Trigonometry

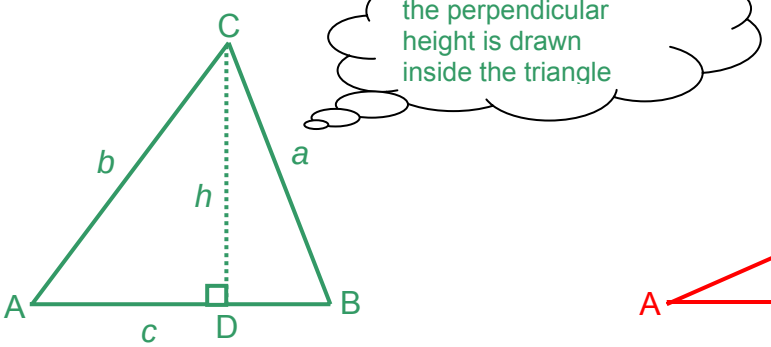
Section 3: Sine and cosine rules

Proof of the Sine Rule

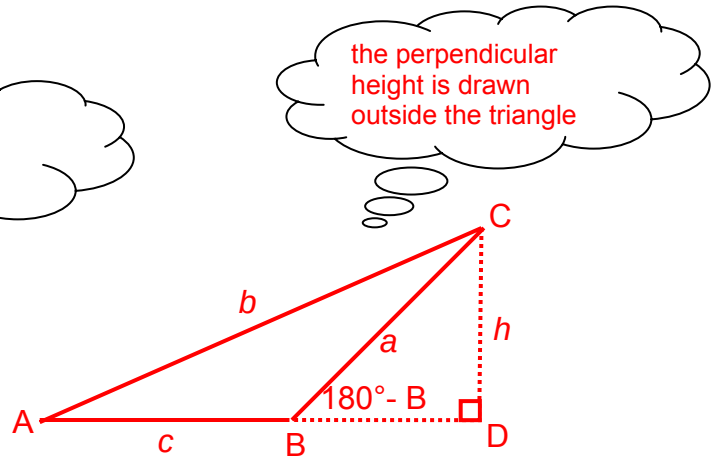
For any triangle ABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

PROOF

There are two possible cases:



Case 1



Case 2

Look at ΔACD.

For both cases: $\sin A = \dfrac{h}{b} \Rightarrow h = b\sin A$ **1**

Look at ΔBCD.

In case 1: $\sin B = \dfrac{h}{a}$

In case 2: $\sin(180° - B) = \dfrac{h}{a}$

But: $\sin(180° - B) = \sin B$ (see unit 2 for why)

So as in case 1: $\sin B = \dfrac{h}{a}$

So in each case: $h = a\sin B$ **2**

Expressions **1** and **2** can be combined to get: $a\sin B = b\sin A$

Rearrange to get: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

Similarly if we had drawn the perpendicular height from A we would have:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

So for any triangle ABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ as required.