

Final value theorem

In mathematical analysis, the **final value theorem** (FVT) is one of several similar theorems used to relate frequency domain expressions to the time domain behavior as time approaches infinity. A final value theorem allows the time domain behavior to be directly calculated by taking a limit of a frequency domain expression, as opposed to converting to a time domain expression and taking its limit.

Mathematically, if

$$\lim_{t \rightarrow \infty} f(t)$$

has a finite limit, then

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$

where $F(s)$ is the (unilateral) Laplace transform of $f(t)$ ^{[1][2]}.

Example where FVT holds

For example, for a system described by transfer function

$$H(s) = \frac{6}{s+2},$$

and so the impulse response converges to

$$\lim_{t \rightarrow \infty} h(t) = \lim_{s \rightarrow 0} \frac{6s}{s+2} = 0.$$

That is, the system returns to zero after being disturbed by a short impulse. However, the Laplace transform of the unit step response is

$$G(s) = \frac{1}{s} \frac{6}{s+2}$$

and so the step response converges to

$$\lim_{t \rightarrow \infty} g(t) = \lim_{s \rightarrow 0} \frac{s}{s} \frac{6}{s+2} = \frac{6}{2} = 3$$

and so a zero-state system will follow an exponential rise to a final value of 3.

Example where FVT does not hold

However, for a system described by the transfer function

$$H(s) = \frac{9}{s^2+9},$$

the final value theorem *appears* to predict the final value of the impulse response to be 0 and the final value of the step response to be 1. However, neither time-domain limit exists, and so the final value theorem predictions are not valid. In fact, both the impulse response and step response oscillate, and (in this special case) the final value theorem describes the average values around which the responses oscillate.

There are two checks performed in Control theory which confirm valid results for the Final Value Theorem:

1. All roots of the denominator of $H(s)$ must have negative real parts.
2. $H(s)$ must not have more than one pole at the origin.

Rule 1 was not satisfied in this example, in that the roots of the denominator are $+j3$ and $-j3$.

Notes

- [1] Wang, Ruye (2010-02-17). "Initial and Final Value Theorems" (http://fourier.eng.hmc.edu/e102/lectures/Laplace_Transform/node17.html). . Retrieved 2011-10-21.
- [2] Alan V. Oppenheim, Alan S. Willsky, S. Hamid Nawab (1997). *Signals & Systems*. New Jersey, USA: Prentice Hall. ISBN 0-13-814757-4.

External links

- http://wikis.controltheorypro.com/index.php?title=Final_Value_Theorem
- http://fourier.eng.hmc.edu/e102/lectures/Laplace_Transform/node17.html Final value for Laplace
- http://www.engr.iupui.edu/~skoskie/ECE595s7/handouts/fvt_proof.pdf Final value proof for Z-transforms

Article Sources and Contributors

Final value theorem *Source:* <http://en.wikipedia.org/w/index.php?oldid=466515677> *Contributors:* DARTH SIDIOUS 2, Einshrek, Endothermic, Geometry guy, Giftlite, Glrx, Michael Hardy, Orborde, Qetuth, Raven1977, TedPavlic, 21 anonymous edits

License

Creative Commons Attribution-Share Alike 3.0 Unported
[//creativecommons.org/licenses/by-sa/3.0/](https://creativecommons.org/licenses/by-sa/3.0/)
