



ME 164 - STATICS OF SOLID MECHANICS / ME 162 - BASIC MECHANICS

D.E.K. Dzebre (PhD)

Department of Mechanical Engineering.

LECTURE 1

Recap of:
Some Fundamental Concepts
Resultants of Forces
Moments of Forces
Equivalent Force Couple Systems

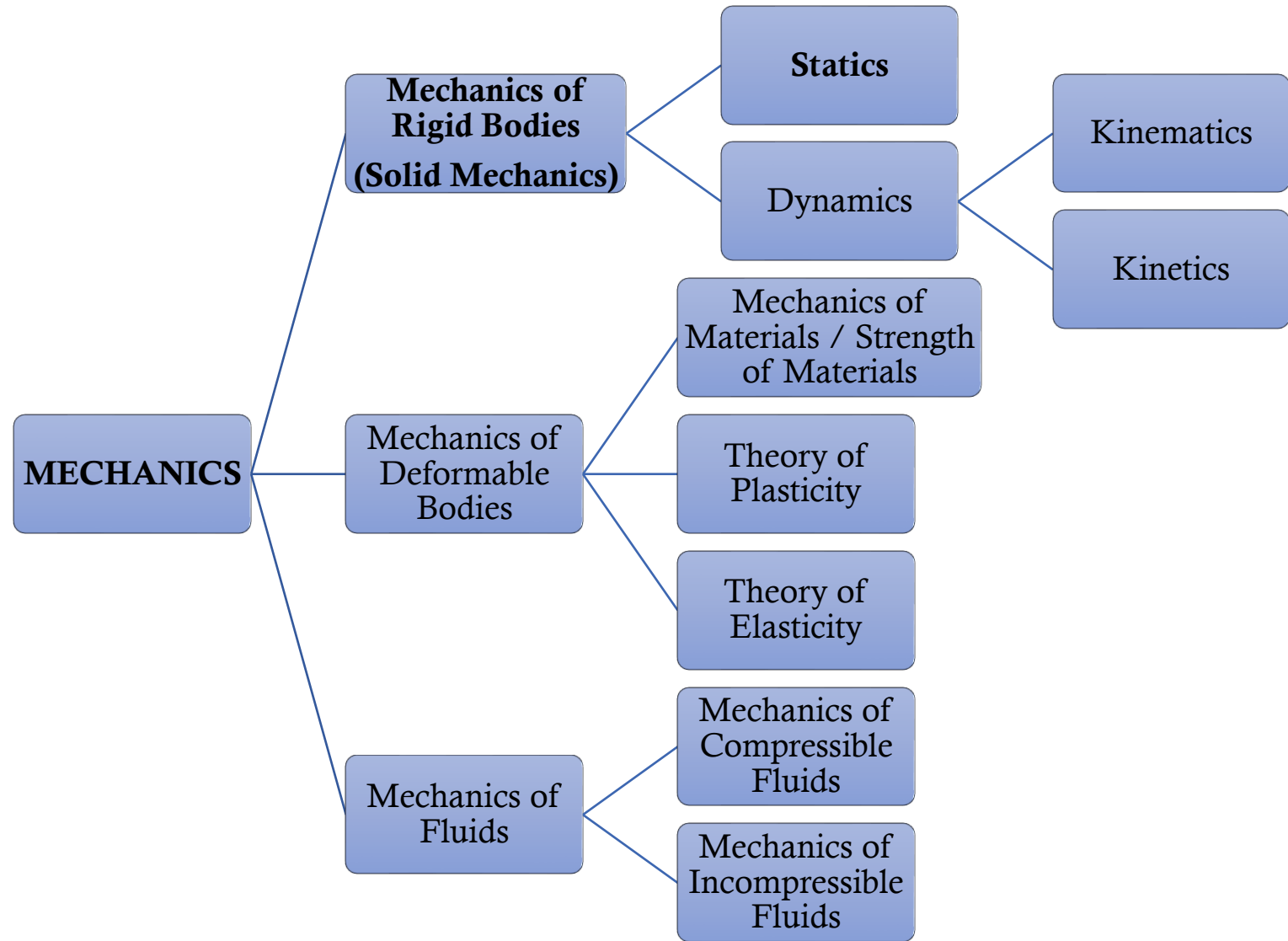
Introduction

- To design → to specify (size, type, shape etc).
- Forces (physical, chemical, etc) are a major consideration in engineering design.
- The study of the interactions between physical forces and bodies constitutes the field of Engineering *Mechanics*



Introduction

**Some Branches of
Engineering Mechanics
(J Antonio,)**



SOME FUNDAMENTAL PRINCIPLES & CONCEPTS



Fundamental Principles & Concepts

➤ Particle

A very small amount of matter which may be assumed to occupy a single point in space. Idealizing bodies as points simplifies problems since body geometry is not considered.

➤ Rigid body

A collection of several particles that remain at a fixed distance from each other, even when under the influence of a load.



Fundamental Principles & Concepts

➤ Space (Length)

This is associated with notion of describing a point in terms of co-ordinates measured from a reference point.

➤ Time

A measure of the succession of two events or the duration of an event. Of significance in dynamics.

➤ Mass

A measure of the inertia of a body, which is its resistance to a change of velocity. The mass of a body affects the gravitational attraction force between it and other bodies.

➤ Force

➤ **Space, Time, and Mass** are **independent** of each other. However, **Force**, is **related to** the **mass** of a body and the variation of its **velocity** with time.



Fundamental Principles & Concepts

Quantities

- Measurements.
 - Time – second (s)
 - Mass – kilogram (kg)
 - Length – metre (m)
 - Force – Newton (N)
- Quantities may be basic (fundamental) or derived.
- They may be scalars or vectors.
- Derived Units of measurement and Relations (formulae) are often in terms of fundamental quantities.

Fundamental Principles & Concepts

Quantity	Dimension	Common SI Units
Length	L	m, cm, mm
Time	T	s
Mass	M	kg
Area	L^2	m^2 , cm^2 , mm^2
Force	ML/T^2 or MLT^{-2}	N
Linear velocity	L/T or LT^{-1}	m/s or ms^{-1}
Linear acceleration	L/T^2 or LT^{-2}	m/s^2 or ms^{-2}
Angular velocity	$1/T$ or T^{-1}	rad/s
Angular acceleration	$1/T^2$ or T^{-2}	rad/s^2
Moment of a force	ML^2/T^2 or ML^2T^{-2}	N.m or N-m
Pressure, Stress	M/LT^2 or $ML^{-1}T^{-2}$	Pa, kPa, MPa
Work and Energy	ML^2/T^2 or ML^2T^{-2}	J, kJ
Power	ML^2/T^3 or ML^2T^{-3}	W, kW
Momentum and linear impulse	ML/T or MLT^{-1}	N.s or N-s



Fundamental Principles & Concepts

Prefixes

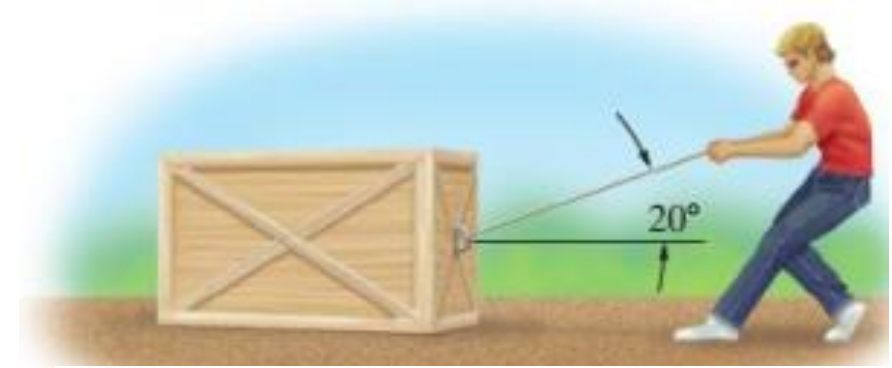
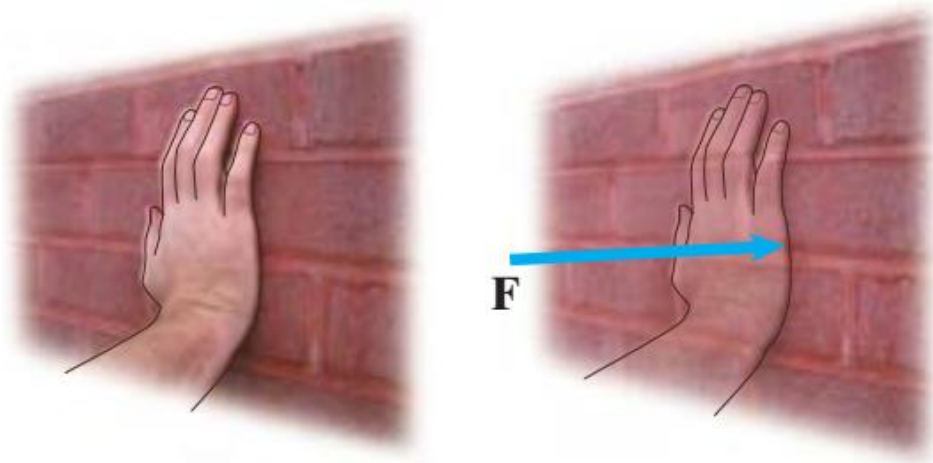
Multiplication Factor	Name of prefix	Symbol of prefix	Example
10^{12}	tera	T	1.23 TJ = 1 230 000 000 000 J
10^9	giga	G	4.53 GPa = 4 530 000 000 Pa
10^6	mega	M	7.68 MW = 7 680 000 W
10^3	kilo	k	5.46 kg = 5 460 g
10^{-2}	centi	c	3.34 cm = 0.0334 m
10^{-3}	milli	m	395 mm = 0.395 m
10^{-6}	micro	μ	65 μ m = 0.000 065 m
10^{-9}	nano	n	34 nm = 0.000 000 034 m

FORCES

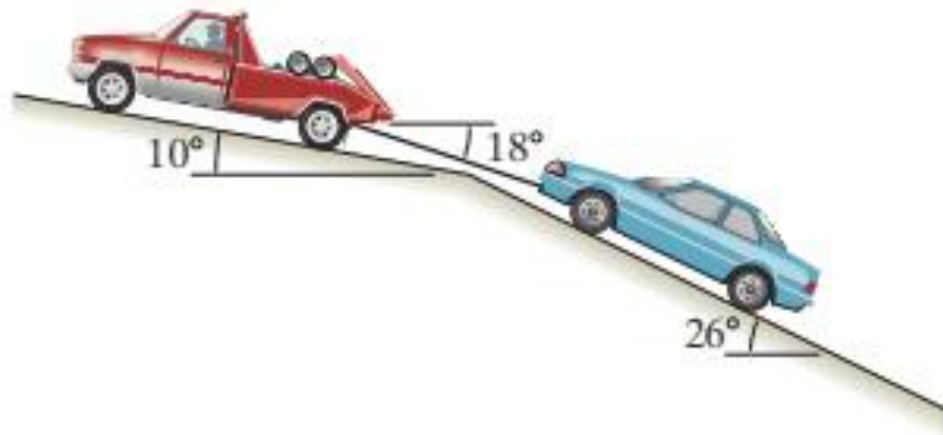
Characteristics and Resultants of Forces



Forces

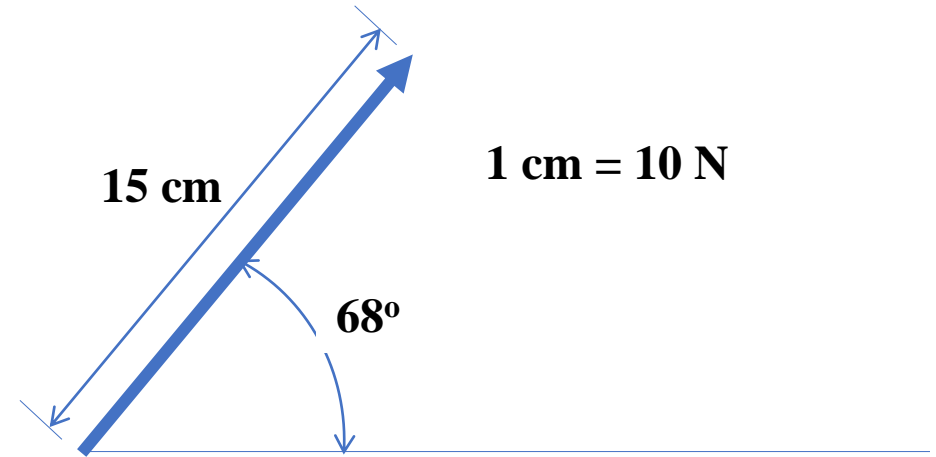
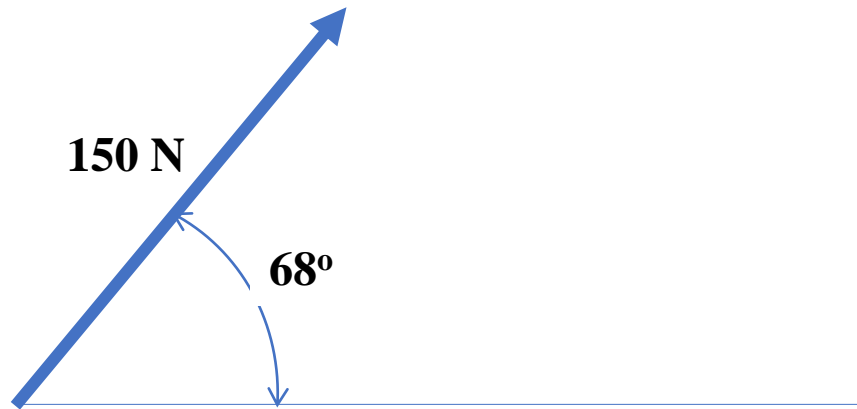


I'm strong



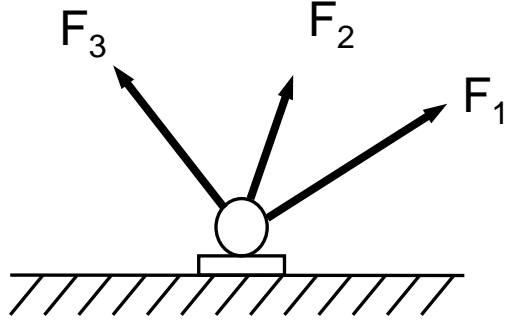
Forces: Some Characteristics

- Forces are vector quantities.

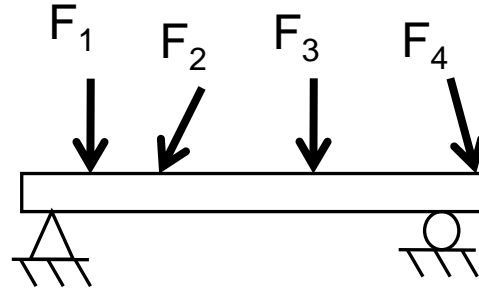


- Forces considered equal if they have the same magnitude and direction.
- Forces are equivalent if they produce the same resultant effect on a rigid body.

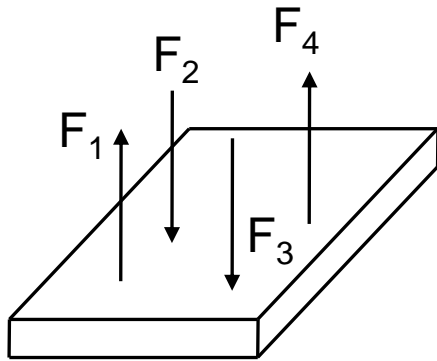
Systems of Forces



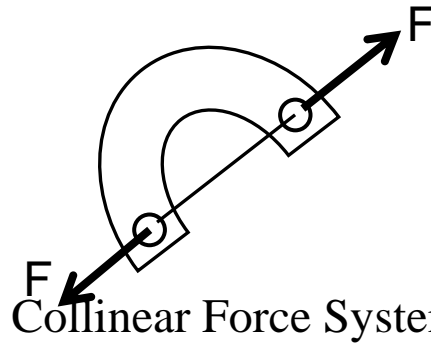
Concurrent Coplanar Force System



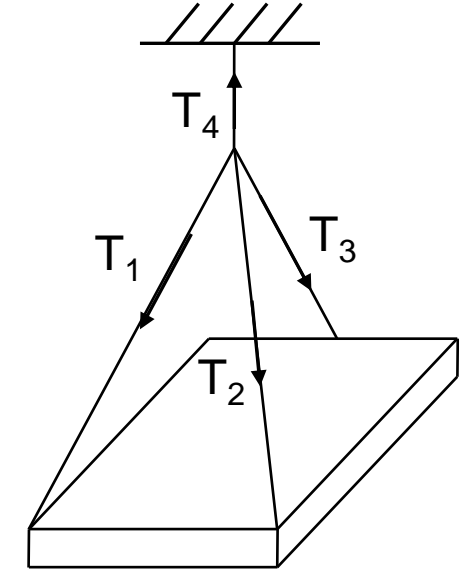
Non-Concurrent
Coplanar Force System



Parallel spatial Force System



Collinear Force System



Concurrent Spatial Force System

A system of forces can be replaced with an equivalent system of forces or a Resultant Force (one equivalent force).

HOW DO WE DETERMINE THE RESULTANT OF A SYSTEM OF FORCES?



The Resultant of a Systems of Forces

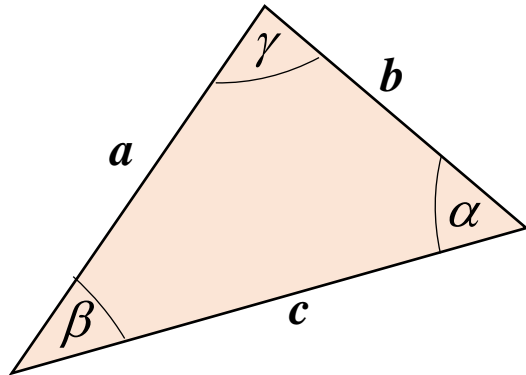
There are different approaches:

- Graphical approach – Parallelogram, Triangle or Polygon rules of vector addition.
- Trigonometric (Force Triangle) approach – Sine and Cosine rules.
- Force Components approach using
 - ✓ Rectangular/Perpendicular components from Pythagoras theorem
 - ✓ Rectangular/perpendicular components from Directional vectors



Resultants of Forces: Force Triangle Approach

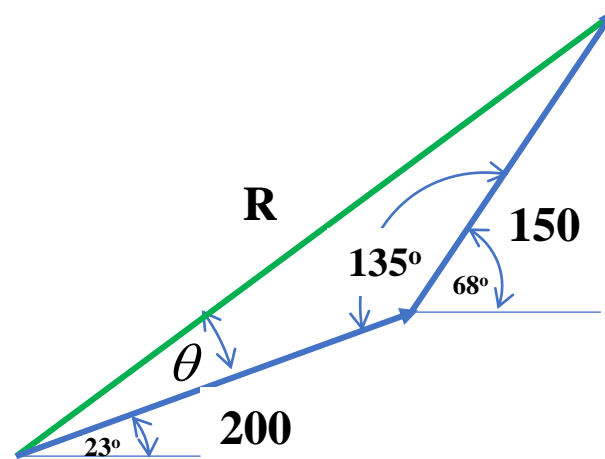
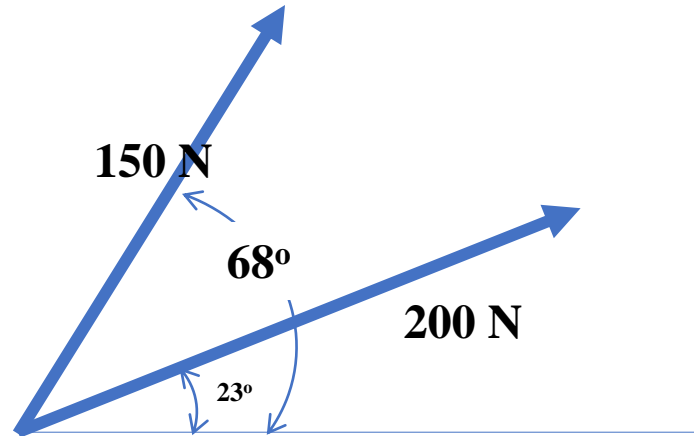
- The sine and cosine laws:



Law of Sines	$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$
Law of cosines	$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos \alpha \\ b^2 &= c^2 + a^2 - 2ca \cos \beta \\ c^2 &= a^2 + b^2 - 2ab \cos \gamma \end{aligned}$

Resultants of Forces: Force Triangle Approach

Example



From the Cosine Rule,

$$R^2 = 200^2 + 150^2 - 2(200)(150)\cos 135^\circ$$

$$R = 323.9 \text{ N.}$$

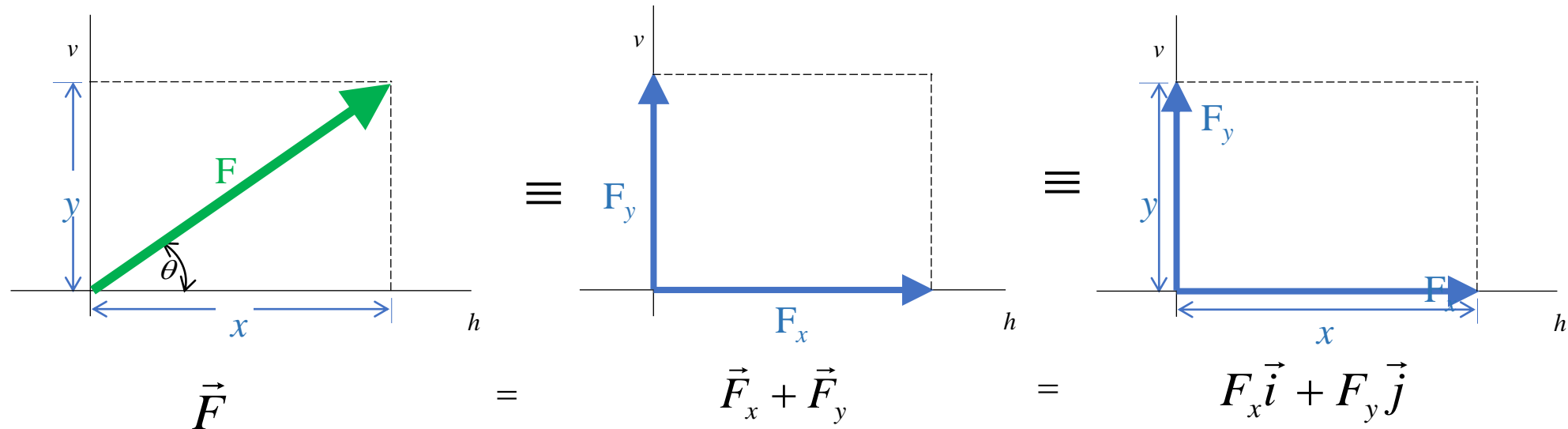
From the Sine Law

$$\frac{323.9 \text{ N}}{\sin 135^\circ} = \frac{150 \text{ N}}{\sin \theta^\circ}, \quad \theta = 19.11^\circ$$



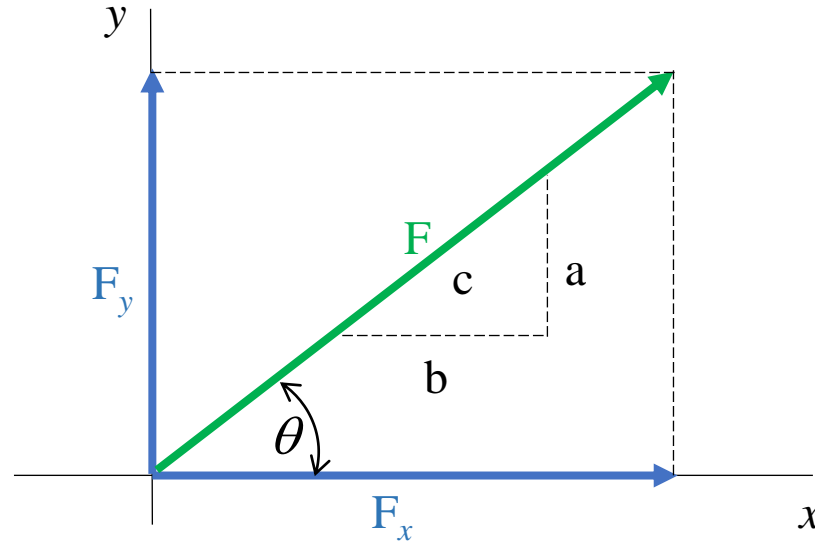
Resultants of Forces: Force Components Approach

- This approach requires the forces to be decomposed/resolved into Rectangular/Cartesian/perpendicular components (along principal directions).



- Like components (along the same direction) are then summed to get the resultant components.
- Magnitude and direction of the resultant force can be obtained through appropriate Trigonometry techniques.

Resultants of Forces: Resolve Forces into Components



$$\begin{aligned}\vec{F} &= \vec{F}_x + \vec{F}_y \\ &= F\vec{i} + F\vec{j}\end{aligned}$$

Pythagoras Theorem

$$\begin{aligned}\vec{F} &= \vec{F}_x + \vec{F}_y \\ \vec{F} &= F \cos \theta + F \sin \theta\end{aligned}$$

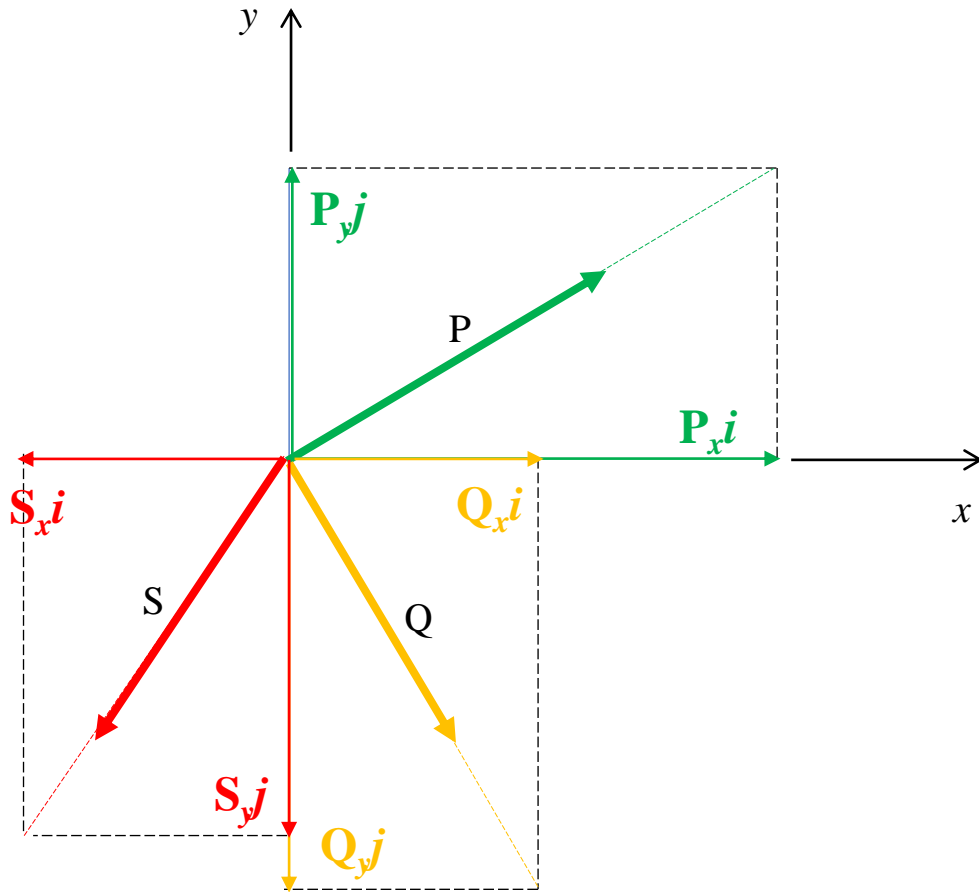
Directional Vectors

$$\begin{aligned}\vec{F} &= F\lambda = F\left(\frac{b\vec{i} + a\vec{j}}{\sqrt{b^2 + a^2}}\right) \\ &= F\left(\frac{b\vec{i}}{\sqrt{b^2 + a^2}}\right) + F\left(\frac{a\vec{j}}{\sqrt{b^2 + a^2}}\right) \\ &= F\vec{i} + F\vec{j}\end{aligned}$$



Resultants of Forces: Resolve Forces into Components

- Sum like components to get the components of the resultant.



$$\vec{R} = \sum F_x + \sum F_y$$

$$\vec{R} = (P_x + Q_x + S_x)\vec{i} + (P_y + Q_y + S_y)\vec{j}$$

The magnitude of the Resultant Force is given by;

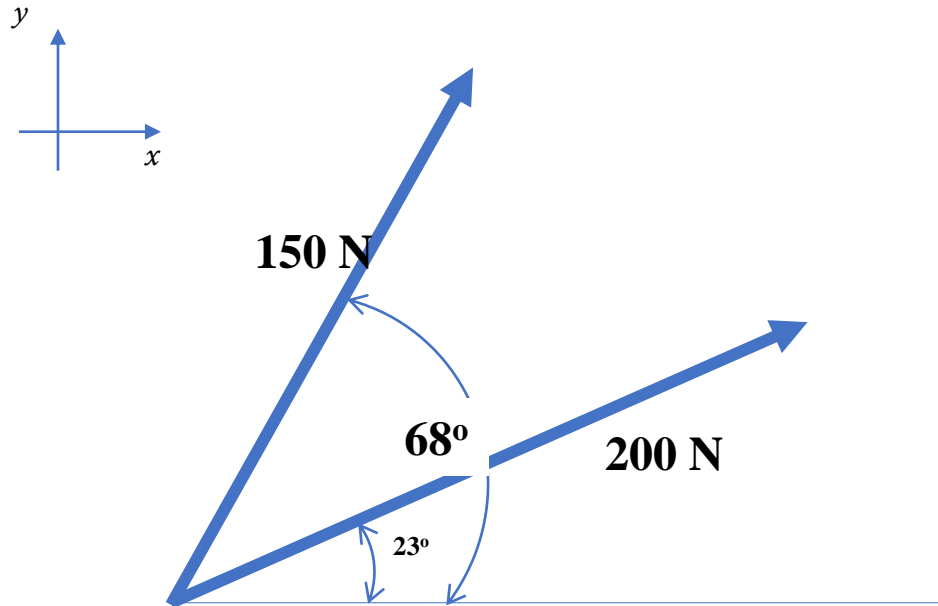
$$R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}$$

And the direction;

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right)$$

Resultants of Forces

Example

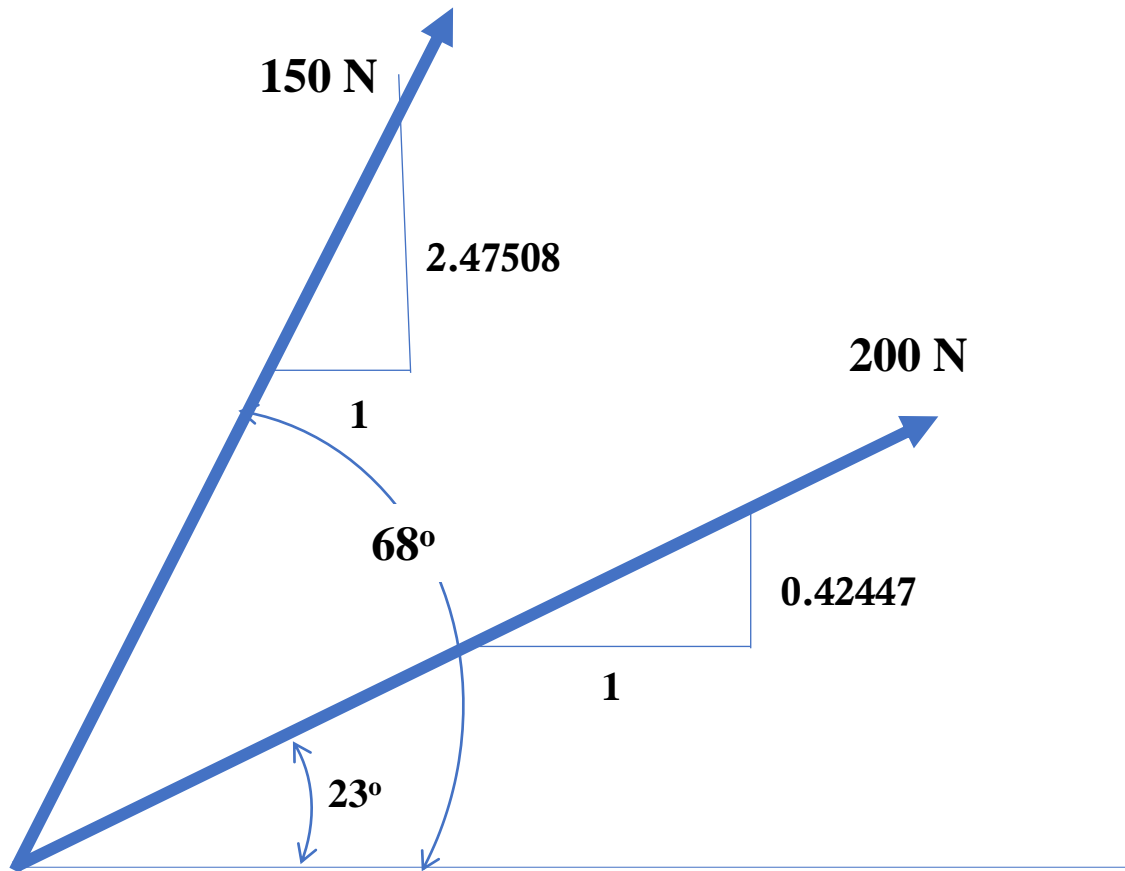


Force	F_x	F_y
200 N	$200 \cos 23^\circ$ N	$200 \sin 23^\circ$ N
150 N	$150 \cos 68^\circ$ N	$150 \sin 69^\circ$ N
Σ	240.292 N	217.224 N

$$\begin{aligned} F &= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \\ &= \sqrt{240.292^2 + 217.224^2} \\ &= 323.924 \text{ N} \end{aligned}$$

Resultants of Forces

Example



Recall:

$$\begin{aligned}\vec{F} &= F \lambda = F \left(\frac{b\vec{i} + a\vec{j}}{\sqrt{b^2 + a^2}} \right) \\ &= F \left(\frac{b\vec{i}}{\sqrt{b^2 + a^2}} \right) + F \left(\frac{a\vec{j}}{\sqrt{b^2 + a^2}} \right)\end{aligned}$$

$$F = \sqrt{\left(\sum F\vec{i} \right)^2 + \left(\sum F\vec{j} \right)^2}$$

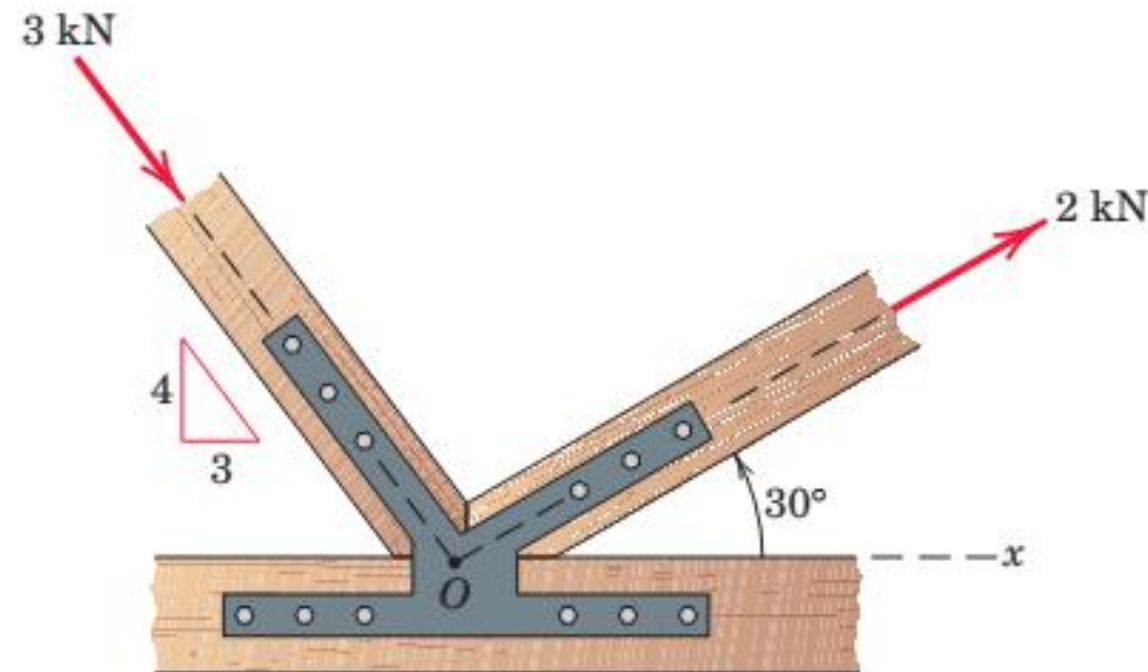
$$\sum F\vec{i} = 150 \left(\frac{1}{2.669} \right) \vec{i} + 200 \left(\frac{1}{2.622} \right) \vec{i} =$$

$$\sum F\vec{j} = 150 \left(\frac{2.475}{2.669} \right) \vec{j} + 200 \left(\frac{0.424}{2.622} \right) \vec{j} =$$

Resultants of Forces

Example

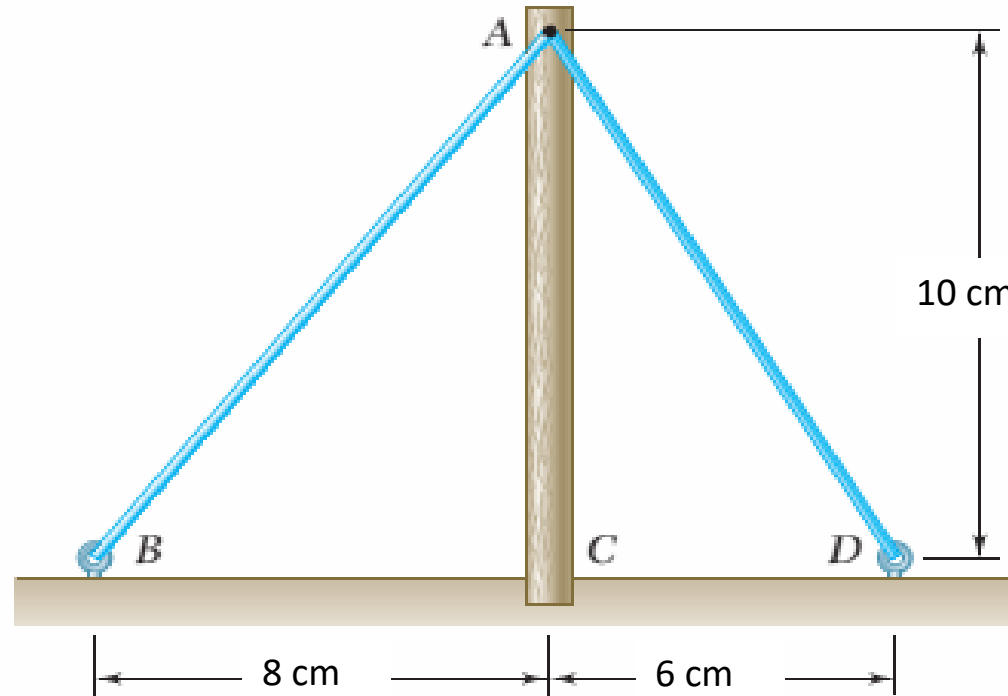
The two structural members, one of which is in tension and other in compression, exert the indicated forces on joint O . determine the magnitude of the resultant \mathbf{R} of the two forces and the angle θ which \mathbf{R} makes with the positive x -axis.



Resultants of Forces

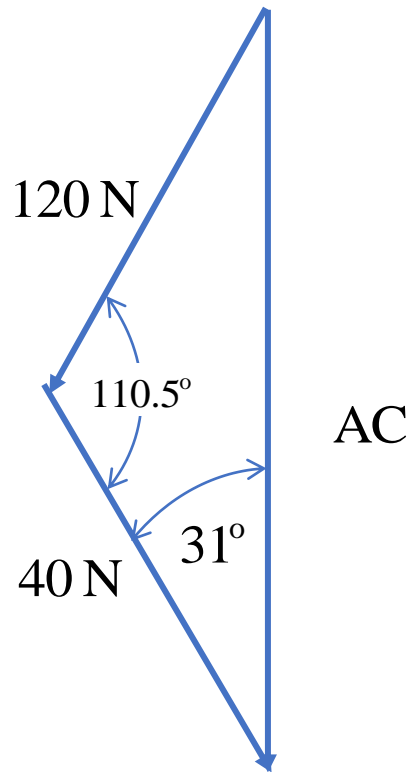
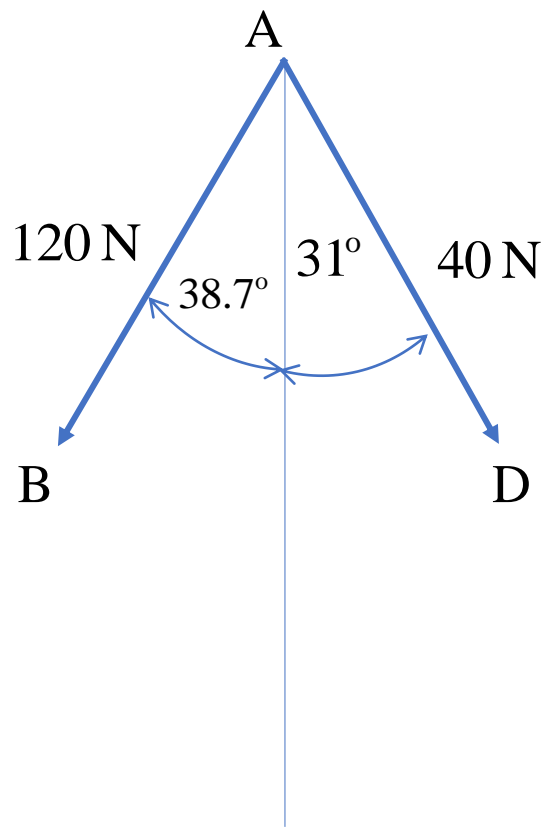
Example

Cables AB and AD help support pole AC. Knowing that the tension is 120 N in AB and 40 N in AD, determine the magnitude of the resultant of the forces exerted by the cables at A.



Resultants of Forces

Solution



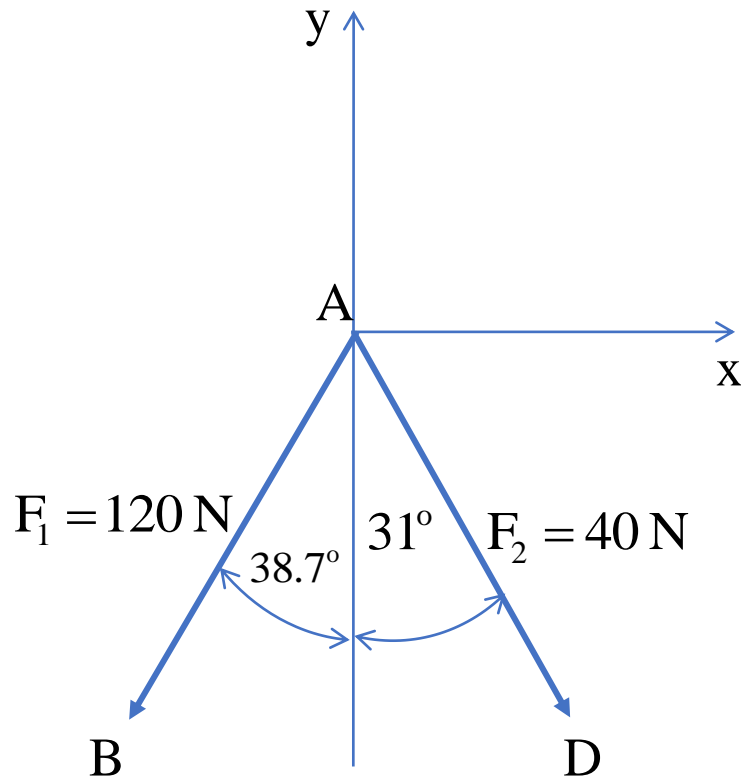
From the Cosine Rule,

$$AC^2 = 40^2 + 120^2 - 2(40)(120)\cos 110.3^\circ$$

$$AC = 139.04 \text{ N.}$$

Resultants of Forces

Example - Solution



$$\vec{R} = \sum \vec{F}_x + \sum \vec{F}_y$$

$$\vec{F}_{1x} = F_1 \sin \theta = 40 \sin 31^\circ = -20.6\text{ N}$$

$$\vec{F}_{1y} = F_1 \cos \theta = 40 \cos 31^\circ = -34.29\text{ N}$$

$$\vec{F}_{2x} = F_2 \sin \theta = 120 \sin 38.7^\circ = 75.03\text{ N}$$

$$\vec{F}_{2y} = F_2 \cos \theta = 120 \cos 38.7^\circ = -93.65\text{ N}$$

$$\vec{R} = (-20.6 + 75.03)_x + (-34.29 - 93.65)_y = 54.43_x - 128.94_y$$

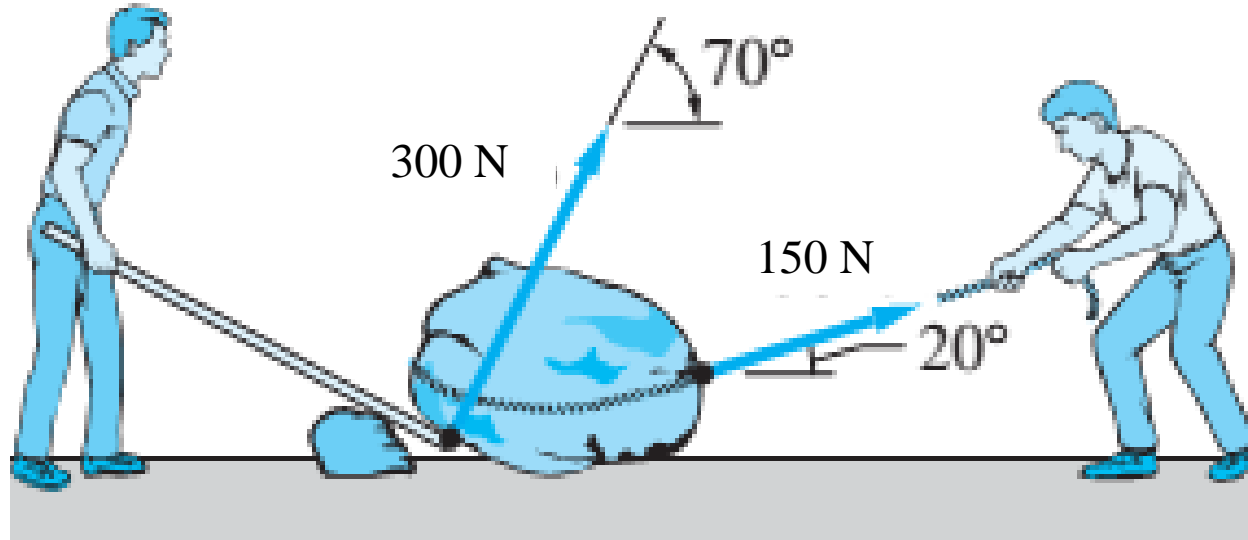
$$R = \sqrt{(54.43^2 + (-128.94)^2)} = 139.96\text{ N}$$

$$\theta_x = \cos^{-1} \left(\frac{F_x}{F} \right) = \cos^{-1} \left(\frac{-54.43}{139.96} \right)$$

Resultants of Forces

Example

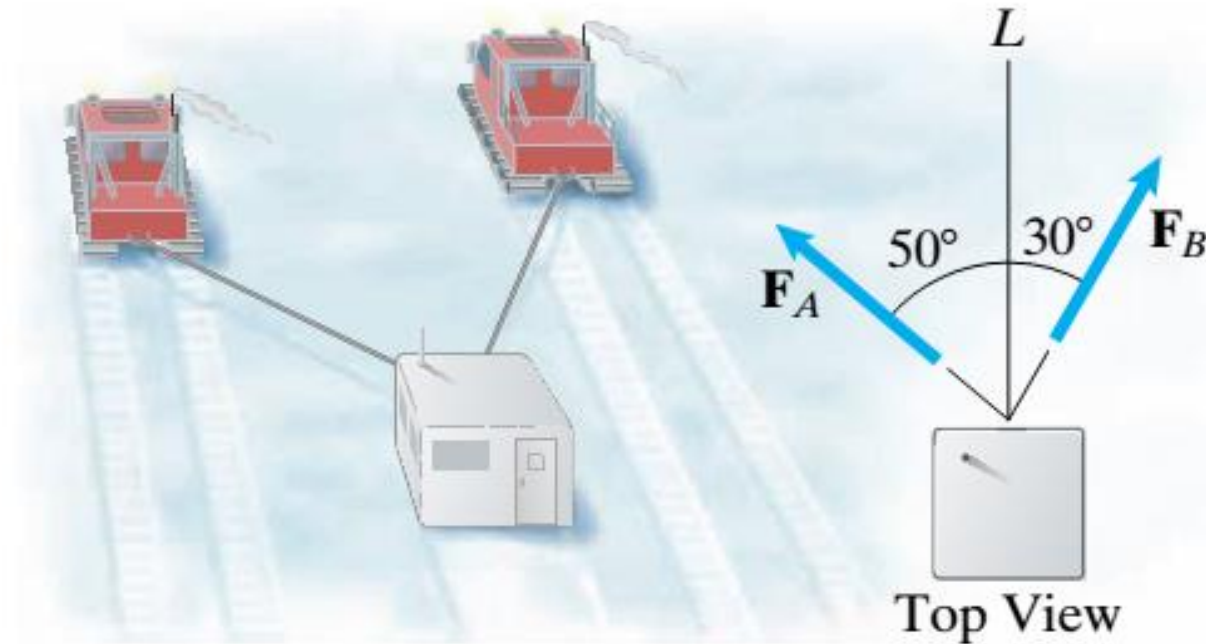
Two men are trying to roll the boulder by applying the forces shown. Determine the magnitude and direction of the force that is equivalent to the two applied forces. (Assume the two forces are acting at the same point.)



Resultants of Forces

Example

Two snow carts tow an emergency shelter. The towing cables are horizontal. The total force $F_A + F_B$ on the shelter is parallel to the direction L and its magnitude is 400 N. Determine the magnitudes of F_A and F_B



Resultants of Forces

$$\rightarrow \sum F_x = F_B \sin 30^\circ - F_A \sin 50^\circ$$

$$+ \uparrow \sum F_y = F_B \cos 30^\circ - F_A \cos 50^\circ$$

Resultant force is vertical. This means;

$$R_x = 0$$

$$R_y = 400 \text{ N}$$

And

$$R_x = \sum F_x$$

$$R_y = \sum F_y$$

So,

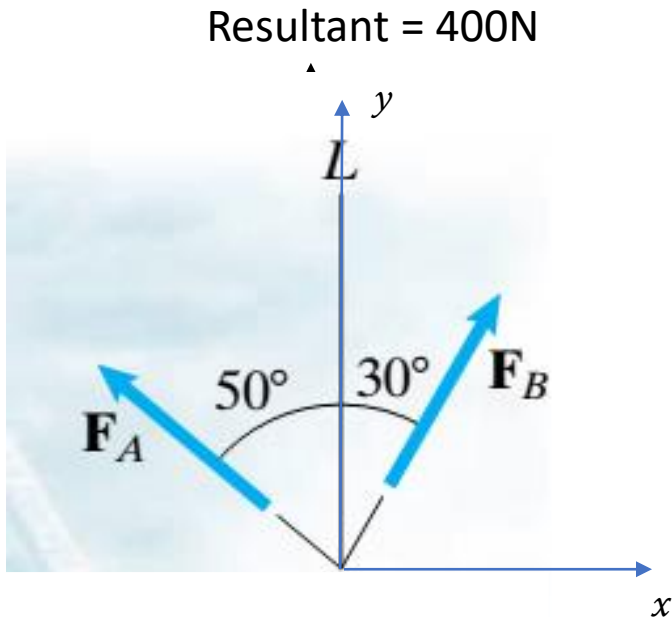
$$F_B \sin 30^\circ - F_A \sin 50^\circ = 0$$

$$F_B \cos 30^\circ - F_A \cos 50^\circ = 400$$

Solve simultaneously to get;

$$F_A = 203 \text{ N}$$

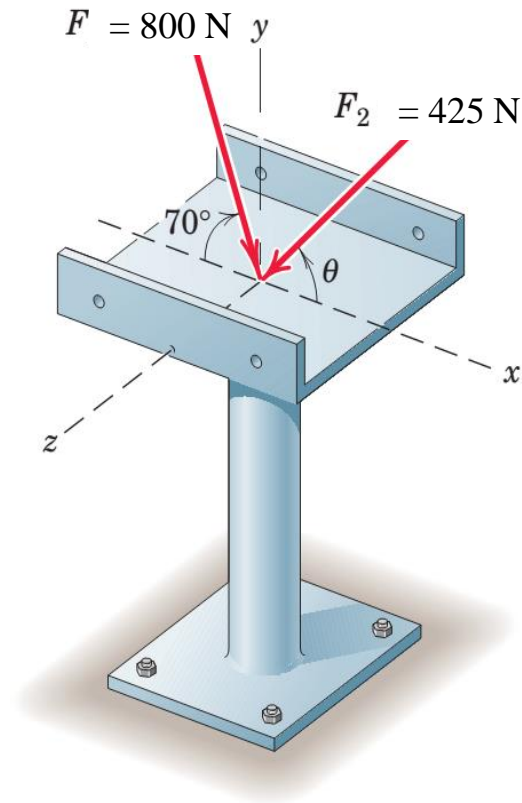
$$F_B = 311 \text{ N}$$



Resultants of Forces

Example

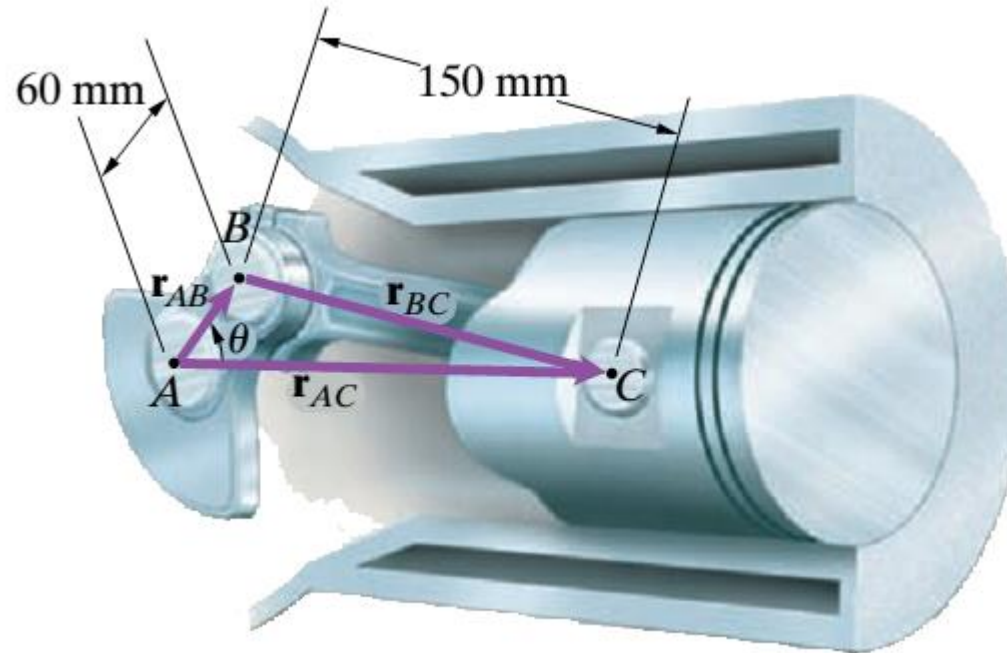
Two forces, the same plane are applied to the construction bracket as shown. Determine the angle θ which makes the resultant of the two forces vertical. Determine the magnitude R of the resultant.



Resultants of Forces

Example

The angle $\theta = 50^\circ$. determine the length of the line representing vector \mathbf{r}_{AC} . (Hint: all three lines lie in the same plane)

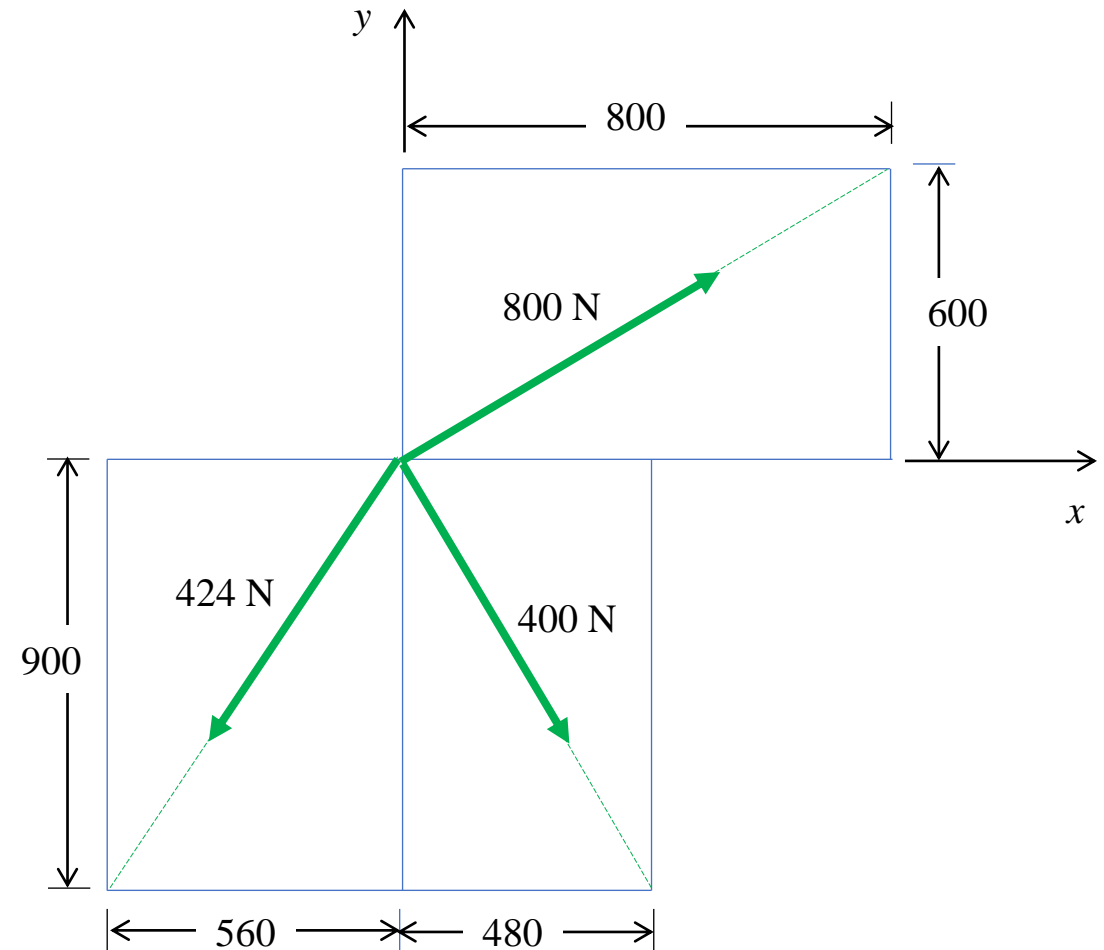


Resultants of Forces

Example

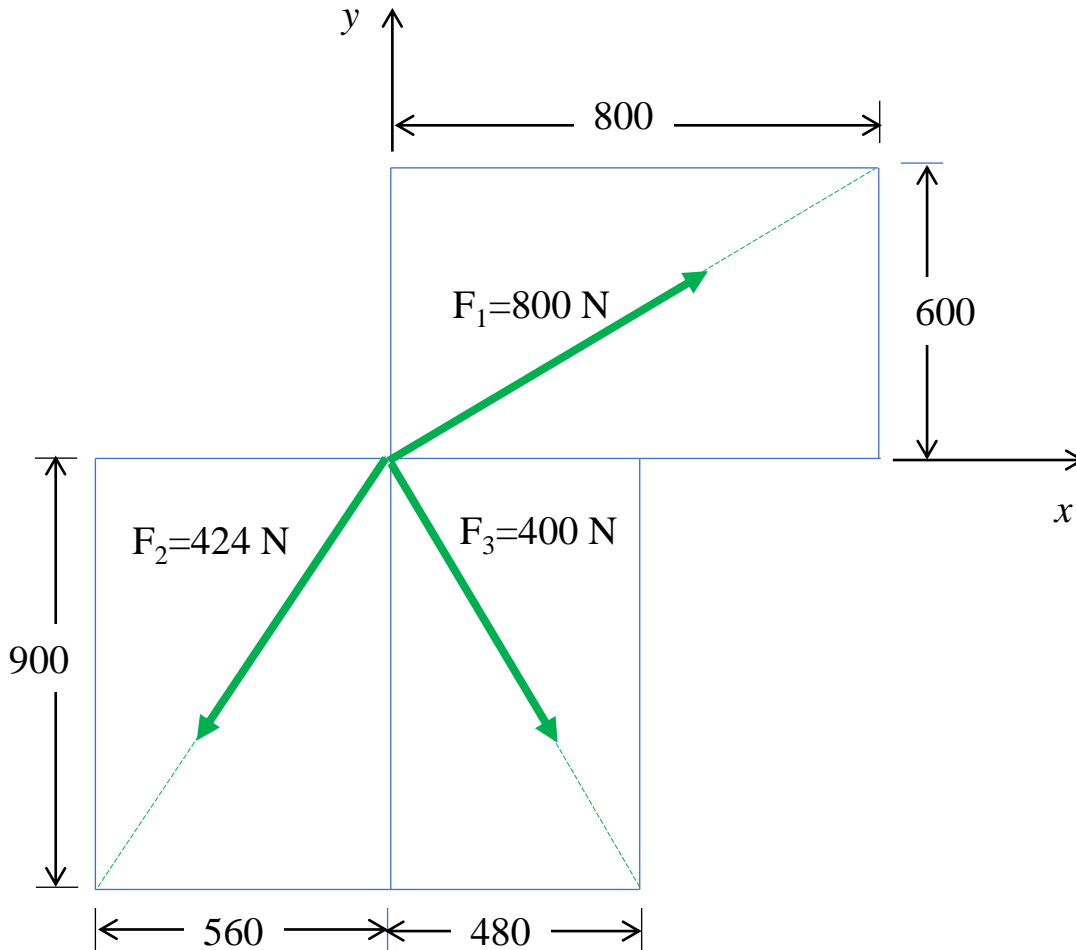
Find the resultant of the forces shown

Dimensions are in mm



Resultants of Forces

Example - Solution



$$\vec{F} = F \lambda = F \left(\frac{x\vec{i} + y\vec{j}}{\sqrt{x^2 + y^2}} \right)$$

$$\vec{F}_1 = 800 \left(\frac{800\vec{i} + 600\vec{j}}{\sqrt{800^2 + 600^2}} \right) = 800 \cdot \frac{800\vec{i}}{1000} + 800 \cdot \frac{600\vec{j}}{1000} = 640\vec{i} + 480\vec{j}$$

$$\vec{F}_2 = 424 \left(\frac{-560\vec{i} - 900\vec{j}}{\sqrt{560^2 + 900^2}} \right) = -224\vec{i} - 360\vec{j}$$

$$\vec{F}_3 = 400 \left(\frac{480\vec{i} - 900\vec{j}}{\sqrt{480^2 + 900^2}} \right) =$$

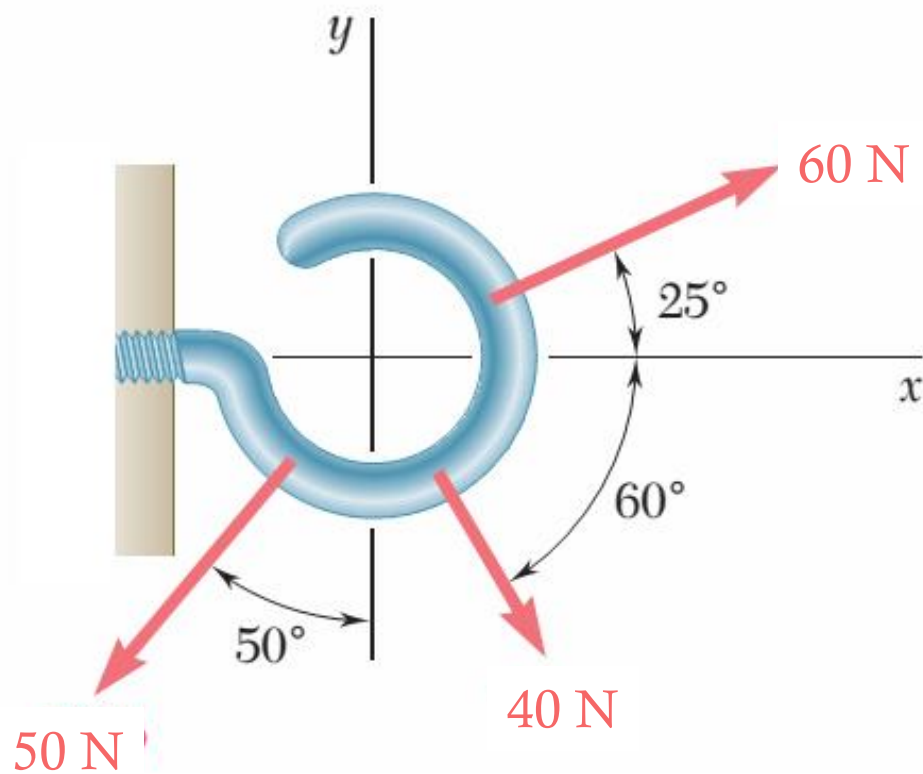
$$\text{Resultant, } \vec{F} = \sum F\vec{i} + \sum F\vec{j} =$$

$$\text{The magnitude of the Resultant, } F = \sqrt{\left(\sum F\vec{i} \right)^2 + \left(\sum F\vec{j} \right)^2}$$

Resultants of Forces

Example

Find the resultant of the forces shown.



$$F = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}$$

$$+ \rightarrow \sum F_x = 60 \cos 25 + 40 \cos 60 - 50 \sin 50 = 36.076 \text{ N}$$

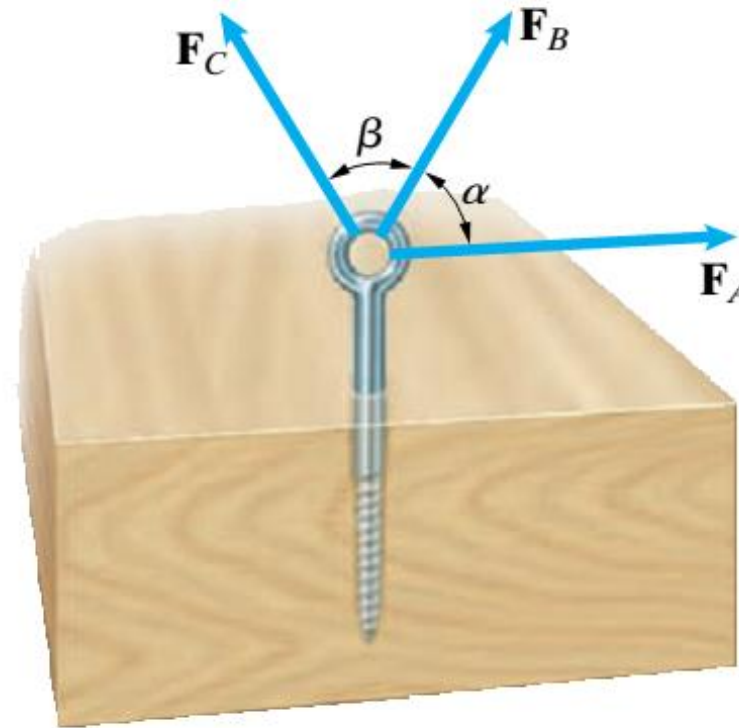
$$+ \uparrow \sum F_y = 60 \sin 25 - 40 \sin 60 - 50 \cos 50 = -41.423 \text{ N}$$

$$F = \sqrt{36.076^2 + (-41.423)^2} = 54.930 \text{ N}$$

Resultants of Forces

Example

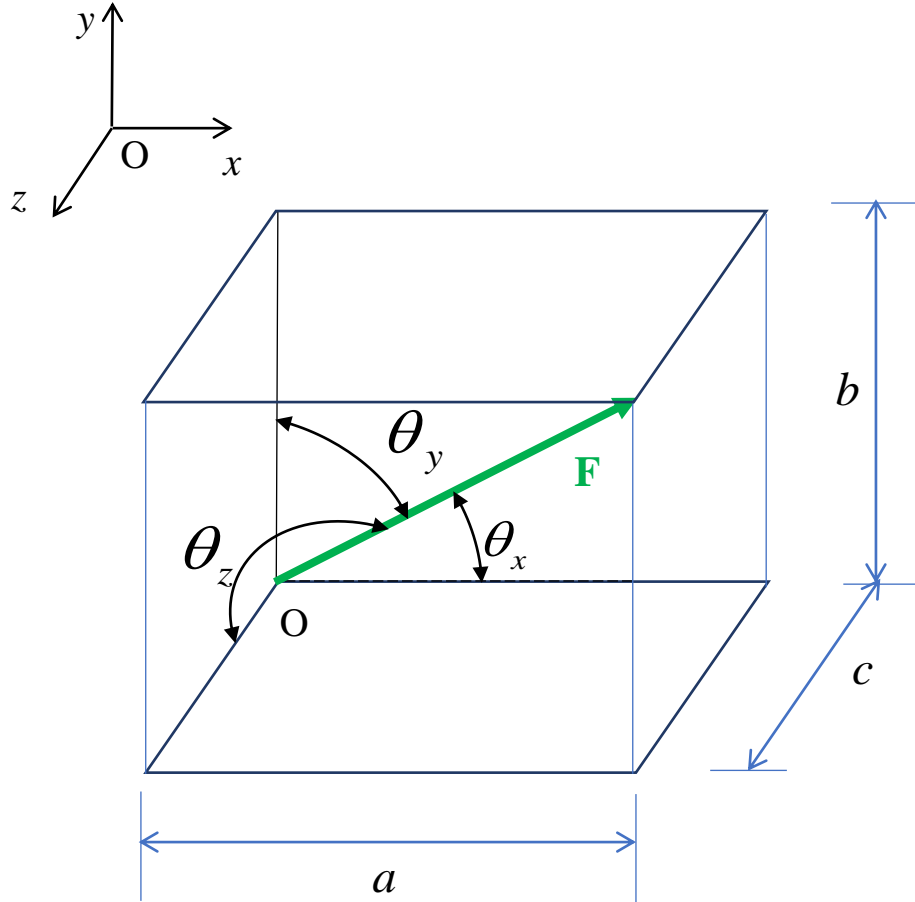
The forces $F_A = 40\text{ N}$, $F_B = 50\text{ N}$, and $F_C = 40\text{ N}$ act on the screw pin as illustrated in the Figure. $\alpha = 50^\circ$ and $\beta = 80^\circ$. Determine the magnitude of the resultant of the three forces on the eye of the screw pin, assuming they are coplanar.



Non-Planar (Spatial/3-D) Forces.



Forces in Space



$$\vec{F} = \vec{F}_x + \vec{F}_y + \vec{F}_z$$

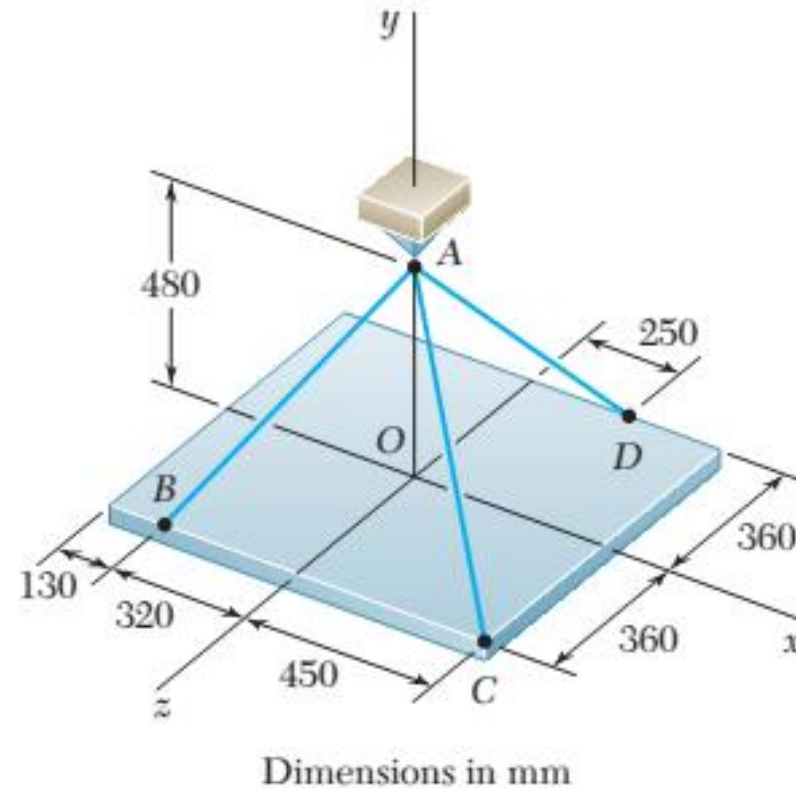
$$\vec{F} = F \cos \theta_x + F \cos \theta_y + F \cos \theta_z$$

$$\begin{aligned}\vec{F} &= F \lambda = F \left(\frac{a\vec{i} + b\vec{j} + c\vec{k}}{\sqrt{a^2 + b^2 + c^2}} \right) \\ &= F\vec{i} + F\vec{j} + F\vec{k}\end{aligned}$$

Forces in Space

Example

A rectangular plate is supported by three cables as shown. Knowing that the tension in cables AC, AB and AD are 60 N, 80 N and 90 N respectively, determine the components of the forces being exerted at C, B and D.



Forces in Space

Soln

For components of force at D, F_{DA} ;

$$\vec{DA} = -250i + 480j + 360k$$

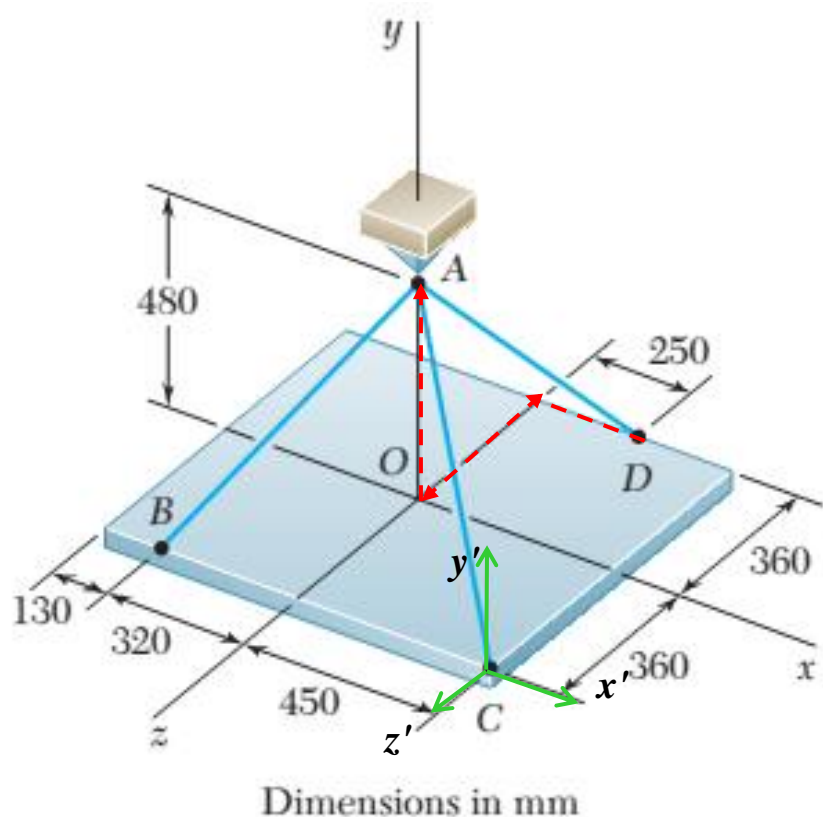
$$\lambda_{DA} = \left(\frac{-250\vec{i} + 480\vec{j} + 360\vec{k}}{(250^2 + 480^2 + 360^2)^{1/2}} \right)$$

$$F_{DA} = 90 \left(\frac{-250\vec{i} + 480\vec{j} + 360\vec{k}}{(250^2 + 480^2 + 360^2)^{1/2}} \right)$$

For components of force at B, F_{BA}

$$\vec{BA} = 320i + 480j - 360k$$

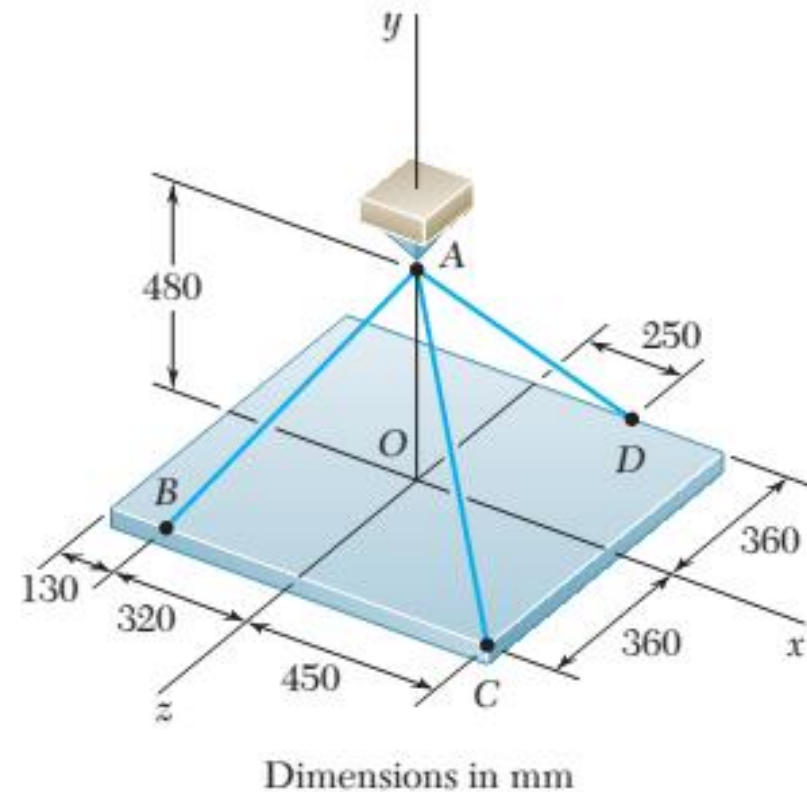
$$\lambda_{BA} =$$



Forces in Space

Example

A rectangular plate is supported by three cables as shown. Knowing that the tension in cables AC, AB and AD are 60 N, 80 N and 90 N respectively, determine the magnitude of a force that can replace the three forces at A.



Moment of Forces

Scalar and vector approaches of calculating Moments

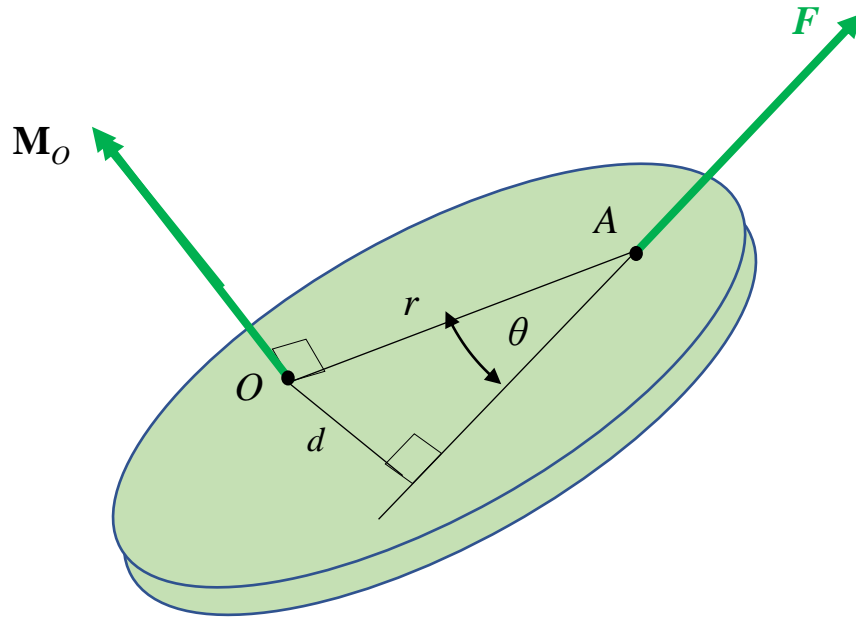
Principles of Moments

Equivalent Force-Couple Systems



Moment of a Force

- Forces have the tendency to cause two types motions in rigid bodies; **translational** and **rotational** motions.
- The tendency of a force to rotate a body is referred to as moment.
- A moment may occur about a point; the *Moment Centre*.



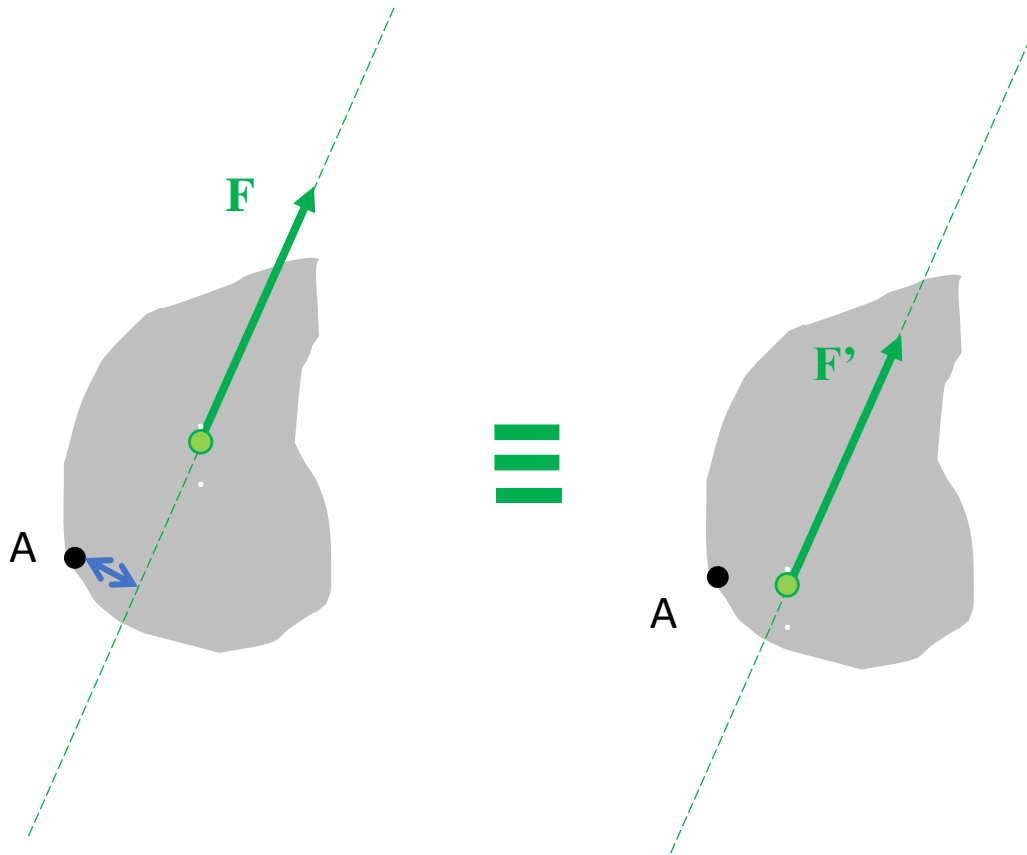
$$\vec{M}_o = \vec{r} \times \vec{F}$$

$$r \sin \theta = d$$

$$M_o = Fr \sin \theta = Fd$$

Moment of a Force

- d is always perpendicular to the Force's line of action, so, the Force is treated as a sliding vector, due to the principle of transmissibility in rigid body mechanics.



If F and F' have the same magnitude,

Moment of F about A = Moment of F' about A

Moment of a Force

➤ Scalar Approach

$$M_o = d \times F$$

- Only the magnitude of the moment is calculated using only the magnitudes of the force and the moment arm, d , defined as the perpendicular distance between the line of action of the force and the moment centre.
- Often used when the moment, d can easily be determined. The sense of the moment is determined by inspection.

➤ Vector Approach

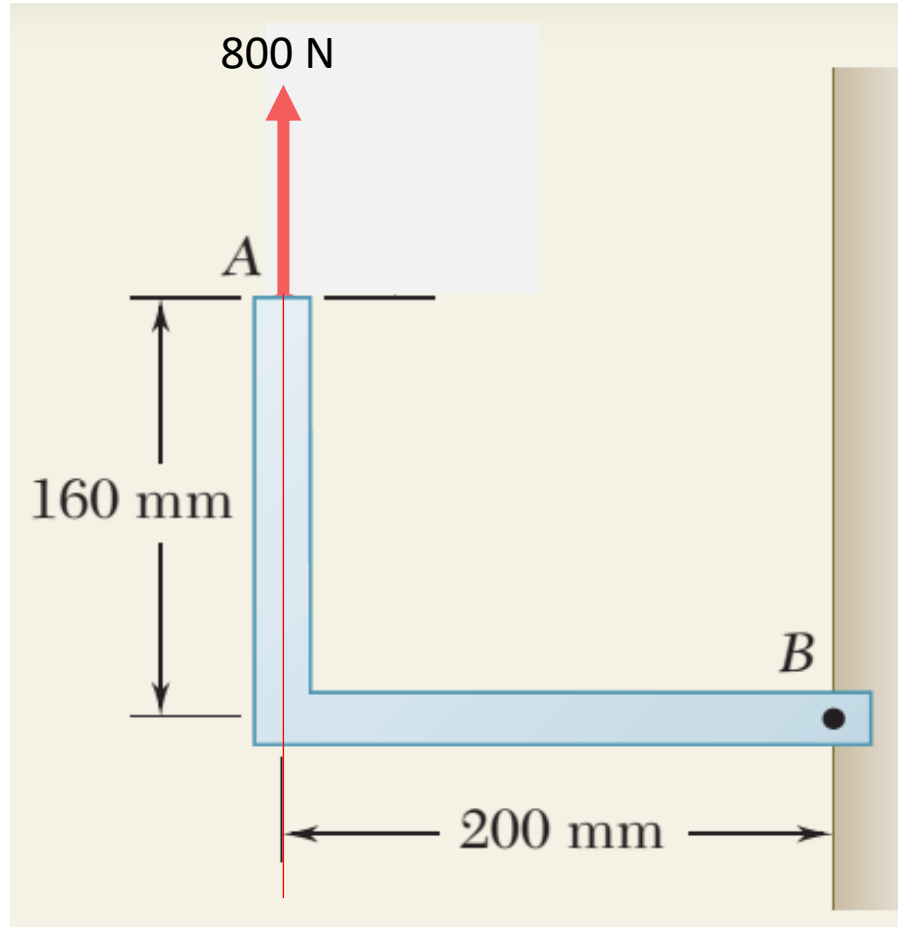
- The position vector for the point of application of the force is multiplied by the components of the force to get the components of the Resultant moment.



Moment of a Force

Example

A force of 800 N acts on a bracket as shown. Determine the moment of the force about B.

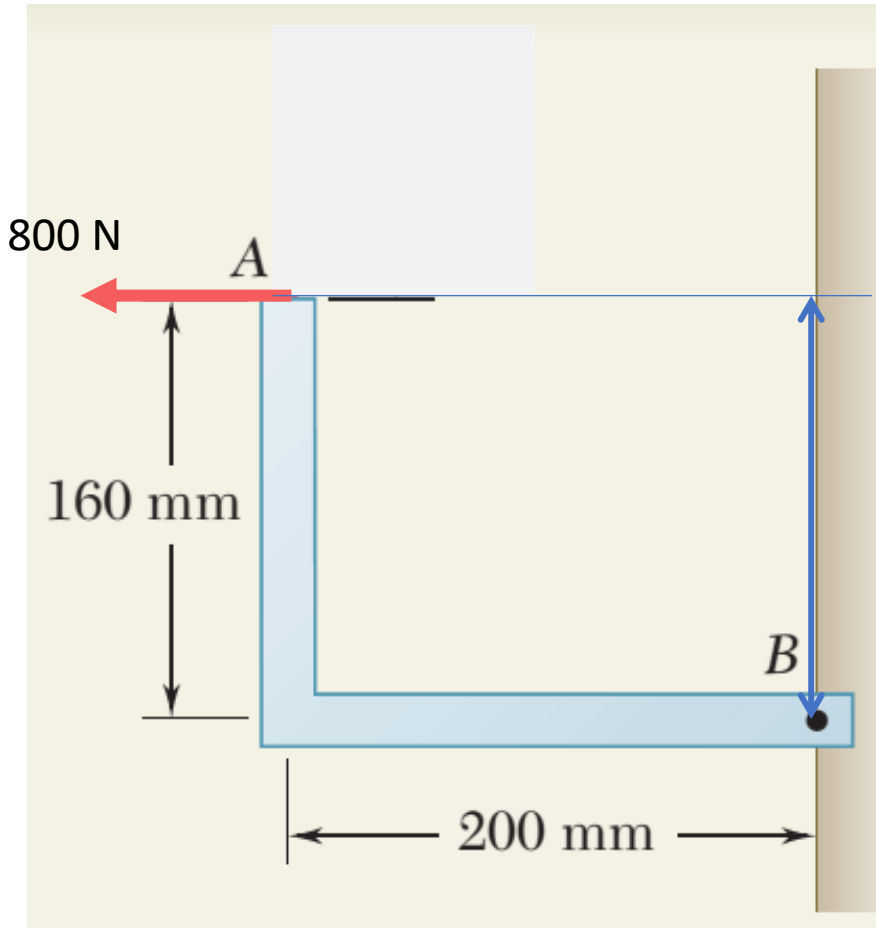


$$\begin{aligned} \curvearrowright M_B &= 0.2 \text{ m} \times 800 \\ &= 160 \text{ Nm} \end{aligned}$$

Moment of a Force

Example

A force of 800 N acts on a bracket as shown. Determine the moment of the force about B.



$$\begin{aligned} \odot \quad M_B &= 0.16m \times 800 \\ &= -128 \text{ Nm} \end{aligned}$$

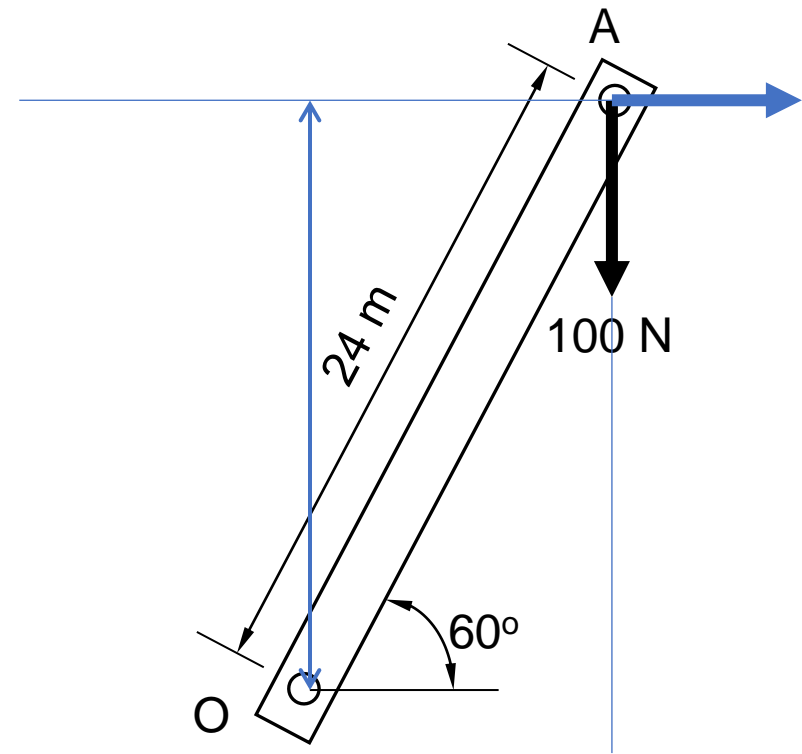
Moment of a Force

Example

A 100-N vertical force is applied to the end of a lever which is attached to a shaft at O .

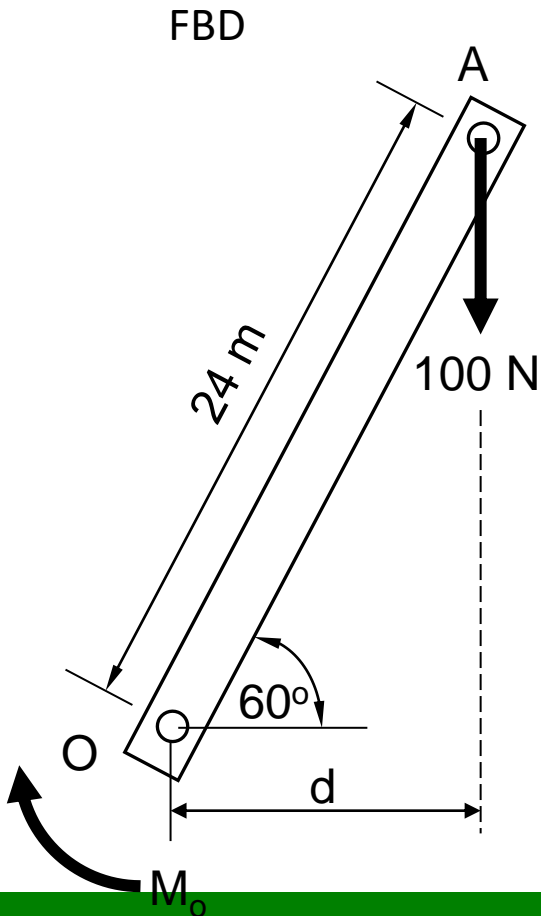
Determine:

- a) moment about O ,
- b) horizontal force at A which creates the same moment,



Moment of a Force

Solution



Moment about O is equal to the product of the force and the perpendicular distance between the line of action of the force and O .

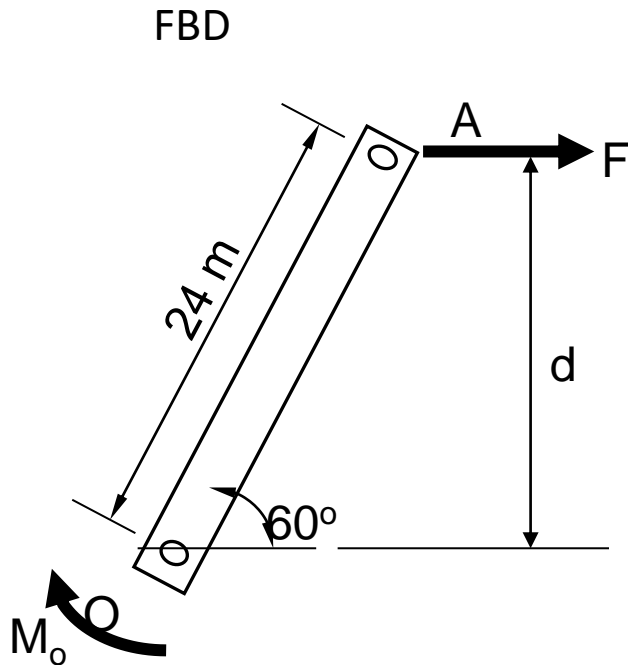
$$M_o = Fd$$

$$d = (24 \text{ m}) \cos 60^\circ = 12 \text{ m.}$$

$$M_o = (100 \text{ N})(12 \text{ m.}) =$$

Moment of a Force

Solution



Horizontal force at A that produces the same moment,

$$d = (24 \text{ m}) \sin 60^\circ = 20.8 \text{ m}$$

$$M_o = Fd$$

$$1200 \text{ Nm.} = F(20.8 \text{ m})$$

$$F = \frac{1200 \text{ Nm}}{20.8 \text{ m.}}$$

=

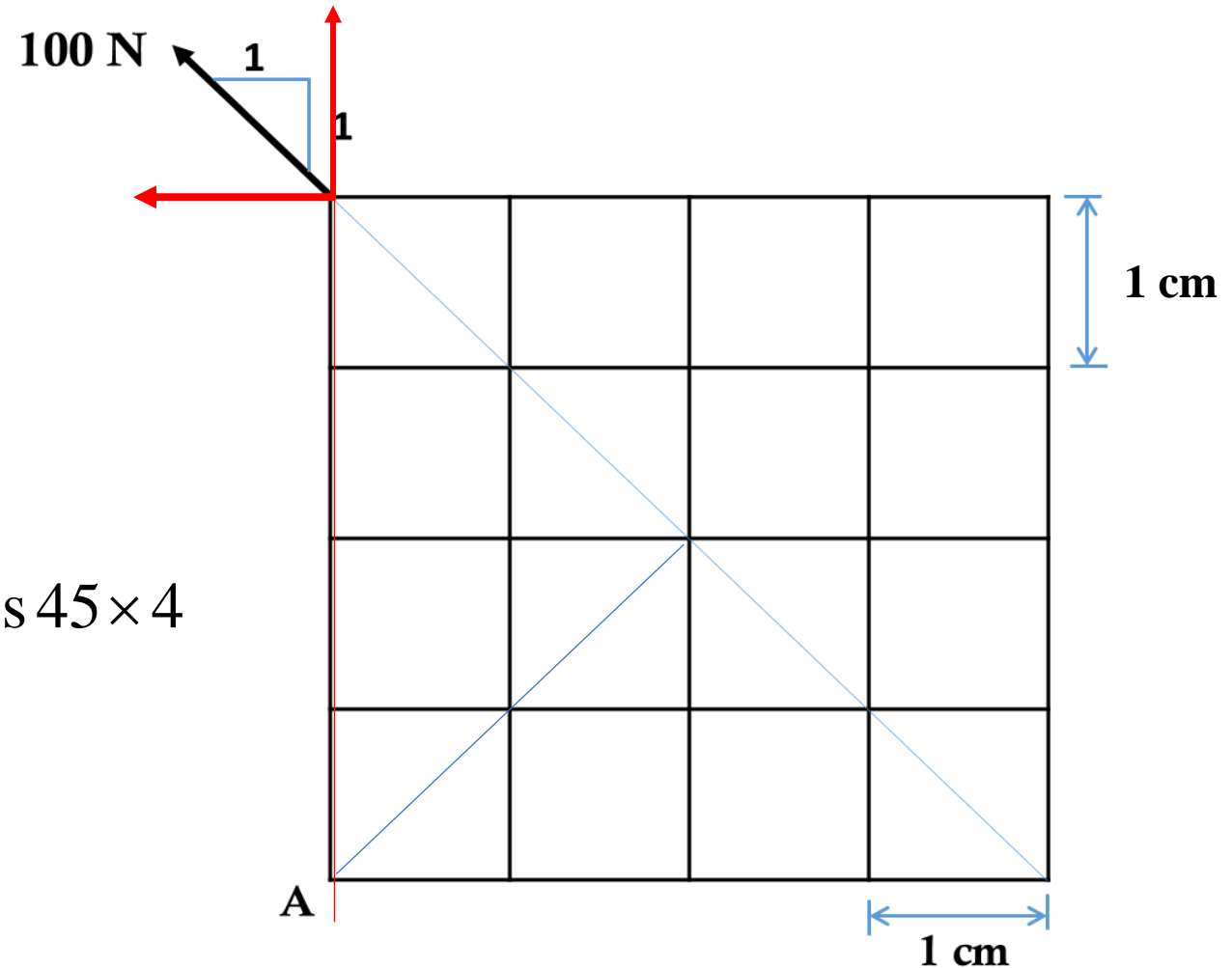
The Principle of Moments (Varignon's Theorem)

The moment of a force about a given point, is *equal and equivalent* to the sum of the moments of an equivalent system of forces in the same plane, about the same point.

The Principle of Moments (Varignon's Theorem)

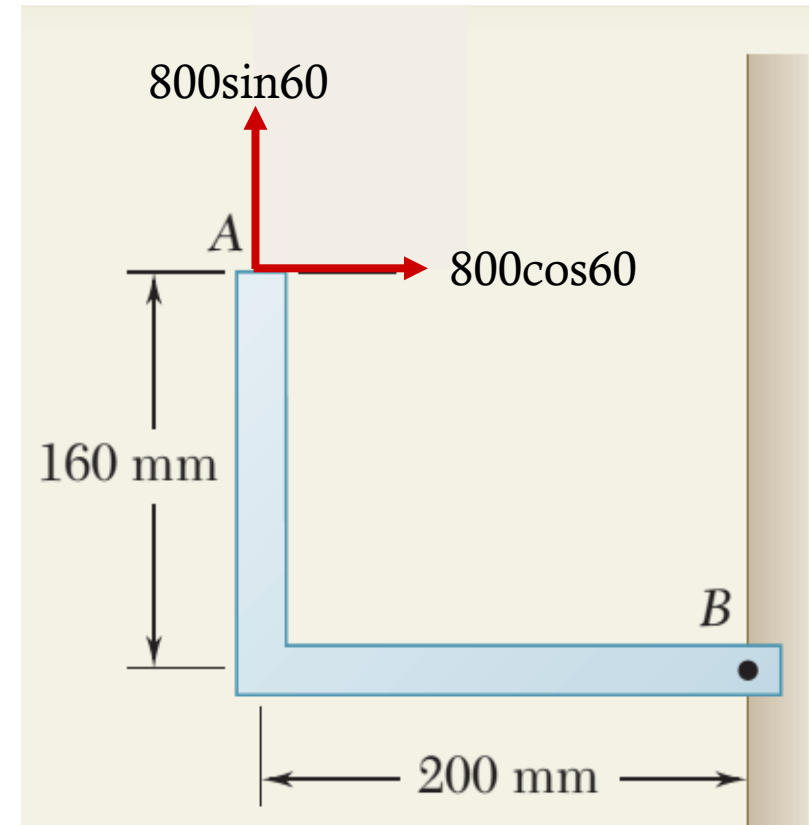
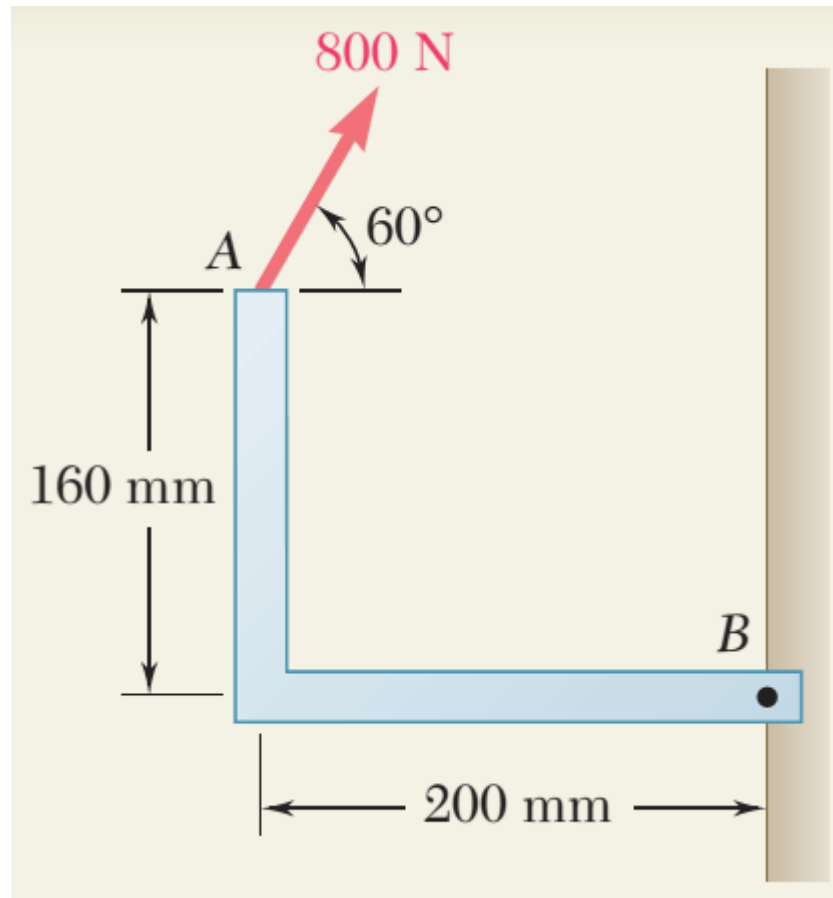
Determine the magnitude and sense of the moment due to the 100 N force shown below about A.

$$\curvearrowright + M_A = 100 \times \left(\frac{\sqrt{4^2 + 4^2}}{2} \right) = 100 \cos 45 \times 4$$



The Principle of Moments (Varignon's Theorem)

Determine the moment of the 800N force about B using the principle of moments,.



$$\begin{aligned} \odot M_B &= (0.16m \times 800 \cos 60) + (0.2m \times 800 \sin 30) \\ &= 202.56 \text{ Nm} \end{aligned}$$

Moment of a Force: Vector Formulation

- The position vector components for the point of application of the force (from the moment centre) are multiplied by the force vector components to get the of the Resultant moment components.
- Multiplication of the position and force vectors may be done in one of two ways;
 - ✓ Matrix approach
 - ✓ Expansion and simplification

Moment of a Force: Vector Formulation

- Vectors are expressed in components, arranged in a matrix form, and the determinant of the matrix taken.

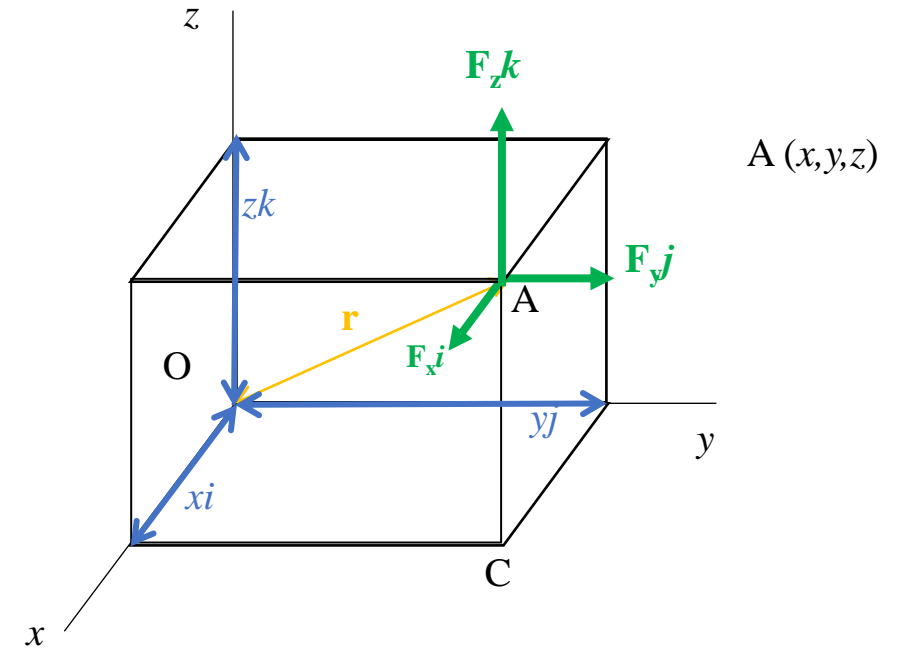
$$\text{If } \vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \text{ and } \vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

Expressing as a matrix,

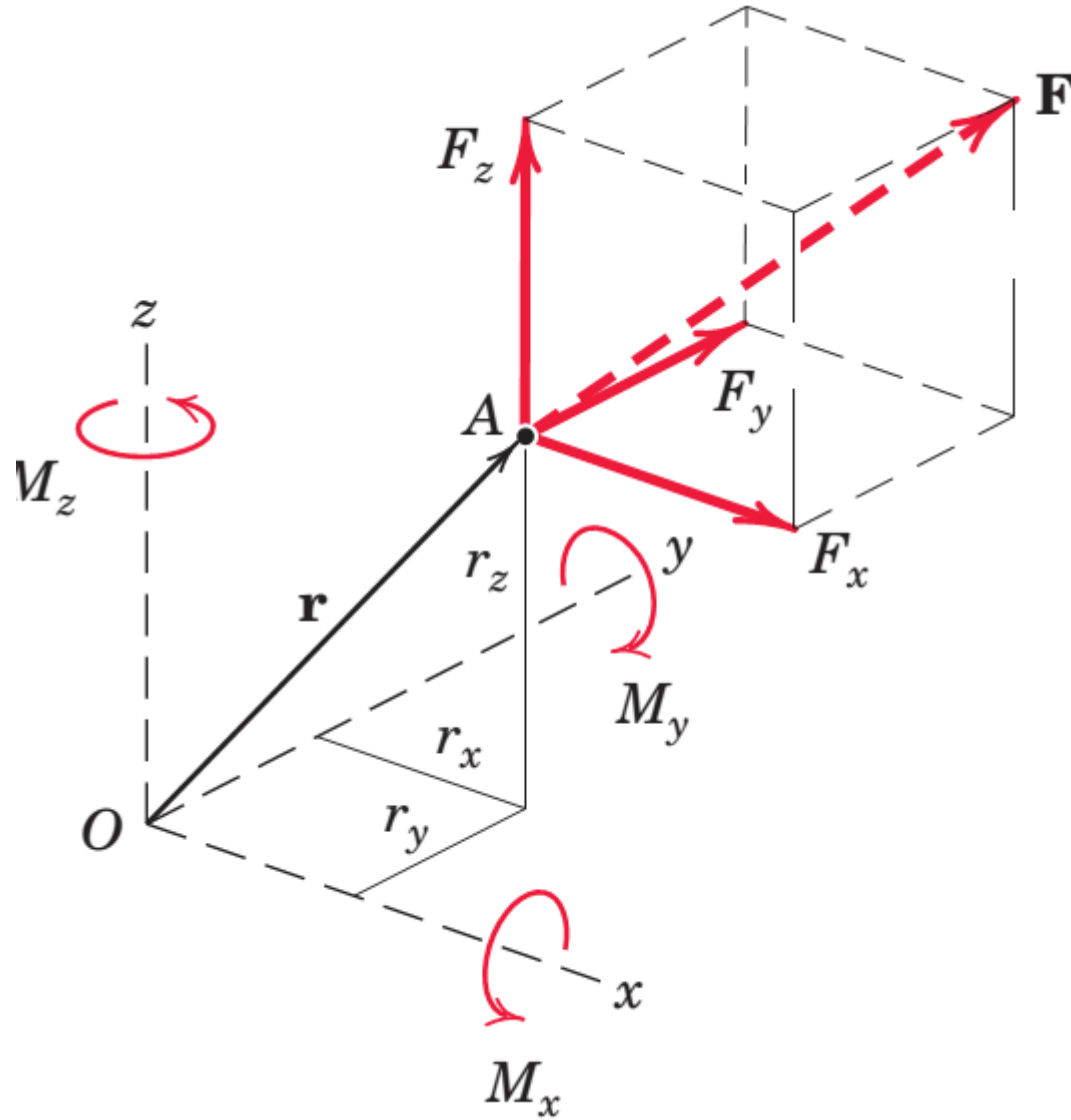
$$\vec{M}_O = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

Taking the determinant of the matrix,

$$\begin{aligned} \vec{M}_O &= (yF_z - zF_y)\vec{i} - (xF_z - zF_x)\vec{j} + (xF_y - yF_x)\vec{k} \\ &= M_x\vec{i} + M_y\vec{j} + M_z\vec{k} \end{aligned}$$



Moment of a Force: Vector Formulation



Directions of component moments

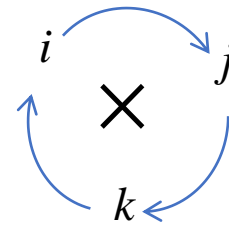
Moment of a Force: Vector Formulation

➤ Alternatively, a sort of expansion and simplification is done.

$$\begin{aligned}
 M_O &= \vec{r} \times \vec{F} \\
 &= (x\vec{i} + y\vec{j} + z\vec{k}) \times (F_x\vec{i} + F_y\vec{j} + F_z\vec{k}) \\
 &= [(x\vec{i} + y\vec{j} + z\vec{k}) \times F_x\vec{i}] + [(x\vec{i} + y\vec{j} + z\vec{k}) \times F_y\vec{j}] + [(x\vec{i} + y\vec{j} + z\vec{k}) \times F_z\vec{k}]
 \end{aligned}$$

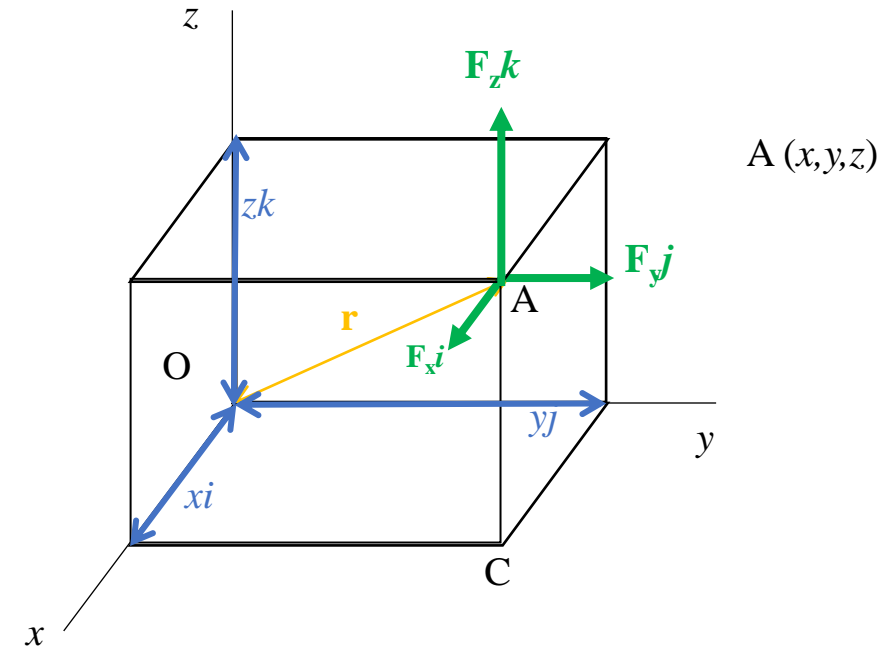
But

$$\begin{aligned}
 \vec{i} \times \vec{i} &= 0 & j \times i &= -k & k \times i &= j \\
 i \times j &= k & j \times j &= 0 & k \times j &= -i \\
 i \times k &= -j & j \times k &= i & k \times k &= 0
 \end{aligned}$$



Therefore,

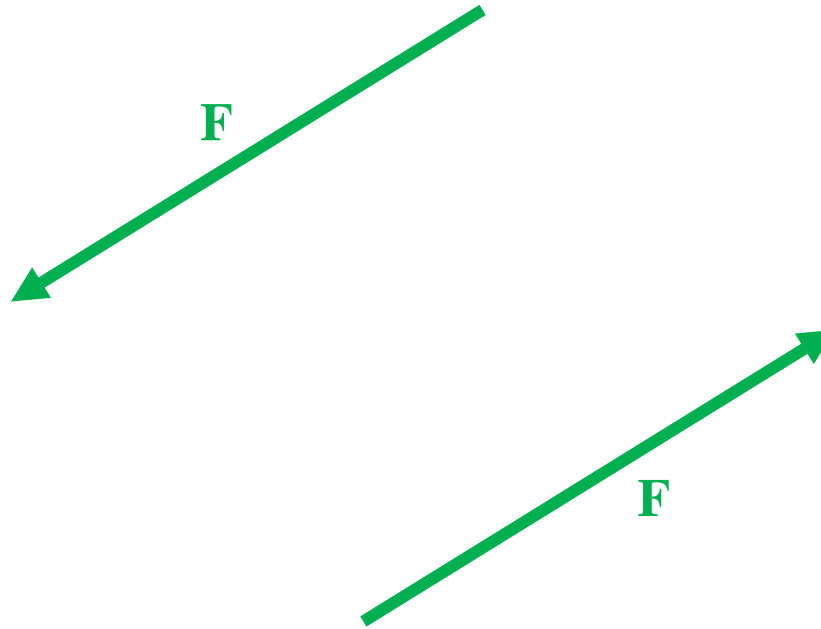
$$\begin{aligned}
 M_O &= -yF_x\vec{k} + zF_x\vec{j} + xF_y\vec{k} - zF_y\vec{i} - xF_z\vec{j} + yF_z\vec{i} \\
 &= (yF_z - zF_y)\vec{i} - (xF_z - zF_x)\vec{j} + (xF_y - yF_x)\vec{k} \\
 &= M_x\vec{i} + M_y\vec{j} + M_z\vec{k}
 \end{aligned}$$



Couples and Equivalent Force-Couple Systems

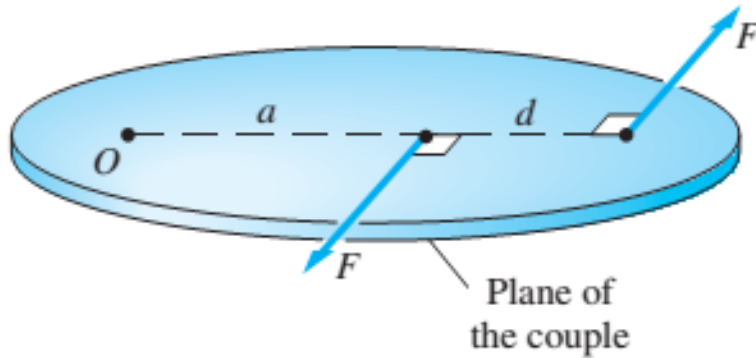
Couples

- This refers to two parallel, non-collinear, (but coplanar) forces that are equal in magnitude and opposite in direction.



- It is a free vector that can be applied anywhere

Couples

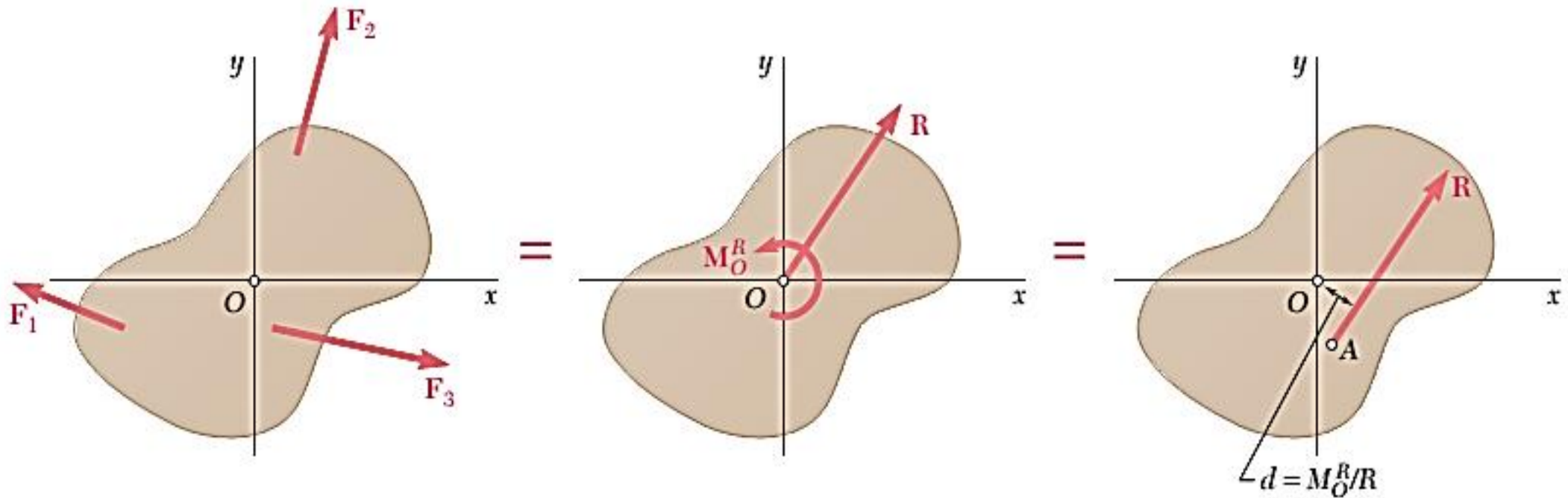


$$\curvearrowleft + \quad M_O = F(a + d) - F(a) = Fd$$

- Two couples are considered equivalent if
 - their magnitude is of the same magnitude
 - they lie in the same plane
 - tend to cause rotation in the same direction
- Couples are vectors
- They also obey the Principle of Moments

Equivalent Force-Couple Systems

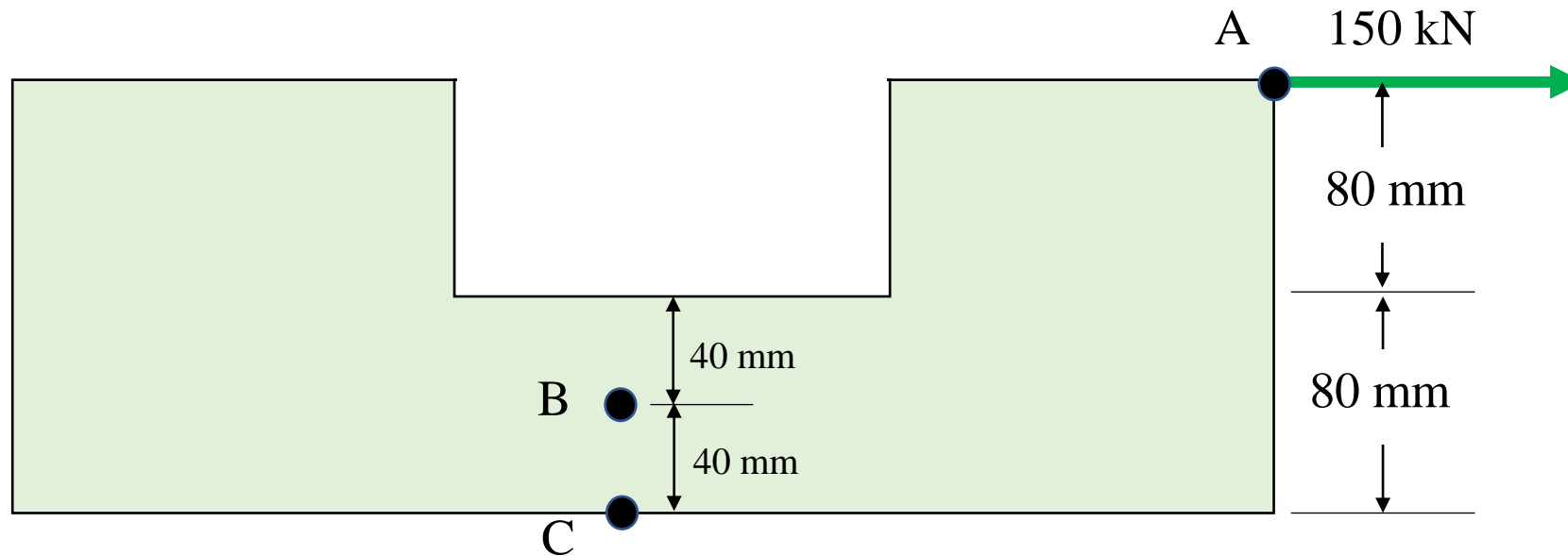
- A system of forces act on a body, **must** be reduced to a force-couple system.
- The force-couple system comprises a resultant force (evaluated with the particle idealization at a desired point) and a resultant moment about that desired point.



Equivalent Force-Couple Systems

Example

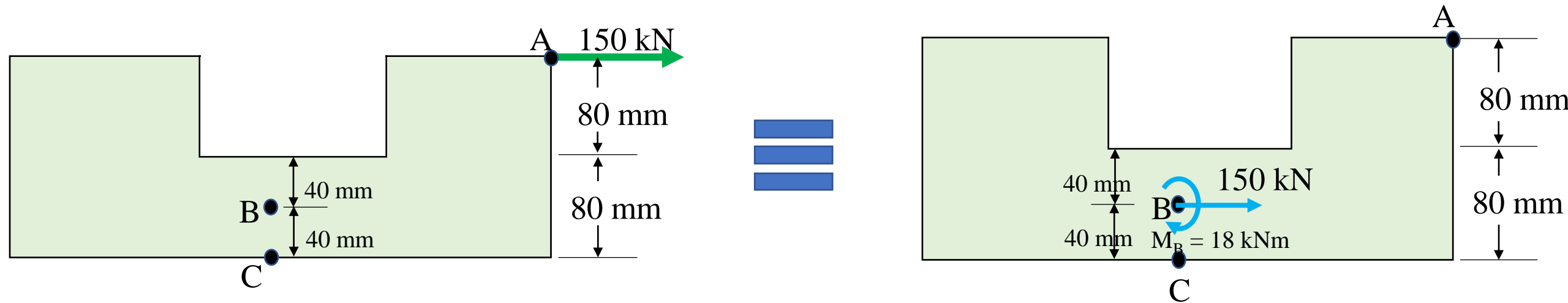
For the machine part shown in the Figure below, replace the applied load of 150 kN acting at point A with an equivalent force-couple system with the force acting at point B.



Equivalent Force-Couple Systems

Solution

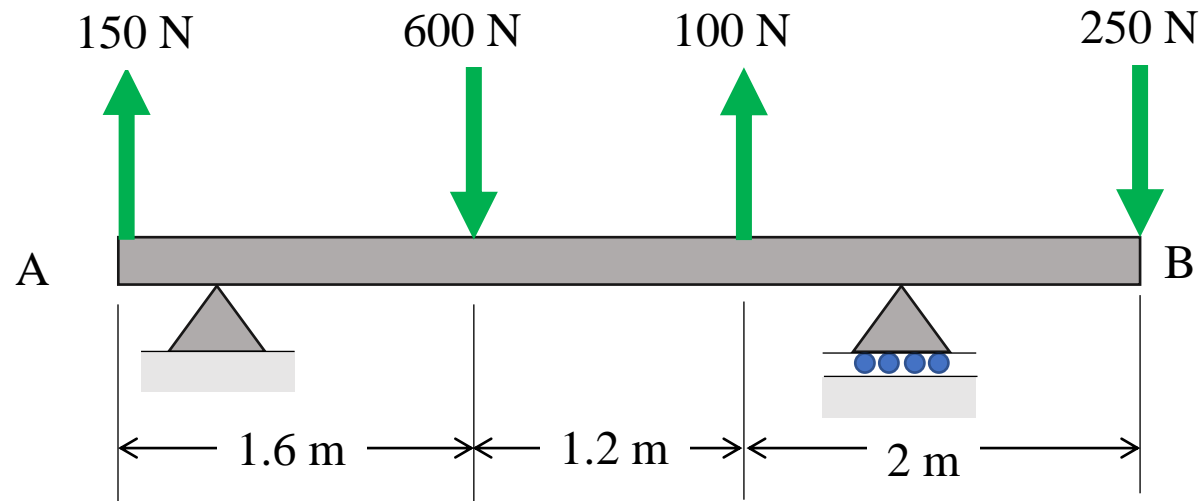
$$\curvearrowright M_B = -150(0.08 \text{ m} + 0.04 \text{ m}) = -18 \text{ kNm}$$



Equivalent Force-Couple Systems

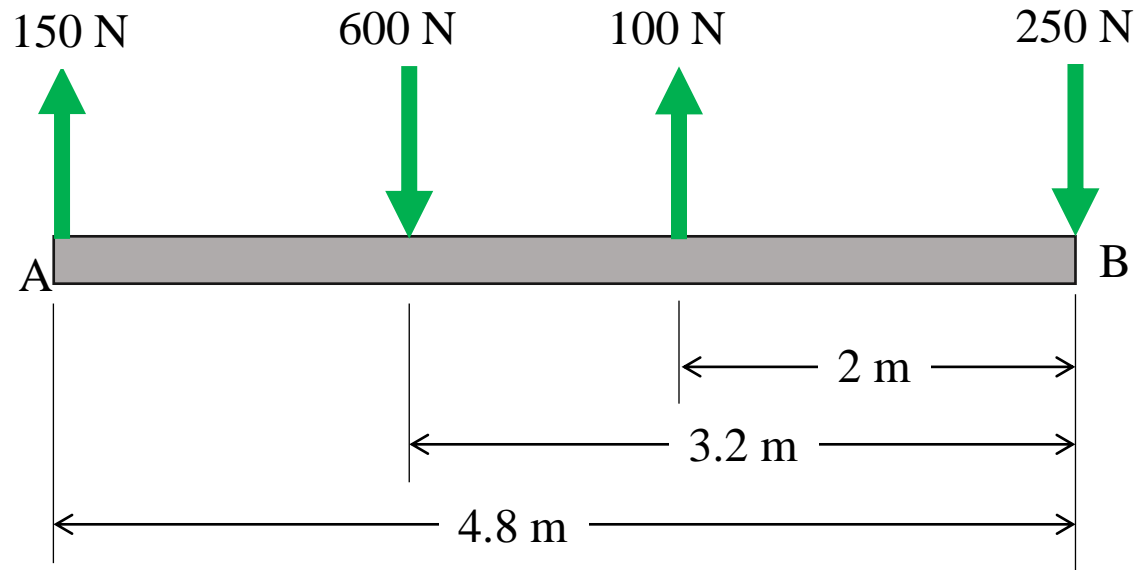
Example

For the beam, reduce the system of forces shown to (a) an equivalent force-couple system at A, (b) an equivalent force couple system at B. (Ignore the support reactions)



Equivalent Force-Couple Systems

Solution (@ B)



The Resultant force will be

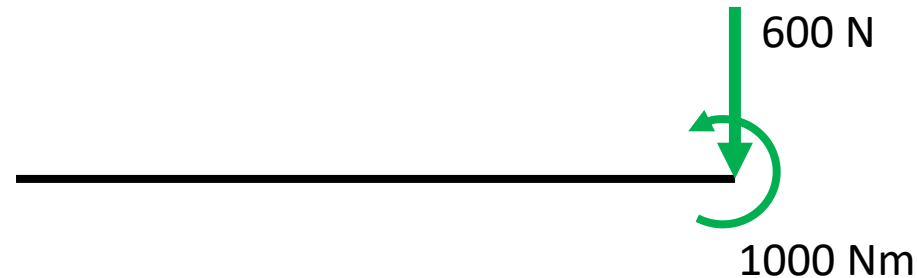
$$\begin{aligned} + \uparrow R &= \sum F \\ &= (150 \text{ N}) - (600 \text{ N}) + (100 \text{ N}) - (250 \text{ N}) \\ &= (-600 \text{ N}) \end{aligned}$$

The Resultant Moment

$$\begin{aligned} \curvearrowright \vec{M}_B &= \sum (r \times F) \\ &= (250 \text{ N} \times 0 \text{ m}) - (100 \text{ N} \times 2 \text{ m}) + (600 \text{ N} \times 3.2 \text{ m}) - (150 \text{ N} \times 4.8 \text{ m}) \\ &= 1000 \text{ Nm} \end{aligned}$$

Equivalent system

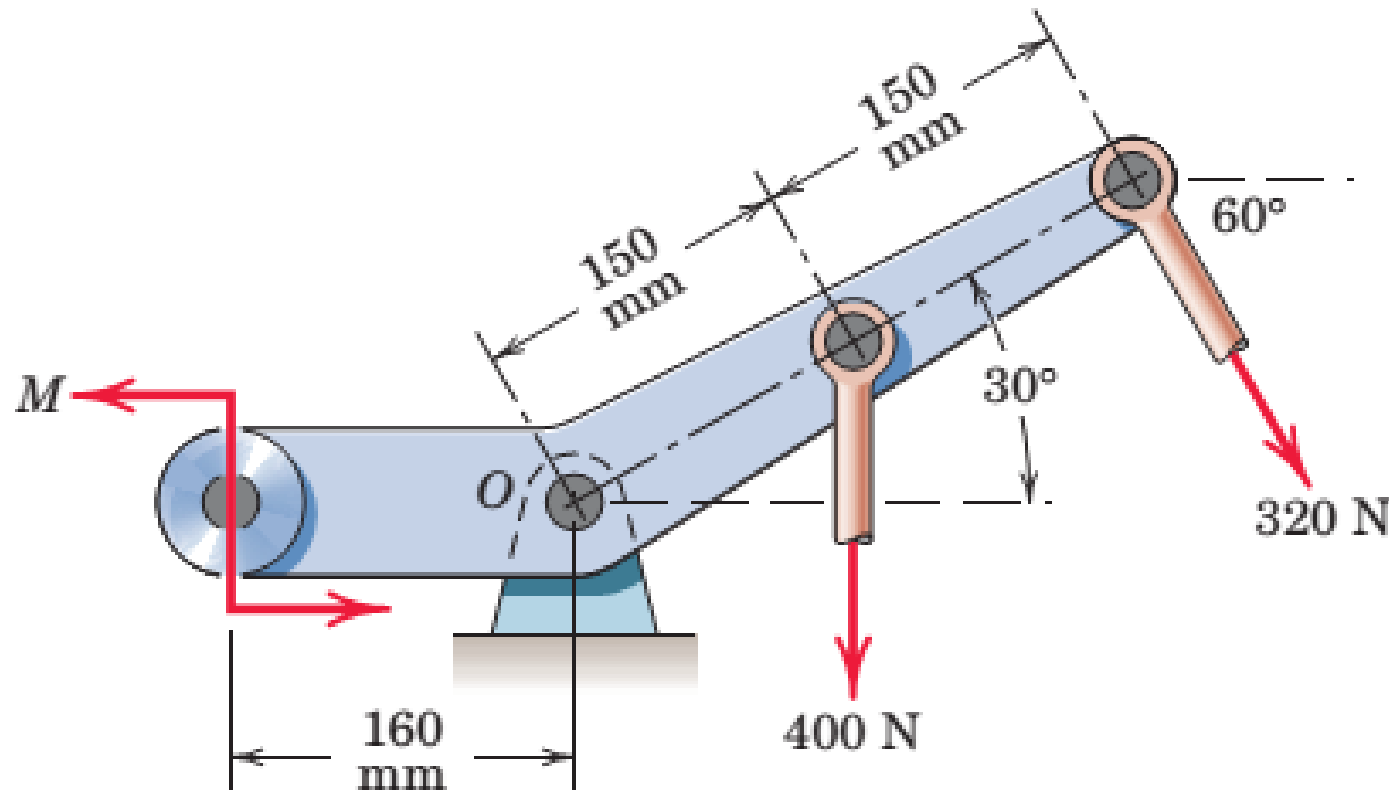
=



Equivalent Force-Couple Systems

Example

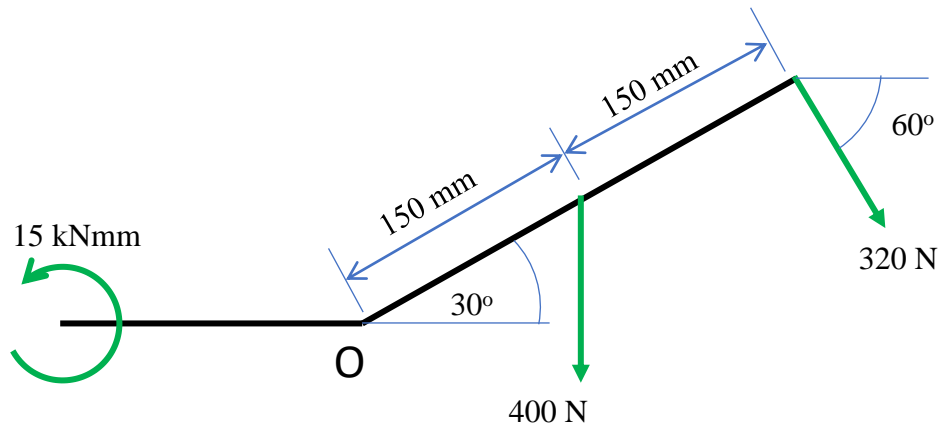
Replace the system of forces and couple acting on the arm to an equivalent force couple system at O . Take M to be 15 kNmm and ignore support reactions.



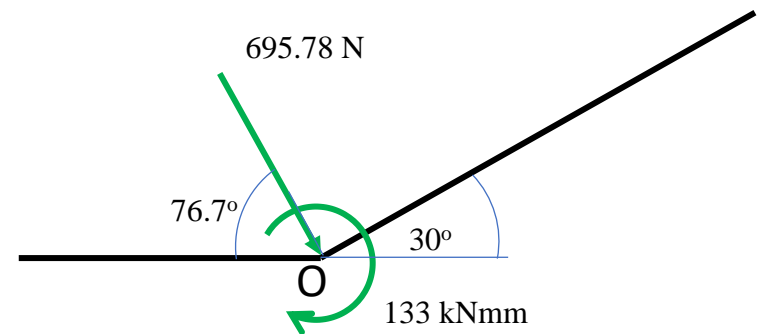
Equivalent Force-Couple Systems

Soln.

FBD



Equivalent system



Equivalent Force-Couple Systems

Example

Four tugboats are used to bring an ocean liner to its pier. Each tugboat exerts a 5000-lb force in the direction shown. Determine the equivalent force-couple system at the foremast O. Also determine the angle the resultant force makes with the horizontal as well as the direction of rotation of the moment.

