DC TRANSIENTS IN R-C CIRCUITS

- Consider a simple series R-C circuit connected through a switch ‘S’ to a constant voltage source V_S as shown in Fig. 3.10
- The switch ‘S’ is closed at time ‘t=0’.
- Let V_C (t) be the capacitor voltage and i(t) be the current flowing through the circuit at any instant of time ‘t’ after closing the switch

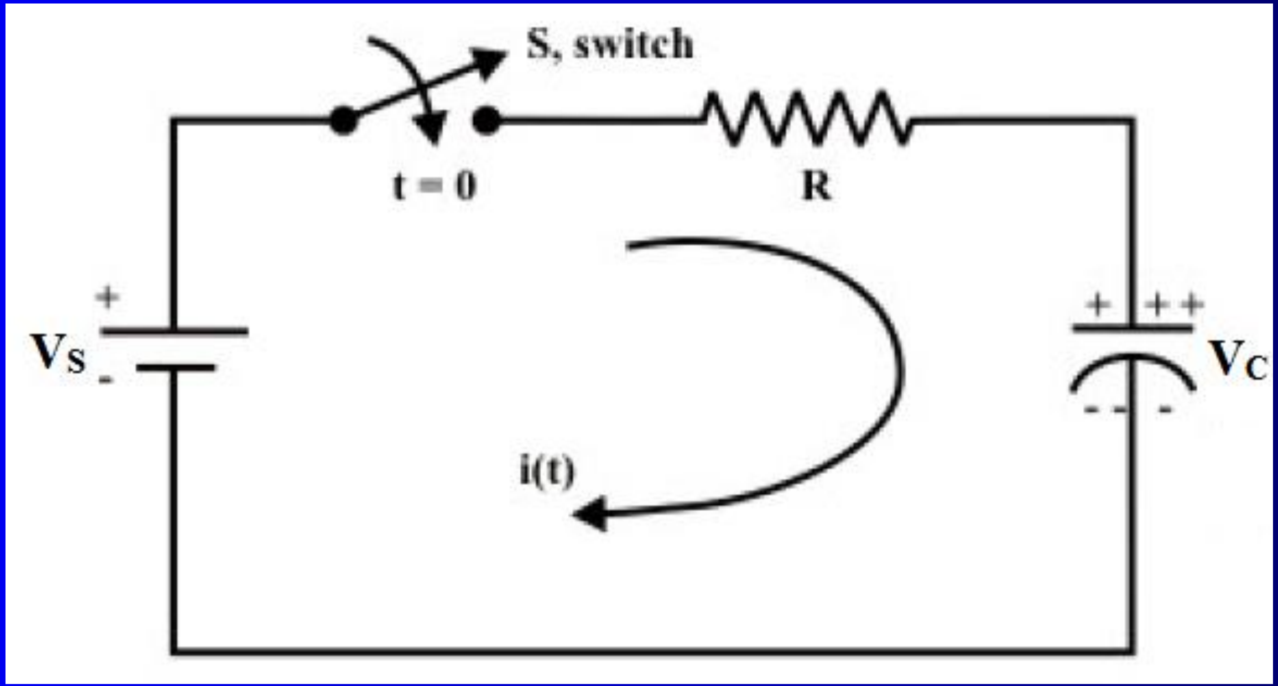

Fig. 3.10: Charging of a RC Circuit

- The KVL equation around the loop can be written as

$$V_S = Ri(t) + v_c(t) \quad \Rightarrow \quad V_S = RC\frac{dv_c(t)}{dt} + v_c(t) \qquad 3.23$$

- The solution of the above first-order differential equation due to forcing function V_S is given by
- $V_c(t) = V_{cn}(t) + V_{cf}(t) = A_1 e^{\alpha t} + A$ 3.24
- The equation (3.23) simplifies into the differential equation,

$$\frac{dv_c(t)}{dt} + \frac{v_c(t)}{RC} = \frac{V_S}{RC} \qquad 3.25$$

- Move the second term to the right hand side and then divide by the numerator

$$\frac{dv_c(t)}{dt}\frac{1}{v_c(t) - V_S}dt = -\frac{1}{RC}dt \qquad 3.26$$

- The indefinite integral resolves to the following form

$$In(v_c(t) - V_S) = -\frac{t}{RC} + D \qquad 3.27$$

D is a constant of integration. Removing the natural log and solving for $v_c(t)$

$$v_c(t) = V_S + e^D e^{-\left(\frac{t}{RC}\right)} \qquad 3.28$$

- The constant e^D, represented by A, can be found at time t=0.

$$\boldsymbol{e^D = A = v_c(0) - V_S} \qquad 3.29$$

- Assuming the voltage across the capacitor before closing the circuit is zero, it means $v_c(0) = 0$
- Hence, $A = -V_S$
- Equation (3.28) becomes

$$\boldsymbol{v_c(t) = V_S\left(1 - e^{-\left(\frac{t}{\tau}\right)}\right)} \qquad 3.30$$

Where the time constant, $\tau = RC$

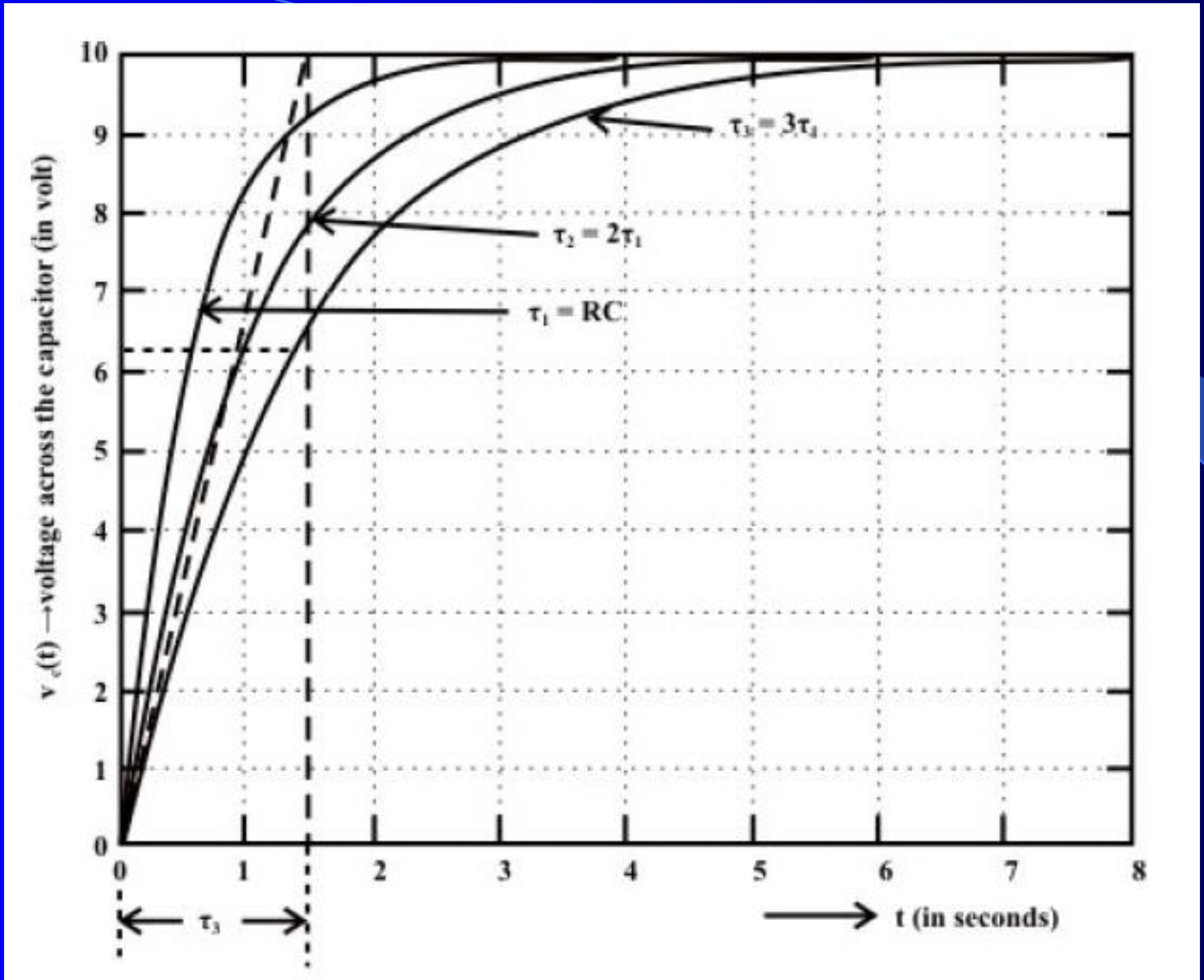

Fig. 3.11: Growth of capacitor voltage (assumed initial capacitor voltage is zero)

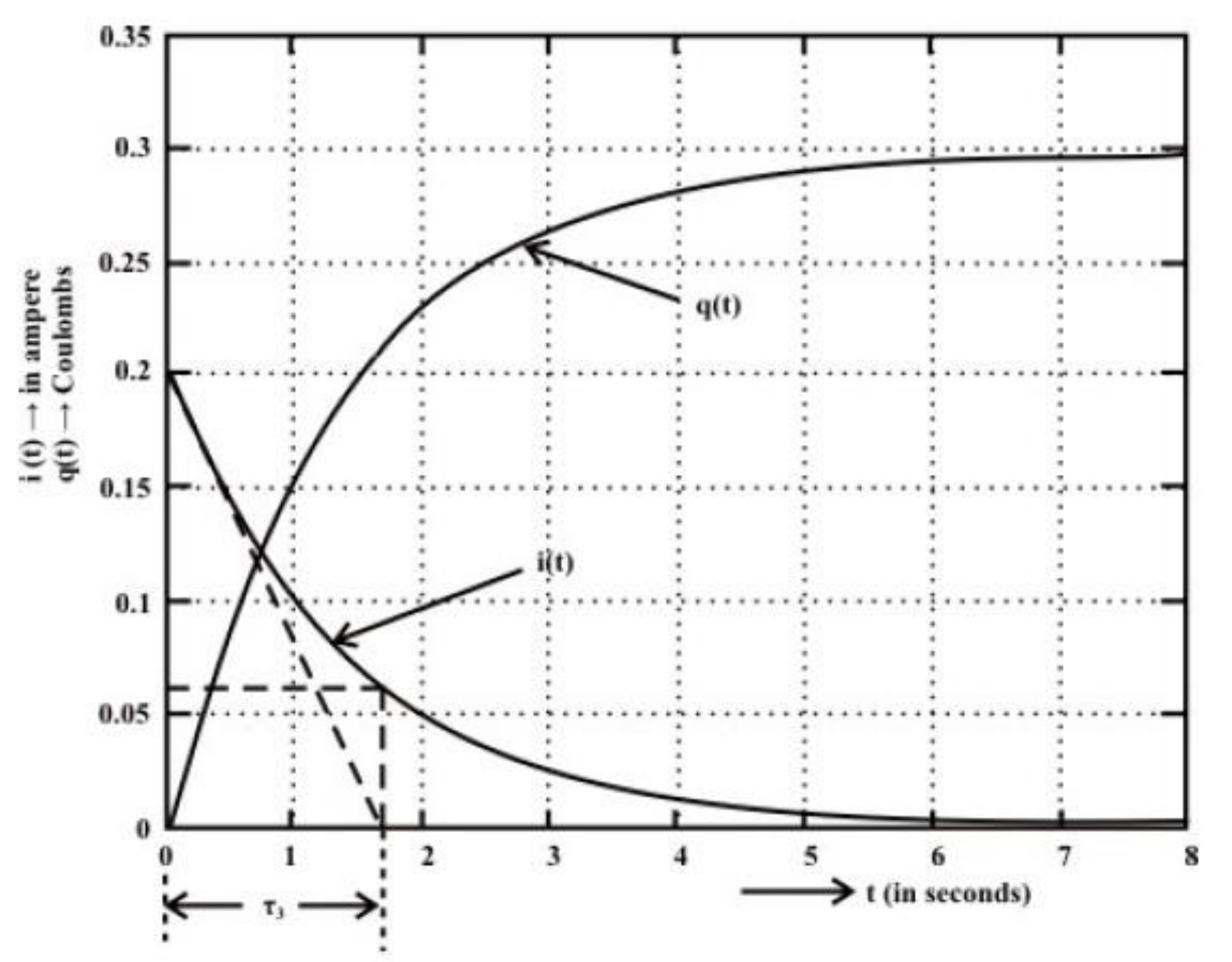

Fig. 3.12: System response due to the forcing function V_S (assumed capacitor initial voltage V_0=0

- Voltage across the resistance of Fig. 3.10 is

$$V_R(t) = V_S - V_C(t) = V_S e^{-\frac{t}{\tau}} \qquad 3.31$$

- Charging current through the capacitor

$$i(t) = \frac{v_R}{R} = \frac{V_S}{R} e^{-\frac{t}{\tau}} \qquad 3.32$$

- **Now if the capacitor has some initial charge and hence initial voltage, the capacitor voltage at the time of switching, $V_C(0)$ will be finite and not zero**
- **i.e. $V_C(0) \neq 0$**
- **Assume $V_C(0) = V_0$**
- The expression of the voltage across the capacitor becomes

$$V_c(t) = V_S\left(1 - e^{-\frac{t}{\tau}}\right) + V_0 e^{-\frac{t}{\tau}} \qquad 3.33$$

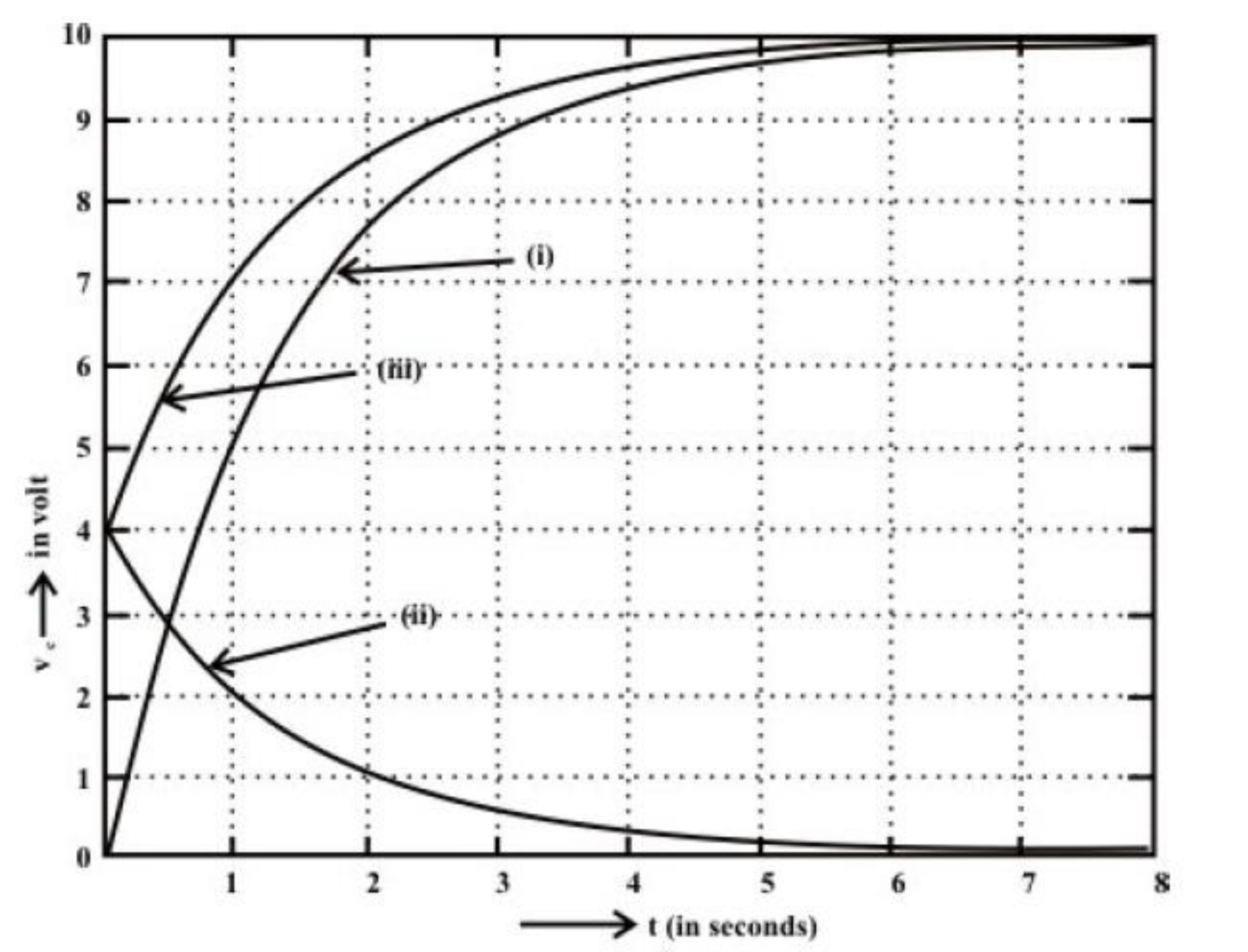

Fig. 3.13: Voltage across the capacitor due to (i) the forcing function V_S acting alone (ii) discharge of capacitor initial voltage V_0 (iii) Combined effect of (i) and (ii)

Discharging of a Capacitor or Fall of a Capacitor Voltage in DC Circuits

- Fig. 3.14 shows that the switch 'S' is closed at position '1' for sufficiently long time and the circuit has reached its steady-state condition.
- At 't=0' the switch 'S' is opened and kept in position '2' and remains there.
- Our job is to find the expressions for (i) voltage across the capacitor V_C
  (ii) voltage across the resistance V_R
  (iii) current through the capacitor (discharging current)

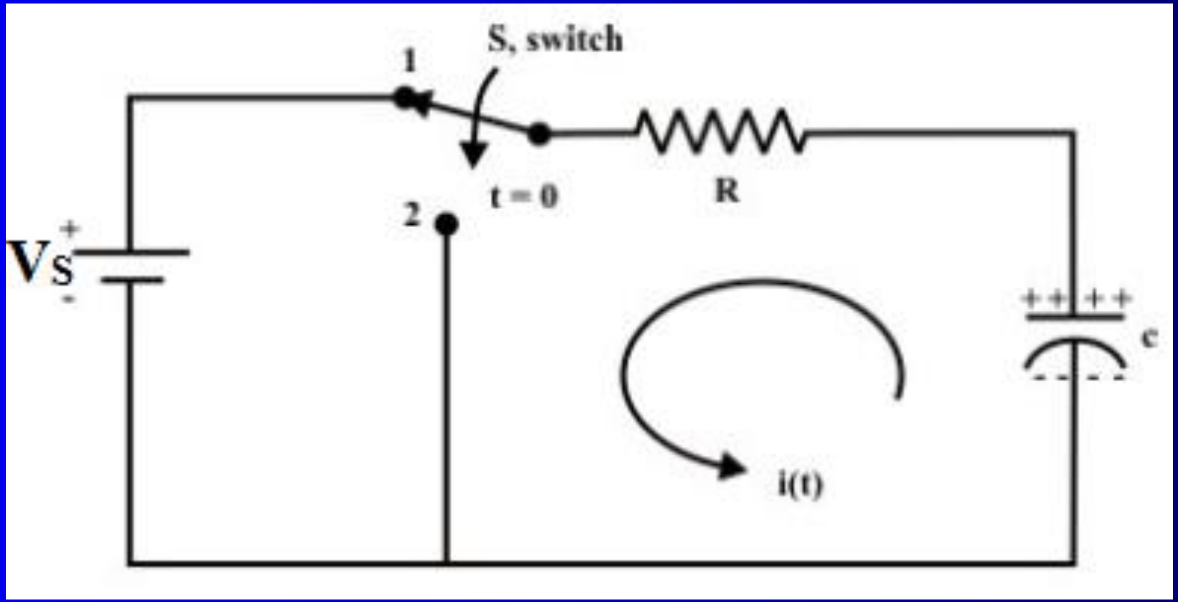

Fig. 3.14: Discharging of Capacitor voltage Circuit

Discharging of a Capacitor or Fall of a Capacitor Voltage in DC Circuits

- For $t < 0$, the switch ‘S’ in position 1.
- The capacitor acts like an open circuit to dc, but the voltage across the capacitor is same as the supply voltage V_S.
- Since, the capacitor voltage cannot change instantaneously, this implies that $\boldsymbol{V_C(0^-) = V_C(0^+) = V_S}$
- When the switch is closed in position ‘2’, the current will flow through the circuit until capacitor is completely discharged through the resistance R
- In other words, the discharging cycle will start at $t = 0$.
- Now applying KVL around the loop, we get

$$\boldsymbol{RC\frac{dV_C(t)}{dt} + V_C(t) = 0} \qquad 3.34$$

- The solution of input free differential equation (3.34) is given by

$$\boldsymbol{V_C(t) = Ae^{\alpha t}} \qquad 3.35$$

- Where $\alpha = -\frac{1}{RC}$
- The constant $\boldsymbol{A}$ is obtained using the initial condition of the circuit in equation (3.35)

- Note, at ‘$t = 0$’ (when the switch is just closed in position ‘2’) the voltage across the capacitor $V_c(0) = V_S$
- Using this condition in equation 3.35, we get

$$V_C(0) = V_S = A \qquad \Rightarrow \qquad A = V_S$$

- Hence the discharging voltage across the capacitor is

$$V_C(t) = V_S e^{-\frac{t}{RC}} \tag{3.36}$$

- Voltage across the resistance is

$$V_R(t) = -V_C(t) = -V_S e^{-\frac{t}{RC}} \tag{3.37}$$

Discharging current through the capacitor is

$$i(t) = \frac{V_R}{R} = -\frac{V_S}{R} e^{-\frac{t}{RC}} \tag{3.38}$$

- An inspection of the above exponential terms of equations from (3.36) to (3.38) reveals that the time constant of circuit is given by

$$\tau = RC\ (sec)$$

- This means that at time $t = \tau$, the capacitor's voltage drops to 36.8% of its initial value (see fig. 3.15(a)).
- For all practical purposes, the dc transient is considered to end after a time span of 5τ.
- At such time steady state condition is said to be reached.
- Plots of above equations as a function of time are depicted in fig. 3.15(a) and fig. 3.15(b) respectively.

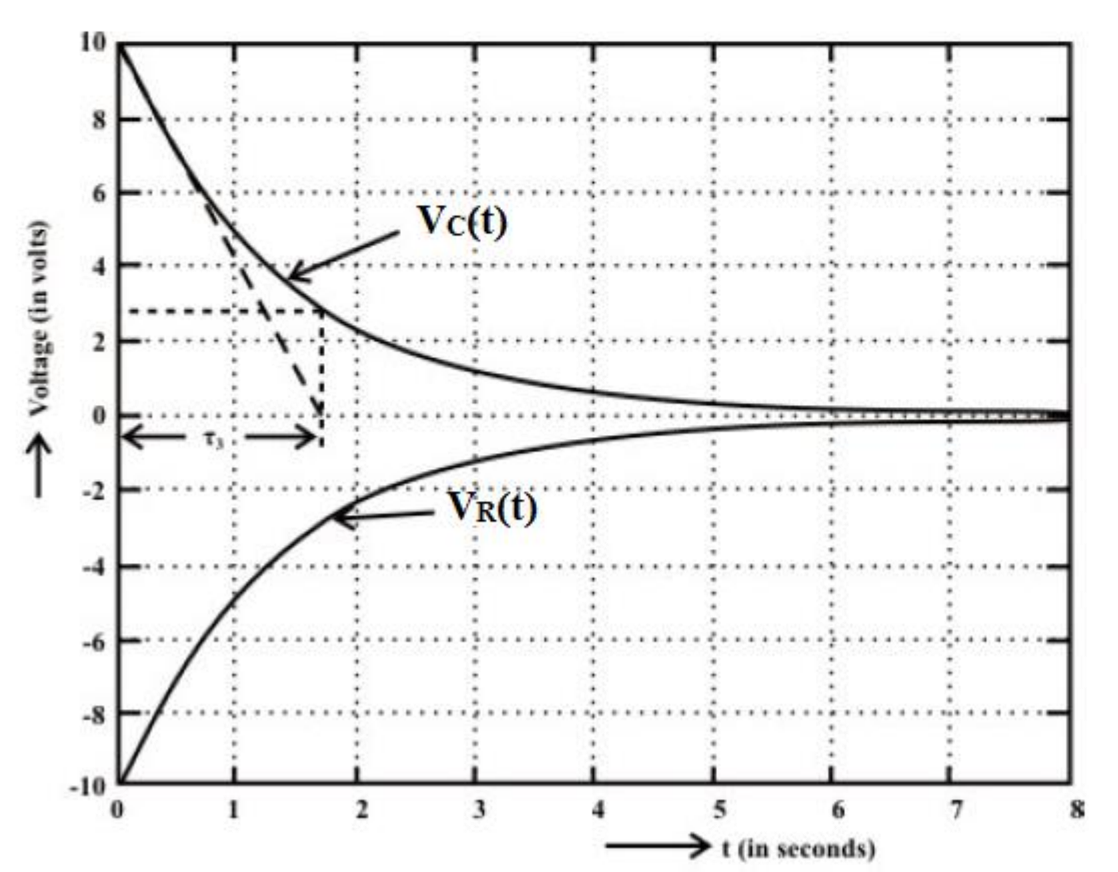

Fig. 3.15(a): Discharge of capacitor voltage with time

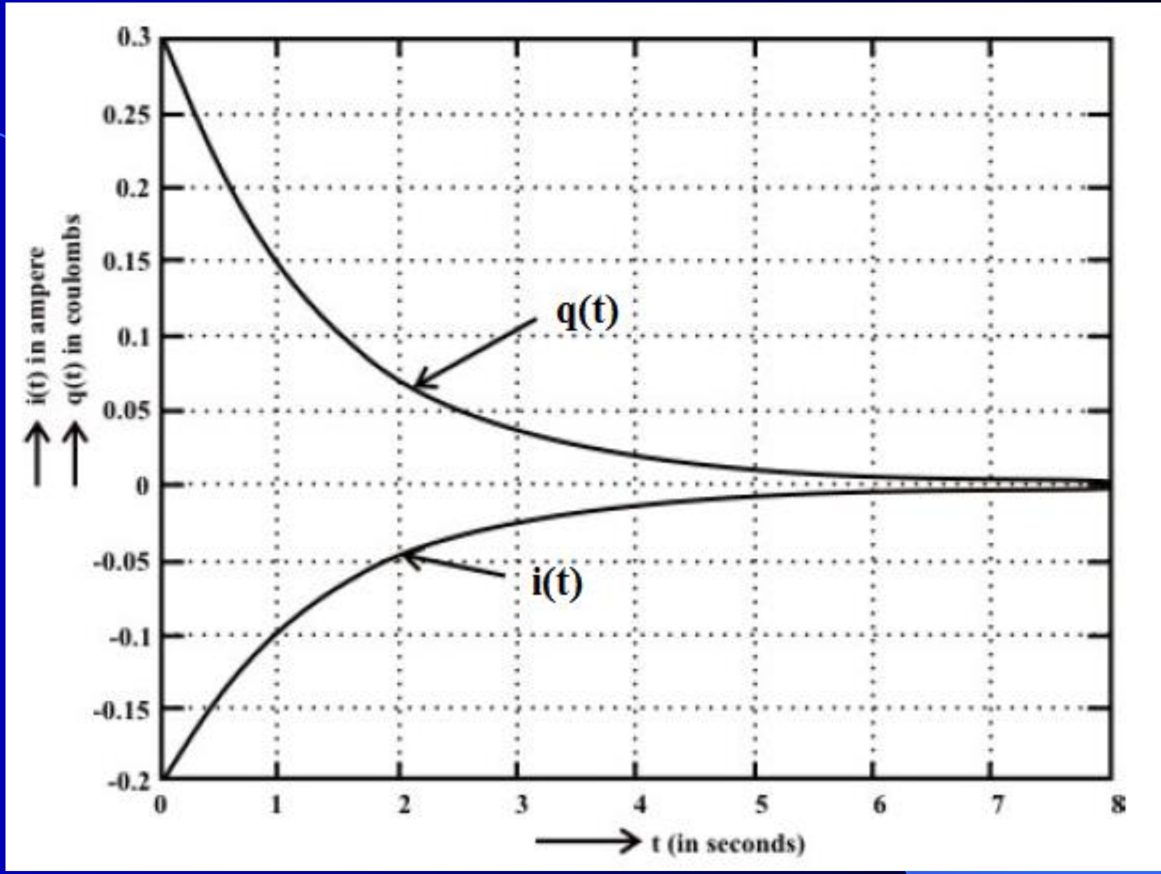

Fig. 3.15(b): System response due to capacitor discharge

- **Example 3.3:**
- The switch ‘S’ shown in Fig. 3.16 is kept open for a long time and then it is closed at time ‘t=0’. Find (i) $V_C(0^-)$ (ii) $V_C(0^+)$ (iii) $i_C(0^-)$ (iv) $i_c(0^+)$ (vi) find the time constants of the circuit before and after the switch is closed

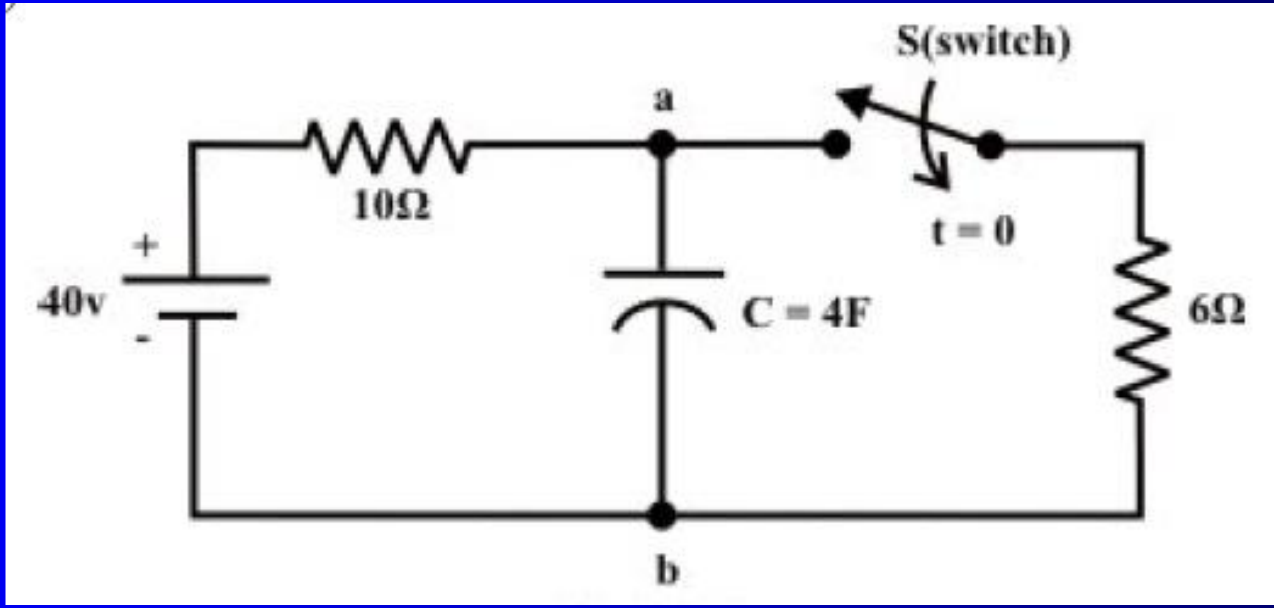

Fig. 3.16: See Example 3.3

- **Solution:**
- (i) $V_C(0^-) = 40$ V
- (ii) $V_C(0^+) = 40V$

  As we know the voltage across the capacitor cannot change instantaneously. Therefore, the voltage across the capacitor just before the switch is closed is same as voltage across the capacitor just after the switch is closed (note the terminal 'a' is positively charged.
- (iii) $i_C(0^-) = 0$.
- (iv) $i_C(0^+) = 0$
- (vi) Time constant of the circuit before the switch was closed = τ = RC = 10 x 4 = 40 sec

  Time constant of the circuit after the switch is closed $\tau = R_{th}C = \frac{10\ x\ 6}{10+6} x\ 4 = 15\ sec$. (replace the part of the circuit than contains only independent sources and resistive elements by an equivalent Thevenin's circuit.

- Example 3.4:
- The circuit shown in Fig. 3.16 is switched on at time t=0.
- (i) How long does it take for the capacitor to attain 70 % of its final voltage? Assume the capacitor is not charged initially
- (ii) Find also the time constant (τ) of the circuit after the switch is closed

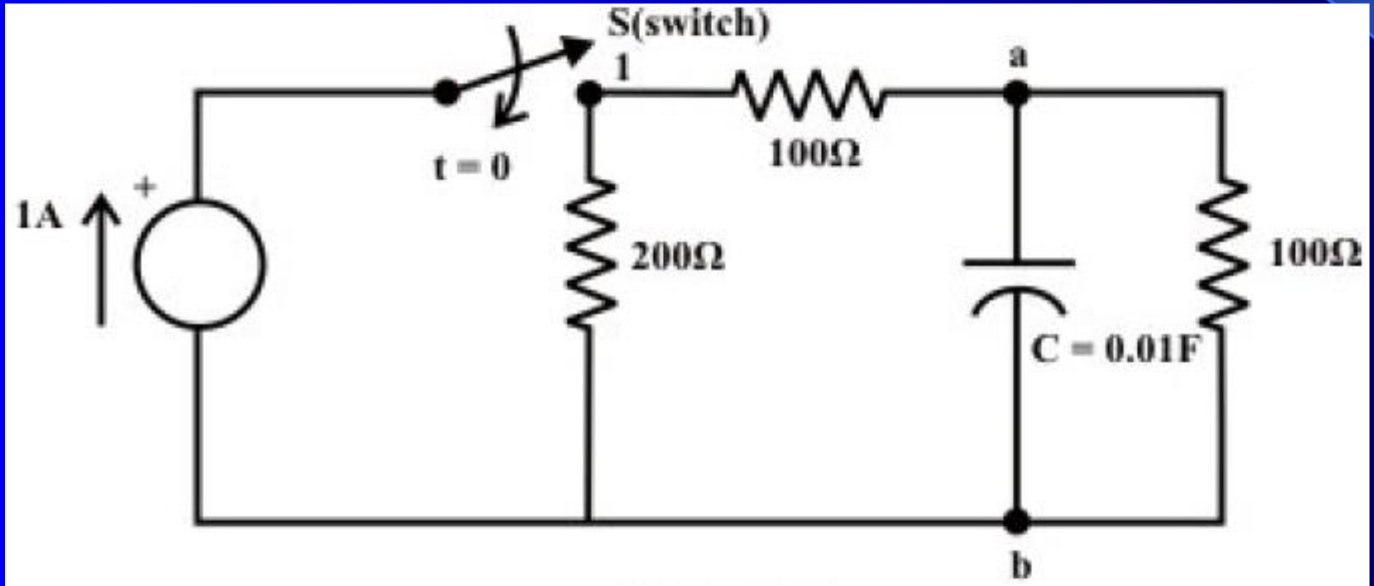

Fig. 3.16: See Example 3.4

- Solution:
- The circuit containing only resistive elements and independent current source (i.e., non-transient part of the circuit) is converted to an equivalent voltage source which is shown in fig.3.17(a).

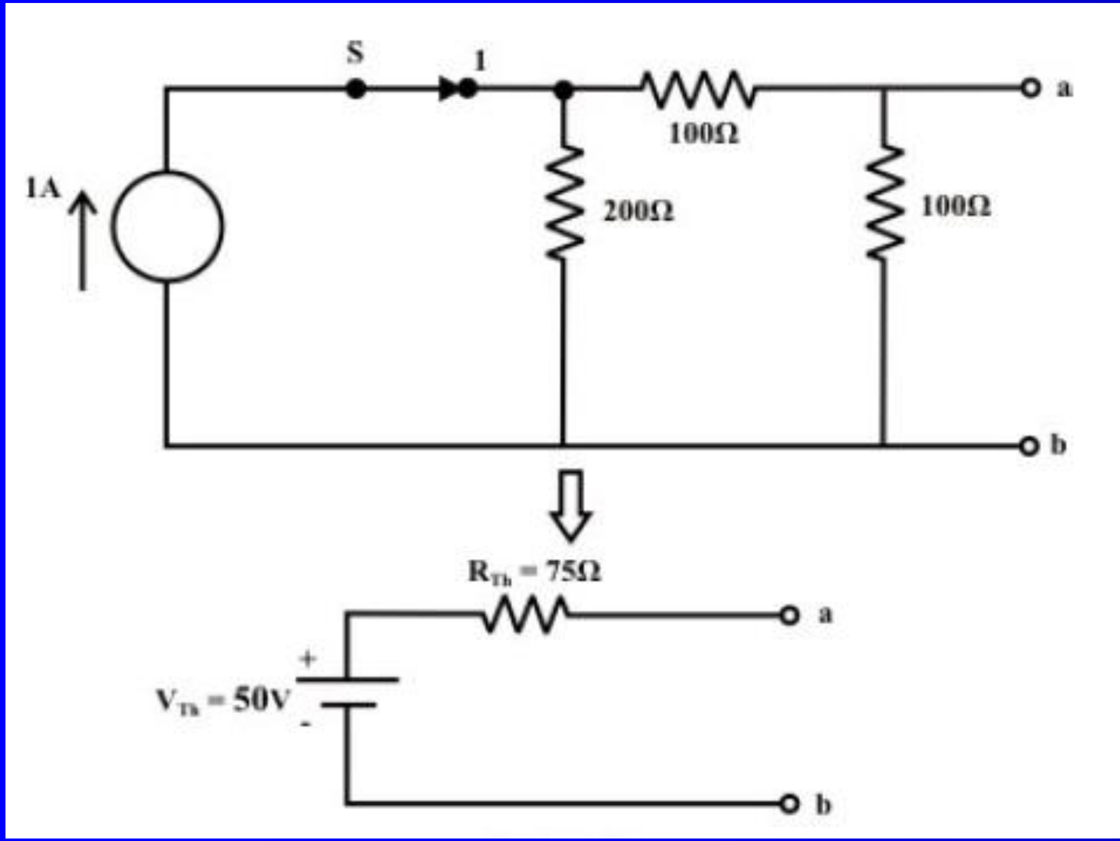

Fig. 3.17(a):

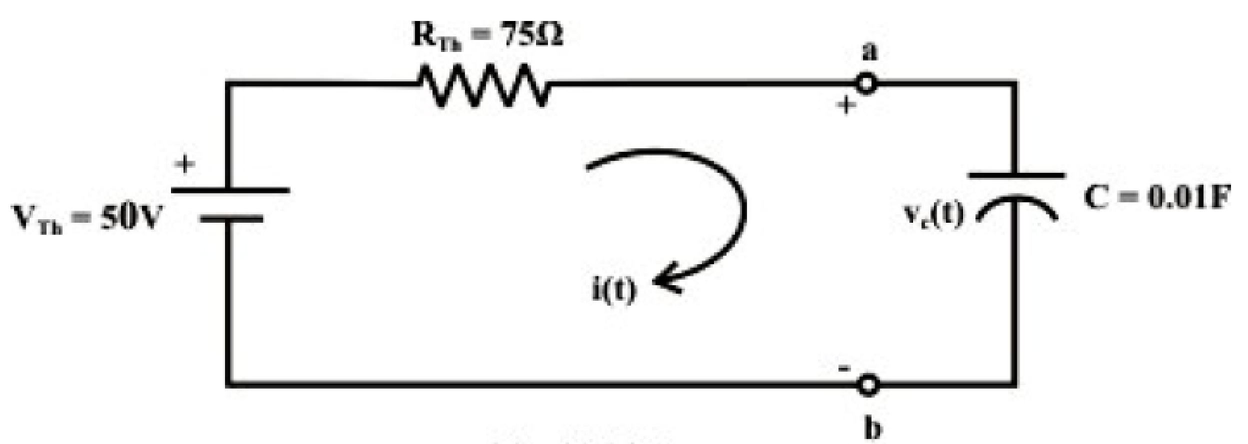

Fig. 3.17(b): Thevenin Equivalent Circuit

The parameters of Thevenin's equivalent circuit are:

$$V_{Th} = \frac{200}{200+100+100} \text{ x1 x 100 = 50V}$$

$$R_{Th} = \frac{100\ x300}{100+300} = 75\ \Omega$$

- The time constant of the circuit is

$$\tau = RC = 75\ x\ 0.01 = 0.75\ sec$$

- The capacitor voltage expression for the circuit is

$$\boldsymbol{v_c(t) = V_S\left(1 - e^{-\left(\frac{t}{\tau}\right)}\right) = 50(1 - e^{-\frac{t}{0.75}})}$$

- Let 't' be the time required for the capacitor voltage to reach 70% of its final voltage
- Hence

$$50\ x\ 0.7 = 35 = 50(1 - e^{-1.33t}) \Rightarrow t = 0.91\ sec$$

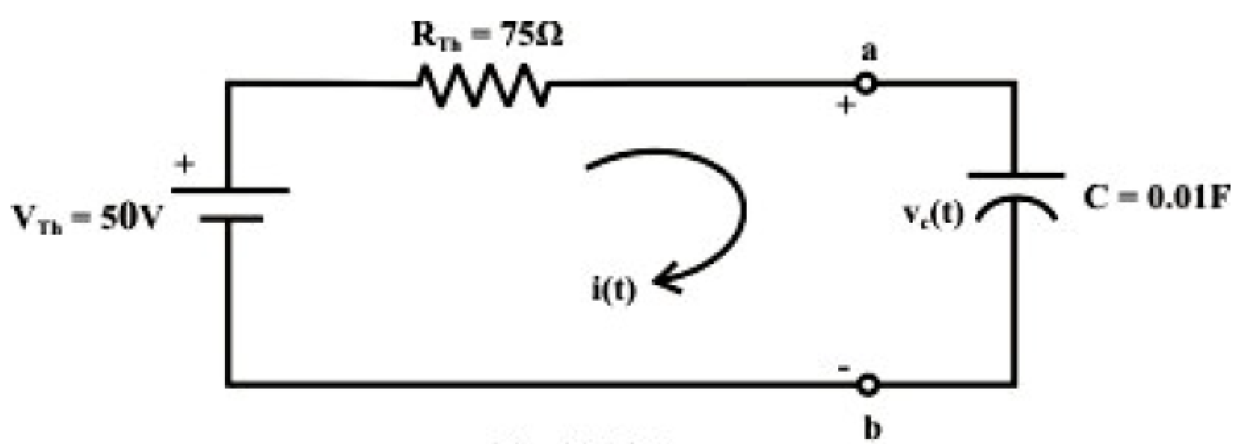

Fig. 3.17(b): Thevenin Equivalent Circuit