SOUTHERN COLLEGE
Department of Electrical and Electronics Engineering
1st Semester Final Examination 2004

Introduction to Electromagnetics

08 MAR 2004

Class: DEE02-A + DEE02-B + DEE02-C
Lecturer: Dr. Ting Chek Ming

Time Allowed: 2½ Hours

Answer <u>all</u> questions. All answers must be written in the answer booklet provided.

Question #1 (20%)

As shown in Figure 1, a cylindrical capacitor consists of an inner conductor of radius ***a*** and an outer conductor whose inner radius is ***b***. The space between the conductors is filled with a dielectric of permittivity ε, and the length of the capacitor is **L**. Assume charges **+Q** and **–Q** on the surface of the inner conductor and the inner surface of the outer conductor, respectively.

Determine

a) $\vec{\mathbf{E}}$ field for $r < a$,

b) $\vec{\mathbf{E}}$ field for $a < r < b$,

c) $\mathbf{V}_{ab}$ (neglect fringing), and

d) the capacitance of this capacitor.

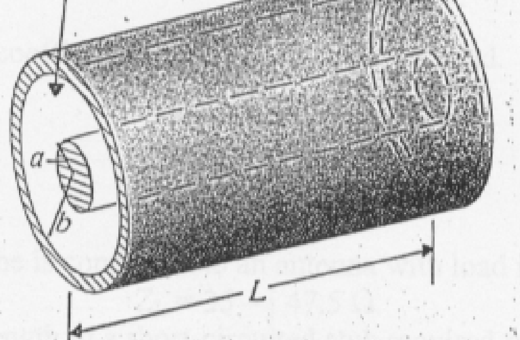

Figure 1

Question #2 (25%)

(a) A vector field

$$\vec{D} = \frac{\hat{a}_R (\cos^2 \Phi)}{R^3}$$

exists in the region between two spherical shells defined by R = 1 and R = 2. Evaluate

(i)

$$\oint \vec{D} \cdot d\vec{s}$$

(ii)

$$\int \nabla \cdot \vec{D}\, dv$$

(b) Assuming that a cloud of electrons confined in a region between two spheres of radii 2 and 5 cm has a charge density of

$$\frac{-3 \times 10^{-8}}{R^4} \cos^2 \Phi \quad (\mathrm{C/m^3}),$$

find the total charge contained in the region.

Question #3 (25%)

(a) A 100-Ω transmission line is connected to a load consisting of a 50-Ω resistor in series with a 10-pF capacitor. Find the reflection coefficient at the load for a 100-MHz signal.

(b) Find the reflection coefficient Γ for a purely reactive load.

Question #4 (30%)

A 50-Ω transmission line is connected to an antenna with load impedance

$$Z_L = 35 - j\,47.5\ \Omega.$$

Find the position and length of a short-circuited stub required to match the line. (Note: Use the Smith chart provided. All workings and markings are to be shown clearly)

TABLE **Parameters of rectangular, cylindrical, and spherical coordinate systems**

Coordinate system	Coordinates	Range	Unit vectors	Length elements	Coordinate surfaces	
Rectangular	x	$-\infty$ to $+\infty$	$\hat{a}_x$	dx	Plane	x = constan
	y	$-\infty$ to $+\infty$	$\hat{a}_y$	dy	Plane	y = constan
	z	$-\infty$ to $+\infty$	$\hat{a}_z$	dz	Plane	z = constan
Cylindrical	r	0 to ∞	$\hat{a}_r$	dr	Cylinder	r = constan
	ϕ	0 to 2π	$\hat{a}_\phi$	$r\,d\phi$	Plane	ϕ = constan
	z	$-\infty$ to $+\infty$	$\hat{a}_z$	dz	Plane	z = constan
Spherical	R	0 to ∞	$\hat{a}_R$	dR	Sphere	R = constant
	θ	0 to π	$\hat{a}_\theta$	$R\,d\theta$	Cone	θ = constant
	ϕ	0 to 2π	$\hat{a}_\phi$	$R\sin\theta\,d\phi$	Plane	ϕ = constant

Table : Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2+y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ $\hat{z} = \hat{z}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{z}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2+y^2+z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2+y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$ $\hat{\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{x} = \hat{R}\sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{R}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2+z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r}\sin\theta + \hat{z}\cos\theta$ $\hat{\theta} = \hat{r}\cos\theta - \hat{z}\sin\theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{r} = \hat{R}\sin\theta + \hat{\theta}\cos\theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

TABLE
Three Basic Orthogonal Coordinate Systems

Coordinate System Relations		Cartesian Coordinates (x, y, z)	Cylindrical Coordinates (r, ϕ, z)	Spherical Coordinates (R, θ, ϕ)
Base vectors	$\mathbf{a}_{u_1}$	$\mathbf{a}_x$	$\mathbf{a}_r$	$\mathbf{a}_R$
	$\mathbf{a}_{u_2}$	$\mathbf{a}_y$	$\mathbf{a}_\phi$	$\mathbf{a}_\theta$
	$\mathbf{a}_{u_3}$	$\mathbf{a}_z$	$\mathbf{a}_z$	$\mathbf{a}_\phi$
Metric coefficients	h_1	1	1	1
	h_2	1	r	R
	h_3	1	1	$R\sin\theta$
Differential volume	dv	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$