EE 287 CIRCUIT THEORY

GIDEON ADOM-BAMFI

What to expect?

Impedance Parameters

Admittance Parameters

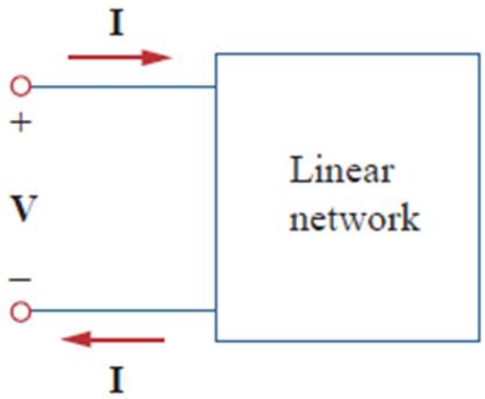
Interconnection of Networks

Hybrid Parameters

Transmission Parameters

Introduction to Two Port Networks

- A pair of terminals through which a current may enter or leave a network is known as a **PORT.**



- Two terminal devices or elements such as resistors, capacitors and inductors result in **one-port networks.**

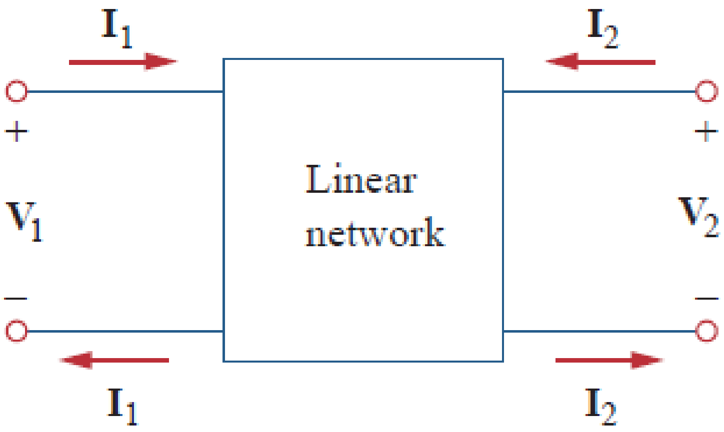
- Most of the circuits we have dealt with so far are two-terminal or one-port circuits and can be modelled using Thevenin or Norton equivalent circuits.

Introduction to Two Port Networks II

- The majority of devices (op amps, transistors, transformers) and electric systems have two pairs of terminals.

- These devices are known as **two-port networks.**

- The standard configuration of a two-port network is as shown below:



Why Two Port Networks?

- A two-port network is an electrical network with two separate ports for input and output.

- Thus a two-port network has two terminal pairs acting as access points.

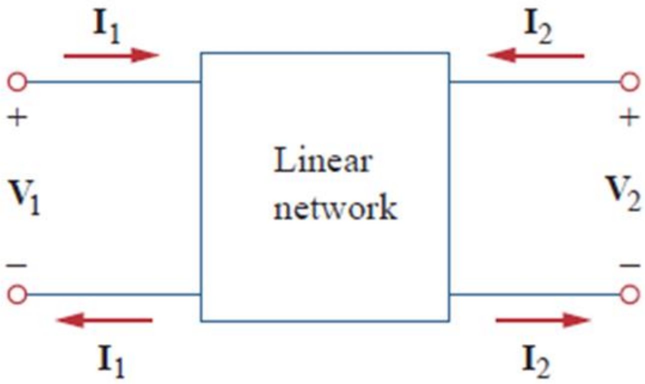
Our study of two-port networks is for at least two reasons:

1. Such networks are useful in communications, control systems, power systems and electronics.

2. Knowing the two-port parameters of a network or system enables us to treat it as ‘black box’ when embedded within a larger network.

Two Port Parameters

- To characterize a two-port network requires that we relate the four port variables V_1, V_2, I_1, and I_2.



- Depending on which two of the four port variables are given, there exists different ways to describe the relationship between these variables.

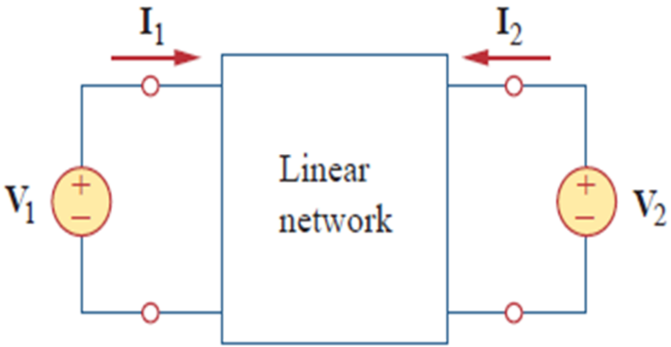
- The relationship between voltages and currents are described in terms of quantities known as **parameters.**

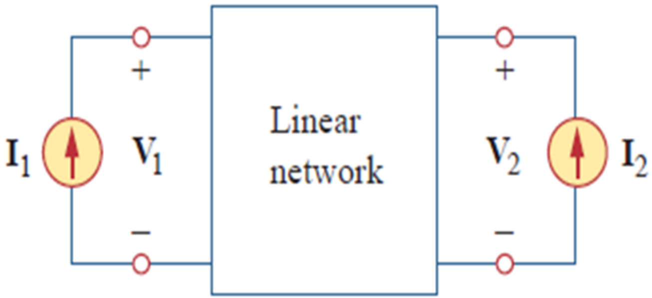
Two Port Parameters

Our goal in this lecture is to learn how to find the characteristic parameters of a two-port network:

- Impedance Parameters
- Admittance Parameters
- Hybrid Parameters
- Transmission Parameters

A two-port network may be voltage-driven or current driven





1

Impedance Parameters

Impedance Parameters

- The impedance parameters are obtained by expressing the terminal voltages in terms of the terminal currents.
- Given currents I_1 and I_2, voltages V_1 and V_2 are derived as:

$$\mathbf{V_1 = z_{11}I_1 + z_{12}I_2}$$
$$\mathbf{V_2 = z_{21}I_1 + z_{22}I_2}$$

or in matrix form as:

$$\begin{bmatrix} \mathbf{V_1} \\ \mathbf{V_2} \end{bmatrix} = \begin{bmatrix} \mathbf{z_{11}} & \mathbf{z_{12}} \\ \mathbf{z_{21}} & \mathbf{z_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{I_1} \\ \mathbf{I_2} \end{bmatrix}$$

where

The z terms are called the **impedance parameters** or simply **z parameters**, and have units of ohms.

Impedance Parameters

The values of the parameters can be evaluated by setting

$I_1 = 0$ (input port open-circuited)

$I_2 = 0$ (output port open-circuited)

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2 = 0} \qquad z_{12} = \frac{V_1}{I_2}\bigg|_{I_1 = 0}$$

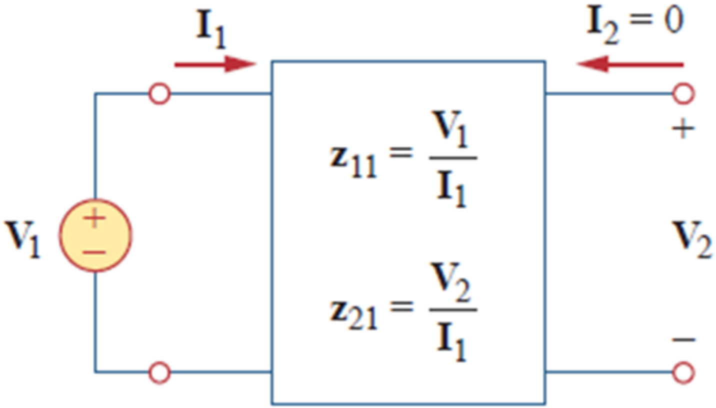
$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2 = 0} \qquad z_{22} = \frac{V_2}{I_2}\bigg|_{I_1 = 0}$$

Since the z-parameters are obtained by open-circuiting the input or output port

They are also called the **open-circuit impedance parameters.**

Impedance Parameters

We obtain z_{11} and z_{21} by connecting a voltage V_1(or current source I_1) to port 1 with port 2 open circuited and finding I_1 and V_2

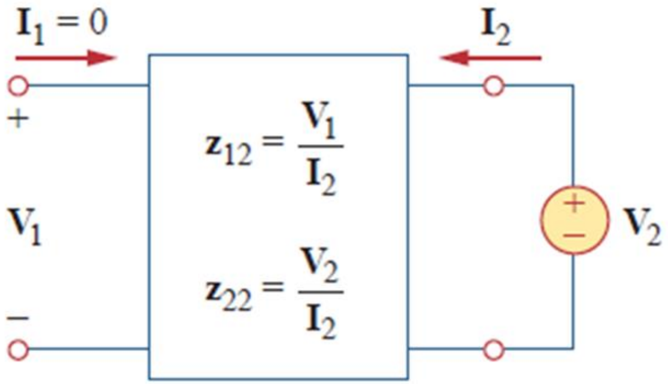


We then get

$$z_{11} = \frac{V_1}{I_1} \qquad z_{21} = \frac{V_2}{I_1}$$

Impedance Parameters

Similarly, we obtain z_{12} and z_{22} by connecting a voltage V_2 (or a current source I_2) to port 2 with port 1 open-circuited and finding I_2 and V_1



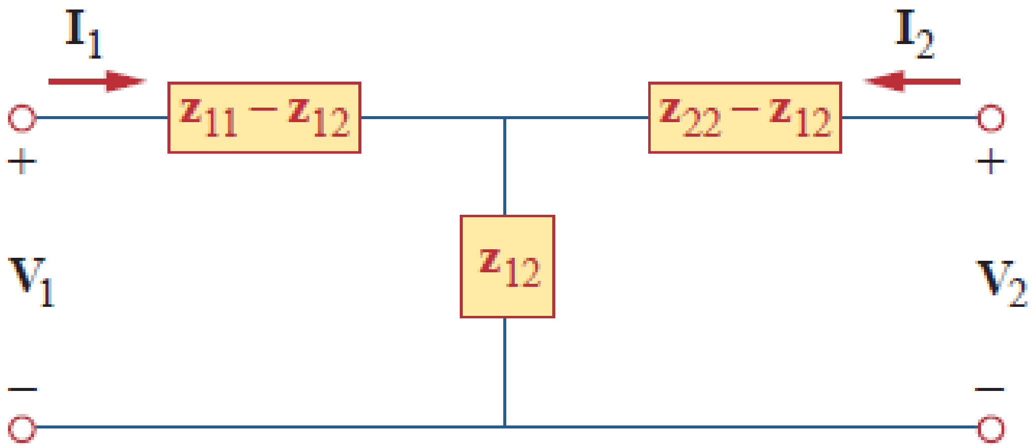
We then get

$$z_{12} = \frac{V_1}{I_2} \qquad z_{22} = \frac{V_2}{I_2}$$

T-Equivalent Circuit

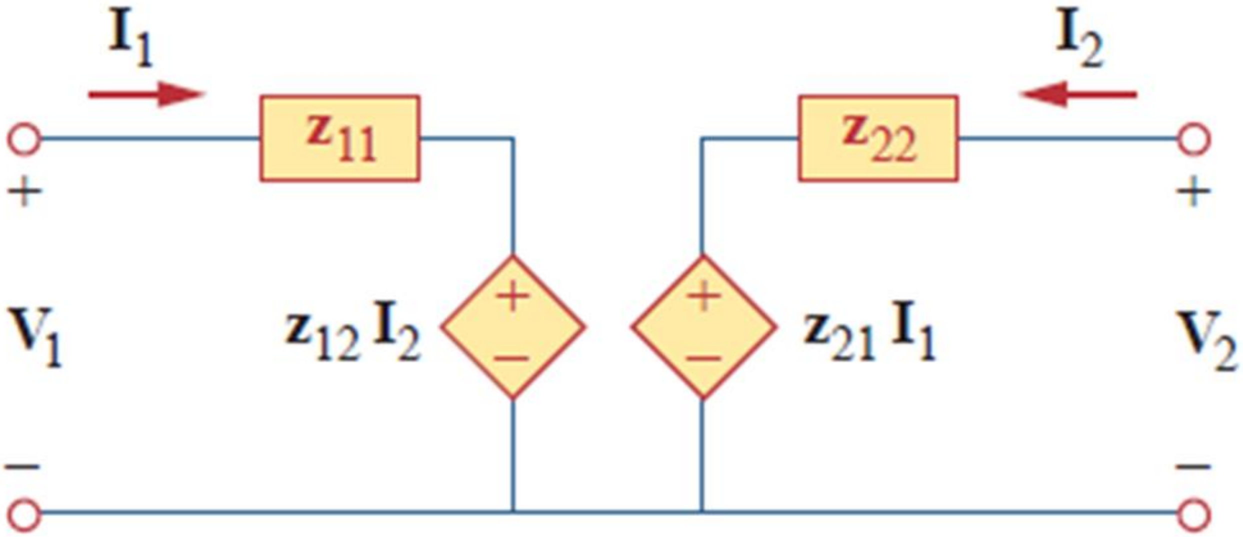
- Once we know what the impedance parameters are, we can model the two-port network with an equivalent circuit.

- A reciprocal network ($z_{12} = z_{21}$) can be replaced by the T-equivalent circuit below:



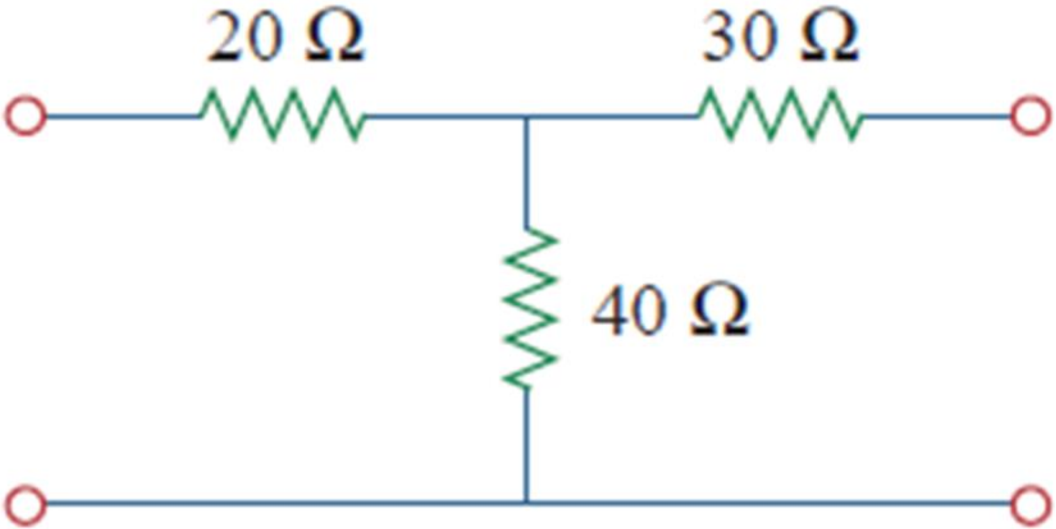
T-Equivalent Circuit

- If the network is not reciprocal, a more general equivalent network is used:



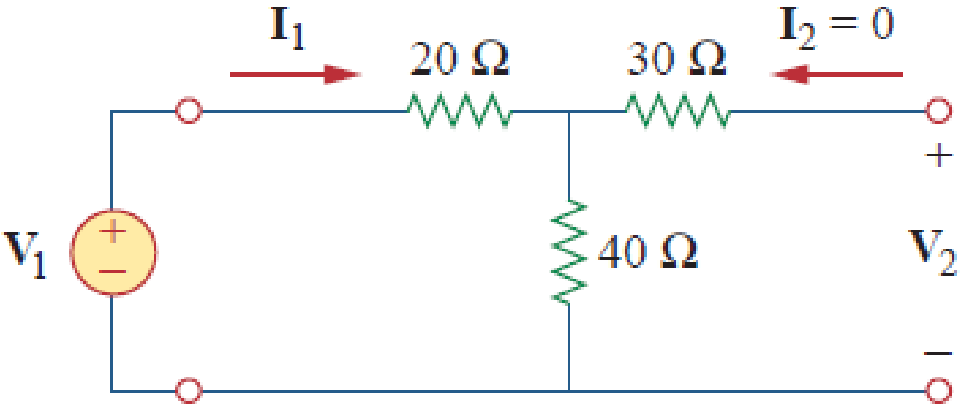
Question 1

Determine the z parameters for the circuit below:



Solution

To determine z_{11} and z_{21}, we apply a voltage source V_1 to the input port and leave the output open as shown below:



KVL for the first mesh yields: $-V_1 + 20I_1 + 40I_1 = 0$

Then

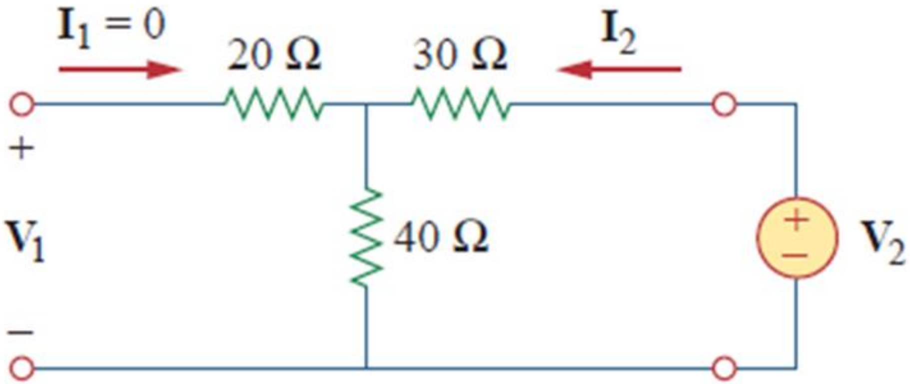
$$z_{11} = \frac{V_1}{I_1} = \frac{(20 + 40)I_1}{I_1} = 60\ \Omega$$

That is z_{11} is the input impedance at port 1

The transfer impedance z_{21} is:

$$z_{21} = \frac{V_2}{I_1} = \frac{40 I_1}{I_1} = 40\ \Omega$$

To find z_{12} and $z_{22,}$ we apply voltage source V_2 to the output port and leave the input port open as shown below:

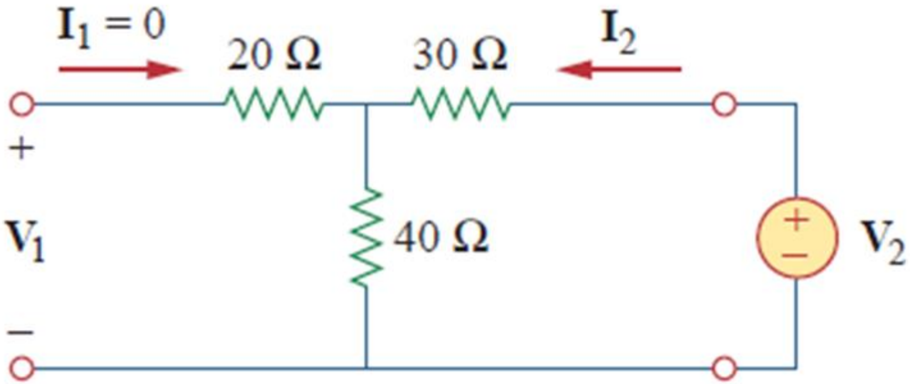


That is z_{11} is the input impedance at port 1

The transfer impedance z_{21} is:

$$z_{21} = \frac{V_2}{I_1} = \frac{40I_1}{I_1} = 40\ \Omega$$

To find z_{12} and $z_{22,}$ we apply voltage source V_2 to the output port and leave the input port open as shown below:



Then

$$z_{12} = \frac{V_1}{I_2} = \frac{40I_2}{I_2} = 40\ \Omega$$

and

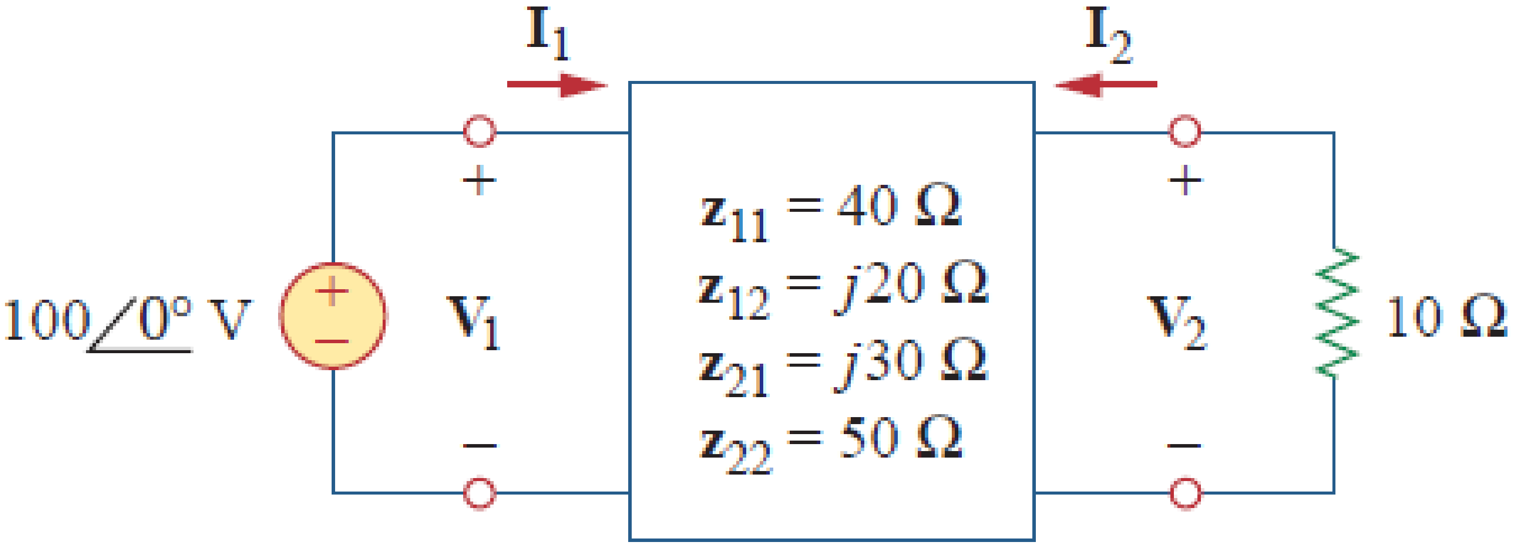
$$z_{22} = \frac{V_2}{I_2} = \frac{(30+40)I_2}{I_2} = 70\ \Omega$$

Thus

$$[z] = \begin{bmatrix} 60\Omega & 40\Omega \\ 40\Omega & 70\Omega \end{bmatrix}$$

Question 2

Find I_1 and I_2 in the circuit below



Solution

We substitute the given z parameters in the matrix shown below:

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

and get

$$\mathrm{V}_1 = 40\mathrm{I}_1 + j20\mathrm{I}_2$$
$$\mathrm{V}_2 = j30\mathrm{I}_1 + 50\mathrm{I}_2$$

Since we are looking for I_1 and I_2, we substitute

$$\mathrm{V}_1 = 100\angle 0°, \qquad \mathrm{V}_2 = 10\mathrm{I}_2$$

Into the above the equations above and get the equations on the next slide.

Solution

The equation becomes

$$100 = 40I_1 + j20I_2 \qquad (1)$$

and

$$-10I_2 = j30I_1 + 50I_2 \quad \Rightarrow I_1 = j2I_2 \qquad (2)$$

Substituting (2) into (1)

$$100 = j80I_2 + j20I_2 \qquad \Rightarrow \qquad I_2 = \frac{100}{j100} = -j$$

From (2)

$$I_1 = j2(-j) = 2$$

Thus

$$I_1 = 2A\,\angle 0°,\;\; I_2 = 1A\,\angle -90°$$

2
Admittance Parameters

Admittance Parameters

- Admittance parameters are obtained by expressing the terminal currents in terms of the terminal voltages.

- That is given voltages V_1 and V_2, currents I_1 and I_2 are derived:

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

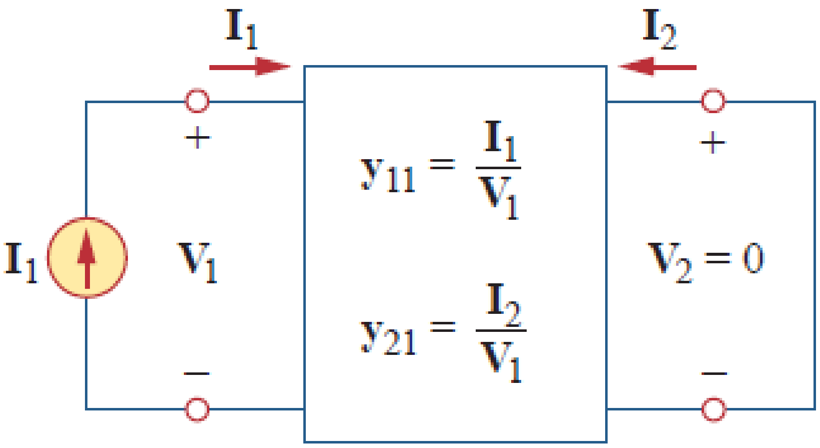
or in matrix form

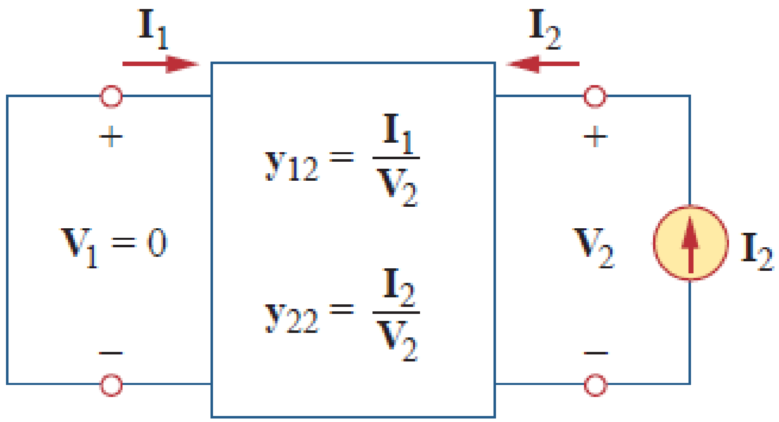
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Admittance Parameters

- The y terms are known as the **admittance parameters** or y parameters and have units of ‘siemens’.

- The values of the parameters can be obtained by setting $V_1 = 0$ (input port short-circuited) or $V_2 = 0$ (output port short-circuited)





Admittance Parameters

Thus

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2 = 0} \qquad y_{12} = \frac{I_1}{V_2}\bigg|_{V_1 = 0}$$

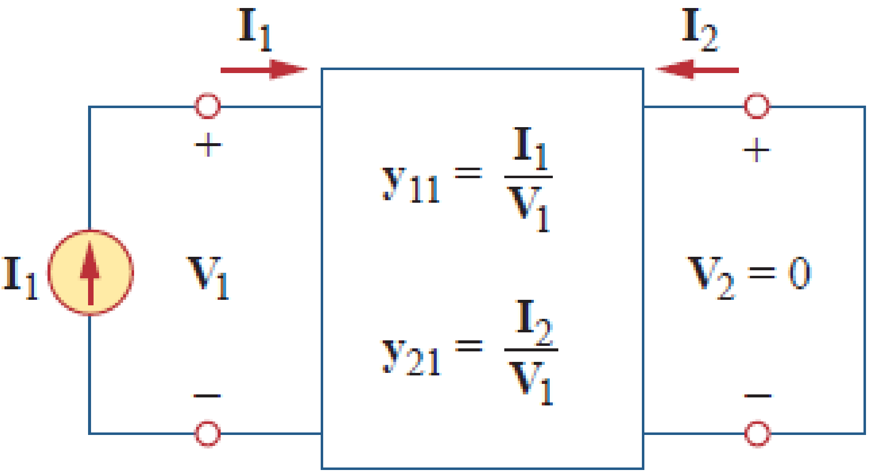
$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2 = 0} \qquad y_{22} = \frac{I_2}{V_2}\bigg|_{V_1 = 0}$$

Since the y parameters are obtained by short-circuiting the input or out put port

They are also called **short-circuiting admittance parameters.**

Admittance Parameters

We obtain y_{11} and y_{21} by connecting a current I_1 to port 1 and short-circuiting port 2

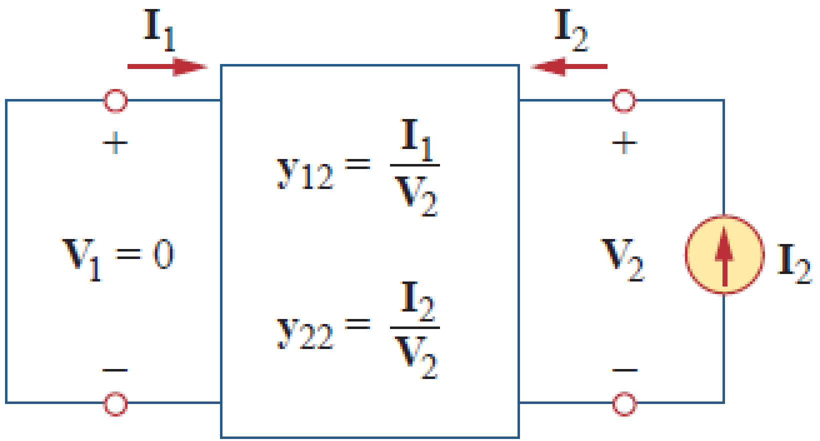


Find V_1 and I_2 and then calculate

$$y_{11} = \frac{I_1}{V_1}, \qquad y_{21} = \frac{I_2}{V_1}$$

Admittance Parameters

Similarly, we obtain y_{12} and y_{22} by connecting a current source I_2 to port 2 and short-circuiting port 1.

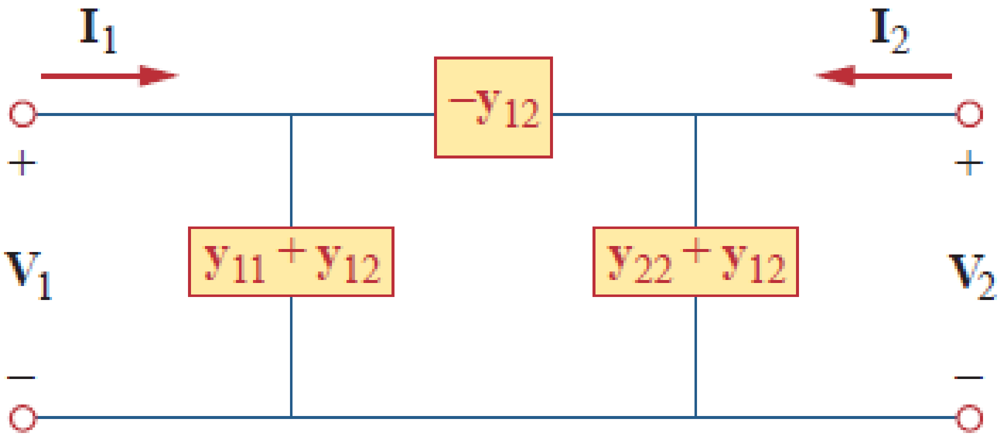


Find I_1 and V_2 and then calculate:

$$y_{12} = \frac{I_1}{V_2}, \qquad y_{22} = \frac{I_2}{V_2}$$

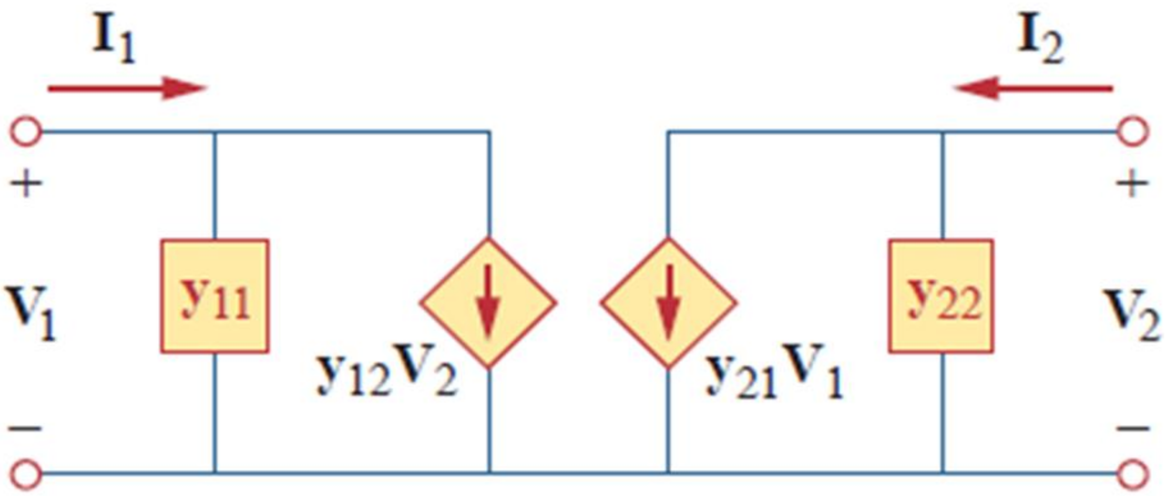
π-Equivalent Circuit

- For a two-port network that is linear and has no dependent sources
- The transfer admittances are equal ($y_{12} = y_{21}$)
- A reciprocal network ($y_{12} = y_{21}$) can be modelled by the π-equivalent circuit as shown below:



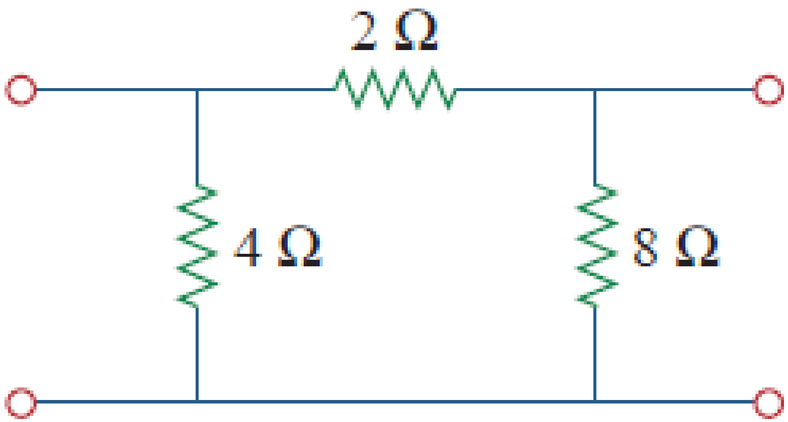
π-Equivalent Circuit

- If the network is not reciprocal, a more general equivalent network is shown below:



Question 1

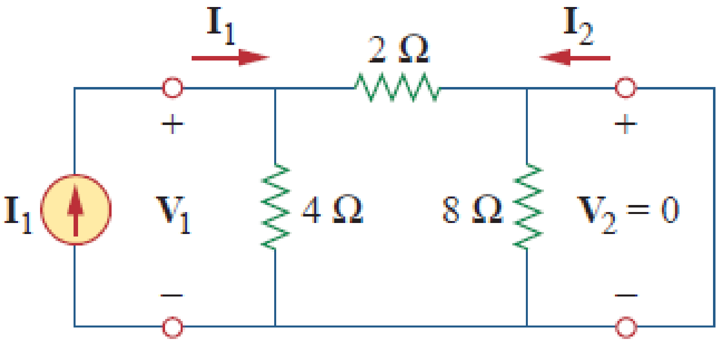
Obtain the y parameters for the π network shown below:



Solution

To find y_{11} and y_{21}

Short-circuit the output port and connect a current source I_1 to the input port as shown below:



Since the 8-Ω resistor is short-circuited, the 2- Ω resistor is in parallel with the 4- Ω

Hence

$$V_1 = I_1(4//2) = \frac{4}{3} I_1 \qquad y_{11} = \frac{I_1}{V_1} = \frac{I_1}{\frac{4}{3} I_1} = 0.75\text{S}$$

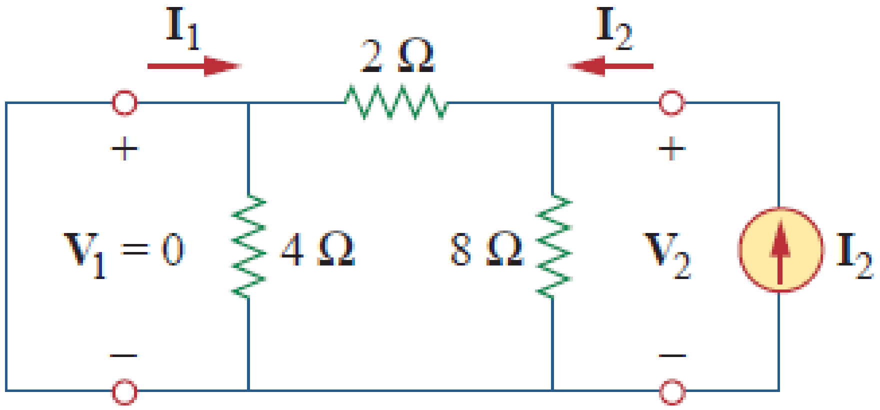
Solution

By current division

$$-I_2 = \frac{4}{4+2} I_1 = \frac{2}{3} I_1 \quad y_{21} = \frac{I_2}{V_1} = \frac{-\frac{2}{3}I_1}{\frac{4}{3}I_1} = -0.5S$$

To get y_{12} and y_{22}

Short-circuit the input port and connect a current source I_2 to the output port as shown below:



Solution

The 4-Ω resistor is short-circuited so the 2- Ω and the 8- Ω resistors are in parallel

$$V_2 = I_2(8//2) = \frac{8}{5}I_2 \qquad y_{22} = \frac{I_2}{V_2} = \frac{I_2}{\frac{8}{5}I_2} = \frac{5}{8} = 0.625S$$

By current division

$$-I_1 = \frac{8}{8+4}I_2 = \frac{4}{5}I_2 \qquad y_{12} = \frac{I_1}{V_2} = \frac{-\frac{4}{5}I_2}{\frac{8}{5}I_2} = -\,0.5S$$

3
Hybrid Parameters

Hybrid Parameters

- This set of parameters is based on making V_1 and I_2 the dependent variable
- That is given V_2 and I_1, V_1 and I_2 are derived

Thus we obtain

$$\mathbf{V_1 = h_{11}I_1 + h_{12}V_2}$$
$$\mathbf{I_2 = h_{21}I_1 + h_{22}V_2}$$

Or in matrix form

$$\begin{bmatrix}\mathbf{V_1}\\ \mathbf{I_2}\end{bmatrix} = \begin{bmatrix}\mathbf{h_{11}} & \mathbf{h_{12}}\\ \mathbf{h_{21}} & \mathbf{h_{22}}\end{bmatrix}\begin{bmatrix}\mathbf{I_1}\\ \mathbf{V_2}\end{bmatrix} = [\mathbf{h}]\begin{bmatrix}\mathbf{I_1}\\ \mathbf{V_2}\end{bmatrix}$$

Hybrid Parameters

- The h terms are known as the **hybrid parameters** or simply the h parameters.
- The h parameters are very useful for describing electronic devices such as transistors.
- It is much easier to measure experimentally the h parameters of such devices than to measure the z or y parameters
- The values of the parameters are determined as:

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2 = 0} \qquad h_{12} = \frac{V_1}{V_2}\bigg|_{I_1 = 0}$$

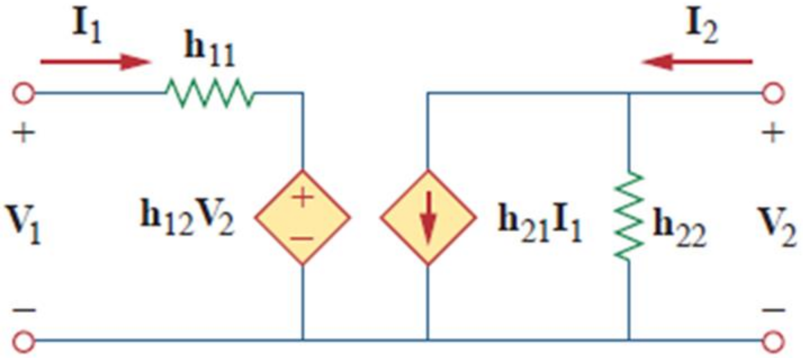
$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2 = 0} \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1 = 0}$$

Procedure for calculating the h parameters

- We apply a voltage or current source to the appropriate port.
- Short-circuit or open-circuit the other port, depending on the parameter of interest, and perform regular circuit analysis
- For reciprocal networks

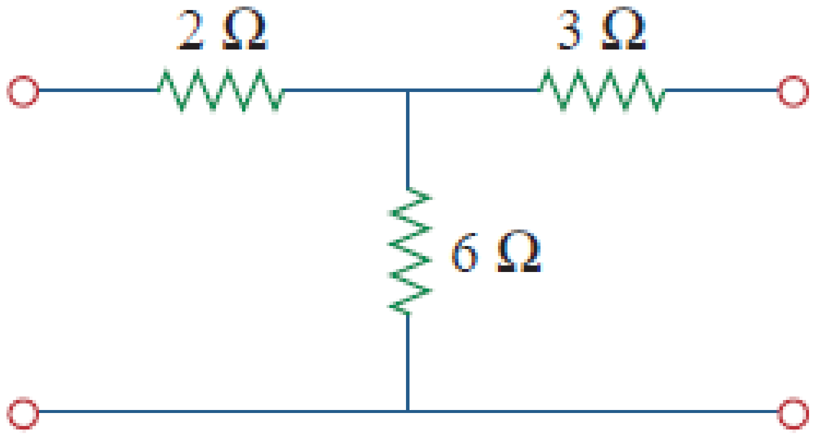
$$h_{12} = - h_{21}$$

- The figure below shows the hybrid model of a two-port network



Question 1

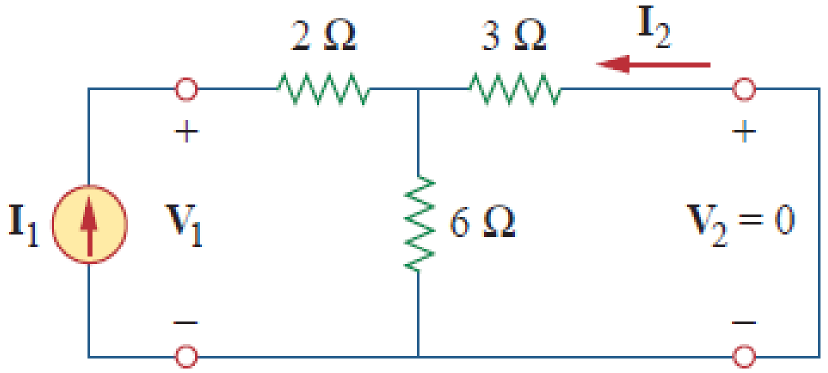
Find the hybrid parameters for the two-port network shown below



Solution

To find h_{11} and h_{21}

We short circuit the output port and connect a current source I_1 to the input port as shown below:



From the figure above

$$V_1 = I_1(2+3//6) = 4I_1$$

Solution

Hence

$$h_{11} = \frac{V_1}{I_1} = 4\Omega$$

By current division

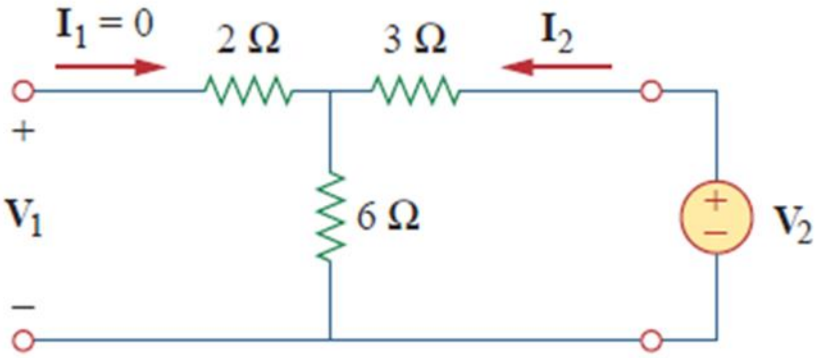
$$-I_2 = \frac{6}{6+3}\, I_1 = \frac{2}{3}\, I_1$$

Hence

$$h_{21} = \frac{I_2}{I_1} = -\frac{2}{3}$$

Solution

To obtain h_{12} and h_{22}, we open-circuit the input port and connect a voltage source V_2 to the output port as shown in the figure below



By voltage division

$$V_1 = \frac{6}{6+3} V_2 = \frac{2}{3} V_2$$

Solution

Hence

$$h_{12} = \frac{V_1}{V_2} = \frac{2}{3}$$

$V_2 = (3 + 6)I_2 = 9I_2$

Thus

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{9}\,S$$

4
Transmission Parameters

Transmission Parameters

- Another set of parameters relates the variables at the input port to those at the output port.
- The direction of the output current is reversed

Thus

$$\mathbf{V_1 = AV_2 - BI_2}$$

$$\mathbf{I_1 = CV_2 - DI_2}$$

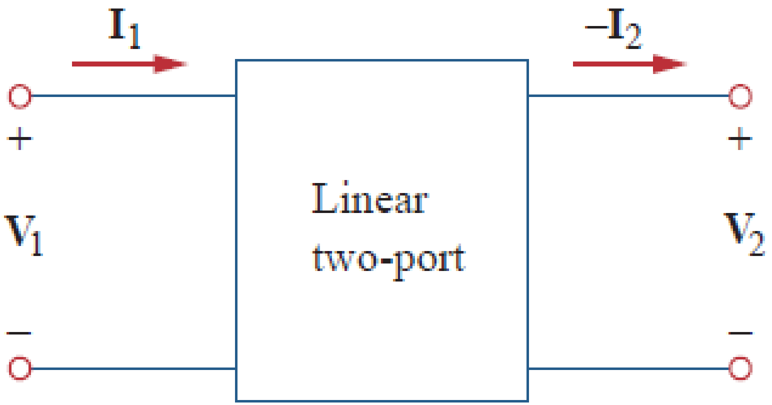
Or in matrix form

$$\begin{bmatrix} \mathbf{V_1} \\ \mathbf{I_1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V_2} \\ \mathbf{-I_2} \end{bmatrix} = [\mathbf{T}] \begin{bmatrix} \mathbf{V_2} \\ \mathbf{-I_2} \end{bmatrix}$$

These equations relate the input variables (V_1 and I_1) to the output variables (V_2 and $-I_2$)

Transmission Parameters

- Notice that in computing the transmission parameters, - I_2 is used rather than I_2, because the current is considered to be leaving the network as shown in the circuit below:



- This is done merely for conventional reasons
- When you cascade two-ports (output to input) it is most logical to think of I_2 as leaving the two-port

Transmission Parameters

- It is also customary in the power industry to consider I_2 as leaving the two-port.

- The two-port parameters provide a measure of how a circuit transmits voltage and current from a source to a load.

- They are useful in the analysis of transmission lines.

- Also known as ABCD parameters and are used in the design of telephone, microwave networks and radars.

Transmission Parameters

- The transmission parameters are determined as:

$$A = \frac{V_1}{V_2}\bigg|_{I_2 = 0} \qquad B = -\frac{V_1}{I_2}\bigg|_{V_2 = 0}$$

$$C = \frac{I_1}{V_2}\bigg|_{I_2 = 0} \qquad D = -\frac{I_1}{I_2}\bigg|_{V_2 = 0}$$

Specifically,

A = Open-circuit voltage ratio

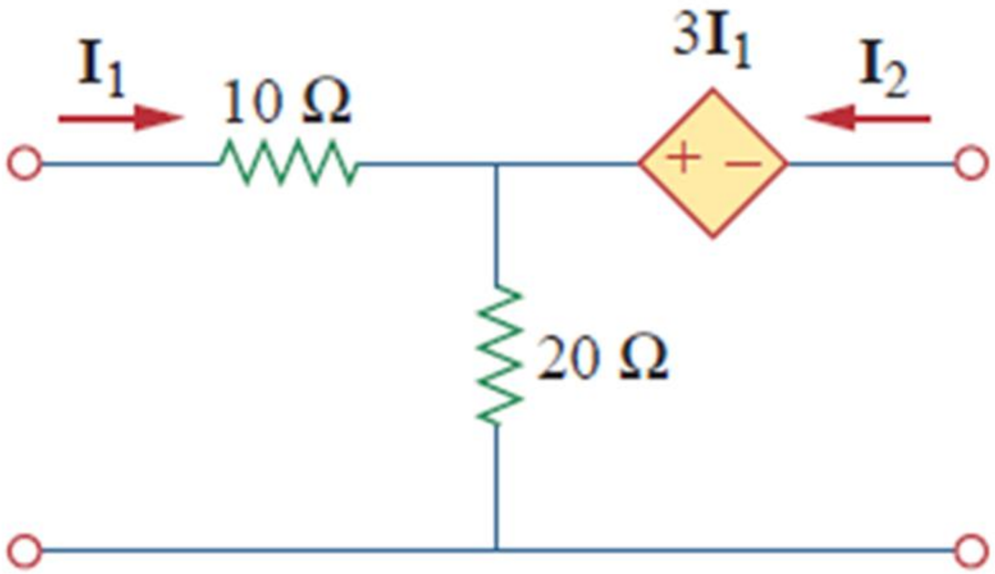
B = Negative short-circuit transfer impedance

C = Open-circuit transfer admittance

D = Negative short-circuit current ratio

Example 1

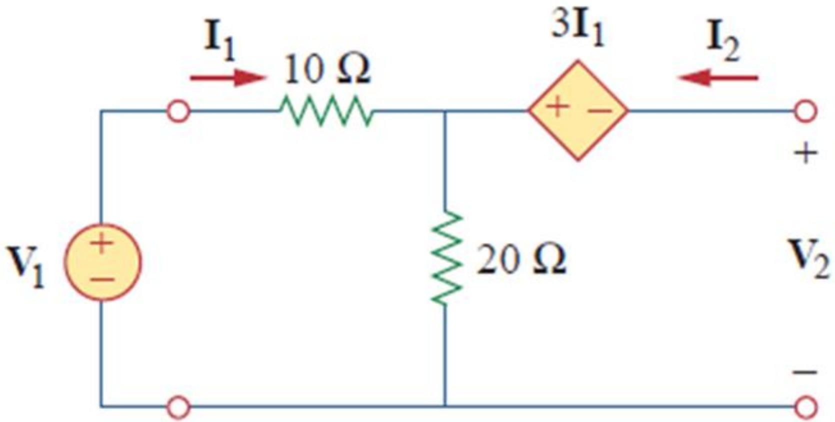
Find the transmission parameters for the two-port network in the figure shown below



Solution

To determine A and C

We leave the output port open as shown below so that $I_2 = 0$ and place a voltage source V_1 at the input port



we have

$$V_1 = (10 + 20)I_1 = 30I_1 \qquad \text{and} \qquad V_2 = 20I_1 - 3I_1 = 17I_1$$

Solution

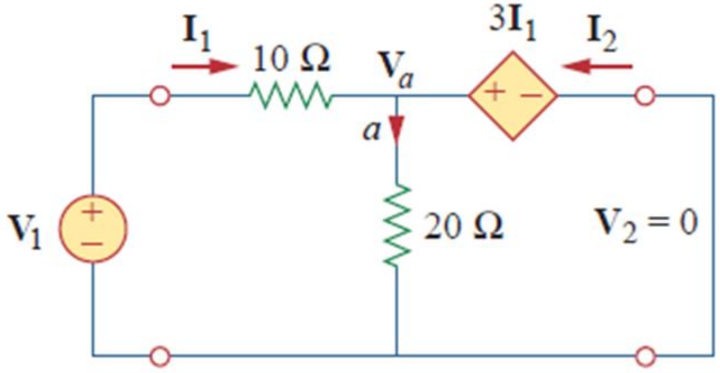
Thus

$$A = \frac{V_1}{V_2} = \frac{30I_1}{17I_1} = 1.765$$

$$C = \frac{I_1}{V_2} = \frac{I_1}{17I_1} = 0.0588S$$

To obtain B and D

We short-circuit the output port so that $V_2 = 0$ and place a voltage source V_1 at the input port. See figure below



Solution

KCL at node a gives

$$\frac{V_1 - Va}{10} - \frac{V_a}{20} + I_2 = 0$$

But

$V_a = 3I_1$ and $I_1 = \dfrac{V_1 - Va}{10}$

Combining these equations

$V_a = 3I_1$ $V_1 = 13I_1$

Substituting them in the KCL equation yields

$$I_1 - \frac{3I_1}{20} + I_2 = 0 \quad \Rightarrow \quad \frac{17}{20} I_1 = -I_2$$

Solution

Therefore

$$D = -\frac{I_1}{I_2} = \frac{20}{17} = 1.176$$

and

$$B = -\frac{V_1}{I_2} = \frac{-13\,I_1}{\left(-\frac{17}{20}\right)I_1} = 15.29\Omega$$

5
Interconnection of Networks

Interconnection of networks

- A large complex network may be divided into subnetworks for the purpose of analysis and design.

- The subnetworks are modelled as two-port networks interconnected to form the original network.

- The two-port networks may therefore be regarded as building blocks that can be interconnected to form a complex network.

- The interconnection can be in series, in parallel, or in cascade although the interconnected network can be described by any of the six parameters.

- A certain set of parameters may have a definite advantage.

Interconnection of networks

For example,

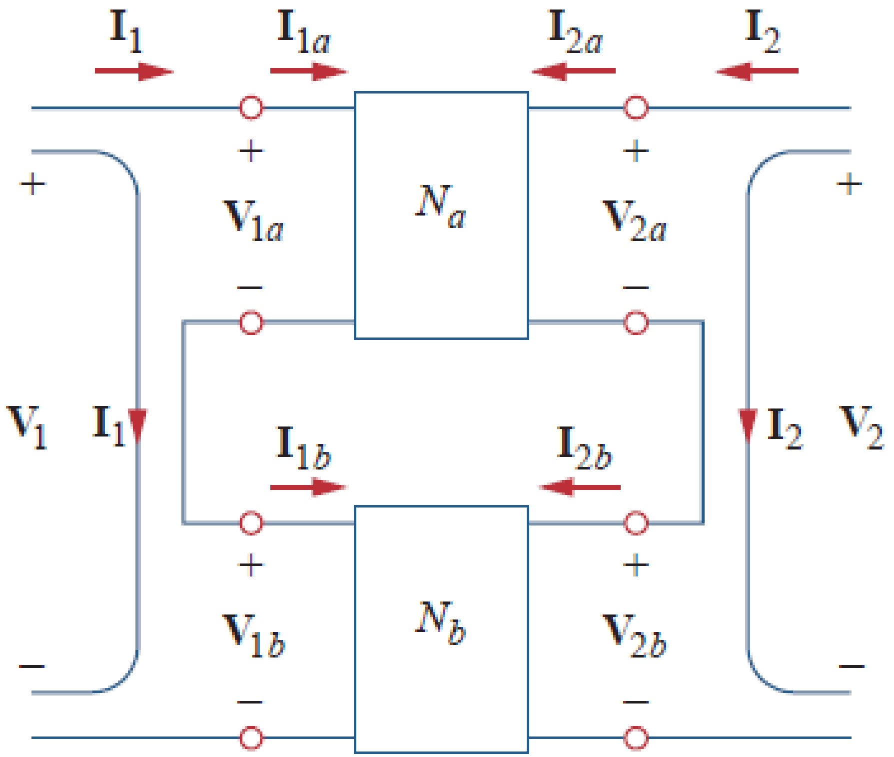
- When the networks are in series, their individual z parameters add up to give the z parameters of the large network.

- When they are in parallel, their individual y parameters add up to give the y parameters of the larger network.

- When they are cascaded, their individual transmission parameters can be multiplied together to give the transmission parameters of the larger network.

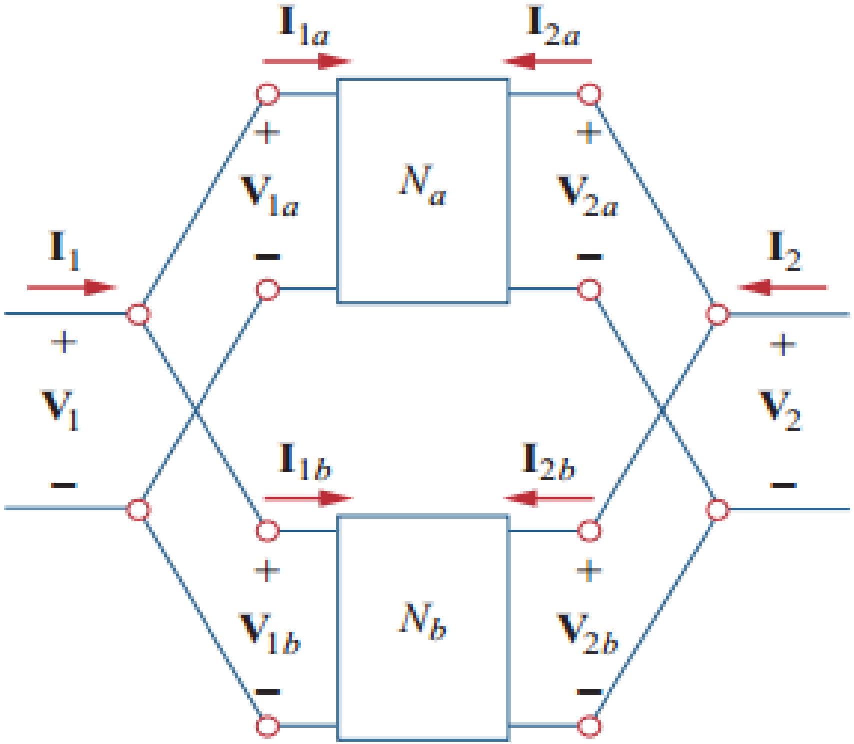
Series Connection of 2 Two-Port Networks

Consider the series connection of two two-port networks as shown below



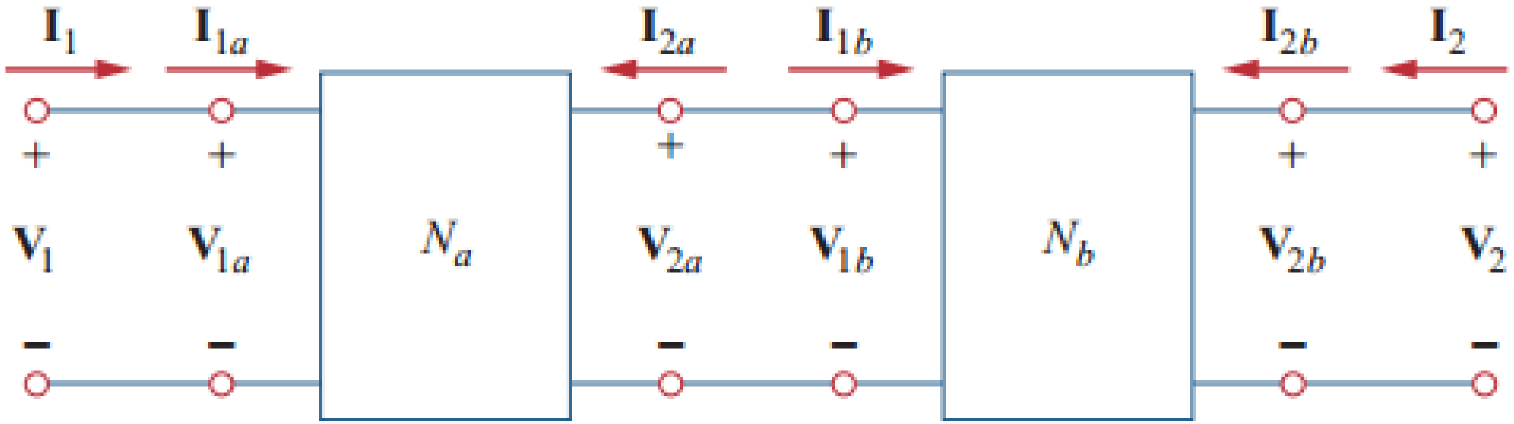
Parallel Connection of 2 Two-Port Networks

Consider the parallel connection of two two-port networks as shown below



Cascade Connection of 2 Two-Port Networks

Consider the cascade connection of two two-port networks as shown below



Thanks!

Any questions?