Nuclear Physics

Basics

Elements are defined by the number of protons in the nuclei of their individual atoms. For example, oxygen atoms always have 8 protons, iron has 26, and so forth.
But more than protons make up the nuclei of atoms. Uncharged particles called neutrons account for more than half of the mass of atoms.
For each element there are different isotopes, atoms with the same number of protons but different numbers of neutrons. For example, carbon atoms always have six protons, but may have six, seven, or eight neutrons. Some isotopes are stable, but others are radioactive.

An isotope will be radioactive if its nuclei are unstable. Large atomic nuclei, with more than 83 protons and their associated complement of neutrons, are inherently unstable. Uranium and plutonium are examples of such elements. Small atomic nuclei may also be radioactive if the ratio of neutrons to protons exceeds certain limits. Even tiny hydrogen, the smallest of atoms, has a radioactive isotope.

Atoms found in nature are either stable or unstable. An atom is stable if the forces among the particles that make up the nucleus are balanced. An atom is unstable (radioactive) if these forces are unbalanced if the nucleus has an excess of internal energy. Unstable atoms are called *radionuclides*. The instability of a radionuclide's nucleus may result from an excess of either neutrons or protons. An unstable nucleus will continually vibrate and contort and, sooner or later, attempt to reach stability by some combination of means:

- ejecting neutrons, and protons
- converting one to the other with the ejection of a beta particle or positron

the release of additional energy by photon (i.e., gamma ray) emission.

When nuclei are unstable, the structure of the nucleus may change with a known probability called is half life. A half life is the period of time it takes half the nuclei present in an isotope to change. They may be very short or very long, depending on the isotope.
When the nuclei change in structure, energy and/or subatomic particles are given off. These occurrences are referred to as radioactive decay.

4.0 Decay of the Unstable Nuclei

4.1 Nuclear Decay

When the unstable nuclear decays, decay products like γ-rays (high energy photons), α-particles (helium nuclei), β^- particles (electrons) and β^+ particles (positrons) are produced. In nuclear decay reactions the following laws are obeyed:

1. Conservation of mass-energy.
2. Conservation of charge
3. Conservation of linear and angular momenta
4. Conservation of nucleons

4.2 Law of radioactive Decay

In a typical radioactive decay an initial nucleus (a parent) decays by emitting a particle forming a new nucleus (a daughter). Generally,

$$N = N_0 e^{-\lambda t} \tag{4.1}$$

where N_0 is the unstable parent nucleus, N is the daughter nucleus, λ is the decay or disintegration constant and t is time.

4.2.1 Half-Life $T_{1/2}$

This is defined as the time interval required for the number of parent nuclei present at the beginning to be reduced by a factor of one-half. Thus, using equation (4.1)

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \tag{4.2}$$

4.2.2 Average of Mean Life time T_m

This is defined as

$$T_m = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} \tag{4.3}$$

Equation (4.3) is derived as follows:

$$T_m = \frac{\int_{N_0}^{0} t dN}{\int_{N_0}^{0} dN} = \frac{1}{-N_0} \int_{N_0}^{0} t dN \tag{4.4}$$

Now taking the first time differential of equation (4.1) gives

$$dN = -\lambda N_0 e^{-\lambda t} dt \tag{4.5}$$

Using (4.5) in (4.4) and changing the limits N_0,0 to ∞,0 in terms of the time variable t gives

$$T_m = \frac{1}{-N_0} \int_0^{\infty} t\left(-\lambda N_0 e^{-\lambda t} dt\right) = \lambda \int_0^{\infty} t e^{-\lambda t} dt = \lambda\left(\frac{1}{\lambda^2}\right) = \frac{1}{\lambda} \tag{4.6}$$

4.2.3 Activity

The activity (i.e. the absolute value of the rate of disintegration) of a nucleus is defined as

$$Activity = \left|\frac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t} = \lambda N \qquad (4.7)$$

Activity is measured in the unit of curie (Ci). 1 Ci = 3.7×10^{10} Bequerel (Bq) or disintegrations per second. 1 Bq = 1 disintegration per second.

Worked Examples

1. What is the activity of 1g of ${}^{226}_{88}Ra$ whose half life is 1622 year?

The number of atoms of 1 g of radium is given by

$$N = (1g)\left(\frac{1g-mole}{226g}\right)\left(6.025\times10^{23}\,\frac{atoms}{g-mole}\right) = 2.666\times10^{21}$$

The decay constant is related to the half-life by

$$\lambda = \left(\frac{0.693}{T_{1/2}}\right) = \left(\frac{0.693}{1622y}\right)\left(\frac{1y}{365d}\right)\left(\frac{1d}{8.64\times10^4\,s}\right) = 1.355\times10^{-11}\,s^{-1}$$

The activity is then found from:

Activity = λN=(1.355×10^{-11} s $^{-1}$)(2.666×10^{21}) = 3.612×10^{10} disintegrations/s

2. Over what distance in free space will the intensity of a 5 eV neutron beam be reduced by a factor of one-half? ($T_{1/2}$ =12.8 min)

The speed of the neutrons in the beam is found from the relation ½mv^2 = K. Hence,

$$\frac{1}{2}\left(1.67\times10^{-27}\,kg\right)v^2 = (5eV)\left(\frac{1.6\times10^{-19}\,J}{1eV}\right), \qquad v = 31.0 \text{ km/s}$$

During a time $T_{1/2}$ = 12.8 min, half the neutrons will have decayed from the beam. The distance travelled by the undecayed neutrons during this time is:

d = vt = (31.0 km/s)(12.8 min)(60s/min) = 23,800 km.

4.3 Gamma Decay

This occurs when a nucleus in an excited energy state makes a transition to a lower energy state and accordingly emits a γ-ray in the process. If the nucleus makes a transition from a higher energy state E_u to a lower energy state E_l then

$$E_u - E_l = h\nu \qquad (4.8)$$

The gamma ray carries neither charge nor mass, hence the charge and atomic number of the nucleus remains the same before and after a gamma decay. If we represent ${}^A_Z X^*$ as the excited state of the nucleus ${}^A_Z X$, then after emitting a γ-ray we have

$${}^A_Z X^* \rightarrow {}^A_Z X + \gamma \qquad (4.9)$$

Excited nuclei are called *isomers* and the excited states are referred to as *isomeric states*.

4.4 Alpha Decay

The α-particle is a helium nucleus. After its decay the parent nucleus loses 2 protons and two neutrons. Thus, the atomic number of the parent decreases by 2 while the mass number decreases by 4. Applying charge and nucleon conservation laws alpha decay can be written as

$${}^A_Z Y \rightarrow {}^{A-4}_{Z-2} X + {}^4_2 He \qquad (4.10)$$

If the parent nucleus is initially at rest then, energy conservation implies that

$$M_p c^2 = M_D c^2 + M_\alpha c^2 + K_D + K_\alpha \qquad (4.11)$$

where K_D, K_α are the kinetic energies of the daughter nuclei and M_P, M_D and M_α are the masses of the parent, daughter nuclei and alpha particle respectively. Since the kinetic energy can never be negative, alpha decay occurs if and only if

$$M_p \geq M_D + M_\alpha \qquad (4.12)$$

4.5 Beta Decay and the Neutrino

In the process of electron capture and also when an unstable nucleus emits a β^- particle (electron) or β^+ particle (positron) the charge (Ze) of the parent nucleus changes but the number of nucleons A remains unaltered. In each of the mentioned processes an extra particle called the neutrino (ν) is produced as one of the decay products. A neutrino has a rest mass $\approx$ 0, electric charge = 0, intrinsic spin = ½ and travels with the speed of light c.

$$n \rightarrow p + e^{-} + \bar{\nu} \tag{4.13}$$

Generally a β^{-}-decay is represented by

$$^{A}_{Z}P \rightarrow {}^{A}_{Z+1}X + e^{-} + \bar{\nu} \tag{4.14}$$

Conservation of energy implies

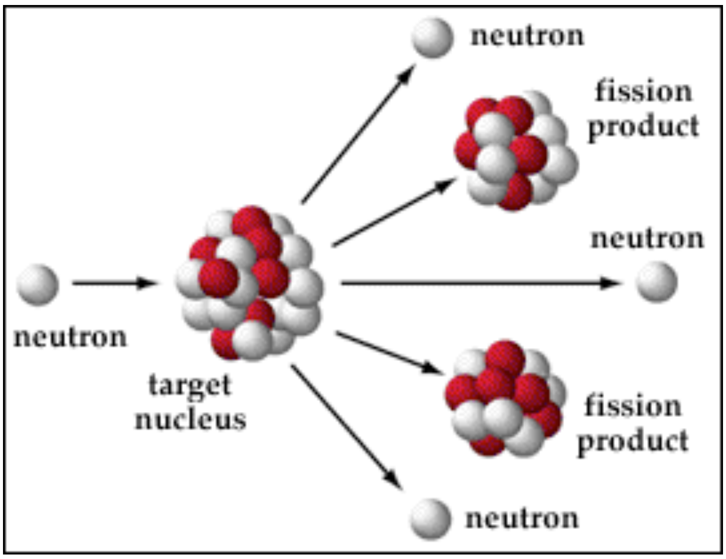
$$M_{p}c^{2} = M_{D}c^{2} + M_{e}c^{2} + K_{total} \tag{4.15}$$

This gives the disintegration energy Q as

$$Q = K_{total} = \left(M_{p} - M_{D} - m_{e}\right)c^{2} \tag{4.16}$$

4.6 Nuclear Fission

Fission is an artificially induced nuclear phenomenon by which a stable and massive nucleus is bombarded by appropriate energy radiations or particles leading to consequent nuclear instability and disintegration into two smaller nuclides of approximately the same order of mass as well as the emission of ionizing radiations or particles with the release of nuclear energy.



Fission is one of the artificial processes by which man harnesses nuclear energy. A typical fission reaction is

$$^{235}_{92}U + {}^{1}_{0}n \rightarrow \left[{}^{236}_{92}U\right] \rightarrow {}^{A_1}_{z_1}X + {}^{A_2}_{z_2}Y + \varepsilon\,{}^{1}_{0}n \tag{4.17}$$

where ε is an integer. The number of neutrons produced in the fissioning of a particular nucleus will depend on the final fragments produced.

4.6.1 Examples of Fission Reactions

$$^{235}_{92}U(n,3n)^{141}_{56}Ba,^{92}_{36}Kr \qquad (4.18)$$

$$^{118}_{50}Sn(p,4n)^{66}_{31}Ga,^{49}_{20}Ca \qquad (4.19)$$

4.6.2 Equivalence of mass and energy ($E=mc^2$)

1. An atomic particle at rest possess a rest mass energy E_0 given by

$$E_0 = m_0 c^2 \qquad (4.20)$$

   where m_0 is the rest mass and c is the speed of electromagnetic waves in vacuum (3.0×10^8m/s).

2. A dynamic particle possesses both kinetic energy and rest mass energy, the sum of which is known as dynamic energy E. Mathematically,

$$E = mc^2 = E_R + E_K \qquad (4.21)$$

   where m is dynamic mass, E_R = rest mass energy and E_K = kinetic mass energy

3. Hence, for moving atomic particles,

$$E = m_0 c^2 + E_K \qquad (4.22)$$

   But for photons $E = hv$ since photons have zero rest masses as they are quanta of electromagnetic radiations moving with the speed of light (c = 3.0×10^8m/s).

4.6.3 Nuclear Binding Energy and Mass Defect

4.6.3.1 Nuclear Binding Energy

The nuclear binding energy is defined as the finite magnitude of energy that must be supplied to an atomic nucleus (nuclide) to disintegrate it completely into its consequent nucleons. The nuclear binding energy is also equal to the amount of energy released when a finite array of nucleons aggregate into a composite nucleus.

4.6.3.2 Mass Defect

It has been experimentally proven that when an atomic nucleus disintegrates into its constituent nucleons, the total mass (i.e. rest mass) of the nucleons is usually more than that of the initial nucleus or nuclide. This difference in mass is known as the mass defect. Thus,

$$\text{Mass Defect} = \sum(\text{Mass of Nucleons}) - \sum(\text{Mass of Nucleus}) \qquad (4.23)$$

The nuclear binding energy is a consequence of the mass defect. The example below illustrates this point. Consider the reaction

$$X \rightarrow a + b \qquad (4.24)$$

where X s the initial nuclide and a and b are nucleons. Then sum of the masses of nucleons = $m_a + m_b$ and mass of X is m_x. But

$$(m_a + m_b) > m_x$$
$$\Rightarrow \quad \text{Mass Defect} = (m_a + m_b) - m_x \text{ or } \left|\mathrm{m_x} - (m_a + m_b)\right| \qquad (4.25)$$

Sample Calculation

For the reaction,

$${}_2^4He \rightarrow 2n + 2p$$
$m\left({}_2^4He\right) = 4.00154u$, $m(n) = 1.0073u, m(p) = 1.0087u$, The mass defect is computed as follows:

Mass of nucleons = 2(1.0073u) + 2(1.0087u) = 4.032u

Mass defect = 4.032 u - 4.00154 u = 0.03046 u

But E = mc^2

Hence, mass defect = 0.03046 × c^2 = 0.03046 × 931 Mev

4.6.4 The Energy released by Nucleons during Fission

Energy released from nucleons in a nuclear reaction usually involves U-235, U-233, Pu-239 and neutrons known as thermal neutrons. Thermal neutrons have energies of the order of 0.025 eV. For energy release by fission, the following conditions must be met:

1. *Bombarding Particles of Appropriate Energies*: The bombarding particles used in most cases is the neutron, since it has no charge and hence possesses greater penetrability into matter due to the less coulombic repulsive force it experiences. These neutrons (thermal neutrons) can produce fission in any of the following nuclides: U-235, U-233, Pu-239. Thermal neutrons can be produced through other reactions, for example, bombarding Beryllium with α-particles to release neutrons. The neutrons have energies of a few MeV and are called fast neutrons. These are thermalised (slowed down) in the moderator of a nuclear reactor so that they can have energies of the order of 0.025 eV in order to produce fission.
2. *The Type of Fuel Used:* The fuel used is usually made of heavy or massive nuclides which are capable of being fissioned by thermal neutrons. Examples of such nuclides are U-235, U-233, Pu-239. However, U-233 cannot be fissioned by thermal neutrons. Unfortunately the Uranium obtained from nature (i.e. natural Uranium) is in the form Uranium Hexafluoride (UF_6) which is a mixture of U-238 and U-235. Since only U-235 is a fissionable material it must be separated from the ore by methods which include:

i. Gasifying through a porous Membrane: Gasifying the ore by separation using the equation $v = \sqrt{3RT/m}$. The gasified product is passed through a long porous membrane and through the process lighter U-235 which diffuses faster than U-238 is obtained. This method is not very economical and efficient.

ii. Electromagnetic Method: Here the gasifyed product is released into a mass spectrometer where the gaseous atoms separate along trajectories which are proportional to their mass-to-charge ratios. This method is not convenient, efficient or economical. The natural ore contains 5% fissionable U-235 and 95% non-fissionable U-238. Taking into account these fractions and the problems encountered in separating the U-235 from the ore, nuclear physicist often used large quantities of the fuel comprising of U-235 and U-238. In effect U-238 behaves as a fertile material which can capture a neutron to become Pu-239 which is a fissionable material.

3. *Control Rods:* The control rods are components of the reactor which are used to regulate the neutron flux and consequently the instantaneous power output of the reactor. They are usually made of elements such as Boron (B) and Hafnium (Hf) which have high absorption cross-sections for thermal neutrons. The function of the control rods are to absorb some calculated number of neutrons if the reactor is becoming super-critical or more than the desired threshold. On the other hand if the reaction is becoming sub-critical with the power falling low, then, the control rods are removed from the reactor.

4. *Coolants:* These are components of the nuclear reactor which prevents it from experiencing high thermal regimes so as to cause the core components to melt. The coolants then extract thermal energy from the reactor core and inject it into the circulating water. The coolants must have high thermal capacity melting point or boiling point and a low neutron absorption cross-section. eg. Molten sodium, heavy water (D_20)
5. *Moderators*: These are components of the nuclear reactor which degrade the energy of the fast neutrons into 0.025 eV. Generally moderators convert the fast fission neutrons to slow thermal neutrons. Hence, a moderator must b e an element with high collision section (probability), low neutron capture cross-section and high melting point e.g. carbon, deuterium.

6. *Reflectors*: These are reactor components which help to maintain the neutron economy by preventing the escape of fission neutrons from the reactor core. Materials used for reflectors must have high neutron capture (absorption) cross-section e.g. Be, BeO.

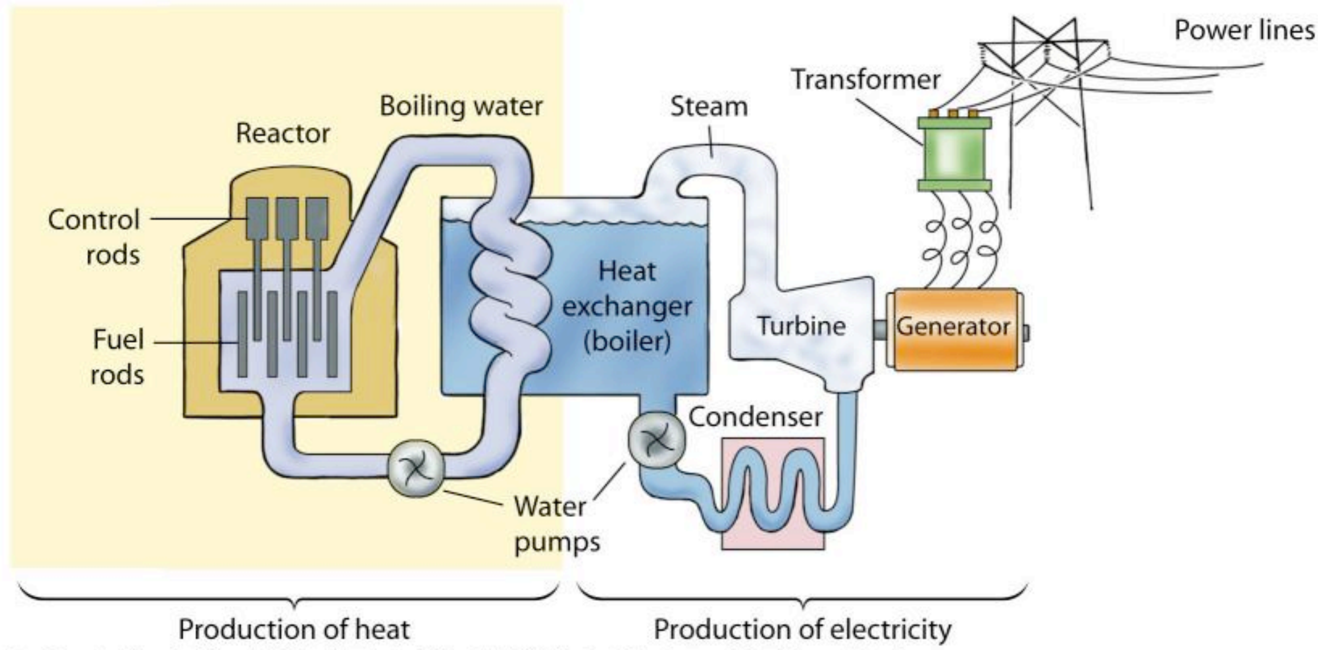
7. *Waste Material*: A nuclear reactor must have an efficient system that handles the dangerous radio-active products (the fissioned fragments) produced in the core of the reactor. Normally these waste products are encased in thick concrete blocks before being finally carried away for dumping in the sea or subterranean environments such as disused mines.

8. *The Shield*: A reactor is usually provided with a radiation shield in the form of thick concrete Pb walls so as to absorb the escaping radiation from the reactor core.

9. *Canning or Cladding Material*: In most cases the fuel or reactor core must be enclosed in some canning material such as Al or stainless steel.

4.6.5 Mechanics of Energy Released in Fuel

The neutrons released by the auxiliary neutron source such as (α,n) reactions in beryllium are first thermalised by the atoms of the moderator material. The thermal neutrons then bombard the atoms (nuclides) of the fuel (U-235) these fissioning them and also producing about 2.5 times fast neutrons so that they can also produce fission in the U-235 of the fuel. In a very short time, a self sustaining chain reaction occurs. By means of control rods the reaction is prevented from becoming super-critical or sub-critical. Now each fission produces energy of the order of 200 MeV. Of this energy about 167 MeV or more than 75% appears as the kinetic energy of the fissioned fragments e.g. Ba and about 5 MeV appears as kinetic energy of the fast fission neutrons while about 10 MeV appear as energy of the gamma radiations. About 5 MeV appears as kinetic energy of β-particles and about 11 MeV of the energy elapsed by Fermi's neutrinos. Apart from the energy carried by the Fermi's neutrinos, all the energies carried by the other cases can be regarded as thermal energy in the component of the reactor-core either as fuel, moderator, coolants, and reflector or cladding materials. By means of thermo-resistant coils, water is circulated around the reactor coils so as to act as to extract thermal energy from the system. This water is then converted into steam and conveyed away by motor-powered systems. The steam is used to rotate turbines which are positioned in dense homogeneous magnetic fields provided by gigantic electro-magnets.



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4.7 Nuclear Fusion

Fusion is a nuclear phenomenon by which two small masses of high atomic nuclides under controlled thermo-nuclear conditions aggregate into a composite (single) atomic nuclide with the consequent release of nuclear energy. Fusion is one of the methods by which energy can be obtained from the nucleus. The fusion process is termed as thermo-nuclear process because it requires an initial input of thermal energy of a very great magnitude and consequently requires super-high temperatures (of the order of 10^2 K) for ignition. Nevertheless, the fusion process after it has been triggered produces a large avalanche of nuclear energy. A few examples of fusion reaction are listed below:

$${}^{3}_{1}H + {}^{1}_{1}H \rightarrow {}^{4}_{2}He + \gamma \quad \text{or} \quad T(p,\gamma)^{4}_{2}He$$

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n \quad \text{or} \quad D(d,n)^{3}_{2}He$$

$${}^{6}_{3}Li(d,\alpha)^{4}_{3}He$$

$${}^{7}Li(p,\alpha)^{4}He$$

$${}^{7}Li(d,n)^{8}Be$$

4.8 Theoretical Background for the Computation of Q Value

A particle undergoing a translational motion in space-time continuum must possess both rest mass energy and kinetic energy. In such a case the total energy E_T of the particle is given by

$$E_T = E_0 + E_K \tag{4.26}$$

4.8.1 General case

$$X(a,b)Y,Z$$

Assume that in the above reaction all the particle i.e. *a,b* and the nuclides possess non-zero translational kinetic energy (mass energy). The total energies of the individual nuclides and particles are calculated as follows:

X: $E_K^X + m_0^X c^2$, a: $E_K^a + m_0^a c^2$, b: $E_K^b + m_0^b c^2$, Y: $E_K^Z + m_0^Z c^2$

where m_0 is in kg and c is in ms^{-1}.

Conservation of mass in nuclear reactions implies that the energy of the interacting system is equal to the energy of the resultant system. Thus,

$$E_K^X + m_0^X c^2 + E_K^a + m_0^a c^2 = E_K^Y + m_0^Y c^2 + E_K^Z + m_0^Z c^2 + E_K^b + m_0^b c^2 \qquad (4.27)$$

$$\Rightarrow \qquad \left(m_0^Y c^2 + m_0^Z c^2 + m_0^b c^2\right) - \left(m_0^X c^2 + m_0^a c^2\right) = \left(E_K^b + E_K^Y + E_K^Z\right) - \left(E_K^X + E_K^a\right) = Q$$

(4.28)

But 1u (atomic mass unit amu) = 931 MeV

$$\therefore \qquad Q = \left[\left(E_K^b + E_K^Y + E_K^Z\right) - \left(E_K^X + E_K^a\right)\right] \times 931 \text{ MeV}$$

WORK EXAMPLES

${}^4_2H$ is the most abundant isotope of helium. Its mass is 6.6447x 10^{-27}kg. What is
a) The mass defect?
b) The binding energy of the nucleus in joules?
c) The binding energy of the nucleus in electron volts?
Solution
a) Mass of component parts m = 2p+2n
= 2(1.672623 x 10^{-27}) + 2(1.674929 x 10^{-27})
m = 6.6950 x 10^{-27}kg

Mass defect = 6.6950 x 10^{-27}kg - 6.6447x 10^{-27}kg
= 5.03 x 10^{-29}kg

b) Binding energy using E =mc^2
E = [5.03 x 10^{-29}kg] x [3 x 10^8]^2
E = 4.53 x 10^{-12} Joules

c) Binding energy = 4.53 x 10^{-12} x 1.60 x 10^{-19}
= 2.83 x 10^7 eV
[= 28.3 MeV]

Questions:

1) ${}^{238}_{92}U$ decays into ${}^{234}_{90}Th$ and an alpha particle
a) Write down the full decay equation
b) How much energy is released.

Mass of ${}^{238}_{92}U$ = 238.0508u
Mass of ${}^{234}_{90}Th$ = 234.0426u
Mass of ${}^4_2\alpha$ = 4.0026u

2) Calculate the mass defect and binding energy the nuclide ${}^{10}_5B$ where the mass of ${}^{10}_5B$ atom = 10.0129 u

3) Oxygen has an unstable isotope O-17 that has a mass of 17.00454. If the mass of a neutron is 1.00898 u and the mass of a proton is 1.00814 u, calculate the binding energy of the oxygen nucleus in MeV.

4) A thorium atom of mass 232.038 u decays by the emission of an alpha particle to a radium atom of mass 228.031 u. If the alpha particle has a mass of 4.003 u, how much energy in J is released in the process ?

5) The fusion reaction below is one of the final stages in the fusion process that occurs in the Sun.

$$^{2}_{1}H \quad + \quad ^{3}_{1}H \quad \rightarrow \quad ^{4}_{2}He \quad + $$

(a) Complete the reaction identifying the missing particle.

(b) Calculate the energy released in the fusion reaction using the following information (you will also need the mass of the other particle).

$^{2}_{1}H$ = 3.345×10^{-27} Kg

$^{3}_{1}H$ → 5.008×10^{-27} Kg

$^{4}_{2}He$ 6.647×10^{-27} Kg

Solutions.

1)
a) $^{238}{}_{92}U \rightarrow {}^{234}{}_{90}Th + {}^{4}{}_{2}\alpha$

b) First calculate mass change

238.0508u - (234.0426u + 4.0026u)
mass change = 5.6×10^{-3}u
Convert to kg = 5.6×10^{-3}u x 1.6605×10^{-27}kg

Mass defect = 9.2988×10^{-30}

Energy released E = mc^2
= $9.2988 \times 10^{-30} \times (3 \times 10^{8})^2$
= 8.36892×10^{-13} J

2) Calculate the mass defect and binding energy the nuclide $^{10}{}_{5}B$ where the mass of $^{10}{}_{5}B$ atom = 10.0129 u

$^{10}{}_{5}B$ has 5 protons and 5 neutrons

Total mass of nucleons = mass of protons + mass of neutrons
= 5 [1.007276u] + 5 [1.008665u]
= 5.03638u + 5.043325
= 10.079705u

Mass defect = Mass of nucleons - mass of $^{10}{}_{5}B$ nucleus
= 10.079705u - 10.0129 u
= 0.066805

Mass defect in Kg = 1.1093×10^{-28} Kg

Binding Energy E = mc^2
= $1.1093 \times 10^{-28} \times (3 \times 10^8)^2$
= 9.9836×10^{-12} J

Binding Energy in eV= 9.9836×10^{-12} J / 1.6×10^{-19}
= 6.2398×10^7 eV
= 624 MeV

3) O-17 $^{17}{}_{8}O$ has 8 protons in the nucleus and 9 neutrons

Total mass of nucleons = mass of protons + mass of neutrons
= 8 [1.007276u] + 9 [1.008665u]
= 8.058208u + 9.077985u
= 17.136193u

Mass defect = Mass of nucleons - mass of O17 nucleus
= 17.136193u - 17.00454u
= 0.131653u

Mass defect in Kg = $0.131653 \times 1.6605 \times 10^{-27}$
= 2.186×10^{-28} Kg

Binding Energy E = mc^2
= $2.186 \times 10^{-28} \times (3 \times 10^8)^2$
= 1.9675×10^{-11} J

Binding Energy in eV= 1.9675×10^{-11} J / 1.6×10^{-19}
= 1.2297×10^8 eV
= 123 MeV

4)A thorium atom of mass 232.038 u decays by the emission of an alpha particle to a radium atom of mass 228.031 u. If the alpha particle has a mass of 4.003 u, how much energy in J is released in the process ?

Write out the rection first (words will do here)

Thorium $\rightarrow$ Radium + alpha particle

Calculate mass of products and reactants in terms of u

Reactants	Products
232.038u	228.031 + 4.003
232.038u	232.034

Calculate the difference = 232.038 - 232.034
= 0.004u

Energy released $E = mc^2$
$= 0.004 \times 1.66 \times 10^{-27} \times (3 \times 10^{8})^{2}$
$= 5.976 \times 10^{-13}$ J

5)
(a)

$${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n$$

(b) Calculate mass of products and reactants in Kg

Reactants	**Products**
$3.345 \times 10^{-27} + 5.008 \times 10^{-27}$ Kg	6.647×10^{-27} Kg + mass of neutron
8.353×10^{-27}	6.647×10^{-27} + $1.6605 \times 10^{-27} \times 1.008665$

Mass difference $= 8.353 \times 10^{-27} - 8.321888 \times 10^{-27}$
$= 3.1112 \times 10^{-29}$

Energy released $E = mc^2$
$= 3.1112 \times 10^{-29} \times (3 \times 10^{8})^{2}$
$= 2.80 \times 10^{-12}$ J

4.9 Accelerated charges and Bremsstrahlung

4.9.1 Radiation from accelerated, charged particles

A charged particle undergoing acceleration radiates photons. A ready example of this is when electrons moving back and forth in antennae produce electromagnetic radiation, such as transmitted by radio stations.

The power in electromagnetic radiation emitted by a particle of charge q with an acceleration a is given by Larmor's formula

$$P = \frac{2q^2a^2}{3c^2} \qquad (4.29)$$

The radiation has some very interesting properties:

- the emitted power, P, is proportional to the square of the charge q^2 and the square of the acceleration a^2.
- the photons are emitted in a characteristic **dipolar** form

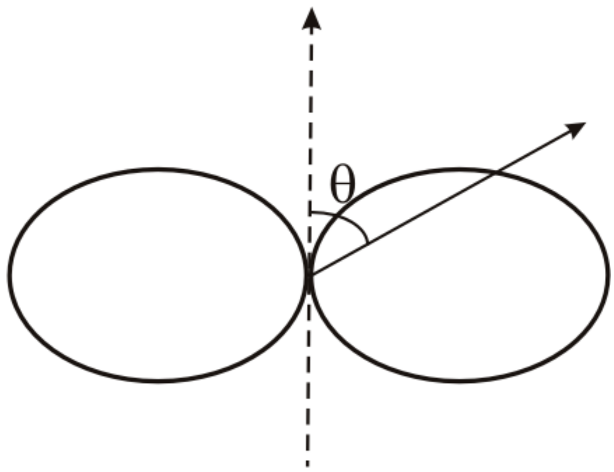

Fig 4.1. Dipolar emission from an accelerated charge. Maximum emission takes place perpendicular to the direction of the acceleration, and is proportional to $\sin^2\theta$.

$$P \ \alpha \ \sin^2\theta \qquad (4.30)$$

where θ is the angle between the direction of the acceleration and the emission. Thus there is no emission along the acceleration vector and maximum emission takes place perpendicular to the acceleration vector.

4.9.2 Bremsstrahlung from an electron passing a charged particle

Bremsstrahlung, or braking radiation, is emitted when a charged particle moves in an electric field, **E**. The particle emits energy in the form of electromagnetic radiation, at the expense of its kinetic energy, hence the name ``braking radiation".

The major astrophysically relevant example of bremsstrahlung, is when an electron, e^- with velocity v, passes a charge consisting of Z protons, with total charge Ze^+. The impact parameter b of the interaction is the distance of closest approach.

The electron is accelerated during its interaction, and since the acceleration is not uniform it emits photons with a range of wavelengths, i.e. a spectrum. The power emitted can be computed from Larmor's formula after the acceleration as a function of time has been determined, with the result that a **flat spectrum** in frequency with an **upper cutoff** ω_{cut}), which is related to the interaction time, $\Delta t = v/b$, or interaction frequency $\omega = 1/\Delta t = b/v$, is produced.

The intensity in the flat part of the spectrum, where ($\omega < \omega_{cut}$) is given by

$$I = \frac{8Z^2e^6}{3\pi . c^3 m_e^{\ 2} v^2 b^2} \qquad (4.31)$$

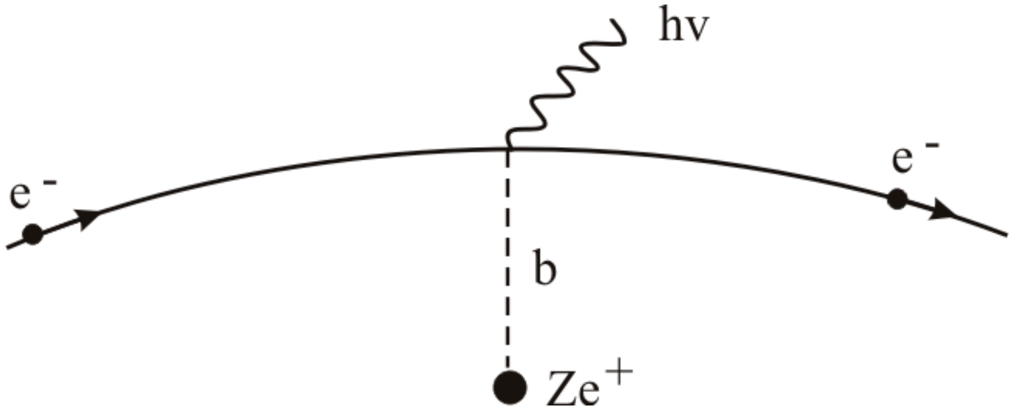

Fig. 4.2: Bremsstrahlung radiation. An electron e^- passes an ion with charge Ze+ with an impact parameter, b. Forces acting on the charge during the passage cause the emission of photons, hν.

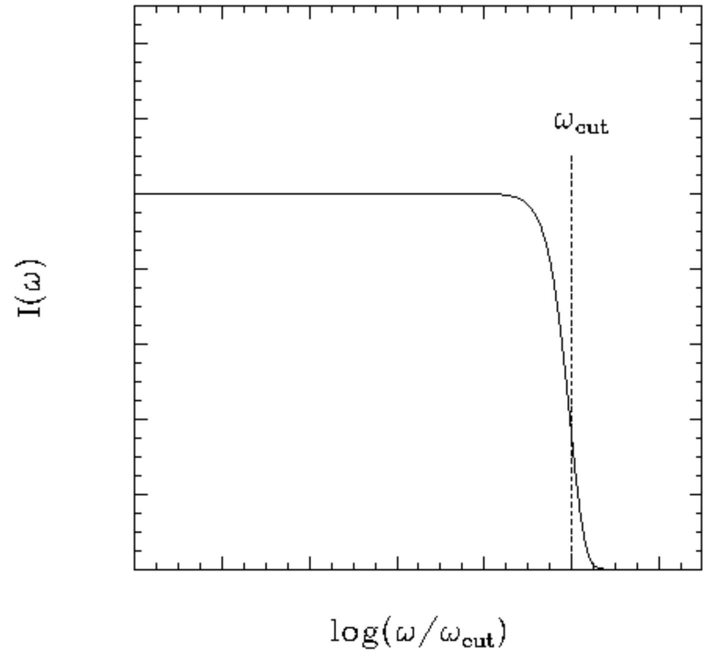

Fig. 4.3: Spectrum produced in the Bremsstrahlung process. The spectrum is flat up to a cutoff frequency ω_{cut}, and falls off exponentially at higher frequencies.

5.0 Radioactive Dating

5.1 Radiocarbon (^{14}C) formation and decay

1. Radiocarbon is formed by interaction of cosmic ray spallation products with stable N gas.

$$ {}^{1}_{0}n + {}^{14}_{7}N \rightarrow {}^{14}_{6}C + {}^{1}_{1}H \tag{5.1}$$

2. Radiocarbon subsequently decays by β- decay back to 14N with a half-life of 5730 y.

$$ {}^{14}_{6}N \rightarrow {}^{14}_{7}C + \beta^{-} + \bar{\nu} + Q \tag{5.2}$$

The activity of radiocarbon in the atmosphere represents a balance of its production, its decay, and its uptake by the biosphere, weathering, etc.

5.2 Radiocarbon Dating

- As plants uptake C through photosynthesis, they take on the ^{14}C activity of the atmosphere.

- Anything that thrives on this C will also have atmospheric ^{14}C activity (including you and I).
- If something stops actively exchanging C (it dies, is buried, etc), that ^{14}C begins to decay.

$$A = A_0 e^{-\lambda t} \tag{5.3}$$

where present-day, pre-bomb, ^{14}C activity = 13.56dpm/g C. So all that one needs to know to calculate in order to calculate an age is A_0, which to first order is 13.56dpm/g. However, small variations (several percent) in atmospheric ^{14}C in the past lead to dating errors of up to 20%! The sources of variability are the changes in:

1. geomagnetic field strength
2. solar activity
3. carbon cycle

5.3 Radiocarbon Measurements and Reporting

1. Radiocarbon dates are determined by measuring the ratio of ^{14}C to ^{12}C in a sample, relative to a standard, usually in an accelerator mass spectrometer.

standard = oxalic acid that represents activity of 1890 wood. ^{14}C ages are reported as "^{14}C years BP" where BP (before present) is 1950.

2. Most living things do not uptake C in atmospheric ratios - i.e. they **fractionate** carbon, (lighter ^{12}C preferentially used), must correct for this fractionation because it affects the $^{14}C/^{12}C$ ratio.

Researchers collect the $^{13}C/^{12}C$ ratio, use it to correct for "missing" ^{14}C

$$\boxed{\delta^{13}C = \left[\frac{\left(^{13}C/^{12}C\right)_{spl} - \left(^{13}C/^{12}C\right)_{std}}{\left(^{13}C/^{12}C\right)_{std}} - 1\right] * 1000}$$

So the less ^{13}C a sample has, the less ^{14}C it has, and so the uncorrected ^{14}C age will be _______ than the calendar age?

$$\boxed{A_{corr} = A_{meas}\left[1 - \frac{2(25+\delta^{13}C_{PDB})}{1000}\right] dpm/g}$$

Samples are "normalized" to a $\delta^{13}C_{PDB}$ value of -25 ppt.