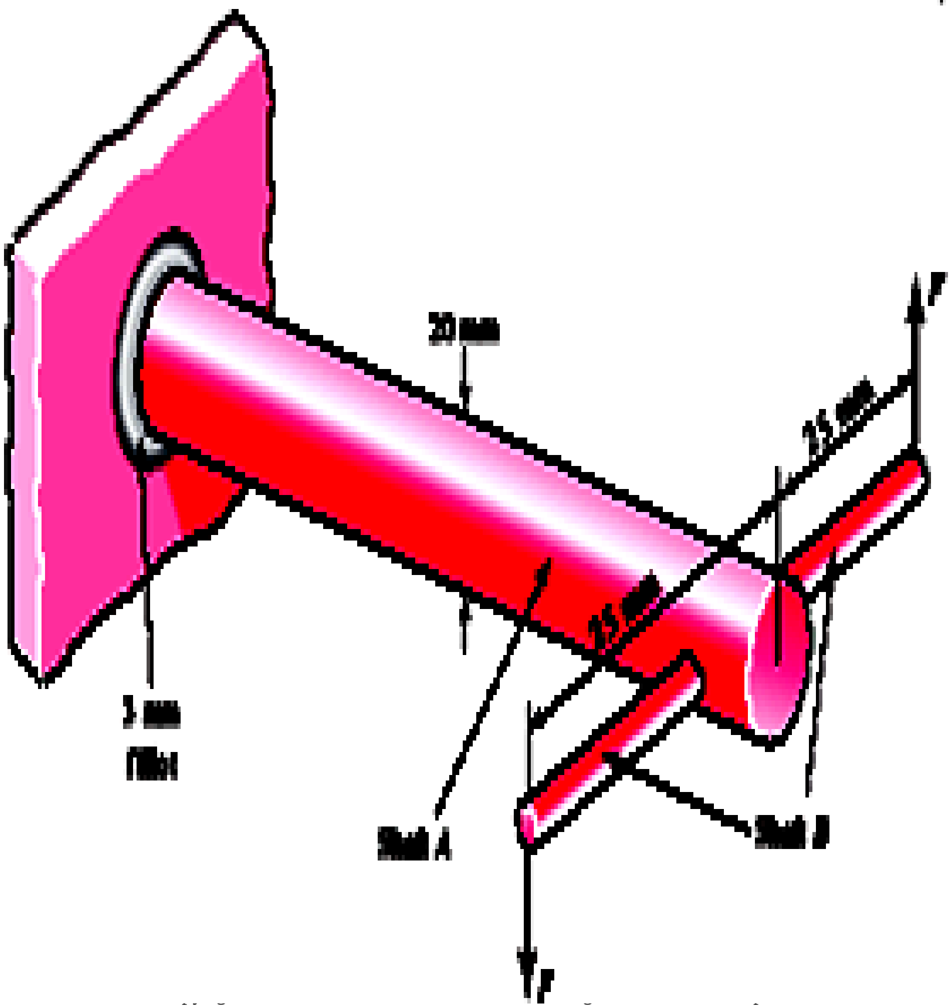
$P_m = \frac{1}{2}(3600 + (-2250)) = 675$ N

$P_a = \frac{1}{2}(3600 - (-2250)) = 2925$ N

Example 4-12

The force F is 2 kN (repeated). Find the mean and alternating stresses in the Member A.





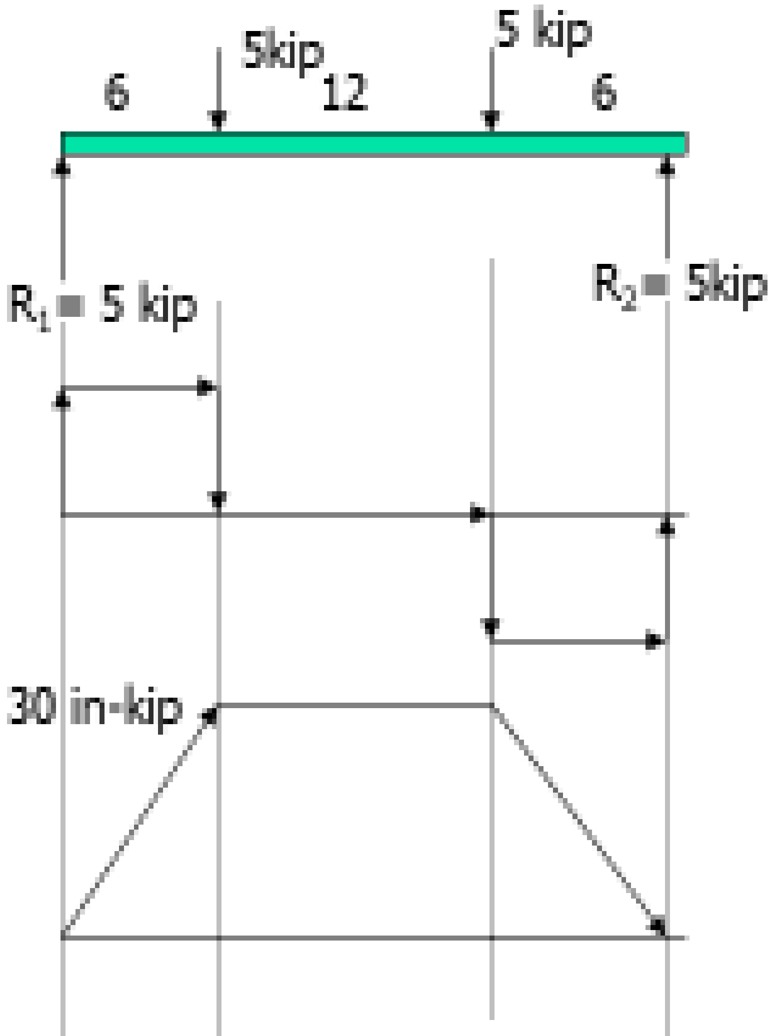
$$F_m = F_a = ½(F_{max} - F_{min})$$
$$F_m = ½(2 + 0) = 1 \text{ kN}$$
$$F_a = ½(2 - 0) = 1 \text{ kN}$$

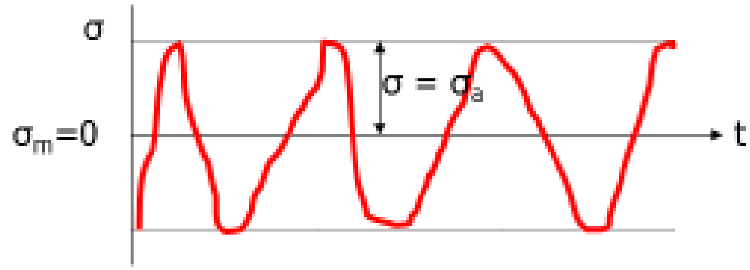
$$T_m = 50F_m = 50 \times 10^3 \text{ N-mm}$$
$$T_a = 50F_a = 50 \times 10^3 \text{ N-mm}$$

$$\tau_m = 16\, T_m / \pi d^3 = 32 \text{ MPa}$$
$$\tau_a = 16\, T_a / \pi d^3 = 32 \text{ MPa}$$

Example 4-13

The rotating shaft is under bending loads as shown. Find the mean and alternating stresses in the shaft.





From the BMD, M = 30 x 10^3 in-Ib

Rotating shaft → Reversed Bending

$\sigma_m = 0$

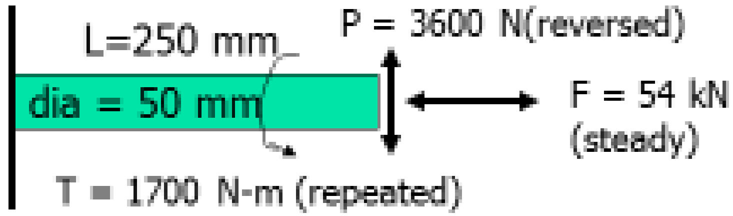
$\sigma_a = 32M_a/\pi d^3$

$= 32 \times 30 \times 10^3/\pi d^3$ psi

$= 306/d^3$ ksi

Example 4-14

Find the equivalent von-Mises mean and alternate stresses.



1. Find: $P_m = 0 \quad P_a = 3600$ N

$F_m = 54{,}000$ N $\quad F_a = 0$

Calculate: $M_m = P_m L = 0$

$M_a = P_a L = 900\text{x}10^3$ N-mm

Calculate: $\sigma_m = F_m/A + M_m/Z$

$= 4F_m/\pi d^2 + 0$

$= 28$ N/mm^2

Calculate $\sigma_a = F_a/A + M_a/Z$

$= 0 + 32M_a/\pi d^3 = 73$ N/mm^2

2. Find: $T_m = 850\text{ x}10^3$ N-mm

$T_a = 850\text{ x}10^3$ N-mm

Calculate: $\tau_m = T_m/Z$

$= 16T_m/\pi d^3$

$= 35$ MPa

$\tau_a = T_a/Z$

$= 16T_a/\pi d^3$

$= 35$ MPa

3. Calculate equivalent von-Mises Normal Stresses:

$\sigma_m' = (\sigma_m^2 + 3\tau_m^2)^{½}$

$= 67$ Mpa

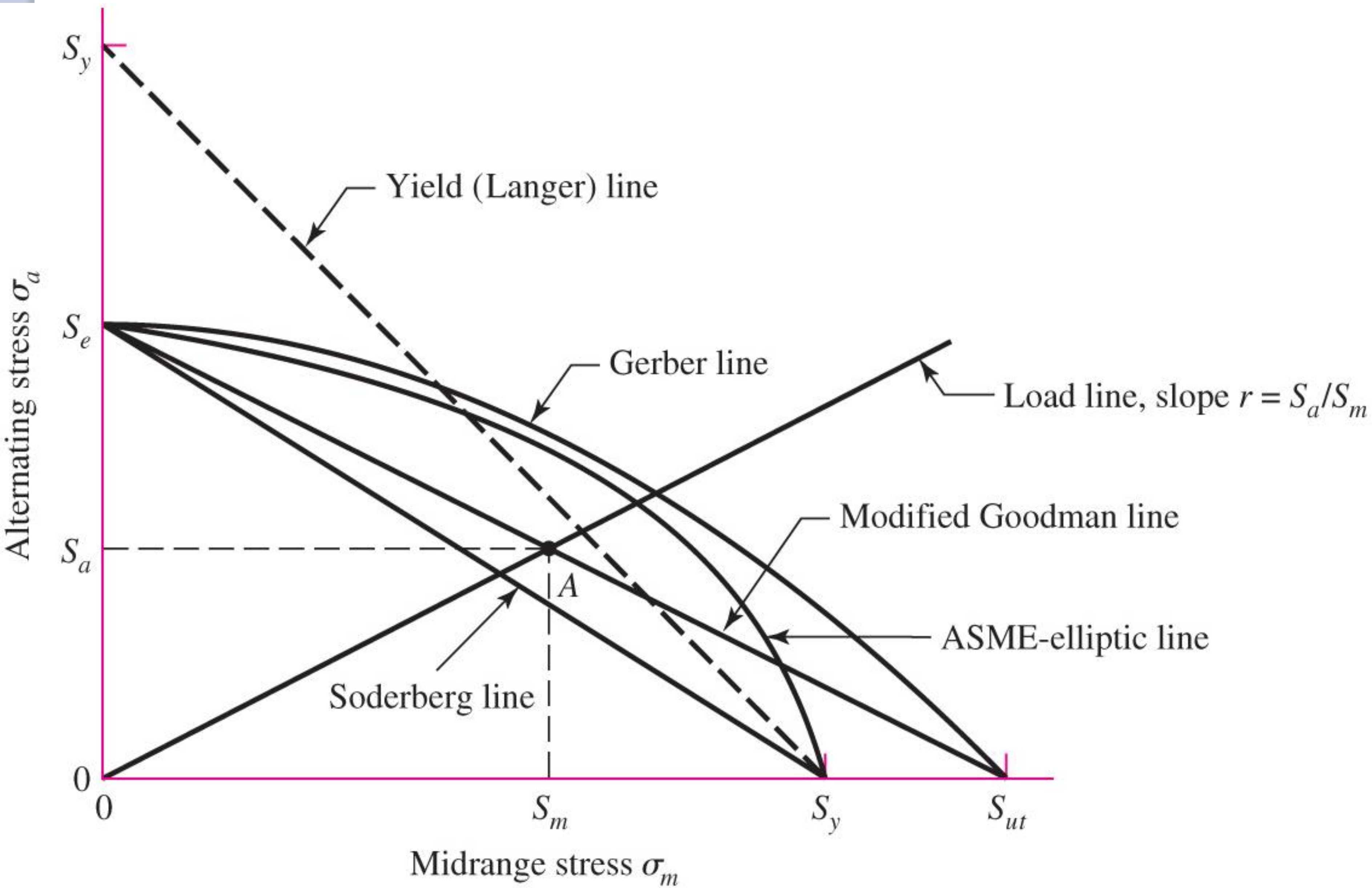
$\sigma_a' = (\sigma_a^2 + 3\tau_a^2)^{½}$

$= 95$ MPa

Failure Criteria

- Five commonly used failure criteria are
  1. Gerber passes through the data
  2. ASME-elliptic passes through data and incorporates rough yielding check
  3. Modified Goodman is linear, so simple to use for design. It is more conservative than Gerber.
  4. Soderberg provides a very conservative single check of both fatigue and yielding
  5. Langer line represents standard yield check.
- It is equivalent to comparing maximum stress to yield strength.

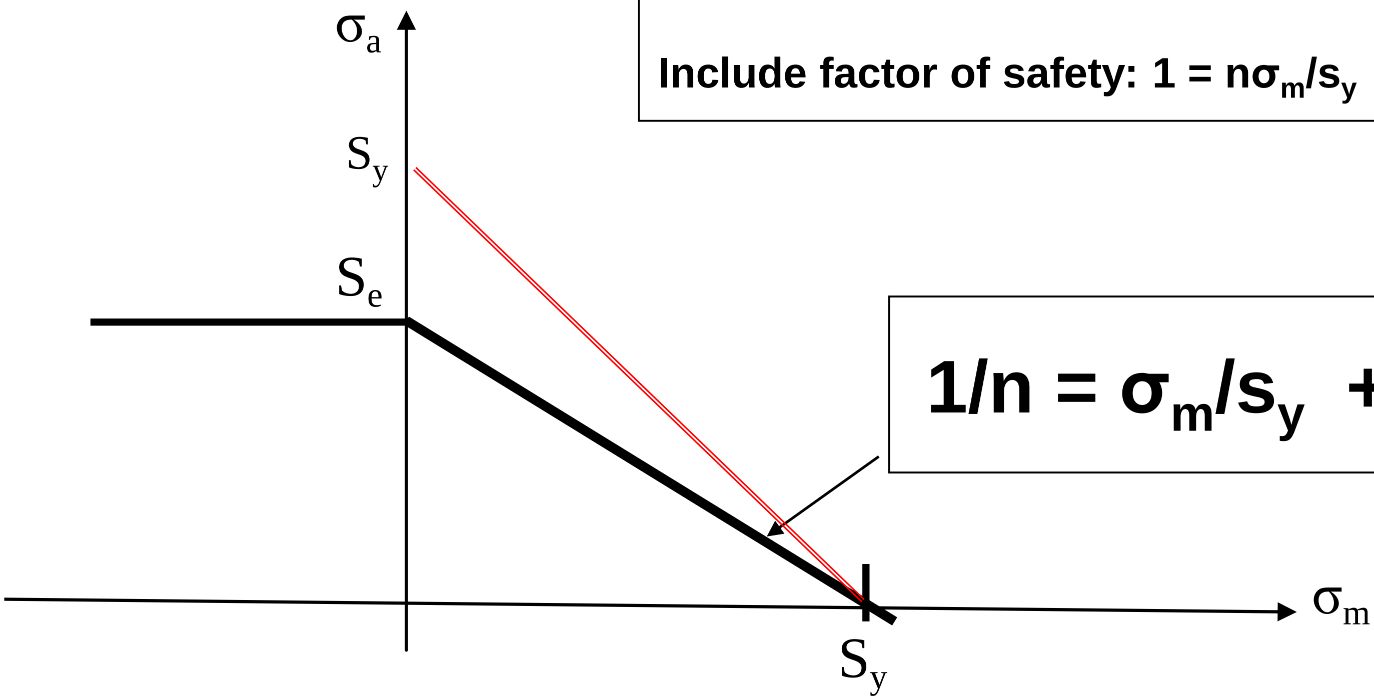
Failure Criteria



Soderberg Criterion

Equation of the Line: $1 = \sigma_m/s_y + \sigma_a/s_e$

Include factor of safety: $1 = n\sigma_m/s_y + n\sigma_a/s_e$

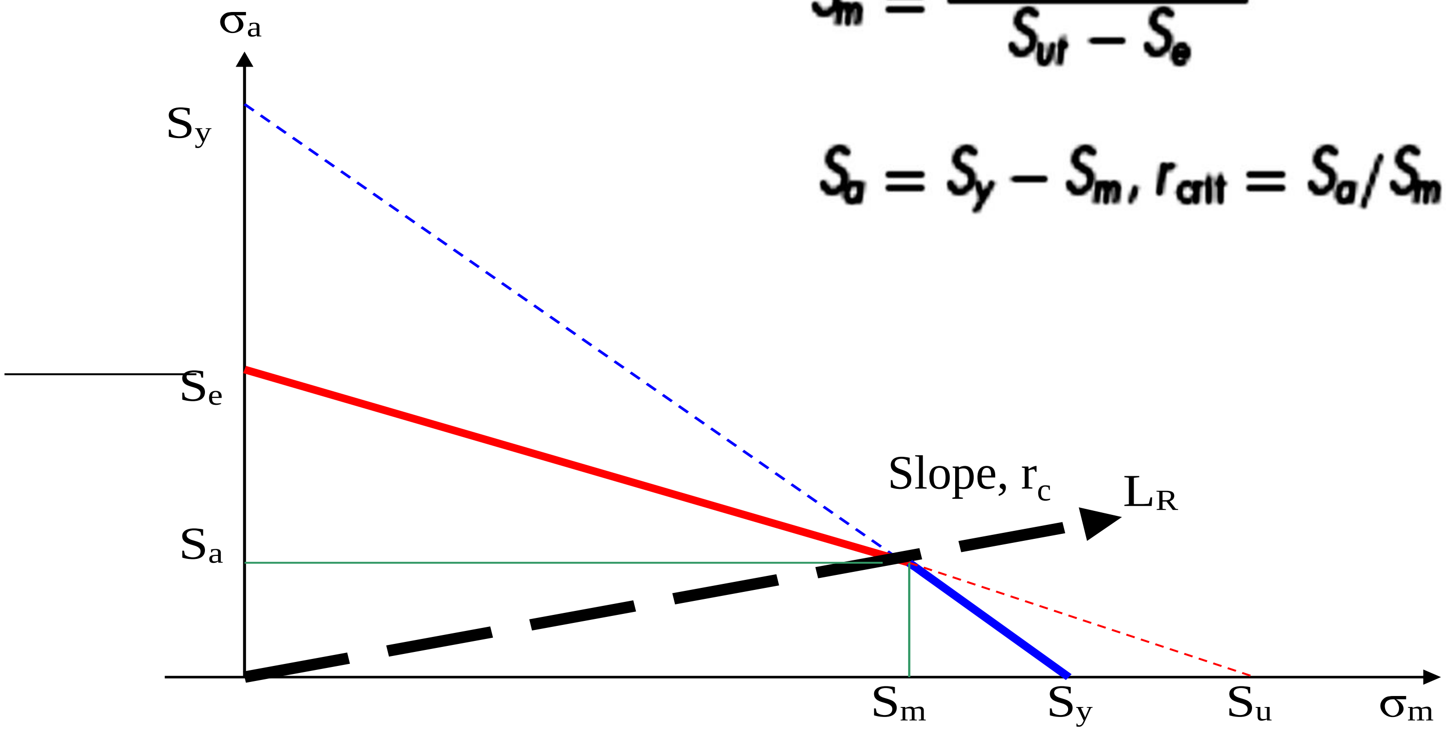


$$1/n = \sigma_m/s_y + \sigma_a/s_e$$

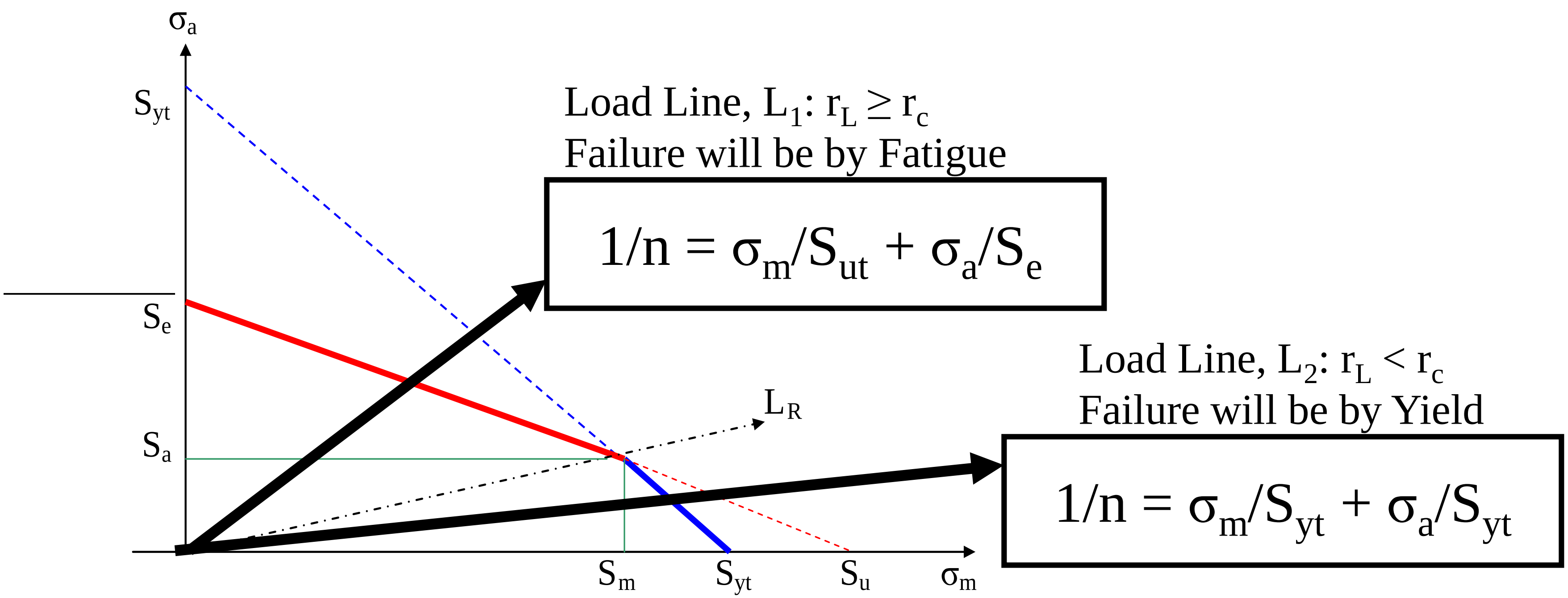
Modified Goodman Criterion

$$S_m = \frac{(S_y - S_e)\, S_{ut}}{S_{ut} - S_e}$$

$$S_a = S_y - S_m,\ r_{crit} = S_a / S_m$$

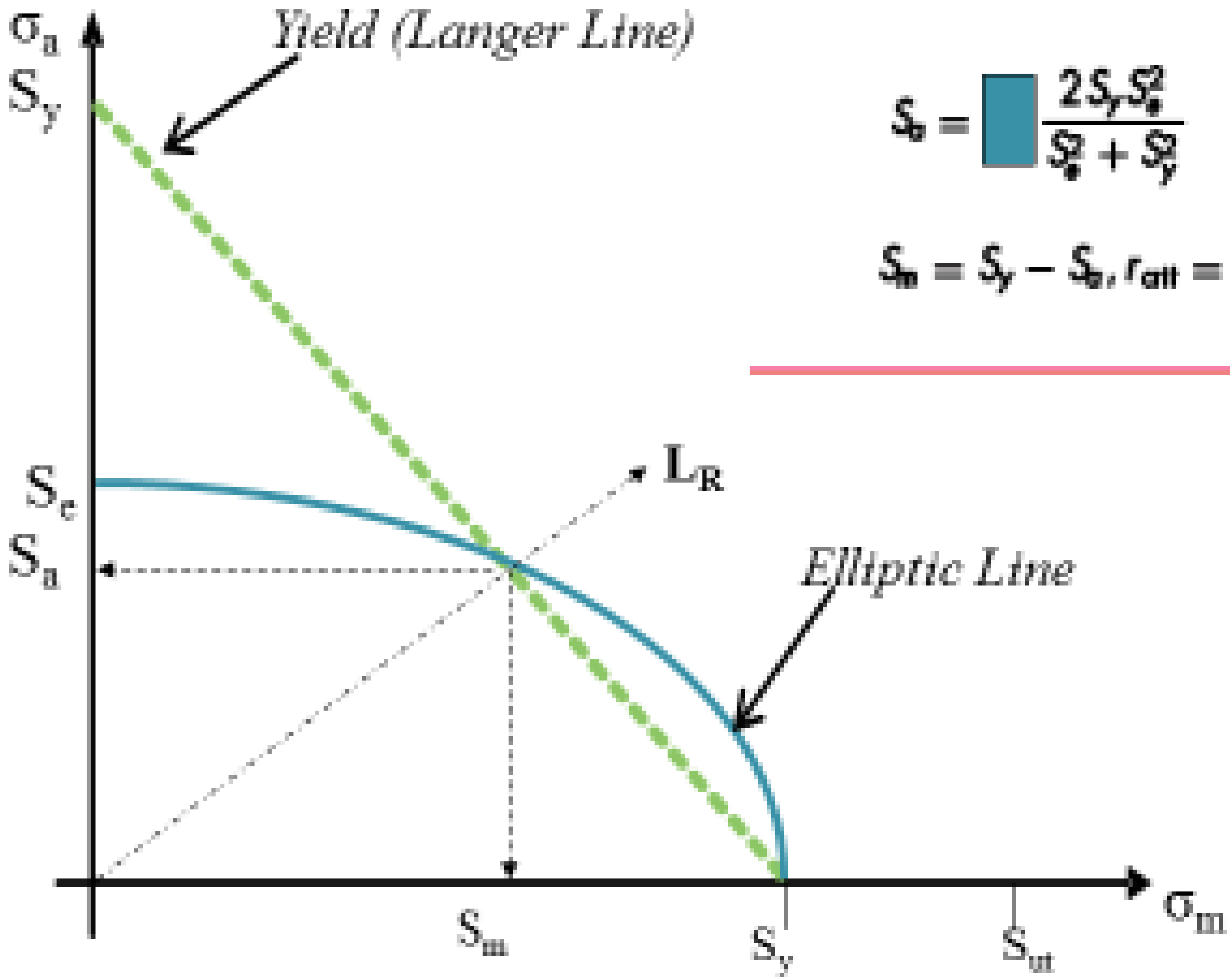


Modified Goodman Criterion



NOTE: Slope of load Line, $r_L = \sigma_a / \sigma_m$

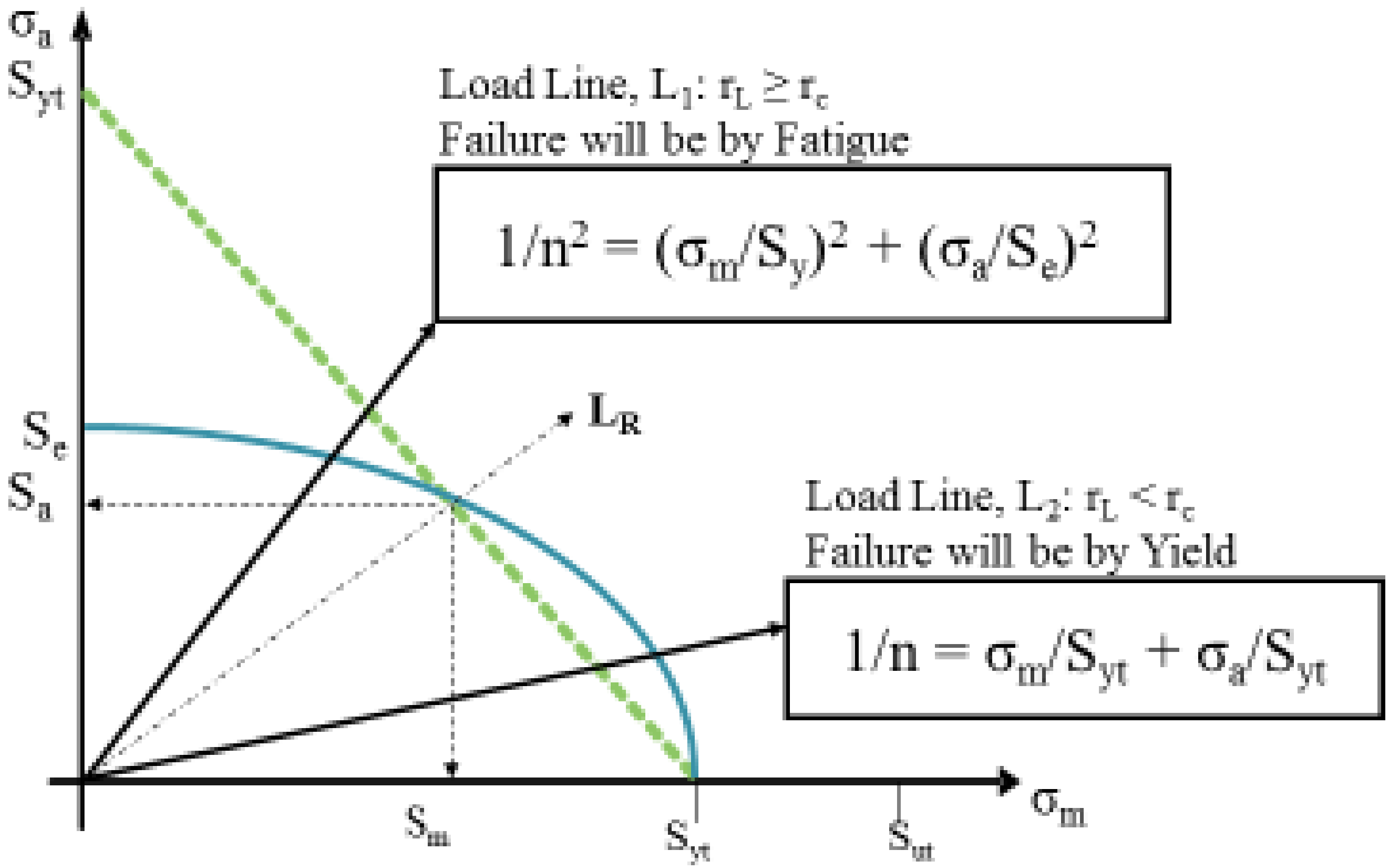
Elliptic Criterion



$$S_a = \frac{2 S_y S_e^2}{S_e^2 + S_y^2}$$

$$S_m = S_y - S_a,\ r_{crit} = S_a / S_m$$

Elliptic Criterion



Equations for the Failure Criteria

- Intersecting a constant slope load line with each failure criteria produces design equations
- *n* is the design factor or factor of safety for infinite fatigue life

$$\textbf{Soderberg} \quad \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n} \qquad (6\text{–}45)$$

$$\textbf{mod-Goodman} \quad \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \qquad (6\text{–}46)$$

$$\textbf{Gerber} \quad \frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1 \qquad (6\text{–}47)$$

$$\textbf{ASME-elliptic} \quad \left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1 \qquad (6\text{–}48)$$

Modification of Design Equations for Definite Life (N)

Soderberg/Fatigue

$$\boxed{1/n = \sigma_m/s_y + \sigma_a/\mathbf{s_f}}$$

Goodman/Fatigue

$$\boxed{1/n = \sigma_m/S_{ut} + \sigma_a/\mathbf{S_f}}$$

ASME/Fatigue

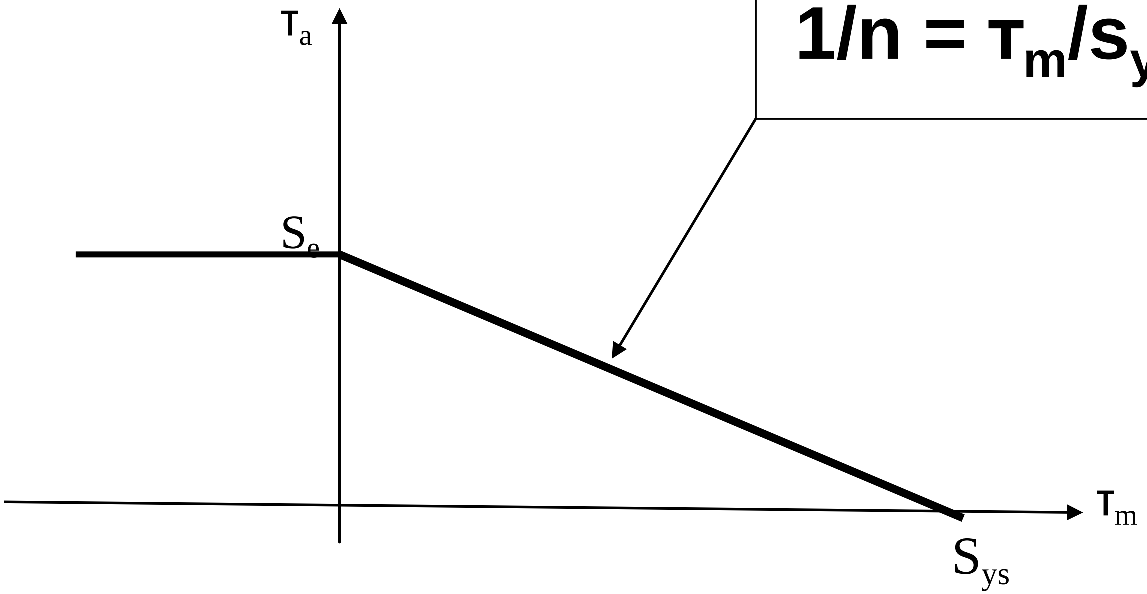
$$\boxed{1/n^2 = (\sigma_m/S_y)^2 + (\sigma_a/\mathbf{S_f})^2}$$

NOTE: $S_f = aN^b$

Modification of Soderberg -Shear Stress

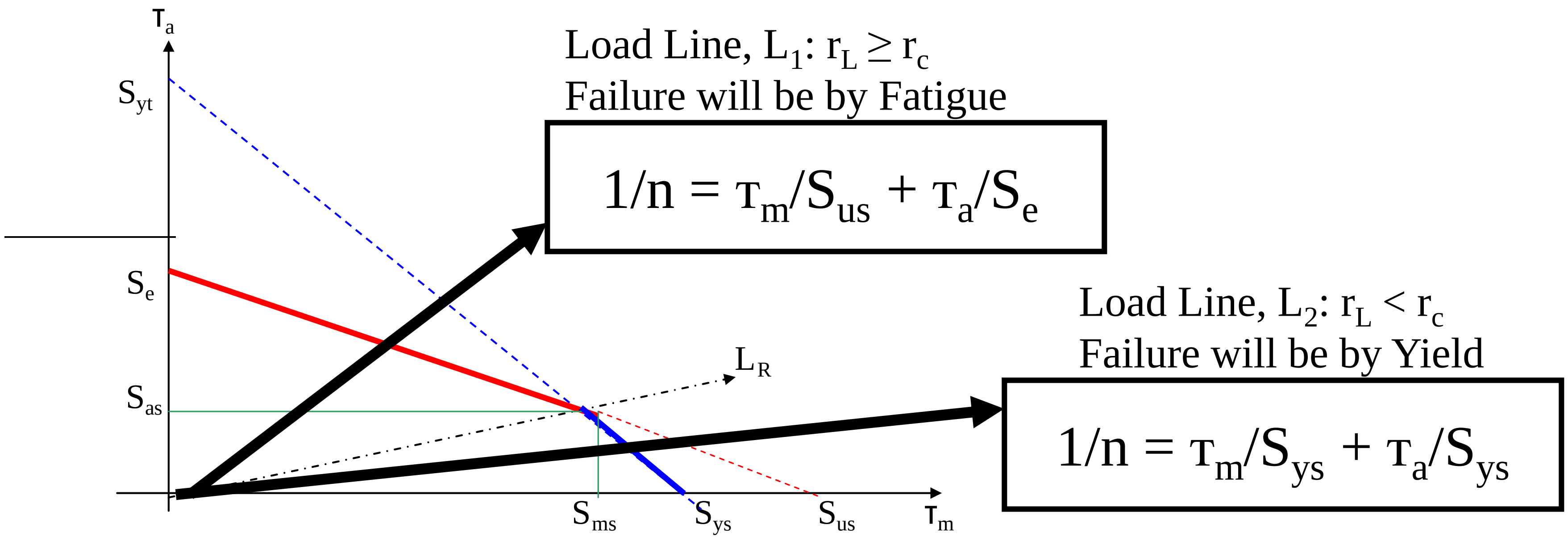
Axes → τ_a versus τ_m

$$1/n = \tau_m/s_{ys} + \tau_a/s_e$$



$S_{ys} = 0.577 S_y$

Modification of Goodman - Shear Stress



NOTE: Slope of load Line, $r_L = \tau_a / \tau_m$

Use S_{us} and S_{ys} to find S_{as} and S_{ms}

Slope of reference line, $r_c = S_{as} / S_{ms}$

$\mathbf{S_{ys} = 0.577 S_y}$

$\mathbf{S_{us} = 0.8 S_{ut}}$

Notched Parts – Corrected Design Equations

Soderberg/Fatigue → $\boxed{1/n = \sigma_m/s_y + K_f\sigma_a/s_e}$

NOTE:
$S_f = aN^b$
Replaces S_e

Goodman/Fatigue → $\boxed{1/n = \sigma_m/S_{ut} + K_f\,\sigma_a/S_e}$

ASME/Fatigue → $\boxed{1/n^2 = (\sigma_m/S_y)^2 + (K_f\,\sigma_a/S_e)^2}$

Langer/Yield → $\boxed{1/n = \sigma_m/S_y + K_f\,\sigma_a/S_y}$

Summarizing Tables for Failure Criteria

- Tables summarize the pertinent equations for Modified Goodman, Gerber, ASME-elliptic, and Langer failure criteria
- The first row gives fatigue criterion
- The second row gives yield criterion
- The third row gives the intersection of static and fatigue criteria
- The fourth row gives the equation for fatigue factor of safety
- The first column gives the intersecting equations
- The second column gives the coordinates of the intersection

Summarizing Tables for Failure Criteria

1 UNS No.	2 SAE and/or AISI No.	3 Processing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
G10450	1045	HR	570 (82)	310 (45)	16	40	163
		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197
G10600	1060	HR	680 (98)	370 (54)	12	30	201
G10800	1080	HR	770 (112)	420 (61.5)	10	25	229
G10950	1095	HR	830 (120)	460 (66)	10	25	248

Summarizing Table for Modified Goodman

Intersecting Equations	Intersection Coordinates
$\dfrac{S_a}{S_e} + \dfrac{S_m}{S_{ut}} = 1$	$S_a = \dfrac{r S_e S_{ut}}{r S_{ut} + S_e}$
Load line $r = \dfrac{S_a}{S_m}$	$S_m = \dfrac{S_a}{r}$
$\dfrac{S_a}{S_y} + \dfrac{S_m}{S_y} = 1$	$S_a = \dfrac{r S_y}{1 + r}$
Load line $r = \dfrac{S_a}{S_m}$	$S_m = \dfrac{S_y}{1 + r}$
$\dfrac{S_a}{S_e} + \dfrac{S_m}{S_{ut}} = 1$	$S_m = \dfrac{(S_y - S_e)\, S_{ut}}{S_{ut} - S_e}$
$\dfrac{S_a}{S_y} + \dfrac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{\text{crit}} = S_a / S_m$

Fatigue factor of safety

$$n_f = \frac{1}{\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}}}$$

Summarizing Table for Gerber

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_a = \frac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{rS_{ut}}\right)^2}\,\right]$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{rS_y}{1+r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1+r}$
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_m = \frac{S_{ut}^2}{2S_e}\left[1 - \sqrt{1 + \left(\frac{2S_e}{S_{ut}}\right)^2\left(1 - \frac{S_y}{S_e}\right)}\,\right]$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{\text{crit}} = S_a/S_m$

Fatigue factor of safety

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \qquad \sigma_m > 0$$

Summarizing Table for ASME-Elliptic

Intersecting Equations	Intersection Coordinates
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$ Load line $r = S_a/S_m$	$S_a = \sqrt{\frac{r^2 S_e^2 S_y^2}{S_e^2 + r^2 S_y^2}}$ $S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$ Load line $r = S_a/S_m$	$S_a = \frac{r S_y}{1 + r}$ $S_m = \frac{S_y}{1 + r}$
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$ $\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = 0, \ \frac{2 S_y S_e^2}{S_e^2 + S_y^2}$ $S_m = S_y - S_a, r_{\text{crit}} = S_a/S_m$

Fatigue factor of safety

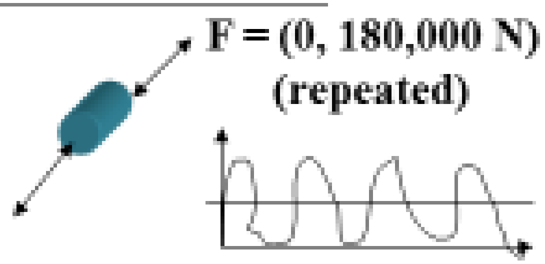
$$n_f = \sqrt{\frac{1}{(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2}}$$

Example 4-15

A connecting rod made of AISI 8650 steel, oil quenched and tempered is subjected to a repeated axial load of 180 kN. The yield strength of the material is 900 MPa, and the tensile strength is 1100 MPa. The following correction factors are applied to the design: Load factor = 0.7; size factor = 0.85; finish factor = 0.8. Determine the required safe diameter of the rod for indefinite life. Use:

a. Soderberg criterion
b. Modified Goodman criterion
c. Elliptic Criterion

Load & Stress Analysis



$F_m = F_{max}/2 = 90{,}000 \text{ N}$

$F_a = F_m \quad = 90{,}000 \text{ N}$

$\sigma_m = 4F_m/\pi d^2 = 115 \times 10^3/d^2$

$\sigma_a = \sigma_m \quad = 115 \times 10^3/d^2 \text{ MPa}$

Slope of Load Line, $r_L = \sigma_a/\sigma_m = 1$

Example 4-15 (Continued)

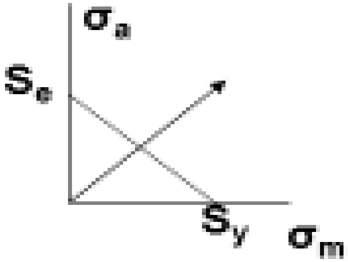
Material Properties

S_y = 900 MPa S_{ut} = 1100 MPa

$$S_e = K_aK_bK_cK_dK_e(0.5S_{ut})$$
$$= (0.8)(0.85)(0.7)(0.5\text{x}1100)$$
$$= 262 \text{ MPa}$$

a. <u>Soderberg Criterion</u>

$$1/n = \sigma_m/s_y + \sigma_a/ s_e$$



Design Equation:

$$1/n = 115 \text{ x } 10^3/900d^2 + 115 \text{ x } 10^3/262d^2$$

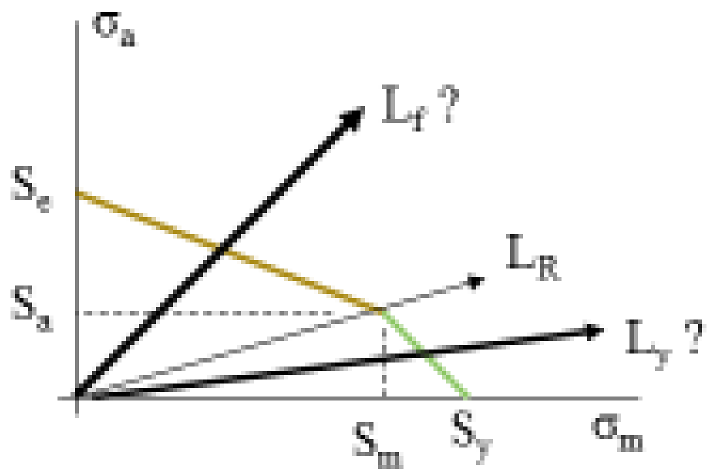
Specify n = 2

Solve d = 33.67 mm.

Use d = 34 mm

Example 4-15 (Continued)

b. Modified Goodman



$S_m = (S_y - S_e)S_{ut} / (S_{ut} - S_e)$
$= 837$ MPa

$S_a = S_y - S_m$
$= 63$ MPa

$r_c = S_a / S_m = 0.075$

Note $r_L = 1.0 > r_c$
Load Line is L_f

Thus Failure will be by fatigue:

$$1/n = \sigma_m/s_{ut} + \sigma_a/s_e$$

Design Equation:

$1/n = 115 \times 10^3/1100d^2 + 115 \times 10^3/262d^2$

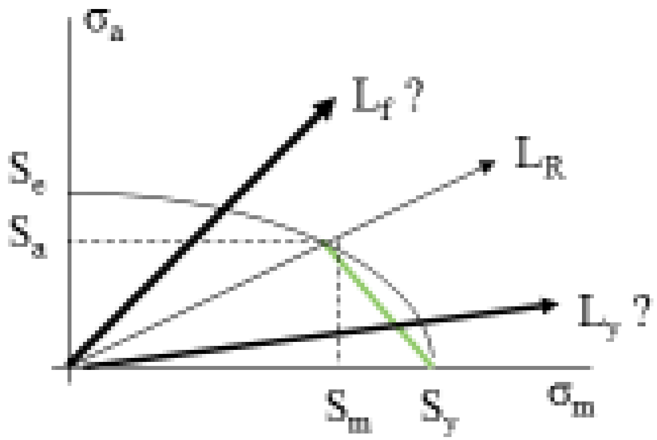
Specify $n = 2$

Solve $d = 32.97$ mm

Use $d = 33$ mm

Example 4-15 (Continued)

c. Elliptic Model



$$S_a = 0,\ \frac{2S_yS_e^2}{S_e^2 + S_y^2}$$

$$S_m = S_y - S_a,\ r_{crit} = S_a/S_m$$

$S_a = 141$ MPa
$S_m = 759$ MPa

$r_c = S_a / S_m = 0.186$
Note $r_L = 1.0 > r_c$
Failure Line = L_f

Thus Failure will be by fatigue:
$1/n^2 = (\sigma_m/s_y)^2 + (\sigma_a/s_e)^2$

Design Equation:
$1/n^2 = (115 \times 10^3/900d^2)^2 + (115 \times 10^3/262d^2)^2$
Specify $n = 2$
Solve $d = 30.24$ mm
Use d = 31 mm

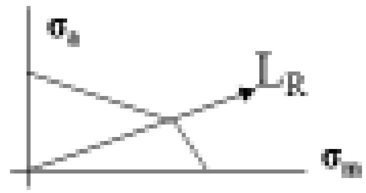
Example 4-16

A 2 inch steel bar undergoes cyclic loading such that σ_{max} = 420 MPa and σ_{min} = 140 MPa. For the material, S_{ut} = 560 MPa and S_y = 450 MPa, fully corrected endurance limit is 280 MPa and f = 0.9. Using Modified Goodman Criterion, find:

a. The design factor for infinite life
b. The design factor for 10^5 life

σ_{max} = 420 MPa
σ_{min} = 140 Mpa
$\sigma_m = (\sigma_{max} + \sigma_{min})/2$
= 280 MPa
$\sigma_a = (\sigma_{max} - \sigma_{min})/2$
= 140 MPa
Load Slope, $r_L = \sigma_a / \sigma_m$
= 0.5

a. Modified Goodman (indefinite life or 10^6 ~):



Calculate S_m = 340 MPa
S_a = 110 MPa
Slope $r_c = 0.3 < r_L$

Failure by Fatigue
$1/n = \sigma_m/S_{ut} + \sigma_a/S_e$
= 280/560 + 140/280
Solve, n = 1 **(verge of failure)**

Example 4-16 (Continued)

Calculation of S_{10^5}

$S_{10^3} = fS_{ut} = 0.9\ S_{ut} = 504$ MPa

$S_f = aN^b$

$a = (S_{10^3})^2 / S_e = 504^2/280 = 907$ MPa

$b = (-1/3) \log [S_{10^3}/ S_e] = -0.085$

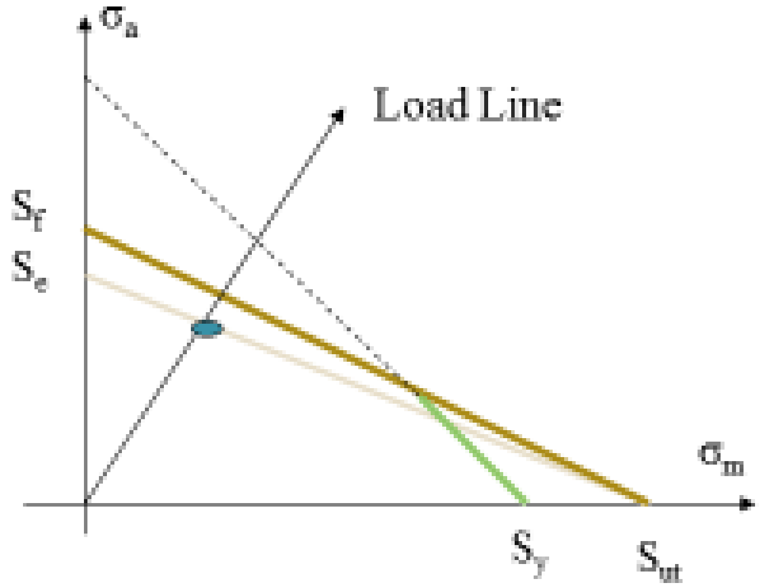
$S_f = 907(10^5)^{-0.085} = 340$ MPa

Modified Goodman:

$1/n = \sigma_m/S_{ut} + \sigma_a/S_f$

$= 280/560 + 140/340$

Solve n = 1.1 (barely safe)



Example 4-17

A commercially ground shaft made from steel transmits a fluctuating torque of T = 1000 ± 250 N-m. The material has an ultimate tensile Strength of 1.2 GPa and a yield strength of 1.0 GPa. Find the factor of safety at a section where the diameter is 30 mm. Use ASME Elliptical Theory.

Stress Analysis

Mean Torque, $T_m = 1000$ N-m

Alternating Torque, $T_a = 250$ N-m

Mean Shear stress, $\tau_m = 16T_m/\pi d^3$
$= 187$ MPa

Alt. Shear stress, $\tau_a = 16T_a/\pi d^3$
$= 47$ MPa

Sloe of Load Line, $r_L = \tau_a/\tau_m = 0.25$

Material Properties

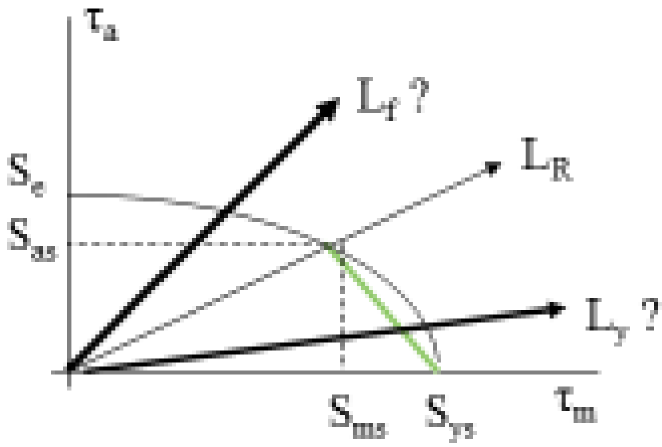
$S_y = 1000$ MPa $\quad S_{ut} = 1200$ MPa

$S_{ys} = 0.577\ S_y = 577$ MPa

$S_{us} = 0.8\ S_{ut} = 960$ MPa

$S_e = K_aK_bK_cK_dK_e(0.5S_{ut})$
$= (0.865)(0.86)(0.58)(1)(1)(0.5\text{x}1200)$
$= 258$ MPa

Example 4-17 (Continued)



$S_{as} = 2S_{ys}S_e^2/(S_e^2 + S_{ys}^2)$

$S_{ms} = S_{ys} - S_{as}$

$S_{as} = 192$ MPa
$S_{ms} = 385$ MPa

$r_c = S_{as} / S_{ms} = 0.5$

Note $r_L = 0.25 < r_c = 0.5$
Load Line = L_y

Thus Failure will be by YIELD:

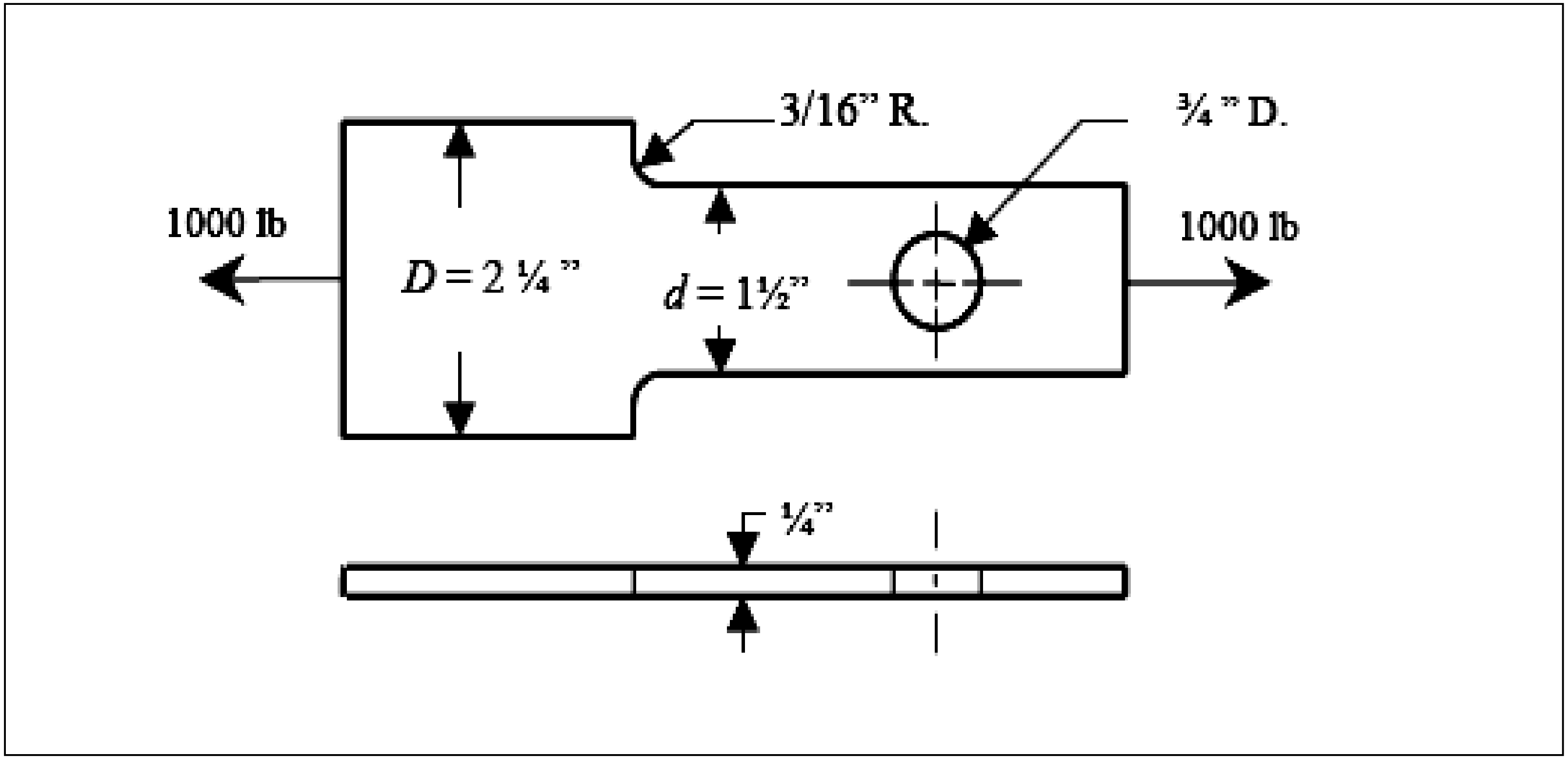
$1/n = (\tau_m/s_{ys}) + (\tau_a/s_{YS})$

$1/n = (187/577) + (47/577)$

Solve $n = 2.46$

Example 4–18

The bar is made of steel with an ultimate tensile strength of 80 ksi and yield strength of 50 ksi. Endurance limit is 19 ksi for the design. The 1000-Ib axial load is completely reversed. Find the factor of safety at the shoulder and the hole against **fatigue failure**.



Example 4–18 (Shoulder)

Fatigue Stress Concentration Factor

$D/d = 2.25/1.5 = 1.5$

$r/d = 0.1875/1.5 = 0.125$

$K_t = 1.9$ (from chart)

Notch sensitivity factor, $q = 0.86$ (chart)

$K_f = 1 + q(K_t - 1) = 1.77$

Stress Calculation

Axial Stress, $\sigma = F/A$

Completely reversed, $\sigma_m = 0$

$\sigma = \sigma_a = F/A = F/dt$

$= 1000/(1.5 \text{ x } 0.25)$

$= 2{,}667 \text{ Ib/in}^2$

$= 2.667$ ksi

Modified Goodman

$1/n = \sigma_m/S_{ut} + K_f\, \sigma_a/\mathbf{S_e}$

$\mathbf{= 0 + 1.77x2.667/19}$

Solve **n = 4.02**

Example 4–18 (Hole)

Fatigue Stress Concentration Factor

w = 1.5 d = ¾ in t= 0.25 in

d/w = 0.5

K_t = 2.19 (from chart)

Notch sensitivity factor, q = 0.86 (chart)

$K_f = 1 + q(K_t - 1) = 2.023$

Modified Goodman

$1/n = \sigma_m/S_{ut} + K_f\, \sigma_a/\mathbf{S_e}$

$\mathbf{= 0 + 2.023x5.333/19}$

Solve **n = 1.76**

Stress Calculation

Axial Stress, σ = F/A

Completely reversed, $\sigma_m = 0$

$\sigma = \sigma_a = F/A = F/(w\text{-}d)t$

$= 1000/(1.5\text{-}0.75)\text{x}0.25$

= 5,333 Ib/in^2

= 5.333 ksi

Note:

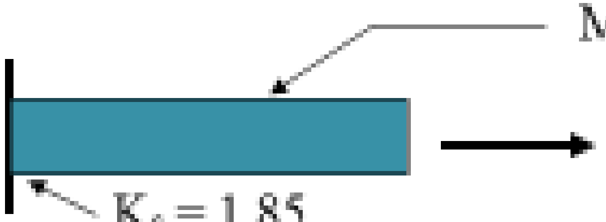
n based on shoulder = 4.02

n based on hole = 1.76

Factor of safety for the part is

n = 1.76 (based on the hole)

Example 4–19



Material: AISI 1050 CD Steel; S_{ut} = 690 MPa, S_y = 580 MPa

Find factor of safety against (a) yield and (ii) against fatigue for infinite life

---

Stress Analysis (Repeated Load)

$F_m = F_a = 72000/2 = 36{,}000$ N

$\sigma_m = \sigma_a = 4F_m/\pi d^2 = 32$ MPa

Material Properties

$S_e = K_a K_b K_c K_d K_e (0.5 S_{ut})$

$= (0.798)(1)(0.85)(1)\ (1)(0.5\text{x}690)$

$= 234$ MPa

Example 4-19 (Continued)

Factor of Safety Against **Yield**:
Langer Yield

$1/n = \sigma_m/S_y + K_f\,\sigma_a/S_y$

$1/n = 32/580 + 1.85 \times 32/580$

Solve $n = 6.4$

Factor of Safety Against **Fatigue**:
Modified Goodman

$1/n = \sigma_m/S_{ut} + K_f\,\sigma_a/S_e$
$1/n = 32/690 + 1.85 \times 32/234$

Solve $n = 3.3$

NOTE:
Factor of safety against yield = 6.4
Factor of safety against fatigue = 3.3

Therefore,
Factor of safety for the part = 3.3

Example 4–20

A 1.5-in-diameter bar has been machined from an AISI 1050 cold-drawn bar. This part is to withstand a fluctuating tensile load varying from 0 to 16 kip. Because of the ends, and the fillet radius, a fatigue stress-concentration factor K_f is 1.85 for 10^6 or larger life. Find S_a and S_m and the factor of safety guarding against fatigue and first-cycle yielding, using (*a*) the Gerber fatigue line and (*b*) the ASME-elliptic fatigue line.

Solution

We begin with some preliminaries. From Table A–20, $S_{ut} = 100$ kpsi and $S_y = 84$ kpsi. Note that $F_a = F_m = 8$ kip. The Marin factors are, deterministically,

$k_a = 2.70(100)^{-0.265} = 0.797$: Eq. (6–19), Table 6–2, p. 296

$k_b = 1$ (axial loading, see k_c)

$k_c = 0.85$: Eq. (6–26), p. 298

$k_d = k_e = k_f = 1$

$S_e = 0.797(1)0.850(1)(1)(1)0.5(100) = 33.9$ kpsi: Eqs. (6–8), (6–18), p. 290, p. 295

Example 4–20 (continued)

The nominal axial stress components σ_{ao} and σ_{mo} are

$$\sigma_{ao} = \frac{4F_a}{\pi d^2} = \frac{4(8)}{\pi 1.5^2} = 4.53 \text{ kpsi} \qquad \sigma_{mo} = \frac{4F_m}{\pi d^2} = \frac{4(8)}{\pi 1.5^2} = 4.53 \text{ kpsi}$$

Applying K_f to both components σ_{ao} and σ_{mo} constitutes a prescription of no notch yielding:

$$\sigma_a = K_f \sigma_{ao} = 1.85(4.53) = 8.38 \text{ kpsi} = \sigma_m$$

(*a*) Let us calculate the factors of safety first. From the bottom panel from Table 6–7 the factor of safety for fatigue is

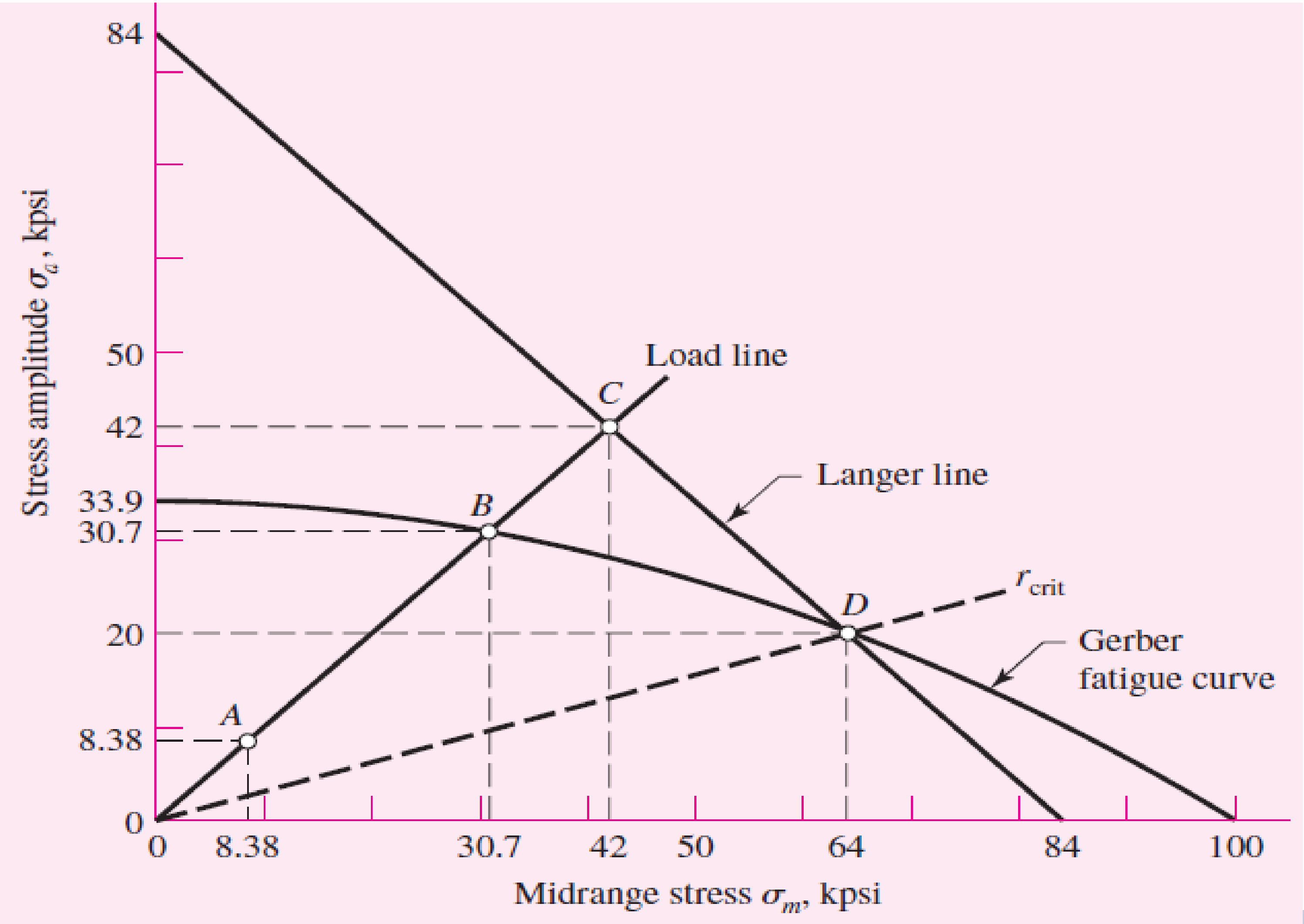
$$n_f = \frac{1}{2}\left(\frac{100}{8.38}\right)^2 \left(\frac{8.38}{33.9}\right) \left\{-1 + \sqrt{1 + \left[\frac{2(8.38)33.9}{100(8.38)}\right]^2}\right\} = 3.66$$ Answer

From Eq. (6–49) the factor of safety guarding against first-cycle yield is

$$n_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{84}{8.38 + 8.38} = 5.01$$ Answer

Example 4–20 (continued)

Thus, we see that fatigue will occur first and the factor of safety is 3.68. This can be seen in Fig. 6–28 where the load line intersects the Gerber fatigue curve first at point B. If the plots are created to true scale it would be seen that $n_f = OB/OA$.



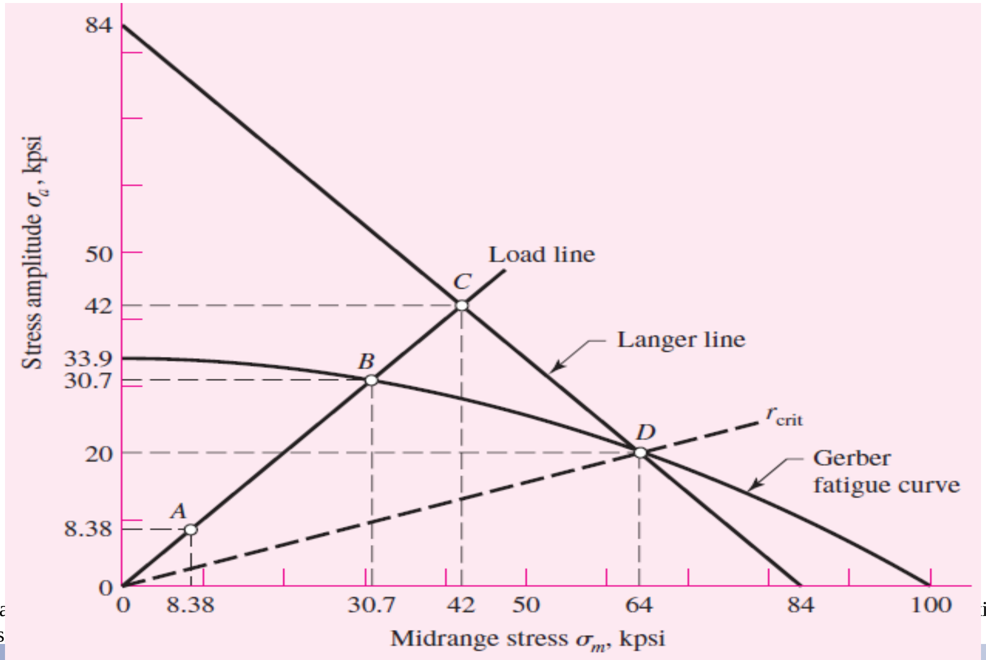
Example 4–20 (continued)

From the first panel of Table 6–7, $r = \sigma_a/\sigma_m = 1$,

$$S_a = \frac{(1)^2 100^2}{2(33.9)}\left\{-1+\sqrt{1+\left[\frac{2(33.9)}{(1)100}\right]^2}\right\} = 30.7\ \text{kpsi} \qquad \text{Answer}$$

$$S_m = \frac{S_a}{r} = \frac{30.7}{1} = 30.7\ \text{kpsi} \qquad \text{Answer}$$

As a check on the previous result, $n_f = OB/OA = S_a/\sigma_a = S_m/\sigma_m = 30.7/8.38 = 3.66$ and we see total agreement.



Example 4–20 (continued)

We could have detected that fatigue failure would occur first without drawing Fig. 6–28 by calculating r_{crit}. From the third row third column panel of Table 6–7, the intersection point between fatigue and first-cycle yield is

$$S_m = \frac{100^2}{2(33.9)}\left[1 - \sqrt{1 + \left(\frac{2(33.9)}{100}\right)^2\left(1 - \frac{84}{33.9}\right)}\right] = 64.0 \text{ kpsi}$$

$$S_a = S_y - S_m = 84 - 64 = 20 \text{ kpsi}$$

The critical slope is thus

$$r_{\text{crit}} = \frac{S_a}{S_m} = \frac{20}{64} = 0.312$$

which is less than the actual load line of $r = 1$. This indicates that fatigue occurs before first-cycle-yield.

Example 4–20 (continued)

(*b*) Repeating the same procedure for the ASME-elliptic line, for fatigue

$$n_f = \sqrt{\frac{1}{(8.38/33.9)^2 + (8.38/84)^2}} = 3.75$$ Answer

Again, this is less than $n_y = 5.01$ and fatigue is predicted to occur first. From the first row second column panel of Table 6–8, with $r = 1$, we obtain the coordinates S_a and S_m of point *B* in Fig. 6–29 as

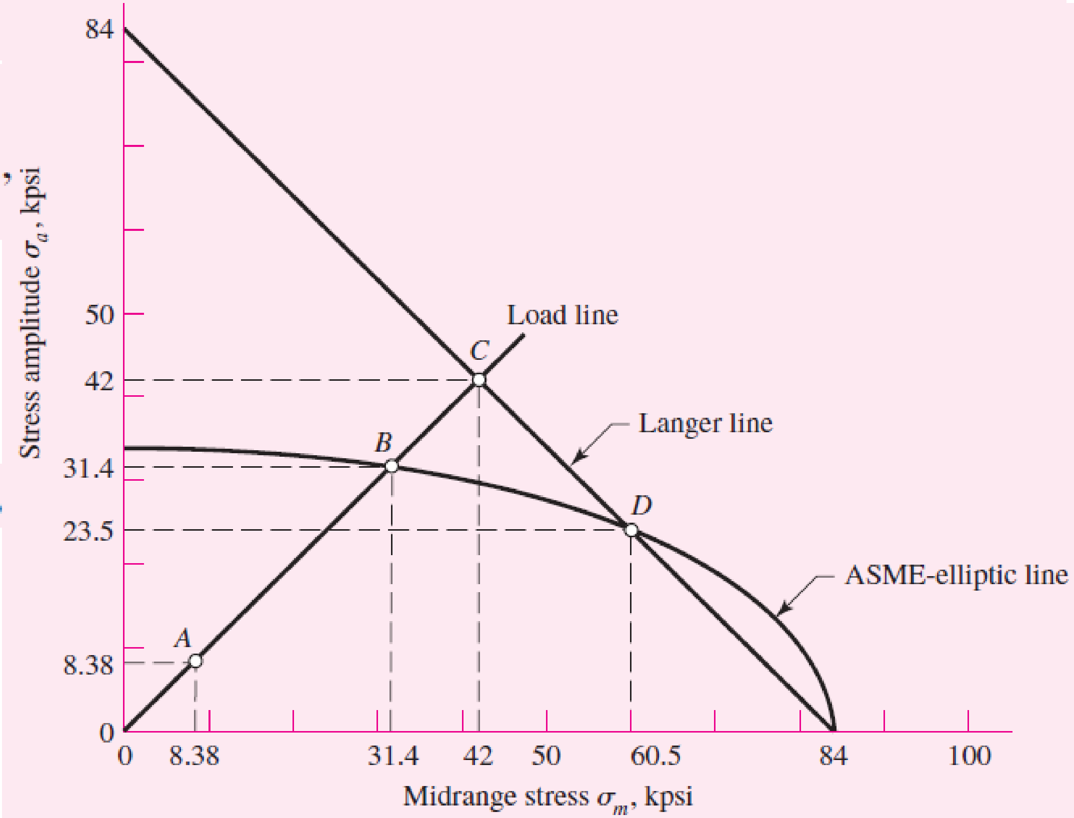
$$S_a = \sqrt{\frac{(1)^2 33.9^2 (84)^2}{33.9^2 + (1)^2 84^2}} = 31.4 \text{ kpsi},$$

$$S_m = \frac{S_a}{r} = \frac{31.4}{1} = 31.4 \text{ kpsi}$$

Answer

To verify the fatigue factor of safety,

$$n_f = S_a/\sigma_a = 31.4/8.38 = 3.75.$$



Example 4–20 (continued)

As before, let us calculate r_{crit}. From the third row second column panel of Table 6–8,

$$S_a = \frac{2(84)33.9^2}{33.9^2 + 84^2} = 23.5\ \text{kpsi}, \quad S_m = S_y - S_a = 84 - 23.5 = 60.5\ \text{kpsi}$$

$$r_{\text{crit}} = \frac{S_a}{S_m} = \frac{23.5}{60.5} = 0.388$$

which again is less than $r = 1$, verifying that fatigue occurs first with $n_f = 3.75$.

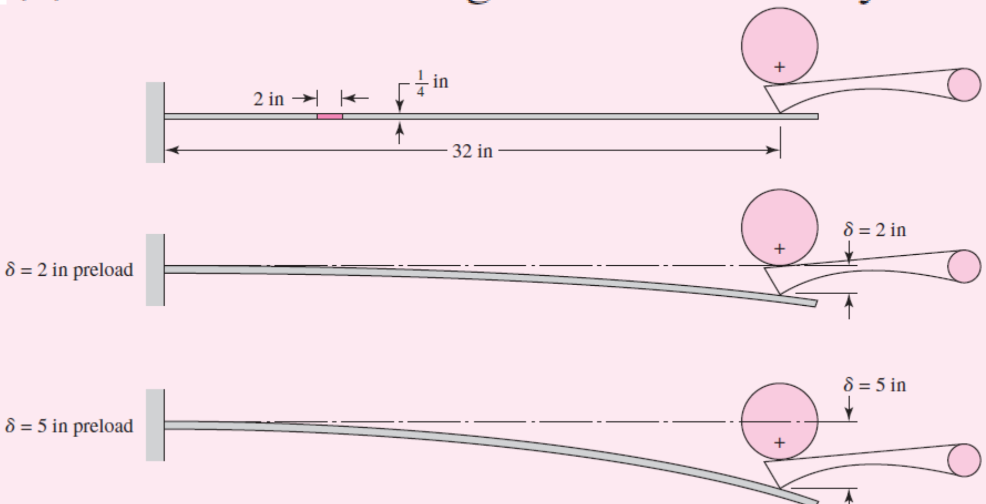
The Gerber and the ASME-elliptic fatigue failure criteria are very close to each other and are used interchangeably. The ANSI/ASME Standard B106.1M–1985 uses ASME-elliptic for shafting.

Example 4–21

A flat-leaf spring is used to retain an oscillating flat-faced follower in contact with a plate cam. The follower range of motion is 2 in and fixed, so the alternating component of force, bending moment, and stress is fixed, too. The spring is preloaded to adjust to various cam speeds. The preload must be increased to prevent follower float or jump. For lower speeds the preload should be decreased to obtain longer life of cam and follower surfaces. The spring is a steel cantilever 32 in long, 2 in wide, and $\frac{1}{4}$ in thick, as seen in Fig. 6–30*a*. The spring strengths are $S_{ut} = 150$ kpsi, $S_y = 127$ kpsi, and $S_e = 28$ kpsi fully corrected. The total cam motion is 2 in. The designer wishes to preload the spring by deflecting it 2 in for low speed and 5 in for high speed.

(*a*) Plot the Gerber-Langer failure lines with the load line.

(*b*) What are the strength factors of safety corresponding to 2 in and 5 in preload?



(*a*)

Example 4–21 (continued)

We begin with preliminaries. The second area moment of the cantilever cross section is

$$I = \frac{bh^3}{12} = \frac{2(0.25)^3}{12} = 0.00260\ \text{in}^4$$

Since, from Table A–9, beam 1, force F and deflection y in a cantilever are related by $F = 3EIy/l^3$, then stress σ and deflection y are related by

$$\sigma = \frac{Mc}{I} = \frac{32Fc}{I} = \frac{32(3EIy)}{l^3}\frac{c}{I} = \frac{96Ecy}{l^3} = Ky$$

where $K = \dfrac{96Ec}{l^3} = \dfrac{96(30 \cdot 10^6)0.125}{32^3} = 10.99(10^3)\ \text{psi/in} = 10.99\ \text{kpsi/in}$

Now the minimums and maximums of y and σ can be defined by

$$y_{\min} = \delta \qquad y_{\max} = 2 + \delta$$

$$\sigma_{\min} = K\delta \qquad \sigma_{\max} = K(2 + \delta)$$

Example 4–21 (continued)

The stress components are thus

$$\sigma_a = \frac{K(2+\delta) - K\delta}{2} = K = 10.99 \text{ kpsi}$$

$$\sigma_m = \frac{K(2+\delta) + K\delta}{2} = K(1+\delta) = 10.99(1+\delta)$$

For $\delta = 0$, $\quad \sigma_a = \sigma_m = 10.99 = 11$ kpsi

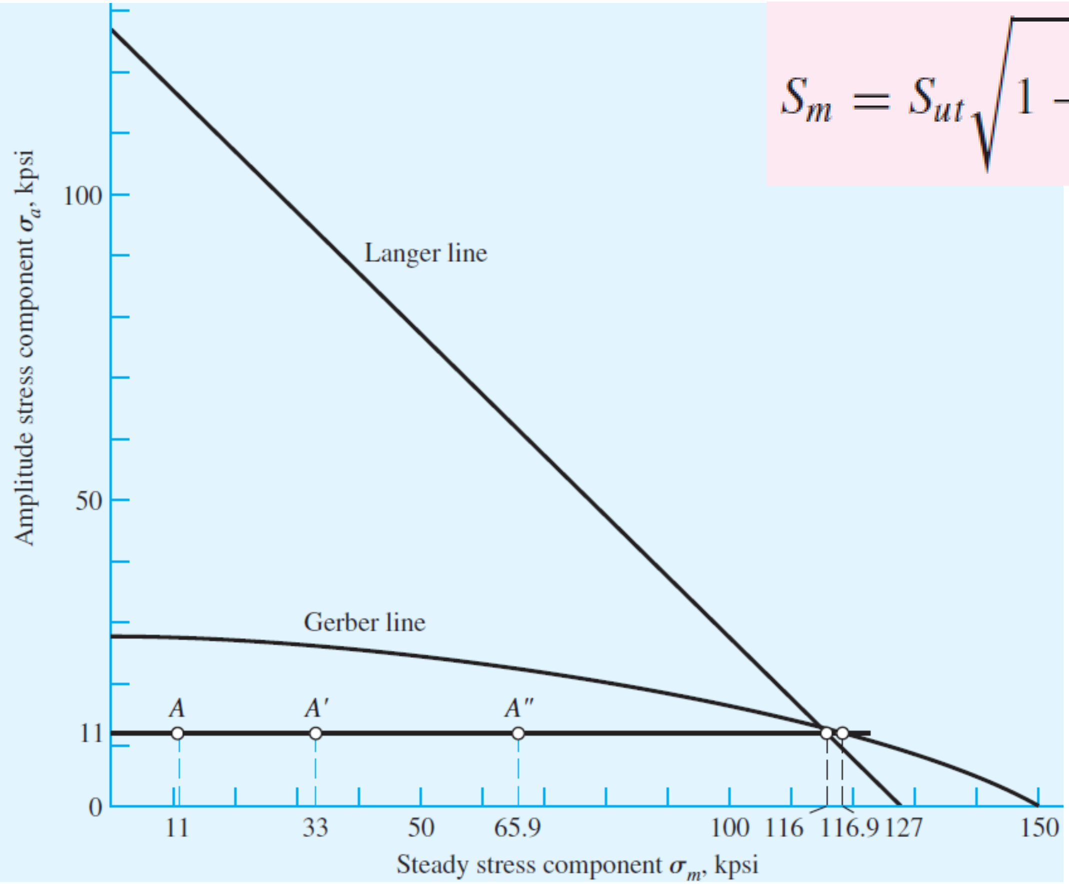
For $\delta = 2$ in, $\quad \sigma_a = 11$ kpsi, $\sigma_m = 10.99(1+2) = 33$ kpsi

For $\delta = 5$ in, $\quad \sigma_a = 11$ kpsi, $\sigma_m = 10.99(1+5) = 65.9$ kpsi

Example 4–21 (continued)

(*a*) A plot of the Gerber and Langer criteria is shown in Fig. 6–30*b*. The three preload deflections of 0, 2, and 5 in are shown as points A, A', and A''. Note that since σ_a is constant at 11 kpsi, the load line is horizontal and does not contain the origin. The intersection between the Gerber line and the load line is found from solving Eq. (6–42) for S_m and substituting 11 kpsi for S_a:

$$S_m = S_{ut}\sqrt{1 - \frac{S_a}{S_e}} = 150\sqrt{1 - \frac{11}{28}} = 116.9 \text{ kpsi}$$



Example 4–21 (continued)

The intersection of the Langer line and the load line is found from solving Eq. (6–44) for S_m and substituting 11 kpsi for S_a:

$$S_m = S_y - S_a = 127 - 11 = 116 \text{ kpsi}$$

The threats from fatigue and first-cycle yielding are approximately equal.

(*b*) For $\delta = 2$ in,

$$n_f = \frac{S_m}{\sigma_m} = \frac{116.9}{33} = 3.54 \qquad n_y = \frac{116}{33} = 3.52$$ Answer

and for $\delta = 5$ in,

$$n_f = \frac{116.9}{65.9} = 1.77 \qquad n_y = \frac{116}{65.9} = 1.76$$ Answer

Example 4–22

A steel bar undergoes cyclic loading such that $\sigma_{max} = 60$ kpsi and $\sigma_{min} = -20$ kpsi. For the material, $S_{ut} = 80$ kpsi, $S_y = 65$ kpsi, a fully corrected endurance limit of $S_e = 40$ kpsi, and $f = 0.9$. Estimate the number of cycles to a fatigue failure using:

(*a*) Modified Goodman criterion.

(*b*) Gerber criterion.

Solution

From the given stresses,

$$\sigma_a = \frac{60 - (-20)}{2} = 40 \text{ kpsi} \qquad \sigma_m = \frac{60 + (-20)}{2} = 20 \text{ kpsi}$$

(*a*) For the modified Goodman criterion, Eq. (6–46), the fatigue factor of safety based on infinite life is

$$n_f = \frac{1}{\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}}} = \frac{1}{\dfrac{40}{40} + \dfrac{20}{80}} = 0.8$$

Example 4–22 (continued)

This indicates a finite life is predicted. The *S-N* diagram is only applicable for completely reversed stresses. To estimate the finite life for a fluctuating stress, we will obtain an equivalent completely reversed stress that is expected to be as damaging as the fluctuating stress. A commonly used approach is to assume that since the modified Goodman line represents all stress situations with a constant life of 10^6 cycles, other constant-life lines can be generated by passing a line through $(S_{ut}, 0)$ and a fluctuating stress point (σ_m, σ_a). The point where this line intersects the σ_a axis represents a completely reversed stress (since at this point $\sigma_m = 0$), which predicts the same life as the fluctuating stress.

This completely reversed stress can be obtained by replacing S_e with σ_{rev} in Eq. (6–46) for the modified Goodman line resulting in

$$\sigma_{\text{rev}} = \frac{\sigma_a}{1 - \dfrac{\sigma_m}{S_{ut}}} = \frac{40}{1 - \dfrac{20}{80}} = 53.3 \text{ kpsi}$$

Example 4–22 (continued)

From the material properties, Eqs. (6–14) to (6–16), p. 293, give

$$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.9(80)]^2}{40} = 129.6 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{0.9(80)}{40}\right] = -0.0851 \qquad (1)$$

$$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{\sigma_{\text{rev}}}{129.6}\right)^{-1/0.0851}$$

Substituting σ_{rev} into Eq. (1) yields

$$N = \left(\frac{53.3}{129.6}\right)^{-1/0.0851} = 3.4(10^4) \text{ cycles} \qquad \text{Answer}$$

Example 4–22 (continued)

(*b*) For Gerber, similar to part (*a*), from Eq. (6–47),

$$\sigma_{\text{rev}} = \frac{\sigma_a}{1 - \left(\dfrac{\sigma_m}{S_{ut}}\right)^2} = \frac{40}{1 - \left(\dfrac{20}{80}\right)^2} = 42.7 \text{ kpsi}$$

Again, from Eq. (1),

$$N = \left(\frac{42.7}{129.6}\right)^{-1/0.0851} \doteq 4.6(10^5) \text{ cycles} \qquad \text{Answer}$$

Comparing the answers, we see a large difference in the results. Again, the modified Goodman criterion is conservative as compared to Gerber for which the moderate difference in S_f is then magnified by a logarithmic S, N relationship.

Fatigue Criteria for Brittle Materials

- For many brittle materials, the first quadrant fatigue failure criteria follows a concave upward Smith-Dolan locus,

$$\frac{S_a}{S_e} = \frac{1 - S_m/S_{ut}}{1 + S_m/S_{ut}}$$

- Or as a design equation,

$$\frac{n\sigma_a}{S_e} = \frac{1 - n\sigma_m/S_{ut}}{1 + n\sigma_m/S_{ut}}$$

- For a radial load line of slope *r*, the intersection point is

$$S_a = \frac{rS_{ut} + S_e}{2}\left[-1 + \sqrt{1 + \frac{4rS_{ut}S_e}{(rS_{ut} + S_e)^2}}\right]$$

- In the second quadrant,

$$S_a = S_e + \left(\frac{S_e}{S_{ut}} - 1\right)S_m \qquad -S_{ut} \le S_m \le 0 \quad \text{(for cast iron)}$$

Fatigue Criteria for Brittle Materials

- Table gives properties of gray cast iron, including endurance limit
- The endurance limit already includes k_a and k_b
- The average k_c for axial and torsional is 0.9

Example 4–23

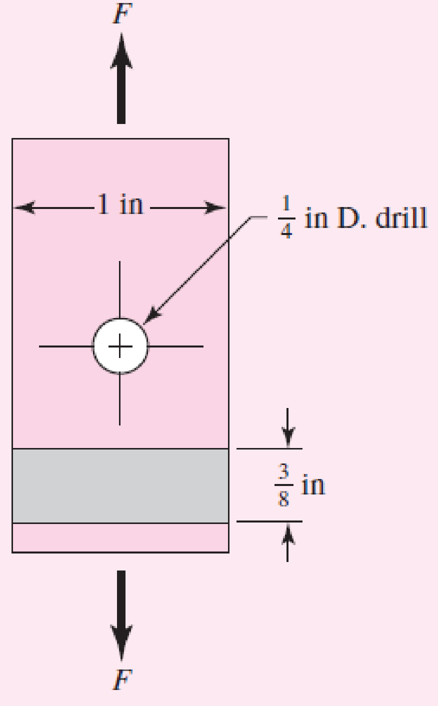
A grade 30 gray cast iron is subjected to a load F applied to a 1 by $\frac{3}{8}$-in cross-section link with a $\frac{1}{4}$-in-diameter hole drilled in the center as depicted in Fig. 6–31*a*. The surfaces are machined. In the neighborhood of the hole, what is the factor of safety guarding against failure under the following conditions:

(*a*) The load $F = 1000$ lbf tensile, steady.

(*b*) The load is 1000 lbf repeatedly applied.

(*c*) The load fluctuates between -1000 lbf and 300 lbf without column action.

Use the Smith-Dolan fatigue locus.



Example 4–23 (continued)

Some preparatory work is needed. From Table A–24, $S_{ut} = 31$ kpsi, $S_{uc} = 109$ kpsi, $k_a k_b S'_e = 14$ kpsi. Since k_c for axial loading is 0.9, then $S_e = (k_a k_b S'_e)k_c = 14(0.9) = 12.6$ kpsi. From Table A–15–1, $A = t(w - d) = 0.375(1 - 0.25) = 0.281\ \text{in}^2$, $d/w = 0.25/1 = 0.25$, and $K_t = 2.45$. The notch sensitivity for cast iron is 0.20 (see p. 304), so

$$K_f = 1 + q(K_t - 1) = 1 + 0.20(2.45 - 1) = 1.29$$

(*a*) Since the load is steady, $\sigma_a = 0$, the load is static. Based on the discussion of cast iron in Sec. 5–2, K_t, and consequently K_f, need not be applied. Thus, $\sigma_m = F_m/A = 1000(10^{-3})/0.281 = 3.56$ kpsi, and

$$n = \frac{S_{ut}}{\sigma_m} = \frac{31.0}{3.56} = 8.71 \qquad \text{Answer}$$

Example 4–23 (continued)

(*b*)

$$F_a = F_m = \frac{F}{2} = \frac{1000}{2} = 500\ \text{lbf}$$

$$\sigma_a = \sigma_m = \frac{K_f F_a}{A} = \frac{1.29(500)}{0.281}(10^{-3}) = 2.30\ \text{kpsi}$$

$$r = \frac{\sigma_a}{\sigma_m} = 1$$

From Eq. (6–52),

$$S_a = \frac{(1)31 + 12.6}{2}\left[-1 + \sqrt{1 + \frac{4(1)31(12.6)}{[(1)31 + 12.6]^2}}\right] = 7.63\ \text{kpsi}$$

$$n = \frac{S_a}{\sigma_a} = \frac{7.63}{2.30} = 3.32$$

Answer

Example 4–23 (continued)

(c) $F_a = \frac{1}{2}|300 - (-1000)| = 650\ \text{lbf}$ $\sigma_a = \frac{1.29(650)}{0.281}(10^{-3}) = 2.98\ \text{kpsi}$

$F_m = \frac{1}{2}[300 + (-1000)] = -350\ \text{lbf}$ $\sigma_m = \frac{1.29(-350)}{0.281}(10^{-3}) = -1.61\ \text{kpsi}$

$$r = \frac{\sigma_a}{\sigma_m} = \frac{3.0}{-1.61} = -1.86$$

From Eq. (6–53), $S_a = S_e + (S_e/S_{ut} - 1)S_m$ and $S_m = S_a/r$. It follows that

$$S_a = \frac{S_e}{1 - \frac{1}{r}\left(\frac{S_e}{S_{ut}} - 1\right)} = \frac{12.6}{1 - \frac{1}{-1.86}\left(\frac{12.6}{31} - 1\right)} = 18.5\ \text{kpsi}$$

$$n = \frac{S_a}{\sigma_a} = \frac{18.5}{2.98} = 6.20$$ Answer

Example 4–23 (continued)

Figure 6–31*b* shows the portion of the designer's fatigue diagram that was constructed.