Tutorial Problems in FLUID MECHANICS

with answers and specimen solutions

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CONTENTS

Preface

These tutorial problems are intended primarily for foreign students who study the subject FLUID MECHANICS in English medium at the Faculty of Mechanical Engineering, University of Miskolc. It is hoped, however, that it may also be of some service to other engineering students.

The material is reasonably self-contained together with the unpublished lecture notes of the author but references are also given for further reading. The choice of notations largely follows the recommendations of the International Standards Organisation, and the international system of units (**SI**) is used throughout the material. The number of problems involved is relatively low, but more or less detailed specimen solutions are given to all problems. The author encourages students to try and solve a lot of problems by their own. I would like to emphasize physical understanding to make students aware of the variety of phenomena that occur in real fluid flow situations.

The study aid contains seven chapters for wording the problems and further seven for the answers and specimen solutions. A list of symbols used and a copy of the Moody diagram are also included.

The acceleration due to gravity should be taken g = 9.81 m/s^2 and the density of water is to be chosen to be 1000 kg/m^3 if it is not stated otherwise.

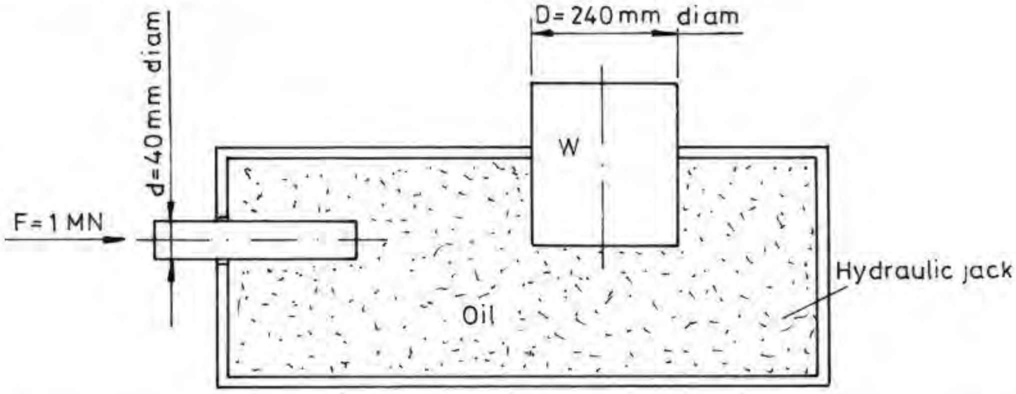
Figures are not to scale in this study aid.

The author gratefully acknowledges the financial support of the TEMPUS JEP-1501.

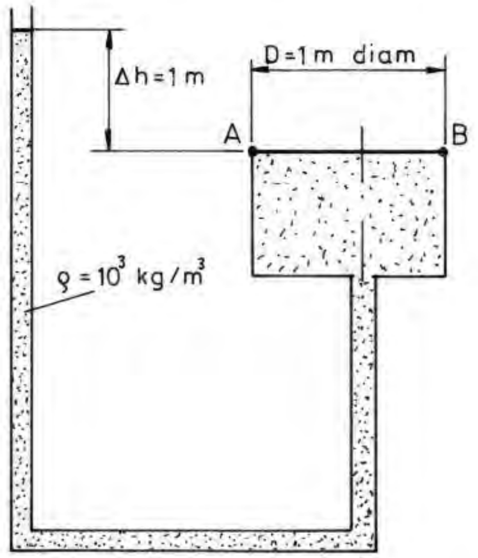
László Baranyi

1 FLUID STATICS, RELATIVE EQUILIBRIUM OF MOVING FLUIDS

1.1 Determine the gravity force W that can be sustained by the force F acting on the piston of the Figure.

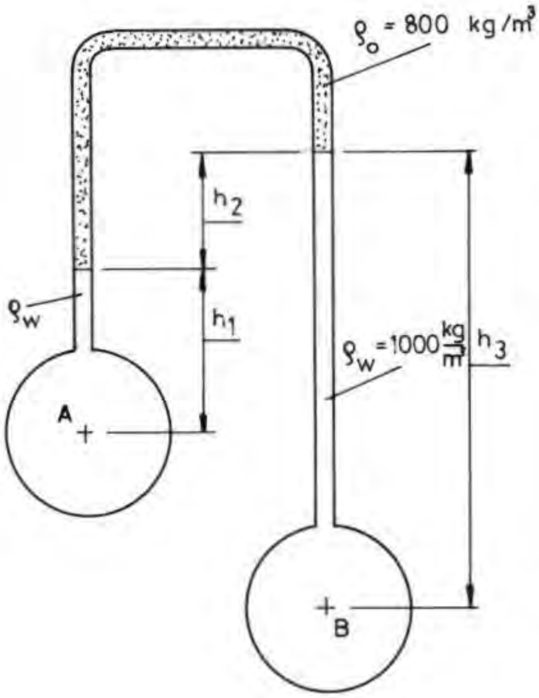


1.2 Neglecting the mass of the container find the force F tending to lift the circular top AB.

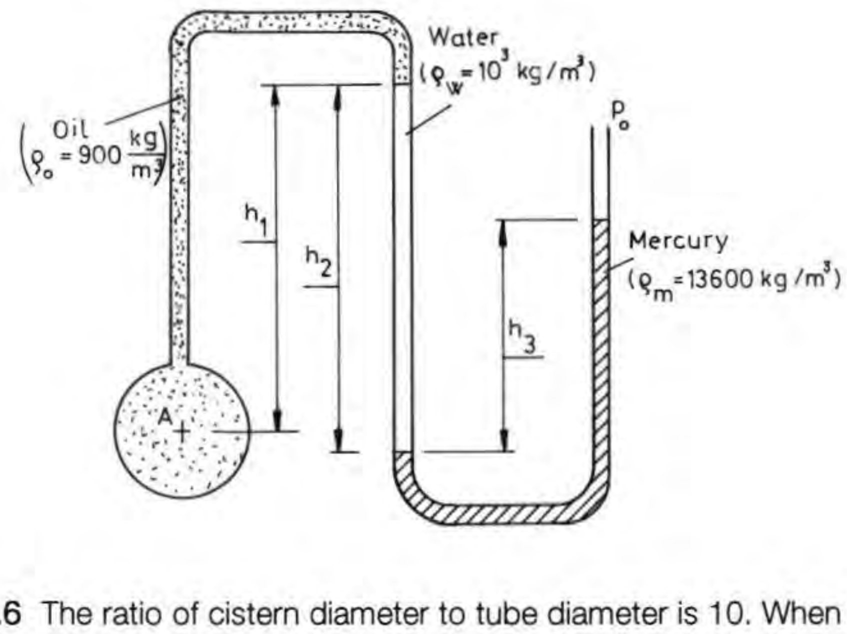


1.3 For isothermal air at 0 °C, determine the pressure and density at H = 3000 m when the pressure is p_o = 0.1 MPa abs at sea level (gas constant R = 287 Nm/(kgK)).

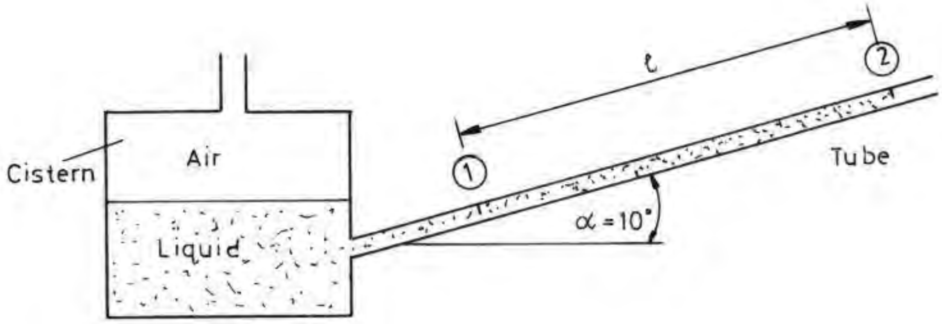
1.4 In the Figure the liquids at A and B are water (ρ_w) and the manometer liquid is oil (ρ_o).
h_1 = 300 mm; h_2 = 200 mm; h_3 = 600 mm
Find pressure difference p_A - p_B .



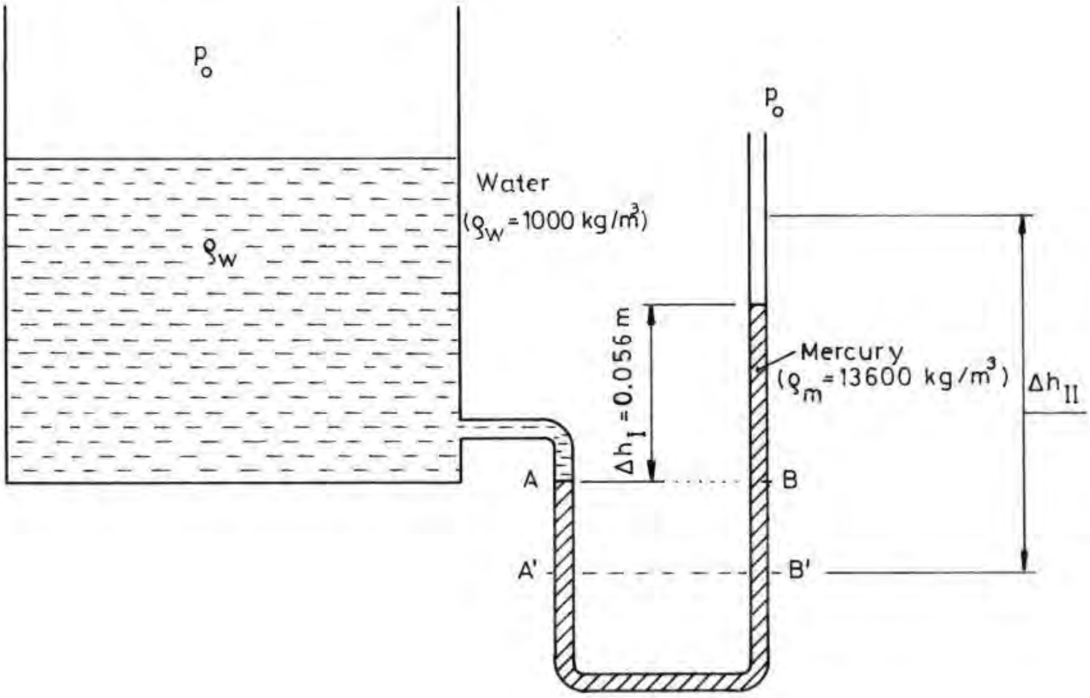
1.5 $h_1 = 1.3$ m; $h_2 = 1.5$ m; $h_3 = 1$ m
Find the gauge pressure in point A.



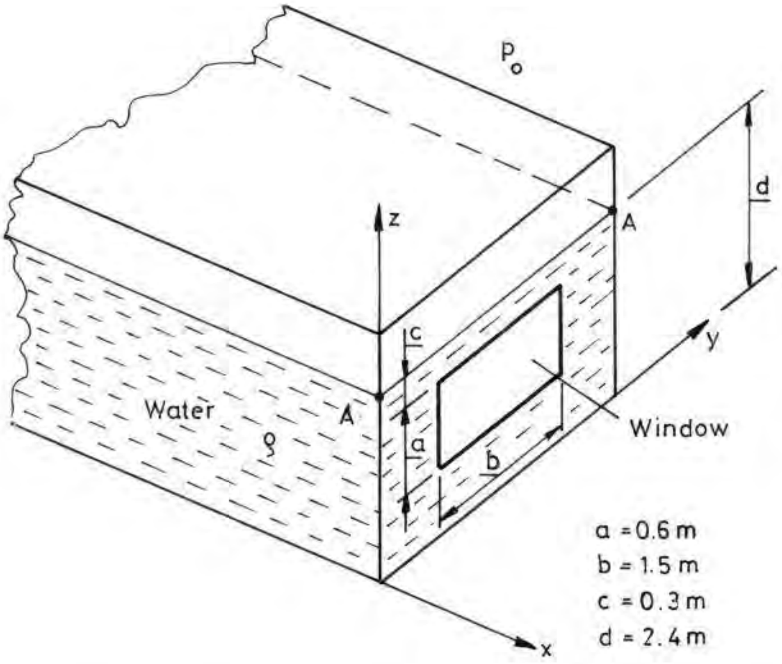
1.6 The ratio of cistern diameter to tube diameter is 10. When the air in the tank is at atmospheric pressure, the free surface in the tube is at position 1. When the cistern is pressurized, the liquid in the tube moves 20 mm (l = 200 mm) up the tube from position 1 to position 2. What is the cistern pressure that causes this deflection? The density of the liquid is $\rho = 800$ kg/m³ .



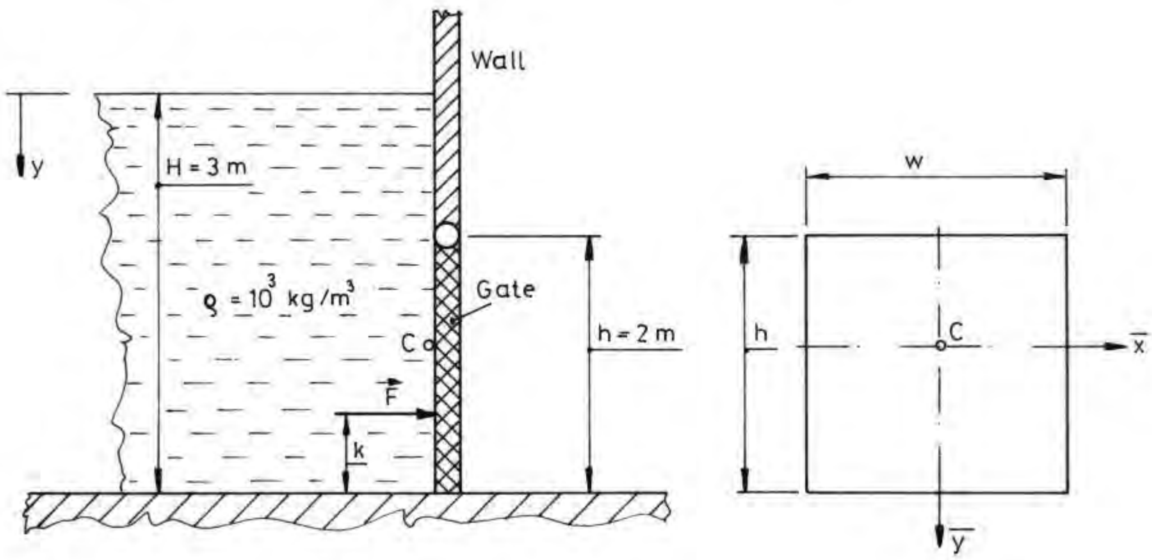
1.7 Find the gauge difference Δh_{II} in the U-tube if the water level in the container is raised by ΔH = 1.5 m (see the Figure).



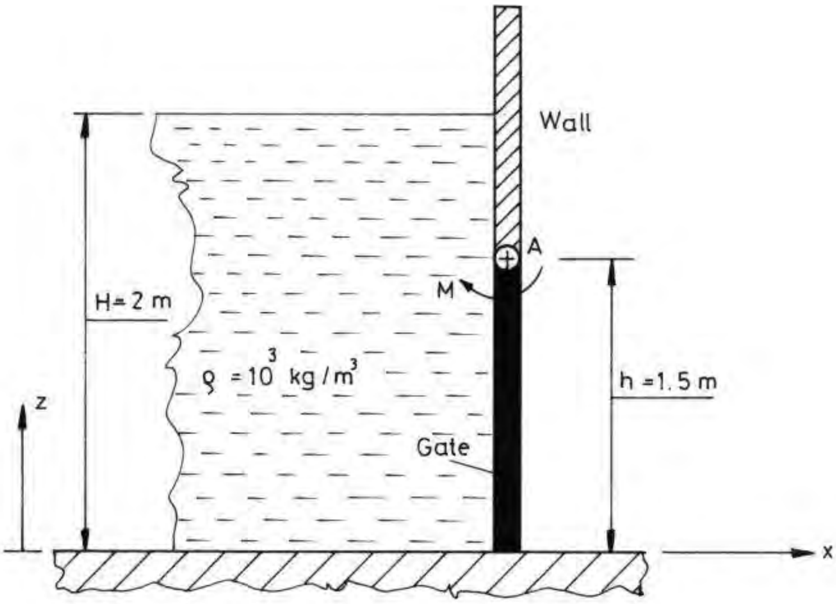
1.8 An aquarium at Marineland has a window as shown.
Find:
(*a*) The resultant force from seawater (ρ = 1015 kg/m³) on the window, F
(*b*) The line of action of the resultant force, in metre below the water surface, k.



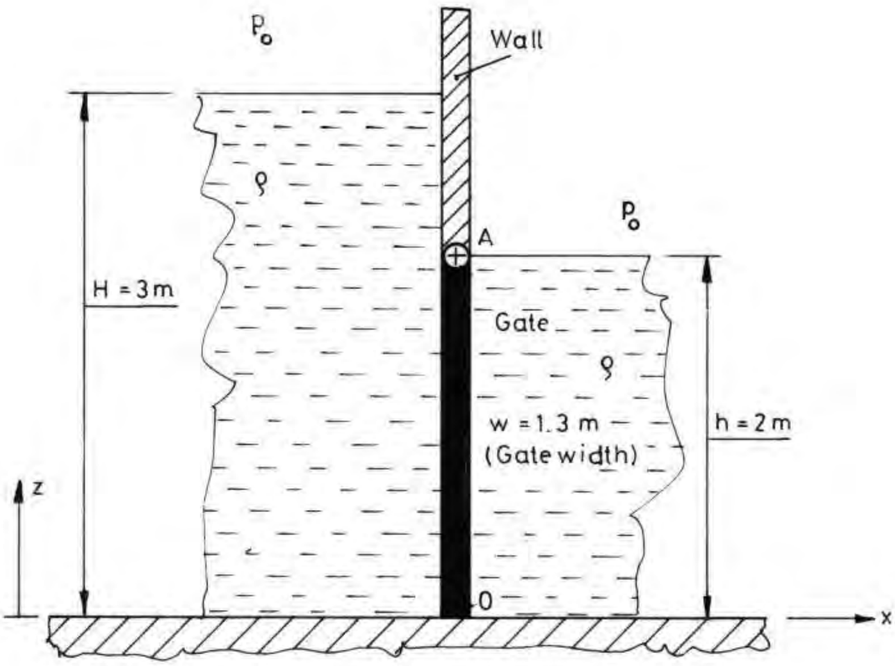
1.9 Locate the pressure centre for the gate in the Figure. The gate is 1.3 m wide. (w = 1.3 m)



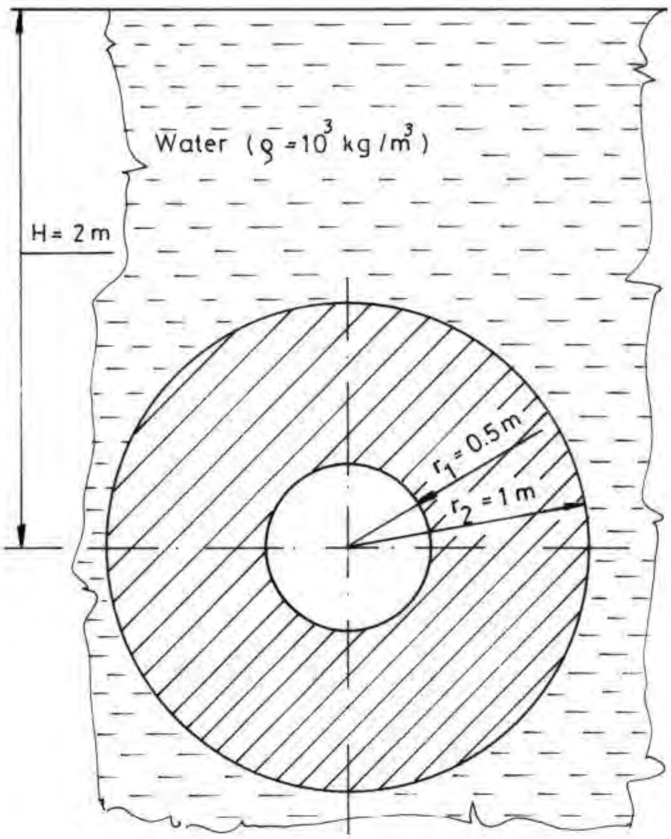
1.10 Determine the moment M at A required to hold the gate.
(the gate is 1.2 m wide; w = 1.2 m)



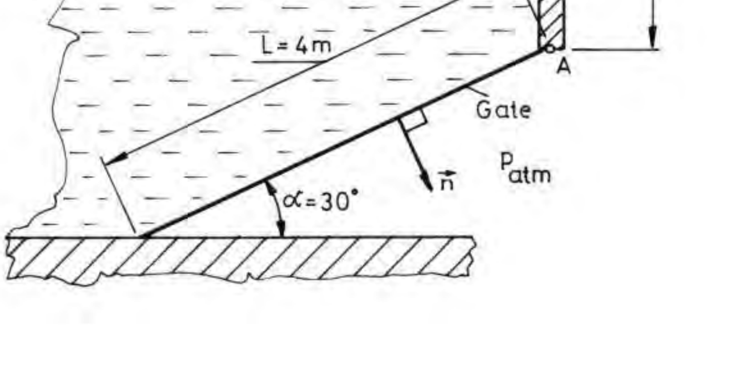
1.11 Find the resultant force due to water on both sides of the gate including its line of action.



1.12 Calculate the force exerted by water on one side of the vertical annular area shown in the Figure.



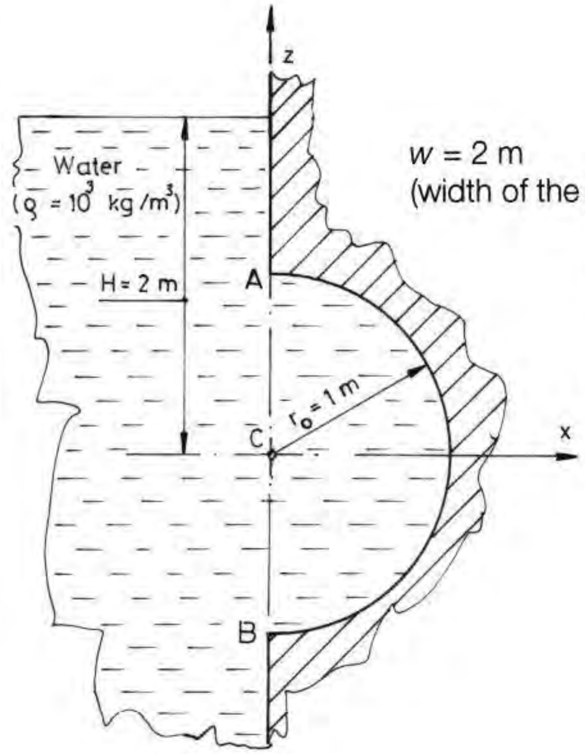
1.13



Given: rectangular gate, hinged along A, width $w = 5$ m.

Find: resultant force, $\vec{F}$, of the water on the gate, and its line of action.

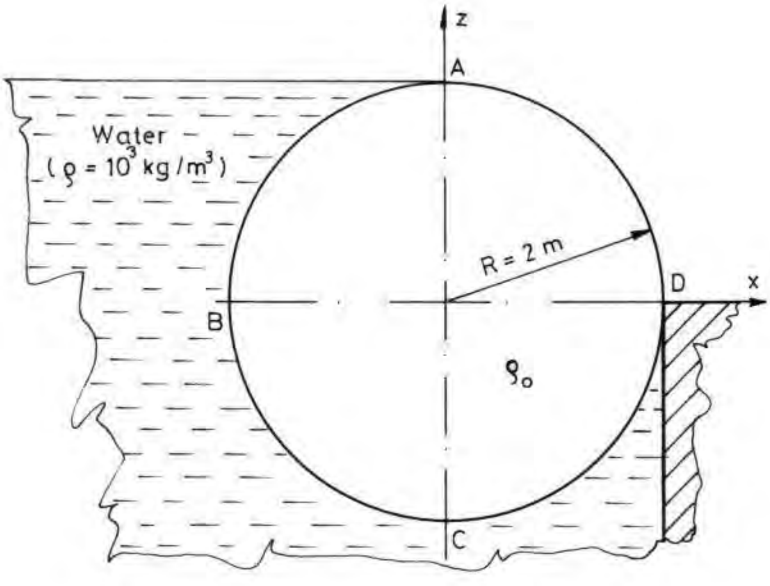
1.14



$w = 2$ m
(width of the pool)

Determine the resultant thrust exerted by the water on the cylindrical surface AB, and also the line of action of the thrust.

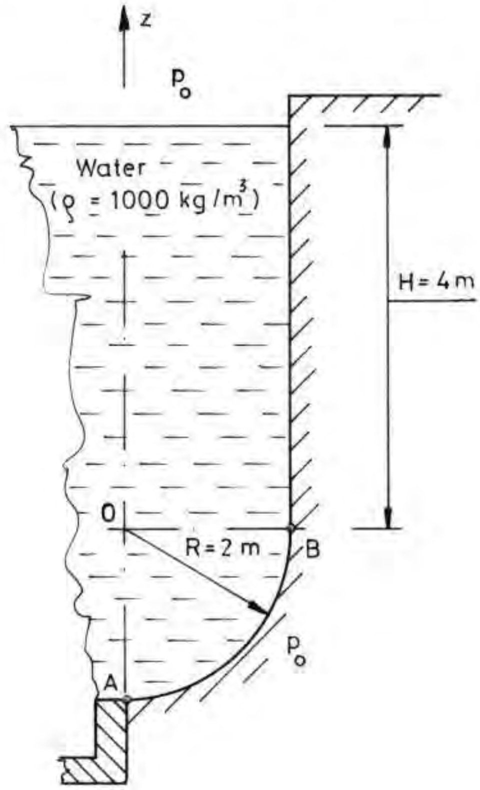
1.15



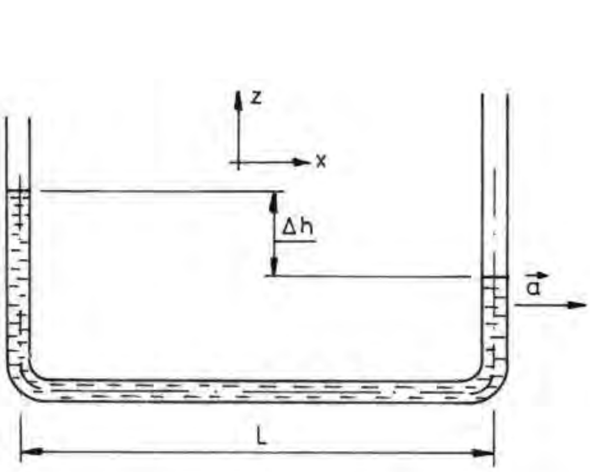
A cylindrical barrier (see the Figure) holds water as shown. The contact between cylinder and wall is smooth.

Determine
(a) the force per metre pushing the cylinder against the wall,
(b) the density of the cylinder (ρ_o)

1.16 Water is supported by the surface shown in the Figure. If atmospheric pressure prevails on the side of AB, determine the magnitude and line of action of the resultant hydrostatic force on AB per unit length.

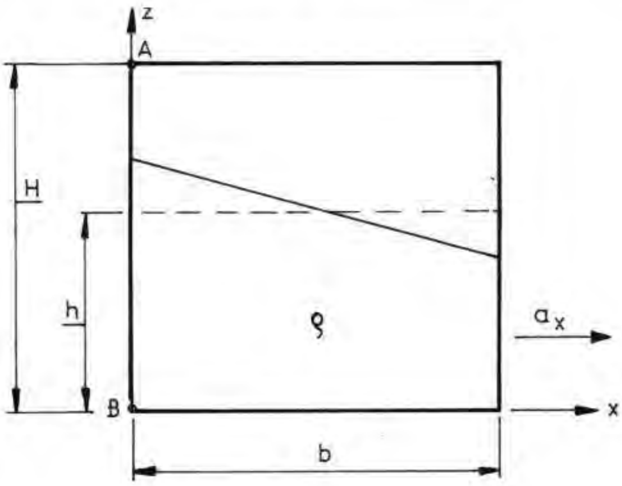


1.17 A crude accelometer can be made liqiud-filled U-tube as shown. Find the acceleration for L = 0.4 m and Δh = 0.2 m.



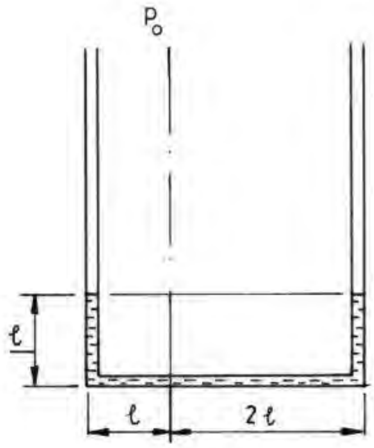
1.18 Tank partially filled with water (to a depth of h) subject to constant linear acceleration, a_x. Tank height is H; length parallel to direction of motion is b. Width perpendicular to direction of motion is c.
$H = 0.3$ m; $b = 0.3$ m; $c = 0.2$ m; $\rho = 10^3$ kg/m^3

Find: (*a*) Shape of the free surface
(*b*) Allowable water height, h_{max} to avoid spilling as a function of a_x
(*c*) Water force on side A-B of the tank for $h = 0.1$ m and $a_x = g/2$

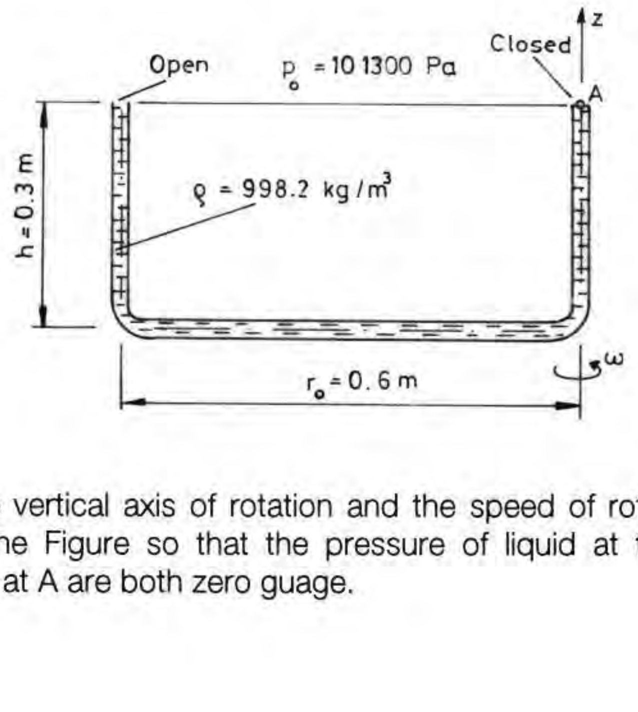


1.19 A cubical box 1.3 m on an edge is open at the top and filled with water. When it is accelerated upward 2.5 m/s^2, find the magnitude of water force on side of the box ($\rho = 1000$ kg/m^3).

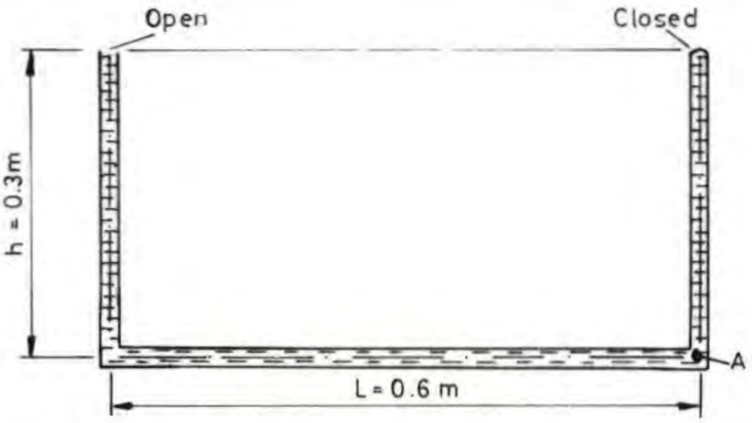
1.20 When the U-tube is not rotated, the water stands in the tube as shown. If the tube is rotated about the eccentric axis at a rate of ω = 8/s, what are the new levels of water in the tube? ℓ = 0.18 m; Neglect capillarity effects.



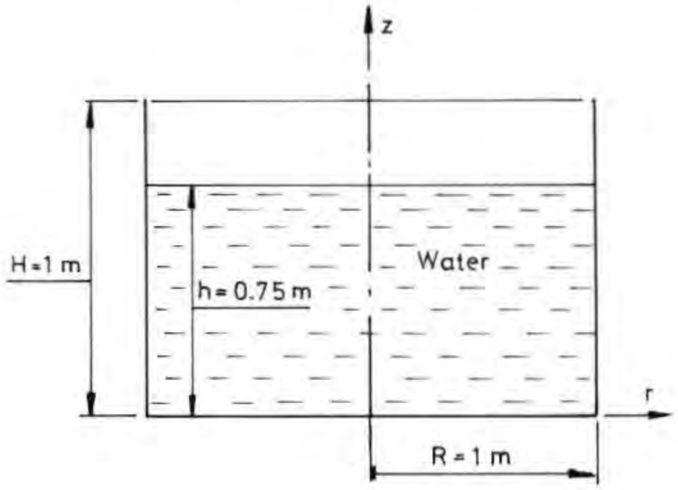
1.21 The U-tube is rotated about a vertical axis through A at such a speed that the water in the tube begins to vaporize at the closed end (A), which is at 20 °C (p_v = 2500 Pa).
What is the angular velocity?



1.22 Locate the vertical axis of rotation and the speed of rotation of the U-tube shown in the Figure so that the pressure of liquid at the midpoint of the U-tube and at A are both zero guage.



1.23 Water stands in this cylindrical tank as shown when no rotation occurs. If the system is rotated about the axis of symmetry of the tank, determine the allowable angular velocity, ω_1, to avoid spilling.



1.24 A cylindrical container of radius R = 0.5 m and height H = 1.5 m with axis vertical is open at the top and totally filled with water ($\rho = 10^3$ kg/m^3).

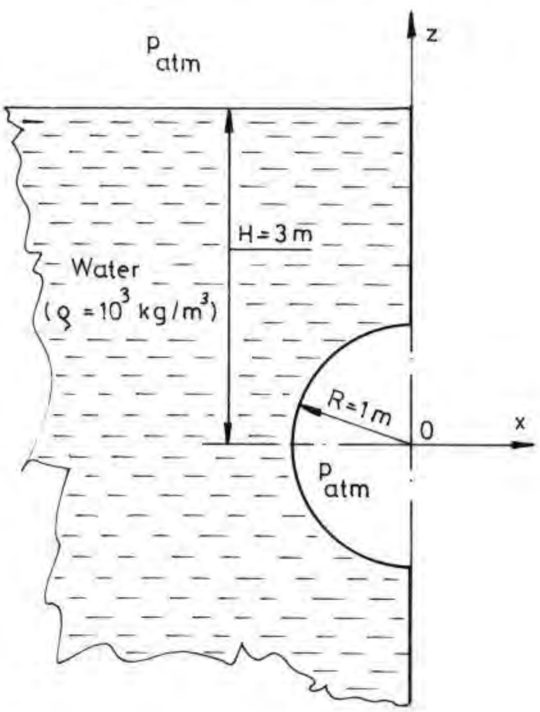
Determine

(*a*) The maximum rate at which the container can be rotated before the liqiud free surface just touches the bottom of the tank,

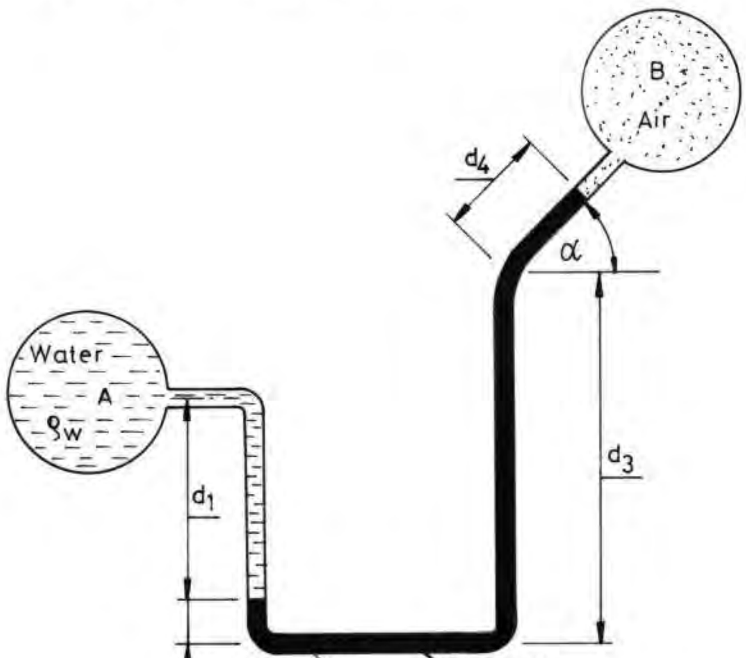
(*b*) The force acting on the bottom of the tank (neglect the weight of the tank).

1.25 A circular cylinder of radius r_o and height h_o with vertical axis is open at the top and filled with liquid. At what angular velocity must it rotate so that half the area of the bottom is exposed?
(h_o = 1 m, r_o = 0.5 m)

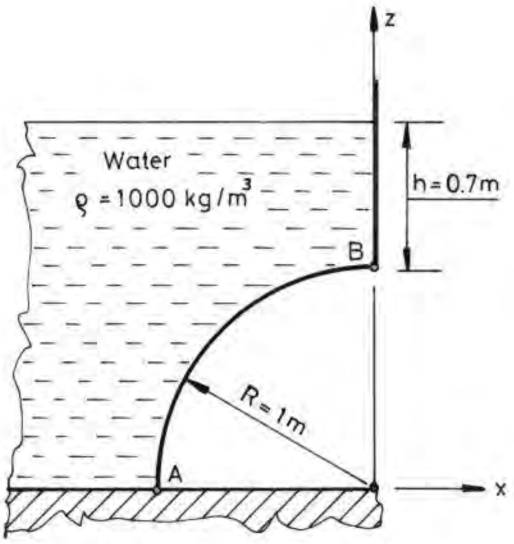
1.26 Determine the magnitude of the resultant force acting on the spherical surface and explain why the line of action goes through the centre O.



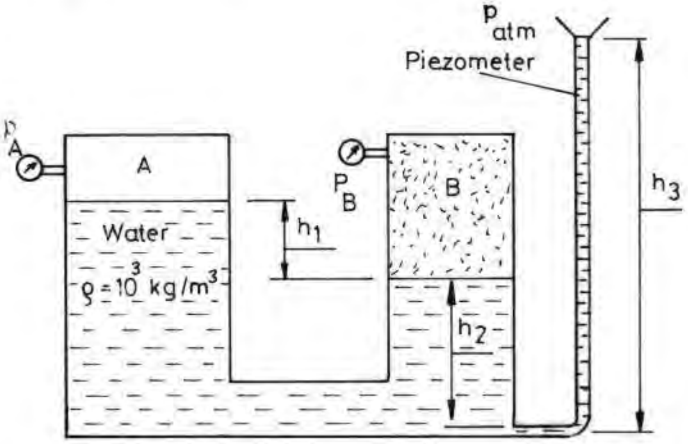
1.27 Find the difference in pressure between tanks A and B if
d_1 = 300 mm, d_2 = 1500 mm, d_3 = 460 mm, d_4 = 200 mm, α = 45°, $\rho_w = 10^3$ kg/m³, $\rho_m = 13.6 \times 10^3$ kg/m³.



1.28 What is the resultant force from the fluid acting on the door AB which is a quarter circle? The width of the door is w = 1.3 m. Give the elevation above the ground of the pressure centre.



1.29 An open tube is connected to a tank. The water rises to a height of h_3 = 900 mm in the tube. A tube used in this way is called a piezometer. What are the gauge pressures p_A and p_B of the air above the water. Neglect capillarity effects in the tube.
h_1 = 200 mm, h_2 = 400 mm.



2 KINEMATICS OF FLUIDS

2.1 Which of the following motions are kinematically possible for an incompressible fluid? (k is constant)

(a) $v_x = kx;\ v_y = -ky;\ v_z = 0$

(b) $v_x = kx;\ v_y = -ky;\ v_z = kz$

(b) $v_x = kx;\ v_y = ky;\ v_z = -2kz$

(c) $v_x = \dfrac{kx}{x^2+y^2};\ v_y = \dfrac{ky}{x^2+y^2};\ v_z = 0$

2.2 The velocity potential of a two-dimensional motion is $\Phi = kxy$. Find the streamlines.

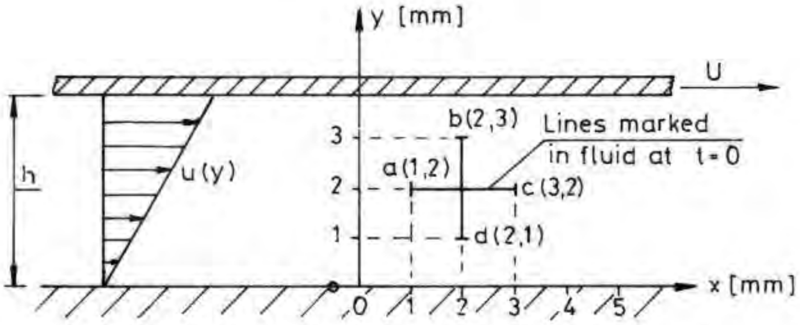
2.3 Find the streamlines when $v_x = ax;\ v_y = -ay;\ v_z = c$.

2.4 Parametric equation for the position of a particle in a flow field are given as

$$x_p = c_1 e^{at} \quad \text{and} \quad y_p = c_2 e^{-bt}.$$

Find the equation of the pathline for a location at $(x,y) = (1,2)$ at $t = 0$. Compare with a streamline through the same point.

2.5



A viscous flow in the narrow gap between parallel plates is shown. The velocity field in the narrow gap is given by $\vec{v} = U\, y / h \vec{i}$, where U = 4 mm/s and h = 4 mm. At t = 0 two lines, ac and bd, are marked in the fluid as shown. Evaluate the positions of the marked points at t = 1.5 sec and sketch for comparison. Calculate the rate of angular deformation and the rate of rotation of a fluid particle in this velocity field.

2.6 Consider the flow described by the velocity field

$$\vec{v} = x(1 + At)\vec{i} + y\,\vec{j}$$

with A = 0.5 sec^{-1}. For the point, (1,1,0), calculate

(*a*) the streamline through the point at $t = 0$, and
(*b*) the pathline traced out by the particle that passes through the point at this instant.

2.7 A three-dimensional flow is described by the velocity field

$$\vec{v} = v_o\left[x^2\vec{i} + y\vec{j} + (z + 1)\vec{k}\right]$$

where v_o is a constant.

Determine:
(*a*) the derivative tensor of $\vec{v}$,
(*b*) the acceleration
(*c*) the velocity potential (provided it exists)

2.8 Consider the flow described by the velocity field

$$\vec{v} = -\alpha y\vec{i} + \beta x\vec{j}$$

where α and β are positive constants. Calculate

(*a*) the streamlines,
(*b*) the acceleration field,
(*c*) the rate of rotation,
(*d*) the rate of volume dilation.
(*e*) Determine the relation between the parameters α and β for the case when the fluid elements do not undergo angular deformation.

2.9 The velocity field of a fully developed laminar flow in a pipe is described as

$$v_x = v_y = 0$$
$$v_z = A\left[R^2 - \left(x^2 + y^2\right)\right] \qquad \left(where\ \ x^2 + y^2 \le R^2\right)$$

with A = 5 x 10^4 s / m and R = 0.02 m.

Determine:
(*a*) the symmetric part of the derivative tensor of $\vec{v}$,
(*b*) the acceleration field,
(c) rate of linear and angular deformation,
(*d*) rate of rotation,
(e) velocity potential (if exists).

2.10 A two-dimensional flow is described by the velocity field

$$\vec{v} = -\omega y\vec{i} + \omega x\vec{j}$$

with ω = 10 s^{-1}. Calculate

(*a*) the sheamlines,
(*b*) the acceleration field,
(*c*) the vorticity vector,
(*d*) the velocity potential (provided it exists),
(*e*) Determine the motion of a fluid element.
(*f*) Is the flow steady or unsteady?
(*g*) Is this motion kinematically possible for an incompressible fluid?

2.11 Given the velocity field

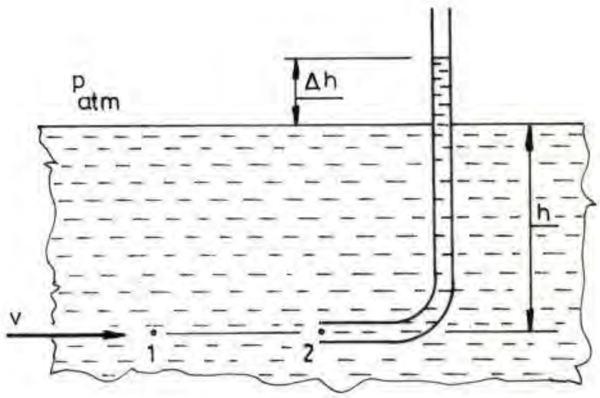
$$\vec{v} = 10\vec{i} + \left(x^2 + y^2\right)\vec{j} - 2xy\vec{k} \quad [\mathrm{m/s}]$$

What is the acceleration of a particle at position (3,1,0) m?

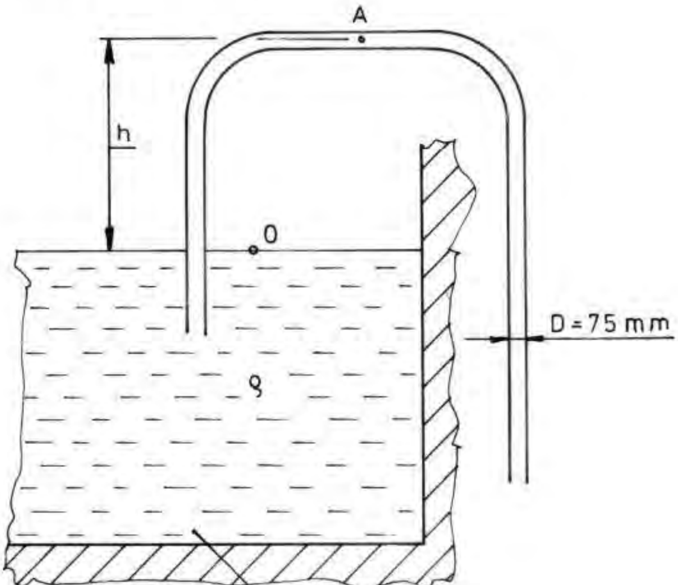
2.12 Given the velocity field

$$\vec{v} = 10x^2y\vec{i} + 20(yz + x)\vec{j} + 13\vec{k} \quad [\mathrm{m/s}]$$

What is the strain rate tensor (symmetric part of derivative tensor $\underline{\underline{D}}$) at P(6,1,2) m?



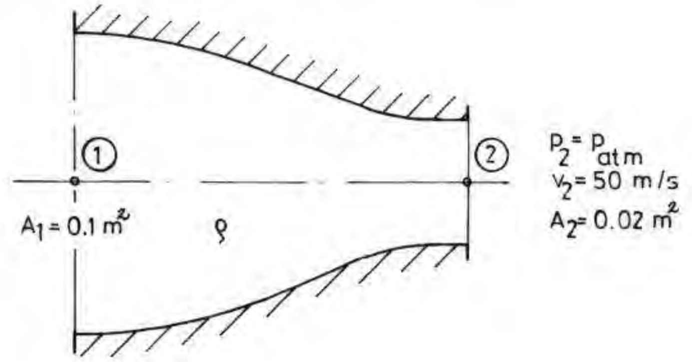
3.4 Water may be considered to flow without friction through the siphon. The water flow rate is 0.03 m^3/sec, its temperature is 20 °C, and the pipe diameter is 75 mm. Compute the maximum allowable height *h*, so that the pressure at point A is above the vapour pressure of the water. ($p_v = 2.5$kPa; $\rho = 998.2\ \mathrm{kg/m^3}$; $p_{atm} = 100.5\ \mathrm{kPa}$)



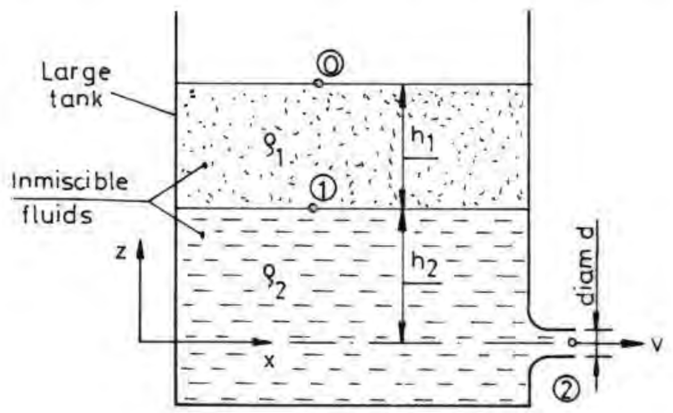
3.5 A smoothly contoured nozzle is connected to the end of a garden nose. At the nozzle inlet where the velocity is negligible, the water pressure is 160 kPa (gauge). Pressure at the nozzle exit is atmospheric. Assuming that the water remains in a single stream that has negligible aerodynamic drag, estimate the maximum height above the nozzle outlet that the stream could reach.

3 APPLICATON OF BERNOULLI'S EQUATION

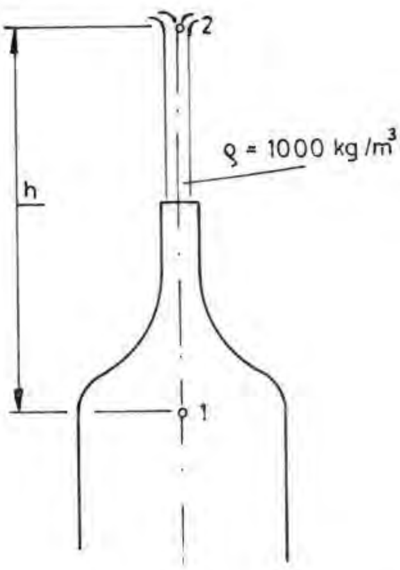
3.1 Air flows steadily and at low speed through a horizontal nozzle, discharging to the atmosphere. At the nozzle inlet, the area is 0.1 m^2. At the nozzle exit the area is 0.02 m^2. The flow is essentially incompressible, and frictional effects are negligible. Determine the gauge pressure required at the nozzle inlet to produce an outlet speed of 50 m/s. (ρ=1.23 kg/m^3)



3.2 Neglecting losses determine the discharge Q in the Figure.
(ρ_1 =750 kg/m^3; ρ_2 =1000 kg/m^3; h_1=1 m; h_2=1.5 m; d=0.1 m)

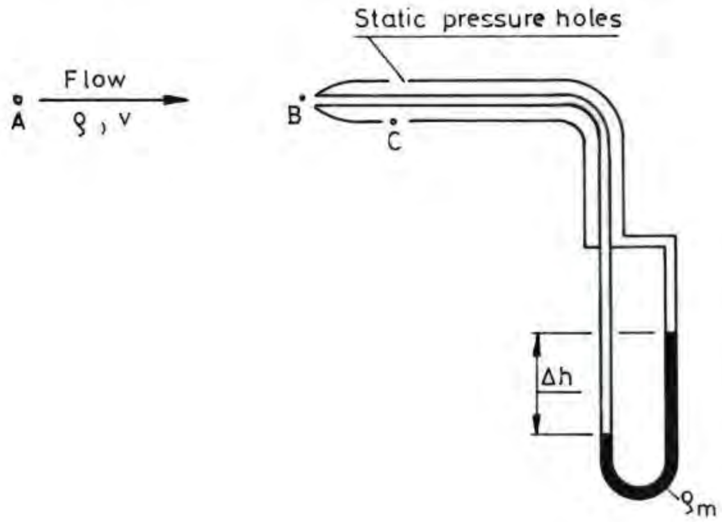


3.3 A glass tube with a 90° bend is open at both ends. It is inserted into a flowing stream of oil so that one opening is directed upstream and the other is directed upward. Oil inside the tube is 50 mm higher than the surface of the flowing oil. Determine the velocity measured by the tube.



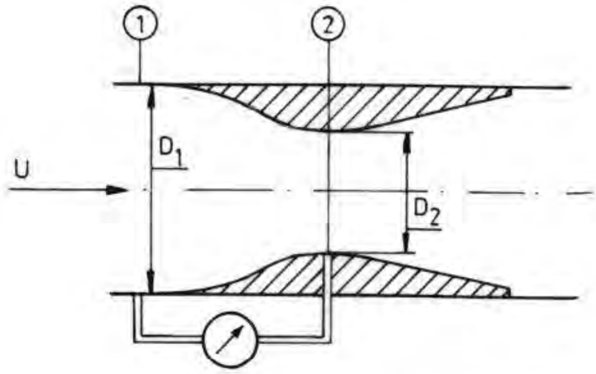
3.6 Two probes are often combined, as in the pitot-static tube in the Figure. The inner tube is used to measure the the stagnation pressure at point B while the static pressure at C is sensed by the small holes in the outer tube. In flow fields where the static pressure variation in the streamwise direction is small, the pitot-static tube may be used to infer the velocity at point A in the flow, by assuring $p_A = p_C$. (Note that when $p_A \neq p_C$, this procedure will give erroneous results).

($\rho = 1000\ \mathrm{kg/m^3}$; $\rho_m = 13600\ \mathrm{kg/m^3}$; $\Delta h = 300\mathrm{mm}$)

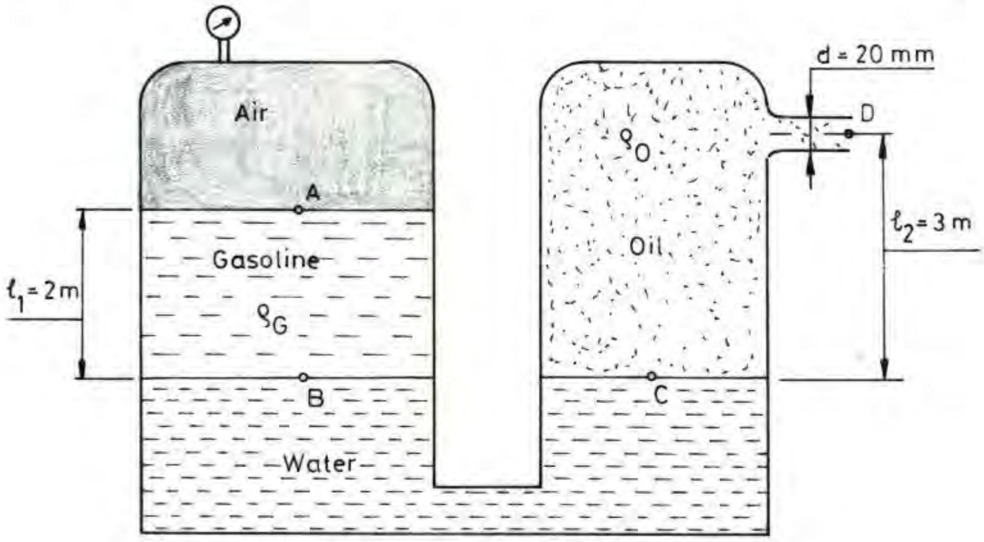


3.7 Water flows in a circular pipe. At one section the diameter is 0.3 m, the static pressure is 260 kPa (gauge), the velocity is 3 m/s, and the elevation is 10 m above ground level. The elevation at a section downstream is 0 m, and the pipe diameter is 0.15 m. Find the gauge pressure at the downstream section if frictional effects may be neglected.

3.8 An airspeed indicator used frequently on World War II aircraft consisted of a converging-diverging duct as shown in the Figure. The diameter of the duct at **2** is three-quarters of the entrance diameter at **1**. The differential pressure gauge records a pressure of 4000 Pa, and the density of the air is 1 kg/m^3. The air flow is steady, incompressible, inviscid and irrotational. Determine the airspeed *U*.



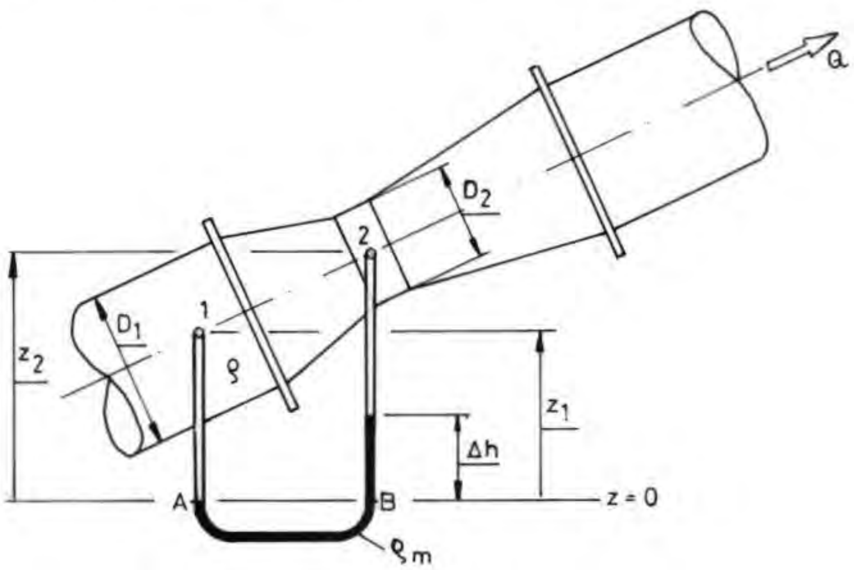
3.9 A large tank contains air, gasoline ($\rho_G = 680\ \mathrm{kg/m^3}$), light oil ($\rho_o = 800\ \mathrm{kg/m^3}$) and water. The pressure *p* of the air is 150 kPa gauge. If we neglect friction, what is the mass flow $\dot{m}$ of oil from a 20-mm-diameter jet?



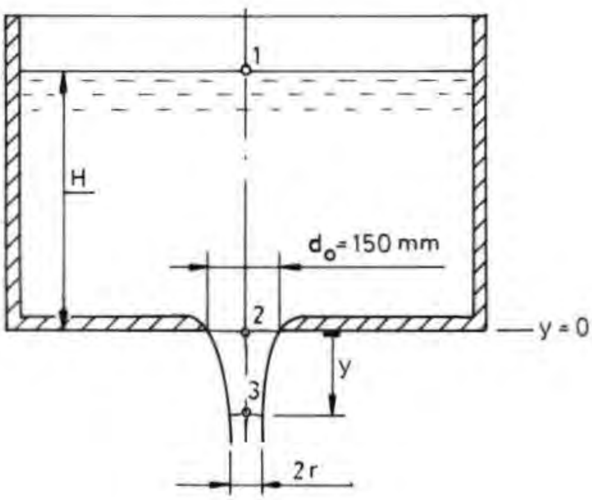
3.10 For the venturi meter and manometer installation shown in the Figure determine the volume rate of flow for the manometer reading Δh.

Data: $D_1 = 200\text{mm}$; $D_2 = 150\text{mm}$; $z_1 = 1\text{m}$; $z_2 = 1.3\text{m}$; $\Delta h = 0.2\text{m}$;

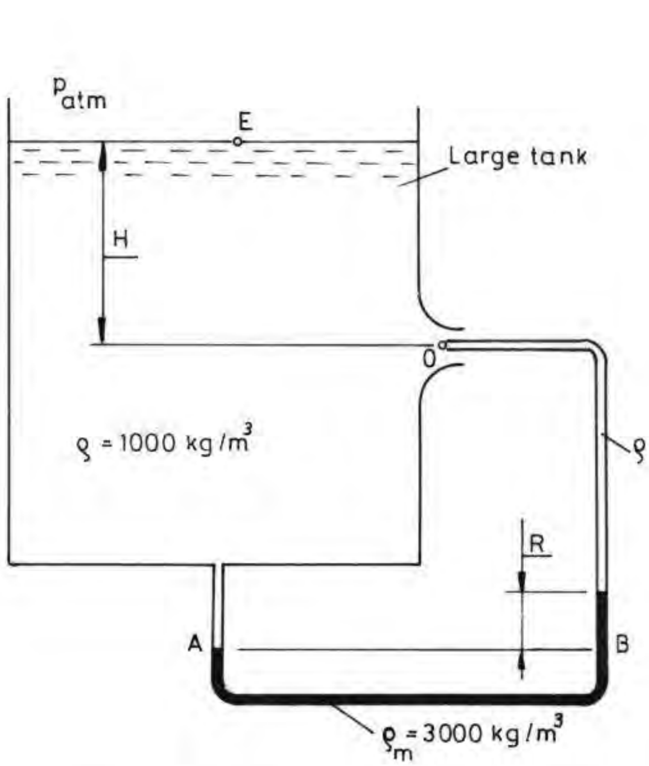
$\rho = 1000\ \text{kg/m}^3$; $\rho_m = 13600\ \text{kg/m}^3$



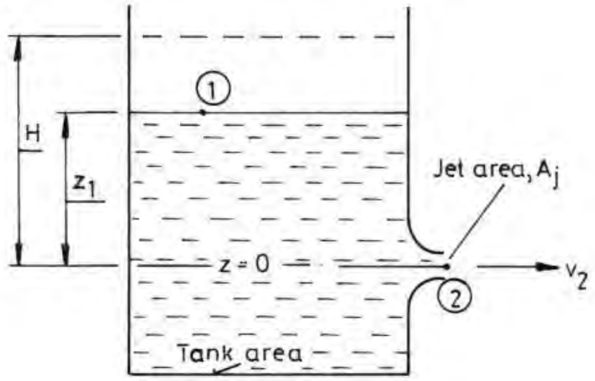
3.11 Neglecting losses and surface tension effects, derive an equation for the water surface r of the jet of the Figure in terms of y/H.



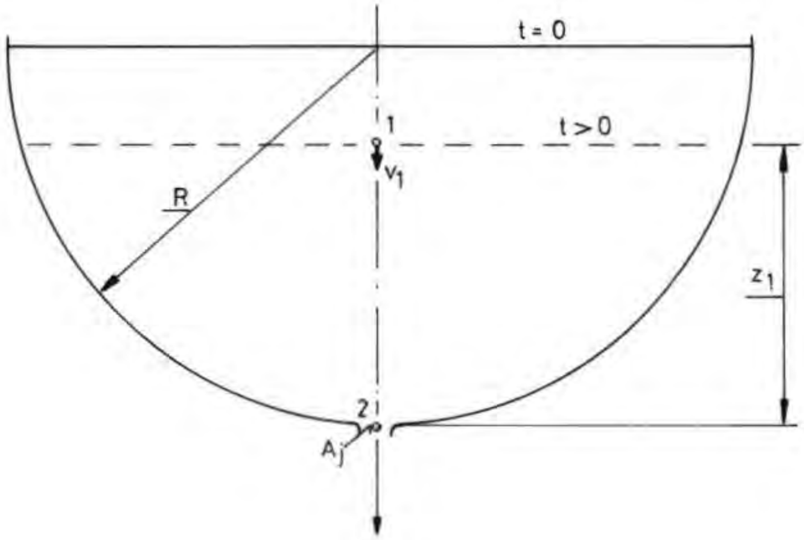
3.12 Neglecting losses, calculate R in terms of H for the Figure.



3.13 The tank shown in the Figure has a well-rounded orifice with area $A_j = 7 cm^2$. At the time t=0, the water level is at height H=2 m. Develop an expression for the water height, z_1, at any later time, t. The cross-sectional area of the tank is $A_t = 0.7\ m^2$. You can neglect frictional effects, and the quasi-steady form of the Bernoulli's equation can be used.

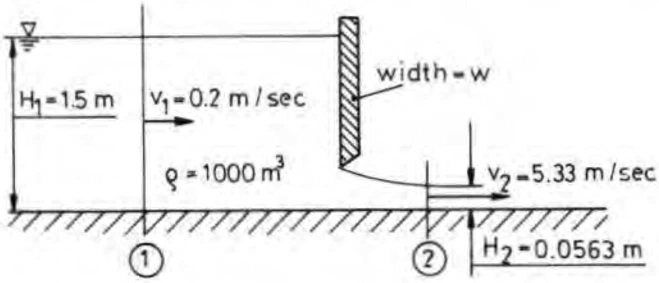


3.14 The tank with hemisphere shape has a well rounded orifice with area $A_j = 0.01\ m^2$. At time t=0 the water level is at height R=2 m. Develop an expression for the water height, z_1 , at any later time, t. Determine time T_1 belonging to $z_1 = R/2$. You can neglect the unsteady term in the Bernoulli's equation.

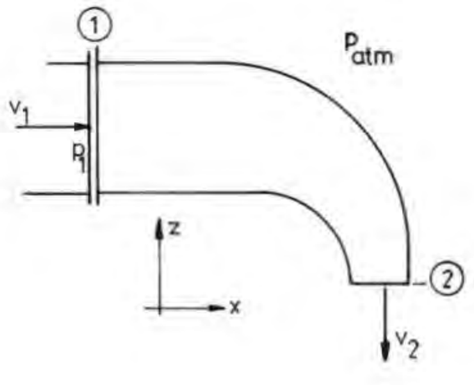


4 MOMENTUM EQUATION

4.1 Water in an open channel flows under a sluice gate as shown in the sketch. The flow is incompressible and uniform at section ① and ②. Hydrostatic pressure distributions may be assumed at sections ① and ② because the flow streamlines are essentially straight there. Determine the madnitude and direction of the force (per unit width) exerted on the gate by the flow.

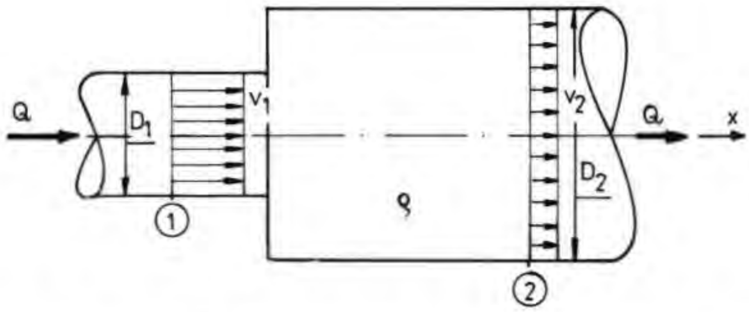


4.2 Water flows steadily through the reducing elbow in the diagram. At the inlet to the elbow the absolute pressure is 221 kPa and the cross-sectional area is 0.01 m². At the outlet the cross-sectional area is 0.0025 m² and the velocity is 16 m/s. The pressure at the outlet is atmospheric (p_{atm} = 101 kPa). Determine the force required to hold the elbow in place. (You can neglect the weight of the elbow and that of the water in it).



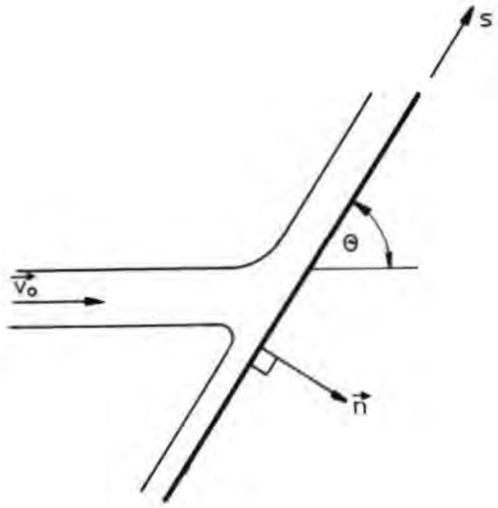
4.3 Calculate the pressure loss at abrupt enlargement of the cross-section (Borda-Carnot head loss) shown in the Figure.

Data: D_1 = 100 mm, D_2 = 200 mm
v_1 = 5 m/s, ρ = 1000 kg/m³

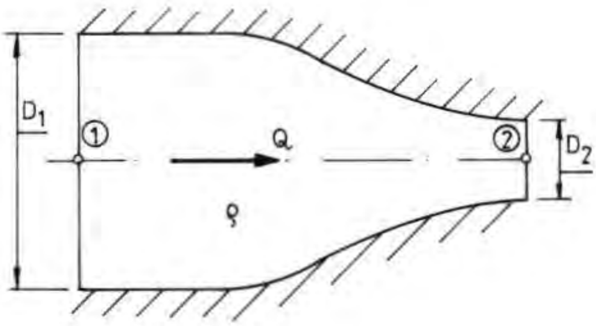


4.4 Fluid issues from a long slot and strikes against a smooth inclined flat plate (see the Figure). Determine the division of flow (Q_1 and Q_2) and the force R exerted on the plate, neglecting loss due to impact.

Data: $\Theta = 60°$, $v_o = 10$ m/s, $Q_o = 0.1$ m³/s, $\rho = 1000$ kg/m³

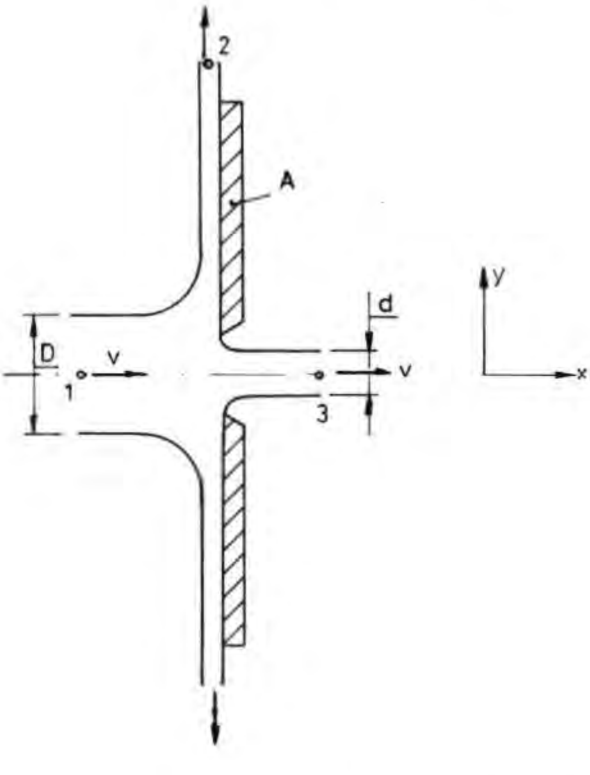


4.5



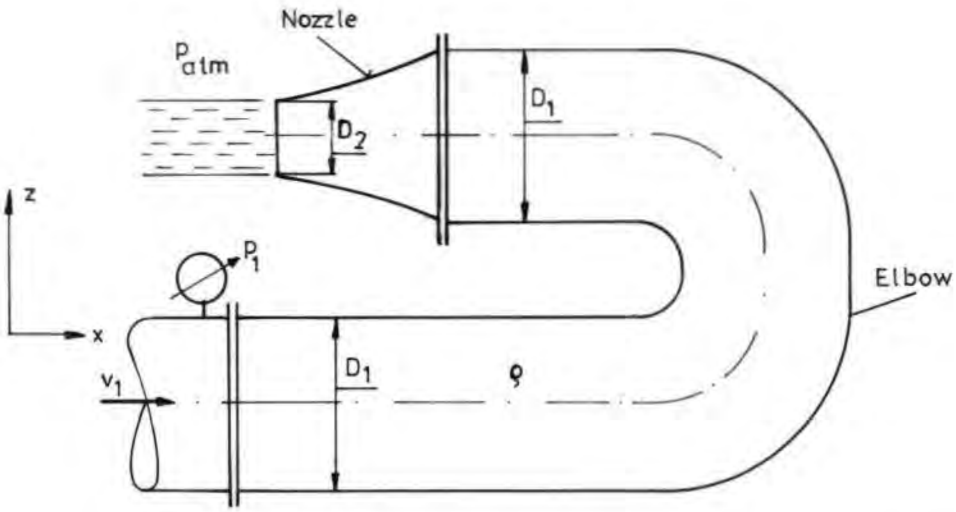
Water (ρ = 1000 kg/m³) flows steadily through a horizontal nozzle, discharging to the atmosphere. At the nozzle inlet the diameter is D_1, at the nozzle outlet the diameter is D_2. Derive an expression for the minimum gauge pressure required at the nozzle inlet to produce a given flow rate, Q. Evaluate the inlet gauge pressure if D_1 = 75 mm, D_2 = 25 mm, and the desired flow rate is *0.01 m³/s*. Determine the force required to hold the nozzle in place.

4.6 Plate *A* is 500 mm in diameter and has a sharp-edged orifice at its centre. A water jet strikes the plate concentrically with a speed of 30 m/s. With the plate held stationary, what external force is needed to hold the plate in place if the jet issuing from the orifice also has a speed of 30 m/s? The diameters of the jets are D = 100 mm and d = 40 mm (ρ = 1000 kg/m³).

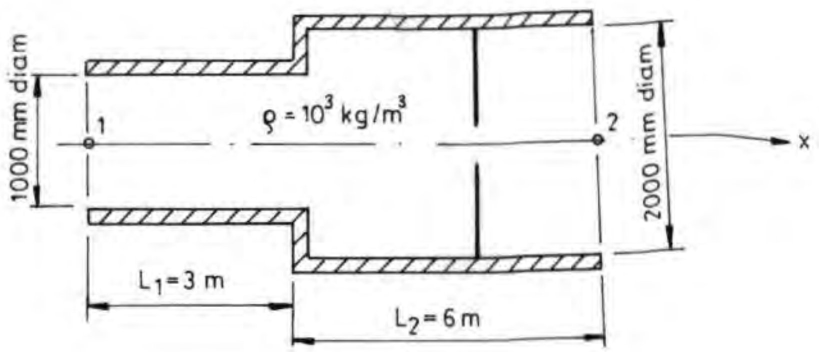


4.7 What is the force on the elbow-nozzle assembly from the water? The water issues out as a free jet from the nozzle. The interior volume of the nozzle elbow assembly is 0.1 m³. Give answer in terms of p_{atm}. (the effect of atmosphere should be taken into consideration)

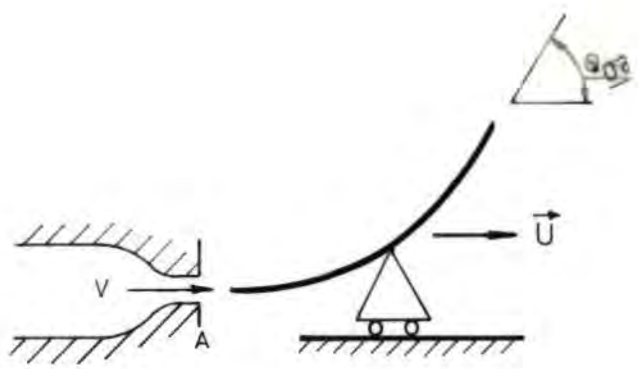
Data: V_1 = 1.5 m/s
D_1 = 0.3 m
D_2 = 0.15 m
P_1 = 100 kPa (gauge)
ρ = 10³ kg/m³



4.8 Water fills the piping system of the Figure. At one instant p_1 = 70 kPa, p_2 = 0 kPa, v_1 = 3 m/s and the flow rate is increasing by $\dot{Q} = 0.0032 \ \ m^3/s^2$. Find the force F_x required to hold the piping system stationary at the mentioned istant.

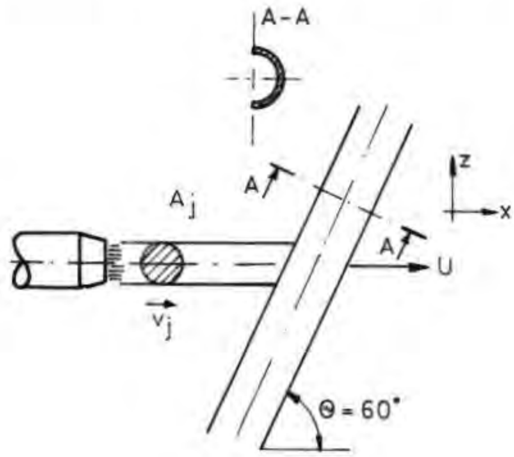


4.9

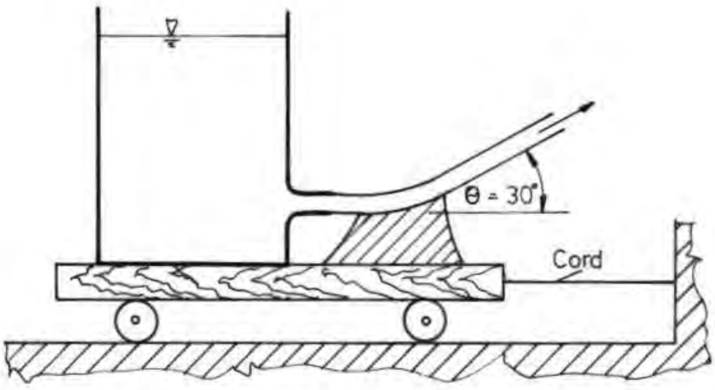


The sketch shows a vane with turning angle of 60°. The vane moves at constant speed, U = 6 m/sec, and receives a jet of water (ρ = 1000 kg/m^3) that leaves a stationery nozzle with speed, V = 25 m/sec. The nozzle has an exit area of 0.002 m^2. Determine the force that must be applied to maintain the vane speed constant.

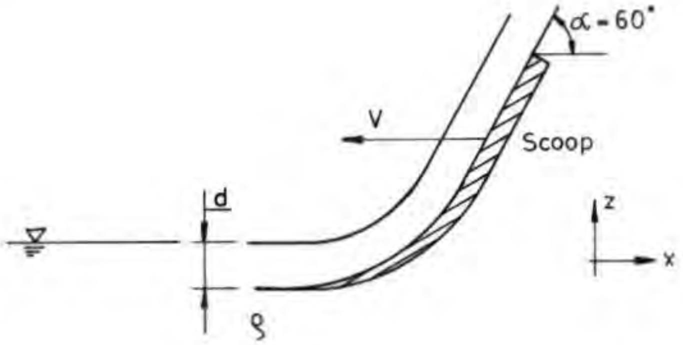
4.10 A jet of water of area A_j = 1290 mm^2 and speed v_j of 18 m/s impinges on a trough which is moving at a speed u of 3 m/s. If the water divides so that two-thirds goes up and one-third goes down, what is the force on the trough?



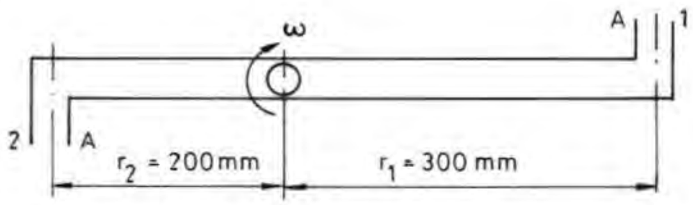
4.11 Water issues from a large tank through a 1300 mm^2 nozzle at a velocity of 3 m/s relative to the cart to which the tank is attached. The jet then strikes a trough (long open vessel) which turns the direction of flow by an angle of 30°, as is shown in the Figure. Assuming steady flow, determine the thrust on the cart which is held stationary relative to the ground by the cord. (You can neglect friction between the ground and the cart wheels.)



4.12 Assume that the scoop shown which is 200 mm wide (w = 200 mm), is used as a braking device for studying decelerating effects such as those on space vehicles. If the scoop is attached to a 1000 kg sled that is initially haveling horizontally at the rate of 100 m/s, what will be the initial deceleration of the sled? The scoop dips into the water (ρ = 1000 kg/m^3) 80 mm (d = 80 mm).



4.13 The sprinkler of the Figure discharges 0.3 l/s water through each nozzle. Neglecting friction, find its speed of rotation. The area of each nozzle opening is 100 mm^2.



5 PLANE POTENTIAL FLOW

5.1 The 2-D stream function for a flow is

$$\Psi = 9 + 6x - 4y + 7xy$$

Find the velocity potential.

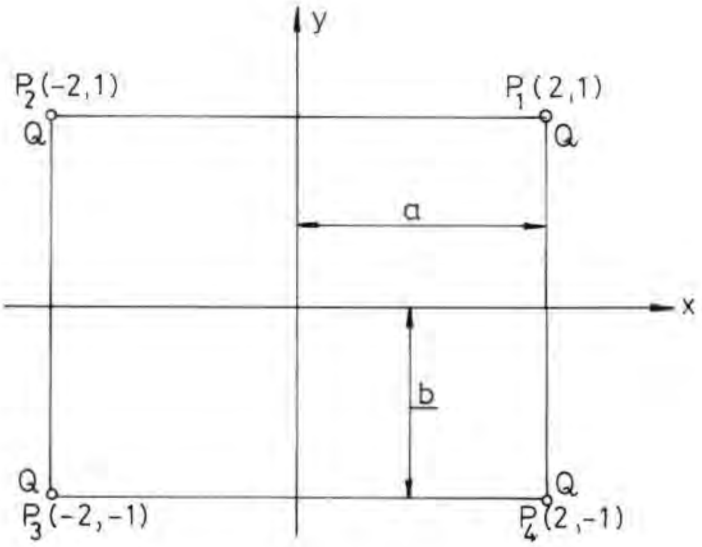
5.2 A velocity potential in two-dimensional flow is

$$\Phi = y + x^2 - y^2$$

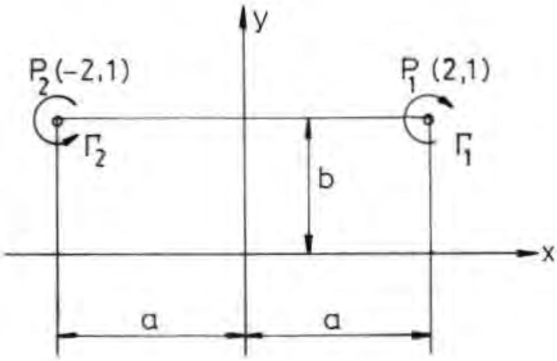
Find the stream function for this flow.

5.3 The x component of velocity is $v_x = x^2 + z^2 + 5$, and the y component is $v_y = y^2 + z^2$. Find the z component of velocity that satisfies continuity for incompressible fluid.

5.4 Four sources of equal strength Q = 1m³/s per m are located at points P_1, P_2, P_3 and P_4, as shown in the Figure. Find the velocity distributions along x and y axes.



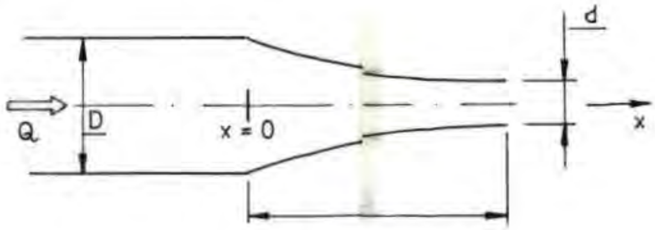
5.5 Two infinite straight vortexes of equal strength $|\Gamma| = \pi$ m^2/s are located at points P_1 and P_2, as shown in the Figure. Vortex at point P_1 has the clockwise direction of spin, while those at P_2 has the reversed direction of spin.
Find the velocity distribution along y axis.



5.6 In an infinite 2-D flow field a sink of strength -3 m³/s per metre is located at the origin and another of strength -4 m³/s per metre at (2 m, 0). What is the magnitude and direction of velocity at (0,2 m)? Where is the stagnation point?

5.7 A source with strength 0.2 m³/(sm) and a vortex with strength 1 m²/s are located at the origin. Determine the equations for velocity potential and stream function. What are the velocity components at x = 1 m, y = 0.5 m?

5.8 The nozzle in the Figure is shaped such that the velocity of flow varies linearly from the base of the nozzle to its tip. Assuming one-dimensional steady flow, what is the convective acceleration midway between the base and tip if diameters D an d are 0.9 m and 0.3 m, respectively, the nozzle length L is 5.4 m, and the discharge is 0.7 m³/s? Also, what is the local acceleration midway between the base and tip?



5.9 The x component of velocity of a 2-D incompressible flow is given by $v_x = Axy$, where A is a constant. What is a possible v_y component? What must the y component be if the flow is irrotational?

5.10 The nose of a solid strut 100 mm wide is to be placed in an infinite two-dimensional air stream of velocity 15 m/s and density 1.23 kg/m^3 and is to be made in the shape of a half body. Determine the strength of the corresponding source, the distance between the stagnation point and the source, the equation of the surface in rectangular co-ordinates based on the source as origin, and the difference in pressure between the stagnation point and the point on the strut where it is 50 mm wide.

5.11 To produce a Rankine oval of length 200 mm and breadth 100 mm in an otherwise uniform infinite two-dimensional stream of 3 m/s (parallel to the length) what strength and positions of source and sink are necessary? What is the maximum velocity outside the oval?

5.12 Consider a uniform flow in the x direction at speed v_∞ superposed on a source of strength Q and at $x = -a$ and a sink of strength $-Q$ at $x = +a$. The streamline along the x axis will split and form an oval shaped region which is called a Rankine oval. First, show that the streamlines that go through points A and B must be $\Psi = 0$. Then show that the width of the oval h satisfies the equation

$$\frac{h}{a} = 2\tan\left(\frac{\pi}{2} - \frac{\pi v_\infty h}{2Q}\right)$$

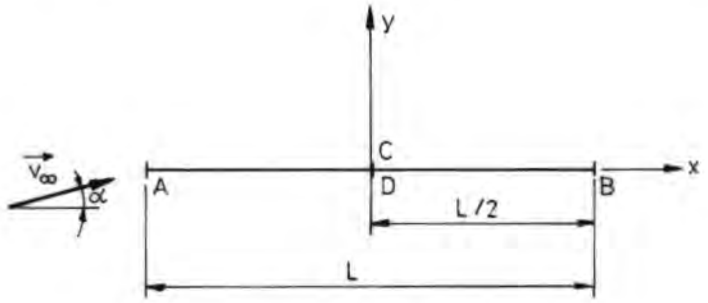
5.13 Select the strength of doublet needed to portray a uniform flow of 20 m/s around a cylinder of radius 2 m.

5.14 A circular cylinder 3 m in diameter rotates at 500 r.p.m. When in an airstream, $\rho = 1.2$ kg/m^3, movig at 120 m/s, what is the lift force per metre of cylinder, assuming 90 percent efficiency in developing circulation from the rotation.

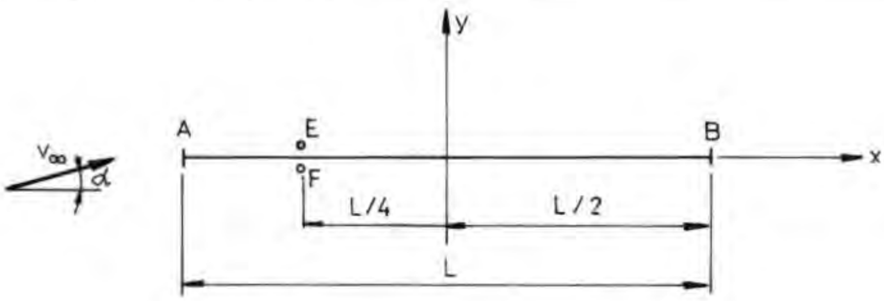
5.15 On a long circular cylinder of radius $R = 1$ m, with its axis perpendicular to an otherwise uniform, infinite, 2-D stream, the stagnation points are at $\varphi = 240°$ and $\varphi = 300°$. The pressure at infinity $p_\infty = 10^5$ Pa and at the stagnation points $p_s = 1.1 \times 10^5$ Pa.
Find
(*a*) the velocity distribution along the surface of the cylinder,
(*b*) the value and position of the maximum velocity,
(*c*) lift force per metre of cylinder ($\rho = 1000$ kg/m^3).

5.16 On a long circular cylinder with its axis perpendicular to an otherwise uniform, infinite, two-dimensional stream, the stagnation points are at $\varphi = 240°$ and $\varphi = 300°$. what is the value of the lift coefficient?

5.17 A flat plate of length L = 2 m is placed in a uniform airstream, ρ = 1.2 kg/m³, moving at v_∞ = 10 m/s. the angle of inclination of the airstream relative to the aerofoil is α = 10° (see the Figure).
Determine the velocity distribution along the upper and lower side of the plate by applying the Kutta-Joukowski condition at point B. Find the pressure difference $\Delta p = p_D - p_C$. Calculate the lift force per metre of plate.

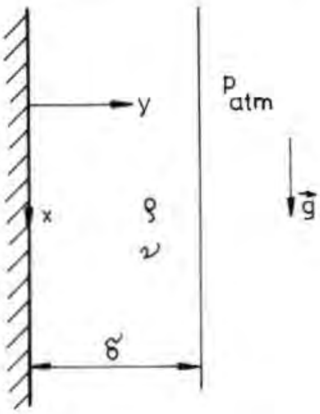


5.18 A flat plate of length L = 1.2 m is placed in a uniform airstream, ρ = 1.2 kg/m³ moving at v_∞ = 12 m/s. The angle of attack α = 15° (see the Figure). Find the difference in velocity and pressure between points E and F, i.e., $\Delta v = v_E - v_F$, $\Delta p = p_F - p_E$, respectively.



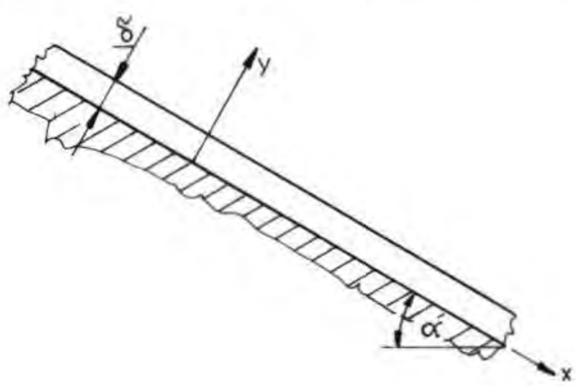
6 LAMINAR FLOW

6.1 Fully developed laminar flow of an incompressible, Newtonian liquid flows down a vertical wall. The thickness, δ, of the liquid film is constant. Since the liquid free surface is exposed to atmospheric pressure, there is no pressure gradient. Find the velocity and shear stress distribution, flow rate (considering width = w), average and maximum velocities, i.e., c and v_{max}, respectively.

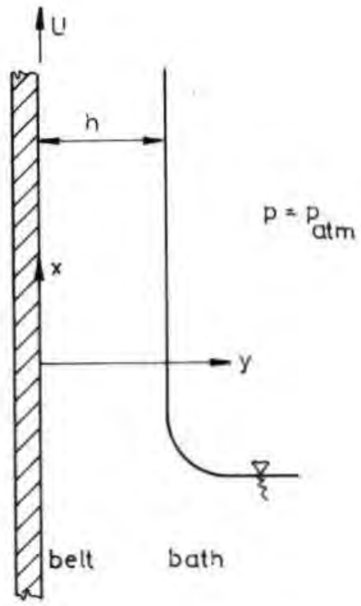


6.2 A viscous oil flows steadily between horizontal parallel plates. The flow is laminar and fully developed. The total gap width between the plates is h = 3 mm. The oil viscosity is η = 0.5 Ns/m^2, and the pressure gradient is -1200 N/m^2/m. Find the magnitude and direction of the shear stress on the upper plate, and the volume flow rate through the channel, per metre of width.

6.3 A film of water ($\rho = 10^3$ kg/m^3 ; $\nu = 10^{-6}$ m^2/s) in steady laminar motion runs down a long slope, inclined $\alpha = 30^\circ$ below the horizontal. The thickness of the film is δ = 0.8 mm. Assume that the flow is fully developed, and at zero pressure gradient. Determine the surface shear stress and the volume flow rate per unit width.

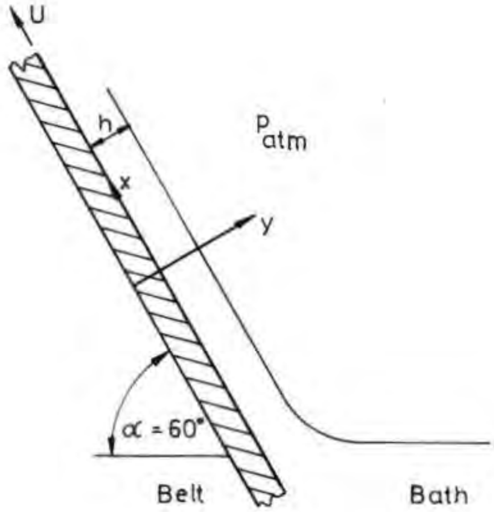


6.4 A continuous belt passing through a chemical bath at speed, U = 0.5 m/s, picks up a liquid film of thickness, h = 1 mm, density, ρ = 900 kg/m^3, and kinematic viscosity, $\upsilon = 10^{-3}$ m^2/s. Gravity tends to make the liquid drain down, but the movement of the belt keeps the liquid from running off completely. Assume that the fluid is fully developed laminar flow with zero pressure gradient, and that the atmosphere produces no shear stress at the outer surface of the film. Determine the velocity profile, the shear stress distribution and the average velocity.



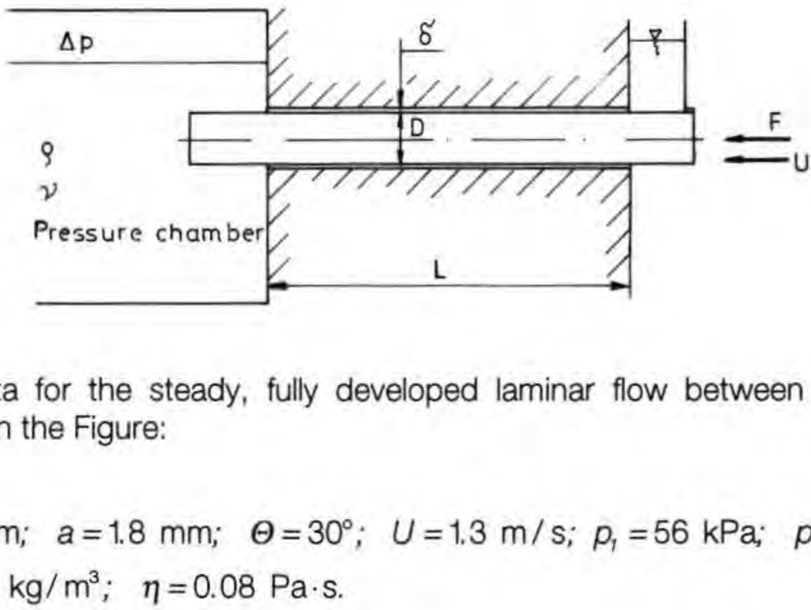
6.5 A crankshaft journal bearing in an car engine is lubricated by SAE 30 engine oil at 210F (η = 0.0096 Ns/m^2). The bearing is 76 mm in diameter, has a diametrical clearance of 0.064 mm and rotates at 3600 rev/min. It is 32 mm long. The bearing is under no load, so the clearance is symmetric. Determine the torque required to turn the journal, and the power dissipated.

6.6 A continuous belt passing through a chemical bath at speed, U = 0.5 m/s, picks up a liquid film of thickness, h = 0.8 mm, density, ρ = 850 kg/m^3, and kinematic viscosity, $\upsilon = 10^{-3}$ m^2/s. Gravity tends to make the liquid drain down, but the movement of the belt keeps the liquid from running off completely. Assume that the fluid is fully developed laminar flow with zero pressure gradient, and that the atmosphere produces no shear stress at the outer surface of the film. Determine the velocity profile, the shear stress distribution and the average velocity.



6.7 In the Figure $U = 0.7$ m/s. Find the rate at which oil is carried into the pressure chamber by the piston and the shear force and total force F acting on the piston.

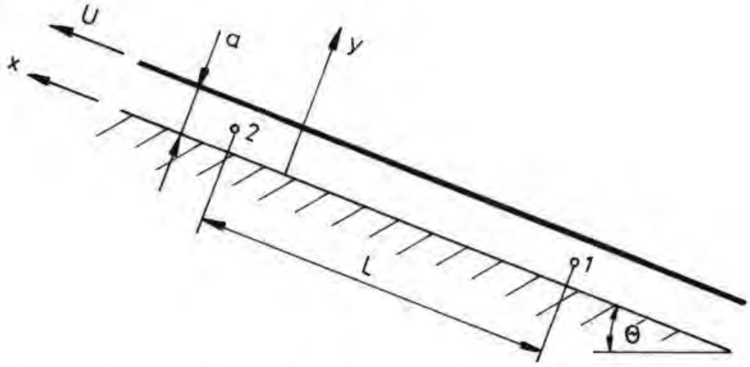
Data: $\rho = 900\ \mathrm{kg/m^3}$; $\upsilon = 0.0005\ \mathrm{m^2/s}$; $D = 50$ mm; $L = 150$ mm; $\delta = 0.05$ mm; $\Delta p = 0.15$ MPa (gauge)



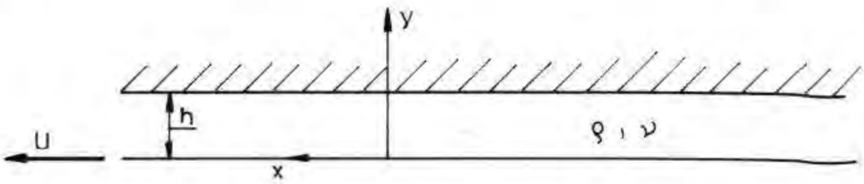
6.8 The data for the steady, fully developed laminar flow between two plates shown in the Figure:

$L = 1.3$ m; $a = 1.8$ mm; $\Theta = 30°$; $U = 1.3$ m/s; $p_1 = 56$ kPa; $p_2 = 42$ kPa; $\rho = 800\ \mathrm{kg/m^3}$; $\eta = 0.08$ Pa·s.

Determine the tangential force per square metre exerted on the upper plate and its direction.



6.9 Water at 60 °C ($\rho = 983\ \text{kg/m}^3$; $\upsilon = 4.5 \times 10^{-7}\ \text{m}^2/\text{s}$) flows between two large flat plates. The lower plate moves to the left at a speed of 0.3 m/s. The plate spacing is h = 3 mm, and the flow is fully developed, steady and laminar. Determine the pressure gradient required to produce zero net flow at a cross section.



6.10 A sealed journal bearing is formed from concentric cylinders. The inner and outer radii are 25 and 26 mm, the journal length is 100 mm, and it turns at 2800 rpm. The gap is filled with oil in laminar motion. The velocity profile is linear across the gap. The torque needed to turn the journal is 0.2 Nm. Calculate the viscosity of the oil. Will the torque increase or decrease with time? Why?

6.11 The basic component of a pressure gauge tester consists of a piston-cylinder apparatus as shown. The piston, 6 mm in diameter, is loaded to develop a pressure of known magnitude. (The piston length is 25 mm). Calculate the mass, M, required to produce 1.5 MPa (gauge) in the cylinder. Determine the leakage flow rate as a function of radial clearance, a, for this load if the density and kinematic viscosity of the fluid are ρ = 900 kg/m^3 and $\upsilon = 5 \times 10^{-4}$ m^2/s , respectively. Specify the maximum allowable radial clearance so the vertical movement of the piston due to leakage will be less than 1 mm/min.

6.12 A simple and accurate viscosimeter can be made from a length of capillary tubing. If the flow rate and pressure drop is measured, and the tube geometry is known, the viscosity can be computed. A test of a certain liquid in a capillary viscosimeter gave the following data:

Flow rate:	880 mm^3/sec
Tube diameter:	0.5 mm
Tube length:	1 m
Density:	999 kg/m^3
Pressure drop:	1.0 MPa

Determine the viscosity of the fluid.

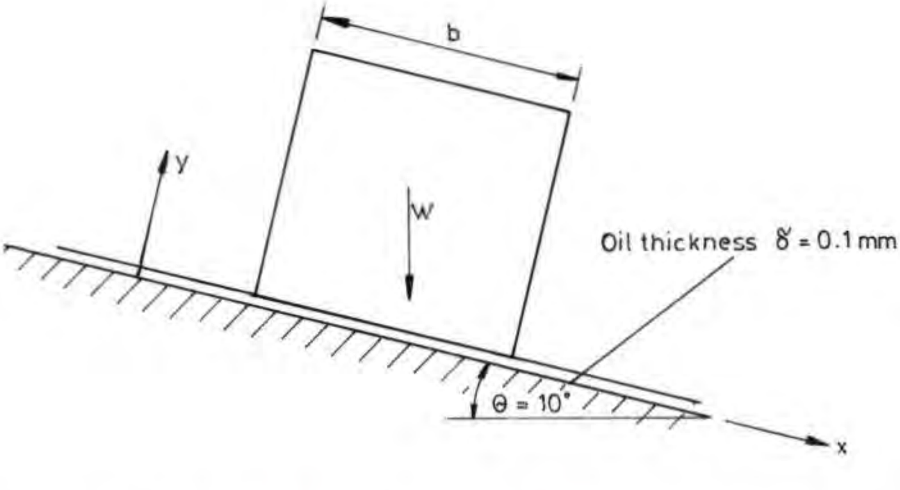
6.13 Consider fully developed laminar flow in the annulus between two concentric pipes. The inner pipe is stationary, and the outer pipe moves in the z direction with speed, *U*. Assume the axial pressure gradient to be zero $(\partial p/\partial z = 0)$. Obtain a general solution for the shear stress τ, as a function of the radius, *r*, in terms of a constant C_1. Obtain a general expression for the velocity profile, *v(r)* in terms of two constants, C_1 and C_2. Evaluate the constants, C_1 and C_2.

6.14 A water injection line is made from smooth capillary tubing with inside diameter, D = 0.25 mm. Determine the maximum volume flow rate at which flow is laminar. Evaluate the pressure drop required to produce this flow rate through a section of tubing with length, $L = 0.75$ m.

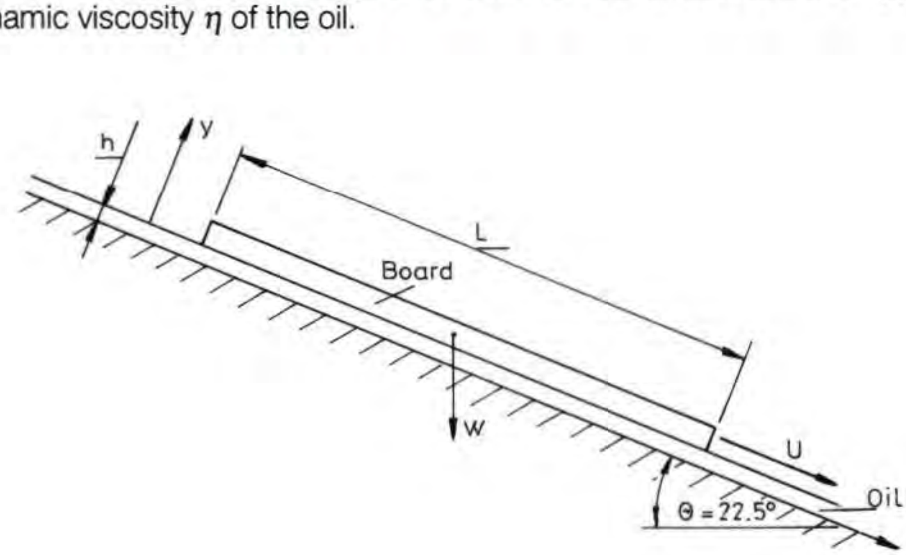
$\left(\rho = 1000\ \text{kg/m}^3;\ \upsilon = 10^{-6}\ \text{m}^2/\text{s}\right)$

6.15 A liquid drug with the viscosity and density of water (ρ = 1000 kg/m^3 and $\upsilon = 10^{-6}$ m^2/s) is to be administered through a hypodermic needle. The inside diameter of the needle is 0.25 mm and its length is 50 mm.
Determine
(*a*) the maximum volume flow rate for which the flow will be laminar,
(*b*) the pressure drop required to deliver the maximum flow rate,
(*c*) the corresponding wall shear stress.

6.16 A cube weighing 200 N and measuring 30 cm on a side is allowed to slide down an inclined surface on which there is a film of oil having a viscosity of 0.01 Ns/m^2. What is the terminal velocity of the block if the oil has a thickness of 0.1 mm?



6.17 A board 1 m by 1 m that weighs 220 N slides down an inclined ramp with a velocity of 0.15 m/s. The board is separated from the ramp by a layer of oil 0.5 mm thick. Neglecting the edge effects of the board, calculate the approximate dynamic viscosity η of the oil.



6.18 A circular horizontal disk with a 20 cm diameter has a clearance of 2 mm from a horizontal plate. What torque is required to rotate the disk at an angular velocity of 100 rad/s when the clearance space contains oil ($\eta = 6$ Ns/m^2)?

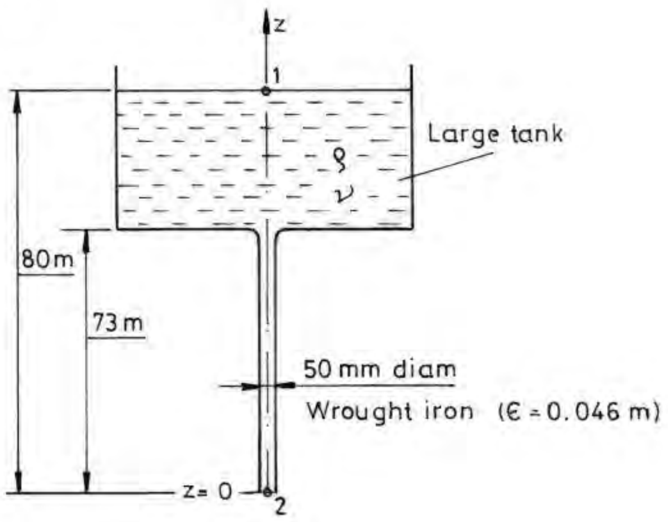
6.19 If a thin oil ($\upsilon = 0.001$ m^2/s) 3.5 mm thick flows down a surface inclined at 30° to the horizontal, what will be the maximum and mean velocity of flow?

6.20 Two vertical parallel plates are placed 2 mm apart. If the pressure decreases at a rate of 9 kPa/m in the positive z direction (vertically upward) in the fluid between the plates, what is the maximum fluid velocity in the z direction? The fluid has a viscosity of 0.1 Ns/m^2 and a density of 850 kg/m^3.

7 TURBULENT FLOW

7.1 60 l/s oil, η = 0.016 Pa·s, ρ = 866.46 kg/m³, is pumped through a 30 cm pipeline of cast iron (Rouhgness height ϵ = 0.25 mm). If each pump produces 560 kPa, how far apart may they be placed?

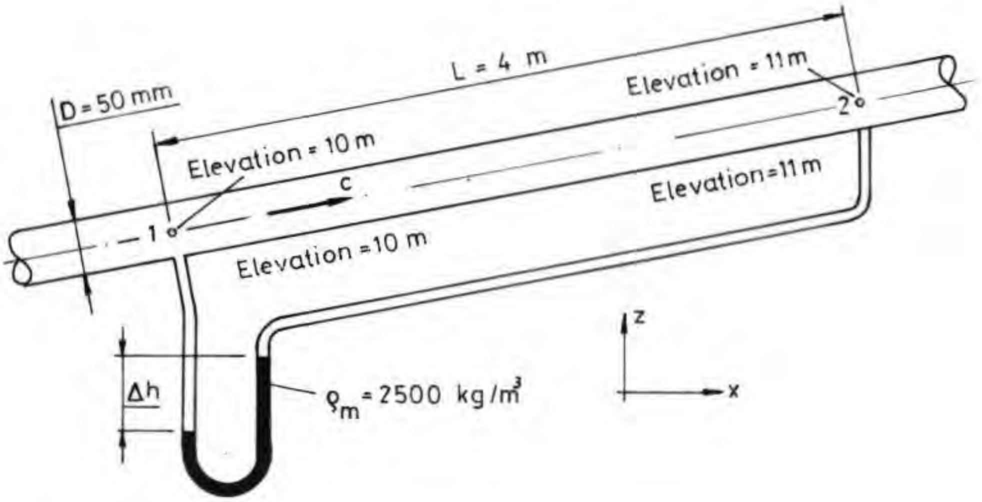
7.2 For water at 65 °C (ρ = 980.6 kg/m³; υ = 4.5 x 10^{-7} m²/s) calculate the discharge for the pipe of the Figure.



7.3 Water $\left(\rho = 998.2\ \text{kg/m}^3;\ \upsilon = 10^{-6}\ \text{m}^2/\text{s}\right)$ is to be pumped through a kilometre of 200-mm-diameter wrought-iron pipe $(\epsilon = 0.046\,\text{mm})$ at the rate of 60 l/s. Calculate the head loss and power required.

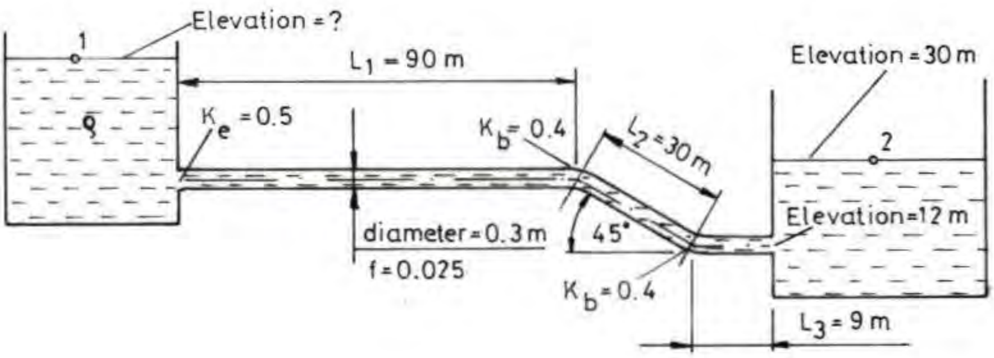
7.4 A racing car weighing 7110 N attains a speed of 386 km/h (v_o) in the first 400 m. Immediately after passing through the timing lights, the driver opens the drag chute. The chute area is 2.32 m² and it has a drag coefficient of 1.2. Air and rolling resistance of the car may be neglected. The local air density is 1.237 kg/m³. Find the time required for the machine to decelerate to 160 km/h (v_1).

7.5 Water ($\rho = 1000\ \text{kg/m}^3$) flows in the pipe shown and the manometer deflects $\Delta h = 800$ mm. What is f for the pipe if $c = 3$ m/s.



7.6 Air $\left(\eta = 1.81\times10^{-5}\ \mathrm{N\cdot s/m^2}\right)$ flows in a 3-cm smooth tube at a rate of 0.012 m^3/s. If $T = 20\ ^{\circ}C$ and $p = 110$ kPa absolute, what is the pressure drop per metre of length of tube?

7.7 Estimate the elevation required in the upper reservoir to produce a water $\left(\rho = 1000\ \mathrm{kg/m^3}\right)$ discharge of 0.283 m^3/s in the system. Where is the point of minimum pressure in the pipe, and what is the magnitude of pressure at that point?



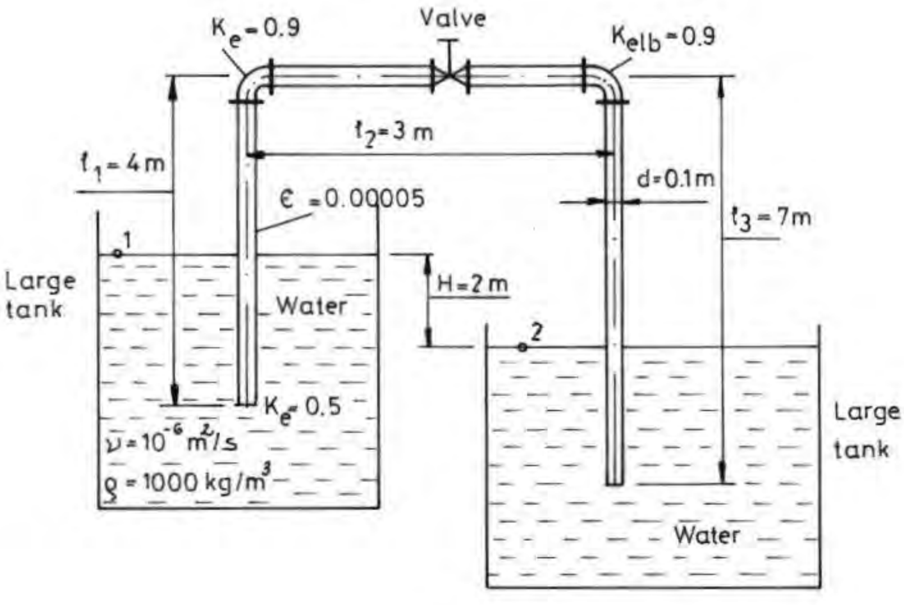
7.8 Points A and B are 1 km apart along a 15-cm new steel pipe $(\epsilon = 4.6 \times 10^{-5}\ \text{m})$. Point B is 20 m higher than A. With flow from A to B of 0.03 m³/s of crude oil $(\rho = 820\ \text{kg/m}^3)$ at 10 °C $(\eta = 0.01\ \text{Ns/m}^2)$, what pressure must be maintained at A if the pressure at B is to be 350 kPa?

7.9 Points A and B are 3 km apart along a 60-cm new cast-iron pipe $(\epsilon = 0.26\ \text{mm})$ carrying water $(\upsilon = 10^{-6}\ \text{m}^2/\text{s})$. Point A is 10 m higher than B. If the pressure at B is 140 kPa greater than that at A, determine the direction and rate of flow.

7.10 What diameter of cast-iron pipe $(\epsilon = 0.26\ \text{mm})$ is needed to carry water $(\upsilon = 10^{-6}\ \text{m}^2/\text{s})$ at a rate of 0.283 m³/s between two reservoirs if the reservoirs are 3.2 km apart and the elevation difference between the water surfaces in the reservoirs is 6 m? (Loss coefficient at entry, $K_e = 0.5$).

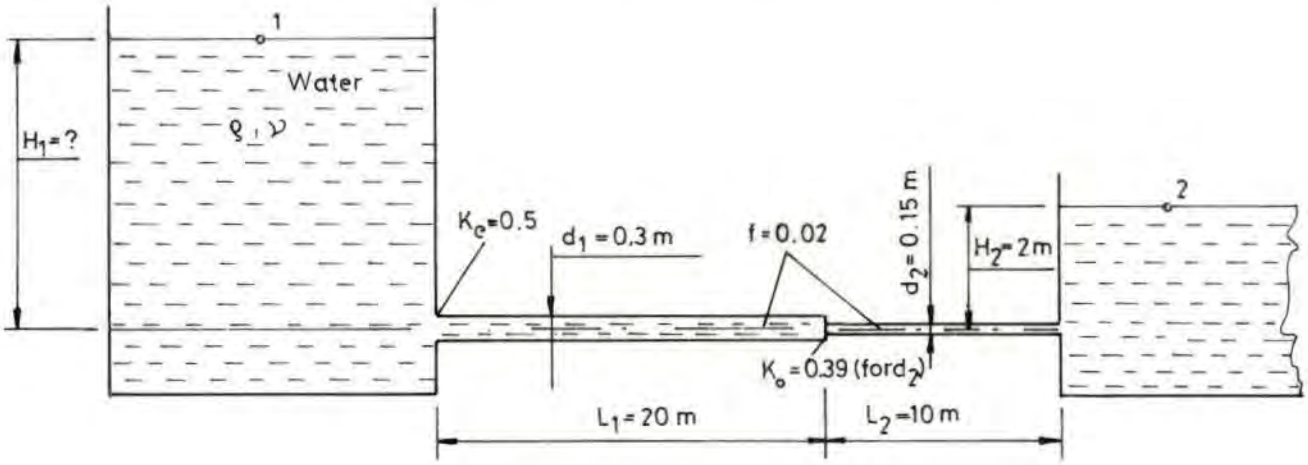
7.11 A pipeline is to be designed to carry crude oil $(\rho = 930\ \text{kg/m}^3;\ \upsilon = 10^{-5}\ \text{m}^2/\text{s})$ with a flow rate of 0.1 m³/s and a head loss per kilometre of 30 m. What diameter of steel pipe $(\epsilon = 0.046\ \text{mm})$ is needed? What power output from a pump is required to maintain this flow?

7.12



Find the loss coefficient K_v of the partially closed valve that is required to reduce the discharge to 50 % of the flow with the value wide open (K_{vo} = 0.2) as shown in the Figure.

7.13 Determine the elevation of the water surface in the upstream reservoir if the discharge in the system is 0.15 m³/s. $\left(\rho = 10^3\ \text{kg/m}^3;\ \ \upsilon = 10^{-6}\ \text{m}^2/\text{s}\ \right)$



7.14 The pressure at a water main is 300 kPa gauge. What size of pipe is needed to carry water $\left(\rho = 10^3\ \text{kg/m}^3;\ \ \upsilon = 10^{-6}\ \text{m}^2/\text{s}\ \right)$ from the main at a rate of 0.025 m³/s to a factory that is 140 m from the main? Assume galvanized-steel pipe (ϵ= 0.2 mm) is to be used and that the pressure required at the factory is 60 kPa gauge at a point 10 m above the main connection.

SOLUTIONS TO CHAPTER 1

1.1 The pressure p can be considered to be constant in the container:

$$p \cong \frac{4F}{d^2\pi} = \frac{4W}{D^2\pi}.$$

Solving for W, yields

$$W = \left(\frac{D}{d}\right)^2 F = 36 \text{ MN}$$

1.2 The gauge pressure $(p_{AB} - p_0)$ at the inner side of the top is

$$p_{A,B} - p_o = \rho g \Delta h$$
$$F = \left(p_{A,B} - p_o\right)A = \rho g \Delta h \frac{D^2\pi}{4} = 7.705\text{x}10^3 \text{ N}$$

1.3 Equation of hydrostatics for this barotropic fluid:

$$gz + P = const$$

where for isothermal change of state $\dfrac{p}{\rho} = \dfrac{p_o}{\rho_o}$

$$P = \int_{p_o}^{p} \frac{dp}{\rho} = \frac{p_o}{\rho_o} \ln \frac{p}{p_o}$$

Hence

$$gz + \frac{p_o}{\rho_o} \ln \frac{p}{p_o} = const$$

Boundary condition: $p = p_o$ at $z = 0$, so $const = 0$

Solving the equation for p yields the variation of pressure with elevation

$$p = p_o \, exp\left(-\frac{g}{RT_o} z\right)$$

Then

$$p_1 = p(z = H) = p_o \ exp\left(-\frac{g}{RT_o}H\right) = 0.687\,\text{kPa}$$

From the perfect gas law

$$\frac{p_1}{\rho_1} = \frac{p}{\rho} = \frac{p_o}{\rho_o} = RT_o$$

Hence

$$\rho_1 = \rho(z = H) = \frac{p_1}{RT_o} = 0.8764 \ \text{kg/m}^3$$

1.4 $p_A - \rho_W g h_1 - \rho_O g h_2 + \rho_W g h_3 = p_B$
$p_A - p_B = -1373.4 \ \text{Pa}$

1.5 $p_A - \rho_O g h_1 + \rho_W g h_2 - \rho_m g h_3 = p_o$
$p_A - p_o = g\ (\rho_o h_1 + \rho_m h_3 - \rho_w h_2) = 1.3018 \times 10^5 \ \text{Pa}$

1.6. Continuity: $\ell\ A_t = \Delta h A_c$. Then $\Delta h = \ell \dfrac{A_t}{A_c} = 2\ \text{mm}$

$p_{cistem} = \rho g(\ell \sin\alpha + \Delta h) = 261.24 \quad \text{Pa}$

1.7 $p_A = p_B$
$p_o + \rho g H = p_o + \rho_m g \Delta h_I$ (1)
where H is the height of the water column above point A.
When the level of the water in the container is raised by ΔH: $p_{A'} = p_{B'}$

$$p_o + \rho g\left[H + \Delta H + \frac{\Delta h_{II} - \Delta h_I}{2}\right] = p_o + \rho_m g \Delta h_{II} \qquad (2)$$

Combining equations (1) and (2), yields

$$\Delta h_{II} = \Delta h_I + \frac{\rho}{\rho_m - \rho/2}\Delta H = 0.1705 \text{ m}$$

1.8
$$\frac{p}{\rho} + gz = const = \frac{p_o}{\rho} + gd$$

Hence the gauge pressure: $p^* = p - p_o = \rho g(d - z)$

(*a*)
$$F = p_c^* A = \rho g(d - z_C)ab;$$

where C is the centroid of area of the window,

$$z_C = d - c - a/2$$

$$F = F_x = 5.3769 \text{ kN}$$

(*b*) The moment of the resultant force is equated to the moment of distributed forces about the axis A-A

$$Fk = \int_A p^*(z)(d-z)dA = \rho gb\left[-\frac{c^3}{3} + \frac{(c+a)^3}{3}\right]$$

whence

$$k = 0.65 \text{ m}$$

1.9

$$y_p = H - k = \frac{I_{\bar{x}}}{A\,y_c} + y_c = \frac{\frac{wh^3}{12}}{wh\left(H - \frac{h}{2}\right)} + H - \frac{h}{2} = \frac{13}{6}\text{m}$$

$$k = H - y_p = 5/6 \text{ m.}$$

1.10 The moment at A have to be equal in magnitude to the moment of the resultant of individual forces acting on the gate.

The gauge pressure distribution in the fluid: $p - p_o = \rho g(H - z)$

$$M = \int_{A_{gate}} (h-z)(p-p_o)dA = \rho g w \int_0^h (h-z)(H-z)dz = \rho g w h^2\left(\frac{H}{2}-\frac{h}{6}\right)$$

$$M = 1.9865 \ \text{x} \ 10^4 \ \text{Nm}$$

1.11 Gauge pressure at the left-hand side (LHS) of the wall: $p - p_o = \rho g(H - z)$

Force at the LHS:

$$\vec{F}_1 = F_1\vec{i} = (p-p_o)_{C_1} A_{gate}\vec{i} = \rho g\left(H - \frac{h}{2}\right)hw\vec{i} = 5.1012 \ \text{x} \ 10^4 \ \vec{i} \ \ [\text{N}]$$

C_1 ...centroid of area on the LHS of the gate (see the Figure)

Line of action at the LHS:

$$h_1 = H - y_{p1} = H - \left(y_{C_1} + \frac{I_{\bar{x}_1}}{A\, y_{C_1}}\right) = H - \left[H - \frac{h}{2} + \frac{\frac{wh^3}{12}}{wh\left(H - \frac{h}{2}\right)}\right] =$$

$$= \frac{h}{2} - \frac{h^2}{12\left(H - \frac{h}{2}\right)} = \frac{5}{6} \ \text{m} \quad \text{(see the Figure)}$$

$I_{\bar{x}_1}$ ...second moment of inertia in centroidal axes

Calculations for the right-hand side (RHS) can be accomplished similarly as above:

Gauge pressure: $p - p_o = \rho g(h - z)$

Force at the LHS: $\vec{F}_2 = -F_2\vec{i} = -(p-p_o)_{C_2} A_{gate}\vec{i} = -\rho g \frac{h}{2} hw\,\vec{i}$

$$\vec{F}_2 = -2.5506 \ \text{x} \ 10^4 \ \vec{i} \ \ [\text{N}]$$

Line of action at the RHS:

$$h_2 = h - y_{p_2} = h - \left(y_{C_2} + \frac{I_{\bar{x}_2}}{A\, y_{C_2}} \right) = h - \left(\frac{h}{2} + \frac{\frac{wh^3}{12}}{wh\frac{h}{2}} \right) = \frac{h}{3} = \frac{2}{3} \text{ m}$$

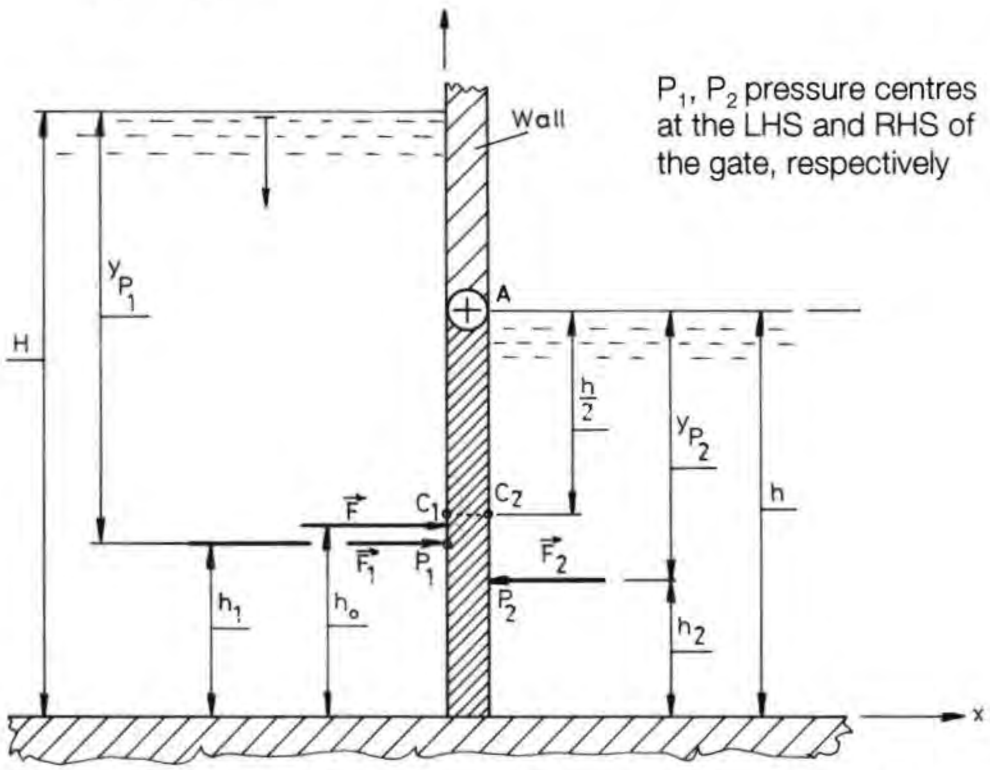
The resultant of the forces $\vec{F}_1$ and $\vec{F}_2$:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = 2.5506 \times 10^4 \, \vec{i} \ \text{[N]}$$

The line of action of the resultant force $\vec{F}$:
The moment of F about O = The sum of the moments of F_1 and F_2:

$$F\, h_o = |F_1| h_1 - |F_2| h_2$$

$$h_o = \frac{|F_1|}{|F|} h_1 - \frac{|F_2|}{|F|} h_2 = 1 \text{ m}$$

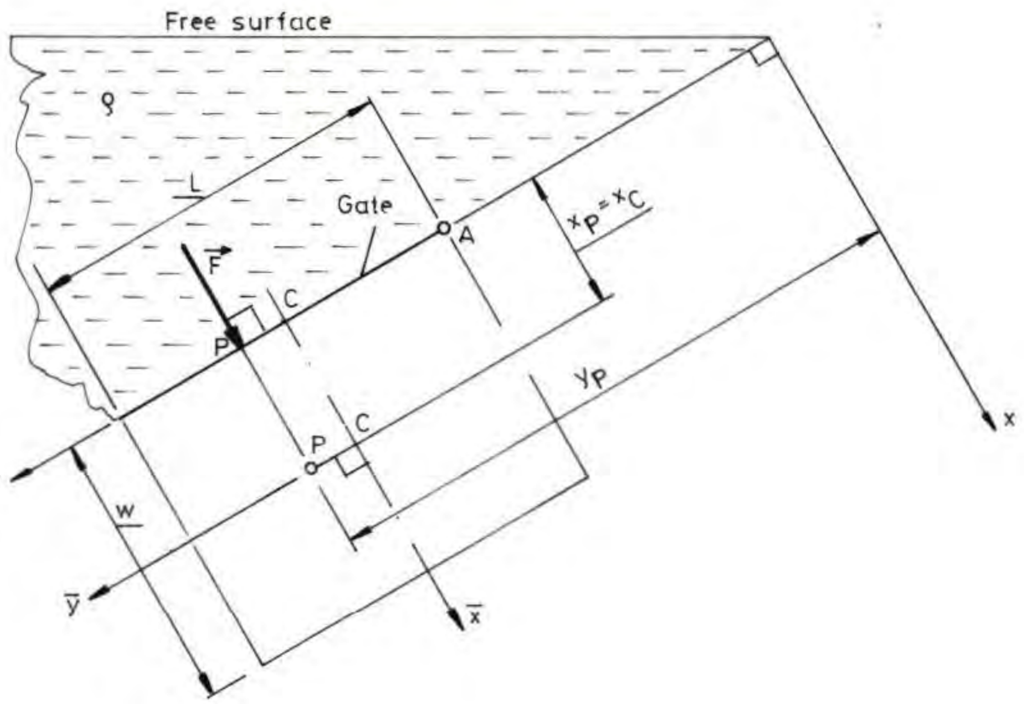


N.B. The same results can be obtained by the integration of elementary forces and their moments.

1.12 $F=(p-p_o)_C\,A=\rho g H\left(r_2^2-r_1^2\right)\pi=4.62285\times 10^4$ N

C...centroid of area

1.13 We introduce new co-ordinate systems:



Force $\vec{F}$ is perpendicular to the surface of the gate; $\vec{F}=F\,\vec{n}$

$$F=(p-p_o)_C A=\rho g\left(H+\frac{L}{2}\sin\alpha\right)Lw=588\text{ kN};$$

$$y_p=y_C+\frac{I_{\bar{x}}}{A\,y_C}=H+\frac{L}{2}\sin\alpha+\frac{\frac{wL^3}{12}}{Lw\left(H+\frac{L}{2}\sin\alpha\right)}=6.222\text{ m}$$

$$y_p-y_C=0.22\text{ m}$$

1.14 F_x= force on the vertical projection of surface AB

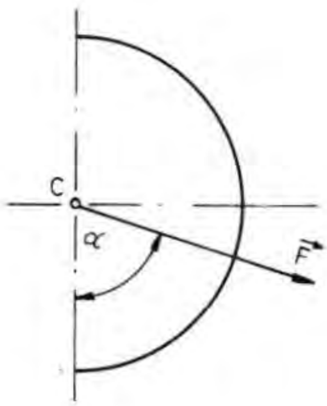
$$F_x = (p - p_o)_C A = \rho g H 2Rw = 7.848 \times 10^4 \text{ N}$$

F_z= the weight of the fluid in the half-cylinder

$$F_z = -W = -\frac{r_o^2 \pi}{2} w \rho g = -3.0819 \times 10^4 \text{ N}$$

$$\vec{F} = (7.848\ \vec{i} - 3.0819\ \vec{k}) \times 10^4 \text{ N}$$

Since all elementary forces $d\vec{F} = (p - p_o) d\vec{A}$ go through point C, the resultant thrust $\vec{F}$ does so, too.



$$\alpha = tan^{-1}\left(\frac{F_x}{|F_z|}\right) = tan^{-1}\left(\frac{F_x}{W}\right) = 68.56^o$$

(see the Figure)

1.15

(*a*) The force exerted against the wall is due to the horizontal force on AB. The horizontal components of force on BC and CD cancel. Hence,

$$F_x = F_{ABx} = \rho g \frac{R}{2} R \cdot 1 = 19620 \text{ N/m}$$

Here $\rho g\ R / 2$ = gauge pressure at the centroid of the projected area,
R . 1 = area of the projection of AB into a vertical plane

(*b*) For equilibrium the weight of the cylinder must be equal to the vertical component of force exerted on it by the water. The vertival force on BCD is

$$F_{zBCD} = \rho g\ 1\left(\frac{\pi R^2}{2} + 2R^2\right)$$

The vertical force on AB is

$$F_{zAB} = -\rho g\, 1\left(R^2 - \frac{R^2\pi}{4}\right).$$

Hence

$$F_z = F_{zBCD} + F_{zAB} = \rho g R^2\left(\frac{3}{4}\pi + 1\right) \qquad (1)$$

The weight of the cylinder is

$$W = \rho_o\, g \cdot 1 \cdot R^2\pi. \qquad (2)$$

Equilibrium:

$$F_z = W \qquad (3)$$

Combining equations, yields

$$\rho_o = \left(\frac{3}{4} + \frac{1}{\pi}\right)\rho = 1068.31\ \mathrm{kg/m^3}$$

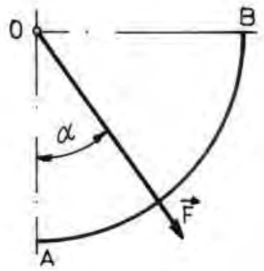
1.16 F_x = force on the vertical projection of surface A B

$$F_x = (p - p_o)_C A = \rho g(H + R/2)R \cdot 1 = 98.1\ \mathrm{kN}$$

F_z = the weight of the fluid above surface A B

$$F_z = -W = -\rho g\left(R^2\pi/4 + RH\right) = -109.299\ \mathrm{kN}$$

Since the elementary forces go through O, the resultant force goes through O, as well.



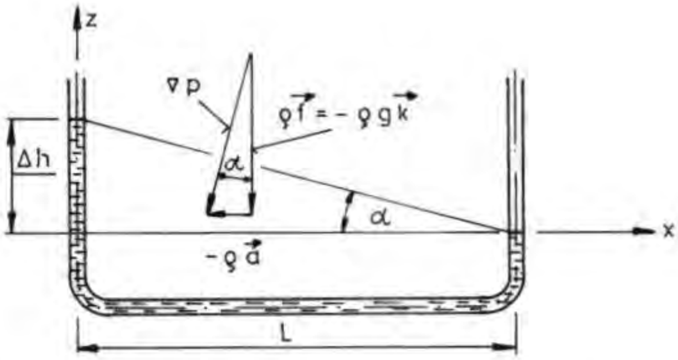
$$\alpha = \tan^{-1}\left(\frac{F_x}{|F_z|}\right) = \tan^{-1}\left(\frac{F_x}{W}\right) = 41.92^\circ$$

1.17

Equation of motion for the relative equilibrium

$$\rho\vec{f} - \nabla p = \rho\vec{a},$$

Hence $$\nabla p = \rho\left(\vec{f} - \vec{a}\right) \qquad (1)$$



1st solution: The free surface is perpendicular to ∇p, hence

$$\tan\alpha = \frac{\Delta h}{L} = \frac{a}{g}$$

$$a = \frac{\Delta h}{L} g = 4.905 \ \text{m/s}^2$$

2nd solution: Equation (1) in componential form:

$$\frac{\partial p}{\partial x} = -\rho a$$

$$\frac{\partial p}{\partial z} = -\rho g$$

Integration of these equations, yields

$$p = -\rho(ax + gz) + const$$

Boundary condition 1: $x = 0$: $z = \Delta h$; $p = p_o$

$$p(x,z) = p_o - \rho\left[ax + g(z - \Delta h)\right]$$

Boundary condition 2: $x = L$: $z = 0$; $p = p_o$

$$a = \frac{\Delta h}{L} g = 4.905 \text{ m/s}^2.$$

1.18

(*a*)

$$p(x,z) = -\rho(a_x x + gz) + const$$

Boundary condition: $x = b/2$: $z = h$, $p = p_o$

Hence

$$p(x,z) = p_o - \rho\left[a_x\left(x - \frac{b}{2}\right) + g(z-h)\right] \tag{1}$$

Free surface: $p(x,z) = p_o$

Hence

$$z = h - \frac{a_x}{g}\left(x - \frac{b}{2}\right) \tag{2}$$

(*b*) Height above the original horizontal water level at $x = 0$ is e.

From (2) $e = \dfrac{ba_x}{2g}$

critical case: $e = H - h_{max}$

Hence $h_{max} = H - \dfrac{ba_x}{2g}$

(c) From (1) $p(x=0,z) - p_o = \rho\left[\dfrac{ba_x}{2} - g(z-h)\right]$

$$e = \frac{ba_x}{2g} = 0.075 \text{ m}$$

$$\vec{F}_{AB} = -\int_A [p(x=0,z) - p_o] dA\, \vec{i} = -\rho c \int_o^{h+e} \left[\frac{ba_x}{2} - g(z-h)\right] dz\, \vec{i} = -30.043\ \vec{i}\ [\text{N}]$$

1.19 Newton's 2nd law: $$\vec{f} - \frac{1}{\rho}\nabla p = \vec{a}$$

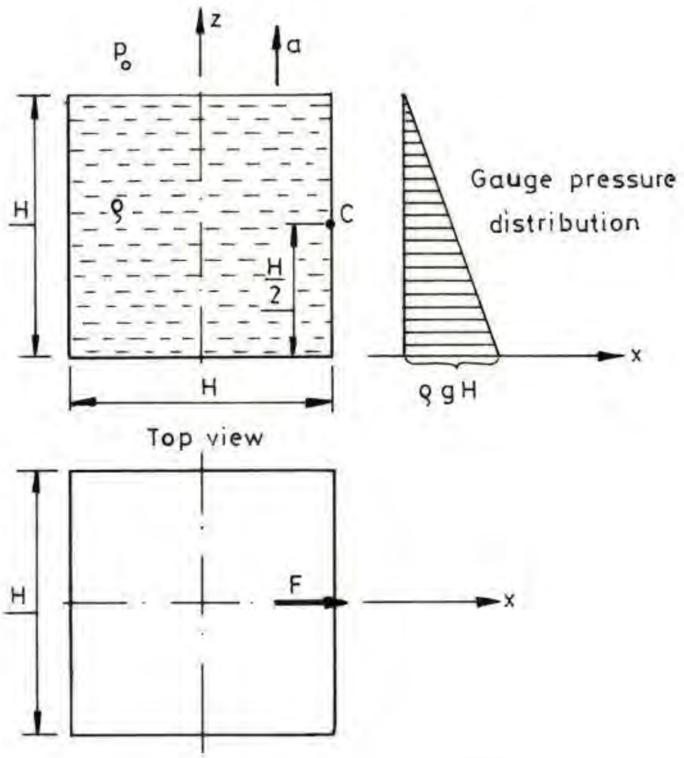
or $$\nabla p = \rho(\vec{f} - \vec{a}) = -\rho(g + a)\vec{k}$$

Integration yields $$p = -\rho(a + g)z + const$$

Boundary condition: $$z = H: \quad p = p_o$$

Hence $$p - p_o = \rho(a + g)(H - z)$$

Force on one side $$F = (p - p_o)_C A = \rho(a + g)\frac{H}{2}H^2 = 13.523 \text{ N}$$



1.20 Equation of motion for relative equilibrium:

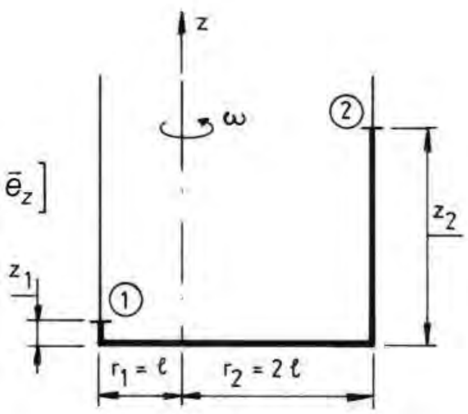
$$\vec{f} - \frac{1}{\rho}\nabla p = \vec{a}$$

$$\vec{f} = -g\,\vec{e}_z; \quad \vec{a} = -r\omega^2\,\vec{e}_r$$

Hence

$$\nabla p = \rho(\vec{f} - \vec{a}) = \rho\left[r\omega^2\vec{e}_r - g\,\vec{e}_z\right]$$

or in componential form



$$\frac{\partial p}{\partial r} = \rho r \omega^2$$

$$\frac{\partial p}{\partial z} = -\rho g$$

Integration yields

$$p(r,z) = \rho\left(\frac{r^2\omega^2}{2} - gz\right) + const$$

Since $p_1 = p_2 = p_o$

$$gz_1 - \frac{r_1^2\omega^2}{2} = gz_2 - \frac{r_2^2\omega^2}{2},$$

on the other hand $z_1 + z_2 = 2\,\ell$ (no spilling).

Hence

$$z_1 = \ell - \frac{3\ell^2\omega^2}{4g} = 0.0215 \text{ m}$$

$$z_2 = 2\ell - z_1 = 0.3385 \text{ m}$$

1.21

$$p(r,z) = \rho\left(\frac{r^2\omega^2}{2} - g\,z\right) + const$$

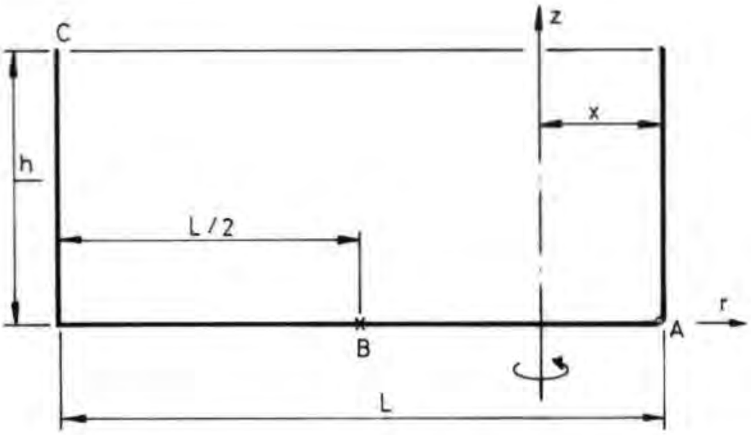
Boundary condition 1: $r = r_o$; $z = h$; $p = p_o$
Hence

$$p(r,z) = p_o + \rho\frac{\left(r^2 - r_o^2\right)\omega^2}{2} + \rho g(h - z)$$

Boundary condition 2: $r = 0$: $z = h$; $p = p_v$
Hence

$$\omega = \frac{1}{r_o}\sqrt{2\frac{p_o - p_v}{\rho}} = 23.449 \text{ s}^{-1}$$

1.22



$$p_A = p_B = p_C = 0 \text{ Pa (gauge)}$$

$$p = \rho\left(\frac{r^2\omega^2}{2} - g\,z\right) + const$$

Boundary condition 1: $r = L - x$: $z = h$; $p = 0$ (gauge)

$$p(r,z) = \rho\left\{g(h-z) + \frac{\omega^2}{2}\left[r^2 - (L-x)^2\right]\right\} \quad (*) \text{ (gauge)}$$

Boundary condition 2: $r = x$: $z = 0$; $p = 0$ (gauge)
Hence from (*)

$$gh + \frac{\omega^2}{2}\left(2Lx - L^2\right) = 0 \tag{1}$$

Boundary condition 3: $r = L/2$: $z = 0$; $p = 0$ (gauge)
Hence from (*)

$$gh + \frac{\omega^2}{2}L\left(x - \frac{3}{4}L\right) = 0 \tag{2}$$

Combining equations (1) and (2), yields

$$x = \frac{L}{4} = 0.15 \text{ m} \qquad \omega = \frac{2}{L}\sqrt{gh} = 5.7184 \text{ s}^{-1}$$

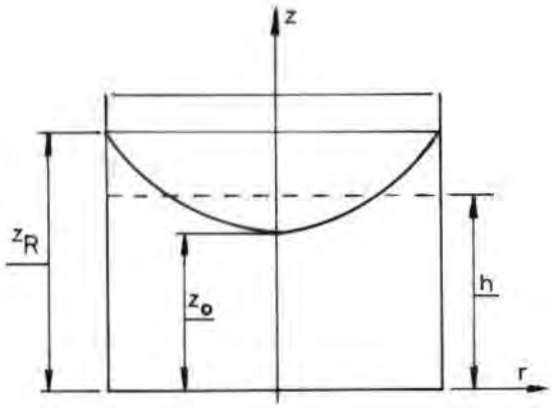
1.23

$$p(r,z) = \rho\left(\frac{r^2\omega^2}{2} - gz\right) + const$$

Boundary condition:
$r = 0$: $z = z_o$; $p = p_o$

Hence

$$p(r,z) = p_o + \rho\left[\frac{r^2\omega^2}{2} - g(z - z_o)\right]$$



Free surface:

$p(r,z) = p_o$, hence $z - z_o = \dfrac{r^2\omega^2}{2g}$ (paraboloid of revolution)

The height of fluid at the wall (r = R):

$$z_R = z_o + \frac{R^2\omega^2}{2g} \tag{1}$$

The volume of the paraboloid of revolution = one half of its circumscribing cylinder (see the Figure)

$$\frac{1}{2}(z_R - z_o)R^2\pi = (h - z_o)R^2\pi \tag{2}$$

Combination of equations (1) and (2), yields

$$z_o = h - \frac{R^2\omega^2}{4g}$$

Hence

$$z_R = h + \frac{R^2\omega^2}{4g} \tag{3}$$

Critical case: $\omega = \omega_1$, $z_R = H$

Thus $$\omega_1 = \frac{2}{R}\sqrt{g(H-h)} = 3.132 \text{ sec}^{-1}$$

1.24 (*a*) see solution of Problem 1.23 ($h = H/2$)

$$\omega_1 = \frac{1}{2}\sqrt{2gH} = 10.845 \text{ sec}^{-1}$$

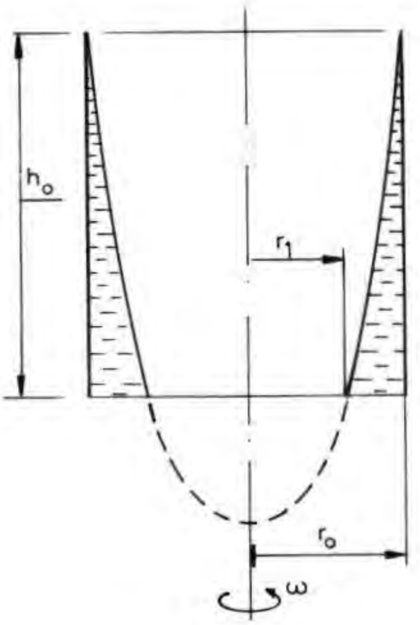
(*b*) Gauge pressure at the bottom: $p(z=0,r) - p_o = \rho g H (r/R)^2$
The integration of the gauge pressure over the bottom yields

$$F = \int_A [p(z=0,r) - p_o]\, dA = \rho g H \int_o^R \left(\frac{r}{R}\right)^2 2\pi r dr = \rho g H \frac{R^2 \pi}{2} = 5.7786 \text{ kN}$$

(acts downward)

(F is equal to the weight of the fluid contained in the tank after rotation, which is one half of the weight of the fluid originally in the container)

1.25



$$p(r,z)=\rho\left(\frac{r^2\omega^2}{2}-gz\right)+const$$

Boundary condition 1: $r=r_o$; $z=h_o$; $p=p_o$

$$p(r,z)=p_o+\rho\left[\frac{\left(r^2-r_o^2\right)\omega^2}{2}+g(h_o-z)\right] \tag{1}$$

$$r_1^2\pi=\frac{1}{2}r_o^2\pi \quad \text{so} \quad r_1=\frac{r_o}{\sqrt{2}}$$

Boundary condition 2: $r=r_1$; $z=0$; $p=p_o$
Substitution into equation (1), yields

$$\omega=\frac{2}{r_o}\sqrt{g\,h_o}=12.528\ \text{sec}^{-1}$$

1.26 $F_x=\rho gH\,R^2\pi=92457.07\ \text{N}$

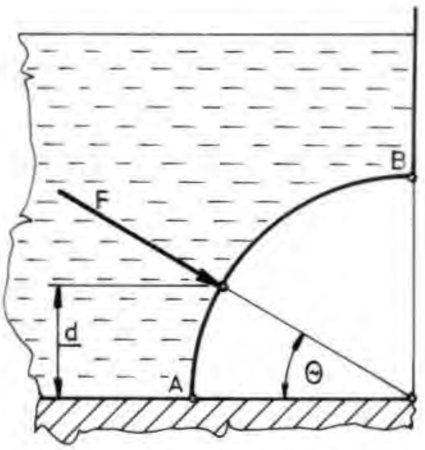
$$F_z=\rho g\frac{1}{2}\frac{4R^3\pi}{3}=20546.02\ \text{N}$$

Force goes through O since the elementary forces are normal to the sphere at all points and hence points to O.

1.27 $p_A+\rho_w g d_1-\rho_m g(d_3+d_4\sin\alpha-d_2)=p_B$

$p_A-p_B=\rho_m g(d_3+d_4\sin\alpha-d_2)-\rho_w g d_1=57283.8\ \text{Pa}$

1.28



$$F_x = \rho g\left(h + \frac{R}{2}\right)Rw = 15.3036 \text{ kN}$$

$$F_z = -W = -\rho g w\left[R(h+R) - \frac{R^2\pi}{4}\right] = -11.6639 \text{ kN}$$

$$F = \sqrt{F_x^2 + F_z^2} = 19.2418 \text{ kN}$$

$$\sin\Theta = \frac{|F_z|}{F}; \quad d = R\sin\Theta = R\frac{|F_z|}{F} = 0.6062 \text{ m}$$

1.29

$$p_{atm} + \rho g(h_3 - h_2) = p_B$$

Hence

$$p_B - p_{atm} = \rho g(h_3 - h_2) = 4905 \text{ Pa}$$

$$p_A + \rho g h_1 = p_B$$

Hence

$$p_A - p_{atm} = p_B - p_{atm} - \rho g h_1 = \rho g(h_3 - h_2 - h_1) = 2943 \text{ Pa}$$

SOLUTIONS TO CHAPTER 2

2.1

$$div\bar{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \quad \text{for incompressible fluid}$$

(*a*) yes $(div\bar{v} = 0)$
(*b*) no $(div\bar{v} \neq 0)$
(*c*) yes
(*d*) yes

2.2

$$\bar{v} = grad\Phi; \qquad v_x = ky; \quad v_y = kx; \quad v_z = 0$$

The differential equation for the streamlines:

$$\frac{dx}{v_x} = \frac{dy}{v_y} \tag{1}$$

Substituting the velocity components into equation (1), and integrating yields the equation of streamlines

$$x^2 - y^2 = const$$

2.3

Basic equation:

$$\frac{dx}{v_x} = \frac{dy}{v_y} = \frac{dz}{v_z} \tag{1}$$

Separating equations (1) into two and substituting the velocity components leads to the system of streamlines

$$xy = k_1 \tag{2}$$

$$x = k_2 \exp\left(\frac{a}{c}z\right) \tag{3}$$

where k_1 and k_2 are arbitrary constants.

N. B. Streamlines are obtained as intersections of cylindrical surfaces (2) and (3).

2.4

It follows from the initial condition that

$$c_1 = 1 \quad \text{and} \quad c_2 = 2$$

hence

$$x_p = e^{at} \quad \text{and} \quad y_p = 2e^{-bt}$$

Eliminating parameter t and dropping subscript p the equation of a pathline

$$y = 2x^{-\frac{b}{a}}$$

$$v_x = \frac{\partial x_p}{\partial t} = ae^{at} = ax$$

$$v_y = \frac{\partial y_p}{\partial t} = -2be^{-bt} = -by$$

Differential equation for streamline

$$\frac{dx}{v_x} = \frac{dy}{v_y}$$

After substitution the velocity components and integration the equation of the streamline through point (1,2) is obtained as

$$y = 2x^{-\frac{b}{a}}$$

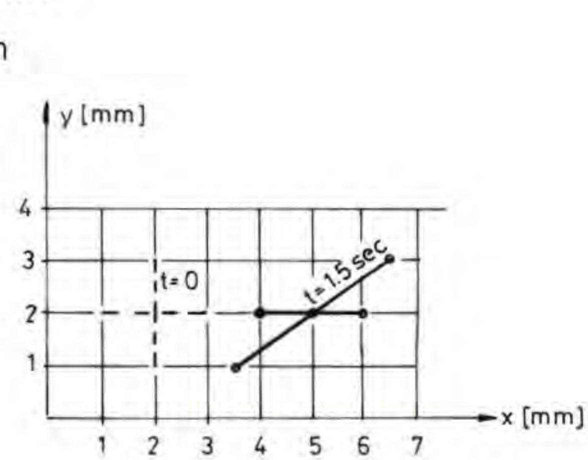
2.5

$$v_y \equiv 0; \quad \Delta x = v_x t$$

$$\Delta x_a = \Delta x_c = v_x(y_a)\Delta t = 3 \text{ mm}$$

$$\Delta x_b = v_x(y_b)\Delta t = 4.5 \text{ mm}$$

$$\Delta x_d = v_x(y_d)\Delta t = 1.5 \text{ m}$$



The rate of angular deformation:

$$\dot{\gamma} = \frac{\partial v_x}{\partial y} + \frac{\partial y}{\partial x} = \frac{u}{h} = 1\ s^{-1}$$

The rate of rotation:

$$\omega_z = \frac{1}{2}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) = -\frac{U}{2h} = -0.5\ \mathrm{s}^{-1}$$

2.6

(*a*) streamline at $t = 0$ through point (1,1,0):

$$y = x$$

(*b*) pathline

$$v_{xp} = x_p(1 + At) = \frac{dx_p}{dt} \quad \rightarrow \quad ln\frac{x_p}{x_{po}} = t + A\frac{t^2}{2} \qquad (1)$$

$$v_{yp} = y_p = \frac{dy_p}{dt} \qquad \rightarrow \quad t = ln\, y_p \qquad (2)$$

The combination of equations (1) an (2) (taking into account that $x_{po} = 1$ and dropping subscript *p*), yields

$$x = exp\left[ln\, y + \frac{A}{2}(ln\, y)^2\right]$$

2.7

(*a*)
$$\underline{\underline{D}} = \bar{v}\,\nabla = \begin{bmatrix} 2v_o x & 0 & 0 \\ 0 & v_o & 0 \\ 0 & 0 & v_o \end{bmatrix}$$

(*b*)
$$\vec{a} = \frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = \underline{\underline{D}}\cdot\vec{v} = v_o^2\left[2x^3\,\vec{i} + y\,\vec{j} + (z+1)\,\vec{k}\right] \ \ \mathrm{m/s}$$

(*c*) $curl\ \bar{v} = \nabla x\, \bar{v} = \bar{0}$, hence $\quad \Phi$ exists

$$\vec{v} = \nabla\Phi \tag{1}$$

Writing equation (1) in componential form, and integrating, yields

$$\Phi = v_o\left(\frac{x^3}{3} + \frac{y^2}{2} + \frac{z^2}{2} + z\right) + const$$

2.8

(*a*)

$$\frac{dx}{v_x} = \frac{dy}{v_y}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1;$$

Normally these are ellipses ($\alpha \neq \beta$)

where $a^2 = \dfrac{2c}{\alpha}$; $b^2 = \dfrac{2c}{\beta}$

They are circles if $\alpha = \beta$

(*b*) $\vec{v} = -\alpha y\,\vec{i} + \beta x\,\vec{j}$

$$\vec{a} = \frac{d\vec{v}}{dt} = \underline{\underline{D}}\cdot\vec{v} = -\alpha\beta\left(x\,\vec{i} + y\,\vec{j}\right) = \vec{a}$$

where

$$\underline{\underline{D}} = \begin{bmatrix} \dfrac{\partial v_x}{\partial x} & \dfrac{\partial v_x}{\partial y} \\ \dfrac{\partial v_y}{\partial x} & \dfrac{\partial v_y}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & -\alpha \\ \beta & 0 \end{bmatrix}$$

(*c*)

$$\vec{\omega} = \frac{1}{2}\,curl\;\vec{v} = \frac{1}{2}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\vec{k} = \frac{1}{2}(\beta + \alpha)\,\vec{k}$$

(*d*) rate of volume dilation : $div\;\vec{v} = \dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_y}{\partial y} + \dfrac{\partial v_z}{\partial z} = 0$

(e) Symmetric part of $\underline{\underline{D}}$:

$$\underline{\underline{S}} = \frac{1}{2}\left(\underline{\underline{D}} + \underline{\underline{D_c}}\right) = \begin{bmatrix} 0 & \frac{\beta-\alpha}{2} \\ \frac{\beta-\alpha}{2} & 0 \end{bmatrix}$$

where $\underline{\underline{D_c}}$ is the conjugate of $\underline{\underline{D}}$.

Hence the rate of angular deformation is: $2\frac{\beta-\alpha}{2} = \beta - \alpha$

If $\alpha = \beta$...there is no angular deformation.

2.9

(a) $$\underline{\underline{S}} = \frac{1}{2}\left(\underline{\underline{D}} + \underline{\underline{D_c}}\right) = \begin{bmatrix} 0 & 0 & -Ax \\ 0 & 0 & -Ay \\ -Ax & -Ay & 0 \end{bmatrix}$$

(b) $$\vec{a} = \frac{d\vec{v}}{dt} = \vec{0}$$

(c) rate of angular deformations are: -2Ax (in x,z plane)
-2Ay (in y,z plane)

rate of linear deformation is $= 0$

(d) $curl\ \vec{v} = \nabla x\ \vec{v} = 2A\left(-y\vec{i} + x\vec{j}\right)$

(e) $curl\ \vec{v} \neq \vec{o} \rightarrow \Phi$ does not exist!

2.10

(a) $x^2 + y^2 = c = R^2$
circles with centres at the origin

(*b*) $\underline{\underline{\vec{a}}} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = \underline{\underline{D}} \cdot \vec{v} = -\omega^2 \left(x\vec{i} + y\vec{j} \right)$

(*c*) $curl\ \vec{v} = \nabla x \vec{v} = 2\omega\vec{k} = 20\vec{k} \quad [s^{-1}]$

(*d*) does not exist $\left(curl\ \vec{v} \neq \vec{0} \right)$

(*e*) Symmetric part of $\underline{\underline{D}}$:

$$\underline{\underline{S}} = \frac{1}{2}\left(\underline{\underline{D}} + \underline{\underline{D_c}} \right) = \underline{\underline{0}}$$

there is no deformation at all.
Skew-symmetric part of $\underline{\underline{D}}$:

$$\underline{\underline{\Omega}} = \frac{1}{2}\left(\underline{\underline{D}} - \underline{\underline{D_c}} \right) = \underline{\underline{D}}$$

so there is rotation

(*f*) steady since $\frac{\partial \vec{v}}{\partial t} = \vec{0}$

(*g*) yes, since $div\ \vec{v} = 0$

2.11

$$\vec{a} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = \frac{\partial \vec{v}}{\partial t} + v_x \frac{\partial \vec{v}}{\partial x} + v_y \frac{\partial \vec{v}}{\partial y} + v_z \frac{\partial \vec{v}}{\partial z}$$

$$\vec{a}(3,1,0) = 80\vec{j} - 80\vec{k} \quad m/s^2$$

SOLUTIONS TO CHAPTER 3

3.1 Basic equations

Bernoulli's equation: $$\frac{p_1}{\rho}+gz_1+\frac{v_1^2}{2}=\frac{p_2}{\rho}+gz_2+\frac{v_2^2}{2} \qquad (1)$$

Equation of continuity: $$A_1v_1=A_2v_2 \qquad (2)$$

Since $z_1=z_2$, the combination of equations (1) and (2) yields

$$p_1-p_{atm}=p_1-p_2=\frac{\rho}{2}(v_2^2-v_1^2)=\frac{\rho}{2}\left[1-\left(\frac{A_1}{A_2}\right)^2\right]v_2^2=1.476\text{ kPa}$$

3.2 Basic equation:

$$\frac{p}{\rho}+gz+\frac{v^2}{2}=const \qquad (1)$$

Application of equation (1) between points 1 and 2:

$$\frac{p_1}{\rho_2}+gh_2+\frac{v_1^2}{2}=\frac{p_{atm}}{\rho_2}+\frac{v^2}{2} \qquad (2)$$

Application of equation (1) between points 0 and 1:

$$\frac{p_{atm}}{\rho_1}+g(h_1+h_2)+\frac{v_0^2}{2}=\frac{p_1}{\rho_1}+gh_2+\frac{v_1^2}{2} \qquad (3)$$

$v_1=v_0=0$ here.
Equation of continuity:

$$Q=\frac{d^2\pi}{4}v \qquad (4)$$

The combination of equations (2)-(4), yields

$$Q = \frac{d^2\pi}{4}\sqrt{2g\left[\frac{\rho_1}{\rho_2}h_1 + h_2\right]} = 0.05218\,\text{m}^3/\text{s}$$

3.3

Apply Bernoulli's equation between points 1 and 2:

$$\frac{p_1}{\rho} + \frac{v^2}{2} = \frac{p_2}{\rho} \tag{1}$$

From the hydrostatic pressure variation:

$$p_1 = p_{atm} + \rho g h \qquad \text{and} \qquad p_2 = p_{atm} + \rho g(h + \Delta h)$$

Hence

$$\frac{p_2 - p_1}{\rho} = g\Delta h \tag{2}$$

Combining equations (1)and (2), yields

$$v = \sqrt{2\frac{p_2 - p_1}{\rho}} = \sqrt{2g\Delta h} = 0.99\text{m}$$

3.4

Bernoulli's equation between points 0 and A:

$$\frac{p_0}{\rho} = \frac{p_A}{\rho} + gh + \frac{v^2}{2} \tag{1}$$

From the equation of continuity:

$$\frac{v^2}{2} = \frac{8Q^2}{D^4\pi^2} \tag{2}$$

The combination of equations (1) and (2), yields

$$h = \frac{p_0 - p_v}{\rho} - \frac{8Q^2}{D^4\pi^2 g} = 7.604\text{m}$$

3.5

Neglecting friction write the Bernoulli's equation between points 1 and 2:

$$\frac{p_1}{\rho} = \frac{p_{atm}}{\rho} + gH$$

whence

$$H = \frac{p_1 - p_{atm}}{\rho g} = 16.310\text{m}$$

3.6

Apply Bernoulli's equation between points A and B:

$$\frac{p_A}{\rho} + \frac{v^2}{2} = \frac{p_B}{\rho} \qquad (1)$$

Apply hydrostatic equation for the two limbs of the U-tube:

$$p_B + \rho g \Delta h = p_C + \rho_m g \Delta h \qquad (2)$$

Taking into account that $p_A = p_C$ and combining equations (1) and (2), yield

$$v = \sqrt{2\left(\frac{\rho_m}{\rho} - 1\right) g \Delta h} = 8.612\text{m/s}$$

3.7

Application of the Bernoulli's equation and the equation of continuity, yields

$$p_2 = p_1 + \rho g(z_2 - z_1) + \frac{\rho}{2}\left[1 - \left(\frac{d_1}{d_2}\right)^4\right] = 290.6\text{kPa}$$

3.8

The combination of the Bernoulli's equation and the equation of continuity written between points **1** and **2**, yields

$$U = \sqrt{\frac{2(p_2 - p_1)}{\rho\left[\left(\frac{D_1}{D_2}\right)^4 - 1\right]}} = 60.851\,\mathrm{m/s}$$

3.9

Bernoulli's equation between points C-D:

$$\frac{p_C}{\rho_o} = \frac{p_D}{\rho_o} + \frac{v^2}{2} + gl_2 \tag{1}$$

Equation of hydrostatics between points A and B:

$$p_B = p_A + \rho_G g l_1 \tag{2}$$

Equation of continuity:

$$\dot{m} = \rho_o v \frac{d^2\pi}{4} \tag{3}$$

Taking into account that $p_B = p_C$ and the combination of equations (1) -(3), yields

$$\dot{m} = \rho_o \frac{d^2\pi}{4}\sqrt{2\left[\frac{p_A - p_D}{\rho_O} + \frac{\rho_G}{\rho_O} g(l_1 - l_2)\right]} = 4.7575\,\mathrm{kg/s}$$

3.10

Bernoulli's equation:

$$\frac{p_1}{\rho} + gz_1 + \frac{v_1^2}{2} = \frac{p_2}{\rho} + gz_2 + \frac{v_2^2}{2} \tag{1}$$

Equation of continuity:

$$Q = A_1 v_1 = A_2 v_2 \tag{2}$$

Equation of hydrostatics for the two limbs of the U-tube:

$$p_1 + \rho g z_1 = p_2 + \rho g(z_2 - \Delta h) + \rho_m g \Delta h \tag{3}$$

The combination of equations (1)-(3), yields

$$Q = \frac{D_1^{\,2}\pi}{4}\sqrt{\frac{\dfrac{\rho_m}{\rho}-1}{\left(\dfrac{D_1}{D_2}\right)^4-1}2g\Delta h} = 0.02653\ \mathrm{m^3/s}$$

3.11

Application of the Bernoulli's equation between points **1** and **2**, and **1** and **3**, yield

$$v_2 = \sqrt{2gH} \qquad (1)$$

$$v_3 = \sqrt{2g(H+y)} \qquad (2)$$

Equation of continuity between points **2** and **3** (neglecting jet contraction):

$$v_2\frac{d_o^{\,2}\pi}{4} = v_3 r^2\pi \qquad (3)$$

Combination of equations (1)-(3) gives

$$r = \frac{d_o}{2}\left(1+\frac{y}{H}\right)^{-0.25}$$

3.12

Bernoulli's equation between points O and E:

$$\frac{p_{atm}}{\rho} + gH = \frac{p_o}{\rho} \qquad (1)$$

Equation of hydrostatics for the two limbs of the U-tube ($p_A = p_B$):

$$p_{atm} + \rho g H = p_o + (\rho_m - \rho)gR \qquad (2)$$

Combination of equations (1) and (2), yields

$$R \equiv 0 \qquad \text{(for an arbitrary value of } H \text{)}$$

3.13

Bernoulli's equation for points 1 and 2, yields

$$\frac{v_1^2}{2} + gz_1 = \frac{v_2^2}{2} \tag{1}$$

Equation of continuity:

$$A_t v_1 = A_j v_2 \tag{2}$$

The definition of v_1:

$$v_1 = -\frac{dz_1}{dt} \tag{3}$$

Combination of equations (1) and (2) gives

$$v_1 = \sqrt{\frac{2g}{\left(\frac{A_t}{A_j}\right)^2 - 1}}\sqrt{z_1} \tag{4}$$

Combination of equations (3) and (4) can be integrated finally to give

$$z_1 = \left[\sqrt{H} - \frac{1}{2}\sqrt{\frac{2g}{\left(\frac{A_t}{A_j}\right)^2 - 1}}\,t\right]^2 = (1.414213562 - 0.002214725t)^2$$

3.14

The method of solution is the same as in Problem 3.13. The only difference is that the cross-sectional area of the tank varies with z_1

$$A_t = \left(2Rz_1 - z_1^2\right)\pi$$

The relation between z_1 and t is obtained as follows

$$t = \frac{2\pi}{15\sqrt{2g}A_j}\left(3z_1^{5/2} - 10Rz_1^{3/2} + 7R^{5/2}\right)$$

$$T_1 = 213.70 \text{ sec}$$

SOLUTIONS TO CHAPTER 4

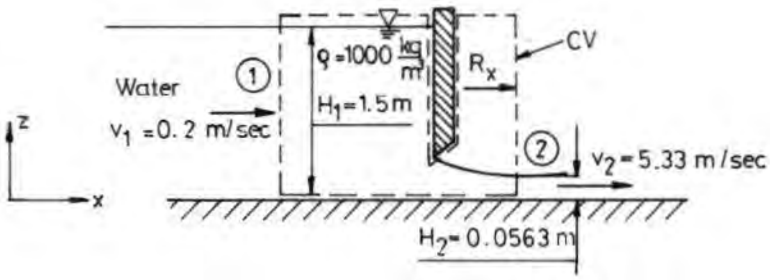
4.1 Choose the control volume (*CV*) (see the Figure)
Basic equation:

$$\int_{CV} \rho \vec{f}\, dV - \int_{CS} p d\vec{A} = \frac{\partial}{\partial t}\int_{CV} \rho \vec{v}\, dV + \int_{CS} \vec{v} \rho \vec{v} \cdot d\vec{A} \tag{1}$$

Assumptions
- steady incompressible flow
- hydrostatic pressure distributions at ① and ②

Let us multiply equation (1) by unit vector $\vec{i}$

$$-\int_{A_1} p d\vec{A} \cdot \vec{i} - \int_{A_2} p d\vec{A} \cdot \vec{i} - R_x = \int_{A_1} v_x \rho \vec{v} \cdot d\vec{A} \tag{2}$$



At section ① $\quad -\int_{A_1} p d\vec{A} \cdot \vec{i} = \int_{A_1} p\, dA = \rho g \int_0^{H_1} (H_1 - z) w dz = \rho g w \frac{H_1^2}{2}$

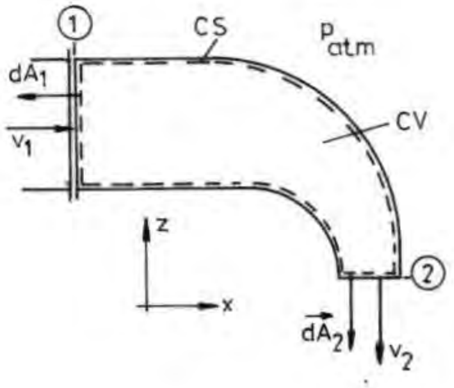
At section ② $\quad -\int_{A_2} p d\vec{A} \cdot \vec{i} = -\int_{A_2} p\, dA = -\rho g \int_0^{H_2} (H_2 - z) w dz = -\rho g w \frac{H_2^2}{2}$

$$\int_{A_1} v_x \rho \vec{v} \cdot d\vec{A} = -\dot{m} v_1; \quad \int_{A_2} v_x \rho \vec{v} \cdot d\vec{A} = \dot{m} v_2$$

Collecting terms, yields

$$\frac{R_x}{w} = \frac{\rho g}{2}\left(H_1^2 - H_2^2\right) - \rho H_1 v_1 (v_2 - v_1) = 9.482 \text{ kN/m}$$

4.2



Choose *CV* as shown by the dashed line

Assumptions: - uniform flow at sections ① and ②
- neglect weight
- incompressible flow

Basic equations:

$$\int_{CV} \rho \vec{f}\, dV - \int_{CS} p\, d\vec{A} = \frac{\partial}{\partial t}\int_{CV} \vec{v}\rho\, dV + \int_{CS} \vec{v}\rho\vec{v}\cdot d\vec{A} \tag{1}$$

$$A_1 v_1 = A_2 v_2 = \frac{\dot{m}}{\rho} \tag{2}$$

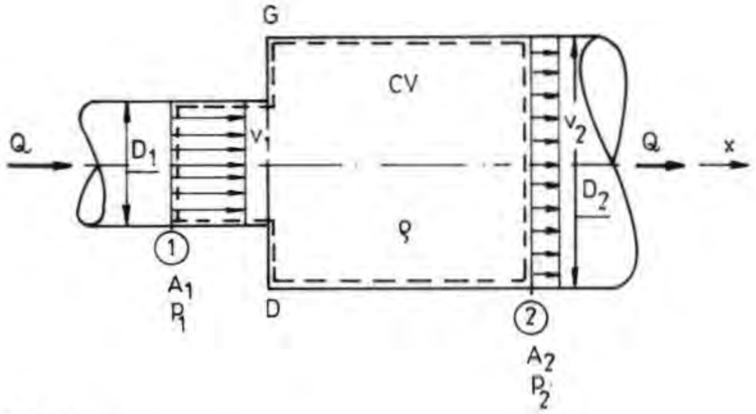
Taking into account the assumptions equation, (1) will have the form

$$-\int_{A_1} p d\vec{A} + \vec{F} = \dot{m}(\vec{v}_2 - \vec{v}_1),$$

Here p means gauge pressure ($p_{2\,gauge} = 0$).
Collecting terms, gives

$$\vec{F} = -\left(p_{1abs} - p_{atm}\right)A_1\vec{i} - \rho A_2 v_2^2\left(\frac{A_2}{A_1}\vec{i} + \vec{k}\right) = -1.36\vec{i} - 0.64\vec{k} \quad [\text{kN}]$$

4.3



Assumptions:
- steady, incompressible flow
- uniform flow at sections ① and ②
- pressure over the annular face GD (experimental evidence) $\cong p_1$
- neglect weight of the fluid in *CV*
- neglect friction along the wall

Basic equations:

$$\int_{CV} \rho \vec{f} dV - \int_{CS} p d\vec{A} = \frac{\partial}{\partial t} \int_{CV} \rho \vec{v} dV + \int_{CS} \vec{v} \rho \vec{v} \cdot d\vec{A} \tag{1}$$

$$A_1 v_1 = A_2 v_2 = Q = \frac{\dot{m}}{\rho} \tag{2}$$

Taking into account assumptions, equation (1) can be written as

$$p_1 A_1 \vec{i} + p_1 (A_2 - A_1) \vec{i} - p_2 A_2 \vec{i} = \dot{m} (\vec{v}_2 - \vec{v}_1)$$

from which

$$p_2 = p_1 + \rho v_2 (v_1 - v_2) \tag{3}$$

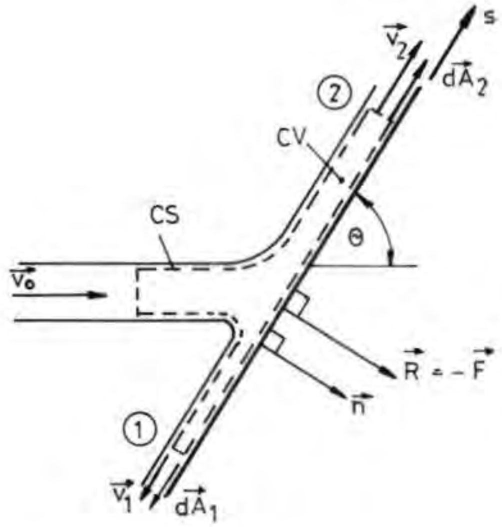
In case of ideal (frictionless) flow the application of Bernoulli's equation between points 1 and 2, yields

$$p_{2\,id} = p_1 + \frac{\rho}{2} \left(v_1^2 - v_2^2 \right) \tag{4}$$

The pressure loss $\Delta p'$ due to abrupt enlargement of the cross-section is

$$\Delta p' = p_{2\,id} - p_2 = \frac{\rho}{2}(v_1 - v_2)^2 = \rho\left(1 - \frac{A_1}{A_2}\right)^2 \frac{v_1^2}{2} = 7.03125 \text{ kPa}$$

4.4



Choose *CV* as shown in the Figure.
Assumptions: $v_1 = v_2 = v_o$ (since elevation differences and friction is neglected)
- steady, incompressible flow

The steady state momentum equation yields

$$\vec{F} = -\dot{m}_o\vec{v}_o + \dot{m}_1\vec{v}_1 + \dot{m}_2\vec{v}_2 \tag{1}$$

where

$$\dot{m}_i = \rho A_i\, v_i = \rho Q_i \quad (i = 0, 1, 2)$$

Equation of continuity:

$$Q_o = Q_1 + Q_2 \tag{2}$$

The combination equation (2) with the *s* component of equation (1), yields

$$Q_1 = Q_o \frac{1 + \cos\Theta}{2} = 0.075 \text{ m}^3/\text{s}$$

$$Q_2 = Q_o \frac{1 - \cos\Theta}{2} = 0.025 \text{ m}^3/\text{s}$$

The force $\vec{R}$ exerted on the plate must be normal to it. For the momentum equation normal to the plate,

$$R = -F = \rho Q_o v_o \sin\Theta = 866.025 \text{ N}$$

4.5

Basic equations:

$$\frac{p_1}{\rho} + g z_1 + \frac{v_1^2}{2} = \frac{p_2}{\rho} + g z_2 + \frac{v_2^2}{2} \quad (1)$$

$$A_1 v_1 = A_2 v_2 = Q \quad (2)$$

$$\int_{CV} \rho \vec{f} dV - \int_{CS} p d\vec{A} = \frac{\partial}{\partial t} \int_{CV} \rho \vec{v} dV + \int_{CS} \vec{v} \rho \vec{v} \cdot d\vec{A} \quad (3)$$

Assumptions:
- steadly, incompressible, frictionless flow
- uniform flow at sections 1 and 2
- the weight of the fluid in *CV* can be neglected.

The combination of equations (1) and (2) yields the inlet gauge pressure p_{1g} as a function of flow rate Q

$$p_{1g} = p_1 - p_{atm} = \frac{\rho Q^2}{2A_1^2}\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right].$$

For the data given:

$$p_{1g} = 2.04944 \times 10^5 \text{ Pa}$$

Taking into account the assumptions, equation (3) will have the form

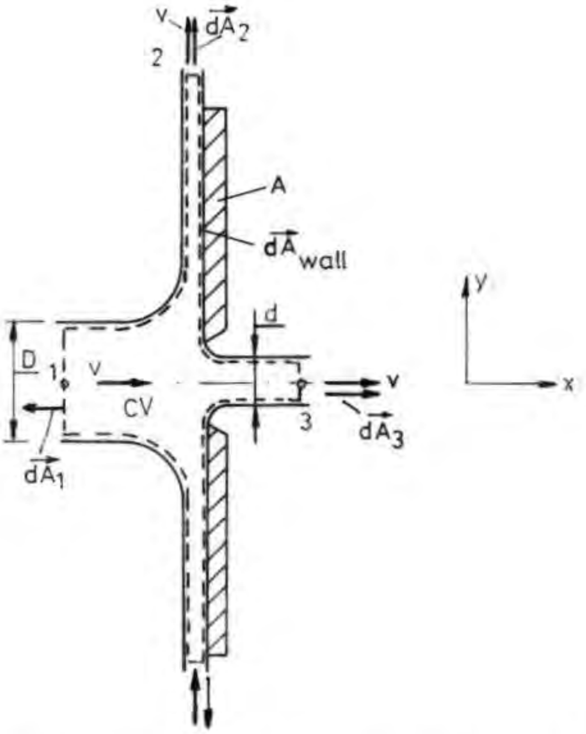
$$-\int_{A_1} p_g d\vec{A} + \vec{F} = \dot{m}(\vec{v}_2 - \vec{v}_1)$$

whence

$$\vec{F} = \left[-p_{1g} A_1 + \rho \frac{4Q^2}{D_1^2 \pi}\left(\frac{D_1^2}{D_2^2} - 1\right)\right]\vec{i}$$

$$F_x = F = -847.77 \text{ N}$$

4.6



Choose *CV* as shown in the Figure. The steady state momentum equation:

$$\vec{F} = \int_{CS} \vec{v}\rho\vec{v} \cdot d\vec{A} \tag{1}$$

Assumptions: $v_1 = v_2 = v_3 = v$
- steady, incompressible frictionless flow
- uniform flow at sections 1 and 3

(1) in more detail

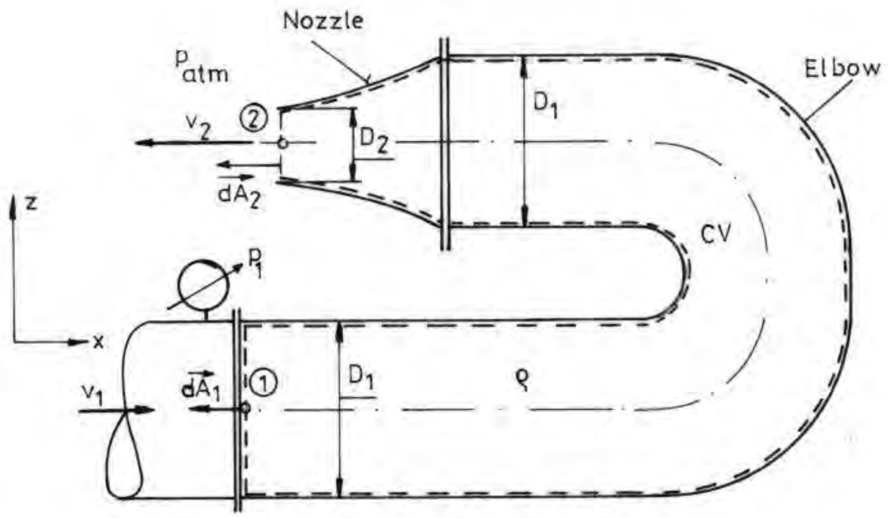
$$\vec{F} = \int_{CS} \vec{v}\rho\vec{v} \cdot d\vec{A} = \vec{v}_1 \int_{A_1} \rho\vec{v} \cdot d\vec{A} + \int_{A_2} \vec{v}\rho\vec{v} \cdot d\vec{A} + \vec{v}_3 \int_{A_3} \rho\vec{v} \cdot d\vec{A}$$

$\vec{F}$ is perpendicular to the wall, i.e., $\vec{F} = F\vec{i}$
The x component of equation (1) gives

$$F = -\rho v^2 \left(D^2 - d^2\right) \frac{\pi}{4} = -5937.61 \text{ N}$$

$$\vec{F} = -5937.61\,\vec{i} \;\; [\text{N}]$$

4.7



Choose *CV* as shown in the Figure.
The application of the steady state momentum equation and equation of continuity, yields

$$\vec{R} = \rho A_1 v_1^2 \left(1 + \frac{A_1}{A_2}\right)\vec{i} - \rho g V \vec{k} + (p_1 - p_{atm})A_1 \vec{i} = 7863.8\,\vec{i} - 981\,\vec{k} \;\; [\mathrm{N}]$$

4.8

The *x* component of the momentum equation for *CV*

$$p_1 A_1 + F_x = \dot{m}(v_2 - v_1) + \int_{CV} \rho \frac{\partial v}{\partial t}\, dV \qquad (1)$$

The integral in (1) can be divided into two parts:

$$\int_{CV} \rho \frac{\partial v}{\partial t} dV = \rho a_1 L_1 A_1 + \rho a_2 L_2 A_2 \qquad (2)$$

where a_1, a_2, A_1, A_2 are accelerations and cross-sectional areas at sections 1 and 2, respectively.

Equation of continuity: $\qquad A_1 v_1 = A_2 v_2 = Q \qquad (3)$

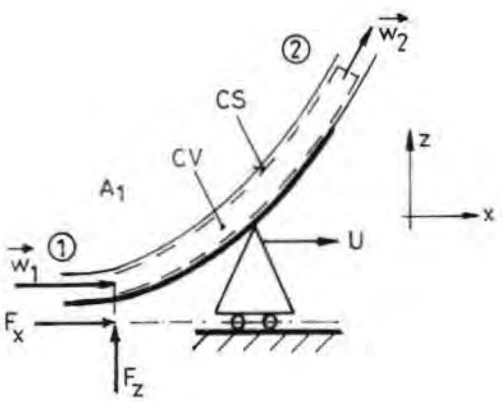
$\frac{d}{dt}(3)$ yields: $\qquad \dot{Q} = A_1 a_1 = A_2 a_2 \qquad (4)$

Collecting terms gives

$$F_x = -p_1A_1 + \rho Q^2\left(\frac{1}{A_2} - \frac{1}{A_1}\right) + \rho\dot{Q}(L_1 + L_2) = -574 \text{ N}$$

4.9

Select control volume (*CV*) moving with the vane at constant velocity, *U*, as shown in the Figure.
CV is inertial



Assumptions:
- Flow is steady relative to the vane
- Magnitude of relative velocity along the vane is constant

$$w = |\vec{w}_1| = |\vec{w}_2| = V - U = 19 \text{ m/s}$$

- Properties are uniform at sections 1 and 2
- The weight of the fluid in the CV may be neglected
- The flow is incompressible
- Frictional effects can be neglected.

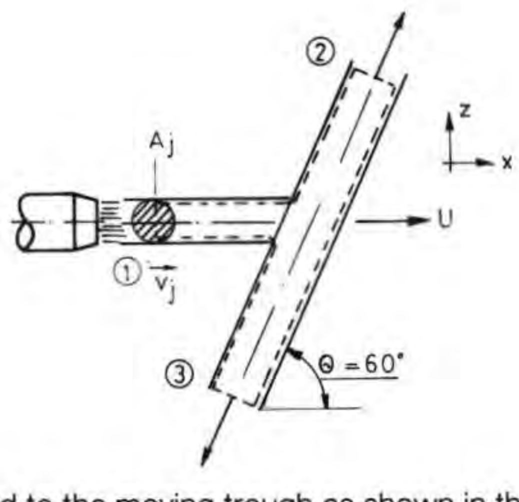
Basic equation:

$$\int_{CV} \rho\vec{f}dV - \int_{CS} pd\vec{A} = \frac{\partial}{\partial t}\int_{CV} \rho\vec{w}\, dV + \int_{CS} \vec{w}\rho\vec{w}\cdot d\vec{A} \tag{1}$$

Taking into account assumptions above, equation (1) can be written as

$$\vec{F} = \rho wA(\vec{w}_2 - \vec{w}_1) = \rho A(V-U)^2\left[(\cos\Theta - 1)\,\vec{i} + \sin\Theta\vec{k}\right] = -361\,\vec{i} + 625.27\,\vec{k} \text{ [N]}$$

4.10



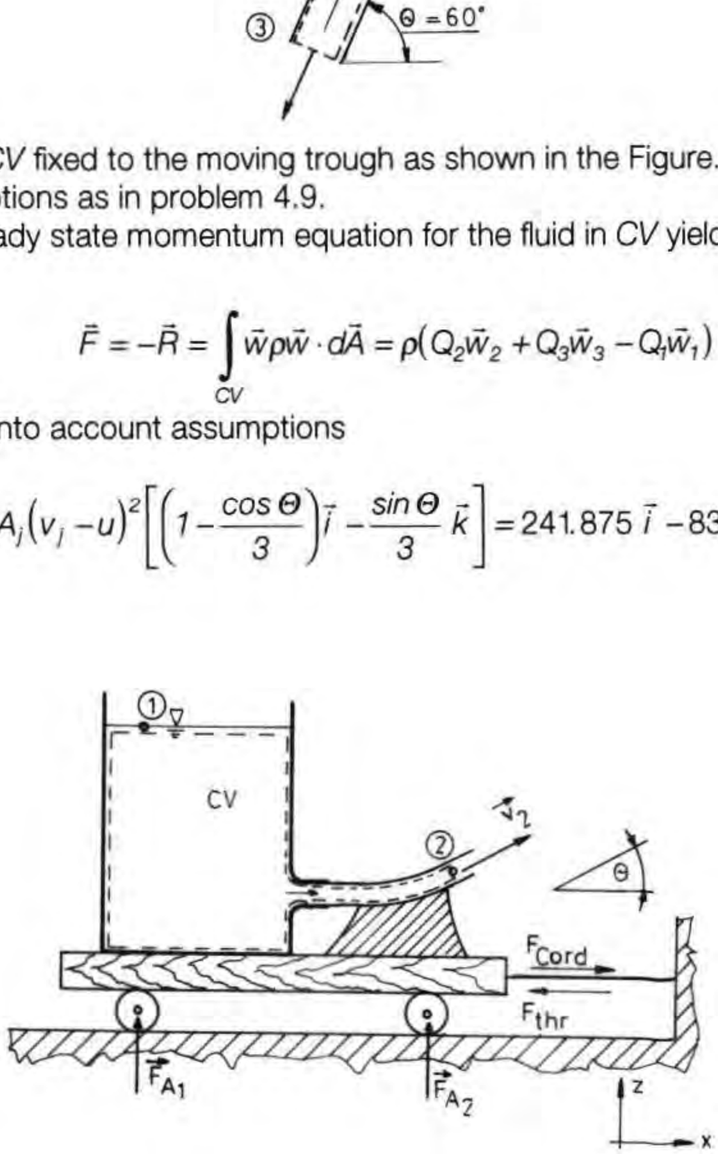
Select *CV* fixed to the moving trough as shown in the Figure.
Assumptions as in problem 4.9.
The steady state momentum equation for the fluid in *CV* yields

$$\vec{F} = -\vec{R} = \int_{CV} \vec{w}\rho\vec{w} \cdot d\vec{A} = \rho\left(Q_2\vec{w}_2 + Q_3\vec{w}_3 - Q_1\vec{w}_1\right)$$

Taking into account assumptions

$$\vec{R} = \rho A_j\left(v_j - u\right)^2\left[\left(1 - \frac{\cos\Theta}{3}\right)\vec{i} - \frac{\sin\Theta}{3}\vec{k}\right] = 241.875\,\vec{i} - 83.788\,\vec{k}\ \text{[N]}$$

4.11



The momentum equation applied for fluid in *CV*:

$$\vec{W} + \vec{F} = \dot{m}(\vec{v}_2 - \vec{v}_1) \tag{1}$$

where $\vec{W}$ is the weight of the fluid in CV
The equilibrium of the cart:

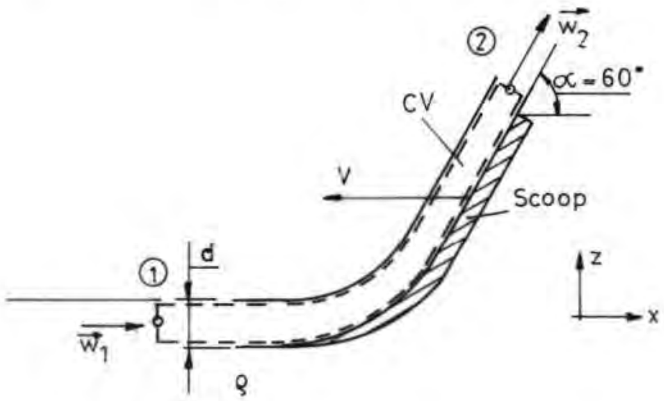
$$\vec{W}_C + \vec{W} + \vec{F}_{A1} + \vec{F}_{A2} - \vec{F} + \vec{F}_{cord} = \vec{o} \tag{2}$$

where W_C ... is the weight of the cart.
The combination of the x components of equations (1) and (2) yields

$$F_{thrust} = -F_{cord} = -F_x = -\dot{m}v_{2x} - \rho A v^2 \cos\Theta = -10.13 \text{ N}$$

4.12

Select *CV* fixed to the scoop (see the Figure).



Assumptions as in Problem 4.9.

The momentum equation for the *CV*:

$$\vec{F} = -\vec{R} = \dot{m}(\vec{w}_2 - \vec{w}_1),$$

where

$$\vec{w}_1 = V\,\vec{i}; \quad \vec{w}_2 = V\left(\cos\alpha\,\vec{i} + \sin\alpha\,\vec{k}\right); \quad \dot{m} = \rho w d V$$

Collecting terms yields:

$$R_x = \rho w d V^2 (1 - \cos\alpha)$$

Applying Newton's 2nd law

$$R_x = -ma$$

gives the initial deceleration of the sled

$$a = -\frac{R_x}{m} = -\frac{\rho w d V^2}{m}(1-\cos\alpha) = -80 \ \ \mathrm{m/s^2}$$

4.13

The fluid entering the sprinkler has no moment of momentum, and no torque is exerted on the system externally; hence the moment of momentum of fluid leaving must be zero. Let ω be the speed of rotation, then the moment of momentum leaving is

$$\rho Q_1 r_1 v_{t1} + \rho Q_2 r_2 v_{t2} = 0 \qquad (1)$$

in which v_{t1} and v_{t2} are the whirl components of the absolute velocities. Then

$$v_{t1} = v_{r1} - \omega r_1 = \frac{Q_1}{A} - \omega r_1; \quad v_{t2} = v_{r2} - \omega r_2 = \frac{Q_2}{A_2} - \omega r_2 \qquad (2)$$

where v_r is the relative velocity.

Substituting equations (2) into (1), yields

$$\omega = \frac{r_1 + r_2}{r_1^2 + r_2^2}\frac{Q}{A} = 11.538 \ \ \mathrm{rad/s}.$$

SOLUTIONS TO CHAPTER 5

5.1 Cauchy-Riemann equations:

$$v_x = \frac{\partial \Psi}{\partial y} = \frac{\partial \Phi}{\partial x} = -4 + 7x \tag{1}$$

$$v_y = -\frac{\partial \Psi}{\partial x} = \frac{\partial \Phi}{\partial y} = -6 - 7y \tag{2}$$

The integration of equations (1) and (2), yields

$$\Phi(x,y) = -4x + \frac{7}{2}\left(x^2 - y^2\right) - 6y + const$$

5.2 Following the procedure shown in the solution of Problem 5.1 we have

$$\Psi(x,y) = 2xy - x + const$$

5.3 From the equation of continuity

$$div\ \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

v_z can be determined:

$$v_z = -2(x+y)z + const$$

5.4 The complex potential and conjugate complex velocity for the four sources

$$W(z) = \frac{Q}{2\pi}\left[ln(z - z_1) + ln(z - z_2) + ln(z - z_3) + ln(z - z_4)\right]$$

$$\bar{v} = \frac{dW}{dz} = \frac{Q}{2\pi}\left[\frac{1}{z - z_1} + \frac{1}{z - z_2} + \frac{1}{z - z_3} + \frac{1}{z - z_4}\right]$$

Substituting the complex co-ordinates of the points $P_1, .., P_4$ and separating the conjugate velocity into real and imaginary parts and finally substituting $y = 0$ and $x = 0$ into these equations to yield

$$v_x(y=0)=\frac{Q}{\pi}\left[\frac{x-a}{(x-a)^2+b^2}+\frac{x+a}{(x+a)^2+b^2}\right]$$

$$v_y(y=0)=0$$
$$v_x(x=0)=0$$

$$v_y(x=0)=\frac{Q}{\pi}\left[\frac{y-b}{a^2+(y-b)^2}+\frac{y+b}{a^2+(y+b)^2}\right]$$

5.5 The complex potential and conjugate complex velocity are

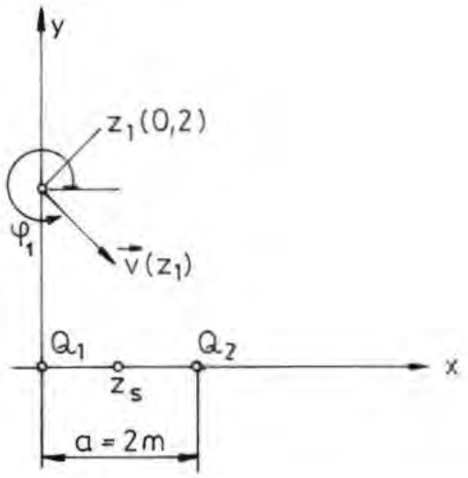
$$W(z)=\frac{i\Gamma}{2\pi}ln(z-z_1)-\frac{i\Gamma}{2\pi}(z-z_2)$$

$$\bar{v}(z)=\frac{dW}{dz}=\frac{i\Gamma}{2\pi}\left(\frac{1}{z-z_1}-\frac{1}{z-z_2}\right)$$

Substitution and separation of the real and imaginary parts, yield

$$v_x(x=0)=0$$
$$v_y(x=0)=\frac{2}{4+(y-1)^2}$$

5.6



$$Q_1=-3\ \mathrm{m^2/s}$$
$$Q_2=-4\ \mathrm{m^2/s}$$

Complex potential:

$$W(z) = \frac{Q_1}{2\pi} \ln z + \frac{Q_2}{2\pi} \ln(z-a) \tag{1}$$

Conjugate complex velocity:

$$\bar{v}(z) = \frac{Q_1}{2\pi}\frac{1}{z} + \frac{Q_2}{2\pi}\frac{1}{z-a} \tag{2}$$

Stagnation point: $\bar{v}(z_s) = 0$

From equation (2): $z_s = \frac{6}{7}$ m

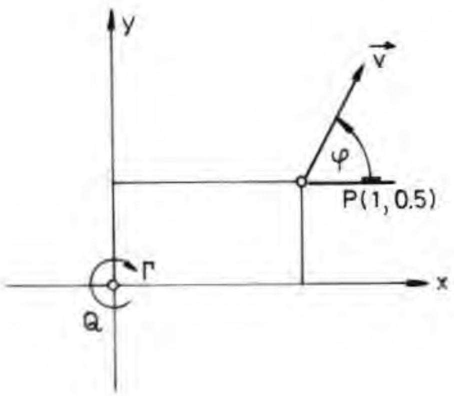
Velocity at z_1

$$\bar{v}(z_1) = \frac{1}{2\pi}(1-2.5i),$$

i.e.,

$$v_x = \frac{1}{2\pi}\ [\mathrm{m/s}];\ v_y = -\frac{2.5}{2\pi}\ [\mathrm{m/s}];\ v = 0.42854\ \mathrm{m/s};\ \varphi_1 = 291.80^\circ$$

5.7



The complex potential:

$$W(z) = \frac{Q + i\Gamma}{2\pi} \ln z = \Phi + i\Psi$$

Separating into real and imaginary parts, yields

$$\Phi(x,y)=\frac{1}{2\pi}\left[Q\ln\sqrt{x^2+y^2}-\Gamma\tan^{-1}\left(\frac{y}{x}\right)\right]$$

$$\Psi(x,y)=\frac{1}{2\pi}\left[Q\tan^{-1}\left(\frac{y}{x}\right)+\Gamma\ln\sqrt{x^2+y^2}\right]$$

The conjugate complex velocity at $z_1 = 1+0.5\,i$

$$\bar{v}_1=\bar{v}(z_1)=\frac{Q+i\,\Gamma}{2\pi}\frac{1}{z_1}=\frac{(Q+i\,\Gamma)\bar{z}_1}{2\pi\,z_1\,\bar{z}_1}=v_{x1}-i\,v_{y1}$$

$$v_{x1}=0.089127\ \text{m/s};\quad v_{y1}=0.114592\ \text{m/s}$$

$$v_1=0.14517\ \text{m/s};\quad \varphi=52.125^{\circ}$$

5.8 The velocity distribution can be written as

$$v(x)=v_{base}-\frac{x}{L}\left(v_{base}-v_{tip}\right)$$

where

$$v_{base}=\frac{4Q}{D^2\pi}\ \text{and}\ v_{tip}=\frac{4Q}{d^2\pi}\tag{1}$$

Acceleration:

$$a(x)=v\frac{dv}{dx}=\frac{v_{base}}{L}\left(v_{tip}-v_{base}\right)+\frac{x}{L^2}\left(v_{tip}-v_{base}\right)^2\tag{2}$$

from which

$$a(x=L/2)=8.9683\ \text{m/s}^2$$

Local acceleration:

$$a_L=\frac{\partial v}{\partial t}=0$$

5.9 From the equation of continuity

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=0$$

$$v_y=-A\frac{y^2}{2}+C(x)$$

where $C(x)$ is an arbitrary function of x.
For irrotationality:

$$\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} = 0$$

whence

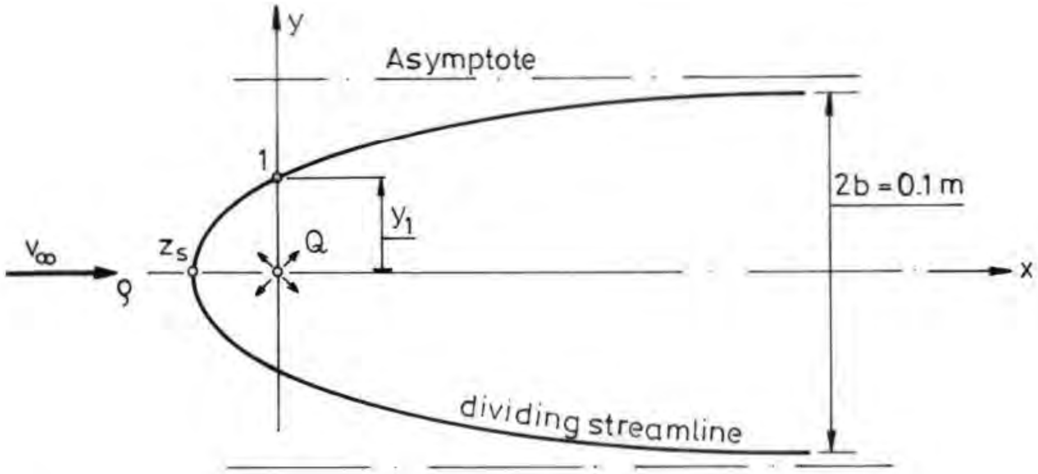
$$v_y = A\frac{x^2}{2} + D(y)$$

where $D(y)$ is an arbitrary function of y.

If we let $D(y) = A\frac{y^2}{2}$ then the equation will also satisty continuity

$$v_y = \frac{A}{2}\left(x^2 - y^2\right)$$

5.10



The complex potential:

$$W(z) = v_\infty z + \frac{Q}{2\pi} ln\, z \tag{1}$$

The stream function:

$$\Psi = v_\infty y + \frac{Q}{2\pi} tan^{-1}\left(\frac{y}{x}\right)$$

or

$$\Psi = v_\infty y + \frac{Q}{2\pi}\varphi \tag{2}$$

where φ is the polar angle.
Stagnation point z_s is on the dividing streamline which separates the 'inner' and 'outer' flow.

At z_s: $\varphi = \pi;\ y = 0$

so the constant Ψ_o for the dividing streamline is

$$\Psi_o = \frac{Q}{2}$$

whence the equation for the dividing streamline is

$$\frac{Q}{2} = v_\infty y + \frac{Q}{2\pi}\varphi \tag{3}$$

When $\varphi \to 0$ then $y \to b$, so from equation (3)

$$Q = 2bv_\infty = 1.5\ \mathrm{m^3/s/m}$$

Differentiating equation (1) yields

$$\bar{v} = v_\infty + \frac{Q}{2\pi z} \tag{4}$$

At the stagnation point $\bar{v}(z_s) = 0$,
so

$$z_s = -\frac{Q}{2\pi\, v_\infty} = -0.01592\ \mathrm{m}$$

Taking into account equation (3) and that

$$\varphi = tan^{-1}\left(\frac{y}{x}\right)$$

the equation of the dividing streamline can be written as

$$x = -y\ cot\left(\frac{2\pi v_\infty}{Q}y\right) = -y\ cot(20\pi y) \tag{5}$$

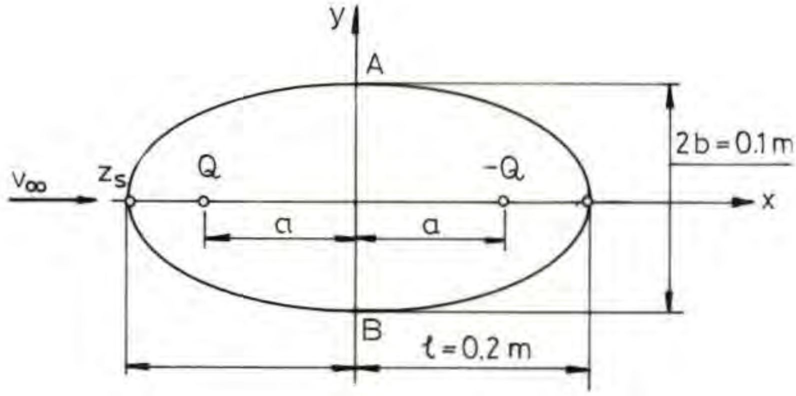
Substituting into equation (5), gives the x co-ordinate of point 1

$$x_1 = 0 \ \ (or\ \varphi_1 = \pi/2)$$

Using the Bernoulli's equation between the stagnation point and point 1, the pressure difference is

$$\Delta p = p_s - p = \frac{\rho}{2}\left(v_\infty^2 + \frac{Q^2}{\pi^2 b^2}\right) = 194.46 \text{ Pa}$$

5.11



Find: *a, Q,* v_{max}

Complex potential:

$$W(z) = v_\infty z + \frac{Q}{2\pi} ln\left(\frac{z+a}{z-a}\right).$$

Conjugate complex velocity:

$$\bar{v}(z) = v_\infty - \frac{Qa}{\pi}\frac{1}{z^2 - a^2} \tag{1}$$

$$\bar{v}\left(z_s = -\frac{\ell}{2}\right) = 0 \rightarrow Q = \frac{\pi}{a}\left(\frac{\ell^2}{4} - a^2\right)v_\infty \tag{2}$$

Flow rate *Q* originated at *z* = -*a* goes through $\overline{AB}$

$$2\int_o^b \bar{v}\big|_{x=0} dy = Q$$

$$2\,bv_\infty + \frac{2Qa}{\pi}\frac{1}{a}\int_0^{b/a}\frac{d(y/a)}{1+(y/a)^2} = 2b\,v_\infty + \frac{2Q}{\pi}\tan^{-1}\left(\frac{b}{a}\right) = Q \qquad (3)$$

Equation (3) can be reshaped

$$b = a\,\cot\left(\frac{b v_\infty \pi}{Q}\right) \qquad (4)$$

Equation (2) and (4) contain two unknowns: Q and a.
The combination of (2) and (4) gives

$$a = b\,\tan\left(\frac{a\,b}{\ell^2/4 - a^2}\right)$$

which by using the method of trial and error gives

$$a = 0.078103\ \text{m}$$

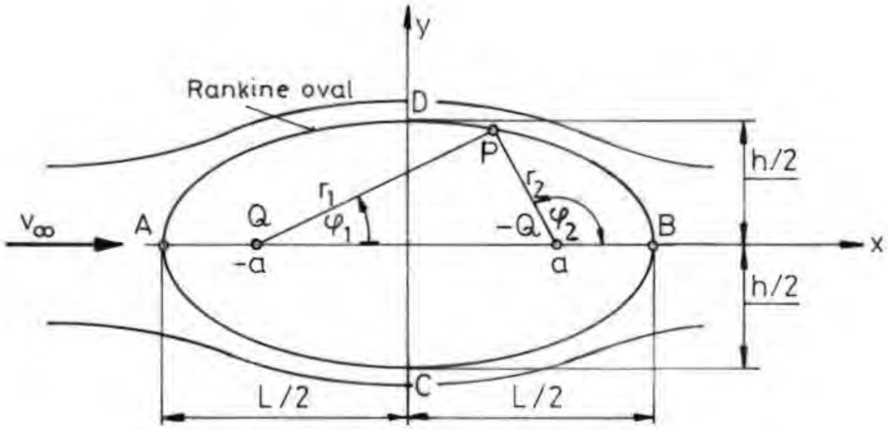
Substituting this result into equation (2) yields

$$Q = 0.470600\ \text{m}^3/\text{s}$$

The maximum velocity outside the oval can be obtained from equation (1):

$$v_{max} = \bar{v}(z = \pm i\,b) = v_\infty + \frac{Qa}{\pi}\frac{1}{a^2+b^2} = 4.3604\ \text{m/s}$$

5.12



$$W(z)=v_\infty z+\frac{Q}{2\pi}\left[ln(z+a)-ln(z-a)\right] \tag{1}$$

Stream function (see the Figure):

$$\Psi=v_\infty y+\frac{Q}{2\pi}(\varphi_1-\varphi_2)$$

At points A and B

$$\psi_A=\Psi(y=0;\ \varphi_1=\varphi_2=\pi)=0;\ \ \Psi_B=\Psi(y=0;\ \varphi_1=\varphi_2=0)=0$$

hence the constant for the Rankine oval is 0 (Ψ = 0)
The derivative of (1) gives

$$\bar{v}=\frac{dW}{az}=v_\infty-\frac{Qa}{\pi}\frac{1}{z^2-a^2}$$

$$\bar{v}(x=0,y)=v_\infty+\frac{Qa}{\pi}\frac{1}{y^2+a^2}=v_x(x=0,y)$$

Equation of continuity (flow rate Q originated at z = -a goes through AB)

$$2\int_0^{h/2}v_x(x=0,y)dy=Q \tag{2}$$

which gives

$$h\,v_\infty+\frac{2Qa}{\pi}\frac{1}{a}\int_0^{h/(2a)}\frac{d(y/a)}{1+(y/a)^2}=h\,v_\infty+\frac{2Q}{\pi}tan^{-1}\left(\frac{h}{2a}\right)=Q$$

The latter equation can easily be reshaped to give

$$\frac{h}{a}=2\ tan\left(\frac{\pi}{2}-\frac{\pi v_\infty h}{2Q}\right)$$

5.13

$$M=R^2v_\infty=80\ \ m^3/s$$

5.14

$F_L=\rho\Gamma v_\infty$

The circulation:

$$\Gamma = \eta D \pi \frac{D}{2} \omega$$

Since $\quad \omega = \frac{2\pi n}{60}$,

$$F_L = \frac{\pi^2 D^2 n \eta \rho v_\infty}{60} = 95.933 \text{ kN/m}$$

5.15

As it is known the velocity around the cylinder can be calculated as

$$v_\varphi(\varphi) = -2v_\infty \sin\varphi - \frac{\Gamma}{2\pi R}$$

(*a*) From Bernoulli's equation between a point at infinity and stagnation point, gives

$$v_\infty = \sqrt{\frac{2(p_s - p\infty)}{\rho}} = \sqrt{20} \text{ m/s}$$

Since

$$v(\varphi_1) = 0 \to \frac{\Gamma}{2\pi R} = -2v_\infty \sin\varphi_1 = \sqrt{60} \text{ m/s}$$

Whence

$$v_\varphi(\varphi) = -2\sqrt{20}\sin\varphi - \sqrt{60}$$

(*b*) $\quad |v_{\max}| = 2\sqrt{20} + \sqrt{60} = 16.6902 \text{ m/s}; \quad \varphi = \frac{\pi}{2}$

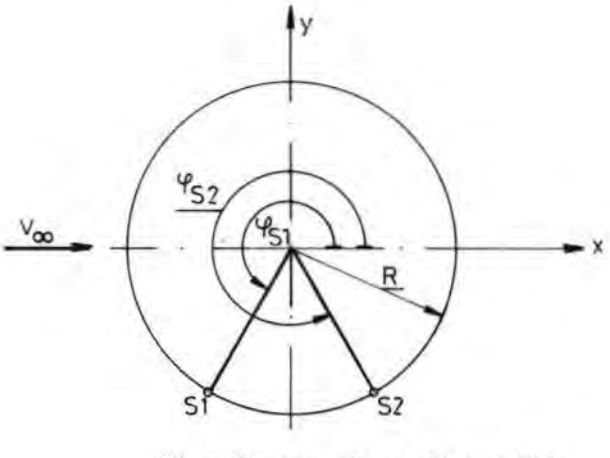
(*c*) $\quad F_L = \rho \Gamma v_\infty = 217.656 \text{ kN/m}$

5.16

$$v_\varphi(\varphi) = -2v_\infty \sin\varphi - \frac{\Gamma}{2\pi R}$$

Since

$$v_\varphi(\varphi_{s1}) = 0 \to \Gamma = -4\pi R v_\infty \sin(\varphi_{s_1})$$



S1 , S2 are stagnation points

Lift coefficient:

$$c_L = \frac{F_L}{\frac{\rho}{2} v_\infty^2 2R} = \frac{\rho \Gamma v_\infty}{\rho R v_\infty^2} = -4\pi \sin(\varphi_{s_1}) = 2\pi\sqrt{3} = 10.8828$$

5.17

The conjugate complex velocity around a circle with radius *a* placed in a uniform stream (see Fig. 1) can be written as

$$\bar{v}_{circle} = \left[2v_\infty \sin(\varphi - \alpha) + \frac{\Gamma}{2\pi a} \right] e^{-i\left(\varphi - \frac{\pi}{2}\right)} \tag{1}$$

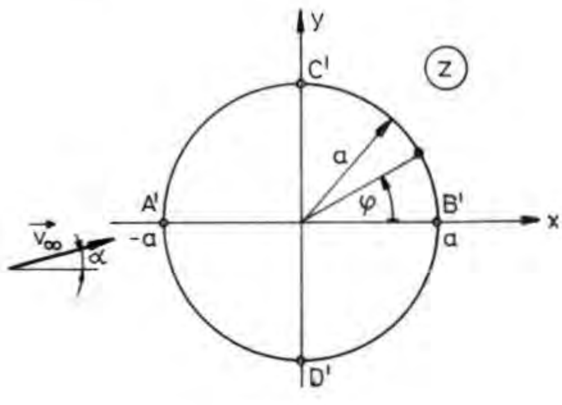


Figure 1

By using the mapping function

$$\zeta = f(z) = z + \frac{a^2}{z} \tag{2}$$

the circle in Figure 1 can be mapped into a plate (see Fig. 2)

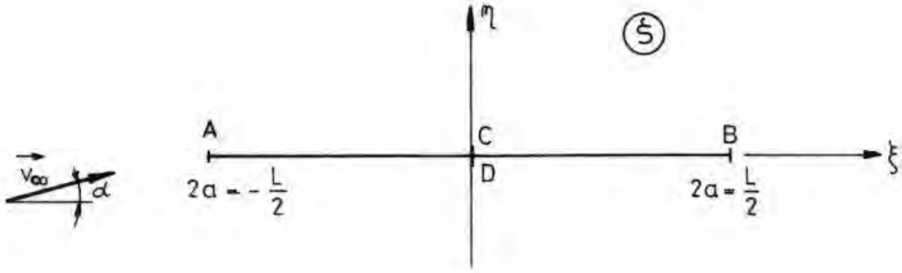


Figure 2

$$\xi = 2x; \quad \eta = 0$$

As it is known

$$\bar{v}_{plate} = \frac{\bar{v}_{circle}}{\left.\frac{df}{dz}\right|_{z_{circle}}} \tag{3}$$

The Kutta condition means that

$$v_{circle}(\varphi = 0) = 0 \rightarrow \frac{\Gamma}{2\pi a} = v_\infty \sin\alpha .$$

The derivative of the mapping function along the circle

$$f'(z_{circle}) = 2 \sin\varphi \; e^{-i\left(\varphi - \frac{\pi}{2}\right)}$$

Hence the velocity around the plate from equation (3) is obtained as

$$\bar{v}_{plate} = v_{plate} = v_\infty \frac{\sin(\varphi - \alpha) + \sin\alpha}{\sin\varphi} \tag{4}$$

v_{plate} becomes infinity at point A ($\varphi = \pi$), but the application of ℓ'Hospital's rule at point B ($\varphi = 0$), gives

$$\lim_{\varphi \to 0} v_{plate} = v_\infty \cos\alpha$$

The pressure difference between points C and D can be obtained from the Bernoulli's equation

$$\Delta p = p_D - p_C = \frac{\rho}{2}\left(v_C^2 - v_D^2\right) = 41.04 \text{ Pa}$$

Here

$$v_C = v_{plate}(\varphi = \pi/2) \text{ and } v_D = v_{plate}(\varphi = 3\pi/2).$$

The lift force

$$F_L = \rho\, \Gamma\, v_\infty = \pi \rho L v_\infty^2 \sin\alpha = 130.9276 \text{ N/m}$$

5.18

Velocity around the plate is given as (see the solution of Problem 5.17)

$$v_{plate} = v_\infty \frac{\sin(\varphi - \alpha) + \sin\alpha}{\sin\varphi}$$

Taking into account the properties of the mapping (the circle into plate) angles belonging to points E and F are easily obtained as

$$\varphi_E = 120^\circ; \quad \varphi_F = 240^\circ.$$

Hence

$$v_E = v_{plate}(\varphi = 120^\circ) = 16.9706 \text{ m/s}$$
$$v_F = v_{plate}(\varphi = 240^\circ) = 6.2117 \text{ m/s}.$$

By using the Bernoulli's equation the pressure difference Δp can also be obtained

$$\Delta p = p_F - p_E = \frac{\rho}{2}\left(v_E^2 - v_F^2\right) = 149.649 \text{ Pa}$$

SOLUTIONS TO CHAPTER 6

6.1 The Navier-Stokes equation for steady, incompressible fluid flow:

$$(\vec{v} \cdot \nabla)\vec{v} = \vec{f} - \frac{1}{\rho}\nabla p + \upsilon \Delta \vec{v} \qquad (1)$$

Here

$(\vec{v} \cdot \nabla)\vec{v} = \vec{0}$ (the flow is supposed to be fully developed)

$\vec{f} = g\vec{i}$ (body force per unit mass)

$\nabla p = \vec{0}$ (see the wording of the text)

$\vec{v} = v(y)\,\vec{i}$

Hence equation (1) reduces to

$$\frac{d^2 v}{dy^2} = -\frac{g}{\nu}$$

the solution of which is

$$v(y) = \frac{g}{2\nu}\left(2y\delta - y^2\right).$$

Here the boundary conditions

$$y = 0: \quad v = 0$$

$$y = 0: \quad \tau = \eta \frac{dv}{dy} = 0$$

were taken into account.
Shear stress distribution:

$$\tau(y) = \eta \frac{dv}{dy} = \rho g(\delta - y)$$

Flow rate:

$$Q = \int_A v dA = \frac{wg\delta^3}{3\upsilon}$$

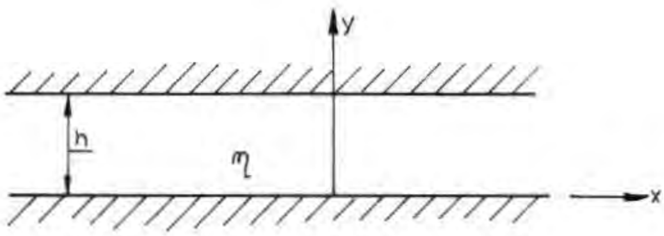
Average velocity:

$$c = \frac{Q}{A} = \frac{g\delta^2}{3\upsilon}$$

The velocity maximum:

$$v_{max} = v(y = \delta) = \frac{g\delta^2}{2\upsilon}$$

6.2



The Navier-Stokes equations for steady, incompressible, fully developed fluid flow is

$$\vec{0} = \vec{f} - \frac{1}{\rho}\nabla p + \upsilon\Delta\vec{v} \tag{1}$$

Here $\vec{f} = -g\ \vec{j}$

$\vec{v} = v_x(y)\ \vec{i}$

Hence the x component of equation (1) is

$$\frac{d^2 v_x}{dy^2} = \frac{1}{\eta}\frac{\partial p}{\partial x} \tag{2}$$

The boundary conditions

$y = 0$: $\quad v_x = 0$

$y = h$: $\quad v_x = 0$

Hence the solution of equation (2):

$$v_x(y) = -\frac{1}{2\eta}\frac{\partial p}{\partial x}\left(hy - y^2\right)$$

The shear stress distribution:

$$\tau(y) = \eta \frac{dv_x}{dy} = -\frac{1}{2}\frac{\partial p}{\partial x}(h - 2y)$$

$$\tau(y = h) = \frac{1}{2}\frac{\partial p}{\partial y} h = -1.8 \ \text{N/m}^2$$

Flow rate per metre of width:

$$\frac{Q}{b} = \int_0^h v_x dy = -\frac{h^3}{12\eta}\frac{\partial p}{\partial x} = 5.4 \times 10^{-6} \ \text{m}^3/\text{s}/\text{m}$$

6.3 The *x* component of the Navier-Stokes equations is reduced to

$$\frac{d^2 v_x}{dy^2} = -\frac{g \sin\alpha}{\upsilon}$$

the solution of which under the boundary conditions

$$y = 0: \quad v_x = 0$$

$$y = \delta: \quad \tau = \eta \frac{dv_x}{dy} = 0$$

is as follows

$$v_x(y) = \frac{g \sin\alpha}{2\upsilon}\left(2y\delta - y^2\right)$$

The shear stress distribution:

$$\tau(y) = \eta \frac{dv_x}{dy} = \rho g \sin\alpha(\delta - y)$$

The flow rate per unit width:

$$Q = \int_A v_x dA = 1 \cdot \int_0 v_x dy = \frac{g\delta^3 \sin\alpha}{3\upsilon} = 1.67424 \times 10^{-3} \ \text{m}^3/\text{s}/\text{m}$$

6.4 The x component of the Navier-Stokes equations is

$$\frac{d^2 v_x}{dy^2} = \frac{g}{\upsilon}. \tag{1}$$

Boundary conditions:

$$y = 0: \quad v_x = U$$

$$y = \mathrm{h}: \quad \tau = \eta \frac{dv_x}{dy} = 0$$

Hence the velocity distribution:

$$v_x(y) = U + \frac{g}{2\upsilon}\left(y^2 - 2hy\right)$$

The shear stress distribution:

$$\tau(y) = \eta \frac{dv_x}{dy} = \frac{g}{\upsilon}(y - h)$$

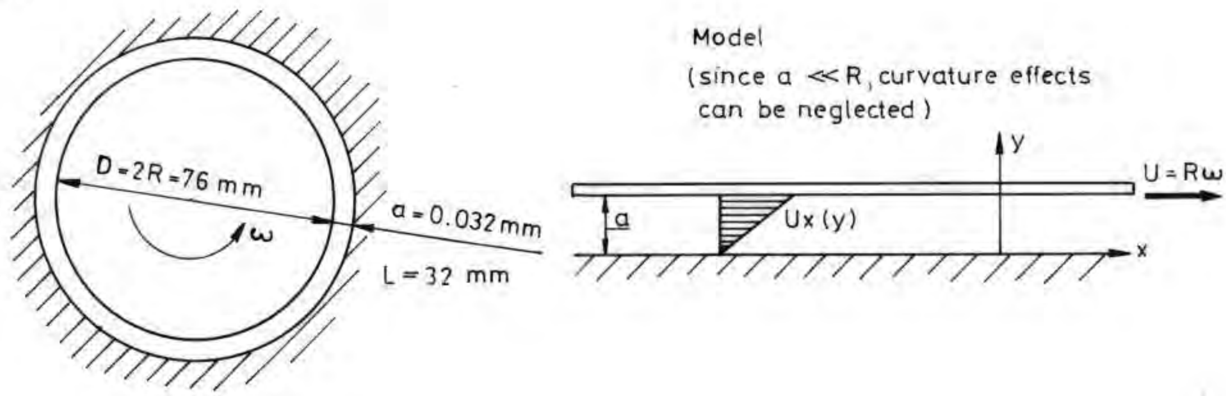
Average velocity:

$$c = \bar{v}_x = \frac{1}{h}\int_o^h v_x(y)dy = U - \frac{gh^2}{3\upsilon} = 0.49673 \ \mathrm{m/s}$$

Control:

$$Re = \frac{ch}{\upsilon} = 0.4967 << \mathrm{Re}_{\mathrm{crit}} = 1500 \rightarrow \text{laminar flow}$$

6.5



Assumptions:

- steady, incompressible, laminar, fully developed flow
- infinite width ($L/a = 32/0.032 = 1000$; so this is a reasonable assumption)
- $\frac{\partial p}{\partial x} = 0$: (flow is symmetric in the actual bearing at no load)

Taking into account the assumptions the solution of the Navier-Stokes equations is

$$v_x(y) = \frac{D}{2}\omega\frac{y}{a}$$

The shear stress

$$\tau = \eta\frac{dv_x}{dy} = \eta\frac{\omega D}{2a} = 4297.7 \ \mathrm{N/m^2}$$

The necessary torque:

$$T = \tau A R = \tau\frac{D^2}{2}\pi L = 1.2478 \ \mathrm{Nm}$$

6.6 The x component of the Navier-Stokes equations:

$$\frac{d^2v_x}{dy^2} = \frac{g}{\upsilon}\sin\alpha$$

Boundary conditions are as in Problem 6.5.
Hence the velocity distribution:

$$v_x(y) = U + \frac{g\sin\alpha}{2\upsilon}\left(y^2 - 2hy\right)$$

Shear stress distribution

$$\tau(y) = \rho g \sin\alpha(y - h)$$

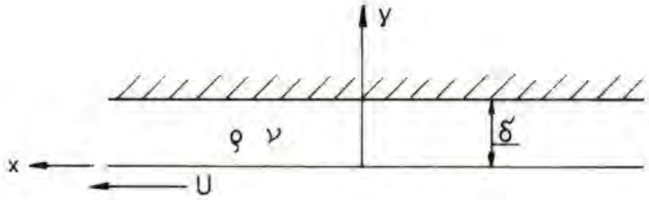
Average velocity:

$$c = \frac{1}{h}\int_0^h v_x(y)dy = U - \frac{gh^2\sin\alpha}{3\upsilon} = 0.4982 \ \mathrm{m/s}$$

Control:

$$Re = \frac{ch}{\upsilon} = 0.4 << \mathrm{Re}_{crit} = 1500 \rightarrow \text{laminar flow}$$

6.7 Since $\delta << D$, the curvature effects can be neglected.
Model:



Boundary conditions:

$$y = 0: \quad v_x = U$$
$$y = \delta: \quad v_x = 0$$

Taking into account these boundary conditions the velocity distribution can be written as

$$v_x(y) = U\left(1 - \frac{y}{\delta}\right) + \frac{1}{2\eta}\frac{\Delta p}{L}\left(y^2 - y\delta\right)$$

The flow rate:

$$Q = D\pi\int_o^\delta v_x(y)dy = D\pi\left[U\frac{\delta}{2} - \frac{\Delta p\delta^3}{12\eta L}\right] = 2.7482\times10^{-6}\ \mathrm{m^3/s} = 2.7482\ \mathrm{cm^3/s}$$

The shear stress at the piston ($y = 0$):

$$\tau_o = -\eta\frac{U}{\delta} - \frac{\Delta p\delta}{2L}$$

The shear force acting on the piston:

$$F_s = \tau_o D\pi L = -D\pi L\left(\eta\frac{U}{\delta} + \frac{\Delta p\delta}{2L}\right) = -149.03\ \mathrm{N}\ (\rightarrow)$$

The pressure force acting upon the piston:

$$F_p = -\frac{D^2\pi}{4}\Delta p = -294.52\ \text{N}\quad (\rightarrow)$$

The total force which is needed to move the piston:

$$F = -F_s - F_p = 443.55\ \text{N}\quad (\leftarrow)$$

6.8 Boundary conditions

$$y = 0:\quad v_x = 0$$
$$y = \text{a}:\quad v_x = U$$

Velocity distribution:

$$v_x(y) = U\frac{y}{a} + \frac{1}{2\upsilon}\left(\frac{1}{\rho}\frac{p_1 - p_2}{L} - g\sin\Theta\right)\left(ay - y^2\right)$$

Shear stress distribution:

$$\tau(y) = \eta\frac{dv_x}{dy} = \eta\frac{U}{a} + \frac{\rho}{2}\left(\frac{1}{\rho}\frac{p_1 - p_2}{L} - g\sin\Theta\right)(a - 2y)$$

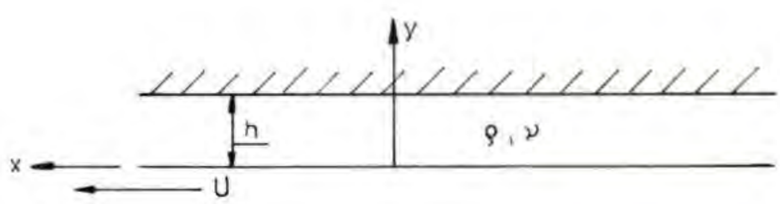
Shear stress acting on the fluid on the upper plate:

$$\tau_o = \tau(y = a) = \eta\frac{U}{a} - \frac{a}{2}\left(\frac{p_1 - p_2}{L} - \rho g\sin\Theta\right) = 51.617\ \text{N/m}^2$$

Force per square metre exerted on the upper plate:

$$\frac{F}{A} = -\tau_o = -51.617\ \text{N/m}^2\ \text{(downward)}$$

6.9



Bounday conditions:

$$y = 0: \quad v_x = U$$
$$y = \text{h}: \quad v_x = 0$$

Velocity distribution:

$$v_x(y) = U\left(1 - \frac{y}{h}\right) - \frac{1}{2\rho\upsilon}\frac{\partial p}{\partial x}\left(hy - y^2\right)$$

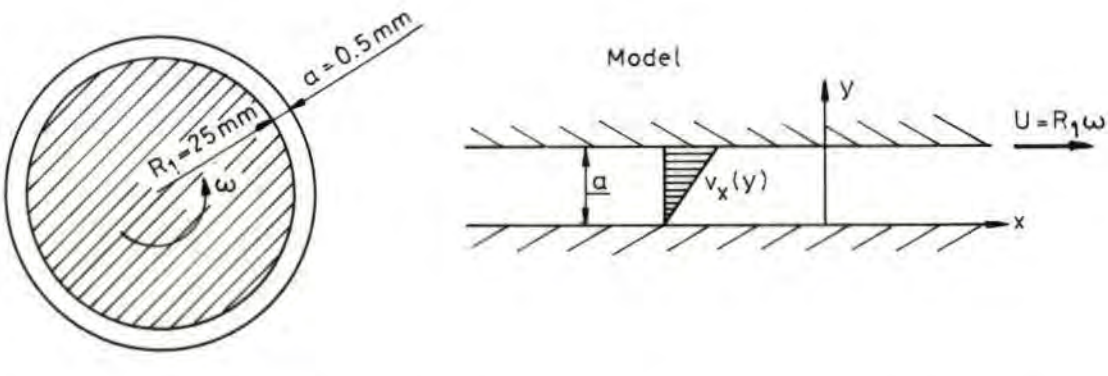
The volume flow rate per unit width must be zero:

$$Q = \int_o^h v(y)dy = U\frac{h}{2} - \frac{1}{2\rho\upsilon}\frac{\partial p}{\partial x}\frac{h^3}{6} = 0$$

Hence

$$\frac{\partial p}{\partial x} = \frac{6\rho\upsilon U}{h^2} = 88.470 \ \text{N/m}^2$$

6.10



The gap width $a = 0.5$ mm is small, so the flow may be modelled as flow between infinite parallel plates.

The velocity distribution:

$$v_x(y) = R_1\,\omega\frac{y}{a}$$

The shear stress:

$$\tau = \eta \frac{dv_x}{dy} = \frac{1}{a} \eta R_1 \omega$$

Torque:

$$T = \tau A R_1 = \eta \frac{R_1 \omega}{a} 2 R_1^2 \pi L \tag{1}$$

Since

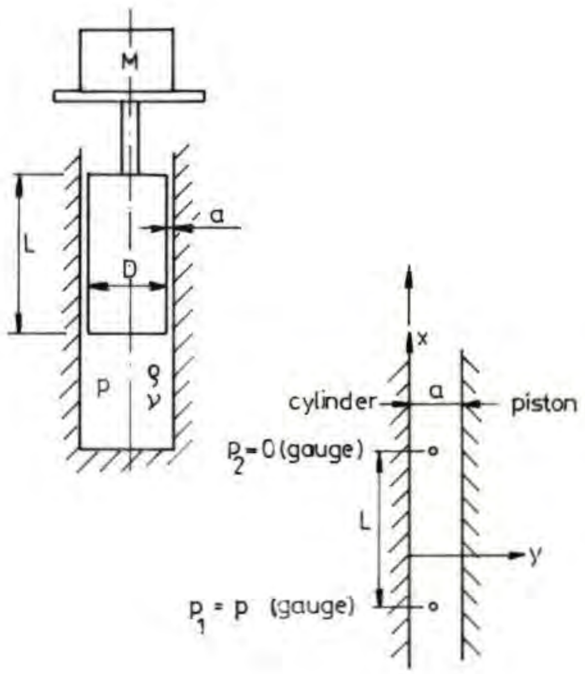
$$\omega = \frac{2\pi n}{60}$$

the viscosity of oil from equation (1) is

$$\eta = \frac{15 a T}{\pi^2 R_1^3 n L} = 0.034739 \ \mathrm{Ns/m^2}$$

Torque will decrease, since the temperature of the oil will increase with time and so its viscosity will decrease.

6.11



Mass producing given gauge pressure in the cylinder:

$$M = \frac{D^2 \pi p}{4g} = 4.323 \text{ kg}$$

Since $a << D$, the curvature effects can be neglected.

Boundary conditions:

$$y = 0, a: \quad v_x = 0$$
$$\left(\text{N.B.: In reality at } x = a: \; v_x = -v_{piston}\right)$$

The velocity distribution

$$v_x = \frac{J}{2\upsilon}\left(ay - y^2\right)$$

where

$$J = \frac{1}{\rho}\frac{p}{L} - g \quad \text{(which is the energy loss per unit mass per unit length)}$$

The leakage flow rate:

$$Q = D\pi \int_o^a v_x(y)dy = \frac{D\pi}{12\upsilon L}\left(\frac{p}{\rho L} - g\right)a^3 = 209408.69 a^3 \; [\text{m}^3/\text{s}] \text{ if a [m]} \tag{1}$$

$$Q \cong \frac{D^2\pi}{4} v_{piston} \tag{2}$$

Hence the combination of equations (1) and (2), yields

$$a_{max} = \sqrt[3]{\frac{3Dv_{piston}\upsilon L}{\frac{p}{\rho L} - g}} = 1.31 \times 10^{-5} \text{m} = 13.1 \; \mu\text{m}$$

6.12 It is known that for laminar incompressible flow in a horizontal pipe

$$Q = \frac{\pi \Delta p D^4}{128 \eta L}$$

From this equation η can be calculated:

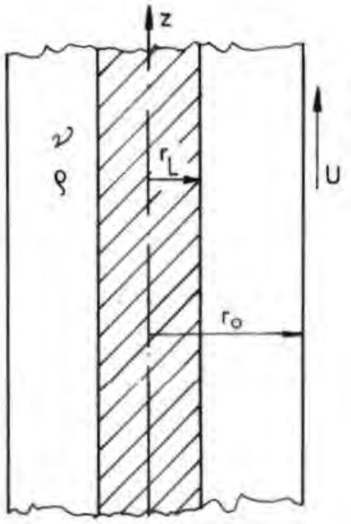
$$\eta = \frac{\pi \Delta p D^4}{128\, QL} = 0.00174\ \mathrm{N/m^2}$$

Checking the Re number:

$$c = \frac{Q}{A} = \frac{4Q}{\pi D^2} = 4.482\ \mathrm{m/s} \quad \text{(average velocity)}$$

$$Re = \frac{\rho c D}{\eta} = 1284.25 < Re_{crit} = 2300 \rightarrow \text{laminar flow}$$

6.13



The z component of the Navier Stokes equations will be

$$0 = \upsilon \Delta v_z$$

which can be written as

$$\frac{\upsilon}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = 0$$

to yield

$$v_z(r) = C_1 \ln r + C_2, \tag{1}$$

where C_1 and C_2 are constants.

The shear stress distribution:

$$\tau(r) = \eta \frac{dv_z}{dr} = \eta \frac{C_1}{r}$$

Constants C_1 and C_2 can be determined by the boundary conditions

$$r = r_i: \quad v_z = 0$$
$$r = r_o: \quad v_z = U$$

to give

$$C_1 = \frac{U}{ln\left(\frac{r_o}{r_i}\right)} \quad \text{and} \quad C_2 = -U\frac{ln(r_i)}{ln\left(\frac{r_o}{r_i}\right)}$$

6.14 The critical Re number can be written as

$$Re_{crit} = 2300 = \frac{c_{crit}\,D}{\upsilon}$$

Critical flow rate:

$$Q_{crit} = \frac{D^2\pi}{4}c_{crit} = \frac{D\pi\upsilon}{4}Re_{crit} = 4.51604\times10^{-7}\ \text{m}^3/\text{s} = 27.096\ \text{cm}^3/\text{min}$$

On the other hand

$$Q = \frac{\pi\,\Delta p\,D^4}{128\rho\upsilon L}$$

whence the pressure drop:

$$\Delta p = \frac{128\ \rho\upsilon LQ}{\pi D^4} = 3.5328\ \text{MPa}$$

6.15

(a) $Q_{max} = \frac{D^2\pi}{4}c_{max} = \frac{D\,\pi\,\upsilon}{4}Re_{crit} = 4.51604\times10^{-7}\ \text{m}^3/\text{s} = 27.096\ \text{cm}^3/\text{min}$

(*b*) $\Delta p = \dfrac{128\ \rho \upsilon L Q}{\pi D^4} = 0.23552$ MPa

(*c*) Since the velocity distribution can be written as

$$v(r) = \frac{\Delta p}{4 \rho \upsilon L}\left(\frac{D^2}{4} - r^2\right)$$

and the shear stress can be obtained as

$$\tau(r) = \eta \frac{dv}{dr} = -\frac{\Delta p}{2L} r$$

$$\tau_{wall} = \left|\tau(r = D/2)\right| = \frac{D \Delta p}{4L} = 294.4\ \mathrm{N/m^2}$$

6.16 From the equilibrium of the cube in *x* direction

$$W \sin \Theta = \tau_{cube} b^2 \tag{1}$$

On the other hand it follows from the Navier-Stokes equations that the velocity distribution of oil below the cube is

$$v_x(y) = U \frac{y}{\delta} \tag{2}$$

where *U* is the constant sliding velocity of the cube.
The shear stress can be determined from equation (2)

$$\tau = \eta \frac{dv_x}{dy} = \frac{\eta U}{\delta} \tag{3}$$

Equating the shear stresses in equations (1) and (3), yields

$$U = \frac{W \delta \sin \Theta}{\eta b^2} = 3.859\ \mathrm{m/s}$$

6.17 As in Problem 6.16:

$$W \sin\Theta = \tau_{board}\, L^2 \tag{1}$$

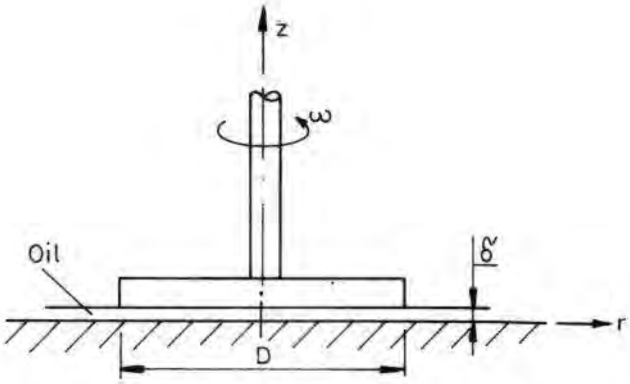
The velocity distribution is linear below the board giving

$$\tau = \eta \frac{d v_x}{dy} = \frac{\eta U}{h} \tag{2}$$

Equating the shear stresses occuring in equations (1) and (2), yields

$$\eta = \frac{H W \sin\Theta}{L^2 U} = 0.28063 \ \mathrm{Ns/m^2}$$

6.18



Cylindrical co-ordinate system should be used. The velocity:

$$\vec{v} = v_\varphi(r,z)\vec{e}_\varphi$$

Boundary conditions:

$$\begin{aligned} z = 0&: \quad v_\varphi = 0 \\ z = \delta&: \quad v_\varphi = r\,\omega \end{aligned} \tag{1}$$

The φ component of the Navier-Stokes equations for this case can be written as

$$0 = \upsilon\left(\frac{\partial^2 v_\varphi}{\partial r^2} + \frac{\partial^2 v_\varphi}{\partial z^2} + \frac{1}{r}\frac{\partial v_\varphi}{\partial r} - \frac{v_\varphi}{r^2}\right) \tag{2}$$

A solution which satisties equation (2) with boundary conditions (1) is

$$v_\varphi(r,z) = r\,\omega\frac{z}{\delta}.$$

The shear stress distribution:

$$\tau_{\varphi z}(r) = \eta \frac{\partial v_\varphi}{\partial z} = \eta \frac{r\omega}{\delta}$$

The torque required:

$$T = \int_A r\tau_{\varphi z}(r)dA = 2\pi \int_0^{D/2} r^2 \tau_{\varphi z}(r)dr = \frac{\pi \eta \omega D^4}{32\delta} = 47.124 \text{ Nm}$$

6.19

$$v_x(y) = \frac{g \sin\alpha}{2\upsilon}\left(2y\delta - y^2\right)$$

$$v_{max} = v_x(y=\delta) = \frac{g\,\delta^2 \sin\alpha}{2\upsilon} = 0.030 \text{ m/s}$$

$$c = \frac{1}{\delta}\int_0^\delta v_x(y)dy = \frac{g\delta^2 \sin\alpha}{3\upsilon} = \frac{2}{3}v_{max} = 0.020 \text{ m/s}$$

6.20 The z component of the Navier-Stokes equations can be written as

$$\frac{d^2 v_z}{dy^2} = \frac{1}{\upsilon}\left(g + \frac{1}{\rho}\frac{\partial p}{\partial z}\right)$$

Taking into account the no-slip condition on the walls we have

$$v_z(y) = \frac{1}{2\upsilon}\left(g + \frac{1}{\rho}\frac{\partial p}{\partial z}\right)\left(y^2 - \delta y\right)$$

$$v_{\max} = v_z(y = \delta/2) = -\frac{\delta^2}{8\upsilon}\left(g + \frac{1}{\rho}\frac{\partial p}{\partial z}\right) = 0.0033075 \text{ m/s (upward)}$$

SOLUTIONS TO CHAPTER 7

7.1 The pressure drop:

$$\Delta p = \rho g h_L = \rho f \frac{L}{D} \frac{8Q^2}{D^4 \pi^2} \tag{1}$$

Reynolds number:

$$Re = \frac{cD}{\upsilon} = \frac{4\rho Q}{\eta D \pi} = 13790.14$$

The relative roughness:

$$\in / D = 8.33 \times 10^{-4}$$

The friction factor f can be obtained from the Moody diagram as a function of Re and $\in/D$

$$f = 0.0293$$

By using this value the distance L to be found can be obtained from equation (1)

$$L = \frac{D^5 \pi^2 \Delta p}{8 \rho f Q^2} = 18.369 \text{ km}$$

7.2 The application of the energy equation between points 1 and 2, gives

$$g(z_1 - z_2) - \frac{c^2}{2} = g h_L \tag{1}$$

where the head loss

$$h_L = f \frac{L}{D} \frac{c^2}{2g} \tag{2}$$

The combination of equations (1) and (2) gives

$$c = \sqrt{\frac{2g\,(z_1 - z_2)}{1 + f\dfrac{L}{D}}} \tag{3}$$

The average velocity c can be calculated by only iteration since the friction factor f also depends on c.

Relative roughness: $\in/D = 0.00092$
1st approximation: the pipe is considered to be completely rough, so

$$f_1 = f(\in/D) = 0.0192 \quad \text{(from Moody diagram)}$$

Substituting this value into equation (3), yields

$$c_1 = 7.353 \ \text{m/s}$$

The Re number:

$$Re_1 = \frac{c_1 D}{\upsilon} = 8.17 \times 10^5$$

The new friction factor from Moody diagram:

$$f_2 = f(Re_1, \in/D) = 0.0195$$

With this

$$c_2 = 7.298 \ \text{m/s}$$

Flow rate

$$Q = \frac{D^2 \pi}{4} c = 0.01433 \ \text{m}^3/\text{s}$$

N.B.: If higher accuracy is expected in c, further iterational steps are needed.

7.3 Relative roughness:

$$\in/D = 0.00023$$

Re number:

$$Re = \frac{cD}{\upsilon} = \frac{4Q}{D\pi\upsilon} = 3.82 \times 10^5$$

Friction factor from Moody chart:

$$f(Re, \in/D) = 0.0154$$

Head loss:

$$h_L = f \frac{L}{D} \frac{8Q^2}{D^4 \pi^2 g} = 14.32 \ \text{m}$$

Power:

$$P = \rho g Q h_L = 8.4107 \ \text{kW}$$

7.4 Writing Newton's second law for the car in the direction of motion gives

$$-F_D = m\frac{dv}{dt} \tag{1}$$

where

$$F_D = \frac{1}{2}c_D \rho v^2 A \tag{2}$$

is the drag force

Substituting (2) into (1), separating variables and integrating, we obtain

$$-\frac{1}{2}c_D \rho \frac{A}{m}\int_0^t dt = \int_{v_o}^{v_1} \frac{dv}{v^2}.$$

Finally,

$$t = \frac{v_o - v_1}{v_1 v_o}\frac{2W}{c_D \rho g A} = 5.545\,s$$

7.5 Energy equation between points ① and ②, gives

$$g(z_1 - z_2) + \frac{p_1 - p_2}{\rho} = g\,h_L \tag{1}$$

where the head loss:

$$h_L = f\frac{L}{D}\frac{c^2}{2g} \tag{2}$$

The application of hydrostatic law to the two legs of the U-tube, gives

$$p_1 + \rho g z_1 = p_2 + \rho g(z_2 - \Delta h) + \rho_m g \Delta h \tag{3}$$

The combination of equations (1) and (2), yields

$$h_L = \left(\frac{\rho_m}{\rho} - 1\right)\Delta h \tag{4}$$

By using equations (2) and (4), f can be obtained

$$f = \left(\frac{\rho_m}{\rho} - 1\right)\frac{D}{L}\frac{2g\Delta h}{c^2} = 0.0327$$

7.6 The average velocity:

$$c = \frac{4Q}{D^2\pi} = 16.977\ \text{m/s}$$

Density of the air:

$$\rho = \frac{p}{RT} = 1.308\ \text{kg/m}^3$$

Reynolds number:

$$Re = \frac{\rho c d}{\eta} = 36\,807$$

From Moody diagram:

$$f = 0.0225$$

Hence the pressure drop:

$$\Delta p = \rho f \frac{L}{d}\frac{c^2}{2} = 141.37\ \text{Pa}$$

7.7 Write the energy equation between water surfaces of the reservoirs

$$z_1 - z_2 = h_L \tag{1}$$

where

$$h_L = \left(K_e + f\frac{L_1 + L_2 + L_3}{d} + 2K_b + K_{Exit}\right)\frac{c^2}{2g} \tag{2}$$

and

$$c = \frac{4Q}{d^2\pi} \tag{3}$$

The combination of equations (1)-(3), yields

$$z_1 = z_2 + \left(K_e + f\frac{L_1 + L_2 + L_3}{d} + 2K_b + K_{Exit}\right)\frac{8Q^2}{d^4\pi^2 g} = 41.662\ \text{m}$$

To determine the magnitude of the minimum pressure, write the energy equation from the upstream reservoir to just downstream of the first bend:

$$g(z_1 - z_b) - \frac{p_b}{\rho} - \frac{c^2}{2} = \left(K_2 + f\frac{L_1}{d} + K_b\right)\frac{c^2}{2} \tag{4}$$

Since

$$z_b = 12\text{ m} + L_2 \sin 45^\circ = 33.213\text{ m}$$

the substitution into equation (4) gives

$$p_b = \rho g(z_1 - z_b) - \rho\left(K_e + f\frac{L_1}{d} + K_b + K_{exit}\right)\frac{8Q^2}{d^4\pi^2} = -2263.9\text{ Pa (gauge)}$$

7.8

$$Re = \frac{cD}{\upsilon} = \frac{4Q\rho}{D\pi\eta} = 20881.1$$

$\in /D = 3.07 \times 10^{-4}$

$f(Re, \in /D) = 0.027$ (from Moody diagram)

Energy equation between points A and B

$$h_L = f\frac{L}{D}\frac{c^2}{2g} = f\frac{L}{D}\frac{8Q^2}{D^4\pi^2 g} = 26.4406\text{ m}$$

whence

$$p_A = p_B + \rho g(z_B - z_A) + \rho g h_L = 723.6\text{ kPa}$$

7.9 The specific energy of fluid at points A and B:

$$e_A - \frac{c^2}{2} = \frac{p_A}{\rho} + g z_A; \quad e_B - \frac{c^2}{2} = \frac{p_B}{\rho} + g z_B$$

From these equation

$$e_B - e_A = \frac{p_B - p_A}{\rho} + g(z_B - z_A) = 41.9\text{ Nm/kg} \tag{1}$$

Since $e_B > e_A$, fluid flows from B to A.

Energy equation between points B and A:

$$\frac{p_B}{\rho} + g z_B = \frac{p_A}{\rho} + g z_A + g h_L \tag{2}$$

where

$$h_L = f\frac{L}{D}\frac{8Q^2}{D^4\pi^2 g} \tag{3}$$

Comparison of equation (1) and (2), yields

$$g\ h_L = e_B - e_A = 41.9\ \text{Nm/kg}$$

The flow rate can be obtained from equation (3):

$$Q = \sqrt{\frac{D^5\pi^2 g\, h_L}{8Lf}} \tag{4}$$

$\in/D = 0.000433$

$$Re = \frac{cD}{\upsilon} = \frac{4Q}{D\pi\upsilon} \tag{5}$$

First trial (Re is unknown)

$f_1 = f(\in/D) = 0.0162$ (Moody diagram)

From (4): $Q_1 = 0.2876\ \text{m}^3/\text{s}$

From (5): $Re_1 = \dfrac{4Q_1}{D\pi\upsilon} = 6.103\times10^5$

Second trial

$f_2 = f(Re_1, \in/D) = 0.017$

From (4): $Q_2 = 0.2807\ \text{m}^3/\text{s}$

The result of the third trial:

$$Q \cong Q_3 = 0.280\ \text{m}^3/\text{s}$$

7.10

$$\frac{p_1}{\rho} + g\ z_1 = \frac{p_2}{\rho} + g\,z_2 + g\ h_L \tag{1}$$

where

$$g\ h_L = \left(f\frac{L}{D} + K_e + K_{exit} \right) \frac{8Q^2}{D^4 \pi^2}$$

$$p_1 = p_2 = p_{atm}$$

The combination of (1) and (2), yields

$$g(z_1 - z_2) = \left(f\frac{L}{D} + K_e + K_{exit} \right) \frac{8Q^2}{D^4 \pi^2} \tag{3}$$

$$Re = \frac{4Q}{D\pi\upsilon} \tag{4}$$

Neglect minor losses and for the first trial assume f = 0.02; then from (3):

$$D \cong \sqrt[5]{\frac{8fLQ^2}{\pi^2 g(z_1 - z_2)}} = 0.5885 \text{ m}$$

Then $Re = \dfrac{4Q}{D\pi\upsilon} = 6.123 \times 10^5$

$\in / D = 4.4 \times 10^{-4}$

$f(Re, \in / D) = 0.0172$

2nd trial: $D = 0.5710$ m.

Use next commericial size larger than 0.571 m.

7.11

$$h_L = f\frac{L}{D}\frac{c^2}{2g} = \frac{8f\,LQ^2}{D^5 \pi^2 g} = 0.03 \text{ L}$$

whence

$$D = \sqrt[5]{\frac{8Q^2 f}{0.03\ \pi^2 g}} \tag{1}$$

$$Re = \frac{cD}{\upsilon} = \frac{4Q}{D\pi\upsilon} \tag{2}$$

Assume $f = 0.015$

From (1) $D = 0.2105$ m

Then $\in / D = 0.00022$; $Re = 6.05 \times 10^4$, so $f = 0.021$

Try again; from (1) $D = 0.225$ mm

Then $\in / D = 0.0002$; $Re = 5.66 \times 10^4$, so $f = 0.0212$

From (1) $D = 0.226$ mm

Use next commercial size larger than 226 mm.

Still assume $\dfrac{h_L}{L} \cong 0.03$

Then $P = \rho g Q h_L = 27.37$ kW (for 1 km long pipeline)

7.12 The energy equation between points 1 and 2, yields

$$\Delta H = h_L = \left(f \frac{\ell_1 + \ell_2 + \ell_3}{\alpha} + K_e + 2K_{elb} + K_{vo} + K_{exit} \right) \frac{c^2}{2g} \qquad (1)$$

Assume $f = 0.015$

Then from (1) $c = 2.647$ m/s.

so $$Re = \frac{cd}{\upsilon} = 2.65 \times 10^5;$$

$f = 0.0186$ (from Moody diagram)

Then from (1) $c = 2.535$ m/s

$$\mathrm{Re} = 2.535 \times 10^5, \quad (\mathrm{O.K.})$$

For 1/2 Q: $Re_1 = 1.268 \times 10^5$; $c_1 = 1.268$ m/s

$f = 0.021$ (Moody diagram)

Writing K_v instead of K_{vo} in equation (1), yields

$$K_v = \frac{2g\,\Delta H}{c_1^2} - \left(f_1 \frac{\ell_1 + \ell_2 + \ell_3}{d} + K_e + 2K_{elb} + K_{exit} \right) = 18.176$$

7.13 The energy equation between points 1 and 2 gives

$$g(H_1 - H_2) = g h_L = \left(f\frac{L_1}{d_1} + K_e \right)\frac{c_1^2}{2} + \left(K_c + f\frac{L_2}{d_2} + K_{exit} \right)\frac{c_2^2}{2} \tag{1}$$

Taking into account the equation of continuity, we obtain

$$H_1 = H_2 + \frac{8Q^2}{g\pi^2}\left[\left(f\frac{L_1}{d_1} + K_e \right)\frac{1}{d_1^4} + \left(K_c + f\frac{L_2}{d_2} + K_{exit} \right)\frac{1}{d_2^4} \right] = 12.42\text{ m} \tag{2}$$

7.14 Energy equation between the water main (m) and the factory (f)

$$\frac{p_m}{\rho} + g\, z_m = \frac{p_f}{\rho} + g\, z_f + g h_L \tag{1}$$

where the head loss:

$$h_L = f\frac{L}{d}\frac{8Q^2}{d^4\pi^2 g} \tag{2}$$

The combination of equations (1) and (2) gives

$$d = \sqrt[5]{\frac{8f\,LQ^2}{\pi^2\left[\dfrac{p_m - p_f}{\rho} + g(z_m - z_f) \right]}} \tag{3}$$

Assume $f = 0.02$; Then from (3)
$d = 0.100$ m

Then $\in/d = 0.002$; $\mathrm{Re} = \frac{\mathrm{cd}}{\upsilon} = \frac{4Q}{d\pi\upsilon} = 3.18\times10^5$
$f = 0.024$ (Moody diagram)
Try again:
from equation (3): $d = 0.1037$ m

Use next commercial size.

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APPENDIX
A
NOTATION

Symbol	Quantity	Unit
a	Acceleration	m/s^2
a	Distance	m
A	Area	m^2
b	Distance	m
c	Distance	m
c_L	Lift coefficient	-
D	Diameter	m
$\underline{\underline{D}}$	Derivative tensor of $\vec{v}$	1/s
d	Diameter	m
d	Distance	m
e	Specific energy	Nm/kg
e	Distance	m
$\vec{f}$	Specific body force vector	m/s^2
f	friction factor	-
$\vec{F}$	Force vector	N
g	Acceleration of gravity	m/s^2
h	Vertical distance	m
H	Vertical distance	m
h_L	Head loss	m
I	Second moment of area	m^4
J	Energy loss per unit mass per unit length	m/s^2
k	Distance	m
K	Minor loss coefficient	-
L	Lift force	N
L	Length	m
l	Length	m
ℓ	Length	m
M	Strength of a doublet	m^3/s
M	Moment	Nm
m	Mass	kg
$\dot{m}$	Mass flow rate	kg/s
$\vec{n}$	Normal unit vector	-
n	Speed of rotation	rev/min
p	Pressure	N/m^2,Pa
P	Pressure potential	N/m, Pa m

P	Power	W
Q	Volume flow rate	m^3/s
Q	Strength of a source/sink	$m^3/s/m$
r	Radius	m
R	Radius	m
R	Distance	m
R	Gas constant	Nm/(kgK)
$\vec{R}$	Force vector	N
Re	Reynolds number	-
s	Distance	m
$\underline{\underline{S}}$	Symmetric part of $\underline{\underline{D}}$	1/s
t	Time	s
T	Time	s
T	Temperature	°C
T	Torque	Nm
u	Velocity, velocity component	m/s
U	Velocity	m/s
$\vec{v}$	Velocity vector	m/s
$\overline{v}(z)$	Complex conjugate velocity	m/s
v	Velocity, velocity component	m/s
V	Velocity	m/s
V	Volume	m^3
w	Relative velocity	m/s
w	Width	m
W	Weight	N
$W(z)$	Complex potential	m^2/s
y	Distance, depth	m
y_C	Distance to centroid of area	m
y_p	Distance to pressure centre	m
z	Vertical distance	m
z	Complex position vector	m
W(z)	Complex potential	m/s^2
α	Angle	-
β	Angle	-
$\dot{\gamma}$	Rate of angular deformation	1/s
δ	Thickness	m
Γ	Circulation	m^2/s
Γ	Strength of the potential vortex	m^2/s
∇	Nabla operator	1/m
Δ	Laplace operator	$1/m^2$
Δh	Manometer reading	m
Δp	Pressure drop	N/m^2

ϵ	Roughness height	m
η	Viscosity	Pa·s
Θ	Angle	-
υ	Kinematic viscosity	m^2/s
Φ	Velocity potential	m^2/s
ρ	Density	kg/m^3
τ	Shear stress	N/m^2
Ψ	Stream function, two dimensions	m^2/s
$\vec{\omega}$	Angular velocity vector	rad/s
ω	Rate of rotatation	rad/s
$\underline{\underline{\Omega}}$	Skew-symmetric part of $\underline{\underline{D}}$	1/s

APPENDIX B
Moody diagram