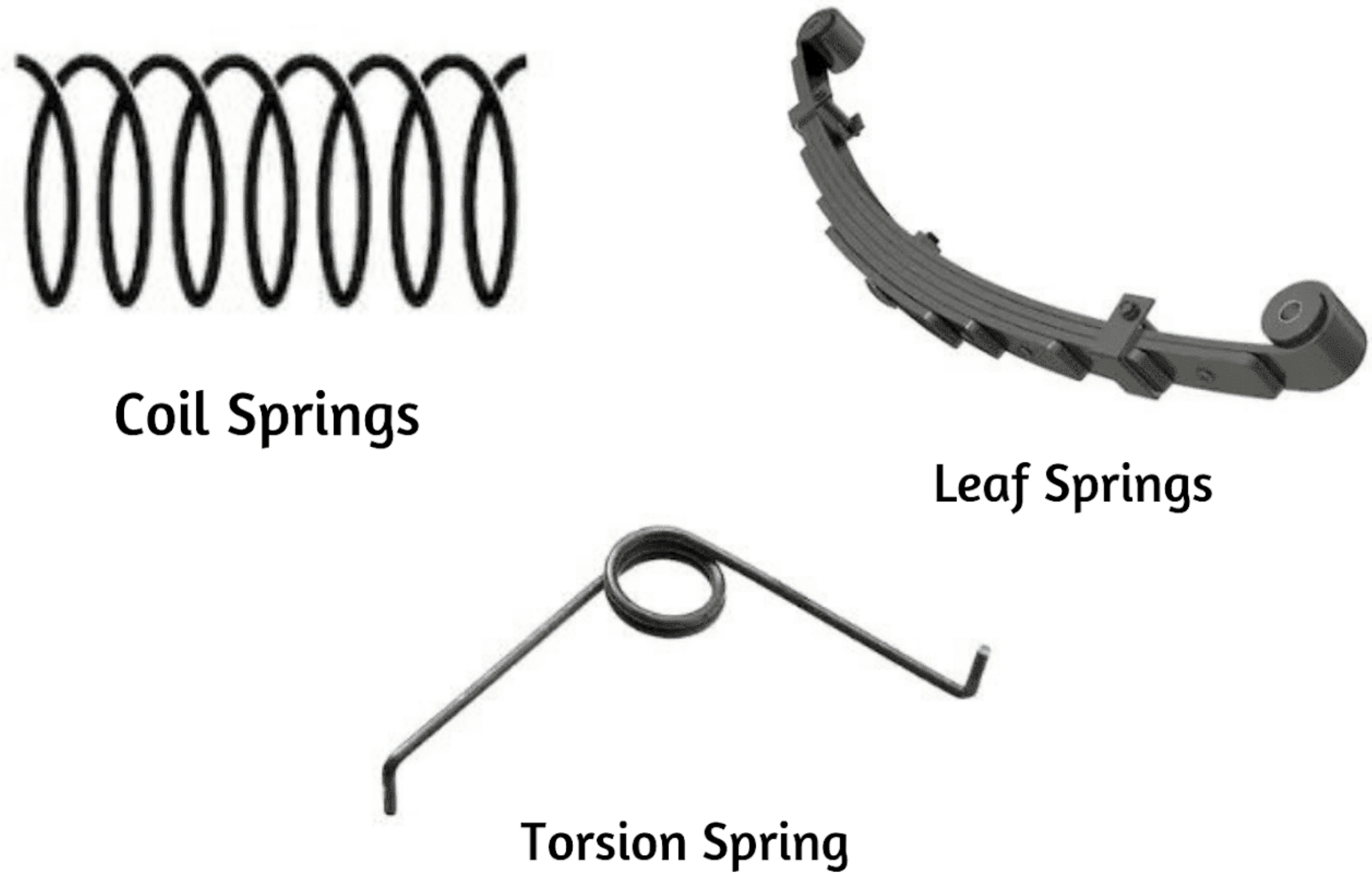
Mechanical Springs

Types of Springs



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Types of Springs Cont'd

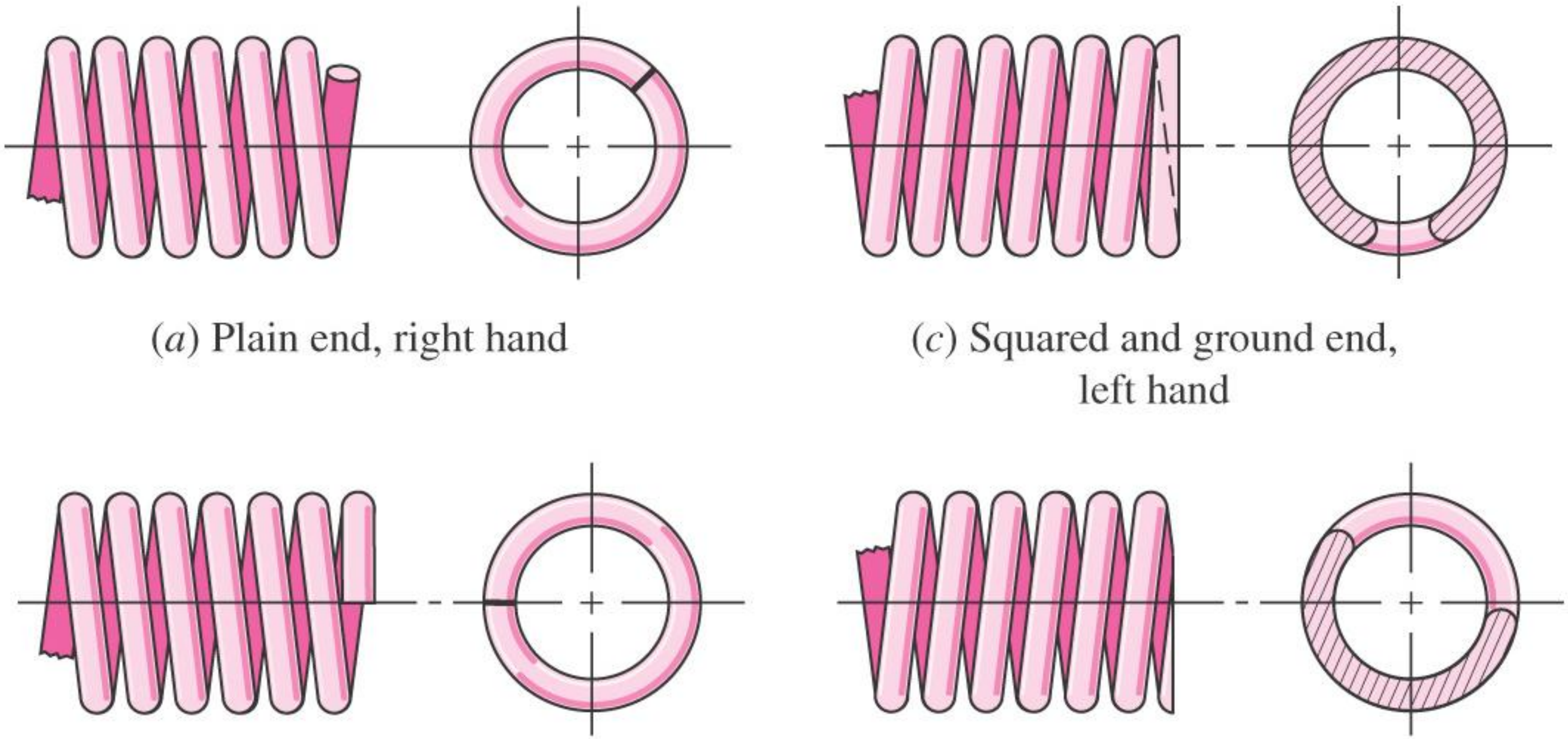
Application of Mechanical Springs

Their main **applications** are

1) To exert force,
2) To provide flexibility and
3) To store or absorb energy.

A helical **spring**, also known as a coil **spring**, is a **mechanical** device, which is typically used **to store energy and subsequently release it,** to **absorb shock,** or **to maintain a force between contacting surfaces.**

Ends of Compression Springs



(*a*) Plain end, right hand

(*c*) Squared and ground end, left hand

(*b*) Squared or closed end, right hand

(*d*) Plain end, ground, left hand

Fig. 10–2

Formulas for Compression Springs With Different Endsbh

Table 10–1

Term	Type of Spring Ends			
	Plain	Plain and Ground	Squared or Closed	Squared and Ground
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

N_a is the number of active coils

Some Common Spring Steels

- **Hard-drawn wire** (0.60-0.70*C*)
  - Cheapest general-purpose
  - Use only where life, accuracy, and deflection are not too important
- **Oil-tempered wire** (0.60-0.70*C*)
  - General-purpose
  - Heat treated for greater strength and uniformity of properties
  - Often used for larger diameter spring wire
- **Music wire** (0.80-0.95*C*)
  - Higher carbon for higher strength
  - Best, toughest, and most widely used for small springs
  - Good for fatigue

Some Common Spring Steels

- **Chrome-vanadium**
  - Popular alloy spring steel
  - Higher strengths than plain carbon steels
  - Good for fatigue, shock, and impact
- **Chrome-silicon**
  - Good for high stresses, long fatigue life, and shock

Strength of Spring Materials

- With small wire diameters, strength is a function of diameter.
- A graph of tensile strength vs. wire diameter is almost a straight line on log-log scale.
- The equation of this line is

$$S_{ut} = \frac{A}{d^m} \tag{10–14}$$

where A is the intercept and m is the slope.

- Values of A and m for common spring steels are given in Table 10–4.

Constants for Estimating Tensile Strength

$$S_{ut} = \frac{A}{d^m} \qquad (10\text{–}14)$$

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, MPa · mm^m	Relative Cost of Wire
Music wire*	A228	0.145	0.004–0.256	201	0.10–6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032–0.437	169	0.8–11.1	2005	3.1
Chrome-silicon wire‖	A401	0.108	0.063–0.375	202	1.6–9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013–0.10	169	0.3–2.5	1867	7.6–11
		0.263	0.10–0.20	128	2.5–5	2065	
		0.478	0.20–0.40	90	5–10	2911	
Phosphor-bronze wire**	B159	0	0.004–0.022	145	0.1–0.6	1000	8.0
		0.028	0.022–0.075	121	0.6–2	913	
		0.064	0.075–0.30	110	2–7.5	932	

Table 10–4

TABLE A-28

Table A–28

Decimal Equivalents of Wire and Sheet-Metal Gauges* (All Sizes Are Given in Inches)

Name of Gauge:	American or Brown & Sharpe	Birmingham or Stubs Iron Wire	United States Standard†	Manu-facturers Standard	Steel Wire or Washburn & Moen	Music Wire	Stubs Steel Wire	Twist Drill
Principal Use:	Nonferrous Sheet, Wire, and Rod	Tubing, Ferrous Strip, Flat Wire, and Spring Steel	Ferrous Sheet and Plate, 480 lbf/ft^3	Ferrous Sheet	Ferrous Wire Except Music Wire	Music Wire	Steel Drill Rod	Twist Drills and Drill Steel
7/0			0.500		0.490			
6/0	0.580 0		0.468 75		0.461 5	0.004		
5/0	0.516 5		0.437 5		0.430 5	0.005		
4/0	0.460 0	0.454	0.406 25		0.393 8	0.006		
3/0	0.409 6	0.425	0.375		0.362 5	0.007		
2/0	0.364 8	0.380	0.343 75		0.331 0	0.008		
0	0.324 9	0.340	0.312 5		0.306 5	0.009		
1	0.289 3	0.300	0.281 25		0.283 0	0.010	0.227	0.228 0
2	0.257 6	0.284	0.265 625		0.262 5	0.011	0.219	0.221 0
3	0.229 4	0.259	0.25	0.239 1	0.243 7	0.012	0.212	0.213 0
4	0.204 3	0.238	0.234 375	0.224 2	0.225 3	0.013	0.207	0.209 0
5	0.181 9	0.220	0.218 75	0.209 2	0.207 0	0.014	0.204	0.205 5
6	0.162 0	0.203	0.203 125	0.194 3	0.192 0	0.016	0.201	0.204 0
7	0.144 3	0.180	0.187 5	0.179 3	0.177 0	0.018	0.199	0.201 0
8	0.128 5	0.165	0.171 875	0.164 4	0.162 0	0.020	0.197	0.199 0
9	0.114 4	0.148	0.156 25	0.149 5	0.148 3	0.022	0.194	0.196 0
10	0.101 9	0.134	0.140 625	0.134 5	0.135 0	0.024	0.191	0.193 5
11	0.090 74	0.120	0.125	0.119 6	0.120 5	0.026	0.188	0.191 0
12	0.080 81	0.109	0.109 357	0.104 6	0.105 5	0.029	0.185	0.189 0
13	0.071 96	0.095	0.093 75	0.089 7	0.091 5	0.031	0.182	0.185 0
14	0.064 08	0.083	0.078 125	0.074 7	0.080 0	0.033	0.180	0.182 0
15	0.057 07	0.072	0.070 312 5	0.067 3	0.072 0	0.035	0.178	0.180 0
16	0.050 82	0.065	0.062 5	0.059 8	0.062 5	0.037	0.175	0.177 0
17	0.045 26	0.058	0.056 25	0.053 8	0.054 0	0.039	0.172	0.173 0

(Continued)

TABLE A-28 (CONTINUED)

Table A–28

Decimal Equivalents of Wire and Sheet-Metal Gauges* (All Sizes Are Given in Inches) (*Continued*)

Name of Gauge:	American or Brown & Sharpe	Birmingham or Stubs Iron Wire	United States Standard†	Manu-facturers Standard	Steel Wire or Washburn & Moen	Music Wire	Stubs Steel Wire	Twist Drill
Principal Use:	Nonferrous Sheet, Wire, and Rod	Tubing, Ferrous Strip, Flat Wire, and Spring Steel	Ferrous Sheet and Plate, 480 lbf/ft^3	Ferrous Sheet	Ferrous Wire Except Music Wire	Music Wire	Steel Drill Rod	Twist Drills and Drill Steel
18	0.040 30	0.049	0.05	0.047 8	0.047 5	0.041	0.168	0.169 5
19	0.035 89	0.042	0.043 75	0.041 8	0.041 0	0.043	0.164	0.166 0
20	0.031 96	0.035	0.037 5	0.035 9	0.034 8	0.045	0.161	0.161 0
21	0.028 46	0.032	0.034 375	0.032 9	0.031 7	0.047	0.157	0.159 0
22	0.025 35	0.028	0.031 25	0.029 9	0.028 6	0.049	0.155	0.157 0
23	0.022 57	0.025	0.028 125	0.026 9	0.025 8	0.051	0.153	0.154 0
24	0.020 10	0.022	0.025	0.023 9	0.023 0	0.055	0.151	0.152 0
25	0.017 90	0.020	0.021 875	0.020 9	0.020 4	0.059	0.148	0.149 5
26	0.015 94	0.018	0.018 75	0.017 9	0.018 1	0.063	0.146	0.147 0
27	0.014 20	0.016	0.017 187 5	0.016 4	0.017 3	0.067	0.143	0.144 0
28	0.012 64	0.014	0.015 625	0.014 9	0.016 2	0.071	0.139	0.140 5
29	0.011 26	0.013	0.014 062 5	0.013 5	0.015 0	0.075	0.134	0.136 0
30	0.010 03	0.012	0.012 5	0.012 0	0.014 0	0.080	0.127	0.128 5
31	0.008 928	0.010	0.010 937 5	0.010 5	0.013 2	0.085	0.120	0.120 0
32	0.007 950	0.009	0.010 156 25	0.009 7	0.012 8	0.090	0.115	0.116 0
33	0.007 080	0.008	0.009 375	0.009 0	0.011 8	0.095	0.112	0.113 0
34	0.006 305	0.007	0.008 593 75	0.008 2	0.010 4		0.110	0.111 0
35	0.005 615	0.005	0.007 812 5	0.007 5	0.009 5		0.108	0.110 0
36	0.005 000	0.004	0.007 031 25	0.006 7	0.009 0		0.106	0.106 5
37	0.004 453		0.006 640 625	0.006 4	0.008 5		0.103	0.104 0
38	0.003 965		0.006 25	0.006 0	0.008 0		0.101	0.101 5
39	0.003 531				0.007 5		0.099	0.099 5
40	0.003 145				0.007 0		0.097	0.098 0

*Specify sheet, wire, and plate by stating the gauge number, the gauge name, and the decimal equivalent in parentheses.

†Reflects present average and weights of sheet steel.

Estimating Torsional Yield Strength

- Since helical springs experience shear stress, shear yield strength is needed.
- If actual data is not available, estimate from tensile strength
- Assume yield strength in shear is between 60-90% of tensile strength

$$0.6S_{ut} \le S_{sy} \le 0.9S_{ut}$$

- Assume the von-Mises theory can be employed to relate the shear strength to the normal strength.

$$S_{sy} = 0.577S_y$$

- This results in

$$0.35S_{ut} \le S_{sy} \le 0.52S_{ut} \qquad (10\text{–}15)$$

Mechanical Properties of Some Spring Wires (Table 10–5)

Material	Elastic Limit, Percent of S_{ut} Tension	Elastic Limit, Percent of S_{ut} Torsion	Diameter d, in	E Mpsi	E GPa	G Mpsi	G GPa
Music wire A228	65–75	45–60	<0.032	29.5	203.4	12.0	82.7
			0.033–0.063	29.0	200	11.85	81.7
			0.064–0.125	28.5	196.5	11.75	81.0
			>0.125	28.0	193	11.6	80.0
HD spring A227	60–70	45–55	<0.032	28.8	198.6	11.7	80.7
			0.033–0.063	28.7	197.9	11.6	80.0
			0.064–0.125	28.6	197.2	11.5	79.3
			>0.125	28.5	196.5	11.4	78.6
Oil tempered A239	85–90	45–50		28.5	196.5	11.2	77.2
Valve spring A230	85–90	50–60		29.5	203.4	11.2	77.2
Chrome-vanadium A231	88–93	65–75		29.5	203.4	11.2	77.2
A232	88–93			29.5	203.4	11.2	77.2
Chrome-silicon A401	85–93	65–75		29.5	203.4	11.2	77.2
Stainless steel							
A313*	65–75	45–55		28	193	10	69.0
17-7PH	75–80	55–60		29.5	208.4	11	75.8
414	65–70	42–55		29	200	11.2	77.2
420	65–75	45–55		29	200	11.2	77.2
431	72–76	50–55		30	206	11.5	79.3
Phosphor-bronze B159	75–80	45–50		15	103.4	6	41.4
Beryllium-copper B197	70	50		17	117.2	6.5	44.8
	75	50–55		19	131	7.3	50.3
Inconel alloy X-750	65–70	40–45		31	213.7	11.2	77.2

Maximum Allowable Torsional Stresses

Table 10–6

Maximum Allowable Torsional Stresses for Helical Compression Springs in Static Applications

Source: Robert E. Joerres, "Springs," Chap. 6 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), *Standard Handbook of Machine Design,* 3rd ed., McGraw-Hill, New York, 2004.

Material	Maximum Percent of Tensile Strength: Before Set Removed (includes K_W or K_B)	Maximum Percent of Tensile Strength: After Set Removed (includes K_s)
Music wire and cold-drawn carbon steel	45	60–70
Hardened and tempered carbon and low-alloy steel	50	65–75
Austenitic stainless steels	35	55–65
Nonferrous alloys	35	55–65

Helical Spring

- Helical coil spring with round wire
- Equilibrium forces at cut section anywhere in the body of the spring indicates direct shear and torsion

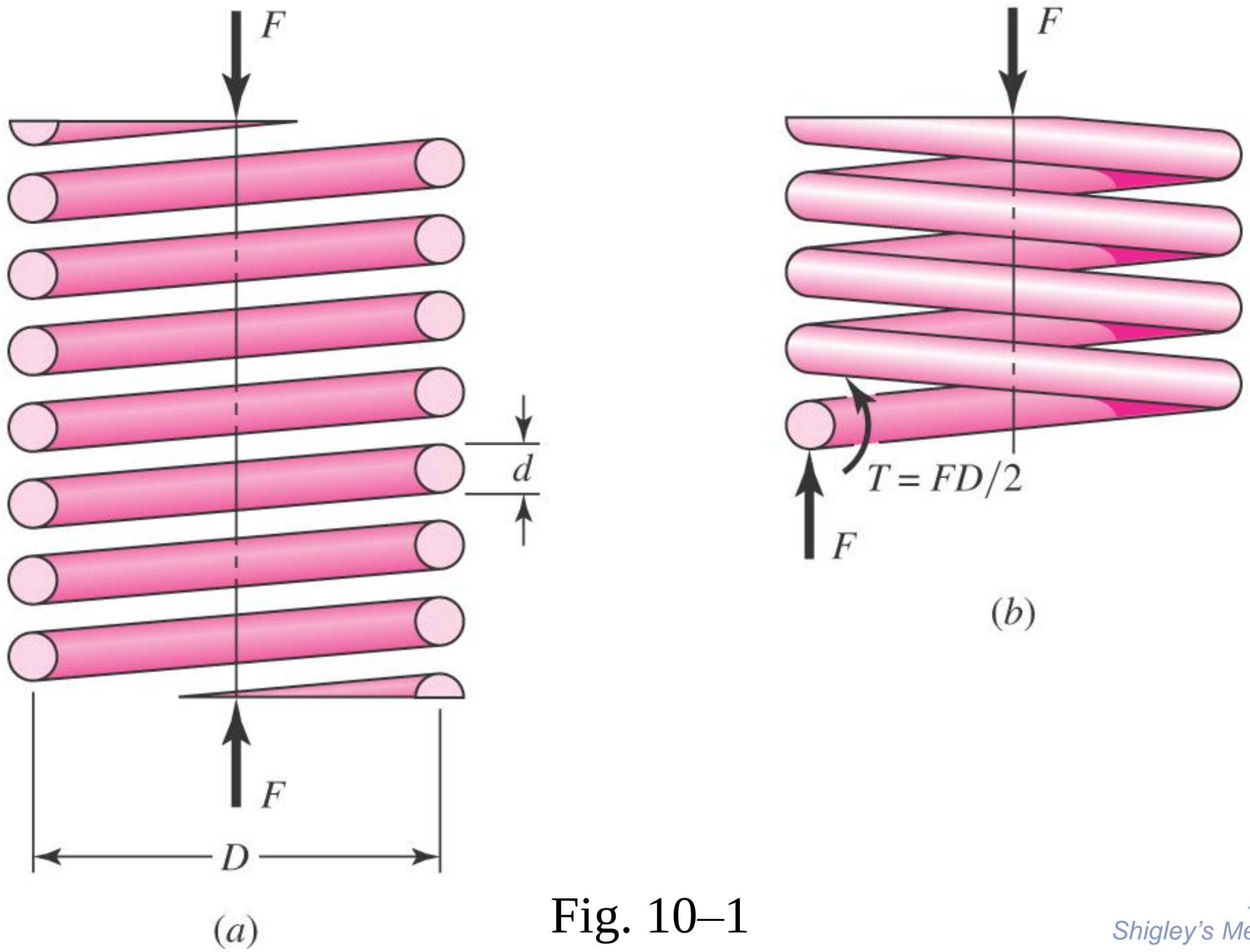


Fig. 10–1

Stresses in Helical Springs

- Torsional shear and direct shear
- Additive (maximum) on inside fiber of cross-section

$$\tau_{\max} = \frac{Tr}{J} + \frac{F}{A}$$

- Substitute terms

$$\tau_{\max} = \tau,\ T = FD/2,\ r = d/2,$$

$$J = \pi d^4/32 \qquad A = \pi d^2/4$$

$$\tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

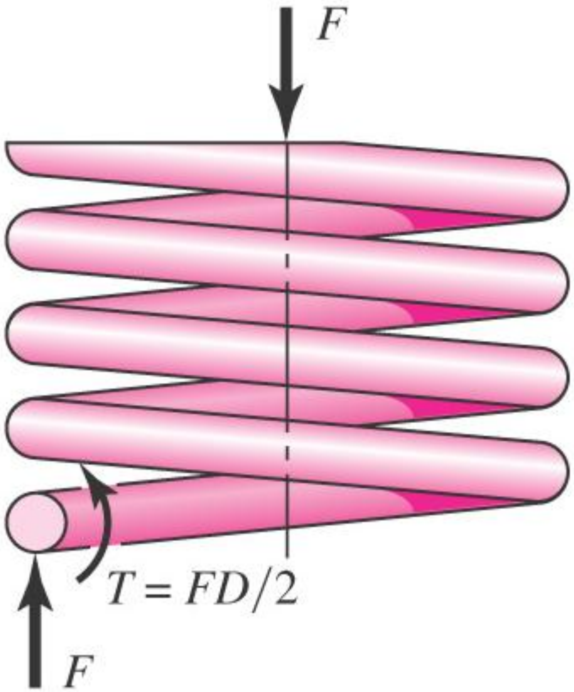


Fig. 10–1*b*

Stresses in Helical Springs

$$\tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

Factor out the torsional stress

$$\tau = \left(1 + \frac{d}{2D}\right)\left(\frac{8FD}{\pi d^3}\right)$$

Define *Spring Index* $C = \dfrac{D}{d}$ (10–1)

Define *Shear Stress Correction Factor*

$$K_s = 1 + \frac{1}{2C} = \frac{2C+1}{2C} \qquad (10\text{–}3)$$

Maximum shear stress for helical spring

$$\tau = K_s \frac{8FD}{\pi d^3} \qquad (10\text{–}2)$$

Curvature Effect

- Stress concentration type of effect on inner fiber due to curvature
- Can be ignored for static, ductile conditions due to localized cold-working
- **Can account for effect by replacing K_s with *Wahl factor* or *Bergsträsser factor* which account for both direct shear and curvature effect**

$$K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C} \qquad (10\text{–}4)$$

$$K_B = \frac{4C+2}{4C-3} \qquad (10\text{–}5)$$

- We will use K_B

$$\tau = K_B \frac{8FD}{\pi d^3} \qquad (10\text{–}7)$$

Deflection of Helical Springs

Using Castigliano's method to relate force and deflection, it can be shown that the deflection (y) is given as:

$$y = \frac{\partial U}{\partial F} = \frac{8FD^3N}{d^4G} + \frac{4FDN}{d^2G} \qquad (10\text{–}8)$$

$$y = \frac{8FD^3N}{d^4G}\left(1 + \frac{1}{2C^2}\right) \approx \frac{8FD^3N}{d^4G}$$

$$k \approx \frac{d^4G}{8D^3N} \qquad (10\text{–}9)$$

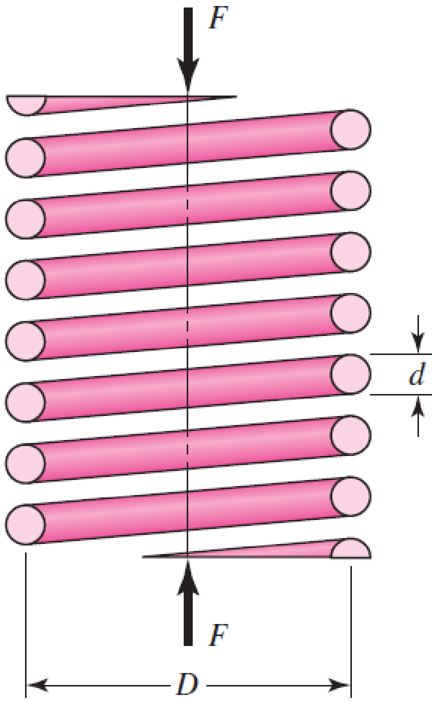


Fig. 10–1*a*

Set Removal

- *Set removal* or *presetting* is a process used in manufacturing a spring to induce useful residual stresses.
- The spring is made longer than needed, then compressed to solid height, intentionally exceeding the yield strength.
- This operation *sets* the spring to the required final free length.
- Yielding induces residual stresses opposite in direction to those induced in service.
- 10 to 30 percent of the initial free length should be removed.
- Set removal is not recommended when springs are subject to fatigue.

Critical Deflection for Stability

- Buckling type of instability can occur in compression springs when the deflection exceeds the *critical deflection* y_{cr}

$$y_{cr} = L_0 C'_1 \left[1 - \left(1 - \frac{C'_2}{\lambda_{eff}^2} \right)^{1/2} \right] \qquad (10\text{–}10)$$

- L_{eff} is the *effective slenderness ratio*

$$\lambda_{eff} = \frac{\alpha L_0}{D} \qquad (10\text{–}11)$$

- α is the *end-condition constant*, defined on the next slide
- C'_1 and C'_2 are elastic constants

$$C'_1 = \frac{E}{2(E - G)}$$

$$C'_2 = \frac{2\pi^2 (E - G)}{2G + E}$$

End-Condition Constant

- The α term in Eq. (10–11) is the *end-condition constant.*
- It accounts for the way in which the ends of the spring are supported.
- Values are given in Table 10–2.

End Condition	Constant α
Spring supported between flat parallel surfaces (fixed ends)	0.5
One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

*Ends supported by flat surfaces must be squared and ground.

Table 10–2

Absolute Stability

- Absolute stability occurs when, in Eq. (10–10),

$$C_2' / \lambda_{\text{eff}}^2 > 1$$

- This results in the condition for absolute stability

$$L_0 < \frac{\pi D}{\alpha}\left[\frac{2(E-G)}{2G+E}\right]^{1/2} \qquad (10\text{–}12)$$

- **For steels, this turns out to be**

$$L_0 < 2.63\frac{D}{\alpha} \qquad (10\text{–}13)$$

Example 1- Helical Compression Spring Fundamentals

A helical compression spring is made of no. 16 music wire. The outside coil diameter of the spring is $\frac{7}{16}$ in. The ends are squared and there are $12\frac{1}{2}$ total turns.

(*a*) Estimate the torsional yield strength of the wire.

(*b*) Estimate the static load corresponding to the yield strength.

(*c*) Estimate the scale of the spring.

(*d*) Estimate the deflection that would be caused by the load in part (*b*).

(*e*) Estimate the solid length of the spring.

(*f*) What length should the spring be to ensure that when it is compressed solid and then released, there will be no permanent change in the free length?

(*g*) Given the length found in part (*f*), is buckling a possibility?

(*h*) What is the pitch of the body coil?

Example 1 - Solution

(*a*) From Table A–28, the wire diameter is $d = 0.037$ in. From Table 10–4, we find $A = 201\ \text{kpsi} \cdot \text{in}^m$ and $m = 0.145$. Therefore, from Eq. (10–14)

$$S_{ut} = \frac{A}{d^m} = \frac{201}{0.037^{0.145}} = 324\ \text{kpsi}$$

Then, from Table 10–6,

$$S_{sy} = 0.45 S_{ut} = 0.45(324) = 146\ \text{kpsi} \qquad \text{Answer}$$

Example 1 – Solution Cont'd

(*b*) The mean spring coil diameter is $D = \frac{7}{16} - 0.037 = 0.400$ in, and so the spring index is $C = 0.400/0.037 = 10.8$. Then, from Eq. (10–6),

$$K_B = \frac{4C + 2}{4C - 3} = \frac{4\,(10.8) + 2}{4\,(10.8) - 3} = 1.124$$

Now rearrange Eq. (10–7) replacing τ with S_{sy}, and solve for F:

$$F = \frac{\pi d^3 S_{sy}}{8K_B D} = \frac{\pi (0.037^3) 146(10^3)}{8(1.124)\,0.400} = 6.46 \text{ lbf}$$

Answer

Example 1 – Solution Cont'd

(*c*) From Table 10–1, $N_a = 12.5 - 2 = 10.5$ turns. In Table 10–5, $G = 11.85$ Mpsi, and the scale of the spring is found to be, from Eq. (10–9),

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.037^4\,(11.85)10^6}{8(0.400^3)10.5} = 4.13 \text{ lbf/in}$$ Answer

(*d*)

$$y = \frac{F}{k} = \frac{6.46}{4.13} = 1.56 \text{ in}$$ Answer

Example 1 – Solution Cont'd

(*e*) From Table 10–1,

$$L_s = (N_t + 1)d = (12.5 + 1)0.037 = 0.500 \text{ in}$$ Answer

(*f*) $$L_0 = y + L_s = 1.56 + 0.500 = 2.06 \text{ in.}$$ Answer

(*g*) To avoid buckling, Eq. (10–13) and Table 10–2 give

$$L_0 < 2.63\frac{D}{\alpha} = 2.63\frac{0.400}{0.5} = 2.10 \text{ in}$$ Answer

Mathematically, a free length of 2.06 in is less than 2.10 in, and buckling is unlikely. However, the forming of the ends will control how close α is to 0.5. This has to be investigated and an inside rod or exterior tube or hole may be needed.

(*h*) Finally, from Table 10–1, the pitch of the body coil is

$$p = \frac{L_0 - 3d}{N_a} = \frac{2.06 - 3(0.037)}{10.5} = 0.186 \text{ in}$$ Answer

Helical Compression Spring Design for Static Service

- Limit the design solution space by setting some practical limits
- Preferred range for spring index (Constraint)

$$4 \le C \le 12 \qquad (10\text{–}18)$$

  - Lower values are difficult to manufacture due to cracking issues
  - The higher index tends to tangle causing packing problems

- Preferred range for number of active coils (Constraint)

$$3 \le N_a \le 15 \qquad (10\text{–}19)$$

Helical Compression Spring Design for Static Service

- To achieve best linearity of spring constant, preferred to limit operating force to the central 75% of the force-deflection curve between $F = 0$ and $F = F_s$ (closure force)
- Limit the maximum operating force to $F_{max} \leq 7/8\ F_s$
- Define *fractional overrun to closure* as ξ where

$$F_s = (1 + \xi) F_{\max} \qquad (10\text{–}17)$$

- This leads to

$$F_s = (1 + \xi) F_{\max} = (1 + \xi)\left(\frac{7}{8}\right) F_s$$

- Solving the outer equality for ξ, $\xi = 1/7 = 0.143$ □0.15
- Thus, it is recommended that fractional overrun:

$$\xi \geq 0.15 \qquad (10\text{–}20)$$

Summary of Recommended Design Conditions

- **The following design conditions are recommended for helical compression spring design for static service**

$$4 \leq C \leq 12 \tag{10–18}$$

$$3 \leq N_a \leq 15 \tag{10–19}$$

$$\xi \geq 0.15 \tag{10–20}$$

$$n_s \geq 1.2 \tag{10–21}$$

$$L_0 < \frac{\pi D}{\alpha}\left[\frac{2(E-G)}{2G+E}\right]^{1/2} \tag{10–12}$$

where n_s is the factor of safety at solid height.

Finding Spring Index (C) for As-Wound Branch

- From Eqs. (10–3) and (10–17),

$$\frac{S_{sy}}{n_s} = K_B \frac{8F_s D}{\pi d^3} = \frac{4C+2}{4C-3}\left[\frac{8(1+\xi)F_{\max}C}{\pi d^2}\right] \quad (a)$$

- Let

$$\alpha = \frac{S_{sy}}{n_s} \quad (b)$$

$$\beta = \frac{8(1+\xi)F_{\max}}{\pi d^2} \quad (c)$$

- Substituting (*b*) and (*c*) into (*a*) yields a quadratic in *C*.

$$C = \frac{2\alpha-\beta}{4\beta} + \sqrt{\left(\frac{2\alpha-\beta}{4\beta}\right)^2 - \frac{3\alpha}{4\beta}} \quad (10\text{–}23)$$

Figure of Merit for High Volume Production

- For high volume production, the **figure of merit** (*fom*) may be the cost of the wire.
- The *fom* would be proportional to the relative material cost **(Table 10-4),** weight density, and volume

$$\text{fom} = -(\text{relative material cost})\frac{\gamma \pi^2 d^2 N_t D}{4} \qquad (10\text{–}22)$$

Design Flowchart for Static Loading

Choose d

Over-a-rod	Free		In-a-hole
As-wound or set	As-wound	Set removed	As-wound or set
$D = d_{\mathrm{rod}} + d + \mathrm{allow}$	$S_{sy} = \mathrm{const}(A)/d^{m\dagger}$	$S_{sy} = 0.65A/d^m$	$D = d_{\mathrm{hole}} - d - \mathrm{allow}$

$$C = \frac{2\alpha - \beta}{4\beta} + \sqrt{\left(\frac{2\alpha - \beta}{4\beta}\right)^2 - \frac{3\alpha}{4\beta}}$$

$$\alpha = \frac{S_{sy}}{n_s} \qquad \beta = \frac{8(1+\xi)F_{\max}}{\pi d^2}$$

$$D = Cd$$

$$D = \frac{S_{sy}\pi d^3}{8n_s(1+\xi)F_{\max}}$$

Continue on next slide

Design Flowchart for Static Loading

Continued from previous slide

$$C = D/d$$

$$K_B = (4C + 2)/(4C - 3)$$

$$\tau_s = 8K_B(1 + \xi)F_{\max}D/(\pi d^3)$$

$$n_s = S_{sy}/\tau_s$$

$$\mathrm{OD} = D + d$$

$$\mathrm{ID} = D - d$$

$$N_a = Gd^4 y_{\max}/(8D^3 F_{\max})$$

N_t: Table 10–1

L_s: Table 10–1

L_O: Table 10–1

$$(L_O)_{\mathrm{cr}} = 2.63D/\alpha$$

$$\mathrm{fom} = -(\text{rel. cost})\,\gamma\pi^2 d^2 N_t D/4$$

Design Flowchart for Static Loading

Print or display: d, D, C, OD, ID, N_a, N_t, L_s, L_O, $(L_O)_{cr}$, n_s, fom

Build a table, conduct design assessment by inspection

Eliminate infeasible designs by showing active constraints

Choose among satisfactory designs using the figure of merit

Example 2 – Helical Compression Spring Design

A music wire helical compression spring is needed to support a 10 kg load while compressed by 50 mm. Because of assembly considerations, the solid height cannot exceed 25 mm and the free length cannot be more than 105 mm. Design the spring Provide all the necessary parameters.

Example 3- Design of Helical Compression Spring

Design a compression spring with plain ends using hard-drawn wire. The deflection is to be 55 mm when the force is 80 N and to close solid when the force is 110 N. Upon closure, use a design factor of 1.2 guarding against yield. Select the smallest possible gauge W&M (Washburn & Moen) wire.

Critical Frequency of Helical Springs

- When one end of a spring is displaced rapidly, a wave called a *spring surge* travels down the spring.
- If the other end is fixed, the wave can reflect back.
- If the wave frequency is near the natural frequency of the spring, resonance may occur resulting in extremely high stresses.
- Catastrophic failure may occur, as shown in this valve-spring from an over-revved engine.

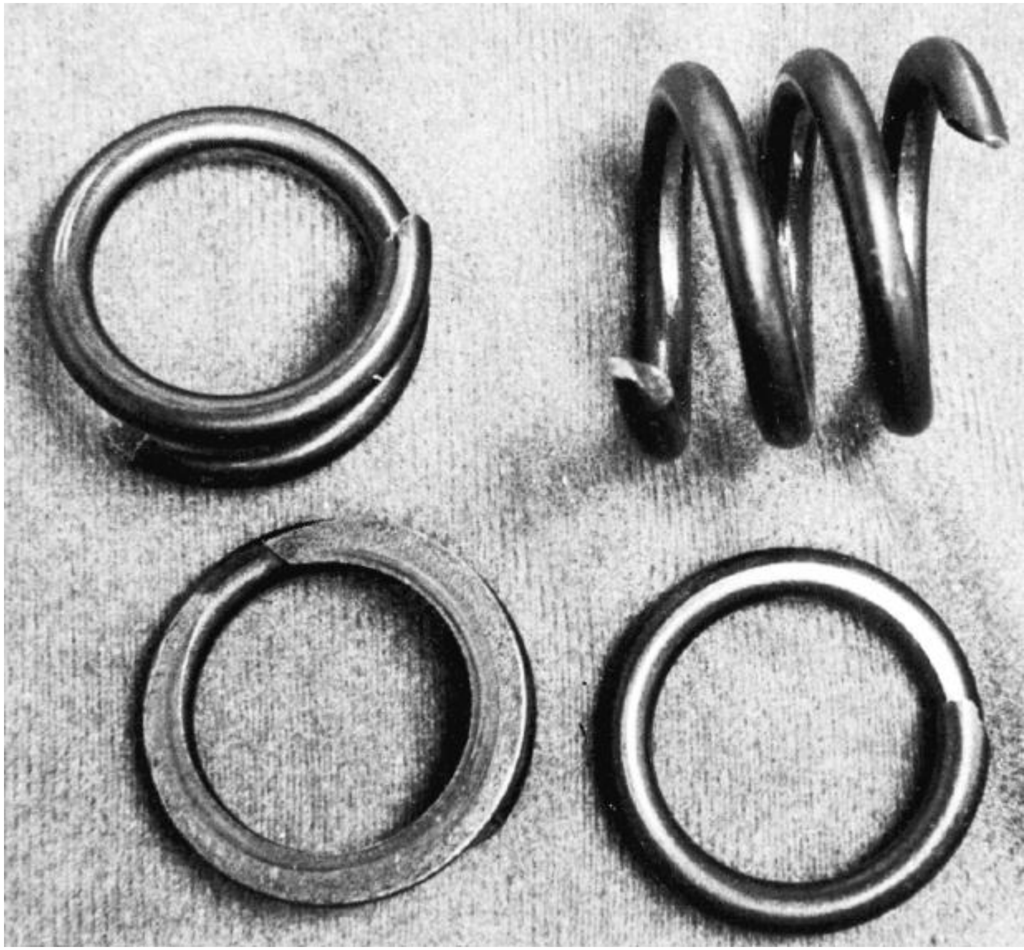


Fig. 10–4

Critical Frequency of Helical Springs

- The governing equation is the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{W}{kgl^2}\frac{\partial^2 u}{\partial t^2} \tag{10–24}$$

where k = spring rate
g = acceleration due to gravity
l = length of spring between plates
W = weight of spring
x = coordinate along length of spring
u = motion of any particle at distance x

Critical Frequency of Helical Springs

- The solution to this equation is harmonic and depends on the given physical properties as well as the end conditions.
- The harmonic, natural, frequencies for a spring placed between two flat and parallel plates, in radians per second, are

$$\omega = m\pi\sqrt{\frac{kg}{W}} \qquad m = 1, 2, 3, \ldots$$

- **In cycles per second, or hertz,**

$$f = \frac{1}{2}\sqrt{\frac{kg}{W}} \tag{10–25}$$

- With one end against a flat plate and the other end free,

$$f = \frac{1}{4}\sqrt{\frac{kg}{W}} \tag{10–26}$$

Critical Frequency of Helical Springs

- The weight of a helical spring is

$$W = AL\gamma = \frac{\pi d^2}{4}(\pi D N_a)(\gamma) = \frac{\pi^2 d^2 D N_a \gamma}{4} \qquad (10\text{–}27)$$

- The fundamental critical frequency should be greater than 15 to 20 times the frequency of the force or motion of the spring.
- If necessary, redesign the spring to increase *k* or decrease *W*.

Stresses for Fatigue Loading

- From the standard approach, the alternating and midrange forces are

$$F_a = \frac{F_{\max} - F_{\min}}{2} \qquad (10\text{–}31a)$$

$$F_m = \frac{F_{\max} + F_{\min}}{2} \qquad (10\text{–}31b)$$

- The alternating and midrange stresses are

$$\tau_a = K_B \frac{8F_a D}{\pi d^3} \qquad (10\text{–}32)$$

$$\tau_m = K_B \frac{8F_m D}{\pi d^3} \qquad (10\text{–}33)$$

Torsional Modulus of Rupture

- The torsional modulus of rupture S_{su} will be needed for the fatigue diagram.
- Lacking test data, the recommended value is

$$S_{su} = 0.67 S_{ut} \tag{10-30}$$

Example 4 Fatigue Loading

A music wire helical compression spring with infinite life is needed to resist a dynamic load that varies from 5 to 20 lbf at 5 Hz while the end deflection varies from $\frac{1}{2}$ to 2 in. Because of assembly considerations, the solid height cannot exceed 1 in and the free length cannot be more than 4 in. The springmaker has the following wire sizes in stock: 0.069, 0.071, 0.080, 0.085, 0.090, 0.095, 0.105, and 0.112 in.

Example 4

The a priori decisions are:

- Material and condition: for music wire, $A = 201$ kpsi $\cdot$ in^m, $m = 0.145$, $G = 11.75(10^6)$ psi; relative cost is 2.6
- Surface treatment: unpeened
- End treatment: squared and ground
- Robust linearity: $\xi = 0.15$
- Set: use in as-wound condition
- Fatigue-safe: $n_f = 1.5$ using the Sines-Zimmerli fatigue-failure criterion
- Function: $F_{\min} = 5$ lbf, $F_{\max} = 20$ lbf, $y_{\min} = 0.5$ in, $y_{\max} = 2$ in, spring operates free (no rod or hole)
- Decision variable: wire size d

The figure of merit will be the cost of wire to wind the spring, Eq. (10–22) without density. The design strategy will be to set wire size d, build a table, inspect the table, and choose the satisfactory spring with the highest figure of merit.

Example 4

Set $d = 0.112$ in. Then

$$F_a = \frac{20 - 5}{2} = 7.5 \text{ lbf} \qquad F_m = \frac{20 + 5}{2} = 12.5 \text{ lbf}$$

$$k = \frac{F_{\max}}{y_{\max}} = \frac{20}{2} = 10 \text{ lbf/in}$$

$$S_{ut} = \frac{201}{0.112^{0.145}} = 276.1 \text{ kpsi}$$

$$S_{su} = 0.67(276.1) = 185.0 \text{ kpsi}$$

$$S_{sy} = 0.45(276.1) = 124.2 \text{ kpsi}$$

Example 4

From Eq. (10–28), with the Sines criterion, $S_{se} = S_{sa} = 35$ kpsi. Equation (10–23) can be used to determine C with S_{se}, n_f, and F_a in place of S_{sy}, n_s, and $(1+\xi)F_{\max}$, respectively. Thus,

$$\alpha = \frac{S_{se}}{n_f} = \frac{35\ 000}{1.5} = 23\ 333\ \text{psi}$$

$$\beta = \frac{8F_a}{\pi d^2} = \frac{8(7.5)}{\pi(0.112^2)} = 1522.5\ \text{psi}$$

$$C = \frac{2(23\ 333) - 1522.5}{4(1522.5)} + \sqrt{\left[\frac{2(23\ 333) - 1522.5}{4(1522.5)}\right]^2 - \frac{3(23\ 333)}{4(1522.5)}} = 14.005$$

Example 4

$$D = Cd = 14.005(0.112) = 1.569 \text{ in}$$

$$F_s = (1+\xi)F_{\text{max}} = (1+0.15)20 = 23 \text{ lbf}$$

$$N_a = \frac{d^4 G}{8D^3 k} = \frac{0.112^4(11.75)(10^6)}{8(1.569)^3 10} = 5.98 \text{ turns}$$

$$N_t = N_a + 2 = 5.98 + 2 = 7.98 \text{ turns}$$

$$L_s = dN_t = 0.112(7.98) = 0.894 \text{ in}$$

$$L_0 = L_s + \frac{F_s}{k} = 0.894 + \frac{23}{10} = 3.194 \text{ in}$$

$$\text{ID} = 1.569 - 0.112 = 1.457 \text{ in}$$

$$\text{OD} = 1.569 + 0.112 = 1.681 \text{ in}$$

$$y_s = L_0 - L_s = 3.194 - 0.894 = 2.30 \text{ in}$$

$$(L_0)_{\text{cr}} < \frac{2.63D}{\alpha} = 2.63\frac{(1.569)}{0.5} = 8.253 \text{ in}$$

Example 4

$$K_B = \frac{4(14.005) + 2}{4(14.005) - 3} = 1.094$$

$$W = \frac{\pi^2 d^2 D N_a \gamma}{4} = \frac{\pi^2 0.112^2 (1.569) 5.98 (0.284)}{4} = 0.0825 \text{ lbf}$$

$$f_n = 0.5\sqrt{\frac{386k}{W}} = 0.5\sqrt{\frac{386(10)}{0.0825}} = 108 \text{ Hz}$$

Example 4

$$\tau_a = K_B \frac{8F_a D}{\pi d^3} = 1.094 \frac{8(7.5)1.569}{\pi 0.112^3} = 23\,334 \text{ psi}$$

$$\tau_m = \tau_a \frac{F_m}{F_a} = 23\,334 \frac{12.5}{7.5} = 38\,890 \text{ psi}$$

$$\tau_s = \tau_a \frac{F_s}{F_a} = 23\,334 \frac{23}{7.5} = 71\,560 \text{ psi}$$

$$n_f = \frac{S_{sa}}{\tau_a} = \frac{35\,000}{23\,334} = 1.5$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{124\,200}{71\,560} = 1.74$$

$$\text{fom} = -(\text{relative material cost})\pi^2 d^2 N_t D/4$$

$$= -2.6\pi^2(0.112^2)(7.98)1.569/4 = -1.01$$

Example 4

Inspection of the results shows that all conditions are satisfied except for $4 \leq C \leq 12$. Repeat the process using the other available wire sizes and develop the following table:

d:	0.069	0.071	0.080	0.085	0.090	0.095	0.105	0.112
D	0.297	0.332	0.512	0.632	0.767	0.919	1.274	1.569
ID	0.228	0.261	0.432	0.547	0.677	0.824	1.169	1.457
OD	0.366	0.403	0.592	0.717	0.857	1.014	1.379	1.681
C	4.33	4.67	6.40	7.44	8.53	9.67	12.14	14.00
N_a	127.2	102.4	44.8	30.5	21.3	15.4	8.63	6.0
L_s	8.916	7.414	3.740	2.750	2.100	1.655	1.116	0.895
L_0	11.216	9.714	6.040	5.050	4.400	3.955	3.416	3.195
$(L_0)_{cr}$	1.562	1.744	2.964	3.325	4.036	4.833	6.703	8.250
n_f	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50
n_s	1.86	1.85	1.82	1.81	1.79	1.78	1.75	1.74
f_n	87.5	89.7	96.9	99.7	101.9	103.8	106.6	108
fom	−1.17	−1.12	−0.983	−0.948	−0.930	−0.927	−0.958	−1.01

Example 4

The problem-specific inequality constraints are

$$L_s \leq 1 \text{ in}$$

$$L_0 \leq 4 \text{ in}$$

$$f_n \geq 5(20) = 100 \text{ Hz}$$

The general constraints are

$$3 \leq N_a \leq 15$$

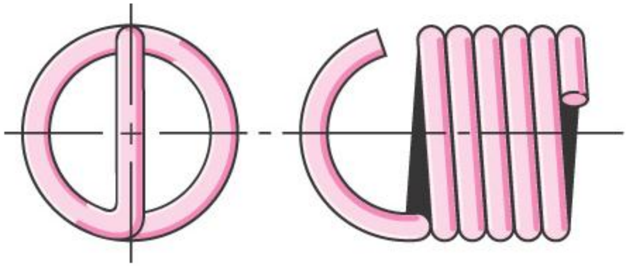
$$4 \leq C \leq 12$$

$$(L_0)_{\text{cr}} > L_0$$

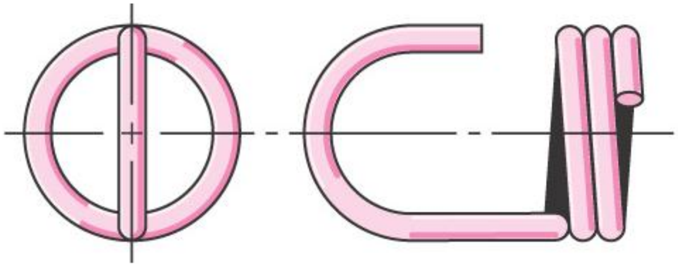
We see that none of the diameters satisfy the given constraints. The 0.105-in-diameter wire is the closest to satisfying all requirements. The value of $C = 12.14$ is not a serious deviation and can be tolerated. However, the tight constraint on L_s needs to be addressed. If the assembly conditions can be relaxed to accept a solid height of 1.116 in, we have a solution. If not, the only other possibility is to use the 0.112-in diameter and accept a value $C = 14$, individually package the springs, and possibly reconsider supporting the spring in service.

Extension Springs

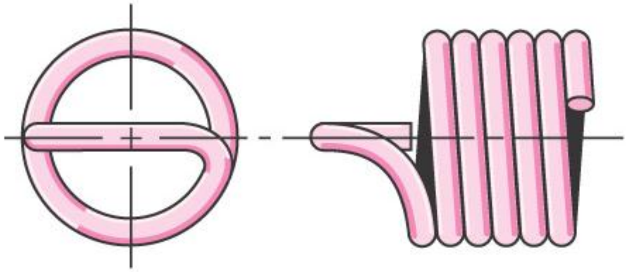
- Extension springs are similar to compression springs within the body of the spring.
- To apply tensile loads, hooks are needed at the ends of the springs.
- Some common hook types:



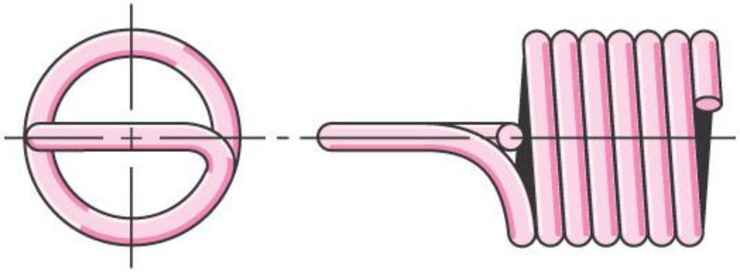
(*a*) Machine half loop–open



(*b*) Raised hook



(*c*) Short twisted loop



(*d*) Full twisted loop

Fig. 10–5

Terminology of Extension Spring Dimensions

- The free length is measured inside the end hooks.

$$L_0 = 2(D - d) + (N_b + 1)d = (2C - 1 + N_b)d \qquad (10\text{–}39)$$

- The hooks contribute to the spring rate. This can be handled by obtaining an equivalent number of active coils.

$$N_a = N_b + \frac{G}{E} \qquad (10\text{–}40)$$

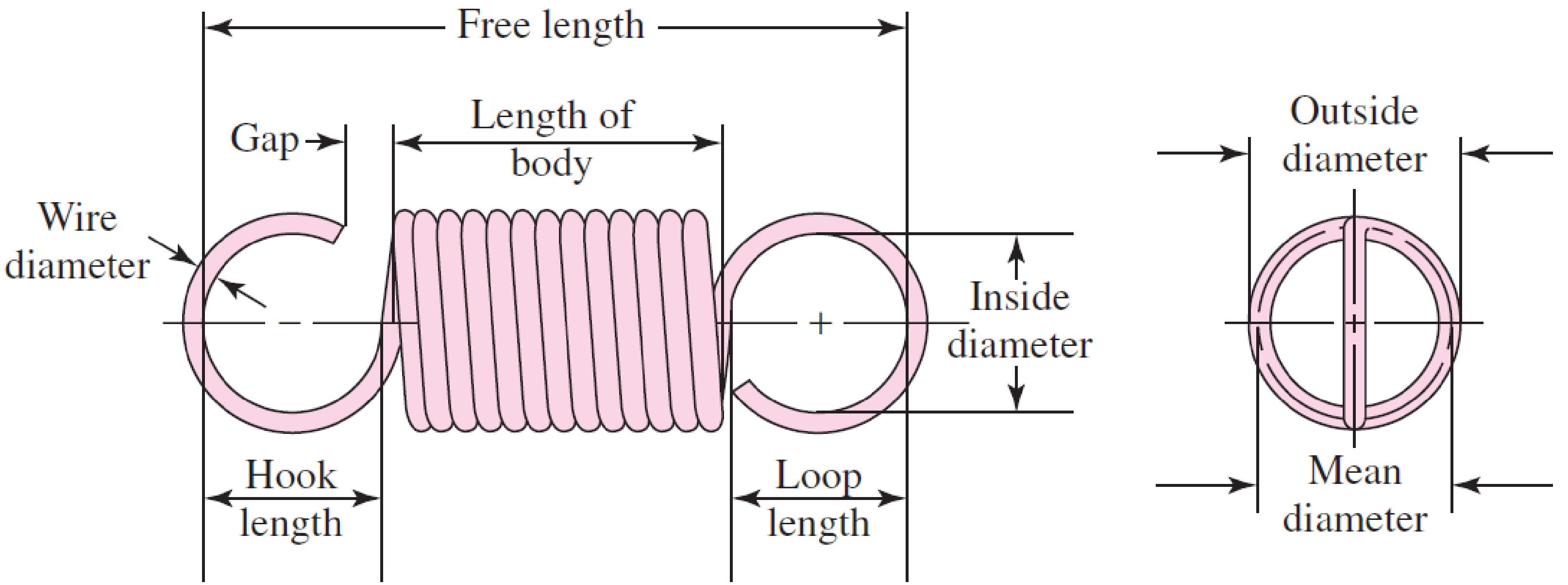


Fig. 10–7 (*b*)

Helical Coil *Torsion* Springs

- Helical coil springs can be loaded with torsional end loads.
- Special ends are used to allow a force to be applied at a distance from the coil axis.
- Usually used over a rod to maintain alignment and provide buckling resistance.

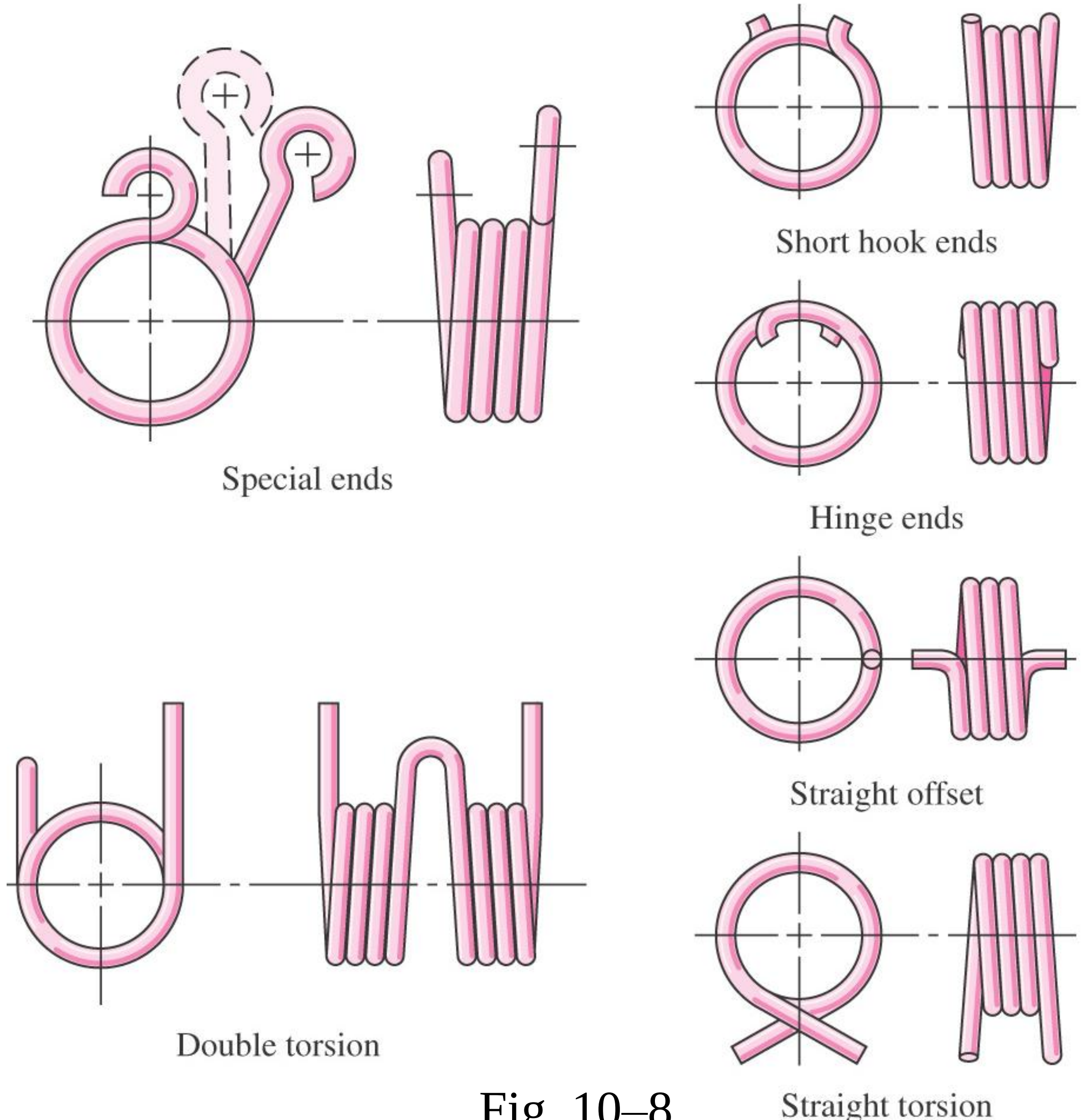


Fig. 10–8

Stress in Torsion Springs

- The coil of a torsion spring experiences bending stress (despite the name of the spring).
- Including a stress-correction factor, the stress in the coil can be represented by

$$\sigma = K\frac{Mc}{I}$$

- The stress-correction factor at inner and outer fibers has been found analytically for round wire to be

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} \qquad K_o = \frac{4C^2 + C - 1}{4C(C+1)} \tag{10–43}$$

- K_i is always larger, giving the highest stress at the inner fiber.
- With a bending moment of $M = Fr$, for round wire the bending stress is

$$\sigma = K_i\frac{32Fr}{\pi d^3} \tag{10–44}$$

Spring Rate for Torsion Springs

- Angular deflection is commonly expressed in both radians and revolutions (turns).
- If a term contains revolutions, the variable will be expressed with a prime sign.
- The spring rate, if linear, is

$$k' = \frac{M_1}{\theta'_1} = \frac{M_2}{\theta'_2} = \frac{M_2 - M_1}{\theta'_2 - \theta'_1} \qquad (10\text{–}45)$$

where moment M can be expressed as Fl or Fr.

Deflection in the Body of Torsion Springs

- Use Castigliano's method to find the deflection in radians in the body of a torsion spring.

$$U = \int \frac{M^2\,dx}{2EI}$$

- Let $M = Fl = Fr$, and integrate over the length of the body-coil wire. The force F will deflect through a distance $r\theta$.

$$r\theta = \frac{\partial U}{\partial F} = \int_0^{\pi D N_b} \frac{\partial}{\partial F}\left(\frac{F^2 r^2\,dx}{2EI}\right) = \int_0^{\pi D N_b} \frac{F r^2\,dx}{EI}$$

- Using I for round wire, and solving for θ,

$$\theta = \frac{64 F r D N_b}{d^4 E} = \frac{64 M D N_b}{d^4 E}$$

Deflection in the Ends of Torsion Springs

- The deflection in the ends of the spring must be accounted for.
- The angle subtended by the end deflection is obtained from standard cantilever beam approach.

$$\theta_e = \frac{y}{l} = \frac{Fl^2}{3EI} = \frac{Fl^2}{3E(\pi d^4/64)} = \frac{64Ml}{3\pi d^4 E} \qquad (10\text{–}46)$$

Deflection in Torsion Springs

- The total angular deflection is obtained by combining the body deflection and the end deflection.
- With end lengths of l_1 and l_2, combining the two deflections previously obtained gives,

$$\theta_t = \frac{64MDN_b}{d^4E} + \frac{64Ml_1}{3\pi d^4E} + \frac{64Ml_2}{3\pi d^4E} = \frac{64MD}{d^4E}\left(N_b + \frac{l_1 + l_2}{3\pi D}\right) \qquad (10\text{–}47)$$

Equivalent Active Turns

- The equivalent number of active turns, including the effect of the ends, is

$$N_a = N_b + \frac{l_1 + l_2}{3\pi D} \qquad (10\text{–}48)$$

Spring Rate in Torsion Springs

- The spring rate, in torque per radian

$$k = \frac{Fr}{\theta_t} = \frac{M}{\theta_t} = \frac{d^4 E}{64 D N_a} \tag{10–49}$$

- The spring rate, in torque per turn

$$k' = \frac{2\pi d^4 E}{64 D N_a} = \frac{d^4 E}{10.2 D N_a} \tag{10–50}$$

- To compensate for the effect of friction between the coils and an arbor, tests show that the 10.2 should be increased to 10.8.

$$k' = \frac{d^4 E}{10.8 D N_a} \tag{10–51}$$

- Expressing Eq. (10–47) in revolutions, and applying the same correction for friction, gives the total angular deflection as

$$\theta_t' = \frac{10.8 M D}{d^4 E}\left(N_b + \frac{l_1 + l_2}{3\pi D}\right) \tag{10–52}$$

Decrease of Inside Diameter

- The diametral clearance Δ between the body coil and the pin of diameter D_p is

$$\Delta = D' - d - D_p = \frac{N_b D}{N_b + \theta'_c} - d - D_p \qquad (10\text{–}55)$$

- Solving for N_b,

$$N_b = \frac{\theta'_c(\Delta + d + D_p)}{D - \Delta - d - D_p} \qquad (10\text{–}56)$$

- This gives the number of body turns necessary to assure a specified diametral clearance.

Belleville Springs

- The *Belleville spring* is a coned-disk spring with unique properties
- It has a non-linear spring constant
- With $h/t \geq 2.83$, the S curve can be useful for snap-acting mechanisms
- For $1.41 \leq h/t \leq 2.1$ the flat central portion provides constant load for a considerable deflection range

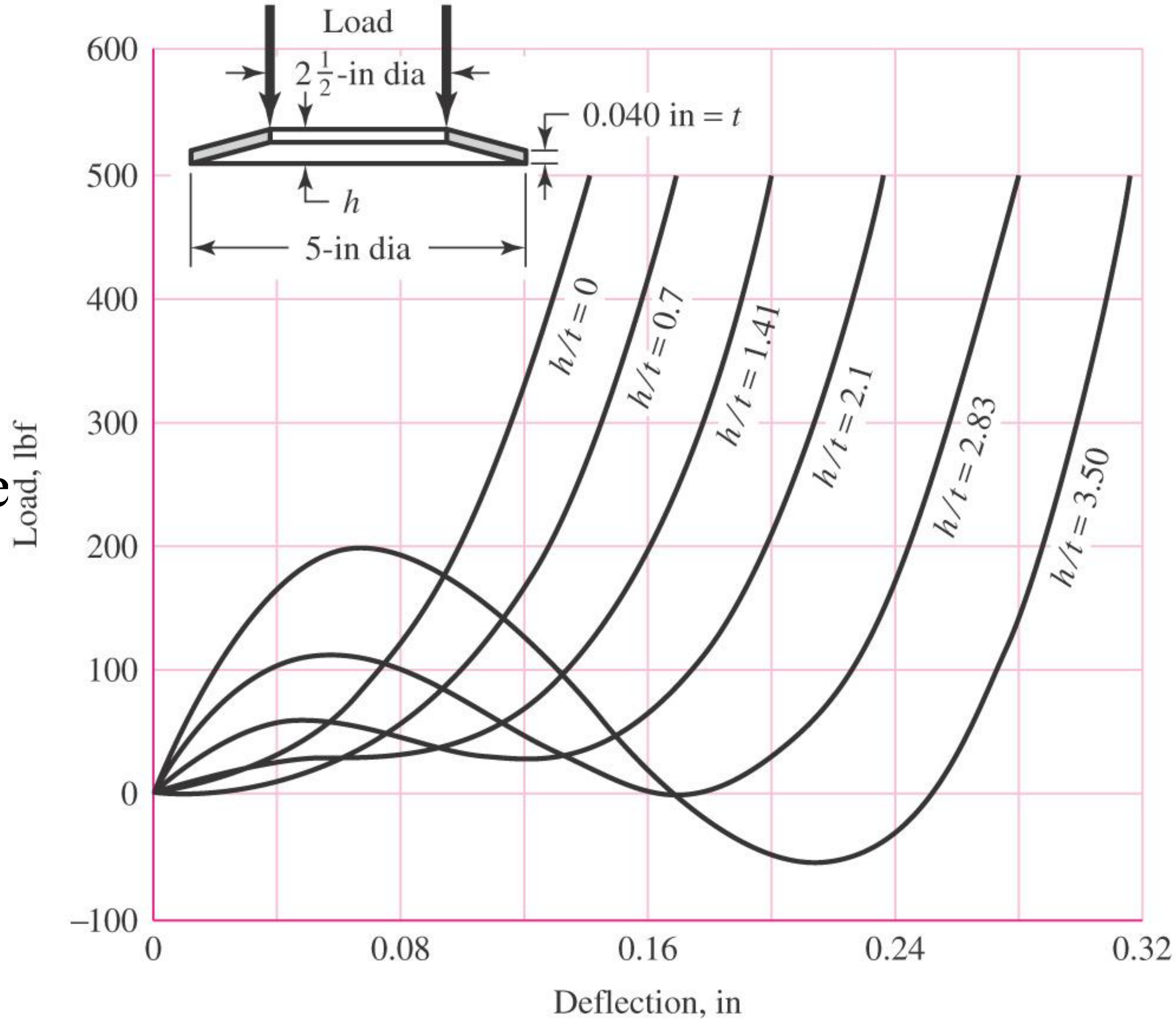


Fig. 10–11

Constant-Force Springs

- The extension spring shown is made of slightly curved strip steel, not flat.
- The fore required to uncoil it remains constant.
- Known as a *constant-force spring*.

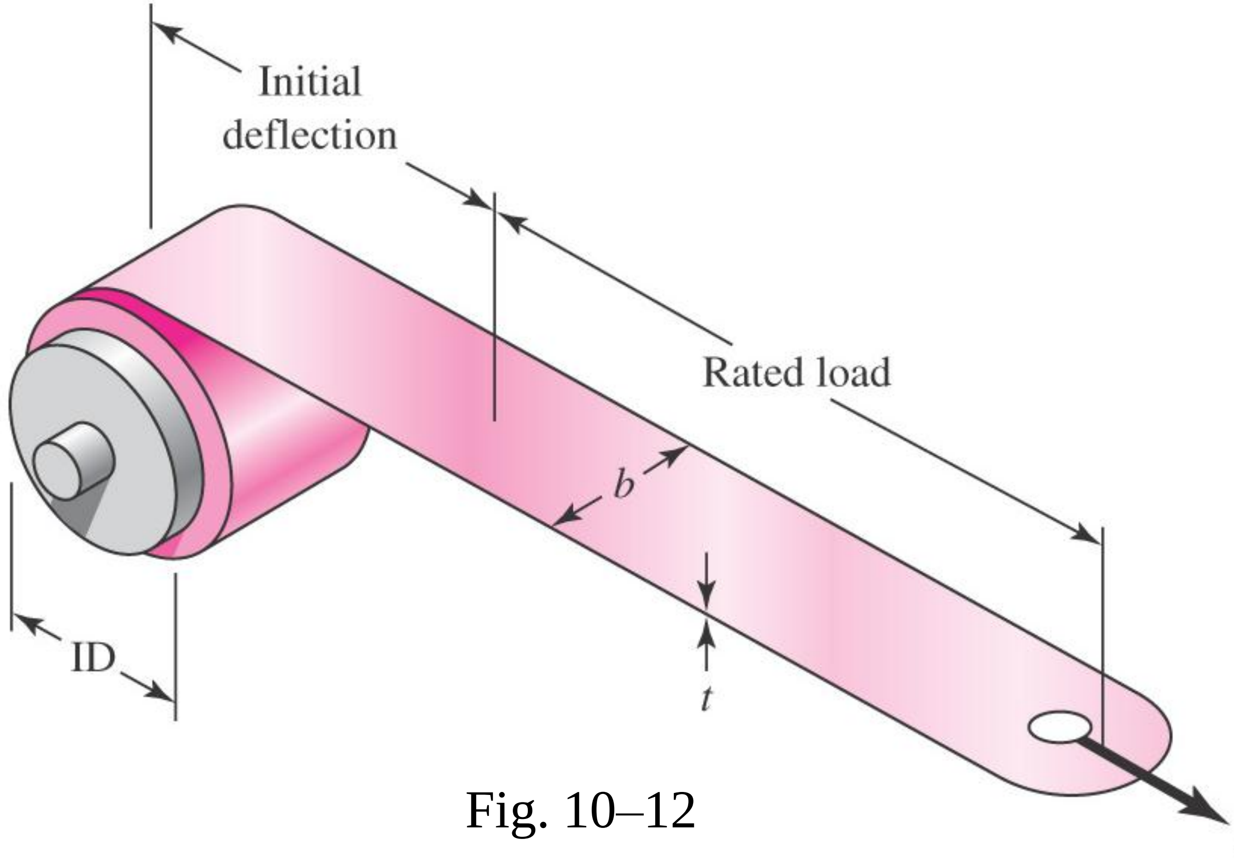
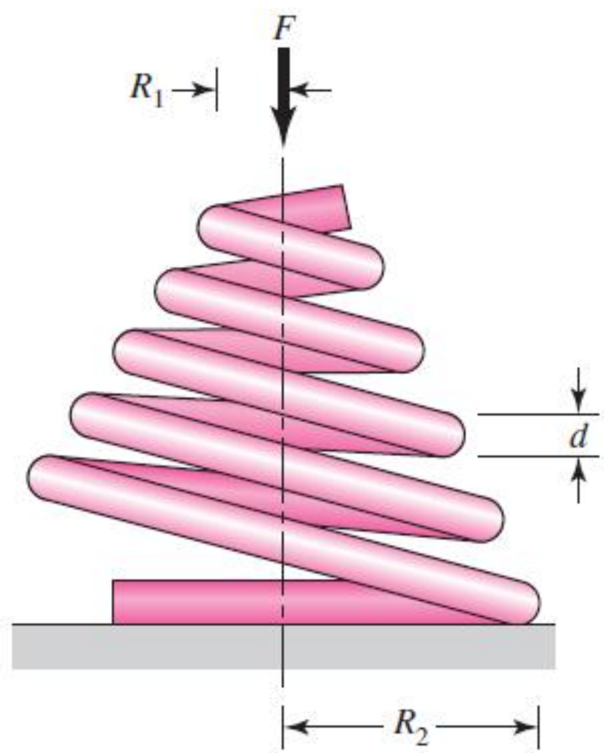


Fig. 10–12

Conical Spring

- A *conical spring* is wound in the shape of a cone.
- Most are compression springs, made with round wire.
- The principal advantage is that the solid height is only a single wire diameter.



Volute Spring

- A *volute spring* is a conical spring made from a wide, thin strip, or "flat", of material wound on the flat so that the coils fit inside one another.
- Since the coils do not stack on each other, the solid height is the width of the strip.
- A variable-spring scale is obtained by permitting the coils to contact the support.
- As deflection increases (in compression), the number of active coils decreases.

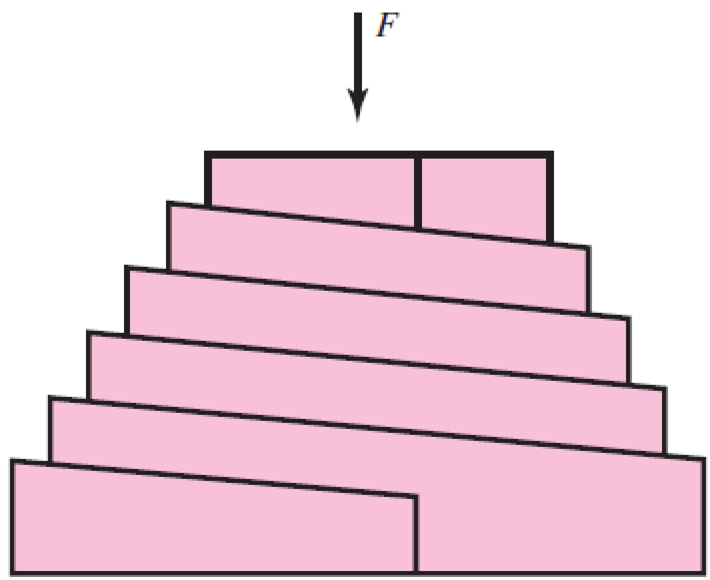


Fig. 10–13*a*

Constant-Stress Cantilever Spring

- A uniform-section cantilever spring made from flat stock has stress which is proportional to the distance *x*.

$$\sigma = \frac{M}{I/c} = \frac{Fx}{I/c} \qquad (a)$$

- It is often economical to proportion the width *b* to obtain uniform stress, independent of *x*.

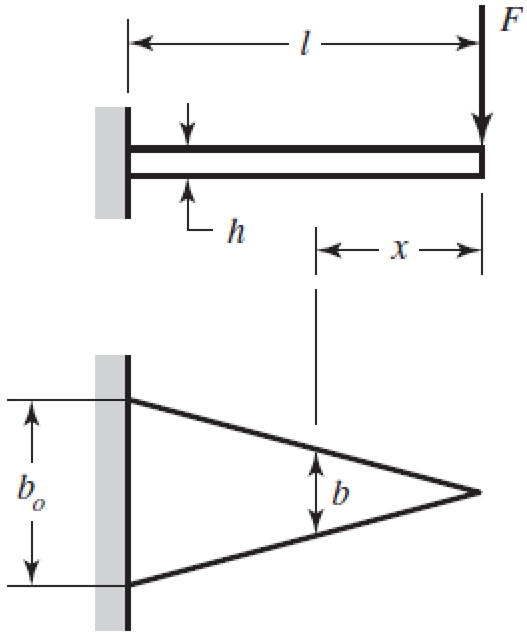


Fig. 10–13*b*