CHAPTER ONE ASSIGNMENT 1

1. Determine the domain of the following function;
   $f(x,y) = ln(9 - x^2 - 9y^2)$

2. Show that the function f does not have a limit at $(0,0)$ by examining the limits of f as $(x,y) \to (0,0)$ along the line $y = x$ and along the parabola $y = x^2$. The function is given by
   $f(x,y) = \dfrac{x^2 y}{x^4 + y^2}, \quad (x,y) \neq (0,0).$

3. Show that the function f does not have a limit at $(0,0)$ by examining the limits of f as $(x,y) \to (0,0)$ along the curve $y = kx^2$ for different values of k. The function is given by
   $f(x,y) = \dfrac{x^2}{x^2 + y}, \quad x^2 + y \neq 0$

   For Problems 3-7 compute the limits of the functions $f(x,y)$ as $(x,y) \to (0,0)$. You may assume that polynomials, exponentials, logarithmic, and trigonometric functions are continuous.

4. $f(x,y) = x^2 + y^2$

5. $f(x,y) = \dfrac{x}{x^2 + 1}$

6. $f(x,y) = \dfrac{x + y}{(\sin y)} + 2$

7. $f(x,y) = \dfrac{\sin(x^2 + y^2)}{x^2 + y^2}$ [Hint: You may assume that $lim_{t \to 0} \dfrac{(\sin t)}{t}$=1].

   For the functions in Problems 8-10, show that $lim_{(x,y) \to (0,0)}$ does not exist.

8. $f(x,y) = \dfrac{x + y}{x - y}, \quad x \neq y$

9. $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$

10. $f(x,y) = \dfrac{xy}{\mid xy \mid}, \quad x \neq 0$ and $y \neq 0$