

assumption

no flow goes through a streamline.

$$\dot{S} = (V_{out} - V_{in}) \int_{-h}^h u \rho \, dy ;$$

From continuity equation

$$\int_{-h}^h V_{in} \rho \, dy = \int_{-h}^h V_{out} \rho \, dy$$

Hence

$$\dot{S} = (V_{out} - V_{in}) \int_{-h}^h u \rho \, dy$$

Since to Newton's 3rd law we can evaluate the force  $F$  as the drag.

$$F = -\dot{S}$$

Momentum Equation

$$\dot{I}_{in} + \dot{I}_{in} - \dot{I}_{out} = 0$$

$$-\dot{S} = \dot{I}_{in}$$

$$\dot{I}_{in} = \int_{-h}^h \rho V_{in}^2 \, dy$$

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$$\dot{I}_{out} = \int_{-h}^h \rho V_{out}^2 \, dy$$

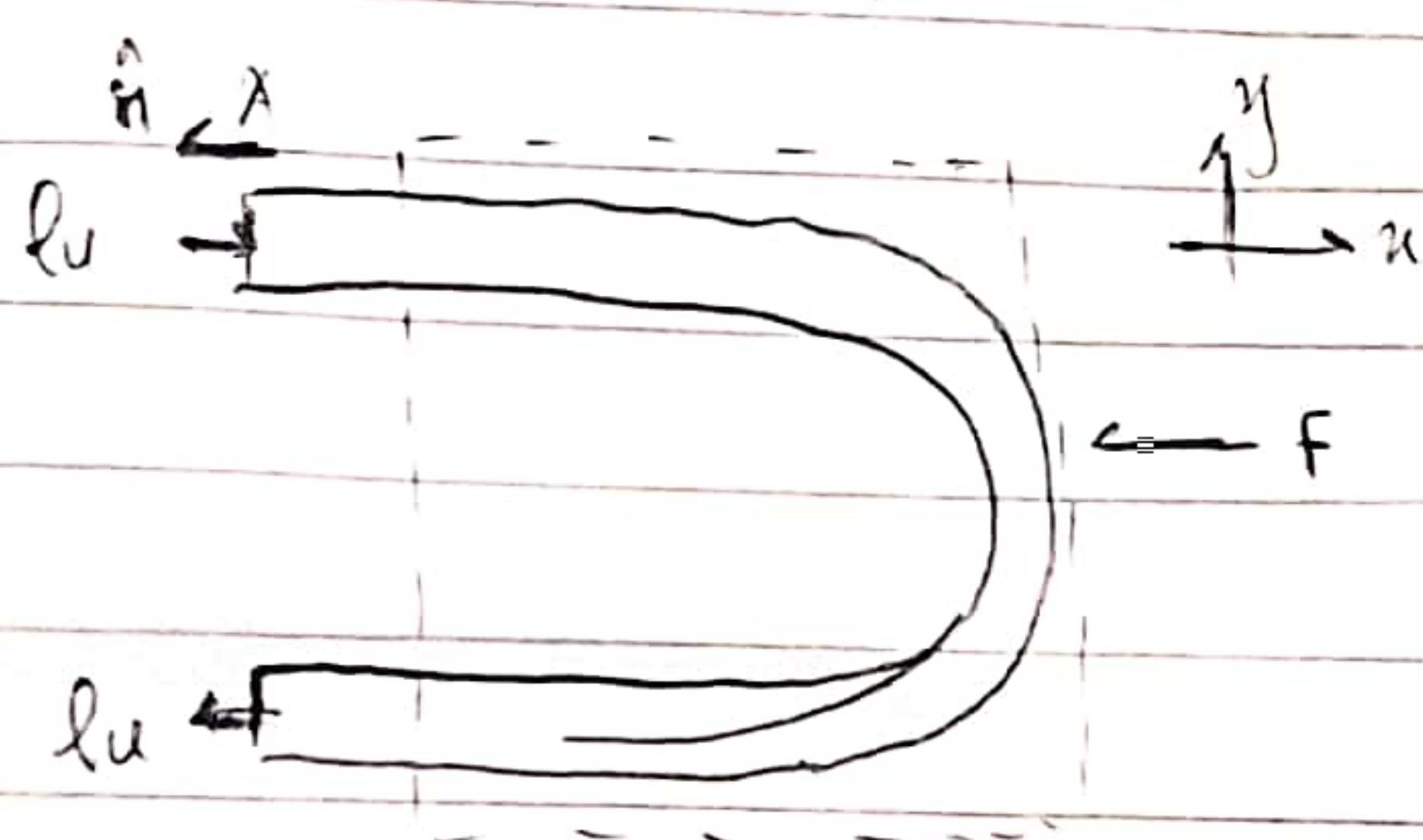
$$\dot{S} = \int_{-h}^h \rho u \, dy$$

$$\dot{I}_{out} = \int_{-h}^h \rho u^2 \, dy$$

$$-\dot{S} + \int_{-h}^h \rho V_{in}^2 \, dy - \int_{-h}^h \rho V_{out}^2 \, dy = 0$$

$$\dot{S} = \int_{-h}^h \rho V_{in}^2 \, dy - \int_{-h}^h \rho V_{out}^2 \, dy$$

Question Two



$$\dot{I}_{in} + \dot{I}_{in} - \dot{I}_{out} = 0$$

$$\dot{I}_{in} = -F$$

$$\dot{I}_{in} = \int_{-A}^A \rho u^2 \, dy$$

$$\dot{S} = \rho u A$$

$$\dot{I}_{in} = \rho u^2 A$$

$$\dot{I}_{out} = \int_{-A}^A \rho u^2 \, dy$$

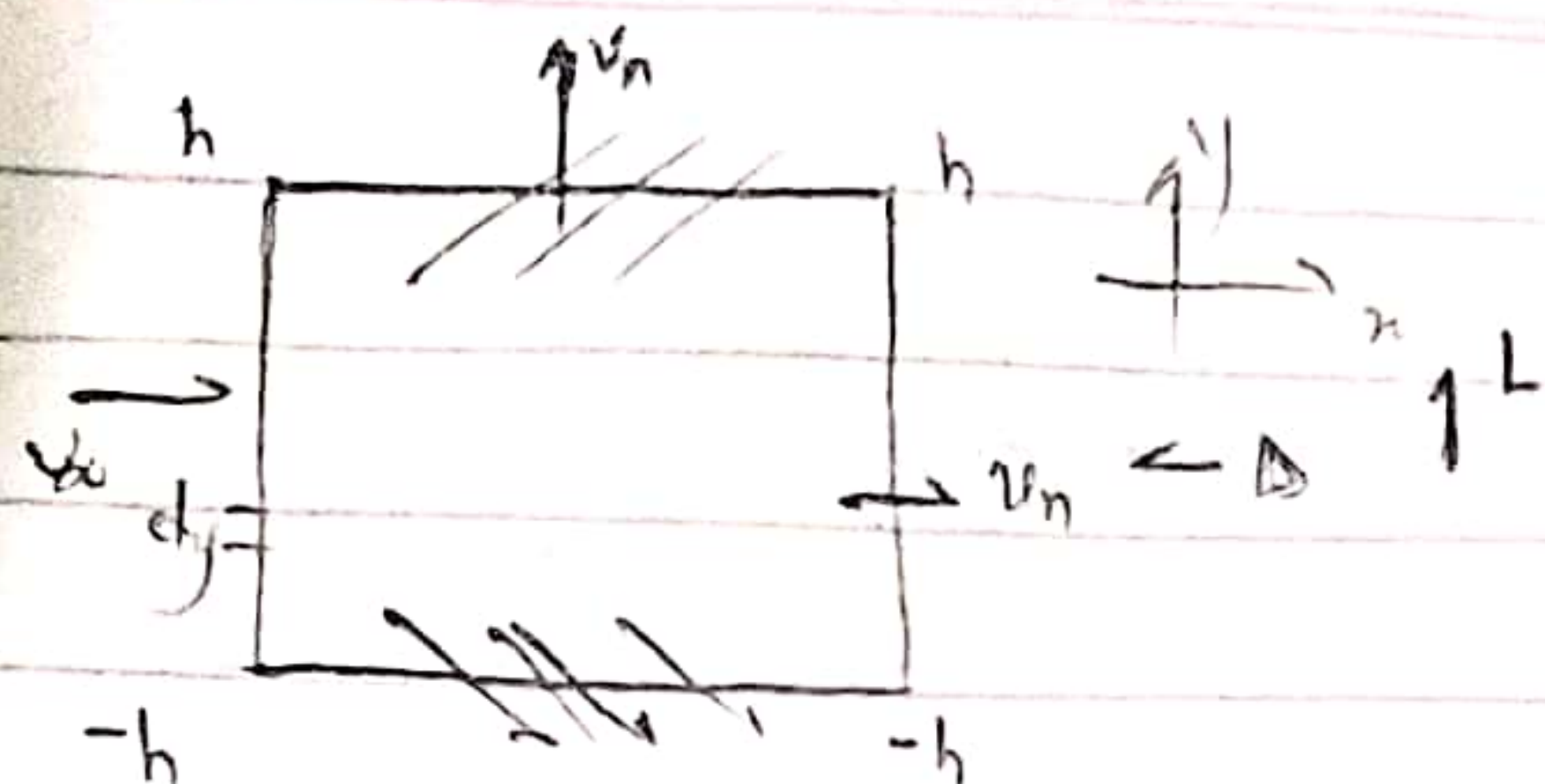
$$\dot{S} = +\rho u A$$

$$\dot{I}_{out} = -\rho u^2 A$$

$$F = -2 \rho u^2 A$$



### QUESTION THREE



Momentum equation

$$\sum F_x + I_{N_x} - I_{out} = 0$$

or

$$\sum F_x + I_{N_x} - I_{out} = 0$$

$$\sum F_x = -F$$

$$I_{N_x} = \int_{-h}^h \rho v_0^2 dy$$

$$I_{out} = \int_{-h}^h \rho v_1^2 dy$$

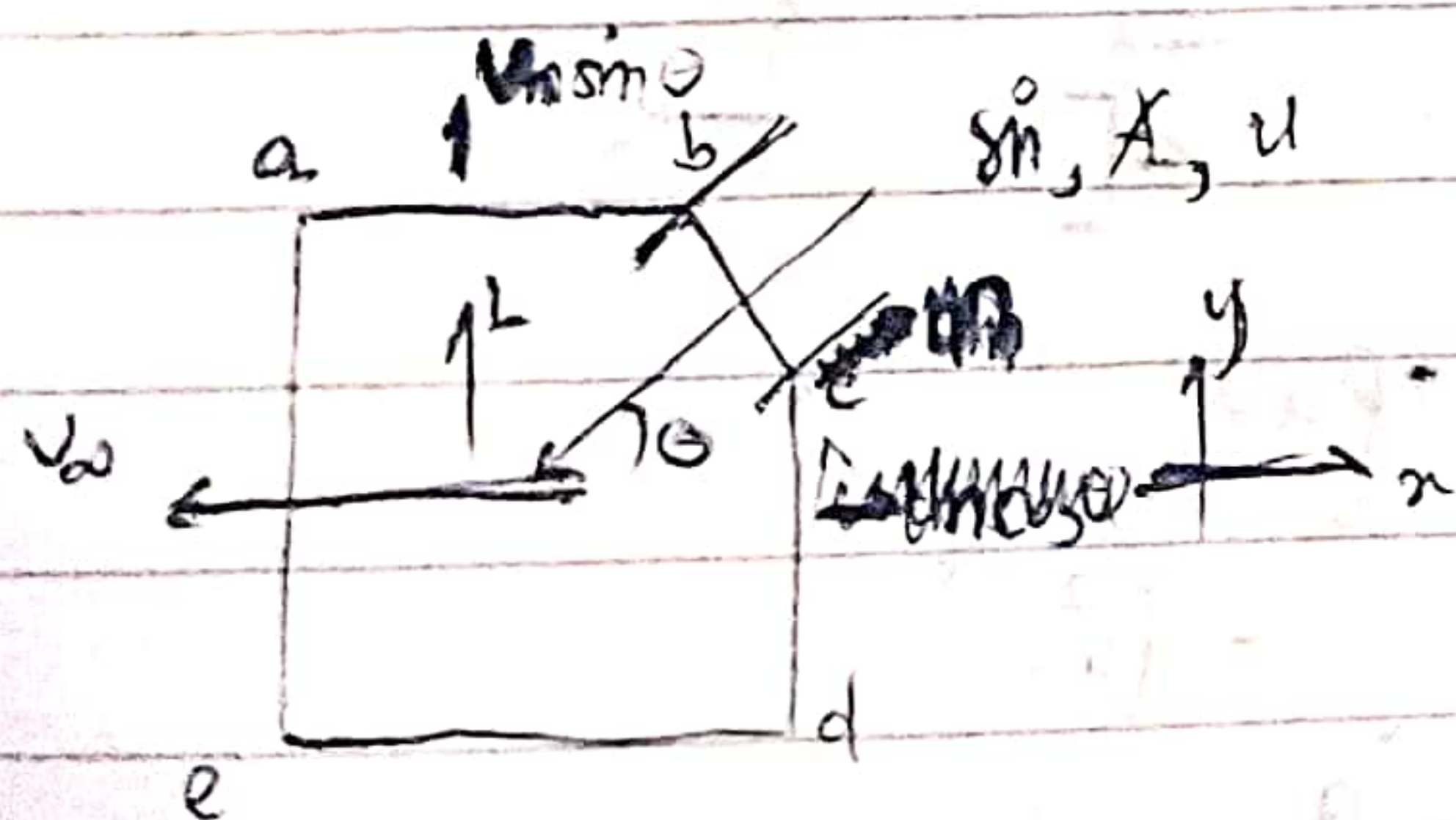
Continuity equation

$$\int_{-h}^h \rho v_0^2 dy = \int_{-h}^h \rho v_1^2 dy$$

$$\Rightarrow -F + (v_0 - v_1) \int_{-h}^h \rho v_1 dy = 0$$

$$F = (v_0 - v_1) \int_{-h}^h \rho v_1 dy$$

### QUESTION FOUR



$$\sum F_y + I_{N_y} - I_{out} = 0$$

$$I_{N_y} = -\dot{m} \sin \theta v_1$$

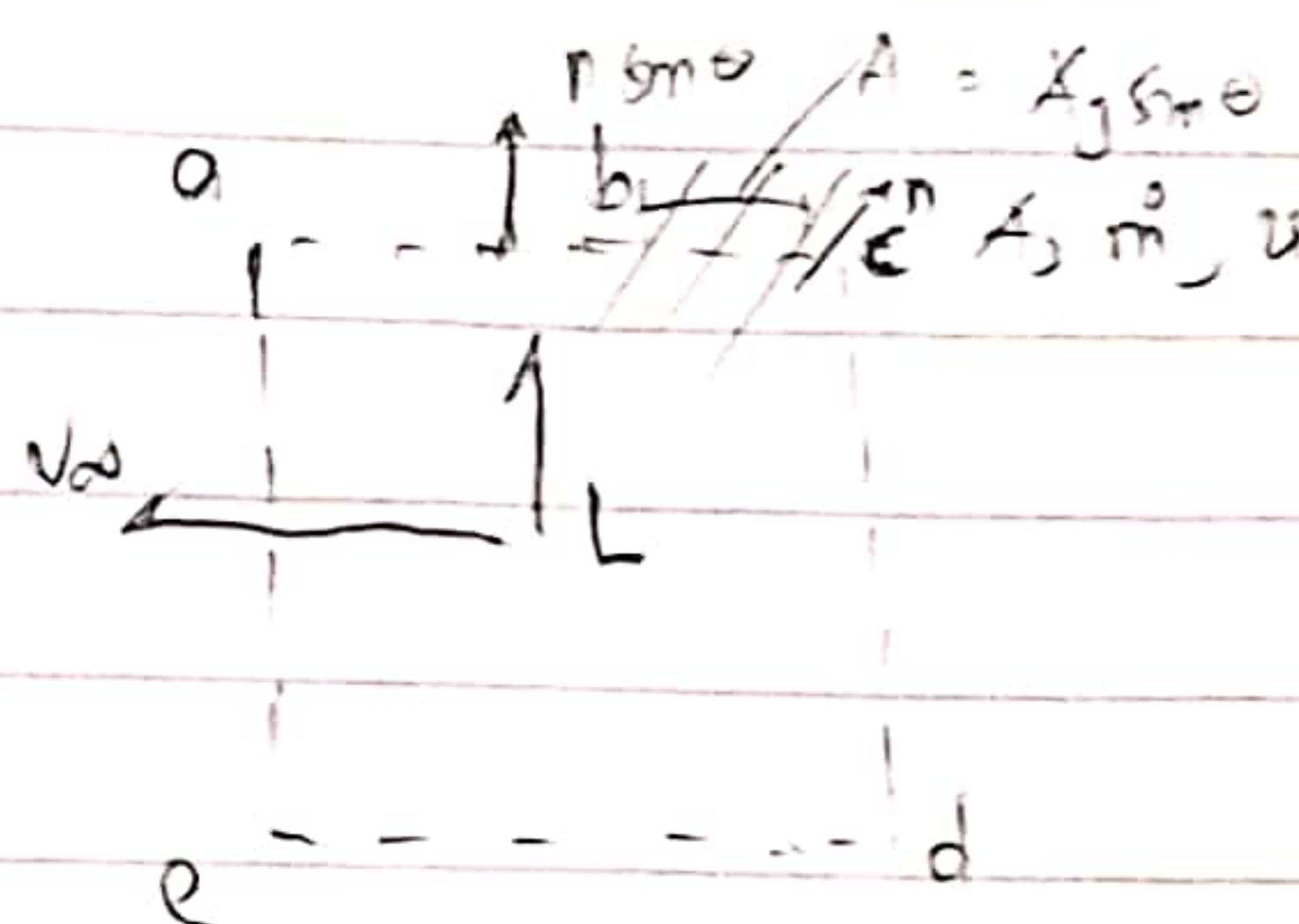
Assuming incompressible and steady conditions

$$I_{out} = \int_a^b dx \rho v_1 \sin^2 \theta - \int_c^d dx \rho v_1 \sin^2 \theta$$

$$\sum F_y = L$$

$$L = \dot{m} \sin \theta v_1 + \left[ \int_a^b dx \rho v_1 \sin^2 \theta - \int_c^d dx \rho v_1 \sin^2 \theta \right]$$

### QUESTION FIVE



$$\sum F_y + I_{N_y} - I_{out} = 0$$

$$\sum F_y = L$$

$$I_{N_y} = \rho v_1 A_y \sin \theta \times -v_1$$

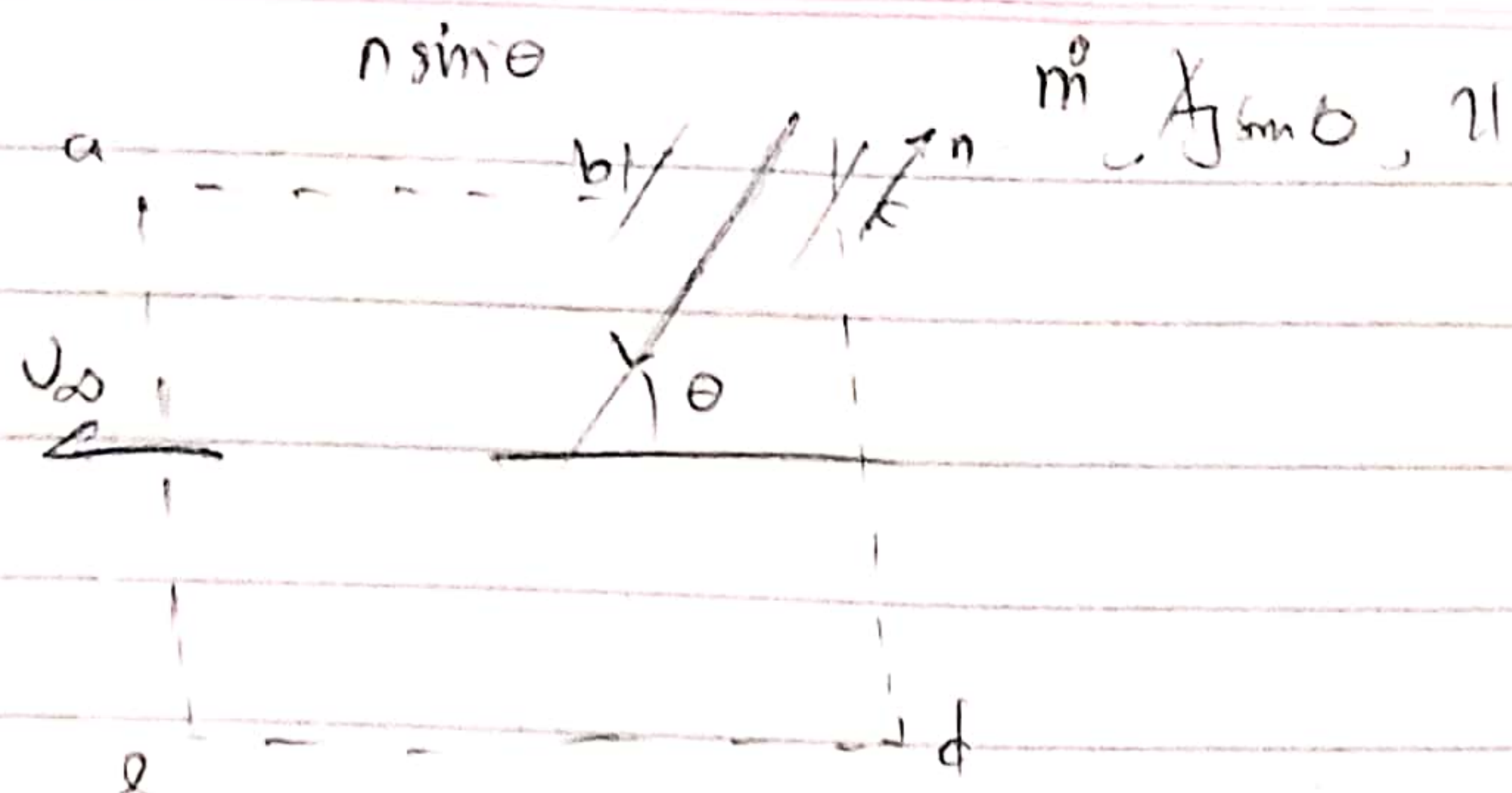
$$= -\dot{m} \sin \theta v_1$$

$$I_{out} = \left[ \int_a^b dx v_1 \rho \sin^2 \theta - \int_c^d dx v_1 \rho \sin^2 \theta \right]$$

$$L = \dot{m} \sin \theta v_1 + \left[ \int_a^b dx \rho v_1 \sin^2 \theta - \int_c^d dx \rho v_1 \sin^2 \theta \right]$$



Question Six



$$I_{xy} + I_H - I_{out} = 0$$

$$I_H = -8n \sin \theta \sin$$

$$I_{out} = \left[ \int_a^b \rho n \sin^2 \theta dx - \int_e^d \rho n \sin^2 \theta dx \right]$$

$$L = -I_H + I_{out}$$

$$= 8n \sin \theta \sin + \left[ \int_a^b \rho n \sin^2 \theta dx - \int_e^d \rho n \sin^2 \theta dx \right]$$