ME 261 Dynamics of Solid Mechanics
Unit 2

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KINETICS OF PARTICLES

Equation of Motion

From Newton's Second Law to a particle of mass m and using a suitable choice of units, we can write

$$\boldsymbol{F} = m\boldsymbol{a}$$

$$F_x i + F_y j + F_z k = m(a_x i + a_y j + a_z k)$$

$$F_x = ma_x; F_y = ma_y; \text{ and } F_z = ma_z$$

When more than one force acts on a particle, the resultant force is determined by a vector summation of all the forces; i.e.,

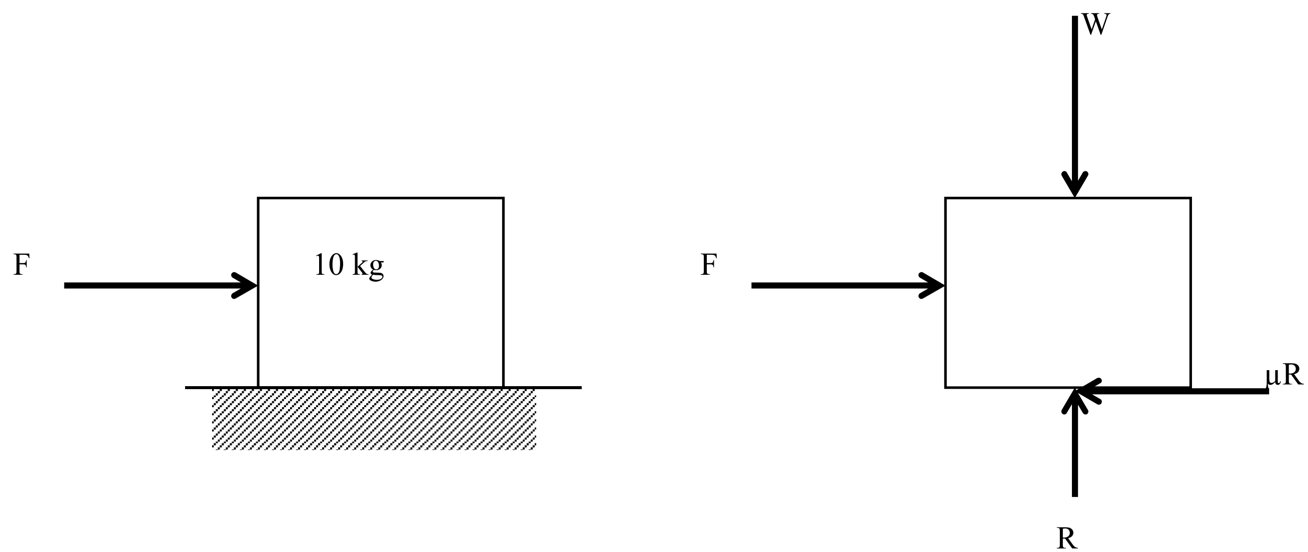
$$F = \sum F$$

For this more general case, the equation of motion may be written as

$$\sum F = ma$$

Example 2-1: Equation fo Motion for Particles

The 5-kg particle shown is being pushed on a rough horizontal surface by a horizontal force F with an acceleration of 2 ms^{-2}. What is the magnitude of F if the coefficient of friction is $\mu = 0.25$?



(a) (b)

Solution

The free-body diagram of the particle is shown in Figure (b). Since there is no acceleration in the y-direction, the sum of the forces in that direction must equal zero, i.e.

$$5x9.81 - R = 0$$

$$R = 49.05 \text{ N}$$

Summing forces in the x-direction and equating to the product of mass and acceleration, we have

$F - \mu R = ma_x$ $\qquad F = ma_x + \mu R$

$= 5(2) + 0.25(49.05)$

$\underline{\underline{F = 22.6 \text{ N}}}$

D'Alembert's Principle

The equation of motion may be written in the alternative form

$$\boldsymbol{F}-(m\boldsymbol{a}) = 0$$

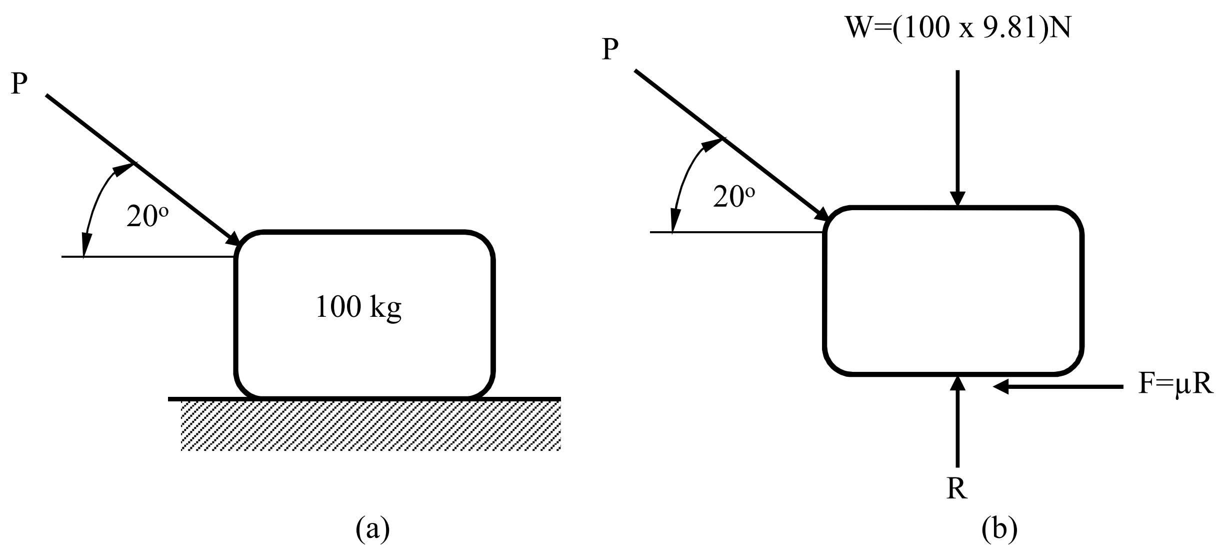
- This equation corresponds to the equilibrium equation of a particle in static equilibrium under the action of two forces of value F and -ma.

- Thus the dynamics problem of a particle having an acceleration ***a*** under the action of an external resultant force ***F***, may, be reformulated as a statics problem if, in addition to the external force F, we apply a fictitious force -*m**a*** to the particle.

- The fictitious force -*m**a*** is referred to as the inertia force or reversed effective force or d'Alembert force.

Example 2-2: Equation of Motion for Particles

A 100-kg block rests on a horizontal plane as shown. Determine the magnitude of the force P required to give the block an acceleration of 3 m/s^2 to the right if the kinetic coefficient of friction between the block and the plane is $\mu = 0.3$.



solution

From the freebody diagram, we sum forces along the vertical and horizontal axes. Since there is no motion along the vertical axis, a_y=0

$$+\uparrow \sum F_y = ma_y = 0$$

$$R - P\sin 20 - 981 = 0 \quad \text{......1}$$

$$+\rightarrow \sum F_x = ma_x = 100(3)$$

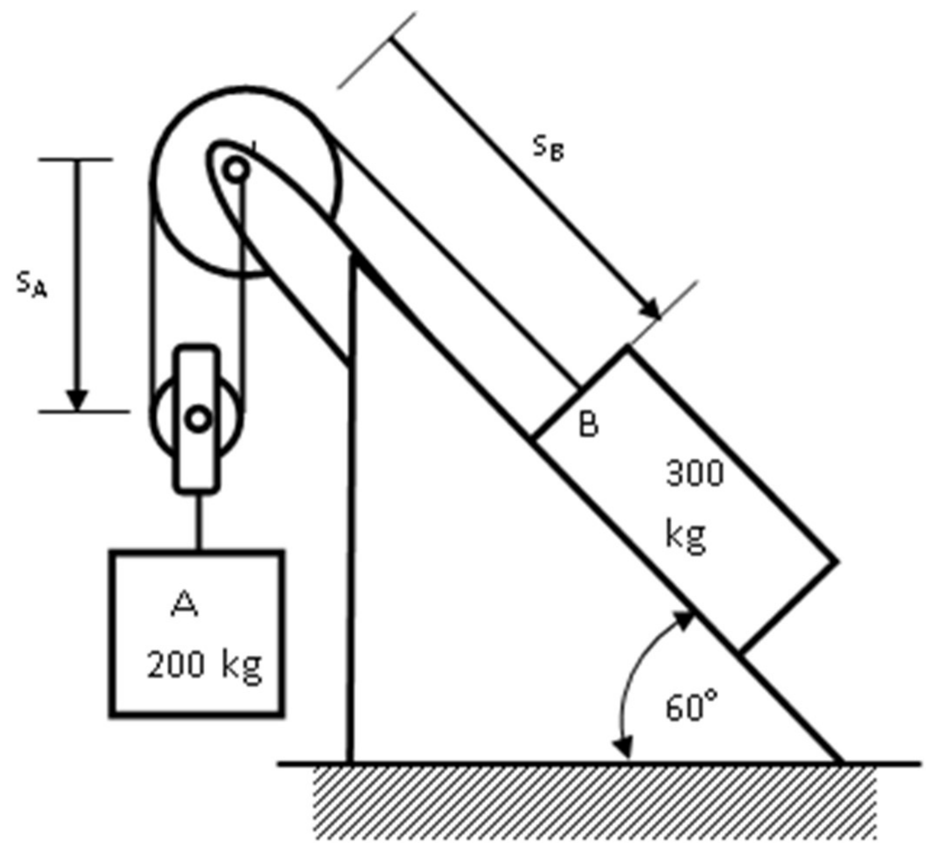
$$P\cos 20 - \mu R = P\cos 20 - 0.3R = 300 \quad \text{......2}$$

Solving Equations (1) and (2) simultaneously,

<u>P=710 N</u>

Example 2-3: Equation of Motion of Particles

Blocks A and B of masses 200 kg and 300 kg respectively, are released from rest simultaneously as shown in Figure E2-2. If the coefficient of kinetic friction between block B and the inclined surface is 0.3, determine the tension in the cable and the acceleration of each block. Neglect the masses of the pulleys and the cable, and assume that the pulley is frictionless.



Solution

Kinematics

The displace $2s_A + s_B = L$

Differentiating the above equation twice, we have

$$2a_A + a_B = 0 \qquad (1)$$

Kinetics $\sum F_x = m_A a_A$

Applying the equation of motion to block A using Figure E2-2(a) $\sum F_x = m_B a_B$

$$-2T + 1962 = 200\ a_A \qquad (2)$$

Applying the equation of motion to block B using Figure E2-2(b)

$$-T - 0.3\ N + 2943 \sin 60° = 300\ a_B = -600 a_A \qquad (3)$$

$$\sum F_y = 0$$

$$N - 2943 \cos 60° = 0 \quad or \quad N = 1471.5\ N$$

Solution

Solving equations (1) to (3) simultaneously gives

$$\underline{\underline{a_A = -1.61\ m/s^2}} \quad \underline{\underline{a_B = 3.22\ m/s^2}} \quad \underline{\underline{T = 1140\ N}}$$

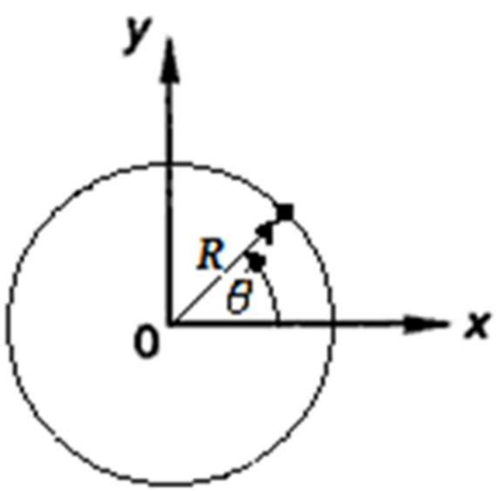
Motion in a Circular Path

Consider a particle of mass m moving in a circular path of radius r The position vector of the body at any time t is given by

$$R = x\mathbf{i} + y\mathbf{j} = r(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$$

The velocity is given by

$$v = r\omega(\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$$



Finally, the acceleration vector

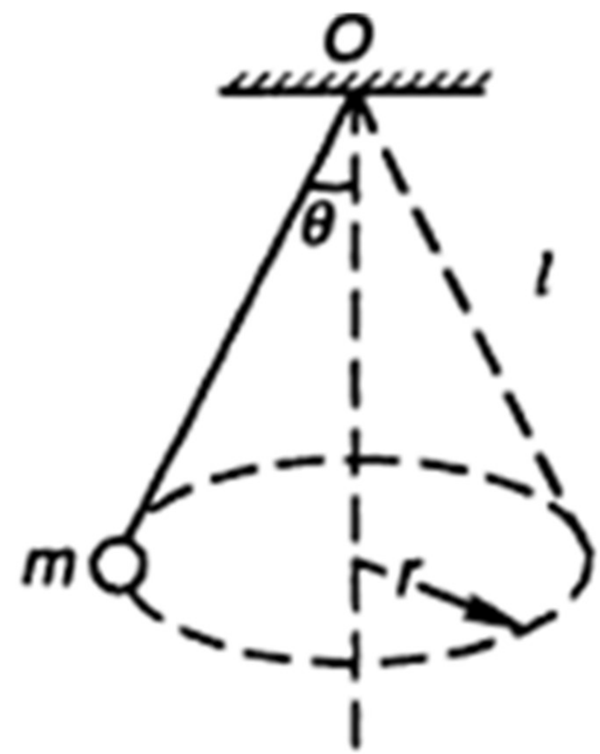
$$\mathbf{a} = d\mathbf{v}/dt = r\alpha(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}) - r\omega^2(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$$

The acceleration is seen to be made up of two components:

- a tangential component of magnitude rα (whose actual direction depends on the sign of α), and
- a radially inward (centripetal) component of magnitude $r\omega^2 = v^2/r$

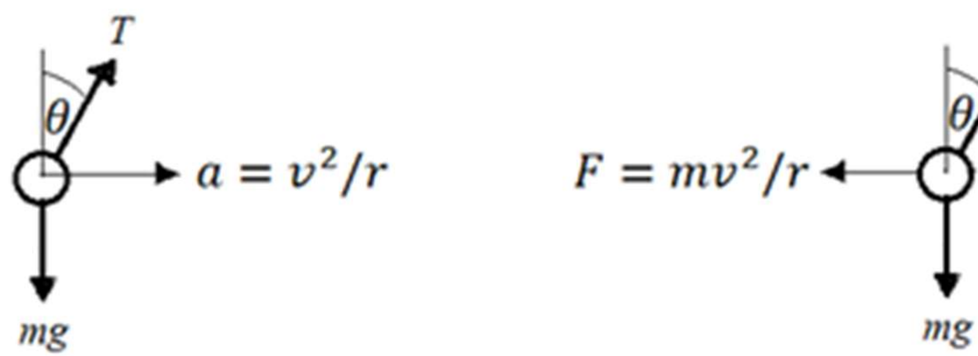
Example 2-4: Equation of Motion for Particles in a Circular Path

The figure shows a particle of mass m suspended by a light string of length l from a fixed point O and describing a horizontal circle with constant speed v m/s. If the angle θ is 30° and l is 1 m, determine the speed v.



Solution

We shall solve the problem using the two alternative methods: Equation of motion and d'Alembert's principle.



(a) (b)

Alternative I: Equation of motion method

Summing forces towards the centre of the circle and equating to mass times the acceleration. We have

$$T \cos \theta - mg = 0$$

$$T \sin \theta = ma_n = m\left(\frac{v^2}{l \sin \theta}\right)$$

(2)

Solving the two equations simultaneously, we have: <u>v=1.68 m/s</u>

Solution

Alternative II: d'Alembert's principle

We can, therefore, sum forces in the vertical and horizontal directions and equate to zero. This will give the following equations:

$T\cos\theta - mg = 0$ (3)

$T\sin\theta = mv^2/l\sin\theta$ (4)

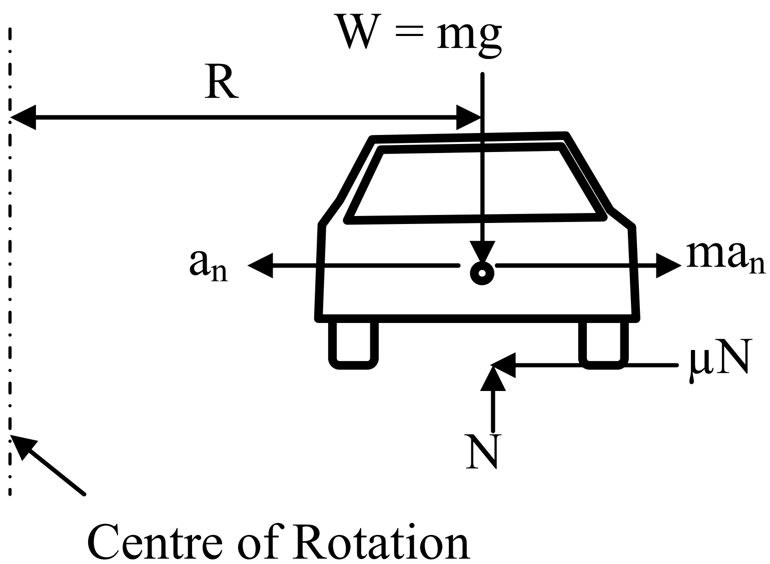
These equations are identical to equations (1) and (2) and may be solved to obtain the required answer, v = 1.68 m/s.

Example 2-5: Equation of Motion for Particles in a circular path

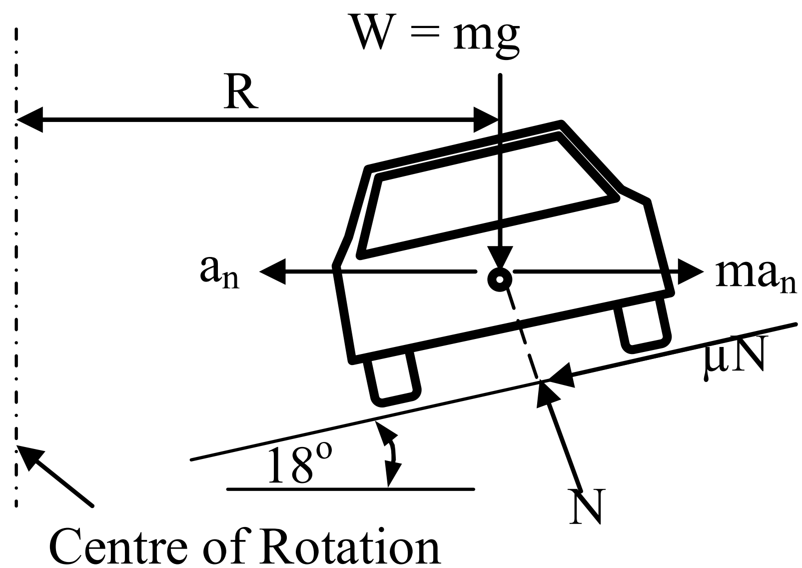
Determine the speed of a highway curve of radius R = 120 m if

a. the coefficient of friction between the tyres and road is μ=0.4 and the road is unbanked
b. the road banking is 18^o,neglecting lateral friction force.
c. The coefficient of friction between the tyres and road is μ=0.4 and the road banking 18^o.

Example 2-5: Equation of Motion for Particles in a circular path



Unbanked road



Banked road

Free-body Diagrams

Solution

(a) From the free-body diagram of the vehicle on an unbanked curve road surface.

$$\sum F_y = ma_y = m(0) \quad N - mg = 0 \quad N = mg \qquad (1)$$

$$\sum F_n = ma_n \qquad \mu N = ma_n = m\left(\frac{v^2}{R}\right) \qquad (2)$$

Substituting (1) into (2) and solving for v, we have

$$\mu mg = m\left(\frac{v^2}{R}\right) \qquad v = \sqrt{\mu g R} = \sqrt{(0.4)(9.81)(120)} \qquad v = 21.7 \text{ m/s}$$

Solution

(b) Shown in Figure E2-5(b) is the free-body diagram of the vehicle on a bank curved road surface.

$$\sum F_y = ma_y = m(0) \qquad mg = N\cos\theta - \mu N\sin\theta \tag{3}$$

$$\sum F_n = ma_n \qquad m\left(\frac{v^2}{R}\right) = N\sin\theta + \mu N\cos\theta \tag{4}$$

Note that if μ=0, Equations (3) and (4) reduce to (1) and (2), respectively.

Dividing (4) by (3) and making *v* the subject, we have

$$v = \sqrt{Rg\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}} \tag{5}$$

If μ =0 then the above equation reduces to

$$v = \sqrt{Rg\frac{\sin\theta + 0}{\cos\theta - 0}} = \sqrt{Rg\tan\theta} = \sqrt{(120)(9.81)\tan(18)} \qquad \underline{\underline{v = 19.6\text{ m/s}}}$$

(c) Using (5), $v = \sqrt{(120)(9.81)\dfrac{\sin 18 + 0.4\cos 18}{\cos 18 - 0.4\sin 18}}$ $\qquad \underline{\underline{v = 31.3\text{ m/s}}}$

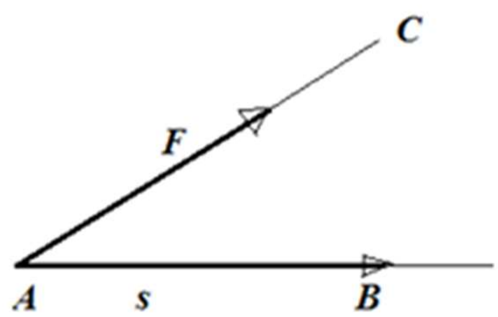
Work, Energy and Power

- A force is said to do work when it moves its point of application, and the work done is defined as the product of the force and the component of the displacement of its point of application in the direction of the force.

- Work is a scalar quantity and has the unit of newton-metre (N.m), which is given the special name of joule (J).

- joule is the work done by a force of 1 N when its point of application moves through a distance of 1 m in the direction of the force.

Work



Let a constant force F acting in the direction AC move its point of application from A to B where the vector AB is s, Figure 2-2. Then the work done by the force is the dot product (i.e scalar product) of the force and displacement vectors.

W = F. s = |F| x AB cos θ

It can be deduced from the above equation that

(a) If the force and displacement are in the same direction, θ = 0 and the work done is equal to Fs.

(b) If the force acts at right-angles to the displacement vector-the work done is zero.

If the force acting on a body changes continuously as its point of application moves along a given path, the work done is

$$W = \int F ds$$

where F is the force and ds is a displacement on the path along which the point of application of the force moves.

Energy

- The energy of a body is its capacity for doing work.
- Energy is measured in the same units as those of work, i.e. in joules.
- The energy possessed by a body may be in a variety of forms, such as electrical, mechanical, thermal, wave, wind, geothermal and solar energy.
- In Engineering Mechanics, we are usually concerned with mechanical energy which may be subdivided into kinetic energy and potential energy.

Kinetic and Potential Energies

Kinetic Energy

The kinetic energy of a body is the energy that it possesses by virtue of its motion. It is measured by the amount of work which the body would do in coming to rest

$$U = \int F ds = \int (ma) ds = \int \left(mv \frac{dv}{ds} \right) ds$$

$$U = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$$

Potential Energy

The potential energy of a body is the energy that it possesses by virtue of its position or configuration

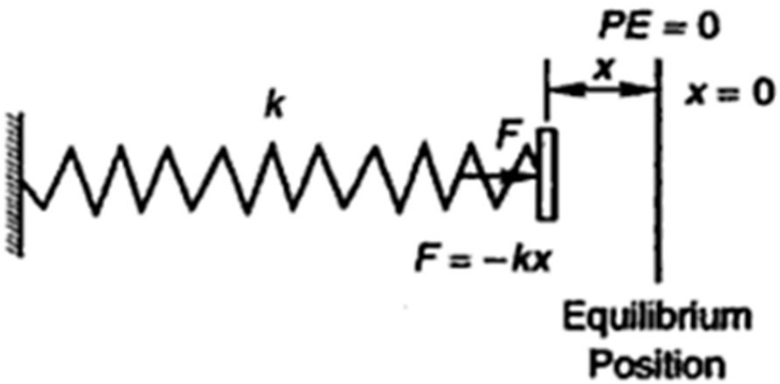
$$U_{1\to2} = -W(y_2 - y_1)$$

$$U_{1\to2} = -W\Delta y$$

$$U_{1\to2} = -mg\Delta y$$

Energy

Elastic Potential Energy of Spring



$$work = \int_0^x kx dx = \frac{1}{2}kx^2$$

If a spring is compressed from x_1 to x_2, then the spring energy stored is given by

$$U = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

The Principle of Conservation of Energy

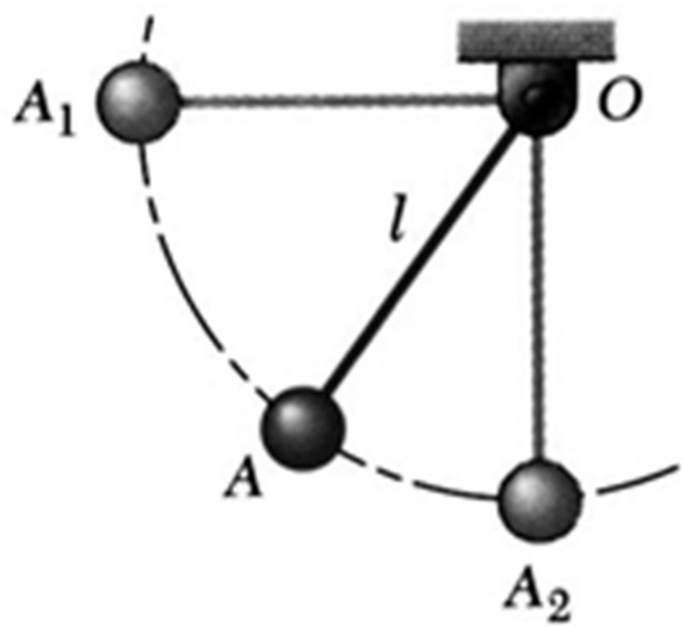
- It states that ***Energy can neither be created nor destroyed, although it may be converted from one form into another*.**
- The principle of conservation of energy may be stated mathematically as

$$KE_1 + PE_1 + PEe_1 = KE_2 + PE_2 + PEe_2 + E_{loss}$$

- Where KE = kinetic energy, PE = potential energy, PEe =potential elastic energy of a spring element, E_{loss} = energy loss. For a conservative system (i.e system without friction), the energy loss should be set to zero.

Example 2-6: Principle of Conservation of Energy

Determine velocity of pendulum bob at A_2 if it is released from rest at A_1. The length of string AO is l =0.5 m.



Solution

Apply the principle of conservation of energy given by Equation 2-13, we have

$$KE_1 + PE_1 + PEe_1 = KE_2 + PE_2 + PEe_2 + E_{loss} \qquad (1)$$

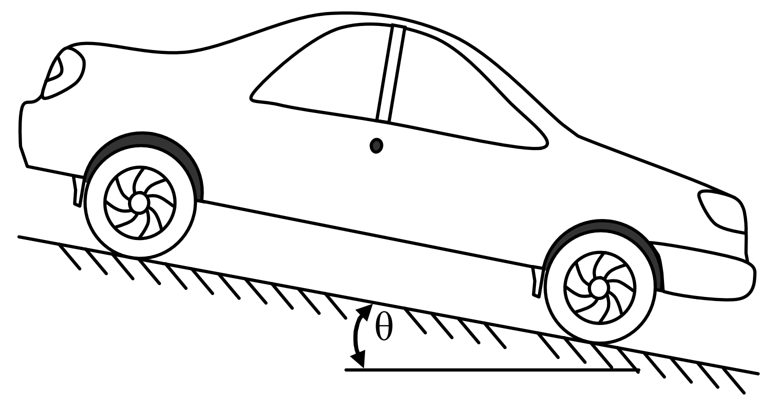
a. There is no elastic element (or spring), therefore PEe_1=PEe_2=0
b. Since the pendulum bulb is released from rest, KE_1 =0
c. Since the pendulum bulb will be moving when it gets to A_2, KE_2 NOT equal to zero.
d. If point A_2 is set as reference, then PE_1= mgl and PE_2=0

Substituting the above into Equation (1), we have

$$0 + mgl + 0 = \frac{1}{2}mv^2 + 0 + 0 + 0 \qquad v = \sqrt{2gl} = \sqrt{2(9.81)(0.5)} \qquad \underline{\underline{v = 3.13\,\text{m/s}}}$$

Example 2-7: Principle of Conservation of Energy

An car weighing 1200 kg is driven down a hill of inclination 10^o to the horizontal at a speed of 80 km/h. When the brakes are applied, all the wheels lock and slide without rolling and the coefficient of friction between the tyres and the road surface is 0.4. Determine the distance traveled by the car before coming to a stop.



Solution

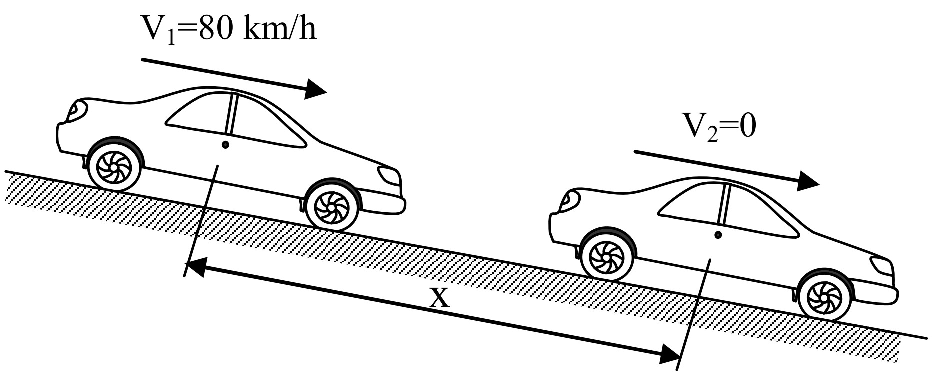
$$+\uparrow \sum F_{normal} = 0 \qquad N = W\cos\theta = mg\cos\theta \;\; (1)$$

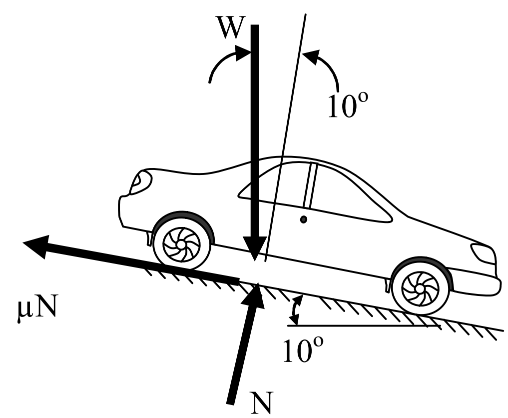
Frictional force is given by: $F = \mu N = \mu W\cos\theta = \mu mg\cos\theta$

Applying Equation 2-12 to tangential direction, we have

$$(+\rightarrow) \qquad mv_1 + \sum\int_{t_1}^{t_2} F\,dt = mv_2 \qquad mv_1 + (-\mu mg\cos\theta + mg\sin\theta)t = mv_2$$

$$(1200)\left(80\frac{1000}{60^2}\right) + (1200)(9.81)[-(0.4)\cos 10 + \sin 10]t = 0 \qquad \underline{\underline{t = 10.3\text{ s}}}$$





Solution

Frictional force is given by: $F = \mu N = \mu W \cos\theta = \mu mg \cos\theta$

Energy loss is given by: $E_{loss} = Fx = (\mu N)x$ $\qquad E_{loss} = \mu(mg\cos)x \qquad (2)$

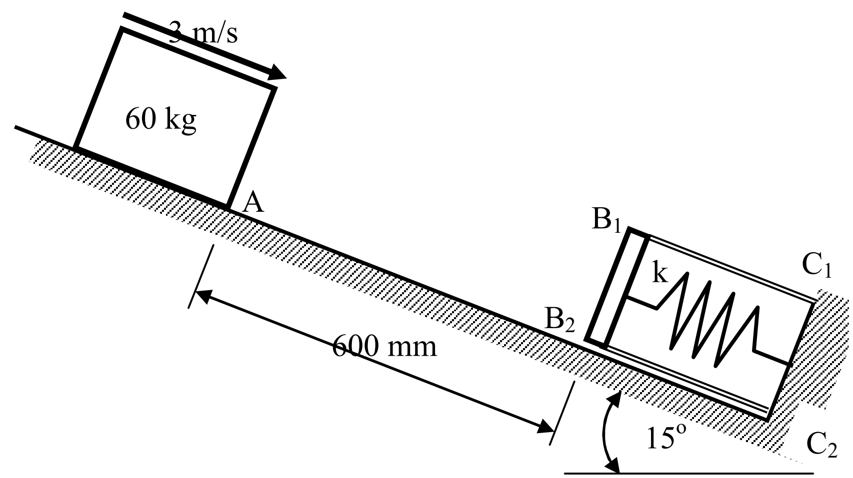
Substituting the above into Equation (1), we have

$$\frac{1}{2}mv_1^{\,2} + mgx\sin 10 + 0 = 0 + 0 + 0 + \mu(mg\cos)x \qquad \Rightarrow x = \frac{v_1^{\,2}}{2g(\mu\cos\theta - \sin\theta)}$$

$$x = \frac{\left(80\text{x}\,\frac{1000}{60^2}\right)^2}{2(9.81)[(0.4)\cos 10 - \sin 10]} \qquad \underline{\underline{x = 114\,\text{m}}}$$

Example 2-9: Principle of Conservation of Energy

- A spring of stiffness *k* = 20 kN/m is used to stop a 60 kg package which is sliding on a surface inclined 15^o to the horizontal shown in Figure E2-9. The package has a velocity of 3 m/s at position A. The spring is held by cables so that it is initially compressed.
- Determine the coefficient of kinetic friction between the package and surface so that it is initially compressed 120 mm and the maximum deflection of the spring is 60 mm.
- Determine the maximum deflection of the spring
  - if the spring is initially uncompressed and the inclined surface is smooth (i.e frictionless)
  - if the spring is initially uncompressed and the coefficient of friction between the inclined surface and the package is 0.3.
  - if the spring is initially compressed 100 mm and the coefficient of friction between the inclined surface and the package is 0.3.



Solution

Using the free-body diagram of Figure E2-9(a)

$$+\uparrow \sum F_{normal} = 0 \qquad\qquad N = W\cos\theta = mg\cos\theta$$

Frictional force acting on the package is given by

$$F = \mu(N) = \mu(mg\cos\theta)$$

Energy loss due to friction between the package and the inclined surface is given by:

$$E_{loss} = F(0.6 + x) = (\mu N)(0.6 + x) \qquad\qquad E_{loss} = \mu(mg\cos\theta)(0.6 + d) \qquad (1)$$

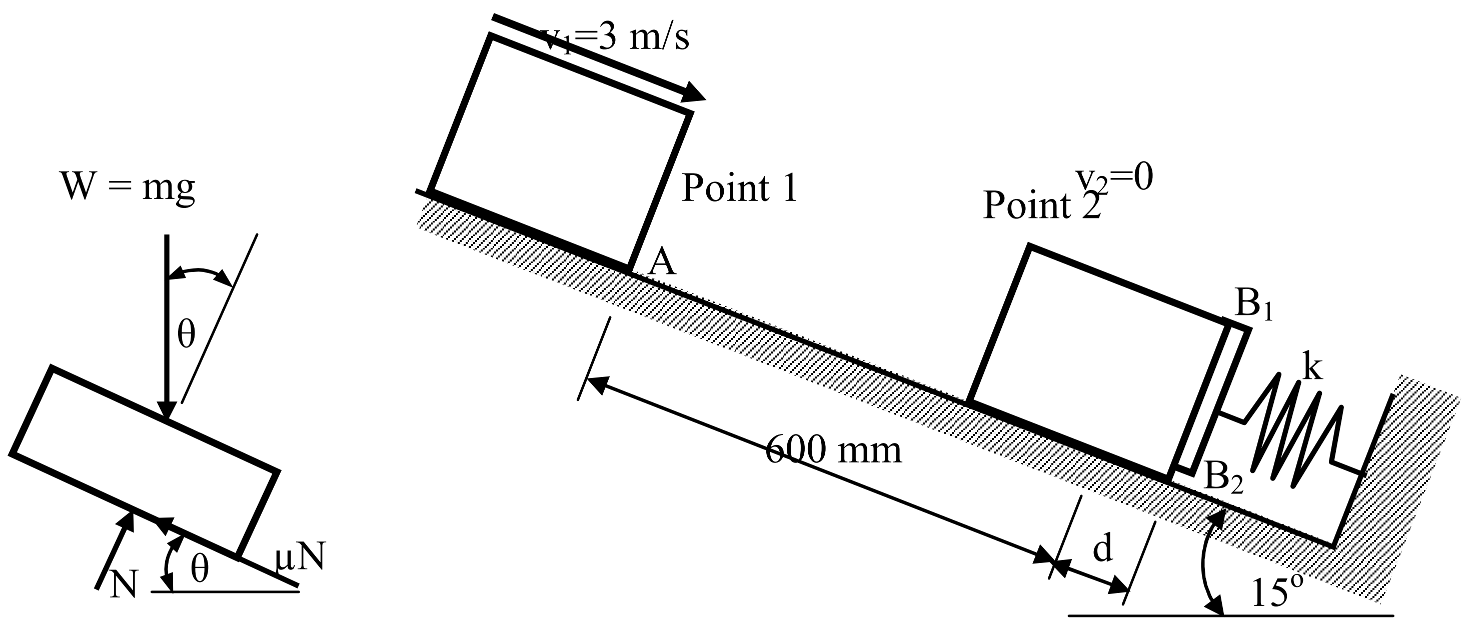
where d is equal to the maximum deflection of the spring from the initial compression. If point 2 is set as reference, then $PE_2=0$ and

$$PE_1 = mgh = mg(0.6 + d)\sin\theta \qquad (2)$$

Solution

Let x_1 and x_2 be the compression (or extension) of the spring at points 1 and 2, which are actually deflections of the spring from its natural length. The word *deflection* in the question is the compression (or extension) from the initial compression (or extension) length. Since d is deflection of the spring from point 1 to 2, $x_2=x_1+d$. Thus

$$KE_1 = \frac{1}{2}kx_1^2 \qquad \text{and} \qquad KE_2 = \frac{1}{2}kx_2^2 \tag{3}$$



Solution

Applying principle of conservation of energy between point 1 and 2 in Figure E2-9(b) and substituting Equations 1 to 3 into the energy conservation equation, we have

$$KE_1 + PE_1 + PEe_1 = KE_2 + PE_2 + PEe_2 + E_{loss}$$

$$\frac{1}{2}mv_1^{\,2} + mg(0.6+d)\sin\theta + \frac{1}{2}kx_1^{\,2} = 0 + 0 + \frac{1}{2}kx_2^{\,2} + \mu(mg\cos\theta)(0.6+d) \qquad (4)$$

(a) From Equation (4), we have

$$\mu = \frac{\frac{1}{2}mv_1^{\,2} + mg(0.6+d)\sin\theta - \frac{1}{2}k\left(x_2^{\,2} - x_1^{\,2}\right)}{mg(0.6+d)\cos\theta}$$

$$\mu = \frac{\frac{1}{2}(60)(3)^2 + (60)(9.81)(0.6+0.06)\sin 15 - \frac{1}{2}\left(20\text{x}10^3\right)\left[(0.12+0.06)^2 - (0.12)^2\right]}{(60)(9.81)(0.6+0.06)\cos 15}$$

$$\underline{\underline{\mu = 0.508}}$$

Solution

(b) $$\frac{1}{2}mv_1^{\,2} + mg(0.6+d)\sin\theta + \frac{1}{2}kx_1^{\,2} = 0 + 0 + \frac{1}{2}kx_2^{\,2} + \mu(mg\cos\theta)(0.6+d)$$

Expanding and simplifying the above equation, we have

$$Ad^2 + Bd + C = 0 \qquad (5)$$

where: $A = \frac{1}{2}k$, $\quad B = kx_1 + mg(\mu\cos\theta - \sin\theta) \qquad C = 0.6mg(\mu\cos\theta - \sin\theta) - \frac{1}{2}mv_1^{\,2}$

(i) From the question, $x_1=0$ and $\mu=0$.Using Equation (4), we have

$$A = \frac{1}{2}k = \frac{1}{2}\left(20\text{x}10^3\right) \qquad A = 10000$$

$$B = kx_1 + mg(\mu\cos\theta - \sin\theta) = \left(20\text{x}10^3\right)(0) + (60)(9.81)((0)\cos 15 - \sin 15) \qquad B = -152.34$$

$$C = 0.6mg(\mu\cos\theta - \sin\theta) - \frac{1}{2}mv_1^{\,2} = 0.6(60)(9.81)((0)\cos 15 - \sin 15) - \frac{1}{2}(60)(3)^2$$

$$C = -361.4$$

Solution

Then, Equation (5) reduces to

$$10000d^2 - 152.34d - 361.4 = 0$$

Solving the above equation gives

$$d = 0.198\,\text{m} \qquad\qquad \underline{\underline{d = 198\,\text{mm}}}$$

Note that the negative solution is disregarded as it will mean that the spring will extend (which is false) instead of compression.

(ii) From the question, $x_1=0$ and $\mu=0.3$. Using Equation (4), we have

$$A = \frac{1}{2}k = \frac{1}{2}\left(20\text{x}10^3\right) \qquad A = 10000$$

$$B = kx_1 + mg(\mu\cos\theta - \sin\theta) = \left(20\text{x}10^3\right)(0) + (60)(9.81)[(0.3)\cos 15 - \sin 15]$$

$$B = 18.222$$

Solution

$$C = 0.6mg(\mu\cos\theta - \sin\theta) - \frac{1}{2}mv_1^{\,2} = 0.6(60)(9.81)[(0.3)\cos 15 - \sin 15] - \frac{1}{2}(60)(3)^2$$

$$C = -259.067$$

Then, Equation (5) becomes $10000d^2 + 18.222d - 259.067 = 0$

Solving the above equation and disregarding the negative solution, we have

$$d = 0.160\,\text{m} \qquad \underline{\underline{d = 160\,\text{mm}}}$$

(iii) From the question, $x_1 = 100$ and $\mu = 0.3$. Using Equation (4), we have

$$A = \frac{1}{2}k = \frac{1}{2}\left(20\text{x}10^3\right) \qquad A = 10000$$

$$B = kx_1 + mg(\mu\cos\theta - \sin\theta) = \left(20\text{x}10^3\right)(0.1) + (60)(9.81)[(0.3)\cos 15 - \sin 15]$$

$$B = 2018.222$$

$$C = 0.6mg(\mu\cos\theta - \sin\theta) - \frac{1}{2}mv_1^{\,2} = 0.6(60)(9.81)[(0.3)\cos 15 - \sin 15] - \frac{1}{2}(60)(3)^2$$

$$C = -259.067$$

Then, Equation (5) becomes $10000d^2 + 2018.222d - 259.067 = 0$

Solving the above equation and disregarding the negative solution, we have

$$d = 0.089\,\text{m} \qquad \underline{\underline{d = 89\,\text{mm}}}$$

Power

- Power is define as the time rate at which work is done or energy is transform from one form to another. If *ΔU* is the work done during the time interval *Δt*, then the average power during that time interval is

$$\text{Average Power} = \frac{\Delta U}{\Delta t}$$

$$\text{Power, P} = \lim_{\Delta t \to 0} \frac{\Delta U}{\Delta t} \qquad \text{P} = \frac{dU}{dt}$$

Substituting the scalar product *F.dr* for *dU*, we can write

$$P = \frac{\left(\int F.dr\right)}{dt} = \int F.\frac{dr}{dt} \qquad P = F.v$$

- Hence, power is a scalar, where in this formulation v represents the velocity of the particle which is acted upon by the force F.
- The basic units of power used in the SI and Foot-Pound-second (FPS) systems are the watt (W) and horsepower (hp), respectively. For conversion between the two systems of units, 1 hp = 746 W.

Efficiency

The mechanical efficiency of a machine is defined as the ratio of the useful power output produced by the machine to the input power supplied to the machine

$$\eta = \frac{\text{power output}}{\text{power input}}$$

If energy supplied to the machine occurs during the same time interval at which it is drawn, then the efficiency may also be expressed in terms of the ratio

$$\eta = \frac{\text{energy output}}{\text{energy input}}$$

For real machines, efficiency is always less than 1 (or 100%).

Principle of Impulse and Momentum

The equation of motion for a particle of mass *m* can be written as

$$\sum F = ma = m\frac{dv}{dt}$$

The above equation is a statement of Newton's Second Law. Rearranging the terms and integrating between the limits v = v_1 at t =t_1 and v = v_2 at t =t_2 we have

$$\sum \int_{t_1}^{t_2} F dt = m \int_{v_1}^{v_2} dv \qquad \sum \int_{t_1}^{t_2} F dt = mv_2 - mv_1$$

Thus, the rate of change of linear momentum of a particle is equal to the resultant of the forces acting on the particle.

Principle of Conservation of Linear Momentum for a System of Particles

When the sum of the external impulses acting on a system of particles is zero, Equation 2-25 reduces to a simplified form, which is

$$\sum m_i (v_i)_1 = \sum m_i (v_i)_2$$

The above equation may be expanded as

$$m_1 u_1 + m_2 u_2 + .. + m_n u_n = m_1 v_1 + m_2 v_2 + .. + m_n v_n$$

This equation is referred to as the *conservation of linear momentum*. It states that the total linear momentum for a system of particles remains constant during the time period t_1 to t_2. The conservation of linear momentum is often applied when particles collide or interact.