18.06 Strang, Edelman, Huhtanen Quiz 3 December 5, 2001

Your name is: ______________________________

For full credit, carefully explain your reasoning, as always!

1 (36 pts.) Let A be the square matrix

$$A = \begin{bmatrix} 2 & 1 \\ x & y \end{bmatrix}.$$

(a) With $x = 2$ and $y = 1$ diagonalize A. That is, compute $A = S\Lambda S^{-1}$, where Λ is a diagonal matrix. (12p)

(b) With $y = 2$ pick x so that S can be orthogonal in a diagonalization of A. Compute then one such S. (12p)

(c) If $y = 2$, can you find $x > 0$ such that A and $\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$ are similar? (Hint: look at the eigenvalues.) (12p)

2 (32 pts.) (a) Choose x and y so that

$$M = \begin{bmatrix} 1/2 & x \\ y & 1/4 \end{bmatrix}$$

is a Markov matrix. (4p)

Compute the steady state eigenvector x_1 of unit length. (That is, $\|x_1\| = 1$). (8p)

(b) Is

$$A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$$

positive definite? (4p)

Find the singular value decomposition of A. (16p)

3 (32 pts.) Let

$$A = \begin{bmatrix} 5 & 4 & 3 \\ -1 & 0 & -3 \\ 1 & -2 & 1 \end{bmatrix}$$

and

$$X = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 0 \\ -1 & 1 & 0 \end{bmatrix} \text{ so that } X^{-1} = \begin{bmatrix} 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 0 \end{bmatrix}$$

(a) Compute $M = X^{-1}AX$. (4p)

What are the eigenvalues of A? (4p)

How many linearly independent eigenvectors does A have? (4p)

Is A diagonalizable? (4p)

(b) Let

$$B = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

Compute e^{Bt} explicitly. (12p)

Compute $\lim_{t\to\infty} e^{Bt}x$. (4p)