INSTITUTE OF DISTANCE LEARNING

KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI, GHANA

ME 355
STRENGTH OF MATERIALS II

UNIT 4

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Introduction

- ***BENDING OF CURVED BARS***

DEFLECTION OF BEAMS

Learning Objectives

- After reading this unit you should be able to:
- Derive the equations used to compute the stresses in bars of small radius of curvature
- Calculate the stresses formed in bars of small radius of curvature for particular cross-section
- Derive the equations used to compute the deflection in beams of small radius of curvature
- Calculate the deflection in beams of small radius of curvature

STRESSES IN CURVED BARS

In the· theory of simple bending,
we have discussed:

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Assumptions for the Stresses in the Bending of Curved Bars

The stresses in the bending of curved bars are determined on the following assumptions:

1. The bar material is stressed within the elastic limit, and thus obeys Hooke's law.
2. The transverse sections, which were plane before bending, remain plane after bending also.
3. The longitudinal fibres of the bar, parallel to the central axis, exert no pressure on each other.
4. The transverse cross-section has at least one axis of symmetry, and the bending moment lies on this plane.
5. The value of *E* (*i.e.*, modulus of elasticity) is the same in tension and compression.

Types of Curved Bars on the Basis of Initial Curvature

- The curved bars may be broadly grouped into the following two categories on the basis of their initial curvature:
  I. Bars with a small initial curvature, and
  II. Bars with a large initial curvature.

- The main characteristic of the above division is the ratio of the depth of bar section *(h)* to the initial radius of curvature *(R)*.

- If this ratio *(i.e.,* h/R) is 0.2 or less, the bar is considered to be of small initial curvature.

- But if this ratio is more than 0.2, the bar is considered to be of large radius of curvature.

Bars with a Small Initial Curvature

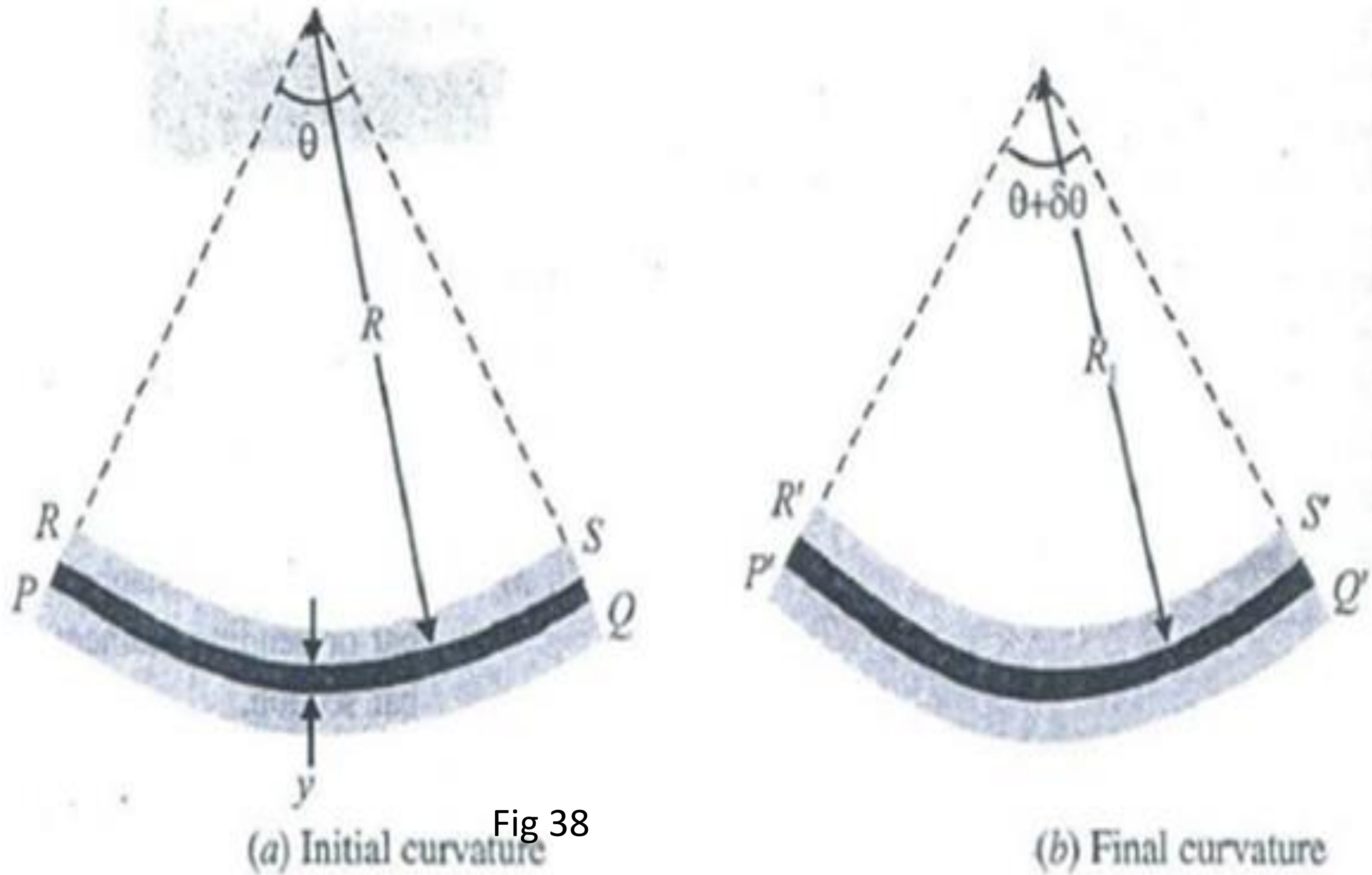


Fig 38

- Let the bar be given more curvature after the application of the end moments as shown in Fig. *(b) above.*

- Let

R Initial radius of curvature of the bar

R_1 Final radius of curvature

θ Initial angle subtended at the centre by the bar

$(\theta + \delta\theta)$ Final angle subtended at the centre of the bar.

The change in length

$$\delta l = P'Q' - PQ$$

Therefore, strain

$$\varepsilon = \frac{\delta l}{PQ} = \frac{P'Q' - PQ}{PQ} \ldots\ldots\ldots.(i)$$

$$\varepsilon = \frac{(R_1 + y)(\theta + \delta\theta) - (R + y)\theta}{(R + y)\theta}$$

$$\varepsilon = \frac{R_1(\theta + \delta\theta) + y.\delta\theta - R\theta}{(R + y)\theta} \ldots.(ii)$$

From the geometry of the bar,

$$RS = R\theta.$$

$$R'S' = R_1(\theta + \delta\theta)$$

But,

$$RS = .R'S'$$

$$\Rightarrow R\theta = R_1(\theta + \delta\theta)$$

Since *y* <<*R*,

Then, *(R* + y) = *R*

Also

$$\frac{\delta\theta}{\theta} = \left(\frac{R - R_1}{R_1}\right)$$

Therefore

$$\varepsilon = y\left(\frac{1}{R_1} - \frac{1}{R}\right)$$

Therefore,

$$\varepsilon = \left(\frac{y}{R + y}\right)\left(\frac{R - R_1}{R}\right)$$

Hence,

$$\sigma = E\varepsilon = Ey\left(\frac{1}{R_1} - \frac{1}{R}\right)$$

- *Example 4-1: A steel bar 50 mm in diameter, is formed into a circular arc of 4 m radius and support an angle of 90°. A couple is applied at each end of the bar, which changes the slope to 95° at one end relative to the other. Calculate the maximum bending stress due to the couple. Take E as 200 Gpa*

Solution

Given: Diameter of bar (d) = 50 mm; Radius of arc (R) = 4 m = 4000 mm; Initial angle subtended at the centre (ϑ) = 90°; Final angle subtended at the centre ($\vartheta + \delta\vartheta$) = 95° and modulus of elasticity (E) = 200 GPa = 200 x 10^3 N/mm^2

Distance between centre line of bar and extreme fibre

$$y = \frac{d}{2} = \frac{50}{2} = 25\,mm$$

$$\delta\theta = 95^o - 90^0 = 5^o$$

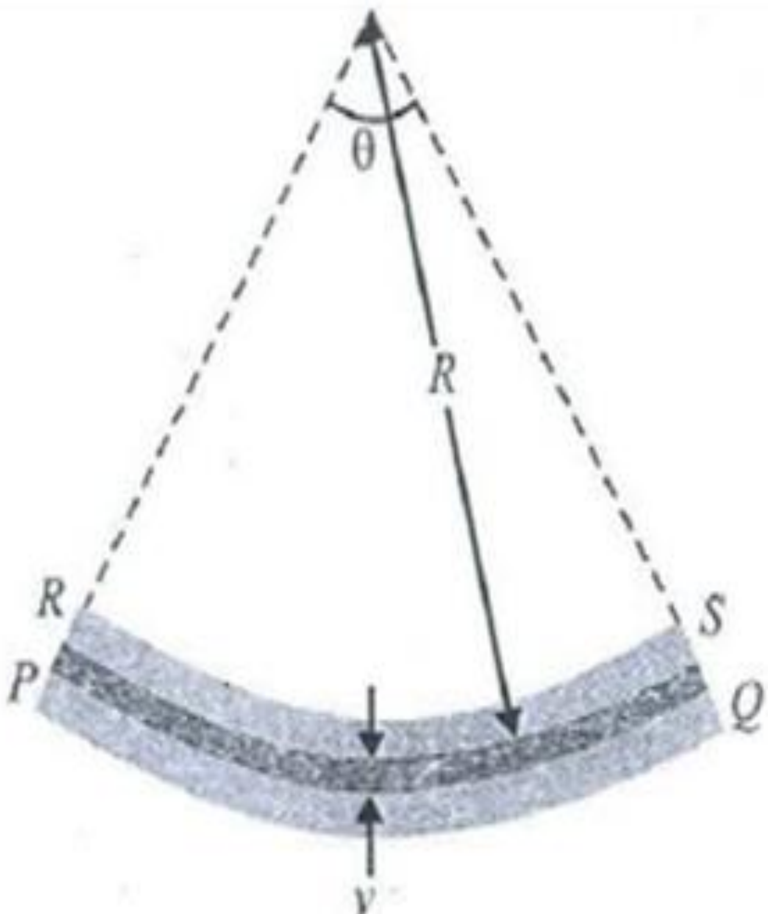
$$\frac{\delta\theta}{\theta} = \frac{(R - R_1)}{R_1} \Rightarrow \frac{5}{90} = \frac{(4000 - R_1)}{R_1}$$

$$5R_1 = 360000 - 90R_1$$

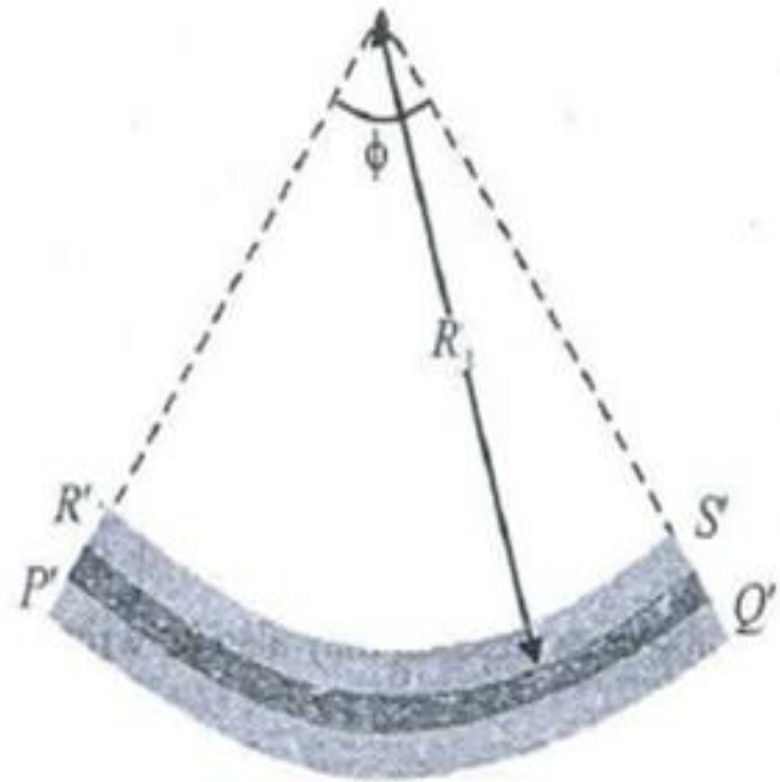
$$\therefore R_1 = \frac{360000}{95} = 3789\,mm$$

$$\sigma = E\varepsilon = Ey\left(\frac{1}{R_1} - \frac{1}{R}\right) = (200x10^3)(25)\left[\frac{1}{3789} - \frac{1}{4000}\right] = 69.6\,N/mm^2$$

Bars with a Large Initial Curvature



(a) Initial curvature



(b) Final curvature

- Let

R Initial radius of curvature of the bar
R_1 Final radius of curvature
θ Initial angle subtended at the centre by the bar,
φ Final angle subtended at the centre of the bar
σ_0 Bending stress in the centroidal fibre R'S'
σ Bending stress in the fibre P'Q' and
dA Area of fibre *P*'Q'.

- Now consider a layer *PQ,* which has been bent up to P'Q' after bending.
- Let *y* be the distance of the layer *PQ* from *RS,* the neutral axis of the bar.
- We know that increase in the length of the bar.

Neutral Axis

$$\varepsilon_0 = \frac{\delta l}{RS} = \frac{R'S' - RS}{RS} = \frac{R'S'}{RS} - 1 ..(i)$$

$$\varepsilon_0 + 1 = \frac{R'S'}{RS} = \frac{R_1\phi}{R\theta} ...(ii)$$

Increase in the length of the bar at a distance *y* from the centroidal axis

$$\varepsilon = \frac{\delta l}{PQ} = \frac{P'Q' - PQ}{PQ} = \frac{P'Q'}{PQ} - 1$$

$$\varepsilon + 1 = \frac{P'Q'}{PQ} = \frac{(R_1 + y)\phi}{(R+1)\theta} ...(iii)$$

Dividing equation (iii) by (ii),

$$\frac{\varepsilon + 1}{\varepsilon_0 + 1} == \left(\frac{R_1 + y}{R + y}\right)\left(\frac{R}{R_1}\right)$$

$$\varepsilon = \varepsilon_0 + = \frac{(\varepsilon_0 + 1)y\left(\frac{1}{R_1} - \frac{1}{R}\right)}{1 + \frac{y}{R}} ..(iv)$$

- The bending stress in the fibre P'*Q'*,

$$\sigma = E.\varepsilon = E\left[\varepsilon_0 + = \frac{(\varepsilon_0 + 1)y\left(\frac{1}{R_1} - \frac{1}{R}\right)}{1 + \frac{y}{R}}\right] ..(v)$$

- The force in an element of area *dA* at a distance *y* of from the centroidal axis,

$$dP = \sigma dA = E\left[\varepsilon_0 + = \frac{(\varepsilon_0 + 1)y\left(\frac{1}{R_1} - \frac{1}{R}\right)}{1 + \frac{y}{R}}\right] dA$$

The total normal force

$$P = E\varepsilon_0 A + E(\varepsilon_0 + 1)\left(\frac{1}{R_1} - \frac{1}{R}\right)\int \frac{y}{1+\left(y/R\right)} dA$$

Since the beam is in equilibrium, therefore the total normal force on the cross-section is zero.

$$P = E\varepsilon_0 A + E(\varepsilon_0 + 1)\left(\frac{1}{R_1} - \frac{1}{R}\right)\int \frac{y}{1+\left(y/R\right)} dA = 0..(vi)$$

Let us find out the value of separately

$$\int \frac{y}{1+\left(y/R\right)} dA$$

Therefore,

$$\int \frac{y}{1+\left(y/R\right)} dA = \int y.dA - \int \frac{y^2}{R+y} dA$$

Since First moment of area

$$\int y.dA$$

Hence

$$\int \frac{y}{1+\left(y/R\right)} dA = -\frac{Ah^2}{R} \quad ...(vii)$$

h^2 is the constant of the section; *h* is called the link radius.

Therefore,

$$E\varepsilon_0 A + E\left(\varepsilon_0 + 1\right)\left(\frac{1}{R_1} - \frac{1}{R}\right)\left(\frac{Ah^2}{R}\right) = 0$$

$$\varepsilon_0 = (\varepsilon_0 + 1)\left(\frac{1}{R_1} - \frac{1}{R}\right)\left(\frac{h^2}{R}\right) \ldots (viii)$$

The total moment of the section

$$M = \int y.\sigma.dA = \int y.E.\varepsilon.dA$$

From (vi) $\varepsilon_0 = \dfrac{M}{EAR}$

The bending stress

$$\sigma == \left[\frac{M}{EAR} + \frac{MRy}{EAh^2(R+y)}\right]$$

$$\sigma = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right]$$

Link Radius for Standard Sections

From Eq. (vii)

$$\int \frac{y^2}{R+y} dA = \frac{Ah^2}{R}$$

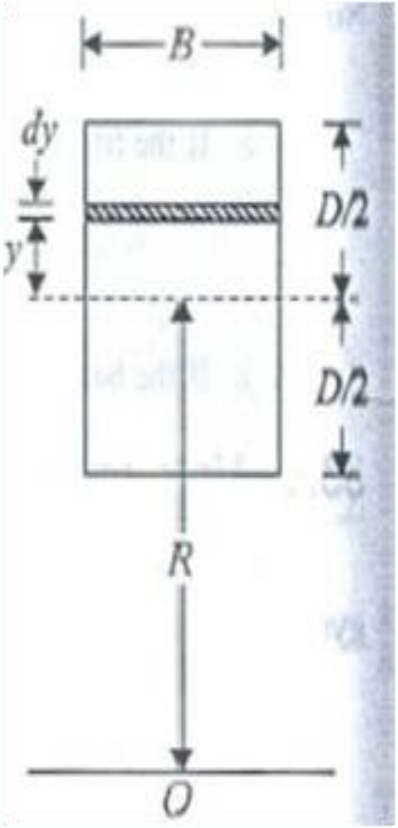
Simplifying

$$h^2 = \frac{R}{A} \int \frac{y^2}{R+y} dA$$

Hence

$$h^2 = \frac{R^3}{A}\left(\int \frac{dA}{R+y}\right) - R^2$$

Value of Link Radius for a Rectangular Section



Therefore area of the strip,

$$dA = b.dy..(i)$$

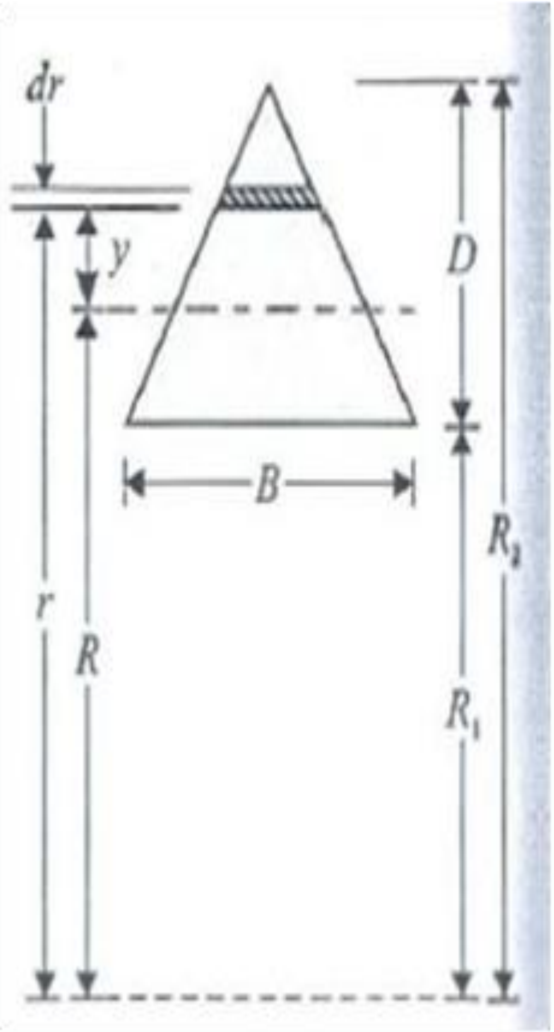
We know that

$$h^2 = \frac{R^3}{A}\left(\int \frac{dA}{R+y}\right) - R^2$$

Substituting the value of dA and simplifying,

$$h^2 = \frac{R^3}{BxD}\int_{-\frac{D}{2}}^{\frac{D}{2}} \frac{Bdy}{R+y} - R^2 == 2.3\frac{R^3}{D}\log\left(\frac{2R+D}{2R-D}\right) - R^2$$

Value of link Radius for Triangular Section



From geometry, the width of the bar

$$b = \frac{B}{D}(R_2 - r)$$

Area of strip

$$dA = \frac{B}{D}(R_2 - r)dr$$

We know that

$$h^2 = \frac{R^3}{A}\left(\int \frac{dA}{R+y}\right) - R^2$$

But

$$R + y = r$$

Substituting the value of dA and integrating,

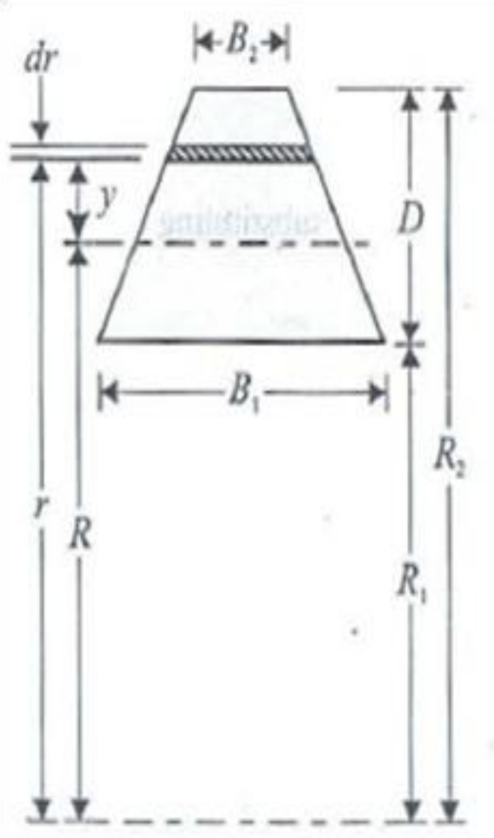
$$h^2 = \frac{R^3}{A}\int_{R_1}^{R_2} \frac{\frac{B}{D}(R_2 - r)dr}{R + y}$$

Hence,

$$h^2 == \frac{R^3}{A} x \frac{B}{D}\left[2.3R_2 \log\frac{R_2}{R_1} - D\right] - R^2$$

Value of link Radius for a Trapezoidal Section

From geometry, the width of the bar



$$b. = B_2 + \left(\frac{B_1 - B_2}{D}\right)(R_2 - r)$$

Area of strip

$$dA = b.dr = \left[B_2 + \left(\frac{B_1 - B_2}{D}\right)(R_2 - r)\right]dr$$

We know that

$$h^2 = \frac{R^3}{A}\left(\int \frac{dA}{R+y}\right) - R^2$$

But

$$R + y = r$$

Therefore

$$h^2 = B_2 \int_{R_1}^{R_2} \frac{dr}{r} + \left(\frac{B_1 - B_2}{D}\right) R_2 \int_{R_1}^{R_2} \frac{dr}{r} - \left(\frac{B_1 - B_2}{D}\right) \int_{R_1}^{R_2} \frac{rdr}{r} - R^2$$

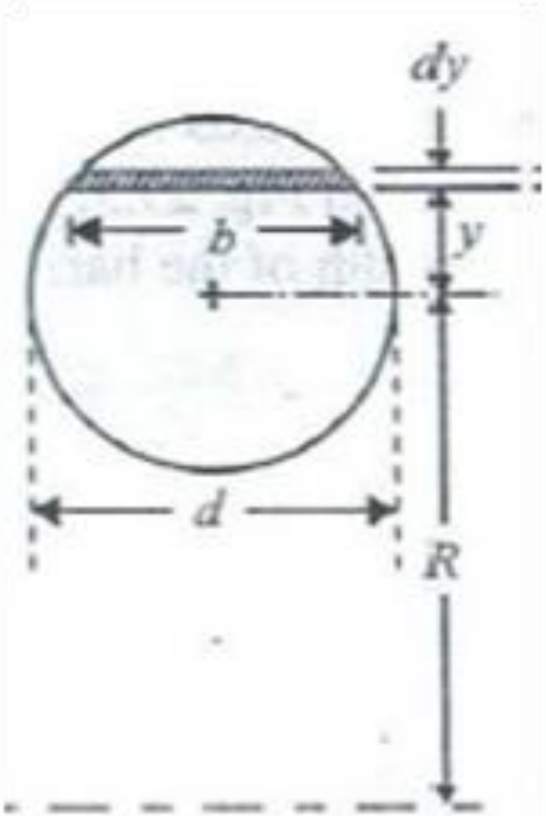
Hence

$$h^2 = \frac{R^3}{A}\left\{2.3\log\frac{R_2}{R_1}\left[B_2 + \frac{(B_1 - B_2)R_2}{D}\right] - (B_1 - B_2)\right\} - R^2$$

NOTE:

- If we substitute $A = BD$; $R_2 = R + \frac{D}{2}$; $R_1 = R - \frac{D}{2}$ and $B_1 = B_2$
- We shall obtain the equation as obtained in rectangular section
- If we substitute $A = \frac{BD}{2}$; $R_2 = R + \frac{D}{2}$ $R_1 = R - \frac{D}{2}$ and $B_2 = 0$,
- We shall obtain the equation as obtained in triangular section.

Value of link Radius for a Circular Section



From geometry, the width of the bar

$$b = 2\left[\sqrt{\left(\frac{d}{2}\right)^2 - y^2}\right] = 2x\sqrt{\left(\frac{d^2}{4} - y^2\right)}$$

Area of strip

$$dA = bdy = 2x\sqrt{\left(\frac{d^2}{4} - y^2\right)}dy$$

We know that

$$h^2 = \frac{R^3}{A}\left(\int \frac{dA}{R+y}\right) - R^2$$

Value of link Radius for a Circular Section

Substituting the value of dA and integrating,

$$h^2 = \frac{R^3}{\frac{\pi}{4}d^2}\int_{-\frac{D}{2}}^{\frac{D}{2}} \frac{2x\sqrt{\left(\frac{d^2}{4} - y^2\right)}}{R+y}dy - R^2 = \frac{d^2}{16} + \frac{1}{8}x\frac{d^4}{16R^2}$$

Example 4-2:

- *A curved bar of square section, 3-cm sides and radius of curvature 4½ cm is initially unstressed. If a bending moment of 300 Nm is applied to the tending to straighten it, find the stresses at the inner and outer face.*

Given: Beam width (B) = 30 mm; Beam depth (*D*) = 30 mm; Radius of beam *(R)* = 45 mm and bending moment *(M)* = 3×10^5 N-mm.

Area, *A* = 30 x 30 = 900 mm^2

Distance between centre Line and extreme fibre

$$y = \frac{D}{2} = \frac{30}{2} = 15\,mm$$

Link radius

$$h^2 = 2.3\frac{R^3}{D}\log\left(\frac{2R+D}{2R-D}\right) - R^2 = (2.3)\left(\frac{45^3}{30}\right)\log\left[\frac{2(45)+30}{2(45)-30}\right] - 45^2 = 78\,mm$$

Maximum stress at bottom surface

$$y = +15\,mm$$

At the bottom

$$\sigma = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right] = \frac{3x10^5}{(900)(45)}\left[1 + \frac{(45^2)(15)}{(78)(45+15)}\right]$$

$$\sigma = 7.4[1 + 6.49] = 55.4\ N/mm^2$$

Maximum stress at top surface

At the top

$$y = -15\,mm$$

$$\sigma = \frac{M}{AR}\left[1 - \frac{R^2 y}{h^2(R+y)}\right] = \frac{3x10^5}{(900)(45)}\left[1 + \frac{(45^2)(15)}{(78)(45+(-15))}\right]$$

$$\sigma = 7.4[1 - 12.98] = -88.7\ N/mm^2$$

Example 4-3:

- *A crane hook whose horizontal cross-section is trapezoidal, 50 mm wide at the inside and 25 mm wide at the outside, thickness 50 mm, carries a vertical load of 9800 N whose line of action is 38 mm from the inside edge of this action. The centre of curvature is 50 mm from the inside edge and Radius of curvature of the bar section R = 72 mm. Calculate the maximum tensile and compressive stresses set up.*

Solution

Given: Base width of the bar section, B_1 = 50 mm; Top width of the bar section, B_2 = 25 mm; Depth of the bar section, D = 50 mm; Distance from line of action (x) = 38 mm; Force (F) = 9800 N

Distance between centre Line and extreme fibre,

$$y_1 = \frac{B_1 + 2B_2}{B_1 + B_2}\left(\frac{D}{3}\right) = \left(\frac{50 + 2x25}{50 + 25}\right)\left(\frac{50}{3}\right) = 22.22\ mm$$

$$y_2 = D - y_1 = 50 - 22.22 = 27.78\ mm$$

$$A = \frac{D}{2}(B_1 + B_2) = \frac{50}{2}(50 + 25) = 1875\, mm^2$$

- From the geometry of the hook section, we find that
- Radius of inner edge, $R_1 = 50$ mm
- Radius of outer edge, $R_2 = 50 + 50 = 100$ mm
- Radius of central line, $R = 50 + 22.22 = 72.22$ mm

Link radius

$$h^2 = \frac{R^3}{A}\left\{2.3\log\frac{R_2}{R_1}\left[B_2 + \frac{(B_1 - B_2)R_2}{D}\right] - (B_1 - B_2)\right\} - R^2$$

$$\Rightarrow h^2 = \frac{72.22^3}{1875}\left\{2.3\log\frac{100}{50}\left[25 + \frac{(50-25)(100)}{50}\right] - (50-25)\right\} - 72.22^2 = 158.24$$

Maximum stress at outside edge

$$y = +27.78\ mm$$

At the outside edge

$$\sigma = \frac{W}{A}\left\{1+\frac{x}{R}\left[1+\frac{R^2 y_2}{h^2(R+y_2)}\right]\right\} = \frac{9800}{1875}\left\{1+\frac{38}{72.22}\left[1+\frac{(72.22^2)(27.78)}{(158.24)(72.22+27.78)}\right]\right\}$$

$$\Rightarrow \sigma = 33.18 = N/mm^2$$

Maximum stress at inside edge

At the inside edge

$$y = -22.22\ mm$$

$$\sigma = \frac{W}{A}\left\{1+\frac{x}{R}\left[1+\frac{R^2 y_2}{h^2(R+y_2)}\right]\right\}$$

$$= \frac{9800}{1875}\left\{1+\frac{38}{72.22}\left[1-\frac{(72.22^2)(22.22)}{(158.24)(72.22-22.22)}\right]\right\} = -37.56 = N/mm^2$$

- *Example 4-4:*
- *A beam of rectangular section 20 mm x 40 nun has its centre line curved to a radius of 50 mm. The beam is subjected to a bending moment of 4 x 10^5 N.mm. Determine the intensity of maximum stresses in the beam*

Solution

Given: Beam width = 20 mm; Beam depth (D) = 40 mm; Radius of beam (R) = 50 mm and bending moment (M) = 4 x 10^5 N-mm.

Area, A = 40 x 20 = 800 mm^2

Distance between centre Line and extreme fibre, $y = \frac{D}{2} = \frac{40}{2} = 20\,mm$

Link radius

$$h^2 = 2.3\frac{R^3}{D}\log\left(\frac{2R+D}{2R-D}\right) - R^2 = (2.3)\left(\frac{50^3}{40}\right)\log\left[\frac{2(50)+40}{2(50)-40}\right] - 50^2 = 145\,mm$$

- *Maximum stress at bottom surface*
- At the bottom

$$y = +20\ mm$$

- The stress is given by

$$\sigma = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2 (R+y)}\right] = \frac{4x10^5}{(800)(50)}\left[1 + \frac{(50^2)(20)}{(145)(50+20)}\right] = 59.26\ N/mm^2$$

- *Maximum stress at top surface*
- At the top $y = -20\ mm$

- The stress is given by

$$\sigma = \frac{M}{AR}\left[1 - \frac{R^2 y}{h^2 (R+y)}\right] = \frac{4x10^5}{(800)(50)}\left[1 + \frac{(50^2)(20)}{(145)(50+(-20))}\right] = -104.94\ N/mm^2$$

- *Example 4-5:*
- *A beam of circular section of diameter 20 mm has its centre line curved to a radius of 50 mm. Find the intensity of maximum stresses in the beam, when subjected to a moment of 5 kN-mm.*

Solution

Given: Diameter of section (d) = 20 mm; Radius of curvature (R) = 50 mm and moment (M) = 5 kN-mm = 5 x 10^3 N-mm

Area $$A = \frac{\pi}{4} x d^2 = \frac{\pi}{4} x 20^2 = 100\pi \ mm^2$$

The distance between centre line and extreme fibre

$$y = \frac{d}{2} = \frac{20}{2} = 10\ mm$$

Link radius

$$h^2 = \frac{d^2}{16} + \frac{1}{8} x \frac{d^4}{16R^2} = \frac{20^2}{16} + \frac{1}{8} x \frac{20^4}{16x50^2} = 25.05\ mm$$

$$y = +10\ mm$$

Maximum stress at bottom surface

At the bottom

$$\sigma = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right] = \frac{5x10^5}{(100\pi)(50)}\left[1 + \frac{(50^2)(10)}{(25.05)(50+10)}\right] = 5.61\ N/mm^2$$

$$y = -10\ mm$$

Maximum stress at top surface

At the top

$$\sigma = \frac{M}{AR}\left[1 - \frac{R^2 y}{h^2(R+y)}\right] = \frac{5x10^5}{(100\pi)(50)}\left[1 + \frac{(50^2)(10)}{(25.05)(50+(-10))}\right] = -4.98\ N/mm^2$$

Application of Bars with a Large Initial Curvature

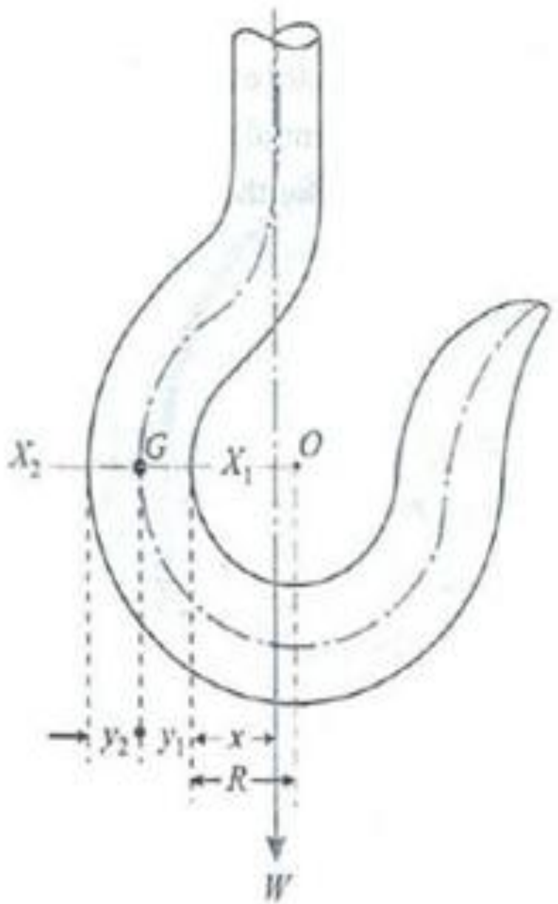
- We have already discussed in that the values of bending stress due to moment in bars with a large initial curvature.

- The results of this may be applied for finding the stresses in

  1. Crane Hooks,
  2. Rings and
  3. Chain links

  when it is subjected to a load.

Crane Hooks

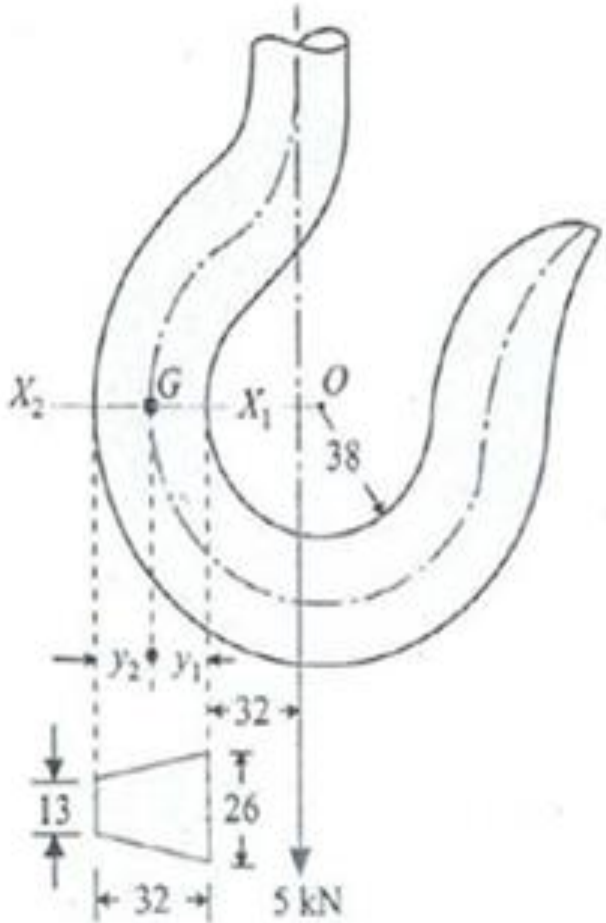


The total stress at X_1 and X_2

$$\sigma = \frac{W}{A}\left\{1 + \frac{x}{R}\left[1 + \frac{R^2 y_2}{h^2(R + y_2)}\right]\right\}$$

Example 4-6:

A crane hook carries a load of 5 kN the line of load being at a horizontal distance of 32 mm from the inside edge of a horizontal section through the centre · of curvature; and the centre of curvature being 38 mm from the same edge. The horizontal section is a trapezium whose parallel sides are 13 mm and 26 mm and height is 32 mm. Determine the greatest tensile and compressive stresses in the hook as shown in Fig.



- **Solution**
- Given: Load (W) = 5 kN = 5 x 10^3 N; Distance between the centre line and inner edge *(x)* = 32 mm; Distance between centre of curvature and inner edge = 38 mm; Outer width (B_2) = 13 mm; Inner width (B_1) = 26 mm and depth *(D)* = 32 mm.
- Distance between centre Line and extreme fibre,

$$y_1 = \frac{B_1 + 2B_2}{B_1 + B_2}\left(\frac{D}{3}\right) = \left(\frac{26 + 2x13}{26 + 13}\right)\left(\frac{32}{3}\right) = 14.2\ mm$$

- and

$$y_2 = D - y_1 = 32 - 14.2 = 17.8\ mm$$

Area $$A = \frac{D}{2}(B_1 + B_2) = \frac{32}{2}(26 + 13) = 624\ mm^2$$

Radius of inner edge, R_1 = 38 mm

Radius of outer edge, R_2 = 38 + 32 = 70 mm

Radius of central line, R = 38 + 14.2 = 52.2 mm

Link radius

$$h^2 = \frac{R^3}{A}\left\{2.3\log\frac{R_2}{R_1}\left[B_2 + \frac{(B_1 - B_2)R_2}{D}\right] - (B_1 - B_2)\right\} - R^2$$

$$\Rightarrow h^2 = \frac{52.22^3}{1875}\left\{2.3\log\frac{70}{38}\left[25 + \frac{(50-25)(100)}{50}\right] - (50-25)\right\} - 52.22^2 = 158.24$$

Maximum stress at outside edge

At the outside edge $y = +17.8\ mm$

$$\sigma = \frac{W}{A}\left\{1 + \frac{x}{R}\left[1 + \frac{R^2 y_2}{h^2(R + y_2)}\right]\right\}$$

$$= \frac{5x10^3}{624}\left\{1 + \frac{38}{52.2}\left[1 + \frac{(52.2^2)(17.8)}{(68)(52.2+17.8)}\right]\right\} = 62.95\ N/mm^2$$

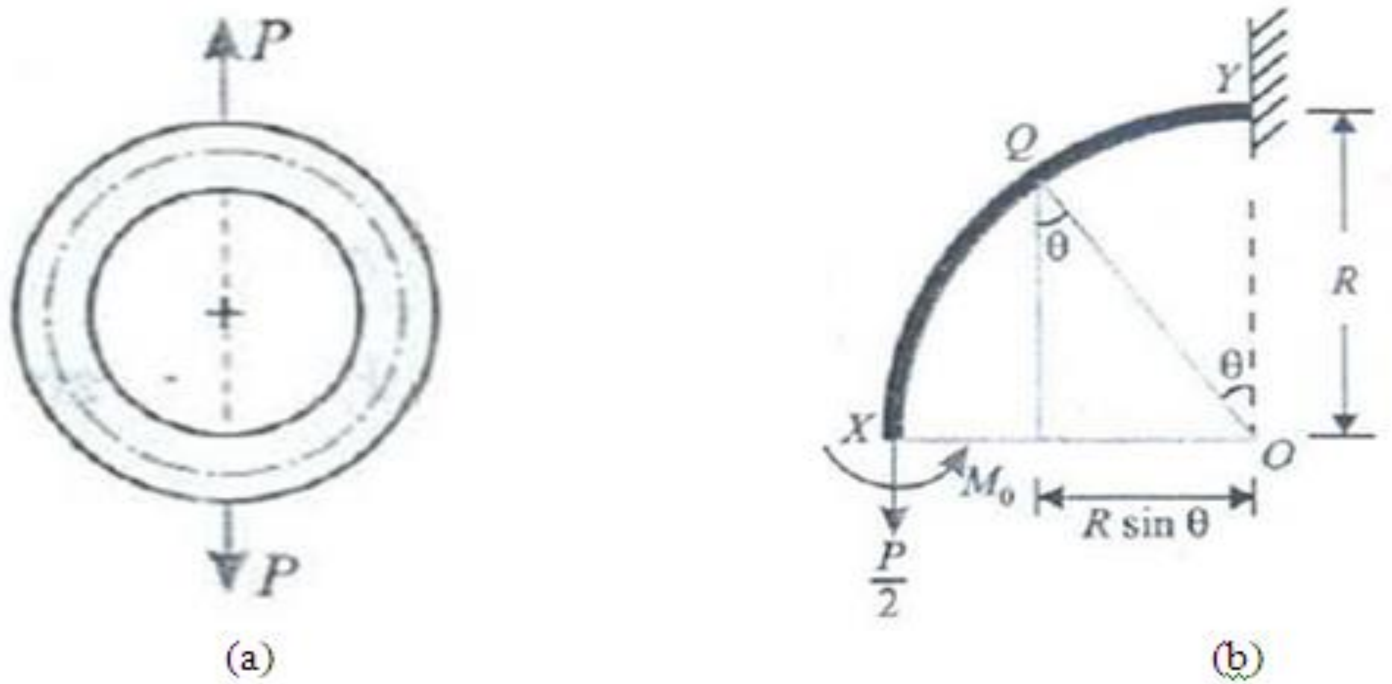
Maximum stress at inside edge

At the inside edge $y = -14.2\ mm$

$$\sigma = \frac{W}{A}\left\{1 + \frac{x}{R}\left[1 + \frac{R^2 y_2}{h^2\left(R + y_2\right)}\right]\right\}$$

$$= \frac{5x10^3}{624}\left\{1 + \frac{38}{52.2}\left[1 - \frac{\left(52.2^2\right)\left(14.2\right)}{\left(68\right)\left(52.2 - 14.2\right)}\right]\right\} = -60.58\, N/mm^2$$

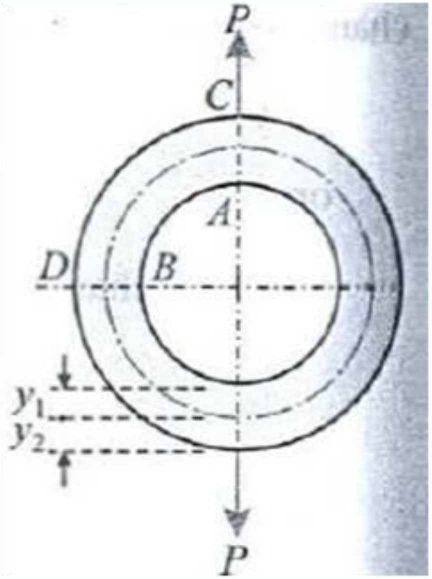
Rings



The stress at point *Q* is

$$\sigma = \sigma_0 + \sigma = \frac{P\sin\theta}{2A} + \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right]$$

Stress at the bottom (point A)



$$\sigma = \frac{P}{\pi A}\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R - y_1}\right]$$

Stress at the inner (point B)

$$\sigma = \frac{P}{2A} - \frac{0.182P}{A}\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R - y_1}\right]$$

Stress at the top (point C)

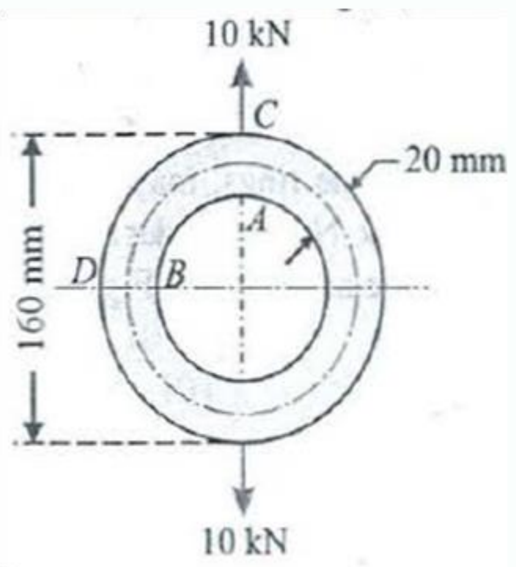
$$\sigma = \frac{P}{\pi A}\left[1 + \frac{R^2}{h^2} x \frac{y_2}{R + y_2}\right]$$

Stress at the outer (point D)

$$\sigma = \frac{P}{2A} - \frac{0.182P}{A}\left[1 + \frac{R^2}{h^2} x \frac{y_2}{R + y_2}\right]$$

Example 4-7:

A close circular ring made up of 20 mm diameter steel bar is subjected to a pull of 10 kN, whose line of action passes through the centre of the ring. Find the maximum value of tensile and compressive stresses in the ring, if the mean diameter of the ring is 160 mm as shown in Fig.



Solution

Given: Diameter of steel bar *(d)*= 20 mm; Pull *(P)* = 10 kN = .10^4 N and diameter of the ring *(D)*= 160 mm or radius of ring *(R)* = 80 mm.

Area, $$A = \frac{\pi}{4} x d^2 = \frac{\pi}{4} x 20^2 = 100\pi \ mm^2$$

The distance between centre line of the ring and extreme fibre

$$y = y_1 = y_2 = 10 \, mm$$

Link radius $h^2 = \frac{d^2}{16} + \frac{1}{8} x \frac{d^4}{16R^2} = \frac{20^2}{16} + \frac{1}{8} x \frac{20^4}{16x80^2} = 25.5$

Stress at the bottom (point A)

$$\sigma = \frac{P}{\pi A}\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R - y_1}\right]$$

$$= \frac{10^4}{\pi(100\pi)}\left[1 - \frac{80^2}{25.2} x \frac{10}{80 - 10}\right] = -357.83\ N/mm^2$$

Stress at the inner (point B)

$$\sigma = \frac{P}{2A} - \frac{0.182P}{A}\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R - y_1}\right]$$

$$= \frac{10^4}{2x100\pi} - \frac{0.182x10^4}{100\pi}\left[1 - \frac{80^2}{25.2} x \frac{10}{80 - 10}\right] = 220.3\ N/mm^2$$

Stress at the top (point C)

$$\sigma = \frac{P}{\pi A}\left[1 + \frac{R^2}{h^2} x \frac{y_2}{R + y_2}\right]$$

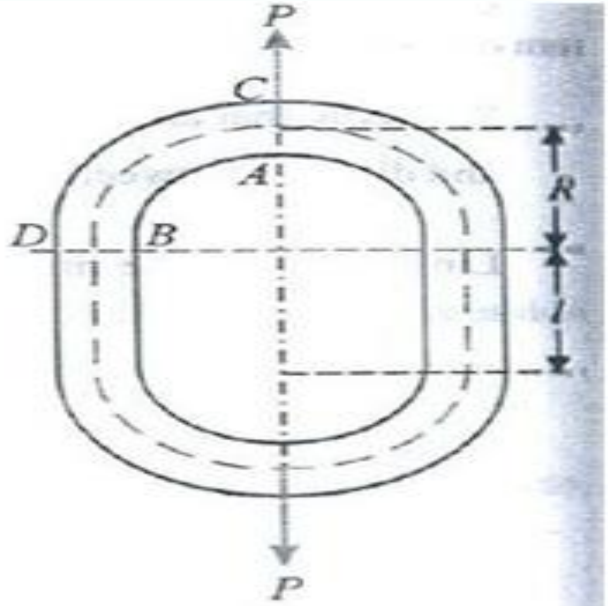
$$= \frac{10^4}{\pi(100\pi)}\left[1 + \frac{80^2}{25.2} x \frac{10}{80+10}\right] = 296\ N/mm^2$$

Stress at the outer (point D)

$$\sigma = \frac{P}{2A} - \frac{0.182P}{A}\left[1 + \frac{R^2}{h^2} x \frac{y_2}{R + y_2}\right]$$

$$= \frac{10^4}{2x100\pi} - \frac{0.182x10^4}{100\pi}\left[1 + \frac{80^2}{25.2} x \frac{10}{80+10}\right] = -153.4\ N/mm^2$$

Chain Links



Circular portion

$$M = M_0 + \frac{PR}{2}(1 - \sin\theta)$$

Straight portion

$$M = \frac{PR}{2}\left(\frac{l + 2R}{l + \pi R}\right) + \frac{PR}{2}$$

Stress at the bottom (point A)

$$\sigma_A = \frac{P}{2A}\left(\frac{l + 2R}{l + \pi R}\right)\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R - y_1}\right]$$

Stress at the inner (point B)

$$\sigma = \frac{P}{2A} - \frac{PR}{2A}\left(\frac{\pi - 2}{l + \pi R}\right)\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R - y_1}\right]$$

Stress at the top (point C)

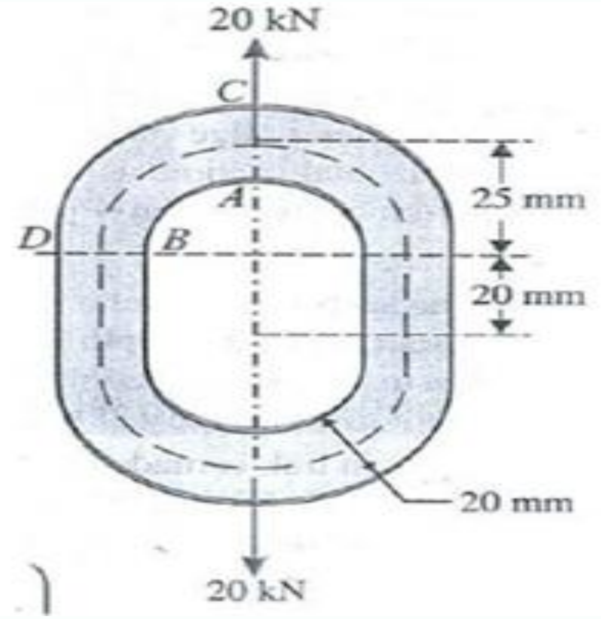
$$\sigma = \frac{P}{2A}\left(\frac{l + 2R}{l + \pi R}\right)\left[1 + \frac{R^2}{h^2} x \frac{y_2}{R + y_2}\right]$$

Stress at the outer (point D)

$$\sigma = \frac{P}{2A} - \frac{PR}{2A}\left(\frac{\pi - 2}{l + \pi R}\right)\left[1 + \frac{R^2}{h^2} x \frac{y_2}{R + y_2}\right]$$

Example 4-8:

A chain link is made of 20 mm diameter round steel with mean radius of circular ends 25 mm, the length of straight portion being 20 mm. Determine the values of maximum tensile and compressive stresses, when the link is subjected to a pull of 20 kN at its ends as shown in Fig.



Solution

Given: Diameter of steel bar *(d)* = 20 mm; Radius of link *(R)* = 25 mm; Length of straight portion (*l*) = 20 mm and pull *(P)* = 20 kN = 2 x 10^4 N

Area $$A = \frac{\pi}{4} x d^2 = \frac{\pi}{4} x 20^2 = 100\pi \ mm^2$$

The distance between centre line of the ring and extreme fibre,

$$y = y_1 = y_2 = 10\, mm$$

Link radius

$$h^2 = \frac{d^2}{16} + \frac{1}{8} x \frac{d^4}{16R^2} = \frac{20^2}{16} + \frac{1}{8} x \frac{20^4}{16x25^2} = 27$$

Stress at the bottom (point A)

$$\sigma_A = \frac{P}{2A}\left(\frac{l+2R}{l+\pi R}\right)\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R-y_1}\right]$$

$$= \frac{2x10^3}{2x100\pi}\left(\frac{20+2x25}{20+\pi x25}\right)\left[1 - \frac{25^2}{27} x \frac{10}{(25-10)}\right] = -326.2\ N/mm^2$$

Stress at the inner (point B)

$$\sigma_B = \frac{P}{2A} - \frac{PR}{2A}\left(\frac{\pi-2}{l+\pi R}\right)\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R-y_1}\right]$$

$$= \frac{2x10^3}{2x100\pi} - \frac{(2x10^3)x25}{2x100\pi}\left(\frac{\pi-2}{20+\pi x25}\right)\left[1 - \frac{25^2}{27} x \frac{10}{(25-10)}\right] = 164.8\ N/mm^2$$

Stress at the top (point C)

$$\sigma = \frac{P}{2A}\left(\frac{l+2R}{l+\pi R}\right)\left[1+\frac{R^2}{h^2}x\frac{y_2}{R+y_2}\right]$$

$$= \frac{2x10^3}{2x100\pi}\left(\frac{20+2x25}{20+\pi x25}\right)\left[1+\frac{25^2}{27}x\frac{10}{25+10}\right] = 172.2\ N/mm^2$$

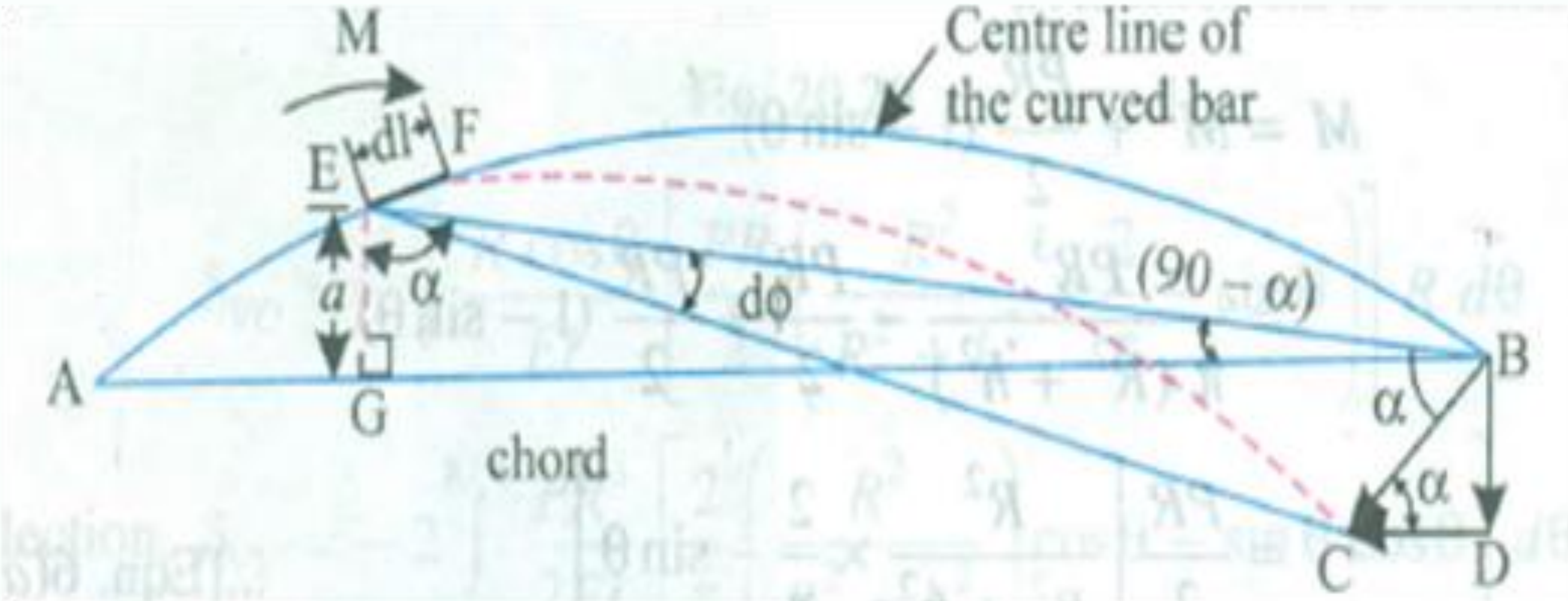
Stress at the outer (point D)

$$\sigma = \frac{P}{2A}-\frac{PR}{2A}\left(\frac{\pi-2}{l+\pi R}\right)\left[1+\frac{R^2}{h^2}x\frac{y_2}{R+y_2}\right]$$

$$= \frac{2x10^3}{2x100\pi}-\frac{\left(2x10^3\right)x25}{2A}\left(\frac{\pi-2}{20+\pi R}\right)\left[1+\frac{25^2}{27}x\frac{10}{(25+10)}\right] = -38.3\ N/mm^2$$

Thus the maximum tensile stress will occur at C equal to 172.2 N/mm^2 and maximum compressive will occur at A equal to 326.3 N/mm^2.

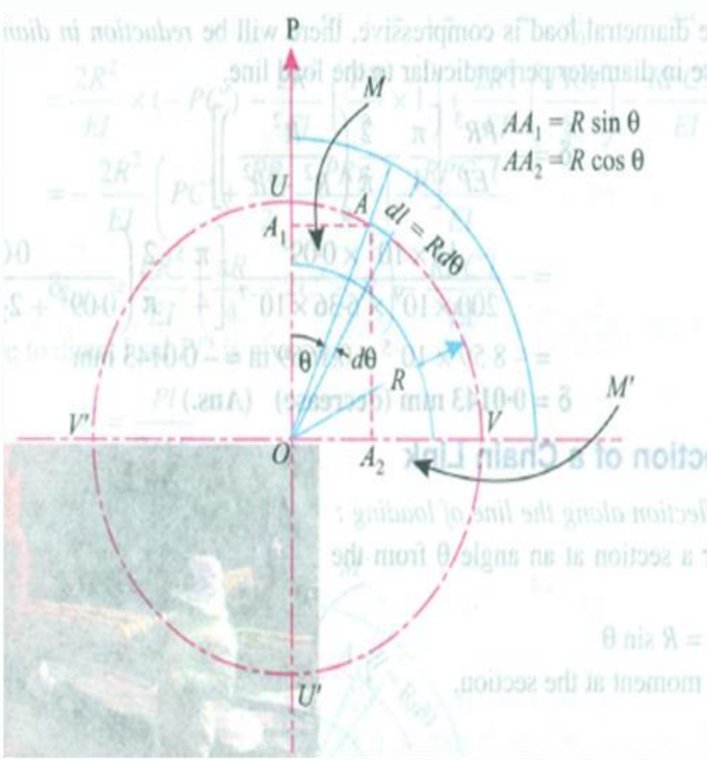
DEFLECTION OF CURVED BARS



Components of displacement BC are:

I. BD perpendicular to the chord AB, and
II. DC parallel to the chord AB ,i.e. , the line joining the ends of the centre line of the curved beam considered,

Deflection of a Closed Ring



Deflection along the Load Line

$$\delta_{UU'} == \frac{PR^3}{EI}\left[\frac{\pi}{4} - \frac{2}{\pi}\left(\frac{R^2}{R^2 + h^2}\right)\right]$$

Deflection Perpendicular to the Load Line

$$\delta_{VV'} == \frac{PR^3}{EI}\left[\frac{1}{2} - \frac{2}{\pi}\left(\frac{R^2}{R^2 + h^2}\right)\right]$$

Example 4-9:

A ring with a circular cross=section of 60 mm in diameter and a mean radius of 90 mm is subjected to a compressive load of 15kN. Calculate the deflection of the ring along and perpendicular to the load line. Take: E = 200 Gpa

Solution

Given: Diameter of the circular cross-section, (d) = 60 mm; Mean radius of the ring, (R) = 90 mm; Compressive load, (P) = 15kN=15x10^3; Young's modulus, (E) = 200 GPa = 200x10^3 N/mm^2

Moment of inertia,

$$I = \frac{\pi}{64} x d^4 = \frac{\pi}{64} x 60^2 = 636 x 10^3 \ mm^4$$

Link Radius

$$h^2 = \frac{d^2}{16} + \frac{d^4}{128R^2} = \frac{60^2}{16} + \frac{60^4}{128(90)^2} = 237.5 \, mm^2$$

- Since the diametral load is compressive, there will be reduction in diameter along the load line and increase in diameter perpendicular to the load line.
- ***Deflection of the Ring along the Load Line***

$$\delta = \frac{PR^3}{EI}\left[\frac{\pi}{4} - \frac{2}{\pi}\left(\frac{R^2}{R^2 + h^2}\right)\right]$$

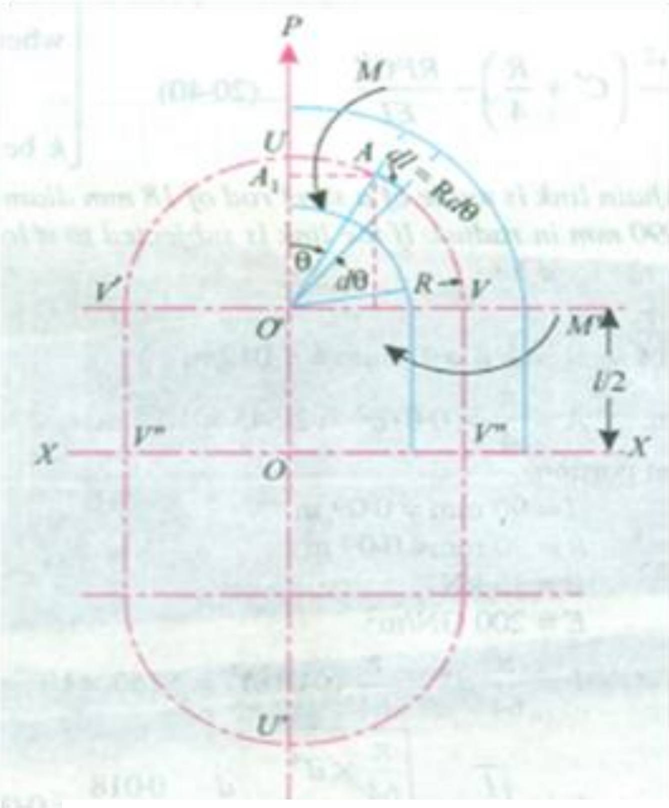
$$= \frac{\left(-15x10^3\right)x(90)^3}{\left(200x10^3\right)x\left(636x10^3\right)}\left[\frac{\pi}{4} - \frac{2}{\pi}\left(\frac{90^2}{90^2 + 237.5}\right)\right] = -0.0143mm$$

- ***Deflection Perpendicular to the Load Line***

$$\delta = \frac{PR^3}{EI}\left[\frac{1}{2} - \frac{2}{\pi}\left(\frac{R^2}{R^2 + h^2}\right)\right]$$

$$= \frac{\left(-15x10^3\right)x(90)^3}{\left(200x10^3\right)x\left(636x10^3\right)}\left[\frac{1}{2} - \frac{2}{\pi}\left(\frac{90^2}{90^2 + 237.5}\right)\right] = 0.0103mm$$

Deflection of a Chain Link



$$C' = \frac{\frac{R^2}{\pi} - \frac{R^2}{2} - \frac{h^2}{2}}{R + \frac{lh^2}{\pi k^2} + \frac{h^2}{R}}$$

Deflection Perpendicular to the Load Line

$$\delta_{VV'} == -\frac{2PR^2}{EI}\left[C' + \frac{R}{4}\right] - \frac{RPC'l}{EI}$$

Deflection along the Load Line

$$\delta_{UU'} == \frac{PR^3}{EI}\left[\frac{\pi R}{4} - 2C' - R\right] - \frac{RPC'l}{EI} + \frac{Pl}{2AE}$$

Example 4-10:

A chain link is made of a steel rod of a steel rod of 18 mm diameter with straight portion 90 mm in length and ends 90 mm in radius. If the link is subjected to a load of 15kN calculate the deflection of the link along and perpendicular to the load line. Take E = 200 Gpa

Solution

Given: Diameter of steel rod, (d) = 18 mm; Length of the straight portion, (*l*) = 90 mm; Radius of curvature, (R) = 90 mm; Load, (P) = 15 kN = $15x10^3$ N; Young's modulus, (E) = 200 GPa = 200 $x10^3$ N/mm^2

Area,
$$A = \frac{\pi}{4} x d^2 = \frac{\pi}{4} x 18^2 = 254.5\, mm^2$$

Moment of inertia
$$I = \frac{\pi}{64} x d^4 = \frac{\pi}{64} x 18^4 = 5153.67\, mm^4$$

Radius of gyration
$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{5153.67}{237.5}} = 4.66\, mm$$

- **Link radius**

$$h^2 = \frac{d^2}{16} + \frac{d^4}{128R^2} = \frac{18^2}{16} + \frac{18^4}{128(90)^2} = 20.35\, mm^2$$

$$C' = \frac{\dfrac{R^2}{\pi} - \dfrac{R^2}{2} - \dfrac{h^2}{2}}{R + \dfrac{lh^2}{\pi k^2} + \dfrac{h^2}{R}} = \frac{\dfrac{90^2}{\pi} - \dfrac{90^2}{2} - \dfrac{20.35}{2}}{90 + \dfrac{90x20.35}{\pi x(4.66)^2} + \dfrac{20.35}{90}} = 12.45\, mm$$

- **Deflection along the load line**

$$\delta_{UU'} = \frac{PR^3}{EI}\left[\frac{\pi R}{4} - 2C' - R\right] - \frac{RPC'l}{EI} + \frac{Pl}{2AE}$$

$$= \frac{(15x10^3)R^3}{(200x10^3)x5153.67}\left[\frac{\pi x90}{4} - 2x12.45 - 90\right] - \frac{90x(15x10^3)x12.45x90}{(200x10^3)x5153.67} + \frac{(15x10^3)x90}{2x254.5x(200x10^3)} = 2.14\, mm$$

- **Deflection perpendicular to line of loading**

$$\delta_{VV'} = \frac{2PR^2}{EI}\left[C' + \frac{R}{4}\right] - \frac{RPC'l}{EI} = -\frac{2x\left(15x10^3\right)\left(90\right)^2}{\left(200x10^3\right)x5153.67}\left[12.45 + \frac{90}{4}\right] - \frac{90x\left(15x10^3\right)x12.45x90}{\left(200x10^3\right)x5153.67} = -9.7\ mm$$

Sample Questions

Problem 6: *Fig. 7 shows a crane hook lifting a load of 150 kN. Determine the compressive and tensile stresses in the critical section of the crane hook.*

Problem 9: A *ring is made of round steel bar 30 mm diameter and the mean radius of the ring is 180 mm. Calculate the maximum tensile and compressive stresses in the material of the ring if it is subjected to a pull of 12 kN.*

Problem 10: *A chain link (Fig. 10) is made of round steel rod of 15 mm diameter. If R = 45 mm, l = 75 mm and load applied is 1·5 kN determine the maximum compressive stress in the link and tensile stress at the same section.*