FORCES & MOMENTS
Moment of a Force

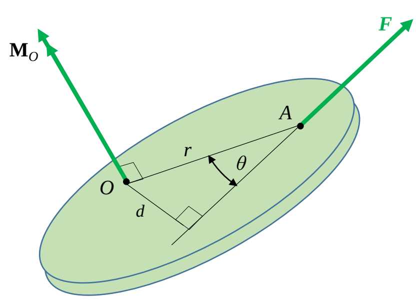
- Forces have the tendency to cause two types motions in rigid bodies; translational and rotational motions (also known as *Torque*).

- The tendency of a force to rotate a body is referred to as moment, given by the product of the force and the perpendicular distance between it's line of action and the point or axis that the body is rotating about.

- It follows that a moment may occur about a point or an axis.

FORCES & MOMENTS
Moment of a Force

- Moment about a point tends to rotate a body about that point, the ***Moment Centre***.

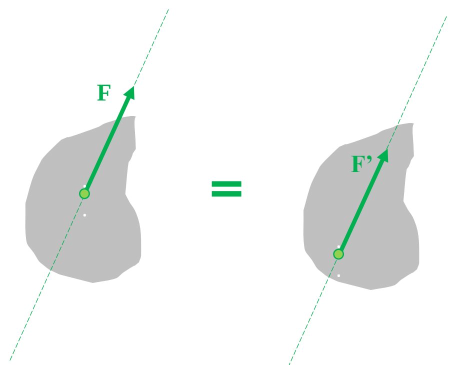


$$\vec{M}_o = \vec{r} \times \vec{F}$$
$$r\sin\theta = \mathrm{d}$$
$$M_o = Fr\sin\theta = Fd$$

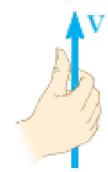
- *d* must be in a specific direction (determined by the line of action of the force). As such, it can be represented by a vector.

FORCES & MOMENTS
Moment of a force

- *d* is always perpendicular to the Force's line of action, so, the Force is treated as a sliding vector, due to the principle of transmissibility in rigid body mechanics.

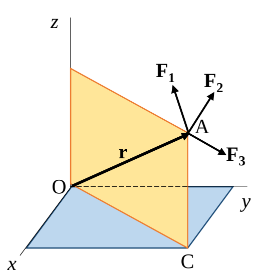


- The direction of the moment is determined by the right hand rule.

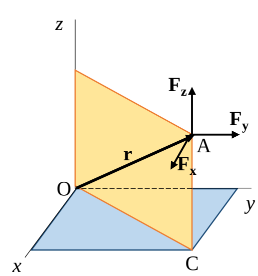


FORCES & MOMENTS
Principle of Moments (Varignon's Theorem)

- States that the moment about a given point *O* of the resultant of several concurrent forces is equal to the sum of the moments of the various forces about the same point *O*.
- In effect, moments of force components can be taken to get the components of a resultant moment.



$$M_O = \sum (r \times F)$$
$$= (r \times F_1) + (r \times F_2) + (r \times F_3)$$
$$= r \times (F_1 + F_2 + F_3)$$
$$= r \times R$$



$$M_O = \sum (r \times F)$$
$$= (r \times F_x) + (r \times F_y) + (r \times F_z)$$
$$= r \times (F_x + F_y + F_z)$$
$$= r \times R$$

FORCES & MOMENTS
Calculating the Moment of a force

- Scalar Approach

  $M_o = Fd$

  - Only the magnitude of the moment is calculated using only the magnitudes of the force and the moment arm, *d*.
  - Used when the moment, *d* can easily be determined. The sense of the moment is determined by inspection.

- Vector Approach
  - The force and the distance from the moment center to the line of action of the force are both expressed as vectors and their cross product determined.

FORCES & MOMENTS
Calculating the Moment of a force

- Multiplying Vectors
  - Vectors are expressed in components, arranged in a matrix form, and the determinant of the matrix taken.

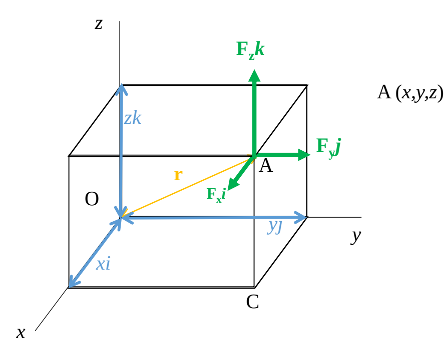
If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$

Expressing as a matrix,

$$\vec{M}_O = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

Taking the determinant of the matrix,

$$\vec{M}_O = (yF_z - zF_y)i - (xF_z - zF_x)j + (xF_y - yF_x)k$$
$$= \vec{M}_x i + \vec{M}_y j + \vec{M}_z k$$



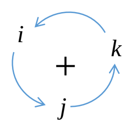
FORCES & MOMENTS
Calculating the Moment of a force

- Multiplying Vectors
  - Alternatively, the vector components are multiplied.

$$M_O = r \times F$$
$$= \left(x\vec{i} + y\vec{j} + z\vec{k}\right) \times \left(F_x\vec{i} + F_y\vec{j} + F_z\vec{k}\right)$$
$$= \left[\left(x\vec{i} + y\vec{j} + z\vec{k}\right) \times F_x\vec{i}\right] + \left[\left(x\vec{i} + y\vec{j} + z\vec{k}\right) \times F_y\vec{j}\right] + \left[\left(x\vec{i} + y\vec{j} + z\vec{k}\right) \times F_z\vec{k}\right]$$

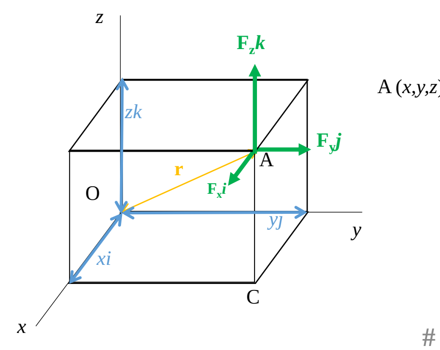
But

$$i \times i = 0 \quad j \times i = -k \quad k \times i = j$$
$$i \times j = k \quad j \times j = 0 \quad k \times j = -i$$
$$i \times k = -j \quad j \times k = i \quad k \times k = 0$$

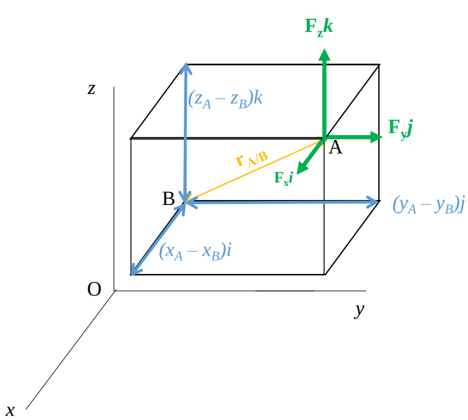


Therefore,

$$M_O = -yF_x k + zF_x j + xF_y k - zF_y i - xF_z j + yF_z i$$
$$= \left(yF_z - zF_y\right)i - \left(xF_z - zF_x\right)j + \left(xF_y - yF_x\right)k$$



FORCES & MOMENTS
Calculating the Moment of a force at a certain point about an arbitrary point



In this case,

$$F = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} \text{ and}$$
$$\vec{r}_{A/B} = \vec{x}_{A/B} + \vec{y}_{A/B} + \vec{z}_{A/B}$$
$$= \left(x_A - x_B\right)\vec{i} + \left(y_A - y_B\right)\vec{j} + \left(z_A - z_B\right)\vec{k}$$

Expressed as a matrix,

$$M_B = r \times F = \begin{vmatrix} i & j & k \\ (x_A - x_B) & (y_A - y_B) & (z_A - z_B) \\ F_x & F_y & F_z \end{vmatrix}$$

Take the determinant of the matrix to find components of the Moment

FORCES & MOMENTS
Calculating the Moment of a force

Example

A 100-lb vertical force is applied to the end of a lever which is attached to a shaft at *O*.

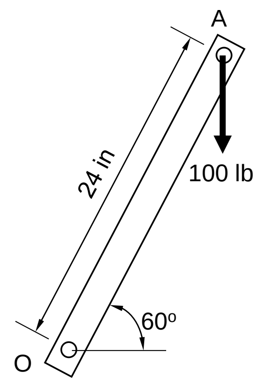
Determine:

a) moment about *O*,

b) horizontal force at *A* which creates the same moment,

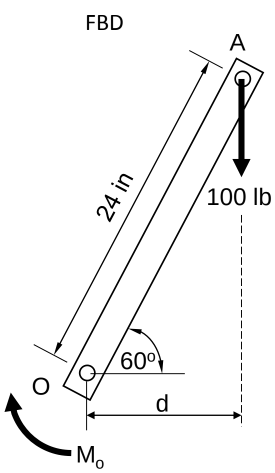
c) smallest force at A which produces the same moment,

d) location for a 240-lb vertical force to produce the same moment,



FORCES & MOMENTS
Calculating the Moment of a force

Solution



Moment about *O* is equal to the product of the force and the perpendicular distance between the line of action of the force and *O*.

$$M_O = Fd$$

$$d = (24\,\text{in.})\cos 60° = 12\text{ in.}$$

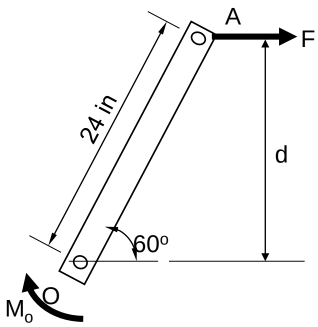
$$M_O = (100\text{ lb})(12\text{ in.}) =$$

Since the force tends to rotate the lever clockwise, the moment vector is into the plane of the paper.

FORCES & MOMENTS
Calculating the Moment of a force

Solution

FBD



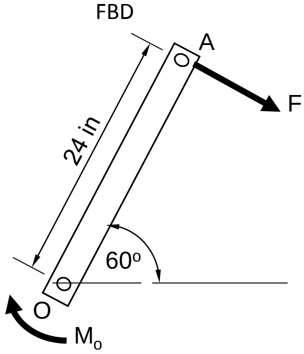
Horizontal force at *A* that produces the same moment,

$$d = (24\text{ in.})\sin 60° = 20.8\text{ in.}$$
$$M_O = Fd$$
$$1200\text{ lb}\cdot\text{in.} = F(20.8\text{ in.})$$
$$F = \frac{1200\text{ lb}\cdot\text{in.}}{20.8\text{ in.}}$$
$$=$$

FORCES & MOMENTS
Calculating the Moment of a force

Solution

FBD

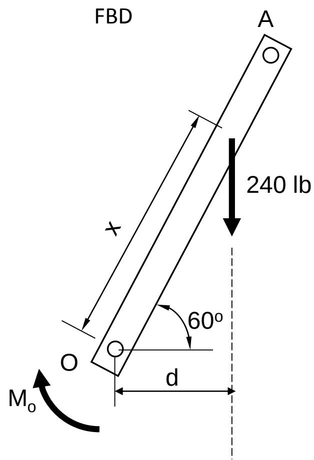


The smallest force *A* to produce the same moment occurs when the perpendicular distance is a maximum or when *F* is perpendicular to *OA*.

$$M_O = Fd$$
$$1200\text{ lb}\cdot\text{in.} = F(24\text{ in.})$$
$$F = \frac{1200\text{ lb}\cdot\text{in.}}{24\text{ in.}}$$
$$=$$

FORCES & MOMENTS
Calculating the Moment of a force

Solution



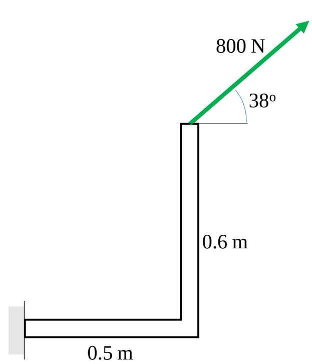
The point of application of a 240 lb force to produce the same moment,

$$M_O = Fd$$
$$1200\,\text{lb}\cdot\text{in.} = (240\,\text{lb})d$$
$$d = \frac{1200\,\text{lb}\cdot\text{in.}}{240\,\text{lb}} = 5\,\text{in.}$$
$$OB = 5\cos^{-1}60° =$$

FORCES & MOMENTS
Calculating the Moment of a force

Example

Determine the moment of the 800 N force about point A.



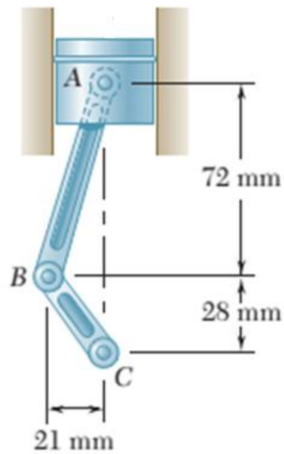
Ans: $\circlearrowleft + \; M_A = -131.99\text{ Nm}$

FORCES & MOMENTS
Calculating the Moment of a force

Example

It is known that the connecting rod AB exerts on the crank BC a 1.5 kN force directed down and to the left along the centerline of AB.

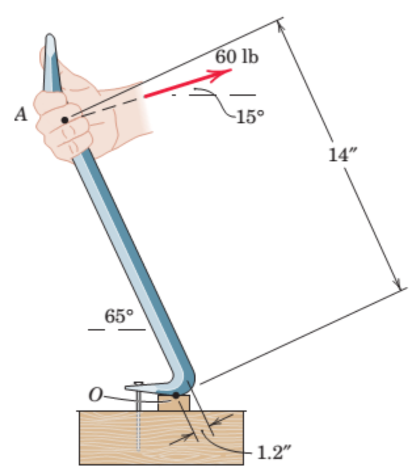
- Determine the moment of the force about C.
- Determine the moment of that same force about C if it were being exerted at B, parallel to the centreline AC.



FORCES & MOMENTS
Calculating the Moment of a force

Example

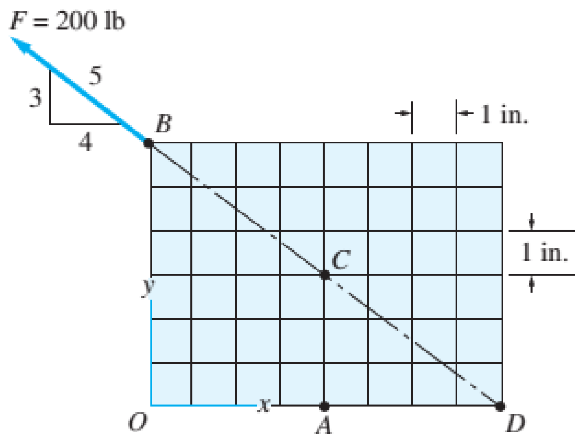
A crowbar is used to remove a nail as shown. Determine the moment of the 60 lb force about the point *O* of contact between the crowbar and the small support block.



FORCES & MOMENTS
Calculating the Moment of a force

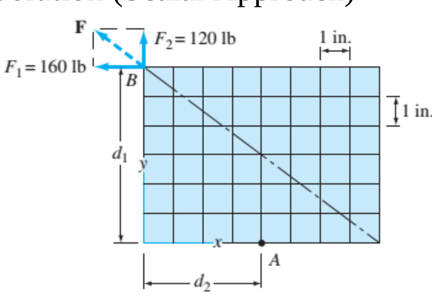
Example

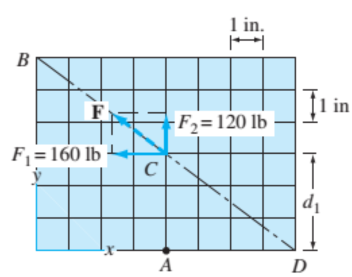
Determine the moment of the force F about point A.



FORCES & MOMENTS
Calculating the Moment of a force

Solution (Scalar Approach)





$$\circlearrowleft + \quad M_A = F_1 d_1 - F_2 d_2$$
$$= 160(6) - 120(4) = 480 \text{ lb} \cdot \text{in.}$$

$$\mathbf{M}_A = 480\mathbf{k} \text{ lb} \cdot \text{in.}$$

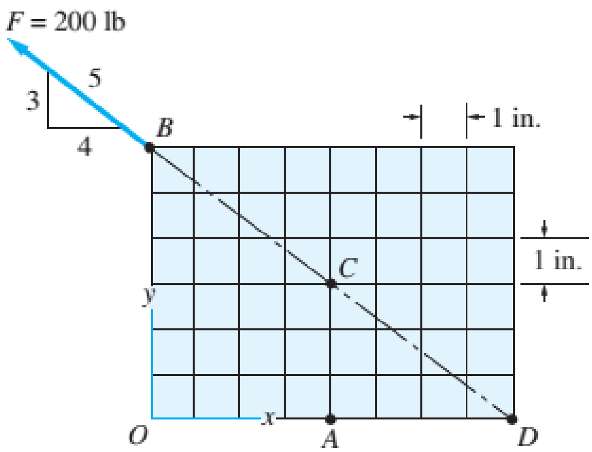
OR

The vector is treated as a sliding vector and moved to point C

$$\circlearrowleft + \quad M_A = F_1 d_1 = 160(3) = 480 \text{ lb} \cdot \text{in.}$$

FORCES & MOMENTS
Calculating the Moment of a force

Solution (Vector Approach)



$$\vec{F} = -\left(\frac{4}{5}\right)200i + \left(\frac{3}{5}\right)200j$$
$$= -160i + 120j$$

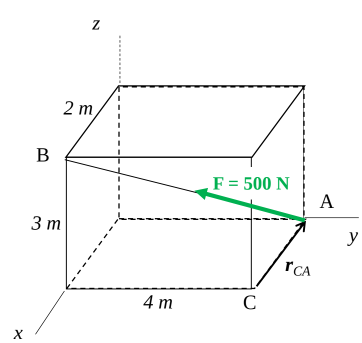
$$\vec{r} = \vec{r}_{AB} = r_B - r_A = (0-4)i + (6-0)j = -4i + 6j$$

$$\vec{M}_A = \vec{r} \times \vec{F} = \vec{r}_{AB} \times \vec{F} = \begin{vmatrix} i & j & k \\ -4 & 6 & 0 \\ -160 & 120 & 0 \end{vmatrix}$$
$$= 480k \text{ lb.in}$$

FORCES & MOMENTS
Calculating the Moment of a force

Example

Determine the moment of the force *F*, about point *C* the perpendicular distance between C and the line of action of F.

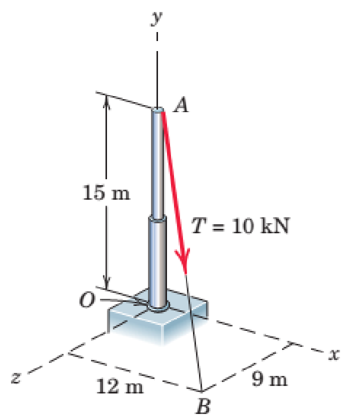


Mc=556.58j+742.12kNm
=927.64 N.m

FORCES & MOMENTS
Calculating the Moment of a force

Example

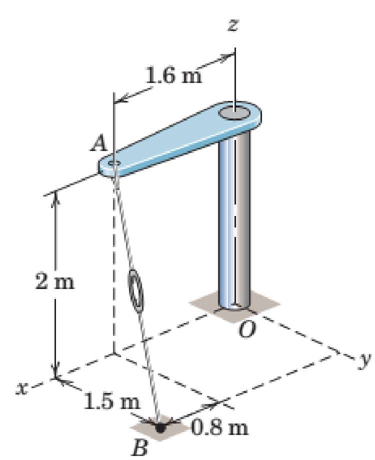
A tension of magnitude 10 kN is applied to the cable attached to the top A of the rigid mast and secured to the ground at B. Determine the moment of T about the point O.



FORCES & MOMENTS
Calculating the Moment of a force

Example

The turn buckle is tightened until the tension is cable AB is 2.4 kN. Determine the moment about point O of the cable force acting on point A and the magnitude of this moment.

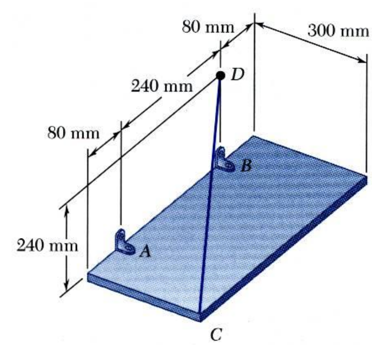


-2.74i+4.39j+2.19k kN.m

FORCES & MOMENTS
Calculating the Moment of a force

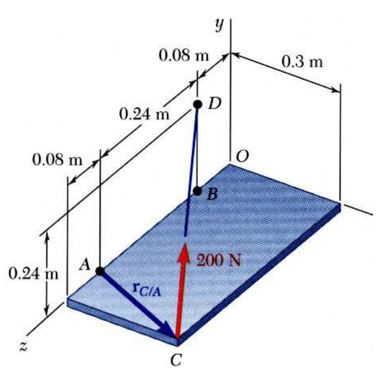
Example

The rectangular plate is supported by the brackets at *A* and *B* and by a wire *CD*. Knowing that the tension in the wire is 200 N, determine the moment about *A* of the force exerted by the wire at *C*.



FORCES & MOMENTS
Calculating the Moment of a force

Solution



$$\vec{M}_A = \vec{r}_{C/A} \times \vec{F}$$

$$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = (0.3\,\text{m})\vec{i} + (0.08\,\text{m})\vec{j}$$

$$\vec{F} = F\vec{\lambda} = (200\,\text{N})\frac{-(0.3\,\text{m})\vec{i} + (0.24\,\text{m})\vec{j} - (0.32\,\text{m})\vec{k}}{0.5\,\text{m}}$$
$$= -(120\,\text{N})\vec{i} + (96\,\text{N})\vec{j} - (128\,\text{N})\vec{k}$$

$$\vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.3 & 0 & 0.08 \\ -120 & 96 & -128 \end{vmatrix}$$

$$\vec{M}_A = -(7.68\,\text{N}\cdot\text{m})\vec{i} + (28.8\,\text{N}\cdot\text{m})\vec{j} + (28.8\,\text{N}\cdot\text{m})\vec{k}$$