KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY
CHEMICAL ENGINEERING DEPARTMENT
CHE 251: CHEMICAL PROCESS CALCULATIONS
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LECTURE 3: INTRODUCTION TO ENGINEERING CALCULATIONS II

Scientific Notation, Significant Figures, and Precision
Dimensional Homogeneity and Dimensionless Quantities

Learning Objectives

At the end of the lecture the student is expected to able to understand or do the following:

- Scientific notation
- Significant figures
- Employ an appropriate number of significant figures in calculations
- Apply the concepts of dimensional consistency to determine the validity of an equation or function

Scientific Notation

Both very large and very small numbers are commonly encountered in process calculations. A convenient way to represent such numbers is to use **scientific notation,** in which a number is expressed as the product of another number (usually between 0.1 and 10) and a power of 10.

Examples: 123,000,000 = 1.23×10^8 (or 0.123×10^9)
0.000028 = 2.8×10^{-5} (or 0.28×10^{-4})
Avogadro's number: 602,213,670,000,000,000,000,000 = 6.0221367×10^{23}

Significant Figures

The **significant figures** of a number are the digits from the first nonzero digit on the left to either
(a) the last digit (zero or nonzero) on the right if there is a decimal point, or
(b) the last nonzero digit of the number if there is no decimal point.

For example,
2300 or 2.3×10^3 has two significant figures.
2300. or 2.300×10^3 has four significant figures.
2300.0 or 2.3000×10^3 has five significant figures.
23,040 or 2.304×10^4 has four significant figures.
0.035 or 3.5×10^{-2} has two significant figures.
0.03500 or 3.500×10^{-2} has four significant figures.

(Note: The number of significant figures is easily shown and seen if scientific notation is used.)

The number of significant figures in the reported value of a measured or calculated quantity provides an indication of the precision with which the quantity is known: the more significant figures, the more precise is the value.

Generally, if you report the value of a measured quantity with three significant figures, you indicate that the value of the third of these figures may be off by as much as a half-unit. Thus, if you report a mass as 8.3 g (two significant figures), you indicate that the mass lies somewhere between 8.25 and 8.35 g, whereas if you give the value as 8.300 g (four significant figures) you indicate that the mass lies between 8.2995 and 8.3005 g.

Note, however, that this rule applies only to measured quantities or numbers calculated from measured quantities. If a quantity is known precisely-like a pure integer (2) or a counted rather than measured quantity (16 oranges)-its value implicitly contains an infinite number of significant figures (5 cows really means 5.0000 ... cows).

When two or more quantities are combined by multiplication and/or division, the number of significant figures in the result should equal the lowest number of significant figures of any of the multiplicands or divisors.

If the initial result of a calculation violates this rule, you must round off the result to reduce the number of significant figures to its maximum allowed value, although if several calculations are to be performed in sequence it is advisable to keep extra significant figures of intermediate quantities and to round off only the final result.

$$\overset{(3)}{(3.57)}\overset{(4)}{(4.286)} = \overset{(7)}{15.30102} \Longrightarrow \overset{(3)}{15.3}$$

$$(\overset{(2)}{5.2} \times 10^{-4})(\overset{(4)}{0.1635} \times 10^{7})/\overset{(3)}{(2.67)} = \overset{(9)}{318.426966} \Longrightarrow \overset{(2)}{3.2} \times 10^{2} = \overset{(2)}{320}$$

(The raised quantities in parentheses denote the number of significant figures in the given numbers.)

The rule for addition and subtraction concerns the position of the last significant figure in the sum-that is, the location of this figure relative to the decimal point.

The rule is: *When two or more numbers are added or subtracted, the positions of the last significant figures of each number relative to the decimal point should be compared. Of these positions, the one farthest to the left is the position of the last permissible significant figure of the sum or difference*.

$$\begin{array}{r} 1530 \\ -\ 2.56 \\ \hline 1527.44 \end{array} \Longrightarrow 1530$$

$$1.0000 + 0.036 + 0.22 = 1.2560 \Longrightarrow 1.26$$

$$2.75 \times 10^6 + 3.400 \times 10^4 = (2.75 + 0.03400) \times 10^6$$

$$= 2.784000 \times 10^6 \Longrightarrow 2.78 \times 10^6$$

Finally, a rule of thumb for rounding off numbers in which the digit to be dropped is a 5 is always to make the last digit of the rounded-off number even:
1.35 ===> 1.4
1.25 ===> 1.2

An **exact number** can be used as if it had an infinite number of significant figures. For example, in the units relationship 1 atm = 14.696 psi, the number 1 is assumed to be an exact number, i.e., it does not restrict the number of significant figures in any calculation using this units conversion factor. This means that 1 atm is really 1.00000….. atm, however, 14.696 is listed to only 5 significant figures. The numbers of significant figures listed in the text for unit conversions are sufficient for most chemical engineering calculations. The first number listed is assumed "exact". Sometimes it is not obvious which other numbers are assumed to be exact (e.g., 760 mm Hg, 1.01325 bar, etc.). If in doubt, and if it is important to your engineering calculation, you should check with an appropriate reference.

Examples:	1 ft = 12 in	(both exact numbers)
	1 m = 100 cm	(both exact numbers)
	1 in = 2.54 cm	(1 exact, 2.54 to 3 sig. figs.)
	1 bar = 10^5 Pa	(both exact numbers)
	1 W = 1 N·m	(both exact numbers)
	1 W = $9.486\text{x}10^{-4}$ Btu/s	(1 exact, 9.486 to 4 sig. figs.)

Dimensional Homogeneity and Dimensionless Quantities

Every valid equation must be dimensionally homogeneous: that is, all additive terms on both sides of the equation must have the same dimensions.

Consider the equation

$$u(m/s) = u_o(m/s) + g(m/s^2\ )t(s) \qquad (3\text{-}1)$$

This equation is dimensionally homogeneous, since each of the terms u, u_o, and gt has the same dimensions (length/time). On the other hand, the equation $u = u_o + g$ is not dimensionally homogeneous and therefore cannot possibly be valid.

Equation 3-1 is both dimensionally homogeneous *and* consistent in its units, in that each additive term has the units m/s. If values of U_o, g, and t with the indicated units are substituted into the equation, the addition may be carried out to determine the value of u.

If an equation is dimensionally homogeneous but its additive terms have inconsistent units, the terms (and hence the equation) may be made consistent simply by applying the appropriate conversion factors.

For example, suppose that in the dimensionally homogeneous equation $U = U_o + gt$ it is desired to express the time (t) in minutes and the other quantities in the units given above. The equation can be written as

$$u(m/s) = \mathrm{u_o(m/s)} + g(m/s2)t(min)(60\ s/min)$$
$$= U_o + 60gt$$

Each additive term again has units of *m/s*, so the equation is consistent. The converse of the given rule is not necessarily true-an equation may be dimensionally homogeneous and invalid. For example, if M is the mass of an object, then the equation $M = 2M$ is dimensionally homogeneous, but it is also obviously incorrect except for one specific value of M.

Example 1 (dimensional homogeneity)

Consider the equation D(ft) = 3(s) - 4

1. If the equation is valid, what are the dimensions of the constants 3 and 4?

2. If the equation is consistent in its units, what are the units of 3 and 4?

3. Derive an equation for distance in meters in terms of time in minutes.

Solution

1. For the equation to be valid, it must be dimensionally homogeneous, so that each term must have the dimension of length. The constant 3 must therefore have the dimension 'length/time', and 4 must have the dimension 'length'.

2. For consistency, the constants must be 3 ft/s and 4 ft

3. Define new variables *D'(m)* and ('(min). The equivalence relations between the old and new variables are

$$D(\text{ft}) = \frac{D'(\text{m})}{} \left| \frac{3.2808\ \text{ft}}{1\ \text{m}} \right. = 3.28D'$$

$$t(s) = \frac{t'(\text{min})}{} \left| \frac{60\ \text{s}}{1\ \text{min}} \right. = 60t'$$

.

Substitute these expressions in the given equation

$$3.28D' = (3)(60t') + 4$$

and simplify by dividing through by 3.28

$$D'\ (m) = 55t'\ (min) + 1.22$$

General procedure for rewriting an equation in terms of new variables having the same dimensions but different units:

1. Define new variables (e.g., by affixing primes to the old variable names) that have the desired units.
2. Write expressions for each old variable in terms of the corresponding new variable.
3. Substitute these expressions in the original equation and simplify.

Example 2 (dimensional homogeneity)

Your handbook shows that microchip etching roughly follows the relation

$$d = 16.2 - 16.2e^{-0.021t} \quad t < 200$$

where d is the depth of the etch in microns (micrometers, μm) and t is the time of the etch in seconds. What are the units associated with the numbers 16.2 and 0.021? Convert the relation so that d becomes expressed in inches and t can be used in minutes.

Solution

After you inspect the equation that relates d as a function of t, you should be able to reach a decision about the units associated with each term on the right hand side of the equation. Both values of 16.2 must have the associated units of microns (μm). The exponential term must be dimensionless so that 0.021 must have the associated units of s^{-1}. To carry out the conversion, look up suitable conversion factors from a reference book and multiply so that the units are converted from 16.2 μm to inches, and 0.021 t/s to t/min

$$d_{in} = \frac{16.2\ \mu m}{} \left| \frac{1\ m}{10^6\ \mu m} \right| \frac{39.27\ in.}{1\ m} \left[1 - \exp \frac{-0.021}{s} \left| \frac{60 s}{1\ min} \right| \frac{t_{min}}{} \right]$$

$$= 6.38 \times 10^{-4}(1 - e^{-1.26 t_{min}})\ \text{inches}$$

Dimensionless Numbers and Groups

A **dimensionless quantity** can be a pure number (2, 1.3, 2/5) or a multiplicative combination of variables with no net dimensions. Dimensionless groups are dimensionless numbers formed by a combination of variables; as a result of theory or experiment.

For example

$$\text{Reynolds number} = \frac{Dv\rho}{\mu} = N_{RE}$$

where D is the pipe diameter, say in cm; ν is the fluid velocity, say in cm/s; ρ is the fluid density, say in g/cm^3; and μ is the viscosity, say in centipoise, units that can be converted to g/(cm)(s). Introducing the consistent set of units for D, ν, ρ, and μ into $D\nu\rho / \mu$, you will find that all the units cancel out so that the numerical value of 1 is the result of the cancellation of the units.

$$\frac{\cancel{cm}}{} \left| \frac{\cancel{cm}}{\cancel{s}} \right| \frac{\cancel{g}}{\cancel{cm^3}} \left| \frac{(\cancel{cm})\ (\cancel{s})}{\cancel{g}} \right.$$

Exponents (such as the 2 *in* X^2*), transcendental functions (such as* log, exp ↔ *e*, and sin), *and arguments of transcendental functions (such as the* X *in* sin X*) must be dimensionless quantities.*

For example, 10^2 makes perfect sense, but $10^{2\ \text{ft}}$ is meaningless, as is log (20 s) or sin (3 dynes).