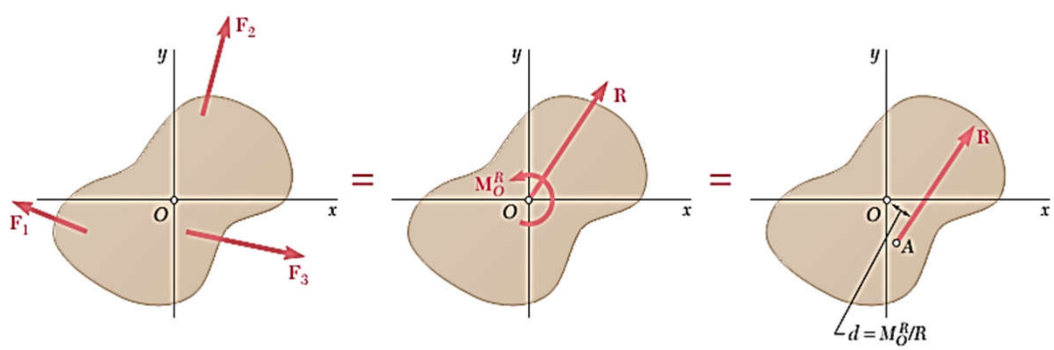
Equivalent Force-Moment Systems

Equivalent Force-Moment Systems

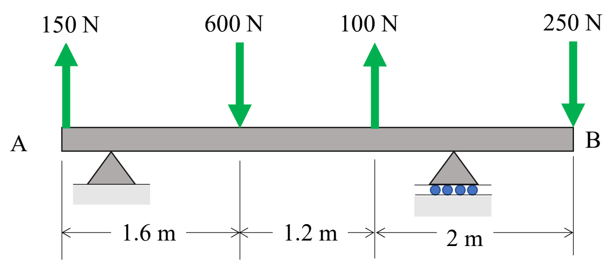
- A system of forces act on a body, **can** be reduced to a force-couple system.
- The force-couple system comprises a resultant force (evaluated with the particle idealization at a desired point) and a resultant moment about that desired point.



Equivalent Force-Moment Systems

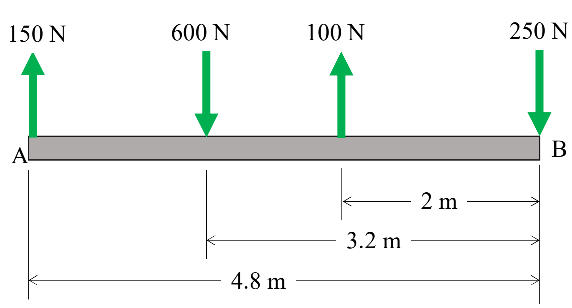
Example

For the beam, reduce the system of forces shown to (a) an equivalent force-couple system at A, (b) an equivalent force couple system at B. (Ignore the support reactions)



Equivalent Force-Moment Systems

Solution (@ B)



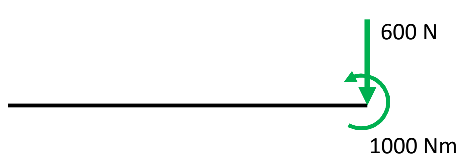
The Resultant force will be

$$+\uparrow R = \sum F$$
$$= (150\ \mathrm{N}) - (600\ \mathrm{N}) + (100\ \mathrm{N}) - (250\ \mathrm{N})$$
$$= (-600\ \mathrm{N})$$

The Resultant Moment

$$\circlearrowleft\ \vec{M}_B = \sum (r \times F)$$
$$= (250N \times 0m) - (100N \times 2m) + (600N \times 3.2m) - (150N \times 4.8m)$$
$$= 1000Nm$$

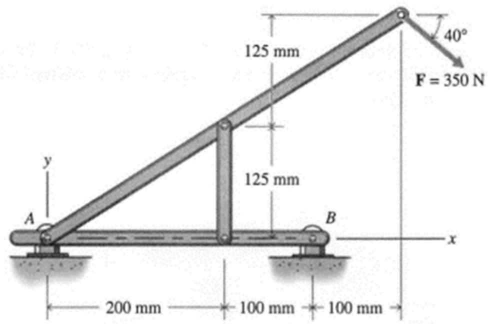
Equivalent system =



Equivalent Force-Moment Systems

Example

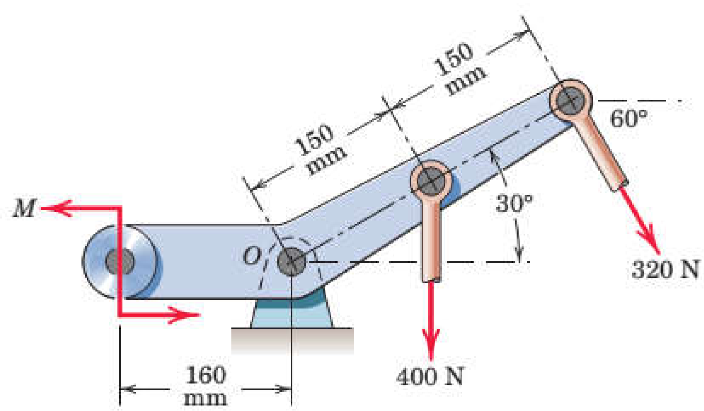
Replace the 350-N force shown below by a force and a couple at point B.



Equivalent Force-Moment Systems

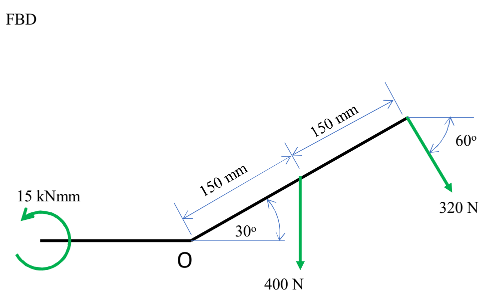
Example

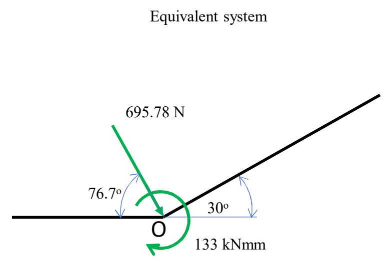
Reduce the system of forces and couple acting on the arm to an equivalent force couple system at *O*. Take *M* to be 15 kNmm and ignore support reactions.



Equivalent Force-Moment Systems

Soln.

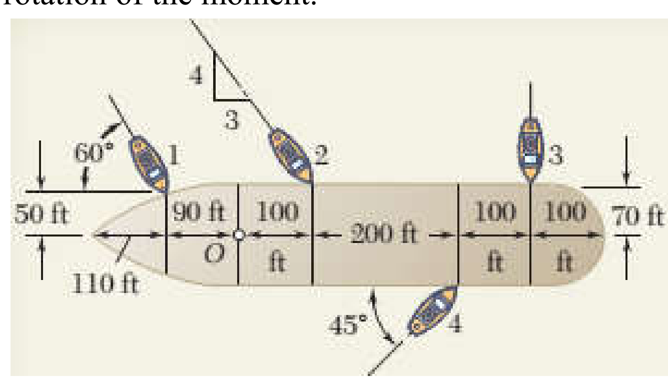




Equivalent Force-Moment Systems

Example

Four tugboats are used to bring an ocean liner to its pier. Each tugboat exerts a 5000-lb force in the direction shown. Determine the equivalent force-couple system at the foremast O. Also determine the angle the resultant force makes with the horizontal as well as the direction of rotation of the moment.



R = 13.33 klb (@ 47.3° to +ve x axis)
M^R_O = 1035 klb.ft

Centroids

Centroids

- It is sometimes necessary in mechanics problems to determine the centre of bodies.
- This central point is defined as that point a physical quantity under consideration may be assumed to be centred.
- The central point may have different terminologies for different physical quantities.

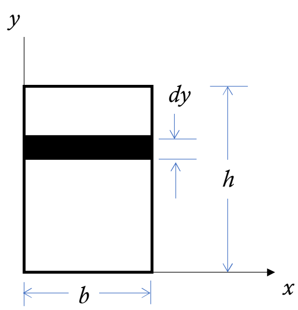
Terminology	Physical Entity
Centroid	Length of a curve
Centroid	Area of a surface
Centroid	Volume of a body
Centre of a mass	Mass of a body
Centre of gravity	Gravitational force on a body

- All the terms mentioned above can be determined analytically using an integral of moments technique.
- But a much simpler summation of first moments (geometric decomposition) approach will be the focus of this course.

Centroid of a Plane area

- The centroid of a plane area is the point of intersection of any two lines that divide the area into two equal halves (in a plane).
- The first moment of area about an axis is the product of the area and the perpendicular distance between its centre and the axis.

$$Q_x = A\bar{y}$$



$$dA = bdy$$

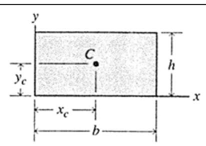
$$Q_x = \int_A ydA = \int_0^h y(bdy) = b\left[\frac{y^2}{2}\right]_0^h = \frac{bh^2}{2}$$

$$\bar{y} = \frac{Q_x}{A} = \frac{bh^2}{2} \times \frac{1}{bh} = \frac{h}{2}$$

Centroids of Some Common Plane Shapes

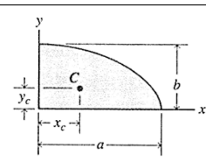
Rectangular area

$A = bh$, $x_C = \dfrac{b}{2}$, $y_C = \dfrac{h}{2}$



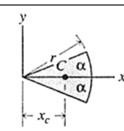
Quadrant of an ellipse

$A = \dfrac{\pi ab}{4}$, $x_C = \dfrac{4a}{3\pi}$, $y_C = \dfrac{4b}{3\pi}$



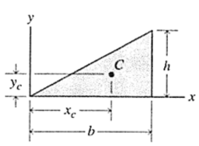
Circular sector

$A = r^2\alpha$, $x_C = \dfrac{2r \sin \alpha}{3\alpha}$, $y_C = 0$



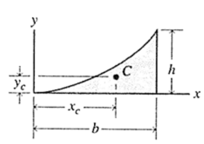
Triangular area

$A = \dfrac{bh}{2}$, $x_C = \dfrac{2b}{3}$, $y_C = \dfrac{h}{3}$



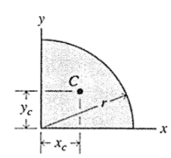
Parabolic spandrel

$A = \dfrac{bh}{3}$, $x_C = \dfrac{3b}{4}$, $y_C = \dfrac{3h}{10}$



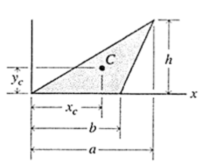
Quadrant of a circle

$A = \dfrac{\pi r^2}{4}$, $x_C = \dfrac{4r}{3\pi}$, $y_C = \dfrac{4r}{3\pi}$



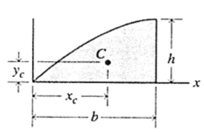
Triangular area

$A = \dfrac{bh}{2}$, $x_C = \dfrac{a+b}{3}$, $y_C = \dfrac{h}{3}$



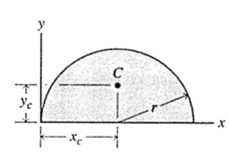
Quadrant of a parabola

$A = \dfrac{2bh}{3}$, $x_C = \dfrac{5b}{8}$, $y_C = \dfrac{2h}{5}$



Semicircular area

$A = \dfrac{\pi r^2}{2}$, $x_C = r$, $y_C = \dfrac{4r}{3\pi}$



Note: $y_C = \bar{y}, \quad x_C = \bar{x}$

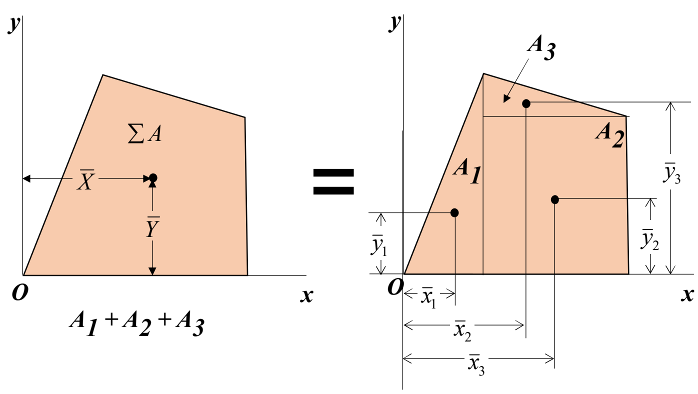
How to determine the Centroid of a Composite Plane area

- The centroid of a plane area is the point of intersection of any two lines that divide the area into two equal halves (in a plane)

- With the Geometric Decomposition approach,
  - We think of the area as a composite area comprising several smaller elemental areas (of various geometries)
  - We then determine the size and location of centroids from the *x* and *y* axes for each elemental area.
  - Calculate the first moments of area about the *x* and *y* axes for each elemental area, and then for the composite area (by summation).
  - The centroid of the composite area is calculated as; $\bar{X} = \frac{\sum Q_y}{\sum A} = \frac{\sum \bar{x}A}{\sum A}$ $\bar{Y} = \frac{\sum Q_x}{\sum A} = \frac{\sum \bar{y}dA}{\sum A}$

$$Centroid\ is\ (\bar{X}, \bar{Y})$$

How to determine the Centroid of a Plane area



$$\bar{X} = \frac{Total\ First\ Moment\ of\ each\ Area\ about\ y\text{-}axis,\ Q_y}{Total\ Area}$$
$$= \frac{\sum \bar{x}dA}{\sum A} = \frac{(\bar{x}_1 A_1) + (\bar{x}_2 A_2) + (\bar{x}_3 A_3)}{A_1 + A_2 + A_3}$$

$$\bar{Y} = \frac{Total\ First\ Moment\ of\ each\ Area\ about\ x\text{-}axis,\ Q_x}{Total\ Area}$$
$$= \frac{\sum \bar{y}dA}{\sum A} = \frac{(\bar{y}_1 A_1) + (\bar{y}_2 A_2) + (\bar{y}_3 A_3)}{A_1 + A_2 + A_3}$$

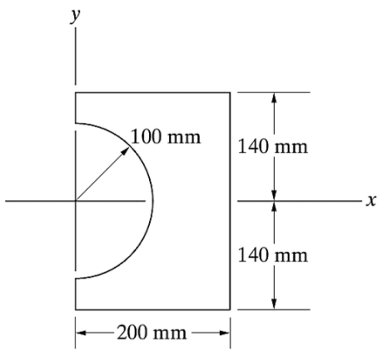
$$Centroid\ is\ (\bar{X}, \bar{Y})$$

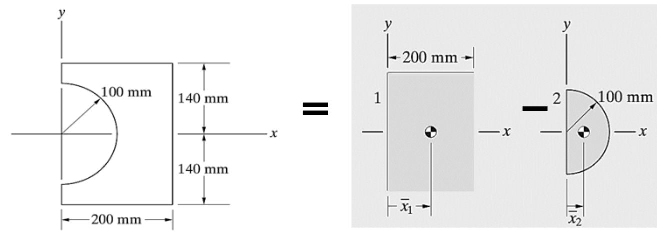
Note:
THE ELEMENTAL AREA CENTROID VALUES MAY BE NEGATIVE OR POSITIVE DEPENDING ON THE LOCATION OF THE ORIGIN OF THE COMPOSITE AREA BEING CONSIDERED.

Centroid of a Plane area

Example

For the plane area shown, determine the location of the centroid.





	$\bar{x}_i$ (mm)	A_i (mm^2)	$\bar{x}_i A_i$ (mm^3)
Part 1 (rectangle)	100	(200)(280)	(100)[(200)(280)]
Part 2 (cutout)	$\frac{4(100)}{3\pi}$	$-\frac{1}{2}\pi(100)^2$	$-\frac{4(100)}{3\pi}\left[\frac{1}{2}\pi(100)^2\right]$

Calculate the Centroid The x coordinate of the centroid is

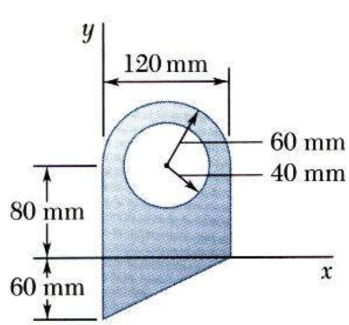
$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2}{A_1 + A_2} = \frac{(100)[(200)(280)] - \frac{4(100)}{3\pi}\left[\frac{1}{2}\pi(100)^2\right]}{(200)(280) - \frac{1}{2}\pi(100)^2} = 122 \text{ mm}$$

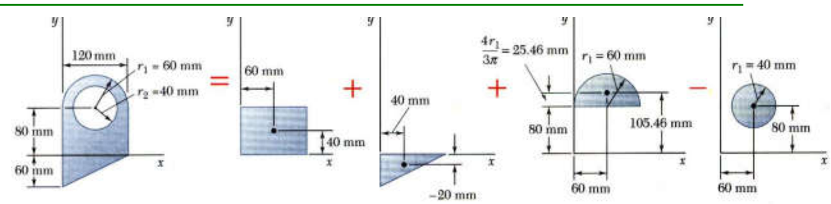
Because of the symmetry of the area, $\bar{y} = 0$.

Centroid of a Plane area

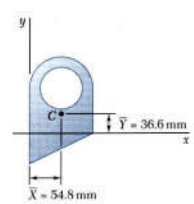
Example

For the plane area shown, determine the first moments with respect to the x and y axes and the location of the centroid.





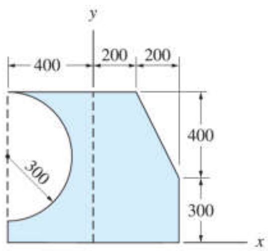
Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$



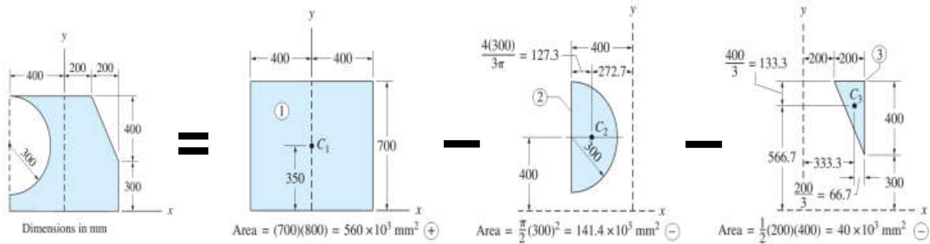
Centroid of a Plane area

Example

For the plane area shown, determine the location of the centroid.



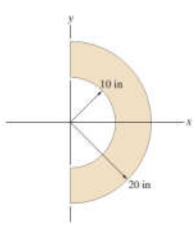
Dimensions in mm

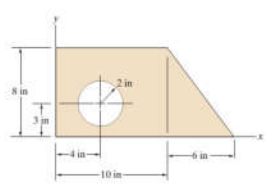


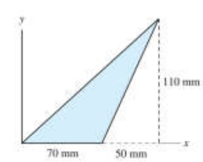
Shape	Area A (mm^2)	$\bar{x}$ (mm)	$A\bar{x}$ (mm^3)	$\bar{y}$ (mm)	$A\bar{y}$ (mm^3)
1 (Rectangle)	$+560.0 \times 10^3$	0	0	+350	196.0×10^6
2 (Semicircle)	-141.4×10^3	−272.7	$+38.56 \times 10^6$	+400	-56.56×10^6
3 (Triangle)	-40.0×10^3	+333.3	-13.33×10^6	+566.7	-22.67×10^6
Σ	$+378.6 \times 10^3$	···	$+25.23 \times 10^6$	···	$+116.77 \times 10^6$

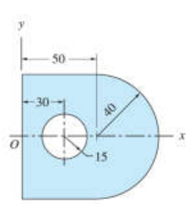
Centroid of a Plane area

Determine the co-ordinates of the centroids of the plane areas shown.

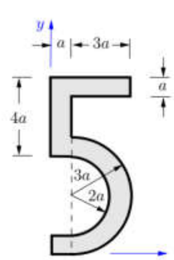




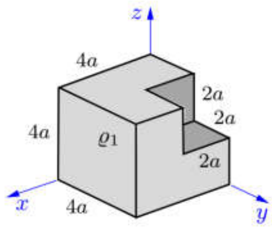


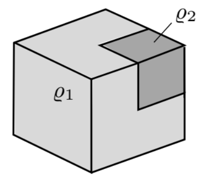


Dimensions in mm



Centroid of a Volume





i	x_i	y_i	z_i	V_i	x_iV_i	y_iV_i	z_iV_i
1	$2a$	$2a$	$2a$	$64a^3$	$128a^4$	$128a^4$	$128a^4$
2	a	$3a$	$3a$	$-8a^3$	$-8a^4$	$-24a^4$	$-24a^4$
		$\sum$		$56a^3$	$120a^4$	$104a^4$	$104a^4$

$$\rightarrow \quad \underline{\underline{x_c}} = \frac{120}{56}\,a = \underline{\underline{\frac{15}{7}\,a}}\,, \qquad \underline{\underline{y_c = z_c}} = \frac{104}{56}\,a = \underline{\underline{\frac{13}{7}\,a}}\,.$$

Static Equilibrium Of Particles And Rigid Bodies

Static Equilibrium
Procedure for analyzing static equilibrium problems
Free Body Diagrams