EE 287 Circuit Theory

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COURSE SYLLABUS

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Course Topics: Network Theorems – Superposition, Nodal Analysis, Mesh Analysis, Millman’s Theorem; AC Thevenin and Maximum Power Transfer Theorem; Transient Circuits and Analysis; Two-Port Networks; AC Response

Textbooks:

1. ***Engineering Circuit Analysis*, William H. Hayt, and Jack E. Kemmerly, International Student Edition**
2. ***Shaum’s Outline of Theory and Problems of Basic Circuit Analysis*, John O’malley, Ph.D**

COURSE SYLLABUS

Grading:

Exams	70%
Homework	10%
Midsemester Exams	15%
Attendance	5 %

Software: Multisim

INTRODUCTION
Types of Active Elements

- Independent Voltage Source: It is a two-terminal element that maintains a specified voltage between its terminals regardless of the current through it.

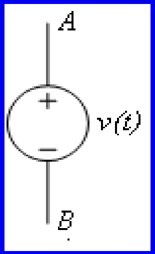


Figure 1 General Symbol

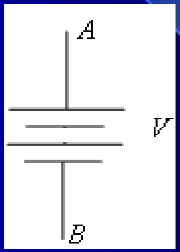


Figure 2 Constant Voltage

- Figure 1 shows the symbol for time varying voltage (General symbol).

- The voltage *v(t)* is referenced positive at A**.**

- Independent Current Source: It is a two-terminal element that maintains a specified current regardless of the voltage across its terminals.

- Figure 3 shows the general symbol, and the arrow indicates the direction of the current source when it is positive.

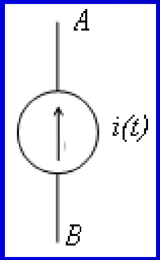
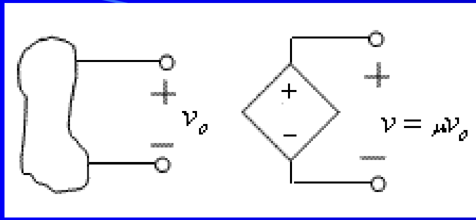


Figure 3 General Symbol

- Two Dependent or Controlled Voltage Sources: A dependent or controlled voltage source is a voltage source whose terminal voltage depends on, or is controlled by a voltage or a current at a specified location in the circuit.

- The two types are:
  - Voltage-Controlled Voltage Source (VCVS) controlled by a voltage.
  - Current-Controlled Voltage Source (CCVS) controlled by a current.



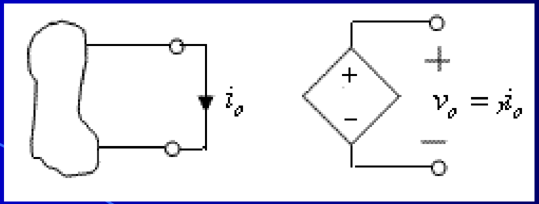


Figure 4 VCVS

Figure 5 CCVS

<u>Two Dependent or Controlled Current Sources</u>: A dependent or controlled current source is a current source whose current depends on, or is controlled by a voltage or current at a specified location in the circuit.

- The two types are:
  - Voltage-Controlled Current Source (VCCS) controlled by voltage.
  - Current-Controlled Current Source (CCCS) controlled by current.

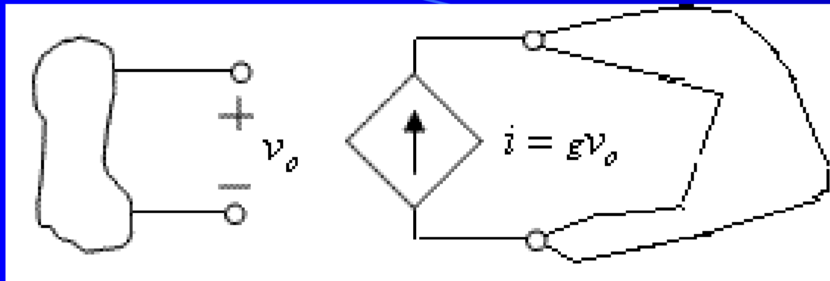


Figure 6 VCCS

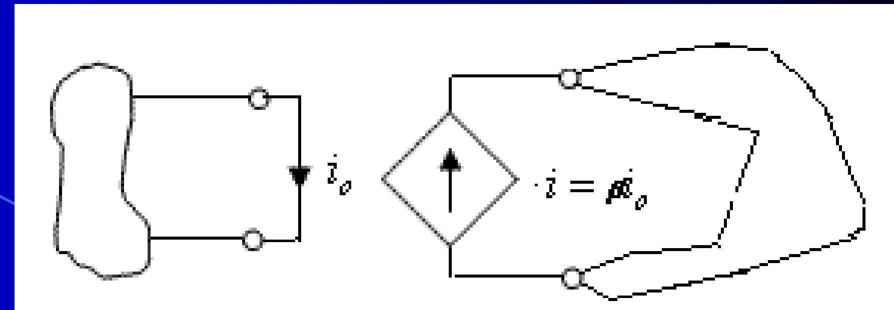
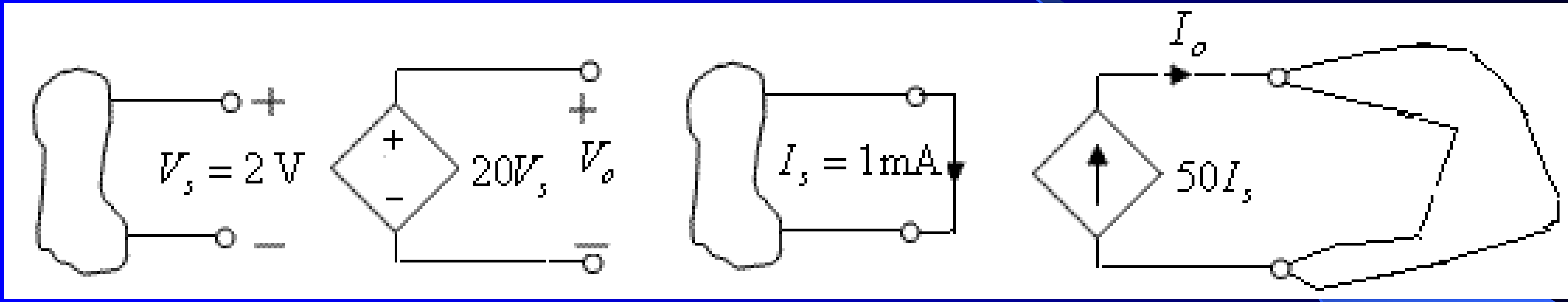


Figure 7 CCCS

- The dependent sources are very important because they form part of the mathematical models used to describe the behaviour of many electronic circuit devices.

- Example 1: For the networks given in the figure below, find V_o and I_o.



- Solution:

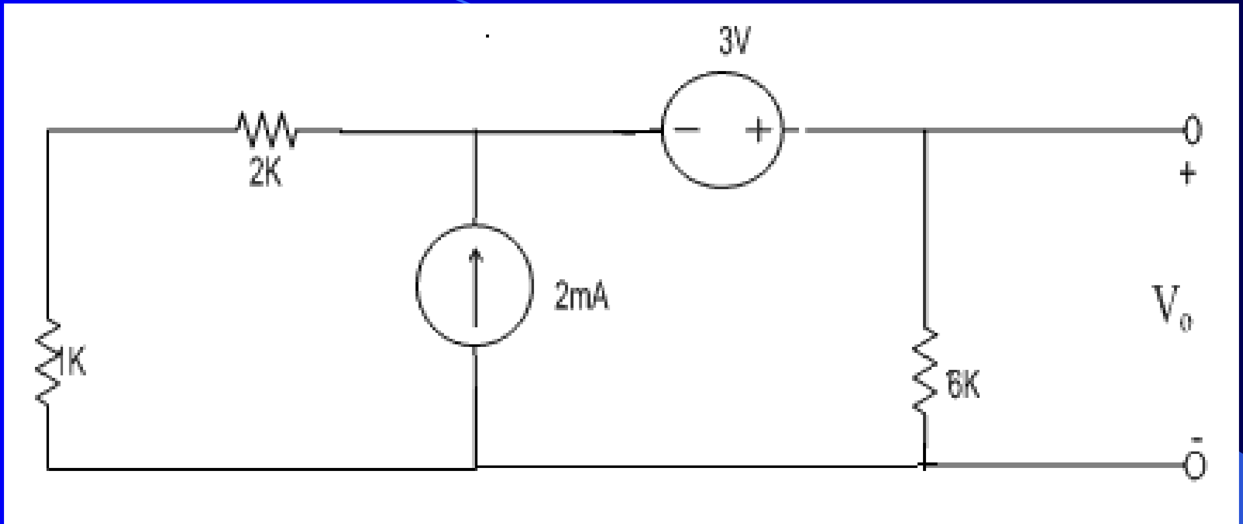
$$V_o = 20V_s = 20\times2 = 40 \text{ V}$$

$$I_o = 50I_s = 50\times1 = 50 \text{ mA}$$

Superposition Theorem

- Superposition theorem states that the current through, or the voltage across an element in a linear network is equal to the algebraic sum of the currents or voltages produced by each source acting alone.
- Superposition can be used for linear circuits containing dependent sources. However, it is not useful in this case because the dependent source is never made zero.

- Example 2: Use superposition to find V_O



Solution:

<u>Current source acting alone:</u>

Current in the 6 k: use current divider rule

$$I_o' = \frac{(1+2)}{(1+2)+6} \times 2 = \frac{3\times 2}{9} = \frac{2}{3}\,\mathrm{mA}$$

$$V_o' = \frac{2}{3}\,\mathrm{mA} \times 6\,\mathrm{k} = 4\,\mathrm{V}$$

Voltage source acting alone:

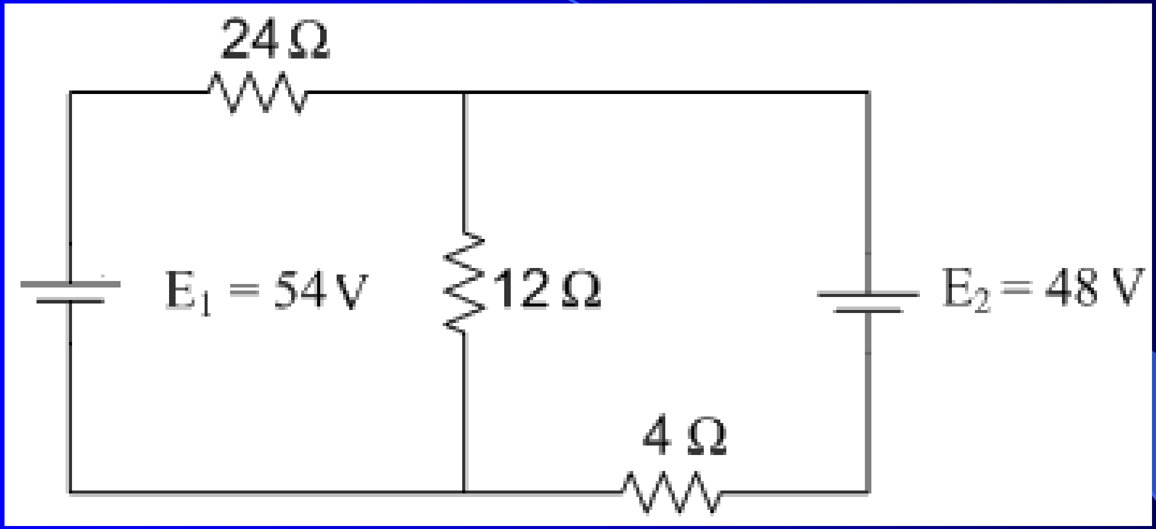
All resistances are in series when the current source is deactivated

$$I_o'' = \frac{3}{1+2+6} = \frac{3}{9}\ \text{mA}$$

$$V_o'' = \frac{3}{9}\,\text{mA} \times 6\,\text{k} = 2\,\text{V}$$

$$V_o = V_o' + V_o'' = 4 + 2 = 6\,\text{V}$$

- Example 3: Using the superposition principle, determine the current through the 4-Ω resistor of Fig. 9



With the 54-V source acting

total resistance = $24 + 12//4 = 24 + \frac{12 \times 4}{16} = 27\ \Omega$

and total current = $\frac{54}{27} = 2\ \text{A}$

By current divider rule, current in the 4-Ω resistor

$$I' = \frac{12}{12+4}\times 2 = \frac{12\times 2}{16} = 1.5\,\text{A}$$

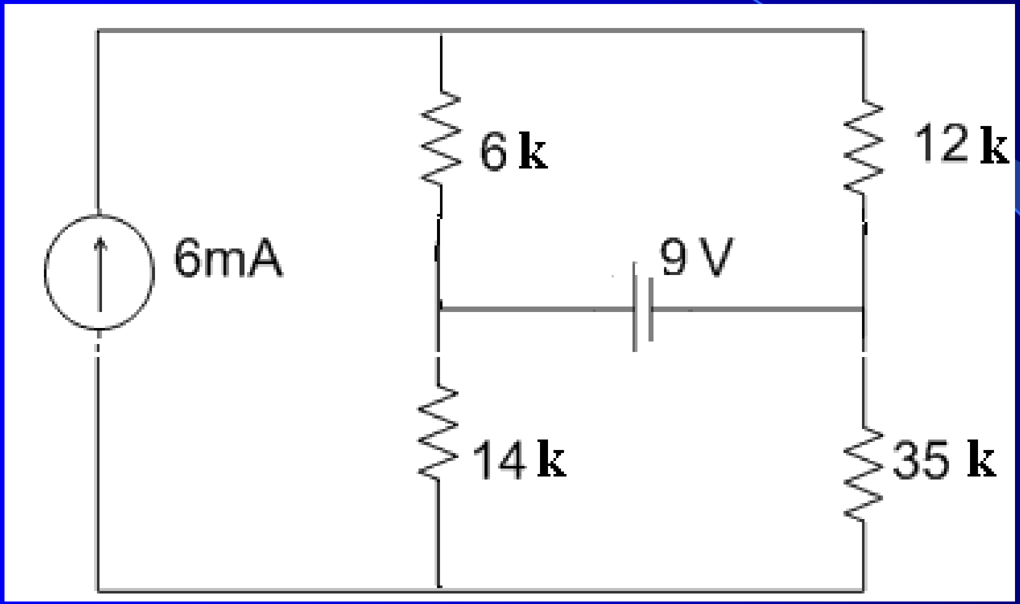
48-V source acting alone

$$\text{total resistance} = 4 + 24//12 = 4 + \frac{24\times 12}{36} = 12\,\Omega$$

$$\text{total current} = \text{current in the } 4-\Omega \text{ resistor } I'' = \frac{48}{12} = 4\,\text{A}$$

$$I = I'' - I' = 4 - 1.5 = 2.5\,\text{A}$$

Example 4: Using the principle of superposition, find the current through the 12-k resistor



<u>The current source acting alone:</u>

6k and 12 k are in parallel

$$I' = \frac{6}{12+6} \times 6 = \frac{6 \times 6}{18} = 2$$

The 9-V source acting alone

$$I'' = \frac{9}{6+12} = \frac{9}{18} = 0.5\,\mathrm{mA}$$

$$I = I' + I'' = 2 + 0.5 = 2.5\,\mathrm{mA}$$

Millman's Theorem

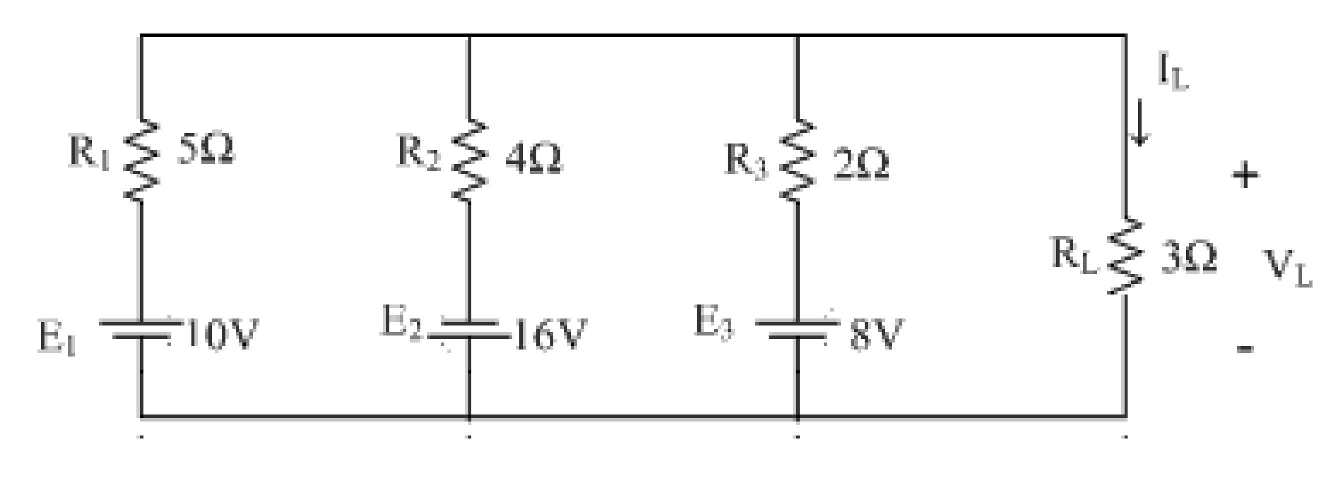
- This theorem may be used to reduce any number of parallel voltage sources to one.
- This would permit finding the current through or voltage across a load resistor.
- In general, Millman's theorem states the following: Any number of parallel voltage sources can be reduced to a single voltage source whose internal resistance R_{eq} and emf E_{eq} are given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots\ldots\ldots \frac{1}{R_N} \text{ and}$$

$$E_{eq} = \left[\pm \frac{E_1}{R_1} \pm \frac{E_2}{R_2} \pm \ldots\ldots\ldots \pm \frac{E_N}{R_N} \right] \times R_{eq}$$

Use the plus for sources supplying energy in one direction and the minus for sources supplying energy in the opposite direction

- Example 5: Using Millman's theorem, find the current through and the voltage across the resistor R_L

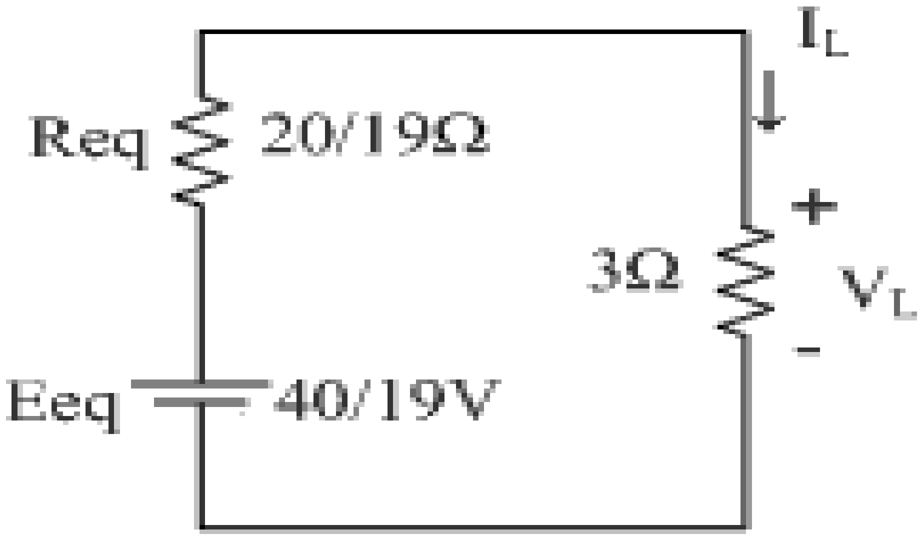


Solution:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_{eq}} = \frac{1}{5} + \frac{1}{4} + \frac{1}{2} = \frac{4+5+10}{20} = \frac{19}{20}.$$

Hence $R_{eq=}\frac{20}{19}\Omega$

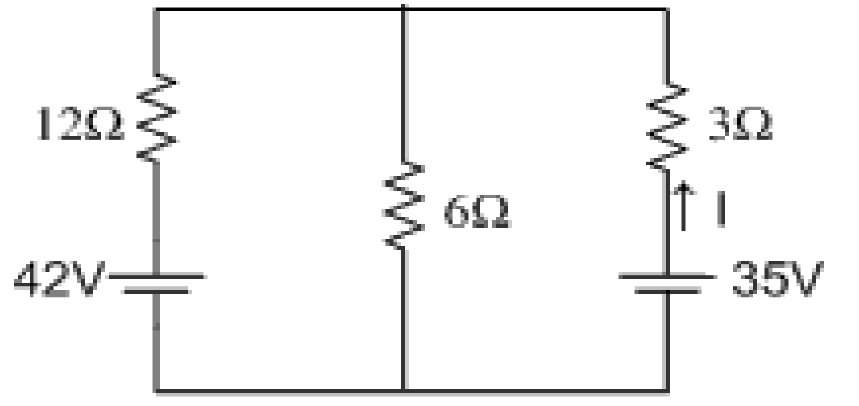
$$E_{eq} = \left[\frac{E_1}{R_1} - \frac{E_2}{R_2} + \frac{E_3}{R_3}\right] R_{eq} = \left[\frac{10}{5} - \frac{16}{4} + \frac{8}{2}\right] \times \frac{20}{19} = \frac{40}{19}\text{V}$$



$$I_L = \frac{40}{19}\left[\frac{1}{\frac{20}{19}+3}\right] = \frac{40}{77} = 0.519\ \text{A} \ \text{ and}$$

$$V_L = I_L V_L = \frac{40}{77} \times 3 = 1.558\ \text{V}$$

- *Example 6: Using Millman's theorem find that current through and voltage across the 6-Ω resistor*

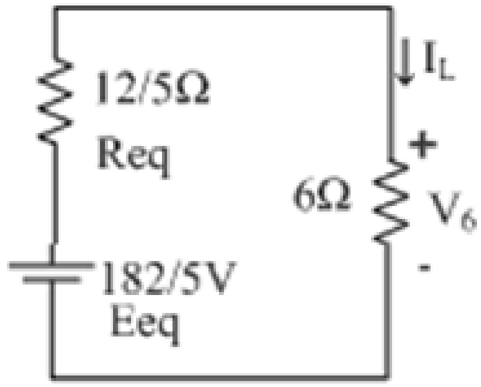


Solution

$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{3} = \frac{5}{12}$$

$$R_{eq} = \frac{12}{5}\Omega = 2.4\Omega$$

$$E_{eq} = \left[\frac{42}{12} + \frac{35}{5}\right] \times \frac{12}{5} = \frac{182}{5}$$



$$I = \frac{182}{5}\left[\frac{1}{\frac{12}{5} + 6}\right] = \frac{13}{3} = 4.33\text{ A}$$

$$V_6 = 6I = 6 \times \frac{13}{3} = 26\text{ V}$$

Nodal Analysis

- The nodal analysis is a method used to solve circuits containing multiple nodes and loops.

- This is based on Kirchoff's Current Law (KCL) and Kirchoff's Voltage Law (KVL).

Definition of terms used

- A branch: It is a portion of a circuit containing a single element or more elements in series
- A node: It is a point of connection of two or more branches or circuit elements
- Note that all the connecting wire in unbroken contact with the point is part of the node.

General approach to nodal analysis

The nodal analysis is applied as follows:

(a) Determine the number of nodes within the network, N.

(b) Select a reference node and label the others as V_1, V_2, V_3,.....V_{N-1}

- A reference node is often chosen to be the node to which the largest number of branches is connected.
- In a practical electronic circuit, this usually corresponds to the chassis or ground line.

General approach to nodal analysis

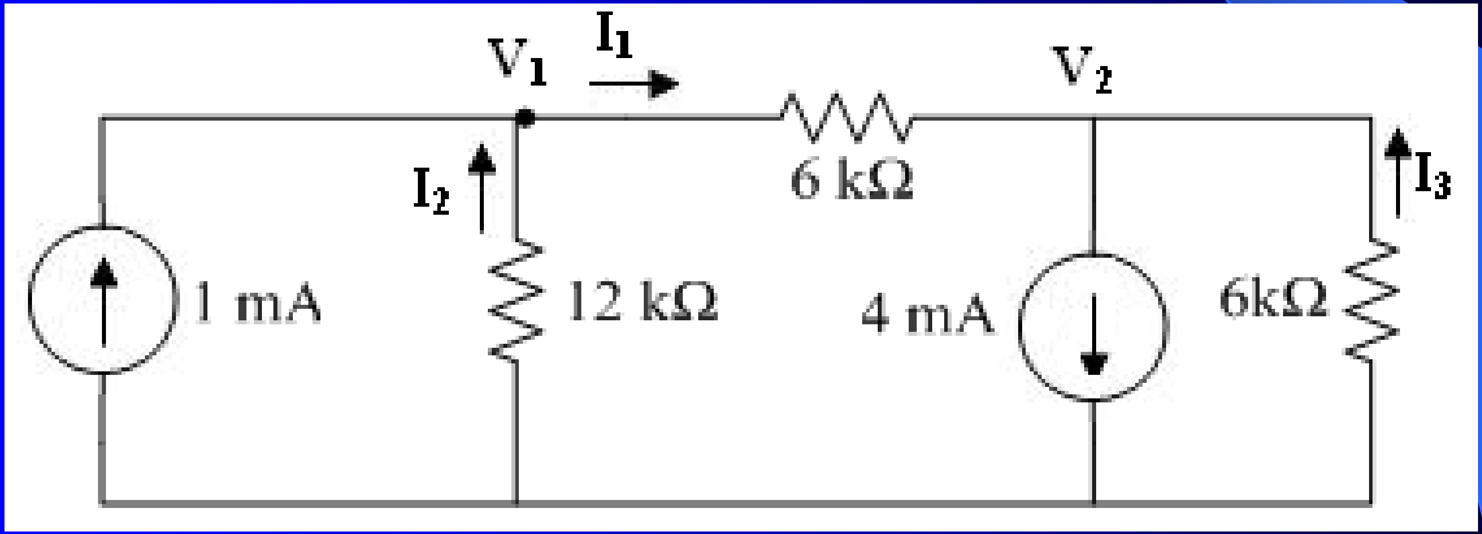
- In many cases such as in electric power systems, the chassis is shorted to the earth itself making its potential zero. For this reason, the reference node is frequently referred to as ground or the ground node.

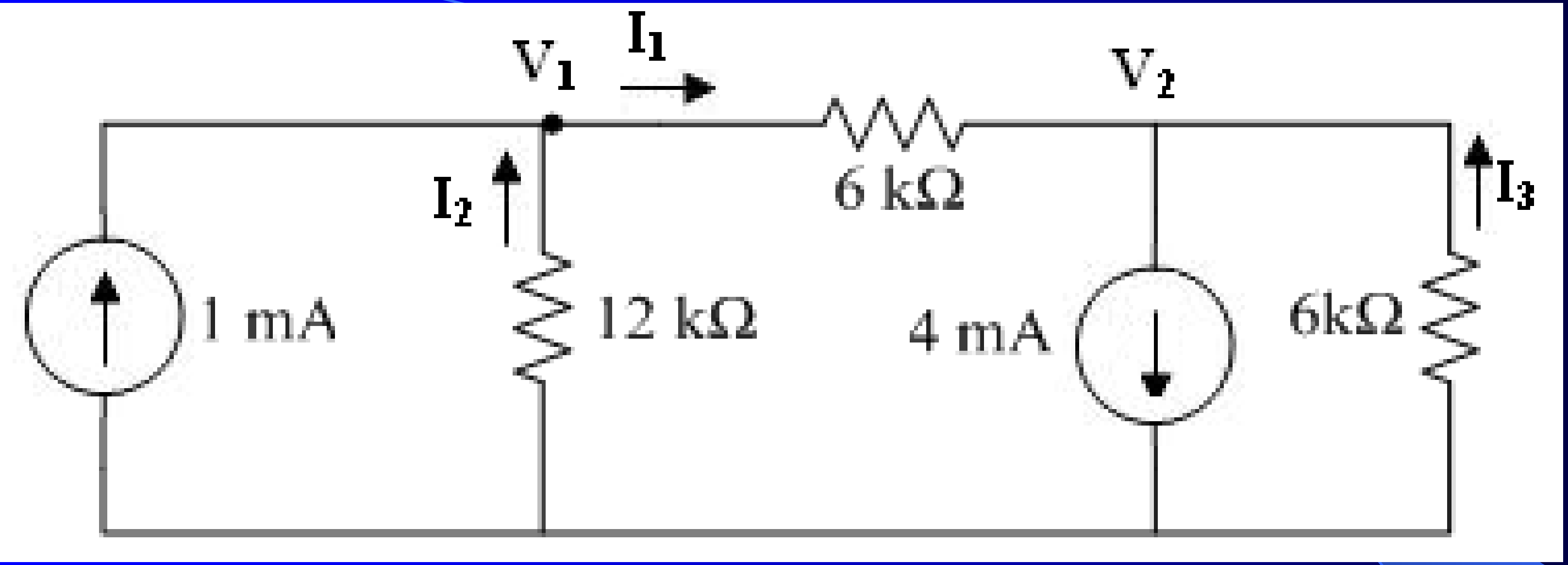
(c) Apply KCL at each node except the reference, assuming that all unknown currents leave the node for each application of KCL.

(d) Solve the resulting equations of the nodal or node voltages.

Circuits containing only independent current sources

- General Approach
- Example 7: Apply nodal analysis to the network



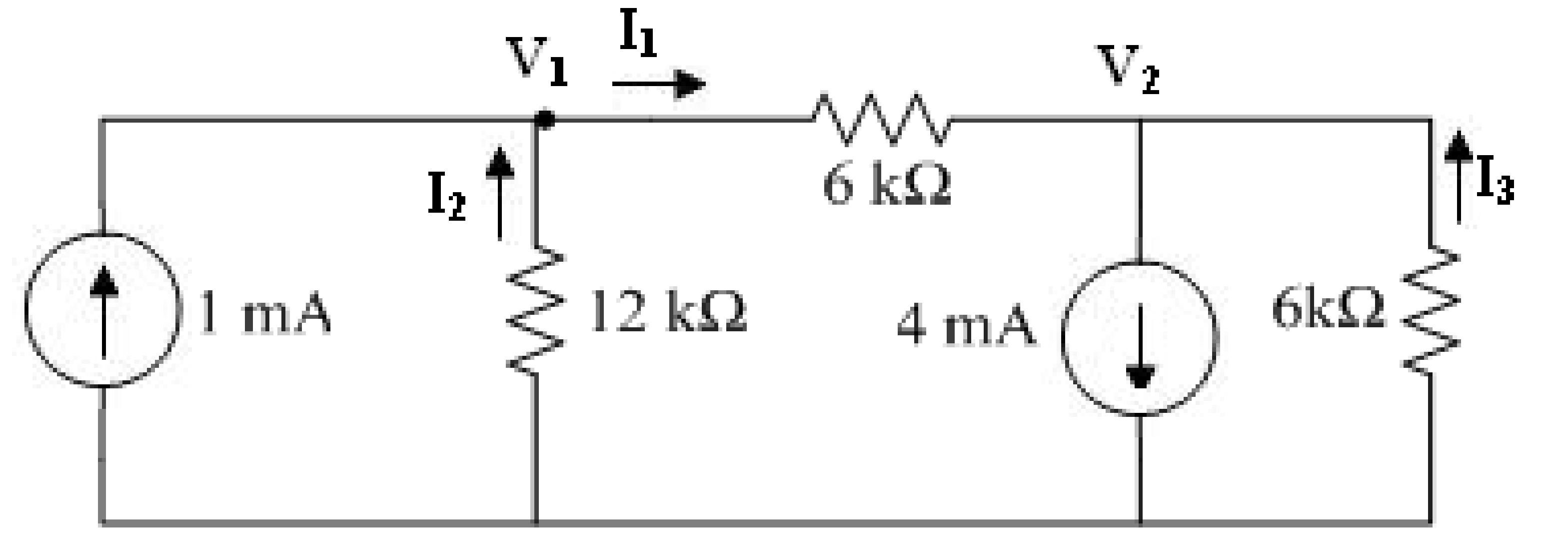


Apply KCL at node 1:

$$1 = \frac{V_1}{12} + \frac{V_1 - V_2}{6} \text{ or } 1 = V_1\left[\frac{1}{12} + \frac{1}{6}\right] - \frac{V_2}{6} \text{ or } 1 = \frac{V_1}{4} - \frac{V_2}{6} \quad (1)$$

Apply KCL at node 2:

$$-4 = \frac{V_2}{6} + \frac{V_2 - V_1}{6} \text{ or } -4 = -\frac{V_1}{6} + V_2\left[\frac{1}{6} + \frac{1}{6}\right] \text{ or } -4 = -\frac{V_1}{6} + \frac{V_2}{3}$$



$$(1)\times 2+(2):-2=\frac{V_1}{3}\Rightarrow V_1=-6\,\mathrm{V}$$

From (1) $\frac{V_2}{6}=\frac{V_1}{4}-1=-\frac{6}{4}-1=-\frac{5}{2}\Rightarrow V_2=-\frac{5}{2}\times 6=-15\,\mathrm{V}$

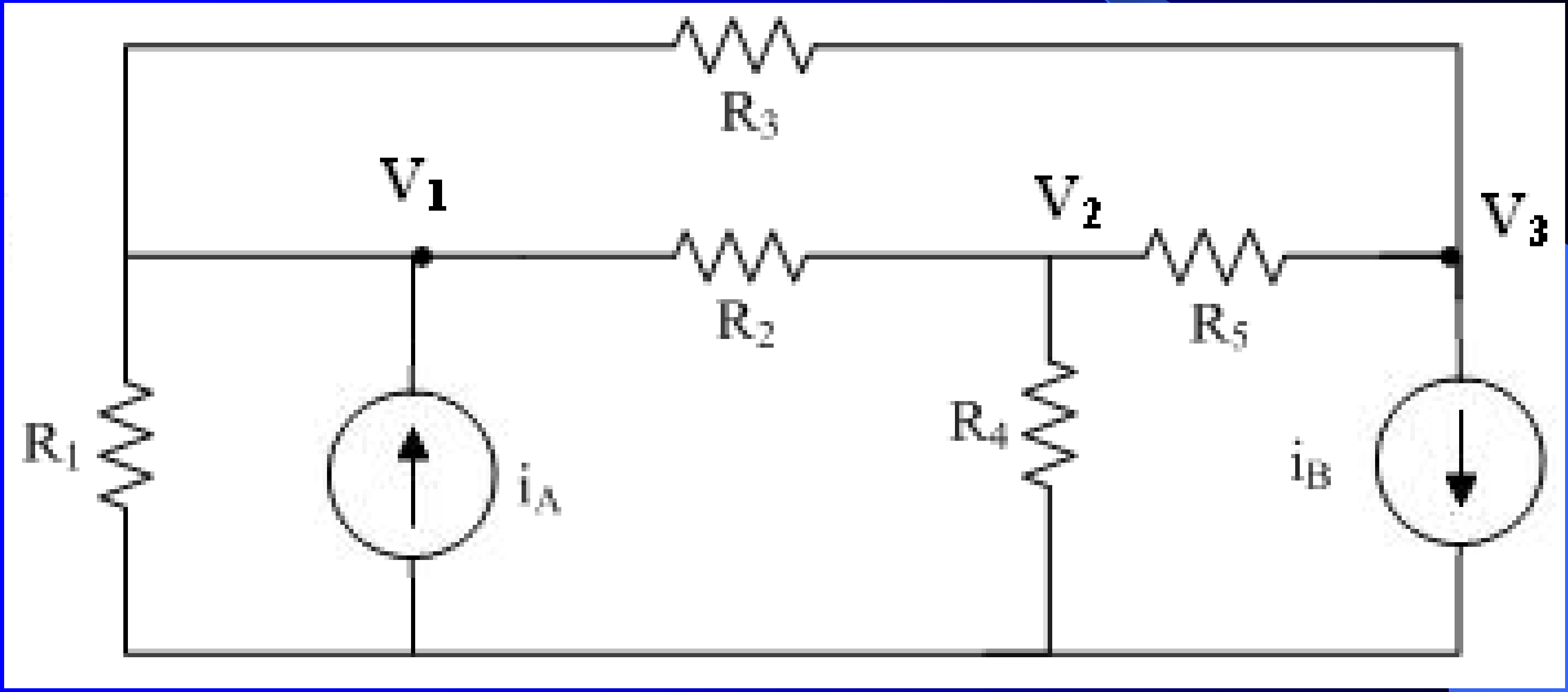
$$\text{Current }\ I_2=\frac{0-V_1}{12}=\frac{0-(-6)}{12}=\frac{6}{12}=0.5\,\mathrm{mA}$$

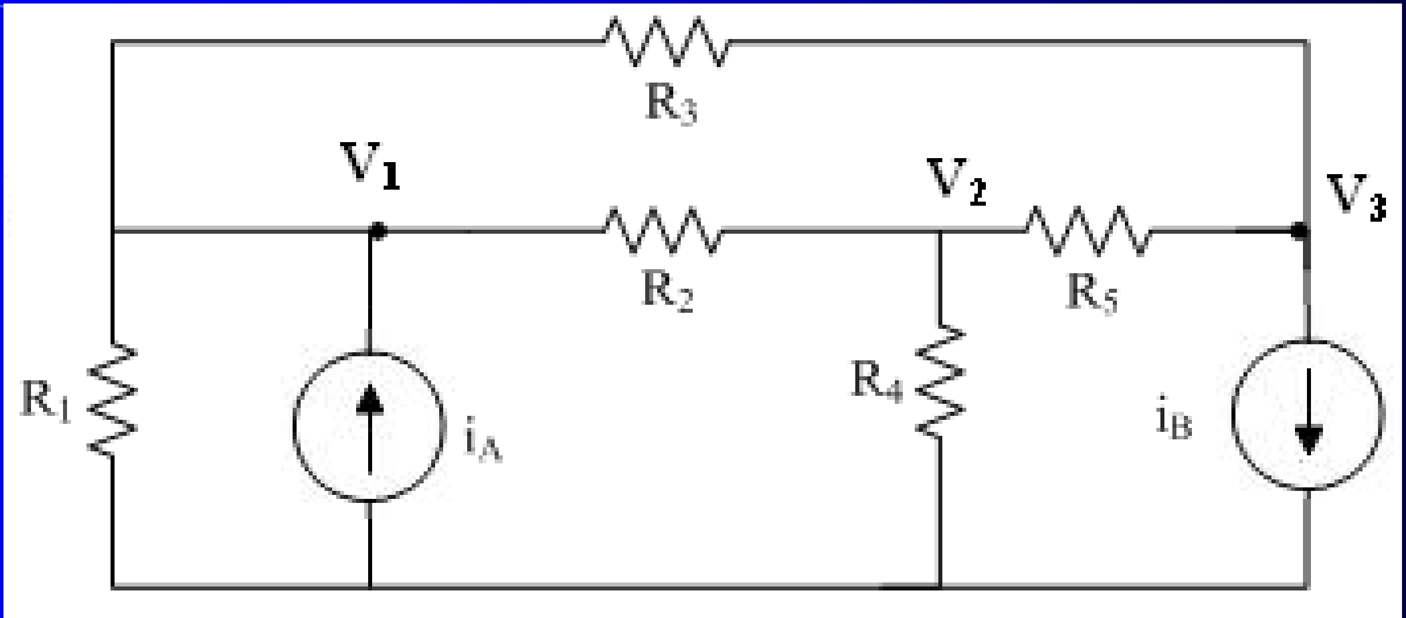
$$I_1=1+I_2=1+0.5=1.5\,\mathrm{mA}\text{ or use }\frac{V_1-V_2}{6}=\frac{9}{6}=1.5\,\mathrm{mA}$$

$$I_3=4-I_1=4-1.5=2.5\,\mathrm{mA}\text{ or use }\frac{0-V_2}{6}=\frac{15}{6}=\frac{5}{2}=2.5\,\mathrm{mA}$$

Circuits containing only independent current sources

- General Approach Cont'd
- Example 8: Apply nodal analysis to the network





Solution:

$$i_A = \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + \frac{v_1 - v_3}{R_3} = \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] v_1 - \frac{1}{R_2}\, v_2 - \frac{1}{R_3} v_3$$

$$0 = \frac{v_2}{R_4} + \frac{v_2 - v_1}{R_2} + \frac{v_2 - v_3}{R_5} = -\frac{1}{R_2} v_1 + \left[\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right] v_2 - \frac{1}{R_5} v_3$$

$$-i_B = \frac{v_3 - v_2}{R_5} + \frac{v_3 - v_1}{R_3} = -\frac{1}{R_3} v_1 - \frac{1}{R_5}\, v_2 + \left[\frac{1}{R_3} + \frac{1}{R_5}\right] v_3$$

In matrix form, the equations are as follows:

$$\begin{bmatrix} i_A \\ 0 \\ -i_B \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & -\frac{1}{R_3} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5} & -\frac{1}{R_5} \\ -\frac{1}{R_3} & -\frac{1}{R_5} & \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

The equation is in the form

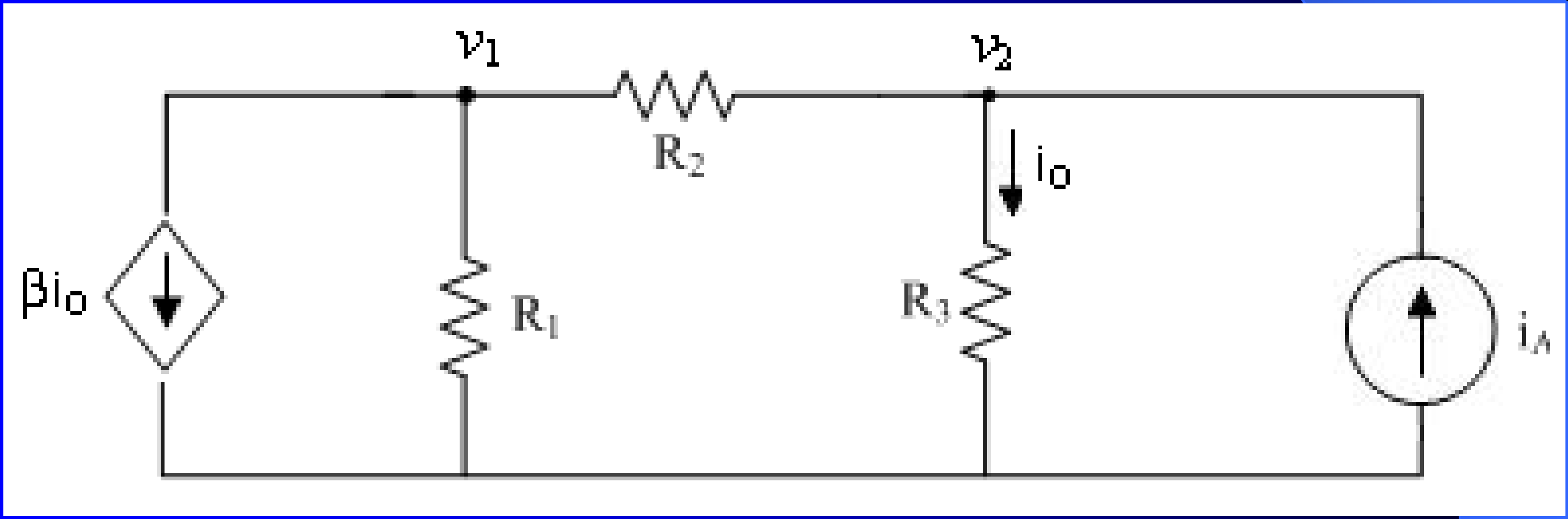
$$[i] = [G][v]$$

The matrix G is called conductance or admittance matrix

Circuits containing dependent current sources

- General approach is used

Example 9: Obtain the nodal equations for the network given below



Solution

Apply KCL at node 1:

$$-\beta i_0 = \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} \text{ or } -\beta \frac{v_2}{R_3} = \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

Or

$$0 = \left[\frac{1}{R_1} + \frac{1}{R_2}\right] v_1 - \left[\frac{1}{R_2} - \frac{\beta}{R_3}\right] v_2$$

Apply KCL at node 2:

$$i_A = \frac{v_2}{R_3} + \frac{v_2 - v_1}{R_2} = -\frac{1}{R_2} v_1 + \left[\frac{1}{R_2} + \frac{1}{R_3}\right] v_2$$

$$0 = \left[\frac{1}{R_1} + \frac{1}{R_2}\right] v_1 - \left[\frac{1}{R_2} - \frac{\beta}{R_3}\right] v_2$$

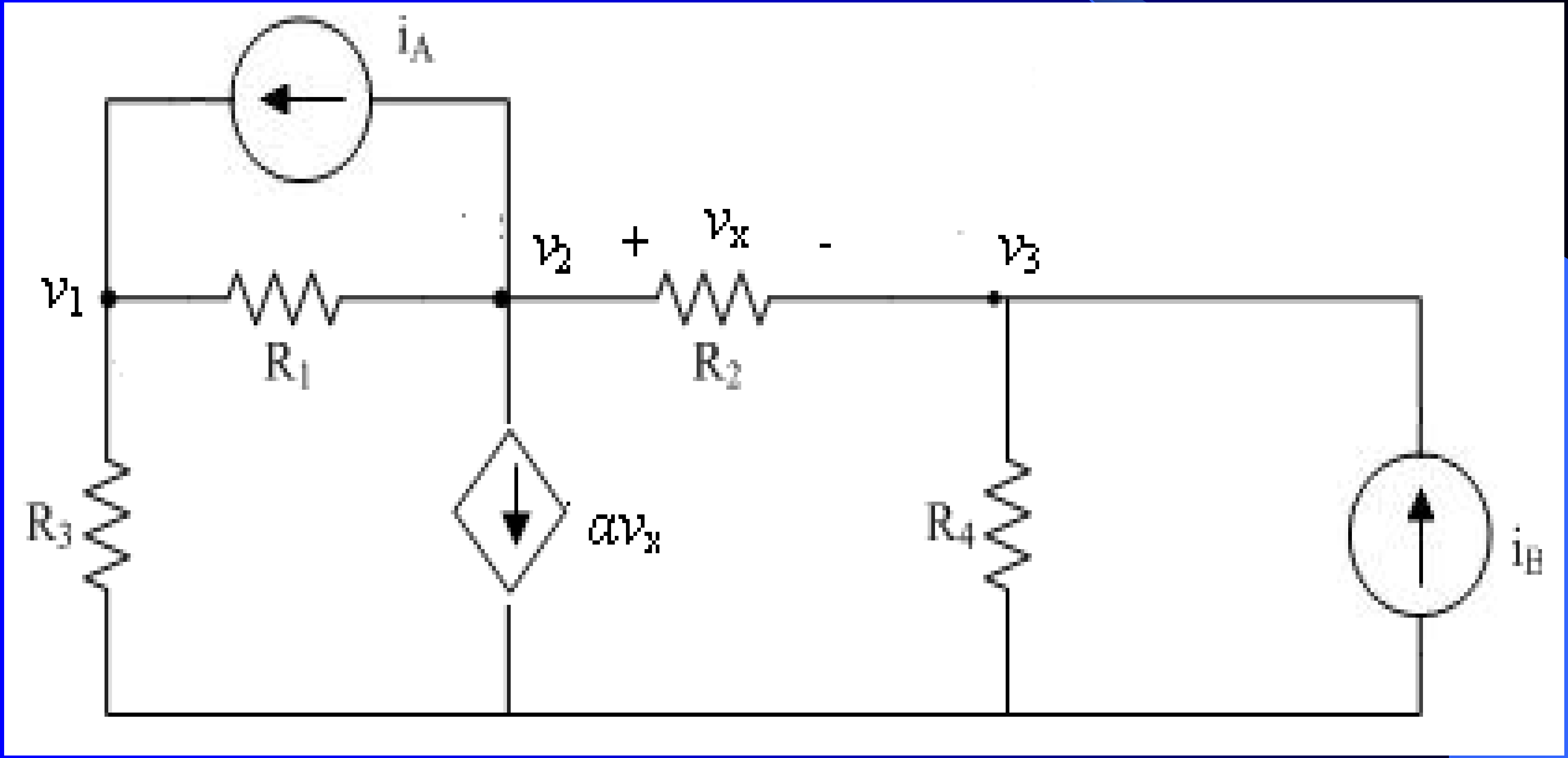
$$i_A = -\frac{1}{R_2} v_1 + \left[\frac{1}{R_2} + \frac{1}{R_3}\right] v_2$$

In matrix form, we have:

$$\begin{bmatrix} 0 \\ i_A \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{R_2}\right) & -\left(\frac{1}{R_2} - \frac{\beta}{R_3}\right) \\ -\frac{1}{R_2} & \left(\frac{1}{R_2} + \frac{1}{R_3}\right) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Circuits containing dependent current sources

Example 10: Obtain the nodal equations for the network



Solution:

Apply KCL at node 1:

$$i_A = G_3 v_1 + G_1(v_1 - v_2) \text{ or } i_A = (G_1 + G_3)v_1 - G_1 v_2$$

Apply KCL at node 2:

$$-i_A - \alpha v_x = G_1(v_2 - v_1) + G_2(v_2 - v_3)$$

$$-i_A = G_1(v_2 - v_1) + G_2(v_2 - v_3) + \alpha(v_2 - v_3)$$

$$-i_A = -G_1 v_1 + (G_1 + G_2 + \alpha)v_2 - (G_2 + \alpha)v_3$$

Apply KCL at node 3

$$i_B = G_2(v_3 - v_2) + G_4 v_3 = -G_2 v_2 + (G_2 + G_4)v_3$$

At node 1:

$$i_A = (G_1 + G_3)v_1 - G_1 v_2$$

At node 2:

$$-i_A = -G_1 v_1 + (G_1 + G_2 + \alpha)v_2 - (G_2 + \alpha)v_3$$

At node 3

$$i_B = -G_2 v_2 + (G_2 + G_4)v_3$$

In matrix form, we have:

$$\begin{bmatrix} i_A \\ -i_A \\ i_B \end{bmatrix} = \begin{bmatrix} G_1 + G_3 & -G_1 & 0 \\ -G_1 & G_1 + G_2 + \alpha & G_2 + \alpha \\ 0 & -G_2 & G_2 + G_4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Circuits containing independent voltage sources

- The presence of voltage sources reduces the number of equations and the number of unknowns by one per voltage source.

- One of the following two approaches may be used:
  - Using the concept of supernode
  - Converting voltage sources to current sources. This is applicable when the independent voltage sources have resistances in series

Using the concept of supernode

- Voltage sources are enclosed in separate regions. Each closed region is called a supernode.

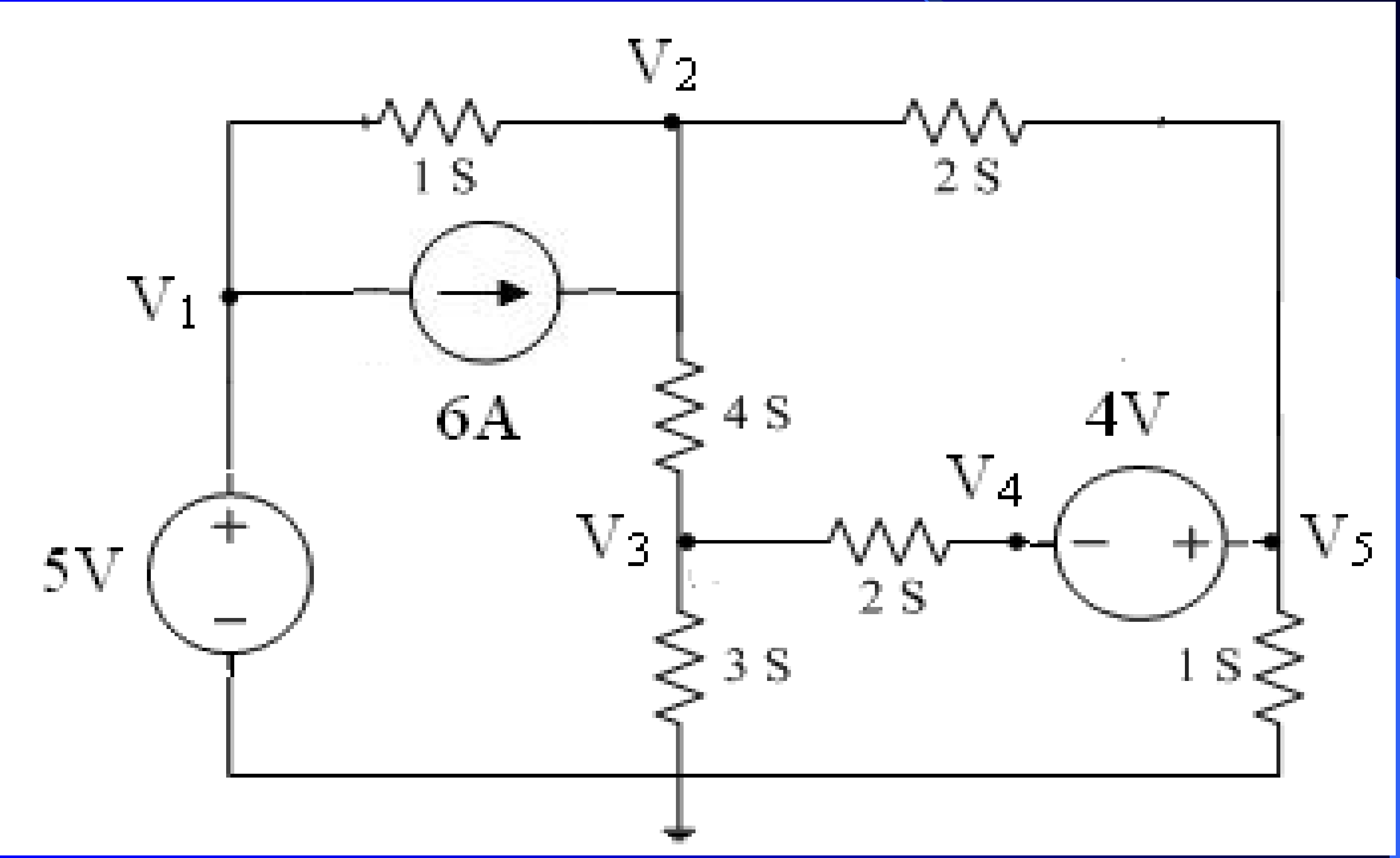
- A supernode contains two nodes which may consist of
  - a non-reference node and a reference node or
  - two non–reference nodes.

Using the concept of supernode cont'd

- To obtain the nodal equations, we apply KCL to
  - all supernodes not containing the reference node and
  - to all other reference nodes.
- KCL is applied to a supernode using the generalized form of KCL which states that algebraic sum of currents entering a closed region is zero.

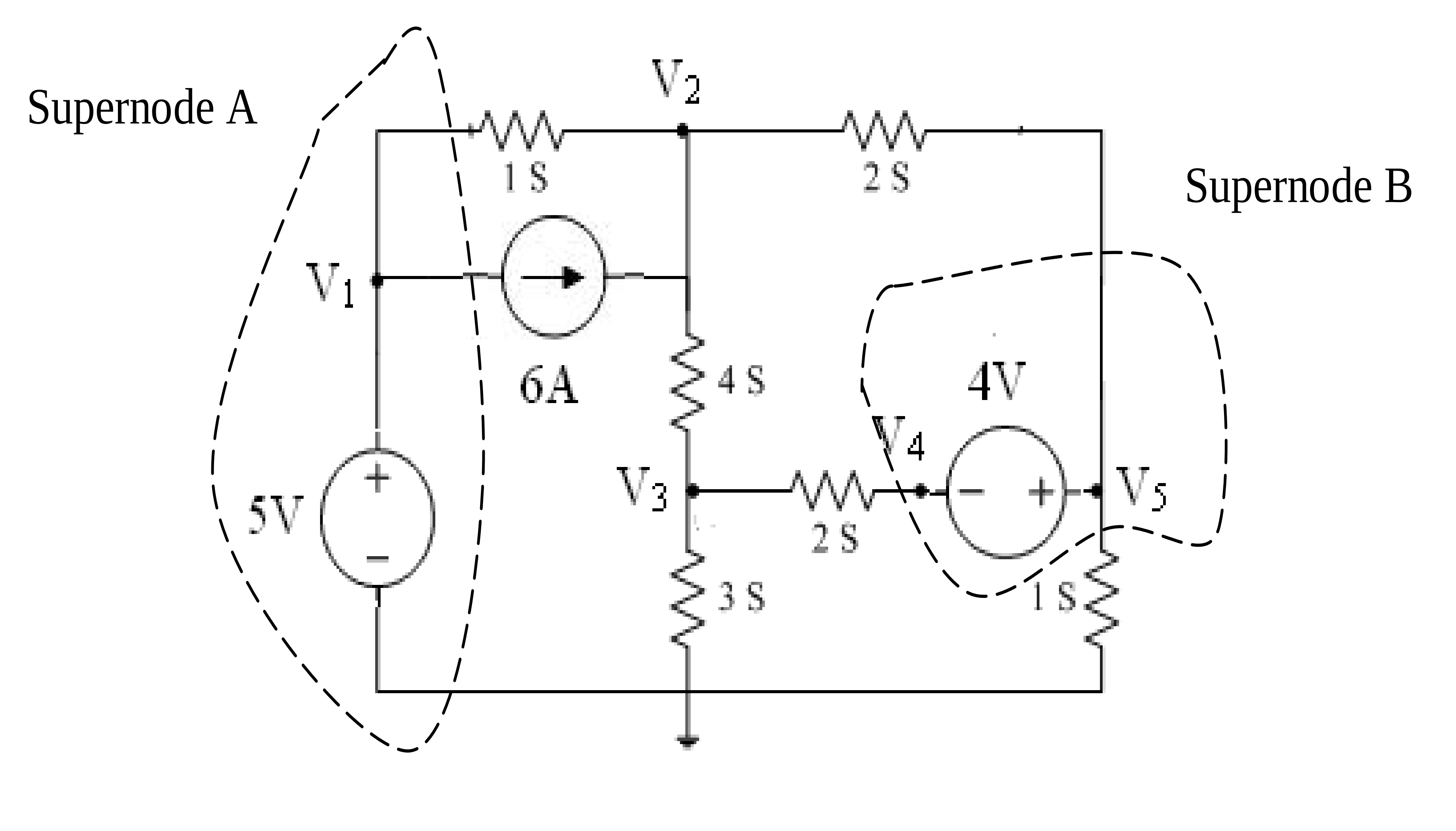
Using the concept of supernode cont'd

Example 11: Obtain the nodal equations for the network given below



Solution

- Two supernodes A and B are shown.



Voltage V_1 at node 1 in supernode A (containing the reference node) may be determined immediately as

$$V_1 = 5 + 0 = 5 \text{ V}$$

Voltage V_5 at node 5 in supernode B can be expressed in terms of V_4 at the other non-reference node 4 in the supernode as

$$V_5 = V_4 + 4$$

The number of nodal equations required $= 5 - 2 = 3$

Apply KCL to non-reference node 2:

$$6 = 1(V_2 - 5) + 4(V_2 - V_3) + 2(V_2 - V_4 - 4)$$

$$19 = 7V_2 - 4V_3 - 2V_4$$

Apply KCL to non–reference node 3:

$$0 = 3V_3 + 4(V_3 - V_2) + 2(V_3 - V_4)$$

$$0 = -4V_2 + 9V_3 - 2V_4$$

Apply KCL to supernode B:

$$0 = 2(V_4 - V_3) + 1(V_4 + 4) + 2[(V_4 + 4) - V_2]$$

$$-12 = -2V_2 - 2V_3 + 5V_4$$

At node 2:

$$19 = 7V_2 - 4V_3 - 2V_4$$

At node 3:

$$0 = -4V_2 + 9V_3 - 2V_4$$

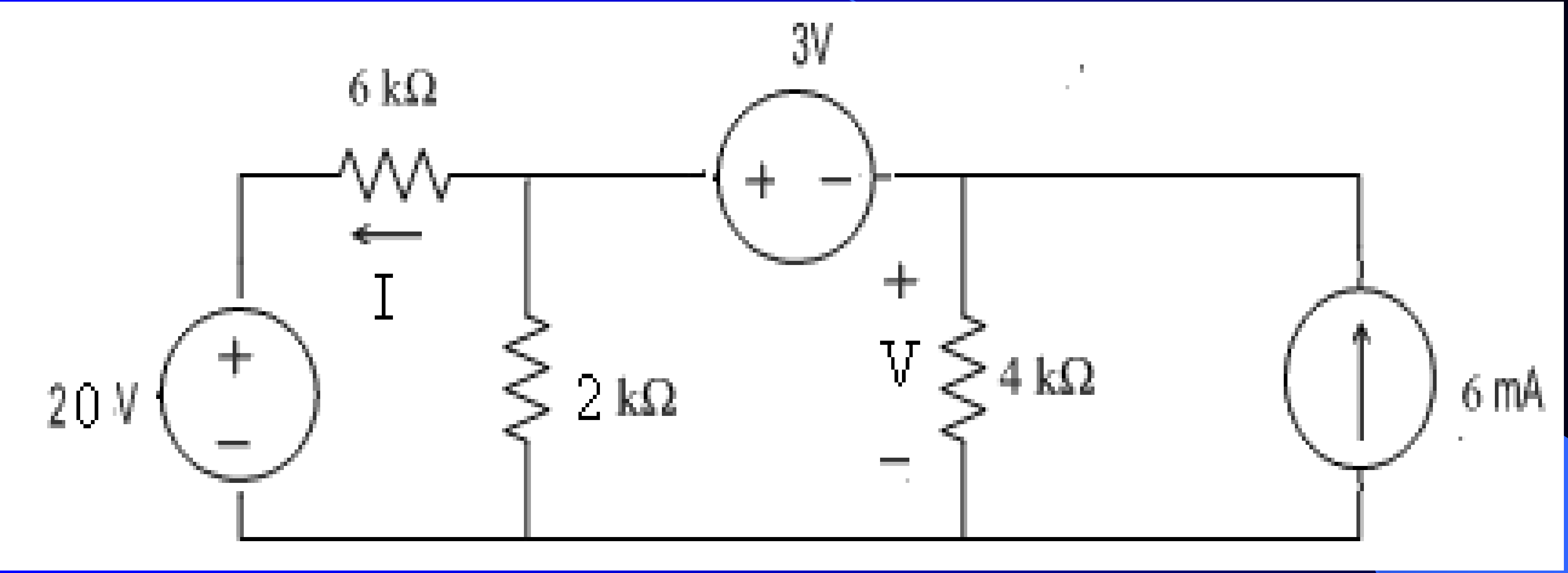
At supernode B:

$$-12 = -2V_2 - 2V_3 + 5V_4$$

In matrix form, we have:

$$\begin{bmatrix} 19 \\ 0 \\ -12 \end{bmatrix} = \begin{bmatrix} 7 & -4 & -2 \\ -4 & 9 & -2 \\ -2 & -2 & 5 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

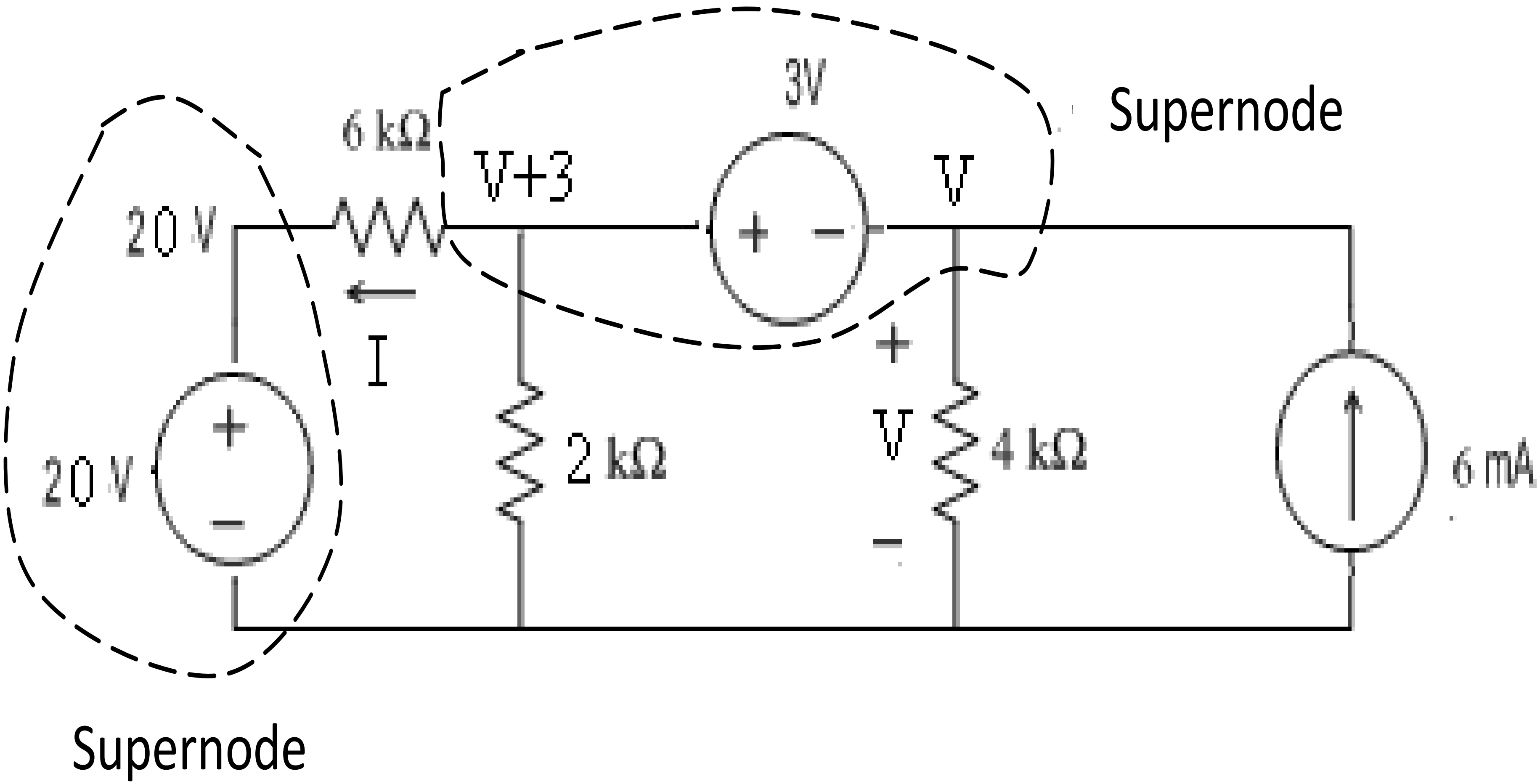
Example 12: Find V and I in the circuit using nodal analysis



Solution

- Network has two supernodes and we must apply KCL to the supernode which does not contain reference node.

Solution



$$6 = \frac{V}{4} + \frac{V+3}{2} + \frac{[(V+3) - \ 20]}{6}$$

$$6 \ \times 12 = 3V + 6(V+3) + \ 2[(V+3) - \ 20]$$

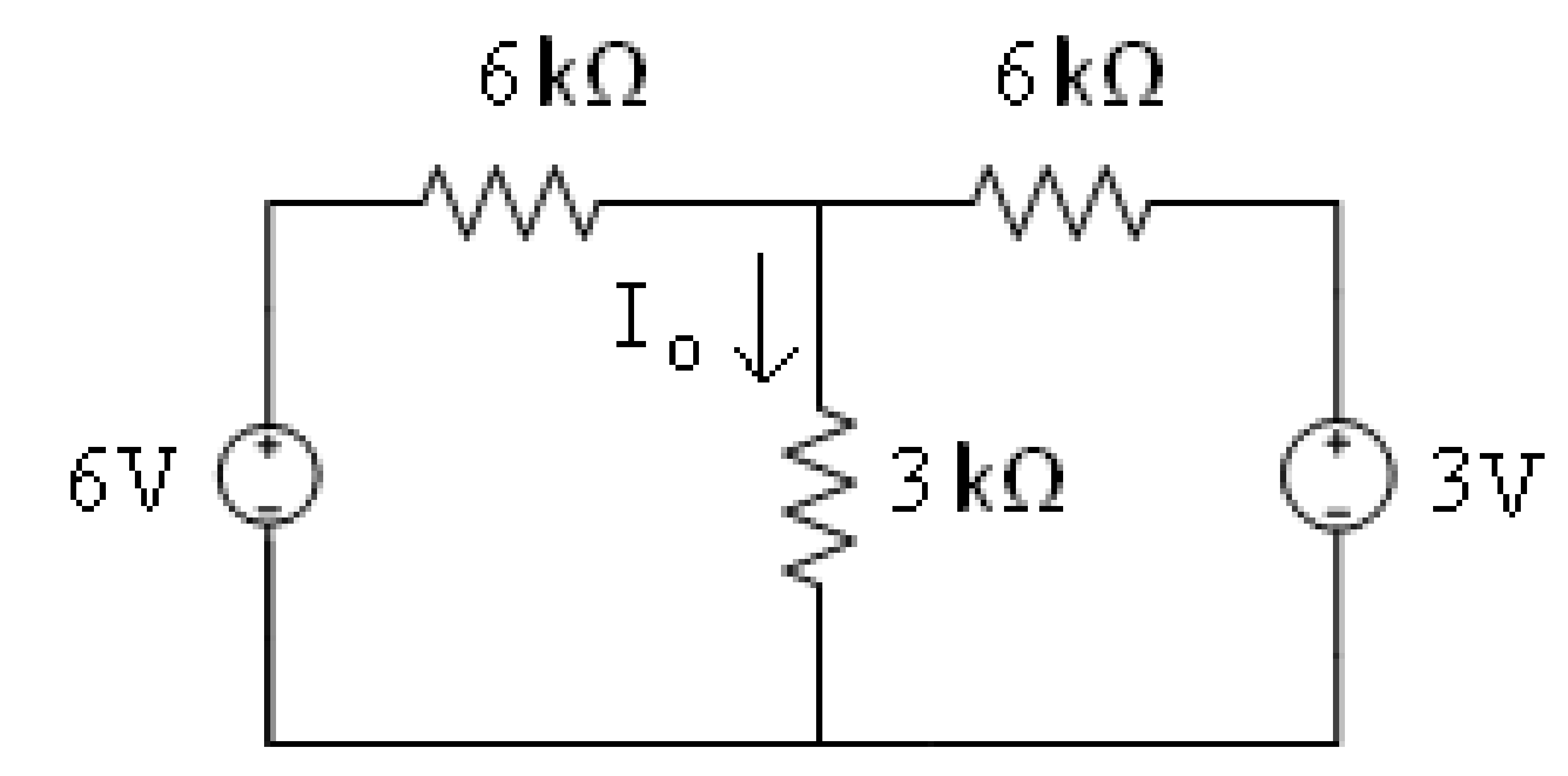
$72 = 11V - 16$ or $11V = 88$ or $V = 8$ volts

The current

$$I = \frac{[(V+3) - \ 20]}{6} = \frac{[(8+3) - \ 20]}{6}$$

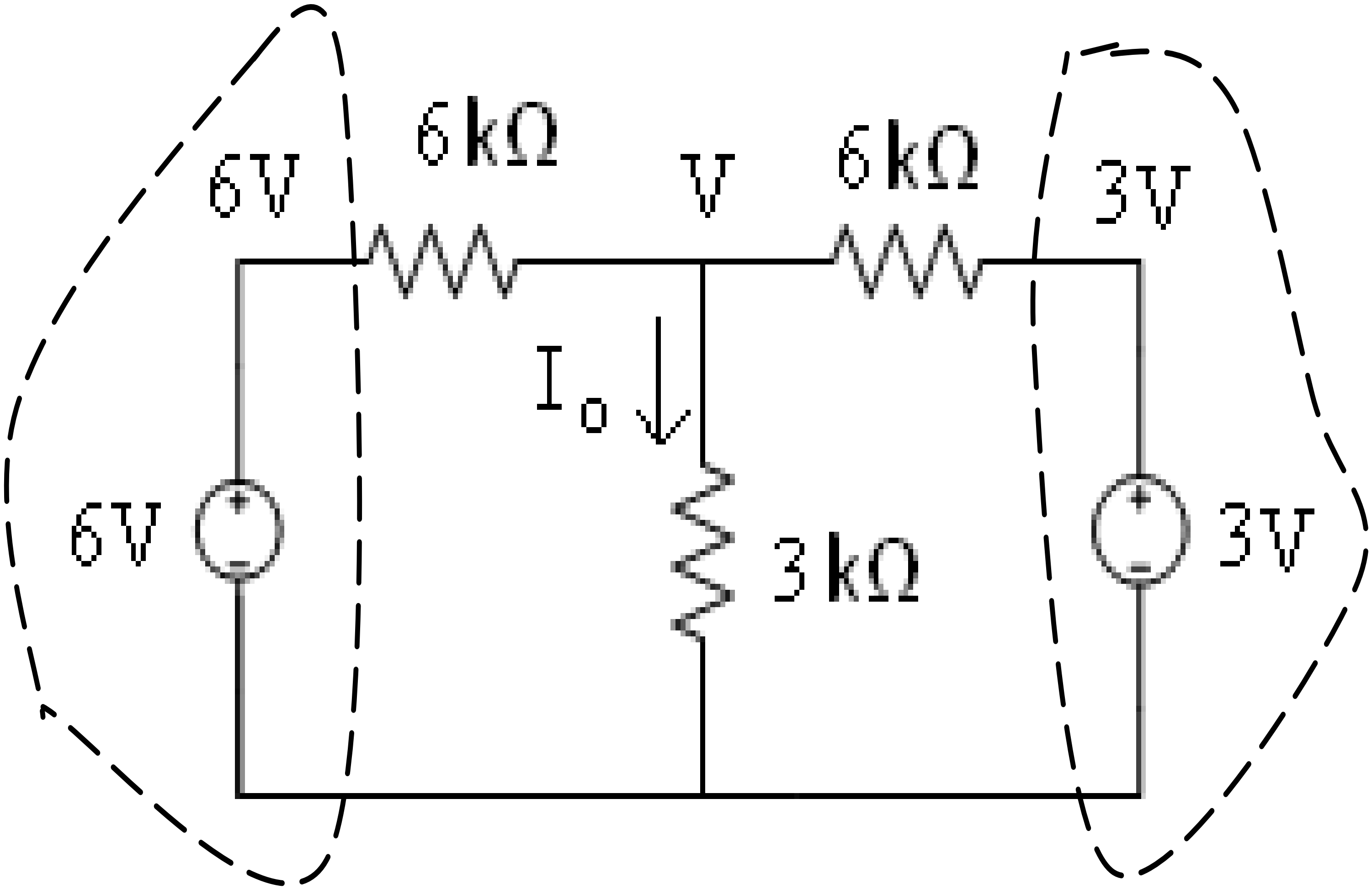
$$= -\frac{9}{6} = 1.5 \text{ mA}$$

Example 13: Find the current I_0 in the network



Solution

- Circuit has two supernodes all containing reference node and one non-reference node.


6kΩ
6V
V
6kΩ
3V
I o
6V
3kΩ
3V

Applying KCL to the non– reference node, we obtain:

$$0 = \frac{V-6}{6} + \frac{V}{3} + \frac{V-3}{6}$$

$$0 = (V-6) + 2V + V - 3$$

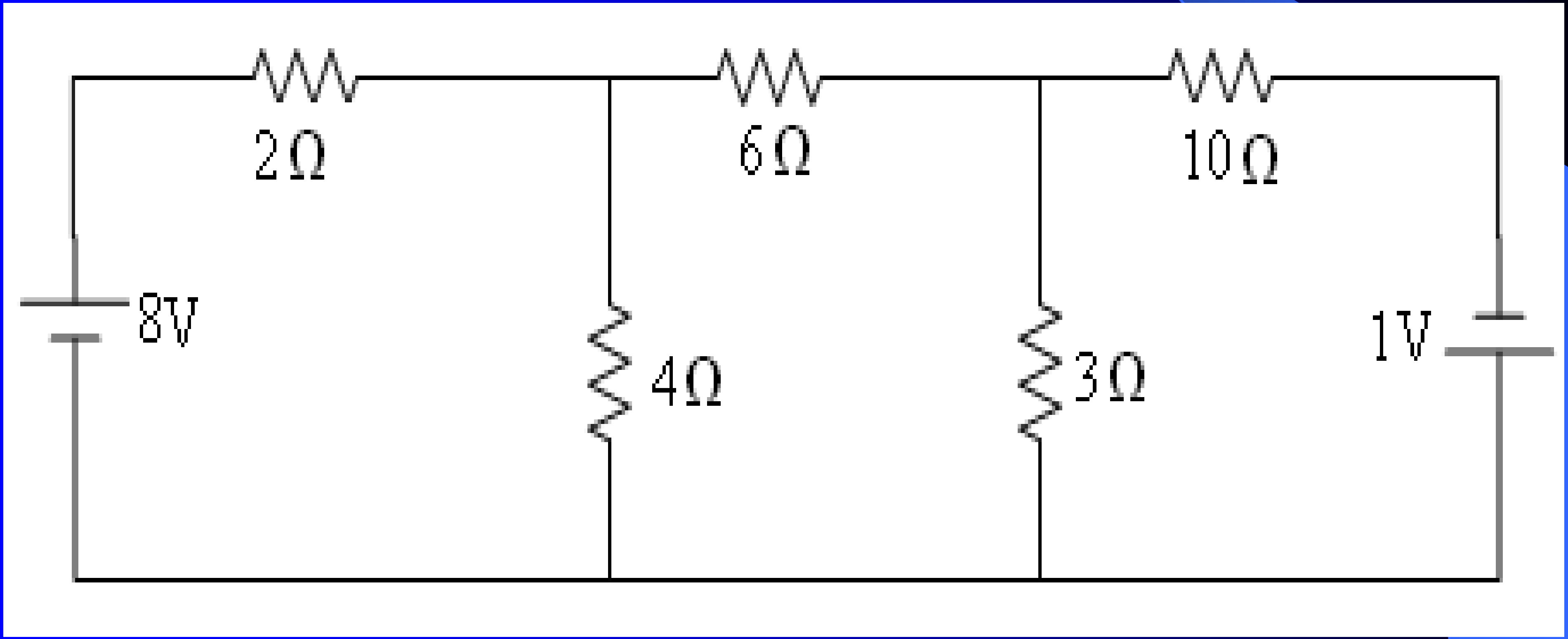
$$0 = 4V - 9 \Rightarrow V = \frac{9}{4}$$

The current

$$I_0 = \frac{V}{3} = \frac{9}{4\times 3} = \frac{3}{4} = 0.75 \text{ mA}$$

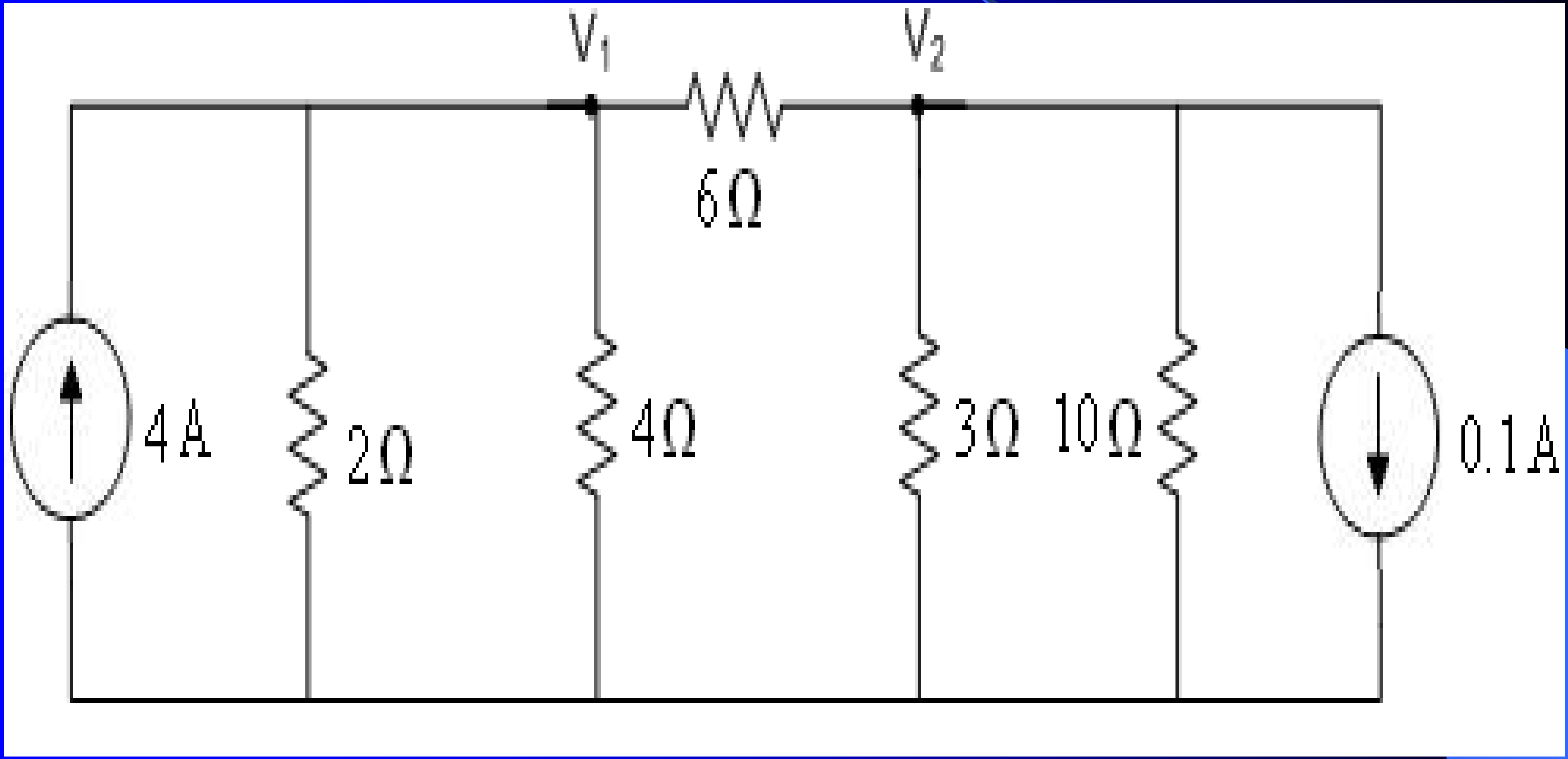
Converting voltage sources to current Sources

Example 14: Find the voltage across the 3-Ω resistor by nodal analysis.



Solution:

- Converting sources and choosing nodes, we obtain



By inspection, we have

$$4 = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6}\right)V_1 - \frac{1}{6}V_2$$

$$-0.1 = -\frac{1}{6}V_1 + \left(\frac{1}{6} + \frac{1}{3} + \frac{1}{10}\right)V_2$$

or

$$4 = \frac{11}{12}V_1 - \frac{1}{6}V_2$$

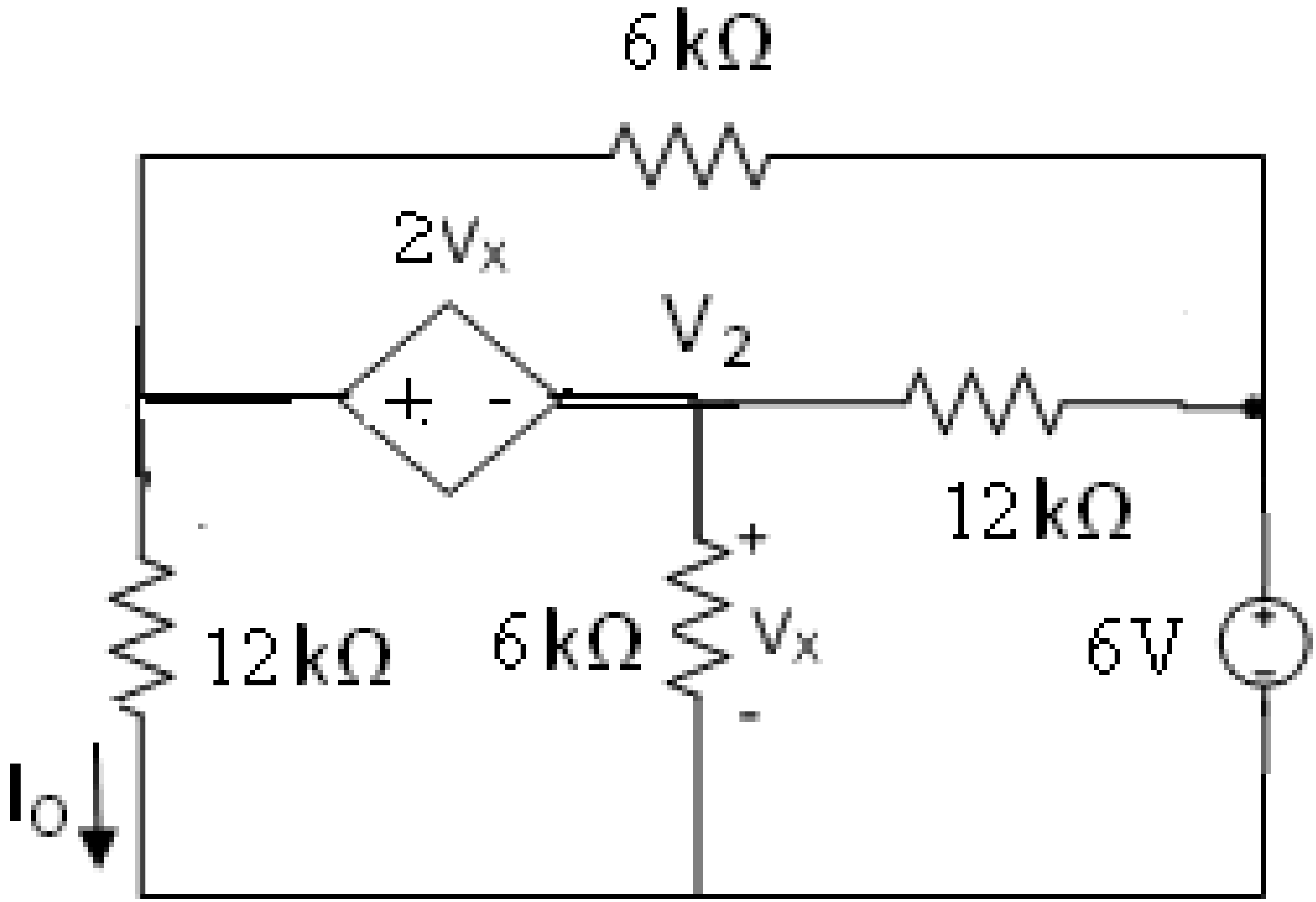
$$-0.1 = -\frac{1}{6}V_1 + \frac{3}{5}V_2$$

Solving the two equations simultaneously, we obtain voltage across the 3-Ω resistor = $V_2 = 1.101$ V

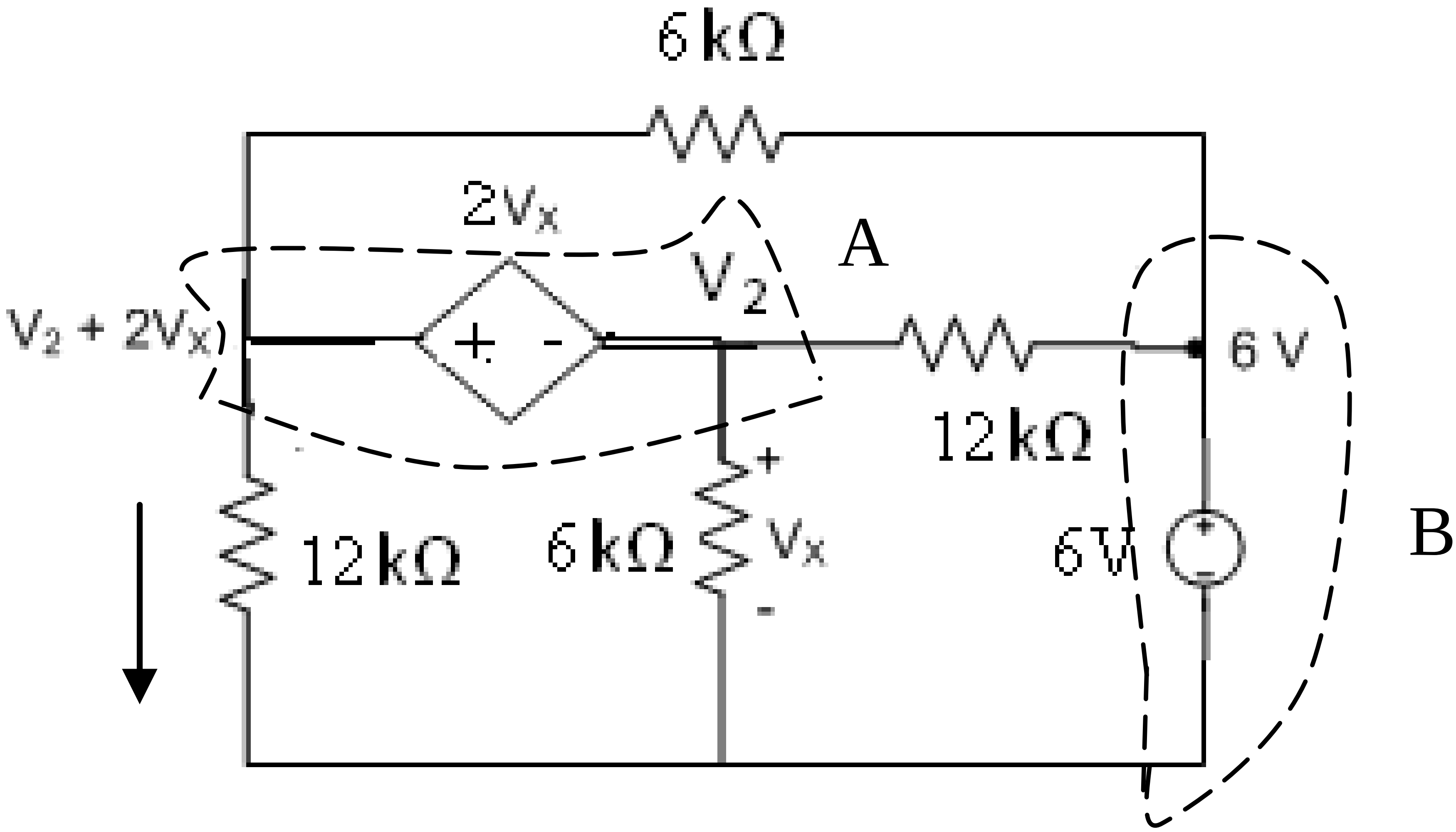
Circuits containing dependent voltage sources

- Treat them in the same manner as circuits containing independent voltage sources.
- When writing the circuit equations, first treat the dependent source as though it were an independent source
- and then write the controlling equations and substitute.

Example 15: Find the current I_0 in the network



Solution:



Apply KCL to the supernode A:

$$0 = \frac{V_2 + 2V_x}{12} + \frac{(V_2 + 2V_x) - 6}{6} + \frac{V_2}{6} + \frac{V_2 - 6}{12}$$

$$0 = \left(\frac{1}{12} + \frac{1}{6} + \frac{1}{6} + \frac{1}{12}\right)V_2 + \left(\frac{2}{12} + \frac{2}{6}\right)V_x - 1 - \frac{6}{12}$$

$$0 = \frac{3}{6}V_2 + \frac{3}{6}V_x - \frac{18}{12}$$

Controlling equation: $V_x = V_2$

Substituting into the nodal equation, we obtain:

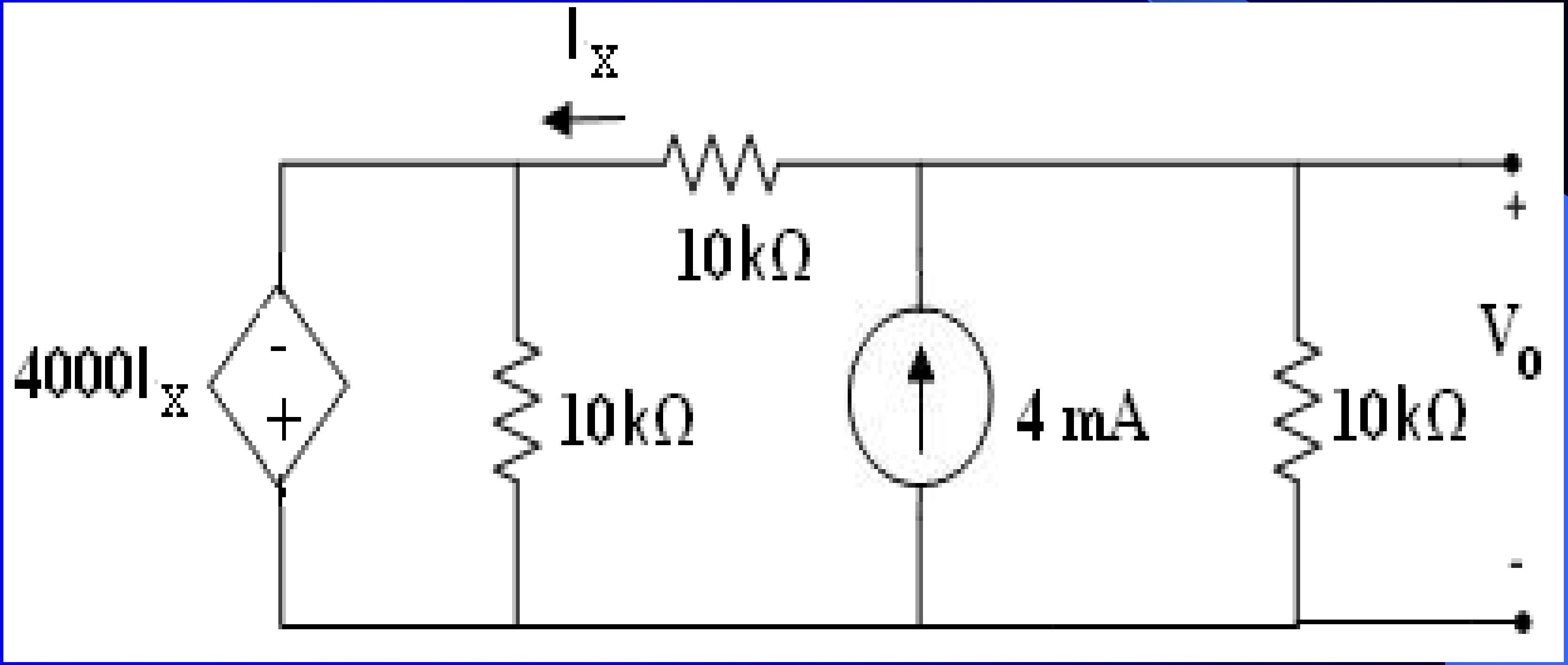
$$0 = \frac{3}{6}V_2 + \frac{3}{6}V_2 - \frac{18}{12} \text{ or } \frac{18}{12} = \frac{6}{6}V_2 \text{ or } V_2 = \frac{3}{2}\text{ V}$$

Current $\mathrm{I}_0 = \dfrac{V_2 + 2V_x}{12}$

$$I_o = \frac{3V_2}{12} = \frac{3}{12} \times \frac{3}{2} = \frac{3}{8}\text{ mA}$$

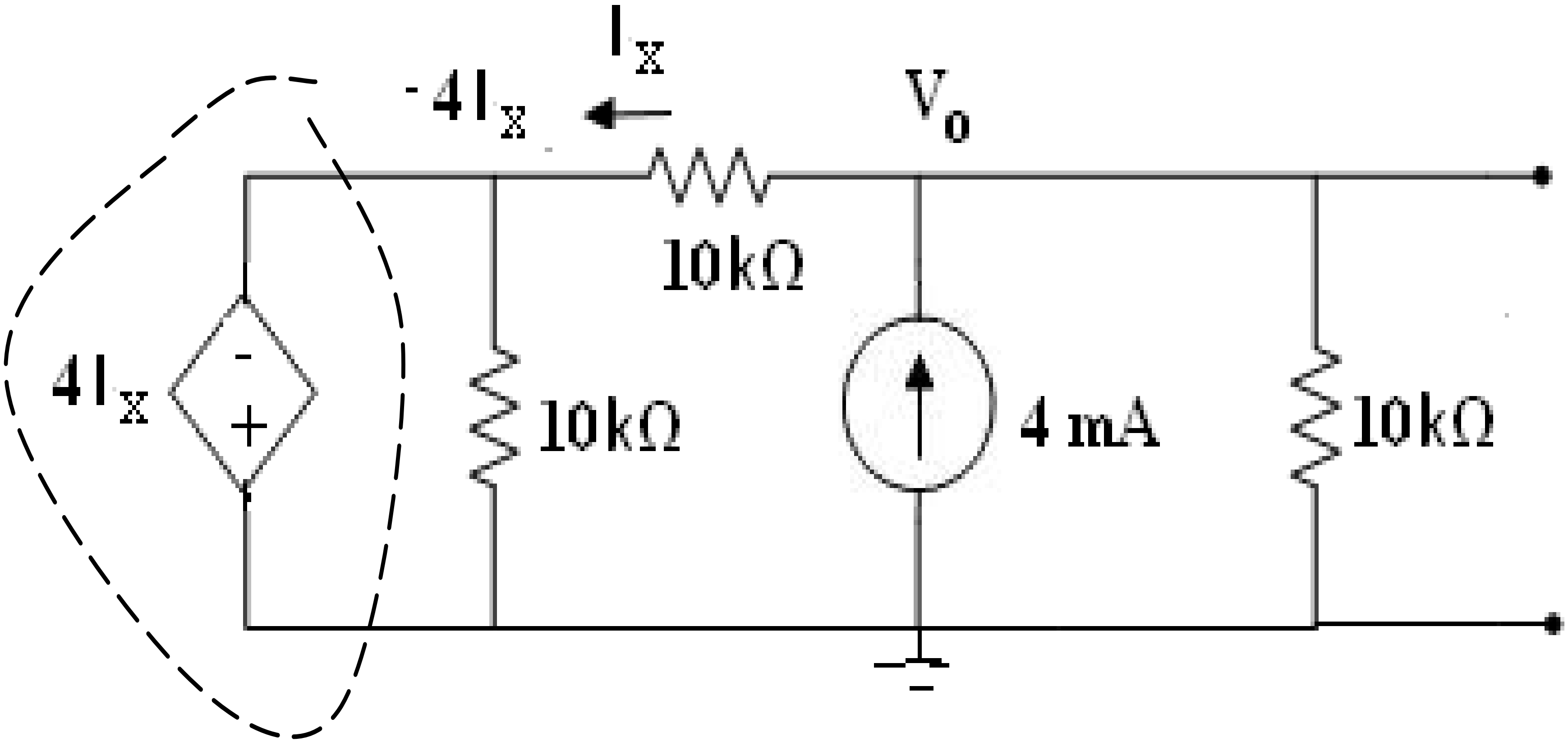
Circuits containing dependent voltage sources cont’d

Example 16: Find V_0 in the network using nodal analysis



Solution

With currents in mA, the CCVS becomes $4I_x$.



Applying KCL to the non-reference node V_0, we obtain

$$4 = \frac{V_0}{10} + \frac{V_0 - (-4I_x)}{10}$$

or

$$4 = \frac{V_0}{5} + \frac{4I_x}{10} \text{ or } 20 = V_0 + 2I_x$$

$$V_0 = 20 - 2I_x$$

The controlling equation is given by

$$I_x = \frac{V_0 - -4I_x}{10} = \frac{V_0 + 4I_x}{10} \text{ or}$$

$$10I_x = V_0 + 4I_x \text{ or } 6I_x = V_0 \text{ or } 2I_x = \frac{V_0}{3}$$

Substituting it in the nodal equation,

$$V_0 = 20 - 2I_x$$

we obtain

$$V_0 = 20 - \frac{V_0}{3} \text{ or } V_0 \times \frac{4}{3} = 20 \text{ or } V_0 = 15\text{ V}$$

Mesh Analysis

- Mesh analysis uses KVL to determine currents in the circuit.
- Once the currents are known, Ohm's law can be used to calculate voltages.
- N independent equations are required if the circuit contains N meshes.

Mesh Analysis Cont'd

- We assume that the circuits are planar.

- In the case of non–planar circuits, we cannot define meshes and mesh analysis cannot be performed.

- Mesh analysis is thus not as general as nodal analysis, which has no topological restrictions.

Definition of terms used

- A loop: It is a closed path through a circuit in which no node is encountered more than once.
- A mesh: A mesh is a special kind of loop that does not contain any loop within it.
- We note that as we traverse the path of a mesh, we do not encircle any circuit element.

Definition of terms used cont'd

- A planar circuit: It is a circuit that can be drawn on a plane surface with no crossovers, i.e.,

  no element or connecting wire crosses another element or connecting wire.
- In planar circuits meshes appear as windows.
- A non–planar circuit: It is a circuit that is not planar.

General approach to mesh analysis

We follow the systematic approach given below:

(a) Place a loop current in the clockwise direction within each mesh or window of the network.

(b) Apply KVL around each mesh in the clockwise direction.

(c) Solve the resulting simultaneous equations for the loop currents.

Circuits containing only independent voltage sources

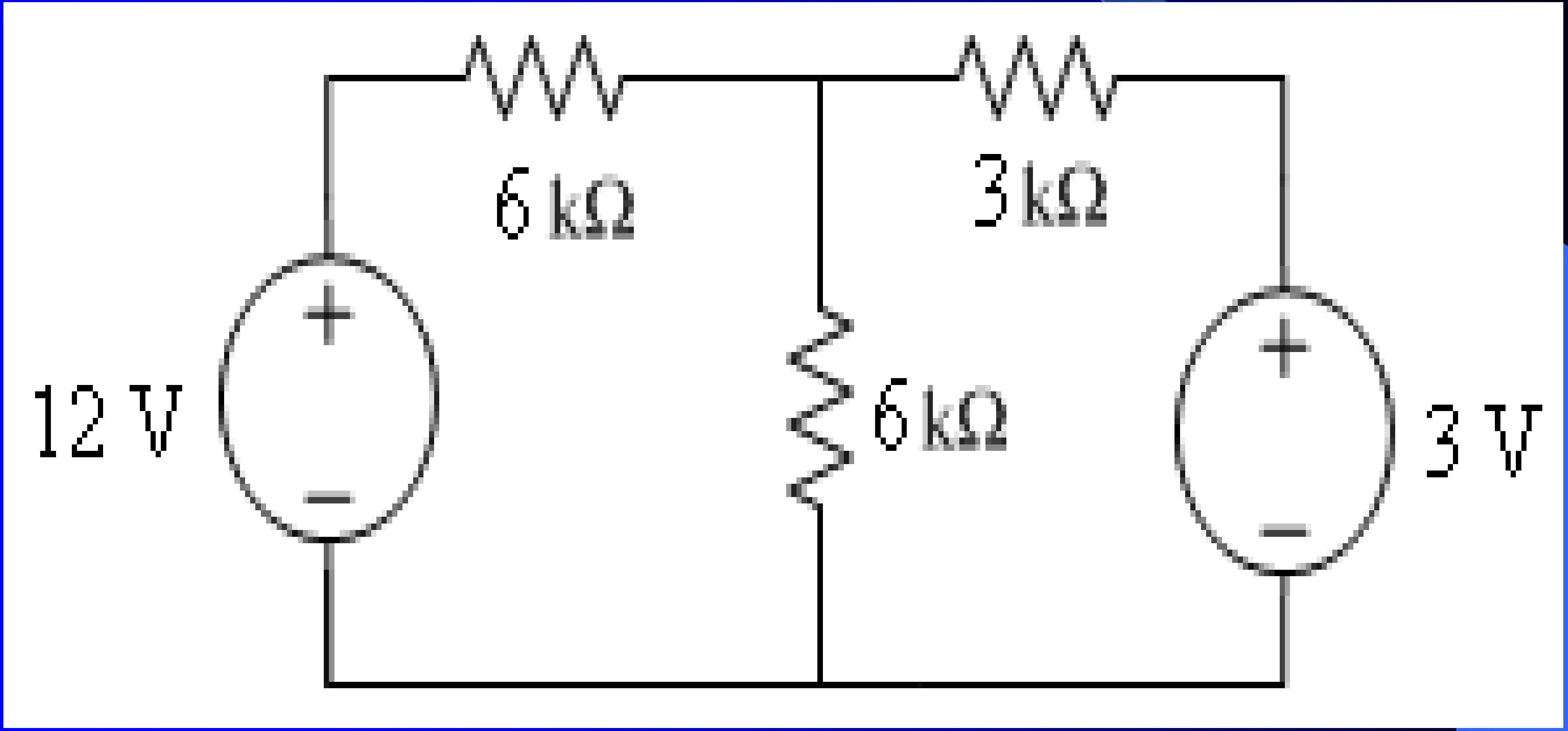
- Either of the following approaches can be used:

  - General approach

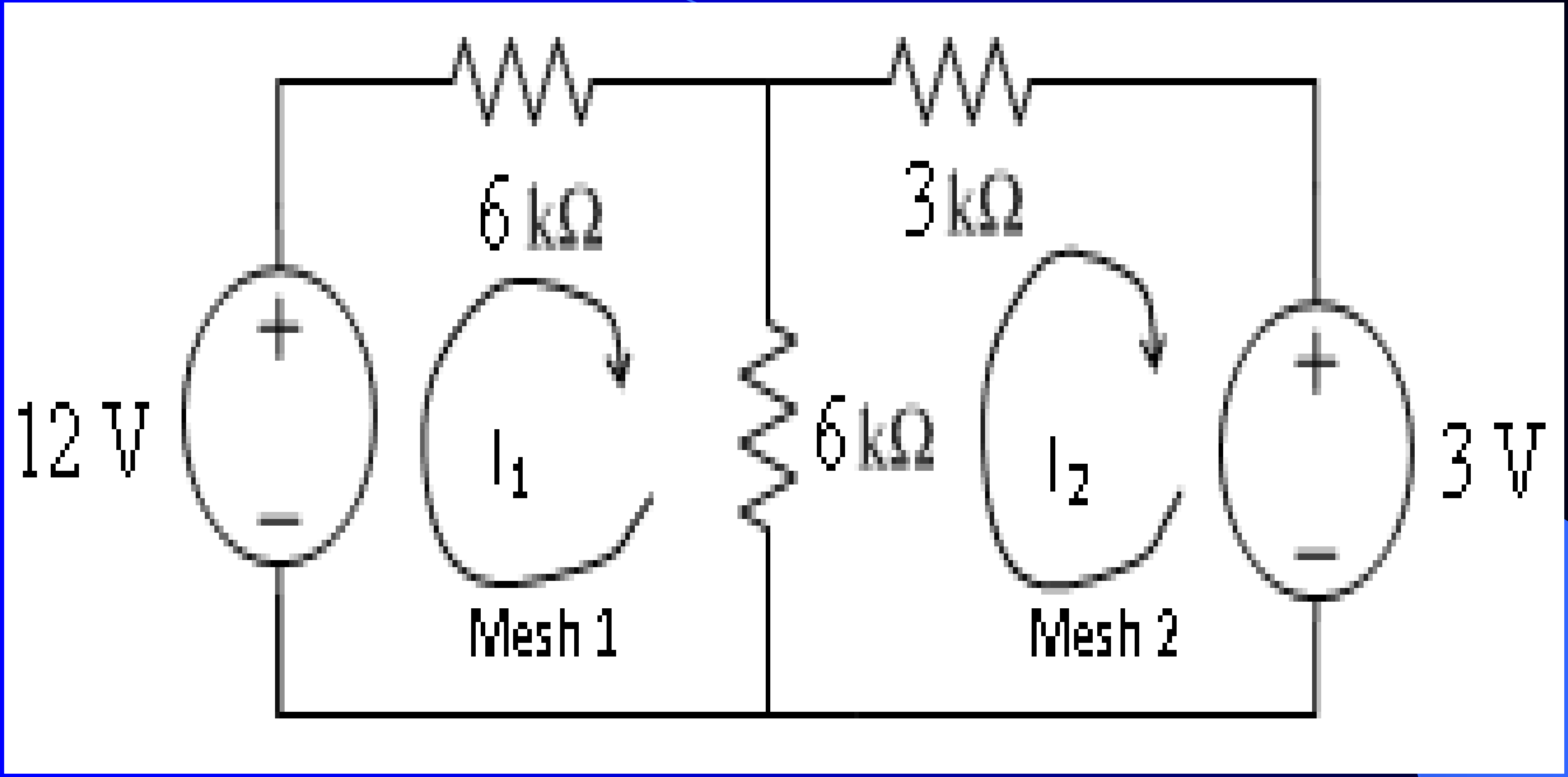
  - Format approach

General Approach

Example 17: Find the current through each branch of the network



Solution



Apply KVL to mesh 1:

$$12 = 6I_1 + 6(I_1 - I_2) = 12I_1 - 6I_2 \text{ or}$$

$$12 = 12I_1 - 6I_2 \quad (1)$$

Apply KVL to mesh 2:

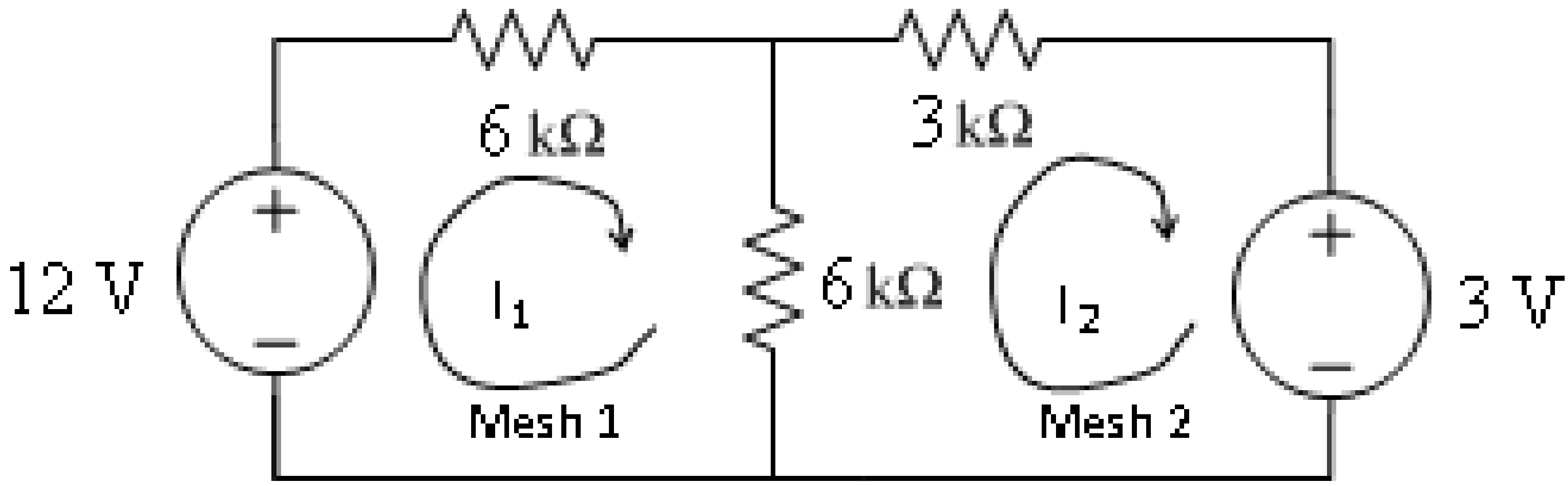
$$-3 = 3I_2 + 6(I_2 - I_1) \quad \text{or}$$

$$-3 = -6I_1 + 9I_2 \quad (2)$$

(1) + 2 x (2) gives

$$6 = 12I_2 \text{ or } I_2 = \frac{6}{12} = 0.5 \text{ mA and from (2)}$$

$$6I_1 = 9I_2 + 3 = 9 \times 0.5 + 3 \text{ or } I_1 = \frac{7.5}{6} = 1.25 \text{ mA}$$



$I_2 = 0.5\ \text{mA}$

$I_1 = 1.25\ \text{mA}$

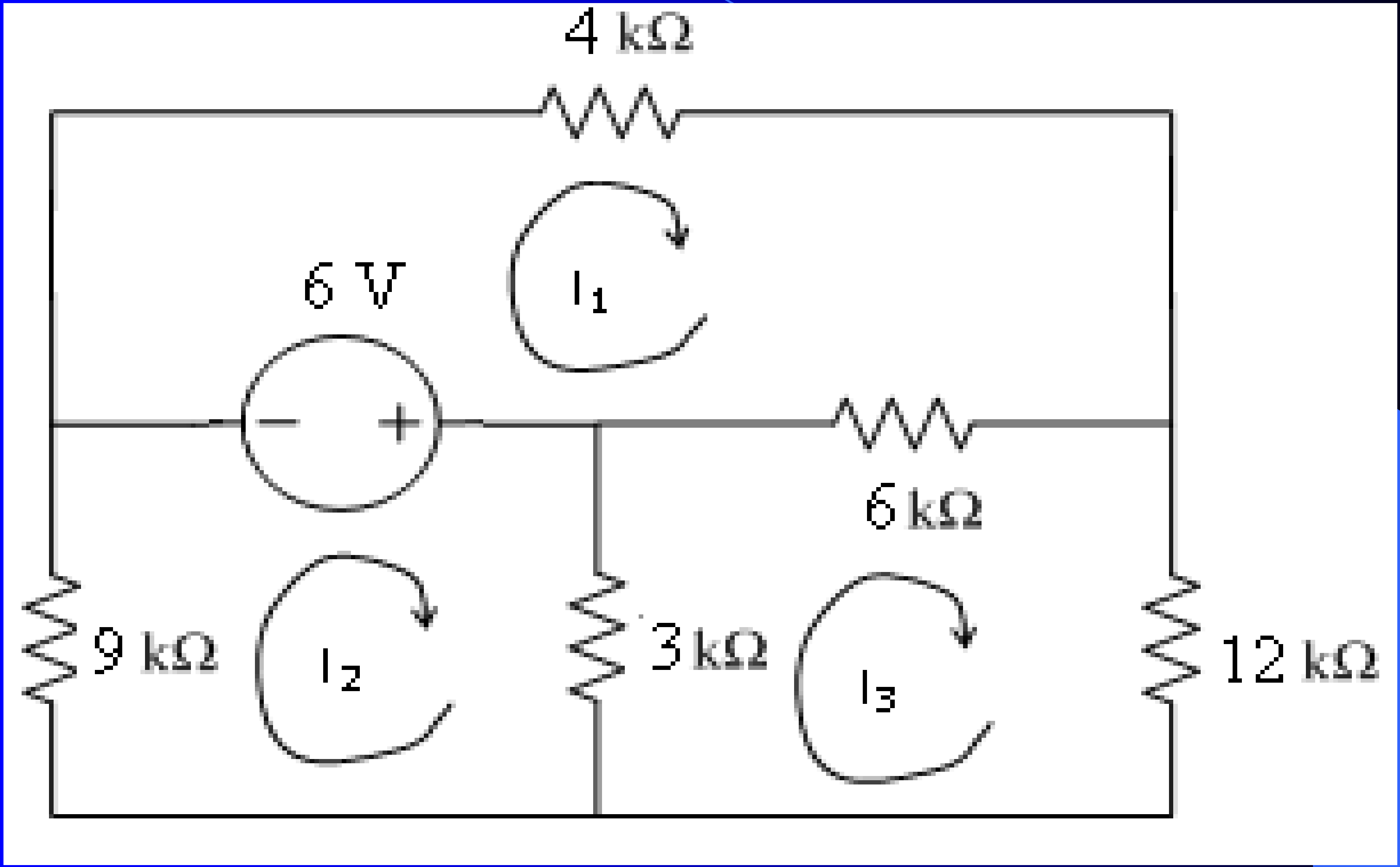
Current in the left outer branch = $I_1 = 1.25\ \text{mA}$

Current in the middle branch = $I_1 - I_2$

$= 1.25 - 0.5 = 0.75\ \text{mA}$

Current in the right outer branch = $I_2 = 0.5\ \text{mA}$

- Example 18: Find the current through each branch



Solution:

Loop 1: $-6 = 4I_1 + 6(I_1 - I_3)$ or

$-6 = 10I_1 - (0)I_2 - 6I_3$

Loop 2: $6 = 3(I_2 - I_3) + 9I_2$ or

$6 = (0)I_1 - 12I_2 - 3I_3$

Loop 3: $0 = 6(I_3 - I_1) + 12I_3 + 3(I_3 - I_2)$ or

$0 = -6I_1 - 3I_2 + 21I_3$

Solution:

Loop 1: $-6 = 10I_1 - (0)I_2 - 6I_3$

Loop 2: $6 = (0)I_1 - 12I_2 - 3I_3$

Loop 3: $0 = -6I_1 - 3I_2 + 21I_3$

In the matrix form, we have:

$$\begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 & -6 \\ 0 & 12 & -3 \\ -6 & -3 & 21 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Circuits containing current sources

- As in the case of nodal analysis with voltage sources, the presence of current sources reduces the number of unknowns in mesh analysis by one per current source.

- Either of the following approaches can be used:

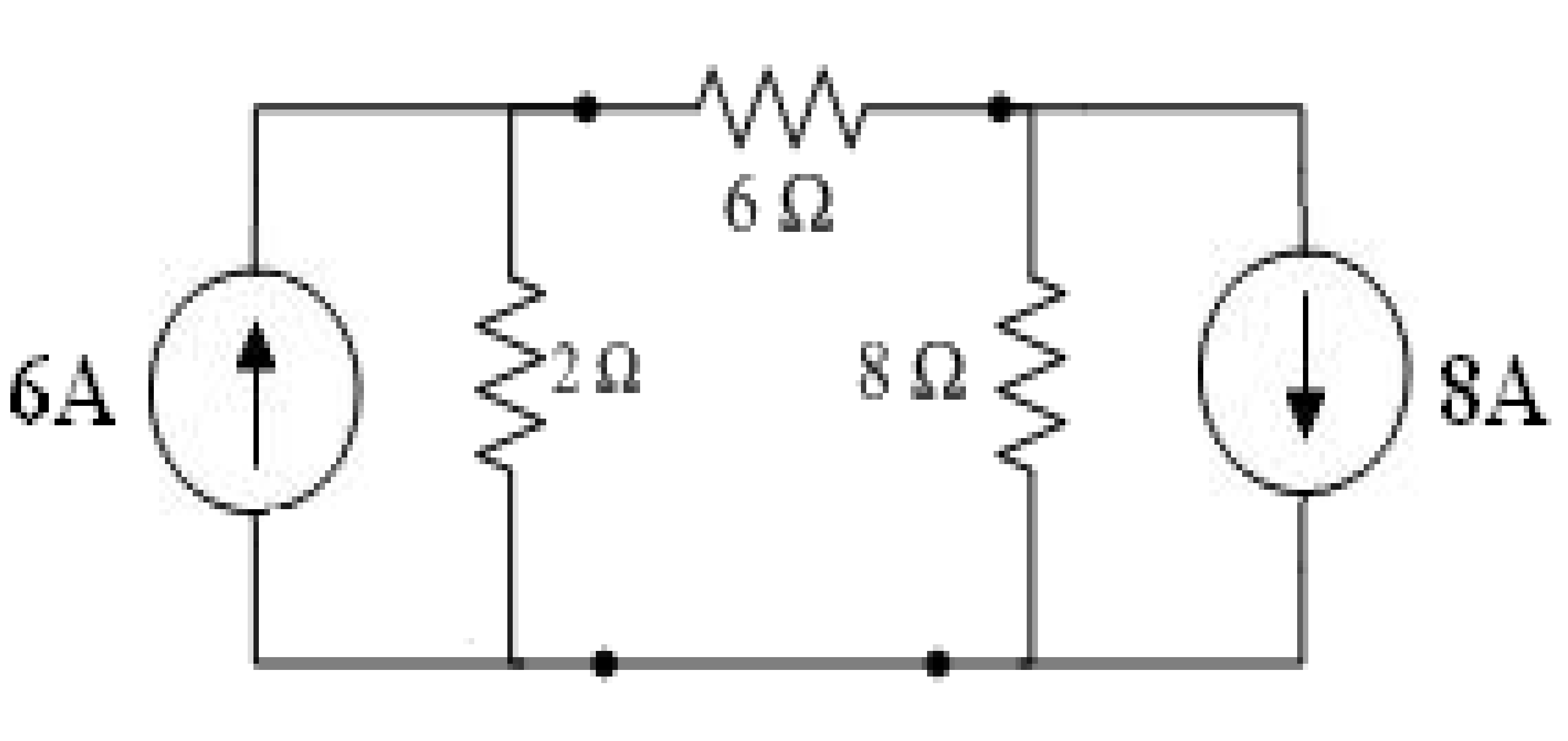
Circuits containing current sources cont'd

Approach :

- We convert current sources to voltage sources and proceed as before.
- The sources must have resistances connected in parallel

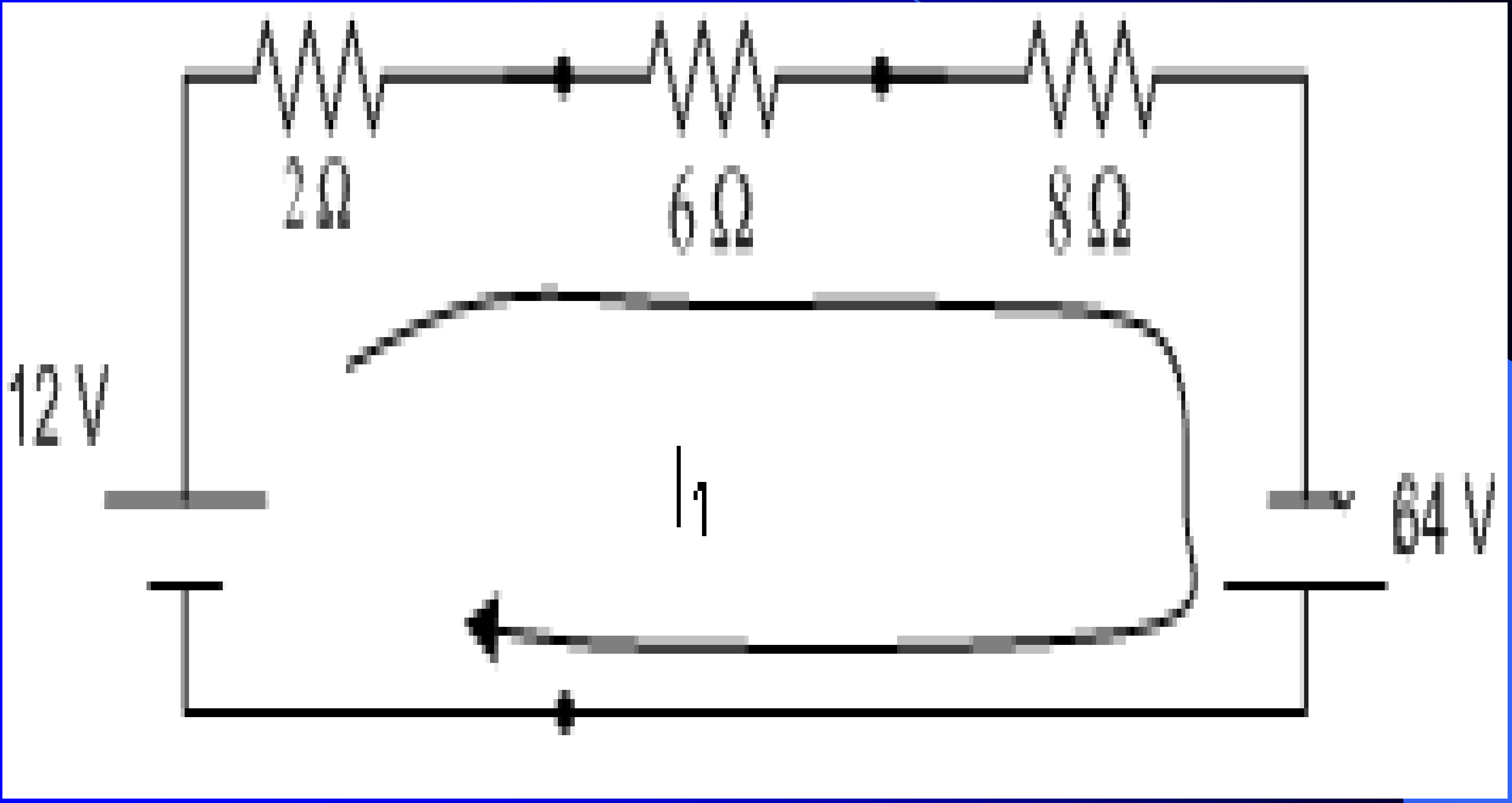
Current sources are converted to voltage sources.

Example 19: Use mesh analysis to determine the currents for the network



Solution

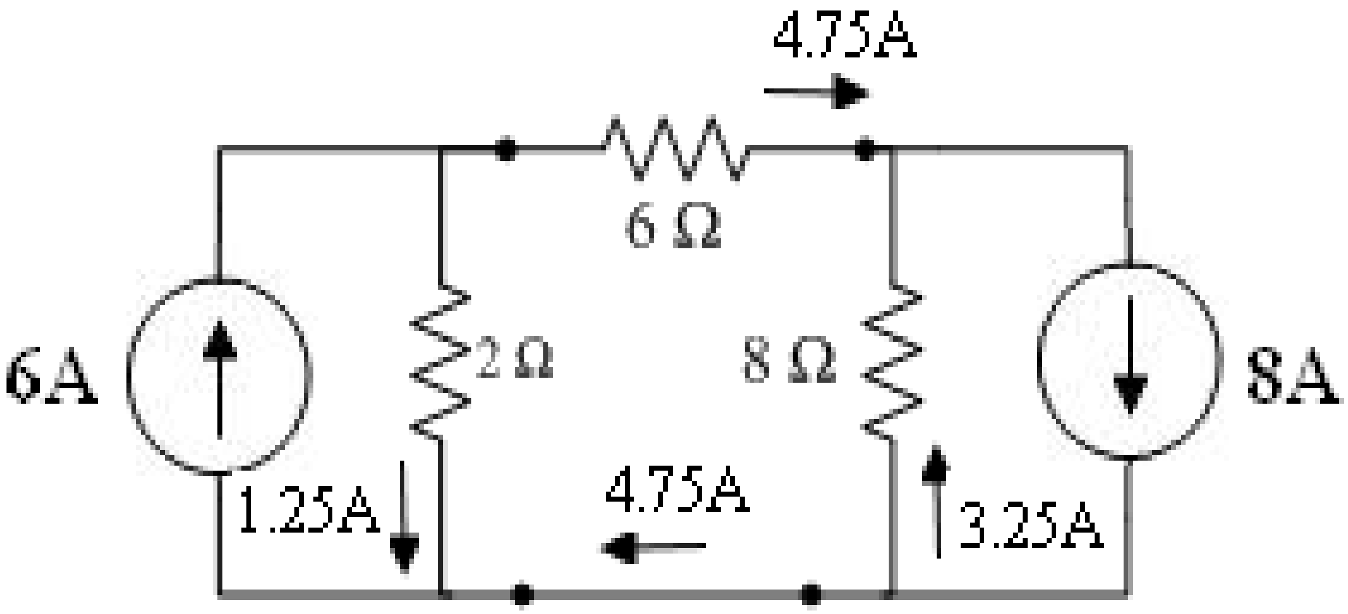
- Convert current sources to voltage sources.



$$12 + 64 = (2 + 6 + 8)I_1$$

$$76 = 16I_1$$

$$I_1 = \frac{76}{16} = 4.75\,\mathrm{A}$$



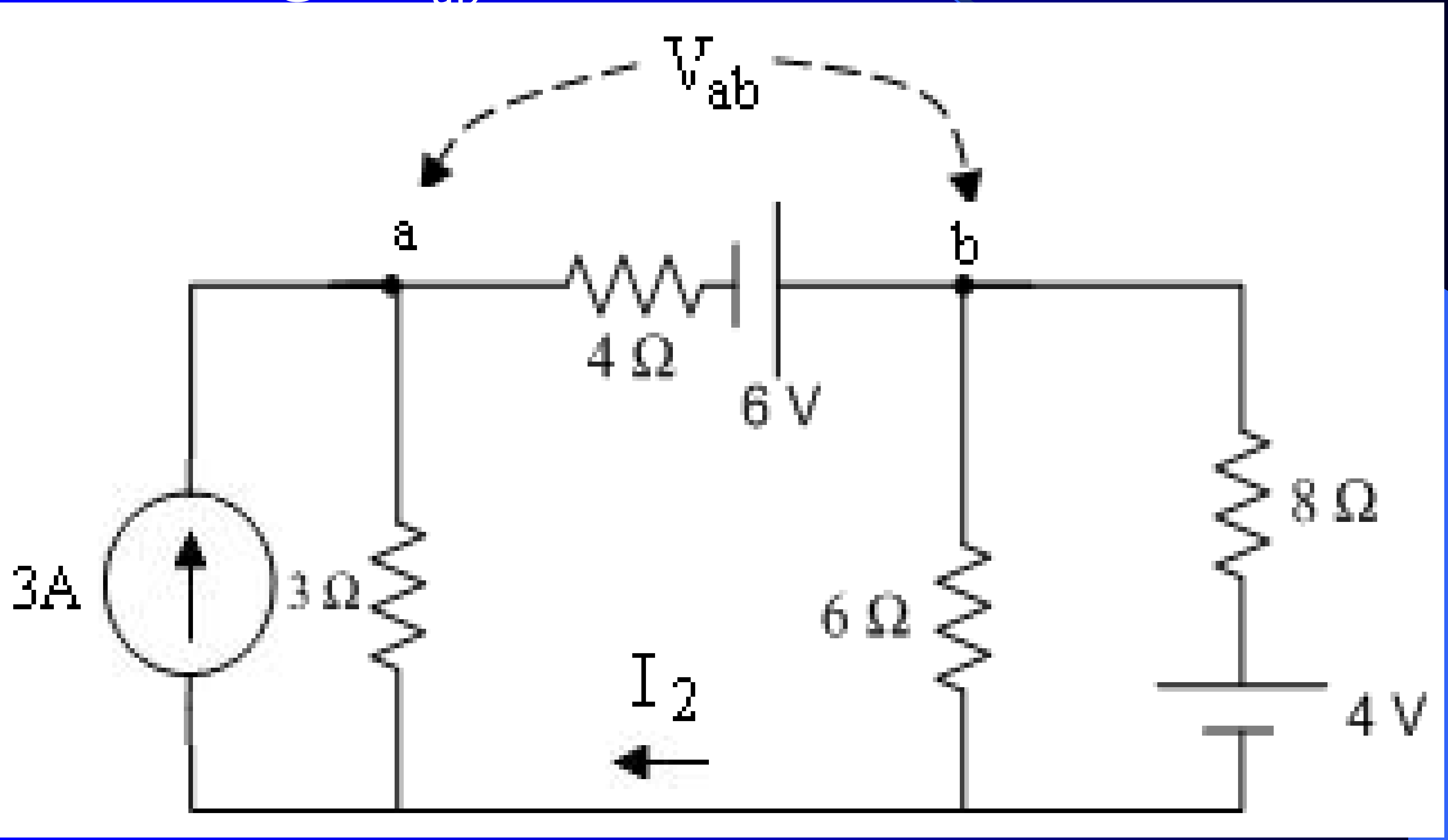
The currents in the various branches of the circuit :

Current in 6-Ω resistor = 4.75A

Current in 2-Ω resistor $= 6 - 4.75 = 1.25\ A$

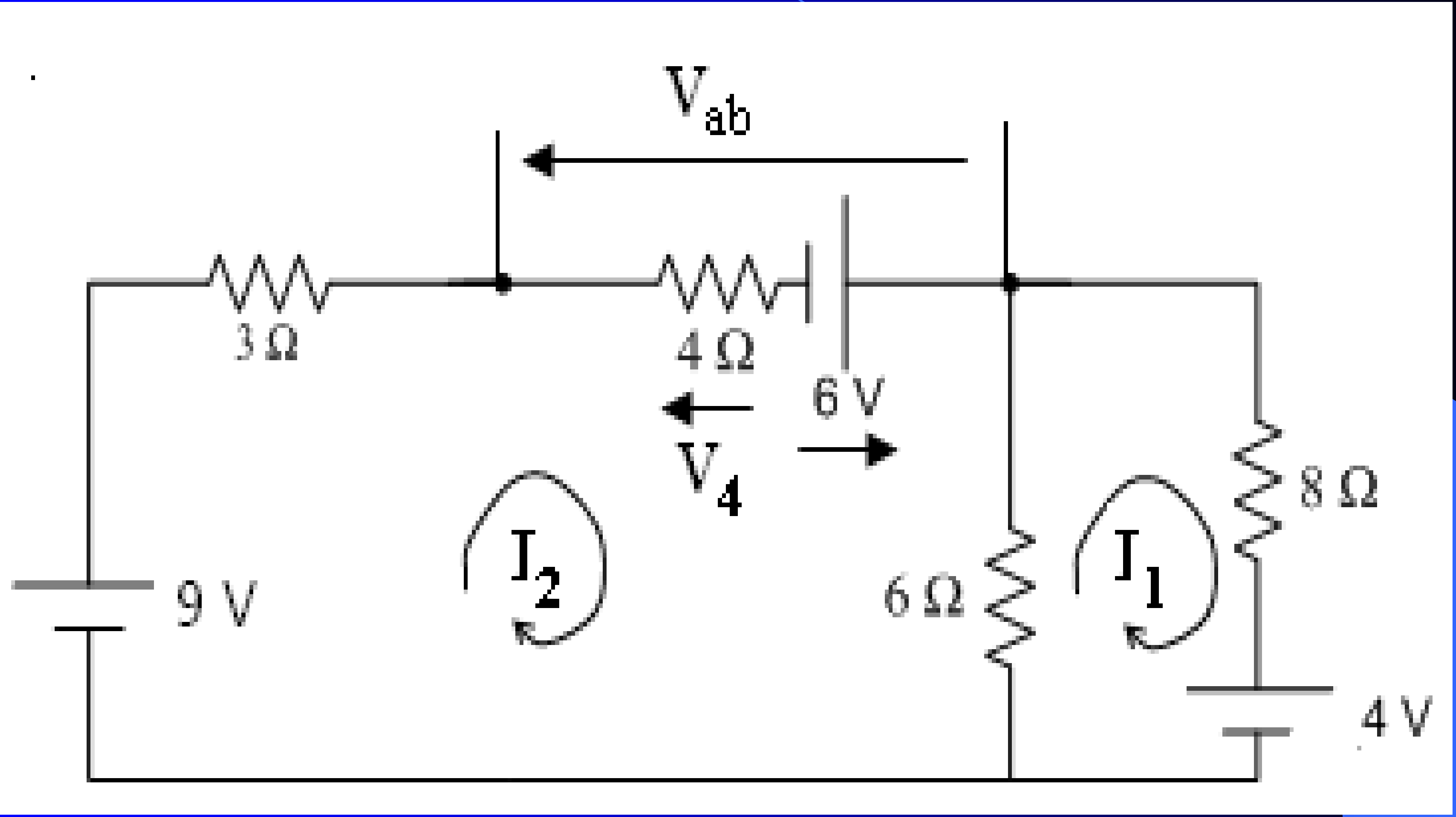
Current in 8-Ω resistor $= 8 - 4.75 = 3.25\ A$

Example 20: For the network , determine the current I_2 using mesh analysis and then find the voltage V_{ab}.



Solution:

Convert current source to voltage source:



By inspection:

$$-4 = 14I_1 - 6I_2$$

$$15 = -6I_1 + 13I_2$$

Solving the two simultaneous equations, we obtain

$I_1 = 0.26$ A and $I_2 = 1.274$ A

Voltage $V_{ab} = V_4 - 6$

$$= 4I_2 - 6$$

$$= 4 \times 1.274 - 6$$

$$= 0.904 \text{ V}$$