RESONANT CIRCUITS

SERIES RESONANT CIRCUITS

- Consider the series RLC circuit shown below with the ac supply $V_s = V_m \sin \omega t$
- The input impedance is given by:

$$\boldsymbol{Z} = \mathbf{R} + \mathbf{j}(\omega \boldsymbol{L} - \frac{\mathbf{1}}{\omega \boldsymbol{C}})$$

- The magnitude of the circuit current (I) is :

$$|\mathbf{I}| = \frac{\boldsymbol{V_m}}{\sqrt{\boldsymbol{R}^2 + \left(\omega \boldsymbol{L} - \frac{\mathbf{1}}{\omega \boldsymbol{C}}\right)^{\mathbf{2}}}}$$

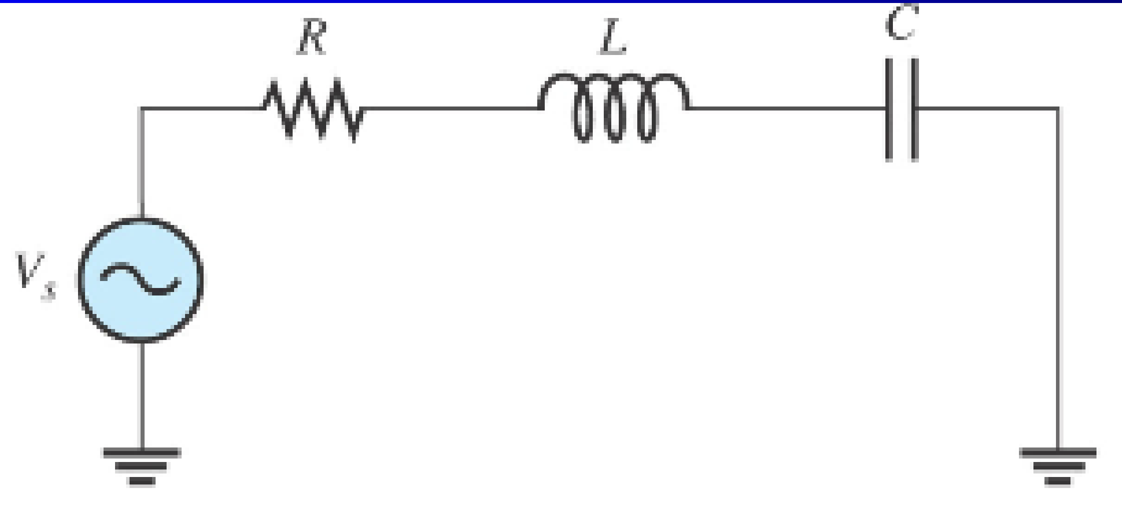


Fig. 4.1: Series resonant circuit

- A series RLC circuit is:
- Capacitive when $X_C > X_L$
- Inductive when $X_L > X_C$
- Resonant when $X_C = X_L$
- At resonance $Z_{tot} = R$

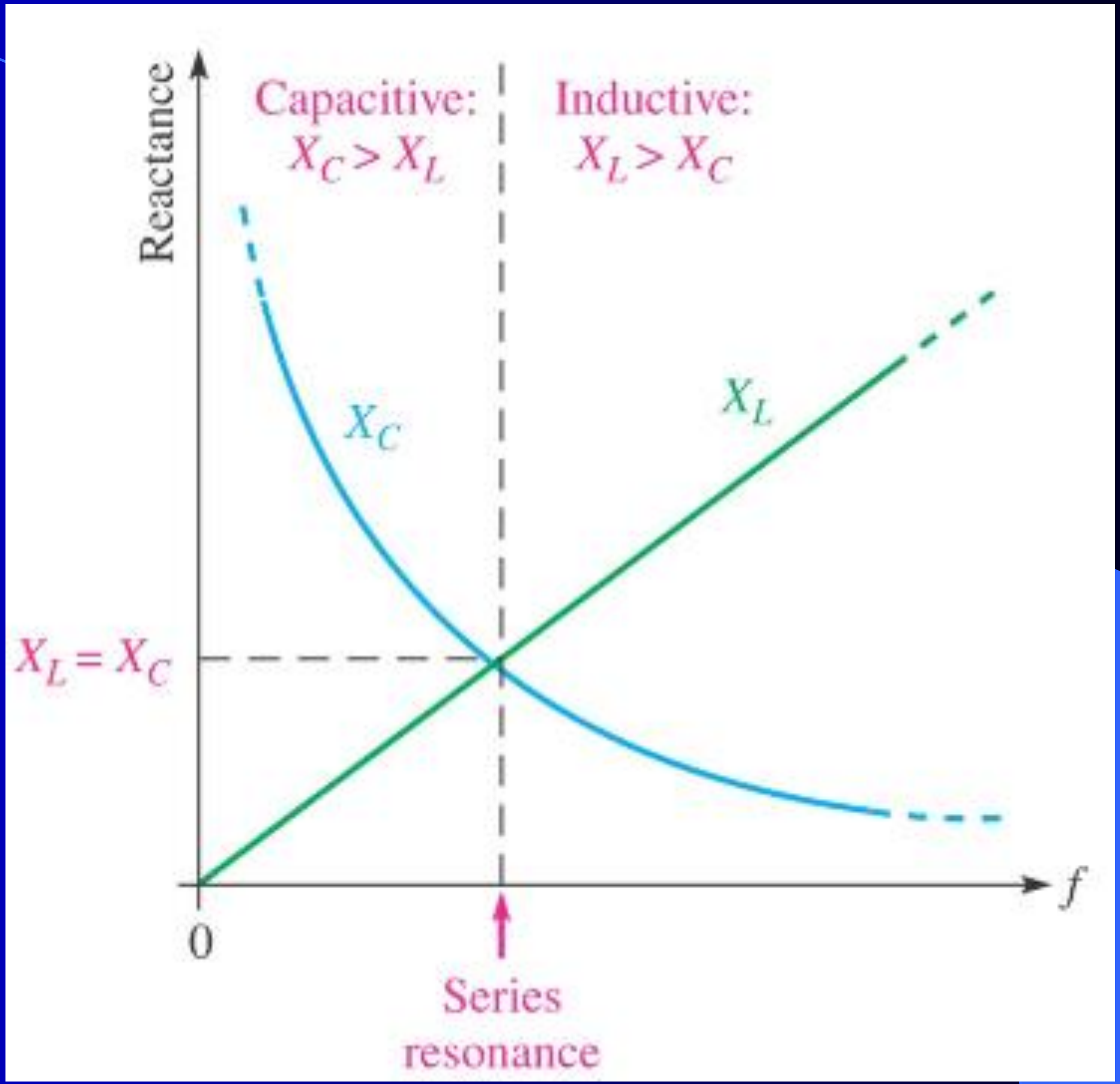
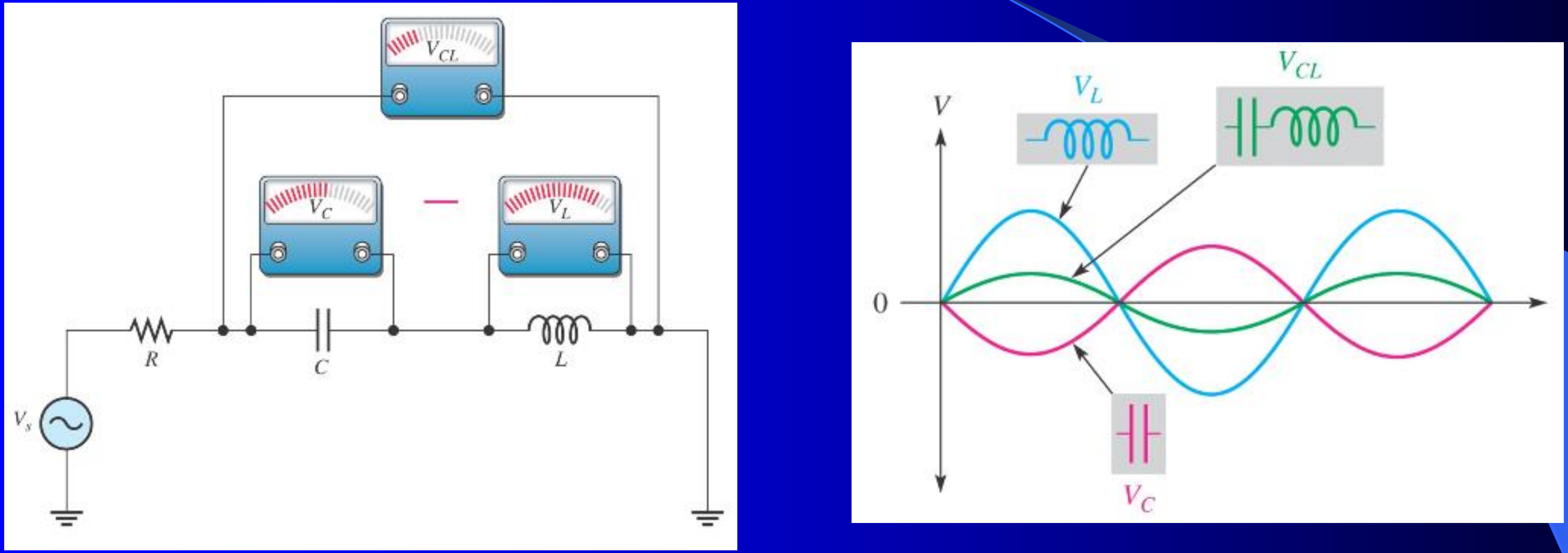


Fig. 4.2: Reactance variation with frequency

VOLTAGE ACROSS THE SERIES COMBINATION OF L AND C

- In a series RLC circuit, the capacitor voltage and the inductor voltage are always 180° out of phase with each other
- Because they are 180° out of phase, V_C and V_L subtract from each other
- The voltage across L and C combined is always less than the larger individual voltage across either element
- This is illustrated in Fig. 4.3.



(a) (b)

Fig. 4.3: (a) Measurement of voltages across C, L and CL combination, (b) waveforms of voltages across C, L and CL combination

SERIES RESONANCE

- Resonance is a condition in a series RLC circuit in which the capacitive and inductive reactances are equal in magnitude, i.e. $\boldsymbol{X_L = X_C}$
- The result is a purely resistive impedance
- The frequency at which resonance occurs in the series RLC circuit is called the resonant or natural frequency (f_r)
- It is derived as:

$$X_L = X_C$$

$$\Rightarrow 2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$\Rightarrow \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

- The figure below illustrates the condition of a series resonant circuit

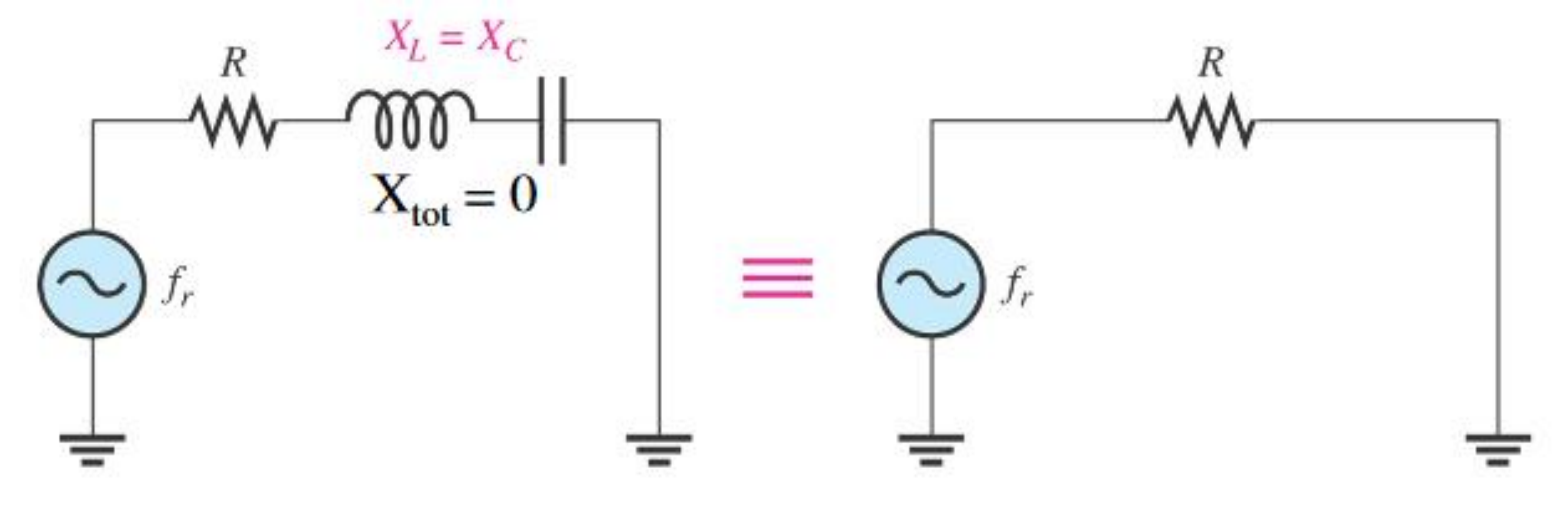


Fig. 4.4: Series resonant circuit

POINTS TO NOTE:

- At resonance,
  - Current (I) is maximum (Fig. 4.4)

i.e. $|\mathbf{I}| = \frac{V_m}{R}$

  - Total Impedance (Z) is minimum (Fig. 4.5)

i.e. $Z = \mathbf{R}$

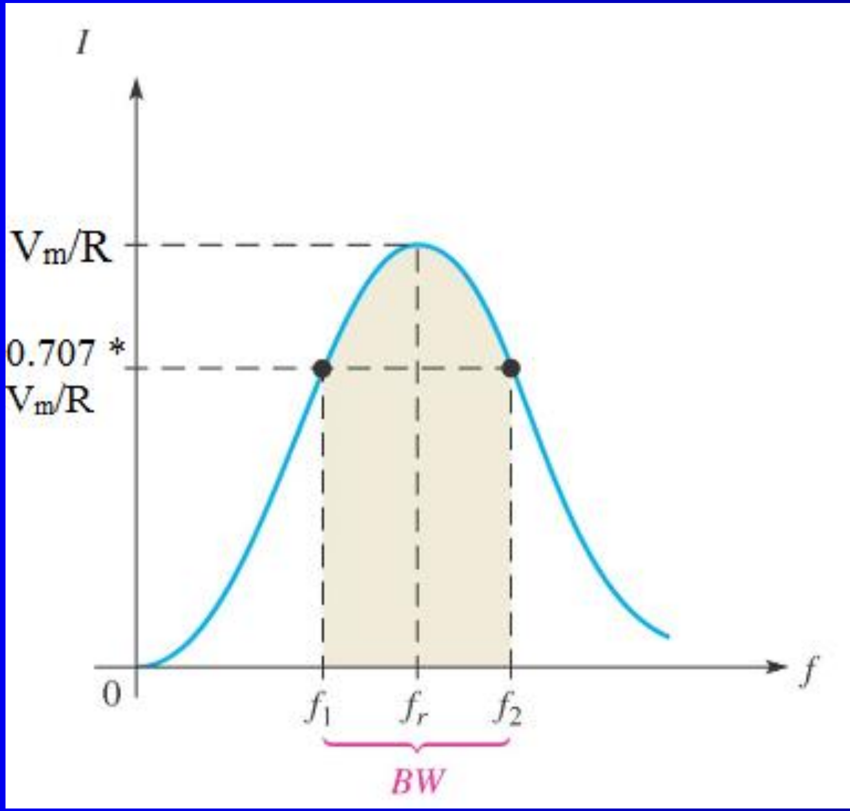


Fig. 4.4: Current variation with frequency

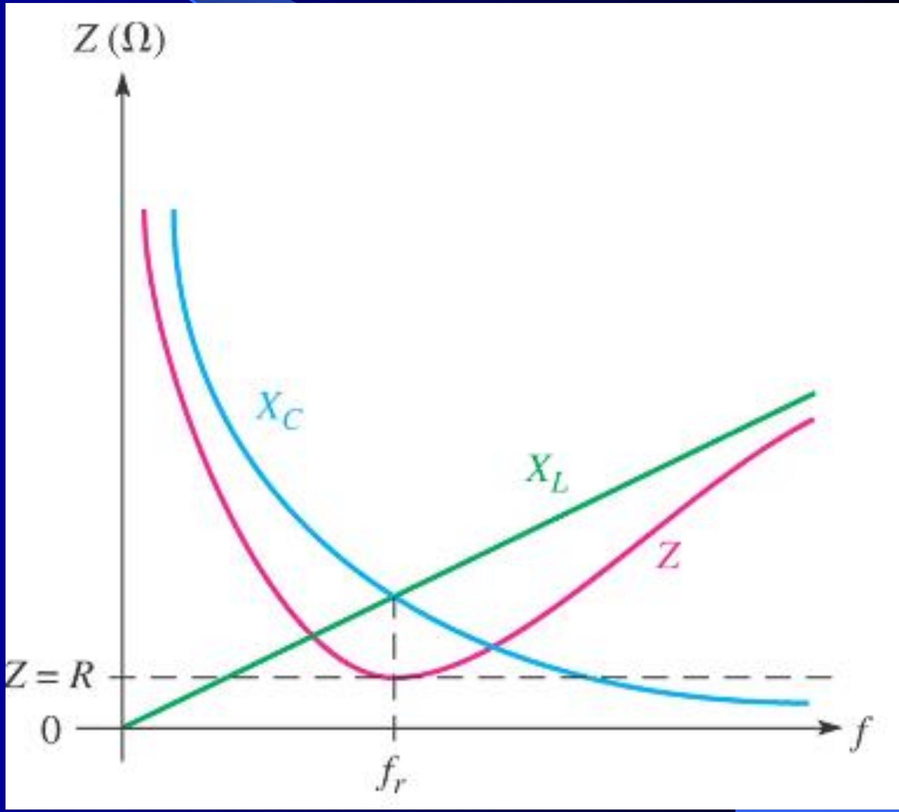
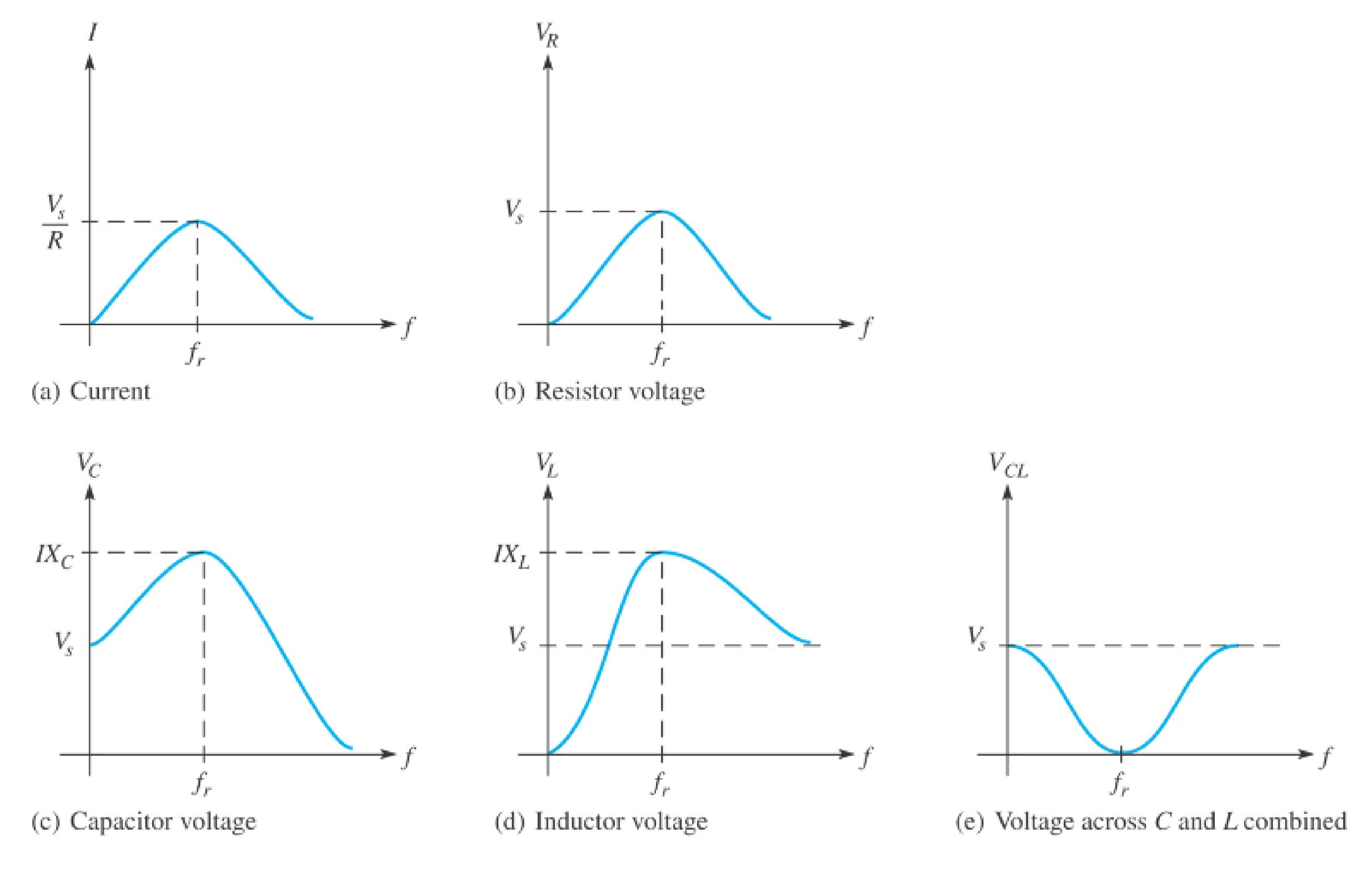


Fig. 4.5 : Impedance variation with frequency

- The figure below shows the generalized current and voltage magnitudes as a function of frequency in a series RLC circuit
- The shapes of the graphs depend on particular circuit values



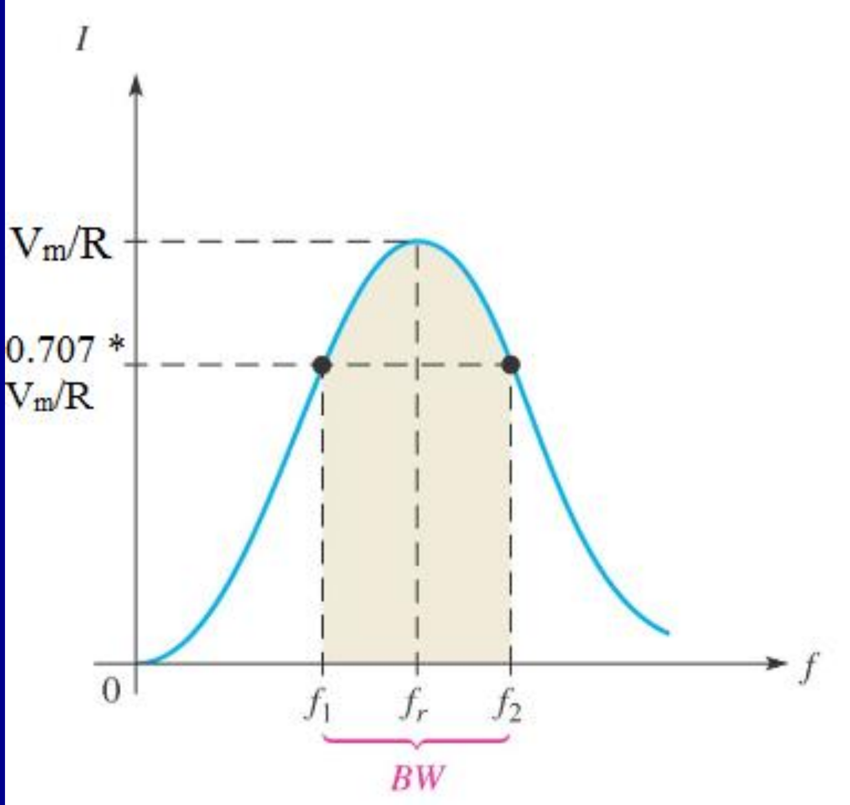
BANDWIDTH OF SERIES RESONANT CIRCUIT

- The frequency variation of current in a series resonant circuit is shown below
- f_1 and f_2 are called the lower and upper cut-off points respectively and they represent the frequencies at which the current in the circuit is 0.707 times the maximum current V_m/R
- They are also called the **-3dB** or **half-power frequencies** because the power at these frequencies is half the peak power at resonant frequency.
- The two half-power frequencies are related to the resonant frequency by

$$f_r = \sqrt{f_1 f_2}$$

- The bandwidth of the circuit is given by:

$$BW = f_2 - f_1 = \frac{R}{2\pi L}$$



QUALITY FACTOR OF SERIES RESONANT CIRCUIT

- The quality factor (Q) of the series resonant circuit is given as:

$$Q = \frac{2\pi f_r L}{R} = \frac{1}{2\pi f_r RC} = \frac{1}{R}\sqrt{\left(\frac{L}{C}\right)}$$

- Quality factor indicates the energy stored relative to the amount of energy loss in a resonant circuit
- A circuit with a higher Q has a low level of damping

- Using Q, we can write the bandwidth as:

$$BW = \frac{f_r}{Q}$$

SELECTIVITY OF SERIES RESONANT CIRCUIT

- Selectivity defines how well a resonant circuit responds to a certain frequency and discriminates against all other frequencies
- The narrower the bandwidth, the greater the selectivity
- This is related to the Quality (Q) Factor (performance) of the inductor at resonance.
- A higher Q Factor produces a tighter bandwidth
- This is illustrated in Fig. 4.7

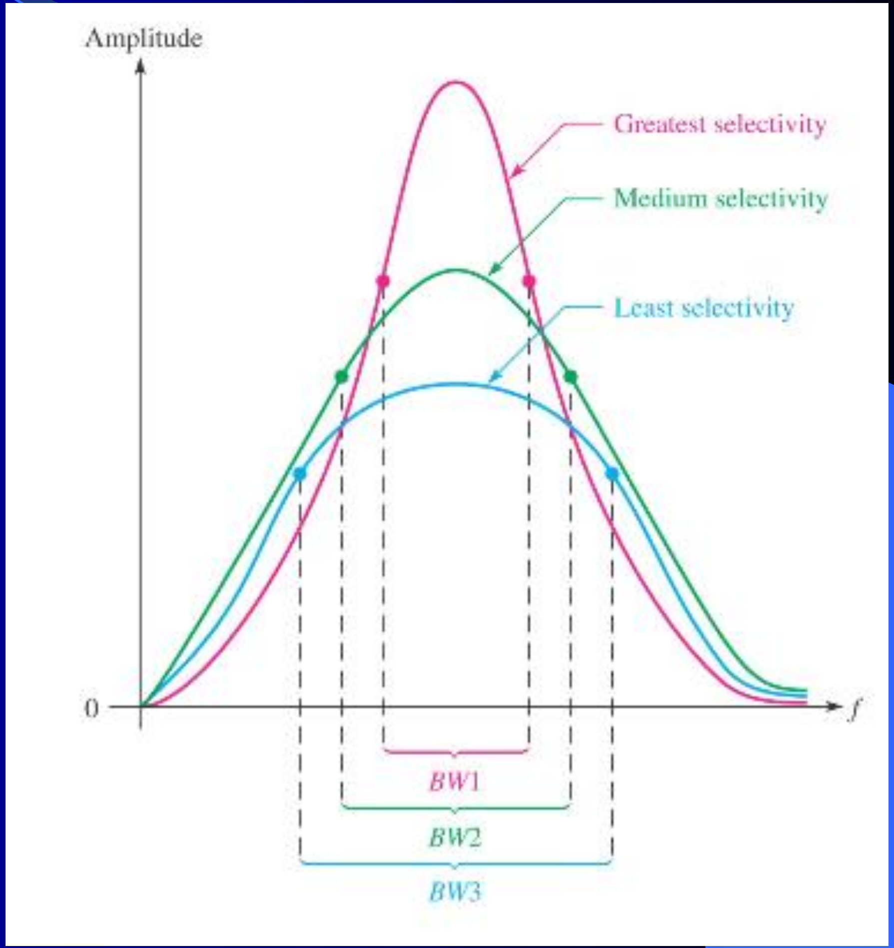


Fig. 4.7: Comparative selectivity curves of current vs frequency

Example 4.1:

- A series RLC resonant circuit has a resonant frequency admittance of 2 x 10^{-2} S. The Q of the circuit is 50, and the resonant frequency is 10000 rad/sec. Calculate the values of R, L and C. Find the bandwidth

- Solution:

- $R = 1/G = \frac{1}{0.02} = 50\ ohms$

- $Q = \frac{2\pi f_r L}{R} = \frac{10000L}{R},$

- knowing Q and R, we find L=0.25 H

- C= $\frac{Q}{2\pi f_r R} = \frac{50}{10000\ x\ 50} = 100\ uF$

- $BW = \frac{f_r}{Q} = \frac{(10000/2\pi)}{50} = 31.8\ Hz$