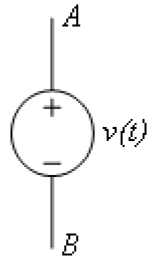
Chapter One
Review of Network Theorems

1. Types of Active Elements

(a) <u>Independent Voltage Source</u>: It is a two-terminal element that maintains a specified voltage between its terminals regardless of the current through it.



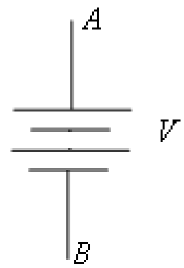


Figure 1 General Symbol   Figure 2 Constant Voltage

Figure 1 shows the symbol for time varying voltage (General symbol). The voltage *v(t)* is referenced positive at A.

(b) <u>Independent Current Source</u>: It is a two-terminal element that maintains a specified current regardless of the voltage across its terminals. Figure 3 shows the general symbol, and the arrow indicates the direction of the current source when it is positive.

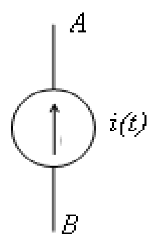


Figure 3 General Symbol

(c) <u>Two Dependent or Controlled Voltage Sources</u>: A dependent or controlled voltage source is a voltage source whose terminal voltage depends on, or is controlled by a voltage or a current at a specified location in the circuit. The two types are:

(i) Voltage-Controlled Voltage Source (VCVS) controlled by a voltage.

(ii) Current-Controlled Voltage Source (CCVS) controlled by a current.

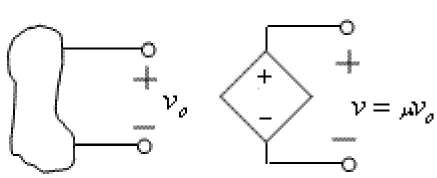


Figure 4 VCVS

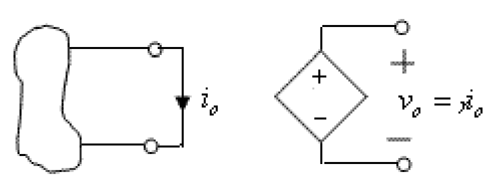


Figure 5 CCVS

(d) <u>Two Dependent or Controlled Current Sources</u>: A dependent or controlled current source is a current source whose current depends on, or is controlled by a voltage or current at a specified location in the circuit. The two types are:

(i) Voltage-Controlled Current Source (VCCS) controlled by voltage.

(ii) Current-Controlled Current Source (CCCS) controlled by current.

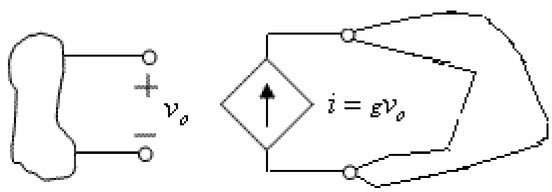


Figure 6 VCCS

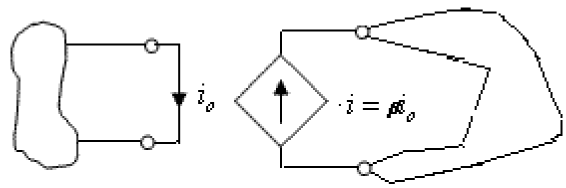
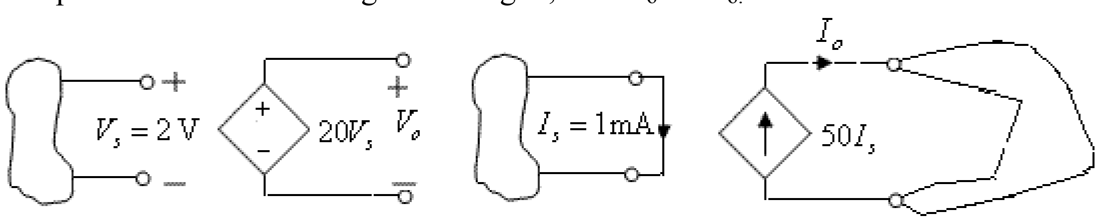


Figure 7 CCCS

The dependent sources are very important because they form part of the mathematical models used to describe the behaviour of many electronic circuit devices.

Example 1: For the networks given in Fig. 7, find V_o and I_o.



Solution:

(a) $V_o = 20V_s = 20 \times 2 = 40$ V

(b) $I_o = 50I_s = 50 \times 1 = 50$ mA

2. Superposition Theorem

Superposition theorem states that the current through, or the voltage across an element in a linear network is equal to the algebraic sum of the currents or voltages produced by each source acting alone.

Superposition can be used for linear circuits containing dependent sources. However, it is not useful in this case because the dependent source is never made zero.

Example 2: Use superposition to find V_O in the circuit in Fig. 8.

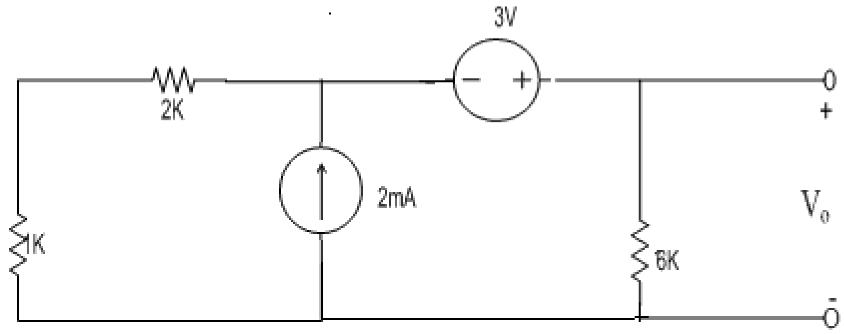


Figure 8 Network for Example 2

Solution: With the current source acting alone, current in the 6 k, using the current divider rule, is given by

$$I_o' = \frac{(1+2)}{(1+2)+6} \times 2 = \frac{3 \times 2}{9} = \frac{2}{3} \text{mA and}$$

$$V_o' = \frac{2}{3} \text{mA} \times 6\,\text{k} = 4\,\text{V}$$

With the voltage source acting alone, current in the 6 k is given by

$$I_o'' = \frac{3}{1+2+6} = \frac{3}{9}\ \text{mA}\ \text{and}$$

$$V_o'' = \frac{3}{9} \text{mA} \times 6\,\text{k} = 2\,\text{V}$$

Therefore $V_o = V_o' + V_o'' = 4 + 2 = 6\,\text{V}$

Example 3: Using the superposition principle, determine the current through the 4-Ω resistor of Fig. 9

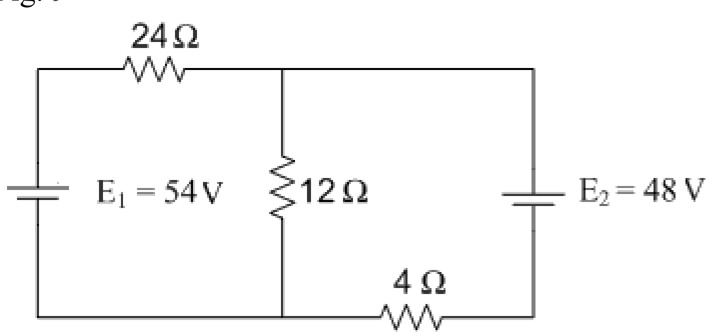


Figure 9 Network for Example 3

Solution: With the 54-V source acting alone,

$$\text{total resistance} = 24 + 12//4 = 24 + \frac{12 \times 4}{16} = 27\ \Omega \text{ and}$$

$$\text{total current} = \frac{54}{27} = 2\ \text{A}$$

Using the current divider rule, current in the 4-Ω resistor

$$\text{I}' = \frac{12}{12+4} \times 2 = \frac{12 \times 2}{16} = 1.5\ \text{A}$$

With the 48-V source acting alone,

$$\text{total resistance} = 4 + 24//12 = 4 + \frac{24 \times 12}{36} = 12\ \Omega$$

$$\text{total current} = \text{current in the } 4\text{-}\Omega \text{ resistor I}'' = \frac{48}{12} = 4\ \text{A}$$

Since I' and I'' are not in the same direction through the 4-k resistor, the actual current $\text{I} = \text{I}'' - \text{I}' = 4 - 1.5 = 2.5\ \text{A}$ (actual current is in the direction of I'')

Example 4: Using the principle of superposition, find the current through the 12-k resistor in Fig. 10

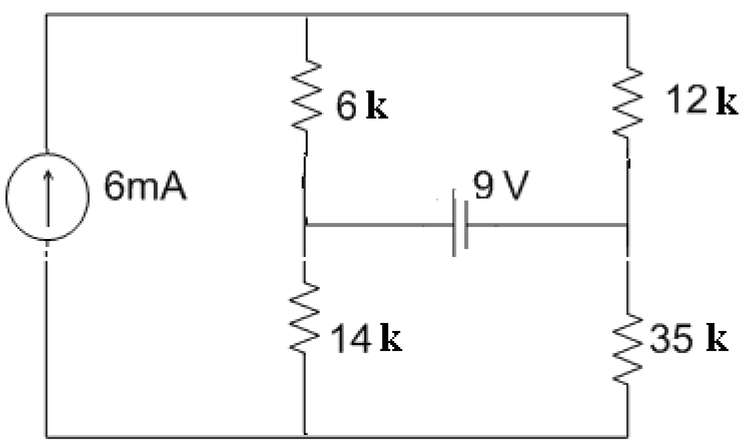


Figure 10 Network for Example 4

Solution: With the current source acting alone, the 6-k and 12-k resistors are in parallel and then in series with the 14-k and 35-k resistors which are also in parallel. Therefore, the current in the 12-k resistor, using the current divider rule is

$$\text{I}' = \frac{6}{12+6} \times 6 = \frac{6 \times 6}{18} = 2\ \text{mA}$$

With the 9-V source acting alone, 6-k and 12-k resistors are in series and are directly across the 9-V source. Therefore, the current in the 12-k resistor

$$I'' = \frac{9}{6+12} = \frac{9}{18} = 0.5\,\text{mA}$$

Since I' and I'' have the same direction through the 12-k resistor, the desired or actual current $I = I' + I'' = 2 + 0.5 = 2.5$ mA.

3. Thevenin's Theorem

Thevenin's theorem states that any two-terminal linear dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistance as shown in Fig. 11

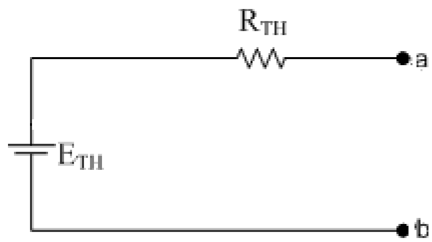


Figure 11 Thevenin Equivalent Circuit

(a) Measurement of circuit parameters: The two popular methods are:

(i) *Direct Measurement of E_{TH} and R_{TH}*

- Measure the open-circuit voltage V_{OC} using a voltmeter, $E_{TH} = V_{OC}$
- Connect a variable resistor across the terminal and vary it till the voltmeter still across the terminal read $\frac{V_{OC}}{2}$
- Measure the value of the variable resistance in circuit with an ohmmeter. With the terminal voltage $= \frac{V_{OC}}{2}$, the values of R_{TH} and the external resistance are the same.

(ii) *Measuring V_{OC} and I_{SC}*

- Measure V_{OC}: $E_{TH} = V_{OC}$
- Measure the current through a short-circuit placed between the terminal with an ammeter I_{SC}: $R_{TH} = \frac{V_{OC}}{I_{SC}}$

(b) Circuits containing ONLY independent sources

Example 5: Find the Thevenin equivalent circuit for the network shown in Fig. 12

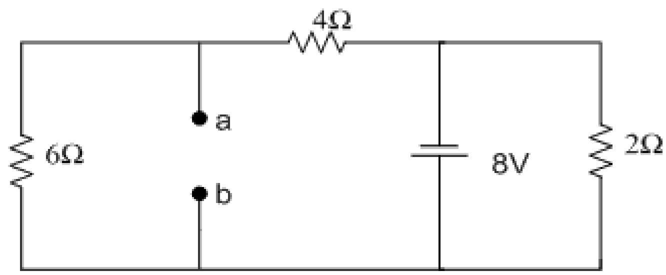


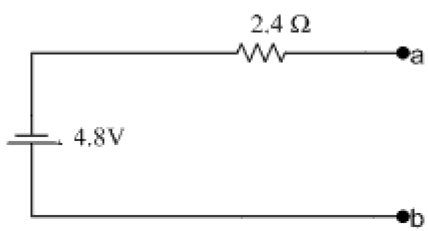
Figure 12 Network for Example 5

Solution: $E_{TH} = V_{ab}$ = voltage across the 6-Ω resistance

By voltage divider rule $V_{ab} = -\frac{6}{10} \times 8 = -4.8\text{ V}$

$R_{TH} = 6//4 = \frac{6 \times 4}{10} = 2.4\ \Omega$

We note that the 2- Ω resistance is short-circuited when the source is deactivated.



Example 6: Find the Thevenin equivalent circuit for the network external to resistance R_L of Fig. 13

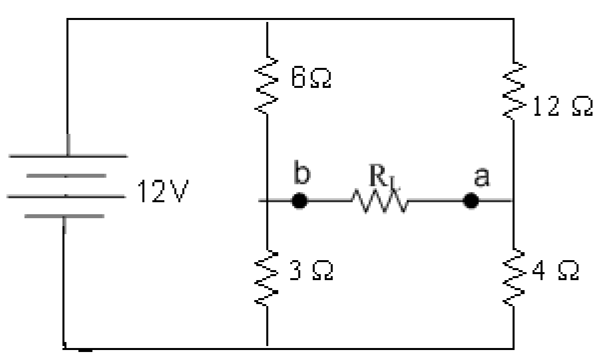


Figure 13 Network for Example 6

Solution: With the R_L disconnected the voltage across the 6-Ω resistance using the voltage divider rule

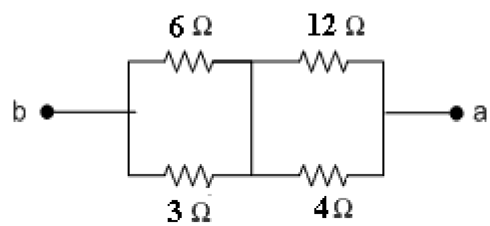
$V_1 = \frac{6}{6 + 3} \times 12 = \frac{6 \times 12}{9} = 8\text{ V}$

The voltage across the 12-Ω resistance using the voltage divider rule

$$V_2 = \frac{12}{12\ +\ 4} \times 12 = \frac{12\ \times\ 12}{16} = 9\ \text{V}$$

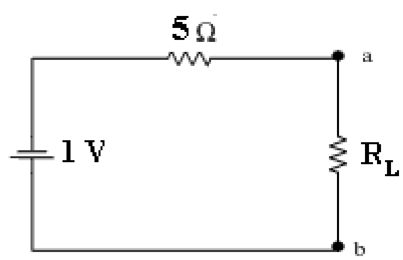
Therefore $E_{TH} = V_{ab} = V_1 - V_2 = 8\ \text{V} - 9\ \text{V} = -1\ \text{V}$

Circuit for R_{TH}:



$$R_{TH} = 6//3\ + 12//4 = \frac{6 \times 3}{9} + \frac{12 \times 4}{6} = 2 + 3 = 5\Omega$$

Substituting the Thevenin equivalent circuit for the network external to the resistance R_L, we obtain



Example 7: Find Thevenin equivalent circuit for the network of Fig. 14

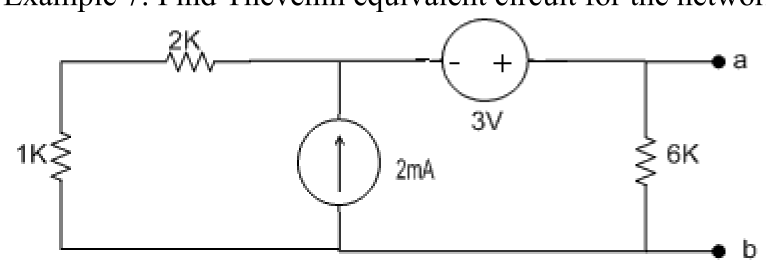


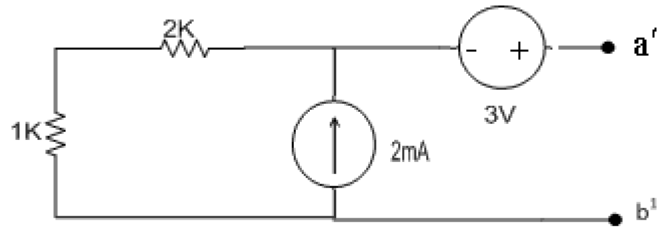
Figure 14 Network for Example 7

Solution: We obtain the solution in two stages.

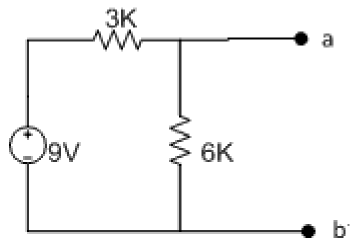
Stage 1: Break the circuit at the 6-k resistor

The voltage $E_{a'b'}$ = voltage across current source or voltage across the resistors + 3 V

$E_{a'b'} = (1+2)\times 2 + 3 = 9\,\text{V}$. $R_{a'b'} = 1 + 2 = 3\,\text{k}$



Substituting the Thevenin equivalent circuit for the network reduced, we obtain



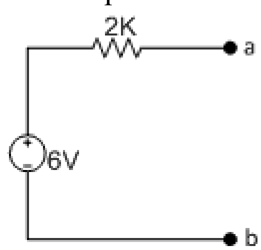
Stage 2: From the above circuit,

E_{TH}=V_{ab} $= \dfrac{6}{3+6} \times 9 = 6$ V (using the voltage divider rule)

and R_{TH} $= \dfrac{3 \times 6}{9} = 2$ k

.

The Thevenin equivalent circuit is given below.



Example 8: Find Thevenin equivalent circuit for the network of Fig. 15

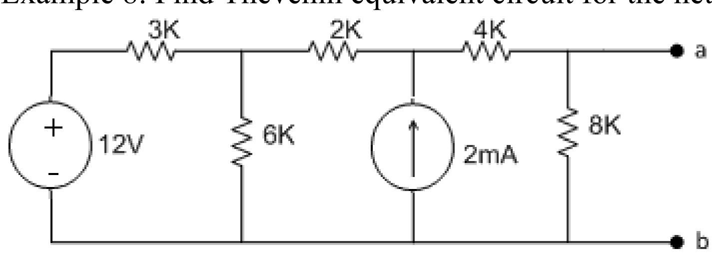
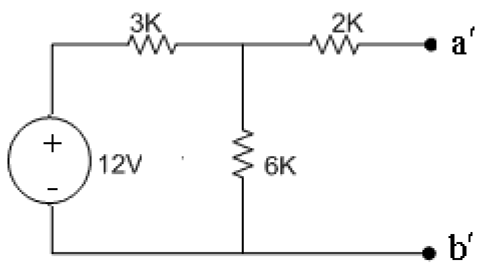


Figure 15 Network for Example 8

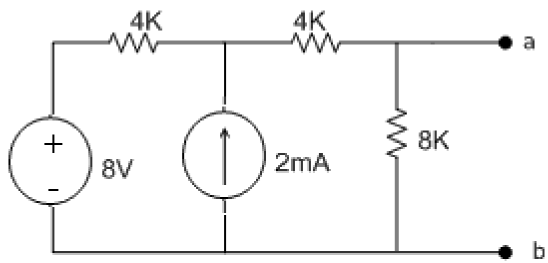
Solution: Break it to the left of the current source



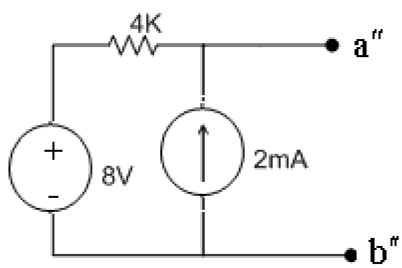
$$E_{a'b'} = \frac{6}{9} \times 12 = 8\ V,$$

$$R_{a'b'} = 2 + \frac{3 \times 6}{9} = 4\,k$$

Replacing the above circuit by its Thevenin equivalent circuit gives



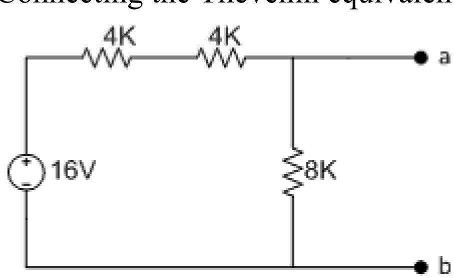
Break the circuit again to the right of the current source



$E_{a''b''} = 8 + (2 \times 4) = 16\,V$ and

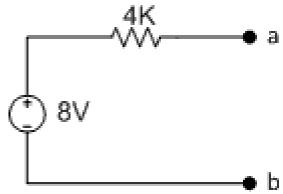
$R_{a''b''} = 4\,k$

Connecting the Thevenin equivalent circuit to the remainder of the circuit produces

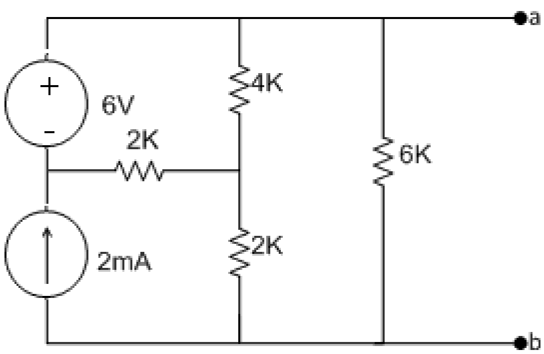


$$E_{TH} = V_{ab} = \frac{8}{8+4+4} \times 16 = 8\ V$$

$R_{TH} = (4+4)//8 = \frac{8+8}{16} = 4\text{k}$



Exercise 1: Find Thevenin equivalent circuit for the network below



Hint: Break at 6 kΩ and apply superposition or Kirchhoff's Laws to find the open-circuit voltage.

Ans: $E_{TH} = \frac{48}{7}\text{V}$ and $R_{TH} = \frac{10}{3}\ \text{k}//6\ \text{k}$

(c) <u>Circuits containing ONLY dependent sources</u>

(i) For this case $E_{TH} = 0$

(ii) To obtain R_{TH} either of the following approaches can be used:

- Apply 1-V source at the terminals, then compute the source current I_S : $R_{TH} = 1/I_S$
- Apply a 1-mA or 1-A current source at the terminals, then compute the source terminal voltage V_S : $R_{TH} = V_S / 1\text{mA}$, in kΩ or V_S / 1A in Ω.

Example 9: Find the Thevenin equivalent circuit of the network in Fig. 16 at the terminals a-b. (Note that controlling voltages and currents at specified locations in a circuit are understood to be in volts and amps respectively if their units are not specified)

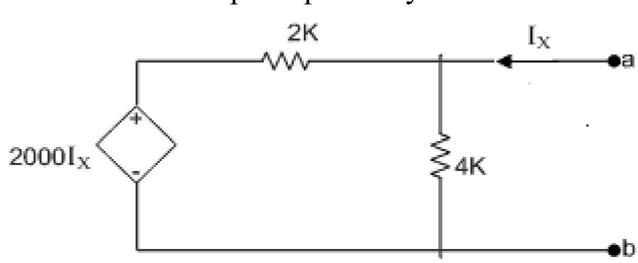
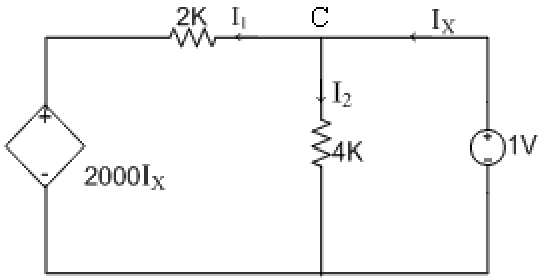


Figure 16 Network for Example 9

Solution:

1st Approach



Let the current be in mA. Then the CCVS = $2I_x$, the current in the 4-k Ω resistance, $I_2 = 1/4$ mA = 0.25 mA and the current in the 2-kΩ resistance

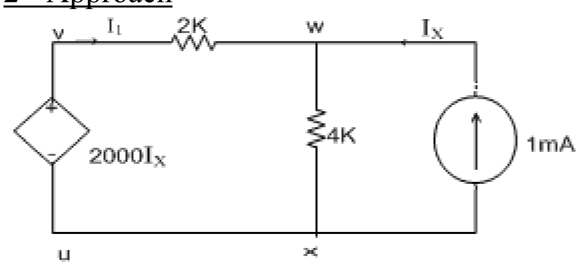
$$I_1 = \frac{1 - 2I_X}{2} = 0.5 - I_X$$

Applying KCL to the node C, we obtain

$$I_X = I_1 + I_2 = 0.5 - I_X + 0.25 \text{ or } 2I_X = 0.75 \text{ or } I_X = \frac{0.75}{2}\,\text{mA}$$

$$R_{TH} = \frac{1}{I_x} = \frac{1}{\left(\frac{0.75}{2}\right)} = \frac{2}{0.75} = \frac{8}{3}\,\text{k}\Omega$$

2nd Approach



CCVS = $2I_x = 2 \times 1 = 2$V. (Note that I_x is in mA)

Applying KVL to loop uvwx, we obtain $2 = 2I_1 + 4\,(I_1 + 1)$, where I_1 is in mA.

From this equation

$$I_1 = \frac{-2}{6} = -\frac{1}{3}\,\text{mA}$$

Hence current in the 4-kΩ resistance $= I_X + I_1 = 1 - \frac{1}{3} = \frac{2}{3}\,\text{mA}$

Voltage across 1-mA current source V_s = voltage across the 4-kΩ resistance $= 4 \times \frac{2}{3} = \frac{8}{3}$ V

Therefore $R_{TH} = \frac{V_s}{1} = \frac{8}{3} \times \frac{1}{1} = \frac{8}{3}\,\text{k}\Omega$

Example 10: Find the Thevenin equivalent of the circuit in Fig. 17 at the terminals A-B. (Note that V_x is in volts and VCCS is in amps.)

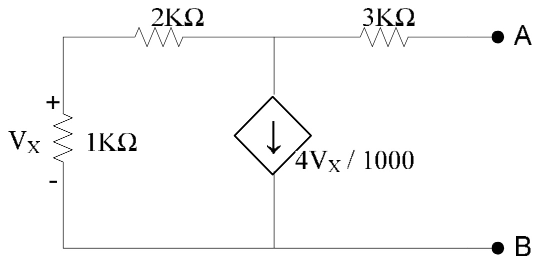
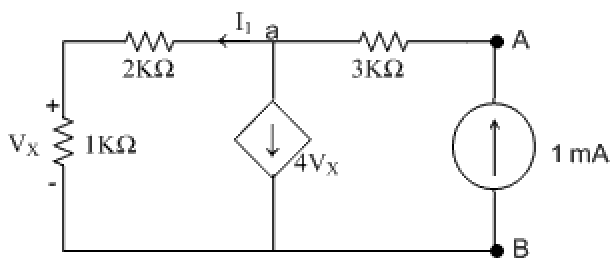


Figure 17 Network for Example 10

Solution: Let all currents be in mA and let's apply 1mA at terminals A-B. Then the circuit becomes



Applying KCL at node a, we obtain $1 = I_1 + 4V_X$

$V_X = 1 \times I_1 = I_1$. Therefore $1 = I_1 + 4\,I_1$, or $I_1 = \frac{1}{5}$ mA

The voltage across the 1-mA current source $= 3 \times 1 + (2+1)I_1 = 3 + \frac{3}{5} = \frac{18}{5}$ V

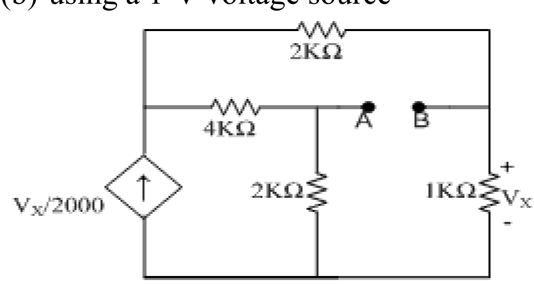
Hence $R_{TH} = \frac{18}{5} \times \frac{1}{1} = \frac{36}{10} = 3.6\ \text{k}\Omega$

Exercise 2: Repeat Example 10 using 1-V source

Exercise 3: Find the Thevenin equivalent of the network given below at terminals A-B

(a) using a 1-mA current source

(b) using a 1-V voltage source



(d) Circuits containing both independent and dependent sources

(i) For these circuits we must calculate both the open-circuit E_{TH} and short-circuit current I_{SC} in order to calculate R_{TH}.

(ii) We note that we CANNOT split a dependent source and its controlling variable if we decide to break the network to find the Thevenin equivalent.

Example 11: Find the Thevenin equivalent of the network in Fig. 18 at the terminals a-b

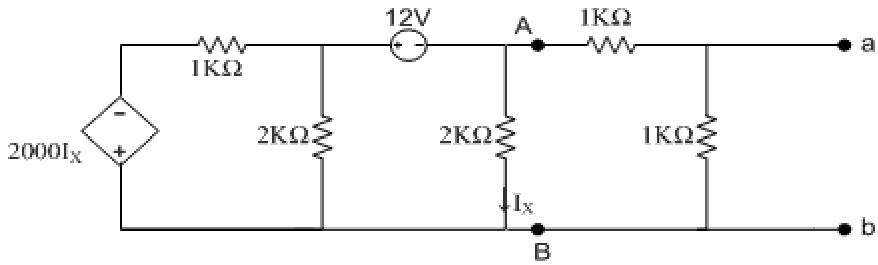
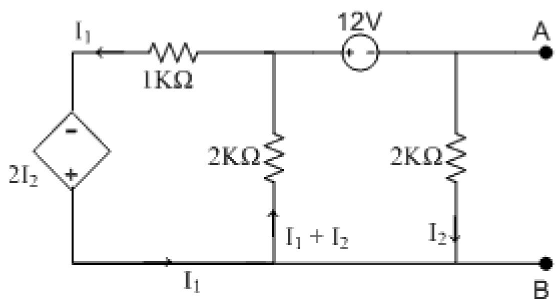


Figure 18 Network for Example 11

Solution: We break the network at points A-B. Let all currents be in mA.



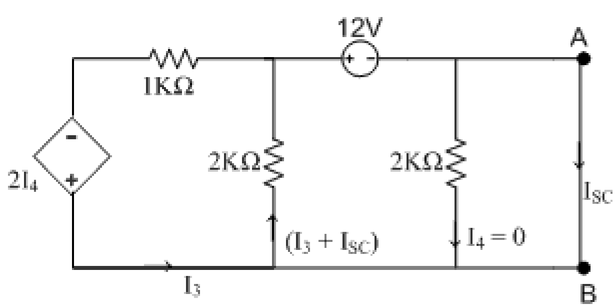
Calculation of open-circuit voltage:

Apply KVL to the left loop: $2I_2 = 2(I_1 + I_2) + I_1$ from which $I_1 = 0$

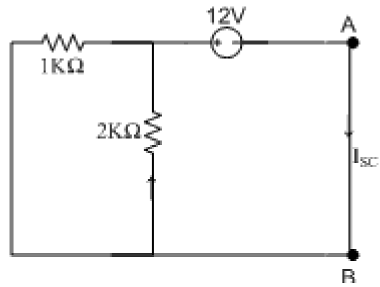
Applying KVL to the right loop: $-12V = 2I_2 + 2(I_1 + I_2) = 4I_2$ from which $I_2 = -3$ mA

$V_{AB} = 2I_2 = 2 \times (-3) = -6V$

Calculation of short-circuit current:



The current $I_4 = 0$ because the 2-kΩ is short-circuited. This also implies that the CCVS = 0. Hence the circuit becomes

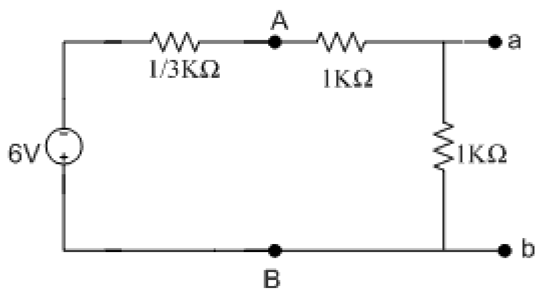


$$I_{SC} = \frac{-12}{1} + \frac{-12}{2} = -18\text{ mA}$$

Calculation of R'_{TH} :

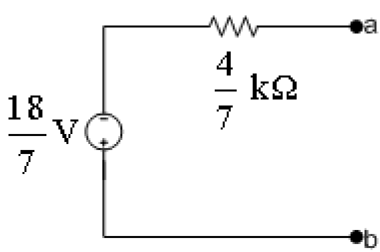
$$R'_{TH} = \frac{V_{AB}}{I_{SC}} = \frac{-6\,V}{-18\,mA} = \frac{1}{3}k\Omega$$

Connect the Thevenin equivalent circuit to the remainder of the circuit at terminal A-B:



$$R_{TH} = \left(1 + \frac{1}{3}\right) //1 = \frac{4}{7}\text{ k}\Omega$$

And using voltage divider rule $E_{TH} = V_{ab} = -6 \times \dfrac{1}{1 + 1 + \frac{1}{3}} = -\dfrac{18}{7}\text{V}$



Example 12: Find the Thevenin equivalent of the circuit external to R_L of Fig. 19

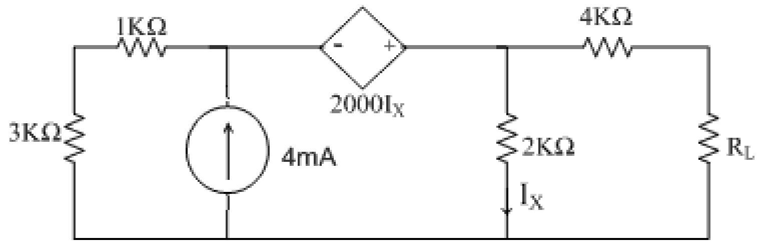
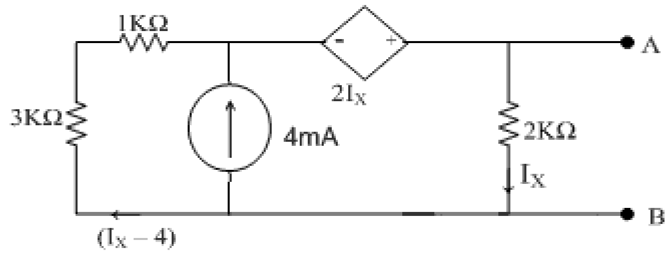


Figure 19 Network for Example 12

Solution: We break it to the right of the 2-kΩ resistor. We let currents be in mA.

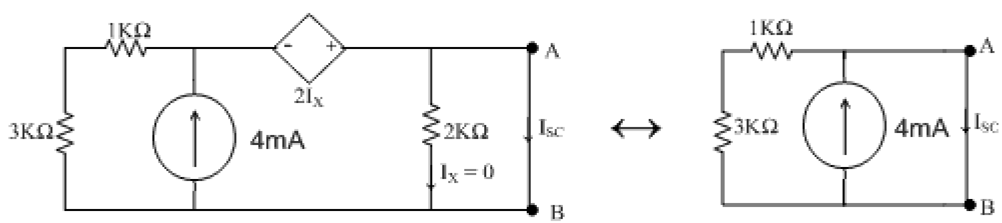


Calculation of V_{AB}:

Apply KVL to loop, $2I_X = 2I_X + (3+1)(I_X - 4)$ or $16 = 4I_X$ which yields $I_X = 4mA$

$V_{AB} = 2I_X = 2 \times 4 = 8V$

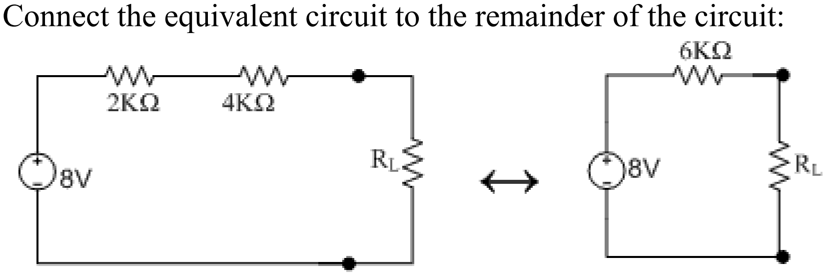
Calculation of short-circuit current, I_{SC}:



I_{SC} = 4mA. (Note: it is here that we find the advantage of breaking the network to the right of the 4-kΩ resistor)

$$R_{TH} = \frac{V_{AB}}{I_{SC}} = \frac{8}{4} = 2\ k\Omega$$

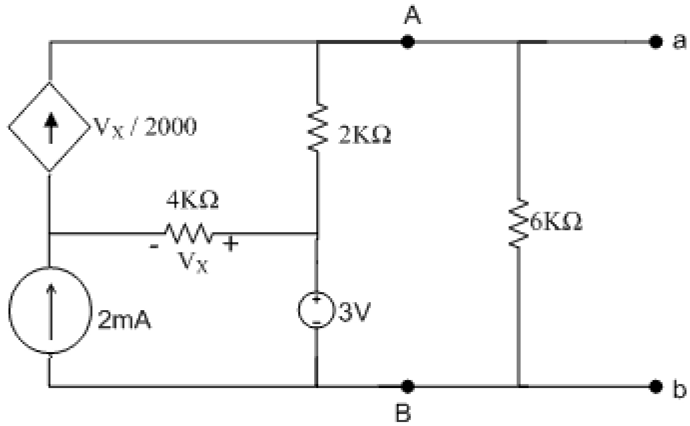
Connect the equivalent circuit to the remainder of the circuit:



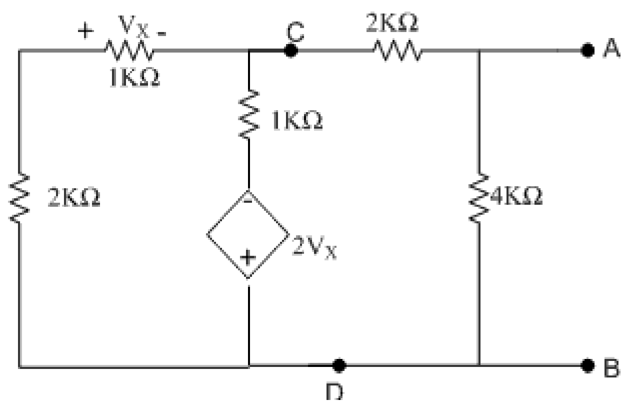
From the above circuit, we have E_{TH} = 8 V and R_{TH} = 6 kΩ

Exercise 4: Find the Thevenin equivalent circuit at terminals a-b of the network shown below

Hint: Break at A-B. Ans:$V_{AB} = 11\ \text{V},\ R_{AB} = 2\ \text{k}\Omega.\ V_{TH} = \frac{33}{4}\text{V}, R_{TH} = 2//6 = 1.5\ \text{k}\Omega$



Exercise 5: Find the Thevenin equivalent of the network shown below at the terminals A-B.



Hint: Break circuit at C and D and apply 1-V source $R_{TH} = \frac{28}{15}\text{k}\Omega$

4. Norton's Theorem

Norton's theorem states that any two terminal linear dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistance as shown in Fig. 20

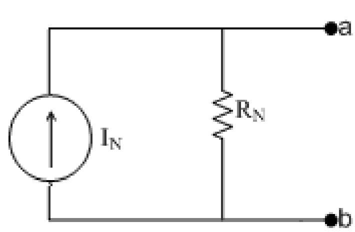


Figure 19 Norton's Equivalent Circuit

(a) Measurement of parameters: Measure $I_N = I_{SC}$ and $R_N = R_{TH}$ in the same way as described for the Thevenin network.
(b) Obtaining equivalent circuits: Norton's Theorem is identical to Thevenin's theorem except that the equivalent circuit is an independent current source in parallel with resistance. Therefore, the above discussions of Thevenin's theorem with respect to the equivalent circuits are also applicable to the Norton's equivalent circuit.

Example 13: Find Norton equivalent circuit at terminals a-b of the circuit shown in Fig. 20

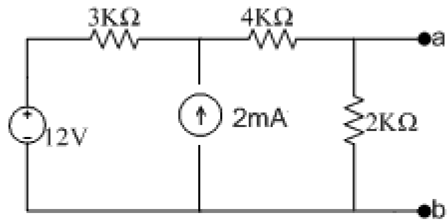
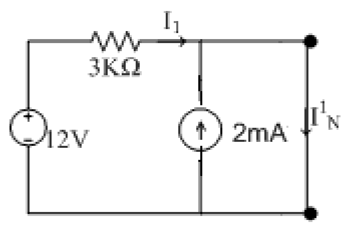


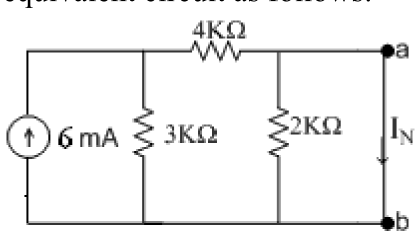
Figure 20 Network for Example 13

Solution: Break the network to the left of the 4- kΩ resistor.



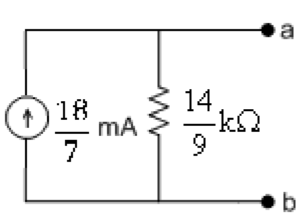
$I'_N = I_1 + 2 = \frac{12}{3} + 2 = 6\,\text{mA}$ and $R'_N = 3\,\text{k}\Omega$

Connect the equivalent circuit to the remainder of the circuit and obtain the required Norton equivalent circuit as follows:



$I_N = \frac{3}{3+4} \times 6 = \frac{18}{7}$ mA because the 2-kΩ is short-circuited.

$R_N = (3+4)//2 = \frac{7 \times 2}{9} = \frac{14}{9}$ kΩ



Example 14: Find Norton equivalent at terminals a-b of the network shown in Fig. 21

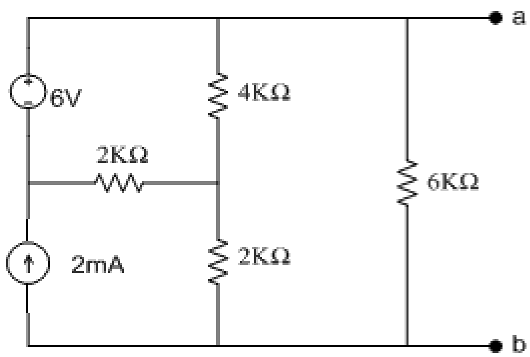
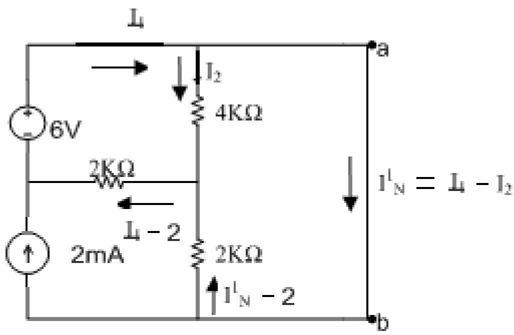


Figure 21 Network for Example 14

Solution: Break the network to the left of the 6-kΩ resistor



Applying KVL to the two loops each having the 6-V source gives

$6 = 4I_2 + 2(I_1 - 2)$ and

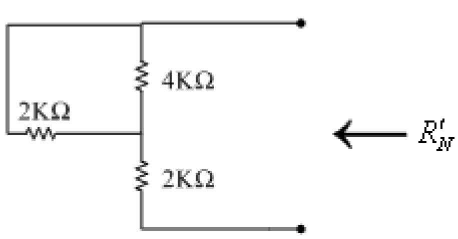
$6 = 2(I_1 - I_2 - 2) + 2(I_1 - 2)$

Solving the two equations simultaneously, we obtain

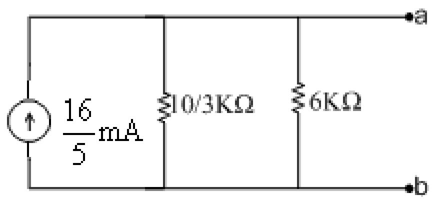
$I_2 = \frac{3}{5}$ mA and $I_1 = \frac{19}{5}$ mA

$I_N{}^1 = I_1 - I_2 = \frac{19}{5} - \frac{3}{5} = \frac{16}{5}$ mA

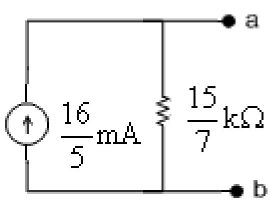
Calculate R'_N



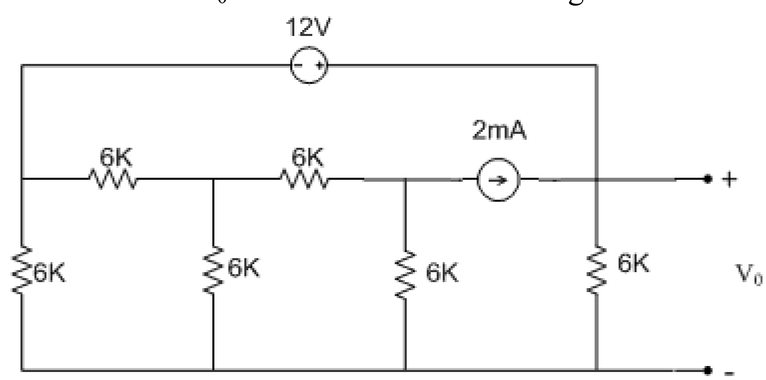
$$R'_N = 2 + 2//4 = 2 + \frac{2 \times 4}{6} = \frac{10}{3}\,\text{k}\Omega$$



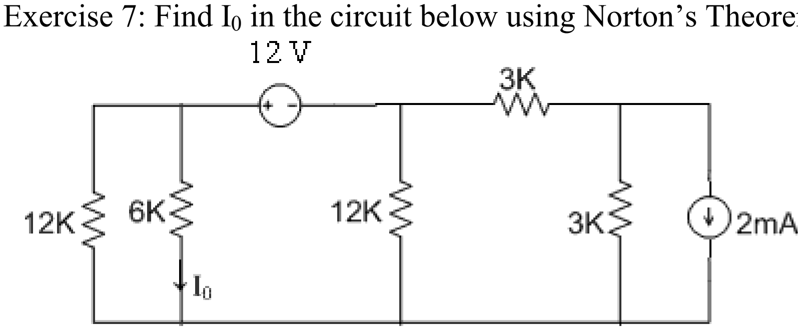
$$R_N = \frac{\frac{10}{3} \times 6}{\frac{10}{3} + 6} = \frac{10 \times 6}{28} = \frac{15}{7} k\Omega$$



Exercise 6: Find V_0 in the network below using Thevenin's theorem



Exercise 7: Find I_0 in the circuit below using Norton's Theorem.



5. Substitution Theorem

It states the following: If the voltage across and the current through any branch of a dc network are known, this branch can be replaced by any combination of elements that will maintain the same voltage across and current through the chosen branch.

The theorem can be stated more simply as this: For any branch equivalence, the terminal voltage and current must be the same.

We note also the following:

(a) It follows from this theorem that a known potential difference and current in a network can be replaced by an ideal voltage source and current source respectively.

(b) For this theorem to be applied, a potential difference or current value must be known or found using some other technique such as Thevenin's or Norton's theorem.

(c) This theorem cannot be used to solve networks with two or more sources that are not in series or parallel.

<u>Example demonstrating the effect of substitution theorem</u>

Consider the circuit of Fig. 22. Equivalent branches for branch a-b obtained by the use of substitution theorem are given in Fig. 23. We note that for each equivalent, the terminal voltage and current are the same. Also, the response of the remainder of the circuit of Fig. 22 is unchanged by substituting any one of the equivalent branches.

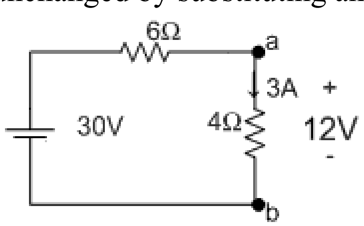


Figure 22: Demonstrating the effect of the substitution theorem

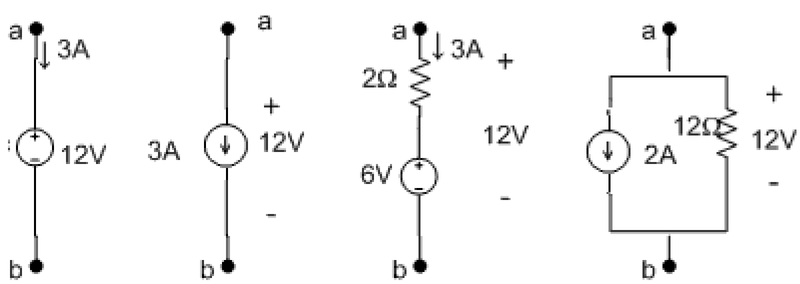


Figure 23: Equivalent branches for branch a-b of Fig. 22

Example demonstrating the effect of knowing a voltage at some point in a complex network

The example shows one application of the theorem. Referring to circuit of Fig. 24, the known potential difference V is replaced by a voltage source thus permitting the isolation of the portion of the network consisting of R_3, R_4, and R_5.

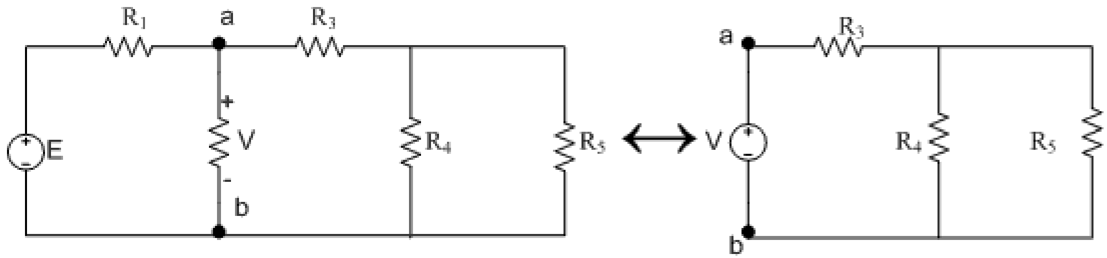
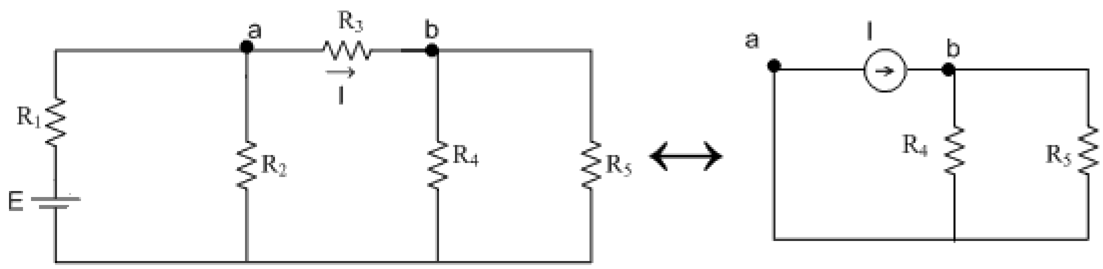


Fig. 25: Demonstrating the effect of knowing a voltage at some point in a complex network

Example demonstrating the effect of knowing the current at some point in a complex circuit

Consider the circuit of Fig. 25. Replacing the current *I* by an ideal current source, permits the isolation of R_4 and R_5.



Example 14: Using the substitution theorem, draw three equivalent branches for the branch a-b of the network of Fig. 26.

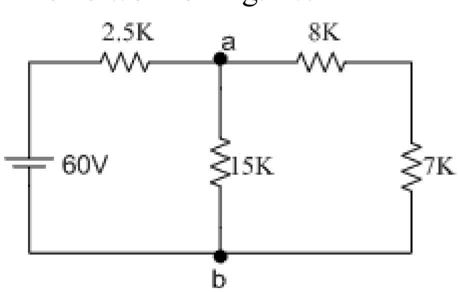
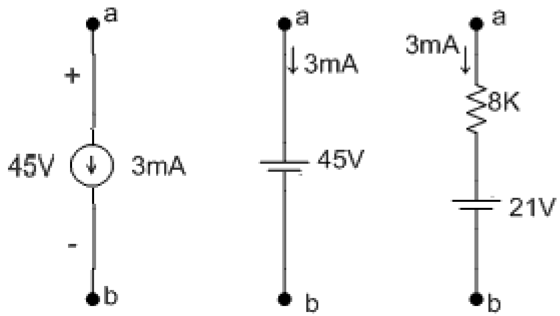


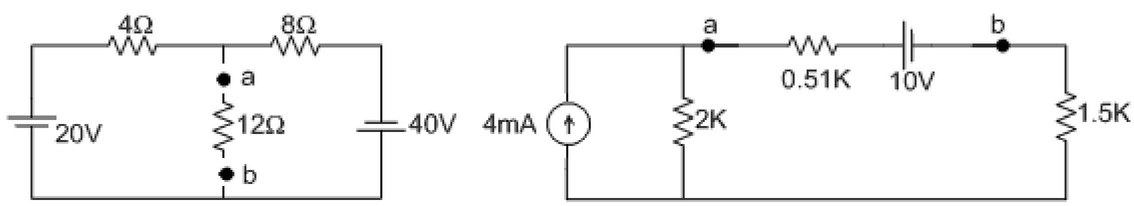
Figure 26: Network for Example 14

Solution: Current in the 15-k resistor is 3 mA and the voltage across it is 45 V. Proposed branches are:



Verify that the currents in other branches of the circuit are unchanged by substituting any of the proposed branches.

Exercise 8: Repeat Example 14 for the networks given below:



6. Reciprocity Theorem

It states the following: The current *I* in any branch of a linear network, due to a single voltage source *E* anywhere else in the circuit, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current *I* was originally measured.

We note also the following:

(a) The theorem can be stated more simply as this: The location of a voltage source and the resulting current anywhere in the circuit may be interchanged without a change in current.

(b) The theorem requires that the polarity of the voltage source corresponds to the direction of the branch current.

(c) The theorem is applicable only to single source networks.

(d) Other ways of stating the theorem are:

- (i) An ideal voltage source and ideal ammeter when connected in two different branches of a linear network may be interchanged without a change in the ammeter reading.
- (ii) An ideal current source and ideal voltmeter when connected across two different branches of a network may be interchanged without a change in the voltmeter reading.

The two statements (i) and (ii) are said to be the dual of each other.

Example 15: Demonstrate the validity of the reciprocity theorem using the networks of Fig. 27

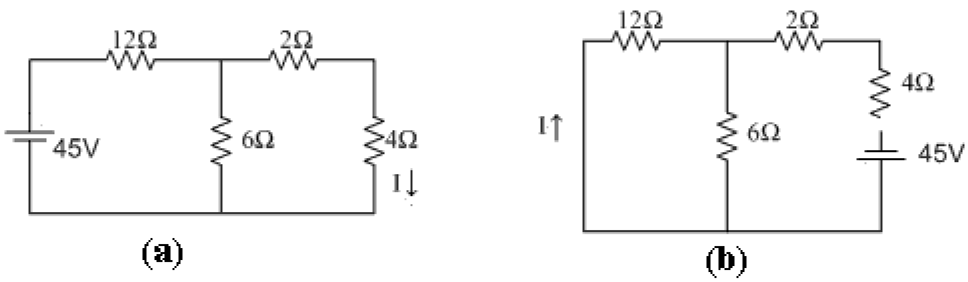


Figure 27: Networks for Example 15

Solution:

Circuit (**a**)

$$R_T = 12 + 6//(2+4) = 12 + \frac{6 \times 6}{12} = 15\ \Omega$$

$$\text{Total current} = \frac{45}{15} = 3\text{ A}, \text{I} = \frac{3}{2} = 1.5\text{ A}$$

Circuit (**b**)

$$R_T = 2 + 4 + 12//6 = 6 + \frac{12 \times 6}{18} = 10\ \Omega$$

$$\text{Total current} = \frac{45}{10} = 4.5\text{ A},\ \text{I} = \frac{6}{6+12} \times 4.5\text{ A} = \frac{6}{18} \times 4.5 = 1.5\text{ A}$$

Example 16: Demonstrate the validity of the reciprocity theorem using the networks of Fig. 28

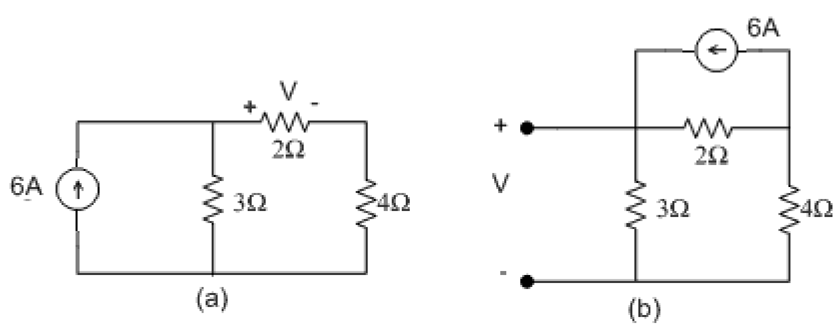


Figure 28: Networks for Example 16

Solution:

Circuit (a)

$$\text{Current in 2- }\Omega\text{ resistor} = \frac{3}{3+6} \times 6 = 2\text{ A. Hence V} = 2 \times 2 = 4\text{ V}$$

Circuit (b)

$$\text{Current in 3- }\Omega\text{ resistor } = \frac{2}{2+(3+4)} \times 6 = \frac{12}{9}\text{. Hence V} = 3 \times \frac{12}{9} = 4\text{ V}$$

Example 17: Use superposition and reciprocity theorems jointly to determine the current I shown in the circuit of Fig. 29

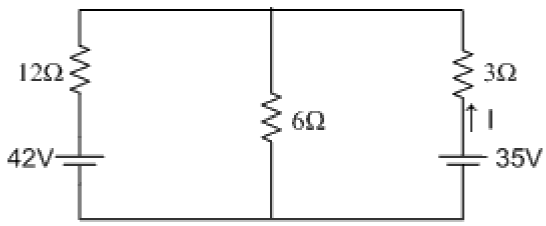
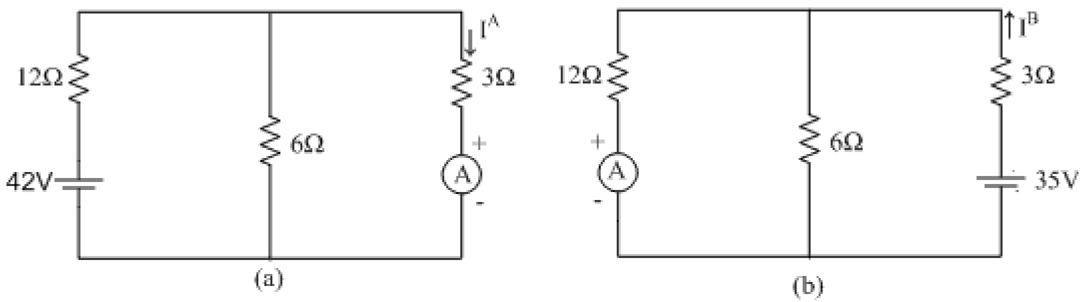


Figure 29: Network for Example 17

Solution: According to superposition I must be obtained by solving the following two circuits for I^A and I^B and the actual current obtained as $I = I^B - I^A$:



<u>Solving circuit (b) for I^B</u>

Total resistance $= 3 + 12//6 = 3 + \dfrac{12 \times 6}{8} = 7\Omega$

The required current I^B = total current $= \dfrac{35}{7} = 5$ A

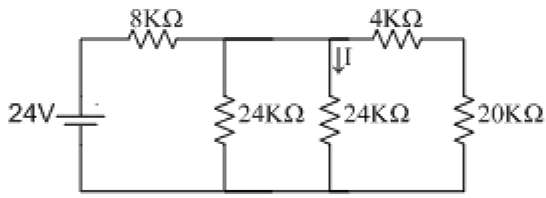
The reciprocity theorem will be applied when solving circuit (a) and to apply it we shall need the reading of the ammeter in circuit (b)

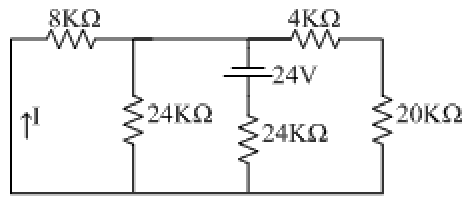
Ammeter reading $= \dfrac{6}{6+12} \times 5 = \dfrac{6}{18} \times 5 = \dfrac{5}{3}$A by current divider rule.

<u>Solving circuit (a) for I^A</u>

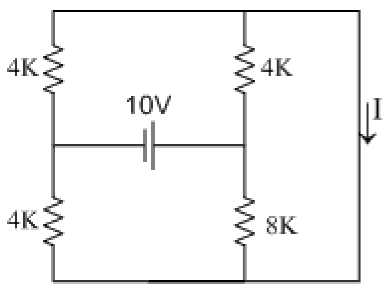
We observe that circuit (a) can be derived from circuit (b) by first interchanging the ammeter and the voltage source 35 V in the circuit and then changing the value 35 to 42. According to reciprocity theorem, when the ammeter and the 35-V source are interchanged, the ammeter should continue to read $\frac{5}{3}$A . If the voltage should change to 42 V, then the reading of the ammeter should by simple proportion also change to $\frac{42}{35} \times \frac{5}{3} = 2\,\text{A}$. Thus the ammeter in circuit (b) reads 2 A, or $I^A = 2$ A. Therefore by superposition $I = I^B - I^A = 5 - 2 = 3$ A.

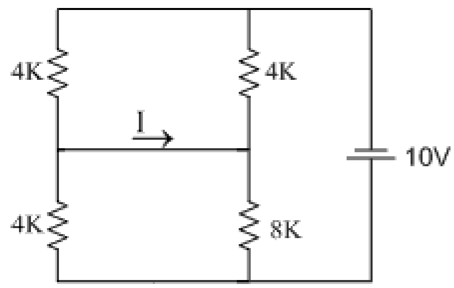
Exercise 9: Verify the reciprocity theorem using the circuits shown below.





Exercise 10: Repeat Exercise 9 for the circuits shown below.





7. Maximum Power Transfer Theorem

It states the following: A load will receive maximum power from a linear dc network when its total resistance is exactly equal to the Thevenin resistance of the network as "seen" by the load.

Example 18: Analysis of a transistor network resulted in the reduced circuit of Fig. 29. Find R_L necessary to transfer maximum power to R_L, and calculate the power P_L under these conditions.

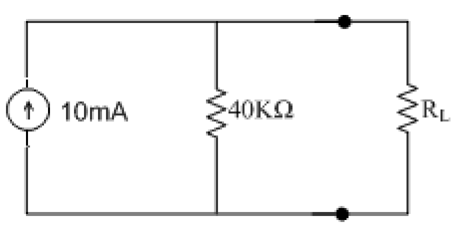


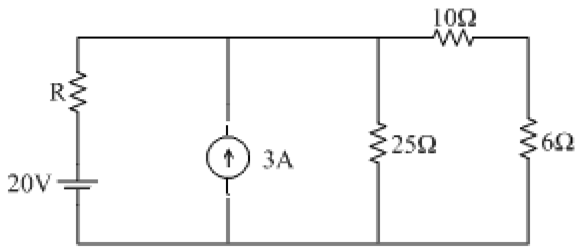
Figure 30: Network for Example 18

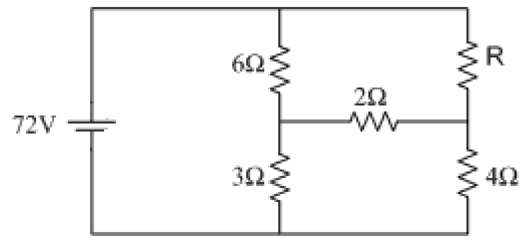
Solution:

$R_L = R_{th} = 40\,k\Omega$, $I_L = 10/2 = 5\,mA$ and

$P_{Lmax} = I_L^2 R_L = (5\times10^{-3})^2 \times 40\times10^3 = 1\,W$

Exercise 11: For each of the networks shown below, find the value of R for maximum power to R and determine the maximum power.





Ans : $9.756\,\Omega, 2.2\,W$ Ans : $2\,\Omega, 450\,W$

8. Millman's Theorem

This theorem may be used to reduce any number of parallel voltage sources to one. This would permit finding the current through or voltage across a load resistor. In general, Millman's theorem states the following: Any number of parallel voltage sources can be reduced to a single voltage source whose internal resistance R_{eq} and emf E_{eq} are given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots\ldots\ldots \frac{1}{R_N} \text{ and}$$

$$E_{eq} = \left[\pm \frac{E_1}{R_1} \pm \frac{E_2}{R_2} \pm \ldots\ldots\ldots \pm \frac{E_N}{R_N} \right] \times R_{eq}$$

The plus and minus signs appear to include those cases where the sources may not be supplying energy in the same direction.

Example 19: Using Millman's theorem, find the current through and the voltage across the resistor R_L of Fig. 31

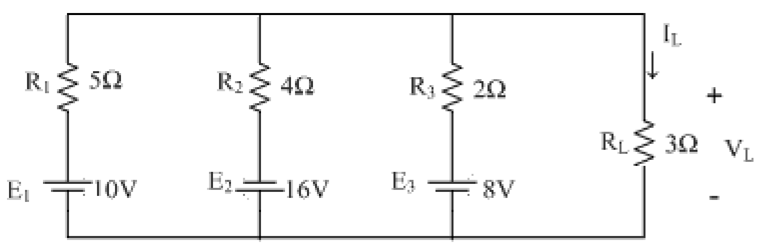


Figure 31: Network for Example 19

Solution:

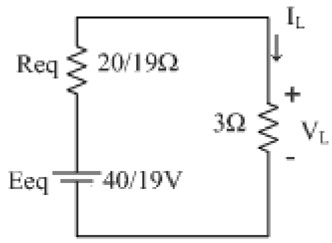
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{5} + \frac{1}{4} + \frac{1}{2} = \frac{4+5+10}{20} = \frac{19}{20}$$

.

Hence $R_{eq} = \dfrac{20}{19}\ \Omega$

$$E_{eq} = \left[\frac{E_1}{R_1} - \frac{E_2}{R_2} + \frac{E_3}{R_3} \right] \times R_{eq} = \left[\frac{10}{5} - \frac{16}{4} + \frac{8}{2} \right] \times \frac{20}{19} = \frac{40}{19} V$$

The equivalent circuit is shown below:



$$I_L = \frac{40}{19}\left[\frac{1}{\frac{20}{19}+3}\right] = \frac{40}{77} = 0.519\,\text{A} \text{ and}$$

$$V_L = I_L V_L = \frac{40}{77} \times 3 = 1.558\ \text{V}$$

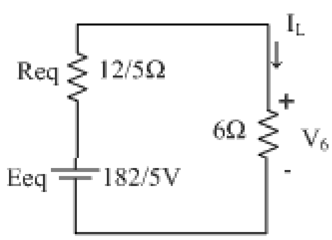
Example 20: Using Millman's theorem, find that current through and voltage across the 6-**Ω** resistor of Fig. 29.

Solution

$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{3} = \frac{1+4}{12} = \frac{5}{12}, R_{eq} = \frac{12}{5}\Omega = 2.4\ \Omega$$

$$E_{eq} = \left[\frac{42}{12} + \frac{35}{5}\right] \times \frac{12}{5} = \left[\frac{42+140}{12}\right] \times \frac{12}{5} = \frac{182}{5} = 36.4\,\text{V}$$

The equivalent circuit is as follows:



$$I = \frac{182}{5}\left[\frac{1}{\frac{12}{5}+6}\right] = \frac{182}{42} = \frac{13}{3} - 4.33\,\text{A}$$

$$V_6 = 6I = 6 \times \frac{13}{3} = 26\ \text{V}$$

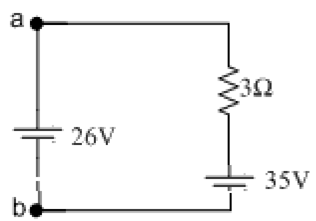
Example 21: For the circuit considered in Example 20, given the current in the 6-**Ω** resistor to be $\frac{13}{3}$ A .find the current supplied by the 35-V source using (a) Kirchoff's Laws (b) Substitution theorem.

Solution

(a) Let the current in the 3-Ω resistor be I_1 and the current in the 6-Ω resistor be I_2. Then applying KVL to the right loop, we obtain

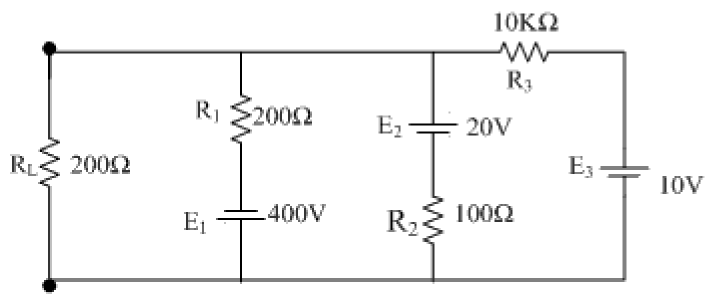
$$35 = 3I_1 + 6I_2 = 3I_1 + 6 \times \frac{13}{3} = 3I_1 + 26.\ \text{Hence } I_1 = \frac{9}{3} = 3\text{ A}$$

(b) The voltage drop across the 6-Ω resistor branch is 26 V. This branch if replaced by 26-V source will permit the right outer branch to be isolated as follows:



Thus the current supplied by the 35-V source $= \frac{35 - 26}{3} = \frac{9}{3} = 3\text{ A}$

Exercise 12: Using Millman's theorem, find the current and the voltage across the resistor R_L in the circuit below.



9. Duality

In a network analysis, we encounter pairs of network equations which exhibit an interesting symmetry of opposites called duality. For these pairs of equations, one may be derived from the other by replacing each symbol by its paired opposite: i by v, v by i, R by G, and G by R. These quantities are said to be duals of each other. That is, R is the dual of G, i is the dual of v. A table of some duals is given below:

Dual Quantities

1	v	i	7	Series	Parallel
2	R	G	8	Mesh	Node
3	Open Circuit	Short Circuit	9	KVL	KCL
4	Independent Voltage Source	Independent Current Source	10	V form	I form
5	VCVS	CCCS	11	Thevenin form	Norton form
6	CCVS	VCCS	12	Voltage divider	Current divider

Examples of dual equations

$v = Ri$	$i = Gv$: Replace v by i, i by v and R by G
$R_S = R_1 + R_2 + \ldots + R_N$ (Series resistance adds rule)	$G_P = G_1 + G_2 + \ldots + G_N$ (Parallel resistance adds rule)
$v = 0$ (Short Circuit)	$i = 0$ (Open Circuit)
Thevenin equivalent circuit (Series voltage source and resistor) Voltage across the load resistance, $V_L = \{R_L /(R_{TH} + R_L) \times E_{TH}\}$	Norton equivalent circuit (Parallel current source and resistor) Current through the load resistance, $I_L = \{G_L / (G_N + G_L) \times I_N\}$
Voltage across one of series connected resistances $V_R = \{R_1 /(R_1 + R_2 + \ldots + R_N) \times V\}$	Current through one of parallel connected conductances $I_R = \{G_1 /(G_1 + G_2 + \ldots + G_N) \times I\}$

Uses of duality

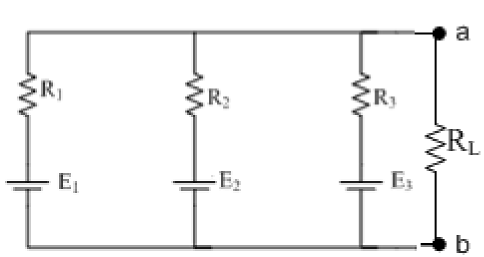
They include the following:

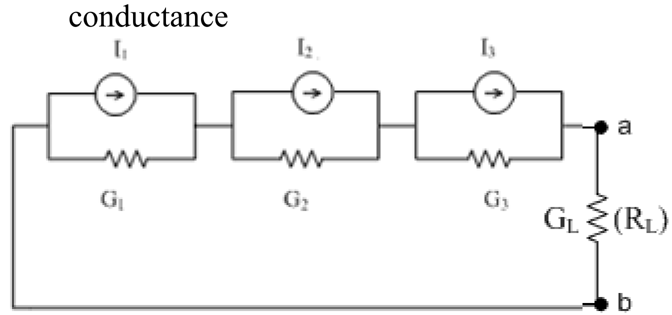
- It may be used to generalize a theoretical result immediately to its dual without further proof.
- It may be used to produce circuits whose currents have certain desirable properties from other circuits whose voltages have those properties or vice versa.
- It permits us to use one general method to deal with dual circuits.

Example 22: Obtain the dual of Millman's theorem

Solution

Millman's Theorem	Dual
Parallel connected voltage sources and resistor	Series connected current sources and conductance



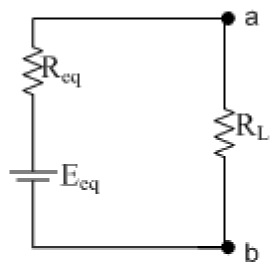


$$E_{eq} = \left[\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3}\right] \times R_{eq}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

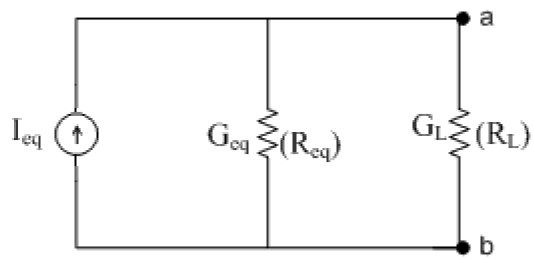
$$I_{eq} = \left[\frac{I_1}{G_1} + \frac{I_2}{G_2} + \frac{I_3}{G_3}\right] \times G_{eq}$$

$$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} = R_1 + R_2 + R_3$$



Voltage across load $= \dfrac{R_L}{R_{eq} + R_L} E_{eq}$

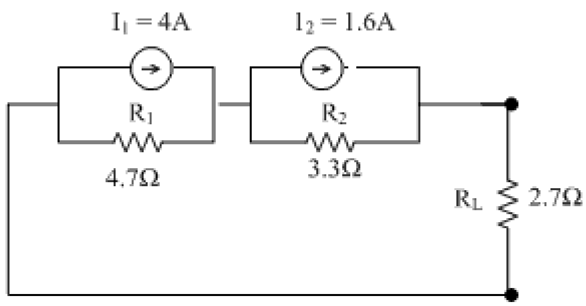
To prove, use Norton's Theorem



Current through load $= \dfrac{G_L}{G_{eq} + G_L} I_{eq}$

To prove, use Thevenin's Theorem

Exercise 13: using the dual of Millman's theorem, find the current through and voltage across the resistor R_L in the circuit below. Ans: 2.25 A, 6.075 V



Exercise 14: For the circuit below:

(a) Find the Thevenin equivalent circuit for that portion of the network to the left of the base (B) terminal.

(b) Using the fact that $I_C = I_E$ and $V_{CE} = 8$ V, determine the magnitude of I_E.

(c) Using the results of parts (a) and (b), calculate I_B if $V_{BE} = 0.7$ V.

(d) What is the voltage V_C?