Chapter 5

TWO-PORT NETWORKS

Introduction

- Two-port networks have two ports or two pairs of terminals of interest.
- Two-port analysis allows us to focus on the relationships between the voltage and current variables of the two ports.
- Thus, it allows us to collapse the many equations of nodal and mesh to a single pair of equations.

Introduction cont'd

- In other words the two-port network is treated as a black box modelled by the relationships between the four variables.
- The relationships between the two voltages and the two currents are described in terms of quantities known as parameters.
- The popular two-port parameters are admittance, impedance, hybrid and transmission.

Introduction cont'd

- Knowing the two-port parameters of a network or system permits us to describe its operation when it is connected into a larger network (System Analysis).
- Two-port networks are used to model devices in electronics such as transistors and op-amps and electrical components such as transformers and transmission lines.

Two-port network

- It has input (on the left) and output (on the right) ports for external connection.
- Each port is assumed to satisfy the port condition (i.e. the current into the network at one terminal of a port is equal to the current flowing out the other terminal of the port or KCL must be satisfied at each port).
- See next slide for the diagram.
- It is customary to label the voltages and currents as shown.

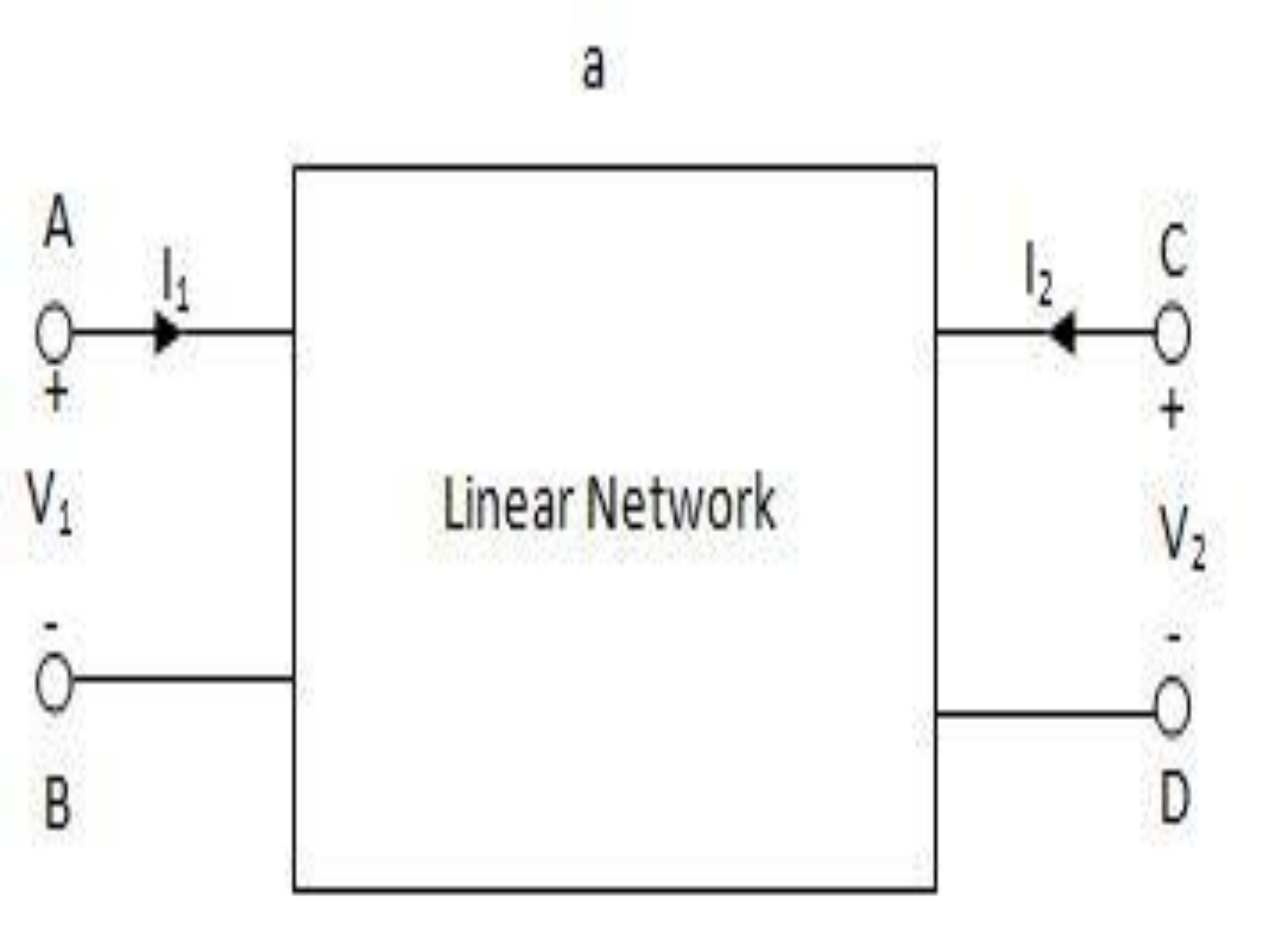
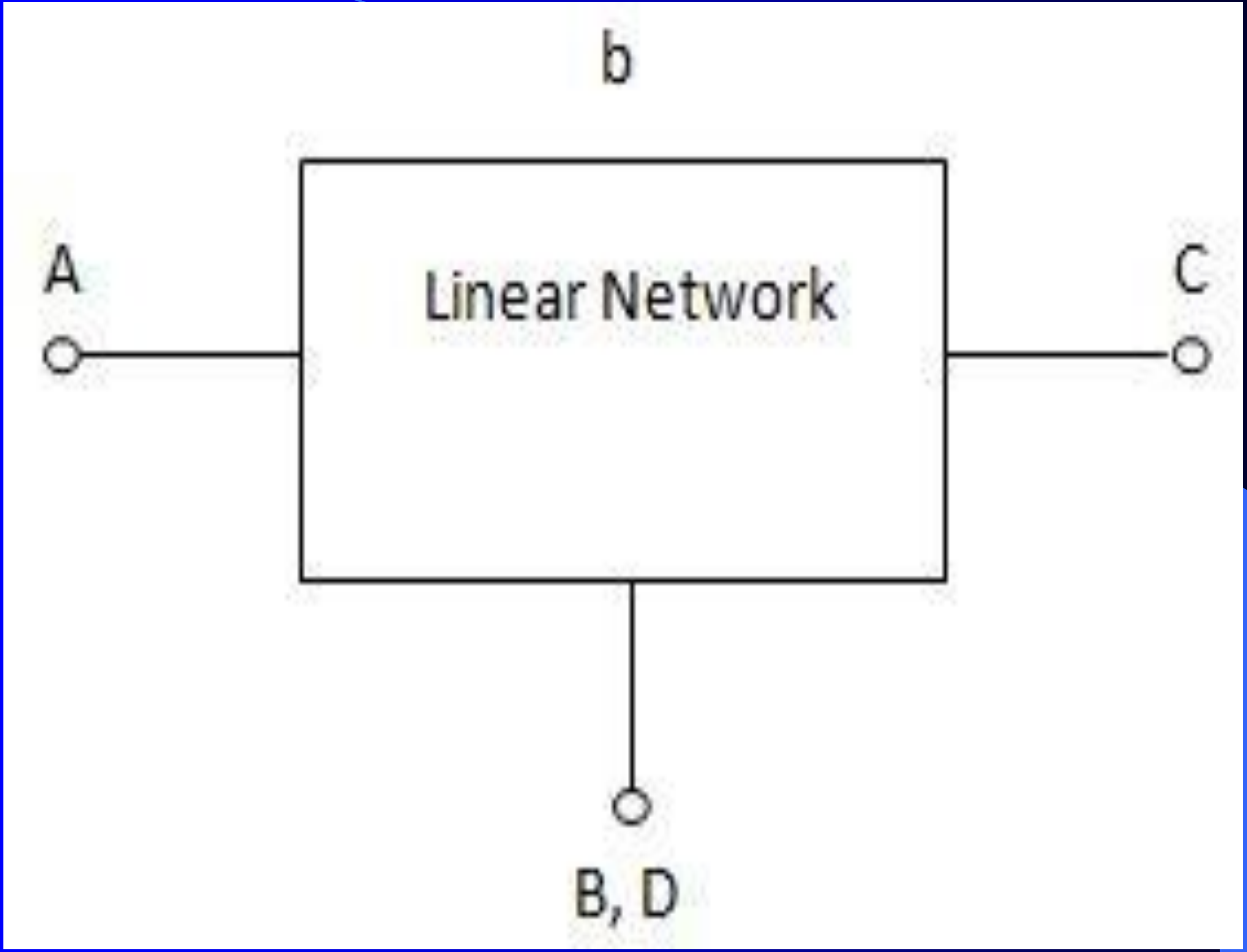


Diagram of Two-Port Network

- For some devices the two–port configuration may appear as 3–terminal device having one terminal common to both the input and output . See the next slide.
- The network consists of R, L and C elements, transformers, op – amps, dependent source but no independent sources.
- It is considered to be linear.


b
A
Linear Network
C
B, D

Admittance Parameters (Y–parameters)

- Since the network is linear and contains no independent sources, the principle of superposition can be applied to determine the current, I_1.
- Applying this principle, I_1 which is the sum of two components, one due to V_1 and the other due to V_2 is given by

$$I_1 = y_{11}V_1 + y_{12}V_2$$

- Similarly, I_2 is given by

$$I_2 = y_{21}V_1 + y_{22}V_2$$

- Therefore, the two equations that describe the two–port network are:

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

- If the y-parameters are known, the input/output operation of the two–port is completely defined.

Admittance parameters cont'd

- The y- parameters can be determined as follows:

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{(short–circuit input admittance)}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{(short–circuit transfer admittance)}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \quad \text{(short–circuit transfer admittance)}$$

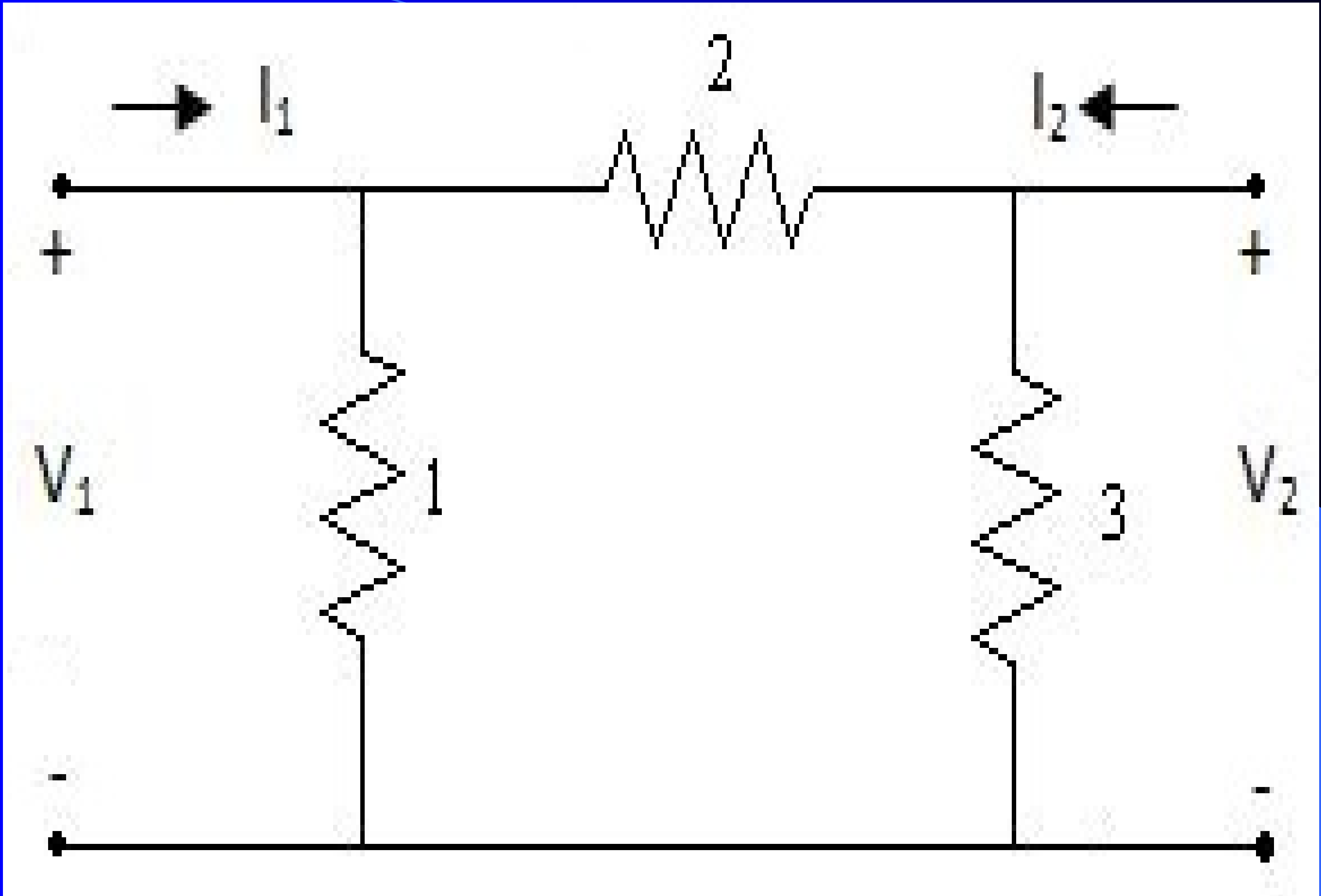
$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \quad \text{(short–circuit output admittance)}$$

Admittance parameters cont'd

- As a group the y–parameters are referred to as the short–circuit admittance parameters.

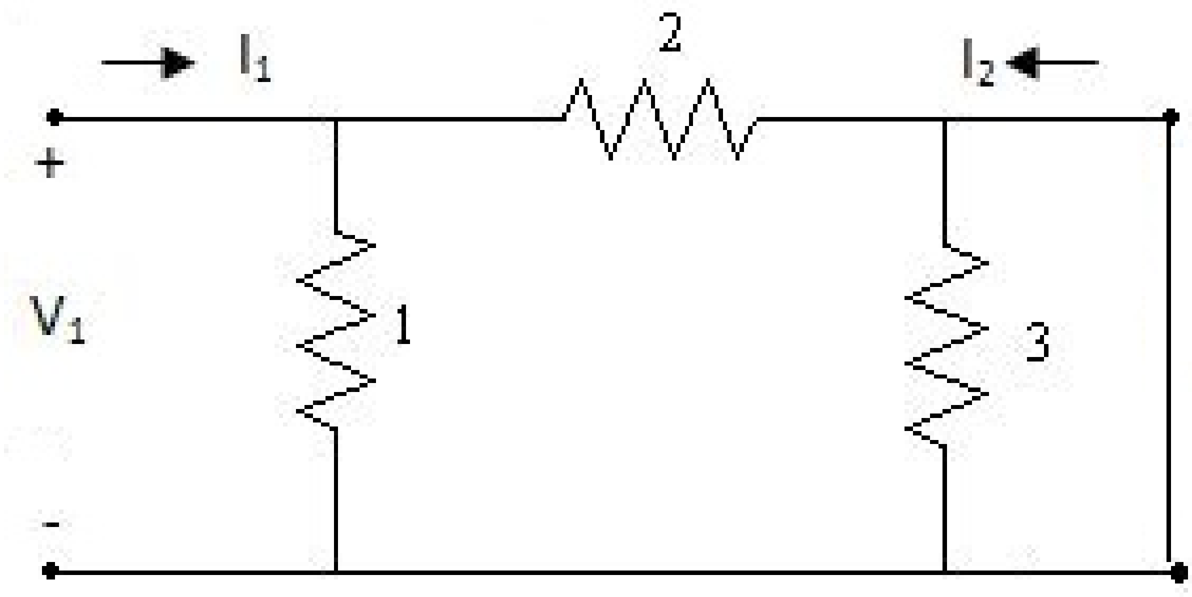
Example 1

Determine the y–parameters for the two-port circuit shown on the next slide.



I1
2
I2
+
+
V1
1
3
V2
-
-

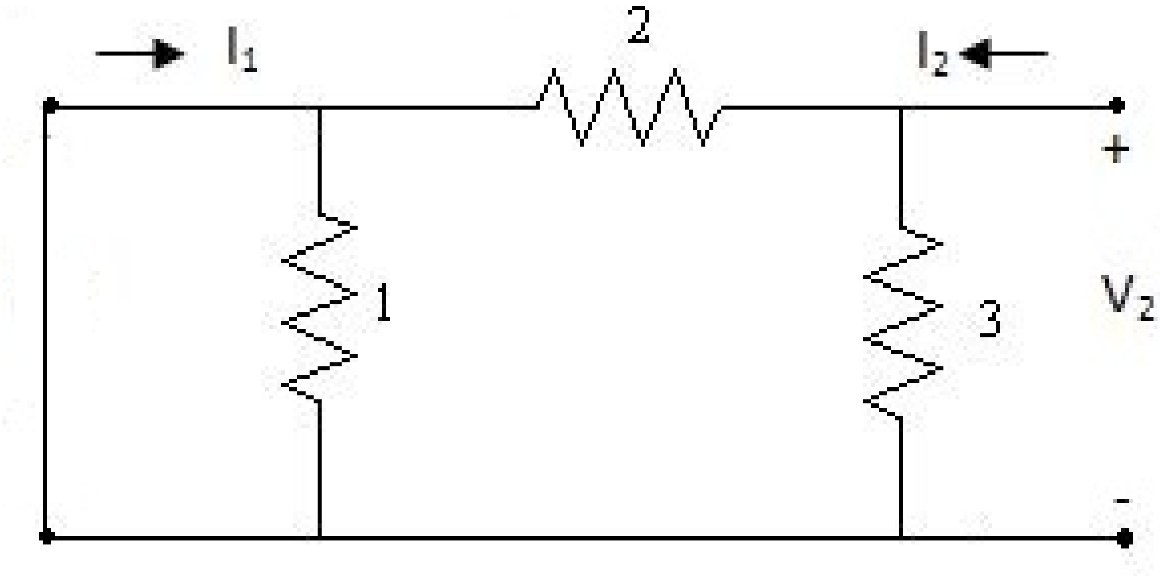
When the output is short–circuited:



$$I_1 = \frac{V_1}{1} + \frac{V_1}{2} = V_1\left(1 + \frac{1}{2}\right) = \frac{3V_1}{2} \Longrightarrow y_{11} = \frac{3}{2}s$$

$$I_2 = -\frac{V_1}{2} = \left(-\frac{1}{2}\right)V_1 \Longrightarrow y_{21} = -\frac{1}{2}s$$

When the input is short–circuited:



$$I_2 = \frac{V_2}{3} + \frac{V_2}{2} = V_2\left(\frac{2}{6} + \frac{3}{6}\right) = \frac{5}{6}V_2 \Rightarrow y_{22} = \frac{5}{6}s$$

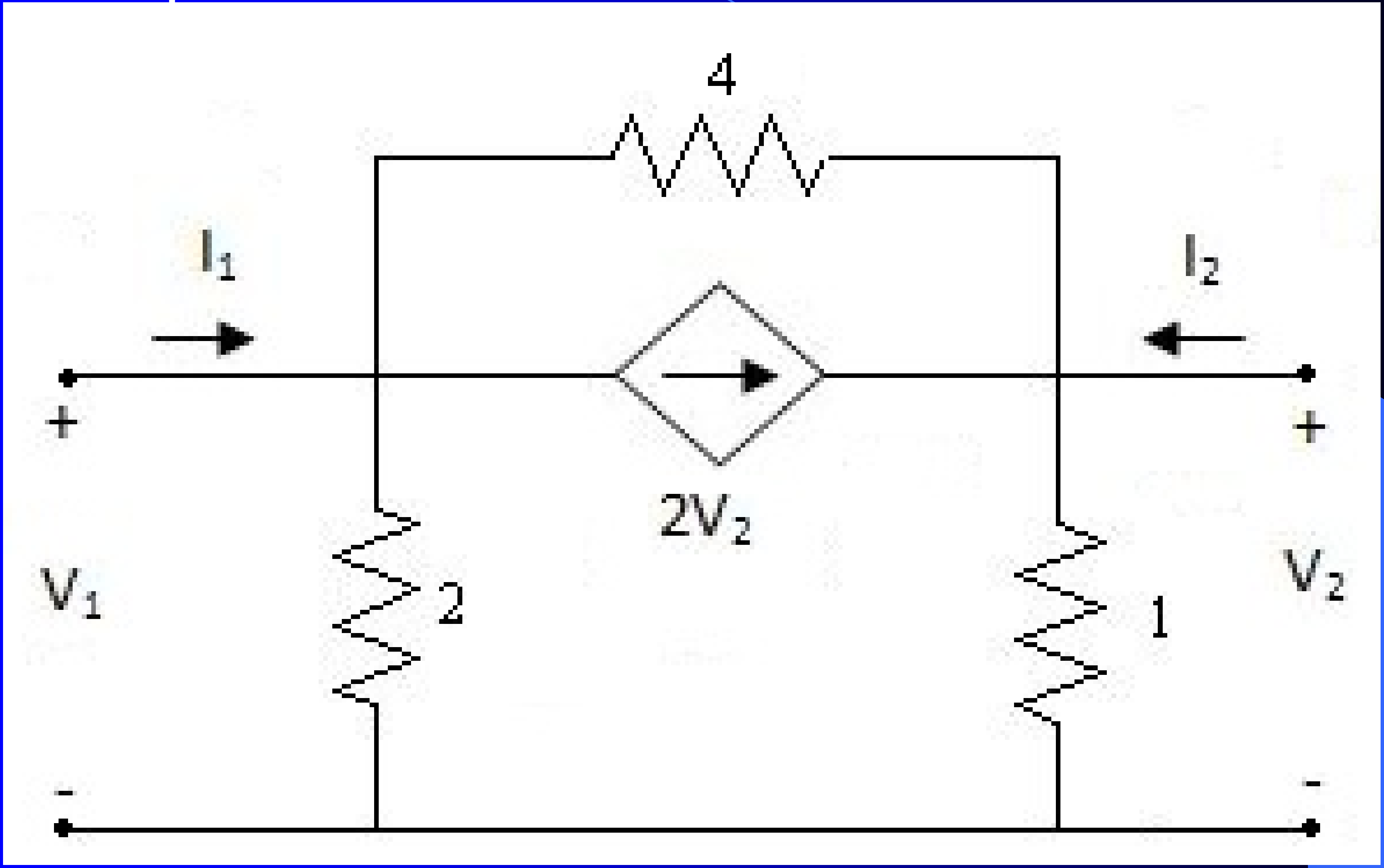
$$I_1 = -\frac{V_2}{2} \Rightarrow \quad \boldsymbol{y_{12} = -\frac{1}{2}s}$$

In the matrix form

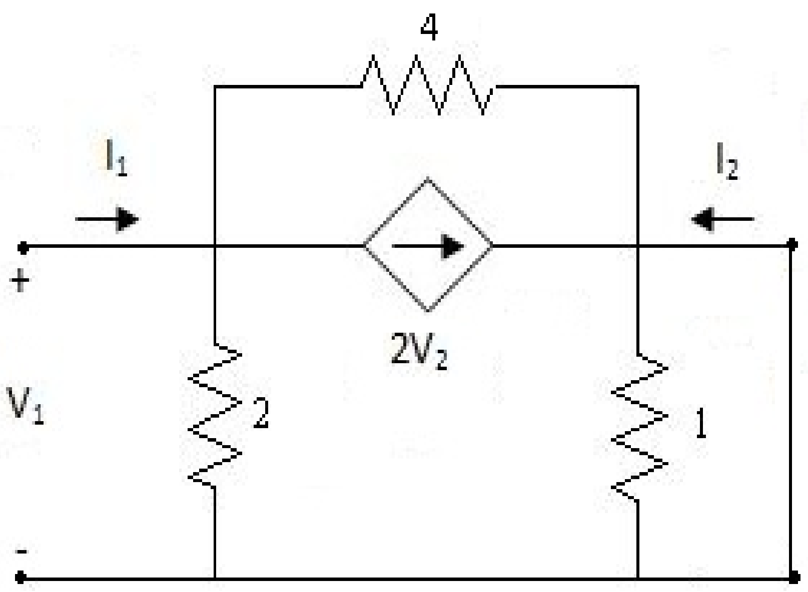
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{6} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

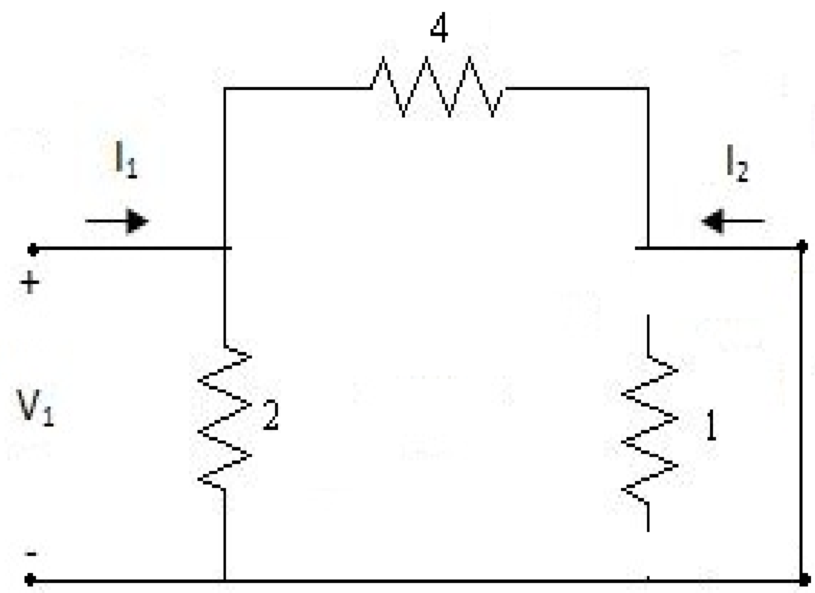
Note: Nodal analysis could have been used.

Example 2: Find the y–parameters for the two–port circuit:



When the output is short–circuited, the controlled current source carries no current, i.e., that branch is open.

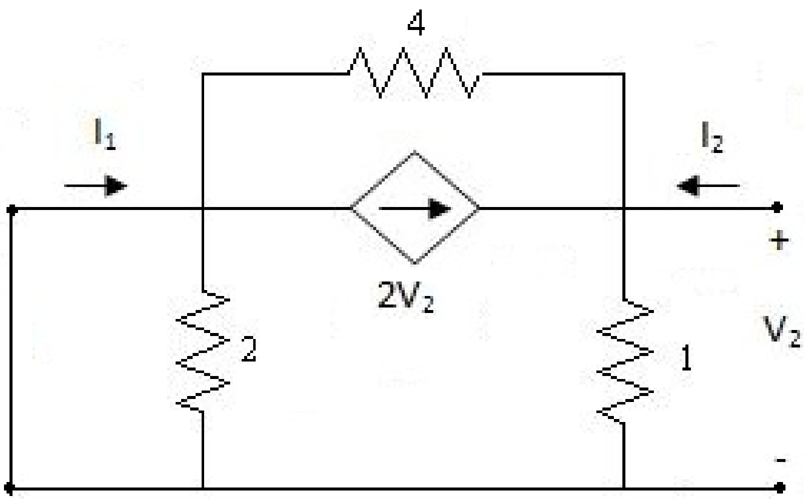


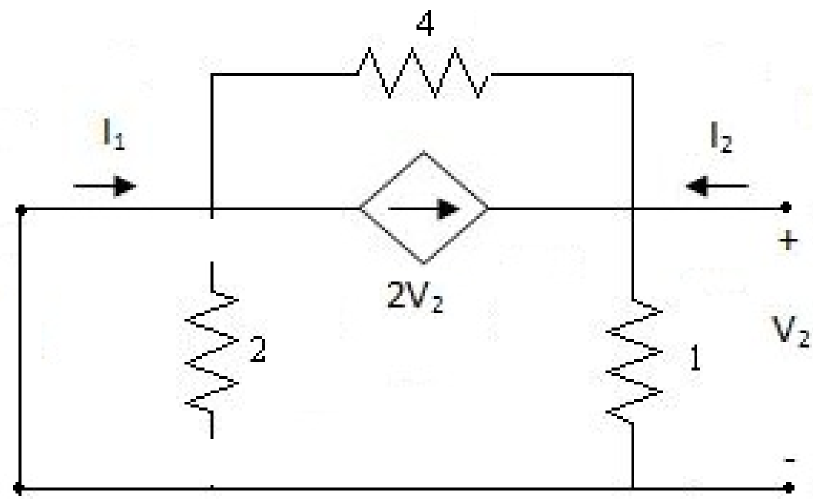


$$I_1 = \frac{V_1}{2} + \frac{V_1}{4} = \left(\frac{1}{2} + \frac{1}{4}\right)V_1 = \frac{3}{4}V_1 \Rightarrow y_{11} = \frac{3}{4}$$

$$I_2 = -\frac{V_1}{4} \Rightarrow y_{21} = -\frac{1}{4}$$

When the input is short–circuited:





$$I_2 = \frac{V_2}{1} + \frac{V_2}{4} - 2V_2 = \left(1 + \frac{1}{4} - 2\right)V_2 = -\frac{3}{4}V_2 \Rightarrow y_{22} = -\frac{3}{4}$$

$$I_1 = -\frac{V_2}{4} + 2V_2 = \left(-\frac{1}{4} + 2\right)V_2 = \frac{7}{4}V_2 \Rightarrow \boldsymbol{y_{12} = -\frac{7}{4}S}$$

$$\begin{bmatrix} I_1 \\ \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{7}{4} \\ -\frac{1}{4} & -\frac{3}{4} \end{bmatrix} \begin{bmatrix} V_1 \\ \\ V_2 \end{bmatrix}$$

Impedance Parameters (Z–parameters)

- By means of superposition, the input and output voltages can be expressed as the sum of two components, one due to I_1 and the other due to I_2:

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

- The z- parameters can be derived as follows (See next slide):

The z- parameters can be derived as follows:

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{(open–circuit input impedance)}$$

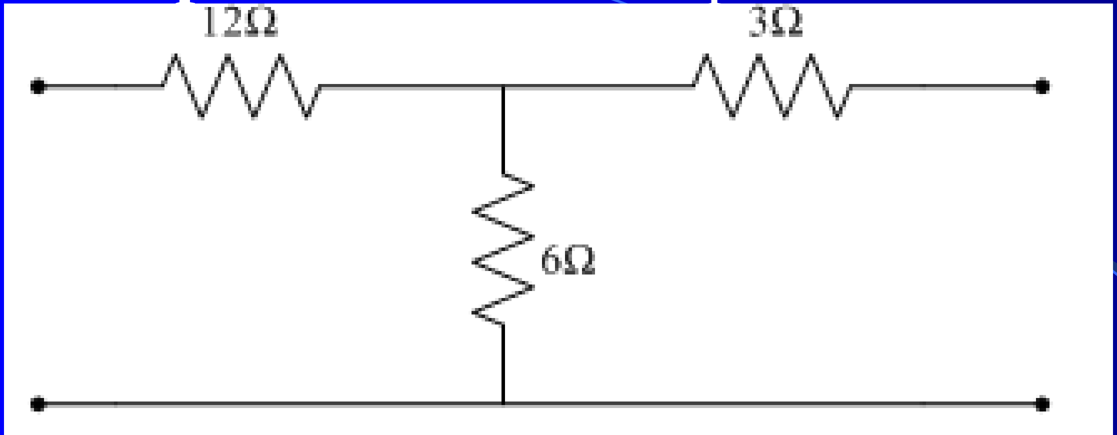
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{(open–circuit transfer impedance)}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \quad \text{(open–circuit transfer impedance)}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \quad \text{(open–circuit output impedance)}$$

The z-parameters are called open–circuit impedance parameters.

- **Example 3:** Find the z–parameters for the network :



With the output open-circuited

$$Z_{11} = \frac{V_1}{I_1} = 12 + 6 = 18\,\Omega \quad Z_{21} = \frac{V_2}{I_1} = \frac{6I_1}{I_1} = 6\Omega$$

With the input open-circuited

$$Z_{22} = \frac{V_2}{I_2} = 3 + 6 = 9\,\Omega \quad Z_{12} = \frac{V_1}{I_2} = \frac{6I_2}{I_2} = 6\,\Omega$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Verify using loop equations (or mesh equations)

Hybrid Parameters (H–parameters)

- The parameters are known as hybrid because they have a mixture of units.
- They are used extensively to analyse transistor networks.
- The two–port equations are:

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

The parameters are determined as follows:

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V2=0}$$ (short–circuit input impedance)

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I1=0}$$ (open–circuit reverse voltage gain)

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V2=0}$$ (short–circuit forward current gain)

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I1=0}$$ (open–circuit output admittance)

In transistor network analysis, the parameters h_{11}, h_{12}, h_{21} and h_{22} are normally labelled h_i, h_r, h_f and h_o.

Inverse Hybrid Parameters (g–parameters)

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

g_{11} = Open–circuit input admittance

g_{12} = Short–circuit reverse current gain

g_{21} = Open–circuit forward voltage gain

g_{22} = Short–circuit output impedance