KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY
CHEMICAL ENGINEERING DEPARTMENT
CHE 252: CHEMICAL PROCESS CALCULATIONS II

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LECTURE 3: ENERGY BALANCES ON OPEN SYSTEMS AT STEADY STATE

Learning Objectives

At the end of the lecture the student is expected to be able to do the following:

- Define the terms flow work, shaft work
- Write the energy balance for an open process system in terms of enthalpy and shaft work and state the conditions under which each of the five terms can be neglected.
- Given a description of an open process system, simplify the energy balance and solve it for whichever term is not specified in the process description.

3.1 Flow Work and Shaft Work

An open process system by definition has mass crossing its boundaries as the process occurs. Work must be done on such a system to push mass in, and work is done on the surroundings by mass that emerges. Both work terms must be included in the energy balance.

The net rate of work done by an open system on its surroundings may be written as

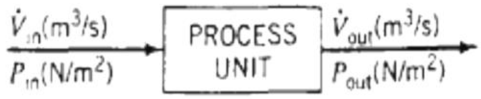
$$\dot{W} = \dot{W}_s + \dot{W}_{fl} \qquad (3.1)$$

where

$\dot{W}_s$= **shaft work,** or rate of work done by the process fluid on a moving part within the system (e.g., a pump rotor)

$\dot{W}_{fl}$ = **flow work,** or rate of work done by the fluid at the system outlet minus the rate of work done on the fluid at the system inlet

To derive an expression for $\dot{W}_{fl}$ we initially consider the single-inlet-single-outlet system shown below:



Fluid at a pressure P_{in} (N/m^2) enters a pipe at a volumetric flow rate $\dot{V}_{in}$ (m^3/s) and exits at a pressure P_{out} (N/m^2) and volumetric flow rate $\dot{V}_{out}$ (m^3/s). The fluid that enters the system has work done on it by the fluid just behind it at a rate

$$\dot{W}_{in}\left(N.\frac{m}{s}\right) = P_{in}\left(\frac{N}{m^2}\right)\dot{V}_{in}\left(\frac{m^3}{s}\right)$$

while the fluid leaving the system performs work on the surroundings at a rate

$$\dot{W}_{out} = P_{out}\dot{V}_{out}$$

The net rate at which work is done by the system at the inlet and outlet is therefore

$$\dot{W}_{fl} = P_{out}\dot{V}_{out} - P_{in}\dot{V}_{in}$$

If several input and output streams enter and leave the system, the $P\dot{V}$ products for each stream must be summed to determine $\dot{W}_{fl}$.

3.2 The Steady-State Open-System Energy Balance

The first law of thermodynamics for an open system at steady state has the form

Input = Output

Input" here signifies the total rate of transport of kinetic energy, potential energy, and internal energy by all process input streams plus the rate at which energy is transferred in as heat, and "output" is the total rate of energy transport by the output streams plus the rate at which energy is transferred out as work.

If $\dot{E}_j$ denotes the total rate of energy transport by the j^{th} input or output stream of a process, and $\dot{Q}$ and $\dot{W}$ are again defined as the rates of flow of heat into and work out of the process,

$$\dot{Q} + \sum_{\substack{\text{input}\\\text{streams}}} \dot{E}_j = \sum_{\substack{\text{output}\\\text{streams}}} \dot{E}_j + \dot{W}$$

$$\Downarrow$$

$$\sum_{\substack{\text{output}\\\text{streams}}} \dot{E}_j - \sum_{\substack{\text{input}\\\text{streams}}} \dot{E}_j = \dot{Q} - \dot{W} \qquad (3.2)$$

If $\dot{m}_j$, $\dot{E}_{kj}$, $\dot{E}_{pj}$, and $\dot{U}_j$ are the flow rates of mass, kinetic energy, potential energy, and internal energy for the j^{th} process stream, then the total rate at which energy is transported into or out of the system by this stream is

$$\dot{E}_j = \dot{U}_j + \dot{E}_{kj} + \dot{E}_{pj}$$

$$\Big\Downarrow \begin{array}{l} \dot{U}_j = \dot{m}_j \hat{U}_j \\ \dot{E}_{kj} = \dot{m}_j u_j^2/2 \\ \dot{E}_{pj} = \dot{m}_j g z_j \end{array}$$

$$\dot{E}_j = \dot{m}_j \left(\hat{U}_j + \frac{u_j^2}{2} + g z_j \right) \qquad (3.3)$$

where u_j is the velocity of the j^{th} stream and z_j is the height of this stream relative to a reference plane at which $E_p = 0$.

The total work $(\dot{W})$ done by the system on its surroundings equals the shaft work $(\dot{W}_s)$ plus the flow work $(\dot{W}_{fl})$. If $\dot{V}_j$ is the volumetric flow rate of the j^{th} stream and P_j is the pressure of this stream as it crosses the system boundary,

$$\dot{W}_{fl} = \sum_{\substack{\text{output}\\ \text{streams}}} P_j \dot{V}_j - \sum_{\substack{\text{input}\\ \text{streams}}} P_j \dot{V}_j$$

$$\Big\Downarrow \; \dot{V}_j = \dot{m}_j \hat{V}_j$$

$$\dot{W} = \dot{W}_s + \sum_{\substack{\text{output}\\ \text{streams}}} \dot{m}_j P_j \hat{V}_j - \sum_{\substack{\text{input}\\ \text{streams}}} \dot{m}_j P_j \hat{V}_j \qquad (3.4)$$

Substituting the expression for $\dot{E}_j$ of Equation 3.3 and that for $\dot{W}$ of Equation 3.4 into Equation 3.2 and bringing the PV terms to the left side yields

$$\sum_{\substack{\text{output}\\ \text{streams}}} \dot{m}_j \left[\hat{U}_j + P_j \hat{V}_j + \frac{u_j^2}{2} + g z_j \right] - \sum_{\substack{\text{input}\\ \text{streams}}} \dot{m}_j \left[\hat{U}_j + P_j \hat{V}_j + \frac{u_j^2}{2} + g z_j \right] = \dot{Q} - \dot{W}_s \qquad (3.5)$$

Equation 3.5 could be used for all steady-state open system energy balance problems.
As a rule, however, the term $\hat{U}_j + P_j \hat{V}_j$ is combined and written as $\hat{H}_j$, the variable previously defined as the specific enthalpy. In terms of this variable, Equation 3.5 becomes

$$\sum_{\substack{\text{output}\\\text{streams}}} \dot{m}_j \left(\hat{H}_j + \frac{u_j^2}{2} + gz_j\right) - \sum_{\substack{\text{input}\\\text{streams}}} \dot{m}_j \left(\hat{H}_j + \frac{u_j^2}{2} + gz_j\right) = \dot{Q} - \dot{W}_s \tag{3.6}$$

Finally, let us use the symbol Δ to denote total output minus total input, so that

$$\begin{aligned}
\Delta\dot{H} &= \sum_{\substack{\text{output}\\\text{streams}}} \dot{m}_j\hat{H}_j - \sum_{\substack{\text{input}\\\text{streams}}} \dot{m}_j\hat{H}_j \\
\Delta\dot{E}_k &= \sum_{\substack{\text{output}\\\text{streams}}} \dot{m}_j u_j^2/2 - \sum_{\substack{\text{input}\\\text{streams}}} \dot{m}_j u_j^2/2 \\
\Delta\dot{E}_p &= \sum_{\substack{\text{output}\\\text{streams}}} \dot{m}_j g z_j - \sum_{\substack{\text{input}\\\text{streams}}} \dot{m}_j g z_j
\end{aligned} \tag{3.7}$$

In terms of these quantities, Equation 3.6 becomes

$$\boxed{\Delta\dot{H} + \Delta\dot{E}_k + \Delta\dot{E}_p = \dot{Q} - \dot{W}_s} \tag{3.8}$$

Equation 3.8 states that the net rate at which energy is transferred to a system as heat and/or shaft work ($\dot{Q} - \dot{W}_s$) equals the difference between the rates at which the quantity (enthalpy + kinetic energy + potential energy) is transported into and out of the system ($\Delta\dot{H} + \Delta\dot{E}_k + \Delta\dot{E}_p$).

This equation will be used as the starting point for most energy balance calculations on open systems at steady state.

Notice that if a process has a single input stream and a single output stream and there is no accumulation of mass in the system (so that $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$), the expression for $\Delta\dot{H}$ simplifies to

$$\Delta\dot{H} = \dot{m}(\hat{H}_{out} - \hat{H}_{in}) = \dot{m}\Delta\hat{H} \tag{3.9}$$

Also notice that if a specific variable has the same value for all input and output streams, the corresponding term of Equation 3.8 drops out. For example, if $\hat{H}_j$ is the same for all streams, then from Equation 3.7a

$$\Delta\dot{H} = \hat{H}\left[\sum_{\substack{\text{output}\\\text{streams}}} \dot{m}_j - \sum_{\substack{\text{input}\\\text{streams}}} \dot{m}_j\right]$$

But from a total mass balance the quantity in brackets (which is simply total mass in minus total mass out) equals zero, and hence $\Delta\dot{H} = 0$, as claimed.

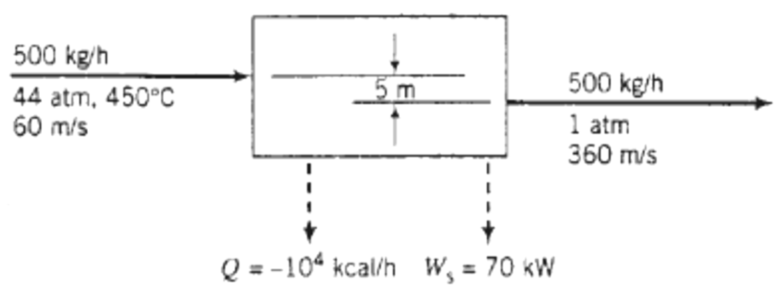
How would you simplify Equation 3.8 in each of the following cases?
1. There are no moving parts in the system.
2. The system and its surroundings are at the same temperature.
3. The linear velocities of all streams are the same.
4. All streams enter and leave the process at a single height.

Example 3.1

Energy Balance on a Turbine

Five hundred kilograms per hour of steam drives a turbine. The steam enters the turbine at 44 atm and 450°C at a linear velocity of 60 m/s and leaves at a point 5 m below the turbine inlet at atmospheric pressure and a velocity of 360 m/s. The turbine delivers shaft work at a rate of 70 kW, and the heat loss from the turbine is estimated to be 104 kcal/h. Calculate the specific enthalpy change associated with the process.

Solution



$$\Delta \dot{H} = \dot{Q} - \dot{W}_s - \Delta \dot{E}_k - \Delta \dot{E}_p$$

Normally, heat, work, and kinetic and potential energy terms are determined in different units. To evaluate $\Delta\dot{H}$, we will convert each term to kW (kJ/s), first noting that $\dot{m}$= (500 kg/h/3600 s/h) = 0.139 kg/s.

$$\Delta \dot{E}_k = \frac{\dot{m}}{2}(u_2^2 - u_1^2) = \frac{0.139 \text{ kg/s}}{2} \left| \frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2} \right| \frac{(360^2 - 60^2)\text{ m}^2}{\text{s}^2} \left| \frac{1 \text{ W}}{1 \text{ N}\cdot\text{m/s}} \right| \frac{1 \text{ kW}}{10^3 \text{ W}}$$

$$= 8.75 \text{ kW}$$

$$\Delta \dot{E}_p = \dot{m}g(z_2 - z_1) = \frac{0.139\ \text{kg/s}}{} \left| \frac{9.81\ \text{N}}{\text{kg}} \right| \frac{(-5)\ \text{m}}{} \left| \frac{1\ \text{kW}}{10^3\ \text{N}\cdot\text{m/s}} \right. = -6.81 \times 10^{-3}\ \text{kW}$$

$$\dot{Q} = \frac{-10^4\ \text{kcal}}{\text{h}} \left| \frac{1\ \text{J}}{0.239 \times 10^{-3}\ \text{kcal}} \right| \frac{1\ \text{h}}{3600\ \text{s}} \left| \frac{1\ \text{kW}}{10^3\ \text{J/s}} \right. = -11.6\ \text{kW}$$

$$\dot{W}_s = 70\ \text{kW}$$

$$\Downarrow$$

$$\Delta \dot{H} = \dot{Q} - \dot{W}_s - \Delta \dot{E}_k - \Delta \dot{E}_p = -90.3\ \text{kW}$$

$$\Delta \dot{H} = \dot{m}(\hat{H}_2 - \hat{H}_1)$$

$$\Downarrow$$

$$\hat{H}_2 - \hat{H}_1 = \Delta \dot{H} / \dot{m}$$

$$= \frac{-90.3\ \text{kJ/s}}{0.139\ \text{kg/s}} = \boxed{-650\ \text{kJ/kg}}$$