ASSIGNMENT 3.

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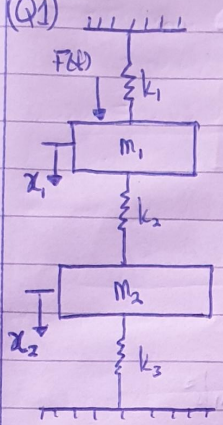
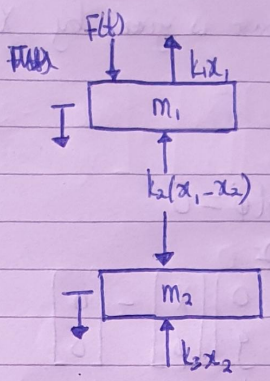
INDEX NUMBER:- 3624118

CLASS:- MECHANICAL ENGINEERING 3

(Q1)





m_1

$-k_1x_1 - k_2(x_1-x_2) + F(t) = m_1\ddot{x}_1$

$m_1\ddot{x}_1 + k_1x_1 + k_2x_1 - k_2x_2 = F(t)$

$m_1\ddot{x}_1 + (k_1+k_2)x_1 + (-k_2)x_2 = F(t)$

m_2

$k_2(x_1-x_2) - k_3x_2 = m_2\ddot{x}_2$

$m_2x_2 + (-k_2)x_1 + (k_2+k_3)x_2 = 0$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 \\ 0 & 17 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 50000 & -20000 \\ -20000 & 36000 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 15000\cos 3t \\ 0 \end{bmatrix}$$

$|k - m\lambda| = 0$

$$\begin{vmatrix} 50000-7\lambda & -20000 \\ -20000 & 36000-17\lambda \end{vmatrix} = 0$$

$119\lambda^2 - 1102\times10^3\lambda + 1400\times10^6 = 0$

$\lambda = 1519.86,\ 7740.64$

$\omega_n = \sqrt{\lambda}$

$\omega_n = 38.99\ \text{rad/s},\ 87.98\ \text{rad/s}$

$(k-m\lambda) = F$ [The operating frequency was 3 rad/s, so its corresponding λ will be 9]

$$\begin{bmatrix} 50000-7(9) & -20000 \\ -20000 & -36000-17(9) \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 15000 \\ 0 \end{bmatrix}$$

Solving this system of equations gives,

$X_1 = 38.68\text{cm}$ and $X_2 = 21.58\text{cm}$

∴ The steady-state response is given by $X_p = \begin{bmatrix} 38.68 \\ 21.58 \end{bmatrix} \cos 3t$ cm

(Q3) $(M+m)\ddot{x} + 2kx + ml\ddot{\phi} = 0$

$\ddot{x} + l\ddot{\phi} + g\phi = 0$

$$\begin{bmatrix} M+m & ml \\ 1 & l \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} 2k & 0 \\ 0 & g \end{bmatrix}\begin{bmatrix} x \\ \phi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$M = 100\text{kg}$, $m = 10\text{kg}$, $k = 2000\text{N/m}$, $l = 2\text{m}$, $g = 9.81\text{m/s}^2$

$$\begin{bmatrix} 110 & 20 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} 4000 & 0 \\ 0 & 9.81 \end{bmatrix}\begin{bmatrix} x \\ \phi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$|k - m\lambda| = 0$

$$\begin{vmatrix} 4000-110\lambda & 0-20\lambda \\ 0-\lambda & 9.81-2\lambda \end{vmatrix} = 0$$

$200\lambda^2 - 9079.1\lambda + 39240 = 0$

$\lambda = 40.558,\ 4.838$

$\omega_n = \sqrt{\lambda}$

$\omega_n = 6.37\text{rad/s},\ 2.20\text{rad/s}$

$(k - m\lambda)X = 0$

For $\lambda = 40.558$

$$\begin{bmatrix} -461.38 & -811.16 \\ -40.558 & -71.306 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-461.38X_1 - 811.16X_2 = 0$

$-461.38X_1 = 811.16X_2$

$\frac{X_1}{X_2} = -\frac{1.758}{1}$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -1.758 \\ 1 \end{bmatrix}$$

For $\lambda = 4.838$

$$\begin{bmatrix} 3467.82 & -96.76 \\ -4.838 & 0.134 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$3467.82X_1 - 96.76X_2 = 0$

$3467.82X_1 = 96.76X_2$

$$\frac{X_1}{X_2} = \frac{35.839}{1}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 35.839 \\ 1 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} -1.758 & 35.839 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} A\sin(6.37t + \phi_1) \\ B\sin(2.20t + \phi_2) \end{bmatrix}$$

at $t = 0$

$$x(0) = \begin{bmatrix} -1.758 & 35.839 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} A\sin\phi_1 \\ B\sin\phi_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$A\sin\phi_1 = -0.0266\text{cm}$

$B\sin\phi_2 = 0.0266\text{cm}$

$$\dot{x}(0) = \begin{bmatrix} -1.758 & 35.839 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 6.37A\cos\phi_1 \\ 2.20B\cos\phi_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$6.37A\cos\phi_1 = 0.0532\text{cm}$

$2.2B\cos\phi_2 = -0.0532\text{cm}$

$A\cos\phi_1 = 0.008352\text{cm}$

$B\cos\phi_2 = 0.02418\text{cm}$

$A = \sqrt{(-0.0266)^2 + (0.008352)^2} = 0.02788\text{cm}$

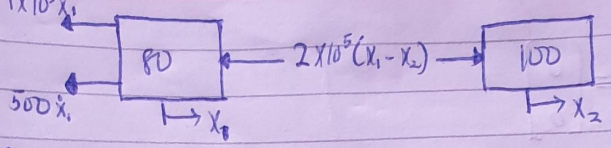
$\phi_1 = \tan^{-1}\left(\frac{-0.0266}{0.008352}\right) = -72.57^\circ = -0.403\pi$

$B = \sqrt{(0.0266)^2 + (0.02418)^2} = 0.0359\text{cm}$

$\phi_2 = \tan^{-1}\left(\frac{0.0266}{-0.02418}\right) = 132.27^\circ = 0.735\pi$

$\therefore$ The response, $x(t) = \begin{bmatrix} -1.758 & 35.839 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0.02788\sin(6.37t - 0.403\pi) \\ 0.0359\sin(2.20t + 0.735\pi) \end{bmatrix}$ cm

(Q2)



$80\ddot{x}_1 + 500\dot{x}_1 + 3\times10^5 x_1 - 2\times10^5 x_2 = 0$

$100\ddot{x}_2 - 2\times10^5 x_1 + 2\times10^5 x_2 = 0$

$$\begin{bmatrix} 80 & 0 \\ 0 & 100 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 500 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 3\times10^5 & -2\times10^5 \\ -2\times10^5 & 2\times10^5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$|k - m\lambda| = 0$

$$\begin{vmatrix} 3\times10^5 - 80\lambda & -2\times10^5 \\ -2\times10^5 & 2\times10^5 - 100\lambda \end{vmatrix} = 0$$

$\lambda^2 - 5750\lambda + 2.5\times10^6 = 0$

$\lambda = 5276.172,\ 473.828$

$\omega_n = \sqrt{\lambda}$

$\omega_n = 72.64 \text{ rad/s},\ 21.77 \text{ rad/s}$

$(k - m\lambda)X = 0$

$\lambda = 5276.172$

$$\begin{bmatrix} -122.09\times10^3 & -200\times10^3 \\ -200\times10^3 & -327.62\times10^3 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -1.638 \\ 1 \end{bmatrix}$$

$\lambda = 473.828$

$$\begin{bmatrix} 2.6209\times10^3 & -2\times10^5 \\ -2\times10^5 & 1.5262\times10^5 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0.763 \\ 1 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} -1.638 & 0.763 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} A\sin(72.64t + \phi_1) \\ B\sin(21.77t + \phi_2) \end{bmatrix}$$

$$x(0) = \begin{bmatrix} -1.638 & 0.763 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} A\sin\phi_1 \\ B\sin\phi_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$A\sin\phi_1 = 0.5369$

$B\sin\phi_2 = 2.4631$

$$\dot{x}(t) = \begin{bmatrix} -1.638 & 0.763 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 72.64A\cos(72.64t + \phi_1) \\ 21.77B\cos(21.77t + \phi_2) \end{bmatrix}$$

$$\dot{x}(0) = \begin{bmatrix} -1.638 & 0.763 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 72.64A\cos\phi_1 \\ 21.77B\cos\phi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$21.77B\cos\phi_2 = -68.22$

$A\cos\phi_1 = -0.004375$

$B\cos\phi_2 = -0.0313$

$A = \sqrt{(0.5369)^2 + (-0.004375)^2} = 0.5369\text{cm}$

$\phi_1 = \tan^{-1}\left(\frac{0.5369}{-0.004375}\right) = \pi/2$

$B = \sqrt{(2.4361)^2 + (-0.0313)^2} = 2.4633\text{cm}$

$\phi_2 = \tan^{-1}\left(\frac{2.4633}{-0.0313}\right) = \pi/2$

$\therefore$ The response, $X(t) = \begin{bmatrix} -1.638 & 0.763 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0.5369\sin(72.64t + \pi/2) \\ 2.4633\sin(21.77t + \pi/2) \end{bmatrix}$ cm