

2 ENERGY AND THE FIRST LAW OF THERMODYNAMICS

2.0 INTRODUCTION

Energy is simply defined as the capacity to do work. It is required for all aspects of productive and social activities. Energy is expended in agriculture for land clearing, planting, plant protection, harvesting and processing of produce; energy in the form of petrol and diesel oil is required for automobiles and farm tractors; energy in the form of sunshine is required to dry agricultural produce; nuclear energy is used to generate electricity. In whatever form of application, the essential practical characteristic of energy is that it can be *transformed* from one form to another and *transferred* between systems by *work* and *heat* transfer. The total amount of energy is *conserved* in all transformations and transfers. The purpose of this chapter is to organise the above ideas into suitable forms for engineering analysis.

2.1 MECHANICAL CONCEPTS OF ENERGY

2.1.1 Work and Kinetic Energy

The curved line in Fig. 2.1 represents the path of a body of mass m (closed system) moving relative to the x-y coordinate frame shown. Let us consider the body as it moves from $s = s_1$, where the magnitude of its velocity is V_1 , to $s = s_2$, where its velocity is V_2 . If we assume that the only interaction between the body and its surroundings involves the force F , then by Newton's 2nd Law, the magnitude of the component F_s is related to the change in the magnitude of V by

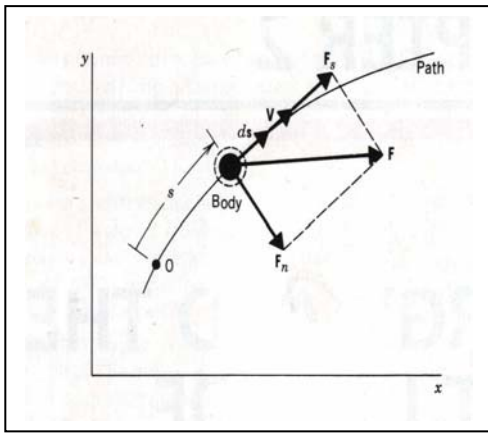


Fig. 2.1 Forces acting on a moving system

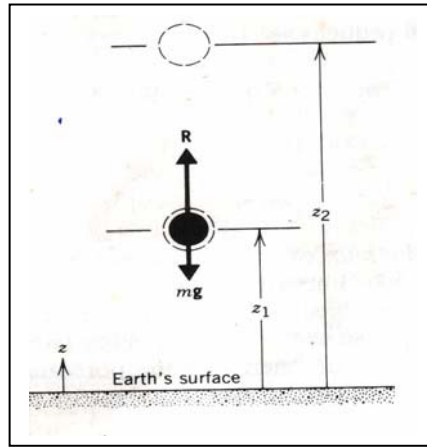


Fig. 2.2 Illustration of potential energy concept

$$F_s = m \frac{dV}{dt} = m \frac{dV}{ds} \frac{ds}{dt} \quad (2.1)$$

where $V = ds/dt$. Rearranging Eq. 2.1 and integrating from s_1 and s_2 gives

$$\int_{V_1}^{V_2} mV dV = \int_{s_1}^{s_2} F_s ds \quad (2.2)$$

The integral on the left of Eq. 2.2 is evaluated as follows:

$$\int_{V_1}^{V_2} mV dV = \left[\frac{1}{2} mV^2 \right]_{V_1}^{V_2} = \frac{1}{2} m (V_2^2 - V_1^2) \quad (2.3)$$

The quantity $\frac{1}{2}mV^2$ is the *kinetic energy*, KE, of the body. Kinetic energy is a scalar quantity.

The *change* in kinetic energy, ΔKE , of the body is $\Delta KE = KE_2 - KE_1 = \frac{1}{2}m(V_2^2 - V_1^2)$.

The integral on the right side of Eq. 2.2 is the work of the force F_s as the body moves from s_1 to s_2 along the path. Work is also a scalar quantity.

Combining Eqs. 2.2 and 2.3 we have

$$\frac{1}{2}m(V_2^2 - V_1^2) = \int_{s_1}^{s_2} F_s \cdot ds \quad (2.4)$$

Equation 2.4 states that the work of the resultant force on the body equals the change in its kinetic energy. When the resultant force accelerates the body, the work done on the body can be considered a *transfer* of energy *to* the body, where it is *stored* as the kinetic energy.

Kinetic energy can be assigned a value knowing only the mass of the body and the magnitude of its instantaneous velocity relative to a specified coordinate frame, without regard for how this velocity was attained. Hence, kinetic energy is a *property* of the body. Since kinetic energy is associated with the body as a whole, it is an *extensive* property of the body.

The units of kinetic energy are the same as that of work. The SI unit of work is the Newton-metre, N m, called the joule, J.

2.1.2 Potential Energy

Consider a body of mass m that moves vertically from an elevation z_1 to an elevation z_2 relative to the surface of the earth, as shown in Fig. 2.2. Two forces are shown acting on the system: a downward force due to gravity with magnitude mg and a vertical force with magnitude R representing the resultant of all *other* forces acting on the system.

In accordance with Eq. 2.4, the total work equals the change in kinetic energy. That is,

$$\frac{1}{2}m(V_2^2 - V_1^2) = \int_{z_1}^{z_2} R dz - \int_{z_1}^{z_2} mg dz \quad (2.5)$$

Assuming acceleration due to gravity to be constant we obtain

$$\frac{1}{2}m(V_2^2 - V_1^2) + mg(z_2 - z_1) = \int_{z_1}^{z_2} R dz \quad (2.6)$$

The quantity mgz is called the gravitational potential energy. Like kinetic energy, potential energy is an *extensive property*.

Equation 2.6 states that when a resultant force causes the elevation of a body to be increased, the body to be accelerated, or both, the work done by the force can be considered a *transfer* of energy *to* the body, where it is stored as gravitational potential energy and/or kinetic energy. If we consider a body on which the only force acting is that due to gravity, the right side of equation (2.6) vanishes and it reduces to

$$\frac{1}{2}m(V_2^2 - V_1^2) + mg(z_2 - z_1) = 0 \quad (2.7)$$

or

$$\frac{1}{2}mV_2^2 + mgz_2 = \frac{1}{2}mV_1^2 + mgz_1 \quad (2.8)$$

Equation 2.8 also illustrates that energy can be transformed from one form to another: For the object falling under the influence of gravity only, the potential energy would decrease as the kinetic energy increases by an equal amount.

2.2 ENERGY TRANSFER BY WORK

The work W done *by*, or *on*, a system evaluated in terms of macroscopically observable forces and displacements is

$$W = \int_{s_1}^{s_2} F_s \cdot ds \quad (2.7)$$

This definition is important in thermodynamics however; a broader interpretation of thermodynamic definition of work is now presented.

A particular interaction is categorised as work interaction if it satisfies the following criterion: *Work is done by a system on its surroundings if the sole effect on everything external to the system could have been the raising of a weight.* Note that the raising of a weight is, in effect, force acting through a distance. The test of whether a work interaction has taken place is not that the elevation of a weight has actually taken place, or that a force has actually acted through a distance, but that the sole effect *could have been* an increase in the elevation of a weight.

Work is a means of transferring energy. Accordingly, the term work does not refer to what is being transferred between systems or what is stored within the system. Energy is transferred and stored when work is done.

2.2.1 Sign convention and notation

$W > 0$: work is done *on* the system

$W < 0$: work is done *by* the system

The value of W depends on the details of the interactions taking place between the system and surroundings during a process and not just the initial and the final states. Hence, work is *not a property* of a system and its differential is *inexact* and it is expressed as δW , but the differential of every property is exact and is represented by a total derivative.

2.2.2 Expansion or Compression Work

Let us evaluate the work done by the closed system shown in Fig 2.3 consisting of a gas (or liquid) contained in a piston-cylinder assembly as the gas expands. During the process the gas pressure exerts a normal force on the piston. Let p denote the pressure acting at the interface between the gas and the piston. The force exerted by the gas on the piston is simply the product pA , where A is the area of the piston face. The work done by the system as the piston is displaced a distance dx is

$$\delta W = -pA dx = -p dV \quad (2.8)$$

Since dV is positive when the volume increases, the work at the moving boundary is negative when the gas expands. For compression, dV is negative, and so the work done from Eq. 2.8 is positive. For a change in volume from V_1 to V_2 , the work is obtained by integrating Eq. 2.8.

$$W = - \int_{V_1}^{V_2} p dV \quad (2.9)$$

Although Eq. 2.9 is derived for the case of a gas (or liquid) in a piston-cylinder assembly, it is applicable to systems of any shape provided the pressure is uniform with position over the moving boundary. To perform the integral of Eq. 2.9 requires a relationship between the gas pressure at the moving boundary and the system volume. Where there is the lack of pressure-volume relationship to merit the application of Eq. 2.9, the work can be determined from an energy balance (See section 2.5).

2.2.3 Work in Quasi-equilibrium Expansion or Compression processes

The quasi-static work given by equation (2.9) can be obtained from a graph of the process on a p - V diagram as shown in Fig. 2.4. The work done W is the area under the curve on the p - V diagram. The curve is obtained by plotting the variation of pressure with volume during the process and it depends on the details of the process as defined by the particular curve and not just on the end states hence work is **not** a property.

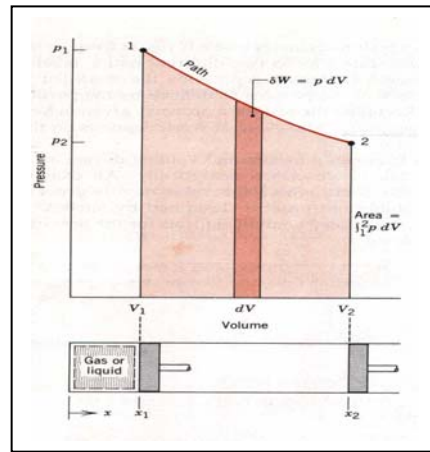
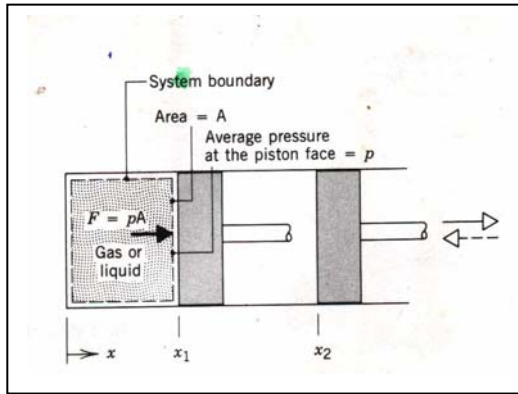


Fig.2.3 Expansion or compression of a gas or liquid **Fig. 2.4** Work of quasi-equilibrium process

The relationship between pressure and volume during an expansion or compression process also can be described analytically. An example is provided by the expression $pV^n = \text{constant}$, where the value of n is a constant for the particular process. A quasi-equilibrium process described by such an expression is called a *polytropic process*.

Note that

$$W = - \int_{V_1}^{V_2} p dV = - \int_{V_1}^{V_2} \frac{\text{constant}}{V^n} dV = - \text{constant} \left(\frac{V_2^{1-n} - V_1^{1-n}}{1-n} \right) \quad (2.10)$$

The constant in this expression can be evaluated at either end state: **i.e.** $\text{constant} = p_1 V_1 = p_2 V_2$. The work expression then becomes

$$W = - \frac{(p_2 V_2^n) V_2^{1-n} - (p_1 V_1^n) V_1^{1-n}}{1-n} = \frac{p_2 V_2 - p_1 V_1}{n-1} \quad (2.11)$$

This expression is valid for all values of n except $n = 1$. The case $n = 0$ is also treated under special cases (See 2.2.4).

2.2.4 Special cases of the polytropic process:

- (a) The case of $n = 1$: In this case, the pressure-volume relationship is $pV = \text{constant}$. The work done is

$$W = -\text{constant} \int_{V_1}^{V_2} \frac{dV}{V} = -(\text{constant}) \ln \frac{V_2}{V_1} = -(p_1 V_1) \ln \frac{V_2}{V_1} \quad (2.12)$$

- (b) The case of $n = 0$: The pressure-volume relation reduces to $p = \text{constant}$, and the integral becomes

$$W = -p(V_2 - V_1) \quad (2.13)$$

- (c) The pressure-volume relation for some processes may also be of the form $p = aV + b$. The work expression, in this case, is given by

$$W = - \int_{V_1}^{V_2} p dV = - \int_{V_1}^{V_2} (aV + b) dV = - \left[a \frac{V^2}{2} + bV \right]_{V_1}^{V_2} \quad (2.14)$$

By inserting the upper and lower limits into Eq. 2.14, the work done is evaluated. It can be shown that the expression for the work done reduces to

$$W = \left(\frac{p_1 + p_2}{2} \right) (V_2 - V_1) \text{ where the subscripts refer 1 and 2 refer to the initial and final states respectively.}$$

The latter expression is equivalent to the area under the p - V diagram for the process 1-2.

It must be noted that in whatever process, the work done can be evaluated as the area under the p - V diagram. The area interpretation of work in a quasi-equilibrium expansion or compression process allows for a simple demonstration of the idea that work depends on the process and therefore it is not a property. This can be confirmed with reference to Fig. 2.5.

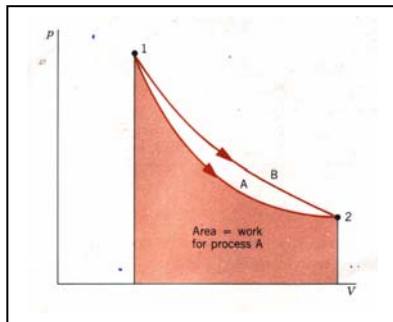


Fig. 2.5 Illustration that work depends of process

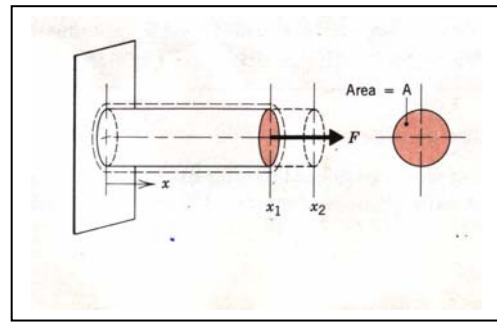


Fig. 2.6. Elongation of solid bar

2.2.4 Further examples of work in quasi-equilibrium processes

Extension of a Solid Bar: Consider of system of a solid bar, as shown in Fig. 2.6. The bar is fixed at $x = 0$, and a force F is applied at the other end. The force $F = \sigma A$, where A is the cross-sectional area of the bar and σ the normal stress acting at the end of the bar. The work done as the end of the bar moves a distance dx is given by $\delta W = +\sigma A dx$. The plus sign is required because work is done *on* the bar when dx is positive.

Stretching of a Liquid Film: Fig. 2.7 shows a system consisting of a liquid film suspended on a wire frame. The two surfaces of the film support the thin liquid layer inside by the effect of *surface tension*. Denoting the *surface tension acting at the movable wire* by τ , the force F indicated on the figure can be expressed as $F = 2l\tau$, where the factor 2 is introduced because two film surfaces act at the wire. If the movable wire is displaced by dx , the work done is given by $\delta W = +2l\tau dx$. The plus sign is required because work is done *on* the system when dx is positive.

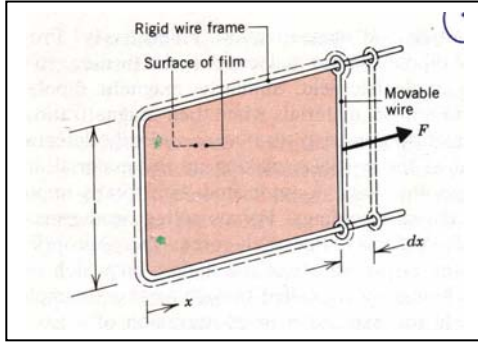


Fig. 2.7. Stretching of a liquid film

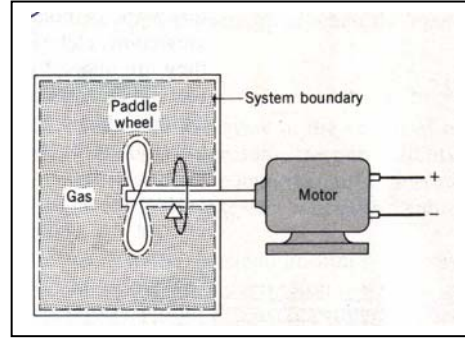


Fig. 2.8. Power transmitted to a gas by a paddle wheel

Power Transmitted by a shaft: Consider a shaft rotating with angular velocity ω and exerting a torque τ on its surroundings. The torque can be expressed in terms of a tangential force F_t and radius R : $\tau = F_t R$. The velocity at the point of application of the force is $V = R\omega$, where ω is in radians per unit time.

The rate of energy transfer by work is called *power* and is denoted by \dot{W} . The rate of energy transfer by work is the product of the force and the velocity at the point of application of the force

$$\dot{W} = F \cdot V \quad (2.15a)$$

Therefore the expression for the *power* transmitted from the shaft to the surroundings is given by

$$\dot{W} = F_t V = (\tau/R)(R\omega) = \tau\omega \quad (2.15b)$$

A related case involving a gas stirred by a paddle wheel is shown in Fig. 2.8.

Electrical work: Shown in Fig. 2.9 is a system consisting of an electrolytic cell. The cell is connected to an external circuit through which an electric current is flowing. Work is done on the system whenever electrons cross the boundary of the system in response to the force associated with an electric field. The flow of electrons is manifested as the current, i , driven by the electric potential difference, E , existing across the terminals a and b . It can be envisioned that the current is supplied to an electric motor that lifts a mass in the surroundings.

The rate at which work is done, or power, is $\dot{W} = Ei$.

Since the current i equals dZ/dt , the work can be expressed in differential form as $\delta W = E dZ$.

When the power is evaluated in terms of the watt, and the unit of current is the ampere, the unit of electric potential is the volt, defined as 1 watt per ampere.

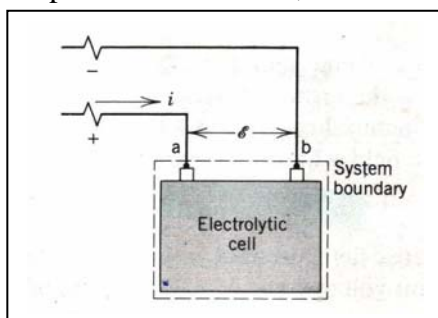


Fig. 2.10 Electrolytic cell used to discuss electrical work

2.3 ENERGY TRANSFER BY HEAT

The quantity denoted by Q accounts for the amount of energy transferred to a closed system during a process by means other than work. Such an energy transfer is induced only as a result of a temperature difference between the system and its surroundings and occurs only in the direction of decreasing temperature. This means of energy transfer is called *energy transfer by heat*.

The symbol Q denotes an amount of energy transferred across the boundary of a system in a heat interaction with the system's surroundings. Heat transfer *into* a system is taken to be *positive*, and heat transfer *from* the system is taken as *negative*.

2.3.1 Sign convention for Q

$$\begin{aligned} Q > 0: & \text{heat transfer to the system} \\ Q < 0: & \text{heat transfer from the system} \end{aligned}$$

Like work, heat is not a property and its differential is δQ . The amount of energy transfer by heat for a process is given by the integral

$$Q = \int_1^2 \delta Q = Q_{12}$$

The principal modes of heat transfer are conduction, convection and radiation.

2.3.2. Heat transfer modes

Energy transfer by conduction can take place in solids, liquids, and gases. Fourier's Law quantifies the time rate of energy transfer by conduction macroscopically. As an illustration, consider the system shown in Fig 2.11, in which the temperature distribution is $T(x)$. The time rate at which energy enters the system by conduction through a plane area A perpendicular to the coordinate x is given by Fourier's law as

$$\dot{Q} = -k A \frac{dT}{dx} \quad (2.16a)$$

The proportionality factor k , which may vary with position, is a property of the material called the thermal conductivity. The minus in Eq. 2.17 is a consequence of the requirement that energy flow is in the direction of decreasing temperature.

Thermal radiation is emitted by matter as a result of the changes in the electronic configurations of the atoms or molecules within it. Unlike conduction, thermal radiation requires no intervening medium to propagate and can even take place in a vacuum. Solid surfaces, gases, and liquids all emit, absorb, and transmit thermal radiation to varying degrees. The rate at which energy is emitted, \dot{Q}_e , from a system with a surface A is quantified macroscopically by a modified form of the *Stefan-Boltzmann law* as

$$\dot{Q}_e = \varepsilon \sigma A T_s^4 \quad (2.16b)$$

T_s is the temperature of the surface, ε is a property of the surface that indicates how effectively the surface radiates ($0 \leq \varepsilon \leq 1$) and σ is the Stefan-Boltzmann constant.

Energy transfer between a solid surface at one temperature and an adjacent moving gas or liquid at another temperature is called convection. The rate of energy transfer from the system to

the fluid can be quantified by the following empirical expression called the Newton's Law of cooling

$$\dot{Q} = h A (T_b - T_f) \quad (2.16c)$$

Refer to Fig. 2.12 to schematic representation of this mode of energy transfer by heat. The proportionality factor in the Newton's law of cooling is called the *heat transfer coefficient* and is *not* a thermodynamic property. It is an empirical parameter that incorporates into heat transfer relationship the nature of the fluid flow pattern near the surface, the fluid properties, and the geometry of the system.

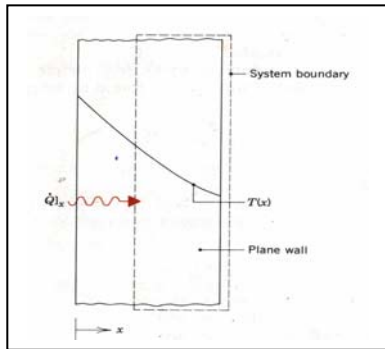


Fig. 2.11 Illustration of Fourier's conduction law

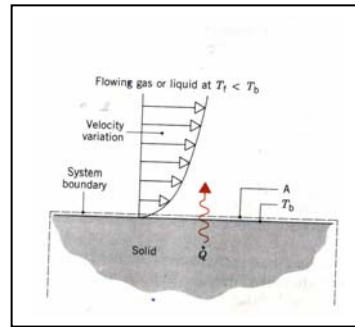


Fig. 2.12 Illustration of Newton's Law of cooling

2.4 FIRST LAW OF THERMODYNAMICS

Under this section, the various forms of the First Law Thermodynamics (Non flow energy equation, steady flow energy equation and the general energy equation) will be studied and applied to solve thermodynamic problems.

2.4.1 Conservation of Energy and Matter

The Principle of Conservation of Energy states "Energy is neither created nor destroyed"

The Principle of Conservation of Matter states "Matter is neither created nor destroyed"

2.4.2 First Law of Thermodynamics for Closed Systems

Because of the historical association of the First Law with heat engine cycles, namely cycles producing a net amount of work on the surroundings, the law is usually stated in the form:

When any closed system is taken through a cycle, the net work delivered to the surroundings is proportional to the net heat taken from the surroundings.

However, the converse is also true, namely:

When a closed system is taken through a cycle, the work done on the system is proportional to the net heat delivered to the surroundings.

In its latter form, the statement refers to heat pump and refrigerator cycles.

Corollary 1

There exists a property of a closed system such that a change in its value is equal to the sum of the net heat and work transfers during any change of state. This property is the energy, E , of the system.

Thus, the First Law can be stated mathematically as

$$\sum_1^2 \delta Q + \sum_1^2 \delta W = \Delta E \quad (2.17)$$

However, in thermodynamics we are often concerned with stationary closed systems in which case the total energy, E , equals the internal energy, U , of the system. The First Law is, therefore, expressed mathematically in a form referred to as the *non-flow energy equation* (NFEE)

$$Q_{12} + W_{12} = \Delta U = U_2 - U_1 \quad (2.18)$$

Where, $\sum_1^2 \delta Q = Q_{12}$ and $\sum_1^2 \delta W = W_{12}$.

Equation 2.17 may also be expressed in the differential form as

$$\delta Q + \delta W = dU \quad (2.19)$$

The first law of thermodynamics establishes the internal energy as a property of the system. As a property, the internal energy of a closed system in any given state would have only one value independent of the process through which the system arrived at the state.

For an isolated system the heat and work transfer are zero. It follows from equation (2.19), that the change in internal energy must be zero, which means that the internal energy of an isolated system is constant.

Corollary 2

The internal energy of a closed system remains unchanged if the system is isolated from the surroundings.

That for isolated system $W = 0$, $Q = 0$ and it implies that $U_1 = U_2$.

All what happens in this case is spontaneous redistribution of energy between parts of the system, which continues until a state of equilibrium is reached; there is no change in total quantity of energy within the system during the process. This corollary is the Principle of Conservation of Energy.

2.4.3 First Law for a cyclic Process

When a system undergoes a cycle all the thermodynamic properties, including internal energy; return to their initial values. The change in internal energy is, therefore, zero for the cyclic process. It follows from the First Law Thermodynamics that

$$\sum \delta Q + \sum \delta W = 0 \quad \text{or} \quad \oint \delta Q = - \oint \delta W. \quad (2.20)$$

That is the cyclic sum of the heat transfers and the work transfers should be zero. We can deduce from Eq. 2.20 that if a net amount of heat is not supplied by the surroundings during a cycle, no net amount of work can be delivered by the system. One can conclude from Equation (2.20) that whenever a system operates in a cycle and the net heat transfer equals zero, then the net work transfer must also be zero. A machine that operates in contravention to this conclusion is impossible. Such a machine is described as *perpetual motion machine of the first kind*, which is a machine that will produce a continuous supply of work without absorbing energy from the surroundings. It is, therefore, impossible for a closed system to operate in a cycle and do a net amount of work without heat being transferred to the system and, therefore, it is impossible for a perpetual motion machine of the first kind to exist. This fact is presented as the third corollary of the First Law:

Corollary 3

A perpetual motion machine of the first kind is impossible

However, it is always possible to devise a machine to deliver a limited quantity of work without requiring a source of energy from the surroundings. For example, a gas compressed behind a piston will expand and do some amount of work at the expense of internal energy of the gas. Note that such a device cannot produce work continuously.

2.4.4 Energy analysis of cycles

In this section, the energy concepts developed are illustrated further by application to systems undergoing cycles. Recall that when a system at a given initial state goes through a sequence of processes and finally returns to that state, the system has executed a thermodynamic cycle. In this section, cycles are considered from the point of view of conservation of energy.

The energy balance for any system undergoing a thermodynamic cycle takes the form

$$\Delta E_{\text{cycle}} = Q_{\text{cycle}} + W_{\text{cycle}} \quad (2.21)$$

where Q_{cycle} and W_{cycle} represent the *net* amounts of energy transfer by heat and work, respectively, for the cycle. Since the system is returned to its initial state after the cycle, there is *no* net change in its energy. Therefore Eq. 2.22 reduces to $Q_{\text{cycle}} = -W_{\text{cycle}}$ (as Eq. 2.20)

For a power cycles the system delivers a net work transfer of energy to their surroundings during each cycle for a net heat transfer to the system. The performance of a system undergoing a power cycle can be described in terms of the extent to which the energy added by heat Q_{in} , is *converted* to a net work output, W_{cycle} . The extent of the energy conversion from heat to work is expressed by the following ratio commonly called the *thermal efficiency*:

$$\eta = - \frac{W_{\text{cycle}}}{Q_{\text{in}}} \text{ (Power cycle)} \quad (2.22a)$$

The negative is introduced to make the thermal efficiency positive since W_{cycle} is the net work done by the system on its surroundings and is considered negative according to the sign convention of work that we have adopted in this course.

For refrigeration cycles and heat pump, Q_{in} is transferred by heat into the system undergoing the cycle from a cold body, and Q_{out} is the energy discharged by heat transfer from the system to the hot body. To accomplish these energy transfers requires a net work input,

W_{cycle} . The performance of a refrigeration or heat pump cycle is described as the ratio of the amount of *energy received by the system* undergoing the cycle from the cold body, Q_{in} , to the net work transfer of energy into the system to accomplish this effect, W_{cycle} . This parameter is called the coefficient of performance and is given by

$$\beta = \frac{Q_{\text{in}}}{W_{\text{cycle}}} \quad (\text{Refrigeration cycle}) \quad (2.22b)$$

For household refrigerator, Q_{out} is discharged to space in which the refrigerator is located. W_{cycle} is usually provided in the form of electricity to run the motor that drives the refrigerator.

For a heat pump the coefficient of performance is

$$\gamma = \frac{Q_{\text{out}}}{W_{\text{cycle}}} \quad (\text{heat pump}) \quad (2.22c)$$

2.5 APPLICATION OF FIRST LAW TO NON-FLOW PROCESSES (CLOSED SYSTEMS)

2.5.1 Constant pressure process

For a closed system undergoing a constant pressure process between states 1 and 2, the First Law can be written as

$$\int_1^2 \delta Q = U_2 - U_1 - \int_1^2 \delta W \quad (2.23)$$

Or

$$\int_1^2 \delta q = u_2 - u_1 - \int_1^2 p dv = u_2 - u_1 + p(v_2 - v_1) = (u_2 + p v_2) - (u_1 + p v_1) = h_2 - h_1 \quad (2.24)$$

$$\text{i.e. } \boxed{Q_{12} = H_2 - H_1} \quad (2.25)$$

Thus, for a closed system undergoing a constant pressure process the heat transferred is given by the change in the enthalpy of the system.

$$\text{Hence, } \boxed{h = u + pv} \quad (2.26)$$

Enthalpy is particularly useful in the analysis of closed systems undergoing a constant pressure process and open systems undergoing all kinds of process (as will be seen in section 2.6 of this chapter) because it includes the flow work given by the term “ pv ” as well as the internal energy of the system.

2.5.2 Polytropic process

The relation between pressure and volume is given by $p v^n = \text{constant}$, where n is the polytropic index. The expression for the work transfer can be expressed by any of the relations derived in section 2.2.4 depending on the relationship between p and v .

The energy transferred by heat between states 1 and 2 is given as $Q_{12} + W_{12} = U_2 - U_1$. When the relevant values of the work transfer and the internal energy are inserted the energy transferred by heat can be evaluated.

2.5.3 Adiabatic process

This is the process undergone by a system thermally insulated from its surroundings. Work is done at the expense of its internal energy. For such a process since $Q = 0$, from the First Law

$$\boxed{W_{12} = U_2 - U_1} \quad (2.27)$$

In adiabatic expansion, there is decrease in the internal of the system whereas in adiabatic compression there is increase in the internal energy of the system.

2.5.4 Isothermal process

This is a constant temperature process. From the First Law

$$\delta q + \delta w = du \quad (2.28)$$

But for a reversible process $dq = T ds$ and since T is constant for an isothermal process it is possible to integrate this expression directly. Thus, for a reversible isothermal process $Q = T(s_2 - s_1)$. Therefore, $\boxed{W_{12} = u_2 - u_1 - Q_{12}}$, in general for an isothermal process.

For a simple compressible substance, $\boxed{du = c_v dT}$ and therefore the *change* in internal energy for a simple compressible substance is zero. Therefore, for simple compressible substance undergoing an isothermal process between states 1 and 2.

$$Q_{12} = -W_{12} \quad (2.29)$$

2.5.5 Constant volume process

Applying the First Law and noting that $W = 0$ since $dV = 0$, we have

$$\boxed{Q_{12} = U_2 - U_1} \quad (2.30)$$

In the differential form, $\delta Q = du$. If the heat addition process is reversible then we write the differential in the form for a constant volume process in the form $\boxed{dQ = du}$

2.6 FIRST LAW OF THERMODYNAMICS FOR OPEN SYSTEMS

2.6.1 General Energy Equation

Figure 2.13 is the sketch of a typical open system. In practice an open system may have several inlets and outlets but it is usual to have one inlet and one outlet. It is also possible for the actual system boundaries to move but for most engineering applications the boundaries may be fixed. Considering an open system with fixed boundaries we pursue the analysis.

We consider the events that take place over a short time interval dt . At the beginning of the time interval (i.e. at the time t) the mass of the fluid in the open system is $m_{s,t}$ and a small mass of fluid dm_i is just about to enter the system. During this time interval, the small mass of fluid, dm_i , enters the system and another small mass of fluid dm_o , leaves the system such that the mass of fluid at the end of the interval is $m_{s,t+dt}$. During this time interval, small amounts of heat and work, δQ and δW , cross the system boundaries and the properties of the fluid inside the system may change such that the energy of the system also changes from $E_{s,t}$ to $E_{s,t+dt}$. Figure 2.13 shows the imaginary closed system at the beginning and end of the time interval. It is assumed that all properties are uniform at the inlet and outlet portions of the system. From the First Law we can write

$$\delta Q + \delta W = E_{x,t+dt} - E_{x,t} \quad (2.31)$$

where the subscript x refers to the imaginary closed system.

At the time t the total energy of the imaginary closed system is given by

$$E_{x,t} = E_{s,t} + dm_i(u_i + \frac{1}{2}C_i^2 + g z_i) \quad (2.32)$$

and at the time $t + dt$ the total energy is

$$E_{x,t+dt} = E_{s,t+dt} + dm_o(u_o + \frac{1}{2}C_o^2 + g z_o) \quad (2.33)$$

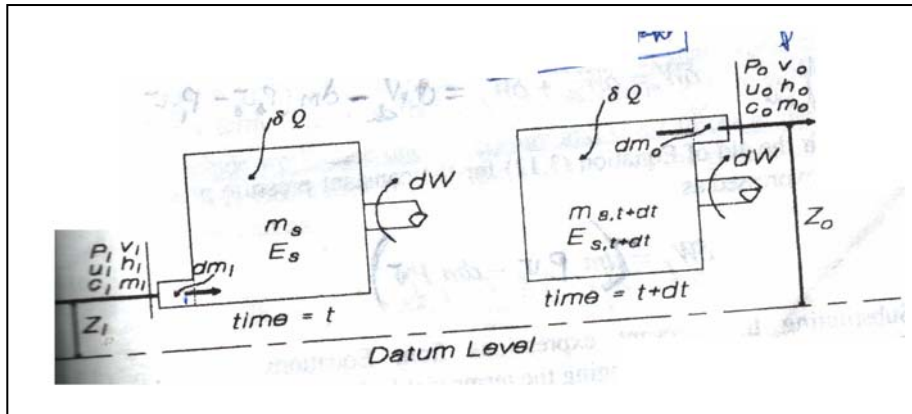


Fig. 2.13 Diagram showing an imaginary closed system at the beginning and end of a differential change of state.

The work done during the time interval is the sum of the shaft work and the flow work, which is the work, required to push the small mass dm_i into the original system and push dm_o out. The work transfer term in equation 2.31 is therefore given by

$$\delta W = \delta W_{sh} + \delta W_f = \delta W_{sh} - \delta m(p_o v_o - p_i v_i) \quad (2.34)$$

Substituting Eqs. 2.32 – 2.34 into Eq. 2.31 yields

$$\delta Q + \delta W_{sh} = (E_{s,t+dt} - E_{s,t}) + dm_o(u_o + p_o v_o + \frac{1}{2}C_o^2 + g z_o) - dm_i(u_i + p_i v_i + \frac{1}{2}C_i^2 + g z_i) \quad (2.35)$$

Substituting for enthalpy and dividing by dt yields

$$\frac{\delta Q}{dt} + \frac{\delta W_{sh}}{dt} = \frac{(E_{s,t+dt} - E_{s,t})}{dt} + \frac{dm_o}{dt} (h_o + \frac{1}{2} C_o^2 + g z_o) - dm_i (h_i + \frac{1}{2} C_i^2 + g z_i) \quad (2.36)$$

In the limit as dt approaches zero, Eq. 2.36 becomes

$$\dot{Q} + \dot{W}_{sh} = \frac{dE_s}{dt} + \sum_{outlets} \dot{m}_o (h_o + \frac{1}{2} C_o^2 + g z_o) - \sum_{inlets} \dot{m}_i (h_i + \frac{1}{2} C_i^2 + g z_i) \quad (2.37)$$

Finally, it is necessary to account for the possibility of moving boundaries which may be rotating or undergoing a displacement. Since when the boundary moves more work terms may be considered the shaft work in equation (2.37) is replaced by a more general work term \dot{W} . With this substitution, Eq. 2.37 becomes

$$\dot{Q} + \dot{W} = \frac{dE_s}{dt} + \sum_{outlets} \dot{m}_o (h_o + \frac{1}{2} C_o^2 + g z_o) - \sum_{inlets} \dot{m}_i (h_i + \frac{1}{2} C_i^2 + g z_i) \quad (2.38)$$

2.6.2 Steady-Flow Energy Equation (SFEE)

The assumptions for the SFEE may be summarised as follows:

1. The boundaries of the system are fixed
2. The mass flow rate of the fluid is constant and is the same at inlet and outlet of the system ($\dot{m}_o = \dot{m}_i = \dot{m}$)
3. The properties of the fluid at any point in the system remain constant with time ($\frac{dE_s}{dt} = 0$)
4. Heat and work transfer cross the boundaries of the system at uniform rates.

Following the above assumptions, the SFEE becomes

$$\dot{Q} + \dot{W} = \dot{m} (h_o - h_i) + \frac{1}{2} \dot{m} (C_o^2 - C_i^2) + \dot{m} g (z_o - z_i) \quad (2.39)$$

Note when the flow is one-dimensional, the mass flow rate becomes $\dot{m} = \rho A V$ and in terms of specific volume, v , the relation is $\dot{m} = \frac{A V}{v}$.

2.6.3 Practical applications of the SFEE

2.6.3.1 Nozzles and diffusers

A nozzle is a flow passage of varying cross-sectional area in which the velocity of the gas or liquid increases in the direction of flow. In a diffuser, the liquid decelerates in the direction of flow. Figure 2.14 shows a nozzle in which the cross-sectional area decreases in the direction of flow and a diffuser in which the walls of the flow passage diverge. For nozzles and diffusers, the only work is the *flow work* at locations where the mass enters and exits the control volume and so the term W drops out of the energy rate equation for these devices. The change in the potential energy from the inlet to exit is negligible under most conditions. By combining these into a single expression and dropping the potential energy change from inlet to exit

$$\dot{Q}_{12} = \dot{m} (h_2 - h_1) + \dot{m} \frac{C_2^2 - C_1^2}{2} \quad (2.40)$$

The subscripts 1 and 2 denote the inlet and exit, respectively.

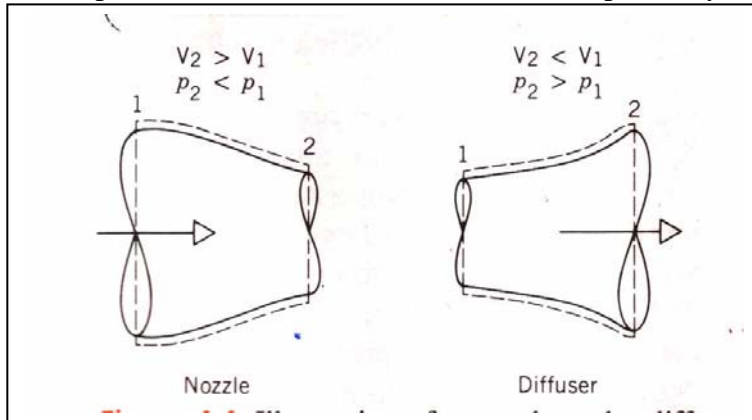


Fig. 2.14 Illustration of a nozzle and a diffuser

2.6.3.2 Turbines, compressors, fans and pumps

A turbine is a device in which work is developed as a result of a gas or liquid passing through a set of blades attached to a shaft free to rotate. A schematic of an axial-flow steam or gas turbine is shown in Figure 2.15. Turbines are widely used in vapour power plants, gas turbines power plants, and aircraft engines. In these applications, superheated steam or gas enters the turbine and expands to a lower exit pressure as work is developed. Water turbines are used in electric power plants.

Compressors, fans and pumps, on the other hand, are devices that require mechanical power to compress fluid or, simply put, to move a fluid from a low pressure to a relatively high-pressure zone.

The processes that take place inside turbines, compressors, fans and pumps are usually assumed to be adiabatic and therefore $\dot{Q} = 0$. Secondly, kinetic and potential energy terms are assumed to be negligible compared to the other terms in the SFEE. The resulting equation for this class of thermodynamic application is

$$\dot{W}_{12} = \dot{m} (h_2 - h_1) \quad (2.41)$$

Note that turbines are work-producing devices, while compressors, fans, and pumps are work-requiring devices.

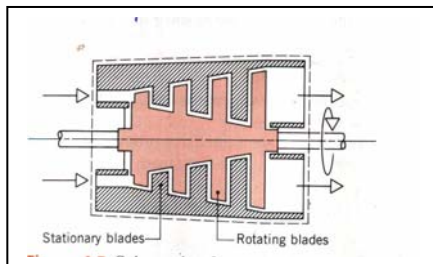


Fig. 2.15a Schematic of an axial flow turbine

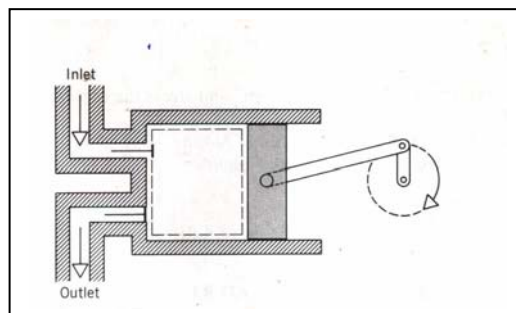


Fig. 2.15b Reciprocating compressor

2.6.3.3 Throttling devices

A throttling device is a restriction in a fluid flow channel, which is designed to effect a significant pressure drop with no work transfer and no change in potential and kinetic energies. In this regard, the ordinary valve in pipeline may be considered as a throttling device. The throttling process takes place over a short distance so that heat transfer is also considered negligible. The SFEE then reduces to

$$h_2 = h_1 \quad (2.42)$$

That is a throttling process is an isentropic process. Throttling valves are used extensively to reduce the pressure refrigerant flowing through the valve. In the vapour-compression refrigerator a valve is used to reduce the pressure of the refrigerant from the pressure at the exit of the *condenser* to the power pressure existing in the *evaporator*. Schematic representations of throttling devices are illustrated in Figure 2.16.

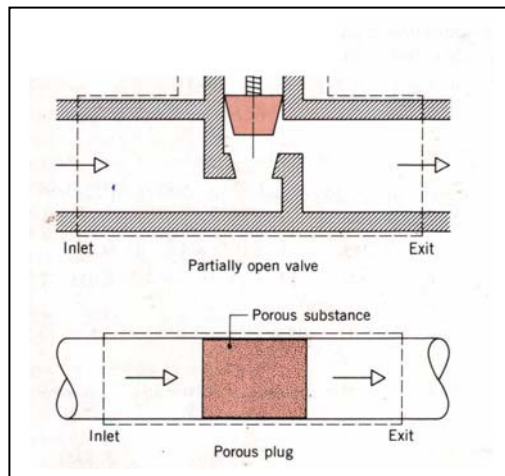


Fig. 2.16 Throttling devices

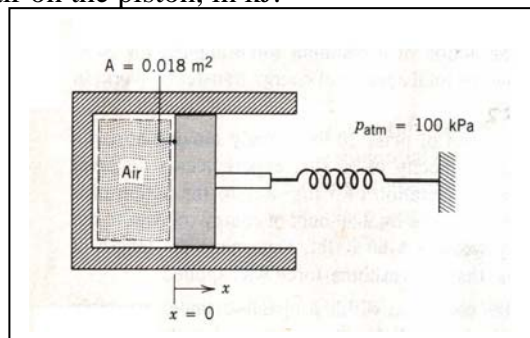
TUTORIAL SET 2

1. A 5 kg of steam is contained within a piston-cylinder assembly. The steam undergoes an expansion from state 1, where the specific internal energy is $u_1 = 2709$ kJ/kg to state 2, where $u_2 = 2659.6$ kJ/kg. During this process, there is heat transfer of energy to the steam with a magnitude of 80 kJ. Also, a paddle wheel transfers energy to the steam by work in the amount of 18.5 kJ. There is no significant change in the kinetic or potential energy of the steam. Determine the amount of energy transfer by work from the steam to the piston during the process, in kJ. **[- 350 kJ]**

2. A gas is compressed from $V_1 = 0.09$ m³ to $V_2 = 0.03$ m³. The relation between pressure and volume during the process is $p = -14 V + 2.44$, where the units of p and V are bar and m³, respectively. For the gas, find the work, in kJ. **[+ 9.6 kJ]**

3. A gas expands from an initial state where $p_1 = 500$ kPa and $V = 0.1$ m³ to a final state where $p_2 = 100$ kPa. The relationship between pressure and volume during the process is $pV = \text{constant}$. Sketch the process on a p - V diagram and determine the work, in kJ.

4. Air is trapped in a piston-cylinder assembly oriented horizontally as shown in Fig. 1. Initially, $p_1 = 100$ kPa, $V_1 = 2 \times 10^{-3}$ m³, and the face of the piston is at $x = 0$. The spring exerts no force on the piston in the initial position. The atmospheric pressure is 100 kPa, and the area of the piston face is 0.018 m². The air expands slowly until its volume is $V_2 = 3 \times 10^{-3}$ m³. During the process, the spring exerts a force on the piston that varies with x according to $F = kx$, where $k = 1.62 \times 10^3$ N/m. There is no friction between the piston and the cylinder wall. Determine the final pressure of the air, in kPa, and the work done by the air on the piston, in kJ.



- 5 Each line in the table below gives information about a process of a closed system. Every entry has the same energy units. By adopting a sign convention for which heat and work inflows into the system are considered positive, copy and complete the table below.

Process	Q	W	U_1	U_2	ΔU
a	+ 50	+ 20	+ 20		
b		- 20		+ 50	+ 30
c	- 25	+ 80		+ 160	
d		+ 90	+ 50		0
e		- 150	+ 20		- 100

6. Gas undergoes a thermodynamic cycle consisting of the following processes:

Process 1 – 2 constant pressure $p = 1.4$ bars, $V_1 = 0.028 \text{ m}^3$, $W_{12} = -10.5 \text{ kJ}$

Process 2 – 3 compression with $pV = \text{constant}$, $U_3 = U_2$

Process 3 – 1 constant volume, $U_1 - U_3 = -26.4 \text{ kJ}$

There are no significant changes in the kinetic or potential energy.

- (i) Sketch the cycle on a $p - V$ diagram.
- (ii) Calculate the net work for the cycle, in kJ.
- (iii) Calculate the heat transfer for process 1 – 2, in kJ.
- (iv) Determine the pressure of the gas at state 3.

7. Water vapour in a piston-cylinder assembly undergoes a process from saturated vapour at 150°C to a pressure of 3 bars. During the process, the pressure and specific volume are related by $pv^{1.2} = \text{constant}$. Neglecting kinetic and potential energy effects, determine the heat transfer and the work per unit mass of water vapour, each in kJ/kg.

8. A gas undergoes a thermodynamic cycle consisting of three processes beginning at an initial state where $p_1 = 1$ bar, $V_1 = 1.5 \text{ m}^3$, and $U_1 = 512 \text{ kJ}$. The processes are as follows:

Process 1-2: compression with $pV = \text{constant}$ to $p_2 = 2$ bar, $U_2 = 690 \text{ kJ}$

Process 2-3: $W_{23} = 0$, $Q_{31} = -150 \text{ kJ}$

Process 3-1: $W_{31} = -50 \text{ kJ}$

There are no significant changes in kinetic or potential energy. Determine the heat transfers Q_{12} and Q_{31} , each in kJ.

9. Steam enters a nozzle operating at steady state with $p_1 = 40$ bars, and a velocity of 10 m/s. The steam flows through the nozzle with negligible heat transfer and no significant change in potential energy. At the exit, $p_2 = 15$ bars, and the velocity is 665 m/s. The mass flow rate is 2 kg/s. Determine the exit area of the nozzle. [Ans. $4.89 \times 10^{-4} \text{ m}^2$]

10. Air is contained in a rigid well-insulated tank with a volume of 0.2 m^3 . The tank is fitted with a paddle wheel that transfers energy to the air at a constant rate of 4 W for 20 min. The initial density of the air is 1.2 kg/m^3 . If no changes in kinetic or potential energy occur, determine

- (a) The specific volume at the final state, in m^3/kg .
- (b) The change in specific internal energy of the air, in kJ/kg.

[0.833 m^3/kg , 20 kJ/kg]

11. The mass flow rate into a steam turbine is 1.5 kg/s and heat transfer from the turbine is estimated at 8.5 kW. The properties of the steam at inlet and outlet of the turbine are as follows:

	<u>Inlet</u>	<u>Outlet</u>
Enthalpy	3137.7 kJ/kg	2675.5 kJ/kg
Velocity	80 m/s	200 m/s
Elevation	6 m	3 m

What is the power output of the turbine?

[658.9 kW]

12. A cylinder-piston arrangement containing a gas has a spring, which makes contact with the piston. As the gas is heated the piston moves outward and compresses the spring such

that the spring force is proportional to the displacement of the piston. The initial volume and pressure of the gas are 0.1 m^3 and 1 bar , respectively and the final pressure is 3 bar . If the volume of the gas doubles during the process and the internal energy also increases by 30 kJ , calculate the heat transferred during the process. **[50 kJ]**

13. A cylinder-piston arrangement containing a gas has a spring located a certain distance above the piston. Initially, the volume and pressure of the gas are $1 \times 10^{-3} \text{ m}^3$ and 150 kPa , respectively, and the gas pressure just balances the external atmospheric pressure and the cylinder piston weight. The gas is heated until its volume doubles and at this point the piston just makes contact with the spring. The gas is then heated further such that the change in gas pressure is directly proportional to the displacement of the spring. The final pressure and volume are 300 kPa and $3 \times 10^{-3} \text{ m}^3$, respectively.

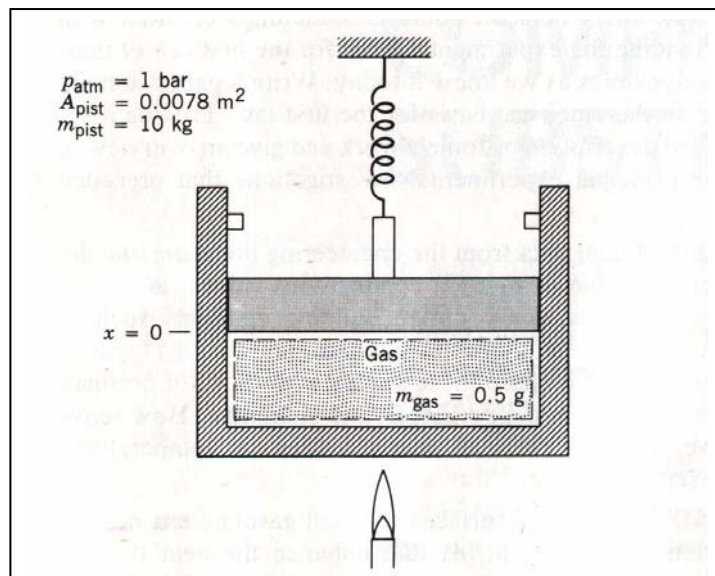
- Draw the p - V diagram for the two heating processes.
- Show that for the second heating process (i.e. when the piston is in contact with the spring) the relationship between p and V is given by

$$p = 1.5 \times 10^5 (V - 10^{-3})$$
 where p is in kPa and V is in m^3 .
- Use an analytical approach to calculate the work done in the two processes and check your answers using the graphical approach.
- Determine the overall change in internal energy of the gas if the total amount of heat transferred is 765 kJ .

[-150 J, -225 J, 300 J]

14. A gas contained within a piston-cylinder assembly is shown in Fig. 4. Initially, the piston face is at $x = 0$, and the spring exerts no force on the piston. As a result of heat transfer, the gas expands, raising the piston until it hits the stops. At this point the piston face is located at $x = 0.05 \text{ m}$, and the heat transfer ceases. The force exerted by the spring on the piston as the gas expands varies linearly with x according to $F = kx$ where $k = 10,000 \text{ N/m}$. Friction between the piston and the cylinder wall can be neglected. The acceleration due to gravity is $g = 9.81 \text{ m/s}^2$. Additional information is given on Fig. 4.

- What is the initial pressure of the gas, in kPa ?
- Determine the work done by the gas on the piston, in J .
- If the specific internal energies of the gas at the initial and final states are 214 and 337 kJ/kg , respectively, calculate the heat transfer, in J .



15. For a power cycle the total heat transfer to the cycle is, Q_{in} is 500 MJ. What is the net work delivered, in MJ, if the thermal efficiency is 30 %. **[150 MJ]**
16. The coefficient of performance of a heat pump cycle is 3.5 and the net work *input* is 500 kJ. Determine the heat transfers Q_{in} and Q_{out} , in kJ. **[12,500 kJ; - 17,500 kJ]**
17. A block of mass 10.0 kg is pushed along an incline a distance of 5.0 m as its centre of gravity is elevated by 3.0 m. The block is acted on by a constant force R parallel to the incline having a magnitude of 70 N and by the force of gravity. Assume frictionless surfaces and $g = 9.8 \text{ m/s}^2$. Determine in J.
 (a) the work done on the block by the constant force R .
 (b) the change in potential energy of the block.
 (c) The change in kinetic energy of the block.
[350 J; 294 J; 56 J]
18. A system with a mass of 10 kg, initially at rest, experiences a constant horizontal acceleration of 4 m/s^2 due to the action of a resultant force applied for 20 s. Determine the total amount of energy transfer by work, in kJ. **[32 kJ]**
19. An object whose mass is 40 kg falls freely under the influence of gravity from an elevation of 100 m above the earth's surface. The initial velocity is directed downward with a magnitude of 100 m/s. Ignoring the effect of air resistance, what is the magnitude of the velocity, in m/s, of the object just before it strikes the earth? The acceleration of gravity is $g = 9.81 \text{ m/s}^2$.
[109.4 m/s]
20. A gas contained in piston-cylinder assembly expands in a constant-pressure process at 4 bar from $V_1 = 0.15 \text{ m}^3$ to a final volume of $V_2 = 0.36 \text{ m}^3$. Calculate the work, in kJ.