1. Which of the following quadric surface represents hyperbolic paraboloid? $(1, 1, 1)$

A. $x^2 + y^2 + 2z^2 - 2xy - 4xz - 4yz + 4x = 8$
B. $4x^2 + 9y^2 + z^2 + 8x - 18y - 4z = 19$ ✗
C. $5y^2 + 20y + z - 23 = 0$ ✗
D. $z = 4xy$

2. Which of the following quadric surface represents parabolic cylinder? $(1, 0, 2)$

A. $x^2 + y^2 + 2z^2 - 2xy - 4xz - 4yz + 4x = 8$
B. $4x^2 + 9y^2 + z^2 + 8x - 18y - 4z = 19$ ✗
C. $5y^2 + 20y + z - 23 = 0$
D. $z = 4xy$

3. Which of the following quadric surface is an ellipsoid? $(3, 0, 0)$

A. $x^2 + y^2 + 2z^2 - 2xy - 4xz - 4yz + 4x = 8$
B. $4x^2 + 9y^2 + z^2 + 8x - 18y - 4z = 19$
C. $5y^2 + 20y + z - 23 = 0$
D. $z = 4xy$

$2dxy = -2$

4. Find the flux of the vector field $F(x, y, z) = \frac{1}{3}x^3\mathbf{i} + \frac{1}{3}y^3\mathbf{j} + \frac{1}{3}z^3\mathbf{k}$ over the surface ∂Q, where Q is the solid bounded by $z = 4 - x^2 - y^2$, and $xy-$plane.

A. 192π cubic units
B. 96π cubic units
C. 32π cubic units
D. 16π cubic units

$$\begin{vmatrix} 1 & -1 & -2 \\ -1 & 1 & -2 \\ -2 & -2 & 2 \end{vmatrix}$$

$\operatorname{div} F = \frac{1}{3} x^2 + y^2 + z^2$

$z = 4 - x^2 - y^2$

5. The Stokes' theorem states that ....

A. $\int_{\partial s} F(x, y, z) \cdot dr = \int\int_s (\nabla \cdot F) \cdot nds$
B. $\int_{\partial s} F(x, y, z) \cdot dr = \int\int_s (\nabla \times F) \cdot nds$
C. $\int_{\partial s} F(x, y, z) \cdot dr = \int\int_s (\nabla F) \cdot nds$
D. $\int_{\partial s} F(x, y, z) \cdot dr = \int\int_s (\Delta F) \cdot nds$

$\sqrt{1 - x^2 - y^2}$

6. Evaluate $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{1-\sqrt{1-x^2-y^2}}^{1-\sqrt{1-x^2-y^2}} \sqrt{(x^2 + y^2 + z^2)} dzdydx$.

$1 -$

$\sqrt{1 - x^2 - y^2}$

..edict Barnes

$$\int_0^{2\pi}\int_0^{2}\int_0^{4-x^2-y^2} r\,dz\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^{2}\left[4-x^2-y^2\right] r\,dr\,d\theta$$

A. $\frac{\pi}{2}$

B. π

C. 2π

D. 3π

7. Find $\int\int\int_Q 2xe^y \sin(z)dv$, where Q is the rectangle defined by $Q = \{(x, y, z)|1 \le x \le 2, 0 \le y \le 1,\ and\ 0 \le z \le \pi\}$.

$$\int_0^{2\pi}\int_0^{2}(r^2+3)r\,dr\,d\theta$$

A. $6e$

B. $6(e-1)$

C. $6(e+1)$

D. $6(e+2)$

$$\int_0^{2\pi}\int_0^{2}\int_0^{4-r^2} r\,dz\,dr\,d\theta$$

$(4-r^2)\ -4$

$-r^2$

8. Find $\int\int_{\mathbf{R}} (x^2 + y^2 + 3)dA$, where $\mathbf{R}$ is the circle of radius 2 centred at the origin.

$$\int_0^{2\pi}\int_0^{2}\left[z\right]_0^{4-r^2}$$

$-\frac{r^3}{3}\quad -\frac{r^4}{4}$

A. 10π square units

B. 12π square units

C. 15π square units

D. 20π square units

$$\int_0^{2\pi}\int_0^{2}\left[4-r^2\right] r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^{2} 4r-r^3\,dr\,d\theta$$

9. Evaluate $\int_{-1}^{1}\int_0^{\sqrt{1-x^2}} x^2(x^2 + y^2)^2 dydx$.

$$\int_0^{2\pi}\left[2r^2-\frac{r^4}{4}\right]_0^2$$

A. $\frac{\pi}{2}$ square units

B. $\frac{\pi}{4}$ square units

C. $\frac{\pi}{8}$ square units

D. $\frac{\pi}{16}$ square units

$$\int_0^{2\pi} 4\,d\theta \quad 8\pi$$

10. Which of the following is **not** a critical point of $f(x, y) = xe^{\frac{-x^2}{2} - \frac{y^3}{3} + y}$?

$$\int_0^{2\pi}\int_0^{2}\int_4^{4-x^2-y^2}$$

A. $(1, 1)$

B. $(-1, 1)$

C. $(0, -1)$

D. $(-1, -1)$

$y = \sqrt{1-x^2}$

$y^2 = 1-x^2$

$1 = x^2+y^2$

11. Find the binormal vector for the curve traced out by $\mathbf{r(t)} = \sin(2t)\mathbf{i} + \cos(2t)\mathbf{j} + t\mathbf{k}$.

$-1(y^2+1) = 0$

$(-y^2+1)\,x\,e^{-\frac{x^2}{2}+\frac{y^3}{3}+y}$

$= 0 \quad 1 = 0$

$$\int_0^{2\pi}\int_0^{1}(r\cos\theta)^2(r^2)\,r\,dr\,d\theta$$

A. $\frac{1}{\sqrt{5}}\Big(\cos(2t)\mathbf{i}+\sin(2t)\mathbf{j}+2\mathbf{k}\Big)$

B. $\frac{1}{\sqrt{5}}\Big(\cos(2t)\mathbf{i}-\sin(2t)\mathbf{j}-2\mathbf{k}\Big)$

C. $\frac{1}{\sqrt{5}}\Big(\cos(2t)\mathbf{i}-\sin(2t)\mathbf{j}+2\mathbf{k}\Big)$

D. $-\frac{1}{\sqrt{5}}\Big(\cos(2t)\mathbf{i}+\sin(2t)\mathbf{j}+2\mathbf{k}\Big)$

12. Evaluate $\lim_{(x,y)\to(0,0)}\frac{2xy}{x^2+2y^2}$.

A. 2

B. 1

C. 0

D. does not exist

11 $\langle \cos 2t, -\sin 2t, 1\rangle$

$x+y \neq 0$

$x \geq y$

$\frac{2x^2}{x^2+2x^2}$ $\frac{2x^2}{3}$

13. Evaluate $\lim_{(x,y)\to(0,0)}\frac{2x^2\sin(y)}{2x^2+y^2}$.

A. 2

B. 1

C. 0

D. does not exist

$\lim \left|\frac{2xy}{x^2}-0\right|$

14. Find the domain of $f(x,y)=\frac{1}{(x+y)}$.

$x+y \geq 0$

A. $x=-y$

B. $x\neq -y$

C. $x\neq y$

D. $x>y$ *or* $y<x$

$x+y \geq 0$ $x \neq -y$

$x > y$

$\frac{2y}{x}$ $\frac{0}{y^2}$

$\left|\frac{2x^2\sin y}{2x^2}-0\right|$

$|\sin y|$

15. Find the range of $f(x,y)=\cos(x^2+y^2)$.

A. $-1\leq f(x,y)\leq 1$

B. $0\leq f(x,y)\leq 1$

C. $0\leq f(x,y)\leq\infty$

D. $-\infty\leq f(x,y)\leq\infty$

$x^2+y^2 \leq$

16. Find the linear approximation of $f(x,y)=\sin(x)\cos(y)$ at a point $(0,\pi)$.

$f_x = \cos x \cos y = -1$

$f_y = -\sin x \sin y = 0$

$f \quad 1(x-0) + 0(x-\pi)$

$$\begin{pmatrix}4\\0\\2\end{pmatrix}-\begin{pmatrix}2\\1\\0\end{pmatrix}=\begin{pmatrix}2\\-1\\2\end{pmatrix}$$

$$t\begin{pmatrix}2\\-1\\2\end{pmatrix}+\begin{pmatrix}2\\1\\0\end{pmatrix}$$

A. $-x+1$

B. $x+1$

C. x

D. $-x$

$2t+2=x$

$-t+1 \quad y$

$2t \quad 0 \quad z$

17. Given that $f(x,y)=x^2y-4y^3$, find $f_y(2,1)$.

A. 4

B. 6

C. -8

D. -3

$f_y = x^2-12y^2 \qquad 6 \quad -8$

$(2)^2-12(1)^2 \qquad \frac{6}{5} \qquad 6\left(\frac{3}{5}\right)+(-8$

$-8y \qquad 8y \qquad 2$

18. If $f(x,y)=x^2+y^2$, find the directional derivative in the direction of $\mathbf{u}=3\mathbf{i}-4\mathbf{j}$.

A. $\frac{14}{5}$

B. $\frac{13}{5}$

C. $\frac{11}{5}$

D. $\frac{7}{5}$

$f_x = 2x \qquad f_y = 2y = -8$

$\frac{6}{}\left(\frac{3}{5}\right)+(6 \qquad \frac{6}{}\ \frac{-8}{\sqrt{5}}$

$2x\left(6\left(\frac{3}{\sqrt{5}}\right)+(-8)(\right.$

$6\left(\frac{3}{5}\right)+(-8)\left(\frac{-4}{\sqrt{5}}\right)\sqrt{5}$

$\frac{18}{\sqrt{5}}+$

19. Given that $f(x,y)=x^2+y^2$, find the minimum rate of change at point $(1,3)$.

$6\left(\frac{3}{5}\right)+(8)\left(\frac{-4}{5}\right) \qquad \frac{50}{5}$

A. $-3\sqrt{10}$

B. $-2\sqrt{10}$

C. $2\sqrt{10}$

D. $3\sqrt{10}$

$\frac{18}{5}+\frac{32}{5}$

grad $= 2xi+2yj$

$2+6$

20. Find the gradient of $f(x,y,z)=3x^2y-z\cos(x)$, at point $(0,2,-1)$.

A. $(1,1,1)$

B. $(0,0,-1)$

C. $(0,1,0)$

D. $(-1,1,0)$

$2x\ i+2yi$

$(6i-8j)\left(\frac{3i}{\sqrt{5}}-\frac{4j}{\sqrt{5}}\right)$

21. Find $\int_c(4xz+2y)dx$, where c is the line segment from $(2,1,0)$ to $(4,0,2)$.

$f_x = (6xy+z\sin x)$

$f_y = 6\ 3$

$\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$ $\frac{\partial}{\partial z}$

x^2y $3x - yz$ z^3

A. $\frac{42}{3}$

B. $\frac{-42}{3}$

C. $\frac{86}{3}$

D. $\frac{-86}{3}$

$i \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x - yz & z^3 \end{vmatrix} + j \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ x^2y & z^3 \end{vmatrix} + K \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x^2y & 3x - yz \end{vmatrix}$

22. Compute $curl\mathbf{F}$, if $F = x^2y\mathbf{i} + (3x - yz)\mathbf{j} + z^3\mathbf{k}$.

A. $y\mathbf{i} + (3 - x^2)\mathbf{k}$

B. $y\mathbf{i} + 3\mathbf{j} - x^2\mathbf{k}$

C. $y\mathbf{i} + 3z^2\mathbf{j} + (3 - x^2)\mathbf{k}$

D. $y\mathbf{i} - 2xy\mathbf{j} + (3 - x^2)\mathbf{k}$

$i(0 + y) + j(0 - 0) + (3 - x^2)$

23. Find $\int\int_s (\nabla \times F) \cdot nds$, where s is the portion of the tetrahedron bounded by $x + y + 2z = 2$, and the coordinate planes with $z > 0$, $\mathbf{n}$ upward, $F = (zy^4 - y^2)\mathbf{i} + (y - x^3)\mathbf{j} + z^2\mathbf{k}$.

A. $\frac{-4}{3}$

B. -1

C. $\frac{-2}{3}$

D. $\frac{-1}{3}$

$yi + (3 - x^2)K$

$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & 3x - yz & z^3 \end{vmatrix}$

24. Find $div\mathbf{F}$, if $\mathbf{F} = (x^3 - y)\mathbf{i} + z^5\mathbf{j} + e^y\mathbf{k}$.

A. $2xy - z + 3z^2$

B. $5xy + z^2$

C. $3x^2 + 5z^4 + e^y$

D. $3x^2$

$3x^2$ $ds = \sqrt{(2)^2 + (1)^2 + 1}$

$2x + y = 2$

$y = 2-$

25. Find $\int\int_s 3zds$, where the surface s is the portion of the plane $2x + y + z = 2$, lying in the first octant.

A. $\sqrt{6}$

B. $2\sqrt{6}$

C. $3\sqrt{6}$

D. $4\sqrt{6}$

$\int \sqrt{6}\, dy$

$i \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x - yz & z^3 \end{vmatrix} - j \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ x^2y & z^3 \end{vmatrix} + K \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x^2y & 3x - y \end{vmatrix}$

26. The Divergence theorem states that ....

$i(0-0) - j(0-0) + K(3 -$

$\int \int^{2-2x} 2 - y - 2x \, dy$

$\int_0 \int_0$ 0 $z = \cancel{2x}$ $2 - 2x - y$

$2 \cdot \sqrt{(2)^2 + (-1)^2 + 1}$

$\sqrt{4 + 1 + 1}$

$\sqrt{6}$

A. $\int\int\int_Q F \cdot nds = \int\int_Q \nabla \cdot F(x,y,z)dv$

B. $\int\int_{\partial s} F \cdot nds = \int\int\int_Q \nabla F(x,y,z)dv$

C. $\int\int_{\partial s} F \cdot nds = \int\int\int_Q \nabla \cdot F(x,y,z)dv$

D. $\int\int_{\partial s} F \cdot nds = \int\int\int_Q \nabla \times F(x,y,z)dv$

$y = 2 - 2x$

$\int_0^2 \int_0 \cancel{z}(2 - 2x - y) \sqrt{6}\, dy\, dx$

27. Evaluate $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{x^2+z^2} (x^2 + z^2)dydzdx$.

A. $\frac{2\pi}{5}$ cubic units

B. $\frac{32\pi}{5}$ cubic units

C. 120π cubic units

D. 243π cubic units

$(x^2 + z^2)\ (x^2 + z^2 - 6)$

28. Find $\int\int\int_Q e^z dv$, where Q is the region inside $x^2 + y^2 = 9, z = x^2 + y^2$ and $z = 0$.

$\sqrt{6} \int_0^2 \left[2y - 2xy - y^2 \right]_0^{2-2x}$

A. $\pi(e^6 + 4)$ cubic units

B. 20π cubic units

C. 32π cubic units

D. $\pi(e^9 - 10)$ cubic units

29. Find $\int\int\int_Q zdv$, where Q is the region between $z = \sqrt{x^2 + y^2}$, and $z = \sqrt{4 - x^2 - y^2}$.

$2(2-2x) - 2x(2-2x) - (2-2x)^2$

$4 - 4x - 4x + 4x^2 - (4 - 8x$

A. π

B. 2π

C. 3π

D. 4π

$\cancel{4} - 4x - 4x + 4x^2 - (\cancel{4} - 8x + 4x^2)$

30. Convert the spherical coordinates $(4, \frac{\pi}{2}, 0)$ to cartesian coordinates.

$4 - 4x - 4x - \cancel{4} - 8x\ \sqrt{6} \int_0^2 - 16x$

$8x^2$

A. $(0, 0, 4)$

B. $(0, 1, 1)$

C. $(4, 0, 0)$

D. $(4, 1, 0)$

$\sqrt{}$

31. Find an arc length parameterization of the circle of radius 4 centred at the origin.

$x = \rho \sin\phi \cos\theta$

$x = 4 \sin \frac{\pi}{2} \cos 4$

$\int_0 \int_0^{2} \int_4^{r^2} 3r^2 \, dr$ $(3r^4 - 12r^2)$

$3r^2(r^2) - 3r^2(4)$ $\int_0^1$ $3r^5 - 12r^3$

$y - 2 = \frac{1}{4}(x-5)$

$\frac{1}{2}r^6 - 3r^4$ $4y - 8 = x - 5$

$4y = x + 3$

$y = \frac{1}{4}x + 3$

A. $x = 4\cos(t), \ y = 4\sin(t)$

B. $x = 4\cos(\frac{s}{4}), \ y = 4\sin(\frac{s}{4})$, where $0 \le s \le 2\pi$

C. $x = 4\cos(t), \ y = 4\sin(t)$, where $0 \le t \le 1$

D. $x = 4\cos(2\pi + t), \ y = 4\sin(2\pi + t)$

32. Find the flux of the vector field $\mathbf{F} = x^3\mathbf{i} + (y^3 - z)\mathbf{j} + xy^2\mathbf{k}$, over the surface ∂Q, where Q is bounded by $z = x^2 + y^2$ and $z = 4$.

-16

A. 60π square units

B. 53π square units

C. 41π square units

D. 32π square units

$3x^2 + 3y^2$ $\binom{5}{2} + t\binom{-4}{-1}$

$3r^2$ $3x^2 + 3y^2 +$ $5 - 4t$ $2 - t$

33. Find $\int_c (3x + y)ds$, where c is the line segment from $(5, 2)$ to $(1, 1)$.

$\int_0^{2} 3r^3 \, dr \, d\theta$

A. $\frac{21}{2}\sqrt{17}$

B. $7\sqrt{17}$

C. $3\sqrt{17}$

D. $2\sqrt{17}$

$y = \frac{1-2}{1-5} = \frac{-1}{4}$ $\binom{1}{1} - \binom{5}{2} = \binom{-4}{-1}$

$\frac{3r^4}{4}\Big]_0^2$

34. Calculate the workdone by the vector field, if force $\mathbf{F} = y\mathbf{i} - x\mathbf{j}$ acting on an objects moves along the parabola $y = x^2 - 1$ from $(1, 0)$ to $(-2, 3)$.

$2 \,/\, (2\pi - 0)$

A. $6J$

B. $4J$

C. $2J$

D. $1J$

$\int_0^{2\pi} \int_0^{2} \int_4^{r^2} (3x^2 + 3y^2) \, dz \, dr \, d\theta$

$(r^2 - 4)(3r^2)$

$3r^4 - 12$

35. Find the volume of the solid bounded by the graphs of $z = 4 - y^2$, $x + z = 4$, $x = 0$, and $z = 0$.

$3r^5 - 12$

$3(r^2)(r^2 - 4)$

A. 128 cubic units

B. $\frac{128}{3}$ cubic units

C. $\frac{128}{5}$ cubic units

D. $\frac{128}{7}$ cubic units

$\int_0^{2\pi} \int_0^{2} 3r^4 - 12r^2 \, r \, dr \, d\theta$

$\frac{1}{6}r^6 - \frac{1}{4}$

36. Find the point on the ellipse that is closest to the origin, if the plane $x + y + z = 12$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse.

$3r^5 - 12r^3 \, dr \, d\theta$

$\frac{1}{6}r^6 - \frac{1}{4}r^4\Big]_0^2$

$\frac{1-2}{1-5} = \frac{1}{4}$

$\frac{3x^2}{2} - \frac{3x}{2} \Big|_1^5$

$2 \quad \frac{75}{2}$

A. $(-3, 2, 1)$
B. $(-3, -3, 18)$
C. $(1, 4, 6)$
D. $(2, 2, 8)$

37. Find $\int\int_{\mathbf{R}} \sqrt{(4 - x^2 - y^2)}dA$, where $\mathbf{R} = \{(x, y)|x^2 + y^2 \leq 4,\ x \geq 0\}$.

A. $\frac{4\pi}{3}$ square units
B. $\frac{8\pi}{3}$ square units
C. $\frac{16\pi}{3}$ square units
D. $\frac{32\pi}{3}$ square units

$y - 2 = \frac{1}{4}(x - 5)$

$4y - 8 = x - 5$

$\frac{dy}{dx} + d$

38. Find $\int\int_{\mathbf{R}} \sqrt{(4 - x^2 - y^2)}dA$, where $\mathbf{R} = \{(x, y)|x^2 + y^2 \leq 4,\ x, y \geq 0\}$.

$\int_0^{\pi/2}\int_0^2$

$ds = \int^{2\pi}$

A. $\frac{4\pi}{3}$ square units
B. $\frac{8\pi}{3}$ square units
C. $\frac{16\pi}{3}$ square units
D. $\frac{32\pi}{3}$ square units

39. Find $\int\int_{\mathbf{R}} \sqrt{(4 - x^2 - y^2)}dA$, where $\mathbf{R} = \{(x, y)|x^2 + y^2 \leq 4,\ y \geq 0\}$.

A. $\frac{4\pi}{3}$ square units
B. $\frac{8\pi}{3}$ square units
C. $\frac{16\pi}{3}$ square units
D. $\frac{32\pi}{3}$ square units

$\int_0^{2\pi}\int_0^3\int_0^{r^2} e^z \, dz \, r \, dr \, d\theta$

$\int_0^{2\pi}\int_0^3 \left[e^z\right]_0^{r^2} r$

$\int_0^{2\pi}\int_0^3 r e^{r^2} \, dr \, d\theta$

$\int_0^{2\pi} \left[\frac{1}{2} e^{r^2}\right]$

40. Find $\int\int_{\mathbf{R}} \sqrt{(4 - x^2 - y^2)}dA$, where $\mathbf{R} = \{(x, y)|x^2 + y^2 \leq 4,\ x^2 + y^2 \leq 4\}$.

A. $\frac{4\pi}{3}$ square units
B. $\frac{8\pi}{3}$ square units
C. $\frac{16\pi}{3}$ square units
D. $\frac{32\pi}{3}$ square units

41. Find $\int\int_D e^{-x^2-y^2}dA$, where D is the region bounded by the semicircle $y = \sqrt{4 - x^2}$, and $y-$axis.

$\int_0^{2\pi} \left[\frac{1}{2} e^9 - \right.$

$x^2 = 4$

$y^2 = 4 - x^2$

$2\pi\left(\frac{1}{2} e^9 - \frac{1}{2}\right)$

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$\int\int_{\pi/2}^{3\pi/2} e^{-r^2}\, r\,dr\,d\theta$

$\frac{1}{2}\pi\left(-\frac{1}{2}e^{-4}+\frac{1}{2}\right)$

A. $\frac{\pi}{2}(e^{-4}-1)$ square units
B. $\frac{\pi}{2}(1-e^{-4})$ square units
C. $\frac{\pi}{4}(1-e^{-4})$ square units
D. $\frac{\pi}{4}(e^{-4}-1)$ square units

42. Find $\int\int_D e^{-x^2-y^2}\,dA$, where D is the region bounded by $y=\sqrt{4-x^2}$, and $x, y \leq 0$.

$\left[-\frac{1}{2}e^{-r^2}\right]_0^2$

A. $\frac{\pi}{2}(e^{-4}-1)$ square units
B. $\frac{\pi}{2}(1-e^{-4})$ square units
C. $\frac{\pi}{4}(1-e^{-4})$ square units
D. $\frac{\pi}{4}(e^{-4}-1)$ square units

43. Find $\int\int_D (x^2+y^2)dA$, where $D=\{(x,y)|0\leq y\leq 4,\ \frac{1}{2}\leq x\leq\sqrt{y}\}$.

$\frac{1}{4}\pi(1-e^{-4})$

A. $\frac{72}{35}$ square units
B. $\frac{180}{35}$ square units
C. $\frac{216}{35}$ square units
D. $\frac{432}{35}$ square units

44. Find the volume lying inside the sphere $x^2+y^2+z^2=2z$, and inside the cone $z^2=x^2+y^2$.

A. 4π
B. 2π
C. π
D. $\frac{\pi}{2}$

45. Evaluate $\int_{-2}^{2}\int_0^{4-x^2}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dzdydx$.

A. 16π sq. units
B. 8π sq. units
C. 4π sq. units
D. 2π sq. units

46. Which of the following point gives a maximum value $f(x,y)=4xy$, subject to $x^2+y^2=8$

$\int_0^4\int_{1/2}^{\sqrt{y}}(x^2+y^2)\,dA$

$z^2 + 2z + 1 = 0$

$4xy = 2y^2\lambda$ $\quad r^2 \quad$ $x^2 - y^2 =$

$4x = 2y\lambda$

$\lambda = \frac{x}{y}$

$\int_0^{2\pi}\int_0^{\pi}\int_0^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

A. $(-2,-2)$
B. $(2,-2)$
C. $(-2,2)$
D. $(1,1)$

47. All the following points are saddle of $f(x,y) = (x^2 - y^3)e^{-x^2-y^2}$ except?

$4xy = 2x^2\left(\frac{2x}{y}\right)$

A. $(0,0)$
B. $\left(\frac{\sqrt{19}}{3\sqrt{3}}, \frac{-2}{3}\right)$
C. $\left(-\frac{\sqrt{19}}{3\sqrt{3}}, \frac{-2}{3}\right)$
D. $\left(0, \frac{-1}{2}\sqrt{6}\right)$

-3 $\quad \begin{pmatrix}2\\-3\end{pmatrix} - \begin{pmatrix}-2\\-1\end{pmatrix} \quad \begin{pmatrix}4\\-2\end{pmatrix}$

ρ^3 $\quad f_x = \frac{\rho^3}{3} \cos(x+y)$ $\quad f_y = \cos(x+y)$

48. Find an equation of the tangent plane to $z = \sin(x+y)$ at the point $(\pi, \pi, 0)$.

$\int_0^4 \int r^2 \cos^2\theta$

A. $(x+\pi) + (y+\pi) + z = 0$
B. $(x-\pi) + (y-\pi) - z = 0$
C. $(x+\pi) + (y-\pi) - z = 0$
D. $(x-\pi) + (y-\pi) + z = 0$

π $\quad 2x + 2y$ $\quad \frac{8}{3}\int_0^{2\pi}\int_0^{\pi} \sin\phi$ $\quad -\cos\phi \,\Big]$

49. Given that $f(x,y) = xe^{xy^2} + \cos(y^2)$, find the gradient function at the point (x,y).

$1+1$ $\quad \frac{16}{3}\, 2\pi$

A. $(e^{xy^2} + xy^2e^{xy^2})\mathbf{i} + (2x^2e^{xy^2} - 2y\sin(y^2))\mathbf{j}$
B. $(e^{xy^2} + xy^2e^{xy^2})\mathbf{i} + (2x^2ye^{xy^2} - 2y\sin(y^2))\mathbf{j}$
C. $(e^{xy^2} + xy^2e^{xy^2})\mathbf{i} + (2ye^{xy^2} + 2y\sin(y^2))\mathbf{j}$
D. $(e^{xy^2} + xy^2e^{xy^2})\mathbf{i} + (x^2ye^{xy^2} - 2y\sin(y^2))\mathbf{j}$

$\begin{pmatrix}2\\-1\end{pmatrix} - \Big($ $\quad \begin{pmatrix}2\\-3\end{pmatrix} - \begin{pmatrix}-2\\-1\end{pmatrix} \quad \begin{pmatrix}4\\-2\end{pmatrix}$

$z - \pi = 1(x-\pi) + 1($

50. Given that $f(x,y) = x^2 - 2xy + y^2$ at the point $(-2,-1)$ in the direction from $A(-2,-1)$ to $B(2,-3)$.

$1(y$ $\quad -2\,(4)$ $\quad \frac{}{\sqrt{6}}$

$4y = \lambda(2x)$

$4x = \lambda(2y)$ $\quad z$

A. $\frac{3}{5}\sqrt{5}$
B. $\frac{-3}{5}\sqrt{5}$
C. $\frac{6}{5}\sqrt{5}$
D. $\frac{-6}{5}\sqrt{5}$

$-2\left(\frac{4}{2\sqrt{5}}\right) + 2\left(\frac{-2}{\sqrt{5}}\right)$ $\quad 2x - 2y$ $\quad -4 - (-2)$

$4xy = \lambda 2x^2$ $\quad z - \pi = x -$

$4xy = \lambda 2y^2$ $\quad z - \pi = x +$

$\begin{pmatrix}-3\\ \end{pmatrix} - \begin{pmatrix}-2\\-2\end{pmatrix}$ $\quad (x-\pi) +$

51. Given that $f(x, y) = x^2 e^y$, $x(t) = t^2 - 1$, and $y = \sin(t)$, find $\frac{df}{dt}$.

A. $4t(t^2-1)e^{\sin(t)} + (t^2-1)^2 e^{\sin(t)} \cos(t)$

B. $4t(t^2+1)e^{\sin(t)} + (t^2-1)^2 \cos(t) e^{\sin(t)}$

C. $4t(t^2-1)e^{\sin(t)} - (t^2-1)e^{\sin(t)} \cos(t)$

D. $4t(t^2-1)e^{\sin(t)} + (t^2-1)e^{\sin(t)} \cos(t)$

$f_y = 2 + t(-4)$

$2 - 4t$

$z = 1 + t($

52. Find an equation of the normal line to $z = 6 - x^2 - y^2$ at the point $(1, 2, 1)$.

$z = 1 + t$

$f_x = -2x$

$f_y = -2y$

$1 + t(-2)$

$1 - 2t$,

A. $x = 1 + 2t,\ y = 2 - 4t,\ z = 1 - t$

B. $x = 1 - 2t,\ y = 2 + 4t,\ z = 1 - t$

C. $x = 1 - 2t,\ y = 2 - 4t,\ z = 1 - t$

D. $x = 1 + 2t,\ y = 2 + 4t,\ z = 1 + t$

53. Given that $f(x, y) = 2x + e^{x^2 - y}$, find the linear approximation of $f(x, y)$ at the point $f(0, 0)$.

$\frac{\partial f}{\partial y} \times \frac{\partial y}{\partial t} + \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t}$

A. $1 + 2x - y$

B. $1 + 2x + y$

C. $1 - 2x - y$

D. $1 - 2x + y$

$x^2 e^y \cdot \cos t$

54. Given that $f(x, y, z) = \frac{2}{\sqrt{x^2+y^2+z^2}}$, find $\frac{\partial f}{\partial y}$.

$+ 2x e^y (2t)$

A. $\frac{-2y}{(x^2+y^2+z^2)^{\frac{1}{2}}}$

B. $\frac{-2y}{(x^2+y^2+z^2)^{\frac{3}{2}}}$

C. $\frac{2y}{(x^2+y^2+z^2)^{\frac{3}{2}}}$

D. $\frac{2y}{(x^2+y^2+z^2)^{\frac{1}{2}}}$

$z - 1 = 2x - y$

$z = 2x - y + 1$

$(t^2-1)^2 e^{\sin t} \cos(t)$

$z = 1 + 2x$

$+ 2(t^2-1) e^{\sin t} (2t)$

$4t(t^2-1) e^{\sin t}$

$+ (t^2-1)^2 e^{\sin}$

55. Evaluate $\lim_{(x,y)\to(0,0)} \frac{x(\cos(y)-1)}{x^3+y^3}$.

A. 0

B. 1

C. 2

$f_x = 2$

$f_x = 2 + 2x e^{x^2-y}$

$f_y = -1 e^{x^2-y}$

-1

12

$2(x-0) + -1(y-0)$

$2x - y$

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$\frac{0}{x^3}$ $\frac{x}{x^3}$ $2(x^2+y^2+z^2)^{-1/2}$

$2 \cdot -\frac{1}{2}(2y)\left((x^2+y^2+z^2)^{3/2}\right.$

D. does not exist

56. Evaluate $\int_3^9 \int_0^1 \int_2^4 \ln(y)dxdydz$.

$\int_0^9 \int_0^1 \ln y(2)\,dy\,dz$ $-\ 2y$

A. 0

B. 1

C. 2

D. does not exist

$x=4$ y $x=2$ $y=1$

$z=$ $y=0,$

$\frac{2t}{\sqrt{4t^2+1}}i +$

57. Find the unit tangent to the curve defined by $r(t) = t^2\mathbf{i} + t\mathbf{j}$.

A. $\frac{-2t}{\sqrt{4t^2+1}}\mathbf{i} + \frac{1}{\sqrt{4t^2+1}}\mathbf{j}$

B. $\frac{2t}{\sqrt{4t^2+1}}\mathbf{i} + \frac{1}{\sqrt{4t^2+1}}\mathbf{j}$

C. $\frac{2t}{\sqrt{4t^2+1}}\mathbf{i} - \frac{1}{\sqrt{4t^2+1}}\mathbf{j}$

D. $\frac{t}{\sqrt{4t^2+1}}\mathbf{i} + \frac{1}{\sqrt{4t^2+1}}\mathbf{j}$

$x\ (4-2)$ $d=$ $\frac{\langle 2t \quad 1\rangle}{\sqrt{2t^2+(1)^2}}$

58. Which of the following is an Hessian matrix of $f(x,y) = x^2y - y^3 + \ln(x)$?

$2\int_3^9 \int_0^1 \ln y\ dy\,dy$

A.
$$\begin{bmatrix} 2y+\frac{1}{x} & 2x \\ 2x & -6y \end{bmatrix}$$

$\frac{1}{y}$ 1

B.
$$\begin{bmatrix} 2y+\frac{1}{x^2} & 2x \\ 2x & -6y \end{bmatrix}$$

$2(9-3)$

C.
$$\begin{bmatrix} 2y-\frac{1}{x^2} & 2x \\ 2x & -6y \end{bmatrix}$$

f_y
$f_{xy} = \sin(xy)$ $\sin(xy)$ $+ x\cos xy$
$+ xy\cos xy$ $\sin(xy)$ y

$f_y = -2x + 2y$

D.
$$\begin{bmatrix} 2y-\frac{1}{x^2} & 2x \\ 2x & 6y \end{bmatrix}$$

f_{yx} $-2y$ $-2x$ $\sin(xy) + xy\cos xy$

$-2(-2) + 2(-1)$

$4 - 2 = 2$

59. Which of the following vector field is **not** conservative?

$f_{xy} = -2y$

A. $2\mathbf{i} + \cos(x)\mathbf{j}$

$f_{yx} = 0$ $f_{xy} = -\sin$ $-2x$ $\ 2e^{xy^2} + xy^2e^{xy^2}$

B. $(x - 2xy)\mathbf{i} + (y^2 - x^2)\mathbf{j}$

$f_x = -2y$ $f_{yx} = -2x$

C. $y\mathbf{i} - x\mathbf{j}$ $=2x$ $f_y = 2y$ $-2y$ $-2x$ $-2x+2y$ $f_{xy} = 1$ $f_{yx} = -1$

D. $y\sin(xy)\mathbf{i} + x\sin(xy)\mathbf{j}$

f_y $-2y$ $y\sin xy + yx\cos yxy$ $x^2e^{xy^2} + 2y\sin(y^2)$

$\begin{pmatrix}-1\end{pmatrix}$ 1

$\sin xy + y$ $2yx\,e^{xy^2} + 2y$

$\begin{pmatrix}-3 \\ 2\end{pmatrix} - \begin{pmatrix}-2 \\ -1\end{pmatrix}$ 2 $=-2$

$\frac{2}{\sqrt{10}}$ $+\ 4$ $\frac{}{\sqrt{10}}i + \frac{2}{\sqrt{10}}j$

60. Given a conservative field $F = (4x - z)\mathbf{i} + (3y + z)\mathbf{j} + (y - x)\mathbf{k}$, find the potential function.

A. $2x^2 - xz + \frac{3}{2}y^2 + c$, where c is a constant of integration

B. $2x^2 - xz + \frac{3}{2}y^2 + yz + c$, where c is a constant of integration

C. $2x^2 + xz + \frac{1}{2}y^2 + yz + c$, where c is a constant of integration

D. $2x^2 + x + y^2 + yz + c$, where c is a constant of integration

$f_x = 4x - z$ $\qquad f_x = 4x - z$

$f_y = 3y + z$ $\qquad f_y = 3$

$3y$

$f_z = y - z$

$f(x, y) = 2x^2 - zx + g(xy, z)$

$+$

60. Given a conservative field $F = (4x - z)\mathbf{i} + (3y + z)\mathbf{j} + (y - x)\mathbf{k}$, find the potential function.

A. $2x^2 - xz + \frac{3}{2}y^2 + c$, where c is a constant of integration

B. $2x^2 - xz + \frac{3}{2}y^2 + yz + c$, where c is a constant of integration

C. $2x^2 + xz + \frac{1}{2}y^2 + yz + c$, where c is a constant of integration

D. $2x^2 + x + y^2 + yz + c$, where c is a constant of integration

$f_x = 4x - z$ $\quad f_x = 4x - z$

$f_y = 3y + z$ $\quad f_y = 3$

$3y$

$f_z = y\,z$

$f(x, y) = 2x^2 - zx + g(xy, z)$