Background
Simultaneous Linear Equations

1. Given [A] = $\begin{bmatrix} 6 & 2 & 3 & 9 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 6 \end{bmatrix}$ then [A] is a ______________ matrix.

(A) diagonal
(B) identity
(C) lower triangular
(D) upper triangular

2. A square matrix [A] is lower triangular if
(A) $a_{ij} = 0, j > i$
(B) $a_{ij} = 0, i > j$
(C) $a_{ij} \neq 0, i > j$
(D) $a_{ij} \neq 0, j > i$

3. Given

$$[A] = \begin{bmatrix} 12.3 & -12.3 & 20.3 \\ 11.3 & -10.3 & -11.3 \\ 10.3 & -11.3 & -12.3 \end{bmatrix}, \ [B] = \begin{bmatrix} 2 & 4 \\ -5 & 6 \\ 11 & -20 \end{bmatrix}$$

then if
$[C] = [A]\ [B]$, then
$c_{31}=$ ______________________

(A) -58.2
(B) -37.6
(C) 219.4
(D) 259.4

4. The following system of equations has ____________ solution(s).

$x + y = 2$

$6x+6y = 12$

(A) infinite
(B) no
(C) two
(D) unique

5. Consider there are only two computer companies in a country. The companies are named *Dude* and *Imac*. Each year, company *Dude* keeps $1/5^{th}$ of its customers, while the rest switch to *Imac*. Each year, *Imac* keeps $1/3^{rd}$ of its customers, while the rest switch to *Dude*. If in *2003*, *Dude* had $1/6^{th}$ of the market and *Imac* had $5/6^{th}$ of the market, what will be share of *Dude* computers when the market becomes stable?

(A) 37/90
(B) 5/11
(C) 6/11
(D) 53/90

6. Three kids - Jim, Corey and David receive an inheritance of \$2,253,453. The money is put in three trusts but is not divided equally to begin with. Corey's trust is three times that of David's because Corey made an *A* in Dr. Kaw's class. Each trust is put in an interest generating investment. The three trusts of Jim, Corey and David pays an interest of 6%, 8%, 11%, respectively. The total interest of all the three trusts combined at the end of the first year is \$190,740.57. The equations to find the trust money of Jim (*J*), Corey (*C*) and David (*D*) in a matrix form is

(A) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 0.06 & 0.08 & 0.11 \end{bmatrix}\begin{bmatrix} J \\ C \\ D \end{bmatrix} = \begin{bmatrix} 2{,}253{,}453 \\ 0 \\ 190{,}740.57 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0.06 & 0.08 & 0.11 \end{bmatrix}\begin{bmatrix} J \\ C \\ D \end{bmatrix} = \begin{bmatrix} 2{,}253{,}453 \\ 0 \\ 190{,}740.57 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 6 & 8 & 11 \end{bmatrix}\begin{bmatrix} J \\ C \\ D \end{bmatrix} = \begin{bmatrix} 2{,}253{,}453 \\ 0 \\ 190{,}740.57 \end{bmatrix}$

d $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 6 & 8 & 11 \end{bmatrix}\begin{bmatrix} J \\ C \\ D \end{bmatrix} = \begin{bmatrix} 2{,}253{,}453 \\ 0 \\ 19{,}074{,}057 \end{bmatrix}$

Gaussian Elimination

1. The goal of forward elimination steps in Naïve Gauss elimination method is to reduce the coefficient matrix to a (an) ____________ matrix.

   (A) diagonal
   (B) identity
   (C) lower triangular
   (D) upper triangular

2. Division by zero during forward elimination steps in Naïve Gaussian elimination of the set of equations [A][X]=[C] implies the coefficient matrix [A] is

   (A) invertible
   (B) nonsingular
   (C) not determinable to be singular or nonsingular
   (D) singular

3. Using a computer with four significant digits with chopping, Naïve Gauss elimination solution to

$$0.0030x_1 + 55.23x_2 = 58.12$$
$$6.239x_1 - 7.123x_2 = 47.23$$

is

   (A) $x_1 = 26.66$; $x_2 = 1.051$
   (B) $x_1 = 8.769$; $x_2 = 1.051$
   (C) $x_1 = 8.800$; $x_2 = 1.000$
   (D) $x_1 = 8.771$; $x_2 = 1.052$

4. Using a computer with four significant digits with chopping, Gaussian elimination with partial pivoting solution to

$$0.0030x_1 + 55.23x_2 = 58.12$$
$$6.239x_1 - 7.123x_2 = 47.23$$

is

   (A) $x_1 = 26.66$; $x_2 = 1.051$
   (B) $x_1 = 8.769$; $x_2 = 1.051$
   (C) $x_1 = 8.800$; $x_2 = 1.000$
   (D) $x_1 = 8.771$; $x_2 = 1.052$

5. At the end of forward elimination steps of Naïve Gauss Elimination method on the following

equations

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 4.2857\times10^{7} & -5.4619\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857\times10^{7} & -3.6057\times10^{5} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887\times10^{3} \\ 0 \\ 0.007 \\ 0 \end{bmatrix}$$

the resulting equations in the matrix form are given by

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 0 & 3.7688\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ 0 & 0 & -26.9140 & 0.579684 \\ 0 & 0 & 0 & 5.62500\times10^{5} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887\times10^{3} \\ 7.887\times10^{3} \\ 1.19530\times10^{-2} \\ 1.90336\times10^{4} \end{bmatrix}$$

The determinant of the original coefficient matrix is

(A) 0.00

(B) 4.2857×10^{7}

(C) 5.486×10^{19}

(D) -2.445×10^{20} ANSWER

6. The following data is given for the velocity of the rocket as a function of time. To find the velocity at t=21 s, you are asked to use a quadratic polynomial, $v(t)=at^2+bt+c$ to approximate the velocity profile.

t	*(s)*	0	14	15	20	30	35
v(t)	*m/s*	0	227.04	362.78	517.35	602.97	901.67

The correct set of equations that will find *a*, *b* and *c* are

(A) $\begin{bmatrix} 176 & 14 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 227.04 \\ 362.78 \\ 517.35 \end{bmatrix}$

(B) $\begin{bmatrix} 225 & 15 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 362.78 \\ 517.35 \\ 602.97 \end{bmatrix}$ ANSWER

(C) $\begin{bmatrix} 0 & 0 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 362.78 \\ 517.35 \end{bmatrix}$

(D) $\begin{bmatrix} 400 & 20 & 1 \\ 900 & 30 & 1 \\ 1225 & 35 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 517.35 \\ 602.97 \\ 901.67 \end{bmatrix}$

LU Decomposition Method

1. LU decomposition method is computationally more efficient than Naïve Gauss elimination for solving

(A) a single set of simultaneous linear equations
(B) multiple sets of simultaneous linear equations with different coefficient matrices and same right hand side vectors.
(C) multiple sets of simultaneous linear equations with same coefficient matrix and different right hand side vectors.
(D) less than ten simultaneous linear equations.

2. The lower triangular matrix [L] in the [L][U] decomposition of matrix given below

$$\begin{bmatrix} 25 & 5 & 4 \\ 10 & 8 & 16 \\ 8 & 12 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

is

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0.40000 & 1 & 0 \\ 0.32000 & 1.7333 & 1 \end{bmatrix}$ ANSWER

(B) $\begin{bmatrix} 25 & 5 & 4 \\ 0 & 6 & 14.400 \\ 0 & 0 & -4.2400 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 10 & 1 & 0 \\ 8 & 12 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0.40000 & 1 & 0 \\ 0.32000 & 1.5000 & 1 \end{bmatrix}$

3. The upper triangular matrix [U] in the [L][U] decomposition of matrix given below

$$\begin{bmatrix} 25 & 5 & 4 \\ 0 & 8 & 16 \\ 0 & 12 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

is

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0.40000 & 1 & 0 \\ 0.32000 & 1.7333 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 25 & 5 & 4 \\ 0 & 6 & 14.400 \\ 0 & 0 & -4.2400 \end{bmatrix}$

(C) $\begin{bmatrix} 25 & 5 & 4 \\ 0 & 8 & 16 \\ 0 & 0 & -2 \end{bmatrix}$ ANSWER

(D) $\begin{bmatrix} 1 & 0.2000 & 0.16000 \\ 0 & 1 & 2.4000 \\ 0 & 0 & -4.240 \end{bmatrix}$

4. For a given 2000×2000 matrix [A], assume that it takes about 15 seconds to find the inverse of [A] by the use of the [L][U] decomposition method, that is, finding the [L][U] once, and then doing forward substitution and back substitution 2000 times using the 2000 columns of the identity matrix as the right hand side vector. The approximate time, in seconds, that it will take to find the inverse if found by repeated use of Naive Gauss Elimination method, that is, doing forward elimination and back substitution 2000 times by using the 2000 columns of the identity matrix as the right hand side vector is

(A) 300
(B) 1500
(C) 7500
(D) 30000

5. The algorithm in solving the set of equations [A][X] = [C], where [A] = [L][U] involves solving [L][Z] = [C] by forward substitution. The algorithm to solve [L][Z]=[C] is given by

(A) $z_1 = c_1 / l_{11}$

for i from 2 to n do
  sum = 0
    for j from 1 to i do
      sum = sum + $l_{ij} * z_j$
    end do
  $z_i = (c_i - \text{sum}) / l_{ii}$
end do

(B) $z_1 = c_1 / l_{11}$

for i from 2 to n do
  sum = 0
    for j from 1 to (i-1) do
      sum = sum + $l_{ij} * z_j$
    end do
  $z_i = (c_i - \text{sum}) / l_{ii}$
end do

(C) $z_1 = c_1 / l_{11}$

for i from 2 to n do
    for j from 1 to (i-1) do
      sum = sum + $l_{ij} * z_j$
    end do
  $z_i = (c_i - \text{sum}) / l_{ii}$
end do

(D) for i from 2 to n do
  sum = 0
    for j from 1 to (i-1) do
      sum = sum + $l_{ij} * z_j$
    end do
  $z_i = (c_i - \text{sum}) / l_{ii}$
end do

6. To solve boundary value problems, finite difference method is used resulting in simultaneous linear equations with tri-diagonal coefficient matrices. These are solved using the specialized [L][U] decomposition method. The set of equations in matrix form with a tri-diagonal coefficient matrix for

$$\frac{d^2y}{dx^2} = 6x - 0.5x^2, \; y(0) = 0, \; y(12) = 0,$$

using finite difference method with a second order accurate central divided difference method and a step size of $h = 4$ is

(A) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.0625 & 0.125 & 0.0625 & 0 \\ 0 & 0.0625 & 0.125 & 0.0625 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 16.0 \\ 16.0 \\ 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.0625 & -0.125 & 0.0625 & 0 \\ 0 & 0.0625 & -0.125 & 0.0625 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 16.0 \\ 16.0 \\ 0 \end{bmatrix}$ ANSWER

(C) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0.0625 & -0.125 & 0.0625 & 0 \\ 0 & 0.0625 & -0.125 & 0.0625 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 16.0 \\ 16.0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0.0625 & 0.125 & 0.0625 & 0 \\ 0 & 0.0625 & 0.125 & 0.0625 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 16.0 \\ 16.0 \end{bmatrix}$

Gauss-Seidel Method of Solving

1. A square matrix $[A]_{nxn}$ is diagonally dominant if

(A) $|a_{ii}| \geq \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|, \quad i = 1, 2, \ldots, n$

(B) $|a_{ii}| \geq \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|, \; i = 1, 2, \ldots, n$ and $|a_{ii}| > \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|,$ for any $i = 1, 2, \ldots, n$

(C) $|a_{ii}| \geq \sum_{j=1}^{n} |a_{ij}|, \; i = 1, 2, \ldots, n$ and $|a_{ii}| > \sum_{j=1}^{n} |a_{ij}|,$ for any $i = 1, 2, \ldots, n$

(D) $\left|a_{ii}\right| \geq \sum_{j=1}^{n}\left|a_{ij}\right|, \; i = 1, 2, \ldots, n$

2. Using [x_1 x_2 x_3] = [1 3 5] as the initial guess, the value of [x_1 x_2 x_3] after three iterations in Gauss-Seidel method for

$$\begin{bmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 6 \end{bmatrix}$$

is

(A) [-2.8333 -1.4333 -1.9727]
(B) [1.4959 -0.90464 -0.84914]
(C) [0.90666 -1.0115 -1.0242]
(D) [1.2148 -0.72060 -0.82451]

3. To ensure that the following system of equations,

$$\begin{aligned} 2x_1 + 7x_2 - 11x_3 &= 6 \\ x_1 + 2x_2 + x_3 &= -5 \\ 7x_1 + 5x_2 + 2x_3 &= 17 \end{aligned}$$

converges using Gauss-Seidel Method, one can rewrite the above equations as follows:

(A) $$\begin{bmatrix} 2 & 7 & -11 \\ 1 & 2 & 1 \\ 7 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ 17 \end{bmatrix}$$

(B) $$\begin{bmatrix} 7 & 5 & 2 \\ 1 & 2 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ -5 \\ 6 \end{bmatrix}$$

(C) $$\begin{bmatrix} 7 & 5 & 2 \\ 1 & 2 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ 17 \end{bmatrix}$$

(D) The equations cannot be rewritten in a form to ensure convergence.

4. For $\begin{bmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 22 \\ 7 \\ -2 \end{bmatrix}$ and using $[x_1 \quad x_2 \quad x_3] = [1 \quad 2 \quad 1]$ as the initial guess, the values of $[x_1 \quad x_2 \quad x_3]$ are found at the end of each iteration as

Iteration #	x_1	x_2	x_3
1	0.41666	1.1166	0.96818
2	0.93989	1.0183	1.0007
3	0.98908	1.0020	0.99930
4	0.99898	1.0003	1.0000

At what first iteration number would you trust at least 1 significant digit in your solution?

(A) 1
(B) 2
(C) 3
(D) 4

5. The algorithm for the Gauss-Seidel Method to solve [A] [X] = [C] is given as follows for using nmax iterations. The initial value of [X] is stored in [X].

(A)

```
Sub Seidel(n, a, x, rhs, nmax)
For k = 1 To nmax
For i = 1 To n
For j = 1 To n
If (i <> j) Then
Sum = Sum + a(i, j) * x(j)
endif
Next j
x(i) = (rhs(i) - Sum) / a(i, i)
Next i
Next k
End Sub
```

(B)

```
Sub Seidel(n, a, x, rhs, nmax)
For k = 1 To nmax
For i = 1 To n
Sum = 0
For j = 1 To n
If (i <> j) Then
Sum = Sum + a(i, j) * x(j)
endif
Next j
x(i) = (rhs(i) - Sum) / a(i, i)
Next i
Next k
End Sub
```

(C)

```
Sub Seidel(n, a, x, rhs, nmax)
For k = 1 To nmax
For i = 1 To n
Sum = 0
For j = 1 To n
Sum = Sum + a(i, j) * x(j)
Next j
x(i) = (rhs(i) - Sum) / a(i, i)
Next i
Next k
End Sub
```

(D)

```
Sub Seidel(n, a, x, rhs, nmax)
For k = 1 To nmax
For i = 1 To n
Sum = 0
For j = 1 To n
If (i <> j) Then
Sum = Sum + a(i, j) * x(j)
endif
Next j
x(i) = rhs(i)  / a(i, i)
Next i
Next k
End Sub
```

6. Thermistors measure temperature, have a nonlinear output and are valued for a limited range. So when a thermistor is manufactured, the manufacturer supplies a resistance vs. temperature curve. An accurate representation of the curve is generally given by

$$\frac{1}{T} = a_0 + a_1 \ln(R) + a_2 \{\ln(R)\}^2 + a_3 \{\ln(R)\}^3$$

where T is temperature in Kelvin, R is resistance in ohms, and a_0, a_1, a_2, a_3 are constants of the calibration curve.

Given the following for a thermistor

R	**T**
ohm	$°C$
1101.0	25.113
911.3	30.131
636.0	40.120
451.1	50.128

the value of temperature in $°C$ for a measured resistance of 900 ohms most nearly is

(A) 30.002
(B) 30.472
(C) 31.272
(D) 31.445

Cholesky and LDL^T Decomposition

1. For a given set of simultaneous linear equations

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} -0.5 \\ 5 \\ -5 \\ 1.5 \end{bmatrix}$$

The Cholesky factorized matrix $[U]$ can be computed as

(A) $\begin{bmatrix} 1.414 & 0.7071 & 0 & 0.3536 \\ 0 & 1.225 & -0.8165 & 0.2041 \\ 0 & 0 & 1.155 & -0.7217 \\ 0 & 0 & 0 & 0.5590 \end{bmatrix}$

(B) $\begin{bmatrix} 1.414 & -0.7071 & 0 & 0.3536 \\ 0 & 1.225 & -0.8165 & 0.2041 \\ 0 & 0 & 1.155 & -0.7217 \\ 0 & 0 & 0 & 0.5590 \end{bmatrix}$ ANSWER B

(C) $\begin{bmatrix} 1.414 & 0.7071 & 0 & -0.3536 \\ 0 & 1.225 & -0.8165 & 0.2041 \\ 0 & 0 & 1.155 & -0.7217 \\ 0 & 0 & 0 & 0.5590 \end{bmatrix}$

(D) $\begin{bmatrix} 1.414 & 0.7071 & 0 & 0.3536 \\ 0 & 1.225 & -0.8165 & 0.2041 \\ 0 & 0 & -1.155 & -0.7217 \\ 0 & 0 & 0 & 0.5590 \end{bmatrix}$

2. For a given set of simultaneous linear equations

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} -0.5 \\ 5 \\ -5 \\ 1.5 \end{bmatrix}$$

the forward solution vector $[y]$ can be computed as

(A) $\vec{y}^T = \{0.5363, 38.784, -15.877, 0.5590\}$
(B) $\vec{y}^T = \{0.5363, -15.877, 38.784, 0.5590\}$
(C) $\vec{y}^T = \{-3.536, -1.5877, 3.878, 0.5590\}$
(D) $\vec{y}^T = \{-0.3536, 3.8784, -1.5877, -0.5590\}$ ANSWER

3. For a given set of simultaneous linear equations

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} -0.5 \\ 5 \\ -5 \\ 1.5 \end{bmatrix}$$

The backward solution vector $[x]$ can be computed as

(A) $\vec{x}^T = \{1, 2, -2, -1\}$ ANSWER
(B) $\vec{x}^T = \{1, 2, 2, -1\}$
(C) $\vec{x}^T = \{-1, 2, -2, 1\}$
(D) $\vec{x}^T = \{1, 2, 2, 1\}$

4. The determinant of

$$[A] = \begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix}$$

most nearly is

(A) -5
(B) 5
(C) -50
(D) 1.25

5. Based on the given matrix $[A]$, and assuming the reordering algorithm will produce the following mapping IPERM (new equation #) = {old equation #}, such as

$$IPERM \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 4 \\ 2 \end{pmatrix},$$

the non zero off-diagonal term A(old row 4, old column 1) = 0.5 will move to the following new location of the new matrix $[A^*]$

(A) A^*(new row 3, new column 1)
(B) A^*(new row 1, new column 3)
(C) A^*(new row 3, new column 2)
(D) A^*(new row 2, new column 2)

6. Based on the given matrix $[A]$, and the given reordering mapping

$$IPERM \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 4 \\ 2 \end{pmatrix},$$

the non-zero diagonal term $A(4,4) = 1$ will move to the following new location of the new matrix $[A^*]$

(A) $A^*(1,1) = 1$
(B) $A^*(2,2) = 1$
(C) $A^*(3,3) = 1$ ANSWER
(D) $A^*(4,4) = 1$

7. For a given set of simultaneous linear equations, and using LDL^T algorithm,

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0.5 \end{bmatrix},$$

the lower triangular matrix $[L]$ can be computed as

(A) $[L]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & -0.6667 & 1 & 0 \\ 0.25 & 0.1667 & -0.625 & 1 \end{bmatrix}$ ANSWER

(B) $[L]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & 0.6667 & 1 & 0 \\ 0.25 & 0.1667 & -0.625 & 1 \end{bmatrix}$

(C) $[L]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & -0.6667 & 1 & 0 \\ 0.25 & 0.1667 & 0.625 & 1 \end{bmatrix}$

(D) $[L]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 0 & -0.6667 & 1 & 0 \\ 0.25 & 0.1667 & -0.625 & 1 \end{bmatrix}$

8. For a given set of simultaneous linear equations, and using LDL^{T} algorithm,

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0.5 \end{bmatrix},$$

the diagonal matrix $[D]$ can be computed as:

(A) $[D]=\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.3333 & 0 \\ 0 & 0 & 0 & 0.3125 \end{bmatrix}$

(B) $[D]=\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -1.5 & 0 & 0 \\ 0 & 0 & 1.3333 & 0 \\ 0 & 0 & 0 & 0.3125 \end{bmatrix}$

(C) $[D]=\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.3333 & 0 \\ 0 & 0 & 0 & -0.3125 \end{bmatrix}$

(D) $[D]=\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.3333 & 0 \\ 0 & 0 & 0 & 0.3125 \end{bmatrix}$ ANSWER

9. For a given set of simultaneous linear equations, and using LDL^T algorithm,

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0.5 \end{bmatrix},$$

the forward solution for the unknown vector $[z]$ in $[L][z]=[b]$ can be computed as

(A) $\{z\}^T=\{-2, 0, 1, 0.625\}$

(B) $\{z\}^T=\{2, 0, 1, 0.625\}$ ANSWER

(C) $\{z\}^T=\{2, 0, -1, 0.625\}$

(D) $\{z\}^T=\{2, 0, 1, -0.625\}$

10. For a given set of simultaneous linear equations, and using LDL^T algorithm,

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0.5 \end{bmatrix},$$

the <u>diagonal scaling</u> solution for the unknown vector $[y]$ in $[D][y]=[z]$ can be computed as:

(A) $\{y\}^T=\{-1, 0, 0.75, 2\}$

(B) $\{y\}^T=\{1, 0, -0.75, 2\}$

(C) $\{y\}^T=\{1, 0, 0.75, -2\}$

(D) $\{y\}^T=\{1, 0, 0.75, 2\}$ ANSWER

11. For a given set of simultaneous linear equations, and using LDL^T algorithm,

$$[A][x]=[b]$$

where

$$[A]=\begin{bmatrix} 2 & -1 & 0 & 0.5 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0.5 & 0 & -1 & 1 \end{bmatrix} \text{ and } [b]=\begin{bmatrix} 2 \\ -1 \\ 1 \\ 0.5 \end{bmatrix},$$

the backward solution for the original unknown vector $[x]$, in $[L]^T[x]=[y]$, can be computed as

(A) $\{x\}^T=\{1, 1, 2, 2\}$ ANSWER

(B) $\{x\}^T=\{2, 1, 2, 1\}$

(C) $\{x\}^T=\{1, 1, 2, 1\}$

(D) $\{x\}^T=\{2, 2, 2, 1\}$

12. Given the following 6×6 matrix $[A]$, which is assumed to be SPD:

$$[A]=\begin{bmatrix} \times & 0 & \times & 0 & \times & 0 \\ & \times & 0 & \times & 0 & 0 \\ & & \times & 0 & \times & \times \\ & & & \times & 0 & 0 \\ & Sym & & & \times & 0 \\ & & & & & \times \end{bmatrix}$$

where $\times$ = a nonzero value (given)

0 = a zero value (given)

Based on the numerically factorized formulas shown in Equations 6-7 of Chapter 04.11, or even more helpful information as indicated in Figure 1 of Chapter 04.11, and given

* = a nonzero value (computed, at the same location as the original nonzero value of $[A]$)

0 = a zero value

F = a nonzero fill-in-term (computed)

and

$U(5,6)=F$

$A(5,6)=0$

the symbolically factorized upper-triangular matrix can be obtained as

(A) $[U]=\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ & * & 0 & * & 0 & 0 \\ & & * & 0 & * & * \\ & & & * & 0 & 0 \\ & & & & * & F \\ & & & & & * \end{bmatrix}$ ANSWER

(B) $[U]=\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ & * & 0 & F & 0 & 0 \\ & & * & 0 & * & * \\ & & & * & 0 & 0 \\ & & & & * & F \\ & & & & & * \end{bmatrix}$

(C) $[U]=\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ & * & 0 & F & 0 & 0 \\ & & * & 0 & F & * \\ & & & * & 0 & 0 \\ & & & & * & F \\ & & & & & * \end{bmatrix}$

D $[U]=\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ & * & 0 & F & 0 & 0 \\ & & * & 0 & F & F \\ & & & * & 0 & 0 \\ & & & & * & F \\ & & & & & * \end{bmatrix}$

Background on Interpolation

1. The number of polynomials that can go through two fixed data points (x_1, y_1) and (x_2, y_2) is
   (D)0
   (E)1
   (F)2
   (G)Infinite D ANSWER

2. A unique polynomial of degree ___________________ passes through $n+1$ data points.
   (A) $n+1$
   (B) $n+1$ or less
   (C) n
   (D) n **or less**

3. The following function(s) can be used for interpolation:

(A) polynomial
(B) exponential
(C) trigonometric
(D) all of the above

4. Polynomials are the most commonly used functions for interpolation because they are easy to

(A) evaluate
(B) differentiate
(C) integrate
(D) evaluate, differentiate and integrate

5. Given $n+1$ data points $(x_0, y_0),(x_1, y_1),......,(x_{n-1}, y_{n-1}),(x_n, y_n)$, assume you pass a function $f(x)$ through all the data points. If now the value of the function $f(x)$ is required to be found outside the range of the given x-data, the procedure is called

(A) extrapolation
(B) interpolation
(C) guessing
(D) regression

6. Given three data points (1,6), (3,28), and (10, 231), it is found that the function $y = 2x^2 + 3x + 1$ passes through the three data points. Your estimate of y at $x = 2$ is most nearly

(A) 6
(B) 15
(C) 17
(D) 28

Direct Method of Interpolation

1. A unique polynomial of degree ________________ passes through $n+1$ data points.

(E) $n+1$
(F) $n+1$ or less
(G) n
(H) n **or less**

2. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

The velocity in m/s at $16\,\text{s}$ using linear polynomial interpolation is most nearly

(I) 27.867
(J) 28.333
(K) 30.429
(L) 43.000

3. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

The velocity in m/s at $16\,\text{s}$ using quadratic polynomial interpolation is most nearly

(M) 27.867
(N) 28.333
(O) 30.429
(P) 43.000

4. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

Using quadratic interpolation, the interpolant

$$v(t) = 8.667t^2 - 349.67t + 3523, \quad 18 \le t \le 24$$

approximates the velocity of the body. From this information, the time in seconds at which the velocity of the body is 35 m/s during the above time interval of $t = 18\,\text{s}$ to $t = 24\,\text{s}$ is

(Q) 18.667
(R) 20.850

(S) 22.200
(T) 22.294

5. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

One of the interpolant approximations for the velocity from the above data is given as

$$v(t) = 8.6667t^2 - 349.67t + 3523, \quad 18 \le t \le 24$$

Using the above interpolant, the distance in meters covered by the body between $t = 19$ s and $t = 22$ s is most nearly

(U) **10.337**
(V) 88.500
(W) 93.000
(X) 168.00

6. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

If you were going to use quadratic interpolation to find the value of the velocity at $t = 14.9$ seconds, what three data points of time would you choose for interpolation?

A) 0, 15, 18
B) 15, 18, 22
C) 0, 15, 22
D) 0, 18, 24

Newton's Divided Difference Polynomial Method

1. If a polynomial of degree n has $n+1$ zeros, then the polynomial is
(A) oscillatory
(B) zero everywhere
(C) quadratic
(D) not defined

2. The following x, y data is given.

x	15	18	22
y	24	37	25

The Newton's divided difference second order polynomial for the above data is given by

$$f_2(x) = b_0 + b_1(x-15) + b_2(x-15)(x-18)$$

The value of b_1 is most nearly
(A) -1.0480
(B) 0.14333
(C) 4.3333
(D) 24.000

3. The polynomial that passes through the following x, y data

x	18	22	24
y	?	25	123

is given by

$8.125x^2 - 324.75x + 3237, \quad 18 \le x \le 24$

The corresponding polynomial using Newton's divided difference polynomial is given by

$f_2(x) = b_0 + b_1(x-18) + b_2(x-18)(x-22)$

The value of b_2 is most nearly

(A) 0.25000

(B) 8.1250

(C) 24.000

(D) not obtainable with the information given

4. Velocity vs. time data for a body is approximated by a second order Newton's divided difference polynomial as

$v(t) = b_0 + 39.622(t-20) + 0.5540(t-20)(t-15), \quad 10 \le t \le 20$

The acceleration in $_2$ m/s at $t = 15$ is

(A) 0.5540

(B) 39.622

(C) 36.852

(D) not obtainable with the given information

5. The path that a robot is following on a $x-y$ plane is found by interpolating the following four data points as

x	2	4.5	5.5	7
y	7.5	7.5	6	5

$y(x) = 0.1524x^3 - 2.257x^2 + 9.605x - 3.900$

The length of the path from $x = 2$ to $x = 7$ is

(A)

$$\sqrt{(7.5-7.5)^2 + (4.5-2)^2} + \sqrt{(6-7.5)^2 + (5.5-4.5)^2} + \sqrt{(5-6)^2 + (7-5.5)^2}$$

(B)

(C) $\int_2^7 \sqrt{1 + (0.4572x^2 - 4.514x + 9.605)^2}\,dx$ ANSWER

D) $\int_2^7 \sqrt{1 + (0.4572x^2 - 4.514x + 9.605)^2}\,dx$

6. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

If you were going to use quadratic interpolation to find the value of the velocity at $t = 14.9$ seconds, the three data points of time you would choose for interpolation are

(A) 0, 15, 18
(B) 15, 18, 22
(C) 0, 15, 22
(D) 0, 18, 24

Lagrange Method of Interpolation

1. A unique polynomial of degree _______________ passes through $n+1$ data points.
   - A) $n+1$
   - B) n
   - **C) n or less**
   - D) $n+1$ or less

2. Given the two points $[a, f(a)], [b, f(b)]$, the linear Lagrange polynomial $f_1(x)$ that passes through these two points is given by
   - A) $f_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{a-b} f(b)$
   - B) $f_1(x) = \frac{x}{b-a} f(a) + \frac{x}{b-a} f(b)$
   - C) $f_1(x) = f(a) + \frac{f(b) - f(a)}{b-a}(b-a)$
   - D) $f_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$ ANSWER

3. The Lagrange polynomial that passes through the 3 data points is given by

x	15	18	22
y	24	37	25

$$f_2(x) = L_0(x)(24) + L_1(x)(37) + L_2(x)(25)$$

The value of $L_1(x)$ at $x = 16$ is most nearly
   - A) –0.071430
   - **B) 0.50000**
   - C) 0.57143
   - D) 4.3333

4. The following data of the velocity of a body is given as a function of time.

Time (s)	10	15	18	22	24
Velocity (m/s)	22	24	37	25	123

A quadratic Lagrange interpolant is found using three data points, $t = 15$, 18 and 22. From this information, at what of the times given in seconds is the velocity of the body 26 m/s during the time interval of $t = 15$ to $t = 22$ seconds.

A) 20.173
B) 21.858
C) 21.667
D) 22.020

5. The path that a robot is following on a x, y plane is found by interpolating four data points as

x	2	4.5	5.5	7
y	7.5	7.5	6	5

$$y(x) = 0.15238x^3 - 2.2571x^2 + 9.6048x - 3.9000$$

The length of the path from $x = 2$ to $x = 7$ is

(A) $\sqrt{(7.5-7.5)^2 + (4.5-2)^2} + \sqrt{(6-7.5)^2 + (5.5-4.5)^2} + \sqrt{(5-6)^2 + (7-5.5)^2}$

(B) $\int_2^7 \sqrt{1 + (0.15238x^3 - 2.2571x^2 + 9.6048x - 3.9000)^2}\,dx$

(C) $\int_2^7 \sqrt{1 + (0.45714x^2 - 4.5142x + 9.6048)^2}\,dx$ **ANSWER**

(D) $\int_2^7 (0.15238x^3 - 2.2571x^2 + 9.6048x - 3.9000)\,dx$

6. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

If you were going to use quadratic interpolation to find the value of the velocity at $t = 14.9$ seconds, what three data points of time would you choose for interpolation?

A) 0, 15, 18
B) 15, 18, 22
C) 0, 15, 22
D) 0, 18, 24

Background

1. Physically, integrating $\int_a^b f(x)dx$ means finding the

   A) area under the curve from a to b
   B) area to the left of point a
   C) area to the right of point b
   D) area above the curve from a to b

2. The mean value of a function $f(x)$ from a to b is given by

   A) $\dfrac{f(a)+f(b)}{2}$

   B) $\dfrac{f(a)+2f\left(\dfrac{a+b}{2}\right)+f(b)}{4}$

   C) $\int_a^b f(x)dx$

   D) $\dfrac{\int_a^b f(x)dx}{b-a}$ **ANSWER**

3. The exact value of $\int_{0.2}^{2.2} xe^x dx$ is most nearly

   A) 7.8036
   B) 11.807
   C) 14.034
   D) 19.611

4. $\int_{0.2}^{2} f(x)dx$ for

$$f(x) = x, \qquad 0 \le x \le 1.2$$
$$= x^2, \quad 1.2 < x \le 2.4$$

is most nearly

   A) 1.9800
   B) 2.6640
   C) 2.7907
   D) 4.7520

5. The area of a circle of radius a can be found by the following integral

A) $\int_0^a \left(a^2 - x^2\right)dx$

B) $\int_0^{2\pi} \sqrt{a^2 - x^2}\,dx$ **ANSWER**

C) $4\int_0^a \sqrt{a^2 - x^2}\,dx$

D) $\int_0^a \sqrt{a^2 - x^2}\,dx$

6. Velocity distribution of a fluid flow through a pipe varies along the radius and is given by $v(r)$. The flow rate through the pipe of radius a is given by

A) $\pi v(a)a^2$ **ANSWER**

B) $\pi \frac{v(0)+v(a)}{2}a^2$

C) $\int_0^a v(r)dr$

D) $2\pi\int_0^a v(r)rdr$

Trapezoidal Rule

1. The two-segment trapezoidal rule of integration is exact for integrating at most ________ order polynomials.

A) first

B) second

C) third

D) fourth

2. The value of $\int_{0.2}^{2.2} xe^x dx$ by using the one-segment trapezoidal rule is most nearly

A) 11.672

B) 11.807

C) 20.099

D) 24.119

3. The value of $\int_{0.2}^{2.2} xe^x dx$ by using the three-segment trapezoidal rule is most nearly

A) 11.672
B) 11.807
C) 12.811
D) 14.633

4. The velocity of a body is given by

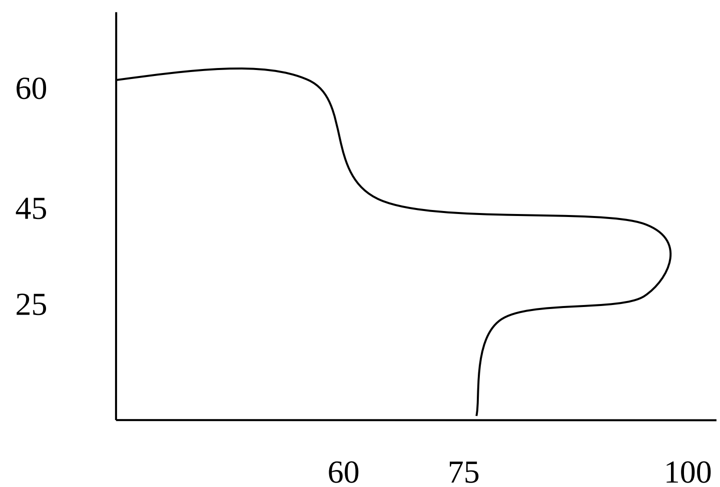
$$v(t) = 2t, \qquad 1 \le t \le 5$$
$$= 5t^2 + 3, \quad 5 < t \le 14$$

where t is given in seconds, and v is given in m/s. Use the two-segment trapezoidal rule to find the distance in meters covered by the body from $t = 2$ to $t = 9$ seconds.

A) 935.00
B) 1039.7
C) 1260.9
D) 5048.9

5. The shaded area shows a plot of land available for sale. The units of measurement are in meters. Your best estimate of the area of the land in m^2 is most nearly

A) 2500
B) 4775
C) 5250
D) 6000



6. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

The distance in meters covered by the body from $t = 12$ s to $t = 18$ s calculated using the trapezoidal rule with unequal segments is

A) 162.90
B) 166.00
C) 181.70
D) 436.50

Simpson's 1/3 Rule

1. The highest order of polynomial integrand for which Simpson's 1/3 rule of integration is exact is
   A) first
   B) second
   C) third
   D) fourth

2. The value of $\int_{0.2}^{2.2} e^x dx$ by using 2-segment Simpson's 1/3 rule most nearly is
   A) 7.8036
   B) 7.8423
   C) 8.4433
   D) 10.246

3. The value of $\int_{0.2}^{2.2} e^x dx$ by using 4-segment Simpson's 1/3 rule most nearly is
   A) 7.8036
   B) 7.8062
   C) 7.8423
   D) 7.9655

4. The velocity of a body is given by

$$v(t) = 2t, \qquad 1 \le t \le 5$$
$$= 5t^2 + 3, \quad 5 < t \le 14$$

where t is given in seconds, and v is given in m/s. Using two-segment Simpson's 1/3 rule, the distance in meters covered by the body from $t = 2$ to $t = 9$ seconds most nearly is
   A) 949.33
   B) 1039.7
   C) 1200.5
   D) 1442.0

5. The value of $\int_{3}^{19} f(x)dx$ by using 2-segment Simpson's 1/3 rule is estimated as 702.039. The estimate of the same integral using 4-segment Simpson's 1/3 rule most nearly is

A) $702.039+\frac{8}{3}\left[2f(7)-f(11)+2f(15)\right]$

B) $\frac{702.039}{2}+\frac{8}{3}\left[2f(7)-f(11)+2f(15)\right]$ **ANSWER**

C) $702.039+\frac{8}{3}\left[2f(7)+2f(15)\right]$

D) $\frac{702.039}{2}+\frac{8}{3}\left[2f(7)+2f(15)\right]$

6. The following data of the velocity of a body is given as a function of time.

Time (s)	4	7	10	15
Velocity (m/s)	22	24	37	46

The best estimate of the distance in meters covered by the body from $t=4$ to $t=15$ using combined Simpson's 1/3 rule and the trapezoidal rule would be

A) 354.70
B) 362.50
C) 368.00
D) 378.80

Romberg Rule

1. If I_n is the value of integral $\int_{a}^{b} f(x)dx$ using n-segment trapezoidal rule, a better estimate of the integral can be found using Richardson's extrapolation as

A) $I_{2n}+\frac{I_{2n}-I_n}{15}$

B) $I_{2n}+\frac{I_{2n}-I_n}{3}$ **ANSWER**

C) I_{2n}

D) $I_{2n}+\frac{I_{2n}-I_n}{I_{2n}}$

2. The estimate of an integral of $\int_{3}^{19} f(x)dx$ is given as 1860.9 using 1-segment trapezoidal rule. Given $f(7) = 20.27$, $f(11) = 45.125$, and $f(14) = 82.23$, the value of the integral using 2-segment trapezoidal rule would most nearly be

A) 787.32
B) 1072.0
C) 1144.9
D) 1291.5

3. The value of an integral $\int_{a}^{b} f(x)dx$ given using 1, 2, and 4 segments trapezoidal rule is given as 5.3460, 2.7708, and 1.7536, respectively. The best estimate of the integral you can find using Romberg integration is most nearly

A) 1.3355
B) 1.3813
C) 1.4145
D) 1.9124

4. Without using the formula for one-segment trapezoidal rule for estimating $\int_a^b f(x)dx$ the true error, E_t can be found directly as well as exactly by using the formula

$$E_t = -\frac{(b-a)^3}{12} f''(\xi),\ a \le \xi \le b$$

for

A) $f(x) = e^x$
B) $f(x) = x^3 + 3x$
C) $f(x) = 5x^2 + 3$ **ANSWER**
D) $f(x) = 5x^2 + e^x$

5. For $\int_a^b f(x)dx$, the true error, E_t in one-segment trapezoidal rule is given by

$$E_t = -\frac{(b-a)^3}{12} f''(\xi),\ a \le \xi \le b$$

The value of ξ for the integral $\int_{2.5}^{7.2} 3e^{0.2x}dx$ is most nearly

A) 2.7998
B) 4.8500
C) 4.9601
D) 5.0327

6. Given the velocity vs. time data for a body

t (s)	2	4	6	8	10	25
v (m/s)	0.166	0.55115	1.8299	6.0755	20.172	8137.5

The best estimate for distance covered in meters between $t = 2$ s and $t = 10$ s by using Romberg rule based on trapezoidal rule results would be

A) 33.456
B) 36.877
C) 37.251
D) 81.350

Gauss Quadrature Rule

1. $\int_{5}^{10} f(x)dx$ is exactly

   A) $\int_{-1}^{1} f(2.5x+7.5)dx$

   B) $2.5\int_{-1}^{1} f(2.5x+7.5)dx$ **ANSWER**

   C) $5\int_{-1}^{1} f(5x+5)dx$

   D) $5\int_{-1}^{1} (2.5x+7.5)f(x)dx$

2. For a definite integral of any third order polynomial, the two-point Gauss quadrature rule will give the same results as the

   A) 1-segment trapezoidal rule
   B) 2-segment trapezoidal rule
   C) 3-segment trapezoidal rule
   D) Simpson's 1/3 rule

3. The value of $\int_{0.2}^{2.2} xe^{x}dx$ by using the two-point Gauss quadrature rule is most nearly

   A) 11.672
   B) 11.807
   C) 12.811
   D) 14.633

4. A scientist uses the one-point Gauss quadrature rule based on getting exact results of integration for functions $f(x)=1$ and x. The one-point Gauss quadrature rule approximation for $\int_a^b f(x)dx$ is

A) $\frac{b-a}{2}\left[f(a)+f(b)\right]$

B) $(b-a)f\left(\frac{a+b}{2}\right)$ **ANSWER**

C) $\frac{b-a}{2}\left[f\left(\frac{b-a}{2}\left\{-\frac{1}{\sqrt{3}}\right\}+\frac{b+a}{2}\right)+f\left(\frac{b-a}{2}\left\{\frac{1}{\sqrt{3}}\right\}+\frac{b+a}{2}\right)\right]$

D) $(b-a)f(a)$

5. A scientist develops an approximate formula for integration as

$$\int_a^b f(x)dx \approx c_1 f(x_1),\text{ where } a \le x_1 \le b$$

The values of c_1 and x_1 are found by assuming that the formula is exact for functions of the form $a_0 x + a_1 x^2$. The resulting formula would therefore be exact for integrating

A) $f(x)=2$

B) $f(x)=2+3x+5x^2$

C) $f(x)=5x^2$ **ANSWER**

D) $f(x)=2+3x$

6. You are asked to estimate the water flow rate in a pipe of radius 2 m at a remote area location with a harsh environment. You already know that velocity varies along the radial location, but you do not know how it varies. The flow rate Q is given by

$$Q=\int_0^2 2\pi rVdr$$

To save money, you are allowed to put only two velocity probes (these probes send the data to the central office in New York, NY via satellite) in the pipe. Radial location, r is measured from the center of the pipe, that is $r=0$ is the center of the pipe and $r=2\text{m}$ is the pipe radius. The radial locations you would suggest for the two velocity probes for the most accurate calculation of the flow rate are

A) 0, 2

B) 1, 2

C) 0, 1

D) 0.42, 1.58