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DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING

B.Sc. Electrical and Electronic Engineering, 2

EE 261: ASYNCHRONOUS AND DC MACHINES

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Course Outline

EE 261 Asynchronous and DC Machines (3 0 3)

Armature Winding and Emf:
Commutator Windings; A.C. Windings; Winding Factors; Emfs Produced by Armature Windings.

D.C. Machines:
Basic Theory; Construction; Emf and Torque Equations; Steady State characteristics of Shunt, Series and Compound Machines; Efficiency; Starters and their Industrial control circuits.

Polyphase Induction Machines:
Basic Theory; Construction; Phasor Diagrams; Equivalent circuits; Determination of equivalent circuit parameters; Efficiency calculation; Torque equations; Circle Diagram; Power factor correction; Starting methods and their industrial control circuits; High torque cage motors; Induction Generator.

Recommended Textbooks

1. John Hindmarsh (1977):Electrical Machines and their Applications, Pergamon Press, Oxford

2. Richard A. Pearman (1994): Electrical Machinery & Transformers, Saunders College Publishing, Harcourt Brace College Publishers, New York.

3. T. Wildi (1988): Electrical Power Technology, Prentice Hall Professional Technical Reference.

4. Robert Stein, William T. Hunt, Jr. (1979): Electric Power System Components, Transformers and rotating Machines, Van Nostrand Reinhold Company, New York

5. B. A. Theraja, A. K. Theraja (2005): A Textbook of Electrical Technology, S. Chand & Company Ltd, New Delhi

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CHAPTER ONE

ARMATURE WINDINGS AND EMF DEVELOPED

1 Armature Windings

Armature windings are the windings of a rotating machine in which motional emfs are induced. These emfs give rise to mechanical/electrical energy conversion. The armature windings consist of coils connected in series or series-parallel circuits. The coils are properly placed so that their individual emfs can be effectively collected.

In this chapter we look at how conductors are placed in rotating machines, how they are interconnected to obtain coils and how the coils are interconnected to give armature windings for effective collection of the emfs induced in the conductors. The expression for the total emf induced in the armature winding is also derived.

2 Armature-conductor connection

Two conductors c_1 and c_1', connected back to back, form a single-turn coil. The two sides of the coils are spaced in such a way that when one of its sides is under the influence of a north pole say, its other side is under the influence of a south pole for most of this time. Multi-turn coils consist of 4 or more conductors.

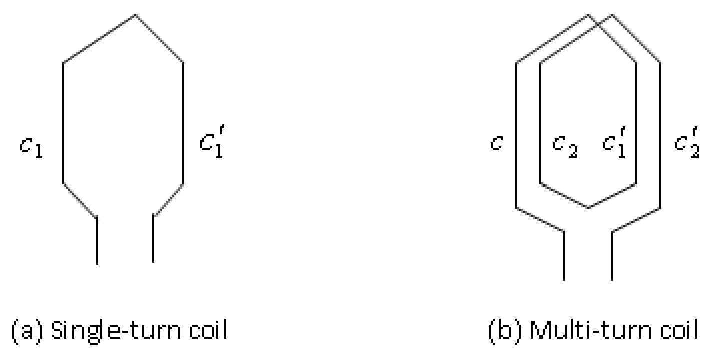


Fig.1 Coil

3 Slots

They are punched uniformly round the gap surface of the armature (i.e. that fixed or rotating member of the machine that carries the armature winding). Coils are placed in the slots. The slots may be open, semi-closed or closed. Slots provide mechanical support for the conductors.

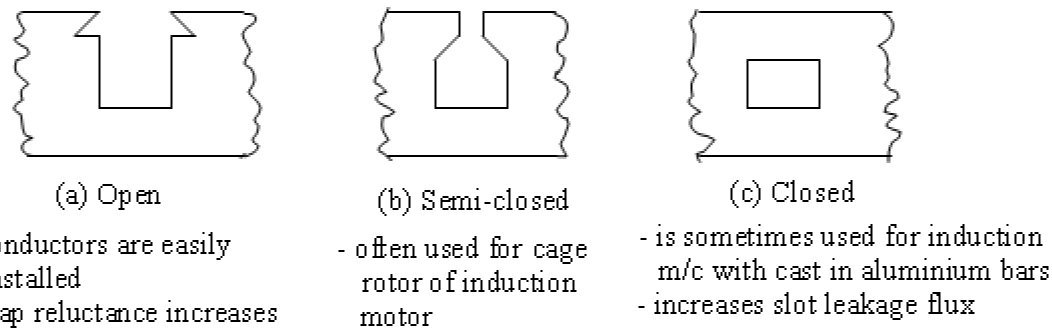


Fig. 2 Slot

4 Single-layer and two-(or double-) layer windings

The way slots are filled with the sides of coils gives two basic types of armature windings:

4.1 Single-layer winding

In this winding each side of one coil occupies a whole slot. The coils contain many insulated turns. Single-layer coils have the advantage that semi-closed or closed slots can be easily used.

4.2 Two-layer winding

Here one side of a coil lies in the top half of a slot and the other side in the bottom half of another slot a pole-pitch or less away. All coils are identical. Open slots are frequently used.

5 Lap and wave winding

The front-end of a two-layer coil can be bent inward or outward to bring about the connection from one coil to another. Inward gives what is called a lap winding and outward gives what is called a wave winding. See Fig.3. In the lap winding, each coil is connected to the next coil lying under the same pair of poles whereas in a wave winding, each coil is connected to a coil lying in approximately the same position under the next pair of poles.

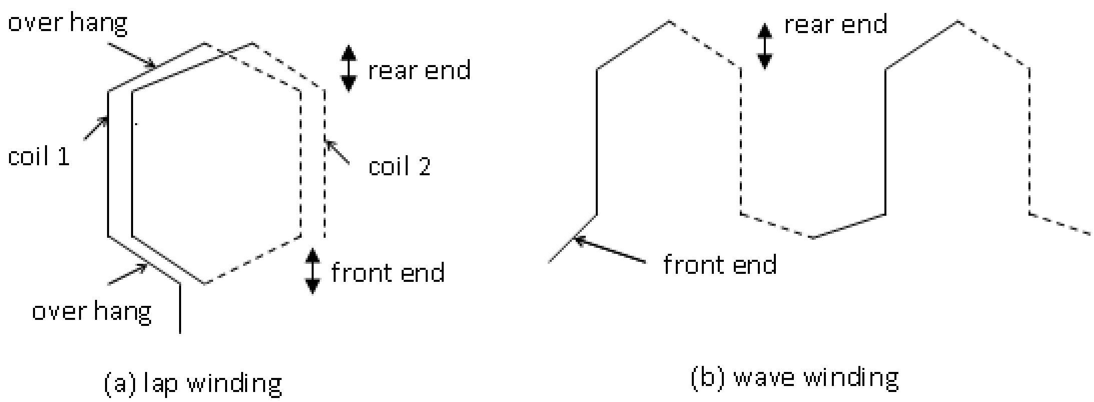


Fig. 3 Types of two-layer winding

6 Common winding terms and symbols

(a) **Conductor:** the active length of wire or strip (bar) in the slot. Z = total number of conductors in a winding.
(b) **Coil:** single-turn coil consists of two conductors separated by a pole-pitch or nearly so connected back to back. In multi-turn coil, many conductors occupying virtually the same magnetic position are included before the ends of the coils are brought out. C = total number of coils.
(c) **Coil side:** one half of a coil lying in one slot.
(d) **Pole pitch:** the distance between the centres of adjacent poles. It is 180 electrical degrees. The symbol is τ.
(e) **Back pitch or coil pitch or coil span:** it is the span of a coil. It is measured in coil sides, sometimes as a fraction of the full pole-pitch or in electrical degrees. The symbol is y_b. If $y_b = 180^o$, the coil is called "full-pitch" coil. If $y_b < 180^o$, the coil is called "fractional-pitch" coil or a "chorded" coil. Fractional-pitch coils use slightly less copper than full-pitch coils, and they also improve commutation.

7 Commutator windings

(a) **The commutator:** it consists of many copper segments insulated (mica is used) from and running parallel to one another. They are clamped around the surface of an insulated cylinder. To each commutator bar or segment is connected a coil junction (i.e. where a coil is joined to another). Hence the number of bars = the number of coils in the winding. The commutator enables us to obtain a steady and direct voltage from alternating emf generated in the armature conductor.
(b) **Type of layer or winding:** two-layer coils are employed.
(c) **Slot arrangement:** we note that
   (a) $u =$ the number of coils per slot and it is also equal to coil sides per layer. From the definition, C = Su
   (b) $u = 1$ is rarely used. $u = 2,3,4$ or even more coil sides/layer are used.

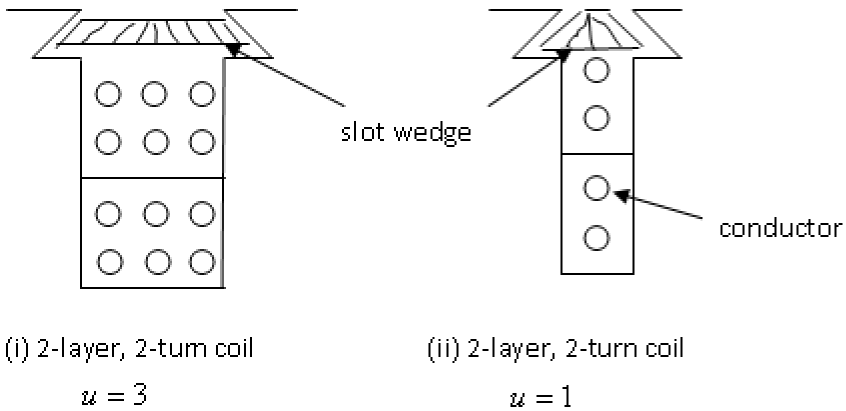


Fig. 4 Slot arrangement

(d) **Other terms used in commutator winding:**

(i) *Slot span*: it is also used to indicate the span of a coil. Measured in slots, it is given by the number of slots or teeth embraced by the coils.

$slot\ span \cong no.of\ slots/total\ no.poles = S/2p$

For all types of dc machine windings slot span can thus be obtained. If quotient is integer, then choosing this number for a coil span, a normal full-pitch winding is obtained but for special cases, slot span can be taken one slot less to obtain slightly chorded winding. If quotient is not an integer, then the nearest integer lower than $S/2p$ is taken.

In actual machine slot span lies between 10 and 15 and it is slightly less than $S/2p$.

(ii) *Coil-end pitch or commutator pitch* y_c: it is the distance between the end connections of a coil. It is measured in commutator segments or bars between the two legs of the coil.

(iii) *Progressive or retrogressive winding*: in progressive winding, winding progresses in the direction in which the coils are wound whereas in retrogressive winding, winding progresses in the opposite direction. An arrow defining the direction in which a coil is wound should point at the lower layer conductor when placed within the coil as shown in Fig. 5.

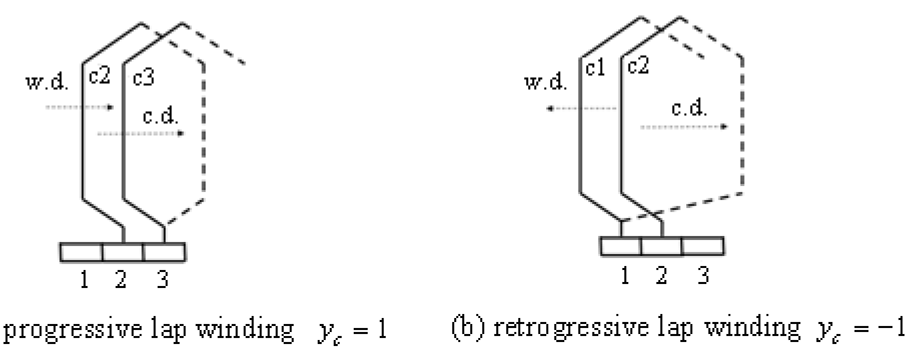


Fig.5 Progressive and retrogressive winding

8 Examples of commutator windings

8.1 Simple lap winding

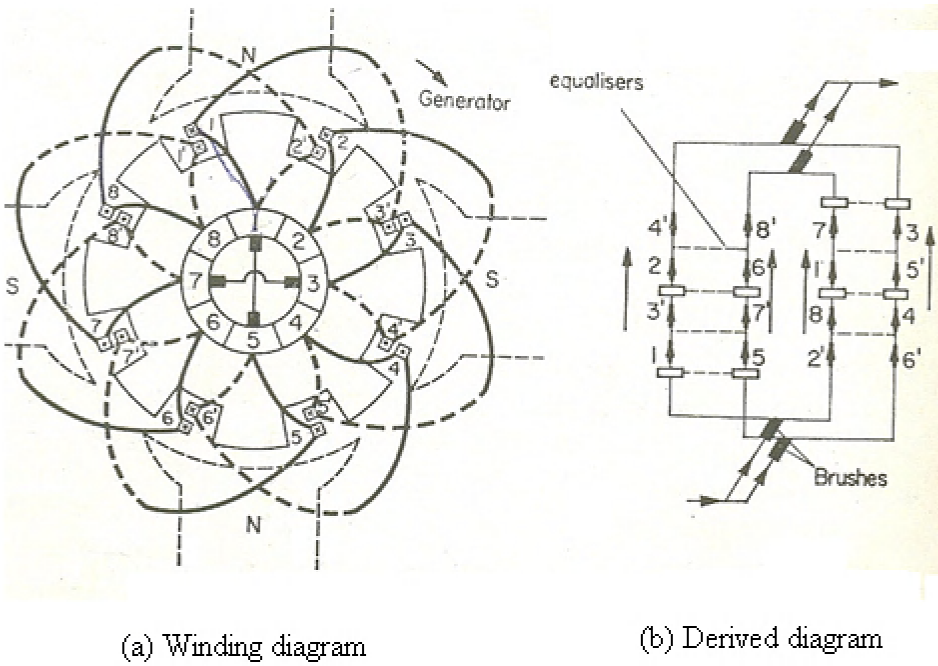


Fig. 6 four-pole simple lap winding

$S = 8, u = 1, p = 2, y_c = +1, C = 8, y_b = 2, \text{slot span} = 2$ (Note that y_b = slot span x u)

(a) **Determining type of machine when direction of rotation is given:** In generators, torque is in the direction opposite to that of the speed, whereas in motors, torque and speed are in the same direction. Therefore, when we know the direction of the speed and the direction of the torque, we can determine the type of machine.

The direction of the torque is determined as shown in Fig. 7. The current in the conductor shown to be flowing into the paper produces a clockwise field around the conductor (from corkscrew rule). This field reacts with the field due to the north pole to make the field stronger on the left-hand side and weaker on the right-hand side of the conductor. This then results in a force towards the right on the conductor.

Exercise 1
The machine in Fig. 6 is operating as a generator with the armature rotating in the clockwise direction. Determine the direction of the current in conductors 1 and 1'.

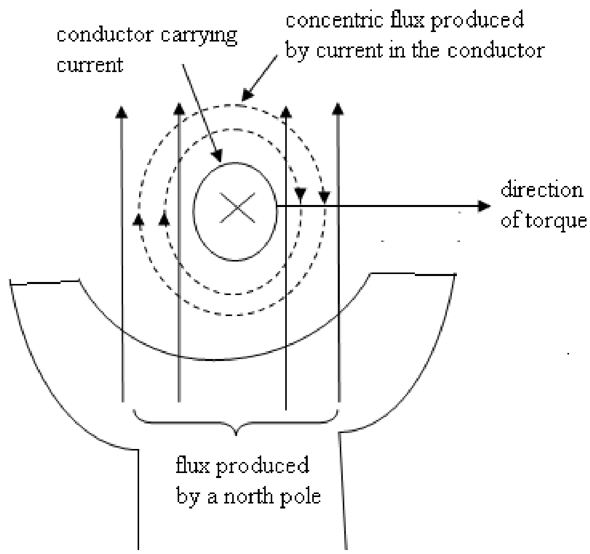


Fig. 7 Finding direction of torque

(b) **Ratio of pole arc to pole-pitch:** In practical machines, this varies from 0.67 to 0.75

(c) **Brushes:**
- The number of brushes = number of poles
- They are equally spaced
- In their normal or neutral position, they short coils which have their sides moving through low-flux inter-polar zone

(d) **Parallel paths:**
- The number of parallel paths = the number of poles
- A parallel path consists of all top-layer conductors under one pole, together with all the bottom-layer conductors under one neighbouring pole. See Fig. 8.

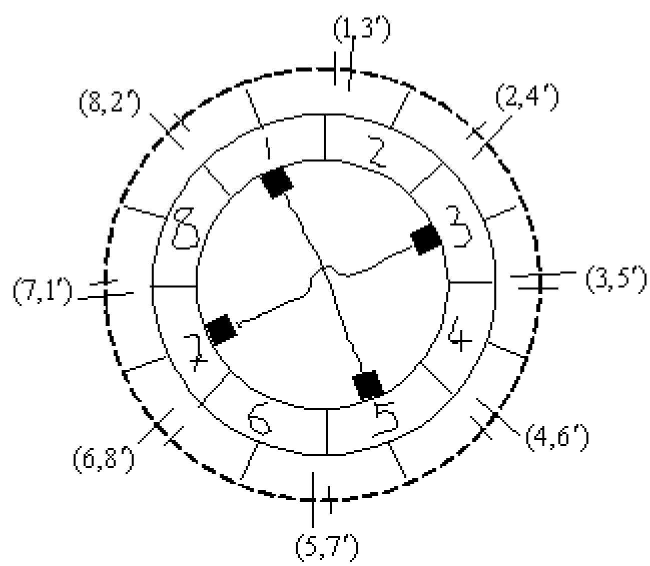


Fig. 8 Diagram showing parallel paths of machine in Fig. 6

(e) **Total emf:** the total emf induced between brushes is not completely constant. Variations arise as rotor changes position. This can be seen from Fig. 6 with the rotor displaced through one-half the commutator segment. The displacement will result in four coils being short-circuited. In practical machine the variations are very small. Hence the number of conductors in series per circuit or parallel path is given by $Z_c = Z/2p$

(f) **Equalizers:**

- Some dc lap winding machines have equalizers. The emfs in the parallel paths are supposed to be the same but they are not because practical machines are not perfectly symmetrical. The equalizers are used so that circulating currents caused by slight discrepancies in the generated emfs in the parallel paths will flow through them rather than through the brushes.
- An equalizer connects together all noses of coils or commutator segments which should be at the same potential or which are exactly similarly situated with respect to the magnetic system
- The number of top layer coil sides which can occupy identical positions relative to the magnetic system = *p*. If the necessary condition is fulfilled for the *p* coil sides to occupy identical positions then the coils in the winding can be divided into *C/p* groups, each group consisting of *p* coils occupying the same identical positions.
- For top layer coil sides to occupy identical positions relative to the magnetic system, the ratio S/p must be an integer. This condition must be fulfilled if equalizers are to be used. In the design of lap windings, it is required that S/p be an integer. We note that when *S/p* is an integer then *C/p = (S/p)u* is also an integer.
- The number of coils connected to an equalizer or an equalizing ring = the number of coils occupying identical positions = *p*.
- Not every coil or commutator segment is equalized with that of correspondingly situated coils. If all coils are to be equalized then *C/p* equalizers will be required. It generally suffices to use 10 to 20 equalizers.
- Equalizers are arranged so that tappings to the various equalizers, which may be connected to commutator segments or noses of the coils, shall be equally spaced.
- $$\text{Pitch of tapping points} = \left(\frac{C}{p}\right) \times \frac{1}{\text{number of equalizers}}. \qquad (1)$$
- The pitch must be an integer.
- With the maximum number of equalizers *C/p* pitch of tapping points = 1
- For example, if $S = 27, p = 3, C = 54$ and number of equalizers $= 9$ then the pitch of tapping points $= \left(\frac{54}{3}\right) \times \frac{1}{9} = 2$. The equalizers will be connected to the commutator segments as shown in Table 1.

Table 1 Connection of commutator bars to equalizers

Equalizer no.	1	2	3	4	5	6	7	8	9
Connected to commutator bar no.	1	3	5	7	9	11	13	[illegible]	17
Connected to commutator bar no.	19	21	23	25	27	29	31	33	[illegible]
Connected to commutator bar no.	37	39	[illegible]	43	45	[illegible]	49	[illegible]	53

Add C/p = 18

$1+(n-1)PTP$

- For the simple lap winding machine given in Fig. 6, the maximum number of equalizers that can be used is $C/p = 8/2 = 4$. If we decide to use 2, then the pitch of tappings is $4/2 = 2$. Refer to Fig. 6 b for the connections of the commutator segments to four equalizers and the Fig. 9 for the connections to two equalizers.

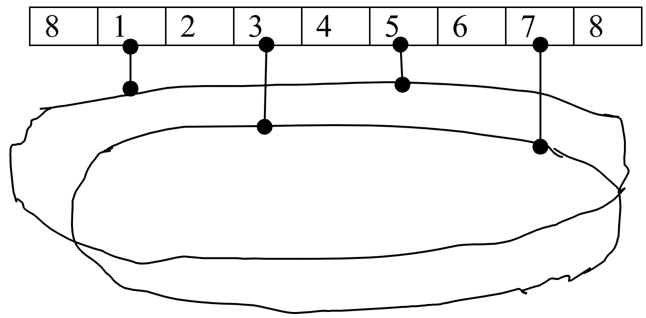


Fig. 9 Connection of commutator bars to 2 equalizers if used on the machine in Fig 6

- Generally, the number of slots used is a multiple of the number of poles so that full-pitch windings are possible. In this case, equalizers can also be used because if *S/2p* is an integer then *S/p* is also an integer.

Exercise 2
The total number of conductors in each parallel path of an 8-pole simple lap winding must be greater than 94 and less than 102. The number of slots per pole must also be greater than 12 and less than 18. Determine

(a) the maximum and minimum number of conductors per slot. Hint: $(Z/S)_{\max} = Z_{\max}/S_{\min}$
(b) the number of conductors per slot
(c) the number of coils per slot if u should be greater than 1.

Exercise 3
A simple lap winding has the following particulars: C = 384, p = 4. Not less than 10 and not more than 15 equalizers must be used. Find the number of equalizers.

8.2 Simple wave winding

(a) A practicable wave winding must satisfy the relation $py_c = C \pm 1$. It is" $+1$ " for progressive winding and " -1 "for retrogressive winding. The relation ensures two things:
  - That after one tour of the armature periphery, the end of the last coil falls on the commutator segment next to the beginning of the first coil and
  - That after several $(= y_c)$ similar tours of the armature, all the commutator segments are occupied and the final coil falls naturally on the first giving a closed winding.

(b) The number of poles commonly used for dc machines is 4, 6 or 8. For 4- and 8-pole machines, C must be odd and since $C = Su$, u and S must also be odd. u is usually 1, 3 or 5. For a 4-pole machine u is usually 3. For 6-pole machine, u is any number not a multiple of 3. u used is 2 or 4. Note that for this number of poles $C = 3y_c \pm 1$.

(c) In this winding, the arrangement is such that all coils carrying current in the same direction at any instant are connected in series. Since coil has either its current in the clockwise or anticlockwise direction, the number of parallel paths is 2.
(d) In general, the number of coils joined in series between adjacent commutator segments = p.
(e) The number of brushes required can be 2 spaced at $1/2p$ of the commutator circumference. Check as follows:
  - The number of commutator segments = the number of coils = C
  - The number of commutator segments embraced by brushes when spaced $1/2p$ of the commutator circumference = $C/2p$
  - The number of coils joined in series between the brushes = $(C/2p) \times p = C/2$
(f) In their normal position, the brushes make contact with conductors which are moving through the low-flux interpolar zone. Their positions can be found by drawing an equivalent ring diagram (this can be drawn with the aid of winding table), finding the commutator bar where two emfs separate (the position of negative brush for generator action) and also finding the commutator bar where two emfs join (the position of positive brush).
(g) The number of brushes used in practice $= 2p$. This is to lower the current per brush and thus keep down the length of the brushes so that short and therefore cheap commutator can be used. They are spaced symmetrically as in lap winding.
(h) When extra brushes are used, a few coils in the low-flux interpolar zone are shorted out but this has little effect on the output voltage of a practical machine.
(i) The progression of wave winding is conveniently indicated in a table called winding table. Consider a simple wave winding which has the following parameters: $S = 5$, $p = 2$, $C = 5$, $u = 1$. For this winding, $y_c = (5 \pm 1)/2 = 2 \text{ or } 3$. We use $y_c = 2$.
Slot span $= S/2p = 5/4 = 1.25 = 1$, 1 being the nearest lower integer.
Coil span $y_b = \text{slot span} \times \text{number of coils/slot (or coil sides/layer)} = \text{slot span} \times u = 1 \times 1 = 1$
The winding table of such a machine will be as shown in Table 2.

Table 2 Winding table for a simple wave winding

Commutator bar number	to	Top conductor number	to	Bottom conductor Number ($+y_b = 1$)	to	Commutator bar number ($+y_c = 2$)
1		1		$1+1=2'$		$1+2=3$
3		3		$3+1=4'$		$3+2=5$
5		5		$5+1=1'$		$5+2=2$
2		2		$2+1=3'$		$2+2=4$
4		4		$4+1=5'$		$4+2=1$

We observe that:
- Starting from commutator bar 1, after one tour of the armature periphery, the end of the last coil falls on commutator bar 5 which is next to commutator bar 1. It falls on the adjacent commutator bar 5 but NOT adjacent commutator bar 1 because the winding is retrogressive. It is retrogressive because $y_c = 2$ is used.
- After the $y_c = 2$ tours of the armature periphery, all the commutator bars are occupied.
- The final coil of the last tour (in this case the second tour) falls on the commutator bar 1 we started with to give a closed winding.

9 Induced voltage in armature conductor

Fig. 10 shows the variation of the emf generated in a conductor while it is moving through two pole pitches. The emf remains constant while the conductor is moving under a pole and then decreases rapidly to zero when it is midway between the pole tips of adjacent poles.

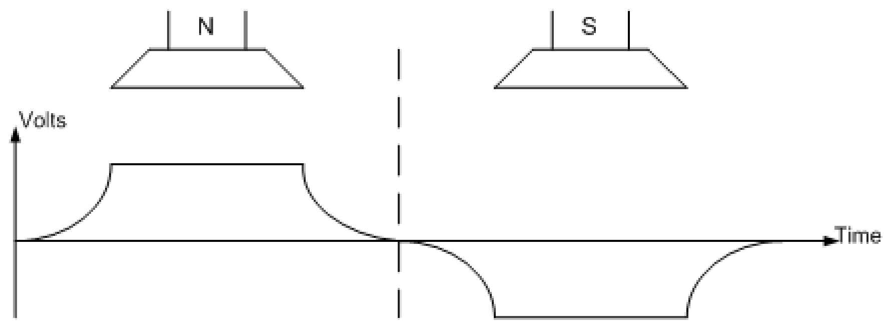


Fig. 10 Waveform of emf generated in a conductor

10 Induced voltage in dc armature windings

The average value of emf in a conductor when it is moving under a pole face is given by

$$E_c = B_{av} L u \tag{2}$$

where
B_{av} = the mean flux density
L = the active length of the conductor
u = the surface speed of the armature

The flux density is given by

$$B_{av} = \frac{\text{Total flux under the poles}}{\text{Total surface area swept through by the active length of the conductor}} = \frac{2p\phi}{2\pi r L} \tag{3}$$

where
r = radius of the armature core
ϕ = flux / pole

Let N = rotor speed in rev/min. Then speed in rad/s $\omega = \dfrac{2\pi N}{60}$ and $u = \omega r = \dfrac{2\pi N r}{60}$ (4)

Substituting equations (3) and (4) into equation (2) yields

$$E_c = \frac{2Np\phi}{60}$$

Let Z = total number of conductors in a winding and $2a$ = number of parallel paths.
Then the number of conductors in series per path $= Z/2a$ and the voltage induced between the positive and negative brushes is given by

$$E = E_c \times \frac{Z}{2a} = \frac{2Np\phi}{60} \times \frac{Z}{2a}$$

$$E = \frac{N}{60} \times \frac{p}{a} \times Z\phi \qquad (5)$$

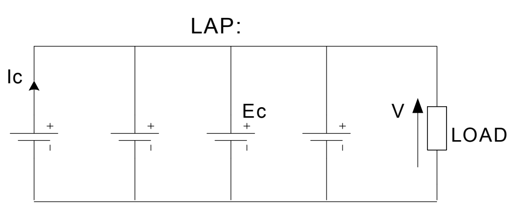
$a = 1$ for simple wave winding and
$a = p$ for simple lap winding

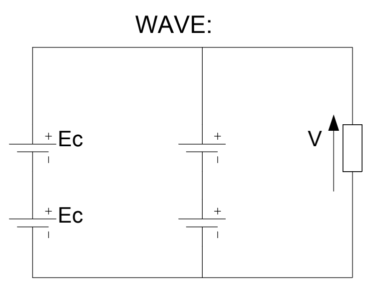
11 Comparing lap and wave winding machines

For a given number of conductors and the same physical size and speed, the wave winding gives a higher terminal voltage and lower current than the lap winding. In general the lap winding is used for low-voltage, heavy-current machine.

Consider a 4-pole machine having 4 conductors. Referring to circuit below, for the lap winding, the terminal voltage V = voltage per path = voltage per conductor = E_c, maximum current I_{max} = 4 x maximum current/path = 4 x rating of a conductor = $4I_c$. Hence maximum power = $4E_cI_c$

Referring to the circuit for the wave winding, $V = 2E_c$ and $I_{max} = 2I_c$. Therefore maximum power = $4E_cI_c$.

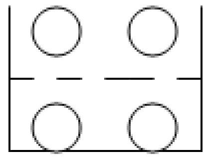




12 Numerical examples on commutator windings

1 A 6-pole simple-wave armature winding has 64 single-turn coils and 32 slots
(a) Draw a slot showing the arrangement of conductors
(b) State whether the winding should be progressive or retrogressive
(c) Calculate the slot span and the coil pitch

Solution
(a) Total number of conductors $= 64 \times 2 = 128$ since a coil has one turn or two conductors.
Number of conductors/slot $= 128/32 = 4$ and $u = C/S = 64/32 = 2$.
Use either the number of conductors/ slot and the number of turns (the number of conductors in a column = the number of turns in a coil x 2) or the number of conductors/slot and u (u = number of conductors in a row) to draw the slot arrangement.



(b) Commutator pitch, $y_c = (64 \pm 1)/3$. Since this must be an integer, it can only be 21. Thus winding can only be retrogressive.

(c) Slot span $= S/2p = 32/6 = 5.33 = 5$, the nearest lower integer.

Coil pitch or coil span $y_b = \text{slot span} \times u = 5 \times 2 = 10$ coil-sides

2 A 4-pole machine has a wave-wound armature with 576 conductors. Each conductor has a cross-sectional area of 5 mm^2 and a mean length of 800 mm. Assuming the resistivity for copper to be 0.0173 μΩm at 20°C and the temperature coefficient of resistance to be 0.004/°C at 0°C, calculate the resistance of the armature winding at its working temperature of 50°C.

Solution

Number of parallel paths = 2

Number of conductors/parallel path $= 576/2 = 288$

Resistance of a conductor at 20°C = $\dfrac{\rho l}{A} = \dfrac{0.0173 \times 800 \times 10^{-3}}{5 \times 10^{-3}}$ = 0.0173 x 16 x 10^{4}μΩ

Resistance per parallel path at 20°C= resistance/conductor x number of conductors/path

=288 x 0.0173 x 16 x 10^{4}μΩ

Resistance of armature winding at 20°C $R_{20} = \dfrac{\text{resistance/path}}{\text{number of paths}} = \dfrac{288 \times 0.0173 \times 16 \times 10^4}{2}$ μΩ

= 0.3986 Ω

Resistance of armature winding at 50°C $R_{50} = \dfrac{R_o\left[1 + \alpha_o t_{50}\right]}{R_o\left[1 + \alpha_o t_{20}\right]} R_{20} = \dfrac{1 + 0.004 \times 50}{1 + 0.004 \times 20} \times 0.3986$

= 0.443Ω

3 A 4-pole wave–connected armature has 51 slots with 12 conductors per slot and is driven at 900 rev/min. If the useful flux per pole is 25 mWb, calculate the value of the generated emf.

Solution

Z = 51 x 12 = 612; a = 1; p = 2; N = 900 rev/min; ϕ = 0.025 Wb

$$E = \frac{N}{60} \times \frac{p}{a} \times Z\phi = \frac{900}{60} \times \frac{2}{1} \times 612 \times 0.025 = 459 \text{ Volts}$$

4 An 8-pole lap–connected armature driven at 550 rev/min is required to generate 260 V. The useful flux per pole is about 0.05 Wb. If the armature has 120 slots, calculate a suitable number of conductors per slot and the corresponding useful flux per pole.

Solution
For an 8-pole lap winding, $a = 4$

$$E = \frac{N}{60}.\frac{p}{a}.Z\phi \Rightarrow 260 = \frac{550}{60}\times\frac{4}{4}\times Z\times 0.05$$

From which $Z = 890$ approximately and the number of conductors per slot = 890/120 = 7.4. This value must be an even number. Hence 8 conductors per slot would be suitable. All other factors remaining the same $Z_{new}\phi_{new} = Z_{old}\phi_{old}$, Thus the flux corresponding to the new total conductor Z_{new}= 8 x 120 would be $\frac{890\times 0.05}{8\times 120} = 0.0464\,\text{Wb/pole}$

5 An eight-pole armature is wound with 480 conductors. The magnetic flux and the speed are such that the average emf generated in each conductor is 2.2 V; each conductor is capable of carrying a full-load current of 100 A. Calculate the terminal voltage on no load, the output current on full load and the total power generated on full load when the armature is (a) lap-connected and (b) wave-connected.

Solution
(a) Lap connected
The number of parallel paths = the number of poles = 8
The number of conductors / path = 480/8 = 60
Terminal voltage on no load = emf/conductor x number of conductors/path
= 2.2 x 60 = 132 V
Output current = full-load current per conductor x number of parallel paths = 100 x 8 = 800A
Total power generated = 800 x 132 = 105.6 kW

(b) Wave connected
The number of parallel paths = 2
The number of conductors/path = 480/2 = 240
Terminal voltage on no load = 2.2 x 240 = 528 V
Output current on full load = 100 x 2 =200 A
Total power generated = 200 x 258 = 105.6 kW

6 The wave-connected armature of a four-pole dc generator is required to generate an emf of 520 V when driven at 660 rev/min. Calculate the flux per pole required if the armature has 144 slots with 2 coil sides per slot (or 1 coil per slot or 1 coil-side per layer), each consisting coil of 3 turns.

Solution
E = 520 V, N = 660 rev/min, Z = 144 x 2 x 3 = 864, p = 2, a =1

$$E = \frac{N}{60}.\frac{p}{a}.Z\phi \Rightarrow 520 = \frac{660}{60}.\frac{2}{1}.864\phi \Rightarrow \phi = 27.3\,\text{mWb}$$

13 3-phase armature windings

The three-phase winding is composed of conductors in a number of evenly spaced slots around the internal surface of the stator. The conductors are arranged in 3 balanced sections. The conductors in a section are interconnected to form a phase winding.

13.1 Definitions

(a) Phase or coil groups: Coils staggered at one-slot interval are connected in series to form a phase or coil group. The voltage of a coil group is given by the phasor sum of the constituent individual coil emfs, which because they are staggered, have emfs displaced in time phase.

(b) Coil voltage: The voltage of a coil is obtained by the product of the number of conductors in the coil and the voltage per conductor when it is full-pitch. If the coil is not full-pitch the product has to be multiplied by coil-span factor

(c) Phase winding: The phase or coil groups may be connected in series, in parallel or in series-parallel to obtain a phase winding. The phase or coil groups of a phase winding are spaced uniformly around the stator. The voltage of a phase winding is the arithmetic sum of the voltages of the phase or coil groups (per parallel, i.e. when they are parallel as well)

(d) Phase spread or phase groupings: The way the conductors or coil-sides are arranged in 3-balanced sections gives what is called 60^o or 120^o phase spread or phase group. Fig. 11 shows the two arrangements. For the 60^o phase spread, the coil-sides over two pole pitches, i.e. 360^o are divided into 6 groups, each group spreading over 60^o. The groups are assigned to the three phases A, B and C in the sequence A, C', B, A', C and B'. For a given phase X, the conductors are interconnected such that the current in the conductors in section X and the current in conductors in section X' flow in opposite directions. In the case of the 120^o phase spread, the coil-sides over two pole pitches are divided into 3 groups, each group spreading over 120^o. For two-layer winding, the groupings of coil-sides given in Fig. 11 represent the groupings of the top-layer coil-sides only.

Coils have return conductors a pole-pitch or less away to give turns. For this reason, the phase spread $\sigma = 120^o$ is possible with two-layer winding but not possible with single-layer winding. With $\sigma = 60^o$, both single- and two-layer windings can be used. In ac winding, u = 1, hence $C = S$ in the two-layer winding.

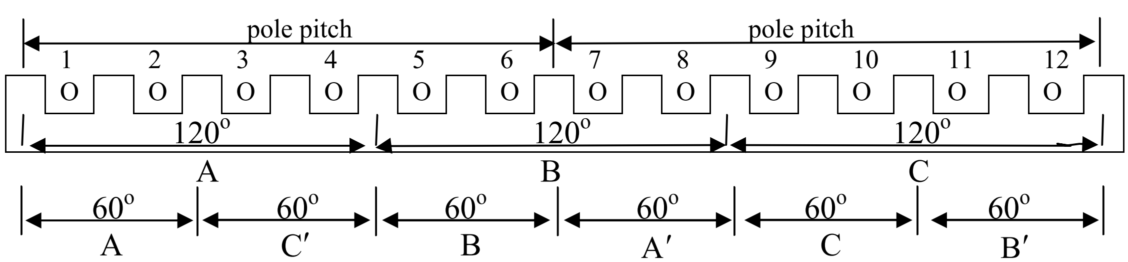


Fig. 11 Phase groupings

13.2 Examples of 3-phase windings

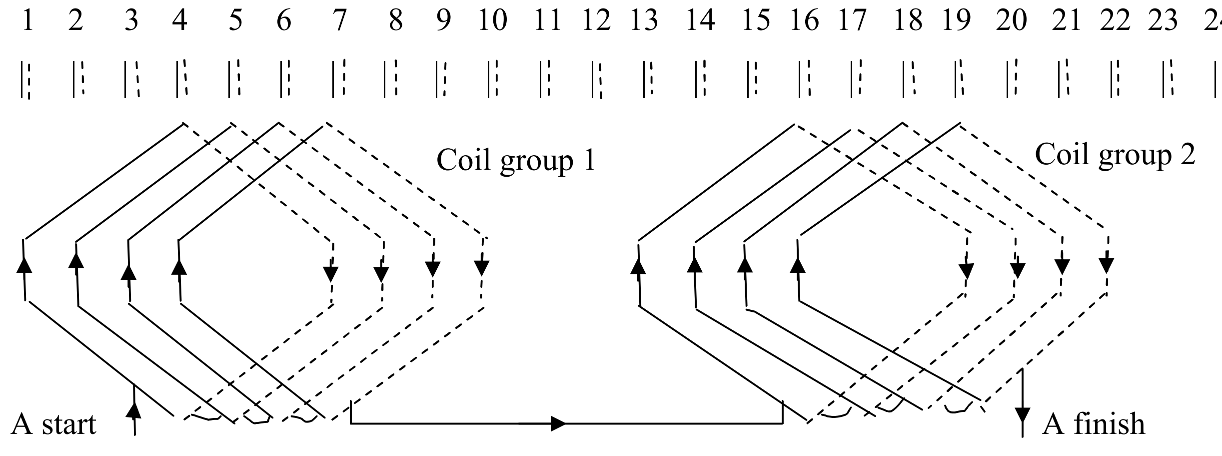


Fig. 12.a 120° groups, 2-layer lap: 2 coil groups are connected in series

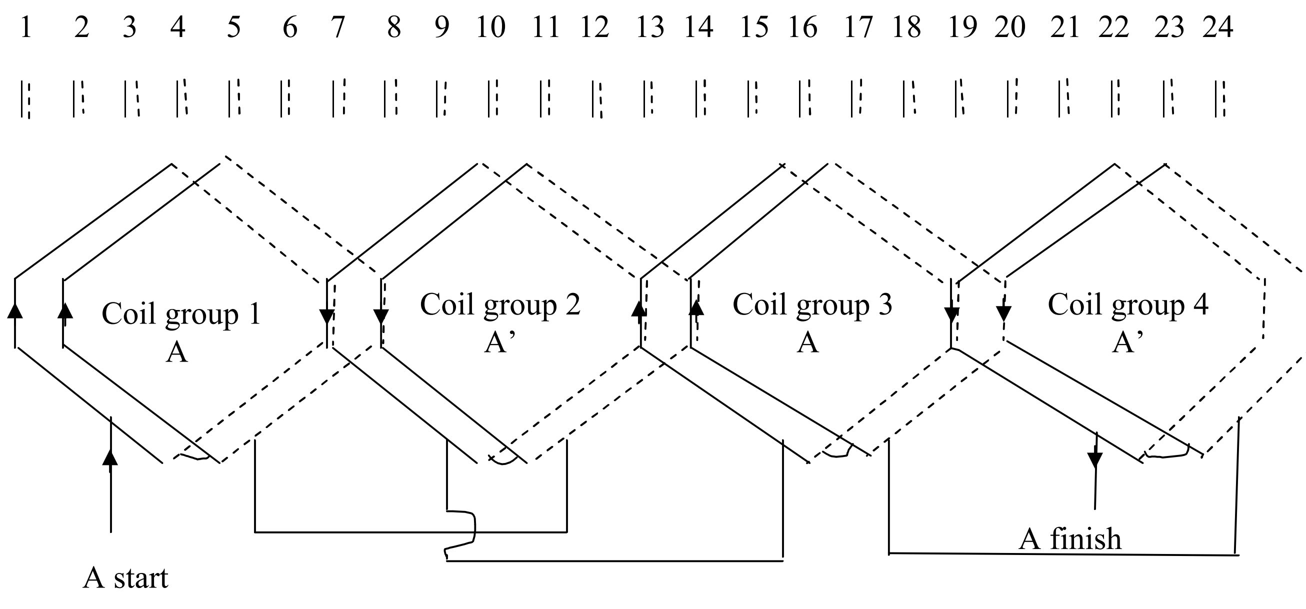


Fig. 12.b 60° groups, 2 layer lap: 4 coil groups are connected in series

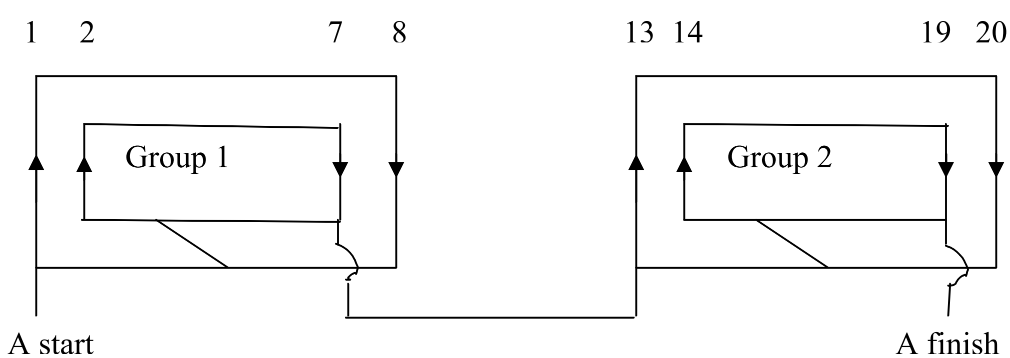


Fig. 12.c 60° groups, 1-layer concentric groups connected in series

The armatures of nearly all synchronous generators and of most induction motors above a few horse powers are wound with 2-layer windings. The most usual phase spread is 60^o. The type of coil used in modern machines is lap.

13.3 Emf induced in armature coil

Fig. 13 shows two pole pitches of 2p-pole machine in which the spatial distribution of the gap flux is sinusoidal. The voltage induced in a conductor moving past the flux, which is considered to be stationary, at speed $u = \omega r$ is given by

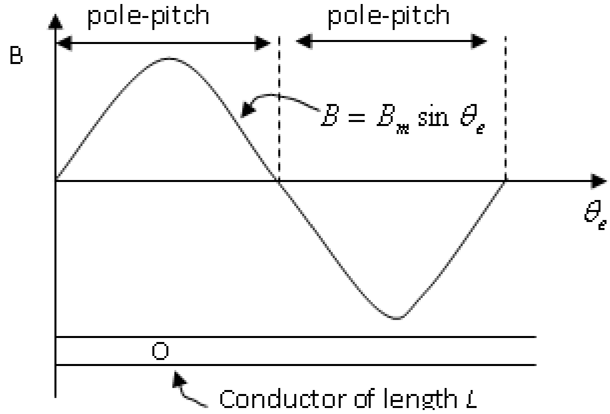


Fig. 13 Flux distribution over a double pole-pitch

$$e = (B_m \sin\theta_e)L(\omega_r r) \tag{6}$$

The angular velocity $\omega_r = \dfrac{d\theta_{mech}}{dt}$ (7)

Hence $\dfrac{d\theta_e}{dt} = p\omega_r$ (8)

If the conductor is located at $\theta_e = \theta_e^o$ when $t = 0$, then its location at the time t is

$$\theta_e = p\omega_r t + \theta_e^o \tag{9}$$

This leads to the following expression for the steady-state voltage as a function of time

$$e = B_m L r \omega_r \sin(p\omega_r t + \theta_e^o)$$

$$e = B_m L r \left(\frac{2\pi f}{p}\right)\sin(2\pi f t + \theta_e^o) \tag{10}$$

where $f = \dfrac{p\omega_r}{2\pi} = pn$ (11)

and n is the speed of the conductor or armature in revolutions per second.

The average flux density in the air gap $B_{av} = \frac{1}{\pi}\int_0^{\pi} B_m \sin\theta_e d\theta = \frac{B_m}{\pi}\left[\cos\theta\right]_{\pi}^{0} = \frac{2B_m}{\pi}$ which implies that

$$B_m = \frac{\pi}{2} B_{av} \tag{12}$$

Let ϕ be the total flux per pole. Then $\phi = B_{av} \times \text{total area under a pole} = B_{av} \times \frac{2\pi rL}{2p}$ which also implies that

$$B_{av} = \frac{p\phi}{\pi rL} \tag{13}$$

It follows from equations (12) and (13) that

$$B_m = \frac{p\phi}{2rL} \tag{14}$$

Substituting equation (14) into equation (10) yields

$$e = \pi f\phi \sin\left(2\pi ft + \theta_e^o\right) \tag{15}$$

Therefore the peak emf of a conductor

$$e_m = \pi f\phi \tag{16}$$

and the rms value will be

$$E = \frac{\pi}{\sqrt{2}} f\phi = 2.22 f\phi \tag{17}$$

For a full-pitch coil of T_c turns the rms coil emf is

$$E_c = 4.44 fT_c\phi \tag{18}$$

13.4 Winding factors

The factors are

Distribution factor

The coil voltages all have the same magnitude and frequency but they are not in phase with each other. The emf of a coil group is the phasor sum of the constituent individual coil emf. The factor defined

$$k_m = \frac{\text{phasor sum}}{\text{arithmetic sum}} \text{of coil emfs} \tag{19}$$

is called the distribution factor. Suppose a coil group to comprise m full-pitch coils and the slot angle is ψ . Then the phase spread $\sigma = m\psi$. It can also be shown that

$$k_m = \frac{\sin\frac{1}{2}m\psi}{m\sin\frac{1}{2}\psi} = \frac{\sin\frac{1}{2}\sigma}{m\sin\frac{1}{2}\frac{\sigma}{m}} \qquad (20)$$

When ψ is small

$$k_m \approx \frac{\sin\frac{1}{2}\sigma}{\frac{1}{2}\sigma} \qquad (21)$$

We note that the angle σ in the denominator is in radians.

The coil group of the m full-pitch coils will produce the rms emf

$$E_{pg} = 4.44mT_c k_m f\phi$$

$$E_{pg} = 4.44T_{pg} k_m f\phi \qquad (22)$$

where $T_{pg} = mT_c$ is the total turns in series per coil group.

Coil span factor

A coil may be under- or over-pitched. Such a coil is said to be chorded. A coil is usually short-pitched. Short-pitched coils have amount of copper in the end-connections reduced and harmonics in the phase emfs reduced or suppressed. A coil pitch of 0.8 to 0.9 per unit is usually employed on 2-layer windings and this reduces the 5th and 7th harmonics considerably. Fig. 14 shows the arrangement of phase A conductors in a 2-layer, 60^o winding over two pole-pitches. There are 6 slots per pole or 2 slots per pole per phase (= m). The arrangement ***W*** is for full pitch coils and the arrangement ***Z*** is for short-pitched or chorded coils. The chorded coils are short-pitched by 1 slot interval which corresponds to $\varepsilon = \frac{1}{6}\times 180^o = 30^o$ electrical.

	1	2		5	6	7	8	
W	a	a	Top layer conductors			a'	a'	Full pitch
	a	a	Bottom layer conductors			a'	a'	
Z	a	a	Top layer conductors			a'	a'	Short pitch
	a		Bottom layer conductors		a'	a'		

Fig. 14 Two arrangements of phase A conductors in 2-layer, 60^o winding

If the coil span is not full pitch, then the voltage of a coil is given by the phasor sum of the conductor emfs in its two sides. Let the coil span depart from its full-pitch value by ε electrical degrees. Then

$$k_e = \frac{\text{phasor sum}}{\text{arithmetic sum}} \text{ of coil side emfs} = \cos\tfrac{1}{2}\varepsilon \qquad (23)$$

The factor k_e is the coil-span factor.

13.5 Phase winding emf

A coil group consisting of short-pitched coils will produce the rms emf

$$E_{pg} = 4.44 T_{pg} k_m k_e f \phi \tag{24}$$

A phase winding consisting of n such coil groups in series will have a phase voltage

$$E_{ph} = 4.44 n T_{pg} k_m k_e f \phi$$

$$E_{ph} = 4.44 T_{ph} k_m k_e f \phi \tag{25}$$

$$E_{ph} = 4.44 T_{ph} k_w f \phi \tag{26}$$

where

T_{ph} is the total turns in series per phase and

$k_w = k_m k_e$ is the winding factor.

13.6 Harmonic voltages

The gap-flux density curve which is not purely sinusoidal can be analysed into fundamental and harmonic densities B_1, B_3, B_5, B_7 etc. The emf of a phase due to nth harmonic is

$$E_{phn} = 4.44 k_{mn} k_{en} n f T_{ph} \phi_n \tag{27}$$

where

$$k_{mn} = \frac{\sin \frac{1}{2} n\sigma}{m \sin \frac{1}{2} \frac{n\sigma}{m}} \tag{28}$$

and

$$k_{en} = \cos \tfrac{1}{2} n\varepsilon \tag{29}$$

A more useful form is

$$E_{phn} = \left(\frac{k_{wn}}{k_{w1}}\right)\left(\frac{B_n}{B_1}\right) E_{ph1} \tag{30}$$

where

$k_{wn} = k_{mn} k_{en}$.

The subscript 1 refers to the fundamental component of the flux. The rms phase emf in this case will be

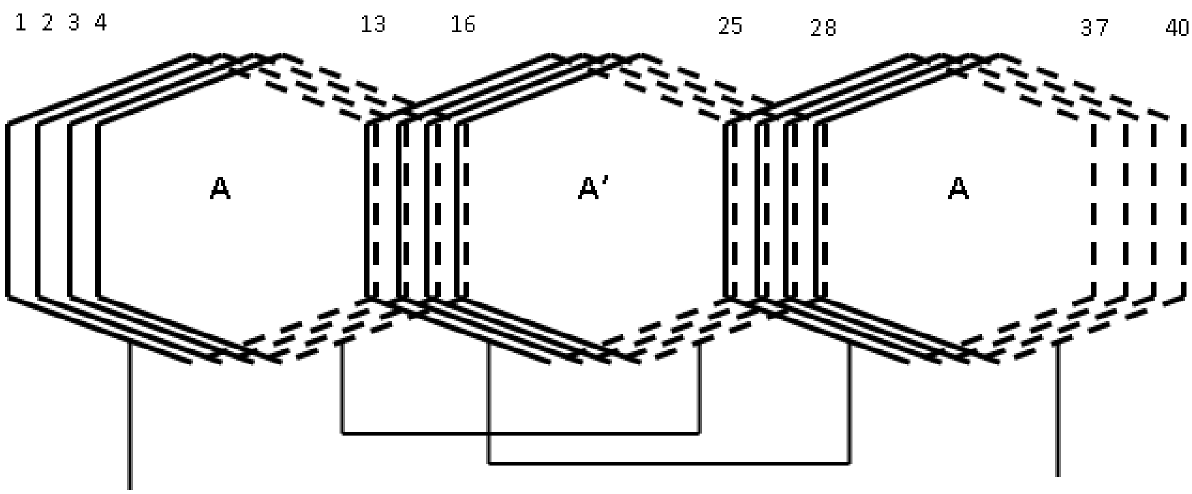
$$E_{ph} = \sqrt{E_{ph1}^2 + E_{ph3}^2 + ...+ E_{phn}^2} \tag{31}$$

The line emf in star connection is $\sqrt{3} E_{ph}$ with E_{ph} calculated as above but omitting all triplen (i.e. all odd multiples of 3) harmonic values.

13.7 Numerical examples on 3-phase armature windings

(1) The stator of a 3-phase, 10-pole induction motor possesses 120 slots. If 60^0 groups, 2-layer lap winding is used, calculate
   (a) The total number of coils
   (b) The number of coils per group
   (c) Indicate the arrangement and slot numbering of three successive groups of a phase all connected in series. The coils are multi-turn and full pitch.

   Solution
   (a) For 2-layer winding, the number of coils = the number of slots = 120
   (b) For 60^o, 2-layer winding, m = number of coils per group = number of slots per pole per phase = 120/(10 x 3) = 4
   (c) Slot span = S/2p = 120/10 = 12. Therefore y_b = slot span x u = 12 x 1 = 12 coil sides. Thus the return conductor for top layer conductor 1 is 13', for top layer conductor 2 is 14', etc. The top layer are grouped as follows, first 4 conductors to A, next 4 conductors to C', next 4 conductors to B, next 4 conductors to A', etc. The arrangement and slot numbering of three successive coil groups of phase A: coil group A starting at 1 and ending at 4, coil group A' starting at 13 and ending at 16 and another coil group A starting at 25 and ending at 28 are shown below:



NB: The connection between coils and coils groups should be well indicated.

(2) A 20-pole stator has 180 slots with single-layer full pitch coils, 6 conductors per slot, and all coils per phase connected in series. The flux per pole is 25 mWb. Compare the 50-Hz phase emfs
   (a) in a 1-ph winding with 5 adjacent slots per pole wound (the others being empty), and
   (b) in a 3-ph star-connected winding with all slots wound.

Solution

Single-phase

The number of slots/pole = 180/20 = 9

Slot angle, $\Psi = 180/9 = 20^0$

m = number of coils per group = 5

Distribution factor $k_m = \dfrac{\sin\frac{1}{2}m\psi}{m\sin\frac{1}{2}\psi} = \dfrac{\sin\frac{1}{2}\times 5\times 20}{5\sin\frac{1}{2}\times 20} = 0.882$

Number of conductors/slots = 6

Number of conductors/pole = 5 x 6 = 30

Total number of conductors = number of poles x number of conductor/pole = 30 x 20= 600

Total number of turns, T_{ph}= total number of conductors/2 = 600/2= 300

Flux/pole Φ = 25 mWb

$E = 4.44k_m fT_{ph}\Phi$ for full-pitch coils

$= 4.44 \times 0.882 \times 50 \times 300 \times 25 \times 10^{-3} = 1470$ V = 1.47 kV

Three-phase

Phase spread, $\delta = 60^0$

m = number of slots per pole per phase = 180/(20 x 3) = 3

Distribution factor = $\dfrac{\sin\frac{1}{2}\sigma}{m\sin\frac{1}{2}\frac{\sigma}{m}} = \dfrac{\sin\frac{1}{2}\times 60^0}{3\sin\frac{1}{2}(\frac{60^0}{3})} = \dfrac{\sin 30^0}{3\sin 10^0} = 0.960$

Total number of conductors/phase = total number of conductors/3 = (180 x 6)/3 = 360

Number of turns/phase= total number of conductors per phase/2 = 360/2 = 180

$E_{ph} = 4.44 \times 0.960 \times 50 \times 180 \times 25 \times 10^{-3} = 960$ V = 0.960 kV

(3) The field form of a 3-phase, 50 Hz, 600-rev/min alternator has spatial flux-density distribution given by the expression: $B = \sin\theta + 0.3\sin 3\theta + 0.2\sin 5\theta$ tesla

The machine has 180 slots, wound with two-layer 3-turn coils. Each coil spans 15 slots, the coils being connected in 60^0 groups. The armature diameter is 125 cm and the core length is 45 cm. Calculate (a) the number of poles (b) the rms phase and line voltages.

Solution

The number of pole-pairs of an alternator running at speed N is $p = \dfrac{60f}{N} = \dfrac{60\times 50}{600} = 5$

Therefore, the number of poles = $2p$ = 5 x 2 = 10

Area under a pole A = total gap surface area of the armature/$2p$ = $\dfrac{125\pi\times 45}{10} = 1765\,\text{cm}^2$

The fundamental flux per pole $\Phi_1 = B_{1av} \times A = (2B_{m1}/\pi) \times A$

$= (2 \times 1)/\pi \times 0.1765 = 0.1122$ Wb

The distribution factors:
The number of slots/pole = 180/10 = 18
Slot angle ψ = 180/18 = 10^0
The number of slots/group, m = number of slot per pole per phase = 18/3 = 6

$$k_{m1} = \frac{\sin\frac{1}{2}n\sigma}{m\sin\frac{1}{2}\frac{n\sigma}{m}} = \frac{\sin\frac{1}{2}\times 60^o}{6\sin\frac{1}{2}\times\frac{60^o}{6}} = \frac{\sin 30^o}{6\sin 5^o} = 0.956 \text{ (n = 1)}$$

$$k_{m3} = \frac{\sin\frac{1}{2}n\sigma}{m\sin\frac{1}{2}\frac{n\sigma}{m}} = \frac{\sin\frac{1}{2}\times 3\times 60^o}{6\sin\frac{1}{2}\times 3\times\frac{60^o}{6}} = \frac{\sin 90^o}{6\sin 5^o} = 0.644 \text{ (n = 3)}$$

$$k_{m5} = \frac{\sin\frac{1}{2}n\sigma}{m\sin\frac{1}{2}\frac{n\sigma}{m}} = \frac{\sin\frac{1}{2}\times 5\times 60^o}{6\sin\frac{1}{2}\times 5\times\frac{60^o}{6}} = \frac{\sin 150^o}{6\sin 25^o} = 0.197 \text{ (n = 5)}$$

Coil span factors
A full pitch coil should span S/2p = 18 slots (from say 1 to 19).
Therefore ε = (18-15) x slot angle = 3 x 10 = 30^0

ke_1= $\cos\frac{1}{2}n\varepsilon$ = cos ½ x 30^0= 0.966 (n = 1)

ke_3= $\cos\frac{1}{2}n\varepsilon$ =cos ½ x 30^0 x 3= 0.707 (n = 3)

ke_5= $\cos\frac{1}{2}n\varepsilon$ =cos ½ x 30^0 x 5= 0.259 (n = 5)

Total number of coils = total number of slots= 180
Total number of turns = total number of coils x turns/coil = 180 x 3
Number of turns/phase = total number of turns/3 = 180 x 3/3 = 180

Eph1= 4.44 x 0.956 x 0.966 x 50 x 180 x 0.1122= 4158 V

$$\mathrm{Eph}_3 = \left(\frac{k_{w3}}{k_{w1}}\right)\left(\frac{B_3}{B_1}\right)E_{ph1} = \frac{0.644\times 0.707\times 0.3}{0.956\times 0.966\times 1}\times \mathrm{Eph}_1 = 615\mathrm{V}$$

$$\mathrm{E}_{\mathrm{ph5}} = \left(\frac{k_{w5}}{k_{w1}}\right)\left(\frac{B_5}{B_1}\right)E_{ph1} = \frac{0.197\times 0.259\times 0.2}{0.956\times 0.966\times 1}\times \mathrm{Eph}_1 = 46\mathrm{V}$$

RMS phase voltage $\mathrm{E}_{\mathrm{ph}} = \sqrt{4158^2 + 615^2 + 46^2} = 4203\,\mathrm{V}$

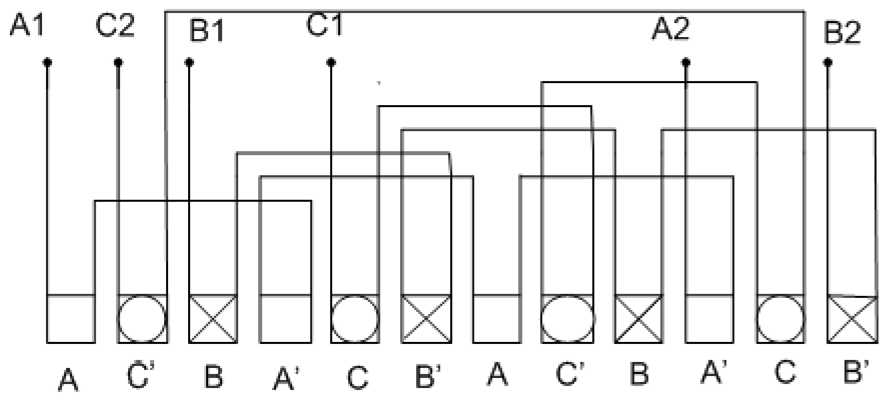
The phase voltage without the third harmonic voltage is $\sqrt{4158^2 + 46^2}$

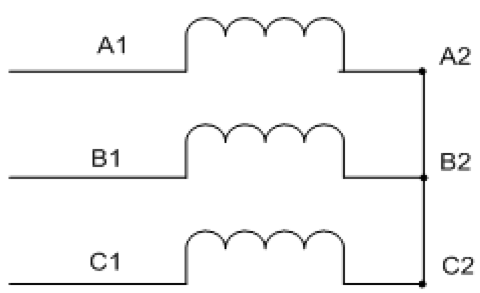
Therefore rms line voltage = $\sqrt{3}\times\sqrt{4158^2 + 46^2} = 7202\,\mathrm{V}$

(4) A stator having 24 slots has to be wound with a 3-phase, 4-pole, 60^0 group, 2-layer winding. Determine (a) the connections between the coil groups and (b) the connections between the phases when (i) star- connected (ii) delta- connected

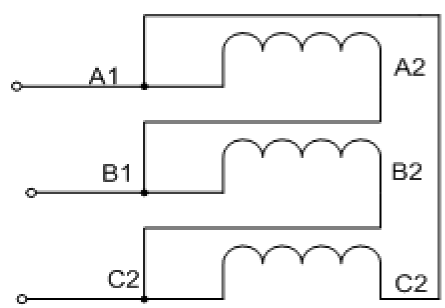
Solution

For a 4-pole machine, the 24 slots is spread over 4 x 180 electrical degrees.
Therefore the total number of groups = (4 x 180)/ 60 = 12 for 60^0 coil groups.
Number of groups/phase = 12/3 = 4.
Number of coils in a group= total number of coils /total number of groups= 24/12= 2





Star-connection

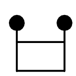


Delta-connection

14 Further Exercises

1. (a) A 6-pole simple-wave armature winding has 64 single-turn coils and 32 slots.
   - (i) Draw a slot showing the arrangement of conductors. [2 marks]
   - (ii) State whether the winding should be progressive or retrogressive. [2 marks]
   - (iii) Calculate the slot span and the coil pitch. [2 marks]
   - (iv) Construct the machine winding table. Stop after the first four rows. [4 marks]

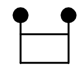
   (b) A 6-pole dc generator has a lap-connected armature with 480 conductors. The resistance of the armature circuit is 0.02 Ω. With an output current of 500 A from the armature, the terminal voltage is 230 V when the machine is driven at 900 rev/min. Calculate the useful flux per pole. [4 marks]
2. (a) A 3-phase induction motor has 12 slots. A 2-layer winding is used. Determine
   - (i) the total number of coils, [1 mark]
   - (ii) the total number of poles if a coil has one of its side in slot 1 and its other side in slot 4 and the coils are full pitch, [1 mark]
   - (iii) the number of coils in a group if 60^o group is used and [1 mark]
   - (iv) the connection between coil groups in the phase "A" winding. Assume all coil groups per phase are in series. Use the following symbol to represent a coil group: [3 marks]

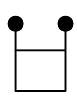
   

   (b) An 8-pole, 3-phase star-connected alternator has 9 slots per pole, 12 conductors per slot and all coils per phase connected in series. If 60^o group, 2-layer lap winding is used, calculate the necessary flux per pole to generate 1,500 V at 50 Hz on open-circuit. The coil span is one pole pitch. [8 marks]
3. (a) What is the object of using equalizers on dc machines? [2 marks]

   (b) The total number of conductors in each parallel path of an 8-pole simple lap winding must be greater than 94 and less than 102. The number of slots per pole must also be greater than 12 and less than 18. Not less than 10 and not more than 15 equalizers must be used. Determine the maximum and minimum number of conductors per slot [1 mark] and proceed to give the following details of a suitable winding:
   - (i) the number of conductors per slot [1 mark]
   - (ii) the number of coil-sides per layer (*u*) which should be greater than 1 and the number of turns in a coil [1 mark]
   - (iii) the number of slots [2 marks]
   - (iv) the number of commutator bars or segments [2 marks]
   - (v) the number of equalizers [1 mark]
   - (vi) the coils which must be connected to equalizer No. 1 and equalizer No. 2. [2 marks]

   (c) The 2-circuit armature of a 4-pole generator has 51 slots, each containing 20 conductors. What will be the voltage generated in the machine when driven at 1500 rev/min, assuming the useful flux per pole to be 0.007 Wb? [3 marks]

4. A 3-phase, 50-Hz, 6-pole star-connected alternator has 972 conductors distributed in 54 slots. 60^o groups, 2-layer winding is used. The coils are chorded by one slot. All coils in a phase are connected in series.
   (a) Determine
      (i) the total number of turns per phase [1 mark]
      (ii) the total number of coil groups per phase [1 mark]
      (iii) the number of coils in a coil group [2 marks]
      (iv) the number of turns in a coil [1 mark]
      (v) the slot which carries the lower side of coil 1 if its upper side lies in slot 1. [2 marks]
   (b) Indicate the connection between all coil groups in phase "A" winding over four pole pitches. Use the following symbol to represent a coil group:  [2 marks]
   (c) Calculate the line voltage on no load, when the flux per pole is 0.01 Wb. [6 marks]
5. (a) The armature of a 4-pole dc generator has 47 slots, each containing 6 conductors. The armature winding is wave-connected. The machine has 141 commutator bars or segments.
      (i) Show how the conductors are arranged in a slot. [3 marks]
      (ii) Draw up the winding table for the machine. Show only the first TWO tours around the armature. Assume the winding to be retrogressive. [9 marks]
      (iii) How many tours are required to complete the winding? [2 marks]
      (iv) Is progressive winding also possible? Why? [2 marks]
   (b) A 6-pole wave-wound dc shunt generator delivers 75 kW at 120 V. The armature has 270 conductors and a total resistance of 0.02 ohms. The flux per pole is 0.01 Wb. Calculate the speed of the machine in rev/min if the field resistance is 60 ohms and the total voltage drop across the brushes is 2 V. [9 marks]
6. (a) A stator having 24 slots is wound with a 3-phase, 4-pole, 2-layer winding. Each coil spans 5 slots, the coils being connected in 60^o groups.
      (i) Draw a coil group, indicating the numbers of all slots containing its coil-sides. Use slot 1 for the first upper coil-side and assume that the coils are multi-turn. [3 marks]
      (ii) Determine the connections between all coil groups. Assume that all coil groups per phase are in series. Use the following symbol to represent a coil group: [7 marks]



      (i) Determine the connections between the phases if the winding is delta-connected. [2 marks]
   (b) A 10-MVA, 11-kV, 50-Hz, 3-phase star-connected alternator is driven at 300 rev/min. The winding is housed in 360 slots and has 6 conductors per slot, the coils spanning five-sixths of a pole pitch. Calculate
      (i) the flux per pole required to give a line voltage of 11 kV on open circuit. [11 marks]
      (ii) the full load current per conductor. [2 marks]

CHAPTER TWO

DC MACHINES

1 Introduction

Though the induction motor is preferable in many applications, the dc motor is the first choice for applications requiring among others wide range of stepless speed control, very precise speed control and rapid acceleration, deceleration and reversal. They are found in papermaking and steel industries, electric traction and hoisting applications.

The dc generator on the other hand has ceded its role as dc power generation to power electronic converters. However, it is still necessary to study the principles involved in the dc generator because dc machines in drives do not only operate as motors but also as generators.

In this chapter, we discuss the principle of operation, construction, characteristics and starting by conventional methods of dc machines.

2 Construction

The general arrangement of a two-pole dc machine is shown in Fig.2. The rotor (or armature) consists of

(a) *The rotor shaft*: this imparts rotation to the armature core and winding.
(b) *The armature core*: It is constructed of iron laminations. The laminations insulated from one another, serve to reduce eddy-current loss. The core is keyed to the shaft and it provides a low reluctance magnetic path between the poles.
(c) *The armature winding*: This consists of coils insulated from each other and from the armature core. The coil sides are placed in slots firmly held in place by fiber slot sticks. If the armature current is below 10 A, round wire is used but for currents exceeding 20 A, rectangular conductors are preferred because they use the slot space better.
(d) *The commutator:* It consists of many tapered copper segments insulated from each other by mica sheets and clamped round the shaft of the machine. To each commutator is connected a coil junction. Hence the number of segments = the number of coils.
(e) *The brushes:* Multi-pole machines have as many brush sets as the number of poles. A brush set is composed of one or more brushes depending on the current that has to be carried. The brush sets are spaced at equal intervals around the commutator. The brushes are made of carbon because it has good electrical conductivity and its softness does not scratch the commutator.

The stator of dc machine consists of

(a) *The yoke or cylindrical frame:* It is made up solid cast steel or iron. It provides a flux return path for the magnetic flux created by the field windings.

(b) *The field windings:* they are electromagnets. Their ampere-turns provide mmfs adequate to produce in the air gap, the flux needed to generate emfs in the conductors. The air gap is the short space between the pole faces and armature teeth under the poles. It ranges from about 1.5 to 5 mm as the machine rating increases from 1 kW to 100 kW. In some machines, the flux is created by permanent magnets.

(c) *The field poles*: they carry the field windings and are excited alternately N and S. The pole shoe is carried to spread the flux more uniformly. DC machine may have 2, 4, 6 or as many as 24 poles. The bigger the machine, the larger the number of poles it will have.

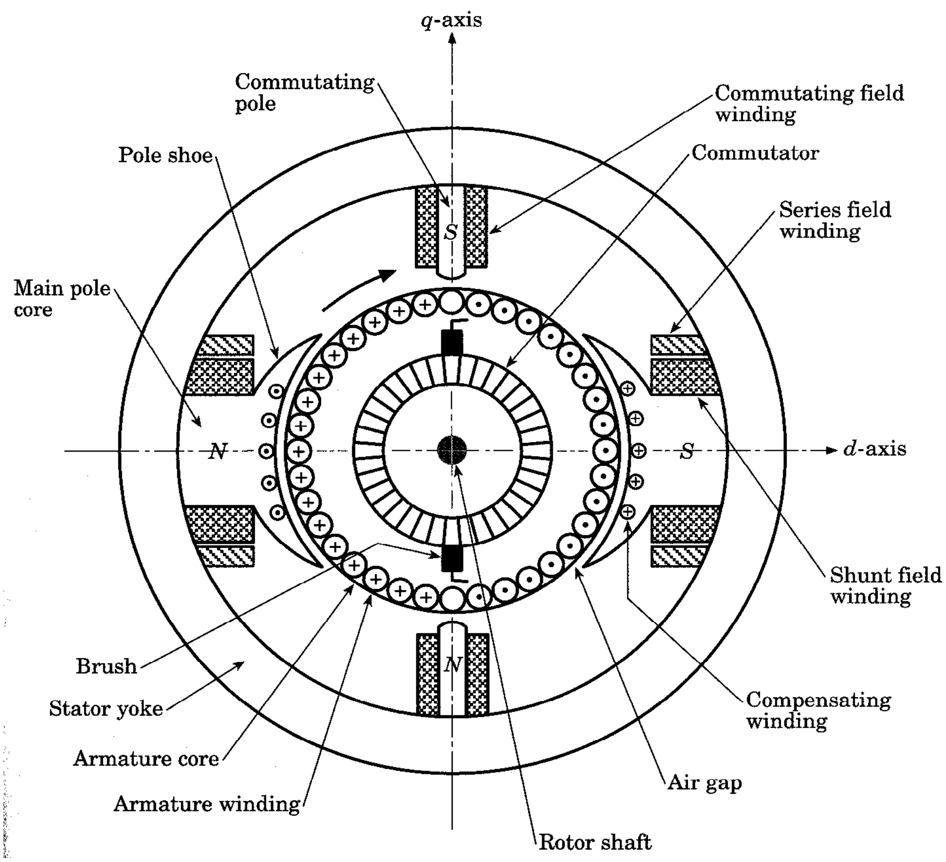


Fig. 2 General arrangement of a 2-pole dc machine

3 Principle of operation of the dc machine

DC machine has two members. One called the field is an electro- or permanent-magnet system providing the working magnetic field in an air gap. The other is the armature which carries the armature windings studied in chapter one. The principle of operation of dc machine is explained using an elementary dc machine shown in Fig. 1

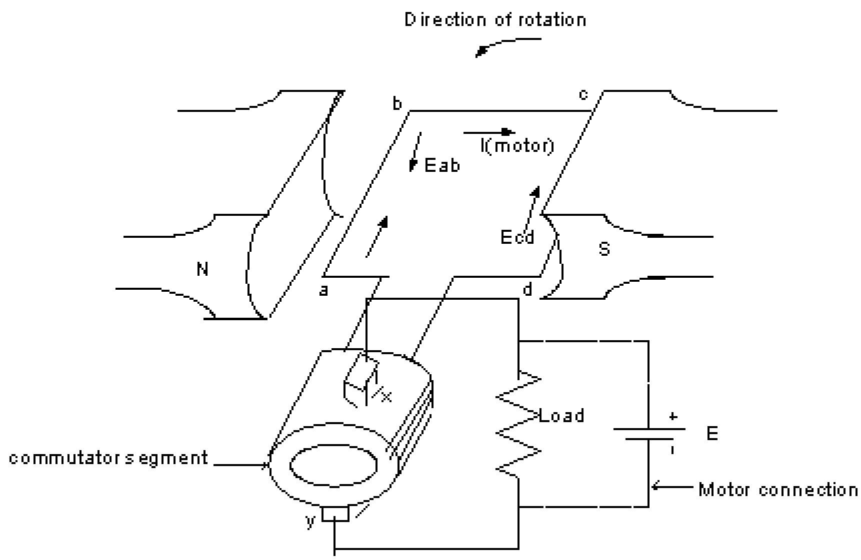


Fig. 1 Elementary dc machine

3.1 Generator action

The rotation is due to an external driving torque. When the coil is in horizontal position as shown, the voltages E_{ab} and E_{cd} are positive. The voltage picked up by the two stationary brushes *x* and *y* is the sum of the two and the brush *x* is positive and *y* is negative. When the coil is in vertical position, no flux is cut and voltage induced in each conductor is zero. At this position, the brushes short the coil. When the coil moves beyond the vertical position, the direction of the voltages in the conductors reverses. At the same time the two commutator halves change contact from one brush to another. Thus brush *x* is always positive and brush *y* negative and the current in the external load always flows in the same direction.

3.2 Motor action

With the current in the coil as shown, the forces on sides *ab* and *cd* are down and upward respectively. The two forces form a couple or a torque which causes the coil to rotate in the anticlockwise direction. In the vertical position, the coil current is cut off but the momentum of the coil carries it past this position. When this occurs, the two commutator halves change contact from one brush to the other. This reverses the current in the coil and consequently the directions of the forces on the conductors. The coil thus continues to rotate in anticlockwise direction for so long as the current is passing.

4 D.C. Generators

A dc generator may have one or more field windings. Each field winding and how it is excited produce a modifying effect on the generator characteristics.

4.1 Methods of excitation

There are two methods:

(a) Separate excitation: the field current is obtained from a separate source such as battery

(b) Self-excitation: the field current is produced by the generator itself. The self-excited generators may be subdivided into 3 groups, namely:

  (i) Shunt-wound generators: the field winding is connected across the armature terminals.

  (ii) Series-wound generators: the field winding is connected in series with the armature winding

  (iii) Compound-wound generators: a combination of shunt and series windings.

4.2 Connection diagrams

The diagrams for the various generators are given in Fig.4.

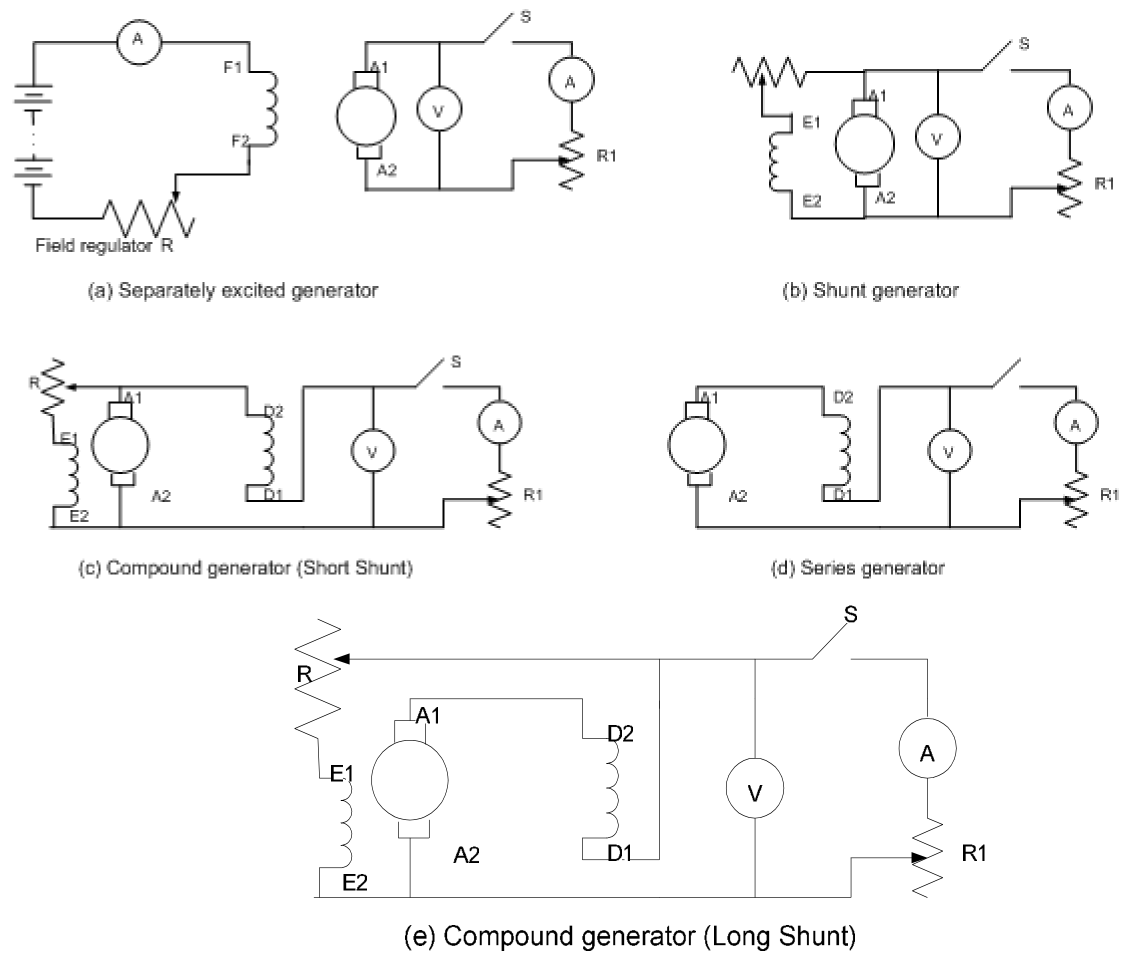


Fig. 4 D.C. Generator connections

4.3 Separately excited generator on no load

The variation of the open-circuit voltage across the armature with the field current I_f at a given speed is shown in Fig. 5.

(a) The curve is called the open-circuit characteristic (O.C.C.). The open-circuit curve for other dc machines is obtained by separately exciting the machine and driving the rotor by a prime mover.
(b) Since the open-circuit voltage E is proportional to ϕ at a given speed, the curve also shows how the flux varies with the field current. For this reason the curve is also referred to as the magnetization curve.
(c) The curve has three portions: a linear portion *oa*, the portion *ab* called the knee of the O.C. curve (in this portion the saturation of the iron begins to be important) and the saturated portion *bc* (here, a large increase in the field current results in a small increase in flux)
(d) The rated voltage of a dc generator is usually a little above the knee of the curve if the curve is obtained at the rated speed.
(e) If the machine has been used before and it is not demagnetised, the emf curve is found to follow the dotted line
(f) *oR* represents the emf generated by residual magnetism in the poles.
(g) At a given field current the generated voltage E is proportional to the speed N.

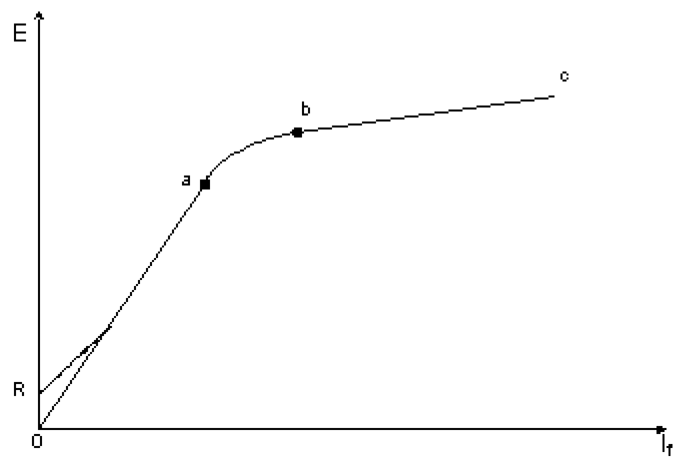


Fig. 5 Open-Circuit Characteristics of a dc generator

4.4 Shunt generator on no load

Refer to circuit in Fig.6. Applying Ohm's law

$$E = R_s I_f \quad (1)$$

where $R_s = R + R_f + R_a$

We also know that at a given speed,

$$E = g(I_f) \quad (2)$$

where $g(I_f)$ is a nonlinear function given by the magnetization curve. The values of E and I_f are determined by the intersection of the curves described by (1) and (2).

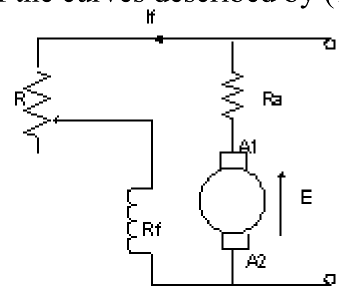


Fig. 6 Shunt generator on no load

The steady state value of E and I_f are reached after a transient build–up process explained below with reference to Fig.7:

(a) At the rated (or a given speed), the voltage across the armature due to residual magnetism is E_1. But this voltage is also across R_s. Thus, the current which flows in the field circuit is I_1

(b) When I_1 flows in the circuit of the generator, an increase in mmf results which aids the residual magnetism in increasing the induced voltage to E_2 as shown in Fig. 7

(c) Voltage E_2 is now impressed across R_s causing a larger current I_2 to flow in the field circuit. An increased mmf due to I_2 produces generated voltage E_3

(d) The process continues until that point where the shunt resistance line crosses the magnetization curve. Here the process stops. The voltage induced, when impressed across R_s, produces a current flow that in turn produces an induced voltage of the same magnitude E_7 as shown in Fig. 7

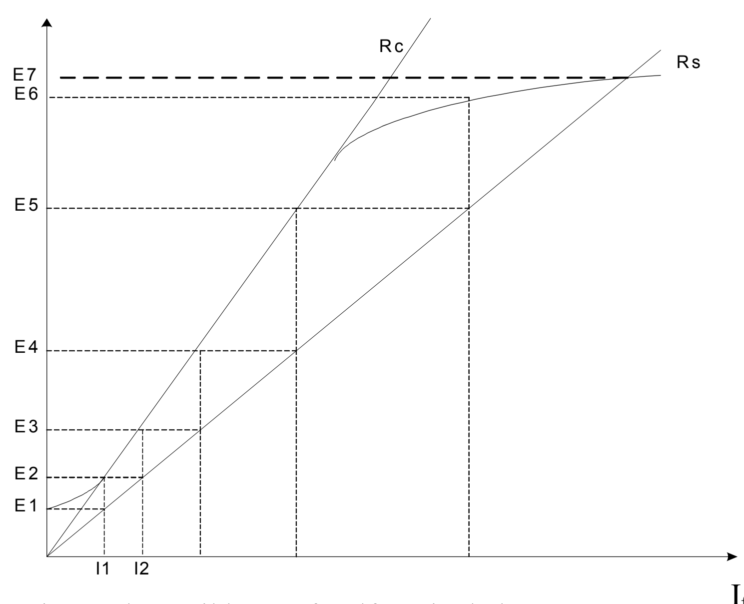


Fig. 7 The Build-Up of Self excited shunt generator

Example 1
A dc machine is connected as a shunt generator. When operated at rated speed of 1500 rev/min, the O.C. curve is approximated by $E = \dfrac{367 I_f}{0.233 + I_f}$

The armature circuit resistance is 0.14 ohms. The shunt field resistance is adjusted to be 459 ohms. Find the no load terminal voltage.

Solution
At no load, $E \approx V$. (V = terminal voltage). Hence
$E = R_f I_f = 459 I_f$
Substituting this in the O.C. characteristic, we obtain $V = E = 260$ V and $I_f = 0.566$ A

4.5 Critical field circuit resistance

It is the field circuit resistance beyond which the shunt generator will fail to excite. The critical field circuit resistance, R_c, is the slope of the tangent to the O.C. curve passing through the origin (See Fig.7). This resistance is proportional to the speed.

4.6 Reasons for failure of shunt generator to build up voltage

There are four reasons
(a) Lack of residual magnetism:
Causes:
Residual magnetism may be lost as a result of mechanical shock in shipment, excessive vibration, extreme heat and inactivity for long periods.

Remedy:
Separately excite the field for a few moments. Where possible, run the machine as a motor; its brushes being of the normal polarity and the direction of rotation also normal (a better method).

(b) Field circuit connections reversed with respect to the armature circuit.
Remedy:
Change over connections from the armature to the field circuit.

(c) Direction of rotation reversed: This also produces reversed armature connections with respect to the field.
Remedy:
Turn it in the right direction.

(d) Field circuit resistance higher than critical filed resistance
Causes:
They are broken lead in any part of the field circuit, too high a brush contact resistance (very rare) and too large a resistance in the field regulator for the particular speed operation.

Example 2
A dc machine has the following details:
Four poles; shunt field resistance 80 Ω; armature resistance 0.4 Ω; number of armature conductors 400. The armature conductors are arranged in two parallel paths and the magnetization curve is as follows:

Field current (A)	0	0.5	1	1.5	2	2.5	3	4
Flux per pole (mWb)	1	6.5	12.8	17.5	20.5	22	23	24

Find the open-circuit voltage to which the machine will excite when driven as a generator at 750 rev/min.

Solution
The generated emf and hence the open-circuit characteristic can be derived from the given data:

$$E = \frac{N}{60}.\frac{p}{a}.Z\phi \Rightarrow E = \frac{750}{60}.\frac{2}{1}.400.\phi = 10^4\phi$$

Tabulate field current and emf values:

Field current (A)	0	0.5	1	1.5	2	2.5	3	4
Generated emf (V)	10	65	128	175	205	220	230	240

The emf may now be plotted against the field current.

Plotting $E = 80I_f$ on the same axes gives an open-circuit terminal voltage of 228 V.

4.7 Load (or External) Characteristics of D.C. Generator

The external characteristic shows the variation of the terminal voltage with the load current at a given speed (usually the rated speed). When a generator is supplying power to a load the voltage-current curve of the load and the load curve of the generator can be plotted on the same graph to obtain the point of steady state operation which is the point of intersection of the two curves.

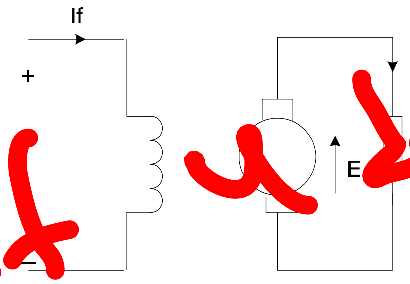
4.7.1 Separately-excited generator

The field current I_f is constant and speed is constant.

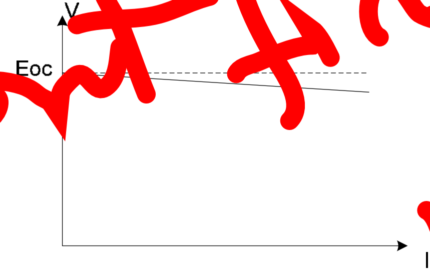
$$V = E - IR_a = E_{oc} - IR_a$$

where E_{oc} = open circuit emf and R_a = armature resistance or total resistance of armature circuit.

Voltage drop from no load to full load is usually less than 10 %



Generator circuit



External characteristic

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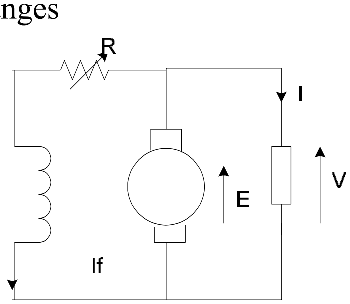
4.7.2 Shunt generator

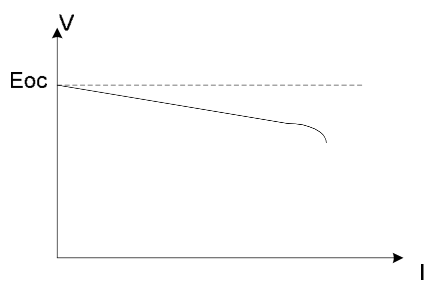
Resistance in the field circuit is constant. Speed is also kept constant.
The fall in terminal voltage is due to
(a) $I_a R_a$ drop and
(b) the reduction in field current.

The voltage drop due to the first results in a decreased field current which in turn results in a decreased air-gap flux ϕ and reduced emf, E. The fall in the terminal voltage is therefore more marked than with the separately excited generator. The voltage drop from no load to full load is about 15 %.

The field regulator can be used to regulate the terminal voltage V manually as the load current I changes



Generator circuit



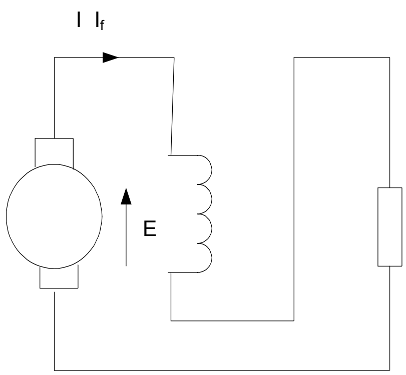
External characteristic

The external characteristic can be obtained following this procedure:
(a) For any value of V, calculate the field current $I_f = V/R_{sh}$ (R_{sh} = total resistance in the field circuit)
(b) From the O.C. curve, determine the corresponding generated emf, E
(c) Calculate the armature current, $I_a = (E - V)/R_a$
(d) Calculate the load current $I = I_a - I_f$

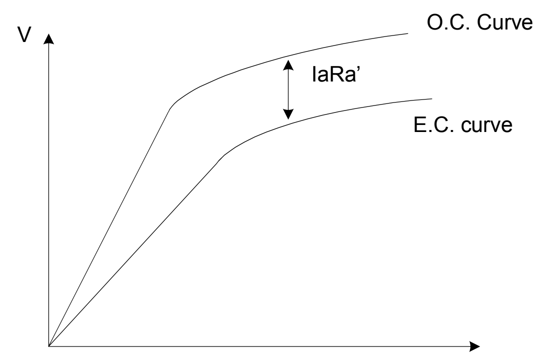
4.7.3 Series Generator

The external characteristic is readily deduced from O.C. curve by subtracting the total armature circuit resistance drop, $I_a R_a'$ (R_a' = total armature circuit resistance)

This generator is unsuitable when the voltage is to be maintained constant or approximately constant over a wide range of load current. Series generators are rarely used as self-excited generators.



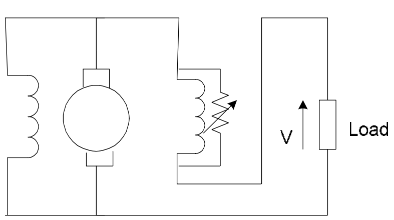
Generator circuit



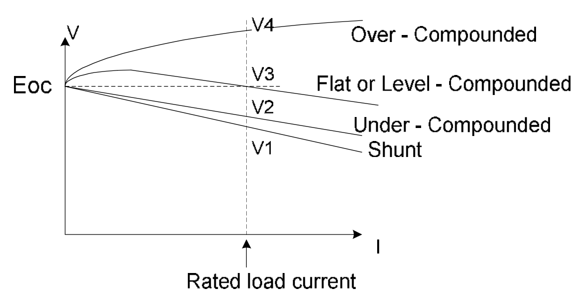
External characteristic (E.C.)

4.7.4 Compound generator

It is a shunt generator (i.e. the shunt field predominates) with an additional field winding in series with the armature to improve its external characteristics. The compounding is either cumulative (the two fields aid each other) or differential (the series field mmf opposes that of the shunt field). To achieve its intended purpose, compounding must be cumulative.



Generator circuit



External characteristics

The level-compounded generator has its terminal voltage practically constant from no load to full load. The over compounding is used when it is necessary to compensate not only for the armature circuit voltage drop but also for the drop in the feeder line between the generator and the load. The degree of compounding (over, level or under) may be adjusted by means of diverter connected across the series field.

Δ + level, Eoc

To determine the series mmf required for a given terminal voltage V when the o.c.c. is given, the following steps are followed (Generator is assumed to be long shunt):

(a) For the given E_{oc} find the corresponding I_{foc}

(b) Use the I_{foc} to find the required field circuit resistance R_f: $R_f = E_{oc}/I_{foc}$

(c) For the given terminal voltage V and R_f obtained in (b) find the actual field current I_{fa} under the loading condition: $I_{fa} = V/R_f$

(d) Calculate I_a from the given rated load current and I_{fa} in (c).

(e) Calculate the emf under the loading condition and use it to obtain the required field current I_{fr} in the shunt circuit from the o.c.c.

(f) Find the required mmf: $F_{req} = N_f I_{fr}$ where N_f = number of turns of shunt field

(g) Find the additional mmf required from (c) and (f). This must be produced by the series field winding. Thus $N_s I_s = F_{add} = N_f I_{fr} - N_f I_{fa} = N_f (I_{fr} - I_{fa}) = N_f \Delta I_f$ where N_s = number of turns of series field.

To obtain the ohmic value of the diverter that will give the terminal voltage V, the following steps are followed:

(a) Determine the series field current I_s required to produce the additional mmf: $I_s = (N_f \Delta I_f)/N_s$.

(b) Determine the value of the diverter:
Diverter current = armature current – series field current: $I_d = I_a - I_s$
Voltage drop across diverter = voltage drop across series field winding: $I_d R_d = I_s R_s$. Thus $R_d = (I_s R_s)/I_d$. R_d and R_s are the resistances of the diverter and the series field respectively.

Example 3.a

It is desired to level-compound a shunt generator so that the terminal voltage at full load of 100 A is equal to that at no load. When connected as a shunt generator the field current had to be increased from 2.75A to 3.3A in order to keep the terminal voltage the same at no load and full load. If the machine is to be connected long-shunt and the turns per pole on the shunt field is 1200, calculate the series turns per pole required. Neglect volt-drop in series winding.

The above machine, after its conversion, had a terminal voltage of 220 V at full-load current and the resistance of the armature; shunt and series fields were 0.12 Ω, 80 Ω and 0.08 Ω respectively. Calculate the value of the emf being generated under these conditions.

Solution

For level-compounding, the actual field current = field current at no load = 2.75 A since $E_{oc} = V$

The required mmf $F_{req} = 3.3 \times 1200$

Additional mmf required $= N_f \Delta I_f = (3.3 - 2.75) \times 1200 = 660\text{AT}$

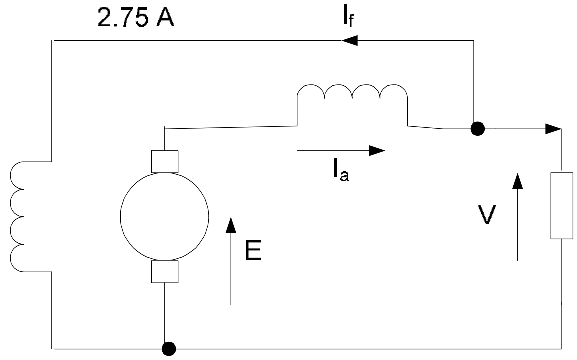
Current in series winding with full load $I_a = 100 + 2.75 = 102.75\text{A}$

Required number of series turns/pole $= \dfrac{660}{102.75} = 6.42 \approx 7\text{turns}$

Referring to the figure below, with long-shunt, $E = V + I_a (R_a + R_s)$ and

$$I_a = I_f + I = \frac{V}{R_f} + I = \frac{220}{80} + 100 = 102.75 \text{ A}$$

Therefore $E = 220 + 102.75(0.12 + 0.08) = 240.6$ V



Example 3.b

A dc shunt generator is rated at 240 V and 400 A armature current. At a speed of 1750 rev/min with the shunt field excited, the OC curve is given by $E = \frac{435 I_f}{1.93 + I_f}$ volts. The armature circuit resistance is 0.0192 Ω and there are 2100 turns per pole. The generator is operated at 1750 rev/min. Find the series-winding mmf required to give an over-compound characteristic at 250 V if the terminal voltage on no load is 240 V

Solution

The no load terminal voltage $V_{NL} \approx E = \frac{435 I_f}{1.93 + I_f}$

Thus $240 = \frac{435 I_f}{1.93 + I_f}$ from which $I_f = \frac{1.93 \times 240}{435 - 240} = 2.375$

The field circuit resistance $R_f = \frac{240}{2.375} = 101\Omega$

The emf under full load condition when the terminal voltage is 250 V is

$E = 250 + 400 \times 0.0192 = 257.68\,\text{V}$

The corresponding field current from the o.c.c. is $I_{fr} = \frac{1.93 \times 257.68}{435 - 257.68} = 2.805\,\text{A}$

At a terminal voltage of 250 V, the shunt field current is $I_{fa} = \frac{250}{101} = 2.475\,\text{A}$

Thus the series-winding mmf required $= 2100 \times (2{,}805 - 2.475) = 693\text{At/pole}$

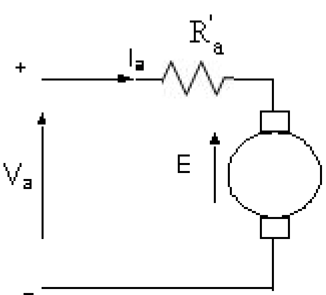
5 D.C. Motors

The speed-torque characteristic of a motor must be adapted to the type of the load it has to drive. This requirement has given rise to three basic types of motors, namely shunt motor, series motor and compound motor.

5.1 Torque of dc motor

Referring to the circuit below, the armature voltage equation is
$V_a = E + I_a R'_a$

Where
V_a = voltage across the armature
I_a = armature current
E = back emf



From the equation, we can obtain
$V_a I_a = EI_a + I_a^2 R'_a$

$V_a I_a$ represents the total electric power supplied to the armature and $I_a^2 R'_a$ represents the loss due to the resistance of the armature circuit. The difference between these two quantities therefore represents the mechanical power developed by the armature.

If T_e is in newton-metres, then mechanical torque developed $= 2\pi T_e N/60$. Hence

$$\frac{2\pi T_e N}{60} = EI_a = \frac{N}{60}.\frac{P}{a}.Z\phi I_a$$

From which

$$T_e = \frac{1}{\pi}\frac{I_a}{2a} Zp\phi \qquad (3)$$

This expression holds equally for generators and motors.

Example 4
A four-pole motor is fed at 440V and takes an armature current of 50A.The resistance of the armature circuit is 0.28 Ω. The armature winding is wave-connected with 888 conductors and the useful flux per pole is 0.023 Wb. Calculate the speed.

Solution

$$440 = E + 50 \times 0.28 \Rightarrow E = 440 - 14 = 426V$$

$$426 = \frac{N}{60}.\frac{2}{1}.888.\phi.0.023 \Rightarrow N = 626 \,\text{rev/min}$$

5.2 Basic equations of dc machines (motors)

$$E = k_1 N\phi \tag{4.a}$$

$$T_e = kI_a\phi \tag{4.b}$$

Or

$$E = k\omega\phi \tag{5.a}$$

$$T_e = kI_a\phi \tag{5.b}$$

5.3 Load characteristics of dc motors

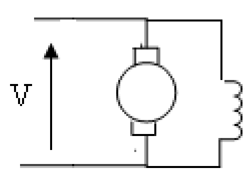
We consider two characteristics.

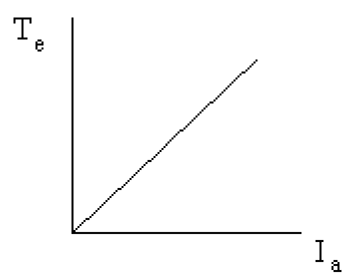
5.3.1 Electromagnetic torque characteristics

It shows the variation of T_e with armature current. The shape of the curve can be deduced from the fundamental torque equation $T_e = k\phi I_a$

Shunt motor

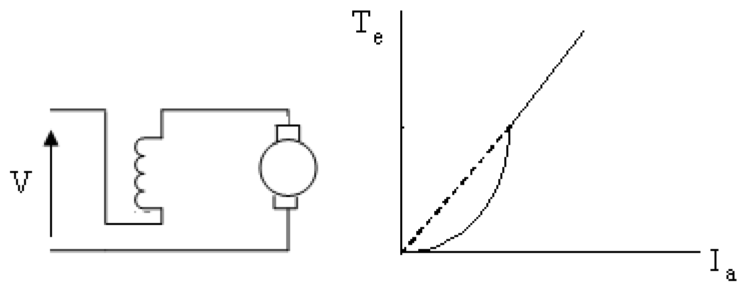
The flux ϕ is constant and the torque equation becomes $T_e = k_2 I_a$ which is a linear relation. The circuit diagram and the curve are shown below.





Series motor

When the magnetic circuit is not saturated ϕ is proportional to I_a and when saturated, $\phi \approx$ constant. Hence the initial portion of the torque-armature curve will be a parabola and ultimately it will merge into a straight line passing through the origin as shown below..



Compound motor

The basic torque equation is $T_e = k(\phi_f + \phi_s)I_a$. The characteristic depends on the relative strengths of the two components.

Fig.8 shows the torque-armature curves of dc motors of the same voltage, power and speed ratings. For a given current below the full-load value the shunt motor exerts the largest torque, but for a given current above that value the series motor exerts the largest torque. The maximum permissible starting current is usually 1.5 to 2.5 times the full-load current. Consequently, where a large starting torque is required such as for hoists, cranes, electric trains etc, the series motor is the most suitable.

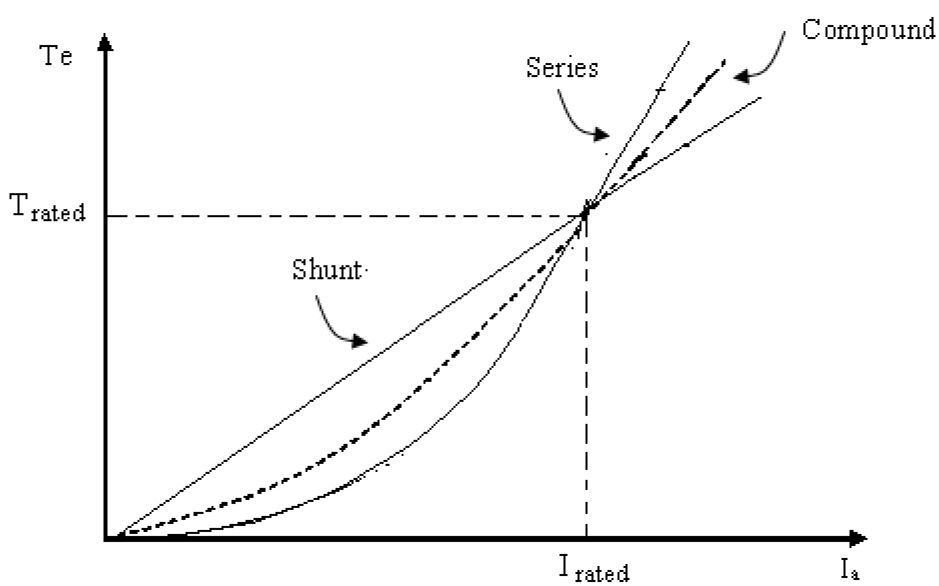


Fig. 8 Comparison of torque characteristics of dc motors

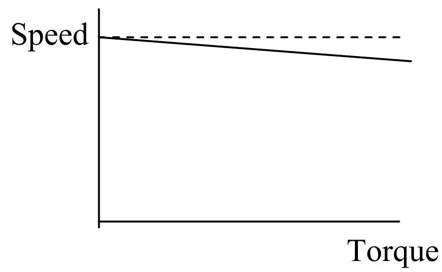
5.3.2 Speed-torque characteristics

These characteristics can be derived from the two basic equations and the armature circuit voltage equation: $E = k\omega\phi$, $T_e = k\phi I_a$ and $E = V_a - I_a R$, where R includes external resistance in the armature circuit.

Shunt motor

$V_a = V$ (applied voltage). Substituting $E = k\omega\phi$, $I_a = T_e / k\phi$ and $V_a = V$ in the armature circuit voltage equation and rearranging, we obtain the relation between speed and torque as

$$\omega = \frac{V}{k\phi} - \frac{RT_e}{(k\phi)^2}$$

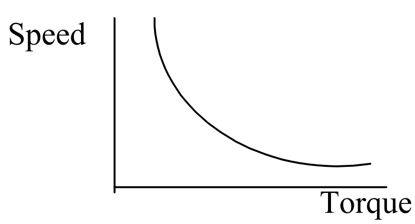


The difference between no-load and full-load speeds has typical values around 5% of full-load speed (In small machines the values are higher, (10 – 15%). Consequently, the dc shunt motor is considered a constant speed drive. A typical application is the lathe machine which requires constant speed and low starting torque.

Series motor

If the resistance in the armature circuit is neglected then $\omega = E / k\phi = V / k\phi$ and if saturation is also neglected we obtain $\omega = V / k'I_a$. This and the torque equation $T_e = k\phi I_a$ lead to the following relation between speed and torque:

$$\omega = \frac{V}{\sqrt{Tek'}}$$



The speed of dc series motor varies widely with the torque. An increase of load calls for a decrease of speed. The series motor is considered a variable speed drive and its field of application is principally determined by this property. Typical cases are electric trains (they run uphill at a lower speed than on a level track), electric cranes and hoists (light loads are lifted quickly and heavy loads more slowly).

At no load the speed of a series motor may rise to dangerously high values. For this reason we never permit a series motor to operate at no load. Series motors are directly coupled or geared to a load as in hoists and cranes. (Note: a series motor belt-coupled risks being run at no load). The extremely excessive speed at no load will give centrifugal forces which could tear the windings out of the armature and destroy the machine.

Fig. 9 shows the speed-torque curves of dc motors of the same voltage, power and speed rating.

Cumulative compound motors are widely used for individual drives of certain machines (e.g. lifts, winches etc) where a speed characteristic of the type possessed by the series motor is required; with the exception that the no-load speed must be limited to a safe value. The speed drop from no load to full load is generally between 10 and 30%.

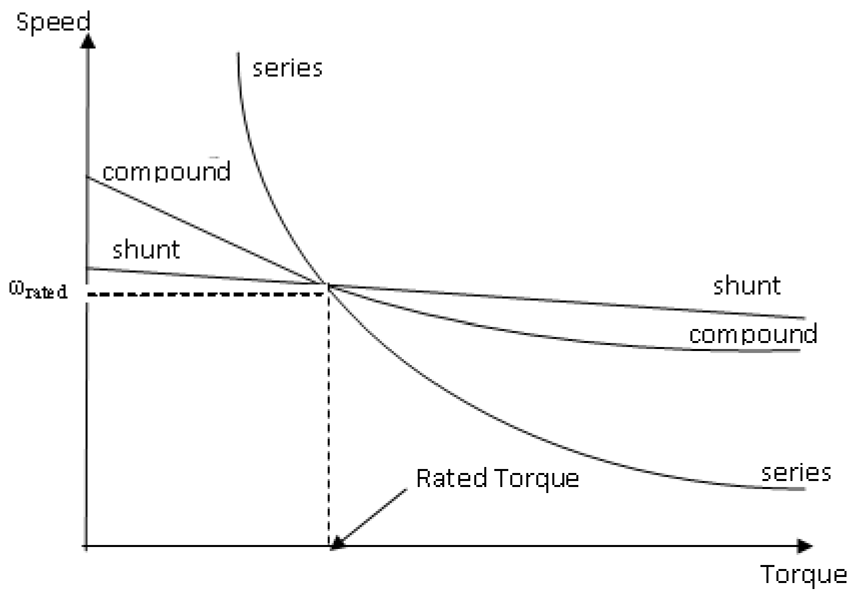


Fig. 9 Comparison of speed- torque curves of dc motors

Example 5
A shunt motor is running at 626 rev/min when taking an armature current of 50 A from a 440-V supply. The armature circuit has resistance of 0.28 Ω. If the flux is suddenly reduced by 5 % find
(a) the maximum value to which the current increases momentarily and the ratio of the corresponding torque to the initial torque
(b) the ultimate steady value of the armature current, assuming the torque due to the load to remain unaltered.

Solution

(a) Initial emf = 440 – 50 x 0.28 = 426V

Immediately after the flux is reduced by 5 %, i.e. before the speed has begun to increase, new emf = 426 x 0.95 = 404.7 V

Thus armature current $= \dfrac{V - E_{new}}{R_a} = \dfrac{440 - 404.7}{0.28} = 126\text{A}$

From $T_e = kI_a\phi$, we have

$$\frac{new\,torque}{original\,torque} = \frac{new\,current}{original\,current} \times \frac{new\;flux}{original\;flux} = \frac{126}{50} \times 0.95 = 2.394$$

Motor accelerates

(b) After the speed and current have attained steady values, the torque will have decreased to the original value, so that $new\,current \times new\;flux = original\;current \times original\;\;flux$

Thus new armature current = 50 x 1/0.95 = 52.6A.

Note that under steady state conditions motor torque = load torque.

Example 6

A series motor runs from a 400 V direct current supply and has a total armature and series field resistance of 0.2 Ω. A variable resistance, *R*, is connected in series with the motor for speed adjustment. With a given load and *R* = 0, the current is 25A and the speed 1000 rev/min. On another load with *R* = 2 Ω, the current is 20A. Calculate the new speed. Assume that the field flux is proportional to the current.

Solution

First condition: $E_1 = 400 - 25 \times 0.2 = 395V$

Second condition $E_2 = 400 - 25 \times 2.2 = 356V$

$$\frac{E_2}{E_1} = \frac{\phi_2 N_2}{\phi_1 N_1} = \frac{I_2 N_2}{I_1 N_1} \Rightarrow N_2 = \frac{E_2}{E_1} \times \frac{I_1}{I_2} \times N_1$$

$$N_2 = \frac{356}{395} \times \frac{25}{20} \times 1000 = 1128\,\text{rev/min}$$

Example 7

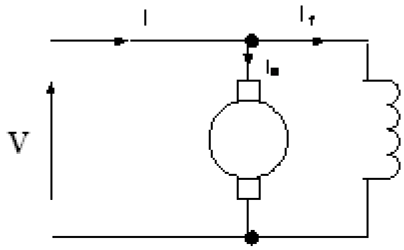
A shunt motor, which has a field resistance of 200 Ω and an armature resistance of 0.8 Ω, takes 26 A from a 200 V supply when running at 500 rev/min on full load. In order to control the speed of the motor a 1.2 Ω resistor is connected in series with the armature. Calculate the speed at which the motor will run when supplying full-load torque.

Solution

Without armature resistor: $I_f = \dfrac{200}{200} = 1A$

$I_{a1} = 26 - 1 = 25A$ and

$E_1 = V - I_{a1}R_a = 200 - 25 \times 0.8 = 180V$



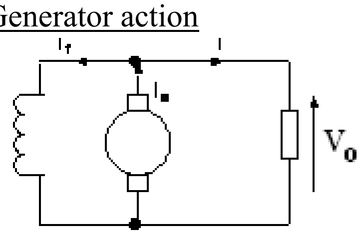
With armature resistor connected:
The torque remains constant and the flux from the field is constant
Therefore from $T_e = k\phi I_a$, armature current has to remain constant. Thus I_{a2} = I_{a1} = 25 A and $E_2 = V - I_{a2}(R_a + R) = 200 - 25(0.8 + 1.2) = 150\ V$
Now $E \propto \phi N$ and ϕ is constant

Therefore $\dfrac{E_2}{E_1} = \dfrac{N_2}{N_1} \Rightarrow N_2 = N_1 . \dfrac{E_2}{E_1} = 500 \times \dfrac{150}{180} = 417$ rev/min

Example 8
A shunt machine has armature and field resistances of 0.04 Ω and 100 Ω respectively. When connected to a 460-V dc supply and driven as a generator at 600 rev/min, it delivers 50 kW. Calculate its speed when running as a motor and taking 50 kW from the same supply.

Show that the direction of rotation of the machine as a generator and as a motor under these conditions is unchanged.

Solution
Generator action



Generator

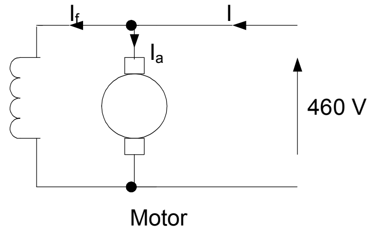
$$I = \frac{50 \times 10^3}{460} = 108.69\text{A}$$

$$I_f = \frac{460}{100} = 4.6\,\text{A}$$

$$I_a = I_f + I = 113.29\text{A}$$

$$E = V_o + I_a R_a = 460 + 113.29 \times 0.04 = 464.53\,\text{V}$$

Motor action



$I = (50\times 10^3/460) = 108.69\text{A}$
$I_f = 4.6\text{A}$
$I_a = I_f - I = 104.09A$
$E = V_0 - I_a R_a = 460 - 104.09\times 0.04 = 455.84\,\text{V}$

$E \propto \phi N$ and ϕ is constant

Thus $\dfrac{E_2}{E_1} = \dfrac{N_2}{N_1} \Rightarrow N_2 = \dfrac{E_2}{E_1}.N_1 = \dfrac{455.84}{464.53}\times 600 = 589\,\text{rev/min}$

6 Efficiency of dc machines

The efficiency is calculated using the formula

$$\eta = \frac{0utput}{Output + losses} = \frac{Input - Losses}{Input} = 1 - \frac{Losses}{Output + losses}$$

The power rating of machine corresponds to the useful output power, electrical or mechanical depending on whether the machine is a generator or motor.

In rotating machines both mechanical and electrical losses are produced.

6.1 Electrical losses in dc machines

They are composed of

(i) Armature circuit copper loss ($I_a^2 R$). R includes all the series connected windings.

(ii) Shunt field circuit copper loss ($V_f I_f$). This will include the regulating resistance loss.

(iii)Brush contact loss (voltage contact drop x I_a). The value of the voltage contact drop depends on the type of brush and the pressure applied. When not given the usual value of 2 V is assumed.

(iv)Iron losses: They are produced in the armature core. They are due to hysteresis and eddy current. Iron losses depend upon flux density, the speed of rotation, the quality of the steel and the size of the armature. For a given machine, if the speed is constant then the iron loss varies approximately as (flux density)^2 and hence as the (applied voltage)^2. Note that eddy current in the core alternates at a frequency $f = pn_r$ (n_r = speed of machine in rev/sec). The core is also magnetized at the same frequency by the field.

6.2 Mechanical losses in dc machines

They are due to (i) brush friction, (ii) bearing friction and (iii) windage.

The friction losses depend on the design of the bearings, brushes and commutators. Windage losses depend on the speed, the design of the cooling fan and on the turbulence produced by revolving parts.

For a given machine, the mechanical loss is constant for a constant speed.

Example 9
A 100-kW, 460-V shunt generator was run as a motor on no load at its rated voltage and speed. The total current was 9.8 A including a shunt current of 2.7 A. The resistance of the armature circuit at normal working temp was 0.1 Ω. Calculate the efficiency at full load.

Solution
Output current at full load $100 \times 1000 / 460 = 217.5\ A$
Thus I_a at full load = $I + I_f$ = 217.5 + 2.7 = 220.2 A
Copper loss in armature circuit at full load = $(220.2)^2$ x 0.11 = 5325 W
Loss in shunt circuit = 2.7 x 460 = 1242 W
Armature current on no load as a motor = 9.8 – 2.7 = 7.1 A
Fixed losses (negecting no load armature loss) = 7.1 x 460 = 3265 W
Input power at full load = output + losses = $100 + 5.325 + 1.242 + 3.265 = 109.832$ kW

$$\text{Efficiency} = \frac{output}{input} = \frac{100}{109.832} = 0.9105 = 91.05\%$$

7 Losses as function of load

As we load a machine (mechanically, if it is a motor, electrically if it is a generator), the armature current increases. Consequently, the armature circuit $I_a^2 R$ and the brush contact loss $2I_a$ will rise. On the other hand, the mechanical and iron losses remain essentially constant as the load increases, unless the speed of the machine changes appreciably.

Since the total losses increase with load and the losses are converted into heat, the temperature of the machine rises progressively as the load increases. However, the temperature must not exceed the limiting temperature, corresponding to the insulation used in the machine. Consequently, there is a limit to the power which a machine may deliver. This temperature-limited power enables us to establish the nominal or rated power of the machine. A machine loaded beyond its nominal rating will usually overheat. The insulation deteriorates more rapidly, which inevitably shortens the service life.

If a machine runs intermittently, it can carry heavy overloads without overheating, provided that the operating time is short.

8 Starting of dc motor

Very small motors are started direct-on-line. For larger machines the armature resistance is small therefore applying full voltage to them when they are stationary results in a highly excessive armature current which can
(a) burn out the armature winding
(b) damage the commutator and brushes owing to heavy sparking
(c) overload the feeder
(d) snap off the shaft due to mechanical shock
(e) damage the driven equipment because of the sudden mechanical hammer blow.

The starting current must initially be limited to 1.5 to 2.5 times the rated current. If the supply is at constant normal voltage the starting current is limited by connecting resistors in series with the armature. A liquid rheostat gives a smooth variation in resistance but robust metallic resistors arranged in series sections that can be cut out successively by manual or automatic operations are usually preferred.

Today, electronic methods are often used to limit the starting current and to provide speed control.

Example 10
A 240-V dc shunt motor has a full-load speed of 750 rev/min and a full-load armature current of 20A. The armature resistance is 1 Ω. The maximum armature current during the starting period is 30 A. Calculate the total resistance of the starter.

Solution
At starting the back emf = 0
Therefore $R_a + R_{st} = \frac{V}{I} = \frac{240}{30} = 8\ \Omega$
Therefore $R_{st} = 8 - 1 = 7\ \Omega$

8.1 Grading starting resistance for shunt and separately excited motors (constant flux)

The steps required to ensure that the starting current varies between specified upper and lower limits I_1 and I_2 (the lower limit is necessary to ensure rapid acceleration) can be obtained as follows:

Let
(1) at switch-on the total armature-circuit resistance R_1 be made up of n series sections: $r_1, r_2, \ldots\ldots\ldots, r_{n-1}$ and $r_n = R_m$ where R_m is the total resistance of the motor. Note that the

number of starter elements $= n-1$ and the start will be made in n steps by cutting out the sections in sequence.

(2) $R_2 = r_2 + r_3 + ... + r_n$, i.e., section 1 is shorted out, $R_3 = r_3 + r_4 + ... + r_n$, i.e., sections 1 and 2 are shorted out, etc

At switch-on, the starting current, $I_1 = V/R_1$

As the motor gains speed and the motor emf builds up, the motor current falls. The resistance r_1 is cut out when the current falls to

$I_2 = (V - E_b)/R_1$ or $E_b = V - I_2 R_1$

When resistor r_1 is cut out, the current jumps back to

$I_1 = (V - E_b)/R_2$ or $E_b = V - I_1 R_2 = V - I_2 R_1$

E_b is the same in the two cases because it is proportional to the speed of the motor and the speed cannot change instantaneously. Hence $R_2/R_1 = I_2/I_1 = k$ or $R_2 = kR_1$

Similarly, $R_3 = kR_2 = k^2 R_1$ and after the nth step $R_n = R_m = k^{n-1} R_1$

The section resistances are

$r_1 = R_1 - R_2 = R_1(1-k)$; $r_2 = R_2 - R_3 = R_2(1-k) = kR_1(1-k) = kr_1, \ldots\ldots.., r_{n-1} = k^{n-2} r_1$, $r_n = R_m$

Both the total resistances R and the section resistances r are in geometrical progression.

Normally either I_1 or I_2 (this is determined by the maximum value of the load torque at starting and during acceleration) and the number of sections of the starter (= n -1) are given and we have to determine the resistance of the sections. From $R_n = R_m = k^{n-1} R_1$, we can obtain

$R_n = R_m = k^{n-1} V/I_1$ or $R_n = R_m = k^{n-1} V/(I_2/k) = k^n V/I_2$

Example 11

Evaluate the five rheostat sections for the starter of a 220 V, 45 kW shunt motor with R_a = 0.045 ohms, the lower current limit being 200 A. What will be the upper current limit?

Solution

For five rheostat sections $n = 5 + 1 = 6$

From $R_m = k^n V/I_2$, $k^6 = \dfrac{R_m I_2}{V} = \dfrac{0.045 \times 200}{220} = 0.0409$ or $k = 0.0409^{\frac{1}{6}} = 0.5869$

$I_1 = I_2/k = 200/0.5869 = 341\text{A}$

$R_1 = V/I_1 = 220/341 = 0.64516\Omega$ and $R_2 = kR_1 = 0.5869 \times 0.64516 = 0.37864\Omega$

Hence $r_1 = R_1 - R_2 = 0.64516 - 0.37864 = 0.267\Omega$. Similarly

$r_2 = kr_1 = 0.5869 \times 0.2652 = 0.156$, $r_3 = kr_2 = 0.092\Omega$, $r_4 = kr_3 = 0.054\Omega$ and $r_5 = kr_4 = 0.031\Omega$

8.2 Grading starting resistance for series motor

In the case of the series motor, the flux changes with the motor current.

Let $k = I_2/I_1$, $f = \phi_1/\phi_2$ and R_m include the field resistance. The flux ϕ_1 produced by I_1 is greater than the flux ϕ_2 produced by I_2.

Adopting a procedure similar to the above case, just before the section 1 is cut out
$E_{b2} = V - I_2 R_1$ and just after cutting it out
$E_{b1} = V - I_1 R_2$
Since the speed cannot change instantaneously,

$$\frac{E_{b1}}{E_{b2}} = \frac{\phi_1}{\phi_2} = f$$

Now $R_2 = \dfrac{V - E_{b1}}{I_1} = \dfrac{V - fE_{b2}}{I_1} = \dfrac{V - fV + fI_2 R_1}{I_1} = (1-f)R_1 + kfR_1$

In general $R_q = (1-f)R_1 + kfR_{q-1}, \quad q = 2,3,..........,n$

The section resistance $r_q = R_q - R_{q+1} = kfr_{q-1}$

Hence $r_2 = kfr_1$, $r_3 = kfr_2 = (kf)^2 r_1, ..., r_{n-1} = (kf)^{n-2} r_1$.

Thus the sections form a geometrical progression but the total resistances do not.
The total resistance in circuit at starting may be expressed as

$R_1 = r_1 + r_2 + ... + r_{n-1} + R_m$ or

$R_1 = r_1(1 + b + b^2 + ... + b^{n-2}) + R_m$ or

$R_1 = r_1\left[\dfrac{1-b^{n-1}}{1-b}\right] + R_m$ where $b = kf$

The value of r_1 is obtained as

$r_1 = R_1 - R_2 = R_1 - (1-f)R_1 - kfR_1 = (f-b)R_1$

Thus $R_1 = R_1(f-b)\left[\dfrac{1-b^{n-1}}{1-b}\right] + R_m$ or

$$1 - \frac{R_m}{R_1} = (f-b)\left[\frac{1-b^{n-1}}{1-b}\right]$$

In general, the values of R_m and $R_1 = V/I_1$ will be known and the number of starter sections $= n-1$ given. It remains then to grade the resistance, which in turn involves the determination of I_2. Since f and b all depend on the I_2, it becomes necessary to evaluate the expression $Y = (f-b)\left[\dfrac{1-b^{n-1}}{1-b}\right]$ for various assumed values of I_2, plot Y against I_2 and from the graph obtain the value of I_2 giving $Y = 1 - \dfrac{R_m}{R_1}$

8.3 Manual starters

They are discussed below

8.3.1 Faceplate starter

An example of the manual starters is the faceplate starter. Fig.10 shows the schematic diagram of a manual faceplate starter for a shunt motor. The current-limiting resistors are R_1, R_2, R_3 and R_4. Conducting arm 1 as it is pulled to the right by means of insulated handle 2 cuts out the resistors successively. Contact M is a dead contact. At this position, the motor circuit is open. The arm is held at the close position by a small electromagnet 4, which is in series with the shunt field. If the supply voltage should fail or the field excitation should be lost, the electromagnet releases the arm allowing it to return to its dead position, under the pull of spring 3. This safety feature prevents the motor from restarting unexpectedly when the supply voltage is re-established and also from being energized when the excitation is lost.

It is obsolete except for small motors up to 5 kW. They are limited electrically because they tend to arc between the sliding surfaces, resulting in rapid wear of the contacting surfaces. They are also not operator-proof and cannot be used in simple applications such as motor reversal or braking.

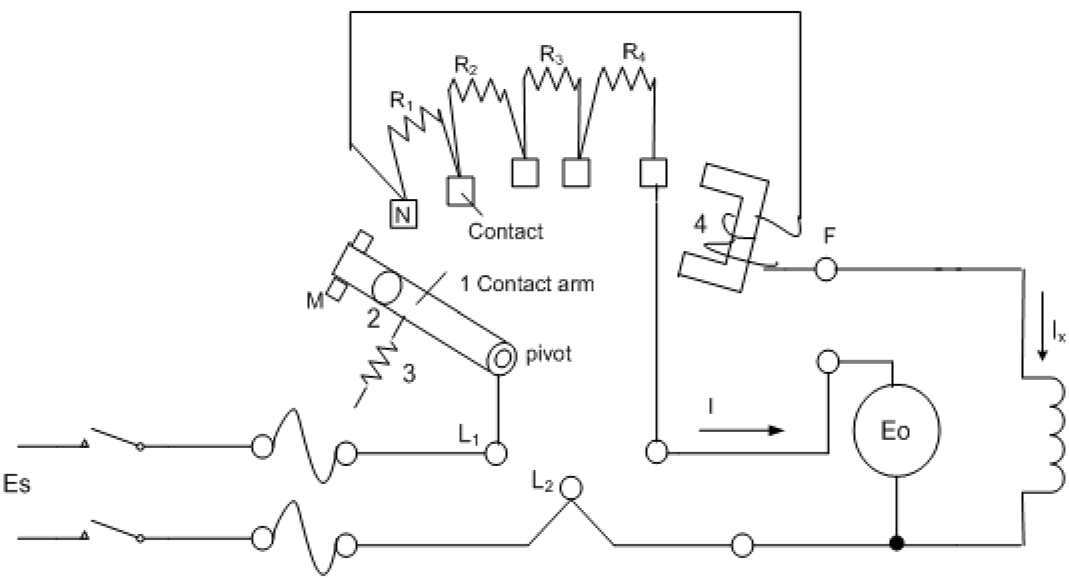


Fig. 10 Manual face-plate starter for a shunt motor

8.3.2 Drum controller

This is an improved manual starter. Stationary contact fingers under spring pressure make contact with a set of contact bars mounted on a cylindrical drum, Fig. 11. The required switching is made by rotating the drum manually. The manual effort needed for operation limits the application to motors of about 100 kW. They are used on crane motors, elevators, machine tools and other applications for start, stop, reverse and speed control.

The controller drum is moved in a series of well-defined steps. When it is moved to position 1, it completes the armature circuit through the full starting resistance, and at the same time power is applied to the shunt field. When it is moved to position 2, r_1 section is short-circuited, to position 3, r_1 and r_2 sections are short-circuited and finally to position 4, the whole starting resistance is short-circuited.

The motor reversal is achieved by reversing the current in the armature circuit (See figure)

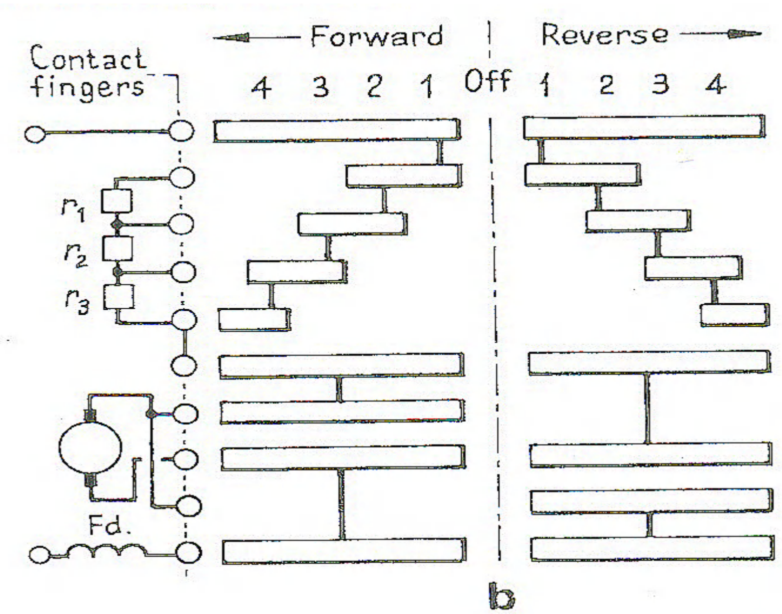


Fig. 11 Drum Controller

8.4 Automatic starters

In modern industrial applications, push-button is used to initiate the starting of most dc motors (both small and large) and automatically controlled contactors used to effect the subsequent switching. There are basically three types of automatic starters: direct-on-line, current-controlled or current-limit and definite-time starters.

8.4.1 Direct-on-line starter

This is used to start small dc motors. These motors draw low starting current so starting resistance is not necessary. A direct-on-line starter using a dual winding contactor is shown in Fig. 12. The two windings (or coils) are known as starting winding and holding winding. During the closing of the contactor when a large magnetic pull is required both windings are energised and in the closing position when a small holding force is required only the holding winding is energised.

Referring to the figure, pressing the start push button, both coils are energised through the start push button and the NC contact M_2 of the dual contactor M. As soon as the contactor M picks up, the NC contact M_2 opens to disconnect the starting coil. At the same time the sealing contact M_1 closes to provide a path around the start push button to keep the holding coil energised.

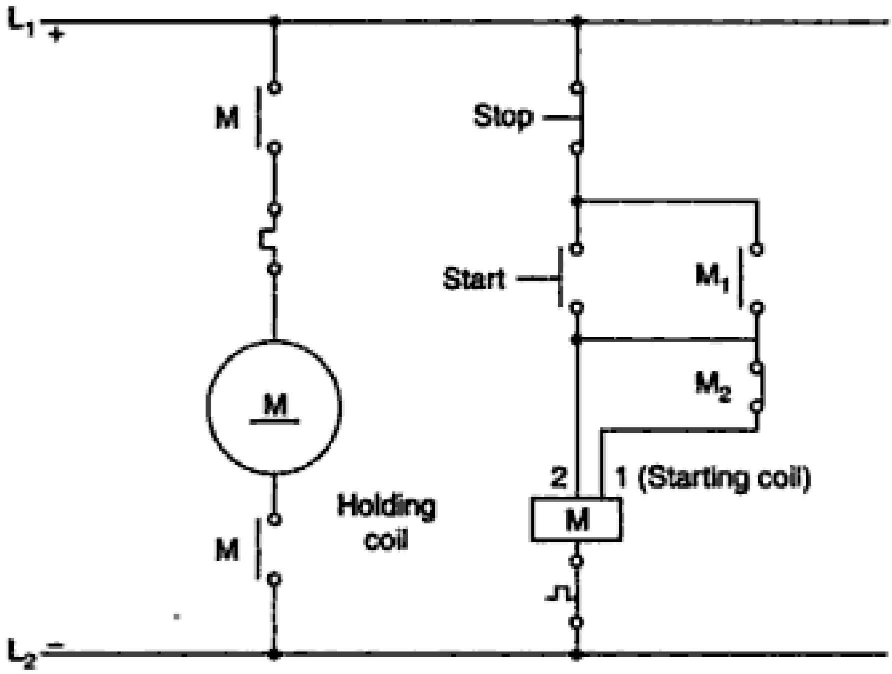


Fig. 12 Direct-on-line starter using a dual winding contactor

8.4.2 Current limit or current controlled starters

In this type of starter, the starting time depends on the motor load. A lightly loaded motor is brought to final speed more quickly than a heavily loaded motor. There are three basic types: series relay, counter emf and lockout type. The first two are discussed.

Series Relay Starter

In this starter, each resistance step is cut out as soon as the current falls to the lower limit. The starting time thus depends on the load. However, on heavy load the sequence is lost if the current fails to drop to its lower limit.

The coils of the series relays used in this starter carry the full armature current. The relays are fast acting: they picked immediately a high armature current is sensed. When the current falls below the setting of a relay, a strong spring opens its contact. A series relay starter diagram for a dc shunt motor is shown in Fig. 13. The starting resistance has three sections: R_1, R_2 and R_3 and the series relays are 1AR, 2AR and 3AR. To follow how the starter circuit works, we need to know that the NO auxiliary contacts of the contactors M, 1A, 2A and 3A close after some delay from the closing of their main or power contacts and that the series relays operate much faster than the contactors.

When the start pushbutton is pressed, the main contactor M picks up, its contact M_1 closes and the motor starts with resistances R_1, R_2 and R_3, and the series relay 1AR in series with the armature circuit. Before auxiliary contacts M_2 and M_3 close, the series relay picks up with the initial current surge and opens its interlock contact $1AR_1$ thus preventing the coil of contactor 1A from being energized. As the motor accelerates the armature current decreases. When the current falls to the minimum current I_2, the relay 1AR drops, its interlock contact $1AR_1$ recloses and thus energizes the coil of contactor 1A, which, in closing, short-circuits the R_1 section of the starting resistance. Now a second current surge takes place with resistance sections R_2 and R_3, and the series relay 2AR in series with the armature circuit. The series relay 2AR has interlock contact $2AR_1$ similar to that of 1AR which prevents the coil of contactor 2A from being energized. Thus the cycle is repeated until the last contactor 3A picks up to connect the motor armature straight across the supply.

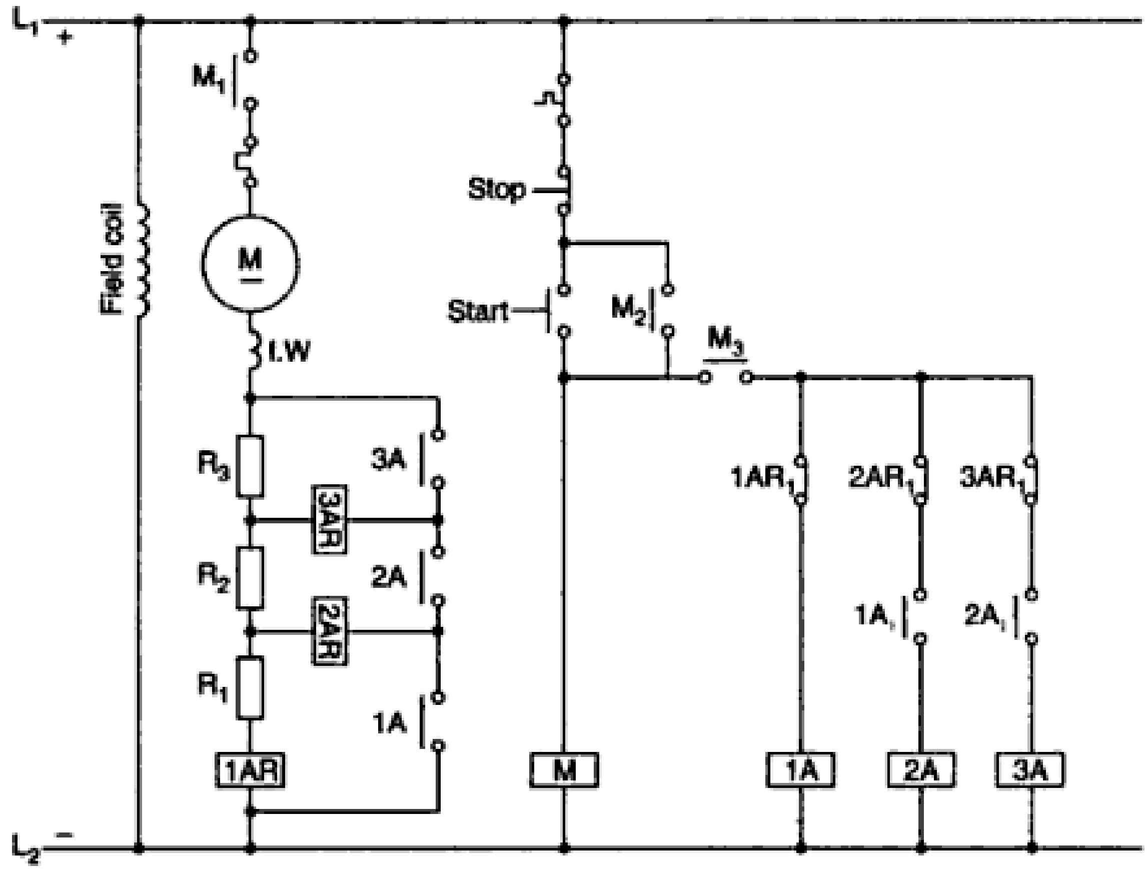


Fig. 13 Series relay starter diagram for dc shunt motor

Counter emf starter

In this starter, voltage relays are connected across the armature and set to operate at successively increasing values of emf to cut out the starting resistance in steps. The starting time is thus load-dependent. The emf for which the contacts open is less than that at which they close so to restart a motor it should be allowed to slow down sufficiently for the emf to fall below the opening level.

A counter emf starter diagram for dc compound motor is shown in Fig. 14. Pressing the start pushbutton energizes contactor M which completes the motor circuit and its own retaining circuit through the contacts M_1 and M_2 respectively. When the armature voltage has risen to, say 50% of rated value, voltage relay1A picks up and its contact 1A closes to cut out the resistance section R_1 through its main contact $1A_1$. Similarly, at, say 80% of rated voltage, voltage relay 2A picks up to cut out resistance section R_2, and finally, at say 90% of rated voltage, voltage relay 3A picks up to cut out the resistance section R_3.

On rising voltage, the voltage relays close relatively slowly and the contacts tend to weld in if this method is used for motors exceeding 5 hp.

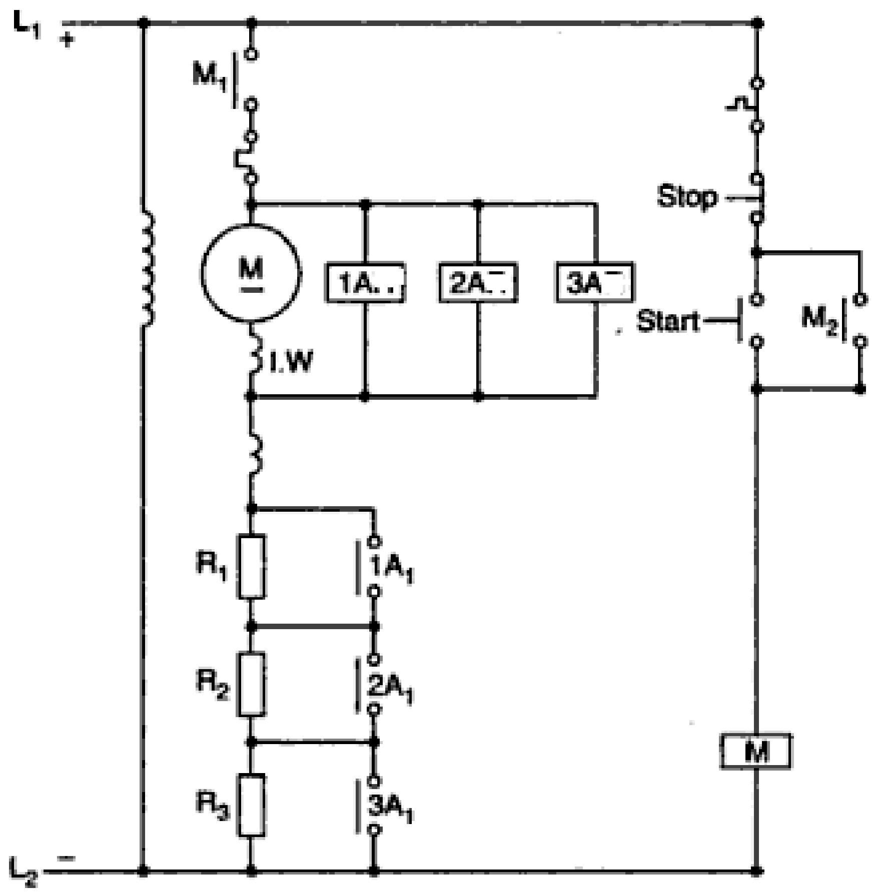


Fig. 14 Counter emf starter diagram for dc compound motor

Example 12
A counter emf starter is to be designed for a 115 V dc shunt motor with R_a = 0.23 ohms, the lower current limit being 29.4 A. (a) Evaluate the starting resistance sections for a three-step starter (b) Determine the upper current limit (c) Determine the voltage settings of the relays.

Solution
(a) For three-step starter $n = 3$

$$\text{From } R_m = k^n V / I_2 \text{ , } k^3 = \frac{R_m I_2}{V} = \frac{0.23 \times 29.4}{115} = 0.0588 \text{ or } k = 0.0588^{\frac{1}{3}} = 0.3889$$

$I_1 = I_2/k = 29.4/0.3889 = 75.6\text{A}$

$R_1 = V/I_1 = 115/75.6 = 1.52\Omega$ and $R_2 = kR_1 = 0.3889 \times 1.52 = 0.59\Omega$

Hence $r_1 = R_1 - R_2 = 1.52 - 0.59 = 0.93\Omega$

And $r_2 = kr_1 = 0.3889 \times 0.93 = 0.36\Omega$

(b) $I_1 = 75.6\text{A}$

(c) A starter section is cut out after the motor has gained speed, the motor emf. has built up and the motor current has fallen to I_2. Consequently, just before section r_1 is cut out the back emf is given by

$E_1 = V - I_2R_1 = 115 - 29.4 \times 1.52 = 70.3V$

The armature voltage sensed by the relay is given by

$V_{a1} = E_1 + I_2R_a = 70.3 + 29.4 \times 0.23 = 77V$

Just before section r_2 is cut out the back emf is given by

$E_2 = V - I_2R_2 = 115 - 29.4 \times 0.59 = 97.7V$

The corresponding armature voltage sensed by the relay is given by

$V_{a2} = E_2 + I_2R_a = 97.7 + 29.4 \times 0.23 = 104V$

8.4.3 Definite-time starters

The relays are set to operate at prefixed intervals after the start is initiated. The disadvantage of this type of starter is that the starting time is independent of the motor load. Despite this disadvantage, they are more widely used than the current-limit starters. This is because the fixed starting time make them useful where a driven machine must always repeat the same cycle of operation in manufacturing process or where several motors in a system must perform in a given time sequence. The delays are timed so that the current does not fall below a lower current limit with the heaviest prospective load. Consequently, on light loads the starting time is likely to be excessive. The timing devices used to establish the required time delays may be pneumatic timers, motor-driven timers, electronic timers, dashpot relay timers, time delay contactors and others.

Fig. 15 shows a definite-time starter diagram for a dc compound motor. The timing device may be pneumatic timers, electronic timers or dashpot relay timers. When the start pushbutton is pressed, the main contactor M and the timer 1T are energized and they get hold through contact M_2. The timer 1T operates after a preset delay to energize contactor 1A through its delayed contact $1T_1$. The contactor 1A picks up and short-circuits the R_1 section of the starting resistance. The timer 2T is energized at the same time as the contactor 1A, and after a preset time it similarly energizes contactor 2A to get the R_2 section of the starting resistance cut out. Finally, just as previously discussed, timer 3T energizes contactor 3A to get the last section of the starting resistance short-circuited. When contactor 3A picks up, it completes its retaining circuit through its NO contact $3A_3$ and de-energizes contactors 1A and 2A and the timers through its NC contact $3A_2$. To keep the starting resistance out of circuit while contactors 1A and 2A are de-energized, the contact $3A_1$ of contactor 3A is connected to short-circuit the entire starting resistance.

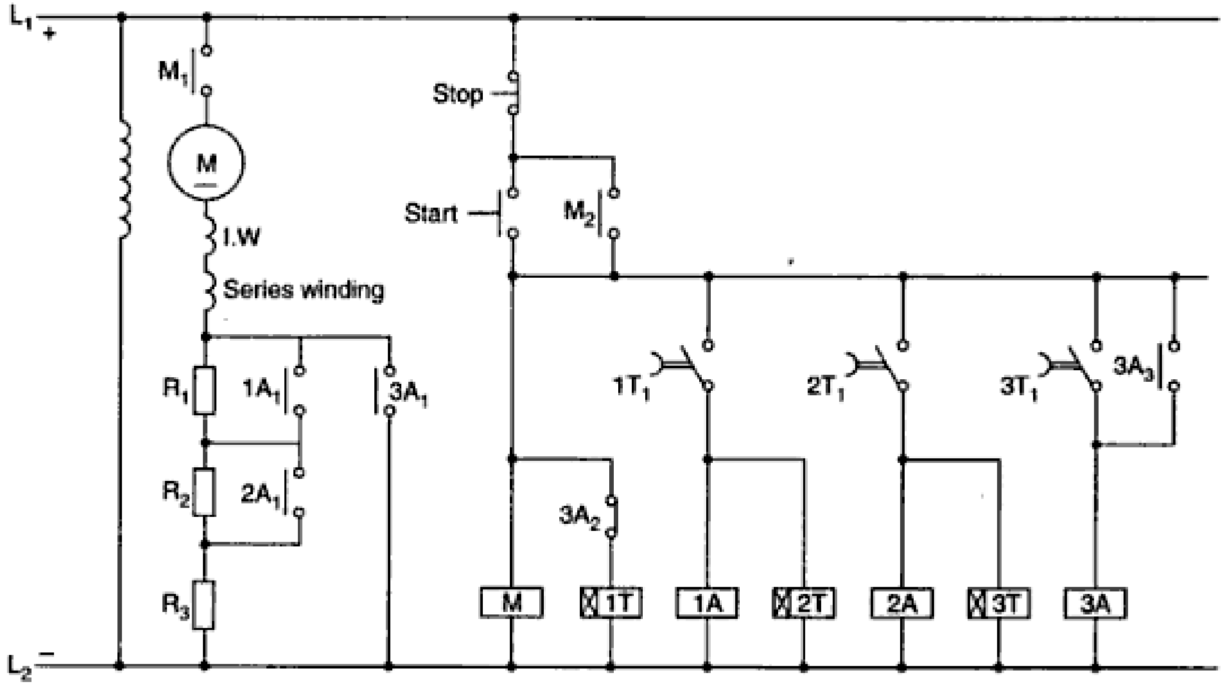


Fig. 15 Definite-time delay starter diagram for dc compound

9 Further Exercises

1. Find the resistances of the four rheostat sections for the starter of a 500 V, 15 kW series motor with $R_s + R_a$ =1.0 ohm. The upper current limit is to be 70 A. The gap flux rises by 15 % between 50 and 70 A, and may be taken as linear over this range.
   (1.93, 1.64, 1.40, 1.19 Ω)
2. The following table relates to the open-circuit curve of a shunt generator running at 750 rev/min.

Generated voltage(V)	10	172	300	360	385	395
Field current(A)	0	1	2	3	4	5

   Determine the no load terminal voltage if the field circuit resistance is 125 Ω. Find also the critical resistance of the shunt field circuit. (354 V)

   If the speed is halved, what is the resultant terminal voltage? At the reduced speed, what value of the field circuit resistance will give a no-load terminal voltage of 175 V?
   (5V, 64.5 Ω)
3. The open-circuit characteristic of a shunt generator when separately excited and running at 1000 rev/min is given by

E	56	112	150	180	200	216	230 V
I_f	0.5	1.0	1.5	2.0	2.5	3.0	3.5 A

   If the generator is shunt-connected and runs at 1100 rev/min with a total field resistance of 80 Ω, determine a) the no-load emf b) the output current when the terminal voltage is 200 V if the armature resistance is 0.1 Ω.
4. A 60-kW, 240-V short-shunt compound generator, operated as a shunt generator, required as increase in field current of 3A to provide an over compounded voltage of 275 V at rated load current of 250A. The shunt field has 200 turns per pole and the series field 5 turns per pole, with resistances of 240 Ω and 0.005 Ω respectively. Calculate the required diverter resistance.
5. Answer the following practical questions:
   (a) How is the induced voltage of a separately excited dc generator affected if (i) the speed increases? (ii) the field current is reduced?
   (b) The terminal voltage of a shunt generator decreases with increasing load. Explain.
   (c) Explain why the output voltage of an over compound generator increases as the load increases.
   (d) What determines the magnitude and polarity of the back emf in a dc motor?
   (e) Explain why the armature current of a shunt motor decreases as the motor accelerates.
   (f) Why is starting resistor needed to bring a motor up to speed?
   (g) Show one way to reverse the direction of rotation of a compound motor.
   (h) What are the essential functions of the field coils, armature, commutator and brushes?
   (i) What conditions must be fulfilled for the self-excitation of a dc shunt generator?

(j) Enumerate the losses in a dc shunt motor and explain how each loss is affected by a change of load.
(k) What is meant by the critical field resistance of a shunt generator?
(l) Give a brief explanation of why the current supplied to dc motor increases as the motor is loaded.
(m) A motor is required to have high starting torque to operate a crane state which is the more suitable motor, a shunt wound or a series wound and give reasons for your choice.

6. (a) From the basic equations of d.c. machines, deduce the shape of the following curves for d.c. shunt motor and sketch them:
   (i) Speed-torque curve; [3 marks]
   (ii) Torque-armature current curve; [1 mark]
   (iii) Speed-armature current curve. [1 mark]
   (b) A d.c. shunt motor takes an armature current of 20 A from a 230-V supply. Resistance of the armature circuit is 0.5 Ω. Calculate the resistance required in series with the armature to halve the speed if
   (i) the load torque is constant [5 marks]
   (ii) the load torque is proportional to the square of the speed. [4 marks]
7. (a) When is a starting resistor necessary for d.c. motors? [2 marks]
   (b) Give two possible damaging effects of highly excessive starting current in d.c. motors. [2 marks]
   (c) Evaluate the five rheostat sections for the starter of a 220-V, 45-kW d.c. shunt motor with $R_a = 0.045\ \Omega$, the lower current limit being 200 A. What is the upper current limit? [10 marks]
8. (a) The two basic equations of dc machines with negligible saturation can be expressed as

$$E = k\omega I_f \quad \text{and} \quad T_e = kI_aI_f$$

   All symbols having their usual meaning. Given any of the two, prove the other. [2 marks]
   (b) A 200-V dc shunt motor with negligible saturation has $R_f = 160\ \Omega$ and $R_a = 0.50\ \Omega$. On no load at a base speed of 800 rev/min the armature current is 4.0 A.
   (i) Calculate the motor constant *k* and the no load torque. [3 marks]
   (ii) The motor drives a load of torque 98 Nm, calculate the armature current and speed. [4 marks]
   (iii) If the motor is to develop 5.0 kW at 1000 rev/min, what resistance must be added to the field circuit? [6 marks]
9. (a) How would you physically identify series and shunt field windings in dc compound machine? [2 marks]
   (b) Explain why the terminal voltage of a dc shunt generator falls when it is loaded. [3 marks]
   (c) Sketch the load characteristics of level- and over-compounded dc generators. State the use of each type. [4 marks]
   (d) Draw the circuit diagrams of short-shunt and long-shunt generators. [2 marks]
   (e) Calculate the number of series turns necessary per pole on a compound generator to give a level characteristic at 500 V between no load and full load of 50 kW. Without any series winding it is found that shunt current for 500 V is 1.5 A on no load and 1.7 A on full load. The machine is connected short-shunt and the number of turns per pole on the shunt field is 2200. [4 marks]

10. (a) Enumerate losses occurring in a d.c. shunt motor, and state how each loss will vary when the load on the machine is varied. [7 marks]

(b) A motor is required to have a high starting torque to operate a crane. State which is the more suitable d.c. motor, shunt wound or series wound and give reasons for your choice. [4 marks]

(c) A 200 V d.c. series motor takes armature current of 20 A when running at 600 rev/min and supplying full load torque. Motor resistance is 0.5 ohms. Calculate

(i) the resistance which must be connected in series with the motor to reduce the speed to 500 rev/min, the load torque remaining the same. [8 marks]

(ii) the speed the motor would run if the torque were halved and the added resistance left in circuit. Assume the magnetic circuit to be unsaturated. [6 marks]

11. (a) Draw the circuit diagram of d.c. shunt generator. [2 marks]

(b) Give FOUR possible reasons why a d.c. shunt generator may fail to excite when driven on no load. [8 marks]

(c) At a speed of 1500 rev/min a separately excited d.c. generator has the following o.c.c.:

emf [V]	24	46	68	89	102	113	122	128	134	139
Field current [A]	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0

(i) Draw the o.c.c. using scales of 1 cm: 10 V and 1 cm: 0.1 A. [2 marks]

(ii) Determine the field circuit resistance required for an open-circuit emf of 120 V to be generated when machine is operated as a shunt generator at 1500 rev/min. [4 marks]

(iii) Determine the speed at which the generator must be driven to give an open-circuit emf of 150 V with the same field circuit resistance. [6 marks]

(iv) Determine the critical field resistance when the generator is run at 750 rev/min. [3 marks]

CHAPTER THREE

3-PHASE INDUCTION MOTORS

1 Introduction

The polyphase induction motor is the most widely used motor in industrial and commercial applications. There are two types, namely slip ring induction motor and squirrel-cage induction motor. The squirrel-cage is more common because of its low-cost, high reliability and high efficiency over a wide range of power outputs. From a constant frequency source, it operates as a constant speed drive. For continuous speed control over a wide speed range, it is supplied from a solid-state variable-frequency converter.

2 Construction

A three-phase induction motor has two main parts: a stationary stator and a revolving rotor. The rotor is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm depending on the power rating of the motor.

2.1 Stator

It consists of a steel frame which encloses a hollow, cylindrical core made up of stacked laminations. A number of slots are punched uniformly round the gap surface of the core. These slots carry the stator 3-phase winding.

2.2 Rotor

It is also composed of punched laminations. These are carefully stacked to create a series of rotor slots to provide space for the rotor winding. Two types of rotor windings are used. The type of winding gives rise to two main classes of motors: squirrel-cage induction motors and wound-rotor or slip-ring induction motors.

2.2.1 Squirrel-cage rotor

Bare copper bars, slightly longer than the rotor are pushed into the slots. The opposite ends are welded to two copper end-rings, so that all bars are short-circuited together. In small and medium-size motors, the bars and end-rings are made of die-cast aluminium moulded together to form an integral block.

2.2.2 Wound or slip-ring rotor

The slots carry three-phase winding similar to the one on the stator. The windings are usually connected in star. The terminals are connected to three-slip rings which turn with the rotor. The revolving slip-rings and associated stationary brushes enable us to connect external resistors in series with the rotor windings. The rotor of an induction motor could be wound for a number of phases different from that for the stator.

3 Principle of operation of the induction motor

A rotating magnetic field is set up when a 3-phase voltage is applied to the stator of an induction motor. The rotating field cuts across the rotor conductors and induces a voltage in all of them. The induced voltage causes large currents to flow in the conductors and these interact with the magnetic field to produce a torque which, according to Lenz's law, drags the rotor along in the direction of the revolving field (According to this law, the induced voltage in the rotor will tend to set up a current whose magnetic field will oppose the motion or change of flux responsible for inducing the voltage).

4 Acceleration of the rotor-slip

As the motor picks up speed, the relative velocity of the field with respect to the rotor diminishes progressively. This causes both the value and the frequency of the rotor induced voltage to decrease because the rotor conductors are cut more slowly. The rotor current, very large at first, decreases rapidly as the motor picks up speed.

The speed will continue to increase, but it will never catch up with the rotating field. In effect, if the rotor did turn at the same speed as the field, the field would no longer cut the rotor conductors and the induced voltage and current would fall to zero. Under these conditions, the force acting on the rotor conductors would also become zero and friction and windage would immediately cause the rotor to slow down.

The rotor speed must always be slightly less than synchronous speed (i.e. the speed of the revolving field) so as to produce a current in the rotor bars sufficiently large to overcome the braking torque. At no load the difference in speed between the rotor and field (called slip) is very small, usually less than 0.1 % of synchronous speed.

5 Production of rotating field by 3-phase currents in 3-phase winding

We now draw the mmf waves for the phases and explain how the rotating field is produced:

- Consider a three-phase machine with a single-layer full-pitch winding having two slots per pole per phase, Fig. 1.a.
- The mmf wave is obtained by application of Ampere's law.
- It is assumed that the reluctance of the iron part of the magnetic circuit is negligible and the conductors are of infinitesimal size.
- To draw mmf for a phase, we start at any arbitrary value for $F(\theta)$, increase or decrease the value by a step at a slot where the phase conductors are located and put the horizontal axis at the end of the operation at a position such that the wave will have an average value of zero.

The magnitude of the step is equal to the ampere conductors in the slot and whether the step increases or decreases depends on the direction of the current.

- The mmf wave is periodic (it repeats over every two poles) and Fourier analysis can be applied to express it as sum of fundamental components and harmonics.
- Ignoring the harmonics, the total mmf can be obtained by summing the mmfs for all phases (Fig. 1. c – 1.d) as follows:

$$F(\theta) = kNi_a \sin\theta + kNi_b \sin(\theta - 120^o) + kNi_c \sin(\theta - 240^o)$$

Substituting $i_a = I_m \sin\omega t$, $i_b = I_m \sin(\omega t - 120^o)$ and $i_c = I_m \sin(\omega t + 120^o)$ into it gives

$$F(\theta) = kNI_m[\sin\omega t \sin\theta + \sin(\omega t - 120^o)\sin(\theta - 120^o) + \sin(\omega t - 120^o)\sin(\theta - 240^o)]$$ or

$$F(\theta) = F_m[\sin\omega t \sin\theta + \sin(\omega t - 120^o)\sin(\theta - 120^o) + \sin(\omega t - 120^o)\sin(\theta - 240^o)]$$

where $F_m = kNI_m$ is the maximum peak value of the magnetic field produced by a phase coil. We note that the peak value of the mmf wave for a phase is varying with time. The angle θ is in electrical degrees or radians. With the aid of trigonometric identity $\sin\theta_1 \sin\theta_2 = [\cos(\theta_1 - \theta_2) - \cos(\theta_1 + \theta_2)]/2$, the resultant mmf can be obtained as

$$F(\theta) = \tfrac{3}{2} F_m \cos(\theta - \omega t)$$

This is equation of a travelling wave having a constant peak value of $\tfrac{3}{2} F_m$. This can be checked with (Refer to Fig.1. f)

$\omega t = 0$ giving $F(\theta) = \tfrac{3}{2} F_m \cos\theta$ and $\omega t = \tfrac{\pi}{2}$ giving $F(\theta) = \tfrac{3}{2} F_m \sin\theta$

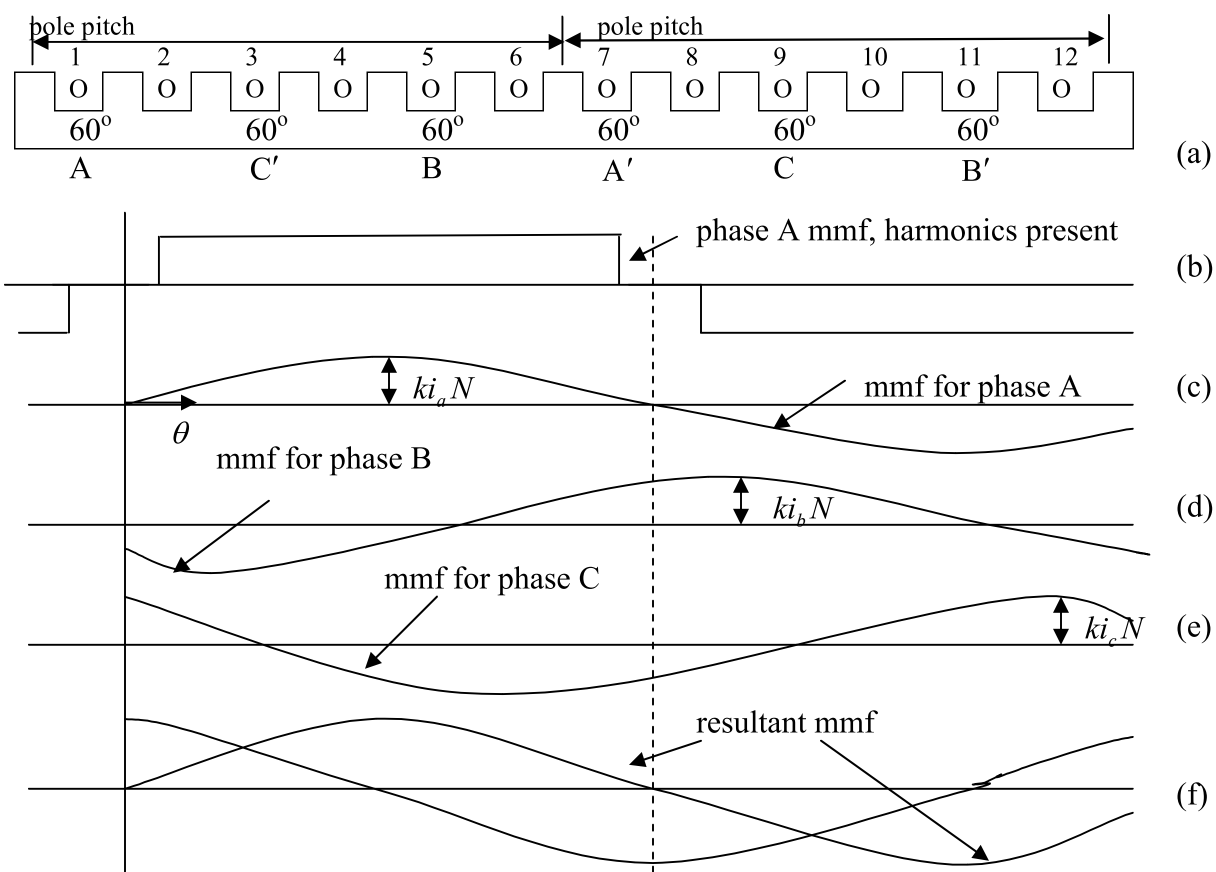


Fig. 1 3-phase stator winding and mmf waves

5.1 Speed of mmf wave

The speed of the wave is obtained by equating the argument of the cosine to a constant and differentiating with respect to time. Thus for a 2p-pole machine, the speed of rotation called synchronous speed $\omega_s = \dfrac{d\theta_{mech}}{dt}$ is given by

$$\frac{d}{dt}(p\theta_{mech} - \omega t) = \frac{d}{dt}(\text{certain constant}) = 0 \text{ or}$$

$$p\frac{d\theta_{mech}}{dt} = p\omega_s = \omega$$

Therefore

$$\omega_s = \frac{\omega}{p} rad/s \;\; or \;\; n_s = \frac{f}{p} \;\; rev/s \;\; or \;\; N_s = \frac{60f}{p} \;\; rev/\min \tag{1}$$

6 Definition of slip

The slip, s of an induction motor is defined as

$$s = \frac{synchronous\,speed - rotor\,speed}{synchronous\,speed} = \frac{n_s - n_r}{n_s} \tag{2}$$

The slip is equal to 1 when the rotor is locked or at standstill. On no load s is usually less than 0.001 and at full load s is usually less than 0.005 for large motors (1000 kW and more) and less than 0.03 for small motors (10 kW or less). For this reason induction motors are considered to be constant speed machines.

7 Rotor frequency

The frequency of the voltage induced in the rotor depends upon the slip. It is given by

$$f_r = sf \tag{3}$$

where $f =$ frequency of the source connected to the stator.

Example 1
An induction motor is excited by a 3-phase, 50-Hz source. If the full-load speed is 1440 rev/min calculate the slip.

Solution
The synchronous speed close to 1440 rev/min is 1500 rev/min. This is obtained if the machine is wound for four poles.
Check $N_s = (60f)/p = (60 \times 50)/2 = 1500\,\text{rev/min}$
Consequently the slip is

$$s = \frac{n_s - n_r}{n_s} = \frac{1500 - 1440}{1500} = 0.04$$

8 Rotor mmf

The polyphase winding of the rotor carries a polyphase current. Thus the rotor also produces a rotating field. The rotor and stator magnetic axes move at the same speed and in the same direction (Were it not so, the machine would not be self-maintaining in rotation).

The stator and rotor windings must be arranged to have the same number of poles, so that their mmf waves can combine to form a resultant rotating mmf wave. A wound-rotor would therefore have to be wound to have the same number of poles as the stator. The cage winding has the advantage of adapting itself readily to any number of the stator poles (Note: a squirrel cage can be viewed as a polyphase winding. Generally with C individual rotor conductors, one per slot, the number of rotor phases is $m = C/2p$. Single turns constitute the windings of each phase. The number of poles of the rotor is entirely determined by the stator).

9 Active power flow through induction motor

The power flow diagram shown in Fig.3 indicates what becomes of the active power P_e that flows into the stator of an induction motor.

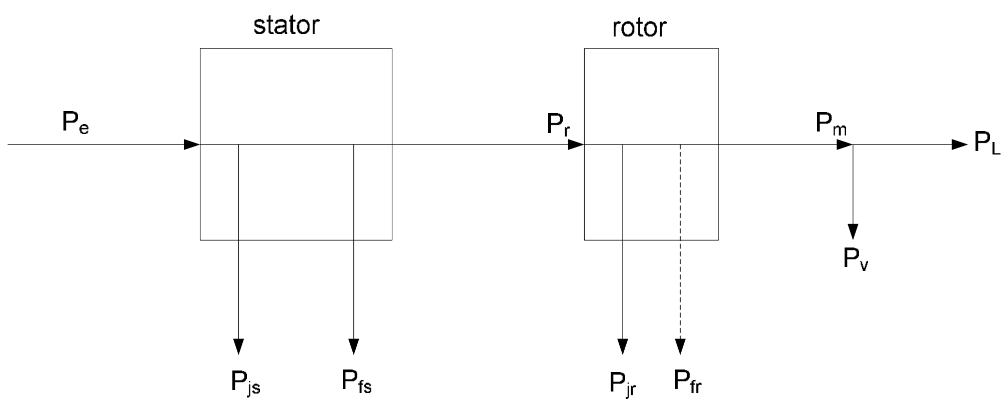


Fig. 3 Power flow diagram

$P_e = input\ \ power$

$P_{js} = stator\ copper\ loss$

$P_{fs} = stator\ \ iron\ \ loss$

$P_r = power\ transferred\ \ from\ stator\ to\ rotor$

$P_{jr} = rotor\ \ copper\ \ loss$

$P_{fr} = rotor\ \ iron\ \ loss\ \ \ (negligible\ because\ f_r = 0)$

$P_m = gross\ output\ power$

$P_L = shaft\ power$

$P_v = friction\ \ and\ windage\ loss$

Gross torque = gross output power/motor speed

Useful or shaft torque = shaft power/motor speed

Loss torque or no load torque = friction and windage loss/motor speed

10 Torque and output power equations

The power transferred from the stator to the rotor and the gross mechanical power are given by

$$P_r = T_e \omega_s \tag{4.a}$$

$$P_m = T_e \omega_r \tag{4.b}$$

where T_e is the electromagnetic or gross torque developed by the motor. If the rotor iron loss is neglected then

$$P_{jr} = P_r - P_m = T_e \frac{\omega_s - \omega_r}{\omega_s}.\omega_s = sP_r \text{, i.e.}$$

$$P_{jr} = sP_r \text{ and} \tag{5.a}$$

$$P_m = P_r - P_{jr} = (1-s)P_r \tag{5.b}$$

Also the gross or electromagnetic torque

$$T_e = \frac{P_r}{\omega_s} = \frac{P_{jr}}{s\omega_s} \text{ in newton-metres} \qquad \text{or} \tag{6.a}$$

$$T_e = \frac{P_{jr}}{s} \text{ in synchronous watts} \tag{6.b}$$

Example 2

The power supplied to a three-phase induction motor is 40 kW and the corresponding stator losses are 1.5 kW, calculate

(a) the total mechanical power developed and the rotor I^2R loss when the slip is 0.04 per unit.
(b) the output power of the motor if the friction and windage losses are 0.8 kW and
(c) the efficiency of the motor. Neglect the rotor iron loss.

Solution

(a) Input power to rotor, $P_r = 40 - 1.5 = 38.5\,kW$

Rotor copper loss $P_{jr} = sP_r = 0.04 \times 38.5 = 1.54kW$

Therefore the total mechanical power developed $P_m = P_r - P_{jr} = 38.5 - 1.54 = 36.96\,kW$

(b) Output power of motor $P_L = P_m - P_v = 36.96 - 0.8 = 36.16\,kW$

(c) Efficiency of motor $\eta = \frac{P_L}{P_e} = \frac{36.16}{40} = 0.904\,pu = 90.4\%$

Example 3

If the speed of the motor of Example 2 is reduced to 40 % of its synchronous speed by means of external rotor resistors, calculate

(a) the total rotor I^2R loss and

(b) the efficiency,
assuming the torque and the stator losses remain unaltered. Also assume that the increase in the iron loss is equal to the reduction in the friction and windage loss.

Solution

(a) New slip, $s = (n_s - n_r)/n_s = (100 - 40)/100 = 0.6\,\text{pu}$

and input power to rotor $P_r = 38.5\,\text{kW}$ (because torque is unaltered)

Total rotor copper loss $P_{jr} = sP_r = 0.6 \times 38.5 = 23.1\,\text{kW}$

(b) Total losses in rotor $= P_{jr} + P_v = 23.1 + 0.8 = 23.9\,\text{kW}$

Therefore output power of motor $= P_r - \textit{total rotor losses} = 38.5 - 23.9 = 14.6\,\text{kW}$

$$\text{Efficiency of motor} = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{stator losses} + \text{P}_\text{r}} = \frac{14.6}{40} = 0.365\,\text{pu} = 36.5\%$$

11 Stator and rotor induced voltages

11.1 Conditions with rotor winding stationary and open-circuited

When a voltage is applied to the stator, it takes current which produces a rotating mmf. The stator rotating mmf then produces a rotating flux. A major part of the flux links the rotor and the remainder is a leakage flux. The mutual flux induces phase voltages:

$$E_1 = 4.44 T_{ph1} k_{w1} f \Phi_m \qquad (7.a)$$

$$E_2 = 4.44 T_{ph2} k_{w2} f \Phi_m \qquad (7.b)$$

in the stator and rotor respectively.

11.2 Conditions with rotor winding closed

When the rotor winding is closed, the voltage induced in the rotor by the mutual flux causes current to circulate. Whether the rotor is locked or released, the mmf of the rotor rotating also at synchronous speed, tend to demagnetize the machine, so that the stator takes equal and opposite mmf component as in transformers. There is established a rotor leakage flux, a reduced mutual flux and an increased stator leakage flux. With the rotor released (the general case) the induced emf in the rotor becomes:

$$E_2 = 4.44 T_{ph2} k_{w2} s f \Phi_m \qquad (8)$$

At standstill $s = 1$and the induced emf is given by (7.b). However, the mutual flux is very small. (Φ_m in the equation is about 50 % of no load value)

12 Torque calculations assuming constant mutual flux

If the flux Φ_m is assumed to be constant, then for a given slip s, the rotor emf per phase

$$E_{2\mathrm{run}} = sE_{2\mathrm{stand}} \tag{9.a}$$

where $E_{2\mathrm{stand}}$ is the rotor emf generated per phase at standstill. It is given by

$$E_{2\mathrm{stand}} = 4.44 T_{ph2} k_{w2} f \Phi_m \tag{9.b}$$

Let

$$\begin{aligned} X_2 &= \text{leakage reactance per phase at standstill} \\ &= 2\pi f \times \text{leakage inductance per phase of rotor winding} \end{aligned} \tag{10.a}$$

Then the rotor leakage reactance/phase for a given slip s

$$X_{2run} = sX_2 \tag{10.b}$$

Therefore the rotor current/phase at a slip s

$$I_{2run} = \frac{E_{2\mathrm{run}}}{\sqrt{R_2^2 + s^2 X_2^2}} = \frac{sE_{2\mathrm{stand}}}{\sqrt{R_2^2 + s^2 X_2^2}}$$

and the power dissipated as $I^2 R$ loss in the rotor circuits

$$P_{jr} = \frac{s^2 E_{2\mathrm{stand}}^2 R_2}{R_2^2 + s^2 X_2^2} \times m_2$$

where R_2 is the resistance/phase of the rotor winding and m_2 is the number of rotor phases. Hence we have

$$T_e = \frac{P_{jr}}{s\omega_s} = \frac{m_2 s E_{2\mathrm{stand}}^2 R_2}{(R_2^2 + s^2 X_2^2)\omega_s}$$

Consequently, for a given synchronous speed and number of rotor phases

$$T_e \propto \frac{sR_2}{R_2^2 + s^2 X_2^2} \tag{11}$$

13 Exact equivalent circuit

The equivalent circuit is arrived at in the same way as for transformer. The circuit on the single-phase or per phase basis is shown in Fig. 3.

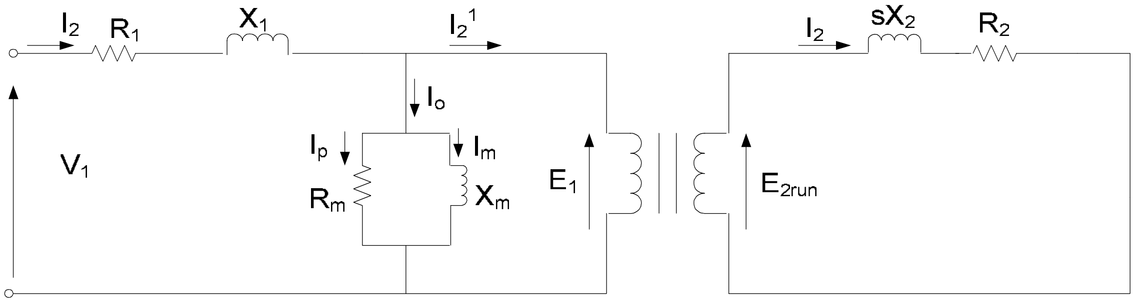


Fig. 3 Exact Equivalent Circuit

The rotor circuit can be redrawn as shown in Fig. 4. The significance of this rotor circuit is that it represents the rotating rotor by a fictitious stationary rotor with a constant reactance (or frequency) but with a variable resistance. This fictitious stationary rotor carries the same current as the actual rotor, and thus, it produces the same mmf wave. With the rotor represented this way, it is now possible to transfer the secondary (rotor) impedance to the primary (stator) side in the same way as for transformer with short-circuit secondary. The equivalent circuit referred to the stator is given in Fig. 5.

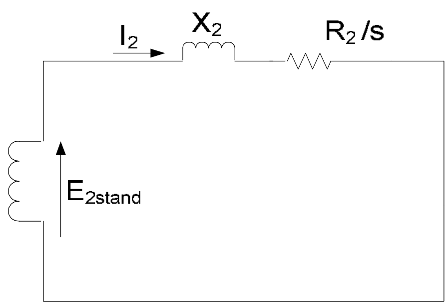


Fig. 4 Equivalent rotor circuit with constant reactance

The voltage ratio

$$a = \frac{E_1}{E_{2stand}} = \frac{T_{ph1}k_{w1}}{T_{ph2}k_{w2}}, \tag{12.a}$$

the current ratio

$$b = \frac{I_2'}{I_2} = \frac{m_2}{m_1}.\frac{1}{a} \quad \text{(This is obtained by equating apparent powers)} \tag{12.b}$$

and the impedance ratio

$$c = \frac{Z_2'}{Z_2} = \frac{a}{b} = \frac{m_1}{m_2}.a^2 \tag{12.c}$$

where m_1 is the number of stator phases and m_2 that of rotor phases.

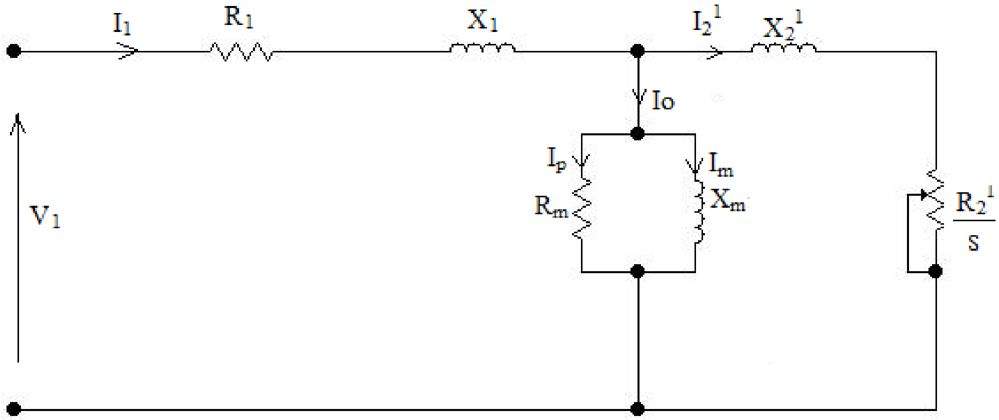


Fig. 5 Exact equivalent circuit

The actual rotor impedance and the ratio by which it is transferred to the stator are of interest mainly to the designer, not the analyst. The study of the motor performance is based on the value of Z_2', not Z_2. Z_2' and other impedances can be obtained from experimental test results.

14 Approximate circuits

A variety of performance characteristics can be estimated simply, and with reasonable accuracy using approximate circuits. Two of such circuits are shown in Fig. 6. The circuit shown in Fig. 6.a is the IEEE recommended equivalent circuit. The core loss is lumped with the mechanical losses. Lumped together the losses are referred to as rotational losses.

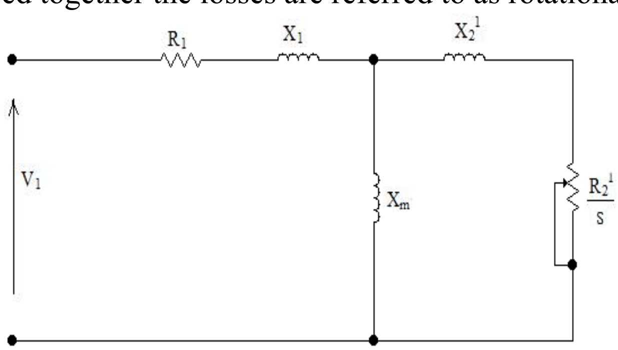


Fig. 6.a Approximate Circuit with R_m omitted

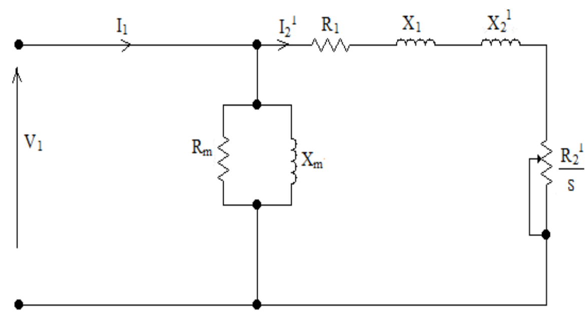


Fig. 6.b Approximate Circuit with the magnetizing branch transferred to the terminals

The voltage drop due to I_o flowing through the stator leakage impedance is appreciable. Its exclusion in Fig.6.b can give rise to errors of 10 % or more in some cases. It is acceptable for motors above 5 hp or 3 kW.

Other approximations can be derived from the exact equivalent circuit by omitting R_1 if $R_1 << X_1 \; or \; R_m \; and \; R_1 \; or \; R_1 \; and \; X_1$. The last one implies that the mutual flux is constant. This gives very large errors at high values of slip.

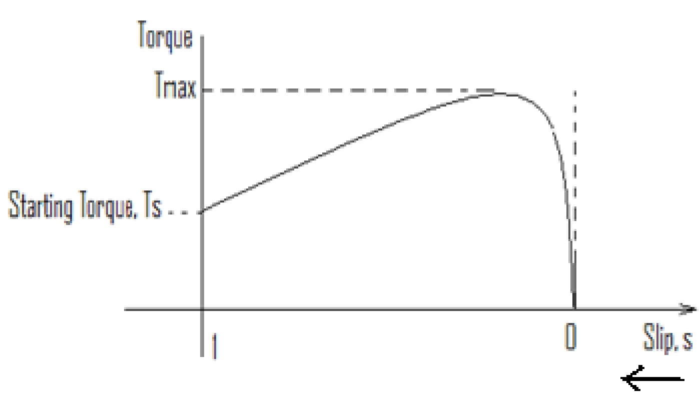
15 Torque calculations

Consider the circuit shown in Fig. 6.b. The torque is given by

$$T_e = \frac{3[I_2']^2 R_2'}{s\omega_S} \; (from \; P_r = T_e \omega_s) \quad \text{Or}$$

$$T_e = \frac{3}{\omega_s} \frac{V_1^2}{\left[\left(R_1 + \frac{R_2'}{s}\right)^2 + \left(X_1 + X_2'\right)^2\right]} \frac{R_2'}{s} \qquad (13)$$

T_{max} is called the breakdown torque



The general shape of the torque-slip curve for an induction motor is in Fig. 7

15.1 Starting torque

At starting $s = 1$ and

$$T_e = \frac{3}{\omega_s} \frac{V_1^2}{\left[(R_1 + R_2')^2 + (X_1 + X_2')^2\right]} R_2' \tag{14}$$

We note that for a given starting torque there are two possible values of rotor resistance.

15.2 Maximum torque

The torque is maximum when the power dissipated in R_2'/s is maximum. Then by maximum power transfer theorem, maximum torque will occur when
$|R_1 + jX| = R_2'/s$ where $X = X_1 + X_2'$. Thus the slip s_m at which maximum torque occurs is given by

$$s_m = R_2' / \left(R_1^2 + X^2\right)^{1/2} \tag{15}$$

When this value of slip is substituted into the general equation for torque (13), the value of maximum torque T_{max} is obtained as

$$T_{\max} = \frac{3V_1^2}{2\omega_s\left[R_1 + \sqrt{R_1^2 + X^2}\right]} \tag{16}$$

We make the following observations:
(i) $s_m \propto$ rotor resistance but T_{max} is independent of it.
(ii) The speed of an induction motor can be controlled by varying the rotor resistance
(iii)The starting torque can be increased by increasing the rotor resistance.

Typical torque-slip curves with variable rotor resistance are shown in Fig. 8

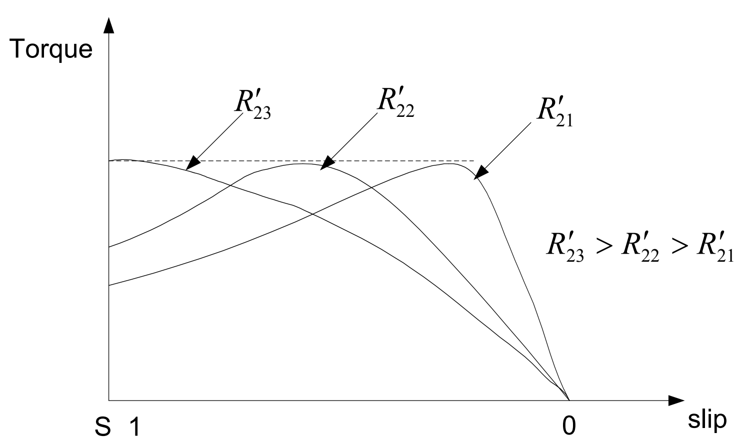


Fig. 8 Effect of rotor resistance on torque curve

Beyond R'_{23} the starting torque starts decreasing. Thus for a given starting torque we notice here too that there are two possible values of rotor resistance. The smaller one is used to give larger accelerating torque at starting.

With a motor having a low-resistance rotor, such as the usual type of cage rotor, the starting torque is small compared with the maximum torque available. A high rotor resistance is desirable because it produces a high starting torque. Unfortunately, it also produces a rapid fall-off in speed with increasing load. Furthermore, because the slip which results at rated torque is high, the rotor copper losses are high, the efficiency is low and the motor tends to get hot.

When a high starting torque is required for a heavy starting duty such as accelerating a high inertia load from standstill to its running speed, the slip-ring is ideal (Fig.9). At starting the variable resistors are set to their highest value. As the motor accelerates, the resistance is gradually reduced until full-load speed is reached whereupon the brushes are short-circuited.

16 Comparison of cage and slip-ring motors

16.1 Advantages of squirrel-cage motors

(i) Cheaper and more robust
(ii) Slightly higher efficiency and power factor
(iii)Explosion proof, since the absence of slip-rings and brushes eliminate risk of sparking

16.2 Advantages of slip-ring motors

(i) The starting torque is much higher and the starting current much lower
(ii) The speed can be varied by means of external rotor resistor

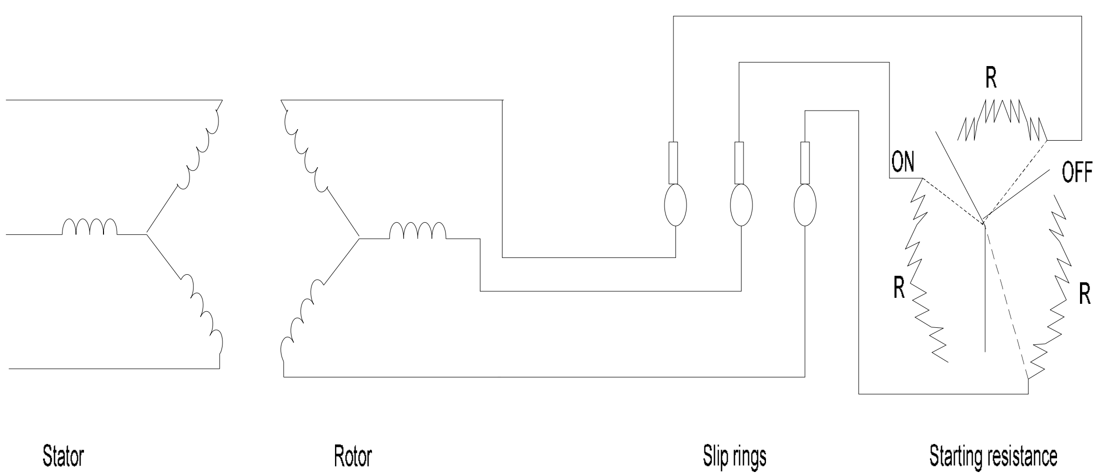


Fig. 9 Slip-ring induction motor

Example 4

A 415-V, 3-phase, 6-pole, 50-Hz, star-connected slip ring induction motor has the following parameters in ohms per phase

$R_1 = 0.04$, $X_1 = 0.15$, $R_2' = 0.05$, $X_2' = 0.15$

Determine

(a) The stator current and the gross torque in Nm when the slip is 0.05 pu.

(b) The maximum gross torque, the slip at which it occurs and the gross output power under these conditions

(c) The value of external resistance to be inserted in the rotor circuit to produce the maximum torque at standstill or starting. Neglect the magnetizing branch.

Solution

(a) Synchronous speed $\omega_s = \dfrac{2\pi f}{p} = \dfrac{2\pi \times 50}{3} = 104.7\,\text{rad/s}$

$$\text{Total impedance} = (R_1 + \frac{R_2'}{s}) + j(X_1 + X_2') = \left(0.04 + \frac{0.05}{0.05}\right) + j(0.15 + 0.15)$$

$$= 1.04 + j0.30 = 1.082\angle 16.09^0\,\Omega$$

$\text{V}_{\text{phase}} = 415/\sqrt{3} = 240\,\text{V}$

Stator current $I = 240\angle 0/.082\angle 16.09 = 222\angle -16.09\text{A}$

Power into rotor $P_r = 3I^2\dfrac{R_2'}{s} = 3\times 222^2\left(\dfrac{0.05}{0.05}\right) = 147.8\text{ kW}$

$$\text{Gross torque} = \frac{P_r}{\omega_s} = \frac{147.8}{104.7} = 1.4\text{ kN}$$

(b) $\text{T}_{\max} = \dfrac{3}{2}\times\dfrac{V_1^2}{\left[R_1 + \sqrt{R_1^2 + X^2}\right]\omega_s} = \dfrac{3}{2}\times\dfrac{240^2}{\left[0.04^2 + \sqrt{0.04^2 + 0.3^2}\right]}\times\dfrac{1}{104.7} = 2.41\text{ kN}$

$s_m = R_2'/\left(R_1^2 + X^2\right)^{1/2} = 0.05/\left(0.04^2 + 0.3^2\right)^{1/2} = 0.16\text{ pu}$

Motor speed $\omega_r = \omega(1-s) = 104.7\times(1-0.16) = 87.948\,\text{rad/s}$

Output power $= T_{\max}\times\omega_r$ at maximum slip $= 2.41\times 87.948 = 212\text{ kW}$

(c) At standstill $s = 1$. Therefore if the maximum occurs at standstill (or at starting), then

$1 = R_2'/\left(R_1^2 + X^2\right)^{1/2}$

or

$R_2' = \left(R_1^2 + X^2\right)^{1/2} = \sqrt{0.04^2 + 0.3^2} = 0.30\,\Omega$

Additional resistance required referred to the stator is $R_{ex}' = 0.30 - 0.05 = 0.25\ \Omega$

17 Determination of equivalent circuit parameters

The parameters of the motor equivalent circuit are determined from the measurement of dc phase resistance and the locked rotor and the no load tests.

17.1 Stator resistance measurement

This is obtained by dc measurement, Fig.10. The dc current is adjusted to approximately rated value so that temperature of winding approximates that of running conditions.

$R_1 = R_{LL}/2$

where $R_{LL} =$ the stator resistance measured between any two terminals.

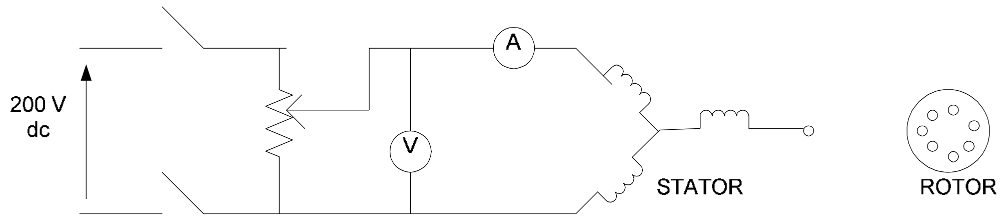


Fig 10. Measurement of stator resistance

17.2 Locked rotor test

The rotor is locked and reduced voltage applied to the stator to circulate in the stator the rated current. The reduced voltage V_{sc} is about 12% to 28% of rated voltage. The effect of the magnetizing branch is therefore usually small enough to be neglected.

The line voltage V_{sc}, current I_{sc} and total power P_{sc} under locked-rotor conditions are measured. The following calculations are then made:

$$R_{e1} = R_1 + R_2' = P_{sc} / 3I_{sc}^2$$

$$X_{e1} = X_1 + X_2' = \sqrt{\left[(V_{sc}/I_{sc})^2 - (R_1 + R_2')^2\right]}$$

The dc value of R_1 is taken as its ac value and this is deducted from R_{e1} to obtain R_2'. Empirical ratios have been developed to separate out X_1 and X_2'. The ratio depends on the motor design class. For both class A (normal starting torque and normal starting current) and class D (high starting torque and high running slip) squirrel-cage motors X_1 and X_2' are equal. X_1 and X_2' for wound- rotor induction motor are also equal.

17.3 No-Load Test

The machine is operated at rated frequency with no shaft load. The no-load current, the line voltage and the total 3-phase active power are measured. At no load the slip is very small hence the resistance $R_{mech} = \left(\frac{1}{s} - 1\right) R_2'$ representing mechanical power is very large compared to X_2' and R_2'. Thus at no load the equivalent circuit becomes simplified as shown in Fig. 11.

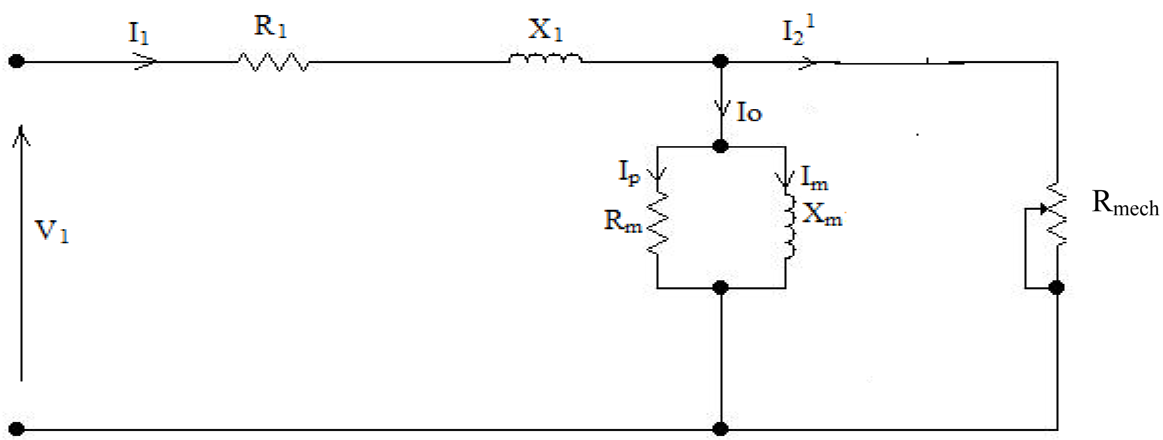


Fig. 11 No load equivalent circuit

The input power at no load represents stator iron loss (P_{fs}), friction and windage or mechanical loss (P_v) and a small stator copper loss $(P_{js}$):

$P_i = P_{fs} + P_v + P_{js}$

The no load current is large as compared to transformer on no load because of the air gap between the stator and rotor. It lies between 30% and 50% of the rated value. Hence the stator copper loss cannot be neglected as in the case of the transformer on no load. The mechanical power (P_v) + stator iron loss (P_{fs}) called rotational losses at a given applied voltage is given by

$P_{fs} + P_v == P_i - 3I_i^2 R_1$

Where R_1 = stator resistance per phase assuming a star connection

The iron loss varies approximately as $(\textit{flux density})^2$ and hence as the $(\textit{applied voltage})^2$. Also the no load speed is practically independent of voltage. Therefore, the mechanical loss can be separated from the stator iron loss, if we plot the power $(P_{fs} + P_v)$ against $(\textit{applied voltage})^2$. The relationship is practically linear over a wide range and extrapolation to zero voltage gives the mechanical loss. The voltage is reduced from 10 % overvoltage to about 20 % of the rated voltage.

The no load power factor is quite low at normal applied voltage. Hence a close estimate of the induced voltage across the magnetizing branch E at normal voltage can be obtained as the algebraic difference between the normal input voltage and the product of the no load current and the stator leakage reactance. The low no-load power factor also means that the magnetizing current I_m is very nearly equal to the reactive component of the no load current, this even being very little different in practice from the no load current itself.

Let us use the subscript '*nl*' to denote condition at normal voltage. Then

$E_{nl}(phase) = V_{nl}(phase) - |I_{nl} X_1|$

$I_p = P_{fs,nl}(phase) / E_{nl}(phase)$

$R_m = E_{nl}(phase) / I_p$

$I_m = I_{nl} \sin\phi_{nl}$

$X_m = E_{nl}(phase) / I_m$

Example 5

The following test data were obtained on a 1330-hp, 6600-V, 50 Hz, 8-pole, 3-phase star-connected induction motor:

	Line Voltage (V)	Line Current (A)	Total Power (kW)
Locked rotor	1400	80	50
No load	6600	40	45

The stator resistance is $1.30\,\Omega$/phase. The mechanical loss is 15 kW. Determine the parameters of the complete equivalent circuit. Assume that X_1 and X_2' are equal.

Solution

$$R_{e1} = \frac{P_{sc}}{3I_{sc}^2} = \frac{50\times10^3}{3\times80^2} = 2.60\,\Omega$$

$$Z_{e1} = \frac{V_{sc}}{I_{sc}} = \frac{1400}{\sqrt{3}} \times \frac{1}{80} = 10.10\,\Omega$$

$$X_{e1} = \sqrt{\left[Z_{e1}^2 - (R_1 + R_2')^2\right]} = \sqrt{10.10^2 - 2.6^2} = 9.76\,\Omega$$

Therefore

$$R_1 = R_{e1} - R_1 = 2.60 - 1.30 = 1.30\,\Omega$$

$$X_1 = X_2' = \frac{X_{e1}}{2} = \frac{9.76}{2} = 4.88\,\Omega$$

The no load stator copper loss $= 3I_{nl}^2 R_1 = 3\times40^2\times1.30 = 6.24\,\text{kW}$

The stator iron loss at full voltage $P_{fs,nl} = P_{nl} - P_v - P_{js,nl} = 45 - 15 - 6.24 = 23.76\,\text{kW}$

The stator emf at full voltage on no load $E_{nl} = V_{nl} - \left| I_{nl} X_1 \right| = \dfrac{6600}{\sqrt{3}} - 40 \times 4.88 = 3615\,\text{V}$

$$I_p = \frac{P_{fs,nl}}{E_{nl}} = \frac{23.76\,kW}{3} \times \frac{1}{3.615\,kV} = 2.19\,\text{A} \text{ and } R_m = \frac{E_{nl}}{I_p} = \frac{3615}{2.19} = 1651\,\Omega$$

No load power factor $\cos\phi_{nl} = \dfrac{P_{nl}}{\sqrt{3} \times V_{nl}(line) \times I_{nl}} = \dfrac{45{,}000}{\sqrt{3} \times 6600 \times 40} = 0.0984$

$$\sin\phi_{nl} = \sqrt{1 - \cos^2\phi_{nl}} = \sqrt{1 - 0.0984^2} = 0.995$$

$$I_m = I_{nl} \sin\phi_{nl} = 40 \times 0.995 = 39.8\,\text{A} \text{ and } X_m = \frac{E_{nl}(phase)}{I_m} = \frac{3615}{39..8} = 91\,\Omega$$

18 Efficiency calculation

The above tests give the parameters of the motors which may be used in either the exact or approximate circuit. The full load slip s_{fl} if not given is obtained from the equation:

$$\left(\frac{1 - s_{fl}}{s_{fl}} \right) \times total\ rotor\ copper\ loss = rated\ power + mechanical\ loss$$

The rotor copper loss is expressed in terms of the applied voltage and the circuit parameters.
The resulting equation is quadratic in s_{fl}. The smallest value gives operating point on the stable portion of the motor main characteristic. When s_{fl} is known, I_2' and I_1 can be calculated from the equivalent circuit, exact or approximate, and then the total losses obtained as follows:

$$Total\ losses = mechanical\ loss + iron\ loss + I_2'^2 R_2 + I_1^2 R_1$$

The full load efficiency is given by $\eta = \dfrac{output}{input} = 1 - \dfrac{total\ losses}{rated\ power + total\ losses}$

19 Current locus diagram for circuit with variable resistance

The voltage equation of the circuit in Fig. 12 is given $\mathbf{V} = (R + jX)\mathbf{I}$ from which we can derive

$$-j\frac{\mathbf{V}}{X} = \mathbf{I} - j\frac{R}{X}\mathbf{I}$$

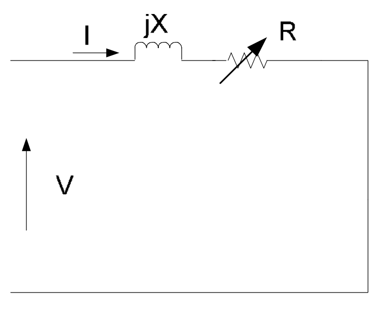


Fig.12 Circuit with varying resistance

The phasor diagram representing the equation is shown in Fig.13. From the geometry of the phasor diagram, where a right-angled triangle is formed over a constant diameter as *R* varies, it can be seen that the current phasor **I** traces out a circle. The current phasor traces a semicircle as *R* varies from zero to ∞.

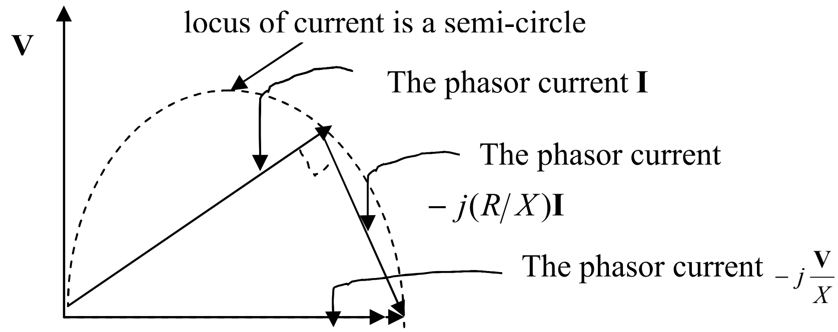


Fig.13 Locus of current phasor with varying resistance

20 Circle diagram

The circle diagram shown in Fig. 14 is used to quickly estimate the machine performance. It is based on the approximate equivalent circuit shown in Fig. 6.b.

(a) Geometrically, the semicircle can be defined by two phasors, if the position and direction of a diameter are known. Thus if I_o is given, it establishes the horizontal diameter and only one other current phasor will be required. Usually these are I_o approximated by I_{nl} obtained from the no load test and the short-circuit current at full-voltage $I_{sc}(s=1)$ obtained from the locked rotor test.

(b) At the point F, $s=-R_2'/R_1$ and $AF=V_1/(X_1+X_2')$

(c) The region between A $(s=0)$ and B $(s=1)$ is the current locus of I_1 and I_2' when the machine is motoring

(d) If the diagram is drawn in terms of line currents, whether the primary is star or delta connected, the vertical or in-phase components represent power to scale. For example, $\sqrt{3}V_1 AH$ is the stator iron loss

(e) All phasors on the diagram are current phasors, V_1 being drawn in for reference purposes

(f) It is possible to convert in-phase currents to power values.

(g) $BC/CD = R_2'/R_1$

(h) At any point such as G, input power$=\sqrt{3}V_1 I_1 \cos\phi_m = \sqrt{3}V_1 Ge$ to scale

$$=\sqrt{3}V_1(Gb+bc+cd+de)$$

where \
Gb = mechanical output power (including mechanical losses)
bc = rotor copper loss
cd = stator copper losses
de = stator iron loss

(i) Efficiency = $(\sqrt{3}V_1 Gb - mechanical\ loss\ if\ known)/\sqrt{3}V_1 Ge$

(j) Slip s = rotor copper loss/power across the gap (P_r) = bc/Gc bc is not measured directly from the diagram but is obtained by the formula $bc = \frac{Ac}{AC} \times BC$

(k) Torque $T_e = P_r$ synchronous watts $= \sqrt{3}V_1 Gc$ to scale or $\sqrt{3}V_1 Gc/2\pi n_s$ N-m

(l) Starting torque = $\sqrt{3}V_1 BC$ to scale

(m) The current phasor corresponding to maximum power factor is tangential to the circle

(n) Let the current scale be $x\,mm : b\,amps$. Then the power scale is $x\,mm : \sqrt{3}V_1 b$

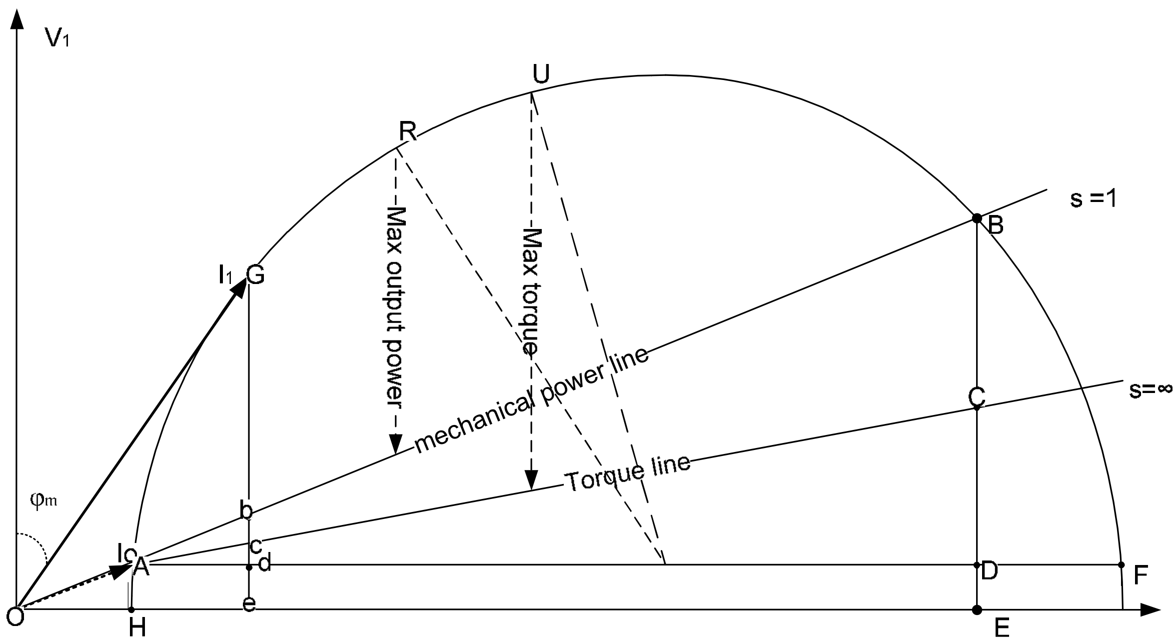


Fig.14 Circle diagram for approximate circuit

Example 6

On no load at 500 V, a certain 3-phase, 50-Hz, 4-pole induction motor takes 16 A at a p.f. of 0.2. With the rotor locked and 125 V applied, it takes 37.5 A at 0.3 p.f.. If the motor is running at full load and supplied at 500 V, the power factor is at its maximum. Estimate (a) the full load current (b) power factor (c) torque (d) speed (e) output power (f) efficiency (g) starting torque and (h) maximum torque. Assume $R_2' = 1.3R_1$

Solution

(i) $I_o = 16A\angle - \cos^{-1} 0.2 = 16A\angle - 78.5^0$

(ii) $I_{sc} = \frac{500}{125} \times 37.5\angle - \cos^{-1}(0.3) = 150A\angle - 72.5^0$

(iii) Choose a scale and use the values of I_o and I_{sc} to construct a circle diagram

(iv) Draw the torque line using the ratio $BC/CD = R_2'/R_1 = 1.3$

(v) The machine is said to operate at the point of maximum power factor. Locate the operating point G by drawing a current phasor from the origin tangential to the circle

(a) The full load current is OG to scale = 50 A

(b) The power factor $= \cos\varphi_m = 0.84$

(c) (i) Measure Gc and multiply by scale to obtain the current value

(ii) Multiply the current value by $\sqrt{3}V_1 = \sqrt{3} \times 500$ to obtain the torque in synchronous watts

(iii) Divide the torque in synchronous watts by the synchronous speed in rad/sec to obtain the torque in N-m

$$\omega_s = 2\pi n_s = {2\pi f}/{p} = {2\pi \times 50}/{2} = 50\pi \text{ rad/sec} \quad Ans = 204.8\,\text{Nm}$$

(d) (i) Obtain bc by using the relation $bc = BC \times {Ac}/{AC}$ (do not measure bc directly)

(ii) Measure also Gc and use the relation $s = {bc}/{Gc}$ to obtain the slip

(iii) The speed $N_r = (1-s)N_s = (1-s) \times \dfrac{60f}{p} = 1415$ rev/min

(e) (i) Measure Gb and multiply by the scale to obtain the current value

(ii) Multiply the result by $\sqrt{3}V_1$ to obtain the output $= 30.2$ kW

(f) Since the mechanical loss is not given $\eta = \dfrac{\sqrt{3}V_1 Gb - mechanical\,loss}{\sqrt{3}V_1 Ge}$ becomes

$$\eta = {Gb}/{Ge} = 83.3\%$$

(g) Measure *BC* and use the procedure described in (c) to obtain $T_s = 131.6$ Nm

(h) Measure *UV* and use the procedure described in (c) to obtain $T_{max} = 332.3$ Nm

21 Starting methods for squirrel cage induction motors

Three-phase squirrel-cage motors are started either by connecting them directly across the line (direct-on-line starting) or by applying reduced voltage (reduced voltage starting) to the stator. The starting method depends upon the capacity of the supply line and the type of mechanical load being driven.

21.1 Direct-on-line starting

The impedance of a polyphase induction motor at standstill (i.e. $s = 1$) is relatively small and when such a machine is switched on directly to the power system, the initial current will be high and at a low-power factor. This high low-power-factor starting current, which is about 5 to 6 times the full load current, can produce a significant voltage drop, which may affect other consumers connected to the same line. Voltage sensitive devices such as incandescent lamps, television sets and high precision machine tools respond badly to such voltage dips. The Supply Authorities protect the interest of neighbouring consumers by limiting the size of cage-motors that can be switched direct on line. The rating of the machine depends on the capacity of the supply. In some cases the limit may be 5 kW, in others over 75 kW on a 415-V supply. It is not uncommon to find cage motors up to 7500 kW switched direct on line in large industrial installations where the capacity of the available supply system is large and 5 kW not switched direct on line. Consultations with the supply authorities for motor applications of high ratings are therefore advisable. This method is simple and inexpensive. It is preferred if the supply system can withstand the high low-power factor currents and the driven equipment can permit it.

Typical circuit diagram for an automatic direct-on-line starter is shown in Fig. 15. Short circuit protection and overload protection is by means of a motor-protective circuit-breaker Q1.
Pressing the pushbutton I, energizes the coil of contactor Q11. The contactor switches on the motor and maintains itself after the button is disenabled via its own auxiliary contact Q11/14-13. In the event of an overload, the motor is de-energized by Q1 which opens both the power and control circuit. The control circuit is opened via Q1/13-14.

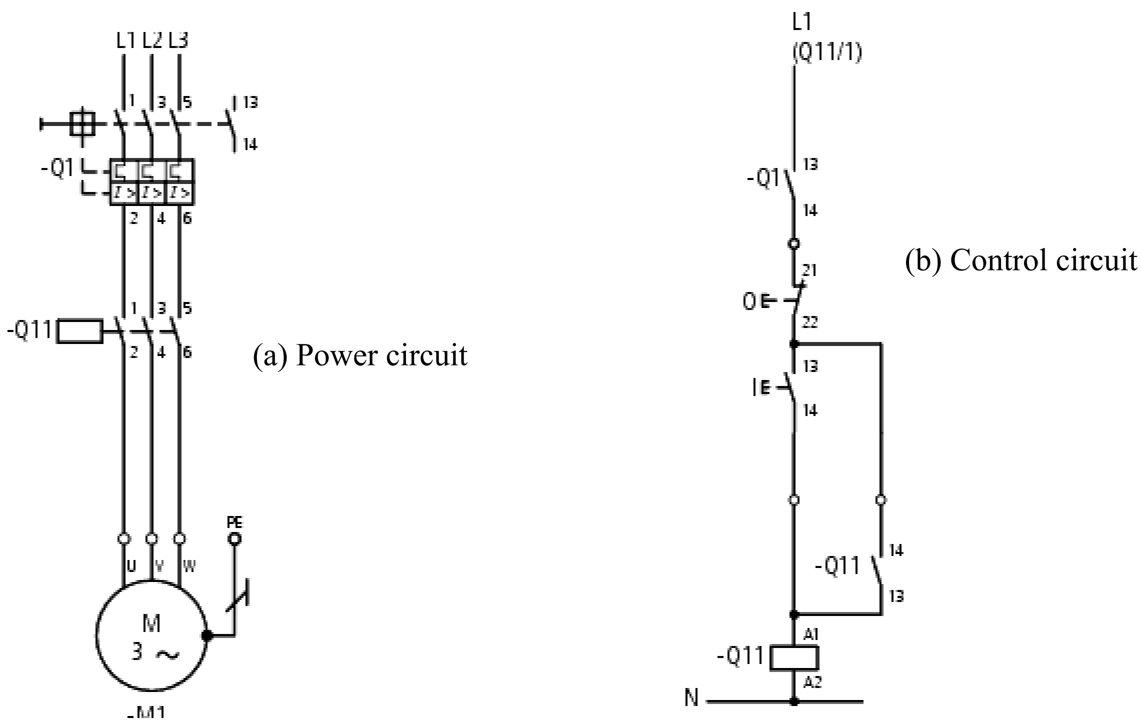


Fig. 15 Typical circuit diagram for automatic direct-on-line starter

21.2 Reduced voltage starting

Some industrial loads have to be started very gradually. Examples are coil winders, printing press and other machines that process fragile products. In other industrial applications a motor cannot be switched direct-on-line because the starting current is too high for the available supply system. In all these cases reduced voltage has to be applied to the motor. In reducing the voltage to a fraction *x times* the normal (i.e. $V_{applied} = xV_{normal}$), the starting current I_s reduces to xI_{sc} (i.e. $I_s = xI_{sc}$) and the starting torque T_s to x^2 *times* the normal (i.e. $T_s = x^2 T_{sc}$). It is necessary while keeping down the starting current to maintain adequate starting torque.

The methods available for reduced-voltage starting are:

21.2.1 Stator resistance starting

This method consists of placing three resistors in series with the stator during the start-up period. The resistors are short circuited after a definite time when the motor runs close to the synchronous speed (usually about 80 % of its normal speed). The voltage drop across the resistance is high at first but gradually diminishes as the current falls. Consequently, the voltage across the motor increases with speed, and electrical and mechanical shock is negligible when full voltage is finally applied. The resistor is short-circuited in one or several steps. The motor is not to be started frequently otherwise the resistors can burn out. The stator resistance starting method may be used for the smooth starting of small machines such as those driving centrifugal oil purifiers.

Inductors may occasionally be used. With this, the starting power-factor becomes still lower though there is less power wastage than when using resistors.

Typical wiring diagram for automatic line resistance starter is shown in Fig. 16. When the start pushbutton is pressed, the start contactor coil S picks up and closes contacts S in the motor power circuit, applying power through the starting resistances to the motor and at the same time sealing the contactor coil circuit via its NO auxiliary contact S_a. The timer is also energized at the same time via NO contact S_a and NO contact S_b. After a preset time the timer closes and through TR_{TC} energizes the RUN contactor, which in turn closes contacts RUN in the power circuit to short-circuit the starting resistances. The RUN contactor when it picks up, disconnects the timing relay and the S contactor via its NC auxiliary contact RUN_b and seal its coil circuit through its NO auxiliary contact RUN_a.

Example 7
A 150-kW, 460-V, 3-phase, 3520-rev/min, 60-Hz induction motor has a locked-rotor torque (T_{sc}) of 600 N-m and a locked rotor current (I_{sc}) of 1400 A. Three resistors are connected in series with the line so as to reduce the voltage across the motor to 0.65 pu. Calculate
(a) The apparent power absorbed by the motor under full-voltage locked-rotor-conditions.
(b) The apparent power absorbed by the motor when the resistors are in circuit.
(c) The apparent power drawn from the line, with the resistors in the circuit.
(d) The locked-rotor torque developed by the motor.
(e) If the locked-rotor power factor of the motor alone is 0.35, calculate the required value and power of the series resistors.

Solution
(a) $S = \sqrt{3}V_L I_L = \sqrt{3} \times 460 \times 1400 = 1114 \text{ kVA}$
(b) The voltage across the motor $V_L = 0.65 \times 460 = 299\,\text{V}$
The current drawn by the motor $I_s = xI_{sc} = 0.65 \times 1400 = 910\,\text{A}$
The apparent power drawn by the motor $S_m = \sqrt{3}V_L I_L$
$= \sqrt{3} \times 299 \times 910 = 471\,\text{kVA}$

(c) The apparent power drawn from the line $S_L = \sqrt{3} \times 460 \times 910 = 724\,\text{kVA}$

(d) The torque $T_s = x^2 T_{sc} = 0.65^2 \times 600 = 252$ N-m

(e) The active power drawn by the motor is $P_m = S_m \cos\phi = 471 \times 0.35 = 165\,\text{kW}$

The reactive power absorbed by the motor $Q_m = \sqrt{S_m^2 - P_m^2} = \sqrt{471^2 - 165^2} = 441$ kVAr

The resistors can only absorb active power in the circuit. Consequently the reactive power supplied by the line must be equal to that absorb by the motor $Q_L = 441$ kVAr

The active power supplied by the line $P_L = \sqrt{S_L^2 - Q_L^2}$

$$= \sqrt{724^2 - 441^2} = 574 \text{ kW}$$

The active power absorbed by the three resistors $P_R = P_L - P_m = 409$ kW

The active power/resistor $P = P_R/3 = 409/3 = 136$ kW

The current in each resistor $I_s = 910$ A

The value of each resistor $R = P/I^2 = 136 \times 10^3/910 = 0.164\ \Omega$

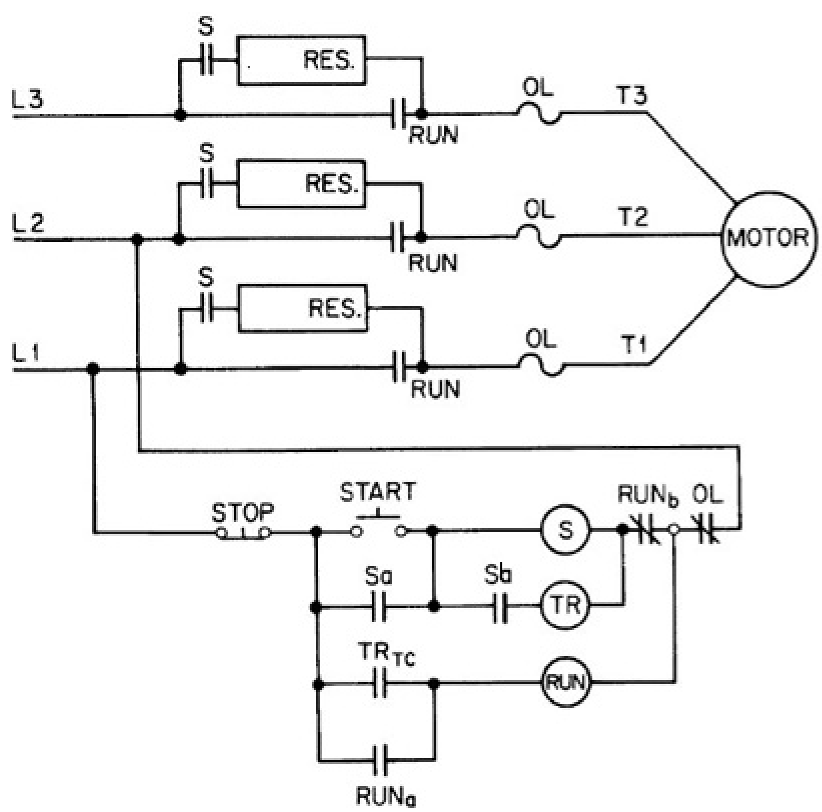


Fig. 16 Typical automatic line resistance starter wiring diagram

21.2.2 Autotransformer starting

Autotransformers are used to reduce the stator applied voltage. The setting of the autotransformers can be predetermined to limit the starting current to any desired value (Commercial autotransformers usually have taps to give output voltage of 0.8, 0.65, 0.5 p.u.). When the motor attains about 80% of its normal speed, full line voltage is applied and the autotransformer switched out of circuit. An autotransformer which reduces the voltage applied to motor to the fraction x will have $T_S = x^2 T_{sc}, I_s = xI_{sc}$ and the starting current on the line side $I_{sL} = x^2 I_{sc}$ (Note that I_{sc} is the starting current of the motor on the line side without the transformer).

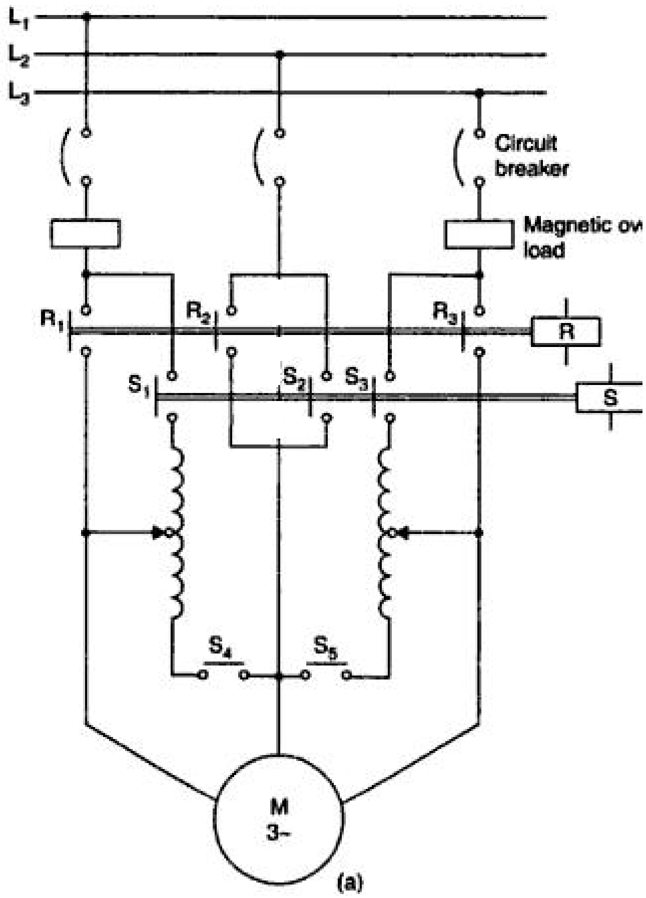
For a given torque, autotransformer starting draws a lower line current and active power than resistance starting does. The disadvantage is that the transition from reduced-voltage to full-voltage is not quite smooth. During the change-over from the starting to the normal running condition, the motor is momentarily disconnected from the supply line. When the motor is reconnected a large transient current of extremely short duration is drawn. This transient surge is hard on the contacts of contactors used and also produces mechanical shock. Another disadvantage is that autotransformers cost more.

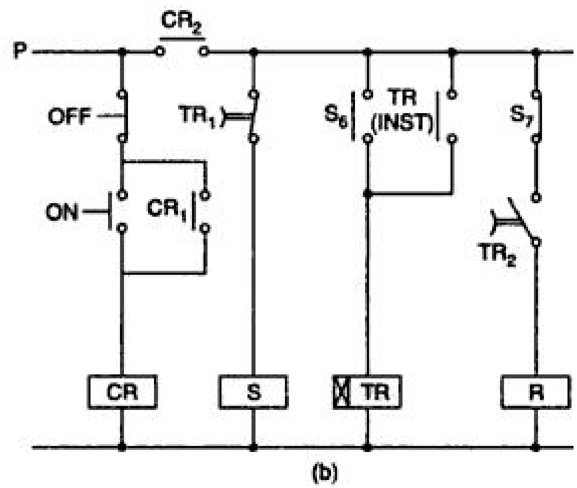
NB: Because the autotransformers and resistors operate for very short periods, they can be wound with much smaller wire than continuously rated devices. This enables us to drastically reduce the size, weight and cost of these components.

Usually two single-phase autotransformers connected in open delta are used. A three-phase autotransformer is considered when the motor must develop maximum starting torque. Typical circuit diagram for the starter using two single-phase transformers is shown in Fig. 17. The starter is open-circuit transition type. It is so called because during the transition from start to run the motor is isolated from the power source.

Pressing the ON pushbutton energizes the control relay CR which in turn gets contactor S energized through NO auxiliary contact CR_2 and NC delayed contact TR_1. When the start contactor is energized, it connects the transformer windings to the neutral point via S4 and S5 and supplies reduced voltage from the autotransformer via S1, S2 and S3. It also energizes the timer through NO contact S_6 and ensures that the R contactor coil circuit remains open while it ON via the NC contact S_7. The timer seals its circuit through its own instantaneous contact TR (INST). After the pre-set time, the timer NC delayed contact TR_1 opens to de-energize the start contactor S and NO delayed contact TR_2 closes to complete the run contactor R coil circuit. However, R coil is not energized until the start contactor drops out and its NC contact S_7 in the coil circuit is closed. The motor is connected directly to the supply when the run contactor R picks up.

The control circuit is designed to ensure that contactor R and contactor S are not ON at the same time because if both are ON at the same time, circulating current will flow in the autotransformer windings between the transformer taps and the terminals connected to the supply and cause overheating of the transformer.





Control circuit

Power circuit

Fig. 17 Typical circuit diagram for autotransformer starter

21.2.3 Star-delta starting

The machine, designed for delta operation, has all the six stator leads brought out to the terminal box. The windings are connected in star during the starting period and in delta during normal running conditions. During starting, $V_{applied/phase} = V_{normal}/\sqrt{3}$. Therefore at starting $I_s/phase = \frac{1}{\sqrt{3}} I_{sc}/phase(delta\,connection)$. This means that the starting line current when star connected is one-third the value for delta connection. The motor behaves as if the autotransformer was employed with $x = 1/\sqrt{3}$. The starting torque $T_s = T_{sc}/3$ where T_{sc} is the starting torque for delta connection.

This method suffers from the same disadvantage as the autotransformer starting in that all three lines must be opened simultaneously during the change-over from the starting to normal running conditions. The method is effective if the starting torque is adequate and cheap so long as the supply voltage does not exceed about 3 kV (At voltages exceeding 3 kV, excessive number of stator turns will be needed for delta running). With this method repeated starting can be obtained.

Fig. 18 shows a typical circuit diagram for automatic star-delta starter (open-circuit transition type). Timer setting is around 10 s. For lightly loaded motor, the setting is slightly less and for heavily loaded motor, it is slightly more.

When the ON pushbutton is pressed, the star contactor S picks up and closes the circuit of the main contactor M coil through its NO auxiliary contact S_1. At the same time, normally closed contact S_2 interrupts the circuit of the delta contactor. The main contactor M gets hold through it NO auxiliary contact M_1 whereas the star contactor S gets hold through both S_2 and M_1. When the main and the star contactors are energized, the motor starts to run star-connected. This continues until the timer times out and opens its NC delayed contact T_1 to de-energize the star contactor S. When the star contactor S drops out, its contact S_1 opens and its contact S_2 closes. The closing of S_2 energizes the delta contactor. When the delta contactor picks up, it opens its NC auxiliary contact D_1 to interrupt the coil circuit of the star contactor S, thus interlocking against the motor being switched on again while running in delta connection. While the motor is running, the main and the delta contactors are energized while the star contactor and the timer are de-energized.

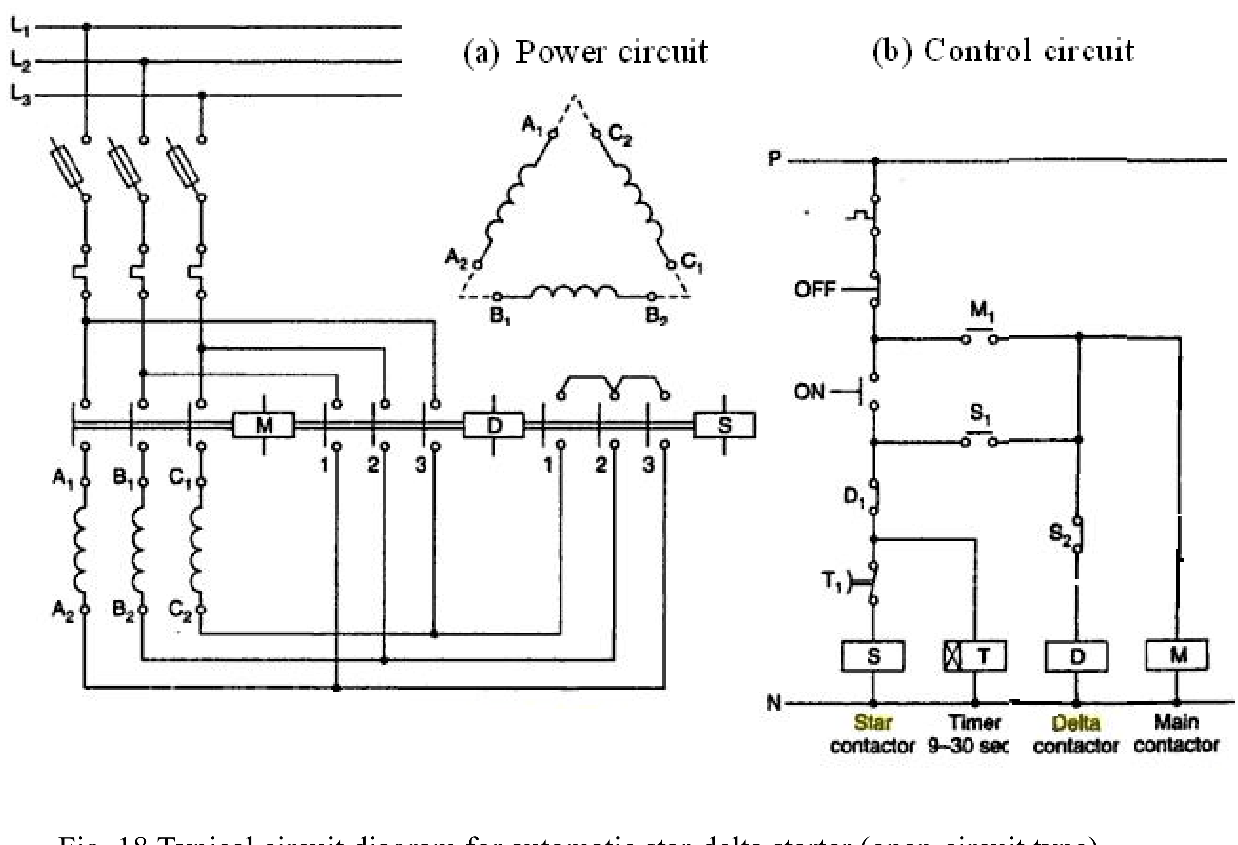


Fig. 18 Typical circuit diagram for automatic star-delta starter (open-circuit type)

Example 8

Calculate the relative values of (a) the starting torque and (b) the starting current of a 3-phase cage-rotor induction motor when started by (a) direct switching (b) a star-delta starter and (c) an autotransformer having 40 % tapping.

Solution

(a) Starting torque: $1:\frac{1}{3}:0.4^2 = 1:0.33:0.16$

(b) Starting current $1:\frac{1}{3}:0.4^2 = 1:0.33:0.16$

21.3 The ratio between starting torque and full-load torque

The ratio between starting torque T_s and full-load torque T_{fl} is usefully expressed in terms of the stator currents. Since the referred rotor current is approximately equal to the input current, and at starting $s = 1$

$$\frac{T_{sc}}{T_{fl}} = \frac{I_{sc}^2 R_2'/1}{I_{fl}^2 R_2'/s_{fl}} \quad \text{from which} \quad T_{sc} = \left(\frac{I_{sc}}{I_{fl}}\right)^2 \times s_{fl} \times T_{fl}$$

In general $T_{sc} = x^2\left(\frac{I_{sc}}{I_{fl}}\right)^2 \times s_{fl} \times T_{fl}$ (11)

Example 9

A 11-kW, 3-phase, 6-pole, 50-Hz, 400-V induction motor runs at 960 rev/min on full load. If it takes 80 A on direct-on-line switching, find the ratio of T_s and T_{fl} in the following cases:

(a) Direct-on-line starting
(b) Star-delta starting
(c) Autotransformer starting with 60 % tapping
(d) Stator resistance starter limiting the starting current to 50 A

Take full-load power factor and efficiency to be 0.834 and 95.6 % respectively

Solution

Full load current $= \frac{output\ power}{\sqrt{3} Applied\ voltage \times efficiency \times pf} = \frac{11000}{\sqrt{3} \times 400 \times 0.956 \times 0.834} = 20\text{A}$

$$N_s = \frac{60f}{p} = \frac{60 \times 50}{3} = 1000 \text{ rev/min}$$

Full-load slip $s_{fl} = \frac{(1000-960)}{1000} = 0.04$

(a) $\frac{T_s}{T_{fl}} = x^2\left(\frac{I_{sc}}{I_{fl}}\right)^2 s_{fl}. = 1 \times \left(\frac{80}{20}\right)^2 \times 0.04 = 0.64$

(b) $\frac{T_s}{T_{fl}} = \left(\frac{1}{\sqrt{3}}\right)^2 \times \left(\frac{80}{20}\right)^2 \times 0.04 = 0.21$

(c) $\frac{T_s}{T_{fl}} = (0.6)^2 \times \left(\frac{80}{20}\right)^2 \times 0.04 = 0.23$

(e) In the stator resistance starter, the current reduces in proportion to the voltage,

$$x = \left(\frac{50}{80}\right) = \frac{5}{8} \text{ . Thus } \frac{T_s}{T_{fl}} = \left(\frac{5}{8}\right)^2 \times \left(\frac{80}{20}\right)^2 \times 0.04 = 0.25$$

22 Starting method for slip-ring induction motor

The starting current is reduced by introducing additional rotor resistance during starting. This can also be made to increase the starting torque available since the slip at which maximum torque occurs, increases as the rotor resistance increases. The external rotor resistance is generally reduced in steps as the machine runs up to speed. The slip-ring motors are suitable for heavy, frequent starting and accelerating duty-cycles.

23 High-torque cage motors

The plain squirrel cage induction motor is simple, robust, low-cost and requires little maintenance. However, it gives a low starting torque with a very large starting current. The most usual type of standard low-resistance cage motor gives 35-45 % of full-load torque with 2.5-3.5 times full-load current, on a start at 60 % of normal voltage

Squirrel cage motors with sophisticated cage arrangements have been designed to obtain high torque cage motors for onerous starting conditions without sacrificing the simplicity, efficiency and cheapness of the cage motor. The cage arrangements in use can be classified in two categories as follows:

23.1 Deep-bar cage

The slot bars are designed with special shapes so that skin effect is pronounced. Some typical bar shapes are shown in Fig 19. One of them shows one slot of a deep-bar rotor with a few typical lines of flux produced by a current in the bar. The upper part of the conductor (nearer to the rotor surface) has fewer flux linkages and consequently lower reactance than the lower part. A time-varying current is thus crowded toward the surface. This phenomenon is known as the skin effect. Any departure from uniform current distribution increases I^2R loss and hence the resistance of the conductor over its dc values. In sinusoidal steady state, skin effect becomes more pronounced with larger current magnitudes, higher frequencies and increased conductor size.

At starting with rotor frequency equal to supply frequency, the effective cage resistance of deep-bar rotor is very high and this improves the rotor power factor, current and torque. In the normal running conditions when the rotor frequency is only a small fraction of the supply frequency the ac resistance is little different from the dc value thus giving low loss and high efficiency.

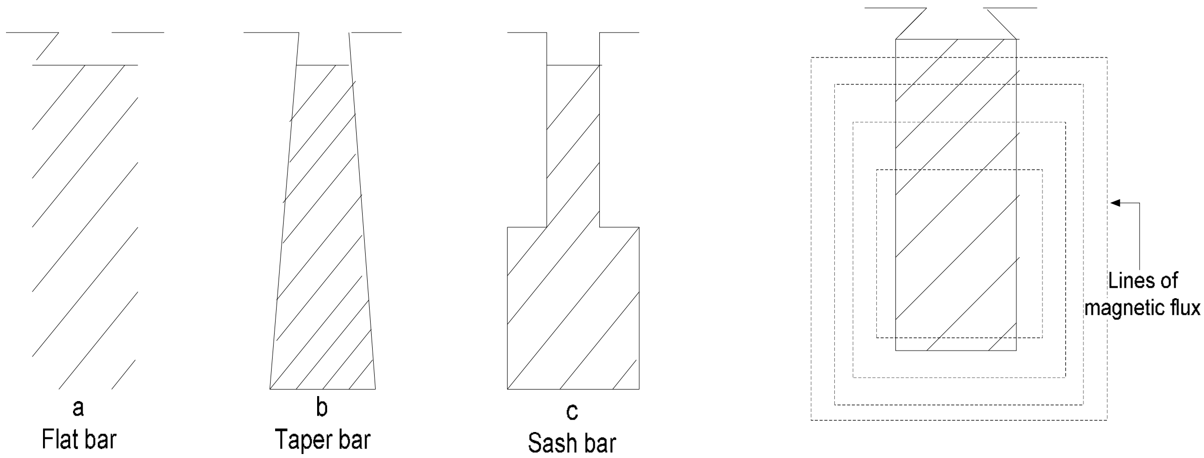
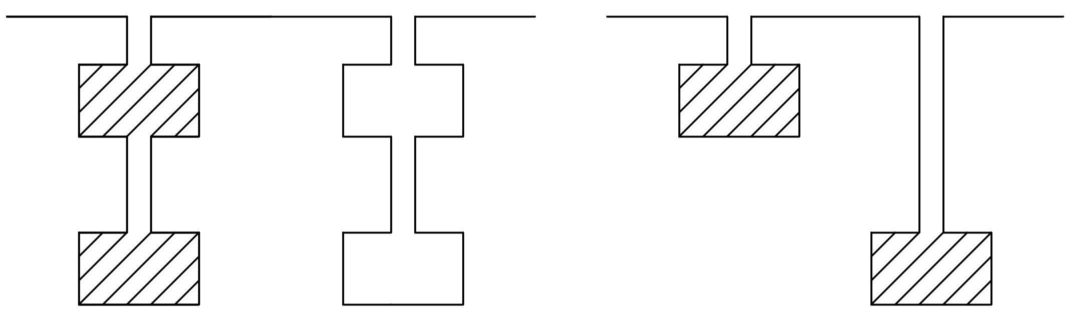


Fig. 19 Deep-bar profiles

23.2 Double-cage rotors

The rotor has two concentric and separate cage windings, the outer and inner, arranged in a slot arrangement that may take a variety of forms, Fig 20. The outer cage which has a low leakage reactance is made with high resistance while the inner cage made with relatively low resistance has a high leakage reactance because it is sunk deep into the iron. At starting, when the frequency of the rotor currents is the same as the supply frequency, the outer cage which has the lower impedance takes on most of the starting current. The outer cage, being of high resistance develops a high starting torque. When the motor is running it is the inner cage that carries most of the current, due to its lower resistance because at the small rotor frequency the leakage reactances do not matter. So the two cage arrangements yield machines which have a high starting torque and a high running efficiency with reasonably small values of starting current.



(a) Dumbell (b) Staggered

Fig. 20 Double cage machine

The torque-speed curve of the machine can be estimated by summing the torques considered to be produced separately by the cages as shown in Fig. 22. By varying the relative values of the resistance and reactance, a wide variety of torque-speed characteristics can be obtained.

The performance of the motor can be determined using the equivalent circuit shown in Fig 21. A simplified circuit is obtained from this by putting x_m and r_m across the stator terminals.

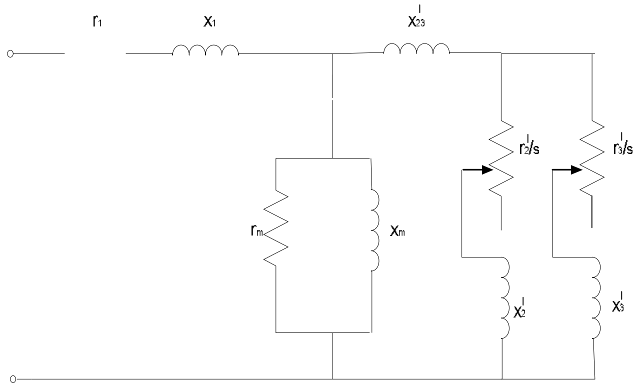


Fig. 22 Equivalent circuit of double cage

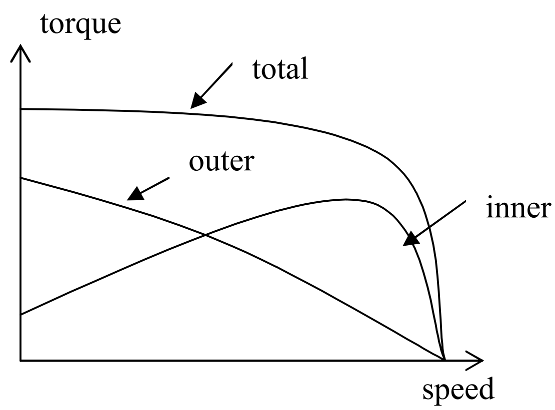


Fig. 22 Torque-speed curve of double-cage motor

Standard high torque motors are available up to about 55 kW for direct-on-line starting with the starting torque at least two times the full-load torque and the starting current about 350 % of the full load current. Standard motors are also available for star-delta starting with T_s = 100 % of full load torque at starting in the star connection, the starting current being 150 to 175 % full load current.

24 High resistance cage motors

In this design efficiency is sacrificed. The cage is made of brass. The starting torque is higher and the locked rotor or starting current lower. They are designed for intermittent operation to prevent overheating. They drive high inertia loads which take a relatively long time to reach full load speed e.g. centrifugal dryers. The large drop in speed with increasing load makes them ideal for driving high impact loads such as punch presses, forge hammers etc that are required with flywheel.

25 Induction generator

An induction machine can be made to generate by raising the rotor speed above the synchronous speed. Like any other generator, an induction generator must be excited. The excitation of an induction generator is by means of the magnetizing current in the stator.

25.1 Generator equivalent circuit

This is the same as that of an induction motor in either the exact or approximate versions. The slip is negative however, so that $-s$ must be substituted for s, if s itself is still to symbolize a positive number. In this case the power transferred to the rotor from the stator $P_r = -I_2'^2 R_2'/s$ and the output power $P_m = -(1+s)I_2'^2 R_2'/s$ (rotational losses must be provided). The negative signifies a reversed power flow.

25.2 Externally-excited induction generator

The excitation determines the flux and the frequency of the induction generator and consequently the synchronous speed above which the machine must run to generate. For proper functioning, the induction generator must therefore be connected to some existing source or supply to supply it with magnetizing current and to fix its frequency. The system also determines its voltage.

The induction generator is useful when the prime mover does not run at constant speed since the stator frequency depends on that of the existing power supply. The utilization of wind power is suitable application for this type of generator. Currently induction generators driven by variable-pitch waterwheel turbines are also used as unattended hydroelectric generating station plants up to 100 kW.

25.3 Self-excited generator

The magnetizing current may be provided by capacitor bank. With this method of excitation, the frequency and the generated voltage are affected by the speed, load and capacitor rating. If the load is inductive the capacitor will have to supply the reactive power required by the load.

25.3.1 Self-excited generator on no load:

On no load the slip is almost zero, and the equivalent circuit becomes as shown in Fig. 23. The frequency $f = pn$ where n is the speed of the generator.

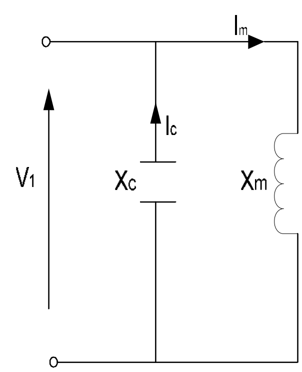


Fig. 23 Self-excited generator on no load

From the circuit, $I_m = I_c$, $V_1 = I_c X_c = I_m X_c$ and $V_1 = I_m X_m$

The value of capacitor required to generate a specified voltage on no load can be determined by solving the last two equations given above. X_m is a nonlinear function of the magnitude of I_m and a direct function of frequency. Therefore the relationship between V_1 and I_m is best expressed by the magnetization curve for various frequencies corresponding to various rotational speeds as shown in Fig. 24.a. The value of the capacitor required to generate a specified voltage at a specified frequency can then be obtained graphically by plotting the voltage V_1 against the magnitude of I_m and plotting also the curve $V_1 = I_m X_c$ and finding the point of intersection (See Fig. 24 c)

Alternatively, we can plot the voltage V_1 against $\dfrac{1}{X_m}$ as in Fig. 24.d and note that at the specified voltage

$$\frac{V_1}{X_c} = \frac{V_1}{X_m}\text{,i.e., } \frac{1}{X_c} = \frac{1}{X_m}$$

It is to be noted that self-excitation is possible

- only when there is sufficient residual magnetism in the rotor
- the capacitive susceptance is at least as great as the minimum unsaturated susceptance.

It follows from the second bullet that there is a critical value of capacitance below which the induction generator will not build up at a given speed. Also there is a critical speed for a given capacitance.

25.3.2 Self-excited generator on load

Fig. 25 shows the equivalent circuit of the generator on load. Under loaded conditions, the generated power $V_1 I_2' \cos\phi_2$ provides the active load power and the loss in R_m. Thus

$$V_1 I_2' \cos\phi_2 = \frac{V_1^2}{R_m} + \frac{V_1^2}{R} \quad or \; I_2 \cos\phi_2 = \frac{V_1}{R_m} + \frac{V_1}{R} \tag{12}$$

where ϕ_2 is the phase angle between V_1 and I_2'.

The capacitor provides all the reactive power absorbed in the system. Thus

$$\frac{V_1^2}{X_c} = \frac{V_1^2}{X_m} + \frac{V_1^2}{X} + V_1 I_2' \sin\phi_2 \quad or \; \frac{V_1}{X_c} = \frac{V_1}{X_m} + \frac{V_1}{X} + I_2' \sin\phi_2 \tag{13}$$

The last equation determines the capacitor for a given load and voltage.

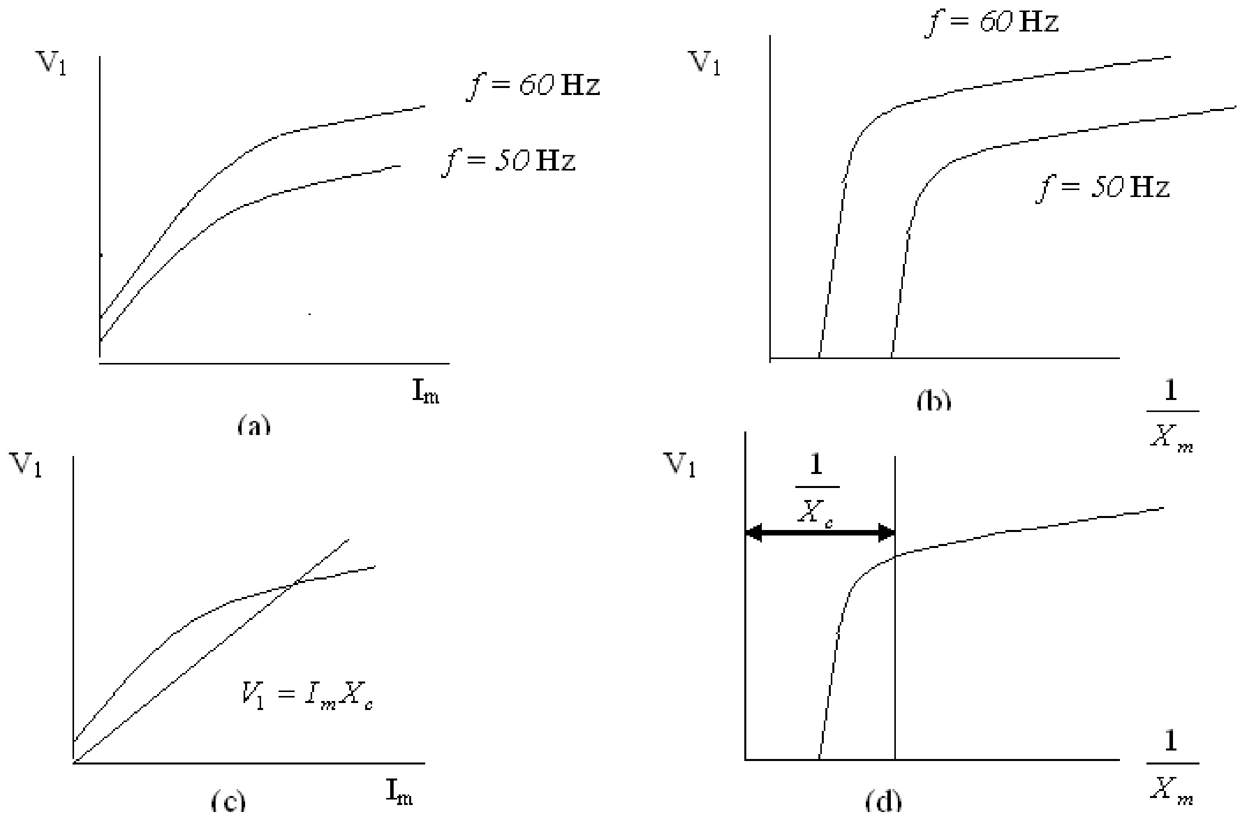


Fig. 24 induction generator characteristic on no load

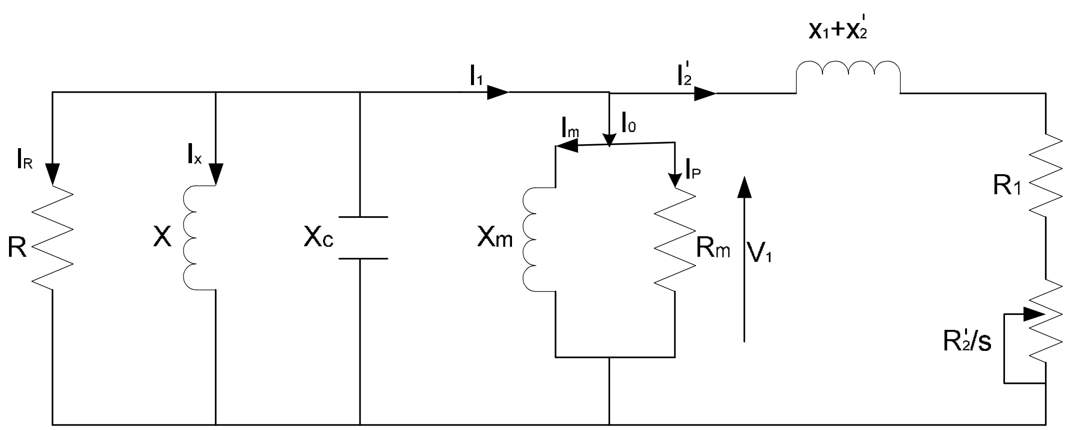


Fig. 25 Self-excited generator on load

Example 10

On rated supply, a 3.0-kV, 50-Hz star-connected induction machine has these phase current components: ideal short-circuit current, $V/(x_1 + x_2') = 750\,A$, core-loss current, $I_p = 10\,A$. The magnetization curve for 50 Hz is:

Line voltage (V)	1450	2500	3000	3400
Magnetizing current (A)	20	40	60	90

For a frequency of 50-Hz, estimate the star-connected capacitance required for the machine (i) just to self-excite (ii) to generate 3.3 kV line on no load (iii) to provide a load current (*125-j20*) A/ph at 3.0 kV line.

Solution

Phase voltage V_1 (V)	840	1440	1730	1960
Magnetizing current I_m (A)	20	40	60	90
Susceptance I_m/V_1 (mS)	24	28	35	46

(i) The minimum capacitance susceptance for self-excitation is 24 mS, so that $C = \dfrac{24\times10^3}{314} = 76\,\mu F$ per phase at 50 Hz

(ii) Using the method of Fig. 24.d $\dfrac{1}{x_c}$ is found to be 43 mS 3300 V line or 1900 V phase. Hence

$C = 137\,\mu F$

(iii) From the circle diagram for induction motor operating with negative slip shown below,

$I_2'\cos\phi = r\sin\alpha$

Therefore, from (12) $r\sin\alpha = I_P + I_R = 10 + 125 = 135\,A$

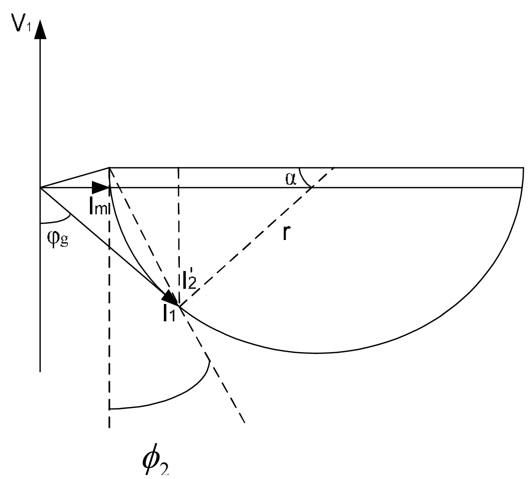
The radius of the circle, $r = \dfrac{V_1}{(x_1 + x_2')} = \dfrac{750}{2} = 375\,A$

Hence $\sin\alpha = \dfrac{I_p + I_R}{r} = \dfrac{135}{375} = 0.36,\ \alpha = 21.1^0$

From the circle diagram, $I_2'\sin\phi = r(1-\cos\alpha) = 375(1-\cos 21.1^0) = 25.14\,\text{A}$

From (13), the capacitor current, $I_c = I_x + I_m + I_2'\sin\phi = 20 + 60 + 25 = 105\,\text{A}$

Therefore the required capacitance, $C = \dfrac{I_c}{\omega V_1} = \dfrac{105\times10^6}{314\times1730} = 190\,\mu\text{F/ph}$



26 Power factor correction of induction motor

Large *kVA* necessitates the installation of larger power plant i.e. generators and associated apparatus (e.g. transformers) and copper of larger cross-sectional area for transmission of power The reactive current also gives rise to additional losses in the copper and voltage drops. For these reasons Utility companies design their rate structure (tariff) in such a way that billing will automatically increase whenever the power factor of a consumer is low.

The full-load power factor of an induction motor rarely exceeds 0.9 (it is worse when motor is under rated), and it may pay to raise the power factor. Capacitors may be employed for that. The capacitor may be connected directly across the stator terminals or indirectly through control equipment as shown in Fig 26. The direct connection which is common because it is practical and economical requires caution to avoid the possibility of momentary self excitation of the motor as an induction generator. If the capacitor rating is too high damage may result to both motor and capacitor, because the motor, while still revolving after disconnection from the supply may act as a generator by self-excitation to produce a voltage higher than the supply voltage. If the motor is switched on again before the speed has fallen to about 80% of the normal running speed, the high voltage will again be superimposed on the supply voltage and there may be a risk for damage for other types of equipment.

The capacitance may have to be limited to a value that makes self excitation impossible. As a general rule, the correct size of capacitor for individual correction of motor should have a kVa*r* rating not exceeding 90 % of the no-load magnetizing *kVA* of the machine. In practice, power factor correction should not be pushed beyond limits shown in Table 1.

Table 1 Reactive power of fixed capacitors for power factor correction of induction motors

Power of motor		3000 rpm	1500 rpm	1000 rpm	750 rpm	500 rpm
Hp	kW	kVAr	kVAr	kVAr	kVAr	kVAr
1	0.74	0.5	0.5	0.6	0.6	-
10	7.46	3	3	4	4	5
100	74.6	25	30	30	30	40
250	184	60	60	60	80	80

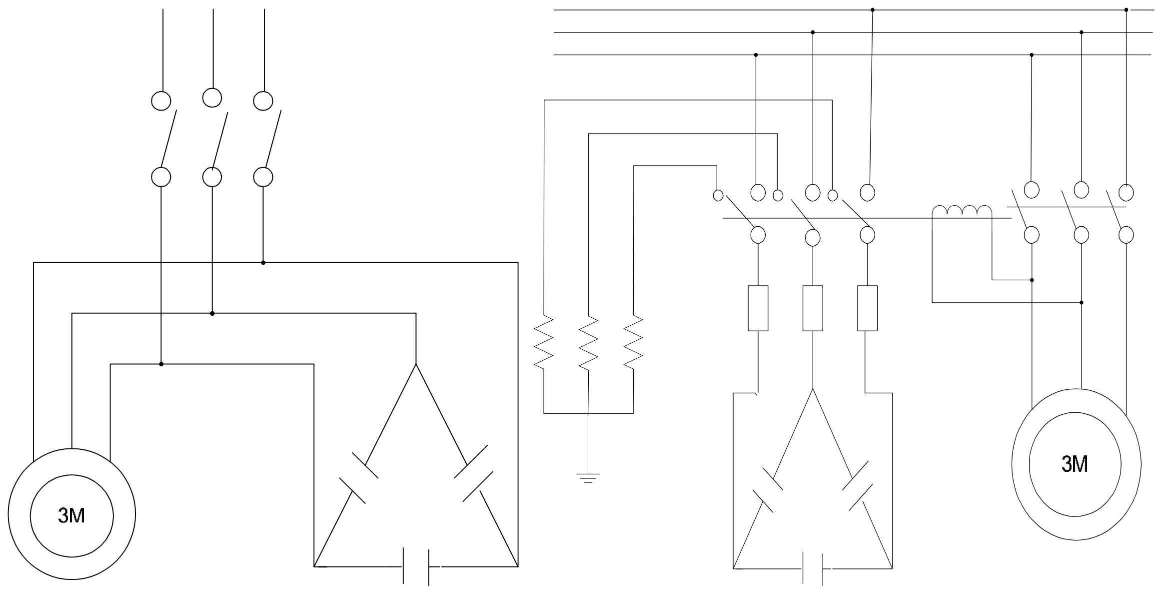
The kVAr required to improve the power factor of a motor from $\cos\phi_1$ to $\cos\phi_2$ can be obtained as follows:

Initial power factor is $\cos\phi_1 = \dfrac{kW}{kVA_1}$ which implies $\tan\phi_1 = \dfrac{kVar_1}{kW}$ $\quad or\, kVar_1 = kW \times \tan\phi_1$

Improved power factor is $\cos\phi_2 = \dfrac{kW}{kVA_2}$ which implies $\tan\phi_2 = \dfrac{kVar_2}{kW}$ $\quad or\, kVar_2 = kW \times \tan\phi_2$

Therefore the capacitor *kVar* required $= kVar_2 - kVar_1 = kW(\tan\phi_1 - \tan\phi_2)$

Where star-delta starting is used, a standard three-terminal delta-connected capacitor as shown in Fig. 27 should be employed which gives maximum power factor correction at the start when the power factor of the motor is low.



(a) Direct (b) Indirect

Fig. 26 Connection of capacitors across induction motor terminals

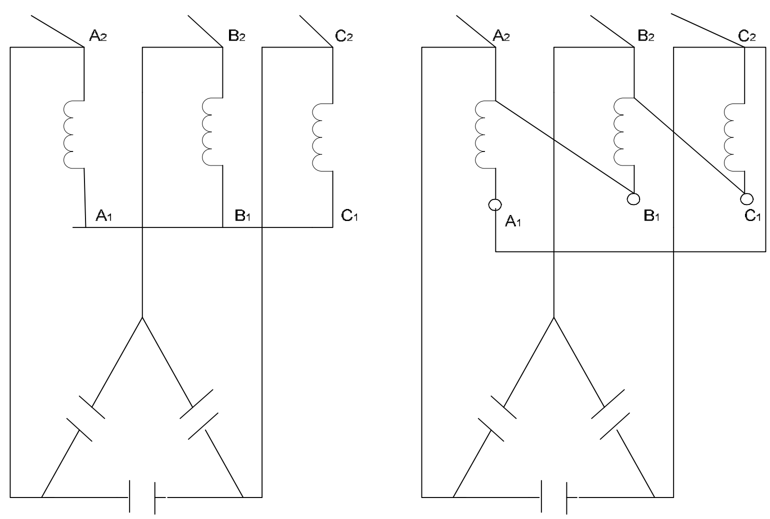


Fig. 27 Connection of capacitors across star-delta induction motor terminals

27 Further Exercises

1. Answer the following practical questions:
   (n) The rotor of an induction motor should never be locked while full voltage is being applied to the stator. Explain
   (o) Why does the rotor of an induction motor turn slower than the revolving field?
   (p) Give two advantages of a slip-ring motor over a squirrel-cage motor.
   (q) How can we change the direction of rotation of a 3-phase induction motor?
   (r) We can bring an induction motor to a quick stop either by plugging it or by exciting the stator from a d.c. source. Which method produces the least amount of heat in the motor? Explain.
2. (a) How can we change the direction of rotation of a 3-phase induction motor? [2 marks]
   (b) A 3-phase, 4-pole, 37-kW, 440-V, 50-Hz, star-connected induction motor has the following parameters per phase:
   $R_1 = 0.10\ \Omega$ $X_1 = 0.35\ \Omega$ $R'_2 = 0.12\ \Omega$ $X'_2 = 0.45\ \Omega$
   At no load the motor draws a line current of 18 A at a power factor of 0.089 lagging. When the motor operates at a slip of 2.5 %, determine
   (i) the input current [6 marks]
   (ii) the power factor [1 mark]
   (iii) the developed torque in N-m. [3 marks]
   (iv) the gross output power [2 marks]
   Consider the magnetizing branch to be connected across the machine terminals.
3. (a) A 3-phase squirrel cage induction motor is started at a reduced voltage. What are the most likely reasons for starting it this way? [2 marks]
   (b) A 3-phase, 2934-rev/min, 150-kW, 415-V, 50-Hz induction motor has a locked rotor torque of 600 N-m and a locked rotor current of 1200 A. Three resistors are connected in series with the line so as to reduce the voltage across the motor to 0.65 per unit. Calculate
   (i) the apparent power absorbed by the motor under full-voltage locked-rotor conditions, [1 mark]
   (ii) the apparent power absorbed by the motor when the resistors are in circuit, [2 marks]
   (iii) the apparent power drawn from the line with the resistors in circuit and [1 mark]
   (iv) the locked-rotor torque developed by the motor. [1 mark]
   (v) If the locked-rotor power factor of the motor alone is 0.35, calculate the required value and power of the series resistors. [5 marks]
   (c) Name *two* advantages and *two* disadvantages of the starter in part (b). [2 marks]
4. (a) Describe the effect of increasing the rotor resistance of a three-phase induction motor on
   (i) the starting torque
   (ii) the starting current
   (iii) the full load efficiency
   and proceed to explain why a slip ring is ideal for a heavy starting duty. [5 marks]
   (b) A 400-V, 50-Hz, 4-pole, 3-phase star-connected slip ring induction motor has the following parameters per phase: $R_1 = 0.20\ \Omega$ $R'_2 = 0.10\ \Omega$ $X_1 + X'_2 = 0.75\ \Omega$
   (c) The motor is used on a crane to start against a load torque of 300 Nm. Find the two possible values of resistance referred to the stator to be added to each rotor phase. [10 marks]

5. (a) Explain why an induction motor cannot develop torque when running at synchronous speed. [3 marks]

(b) Supply authority generally does not permit squirrel cage motors to be started direct-on-line. Why? [2 marks]

(c) A 6-pole, 50-Hz, 11-kW, 3-phase 400-V induction motor runs at 960 rev/min on full load. If it takes 80.0 A on direct-on-line switching, find the ratio of starting torque to full load torque in the following cases:

(i) when started direct-on-line

(ii) when started by star delta starter

(iii) when started by autotransformer starter with 60 % tapping

(iv) when started by resistance starter, limiting the starting current to 50A.

Take the full load power factor and efficiency to be 0.834 and 95.6 % respectively. [10 marks]

6. (a) How can we change the direction of rotation of a 3-phase induction motor? [2 marks]

(b) A 440-V, 3-phase, 50-Hz, 8-pole, star-connected induction motor has the following equivalent parameters per phase: $R_1 = R'_2 = 0.10\ \Omega$ $X_1 = X'_2 = 0.7\ \Omega$ $R_m = 100\ \Omega$ $X_m = 25\ \Omega$. At a slip of 4%, calculate using exact equivalent circuit

(iii) the stator current [8 marks]

(iv) the input power factor [2 marks]

(v) the rotor current referred to the stator [5 marks]

(vi) the torque in N-m [4 marks]

(vii) the efficiency [4 marks]

7. (a) What are the advantages of squirrel-cage motors over slip-ring motors? [5 marks]

(b) A 3-phase, 2934-rev/min, 150-kW, 415-V, 50-Hz induction motor has a locked rotor torque of 600 N-m and a locked rotor current of 1200 A. Three resistors are connected in series with the line so as to reduce the voltage across the motor to 0.50 per unit. Calculate

(i) the apparent power drawn by the motor under full-voltage locked-rotor conditions [1 mark]

(ii) the apparent power absorbed by the motor when the resistors are in circuit [3 marks]

(iii) the apparent power drawn from the line with the resistors in circuit and [2 mark]

(iv) the locked-rotor torque developed by the motor. [2 marks]

(v) If the locked-rotor power factor of the motor alone is 0.35, calculate the required value and power of the series resistors. [8 marks]

(c) Name *two* advantages and *two* disadvantages of the starter in part (b). [4 marks]