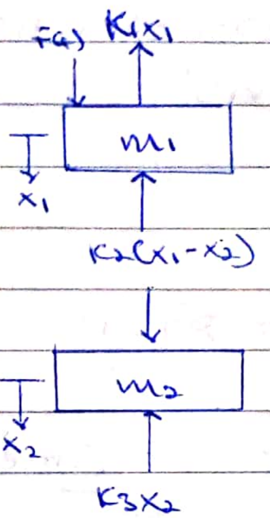
(1)



mass 1

$$m_1\ddot{x}_1 = -k_2(x_1-x_2) - k_1x_1 + F(t)$$

$$m_1\ddot{x}_1 + k_2(x_1-x_2) + k_1x_1 = F(t)$$

$$m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2x_2 = F(t) \quad \text{--- (1)}$$

mass 2

$$m_2\ddot{x}_2 = k_2(x_1-x_2) - k_3x_2$$

$$m_2\ddot{x}_2 + k_2(x_2-x_1) + k_3x_2 = 0$$

$$m_2\ddot{x}_2 - k_2x_1 + (k_2+k_3)x_2 = 0 \quad \text{--- (2)}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\ddot{x} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}x = \begin{bmatrix} F(t) \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 \\ 0 & 17 \end{bmatrix}\ddot{x} + \begin{bmatrix} 50000 & -20000 \\ -20000 & 36000 \end{bmatrix}x = \begin{bmatrix} 15\cos 3t \\ 0 \end{bmatrix}$$

$$\ddot{x} = -\omega^2 x \quad \text{let } \omega^2 = \lambda \quad \therefore \ddot{x} = -\lambda x$$

$$\begin{bmatrix} 7 & 0 \\ 0 & 17 \end{bmatrix}(-\lambda x) + \begin{bmatrix} 50000 & -20000 \\ -20000 & 36000 \end{bmatrix}x = \begin{bmatrix} 15\cos 3t \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} 50000-7\lambda & -20000 \\ -20000 & 36000-17\lambda \end{bmatrix}\right) X = 0$$

$$(50000-7\lambda)(36000-17\lambda) - 400\times10^6 = 0$$

$$180\times10^7 - 850000\lambda - 252000\lambda + 119\lambda^2 - 400\times10^6 = 0$$

$$119\lambda^2 - 1.102\times10^6\lambda + 1.4\times10^9 = 0$$

$$\lambda_1 = 7740.64$$

$$X_2 = 1519.86$$

$$\omega_{n_1} = \sqrt{7740.64} = \underline{\underline{87.98 \text{ rad/s}}}$$

$$\omega_{n_2} = \sqrt{1519.80} = \underline{\underline{38.98 \text{ rad/s}}}$$

Computing steady state Response.

$$\begin{bmatrix} 50-7\lambda & -20 \\ -20 & 36-17\lambda \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 15\cos 3t \\ 0 \end{Bmatrix}$$

$F_0 = 15\text{KN}$ $\omega = 3\text{rad/s}$ $\lambda = \omega^2$; $\omega^2 = 9$

$$\begin{bmatrix} 50-7(9) & -20 \\ -20 & 36-17(9) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 15 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} -13 & -20 \\ -20 & -117 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 15 \\ 0 \end{Bmatrix}$$

Let $[Z(\omega)]$ = Impedance matrix.

$$[Z(\omega)] = \begin{bmatrix} -13 & -20 \\ -20 & -117 \end{bmatrix}$$

$$\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = [Z(\omega)]^{-1} \begin{Bmatrix} 15 \\ 0 \end{Bmatrix}$$

Computing $[Z(\omega)]^{-1}$

$$[Z(\omega)] = \begin{bmatrix} -13 & -20 \\ -20 & -117 \end{bmatrix}$$

$$\det[Z(\omega)] = 1521 - 400 = 1121$$

$$\text{Adjoint } [Z(\omega)] = \begin{bmatrix} -117 & 20 \\ 20 & -13 \end{bmatrix}$$

$$[Z(\omega)]^{-1} = \frac{1}{\det[Z(\omega)]} \begin{bmatrix} -117 & 20 \\ 20 & -13 \end{bmatrix} = \frac{1}{1121} \begin{bmatrix} -117 & 20 \\ 20 & -13 \end{bmatrix}$$

$$[Z(\omega)]^{-1} = \begin{bmatrix} -117/1121 & 20/1121 \\ 20/1121 & -13/1121 \end{bmatrix}$$

$$\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}_0 = \begin{bmatrix} -\frac{117}{1121} & \frac{20}{1121} \\ \frac{20}{1121} & -\frac{13}{1121} \end{bmatrix} \begin{Bmatrix} 15 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 1.5656 \\ 0.2676 \end{Bmatrix}$$

$$\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}_0 = \begin{Bmatrix} 1565.6 \\ 267.6 \end{Bmatrix}$$

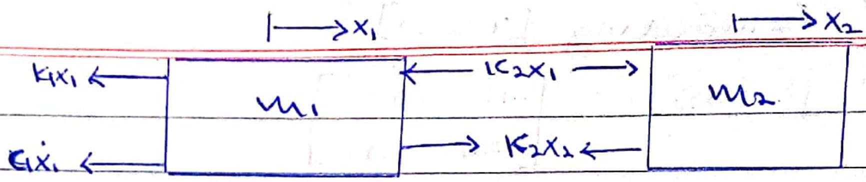
$\therefore$ Responce $= \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}_0 \cos \omega t$

$$x(t) = \begin{Bmatrix} 1565.6 \\ 267.6 \end{Bmatrix} \cos 3t$$

$\therefore$ The steady state responce is given as

$$\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = \begin{Bmatrix} 1565.6 \\ 267.6 \end{Bmatrix} \cos 3t$$

(2)



mass 1

$$m_1\ddot{x}_1 = -k_1x_1 - c_1\dot{x}_1 - k_2x_1 + k_2x_2$$

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + (k_1+k_2)x_1 - k_2x_2 = 0 \quad -(1)$$

mass 2

$$m_2\ddot{x}_2 = -k_2x_2 + k_2x_1$$

$$m_2\ddot{x}_2 - k_2x_1 + k_2x_2 = 0 \quad -(2)$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\ddot{x} + \begin{bmatrix} c_1 & 0 \\ 0 & 0 \end{bmatrix}\dot{x} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}x = 0$$

Computing Natural frequency.

$$\begin{bmatrix} 80 & 0 \\ 0 & 100 \end{bmatrix}\ddot{x} + \begin{bmatrix} 300 & -200 \\ -200 & 200 \end{bmatrix}x = 0$$

$$\left(\begin{bmatrix} 300-80\lambda & -200 \\ -200 & 200-100\lambda \end{bmatrix}\right)x = 0$$

$$(300-80\lambda)(200-100\lambda) - 4\times10^4 = 0$$

$$60000 - 30000\lambda - 16000\lambda + 8000\lambda^2 - 4\times10^4 = 0$$

$$8000\lambda^2 - 46000\lambda + 20000 = 0$$

$$\lambda_1 = 5.27617\times10^3$$

$$\lambda_2 = 0.47383\times10^3$$

$$\omega_{n_1} = \sqrt{5.27617\times10^3} = 72.637\,rad/s$$

$$\omega_{n_2} = \sqrt{0.47383\times10^3} = 21.768\,rad/s$$

$\therefore$ Natural frequencies are.

ω_{n_1} = 72.637 rad/s and 21.768 rad/s

P.T.O

Computing modal matrix

$$\lambda_1 = 473.83$$

$$\begin{bmatrix} 300000 - 80(473.83) & -200000 \\ -200000 & 200000 - 100(473.83) \end{bmatrix} \begin{Bmatrix} u_{11} \\ u_{21} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 262093.6 & -200000 \\ -200000 & 152617 \end{bmatrix} \begin{Bmatrix} u_{11} \\ u_{21} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$262093.6\,u_{11} - 200000\,u_{21} = 0$$

$$u_{11} = 1$$

$$u_{21} = \frac{262093.6}{200000} = 1.31$$

$$\begin{Bmatrix} u_{11} \\ u_{21} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1.31 \end{Bmatrix}$$

$$\lambda_2 = 5276.17$$

$$\begin{bmatrix} 300000 - 80(5276.17) & -200000 \\ -200000 & 200000 - 100(5276.17) \end{bmatrix} \begin{pmatrix} u_{12} \\ u_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} -122093.6 & -200000 \\ -200000 & -327617 \end{bmatrix} \begin{pmatrix} u_{12} \\ u_{21} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-122093.6\,u_{12} - 200000\,u_{22} = 0$$

$$u_{12} = 1$$

$$u_{22} = \frac{-122093.6}{200000} = -0.61$$

$$\begin{pmatrix} u_{12} \\ u_{22} \end{pmatrix} = \begin{pmatrix} 1 \\ -0.61 \end{pmatrix}$$

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modal matrix $[P] = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1.31 & -0.61 \end{bmatrix}$

$V_1 = \begin{pmatrix} u_{11} \\ u_{21} \end{pmatrix} = \begin{pmatrix} 1 \\ 1.31 \end{pmatrix}$ $\quad V_2 = \begin{pmatrix} u_{12} \\ u_{22} \end{pmatrix} = \begin{pmatrix} 1 \\ -0.61 \end{pmatrix}$

Given general response

$A \sin(\omega_n t + \phi)$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} A_1 \sin(\omega_n t + \phi_1) + A_2 \sin(\omega_n t + \phi_2) \\ 1.31 A_1 \sin(\omega_n t + \phi_1) + (-0.61) A_2 \sin(\omega_n t + \phi_2) \end{bmatrix}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} A_1 \sin(21.77t + \phi_1) + A_2 \sin(72.64t + \phi_2) \\ 1.31 A_1 \sin(21.77t + \phi_1) - 0.61 A_2 \sin(72.64t + \phi_2) \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 21.77 A_1 \cos(21.77t + \phi_1) + 72.64 A_2 \cos(72.64t + \phi_2) \\ 28.52 A_1 \cos(21.77t + \phi_1) - 44.31 A_2 \cos(72.64t + \phi_2) \end{bmatrix}$$

Applying conditions.

$x(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ cm $\quad \dot{x}(0) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ cm/s

$1 = A_1 \sin\phi_1 + A_2 \sin\phi_2$ — (1)

$3 = 1.31 A_1 \sin\phi_1 + 0.61 A_2 \sin\phi_2$ — (2)

$0 = 21.77 A_1 \cos\phi_1 + 72.64 A_2 \cos\phi_2$ — (3)

$-1 = 28.52 A_1 \cos\phi_1 - 44.31 A_2 \cos\phi_2$ — (4)

Solving simultaneously.

$\phi_1 = \phi_2 = \frac{\pi}{2}$

$A_1 = 0.127$ cm

$A_2 = 0.8730$ cm

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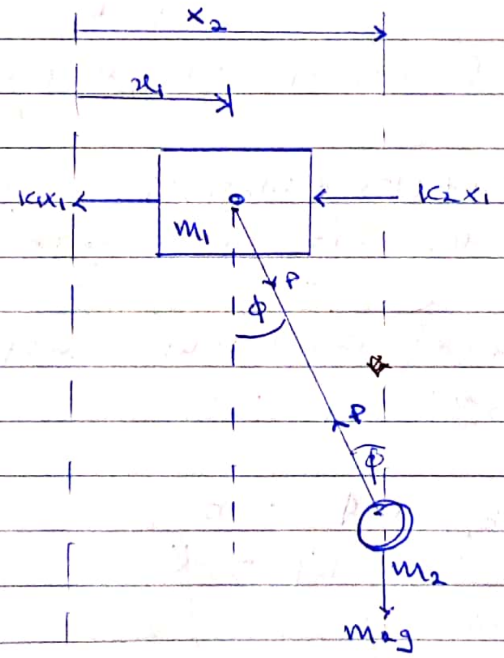
Free Responce

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0.127\sin\left(21.77t + \frac{\pi}{2}\right) + 0.875\sin\left(72.64t + \frac{\pi}{2}\right) \\ 0.166\sin\left(21.77t + \frac{\pi}{2}\right) - 0.534\sin\left(72.6t + \pi/2\right) \end{bmatrix}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0.127\sin\left(21.77t + \pi/2\right) + 0.875\sin\left(72.64t + \pi/2\right) \\ 0.166\sin\left(21.77t + \pi/2\right) - 0.534\sin\left(72.64t + \pi/2\right) \end{bmatrix}$$

3



Equation of motion of trolley

$$m_1\ddot{x}_1 = -K_1x_1 - K_2x_1 + P\sin\phi$$

$$m_1\ddot{x}_1 + (K_1+K_2)x_1 - P\sin\phi = 0 \quad \text{--- (1)}$$

Equation of simple pendulum

$$m_2l^2\ddot{\phi} = m_2gl\sin\phi$$

$$m_2l^2\ddot{\phi} - m_2gl\sin\phi = 0 \quad \text{--- (2)}$$

$$P = m_2g\cos\phi \quad \text{--- (*)} \quad ; \quad P = m_2g$$

$$\sin\phi = \frac{x_2 - x_1}{L} \quad \text{--- (*)} \quad ; \quad \phi = \frac{x_2 - x_1}{L}$$

For small angles
$\cos\phi = 1$ and $\sin\phi = \phi$

Equation of trolley transforms to

$$m_1\ddot{x}_1 + (k_1+k_2)x_1 - m_2 g\cos\phi\sin\phi = 0$$

$$m_1\ddot{x}_1 + (k_1+k_2)x_1 - m_2 g\phi = 0$$

$$m_1\ddot{x}_1 + (k_1+k_2)x_1 - m_2 g\left(\frac{x_2-x_1}{L}\right) = 0$$

$$m_1\ddot{x}_1 \cdot L + L(k_1+k_2)x_1 - m_2 g x_2 + m_2 g x_1 = 0$$

$$m_1\ddot{x}_1 L + (Lk_1 + Lk_2 + m_2 g)x_1 - m_2 g x_2 = 0 \quad \text{— (1)}$$

$$m_1\ddot{x}_1 + (k_1 + k_2 + \frac{m_2 g}{L})x_1 - \frac{m_2 g x_2}{L} = 0 \quad \text{— (1a)}$$

Equation of Pendulum transforms into

$$m_2 L^2\ddot{\phi} - m_2 g L\sin\phi = 0$$

$$m_2 L\ddot{\phi} - m_2 g\sin\phi = 0$$

but $L\ddot{\phi} = \ddot{x}$

$$m_2\ddot{x}_2 + m_2 g\phi = 0$$

$$m_2\ddot{x}_2 + m_2 g\left(\frac{x_2-x_1}{L}\right) = 0$$

$$Lm_2\ddot{x}_2 + m_2 g x_2 + m_2 g x_1 = 0$$

$$Lm_2\ddot{x}_2 + m_2 g x_1 + m_2 g x_2 = 0 \quad \text{— (2)}$$

$$m_2\ddot{x}_2 + \frac{m_2 g}{L}x_1 + \frac{m_2 g}{L}x_2 = 0 \quad \text{— (2a)}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \ddot{x} + \begin{bmatrix} k_1+k_2+\frac{m_2 g}{L} & -\frac{m_2 g}{L} \\ -\frac{m_2 g}{L} & \frac{m_2 g}{L} \end{bmatrix} x = 0$$

$$\begin{bmatrix} 100 & 0 \\ 0 & 10 \end{bmatrix} (-\lambda x) + \begin{bmatrix} 4000+\frac{10\times 9.81}{2} & -\frac{10\times 9.81}{2} \\ -\frac{10\times 9.81}{2} & \frac{10\times 9.81}{2} \end{bmatrix} x = 0$$

$$\left(\begin{bmatrix} 4049.05-100\lambda & -49.05 \\ -49.05 & 49.05-10\lambda \end{bmatrix} \right) x = 0$$

$$(4049.05-100\lambda)(49.05-10\lambda) - 2405.9025 = 0$$

$$198605.9025 - 40490.5\lambda - 4905\lambda + 1000\lambda^2 - 2405.9025 = 0$$

$$1000\lambda^2 - 45395.5\lambda + 196200 = 0$$

$$\lambda_1 = 40.558$$

$$\lambda_2 = 4.838$$

$$\omega_{n1} = \sqrt{4.838} = 2.1995 \text{ rad/s} = \underline{\underline{2.2 \text{ rad/s}}}$$

$$\omega_{n2} = \sqrt{40.558} = \underline{\underline{6.37 \text{ rad/s}}}$$

Computing mode shape.

$$\lambda_1 = 4.838$$

$$\begin{bmatrix} 4049.05-100(4.838) & -49.05 \\ -49.05 & 49.05-10(4.838) \end{bmatrix} \begin{pmatrix} u_{11} \\ u_{21} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} 3565.25 & -49.05 \\ -49.05 & 0.67 \end{bmatrix} \begin{pmatrix} u_{11} \\ u_{21} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$3565.25 u_{11} - 49.05 u_{21} = 0$$

$$u_{11} = 1$$

$$u_{21} = \frac{3565.25}{49.05} = 72.69$$

$$\begin{pmatrix} u_{11} \\ u_{21} \end{pmatrix} = \begin{pmatrix} 1 \\ 72.69 \end{pmatrix}$$

$$\lambda_1 = 40.558$$

$$\begin{bmatrix} 4049.05 - 100(40.558) & -49.05 \\ -49.05 & 49.05 - 10(40.558) \end{bmatrix} \begin{pmatrix} u_{12} \\ u_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} -6.75 & -49.05 \\ -49.05 & -356.53 \end{bmatrix} \begin{pmatrix} u_{12} \\ u_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$u_{12} = 1$$

$$-6.75 u_{12} = 49.05 u_{22}$$

$$u_{22} = -\frac{6.75}{49.05} = -0.138$$

$$\begin{pmatrix} u_{12} \\ u_{22} \end{pmatrix} = \begin{pmatrix} 1 \\ 0.138 \end{pmatrix}$$

$$\text{modal matrix} = \begin{bmatrix} 1 & 1 \\ 72.69 & -0.138 \end{bmatrix}$$

General Response is given as

$A \sin(\omega_n t + \phi)$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} A_1 \sin(\omega_n t + \phi_1) + A_2 \sin(\omega_n t + \phi_2) \\ 72.69 A_1 \sin(\omega_n t + \phi_1) - 0.138 A_2 \sin(\omega_n t + \phi_2) \end{bmatrix}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} A_1 \sin(2.2t+\phi_1) + A_2 \sin(6.37t+\phi_2) \\ 72.69 A_1 \sin(2.2t+\phi_1) - 0.138 A_2 \sin(6.37t+\phi_2) \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 2.2 A_1 \cos(2.2t+\phi_1) + 6.37 A_2 \cos(6.37t+\phi_2) \\ 159.918 A_1 \cos(2.2t+\phi_1) - 0.879 A_2 \cos(6.37t+\phi_2) \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 2.2 A_1 \cos(2.2t+\phi_1) + 6.37 A_2 \cos(6.37t+\phi_2) \\ 159.918 A_1 \cos(2.2t+\phi_1) - 0.879 A_2 \cos(6.37t+\phi_2) \end{bmatrix}$$

Applying initial conditions.

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{cm} \qquad \dot{x}(0) = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \text{cm/s}$$

~~1 = A1 sin (2.2t~~

$$1 = A_1 \sin\phi_1 + A_2 \sin\phi_2 \quad \text{--- (1)}$$

$$0 = 72.69 A_1 \sin\phi_1 - 0.138 A_2 \sin\phi_2 \quad \text{--- (2)}$$

$$-2 = 2.2 A_1 \cos\phi_1 + 6.37 A_2 \cos\phi_2 \quad \text{--- (3)}$$

$$0 = 159.918 A_1 \cos\phi_1 - 0.879 A_2 \cos\phi_2 \quad \text{--- (4)}$$