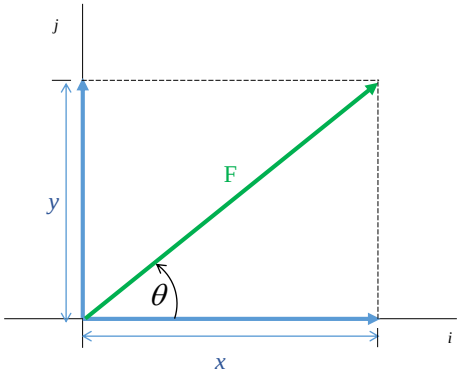
FORCES & MOMENTS

Resolving Forces in a Plane Into Rectangular Components – Unit Vector Approach

- The force is expressed as a product of its magnitude and it's unit vector.



$$\vec{F} = F\lambda = F\left(\frac{xi + yj}{\sqrt{x^2 + y^2}}\right)$$

$$= F\left(\frac{xi}{\sqrt{x^2 + y^2}}\right) + F\left(\frac{yj}{\sqrt{x^2 + y^2}}\right)$$

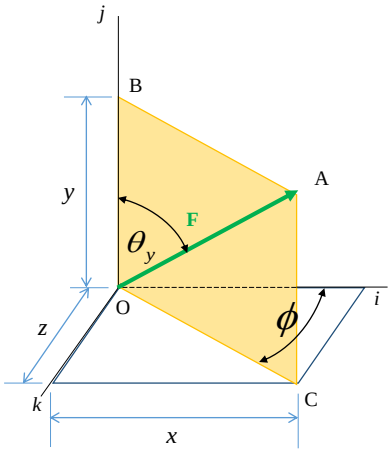
$$= F\vec{i} + F\vec{j}$$

$$\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right)$$

FORCES & MOMENTS

Resolving Forces in Space Into Rectangular Components – Unit Vector Approach

- For 3-Dimensions,



$$\vec{F} = F\lambda = F\left(\frac{xi + yj + zk}{\sqrt{x^2 + y^2 + z^2}}\right)$$

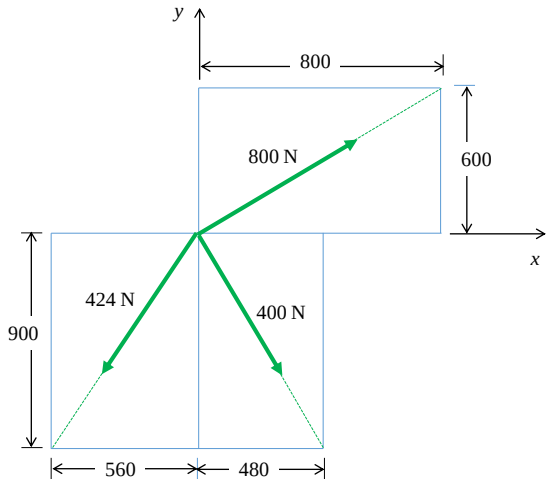
$$= F\vec{i} + F\vec{j} + F\vec{k}$$

$$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right)$$

FORCES & MOMENTS

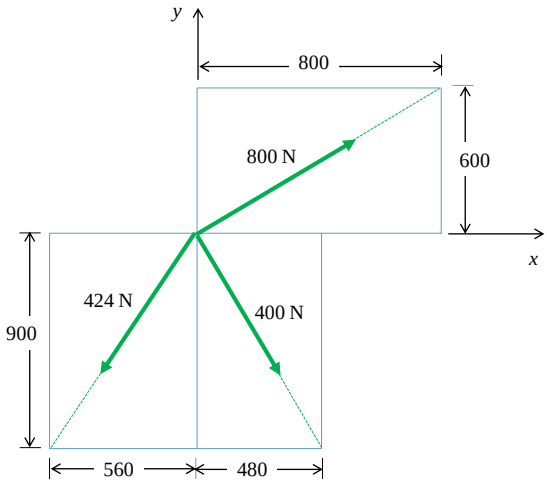
Example 2.6

Find the components of the forces shown using the unit vector approach.



FORCES & MOMENTS

Example 2.6-Solution



$$\vec{F} = F\lambda = F\left(\frac{xi + yj}{\sqrt{x^2 + y^2}}\right)$$

$$\vec{F}_1 = 800\left(\frac{800i + 600j}{\sqrt{800^2 + 600^2}}\right) = 800.\frac{800i}{1000} + 800.\frac{600j}{1000} = 640i + 480j$$

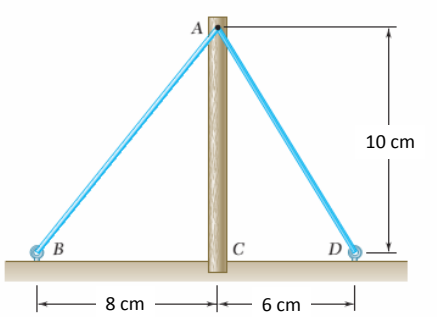
$$\vec{F}_2 = 424\left(\frac{-560i - 900j}{\sqrt{560^2 + 900^2}}\right) = -224i - 360j$$

$$\vec{F}_3 = 400\left(\frac{480i - 900j}{\sqrt{480^2 + 900^2}}\right) =$$

FORCES & MOMENTS

Example

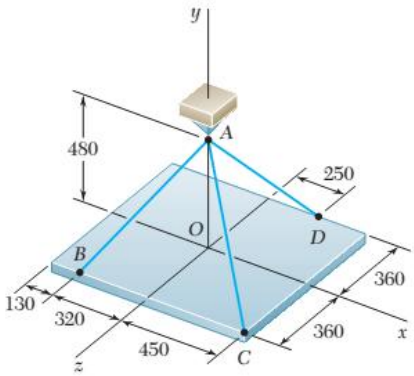
The cable stays AB and AD help support pole AC. Knowing that the tension is 120 N in AB and 40 N in AD, determine the components of the forces in the stays using the force unit vector approach.



FORCES & MOMENTS

Example 2.8

A rectangular plate is supported by three cables as shown. Knowing that the tension in cables AC, AB and AD are 60 N, 80 N and 90 N respectively, determine the components of the force being exerted at C, B and D.

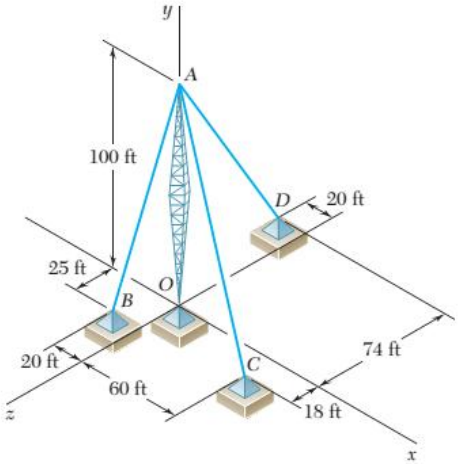


Dimensions in mm

FORCES & MOMENTS

Example 2.7

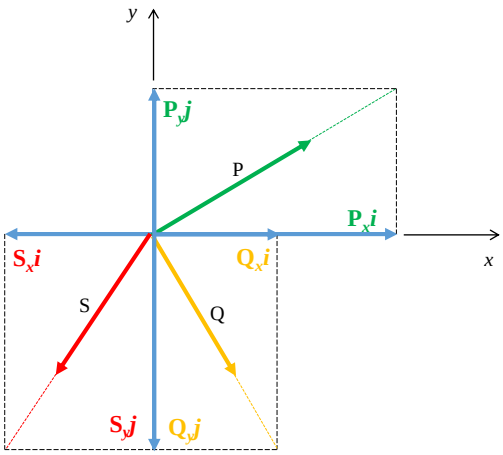
A transmission tower is held by three guy wired anchored by bolts B, C and D. If the tension in wire AD is 315 lb, determine the components of the force exerted by the wire on the bolt at D.



FORCES & MOMENTS

Resultants by Summing Components

- Sum all like components to get the components of the resultant.



$$\vec{R} = \vec{R}_x + \vec{R}_y = \sum F_x + \sum F_y$$

$$\vec{R} = (P_x + Q_x + S_x)\vec{i} + (P_y + Q_y + S_y)\vec{j}$$

The magnitude of the Resultant Force is given by;

$$R = \sqrt{R_x^{\,2} + R_y^{\,2}}$$

And the direction;

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right)$$

FORCES & MOMENTS
Resultants by Summing Components

- Like components are summed to obtain the components of the resultant.

$$\vec{R} = \vec{R}_x + \vec{R}_y + \vec{R}_z = \sum F_x + \sum F_y + \sum F_z$$

$$\vec{R} = (\vec{P}_x + \vec{Q}_x + \vec{S}_x) + (\vec{P}_y + \vec{Q}_y + \vec{S}_y) + (\vec{P}_z + \vec{Q}_z + \vec{S}_z)$$

OR

$$\vec{R} = (P + Q + S)\vec{i} + (P + Q + S)\vec{j} + (P + Q + S)\vec{k}$$

The magnitude of the Resultant Force is given by;

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{R_i^2 + R_j^2 + R_k^2}$$

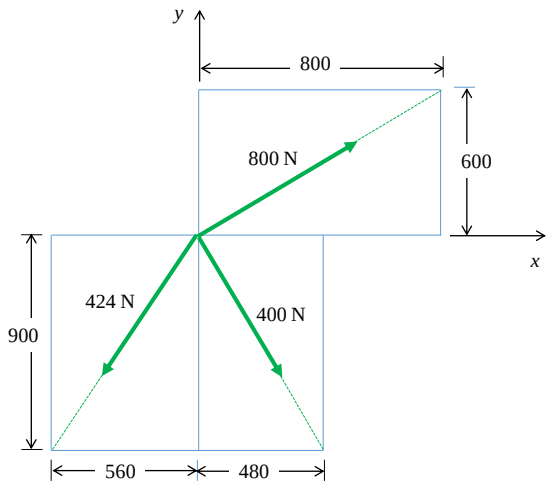
And the direction;

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right), \quad \theta_y = \cos^{-1}\left(\frac{R_y}{R}\right), \quad \theta_z = \cos^{-1}\left(\frac{R_z}{R}\right)$$

FORCES & MOMENTS

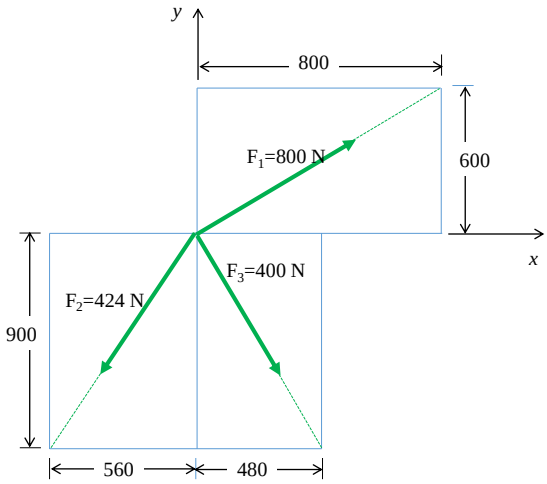
Example

Find components of the forces shown using the unit vector approach.



FORCES & MOMENTS

Example Solution



$$\vec{F} = F\lambda = F\left(\frac{xi + yj}{\sqrt{x^2 + y^2}}\right)$$

$$\vec{F}_1 = 800\left(\frac{800i + 600j}{\sqrt{800^2 + 600^2}}\right) = 800.\frac{800i}{1000} + 800.\frac{600j}{1000} = 640i + 480j$$

$$\vec{F}_2 = 424\left(\frac{-560i - 900j}{\sqrt{560^2 + 900^2}}\right) = -224i - 360j$$

$$\vec{F}_3 = 400\left(\frac{480i - 900j}{\sqrt{480^2 + 900^2}}\right) =$$

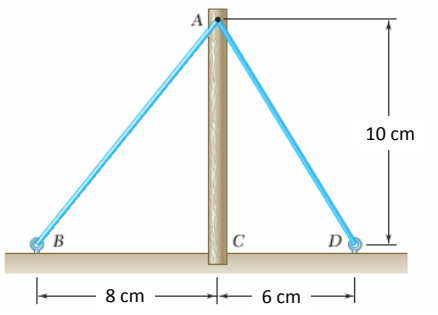
Resultant, $\vec{F} = \sum Fi + \sum Fj =$

The magnitude of the Resultant, $F = \sqrt{\left(\sum Fi\right)^2 + \left(\sum Fj\right)^2}$

FORCES & MOMENTS

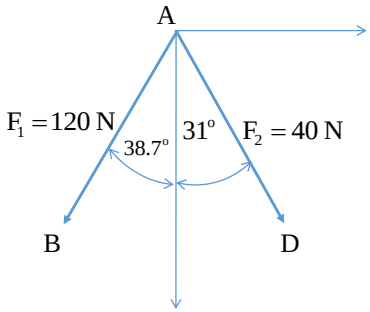
Example

The cable stays AB and AD help support pole AC. Knowing that the tension is 120 N in AB and 40 N in AD, determine the magnitude and direction of the resultant of the forces exerted by the stays at A.



FORCES & MOMENTS

Example - Solution



$$\vec{F} = \sum \vec{F}_x + \sum \vec{F}_y$$

$$\vec{F}_{1x} = F_1 \sin\theta = 40 \sin 31^\circ = 20.6 \text{ N}$$

$$\vec{F}_{1y} = F_1 \cos\theta = 40 \cos 31^\circ = -34.29 \text{ N}$$

$$\vec{F}_{2x} = F_2 \sin\theta = 120 \sin 38.7^\circ = -75.03 \text{ N}$$

$$\vec{F}_{2y} = F_2 \cos\theta = 120 \cos 38.7^\circ = -93.65 \text{ N}$$

$$\vec{F} = (20.6 - 75.03)_x + (-35.29 - 93.65)_y = -54.43_x - 128.94_y$$

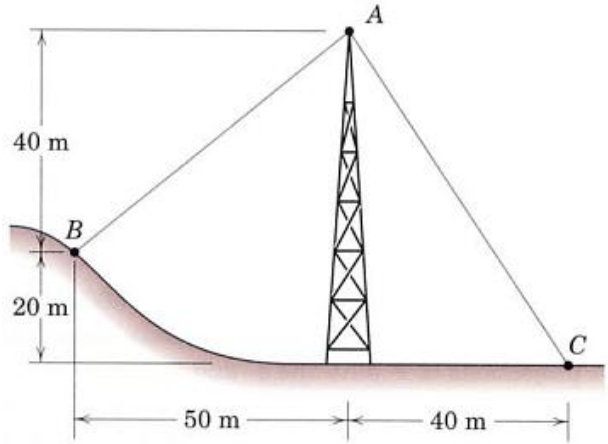
$$F = \sqrt{(-54.43^2 + (-128.94)^2} = 139.96 \text{ N}$$

$$\theta_x = \cos^{-1} = \left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{-54.43}{139.96}\right)$$

FORCES & MOMENTS

Example

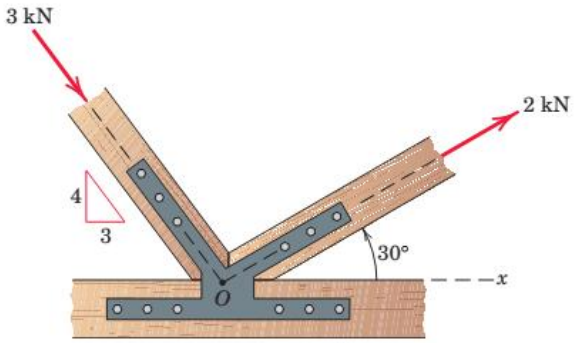
The guy cables AB and AC are attached to the top of the transmission tower. The tension in the cable AC is 8 kN and that of AB is 5 kN. Determine the magnitude R of the resultant of the forces.



FORCES & MOMENTS

Example

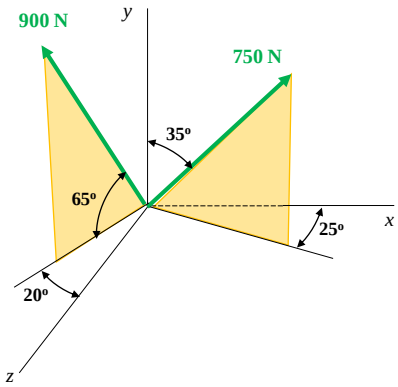
The two structural, one of which is in tension and other in compression, exert the indicated forces on joint *O*. determine the magnitude of the resultant **R** of the two forces and the angle θ which **R** makes with the positive *x*-axis.



FORCES & MOMENTS

Example

Determine the components of the forces shown of the resultant force acting on the point *O* and it's direction measured from the *x* - axis.



FORCES & MOMENTS

Solution

Recall

$\vec{F}_{1y} = 614.36\ \text{N}$

$\vec{F}_{1x} = 389.88\ \text{N}$

$\vec{F}_{1z} = 181.8\ \text{N}$

$\vec{F}_{2y} = 815.68\ \text{N}$

$\vec{F}_{2x} = -130.09\ \text{N}$

$\vec{F}_{2z} = 357.42\ \text{N}$

$$\vec{F} = \sum F_x + \sum F_y + \sum F_z$$
$$= (389.88 - 130.09)_x + (614.36 + 815.68)_y + (181.8 + 357.42)_z$$
$$= 259.79_x + 1430.04_y + 539.22_z$$
$$F = \sqrt{(259.79^2 + (1430.04)^2 + (539.22)^2} = 1550.25\ \text{N}$$
$$\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{259.79}{1550.25}\right) = 80.4^o$$
$$\theta_y = 22.7^o \qquad \theta_z = 69.7^o$$