ME 255: Strength of Materials I

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Department of Mechanical Engineering

Faculty of Mechanical and Chemical Engineering

College of Engineering

Course Introduction

ME 255 Strength of Materials is a second year course offered in first semester for the degree programme in Mechanical Engineering

Course Objectives

The objectives of this course are to

1. Determine centroids and centre of gravity of single and composite bodies.
2. Calculate simple stresses and strains of simple determinant and indeterminate structures within the elastic region.
3. Know and calculate thermal stresses and strains in simple determinant and indeterminate structures.
4. Know and calculate torsional stresses and strains in circular solid and hollow shafts.
5. Know the different types of loading and apply Mohr's stress and strain circles in combining stresses.
6. Draw shear force and bending moment diagrams for different beams with different supports.
7. Know how to apply the theories of static failure to compute failure stresses and strains

Course Outline

1. Introduction
2. Tension in Bars
3. Torsion in Shaft
4. Bending in Beams
5. State of Stress and Strain
6. Failure Criteria

Assessment

A. Continuous Assessment (30% of Total)

1. Mid-Semester Exams (1/3 of Conti. Assess.)
2. Quizzes (1/3 of Conti. Assess.)
3. Homework/Assignment (1/3 of Conti. Assess.)
4. Contribution to Class Discussions (Bonus, max 10%, only included if sum of items 1 & 2 is less than 30%)

B. End of Semester Exams (70% of Total)

– Multiple Choice
– True/False
– Numeric Response
– Fill-in Spaces

Note

- You will NOT be allowed to write the End of Semester Exams if you miss at least four lectures without permission.
- All homework /assignments are due exactly two week after the assigned day. No excuse will be tolerated.

Schedule

Assignments

There will be six (6) group assignments, 1 from each lecture.

The date of submission is 2 weeks after each lecture

Quizzes

There will be six (6) quizzes, 1 from each lecture.

Each quiz will take place the second Saturday @ 10 pm after the end of each lecture.

It will be online examination and open from 10 pm to 12 am. It is 1 attempt

It will cover the whole unit comprising of multiple choice, true/false and numeric response

Mid Semester

There will be one (1) mid-semester examination for the whole course.

The date of examination will be announced by the College Examination officer

It will cover the area taught comprising of multiple choice, true/false and numeric response

THANK YOU

LECTURE 1

INTRODUCTION

Lecture Outline

Stress and Strain

Review of Moment of Area

Review of Equilibrium of Rigid Bodies

Classification of Stresses and Strains

- Stresses can be classified based on two conditions.
- These are:
  1. The type of loading in the machine member or the structure. Therefore, the following types of stresses can be identified with the corresponding type of loading;

     a. Axial stress due to axial loading,

     b. Bending stress due to bending in beams,

     c. Shearing stress due to torsion in shafts,

     d. Shearing stress due to transverse or shear load

     e. Hood/Transverse stress in pressure vessels

     f. Axial stress in pressure vessels

Classification of Stresses and Strains

2. How the force-couple acts on the surface of the machine or structure. Thus, we have:

   a. Normal stress acting perpendicular to the surface area

   b. Shearing stress acting tangential to the surface area

   c. Bearing stress acting on connecting members (bolts. rivets, pins)

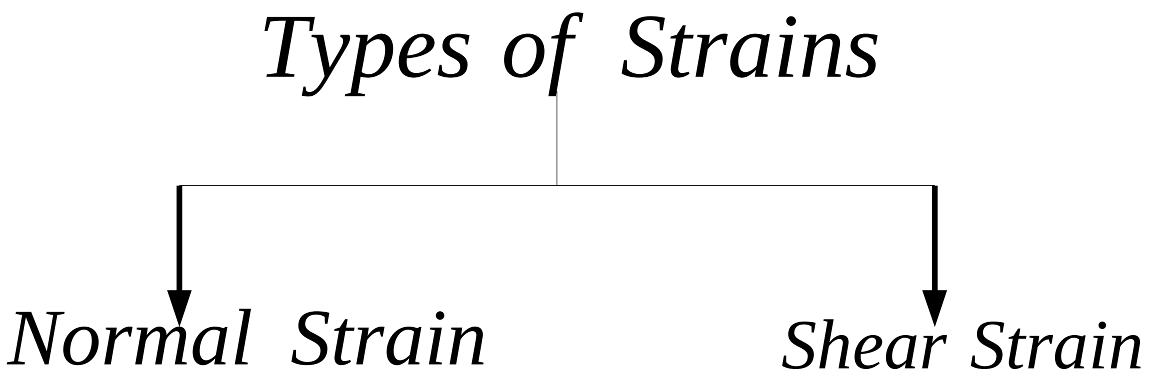
Classification of Stresses and Strains

Types of Stresses

- *Normal Stress*
  - *Axial stress due axial load*
  - *Bending stress due moment*
  - *Axial stress in pressure vessel*
  - *Transverse stress in pressure vessel*
- *Shearing Stress*
  - *Shear stress due transverse load*
  - *Shear stress due torsion*

Classification of Stresses and Strains

Similarly, strains can be classified and presented in the figure below.



Review of Moment of Area

First Moment and Centroid of an Area

First Moment and Centroid of a Composite Area

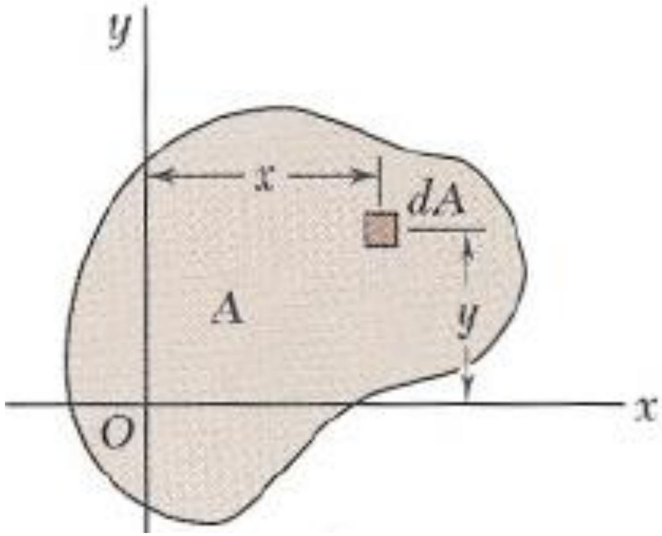
Second Moment of Area and Radius of Gyration

Parallel-Axis Theorem

Moment of Inertia of a Composite Area

First Moment and Centroid of an Area

Consider an area A located in the xy plane as shown in Figure 1-1.



The first moment of area A about x-axis as the integral

$$Q_x = \int_A y dA$$

The first moment of area A about y axis as the integral

$$Q_y = \int_A x dA$$

However,

$$\int_A y dA = A\bar{y}$$

$$\int_A x dA = A\bar{x}$$

First Moment and Centroid of an Area

Comparing equations 1-1 and 1-2 with equations 1-3 and 1-4,

$$Q_x = A\bar{y}$$

$$Q_y = A\bar{x}$$

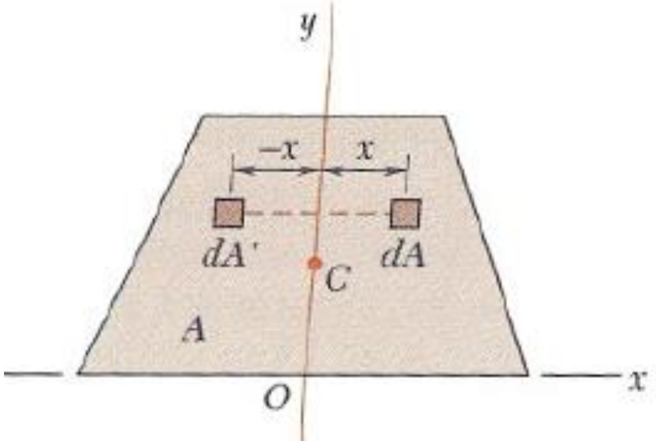
First Moment and Centroid of an Area

- When an area possesses an axis of symmetry,
  - the first moment of area with respect to that axis is zero.
  - It follows that the integral in equation 2-2 is zero, that is

$$Q_y = 0$$

  - It also follows from the equation 1-3 that

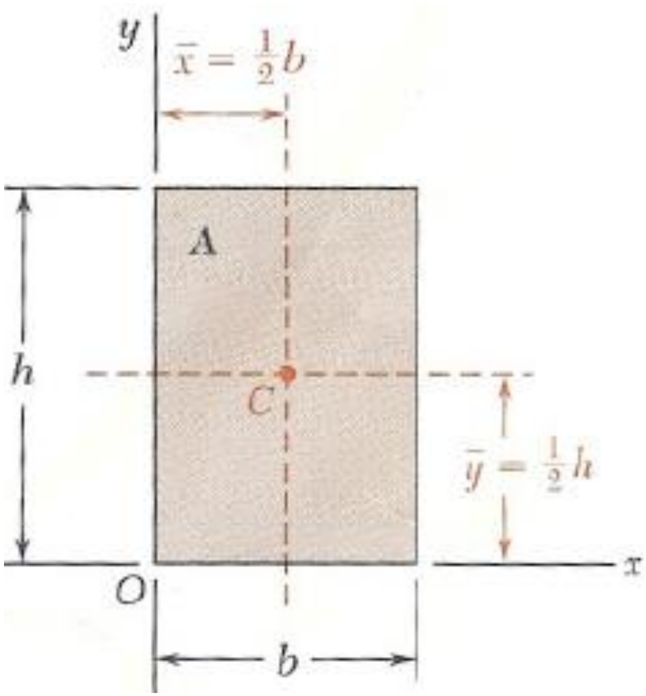
$$\bar{x} = 0$$



Thus, if an area A possesses an axis of symmetry, its centroid C is located on that axis.

Example 1-1

Determine the first moments of area of a rectangular area as shown in Figure E1-1



Solution

We know that

$$A = bh$$

$$\bar{x} = \tfrac{1}{2}b$$

$$\bar{y} = \tfrac{1}{2}h$$

Thus,

$$Q_x = A\bar{y} = (bh)\left(\tfrac{1}{2}h\right) = \tfrac{1}{2}bh^2$$

Similarly,

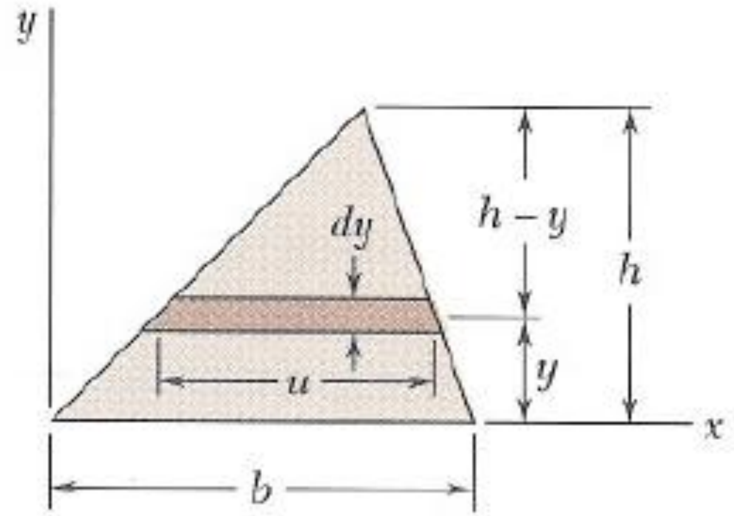
$$Q_x = A\bar{x} = (bh)\left(\tfrac{1}{2}b\right) = \tfrac{1}{2}b^2h$$

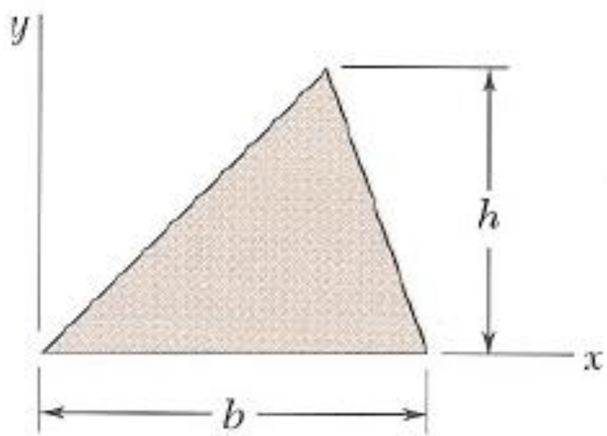
Example 1-2

For the triangular area of Figure E1-2, determine

a. the first moment Q_x of the area

b. the ordinate of the centroid of the area.





(a) From similar triangles, we have

$$\frac{u}{b} = \frac{h-y}{h} \Rightarrow u = b\,\frac{h-y}{h}$$

and

$$dA = u\,dy = b\,\frac{h-y}{h}\,dy$$

Example 1-2 Continues

The first moment of the with respect to the x-axis is

$$Q_x = \int_A y\,dA = \int_0^h yb\frac{h-y}{h}dy = \frac{b}{h}\int_0^h \left(hy - y^2\right)dy$$

$$\Rightarrow Q_x = \frac{b}{h}\left[h\frac{y^2}{2} - \frac{y^3}{3}\right]_0^h = \frac{bh^2}{6}$$

The area of a triangle is

$$A = \tfrac{1}{2}bh$$

From equation 2-5,

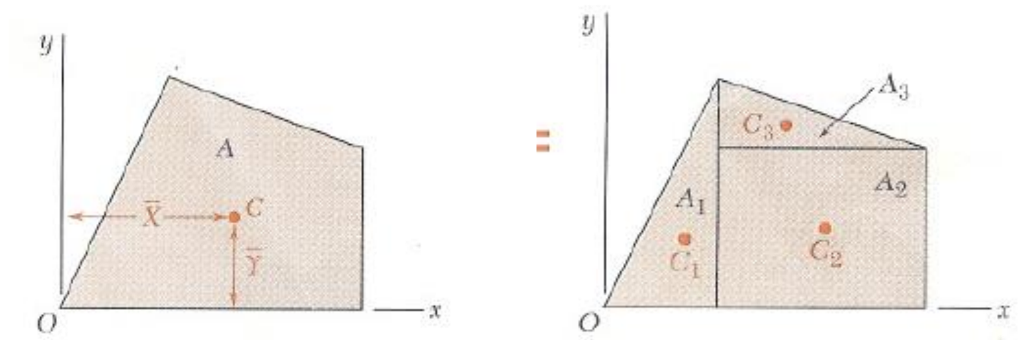
$$Q_x = A\bar{y}$$

$$Q_x = \left(\tfrac{1}{2}bh\right)\bar{y} = \frac{1}{6}bh^2 \Rightarrow \bar{y} = \tfrac{1}{3}h$$

First Moment and Centroid of a Composite Area

Consider an A, such that the trapezoidal area shown in Figure 1-2 which may be divided into simple geometric shapes.

$$Q_x = \sum A_i \bar{y}_i \qquad Q_y = \sum A_i \bar{x}_i$$



$$Q_x = A\bar{Y} \qquad Q_y = A\bar{X}$$

$$A\bar{Y} = \sum_i A_i \bar{y}_i \qquad AX = \sum_i A_i x_i$$

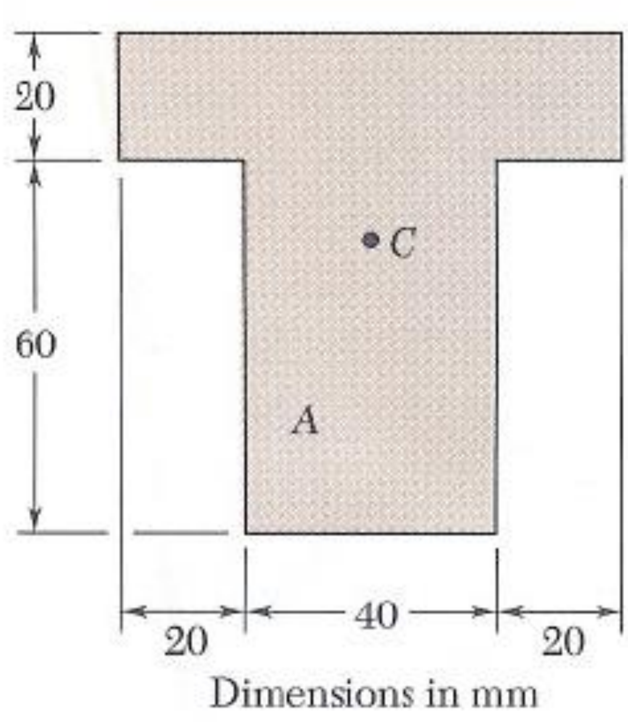
$$Q_x = \int_A y\,dA = \int_{A_1} y\,dA + \int_{A_2} y\,dA + \int_{A_3} y\,dA$$

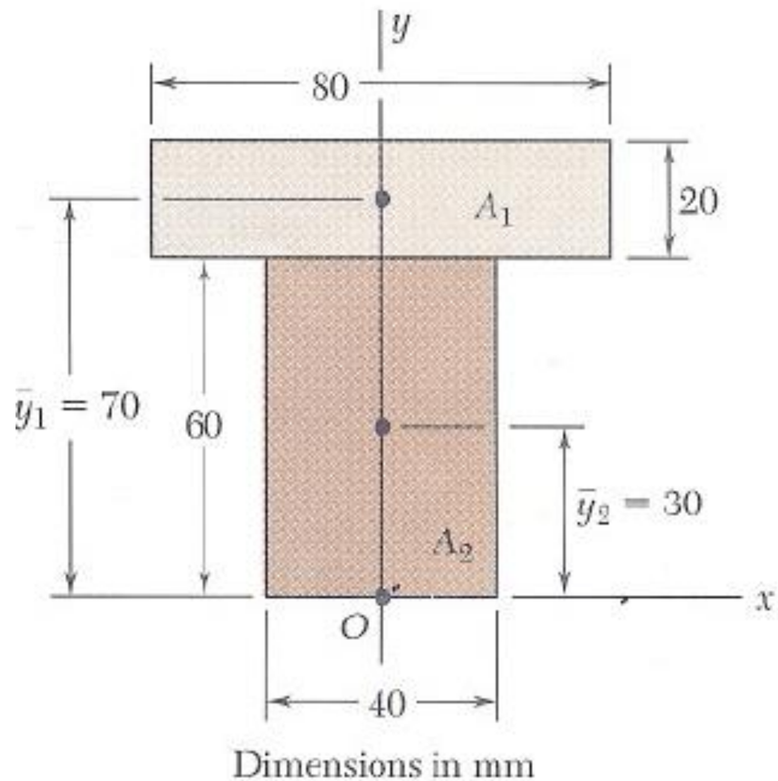
$$Q_x = A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3$$

$$\bar{X} = \frac{\sum_i A_i \bar{x}_i}{\sum_i A_i} \qquad \bar{Y} = \frac{\sum_i A_i \bar{y}_i}{\sum_i A_i}$$

Example 1-3

Determine the first moments through the centroidal axes and hence locate the centroid C of the area A as shown in Figure E1-3.





Example 1-3 Continue

Solution

Selecting the coordinate axes shown below, we note that the centroid C must be located on the y-axis, since this axis is an axis of symmetry, thus,

	A, mm^2	mm	mm^3
A_1	1600	70	**112000**
A_2	2400	30	**72000**
Summation	**4000**		**184000**

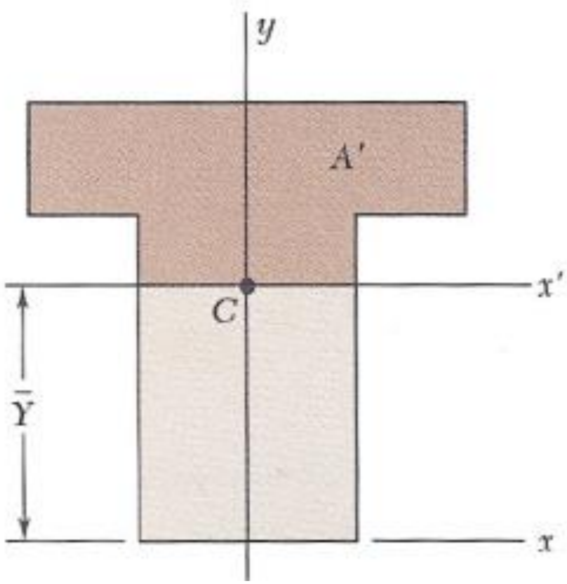
$$\overline{Y} = \frac{\sum_i A_i \overline{y}_i}{\sum_i A_i} = \frac{184000 mm^3}{4000 mm^2} = 46\ mm$$

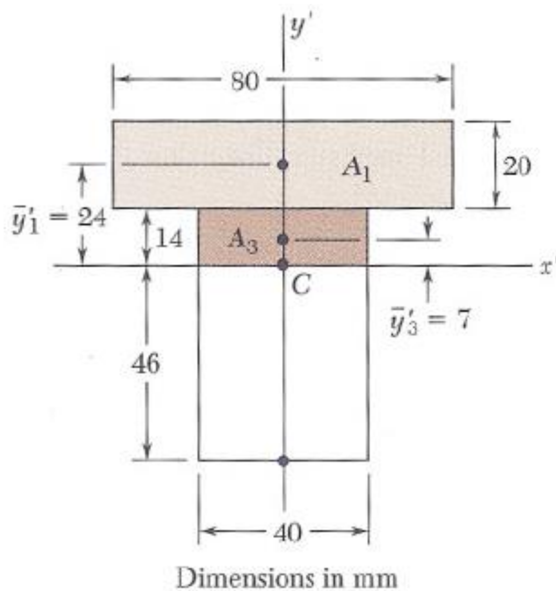
Example 1-4

Determine the first moment of A' with respect to the x' axis as shown in Figure E1-4.

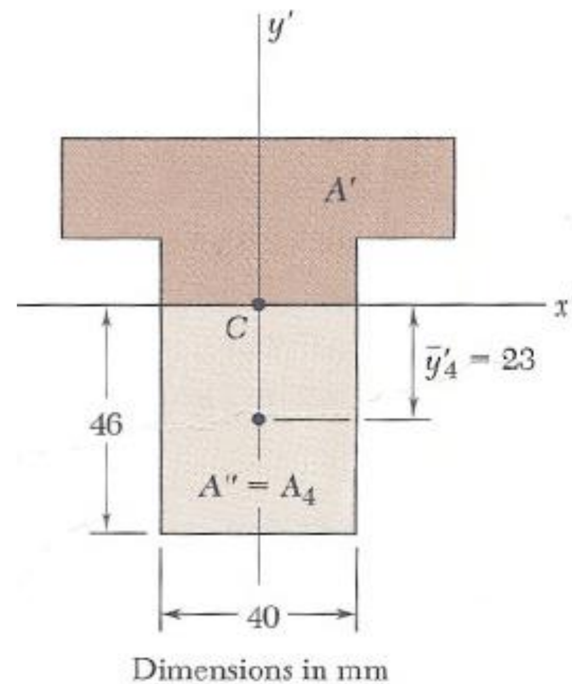
Solution

1. Referring to the area A as in example 2-3, we consider the horizontal x' axis through its centroid C.
2. Such an axis is called a centroidal axis.
3. Denoting by A' the portion of A located above that axis as shown in Figure (a),





(a)



(b)

Example 1-4 Continues

$$Q'_{x'} = A_1 \overline{y'}_1 + A_3 \overline{y'}_3$$

$$Q'_{x'} = (20X80)(24) + (14X40)(7)$$

$$= 42300mm^3$$

Second Moment of Area and Radius of Gyration

- The second moment, or moment of inertia, of the area A with respect to the x-axis is defined

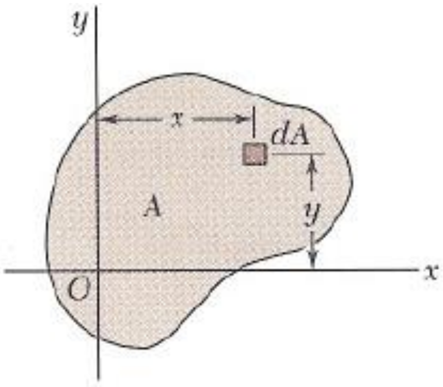
$$I_x = \int_A y^2 dA$$

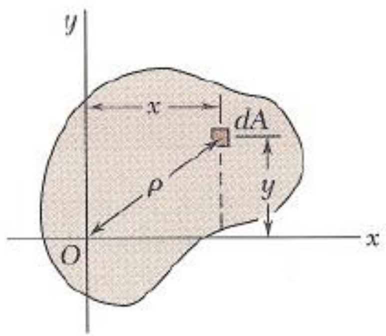
- The polar moment of inertia of the area A with respect to point O as

$$J_0 = \int_A \rho^2 dA$$

- The second moment or moment of inertia, of A with respect to the y axis is defined as:

$$I_y = \int_A y^2 dA$$





Second Moment of Area and Radius of Gyration

- Noting that

$$\rho^2 = x^2 + y^2$$

- we write

$$J_0 = \int_A \rho^2 dA = \left[\begin{array}{l} \int_A \left(x^2 + y^2\right) dA \\ \int_A x^2 dA + \int_A y^2 dA \end{array} \right]$$

-

$$\Rightarrow J_0 = I_x + I_y$$

- The radius of gyration of an area A with respect to the x axis is defined as the quantity r_x that satisfies the relation

$$I_x = r_x^2 A \Rightarrow r_x = \sqrt{\frac{I_x}{A}}$$

- Similarly

$$I_y = r_y^2 A \Rightarrow r_y = \sqrt{\frac{I_y}{A}}$$

- Also,

$$I_0 = r_0^2 A \Rightarrow r_0 = \sqrt{\frac{I_0}{A}}$$

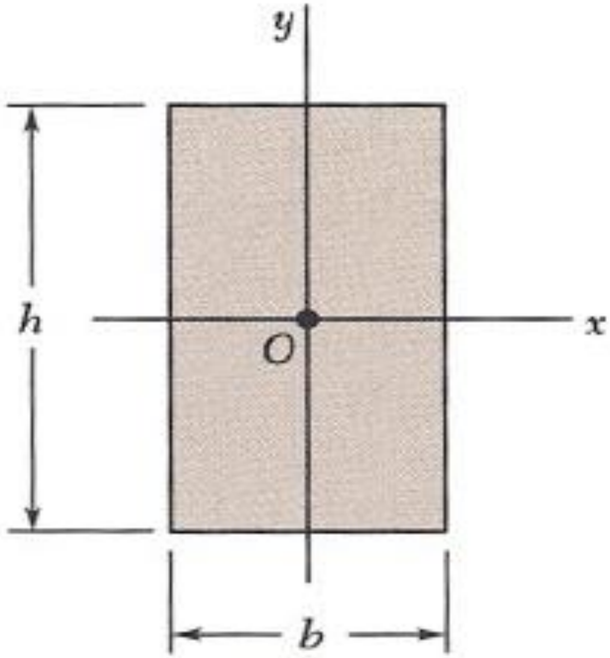
- Then,

$$r_0^2 = r_x^2 + r_y^2$$

Example 1-5

For the rectangular area of Figure E1-5, determine

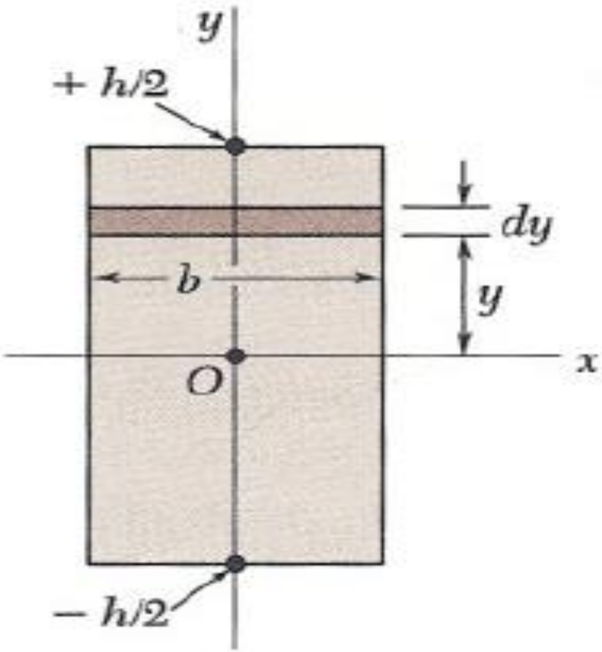
a. the moment of inertia I_x of the area with respect to the centroidal x axis,
b. the corresponding radius of gyration r_x.



Solution

a. Moment of inertia I_x

We select as an element of area a horizontal strip of length b and thickness dy as shown below.



Example 1-5 Continues

Then, $$dI_x = y^2 dA = y^2 (bdy) by^2 dy$$

Integrating
$$I_x = \int_{-h/2}^{+h/2} by^2 dy = \tfrac{1}{3} b \left[y^3 \right]_{-h/2}^{+h/2}$$

$$\Rightarrow I_x = \tfrac{1}{3} b \left(\frac{h^3}{8} + \frac{h^3}{8} \right) = \tfrac{1}{12} bh^3$$

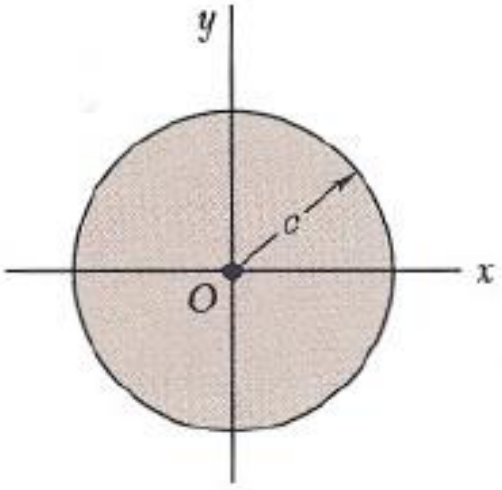
b. ***Radius of Gyration r**$_x$*.

From equation 2-17, we have

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\left(\tfrac{1}{12} bh^3\right)}{(bh)}} = h\sqrt{12}$$

Example 1-6

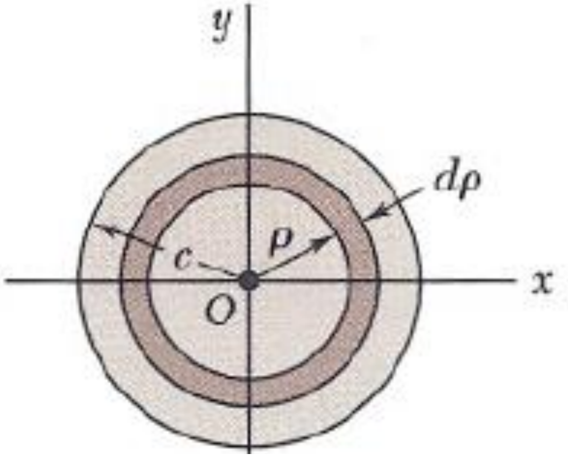
For the circular area of Figure E2-6, determine (a) the polar moment of inertia Jo; (b) the rectangular moment of inertia I_x and I_y.



Solution

a. Polar Moment of Inertia

We select as an element of area and ring of radius p and thickness dρ as shown in the Figure.



The polar moment of inertia of the ring is

$$dJ_0 = \rho^2 dA = \rho^2 \left(2\pi\rho d\rho\right) = 2\pi\rho^3 d\rho$$

Example 1-6 Continues

Integrating in p from 0 to c,

we write

$$J_0 = 2\pi \int_0^c \rho^3 d\rho = 2\pi \left[\frac{\rho^4}{4} \right]_0^c = \tfrac{1}{2} \pi c^4$$

b. Rectangular Moments of Inertia

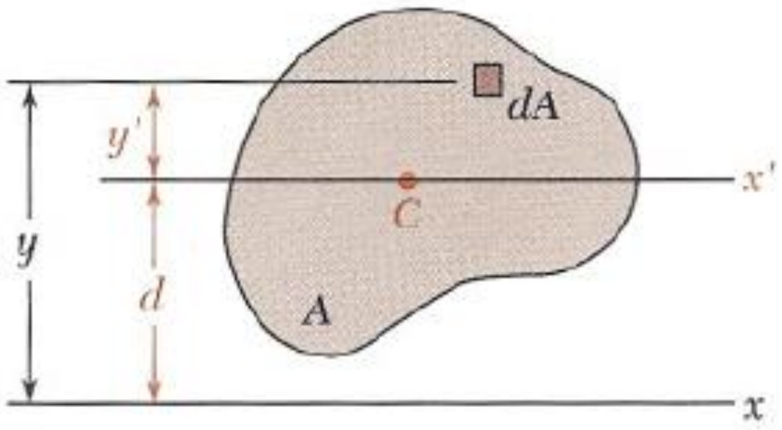
Because of the symmetry of the circular area, we have $I_x = I_y$. Recalling equation 2-15, we write

$$J_0 = I_x + I_y = 2I_x \quad \Rightarrow 2I_x = \tfrac{1}{2} \pi c^2$$

$$I_x = I_y = \tfrac{1}{4} \pi c^4$$

Parallel-Axis Theorem

- Consider the moment of inertia I_x of an area A with respect to an arbitrary x-axis as shown in Figure 1-11



- Denoting by y the distance from an element of area dA to that axis we recall that

$$I_x = \int_A y^2 dA$$

Parallel-Axis Theorem

- Denoting by y' the distance from the element dA to that axis

- we write

$$I_x = \int_A y^2 dA = \int_A (y' + d)^2 dA$$

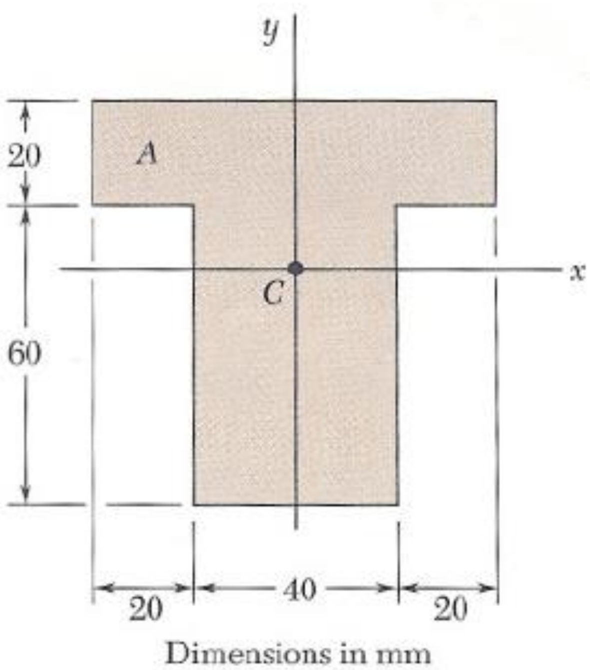
$$I_x = \int_A y'^2 dA + 2d \int_A y dA + d^2 \int_A dA$$

- We have, therefore

$$I_x = I_{CG} + Ad^2$$

Example 1-7

Determine the moment of inertia I_x of the area shown with respect to the centroidal x-axis as shown in Figure E1-7.



Solution

Location of centroid

1. The centroid C of the area must first be located.

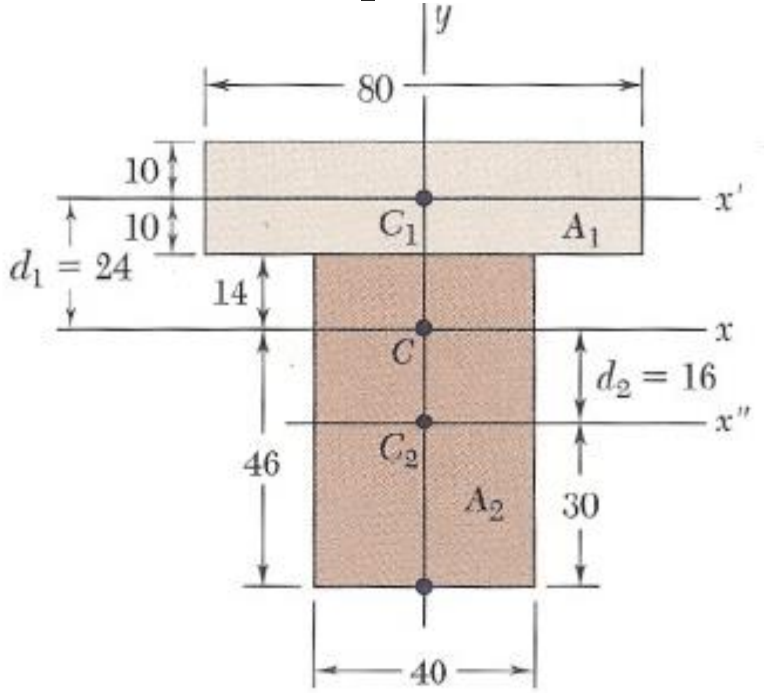
2. However, this has already been done in example 1-3 for the given area.

3. We recall from that example hat c is located 46 mm above the lower edge of the area A.

Example 1.7 Continues

Computation of Moments of Inertia

We divide the area a into the two rectangular areas A_1 and A_2 in the figure below, and compute the moments of inertia of the area with respect to the x-axis.



Rectangular area A_1

To the obtained the moment of inertia $(I_x)_1$ of A_1 with respect to the x axis , we first compute the moments of inertia of A_1 using the parallel axis theorem (Eq.), we have

$$(I_x)_1 = \tfrac{1}{12} bh^3 + A_1 d_1^2$$

$$= \left\{ \left[\tfrac{1}{12}(80)(20)^3\right] + \left[(80)(20)(24)^2\right] \right\}$$

$$= 975000 mm^4$$

Example 1.7 Continues

Rectangular Area A_2

Computing the moments of inertia of A_2 with respect to its centroidal axis, and using the parallel axis theorem to transfer it to the x-axis we have $(I_x)_2 = \frac{1}{12}bh^3 + A_2 d_2^2$

$$= \left\{ \begin{array}{l} \left[\frac{1}{12}(40)(60)^3 \right] \\ + \left[(40)(60)(16)^2 \right] \end{array} \right\}$$

$$= 1334000 mm^4$$

Entire Area A

Adding the values computed for the moments of inertia of A_1 and A_2 with respect to the x axis, we obtain the moment of inertia I_x of the entire area:

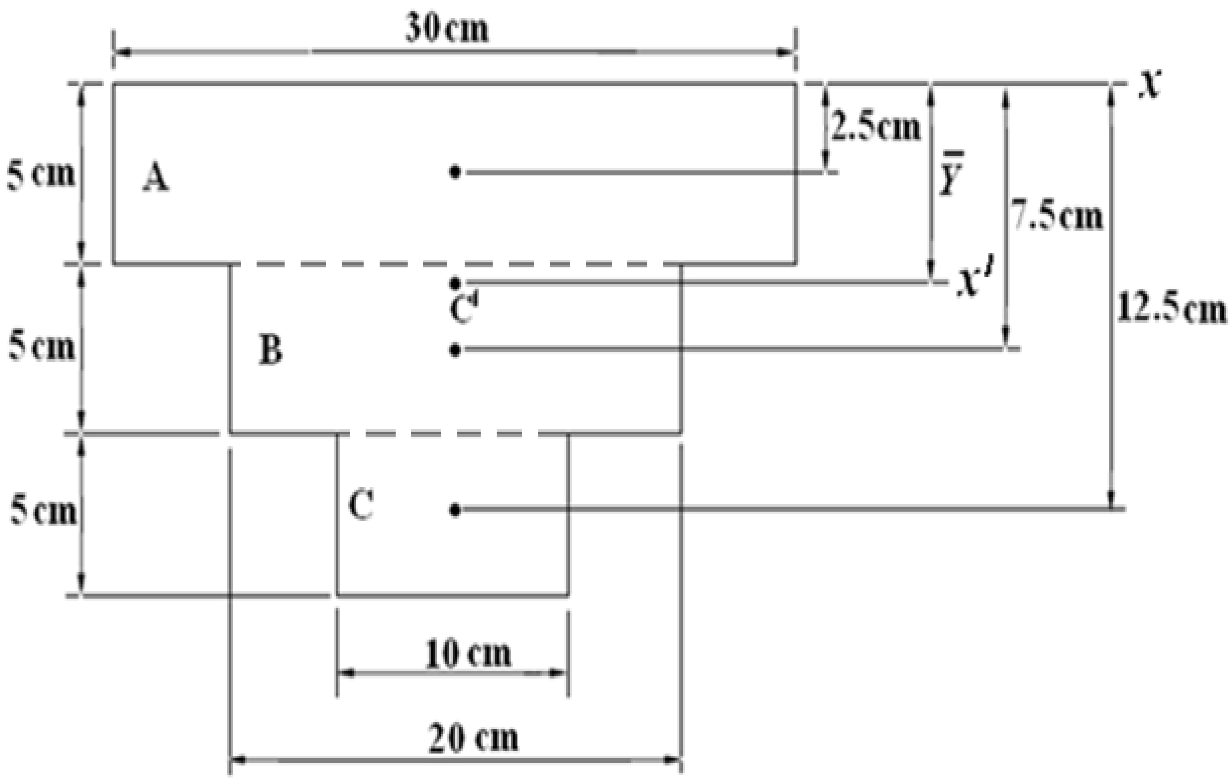
$$I_x = (I_x)_1 + (I_x)_2$$

$$I_x = 975000 + 1334000 mm^4$$

$$\Rightarrow I_x = 2.31 X 10^6 \ mm^4$$

Example 1-8

Determine the centroid and the moment of inertia of the composite area in Figure E 1-8.



Example 1.8 Continues

Section	Area [cm^2]	$\bar{Y}$ [cm]	$\bar{Y}$A [cm^3]
A	5 x 30 = 150	2.5	375
B	5 x 20 = 100	7.5	750
C	5 x 10 = 50	12.5	625
Sum	300		1750

Example 1-8 Continues

$$\bar{Y}\sum A = \sum \bar{Y}\,\mathrm{A} \Rightarrow \bar{Y} = \frac{\sum \bar{Y}\mathrm{A}}{\sum \mathrm{A}} = \frac{1750}{300} = 5.833333 \approx 5.83 \text{ cm}$$

$$I_{x'} = \sum_{i=1}^{3}(I + Ad^2) = \sum\left(\frac{bh^3}{12} + Ad^2\right)$$

$$= \frac{30\times5^3}{12} + 30\times5\times(5.83 - 2.5)^2 + \frac{20\times5^3}{12} + 20\times5\times(7.5 - 5.83)^2 +$$

$$+ \frac{10\times5^3}{12} + 10\times5\times(12.5 - 5.83)^2$$

$$= (1975.8350 + 487.2233 + 2328.6117) \text{ cm}^4$$

$$= 4791.67 \text{ cm}^4$$

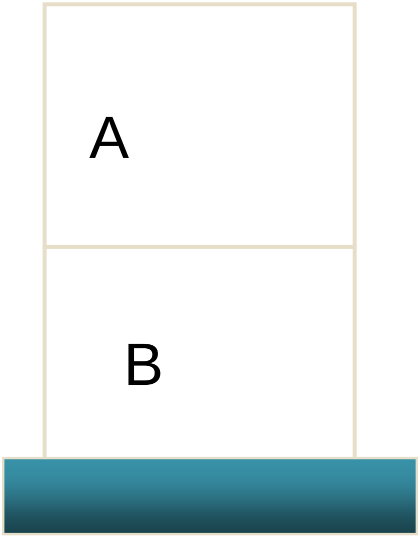
Equilibrium of Rigid Bodies

Free-body Diagram

Reactions at Supports and Connections for 2D Bodies

Equilibrium Conditions Two-force body

Free-Body Diagram



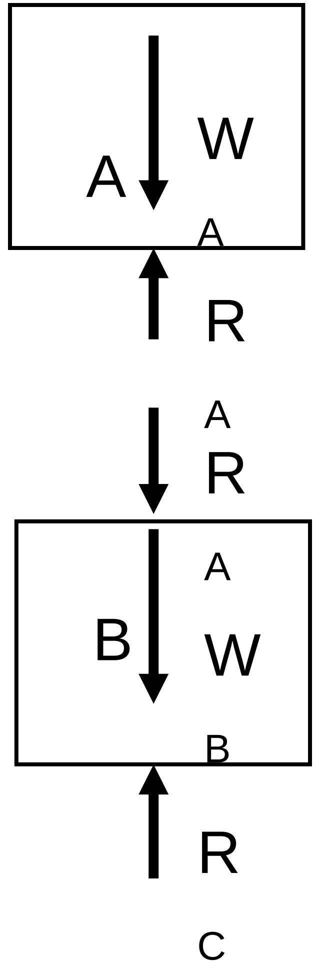
Steps for Drawing a Free-body Diagram

1. Decide which body (or group of bodies) is under consideration , imagine it to be isolated from all other bodies, and sketch the outlined shape of the body.
2. Indicate by means of arrows all external forces and moments acting on the body. This should include (a) the weight of the body, (b) all external forces (c) reactions at supports and other contacts with other bodies.
3. For each unknown force, indicate its point of application and assumed a direction, if it is unknown.
4. Include the dimensions and angles needed for computing moments of forces and resolving forces.

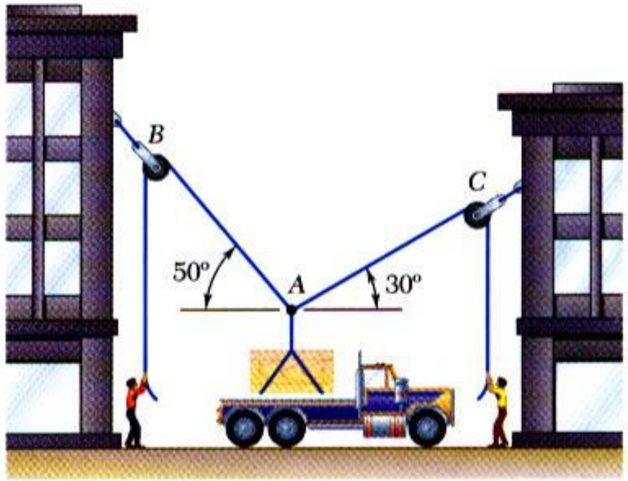
Free-Body Diagram

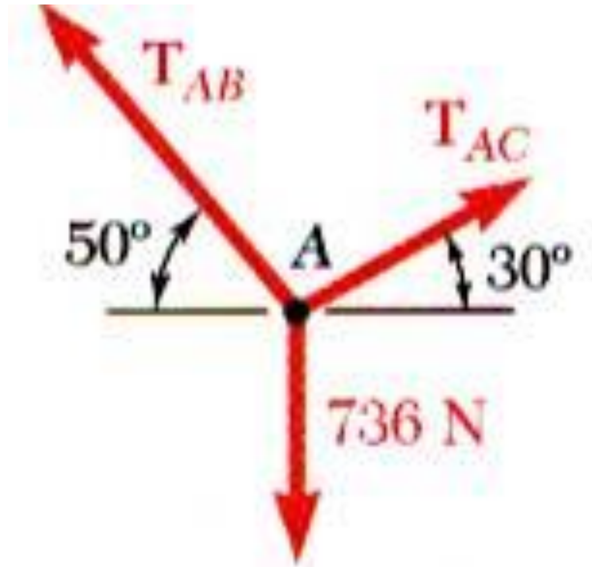
5. The weight of a body always acts vertically downward through the centre of gravity of the body.

6. Forces acting at joints should considered as internal forces and should not be shown if the joint is not separated. Once the joint is separated, the forces acting at the joints must consider as external forces and indicated on the free-body diagram



Free-Body Diagram





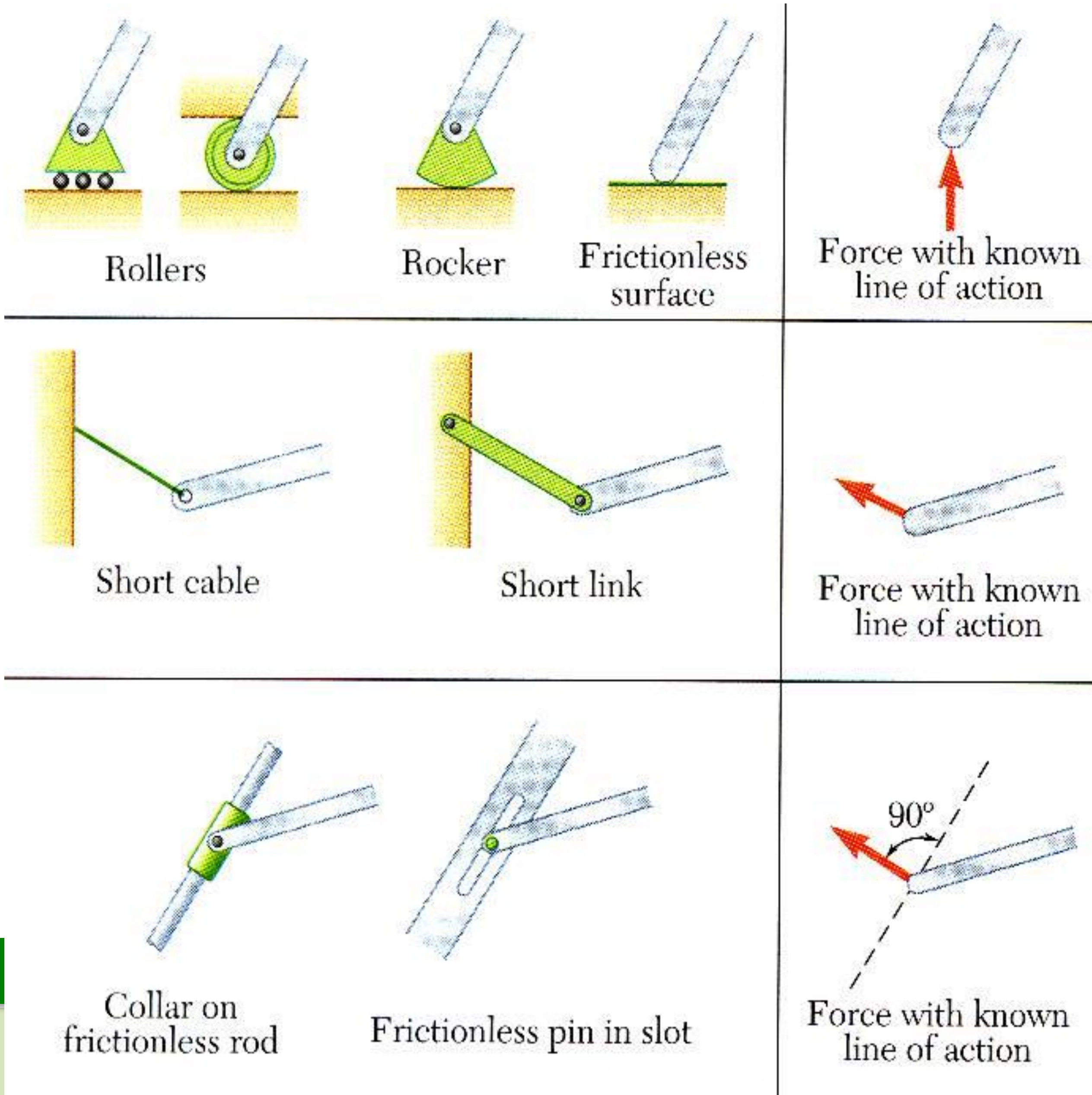
Space Diagram

A sketch showing the physical conditions of the problem.

Free-Body Diagram

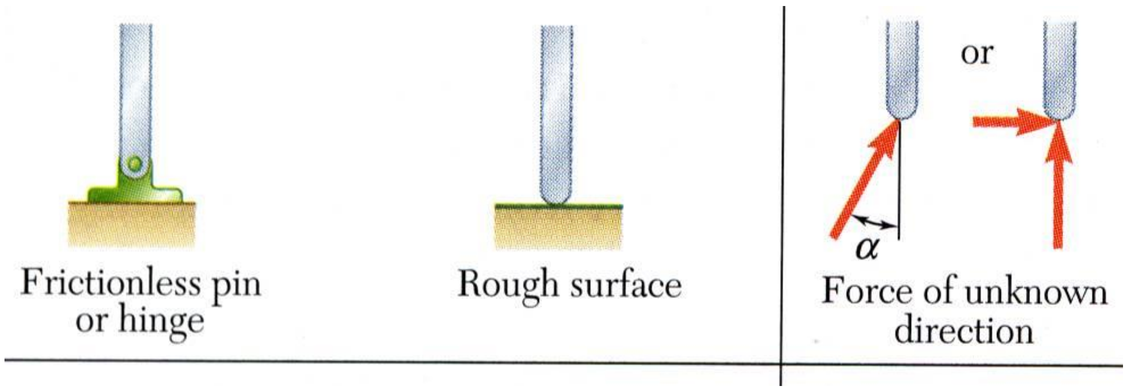
A sketch showing only the forces on the selected particle.

Reactions at Supports and Connections for a 2D Structure



Reactions equivalent to a force with known line of action.

Reactions at Supports and Connections for a 2D Structure



Reactions equivalent to a force of unknown direction and magnitude.

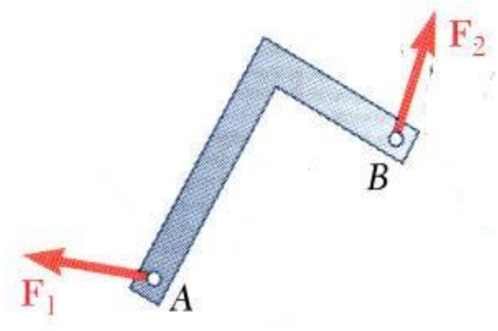
Reactions equivalent to a force of unknown direction and magnitude and a couple of unknown magnitude

Equilibrium Conditions for a Particle

A particle is in equilibrium if

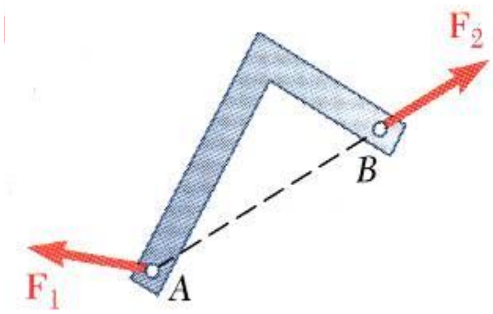
1. it is at rest relative to an initial reference frame
2. the body moves with constant velocity along a straight line relative to an initial frame

Equilibrium of a Two-Force Body



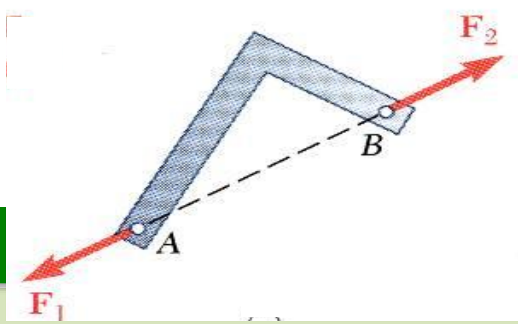
Consider a plate subjected to two forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$

For static equilibrium, the sum of moments about *A* must be zero. The moment of $\boldsymbol{F_2}$ must be zero.



It follows that the line of action of $\boldsymbol{F_2}$ must pass through *A*.

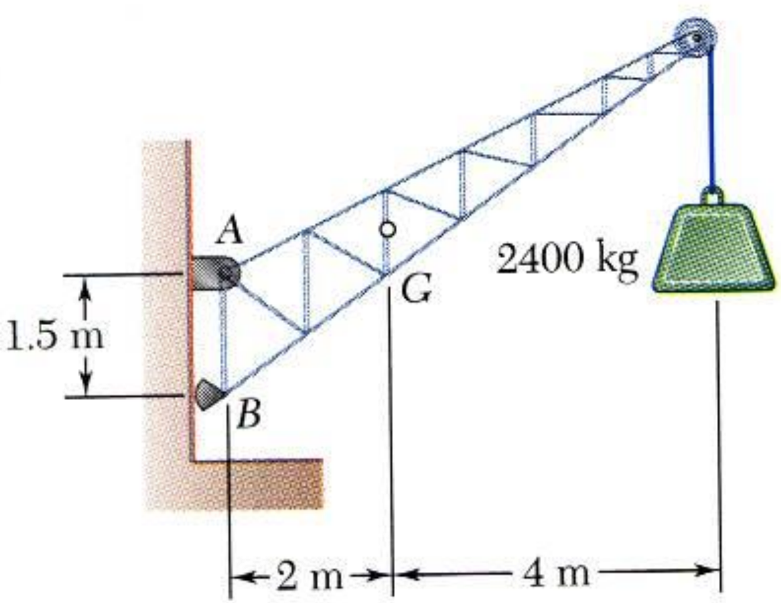
Similarly, the line of action of $\boldsymbol{F_1}$ must pass through *B* for the sum of moments about *B* to be zero.

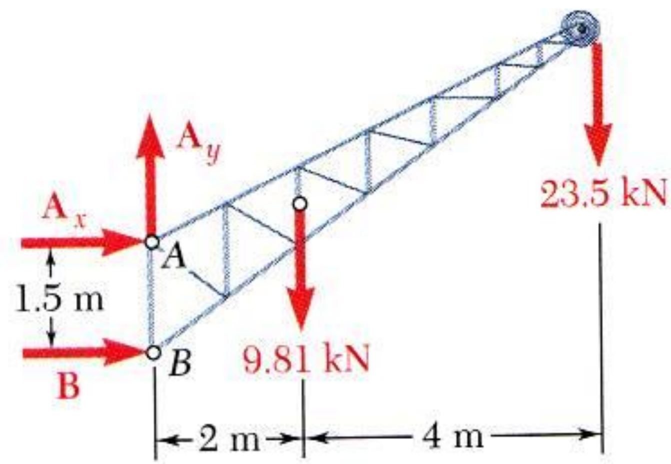


Requiring that the sum of forces in any direction be zero leads to the conclusion that $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ must have equal magnitude but opposite sense

Example 1-9

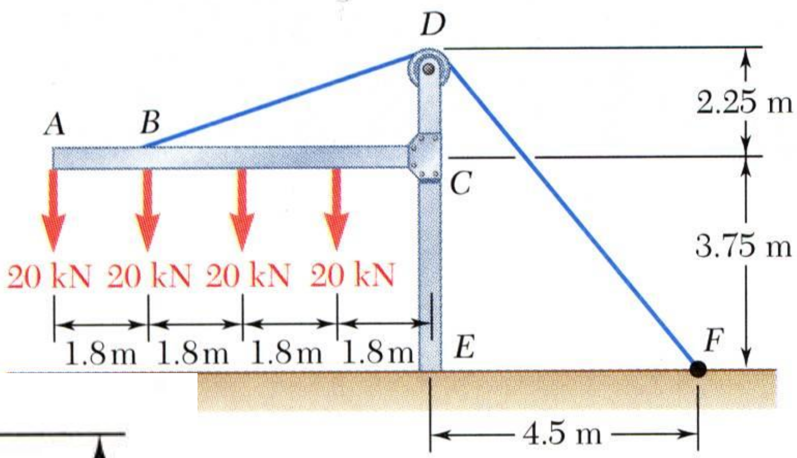
A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*. Determine the components of the reactions at *A* and *B*.

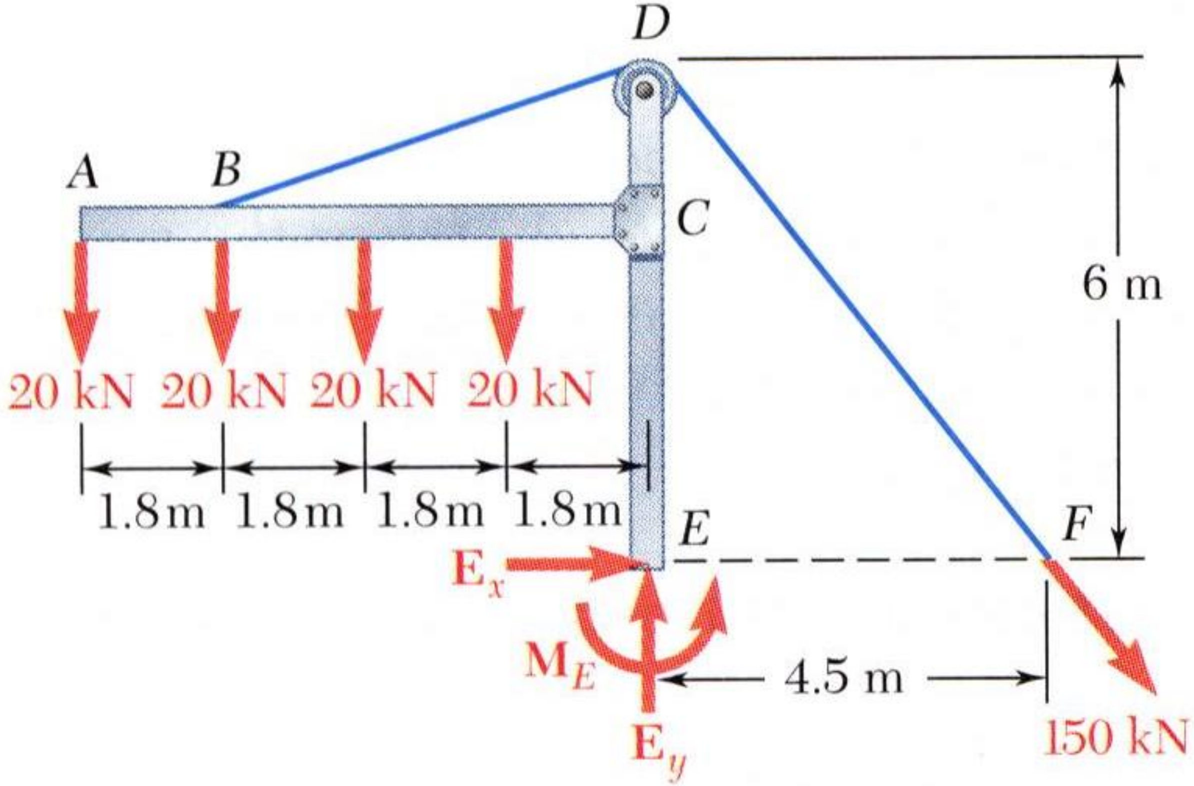




Example 1-10

The frame supports part of the roof of a small building. The tension in the cable is 150 kN. Determine the reaction at the fixed end *E*.





THANK YOU

LECTURE 2

TENSION IN STRUCTURAL MEMBERS

Lecture Outline

Stress and Strain within the Elastic Limits

Thermal Deformation

Variable Load

Tensile Test

Stress and Strain within the Elastic Limits

- By considering engineering structures as deformable and analyzing the deformations in their various members, it will be possible for as to

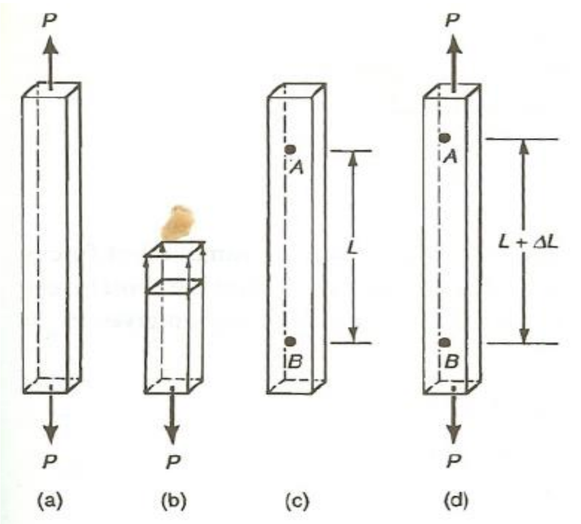
  i. compute forces which are statically indeterminate, indeterminate within the framework of statics.

  ii. determine the actual distribution of stresses within a member it is thus, necessary to analyze the deformation which take place in that member.

- We will consider the deformations of a structural member such as a rod, bar, or plate under axial loading

Stress and Strain within the Elastic Limits

- If a force P is applied to a member as shown in Figure 2-1 (a),



- The force per unit area is called the stress and is given the symbol **σ**.
- Thus,

$$\sigma = P/A$$

- The ultimate stress is therefore

$$\sigma_{ult} = \frac{P_{ult}}{A}$$

- When the specimen is subjected to load, P very beyond the ultimate, necking of the material occurs and then rapture.

Stress and Strain within the Elastic Limits

- Assuming the area of the neck is A_n, the yielding stress is given as

$$\sigma_y = \frac{P_y}{A_n}$$

Example 2-1

A tensile is carried out on a bar of mild steel of diameter 2 cm. The bar yield under a load of 80 kN. It reaches a maximum load of 150 kN, and breaks finally at a load of 70 kN. Find the following:

a. the tensile stress at the yield point
b. the ultimate tensile stress
c. the average stress at the breaking point if the diameter of the fractured neck is 1 cm.

Solution

The initial cross-section of the bar is

$$A_i = \frac{\pi}{4} d_i^2 = \frac{\pi}{4}(0.02)^2 = 0.000314 m^2$$

Example 2-1 Continues

a. the average tensile stress at yielding point

$$\sigma_y = \frac{P_y}{A_i} = \frac{80000}{0.000314} = 254MPa$$

b. the ultimate stress is the stress at the maximum load,

$$\sigma_u = \frac{p_{\max}}{A_i} = \frac{150000}{0.000314} = 477MPa$$

c. the cross-sectional area in the fractured neck is

$$A_f = \frac{\pi}{4} d_f^2 = \frac{\pi}{4}(0.01)^2 = 0.0000785m^2$$

The average stress at the breaking point is

$$\sigma_f = \frac{P_f}{A_f} = \frac{70000}{0.0000785} = 892MPa$$

Deformation under Tension & Compression

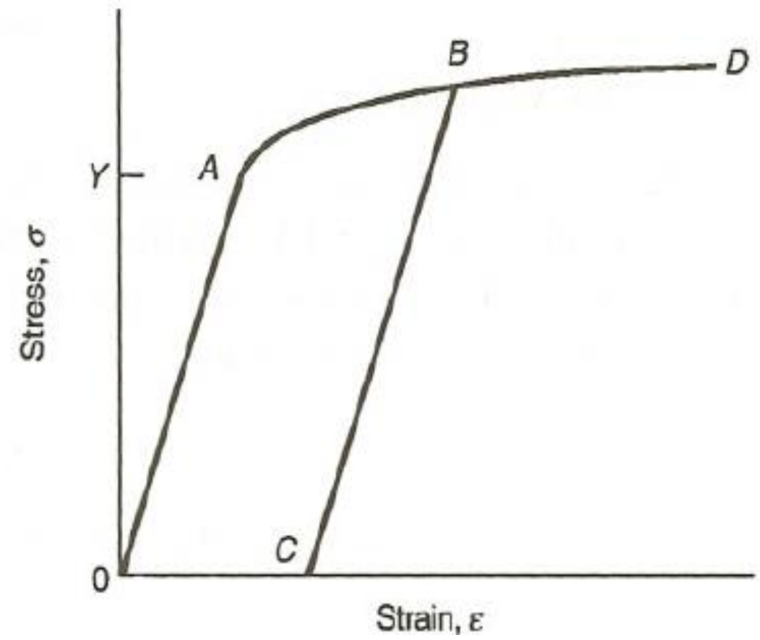
- A quantity measuring the intensity of deformation and bring independent of the original length L is the strain **ε**, defined as:

$$\varepsilon = \Delta L / L = \delta / L$$

- where **ΔL** is denoted as δ.

- The relationship between stress and strain is determined experimentally.

- A typical plot of stress versus strain is shown in Figure 2-2.



Modulus of Elasticity

- The region of greatest concern is that below the yield point.

- The slope of the line between O and A is called the modulus of elasticity and is given the symbol **E**, so $\sigma = E\varepsilon$

- The relation of the applied force in a member to its axial deformation can be found by inserting the definitions of the stress [Eq. (2-1)] and the axial strain [Eq. (2-4)] into Hooke's Law [Eq.(2-5)],which gives

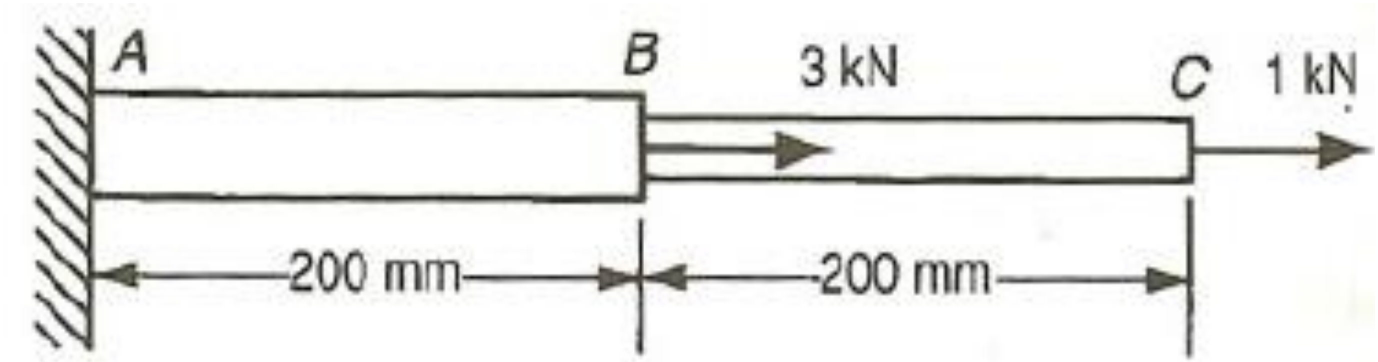
$$\frac{P}{A} = E\frac{\delta}{L} \qquad \delta = \frac{PL}{AE}$$

In the examples that follow, wherever it is appropriate, the three steps of Equilibrium, Force-Deformation, and Compatibility will be explicitly stated.

Example 2-2

The steel rod shown in Figure E2-2 is fixed to a wall at its left end. It has two applied forces. The 3 kN force is applied at the point B and the 1 kN force is applied at the point C. The area of the rod between A and B is 1000 mm^2, and the area of the rod between B and C is 500 mm^2. Take E= 210 GPa. Find:

a. the stress in each section of the rod and
b. the horizontal displacement at the points B and C

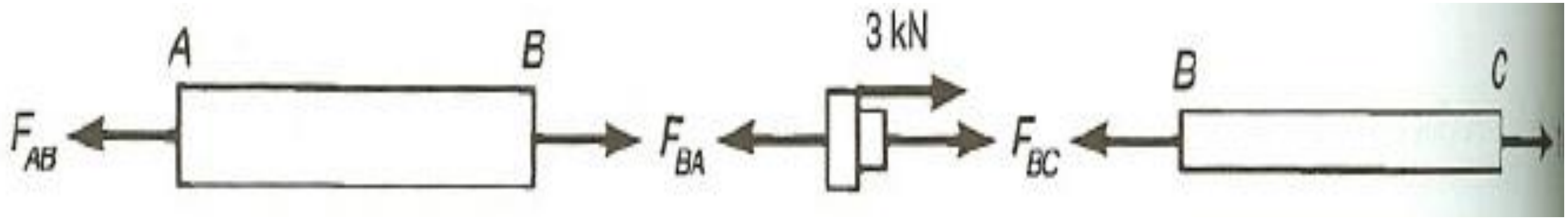


Example 2-2 Continues

Solution

I: Equilibrium

Draw free-body diagrams for each section of the rod.



From a summation of forces on the member BC, F_{BC}=1 kN.

Summing forces on the centre free-body diagram, F_{BA} = 3+1=4 kN

Summing forces on the left free-body diagram gives F_{AB} = F_{BA} = 4 kN.

The stresses then are:

$$\sigma_{AB} = 4kN/1000mm^2 = 4MPa$$

$$\sigma_{BC} = 1kN/500mm^2 = 2MPa$$

Example 2-2 Continues

II: Force-Deformation

$$\delta_{AB} = \left(\frac{PL}{AE}\right)_{AB} = \left[\frac{(4kN)(200mm)}{(1000mm^2)(210GPa)}\right]$$

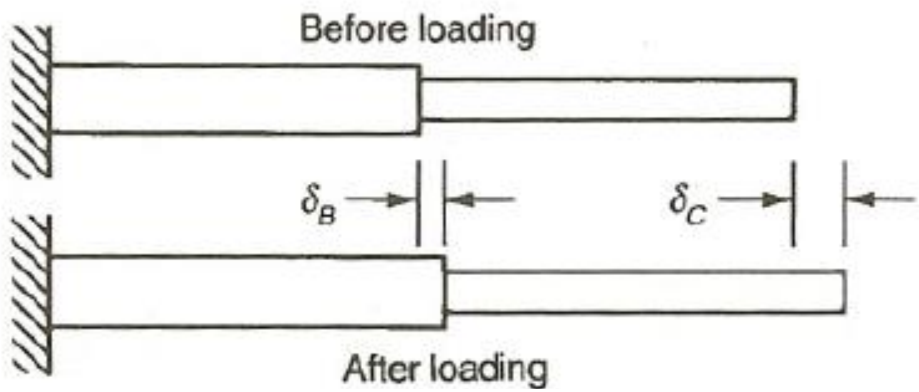
$$\delta_{AB} = 3.81x10^{-3}mm$$

$$\delta_{BC} = \left(\frac{PL}{AE}\right)_{AB} = \left[\frac{(1kN)(200mm)}{(500mm^2)(210GPa)}\right]$$

$$\delta_{BC} = 1.91x10^{-3}mm$$

III: Compatibility

Draw the body before loading and after loading



It is then obvious that

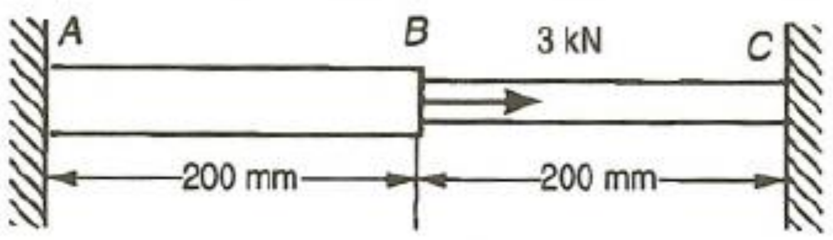
$$\delta_B = \delta_{AB} = 3.81X10^{-3}mm$$

$$\delta_C = \delta_{AB} + \delta_{BC} = [3.81 + 1.91]x10^{-3} = 5.71x10^{-3}\ mm$$

Example 2-3

Consider the same steel rod as in example 2-2 except that now the right end is fixed to a wall as well as the left (Figure E2-3). It is assumed that the rod is built into the walls before the load is applied. Find:

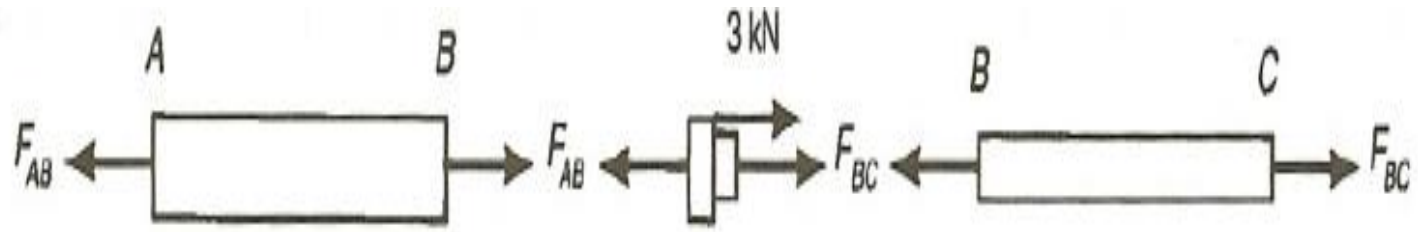
a. the stress in each section of the rod, and
b. the horizontal displacement at point B.



Solution

I: Equilibrium

Draw free-body diagrams for each section of the rod.



Example 2-3 Continues

Summing forces in the horizontal direction on the centre free-body diagram

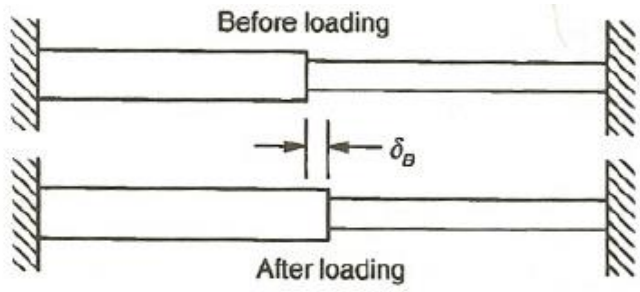
$$-F_{AB} + F_{BC} + 3 = 0 \qquad \textbf{a}$$

II: Force-Deformation

$$\delta_{AB} = \left(\frac{PL}{AE}\right)_{AB} = \frac{F_{AB}L_{AB}}{A_{AB}E} \qquad \textbf{b}$$

$$\delta_{BC} = \left(\frac{PL}{AE}\right)_{BC} = \frac{F_{BC}L_{BC}}{A_{BC}E} \qquad \textbf{c}$$

III: Compatibility



$$\delta_C = \delta_{AB} + \delta_{BC}$$

But, $\delta_C = 0$

$$\therefore \delta_{AB} = -\delta_{BC} \Rightarrow \frac{F_{AB}L_{AB}}{A_{AB}E} = \frac{F_{BC}L_{BC}}{A_{BC}E}$$

Example 2-3 Continues

Since $L_{AB} = L_{BC}$ and $A_{AB} = 2A_{BC}$,
then

$$F_{AB} = -2F_{BC}$$

Inserting this relation into the equilibrium equation (Eq. a),

$$\Rightarrow -F_{AB} + F_{BC} + 3 = 2F_{BC} + F_{BC} + 3 = 0$$

$$\Rightarrow F_{BC} = -1kN$$ and $$F_{AB} = 2kN$$

The stresses are $$\sigma_{AB} = 2kN/1000\,mm^2 = 2MPa$$

$$\sigma_{BC} = -1kN/500\,mm^2 = -2MPa$$

Example 2-3 Continues

The displacement at B is given by

$$\delta_B = \delta_{AB} = \left[\left(\frac{PL}{AE} \right)_{AB} \right] = \left[\frac{F_{AB} L_{AB}}{A_{AB} E} \right]$$

$$\delta_B = \left[\frac{(2)(200)}{(1000)(210)} \right] mm = 19.5x10^{-3}\ mm$$

Poisson's Ratio

- The ratio of the magnitude of the lateral strain to the magnitude of the longitudinal strain is called Poisson's Ratio, v

$$\upsilon = -\frac{Lateral\ \ strain}{Longitudinal\ \ strain}$$

- Poisson's Ratio is a dimensionless material property that never exceeds 0.5.

- Typical values for steel, aluminum, and copper are 0.30, 0.33 and 0.34, respectively.

Example 2-4

A circular aluminum rod 10 mm in diameter is loaded with an axial force of 2 kN. What is the decrease in diameter of the rod? Take E = 70 GPa and ν = 0.33.

Solution

The stress is $$\sigma = P/A = (2)/(\pi 5^2) = 25.5 MPa$$

The longitudinal strain is

$$\varepsilon_{lon} = \sigma/E = 25.5/70000 = 0.000364$$

The lateral strain is

$$\varepsilon_{lat} = -\upsilon \varepsilon_{lon} = -(0.33)(0.000364) = -0.00012$$

The decrease in diameter is then

$$\Delta d = -d\varepsilon_{lat} = -(10)(-0.00012) = 0.0012\ mm$$

Thermal Deformations

- When a material is heated, expansion forces are created.

- If it is free to expand, the thermal strain is

$$\varepsilon_t = \alpha\left(t - t_0\right)$$

- For problems where the load is purely axial, this becomes

$$\varepsilon_T = \frac{\sigma}{A} + \alpha\left(t - t_0\right)$$

- The deformation δ is found by multiplying the strain by the length, L

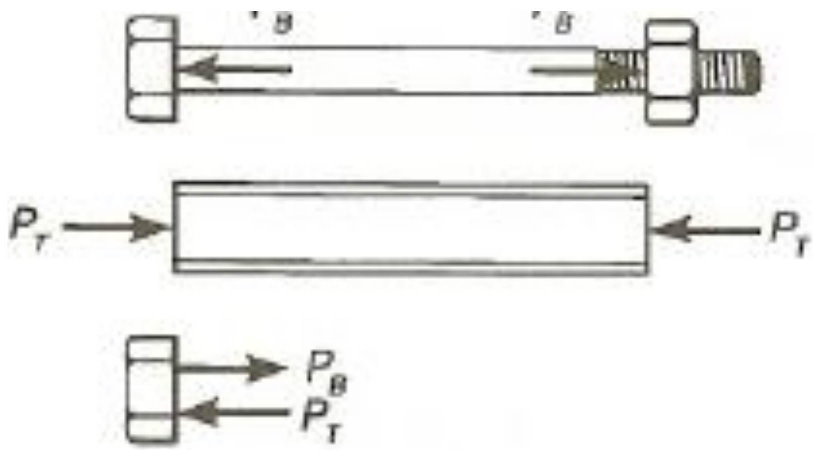
$$\delta = \frac{PL}{AE} + \alpha L\left(t - t_0\right)$$

Example 2-5

The steel bolt is put through the aluminum tube as shown in Figure E2-5. The nut is made just tight. The entire assembly is then raised in temperature by 60^0C. Because aluminum expands more than steel, the bolt will be put in tension and the tube in compression. Find the force in the bolt and the tube. For the steel bolt, take E = 210 GPa, α = 12

I: Equilibrium

Draw free-body diagrams for each section of the rod.



From equilibrium of the head of the bolt, it can be seen that $P_B = P_T$.

Example 2-5 Continues

II: Force-Deformation

$$\delta_B = \left(\frac{PL}{A_B E_B}\right) + \alpha_B L(t - t_0)$$

$$\delta_T = -\left(\frac{PL}{A_T E_T}\right) + \alpha_T L(t - t_0)$$

The minus sign in the second expression occurs because the tube is in compression

III: Compatibility

The tube and the bolt must both expand the same amount, therefore,

$$\delta_B = \delta_T = \left[\frac{P(100)}{(32)(210E+9)} + (12E-06)(100)(60)\right]$$

$$\delta_B = \left[\frac{P(100)}{(64)(69E+9)} + (23E-06)(100)(100)\right]$$

Solving for P gives P = 1.759 kN

Variable Load

- For variable load, Eq. (2-6) holds only over an infinitesimally small length L= dx and Eq. (2-6) then becomes

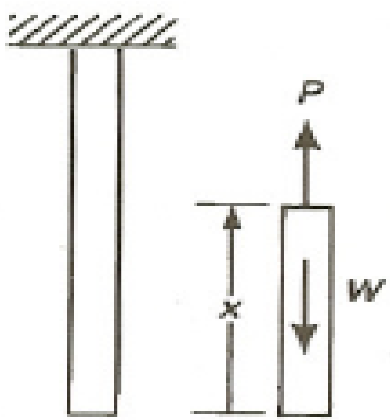
$$d\delta = \frac{P(x)}{AE} dx$$

- or equivalently

$$\delta = \int_0^L \frac{P(x)}{AE} dx$$

Example 2-6

An aluminum rod is hanging from one end. The rod is 1m long and has a square cross-section 20 mm by 20 mm. Find the total extension of the rod resulting from its own weight. Take E = 70 GPa and the unit weight γ =27 KN\m.



Solution

I: Equilibrium

Draw a free-body diagram.

The weight of the section shown in Figure E2-6 is

$$W = \gamma V = \gamma A x = P$$

which clearly yields P as a function of x, and Eq.(2-11) gives

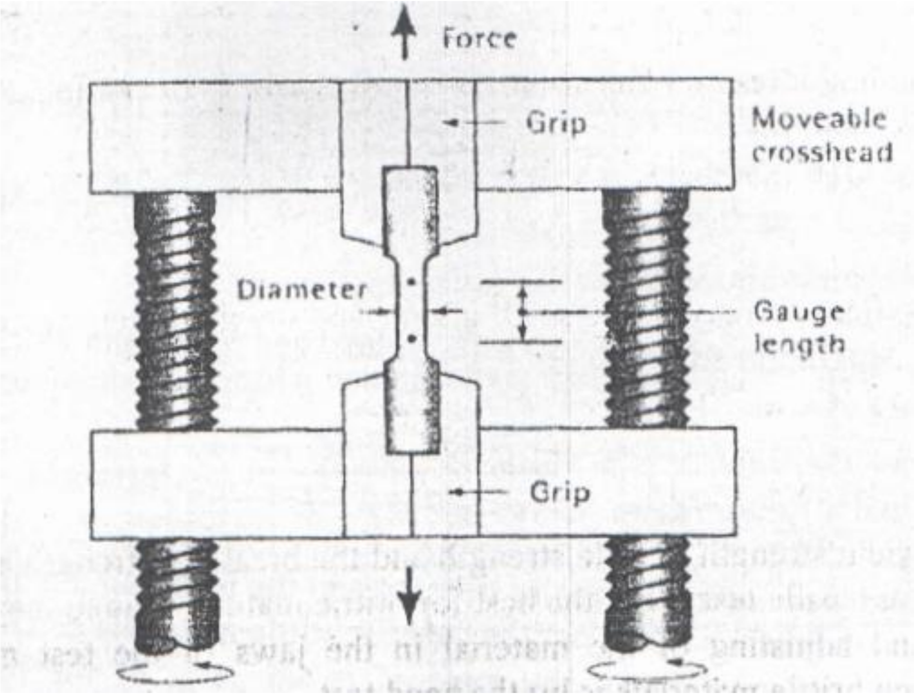
$$\delta = \int_0^L \frac{\gamma A x}{AE} dx = \frac{\gamma}{E}\int_0^L x\,dx = \frac{\gamma L^2}{2E}$$

$$\delta = \frac{(27E+3)(1)^2}{(2)(70E+9)} = 0.1929\,\mu m$$

Tensile Test

The tensile test measures the resistance of a material to a static or slowly changing force.

The test set-up is as shown in Fig. 2-3



The stress $$\sigma = F/A_0$$

The strain, $$\varepsilon = \frac{l - l_0}{l_0}$$

where

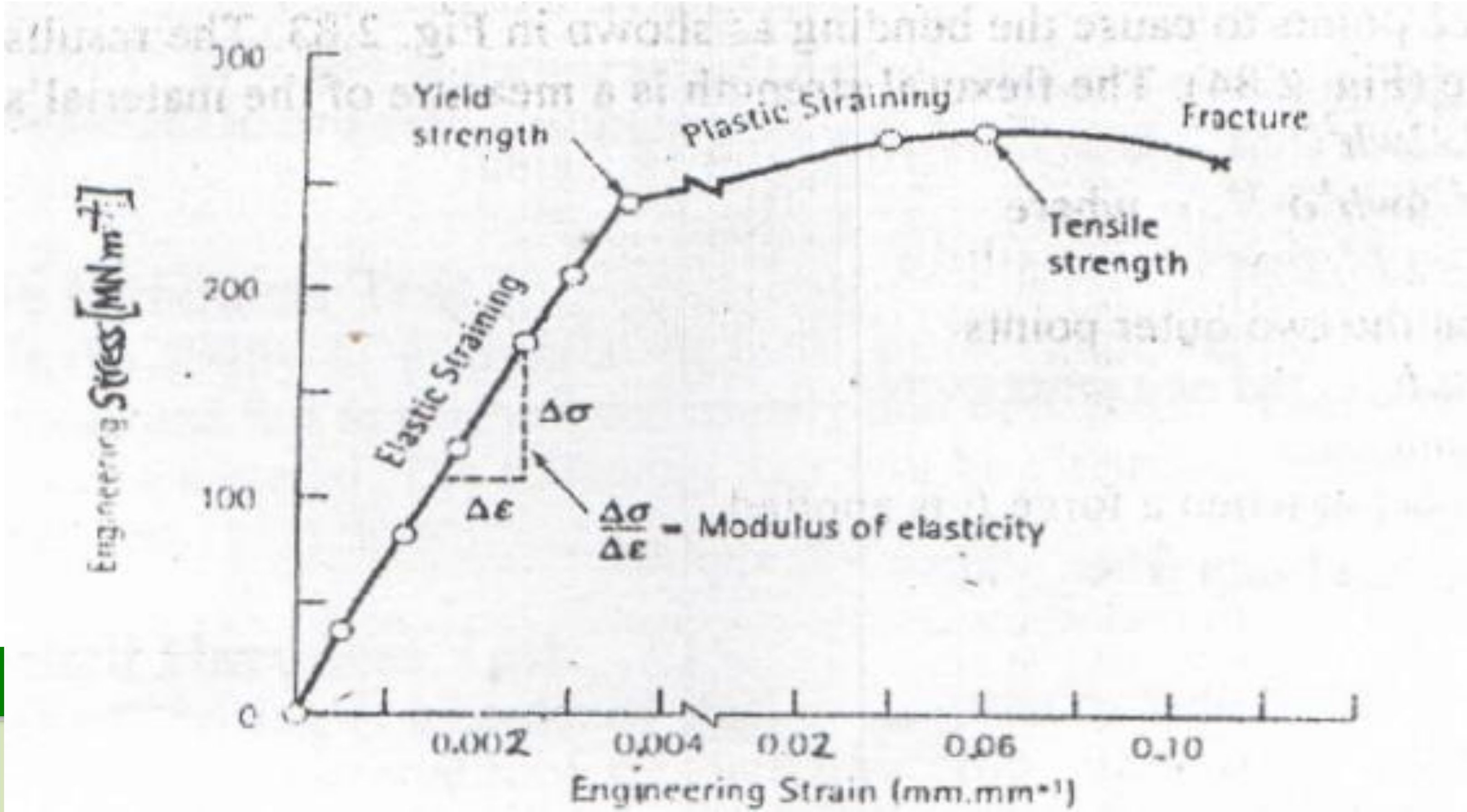
L_0 Original distance between the gauge marks before the test

l Distance between the gauge marks after the force F is applied

A_0 Original cross-sectional area before the test

Example 2-7

An aluminium rod is to withstand an applied tensile force of 300 kN. The maximum allowable stress on the rod is fixed at 170 x 10^3 kNm^{-2}, to ensure sufficient factor of safety. If the rod should be at least 4 m long, but must not deform more than 8 mm when the force is applied, design a rod for this purpose.



Example 2-7 Continues

Solution

Under normal circumstances, the expected cross-sectional area:

$$A_0 = F/\sigma = (300)/(170x10^3) = 1.7647x10^{-3}\ m^2 = 1764.7\ mm^2$$

The rod should have a cross-sectional area of 1765 mm^2.

For a cylindrical rod,

$$d = \sqrt{A_0/\pi} = \sqrt{1764.7/\pi} = 47.4\ mm$$

From Fig. E3-7, the strain expected for the stress 170 x 10^3 kNm^{-2} is 0.0025.

For the cross-sectional area above, the length of the rod is:

$$\varepsilon = \Delta l/l = 8/l \Rightarrow l = 8/\varepsilon = 8/0.0025 = 3200\ mm = 3.2\ m\ (< 4\ m)$$

Example 2-7 Continues

The minimum length of the rod is given as 4 m.

To keep this, the cross-sectional area should be larger

$$\varepsilon = \Delta l / l = 8/4000 = 0.002$$

From Fig. E3-7, the corresponding stress for the strain of 0.002 is 140 x 10^3 kNm^{-2} which is less than the given maximum of 170 x 10^3 kNm^{-2}. The minimum cross-sectional area then, is

$$A_0 = F/\sigma = (300)/(140x10^3) = 2.143x10^{-3}\ m^2 = 2143\,mm^2$$

Example 2-7 Continues

To satisfy, the conditions, maximum stress of 170 x 10^3 kNm^{-2} and maximum elongation of 8 mm: A_0 = 2143 mm^2.

Therefore the Diameter, d is

$$d = \sqrt{A_0/\pi} = \sqrt{2143/\pi} = 52.24\ mm$$

THANK YOU

LECTURE 3

TORSION IN SHAFT

Lecture Outline

Circular Shafts

Hollow, Thin- Walled Shafts

Stresses in Thin-Walled Pressure vessel

Introduction

- Torsion refers to the twisting of long members.
- Torsion can occur with members of any cross-sectional shape, but the most common is the circular shaft.
- Another fairly common shaft configuration, which has a simple solution, is the hollow, thin-walled shaft.

Circular Shafts

Figure 3-1 shows a circular shaft before loading; r-θ-z cylindrical coordinates system is also shown.

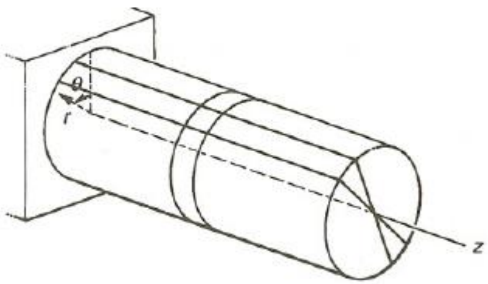
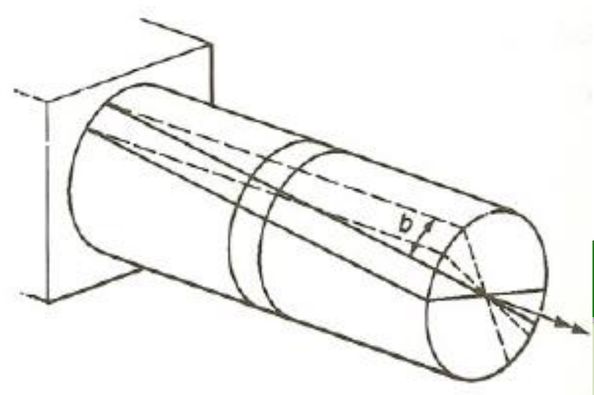


Figure 3-2 shows the shaft after loading with a torque, **T**.



- The shear strain for this special case can be expressed as

$$\gamma_{\phi z} = r\frac{\phi}{z}$$

- For the general case where Φ is not a linear function of z the shear strain can be expressed as

$$\gamma_{\phi z} = r\frac{d\phi}{dz}$$

Circular Shafts

- The application of Hooke's law gives

$$\tau_{\phi z} = G\gamma_{\phi z} = Gr\frac{d\phi}{dz}$$

- The torque at the distance z along the shaft is found by summing the contributions of the shear stress at each point in the cross-section by means of integration.

$$T = \int_A \tau_{\phi z} r dA = G\frac{d\phi}{dz}\int_A r^2 dA = GJ\frac{d\phi}{dz}$$

- For a solid shaft with an outer radius of r the polar moment of inertia is

$$J = \tfrac{1}{2}\pi r^4$$

- For a hallow circular shaft with outer radius r and inner radius the polar moment of inertia is

$$J = \tfrac{1}{2}\pi\left(r_o^{\,4} - r_i^{\,4}\right)$$

Circular Shafts

- Equation (3-3) can be combined with equation (3-4) to give

$$\tau_{\phi z} = \frac{Tr}{J}$$

- The maximum shear stress is

$$\tau_{\phi z \max} = \frac{T_{\max} r_o}{J}$$

- The angle Φ, of twist is

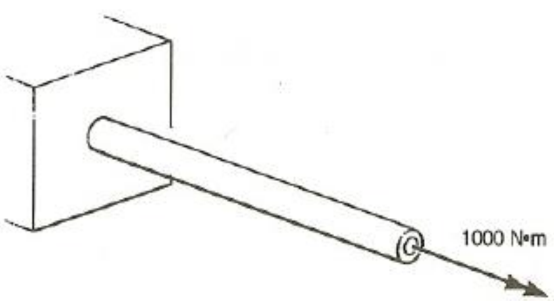
$$\phi = \int_0^L \frac{T}{GJ} dz$$

- Thus

$$\phi = \frac{TL}{GJ}$$

Example 3-1

The hallow circular steel shaft shown in Figure E3-1 has an inner diameter of 25 mm, an outer diameter of 50 mm, and a length of 600 mm. It is fixed at the left end and subjected to a torque of 1400 N-m as shown in Figure E3-1. Find the maximum shear stress in the shaft and the angle of twist at the right end. Take G= 84 GPa.



Solution

I: Data Given

d_i = 25 mm; d_o = 50 mm; L = 600 mm;

T = 1400 N-m

II: Solve

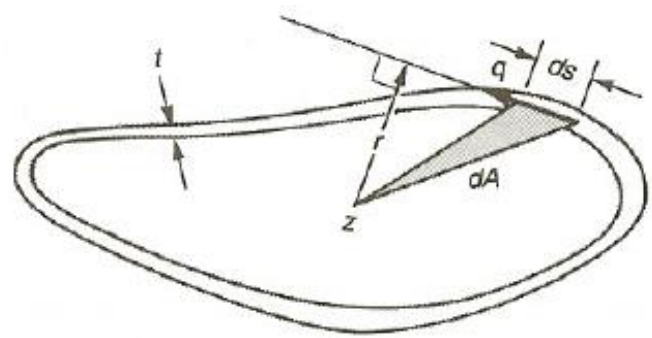
$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = 575E - 03m^4$$

$$\tau_{\phi z} = \frac{Tr_o}{J} = 60.8MPa$$

$$\phi = \frac{TL}{GJ} = 0.0174rad$$

Hollow, Thin- Walled Shafts

- Figure 3-2 shows the cross-section of the thin walled tube of non-constant thickness.



- The torque produced by q over the element ds is

$$dT = qrds = \tau_{sz}trds$$

- The total torque is therefore,

$$T = \oint \tau_{sz}trds = \tau_{sz}t\oint rds$$

- The area dA is the area of the triangle of base ds and height r:

$$dA = \tfrac{1}{2}(base)(height)$$

$$= \tfrac{1}{2}rds$$

Hollow, Thin- Walled Shafts

- So that

$$\oint r\,ds = 2Am$$

- where Am is the area enclosed by the wall (including the hole).

- The expressions for the torque is

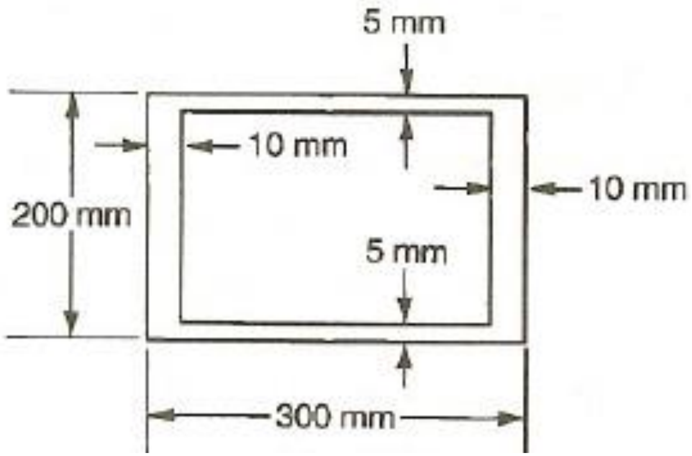
$$T = (\tau_{sz} t)(2Am) = 2Amt\tau_{sz}$$

- Hence the definition of the shear can be expressed as

$$\tau_{sz} = \frac{T}{2Amt}$$

Example 3-2

A torque of 10 kN-m is applied to the thin-walled rectangular steel shaft whose cross-section is shown in Figure E3-2. The shaft has wall thickness of 5 mm and 10 mm. Find the maximum shear stress in the shaft.



Solution

I: Data Given

T = 10 kN-m

II: Solve

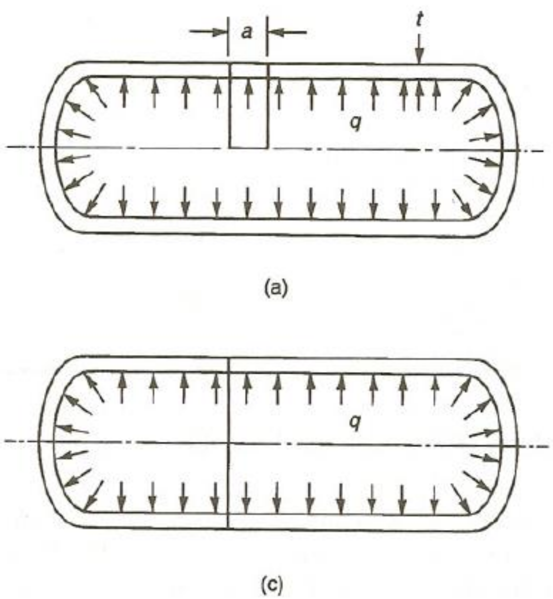
Am = (200-5)(300-10) = 56,550 mm^2

The maximum shear stress will occur in the thinnest section, so t = 5 mm.

$$\tau_{sz} = \frac{T}{2Amt} = \frac{10000}{(0.05655)(0.005)} = 17.68 MPa$$

Stresses in Thin-Walled Pressure Vessel

- Consider the thin-walled circular cylinder subjected to a uniform internal pressure, as shown in Figure 3-3.



- Summing forces in the vertical direction gives

$$2qra - 2\sigma_t ta = 0$$

- This implies $$\sigma_t = \frac{qr}{t}$$

- Similarly, $$\sigma_a = \frac{qr}{2t}$$

- where
- r is the radius
- t is the thickness

Example 3-3

Consider a cylindrical pressure vessel with a wall thickness of 25 mm, an internal pressure of 1.4 MPa, and an outer diameter of 1.2 m. Find the axial and tangential stresses.

Solution

I: Data Given

q = 1.4 MPa; r = 1/2d = 575 mm; t = 25 mm

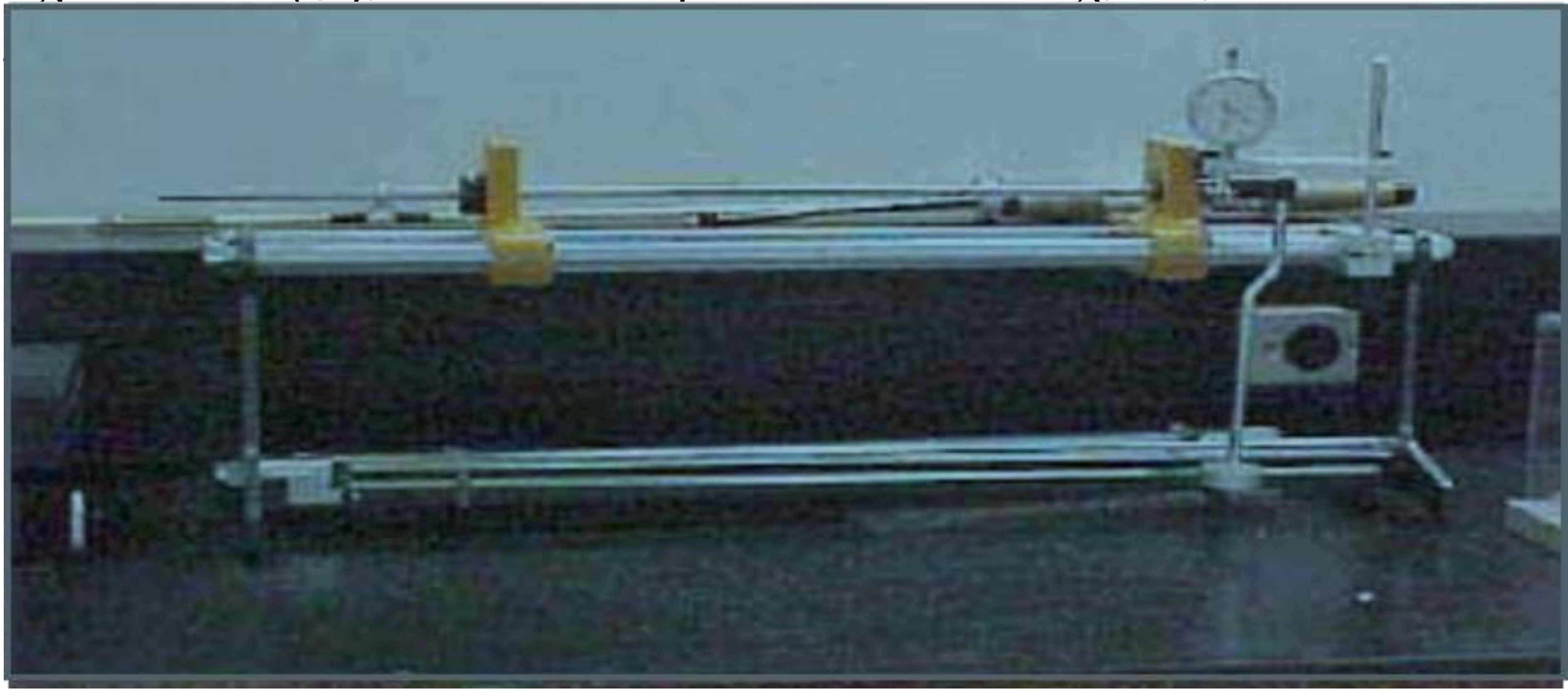
II: Solve

$$\sigma_t = \frac{qr}{t} = \frac{(1.4MPa)(575mm)}{25mm} = 32.2MPa$$

$$\sigma_a = \frac{qr}{2t} = \frac{(1.4MPa)(575mm)}{(2)(25mm)} = 16.1MPa$$

Torsional Testing

Torsional testing is to determine Shear Modulus of Elasticity (G) of some Steel, Aluminum and Brass circular shafts and develop a relationship among the Torque (T0 and Clamping length (L) and the angle of twist (θ). The test set-up is as shown in Fig. 4-4.



Torsional Testing

The stress

$$\tau_{\phi z} = \frac{Tr}{J}$$

The angle Φ, of twist is

$$\phi = \frac{TL}{GJ}$$

The strain,

$$\gamma_{\phi z} = r\frac{\phi}{z}$$

Example 3-4

A bar of metal 25 mm in diameter is tested on a length of 250 mm. A torsion test gave the following results (Table E3-4).

Torque (kN m)	0.051	0.152	0.253	0.354
Angle of twist (degrees)	0.24	0.71	1.175	1.642

Represent these results in graphical form and hence determine the modulus of rigidity, *G*, for the metal.

Solution

From

$$\phi = \frac{TL}{GJ}$$

Then

$$T = \frac{GJ}{L}\phi$$

Torque	Angle of Twist	
kNm	Degrees	Radians
0.051	0.240	0.004189
0.152	0.710	0.012393
0.253	1.175	0.02051
0.354	1.642	0.028662

Example 3-4 Continues

Plotting of T against φ,
The slope is $\dfrac{GJ}{L}$

Hence,

$$G = \frac{slope\ x\ L}{J}$$

But, $J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(25)^4$

$$\Rightarrow J = 38349.52\ mm^4$$

From the plot, the slope is 12.335×10^6 N/mm^2

Example 3-4 Continues

This implies

$$G = \frac{12.335\,x\,10^6(250)}{38349.52} = 80411\,N/mm^2$$

Hence, the modulus of rigidity, *G is 80411* N/mm^2

THANK YOU

LECTURE 4

BENDING IN BEAMS

Lecture Outline

Shear and Bending Moment Diagrams

Relation between the Shear force and the

Bending Moment

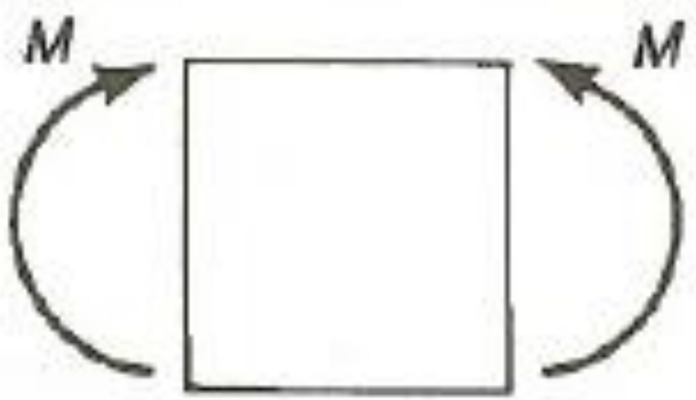
Stresses in Beams

Bend Test

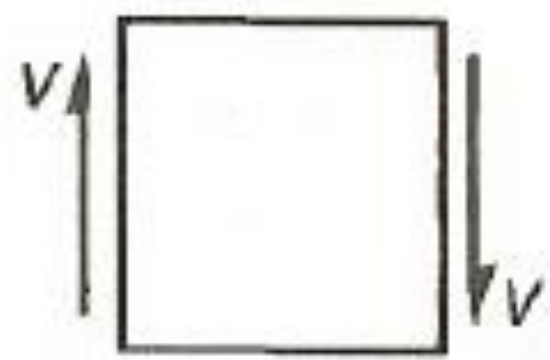
SHEAR AND BENDING IN BEAMS

- Shear and moment diagrams are plots of the shear forces and bending moments respectively along the length of a beam.

- The most common sign conversion for the shear force and bending moment in beams is shown in Figure 4-1



Positive bending moment



Positive shear

Shearing Force and Bending Moment Diagram

One method of determining the shear force and bending moment diagrams is by the following steps:

1. The free body is first drawn and determine the reactions from equilibrium of the entire beam.

2. The bending moment along the beam is then calculated with emphasis on the extreme and critical points.

3. Cut the beam at an arbitrary point.

4. Show the unknown shear force and bending moment on the cut using the positive sign convention shown in Figure 4-1.

Shearing Force and Bending Moment Diagram

5. Sum forces in the vertical direction to determine the unknown shear.

6. Sum moments about the cut to determine the unknown moment.

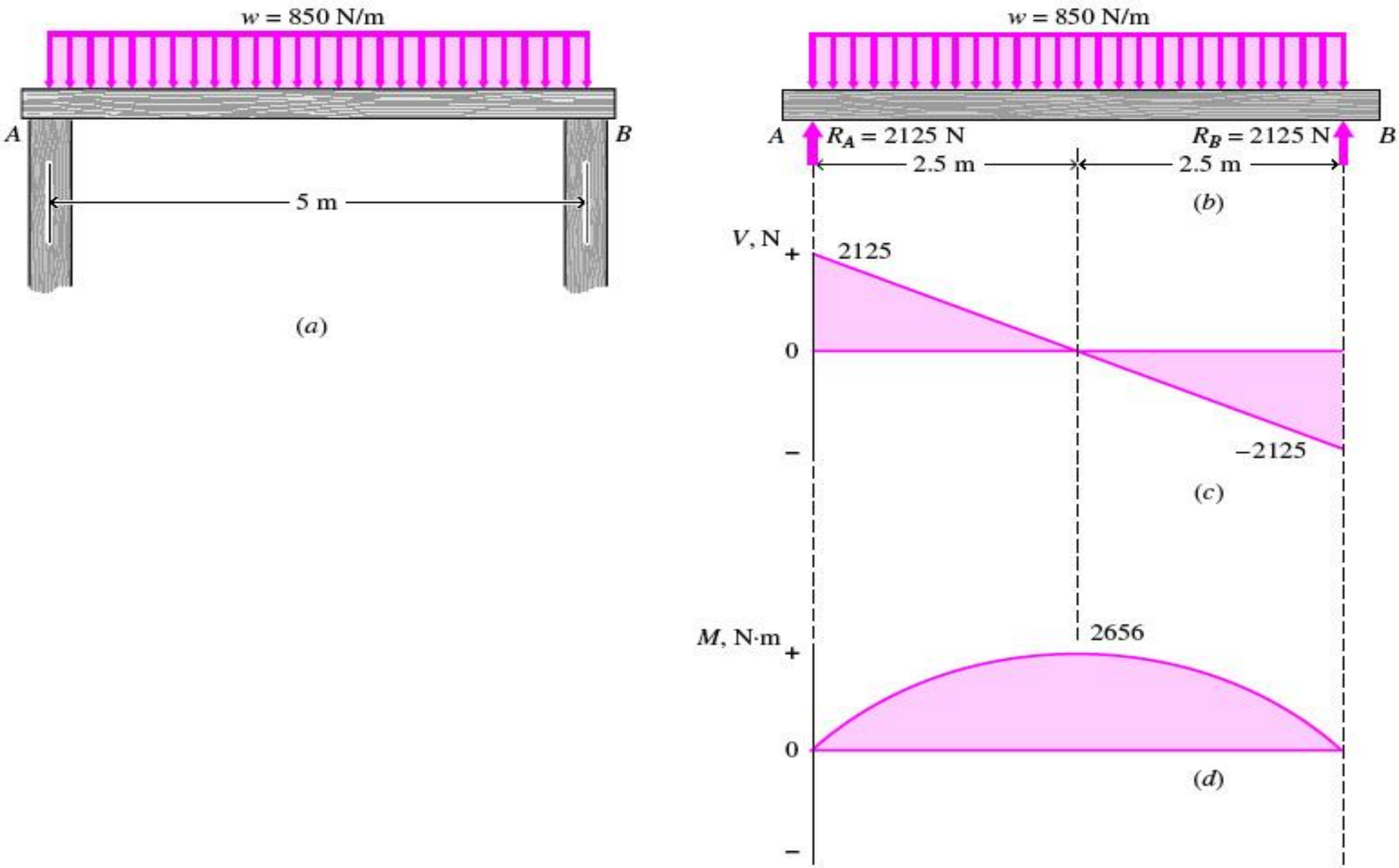
7. The calculated forces and moments are used in the drawing of the diagrams.

8. Figure 4-2 (a) shows a beam supported at the ends *A* and *B*.

9. Figure 4-2 (b) shows the relevant free body diagram with calculated reactions.

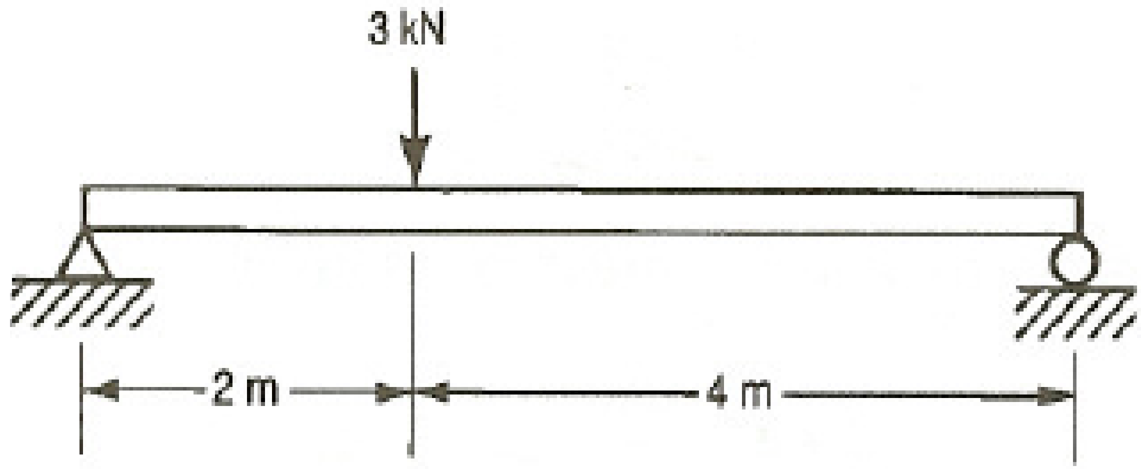
Shearing Force and Bending Moment Diagram

10. Figure 4-2 (c) and (d) shows respectively the shearing force and bending moment diagrams



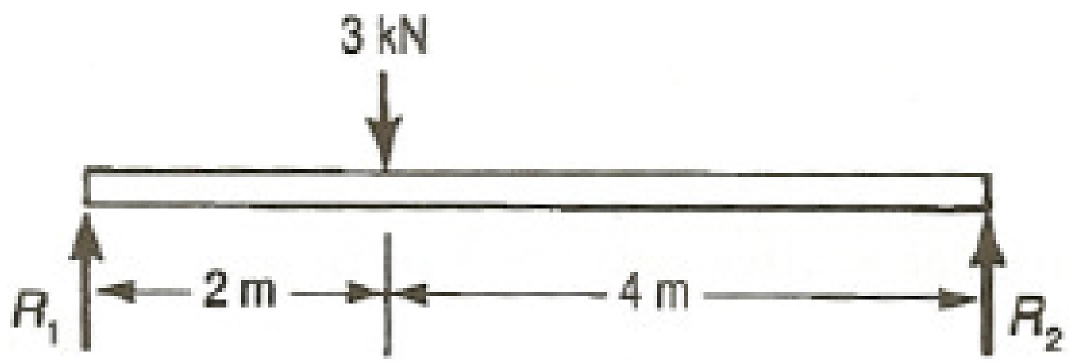
Example 4-1

For the beam shown in Figure E4-1, plot the shear and bending moment diagram.



Solution

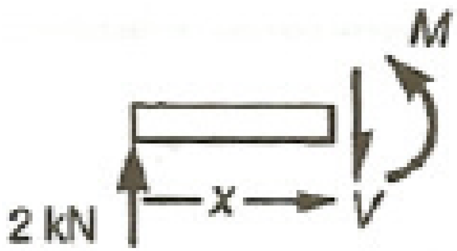
First solve for the unknown reactions using the free-body diagram of the beam shown below



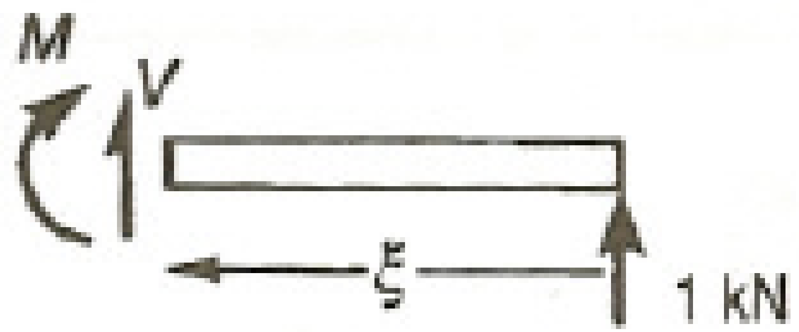
Example 4-1 Continues

To find the reactions:

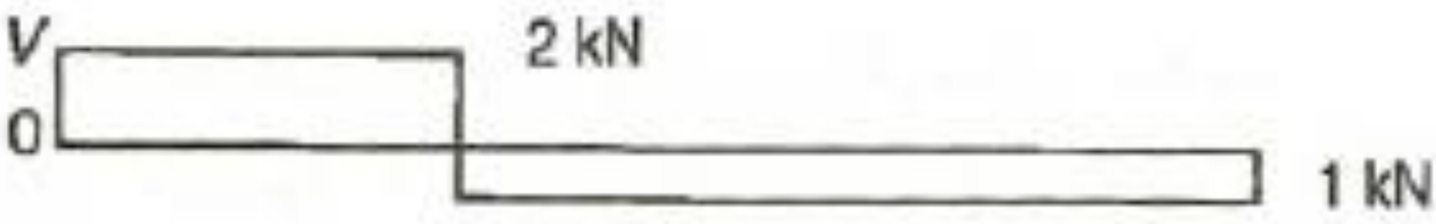
Cut the beam between the x = 0 and x = 2 m and the load as shown below.



Repeat the procedure by making a cut between x = 2 and the right end of the beam as shown in below



Example 4-1 Continues



(d)

(e)

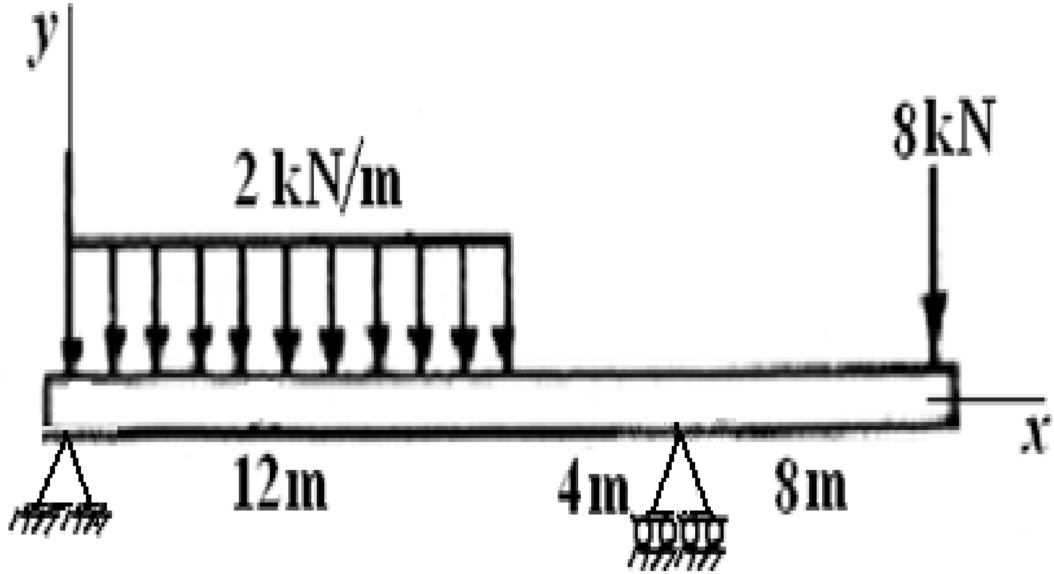
Example 4-2

For the beam in Figure E 4-1, draw:

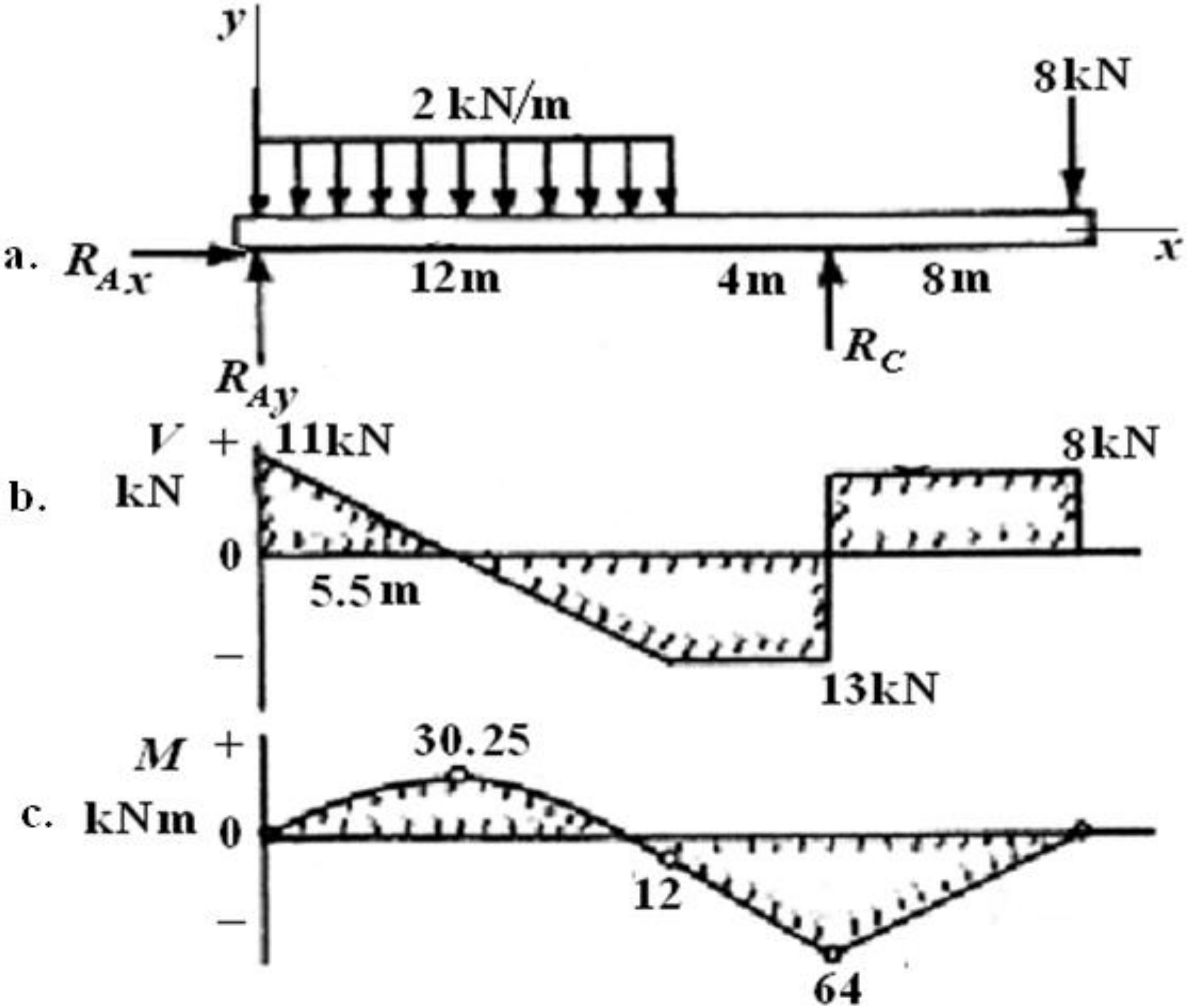
a) a free body diagram of the beam

b) the shearing force diagram of the beam

c) the bending moment diagram of the beam.

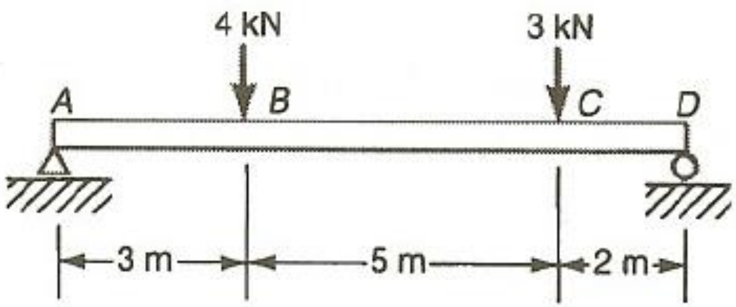


Example 4-2 Continues



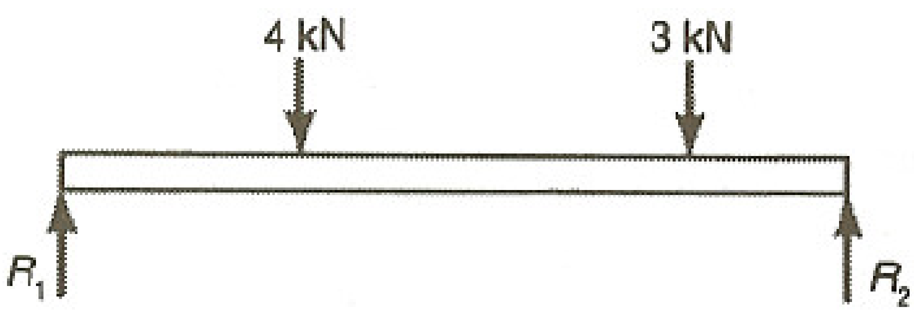
Example 4-3

Draw the shear and moment diagram shown in Figure E5-3.



Solution

Draw the free-body diagram of the beam.

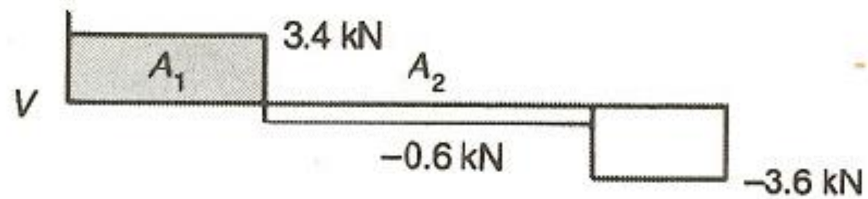


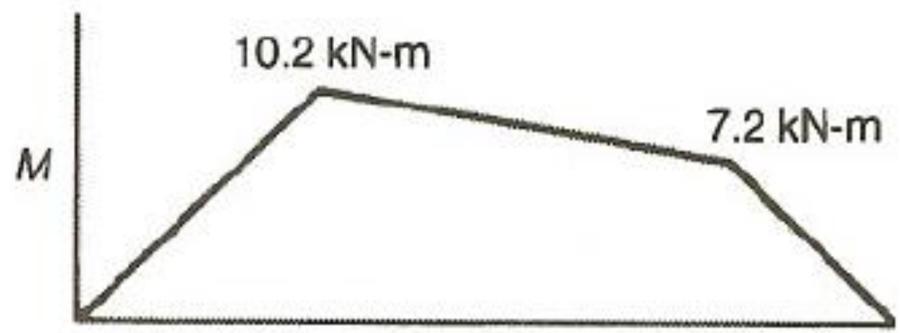
Summation of the moments about the right end

$$10R_1 = (4)(7) + (3)(2) = 34 \Rightarrow R_1 = 3.4kN$$

Summation of forces in the vertical direction

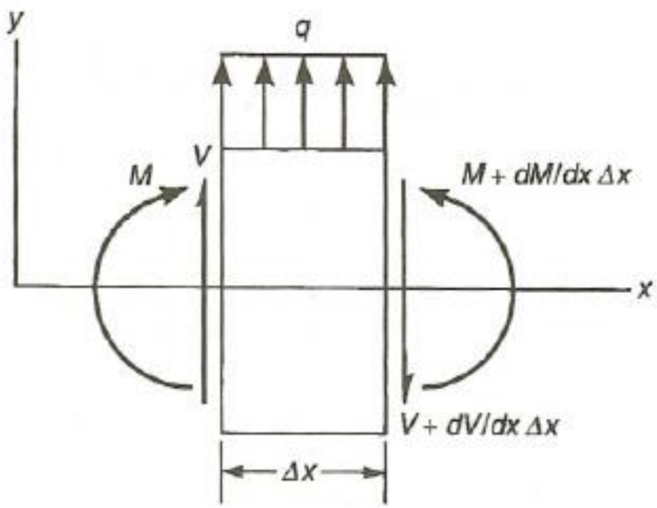
$$R_2 = 7 - R_1 = 7 - 3.4 = 3.6kN$$





Relation between the Shear force and the Bending Moment

- Let us consider a simple supported beam AB carrying a distributed load w per unit length as shown in Figure 4-3.



- Summation of forces in the y direction gives

$$\Rightarrow V - \left(V + \frac{dV}{dx} \Delta x \right) + q\Delta x = 0$$

- Dividing through by Δx gives

$$\frac{dV}{dx} = qdx$$

- Integrating Eq. (2-34) gives

$$V_2 - V_1 = \int_{x_1}^{x_2} qdx$$

Relation between the Shear force and the Bending Moment

- Summation of moments and neglecting higher order terms gives

$$\left(M + \frac{dM}{dx}\Delta x\right) - M - V\Delta x = 0$$

- Dividing through by Δx gives

$$\frac{dM}{dx} = V(x)$$

- Integrating gives

$$M_2 - M_1 = \int_{x_1}^{x_2} V dx$$

Step 1: Express w in terms of the length x;

Thus

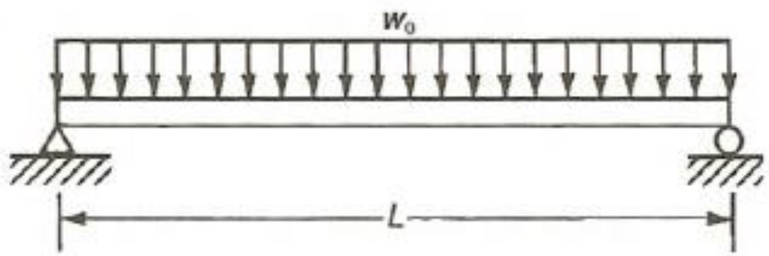
$$q(x) = f(x)$$

Step 2: Obtain V(x) by integrating q(x) within the interval.

Step 3: Find M(x) by determining the area under the shear force diagram or integrating V(x) within the interval.

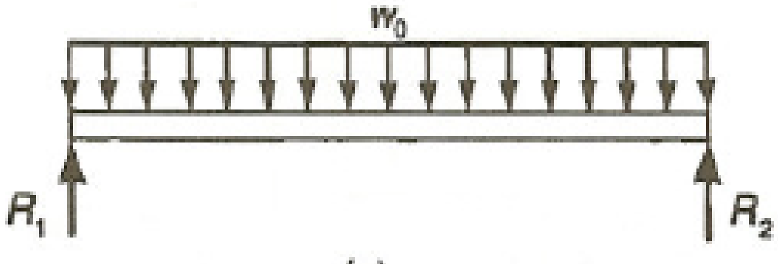
Example 4-4

The simply supported uniform beam shown in Figure E4-4 carries a uniform load of w_0. Plot the shear and moment diagram for this beam.



Solution

As before, the reactions can be found first from the free-body diagram of the beam.



Summing vertical forces then gives.

$$R = R_1 = R_2 = \frac{w_0 L}{2}$$

The load $q = -w_0$ integrating gives

$$V = V_0 - \int_0^x w_0 dx = V_0 - w_0 x$$

Example 4-4 Continues

But

$$V_0 = R = \frac{w_0 L}{2}$$

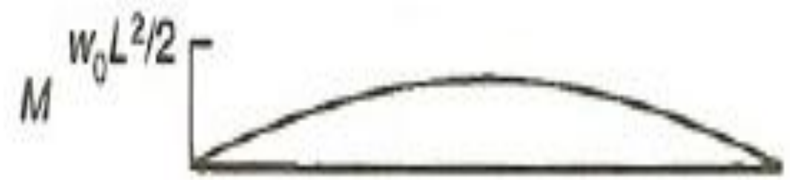
Therefore

$$V = \frac{w_0 L}{2} - w_0 x$$

Integrating the above and setting the moment at x =0 to be zero

$$M == \frac{w_0 x}{2}\left(L - x\right)$$

(b)

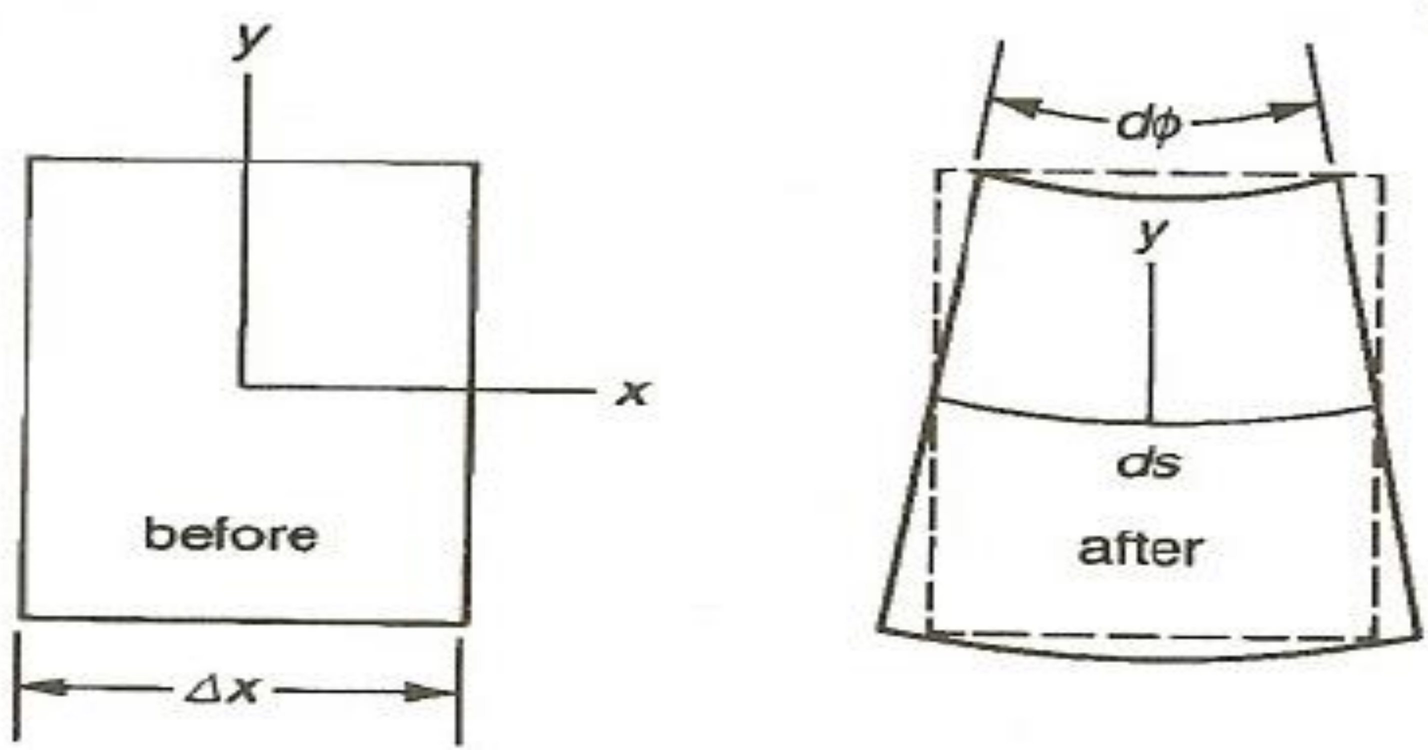


(c)

Stresses in Beams

- The basic assumptions in elementary beam theory is that the cross-section remains plane and perpendicular to the neutral axis as shown in Figure 5-4 when the beam is loaded.

- This assumption is strictly true only for the case of pure bending but gives good results even when shear is taking place.



Stresses in Beams

- The distance y is measured from the neutral axis.
- The strain in the x-direction is $$\Delta L / L$$
- The change in length $$\Delta L = y d\phi$$
- The Strain is $$\varepsilon_x = -y \frac{d\phi}{ds} = -\frac{y}{\rho} = -ky$$
- The stress is $$\sigma_x = -Eky$$

Bending Stress

The axial force and the bending moment can be found by summing the effects of the normal stress σ_y:

$$P = \int_A \sigma_x dA = -Ek\int_A ydA$$

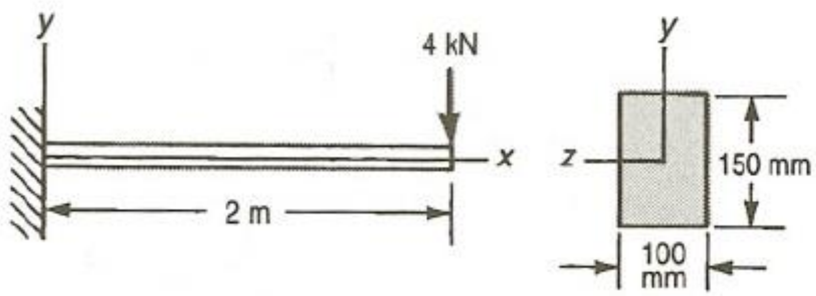
$$M = -\int_A y\sigma_x dA = -Ek\int_A y^2 dA = EIk$$

The bending stress can be expressed as:

$$\sigma_x = \pm\frac{MC}{I} = \pm\frac{M}{S}$$

Example 4-5

A 100 mm x 150 mm wooden cantilever beam is 2 m long It is loaded at its tip with a 4 kN load Find the maximum bending stress in the beam shown in Figure EA13 .The maximum bending moment occurs at the wall and is M_{max} = 8 kN- m.



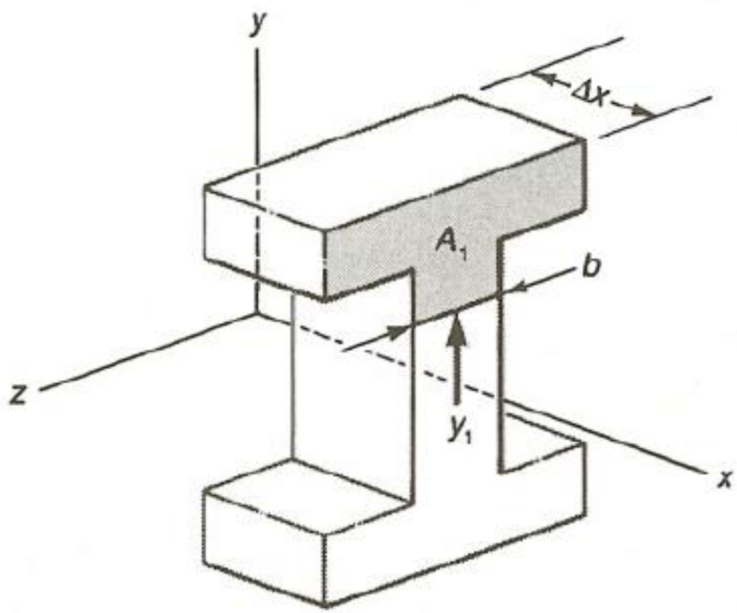
Solution

$$I = \frac{bh^3}{12} = 0.0281m^4$$

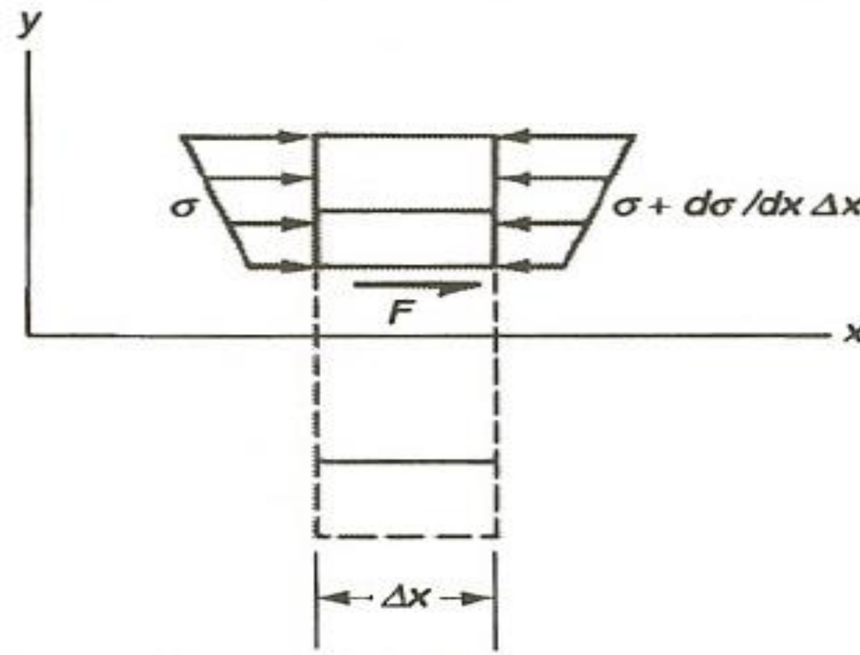
$$\sigma_{x,\max} = \frac{|M|_{\max} C}{I} = 21.3MPa$$

Shear stress

- To find the shear stress, consider the element of length shown in Figure 5-5.



- The bending stresses acting on that element are shown in Figure 5-6



Shear stress

- Summation of forces in x-direction for the free-body diagram shown Figure 5-6

$$-F = q\Delta x = \left[\int_{A_1} \sigma dA - \int_{A_1} \left(\sigma + \frac{d\sigma}{dx} \Delta x \right) dA \right] \qquad q\Delta x = -\int_{A_1} \frac{d\sigma}{dx} \Delta x dA$$

- Substituting bending stress equation into the above equation gives

$$q = \frac{V}{I} \int_{A_1} y dA = \frac{VQ}{I}$$

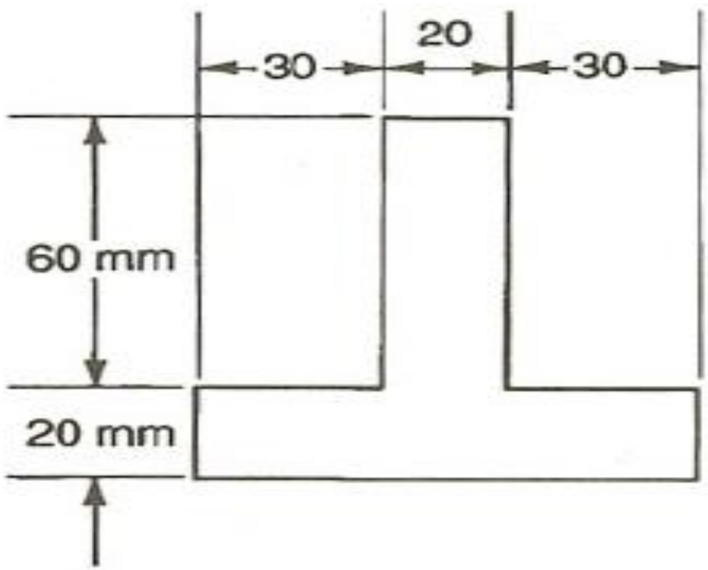
- If the shear stress τ is assumed to be uniform over the thickness b, then τ = q/b and the expression for shear stress is

$$\tau = \frac{VQ}{Ib}$$

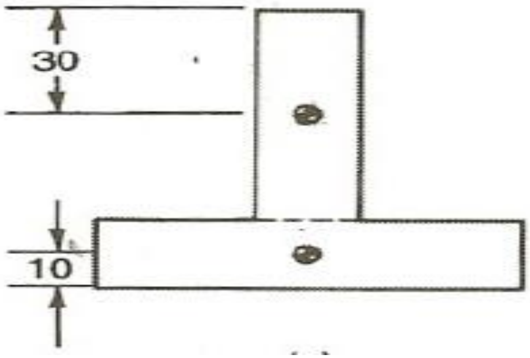
Example 4-6

The cross-section of the beam shown in Figure E4-6 has an applied shear of 10 kN. Find:

a. the shear stress at a point 20 mm below the top of the beam
b. the maximum shear stress from the shear force



The centroids of each of the two sections are also shown in Figure below

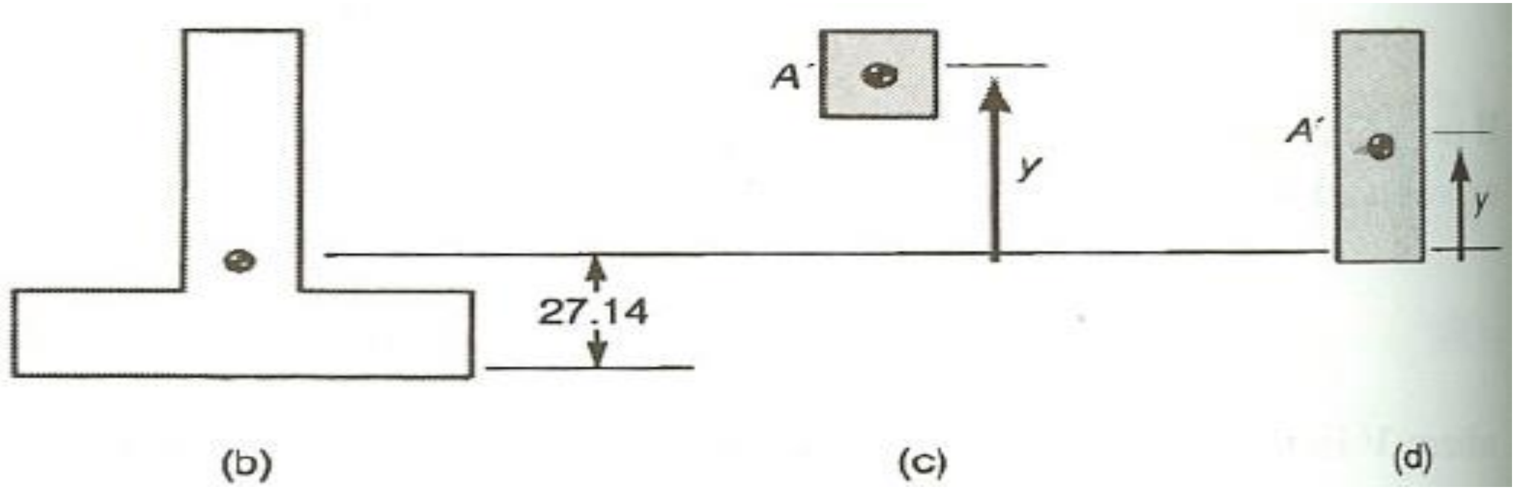


Example 4-6 Continues

The centroid of the entire cross-section is found as follows:

$$\bar{y} = \frac{\sum_{n=1}^{N} \bar{y}_n A_n}{\sum_{n=1}^{N} A_n} = \frac{(60)(20)(30+20)+(80)(20)(10)}{(60)(20)+(80)(20)} = 27.14\ mm$$

The centroid is shown in Figure (b)



Example 4-6 Continues

The second moment of area is

$$I = \sum_{n=1}^{N} I_{CG} + A_n d_n^2 = \left\langle \begin{array}{l} \left\{ \dfrac{(20)(60)^3}{12} + 60(20)[50-27.14]^2 \right\} \\ + \left\{ \dfrac{(80)(20)^3}{12} + 20(80)[10-27.14]^2 \right\} \end{array} \right\rangle = 1,510000\ mm^4$$

For the point 20 mm below the top of the beam, the area A

The value of Q is then

$$Q = Ay = (20)(70-27.14) = 17,140 mm^3$$

The Shear stress is $\tau = \dfrac{VQ}{Ib} = \dfrac{(10^4)(17140)}{(1.51x10^6)(20)} 5.68 MPa$

Example 4-6 Continues

The maximum Q will be at the centroid of the cross-section.

The maximum moment of area Q_{max} is

$$Q = A\bar{y} = (20)(80-27.14)\left[\frac{(80-27.14)}{2}\right] = 27942\ mm^3$$

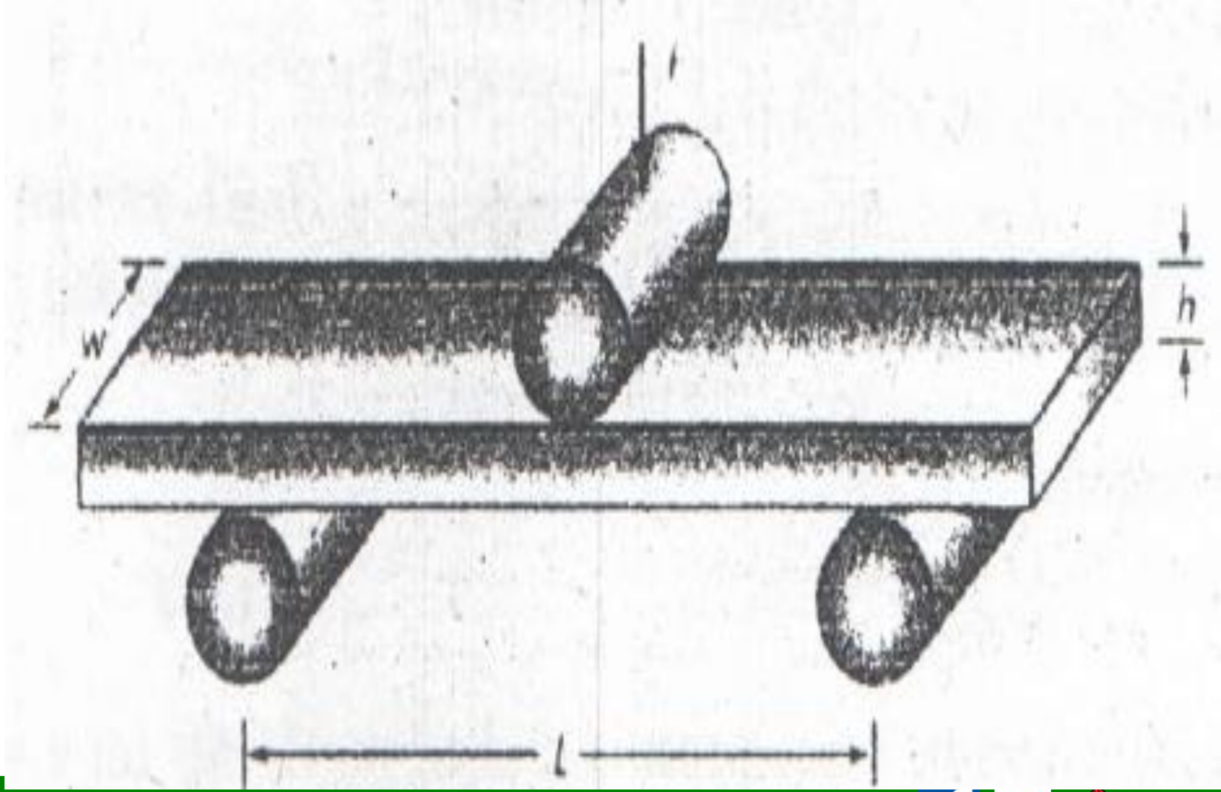
The Shear stress is

$$\tau = \frac{VQ}{Ib} = \frac{(10^4)(27942)}{(1.51x10^6)(20)} = 9.25MPa$$

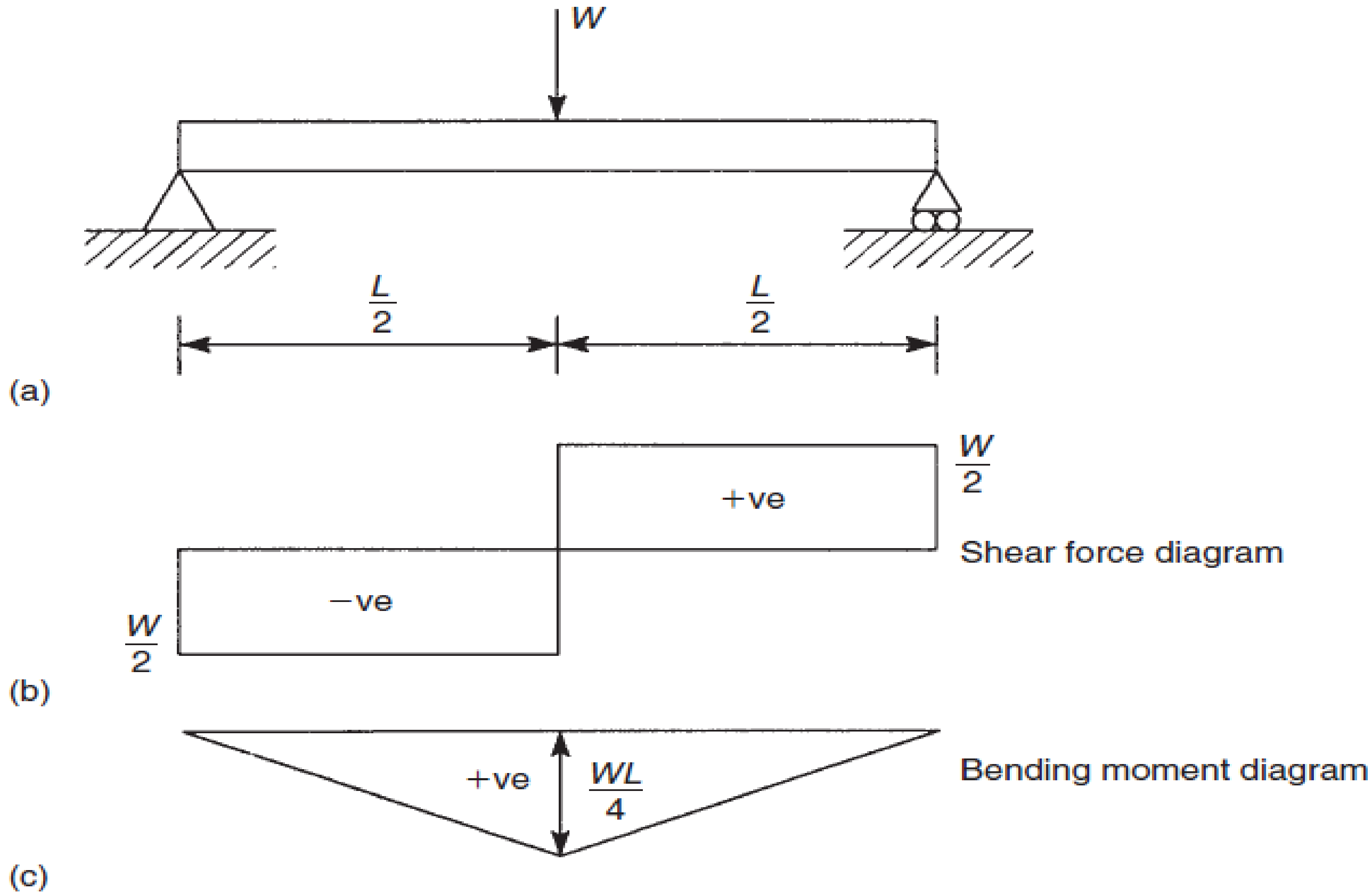
Bend Test

A more convenient way of testing such brittle materials is by the bend test.

A load is applied at three points to cause the bending as shown in Fig. 4-7.

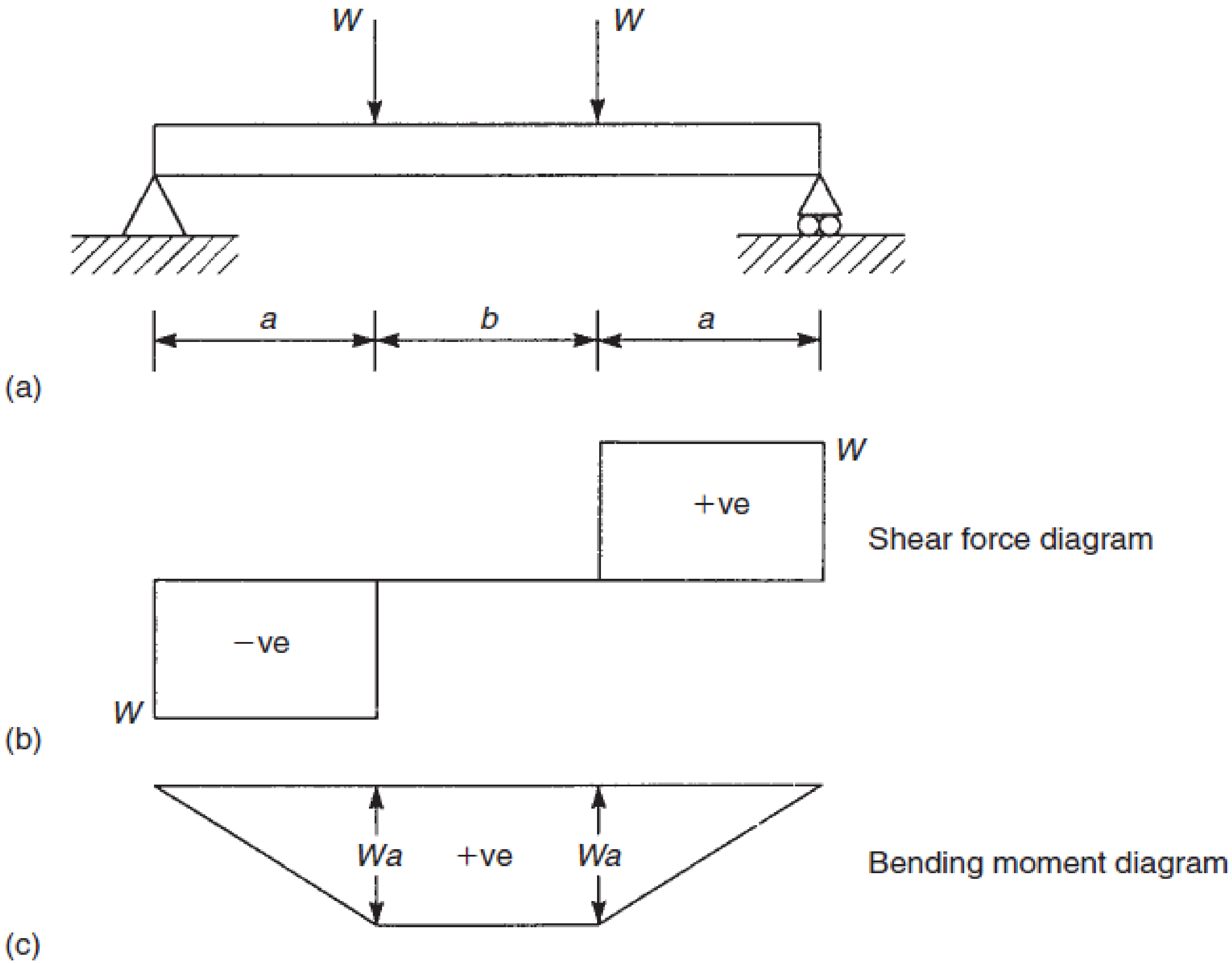


Bend Test



W
L/2
L/2
(a)
W/2
+ve
Shear force diagram
−ve
W/2
(b)
+ve
WL/4
Bending moment diagram
(c)

Bend Test



W
W
a
b
a
(a)
W
+ve
Shear force diagram
−ve
W
(b)
Wa
+ve
Wa
Bending moment diagram
(c)

Bend Test

Considering the simply supported beam AB of the length *l*, and carrying a point load *w* at the centre of beam c as shown in Figure.

Reactions at the supports, $R_A = R_B = \frac{W}{2}$ 1

Considering a section X at a distance x from B. The bending moment at this section is, $M_x = RB(x) = \frac{W}{2}x$ 2

From bending moment equation, $M = EI\frac{d^2y}{dx^2}$ 3

Bend Test

Substituting Eq. (3) into Eq. (2) and integrating twice, the equation of

deflection is EIy = $\frac{Wx^3}{12}$ + $C_1x + C_2$ 4

Solving for C_1 and C_2, and substituting these values in equation (4),

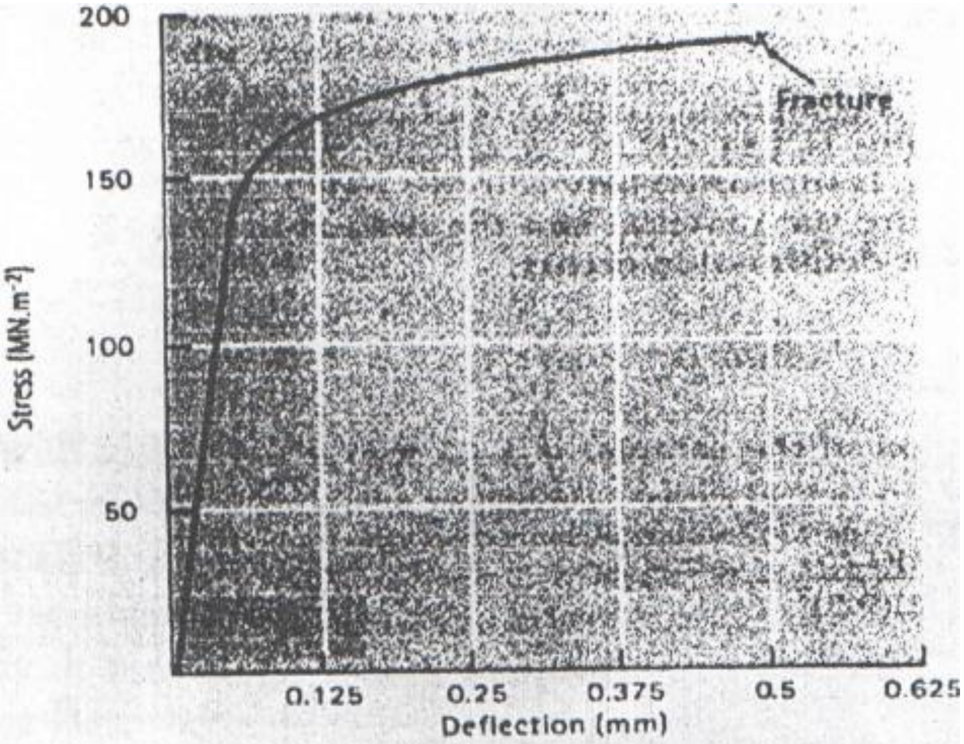
we get EIy = $\frac{Wx^3}{12}$ - $\frac{Wl^2x}{16}$ 5

Maximum deflection occurs at midpoint c, where, x = $\frac{l}{2}$, substituting

into equation (5) give a maximum defection of $\delta = -\frac{wl^3}{48EI}$ 6

Bend Test

The results are recorded a stress-deflection curve (Fig. 4-8).



Material	Tensile Strength (MNm^{-2})	Compressive Strength (MNm^{-2})	Flexural Strength (MNm^{-2})
Polyester – 50% glass fibres	160	225	315
Polyester – 50% glass fibre fabric	260	190	320
Al_2O_3 (99% pure)	210	2625	350
SiC (pressureless-sintered)	175	3920	560

Bend Test

- This is taken to be the maximum direct stress in bending, $\sigma_{x,u}$, corresponding to the ultimate moment M_u, and is assumed to be related to M_u by the elastic relationship

$$\sigma_{u,x} = \frac{M_u}{I} y$$

where:

F the fracture load

L the distance between the two outer points

The flexural strength is a measure of the material's strength.

Flexural strength is $3FL/2wh^2$

Flexural modulus is $FL^3/4wh^3\delta$

w width of the specimen

h height of the specimen

δ the deflection of the beam when a force F is applied

•

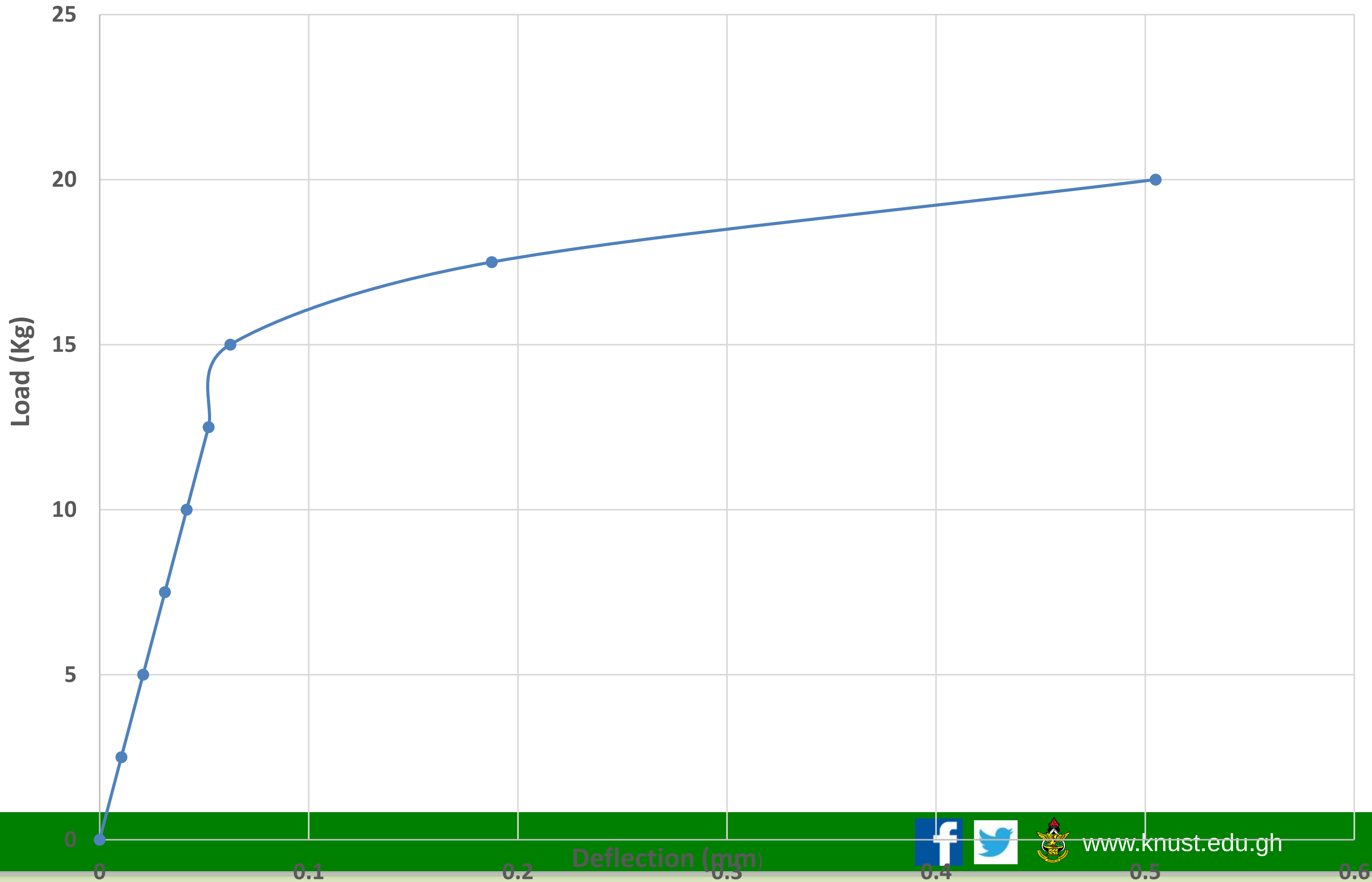
Example 4-7

A square rod of size 10 mm and 200 mm long was passed through the eye of the load holder and then placed on of the support. A dial gauge is set on top of the load holder on the square rod. A load of 2.5 kg was hanged on the specimen and the corresponding reading of the dial gauge was recorded. The load is increased incrementally of 2.5 kg and the corresponding values recorded until the specimen fails. Determine

a. The yield strength;
b. The ultimate strength
c. The modulus of elasticity

Load (Kg)	0	2.5	5	7.5	10	12.5	15	17.5	20
Deflection (mm)	0	0.01042	0.02083	0.03125	0.04167	0.05208	0.0625	0.1875	0.505

Example 4-7 Continues



Example 4-7 Continues

Length = 200 mm; y = 5 mm; I = 833.33 mm^4

Stress $\sigma = \frac{M}{I} y$ but $M = \frac{wl}{4} = \frac{mgl}{4} = 2.5ml$

This implies

$$\sigma = \frac{2.5mly}{I} = \frac{2.5(200)(5)}{833.33} m = 3m$$

Where ***m*** is the load in ***Kg***

a. The yield strength

From the plot, yield load is 12.5 Kg, therefore, the yield stress is 12.5(3) = 37.5 Nmm^{-2}

Example 4-7 Continues

b. The ultimate strength

From the plot, yield load is 20 Kg, therefore, the yield stress is 20(3) = 60 Nmm^{-2}

c. The modulus of elasticity

From the plot, the slope is 240

From eq. 6,

$$w = \frac{48EI}{l^3}\delta$$

This implies

$$Slope = \frac{48EI}{l^3}$$

$$E = \frac{Slope\ x\ l^3}{48I}$$

$$E = \frac{240\left(200^3\right)}{48\left(833.33\right)}$$

$$= 48x10^3\ Nmm^{-2}$$

THANK YOU

LECTURE 5

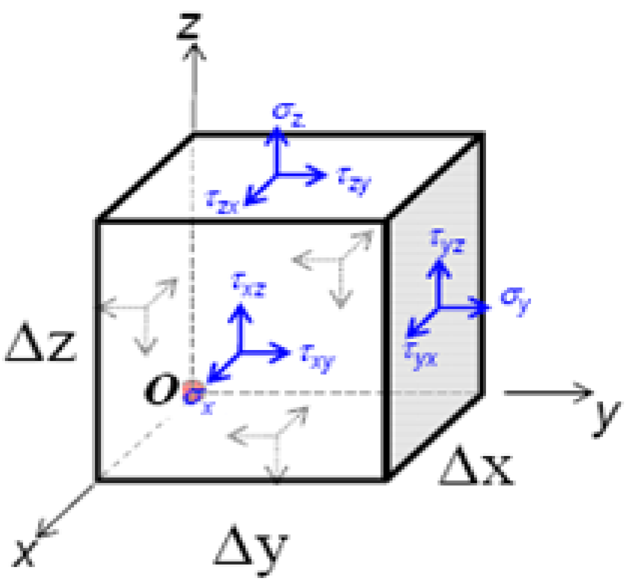
ANALYSIS OF STRESS AND STRAIN

Lecture Outline

1. Transformation of Plane Stress
2. Principal Planes
3. Mohr’s Circle for Plain Stress
4. Steps in the Construction of Mohr’s Circle
5. Mohr’s Circle for Plain Strain

Transformation of Plane Stress

- Figure 5-1(a) shows the stresses in all three directions of the Cartesian coordinate system.



a

- These stresses are put together in a matrix form as shown in Figure 5-1(b) to give the stress tensor.

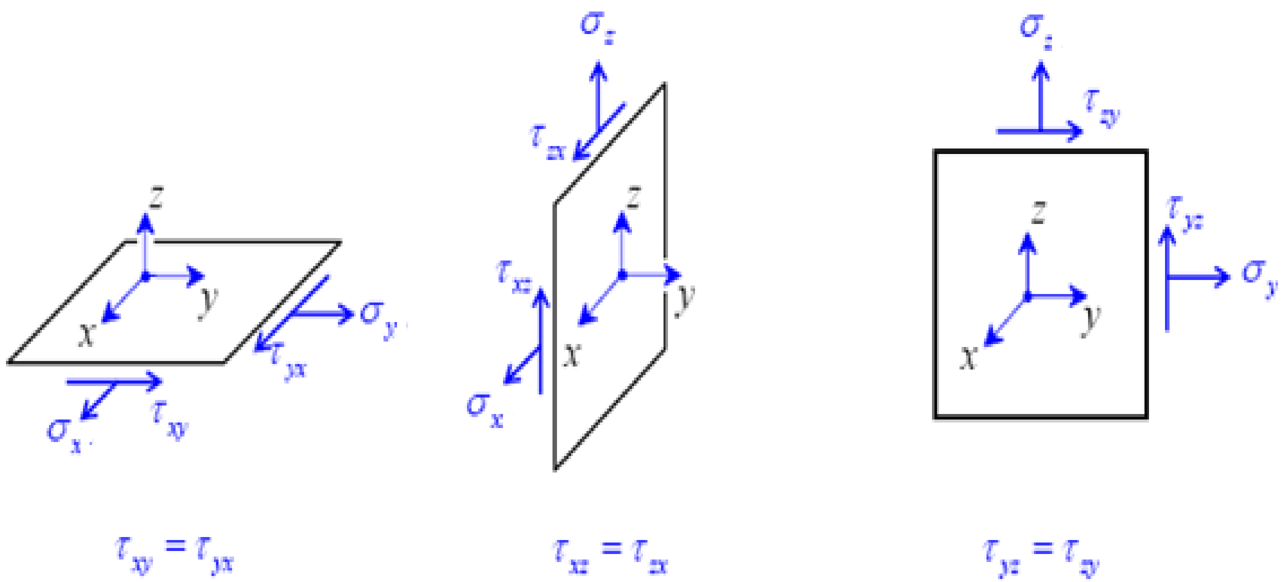
$$\sigma_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

b

Transformation of Plane Stress

- In plane stress condition all stresses in one particular primary direction are equal to zero as illustrated in Figure 5-2.

$$\begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} \sigma_x & 0 & \tau_{xz} \\ 0 & 0 & 0 \\ \tau_{zx} & 0 & \sigma_z \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_y & \tau_{yz} \\ 0 & \tau_{zy} & \sigma_z \end{bmatrix}$$

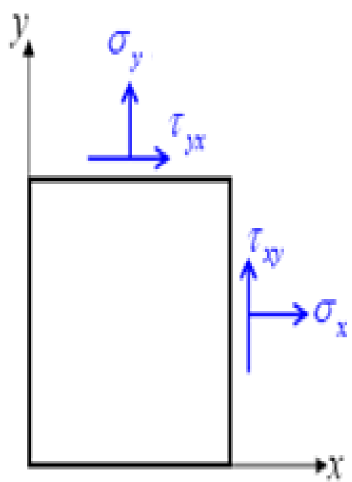


a b c

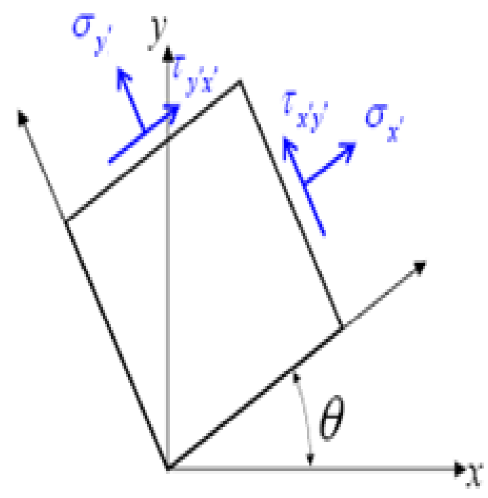
Transformation of Plane Stress

- The stresses in the *x* and *y* directions can be to transformed to the *x'* and *y'* directions inclined at an angle θ to the *x* and *y* axes respectfully.
- By resolving and balancing all forces in the new directions.
- This transforms the stress state in Figure 5-3a to that in Figure 5-3b.

$$\begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \sigma_{x'} & \tau_{x'y'} & 0 \\ \tau_{x'y'} & \sigma_{y'} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



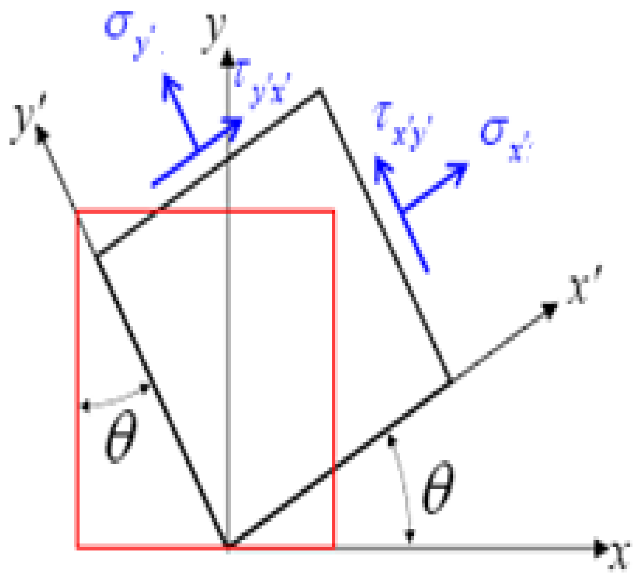
a

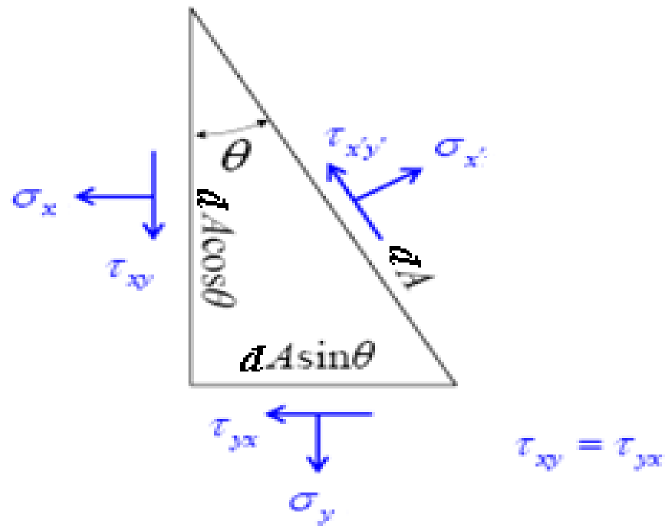


b

Transformation of Plane Stress

- Resolving and summing forces along the x' followed by y' direction using Figure 5-4:





$$\sum F_x = 0 = \begin{Bmatrix} \sigma_{x'}A - \tau_{xy}\left(dA\sin\theta\right)\cos\theta - \tau_{xy}\left(dA\cos\theta\right)\sin\theta \\ -\sigma_x\left(dA\cos\theta\right)\cos\theta - \sigma_y\left(\mathrm{dA}\sin\theta\right)\sin\theta \end{Bmatrix}$$

Transformation of Plane Stress

- Dividing through by dA and solving for $\sigma_{x'}$ and $\tau_{x'y'}$ yields:

$$\sum F_y = 0 = \left\{ \begin{array}{l} \tau_{x'y'}A - \tau_{xy}\left(dA\cos\theta\right)\cos\theta - \tau_{xy}\left(dA\sin\theta\right)\sin\theta \\ -\sigma_x\left(dA\cos\theta\right)\sin\theta - \sigma_y\left(\mathrm{dA}\sin\theta\right)\cos\theta \end{array} \right\}$$

$$\sigma_{x'} = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\cos\theta\sin\theta$$

$$\tau_{x'y'} = \left(\sigma_y - \sigma_x\right)\cos\theta\sin\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$

- Since $\sigma_{y'}$ is 90° away from $\sigma_{x'}$, $\sigma_{y'}$ is:

$$\sigma_{y'} = \sigma_x\cos^2\left(\theta + \tfrac{\pi}{2}\right) + \sigma_y\sin^2\left(\theta + \tfrac{\pi}{2}\right) + 2\tau_{xy}\cos\left(\theta + \tfrac{\pi}{2}\right)\sin\left(\theta + \tfrac{\pi}{2}\right)$$

$$\sigma_{y'} = \sigma_x\sin^2\theta + \sigma_y\cos^2\theta - 2\tau_{xy}\cos\theta\sin\theta$$

Transformation of Plane Stress

- We can rewrite them as

$$\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = \left(\frac{\sigma_y - \sigma_x}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Transformation of Plane Stress

- It is observed that for a plane stress situation the sum of the normal stresses exerted on a cubic element of material is independent of the orientation of that element.

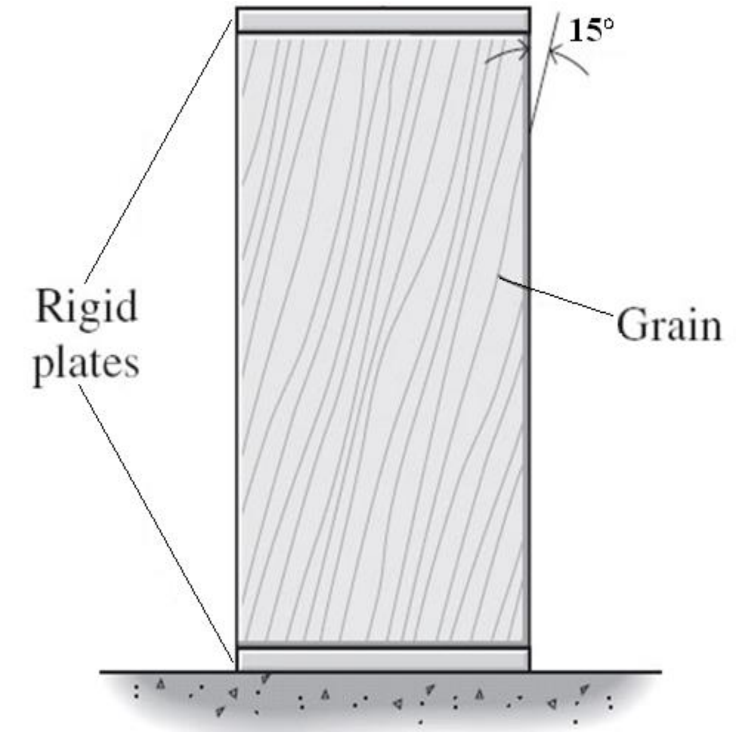
$$\sigma_{x'} + \sigma_{y'} = \left\{ \begin{array}{l} \left[\left(\dfrac{\sigma_x + \sigma_y}{2} \right) + \left(\dfrac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta \right] \\ + \left[\left(\dfrac{\sigma_x + \sigma_y}{2} \right) - \left(\dfrac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta - \tau_{xy} \sin 2\theta \right] \end{array} \right\} = \sigma_x + \sigma_y$$

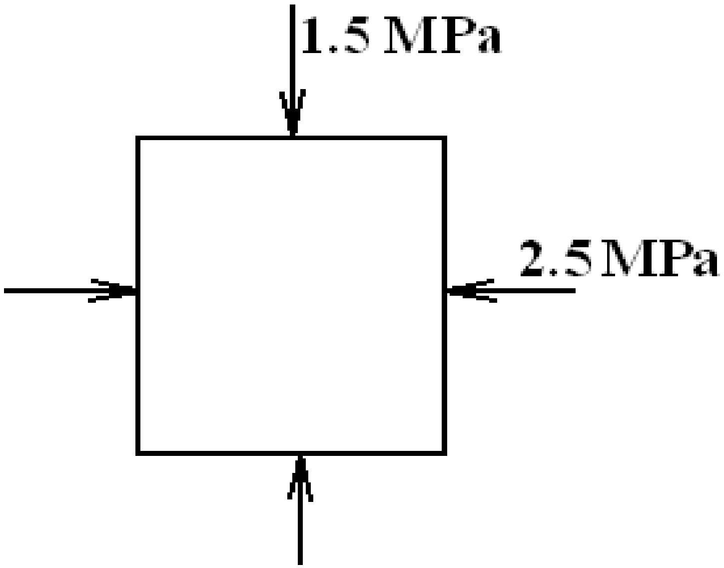
Example 5-1

The grain of a wooden member in a structure makes an angle of 15° to the vertical. For the state of stress shown in Figure Ex 5-1, determine:

a. the in-plane shearing stress parallel to the grain

b. the normal stress perpendicular to the grain





Example 5-1 Continues

a

$$\tau_{xy} = 0 \text{ MPa}, \sigma_x = -2.5 \text{ MPa}, \sigma_y = -1.5 \text{ MPa}, \theta = -15°$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} sin2\theta + \tau_{xy}\, cos2\theta$$

$$= -\frac{(-2.5\,) - (-1.5)}{2} \sin(-30) + 0$$ = -0.25 Mpa

b.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} cos2\theta + \tau_{xy}\, sin2\theta$$

$$= \frac{(-2.5) + (-1.5)}{2} + \frac{(-2.5) - (-1.5)}{2} \cos(-30) + 0$$ = -2.45 MPa

Principal Planes

- The normal stress, $\sigma_{x'}$ (5-7) is maximum or minimum when its derivative is equal to zero:

$$\frac{\delta \sigma_{x'}}{\delta \theta} = 0 = -\left(\frac{\sigma_x - \sigma_y}{2}\right)(2\sin 2\theta) + 2\tau_{xy}\cos 2\theta$$

- Solving for θ yields the plane where maximum and minimum normal stresses occur.

$$\tan 2\theta_{principal} = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

Principal Planes

- Similarly, to find the plane on which the shear stress is maximum, the derivative of $\tau_{x'y'}$ can be used.

- The maximum in-plane shear stress can therefore be found as follows:

$$\frac{\delta\tau_{x'y'}}{\delta\theta} = 0 = -\left(\frac{\sigma_x - \sigma_y}{2}\right)(2\cos 2\theta) - 2\tau_{xy}\sin 2\theta$$

- Solving for θ yields the plane of maximum/minimum shear stress

$$\tan 2\theta_{shear} = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

Principal Planes

- The two principal stresses and the maximum shearing stress are:

$$\sigma_{\max} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

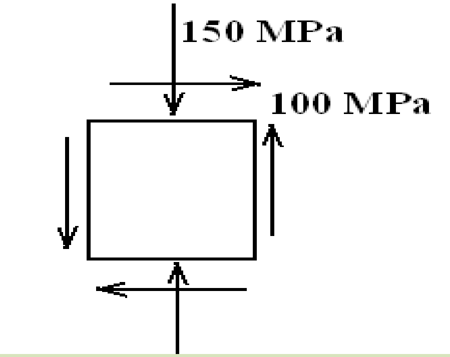
$$\sigma_{\min} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

Example 5-2

For a cast iron material with ultimate tensile strength of 160 MPa, and ultimate compression strength of 320 MPa, having the stress conditions shown in Figure Ex 5-2 determine:

a. The maximum shearing stress
b. The principal stresses



Solution

a.

$$\sigma_x = 0 \text{ MPa}, \sigma_y = -150 \text{ MPa}, \tau_{xy} = 100 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{75^2 + 100^2} = 125\ MPa$$

Example 5-2 Continues

$\tau_{\max} = \mathrm{R}$ =125 MPa

$$\boldsymbol{\sigma_1 = \sigma_{ave} + R = -75 + 125 = 50\ \mathrm{M}P\mathrm{a}}$$

$$\boldsymbol{\sigma_2 = \sigma_{ave} - R = -75 - 125 = -200\ \mathrm{MPa}}$$

Mohr's Circle for Plain Stress

- The Mohr's stress provides a graphical way of representing the transformation of stress equations.

- The equation for the Mohr's circle can be derived by rearranging the stress transformation equations.

$$\sigma_{x'} - \sigma_{average} = \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

- Re-write the equations 5-7 and 5-9 as:

$$\tau_{x'y'} = \left(\frac{\sigma_y - \sigma_x}{2} \right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

Mohr's Circle for Plain Stress

- Squaring both sides of each equation and adding them together yields:

$$\left[\sigma_{x'} - \sigma_{average}\right]^2 + \left(\tau_{xy}\right)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$\left[\sigma_{x'} - \sigma_{average}\right]^2 + \left(\tau_{xy}\right)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + R^2$$

- Equation for a circle expressed in (σ, τ) coordinates with a center at ($\sigma_{average}$, 0), where $\sigma_{average}$ is

$$\sigma_{average} = \left(\frac{\sigma_x + \sigma_y}{2}\right)$$

- The radius of Mohr's circle, R is:

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \tau_{\max}$$

- Therefore

$$\sigma_{\max} = \sigma_{average} + R$$

$$\sigma_{\max} = \sigma_{average} - R$$

$$\tau_{\max} = R$$

Steps in the Construction of Mohr's Circle

1. Draw a state of stress free body diagram. i.e. show the stresses σ_x, σ_y, and τ_{xy} on a cube. Label the vertical plane V and the horizontal plane H.

2. Write the coordinates of points V and H as V(σ_x, - τ_{xy}) and H(σ_y, τ_{xy}). A positive value for σ_{ij} produces a clockwise (CW) moment about the center of the cube (i.e. CW rotation of the cube).

3. Draw the horizontal axis with the tensile normal stress to the right (i.e. positive) and the compressive normal stress to the left (i.e. negative).

Steps in the Construction of Mohr's Circle

4. Locate points *V* and *H* and join the points by drawing a line. Label the point where line *VH* intersects the horizontal axis as *C*, the center of the circle. The center has coordinates *C* ($\sigma_{average}$, 0).

5. Draw Mohr's circle with point *C* as the center and a radius, *R* of length *CH* or

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \tau_{max}$$

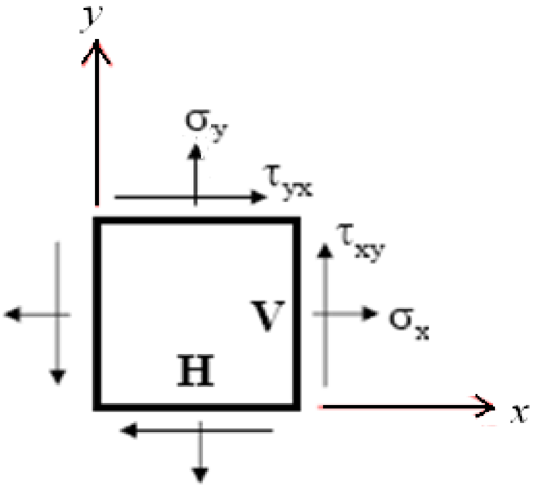
6. The angle between lines CV and Cσ_1 is labeled 2θ because the angles on Mohr's circle are double the actual angle between planes.

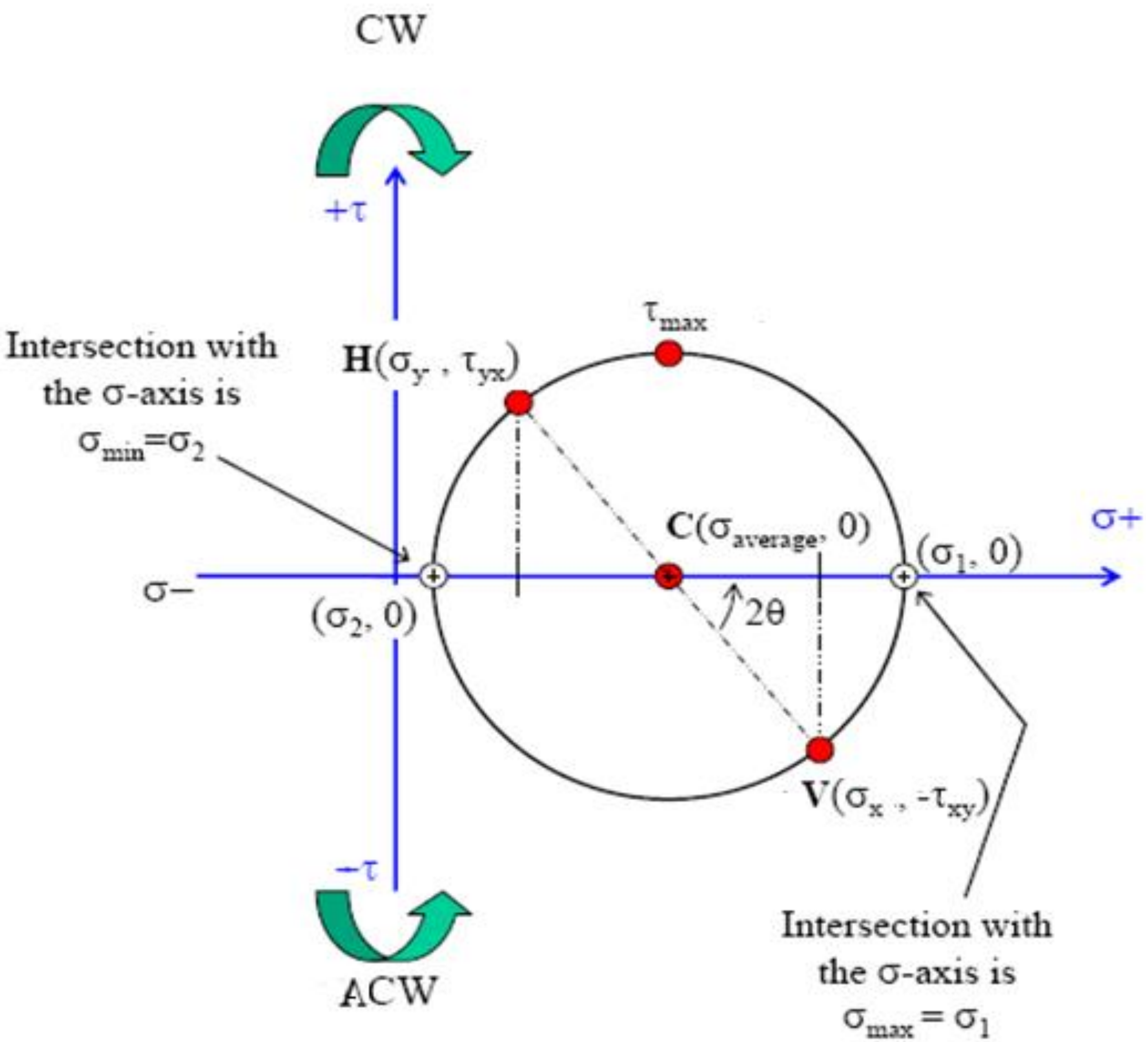
Steps in the Construction of Mohr's Circle

7. To determine the direction of rotation (i.e. the sign) we first record the direction in which we move from point V(σ_x, -τ_{xy}) to point (σ_1, 0) on Mohr's circle. If the direction of rotation is CCW (i.e. towards the positive shear direction), then the sign of θ is positive. If the rotation is CW then the sign of θ is negative.

8. This is illustrated in Figure 5-8.

Steps in the Construction of Mohr's Circle





Example 5-3

Consider a point in a solid that is subjected to the following state of stress:

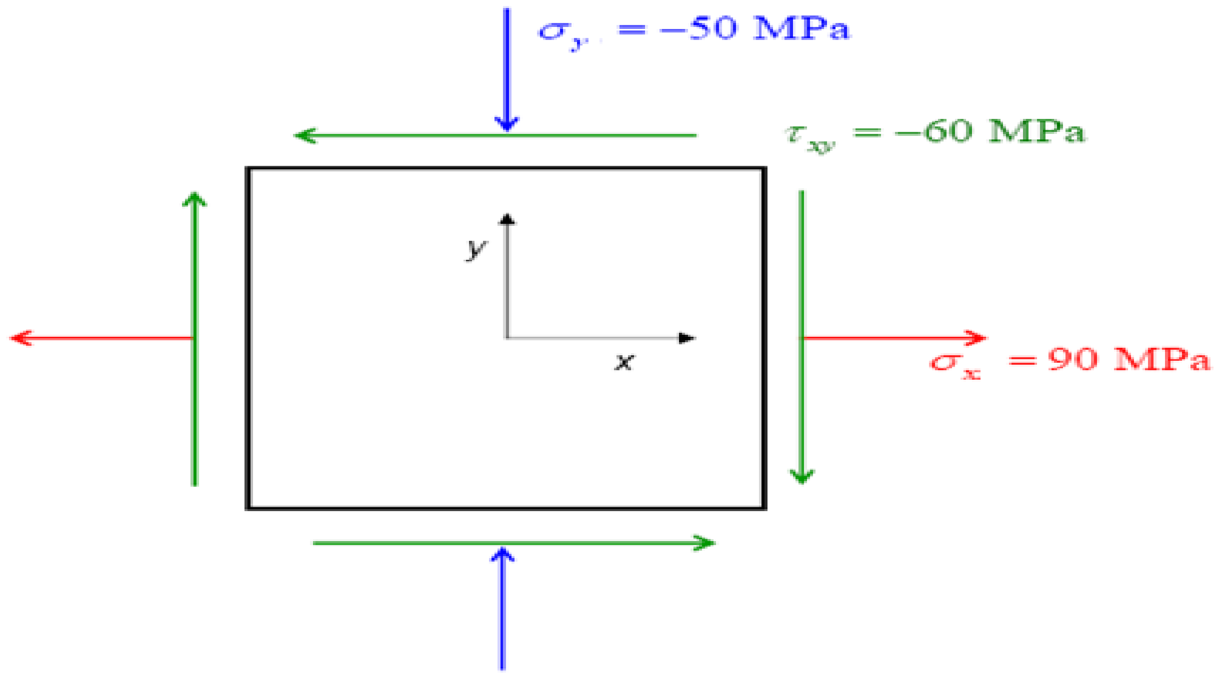
σ_x = 90 MPa;

σ_y = -50 MPa;

τ_{xy} = -60 MPa.

a. Draw a free body diagram representing the stress state.
b. Determine the principal stresses, the maximum in-plane shear stress acting on the point, and the orientation of the principal planes using Mohr's circle.
c. Show the stresses on an appropriate diagram.

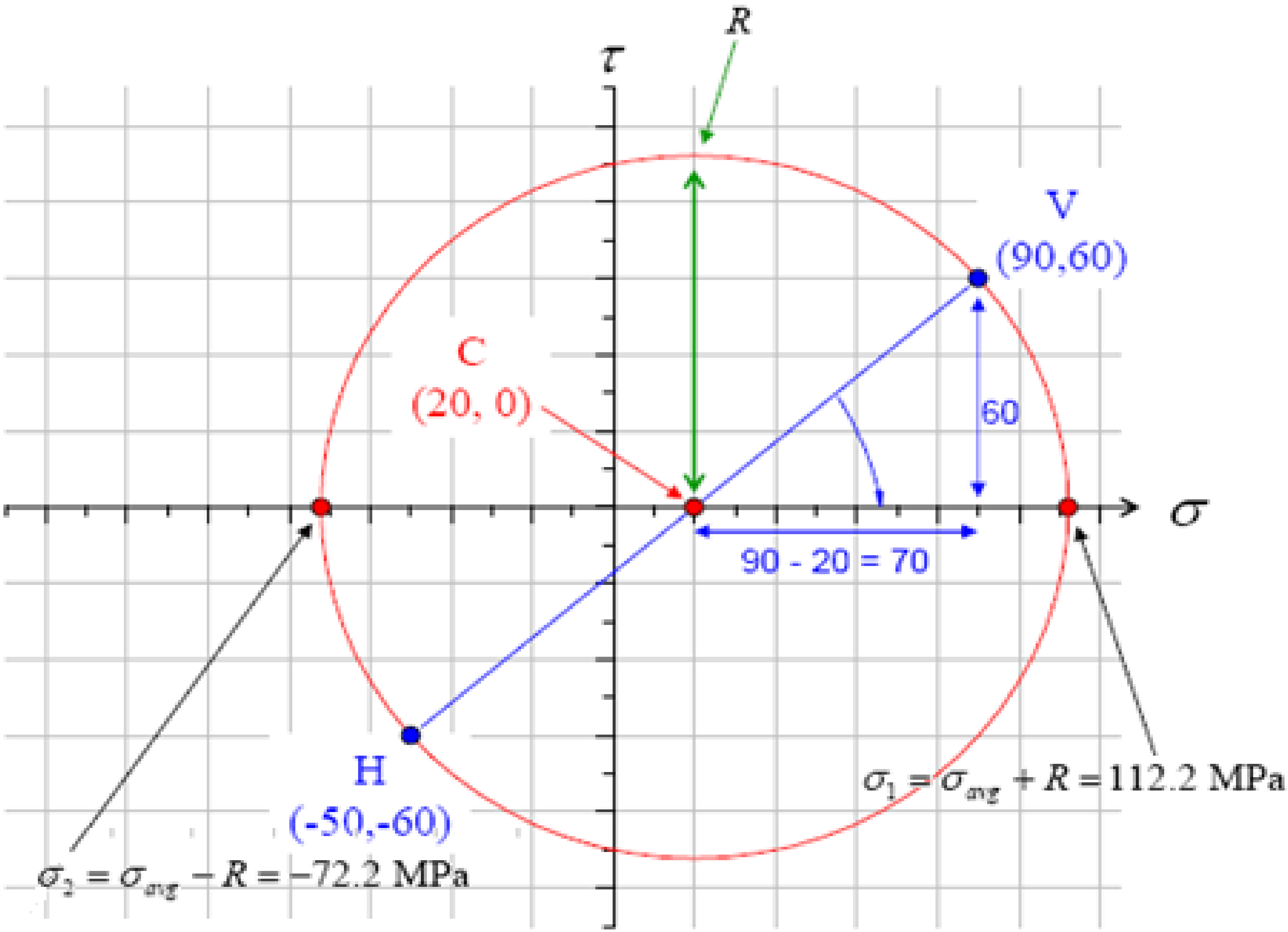
Example 5-3 Continues



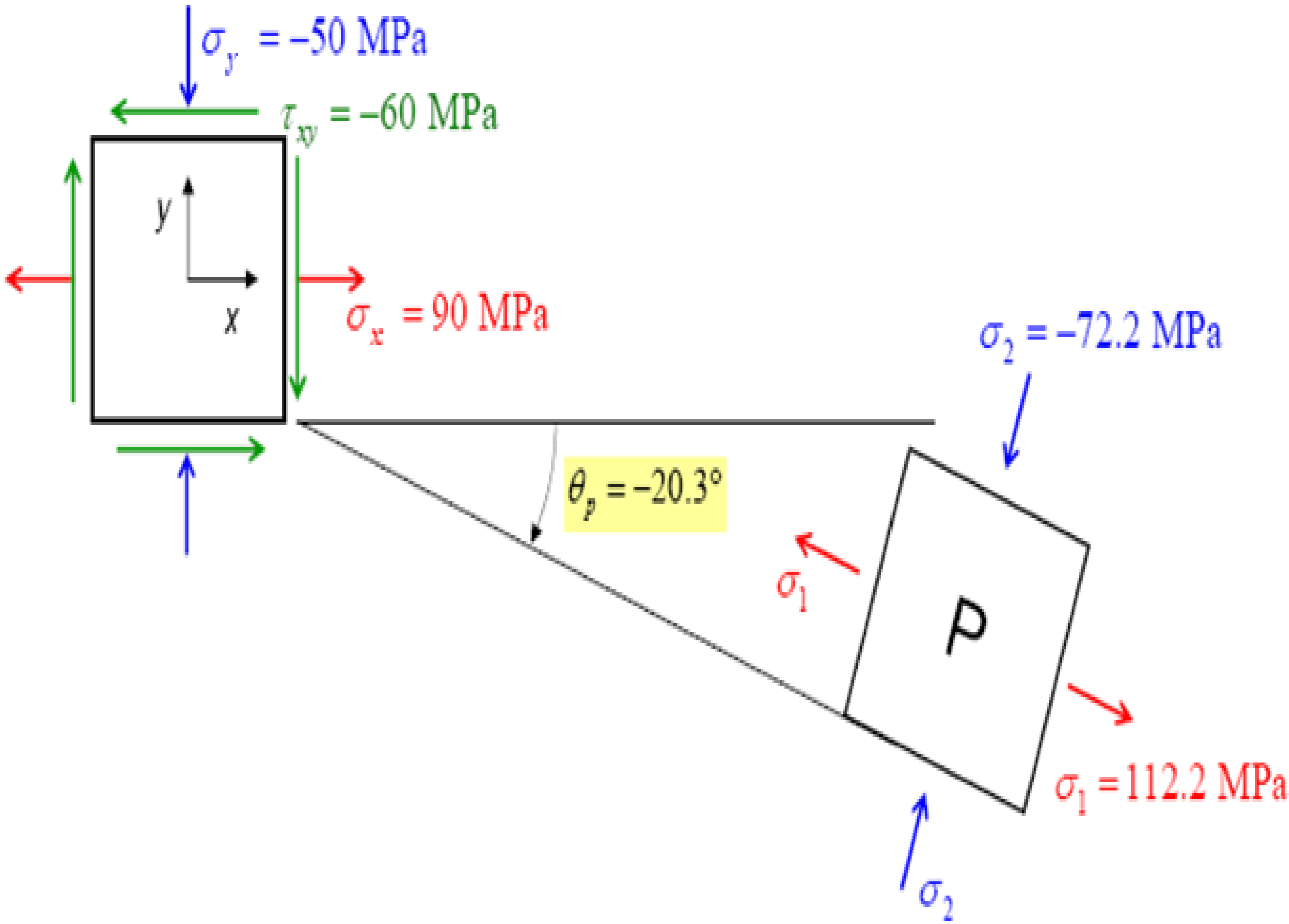
$V = (\sigma_x, -\tau_{xy}) = (90, 60)$

$H = (\sigma_y, +\tau_{xy}) = (-50, -60)$

Example 5-3 Continues



Example 5-3 Continues



Mohr's Strain Circle

- Mohr's strain circle is very close to the stress circle..
- The center is given by: (ε_{ave}, 0),
- where $\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y)$
- The radius of the Mohr's strain circle is

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

- The principal strains are given by: $\varepsilon_1 = \varepsilon_{ave} + R$ and : $\varepsilon_2 = \varepsilon_{ave} - R$
- The maximum shearing strain is: $\gamma_{max} = 2R$
- Mohr's circle of strain can be transformed into a concentric Mohr's circle of for stress by means of the scale transformations:

$$R_\sigma = R_\varepsilon \frac{E}{1+\nu};$$

$$(OC)_\sigma = (OC)_\varepsilon \frac{E}{1-\nu}$$

Example 5-4

Using the values in Example 5-3, Determine the principal strains, the maximum in-plane shear strain acting on the point, and the orientation of the principal planes using Mohr's circle. Take Young's Modulus = 200 GPa and Poisson ratio = 0.3

$$\varepsilon_x = \tfrac{1}{E}\left(\sigma_x - \upsilon\sigma_y\right) = \frac{1}{200x10^3}\left(90 - 0.3[-50]\right) = 525x10^{-6} = 525\mu$$

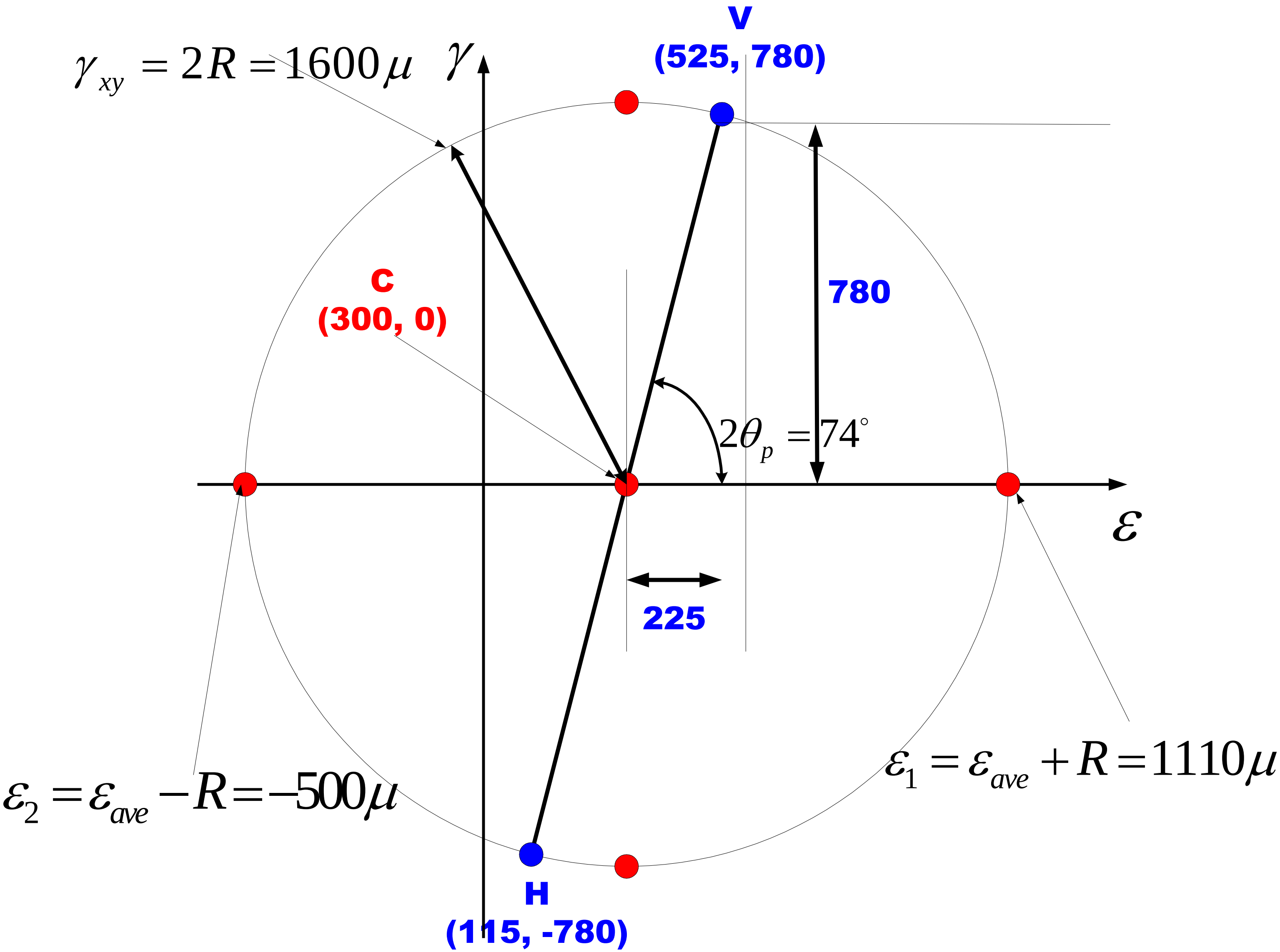
Solution

$$\varepsilon_y = \tfrac{1}{E}\left(\sigma_y - \upsilon\sigma_x\right) = \frac{1}{200x10^3}\left(50 - 0.3[90]\right) = 115x10^{-6} = 115\mu$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} = \frac{2(1+\upsilon)}{E}\tau_{xy} = \frac{2(1+0.3)}{200x10^3}\left(-60\right) = -780x10^{-6} = -780\mu$$

Example 5-4 Continues

V = (ε_x, -γ_{xy}) = (525, 7.8); H = (ε_y, + γ_{xy}) = (115, -7.8)



THANK YOU

LECTURE 6

FAILURE CRITERIA

Lecture Outline

1. Introduction
2. Mohr’s Fracture Criterion
3. Tresca or Maximum Shearing Stress Criterion
4. Von Mises or Maximum-Distortion-Energy Criterion
5. Steps in the Construction of Mohr’s Circle
6. Mohr’s Circle for Plain Strain

Introduction

- Considering the same isotropic polycrystalline metal deformed in a multi-axial stress state.

- We can not simply determine the stress at yielding because stress will vary from point to point.

- Instead we calculate an equivalent stress from the components of the stress tensor and compare it with the critical stress for yielding or failure.

- For an isotropic metal under a uni-axial tension for example, the material deforms elastically up to the yield stress.

- When applied load reaches the critical load (i.e., the yield point), plastic deformation occurs.

Introduction

- The yield or failure criterion could be expressed as:

$$f(\sigma_{ij}) = \sigma_{applied} - \sigma_Y = 0$$

- For an isotropic material, the yield criteria can be expressed in terms of the principal stresses;$f(\sigma_1, \sigma_2, \sigma_3) = 0$

- If the function $f(\sigma_1, \sigma_2, \sigma_3)$ is plotted on an orthogonal σ_1, σ_2, σ_3 axes a yield surface is obtained.

- The yield surface can be used to determine, for each possible state of stress, whether or not a material yields or fails.

- There are many different yield criteria. Only three are going to be considered.

Introduction

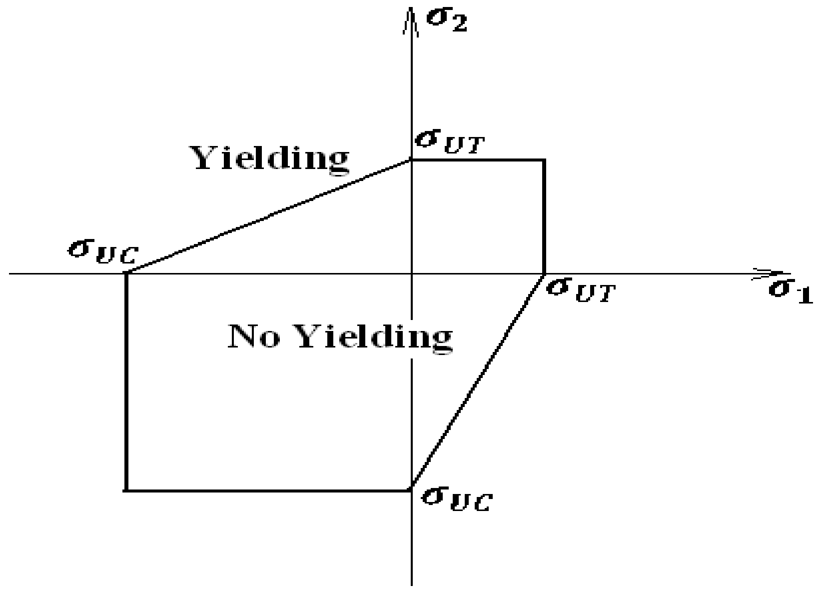
- They are the
  - Mohr’s criterion (for brittle fracture prediction),
  - the maximum shearing stress criterion and the maximum distortion energy criterion (for ductile yielding prediction).

Mohr's Fracture Criterion

- The Mohr's criterion is used to predict the failure of brittle materials.

- The criterion states that if the plot of the principal stresses falls within the envelope in Figure 6-1, then no failure occurs.

- However, if it falls outside the enclosed region, failure will occur.



Mohr's Fracture Criterion

- In other words, if the value of the expression: $\left(\frac{\sigma_1}{U_{UT}} - \frac{\sigma_2}{U_{UC}}\right)$ is less than one, no fracture occurs, else if it is more than one, then fracture would occur.
- i.e., if $\left(\frac{\sigma_1}{U_{UT}} - \frac{\sigma_2}{U_{UC}}\right) > 1$ Yielding occurs.
- But if $\left(\frac{\sigma_1}{U_{UT}} - \frac{\sigma_2}{U_{UC}}\right) < 1$ No Yielding occurs.

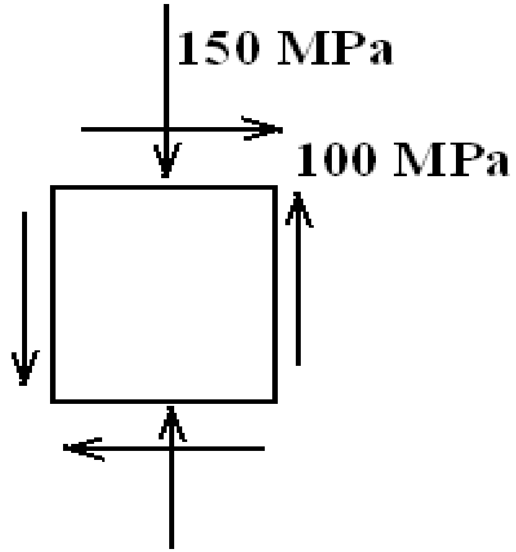
Example 6-1

For a cast iron material with ultimate tensile strength of 160 MPa, and ultimate compression strength of 320 MPa, having the stress conditions shown in Figure Ex 6-1 determine:

a. The maximum shearing stress

b. The principal stresses

c. Check for failure using the Mohr's fracture criterion.



Example 6-1 Continues

- $\sigma_x = 0\ \text{MPa}, \sigma_y = -150\ \text{MPa}, \tau_{xy} = \mathbf{100\ MPa}$
- $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \mathbf{-75\ MPa}$
- $\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{75^2 + 100^2} = \mathbf{125\ MPa}$
- $\sigma_1 = \sigma_{ave} + R = -75 + 125 = \mathbf{50\ MPa}$
- $\sigma_2 = \sigma_{ave} - R = -75 - 125 = \mathbf{-200\ MPa}$
- $\frac{\sigma_1}{\sigma_{UT}} - \frac{\sigma_2}{\sigma_{UC}} = \frac{50}{160} - \frac{(-200)}{320} = 0.9373 < \mathbf{1}$
- Therefore there will be no Rapture or failure.

Tresca or Maximum Shearing Stress Criterion

- Yielding in a material occurs when the maximum shearing stress, τ_{max} reaches a critical value, the shearing yield strength, τ_Y.
- The shearing yield strength is half the tensile yield strength,

$$\left(\boldsymbol{\tau_Y} = \frac{1}{2}\boldsymbol{\sigma_Y}\right).$$

- If the principal stresses are either both positive or both negative,

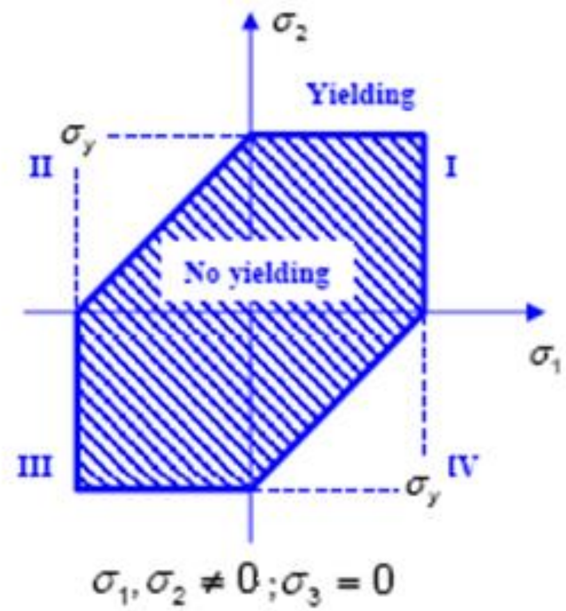
$$\boldsymbol{\tau_{max}} = \frac{1}{2}|\boldsymbol{\sigma_{max}}| = \frac{1}{2}|\boldsymbol{\sigma_1}|$$

- If the maximum stress is positive and the minimum stress is negative.

$$\boldsymbol{\tau_{max}} = \frac{1}{2}|\boldsymbol{\sigma_{max}} - \boldsymbol{\sigma_{min}}| = \frac{1}{2}|\boldsymbol{\sigma_1} - \boldsymbol{\sigma_2}|$$

Tresca or Maximum Shearing Stress Criterion

- Thus in this criterion:
  - If $\tau_{max} > \tau_Y$, **yielding** occurs
  - If $\tau_{max} < \tau_Y$, **no yielding** occurs



- That is if a plot of the principal stresses in the plot shown in Figure 6-2 lies within the shaded region, no yielding occurs, but if it lies outside the enclosed region, then yielding will occur.
- The factor of safety, *N* for this criterion is defined as $N = \dfrac{\tau_Y}{\tau_{max}}$.

Von Mises or Maximum-Distortion-Energy Criterion

- The Von Mises criterion states that, a given structural component is safe as long as the maximum value of the distortion energy per unit volume in that material remains smaller than the distortion energy per unit volume required to cause yield in a tensile-test specimen of the same material.

- Under plane stress, the distortion energy per unit volume in an isotropic material is given by:

$$u_d = \frac{1}{6G}\left({\sigma_1}^2 - \sigma_1\sigma_2 + {\sigma_2}^2\right);\ \sigma_3 = 0.$$

Where $\boldsymbol{\sigma_1}$ and $\boldsymbol{\sigma_2}$ are the principal stresses and G is the modulus of rigidity.

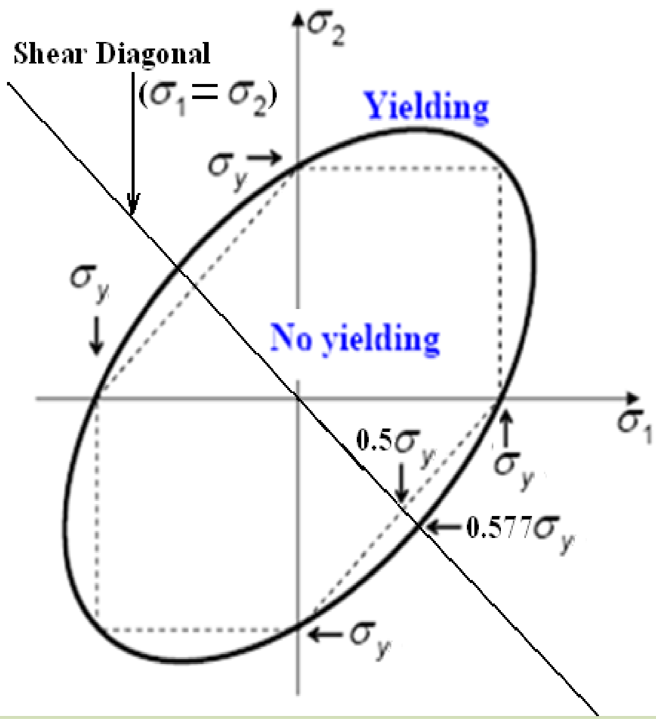
Von Mises or Maximum-Distortion-Energy Criterion

- For yielding to occur, the equivalent stress,

$$\sigma_e = \sqrt{(\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2)} = \sqrt{(\sigma_x{}^2 - \sigma_x\sigma_y + \sigma_y{}^2 + 3\tau_{xy}{}^2)},$$

should be greater than the yield strength.

- Figure 7-3 shows the plot of the safe region of this criterion, which is an ellipse.

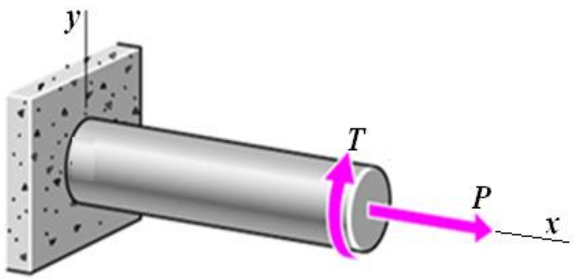


Von Mises or Maximum-Distortion-Energy Criterion

- The dashes in the diagram give the plot of the Tresca criterion as a comparison to the von Mises criterion.
- i.e. if $\sigma_e > \sigma_y$; **yielding** occurs.
- But, if $\sigma_e < \sigma_y$; **No yielding** occurs.
- The factor of safety, *N* for maximum-distortion-energy criterion is defined as $N = \frac{\sigma_Y}{\sigma_e}$.

Example 6-2

- The circular shaft in Figure Ex 6-2 is made with a material with a yield strength of 248 MPa.



Given that the tensile force P is 250 N and the diameter of the shaft is 3.8 cm, determine the maximum torque T, that the shaft can bear without failing.

$\sigma_Y = 248$ MPa

d = 3.8 cm = 0.038 m

$P = 250$ N

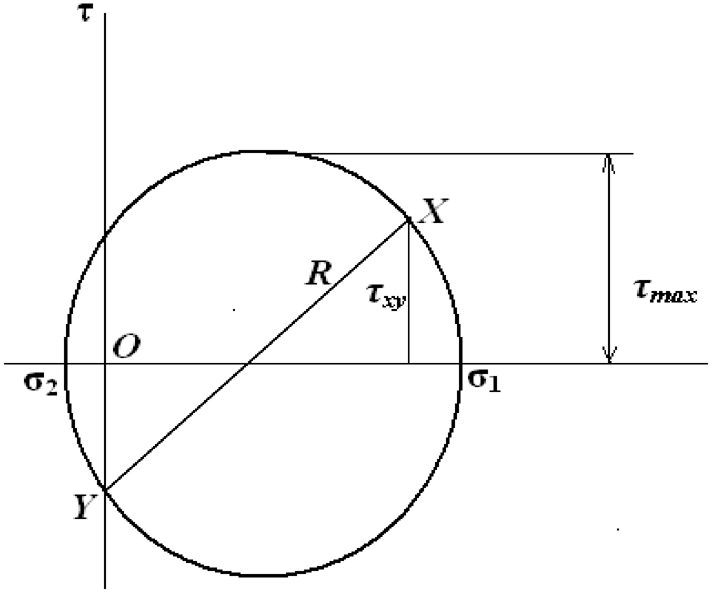
$$\sigma_x = \frac{P}{A} = \frac{4\times 250}{\pi\times(0.038)^2}$$

$= 220.4362$ MPa ≈ 220.44 MPa

$\sigma_y = 0$ MPa

$\tau_{xy} = ? = \dfrac{Tc}{J}$

Example 6-2 Continues



From the sketch of the Mohr's stress circle, the principal stresses have different signs.

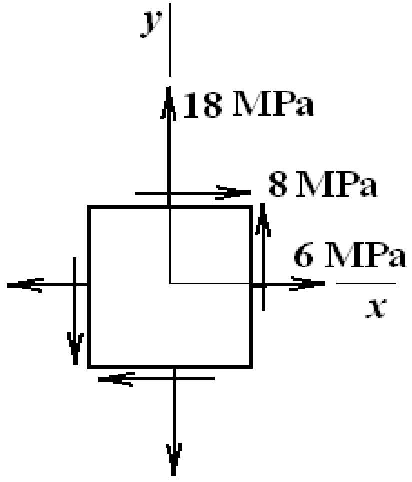
Therefore $\tau_{max} = R = \sqrt{\left(\frac{\sigma_x \;\; \sigma_y}{2}\right)^2 + \tau_{xy}^{\;\;2}} = \sqrt{110.22^2 + \tau_{xy}^{\;\;2}}$

- But for yield to occur, $\tau_{max} = \tau_Y = \frac{\sigma_Y}{2} = \frac{248}{2} = 124$ MPa
- $\Rightarrow 124^2 - 110.22^2 = \tau_{xy}^{\;\;2}$
- $\tau_{xy} = \sqrt{124^2 - 110.22^2} = 56.8115$ MPa
- But $\tau_{xy} = \frac{Tc}{J}$
- $\Rightarrow T = \frac{\tau_{xy} J}{c} = \frac{\tau_{xy}(\pi c^3)}{2} = \frac{56.8115\times 10^6(\pi\times 0.019^3)}{2} = 6.120923 = 6$ kNm

Example 6-3

For the state of stress given in Figure Ex 6-3 for a material with yield strength of 42 MPa, determine:

a. the principal stresses
b. the maximum shearing stress
c. whether the material will yield using the maximum-shearing-stress criterion
d. the factor of safety using the maximum-shearing-stress criterion
e. whether the material will yield using the maximum-distortion-energy criterion
f. the factor of safety using the maximum-distortion-energy criterion



Example 6-3 Continues

$\sigma_x = 6$ MPa, $\sigma_y = 18$ MPa, $\tau_{xy} = 8$ MPa

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(6 + 18) = 12 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x \quad \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{(-6)^2 + 8^{\,2}} = 10 \text{ MPa}$$

a. Principal Stresses

$$\sigma_1 = \sigma_{ave} + R = 12 + 10 = 22 \text{ MPa}$$

$$\sigma_2 = \sigma_{ave} - R = 12 - 10 = 2 \text{ MPa}$$

b. Maximum shearing stress

Since both σ_1 and σ_2 have the same sign (+ve)

$$\tau_{max} = \frac{\sigma_1}{2} = \frac{22}{2} = 11 \text{ MPa}$$

Example 6-3 Continues

c. Failure, using Maximum-shearing-stress criterion

$$\tau_Y = \frac{\sigma_Y}{2} = \frac{42}{2} = 21 \text{ MPa}$$

11 < 21; i.e. $\tau_{max} < \tau_Y$; Therefore there will be no failure

d. *F.S.* using Maximum-shearing-stress criterion

$$F.S. = \frac{\tau_Y}{\tau_{max}} = \frac{21}{11} = 1.91$$

e. Failure using the Maximum-distortion-energy criterion

$$\sigma_E = \sqrt{\sigma_1^{2} + \sigma_2^{2} - \sigma_1 \sigma_2} = \sqrt{22^2 + 2^2 - 22 \times 2} = 21.0 \text{ MPa}$$

21.07 MPa < 42 MPa; i.e. $\sigma_E < \sigma_Y$ therefore there will be no failure

Example 6-3 Continues

f. *F.S.* using Maximum-distortion-energy criterion

$$F.S. = \frac{\tau_Y}{\tau_{max}} = \frac{42}{21.07} = 1.99$$

THANK YOU