

UNIVERSITY OF SASKATCHEWAN

Department of Mathematics & Statistics

Stat 103.3 Final Exam

Date: April 26, 2006

Time: 3 hours

Instructor: Qingde Yang

Closed Book, No Notes

One formula sheet and one Calculator are allowed.

-
- ❖ **PRINT** your name clearly and write your student ID number on the opscan sheet.
 - ❖ Encode your student ID number on the opscan sheet.
 - ❖ All questions are of equal value.
 - ❖ Encode your answers carefully on the opscan sheet. Mark only one answer; otherwise the answer will be counted as incorrect. In case there is more than one answer; mark the best answer.
-

Concerning the events A and B , the following probabilities are given.

$$P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$$

[1]. Find $P(B \bar{A})$ if A and B are mutually exclusive.

[2]. Find $P(B \bar{A})$ if A and B are independent.

Answer Choices for question 1-2 are:

(A) 0.2857 (B) 0.3333 (C) 0.5600 (D) 0.4832 (E) 0.5625 (F) 0.3602 (G) 0.5000
(H) 0.8201 (I) 0.5079 (J) None of the above

A box contains four chips numbered 2, 3, 6, 7. One chip is drawn at random. Let A be the event “an even number is drawn” and B the event “a number greater than 5 is drawn.”

[3]. Find $P(AB)$.

[4]. Find $P(A | B)$.

[5]. Are A and B mutually exclusive events?

[6]. Are A and B independent events?

Answer Choices for question 3-6 are:

(A) No (B) Yes (C) 0.2500 (D) 0.4832 (E) 0.5625 (F) 0.3602
(G) 0.40320 (H) 0.8201 (I) 0.5000 (J) None of the above

Two brothers are evenly matched when playing chess. One Sunday they decide to play until one or the other has won three games.

[7]. Ignoring draws, what is the probability that they play three games?

[8]. Ignoring draws, what is the probability that they play four games?

Answer Choices for question 7-8 are:

(A) 0.2857 (B) 0.3333 (C) 0.5600 (D) 0.3750 (E) 0.2500 (F) 0.1202 (G) 0.5023
(H) 0.8324 (I) 0.7235 (J) None of the above

A man has nine keys in his pocket.

[9]. He samples from his pocket one at a time, with replacement (since he has no place to put keys and his other hand is not free). What is the probability that he will find his car key on the second trial?

[10]. He samples from his pocket one at a time, with replacement. What is the probability that he will need at most three trials to find his car key?

[11]. Suppose he samples without replacement (he puts his keys on a table nearby or drops them on the floor). What is the probability that he will need at most three trials to find his car key?

Answer choices for questions 9 to 11 are:

- | | | | |
|------------|-----------------------|------------|------------|
| (A) 0.1525 | (B) 0.0345 | (C) 0.3333 | (D) 0.4123 |
| (E) 0.2345 | (F) 0.2977 | (G) 0.5 | (H) 0.0988 |
| (I) 0.7430 | (J) None of the above | | |
-

A corporation is reviewing the success rate of its management training program. It finds that 80% of those entering the program are college graduates and that 90% of them successfully complete it, while just 60% of those who are not college graduates successfully complete it.

[12]. What is the probability that an employee who successfully completes the training program is not a college graduate?

Answer choices for questions 12 are:

- | | | | |
|------------|-----------------------|------------|------------|
| (A) 0.0045 | (B) 0.1875 | (C) 0.0344 | (D) 0.1429 |
| (E) 0.0096 | (F) 0.4555 | (G) 0.5601 | (H) 0.6531 |
| (I) 0.1607 | (J) None of the above | | |
-

[13]. In a town, 60% of the citizens are Republicans and 40% are Democrats. In the last election 55% of the Republicans voted and 65% of the Democrats voted. If a voter is randomly selected, what is the probability that the person is a Republican?

Choices for questions 13 are:

- | | | | | | |
|------------|------------|------------------------|------------|------------|------------|
| (A) 0.5346 | (B) 0.3830 | (C) 0.4471 | (D) 0.4665 | (E) 0.2234 | (F) 0.5593 |
| (G) 0.2600 | (H) 0.2941 | (I) None of the above. | | | |
-

Urn I contains five red and eight green balls and urn II contains four red and eleven green balls. An urn is selected at random. Each urn has the same chance of being selected. Then a ball is drawn at random from the selected urn.

[14]. Find the probability of drawing a green ball.

[15]. Find the probability of drawing a red ball from Urn II.

Choices for questions 14-15 are:

- | | | | | | |
|------------|------------|------------------------|------------|------------|------------|
| (A) 0.1942 | (B) 0.1333 | (C) 0.3768 | (D) 0.3767 | (E) 0.3026 | (F) 0.6041 |
| (G) 0.5000 | (H) 0.6744 | (I) None of the above. | | | |
-

A committee of three people is to be chosen from four married couples.

[16]. What is the probability that no two committee members are married to each other?

[17]. What is the probability that the committee contains two women and one man?

[18]. What is the probability that the committee contains two women and one man such that no two committee members are married to each other?

Answer choices for questions 16-18 are:

- | | | | |
|------------|------------|------------|------------|
| (A) 0.2857 | (B) 0.8234 | (C) 0.1080 | (D) 0.5623 |
| (E) 0.1124 | (F) 0.1815 | (G) 0.2143 | (H) 0.4565 |
| (I) 0.3636 | (J) 0.4286 | | |

[19]. In how many ways can the letters of the word **TENNESSEE** be arranged?

[20]. How many different 9-letter sequences in [19] that begin with N and end with E?

[21]. Twenty automobiles enter a race. Eight are made by manufacturer A, seven by manufacturer B, and the rest by manufacturer C. Keeping track of only the manufacturer of the cars, in how many ways can the cars cross the finish line?

[22]. How many five-card hands from a standard deck of 52 cards will contain only cards of the same suit?

Answer choices for questions 19-22 are:

- | | | | |
|----------|-----------|----------|--------------|
| (A) 1060 | (B) 1260 | (C) 5148 | (D) 99768240 |
| (E) 3780 | (F) 34650 | (G) 3205 | (H) 65034560 |
| (I) 420 | (J) 1000 | | |

Let X be the number of female children in a randomly chosen family with three children. Find the probability distribution of X (assume that the birth rates for girls and boys are the same). Using the probability distribution of X to find

[23]. $E(X)$ (Expected value or mean of X)

[24]. $\sigma(X)$

Answer Choices for Questions 23-24:

- | | | | | | | | |
|------------|-----------------------|-------|---------|------------|------------|------------|------------|
| (A) 2.5 | (B) 1.5 | (C) 0 | (D) 3.5 | (E) 0.4140 | (F) 0.8660 | (G) 0.9021 | (H) 0.7324 |
| (I) 0.7500 | (J) None of the above | | | | | | |

A gas (gasoline) station has found that 75% of its customers pay by credit card.

[25]. If this gas station has 12 customers during a one-hour period, what is the probability that exactly nine of these customers pay by credit card?

[26]. During another one-hour period, this gas station has 6 customers. What is the probability that at least 5 of these customers pay by credit card?

[27]. Over a one-week period, this gas station has 2000 customers. What is the probability that the number paying by credit card is *at least* 1505?

Answer Choices for Questions 25 to 27:

- | | | | | | |
|------------|------------|------------|------------|------------|------------|
| (A) 0.4823 | (B) 0.3512 | (C) 0.5900 | (D) 0.2231 | (E) 0.2581 | (F) 0.6568 |
| (G) 0.4435 | (H) 0.6442 | (I) 0.5339 | (J) 0.4090 | | |

Calls come into a telephone exchange at random at a rate of 300 per hour. Assume that calls arrive according to the Poisson distribution. What is the probability that

[28]. Exactly one call comes in during a given one-minute period?

[29]. At least two calls come in during a one-minute period?

In a bus company, people find that, on the average, three drivers per day are absent. Assume that the number of absences follows the Poisson distribution.

[30]. What is the probability that all of the drivers show up for work on a given day?

Answer Choices for Questions 28 to 30:

(A) 0.0337 (B) 0.0351 (C) 0.5900 (D) 0.0498 (E) 0.2581 (F) 0.6568
(G) 0.4435 (H) 0.9596 (I) 0.5339 (J) 0.4090

A basketball player makes repeated shots from the free throw line. Assume his shots are independent Bernoulli trials with $p = 0.7$ (probability that the ball goes through the basket).

[31]. What is the probability that he makes the first basket on the third trial?

[32]. What is the probability that it takes him less than 3 shots to make his first basket?

Answer Choices for Questions 31 to 32:

(A) 0.0370 (B) 0.0351 (C) 0.5900 (D) 0.0498 (E) 0.2581 (F) 0.6568
(G) 0.0630 (H) 0.9100 (I) 0.5339 (J) 0.4090

An urn contains 3 red balls and 7 black balls. If a sample of 4 balls are drawn at random from the box, find

[33]. what is the probability that it will contain three red balls?

[34]. what is the probability that it will contain fewer than two red balls?

Answer Choices for Questions 33 to 34:

(A) 0.6667 (B) 0.4353 (C) 0.2301 (D) 0.4908 (E) 0.2581 (F) 0.0568
(G) 0.0630 (H) 0.2100 (I) 0.5339 (J) 0.0333

The change in depth of a river from one day to the next, measured (in feet) at a specific location, is a continuous uniform random variable X with the following density function:

$$f(x) = \begin{cases} k, & -2 \leq x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

[35]. Determine the value of k .

[36]. Find $P(X > 1)$

Answer Choices for Questions 35 to 36:

(A) 0.6667 (B) 0.2500 (C) 0.2301 (D) 0.4908 (E) 0.2581 (F) 0.0568
(G) 0.0630 (H) 0.2100 (I) 0.5339 (J) 0.0333

According to government reports, the heights of adult male residents are approximately normally distributed with a mean of 69.0 inches and a standard deviation of 2.8 inches.

[37]. What is the proportion of the adult male residents whose heights are greater than 71 inches or less than 67 inches?

[38]. If a clothing manufacturer wants to limit his market to the central 80% of the adult male population, what range of heights should be targeted?

Answer Choices for Questions 37 to 38:

- (A) 0.3457 (B) 0.3567 (C) 0.6801 (D) 0.9308 (E) 0.4778
(F) 65.52 to 72.58 (G) 66.65 to 71.35 (H) 65.87 to 72.13 (I) 64.93 to 73.07
(J) 65.38 to 72.62
-

Response times to emergency calls for an emergency medical organization (EMO) are normally distributed with a mean of 12.5 minutes and a standard deviation of 3.9 minutes.

[39]. What is the probability that this EMO responds in less than 10.0 minutes to a call?

[40]. What is the probability that the response time to a call is between 13.0 and 18.0 minutes?

[41]. The EMO received 7 calls on another day. What is the probability that the response time was under 12.5 minutes on exactly 4 of these calls (and over 12.5 minutes on the other 3)?

Answer Choices for Questions 39 to 41:

- (A) 0.5561 (B) 0.7799 (C) 0.4040 (D) 0.2137 (E) 0.4226
(F) 0.4335 (G) 0.3690 (H) 0.2611 (I) 0.3114 (J) 0.2734

[42]. The 20% of the calls that require the longest response time are always examined after the fact to determine the reason for the extended time. Thus response times of XXX or more are examined, where XXX = ..?

Answer Choices for Questions 42:

- (A) 15.2 (B) 17.3 (C) 16.4 (D) 15.8 (E) 19.5 (F) 17.7 (G) 18.4 (H) 16.8 (I) 14.7 (J) 18.9
-

The magnitude of earthquakes records in a region of North American can be modeled as having an exponential distribution with mean 2.4, as measured on the Richter scale. Find the probability that an earthquake striking this region will

[43]. be less than 3.0 on the Richter scale.

[44]. Fall between 2.0 and 3.0 on the Richter scale.

[45]. Of the next 10 earthquakes to strike this region, what is the probability that at least one will exceed 5.0 on the Richter scale?

Answer Choices for Questions 43 to 45:

- (A) 0.1245 (B) 0.2865 (C) 0.1481 (D) 0.0498 (E) 0.2581 (F) 0.6568
(G) 0.0630 (H) 0.9100 (I) 0.7354 (J) 0.4391
-

The number of eggs a female house fly lays during her lifetime is normally distributed, with a mean of 800 eggs and a standard deviation of 100 eggs. Random samples of size 15 are drawn from this population. Let X_1, X_2, \dots, X_{15} be the numbers of eggs for the selected flies. Let $\bar{X} = \frac{X_1 + X_2 + \dots + X_{15}}{15}$.

[46]. Determine the value of $\mu_{\bar{X}}$.

[47]. Determine the value of $\sigma_{\bar{X}}$.

[48]. Find $P(750 \leq \bar{X} \leq 810)$.

Answer choices for question are:

- | | | | |
|------------|-----------------------|------------|------------|
| (A) 0 | (B) 100 | (C) 800 | (D) 0.4735 |
| (E) 25.82 | (F) 6.67 | (G) 0.3535 | (H) 0.5437 |
| (I) 0.6255 | (J) None of the above | | |
-

***** The End *****