Set 2: Multiple-Choice Questions on Limits and Continuity

1. $\lim_{x \to 2} \frac{x^2 - 4}{x^2 + 4}$ is

   (A) 1 **(B)** 0 **(C)** $-\frac{1}{2}$ **(D)** -1 **(E)** ∞

2. $\lim_{x \to \infty} \frac{4 - x^2}{x^2 - 1}$ is

   (A) 1 **(B)** 0 **(C)** -4 **(D)** -1 **(E)** ∞

3. $\lim_{x \to 3} \frac{x - 3}{x^2 - 2x - 3}$ is

   (A) 0 **(B)** 1 **(C)** $\frac{1}{4}$ **(D)** ∞ **(E)** none of these

4. $\lim_{x \to 0} \frac{x}{x}$ is

   (A) 1 **(B)** 0 **(C)** ∞ **(D)** -1 **(E)** nonexistent

5. $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4}$ is

   (A) 4 **(B)** 0 **(C)** 1 **(D)** 3 **(E)** ∞

6. $\lim_{x \to \infty} \frac{4 - x^2}{4x^2 - x - 2}$ is

   (A) -2 **(B)** $-\frac{1}{4}$ **(C)** 1 **(D)** 2
   (E) nonexistent

7. $\lim_{x \to \infty} \frac{5x^3 + 27}{20x^2 + 10x + 9}$ is

   (A) ∞ **(B)** $\frac{1}{4}$ **(C)** 3 **(D)** 0 **(E)** 1

8. $\lim_{x \to \infty} \frac{3x^2 + 27}{x^3 - 27}$ is

   (A) 3 **(B)** ∞ **(C)** 1 **(D)** -1 **(E)** 0

9. $\lim_{x \to \infty} \frac{2^{-x}}{2^x}$ is

   (A) -1 **(B)** 1 **(C)** 0 **(D)** ∞ **(E)** none of these

10. If $[x]$ is the greatest integer not greater than x, then $\lim_{x \to 1/2} [x]$ is

(A) $\frac{1}{2}$ **(B)** 1 **(C)** nonexistent **(D)** 0 **(E)** none of these

11. (With the same notation) $\lim_{x \to 2} [x]$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** none of these

12. $\lim_{x \to 0} \frac{\tan x}{x}$ is

(A) 0 **(B)** 1 **(C)** π **(D)** ∞ **(E)** The limit does not exist.

13. $\lim_{x \to 0} \frac{\sin 2x}{x}$ is

(A) 1 **(B)** 2 **(C)** $\frac{1}{2}$ **(D)** 0 **(E)** ∞

14. $\lim_{x \to \infty} \sin x$

(A) is nonexistent **(B)** is infinity **(C)** oscillates between -1 and 1
(D) is zero **(E)** is 1 or -1

15. $\lim_{x \to 0} \frac{\sin 3x}{\sin 4x}$ is

(A) 1 **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{4}$ **(D)** 0 **(E)** nonexistent

16. $\lim_{x \to 0} \frac{1 - \cos x}{x}$ is

(A) nonexistent **(B)** 1 **(C)** 2 **(D)** ∞ **(E)** none of these

17. $\lim_{x \to 0} \frac{\sin x}{x^2 + 3x}$ is

(A) 1 **(B)** $\frac{1}{3}$ **(C)** 3 **(D)** ∞ **(E)** $\frac{1}{4}$

18. $\lim_{x \to 0} \sin \frac{1}{x}$ is

(A) ∞ **(B)** 1 **(C)** nonexistent **(D)** -1 **(E)** none of these

19. $\lim_{x \to 0} \frac{\tan \pi x}{x}$ is

(A) $\frac{1}{\pi}$ **(B)** 0 **(C)** 1 **(D)** π **(E)** ∞

20. $\lim_{x \to \infty} x^2 \sin \frac{1}{x}$

(A) is 1 **(B)** is 0 **(C)** is ∞ **(D)** oscillates between -1 and 1
(E) is none of these

21. $\lim_{x\to 0} x\csc x$ is

(A) $-\infty$ **(B)** -1 **(C)** 0 **(D)** 1 **(E)** ∞

22. $\lim_{x\to\infty} \dfrac{2x^2+1}{(2-x)(2+x)}$ is

(A) -4 **(B)** -2 **(C)** 1 **(D)** 2 **(E)** nonexistent

23. $\lim_{x\to 0} \dfrac{|x|}{x}$ is

(A) 0 **(B)** nonexistent **(C)** 1 **(D)** -1 **(E)** none of these

24. $\lim_{x\to\infty} x\sin\dfrac{1}{x}$ is

(A) 0 **(B)** ∞ **(C)** nonexistent **(D)** -1 **(E)** 1

25. $\lim_{x\to\pi} \dfrac{\sin(\pi-x)}{\pi-x}$ is

(A) 1 **(B)** 0 **(C)** ∞ **(D)** nonexistent **(E)** none of these

26. If $[x]$ is the greatest integer in x, then what is $\lim_{x\to -1} [x+1]$?

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** The limit does not exist.

27. Let $f(x) = \begin{cases} x^2-1 & \text{if } x \neq 1, \\ 4 & \text{if } x = 1. \end{cases}$

Which of the following statements, I, II, and III, are true?

I. $\lim_{x\to 1} f(x)$ exists II. $f(1)$ exists III. f is continuous at $x = 1$

(A) only I **(B)** only II **(C)** I and II **(D)** none of them
(E) all of them

29. If $\begin{cases} f(x) = \dfrac{x^2-x}{2x} \text{ for } x \neq 0, \\ f(0) = k, \end{cases}$

and if f is continuous at $x = 0$, then $k =$

(A) -1 **(B)** $-\dfrac{1}{2}$ **(C)** 0 **(D)** $\dfrac{1}{2}$ **(E)** 1

30. Suppose $\begin{cases} f(x) = \dfrac{3x(x-1)}{x^2-3x+2} \text{ for } x \neq 1, 2, \\ f(1) = -3, \\ f(2) = 4. \end{cases}$

Then $f(x)$ is continuous

(A) except at $x = 1$ **(B)** except at $x = 2$ **(C)** except at $x = 1$ or 2
(D) except at $x = 0$, 1, or 2 **(E)** at each real number

Answers for Set 2: Limits and Continuity

1. B	**6.** B	**11.** E	**16.** E	**21.** D	**26.** E
2. D	**7.** A	**12.** B	**17.** B	**22.** B	**27.** C
3. C	**8.** E	**13.** B	**18.** C	**23.** A	**28.** A
4. A	**9.** C	**14.** A	**19.** D	**24.** E	**29.** B
5. D	**10.** D	**15.** C	**20.** C	**25.** A	**30.** B

Set 3: Multiple-Choice Questions on Differentiation

In each of Questions 1–27 a function is given. Choose the alternative that is the derivative, $\frac{dy}{dx}$, of the function.

1. $y = (4x + 1)^2(1 - x)^3$

(A) $(4x + 1)^2(1 - x)^2(5 - 20x)$ **(B)** $(4x + 1)(1 - x)^2(4x + 11)$
(C) $5(4x + 1)(1 - x)^2(1 - 4x)$ **(D)** $(4x + 1)(1 - x)^2(11 - 20x)$
(E) $-24(4x + 1)(1 - x)^2$

2. $y = \frac{2 - x}{3x + 1}$

(A) $-\frac{7}{(3x + 1)^2}$ **(B)** $\frac{6x - 5}{(3x + 1)^2}$ **(C)** $-\frac{9}{(3x + 1)^2}$

(D) $\frac{7}{(3x + 1)^2}$ **(E)** $\frac{7 - 6x}{(3x + 1)^2}$

3. $y = \sqrt{3 - 2x}$

(A) $\frac{1}{2\sqrt{3 - 2x}}$ **(B)** $-\frac{1}{\sqrt{3 - 2x}}$ **(C)** $-\frac{(3 - 2x)^{3/2}}{3}$

(D) $-\frac{1}{3 - 2x}$ **(E)** $\frac{2}{3}(3 - 2x)^{3/2}$

4. $y = \frac{2}{(5x + 1)^3}$

(A) $-\frac{30}{(5x + 1)^2}$ **(B)** $-30(5x + 1)^{-4}$ **(C)** $\frac{-6}{(5x + 1)^4}$

(D) $-\frac{10}{3}(5x + 1)^{-4/3}$ **(E)** $\frac{30}{(5x + 1)^4}$

5. $y = 3x^{2/3} - 4x^{1/2} - 2$

(A) $2x^{1/3} - 2x^{-1/2}$ **(B)** $3x^{-1/3} - 2x^{-1/2}$ **(C)** $\frac{9}{5}x^{5/3} - 8x^{3/2}$

(D) $\frac{2}{x^{1/3}} - \frac{2}{x^{1/2}} - 2$ **(E)** $2x^{-1/3} - 2x^{-1/2}$

6. $y = 2\sqrt{x} - \frac{1}{2\sqrt{x}}$

(A) $x + \frac{1}{x\sqrt{x}}$ **(B)** $x^{-1/2} + x^{-3/2}$ **(C)** $\frac{4x - 1}{4x\sqrt{x}}$

(D) $\frac{1}{\sqrt{x}} + \frac{1}{4x\sqrt{x}}$ **(E)** $\frac{4}{\sqrt{x}} + \frac{1}{x\sqrt{x}}$

7. $y = \sqrt{x^2 + 2x - 1}$

(A) $\frac{x + 1}{y}$ **(B)** $4y(x + 1)$ **(C)** $\frac{1}{2\sqrt{x^2 + 2x - 1}}$

(D) $-\frac{x + 1}{(x^2 + 2x - 1)^{3/2}}$ **(E)** none of these

8. $y = \frac{x}{\sqrt{1 - x^2}}$

(A) $\frac{1 - 2x^2}{(1 - x^2)^{3/2}}$ **(B)** $\frac{1}{1 - x^2}$ **(C)** $\frac{1}{\sqrt{1 - x^2}}$

(D) $\frac{1 - 2x^2}{(1 - x^2)^{1/2}}$ **(E)** none of these

9. $y = \cos x^2$

(A) $2x \sin x^2$ **(B)** $-\sin x^2$ **(C)** $-2 \sin x \cos x$
(D) $-2x \sin x^2$ **(E)** $\sin 2x$

10. $y = \sin^2 3x + \cos^2 3x$

(A) $-6 \sin 6x$ **(B)** 0 **(C)** $12 \sin 3x \cos 3x$
(D) $6(\sin 3x + \cos 3x)$ **(E)** 1

11. $y = \ln \frac{e^x}{e^x - 1}$

(A) $x - \frac{e^x}{e^x - 1}$ **(B)** $\frac{1}{e^x - 1}$ **(C)** $\frac{1}{1 - e^x}$ **(D)** 0

(E) $\frac{e^x - 2}{e^x - 1}$

12. $y = \tan^{-1} \frac{x}{2}$

(A) $\frac{4}{4 + x^2}$ **(B)** $\frac{1}{2\sqrt{4 - x^2}}$ **(C)** $\frac{2}{\sqrt{4 - x^2}}$ **(D)** $\frac{1}{2 + x^2}$

(E) $\frac{2}{x^2 + 4}$

13. $y = \ln(\sec x + \tan x)$

(A) $\sec x$ **(B)** $\dfrac{1}{\sec x}$ **(C)** $\tan x + \dfrac{\sec^2 x}{\tan x}$ **(D)** $\dfrac{1}{\sec x + \tan x}$

(E) $-\dfrac{1}{\sec x + \tan x}$

14. $y = \cos^2 x$

(A) $-\sin^2 x$ **(B)** $2 \sin x \cos x$ **(C)** $-\sin 2x$ **(D)** $2 \cos x$
(E) $-2 \sin x$

15. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(A) 0 **(B)** 1 **(C)** $\dfrac{2}{(e^x + e^{-x})^2}$ **(D)** $\dfrac{4}{(e^x + e^{-x})^2}$

(E) $\dfrac{1}{e^{2x} + e^{-2x}}$

16. $y = \ln(x\sqrt{x^2 + 1})$

(A) $1 + \dfrac{x}{x^2 + 1}$ **(B)** $\dfrac{1}{x\sqrt{x^2 + 1}}$ **(C)** $\dfrac{2x^2 + 1}{x\sqrt{x^2 + 1}}$ **(D)** $\dfrac{2x^2 + 1}{x(x^2 + 1)}$
(E) none of these

17. $y = \ln(x + \sqrt{x^2 + 1})$

(A) $\dfrac{1}{x} + \dfrac{x}{x^2 + 1}$ **(B)** $\dfrac{1}{\sqrt{x^2 + 1}}$ **(C)** 1 **(D)** $\sqrt{x^2 + 1}$

(E) $\dfrac{1}{x} + \dfrac{1}{2\sqrt{x^2 + 1}}$

18. $y = x^2 \sin \dfrac{1}{x}$ $(x \neq 0)$

(A) $2x \sin \dfrac{1}{x} - x^2 \cos \dfrac{1}{x}$ **(B)** $-\dfrac{2}{x} \cos \dfrac{1}{x}$ **(C)** $2x \cos \dfrac{1}{x}$

(D) $2x \sin \dfrac{1}{x} - \cos \dfrac{1}{x}$ **(E)** $-\cos \dfrac{1}{x}$

19. $y = \dfrac{1}{2 \sin 2x}$

(A) $-\csc 2x \cot 2x$ **(B)** $\dfrac{1}{4 \cos 2x}$ **(C)** $-4 \csc 2x \cot 2x$

(D) $\dfrac{\cos 2x}{2\sqrt{\sin 2x}}$ **(E)** $-\csc^2 2x$

20. $y = x^{\ln x} \quad (x > 0)$

(A) $\frac{2}{x}$ **(B)** $2\frac{\ln x}{x}$ **(C)** $\frac{2(\ln x)y}{x}$ **(D)** $\frac{2y}{x}$ **(E)** $(\ln x)x^{\ln x - 1}$

21. $y = x \tan^{-1} x - \ln \sqrt{x^2 + 1}$

(A) 0 **(B)** $\frac{1}{\sqrt{1 - x^2}} - \frac{x}{x^2 + 1}$ **(C)** $\tan^{-1} x$

(D) $\frac{x}{1 + x^2} + \tan^{-1} x - x$ **(E)** $\frac{1 - x}{1 + x^2}$

22. $y = e^{-x} \cos 2x$

(A) $-e^{-x}(\cos 2x + 2 \sin 2x)$ **(B)** $e^{-x}(\sin 2x - \cos 2x)$
(C) $2e^{-x} \sin 2x$ **(D)** $-e^{-x}(\cos 2x + \sin 2x)$ **(E)** $-e^{-x} \sin 2x$

23. $y = \sec^2 \sqrt{x}$

(A) $\frac{\sec \sqrt{x} \tan \sqrt{x}}{\sqrt{x}}$ **(B)** $\frac{\tan \sqrt{x}}{\sqrt{x}}$ **(C)** $2 \sec \sqrt{x} \tan^2 \sqrt{x}$

(D) $\frac{\sec^2 \sqrt{x} \tan \sqrt{x}}{\sqrt{x}}$ **(E)** $2 \sec^2 \sqrt{x} \tan \sqrt{x}$

24. $y = x \ln^3 x$

(A) $\frac{3 \ln^2 x}{x}$ **(B)** $3 \ln^2 x$ **(C)** $3x \ln^2 x + \ln^3 x$ **(D)** $3(\ln x + 1)$
(E) none of these

25. $y = \frac{1 + x^2}{1 - x^2}$

(A) $-\frac{4x}{(1 - x^2)^2}$ **(B)** $\frac{4x}{(1 - x^2)^2}$ **(C)** $\frac{-4x^3}{(1 - x^2)^2}$ **(D)** $\frac{2x}{1 - x^2}$

(E) $\frac{4}{1 - x^2}$

26. $y = \ln\left(\sqrt{2}\, x\right)$

(A) $\frac{\sqrt{2}}{x}$ **(B)** $\frac{1}{\sqrt{2}\, x}$ **(C)** $\frac{1}{2x}$ **(D)** $\frac{1}{x}$ **(E)** $\frac{1}{\sqrt{x}}$

27. $y = \sin^{-1} x - \sqrt{1 - x^2}$

(A) $\frac{1}{2\sqrt{1 - x^2}}$ **(B)** $\frac{2}{\sqrt{1 - x^2}}$ **(C)** $\frac{1 + x}{\sqrt{1 - x^2}}$ **(D)** $\frac{x^2}{\sqrt{1 - x^2}}$

(E) $\frac{1}{\sqrt{1 + x}}$

In each of Questions 32–35, y is a differentiable function of x. Choose the alternative that is the derivative $\frac{dy}{dx}$.

32. $x^3 - xy + y^3 = 1$

(A) $\frac{3x^2}{x - 3y^2}$ **(B)** $\frac{3x^2 - 1}{1 - 3y^2}$ **(C)** $\frac{y - 3x^2}{3y^2 - x}$

(D) $\frac{3x^2 + 3y^2 - y}{x}$ **(E)** $\frac{3x^2 + 3y^2}{x}$

33. $x + \cos(x + y) = 0$

(A) $\csc(x + y) - 1$ **(B)** $\csc(x + y)$ **(C)** $\frac{x}{\sin(x + y)}$

(D) $\frac{1}{\sqrt{1 - x^2}}$ **(E)** $\frac{1 - \sin x}{\sin y}$

34. $\sin x - \cos y - 2 = 0$

(A) $-\cot x$ **(B)** $-\cot y$ **(C)** $\frac{\cos x}{\sin y}$ **(D)** $-\csc y \cos x$

(E) $\frac{2 - \cos x}{\sin y}$

35. $3x^2 - 2xy + 5y^2 = 1$

(A) $\frac{3x + y}{x - 5y}$ **(B)** $\frac{y - 3x}{5y - x}$ **(C)** $3x + 5y$ **(D)** $\frac{3x + 4y}{x}$

(E) none of these

37. If $f(x) = x^4 - 4x^3 + 4x^2 - 1$, then the set of values of x for which the derivative equals zero is

(A) $\{1, 2\}$ **(B)** $\{0, -1, -2\}$ **(C)** $\{-1, +2\}$ **(D)** $\{0\}$
(E) $\{0, 1, 2\}$

38. If $f(x) = 16\sqrt{x}$, then $f'''(4)$ is equal to

(A) $\frac{3}{16}$ **(B)** -4 **(C)** $-\frac{1}{2}$ **(D)** 0 **(E)** 6

39. If $f(x) = \ln x$, then $f^{iv}(x)$ is

(A) $\frac{2}{x^3}$ **(B)** $\frac{24}{x^5}$ **(C)** $\frac{6}{x^4}$ **(D)** $-\frac{1}{x^4}$ **(E)** none of these

40. If a point moves on the curve $x^2 + y^2 = 25$, then, at $(0, 5)$, $\frac{d^2y}{dx^2}$ is

(A) 0 **(B)** $\frac{1}{5}$ **(C)** -5 **(D)** $-\frac{1}{5}$ **(E)** nonexistent

41. If $y = a \sin ct + b \cos ct$, where a, b, and c are constants, then $\frac{d^2y}{dt^2}$ is

(A) $ac^2(\sin t + \cos t)$ **(B)** $-c^2y$ **(C)** $-ay$ **(D)** $-y$
(E) $a^2c^2 \sin ct - b^2c^2 \cos ct$

42. If $f(x) = x^4 - 4x^2$, then $f^{iv}(2)$ equals

(A) 48 **(B)** 0 **(C)** 24 **(D)** 144 **(E)** 16

43. If $f(x) = \frac{x}{(x-1)^2}$, then the set of x's for which $f'(x)$ exists is

(A) all reals **(B)** all reals except $x = 1$ and $x = -1$

(C) all reals except $x = -1$ **(D)** all reals except $x = \frac{1}{3}$ and $x = -1$

(E) all reals except $x = 1$

44. If $y = (x-1)^2e^x$, then $\frac{d^2y}{dx^2}$ is equal to

(A) $e^x(x-1)^2$ **(B)** $e^x(x^2 - 2x - 1)$ **(C)** $e^x(x^2 + 2x - 1)$
(D) $2e^x(x-1)$ **(E)** none of these

45. If $f(x) = e^{-x} \ln x$, then, when $x = 1$, $\frac{df}{dx}$ is

(A) 0 **(B)** nonexistent **(C)** $\frac{2}{e}$ **(D)** $\frac{1}{e}$ **(E)** e

46. If $y = \sqrt{x^2 + 1}$, then the derivative of y^2 with respect to x^2 is

(A) 1 **(B)** $\frac{x^2 + 1}{2x}$ **(C)** $\frac{x}{2(x^2 + 1)}$ **(D)** $\frac{2}{x}$ **(E)** $\frac{x^2}{x^2 + 1}$

47. If $f(x) = \frac{1}{x^2 + 1}$ and $g(x) = \sqrt{x}$, then the derivative of $f(g(x))$ is

(A) $\frac{-\sqrt{x}}{(x^2 + 1)^2}$ **(B)** $-(x + 1)^{-2}$ **(C)** $\frac{-2x}{(x^2 + 1)^2}$ **(D)** $\frac{1}{(x + 1)^2}$

(E) $\frac{1}{2\sqrt{x}(x + 1)}$

50. If $y = x^2 + x$, then the derivative of y with respect to $\frac{1}{1-x}$ is

(A) $(2x + 1)(x - 1)^2$ **(B)** $\frac{2x + 1}{(1 - x)^2}$ **(C)** $2x + 1$ **(D)** $\frac{3 - x}{(1 - x)^3}$
(E) none of these

51. $\lim_{h \to 0} \frac{(1+h)^6 - 1}{h}$ is

(A) 0 (B) 1 (C) 6 (D) ∞ (E) nonexistent

52. $\lim_{h \to 0} \frac{\sqrt[3]{8+h} - 2}{h}$ is

(A) 0 (B) $\frac{1}{12}$ (C) 1 (D) 192 (E) ∞

53. $\lim_{h \to 0} \frac{\ln(e+h) - 1}{h}$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) nonexistent

54. $\lim_{x \to 0} \frac{\cos x - 1}{x}$ is

(A) -1 (B) 0 (C) 1 (D) ∞ (E) none of these

55. The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the mean-value theorem because

(A) $f(0)$ is not defined (B) $f(x)$ is not continuous on $[-8, 8]$
(C) $f'(-1)$ does not exist (D) $f(x)$ is not defined for $x < 0$
(E) $f'(0)$ does not exist

56. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

(A) $f(x)$ must be identically zero
(B) $f'(x)$ may be different from zero for all x on $[a, b]$
(C) there exists at least one number c, $a < c < b$, such that $f'(c) = 0$
(D) $f'(x)$ must exist for every x on (a, b)
(E) none of the preceding is true

57. If c is the number defined by Rolle's theorem, then, for $f(x) = 2x^3 - 6x$ on the interval $0 \leqq x \leqq \sqrt{3}$, c is

(A) 1 (B) -1 (C) $\sqrt{2}$ (D) 0 (E) $\sqrt{3}$

58. If h is the inverse function of f and if $f(x) = \frac{1}{x}$, then $h'(3) =$

(A) -9 (B) $-\frac{1}{9}$ (C) $\frac{1}{9}$ (D) 3 (E) 9

59. Suppose $y = f(x) = 2x^3 - 3x$. If h is the inverse function of f, then $h'(y) =$

(A) $\frac{1}{6y^2 - 3}$ (B) $\frac{1}{6x^2} - \frac{1}{3}$ (C) $\frac{1}{6x^2 - 3}$ (D) $-\frac{6x^2 - 3}{(2x^2 - 3)^2}$
(E) none of these

60. Suppose $y = f(x)$ and $x = f^{-1}(y)$ are mutually inverse functions. If $f(1) = 4$ and $\frac{dy}{dx} = -3$ at $x = 1$, then $\frac{dx}{dy}$ at $y = 4$ equals

(A) $-\frac{1}{3}$ **(B)** $-\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** 3 **(E)** 4

61. Let $y = f(x)$ and $x = h(y)$ be mutually inverse functions. If $f'(2) = 5$, then what is the value of $\frac{dx}{dy}$ at $y = 2$?

(A) -5 **(B)** $-\frac{1}{5}$ **(C)** $\frac{1}{5}$ **(D)** 5

(E) It cannot be determined from the information given.

62. If $f(x) = x^{\sin x}$ for $x > 0$, then $f'(x) =$

(A) $(\sin x)x^{\sin x - 1}$ **(B)** $x^{\sin x}(\cos x)(\ln x)$ **(C)** $\frac{\sin x}{x} + (\cos x)(\ln x)$

(D) $x^{\sin x}\left[\frac{\sin x}{x} + (\cos x)(\ln x)\right]$ **(E)** $x \cos x + \sin x$

63. Suppose $\lim_{x \to 0} \frac{g(x) - g(0)}{x} = 1$. It follows necessarily that

(A) g is not defined at $x = 0$
(B) g is not continuous at $x = 0$
(C) The limit of $g(x)$ as x approaches 0 equals 1
(D) $g'(0) = 1$
(E) $g'(1) = 0$

64. If $\sin(xy) = x$, then $\frac{dx}{dy} =$

(A) $\sec(xy)$ **(B)** $\frac{\sec(xy)}{x}$ **(C)** $\frac{\sec(xy) - y}{x}$

(D) $-\frac{1 + \sec(xy)}{x}$ **(E)** $\sec(xy) - 1$

65. $\lim_{x \to 0^+} x^x =$

(A) 0 **(B)** $\frac{1}{e}$ **(C)** 1 **(D)** e **(E)** none of these

Answers for Set 3: Differentiation

1. C	14. C	27. C	40. D	53. B
2. A	15. D	28. A	41. B	54. B
3. B	16. D	29. D	42. C	55. E
4. B	17. B	30. E	43. E	56. B
5. E	18. D	31. C	44. C	57. A
6. D	19. A	32. C	45. D	58. B
7. A	20. C	33. A	46. A	59. C
8. E	21. C	34. D	47. B	60. A
9. D	22. A	35. B	48. E	61. E
10. B	23. D	36. E	49. B	62. D
11. C	24. E	37. E	50. A	63. D
12. E	25. B	38. A	51. C	64. C
13. A	26. D	39. E	52. B	65. C

Set 4: Multiple-Choice Questions on Applications of Differential Calculus

1. The slope of the curve $y^3 - xy^2 = 4$ at the point where $y = 2$ is

   (A) -2 **(B)** $\frac{1}{4}$ **(C)** $-\frac{1}{2}$ **(D)** $\frac{1}{2}$ **(E)** 2

2. The slope of the curve $y^2 - xy - 3x = 1$ at the point $(0, -1)$ is

   (A) -1 **(B)** -2 **(C)** $+1$ **(D)** 2 **(E)** -3

3. The equation of the tangent to the curve $y = x \sin x$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

   (A) $y = x - \pi$ **(B)** $y = \frac{\pi}{2}$ **(C)** $y = \pi - x$ **(D)** $y = x + \frac{\pi}{2}$
   (E) $y = x$

4. The tangent to the curve of $y = xe^{-x}$ is horizontal when x is equal to

   (A) 0 **(B)** 1 **(C)** -1 **(D)** $\frac{1}{e}$ **(E)** none of these

5. The point on the curve $y = \sqrt{2x + 1}$ at which the normal is parallel to the line $y = -3x + 6$ is

   (A) $(4, 3)$ **(B)** $(0, 1)$ **(C)** $(1, \sqrt{3})$ **(D)** $(4, -3)$
   (E) $(2, \sqrt{5})$

6. The minimum value of the slope of the curve $y = x^5 + x^3 - 2x$ is

   (A) 0 **(B)** 2 **(C)** 6 **(D)** -2 **(E)** none of these

7. The equation of the tangent to the curve $x^2 = 4y$ at the point on the curve where $x = -2$ is

   (A) $x + y - 3 = 0$ **(B)** $y - 1 = 2x(x + 2)$
   (C) $x - y + 3 = 0$ **(D)** $x + y - 1 = 0$
   (E) $x + y + 1 = 0$

8. The equation of the tangent to the hyperbola $x^2 - y^2 = 12$ at the point $(4, 2)$ on the curve is

   (A) $x - 2y + 6 = 0$ **(B)** $y = 2x$ **(C)** $y = 2x - 6$ **(D)** $y = \frac{x}{2}$
   (E) $x + 2y = 6$

9. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

(A) $y = 0$ (B) $y = \pm\sqrt{3}$ (C) $y = \frac{1}{2}$ (D) $y = \pm 3$

(E) none of these

10. If differentials are used for computation, then the best approximation, in cubic inches, to the increase in volume of a sphere when the radius is increased from 3 to 3.1 in. is

(A) 11.3 (B) 11.7 (C) 12.1 (D) 33.9 (E) 39.7

11. When $x = 3$, the equation $2x^2 - y^3 = 10$ has the solution $y = 2$. If differentials are used to compute, then, when $x = 3.04$, y equals approximately

(A) 1.6 (B) 1.96 (C) 2.04 (D) 2.14 (E) 2.4

12. If the side e of a square is increased by 1%, then the area is increased approximately by

(A) $0.02e$ (B) $0.02e^2$ (C) $0.01e^2$ (D) 1% (E) $0.01e$

13. The edge of a cube has length 10 in., with a possible error of 1%. The possible error, in cubic inches, in the volume of the cube is

(A) 3 (B) 1% (C) 10 (D) 30 (E) none of these

14. The function $f(x) = x^4 - 4x^2$ has

(A) one relative minimum and two relative maxima
(B) one relative minimum and one relative maximum
(C) two relative maxima and no relative minimum
(D) two relative minima and no relative maximum
(E) two relative minima and one relative maximum

15. The number of inflection points of the curve in Question 14 above is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

16. The maximum value of the function $y = -4\sqrt{2 - x}$ is

(A) 0 (B) -4 (C) 2 (D) -2 (E) none of these

17. The total number of relative maximum and minimum points of the function whose derivative, for all x, is given by $f'(x) = x(x - 3)^2(x + 1)^4$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) none of these

18. If $x \neq 0$, then the slope of $x \sin \frac{1}{x}$ equals zero whenever

(A) $\tan \frac{1}{x} = x$ (B) $\tan \frac{1}{x} = -x$ (C) $\cos \frac{1}{x} = 0$

(D) $\sin \frac{1}{x} = 0$ (E) $\tan \frac{1}{x} = \frac{1}{x}$

19. On the closed interval $[0, 2\pi]$, the maximum value of the function $f(x) = 4 \sin x - 3 \cos x$ is

(A) 3 (B) 4 (C) $\frac{24}{5}$ (D) 5 (E) none of these

20. If m_1 is the slope of the curve $xy = 2$ and m_2 is the slope of the curve $x^2 - y^2 = 3$, then at a point of intersection of the two curves

(A) $m_1 = -m_2$ (B) $m_1 m_2 = -1$ (C) $m_1 = m_2$ (D) $m_1 m_2 = 1$
(E) $m_1 m_2 = -2$

21. The line $y = 3x + k$ is tangent to the curve $y = x^3$ when k is equal to

(A) 1 or -1 (B) 0 (C) 3 or -3 (D) 4 or -4
(E) 2 or -2

22. The two tangents that can be drawn from the point $(3, 5)$ to the parabola $y = x^2$ have slopes

(A) 1 and 5 (B) 0 and 4 (C) 2 and 10 (D) 2 and $-\frac{1}{2}$
(E) 2 and 4

In Questions 23–26, the motion of a particle on a straight line is given by $s = t^3 - 6t^2 + 12t - 8$.

23. The distance s is increasing for

(A) $t < 2$ (B) all t except $t = 2$ (C) $1 < t < 3$
(D) $t < 1$ or $t > 3$ (E) $t > 2$

24. The minimum value of the speed is

(A) 1 (B) 2 (C) 3 (D) 0 (E) none of these

25. The acceleration is positive

(A) when $t > 2$ (B) for all t, $t \neq 2$ (C) when $t < 2$
(D) for $1 < t < 3$ (E) for $1 < t < 2$

26. The speed of the particle is decreasing for

(A) $t > 2$ (B) $t < 3$ (C) all t (D) $t < 1$ or $t > 2$
(E) none of these

In Questions 27–29, a particle moves along a horizontal line according to the law $s = t^4 - 6t^3 + 12t^2 + 3$.

27. The particle is at rest when t is equal to

(A) 1 or 2 (B) 0 (C) $\frac{9}{4}$ (D) 0, 2, or 3 (E) none of these

28. The velocity, v, is increasing when

(A) $t > 1$ **(B)** $1 < t < 2$ **(C)** $t < 2$ **(D)** $t < 1$ or $t > 2$
(E) $t > 0$

29. The speed of the particle is increasing for

(A) $0 < t < 1$ or $t > 2$ **(B)** $1 < t < 2$ **(C)** $t < 2$
(D) $t < 0$ or $t > 2$ **(E)** $t < 0$

30. The displacement from the origin of a particle moving on a line is given by $s = t^4 - 4t^3$. The maximum displacement during the time interval $-2 \leqq t \leqq 4$ is

(A) 27 **(B)** 3 **(C)** $12\sqrt{3} + 3$ **(D)** 48 **(E)** none of these

31. If a particle moves along a line according to the law $s = t^5 + 5t^4$, then the number of times it reverses direction is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

42. A balloon is being filled with helium at the rate of 4 ft³/min. The rate, in square feet per minute, at which the surface area is increasing when the volume is $\frac{32\pi}{3}$ ft³ is

(A) 4π **(B)** 2 **(C)** 4 **(D)** 1 **(E)** 2π

43. A circular conical reservoir, vertex down, has depth 20 ft and radius of the top 10 ft. Water is leaking out so that the surface is falling at the rate of $\frac{1}{2}$ ft/hr. The rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 ft deep is

(A) 4π **(B)** 8π **(C)** 16π **(D)** $\frac{1}{4\pi}$ **(E)** $\frac{1}{8\pi}$

44. A vertical circular cylinder has radius r feet and height h feet. If the height and radius both increase at the constant rate of 2 ft/sec, then the rate, in square feet per second, at which the lateral surface area increases is

(A) $4\pi r$ **(B)** $2\pi(r + h)$ **(C)** $4\pi(r + h)$ **(D)** $4\pi rh$
(E) $4\pi h$

45. A relative minimum value of the function $y = \frac{e^x}{x}$ is

(A) $\frac{1}{e}$ **(B)** 1 **(C)** -1 **(D)** e **(E)** 0

46. The area of the largest rectangle that can be drawn with one side along the x-axis and two vertices on the curve of $y = e^{-x^2}$ is

(A) $\sqrt{\frac{2}{e}}$ **(B)** $\sqrt{2e}$ **(C)** $\frac{2}{e}$ **(D)** $\frac{1}{\sqrt{2e}}$ **(E)** $\frac{2}{e^2}$

Answers for Set 4: Applications of Differential Calculus

1. D	12. B	23. B	34. A	45. D	55. A
2. A	13. D	24. D	35. C	46. A	56. A
3. E	14. E	25. A	36. E	47. E	57. D
4. B	15. C	26. E	37. A	48. A	58. D
5. A	16. A	27. B	38. B	49. B	59. E
6. D	17. B	28. D	39. D	50. C	60. D
7. E	18. E	29. A	40. D	51. E	61. C
8. C	19. D	30. D	41. E	52. C	62. A
9. D	20. B	31. C	42. C	53. D	63. B
10. A	21. E	32. E	43. B	54. B	64. D
11. C	22. C	33. B	44. C		