ASSIGNMENT 1

ME 362

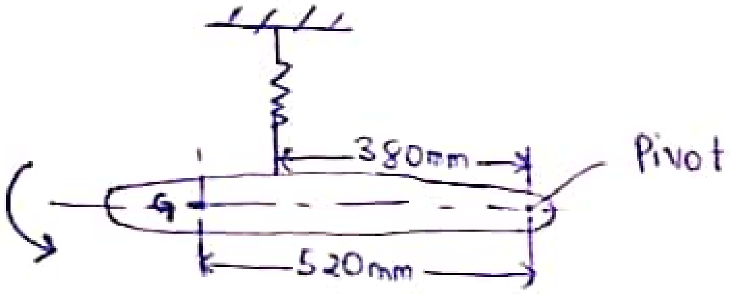
INDEX No: 9383417

Q1 Mass of bar (M_b) = 25kg

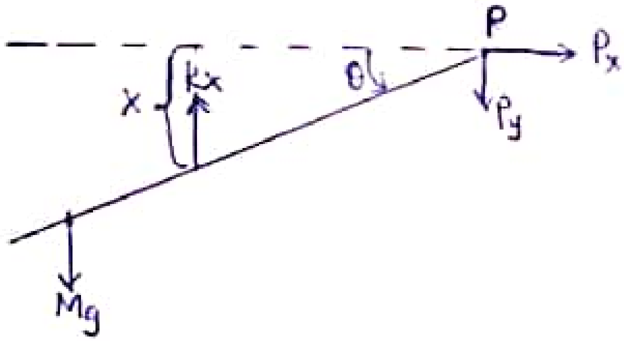
Radius of gyration about c.g (r_G) = 0.235m

Mass of spring (m_s) = 3kg

Stiffness (k) = 520N/m



Free body Diagram



$x = 0.38\theta \quad \dot{x} = 0.38\dot{\theta}$

Equivalent mass of spring (m_{eq}) = $\frac{m_s}{3} = 1$ kg

Moment of inertia of bar about pivot

$I_p = M_b r_G^2 + M_b d^2$ $\quad$ d = distance from P to c.g of bar

$\therefore \; I_p = 25(0.235)^2 + 25(0.520)^2$

$= 8.1406$ kgm^2

Energy of system is conserved

$$P.E + K.E = C$$

$$\Rightarrow \left(\tfrac{1}{2}kx^2\right) + \left(\tfrac{1}{2}I_p\dot{\theta}^2 + \tfrac{1}{2}m_{eq}\dot{x}^2\right) = C$$

Differentiating both sides w.r.t time

$$\Rightarrow I_p\dot{\theta}\ddot{\theta} + (0.38)^2 m_{eq}\dot{\theta}\ddot{\theta} + (0.38)^2 k\theta\dot{\theta} = 0$$

Dividing through by $\dot{\theta}$

$$\Rightarrow \left(I_p + (0.38)^2 m_{eq}\right)\ddot{\theta} + (0.38)^2 k\theta = 0$$ — Equation of motion

$$\therefore \omega_n = \sqrt{\frac{(0.38)^2 k}{I_p + (0.38)^2 m_{eq}}}$$

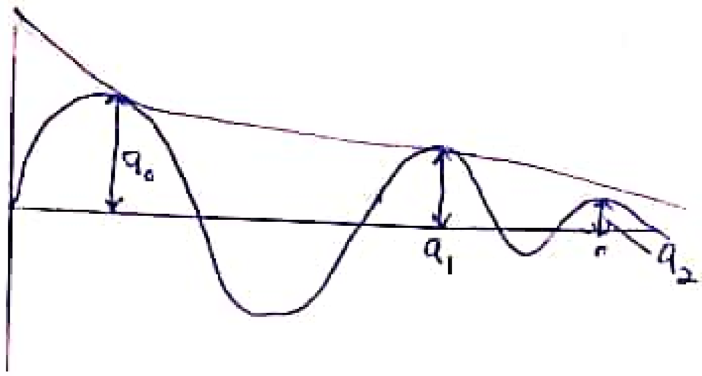
$$\omega_n = \sqrt{\frac{(0.38)^2 \times 520}{8.1406 + (0.38)^2(1)}} = 3.0105 \text{ rad/s}$$

$$\text{Period } (T) = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.0105} = 2.09\text{s}$$

$\therefore$ The natural period of the oscillation is 2.09s

Q2. Mass(m) = 25 kg

Stiffness of spring (k) = 15×10^3 N/m



$$a_2 = \frac{1}{5} a_1$$

logarithmic decrement

$$\delta = \ln\left(\frac{a_1}{a_r}\right)$$

$$\delta = \ln\left(\frac{a_1}{a_2}\right) = \ln(5)$$

Also $\delta = \dfrac{2\pi\xi}{\sqrt{1-\xi^2}}$, ξ = damping ratio

$$\Rightarrow \ln 5 = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$(\ln 5)^2 (1-\xi^2) = (2\pi)^2 \xi^2$$

$$(\ln 5)^2 - \xi^2 (\ln 5)^2 = (2\pi)^2 \xi^2$$

$$\left[4\pi^2 + (\ln 5)^2\right] \xi^2 = (\ln 5)^2$$

$$\therefore \quad \xi = \sqrt{\frac{(\ln s)^2}{4\pi^2 + (\ln s)^2}}$$

$$\xi = 0.2481$$

Also $\xi = \frac{c}{c_c}$, c = Actual damping coefficient
c_c = Critical damping coefficient

$$c = \xi c_c$$

But $c_c = 2\sqrt{mk}$

$$= 2\sqrt{25 \times 15 \times 10^3}$$

$$= 1224.7449 \text{ kg/s}$$

$$\therefore \quad c = 0.2481 \times 1224.7449$$

$$= 303.86 \text{ kg/s}$$

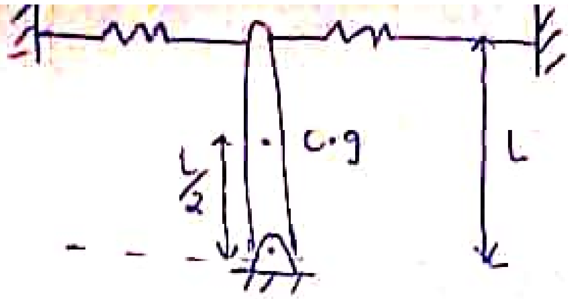
$$\omega_n = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{15 \times 10^3}{25}} = 24.49 \text{ rad/s}$$

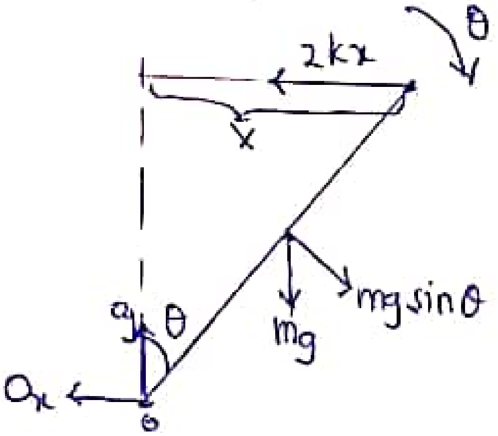
$$\omega_n = 2\pi f$$

$$f = \frac{1}{2\pi} \cdot \sqrt{\frac{15 \times 10^3}{25}} = 3.9 \text{ Hz}$$

Q3.



Free body diagram (After displace bar through angle θ



$x = L\theta$ for small angles of θ

$$\Sigma M_o = I_o \ddot{\theta}$$

$$\Rightarrow -2kxL + mg\sin\theta \frac{L}{2} = I_o \ddot{\theta}$$

$$x = L\theta$$

$$\Rightarrow -2kL^2\theta + mg\sin\theta \frac{L}{2} = I_o \ddot{\theta}$$

For small angles of θ, $\sin\theta = \theta$

$$\Rightarrow -2kL^2\theta + mg\theta\frac{L}{2} = I_o \ddot{\theta}$$

I_o for bar $= \frac{1}{3}mL^2$

Equation of motion:

$$\Rightarrow \frac{mL^2}{3}\ddot{\theta} + 2kL^2\theta - \frac{mgL\theta}{2} = 0$$

$$\frac{mL^2}{3}\ddot{\theta} + \left(2kL^2 - \frac{mgL}{2}\right)\theta = 0$$

The system is stable if $\left(2kl^2 - \frac{mgl}{2}\right) > 0$

$$\Rightarrow 2kl^2 > \frac{mgl}{2}$$

$$4kl > mg$$