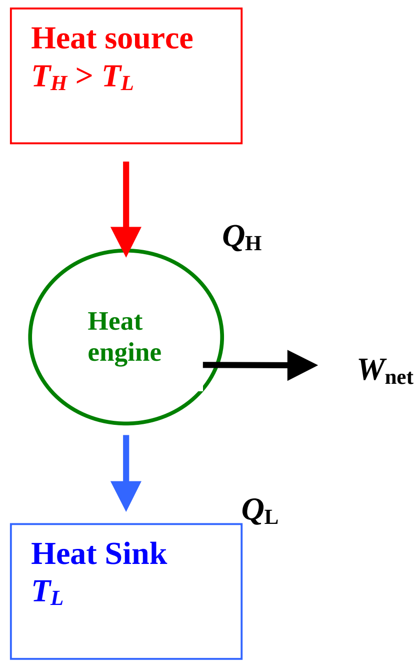
Chapter 9: Vapor and Combined Power Cycles

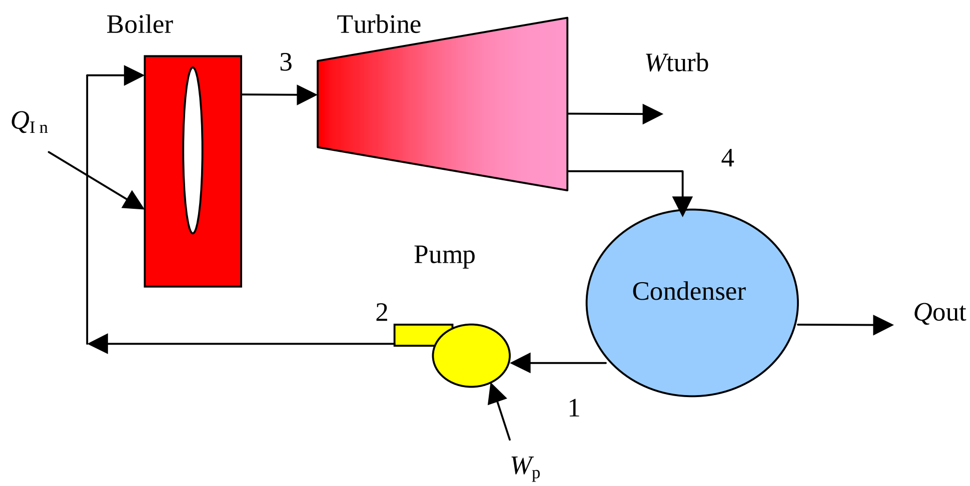
We consider power cycles where the working fluid undergoes a phase change. The best example of this cycle is the steam power cycle where water (steam) is the working fluid.

Carnot Vapor Cycle

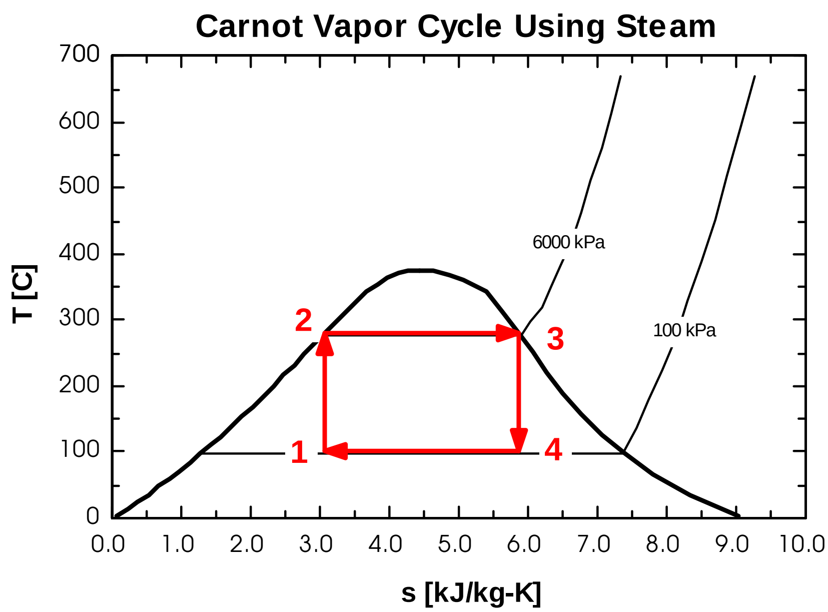


The heat engine may be composed of the following components.

Steam Power Cycle



The working fluid, steam (water), undergoes a thermodynamic cycle from 1-2-3-4-1. The cycle is shown on the following *T*-*s* diagram.



The thermal efficiency of this cycle is given as

$$\eta_{th,\,Carnot} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$= 1 - \frac{T_L}{T_H}$$

Note the effect of T_H and T_L on $\eta_{\text{th, Carnot}}$.

- The larger the T_H the larger the $\eta_{\text{th, Carnot}}$
- The smaller the T_L the larger the $\eta_{\text{th, Carnot}}$

To increase the thermal efficiency in any power cycle, we try to increase the maximum temperature at which heat is added.

Reasons why the Carnot cycle is not used:

- Pumping process 1-2 requires the pumping of a mixture of saturated liquid and saturated vapor at state 1 and the delivery of a saturated liquid at state 2.

- To superheat the steam to take advantage of a higher temperature, elaborate controls are required to keep T_H constant while the steam expands and does work.

To resolve the difficulties associated with the Carnot cycle, the Rankine cycle was devised.

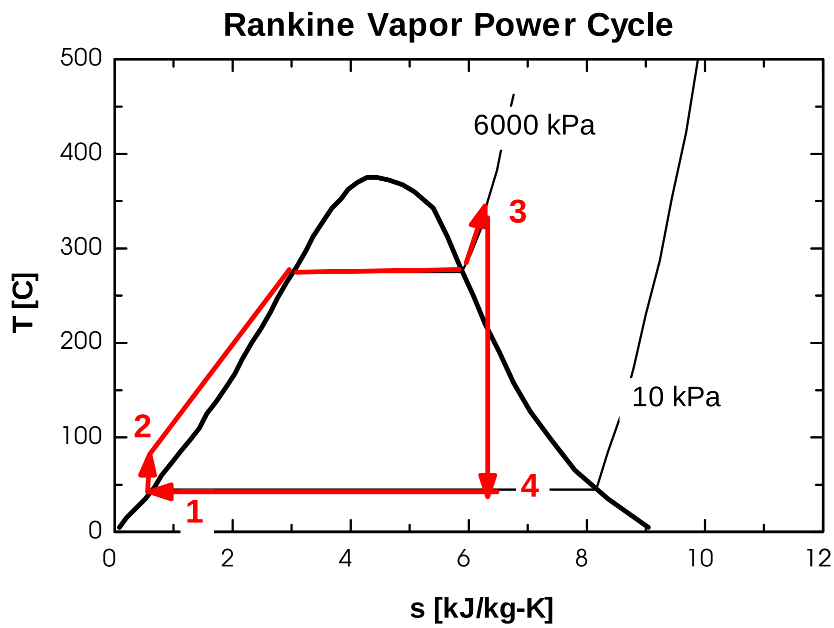
Rankine Cycle

The simple Rankine cycle has the same component layout as the Carnot cycle shown above. The simple Rankine cycle continues the condensation process 4-1 until the saturated liquid line is reached.

Ideal Rankine Cycle Processes

Process	Description
1-2	Isentropic compression in pump
2-3	Constant pressure heat addition in boiler
3-4	Isentropic expansion in turbine
4-1	Constant pressure heat rejection in condenser

The *T-s* diagram for the Rankine cycle is given below. Locate the processes for heat transfer and work on the diagram.



Example 9-1

Compute the thermal efficiency of an ideal Rankine cycle for which steam leaves the boiler as superheated vapor at 6 MPa, 350^{o}C, and is condensed at 10 kPa.

We use the power system and *T*-*s* diagram shown above.
$P_2 = P_3 = 6$ MPa = 6000 kPa
$T_3 = 350^{o}$C
$P_1 = P_4 = 10$ kPa

Pump

The pump work is obtained from the conservation of mass and energy for steady-flow but neglecting potential and kinetic energy changes and assuming the pump is adiabatic and reversible.

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\dot{m}_1 h_1 + \dot{W}_{pump} = \dot{m}_2 h_2$$

$$\dot{W}_{pump} = \dot{m}(h_2 - h_1)$$

Since the pumping process involves an incompressible liquid, state 2 is in the compressed liquid region, we use a second method to find the pump work or the Δh across the pump.

Recall the property relation:

$$dh = T\,ds + v\,dP$$

Since the ideal pumping process 1-2 is isentropic, $ds = 0$.

$$dh = v\,dP$$

$$\Delta h = h_2 - h_1 = \int_1^2 v\,dP$$

The incompressible liquid assumption allows

$$v \cong v_1 = const.$$

$$h_2 - h_1 \cong v_1(P_2 - P_1)$$

The pump work is calculated from

$$\dot{W}_{pump} = \dot{m}(h_2 - h_1) \cong \dot{m}v_1(P_2 - P_1)$$

$$w_{pump} = \frac{\dot{W}_{pump}}{\dot{m}} = v_1(P_2 - P_1)$$

Using the steam tables

$$\left.\begin{array}{l} P_1 = 10\ kPa \\ Sat.\ liquid \end{array}\right\} \left\{\begin{array}{l} h_1 = h_f\ = 191.83\dfrac{kJ}{kg} \\ v_1 = v_f\ = 0.00101\dfrac{m^3}{kg} \end{array}\right.$$

$$
\begin{aligned}
w_{pump} &= v_1(P_2 - P_1) \\
&= 0.00101\frac{m^3}{kg}(6000 - 10)\ kPa\frac{kJ}{m^3 kPa} \\
&= 6.05\frac{kJ}{kg}
\end{aligned}
$$

Now, h_2 is found from

$$
\begin{aligned}
h_2 &= w_{pump} + h_1 \\
&= 6.05\frac{kJ}{kg} + 191.83\frac{kJ}{kg} \\
&= 197.88\frac{kJ}{kg}
\end{aligned}
$$

Boiler

To find the heat supplied in the boiler, we apply the steady-flow conservation of mass and energy to the boiler. If we neglect the potential and kinetic energies, and note that no work is done on the steam in the boiler, then

$$
\begin{aligned}
\dot{m}_2 &= \dot{m}_3 = \dot{m} \\
\dot{m}_2 h_2 + \dot{Q}_{in} &= \dot{m}_3 h_3 \\
\dot{Q}_{in} &= \dot{m}(h_3 - h_2)
\end{aligned}
$$

We find the properties at state 3 from the superheated tables as

$$
\left.\begin{aligned}
P_3 &= 6000\ kPa \\
T_3 &= 350^o C
\end{aligned}\right\}
\begin{cases}
h_3 = 3043.0\dfrac{kJ}{kg} \\
s_3 = 6.335\dfrac{kJ}{kg \cdot K}
\end{cases}
$$

The heat transfer per unit mass is

$$
\begin{aligned}
q_{in} &= \frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 \\
&= (3040.3 - 197.88)\frac{kJ}{kg} \\
&= 2845.2\frac{kJ}{kg}
\end{aligned}
$$

Turbine

The turbine work is obtained from the application of the conservation of mass and energy for steady flow. We assume the process is adiabatic and reversible and neglect changes in kinetic and potential energies.

$$
\begin{aligned}
&\dot{m}_3 = \dot{m}_4 = \dot{m} \\
&\dot{m}_3 h_3 = \dot{W}_{turb} + \dot{m}_4 h_4 \\
&\dot{W}_{turb} = \dot{m}(h_3 - h_4)
\end{aligned}
$$

We find the properties at state 4 from the steam tables by noting $s_4 = s_3$ and asking three questions.

$$
\begin{aligned}
&at\ \ P_4 = 10kPa:\ s_f = 0.6483\frac{kJ}{kg \cdot K};\ s_g = 8.1502\frac{kJ}{kg \cdot K} \\
&is\ \ s_4 < s_f\ ? \\
&is\ \ s_f < s_4 < s_g\ ? \\
&is\ \ s_g < s_4\ ?
\end{aligned}
$$

$$s_4 = s_f + x_4 s_{fg}$$

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.335 - 0.6493}{7.5009} = 0.758$$

$$\begin{aligned} h_4 &= h_f + x_4 h_{fg} \\ &= 191.83 \frac{kJ}{kg} + 0.758(2584.7 - 191.83) \frac{kJ}{kg} \\ &= 2005.6 \frac{kJ}{kg} \end{aligned}$$

The turbine work per unit mass is

$$\begin{aligned} w_{turb} &= h_3 - h_4 \\ &= (3043.0 - 2005.63) \frac{kJ}{kg} \\ &= 1037.4 \frac{kJ}{kg} \end{aligned}$$

The net work done by the cycle is

$$\begin{aligned} w_{net} &= w_{turb} - w_{pump} \\ &= (1037.4 - 6.05) \frac{kJ}{kg} \\ &= 1031.4 \frac{kJ}{kg} \end{aligned}$$

The thermal efficiency is

$$\eta_{th} = \frac{w_{net}}{q_{in}}$$

$$= \frac{1031.4 \frac{kJ}{kg}}{2845.2 \frac{kJ}{kg}}$$

$$= 0.363 \; or \;\; 36.3\%$$

Ways to improve the simple Rankine cycle efficiency:

- Superheat the vapor
  - Average temperature is higher during heat addition.
  - Moisture is reduced at turbine exit (we want x_4 in the above example > 85 percent).

- Increase boiler pressure (for fixed maximum temperature)
  - Availability of steam is higher at higher pressures.
  - Moisture is increased at turbine exit.
- Lower condenser pressure
  - Less energy is lost to surroundings.
  - Moisture is increased at turbine exit.

Extra Assignment

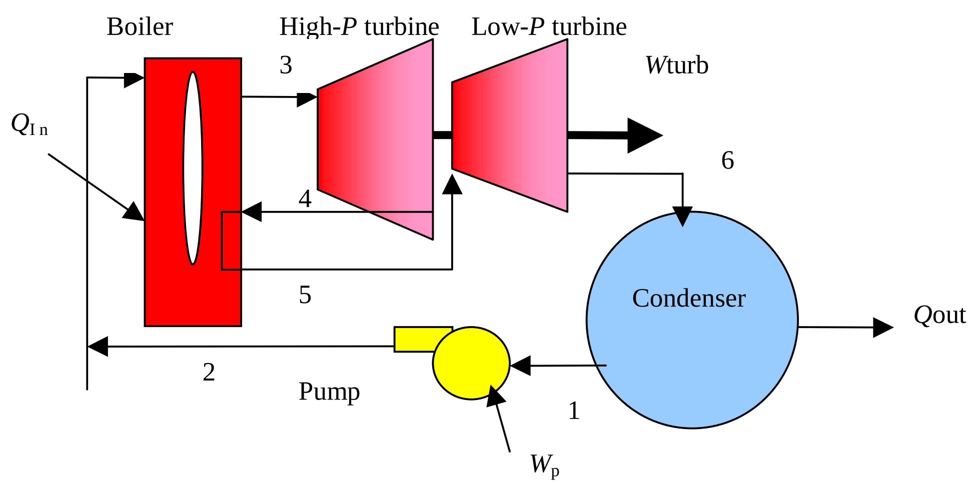
For the above example, find the heat rejected by the cycle and evaluate the thermal efficiency from

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

Reheat Cycle

As the boiler pressure is increased in the simple Rankine cycle, not only does the thermal efficiency increase, but also the turbine exit moisture increases. The reheat cycle allows the use of higher boiler pressures and provides a means to keep the turbine exit moisture (x > 0.85 to 0.90) at an acceptable level.

Steam Power Cycle with Reheat



Rankine Cycle with Reheat

Component	Process		First Law Result
Boiler	Const. P	q_{in}	$= (h_3 - h_2) + (h_5 - h_4)$
Turbine	Isentropic	w_{out}	$= (h_3 - h_4) + (h_5 - h_6)$
Condenser	Const. P	q_{out}	$= (h_6 - h_1)$
Pump	Isentropic	w_{in}	$= (h_2 - h_1) = v_1(P_2 - P_1)$

The thermal efficiency is given by

$$\eta_{th} = \frac{w_{net}}{q_{in}}$$

$$= \frac{(h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)}{(h_3 - h_2) + (h_5 - h_4)}$$

$$= 1 - \frac{h_6 - h_1}{(h_3 - h_2) + (h_5 - h_4)}$$

Let's sketch the T-s diagram for the reheat cycle.

Example 9-2

Compare the thermal efficiency and turbine-exit quality at the condenser pressure for a simple Rankine cycle and the reheat cycle when the boiler pressure is 4 MPa, the boiler exit temperature is 400°C, and the condenser pressure is 10 kPa. The reheat takes place at 0.4 MPa and the steam leaves the reheater at 400°C.

	η_{th}	$x_{turb\ exit}$
No Reheat	35.3%	0.8159
With Reheat	35.9%	0.9664

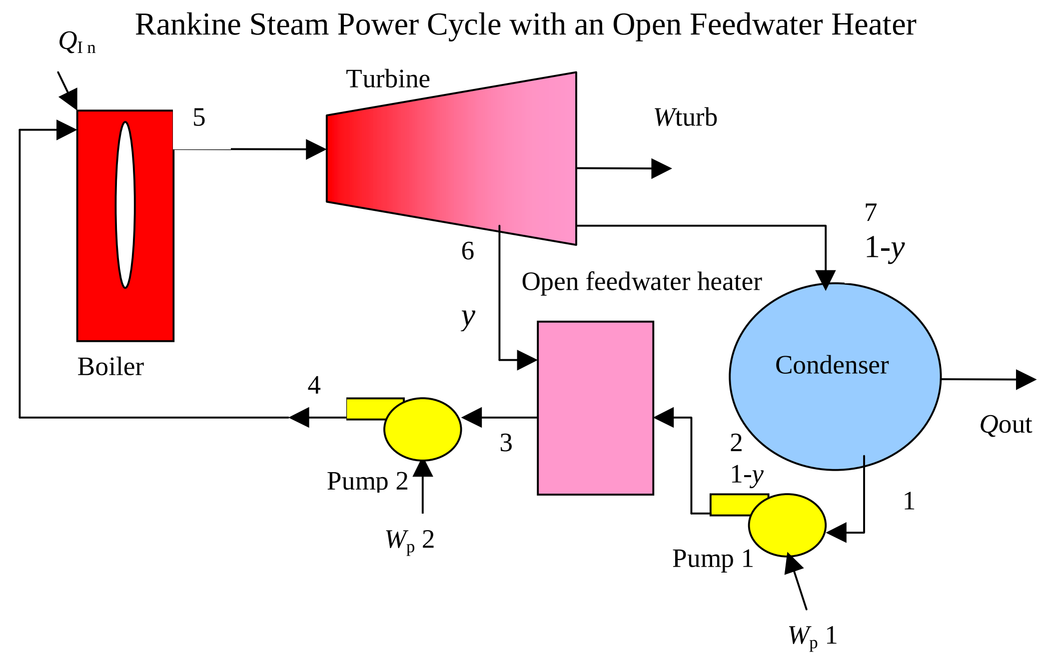
Regenerative Cycle

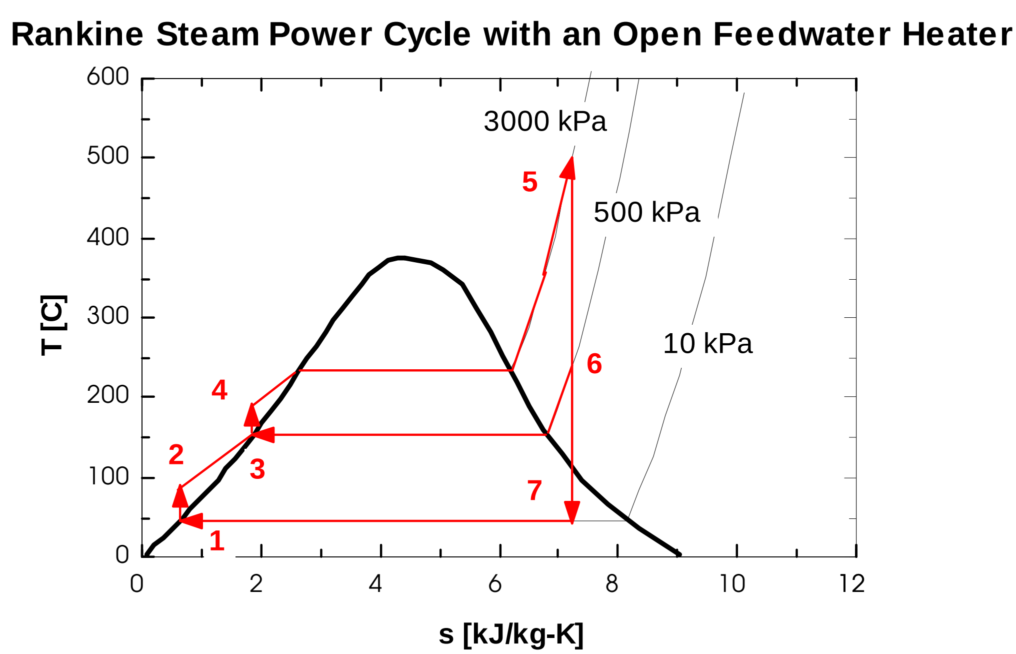
To improve the cycle thermal efficiency, the average temperature at which heat is added must be increased.

One way to do this is to allow the steam leaving the boiler to expand the steam in the turbine to an intermediate pressure. A portion of the steam is extracted from the turbine and sent to a regenerative heater to preheat the condensate before entering the boiler. This approach increases the average temperature at which heat is added in the boiler. However, this reduces the mass of steam expanding in the lower- pressure stages of the turbine, and, thus, the total work done by the turbine. The work that is done is done more efficiently.

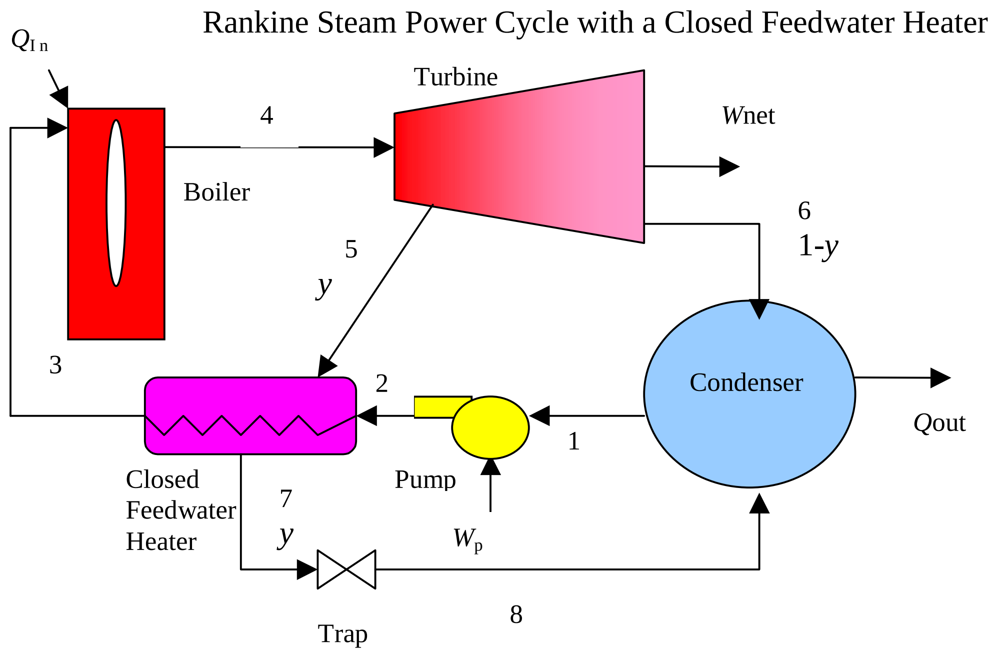
The preheating of the condensate is done in a combination of open and closed heaters. In the open feedwater heater, the extracted steam and the condensate are physically mixed. In the closed feedwater heater, the extracted steam and the condensate are not mixed.

Cycle with an open feedwater heater

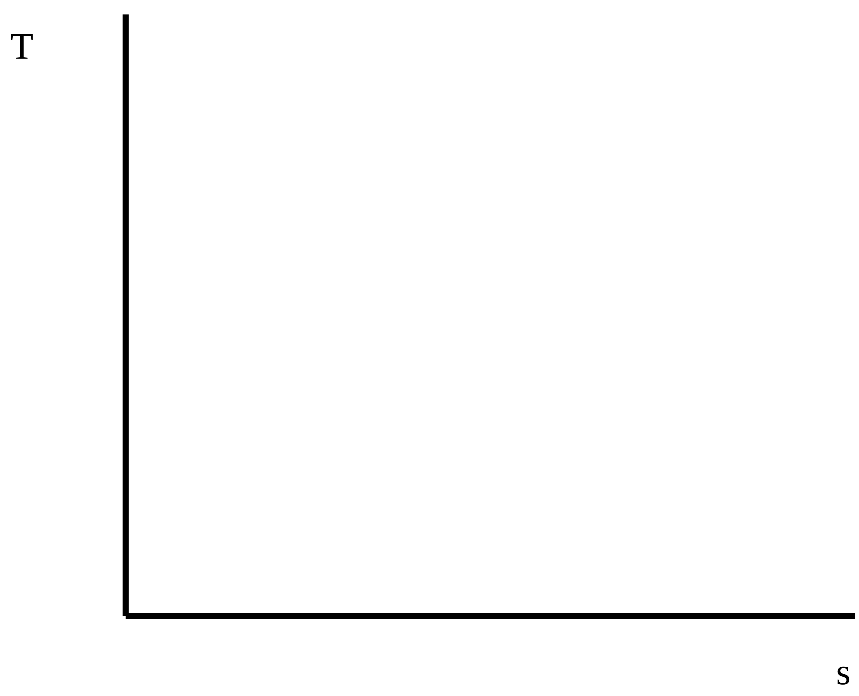




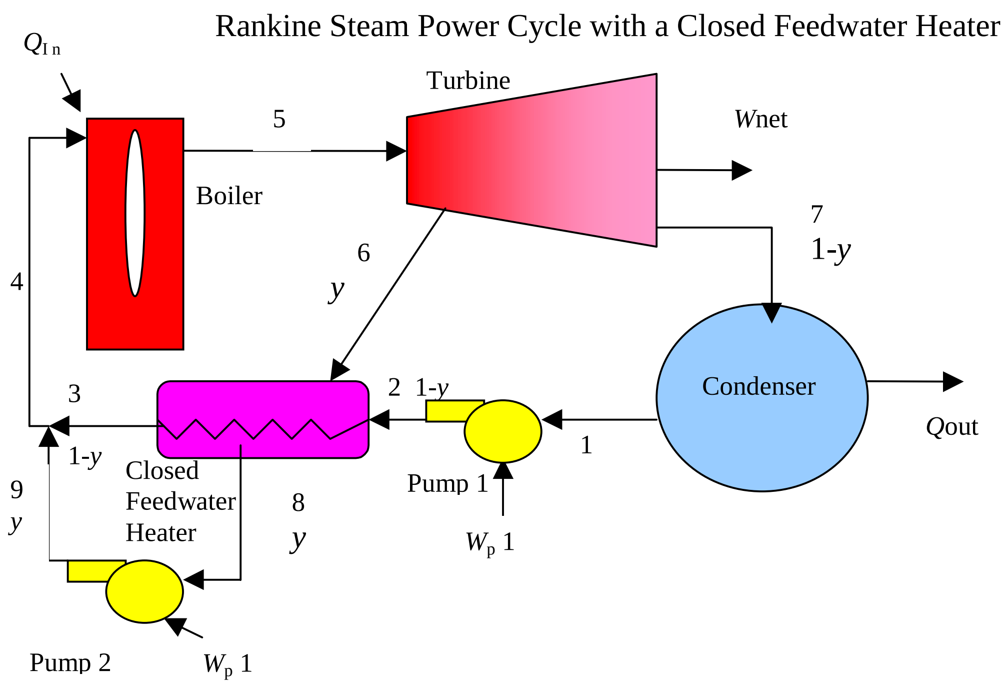
Cycle with a closed feedwater heater with steam trap to condenser



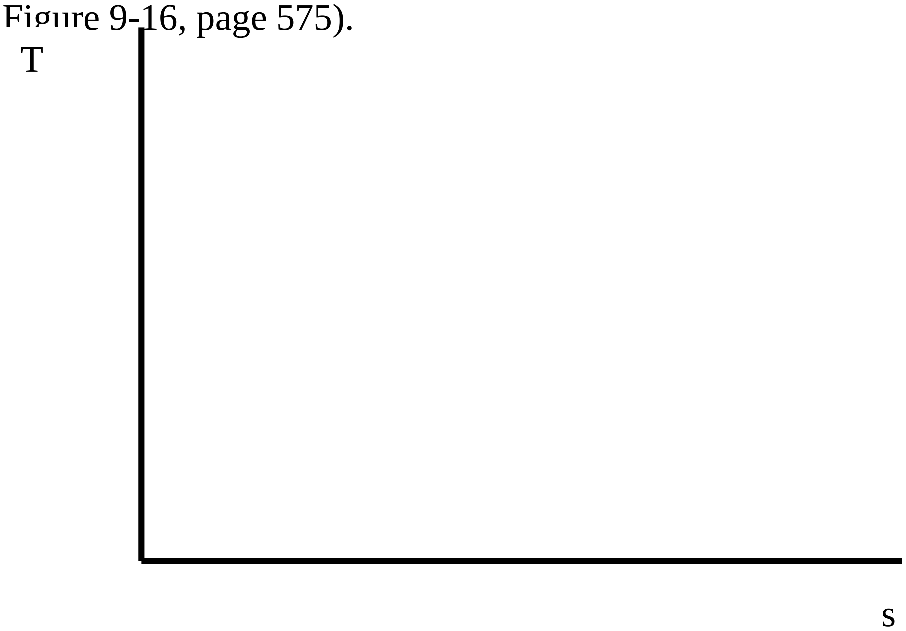
Let’s sketch the *T*-*s* diagram for this closed feedwater heater cycle.



Cycle with a closed feedwater heater with pump to boiler pressure



Let's sketch the *T-s* diagram for this closed feedwater heater cycle (review Figure 9-16, page 575).



Consider the regenerative cycle with the open feedwater heater.

To find the fraction of mass to be extracted from the turbine, apply the first law to the feedwater heater and assume, in the ideal case, that the water leaves the feedwater heater as a saturated liquid. (In the case of the closed feedwater heater, the feedwater leaves the heater at a temperature equal to the saturation temperature at the extraction pressure.)

Conservation of mass for the open feedwater heater:

Let $y = \dot{m}_6 / \dot{m}_5$ be the fraction of mass extracted from the turbine for the feedwater heater.

$$\begin{aligned} \dot{m}_{in} &= \dot{m}_{out} \\ \dot{m}_6 + \dot{m}_2 &= \dot{m}_3 = \dot{m}_5 \\ \dot{m}_2 &= \dot{m}_5 - \dot{m}_6 = \dot{m}_5(1-y) \end{aligned}$$

Conservation of energy for the open feedwater heater:

$$\begin{aligned} \dot{E}_{in} &= \dot{E}_{out} \\ \dot{m}_6 h_6 + \dot{m}_2 h_2 &= \dot{m}_3 h_3 \\ y\dot{m}_5 h_6 + (1-y)\dot{m}_5 h_2 &= \dot{m}_5 h_3 \\ y &= \frac{h_3 - h_2}{h_6 - h_2} \end{aligned}$$

Example 9-3

An ideal regenerative steam power cycle operates so that steam enters the turbine at 3 MPa, 500^{o}C, and exhausts at 10 kPa. A single open feedwater heater is used and operates at 0.5 MPa. Compute the cycle thermal efficiency.

Using the software package the following data are obtained.

State	P	T	h	s	v
	kPa	^{o}C	kJ/kg	kJ/kg·K	m^3/kg
1	10		191.8		0.00101
2	500				
3	500		640.2		0.00109
4	3000				
5	3000	500	3456.5	7.2338	
6	500		2941.6	7.2338	
7	10		2292.7	7.2338	

The work for pump 1 is calculated from

$$\begin{aligned} w_{pump\,1} &= v_1(P_2 - P_1) \\ &= 0.00101\frac{m^3}{kg}(500-10)\ kPa\frac{kJ}{m^3 kPa} \\ &= 0.5\frac{kJ}{kg} \end{aligned}$$

Now, h_2 is found from

$$\begin{aligned} h_2 &= w_{pump\,1} + h_1 \\ &= 0.5\frac{kJ}{kg} + 191.8\frac{kJ}{kg} \\ &= 192.3\frac{kJ}{kg} \end{aligned}$$

The fraction of mass extracted from the turbine for the open feedwater heater is obtained from the energy balance on the open feedwater heater, as shown above.

$$y = \frac{h_3 - h_2}{h_6 - h_2}$$

$$= \frac{(640.2 - 192.3)\frac{kJ}{kg}}{(2941.6 - 192.3)\frac{kJ}{kg}} = 0.163$$

This means that for each kg of steam entering the turbine, 0.163 kg is extracted for the feedwater heater.

The work for pump 2 is calculated from

$$w_{pump\ 2} = v_3(P_4 - P_3)$$

$$= 0.00109\frac{m^3}{kg}(3000 - 500)\ kPa\frac{kJ}{m^3 kPa}$$

$$= 2.7\frac{kJ}{kg}$$

Now, h_4 is found from the energy balance for the pump.

$$E_{out} = E_{in}$$

$$h_4 = w_{pump\ 2} + h_3$$

$$= 2.7\frac{kJ}{kg} + 640.2\frac{kJ}{kg}$$

$$= 643.9\frac{kJ}{kg}$$

Apply the steady-flow conservation of energy to the isentropic turbine.

$$\dot{m}_5 h_5 = \dot{W}_{turb} + \dot{m}_6 h_6 + \dot{m}_7 h_7$$

$$\dot{W}_{turb} = \dot{m}_5 [h_5 - y h_6 - (1-y) h_7]$$

$$w_{turb} = \frac{\dot{W}_{turb}}{\dot{m}_5} = h_5 - y h_6 - (1-y) h_7$$

$$= [3456.5 - (0.163)(2941.6) - (1-0.163)(2292.7)] \frac{kJ}{kg}$$

$$= 1058.0 \frac{kJ}{kg}$$

The net work done by the cycle is

$$\dot{W}_{net} = \dot{W}_{turb} - \dot{W}_{pump\,1} - \dot{W}_{pump\,2}$$

$$\dot{m}_5 w_{net} = \dot{m}_5 w_{turb} - \dot{m}_1 w_{pump\,1} - \dot{m}_3 w_{pump\,2}$$

$$\dot{m}_5 w_{net} = \dot{m}_5 w_{turb} - \dot{m}_5 (1-y) w_{pump\,1} - \dot{m}_5 w_{pump\,2}$$

$$w_{net} = w_{turb} - (1-y) w_{pump\,1} - w_{pump\,2}$$

$$= [1058.0 - (1-0.163)(0.5) - 2.7] \frac{kJ}{kg}$$

$$= 1054.9 \frac{kJ}{kg}$$

Apply the steady-flow conservation of mass and energy to the boiler.

$$\dot{m}_4 = \dot{m}_5$$

$$\dot{m}_4 h_4 + \dot{Q}_{in} = \dot{m}_5 h_5$$

$$\dot{Q}_{in} = \dot{m}_5 (h_5 - h_4)$$

$$q_{in} = \frac{\dot{Q}_{in}}{\dot{m}_5} = h_5 - h_4$$

The heat transfer per unit mass entering the turbine at the high pressure, state 5, is

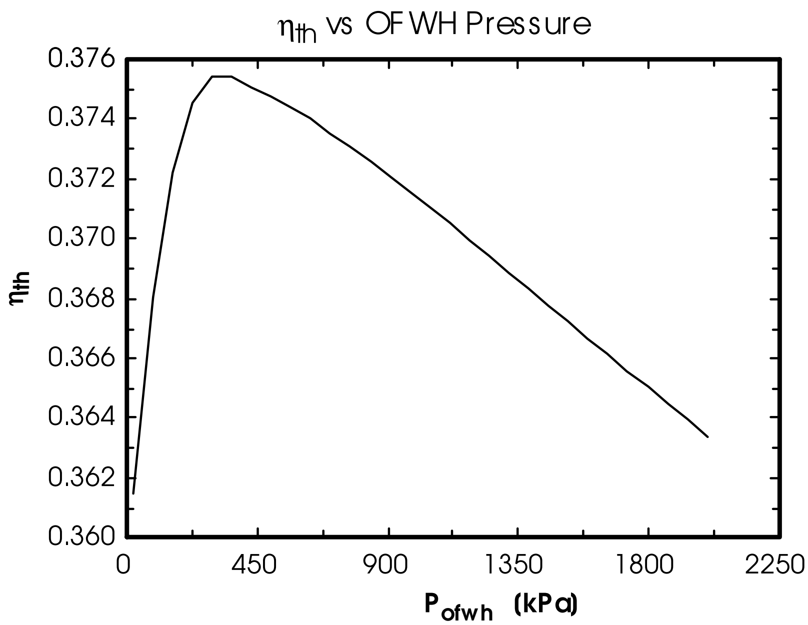
$$\begin{aligned} q_{in} &= h_5 - h_4 \\ &= (3456.5 - 642.9) \frac{kJ}{kg} \\ &= 2813.6 \frac{kJ}{kg} \end{aligned}$$

The thermal efficiency is

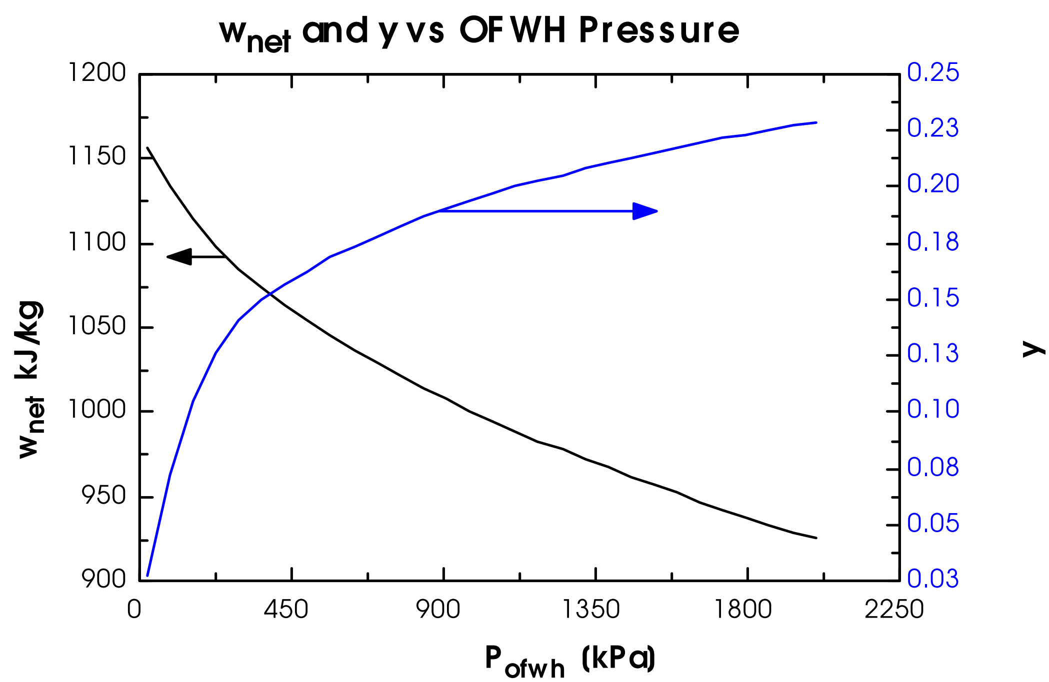
$$\begin{aligned} \eta_{th} &= \frac{w_{net}}{q_{in}} \\ &= \frac{1054.9 \frac{kJ}{kg}}{2813.6 \frac{kJ}{kg}} \\ &= 0.375 \quad or \quad 37.5\% \end{aligned}$$

If these data were used for a Rankine cycle with no regeneration, then η_{th} = 35.6 percent. Thus, the one open feedwater heater operating at 0.5 MPa increased the thermal efficiency by 5.3 percent. However, note that the mass flowing through the lower-pressure stages has been reduced by the amount extracted for the feedwater and the net work output for the regenerative cycle is about 10 percent lower than the standard Rankine cycle.

Below is a plot of cycle thermal efficiency versus the open feedwater heater pressure. The feedwater heater pressure that makes the cycle thermal efficiency a maximum is about 400 kPa.



Below is a plot of cycle net work per unit mass flow at state 5 and the fraction of mass *y* extracted for the feedwater heater versus the open feedwater heater pressure. Clearly the net cycle work decreases and the fraction of mass extracted increases with increasing extraction pressure. Why does the fraction of mass extracted increase with increasing extraction pressure?



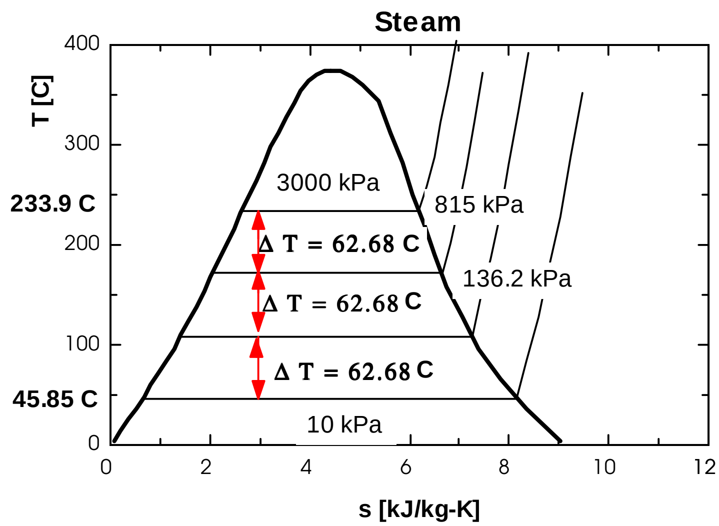
Placement of Feedwater Heaters

The extraction pressures for multiple feedwater heaters are chosen to maximize the cycle efficiency. As a rule of thumb, the extraction pressures for the feedwater heaters are chosen such that the saturation temperature difference between each component is about the same.

$$\Delta T_{cond\ to\ FWH} = \Delta T_{boiler\ to\ FWH},\ etc.$$

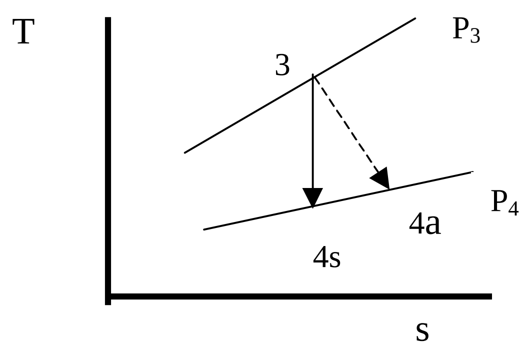
Example 9-4

An ideal regenerative steam power cycle operates so that steam enters the turbine at 3 MPa, 500^{o}C, and exhausts at 10 kPa. Two closed feedwater heaters are to be used. Select starting values for the feedwater heater extraction pressures.



Deviation from Actual Cycles

- Piping losses--frictional effects reduce the available energy content of the steam.
- Turbine losses--turbine isentropic (or adiabatic) efficiency.

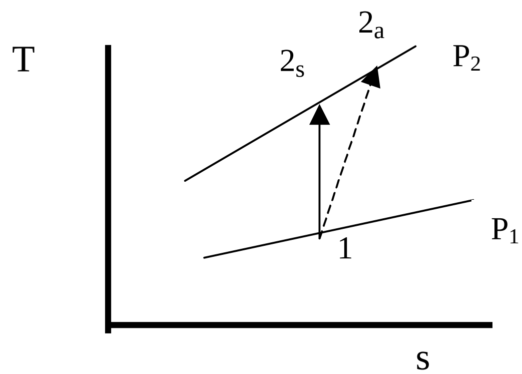


$$\eta_{turb} = \frac{w_{actual}}{w_{isentropic}} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

The actual enthalpy at the turbine exit (needed for the energy analysis of the next component) is

$$h_{4a} = h_3 - \eta_{turb}(h_3 - h_{4s})$$

- Pump losses--pump isentropic (or adiabatic) efficiency.



$$\eta_{pump} = \frac{w_{isentropic}}{w_{actual}} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

The actual enthalpy at the pump exit (needed for the energy analysis of the next component) is

$$h_{2a} = h_1 + \frac{1}{\eta_{pump}}(h_{2s} - h_1)$$

- Condenser losses--relatively small losses that result from cooling the condensate below the saturation temperature in the condenser.