KWAMENKRUMAHUNIVERSITY OF SCIENCE AND TECHNOLOGY

INSTITUTE OF DISTANCE LEARNING

END OF SEMESTER EXAMINATIONS

(FIRST SEMESTER 2013/2014)

MATH 265 Mathematical Methods I

December 2013 **Time: 2 Hours 30 Minutes**

Bsc. Statistics 3

Index Number: 8524912

Faculty/Department: ______

Instructions:

- Answer **ALL** in Section A and **ONE** question from Section B. **Shade on the Scanable Sheet the letter that corresponds to your choice of the correct answer to each Question.**
- Please make sure you have all **Thirteen (12) pages** of questions.
- Write your **Index Number** boldly in the space provided on this front page and every other sheet.
- **Programmable and Graphing Calculators are NOT ALLOWED for this Paper.**

1. Find the domain of the function $f(x,y) = x\ln\left(y^2 - x\right)$.

$(a)\ D = \{(x,y) | x \le y^2\}$

(b) $D = \{(x,y) | x < y^2\}$

(c) $D = \{(x,y) | x > y^2\}$

(d) $D = \{(x$

2. Find the domain of the function $f(x,y) = \dfrac{\sqrt{x+y+1}}{x-1}$.

$(a)\ D = \{(x,y) | x+y+1 \ge 0\}$

(b) $D = \{(x,y) | x+y+1 > 0\}$

(c) $D = \{(x,y) | x+y+1 > 0, x \ne 1\}$

(d) $D = \{(x,y) | x+y+1 \ge 0, x \ne 1\}$

3. Find the range of $h(x,y) = \sqrt{9 - x^2 - y^2}$.

$(a)[0,3)$

(b) $(0,3)$

$(c)\ (0,3]$

$(d)[0,$

4. Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - y^2}{x^2 + y^2}$.

$(a)1$

(b) -1

$(c)0$

(d) Does not exist

5. Find the domain of the function $f(x,y,z) = \ln(z-y) + xy\sin x$.

$(a)\, D = \{(x,y,z) \in \mathrm{R}^3 \,|\, z > y\}$

(b) $D = \{(x,y,z) \in \mathrm{R}^3 \,|\, z \geq y\}$

(c) $D = \{(x,y,z) \in \mathrm{R}^3 \,|\, z < y\}$

(d) $D = \{(x,y,z) \in \mathrm{R}^3 \,|\, z \leq y\}$

6. Evaluate $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$, evaluate $\lim_{(x,y)\to(0,0)} f(x,y)$.

(a) 0

(b) 1/2

(c) 1

(d) Does not exist

7. Thefunction $f(x,y) = \begin{cases} \dfrac{xy^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$, is discontinuous at $(0,0)$. Determine

whether this discontinuity is removable, essential or otherwise.

(a) Removable

(b) Essential

(c) Removable and Essential

(d) Insufficient information

8. if $f(x,y) = \sin\left(\dfrac{x}{1+y}\right)$, calculate $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

$(a)\, \dfrac{\partial f}{\partial x} = \cos\left(\dfrac{x}{1+y}\right).\dfrac{1}{1+y}$ and $\dfrac{\partial f}{\partial x} = -\cos\left(\dfrac{x}{1+y}\right).\dfrac{1}{(1+y)^2}$

(b) $\dfrac{\partial f}{\partial x} = \cos\left(\dfrac{x}{1+y}\right).\dfrac{x}{1+y}$ and $\dfrac{\partial f}{\partial x} = -\cos\left(\dfrac{x}{1+y}\right).\dfrac{x}{(1+y)^2}$

(c) $\dfrac{\partial f}{\partial x} = \cos\left(\dfrac{x}{1+y}\right).\dfrac{1}{1+y}$ and $\dfrac{\partial f}{\partial x} = -\cos\left(\dfrac{x}{1+y}\right).\dfrac{x}{(1+y)^2}$

(d) $\dfrac{\partial f}{\partial x} = \cos\left(\dfrac{x}{1+y}\right).\dfrac{x}{1+y}$ and $\dfrac{\partial f}{\partial x} = -\cos\left(\dfrac{x}{1+y}\right).\dfrac{y}{(1+y)^2}$

9. Find $\frac{\partial f}{\partial x}$ *and* $\frac{\partial f}{\partial y}$ if z is defined implicitly as a function of *x and y* by the equation $x^3 + y^3 + 6xyz + z^3 = 1$.

(a) $\frac{\partial f}{\partial x} = -\frac{x^2 + 2yz}{z^2 + 2xy}$ *and* $\frac{\partial f}{\partial x} = -\frac{y^2 + 2xz}{z^2 + 2xy}$

(b) $\frac{\partial f}{\partial x} = -\frac{x^2 + 2yz}{z^2 + 2xy}$ *and* $\frac{\partial f}{\partial x} = \frac{y^2 + 2xz}{z^2 + 2xy}$

(c) $\frac{\partial f}{\partial x} = \frac{x^2 + 2yz}{z^2 + 2xy}$ *and* $\frac{\partial f}{\partial x} = \frac{y^2 + 2xz}{z^2 + 2xy}$

(d) $\frac{\partial f}{\partial x} = \frac{x^2 + 2yz}{z^2 + 2xy}$ *and* $\frac{\partial f}{\partial x} = \frac{y^2 + 2zy}{z^2 + 2xy}$

10. Calculate f_{xyz}, if $f(x, y, z) = \sin(3x + yz)$.

(a) $-9\cos(3x + yz) - 9\,yz\sin(3x + yz)$

(b) $-9\cos(3x + yz) + 9\,yz\sin(3x + yz)$

(c) $9\cos(3x + yz) + 9\,yz\sin(3x + yz)$

(d) $9y\cos(3x + yz) + 9\,yz\sin(3x + yz)$

11.. Given that $f(x, y) = \ln\left(x + \sqrt{x^2 + y^2}\right)$; find $f_x(3,4)$.

(a) 1/2

(b) 2/5

(c) 1/5

(d) 2/7

12. Given that $f(x, y, z) = \frac{y}{x + y + z}$; find $f_x(2,1,-1)$.

(a) 1/4

(b) 1/5

(c) 1/6

(d) 2/3

13. Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1,1,3)$.

(a) $z = 4x - 2y - 3$

(b) $z = 4x - 2y + 3$

(c) $z = -4x - 2y + 3$

(d) $z = 4x + 2y - 3$

14. If $z = f(x, y) = x^2 + 3xy - y^2$, find the differential dz

(a) $dz = (2x - 3y)dx + (3x - 2y)dy$

(b) $dz = (2x + 3y)dx + (3x - 2y)dx$

(c) $dz = (2x + 3y)dx + (3x + 2y)dy$

(d) $dz = (3x + 2y)dx + (2x + 3y)dy$

16. Find the linearization $L(x, y)$ of the function $f(x, y) = e^{4y}$ at the point $(\pi, 0)$.

(a) $1 + \pi y$

(b) πy

(c) $1 - \pi y$

(d) $\pi y - 1$

17. Find the linear approximation of the function $f(x, y) = \sqrt{20 - x^2 - 7y^2}$ at $(2,1)$ and use it to approximate $f(1.98, 1.08)$.

(a) $-\frac{2}{3}x + \frac{7}{3}y + \frac{20}{3}$; 2.846

(b) $-\frac{2}{3}x - \frac{7}{3}y + \frac{20}{3}$; 2.864

(c) $\frac{2}{3}x + \frac{7}{3}y - \frac{20}{3}$; 2.846

(d) $-\frac{2}{3}x - \frac{7}{3}y + \frac{20}{3}$; 2.846

18. Find the equation of the tangent plane to the given surface $z = e^{x^2+y^2}$ at the specified point $(1,-1,1)$.

(a) $x + y - 2e = 0$

(b) $x + y - 2e^2 = 0$

(c) $x + y + 2e^2 = 0$

(d) $x - y - 2e^{2x} = 0$

19. If $z = x^2 + y^2 + 3xy^4$, where $x = \sin 2t$ and $y = \cos t$, find dz/dt when $t = 0$.

(a) 7

(b) 5

(c) 6

(d) 2

20. If $z = e^x \sin y$, where $x = st^2$ and $y = s^2 t$, find $\partial z / \partial s$.

(a) $t^2 e^{st^2} \sin(s^2 t) + 2ste^{st^2} \cos(s^2 t)$

(b) $te^{st^2} \sin(s^2 t) + 2ste^{st^2} \cos(s^2 t)$

(c) $t^2 e^{st^2} \sin(s^2 t) + 2te^{st^2} \cos(s^2 t)$

(d) $t^2 e^{st^2} \sin(s^2 t) + 2se^{st^2} \cos(s^2 t)$

21. The rate of change of a function in a specified direction is called

a. Derivative

b. Level Curves

c. Level Set

d. Directional derivative

22. Let $z = f(x,y)$, $x = g(s,t)$, and $y = h(s,t)$. Find an expression for $\dfrac{\partial z}{\partial t}$.

a. $\dfrac{\partial z}{\partial t} = \dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial t} - \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial t}$

b. $\dfrac{\partial z}{\partial t} = \dfrac{\partial f}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial t}$

c. $\dfrac{\partial z}{\partial t} = \dfrac{\partial f}{\partial t}\dfrac{\partial x}{\partial f} + \dfrac{\partial f}{\partial y}\dfrac{\partial y}{\partial f}$

d. $\dfrac{\partial z}{\partial t} = \dfrac{\partial f}{\partial x}\dfrac{\partial y}{\partial t} + \dfrac{\partial f}{\partial y}\dfrac{\partial x}{\partial t}$

23. Identify the surface $\rho \sin \varphi = 2$

a.	A cylinder of radius 4 centered on the x – axis
b.	A cylinder of radius 2 centered on the y – axis
c.	A cylinder of radius 2 centered on the z – axis
d.	A cylinder of radius 2 centered on the x – axis

$r = \rho \sin \theta$

$z = \rho \cos \theta$

24. $\iint_R 6xy^2 dA, \quad R = [2,4] \times [1,2]$

a. 84 b. 85

c. 24 d. 25

25. Determine the set where the function $f(x,y) = \ln(x^2 + y^2)$ is continuous.

a. $\{(x,y):(x,y) \in R^2, x^2 + y^2 > 0\}$ b. $\{(x,y):(x,y) \in R^2, x^2 + y^2 < 0\}$

c. $\{(x,y):(x,y) \in R^2, x^2 + y^2 \le 0\}$ d. $\{(x,y):(x,y) \in R^2, x^2 + y^2 \ge 0\}$

26. Find the equation of the tangent plane to $z = \ln(2x + y)$ at $(-1,3)$.

$\frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2}$

a. $z = 2x + y - 1$ b. $z = 2x + y + 1$

c. $z = 2x + 2y - 1$ d. $z = 2x + 2y + 1$

27. Find the equation of the normal line to $x^2 + y^2 + z^2 = 30$ at the point $(1,-2,5)$.

a. $\langle 1+2t, -2-4t, 5+10t \rangle$ b. $\langle 1+2t, 2-4t, 5+10t \rangle$

c. $\langle 1+2t, 2-4t, 5-10t \rangle$ d. $\langle 1-2t, -2-4t, 5+10t \rangle$

28. Find the derivative of $f(x,y,z) = x \cos y + 3z^3 - xz$ at $(1, \pi, 1)$ in the direction of the vector $[3,-2,2]$.

a. $-\frac{10}{\sqrt{17}}$ b. $\frac{10}{\sqrt{17}}$

c. $-\frac{11}{\sqrt{17}}$ d. $\frac{11}{\sqrt{17}}$

29. The critical point (0,1) of $f(x,y) = \frac{1}{3}x^3 - x + xy^2$ is a ....

a. Saddle point b. Absolute Maximum

c. Relative Maximum d. Absolute Minimum

A hen's egg is $6cm$ *in length and has a surface roughly described by the equations* $z = (x^2 + y^2)/2$ *and* $z = 6 - x^2 - y^2$. *Use this information to answer questions* 30 *and* 31.

30. Write the volume of the egg as a double in polar coordinates.

a. $\int_0^{\pi}\int_0^2 \left(6r - \frac{3}{2}r^3\right) drd\theta$ b. $\int_0^{\pi}\int_0^2 \left(6 - \frac{3}{2}r^2\right) drd\theta$

c. $\int_0^{2\pi}\int_0^2 \left(6r - \frac{3}{2}r^3\right) drd\theta$ d. $\int_0^{2\pi}\int_0^2 \left(6 - \frac{3}{2}r^2\right) drd\theta$

31. Find the volume of the egg.

a. $6\pi cm^3$ b. $8\pi cm^3$

c. $12\pi cm^3$ d. $16\pi cm^3$

32. A solid in the first octant is bounded by the surface $z = 9 - y^2$ and the plane $x = 2$. Rewrite the volume of this solid as a double integral.

a. $\int_{-3}^{3}\int_0^2 9 - y^2 \; dxdy$ b. $\int_{-3}^{3}\int_{-2}^2 9 - y^2 \; dxdy$

c. $\int_0^{9}\int_0^2 9 - y^2 \; dxdy$ d. $\int_0^{3}\int_0^2 9 - y^2 \; dxdy$

33. Reverse the order of the integral $\int_0^3\int_{y^2}^9 y\cos(x^2)\; dxdy$.

a. $\int_0^9\int_0^{\sqrt{x}} y\cos(x^2)\; dydx$ b. $\int_0^9\int_0^{\sqrt{x}} y\cos(x^2)\; dydx$

c. $\int_0^9\int_0^{\sqrt{x}} y\cos(x^2)\; dydx$ d. $\int_0^9\int_0^{\sqrt{x}} y\cos(x^2)\; dydx$

34. The function $f(x,y) = \begin{cases} \dfrac{xy^2}{x^2 + y^6} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ is discontinuous at $(0,0)$. Determine whether this discontinuity is removable, essential or otherwise.

a. Removable
b. Essential
c. Removable and Essential.
d. Insufficient information to conclude

35. Evaluate $\lim_{(x,y)\to(0,0)} f(x,y)$, where $f(x,y) = \begin{cases} 1, & \text{if } y = x^2 + 1 \\ 0, & \text{if } y \neq x^2 + 1 \end{cases}$.

a. 0 c. 2

b. 1 d. ∞

36. Let $g(x_1, x_2, x_3) = (x_1x_3, x_2x_3 + 1)$ and $f(x_1, x_2) = (x_1x_2 \cos x_1, x_1 + 3x_2, x_2 - 2x_1^2)$. Find the derivative of $g \circ f \equiv g[f(x_1, x_2)]$ at $(1, -4)$.

a. $\begin{pmatrix} 40\cos 1 + 24\sin 1 & -10\cos 1 \\ 38 & 29 \end{pmatrix}$

b. $\begin{pmatrix} 40\cos 1 + 24\sin 1 & 10\cos 1 \\ 38 & 29 \end{pmatrix}$

c. $\begin{pmatrix} 40\cos 1 - 24\sin 1 & 10\cos 1 \\ 38 & 29 \end{pmatrix}$

d. $\begin{pmatrix} 40\cos 1 - 24\sin 1 & -10\sin 1 \\ 38 & -29 \end{pmatrix}$

37. Find the directional derivatives of $f = 3x^2y + \sin xy$ in the direction $v = \langle 1, 2 \rangle$ at the point $p = (2, 3)$.

a. $7 + 6\cos 60$

b. $70 + 6\cos 6$

c. $6 + 7\cos 60$

d. $60 + 7\cos 6$

Given that $f(x, y) = \sqrt{9 - x^2 - 4y^2}$, answer Questions 38 – 39

38. Find the domain of $f(x, y)$

a. $x^2 + 4y^2 > 9$

b. $x^2 + 4y^2 < 9$

c. $x^2 + 4y^2 \geq 9$

d. $x^2 + 4y^2 \leq 9$

39. Find the range of $f(x, y)$

a. $(0, 3)$

b. $(0, 3]$

c. $[0, 2]$

d. $[0, 3]$

a-

Use the following to answer 40 – 41. $f(x, y) = \left(x^2 + y^2\right)^{1/2}$.

40. $f(x, y)$ is homogeneous function of order:

a. 0

b. 2

c. 1

d. 1/2

41. $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = .$

a. $f(x, y)$

b. $\frac{f(x,y)}{2}$

c. $\frac{x+y}{f(x,y)}$

d. $2f(x, y)$

42. Compute the directional derivatives of the following functions in the indicated direction and from the given point. $f = x^2 + y^2 + z^2$, $v = \langle 1,1,1 \rangle$, $p = (1,2,1)$

a. 7
b. 8
c. 9
d. 10

Compute the partial derivatives with respect to x and y:

43. $f = e^{g(x,y,z)}$, where $z = xy$

a. $f_x = e^g(g_x + g_z x),\ f_y = e^g(g_y + g_z y)$
b. $f_x = e^g(g_z + g_x y),\ f_y = e^g(g_y + g_z x)$
c. $f_x = e^g(g_x + g_z y),\ f_y = e^g(g_z + g_y x)$
d. $f_x = e^g(g_x + g_z y),\ f_y = e^g(g_y + g_z x)$

44. $f = g(x,u)$, where $u = yh(x,y)$

a. $f_x = yg_x + g_u h_x,\ f_y = g_u(h + yh_y)$
b. $f_x = g_x + g_u yh_x,\ f_y = g_u(yh + h_y)$
c. $f_x = g_x + g_u yh_x,\ f_y = g_u(h + yh_y)$
d. $f_x = g_x + g_u yh_x,\ f_y = g_u(h_x + yh_y)$

45. Find the equations of the tangent plane and the normal to the given surface at the point P.
$z = e^{3y} \sin 3x,\quad P(\pi/6, 0, 1)$.

a. $3y - z = 1,\ x = \pi/6\quad y = 3t\quad z = 1 - t$
b. $3y + z = -1,\ x = \pi/6\quad y = 3t\quad z = 1 - t$
c. $3y - z = -1,\ x = \pi/6\quad y = 3t\quad z = 1 + t$
d. $3y - z = -1,\ x = \pi/6\quad y = 3t\quad z = 1 - t$

46. Let S be the surface described by $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$. Determine an equation of the tangent plane to S at the point $\left(1,1,\frac{\pi}{4}\right)$.

a. $z = \frac{\pi}{4} - \frac{1}{2}(x-1) - \frac{1}{2}(y-1)$
b. $z = \frac{\pi}{4} - \frac{1}{2}(x-1) + \frac{1}{2}(y-1)$
c. $z = \frac{1}{2}(x-1) - \frac{1}{2}(y-1)$
d. $z = \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{2}(y-1)$

47. Let $f(x,y)=\dfrac{3}{x}+2xy$ for $x\neq 0$. If the gradient of f at (a, b) has length b, then which of the following equations must a and b satisfy?

a. $3+2a^2b-ab=0$

b. $9+6a^2b+4a^4b^2-a^4b=0$

c. $9+12a^2b+3a^4b^2+4a^6=0$

d. $9-12a^2b+3a^4b^2+4a^6=0$

48. The rate of change of a function in a specified direction is called

a. Derivative

b. Level Curves

c. Level Set

d. Directional derivative

Use the following to answer questions 49 and 50. Let $z=e^{xy}$, $x=2u+v$, $y=u/v$.

49. Find $\dfrac{\partial z}{\partial u}$.

a. $\dfrac{\partial z}{\partial u}=\left[\dfrac{4u}{v}+1\right]e^{(2u+v)(u/v)}$

b. $\dfrac{\partial z}{\partial u}=\left[\dfrac{v}{4u}+1\right]e^{(2u+v)(u/v)}$

c. $\dfrac{\partial z}{\partial u}=\left[\dfrac{4u}{v}-1\right]e^{(2u+v)(u/v)}$

d. $\dfrac{\partial z}{\partial u}=\left[\dfrac{v}{4u}-1\right]e^{(2u+v)(u/v)}$

50. Find $\dfrac{\partial z}{\partial v}$.

b. $\dfrac{\partial z}{\partial v}=\dfrac{2u^2}{v^2}e^{(2u+v)(u/v)}$

b. $\dfrac{\partial z}{\partial v}=-\dfrac{2u^2}{v^2}e^{(2u+v)(u/v)}$

c. $\dfrac{\partial z}{\partial v}=\dfrac{u^2}{2v^2}e^{(2u+v)(u/v)}$

d. $\dfrac{\partial z}{\partial v}=-\dfrac{2v^2}{u^2}e^{(2u+v)(u/v)}$

SECTION B

Answer the following TWO questions

1. What is the spherical coordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$

2. Where does the absolute minimum of $f(x,y) = x^2 + 4y^2 - 2x^2y + 4$ on the rectangle $D = \{(x,y) | -1 \le x \le 1, -1 \le y \le 1\}$ occur?

E. OSEI FRIMPONG

PCU/IDL/289--13