INSTITUTE OF DISTANCE LEARNING

KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI, GHANA

ME 355
STRENGTH OF MATERIALS II

UNIT 3

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Introduction

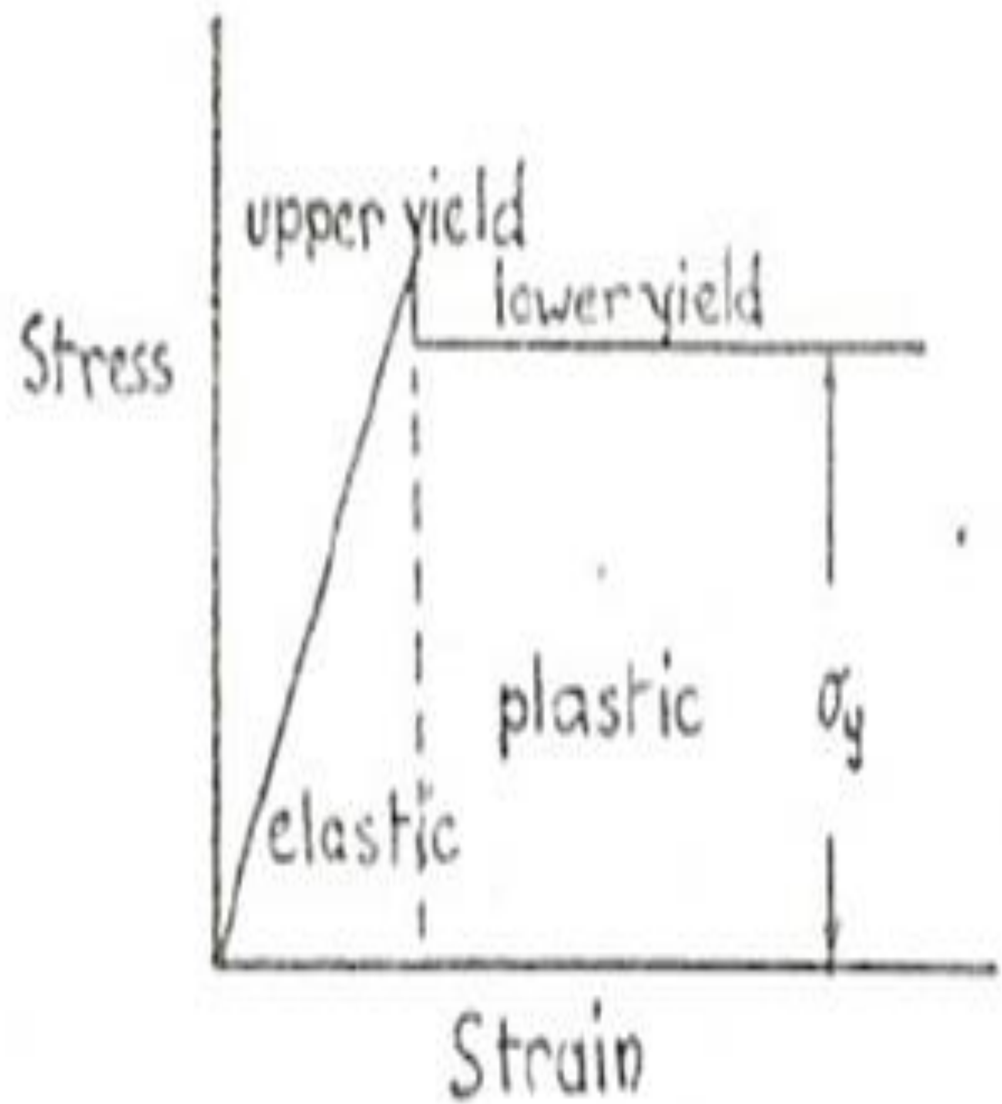
- ***COMPRESSIONS, BENDING AND TORSION UNDER PLASTIC CONDITIONS***

Learning Objectives

- After reading this unit you should be able to:
- Calculate the bending moment required to form a plastic hinge for any particular cross-section
- Determine the distribution of bending moment along the beam at the collapse load
- Compute the shape factor for each type of cross-section
- Calculate the torque required to form a plastic hinge for a circular section

MOMENT UNDER PLASTIC CONDITION

- In the elastic theory of bending, the method of design has been to
  1. calculate the maximum stresses occurring
  2. keep them within the limits of working stresses in tension and compression
- If the load system were increased gradually, yielding would first occur at the extreme fibres of the weakest section
- These fibres are then said to be in the *plastic* state


upper yield
lower yield
Stress
plastic
σ_y
elastic
Strain

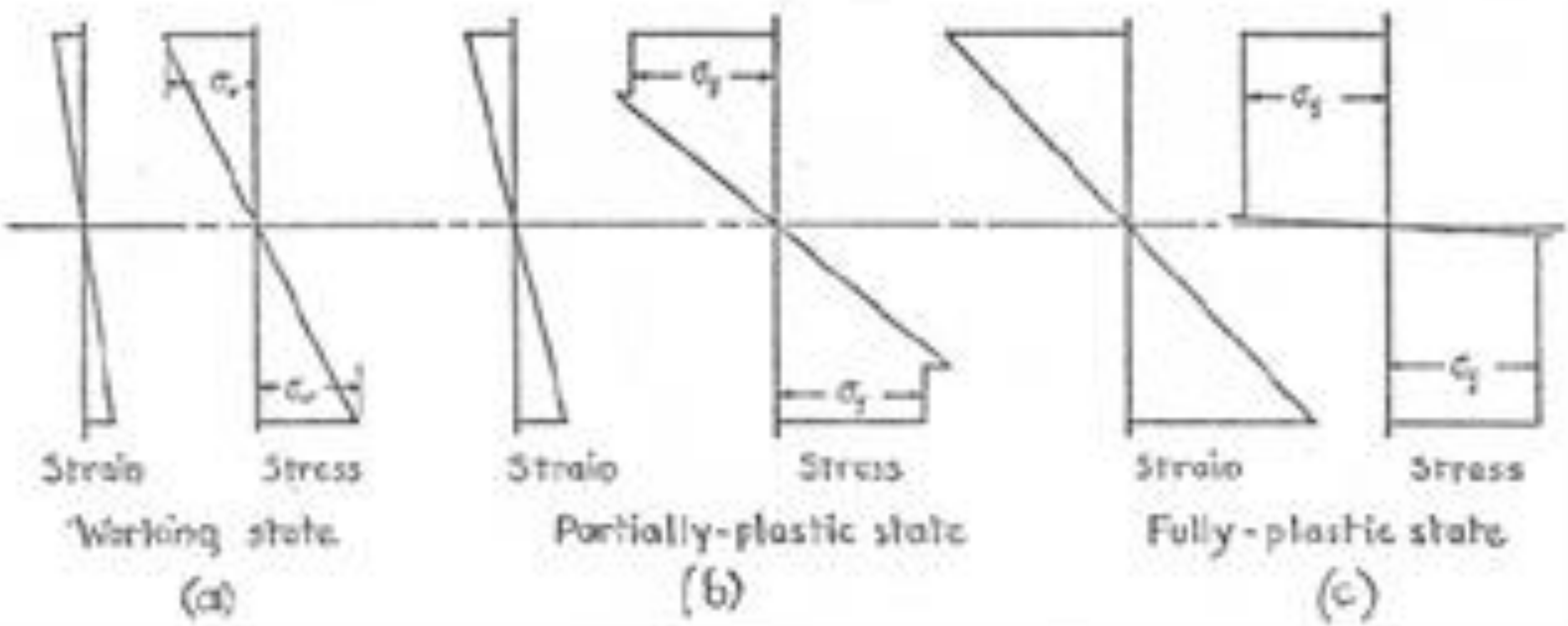
Assumptions in the Plastic Theory

- The yield stress is the same in tension and compression.

- Transverse cross-sections remain plane,
  - so that strain is proportional to the distance from the neutral axis,
  - though in the plastic region stress will be constant, and not proportional to strain.

- When a plastic hinge has developed at any cross-section the moment of resistance at that point
  - remains constant until collapse of the whole structure
  - takes place due to the formation of the required number of further plastic hinges at other points.

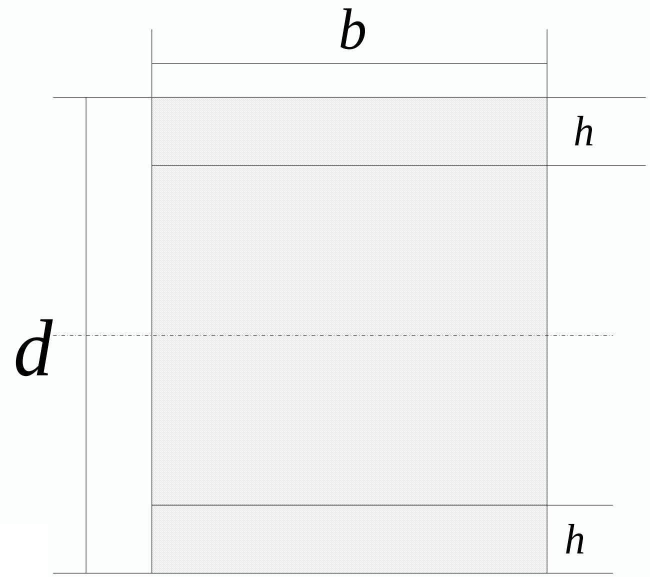
Moment of Resistance at the Plastic Hinges



Case 1: Rectangular Section

The total moment of resistance (Fig b)

$$M = M_{elastic} + M_{plastic} = \tfrac{1}{6}\sigma_y b(d-2h)^2 + \sigma_y bh(d-h)$$



In an elastic range, h = 0, (Fig a)

$$M_y = \sigma_y\left(\frac{I}{y}\right) = \sigma_y\left(\frac{bd^2}{6}\right)$$

For fully plastic, h = d/2, (Fig c)

$$M_p = \frac{3}{2}\left(\frac{\sigma_y bd^2}{6}\right) = \tfrac{1}{4}\sigma_y bd^2 = 1.5M_y$$

$$s = \frac{M_p}{M_y} = 1.5$$

The ratio $M_p \backslash M_y$ is called the *shape factor* , S

Example 3-1:

- *A steel bar of rectangular section 72 mm by 30 mm is used as a simply supported beam on a span of 1.2 m and loaded at mid-span. If the yield stress is 280 N/mm² and the long edges of the section are vertical, find the load when yielding first occurs.*
- *Assuming that a further increase in load causes yielding to spread inwards towards the neutral axis, with the stress in the yielded part remaining at 280 N/mm², find the load required to cause yielding for a depth of 12 mm at the top and bottom of the section at mid-span, and find the length of beam over which yielding has occurred.*

Solution

If W_y is the load at first yield, then:

$$M_y = \left(\frac{bd^2}{6}\right)\sigma_y = \frac{(30)(72)^2(280)}{6} = 7257600\,Nmm..(i)$$

$$M_y = 300W...(ii)$$ *From (i) and (ii)* $$W_y = 24.2kN$$

The outer 12 mm on each side of the neutral axis being under constant stress of 280 N/mm² with no drop of stress at yield.

The moment of resistance

$$M = \tfrac{1}{6}\sigma_y b(d-2h)^2 + \sigma_y bh(d-h)$$

$$= \tfrac{1}{6}(280)(30)(48)^2 + (280)(30)(12)(60)$$

$$= 3225600 + 6048000 = 9273600$$

$$M = 300W = 9273600 \Rightarrow W = 31kN$$

First yield $W_y(0.3) = 0.3(24.2) = 7.26kN$

The length of the beam

Distance *x* from either end under the central load *W*

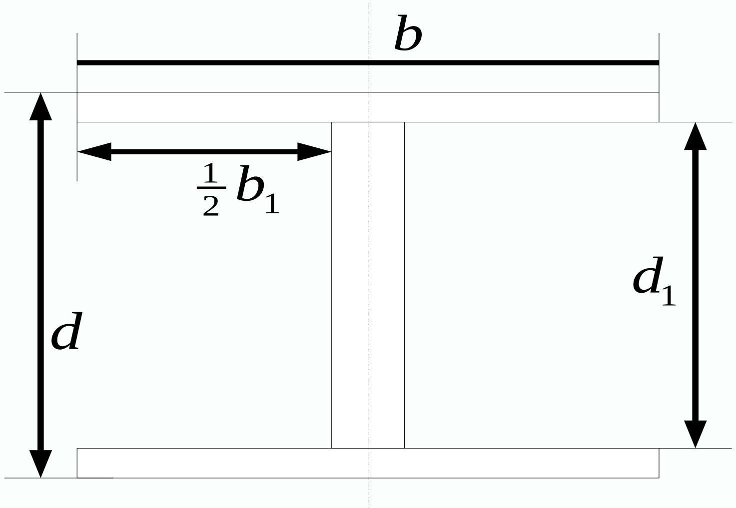
$$1.2 - 2x = 0.264m$$

$$\tfrac{1}{2}Wx = 7.26 \Rightarrow x = 0.468m$$

Case 2: I-Section

In an elastic range, (Fig a)

$$M_y = \sigma_y \left(\frac{bd^3}{12} - \frac{b_1 d_1^3}{12} \right) \left(\frac{2}{d} \right)$$



For fully plastic, (Fig c)

$$M_p = \sigma_y \left(\frac{bd^2}{4} - \frac{b_1 d_1^2}{4} \right)$$

The ratio $M_p \backslash M_y$ is called the *shape factor* , S

$$S = \frac{M_p}{M_y} = 1.5d \left(\frac{bd^2 - b_1 d_1^2}{bd^3 - b_1 d_1^3} \right)$$

- The shape factor will vary slightly with the proportions of flange to web, an average value being about 1.15, as illustrated by the example below.

Example 3-2: A 300 mm by 125 mm I-beam has flanges 13 mm thick and web 8.5 mm thick. Calculate the shape factor and the moment of resistance in the fully plastic state. Take σ_y =250 N/mm^2 and I_x =85 x 10^6 mm^4

- ***Solution***
- At fist yield,

$$M_y = (I/y)\sigma_y = (85.10^6/150)(250) = 141x10^6\ Nmm = 141kNm$$

- In the fully plastic state the stress is equal to 250 N/mm^2 everywhere being tensile on one side and compressive on the other side of the neutral axis.
- By moment of the stress x area products, dividing the web into two parts

$$M_p = \sigma_y\left(\frac{bd^2}{4} - \frac{b_1 d_1^2}{4}\right) = \frac{250}{4}\left[(125)(300)^2 - (116.5)(274)^2\right] = 156kNm$$

- The shape factor

$$S = \frac{M_p}{M_y} = \frac{156}{141} = 1.11$$

Case 3: General

- If A is the total area of cross-section, then it is clear that for pure bending in the fully plastic state the "neutral axis" must divide the area into equal halves.

- If the centroids of these halves are G_1 and G_2 (Figure) at a distance $y_1 + y_2$ apart, then

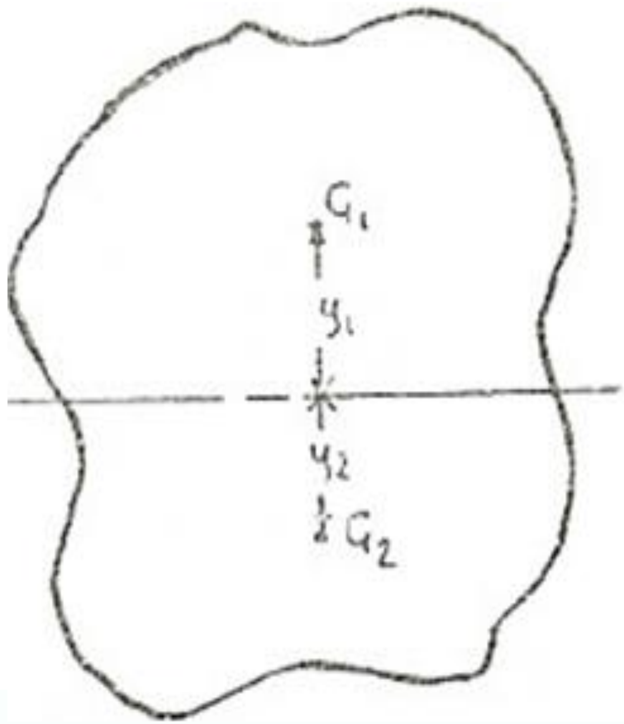
- But a $M_p = \left(\frac{1}{2}\sigma_y A\right)\left(y_1 + y_2\right)$

- where Z is the section modulus.

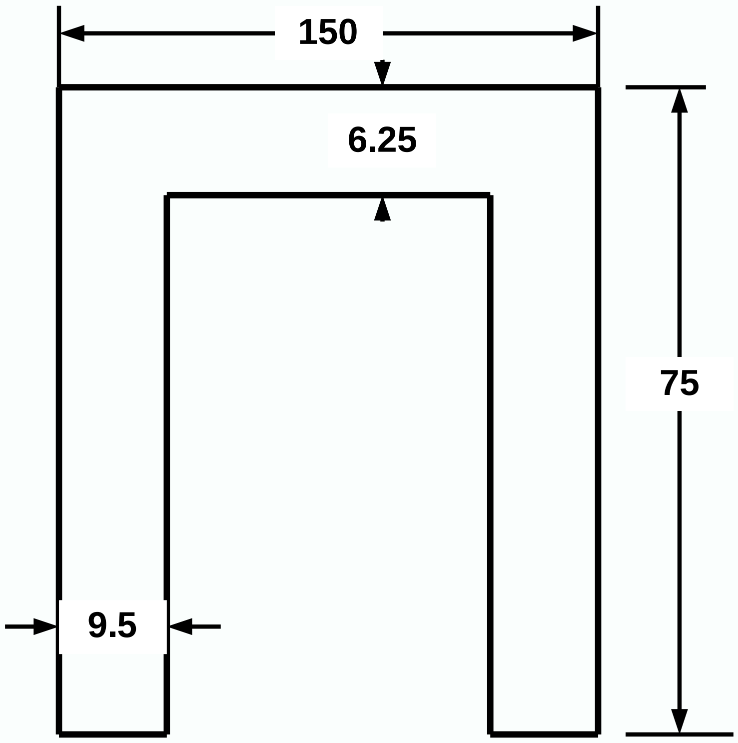
- Hence

$$M_y = Z\sigma_y$$

$$S = \frac{M_p}{M_y} = \frac{A\left(y_1 + y_2\right)}{2Z}$$



- *Example 3-3: Find the shape factor for a 150 mm by 75 mm channel in puree bending with the plane of bending perpendicular to the web of the channel. The dimensions are shown in Fig. 35, Z =12,000 mm³.*



Solution

Area of the channel

$$A_2 = A_3 = 9.5(75 - 6.25) = 653\, mm^2$$

$$A_1 = 150(6.25) = 937.5\ mm^2$$

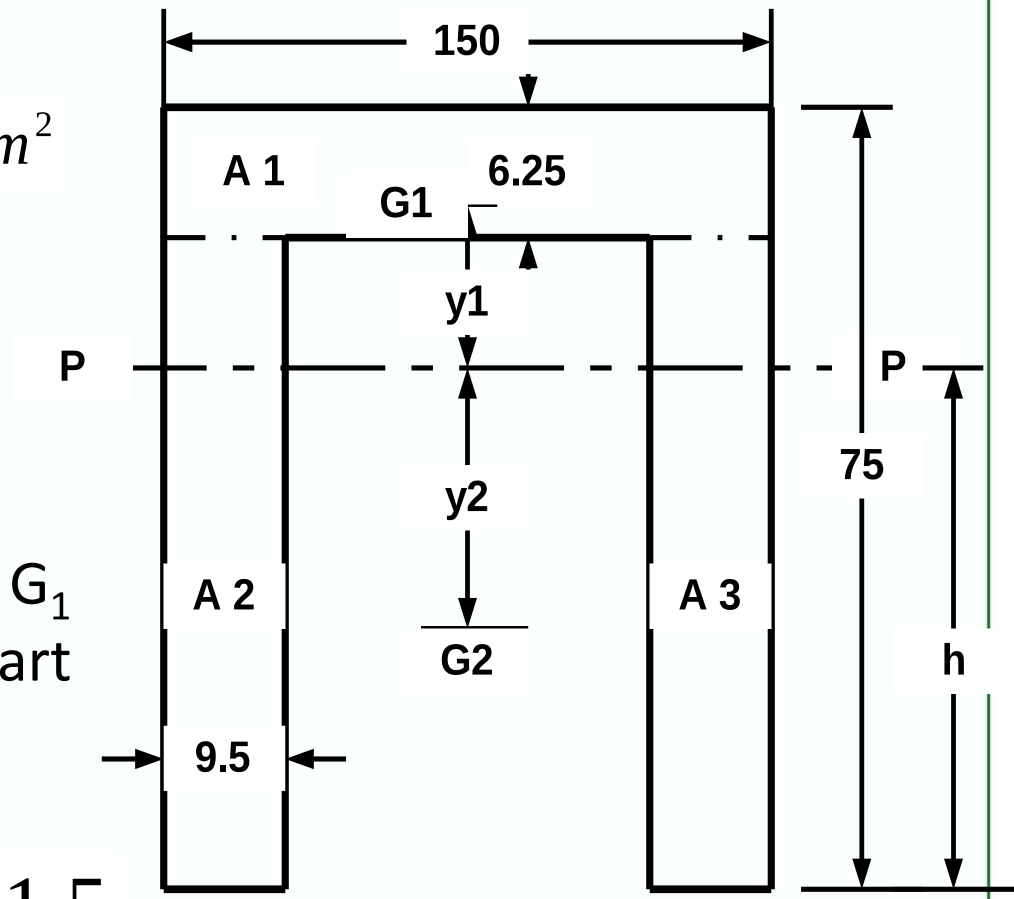
$$A_1 + 2A_2 = 9.5(75 - 6.25) = 937.5 + 2(653) = 2243\, mm^2$$

Neutral axis that divides the area into equal halves

$$A_1 < \tfrac{1}{2}A$$

Since, $\tfrac{1}{2}A = \tfrac{1}{2}(2243) = 1121.5\ mm^2$

Then, the "neutral axis will pass through A_2 and A_3 labeled PP as shown in the Fig



The centroids of these halves as G_1 and G_2 at a distance $y_1 + y_2$ apart are also indicated

Therefore,

$$2(9.5)h = 1121.5$$

$$\Rightarrow h = 59mm$$

The values of y_1 and y_2

$$y_1 = h/2 = 29.5mm$$

$$y_2 == \frac{150(6.25)(16-3.125)+(2)(9.5)(9.75)(4.875)}{150(6.25)+2(9.5)(9.75)} = 11.8mm$$

The shape factor

$$S = \frac{A(y_1 + y_2)}{2Z} = \frac{2243(41.76)}{2(21000)} = 2.23$$

TORSION UNDER PLASTIC CONDITION

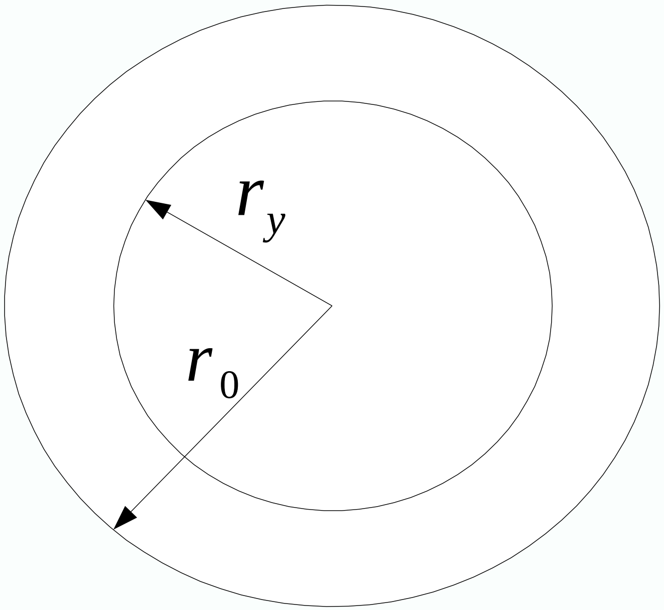
- In this session, we shall study the Torque in both the elastic and plastic zone;

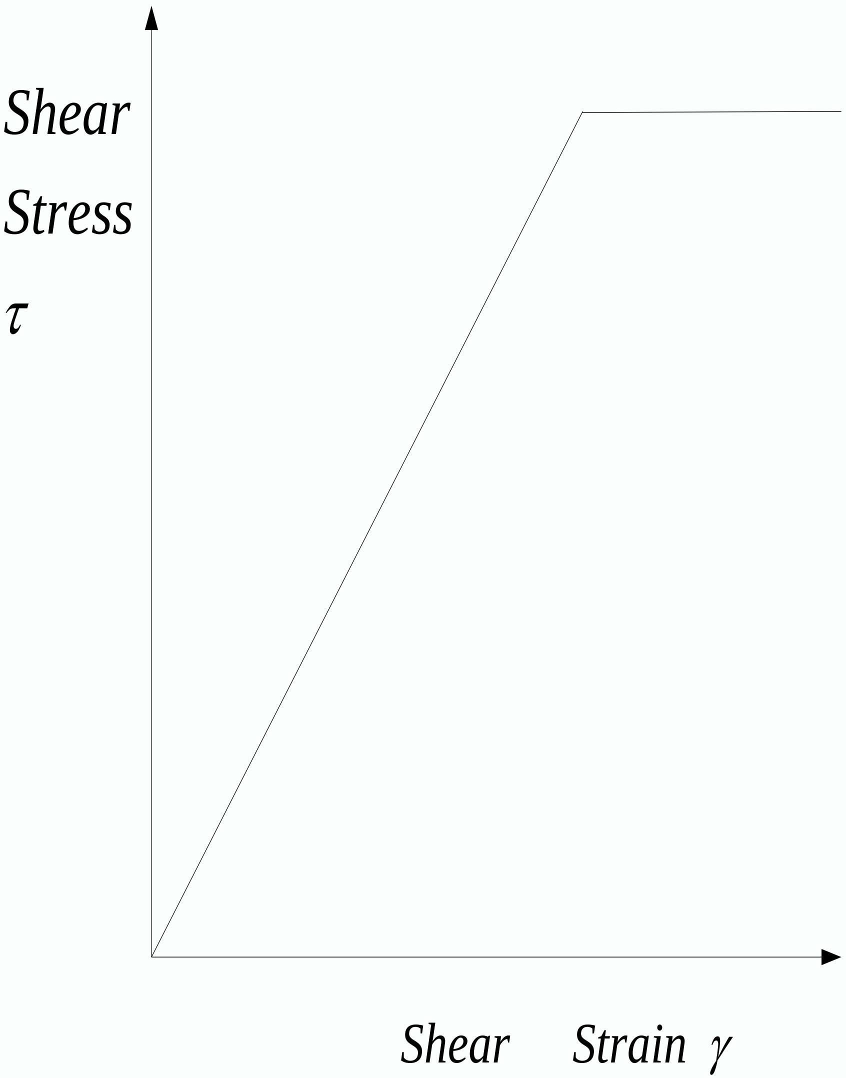
- Assumptions in the plastic theory and

- Moment of Resistance at a plastic hinge

Assumptions in the Plastic Theory

- A plane cross section of the shaft remains plane when in the plastic state.
- Radius remains straight





Torque of Resistance at the Plastic Hinges

The total torque of resistance

$$T_1 = \frac{\tau_y \pi r_y^3}{2}$$

$$T_2 = \int_{r_y}^{r_0} 2\pi r \tau_y . r dr = \frac{2\pi}{3}\tau_y\left(r_0^3 - r_y^3\right)$$

$$T = T_1 + T_2 = \frac{2}{3}\pi\tau_y r_0^3\left(1 - \frac{r_y^3}{4r_0^3}\right)$$

In an elastic range, $r_0 = r_y$,

$$T_y = \frac{\tau_y \pi r_y^3}{2}$$

For fully plastic, $r_y = 0$

$$T_p = \frac{2}{3}\pi\tau_y r_0^3$$

When the fibres at the outer surface of the shaft are about to become plastic, the angle of twist of the shaft is given by

$$\theta_y = \frac{\tau_y L}{G r_0}$$

L the length of the shaft
G Modulus of Rigidity

For elastoplastic condition

$$\theta = \frac{\tau_y L}{G r_y}$$

$$\frac{\theta_y}{\theta} = \frac{r_y}{r_0}$$

$$T = \frac{2}{3}\pi\tau_y r_0^3\left(1 - \tfrac{1}{4}\left(\frac{\theta_y}{\theta}\right)^3\right)$$

The *shape factor* , S

$$s = \tau_p / \tau_y = 4/3$$

- *Example 3-4: A mild steel shaft 40mm in diameter and 250mm in length is subjected to an overload torque of 1800Nm which caused shear yielding of the shaft, 120 MPa. Determine the radial depth to which plasticity has penetrated and the angle of twist. Take G = 80 GPa*

Solution

The Torque

$$T = \frac{2}{3}\pi\tau_y r_0^3\left(1-\tfrac{1}{4}\left(\frac{r_y}{r_0}\right)^3\right) \quad 1800 = \frac{2}{3}\pi\left(120.10^6\right)(0.02)^3\left[1-\frac{r_y^3}{4(0.02)^3}\right]$$

From which

$$r_y = 15mm$$

Hence depth of plastic deformation is 5mm

The shear strain at r_y==15mm is

$$\gamma = \frac{r_y\theta}{L} = \frac{\tau_y}{G} = \frac{120\left(10^6\right)}{80\left(10^9\right)} = 1.5x10^{-3}$$

$$\Rightarrow \theta = \frac{\gamma L}{r_y} = \frac{(0.0015)(0.25)}{0.015} = 0.025rad = 1.43^0$$

Quiz 3

- State the assumptions used to derive the equations used to compute the
  1. Bending moment required to form the plastic hinge
  2. Torque required to form the plastic hinge