ME 274: DESIGN PROJECT II

Lecture 2

Load and Stress Analysis

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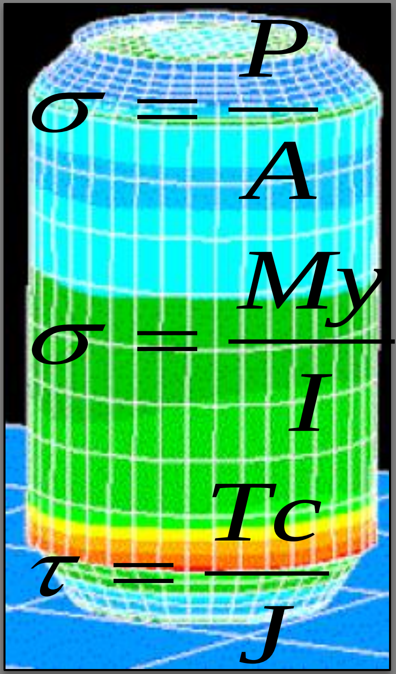


$$\sigma = \frac{P}{A}$$

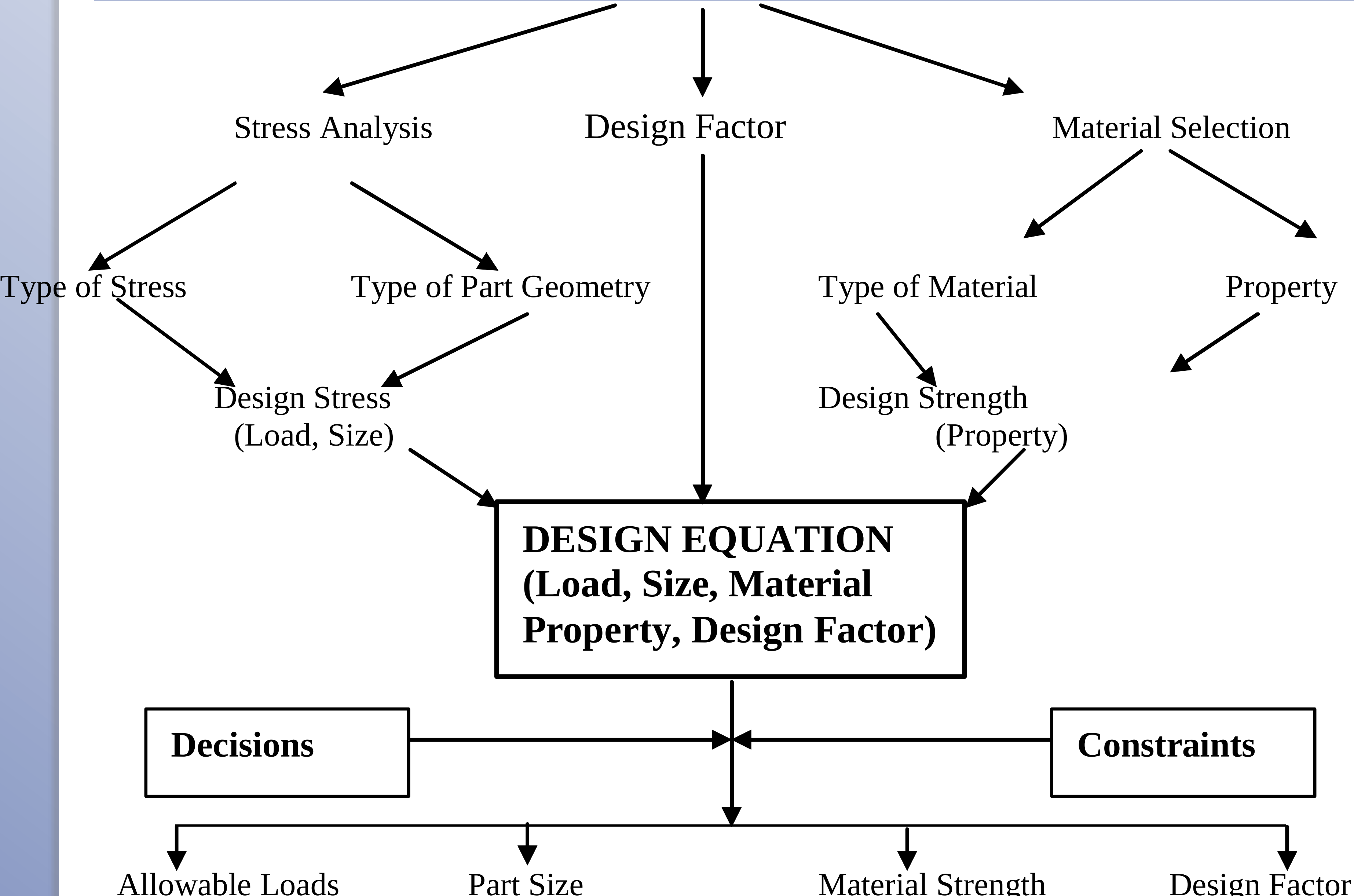
$$\sigma = \frac{My}{I}$$

$$\tau = \frac{Tc}{J}$$

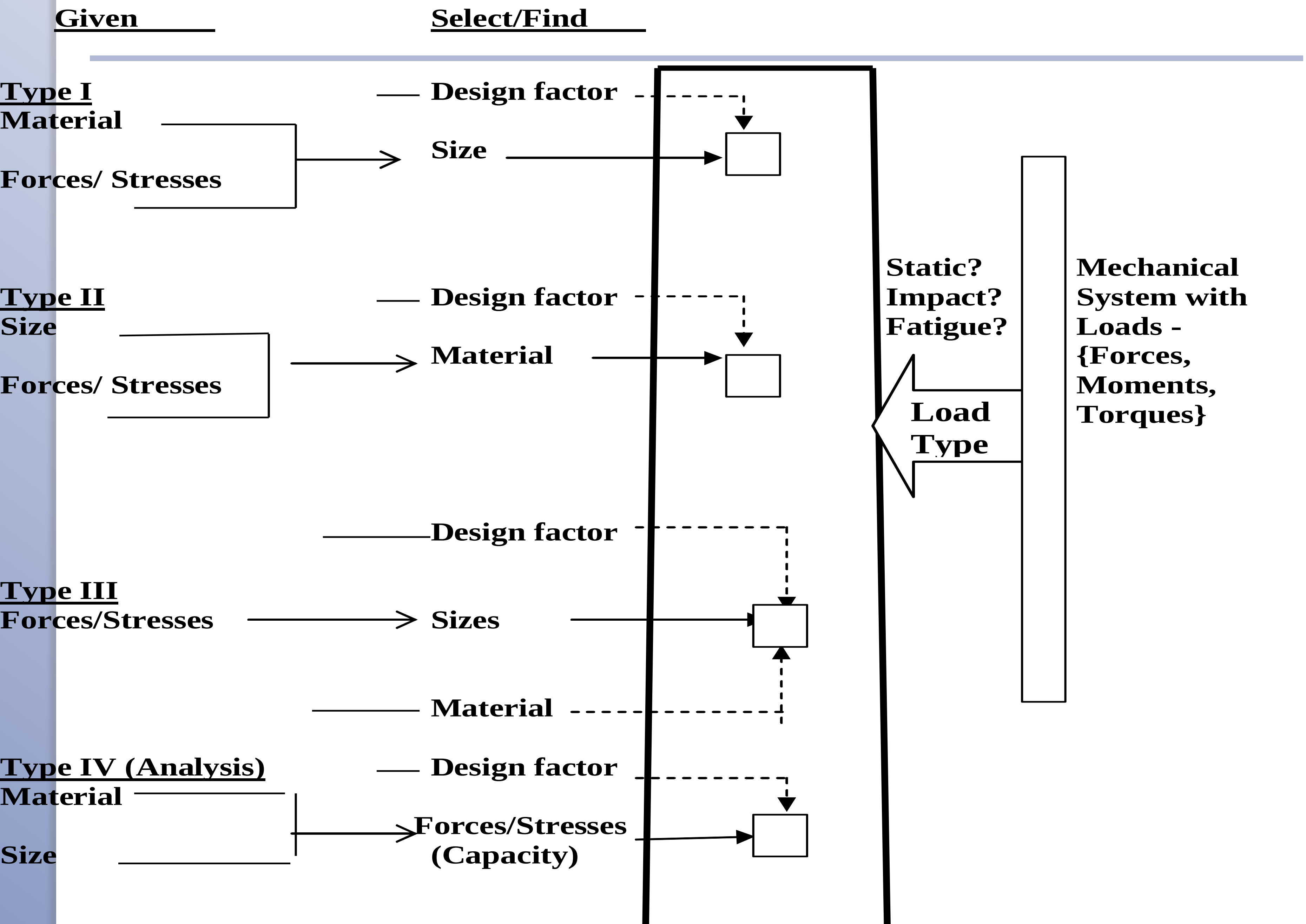
Lecture Outline

- Stress Concentration
- Normal Stress
- Shearing Stress
- Stress Analysis for Pure Loading
- Stress Analysis Combined Loading
- Working Stresses

Design for Strength – Philosophy



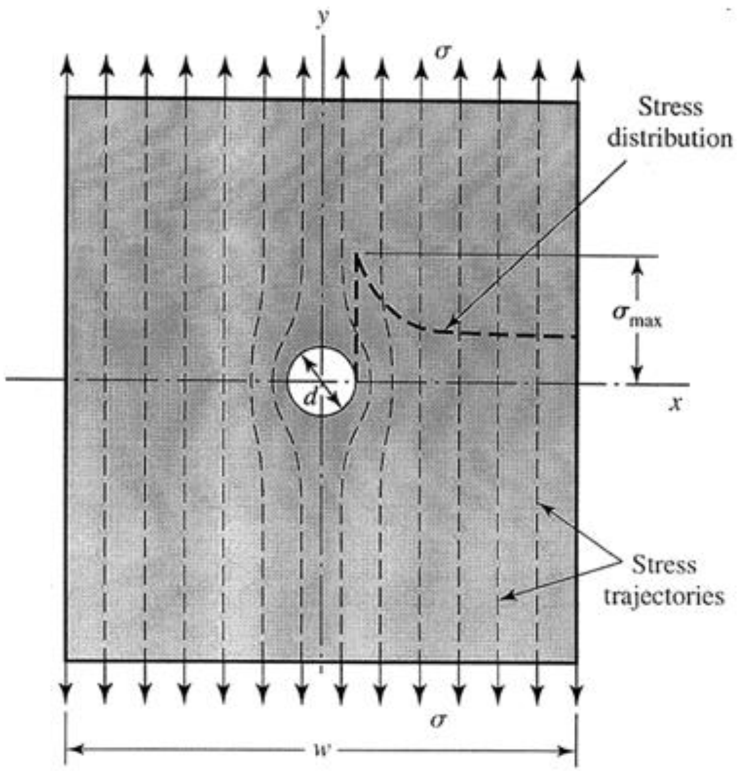
Types of Design Problems



Stress Concentration

- Localized increase of stress near discontinuities
- K_t is Theoretical (Geometric) Stress Concentration Factor

$$K_t = \frac{\sigma_{\max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{\max}}{\tau_0}$$



Theoretical Stress Concentration Factor

- Graphs available for standard configurations
- See Appendix A–15 and A–16 for common examples
- Many more in *Peterson's Stress-Concentration Factors*
- Note the trend for higher K_t at sharper discontinuity radius, and at greater disruption

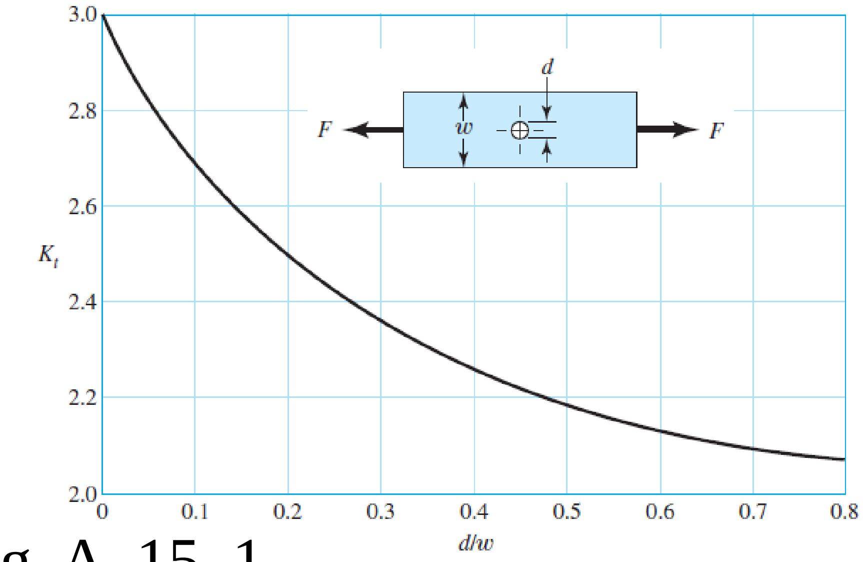


Fig. A–15–1

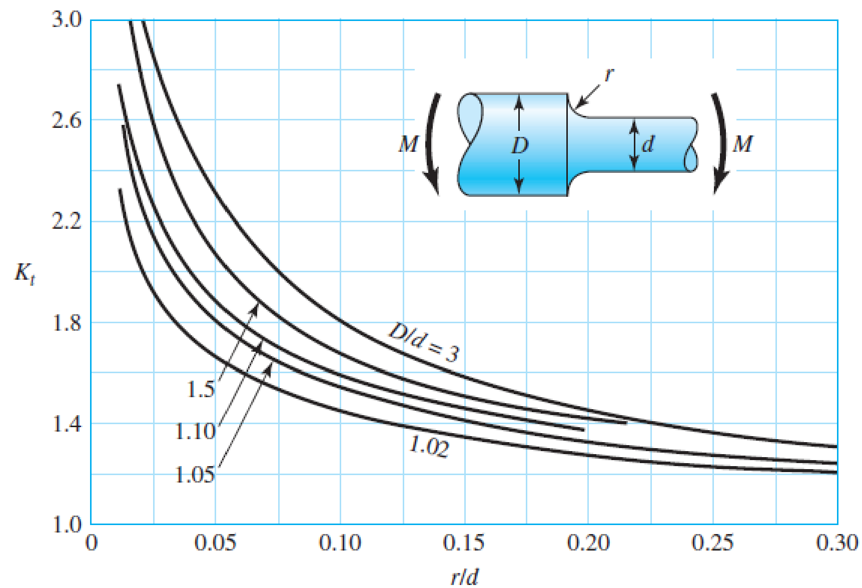


Fig. A–15–9

Stress Concentration for Static and Ductile Conditions

- With static loads and ductile materials
  - Highest stressed fibers yield (cold work)
  - Load is shared with next fibers
  - Cold working is localized
  - Overall part does not see damage unless ultimate strength is exceeded
  - Stress concentration effect is commonly ignored for static loads on ductile materials

Techniques to Reduce Stress Concentration

- Increase radius
- Reduce disruption
- Allow "dead zones" to shape flowlines more gradually

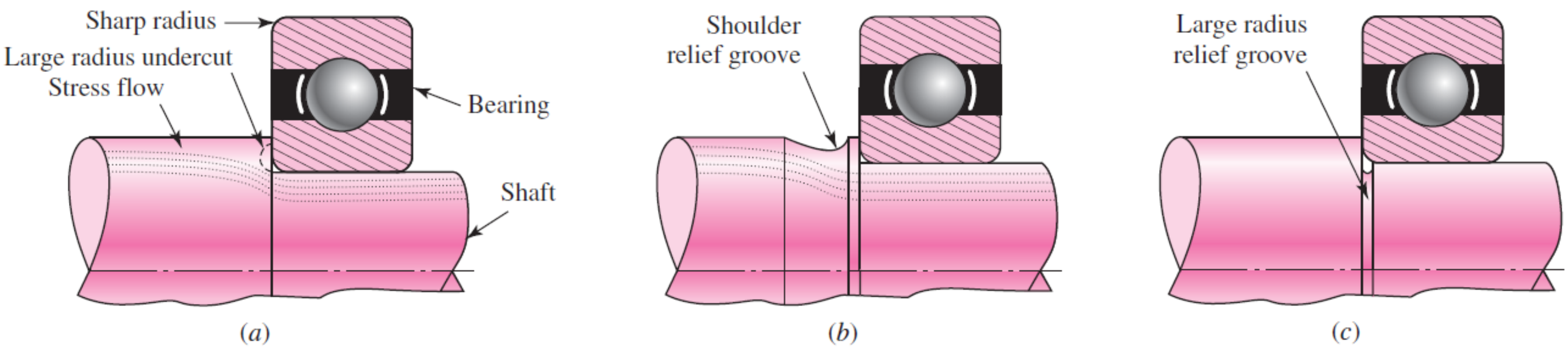


Figure 7–9

Example 2–1

The 2-mm-thick bar shown in Fig. 3–30 is loaded axially with a constant force of 10 kN. The bar material has been heat treated and quenched to raise its strength, but as a consequence it has lost most of its ductility. It is desired to drill a hole through the center of the 40-mm face of the plate to allow a cable to pass through it. A 4-mm hole is sufficient for the cable to fit, but an 8-mm drill is readily available. Will a crack be more likely to initiate at the larger hole, the smaller hole, or at the fillet?

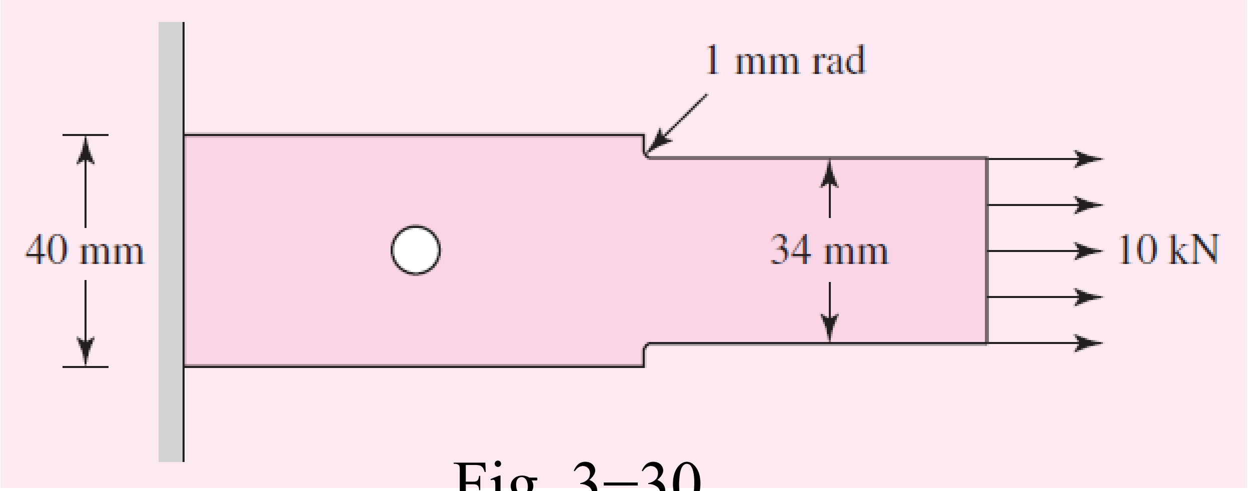


Fig. 3–30

Example 2–1 (continued)

Since the material is brittle, the effect of stress concentrations near the discontinuities must be considered. Dealing with the hole first, for a 4-mm hole, the nominal stress is

$$\sigma_0 = \frac{F}{A} = \frac{F}{(w-d)t} = \frac{10\,000}{(40-4)2} = 139 \text{ MPa}$$

The theoretical stress concentration factor, from Fig. A–15–1, with $d/w = 4/40 = 0.1$, is $K_t = 2.7$. The maximum stress is

$$\sigma_{\max} = K_t \sigma_0 = 2.7(139) = 380 \text{ MPa}$$

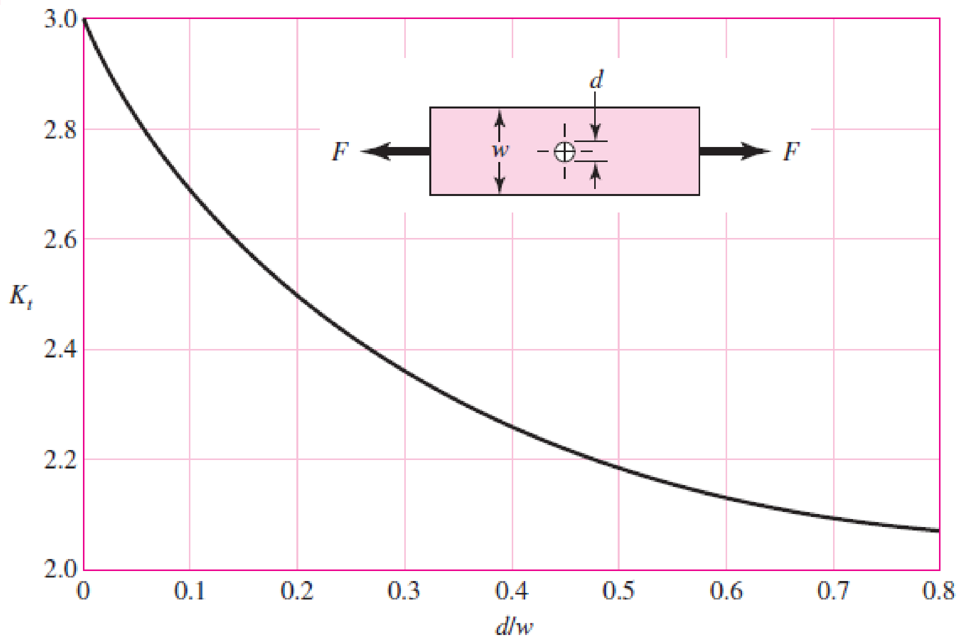


Fig. A−15−1

Example 2–1 (continued)

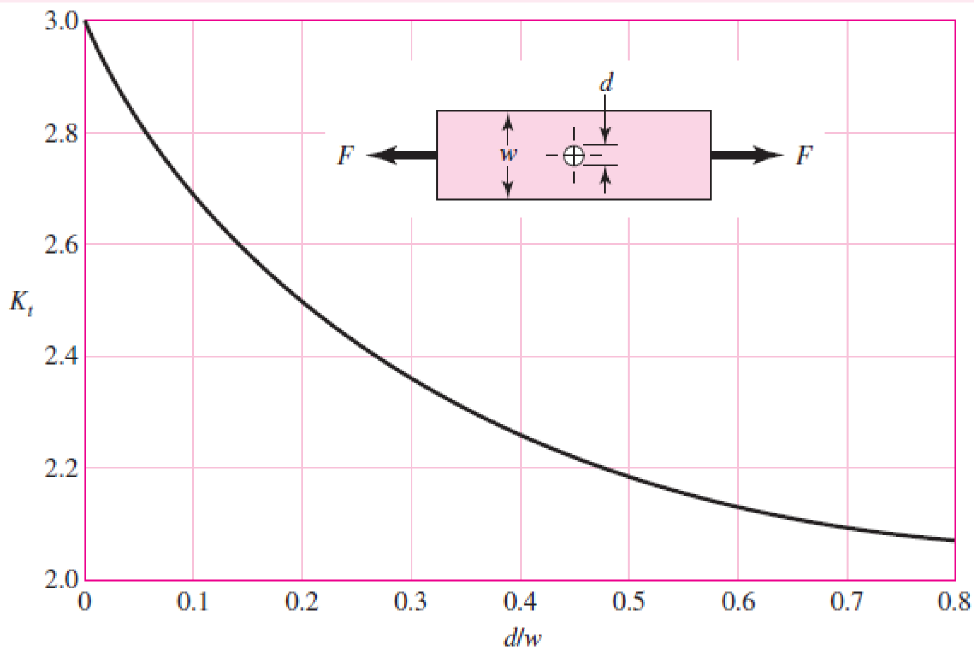
Similarly, for an 8-mm hole,

$$\sigma_0 = \frac{F}{A} = \frac{F}{(w-d)t} = \frac{10\ 000}{(40-8)2} = 156\ \text{MPa}$$

With $d/w = 8/40 = 0.2$, then $K_t = 2.5$, and the maximum stress is

$$\sigma_{\max} = K_t \sigma_0 = 2.5(156) = 390\ \text{MPa}$$

Though the stress concentration is higher with the 4-mm hole, in this case the increased nominal stress with the 8-mm hole has more effect on the maximum stress.



Example 2–1 (continued)

For the fillet,

$$\sigma_0 = \frac{F}{A} = \frac{10\ 000}{(34)2} = 147 \text{ MPa}$$

From Table A–15–5, $D/d = 40/34 = 1.18$, and $r/d = 1/34 = 0.026$. Then $K_t = 2.5$.

$$\sigma_{\max} = K_t\sigma_0 = 2.5(147) = 368 \text{ MPa}$$

The crack will most likely occur with the 8-mm hole, next likely would be the 4-mm hole, and least likely at the fillet.

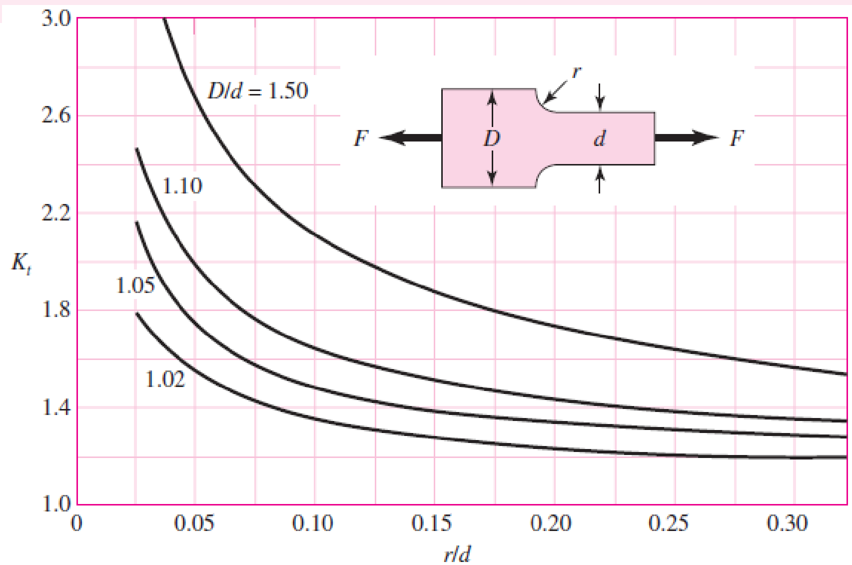


Fig. A–15–5

Normal Stress

Axial Stress

$$\sigma_{axial} = \pm F/A$$

- where F is applied force perpendicular to the cross-section area, A.
- The stress may be in tension (+) or compression (-)

Bending Stress

$$\sigma_{bending} = \pm M/S$$

,

$$M = PL$$

$$S = I/y_{\max}$$

- where S is the Bending Section Modulus and I is the moment of inertia.

Tangential Stress (Thin-walled Pressure Vessel)

$$\sigma_\theta = \begin{bmatrix} pr/t \ \ for\, cylindrical\ vessel \\ pr/2t \ \ for\, spherical\ vessel \end{bmatrix}$$

Axial Stress in (Thin-walled Pressure Vessel)

$$\sigma_{axial} = \begin{bmatrix} pr/2t \ \ for\, cylindrical\ vessel \\ pr/2t \ \ for\, spherical\ vessel \end{bmatrix}$$

- where p is the internal pressure, t is the wall thickness and r is the mean radius

Shearing Stress

Torsional Shearing Stress

- $\tau = T/Z$ and $Z = J/y_{\max}$
- where Z is the Torsional Section Modulus.

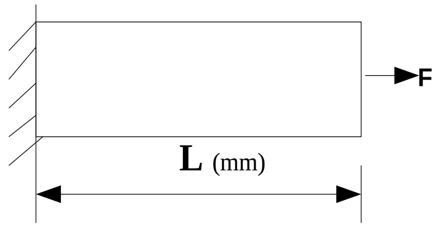
Transverse Shearing Stress

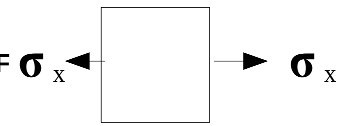
- $$\tau_{xy} = \begin{bmatrix} 3V/2A \text{ for rectangular section} \\ 4V/3A \text{ for solid cylindrical section} \\ 2V/A \text{ for thin wall cylindrical section} \end{bmatrix}$$

- τ_v is zero at maximum bending locations.

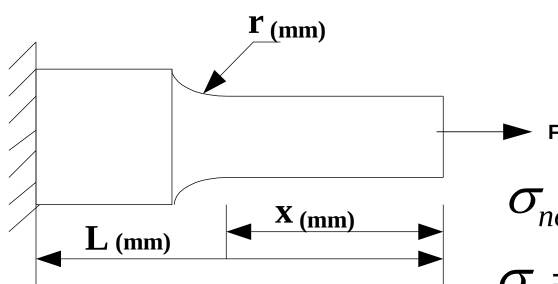
Stress Analysis for Pure Loading

- Pure Axial Load





$$\sigma_x = \sigma_{axial}$$

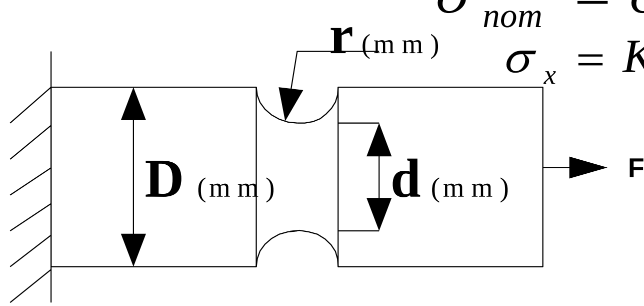


$$\sigma_{nom} = \sigma_{axial}$$

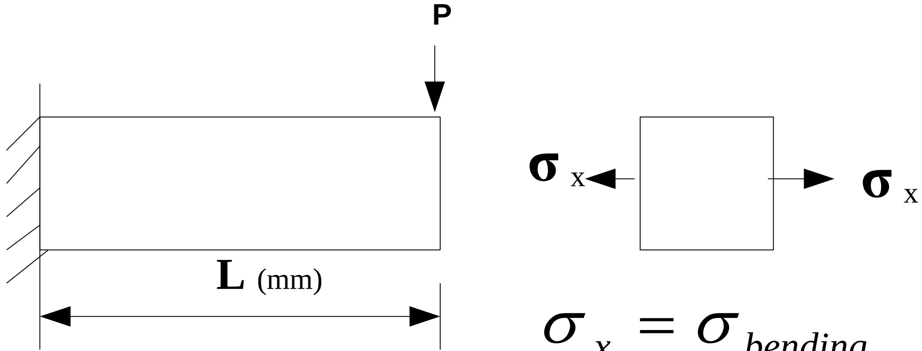
$$\sigma_x = K_t \sigma_{nom}$$

$$\sigma_{nom} = \sigma_{axial}$$

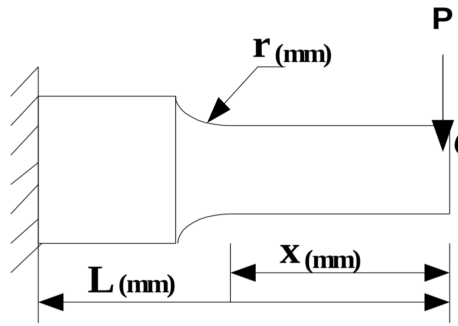
$$\sigma_x = K_t \sigma_{nom}$$



Pure Bending Load



$$\sigma_x = \sigma_{bending}$$

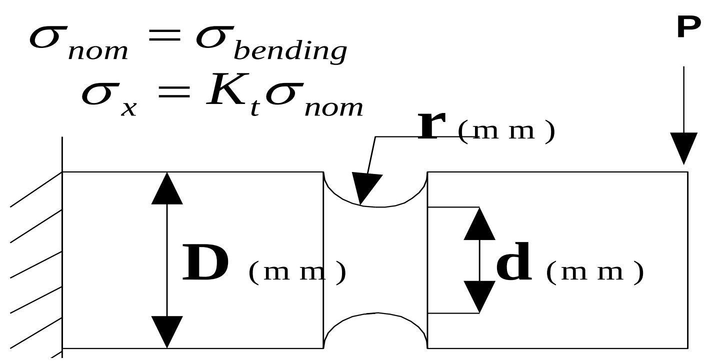


$$\sigma_{nom} = \sigma_{bending}$$

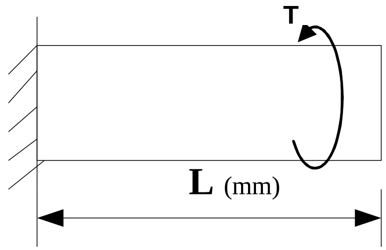
$$\sigma_x = K_t \sigma_{nom}$$

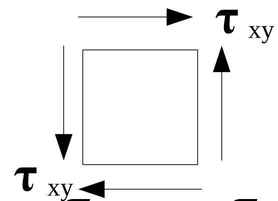
$$\sigma_{nom} = \sigma_{bending}$$

$$\sigma_x = K_t \sigma_{nom}$$



Pure Torsional Load

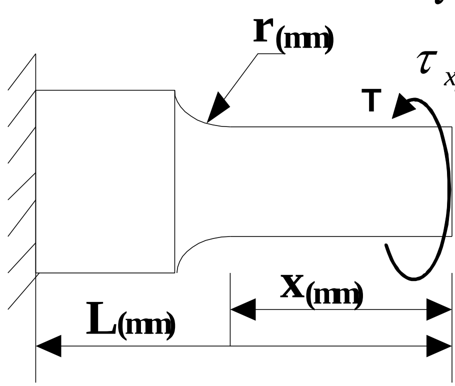


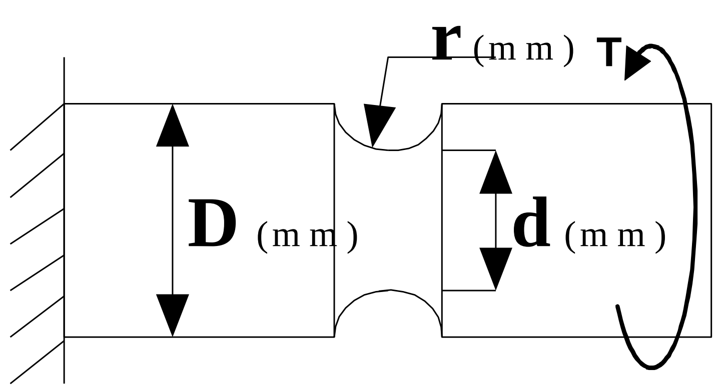


$$\tau_{xy} = \tau$$

$$\tau_{nom} = \tau$$

$$\tau_{xy} = K_t \tau_{xy}$$



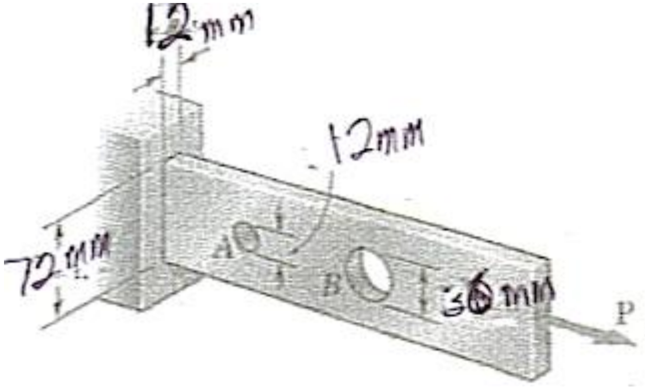


$$\tau_{nom} = \tau$$

$$\tau_{xy} = K_t \tau_{xy}$$

Example 2–2

- *Two holes have been drilled through a long steel bar that is subjected to a concentric axial load as shown. For P= 8.5 kN, determine the maximum values of the stress at A and B.*



- **Solution**
- <u>Section Properties</u>

At hole A, r = 6 mm; d = 72 mm – 12 = 60 mm; t = 12 mm;

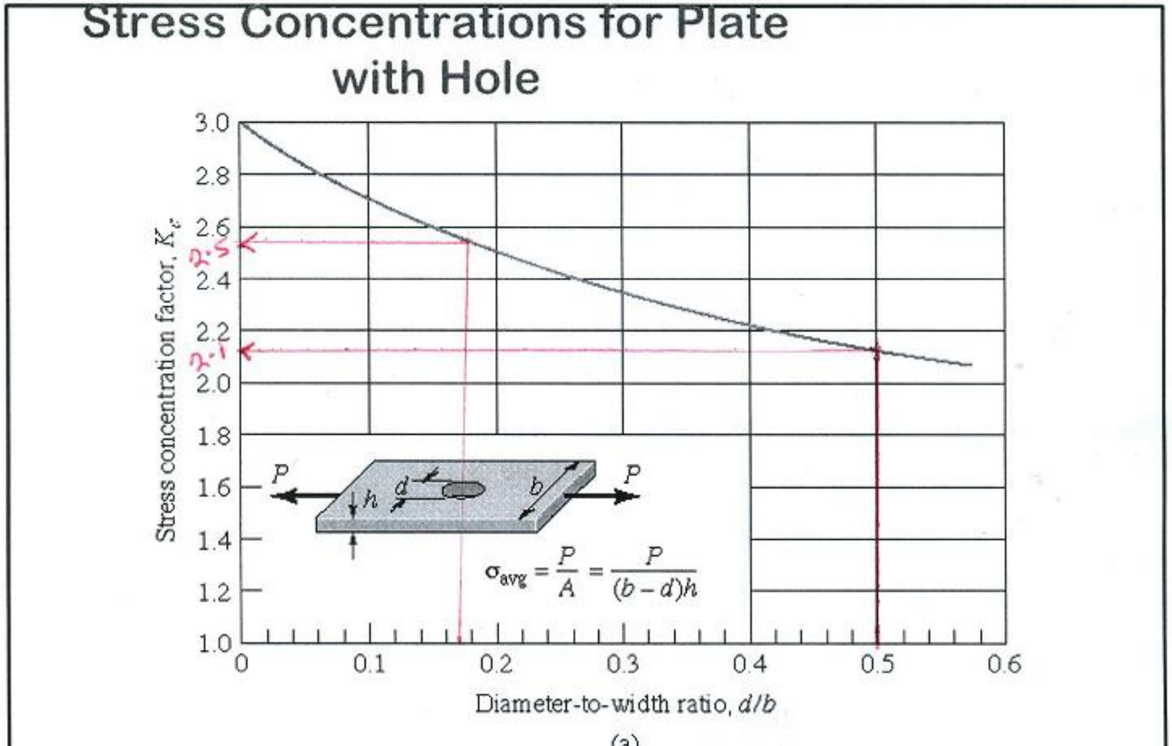
$A_{net} = dt$ = 7.20 x 10^{-4} m^2

At hole B, r = 18 mm;

d = 72 – 36 = 36 mm; t = 12 mm;

$A_{net} = dt$ = 4.32 x 10^{-4} m^2

Example 2–2 (continued)



(a)

Figure 6.2 Stress concentration factor for rectangular plate with central hole. (a) Axial Load. [Adapted from Collins (1981).]

Text Reference: Figure 6.2, page 222

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Example 2–2 (continued)

<u>Force-couple System at A and B</u>

- F = P = 8.5 kN

<u>Axial Stress</u>

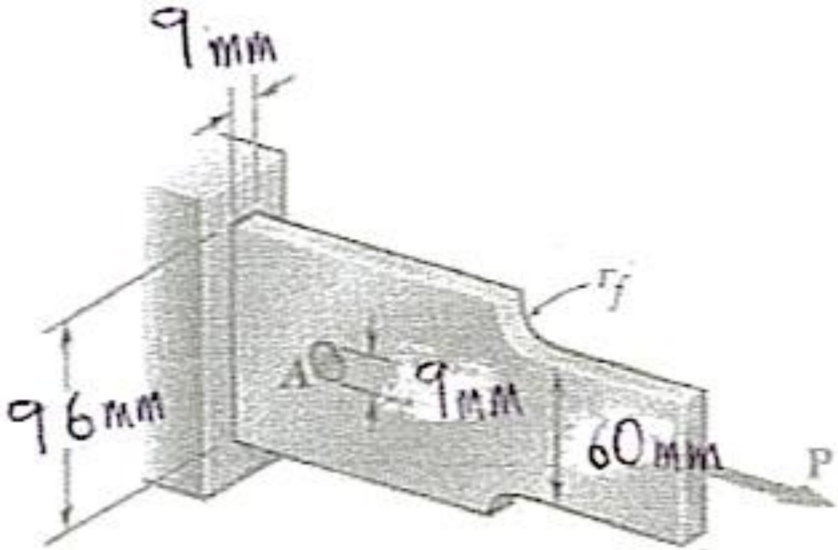
- at A, $\sigma_{nom} = F/A_{net}$ = 11.8 MPa
- r/d = 0.10; From Figure, K = 2.50
- $\therefore \sigma_{\max} = K\sigma_{nom}$ = ~~**31.86 MPa**~~
- at B, $\sigma_{nom} = F/A_{net}$ = 19.77 MPa
- r/d = 0.50; From Fig K = 2.10
- $\therefore \sigma_{\max} = K\sigma_{nom}$ = **41.52 MPa**

Example 3–3

A centric axial load P is applied to the steel bar shown. A hole of 9 mm diameter is drilled through the steel plate. Determine

a. *the radius r_f of the fillets for which the same maximum stress occurs at the hole A and at the fillets*

b. *the corresponding maximum allowable stress if the allowable load* ***P*** *is 7.5 kN*



Example 2–3 (continued)

Stress Concentrations for Plate with Hole

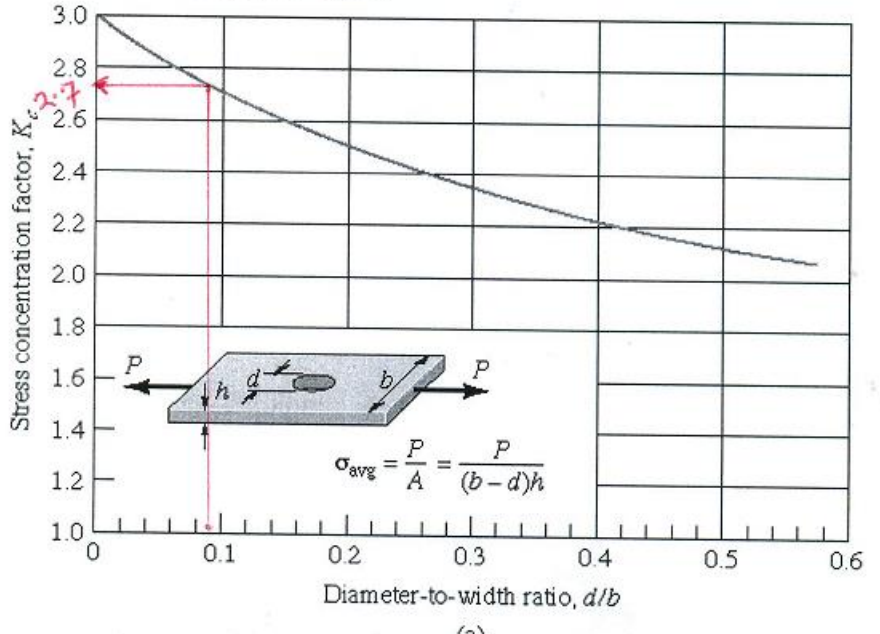


Figure 6.2 Stress concentration factor for rectangular plate with central hole. (a) Axial Load. [Adapted from Collins (1981).]

Text Reference: Figure 6.2, page 222

Example 2–3 (continued)

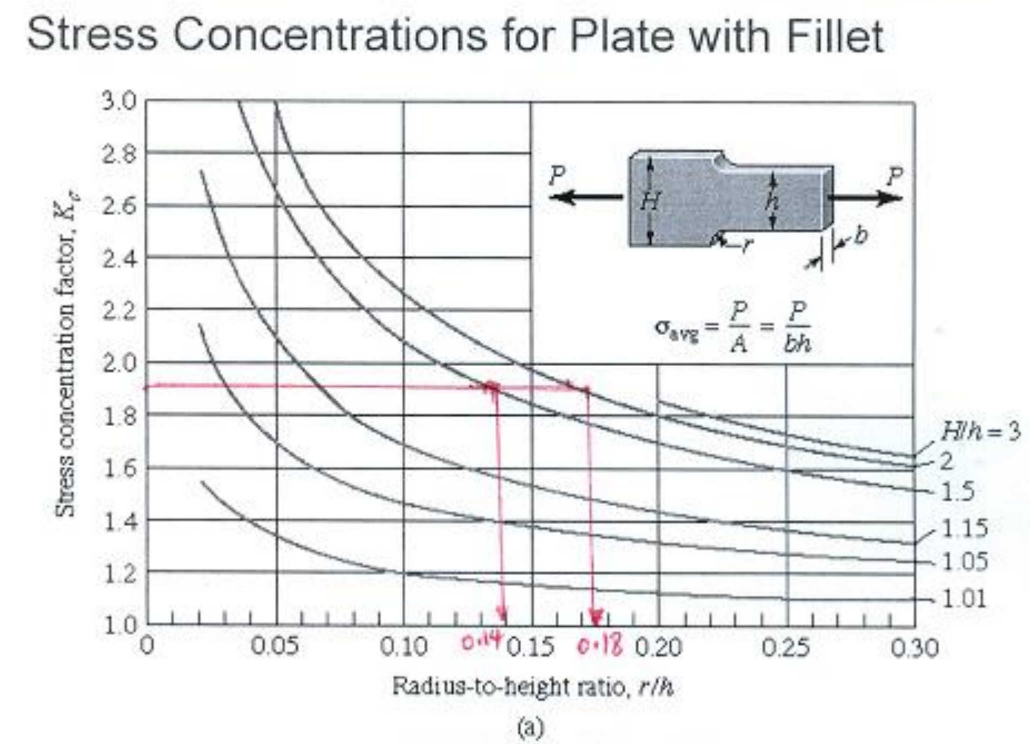


Figure 6.3 Stress concentration factor for rectangular plate with fillet. (a) Axial Load. [Adapted from Collins (1981).]

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Example 2–3 (continued)

- <u>Section Properties</u>
- At the hole ,r = 4.5 mm; d = 96 – 9 = 87 mm; t = 9 mm;
- $A_{net} = dt$ = 7.83 x 10^{-4} m^2
- For fillet, D = 96 mm; d = 60 mm;
- $A_{net} = dt$ = 5.40 x 10^{-4} m^2
- <u>Force-couple System at A and B</u>
- F = P = 7.5 kN
- <u>Axial Stress</u>
- at the hole, = 9.58 MPa
- r/d = ~~0.05;~~ From Fig K = ~~2.82~~
- $\therefore \sigma_{\max} = K\sigma_{nom}$ = ~~**27 MPa**~~

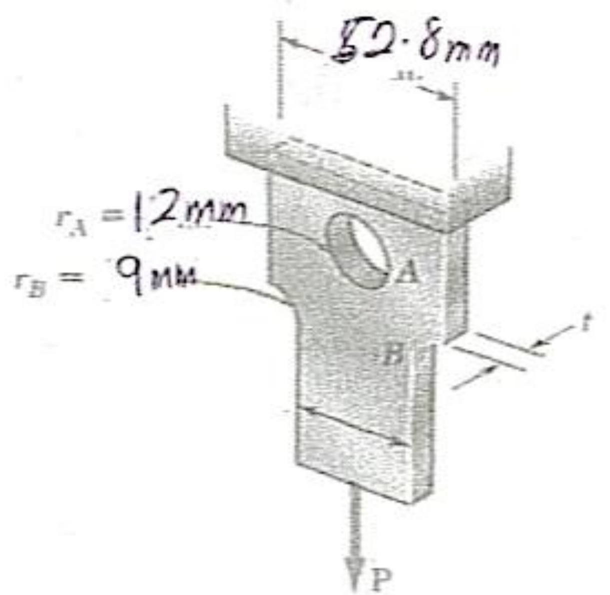
Example 2–3 (continued)

- at the fillet, $\sigma_{nom} = F/A_{net}$ = 13.90 MPa
- D/d = 1.6;

- This implies 13.9K_{fillet} = 27
- Hence, K_{fillet} = 1.945
- Therefore, From Figure. r_f = **10 mm**

Example 2–4

For P=8.5kN, determine the allowable stress if the thickness t is 20 mm.



<u>Section Properties</u>

At the hole A, r = 12 mm;

d = 52.8 – 24 = 28.8 mm; t = 20 mm;

$A_{net} = dt$ = 5.76 x 10^{-4} m^2

At the fillet B, D = 52.8 mm; d = 38.4 mm; r_f = 9 mm;

$A_{net} = dt$ = 3.46 x 10^{-4} m^2

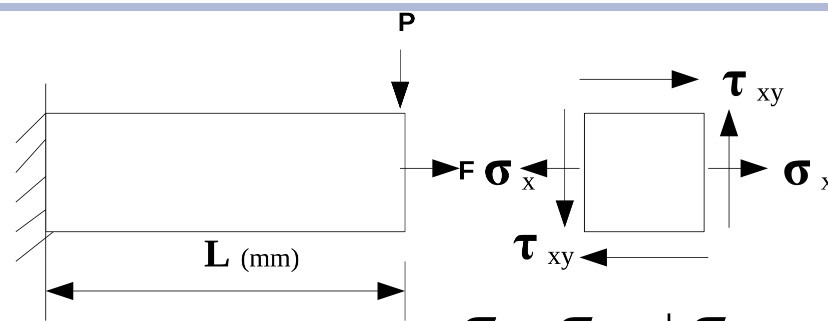
Example 2–4 (continued)

- <u>Force-couple System at A and B</u>
- F = P = 8.5 kN
- <u>Axial Stress</u>
- at the hole A, $\sigma_{nom} = F/A_{net}$ = 14.76 MPa
- $\frac{r}{d}$ = 0.417; From Fig K = 2.22
- $\therefore \sigma_{\max} = K\sigma_{nom}$ = **32.76 MPa**
- at the fillet B, $\sigma_{nom} = F/A_{net}$ = 24.57 MPa
- $D/d = 1.375$ and $r_f/d \approx 0.23$; From Fig K = 1.70
-
- $\therefore \sigma_{\max} = K\sigma_{nom}$ = **55.7 MPa**

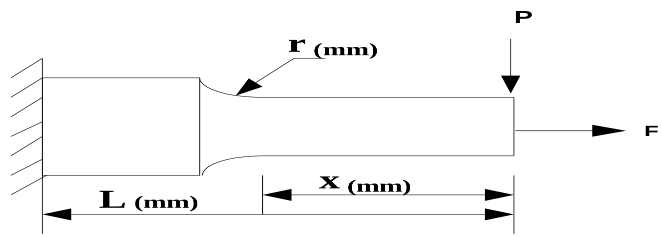
Stress Analysis for Combined Loading

- Axial and Bending
- Axial and Torsional
- Bending and Torsional
- Axial, Bending and Torsional

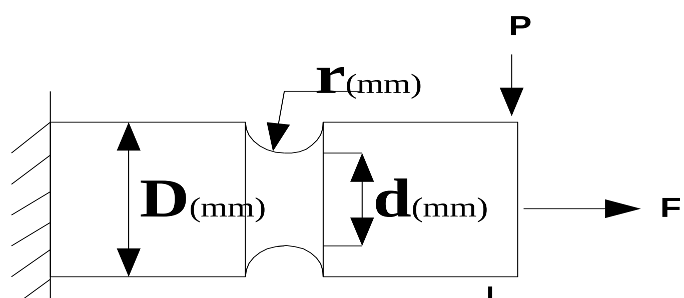
Axial and Bending Load



$$\sigma_x = \sigma_{axial} + \sigma_{bending}$$

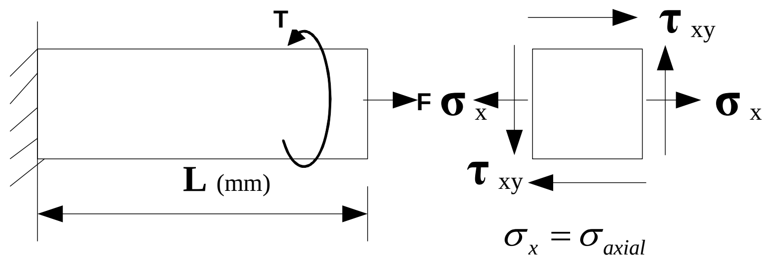


$$\sigma_x = K_t \sigma_{nom} = K_t \left[\sigma_{axial} + \sigma_{bending}\right]$$

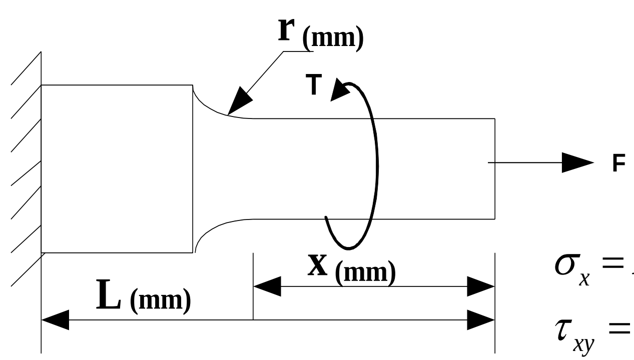


$$\sigma_x = K_t \sigma_{nom} = K_t \left[\sigma_{axial} + \sigma_{bending}\right]$$

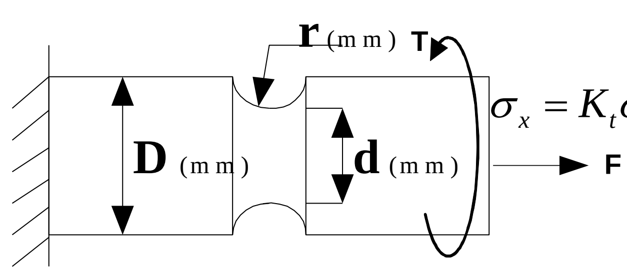
Axial and Torsional Load



$$\sigma_x = \sigma_{axial}$$
$$\tau_{xy} = \tau$$

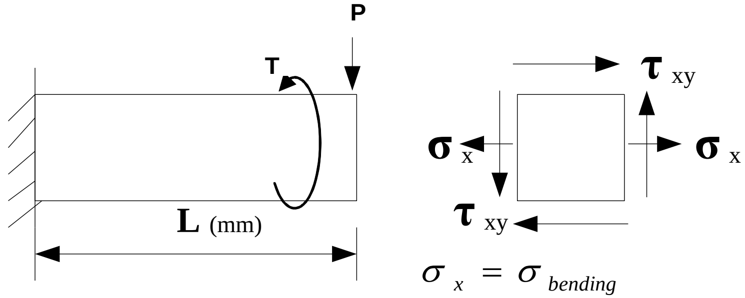


$$\sigma_x = K_t \sigma_{nom} = K_t \left[\sigma_{axial} \right]$$
$$\tau_{xy} = K_t \tau_{nom}$$



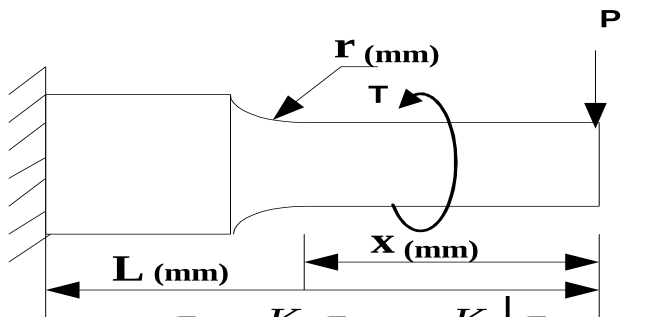
$$\sigma_x = K_t \sigma_{nom} = K_t \left[\sigma_{axial} \right]$$
$$\tau_{xy} = K_t \tau_{nom}$$

Bending and Torsional Load



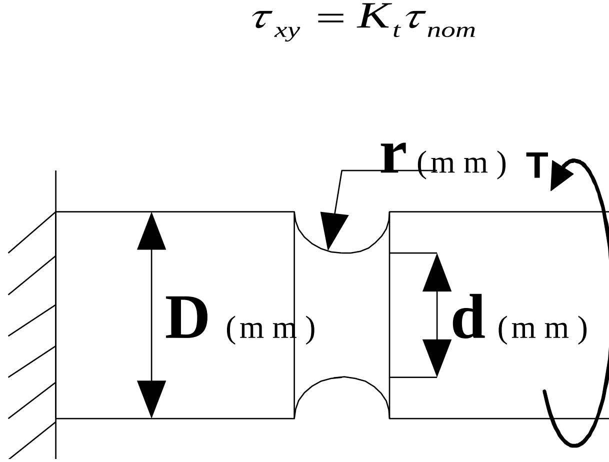
$$\sigma_x = \sigma_{bending}$$

$$\tau_{xy} = \tau$$



$$\sigma_x = K_t \sigma_{nom} = K_t \left\lfloor \sigma_{bending} \right\rfloor$$

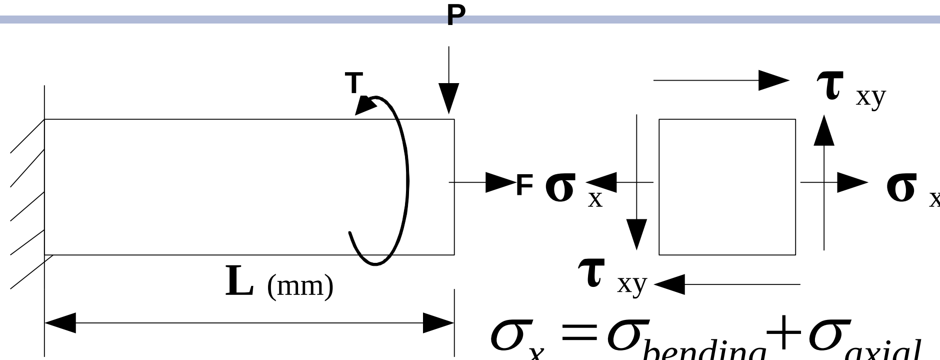
$$\tau_{xy} = K_t \tau_{nom}$$



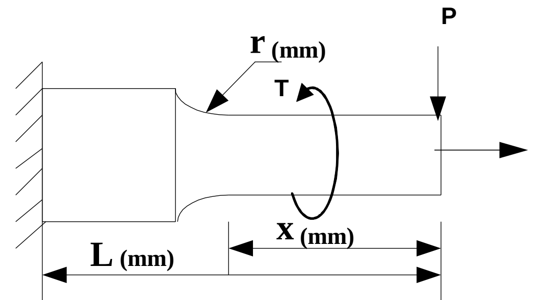
$$\sigma_x = K_t \sigma_{nom} = K_t \left\lfloor \sigma_{bending} \right\rfloor$$

$$\tau_{xy} = K_t \tau_{nom}$$

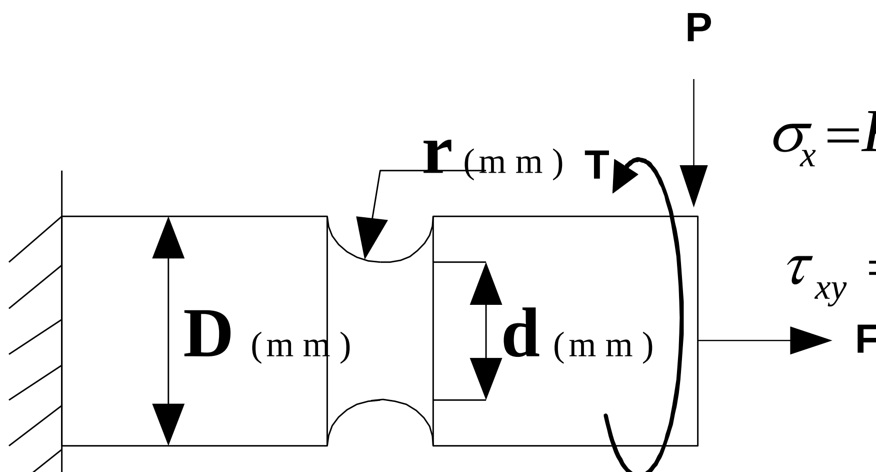
Axial, Bending and Torsion Load



$$\sigma_x = \sigma_{bending} + \sigma_{axial}$$
$$\tau_{xy} = \tau$$



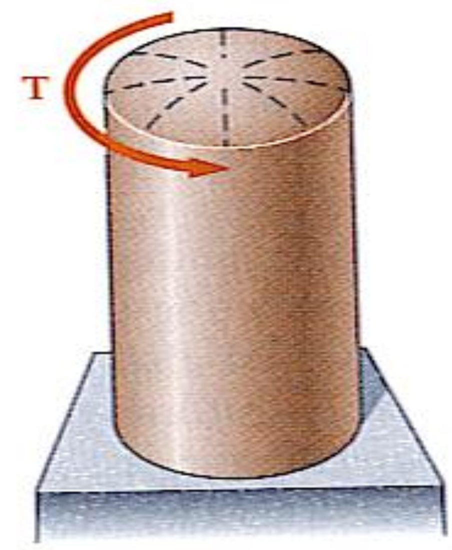
$$\sigma_x = K_t \sigma_{nom} = K_t \left[\sigma_{axial} + \sigma_{bending}\right]$$
$$\tau_{xy} = K_t \tau_{nom}$$



$$\sigma_x = K_t \sigma_{nom} = K_t \left[\sigma_{axial} + \sigma_{bending}\right]$$
$$\tau_{xy} = K_t \tau_{nom}$$

Example 2–5 (continued)

- *A torque of magnitude T = 12kN m is applied to the end of a tank containing compressed air under a pressure of 8 MPa. The tank has a 180-mm inner diameter and a 12-mm wall thickness, determine the normal stress and the shearing stress in the tank.*



Solution

- Section Properties

$$r_i = d_i/2 = 90\,mm$$

$$c = r_o = r_i + t = 102\,mm$$

$$J = \pi/2\left(r_o^4 - r_i^4\right) == 66.97x10^{-6}\,m^4$$

Example 2–5 (continued)

- Force-couple system
- Pressure, p = 8 MPa and T = 12 kN.m

- Stresses in Pressure Vessel
- Tangential:

$$\sigma_x = pr/t = 60\,MPa$$

- Axial:

$$\sigma_y = pr/2t = 30\,MPa$$

- Torsional Shearing Stress
- $$\tau_{xy} = Tc/J = 18.3MPa$$

- Total Stresses
- **σ_x = 60 MPa;**
- **σ_y = 30 MPa;**
- **τ_{xy} = 18.3 MPa;**

Working Stresses

- The average stress : $$\sigma_{avg} = \tfrac{1}{2}(\sigma_x + \sigma_y)$$
- The radius : $$R = \sqrt{\tau_{xy}^2 + \left[\tfrac{1}{2}\left(\sigma_x - \sigma_y\right)\right]^2}$$
- Maximum Principal Stress : $$\sigma_1 = \sigma_{avg} + R$$
- Minimum Principal Stress : $$\sigma = \sigma_{avg} - R$$
- Maximum Shear Stress : $$\tau_{\max} = R$$

Example 2–6

- *Determine the principal stresses, and. the maximum shearing stress in example 4.*

- **Solution**
- From Example 4, the stresses are
- $\sigma_x = 60$ MPa; $\sigma_y = 30$MPa; $\tau_{xy} = 18$MPa;

- <u>Mohr's Circle</u>

$$\sigma_{avg} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 45\ MPa$$

$$R = \sqrt{\tau_{xy}^2 + \left[\tfrac{1}{2}\left(\sigma_x - \sigma_y\right)\right]^2} = 23\ MPa$$

$$\sigma_{\max} = \sigma_{avg} + R = 68\ MPa \qquad \tau_{\max} = R = 23\ MPa$$

$$\sigma_{\min} = \sigma_{avg} - R = 22\ MPa$$