QUESTION 1

A**) Construct a mathematical equations for this cooling tower.**

The centre is taken at the origin with z-axis,
Equation of an hyperboloid will is:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{y^2}=1$$

The traces are then circles on the x and y planes, meaning that a = b

Thus the equation reduces to;

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{c^2}=1$$

The Z-trace gives (since z=0)

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}=1$$

a = 10 because the minimum radius occurs on the x-y plane. We now have

$$\frac{x^2}{10^2}+\frac{y^2}{10^2}=1$$

(with z = 0)

The height of the tower is 40m, and is symmetric with respect to the xy plane so we have 20 above and 20 below. The equation becomes:

$$\frac{x^2}{10^2}+\frac{y^2}{10^2}-\frac{20^2}{c^2}=1$$

$$\frac{x^2}{100}+\frac{y^2}{100}-\frac{400}{c^2}=1$$

$$x^2+\ y^2-\frac{40000}{c^2}=100$$

The z plane intersects the hyperboloid and gives a circle with radius 15 and equation $x^2+\ y=15^2$, we have:

$$100+\frac{40000}{c^2}=225, therefore\ c^2=320, and\ c=8\sqrt{5}$$

The equation of the hyperboloid is then

$$\frac{x^2}{100}+\frac{y^2}{100}-\frac{z^2}{320}=1$$

B)

$$x = \mathrm{acos}(u)\cosh(v)\,, \qquad y = bsin(u)\cosh(v)\,, \qquad z = csinh(v)$$

From the previous question, we had $a = 10, b = 10\ and\ c = 8\sqrt{5}$, therefore

$$\frac{x^2}{100} = \cos^2(u)\cosh^2(v)\,, \qquad \frac{y^2}{100} = sin^2(u)\cosh^2(v)\,, \qquad \frac{z^2}{320} = sinh^2(v)$$

$$\frac{x^2}{100}+\frac{y^2}{100}-\frac{z^2}{320} = cos^2(u)\cosh^2(v) + sin^2(u)\cosh^2(v) - sinh^2(v) \quad (Eq\ 1)$$

$$(\cosh^2(v))\big(cos^2(u) + sin^2(u)\big) - sinh^2(v)$$

We know that $cos^2(u) + sin^2(u) = 1$, so the equation becomes

$$\cosh^2(v) - sinh^2(v) = 1\ (hyperbolic\ identity)$$

The LHS and RHS are the same and are equal to 1, meaning that $cos^2(u)\cosh^2(v) + sin^2(u)\cosh^2(v) - sinh^2(v)\ and\ \frac{x^2}{100}+\frac{y^2}{100}-\frac{z^2}{320}$ lie on the same equation.

C) **A colleague at the same institution want to construct the cooling tower using an hyperbolic cylinder, give reasons for your result in Q1(A) as the best model for the design of cooling tower.**

The hyperboloid's shape improves efficiency by helping the upward air flow through an acceleration, which is more difficult to achieve with a cylindrical shape.
The hyperboloid of one sheet is a surface that is continuous and encloses itself. That shape helps the purpose of the cooling tower. Furthermore, since some cooling towers operates by cooling water to a lower temperature, the shape of the hyperboloid helps rapid conversion and dissipation.

<u>QUESTION 2</u>

A) Show that the rate of change of the mass m of the fluid contained in a region Q is

$$\boldsymbol{\frac{dm}{dt} = \iiint_Q \frac{\partial p}{\partial t}\,dv.}$$

The fluid has a density ρ, a function of time, space and velocity

$$mass\ of\ fluid = volume\ \times density$$

$$volume\ (V) = \iiint dxdydz$$

$$So\ total\ mass = \iiint \rho dxdydz$$

In region Q

$$velocity(V) = \frac{dx}{dt}$$

So the smallest time mass change is given by:

$$dm = \rho \frac{dx}{dt}$$

On the 3 axis, in 3D:

$$dm = \iiint \frac{dx}{dt} dydz\rho$$

The mass charge with time is:

$$\frac{dm}{dt} = \iiint \frac{\delta\rho}{\delta t} \cdot dv$$

B) Suppose further that, if the fluid crosses the boundary, show that

$$\boldsymbol{\frac{dm}{dt} = -\iint_{\partial Q} (pv).nds}$$

$$\frac{dm}{dt} = \iiint \frac{\delta\rho}{\delta t} \cdot dv$$

The mass decreases in the container as it flows across the boundary, so the integral will be negative:

$$\frac{dm}{dt} = -\iiint \frac{\delta\rho}{\delta t} \cdot dv$$

The fluid passes through a cross section, so

$$n \cdot ds = Area$$

$$with\ dA = n \cdot ds$$

$$Volume = height \times dA, therefore:$$

$$\frac{dm}{dt} = -\iint_{\delta Q} (\rho v) n \cdot ds$$

C) Using the result from Q2(A) and Q2(B), show that $\nabla \cdot (\boldsymbol{\rho v}) + \frac{\delta \rho}{\delta t}$=0

From questions a) and b)

$$\iint_{\delta} (\rho v) \cdot nds = \iiint \frac{\delta \rho}{\delta t} \cdot dv$$

Stokes' theorem gives us:

$$\iiint \nabla \cdot (\rho v) dv = -\iiint \frac{\delta \rho}{\delta t} \cdot dv$$

Thus,

$$\nabla \cdot (\rho v) + \frac{\delta \rho}{\delta t} = 0$$

D) Why the continuity for water is given by $\boldsymbol{\nabla \cdot (\rho v) = 0}$

The density of water doesn't depend on time. Also, water is an incompressible fluid, so $\frac{\delta \rho}{\delta t} = 0$

We then have:

$$\nabla \cdot (\rho v) + \frac{\delta \rho}{\delta t} = \nabla \cdot (\rho v) + 0$$

So

$$\nabla \cdot (\rho v) = 0$$