Chapter 3
Nodal and Mesh (Loop) Analyses

1. Introduction

The nodal and mesh analyses are two of the methods used to solve circuits containing multiple nodes and loops. These analyses are based on Kirchoff's Current Law (KCL) and Kirchoff's Voltage Law (KVL).

2. Nodal Analysis

In this analysis, general network equations are obtained by applying KCL at each of the (N – 1) nodes of a network of N nodes. Once the node voltages are known, any branch current can immediately be calculated.

<u>Definition of terms used</u>

(a) A branch: It is a portion of a circuit containing a single element (Not this definition: <u>A branch of a circuit is any portion of the circuit that has one or more elements in series</u>).
(b) A node: It is a point of connection of two or more circuit elements or branches (Not this definition: <u>a node of a circuit of any point of connection of three or more branches</u>). Note that all the connecting wire in unbroken contact with the point is part of the node.

<u>General approach to nodal analysis</u>

The nodal analysis is applied as follows:
(a) Determine the number of nodes within the network, N.
(b) Select a reference node and label the others as V_1, V_2, V_3,.....V_{N-1}
- A reference node is often chosen to be the node to which the largest number of branches is connected.
- In a practical electronic circuit, this usually corresponds to the chassis or ground line.
- In many cases such as in electric power systems, the chassis is shorted to the earth itself making its potential zero. For this reason, the reference node is frequently referred to as ground or the ground node.

(c) Apply KCL at each node except the reference, assuming that all unknown currents leave the node for each application of KCL.
(d) Solve the resulting equations of the nodal or node voltages.

Circuits containing only independent current sources

(a) General Approach

Example 1: Apply nodal analysis to the network of Fig. 1.

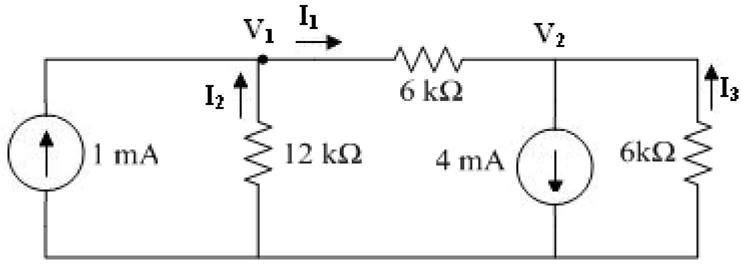


Fig. 1 Network for Example 1

Solution:

Apply KCL at node 1:

$$1 = \frac{V_1}{12} + \frac{V_1 - V_2}{6} \text{ or } 1 = V_1\left[\frac{1}{12} + \frac{1}{6}\right] - \frac{V_2}{6} \text{ or } 1 = \frac{V_1}{4} - \frac{V_2}{6} \quad (1)$$

Apply KCL at node 2:

$$-4 = \frac{V_2}{6} + \frac{V_2 - V_1}{6} \text{ or } -4 = -\frac{V_1}{6} + V_2\left[\frac{1}{6} + \frac{1}{6}\right] \text{ or } -4 = -\frac{V_1}{6} + \frac{V_2}{3} \quad (2)$$

$$(1) \times 2 + (2): -2 = \frac{V_1}{3} \Rightarrow V_1 = -6\text{ V}$$

$$\text{From (1) } \frac{V_2}{6} = \frac{V_1}{4} - 1 = -\frac{6}{4} - 1 = -\frac{5}{2} \Rightarrow V_2 = -\frac{5}{2} \times 6 = -15\text{ V}$$

$$\text{Current } I_2 = \frac{0 - V_1}{12} = \frac{0-(-6)}{12} = \frac{6}{12} = 0.5\text{ mA}$$

$$I_1 = 1 + I_2 = 1 + 0.5 = 1.5\text{ mA or use } \frac{V_1 - V_2}{6} = \frac{9}{6} = 1.5\text{ mA}$$

$$I_3 = 4 - I_1 = 4 - 1.5 = 2.5\text{ mA or use } \frac{0 - V_2}{6} = \frac{15}{6} = \frac{5}{2} = 2.5\text{ mA}$$

Example 2: Write the nodal equations for the network of Fig. 2.

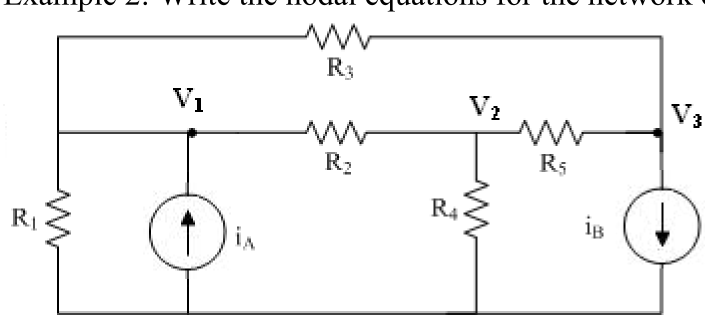


Fig. 2 Network for Example 2

Solution:

$$i_A = \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + \frac{v_1 - v_3}{R_3} = \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] v_1 - \frac{1}{R_2}\, v_2 - \frac{1}{R_3} v_3$$

$$0 = \frac{v_2}{R_4} + \frac{v_2 - v_1}{R_2} + \frac{v_2 - v_3}{R_5} = -\frac{1}{R_2} v_1 + \left[\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right] v_2 - \frac{1}{R_5} v_3$$

$$-i_B = \frac{v_3 - v_2}{R_5} + \frac{v_3 - v_1}{R_3} = -\frac{1}{R_3} v_1 - \frac{1}{R_5}\, v_2 + \left[\frac{1}{R_3} + \frac{1}{R_5}\right] v_3$$

In matrix form, the equations are as follows:

$$\begin{bmatrix} i_A \\ 0 \\ -i_B \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & -\frac{1}{R_3} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5} & -\frac{1}{R_5} \\ -\frac{1}{R_3} & -\frac{1}{R_5} & \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

The equation is in the form

$$[i] = [G][v]$$

The matrix G is called conductance or admittance matrix

(b) Format approach

The conductance or admittance matrix for networks containing only independent current sources is always symmetrical. This matrix equation can be obtained directly by inspection of the circuit diagram as follows:

- The kth row of the current vector on the left side of the equation consists of the net current flowing into node k due to current sources.
- The conductance matrix is obtained as

$$G = \begin{bmatrix} \sum_1 G_{11} & -G_{12} & -G_{13} \\ -G_{21} & \sum_2 G_{22} & -G_{23} \\ -G_{13} & -G_{23} & \sum_3 G_{33} \end{bmatrix}$$

Where

$\sum_n G_{nn}$ is the sum of the conductances at node n and

G_{ij} is the sum of the conductances connecting nodes *i* and *j*.

We note that conductance in series with current sources are ignored since they cannot affect the current.

Example 3: Write the nodal equations for the network in Fig. 3 by inspection.

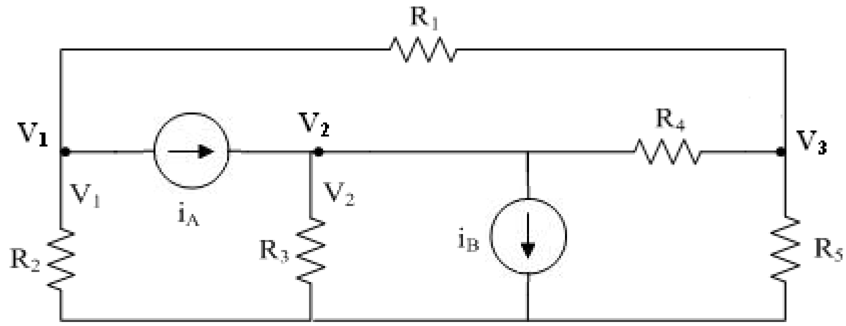
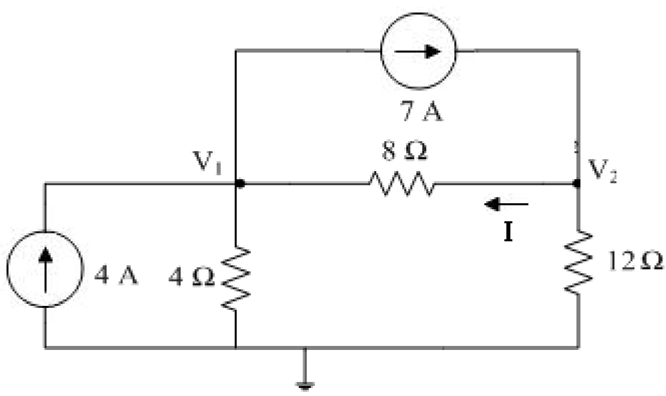


Fig. 3 See Example 3

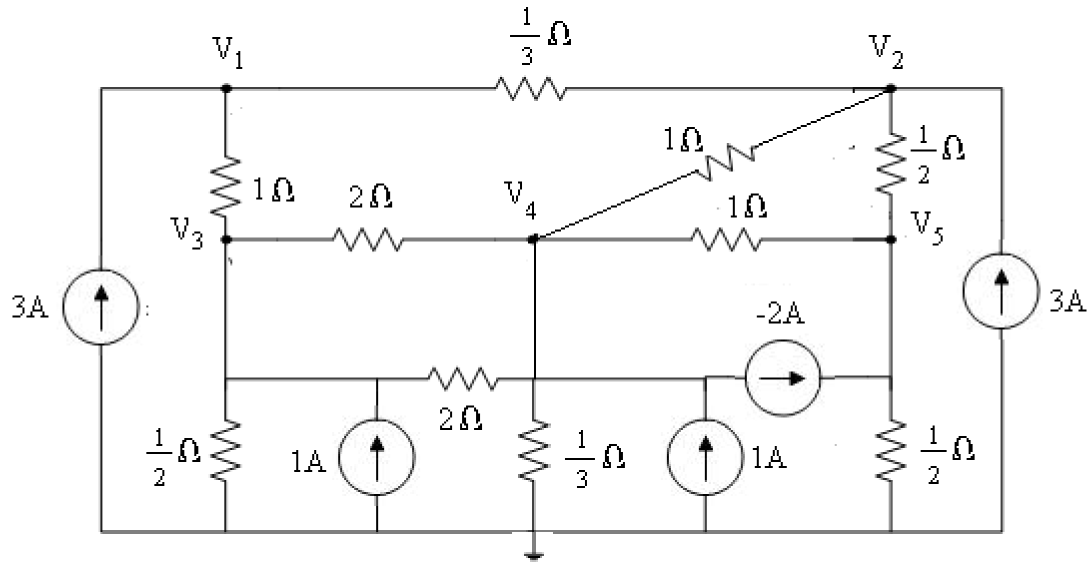
Solution:

$$\begin{bmatrix} -i_A \\ i_A - i_B \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & 0 & -\frac{1}{R_1} \\ 0 & \frac{1}{R_3}+\frac{1}{R_4} & -\frac{1}{R_4} \\ -\frac{1}{R_1} & -\frac{1}{R_4} & \frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Exercise 1: Using the nodal analysis, find V_1, V_2 and I. Use the general approach.
Answers: 4 V; 36 V; 4 A



Exercise 2: Write the nodal equations directly in vector- matrix form. Use the format approach or by inspection.



Answer:

$$\begin{bmatrix}3\\3\\1\\3\\2\end{bmatrix}=\begin{bmatrix}4&-3&-1&0&0\\-3&6&0&-1&-2\\-1&0&4&-1&0\\0&-1&-1&6&-1\\0&-2&0&-1&5\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\\v_4\\v_5\end{bmatrix}$$

<u>Circuits containing dependent current sources</u>

The presence of a dependent current source may destroy the symmetry of the nodal equations.

Example 4: Obtain the nodal equations for the network given in Fig. 4.

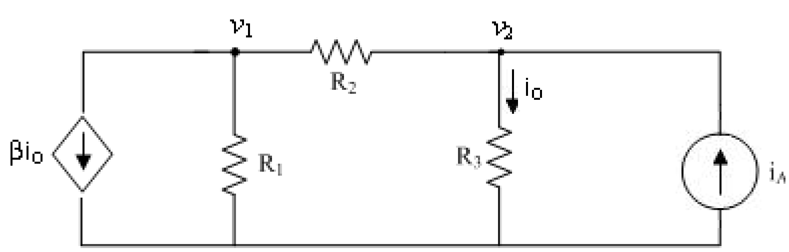


Fig. 4 Network for Example 4

Solution

Apply KCL at node 1:

$$-\beta i_o=\frac{v_1}{R_1}+\frac{v_1-v_2}{R_2}\ \text{or}-\beta\frac{v_2}{R_3}=\frac{v_1}{R_1}+\frac{v_1-v_2}{R_2}$$

Or

$$0 = \left[\frac{1}{R_1} + \frac{1}{R_2}\right] v_1 - \left[\frac{1}{R_2} - \frac{\beta}{R_3}\right] v_2$$

Apply KCL at node 2:

$$i_A = \frac{v_2}{R_3} + \frac{v_2 - v_1}{R_2} = -\frac{1}{R_2} v_1 + \left[\frac{1}{R_2} + \frac{1}{R_3}\right] v_2$$

In matrix form, we have:

$$\begin{bmatrix} 0 \\ i_A \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{R_2}\right) & -\left(\frac{1}{R_2} - \frac{\beta}{R_3}\right) \\ -\frac{1}{R_2} & \left(\frac{1}{R_2} + \frac{1}{R_3}\right) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Exercise 3: Determine the node voltages for the network in Fig. 4 given the following parameters: $\beta = 2, R_2 = 6\ \text{k}\Omega, i_A = 2\ \text{mA}, R_1 = 12\ \text{k}\Omega$ and $R_3 = 3\ \text{k}\Omega$

Answers: $v_1 = -\frac{24}{5}$ V and $v_2 = \frac{12}{5}$ V

Example 5: Obtain the nodal equations for the network in Fig. 5

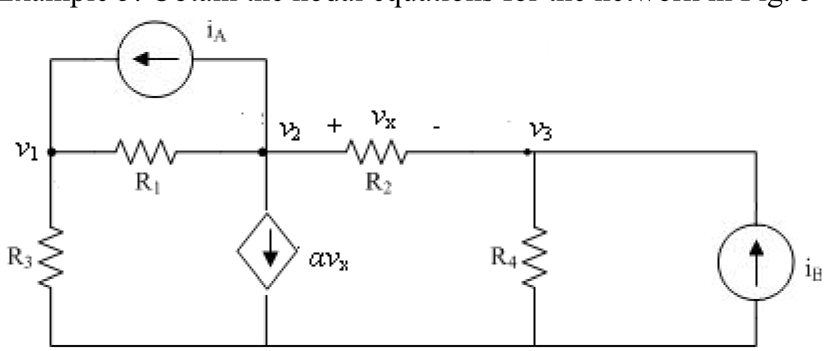


Fig. 5 Network for Example 5

Solution:

Apply KCL at node 1:

$i_A = G_3 v_1 + G_1(v_1 - v_2)$ or $i_A = (G_1 + G_3)v_1 - G_1 v_2$

Apply KCL at node 2:

$$-i_A - \alpha v_x = G_1(v_2 - v_1) + G_2(v_2 - v_3)$$

$$-i_A = G_1(v_2 - v_1) + G_2(v_2 - v_3) + \alpha(v_2 - v_3)$$

$$-i_A = -G_1 v_1 + (G_1 + G_2 + \alpha)v_2 - (G_2 + \alpha)v_3$$

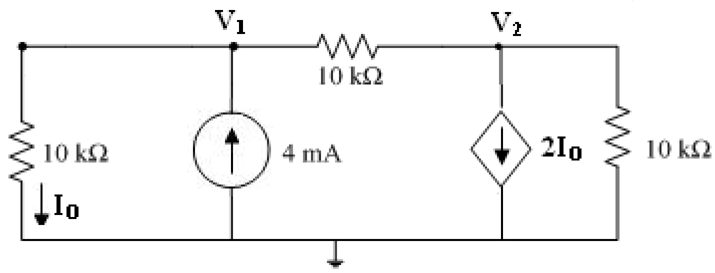
Apply KCL at node 3

$$i_B = G_2(v_3 - v_2) + G_4 v_3 = -G_2 v_2 + (G_2 + G_4)v_3$$

In matrix form, we have:

$$\begin{bmatrix} i_A \\ -i_A \\ i_B \end{bmatrix} = \begin{bmatrix} G_1 + G_3 & -G_1 & 0 \\ -G_1 & G_1 + G_2 + \alpha & G_2 + \alpha \\ 0 & -G_2 & G_2 + G_4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Exercise 4: Find the node voltages in the circuit below:



Answers: $V_1 = 16$ V, $V_2 = -8$ V

<u>Circuits containing independent voltage sources</u>

The presence of voltage sources reduces the number of equations and the number of unknowns by one per voltage source.

(a) Using the concept of supernode

- Voltage sources are enclosed in separate regions. Each closed region is called a supernode.
- A supernode contains two nodes which may consist of a non-reference node and a reference node or two non–reference nodes.
- To obtain the nodal equations, we apply KCL to all supernodes not containing the reference node and to all other reference nodes.
- KCL is applied to a supernode using the generalized form of KCL which states that algebraic sum of currents entering a closed region is zero.

Example 6: Obtain the nodal equations for the network in Fig. 6.

Solution:
Two supernodes A and B are shown in Fig, 6. Voltage V_1 at node 1 in supernode A (containing the reference node) may be determined immediately as

$$V_1 = 5 + 0 = 5 \text{ V}$$

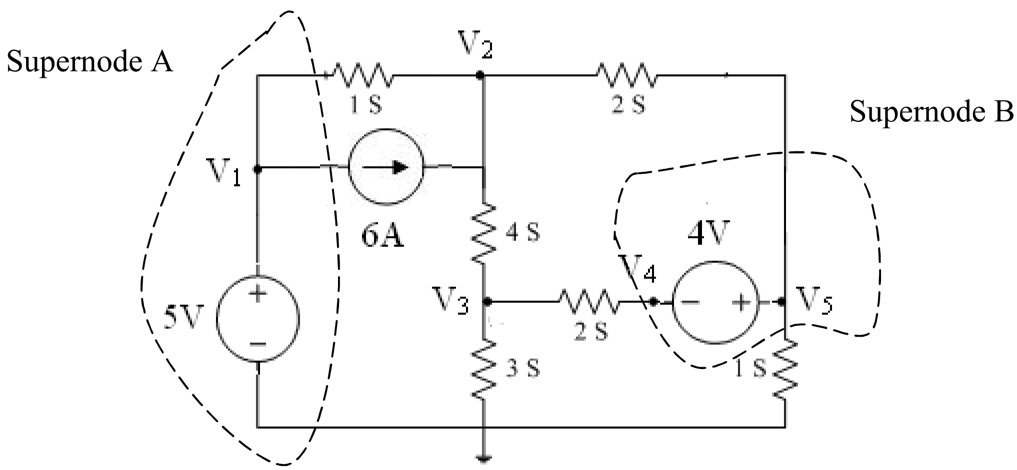


Fig. 6 Network for Example 6

Voltage V_5 at node 5 in supernode B can be expressed in terms of V_4 at the other non-reference node 4 in the supernode as

$$V_5 = V_4 + 4$$

The number of nodal equations required = 5 – 2 = 3

Apply KCL to non-reference node 2:

$$6 = 1(V_2 - 5) + 4(V_2 - V_3) + 2(V_2 - V_4 - 4)$$

$$19 = 7V_2 - 4V_3 - 2V_4$$

Apply KCL to non–reference node 3:

$$0 = 3V_3 + 4(V_3 - V_2) + 2(V_3 - V_4)$$

$$0 = -4V_2 + 9V_3 - 2V_4$$

Apply KCL to supernode B:

$$0 = 2(V_4 - V_3) + 1(V_4 + 4) + 2[(V_4 + 4) - V_2]$$

$$-12 = -2V_2 - 2V_3 + 5V_4$$

In matrix form, we have:

$$\begin{bmatrix} 19 \\ 0 \\ -12 \end{bmatrix} = \begin{bmatrix} 7 & -4 & -2 \\ -4 & 9 & -2 \\ -2 & -2 & 5 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Example 7: Find V and I in the circuit of Fig. 7 using nodal analysis

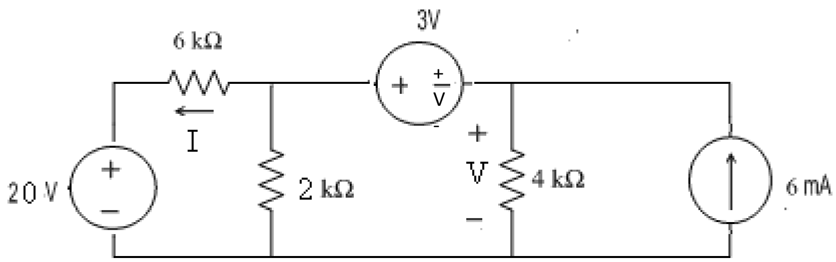
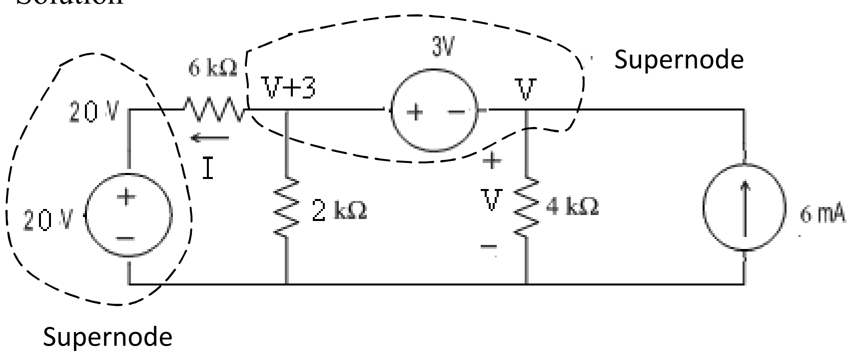


Fig. 7 Network for Example 7

Solution



Network has two supernodes and we must apply KCL to the supernode which does not contain reference node. Thus,

$$6 = \frac{V}{4} + \frac{V+3}{2} + \frac{[(V+3) - 20]}{6}$$

$$6 \times 12 = 3V + 6(V+3) + 2[(V+3) - 20]$$

$$72 = 11V - 16 \text{ or } 11V = 88 \text{ or } V = 8 \text{ volts}$$

The current

$$I = \frac{[(V+3) - 20]}{6} = \frac{[(8+3) - 20]}{6} = -\frac{9}{6} = 1.5 \text{ mA}$$

Example 8: Find the current I_0 in the network in Fig. 8

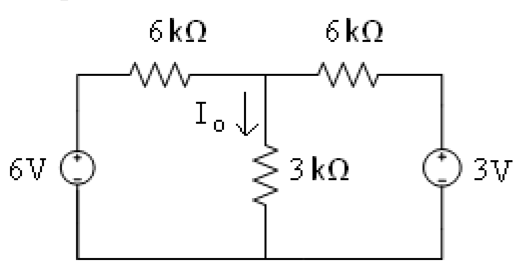
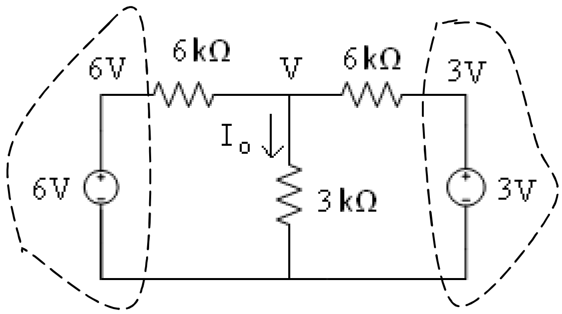


Fig. 8 Network for Example

Solution:



Circuit has two supernodes all containing reference node and one non-reference node. Applying KCL to the non– reference node, we obtain:

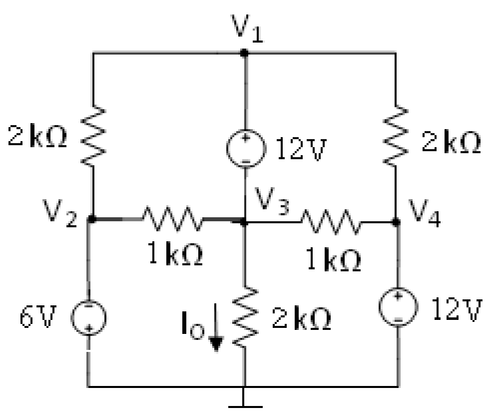
$$0 = \frac{V-6}{6} + \frac{V}{3} + \frac{V-3}{6}$$

$$0 = (V-6) + 2V + V - 3$$

$$0 = 4V - 9 \Rightarrow V = \frac{9}{4}$$

The current $I_0 = \frac{V}{3} = \frac{9}{4 \times 3} = \frac{3}{4} = 0.75 \text{ mA}$

Exercise 5: Determine the current I_0. (Answer: $I_0 = -\frac{3}{7}$ mA)



(b) Circuits containing independent voltage sources having resistances in series

Here, voltage sources can be converted to current sources.

Example 9: Find the voltage across the 3-Ω resistor of Fig. 9 by nodal analysis.

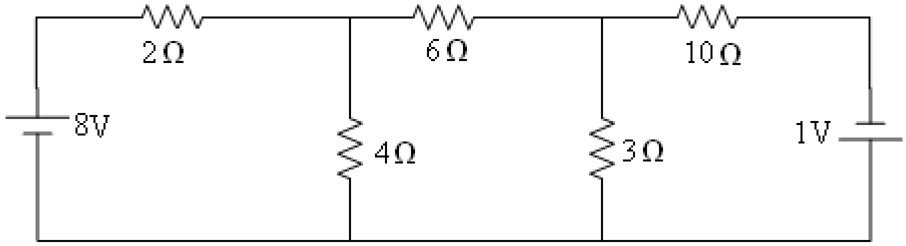
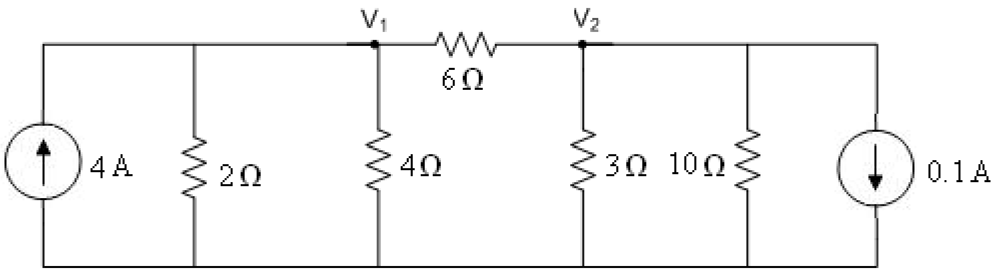


Fig. 9 Network for Example 9

Solution: Converting sources and choosing nodes, we obtain



By inspection, we have

$$4 = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6}\right)V_1 - \frac{1}{6}V_2$$

$$-0.1 = -\frac{1}{6}V_1 + \left(\frac{1}{6} + \frac{1}{3} + \frac{1}{10}\right)V_2$$

or

$$4 = \frac{11}{12}V_1 - \frac{1}{6}V_2$$

$$-0.1 = -\frac{1}{6}V_1 + \frac{3}{5}V_2$$

Solving the two equations simultaneously, we obtain voltage across the 3-Ω resistor $= V_2 = 1.101$ V

Exercise 6: Repeat Example 8 converting the voltage sources to current sources. (Ans: 0.75 mA)

Circuits containing dependent voltage sources

They are treated in the same manner as circuits containing independent voltage sources. When writing the circuit equations, first treat the dependent source as though it were an independent source and then write the controlling equations and substitute.

Example 11: Find the current I_0 in the network in Fig. 11.

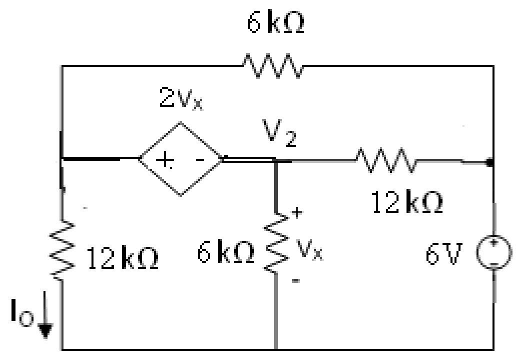
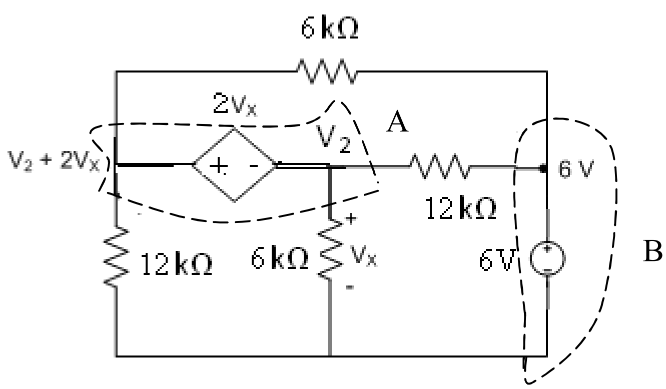


Fig. 11 Network for Example 11

Solution:



Apply KCL to the supernode A:

$$0 = \frac{V_2 + 2V_x}{12} + \frac{(V_2 + 2V_x) - 6}{6} + \frac{V_2}{6} + \frac{V_2 - 6}{12}$$

$$0 = \left(\frac{1}{12} + \frac{1}{6} + \frac{1}{6} + \frac{1}{12}\right)V_2 + \left(\frac{2}{12} + \frac{2}{6}\right)V_x - 1 - \frac{6}{12}$$

$$0 = \frac{3}{6}V_2 + \frac{3}{6}V_x - \frac{18}{12}$$

Controlling equation:

$$V_x = V_2$$

Substituting into the nodal equation, we obtain:

$$0 = \frac{3}{6}V_2 + \frac{3}{6}V_2 - \frac{18}{12} \text{ or } \frac{18}{12} = \frac{6}{6}V_2 \text{ or } V_2 = \frac{3}{2} \text{ V}$$

$$\text{Current } I_0 = \frac{V_2 + 2V_x}{12} = \frac{3V_2}{12} = \frac{3}{12} \times \frac{3}{2} = \frac{3}{8} \text{ mA}$$

Example 12: Find V_0 in the network in Fig. 12 using nodal analysis

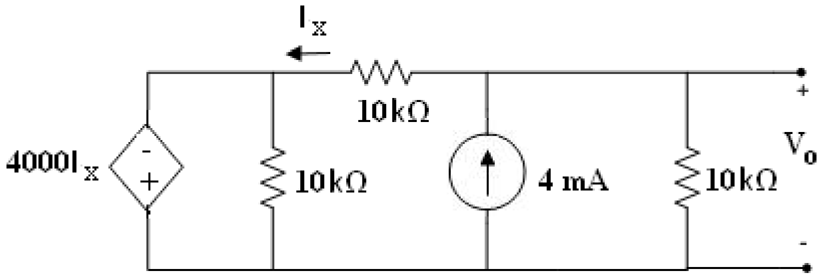
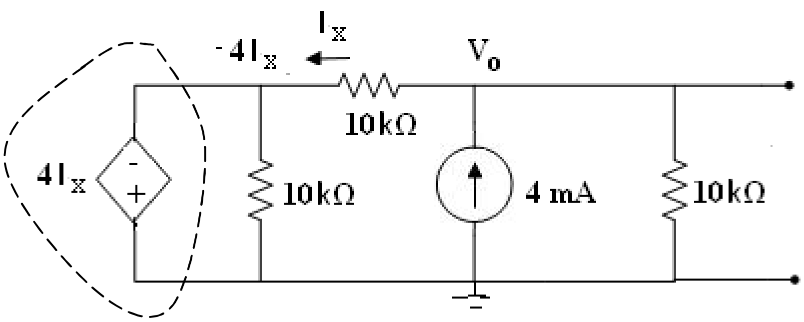


Fig. 12 Network for Example 12

Solution: With currents in mA, the CCVS becomes $4I_x$.



Applying KCL to the non-reference node V_0, we obtain

$$4 = \frac{V_0}{10} + \frac{V_0 - (-4I_x)}{10} \text{ or } 4 = \frac{V_0}{5} + \frac{4I_x}{10}$$

$$4 = \frac{V_0}{5} + \frac{2I_x}{5} \text{ or } 20 = V_0 + 2I_x$$

$$V_0 = 20 - 2I_x$$

The controlling equation is given by

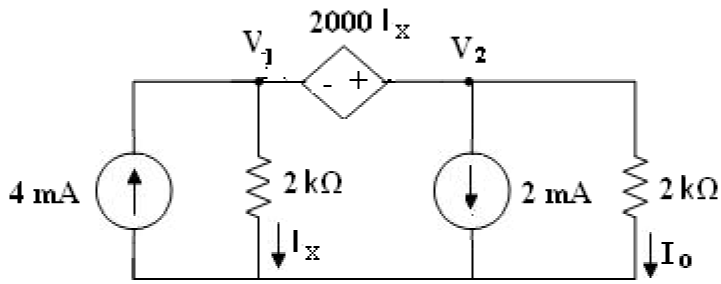
$$I_x = \frac{V_0 - (-4I_x)}{10} = \frac{V_0 + 4I_x}{10} \text{ or } 10I_x = V_0 + 4I_x$$

$$6I_x = V_0 \text{ or } 2I_x = \frac{V_0}{3}$$

Substituting it in the nodal equation, we obtain

$$V_0 = 20 - \frac{V_0}{3} \text{ or } V_0 \times \frac{4}{3} = 20 \text{ or } V_0 = 15 \text{ V}$$

Exercise 7: Find I_0 in the circuit of the figure below using nodal analysis. Answer: I_0 = 4/3 mA.



3. Mesh Analysis

Mesh analysis uses KVL to determine currents in the circuit. Once the currents are known, Ohm's law can be used to calculate voltages. N independent equations are required if the circuit contains N meshes. We assume that the circuits are planar. In the case of non–planar circuits, we cannot define meshes and mesh analysis cannot be performed. Mesh analysis is thus not as general as nodal analysis, which has no topological restrictions.

Definition of terms used

(a) A loop: It is a closed path through a circuit in which no node is encountered more than once.
(b) A mesh: A mesh is a special kind of loop that does not contain any loop within it. We note that as we traverse the path of a mesh, we do not encircle any circuit element.
(c) A planar circuit: It is a circuit that can be drawn on a plane surface with no crossovers, i.e., no element or connecting wire crosses another element or connecting wire. In planar circuits meshes appear as windows.
(d) A non–planar circuit: It is a circuit that is not planar.

General approach

The systematic approach outlined below should be followed:
(a) Place a loop current in the clockwise direction within each mesh or window of the network.
(b) Apply KVL around each mesh in the clockwise direction.
(c) Solve the resulting simultaneous equations for the loop currents.

Circuits containing only independent voltages

(a) General approach

Example 13: Find the current through each branch of the network of Fig. 13.

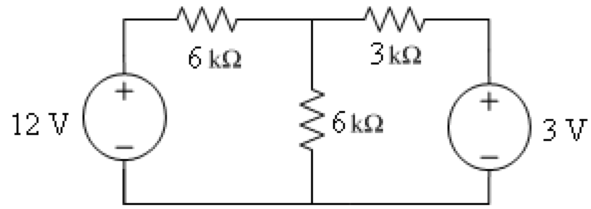
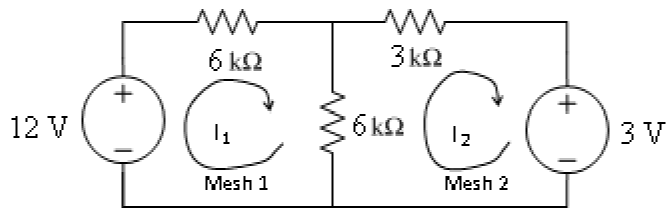


Fig 13 network for Example 13

Solution:



Apply KVL to mesh 1:

$12 = 6I_1 + 6(I_1 - I_2) = 12I_1 - 6I_2$

Or

$12 = 12I_1 - 6I_2$ (1)

Apply KVL to mesh 2:

$-3 = 3I_2 + 6(I_2 - I_1)$

Or

$-3 = -6I_1 + 9I_2$ (2)

(1) + 2 x (2) gives

$6 = 12I_2 \text{ or } I_2 = \dfrac{6}{12} = 0.5 \text{ mA}$

From (2)

$6I_1 = 9I_2 + 3 = 9 \times 0.5 + 3 \text{ or } I_1 = 7.5/6 = 1.25 \text{ mA}$

Current in the left outer branch = $I_1 = 1.25$ mA

Current in the middle branch = $I_1 - I_2 = 1.25 - 0.5 = 0.75$ mA

Current in the right outer branch = $I_2 = 0.5$ mA

Example 14: Write the mesh equations for the network of Fig. 14.

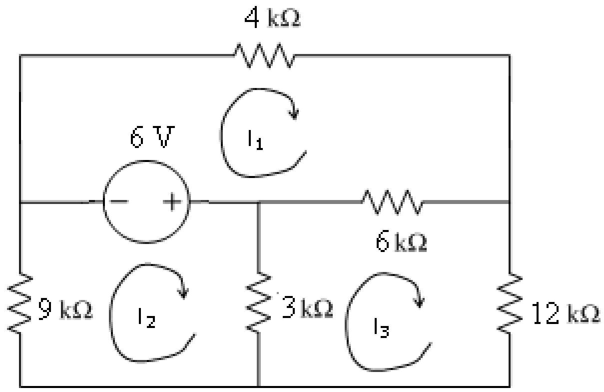


Fig 14 Network for Example 14

Solution:

Loop 1: $-6 = 4I_1 + 6(I_1 - I_3)$ or $-6 = 10I_1 - (0)I_2 - 6I_3$

Loop 2: $6 = 3(I_2 - I_3) + 9I_2$ or $6 = (0)I_1 - 12I_2 - 3I_3$

Loop 3: $0 = 6(I_3 - I_1) + 12I_3 + 3(I_3 - I_2)$ or $0 = -6I_1 - 3I_2 + 21I_3$

In the matrix form, we have:

$$\begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 & -6 \\ 0 & 12 & -3 \\ -6 & -3 & 21 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

(b) Mesh analysis by format approach

Equations can be obtained by inspection as follows:

- The left–hand side of the equation is the algebraic sum of the voltage sources in a corresponding mesh. Voltage is positive if it aids the assumed direction of the loop current.
- For a given mesh *i*, the coefficient of the mesh current I_i is the sum of the resistances through which the mesh current flows and the coefficient of mesh current I_j ($j \neq i$) is the negative of the sum of resistances common to mesh current *i* and mesh current *j* (or resistances common to the mesh *i* and mesh *j*)

Example 15: Obtain the mesh equations for the network of Fig. 15 by inspection.

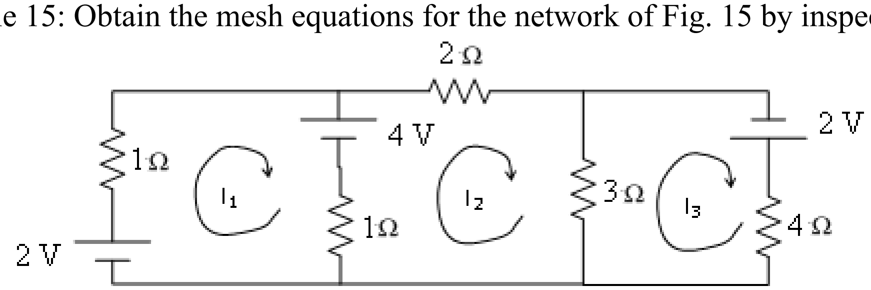


Fig 15 Network for Example 15

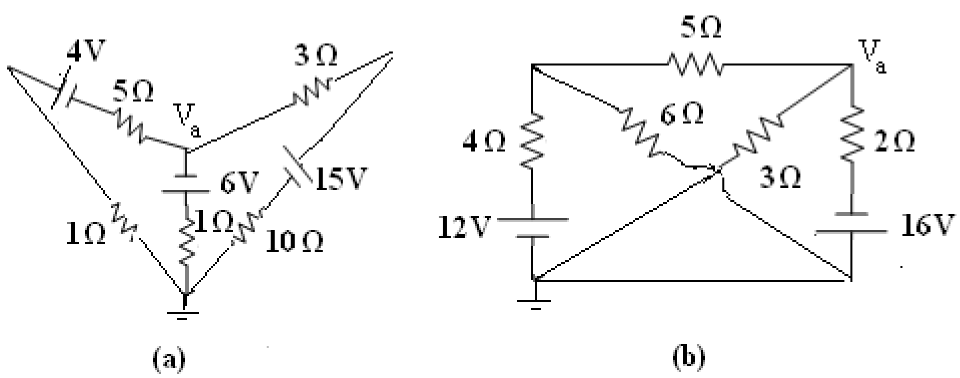
Solution:

$$\begin{bmatrix} -2 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 6 & -3 \\ 0 & -3 & 7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Note that matrix is symmetrical and

$Z_{11} = 1 + 1 = 2$ = sum of resistances through which I_1 flows
$Z_{22} = 1 + 2 + 3 = 6$ = sum of resistances through which I_2 flows
$Z_{33} = 3 + 4 = 7$ = sum of resistances through which I_3 flows
$Z_{12} = -1$ = negative of the sum of resistances common to mesh 1 and 2
$Z_{13} = 0$ = negative of the sum of resistances common to mesh 1 and mesh 3
$Z_{23} = -3$ = negative of the sum of resistances common to mesh 2 and mesh 3

Exercise 8: Using mesh analysis (format approach), determine the current through the 5-Ω resistor for each network given below. Then determine V_a. (Ans: (a) 72.16 mA, -4.433 V; (b) 1.953 A, -7.257 V)



Exercise 9: Repeat Exercise 7 using the general approach.

<u>Circuits containing current sources</u>

As in the case of nodal analysis with voltage sources, the presence of current sources reduces the number of unknowns in mesh analysis by one per current source.

(a) Current sources having resistances in parallel

In such cases, convert sources to voltage sources and proceed as before.

Example 16: Use mesh analysis to determine the currents for the network of Fig. 16.

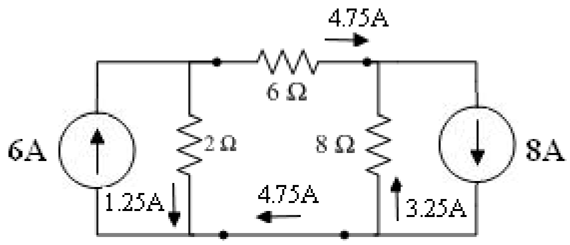
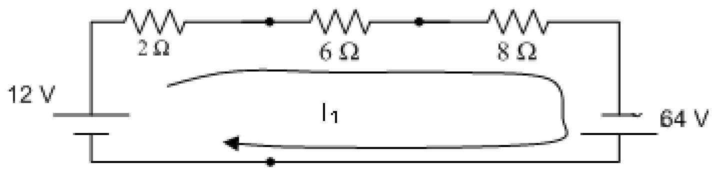


Fig 16 Network for Example 16

Solution

Convert current sources to voltage sources.



$$12 + 64 = (2 + 6 + 8)I_1 \text{ or } 76 = 16I_1 \text{ or } I_1 = \frac{76}{16} = 4.75\ A$$

The currents in the various branches of the circuit (See the original circuit) were then obtained as follows:

Current in 6-Ω resistor = $4.75A$

Current in 2-Ω resistor = $6 - 4.75 = 1.25\ A$

Current in 8-Ω resistor = $8 - 4.75 = 3.25\ A$

Example 17: For the network of Fig. 17, determine the current I_2 using mesh analysis and then find the voltage V_{ab}.

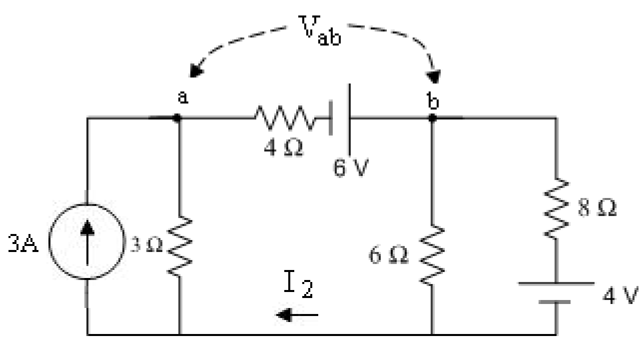
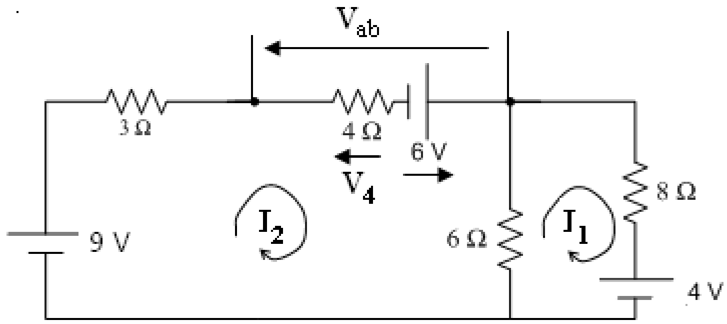


Fig 17 Network for Example 17

Solution:

Convert current source to voltage source:



By inspection:

$-4 = (8 + 6)I_1 - \ 6I_2$ or $-4 = 14I_1 - \ 6I_2$

$9 + 6 = \ -6I_1 + \ (3 + 4 + 6)I_2$ or $15 = \ -6I_1 + \ 13I_2$

Solving the two simultaneous equations, we obtain

$I_1 = 0.26$ A and $I_2 = 1.274$ A

Voltage $V_{ab} \ = \ V_4 - \ 6 = 4I_2 - \ 6 = \ \ 4 \times 1.274 - 6 = 0.904$ V

(b) Using the concept of supermesh

- Assign mesh currents as before, considering current sources as if they were resistors or voltage sources.
- Mentally remove the current sources (i.e. replace with open–circuit equivalents) and apply Kirchoff's voltage law to all the remaining closed paths of the network using the mesh currents just defined.
- Relate the chosen mesh currents of the network to the independent current sources and solve for the mesh currents.

Any resulting window (when the current sources are removed) which includes two or more mesh currents is said to be supermesh or the path of a supermesh current.

Example 18: Using mesh analysis, determine the currents of the network of Fig. 18.

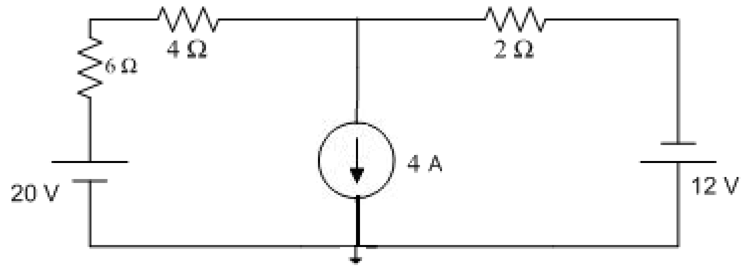
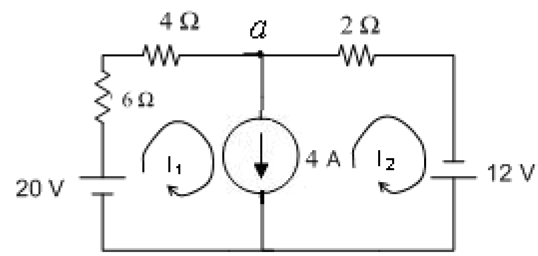
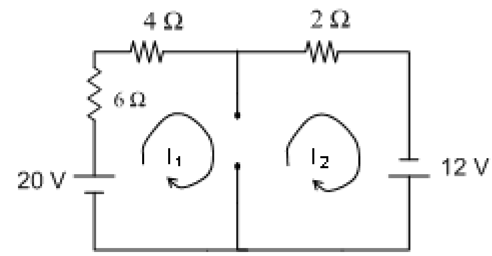


Fig 18 See Example 18

Solution:





Applying KVL to network when current source is removed (it is supposed to be removed mentally in order to save time). Note that the number of mesh equations required = 2 – 1 = 1.

$20 + 12 = (6 + 4)I_1 + 2I_2$ or $32 = 10I_1 + 2I_2$

Relate mesh currents to current source by applying KCL at node *a*.

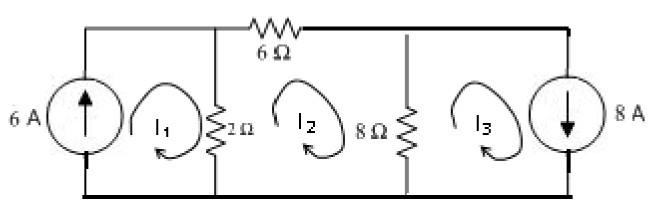
$I_1 = I_2 + 4$

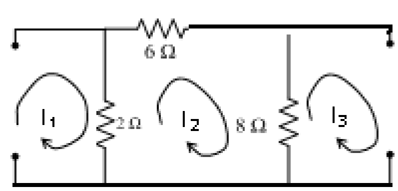
Solving the two equations simultaneously, we obtain

$I_1 = 3.33\ A$ and $I_2 = -0.67\ A$

Example 19: Repeat Example 16 using Supermesh approach

Solution





Number of mesh equations required = 3 – 2 = 1

Applying KVL to supermesh, we obtain:

$0 = 2(I_2 - I_1) + 6I_2 + 8(I_2 - I_3)$

Or

$$0 = -2I_1 + 16I_2 - 8I_3$$

$I_1 = 6\ A$ and $I_3 = 8\ A$

Therefore

$$16I_2 = 2I_1 + 8I_3 = 12 + 64 = 76$$

Or

$$I_2 = \frac{76}{16} = 4.75\ A$$

Example 20: Find the current I_0 in the network in Fig. 19.

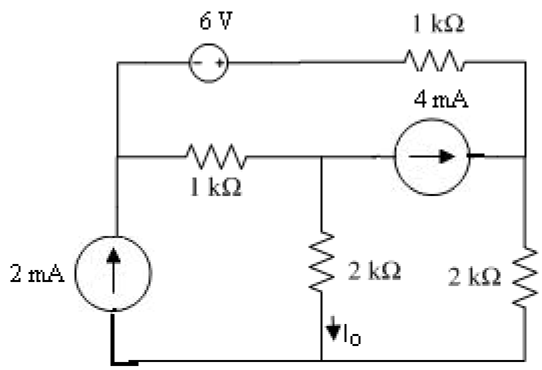


Fig. 19 Network for Example 20

Solution:

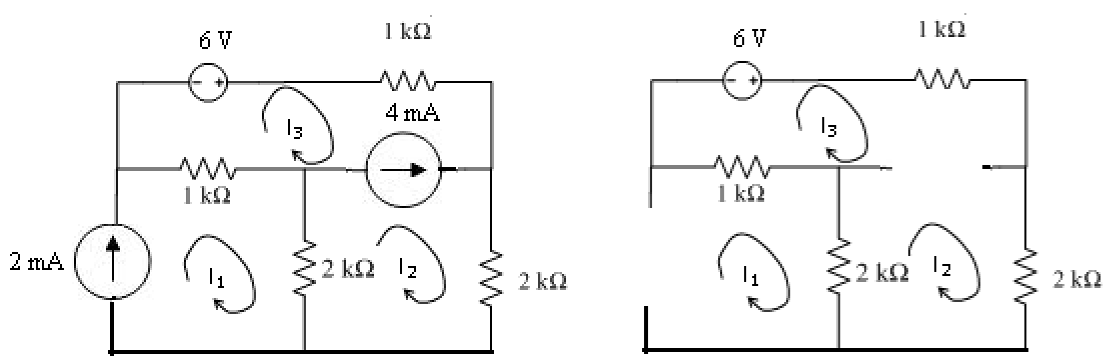
Number of mesh equations required = 3 – 2 = 1

Applying KVL to supermesh, we obtain

$$6 = I_3 + 2I_2 + 2(I_2 - I_1) + (I_3 - I_1)$$

or

$$6 = -3I_1 + 4I_2 + 2I_3$$



Relate mesh currents to independent current sources:

$I_1 = 2$ mA

$I_2 - I_3 = 4$ mA

Or

$I_2 = 4 + I_3$

Substituting the two equations in the mesh equation, we obtain

$6 = -3 \times 2 + 4(4 + I_3) + 2I_3$

Or

$6 = 10 + 6I_3$

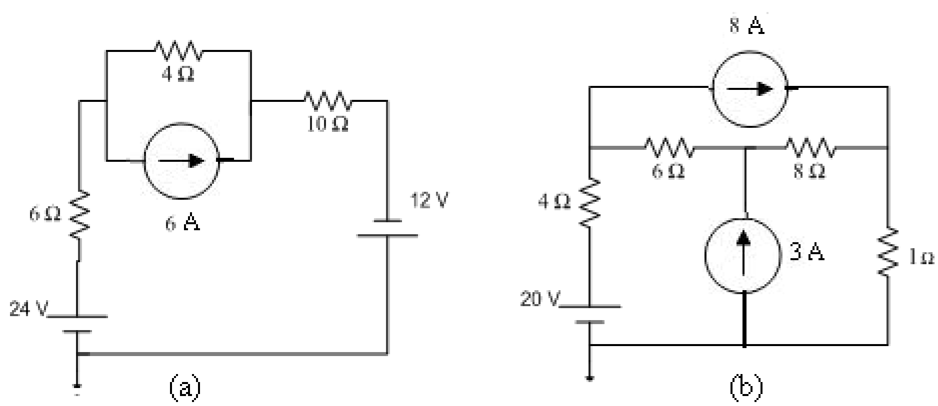
$$I_3 = -\frac{4}{6} = -\frac{2}{3} \text{ mA}$$

Hence

$$I_2 = 4 + I_3 = 4 - \frac{2}{3} = \frac{10}{3} \text{ mA}$$

$$I_0 = I_1 - I_2 = 2 - \frac{10}{3} = -\frac{4}{3} \text{ mA}$$

Exercise 10: Using the supermesh approach, find the current through each element of the networks given below.



Exercise 11: Repeat Exercise 9 (a) with current source converted to voltage source.

<u>Circuits containing dependent sources</u>

The dependent sources are treated as if they were independent sources and the circuits dealt with as we have in the previous cases. Additionally, we write the controlling equations for the dependent sources.

Example 21: Find V_0 in the circuit in Fig. 20

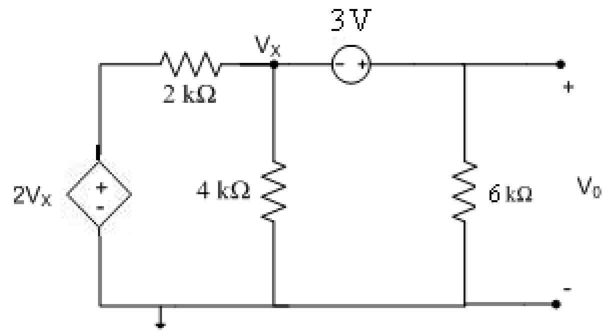
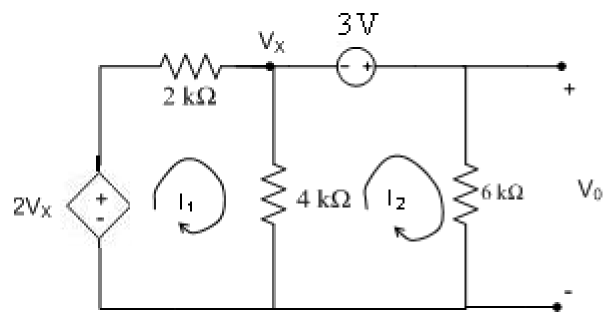


Fig. 20 Network for Example 21

Solution:



Mesh equations by inspection:

$2V_x = 6I_1 - 4I_2$

$3 = -4I_1 + 10I_2$

Controlling equation:

$V_x = 4(I_1 - I_2)$

Substituting this in the first equation, we obtain

$2 \times 4(I_1 - I_2) = 6I_1 - 4I_2$

Or

$0 = -2I_1 + 4I_2$

Multiplying this equation by two and subtracting it from the second equation, we obtain

$3 = 2I_2 \text{ or } I_2 = \frac{3}{2}$

Therefore

$V_0 = 6I_2 = 6 \times \frac{3}{2} = 9 \text{ V}$

Example 22: Find V_0 in the circuit in Fig. 21

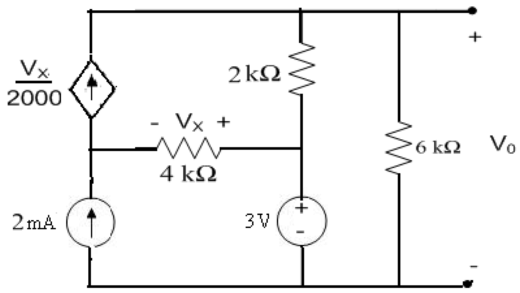
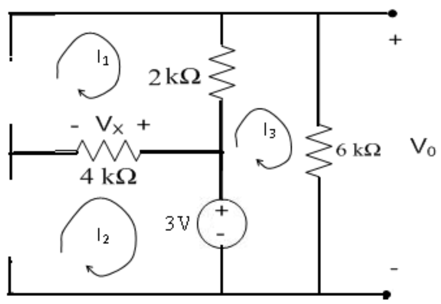


Fig. 21 Network for Example 22

Solution:



Number of mesh equations required = 3 – 2 = 1

Apply KVL to supermesh:

$3 = 2(I_3 - I_1) + 6I_3$ or $3 = 8I_3 - 2I_1$

Relate mesh currents to current sources:

$I_1 = \dfrac{v_x}{2000}\ \mathrm{A} = \dfrac{V_x}{2}\ \mathrm{mA}$ or $2I_1 = V_x$

$I_2 = 2\ \mathrm{mA}$

Controlling equation:

$V_x = 4(I_1 - I_2)$

From the second and fourth equations

$2I_1 = 4(I_1 - I_2)$ or $I_1 = 2(I_1 - I_2)$ or $I_1 = 2I_2$

and from the third equation

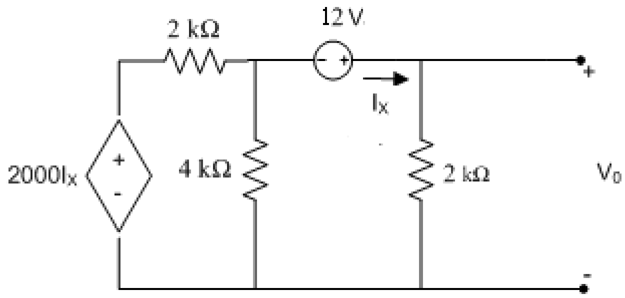
$I_1 = 2I_2 = 2 \times 2 = 4\ \mathrm{mA}$

From the first equation

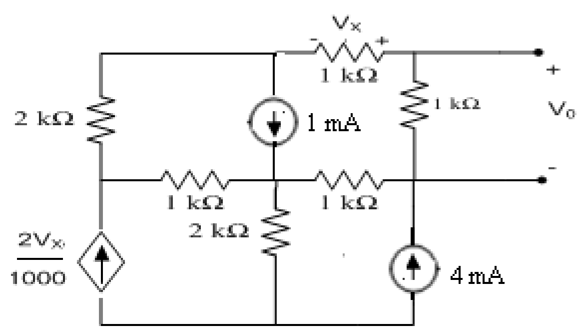
$$8I_3 = 3 + 2I_1 = 3 + 2 \times 4 = 11 \text{ or } I_3 = \frac{11}{8} \text{ mA}$$

$$V_0 = 6I_3 = 6 \times \frac{11}{8} = \frac{33}{4} \text{ V}$$

Exercise 12: Use mesh analysis to find V_0 in the circuit in the figure below. (Ans: $V_0 = 12$V)



Exercise 13: Use mesh analysis to find V_0 in the network given below.



4. **Circuit equations via network topology**

A study of a network topology or the manner in which the elements are interconnected proves to be an effective way of generalizing the methods of nodal and loop (mesh) analyses.

<u>Basic definitions</u>

(a) A network or circuit graph: It is a circuit or network diagram having all elements replaced by featureless lines.

Example 23: Find the graph of the circuit shown in Fig. 22

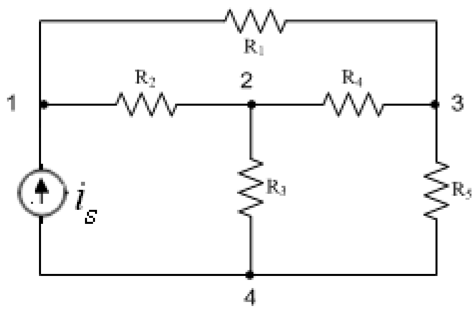
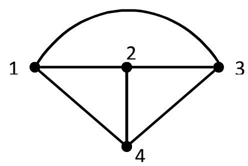


Fig. 22 Network for Example 23

Solution:

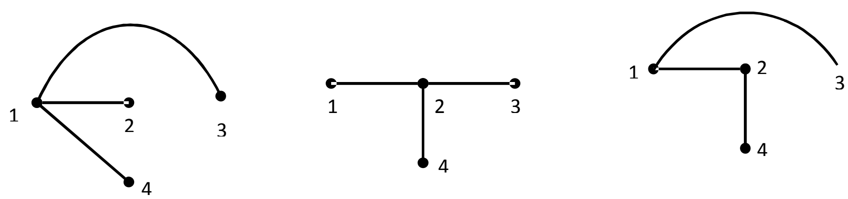


(b) A network graph is said to be connected if there is a path between any two nodes.

(c) A tree is a subgraph having every node connected to every other node via some path without forming a closed path. A given circuit may have several trees.

Example 24: Find the possible trees for the graph of Example 23.

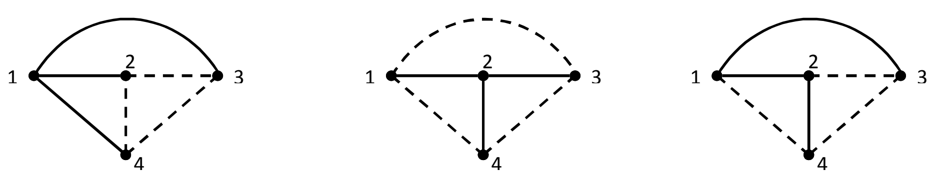
Solution:



(d) A cotree of a specified tree consists of all branches that are not part of the tree. The branches in the tree are called *tree branches* and the branches in the cotree are called *links*.

Example 25: Find the links for the co-trees corresponding to trees of Example 24.

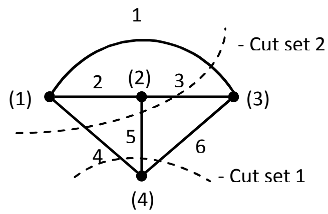
Solution:
The links are shown as dashed.



(e) A cut set is a minimum set of branches that must be cut to divide a graph into two separate parts.

Example 26: Find two cut sets for the graph of Example 23.

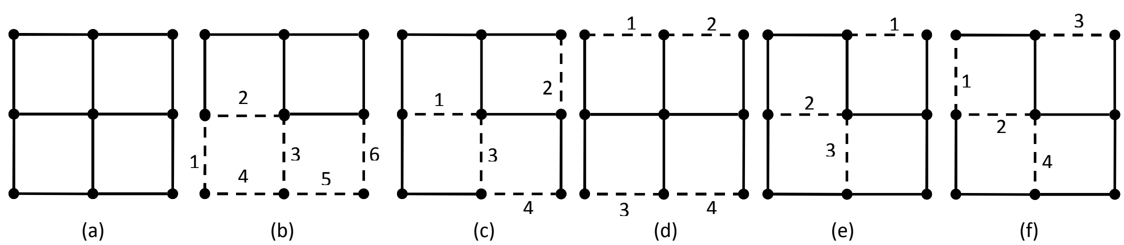
Solution:



- A cut set consisting of branches 4, 5 and 6 separates node 4 from 1, 2 and 3.
- A cut set consisting of branches 1, 3, 4 and 5 separates nodes 3 and 4 from nodes 1 and 2.

(f) Let L = number of links, B = number of branches and N = number of nodes. Then number of tree branches = B – L. Since to construct a tree, we begin by placing a branch between two nodes and each time that we add a branch we add one more node, the number of tree branches = N – 1. Therefore B – L = N – 1 or L =B – N +1.

Example 27: A graph for a network is given in Fig. 26(a). State whether the tree–cotree combinations in Fig. 26(b) to (f) form a valid set of tree branches and links. Compute the proper number of links for the graph in Fig. 26 (a).



Number of links, L= $B - N + 1 = 12 - 9 + 1 = 4$

(b) is no; (c) is yes; (d) is yes; (e) is no; (f) is yes

Using topology to determine the nodal equations for a network

(a) We require (N – 1) linearly independent equations. These can be obtained by determining independent cut–set equations.

(b) The cut–set equations are obtained by applying general KCL to the surface of either part of the circuit divided by the cut set.

(c) We derive the independent cut–set equations from fundamental cut sets and for that matter, a tree. A fundamental cut set is a set containing only a single tree branch. Thus, there are N – 1 fundamental cut–set (or independent) equations.

Example 28: Write the nodal equations for the network in Fig 22 via network topology.

Solution: Circuit is redrawn in Fig 27(a) with assumed directions for currents. A corresponding tree with fundamental cut sets is shown in Fig. 27(b).

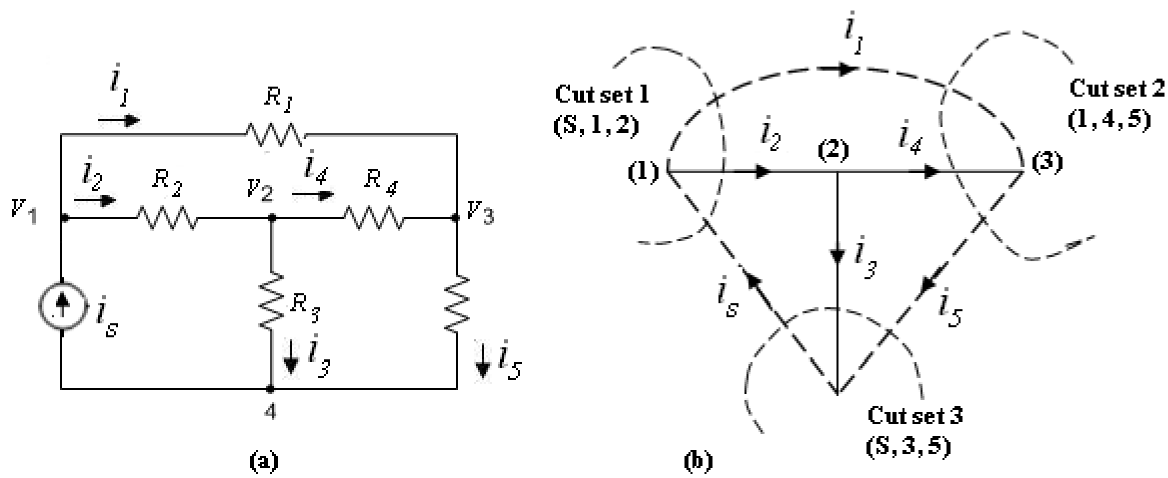


Fig. 27 Network and graph used in Example 28

The cut-set equations are:

cut − set 1: $i_1 + i_2 - i_s = 0$

cut − set 2: $i_1 + i_4 - i_5 = 0$

cut − set 3: $i_s - i_3 - i_5 = 0$

Writing the branch currents in terms of the node voltage yields

$$\frac{v_1 - v_3}{R_1} + \frac{v_1 - v_2}{R_2} - i_s = 0$$

$$\frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_4} - \frac{v_3}{R_5} = 0$$

$$i_s - \frac{v_2}{R_3} - \frac{v_3}{R_5} = 0$$

To help recognize these as nodal equations, obtain a fourth equation as follows:
(first equation) + (third equation) – (second equation):

$i_2 - i_4 - i_3 = 0$

and use first, second and fourth equations rather than first, second and third equations.

Example 29: Write the nodal equations for the network in Fig. 2 using network topology.

Solution: Circuit is redrawn in Fig 28(a) with currents as indicated. A corresponding tree with fundamental cut sets is shown in Fig. 28(b).

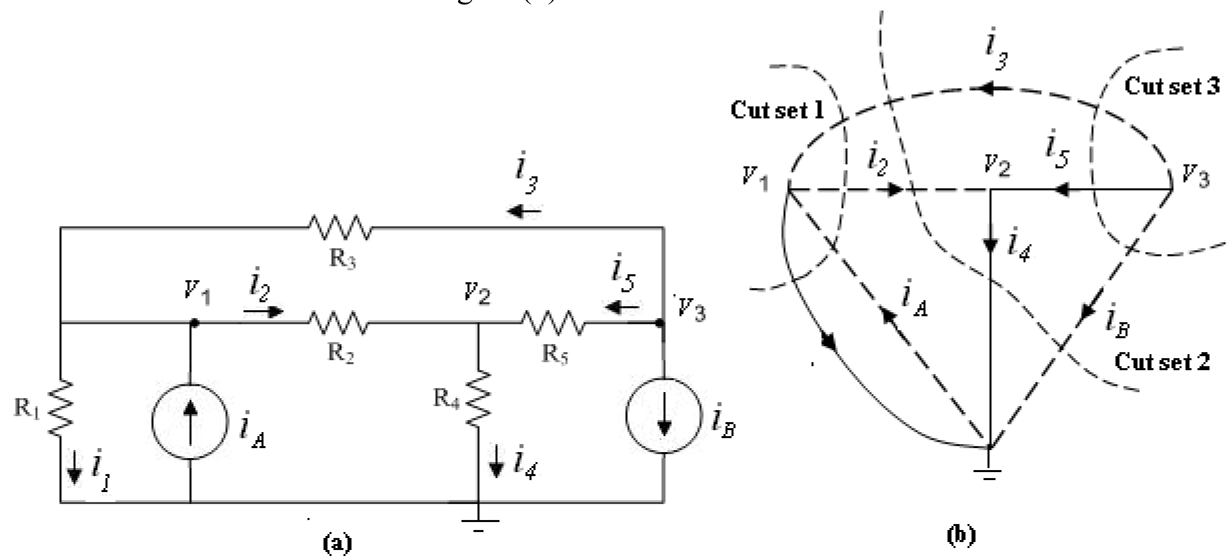


Fig. 27 Network and graph used in Example 29

The cut–set equations are:

Cut – set 1: $i_1 - i_A + i_2 - i_3 = 0$

Cut – set 2: $i_3 - i_2 + i_4 + i_B = 0$

Cut – set 3: $i_3 + i_5 + i_B = 0$

Subtracting the third equation from the second equation and using the resultant equation with the first and third equation yields:

$$i_1 + i_2 - i_3 = i_A$$

$$-i_2 + i_4 - i_5 = 0$$

$$i_3 + i_5 = -i_B$$

Writing the branch currents in terms of the node voltages yields:

$$i_A = \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} - \frac{v_3 - v_1}{R_3} = \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + \frac{v_1 - v_3}{R_3}$$

$$0 = -\frac{v_1 - v_2}{R_2} + \frac{v_2}{R_4} - \frac{v_3 - v_2}{R_5} = \frac{v_2}{R_4} + \frac{v_2 - v_1}{R_2} + \frac{v_2 - v_3}{R_5}$$

$$-i_B = \frac{v_3 - v_1}{R_1} + \frac{v_3 - v_2}{R_5} = \frac{v_3 - v_2}{R_5} + \frac{v_3 - v_2}{R_3}$$

These are identical to the equations derived earlier.

Exercise 14: Given the network in Fig. 29.a use the specified graph for the network in Fig.29.b to write proper set of nodal equations for the network.

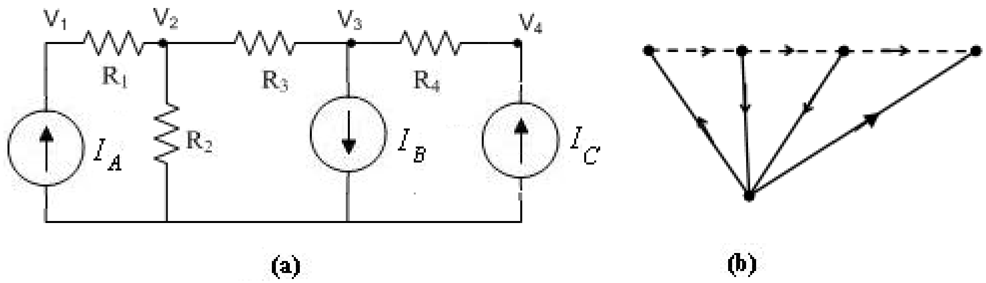
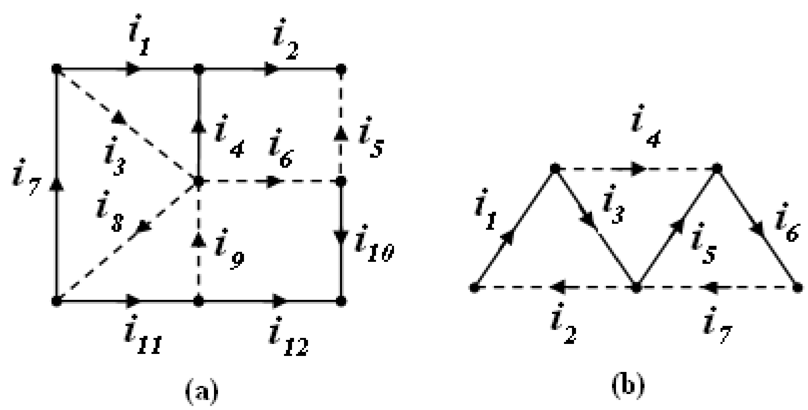


Fig. 29 Network and graph for Exercise 13

Exercise 15: For the graphs below with a specified tree, find an independent set of KCL equations using cut sets.



Ans:
$i_1 - i_2 = 0$
$i_4 + i_3 - i_2 = 0$
$i_4 + i_5 - i_7 = 0$
$i_6 - i_7 = 0$

For circuits containing voltage sources, the following rules govern the formulation of the equations required:

(a) Choose one node as the reference and assign a voltage variable to all other nodes. Let N = total number of nodes.

(b) Form a tree including all voltage sources (dependent and independent). Note that there are $N - 1$ tree branches and equal number of fundamental cut sets. If M is the number of voltage sources then there are M fundamental cut sets each of which contains a voltage source.

(c) For each voltage source, write the corresponding constraint equations. There are M constraint equations.

(d) For each fundamental cut set containing no voltage source, write the corresponding cut-set equation. There are $N - 1 - M$ linear independent nodal equations.

Example 30: Write down the node equations necessary to solve for all the branch currents of network in Fig.30.a

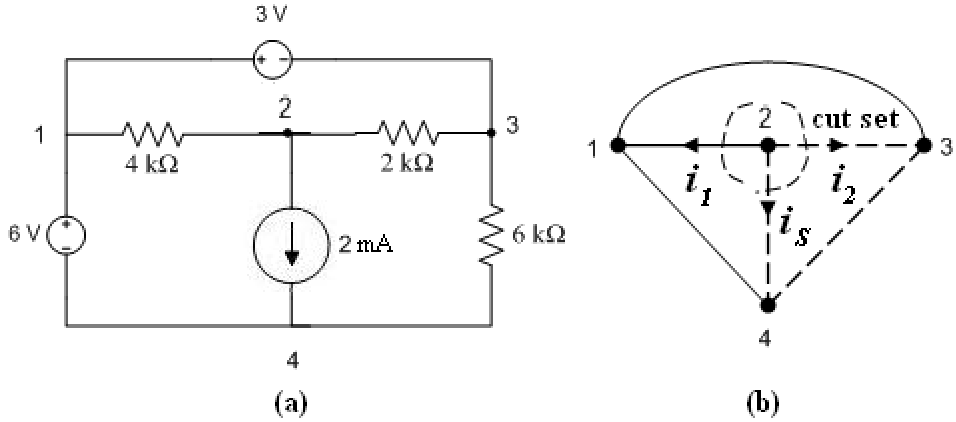


Fig. 30 Network and graph used in Example 30

Solution:
The graph with a specified tree is shown in Fig. 30 (b). Node 4 is chosen as reference.
The constraints equations are

$$v_1 = 6 \text{ V}$$

$$v_1 = v_3 + 3 \Rightarrow v_3 = 3 \text{ V}$$

One cut-set equation is required. It is given by

$$i_1 + i_2 + i_S = 0$$

or

$$\frac{v_2 - v_1}{4} + \frac{v_2 - v_3}{2} + 2 = 0$$

$$v_2 - v_1 + 2(v_2 - v_3) + 8 = 0$$

$$3v_2 = v_1 + 2v_3 - 8 = 6 + 6 - 8 = 4$$

$$v_2 = \frac{4}{3} \text{ V}$$

Exercise 16: Obtain the nodal equations for the network in Fig.6 using network topology.

Using topology to obtain loop equations for a network

If we add one link at a time to a tree, we create a new loop with each link. The loops constructed in this manner, having one link, are called fundamental loops. The number of fundamental loops = number of links = B – N +1. KVL equations written for these loops give B – N + 1 independent KVL equations for the solution of the network.

If there are current sources present in the network, then in general, the number of independent KVL equations, $N_i = B - (N - 1) -$ (Number of current sources)

The following rules govern the formulation of the equations required.

(a) Form a tree excluding all current sources (dependent or independent)

(b) Assign each fundamental loop current variable.

(c) For each fundamental loop containing a current source, write the corresponding constraint equation that relates the loop current to the value of the current source.

(d) For each fundamental loop containing no current source, write the corresponding loop equation. If P = total number of current sources, then
$N_i = B - (N - 1) - P = B - N + 1 - P$

Example 31: Draw a tree for the network in Fig. 30 and write the loop equations necessary to solve for all the unknown voltages.

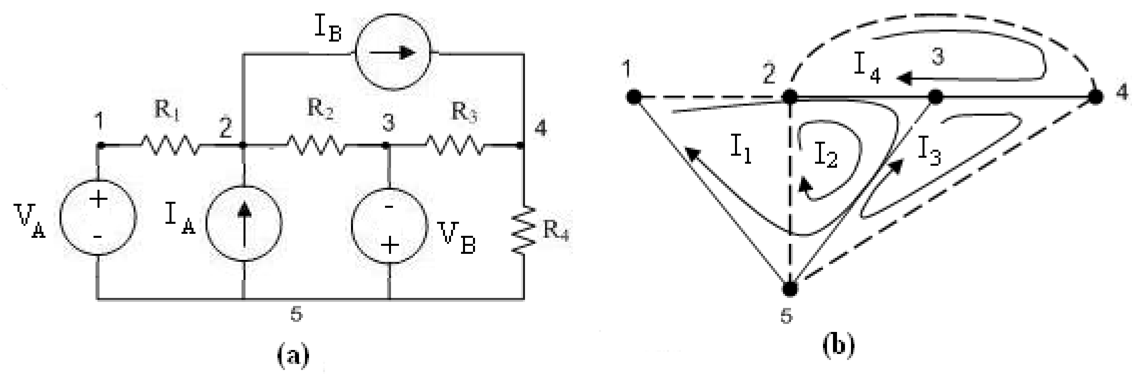


Fig. 30 Network and graph used in Example 31

Solution:

The graph of the network with a specified tree is given in Fig. 30 (b)

The constraint equations are:

$$I_4 = I_B$$

$$I_2 = I_A$$

The loop equations are

$$V_A + V_B = I_1R_1 + (I_1 + I_2 - I_4)R_2$$

$$-V_B = (I_3 - I_4)R_2 + I_3R_4$$

Example 32: For the graphs shown below, with a specified tree, find an independent set of KVL equations using fundamental loops.

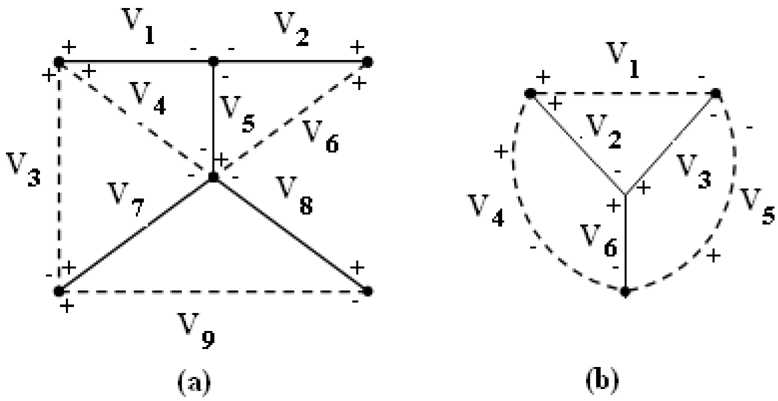
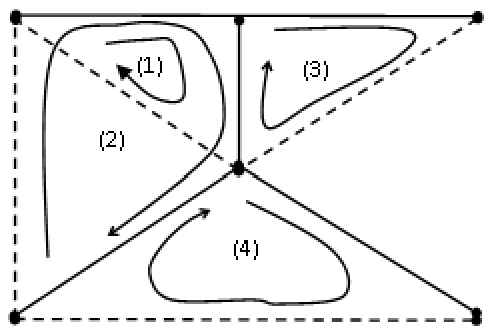


Fig. 31 Graphs for Example 32

Solution:

a) Four fundamental loops required are shown in the graph redrawn below.



The loop equations are:

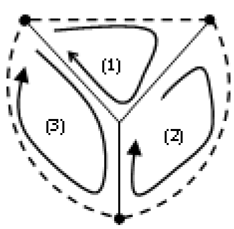
Loop 1: $V_4 - V_1 + V_5 = 0$

Loop 2: $V_3 - V_1 + V_5 + V_7 = 0$

Loop 3: $V_2 - V_6 - V_5 = 0$

Loop 4: $V_9 - V_7 + V_8 = 0$

b) The three fundamental loops required are shown in the graph redrawn below.



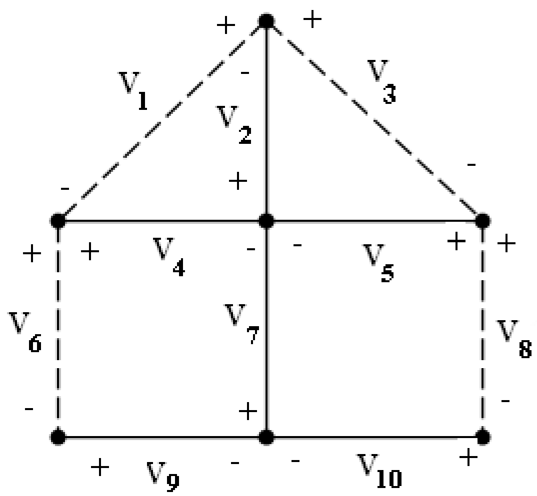
The loop equations are:

Loop 1: $V_2 - V_1 + V_3 = 0$

Loop 2: $V_6 - V_3 + V_5 = 0$

Loop 3: $V_4 - V_2 - V_6 = 0$

Exercise 17: For the graph below, with a specified tree, find an independent set of equations using fundamental loops.



<u>Choosing between nodal, mesh and basic loop analysis</u>

If N – 1 << B – N +1 (circuits with many elements in parallel), nodal analysis will be more efficient. If B – N + 1 << N – 1 (circuits with many elements in series), either basic loop or mesh analysis would be a better choice. For planar circuits, mesh analysis has the edge over basic loop analysis, since no tree is needed.