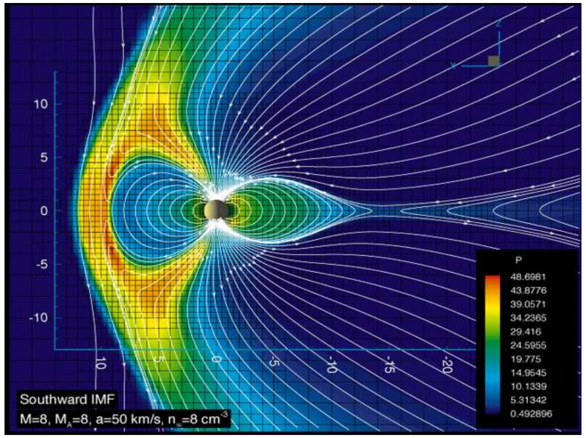

ME 252 Fluid Dynamics 1

Equations of Motion and Energy

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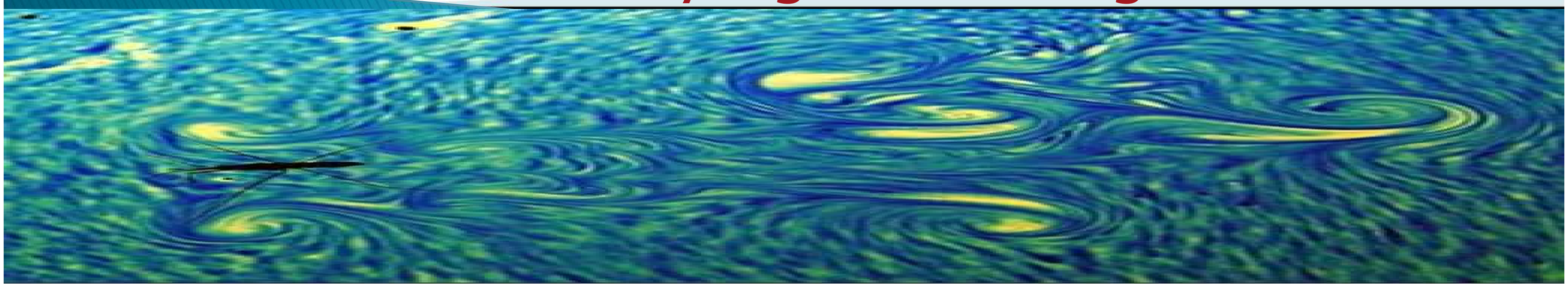


Course Objectives

- Understand the Equation of motion.
- Appreciate Bernoulli Equation and its applications.
- Understand the Equation of Energy.
- Momentum equation and its application.
- Appreciate incompressible flow in pipes and ducts.
- Appreciate issues with Open channel flow

Areas to Cover

- Mass conservation
- Equations of motion and energy.
  - Bernoulli equation
  - Energy Conservation
- Momentum equation and its applications.
- Moment of momentum equations
- Introduction to incompressible flow in pipes and ducts
- Open Channel flow

Conservation of Mass

- The conservation of mass relation for a closed system undergoing a change is expressed as m_{sys} = constant or $dm_{sys}/dt = 0$,
- A statement that the mass of a system remains constant during a process.
- For a control volume (CV), mass balance is expressed in the rate form as

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{CV}}{dt}$$

where m_{in} and m_{out} are the total rates of mass flow into and out of the control volume, respectively, and dm_{CV}/dt is the rate of change of mass within the control volume boundaries

Mass Balance for Steady-Flow Processes

- During a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant).
- Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.
- For steady-flow processes, the interested is not in the amount of mass that flows in or out of a device over time; instead, the amount of mass flowing per unit time, (mass flow rate m)
- The conservation of mass principle for a general steady-flow system with multiple inlets and outlets can be expressed in rate form as

Steady flow: $$\sum_{in} \dot{m} = \sum_{out} \dot{m} \qquad \text{(kg/s)}$$

Special Case: Incompressible Flow

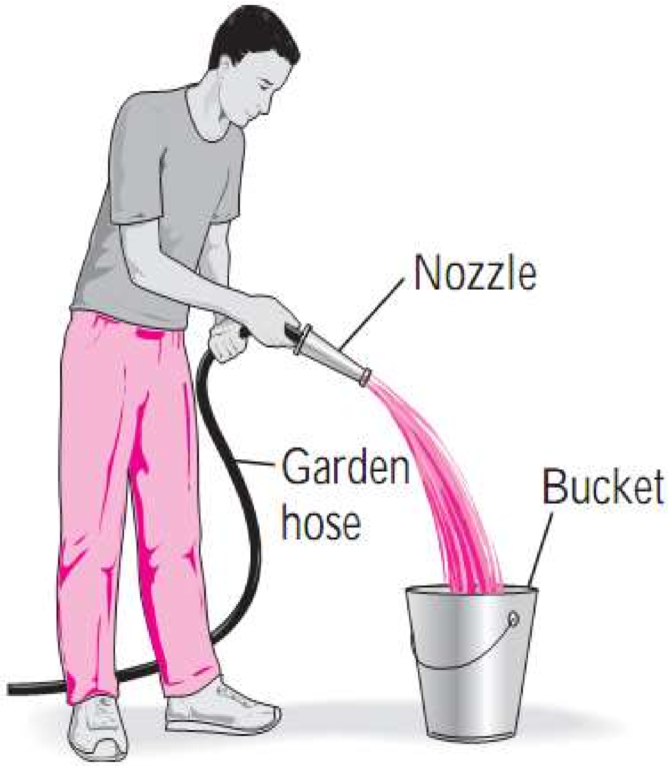
- The conservation of mass relations is simplified when the fluid is incompressible (which is usually the case for liquids).
- The general steady-flow relation gives

Steady, incompressible flow: $$\sum_{\text{in}} \dot{V} = \sum_{\text{out}} \dot{V} \qquad (\text{m}^3/\text{s})$$

Steady, incompressible flow (single stream): $$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$$

EXAMPLE: Water Flow through a Garden Hose Nozzle

A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit. If it takes 50 s to fill the bucket with water, determine (a)the volume and mass flow rates of water through the hose, and (b)the average velocity of water at the nozzle exit.



SOLUTION A garden hose is used to fill a water bucket. The volume and mass flow rates of water and the exit velocity are to be determined.

Assumptions **1** Water is an incompressible substance. **2** Flow through the hose is steady. **3** There is no waste of water by splashing.

Properties We take the density of water to be 1000 kg/m^3 = 1 kg/L.

Analysis (*a*) Noting that 10 gal of water are discharged in 50 s, the volume and mass flow rates of water are

$$\dot{V} = \frac{V}{\Delta t} = \frac{10 \text{ gal}}{50 \text{ s}}\left(\frac{3.7854 \text{ L}}{1 \text{ gal}}\right) = \mathbf{0.757\ L/s}$$

$$\dot{m} = \rho \dot{V} = (1 \text{ kg/L})(0.757 \text{ L/s}) = \mathbf{0.757\ kg/s}$$

(*b*) The cross-sectional area of the nozzle exit is

$$A_e = \pi r_e^2 = \pi(0.4 \text{ cm})^2 = 0.5027 \text{ cm}^2 = 0.5027 \times 10^{-4} \text{ m}^2$$

The volume flow rate through the hose and the nozzle is constant. Then the average velocity of water at the nozzle exit becomes

$$V_e = \frac{\dot{V}}{A_e} = \frac{0.757 \text{ L/s}}{0.5027 \times 10^{-4} \text{ m}^2}\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right) = \mathbf{15.1\ m/s}$$

Discussion It can be shown that the average velocity in the hose is 2.4 m/s. Therefore, the nozzle increases the water velocity by over six times.

Conservation of Momentum

- The product of the mass and the velocity of a body is called the linear momentum or simply "momentum" of the body.
- For a rigid body of mass 'm' moving with a velocity 'V' the momentum is mV
- Newton's second law states that the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass, and that the rate of change of the momentum of a body is equal to the net force acting on the body.
- The momentum of a system remains constant when the net force acting on it is zero, and thus the momentum of such systems is conserved.

EQUATIONS OF MOTION

- According to Newton's second law of motion, the net force F_x acting on a fluid element in the direction of x is equal to mass m of the fluid element multiplied by the acceleration a_x in the x-direction. Thus mathematically,

$$F_x = m.a_x$$

- In the fluid flow, the following forces are present:
  - (i) F_g, gravity force.
  - (ii) F_p, the pressure force.
  - (iii) F_v, force due to viscosity.
  - (iv) F_t, force due to turbulence.
  - (v) F_c, force due to compressibility.

EQUATIONS OF MOTION CONT'D

- Thus the net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$$

If the force due to compressibility is negligible, the resulting net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

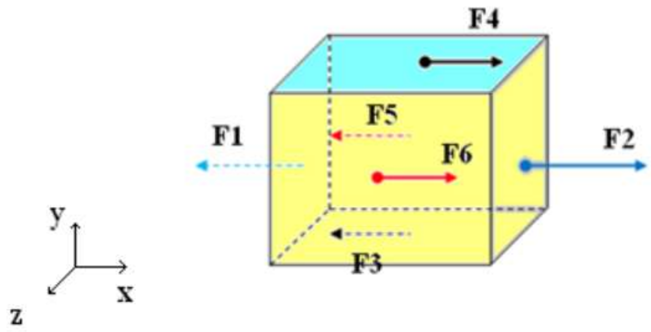
and the resulting equations of motion are called **Reynold's equations of motion**.

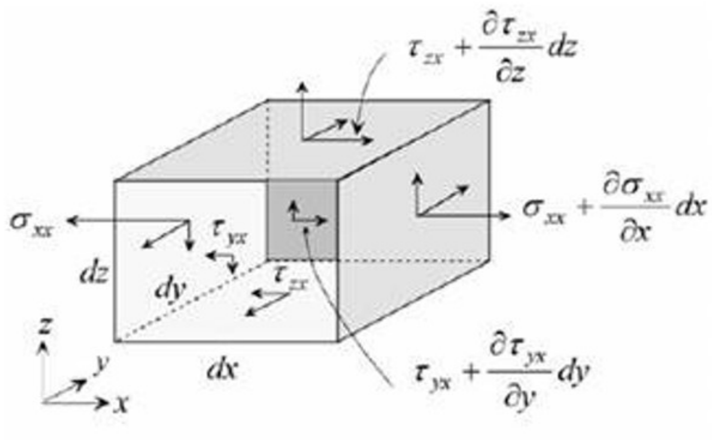
If the flow is assumed to be ideal, viscous force is zero and equations of motion are known as **Euler's equations of motion.**

NAVIER-STOKES EQUATIONS

- Consider a control volume which is a differential element in a fluid continuum.
- Forces acting on it are to include, gravitational, viscous or frictional, and pressure forces to encompass the majority of fluid problems.
- If the continuity and momentum equations are written for all three principal directions, and the fluid is Newtonian with constant properties of density and viscosity, a set of differential equations results.
- The momentum equation written for each principal direction gives what are called the **Navier-Stokes equations**.

Considering the x-component of the net surface force $\sum F_{x,surface}$





$$\mathbf{F}1 = -(\sigma_{xx} - \frac{dx}{2}\frac{\partial \sigma_{xx}}{\partial x})dydz \qquad \mathbf{F}2 = (\sigma_{xx} + \frac{dx}{2}\frac{\partial \sigma_{xx}}{\partial x})dydz$$

$$\mathbf{F}3 = -(\sigma_{yx} - \frac{dy}{2}\frac{\partial \sigma_{yx}}{\partial y})dxdz \qquad \mathbf{F}4 = (\sigma_{yx} + \frac{dy}{2}\frac{\partial \sigma_{yx}}{\partial y})dxdz$$

$$\mathbf{F}5 = -(\sigma_{zx} - \frac{dz}{2}\frac{\partial \sigma_{zx}}{\partial z})dxdy \qquad \mathbf{F}6 = (\sigma_{zx} + \frac{dz}{2}\frac{\partial \sigma_{zx}}{\partial z})dxdy$$

$$\sum F_{x,surface} = \mathbf{F}1 + \mathbf{F}2 + \mathbf{F}3 + \mathbf{F}4 + \mathbf{F}5 + \mathbf{F}6$$

NAVIER-STOKES EQUATIONS CONT'D

- For the general problem in fluid mechanics, assuming we have a Newtonian fluid with constant properties, the governing equation in Cartesian coordinates are:
- (x- direction)
- $\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$
- (y direction)
- $\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$
- (z direction)
- $\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$

NAVIER-STOKES EQUATIONS CONT'D

- These are nonlinear and present difficulties in trying to solve.
- Even a variety of exact solutions for specific flows have been found, the equations have not been solved in general- due primarily to the presence of the nonlinear terms.
- The right-hand side of the equations includes pressure, gravitational or body, and viscous forces.

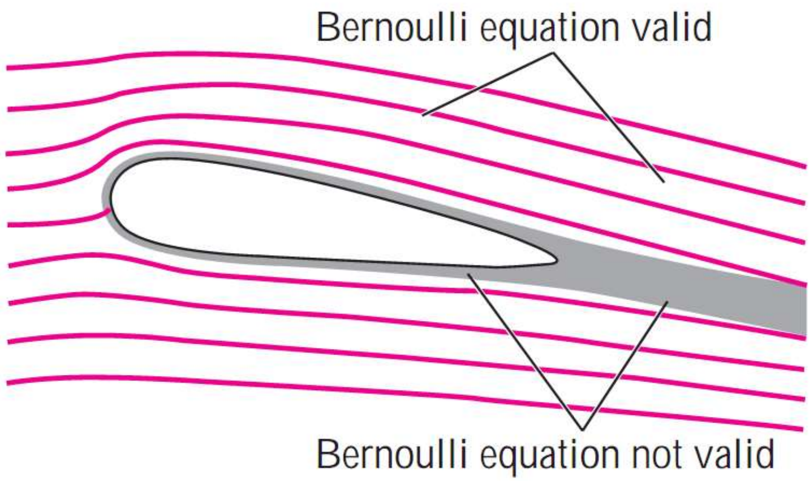
Conservation of Energy

- Energy can be transferred to or from a closed system by heat or work,
- the conservation of energy principle requires that the net energy transfer to or from a system during a process be equal to the change in the energy content of the system.
- Control volumes involve energy transfer via mass flow.
- The conservation of energy principle (energy balance) is expressed as

$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{CV}}{dt}$$

THE BERNOULLI EQUATION

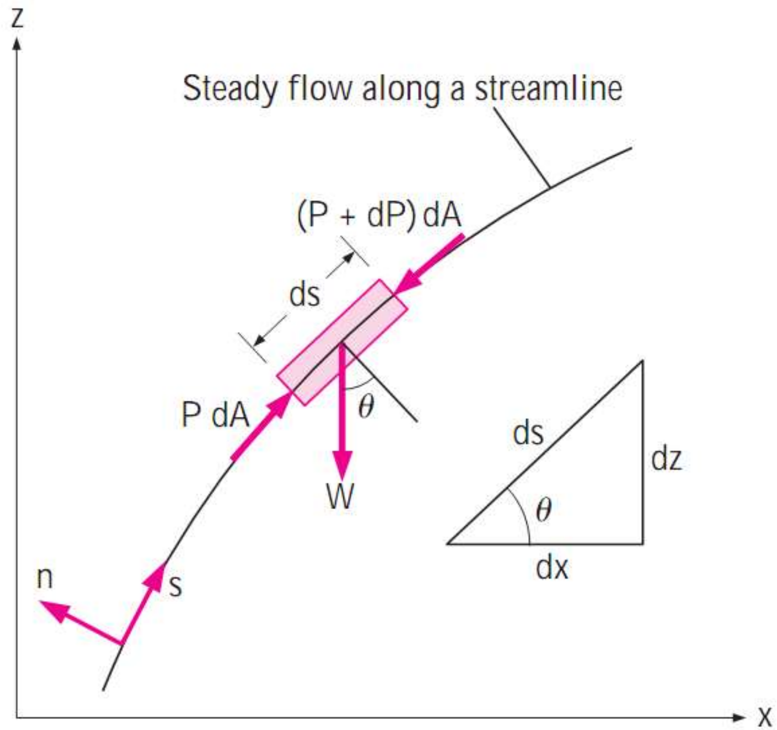
- The Bernoulli equation is an approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible.
- Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics.



Derivation of the Bernoulli Equation

- Consider the motion of a fluid particle in a flow field in steady flow.
- Applying Newton's second law (conservation of linear momentum relation in fluid mechanics) in the s-direction on a particle moving along a streamline gives

$$\sum F_s = ma_s$$



Bernoulli Equation

- In regions of flow where net frictional forces are negligible, the significant forces acting in the s-direction are the pressure (acting on both sides) and the component of the weight of the particle in the s-direction.
- Therefore

$$P\,dA - (P + dP)\,dA - W \sin\theta = mV\,\frac{dV}{ds}$$

where θ is the angle between the normal of the streamline and the vertical z-axis at that point, $m = \rho V = \rho\, dA\, ds$ is the mass, $W = mg = \rho g\, dA\, ds$ is the weight of the fluid particle, and $\sin\theta = dz/ds$.

Bernoulli Equation

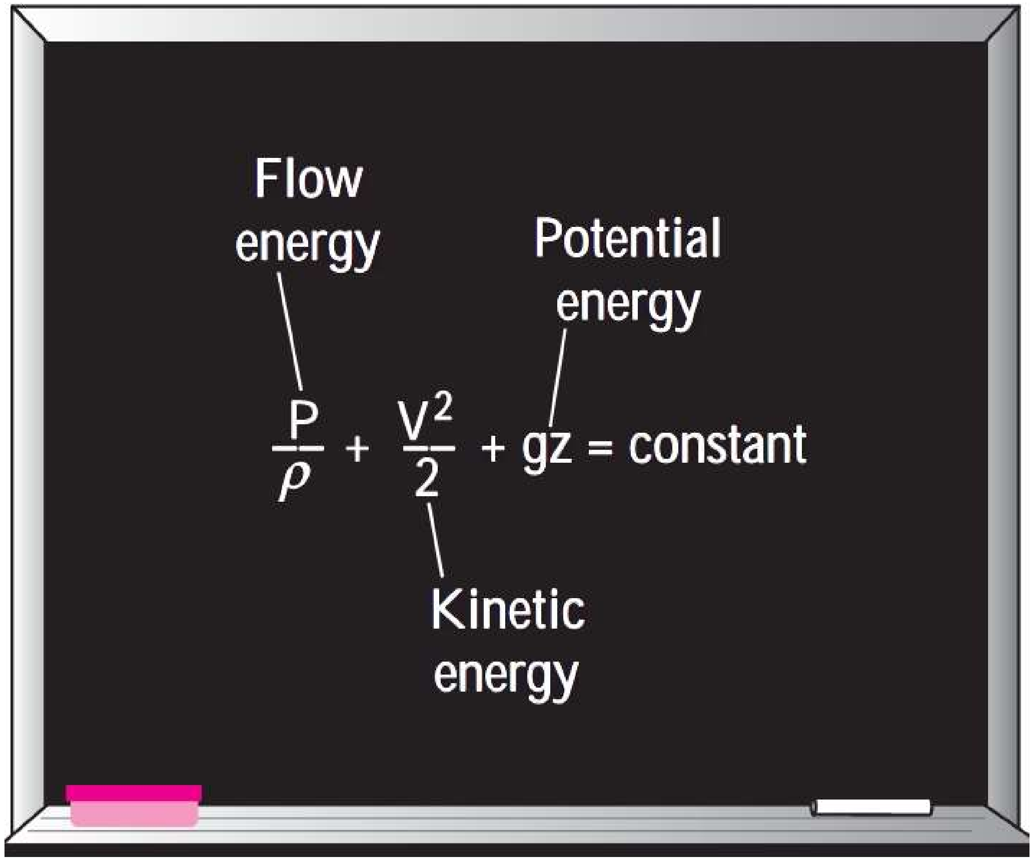
$$-dP\,dA - \rho g\,dA\,ds\,\frac{dz}{ds} = \rho\,dA\,ds\,V\,\frac{dV}{ds}$$

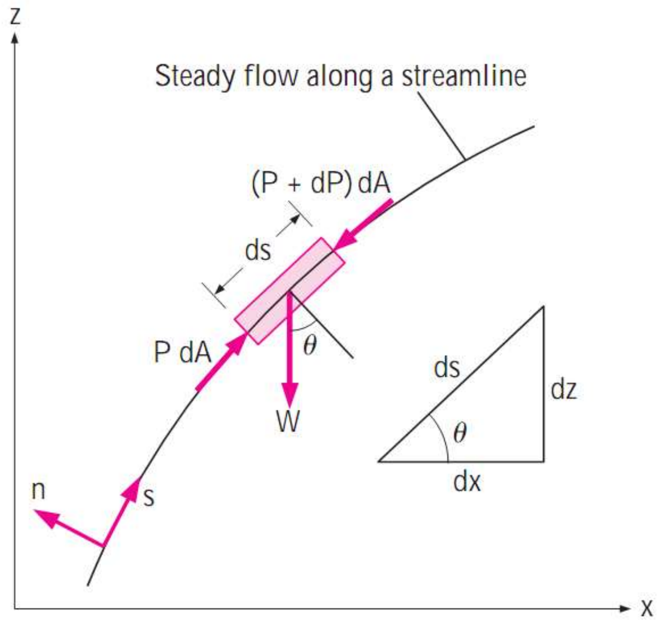
$$\frac{dP}{\rho} + \frac{1}{2}\,d(V^2) + g\,dz = 0$$

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Steady, incompressible flow: $\dfrac{P}{\rho} + \dfrac{V^2}{2} + gz = \text{constant}$

Bernoulli Equation





The value of the constant can be evaluated at any point on the streamline where the pressure, density, velocity, and elevation are known.

Bernoulli Equation

- The Bernoulli equation was first stated in words by the Swiss mathematician Daniel Bernoulli (1700–1782) in a text written in 1738 when he was working in St. Petersburg, Russia.
- It was later derived in equation form by his associate Leonhard Euler in 1755.
- It can also be written between any two points on the same streamline as

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Bernoulli Equation

- Since (per unit mass)
  - $V^2/2$ as kinetic energy,
  - gz as potential energy, and
  - P/ρ as flow energy.
- The Bernoulli equation can be viewed as an expression of mechanical energy balance and can be stated as:

The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when the compressibility and frictional effects are negligible.

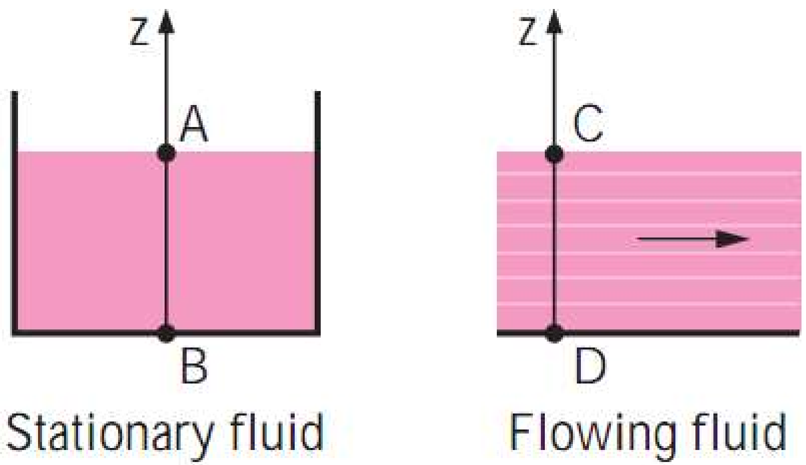
Bernoulli Equation

- The Bernoulli equation can be viewed as the "conservation of mechanical energy principle."
- The Bernoulli equation states that
  - During steady, incompressible flow with negligible friction, the various forms of mechanical energy are converted to each other, but their sum remains constant.
- There is no dissipation of mechanical energy during such flows since there is no friction that converts mechanical energy to sensible thermal (internal) energy.

Force Balance across Streamlines

- A force balance in the direction normal to the streamline yields the following relation applicable across the streamlines for steady, incompressible flow:

$$\frac{P}{\rho} + \int \frac{V^2}{R} dn + gz = \text{constant} \qquad \text{(across streamlines)}$$



$$P_B - P_A = P_D - P_C$$

For flow along a straight line, $R \rightarrow \infty$ and thus relation reduces to $P/\rho + gz =$ constant or $P + \rho gz =$ constant, an expression for the variation of hydrostatic pressure with vertical distance for a stationary fluid body. Therefore, the variation of pressure with elevation in steady, incompressible flow along a straight line is the same as that in the stationary fluid.

Static, Dynamic, and Stagnation Pressures

- The Bernoulli equation states that the sum of the flow, kinetic, and potential energies of a fluid particle along a streamline is constant.
- Therefore, the kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change.
- This phenomenon can be made more visible by multiplying the Bernoulli equation by the density ρ,

$$P + \rho \frac{V^2}{2} + \rho g z = \text{constant (along a streamline)}$$

$$P + \rho \frac{V^2}{2} + \rho g z = \text{constant (along a streamline)}$$

- • P is the static pressure (it does not incorporate any dynamic effects); it represents the actual thermodynamic pressure of the fluid.
- • $\rho V^2/2$ is the dynamic pressure; it represents the pressure rise when the fluid in motion is brought to a stop isentropically.
- • ρgz is the hydrostatic pressure, which is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, i.e., of fluid weight on pressure.

Static, Dynamic, and Stagnation Pressures

- The sum of the static, dynamic, and hydrostatic pressures is called the total pressure.
- The Bernoulli equation states that the total pressure along a streamline is constant.
- The sum of the static and dynamic pressures is called the stagnation pressure, expressed as

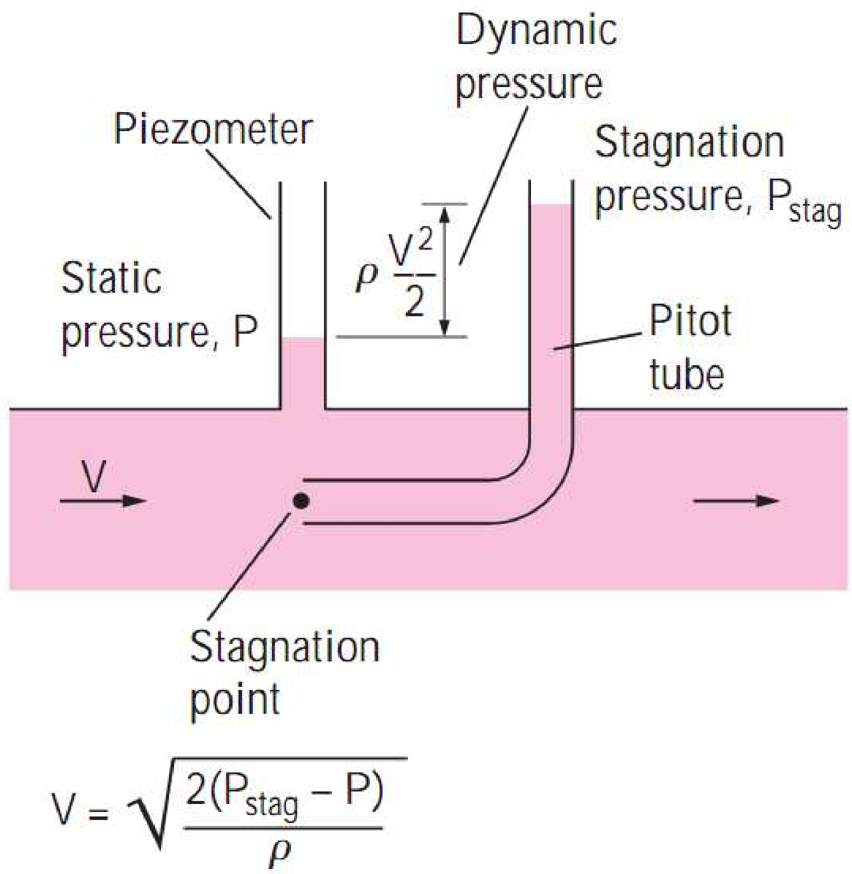
$$P_{stag} = P + \rho \frac{V^2}{2} \qquad \text{(kPa)}$$

Static, Dynamic, and Stagnation Pressures

The stagnation pressure represents the pressure at a point where the fluid is brought to a complete stop isentropically.

When static and stagnation pressures are measured at a specified location, the fluid velocity at that location can be calculated from

$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$



Limitations on the Use of the Bernoulli Equation

- The Bernoulli equation is one of the most frequently used and misused equations in fluid mechanics.
- Its versatility, simplicity, and ease of use make it a very valuable tool for use in analysis, but the same attributes also make it very tempting to misuse.
- It is important to understand the restrictions on its applicability and observe the limitations on its use.

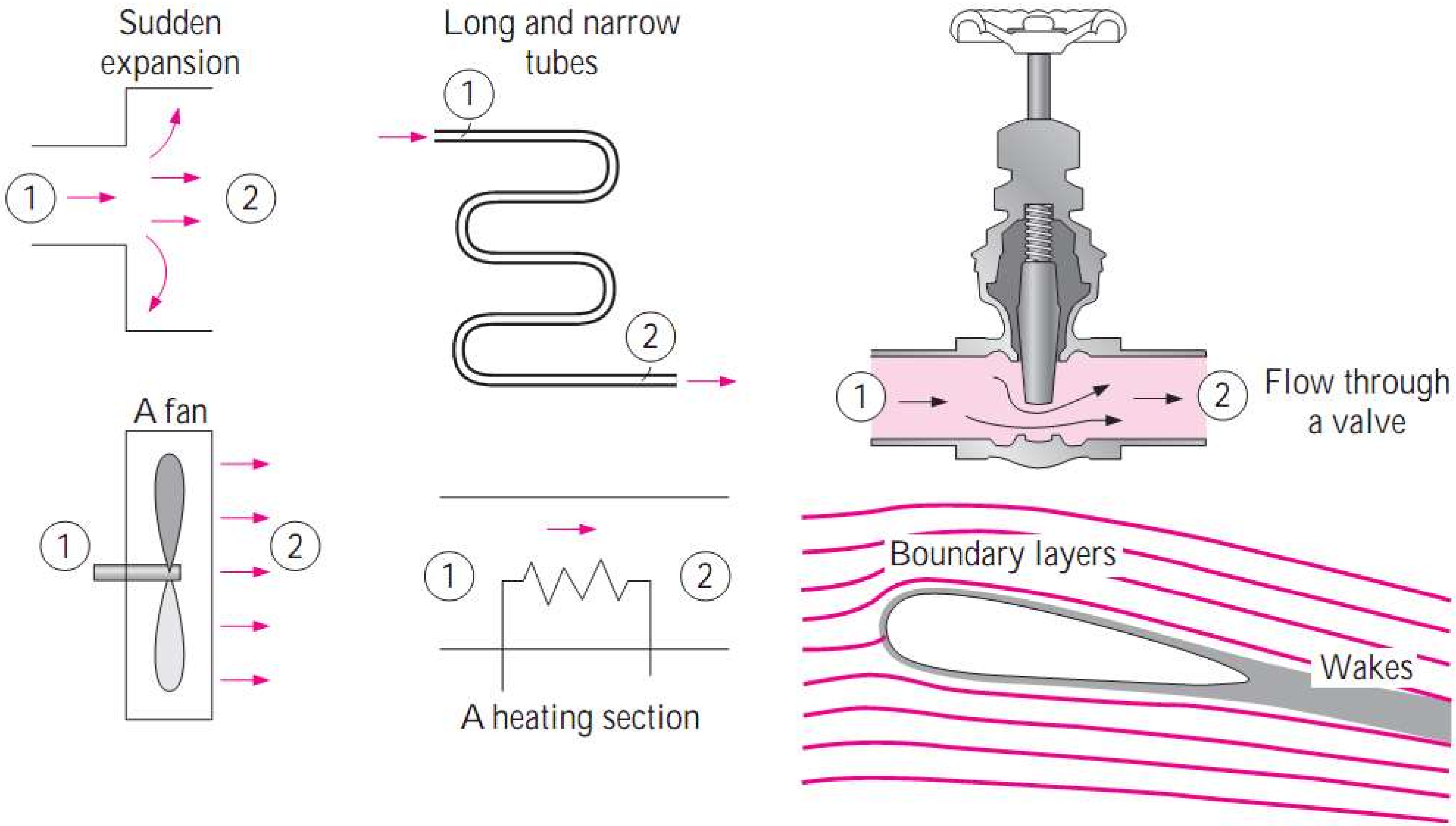
Limitations on the Use of the Bernoulli Equation

- Steady flow: It is applicable to steady flow.
- Frictionless flow: Every flow involves some friction, no matter how small, and frictional effects may or may not be negligible. The situation is complicated even more by the amount of error that can be tolerated.
  - In general, frictional effects are negligible for short flow sections with large cross sections, especially at low flow velocities.
  - Frictional effects are usually significant in long and narrow flow passages, in the wake region downstream of an object, and in diverging flow sections such as diffusers because of the increased possibility of the fluid separating from the walls in such geometries.

Limitations on the Use of the Bernoulli Equation

- No shaft work: The Bernoulli equation was derived from a force balance on a particle moving along a streamline. Therefore, it is not applicable in a flow section that involves a pump, turbine, fan, or any other machine or impeller since such devices destroy the streamlines and carry out energy interactions with the fluid particles.
- Incompressible flow: One of the assumptions used in the derivation of the Bernoulli equation is that ρ = constant and thus the flow is incompressible. This condition is satisfied by liquids and also by gases at Mach numbers less than about 0.3 since compressibility effects and thus density variations of gases are negligible at such relatively low

Limitations on the Use of the Bernoulli Equation



Limitations on the Use of the Bernoulli Equation

- No heat transfer: The density of a gas is inversely proportional to temperature, and thus the Bernoulli equation should not be used for flow sections that involve significant temperature change such as heating or cooling sections.
- Flow along a streamline: Strictly speaking, the Bernoulli equation $P/\rho + V^2/2 + gz = C$ is applicable along a streamline, and the value of the constant C, in general, is different for different streamlines.
    - But when a region of the flow is irrotational, and thus there is no vorticity in the flow field, the value of the constant C remains the same for all streamlines, and, therefore, the Bernoulli equation becomes applicable across streamlines as well.

Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)

- It is often convenient to represent the level of mechanical energy graphically using heights to facilitate visualization of the various terms of the Bernoulli equation.
- This is done by dividing each term of the Bernoulli equation by g to give

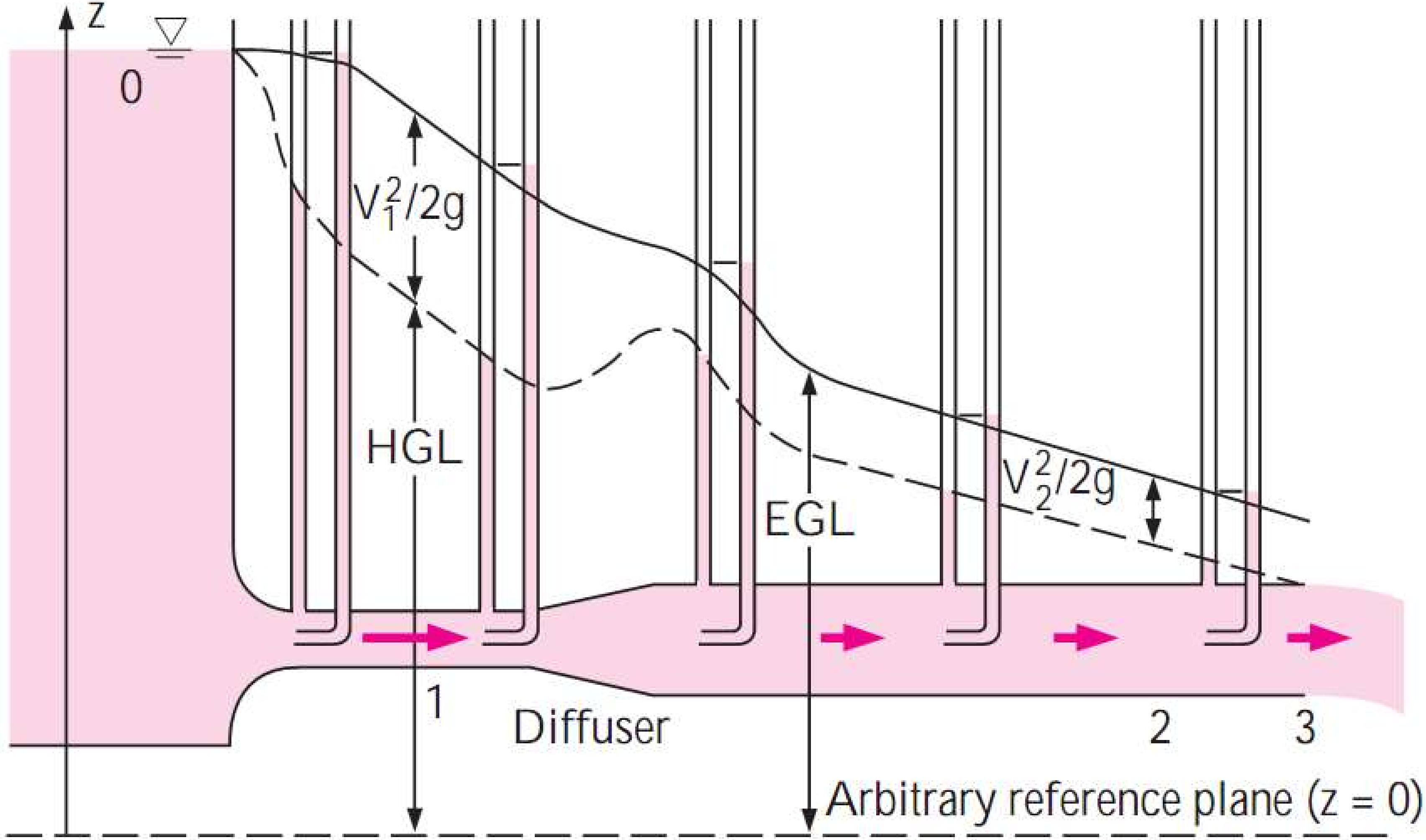
$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant} \qquad \text{(along a streamline)}$$

HGL and EGL

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant}$$

- • $P/\rho g$ is the pressure head; it represents the height of a fluid column that produces the static pressure P.
- • $V^2/2g$ is the velocity head; it represents the elevation needed for a fluid to reach the velocity V during frictionless free fall.
- • z is the elevation head; it represents the potential energy of the fluid.
- H is the total head for the flow.
- The Bernoulli equation can be expressed in terms of heads as:
  - The sum of the pressure, velocity, and elevation heads along a streamline is constant during steady flow when the compressibility and frictional effects are negligible.

HGL and EGL



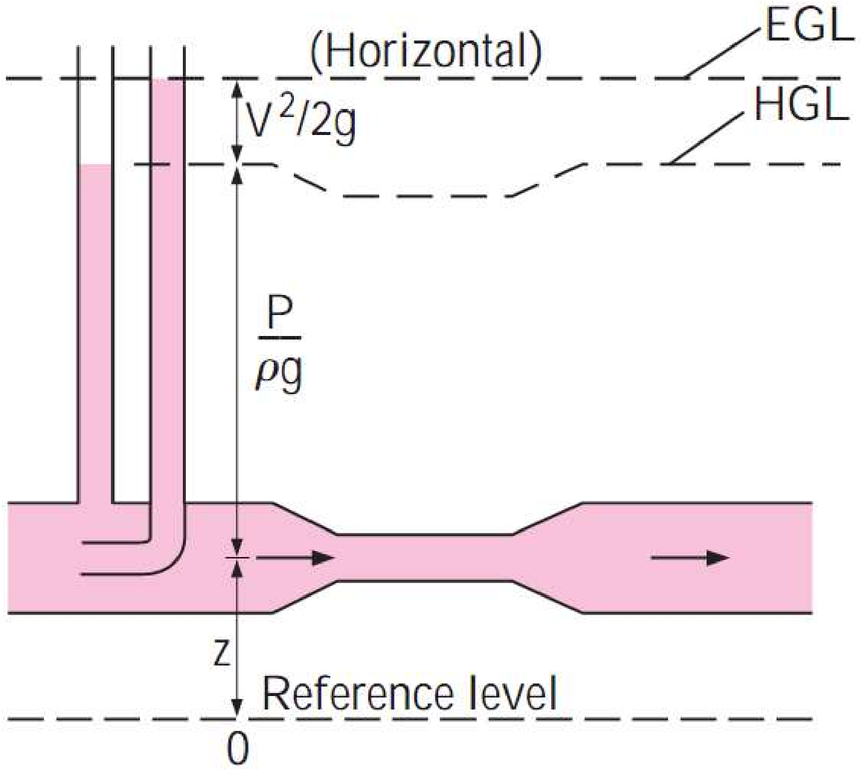
z
0
$V_1^2/2g$
HGL
EGL
$V_2^2/2g$
1
Diffuser
2
3
Arbitrary reference plane (z = 0)

HGL and EGL

- For stationary bodies such as reservoirs or lakes, the EGL and HGL coincide with the free surface of the liquid. The elevation of the free surface z represents both the EGL and the HGL since the velocity is zero and the static pressure (gauge) is zero.
- • The EGL is always a distance $V^2/2g$ above the HGL.
  - These two lines approach each other as the velocity decreases, and
  - they diverge as the velocity increases.
  - The height of the HGL decreases as the velocity increases, and vice versa.

HGL and EGL

- • In an idealized Bernoulli-type flow, EGL is horizontal and its height remains constant. This would also be the case for HGL when the flow velocity is constant.



HGL and EGL

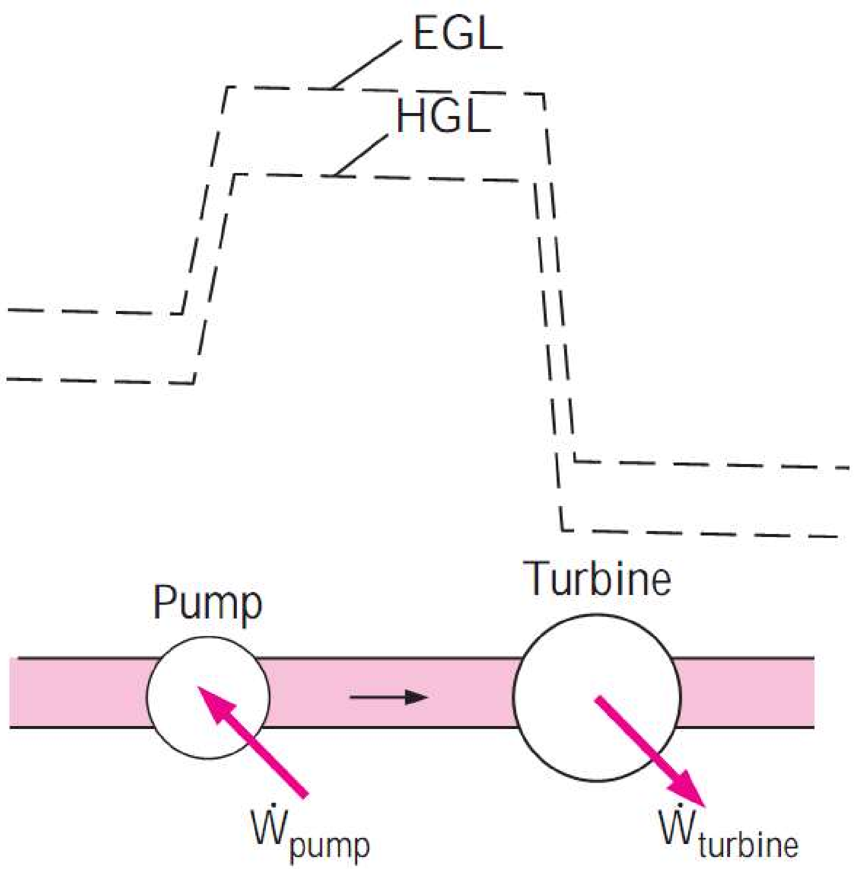
- • For open-channel flow, the HGL coincides with the free surface of the liquid, and the EGL is a distance $V^2/2g$ above the free surface.
- • At a pipe exit, the pressure head is zero (atmospheric pressure) and thus the HGL coincides with the pipe outlet (location 3).

HGL and EGL

- • The mechanical energy loss due to frictional effects (conversion to thermal energy) causes the EGL and HGL to slope downward in the direction of flow.
  - The slope is a measure of the head loss in the pipe.
  - A component that generates significant frictional effects such as a valve causes a sudden drop in both EGL and HGL at that location.

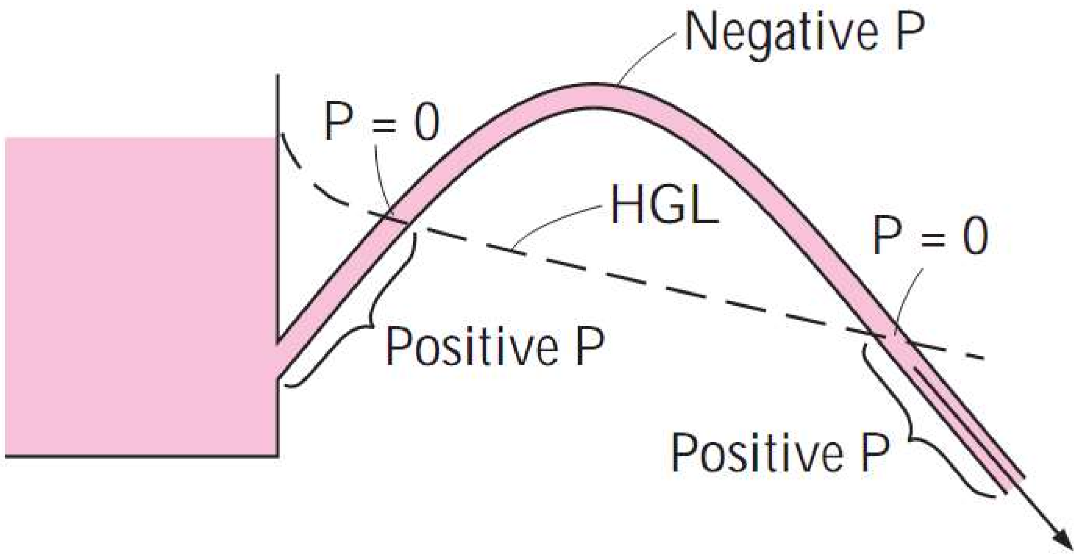
HGL and EGL

- • A steep jump occurs in EGL and HGL whenever mechanical energy is added to the fluid (by a pump, for example).
- Likewise, a steep drop occurs in EGL and HGL whenever mechanical energy is removed from the fluid (by a turbine, for example).



HGL and EGL

- • The pressure (gauge) of a fluid is zero at locations where the HGL intersects the fluid.
  - The pressure in a flow section that lies above the HGL is negative, and
  - The pressure in a section that lies below the HGL is positive.
  - Therefore, an accurate drawing of a piping system and the HGL can be used to determine the regions where the pressure in the pipe is negative (below the atmospheric pressure).



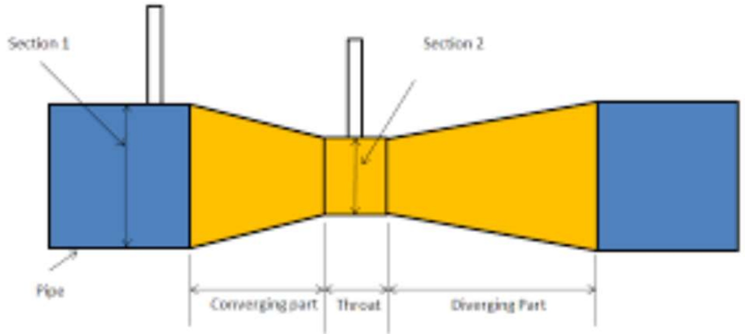
APPLICATIONS OF THE BERNOULLI EQUATION

PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

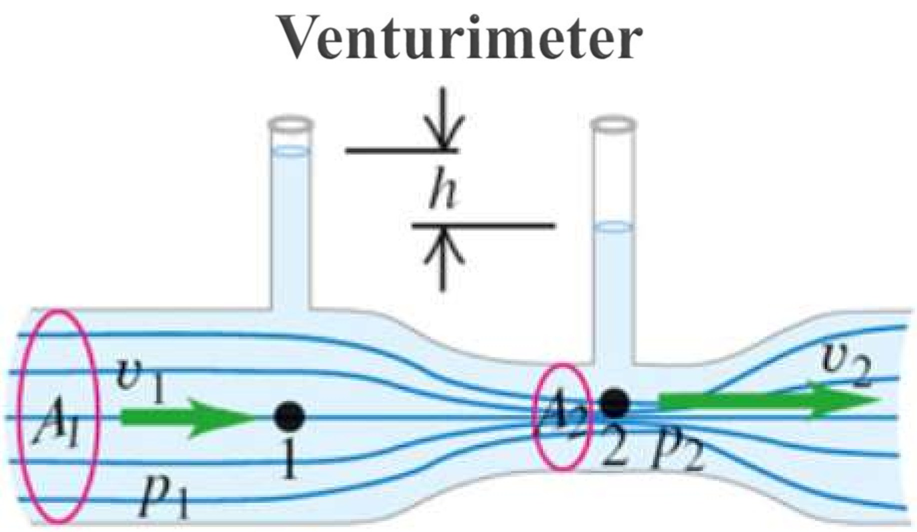
- Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved.
- Typical applications are following measuring devices:
- 1. Venturimeter
- 2. Orifice meter
- 3. Pitot-tube

Venturimeter

- A Venturimeter is a device used for measuring the rate of flow of a fluid flowing through a pipe.
- consists of three parts:
  - (i) A short converging part,
  - (ii) Throat, and
  - (iii) Diverging part..



- It is based on the Principle of Bernoulli's equation
- **Expression for Rate of Flow through the Venturimeter**
- Consider a Venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water)



- $Let\ d_1 = diameter\ at\ inlet\ or\ at\ section\ 1$
- $p_1 = pressure\ at\ section\ 1$
- $v_1 = velocity\ of\ fluid\ at\ section\ 1,\ \ a_1 = area\ at\ section\ 1 = \frac{\pi}{4}{d_1}^2$
- and d_2, p_2, v_2, a_2 are corresponding values at section 2.
- Applying Bernoulli's equation at sections 1 and 2, we get
- $\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2.$ As pipe is horizontal, $z_1 = z_2$

Venturimeter

- $\therefore \frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} \quad or \ \frac{p_1 - p_2}{\rho g} = \frac{{v_2}^2}{2g} - \frac{{v_1}^2}{2g}$
- But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h.
- Thus $\frac{p_1 - p_2}{\rho g} = h$
- Substituting this value into above equation, we get $h = \frac{{v_2}^2}{2g} - \frac{{v_1}^2}{2g}$
- Now applying continuity equation at sections 1 and 2

$$a_1 v_1 = a_2 v_2 \quad or \quad v_1 = \frac{a_2 v_2}{a_1}$$

- Substituting this value in equation (2.6)
- $h = \frac{{v_2}^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{{v_2}^2}{2g}\left[1 - \frac{{a_2}^2}{{a_1}^2}\right] = \frac{{v_2}^2}{2g}\left[\frac{{a_1}^2 - {a_2}^2}{{a_1}^2}\right] \ or \ {v_2}^2 = 2gh\frac{{a_1}^2}{{a_1}^2 - {a_2}^2}$
- $\therefore v_2 = \sqrt{2gh\frac{{a_1}^2}{{a_1}^2 - {a_2}^2}} = \frac{a_1}{\sqrt{{a_1}^2 - {a_2}^2}}\sqrt{2gh}$

Venturimeter

$$\therefore Discharge, Q = a_2 v_2$$

$$= a_2 \frac{a_1}{\sqrt{{a_1}^2 - {a_2}^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{{a_1}^2 - {a_2}^2}} \times \sqrt{2gh} \qquad (2.13)$$

- Equation (2.13) gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge Q_{act} will be less than theoretical discharge.

- $\therefore Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{{a_1}^2 - {a_2}^2}} \times \sqrt{2gh} \qquad (2.14)$

- where C_d = coefficient of Venturimeter and its value is less than 1.
- **Value of 'h' given by differential U-tube manometer**

Venturimeter

- **Case I.** Let the differential manometer contain a liquid which is heavier than the liquid flowing through the pipe. Let
- S_h = specific gravity of the heavier liquid
- S_o = specific gravity of the liquid flowing/ through pipe
- x = difference of the heavier liquid column in U-tube
- Then
- $h = x\left[\frac{S_h}{S_o} - 1\right]$ (2.15)
- **Case II.** If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by
- $h = x\left[1 - \frac{S_l}{S_o}\right]$ (2.16)
  - where S_ℓ = sp. gravity of lighter liquid in U-tube

Venturimeter

- **Case III. Inclined Venturimeter with Differential U-tube manometer.** The above two cases are given for a horizontal Venturimeter. This case is related to inclined Venturimeter having differential U-tube manometer. Let the differential manometer contain heavier liquid. Then h is given as

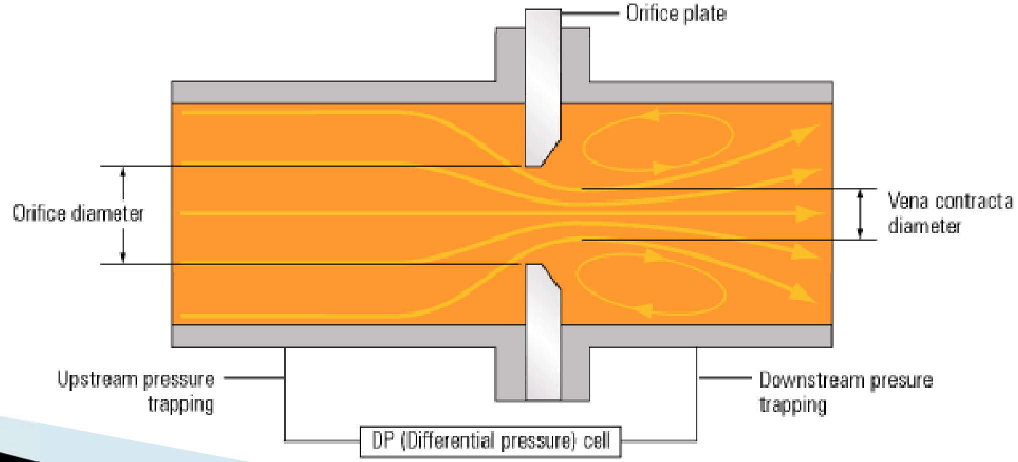
- $$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right] \qquad (2.17)$$

- **Case IV.** Similarly, for inclined Venturimeter in which differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given as

- $$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right] \qquad (2.18)$$

ORIFICE METER OR ORIFICE PLATE

- This is a device used for measuring the rate of flow of a fluid through a pipe.
- It is a cheaper device as compared to Venturimeter.
- It also works on the same principle as that of Venturimeter.
- Consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe.
- The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.



ORIFICE METER OR ORIFICE PLATE

- A differential manometer is connected at section 1, which is at a distance of about 1.5 to 2.0 times the pipe diameter upstream from the orifice plate, and at section 2, which is at a distance of about half the diameter of the orifice on the downstream side from the orifice plate.

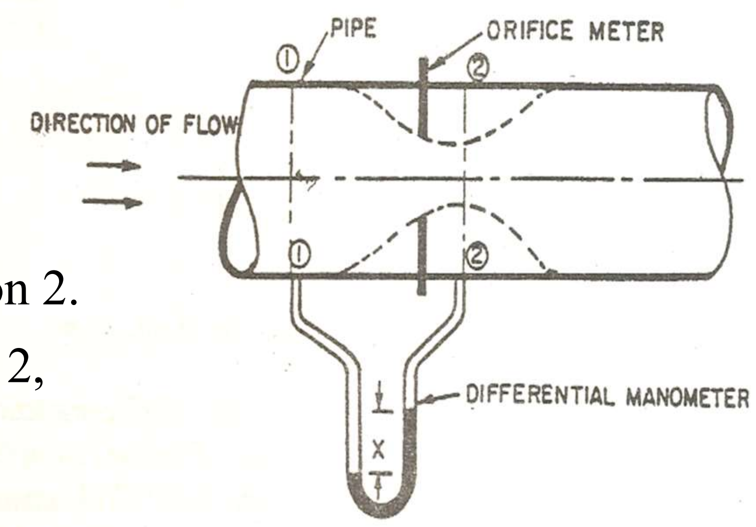
- Let $p_1 = pressure\ at\ section\ 1$;

$v_1 = velocity\ at\ section\ 1$

$a_1 = area\ of\ pipe\ at\ section\ 1$

and p_2, v_2, a_2 are corresponding values at section 2.

Applying Bernoulli's equation at sections 1 and 2,



- $$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2$$

$$or\ \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{{v_2}^2}{2g} - \frac{{v_1}^2}{2g}$$

ORIFICE METER OR ORIFICE PLATE

- $But\ \left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=h=Differential\ head$
- $\therefore h=\frac{{v_2}^2}{2g}-\frac{{v_1}^2}{2g}\quad or\quad 2gh={v_2}^2-{v_1}^2$
- $or\ v_2=\sqrt{2gh+{v_1}^2}\qquad\qquad (i)$
- Now section 2 is at the vena contracta and a_2 represents the area at the vena contracta. If a_o is the area of the orifice, then we have

$$C_c=\frac{a_2}{a_0}$$

- where C_c = coefficient of contraction.
- $\therefore a_2=a_0\times C_c\qquad\qquad (ii)$
- By continuity equation, we have

$$a_1v_1=a_2v_2\quad or\quad v_1=\frac{a_2v_2}{a_1}=\frac{a_oC_c}{a_1}v_2\qquad\qquad (iii)$$

ORIFICE METER OR ORIFICE PLATE CONT'D

- Substituting the value of v_1 in equation (i), we get
- $v_2 = \sqrt{2gh + \frac{{a_0}^2 {C_c}^2 {v_2}^2}{{a_1}^2}}$
- ${v_2}^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 {C_c}^2 v^2$ or ${v_2}^2 \left[1 - \left(\frac{a_0}{a_1}\right)^2 {C_c}^2\right] = 2gh$
- $\therefore v_2 = \frac{\sqrt{2gh}}{\sqrt{\left[1-\left(\frac{a_0}{a_1}\right)^2 {C_c}^2\right]}}$
- $\therefore The\ discharge \qquad Q = a_2 \times v_2 = v_2 \times a_0 C_c$
- $Q = \frac{a_0 C_c \sqrt{2gh}}{\sqrt{\left[1-\left(\frac{a_0}{a_1}\right)^2 {C_c}^2\right]}}$

ORIFICE METER OR ORIFICE PLATE CONT'D

- The above expression is simplified by using

- $C_d = C_c \frac{\sqrt{\left[1-\left(\frac{a_0}{a_1}\right)^2\right]}}{\sqrt{\left[1-\left(\frac{a_0}{a_1}\right)^2 C_c{}^2\right]}}$ $\quad\quad\therefore C_c = C_d \frac{\sqrt{\left[1-\left(\frac{a_0}{a_1}\right)^2 C_c{}^2\right]}}{\sqrt{\left[1-\left(\frac{a_0}{a_1}\right)^2\right]}}$

- Substituting this value of C_c in equation (iv), we get

- $Q = a_0 \times C_d \frac{\sqrt{\left[1-\left(\frac{a_0}{a_1}\right)^2 C_c{}^2\right]}}{\sqrt{\left[1-\left(\frac{a_0}{a_1}\right)^2\right]}} \times \frac{\sqrt{2g}}{\sqrt{\left[1-\left(\frac{a_0}{a_1}\right)^2 C_c{}^2\right]}}$

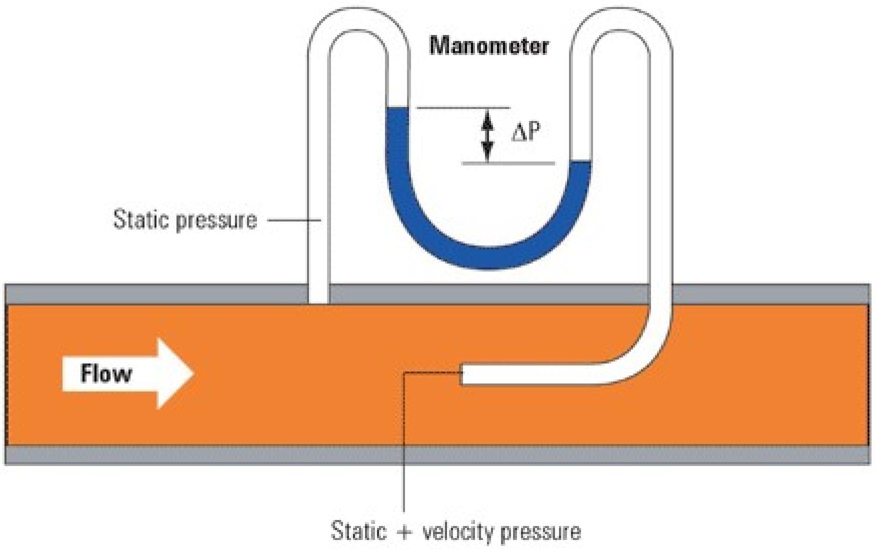
- $= \frac{a_0 C_d \sqrt{2gh}}{\sqrt{\left[1-\left(\frac{a_0}{a_1}\right)^2\right]}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1{}^2 - a_2{}^2}}$

- $Where\ C_d = Co - efficient\ of\ discharge\ for\ orifice\ meter.$

- The co-efficient of discharge for orifice meter is much smaller than that for a Venturimeter.

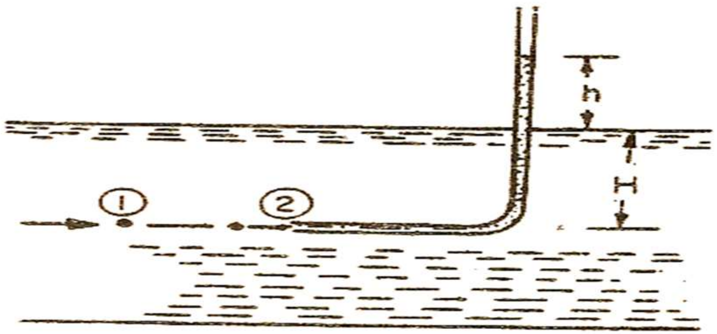
Pitot-tube

- The Pitot-tube is a device used for measuring the velocity of flow at any point in a pipe or a channel.
- It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy.
- In its simplest form, the Pitot-tube consists of a glass tube, bent at right angles.



Pitot-tube

- The lower end, which is bent through 90^o is directed in the upstream direction
- The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of liquid in the tube.
- Consider two points 1 and 2 at the same level in such a way that point 2 is just at the inlet of the pitot-tube and point 1 is far away from the tube.
- Let p_1= Pressure at point 1
- $v_1 = velocity\ of\ flow\ at\ 1$,
- p_2= Pressure at point 2
- $v_2 = velocity\ of\ flow\ at\ point\ 2, which\ is\ zero$
- $H = depth\ of\ tube\ in\ the\ liquid$
- $h = rise\ of\ liquid\ in\ the\ tube\ above\ the\ free\ surface$



Pitot-tube

- Applying the Bernoulli's equations at points (1) and (2), we get
- $\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2$
- $But\ z_1 = z_2$ as points (1) and (2) are on the same line and $v_2 = 0$
- $\frac{p_1}{\rho g} = pressure\ head\ at\ (1) = H$
- $\frac{p_2}{\rho g} = pressure\ head\ at\ (2) = (h + H)$

Pitot-tube

Substituting these values, we get

$$\therefore H + \frac{{v_1}^2}{2g} = (h + H)$$

$$\therefore h = \frac{{v_1}^2}{2g} \quad or \quad v_1 = \sqrt{2gh}$$

- This v_1 is the theoretical velocity. The actual velocity is given by

$$(v_1)_{act} = C_v\sqrt{2gh}$$

- Where $C_v = Co - efficient\ of\ pitot - tube$
- $\therefore Velocity\ at\ any\ point\ \ v = C_v\sqrt{2gh}$ (2.20)

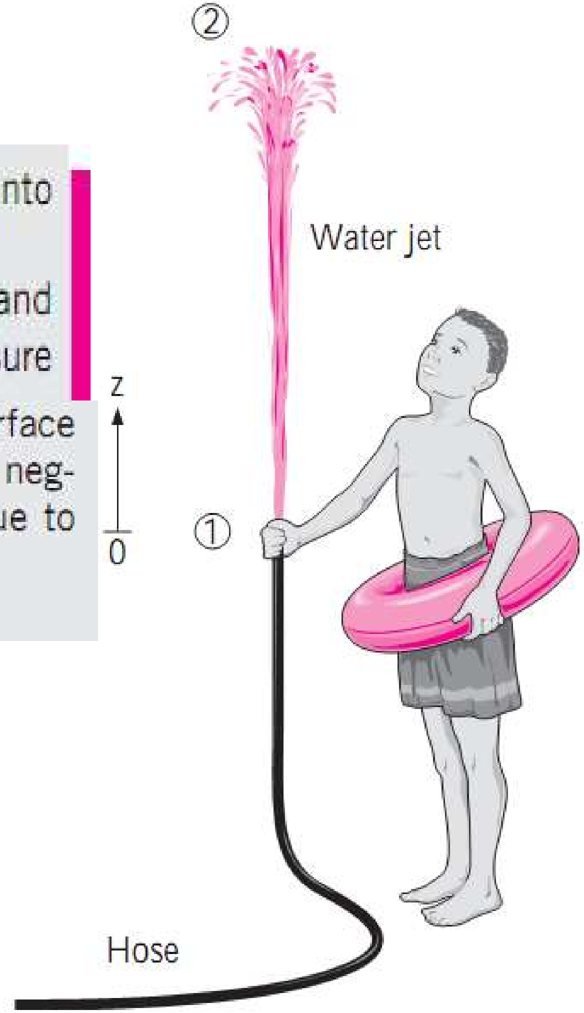
Spraying Water into the Air

Water is flowing from a hose attached to a water main at 400 kPa gauge. A child places his thumb to cover most of the hose outlet, causing a thin jet of high-speed water to emerge. If the hose is held upward, what is the maximum height that the jet could achieve?

SOLUTION Water from a hose attached to the water main is sprayed into the air. The maximum height the water jet can rise is to be determined.

Assumptions **1** The flow exiting into the air is steady, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The water pressure in the hose near the outlet is equal to the water main pressure. **3** The surface tension effects are negligible. **4** The friction between the water and air is negligible. **5** The irreversibilities that may occur at the outlet of the hose due to abrupt expansion are negligible.

Properties We take the density of water to be 1000 kg/m^3.



Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is relatively low ($V_1 \cong 0$) and we take the hose outlet as the reference level ($z_1 = 0$). At the top of the water trajectory $V_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g}\nearrow^{0} + z_1 \nearrow^{0} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}\nearrow^{0} + z_2 \quad \rightarrow \quad \frac{P_1}{\rho g} = \frac{P_{atm}}{\rho g} + z_2$$

Solving for z_2 and substituting,

$$z_2 = \frac{P_1 - P_{atm}}{\rho g} = \frac{P_{1,\,gage}}{\rho g} = \frac{400 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right)\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right)$$

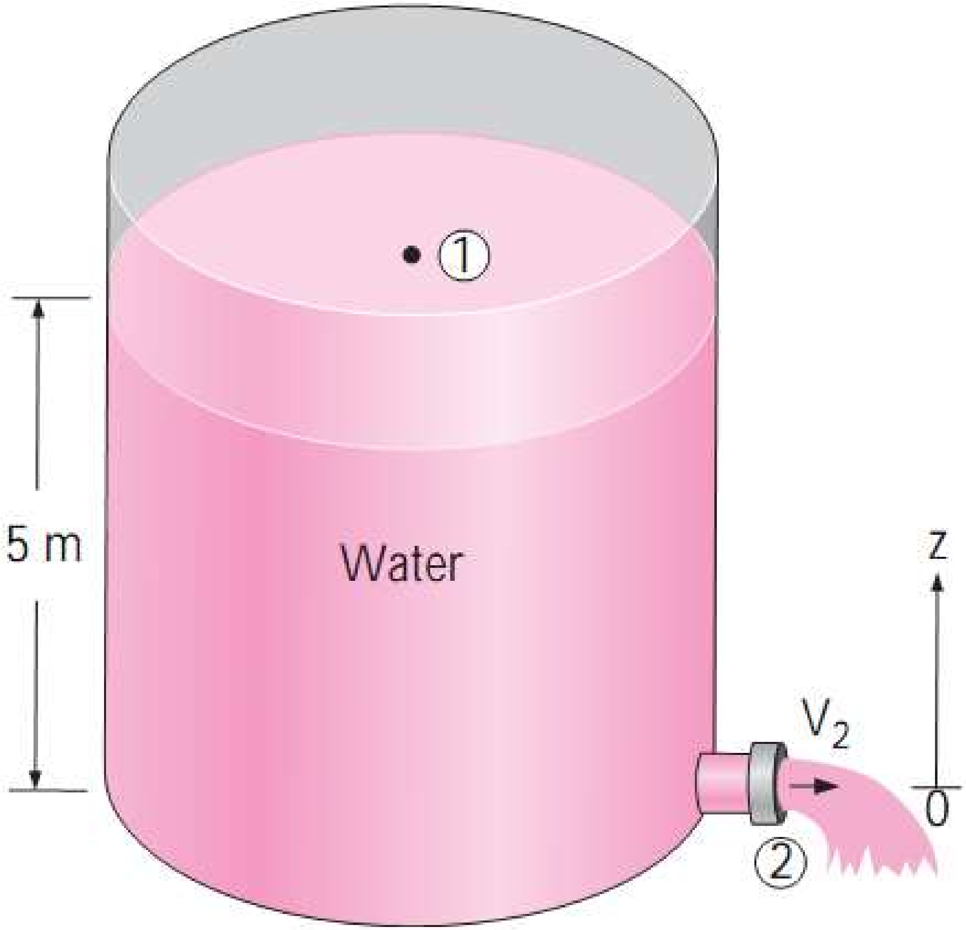
$$= 40.8 \text{ m}$$

Therefore, the water jet can rise as high as 40.8 m into the sky in this case.

Discussion The result obtained by the Bernoulli equation represents the upper limit and should be interpreted accordingly. It tells us that the water cannot possibly rise more than 40.8 m, and, in all likelihood, the rise will be much less than 40.8 m due to irreversible losses that we neglected.

Water Discharge from a Large Tank

A large tank open to the atmosphere is filled with water to a height of 5 m from the outlet tap. A tap near the bottom of the tank is now opened, and water flows out from the smooth and rounded outlet. Determine the water velocity at the outlet.



SOLUTION A tap near the bottom of a tank is opened. The exit velocity of water from the tank is to be determined.

Assumptions **1** The flow is incompressible and irrotational (except very close to the walls). **2** The water drains slowly enough that the flow can be approximated as steady (actually quasi-steady when the tank begins to drain).

Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. We take point 1 to be at the free surface of water so that $P_1 = P_{atm}$ (open to the atmosphere), $V_1 \cong 0$ (the tank is large relative to the outlet), and $z_1 = 5$ m and $z_2 = 0$ (we take the reference level at the center of the outlet). Also, $P_2 = P_{atm}$ (water discharges into the atmosphere). Then the Bernoulli equation simplifies to

$$\cancel{\frac{P_1}{\rho g}} + \frac{V_1^2}{2g}^{\nearrow 0} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + z_2^{\;0} \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Solving for V_2 and substituting,

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81\ \text{m/s}^2)(5\ \text{m})} = 9.9\ \text{m/s}$$

The relation $V = \sqrt{2gz}$ is called the **Toricelli equation.**

Therefore, the water leaves the tank with an initial velocity of 9.9 m/s. This is the same velocity that would manifest if a solid were dropped a distance of 5 m in the absence of air friction drag. (What would the velocity be if the tap were at the bottom of the tank instead of on the side?)

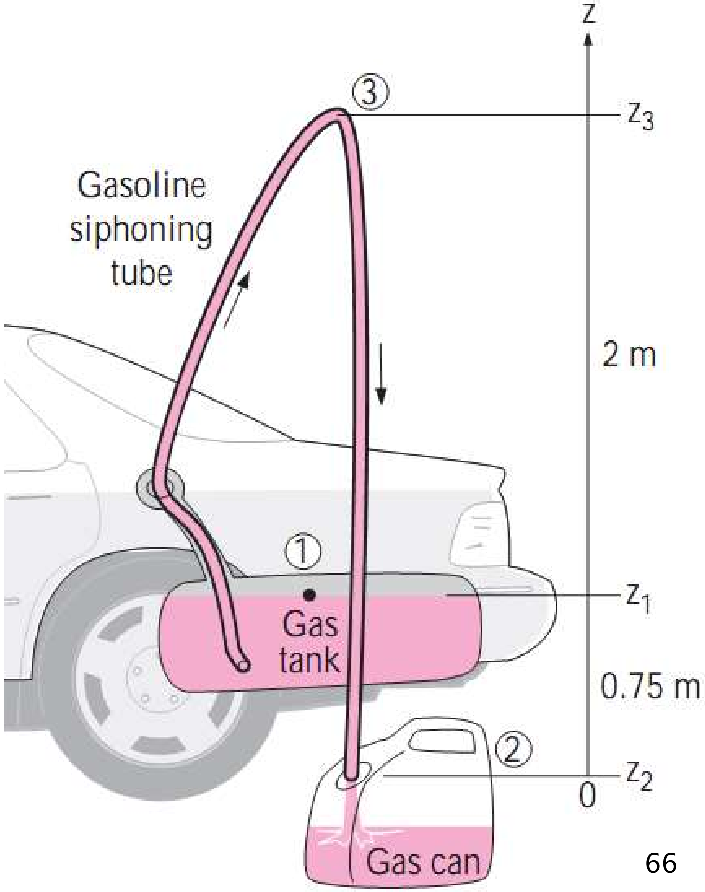
Discussion If the orifice were sharp-edged instead of rounded, then the flow would be disturbed, and the velocity would be less than 9.9 m/s, especially near the edges. Care must be exercised when attempting to apply the Bernoulli equation to situations where abrupt expansions or contractions occur since the friction and flow disturbance in such cases may not be negligible.

Siphoning Out Gasoline from a Fuel Tank

During a trip to the beach (Patm =1 atm = 101.3 kPa), a car runs out of gasoline, and it becomes necessary to siphon gas out of the car of a Good Samaritan. The siphon is a small-diameter hose, and to start the siphon it is necessary to insert one siphon end in the full gas tank, fill the hose with gasoline via suction, and then place the other end in a gas can below the level of the gas tank. The difference in pressure between point 1 (at the free surface of the gasoline in the tank) and point 2 (at the outlet of the tube) causes the liquid to flow from the higher to the lower elevation.

Point 2 is located 0.75 m below point 1 in this case, and point 3 is located 2 m above point 1. The siphon diameter is 4 mm, and frictional losses in the siphon are to be disregarded.

Determine (a) the minimum time to withdraw 4 L of gasoline from the tank to the can and (b) the pressure at point 3. The density of gasoline is 750 kg/m3.



SOLUTION Gasoline is to be siphoned from a tank. The minimum time it takes to withdraw 4 L of gasoline and the pressure at the highest point in the system are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Even though the Bernoulli equation is not valid through the pipe because of frictional losses, we employ the Bernoulli equation anyway in order to obtain a *best-case estimate*. **3** The change in the gasoline surface level inside the tank is negligible compared to elevations z_1 and z_2 during the siphoning period.

Properties The density of gasoline is given to be 750 kg/m^3.

Analysis (*a*) We take point 1 to be at the free surface of gasoline in the tank so that $P_1 = P_{atm}$ (open to the atmosphere), $V_1 \cong 0$ (the tank is large relative to the tube diameter), and $z_2 = 0$ (point 2 is taken as the reference level). Also, $P_2 = P_{atm}$ (gasoline discharges into the atmosphere). Then the Bernoulli equation simplifies to

$$\frac{\cancel{P_1}}{\cancel{\rho} g} + \frac{V_1^2}{2g}^{\nearrow 0} + z_1 = \frac{\cancel{P_2}}{\cancel{\rho} g} + \frac{V_2^2}{2g} + \overset{0}{z_2} \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Solving for V_2 and substituting,

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81\ \text{m/s}^2)(0.75\ \text{m})} = 3.84\ \text{m/s}$$

The cross-sectional area of the tube and the flow rate of gasoline are

$$A = \pi D^2/4 = \pi(5 \times 10^{-3}\ \text{m})^2/4 = 1.96 \times 10^{-5}\ \text{m}^2$$

$$\dot{V} = V_2 A = (3.84\ \text{m/s})(1.96 \times 10^{-5}\ \text{m}^2) = 7.53 \times 10^{-5}\ \text{m}^3/\text{s} = 0.0753\ \text{L/s}$$

Then the time needed to siphon 4 L of gasoline becomes

$$\Delta t = \frac{V}{\dot{V}} = \frac{4\ \text{L}}{0.0753\ \text{L/s}} = \mathbf{53.1\ s}$$

(*b*) The pressure at point 3 can be determined by writing the Bernoulli equation between points 2 and 3. Noting that $V_2 = V_3$ (conservation of mass), $z_2 = 0$, and $P_2 = P_{atm}$,

$$\frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \overset{0}{z_2} = \frac{P_3}{\rho g} + \cancel{\frac{V_3^2}{2g}} + z_3 \quad \rightarrow \quad \frac{P_{atm}}{\rho g} = \frac{P_3}{\rho g} + z_3$$

Solving for P_3 and substituting,

$$P_3 = P_{atm} - \rho g z_3$$

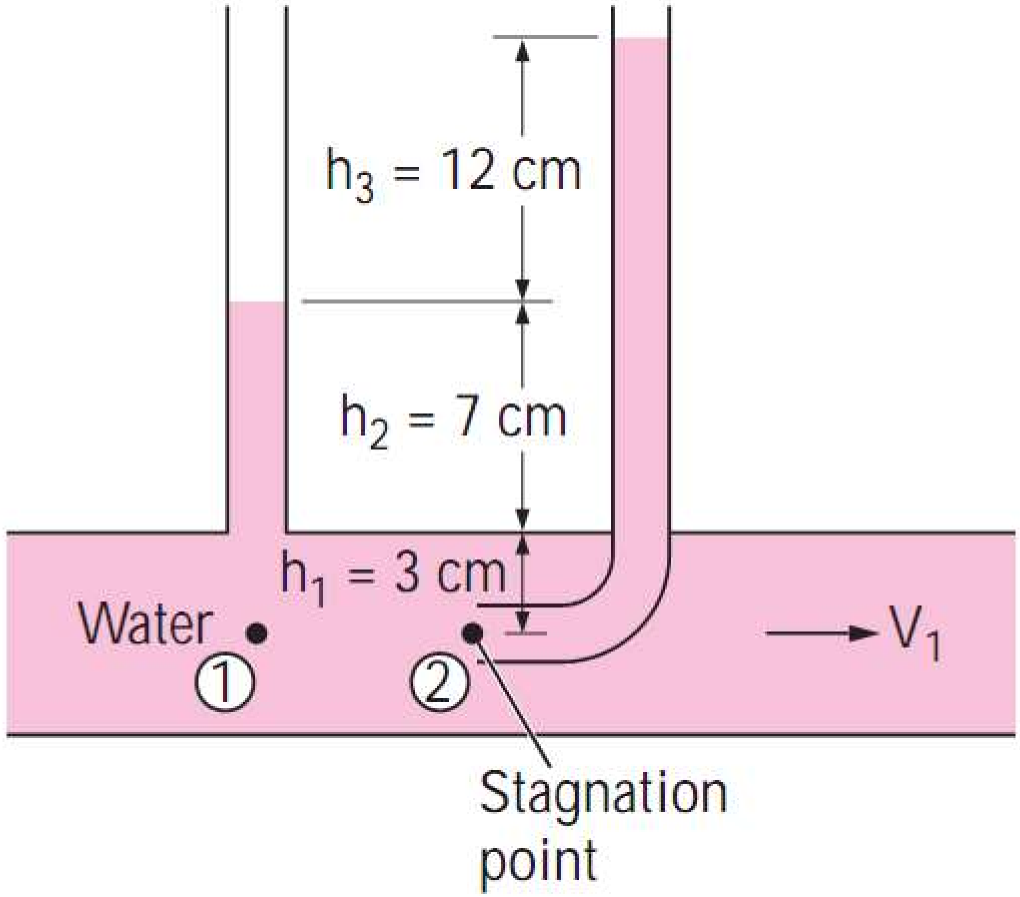
$$= 101.3\ \text{kPa} - (750\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.75\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right)$$

$$= \mathbf{81.1\ kPa}$$

Discussion The siphoning time is determined by neglecting frictional effects, and thus this is the *minimum time* required. In reality, the time will be longer than 53.1 s because of friction between the gasoline and the tube surface. Also, the pressure at point 3 is below the atmospheric pressure. If the elevation difference between points 1 and 3 is too high, the pressure at point 3 may drop below the vapor pressure of gasoline at the gasoline temperature, and some gasoline may evaporate (cavitate). The vapor then may form a pocket at the top and halt the flow of gasoline.

Velocity Measurement by a Pitot Tube

A piezometer and a Pitot tube are tapped into a horizontal water pipe, as shown in Figure, to measure static and stagnation (static + dynamic) pressures. For the indicated water column heights, determine the velocity at the center of the pipe.



SOLUTION The static and stagnation pressures in a horizontal pipe are measured. The velocity at the center of the pipe is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Points 1 and 2 are close enough together that the irreversible energy loss between these two points is negligible, and thus we can use the Bernoulli equation.

Analysis We take points 1 and 2 along the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the tip of the Pitot tube. This is a steady flow with straight and parallel streamlines, and the gage pressures at points 1 and 2 can be expressed as

$$P_1 = \rho g(h_1 + h_2)$$

$$P_2 = \rho g(h_1 + h_2 + h_3)$$

Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}^{\nearrow 0} + \cancel{z_2} \quad \rightarrow \quad \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

Substituting the P_1 and P_2 expressions gives

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} = \frac{\rho g(h_1 + h_2 + h_3) - \rho g(h_1 + h_2)}{\rho g} = h_3$$

Solving for V_1 and substituting,

$$V_1 = \sqrt{2gh_3} = \sqrt{2(9.81 \text{ m/s}^2)(0.12 \text{ m})} = \mathbf{1.53 \text{ m/s}}$$

Discussion Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the Pitot tube.

MECHANICAL ENERGY AND EFFICIENCY

- The mechanical energy can be defined as the form of energy that can be converted to mechanical work completely and directly by an ideal mechanical device such as an ideal turbine.
- Kinetic and potential energies are the familiar forms of mechanical energy.
- Thermal energy is not mechanical energy, since it cannot be converted to work directly and completely (the second law of thermodynamics)

MECHANICAL ENERGY AND EFFICIENCY

- A pump transfers mechanical energy to a fluid by raising its pressure, and
- A turbine extracts mechanical energy from a fluid by dropping its pressure.
- The pressure of a flowing fluid is also associated with its mechanical energy.
- The mechanical energy of a flowing fluid can be expressed on a unit-mass basis as

$$e_{mech} = \frac{P}{\rho} + \frac{V^2}{2} + gz$$

MECHANICAL ENERGY AND EFFICIENCY

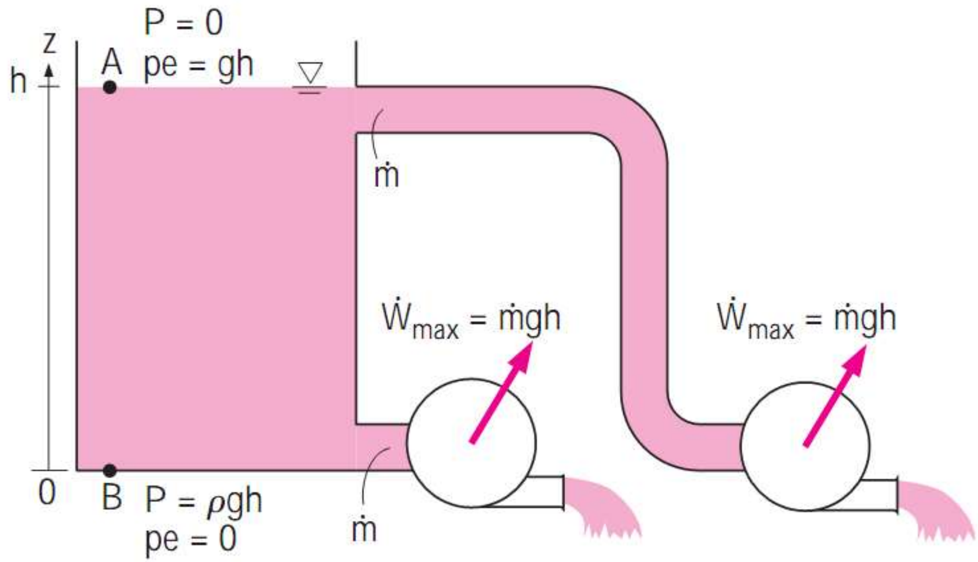
- The mechanical energy change of a fluid during incompressible flow becomes

$$\Delta e_{mech} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \qquad \text{(kJ/kg)}$$

- The mechanical energy of a fluid does not change during flow if its pressure, density, velocity, and elevation remain constant.
- In the absence of any losses, the mechanical energy change represents the mechanical work supplied to the fluid (if $\Delta e_{mech} > 0$) or extracted from the fluid (if $\Delta e_{mech} < 0$).

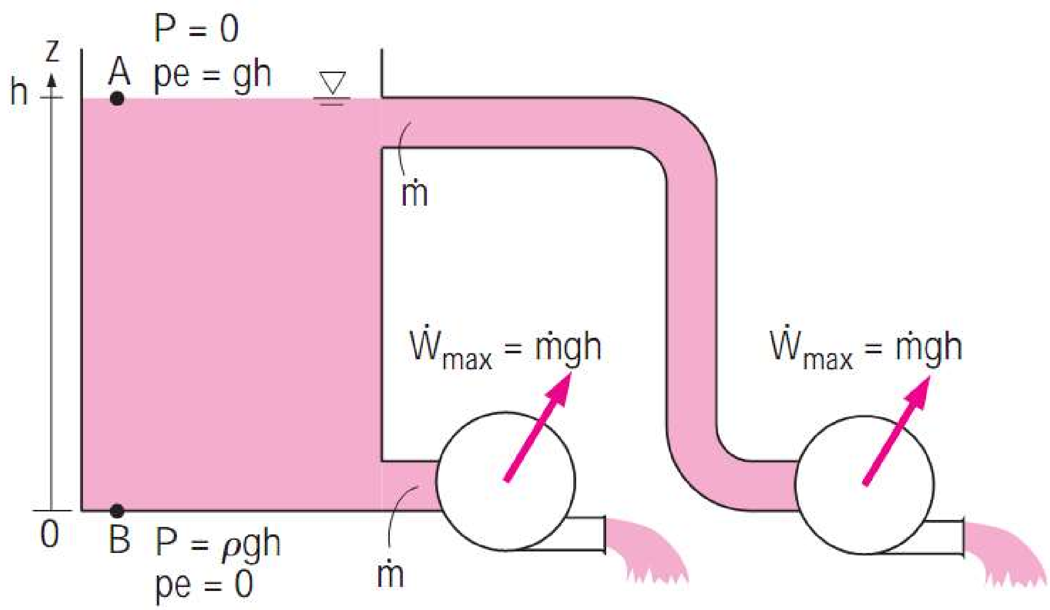
MECHANICAL ENERGY AND EFFICIENCY

- For a container of height, h, filled with water,
- at point A at the free surface
  - The gauge pressure per unit mass $P_A = 0$ and
  - The potential energy per unit mass $pe_A = gh$
- at point B at the bottom
  - The gauge pressure per unit mass $P_B = \rho gh$ and
  - The potential energy per unit mass $pe_B = 0$



MECHANICAL ENERGY AND EFFICIENCY

- An ideal hydraulic turbine would produce the same work per unit mass $W_{turbine} = gh$ whether it receives water (or any other fluid with constant density) from the top or from the bottom of the container.
- The total mechanical energy of water at the bottom is equivalent to that at the top.



MECHANICAL ENERGY AND EFFICIENCY

- The transfer of mechanical energy is usually accomplished by a rotating shaft, and thus mechanical work is often referred to as shaft work.
- A pump or a fan receives shaft work (usually from an electric motor) and transfers it to the fluid as mechanical energy (less frictional losses).
- A turbine, converts the mechanical energy of a fluid to shaft work.
- Mechanical efficiency of a device or process can be defined as

$$\eta_{mech} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy input}} = \frac{E_{mech,\,out}}{E_{mech,\,in}} = 1 - \frac{E_{mech,\,loss}}{E_{mech,\,in}}$$

MECHANICAL ENERGY AND EFFICIENCY

- In fluid systems, we are usually interested in increasing the pressure, velocity, and/or elevation of a fluid.
- This is done by
  - Supplying mechanical energy to the fluid by a pump, a fan, or a compressor (refer to as pumps).
  - Extracting mechanical energy from a fluid by a turbine and producing mechanical power in the form of a rotating shaft that can drive a generator or any other rotary device.

Pump efficiency

- The degree of perfection of the conversion process between the mechanical work supplied or extracted and the mechanical energy of the fluid is expressed by
  - pump efficiency and
  - turbine efficiency,

$$\eta_{pump} = \frac{\text{Mechanical energy increase of the fluid}}{\text{Mechanical energy input}} = \frac{\Delta \dot{E}_{mech,\ fluid}}{\dot{W}_{shaft,\ in}} = \frac{\dot{W}_{pump,\ u}}{\dot{W}_{pump}}$$

where $\Delta E_{mech,\ fluid} = E_{\cdot mech,\ out} - E_{\cdot mech,\ in}$ is the rate of increase in the mechanical energy of the fluid, which is equivalent to the useful pumping power $W_{\cdot pump,\ u}$ supplied to the fluid

Turbine efficiency

$$\eta_{turbine} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy decrease of the fluid}} = \frac{\dot{W}_{shaft,\ out}}{|\Delta \dot{E}_{mech,\ fluid}|} = \frac{\dot{W}_{turbine}}{\dot{W}_{turbine,\ e}}$$

where $|\Delta \dot{E}_{mech,\ fluid}| = E_{mech,\ in} - E_{mech,\ out}$ is the rate of decrease in the mechanical energy of the fluid, which is equivalent to the mechanical power extracted from the fluid by the turbine $W_{turbine,\ e}$, and absolute value to avoid negative values for efficiencies.

Mechanical, Motor & Generator efficiencies

- The mechanical efficiency should not be confused with the motor efficiency and the generator efficiency, which are defined as

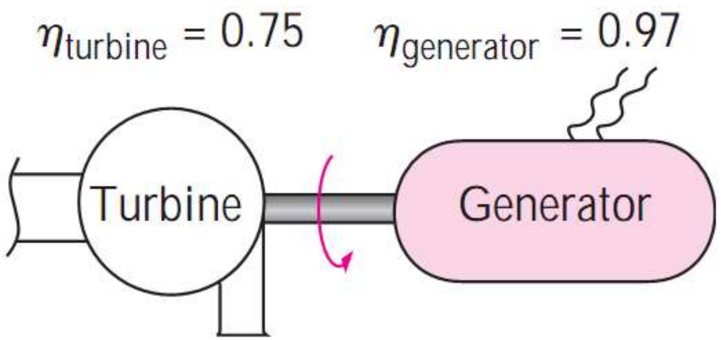
Motor:
$$\eta_{\text{motor}} = \frac{\text{Mechanical power output}}{\text{Electric power input}} = \frac{\dot{W}_{\text{shaft, out}}}{\dot{W}_{\text{elect, in}}}$$

and

Generator:
$$\eta_{\text{generator}} = \frac{\text{Electric power output}}{\text{Mechanical power input}} = \frac{\dot{W}_{\text{elect, out}}}{\dot{W}_{\text{shaft, in}}}$$

Overall Efficiency

- A pump goes with its motor, and
- A turbine goes with its generator.
- the combined or overall efficiency of pump-motor and turbine-generator combinations is



$$\eta_{turbine\text{-}gen} = \eta_{turbine}\eta_{generator} = 0.75 \times 0.97 = 0.73$$

$$\eta_{pump\text{-}motor} = \eta_{pump}\,\eta_{motor} = \frac{\dot{W}_{pump,\,u}}{\dot{W}_{elect,\,in}} = \frac{\Delta\dot{E}_{mech,\,fluid}}{\dot{W}_{elect,\,in}}$$

and

$$\eta_{turbine\text{-}gen} = \eta_{turbine}\,\eta_{generator} = \frac{\dot{W}_{elect,\,out}}{\dot{W}_{turbine,\,e}} = \frac{\dot{W}_{elect,\,out}}{|\Delta\dot{E}_{mech,\,fluid}|}$$

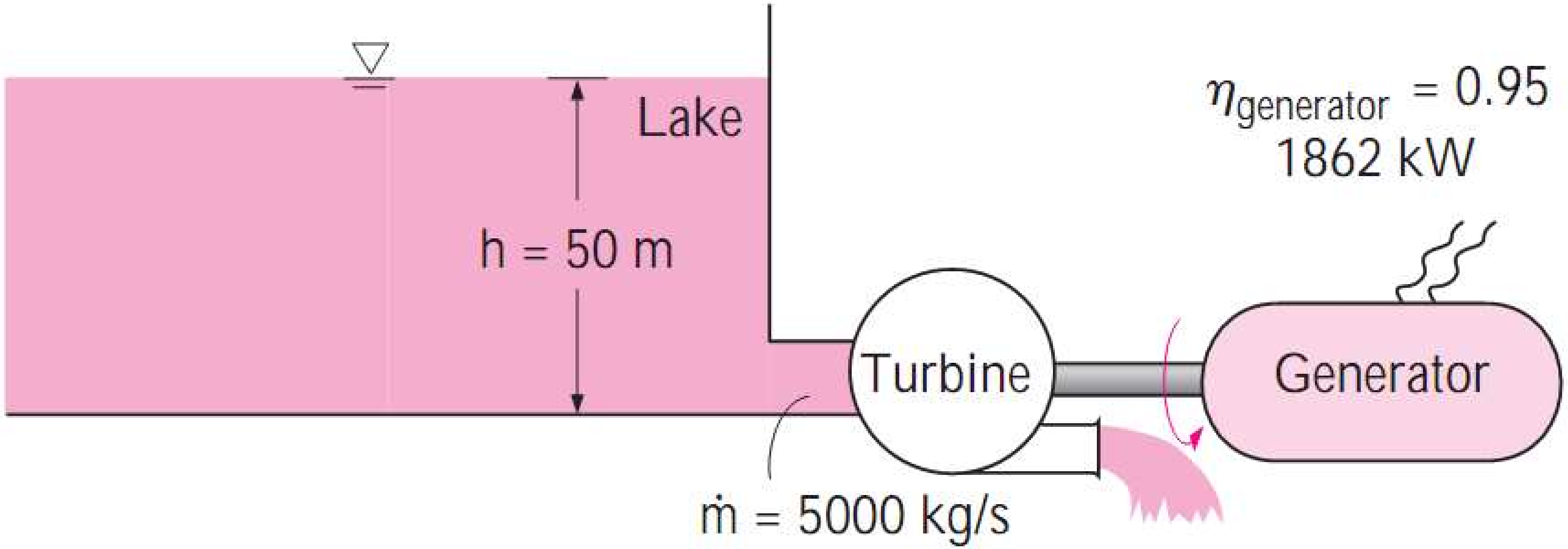
Performance of a Hydraulic Turbine-Generator

The water in a large lake is to be used to generate electricity by the installation of a hydraulic turbine-generator at a location where the depth of the water is 50 m. Water is to be supplied at a rate of 5000 kg/s. If the electric power generated is measured to be 1862 kW and the generator efficiency is 95 percent, determine

(a) the overall efficiency of the turbine-generator,

(b) the mechanical efficiency of the turbine, and

(c) the shaft power supplied by the turbine to the generator.



SOLUTION A hydraulic turbine–generator is to generate electricity from the water of a lake. The overall efficiency, the turbine efficiency, and the shaft power are to be determined.

Assumptions **1** The elevation of the lake remains constant. **2** The mechanical energy of water at the turbine exit is negligible.

Properties The density of water can be taken to be $\rho = 1000\ \text{kg/m}^3$.

Analysis (*a*) We take the bottom of the lake as the reference level for convenience. Then kinetic and potential energies of water are zero, and the change in its mechanical energy per unit mass becomes

$$e_{\text{mech, in}} - e_{\text{mech, out}} = \frac{P}{\rho} - 0 = gh = (9.81\ \text{m/s}^2)(50\ \text{m})\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 0.491\ \text{kJ/kg}$$

Then the rate at which mechanical energy is supplied to the turbine by the fluid and the overall efficiency become

$$|\Delta \dot{E}_{\text{mech, fluid}}| = \dot{m}(e_{\text{mech, in}} - e_{\text{mech, out}}) = (5000\ \text{kg/s})(0.491\ \text{kJ/kg}) = 2455\ \text{kW}$$

$$\eta_{\text{overall}} = \eta_{\text{turbine-gen}} = \frac{\dot{W}_{\text{elect, out}}}{|\Delta \dot{E}_{\text{mech, fluid}}|} = \frac{1862\ \text{kW}}{2455\ \text{kW}} = \mathbf{0.76}$$

(*b*) Knowing the overall and generator efficiencies, the mechanical efficiency of the turbine is determined from

$$\eta_{\text{turbine-gen}} = \eta_{\text{turbine}}\,\eta_{\text{generator}} \rightarrow \eta_{\text{turbine}} = \frac{\eta_{\text{turbine-gen}}}{\eta_{\text{generator}}} = \frac{0.76}{0.95} = \mathbf{0.80}$$

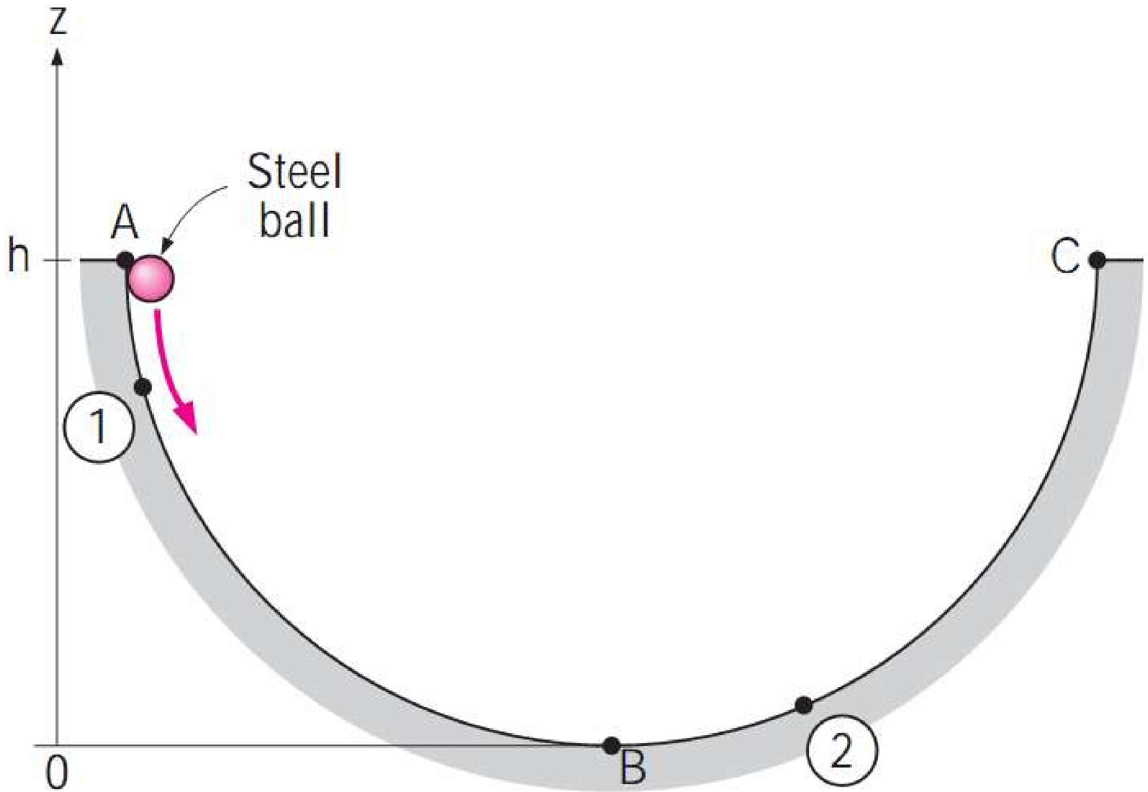
(*c*) The shaft power output is determined from the definition of mechanical efficiency,

$$\dot{W}_{\text{shaft, out}} = \eta_{\text{turbine}}\left|\Delta\dot{E}_{\text{mech, fluid}}\right| = (0.80)(2455\ \text{kW}) = \mathbf{1964\ kW}$$

Discussion Note that the lake supplies 2455 kW of mechanical energy to the turbine, which converts 1964 kW of it to shaft work that drives the generator, which generates 1862 kW of electric power. There are irreversible losses through each component.

Conservation of Energy for an Oscillating Steel Ball

The motion of a steel ball in a hemispherical bowl of radius h shown in Figure is to be analyzed. The ball is initially held at the highest location at point A, and then it is released. Obtain relations for the conservation of energy of the ball for the cases of frictionless and actual motions.



SOLUTION A steel ball is released in a bowl. Relations for the energy balance are to be obtained.

Assumptions The motion is frictionless, and thus friction between the ball, the bowl, and the air is negligible.

Analysis When the ball is released, it accelerates under the influence of gravity, reaches a maximum velocity (and minimum elevation) at point *B* at the bottom of the bowl, and moves up toward point *C* on the opposite side. In the ideal case of frictionless motion, the ball will oscillate between points *A* and *C*. The actual motion involves the conversion of the kinetic and potential energies of the ball to each other, together with overcoming resistance to motion due to friction (doing frictional work). The general energy balance for any system undergoing any process is

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

Then the energy balance for the ball for a process from point 1 to point 2 becomes

$$-w_{friction} = (ke_2 + pe_2) - (ke_1 + pe_1)$$

or

$$\frac{V_1^2}{2} + gz_1 = \frac{V_2^2}{2} + gz_2 + w_{friction}$$

since there is no energy transfer by heat or mass and no change in the internal energy of the ball (the heat generated by frictional heating is dissipated to the surrounding air). The frictional work term $w_{friction}$ is often expressed as e_{loss} to represent the loss (conversion) of mechanical energy into thermal energy.

For the idealized case of frictionless motion, the last relation reduces to

$$\frac{V_1^2}{2} + gz_1 = \frac{V_2^2}{2} + gz_2 \qquad \text{or} \qquad \frac{V^2}{2} + gz = C = \text{constant}$$

where the value of the constant is $C = gh$. That is, *when the frictional effects are negligible, the sum of the kinetic and potential energies of the ball remains constant.*

Discussion This is certainly a more intuitive and convenient form of the conservation of energy equation for this and other similar processes such as the swinging motion of the pendulum of a wall clock. The relation obtained is analogous to the Bernoulli equation derived in Section 5–4.

MECHANICAL ENERGY AND EFFICIENCY

- Most processes encountered in practice involve only certain forms of energy,
- It is more convenient to work with the simplified versions of the energy balance.
- For systems that involve only mechanical forms of energy and its transfer as shaft work, the conservation of energy principle is

$$E_{\text{mech, in}} - E_{\text{mech, out}} = \Delta E_{\text{mech, system}} + E_{\text{mech, loss}}$$

where $E_{\text{mech, loss}}$ represents the conversion of mechanical energy to thermal energy due to irreversibilities such as friction. For a system in steady operation, the mechanical energy balance becomes

$$E_{\text{mech, in}} = E_{\text{mech, out}} + E_{\text{mech, loss}}$$

Steady flow

$$V_1 = V_2$$

$$z_2 = z_1 + h$$

$$P_1 = P_2 = P_{atm}$$

$$\dot{E}_{mech,\ in} = \dot{E}_{mech,\ out} + \dot{E}_{mech,\ loss}$$

$$\dot{W}_{pump} + \dot{m}gz_1 = \dot{m}gz_2 + \dot{E}_{mech,\ loss}$$

$$\dot{W}_{pump} = \dot{m}gh + \dot{E}_{mech,\ loss}$$