Fuid Mechanics
Problem Solving on Bernoulli Equation

Problem 1

A water reservoir, *A*, whose free-surface is kept at a pressure 2 x 10^5 Pa above the atmospheric pressure, discharges to another reservoir, *B*, open to the atmosphere. The water free-surface level at the second reservoir is 0.5 m above the pressurized reservoir *A*. Neglect the energy dissipation in the connecting duct between the two reservoirs. The connecting duct has constant diameter.

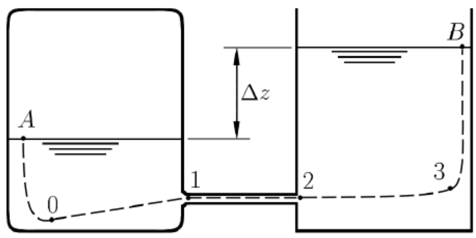

Fig. 1 – Water flow between the two reservoirs, from *A* to *B*.

1. Compute the water velocity in the connecting duct.
2. Would the velocity at the duct exit change if the diameter of the connecting duct is not constant?
3. What would be the pressure difference between the duct inlet and outlet if the duct is horizontal and of constant diameter?
4. The pressure in the duct is imposed by which of the two reservoirs? What is the essential difference between the streamlines in the upstream (0-1) and downstream (2-3) reservoirs that justifies the response to the previous question?
5. Bernoulli equation in *stricto sensu* cannot be applied between a point 2 at duct exit and another point in the same streamline where the velocity can be assumed to be negligible, e.g. 3 or *B*¸ because of significant viscous effects at the duct exit. Determine the error induced by applying Bernoulli equation in *stricto sensu* between the two points mentioned above by applying an energy balance to the water in the discharge reservoir.

Solution

By applying Bernoulli equation along a streamline like the one shown in Fig.1 between a generic point A at the pressurized free-surface and point 2 at the duct exit it results in

$$p_A + \frac{1}{2}\rho v_A^2 + \rho g z_A = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2$$

However, the streamlines at the duct exit (where a jet of water is formed) are parallel to each other, from what results a hydrostatic pressure distribution at the duct exit. Therefore the hydrostatic equation results in $p_2 = p_B + \rho g(z_B - z_2)$. Introducing this result in the first equation the following result will come out:

$$v_2 = \sqrt{2\frac{p_A - p_B}{\rho} + 2g(z_A - z_B)} = 20\ m/s.$$

The mean velocity is constant in a constant diameter duct due to mass conservation.

The exit velocity is not affected by the duct diameter (as we are neglecting viscous losses in the duct). The velocity in the duct changes along the duct if its diameter is not constant.

The discharge reservoir is the one imposing the pressure in the duct. This results from the water leaving the duct as a jet (with straight streamlines). Straight streamlines impose a hydrostatic pressure distribution, with the local pressure given by the pressure at the free-surface and the exit submergence depth. At the inlet the streamlines are curvilinear and locally radially converging into the duct's inlet, thus not imposing a hydrostatic pressure distribution.

The pressure is hydrostatic in all the discharge reservoir, including the duct's exit (point 2) and free-surface (point B). Therefore the pressure forces are compensated by the fluid weight resulting in a zero force that produces zero work (no conversion of kinetic energy in pressure or potential energy). Thus all the kinetic energy is dissipated into heat by the viscous deformation of the fluid. The error in applying Bernoulli equation between points 2 and 3 or B may be calculated by introducing the error Δp in the Bernoulli equation as follows:

$$p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2 = p_B + \frac{1}{2}\rho v_B^2 + \rho g z_B + \Delta p$$

As already shown $p_2 = p_B + \rho g(z_B - z_2)$ from which $\Delta p = \frac{1}{2}\rho v_2^2$. This results means that the kinetic energy at the exit of a duct is fully dissipated by viscous effects sufficiently away from the exit (i.e., there is no kinetic energy recovery – accumulation in pressure or potential energy increase).

Problem 2

A U tube manometer containing water is connected to a nozzle of an air tunnel that discharges to the atmosphere as shown in Fig. 2. The area ratio is $A_2/A_1 = 0.25$. For given operational conditions the level difference in the manometer is $h = 94\,mm$. Take the water density $\rho = 1000\,kg/m^3$ and the air density $\rho_{air} = 1.23\,kg/m^3$.

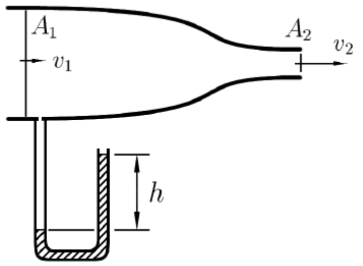

Fig. 2 – One-phase flow in a nozzle

1. What is the average air velocity at the nozzle exit, v_2?
2. Schematically draw the longitudinal pressure distribution along the nozzle.
3. Describe the pressure distribution in the nozzle exit.

Solution

The average air velocity at the nozzle exit is $v_2 = \sqrt{2\frac{\rho}{\rho_a}\frac{gh}{[1-(A_2/A_1)^2]}} = 40\,m/s.$

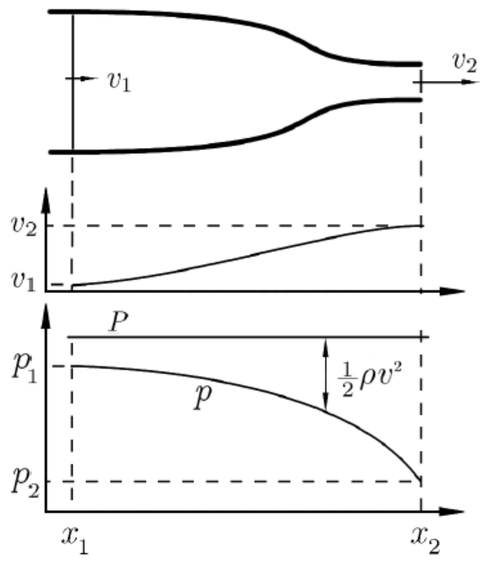

Fig. 3 – Velocity and pressure distribution along the nozzle.

The pressure distribution is shown in Fig.3, where *x* is the longitudinal direction and *P* the total pressure of the flow. The distribution of the relative pressure to the local hydrostatic pressure is the same for all the streamlines; the distribution of the absolute pressure differs from one streamline to the other due to the hydrostatic pressure contribution (the weight of the fluid column between streamlines).

In the exit section, the absolute pressure follows a hydrostatic distribution (it does not change horizontally and decreases linearly with the level). The relative pressure to the local hydrostatic pressure is zero everywhere.