COLLEGE OF ENGINEERING
Kwame Nkrumah University of Science & Technology

KWAME NKRUMAH UNIVERSITY
OF SCIENCE AND TECHNOLOGY

ME 164
STATICS OF SOLID MECHANICS

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INTRODUCTION

- Engineering applies scientific knowledge to practical problems.
- A major component of this application involves designing.
- Design basically means specifying; size, type, shape etc.
- A good knowledge of what is being specified and what it being specified for is a must for every designer.
- In mechanical engineering, a lot of things are designed to withstand forces (among other things).
- The study of the effects of forces on bodies at rest or in motion constitutes the field of ***Mechanics***
- Mechanics is the foundation of most engineering sciences and is an indispensable prerequisite to their study.

INTRODUCTION

Branches of Mechanics

- MECHANICS
  - Rigid Bodies/ Solid Mechanics
    - Statics
    - Dynamics
      - Kinematics
      - Kinetics
  - Deformable Bodies/ Materials/ Strength of Materials
    - Mechanics of Materials
    - Elasticity
    - Plasticity
  - Fluids
    - Compressible Fluids
    - Incompressible Fluids

OUTLINE

- Fundamental Concepts and Principles
- Forces: Characteristics, 2D & 3D Representation, Resultants.
- Equilibrium of Particles and Bodies
- Structural Analysis
- Friction
- Simple Machines
- Method of Virtual Work
- Resultant of distributed line loads, liquid pressure and flexible cables.

READING MATERIALS

- Basic Engineering Mechanics, J. Antonio
- Vector Mechanics for Engineers, Beer *et al*.
- Engineering Mechanics – Statics, R. C. Hibbler.
- Engineering Mechanics – Statics, Pytel and Kiusalaas.
- Any book on Vector Mechanics or Engineering Mechanics.

ASSESSMENT

Class Attendance	5%
Assignments	5%
Pop Quizzes	5%
Mid-Semester Exam	15%
End of Semester Exam	70%
TOTAL	**100%**

FUNDAMENTAL PRINCIPLES

- Newton's Laws of Motion
  - 1st Law – a body will maintain it state of motion (remain at rest or continue to move in a straight line) unless the resultant force on it is not zero.
  - 2nd Law – A body under the influence of a force experiences a proportionate acceleration in the direction of that force.

$$\vec{F} = m\vec{a}$$

  - 3rd Law – Action and Reaction are equal and opposite.

FUNDAMENTAL PRINCIPLES

- Newton's Law of Universal Gravitation
  - Two particles are attracted to each other by a force defined mathematically as;

$$F = G\frac{Mm}{r^2} \qquad W = mg, \quad g = \frac{GM}{R^2}$$

  - *g* varies from place to place on earth.
- Two or more forces acting on a particle may be replaced by a single force, the resultant.

FUNDAMENTAL CONCEPTS

- Space

  This is associated with notion of describing a point in terms of co-ordinates measured from a reference point.

- Particle

  A very small amount of matter which may be assumed to occupy a single point in space. Idealizing bodies as points simplifies problems since body geometry is not considered.

- Rigid body

  A collection of a large number of particles that remain at a fixed distance from each other, even when under the influence of a load.

FUNDAMENTAL CONCEPTS

- Quantities/Dimensions and Units (SI)
  - Time – second (*s*)
  - Mass – kilogram (*kg*)
  - Length – metre (*m*)
  - Force – Newton (*N*)
- Units and Relations (formulae) may be derived, in which case they must be dimensionally homogenous (all the terms in it have the same dimensions).
- Units may also be converted from one system to another.

FUNDAMENTAL CONCEPTS

Quantity	Unit	Symbol	Formula
Acceleration	Meter per second squared	. . .	m/s^2
Angle	Radian	rad	†
Angular acceleration	Radian per second squared	. . .	rad/s^2
Angular velocity	Radian per second	. . .	rad/s
Area	Square meter	. . .	m^2
Density	Kilogram per cubic meter	. . .	kg/m^3
Energy	Joule	J	$N \cdot m$
Force	Newton	N	$kg \cdot m/s^2$
Frequency	Hertz	Hz	s^{-1}
Impulse	Newton-second	. . .	$kg \cdot m/s$
Length	Meter	m	‡
Mass	Kilogram	kg	‡
Moment of a force	Newton-meter	. . .	$N \cdot m$
Power	Watt	W	J/s
Pressure	Pascal	Pa	N/m^2
Stress	Pascal	Pa	N/m^2
Time	Second	s	‡
Velocity	Meter per second	. . .	m/s
Volume			
Solids	Cubic meter	. . .	m^3
Liquids	Liter	L	$10^{-3} m^3$
Work	Joule	J	$N \cdot m$

†Supplementary unit (1 revolution = 2π rad = 360°).
‡Base unit.

Source: Vector Mechanics for Engineers
– Beer *et al*, 2010.

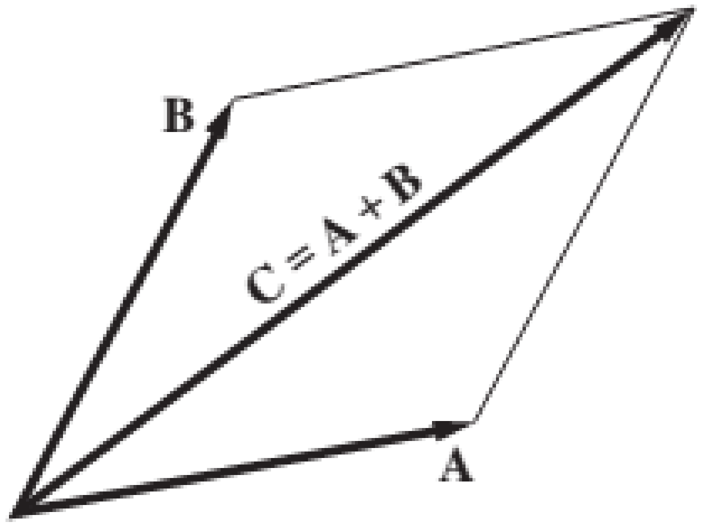
FORCES

- Quantities/dimensions are either scalar or vectors.
- All the basic quantities mentioned are scalars with the exception of Forces.
- As such, forces are essentially treated as vectors in Rigid Body Mechanics.
- The external effects of force(s) on a body/particle depend on:
  - The magnitude of the force(s)
  - The direction of the force(s)
  - The line of action of the force(s)
- Two or more Forces acting on a body/particle (A System of forces) may be collinear, parallel, coplanar or concurrent.
- Two Systems of forces are considered equivalent if they produce the same effect on a rigid body.

FORCES

Some Characteristics of Vectors

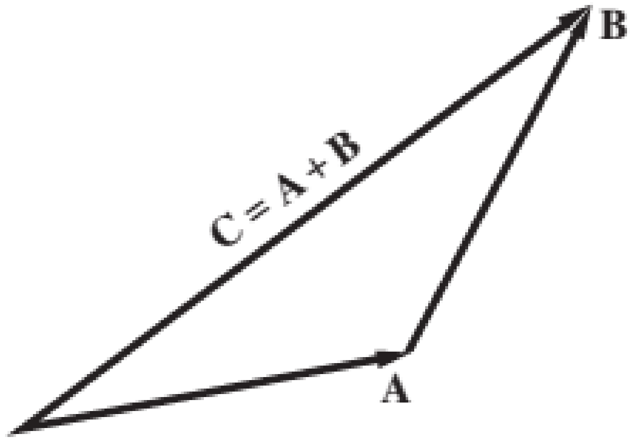
- Vectors may be Fixed or bound, Free or Sliding.
- Vectors are considered equal if they have the same magnitude and direction.
- Scalar multiplication of a vector changes only it’s magnitude, unless the scalar is –ve in which case a change in direction is also produced.
- They obey the parallelogram law of addition:



Source: Engineering Mechanics Statics – Pytel and Kiusalaas

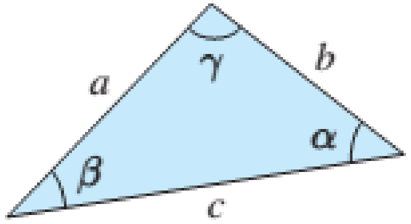
Forces

- They obey the triangle law:



Source: Engineering Mechanics Statics – Pytel and Kiusalaas

- They obey sine and cosine laws:

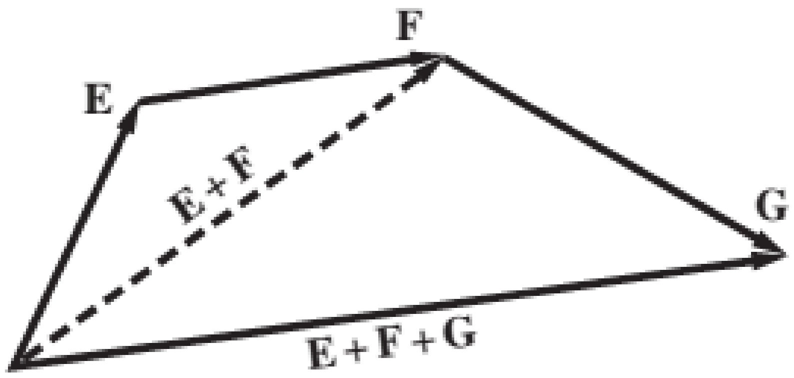


Law of sines	$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$
Law of cosines	$a^2 = b^2 + c^2 - 2bc\cos\alpha$ $b^2 = c^2 + a^2 - 2ca\cos\beta$ $c^2 = a^2 + b^2 - 2ab\cos\gamma$

Source: Engineering Mechanics Statics – Pytel and Kiusalaas

Forces

- Obey the polygon rule of addition.



Source: Engineering Mechanics Statics – Pytel and Kiusalaas

- Successive application of the parallelogram, triangle laws is possible.
- Vector addition is commutative and associative.

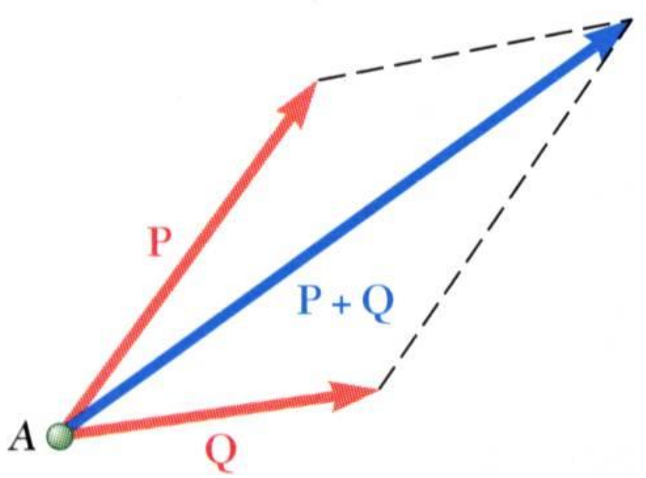
$$\vec{P}+\vec{Q}=\vec{Q}+\vec{P}$$

$$\vec{P}+\vec{Q}+\vec{S}=\left(\vec{P}+\vec{Q}\right)+\vec{S}=\vec{P}+\left(\vec{Q}+\vec{S}\right)$$

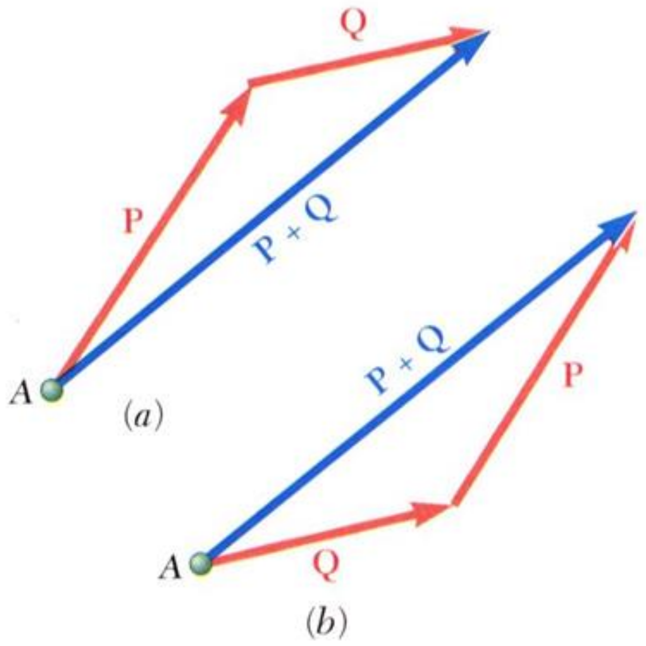
LECTURE 2

Resultant of Forces

- This is the simplest equivalent force system to which a system of forces can be reduced.
- The resultant is the vector sum of all the individual forces acting on the particle or rigid body.



Parallelogram law



Triangle law

Source:
Vector Mechanics for Engineers, Beer *et al.*

FORCES
Resultant of Forces

- The resultant may be determined through the Graphical or Trigonometric Approaches.

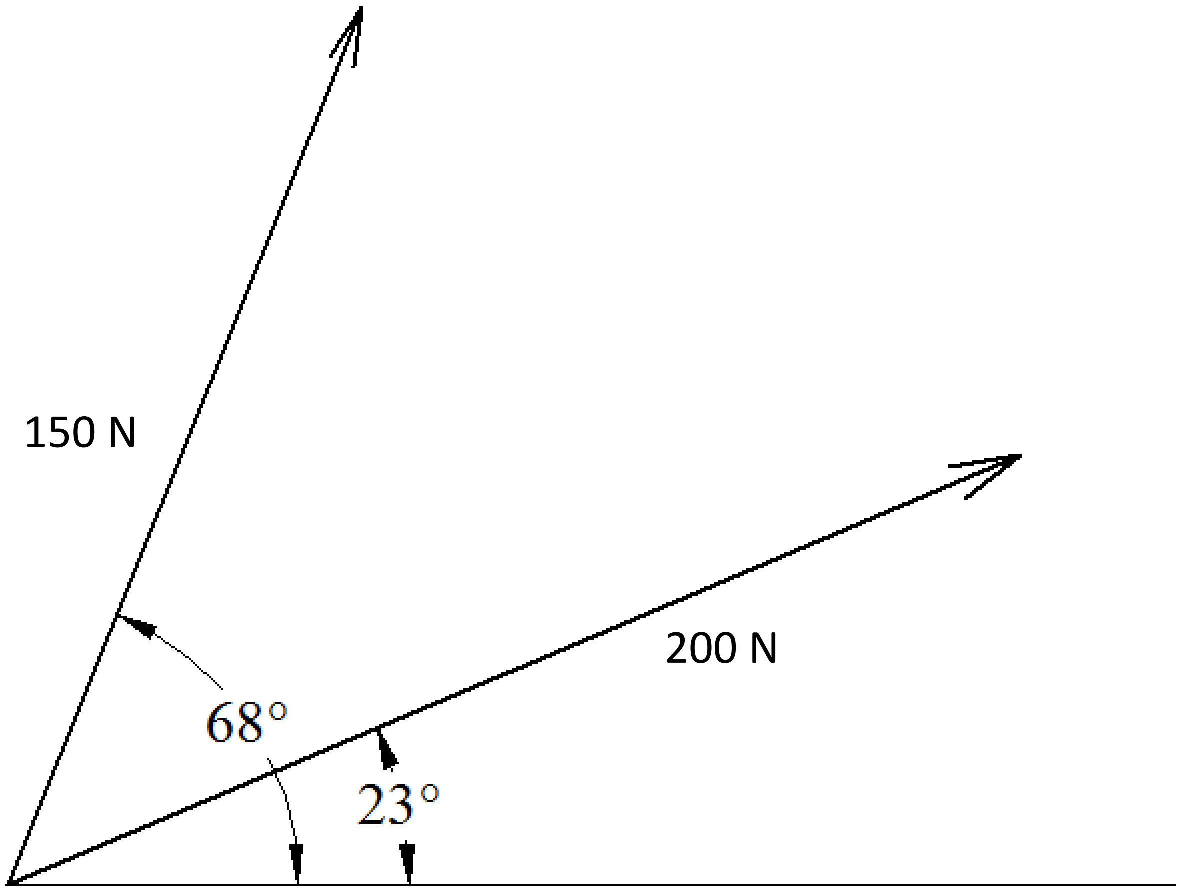
- Graphical approach – Parallelogram, Triangle or Polygon rules of vector addition.

- Trigonometric approach – Sine and Cosine rules.

- Force Components approach.

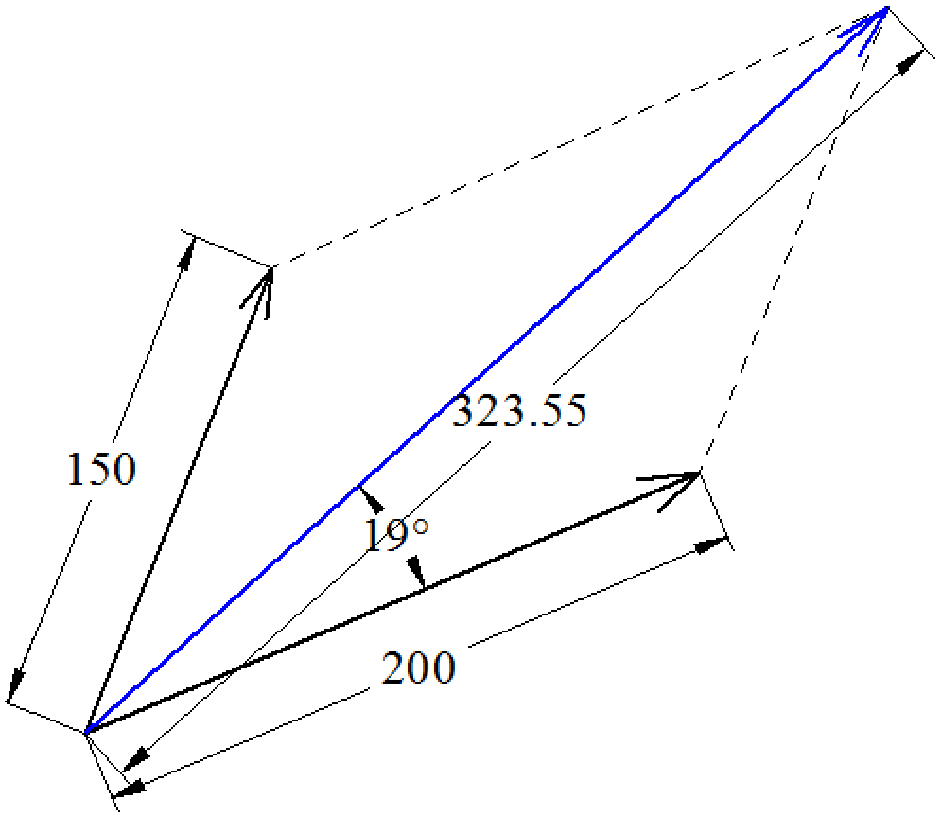
Example 1.1

Determine the Resultant of the forces shown below:

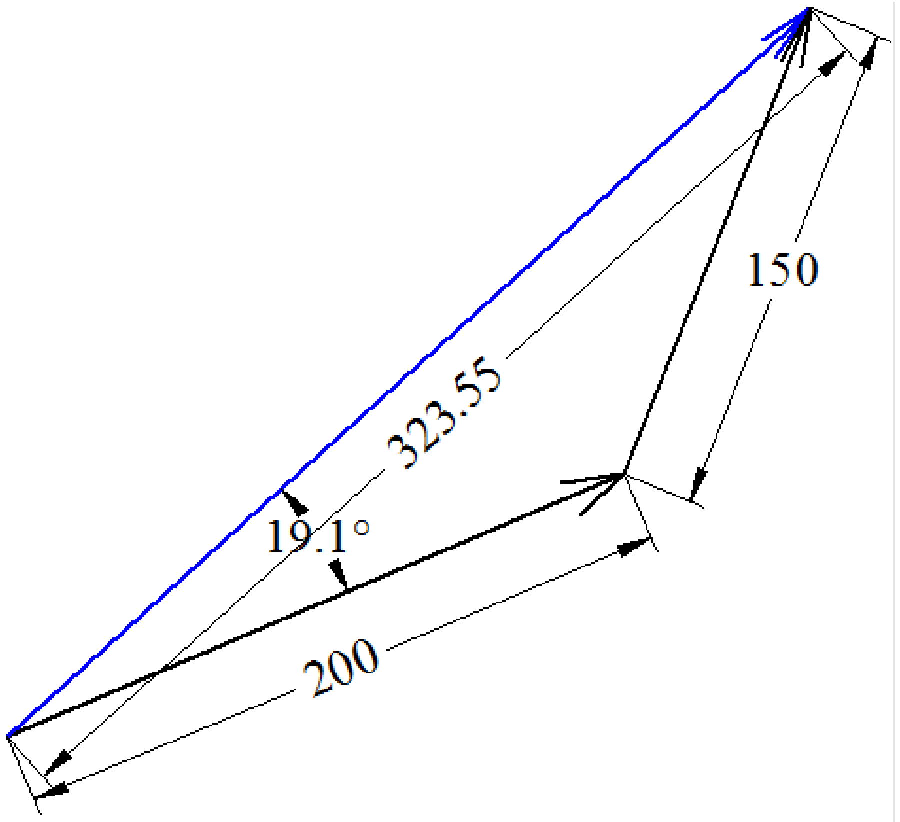


Solution to Example 1.1 – Graphical Approach

1N = 1mm



Parallelogram Law

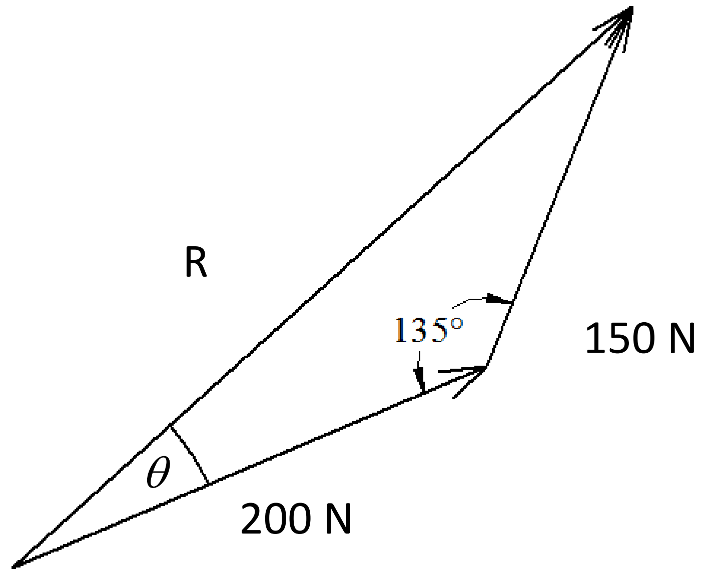


Triangle Law

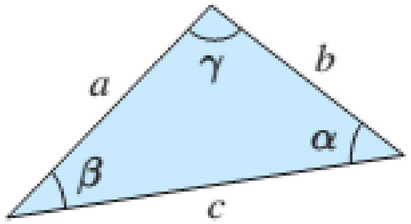
FORCES

Resultant of Forces

Solution to Example 1.1 - Trigonometric Approach



Recall



Law of sines	$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
Law of cosines	$a^2 = b^2 + c^2 - 2bc\cos\alpha$ $b^2 = c^2 + a^2 - 2ca\cos\beta$ $c^2 = a^2 + b^2 - 2ab\cos\gamma$

From the Cosine Rule,

$$R^2 = 200^2 + 150^2 - 2(200)(150)\cos 135^o$$

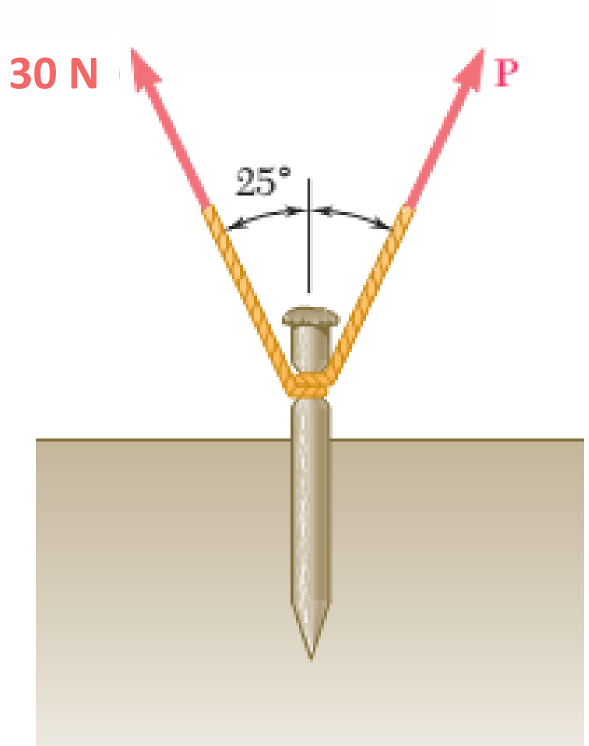
$$R = 323.9\,\text{N}.$$

From the Sine Law

$$\frac{323.9\,\text{N}}{\sin 135^o} = \frac{150\,\text{N}}{\sin\theta^o}, \quad \theta = 19.11^o$$

Example 1.2

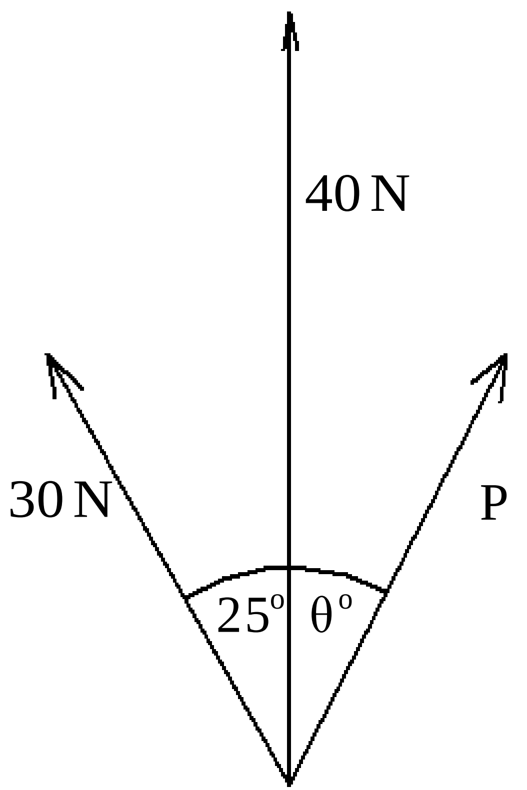
A stake is being pulled out of the ground by means of two ropes as shown below. Knowing the magnitude and direction of the force exerted on one rope, determine the magnitude and direction of the force *P*, that should be exerted on the other rope if the resultant of these two forces is to be a 40 N vertical force. Also determine the angle the 30 N force makes with the unknown force.

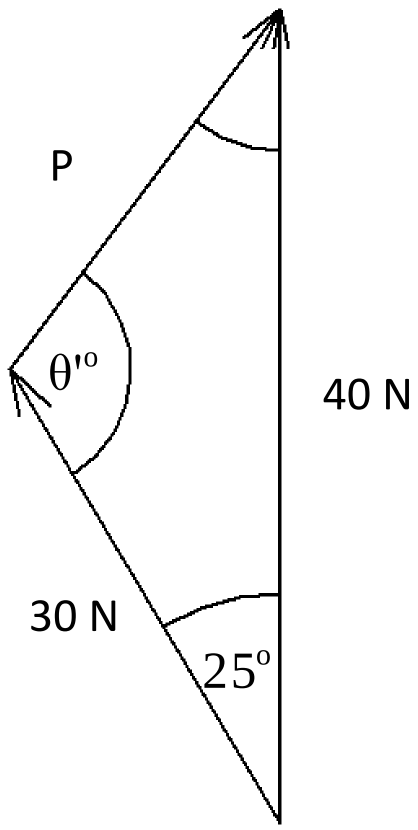


FORCES
Resultant of Forces

Solution to Example 1.2

Free Body Diagram





From the Cosine Rule,

$$P^2 = 40^2 + 30^2 - 2(40)(30)\cos 25^{o}$$

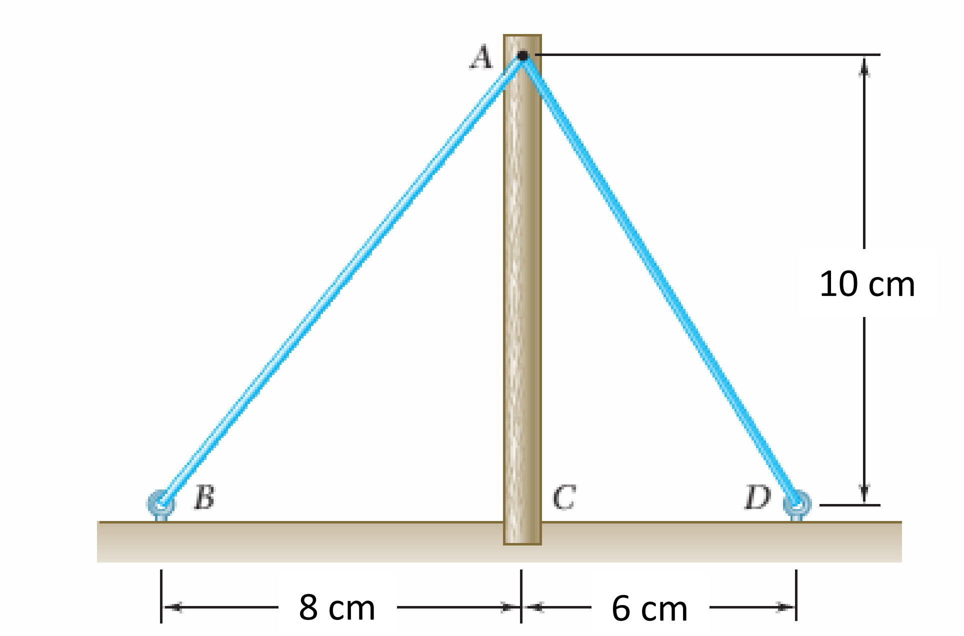
$$P = 18.02\,\text{N}.$$

From the Sine Law

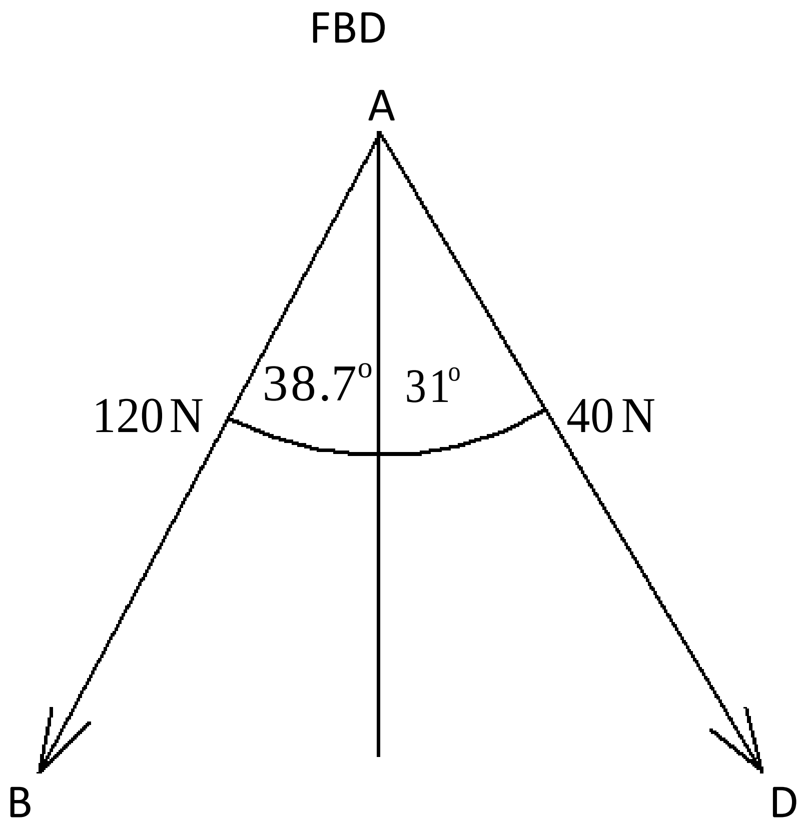
$$\frac{18.02\,\text{N}}{\sin 25^{o}} = \frac{40\,\text{N}}{\sin \theta^{o}}, \quad \theta = 69.74^{o}$$

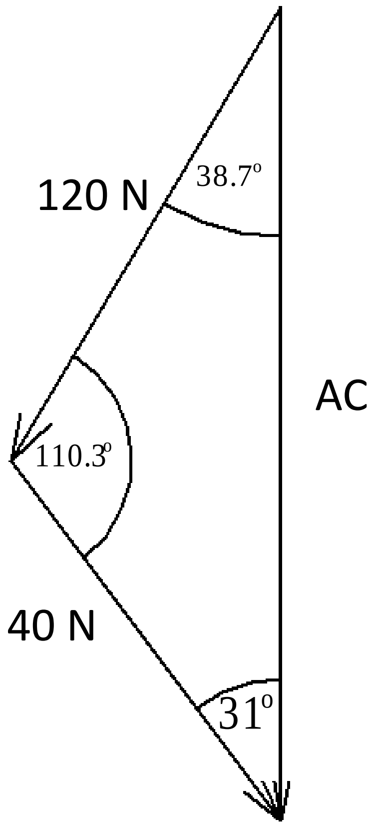
Example 1.3

The cable stays AB and AD help support pole AC. Knowing that the tension is 120 N in AB and 40 N in AD, determine the magnitude of the resultant of the forces exerted by the stays at A.



Solution to Example 1.3





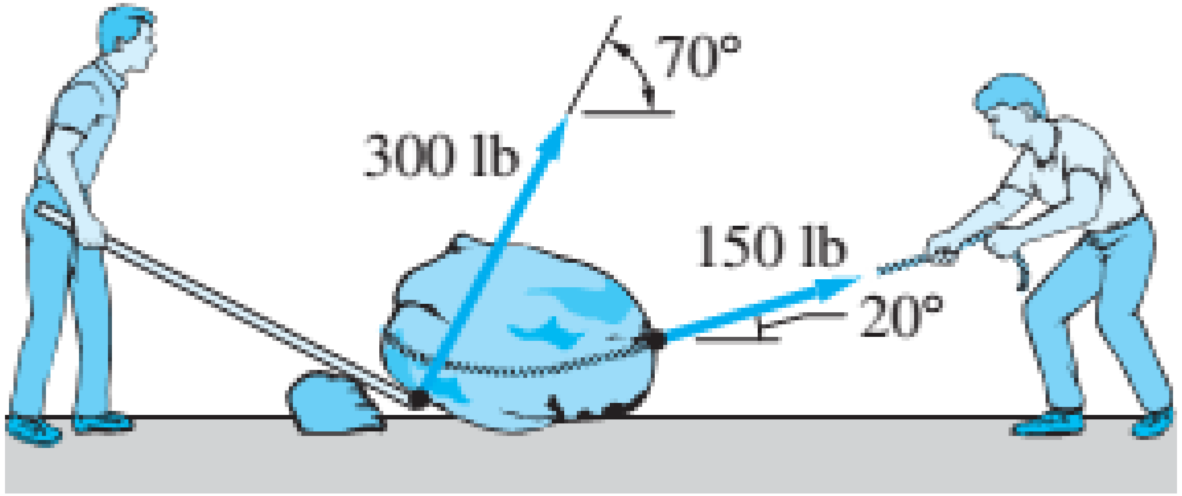
From the Cosine Rule,

$$AC^2 = 40^2 + 120^2 - 2(40)(120)\cos 110.3^o$$

$$AC = 139.04\ \text{N}.$$

Example 1.4

Two men are trying to roll the boulder by applying the forces shown. Determine the magnitude and direction of the force that is equivalent to the two applied forces.



FORCES

Resultant of Forces - Force Components approach

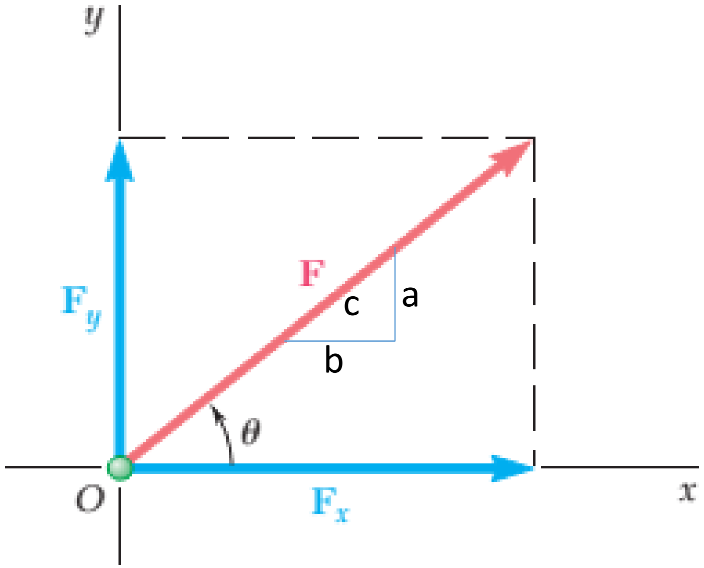
- This approach requires the forces to be resolved into Rectangular or Cartesian components.

- Like components are then summed to get the components of the resultant force.

- Magnitude and direction of the resultant force can be obtained through appropriate Trigonometry techniques

FORCES

Resolving Forces in a Plane Into Rectangular Components – Scalar Approach



$$\vec{F} = \vec{F}_x + \vec{F}_y$$

$$\vec{F}_x = F\cos\theta = F\left(\frac{b}{c}\right)$$

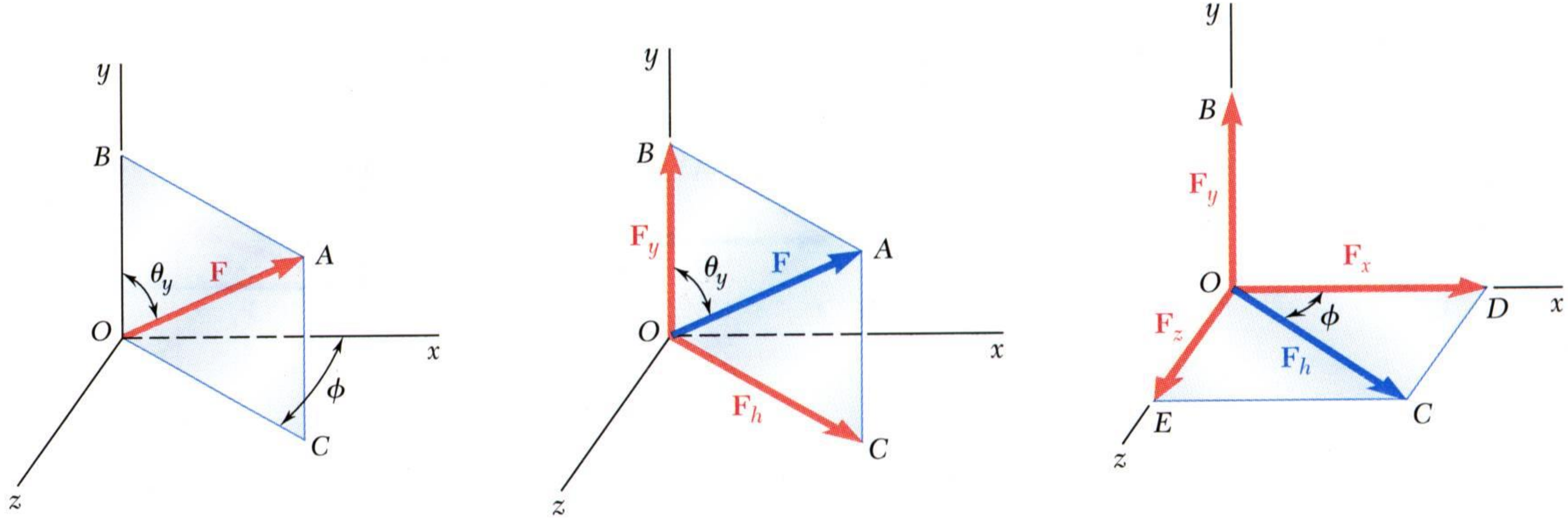
$$\vec{F}_y = F\sin\theta = F\left(\frac{a}{c}\right)$$

$$\theta_x = \cos^{-1} = \left(\frac{F_x}{F}\right) = \cos^{-1} = \left(\frac{b}{c}\right)$$

FORCES

Resolving Forces in Space Into Rectangular Components – Scalar Approach

➢The same idea is extended to forces in space. A third component, *Fz* is introduced.



Diagrams from: Vector Mechanics for Engineers, Beer *et al.*

- Resolve $\vec{F}$ into horizontal and vertical components.

$$F_y = F\cos\theta_y$$

$$F_h = F\sin\theta_y$$

- Resolve F_h into rectangular components

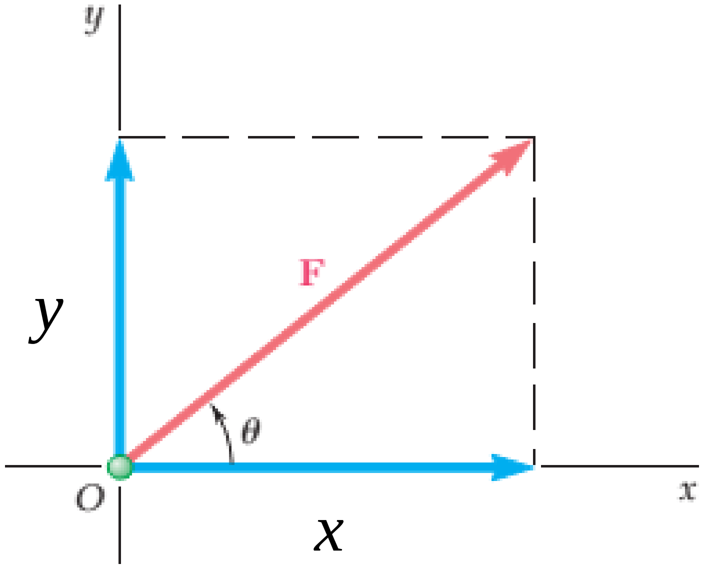
$$F_x = F_h\cos\phi = F\sin\theta_y\cos\phi$$

$$F_z = F_h\sin\phi = F\sin\theta_y\sin\phi$$

FORCES

Resolving Forces in a Plane Into Rectangular Components – Unit Vector Approach

- The force is expressed as a product of its magnitude and it's unit vector.



$$\vec{F} = F\lambda = F\left(\frac{xi + yj}{\sqrt{x^2 + y^2}}\right)$$

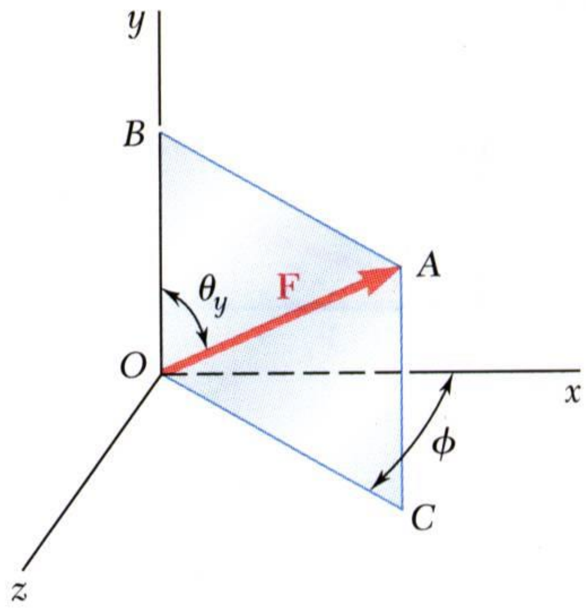
$$= Fx\vec{i} + Fx\vec{j}$$

$$\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right)$$

FORCES

Resolving Forces in Space Into Rectangular Components – Unit Vector Approach

➢ For the Cartesian Unit Vector Approach,



$$\vec{F} = F\lambda = F\left(\frac{xi + yj + zk}{\sqrt{x^2 + y^2 + z^2}}\right)$$

$$= Fx\vec{i} + Fx\vec{j} + Fz\vec{k}$$

$$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right)$$

Resultants by Summing Components

- Like components are summed to obtain the components of the resultant.

$$\vec{R} = \vec{R}_x + \vec{R}_y + \vec{R}_z = \sum F_x + \sum F_y + \sum F_z$$

$$\vec{R} = (\vec{P}_x + \vec{Q}_x + \vec{S}_x) + (\vec{P}_y + \vec{Q}_y + \vec{S}_y) + (\vec{P}_z + \vec{Q}_z + \vec{S}_z)$$

OR

$$\vec{R} = (P_x + Q_x + S_x)\vec{i} + (P_y + Q_y + S_y)\vec{j} + (P_z + Q_z + S_z)\vec{k}$$

The magnitude of the Resultant Force is given by;

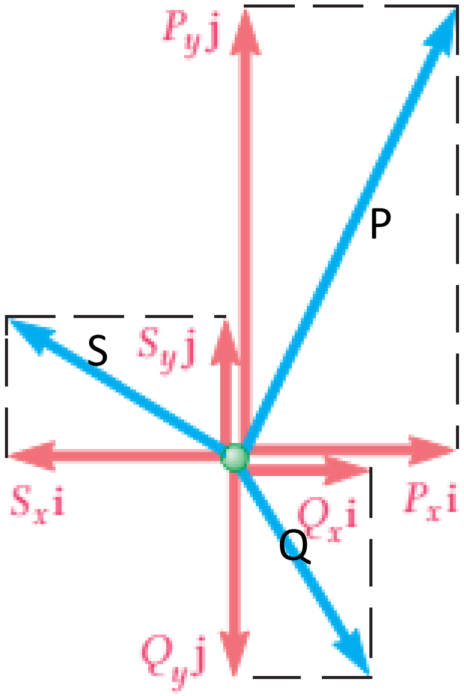
$$R = \sqrt{R_x^{\,2} + R_y^{\,2} + R_z^{\,2}}$$

And the direction;

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right), \quad \theta_y = \cos^{-1}\left(\frac{R_y}{R}\right), \quad \theta_z = \cos^{-1}\left(\frac{R_z}{R}\right)$$

Resultants by Summing Components

- For instance,



$$\vec{R} = \vec{R}_x + \vec{R}_y = \sum F_x + \sum F_y$$

$$\vec{R} = (P_x + Q_x + S_x)\vec{i} + (P_y + Q_y + S_y)\vec{j}$$

The magnitude of the Resultant Force is given by;

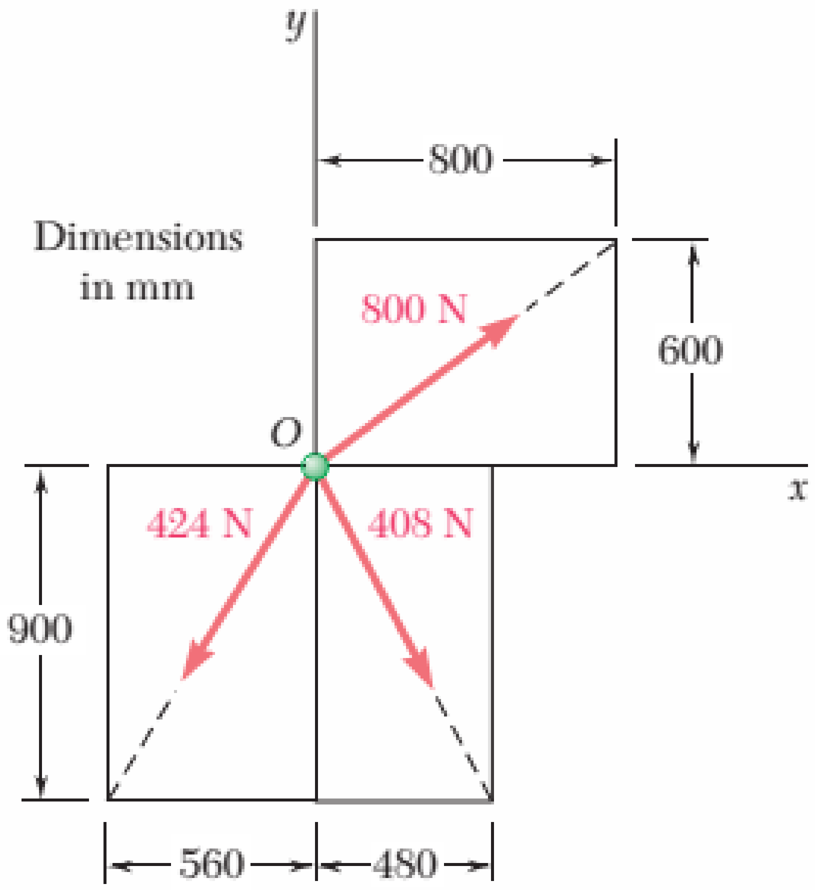
$$R = \sqrt{R_x^{\ 2} + R_y^{\ 2}}$$

And the direction;

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right)$$

Example 2.1

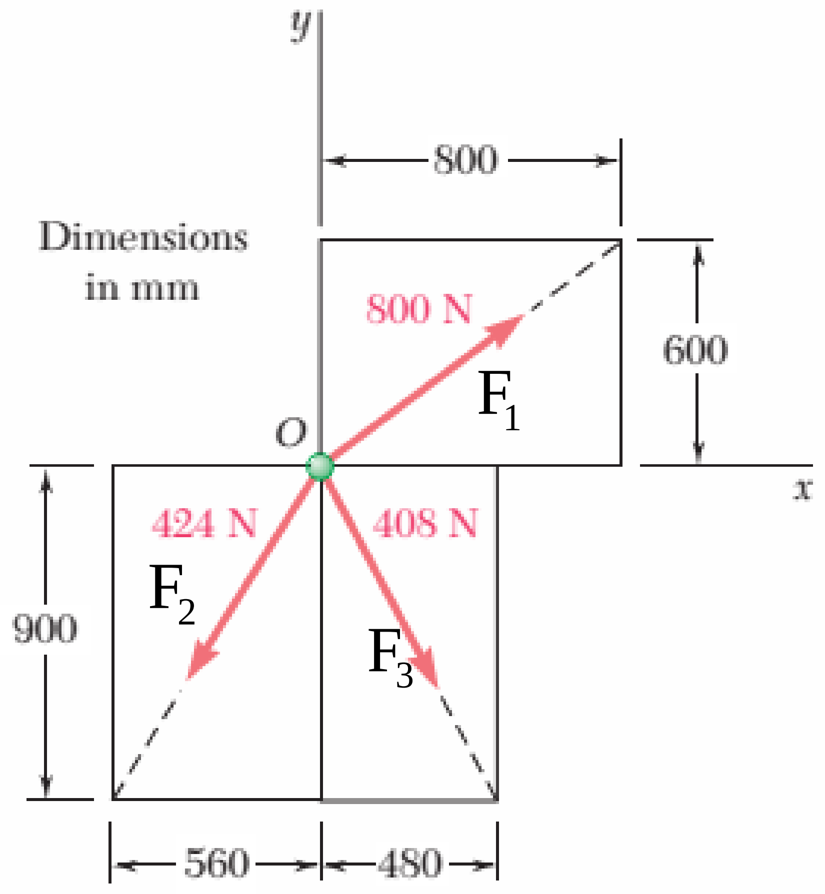
Find resultant of the forces shown using the unit vector approach.



Example 2.1-Solution

Resultants

$$\vec{F} = F\lambda = F\left(\frac{xi + yj}{\sqrt{x^2 + y^2}}\right)$$



$$\vec{F}_1 = 800\left(\frac{800i + 600j}{\sqrt{800^2 + 600^2}}\right) = 800.\frac{800i}{1000} + 800.\frac{600j}{1000} = 640i + 480j$$

$$\vec{F}_2 = 424\left(\frac{-560i - 900j}{\sqrt{560^2 + 900^2}}\right) = -224i - 360j$$

$$\vec{F}_3 = 408\left(\frac{480i - 900j}{\sqrt{480^2 + 900^2}}\right) = 192i - 360j$$

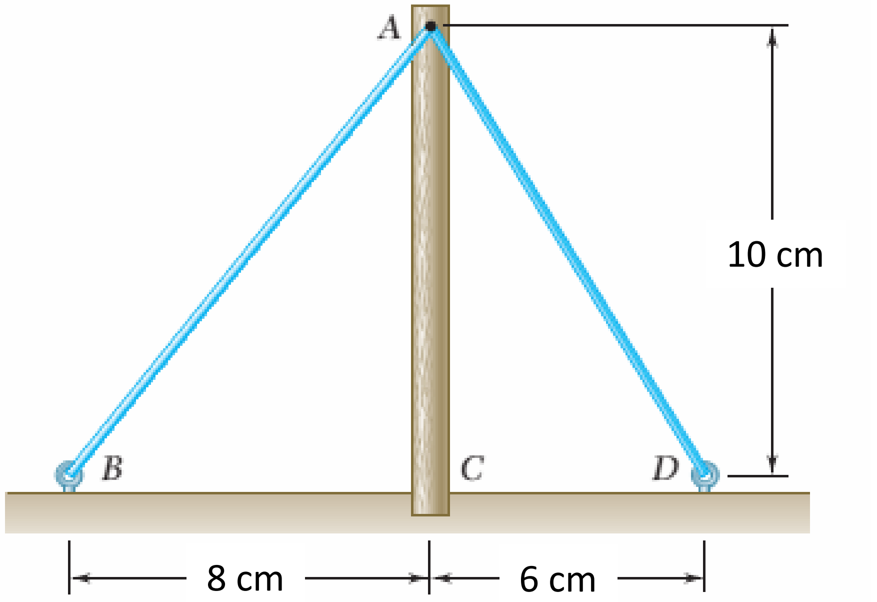
Resultant, $\vec{\mathrm{F}} = \sum Fi + \sum Fj = (640 - 224 + 192)i + (480 - 360 - 360)j$

$= 608i - 240j$

The magnitude of the Resultant, $\mathrm{F} = \sqrt{(608)^2 + (-240)^2} = 558.63\ \mathrm{N}$

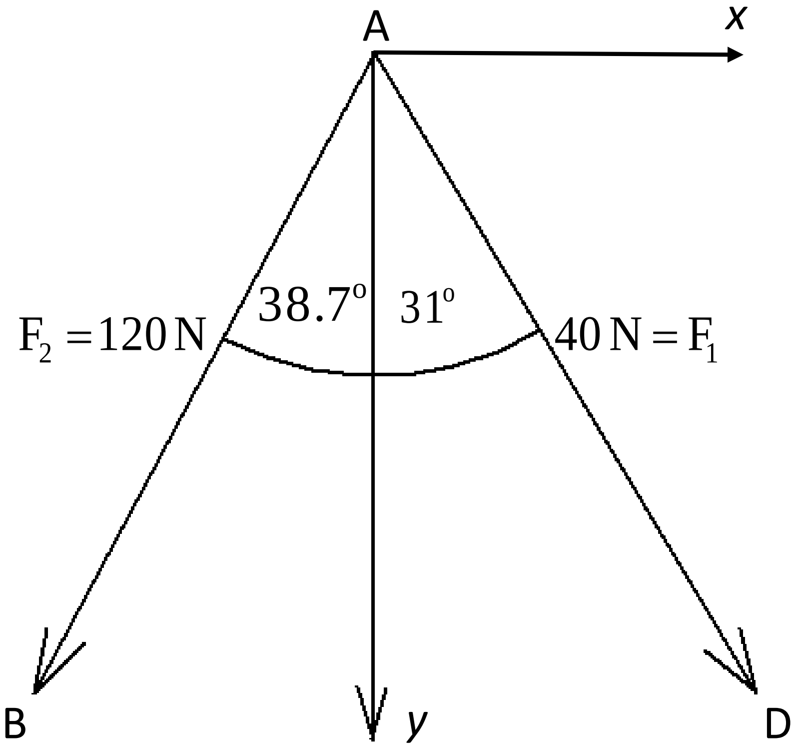
Example 2.2

The cable stays AB and AD help support pole AC. Knowing that the tension is 120 N in AB and 40 N in AD, determine the magnitude and direction of the resultant of the forces exerted by the stays at A.



Resultants

Example 2.2 - Solution



$$\vec{F} = \sum \vec{F}_x + \sum \vec{F}_y$$

$$\vec{F}_{1x} = F_1 \sin\theta = 40 \sin 31^o = 20.6\ N$$

$$\vec{F}_{1y} = F_1 \cos\theta = 40 \cos 31^o = -34.29\,N$$

$$\vec{F}_{2x} = F_2 \sin\theta = 120 \sin 38.7^o = -75.03\,N$$

$$\vec{F}_{2y} = F_2 \cos\theta = 120 \cos 38.7^o = -93.65\,N$$

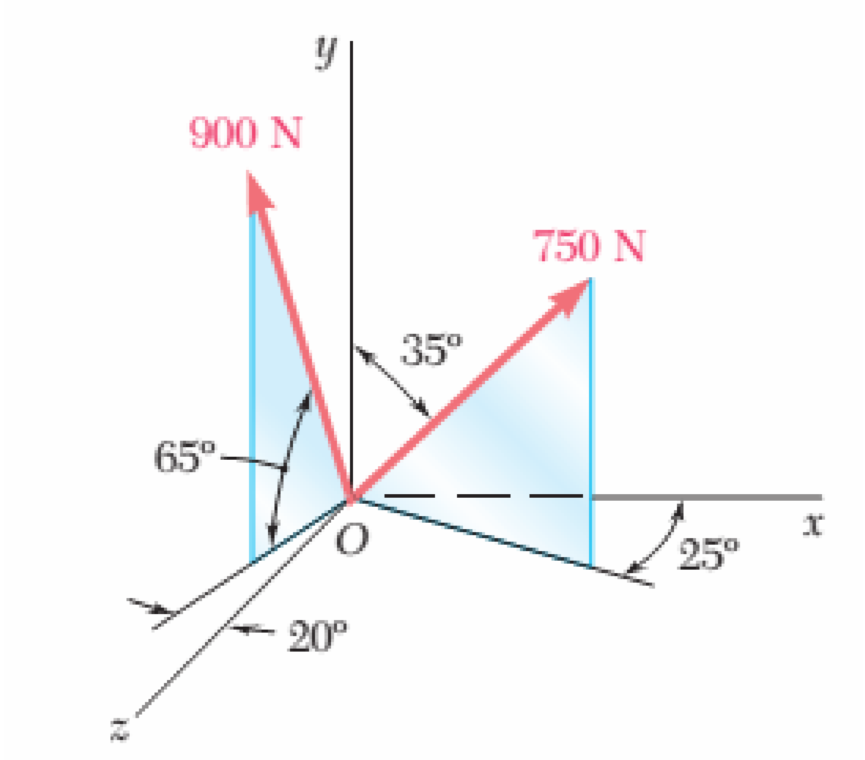
$$\vec{F} = (20.6 - 75.03)_x + (-35.29 - 93.65)_y = -54.43_x - 128.94_y$$

$$F = \sqrt{(-54.43^2 + (-128.94)^2} = 139.96\,N$$

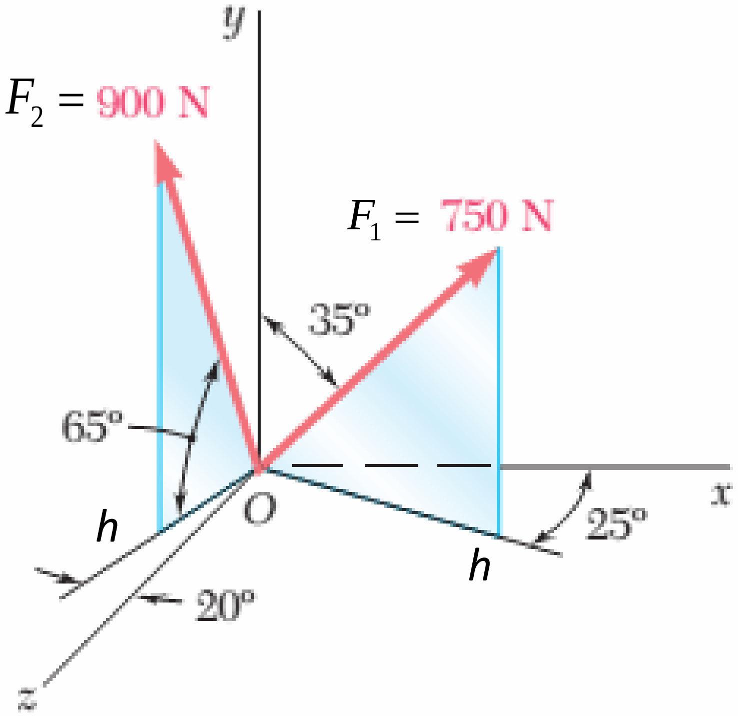
$$\theta_x = \cos^{-1} = \left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{-54.43}{139.96}\right)$$

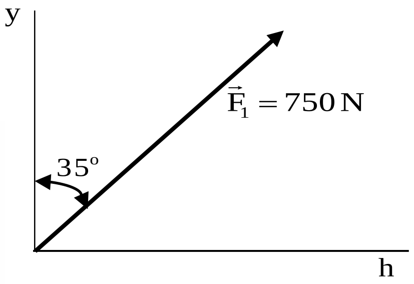
Example 2.3

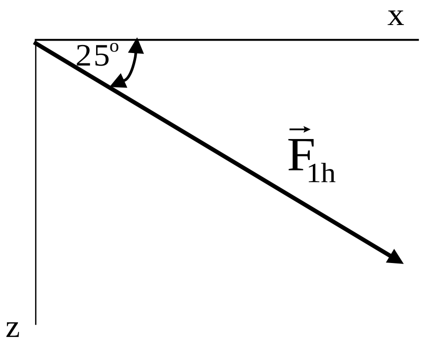
Determine the magnitude of the resultant force acting on the point *O* and it's direction measured from the *x* - axis.



Example 2.3-Solution







$$\vec{F}_{1y} = F_1\cos\theta = 750\cos 35^{\circ} = 614.36\ \text{N}$$

$$\vec{F}_{1h} = F_1\sin\theta = 750\sin 35^{\circ} = 430.18\ \text{N}$$

$$\vec{F}_{1x} = F_{1h}\cos\theta = 430.18\cos 25^{\circ} = 389.88\ \text{N}$$

$$\vec{F}_{1z} = F_{1h}\sin\theta = 430.18\sin 25^{\circ} = 181.8\ \text{N}$$

Applying same principle to the 900 N force,

$$\vec{F}_{2y} = F_2\sin\theta = 900\sin 65^{\circ} = 815.68\ \text{N}$$

$$\vec{F}_{2h} = F\cos\theta = 900\cos 65^{\circ} = 380.36\ \text{N}$$

$$\vec{F}_{2x} = F_{2h}\sin\theta = 380.36\sin 20^{\circ} = -130.09\ \text{N}$$

$$\vec{F}_{2z} = F_{2h}\cos\theta = 380.36\cos 20^{\circ} = 357.42\ \text{N}$$

$$\vec{F} = \sum F_x + \sum F_y + \sum F_z$$

$$= (389.88 - 130.09)_x + (614.36 + 815.68)_y + (181.8 + 357.42)_z$$

$$= 259.79_x + 1430.04_y + 539.22_z$$

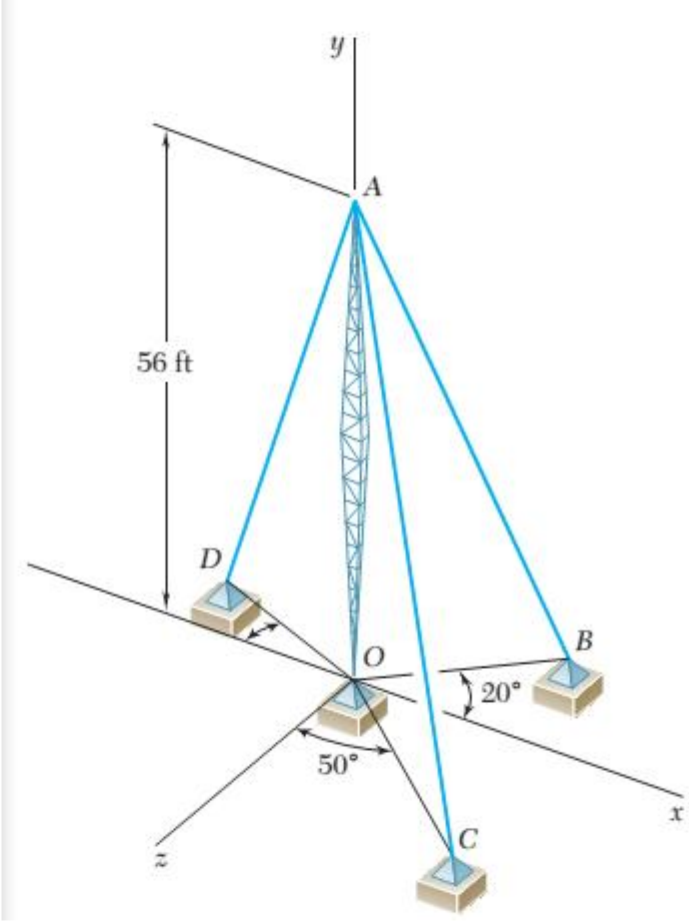
$$F = \sqrt{(259.79^2 + (1430.04)^2 + (539.22)^2} = 1550.25\ \text{N}$$

$$\theta_x = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{259.79}{1550.25}\right) = 80.4^{\circ}$$

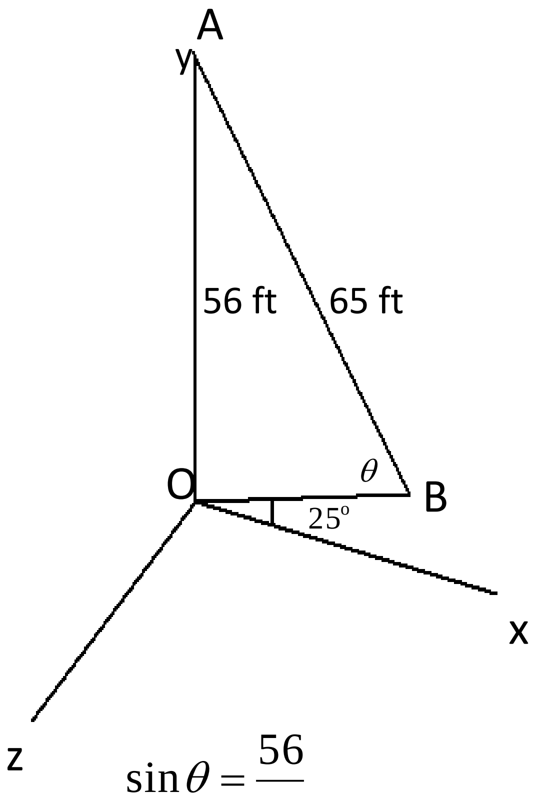
$$\theta_y = 22.7^{\circ} \qquad \theta_z = 69.7^{\circ}$$

Example 2.3

Cable AB is 65 ft long, and the tension in that cable is 3900 lb. Determine (a) the x, y, and z components of the force exerted by the cable on the anchor B.

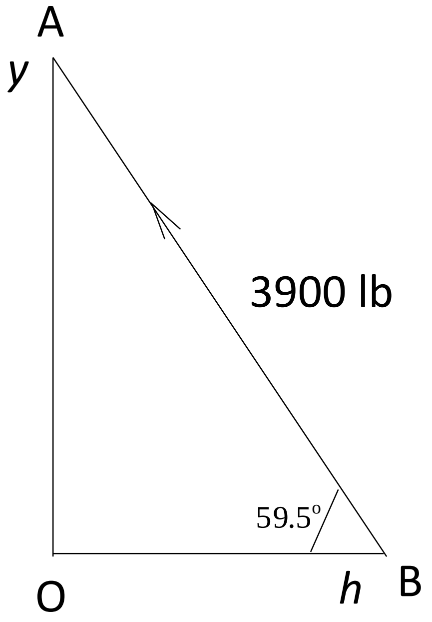


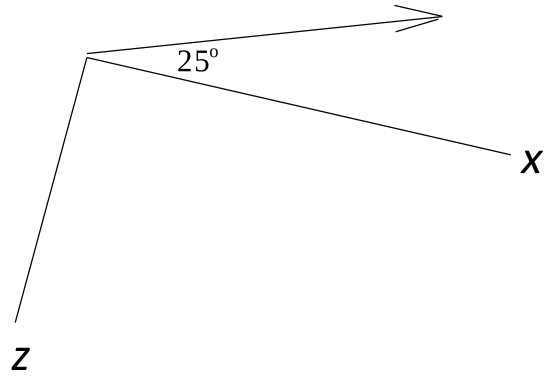
Example 2.3 – Solution



$$\sin\theta = \frac{56}{65}$$

$$\theta = 59.5^{o}$$





$$F_y = 3900\sin 59.5^{o}$$

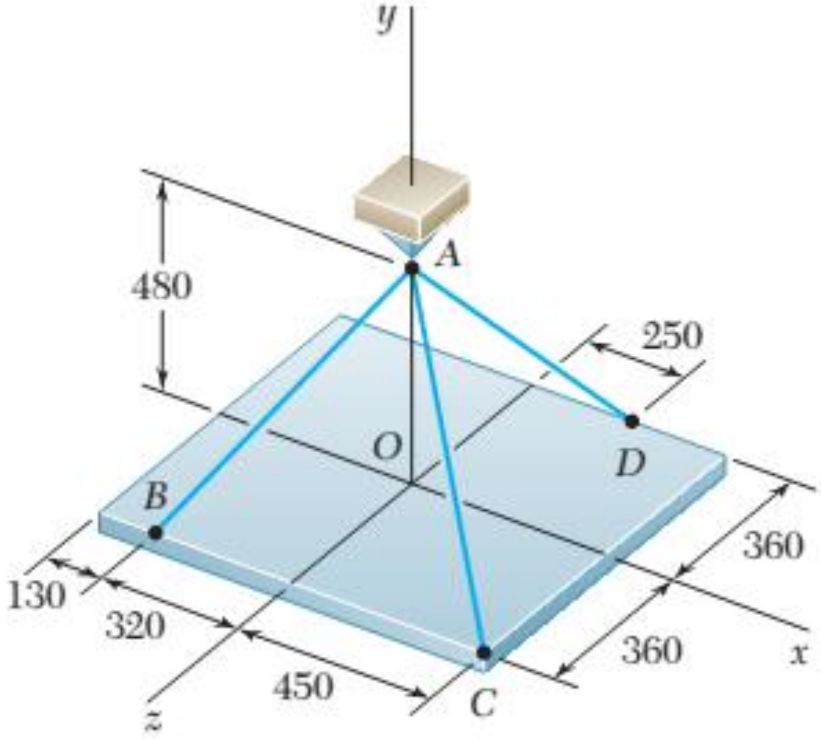
$$F_h = -3900\cos 59.5^{o}$$

$$F_x = \left(-3900\sin 59.5^{o}\right)\cos 25^{o}$$

$$F_z = -\left(-3900\sin 59.5^{o}\right)\sin 25^{o}$$

Example 2.4

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AC is 60 N, determine the components of the force being exerted at C.



Dimensions in mm

Ans :

$$F_c = -36i + 38.4j - 28.8k$$

LECTURE 3

EQUIVALENT FORCE SYSTEMS

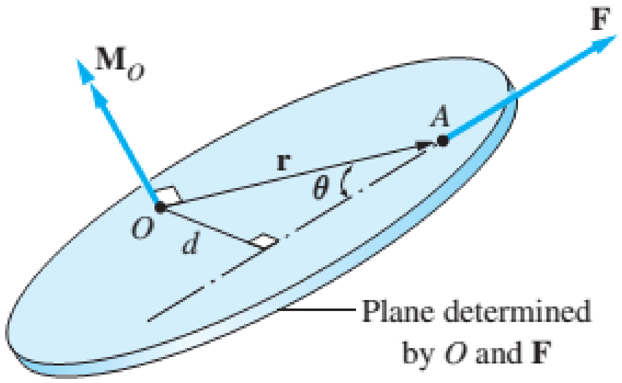
- Forces have the tendency to cause two types motions in rigid bodies; translational and rotational motions (also known as *Torque*).

- The tendency of a force to rotate a body is referred to as moment, given by the product of the force and the perpendicular distance between it's line of action and the point or axis that the body is rotating about.

- It follows that a moment may occur about a point or an axis.

EQUIVALENT FORCE SYSTEMS
Moment of a Force

- Moment about a point tends to rotate a body about that point, known as the moment centre.



$$\vec{M}_o = \vec{r} \times \vec{F}$$

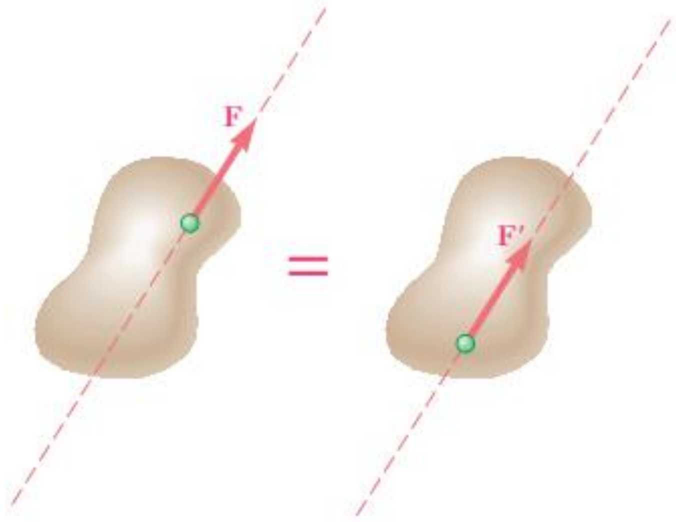
$$r\sin\theta = \mathrm{d}$$

$$M_o = Fr\sin\theta = Fd$$

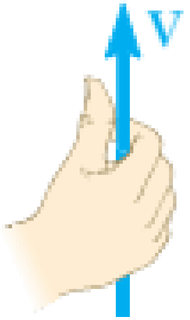
- *d* must be in a specific direction (determined by the line of action of the force). As such, it can be represented by a vector.

Moment of a force

- *d* is always perpendicular to the Force's line of action because the Force is treated as a sliding vector, due to the principle of transmissibility in rigid body mechanics.



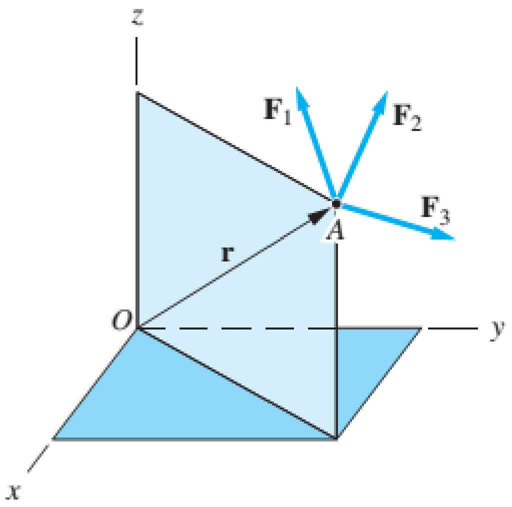
- The direction of the moment is determined by the right hand rule.



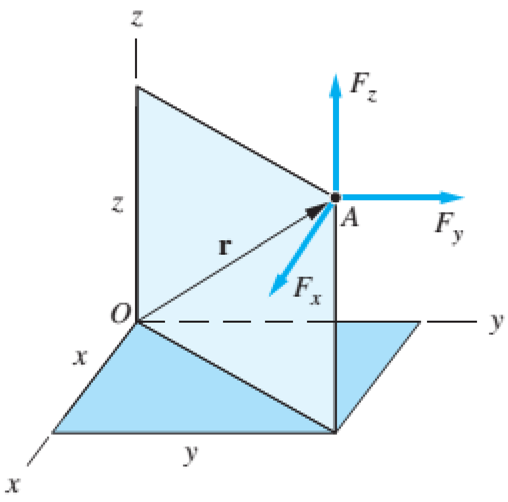
EQUIVALENT FORCE SYSTEMS

Principle of Moments (Varignon's Theorem)

- States that the moment about a given point *O* of the resultant of several concurrent forces is equal to the sum of the moments of the various forces about the same point *O*.
- In effect, moments of force components can be taken to get the components of a resultant moment.



$$
\begin{aligned}
M_O &= \sum (r \times F) \\
&= (r \times F_1) + (r \times F_2) + (r \times F_3) \\
&= r \times (F_1 + F_2 + F_3) \\
&= r \times R
\end{aligned}
$$



$$
\begin{aligned}
M_O &= \sum (r \times F) \\
&= (r \times F_x) + (r \times F_y) + (r \times F_z) \\
&= r \times (F_x + F_y + F_z) \\
&= r \times R
\end{aligned}
$$

EQUIVALENT FORCE SYSTEMS
Calculating the Moment of a force

- Scalar Approach

$$M_o = Fd$$

  - Only the magnitude of the moment is calculated using only the magnitudes of the force and the moment arm, *d*.
  - Used when the moment, *d* can easily be determined. The sense of the moment is determined by inspection.

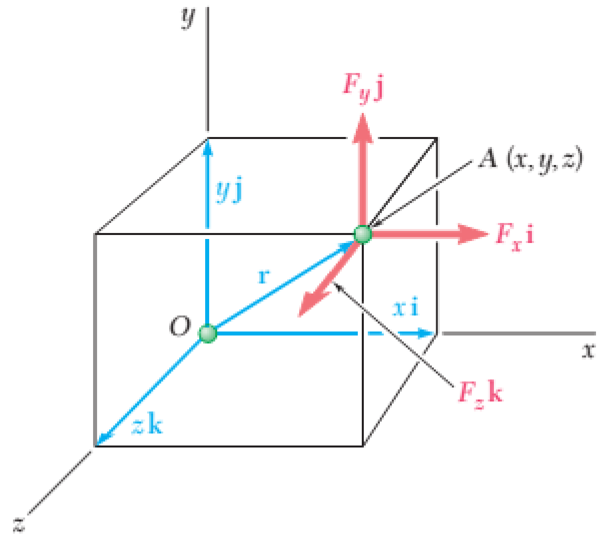
- Vector Approach
  - The force and the distance from the moment center to the line of action of the of the force are both expressed as vectors and their cross product determined.

- Multiplying Vectors
  - Vectors are expressed in components, arranged in a matrix form, and the determinant of the matrix taken.

If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$

Expressing as a matrix,

$$\vec{M}_O = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$



Taking the determinant of the matrix,

$$\vec{M}_O = (yF_z - zF_y)i - (xF_z - zF_x)j + (xF_y - yF_x)k$$

$$= \vec{M}_x i + \vec{M}_y j + \vec{M}_z k$$

- Multiplying Vectors
  - Alternatively, the vector components are multiplied.

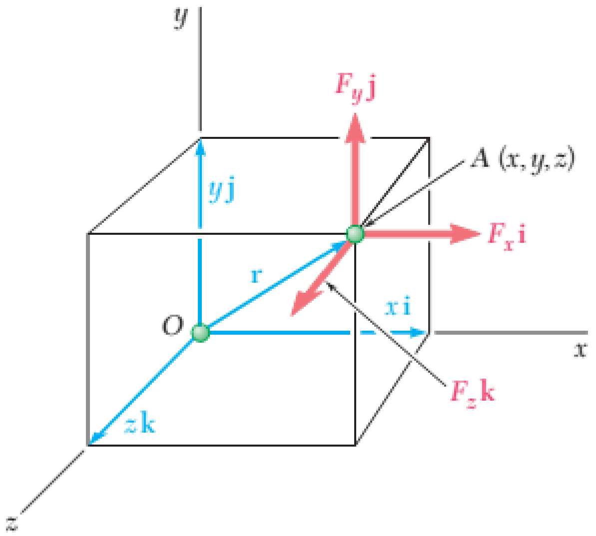
$$M_O = r \times F$$
$$= \left(x\vec{i} + y\vec{j} + z\vec{k}\right) \times \left(F_x\vec{i} + F_y\vec{j} + F_z\vec{k}\right)$$
$$= \left[\left(x\vec{i} + y\vec{j} + z\vec{k}\right) \times F_x\vec{i}\right] + \left[\left(x\vec{i} + y\vec{j} + z\vec{k}\right) \times F_y\vec{j}\right] + \left[\left(x\vec{i} + y\vec{j} + z\vec{k}\right) \times F_z\vec{k}\right]$$

But

$$i \times i = 0 \quad j \times i = -k \quad k \times i = j$$
$$i \times j = k \quad j \times j = 0 \quad k \times j = -i$$
$$i \times k = -j \quad j \times k = i \quad k \times k = 0$$

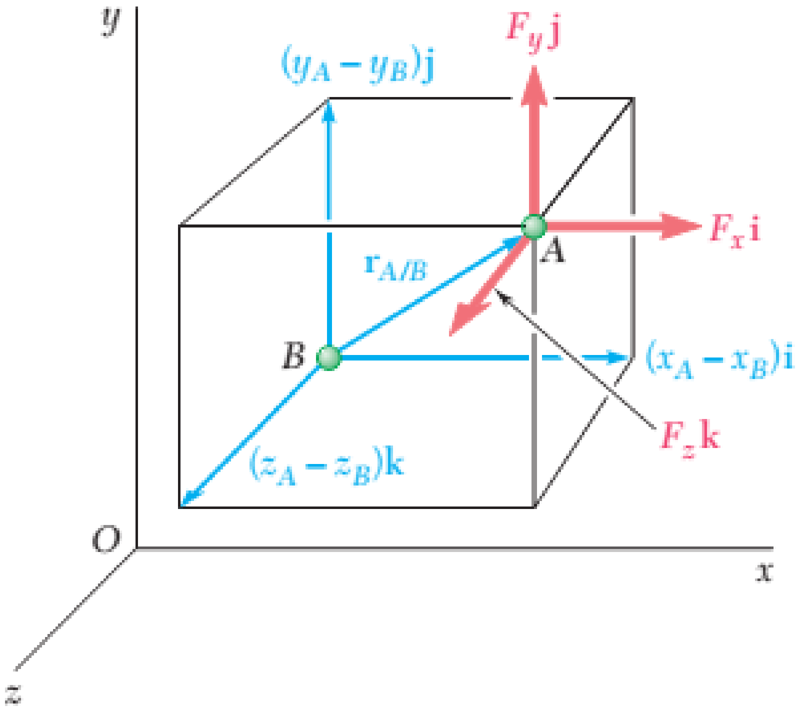
Therefore,

$$M_O = -yF_x k + zF_x j + xF_y k - zF_y i - xF_z j + yF_z i$$
$$= \left(yF_z - zF_y\right)i - \left(xF_z - zF_x\right)j + \left(xF_y - yF_x\right)k$$



EQUIVALENT FORCE SYSTEMS

Calculating the Moment of a force at a certain point about an arbitrary point



In this case,

$F = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$ and

$$\vec{r}_{A/B} = \vec{x}_{A/B} + \vec{y}_{A/B} + \vec{z}_{A/B}$$

$$= (x_A - x_B)\vec{i} + (y_A - y_B)\vec{j} + (z_A - z_B)\vec{k}$$

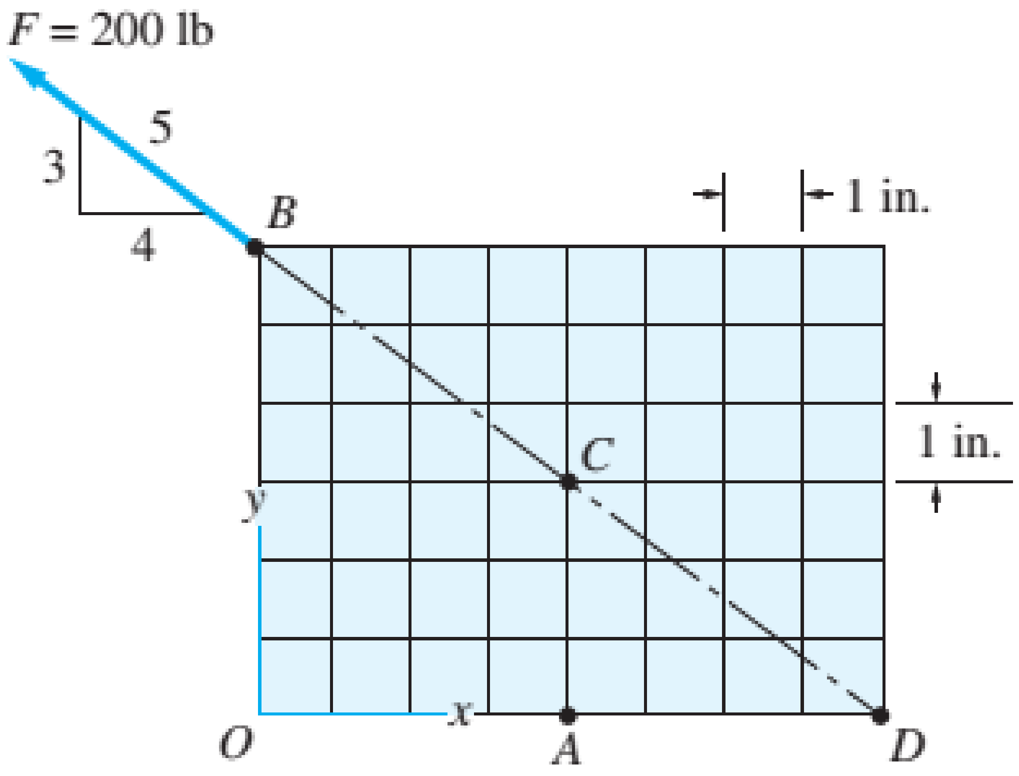
Expressed as a matrix,

$$M_O = r \times F = \begin{vmatrix} i & j & k \\ (x_A - x_B) & (y_A - y_B) & (z_A - z_B) \\ F_x & F_y & F_z \end{vmatrix}$$

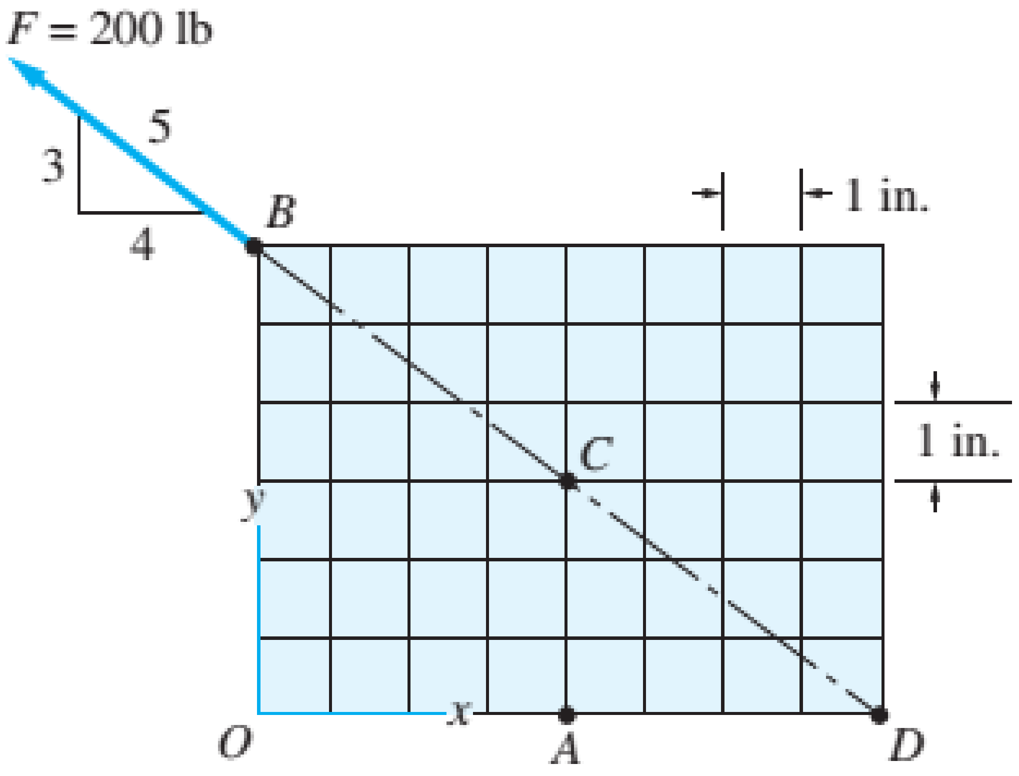
Take the determinant of the matrix to find components of the Moment

Example 3.1

Determine the moment of the force F about point A.



Example 3.1 – Solution (Vector Approach)



$$\vec{F} = -\left(\frac{4}{5}\right)200i + \left(\frac{3}{5}\right)200j$$
$$= -160i + 120j$$

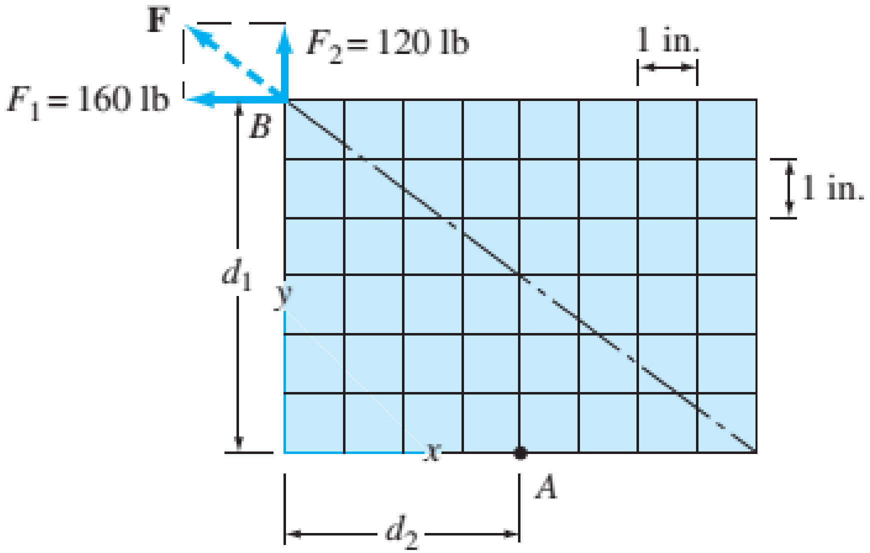
$$\vec{r} = \vec{r}_{AB} = -4i + 6j$$

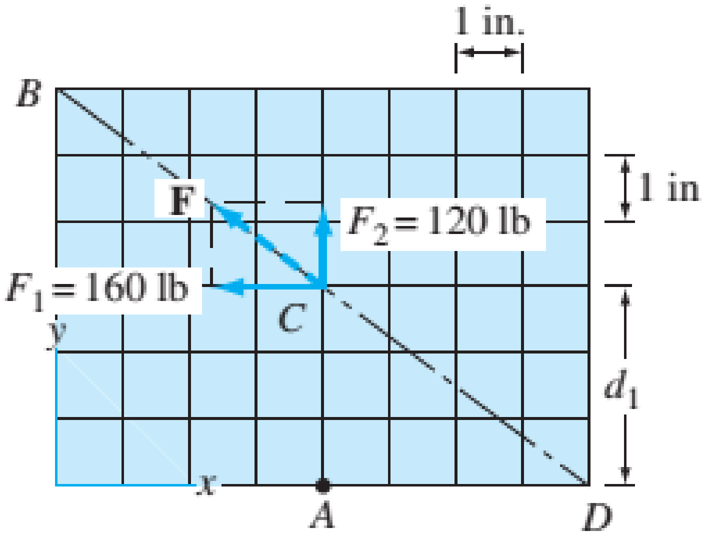
$$\vec{M}_A = \vec{r} \times \vec{F} = \vec{r}_{AB} \times \vec{F} = \begin{vmatrix} i & j & k \\ -4 & 6 & 0 \\ -160 & 120 & 0 \end{vmatrix}$$
$$= 480\text{k lb.in}$$

EQUIVALENT FORCE SYSTEMS

Calculating the Moment of a force

Example 3.1 – Solution (Scalar Approach)





OR

The vector is treated as a sliding vector and moved to point C

$$M_A = F_1 d_1 - F_2 d_2$$

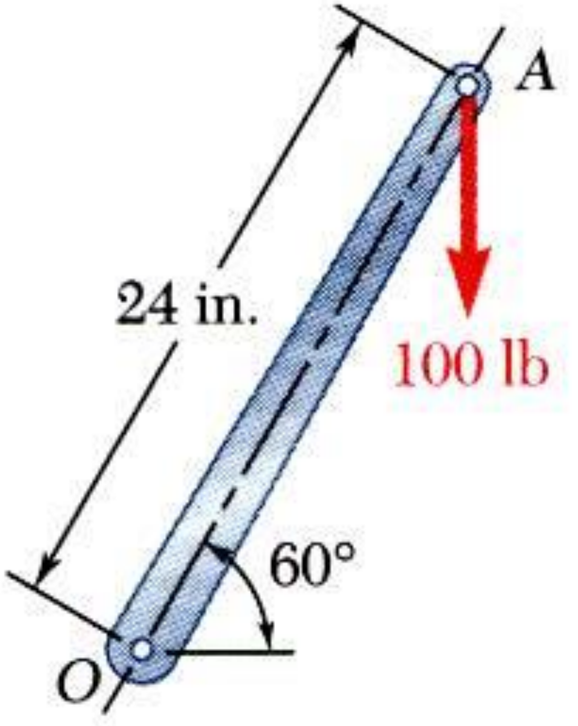
$$= 160(6) - 120(4) = 480 \text{ lb} \cdot \text{in.}$$

$$\mathbf{M}_A = 480\mathbf{k} \text{ lb} \cdot \text{in.}$$

$$M_A = F_1 d_1 = 160(3) = 480 \text{ lb} \cdot \text{in.}$$

Example 3.2

A 100-lb vertical force is applied to the end of a lever which is attached to a shaft at *O*.



Determine:

a) moment about *O*,

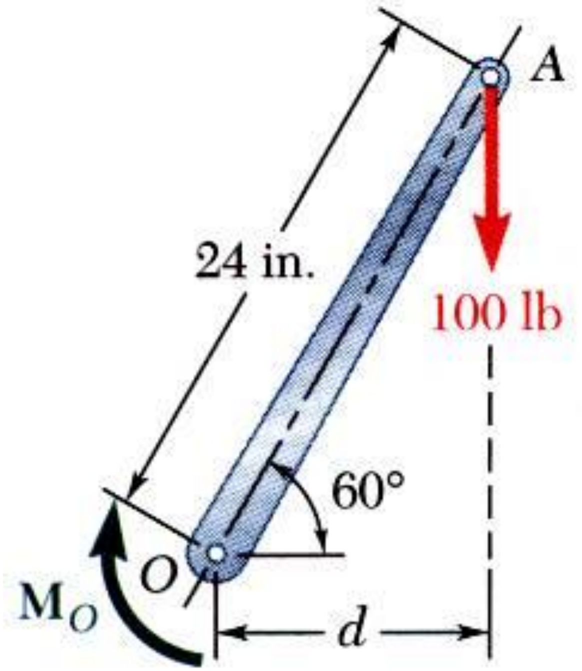
b) horizontal force at *A* which creates the same moment,

c) smallest force at A which produces the same moment,

d) location for a 240-lb vertical force to produce the same moment,

Example 3.2 - Solution

FBD



Moment about *O* is equal to the product of the force and the perpendicular distance between the line of action of the force and *O*.

$$M_O = Fd$$

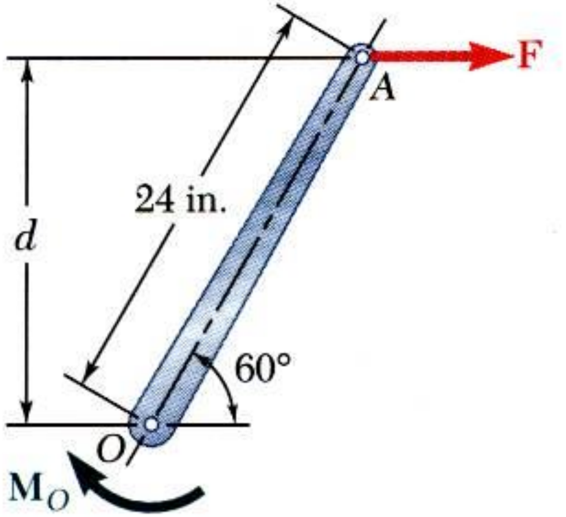
$$d = (24\,\text{in.})\cos 60° = 12\,\text{in.}$$

$$M_O = (100\,\text{lb})(12\,\text{in.}) =$$

Since the force tends to rotate the lever clockwise, the moment vector is into the plane of the paper.

Example 3.2 - Solution

FBD



Horizontal force at *A* that produces the same moment,

$$d = (24\,\text{in.})\sin 60° = 20.8\,\text{in.}$$

$$M_O = Fd$$

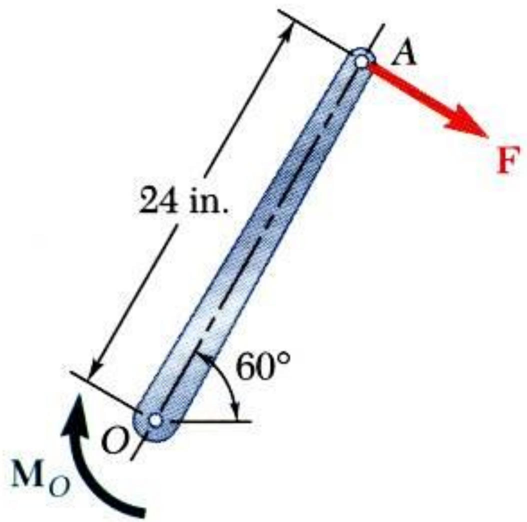
$$1200\,\text{lb}\cdot\text{in.} = F(20.8\,\text{in.})$$

$$F = \frac{1200\,\text{lb}\cdot\text{in.}}{20.8\,\text{in.}}$$

$$=$$

Example 3.2 - Solution

FBD



The smallest force A to produce the same moment occurs when the perpendicular distance is a maximum or when F is perpendicular to OA.

$$M_O = Fd$$

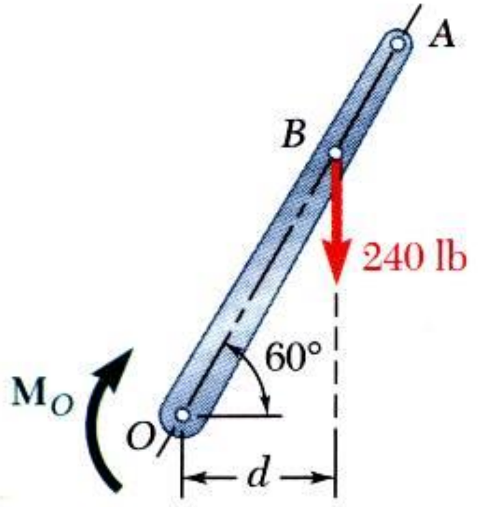
$$1200\,\text{lb}\cdot\text{in.} = F(24\,\text{in.})$$

$$F = \frac{1200\,\text{lb}\cdot\text{in.}}{24\,\text{in.}}$$

$$=$$

Example 3.2 - Solution

FBD



The point of application of a 240 lb force to produce the same moment,

$$M_O = Fd$$

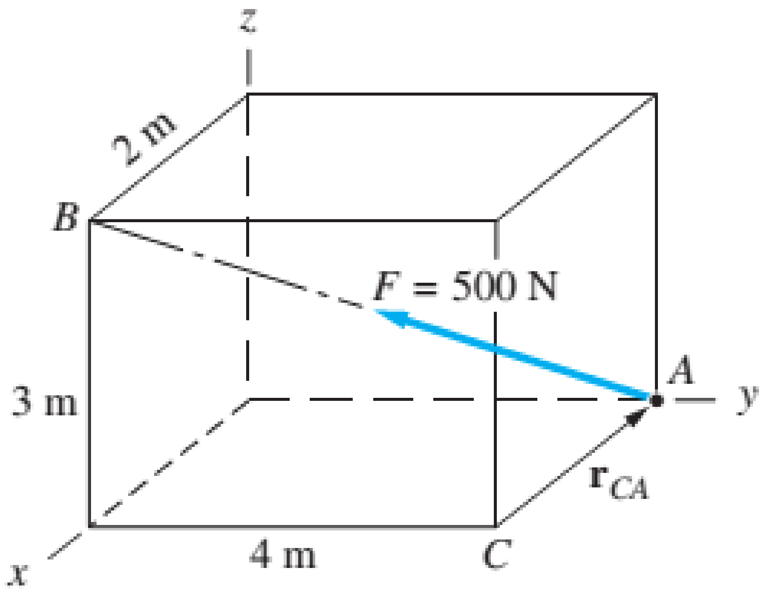
$$1200\,\text{lb} \cdot \text{in.} = (240\,\text{lb})d$$

$$d = \frac{1200\,\text{lb} \cdot \text{in.}}{240\,\text{lb}} = 5\,\text{in.}$$

$$OB = 5\cos^{-1}60° =$$

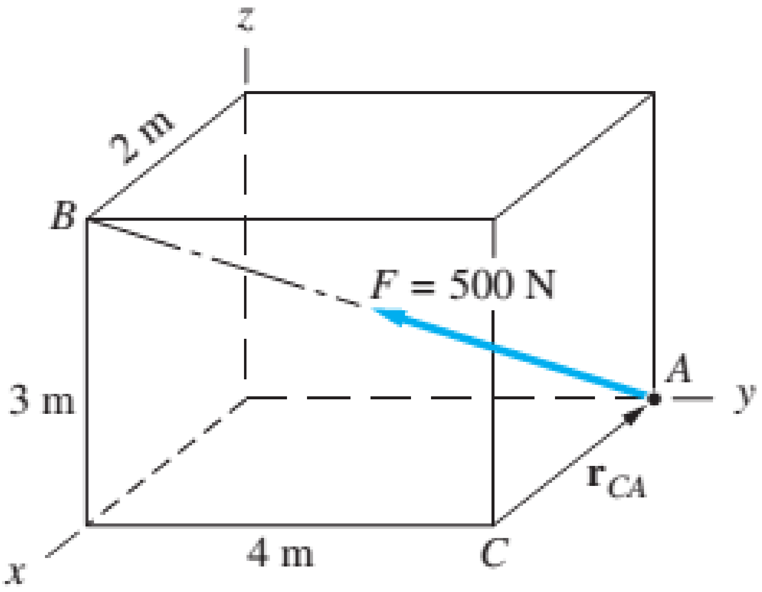
Example 3.3

Determine the moment of the force *F*, about point *C* the perpendicular distance between C and the line of action of F.



Calculating the Moment of a force

Example 3.3 - Solution



$$\vec{M}_A = \vec{r}_{A/C} \times \vec{F}$$

$$\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C = (0\text{ m} - 2\text{ m})\vec{i} + (4\text{ m} - 4\text{ m})\vec{j} + (0\text{ m} - 0\text{ m})\vec{k}$$

$$= -2\text{ m}i$$

$$\vec{F} = F\vec{\lambda} = (500\text{ N})\left(\frac{2i - 4j + 3k}{\sqrt{2^2 + (-4)^2 + 3^2}}\right)$$

$$= (185.53\text{ N})\vec{i} - (371.06\text{ N})\vec{j} + (278.29\text{ N})\vec{k}$$

$$\vec{M}_C = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 0 & 0 \\ 185.53 & -371.06 & 278.29 \end{vmatrix}$$

$$= 556.58j + 742.12k$$

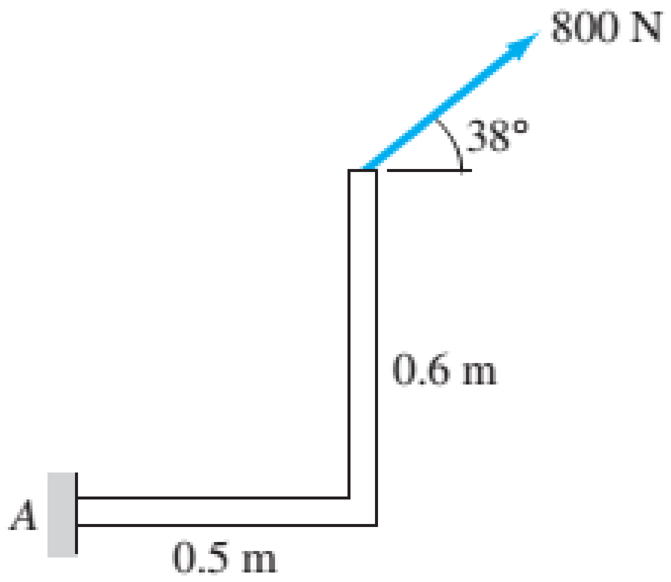
$$M_C = \sqrt{(556.58)^2 + (742.12)^2}$$

$$= 927.64\text{ Nm}$$

$$\text{The perpendicular distance, } d = \frac{M}{F} = \frac{927.64\text{ Nm}}{500\text{ N}} = 1.86\text{ m}$$

Example 3.4

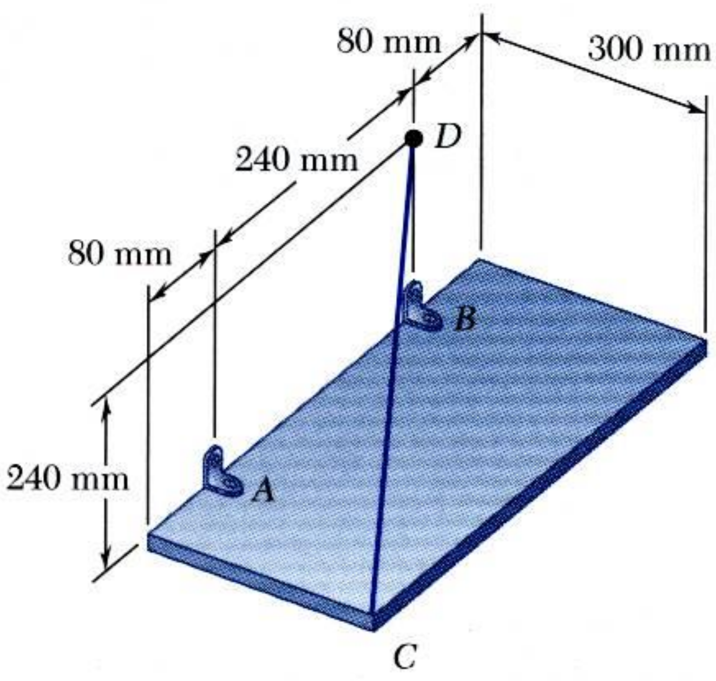
Determine the moment of the 800 N force about point A.



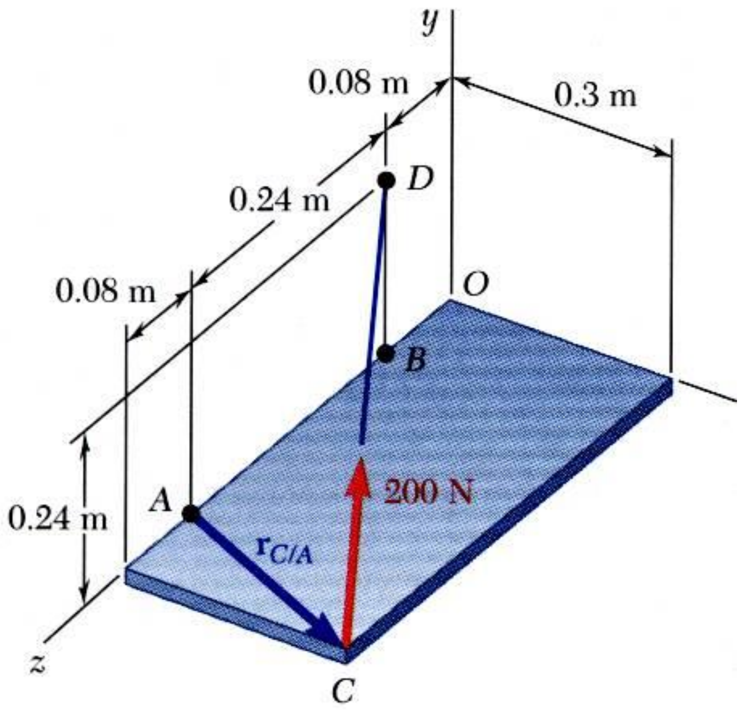
Ans: $\circlearrowleft +$ $M_A = -131.99\,\text{Nm}$

Example 3.5

The rectangular plate is supported by the brackets at *A* and *B* and by a wire *CD*. Knowing that the tension in the wire is 200 N, determine the moment about *A* of the force exerted by the wire at *C*.



Example 3.5 - Solution



$$\vec{M}_A = \vec{r}_{C/A} \times \vec{F}$$

$$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = (0.3\,\mathrm{m})\vec{i} + (0.08\,\mathrm{m})\vec{j}$$

$$\vec{F} = F\vec{\lambda} = (200\,\mathrm{N})\frac{-(0.3\,\mathrm{m})\vec{i} + (0.24\,\mathrm{m})\vec{j} - (0.32\,\mathrm{m})\vec{k}}{0.5\,\mathrm{m}}$$

$$= -(120\,\mathrm{N})\vec{i} + (96\,\mathrm{N})\vec{j} - (128\,\mathrm{N})\vec{k}$$

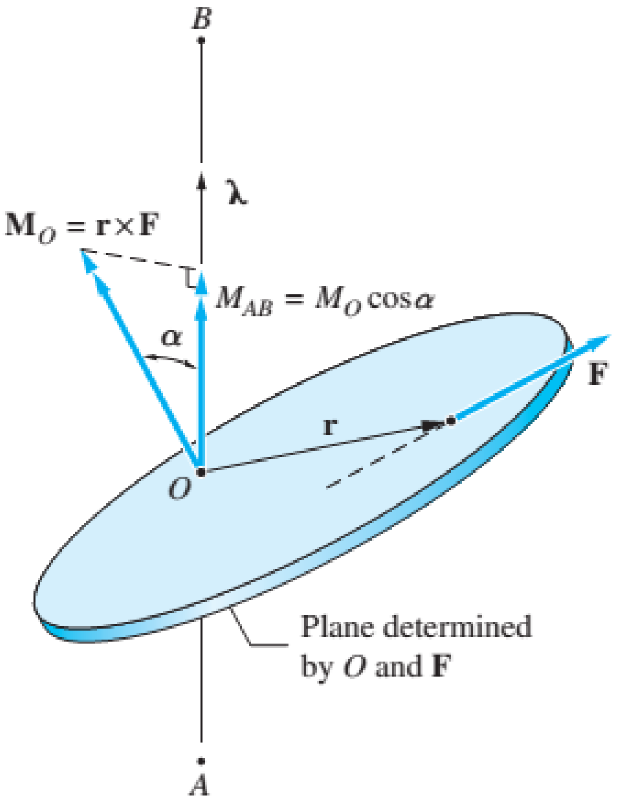
$$\vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.3 & 0 & 0.08 \\ -120 & 96 & -128 \end{vmatrix}$$

$$\vec{M}_A = -(7.68\,\mathrm{N}\cdot\mathrm{m})\vec{i} + (28.8\,\mathrm{N}\cdot\mathrm{m})\vec{j} + (28.8\,\mathrm{N}\cdot\mathrm{m})\vec{k}$$

Moment of a force about an axis

- Calculated by finding the scalar /dot product of a directional vector along the axis of interest, and the moment about a point on the axis.

$$M_{AB} = M_O \cos\alpha$$
$$= M_O \cdot \lambda$$
$$M_O = M_x + M_y + M_z$$



- Scalar product of two vectors

$$M_{AB} = \vec{\lambda} \cdot \vec{M}_O$$
$$= \left(\vec{\lambda}_x i + \vec{\lambda}_y j + \vec{\lambda}_z k\right) \times \left(F_x \vec{i} + F_y \vec{j} + F_z \vec{k}\right)$$
$$= \left[\left(\vec{\lambda}_x i + \vec{\lambda}_y j + \vec{\lambda}_z k\right) \times F_x \vec{i}\right] + \left[\left(\vec{\lambda}_x i + \vec{\lambda}_y j + \vec{\lambda}_z k\right) \times F_y \vec{j}\right] + \left[\left(\vec{\lambda}_x i + \vec{\lambda}_y j + \vec{\lambda}_z k\right) \times F_z \vec{k}\right]$$

But

$$i \times i = 1 \quad j \times i = 0 \quad k \times i = 0$$
$$i \times j = 0 \quad j \times j = 1 \quad k \times j = 0$$
$$i \times k = 0 \quad j \times k = 0 \quad k \times k = 1$$

Therefore,

$$M_{AB} = \lambda_x F_X + \lambda_y F_y + \lambda_z F_z$$

➢ Mixed triple product of three vectors

$$M_{AB} = \lambda \cdot M_O$$

$$M_{AB} = \vec{\lambda} \cdot \left(\vec{r} \times \vec{F}\right)$$

$$= \left(\vec{\lambda}_x i + \vec{\lambda}_y j + \vec{\lambda}_z k\right) \cdot \left[\left(\vec{x} i + \vec{y} j + \vec{z} k\right) \times \left(F_x \vec{i} + F_y \vec{j} + F_z \vec{k}\right)\right]$$

Expressing as a matrix,

$$M_{AB} = \lambda \cdot \left(r \times F\right) = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

Taking the determinant of the matrix,

$$M_O = \left(yF_z - zF_y\right)\lambda_x - \left(xF_z - zF_x\right)\lambda_y + \left(xF_y - yF_x\right)\lambda_z$$

➢ Mixed triple product of three vectors

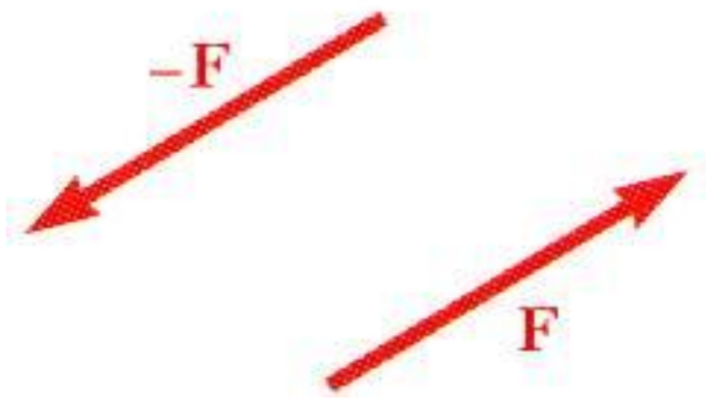
$$M_{BL} = \vec{\lambda} \bullet \vec{M}_B$$

$$= \vec{\lambda} \bullet \left(\vec{r}_{A/B} \times \vec{F} \right)$$

$$\vec{r}_{A/B} = \vec{r}_A - \vec{r}_B$$

EQUIVALENT FORCE SYSTEMS
Couples

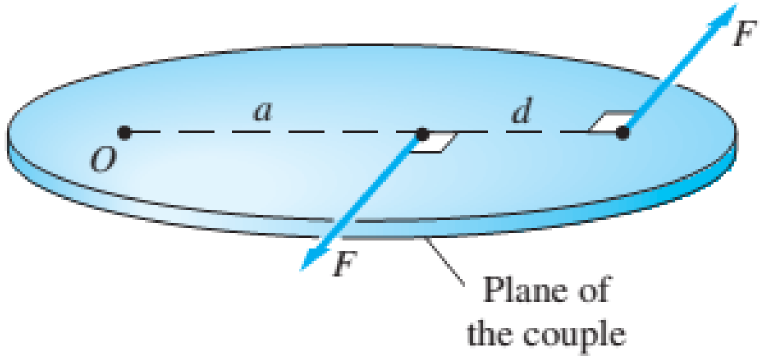
- This refers to two parallel, noncollinear forces that are equal in magnitude and opposite in direction.



- It is a free vector that can be applied anywhere

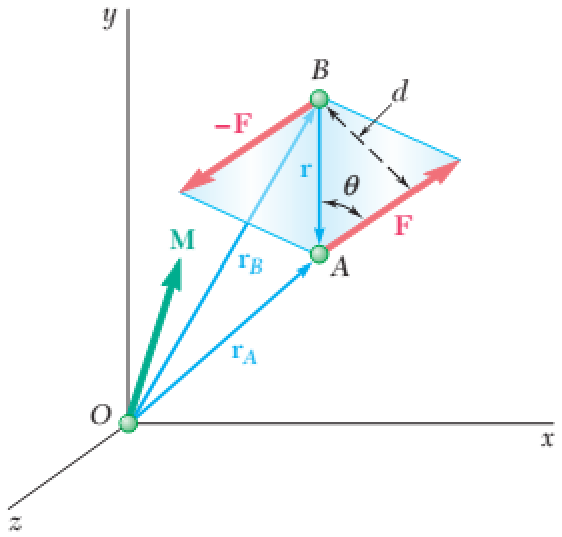
EQUIVALENT FORCE SYSTEMS
Calculating the Moment of a Couple

- Scalar approach



$$\circlearrowleft + \quad M_O = F(a+d) - F(a) = Fd$$

- Vector Approach



$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times \left(-\vec{F}\right)$$
$$= \left(\vec{r}_A - \vec{r}_B\right) \times \vec{F}$$
$$= \vec{r} \times \vec{F}$$
$$M = rF\sin\theta = Fd$$

EQUIVALENT FORCE SYSTEMS
Some Properties of Couples

- Two couples are considered equivalent if
  - their moments is of the same magnitude
  - They lie in the same plane
  - Tend to cause rotation in the same direction

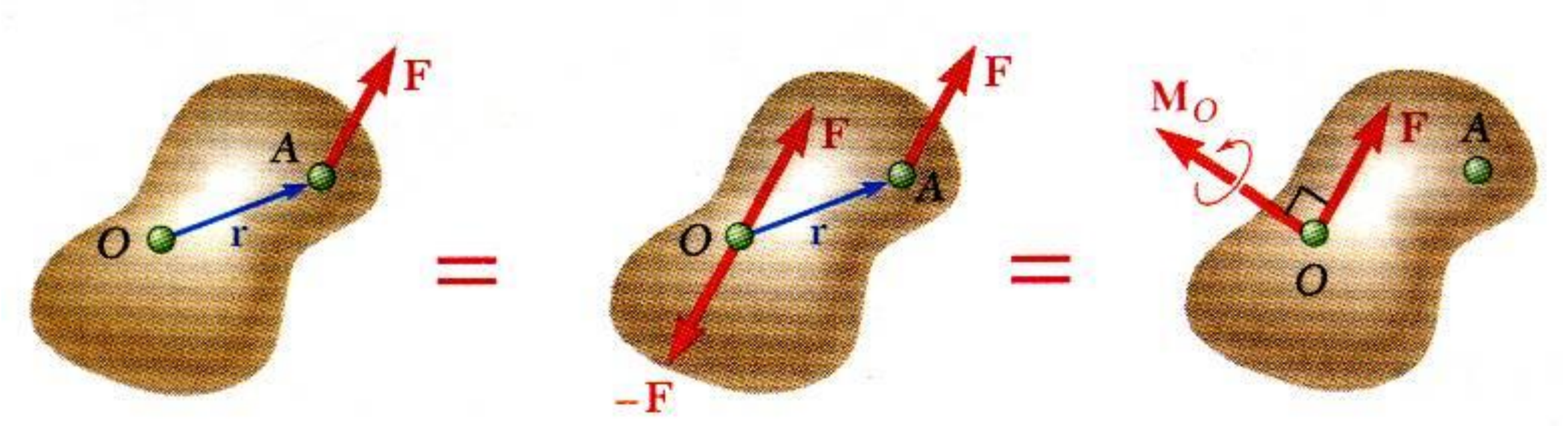
- Couples are vectors

- They obey Varignon’s Theorem

$$\begin{aligned}\vec{M} &= \vec{r}\times\vec{F}_1 + \vec{r}\times\vec{F}_2 \\ &= \vec{M}_1 + \vec{M}_2\end{aligned}$$

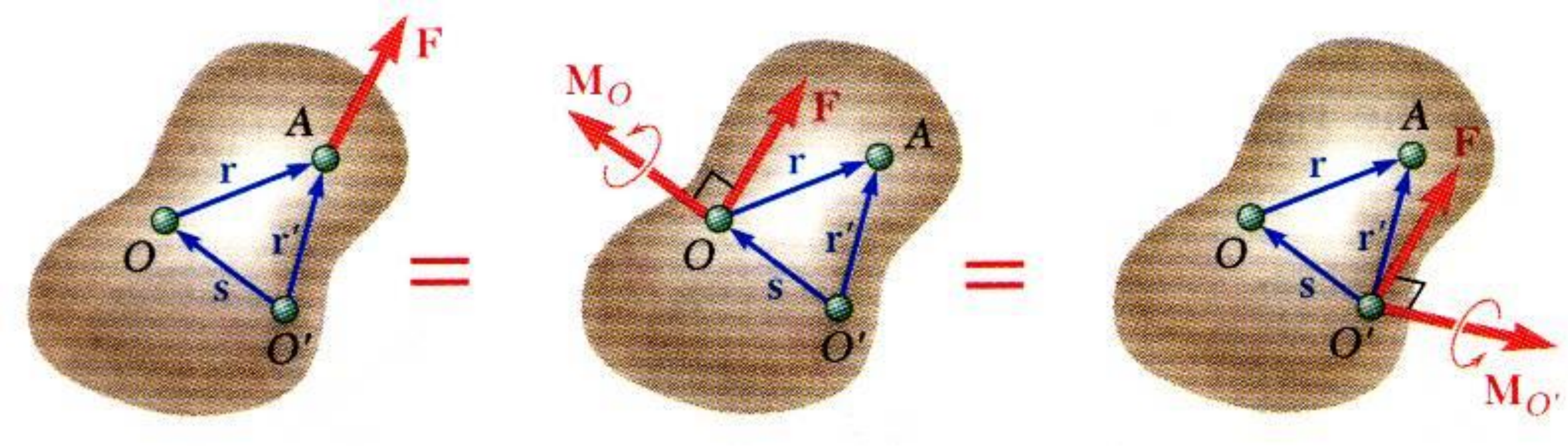
EQUIVALENT FORCE SYSTEMS
Shifting the line of Action of a Force

- This can be done by replacing the force with a force-couple system that acts at the desired point.
- The couple is given by the product of the force and the perpendicular distance between the old and new line of actions.



EQUIVALENT FORCE SYSTEMS
Shifting the line of Action of a Force

- A force-couple system can also be moved to a new position.

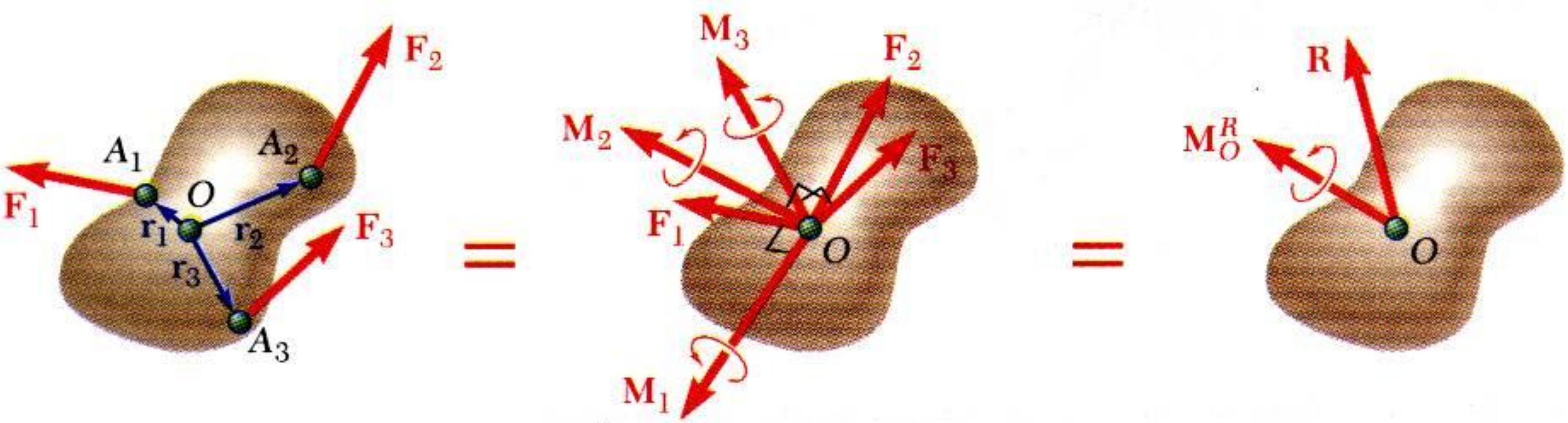


$$\vec{M}_{O'} = \vec{r}'\times\vec{F} = (\vec{r}+\vec{s})\times\vec{F} = \vec{r}\times\vec{F}+\vec{s}\times\vec{F}$$
$$= \vec{M}_{O}+\vec{s}\times\vec{F}$$

EQUIVALENT FORCE SYSTEMS
Reduction of Several forces into a force couple-system

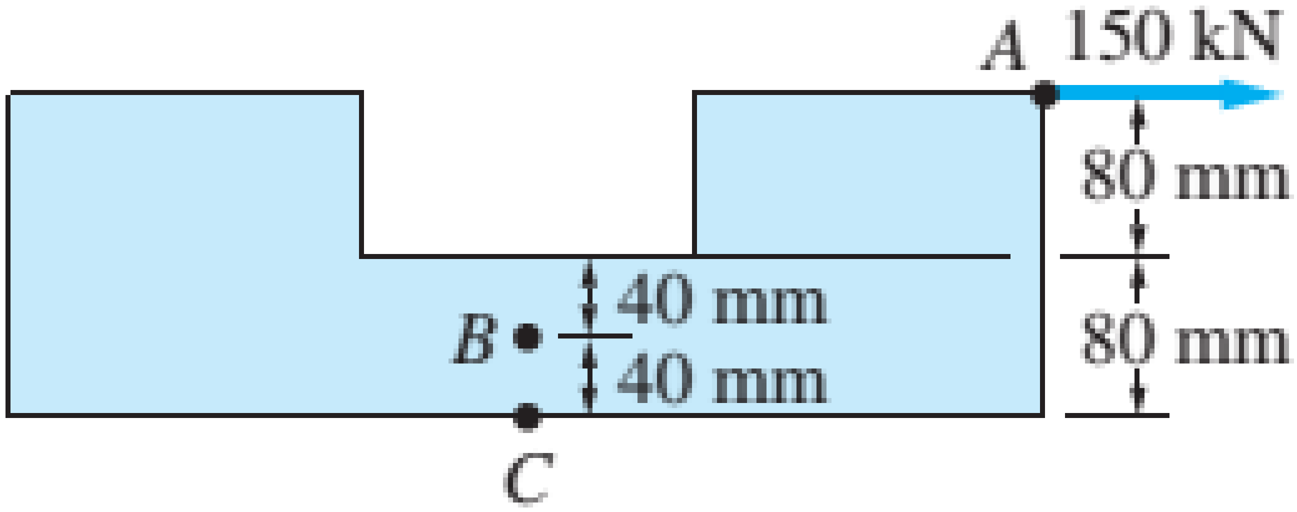
- Where a system of forces act on a body, they can be reduced to a several force-couple systems acting at a desired point.
- The force couple systems can be combined into a resultant force-couple.

$$\vec{R} = \sum \vec{F} \qquad \vec{M}_O^R = \sum \left(\vec{r} \times \vec{F} \right)$$



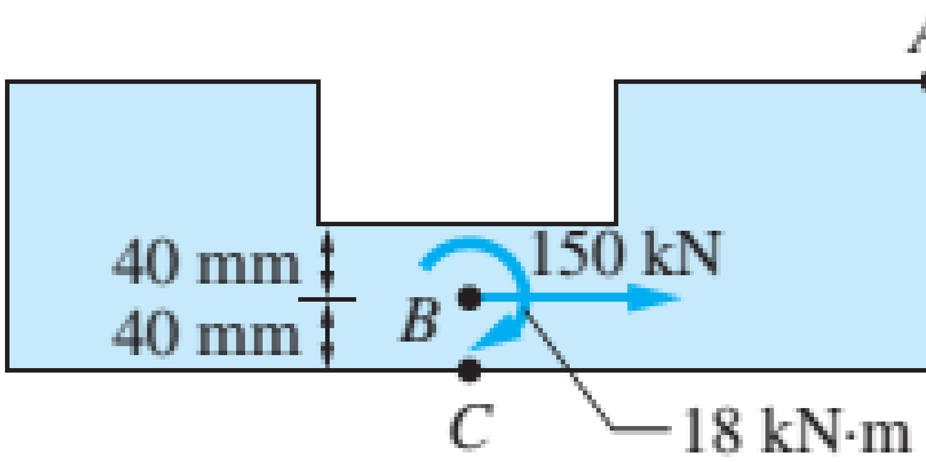
Example 3.6

For the machine part shown in the Figure below, replace the applied load of 150 kN acting at point A by (1) an equivalent force-couple system with the force acting at point B.



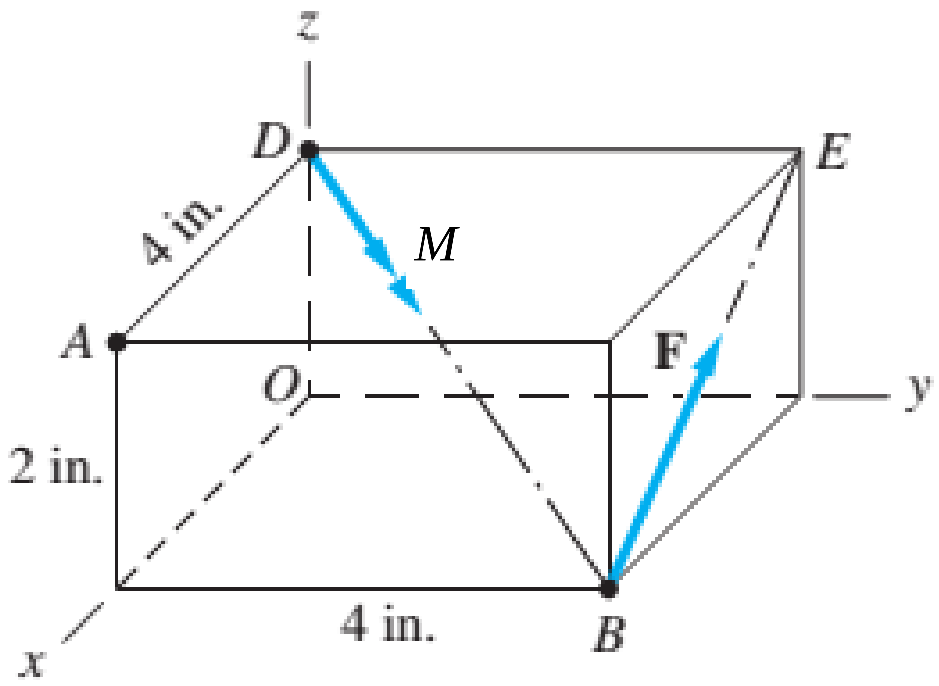
Example 3.6 - Solution

$$\circlearrowleft + \quad M_B = -150(0.08\,\text{m} + 0.04\,\text{m}) = \text{-}18\,\text{kN}$$

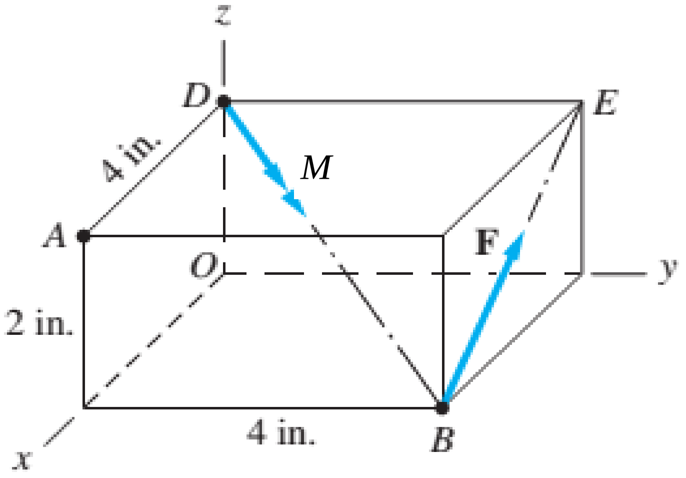


Example 3.7

Replace the force-couple system shown in Fig. (a) with an equivalent force-couple system, with the force acting at point A, given that F=100 lb and M =120 lb·in.



Example 3.7 - Solution



$$\vec{F} = 100\lambda_{BE} = 100\left(\frac{-4i+2k}{\sqrt{\left((-4)^2+2^2\right)}}\right) = -89.44i + 44.72k$$

$$\vec{r}_{AB} = 4j - 2k$$

$$\vec{M}_A = \vec{r}_{AB} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 4 & -2 \\ -89.44 & 0 & 44.72 \end{vmatrix}$$

$$= (178.9i + 178.9j + 357.8k)\,\text{lb.in}$$

$$\vec{M} = 120\lambda_{DB} = 120\left(\frac{4i+4j-2k}{\sqrt{\left(4^2+4^2+(-2)^2\right)}}\right)$$

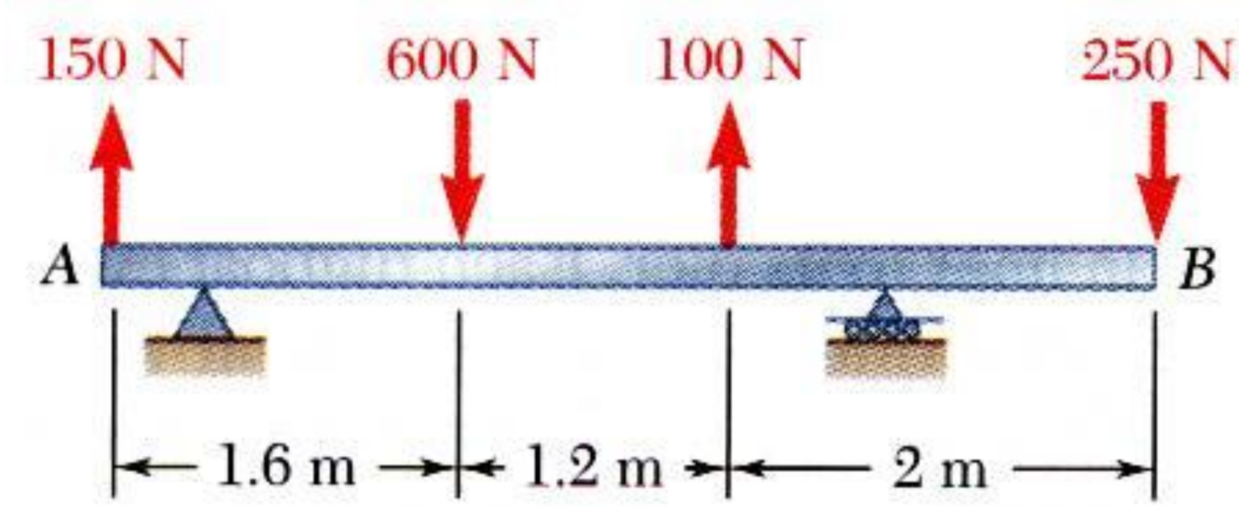
$$= (80i + 80j - 40k)\,\text{lb.in}$$

$$\vec{\mathrm{M}}_{\mathrm{RA}} = \vec{M}_A + \vec{M} = (258.9i + 258.9j + 317.8k)\,\text{lb.in}$$

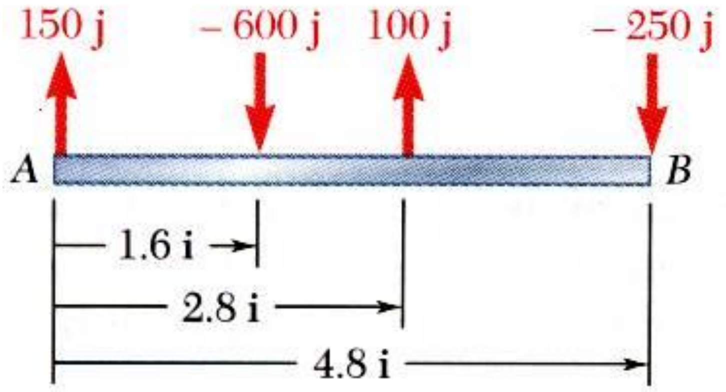
$$\mathrm{M}_{\mathrm{RA}} = 484.8\,\text{lb.in}$$

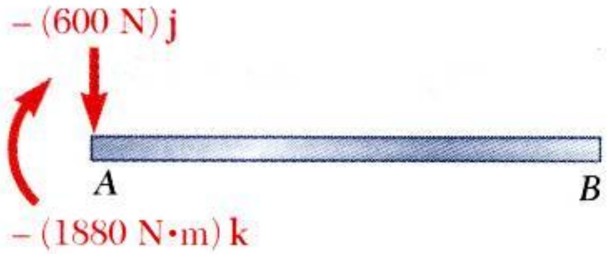
Example 3.8

For the beam, reduce the system of forces shown to (a) an equivalent force-couple system at A, (b) an equivalent force couple system at B. (Ignore the support reactions)



Example 3.8 - Solution





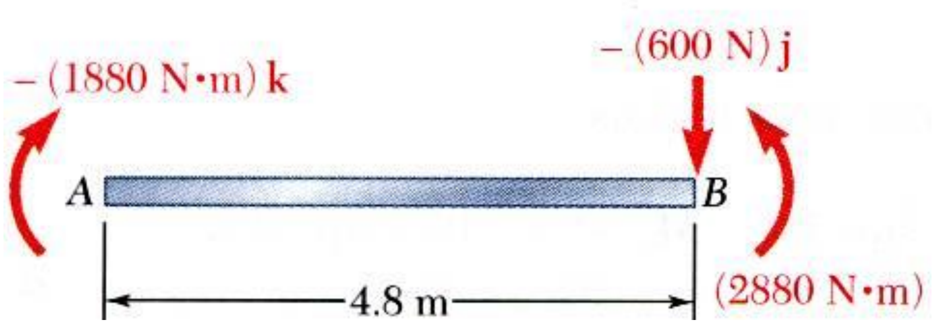
The Resultant force will be

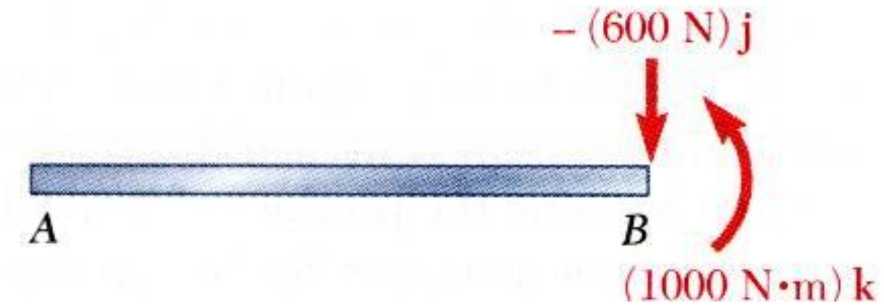
$$\vec{R} = \sum \vec{F}$$
$$= (150\,\text{N})\vec{j} - (600\,\text{N})\vec{j} + (100\,\text{N})\vec{j} - (250\,\text{N})\vec{j}$$
$$= (-600\,\text{N})j$$

The Resultant Moment

$$\vec{M}_{RA} = \sum \left(\vec{r} \times \vec{F}\right)$$
$$= \left(1.6\vec{i}\right) \times \left(-600\vec{j}\right) + \left(2.8\vec{i}\right) \times \left(100\vec{j}\right) + \left(4.8\vec{i}\right) \times \left(-250\vec{j}\right)$$
$$= (-1880\,\text{N.m})\text{k}$$

Example 3.8 - Solution



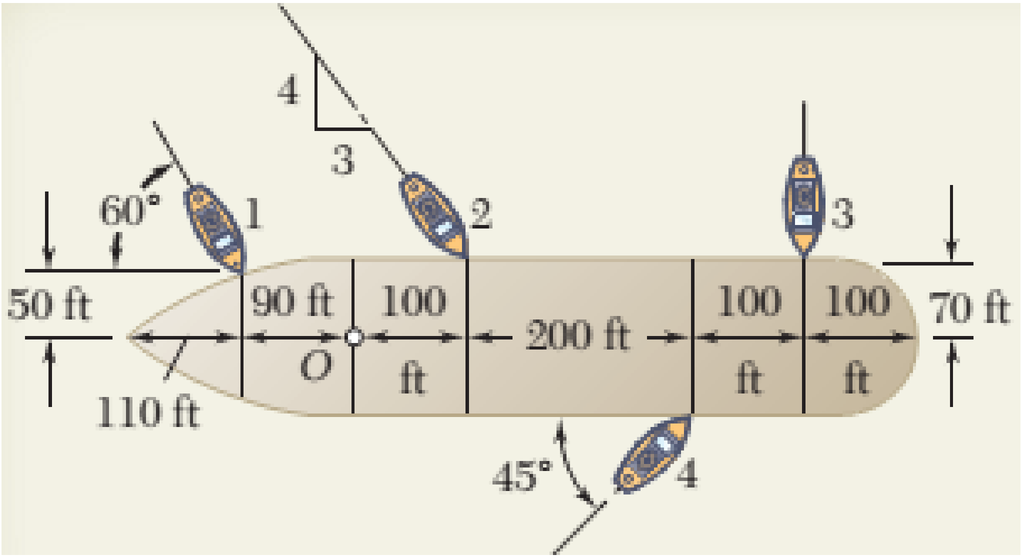


Transferring the force-couple system at A,

$$\vec{M}_{RB} = \vec{M}_{RA} + \vec{r}_{B/A} \times \vec{R}$$
$$= -(1880\,\mathrm{N \cdot m})\vec{k} + (-4.8\,\mathrm{m})\vec{i} \times (-600\,\mathrm{N})\vec{j}$$
$$= -(1880\,\mathrm{N \cdot m})\vec{k} + (2880\,\mathrm{N \cdot m})\vec{k}$$
$$=$$

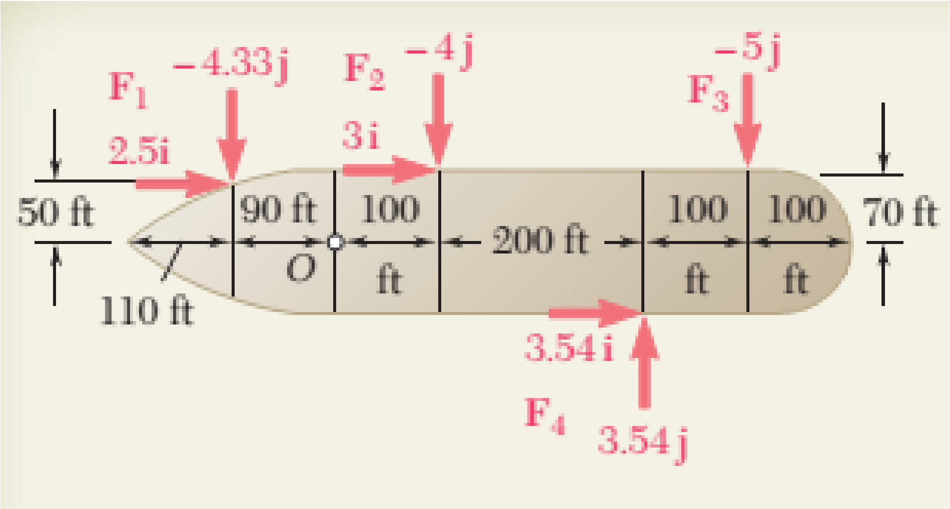
Example 3.9

Four tugboats are used to bring an ocean liner to its pier. Each tugboat exerts a 5000-lb force in the direction shown. Determine the equivalent force-couple system at the foremast O. Also determine the angle the resultant force makes with the horizontal as well as the direction of rotation of the moment.



Example 3.9 - Solution

Resolving the forces being exerted by the tugboats into components,



$$\vec{R} = \sum \vec{F} = \left(9.04\vec{i} - 9.79\vec{j}\right)\text{lb}$$

$$\text{R} = 13.33\,\text{lb},\ \theta_{\text{i}} = 47.3^{\text{O}}$$

$$\vec{M}_{RO} = \sum (\vec{r} \times \vec{F}) = \left(-1035\text{k}\right)\text{lb.in}$$

$$M_{RO} = 1035\text{lb.in}, \quad \text{Clockwise}$$

LECTURE 4

EQUILIBRIUM OF PARTICLES AND RIGID BODIES

EQUILIBRIUM OF PARTICLES & RIGID BODIES

- A particle or body is said to be in equilibrium if the resultant force and moment acting on it is zero.

- This is necessary condition for Newton's first law.

- The equilibrant of a system of forces acting on a particle can easily be obtained applying the already discussed techniques for finding the resultant of forces on a particle.

- Problems on equilibrium often require equations of equilibrium to obtained and solved.

EQUILIBRIUM OF PARTICLES & RIGID BODIES

- Solving problems on equilibrium involve three main steps;
  - Draw a free body diagram for the problem
  - Obtain equations the of equilibrium for the problem
  - Solve the equations and interpret your results

Static Equilibrium of a Particle

- For particles,

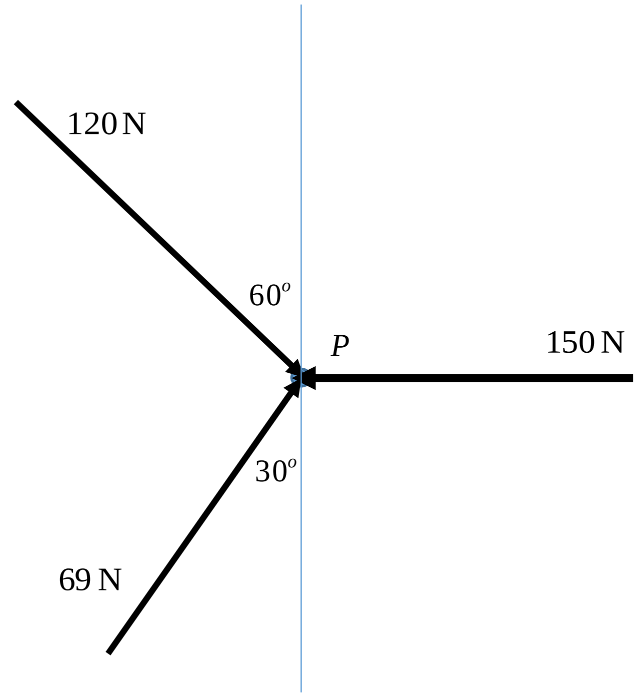
$$\vec{R} = \sum F = 0$$

$$\Rightarrow \sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

Static Equilibrium of a Particle

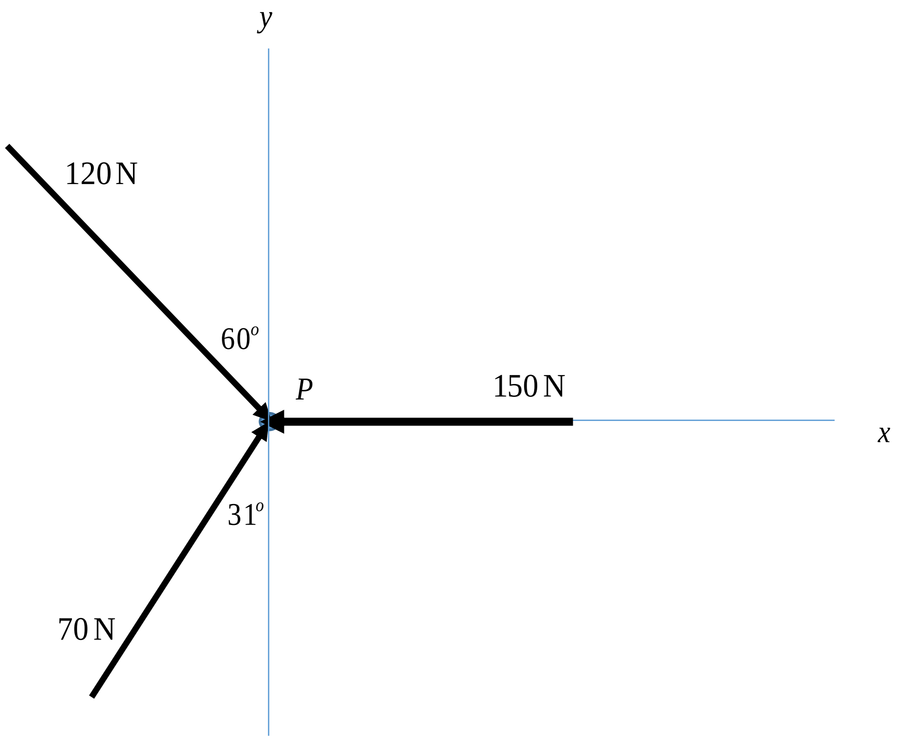
- Example 4.1

Determine if the particle P is in equilibrium under the influence of the forces shown in the diagram.



Static Equilibrium of a Particle

- Example 4.1 - Solution



Equations of Equilibrium

$$\sum F_x = 120\sin 60^o + 70\sin 31^o - 150 = -11.02\ \text{N}$$

$$\sum F_y = -120\cos 60^o + 70\cos 31^o = -0.0017\ \text{N} = 0.00\ \text{N}$$

For equilibrium, $\sum F_x = 0 = \sum F_y$

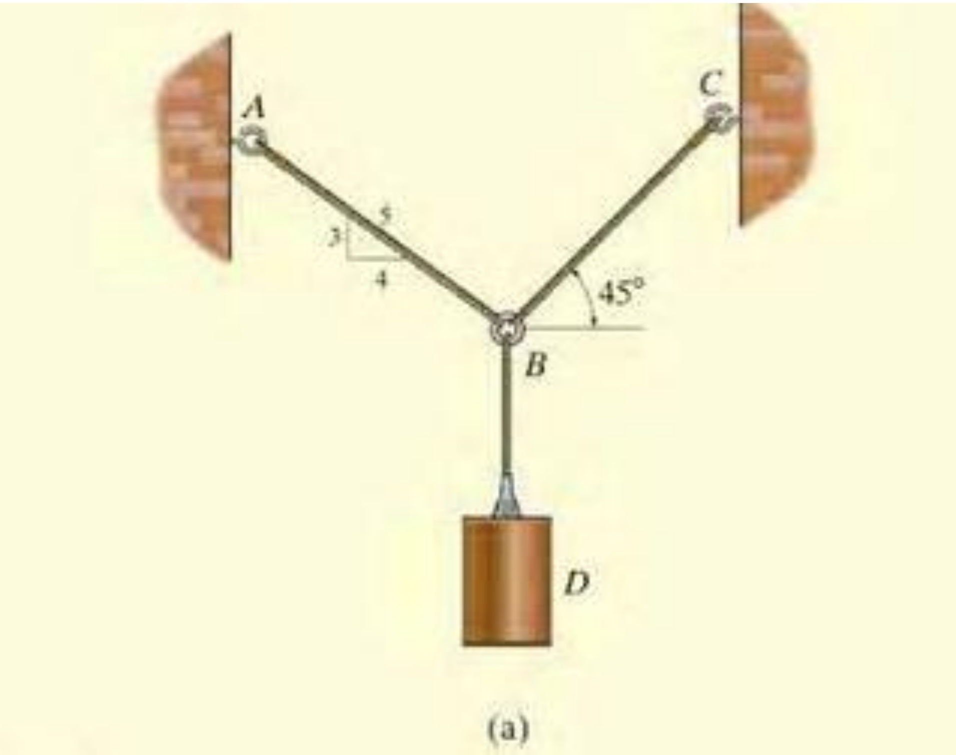
But $\sum F_x \neq 0$

Hence, P is not in equilibrium

Static Equilibrium of a Particle

- Example 4.2

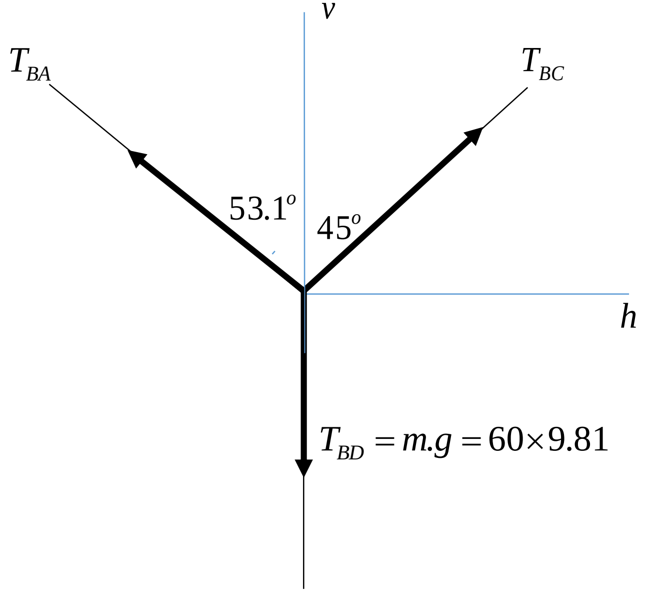
Determine the required tensions in cables BC and BA so they can sustain the 60kg weight.



Static Equilibrium of a Particle

- Example 4.2 - Solution

FBD



Equations of Equilibrium

$$\sum F_h = 0 : T_{BC} \sin 45^o - T_{BA} \sin 53.1^o = 0 \qquad \text{---} \quad (1)$$

$$\sum F_v = 0 : T_{BC} \cos 45^o + T_{BA} \cos 53.1^o - T_{BD} = 0$$

$$= T_{BC} \cos 45^o + T_{BA} \cos 53.1^o = 588.6 \text{ N} \qquad \text{---} \quad (2)$$

Solving (1) and (2) simultaneously,

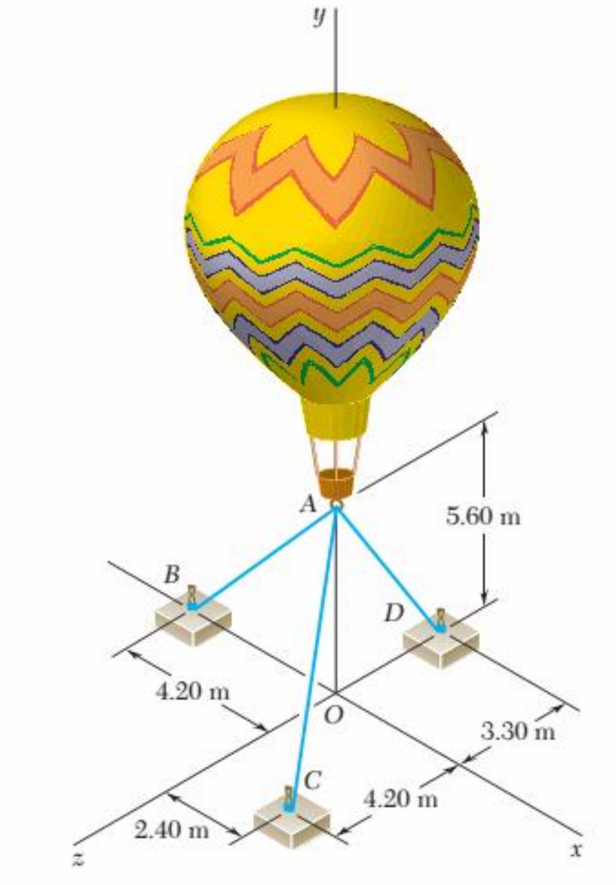
$\mathrm{T_{BC}} = 475.41\mathrm{N}$

$\mathrm{T_{BA}} = 420.43\,\mathrm{N}$

Static Equilibrium of a Particle

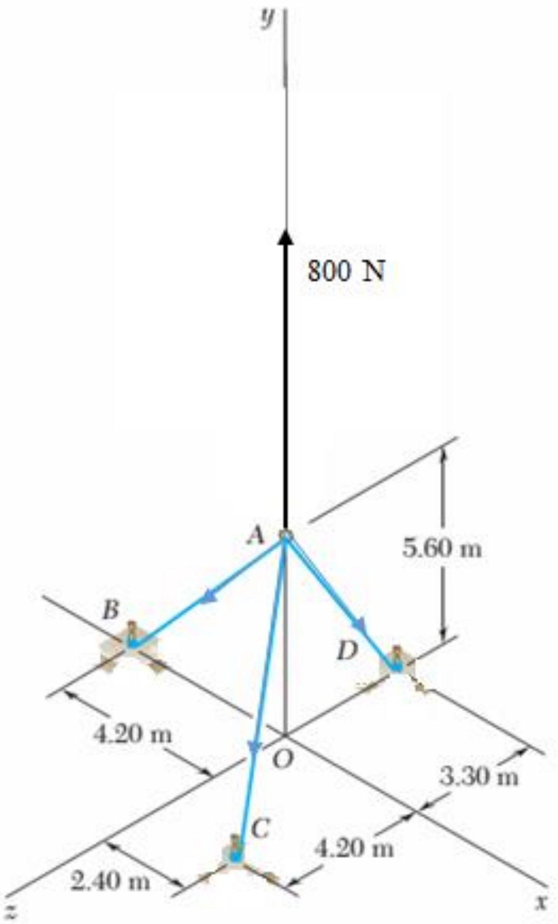
Example 4.3

Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800-N vertical force at A, determine the tension in each cable.



Static Equilibrium of a Particle

Example 4.3 - Solution



Equations of Equilibrium

$$\vec{T}_{AB} = T_{AB}\frac{-4.2i - 5.6j}{\sqrt{4.2^2 + 5.6^2}} = -0.6T_{AB}i - 0.8T_{AB}j$$

$$\vec{T}_{AC} = T_{AC}\frac{2.4i - 5.6j + 4.2k}{\sqrt{2.4^2 + 5.6^2 + 4.2^2}} = 0.37T_{AC}i - 0.87T_{AC}j + 0.65T_{AC}k$$

$$\vec{T}_{AD} = T_{AD}\frac{-5.6j - 3.3k}{\sqrt{5.6^2 + 3.3^2}} = -0.86T_{AD}j - 0.51T_{AD}k$$

$$F_{Ay} = 800\,\mathrm{N}j$$

$$\sum F_x = -0.6T_{AB} + 0.37T_{AC} = 0 \qquad \text{---} \quad (1)$$

$$\sum F_y = -0.8T_{AB} - 0.87T_{AC} - 0.86T_{AD} + 800 = 0 \qquad \text{---} \quad (2)$$

$$\sum F_z = 0.65T_{AC} - 0.51T_{AD} = 0 \qquad \text{---} \quad (3)$$

Solving (1), (2) and (3) simultaneously,

$T_{AB} = 200.6\ \mathrm{N}$

$T_{AC} = 325.3\ \mathrm{N}$

$T_{AD} = 414.6\ \mathrm{N}$

EQUILIBRIUM OF PARTICLES AND BODIES

Static Equilibrium of Rigid Bodies

- For rigid bodies, the sum of moments is considered in addition to the sum of forces.

$$\vec{R} = \sum F = 0$$

$$\Rightarrow \sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\vec{M} = \sum M = 0$$

$$\Rightarrow \sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

- Reaction at supports of rigid bodies must not be ignored in equilibrium analysis.
- A free body diagram is indispensable in solving problems on equilibrium of rigid bodies.
- All support reactions must be accounted on the free body diagram in order for the problem to be solved correctly.

Statically Determinate and Statistically Indeterminate Reactions

- Statically Determinate Problems
  - The number of unknowns in the FBD equals the number of equations obtained.

- Statistically Indeterminate Problems
  - The number of unknowns in the FBD exceeds the number of equations obtained. Solving of such problems is beyond the scope of this course.

EQUILIBRIUM OF PARTICLES AND BODIES

Static Equilibrium of Rigid Bodies

- Drawing a free body diagram

  - Select the extent of the free-body and detach it from the ground and all other bodies.

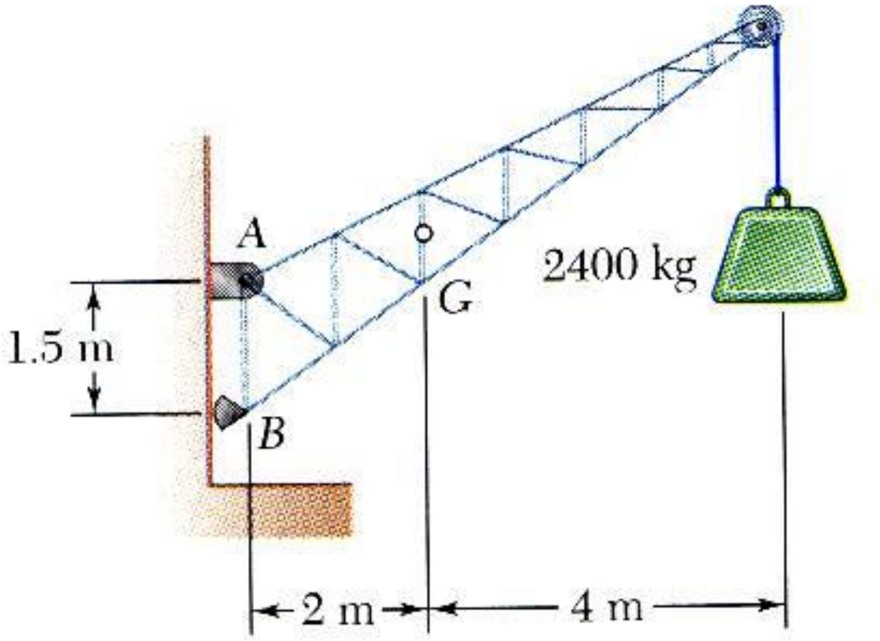
  - Indicate point of application, magnitude, and direction of external forces, including the rigid body weight.

  - Indicate point of application and assumed direction of unknown applied forces. These usually consist of reactions through which the ground and other bodies oppose the possible motion of the rigid body.

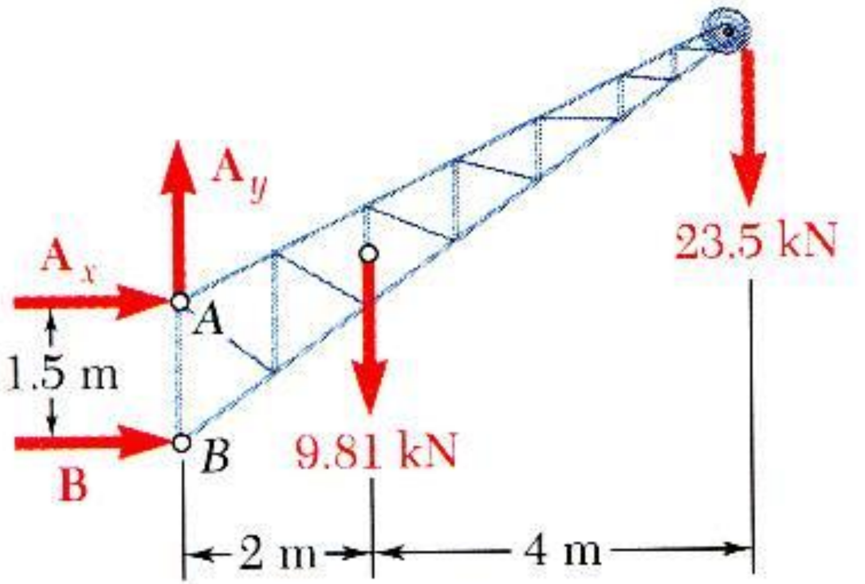
  - Include the dimensions necessary to compute the moments of the forces.

➢Drawing a free body diagram

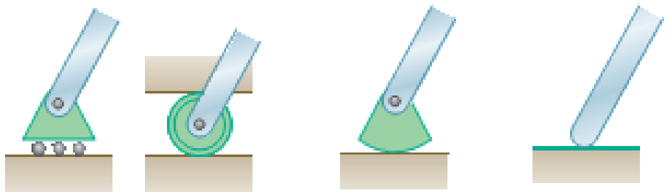
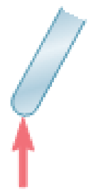
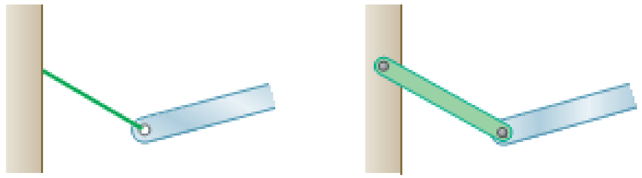
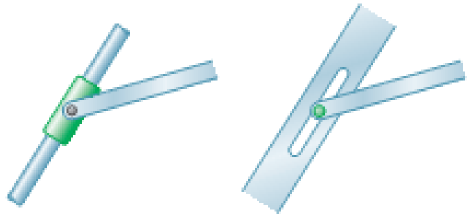
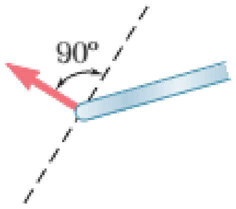
Problem diagram



Free-body diagram



➢Reactions at Supports and Connections for Two-Dimensional Structures

Support or Connection	Reaction	Number of Unknowns
 Rollers, Rocker, Frictionless surface	 Force with known line of action	1
 Short cable, Short link	 Force with known line of action	1
 Collar on frictionless rod, Frictionless pin in slot	 Force with known line of action	1

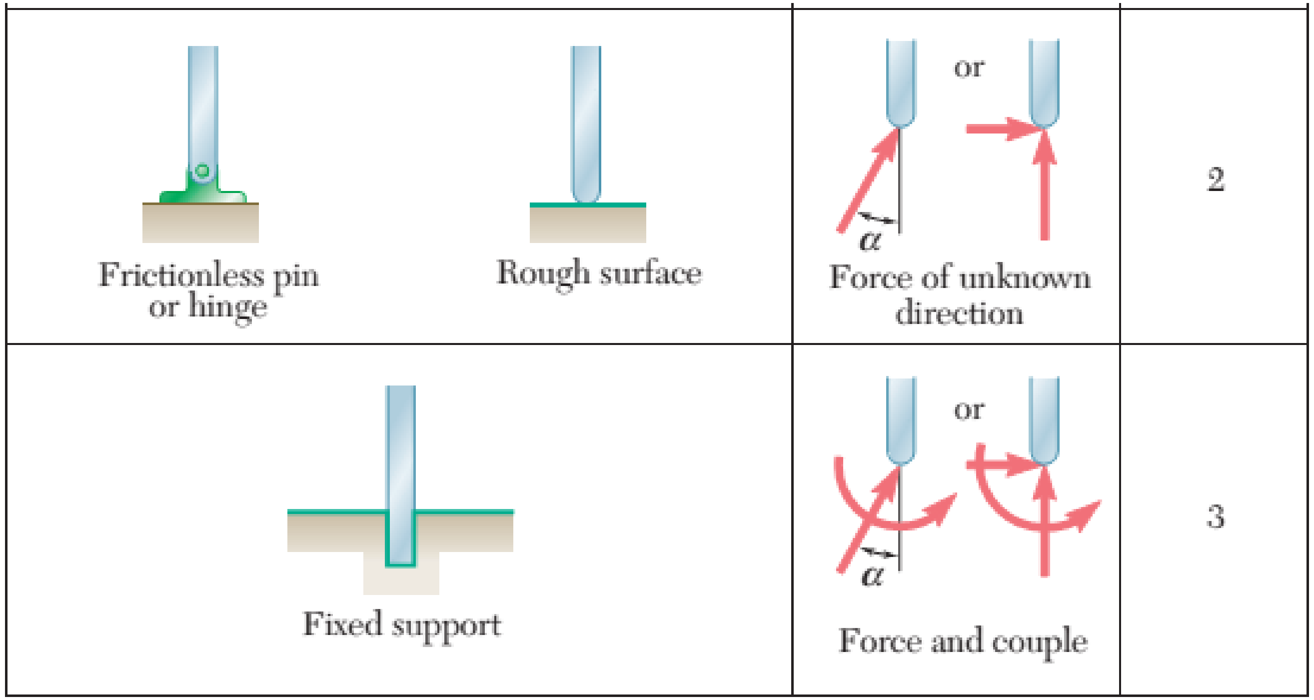
- Reactions equivalent to a force with known line of action.

Source:
Vector Mechanics for Engineers, Beer *et al.*

EQUILIBRIUM OF PARTICLES AND BODIES

Static Equilibrium of Rigid Bodies

➢Reactions at Supports and Connections for Two-Dimensional Structures

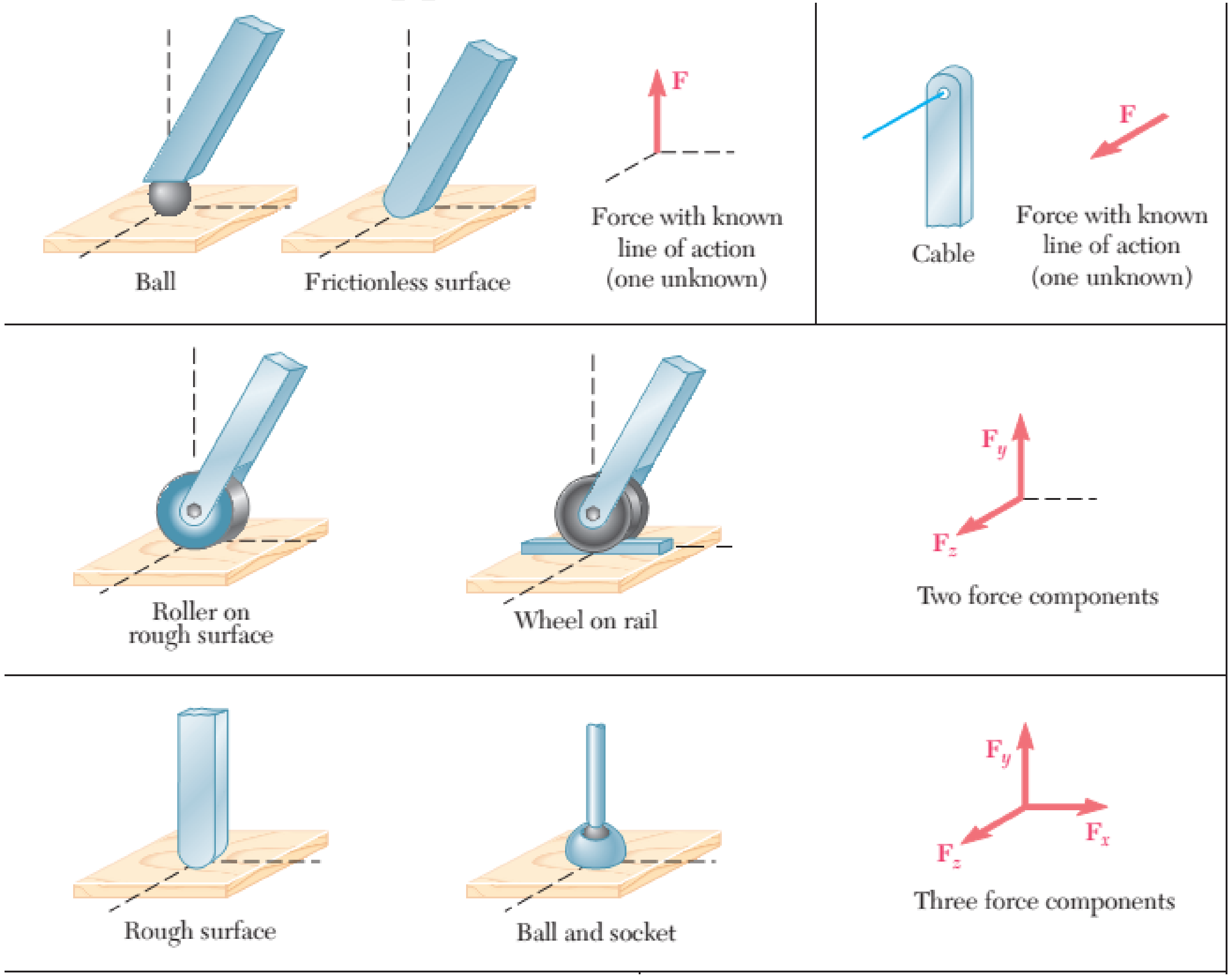


- Reactions equivalent to a force of unknown direction and magnitude.

- Reactions equivalent to a force of unknown direction and magnitude and a couple.of unknown magnitude

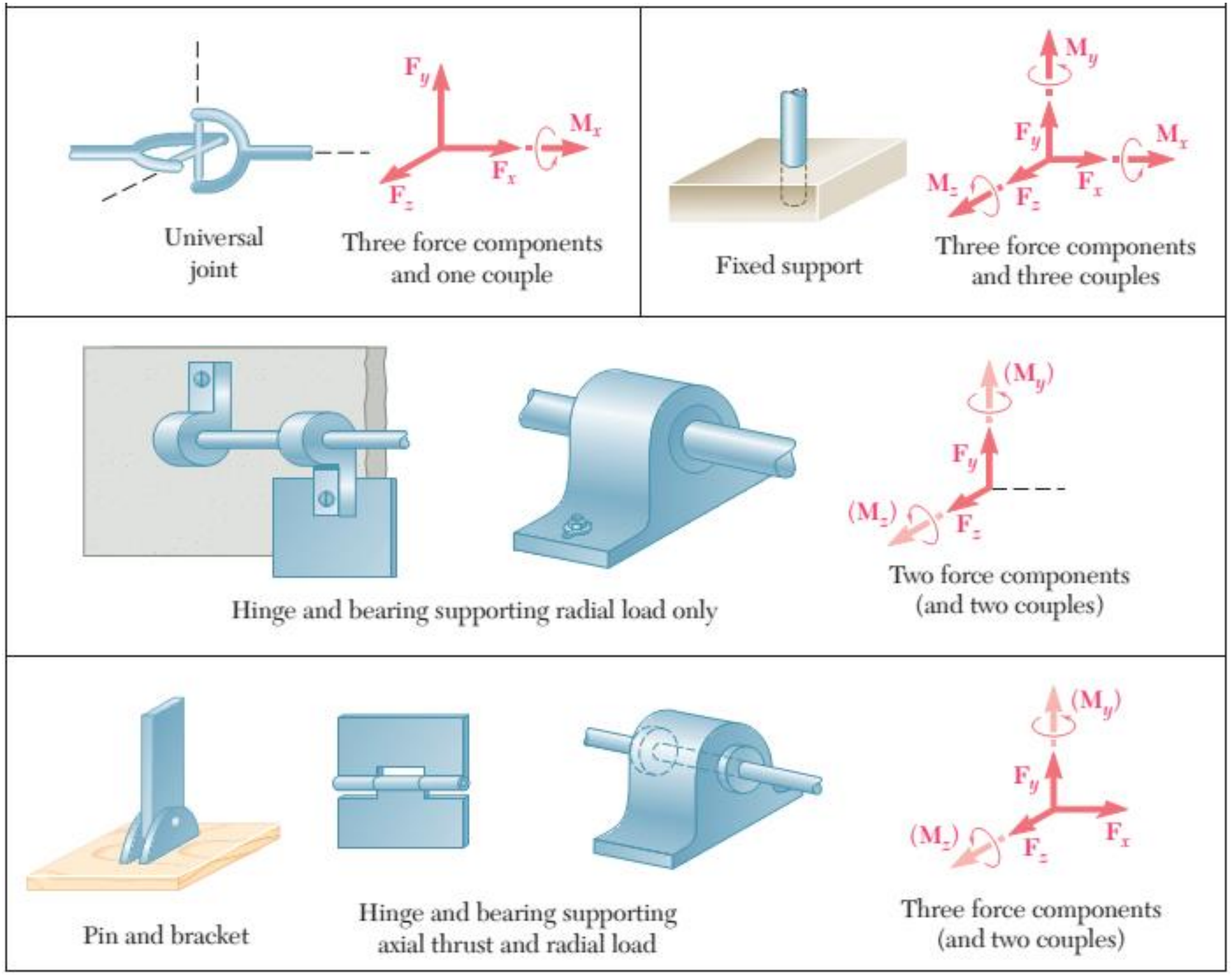
Source:
Vector Mechanics for Engineers, Beer *et al.*

➢Reactions at Supports and Connections for Three-Dimensional Structures



Source:
Vector Mechanics for Engineers, Beer *et al.*

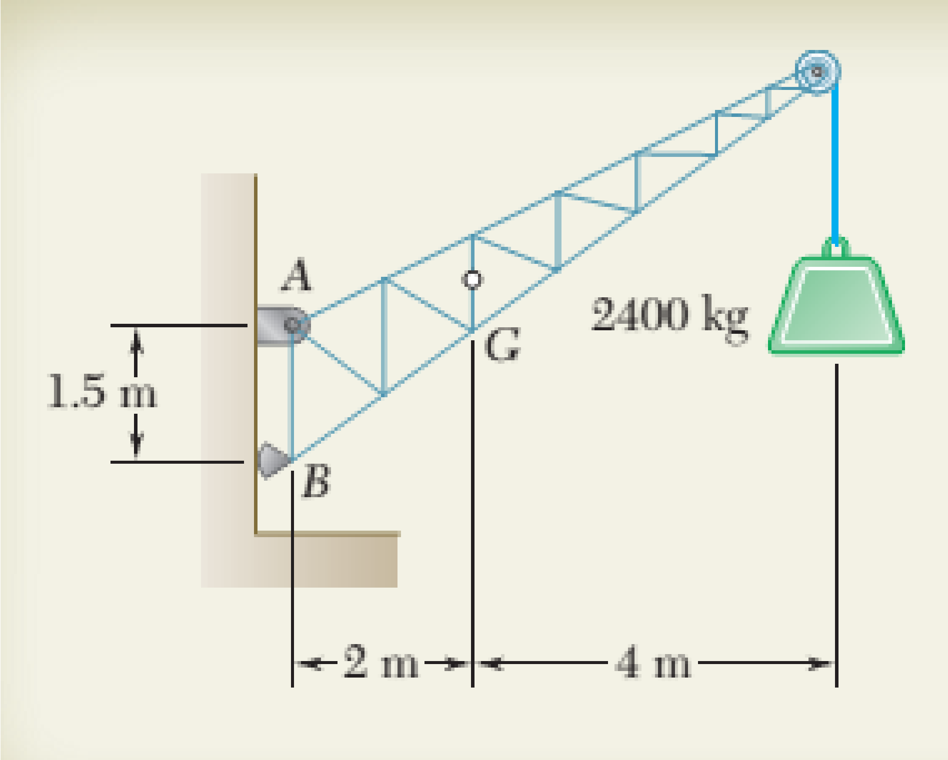
➢Reactions at Supports and Connections for Three-Dimensional Structures



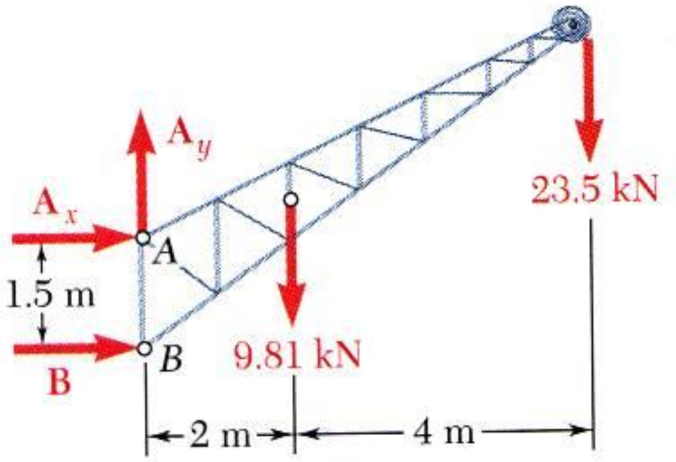
Source:
Vector Mechanics for Engineers, Beer *et al.*

➢Example 4.4

A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G. Determine the components of the reactions at A and B.



➢Example 4.4 - Solution



At A,

$$\sum F_x = 0: \quad A_x + B = 0$$

$$\sum F_y = 0: \quad A_y - 9.81\,\text{kN} - 23.5\,\text{kN} = 0$$

$$A_y = +33.3\,\text{kN}$$

Taking moments about A,

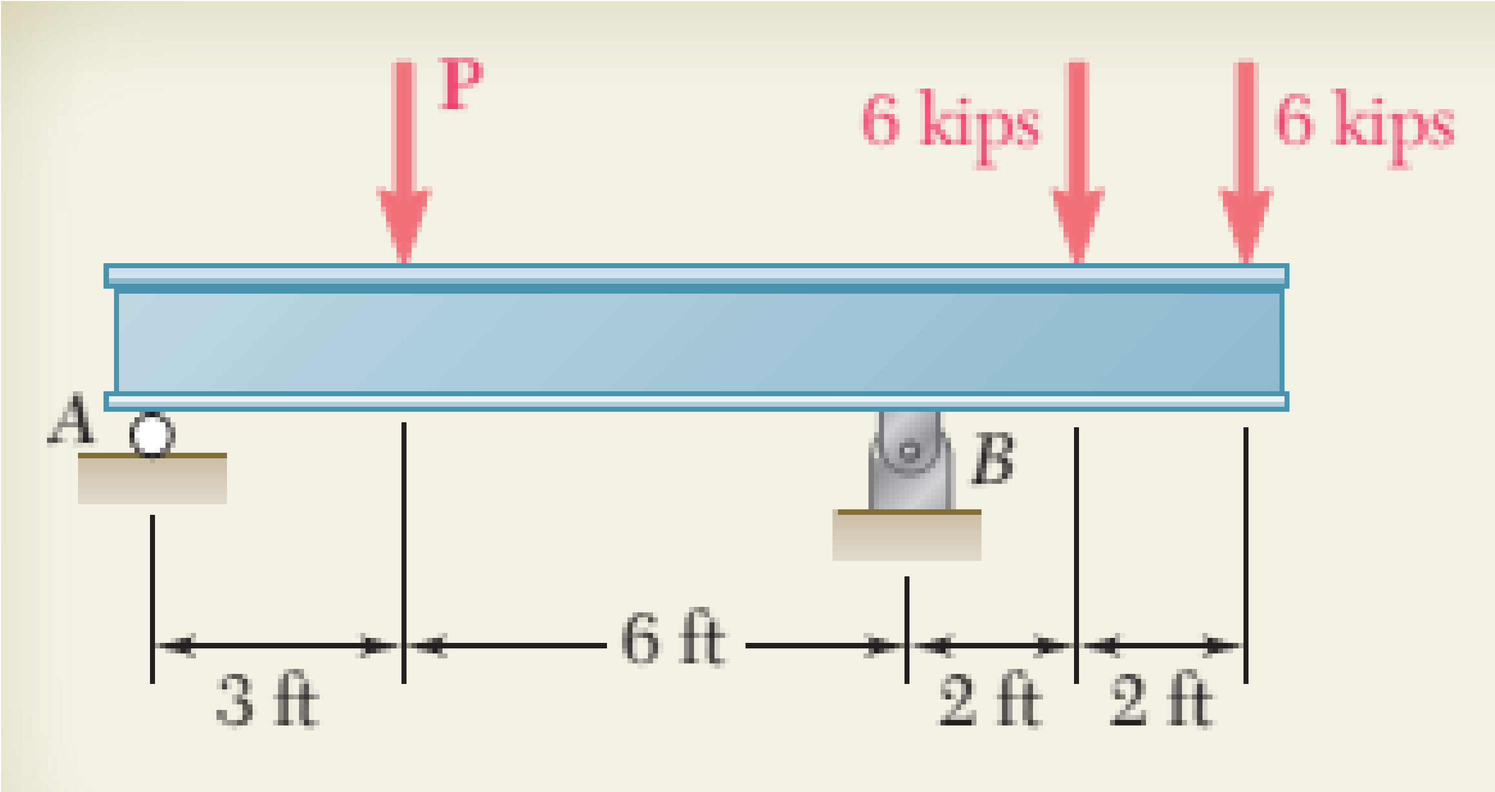
$$\sum M_A = 0: \quad +B(1.5\text{m}) - 9.81\,\text{kN}(2\text{m}) - 23.5\,\text{kN}(6\text{m}) = 0$$

$$B = +107.1\,\text{kN}$$

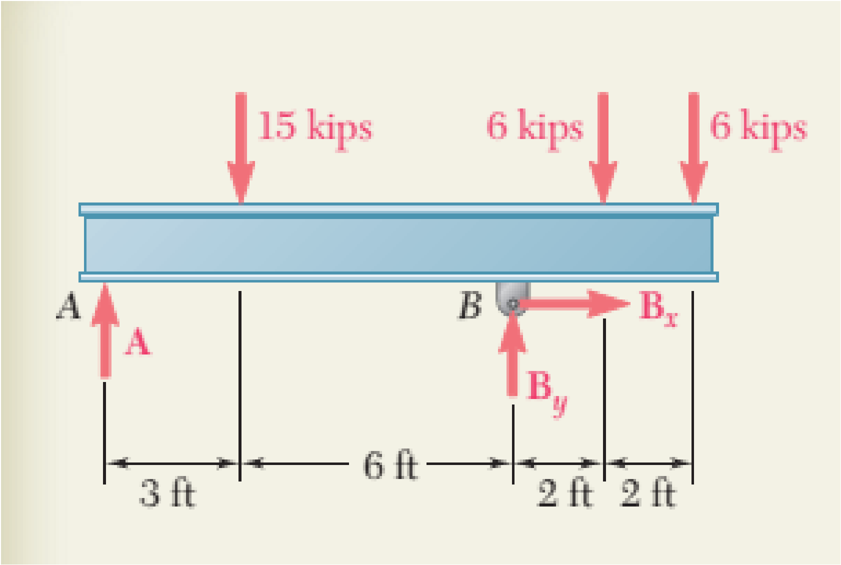
$$\therefore A_x = -107.1\,\text{kN}$$

➢Example 4.5

Three loads are applied to a beam as shown. The beam is supported by a roller at A and by a pin at B. Neglecting the weight of the beam, determine the reactions at A and B when P is 15 kips.



➢Example 4.5 - Solution



$$\sum F_x = 0: \quad B_x = 0$$

$$\sum F_y = 0: \quad A - 15\,\text{kips} + B_y - 6\,\text{kips} - 6\,\text{kips} = 0$$

$$A + B_y = 27\,\text{kips}$$

Taking moments about A (clockwise to be + ve),

$$\sum M_A = 0: \quad (15\,\text{kips})(3\,\text{ft}) + (6\,\text{kips})(11\,\text{ft}) - (B_y)(9\,\text{ft}) + (6\,\text{kips})(13\,\text{ft})$$

$$B_y = -21.0\,\text{kips} = 21.0\,\text{kips (acting upwards)}$$

$$\sum M_B = 0: \quad (A)(9\,\text{ft}) + (-15\,\text{kips})(6\,\text{ft}) + (6)(2\,\text{ft}) + (6\,\text{kips})(4\,\text{ft})$$

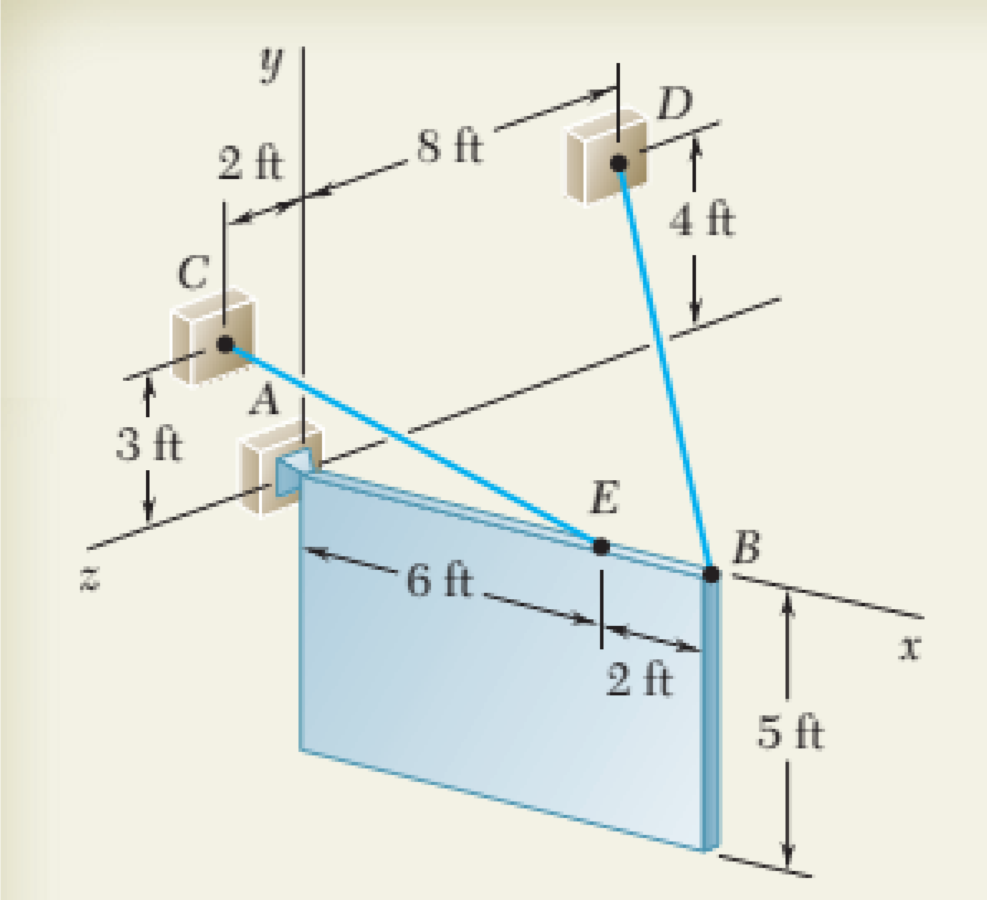
$$\text{A} = -6\,\text{kips} = (6\,\text{kips upwards})$$

OR

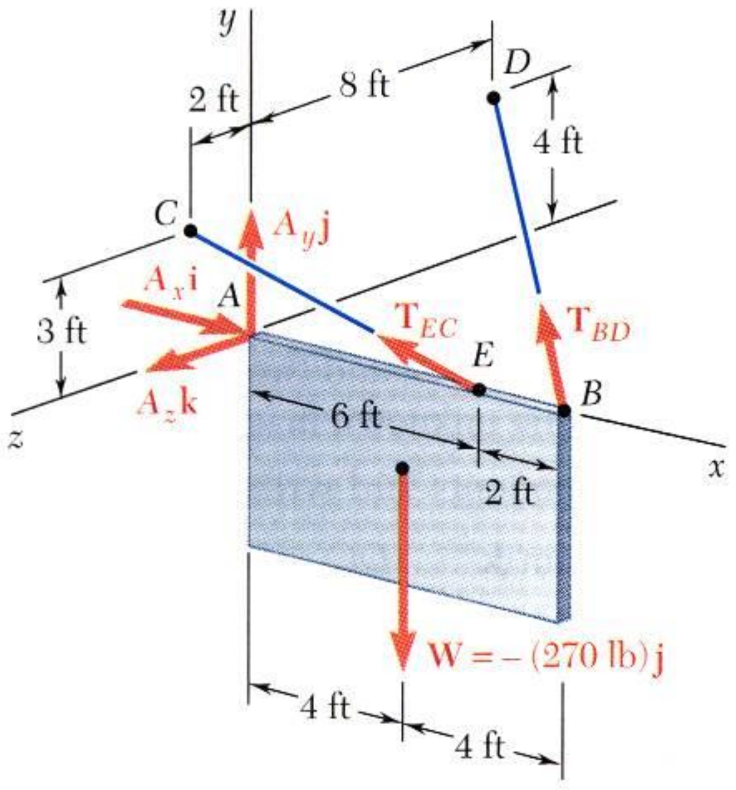
$$A = 27 - B_y = 27\,\text{kips} - 21.0\,\text{kips} = 6\,\text{kips}.$$

➢Example 4.6

A 5 38-ft sign of uniform density weighs 270 lb and is supported by a ball-and-socket joint at A and by two cables. Determine the tension in each cable and the reaction at A.



- Example 4.6



$$\vec{T}_{BD} = -\frac{2}{3}T_{BD}i + \frac{1}{3}T_{BD}j - \frac{2}{3}T_{BD}k$$

$$\vec{T}_{EC} = -\frac{6}{7}T_{EC}i + \frac{3}{7}T_{EC}j - \frac{2}{7}T_{EC}k$$

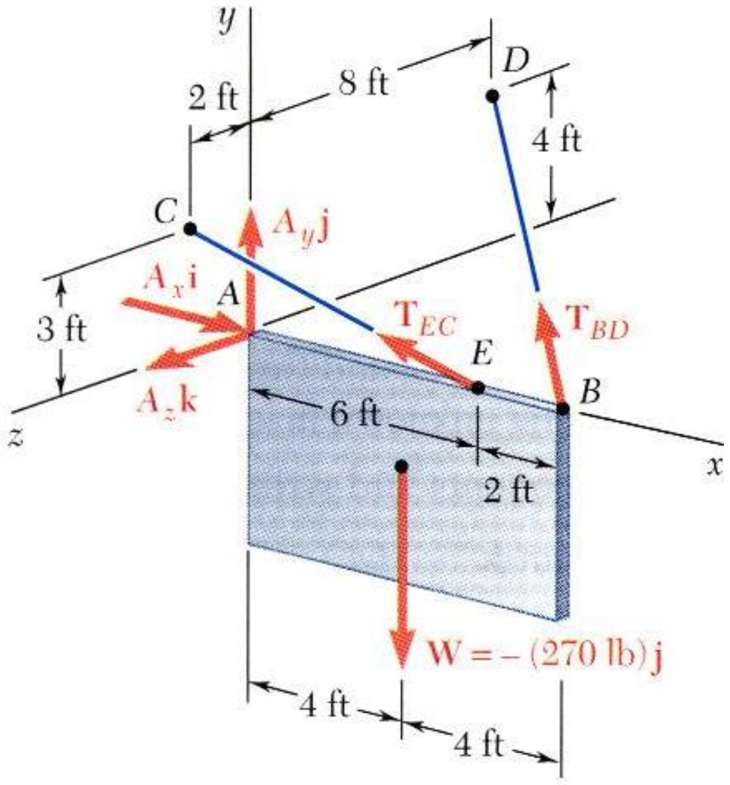
$$W = -270\,\text{lb}j$$

$$\sum F_x = \sum F_i = 0: \quad A_x - \frac{2}{3}T_{BD} - \frac{6}{7}T_{EC} = 0 \qquad ---(1)$$

$$\sum F_y = \sum F_j = 0: \quad A_y + \frac{1}{3}T_{BD} + \frac{3}{7}T_{EC} - 270\,\text{lb} = 0 \qquad ---(2)$$

$$\sum F_z = \sum F_k = 0: A_z - \frac{2}{3}T_{BD} - \frac{2}{7}T_{EC} = 0 \qquad ---(3)$$

- Example 4.6



Taking moments about A,

$$\sum M_B = 0 : (\vec{r}_B)(\vec{T}_{BD}) + \vec{r}_E(\vec{T}_{EC}) + \vec{r}_W(W)$$

$$= (8\text{ ft})i\left(-\frac{2}{3}T_{BD}i + \frac{1}{3}T_{BD}j - \frac{2}{3}T_{BD}k\right) + (6\text{ ft})i\left(-\frac{6}{7}T_{EC}i + \frac{3}{7}T_{EC}j - \frac{2}{7}T_{EC}k\right) + (4\text{ ft})i(-270\,\text{lb}j)$$

$$(2.667T_{BD} + 2.571T_{EC} - 1080\,\text{lb})k + (5.333T_{BD} - 1.714T_{EC})j = 0$$

$$\therefore\quad 2.667T_{BD} + 2.571T_{EC} - 1080\,\text{lb} = 0 \qquad ---(4)$$

$$5.333T_{BD} - 1.714T_{EC} = 0 \qquad ---(5)$$

Solvingb (1) to (5) simultaneously,

$T_{BD} = 101.3\,\text{lb} \qquad T_{EC} = 315\,\text{lb}$

$A_x = 338\,\text{lb}\,i \qquad A_y = 101.2\,\text{lb}j \qquad A_z = -22.5\,\text{lb}\,k$

TAKE HOME ASSIGNMENT 1

LECTURE 5 & 6

STRUCTURAL ANALYSIS:PLANAR TRUSSES AND FRAMES

ANALYSIS OF STRUCTURES

- Structural engineering is the field of engineering mostly associated with the design and analysis of engineering structures.

- Engineering structures basically, are designed to support, transmit or modify forces safely. They normally comprise of a number of members connected to form a main structure.

- There are different types of engineering structures, but our focus will be on trusses and frames.

- Trusses and Frames are designed to transmit forces over long distances and comprise straight, **slender** bars that are joined together to form in most cases a pattern of triangles.

- The loads on trusses are always applied in the joints, while in frames, loads may not necessarily be applied at the joints. Also, members of trusses are normally two force members, whereas in frames, at least one member is a multiforce member.

ANALYSIS OF STRUCTURES

Trusses

- Trusses may be classified as simple or compound.

- In simple trusses the number of joints and number of members is related by

$$m = 2j - 3$$

where m is the number of members and j, the number of joints.

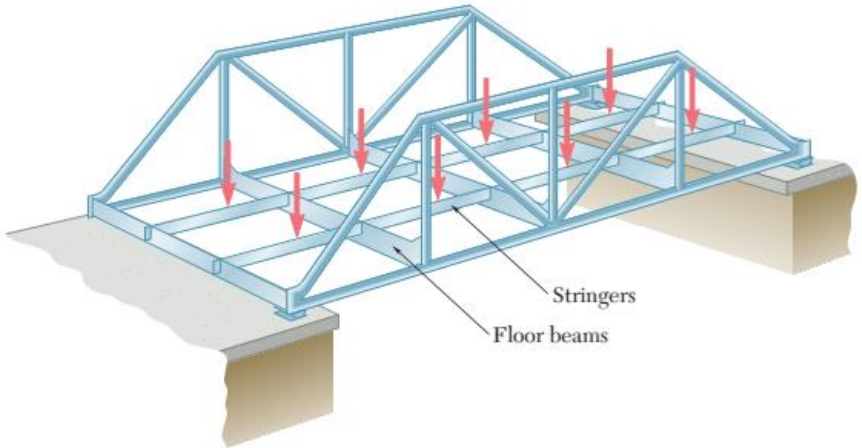
- A truss is therefore often considered simple if its design or construction is such that each time we add 2 new members, they are joined such the number of joints in the truss increases by 1.

- A Truss is designed such that its members are either in tension or compression.

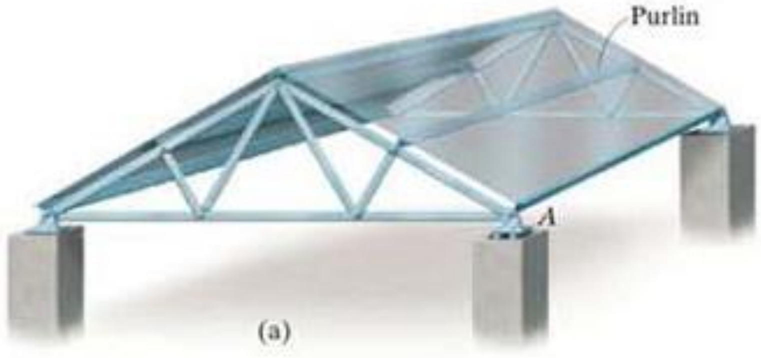
- In designing a truss, it is desirable to know the force each individual member must sustain.

ANALYSIS OF STRUCTURES
Trusses

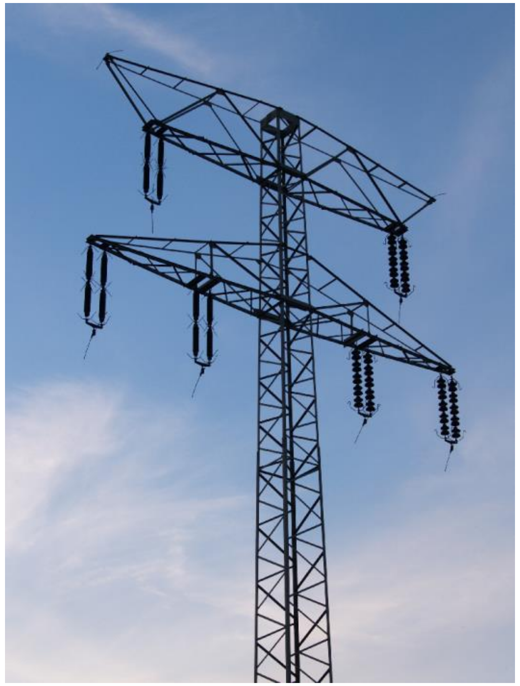
- Common applications are roofs, bridges and power pylons.



Bridge Truss
Source: Mechanics for Engineers by Beet *et al*

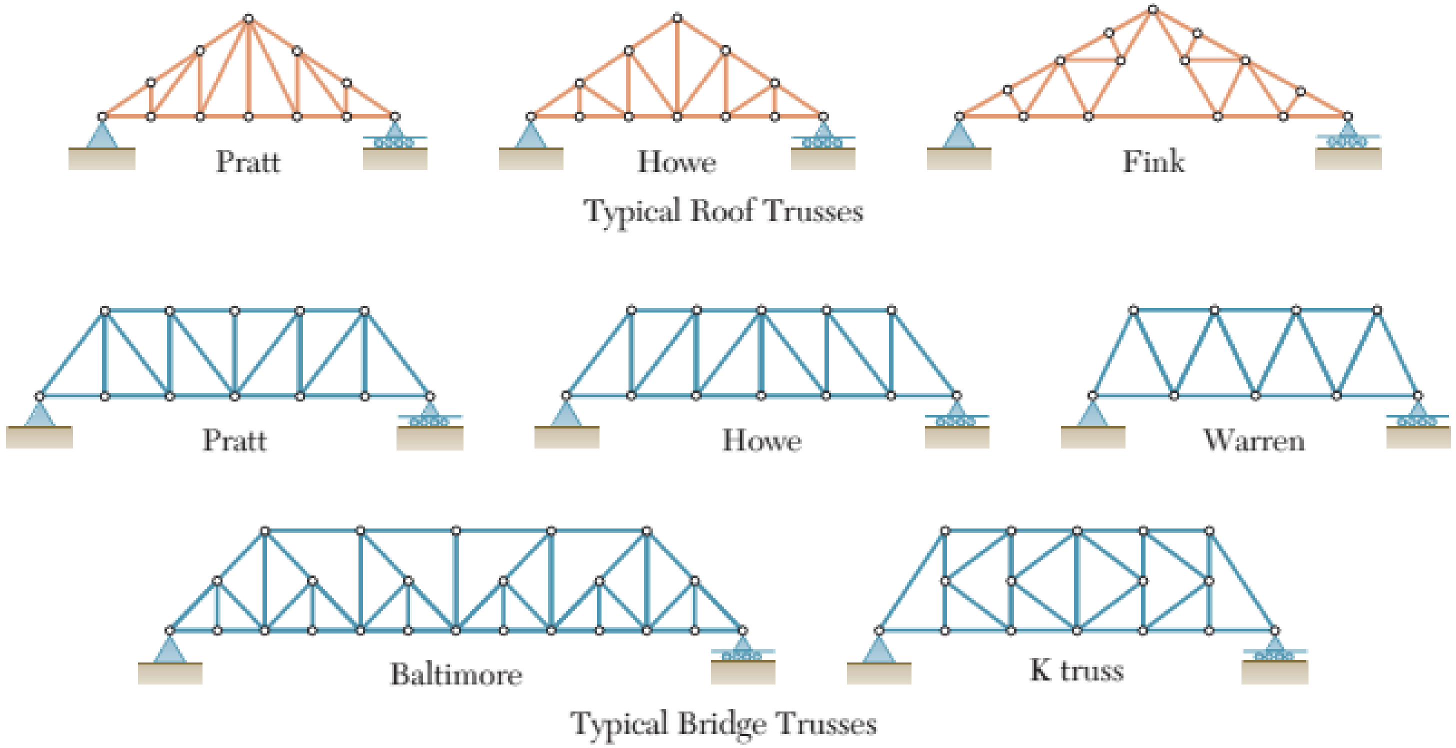


Roof Supporting Truss
Source: Engineering Mechanics Statics by Hiebbler



Power Pylon
Source: http://upload.wikimedia.org/wikipedia/commons/7/7e/Electricity_pylon_power_outage.jpg

- Trusses may be Planar or Space Trusses.



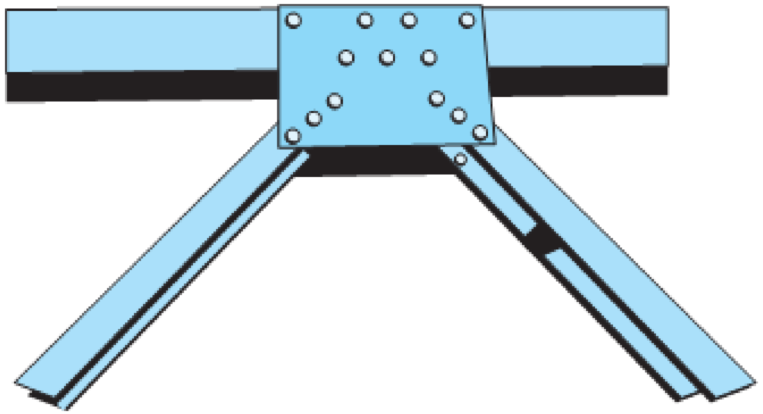
Some Planar Trusses
Source: Beet *et al*

Space Truss
Source
*http://www.picstopin.com/1000/truss/http:%7C%7Cwww*timplex*com%7Ctrusstypess*png/*

Analysis of Planar Trusses

- Assumptions for analysis:
  - The weights of the slender bars/members are negligible.
  - Forces act at the ends of the members such that they are in either tension or compression.
  - All joints are pins.

- There are two approaches to analysis:
  - The method of Joints
  - The method of Sections.



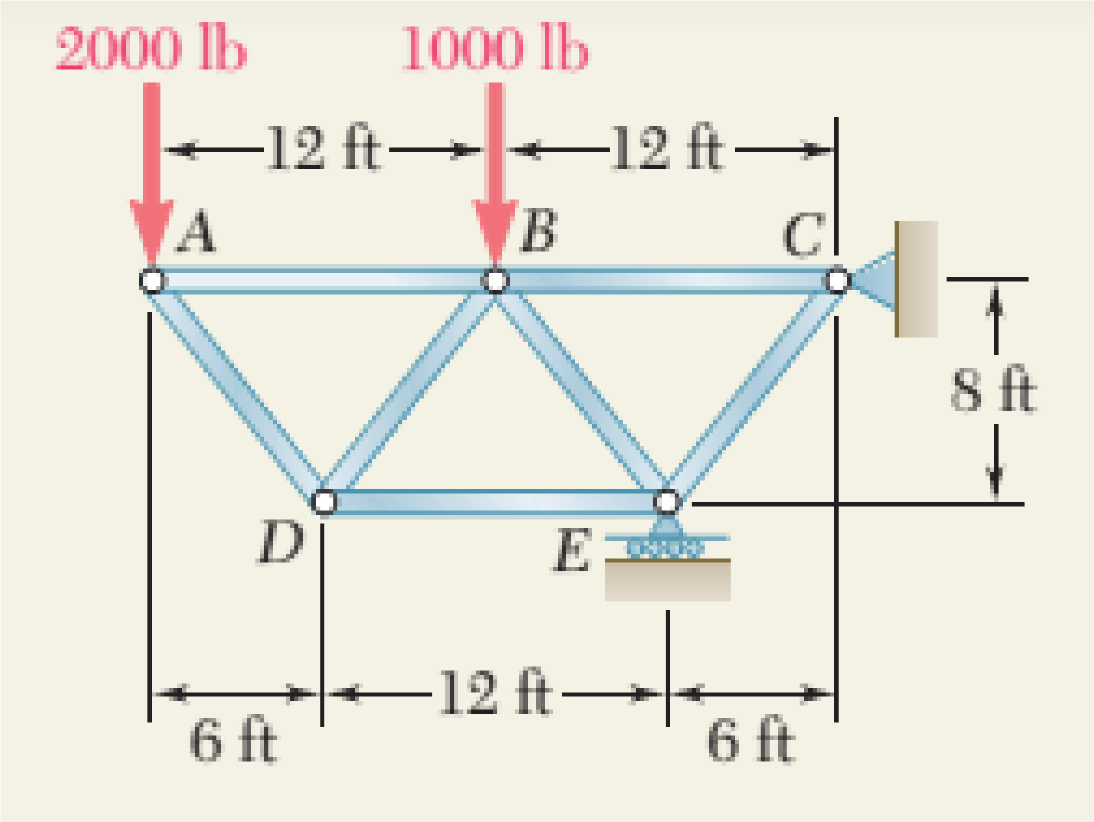
Truss members welded or riveted to a gusset plate
Source: Engineering Mechanics – Statics by Pytel

Analysis of Planar Trusses - The Method of Joints

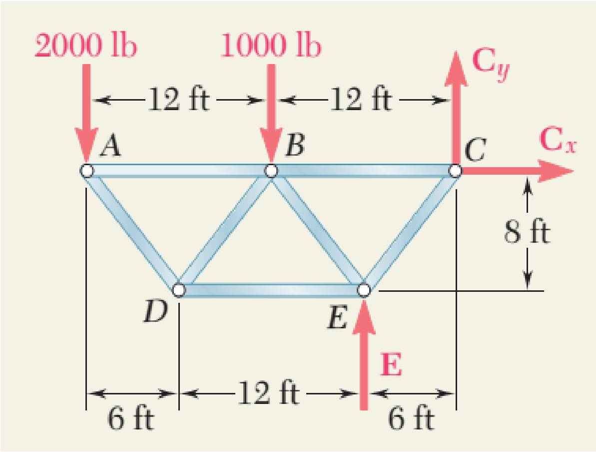
- This involves an equilibrium analysis on each joint to determine the forces being exerted on the end of each member at that joint.
- It is based on the assumption that if the whole truss is in equilibrium, each of its members or joints are also in equilibrium.
- Analysis is done in two main steps:
  - Determine the reactions at the support reactions using the FBD of the entire truss.
  - Conduct an equilibrium analysis at each joint/pin to determine the forces in each member.

➢Example 5.1

Determine the force in each member of the truss shown below using the method of joints.



- Example 5.1 - Solution
  - Equilibrium analysis on the entire truss to determine the support reactions



For equilibrium,

$$\xrightarrow{+} \sum F_x = 0 : C_x = 0$$

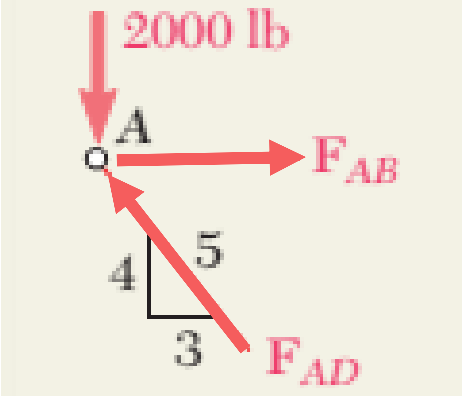
$$+\uparrow \sum F_y = 0 : C_y + E = 3000\text{lb}$$

$$\sum M_C = 0 \text{ (clockwise being positive)} : (6\text{ ft})E - (2000\text{lb})(24\text{ ft}) - (1000\text{lb})(12\text{ ft}) = 0$$

$$E = 10000\text{lb} \uparrow$$

  - Equilibrium analysis on the individual joints

Joint A



For equilibriu m,

$$\xrightarrow{+} \sum F_x = 0 : F_{AB} - \frac{3}{5} F_{AD} = 0 \qquad \text{----}(1)$$

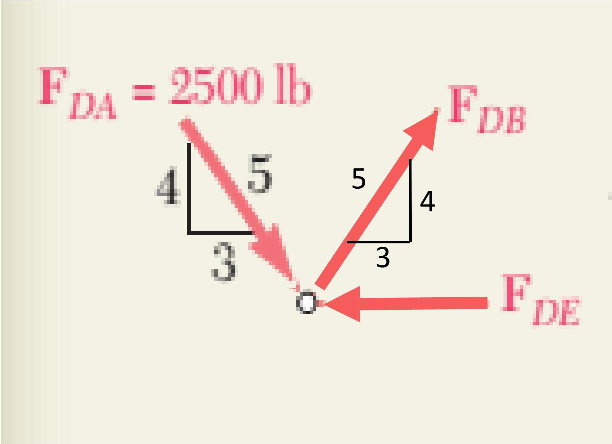
$$+\uparrow \sum F_y = 0 : \frac{4}{5} F_{AD} - 2000\text{ lb} = 0 \qquad \text{----}(2)$$

Solving (1) and (2) simultaneo usly,

$$F_{AD} = 2500\text{ lb} \qquad F_{AB} = 1500\text{ lb}$$

- Example 5.1 - Solution

Joint D



For equilibrium,

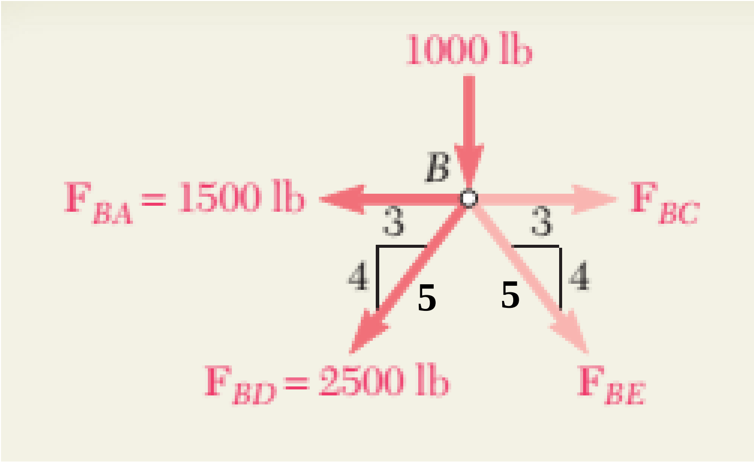
$$+\rightarrow\sum F_x = 0: \frac{3}{5}(2500\text{lb}) - F_{DE} + \frac{3}{5}F_{DB} = 0 \qquad ----(1)$$

$$+\uparrow\sum F_y = 0: \frac{4}{5}F_{DB} - \frac{4}{5}(2000\text{lb}) = 0 \qquad ----(2)$$

Solving (1) and (2) simultaneously,

$F_{DB} = 2500\text{lb} \qquad F_{DE} = 3000\text{lb}$

Joint B



For equilibrium,

$$+\rightarrow\sum F_x = 0: -\frac{3}{5}(2500\text{lb}) - 1500\text{lb} + F_{BC} + \frac{3}{5}F_{BE} = 0 \qquad ----(1)$$

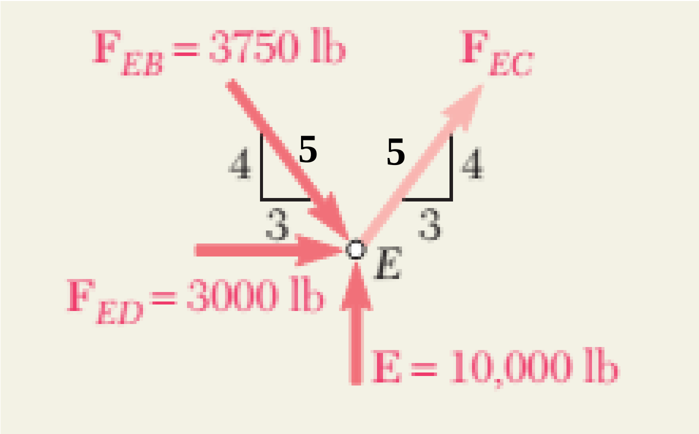
$$+\uparrow\sum F_y = 0: -1000\text{lb} - \frac{4}{5}(2000\text{lb}) - \frac{4}{5}F_{BE} = 0 \qquad ----(2)$$

Solving (1) and (2) simultaneously,

$F_{BE} = 3750\text{lb} \qquad F_{BC} = 5250\text{lb}$

➢Example 5.1 - Solution

Joint E



For equilibriu m,

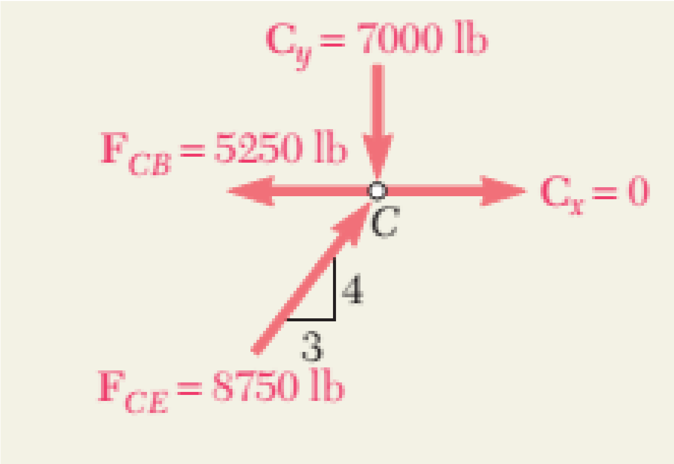
$$+\rightarrow \sum F_x = 0: \frac{3}{5}(3750 \text{ lb}) + 3000 \text{ lb} + \frac{3}{5}F_{EC} = 0 \qquad ----(1)$$

$$+\uparrow \sum F_y = 0: -\frac{4}{5}(3750 \text{ lb}) + \frac{4}{5}F_{EC} + 10000 \text{ lb} = 0 \qquad ----(2)$$

Solving (1) and (2) simultaneously,

$$F_{DB} = 2500 \text{ lb} \qquad F_{DE} = 3000 \text{ lb}$$

Joint C



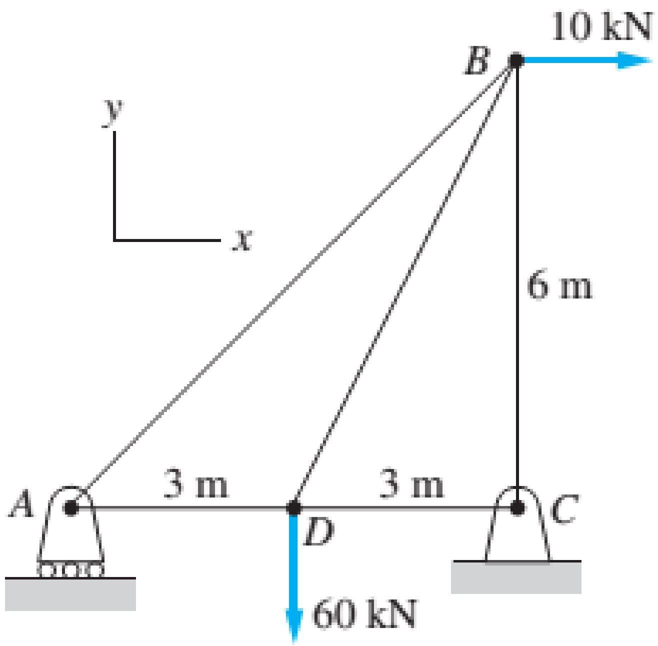
For equilibrium,

$$+\rightarrow \sum F_x = 0: \frac{3}{5}(8750\text{lb}) - 5250\text{lb} + 0 = 0$$

$$+\uparrow \sum F_y = 0: -7000\text{lb} - \frac{4}{5}(8750\text{lb}) = 0$$

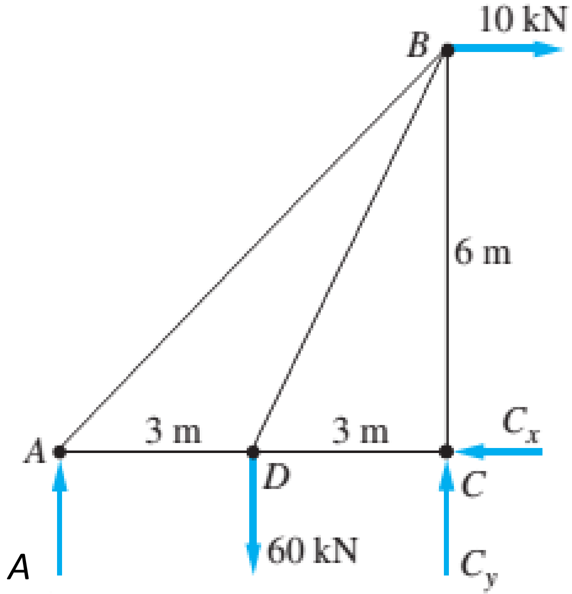
- Example 5.2

Determine the force in each member of the truss shown below using the method of joints. Indicate whether each member is in tension or compression



➢ Example 5.2 - Solution

Equilibrium analysis of the whole structure to determine support reactions



$$\text{For equilibrium,}$$

$$+\rightarrow \sum F_x = 0 : -C_x + 10\ \text{kN} = 0$$

$$+\uparrow \sum F_y = 0 : C_y + A - 60\ \text{kN} = 0$$

$$\sum M_C = 0\ (\text{clockwise being positive}) : A(6\ \text{m}) + (10\ \text{kN})(6\ \text{m}) - (60\ \text{kN})(3\ \text{m}) = 0$$

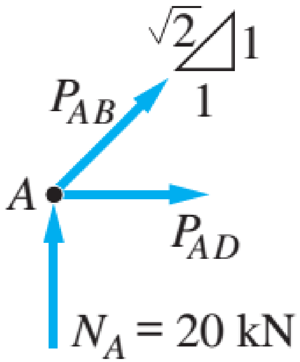
$$A = 20\ \text{kN}$$

$$\therefore C_y = 40\ \text{kN}$$

➢Example 5.2 - Solution

Equilibrium analysis on the joints

Joint A



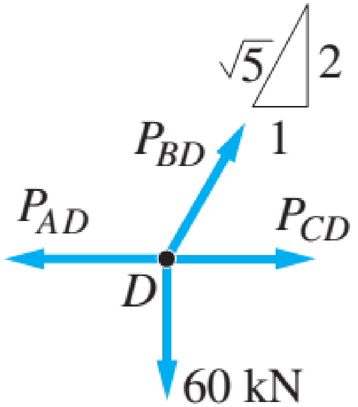
For equilibrium,

$$+\rightarrow\sum F_x = 0: P_{AD} + \frac{1}{\sqrt{2}}P_{AB} = 0 \qquad \text{-----}\ (1)$$

$$+\uparrow\sum F_y = 0: 20\,\text{kN} + \frac{1}{\sqrt{2}}P_{AB} = 0 \qquad \text{------}(2)$$

$$P_{AB} = -28.3\,\text{kN (Compression)} \qquad P_{AD} = 20\,\text{kN (Tension)}$$

Joint D



For equilibriu m,

$$+\rightarrow\sum F_x = 0: -P_{AD} + \frac{1}{\sqrt{5}}P_{BD} + P_{CD} = 0 \qquad \text{-----}\ (1)$$

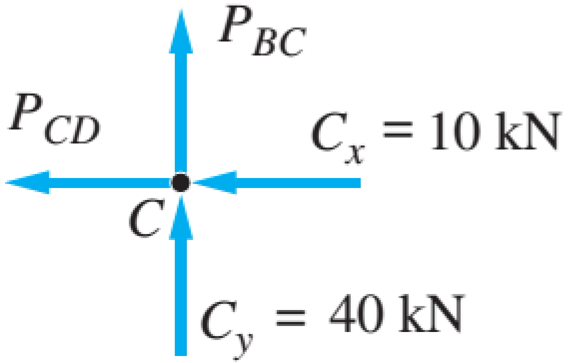
$$+\uparrow\sum F_y = 0: -60\,\text{kN} + \frac{2}{\sqrt{5}}P_{BD} = 0 \qquad \text{------}(2)$$

$$P_{BD} = 67.1\,\text{kN (Tension)} \qquad P_{CD} = -10\,\text{kN (Compression)}$$

➢Example 5.2 - Solution

Equilibrium analysis on the joints

Joint C



For equilibrium,

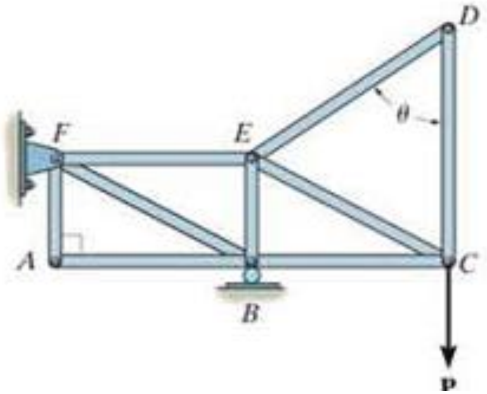
$$+\rightarrow \sum F_x = 0 : -P_{CD} - 10\text{ kN} = 0 \qquad \text{-----} \; (1)$$

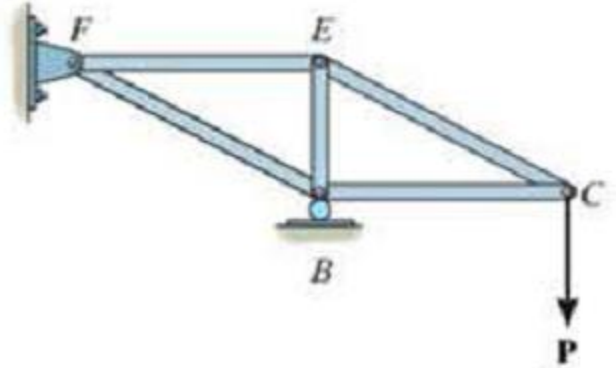
$$+\uparrow \sum F_y = 0 : 40\text{ kN} + P_{BC} = 0 \qquad \text{------} (2)$$

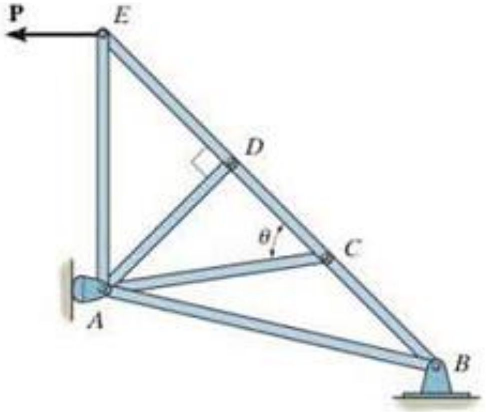
$$P_{BC} = -40\text{ kN (Compression)}$$

Zero Force Members

- It is possible during analysis to realize during analysis that some members do not experience any external forces. Such members are referred to as ***Zero Force Members.***
- Identification of such members can significantly make analysis of planar trusses using the method of joints easier.
- Identification is normally by inspection.
- However, two rules of thumb for identifying such members are;
  1. *if three members from a truss joint for which two of the members are collinear (have the same line of action), the third member may be a zero force member provided no external force or support reaction is applied to the joint.*
  2. *If a joint is formed by only two members and the joint is not subjected to any external load or support reactions, then the two members may be zero force members.*





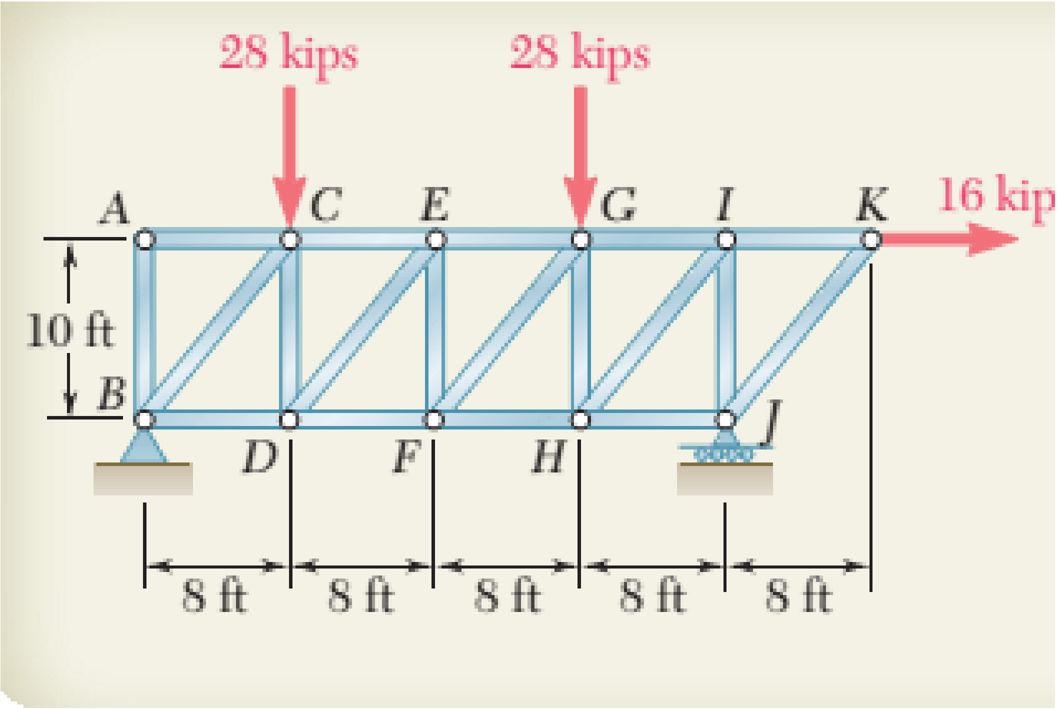


Analysis of Planar Trusses - The Method of Sections

- This method is useful if it is desired to find the forces in only few members.
- It is based on the assumption that if the whole truss is in equilibrium, any section of it must also be in equilibrium.
- Analysis in the following steps:
  - Determine the support reactions using the FBD of the entire truss.
  - Draw a line which divides the truss into two separate portions but does not intersect more than three members. One of the three intersected members must be the member of interest.
  - Use one of the two portions as a free body diagram and determine the force due to the member of interest.

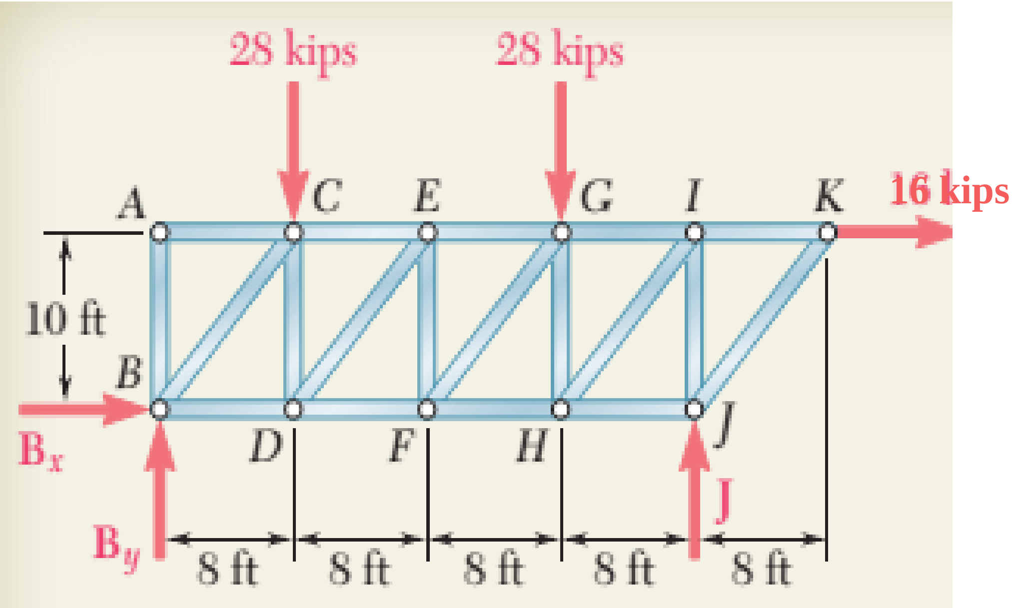
- Example 5.3

Determine the force in members *EF* and *GI* of the truss shown



➢Example 5.3 - Solution

Equilibrium analysis on the whole truss to determine support reactions.



For equilibrium,

$$+\rightarrow \sum F_x = 0: B_x + 16\text{ kips} = 0$$

$$+\uparrow \sum F_y = 0: B_y + J - 28\text{ kips} - 28\text{ kips} = 0$$

$$\sum M_B = 0 \text{ (clockwise being positive)}:$$

$$28\text{ kips}(8\text{ ft}) + 28\text{ kips}(24\text{ ft}) + 16\text{ kips}(10\text{ ft}) - J(32\text{ ft}) = 0$$

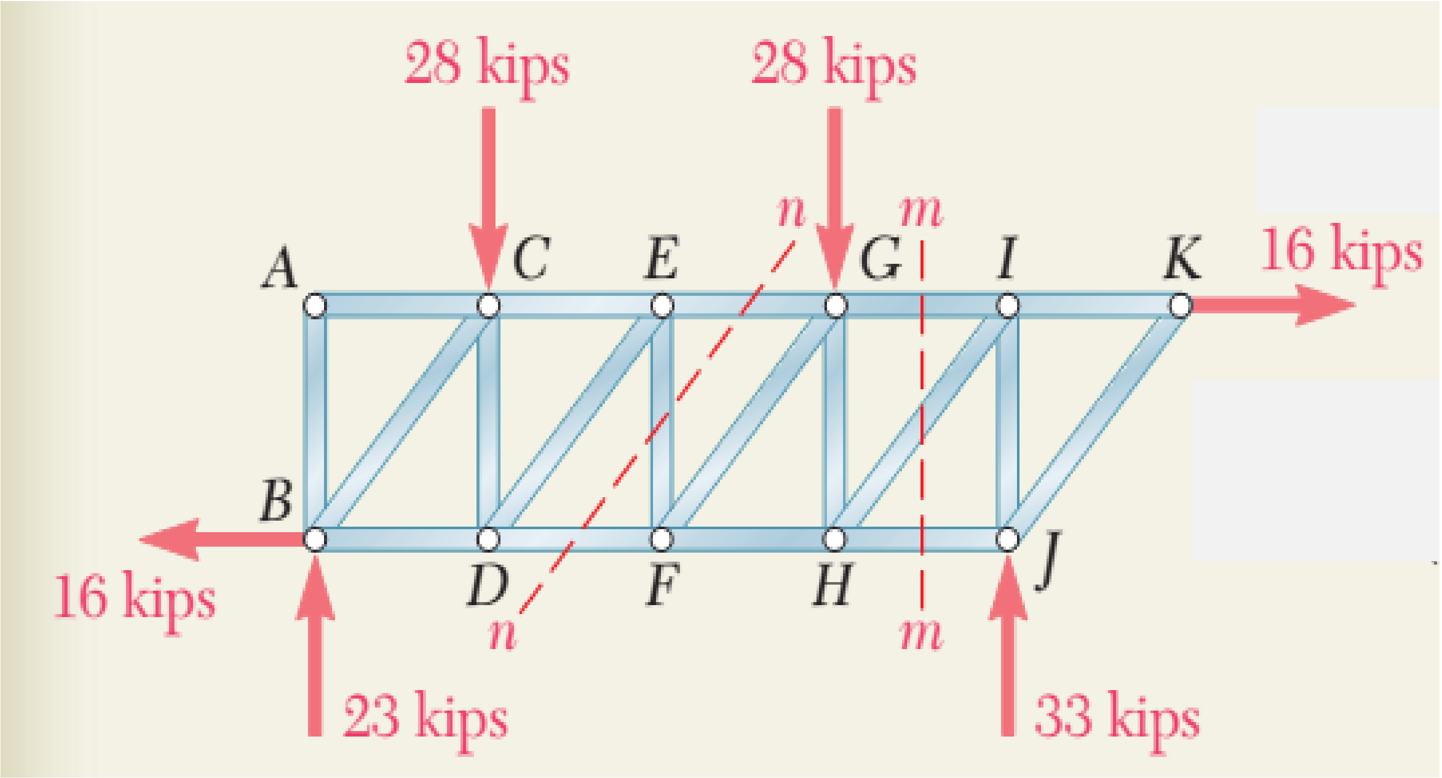
$$J = 33\text{ kips}$$

$$B_x = -16\text{ kips}$$

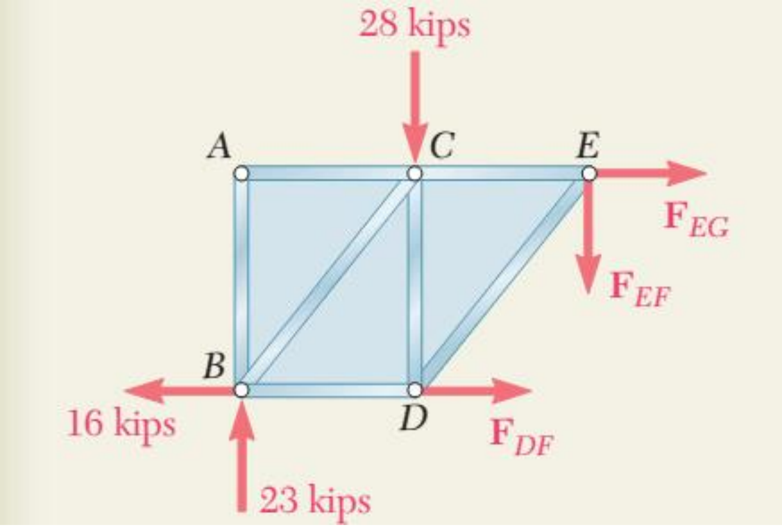
$$B_y = 23\text{ kips}$$

➢Example 5.3 - Solution

Dividing the truss into two with line *nn* (to solve for member *EF*), then with *mm* (to solve for *GI*)



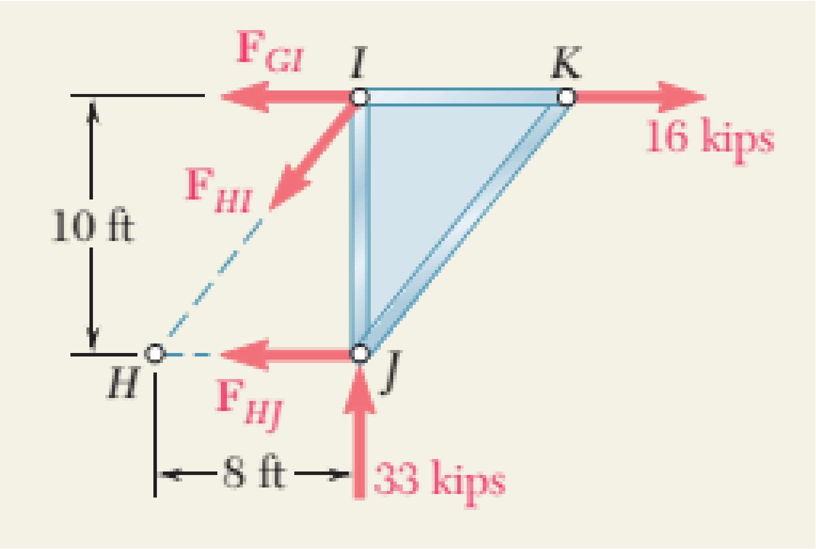
- Example 5.3 - Solution



Performing an equilibrium analysis on the left side of nn to obtain EF,

$$+\uparrow \sum F_y = 0 : 23\text{ kips} - 28\text{ kips} - F_{EF} = 0$$

$$F_{EF} = -5\text{ kips}$$



Performing an equilibrium analysis on the right side of mm to obtain GI,

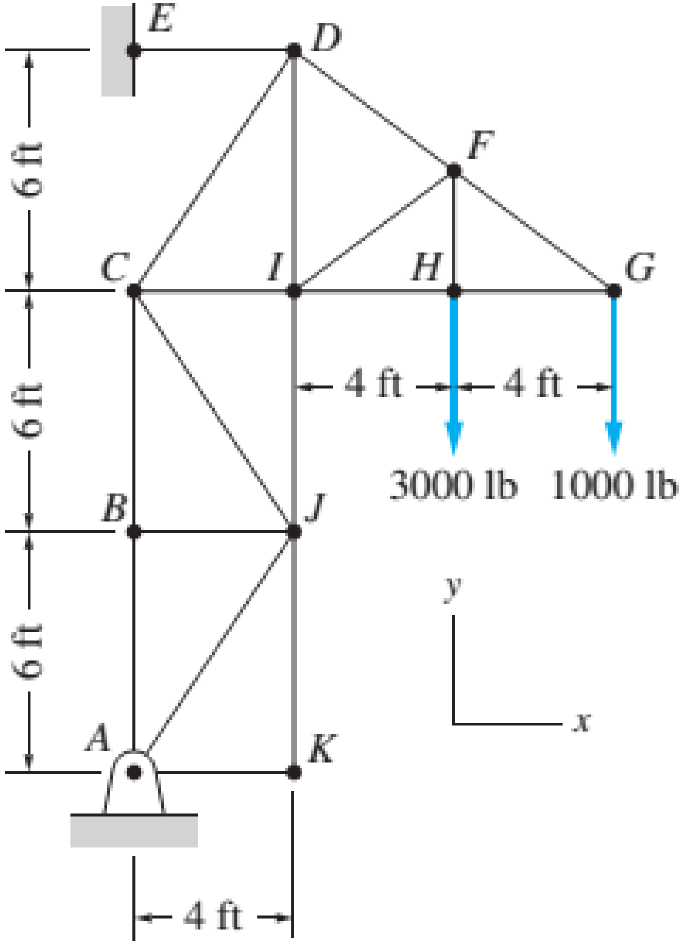
$$\sum M_H = 0 \text{ (clockwise being positive)} :$$

$$-F_{GI}(10\text{ ft}) - 33\text{ kips}(8\text{ ft}) + 16\text{ kips}(10\text{ ft}) = 0$$

$$F_{GI} = -10.4\text{ kips}$$

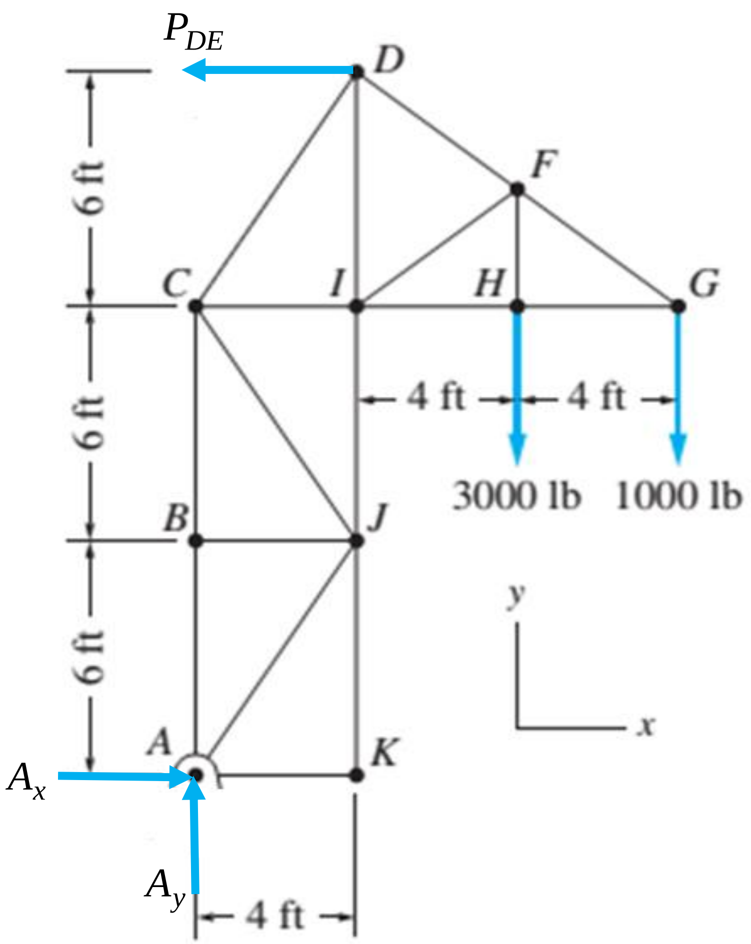
➢Example 5.4

Determine the forces in members *FI* and *JC* of the truss shown. Indicate tension or compression.



➢Example 5.4 - Solution

Equilibrium analysis on the whole truss to determine support reactions gives.



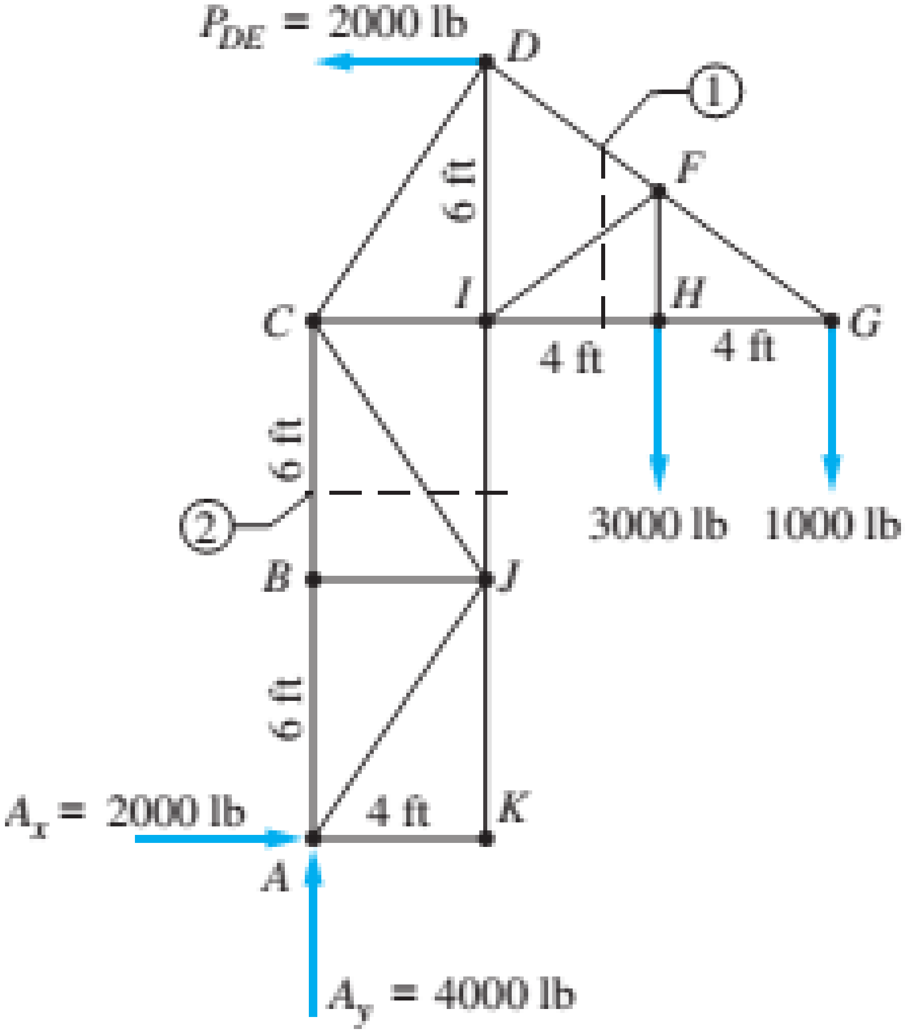
$$P_{DE} = 2000 \text{ lb}$$

$$A_x = 2000 \text{ lb}$$

$$A_y = 4000 \text{ lb}$$

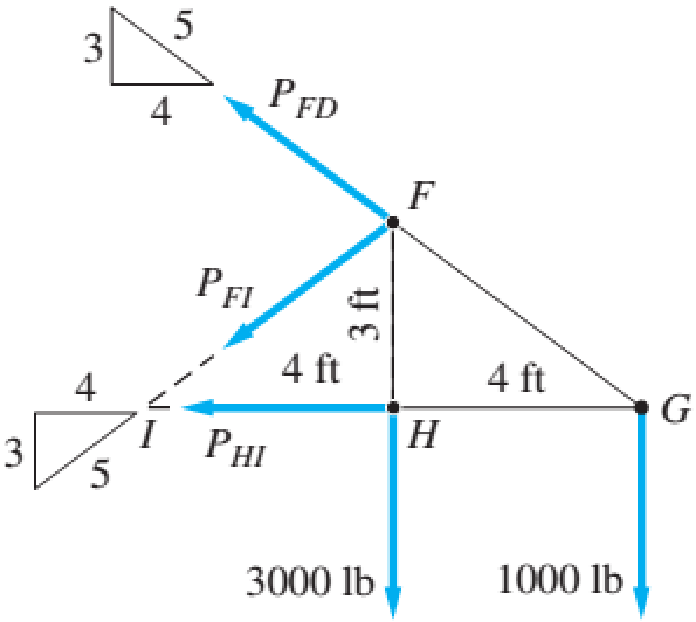
- Example 5.4 - Solution

The truss is divided into 2 with line 1 so *FI* can be determined, and with line 2 so *JC* can be determined.



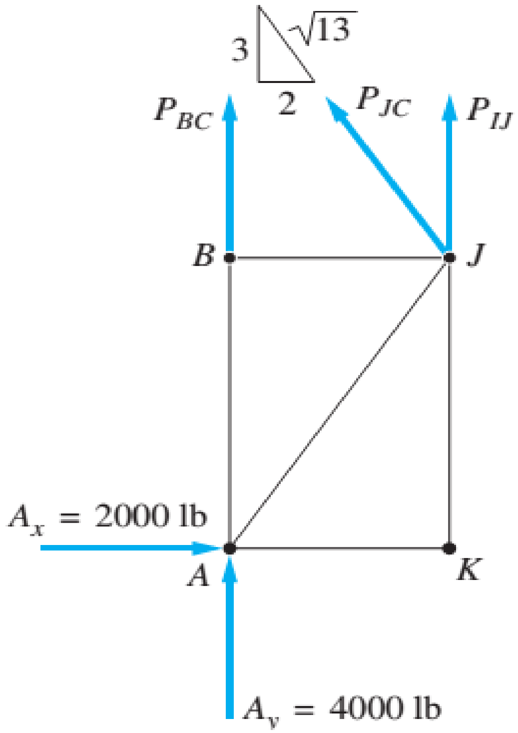
➢Example 5.4 - Solution

For *FI*



$$P_{FI} = -2500 \text{ lb}$$

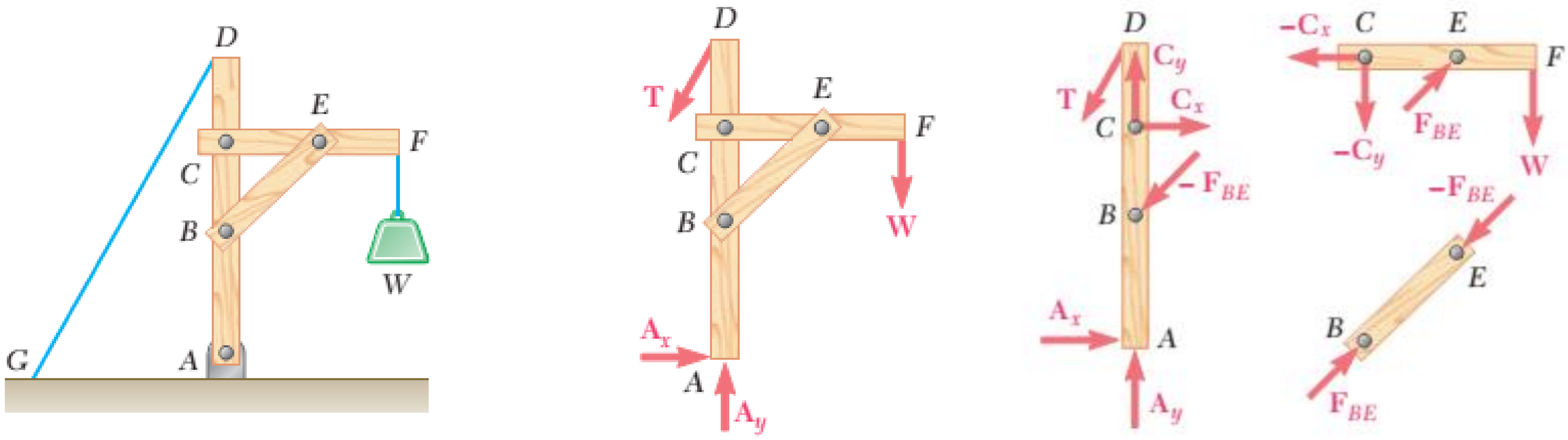
For *JC*



$$P_{JC} = -3610 \text{ lb}$$

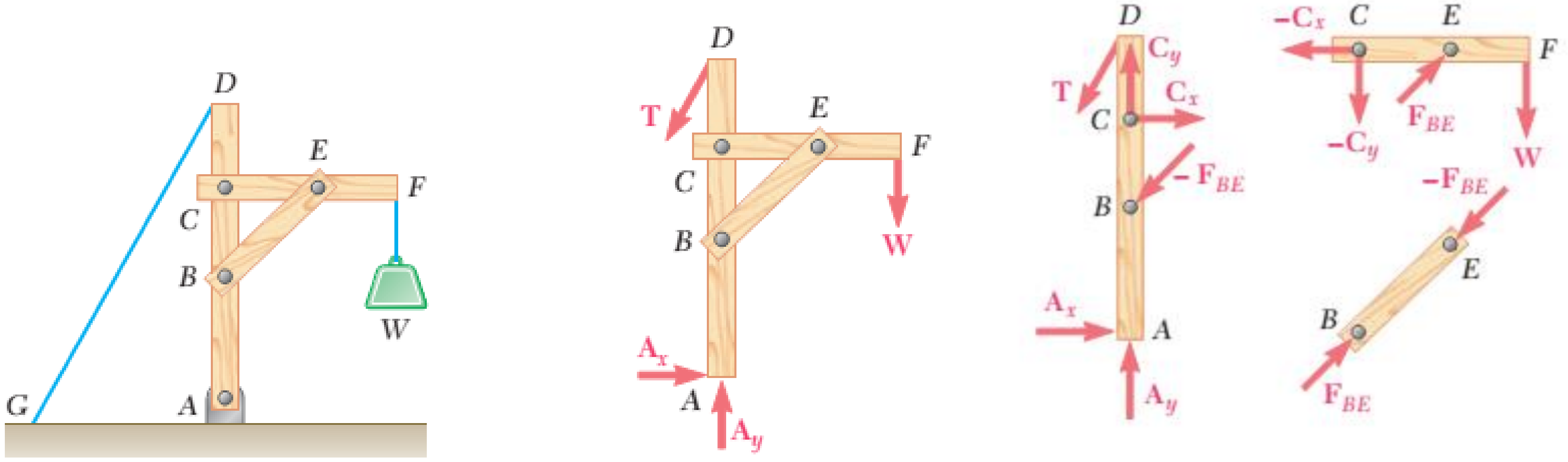
ANALYSIS OF STRUCTURES
Frames

- Frames and Machines are structures in which at least one member is a multiforce member (acted upon by three or more forces).
- While frames are designed to support forces, machines transmit and modify forces.



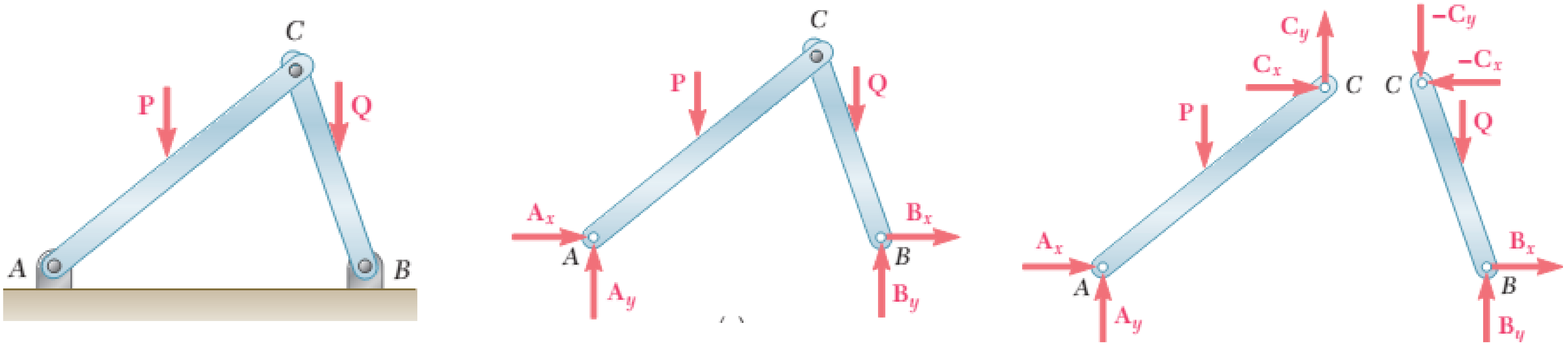
ANALYSIS OF STRUCTURES
Analysis of Frames

- Procedure is similar to what is used in trusses.
- An equilibrium analysis is first carried out on the entire frame.
- An equilibrium is then carried out on the individual members of the frame to determine all forces acting on each of them (It is sometimes easier if this is done first on the two force, then three force members).



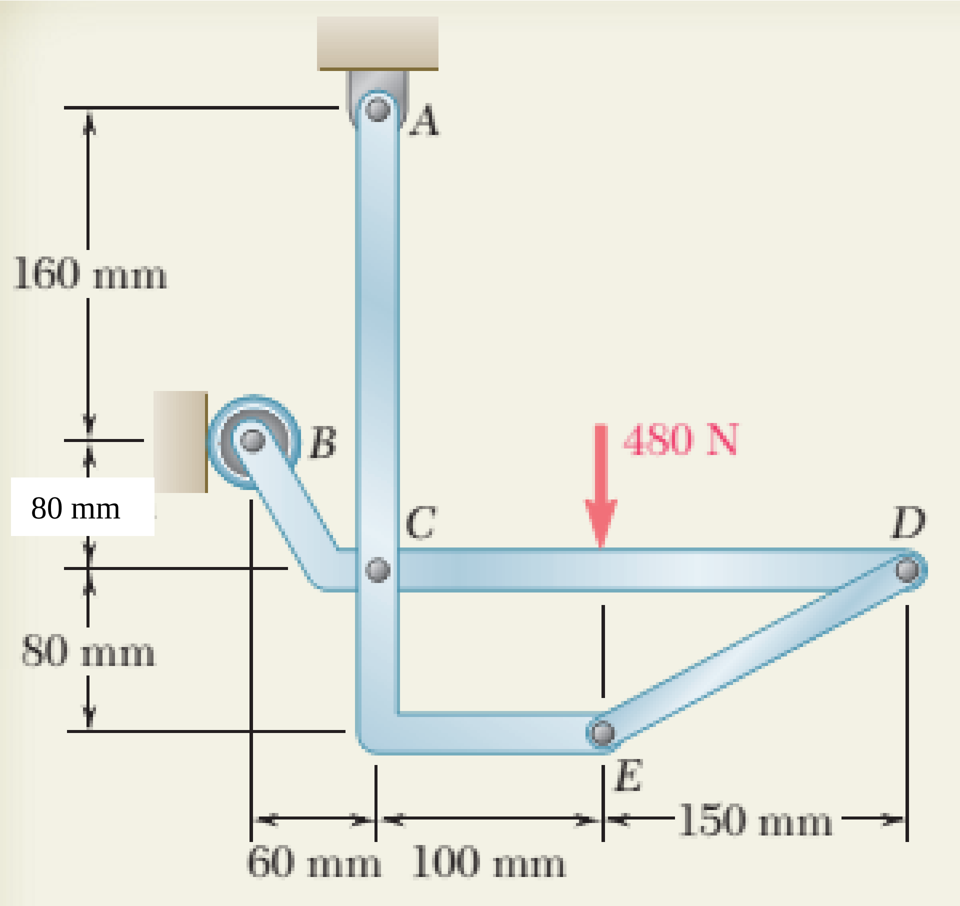
Analysis of Frames

- During analysis of non-rigid frames, the individual members of frame are considered as rigid members.



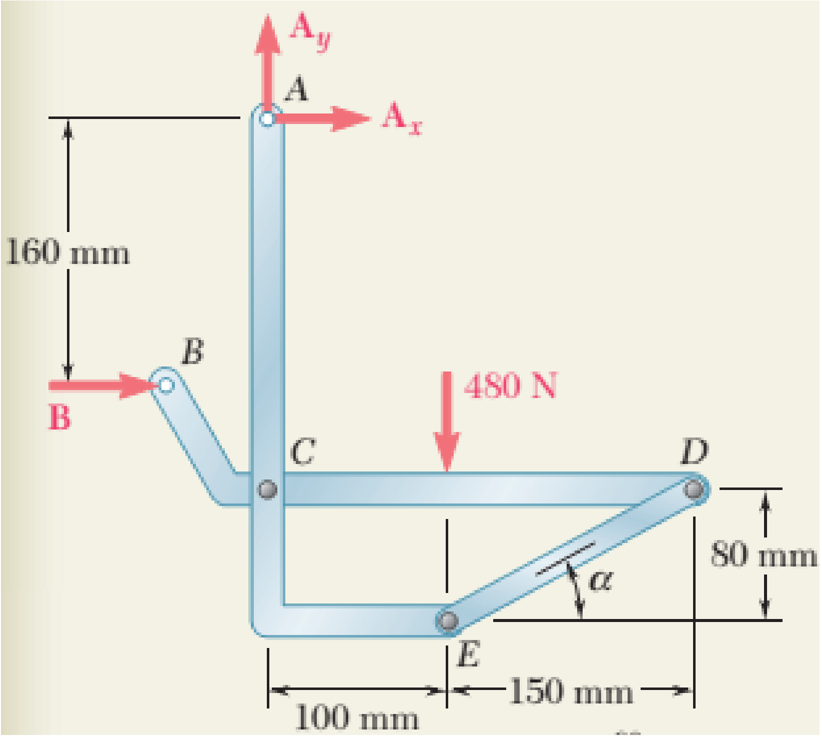
- Example 5.5

In the frame shown, members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*. Is link *DE* tension or compression?



➢Example 5.5 – Solution

Equilibrium analysis on the entire structure



For equilibrium,

$$+\rightarrow \sum F_x = 0 : B + A_x = 0$$

$$+\uparrow \sum F_y = 0 : A_y = 480 \text{ N}$$

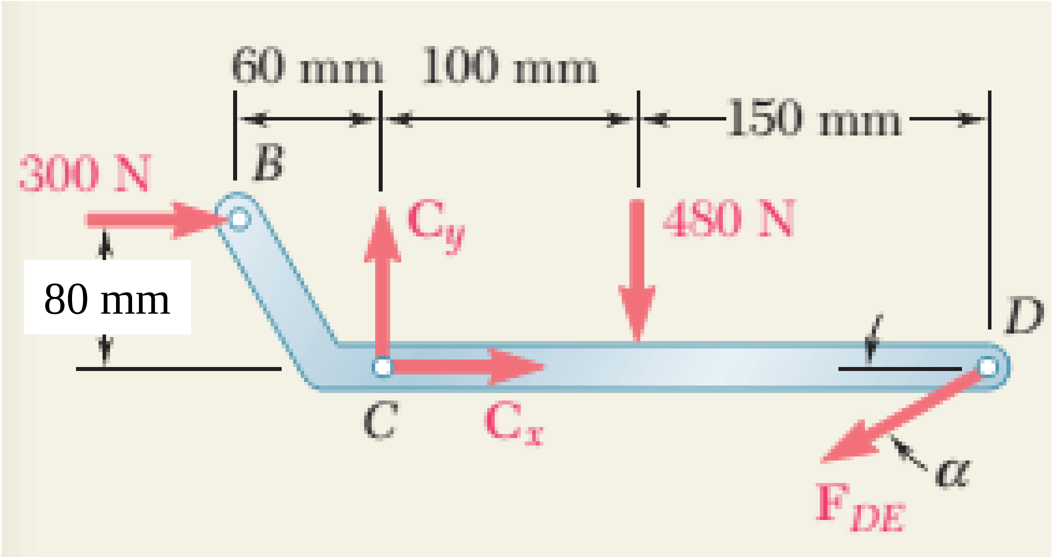
$$+\circlearrowleft \sum M_A = 0 : -B(160 \text{ mm}) + 480 \text{ N}(100 \text{ mm}) = 0$$

$$B = 300 \text{ N}$$

So,

$$A_x = -300 \text{ N}$$

Example 5.5 – Solution (Equilibrium analysis on individual links)



For equilibriu m,

$$+\rightarrow \sum F_x = 0 : 300\text{ N} + C_x - F_{DEx} = 0$$

$$+\uparrow \sum F_y = 0 : C_y - 480\text{ N} - F_{DEy} = 0$$

$$+\circlearrowleft \sum M_C = 0 : 300\text{ N}(80\text{ mm}) + 480\text{ N}(100\text{ mm}) + F_{DEy}(250\text{ mm}) = 0$$

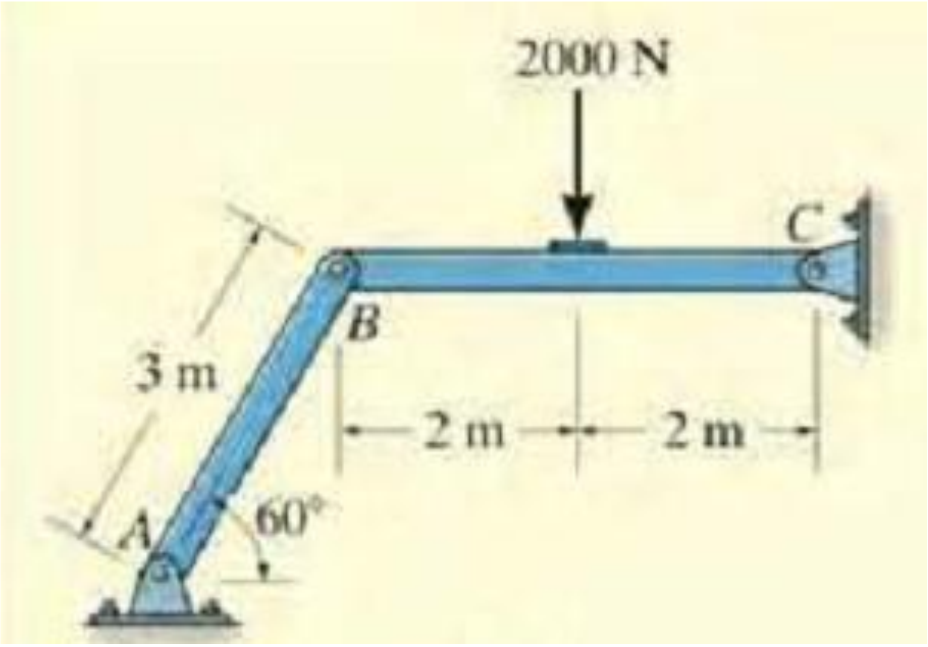
$$F_{DEy} = -264\text{ N} \qquad C_y = 216\text{ N}$$

$$\alpha = \tan^{-1}\left(\frac{80}{150}\right)$$

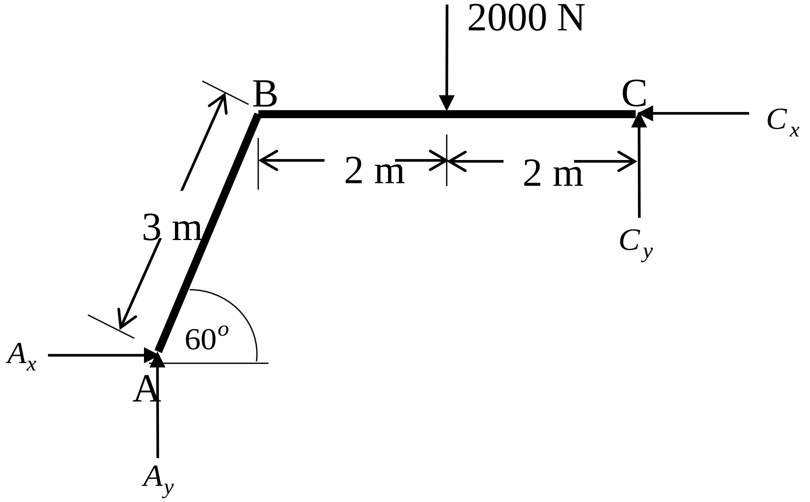
$$F_{DE} = -561\text{ N} \qquad F_{DEx} = -495\text{ N} \qquad C_x = -795\text{ N}$$

- Example 5.6

Determine the horizontal and vertical components of the force which the pin at *C* exerts on member *BC* of the frame shown below. Also determine the forces acting on both members at *B*



- Example 5.6 - Solution



For equilibrium,

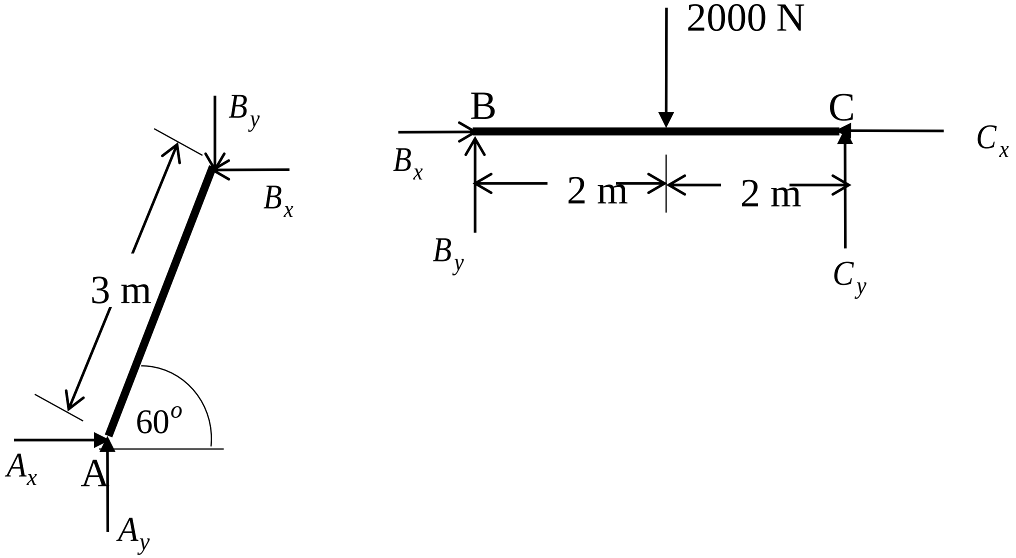
$$+\rightarrow \sum F_x = 0 : A_x - C_x = 0$$

$$+\uparrow \sum F_y = 0 : A_y + C_y - 2000\text{ N} = 0$$

$$+\circlearrowleft \sum M_C = 0 : -A_y(4 + 3\cos 60^o) + A_x(3\sin 60^o) + 2000\text{ N}(2\text{ m}) = 0$$

$$+\circlearrowleft \sum M_A = 0 : -C_y(4 + 3\cos 60^o)\text{m} + C_x(3\sin 60^o)\text{m} + 2000\text{ N}(2 + 3\cos 60^o)\text{m} = 0$$

$$A_x = C_x = 577\text{ N} \qquad A_y = C_y = 1000\text{ N}$$



Taking member BC, for equilibrium,

$$+\rightarrow \sum F_x = 0 : B_x - C_x = 0$$

$$+\uparrow \sum F_y = 0 : B_y + C_y - 2000\text{ N} = 0$$

$$B_x = 577\text{ N}$$

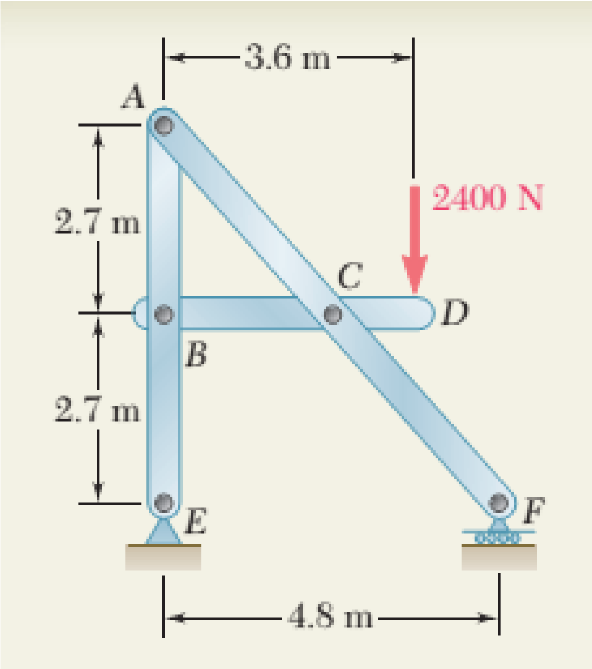
$$B_y = 1000\text{ N}$$

For member AB,

$$B_x = 577\text{ N} \leftarrow$$

$$B_y = 1000\text{ N} \downarrow$$

- Example 5.7
- Determine the components of the forces acting on each member of the frame shown.



$F = 1800\ \text{N}$

$E_y = 600\ \text{N}$

$E_x = 0\ \text{N}$

$A_x = 0\ \text{N}$

$A_y = 1800\ \text{N}$

$B_x = 0\ \text{N}$

$B_y = 1200\ \text{N}$

$C_x = 0\ \text{N}$

$C_y = 1000\ \text{N}$

LECTURE 7

A BRIEF OVERVIEW FRICTION

FRICTION

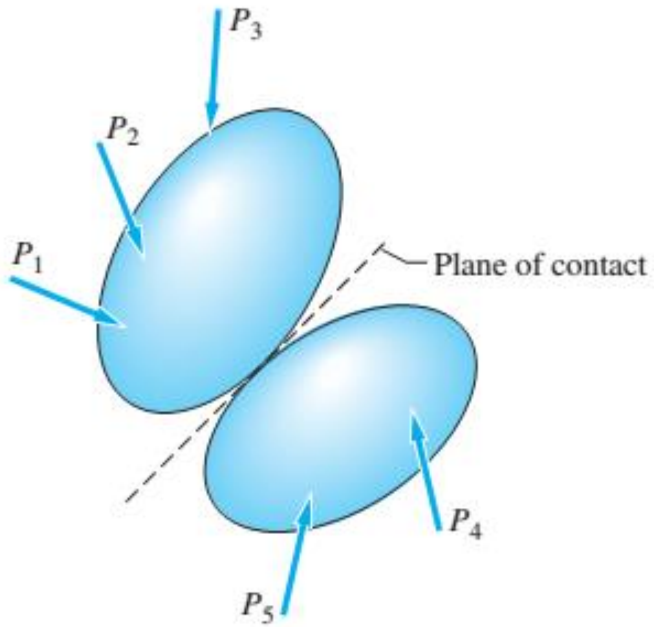
- Friction resists relative sliding motion between two surfaces that are in contact.
- Friction may be helpful; as seen during walking, applications to clutches, belts etc.
- It may also be detrimental; wear in machinery, reduction of efficiency in transmission of power by converting mechanical energy into heat.
- It plays a very important role in a lot of engineering applications.

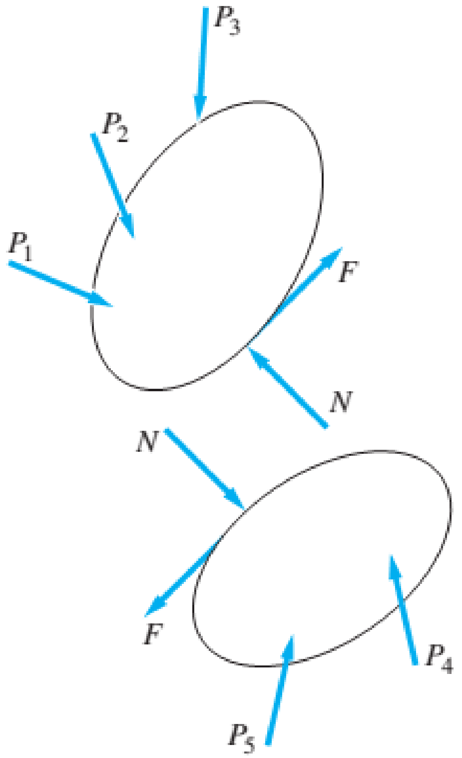
- Friction may be dry or wet.
- *Dry* or *Coloumb* Friction occurs when there is no layer of fluid separating the contacting surfaces.

FRICTION

Coloumb's Theory of Dry Friction

- Has been found to give satisfactory results in practical problems.
- Explanation:
  - Two bodies in contact, will each experience a normal reaction, N and Friction force, F which will act in a direction opposite to that the body wants to move in.





FRICTION
Coloumb's Theory of Dry Friction

- The relation between the Normal and the Friction force can be one of three cases:
  - Static Case: where there is no sliding movement between the two bodies, *N* and *F* are related by

$$F \le F_{\max} = \mu_s N$$

and there will be no movement so long as

$$F \le F_{\max}$$

  - Impending Sliding Case:

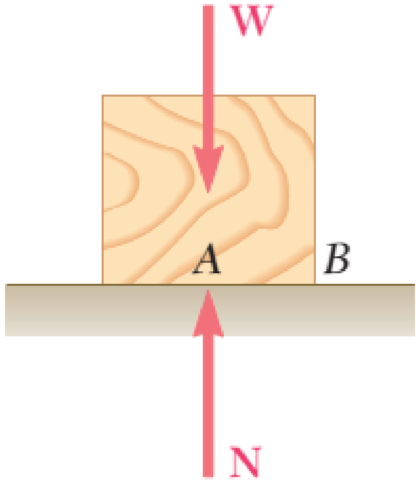
$$F = F_{\max} = \mu_s N$$

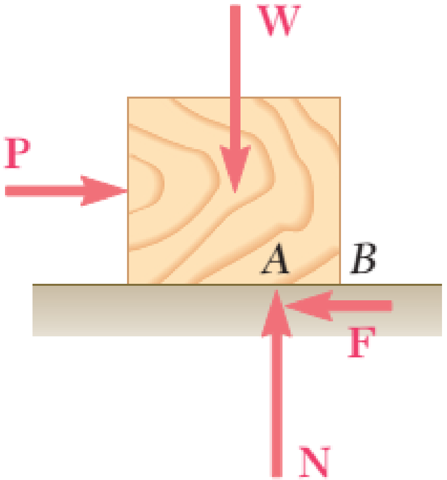
  - Dynamic Case: when the two bodies are already sliding relative to each other,

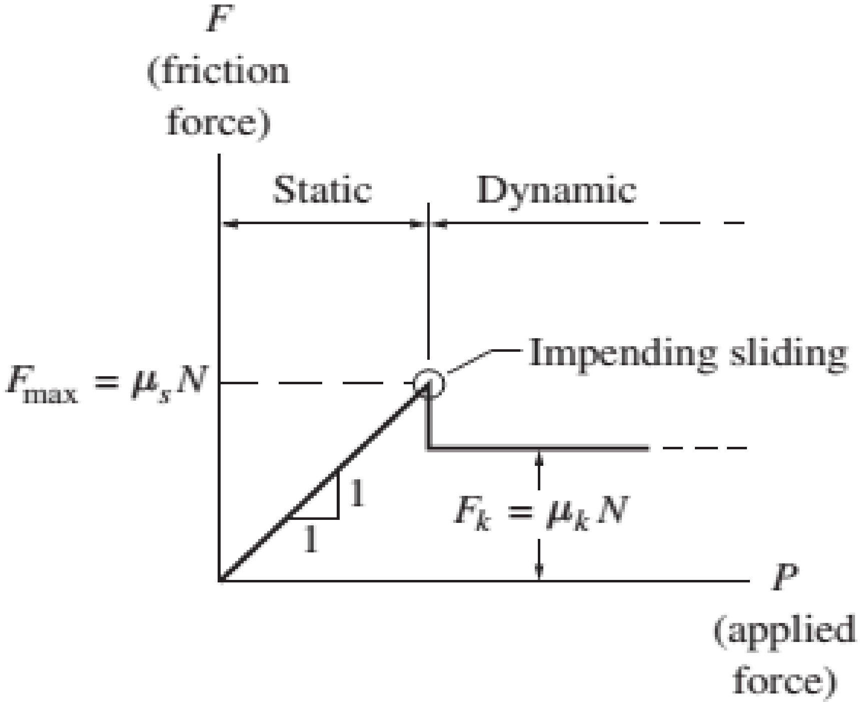
$$F_k = \mu_k N$$

FRICTION
Coloumb's Theory of Dry Friction

- When a force P is applied to a body in the direction shown, the body experiences friction in the direction shown. The body will move when P exceeds F.
- Once the body is moving, the force P required to keep it moving is less than what was required to make it start moving. Hence, Static Friction Force > Dynamic Friction force





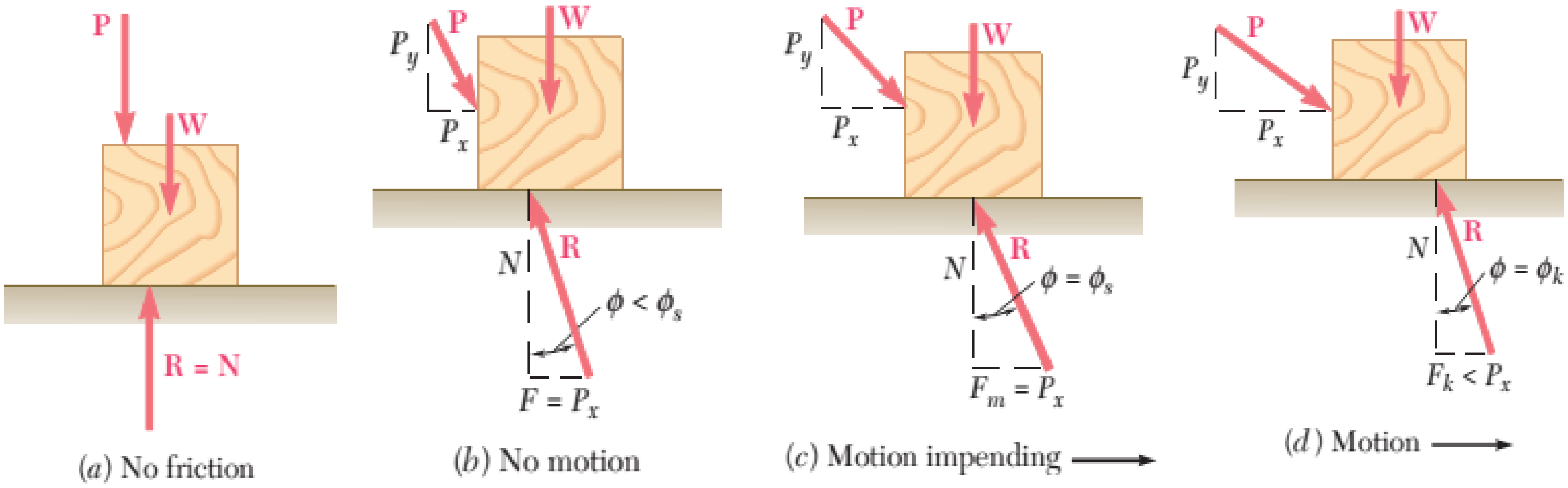


Variation of Friction Force with Applied Force
Source: Engineering Mechanics – Statics by Pytel

FRICTION

Angles of Dry Friction

- The Friction force and Normal Reaction acting on a body can be resolved into a resultant.
- The angle this resultant makes with the normal reaction is referred to as the angle of friction.
- There can be the angle of static friction and the angle of kinetic friction.



$$\tan \phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan \phi_s = \mu_s$$

$$\tan \phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

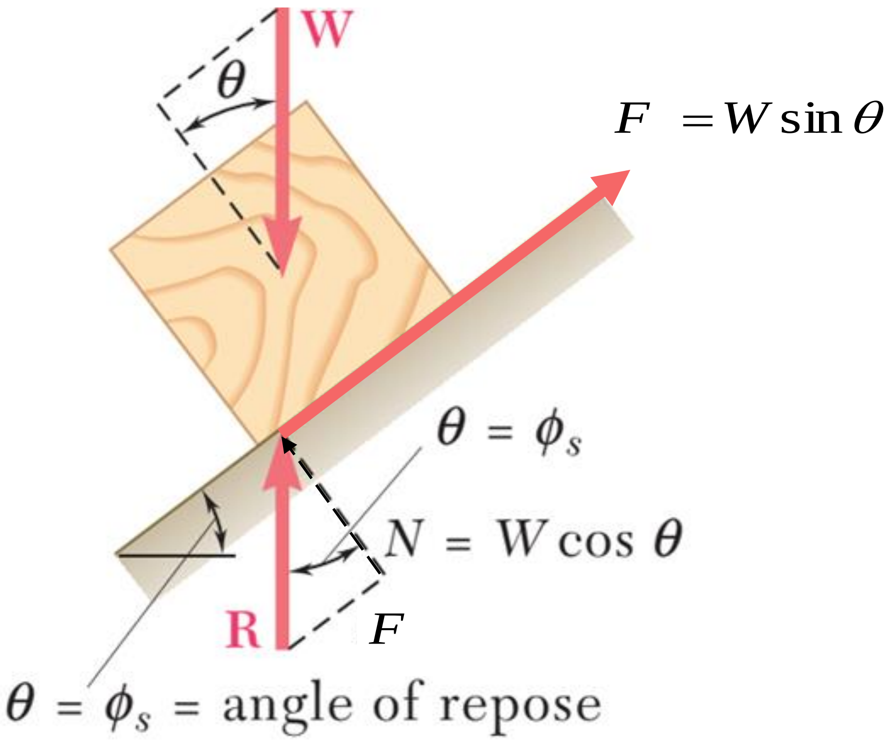
$$\tan \phi_k = \mu_k$$

FRICTION
Angles of Dry Friction

- The co-efficient of static friction between two surfaces is determined experimentally.
- A block of one material is placed on a plane of variable inclination of the second material.
- Inclination is increased until sliding is imminent, in which case $F_s = F_{\max} = \mu_s N$

$$\phi_s = \theta = \frac{F}{N} = \frac{\mu_s N}{N} = \mu_s$$

- θ is known as the *angle of repose*
- $\mu_s = \tan\theta$



Motion impending

FRICTION

Laws of Dry Friction

- Coloumb and other subsequent works, give the following laws of friction:
  - The maximum frictional resistance that can be developed is independent of the size of the areas in contact.
  - The maximum static Friction Force, F_{max} and the dynamic friction force F_k are each proportional to the normal reaction between the surfaces.

$$F_{\max} = \mu_s N$$

$$F_k = \mu_k N$$

where μ_s and μ_k are the static and kinetic co-efficients of friction respectively.

  - The limiting value of Static friction force is greater than the kinetic friction force
  - For low velocities, the kinetic frictional resistance is practically independent of velocity.

FRICTION

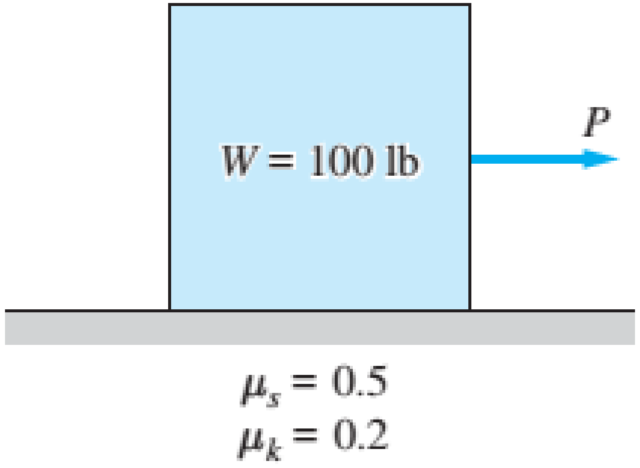
Some Limitations

- Some limitations of Coloumb's Theory and other related works include:
  - The co-efficients of friction are all determined experimentally and are at best approximations. They may vary with environmental conditions, the condition of the surfaces among other factors.
  - The theory of dry friction is applicable only to surfaces that are dry or that contain only a small amount of lubricant. If there is relative motion between the surfaces of contact, the theory is valid for low speeds only.
  - There are situations where the amount of friction between surfaces depends on the area of contact. For example, the traction (friction force) between an automobile tire and the pavement can be increased under certain conditions by letting a small amount of air out of the tire, thus increasing the contact area.

FRICTION

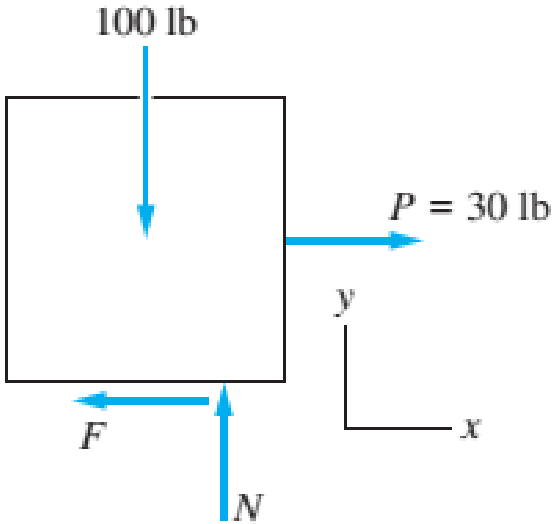
- Problems involving dry friction are solved in pretty much the same way as equilibrium problems.
- Free body diagrams on which forces and dimensions must be correctly indicated.
- Friction forces must always oppose the direction of motion.
- Problems involving dry friction may fall into one of three possible scenarios;
  - It is not known if there is impending sliding
    - Assume equilibrium, solve equilibrium equations for unknowns, check to see if $F \le \mu_s N$. If not, the problem is a dynamic problem in which there is sliding.
  - There is impending sliding, and impending sliding surfaces are known.
    - $F = F_{\max} = \mu_s N$, solve the equilibriums equations for the unknowns.
  - There is impending sliding, but the impending sliding surfaces are not known.
    - Identify surfaces for possible motion, set $F = F_{\max} = \mu_s N$, solve equilibrium equations for unknowns and choose correct surface by comparison of results.

- Example 7.1
- A force of P of 30 lb is applied as shown to a 100 lb block that was initially at rest. Determine if the block will slide or not.



Example 7.1 – Solution

FBD



Assuming equilibrium,

$$+\rightarrow \sum F_x = 0; P - F = 0$$

$$+\uparrow \sum F_y = 0; -100\text{ lb} + N = 0$$

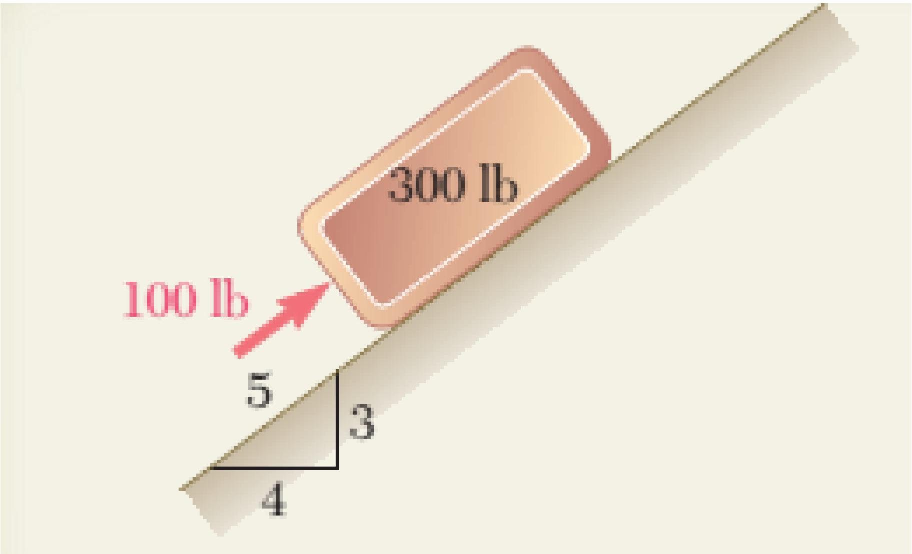
$$F = 30\text{ lb}$$

$$N = 100\text{ lb}$$

$$F_{\max} = \mu_s N = 0.5(100\text{ lb}) = 50\text{ lb}$$

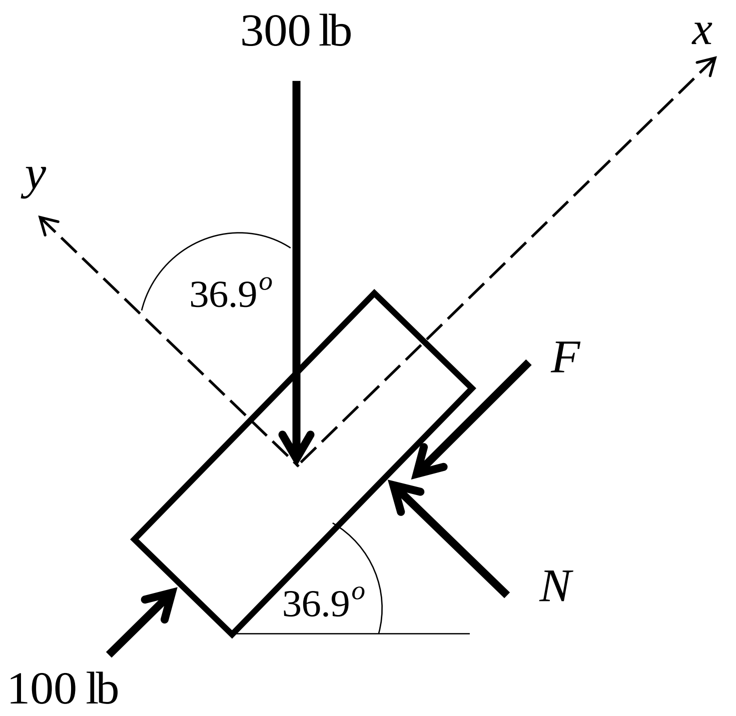
Since $F < F_{\max}$, the body is in static equilibrium and will remain at rest.

- Example 7.2
- A 100-lb force acts as shown on a 300-lb block placed on an inclined plane. The coefficients of friction between the block and the plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine if the block will slide down the plane and also determine value of the friction force.



FRICTION

- Example 7.2 – Solution z



Assuming equilibrium,

$$+\nearrow \sum F_x = 0; 100 \text{ lb} - (300 \sin 36.9) \text{ lb} - F = 0$$

$$+\nwarrow \sum F_y = 0; -(300 \cos 36.9) \text{ lb} + N = 0$$

$$F = -80 \text{ lb}$$

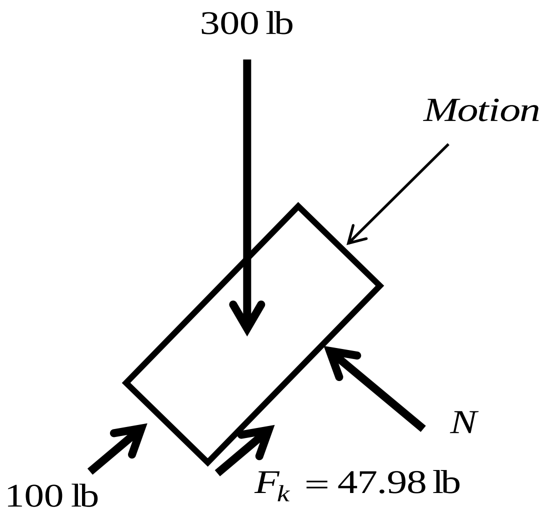
$$N = 239.9 \text{ lb}$$

$$F_{\max} = \mu_s N = 0.25(239.9 \text{ lb}) = 59.98 \text{ lb}$$

Since $|F| > |F_{\max}|$, the body will not be at rest. It will slide down the plane.

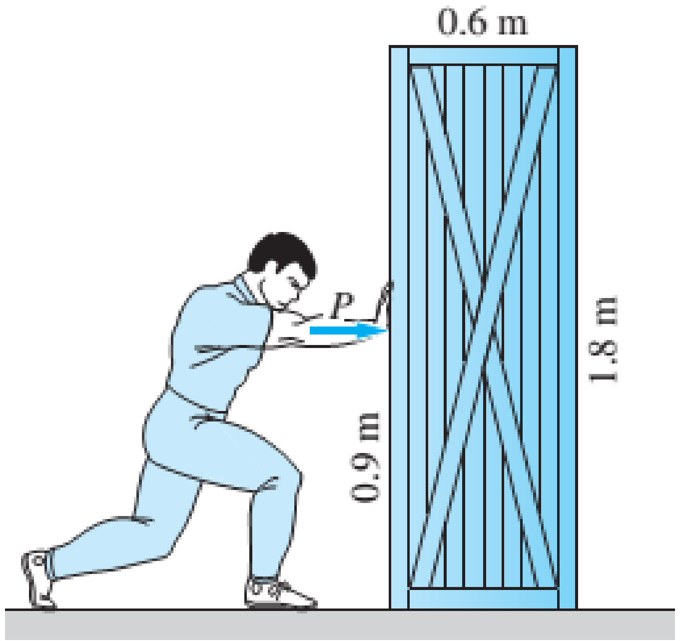
Therefore, Force due to friction is dynamic. It will be given by

$$F_k = \mu_k N = 0.20(239.9 \text{ lb}) = 47.98 \text{ lb}$$



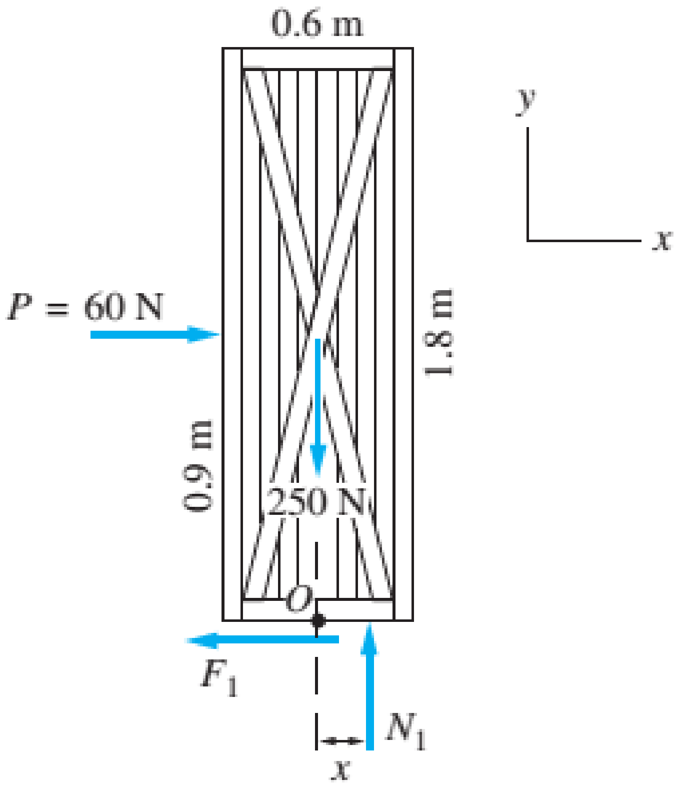
FRICTION

- Example 7.3
- A force of 60 N is being applied to a 250 N crate by a man as shown below. Determine if the crate will remain in static equilibrium. The weight of the crate acts through its geometric center, and the coefficient of static friction between the crate and the floor is 0.3



FRICTION

- Example 7.3 - Solution



Assuming equilibriu m,

$$+\rightarrow \sum F_x = 0; -F_1 + 60\ \text{N} = 0$$

$$+\uparrow \sum F_y = 0; N_1 - 250\ \text{N} = 0$$

$$+\circlearrowleft \sum M_O = 0; -N_1 x + (P)0.9\ \text{m} = 0$$

$$x = 0.216\ \text{m} < 0.3\ \text{m}$$

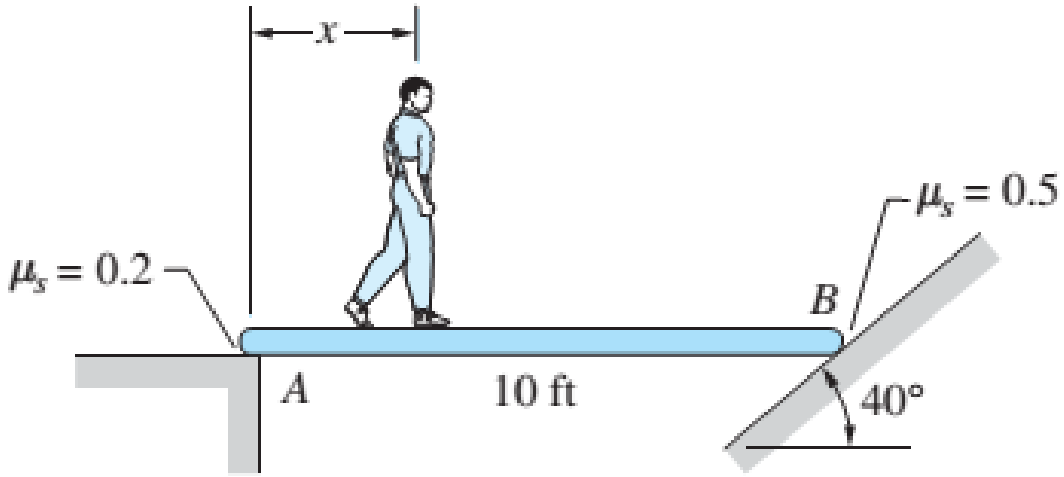
The crate will not tip over

$$F_{1max} = \mu_s N_1 = 0.3(250) = 75\ \text{N}$$

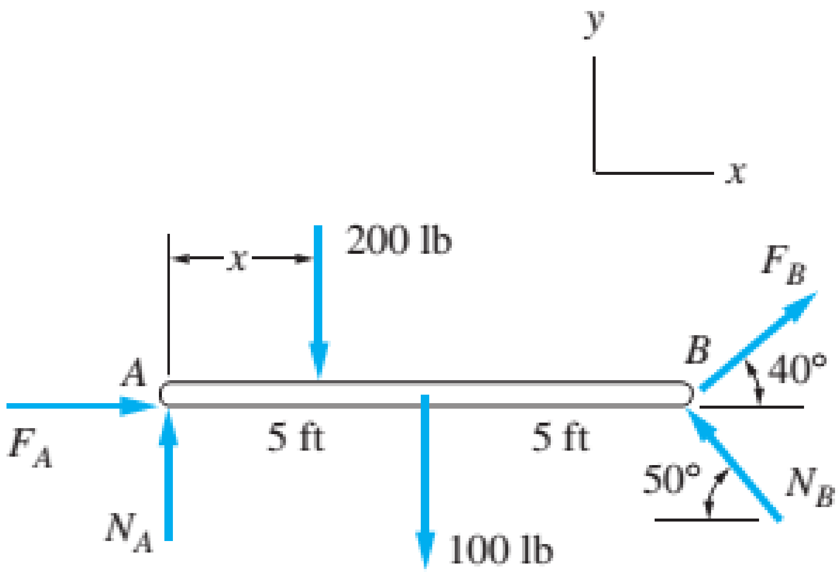
$\text{P} < F_{1max}$

$\therefore$ Crate will not slide

- Example 7.4
- The uniform 100-lb plank is resting on friction surfaces at A and B. The coefficients of static friction are shown in the figure. If a 200-lb man starts walking from A toward B, determine the distance *x* when the plank will start to slide.



➢Example 7.4 – Solution



Assuming equilibrium,

$$+\rightarrow \sum F_x = 0;\ F_A - (N_B \cos 50)\,\text{lb} + (F_B \cos 40)\,\text{lb} = 0$$

$$+\uparrow \sum F_y = 0;\ N_A - 200\text{ lb} - 100\text{ lb} + (N_B \sin 50)\,\text{lb} + (F_B \sin 40)\,\text{lb} = 0 F = -80\text{ lb}$$

$$+\circlearrowleft \sum M_A = 0;\ (200\text{ lb})x + (200\text{ lb})5\text{ ft} - (F_B \sin 40\text{ lb})10\text{ ft} - (N_B \sin 50\text{ lb})10\text{ ft} = 0$$

Subtituting $F_A = 0.2N_A$ and $F_B = 0.5N_B$ and solving,

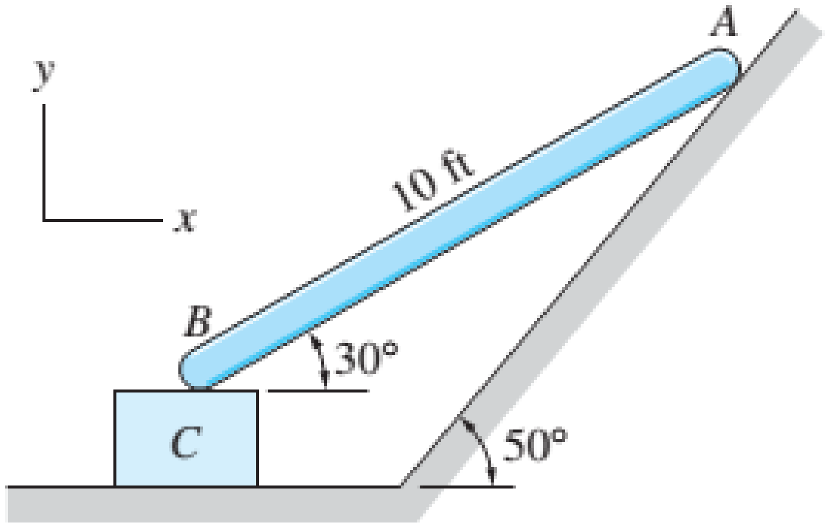
$N_A = 163.3$ lb

$N_B = 125.7$ lb

$x = 4.34$ ft

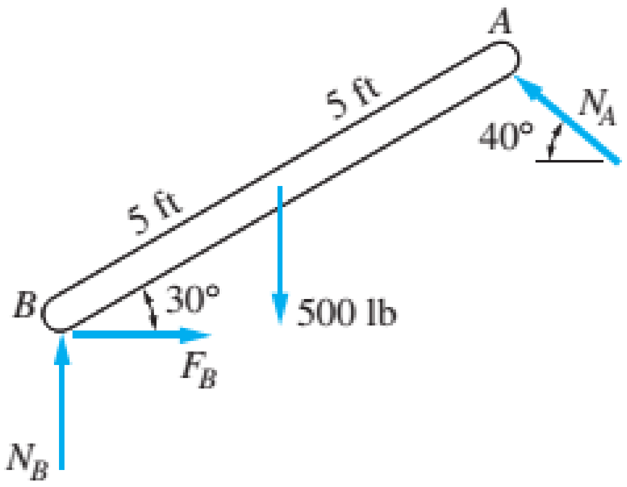
- Example 7.5

- Determine if the system below will be in static equilibrium. The uniform bar AB weighs 500 lb, and the weight of block C is 300 lb. Friction at A is negligible, and the coefficient of static friction is 0.4 at the other two contact surfaces.

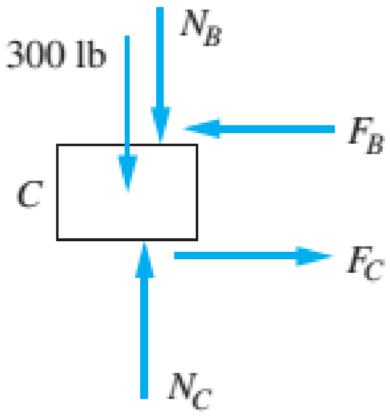


Ans:
The system will not be in equilibrium because $F > F_{max}$ for the block, C.

- Example 7.5-Solution

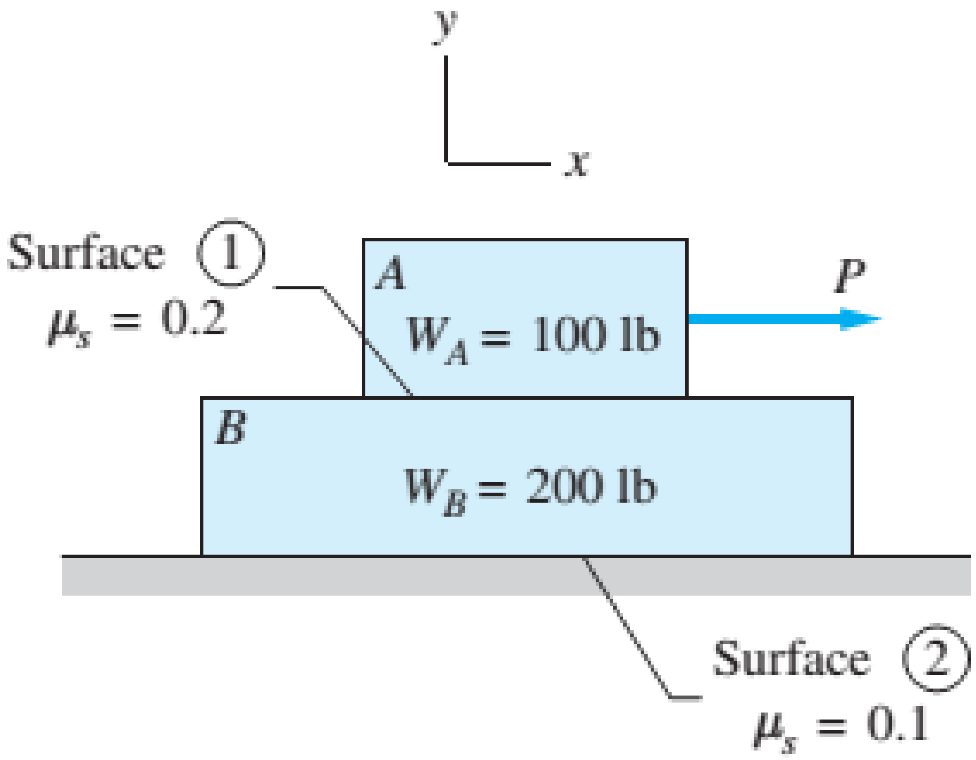


(b)

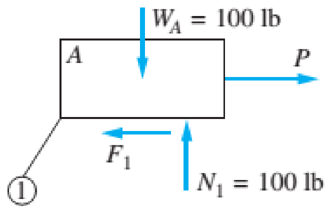


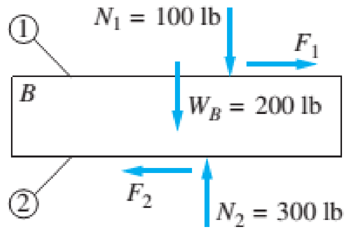
Ans:
The system will not be in equilibrium because F > F_{max} for the block, C.

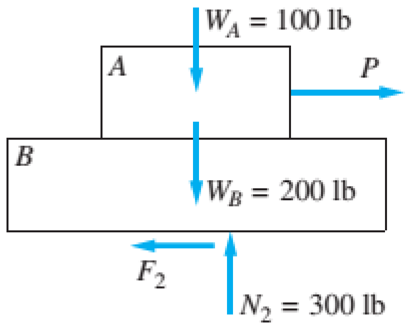
- Example 7.5
- Determine the maximum force P that can be applied to block A without causing either block to move.



- Example 7.5 - Solution







(b)

For surface 1,

$$F_1 = (F_1)_{max} = (\mu_s)_1 N_1 = 0.2(100) = 20 \text{ lb}$$

The FBD of block *A* then gives

$$\Sigma F_x = 0 \quad \xrightarrow{+} \quad P - F_1 = 0$$

$$P = F_1 = 20 \text{ lb}$$

For surface 2,

$$F_2 = (F_2)_{max} = (\mu_s)_2 N_2 = 0.1(300) = 30 \text{ lb}$$

From the FBD of the entire system, Fig. (b), we then obtain

$$\Sigma F_x = 0 \quad \xrightarrow{+} \quad P - F_2 = 0$$

$$P = F_2 = 30 \text{ lb}$$

Hence, the largest force that can be applied without causing motion is 20 lb

TAKE HOME ASSIGNMENT II

LECTURE 8

SIMPLE MACHINES

SIMPLE MACHINES

- Machines make work easier.
- They normally transform input forces into output forces in a more convenient form for a required task.
- Simple machines in mechanics fall into six broad categories:
  - Lever
  - Inclined plane
  - Wedge
  - Screw
  - Wheel and axle
  - Pulley
- Compound machines comprise two or more simple machines that work together to achieve a required task.

SIMPLE MACHINES
Some Terms and Definitions

- Effort – force applied to do work at on some part of the machine.
- Load – external force that is overcome by the effort in doing work.
- Effort and Load Distances – distances effort and load must move through in order for work to be done.
- Mechanical Advantage – ratio of load to effort.
- Velocity Ratio – ratio of effort distance to load distance.
- For an ideal machine, work done by the effort is equal to work that should be done on the load.

$$P \times \text{distance through which } P \text{ moves} = W \times \text{distance through which } W \text{ moves}$$

$$\therefore \frac{W}{P} = \frac{\text{distance through which } P \text{ moves}}{\text{distance through which } W \text{ moves}} = \textit{Velocity Ratio}$$

$$M.A. = V.R.$$

- Ideal machines don't exist in real life.

SIMPLE MACHINES
Some Terms and Definitions

- Efficiency of the machine – ratio of useful work done to work supplied.

$$Efficiency = \frac{Useful\ work\ done\ by\ the\ machine}{work\ input\ to\ the\ machine}$$

If x and y are the distances moved through by the Effort, (P) and the Load, (W) respectively, then

$$Efficiency = \frac{Wy}{Px}$$

$$= \frac{W/P}{x/y} = \frac{Mechanical\ Advantage}{Velocity\ Ratio}$$

Overhauling in Machines

- Overhauling in simple machines refers to a scenario in which the when the effort on the machine is removed, it turns to run back with the load now acting as the effort.
- Machines must be *self-locking* or *self-sustaining* in order for this not to happen
- Machines overhaul only if their efficiency is greater than 50%.
- Proof:

For a machine running a direction assumed to be the foward direction,

$$E_{in} = E_{out} + E_{loss}$$

$$Efficiency = \frac{E_{out}}{E_{in}} = \frac{E_{out}}{E_{out} + E_{loss}} = \frac{1}{1 + \left(\frac{E_{loss}}{E_{out}}\right)}$$

SIMPLE MACHINES
Overhauling in Machines

- If this machine runs backward now, with the effort removed,

$$\text{Work done in lowering the load} = E_{out}$$

$$\text{Energy available for this} = E_{out} - E_{loss}$$

$$\text{If } E_{in} = E_{out} > E_{loss}$$

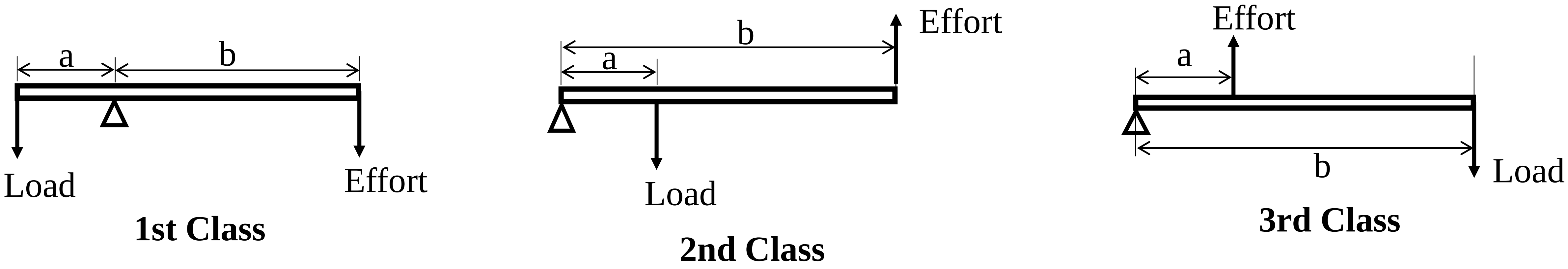
$$Efficiency = \frac{E_{out}}{E_{in}} = \frac{E_{out}}{E_{out} + E_{loss}} = \frac{1}{1+\left(\frac{E_{loss}}{E_{out}}\right)} => 50\%$$

- Else, it means the losses in the system are so great the amount of energy the load supplies to the machine when it is running backwards, is too small and can't overcome them. Hence, the machine can't overhaul, so it will ***self-lock.***

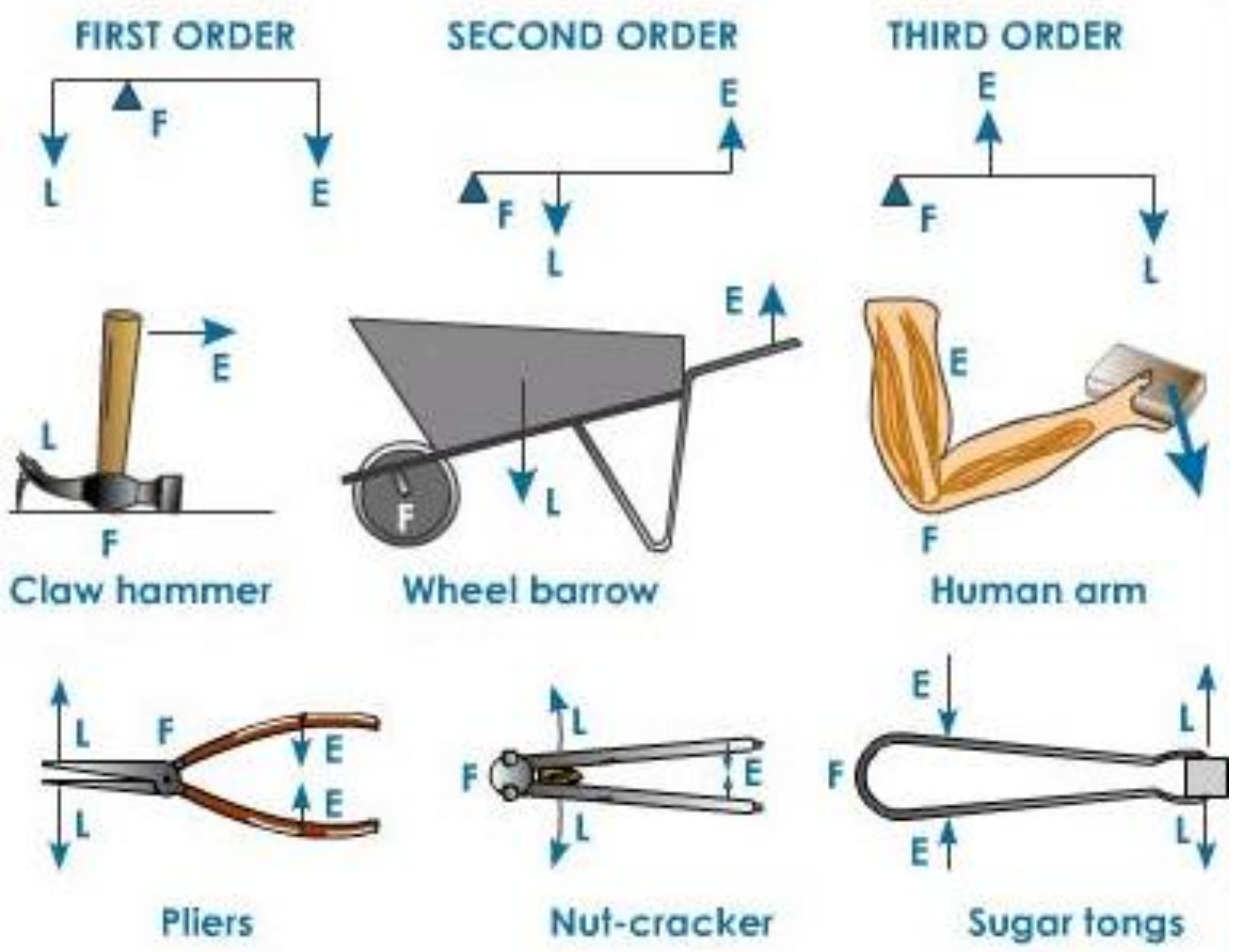
SIMPLE MACHINES

The Lever

- A rigid member that is capable or rotating about a fulcrum such that the effort at one point can overcome a load at another point.
- Classified according to positions of effort and load
- The relationship between load and effort is given by $Wa = Pb$



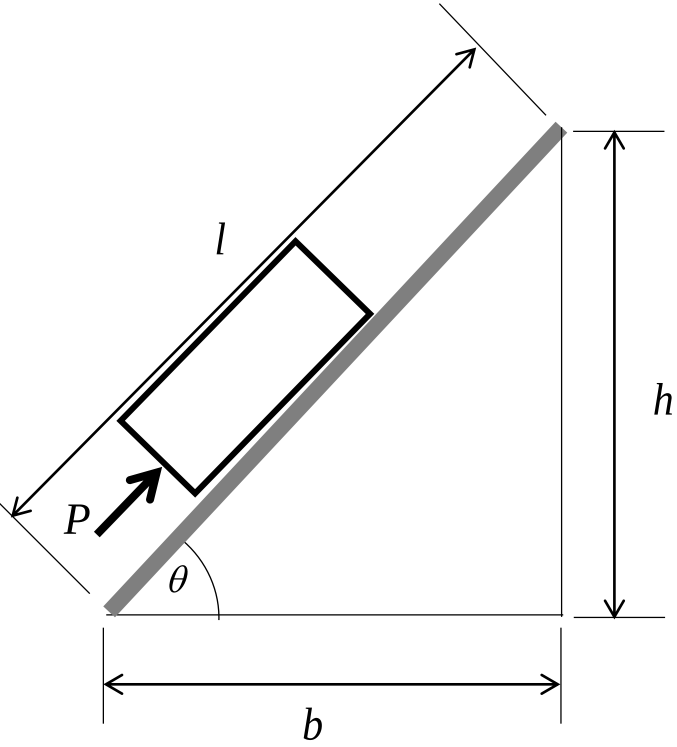
SIMPLE MACHINES
The Lever

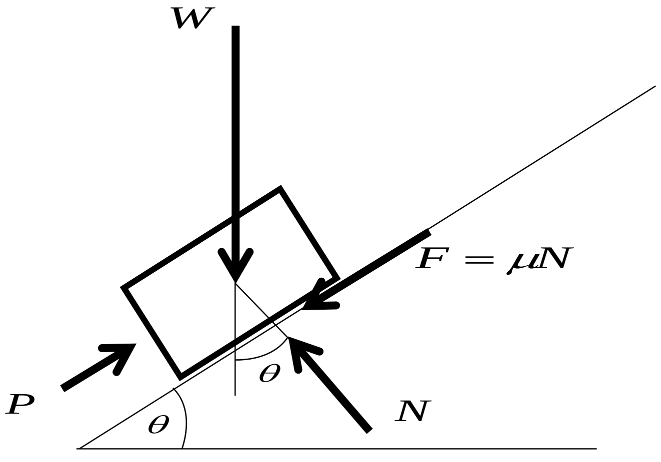


SIMPLE MACHINES

The Inclined Plane

➢Normally used to move heavy bodies through a vertical distance by moving it up an inclined plane.





FBD

Summing forces,

$$+\nearrow \sum F_{parallel} = 0;\ P - W\sin\theta - F = 0$$

$$+\nwarrow \sum F_{perpendicular} = 0;\ W\cos\theta - N = 0$$

$$P = W(\sin\theta + \mu\cos\theta)$$

$$M.A. = \frac{W}{P} = \frac{1}{(\sin\theta + \mu\cos\theta)}$$

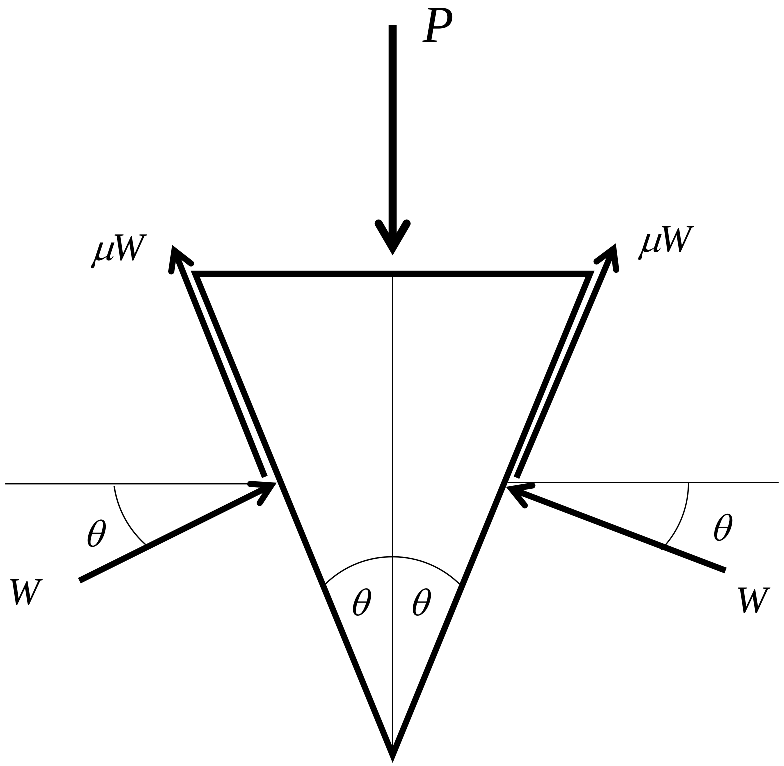
SIMPLE MACHINES
The Inclined Plane

- Example
- A 100 kg crate is being loaded onto the bed of a pickup truck using a ramp. The floor of the truck's bed is 1.2 m from the ground. If the crate is being pushed with a force of 30N, determine the angle of inclination, as well as the length of the ramp. Take the co-efficient of static friction between the crate material and ramp material to be 2.5.

SIMPLE MACHINES

The Wedge

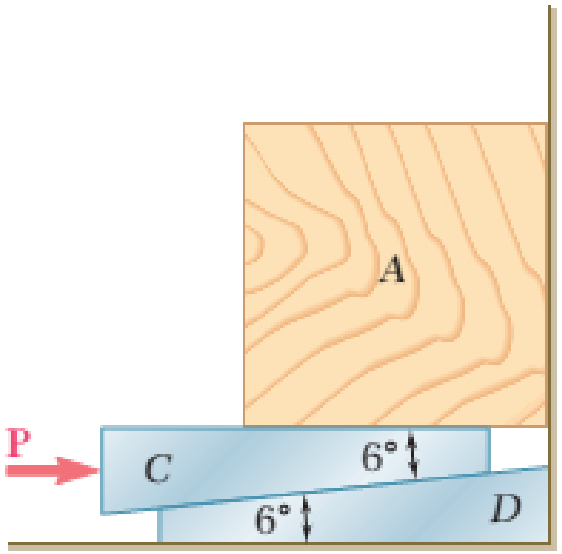
- This is basically two inclined planes set together.

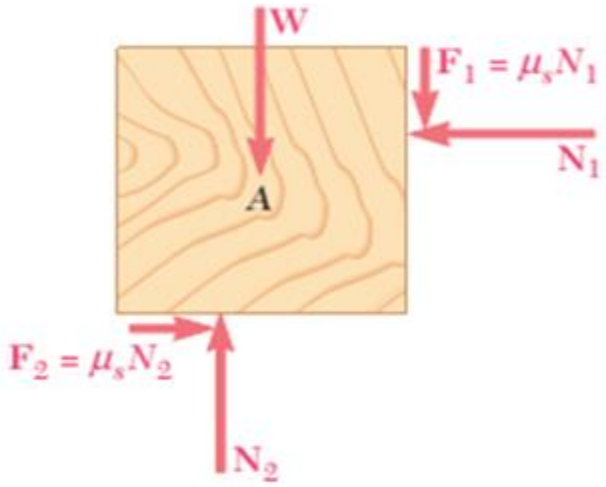


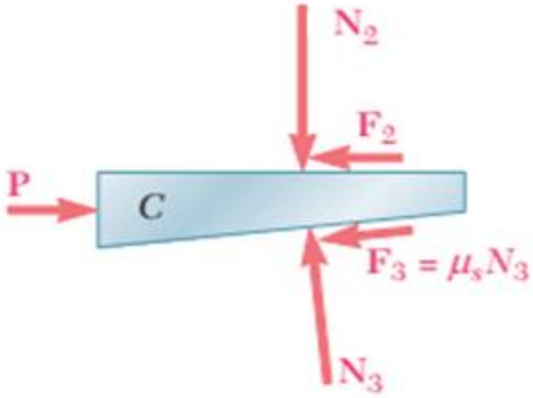
$$P = 2W(\sin\theta + \mu\cos\theta)$$

$$M.A. = \frac{W}{P} = \frac{1}{2(\sin\theta + \mu\cos\theta)}$$

- Wedges are also used to raise loads.







SIMPLE MACHINES

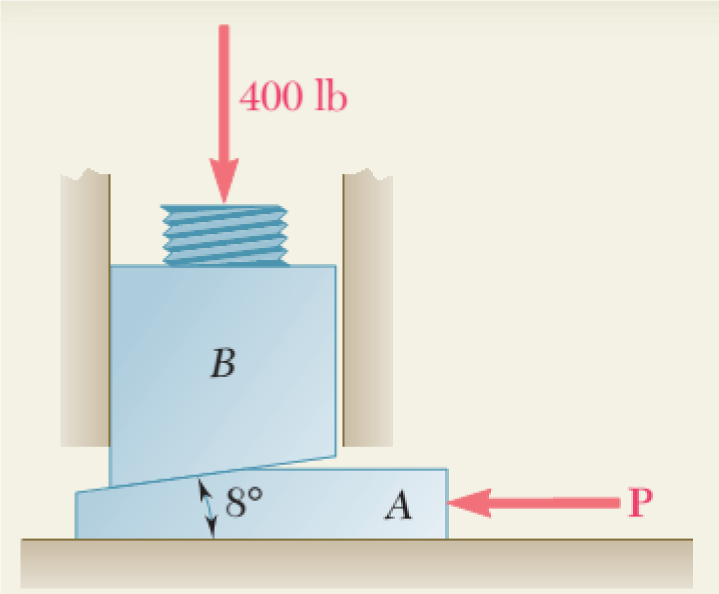
The Wedge

- Example

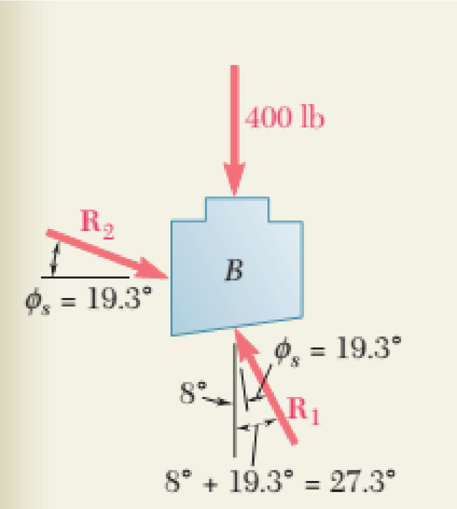
The position of the machine block B is adjusted by moving the wedge A. Knowing that the coefficient of static friction is 0.35 between all surfaces of contact, determine the force P required

(a) to raise block B,

(b) to lower block B.



➢Example – Solution



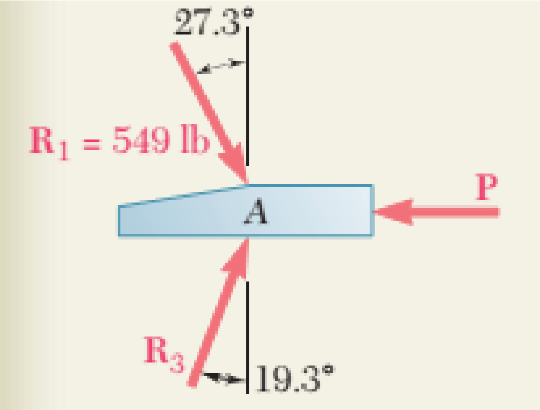
Summing forces,

$$+\rightarrow \sum F_x = 0; -R_1 \sin 27.3^o + R_2 \cos 19.3^o = 0 \qquad ---(1)$$

$$+\uparrow \sum F_y = 0; R_1 \cos 27.3^o - R_2 \sin 19.3^o - 400 \text{ lb} = 0 \qquad ---(2)$$

Solving simultaneously,

$R_1 = 549 \text{ lb}$



Summing forces,

$$+\rightarrow \sum F_x = 0; R_1 \sin 27.3^o + R_3 \sin 19.3^o - P = 0 \qquad ---(1)$$

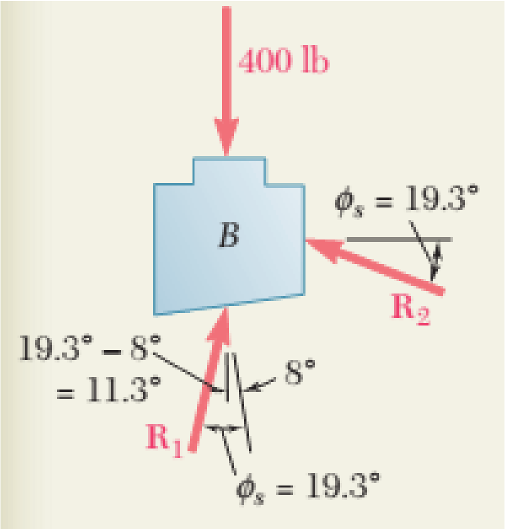
$$+\uparrow \sum F_y = 0; R_3 \cos 19.3^o - R_1 \cos 27.3^o = 0 \qquad ---(2)$$

Solving simultaneously,

$P = 423 \text{ lb}$

SIMPLE MACHINES
The Wedge

➢Example – Solution



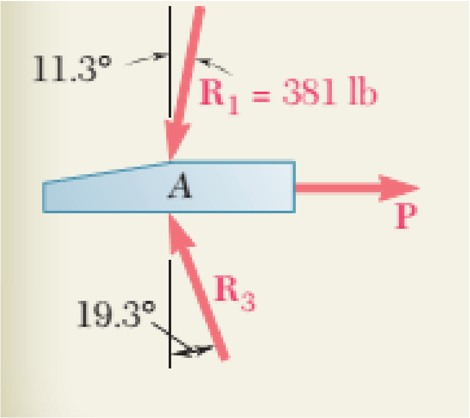
Summing forces,

$$+\rightarrow \sum F_x = 0; R_1 \sin 11.3.^o - R_2 \cos 19.3^o = 0 \qquad ---(1)$$

$$+\uparrow \sum F_y = 0; R_1 \cos 11.3^o + R_2 \sin 19.3^o - 400 \text{ lb} = 0 \qquad ---(2)$$

Solving simultaneously,

$R_1 = 381 \text{ lb}$



Summing forces,

$$+\rightarrow \sum F_x = 0; -R_1 \sin 11.3^o - R_3 \sin 19.3^o + P = 0 \qquad ---(1)$$

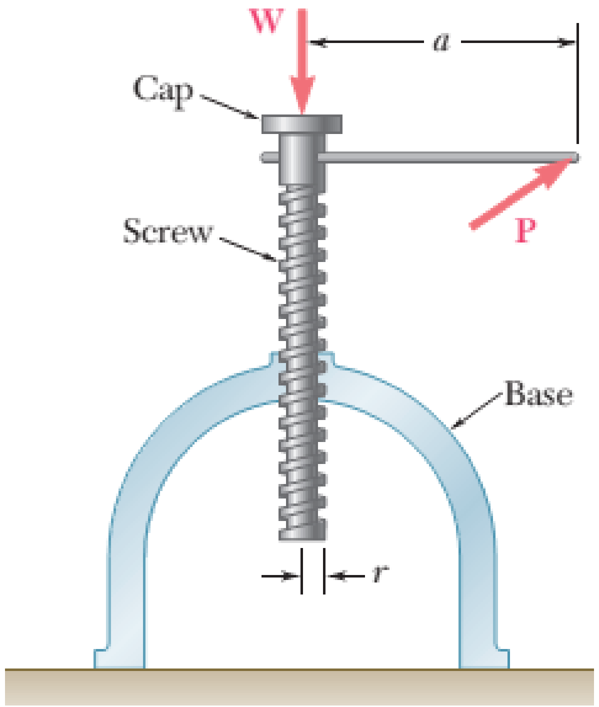
$$+\uparrow \sum F_y = 0; R_3 \cos 19.3^o - R_1 \cos 11.3^o = 0 \qquad ---(2)$$

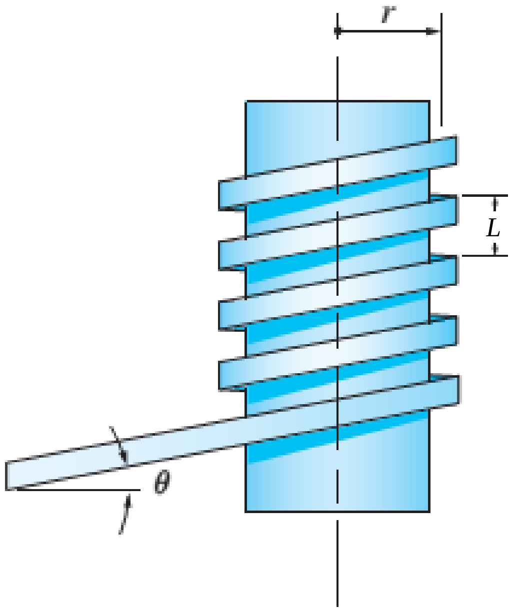
Solving simultaneously,

$P = 206 \text{ lb}$

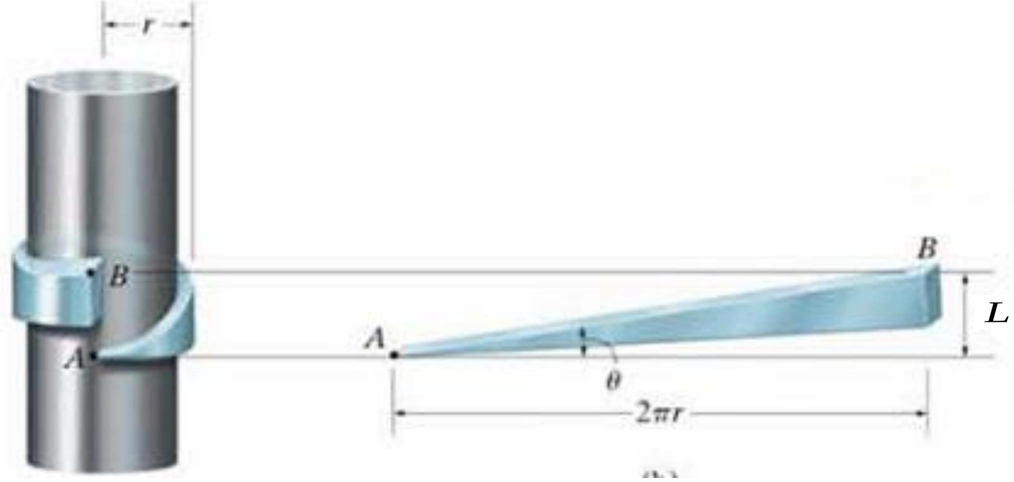
SIMPLE MACHINES
The Screw

- Normally used as fasteners. But square-threaded screws are also often used for transmitting power.
- Can be considered as an inclined plane wrapped around a shaft.
- Often used in jacks, presses, vices, clamps, etc.

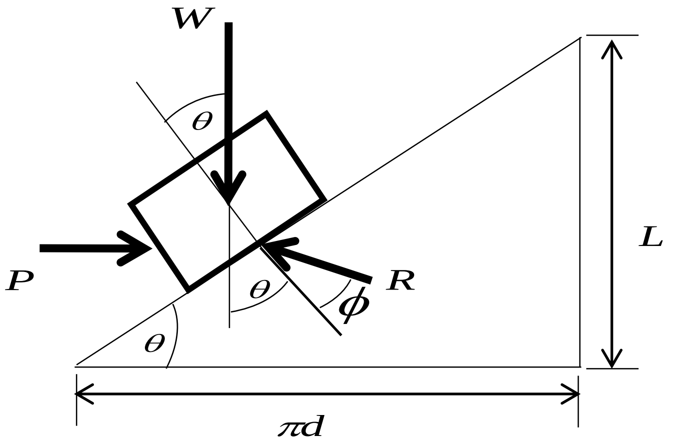




SIMPLE MACHINES
The Screw



- Screwing a nut against the load (moving the load up)



FBD

For equilibrium,

$$+\rightarrow \sum F_x = 0; P - R\sin(\phi+\theta) = 0 \qquad ---(1)$$

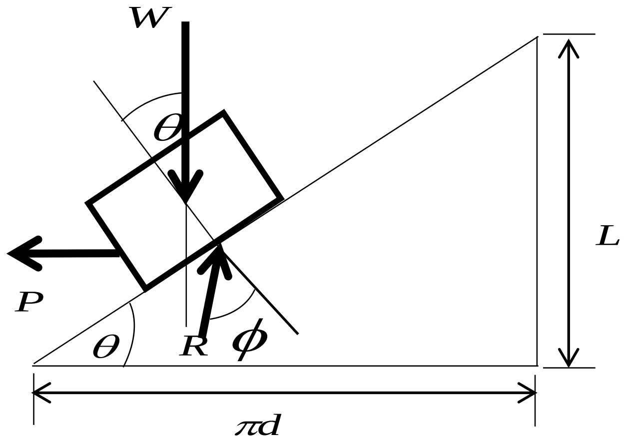
$$+\uparrow \sum F_y = 0; -W + R\cos(\phi+\theta) = 0 \qquad ---(2)$$

Eliminating R and simplifying, we obtain

$$P = W\tan(\phi+\theta)$$

SIMPLE MACHINES
The Screw

- For a load to move down



FBD

For equilibrium,

$$+\rightarrow \sum F_x = 0; -P + R\sin(\phi - \theta) = 0 \quad ---(1)$$

$$+\uparrow \sum F_y = 0; -W + R\cos(\phi - \theta) = 0 \quad ---(2)$$

Eliminating R and simplifying, we obtain

$$P = W\tan(\phi - \theta)$$

- Load won't move down until a force, P is applied as shown above. This is the situation in most practical scenarios, where screws **self-locking**. The loads self-locking screws support don't move down until a force is applied to move them down. Screws are also self locking when *Ø*=*Θ*.
- If , *Ø*<*Θ*, the load moves down on it's own, **without** the application of P as shown above. The screw is then not **self-locking**.

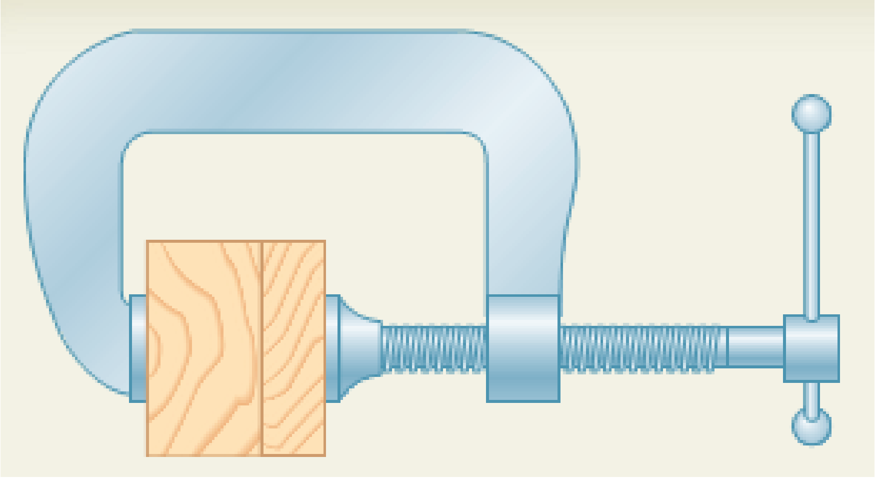
Example

A clamp is used to hold two pieces of wood together as shown. The clamp has a double square thread of mean diameter equal to 10 mm with a pitch of 2 mm. The coefficient of friction between threads is $\mu_s = 0.30$.

If a maximum couple of 40 Nm is applied in tightening the clamp, determine

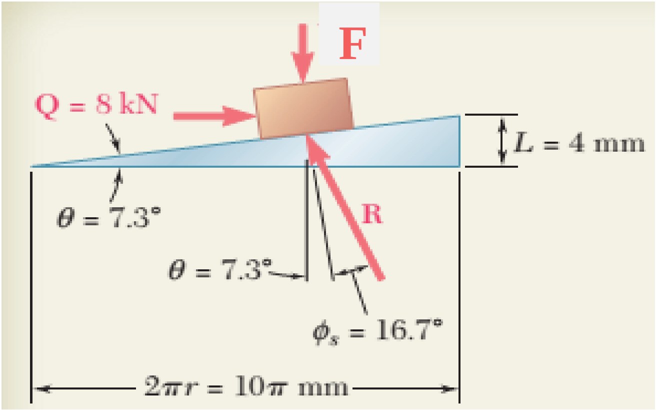
(*a*) the force exerted on the pieces of wood,

(*b*) the couple required to loosen the clamp.



Example – Solution

Force exerted on wood in tightening the clamp



Summing forces,

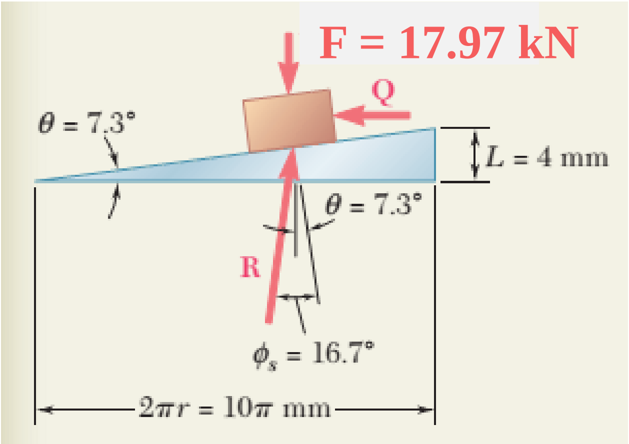
$$+\rightarrow \sum F_x = 0; 8\text{ kN} - R\sin\left(16.7^o + 7.3^o\right) = 0 \qquad ---(1)$$

$$+\uparrow \sum F_y = 0; -F + R\cos\left(16.7^o + 7.3^o\right) = 0 \qquad ---(2)$$

Solving simultaneously,

$F = 17.96\text{ kN}$

To loosen the clamp,



Summing forces,

$$+\rightarrow \sum F_x = 0; Q - R\sin 9.4^o = 0 \qquad ---(1)$$

$$+\uparrow \sum F_y = 0; -17.97\text{ kN} + R\cos 9.4^o = 0 \qquad ---(2)$$

Solving simultaneously,

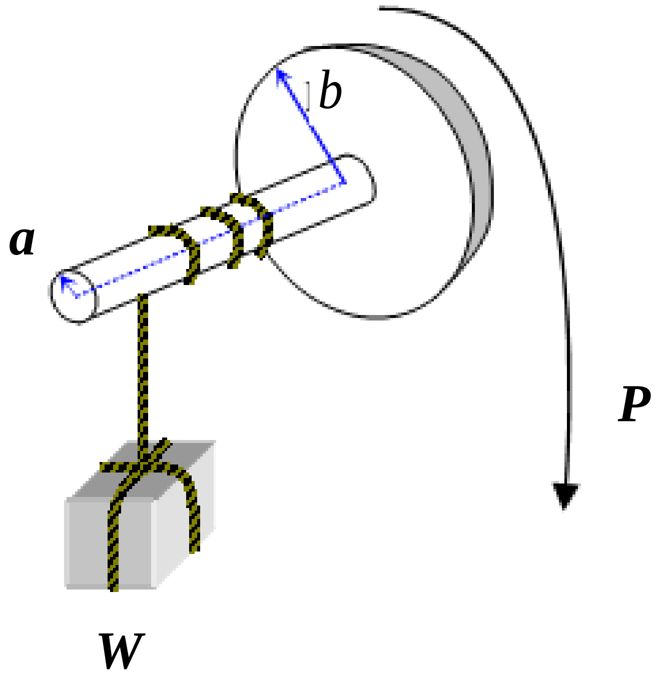
$Q = 2.97\text{ kN}$

Couple Required $= Q \times r = 2.97\text{ kN} \times 0.005\text{ m} = 14.85\text{ Nm}$

The Wheel and Axle

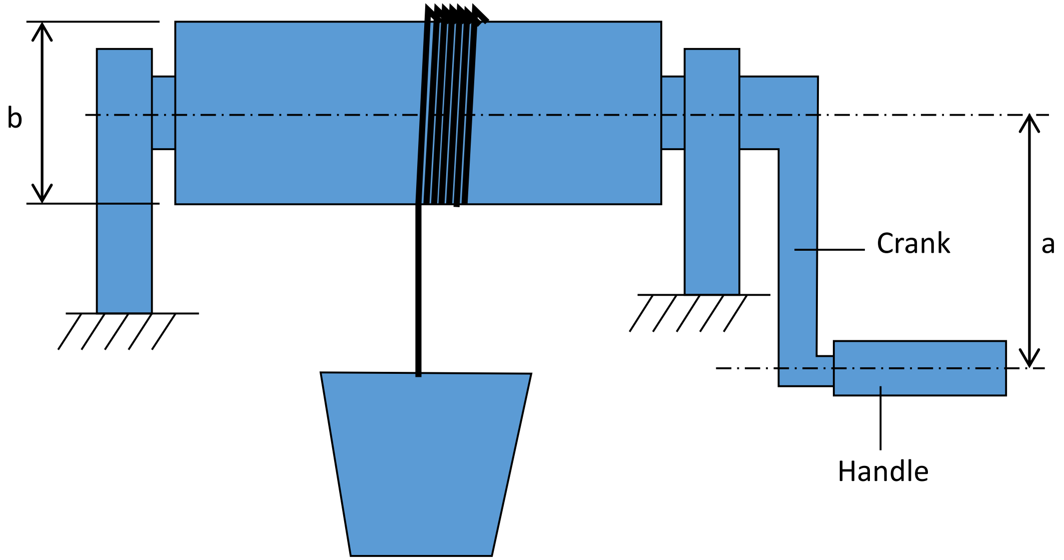
- Comprises a wheel attached to a shaft (axle) of smaller diameter. The axle turns the wheel.
- Effort is applied to the wheel via a rope for instance.
- Load is attached to axle.

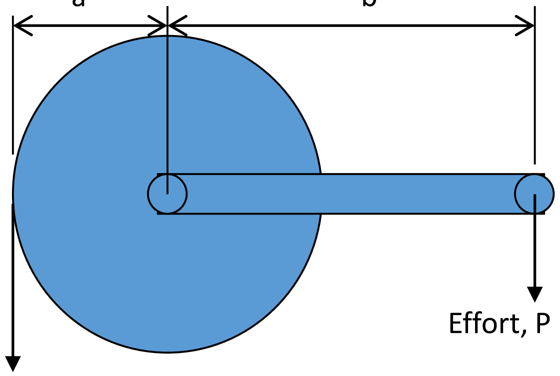
$$Wa = Pb$$



- There is also the differential wheel and axle. This helps achieve a bigger MA

SIMPLE MACHINES
The Wheel and Axle





Free-body diagram

$$Wa = Pb$$

SIMPLE MACHINES

STUDENTS SHOULD READ ON WHEEL AND AXLES AND PULLEYS.

ASSIGNMENT III

SIMPLE MACHINES
The Pulley

- Consists of a wheel which can turn freely on its axle and carries a rope or cable.

SIMPLE MACHINES
The Pulley

- we also have differential pulleys; the *Weston Differential Pulley* for instance.

LECTURE 9

THE METHOD OF VIRTUAL WORK

THE METHOD OF VIRTUAL WORK

- The method of virtual work provides is often used for analyzing mechanisms. Mechanisms as used here refers to bodies with interconnected members that can move relative to each other.
- It is a more direct and convenient approach as compared to the force and moment equilibrium equations approach to analyzing mechanisms.
- It states that:

  "If a body is in equilibrium, then the virtual work of all forces acting on the body is zero for all kinematically admissible virtual displacements of the body from the equilibrium position."

- A similar concept is the Principle of Minimum Potential Energy which determines the positions of equilibrium for a given mechanism.
- Virtual work, δU is the product of a force and the virtual displacement in the direction of that force.
- Work is done when a force causes a linear displacement or a Moment causes an angular displacement.

THE METHOD OF VIRTUAL WORK

Procedure

- Draw a FBD of the system.

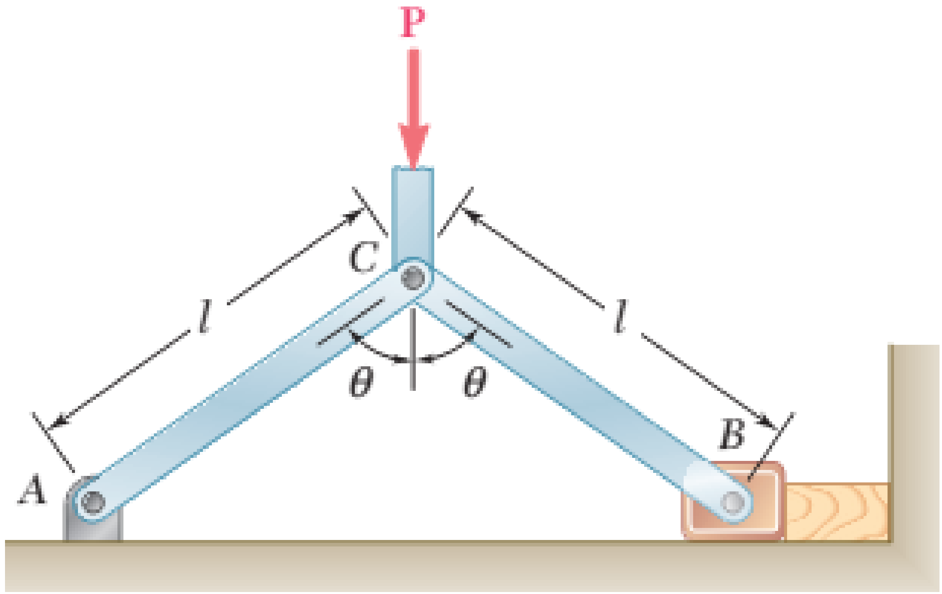
- Subject the FBD to a virtual displacement identify the forces and couples that do admissible work.

- Write the virtual work equation for the system.

- Solve the equation for the unknowns.

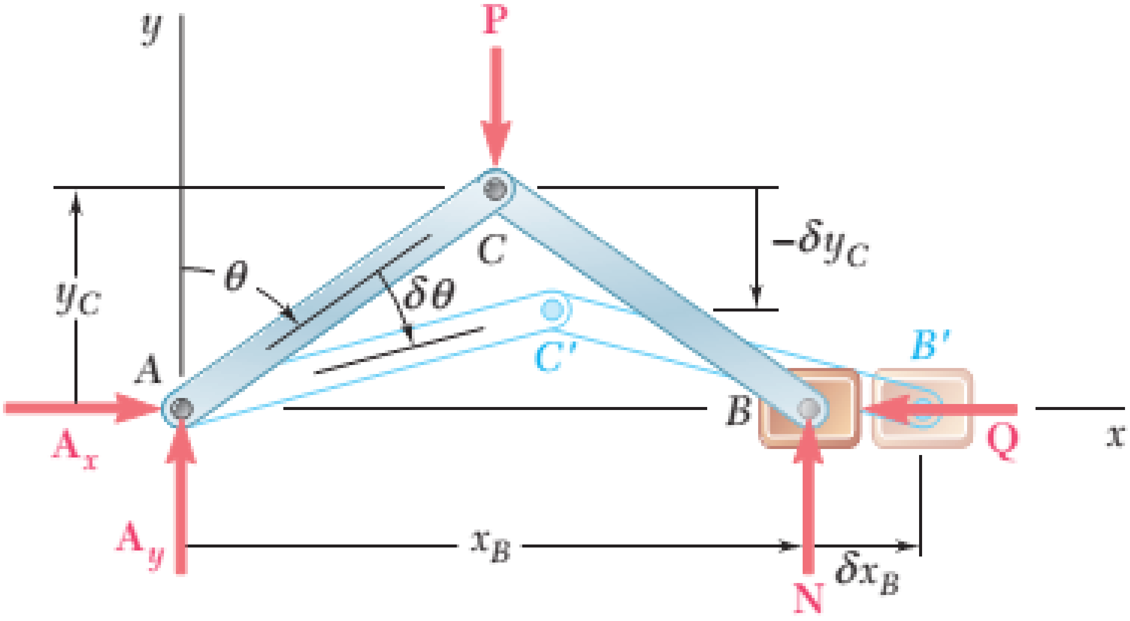
Example 1

Determine a relation between P and force exerted on the piece of wood for the toggle vice shown below. Take the joint at C to be a frictionless pin joint.



THE METHOD OF VIRTUAL WORK

Example 1



$$x_B = 2l\sin\theta$$
$$\delta x_B = 2l\cos\theta\delta\theta$$

$$y_C = l\cos\theta$$
$$\delta y_C = -l\sin\theta\delta\theta$$

Summing virtual worksdone,

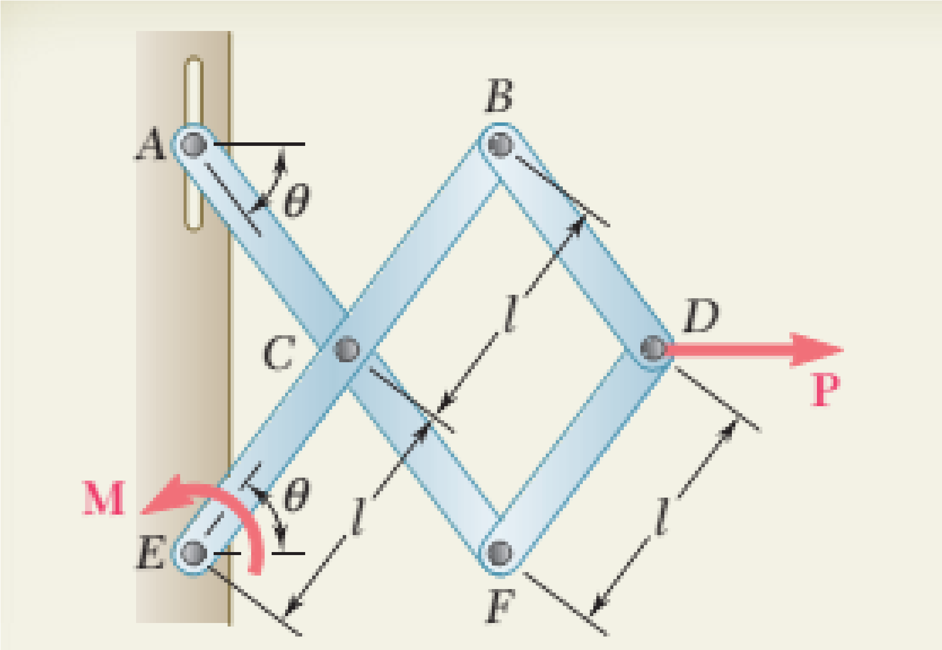
$$\delta U = \delta U_Q + \delta U_P = 0$$
$$= -Q\delta x_B - P\delta y_c$$
$$= -2Ql\cos\theta\delta\theta \quad + Pl\delta\theta\sin\theta$$

Simplifying,

$$Q = \frac{1}{2}P\tan\theta$$

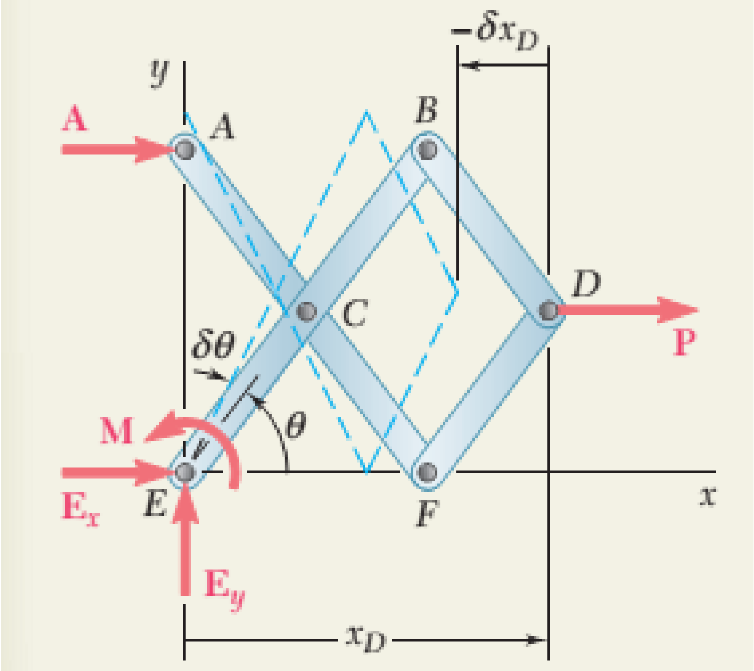
Example 2

- Using the method of virtual work, determine the magnitude of the couple M required to maintain the equilibrium of the mechanism shown.



THE METHOD OF VIRTUAL WORK

Example 2



$$x_D = 3 \times l \cos\theta$$

$$\delta x_D = -3l \sin\theta\delta\theta$$

Summing virtual workdone,

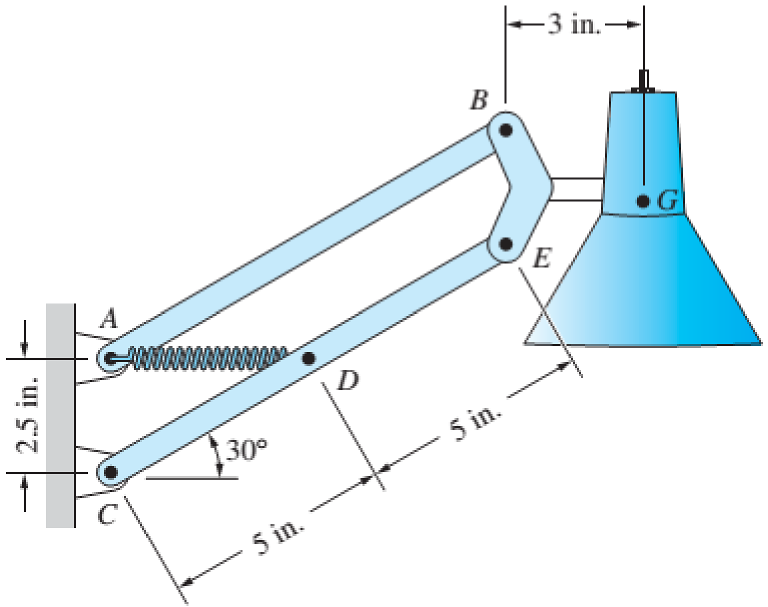
$$\delta U = \delta M_E + \delta U_P = 0$$

$$= M\delta\theta - P(3l \sin\theta\delta\theta) = 0$$

$$M = 3Pl \sin\theta$$

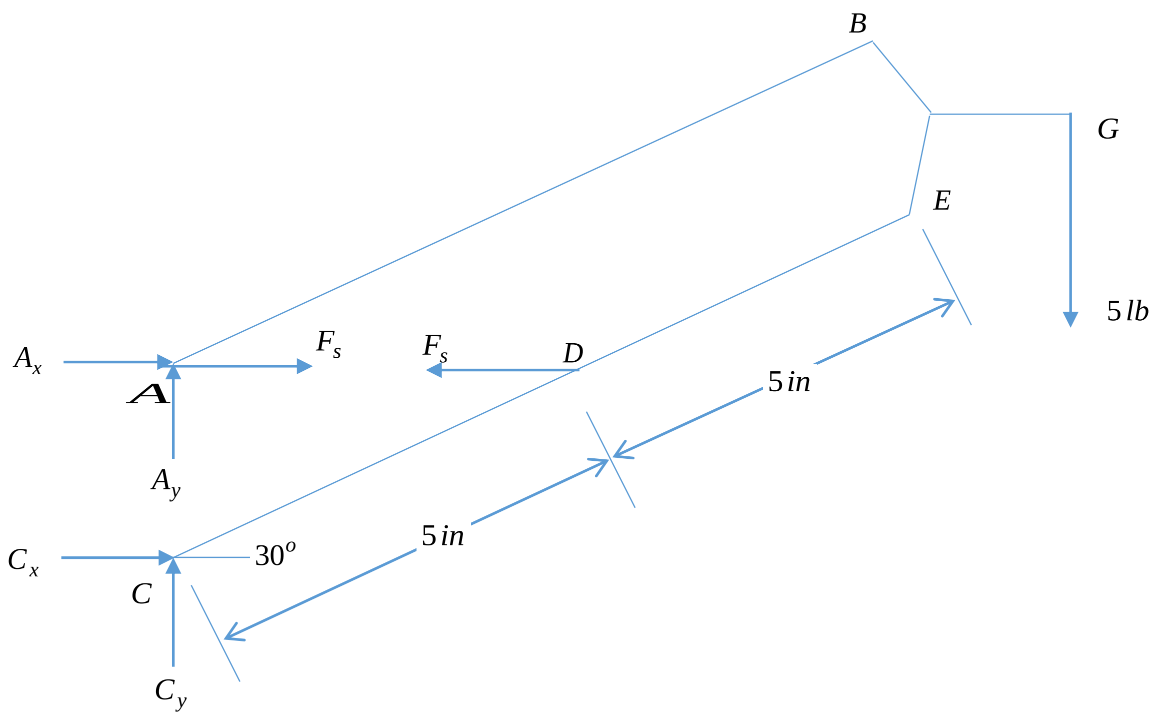
Example 3

The 5-lb lamp, with center of gravity located at G, is supported by the parallelogram linkage of negligible weight. Find the tension in the spring AD when the lamp is in equilibrium in the position shown.



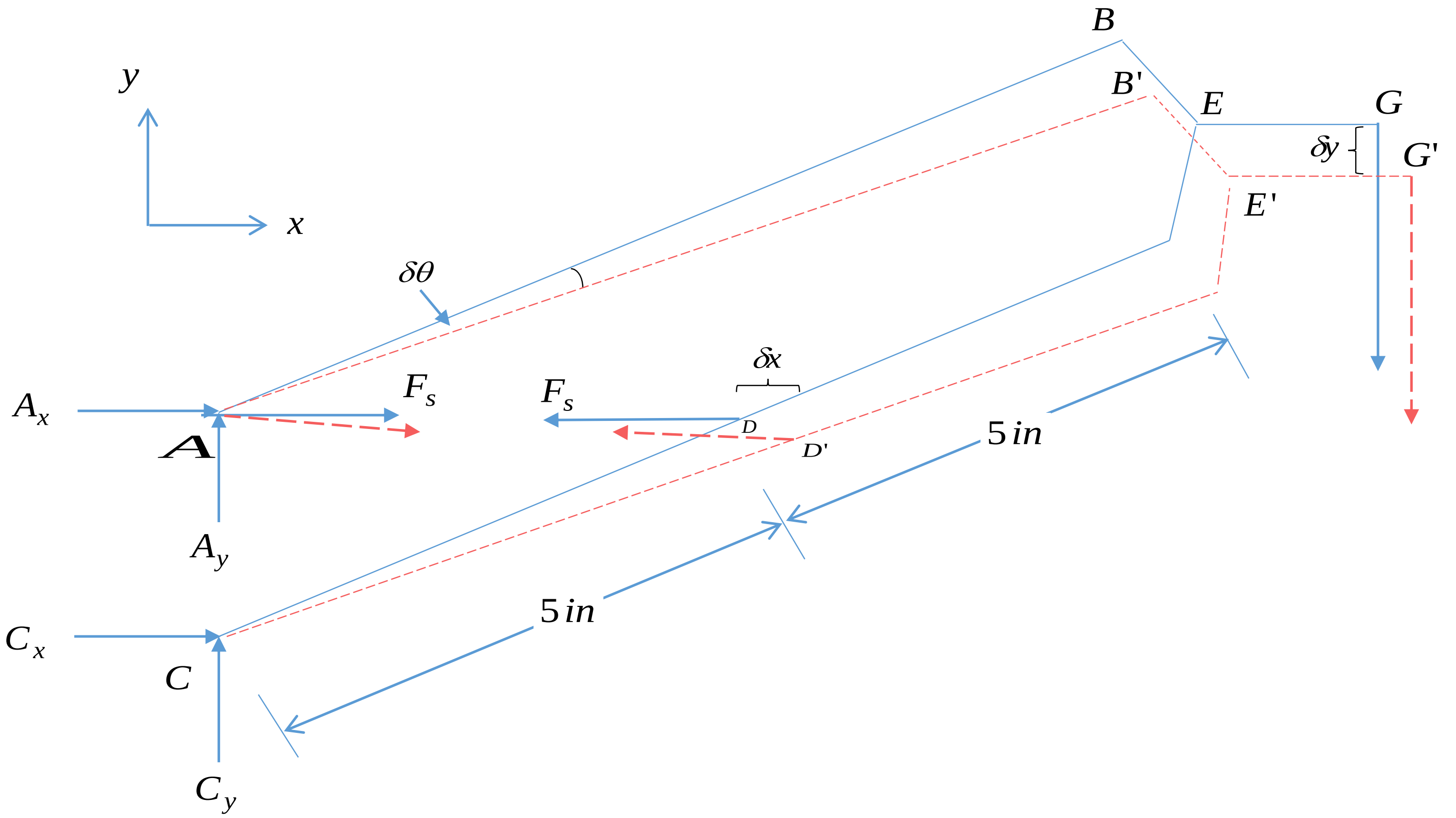
Example 3

FBD



THE METHOD OF VIRTUAL WORK

Example 3



$$x_D = 5\cos\theta$$

$$\delta x_D = -5\sin\theta\,\delta\theta$$

$$y_G = 10\sin\theta + 3$$

$$\delta y_G = 10\cos\theta\,\delta\theta$$

Summing virtual work done,

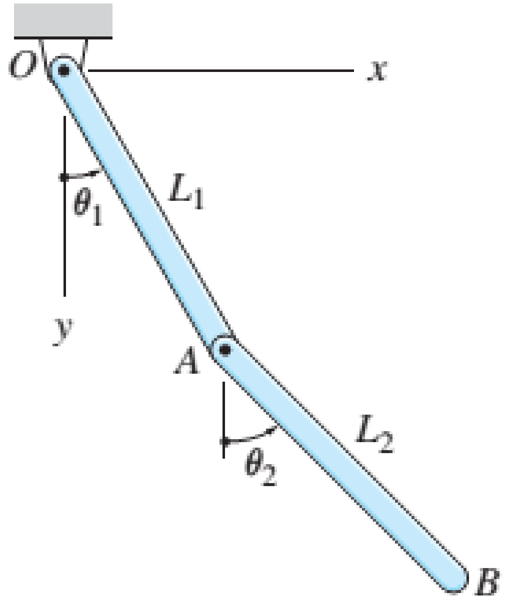
$$\delta U = \delta U_D + \delta U_G = 0$$

$$= \left(-F_s \times -5\sin\theta\,\delta\theta\right) + \left(-5\,lb \times 10\cos\theta\,\delta\theta\right) = 0$$

$$F_s = 17.32\,lb$$

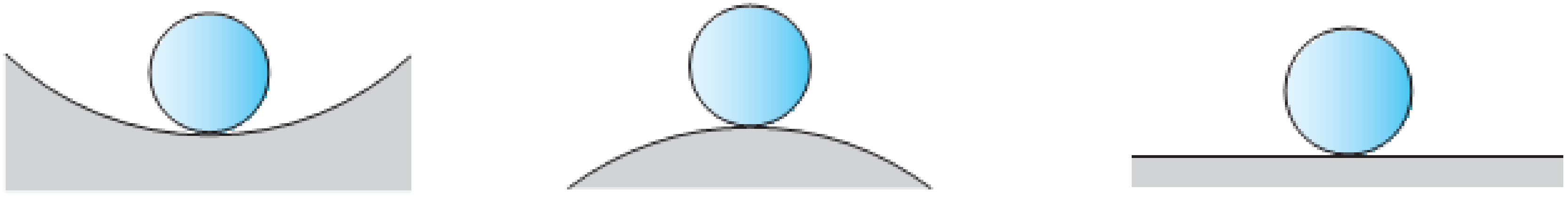
THE METHOD OF VIRTUAL WORK
Multi-Degree of Freedom Problems

- For multi-degree of freedom (DOF) problems, a virtual work equation is written for a virtual displacement of each DOF, (so to speak).
- When a DOF is being considered, all other DOFs are assumed not to occur.
- The resulting equations are then solved for the unknowns.



The Principle of Minimum Potential Energy

- A body can be in stable equilibrium, neutral equilibrium or unstable equilibrium.



(a) Stable (b) Unstable (c) Neutral

- Principle of Minimum Potential Energy helps in determining the position system must be in, in order to be in stable equilibrium.
- It states that “The potential energy of a conservative system is at its minimum in a stable equilibrium position.”

THE METHOD OF VIRTUAL WORK

The Principle of Minimum Potential Energy

- To determine the equilibrium positions of a system as well as their stability, we take derivatives of the total potential energy (PE) of the system w.r.t the position co-ordinate.

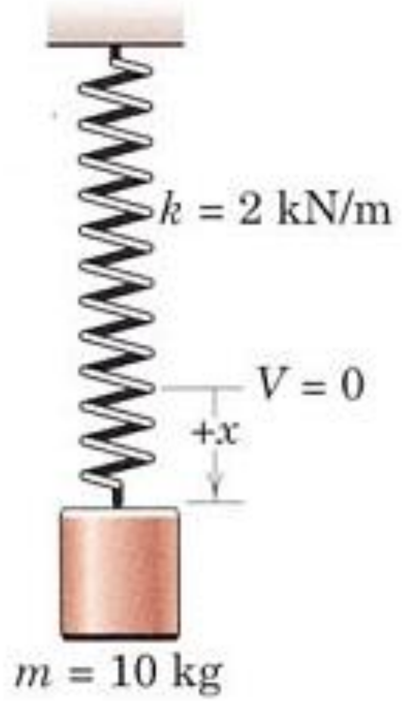
- The total PE mostly comprises Elastic PE and Gravitational PE.

- If V is the total PE of the system and q the position co-ordinate, then the equilibrium position of the system will be given by $\frac{dV}{dq} = 0$

and position is stable if $\frac{d^2V}{dq^2} > 0$

For instance, determine the equilibrium position and it's stability for a 10 kg mass suspended by a spring of stiffness, 2 kN/m.

Solution



$$Total\ system\ P.E. = Elastic\ P.E. + Gravitational\ P.E$$

$$V = V_e + V_g$$

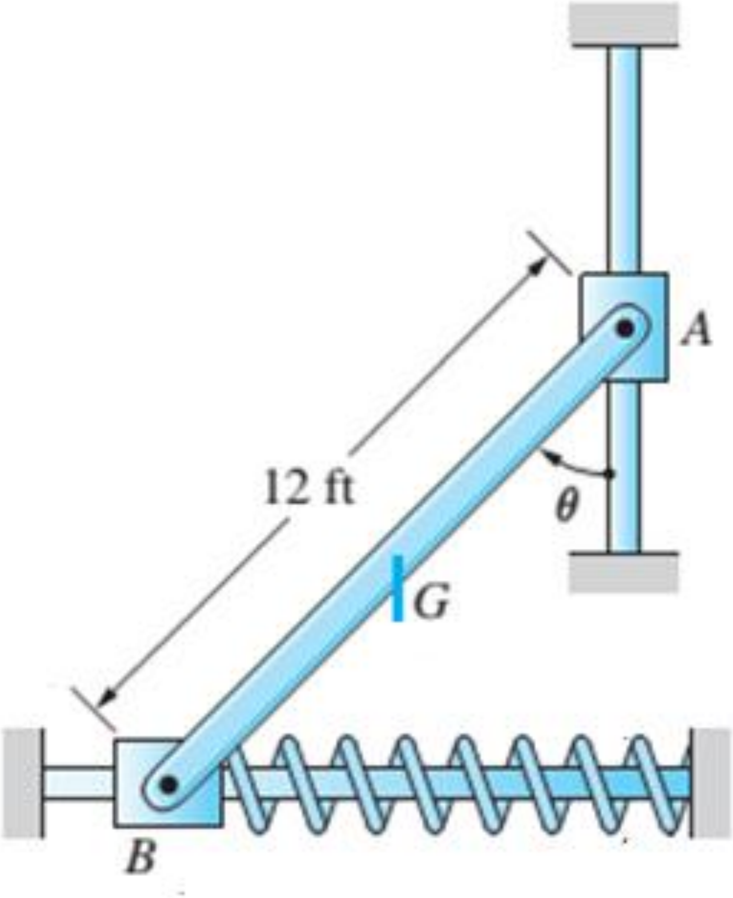
$$= \frac{1}{2}kx^2 - mgx$$

$$\frac{dV}{dx} = kx - mg$$

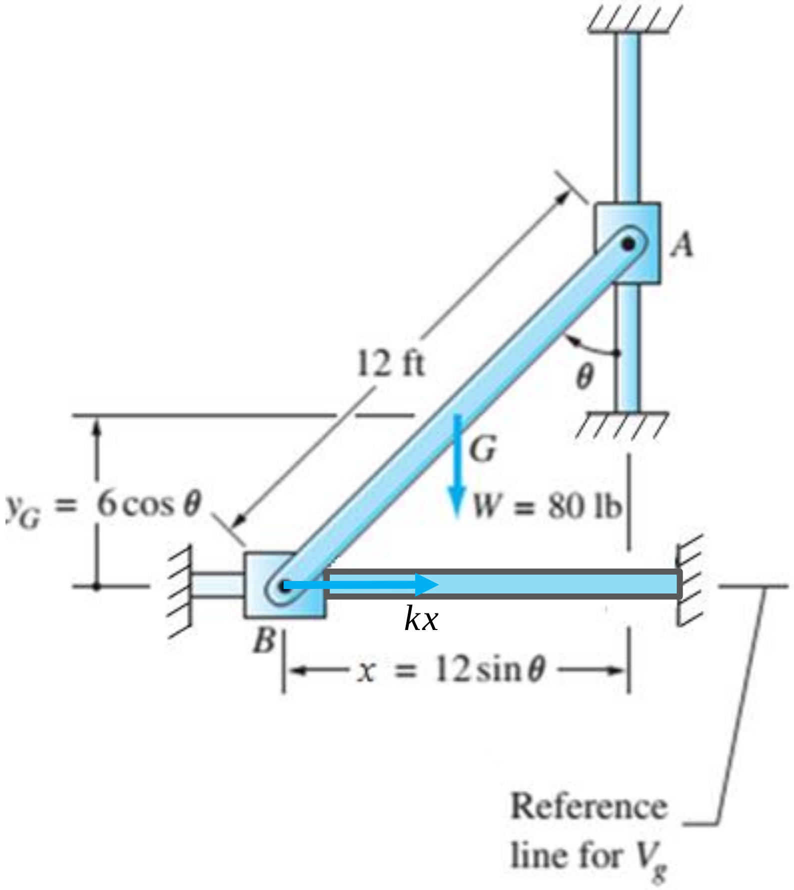
$$\frac{d^2V}{dx^2} = k$$

Therefore the equilibrium value of x *is* 0.049 m *and the equilibrium is stable.*

For the system shown, determine all the equilibrium positions as well as their stability. The homogeneous rod AB weighs 80 lb, and the ideal spring is unstretched when θ=0. Neglect friction and the weights of the sliders at A and B. The stiffness of the spring, k is 6 lb/ft.



Solution:



$$Total\ system\ P.E.\ (V) = Elastic\ P.E.(V_e) + Gravitational\ P.E.(V_g)$$

$$= Wy_G + \frac{1}{2}kx^2$$

$$V = 80(6\cos\theta) + \frac{1}{2}(6)(12\sin\theta)^2 = 480\cos\theta + 432\sin^2\theta\ lb.ft$$

$$\frac{dV}{dx} = 0: -480\sin\theta + 864\sin\theta\cos\theta\ lb.ft = 0$$

$$\sin(-480 + 864\cos\theta) = 0$$

$$which\ gives\ \theta = 0^o\ \ and\ 56.25^o$$

Solution:

$Next,$

$$\frac{d^2V}{dx^2} = -480\cos\theta + 864\left(\cos^2\theta - \sin^2\theta\right) lb.ft$$

$For\ \theta = 0,$

$$\frac{d^2V}{dx^2} = -480\cos 0 + 864\left(\cos^2 0 - \sin^2 0\right) = 384\ lb.ft$$

$Hence, \theta = 0\ is\ a\ stable\ equilibrium\ position$

$For\ \theta = 56.25^0,$

$$\frac{d^2V}{dx^2} = -480\cos 56.25 + 864\left(\cos^2 56.25 - \sin^2 56.25\right) = -597\ lb.ft$$

$Hence, \theta = 56.2556.25\ is\ an\ unstable\ equilibrium\ position$

LECTURE 10

CENTROIDS

CENTROIDS

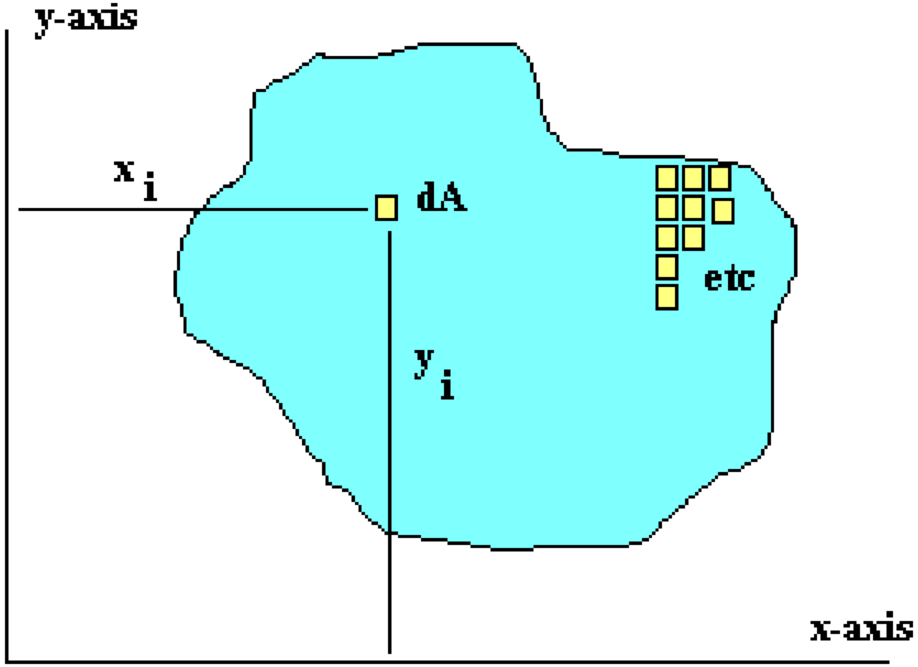
- It is sometimes necessary in mechanics problems to determine the central point of bodies.
- This central point is defined as that point a physical quantity under consideration may be assumed to be centered.
- The central point may have different terminologies for different physical quantities.

Terminology	**Physical Entity**
Centroid	Length of a curve
Centroid	Area of a surface
Centroid	Volume of a body
Centre of a mass	Mass of a body
Centre of gravity	Gravitational force on a body

- All the terms mentioned above can be determined using the summation approach, which will be discussed. An integration approach can also be used.

Determining the Centroid of a Plane Area

- Using the summation approach,
  - We assume the area comprises several smaller elemental areas.
  - We then sum moments of the elemental areas about axes of the origin (1st moments) and divide this by the total area to get the centroid of the area.



$$\bar{x} = \frac{First\ Moment\ of\ Area\ about\ y\text{-}axis,\ Q_y}{Total\ Area} = \frac{\sum x_i dA_i}{\sum A_i}$$

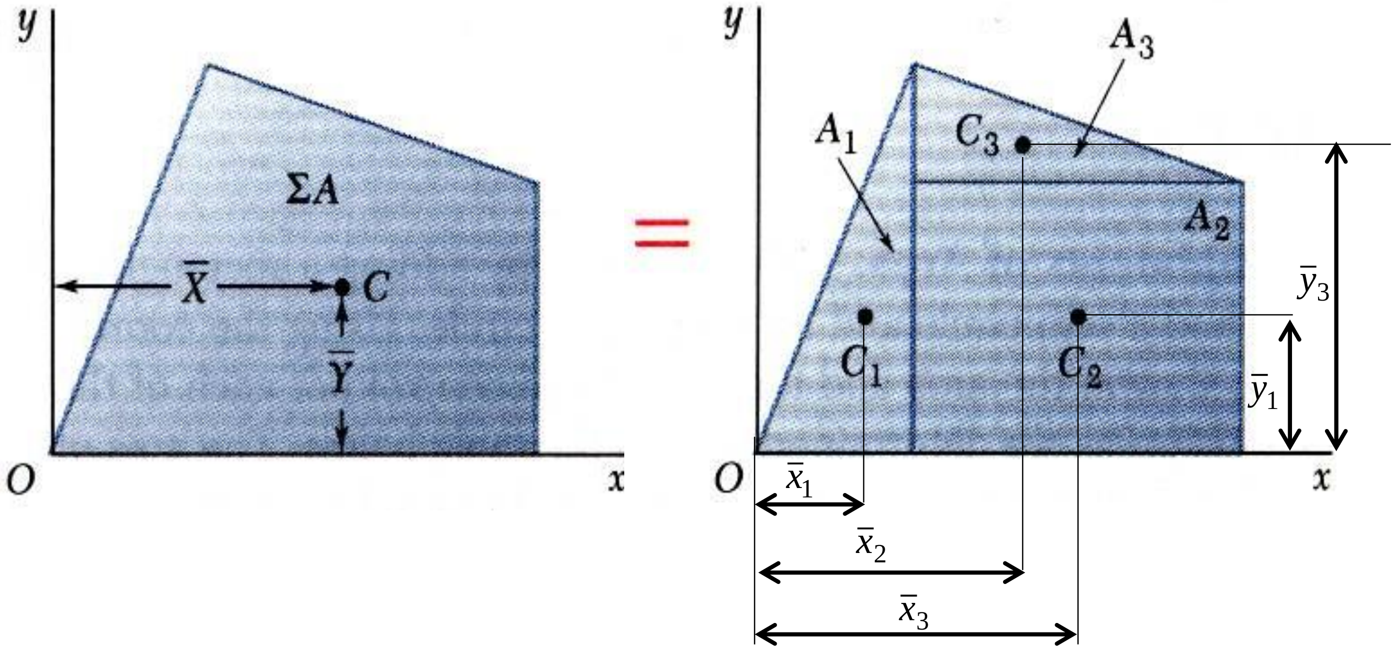
$$\bar{y} = \frac{First\ Moment\ of\ Area\ about\ x\text{-}axis,\ Q_x}{Total\ Area} = \frac{\sum y_i dA_i}{\sum A_i}$$

$$Centroid\ is\ (\bar{X}, \bar{Y})$$

CENTROIDS

Determining the Centroid of a Plane Area

➢The same idea is used in determining the centroids of composite areas.



$$\bar{X} = \frac{\textit{First Moment of each Area about y-axis}, Q_y}{\textit{Total Area}}$$

$$= \frac{\sum \bar{x}_i dA_i}{\sum A_i} = \frac{(\bar{x}_1 A_1) + (\bar{x}_2 A_2) + (\bar{x}_3 A_3)}{A_1 + A_2 + A_3}$$

$$\bar{Y} = \frac{\textit{First Moment of each Area about x-axis}, Q_x}{\textit{Total Area}}$$

$$= \frac{\sum \bar{y}_i dA_i}{\sum A_i} = \frac{(\bar{y}_1 A_1) + (\bar{y}_2 A_2) + (\bar{y}_3 A_3)}{A_1 + A_2 + A_3}$$

$\textit{Note : For the diagram above, } \bar{y}_1 = \bar{y}_2$

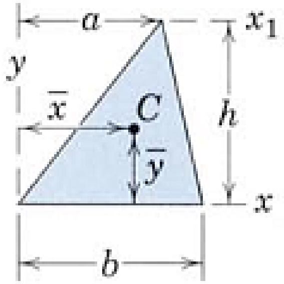
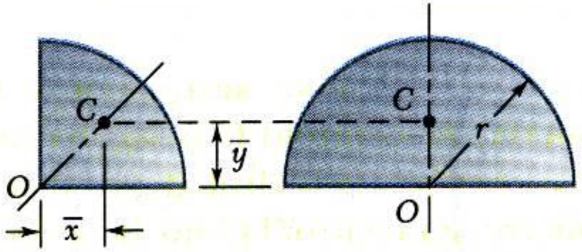
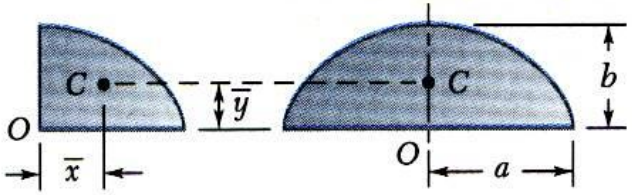
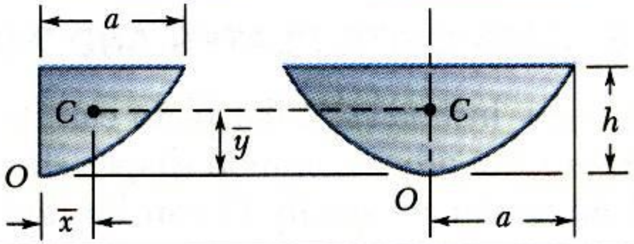
$\textit{Centroid is } (\bar{X}, \bar{Y})$

Note:

THE ELEMENTAL AREA CENTROID VALUES MAY BE NEGATIVE OR POSITIVE DEPENDING ON THE LOCATION OF THE ORIGIN OF THE COMPOSITE AREA BEING CONSIDERED.

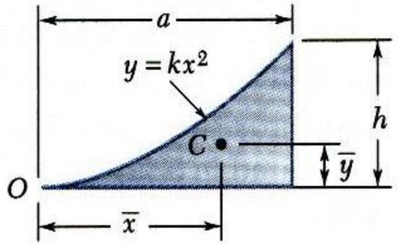
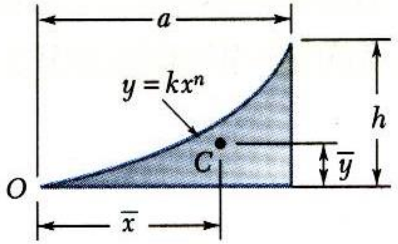
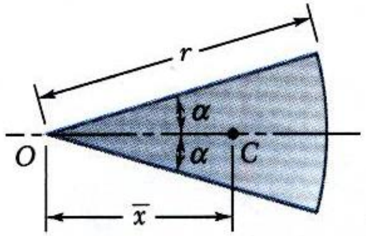
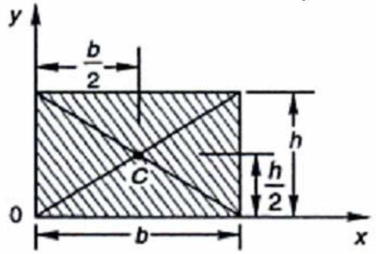
CENTROIDS

Centroids of Common Shapes

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area		$\frac{a+b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$

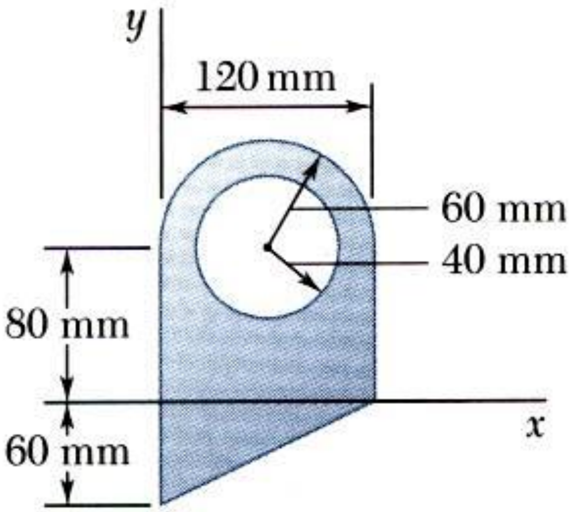
CENTROIDS

Centroids of Common Shapes

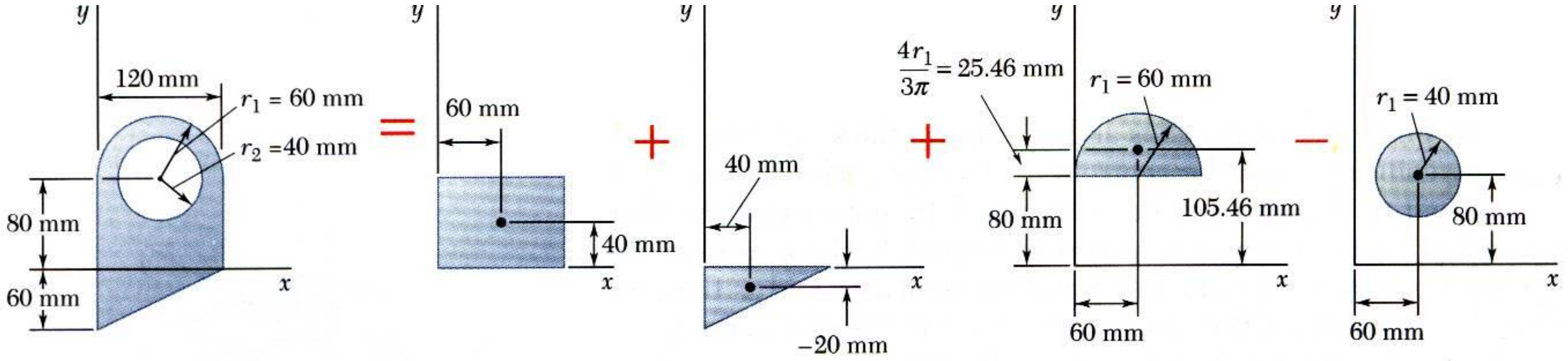
Shape		$\bar{x}$	$\bar{y}$	Area
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2
Rectangle		$b/2$	$h/2$	bh

Example

For the plane area shown, determine the first moments with respect to the *x* and *y* axes and the location of the centroid.



Determining the Centroid of a Plane Area



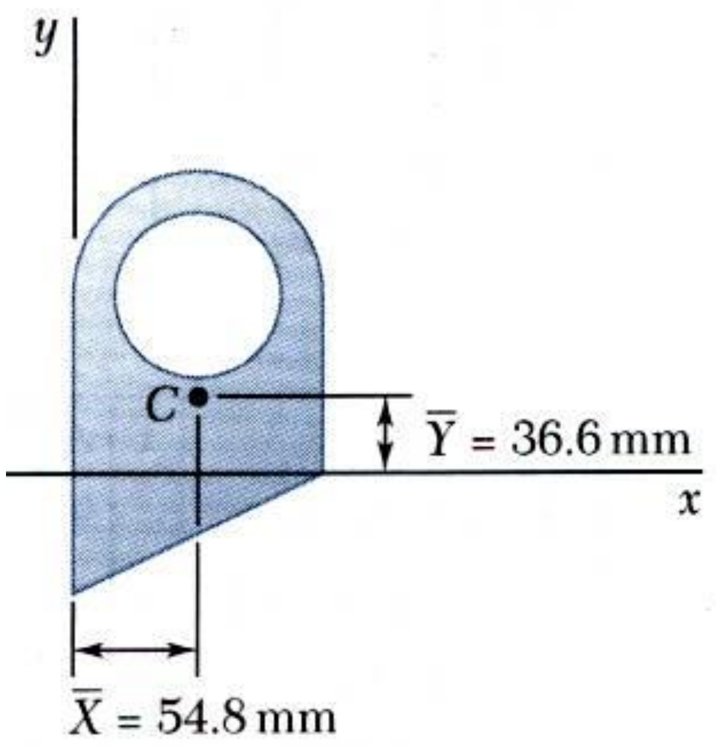
Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

CENTROIDS

Determining the Centroid of a Plane Area

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{+757.7\times 10^3 \text{mm}^3}{13.828\times 10^3 \text{mm}^2}$$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{+506.2\times 10^3 \text{mm}^3}{13.828\times 10^3 \text{mm}^2}$$



CENTROIDS

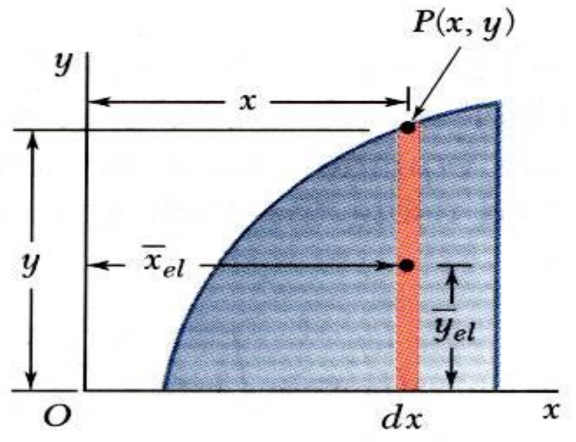
- The same principles are used in determining the centroids of curves and volumes as well as the centers of mass and gravity of bodies.

CENTROIDS

The Integration Approach to determining the centroid of an area

➢This involved the use of an elemental area that is integrated.

Elemental area of *ydx.*



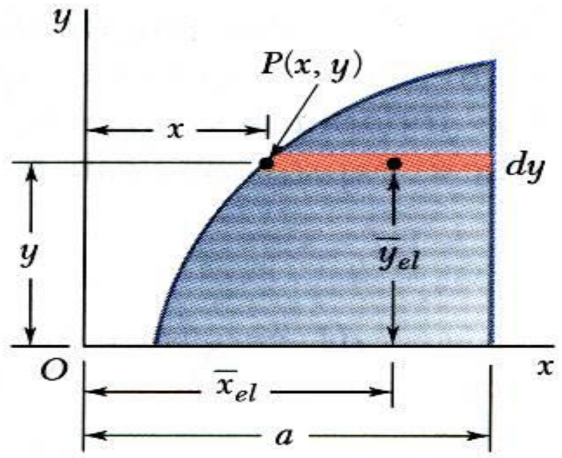
$$Q_x = \bar{x}A = \int \bar{x}_{el}\, dA$$

$$= \int x(ydx)$$

$$Q_y = \bar{y}A = \int \bar{y}_{el}\, dA$$

$$= \int \frac{y}{2}(ydx)$$

Elemental area of *(a-x)dy*



$$Q_x = \bar{x}A = \int \bar{x}_{el}\, dA$$

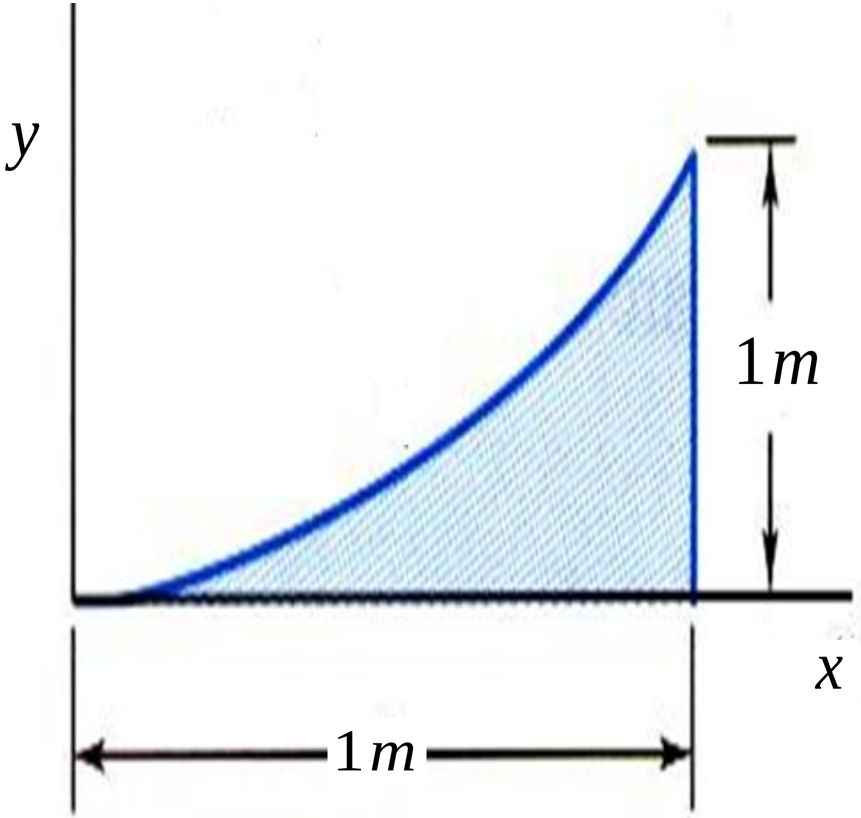
$$= \int \frac{a+x}{2}[(a-x)dy]$$

$$Q_y = \bar{y}A = \int \bar{y}_{el}\, dA$$

$$= \int y[(a-x)dy]$$

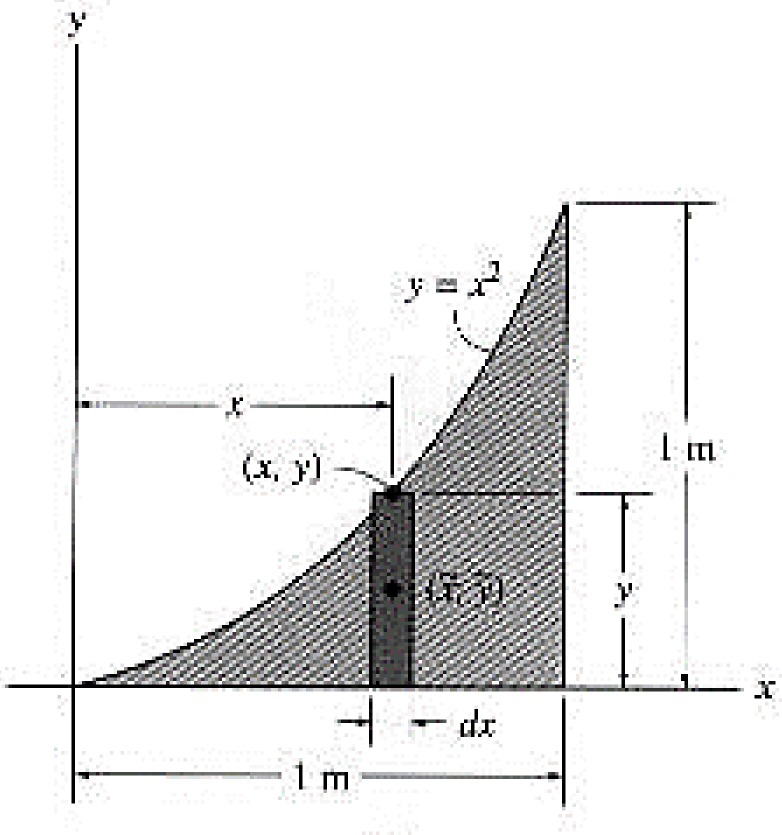
Example

Locate the centroid of the area shown in figure



CENTROIDS

The Integration Approach



(a)

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{\int_0^1 xy\, dx}{\int_0^1 y\, dx} = \frac{\int_0^1 x^3 dx}{\int_0^1 x^2 dx} = \frac{0.250}{0.333} = 0.75$$

$$\bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} = \frac{\int_0^1 (y/2) y\, dx}{\int_0^1 y\, dx} = \frac{\int_0^1 (x^2/2) x^2\, dx}{\int_0^1 x^2 dx} = \frac{0.100}{0.333} = 0.3\text{m}$$

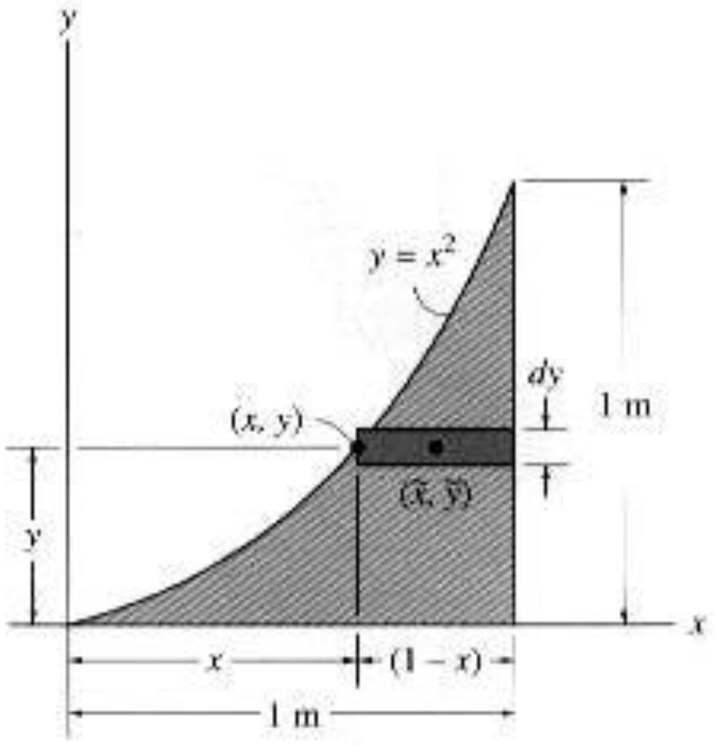
$$Q_x = \bar{x}A = \int \bar{x}_{el}\, dA$$

$$= \int x\left(ydx\right)$$

$$Q_y = \bar{y}A = \int \bar{y}_{el}\, dA$$

$$= \int \frac{y}{2}\left(ydx\right)$$

CENTROIDS
The Integration Approach



$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{\int_0^1 \left[(1+x)/2\right](1-x)\, dy}{\int_0^1 (1-x)\, dy} = \frac{\frac{1}{2}\int_0^1 (1-y)\, dy}{\int_0^1 (1-\sqrt{y})\, dy} = \frac{0.250}{0.333} 0.75\text{m}$$

$$\bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} = \frac{\int_0^1 y(1-x)\, dy}{\int_0^1 (1-x)\, dy} = \frac{\int_0^1 (y - y^{3/2})\, dy}{\int_0^1 (1-\sqrt{y})\, dy} = \frac{0.100}{0.333} = 0.3\text{m}$$

$$Q_x = \bar{x}A = \int \bar{x}_{el}\, dA$$

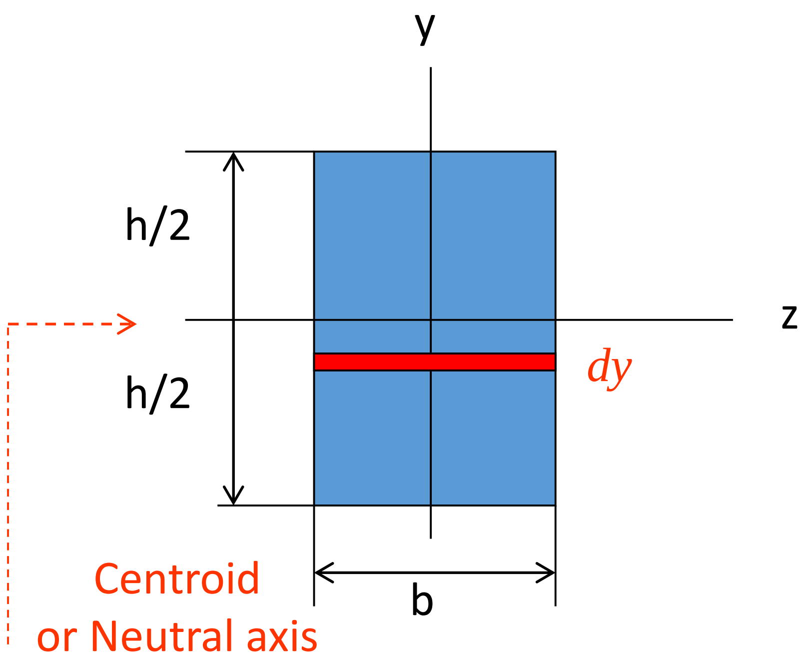
$$= \int x(ydx)$$

$$Q_y = \bar{y}A = \int \bar{y}_{el}\, dA$$

$$= \int \frac{y}{2}(ydx)$$

An Introduction to Moment of Inertia of an Area

- The moment of inertia is a measure of the amount of resistance a cross-section can offer to bending.
- It is the same as the 2nd Moment of Area of a cross-section and referred to as the Polar moment of inertia in circular cross-sections.
- Thus, for the cross-section shown below,

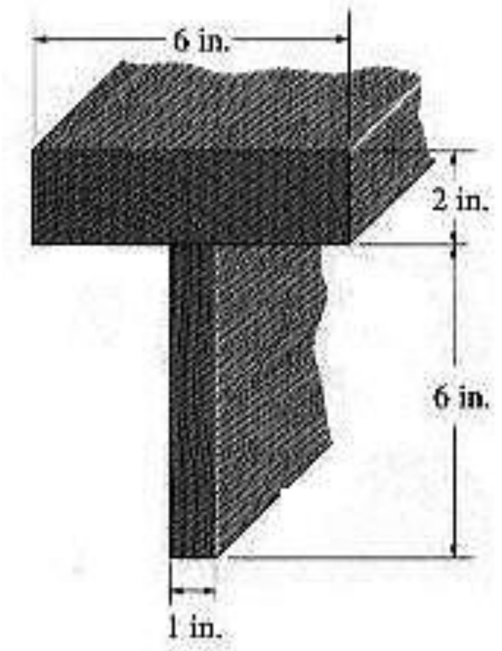


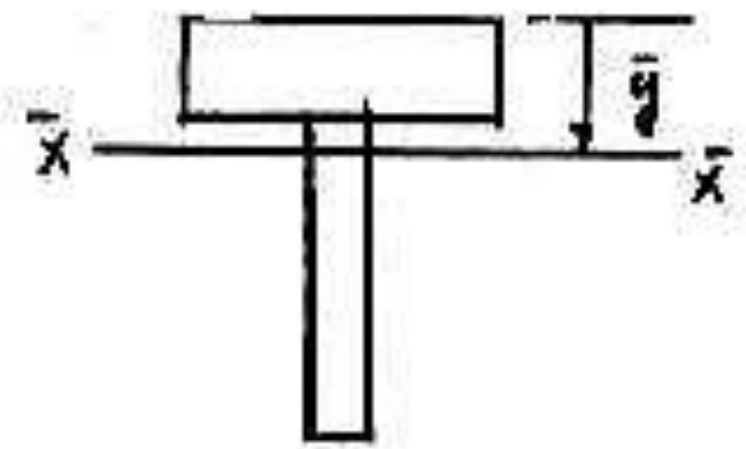
$$I_z = \int y^2 dA = \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 b\,dy = b\frac{y^3}{3}\Bigg|_{-\frac{h}{2}}^{\frac{h}{2}} = \frac{b}{3}\left[\frac{h^3}{8} - \left(-\frac{h^3}{8}\right)\right]$$

$$\therefore I_z = \frac{bh^3}{12}$$

An Introduction to Moment of Inertia of an Area

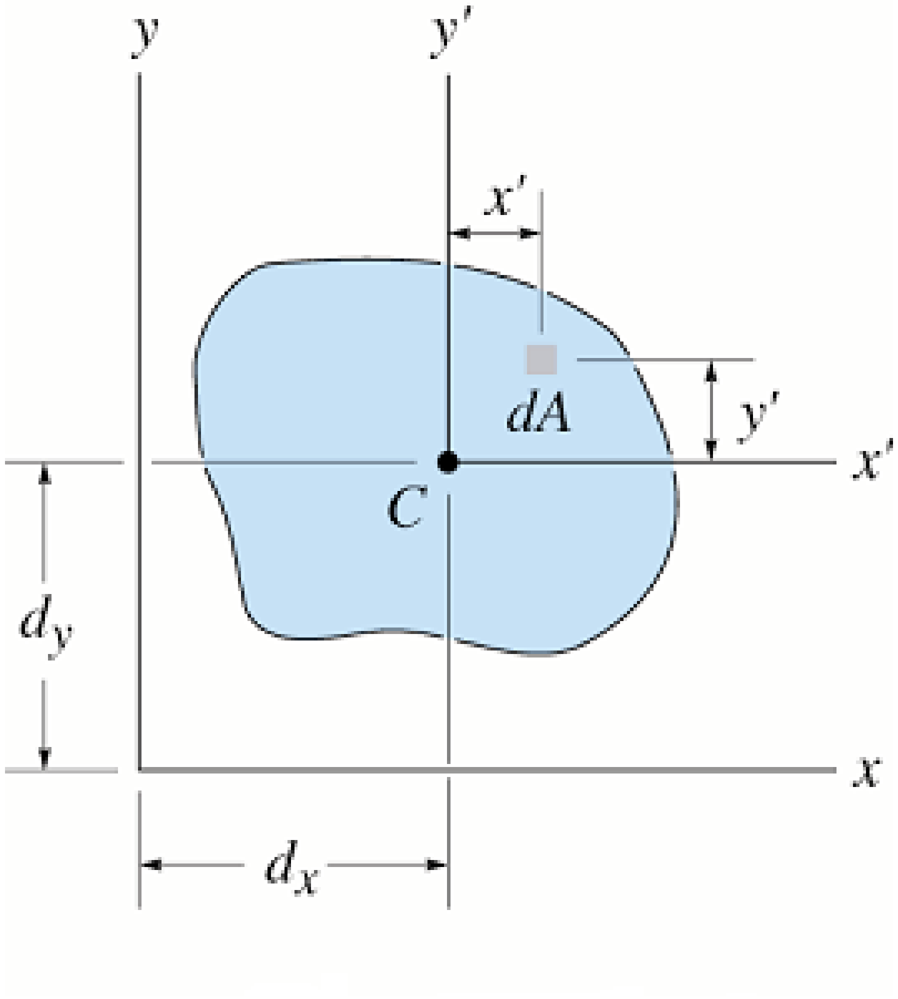
- It is common to have composite cross-sections comprising many built-up sections in which the component parts are not symmetrically distributed about the centroidal axis.
- For instance





- We determine the moment of inertia of such a section by finding the moment of inertia of the component parts about their own centroidal axis, and then applying a transfer formula.
- The resulting moment of Inertia’s are then summed to get the moment of inertia for the composite cross section.

- One such transfer formula is the parallel axis theorem.



$$I_x = \bar{I}_{x'} + A{d_y}^2$$

$$I_y = \bar{I}_{y'} + A{d_x}^2$$

LECTURE 10

RESULTANT OF DISTRIBUTED LINE LOADS, LIQUID PRESSURE AND FLEXIBLE CABLES