KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

INSTITUTE OF DISTANCE LEARNING

(*BSc. ELECTRICAL AND ELECTRONIC ENGINEERING, 3*)

EE 252: ELECTRICAL ENGINEERING MACHINES

[Credit: 2.]

(FOR TOP-UP STUDENTS)

MECHANICAL ENGINEERING

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Course Author

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He returned to Ghana as the Reintegration Counsellor in charge of the German Office for Germany-trained Returning Experts (Rueckkehrerbuero), an office dedicated to ensuring the smooth and comfortable reintegration back into Ghana of Germany-trained Ghanaian experts.

He later joined the Ghana Standards Board (GSB) as Senior Scientific Officer attached to the Metrology Division and worked on the Electricity Meter Testing, Calibration and Verification Project. He has finally settled as Lecturer at the Kwame Nkrumah University of Science and Technology (KNUST), Kumasi, Ghana, since October 2001.

He has been an External Moderator for the National Board for Board for Professional And Technician Examinations (NABPTEX) since 2005, and was a Visiting Lecturer for the National University of Rwanda during the period of 2004 to 2006. He was a member of the team of consultants that conducted survey for the Private Enterprise Foundation (PEF) on the Impact of Power Outages on Some Selected Manufacturing Industries in Ghana.

His research interests are on electrical energy management systems, prepayment metering system, rural electrification, power system quality and application of local material as backfills for improving earthing. He has been the University Electrical Services Consultant since 2004. He has also served as Faculty Exams Officer and is currently the Head of the Electrical Engineering Department.

Course Introduction

Introduction

This course introduces the student to basic electrical machine theory and the energy-conversion processes whereby energy is converted from one system to another via an electromagnetic system. An electrical machine may be either of the *rotary type* (e.g. generator or motor) or the *non-rotary type* (e.g. transformer). For the rotary electrical machines, the rotary motion is realized through the application of electromechanical energy conversion principles. Usually, the energy system coupled to the electrical energy system is a mechanical one. Thus for a generator, mechanical energy is converted to electrical energy, whilst the function of a motor is to convert electrical energy into mechanical energy. However, for *converters*, i.e., the *non-rotary devices*, the *same electrical energy* is transferred from one system to another as in the transformer.

The course will generally address, amongst others, the principles of operation of electromechanical energy conversion machines and converters, their circuit and phasor representations, steady-state performance characteristics and equations, as well as practical methods/techniques of determining losses and efficiency of a transformer.

Learning objectives

After going through this course, you should:

- understand the principles of electromechanical energy conversion in an electromagnetic machine or device
- know the types and characteristics of DC machines
- have basic understanding of polyphase AC machines
- be familiar with the principles of operation and voltage regulation of the transformer, as a non-rotary electrical energy converter
- be able to practically determine the losses associated with the transformer and hence its efficiency.

Course Outline

The course is divided into four units. Each unit is broken down into sessions, each of which will address one or more of the course objectives.

Unit 1: Principles of Electromechanical Energy Conversion

Unit 2: DC Machines

Unit 3: Introduction to Polyphase Machines

Unit 4: Transformers

References

1. Electrical Technology – by Edward Hughes
2. Electric Machinery & Transformer Technology– by R. A. Pearman

EE 252: Electrical Engineering Machines (2 0 2)

Course Content:

Principles of Electromechanical Energy Conversion. Basic Transducers. Single and Double Excitation. DC Machines. Introduction to Polyphase Induction Machines. Transformers.

Unit 1

PRINCIPLES OF ELECTROMECHANICAL ENERGY CONVERSION

Introduction:

An ***electromechanical*** machine is one that links an electrical energy system to a mechanical system by providing a reversible means of energy flow in its magnetic field. The magnetic field is thus the mutual link coupling the two systems. The energy transferred from one system, the input, is temporarily partly stored or lost in the magnetic field and the rest released to the other system, the output. From the law of conservation of energy, it follows that:

Energy input = Energy lost + Increase in stored energy + Energy Output

An electromechanical machine can be operated by either an alternating current or direct current, and can function as a *motor, a generator or a converter* under steady-state or transient conditions. The function of a motor is to transfer electrical energy into mechanical energy whilst a generator converts mechanical energy into electrical energy. Converters transfer electrical energy from one system to another as in the transformer.

The coupling of a mechanical energy system to an electrical system in an electromechanical machine implies that a *mechanical force or torque is associated with linear (translational) or radial (rotary) displacement of its point of action*.

Suppose an infinitesimal electrical energy input produces an infinitesimal increment dx in mechanical displacement. The corresponding increments of mechanical energy output, loss, storage and output are related by the ***energy balance equation***:

$$dW_{elecinput} = dW_{loss} + dW_{field} + dW_{mechoutput} \qquad (1.1)$$

The mechanical power output accounts for both mechanical storages as well as the useful power. The force f_x exerted by the mechanical member in the direction of displacement dx is the rate of change of energy with position and is given by:

$$f_x = \frac{dW_{mechoutput}}{\mathrm{dx}} \qquad (1.2)$$

Objectives

After going through this unit, you should be able to:

- understand the basic principles of electromechanical energy conversion
- explain how mechanical forces can be developed for linear and rotary applications
- explain how these principles are applied in basic transducers

SESSION 1-1 FORCE OF ALIGNMENT AND ITS APPLICATION IN BASIC TRANSDUCERS

An electromechanical system can develop a mechanical force in two ways:

1. By alignment
2. By interaction

1-1.1 Force of Alignment:

The *alignment force may be seen either as a force of attraction or as a lateral force*. This can be illustrated by the arrangement of two poles of ferromagnetic material of opposite polarity and situated opposite one another. A flux passes from the one to the other. The magnetized surfaces are attracted towards one another as indicated in the diagram below.

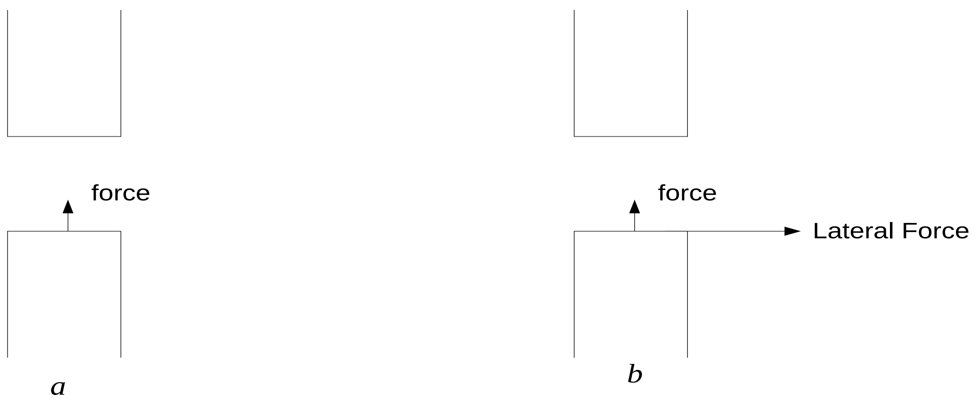


Fig 1.1: Force of Alignment

It can be shown that the ***force of alignment acts in a direction that increases the magnetic stored energy*** in the arrangement.

The following **magnetic equations** are recalled.

$$S = \frac{l}{\mu_0 \mu_r A} \quad ; \quad \Phi = \frac{mmf}{reluctance} = \frac{NI}{S} \quad ; \quad L = \frac{N\Phi}{I} = \frac{N^2}{S} \quad ; \quad W_{fld} = \frac{1}{2} L I^2$$

$$mmf = Hl = IN \quad ; \quad \mathrm{B} = \mu H = \mu_r \mu_0 H \quad ; \quad \mathrm{H} = \frac{B}{\mu_r \mu_0} \quad ; \quad \mathrm{I} = \frac{Hl}{N}$$

where the symbols have their usual meanings.

In the first case (a), it will try to bring the poles together, thus reducing the airgap length l. This causes a decrease in the reluctance S of the air gap in the magnetic circuit and hence will increase the flux Φ and consequently the stored magnetic field energy W_{fld} .

In the second case (b), the poles are not situated opposite one another, and the resultant force tries to achieve maximum stored magnetic energy by two component actions:

i. By attraction of the poles towards one another as before
ii. By aligning the poles laterally.

The force of alignment thus results from the distortions in the magnetic field. It can be applied to produce

1. *linear motion* as in basic transducers (like electromagnetic relays to operate switches, loudspeakers) and
2. *rotary motion* as in a motor.

1-1.2 Basic Transducers

Definition: *Transducers are electromechanical machines that operate at very low power levels, and are used particularly to provide 'signals' with which to activate electronic control devices*.

TRANSDUCERS CONVERT ONE FORM OF ENERGY TO ANOTHER

We will be treating the ***attracted-armature electromagnetic relay*** as a ***translational transducer*** where motion is linear.

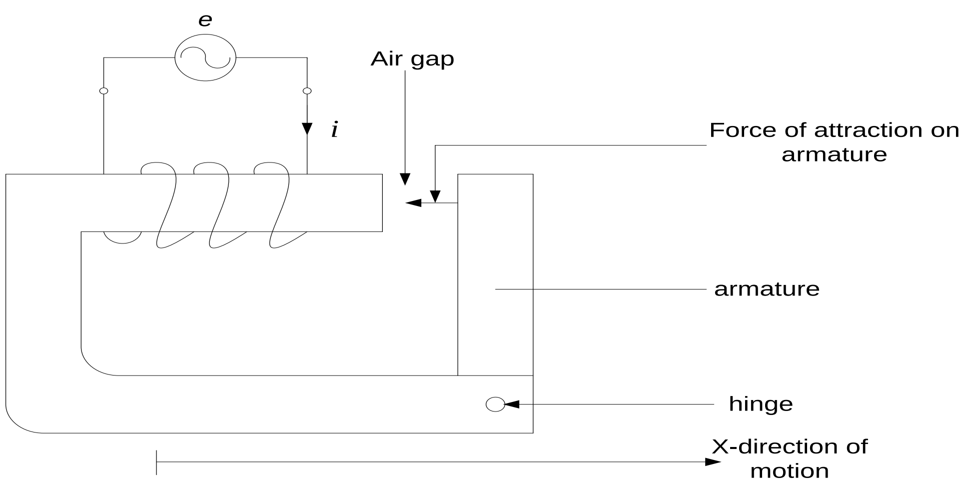


Fig 1.2: Attracted-Armature Electromagnetic Relay Operation

This relay is an example of the so-called *singly-excited* electromechanical machine, that is, only a single coil is excited or energized for its operation.

When the coil is energized by an instantaneous voltage e such that a current i flows, a flux is set up in the relay core and the air gap. The surfaces adjacent to the air gap become magnetized and are attracted, hence pulling the plate in the direction shown.

Let the armature move an infinitesimal distance dx in the x-direction as a result of the electrical energy input. With the movement of the armature, the lengths of the air gaps are decreasing, the reluctance S is also decreasing and hence the inductance L of the air gap is also changing by a small amount resulting in an increase of the stored energy in the magnetic field system.

It must be noted that it is this change in magnetic field energy that produces the force of reaction f over the distance dx.

It can be noted that the velocity is $v = -\frac{dx}{dt}$. Then from the energy balance Eqn (1.1), the general solution of the *power balance* can be found.

$$p_e = p_{loss} + p_f + p_m \qquad (1.3)$$

1-1.3 Alignment Force on Attracted-Armature Relay:

Electrical Power Input p_e :

The electrical power input is given as the product of the instantaneous voltage and current. Thus $p_e = e \times i$

But $e = \frac{d}{dt}(N\Phi) = \frac{d}{dt}(L \cdot i) = L\frac{di}{dt} + i\frac{dL}{dt}$

Thus $$p_e = e \times i = Li\frac{di}{dt} + i^2\frac{dL}{dt} \qquad (1.4)$$

Losses p_{loss}:

Since the change takes place relatively slowly over a short distance, the losses may be assumed negligible $\Rightarrow$ $p_{loss} = 0$.

Field Stored Energy Field p_f :

$$p_f = \frac{dW_f}{dt} = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt} + \frac{1}{2}i^2\frac{dL}{dt} \qquad (1.5)$$

Mechanical Power p_m :

The mechanical power is given by the product of the force f and velocity v. Thus $p_m = f \times v$. The mechanical power can also be obtained from the general power equation (1.3).

$$
\begin{aligned}
p_m &= p_e - p_{loss} - p_f \\
&= \left\{ Li\frac{di}{dt} + i^2\frac{dL}{dt} \right\} - \{o\} - \left\{ Li\frac{di}{dt} + \frac{1}{2}i^2\frac{dL}{dt} \right\} \\
&= \frac{1}{2}i^2\frac{dL}{dt} = \frac{1}{2}i^2\frac{dL}{dx}\cdot\frac{dx}{dt} \\
&= \frac{1}{2}i^2\frac{dL}{dx}\cdot(-v)
\end{aligned} \tag{1.6}
$$

But $p_m = f \times v$. And hence

$$
\begin{aligned}
& p_m = f \times v = \frac{1}{2}i^2\frac{dL}{dx}\cdot(-v) \\
& \Rightarrow f = -\frac{1}{2}i^2\frac{dL}{dx}
\end{aligned} \tag{1.7}
$$

Since most of the stored energy is *retained in the air gap* and all of the energy is stored within the magnetic circuit, this force is also due to the *change in reluctance* of the magnetic circuit. Hence the force is alternatively referred to as the ***reluctance force*.**

It is sometimes convenient and useful to *express the force in terms of the rate of change of reluctance with the air gap length*. Consider the general expression of the force in terms of rate of change of inductance with air gap length.

$$
\begin{aligned}
& f = -\frac{1}{2}i^2\frac{dL}{dx} = -\frac{1}{2}i^2\frac{dL}{dS}\cdot\frac{dS}{dx} \\
& \text{But } L = \frac{N^2}{S} \;\Rightarrow\; \frac{dL}{dS} = -\frac{N^2}{S^2} \\
& \Rightarrow f = \frac{1}{2}\frac{i^2N^2}{S^2}\cdot\frac{dS}{dx} \\
& \qquad = \frac{1}{2}\Phi^2\frac{dS}{dx}
\end{aligned} \tag{1.8}
$$

It must be noted from Eqns (1.5) and (1.6) that the rate of mechanical energy conversion p_m is equal to one part (i.e. second term of Eqn 1.5) of the expression for the rate of field energy storage - that partial differential due to *constant current*.

Thus the force of reaction *f* can be expressed as:

$$
f = \left.\frac{dW_f}{dx}\right|_{i=const} \tag{1.9}
$$

It can also be seen from Eqn (1.4) that the input of electrical energy is divided, one part (first term) being converted to energy stored in the magnetic field and the other part (second term) converted to mechanical energy.

1-1.4 Alignment Force between Parallel Magnetic Surfaces:

Consider the arrangement shown below.

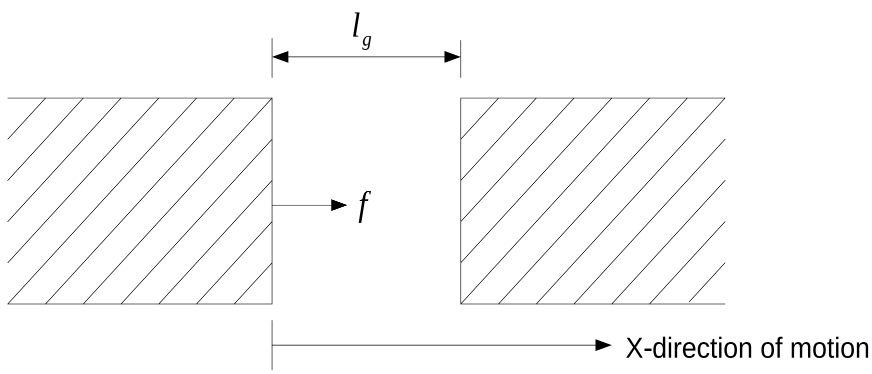


Fig 1.3: Force of Alignment between Parallel Magnetized Surfaces

To deduce the force of alignment between the core surfaces, consider again the Eqn (1.9). Since the remainder of the core is assumed ideal, the total reluctance appears at the air gap. Hence the reluctance is given by:

$$S = \frac{l_g}{\mu_0 \mu_r A}$$

where $\mu_r = 1$ for air and l_g is the particular value of x for the arrangement considered.

Recall the force equation $f = \frac{1}{2}\Phi^2 \frac{dS}{dx}$. We differentiate the reluctance with respect to the distance, and obtain $\frac{dS}{dx} = \frac{dS}{d\,l_g} = \frac{1}{\mu_0 A}$. Thus the force equation becomes

$$\begin{aligned} f &= \frac{1}{2}\Phi^2 \frac{1}{\mu_0 A} \\ &= \frac{1}{2} B^2 A^2 \frac{1}{\mu_0 A} \\ &= \frac{1}{2}\frac{B^2 A}{\mu_0} \end{aligned} \qquad (1.10)$$

Example 1.1

An electromagnet shown below has a core of effective length 600 mm and a cross-sectional area of 500 mm^2. A rectangular block of steel of mass 0.4 kg is attracted by the electromagnet's force of alignment when its 100-turn coils are energized. The magnetic circuit is 200 mm long and the effective cross-sectional area is also 500 mm^2. If the relative permeability of both core and steel block is 700, estimate the coil current. Neglect frictional losses and assume the acceleration due to gravity as $g = 10 ms^{-2}$.

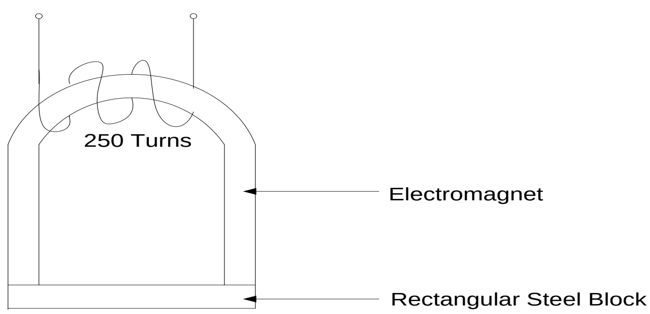


Fig 1.4: An electromagnet

Solution 1.1

There are two air gaps in the magnetic circuit. Hence the force to part the steel block is **double** that for a single air gap.

$$f = 2 \times \frac{B^2 A}{2\mu_0} = 20\,N$$

The flux density is thus given by $B = \sqrt{\frac{20\mu_0}{A}} = \sqrt{\frac{20 \times 4\pi \cdot 10^{-7}}{500 \times 10^{-6}}} = \underline{0.222T}$

The magnetizing force is $H = \frac{B}{\mu_0 \mu_r} = \frac{0.222}{4\pi \cdot 10^{-7} \times 700} = 250\,At/m$

The magnetomotive force is given by

$$\begin{aligned} F &= H \cdot l = 250 \times (600 + 200) \times 10^{-3} = 200\,At \\ &= I \cdot N \end{aligned}$$

Hence the current is $I = \frac{F}{N} = \frac{200}{100} = \underline{2\,A}$

Example 1.2:

A solenoid relay is operated from a *110 V DC* supply and the *5000*-turn coil resistance is *5.5 kΩ*. The core diameter of the relay is *20 mm* and the air gap length is *1.5 mm*. The gap faces may be taken as parallel and the permeability of the ferromagnetic parts as very high. Estimate:

(a) the air gap flux density
(b) the coil inductance
(c) the force on the armature.

Solution 1.2:

(a) The direct current through the armature windings is:

$$I = \frac{V}{R} = \frac{110}{5500} = 20\,mA$$

The magnetomotive force $mmf = I \cdot N = 20 \times 10^{-3} \times 5000 = \underline{100\,At}$

The magnetizing force $H = \frac{mmf}{l_g} = \frac{100}{1.5 \times 10^{-3}} = 0.67 \times 10^5 \; At/m$

The air gap flux density is $B = \mu_0 H = (4\pi \times 10^{-7}) \times (0.67 \times 10^5) = \underline{84\,mT}$

(b) The coil inductance is given by

$$L = \frac{N\Phi}{I} = \frac{N \cdot BA}{I} = \frac{5000 \times \left(84 \times 10^{-3}\right)\left(\dfrac{\pi \times 20^2}{4}\right) \times 10^{-6}}{20 \times 10^{-3}} = \underline{6.56\,H}$$

(c) The magnetic (or reluctance) force on the armature is a reluctance force, and is given by

$$f = -\frac{1}{2} i^2 \frac{dL}{dx}$$

The inductance is inversely proportional to the air gap length. Hence the inductance L_x at a particular air gap length x can be found from the calculated inductance at air gap length of *1.5mm*.

$$\frac{L_x}{L_{1.5}} = \frac{1.5}{x} \quad \Rightarrow L_x = L_{1.5} \times \frac{1.5}{x} \quad \therefore \frac{dL}{dx} = -\frac{9.84}{x^2}$$

For a change in displacement of 1.5 mm, the force is given as

$$f = -\frac{1}{2} i^2 \left.\frac{dL}{dx}\right|_{x=1.5} = -\frac{1}{2} i^2 \left.\left(\frac{-9.84}{x^2}\right)\right|_{x=1.5} = \frac{1}{2}(20\times10^{-3})^2 \left\{\frac{9.84}{(1.5\times10^{-3})^2}\right\} = \underline{0.872\,N}$$

Alternately, the force is given as

$$f = \frac{B^2 A}{2\mu_0} = \frac{(84\times10^{-3})^2 \times \left\{\frac{\pi(20)^2\times10^{-6}}{4}\right\}}{2\times(4\pi\cdot10^{-7})} = \underline{0.882\,N}$$

SESSION 2-1 FORCE OF INTERACTION AND ITS APPLICATION IN ROTARY MACHINES

The force of interaction is applied in the principle of force on a *current-carrying* conductor. This is another form of the principle of maximum stored magnetic energy. The relationship between these principles will be discussed in the treatment of doubly-excited rotating machines.

2-1.1 Force of Interaction on a Current-Carrying Conductor:

Consider a rectangular coil of N turns of dimensions $l\times b$ in which the plane of the coil is at an angle θ to the direction of a uniform magnetic field of density B. The coil is pivoted about the midpoints of the sides b and carries a current I. The illustration is shown below.

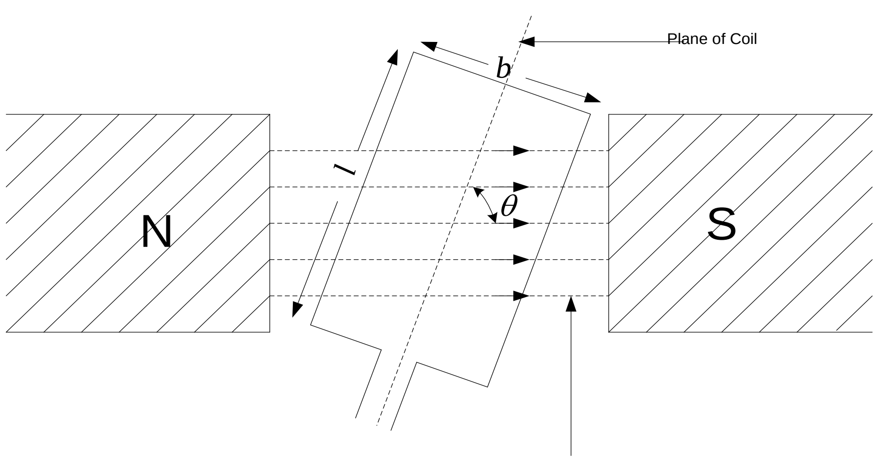


Fig 1.5: Current-Carrying Coil in a Magnetic Field

The flux density is given by the flux Φ passing uniformly and *normally, i.e., at right angles* through the surface area $A = l\times b$. Since the plane is not at right angles to the direction of the field but inclined at an angle θ, the flux through the plane is given by:

$$\Phi = BA\sin\theta \tag{1.11}$$

The passage of the current in the coil produces flux which interacts with the magnetic field to produce *interaction force*. The forces developed in each half of the coil sides b are equal and produce torques of opposing sense, they therefore cancel each other. However, the forces F on each of the coil sides l give rise to a torque T. Because the coil and the field are inclined at an angle θ with one another, the effective length of the coil in the field, i.e., the length presented normally to the field, is $l\sin\theta$.

The force exerted on the N turns is given by the expression:

$$F = BINl\sin\theta \tag{1.12}$$

The torque on the coils is given by

$$\begin{aligned} T &= F \cdot b \\ &= BINl\sin\theta \cdot b \\ &= BANI\sin\theta \end{aligned} \tag{1.13}$$

SESSION 3-1 ROTATING MACHINES – TYPES AND CONSTRUCTIONAL FEATURES

The linear operations of machines like relays are useful for certain modes of application but they have the ***drawback that the motion such linear machines give rise to is not continuous***. Practical linear motions take place over relatively short distances before termination but rotary motion can be maintained almost indefinitely.

A rotating machine, which is an electromechanical energy conversion device, may be used either to convert mechanical energy to electric energy or to convert electric energy to mechanical energy. When a machine is driven mechanically by a prime mover such as a steam turbine, hydraulic turbine or diesel engine, and delivers electric energy at its output, it is called a ***generator***. If electric energy is supplied to the machine and its output is used to drive mechanical devices such as conveyors or machine tools, it is called a ***motor***.

3-1.1 Generator action – Fleming's Right-Hand Rule

The generator action depends on the principle that an emf is induced in a conductor ***cutting*** magnetic lines of flux. There is a definite relation between the direction of flux, the direction of motion of the conductor and the direction of the induced emf. This relation is summed up in **Fleming's Right-Hand-Rule** which states that:

*"Extend the thumb, the forefinger and the middle finger of the **right hand** at right angles to one another. Point the forefinger in the direction of the flux and the thumb in the direction in which the conductor is moving; then the middle finger will point in the direction of the induced emf"*.

Generators are rated as to the ***kilowatts*** *(or megawatts)* they can deliver without overheating at a rated voltage and speed.

3-1.2 Motoring action – Fleming's Left-Hand Rule

The motor action is based on the principle that when a ***current-carrying conductor*** of active length L is placed in a magnetic field of field strength B, it experiences a mechanical force F, whose direction is given by Fleming's Left-Hand Rule and its magnitude is given by the relation $F = BIL$ (in Newton). The ***Fleming's Left-Hand Rule*** may be stated as follows:

"Extend the thumb, the forefinger and the middle finger of the ***left hand*** *at right angles to one another. Now turn the left hand in such a position that the forefinger points in the direction of the field and the middle finger in the direction of the current in the conductor; then the thumb will point in the direction of force or motion of the conductor"*.

Motors are rated as to the ***horsepower*** (or kilowatts) they can deliver without overheating at their rated voltage and speed.

SESSION 4-1 CONSTRUCTIONAL FEATURES OF ROTATING MACHINES

Rotating electrical machines have the following essential constructional parts:

- *Stator* (stationary member)
- *Rotor* (rotating member)
- *Auxiliary equipment* such as
    - slip rings (or collector rings)
    - brushgear/commutator assembly (carbon brushes, split-rings, etc) in the case of DC machines
    - armature shaft bearings
    - starting circuit (for single-phase AC motors)

4-1.1 The Stator Unit

The *stator* unit is the stationary part of the machine and consists of the stator frame or yoke, pole-shoes and pole coils (or stator field coils). Consider the four-pole machine in the figure below:

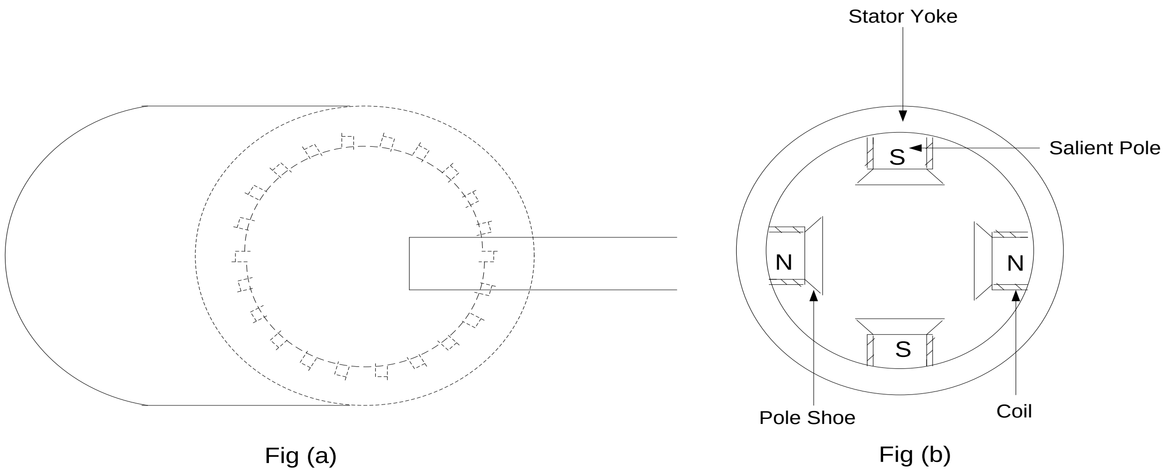


Fig 1.6: The Stator Unit
(a) Stator and one of its Coils (b) Salient-Pole Stator

The *stator frame or yoke* serves the purposes of

i. Providing mechanical support for the poles
ii. Protection for the machine and
iii. Carrying the magnetic field or flux produced by the poles.

In small machines, where weight is of little importance and cheapness is the main consideration, the yoke is made of cast iron. But for large machines, usually cast steel or rolled steel is employed.

The stator yoke carries the stator *pole-cores and pole-shoes*, which serve a double purpose of:

1. spreading out the flux in the air gap, and being of larger cross-sectional area, reducing the reluctance of the magnetic path
2. supporting the field coils (or exciting coils).

Pole-cores and pole-shoes are made of cast iron or cast steel, forged steel or steel laminations to reduce hysteresis and eddy-current loss. Laminated pole-cores, which are used almost universally except on very small machines, have a rectangular cross-section and are held to the frame by bolts. The stator field coils consist of copper wire or strip, and insulated from one another. When the field coils are energised and current is passed through them, they electro-magnetize the poles which produce the necessary flux that is cut by the rotating armature windings.

4-1.2 The Rotor Unit:

The *rotor*, which is the rotating part of the machine, is located on a shaft running on bearings, and is free to rotate between magnetic poles. The *rotor core* is usually cylindrical or drum-shaped, and is built of steel laminations with *slots* to house the *rotor windings*.

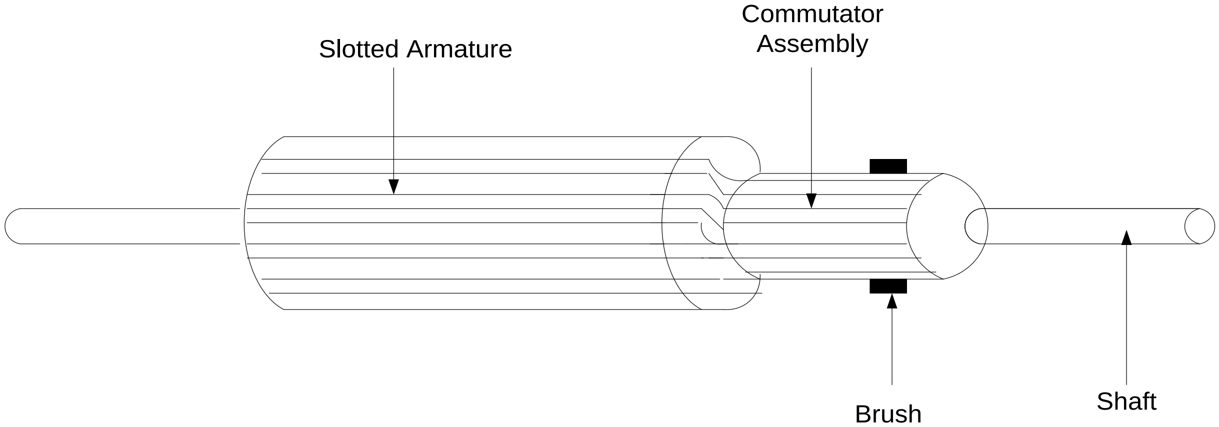


Fig 1.7: The Rotor Unit

Besides housing the rotor windings in *slots* and causing them to rotate to cut the magnetic flux of the magnetic fields, the rotor core also provides a magnetic path of low reluctance to the flux from the poles. The ***rotor windings*** (also referred to as ***armature***) are insulated from each other and placed in rotor slots which are lined with tough insulating material.

4-1.3 The Auxiliary Equipment

The Commutator:

The *commutator or split-rings* is made of copper segments, and has the same functions in the motor as in a generator. Its *purpose is to facilitate the collection of current from the armature windings*. By reversing the alternating current in each conductor as it passes from one N-pole to another S-pole, it thereby rectifies the AC current induced in the armature windings into a continuous and unidirectional current in the external load (as in the case of a generator) or torque (as in the case of a motor).

The Brushgear:

To collect current from a rotating commutator or to feed current to it, use is made of *brushgear*, which consists of brushes, brush holders, brush studs or brush-holder arms, current-collecting busbars, etc

The purpose of brushes is to carry current from the external circuit to the commutator. They are usually made of blocks of *carbon or graphite*, and are rectangular in shape. *Graphite and carbon graphite brushes are self-lubricating and are therefore used widely*. The use of ***copper brushes*** is reserved for machines designed for ***large currents at low voltages***.

The Armature Shaft Bearings:

- Because of their reliability, *ball-bearings* are frequently employed, though for heavy duties, *roller-bearings* are preferable.
- The ball and rollers are lubricated by hard oil for quieter operation and for reducing the wear of the bearings.

EXERCISES ON PRINCIPLES OF ELECTROMECHANICAL ENERGY CONVERSION

Exercise 1.1

(a) What is a basic transducer?

(b) By means of a diagram, briefly explain the principle of operation of an attracted-armature electromagnetic relay.

(c) A solenoid relay operating from a 240 V DC supply has a 4500-turn coil of resistance 8.5 kΩ. The core diameter of the relay is 30 mm and the airgap length is 1.5 mm, the armature being stationary. Assuming the gap surfaces to be parallel and taking $\mu_0 = 4\pi \times 10^{-7}$, determine the following:

 (i) gap flux density
 (ii) inductance of the coil
 (iii) force exerted on the armature

Exercise 1.2

A U-shaped electromagnet shown below has a core of effective length 650 mm and a cross-sectional area of 450 mm^2. A rectangular block of steel of mass 0.65 kg is attracted by the electromagnet's force of alignment when its 300-turn coils are energized. The magnetic circuit is 150 mm long and the effective cross-sectional area is also 450 mm^2. Neglecting frictional losses and assuming the acceleration due to gravity as $g = 10ms^{-2}$, calculate the excitation current. Take the relative permeability of both core and steel block to be 750.

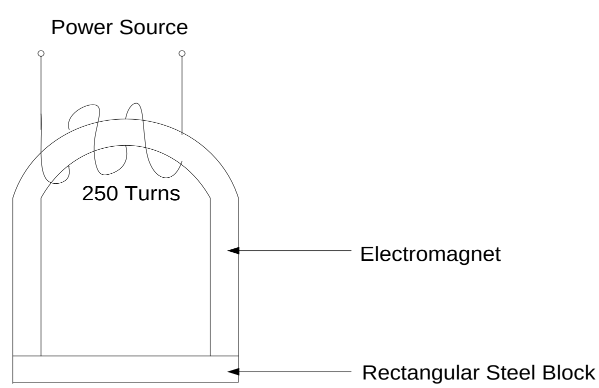


Fig 1.8: Electromagnet

Unit 2
DC MACHINES

Introduction

A DC machine is an alternating current (AC) machine, but furnished with a commutator, which under certain conditions converts AC to DC. The conditions under which the DC machine operates are greatly complicated by the commutator.

Currently, the *most basic type of DC machine* is that of the *commutator type*. Because the field of industrial application of DC is very large, DC machines are produced both as generators and motors, for a large range of outputs, voltages, speeds, etc.

NOTE: The DC motor is very similar to the DC generator in construction. In fact, a machine that runs well as a generator will operate satisfactorily as a motor. They usually differ slightly in detail of design because of the different operating conditions.

For example, as motors often operate in locations in which they are exposed to mechanical damage, dust, moisture or corrosive fumes, motors are often of the *semi-guarded, drip-proof or totally enclosed types*. The generator is, however, usually of the *open type*; that is, the armature and field windings are exposed.

Although the mechanical construction of DC motors and generators is very similar, their functions are different. The function of a generator is to *generate a voltage* when conductors are moved through a field, while the function of a motor is to develop a *twisting effort or torque*.

Objectives

After going through this unit, you should be able to:

- classify and explain the principles of operation of the various DC machines
- understand the steady-state characteristics of DC machines
- deduce performance equations of DC machines and use them to undertake exercises

SESSION 1-2 CLASSIFICATION OF DC MACHINES

On the basis of the excitation systems, DC machines are classified as follows, having different characteristics:

1. ***Separately Excited Machines*** – the field winding is connected to a source of supply other than the armature of its own machine
2. ***Self-Excited Machines*** , which may be subdivided into
    a) *shunt-wound machines* – the field winding is connected across or in parallel with the armature terminals
    b) *series-wound machines* – the field windings are connected in series with the armature winding
    c) *compound-wound machines* – a combination of shunt and series windings

NOTE: ***We'll NOT be treating compound machines into much detail.***

1-2.1 Separately-Excited Machines

The Fig below shows the *connection diagram* of a separately excited machine.

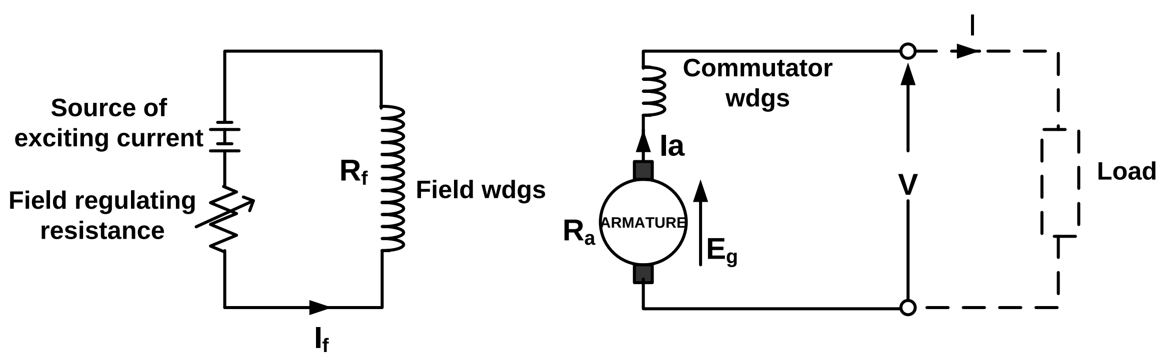


Fig 2.1: ***Separately-Excited Machine***

1-2.2 Self-Excited Machines

There are **three types** of self-excited DC machines, namely, ***shunt, series and compound machines***, each differing from the other in the mode of connection of the field coils to the armature winding. If the field coil is connected across (or in parallel with) the armature, the machine becomes a shunt machine. If it is connected in series with the armature, it becomes a series machine. If both the shunt and series field coils are used, then the machine is a compound machine. See Fig below

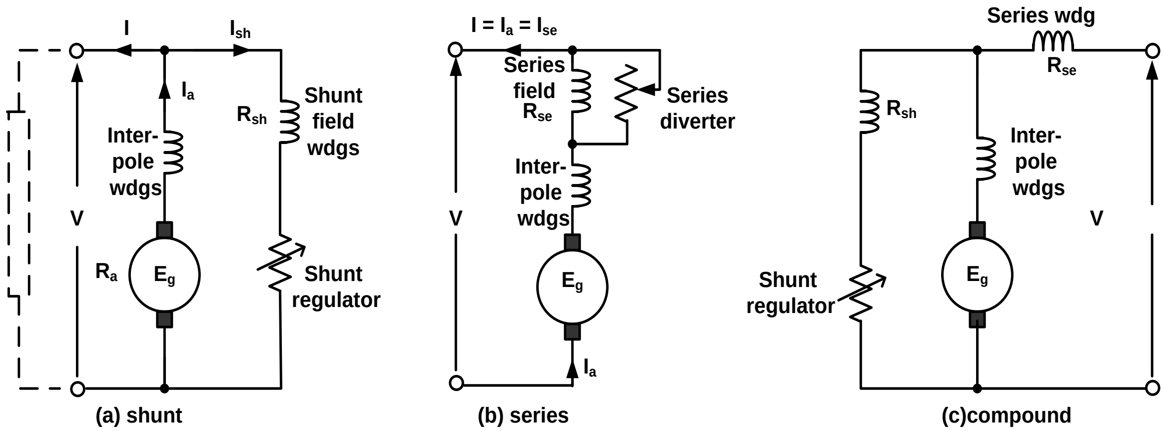


Fig 2.2: ***Self-Excited Machines*** *(Shunt, Series and Compound)*

In machines designed for either ***separate or shunt excitation***, the field coils are made of a large number of turns of relatively small wire, thus providing a high resistance and taking small current. A rheostat is usually included in the field circuit to vary the generated voltage.

In machines designed for ***series excitation***, the field coils are made of a few turns of large wire, so as not reduce voltage drops across the coils, since these coils carry the total load current of the machine.

A generator designed for ***compound excitation*** has both a shunt and a series winding placed on the field poles. Ordinarily, the *series coils are connected to* ***aid*** the shunt coils, and the machine is said to be ***cumulatively compounded***. When the *series winding is connected to* ***oppose*** the shunt, it is ***differentially compounded***. A compound machine is short-shunted when the shunt field is connected directly across the armature as in Fig (a) below, and is long-shunted when the shunt field is connected directly to the line as in Fig (b) below.

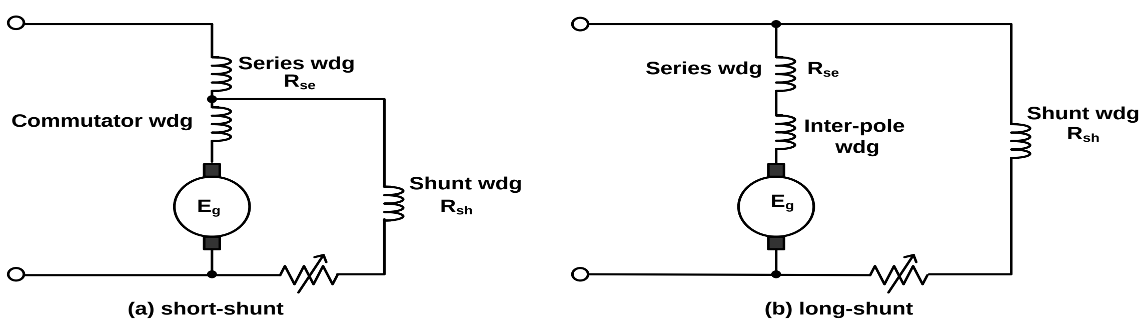


Fig 2.3: (a) Short-Shunted and (b) Long-Shunted ***Compound Machines***

SESSION 2-2 DC GENERATORS – CONSTRUCTIONAL FEATURES AND TYPES

This section deals with DC machines used as generators, although much of what is said concerning generators is equally applicable to motors. *The general types of DC generators are classified based on the method of or type of field excitation used*.

In all generators, the field flux is produced by current flowing in coils placed on the pole pieces. In practice, the emf required to drive current through the field coils may be obtained either from an *independent or separate external source*, such as another generator or from the *armature of the generator itself*. When the field coil is excited from a separate source, the machine is said to be *separately-excited*, and if it uses the DC voltage already available on the armature winding, the machine is said to be *self-excited*.

2-2.1 General Construction of DC Generator

A modern 4-pole DC generator with the main parts named is shown below.

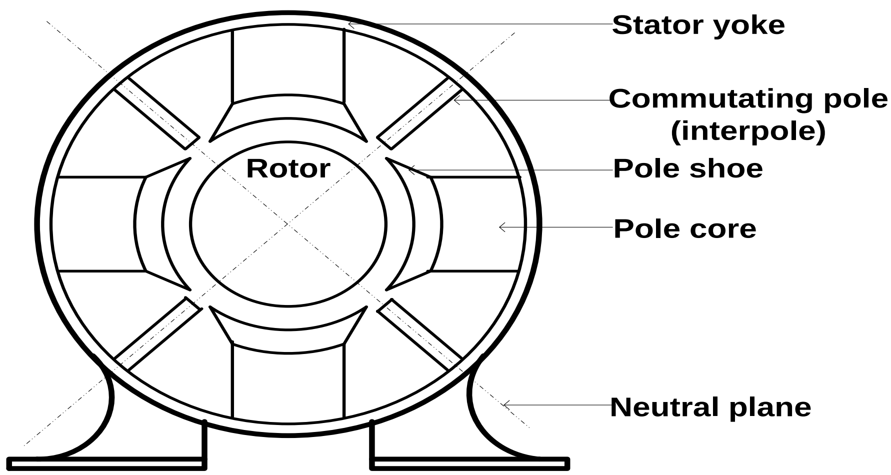


Fig 2.4: Parts of a 4-Pole DC Generator

The flux produced by the field windings of a generator is established in the field yoke, pole cores, air gap and armature core – all of which form what is known as the ***magnetic circuit***. The airgap is the space between the armature surface and the pole face, and varies in length with the size of the machine, but is of the order of 1/16 to ¼ in. The ***electric circuits*** of a DC generator are made up of the armature winding, commutator, brushes and field windings.

Most generators are equipped with small poles called ***interpoles or commutating poles***, which are placed midway between main poles, as shown in the Fig above. *Flux is established in these interpoles only when current flows in the armature circuit, the purpose of the commutating flux being to improve commutation*

The term ***armature*** is generally associated with the ***rotating part (windings)*** of the DC machine.

2-2.2 Calculation of emf induced in armature winding of a generator

The amount of emf induced in a conductor depends on the *rate* at which it is cutting flux.

When an armature is rotated through one revolution, each conductor cuts the magnetic flux emanating from all the N poles and also that entering all the S poles. Consequently,

Let P = number of poles
Φ = useful flux per pole in Webers (Wb), entering or leaving the armature
Z = total number of conductors on armature = number of slots x number of conductors per slot
N = rotational speed of armature in revolutions per minute (r.p.m.)
b = number of parallel paths in armature, *depending on type of armature winding (Lap or Wave)*
E_g = generated emf *per parallel path in armature*.

For a single-turn coil consisting of two (2) sides, the *average* emf generated per conductor is $E_{av} = \frac{d\Phi}{dt}$. In one revolution, each conductor cuts a total flux given by the flux per pole (Φ) *times* the number of poles (P), or $d\Phi = P\Phi$. The number of revolutions per second = $\frac{N}{60}$. Hence time for one revolution, $dt = \frac{60}{N}$.

Hence according to Faraday's Law of Electromagnetic Induction, the average emf generated per conductor is given as $E_{av} = \frac{P\Phi N}{60}$.

In an armature, the total number of conductors (Z) is divided into b paths. The total generated emf (E_g) equals the emf generated in one path. The number of conductors per path is Z/b.

Hence in general, the generated voltage per path is

$$E_g = \frac{P\Phi N}{60} \times \left(\frac{Z}{b}\right) = \frac{P\Phi ZN}{60b} \qquad (2.1)$$

where $b = P$ for Lap winding
$b = 2$ for Wave Winding

For any given generator, all the factors in Eqn (2.1) are fixed values except the flux per pole Φ and the speed *N*. Therefore Eqn (2.1) may be simplified to the form

$$E_g = K_1 \Phi N \qquad (2.2)$$

Example 2.1:

A 60-kW four pole generator has a lap winding placed in 48 armature slots, each slot containing six conductors. The pole flux is 0.08 Wb and the speed of rotation is 1040 rpm.

(a) Determine the generated voltage
(b) What is the current flowing in the armature conductors when the generator delivers full load?

Solution 2.1

(a) The total number of conductors is Z = 48 x 6 = 288. For *lap winding*, the number of parallel paths in the armature circuit equals the number of poles, that is, *b = 4*. Thus the generated emf is

$$E_g = \frac{P\Phi ZN}{60b} = \frac{4 \times 0.08 \times 288 \times 1040}{60 \times 4} = \underline{400V}$$

(b) The output current of the generator is

$$I = \frac{P}{V} = \frac{60{,}000}{400} = 150\,A$$

Since there are four parallel paths, each path must supply 150 ÷ 4 = 37.5 A. Therefore conductor current is <u>37.5 A</u>.

Example 2.2:

A four-pole wave connected armature has 51 slots with 12 conductors per slot and is driven at 900 rpm. If the useful flux per pole is 25 mWb, calculate the value of generated emf.

Solution 2.2

Total number of conductors is Z = 51x12 = 612. For *wave-winding*, the number of parallel paths in the armature circuit is 2, that is, *b = 2*. Thus the generated emf is

$$E_g = \frac{P\Phi ZN}{60b} = \frac{4 \times 25 \times 10^{-3} \times 612 \times 900}{60 \times 2} = \underline{459V}$$

Example 2.3:

An 8-pole lap-connected armature driven at 350 rpm is required to generate 260 V. The useful flux per pole is about 0.05 Wb. If the armature has 120 slots, calculate a suitable number of conductors per slot.

Solution 2.3:

For an 8-pole, lap-winding, the number of parallel paths in the armature is $b = P = 8$

Hence $E_g = \dfrac{P\Phi ZN}{60b} \Rightarrow Z = \dfrac{E_g 60b}{P\Phi N} = \dfrac{260 \times 60 \times 8}{8 \times 0.05 \times 350} = \underline{891.4}$

Thus the number of conductors per slots = 891/120 = 7.43. But this value must be an even number. And so **8 conductors per slot** would be suitable.

2-2.3 Armature Reaction in a Generator:

When the field winding of a generator is energised and no current flows through the armature, the flux in the air gap is uniformly distributed. But when the armature is connected to a load and delivers current, the armature current produces armature flux that *interacts* with the main field flux.

By ***armature reaction*** *is thus meant the influence or distorting effect of the magnetic field set up by the armature current (when armature is on-load and delivering current) on the value and distribution of the main field flux in the air gaps between poles and on the stator core*. The armature reaction in a generator has ***two main effects***:

- It demagnetises or weakens the main flux
- It cross-magnetises the poles.

The first effect leads to reduction in the generated voltage, whilst the second effect leads to sparking at the brushes.

2-2.4 Compensating Windings in Generator

The ***cross-magnetising effect of the armature reaction*** *in a generator* ***may be neutralised*** *by means of* ***compensating windings*** embedded in the pole faces of the generator. In the absence of compensating windings, the flux will be shifting backward and forward with every change in load. This shifting of flux will result in *statically induced* emf in the armature windings.

The magnitude of this statically induced emf will depend on the rapidity of changes in the load and the amount of change. *This statically induced emf may be so high as to strike an arc between the consecutive commutator segments. This induced emf may be so high as to strike an arc between the consecutive commutator segments and further develop into a flashover around the whole commutator, thereby short-circuiting the whole armature*.

These pole-face compensating windings are connected in series with the armature so as to carry the same current as the armature, but the connection is made such that every compensating winding embedded in the field poles carries current in the opposite direction to that of the adjacent armature windings.

Compensating windings are not used on many machines, because of the cost of manufacturing them. ***Compensating windings*** *find greatest application in high speed and high voltage DC machines of large capacity and which are subject to* ***large fluctuations in load****. For example, rolling mill motors and turbo-generators*.

2-2.5 Types of DC Generators

As stated earlier, t*he general types of DC generators are classified based on the method of or type of field excitation used*. Accordingly, DC generators are also sub-divided as:

1. ***Separately Excited Generators*** – the field winding is connected to a source of supply other than the armature of its own machine
2. ***Self-Excited Generators*** , which may be subdivided into
   a) *Shunt Generators* – the field winding is connected across or in parallel with the armature terminals
   b) *Series Generators* – the field windings are connected in series with the armature winding
   c) *Compound Generators* – a combination of shunt and series windings

NOTE: We'll NOT be treating compound machines into much detail.

SESSION 3-2 LOAD CHARACTERISTICS OF DC GENERATORS

The properties of generators are analysed with the aid of characteristics which give the relations between fundamental quantities determining the operation of a generator. These include the voltage V across the generator terminals, the field or exciting current I_f, the armature current I_a and the speed N. The three most important steady-state characteristics of DC generators are:

1. No-Load Saturation Characteristics (E_0 *versus* I_f)
2. Internal Characteristics (E *versus* I_a)
3. External (load) Characteristics (V *versus* I)

NOTE: We'll be looking only at the no-load and load characteristics of the separately-excited and self-excited (shunt and series) generators.

3-2.1 Separately-excited Generator

The separately-excited generator is supplied field excitation from an external independent DC source such as a storage battery or separate DC generator. The connection diagram is already shown under Section 2.1.1.

3-2.1.1 No-Load Characteristics (E_0 *versus* I_f) of Separately-Excited Generator

The no-load characteristic is also known as the *generator magnetisation curve* or *open-circuit characteristics (OCC)* and shows the relationship between the no-load generated emf in armature and the field or exciting current ***at a given constant speed***.

NOTE: ***The shape of the curve is practically the same for all types of generators, whether they are separately excited or self-excite***d. It is just the magnetisation curve for the material of the electromagnets.

The no-load saturation characteristics (or open-circuit characteristics, OCC) is as shown below:

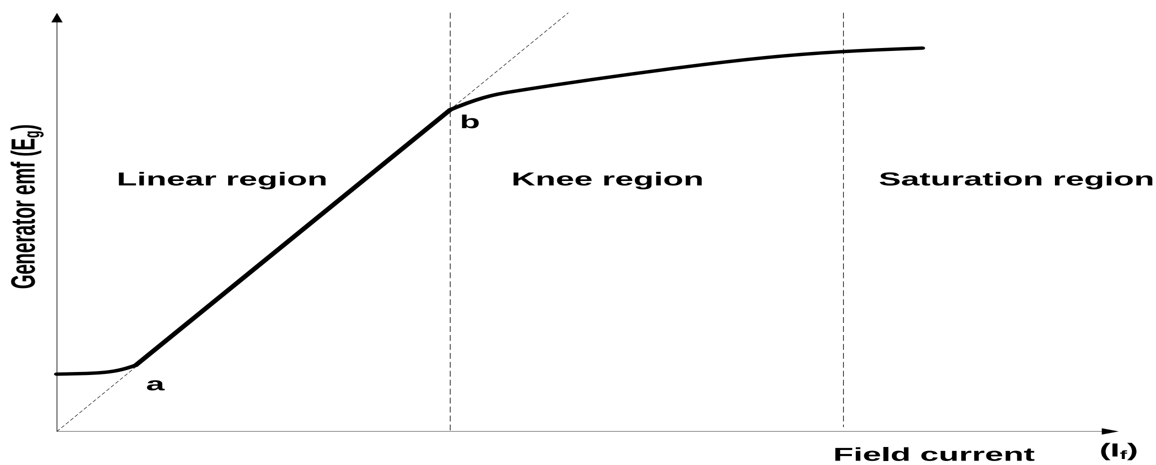


Fig 2.5: Generator Magnetisation Curve

According to the molecular theory of magnetism, the molecules of an unmagnetized piece of iron are not arranged in any definite order. When the iron is magnetised by passing current through a coil placed around the iron core, the molecules become arranged in a definite order. *To arrange the greater part of the molecules in a definite order or to* ***magnetise*** *the iron up to a certain point* requires relatively few ampere-turns of applied mmf.

In this stage of magnetisation, the amount of flux established in the iron is increases almost directly with increases in the ampere-turns applied. However, above this point, which is called *saturation point*, it becomes increasingly difficult to magnetise the iron further, since the unmagnetised molecules become fewer and fewer. Above the saturation point, when much larger increases in ampere-turns are required for corresponding increases in flux in the iron, the iron is said to be ***saturated***.

NOTES:

- It will be noticed that the curved part at point *a* does not start at zero but at some value slightly greater than zero, that is, a small voltage is produced when the field current is zero. This is due to small amount of permanent magnetism in the field poles. It is called ***residual magnetism*** and is usually sufficient to produce 2 or 3 % of normal terminal voltage, although in some special cases it is purposely increased to 10 % or more.
- Beyond this point *a*, the curve to point *b* is practically straight because the flux increases in proportion to the field current.
- After point *b*, saturation becomes perceptible as the curve departs from straight line form.

3-2.1.2 Load Characteristics (V *versus* I) of the Separately-Excited Generator

This characteristic is sometimes referred to as *performance characteristic* or *voltage regulation curve*. ***It gives the relationship between the terminal voltage and the load curren***t. The load-characteristic of the separately-excited generator is shown below:

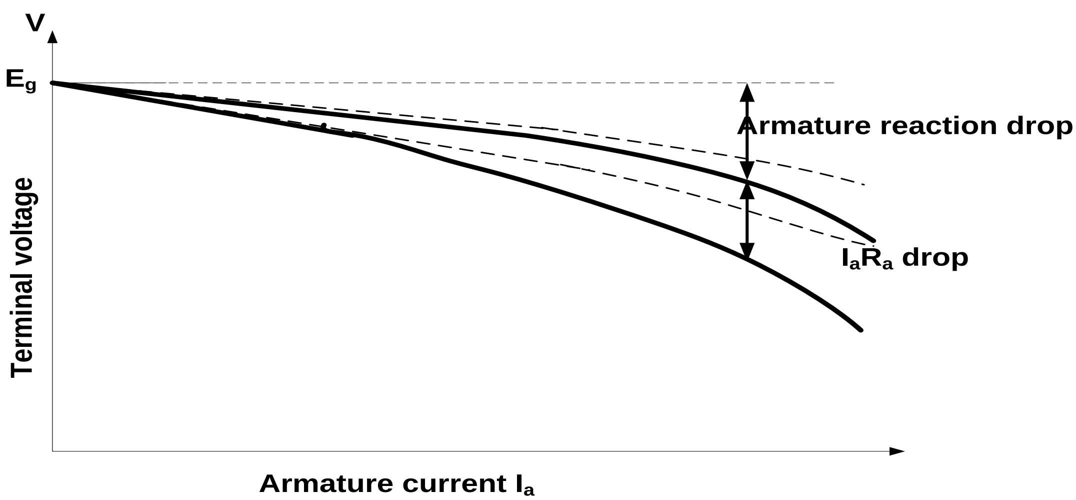


Fig 2.6: Load Characteristics of a Separately-Excited Generator

Suppose the separately excited generator is operated at a constant speed and with a constant field excitation. When it is *unloaded*, the terminal voltage will be equal to the generated voltage E_g . But when it is loaded and delivering current, the terminal voltage will be less than the generated voltage.

NOTE: There are ***two important reasons for the decrease in terminal voltage***:

1. **Armature-reaction drop:** Armature currents establish an mmf that distorts and weaken the main field flux, especially in non-interpole machines, an effect know as armature reaction. Due to armature reaction, the amount of effective field flux is reduced. As the flux per pole is reduced by the armature reaction, the emf generated by cutting this flux is also reduced.

2. The terminal voltage becomes less than the generated emf by an amount equal to the ***IR-drop of the armature winding and brushes***.

The value of the terminal voltage V is obtained by subtracting armature resistance drop $I_a R_a$ from the corresponding values of E.

$$V = E - I_a R_a \tag{2.3}$$

The values of voltage for various load currents may be calculated or may be obtained from an actual test. Readings of current and voltage obtained form a test are taken at constant speed and with the field excitation so adjusted that rated voltage is obtained at full-load.

Example 2.4

A separately excited generator has a generated voltage of 200 V at a speed of 1200 rpm with a field of 2.5 A. Calculate:

a) Generated voltage at 1600 rpm with a field current of 2.5 A
b) Generated voltage at 1000 rpm with a field current of 3.0 A

Solution 2.4

Recall the generated voltage $E_g = K_1 \Phi N$, where $K_1 = \dfrac{PZ}{60b}$ is a constant which varies from one machine to another. Assuming that $\Phi = f(I_f)$, then $E_g = K I_f N$

a) $E_{g2} = E_{g1} \times \dfrac{N_2}{N_1} \times \dfrac{I_{f2}}{I_{f1}} = 200 \times \dfrac{1600}{1200} \times \dfrac{2.5}{2.5} = \underline{\underline{266.67\ V}}$

b) $E_{g2} = E_{g1} \times \dfrac{N_2}{N_1} \times \dfrac{I_{f2}}{I_{f1}} = 200 \times \dfrac{1000}{1200} \times \dfrac{3.5}{2.5} = \underline{\underline{200\ V}}$

3-2.1.3 Applications of Separately Excited Generator

- Separately excited generators are more expensive than self-excited generators as they require a separate source of supply. For this reason, the *use of the separately excited generator is largely confined to experimental and testing laboratories* where such a source is available and a wide variation of output voltage is desirable.
- Separately excited generators are also used where quick and requisite response to control is important, since separate excitation gives a quicker and more precise response to the changes in the resistance of the field circuit.

3-2.2 Shunt Generator

The *shunt generator is a self-excited generator whose field circuit is connected directly across or in parallel with the armature*.

3-2.2.1 Load Characteristic of a Shunt Generator

The load characteristic curve of a shunt generator is shown in the Fig below.

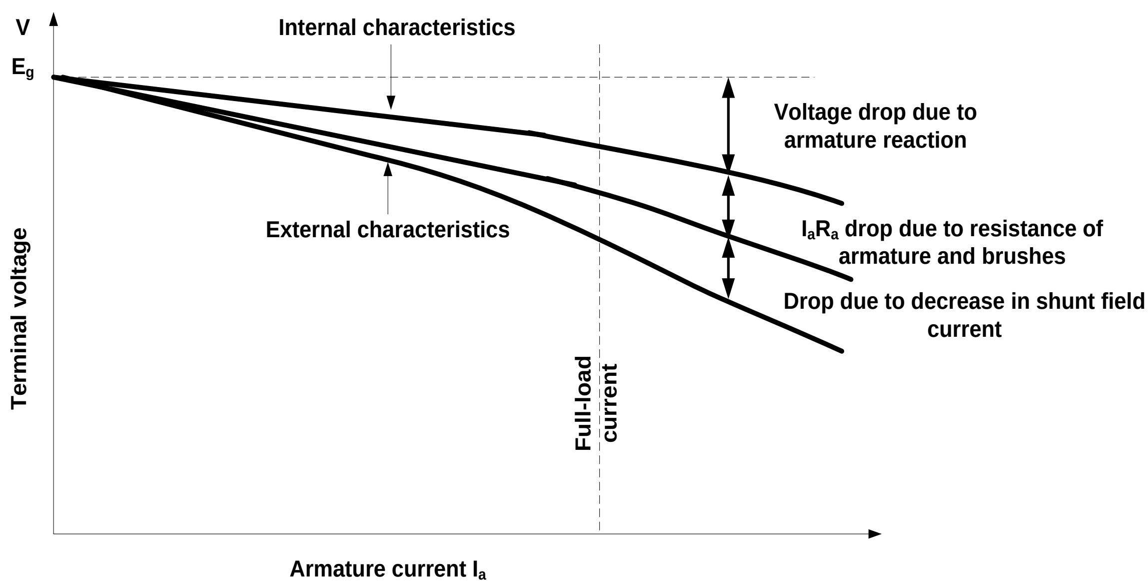


Fig 2.7: Load Characteristic of a ***Shunt Generator***

When the machine is unloaded, the terminal voltage equals the generated emf. As more and more current is delivered, the terminal voltage drops farther and farther from the no-load value. This curve thus shows *the performance of the shunt generator to be similar to that of the separately excited generator, except that the voltage of the shunt generator falls off more rapidly with the addition of load*.

NOTE: Three main factors account for the higher rate of decrease in terminal voltage of the shunt generator. These are

1. Armature reaction
2. Armature circuit resistance drop $I_a R_a$
3. Reduction in field current I_f

As in the separately excited generator, armature reaction and *IR*-drop in the armature circuit cause the terminal voltage to decrease. In the self-excited generator, there is a third factor. Since the field excitation is obtained from the shunt generator itself, the decrease in terminal voltage (because of the above mentioned tow effects) also decreases the voltage applied to the field circuit. This drop causes a decrease in the field current and hence less field flux is produced, which in turn causes a further decrease in the terminal voltage. These three important components of voltage drop are indicated above.

Generally, the external *load-voltage* ***characteristic decreases*** with application of load ***only to a small extent*** *up to its rated load (current) value*. ***Thus the shunt generator is considered as having fairly constant output voltage with application of load***, and in practice, is rarely operated beyond the rated load current value continuously for any appreciable time.

3-2.2.2 Applications of Shunt Generator

- The shunt generator may be used for supplying excitation to AC generators or in other applications where the distance from the generator to its load is short.
- It is also used for *charging storage batteries*.

3-2.2.3 Important Governing Equations of a Shunt Generator

$$I_{sh} = \frac{V}{R_{sh}} \qquad I_a = I_{sh} + I \qquad V = E_g - I_a R_a$$

$$Power\ developed = E_g I_a \qquad Power\ delivered = VI$$

where

I_{sh}	= shunt field current	I_a	= armature current
I	= load current	R_{sh}	= shunt field resistance
R_a	= armature resistance	$E_g = \frac{P\Phi ZN}{60b}$	= generated emf
V	= terminal voltage		

Example 2.5

A shunt generator has a no-load terminal voltage of 254 V. At rated full-load, the terminal voltage is 240 V. Calculate the full-load current, if the field circuit resistance is 30 Ω and the armature resistance is 0.02 Ω.

Solution 2.5

The field current is given as $I_f = \frac{V_f}{R_f} = \frac{V_t}{R_f} = \frac{240}{30} = \underline{8A}$.

Recalling the voltage equation for a shunt generator as $E_g = V_t + I_a R_a$, the armature current is given as $I_a = \frac{E_g - V_t}{R_a} = \frac{254-240}{0.02} = \underline{700A}$.

The full-load current is $I_L = I_a - I_f = 700 - 8 = \underline{\underline{692\ A}}$

Example 2.6

A DC shunt generator delivers a current of 96 A at 240 V to a resistance load. The resistance of the armature is 0.15 Ω and the field resistance is 60 Ω. Assuming a brush drop of 2 V, calculate the

a) current in the armature
b) generated emf

Solution 2.6

a) The load current is already given. The field current is obtained as

$I_f = \frac{V_f}{R_f} = \frac{V_t}{R_f} = \frac{240}{60} = \underline{4A}$. Thus the armature current is

$I_a = I_f + I_L = 4 + 96 = \underline{\underline{100A}}$.

b) The generated voltage is

$E_g = V_t + drops = V_t + I_a R_a + V_{brush} = 240 + (100)(0.15) + 2 = \underline{\underline{257V}}$

3-2.3 Series Generator

The *series generator is a self-excited generator with armature, field windings and load all connected* ***in series***. Thus the field current and field flux are all proportional to the load current which can be very high. This explains why the *series field coil consists of few turns of thick wire capable of carrying the high output current of the machine without overheating*.

3-2.3.1 Load characteristics of Series Generators

The load characteristic of a series generator is as shown in the Fig below:

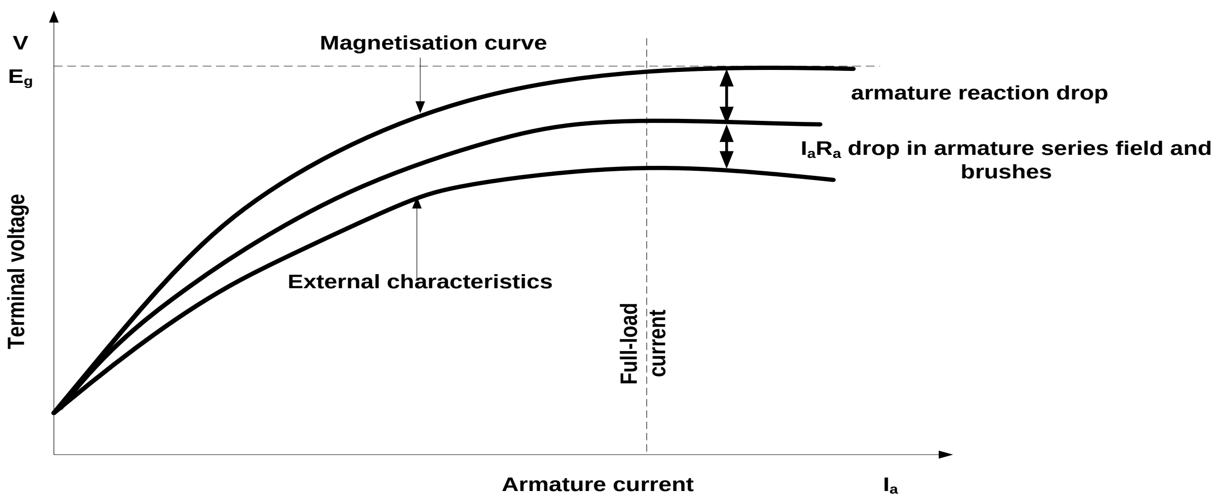


Fig 2.8: Saturation and Load Characteristics of a ***Series Generator***

When the series generator is running without load, there is a small emf generated owing to the residual magnetism. As an external load is applied and more current begins to flow, the terminal voltage of the generator is increased. *This rise in terminal voltage is due to the fact that the armature current flows in the field coil and more armature current means more flux and a higher generated emf*. The terminal voltage continues to increase until the magnetic circuit becomes saturated.

Increasing the armature current beyond the saturation point decreases the net flux in the airgap because of armature reaction. This explains why the terminal voltage starts to decrease as the current exceeds the value corresponding to the saturation point.

NOTE: ***The saturation curve of a series generator which is shown in the Fig above does NOT differ from that of a shunt generator***.

The load characteristic curve is similar in shape to the saturation curve, but lies below it for the following two reasons:

1. The flux per pole is reduced due to *armature reaction*, which in turn causes a reduction in the generated voltage and
2. the resistance of the armature winding, field winding and brushes causes IR - *drop*.

3-2.3.2 Applications of Series Generator

- This type of $V-I$ characteristic of the series generator is rather unstable which limits the use of series generators.
- However, due to the initial rising voltage characteristic, the series generator is suitable for applications where ***voltage boosting*** is required to give an increase of voltage practically proportional to the current.
- For special purposes, such as supplying the field current for regenerative braking of DC locomotives

3-2.3.3 Important Governing Equations of a Series Generator

$$I_a = I_{se} = I \qquad V = E_g - I(R_a + R_{se})$$

$$Power\ developed = E_g I \qquad Power\ delivered = VI$$

$$E_g = \frac{P\Phi ZN}{60b} = K_1 \Phi N \qquad \Phi = f(I)$$

where

I_{se}	= series field current	I_a	= armature current
R_{se}	= series field resistance	I	= load current

EXERCISES ON DC GENERATORS

Exercise 2.1

A four-pole armature is wound with 564 conductors and driven at 800 rpm, the flux per pole being 20 mWb. Calculate the emf generated in the armature if the conductors are connected (a) wave (b) lap.

Exercise 2.2

An eight-pole lap-connected armature has 96 slots with 6 conductors per slot, and is driven at 500 rpm. The useful flux per pole is 0.09 Wb. Calculate the generated emf.

Exercise 2.3

Give TWO applications EACH of a shunt and series generator. A four-pole armature has 624 lap-connected conductors and is driven at 1200 rpm. Calculate the useful flux per pole required to generate an emf of 250 V.

Exercise 2.4

A six-pole armature has 410 wave-connected conductors. The useful flux per pole is 0.025 Wb. Find the speed at which the armature must be driven if the generated emf is to be 485V.

Exercise 2.5

A 6-pole 240 V shunt generator with a wave-wound armature having 140 slots, each slot containing 8 conductors, supplies ten 4 kW resistive loads. Allowing a voltage drop of 15 V in the connection leads between the generator and the load, calculate the speed at which the generator must be driven. Assume a brush drop of 1.5 V. The armature and field resistance are respectively 0.025 Ω and 51 Ω. Take the flux per pole as 40 mWb.

Exercise 2.6

A 6-pole 240 V shunt generator with a wave-wound armature having 140 slots, each slot containing 8 conductors, supplies ten 4 kW resistive loads. Allowing a voltage drop of 15 V in the connection leads between the generator and the load, calculate the speed at which the generator must be driven. Assume a brush drop of 1.5 V. The armature and field resistance are respectively 0.025 Ω and 51 Ω. Take the flux per pole as 40 mWb.

Exercise 2.7

The armature of a DC machine has a resistance of 0.25 Ω and is connected to a 300 V supply. Calculate the e.m.f. generated when it is running:

a) as generator supplying 100 A load
b) as a motor taking 80 A current

SESSION 5-2 DC MOTORS – CONSTRUCTIONAL FEATURES AND PERFORMANCE EQUATIONS

Motors are the commonest and best known of electrical machines. They are classified into two broad groups, depending on whether they are suitable for use on direct-current or alternating current systems. They are the DC and AC motors. There is a third motor known as Universal Motor which can be used on DC or AC supply, and these are used to drive small appliances such as vacuum cleaners, food mixers, etc. Each of the two main groups is subdivided, giving several types of motors, each having its own particular application.

More AC motors are, however, in use than DC motors, and this is due to two factors:

- The majority of our supply systems are AC.
- The AC motor is simpler and cheaper than its DC counterpart.

In this section, the motor principle and the characteristics of the several types of DC motors are discussed.

5-2.1 Motor Principle

Every carrying-current conductor has magnetic field around it, the direction of which may be established by the Fleming's Right-Hand Rule. The strength of the field depends on the amount of current flowing in the conductor. *If this current-carrying conductor with magnetic field around it, is placed in a uniform main magnetic field,* the **resultant or *combined field***.

As stated in Fleming's Left-Hand Rule, there is a definite relation between the direction of the lines in a magnetic field, the direction of the current flowing in the conductor and the direction in which the conductor tends to move. The force exerted on a conductor carrying current while in a magnetic field is directly proportional to the field strength, the active length of the conductor and the current flowing through it.

For a coil of *N* turns of *active length* l , the plane of which coil is inclined at an angle θ to the direction of a uniform magnetic field of density *B*, the force exerted on the *N* turns when a current I flows through the coil may be computed by the equation:

$$F = BINl\sin\theta \qquad (2.4)$$

5-2.2 Back Emf E_b In A Motor

A motor experiences *generator effect* when the supply voltage is applied across the armature via the split-rings or commutator, and current flows in the armature windings. According to *Fleming's Left-Hand Rule*, the *current-carrying armature windings* in the presence of the stator magnetic field will experience either an upward or a downward force, depending on the direction of the current and field polarity.

On moving vertically upward or downward, the armature windings will cut the magnetic flux of the field coils. And so in accordance with Faraday's Laws of Electromagnetic Induction, an emf will be induced in the armature conductors cutting the field flux. This effect is called ***generator effect*** in a motor action.

The direction of the induced emf is determined by *Fleming's Right-Hand Rule*, and is found to be in the opposite direction to the impressed supply voltage. ***This induced counter emf always opposes the applied voltage, and is thus called the counter or back emf*** E_b***.*** It is directly proportional to the armature speed N and the field strength *B*, and the *same in value as the generated voltage* E_g in the armature windings of a generator. Thus

$$E_b = E_g = \frac{\Phi Z N}{60}\left(\frac{P}{b}\right) = K_1 N \Phi \qquad (2.5)$$

where the symbols have their usual meanings.

The *effective voltage* across the motor armature thus equals the applied voltage *V* minus the induced counter voltage (or back emf) E_b. If an armature has R_a ohms resistance, the current flowing through it can be determined by the equation:

$$I_a = \frac{V - E_b}{R_a} \qquad (2.6)$$

5-2.3 Significance of the Back Emf E_b

Suppose that 230 V is applied to a 5-hp, 850-rpm motor of armature resistance of value 0.1 Ω. According to *Ohm's Law*, the armature current will be 230/0.1 = 2300 A. Of course, such a high current would burn out the armature winding. In practice, however, the armature current is not that high, and the restraining action is due to the counter or back emf, as seen in Eqn (2.6).

From Eqn (2.5), the back emf depends, among others, on the armature rotational speed. If the speed is high, E_b is large and hence the armature current I_a, as seen from Eqn (2.6), is small. If the speed is less, then E_b is also less, and hence more current flows which develops motor torque.

The back emf E_b ***therefore acts like a governor, that is, it makes a motor self-regulating so that it draws as much current as is just necessary***.

5-2.4 Voltage Equation of a Motor:

Multiplying both sides of Eqn (2.6) by R_a, and transposing results in

$$V = E_b + I_a R_a \tag{2.7}$$

which is the *fundamental voltage equation of the motor*. The voltage V is thus applied to overcome the back emf and also supply the armature ohmic drop $I_a R_a$. Note that this is the same as the generator equation $V = E_g - I_a R_a$ with the sign of the $I_a R_a$ term changed. From Eqn (2.7), the back emf of a motor is always less than its terminal voltage.

Example 2.7

The armature of a DC machine has a resistance of 0.1 Ω and is connected to a 250 V supply. Calculate the generated voltage when it is running

(a) as a generator giving 80 A
(b) as a motor taking 60 A

Solution 2.7

(a) Voltage drop due to the armature resistance = 80 x 0.1 = 8 V. Thus **as generator**, the generated voltage is $E_g = V + I_a R_a = 250 + 8 = \underline{\underline{258V}}$

(b) Voltage drop due to armature resistance = 20 x 0.1 = 6 V. Thus **as motor**, the generated voltage is $E_g = V - I_a R_a = 250 - 6 = \underline{\underline{244V}}$

5-2.5 Power Relationships and Torque Equations in a DC Motor

If each term of the fundamental motor equation is multiplied by the armature current, the resulting equation is

$$V I_a = E_b I_a + I_a^2 R_a \tag{2.8}$$

The term VI_a represents the electrical power supplied to the armature of the motor, and the term $I_a{}^2 R_a$ represents the power lost as heat due to armature resistance. The remainder of the power, $E_b I_a$, must therefore be the electrical equivalent of mechanical power developed *within* the armature.

NOTE: ***Not all of this developed mechanical power is available at the shaft, for some of it is lost in overcoming friction of the bearings and brushes, windage resistance, hysteresis and eddy-current current in the armature and pole faces***. All these losses produce a mechanical drag on the armature.

The remainder of the power, after subtracting these losses, appears as power available at the shaft for driving the external load. Nevertheless, in practice, it is sufficiently accurate to assume that the power supplied by the shaft is $E_b I_a$.

It may be noted that motor efficiency is given by the ratio of power developed by the armature to its input, that is, $\eta = \frac{E_b I_a}{V I_a} = \frac{E_b}{V}$. Obviously, the higher the value of the back emf E_b as compared to the applied voltage V, the higher the motor efficiency.

5-2.5.1 Developed or Armature Torque T_a of a Motor:

If T_a is the torque (in Nm) is the torque developed ***within*** the armature of a motor that running at N ***revolutions per sec (rps)***), then the power developed is given by the relation:

$$P = T_a \cdot 2\pi N \tag{2.9}$$

But the electrical power converted into mechanical power in the armature is given according to Eqn (2.8) as:

$$P = E_b \cdot I_a \tag{2.10}$$

Hence equating the two equations, we obtain

$$\begin{aligned} &T_a 2\pi N = E_b I_a \\ &\Rightarrow T_a = \frac{E_b I_a}{2\pi N} \end{aligned} \tag{2.11}$$

Substituting the value of the back emf E_b of Eqn (2.5), the torque Eqn (2.11) becomes

$$\begin{aligned} T_a &= \frac{1}{2\pi} \Phi Z \, I_a \left(\frac{P}{b} \right) \\ &= \frac{1}{2\pi} \cdot \frac{I_a}{b} \cdot \Phi Z P \end{aligned} \tag{2.12}$$

Since P, b and Z are constants, the torque equation can be rewritten as:

$$T_a = K_2 \Phi I_a \tag{2.13}$$

where K_2 is a constant. In other words, the torque developed *within* the armature is directly proportional to the flux per pole and to the armature current, ***but the developed torque is independent of the speed***.

5-2.6 Speed of a DC Motor

If a DC motor is unloaded, the retarding torque is small, being due only to windage and friction. The armature will develop a counter emf that will restrict the armature current to a value that will cause the motor to develop a torque equal to the resisting torque. A motor must slow down when an external load is applied to it, because the small no-load current is just sufficient to produce a torque to overcome friction.

An equation suitable for investigating the speed variation in a motor may be obtained by substituting the expression for the back emf E_b of Eqn (2.5) into Eqn (2.6). This gives:

$$I_a = \frac{V - E_b}{R_a} = \frac{V - K_1 \Phi N}{R_a} \qquad (2.14)$$

Solving for the speed N,

$$N = \frac{V - I_a R_a}{K_1 \Phi} = \frac{E_b}{K_1 \Phi} \qquad (2.15)$$

The value of $I_a R_a$ is usually less than 5% of the terminal voltage, so that

$$N = \frac{V}{K_1 \Phi} \qquad (2.16)$$

NOTE: ***In words, the expression for Eqn (2.16) means that the speed of an electric motor is approximately proportional to the voltage applied to the armature and inversely proportional to the flux.***

All methods of controlling the speed involve the use of either or both of these relationships.

Example 2.8

A 4-pole DC motor is fed at 440 V and takes an armature current of 50 A. The resistance of the armature circuit is 0.28 Ω. The armature winding is wave-connected with 888 conductors and the useful flux per pole is 0.023 Wb. Calculate the speed.

Solution 2.8

Using the voltage equation of the DC motor, namely, $V = E_b + I_a R_a$, the generated voltage (which is equal in value to the back emf) is given as

$$E_g = V - I_a R_a = 440 - 50 \times 0.28 = \underline{\underline{426 V}}$$

Substituting in the emf equation $E_g = \frac{\Phi ZNP}{60b}$, the speed is obtained as

$$N = \frac{E_g 60b}{\Phi ZP} = \frac{426 \times 60 \times 2}{0.023 \times 888 \times 4} = \underline{\underline{626 rpm}}$$

Example 2.9

A motor runs at 900 rpm on a 460 V supply. If the machine is connected across a 200 V supply, calculate the new speed. Assume the new flux to be 0.7 times the original flux.

Solution 2.9

We make use of the speed Equation (2.16), $N = \frac{V}{K_1 \Phi}$, and so the relationship between the old speed (conforming to situation with subscript 1) and the new speed (conforming to a new situation with subscript 2) is given as

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} \times \frac{\Phi_1}{\Phi_2} \Rightarrow N_2 = \frac{V_2}{V_1} \times \frac{\Phi_1}{\Phi_2} \times N_1 = \frac{200}{460} \times \frac{\Phi_1}{0.7\Phi_1} \times 900 = \underline{\underline{559 rpm}}$$

SESSION 6-2 DC MOTORS – TYPES, SPEED-TORQUE CHARACTERISTICS AND CONTROL

As in DC generators, the field flux in DC motor is produced by current flowing in coils placed on the pole pieces. The emf required to force current through the field coils may be obtained either from a separate DC source, such as a DC generator, or from the generated voltage in the armature. As in DC generators, DC motors are also divided into three principle types – *shunt, series and compound*, according to the way the field windings are connected.

NOTE: We'll NOT be treating compound motors.

6-2.1 Shunt Motor

The shunt motor is the most common type of DC motor. It is connected in the same way as the shunt generator, that is, with the shunt field directly across the terminals in parallel with the armature circuit. A field rheostat is usually connected in series with the shunt field.

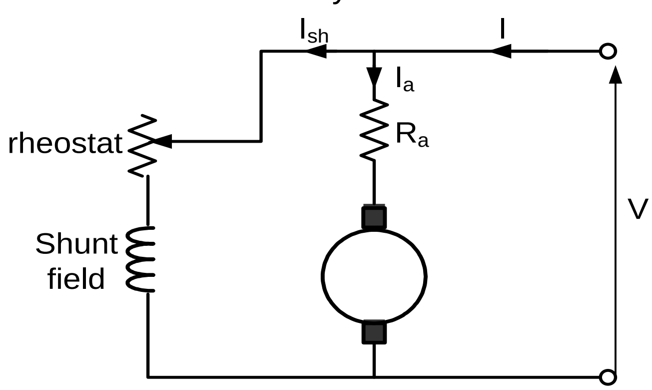


Fig 2.9: Connection Diagram of a ***Shunt Motor***

From this connection, it may be observed that since the line voltage is assumed steady or constant, *the field current and field flux are constant*. For a shunt motor, the supply voltage is the current source, and so input or line current I is related to the shunt (field) current I_{sh} and armature current I_a as follows:

$$I = I_a + I_{sh} \qquad \text{(for shunt motor)} \tag{2.17}$$

Recall that for shunt generator, the current source is the armature, and hence

$$I_a = I + I_{sh} \qquad \text{(for shunt generator)} \tag{2.18}$$

6-2.1.1 Speed and Torque Characteristics of Shunt Motor

Speed Consideration:

A ***shunt motor*** has good speed regulation and is classed as a ***constant-speed motor***, even though its speed does increase slightly with an increase in load. Let us consider the speed of motor in Eqn (2.15).

$$N = \frac{V - I_a R_a}{K_1 \Phi} \qquad (2.15)$$

In a **shunt motor**, the applied voltage V, armature resistance R_a, K_1 and flux per pole Φ are practically constant, and the ***armature current*** I_a ***is the only variable***. But strictly speaking, the flux Φ and back emf E_b decrease with increasing load. When the motor is carrying no load, the value of the armature current I_a is small, because the speed and therefore the back emf are both at a maximum.

NOTE: Care must be taken NEVER to open the field circuit of a shunt motor that is running, *since the sudden loss of the field flux will cause the shunt motor speed* $N \cong \frac{V}{K_1 \Phi}$ *to increase to dangerously high values*.

The speed of the shunt motor may also be changed by means of an adjustable resistance in the armature circuit (so-called *rheostatic control method*), but this method is less efficient than the shunt field flux control method. The method is also objectionable because it causes the motor to have a very poor speed regulation.

Torque Consideration:

The relation between the torque developed by a shunt motor and the current flowing through its armature winding may be analysed easily by referring to the general torque equation of Eqn (2.13)

$$T_a = K_2 \Phi I_a \qquad (2.13)$$

Since for a shunt motor, the field flux Φ is practically constant (although at heavy loads, Φ decreases somewhat due to increased armature reaction), *the torque of the shunt motor will vary directly linearly with the armature current*, as in the equation below.

$$T_a \ \alpha \ I_a \quad \text{(for shunt motor)} \qquad (2.19)$$

The speed and torque characteristics of a shunt motor are shown in Fig below:

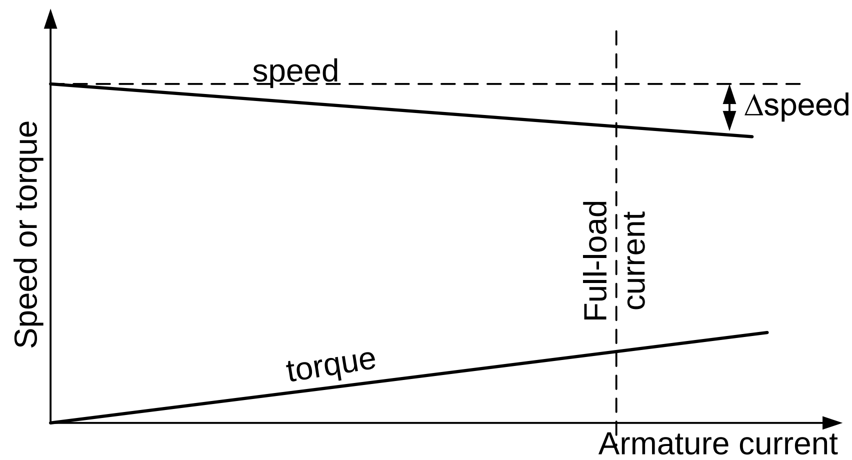


*Fig 2.10: Speed-Load and Torque-Load Characteristics of a **Shunt Motor***

Note that the torque increases in a practically straight-line relationship with an increase in armature current, while the *speed drops slightly* as armature current is increased. The fall in speed is reduced slightly by armature reaction, which causes the flux to be less and the speed to increase correspondingly. In some cases, the armature reaction is sufficient to cause the speed to remain nearly constant or even rise with an increase in load. For these reasons, a ***shunt motor is considered a constant speed motor****, even though the speed usually decreases slightly with an increase in the load*.

6-2.1.2 Applications of Shunt Motor

Because there is no appreciable change is speed of a shunt DC motor from no-load to full-load, it may be connected to loads which are totally and suddenly thrown off without any fear of excessive speed resulting.

Due to the constancy of their speed, ***shunt motors are best suited for constant speed drives***. It meets the requirements for a large range of industrial applications, such as the driving of *machine tools*, *blowers, fans, and line shaft, lathes, wood-working machines and for all other purposes where an approximately constant speed is required*.

6-2.2 Series Motor

In a series motor, the field windings are connected in series with the armature windings and therefore carry the full-load current. As the series field coils must carry the full armature (or load) current, the *field coils are wound with few turns of comparatively large wire*. Any change in load causes a change in armature current and also a change in field flux. Therefore as the load changes, the speed changes. In motors, the supply voltage is the current source. And so because the field windings are connected in series with the armature (see Fig below), the input or line current I is related to the field current I_{se} and armature current I_a in a series DC machine as follows:

$$I = I_a = I_{se} \qquad \text{(for series generator and motor)} \tag{2.20}$$

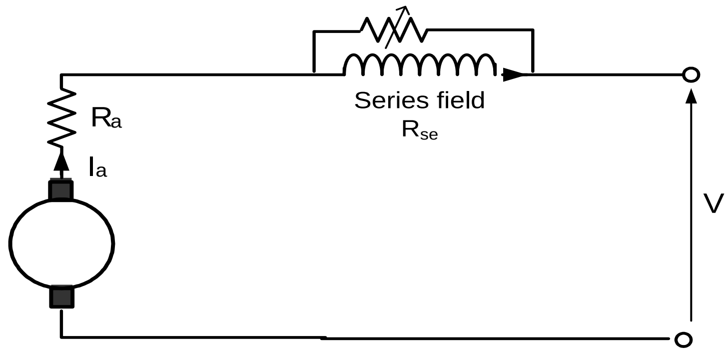


*Fig 2.11: Connection Diagram of **Series Motor***

6-2.2.1 Speed and Torque Characteristics of Series Motor

Speed Consideration:

The fundamental motor speed equation of Eqn (2.16) $N = \dfrac{V - I_a R_a}{K_1 \Phi}$ may also be applied to a series motor, where R_a is considered to include the resistance R_{se} of the series winding. Thus modifying the value of the armature circuit resistance R_a into R_{as} to reflect the total resistance in the *armature circuit*, that is, the sum of the armature resistance R_a and the series field winding resistance R_{se}, we obtain the speed equation for a series motor.
Thus the speed equation of the series motor becomes:

$$N = \frac{V - I_a (R_a + R_{se})}{K_1 \Phi} = \frac{V - I_a R_{as}}{K_1 \Phi} \qquad (2.21)$$

where $R_{as} = R_a + R_{se}$ (2.22)

Hence comparing the shunt motor speed Eqn (2.16) with the series motor speed Eqn (2.21), it is seen that the combined resistance is R_{as}, and therefore the $I_a R_{as}$ drop is slightly higher than in a shunt motor.

In a series motor, the flux Φ increases almost directly with the load. Then since the numerator in the expression $N = \dfrac{V - I_a R_{as}}{K_1 \Phi}$ decreases appreciably, while the denominator increases considerably with an increase in load, it follows that the ***series motor speed decreases very appreciably with increase in load***.

The effect of the armature circuit IR drop is very small compared with the effect of the field flux. *Therefore the speed of a series motor depends almost entirely on the flux in an inverse proportionality – the stronger the field flux, the lower the speed. Likewise, a decrease in load current and therefore in field current and hence field flux, causes an increase in speed*. **Thus the speed of a series motor varies from a very high speed at light loads to a low speed at full-load**.

A series motor does not have a definite no-load speed. As load is removed from the motor, the field flux decreases. If all the load is removed, the flux drops to practically zero and the motor speed $N = \dfrac{V - I_a R_{as}}{K_1 \Phi}$ may become dangerously high.

NOTE: **For this reason**, **as a precaution, the load should never be removed completely from a series motor**.

Torque Consideration:

Recall from Eqn (2.16), $T_a \propto \Phi I_a$, that the torque developed by any DC motor varies with flux Φ and armature current I_a. Recall also that in a shunt motor, the torque is practically proportional to the armature current, since the field flux is almost constant. However, in a series motor, the current through the armature also passes through the field. Therefore up to the point of magnetic saturation, the flux Φ will be almost directly proportional to the armature current, that is, $\Phi \propto I_a$, and the torque is therefore directly proportional to the square of the armature current $I_a{}^2$. Thus

$$T_a = K_3 I_a^2 \qquad \text{(for series motor)} \tag{2.23}$$

At light loads, the armature current I_a and hence the flux Φ is small. But as I_a increases, the torque T_a increases as the square of the current. That is, when the armature current doubles, the torque becomes four times as great.

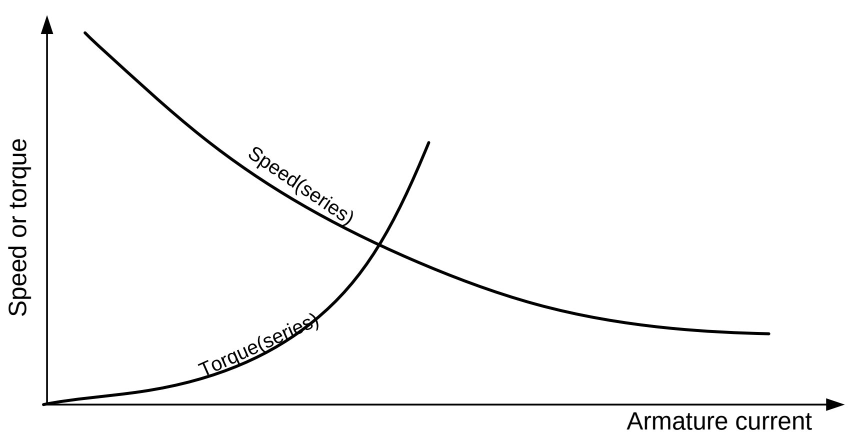


*Fig 2.12: Speed-Load and Torque-Load Characteristics of a **Series Motor***

6-2.2.2 Applications of Series Motor

- It is seen that a drop in speed with increased load is much more prominent in series motors than in shunt motors. Hence *series motors are not suitable for applications requiring constant speed*. *Series motors are used chiefly for widely varying loads, where extreme speed changes are not objectionable*.
- Because a series motor exerts a torque proportional to the square of the armature current, they are used in situations where *high starting torque* is required for accelerating heavy masses quickly as in ***hoists, cranes, and traction purposes*** (electric trains).
- In these cases, the variation of speed with the load is a favourable condition. For instance, in using a crane to lift a heavy load, it is generally desirable to proceed slowly, but in carrying a light load, it is desirable to have increased speed for making rapid headway. The enormous torque of a series motor makes it very suitable for hoisting and other work demanding frequent acceleration under heavy loads.
- In addition to the huge starting torque, there is another unique characteristic of series motors, which makes them especially desirable for ***traction work***. That is, when a load comes on a series motor, it responds by decreasing its speed (and hence E_b) and supplies the increased torque with a small current. On the other hand, a shunt motor under the same conditions would hold its speed nearly constant, and would supply the required increased torque with a large increase of input current.

Example 2.10

A 125 V shunt motor has an armature circuit resistance of 0.2 Ω and a shunt field resistance of 45 Ω. If the line current is 50 A, calculate the

a) generated voltage
b) developed power

Solution 2.10

$$I_f = \frac{V_f}{R_f} = \frac{V_t}{R_f} = \frac{125}{45} = \underline{2.78A}$$

For the motor, the armature current is given as

$$I_a = I_L - I_f = 50 - 2.78 = \underline{47.22\ A}$$

a) The generated voltage (back emf) is

$$E_b = V_t - I_a R_a = 125 - (47.22 \times 0.20) = \underline{\underline{115.55\ V}}$$

b) The developed power is

$$P_a = E_b I_a = 115.55 \times 47.22 = \underline{\underline{5.46\ \text{kW}}}$$

6-2.3 Speed Control of DC Motors

The fact that the speed of a motor varies with the field excitation provides a convenient means for controlling the speed of a DC motor. The field current and, therefore, the field flux may be varied by a field rheostat. Putting in resistance in the field circuit causes a decrease in the field flux and, therefore, an increase in speed. Likewise, a decrease in resistance causes a decrease in speed.

If a motor is able to maintain a nearly constant speed for varying loads, the motor is said to have a good ***speed regulation***. *Speed regulation is the ratio of the loss in speed (between no-load and full-load) to full-load*, and is usually expressed in percent as:

$$SR = \frac{no\text{-}load\ speed\ -\ full\text{-}load speed}{full\text{-}load speed} \times 100 \qquad (2.24)$$

The methods normally used to control the speed of the DC motor originate from the speed equation; namely,

$$N = \frac{V - I_a R_a}{K_1 \Phi} \qquad (2.25)$$

The speed of the DC motor can be controlled in the following three ways by varying:

1. Armature resistance R_a i.e., Armature resistance control (use of a resistor termed ***controller*** in series with the armature)
2. Field flux per pole Φ i.e., Field flux control (use of variable resistor termed ***field regulator***, in series with the shunt winding)
3. Applied voltage V i.e., Voltage control (use of ***thyristors*** fired at appropriate times or angles in power electronic circuits)

Example 2.11

A 600 V shunt motor has an armature resistance of 0.14 Ω. At no-load, when running at 800 rpm, the armature current is 12 A. The full-load armature current is 225 A. Calculate the full-load speed when the filed pole flux is

a) constant
b) reduced to 95 % of its original value by armature reaction
c) reduced to 85 % of its original value by adjusting the field current

Solution 2.11

At no-load, the generated voltage (back emf) is obtained from the general motor voltage equation

$$E_{nl} = V_t - I_{a,nl} R_a = 600 - (12 \times 0.14) = \underline{598.32\ V}$$

At full-load, the generated voltage (back emf) is obtained from the general motor voltage equation

$$E_{fl} = V_t - I_{a,fl} R_a = 600 - (225 \times 0.14) = \underline{568.50\ V}$$

Recall the general relationship between generated voltage, flux and speed and linked by the constant K as $E = K\Phi N \quad \Rightarrow N = \dfrac{E}{K\Phi}$

a) Since the **flux Φ is constant**, the speed is proportional to the generated voltage. Thus $\dfrac{N_{fl}}{N_{nl}} = \dfrac{E_{fl}}{E_{nl}} \quad \Rightarrow N_{fl} = \dfrac{E_{fl}}{E_{nl}} \times N_{nl} = \dfrac{568.50}{598.32} \times 800 = \underline{\underline{760\ rpm}}$

b) When the field pole flux has been reduced to 95%,

$$\frac{N_{fl}}{N_{nl}} = \frac{E_{fl}}{E_{nl}} \times \frac{\Phi_{\mathrm{nl}}}{\Phi_{\mathrm{fl}}} \quad \Rightarrow N_{fl} = \frac{E_{fl}}{E_{nl}} \times \frac{\Phi_{\mathrm{nl}}}{\Phi_{\mathrm{fl}}} \times N_{nl} = \frac{568.50}{598.32} \times \frac{\Phi_{\mathrm{nl}}}{0.95\Phi_{\mathrm{fl}}} \times 800 = \underline{\underline{800.14\ rpm}}$$

c) When the field pole flux has been reduced to 80%,

$$\frac{N_{fl}}{N_{nl}} = \frac{E_{fl}}{E_{nl}} \times \frac{\Phi_{\mathrm{nl}}}{\Phi_{\mathrm{fl}}} \quad \Rightarrow N_{fl} = \frac{E_{fl}}{E_{nl}} \times \frac{\Phi_{\mathrm{nl}}}{\Phi_{\mathrm{fl}}} \times N_{nl} = \frac{568.50}{598.32} \times \frac{\Phi_{\mathrm{nl}}}{0.80\Phi_{\mathrm{fl}}} \times 800 = \underline{\underline{950\ rpm}}$$

Example 2.12

A shunt motor supplied at 250 V runs at 900 rpm when the armature current is 30 A. The resistance of the armature circuit is 0.4 Ω. Calculate the resistance required in series with the armature to reduce the speed to 600 rpm, assuming that the armature current is then 20 A.

Solution 2.12

Initial generated emf is given as $E_1 = V_1 - I_{a1} R_{a1} = 250 - 30 \times 0.4 = \underline{\underline{238V}}$

Since the excitation remains constant, the **flux Φ is constant**, and the speed is proportional to the generated voltage. Thus the new emf generated at 600 rpm is given as

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \Rightarrow E_2 = \frac{N_2}{N_1} \times E_1 = \frac{600}{900} \times 238 = \underline{\underline{158.7V}}$$

Hence the new IR-voltage drop due to the **total resistance** of armature circuit is obtained from the formula

$$E_2 = V_2 - I_{a2} R_{a2} \Rightarrow I_{a2} R_{a2} = V_2 - E_2 = 250 - 158.7 = \underline{\underline{91.3V}}$$

The total resistance of the armature circuit is thus

$$I_{a2}R_{a2} = 91.3 \Rightarrow R_{a2} = \frac{91.3}{I_{a2}} = \frac{91.3}{20} = \underline{\underline{4.567\Omega}}$$

Therefore the additional resistance required in the armature circuit is

$$\Delta R = R_{a2} - R_{a1} = 4.567 - 0.4 = \underline{\underline{4.17\Omega}}$$

EXERCISES ON DC MOTORS

Exercise 2.7

(a) State THREE methods of speed control of DC motors.

(b) When operating on full-load, a 250 V shunt motor drives a shaft at 950 rpm. The armature takes a current of 80 A and has resistance of 0.15 Ω. Determine the speed of the motor when a 1.5 Ω resistor is connected in series with the armature, the flux and torque remaining constant.

Exercise 2.8

An eight pole 1250 rpm lap-wound motor has 140 slots in its armature, each slot containing 6 conductors, and draws an armature current of 60 A from a 250 V DC supply. If the resistance of the armature circuit is 0.15 Ω, calculate the following:

(i) useful flux per pole

(ii) gross torque developed by the armature.

Exercise 2.9

A 4-pole motor has its armature *lap-wound* with 130 slots, each slot containing 8 conductors, and runs at 1000 rpm when taking an armature current of 50 A from a 250 V DC supply. If the resistance of the armature circuit is 0.2 Ω, calculate the following:

(i) useful flux per pole

(ii) gross torque developed by the armature.

Unit 3

INTRODUCTION TO POLYPHASE AC MACHINES

Introduction

AC machines may be either generators or motors. For the purpose of this course syllabus, the focus in this section will thus be on AC motors.

Most of the motors used in industry and the home are of the alternating-current type. Industrial motors are usually designed for three-phase operation and the domestic motors are designed for the single-phase systems used in the home. The size of single-phase motors used in industry seldom exceeds 7 kW.

Three-phase motors are fairly simple in construction and have the advantage of being self-starting. The single-phase motor is not self-starting and requires to be wound in a special way and fitted with special starting arrangements. This makes single-phase motors expensive and larger than a three-phase motor of equivalent rating.

One minor disadvantage of the AC motor is that it runs at a fixed speed in standard motors. The speed depends on the number of poles and the frequency of the supply system. The relationship between speed N_s, number of poles P and frequency f is given by the formula $N_s = \frac{120f}{P}$.

The speed of some industrial motors is changed either by

- pole-changing in which the motor stator windings are switched to give different number of poles. This pole-changing system gives only different values of *fixed speeds*.
- frequency change through frequency changers. The frequency-changing system, however, gives much finer *variable speeds*.

The AC motor, like DC motors, consists of a stationary part, the stator, and a rotating part, called the rotor, on which are wound a set of current-carrying coils that are both excited. Such motors are thus *doubly-excited* rotating machines.

The stator is built up of laminated steel plates. Laminations are used, rather than a solid core, to minimize the eddy-current losses associated with AC magnetic circuits. The stator carries the field winding that is distributed in slots around the core, and the winding is retained in the slots by wooden or fibre wedges.

The rotating part, the rotor, is also made up of laminated plates mounted on the motor shaft, and the rotor can be of the squirrel cage or wound-rotor design. The rotor is located on a shaft running on bearings, and is free to rotate between magnetic poles.

As already mentioned, AC motors may be of the synchronous, induction or commutator type.

Synchronous: If one winding is made to carry a steady direct current (DC) and the other an alternating current (AC) whose *frequency equals the angular velocity of the rotating part*, the device is a *synchronous* machine.

Induction: If one winding is energised by an AC, and the other is *short-circuited*, the device is called an *induction machine*.

Commutator: If one winding carries a steady DC, and the other carries a current that reverses each half revolution of the rotor, a non-zero average torque will result. The reversal is accompanied by a commutator. *Commutator machines* produce exceptionally good torque and speed control but are most costly to build and maintain.

Objectives

After going through this unit, you should be able to:

- differentiate between the various AC polyphase machines
- know the types and understand the principles of operation of induction machines
- solve simple exercises on slip

SESSION 1-3 CONSTRUCTIONAL FEATURES OF INDUCTION MOTORS

1-3.1 Polyphase Induction Motors

Because of its *simple, rugged and cheap construction as well as good operating characteristics that suit most industrial requirements*, the induction motor is the most commonly used type of AC machines. The polyphase induction motors used in industrial applications are practically without exception three-phase, thus corresponding to the number of phases in commercial power systems. In conventional induction motors, the stator is connected to the AC supply, and the ***rotor winding is short-circuited*** for many applications, or it may be closed through external resistances. Current is induced in the rotor winding by transformer action from the stator. Because of this, the stator is sometimes referred to as the primary and the rotor as the secondary of the induction motor.

While the synchronous motor has certain *advantages* – such as practically absolute constant speed, the ability to generate reactive power with an overexcited field and low cost of slow-speed motors – it has the *disadvantages* of requiring a DC source (exciter) for its field excitation, lack of flexible speed control and relatively higher cost for high-speed motors.

The polyphase induction motor, however, requires no means for its excitation other than the AC line. It is economical to build for higher speeds, and one type (i.e., the wound-rotor induction motor) lends itself to a fair degree of speed control. The induction motor runs below synchronous speed and is known as an ***asynchronous machine***.

1-3.2 Construction of Polyphase Induction Motors

The stator core has no salient or projecting poles, and is built of slotted laminations that are supported in a stator frame of cast iron or fabricated steel plate. Since the induction motor has no salient poles, it has an airgap that is uniform except for the presence of the slots. *Laminations are used rather than a solid core, to minimize the eddy current losses associated with AC magnetic circuits.*

The polyphase induction motor falls into two general categories, depending on the kind of rotor used:

1. the squirrel-cage rotor and
2. the wound-rotor.

1-3.2.1 Squirrel-Cage Induction Motor

The rotor of a squirrel-cage induction motor is also laminated and slotted to contain the rotor windings. The rotor conductors are placed parallel, or approximately parallel, to the shaft and embedded in the surface of the core. Actually, the conductors of the squirrel-cage rotor are ***bars of copper or aluminium***, known as ***rotor bars***. At each end of the rotor, the rotor conductors are all ***short-circuited*** by continuous *end rings*. In the larger motors, the rotor bars, instead of being cast, are *wedged into the rotor slots and are then welded securely to the end rings*.
The rotor conductors and their end rings are similar to a revolving squirrel cage, thus explaining the name.

Squirrel-cage rotor bars are not always placed parallel to the motor shaft but are sometimes skewed. This results in a more uniform torque and also reduces the magnetic humming noise when the motor is running. The Fig below shows a simplified squirrel-cage rotor.

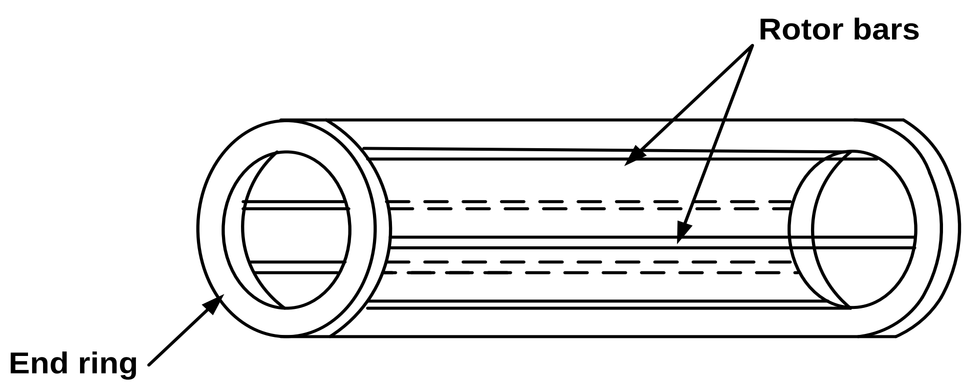


Fig 3.1: Simplified Squirrel-Cage Rotor

It must be noted that with the squirrel-cage rotor where the rotor bars (windings) are short-circuited in end rings, there are no external connections, which eliminates the use of slip rings and brushes.

1-3.2.2 Wound Rotor (or Slip-Ring) Induction Motor

The wound-rotor or slip-ring motor has a three-phase stator winding similar to that of the squirrel-cage motor. The wound-rotor motors differ from the squirrel-cage motor in the rotor construction. As the name implies, the rotor is wound with an *insulated winding* similar to the stator winding. The rotor phase windings are *star-connected* and terminate in ***slip-rings*** mounted on the rotor shaft. Wound-rotor induction motors are thus called ***slip-ring motors***. Brushes ride on the slip rings.

The rotor winding is not connected to a supply, the *slip rings and brushes merely providing a means of connecting an external variable-speed-control resistance into the rotor circuit, particularly for starting*. During normal running, the slip rings are short-circuited. Thus the principle of operation is the same for both squirrel-cage and wound-rotor types of induction machine. *Wound-rotor motors are less extensively used than squirrel-cage motors because of their higher first cost and greater maintenance costs*.

SESSION 2-3 PRINCIPLE OF OPERATION OF INDUCTION MOTORS

The *basic principle of operation of three-phase induction motors* is explained below. In a DC motor, current is drawn from the supply and conducted into the armature conductors through brushes and commutator. When the armature conductors carry current in the magnetic field established by the field circuit, a force is exerted on the conductors which tend to move them at right angles to the field.

In an induction motor, there is no electrical connection to the rotor, the rotor currents being induced currents. However, the same condition exists as in the DC motor, that is, the rotor conductors carry current in a magnetic field and thereby have a force exerted upon them tending to move them at right angles to the field.

When the stator winding is energized from a three-phase supply, a rotating magnetic field is established which rotates at synchronous speed $N_s = \frac{120f}{P}$. As the field sweeps across the rotor conductors, an emf is induced in these conductors just as an emf is induced in the secondary winding of a transformer by the flux of the primary currents. The rotor circuit being complete, either through end rings or an external resistance, the induced emf causes a current to flow in the rotor conductors. The rotor conductors carrying current in the stator magnetic field thus have a force exerted upon them.

If the developed torque is great enough to overcome the resisting torque of the load, the motor will accelerate in the same direction as the rotating stator field. It should be remembered that there is no electrical connection between the stator and the rotor. The motor action is due to rotor induced voltages, and so the motor is often referred to as *induction motor*.

SESSION 3-3 SPEED AND SLIP OF INDUCTION MOTORS

An induction motor cannot run at synchronous speed at which the stator magnetic field rotates. *If it were possible, by some means, for the rotor to attain synchronous speed, the rotor would then be* ***stationary*** *with respect to the rotating magnetic flux*. There would be no relative movement between the rotor conductors and the field, and hence no emf would be induced in the rotor winding, no rotor current would flow and therefore there would be no torque developed. ***Thus it is impossible for a normal induction motor to run at synchronous speed***. And so the rotor speed even at no-load must be slightly less than synchronous speed in order that current may be induced in the rotor, thereby producing a torque.

This *difference between the actual rotor speed* N_r *and synchronous speed* N_s *is called the* ***slip*** *of the motor*, and is of the order of 5% of the synchronous speed of standard motors at full-load. Thus

$$Slip = Synchronous\ speed - Rotor\ speed = N_s - N_r$$

The slip is more commonly expressed as percent of the synchronous speed. The per unit or percent slip is calculated as follows:

$$Percent\ slip\ s = \frac{Synchronous\ speed - Rotor\ speed}{Synchronous\ speed} \times 100 \quad = \frac{N_s - N_r}{N_s} \tag{3.1}$$

$$\Rightarrow N_r = N_s(1-s)$$

Example 3.1:

A 4-pole 50 Hz squirrel-cage motor has full-load speed of 1440 rpm. What is the percent slip at full-load?

Solution 3.1:

The synchronous speed is $N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500\,rpm.$

Slip in rpm : $N_s - N_r = 1500 - 1440 = 60\,rpm$

$\therefore$Percent slip : $s = \frac{N_s - N_r}{N_s} = \frac{60}{1500} \times 100 = \underline{4\,\%}$

Unit 4
TRANSFORMERS

Introduction

A *transformer* is a stationery AC voltage-changing device by means of which electrical energy in one circuit is transformed or transferred to electrical energy of the same frequency in another circuit. The transformation may involve either the *stepping up or stepping down* of the supply voltage. Lamps, motors, etc., are designed to be used on low voltage partly for the sake of safety. However, power is never transmitted any great distance at a low voltage, because of large power (I^2R) losses in the line, or because of heavy investments in copper wire.

Power is usually generated at higher voltages (13.8 kV at VRA), and stepped up at the generator station (to 161 kV by VRA) before being transmitted along lines over long distances to load centers or regions of consumption. There, the power is stepped down for the load.

Transformers have no moving parts, and are simple, rugged, and durable in construction, thus requiring little attention. They also have a very high efficiency (as high as 99%). As a static electric device, it consists of a winding, or two or more coupled windings, with or without a magnetic core. It transfers power by **electromagnetic induction** between the circuits at the same frequency, but usually at changed values of voltage and current (either serving as step-up or step-down function).

The generation of electricity requires motion between a magnetic field and conductors. In power plants, this motion is supplied by moving the wires through a fixed (or DC) magnetic field. A transformer, however, depends on a constantly changing magnetic field to transmit power. The wires are fixed, and the magnetic field moves. This **continual** movement of oscillating magnetism is set up by the 50 Hz AC power.

Objectives

After going through this unit, you should be able to:

- explain the principles of operation of the transformer
- deduce and apply the basic relationships of transformer parameters
- understand the basic circuits and phasor diagram representation of the transformer under load and no-load conditions
- practically determine the losses, voltage regulation and efficiency of the transformer
- familiar with cooling methods and designations
- undertake simple exercises on transformers

SESSION 1-4 PRINCIPLES OF OPERATION AND BASIC RELATIONSHIPS OF TRANSFORMER PARAMETERS

1-4.1 Electromagnetic Induction – Lenz's Law

Whenever **a conductor moves *relative* to a magnetic field**, electric potential, and therefore current, is **induced** in the conductor. In more sophisticated language, electric potential is induced in a conductor whenever flux through the conductor changes. A simple generator is a conductor loop rotating in a magnetic field. All functional generators, no matter how large, are based on this concept.

The average induced electric potential, regardless of how it is produced, is proportional to the rate of change of flux linkage, and is given by Lenz Law as:

$$e = -\frac{d}{dt}(N\Phi) = -N\frac{d\Phi}{dt} \qquad (4.1)$$

where e = induced instantaneous voltage
N = number of loops or turns
Φ = inducing flux

*The **minus sign indicates** that the **instantaneous emf acts** in a way **to oppose the flux inducing it***. This was discovered by Lenz, and referred to as **Lenz's Law**.

1-4.2 Principle of Operation of the Transformer

The transformer is a typical application of ***Faraday's Law of Electromagnetic Induction***. For the purposes of understanding the main principle of operation of transformers, we would be considering the construction and principle of operation of a single-phase *two-winding* transformer. In its simplest form, the two-winding transformer consists of two *coils* of wire P and S insulated from each other and wound on a common *iron core*. The core may take one of many forms, such as a toroid or a rectangular frame.

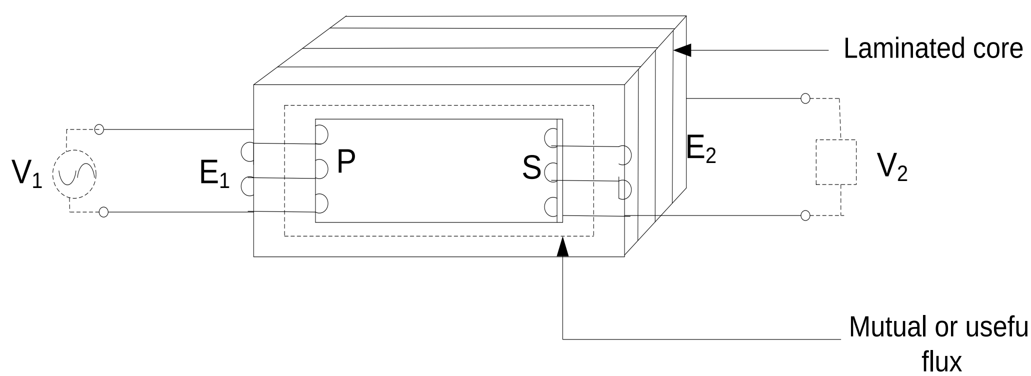


Fig 4.1: Schematic Diagram of Transformer

Each of the coils P and S is connected to an external circuit. The coil P, referred to as the *primary windings*, is connected to an electrical source, an AC supply whose voltage or current varies with time. In line with Faraday's observation, the alternating *primary current* which flows in the primary windings P generates an alternating magnetic flux, known as *useful or mutual flux* that *links or cuts* not only the primary windings P but also the *secondary windings* S.

Consequently, the generated ***flux linkage*** (i.e., the product of flux and number of turns, $\lambda = N\Phi$) induces an alternating emf in the secondary windings. Because the secondary coil is in circuit, it will also carry a time-varying current, the *secondary current*. This secondary current will also contribute to the magnetic field, so that the total magnetic field is created by contributions from both currents. The two coils are thus said to be *magnetically or inductively coupled*.

On the windings:

The windings are in the form of a coil so as to ensure that the magnetic field induced in the winding as a result of the time-varying current is concentrated and enhanced. For a given coil, the *inductance* L depends on a number of factors such as the:

1. number of turns N in the windings
2. permeability (property) of the medium surrounded by the coil
3. geometry of the windings (e.g. *toroid* with circular or rectangular cross-section, *solenoid*, etc.)

Depending on how the coils are wound on the core and on the permeability of the core, some of the alternating magnetic flux generated by the current in the primary coil will not link the second coil. Similarly, it is possible that not all the flux generated by the second coil will link the primary coil. In this situation where the induced flux avoids linkage with the other winding, we talk of *leakage flux*.

On the core:

For the type of transformers used in electricity supply where large values of *inductances* are required, it is necessary that the ***magnetic flux linkage be as large as possible***. This means that the paths of the magnetic lines of force must be through a ***core of ferromagnetic material* like *iron***. This explains why most transformers have iron cores.

For use at power and audio frequencies, the cores are ***laminated*** to reduce hysteresis and eddy-current losses, and also to ensure that all but a small fraction of the magnetic flux links with both primary and secondary coils. Because the self and mutual inductances are very high, a small current in one coil suffices to produce a large flux linkage, $\lambda = N\Phi = LI$, through the low-reluctance core.

Laminated iron is, however, not practicable at high frequencies because of the significant loss due to eddy-currents and hysteresis. At high frequencies, the eddy-currents may radically alter the flux distribution in the core. Instead, cores of powdered iron and ferrite are commonly used at high frequencies.

1-4.3 Basic Relationships of Transformer Parameters

In considering the basic theory, we would be making a few *assumptions*.

1. Firstly, if the applied or source voltage to the primary windings is **sinusoidal**, it is assumed that the emf induced in the secondary windings is also sinusoidal.

2. Secondly, we will also assume that all the flux generated by *each* current links *both* primary and secondary coils - that is, the flux linking each coil is the same, so that there is **no flux leakage**.

Let N_1 = number of turns on the primary P
N_2 = number of turns on the secondary S
Φ_{max} = maximum value of the flux linking P and S (in webers, Wb)

Since the flux varies sinusoidally, we have for the flux at any instant

$$\Phi = \Phi_{max} \sin \omega t \tag{4.2}$$

Hence the instantaneous electromotive force (emf) e_2 induced in the secondary coil of N_2 turns linked by this flux is given by:

$$\begin{aligned} e_2 &= -\text{rate of change of flux linkage} \\ &= -\frac{d}{dt}\left[N_2 \Phi\right] \\ &= -\Phi_{max}\,\omega\, N_2 \cos \omega t \end{aligned} \tag{4.3}$$

The minus sign indicates that the instantaneous emf acts in a way to oppose the flux inducing it. Neglecting it, the maximum value of the induced emf e_2 is thus:

$$\begin{aligned} E_{2\max} &= \Phi_{max}\,\omega\, N_2 \\ &= 2\pi f\ \Phi_{max}\,\omega\, N_2 \end{aligned} \tag{4.4}$$

The r.m.s. value of the secondary voltage can be obtained from the maximum value as:

$$\begin{aligned} E_2 &= \frac{E_{2\max}}{\sqrt{2}} = \frac{2\pi f\ \Phi_{max} N_2}{\sqrt{2}} \\ &= 4.44 f\ \Phi_{max} N_2 \\ &= 4.44 f\ B_{max} A N_2 \end{aligned} \tag{4.5}$$

Similarly, at the primary side,

$$E_1 = 4.44 f\ \Phi_{max}\ N_1 = 4.44 f\ B_{max}\ A N_1 \tag{4.6}$$

Combining the Equations (4.5) and (4.6), we obtain the ratio:

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = n \quad \text{or} \quad E_2 = nE_1 \tag{4.7}$$

This is called the ***voltage transformation or turns ratio***.

We can see from Equation (4.7) that, for a transformer in which the same flux links both the primary and secondary, and in which there are no voltage drops in the windings, the ***emf transformation ratio is equal to the turns ratio***. Depending upon the turns ratio, the transformer can either step up (*n>1*) or step down (*n<1*) voltage.

SESSION 2-4 PHASOR DIAGRAM AND EQUIVALENT CIRCUIT REPRESENTATION OF THE TRANSFORMER UNDER NO-LOAD AND ON-LOAD CONDITIONS

2-4.1 Operation of Transformer on *No-Load*

A transformer is said to be operated on "*no-load*" when the secondary winding is *open-circuited*. The secondary current is consequently zero and it is clear that the secondary winding can have no effect whatsoever on the magnetic flux in the core or the current in the primary. Hence the primary will act like a highly inductive circuit, its flux contribution coming only from the no-load current I_0 from the supply voltage.

See the *phasor diagram* representation in the figure below.

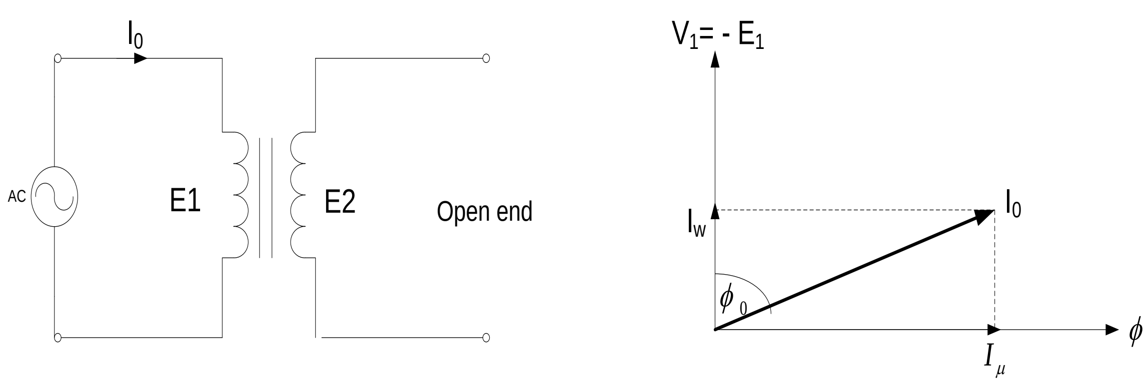


Fig 4.2: No-Load Current of the Ideal Transformer

The no-load current I_0 has two components, the reactive or magnetizing component I_μ and the active or power component I_w, and serves two functions:

2-4.1.1 The Magnetizing Component I_μ

The magnetizing component I_μ is needed to produce the m.m.f ($mmf = NI$) necessary to generate the flux mutual or useful Φ. This flux is related to the primary applied voltage V_1 (or primary induced e.m.f E_1) and the primary turns N_1 by Eqn (4.6):

$$\Phi_{\max} = \frac{E_1}{4.44 f\ N_1} \tag{4.8}$$

The reactive or magnetizing component is in quadrature lagging V_1 , and it must be remembered that whenever a magnetic flux is set up by an alternating current, that component of the current whose function is to produce the necessary mmf is a quadrature lagging component.

2-4.1.2 The Active Component I_w:

The active or power component I_w is needed to convey the power necessary to supply the *core losses* resulting from the alternating magnetization. The core losses are of two types, namely, *hysteresis and eddy-current losses*.

The active and magnetizing components I_w and I_μ of the no-load current can be calculated from the no-load phasor diagram.

$$I_w = I_0 \cos\varphi_0 \tag{4.9}$$

$$I_\mu = I_0 \sin\varphi_0 \tag{4.10}$$

$$I_0 = \sqrt{\left(I_\mu^2 + I_w^2\right)} \tag{4.11}$$

$$\text{Core loss} = I_c V_1 \tag{4.12}$$

where V_1 is the supply voltage.

The no-load current I_0 is thus the vector addition of two components at right angles, I_w in phase with E_1 and I_μ in ***quadrature l***agging, as shown in the figure above.

2-4.2 Operation of Transformer Under Load

Consider the secondary winding of an **ideal** transformer connected to a complex load impedance $Z_2 = R_2 + jX_2$. *By an ideal transformer, the windings are assumed to have no impedance. Thus the voltage drop across the windings are zero and so the secondary terminal voltage V_2 is equal to the no-load induced emf E_2.*

The secondary current that flows is:

$$I_2 = \frac{E_2}{Z_2}\angle\varphi \qquad (4.13)$$

The value of φ, that is the angle at which I_2 leads or lags E_2, depends on the resistance and reactance of the load. Before the flow of the secondary current I_2 the core carries the flux Φ whose value is given by the emf Eqn (4.8), and the primary winding first carries the no-load current I_0. Under these no-load conditions, the self-induced emf E_1 exactly balances the applied voltage V_1.

When a load is connected to the secondary side, a current I_2 flows due to the induced voltage E_2, and sets up a secondary mmf $N_2 I_2$. This mmf produces a *secondary flux Φ_2 that has the same magnetic path as and in the opposite direction to the useful flux Φ originally set up in the primary due to the no-load current*, and links with the primary winding.

This secondary flux Φ_2 tends to reduce the mutual flux Φ. But the slightest decrease in the mutual flux due to the secondary current would cause a corresponding decrease in the primary induced voltage E_1 and hence $V_1 = -E_1$. If the applied voltage, however, is constant, then the primary induced emf E_1 and thus mutual flux Φ in the core must also remain constant.

This can happen, only when the primary draws more current I_1' from the source to neutralize the demagnetizing effect of the mmf $N_2 I_2$.

Thus the load current I_2 causes the primary to take more current I_1' in addition to the no-load current I_0 such that

$$N_1 I_1' = N_2 I_2 \qquad (4.14)$$

The core flux in an ideal transformer thus remains constant and is independent of the load current. Eqn (4.14) implies that the compensating primary mmf $N_1 I_1'$ and the secondary mmf $N_2 I_2$ must be equal at every instant and the two currents I_2 and I_1' must be in phase opposition.

This component I_1' of the primary current which neutralizes the demagnetizing effect of I_2 is termed the *load component of the primary current and is drawn opposite to the load current* I_2 *in the phasor diagram*.

The total primary current I_1 is thus the ***phasor sum*** of the no-load current and the load component I_1'.

$$\underline{I_1} = \underline{I_0} + \underline{I_1}' \tag{4.15}$$

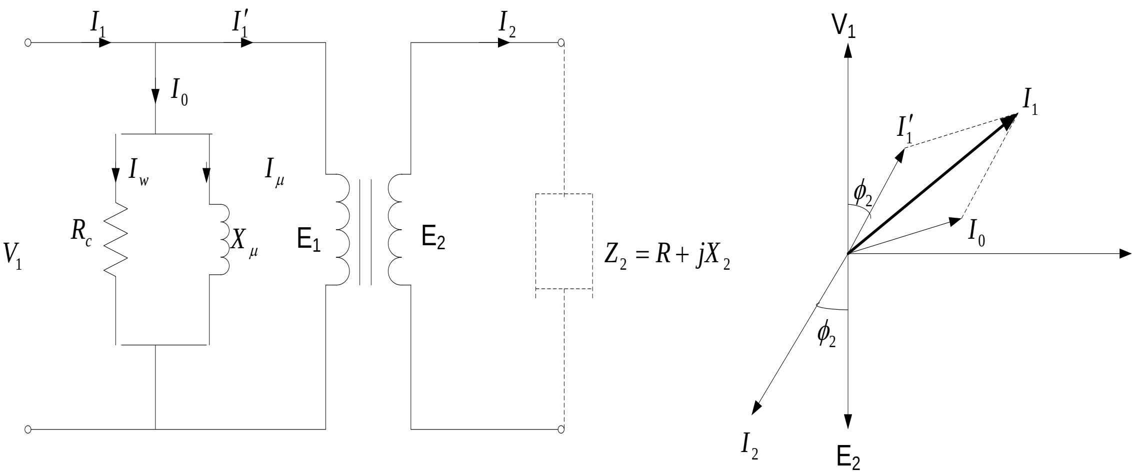


Fig 4.3: Phasor Diagram of Transformer on Inductive Load

From the equality of the two mmfs, we have

$$N_1 I_1' = N_2 I_2 \qquad \text{or} \qquad \frac{I_1'}{I_2} = \frac{N_2}{N_1} \tag{4.16}$$

Comparing with $\dfrac{E_1}{E_2} = \dfrac{N_1}{N_2}$, we obtain

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1'} \tag{4.17}$$

It can be seen from Eqn (4.17) that the **current transformation ratio is the inverse of the voltage transformation ratio**.

Since there are no internal drops (the case of an *ideal* transformer), $\underline{E_2}$ is in ***phase opposition*** to $\underline{E_1}$, and $\underline{I_2}$ is in phase opposition to $\underline{I_1'}$ and since the total primary current is the vector sum of its two components $\underline{I_1} = \underline{I_0} + \underline{I_1}'$, the vector diagram is as shown in figure above.

Example 4.1

A 250 kVA 11,000 V/400 V 50 Hz single-phase transformer has 80 turns on the secondary side. Calculate the following:

a) approx. values of the primary and secondary currents
b) approx. number of primary turns
c) maximum value of the flux

Solution 4.1

(i) The full-load primary and secondary currents are

$$I_1 = \frac{S}{V_1} = \frac{250 \times 1000}{11{,}000} = \underline{\underline{22.7A}}$$

$$I_2 = \frac{S}{V_2} = \frac{250 \times 1000}{400} = \underline{\underline{625A}}$$

(ii) The number of primary turns is found from the Eqn (4.17)

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \Rightarrow N_1 = \frac{E_1}{E_2} N_2 = \frac{11{,}000 \times 80}{400} = \underline{\underline{2{,}200}}$$

(iii) The maximum value of the flux can be obtained fro Eqn (4.5). Thus

$$E_2 = 4.44 f N_2 \Phi_{\max} \Rightarrow \Phi_{\max} = \frac{E_2}{4.44 f N_2} = \frac{400}{4.44 \times 50 \times 80} = \underline{\underline{22.5mWb}}$$

2-4.3 Equivalent Circuits of The Transformer:

The equivalent circuit is the representation of the physical device by a mathematical model using mathematical equations. The existence of a *magnetic or inductive coupling* in the transformer has already been explained. It will be recalled that in the treatment of the ideal transformer, the windings were assumed to possess no resistance.

In an actual transformer, however, the windings will inevitably have some resistance, and a small magnetizing current is required to maintain the flux in the core. Furthermore, not all the flux produced by the primary windings cut or link the secondary windings, and the leakage flux can be considerable.

This leakage flux would induce a back voltage in the primary, thus introducing a primary reactance drop. Similarly, the secondary current would set up a flux that would not cut the primary. Hence it would not be neutralized by the primary flux, and would produce a reactance drop in the secondary. The effect of these leakage fluxes would be the same as though a reactance were connected in series with each winding of a transformer that had no flux leakage.

Thus a model of the physical transformer should include winding resistance and reactance, as well as the magnetizing current. Thus in each winding of a transformer, there is an IX drop due to the reactance produced by the flux leakage and an IR drop due to resistance. The IX drop may be made small by reducing the flux leakage to a minimum, which may be done by placing primary and secondary windings on the same leg of the iron core, with the necessary insulation between them.

Furthermore, there will be an inevitable *capacitive effect* among the turns of the coils (represented by the capacitances C_1, C_2 and C_M), although windings are designed to minimize the capacitance.

These characteristics of a physical device lead us to represent the transformer as in the figure below.

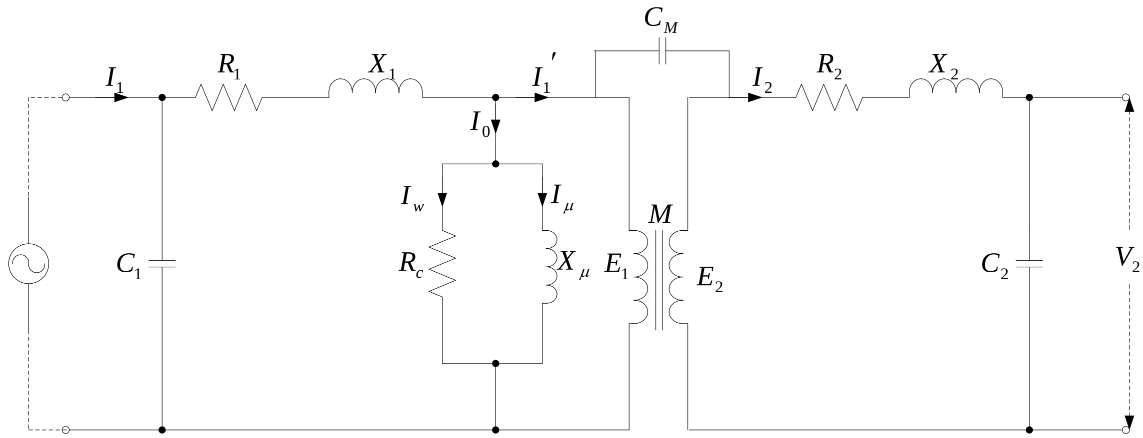


Fig 4.4: Equivalent Circuit of a ***Real*** *Transformer*

This representation gives the ideal windings with all the imperfections brought out of the windings.

- R_1 and R_2 are the resistances of the primary and secondary windings respectively.
- X_1 and X_2 are the reactances due to the leakage fluxes in their respective windings.
- The resistance R_w is a *fictitious* one that allows a current I_w, the active component of the no-load current I_0, to flow through it. The product $I_w^{\,2}R_w$ accounts for the heating or core-loss of the transformer.
- X_μ allows the magnetizing current I_μ to flow through it to produce the core flux.
- I_1 and I_2 are the full-load (rated) currents flowing in the primary and secondary windings respectively.

2-4.3.1 Approximate Equivalent Circuit of a *Low-Frequency* Transformer:

For high enough frequencies, the effect of these inter-turn capacitances (C_1 and C_2) and the inter-winding capacitance (C_M) can be quite significant. The effect of the capacitances may, however, be negligible at ***low frequencies*** of operation (reactance of a capacitance is inversely proportional to the frequency).

For our purpose, we will be considering low frequency operations of the transformer (power and audio transformers), and so the capacitances will not be considered.

Furthermore, since the no-load current I_0 is comparatively small (i.e., about 2.5% of the full-load current), the shunt branch can be neglected.

The equivalent circuit of the transformer (figure above) can thus be approximated to that of the figure below.

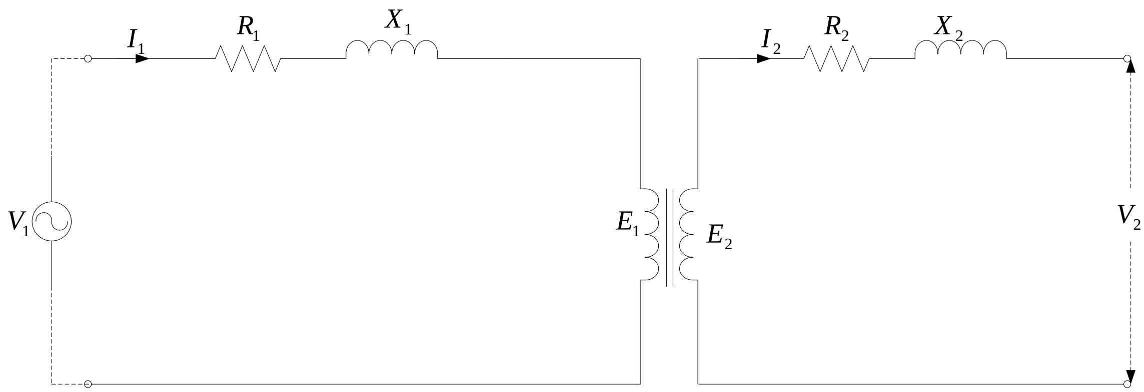


*Fig 4.5: Approx. Equivalent Circuit of a **Low-Frequency** Transformer*

2-4.3.2 Simplified Approximate Equivalent Circuit of Low-Frequency Transformer:

To simplify calculations, the resistance R_2 and X_2 of the secondary windings can be ***referred*** to the primary to obtain R_2' and X_2'. The simplified approximate equivalent circuit is as shown in the figure below.

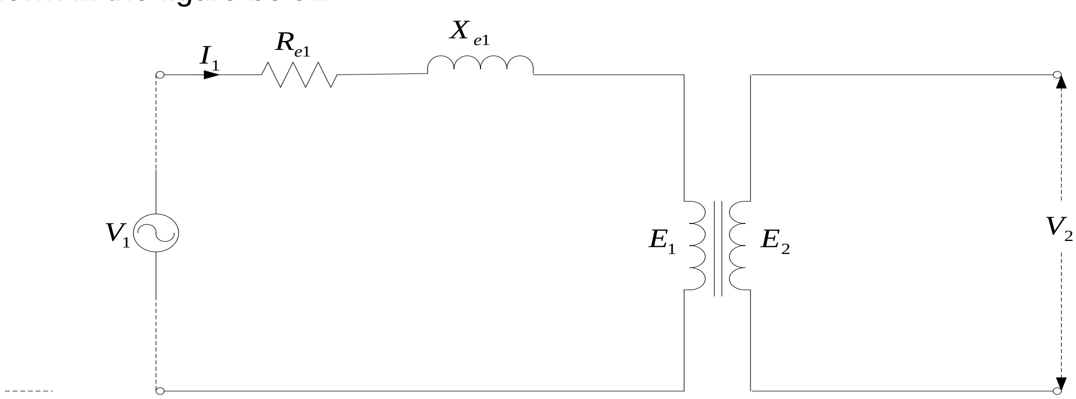


Fig 4.6: Simplified Approx. Equivalent Circuit of Low-Frequency Transformer

NOTE: ***In such a referral, the value of the new resistance and reactance*** R_2' ***and*** X_2' ***must be such that the power taken by*** R_2' ***while in the primary is equal to the power taken by*** R_2 ***in the secondary***, i.e.,

$$I_1^2 R_2' = I_2^2 R_2$$

$$\Rightarrow R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2 = \left(\frac{N_1}{N_2}\right)^2 R_2 \tag{4.18}$$

Similarly,

$$X_2' = \left(\frac{I_2}{I_1}\right)^2 X_2 = \left(\frac{N_1}{N_2}\right)^2 X_2 \tag{4.19}$$

and

$$Z_2' = \left(\frac{I_2}{I_1}\right)^2 Z_2 = \left(\frac{N_1}{N_2}\right)^2 Z_2 \tag{4.20}$$

Hence the impedance of the circuit is ***increased*** by an amount (*referred to the primary*) equal to

$$\left(\frac{N_1}{N_2}\right)^2 Z_2 \tag{4.21}$$

where Z_2 is the impedance of the secondary terminal (i.e., that of the secondary load and windings).

The total equivalent resistances and reactances referred to the primary are given by:

$$\begin{aligned} R_{e1} &= R_1 + R_2' \\ &= R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2 \end{aligned} \tag{4.22}$$

$$\begin{aligned} X_{e1} &= X_1 + X_2' \\ &= X_1 + \left(\frac{N_1}{N_2}\right)^2 X_2 \end{aligned} \tag{4.23}$$

$$Z_{e1} = \sqrt{\left(R_{e1}^2 + X_{e1}^2\right)} \tag{4.24}$$

SESSION 3-4 VOLTAGE REGULATION AND TRANSFORMER LOSSES

3-4.1 Voltage Regulation

By *voltage regulation*, we are referring to measures to ensure that *voltage variation* at the *secondary* terminals is *reduced to practically acceptable limits*.

It is a known fact that there is an increase in voltage when the load connected is decreased. Similarly, there is a voltage dip when more load is connected to the secondary terminal.

The voltage regulation *V.R.* is defined as the change in secondary terminal voltage expressed as a percentage (or p.u.) of the secondary rated voltage under no-load conditions:

$$\begin{aligned} V.R. &= \frac{\text{no - load secondary voltage - full load secondary voltage}}{\text{no - load secondary voltage}} \\ &= \frac{E_2 - V_2}{E_2} \end{aligned}$$

Thus the secondary terminal voltage is given as

$$V_2 = E_2(1 - V.R.) \qquad (4.25)$$

A distribution transformer should have a small value of *V.R.* (i.e. good voltage regulation), so that the terminal voltage does not vary widely as the load changes. For a transformer of large value of *V.R.* (i.e. poor voltage regulation), the terminal voltage will decrease appreciably with increase in load, and this has detrimental effect on the operation of the load (fluorescent tubes, T.V. sets, refrigerator motors, etc.), since these are designed to operate satisfactorily at constant voltage.

The secondary induced emf is related to the primary induced emf by the relation $E_2 = \frac{N_2}{N_1} \times E_1$. But at *no-load*, the induced primary emf E_1 is equal to the applied voltage V_1, *since the primary impedance voltage drop due to the comparatively small no-load current can be neglected*. Hence $E_2 = \frac{N_2}{N_1} \times V_1$.

Substituting this value of E_2 into Eqn (4.25), the expression for the *V.R.* can also be written in terms of the primary quantities as:

$$\begin{aligned} V.R. &= \frac{V_1(N_2/N_1) - V_2}{V_1(N_2/N_1)} \\ &= \frac{V_1 - V_2(N_1/N_2)}{V_1} \end{aligned} \qquad (4.26)$$

Thus $V.R. \cong \dfrac{V_1 - E_1}{V_1}$ (4.27)

The voltage regulation of a transformer can be obtained from its simplified approximate equivalent referred either to the primary or the secondary.

3-4.1.1 Voltage Regulation In Terms of Transformer Winding Parameters

Consider the equivalent circuit referred to the primary and the phasor diagram for a *lagging* (inductive) load.

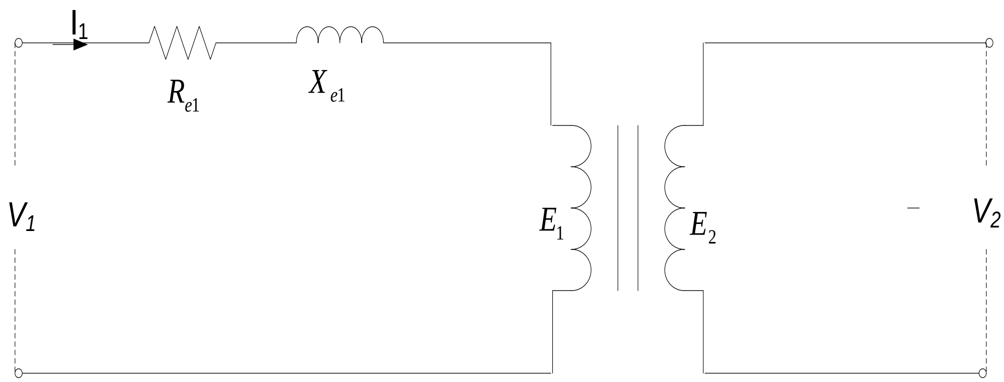
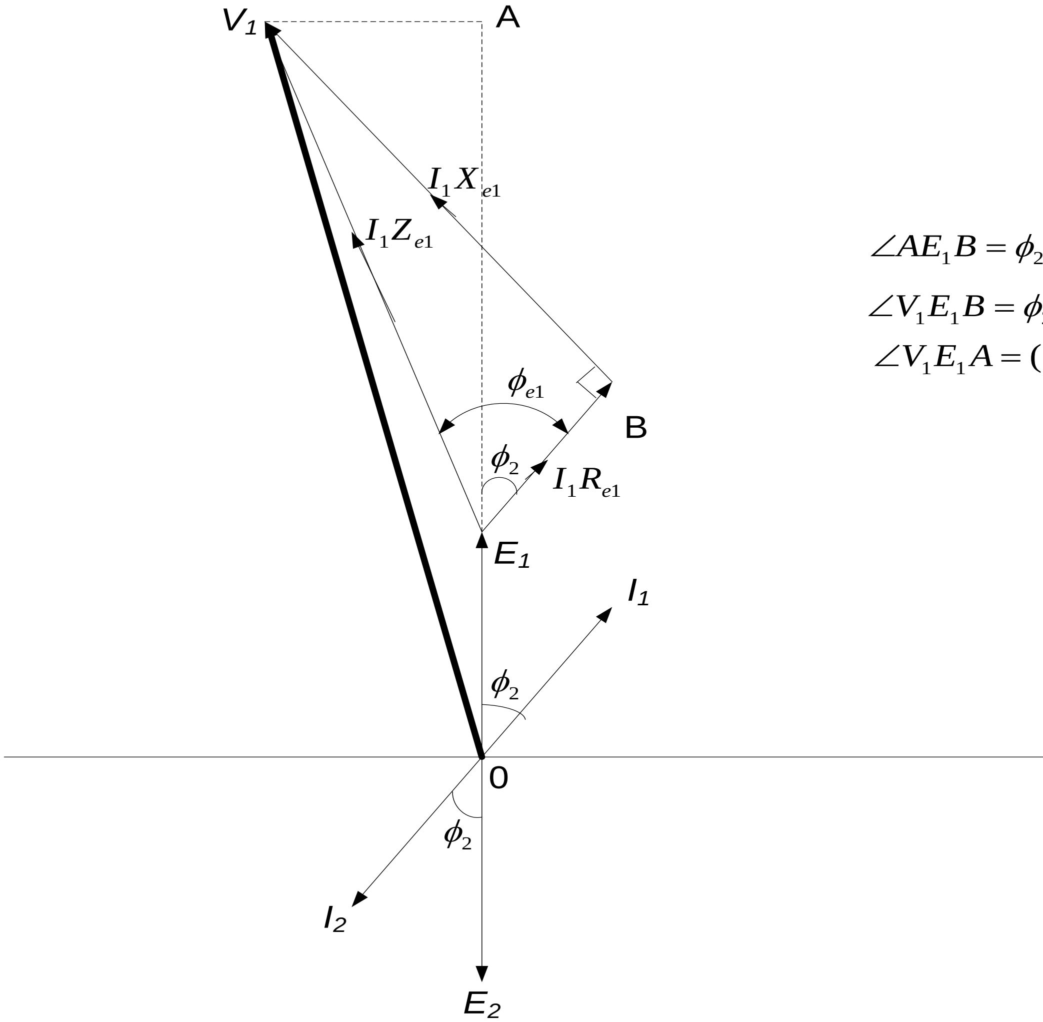


Fig 4.7: Equivalent Circuit with Secondary Referred to the Primary

The voltage equation at the primary side is given as

$$V_1 = E_1 + I_1 Z_{e1} \quad (4.28)$$

The vector diagram is as below.



$$\angle AE_1B = \phi_2$$
$$\angle V_1E_1B = \phi_2$$
$$\angle V_1E_1A = (\phi_{e1} - \phi_2)$$

Fig 4.8: Phasor Diagram for Circuit in Figure Above

From the phasor diagram,

$$V_1^2 = (E_1 + E_1A)^2 + (V_1A)^2$$

But V_1A is negligible compared to $(E_1 + E_1A)$

$$V_1 = E_1 + E_1A$$
$$\Rightarrow V_1 - E_1 = E_1A$$
$$\Rightarrow \frac{V_1 - E_1}{V_1} = \frac{E_1A}{V_1} = V.R.$$

Therefore

$$
\begin{aligned}
V.R. &\approx \frac{E_1 A}{V_1} \\
&= \frac{I_1 Z_{e1} \cos(\varphi_{e1} - \varphi_2)}{V_1} \\
&= \frac{I_1 Z_{e1}}{V_1}\left[\cos\varphi_{e1}\cos\varphi_2 + \sin\varphi_{e1}\sin\varphi_2\right] \\
&= \frac{I_1}{V_1}\left[Z_{e1}\cos\varphi_{e1}\cos\varphi_2 + Z_{e1}\sin\varphi_{e1}\sin\varphi_2\right]
\end{aligned}
\qquad (4.29)
$$

$$V.R. = \frac{I_1}{V_1}\left[R_{e1}\cos\varphi_2 + X_{e1}\sin\varphi_2\right]$$

where $\cos\varphi_2$ is the power factor of the inductive load.

It can be shown that for a ***leading*** power factor, expression for the voltage regulation will be (φ_2 may be replaced by $-\varphi_2$):

$$V.R. = \frac{I_1}{V_1}\left[R_{e1}\cos\varphi_2 - X_{e1}\sin\varphi_2\right]$$ *:* *for* ***leading*** *power factor*

The $V.R.$ could also be expressed in terms of the equivalent secondary values.

And so more generally, the voltage regulation could be expressed as

$$V.R. = \frac{I}{V}\left[R_e\cos\varphi \pm X_e\sin\varphi\right]$$

$+$ *sign: lagging power factor*

$-$ *sign: leading power factor*

Accordingly, depending upon the lag or lead of the *p.f.*, the *secondary full-load terminal voltage* is given by

$$V_2 = E_2(1 \pm V.R.) \qquad (4.30)$$

Example 4.2

A 100 kVA transformer has 400 turns on the primary and 80 turns on the secondary. The primary and secondary resistances are 0.3 Ω and 0.01 Ω respectively, and the corresponding leakage reactances are 1.1 Ω and 0.035 Ω respectively. The supply voltage is 2,200 V. Calculate the following:

a) Equivalent impedance referred to the primary circuit
b) Voltage regulation and the secondary terminal voltage for full-load having power factor of (i) 0.8 lagging (ii) 0.8 leading

Solution 4.2

(a) From Eqn (6.22), the equivalent resistance referred to the primary side is

$$R_{e1} = R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2 = 0.3 + \left(\frac{400}{80}\right)^2 \times 0.01 = \underline{\underline{0.55\Omega}}$$

From Eqn (6.23), the equivalent leakage reactance referred to the primary side is

$$X_{e1} = X_1 + \left(\frac{N_1}{N_2}\right)^2 X_2 = 1.1 + \left(\frac{400}{80}\right)^2 \times 0.035 = \underline{\underline{1.975\Omega}}$$

From Eqn (6.24), the equivalent impedance referred to the primary side is

$$Z_{e1} = \sqrt{(R_{e1})^2 + (X_{e1})^2} = \sqrt{(0.55)^2 + (1.975)^2} = \underline{\underline{2.05\Omega}}$$

b) (i) Since $\cos\phi_2 = 0.8$, therefore $\sin\phi_2 = 0.6$

Full-load primary current is $I_1 = \frac{S}{V_1} = \frac{100 \times 1000}{2,200} = \underline{45.45A}$

Hence the voltage regulation for a power factor of **0.8 lagging** is

$$VR = \frac{I_1(R_{e1}\cos\phi + X_{e1}\sin\phi)}{V_1} = \frac{45.45(0.55 \times 0.8 + 1.975 \times 0.6)}{2,200} = \underline{\underline{0.0336 = 3.36\%}}$$

Secondary terminal voltage on ***no-load*** is equal to the induced emf , and is

$$E_2 = E_1 \frac{N_2}{N_1} = 2,200 \times \frac{80}{400} = \underline{440V}$$

Thus the secondary terminal voltage ***on full-load*** is given as

$$V_2 = E_2(1 - VR) = 440(1 - 0.0336) = \underline{\underline{425.2V}}$$

(ii) the voltage regulation for a power factor of **0.8 leading** is

$$VR = \frac{I_1\left(R_{e1}\cos\phi + X_{e1}\sin\phi\right)}{V_1} = \frac{45.45\left(0.55\times 0.8 - 1.975\times 0.6\right)}{2{,}200} = \underline{\underline{-0.0154 = -1.54\%}}$$

Secondary terminal voltage on ***no-load*** is equal to the induced emf , and is

$$E_2 = E_1\frac{N_2}{N_1} = 2{,}200\times\frac{80}{400} = \underline{440V}$$

Thus the secondary terminal voltage ***on full-load*** is given as

$$V_2 = E_2(1 + VR) = 440(1 + 0.0154) = \underline{\underline{447V}}$$

3-4.2 Transformer Losses

The losses in the transformer could be classified into two, namely, the *core or iron losses* and the *copper losses*, though these amount to only a small portion of the total input power (1% to 15%). Since the losses in a transformer are very low compared with the output, the efficiency is very high, varying from 85% to 99% (the larger the transformer, the higher the efficiency).

3-4.2.1 Iron Losses

The iron losses are due to ***hysteresis*** and ***eddy current losses*** in the core of the transformer. For a better understanding of the iron losses, consider briefly how these losses occur in the core.

3-4.2.2 Hysteresis Loss P_{hys} :

The alternating induced flux in the core causes an alternating magnetisation of the core at a frequency of the time-changing magnetic flux. These alternate cycles of magnetisation can be represented by the so-called *hysteresis loop* on a B-H curve. The iron subsequently experiences *hysteresis losses*. The B-H curve is shown below.

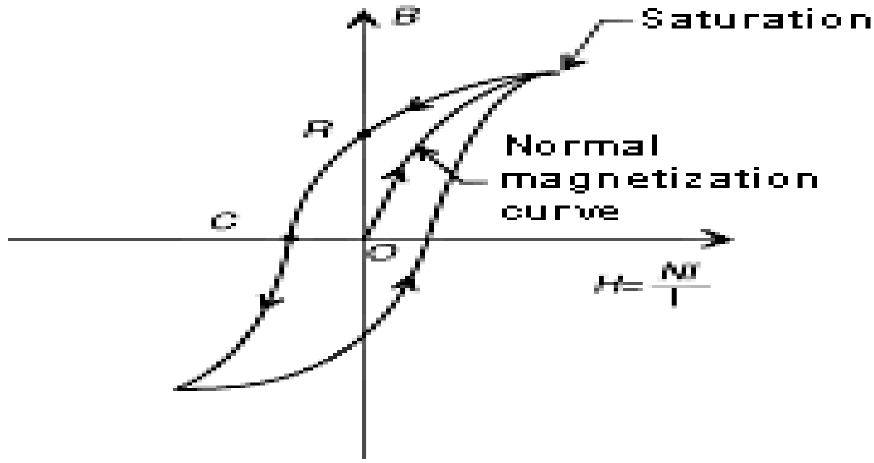


Fig 4.9: Diagram of B-H curve

If a coil on an iron core is energized, a flux will be set up which will increase in the manner suggested by the B-H curve in the figure above, and is termed *saturation or magnetization or hysteresis curve*. When the flux in the coil is decreased, the flux density will not decrease along the line *normal magnetization curve*, but will decrease less rapidly as indicated by the line RC. The magnetization force is zero when R is reached, but the flux density is NOT zero. This flux, which remains after the magnetization force has been removed, is called ***residual magnetism***.

At C, the residual magnetism has been reduced to zero by reversing the coil current and a corresponding mmf is produced. As this reversed coil current is increased further, the flux density will build up as shown in the diagram. If the coil current is reduced to zero, the flux density will decrease along the curve and will again reach *a non-zero flux density* at zero magnetization. To reduce the flux to zero, it is necessary to supply a magnetizing force. While increasing the magnetization action to *the maximum*, the flux density will return to its maximum value. This closes the so-called ***hysteresis loop***. It is seen that the flux lags behind the magnetizing force, and this lagging effect is called *hysteresis*.

In DC and AC machines, the flux is continually changing in this way, and as a result some power is lost as heat. The magnitude of the hysteresis loss is dependent on the grade of the ferromagnetic material used for the core. It is also proportional to the frequency f.

If V_c is the volume of the core, the hysteresis loss is obtained by the **Steinmetz empirical formular**:

$$P_{hys} = k_h V_c f B_m^{1.6} \tag{4.31}$$

where k_h is the *Steinmetz or hysteresis constant* whose value depends on the ferromagnetic material used for the core. For instance, k_h for silicon steel is between 100 and 200.

3-4.2.3 Eddy-Current Loss P_{eddy} :

Since the iron is electrically conducting, a *solid core* in the presence of an induced emf would constitute electrical paths of very low resistance and consequently currents would circulate within the core. These circulating currents inside the solid conducting core, arising out of the induced emf, are referred to as *eddy-currents*. These eddy-currents result in so-called *eddy-current loss*, and produces heat which lowers the efficiency and raises the temperature of the windings, lowering its output capacity.

In practice, however, the effects of the eddy-currents are greatly reduced through a process of ***lamination*** of the core. In the lamination process, the core is made of very thin slices of iron sheets, which are insulated electrically from one another. The magnetic flux is not affected by the laminations, since the magnetic lines flow perpendicular to the electric current.

Suppose there are *n* slices, then the emf acting around an eddy-current path is reduced to 1/*n*th of the previous value. Also the cross-section of an eddy-current path is reduced to 1/*n*th, with the resistance being correspondingly increased to *n* times.

Consequently, the eddy-current is reduced. By making *n* large, that is, by using thinner sheet materials, the eddy-current can be reduced to a practicable value. To reduce the eddy current still further, the core material should have as high an electrical resistance as possible. The addition of a small percentage of silicon has the effect of increasing the electrical resistance. Unfortunately, however, silicon sheet steels are expensive.

The eddy-current loss P_{eddy} can be calculated from the equation:

$$P_{eddy} = \frac{k_{eddy} A^2 f^2 B_m^2}{\rho} \tag{4.32}$$

where A = cross-sectional area
k_{eddy} = constant
ρ = resistivity of the ferromagnetic core material

The eddy-current power loss is therefore proportional to the square of the cross-sectional area normal to the direction of the field, the square of the maximum flux density, the square of the frequency and inversely proportional to the resistivity of the ferromagnetic material from which the core is made.

To minimize the eddy-current loss, only the cross-sectional area and the resistivity can be varied.

3-4.2.4 Calculation of Core Losses:

The total core or iron loss P_{Fe} is thus the sum of the hysteresis loss P_{hys} and the eddy-current loss P_{eddy}.

$$P_{Fe} = P_{hys} + P_{eddy} \tag{4.33}$$

The active component I_w of the no-load current which conveys this power is ***in phase*** with the applied voltage V_1 and is given by:

$$I_w = \frac{P_{Fe}}{V_1} \tag{4.34}$$

NOTE: The iron losses can be fairly determined from the Open-Circuit Test.

3-4.2.5 Copper Losses

The copper losses P_{Cu} are due to the resistance present in the primary and secondary windings. *The copper losses vary as the load current varies*. The total copper losses are obviously the sum of the copper losses in the primary and secondary windings.

Total copper losses = Primary winding losses + Secondary winding losses

$$P_{cu} = I_1^2 R_1 + I_2^2 R_2 \tag{4.35}$$

To simplify the calculations, we ***refer*** the secondary resistance R_2 to the primary using the Eqn (4.18), in which case only the primary current I_1 will be needed.

$$\begin{aligned} P_{cu} &= I_1^2 (R_1 + R_2') \\ &= I_1^2 \left[R_1 + \left(\frac{I_2}{I_1} \right)^2 R_2 \right] \\ &= I_1^2 \left[R_1 + \left(\frac{N_1}{N_2} \right)^2 R_2 \right] \\ P_{cu} &= I_1^2 R_{e1} \end{aligned} \tag{4.36}$$

where $R_{e1} = R_1 + R_2'$ and $R_2' = \left(\frac{N_1}{N_2} \right)^2 R_2 = \left(\frac{I_2}{I_1} \right)^2 R_2$.

Similarly, using the quantities referred to the secondary side, the copper losses may be determined as $P_{cu} = I_2^2 R_{e2}$, where $R_{e2} = R_2 + R_1^{'}$ and $R_1^{'} = \left(\frac{N_2}{N_1}\right)^2 R_1 = \left(\frac{I_1}{I_2}\right)^2 R_1$.

NOTE: The copper losses can be fairly determined from the Short-Circuit Test.

SESSION 4-4 TRANSFORMER TESTS, EFFICIENCY AND COOLING METHODS

4-4.1 Tests Performed On Transformer

Two main tests known as the ***Open-Circuit*** *(No-Load)* and ***Short-Circuit Tests*** are performed on the transformer. These tests are performed to determine the voltage regulation *V.R.*, the efficiency η and other equivalent circuit parameters of the transformer without actually loading it. The power required during these two tests is approximately equal to the appropriate power loss occurring in the transformer.

The ***advantages*** with these tests are that the ***parameters*** of a very large machine, having a rating of say over *10 MVA*, ***can be determined*** relatively in a ***less expensive*** way, when compared with an actual test performance with an artificial load. Furthermore, the power required to perform these tests is very small as compared to the full-load output of the transformer.

4-4.1.1 Open-Circuit Test on Transformer:

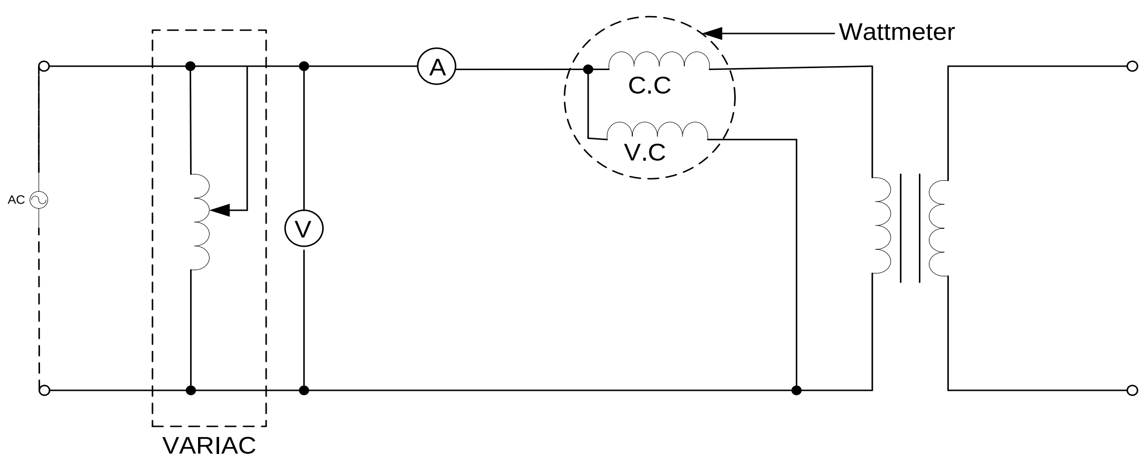


Fig 4.10: Open-Circuit (No-Load) Test Performed on Transformer

The secondary is left open and a *rated or full-load voltage* is applied through a *VARIAC* to the primary of the transformer. The following measurements are made:

1. The ammeter measures the no-load current I_0 in the primary, which is about *2.5 %* the rated or full-load current.
2. The voltmeter measures the rated or full-load voltage $V_1 = V_{oc}$.
3. The wattmeter measures the open-circuit power P_{oc}. Since the secondary current I_2 is zero, the only copper losses are due to the no-load current I_0 in the primary.

The core or iron loss is given as:

$$P_{Fe} = P_{oc} - R_1 I_0^2 \tag{4.37}$$

Since the total copper loss $R_1 I_0^2$ is small (no-load current is very small) compared with the open-circuit power P_{oc}, the core or iron loss is ***approximately equal*** to the open-circuit power.

$$P_{Fe} \cong P_{oc} \tag{4.38}$$

NOTE.: The no-load losses therefore give an indication of the core or iron losses.

4-4.1.2 Calculation of ***Branch Impedance Parameters*** Using ***No-Load Test Data***:

$$\text{No - load p.f.} = \cos\varphi_0 = \frac{P_{oc}}{V_{oc} I_0}$$

$$I_w = I_0 \cos\varphi_0 \qquad ; \qquad I_\mu = I_0 \sin\varphi_0$$

$$R_c = \frac{V_{oc}}{I_c} = \frac{V_{oc}}{I_0 \cos\varphi_0} \qquad ; \qquad X_\mu = \frac{V_{oc}}{I_\mu} = \frac{V_{oc}}{I_0 \sin\varphi_0}$$

4-4.1.3 Short-Circuit Test On Transformer:

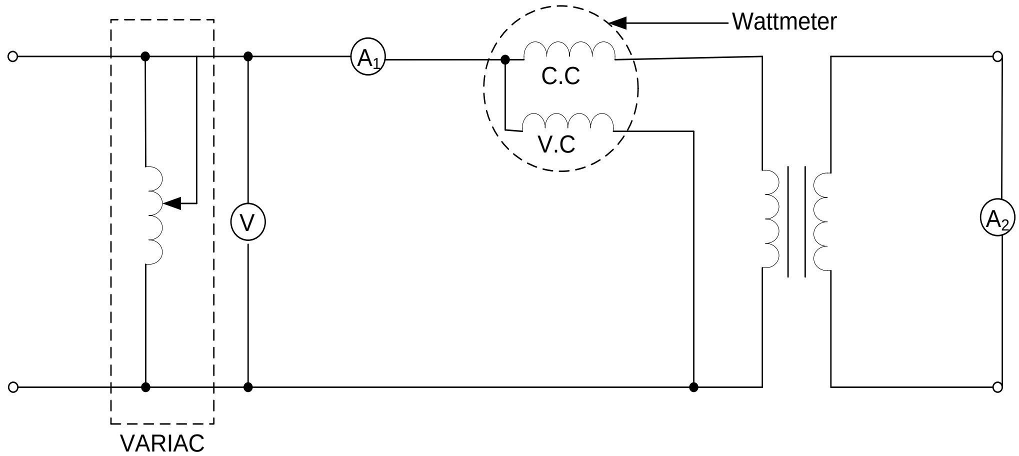


Fig 4.11: Short-Circuit Test Performed On Transformer

The secondary of the transformer is shorted through an ammeter and a *reduced voltage* is applied through a *VARIAC* to the primary of the transformer *until the rated or full-load current flows in the primary*. Since in a transformer the compensating primary mmf is almost equal to the secondary mmf ($N_1 I_1' = N_2 I_2$), a rated current in the primary causes rated current to flow in the secondary. A primary voltage of 2% to 12% is sufficient to circulate the rated primary and secondary currents in the short-circuit test.

The following measurements are made:

1. The ammeters A_1 and A_2 measure the primary and secondary currents I_1 $(=I_{sc})$ and I_2 respectively.

2. The voltmeter measures the short-circuit voltage V_{sc}.

3. The wattmeter measures the short-circuit power P_{sc}. This gives the full-load copper losses, if the currents flowing in the primary and secondary are rated or full-load currents. These are obtained by adjusting the variac.

4-4.1.4 Calculation of ***Equivalent Impedance*** Values Using ***Short-Circuit Test Data***:

$$\text{Short-circuit p.f.} = \cos\varphi_{sc} = \frac{P_{sc}}{V_{sc} I_{sc}}$$

$$R_{e1} = \frac{P_{sc}}{I_{sc}^2} \quad ; \quad Z_{e1} = \frac{V_{sc}}{I_{sc}} \qquad \Rightarrow X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2}$$

The *V.R.* of the transformer can be calculated from the short-circuit measurements. Referring to Eqn (35),

$$V.R. = \frac{I_1 Z_{e1}}{V_1}\cos(\varphi_{sc} - \varphi_2) \qquad (4.39)$$

where I_1 = ***rated current*** *at where the measurement is being made*

V_1 = ***rated voltage*** *at where the measurement is being made*

If Z_{e1} is the equivalent impedance referred to the primary circuit, and V_{sc} *is the primary applied voltage on the short-circuit test* **when rated (full-load) currents are flowing** *in the primary and secondary windings*, then

$$I_1 Z_{e1} = V_{sc} \qquad (4.40)$$

In other words, if a ***rated current*** I_1 flows with the application of the short-circuit test applied voltage V_{sc}, the *V.R.* can then be calculated as:

$$V.R. = \frac{V_{sc}}{V_1}\cos(\varphi_{sc} - \varphi) \qquad (4.41)$$

cos φ is the load power factor.

NOTES:

1. *Short-circuit losses give an indication of the copper losses.*
2. *The equation of* $V.R. = \frac{V_{sc}}{V}\cos(\varphi_{sc} - \varphi)$ *involving the applied short-circuit test voltage* V_{sc} *can be used for the calculation of voltage regulation,* ***only when rated or full-load current flows*** *with the applied short-circuit test voltage* V_{sc}*.*
3. *Otherwise the more general equation of* $V.R. = \frac{I}{V}[R_e\cos\varphi \pm X_e\sin\varphi]$ *must be used in calculating the voltage regulation.*

Example 4.3

Tests performed on a $10\,kVA$ $200/400V$ $50\,Hz$ single-phase transformer yielded the following test results:

No-load test: $200V$, $1.3\,A$, $120W$ on LV side.
Short-circuit test: $22V$, $30\,A$, $200W$ on the HV side.

a) Calculate the magnetizing current and the component corresponding to core loss at normal frequency and voltage
b) Calculate the magnetizing-branch impedances
c) Find the percentage voltage regulation when supplying full-load at *0.8 p.f.* leading

Solution 4.3

It must be recalled that from the **open-circuit** or **no-load test**, the iron or core losses P_{Fe} (equivalent to the measured power) are obtained, as well as the parameters of the shunt or magnetizing branch impedances, namely R_c, I_w, X_μ, I_μ.

From the **short-circuit test** where a **reduced voltage is applied**, the rated or full-load copper losses $P_{Cu,fl}$ are obtained. The rated copper losses is equivalent to the measured power under short-circuit conditions, ***if and only if***, the measured current was a rated or full-load current. If the measured current is not rated, then the measured power does not correspond to the full-load copper losses, from which the full-load copper losses must be calculated. Furthermore, the other parameters of the transformer can be deduced, namely, the equivalent resistance R_e and reactance X_e *referred to the side where the short-circuit test was conducted*. With the knowledge of these parameters, the voltage regulation can then be calculated at any power factor load.

Having deduced the copper and iron losses, the efficiency of the transformer at whatever loading situation can be calculated.

(a) The open-circuit power is $P_{oc} = V_{oc} I_o \cos\varphi_o$. The open-circuit power factor is

$$\cos\varphi_0 = \frac{P_{oc}}{I_0 V_{oc}} = \frac{120}{1.3 \times 200} = \underline{0.462}$$

$$\Rightarrow \sin\varphi_0 = \sqrt{\left(1 - \cos^2\varphi_0\right)} = \underline{0.886}$$

(i) The magnetizing current is given as

$$I_\mu = I_0 \sin\varphi_0 = 1.3 \times 0.886 = \underline{\underline{1.15\,A}}$$

(ii) The component of current corresponding to core loss is given as,

$$I_w = I_0 \cos\varphi_0 = 1.3 \times 0.462 = \underline{\underline{0.60\,A}}$$

b) *Calculation of Magnetizing branch impedances (Using No-Load Test Data):*

$$R_c = \frac{V_{oc}}{I_0 \cos\varphi_0} = \frac{V_{oc}}{I_w} = \frac{200}{0.6} = \underline{\underline{333\Omega}}$$

$$X_\mu = \frac{V_{oc}}{I_0 \sin\varphi_0} = \frac{V_{oc}}{I_\mu} = \frac{200}{1.15} = \underline{\underline{174\Omega}}$$

c) *Percentage voltage regulation at 0.8 leading p.f. (Using Short-Circuit Test Data):*

Total impedance referred to the HV (secondary) side where short-circuit test was performed is

$$Z_{e2} = \frac{V_{sc}}{I_{sc}} = \frac{22}{30} = \underline{\underline{0.733\Omega}}$$

Total resistance referred to the HV side is

$$R_{e2} = \frac{P_{sc}}{{I_{sc}}^2} = \frac{200}{30^2} = \underline{\underline{0.222\Omega}}$$

Therefore total reactance referred to the HV side is

$$X_{e2} = \sqrt{\left({Z_{e2}}^2 - {R_{e2}}^2\right)} = \sqrt{\left(0.733^2 - 0.222^2\right)} = \underline{\underline{0.698\Omega}}$$

Full-load current on HV side is

$$I_2 = \frac{S}{V_2} = \frac{10{,}000}{400} = \underline{25\,A}$$

The voltage regulation at *0.8* ***leading*** *p.f.* is calculated using the equivalent resistance and reactance parameters referred to the HV (secondary) side

$$VR = \frac{I_2}{E_2}\left[R_{e2}\cos\varphi - X_{e2}\sin\varphi\right]$$

$$= \frac{25}{400}\left[(0.222 \times 0.8) - (0.698 \times 0.6)\right]$$: negative sign for ***leading*** power factor load

$$= -0.015 = \underline{\underline{-1.5\,\%}}$$

Thus the actual terminal voltage is

$$V_2 = E_2(1 - VR) = 400 \times \left[1 - (-0.015)\right] = \underline{\underline{406V}}$$

$\Rightarrow$ *Voltage rise of 6V due to the leading power factor.*

4-4.2 Efficiency of Transformer

The ordinary transformer efficiency is given by

$$\begin{aligned}\eta &= \frac{\text{output power}}{\text{input power}} \\ &= \frac{\text{output power}}{\text{output power} + \text{losses}} \\ &= \frac{\text{input power - losses}}{\text{input power}} \\ &= 1 - \frac{\text{losses}}{\text{input power}} \\ &= 1 - \frac{\text{losses}}{\text{output power} + \text{losses}}\end{aligned} \tag{4.42}$$

The losses in the transformer may be categorized into *three* kinds, namely:

1. The *core or iron losses*. Since the mutual flux in a constant voltage transformer remains constant, this loss remains constant. In practice, however, it may vary by 1 to 2% between the no-load and full-load. We will denote it by P_{Fe}
2. The *ohmic copper losses* due to the actual ohmic resistance of the windings. It is denoted by P_{Cu} .

   $$P_{cu} = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{e1} = I_2^2 R_{e2} \tag{4.43}$$

3. The *additional load losses*. It will be remembered that the transformer sets up alternating leakage fluxes when on load. These fluxes in turn set up induced eddy-currents in any metal within their influence, e.g., tank, steel structure, or even in the copper windings themselves. The losses due to these eddy-currents only occur on load, hence the name *additional load losses*. *It is sometimes convenient to regard this loss as part of the actual ohmic copper loss*.

$$Output\ power = V_2 I_2 \cos\varphi_2 = S_n \times p.f. \tag{4.44}$$

$$\begin{aligned}Losses &= Iron\ losses(fixed) + Copper\ losses(\text{var}\,iable) \\ &= P_{Fe} + P_{Cu} \\ &= P_{Fe} + I_2^2 R_{e2}\end{aligned} \tag{4.45}$$

Therefore the efficiency can be deduced from Eqn (4.42) as:

$$\begin{aligned}\eta &= \frac{V_2 I_2 \cos\varphi_2}{V_2 I_2 \cos\varphi_2 + P_{Fe} + I_2^{\,2} R_{e2}} \\ &= \frac{S_n \cdot p.f}{S_n \cdot p.f + P_{Fe} + P_{Cu,fl}}\end{aligned} \tag{4.46}$$

This is the ***full-load (or rated) efficiency*** of the transformer.

Example 4.4

The following results were obtained on a *50 kVA* transformer:

O.C test: primary voltage *3300 V*, secondary voltage, *415 V*, primary power, *430W*
S.C test: primary voltage *124 V*, primary current *15.3 A*, primary power *525W*, secondary current, *rated value*.
Calculate:

(a) the efficiency at full-load for *0.7 lagging p.f*.
(b) the voltage regulations for lagging and leading p.f.s of *0.7*
(c) the secondary terminal voltages corresponding to situation (b).

Solution 4.4

(a) The normal full-load efficiency load is given by

The copper losses are obtained from the S/C tests under *rated conditions*, whilst the iron losses are obtained from the no-load or O/C test.

$$\begin{aligned}\eta &= 1 - \frac{\text{losses}}{\text{input power}} = 1 - \frac{\text{losses}}{\text{output power} + \text{losses}} = 1 - \frac{430 + 525}{(50{,}000 \times 0.7) + 430 + 525} \\ &= 0.9734 = \underline{\underline{97.34\%}}\end{aligned}$$

Alternatively

$$\eta = \frac{S_n \times pf}{S_n \times pf + P_{Fe} + P_{Cu}} = \frac{50{,}000 \times 0.7}{(50{,}000 \times 0.7) + 430 + 525} = \underline{\underline{97.34\%}}$$

(b) Since rated currents were measured in the S/C test, the voltage regulation is given by the equation

$$V.R. = \frac{V_{sc}}{V_1} \cos\left(\varphi_{sc} - \varphi\right)$$

But $\cos\phi_{sc}. = \dfrac{P_{sc}}{V_1 I_{sc}} = \dfrac{525}{124\times15.3} = 0.2765 \Rightarrow \phi_{sc} = \cos^{-1} 0.2765 = \underline{74^0}$

Again, $\cos\phi_2. = 0.7 \Rightarrow \phi_2 = \cos^{-1} 0.7 = \underline{45.5^0}$

Thus for lagging 0.7 power factor, $V.R. = \dfrac{V_{sc}}{V_1}\cos\left(\varphi_{sc} - \varphi\right) = \dfrac{124}{3300}\cos\left(74^0 - 45.5^0\right) = \underline{\underline{0.033}}$

And for leading 0.7 power factor, $V.R. = \dfrac{V_{sc}}{V_1}\cos\left(\varphi_{sc} + \varphi\right) = \dfrac{124}{3300}\cos\left(74^0 + 45.5^0\right) = \underline{\underline{-0.0185}}$

(c) The secondary voltage is given by the equation

$$V_2 = E_2(1 - VR)$$

For lagging load, $V_2 = E_2(1 - VR) = 415\times\left[1 - (0.033)\right] = \underline{\underline{401.3V}}$

For leading load, $V_2 = E_2(1 - VR) = 415\times\left[1 - (-0.0185)\right] = \underline{\underline{422.7V}}$

4-4.3 Cooling of Transformer

No-load losses and load losses are the two significant sources of heating considered in thermal modelling of power transformers. Cooling of transformers is done to prevent rapid deterioration of the insulating materials.

The basic method for cooling transformers is transferring heat from the core and windings to the insulating coolant such as oil. The *wasted energy* in the form of heat generated in the transformers due to the foregoing iron and copper losses *must be carried away* to *prevent excessive temperature rise* and *injury to the insulation around the conductors.* Overheating of the transformer core can thus lead to damage. Overheating of the windings and its attendant temperature rise lead to *accelerated ageing* of the insulation and thus reduction in the life of the transformer

4-4.3.1 Cooling Medium and Method

The cooling medium *air or gas* is used in a *dry-type* transformer, whilst *mineral oil* is used as cooling agent in *oil-immersed* transformers. The coolant obviously serves a double purpose:

- transporting the heat from the winding to the places of dissipation, either to ambient air or surface of a tank, and
- insulating the primary from the secondary.

Small transformers rated at less than 5 kVA are generally air-cooled, that is, the heat produced is carried away by the surrounding natural air. Small- and medium-size power or distribution transformers are generally cooled by housing them in especially designed tanks filled with oil or synthetic non-flammable liquids.

The oil and synthetic liquids are good insulators, unless carbonized or contaminated with moisture.

For larger transformers, oil cooling is preferred, especially where high voltages are in use. Oil seems to have the following advantages over air as a cooling medium:

1. Oil has a *larger specific heat capacity* than air, so that it absorbs larger quantities of heat for the same temperature rise.
2. Oil has a *greater heat conductivity* than air, and thus enables the heat to be transferred to the oil much more quickly.
3. Oil has *higher breakdown strength* than air (about six times), thus offering enhanced reliability at high voltages.

4-4.3.2 Cooling Arrangements and Designations

Both the IEEE and the IEC established standard designations for the various cooling modes of transformers. The IEEE has adopted the IEC designations. The designation completely describes the cooling method for the transformer, and the cooling method impacts the response of the transformer insulating oil to overload conditions.

The Table below describes the various cooling designations for dry-type and oil-immersed transformers.

Table 1: Various Cooling Designations for Dry-type and Oil-immersed Transformer

<table>
<tr><th colspan="3">DRY-TYPE TRANSFORMERS</th></tr>
<tr><td colspan="3">Natural Cooling: type AN
Forced Cooling: type AF</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><th colspan="3">OIL-IMMERSED TRANSFORMERS</th></tr>
<tr><th>Oil Circulation</th><th>Cooling Method</th><th>IEC Abbreviation</th></tr>
<tr><td rowspan="2">Natural Thermal Head Only</td><td>Air natural</td><td>ONAN</td></tr>
<tr><td>Air blast</td><td>ONAF</td></tr>
<tr><td rowspan="3">Forced Oil Circulation by Pumps</td><td>Air natural</td><td>OFAN</td></tr>
<tr><td>Air blast</td><td>OFAF</td></tr>
<tr><td>Water forced</td><td>OFWF</td></tr>
</table>

Oil-immersed transformers are built with **mixed** natural cooling and forced cooling. The abbreviations for such mixed cooling arrangements are ONAN/ONAF, ONAN/OFAN, ONAN/OFAF, ONAN/OFWF, etc, where “O” is used to indicate that the insulating liquid is mineral oil to BS 148. Where a synthetic insulant is employed, “L” is substituted.

EXERCISES ON TRANSFORMERS

Exercise 4.1

A 10 kVA 1-phase transformer has a voltage ratio 1100/250 V. On no-load and at normal voltage (1100 V) and frequency, the input current is 0.75 A at a power factor of 0.2 lagging. With the secondary shorted, the full-load currents flow when the primary applied voltage is 77 V, the power input being 240 W. Calculate:

a) the transformer equivalent resistances and reactances referred to the secondary side
b) the voltage regulation for a load power factor of 0.8 lagging and leading
c) the corresponding secondary terminal voltages

Exercise 4.2

The following results were obtained on a *50 kVA* transformer:

O.C test: primary voltage *3300 V*, secondary voltage, *415 V*, primary power, *430W*
S.C test: primary voltage *124 V*, primary current *15.3 A*, primary power *525W*, secondary current, rated value.
Calculate

(i) the efficiency at full-load for *0.7 lagging p.f*.
(ii) the voltage regulations for lagging and leading p.f.s of *0.7*
(iii) the secondary terminal voltages corresponding to situation (b).

Exercise 4.3

The following results were obtained on a *5 kVA* transformer:

O.C test: The voltage was raised to normal rated voltage, and the meter readings were 200 V, 24 W and 1.2 A
S.C test: The voltage was raised until the transformer full-load current was flowing. The meter readings were 6.4 V, 28 W, 15 A.

From the results, obtain the following

a) no-load current and its power factor
b) iron losses of the transformer at normal frequency and voltage
c) full-load copper losses
d) transformer resistances R_m and R_{e1}, and reactances X_m and X_{e1}
e) efficiency of the transformer at full-load at a power factor of 0.8