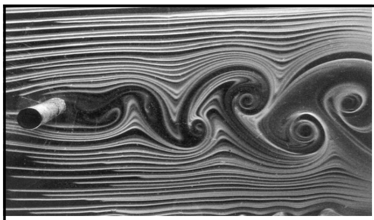
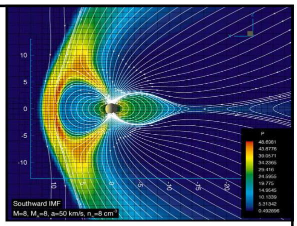
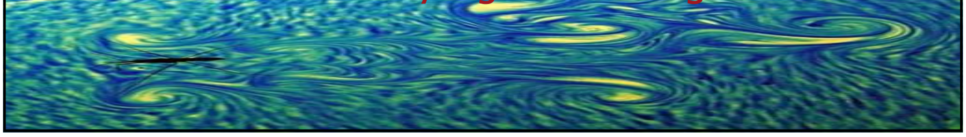




ME 252 Fluid Dynamics 1

Energy, Momentum and Moment of momentum equations and their applications.

Dr. Yesuenyeagbe A.K. Fiagbe



Course Objectives

- Understand the Equation of motion.
- Appreciate Bernoulli Equation and its applications.
- Understand the Equation of Energy.
- Momentum equation and its application.
- Appreciate incompressible flow in pipes and ducts.
- Appreciate issues with Open channel flow

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Areas to Cover

- Mass conservation
- Equations of motion and energy.
  - Bernoulli equation
  - Energy Conservation
- Momentum equation and its applications.
- Moment of momentum equations
- Introduction to incompressible flow in pipes and ducts
- Open Channel flow

3

GENERAL ENERGY EQUATION

- One of the most fundamental laws in nature is the first law of thermodynamics, also known as the conservation of energy principle,
- It states that energy can be neither created nor destroyed during a process; it can only change forms.
- Therefore, every bit of energy must be accounted for during a process.

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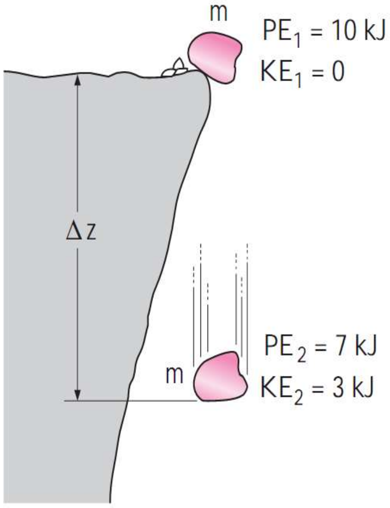
GENERAL ENERGY EQUATION

A rock falling off a cliff, for example, picks up speed as a result of its potential energy being converted to kinetic energy.
Experimental data show that the decrease in potential energy equals the increase in kinetic energy when the air resistance is negligible, thus confirming the conservation of energy principle.



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GENERAL ENERGY EQUATION

- The change in the energy content of a system is equal to the difference between the energy input and the energy output, and
- The conservation of energy principle for any system can be expressed as

$$E_{in} - E_{out} = \Delta E$$

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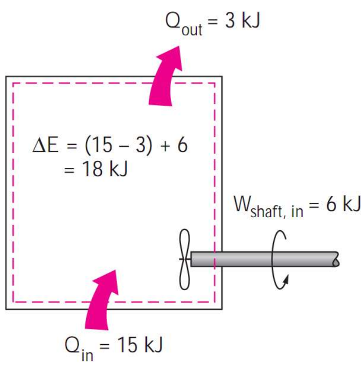
GENERAL ENERGY EQUATION

- The energy content of a fixed quantity of mass (a closed system) can be changed by two mechanisms:
  - heat transfer, Q, and
  - work transfer, W.
- The conservation of energy for a fixed quantity of mass can be expressed in rate form as



$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{dE_{sys}}{dt} \quad \text{or} \quad \dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt}\int_{sys} \rho e \, dV$$

GENERAL ENERGY EQUATION

$$\dot{Q}_{\text{net in}} = \dot{Q}_{in} - \dot{Q}_{out}$$

is the **net rate of heat transfer** to the system (negative, if from the system),

$$\dot{W}_{\text{net in}} = \dot{W}_{in} - \dot{W}_{out}$$

is the **net power input** to the system in all forms (negative, if power output) and

$$dE_{sys}/dt$$

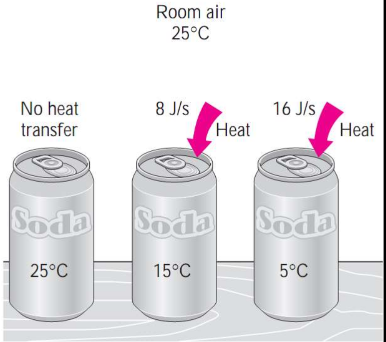
is the rate of change of the total energy content of the system.

Energy Transfer by Heat, Q

- For single-phase substances, a change in the thermal energy of a given mass results in a change in temperature,
- Temperature is a good representative of thermal energy.
- Thermal energy tends to move naturally in the direction of decreasing temperature,
- Transfer of thermal energy from one system to another as a result of a temperature difference is called **heat transfer**.

Energy Transfer by Heat, Q

- The time rate of heat transfer is called **heat transfer rate, Q.**
- Once temperature equality is established, heat transfer stops.
- There cannot be any heat transfer between two systems (or a system and its surroundings) that are at the same temperature.



Energy Transfer by Heat, Q

- A process during which there is no heat transfer is called an **adiabatic process.**
- There are two ways a process can be adiabatic:
  - By insulation so that only a negligible amount of heat can pass through the system boundary, or
  - By maintaining both the system and the surroundings at the same temperature.
- Even though there is no heat transfer during an adiabatic process, the energy content and thus the temperature of a system can still be changed by other means such as work transfer.

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Energy Transfer by Work, W

- An energy interaction is **work** if it is associated with **a force** acting through **a distance.**
- The time rate of doing work is called power and is denoted by W.
- Car engines and hydraulic, steam, and gas turbines produce work
- compressors, pumps, fans, and mixers consume work.

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Energy Transfer by Work, W

- Work-consuming devices transfer energy to the fluid, and thus increase the energy of the fluid.
- A fan in a room, for example, mobilizes the air and increases its kinetic energy.
- The electric energy, a fan consumes is first converted to mechanical energy by its motor that forces the shaft of the blades to rotate.

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Energy Transfer by Work, W

- A system may involve numerous forms of work, and the total work can be expressed as

$$W_{total} = W_{shaft} + W_{pressure} + W_{viscous} + W_{other}$$

  - where W_{shaft} is the work transmitted by a rotating shaft,
  - $W_{pressure}$ is the work done by the pressure forces on the control surface,
  - $W_{viscous}$ is the work done by the normal and shear components of viscous forces on the control surface, and
  - W_{other} is the work done by other forces such as electric, magnetic, and surface tension, which are insignificant for simple compressible systems

$W_{viscous}$ may be neglected since it is usually small relative to other terms in control volume analysis.

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Shaft Work

- Many flow systems involve a machine such as a pump, a turbine, a fan, or a compressor whose shaft protrudes through the control surface, and the work transfer associated with all such devices is simply referred to as shaft work W_{shaft}
- The power transmitted via a rotating shaft is proportional to the shaft torque T_{shaft} and is expressed as

$$\dot{W}_{shaft} = \omega T_{shaft} = 2\pi \dot{n} T_{shaft}$$

  - where ω is the angular speed of the shaft in rad/s and
  - n. is defined as the number of revolutions of the shaft per unit time, often expressed in rev/min

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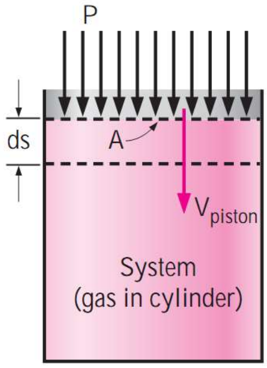
Work Done by Pressure Forces

- Consider a gas being compressed in the piston-cylinder device.
- When the piston moves down a differential distance ds under the influence of the pressure force PA, (A is the cross-sectional area of the piston)
- The boundary work done on the system is $dW_{boundary} = PA\, ds$.
- Dividing both sides of this relation by the differential time interval dt gives the time rate of boundary work (i.e., power),
- where $V_{piston} = ds/dt$ is the piston velocity, (velocity of the moving boundary at the piston face)



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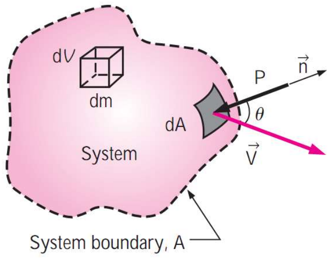
Work Done by Pressure Forces

- Consider a material chunk of fluid (a system) of arbitrary shape, which moves with the flow and is free to deform under the influence of pressure.
- Pressure always acts inward and normal to the surface,
- Pressure force acting on a differential area dA is PdA.
- the time rate at which work is done by pressure forces on this differential part of the system is



$$\delta \dot{W}_{pressure} = -P\, dA\, V_n = -P\, dA(\vec{V} \cdot \vec{n})$$

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Work Done by Pressure Forces

- The total rate of work done by pressure forces is obtained by integrating $dW_{pressure}$ over the entire surface A,

$$\dot{W}_{pressure,\ net\ in} = -\int_A P(\vec{V} \cdot \vec{n})\, dA = -\int_A \frac{P}{\rho}\rho(\vec{V} \cdot \vec{n})\, dA$$

the net power transfer can be expressed as

$$\dot{W}_{net\ in} = \dot{W}_{shaft,\ net\ in} + \dot{W}_{pressure,\ net\ in} = \dot{W}_{shaft,\ net\ in} - \int_A P(\vec{V} \cdot \vec{n})\, dA$$

for a closed system the rate form of the conservation of energy relation becomes

$$\dot{Q}_{net\ in} + \dot{W}_{shaft,\ net\ in} + \dot{W}_{pressure,\ net\ in} = \frac{dE_{sys}}{dt}$$

Total Energy Equation

- the Reynolds transport theorem can be stated as

$$\frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{CV} e\rho \, dV + \int_{CS} e\rho(\vec{V}_r \cdot \vec{n})A$$

$V_r = V - V_{CS}$ is the fluid velocity relative to the control surface, and the product $\rho(Vr \cdot n)dA$ represents the mass flow rate through area element dA into or out of the control volume.

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \frac{d}{dt}\int_{CV} e\rho \, dV + \int_{CS} e\rho(\vec{V}_r \cdot \vec{n})\, dA$$

$$\begin{pmatrix}\text{The net rate of energy}\\ \text{transfer into a CV by}\\ \text{heat and work transfer}\end{pmatrix} = \begin{pmatrix}\text{The time rate of}\\ \text{change of the energy}\\ \text{content of the CV}\end{pmatrix} + \begin{pmatrix}\text{The net flow rate of}\\ \text{energy out of the control}\\ \text{surface by mass flow}\end{pmatrix}$$

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Total Energy Equation

- Recalling $\dot{W}_{\text{pressure, net in}} = -\int_A P(\vec{V} \cdot \vec{n})\, dA = -\int_A \frac{P}{\rho}\rho(\vec{V} \cdot \vec{n})\, dA$

- Then

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{CV} e\rho \, dV + \int_{CS}\left(\frac{P}{\rho} + e\right)\rho(\vec{V}_r \cdot \vec{n})\, dA$$

The term $P/\rho = Pv = w_{flow}$ is the flow work, which is the work associated with pushing a fluid into or out of a control volume per unit mass.
Pressure work for fixed control volumes can exist only along the imaginary part of the control surface where the fluid enters and leaves the control volume, i.e., inlets and outlets.

Total Energy Equation

- For a fixed CV Vr=V

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{CV} e\rho \, dV + \int_{CS}\left(\frac{P}{\rho} + e\right)\rho(\vec{V}\cdot\vec{n})\, dA$$

If $P/\rho + e$ is nearly uniform across an inlet or outlet, we can simply take it outside the integral.

And

$$\dot{m} = \int_{A_c} \rho(\vec{V}\cdot\vec{n})\, dA_c$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{CV} e\rho \, dV + \sum_{\text{out}} \dot{m}\left(\frac{P}{\rho} + e\right) - \sum_{\text{in}} \dot{m}\left(\frac{P}{\rho} + e\right)$$

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Total Energy Equation

- But $e = u + V^2/2 + gz$ is the total energy per unit mass for both the control volume and flow streams.
- Therefore,

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{CV} e\rho \, dV + \sum_{\text{out}} \dot{m}\left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz\right)$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{CV} e\rho \, dV + \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

where we used the definition of enthalpy

$$h = u + Pv = u + P/\rho$$

the subscript "net in" stands for "net input," and thus any heat or work transfer is positive if to the system and negative if from the system.

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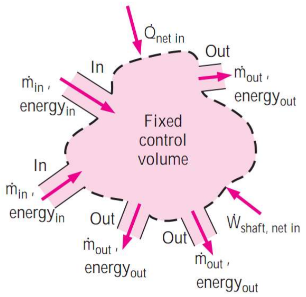
ENERGY ANALYSIS OF STEADY FLOWS

- For steady flows, the time rate of change of the energy content of the control volume is zero, therefore

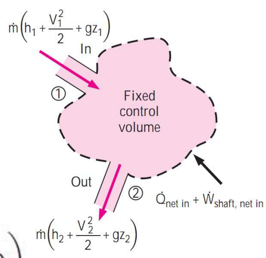


$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

It states that the net rate of energy transfer to a control volume by heat and work transfers during steady flow is equal to the difference between the rates of outgoing and incoming energy flows with mass.

ENERGY ANALYSIS OF STEADY FLOWS

- For many practical problems there is just one inlet and one outlet.
- The mass flow rate remains constant.



$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right)$$

on a unit-mass basis

$$q_{\text{net in}} + w_{\text{shaft, net in}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$q_{\text{net in}} = Q_{\text{net in}}/m$ is the net heat transfer to the fluid per unit mass and $w_{\text{shaft, net in}} = W_{\text{shaft, net in}}/m$ is the net shaft work input to the fluid per unit mass.

ENERGY ANALYSIS OF STEADY FLOWS

- Using the definition of enthalpy **h = u + P/ρ** and rearranging, the steady-flow energy equation becomes

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + (u_2 - u_1 - q_{\text{net in}})$$

For an Ideal flow (no mechanical energy loss)

$$q_{\text{net in}} = u_2 - u_1$$

Mechanical energy loss:

$$e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}}$$

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ENERGY ANALYSIS OF STEADY FLOWS

- The steady-flow energy equation on a unit-mass basis can be written conveniently as a mechanical energy balance as

$$e_{\text{mech, in}} = e_{\text{mech, out}} + e_{\text{mech, loss}}$$

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + e_{\text{mech, loss}}$$

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}} + e_{\text{mech, loss}}$$

$$\dot{m}\left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}}$$

Mechanical energy loss

- The $E_{mech, loss}$ is the total mechanical power loss, consisting of pump and turbine losses as well as the frictional losses in the piping network

$$\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, pump}} + \dot{E}_{\text{mech loss, turbine}} + \dot{E}_{\text{mech loss, piping}}$$

By convention, irreversible pump and turbine losses are treated separately from irreversible losses due to other components of the piping system. Thus the energy equation can be expressed in terms of heads as

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L$$

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Mechanical energy loss

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L$$

$$h_{\text{pump, u}} = \frac{w_{\text{pump, u}}}{g} = \frac{\dot{W}_{\text{pump, u}}}{\dot{m}g} = \frac{\eta_{\text{pump}}\dot{W}_{\text{pump}}}{\dot{m}g}$$

$$h_{\text{turbine, e}} = \frac{w_{\text{turbine, e}}}{g} = \frac{\dot{W}_{\text{turbine, e}}}{\dot{m}g} = \frac{\dot{W}_{\text{turbine}}}{\eta_{\text{turbine}}\dot{m}g}$$

$$h_L = \frac{e_{\text{mech loss, piping}}}{g} = \frac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g}$$

is the irreversible head loss between 1 and 2 due to all components of the piping system other than the pump or turbine.

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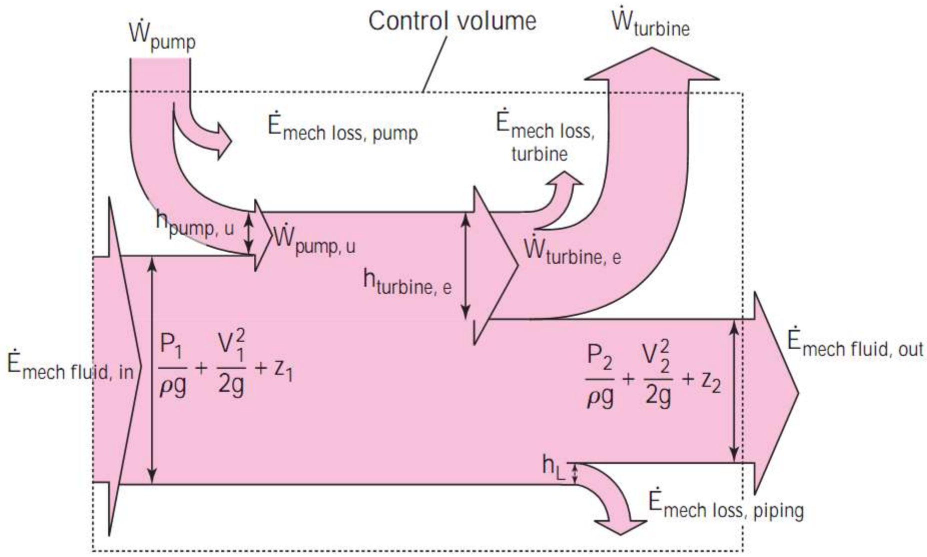
Energy Balance



Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction

- When piping losses are negligible, there is negligible dissipation of mechanical energy into thermal energy,
- thus $h_L = e_{mech\ loss,\ piping} / g = 0$,
- $h_{pump,\ u} = h_{turbine,\ e} = 0$ when there are no mechanical work devices such as fans, pumps, or turbines.
- Then we have

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \qquad \text{or} \qquad \frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$$

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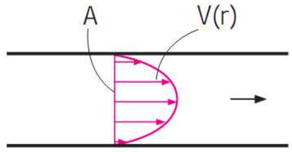
Kinetic Energy Correction Factor, α

- The average flow velocity V_{avg} was defined such that the relation $\rho V_{avg} A$ gives the actual mass flow rate.
- However, the kinetic energy of a fluid stream obtained from $V^2/2$ is not the same as the actual kinetic energy of the fluid stream since the square of a sum is not equal to the sum of the squares of its components.
- This error can be corrected by replacing the kinetic energy terms $V^2/2$ in the energy equation by $\alpha V_{avg}^2 / 2$,
- where α is the kinetic energy correction factor.



$$\dot{m} = \rho V_{avg} A, \qquad \rho = \text{constant}$$

$$\dot{KE}_{act} = \int ke\,\delta\dot{m} = \int_A \frac{1}{2} V^2(r)[\rho V(r)\, dA]$$

$$= \frac{1}{2}\rho \int_A V^3(r)\, dA$$

$$\dot{KE}_{avg} = \frac{1}{2}\dot{m}V_{avg}^2 = \frac{1}{2}\rho A V_{avg}^3$$

$$\alpha = \frac{\dot{KE}_{act}}{\dot{KE}_{avg}} = \frac{1}{A}\int_A \left(\frac{V(r)}{V_{avg}}\right)^3 dA$$

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Kinetic Energy Correction Factor, α

- By using equations for the variation of velocity with the radial distance, it can be shown that the
  - correction factor is 2.0 for fully developed laminar pipe flow,
  - and it ranges between 1.04 and 1.11 for fully developed turbulent flow in a round pipe.
- The kinetic energy correction factors are often ignored (i.e., α is set equal to 1) in an elementary analysis since
  - (1) most flows encountered in practice are turbulent, for which the correction factor is near unity, and
  - (2) the kinetic energy terms are often small relative to the other terms in the energy equation, and multiplying them by a factor less than 2.0 does not make much difference.
- Besides, when the velocity and thus the kinetic energy are high, the flow turns turbulent.

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$$\dot{m}\left(\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{pump} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{turbine} + \dot{E}_{mech,\ loss}$$

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump,\ u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine,\ e} + h_L$$

- If the flow at an inlet or outlet is fully developed turbulent pipe flow, $\alpha = 1.05$ is recommended as a reasonable estimate of the correction factor.
- This leads to a more conservative estimate of head loss, and it does not take much additional effort to include α in the equations.

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Pumping Power and Frictional Heating in a Pump

The pump of a water distribution system is powered by a 15-kW electric motor whose efficiency is 90 percent. The water flow rate through the pump is 50 L/s. The diameters of the inlet and outlet pipes are the same, and the elevation difference across the pump is negligible. If the pressures at the inlet and outlet of the pump are measured to be 100 kPa and 300 kPa (absolute), respectively, determine (a) the mechanical efficiency of the pump and (b) the temperature rise of water as it flows through the pump due to the mechanical inefficiency.

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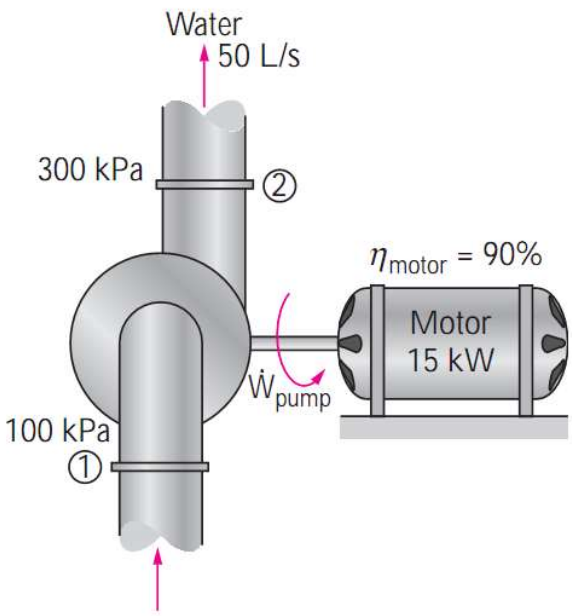

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SOLUTION The pressures across a pump are measured. The mechanical efficiency of the pump and the temperature rise of water are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The pump is driven by an external motor so that the heat generated by the motor is dissipated to the atmosphere. **3** The elevation difference between the inlet and outlet of the pump is negligible, $z_1 \cong z_2$. **4** The inlet and outlet diameters are the same and thus the inlet and outlet velocities and kinetic energy correction factors are equal, $V_1 = V_2$ and $\alpha_1 = \alpha_2$.

Properties We take the density of water to be 1 kg/L = 1000 kg/m^3 and its specific heat to be 4.18 kJ/kg · °C.

Analysis (*a*) The mass flow rate of water through the pump is

$$\dot{m} = \rho \dot{V} = (1 \text{ kg/L})(50 \text{ L/s}) = 50 \text{ kg/s}$$

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The motor draws 15 kW of power and is 90 percent efficient. Thus the mechanical (shaft) power it delivers to the pump is

$$\dot{W}_{\text{pump, shaft}} = \eta_{\text{motor}}\dot{W}_{\text{electric}} = (0.90)(15\text{ kW}) = 13.5\text{ kW}$$

To determine the mechanical efficiency of the pump, we need to know the increase in the mechanical energy of the fluid as it flows through the pump, which is

$$\Delta\dot{E}_{\text{mech, fluid}} = \dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2\frac{V_2^2}{2} + gz_2\right) - \dot{m}\left(\frac{P_1}{\rho} + \alpha_1\frac{V_1^2}{2} + gz_1\right)$$

Simplifying it for this case and substituting the given values,

$$\Delta\dot{E}_{\text{mech, fluid}} = \dot{m}\left(\frac{P_2 - P_1}{\rho}\right) = (50\text{ kg/s})\left(\frac{(300 - 100)\text{ kPa}}{1000\text{ kg/m}^3}\right)\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right) = 10\text{ kW}$$

Then the mechanical efficiency of the pump becomes

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{pump, shaft}}} = \frac{\Delta\dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{pump, shaft}}} = \frac{10\text{ kW}}{13.5\text{ kW}} = 0.741 \quad \text{or} \quad 74.1\%$$

(*b*) Of the 13.5-kW mechanical power supplied by the pump, only 10 kW is imparted to the fluid as mechanical energy. The remaining 3.5 kW is converted to thermal energy due to frictional effects, and this "lost" mechanical energy manifests itself as a heating effect in the fluid,

$$\dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump, shaft}} - \Delta\dot{E}_{\text{mech, fluid}} = 13.5 - 10 = 3.5\text{ kW}$$

The temperature rise of water due to this mechanical inefficiency is determined from the thermal energy balance, $\dot{E}_{\text{mech, loss}} = \dot{m}(u_2 - u_1) = \dot{m}c\Delta T$. Solving for ΔT,

$$\Delta T = \frac{\dot{E}_{\text{mech, loss}}}{\dot{m}c} = \frac{3.5\text{ kW}}{(50\text{ kg/s})(4.18\text{ kJ/kg}\cdot{}^\circ\text{C})} = 0.017^\circ\text{C}$$

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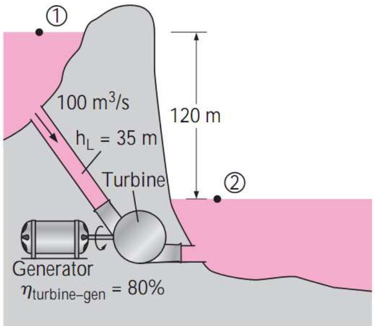
Therefore, the water will experience a temperature rise of 0.017°C due to mechanical inefficiency, which is very small, as it flows through the pump.

Discussion In an actual application, the temperature rise of water will probably be less since part of the heat generated will be transferred to the casing of the pump and from the casing to the surrounding air. If the entire pump motor were submerged in water, then the 1.5 kW dissipated to the air due to motor inefficiency would also be transferred to the surrounding water as heat. This would cause the water temperature to rise more.

38

Hydroelectric Power Generation from a Dam

In a hydroelectric power plant, 100 m^3/s of water flows from an elevation of 120 m to a turbine, where electric power is generated. The total irreversible head loss in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 m. If the overall efficiency of the turbine–generator is 80 percent, estimate the electric power output.



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SOLUTION The available head, flow rate, head loss, and efficiency of a hydroelectric turbine are given. The electric power output is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Water levels at the reservoir and the discharge site remain constant.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The mass flow rate of water through the turbine is

$$\dot{m} = \rho\dot{V} = (1000\ kg/m^3)(100\ m^3/s) = 10^5\ kg/s$$

We take point 2 as the reference level, and thus $z_2 = 0$. Also, both points 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$) and the flow velocities are negligible at both points ($V_1 = V_2 = 0$). Then the energy equation for steady, incompressible flow reduces to

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + \overset{0}{h_{pump,u}} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + \cancelto{0}{z_2} + h_{turbine,e} + h_L \rightarrow$$

$$h_{turbine,e} = z_1 - h_L$$

Substituting, the extracted turbine head and the corresponding turbine power are

$$h_{turbine,e} = z_1 - h_L = 120 - 35 = 85\ m$$

$$\dot{W}_{turbine,e} = \dot{m}gh_{turbine,e} = (10^5\ kg/s)(9.81\ m/s^2)(85\ m)\left(\frac{1\ kJ/kg}{1000\ m^2/s^2}\right) = 83{,}400\ kW$$

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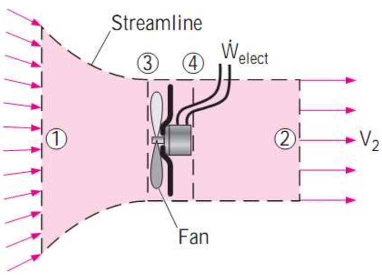
Therefore, a perfect turbine–generator would generate 83,400 kW of electricity from this resource. The electric power generated by the actual unit is

$$\dot{W}_{electric} = \eta_{turbine\text{-}gen}\dot{W}_{turbine,\,e} = (0.80)(83.4\ \text{MW}) = 66.7\ \text{MW}$$

Discussion Note that the power generation would increase by almost 1 MW for each percentage point improvement in the efficiency of the turbine–generator unit.

Fan Selection for Air Cooling of a Computer

A fan is to be selected to cool a computer case whose dimensions are 12 cm X 40 cm X 40 cm. Half of the volume in the case is expected to be filled with components and the other half to be air space. A 5-cm-diameter hole is available at the back of the case for the installation of the fan that is to replace the air in the void spaces of the case once every second. Small low-power fan-motor combined units are available in the market and their efficiency is estimated to be 30 percent. Determine (a) the wattage of the fan-motor unit to be purchased and (b) the pressure difference across the fan. Take the air density to be 1.20 kg/m^3.



SOLUTION A fan is to cool a computer case by completely replacing the air inside once every second. The power of the fan and the pressure difference across it are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Losses other than those due to the inefficiency of the fan–motor unit are negligible ($h_L = 0$). **3** The flow at the outlet is fairly uniform except near the center (due to the wake of the fan motor), and the kinetic energy correction factor at the outlet is 1.10.

Properties The density of air is given to be 1.20 kg/m^3.

Analysis (*a*) Noting that half of the volume of the case is occupied by the components, the air volume in the computer case is

$$V = (\text{Void fraction})(\text{Total case volume})$$
$$= 0.5(12\text{ cm} \times 40\text{ cm} \times 40\text{ cm}) = 9600\text{ cm}^3$$

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Therefore, the volume and mass flow rates of air through the case are

$$\dot{V} = \frac{V}{\Delta t} = \frac{9600\text{ cm}^3}{1\text{ s}} = 9600\text{ cm}^3/\text{s} = 9.6 \times 10^{-3}\text{ m}^3/\text{s}$$

$$\dot{m} = \rho\dot{V} = (1.20\text{ kg/m}^3)(9.6 \times 10^{-3}\text{ m}^3/\text{s}) = 0.0115\text{ kg/s}$$

The cross-sectional area of the opening in the case and the average air velocity through the outlet are

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.05\text{ m})^2}{4} = 1.96 \times 10^{-3}\text{ m}^2$$

$$V = \frac{\dot{V}}{A} = \frac{9.6 \times 10^{-3}\text{ m}^3/\text{s}}{1.96 \times 10^{-3}\text{ m}^2} = 4.90\text{ m/s}$$

We draw the control volume around the fan such that both the inlet and the outlet are at atmospheric pressure ($P_1 = P_2 = P_{atm}$), as shown in Fig. 5–56, and the inlet section 1 is large and far from the fan so that the flow velocity at the inlet section is negligible ($V_1 \cong 0$). Noting that $z_1 = z_2$ and frictional losses in flow are disregarded, the mechanical losses consist of fan losses only and the energy equation (Eq. 5–76) simplifies to

$$\dot{m}\left(\cancel{\frac{P_1}{\rho}} + \alpha_1\frac{V_1^2}{2}^{\nearrow 0} + \cancel{gz_1}\right) + \dot{W}_{fan} = \dot{m}\left(\cancel{\frac{P_2}{\rho}} + \alpha_2\frac{V_2^2}{2} + \cancel{gz_2}\right) + \overset{0}{\dot{W}_{turbine}} + \dot{E}_{mech\ loss,\ fan}$$

Solving for $\dot{W}_{fan} - \dot{E}_{mech\ loss,\ fan} = \dot{W}_{fan,\ u}$ and substituting,

$$\dot{W}_{fan,\ u} = \dot{m}\alpha_2\frac{V_2^2}{2} = (0.0115\text{ kg/s})(1.10)\frac{(4.90\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 0.152\text{ W}$$

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Then the required electric power input to the fan is determined to be

$$\dot{W}_{elect} = \frac{\dot{W}_{fan,\,u}}{\eta_{fan-motor}} = \frac{0.152\ W}{0.3} = 0.506\ W$$

Therefore, a fan–motor rated at about a half watt is adequate for this job.
(*b*) To determine the pressure difference across the fan unit, we take points 3 and 4 to be on the two sides of the fan on a horizontal line. This time again $z_3 = z_4$ and $V_3 = V_4$ since the fan is a narrow cross section, and the energy equation reduces to

$$\dot{m}\frac{P_3}{\rho} + \dot{W}_{fan} = \dot{m}\frac{P_4}{\rho} + \dot{E}_{mech\ loss,\ fan} \quad \rightarrow \quad \dot{W}_{fan,\,u} = \dot{m}\frac{P_4 - P_3}{\rho}$$

Solving for $P_4 - P_3$ and substituting,

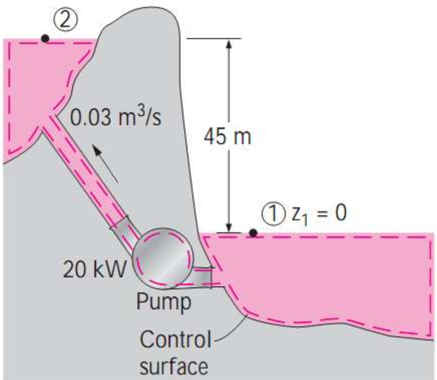
$$P_4 - P_3 = \frac{\rho \dot{W}_{fan,\,u}}{\dot{m}} = \frac{(1.2\ kg/m^3)(0.152\ W)}{0.0115\ kg/s}\left(\frac{1\ Pa \cdot m^3}{1\ Ws}\right) = 15.8\ Pa$$

Therefore, the pressure rise across the fan is 15.8 Pa.

Discussion The efficiency of the fan–motor unit is given to be 30 percent, which means 30 percent of the electric power $\dot{W}_{electric}$ consumed by the unit is converted to useful mechanical energy while the rest (70 percent) is "lost" and converted to thermal energy. Also, a more powerful fan is required in an actual system to overcome frictional losses inside the computer case. Note that if we had ignored the kinetic energy correction factor at the outlet, the required electrical power and pressure rise would have been 10 percent lower in this case (0.460 W and 14.4 Pa, respectively).

Head and Power Loss During Water Pumping

Water is pumped from a lower reservoir to a higher reservoir by a pump that provides 20 kW of useful mechanical power to the water. The free surface of the upper reservoir is 45 m higher than the surface of the lower reservoir. If the flow rate of water is measured to be 0.03 m³/s, determine the irreversible head loss of the system and the lost mechanical power during this process.



SOLUTION Water is pumped from a lower reservoir to a higher one. The head and power loss associated with this process are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The elevation difference between the reservoirs is constant.

Properties We take the density of water to be 1000 kg/m³.

Analysis The mass flow rate of water through the system is

$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(0.03\ \text{m}^3/\text{s}) = 30\ \text{kg/s}$$

We choose points 1 and 2 at the free surfaces of the lower and upper reservoirs, respectively, and take the surface of the lower reservoir as the reference level ($z_1 = 0$). Both points are open to the atmosphere ($P_1 = P_2 = P_{atm}$) and the velocities at both locations are negligible ($V_1 = V_2 = 0$). Then the energy equation for steady incompressible flow for a control volume between 1 and 2 reduces to

$$\dot{m}\left(\cancel{\frac{P_1}{\rho}} + \alpha_1 \frac{V_1^2}{2}^{\nearrow 0} + gz_1^{\nearrow 0}\right) + \dot{W}_{pump}$$

$$= \dot{m}\left(\cancel{\frac{P_2}{\rho}} + \alpha_2 \frac{V_2^2}{2}^{\nearrow 0} + gz_2\right) + \dot{W}_{turbine}^{\ 0} + \dot{E}_{mech,\ loss}$$

$$\dot{W}_{pump} = \dot{m}gz_2 + \dot{E}_{mech,\ loss} \quad \rightarrow \quad \dot{E}_{mech,\ loss} = \dot{W}_{pump} - \dot{m}gz_2$$

Substituting, the lost mechanical power and head loss are determined to be

$$\dot{E}_{mech,\ loss} = 20\ \text{kW} - (30\ \text{kg/s})(9.81\ \text{m/s}^2)(45\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right)$$

$$= 6.76\ \text{kW}$$

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Noting that the entire mechanical losses are due to frictional losses in piping and thus $\dot{E}_{mech,\ loss} = \dot{E}_{mech,\ loss,\ piping}$, the irreversible head loss is determined to be

$$h_L = \frac{\dot{E}_{mech\ loss,\ piping}}{\dot{m}g} = \frac{6.76\ \text{kW}}{(30\ \text{kg/s})(9.81\ \text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)\left(\frac{1000\ \text{N}\cdot\text{m/s}}{1\ \text{kW}}\right) = 23.0\ \text{m}$$

Discussion The 6.76 kW of power is used to overcome the friction in the piping system. Note that the pump could raise the water an additional 23 m if there were no irreversible head losses in the system. In this ideal case, the pump would function as a turbine when the water is allowed to flow from the upper reservoir to the lower reservoir and extract 20 kW of power from the water.

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NEWTON'S LAWS AND CONSERVATION OF MOMENTUM

- Newton's laws are relations between motions of bodies and the forces acting on them.
- Newton's first law states that **a body at rest remains at rest, and a body in motion remains in motion at the same velocity in a straight path when the net force acting on it is zero.**
- Therefore, a body tends to preserve its state of inertia.

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NEWTON'S LAWS AND CONSERVATION OF MOMENTUM

- Newton's second law states that **the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass.**
- Newton's third law states that **when a body exerts a force on a second body, the second body exerts an equal and opposite force on the first.**
- Therefore, the direction of an exposed reaction force depends on the body taken as the system.

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NEWTON'S LAWS AND CONSERVATION OF MOMENTUM

- For a rigid body of mass m, Newton's second law is expressed as

$$\vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d(m\vec{V})}{dt}$$

❑The product of the mass and the velocity of a body is called the linear Momentum of the body.
❑The momentum of a rigid body of mass m moving with a velocity V is mV.
❑Then Newton's second law can also be stated **as the rate of change of the momentum of a body is equal to the net force acting on the body.**

NEWTON'S LAWS AND CONSERVATION OF MOMENTUM

- The momentum of a system remains constant when the net force acting on it is zero,
- This is known as the conservation of momentum principle.

NEWTON'S LAWS AND CONSERVATION OF MOMENTUM

- The counterpart of Newton's second law for rotating rigid bodies is expressed as $M = I\alpha$
  - where M is the net moment or torque applied on the body,
  - I is the moment of inertia of the body about the axis of rotation, and
  - α is the angular acceleration.
- It can also be expressed in terms of the rate of change of angular momentum dH/dt as

$$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

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NEWTON'S LAWS AND CONSERVATION OF MOMENTUM

- ω is the angular velocity.
- For a rigid body rotating about a fixed x-axis, the angular momentum equation can be written in scalar form as

$$M_x = I_x\frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

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NEWTON'S LAWS AND CONSERVATION OF MOMENTUM

- The angular momentum equation can be stated as **the rate of change of the angular momentum of a body is equal to the net torque acting on it.**
- The total angular momentum of a rotating body remains constant when the net torque acting on it is zero,
- Thus, the conservation of angular momentum principle expressed as $I\omega$ = constant.

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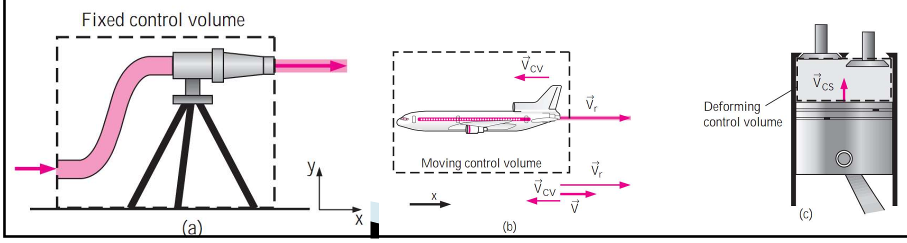
Types of control volume

- Fixed Control Volume
  - To determine the reaction force acting on a tripod holding the nozzle of a hose. This is a fixed control volume, and the water velocity relative to a fixed point on the ground is the same as the water velocity relative to the nozzle exit plane.
- Moving Control Volume
  - To determine the thrust developed by the jet engine of an airplane cruising at constant velocity, for example, The control volume moves with velocity V_{CV}, which is identical to the cruising velocity of the airplane relative to a fixed point on earth.
- Deforming Control Volume
  - To analyze the purging of exhaust gases from a reciprocating internal combustion engine, a deforming control volume is used since part of the control surface moves relative to other parts.



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FORCES ACTING ON A CONTROL VOLUME

- The forces acting on a control volume consist of
  - **Body forces** that act throughout the entire body of the control volume (such as gravity, electric, and magnetic forces) and
  - **Surface forces** that act on the control surface (such as pressure and viscous forces and reaction forces at points of contact).
- In control volume analysis, the sum of all forces acting on the control volume at a particular instant in time is represented by ΣF expressed as

$$\sum \vec{F} = \sum \vec{F}_{body} + \sum \vec{F}_{surface}$$

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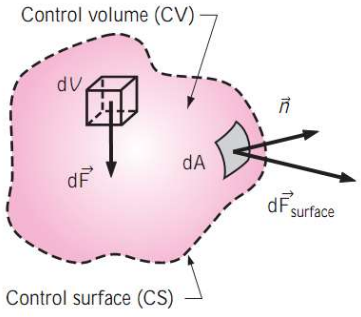
FORCES ACTING ON A CONTROL VOLUME

- Body forces act on each volumetric portion of the control volume.
  - There must be a volume integral to account for the net body force on the entire control volume.
- Surface forces act on each portion of the control surface.
  - There must be an area integral to obtain the net surface force acting on the entire control surface.



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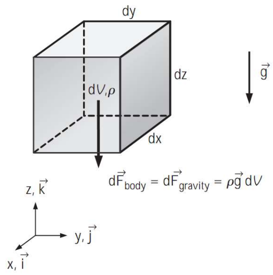
Body force

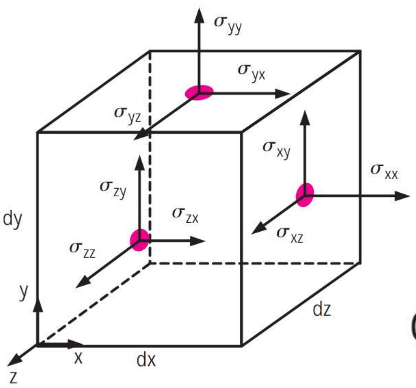
- The most common body force is that of gravity, which exerts a downward force on every differential element of the control volume.
- While other body forces, such as electric and magnetic forces, may be important in some analyses, they are neglected.
- The differential body force $dF_{body} = dF_{gravity}$ acting on the small fluid element and is simply its weight,



$$d\vec{F}_{gravity} = \rho\vec{g}\, dV \qquad \sum \vec{F}_{body} = \int_{CV} \rho\vec{g}\, dV = m_{CV}\vec{g}$$

Surface forces

- Surface forces are not as simple to analyze since they consist of both normal and tangential components.



$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

$$d\vec{F}_{surface} = \sigma_{ij} \cdot \vec{n}\, dA$$

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Surface forces

- The diagonal components of the stress tensor, σ_{xx}, σ_{yy}, and σ_{zz}, are called normal stresses
  - composed of pressure (which always acts inwardly normal) and viscous stresses.
- The off-diagonal components, σ_{xy}, σ_{zx},etc. are shear stresses
  - Shear stresses are composed entirely of viscous stresses.

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

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Surface forces

- Surface force acting on a differential surface element

$$d\vec{F}_{surface} = \sigma_{ij} \cdot \vec{n}\, dA$$

- Finally, integrating over the entire control surface,

$$\sum \vec{F}_{surface} = \int_{CS} \sigma_{ij} \cdot \vec{n}\, dA$$

Total surface force acting on control surface

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Total Force on CV

$$\sum \vec{F} = \sum \vec{F}_{body} + \sum \vec{F}_{surface} = \int_{CV} \rho \vec{g}\, dV + \int_{CS} \sigma_{ij} \cdot \vec{n}\, dA$$

$$\underbrace{\sum \vec{F}}_{\text{total force}} = \underbrace{\sum \vec{F}_{gravity}}_{\text{body force}} + \underbrace{\sum \vec{F}_{pressure} + \sum \vec{F}_{viscous} + \sum \vec{F}_{other}}_{\text{surface forces}}$$

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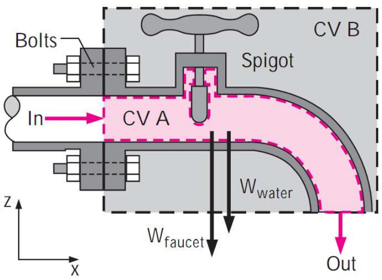
Choice of CV



- CV A (the color red control volume).
  - With this control volume, there are pressure forces that vary along the control surface, there are viscous forces along the pipe wall and at locations inside the valve, and there is a body force, namely, the weight of the water in the control volume.
  - Fortunately, to calculate the net force on the flange, we do not need to integrate the pressure and viscous stresses all along the control surface. Instead, we can lump the unknown pressure and viscous forces together into one reaction force, representing the net force of the walls on the water.
  - This force, plus the weight of the faucet and the water, is equal to the net force on the flange. (We must be very careful with our signs, of course.)

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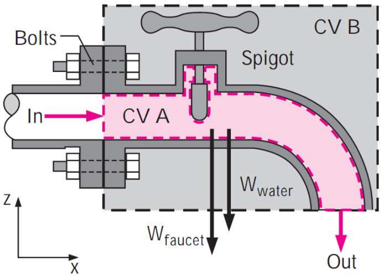
Choice of CV



- Often it is more convenient to slice the control surface through solid objects such as walls, struts, or bolts as illustrated by CV B (the gray control volume)
- A control volume may even surround an entire object, like the one shown here.
- Control volume B is a wise choice because we are not concerned with any details of the flow or even the geometry inside the control volume.
- We assign a net reaction force acting at the portions of the control surface that slice through the flange.
- Then, the only other things we need to know are the gauge pressure of the water at the flange (the inlet to the control volume) and the weights of the water and the faucet assembly.
- The pressure everywhere else along the control surface is atmospheric (zero gage pressure) and cancels out.

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THE LINEAR MOMENTUM EQUATION

- Newton's second law for a system of mass m subjected to a net force F is expressed as

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}(m\vec{V})$$

Noting that both the density and velocity may change from point to point within the system

$$\sum \vec{F} = \frac{d}{dt}\int_{sys} \rho\vec{V}\, dV$$

the Reynolds transport theorem can be expressed for linear momentum as

$$\frac{d(m\vec{V})_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, dV + \int_{CS} \rho\vec{V}\,(\vec{V}_r \cdot \vec{n})\, dA$$

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THE LINEAR MOMENTUM EQUATION

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \int_{CS} \rho \vec{V}(\vec{V}_r \cdot \vec{n})\, dA$$

$$\begin{pmatrix}\text{The sum of all}\\ \text{external forces}\\ \text{acting on a CV}\end{pmatrix} = \begin{pmatrix}\text{The time rate of change}\\ \text{of the linear momentum}\\ \text{of the contents of the CV}\end{pmatrix} + \begin{pmatrix}\text{The net flow rate of}\\ \text{linear momentum out of the}\\ \text{control surface by mass flow}\end{pmatrix}$$

$V_r = V - V_{CS}$ is the fluid velocity relative to the control surface (for use in mass flow rate calculations at all locations where the fluid crosses the control surface), and V is the fluid velocity as viewed from an inertial reference frame.

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THE LINEAR MOMENTUM EQUATION

- For a fixed control volume (no motion or deformation of control volume), $V_r = V$ and the linear momentum equation becomes

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \int_{CS} \rho \vec{V}(\vec{V} \cdot \vec{n})\, dA$$

Note: the momentum equation is a vector equation, and thus each term should be treated as a vector. Also, the components of this equation can be resolved along orthogonal coordinates (such as x, y, and z in the Cartesian coordinate system) for convenience.

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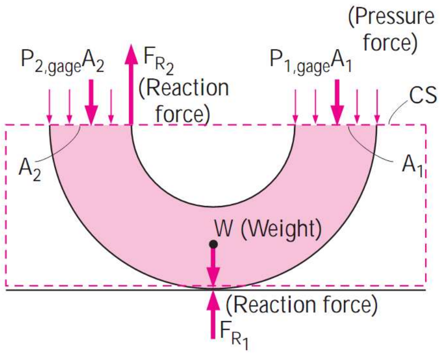
THE LINEAR MOMENTUM EQUATION

- The force F in most cases consists of weights, pressure forces, and reaction forces.
- The momentum equation is commonly used to calculate the forces (usually on support systems or connectors) induced by the flow.



An 180° elbow supported by the ground

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Special Cases

- During steady flow, the amount of momentum within the control volume remains constant,
- The time rate of change of linear momentum of the contents of the control volume is zero

$$\sum \vec{F} = \int_{CS} \rho \vec{V} \, (\vec{V}_r \cdot \vec{n}) \, dA$$

The mass flow rate m into or out of the control volume across an inlet or outlet at which ρ is nearly constant is

$$\dot{m} = \int_{A_c} \rho(\vec{V} \cdot \vec{n}) \, dA_c = \rho V_{avg} A_c$$

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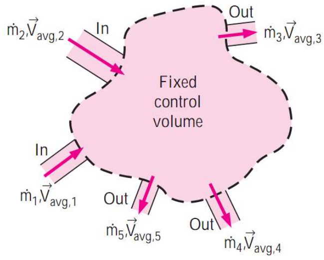
Special Cases

- Momentum flow rate across a uniform inlet or outlet

$$\int_{A_c} \rho \vec{V}(\vec{V} \cdot \vec{n})\, dA_c = \rho V_{avg}\, A_c \vec{V}_{avg} = \dot{m}\vec{V}_{avg}$$



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Momentum-Flux Correction Factor, β

- Unfortunately, the velocity across most inlets and outlets of practical engineering interest is not uniform.
- Nevertheless, it turns out that we can still convert the control surface integral into algebraic form, but a dimensionless correction factor β, called the momentum-flux correction factor, is required,
- for a fixed control volume we have

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \sum_{out} \beta \dot{m}\vec{V}_{avg} - \sum_{in} \beta \dot{m}\vec{V}_{avg}$$

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Momentum-Flux Correction Factor, β

$$\beta = \frac{\int_{A_c} \rho V(\vec{V}\cdot\vec{n})\, dA_c}{\dot{m}V_{avg}} = \frac{\int_{A_c} \rho V(\vec{V}\cdot\vec{n})\, dA_c}{\rho V_{avg} A_c V_{avg}}$$

$$\beta = \frac{1}{A_c}\int_{A_c}\left(\frac{V}{V_{avg}}\right)^2 dA_c$$

❑For fully developed laminar pipe flow β may be above 1.5 (not very close to unity), and ignoring β could potentially lead to significant error.
❑For fully developed turbulent pipe flow, β ranges from about 1.01 to 1.04 (so close to unity), many practicing engineers completely disregard the momentum flux correction factor.

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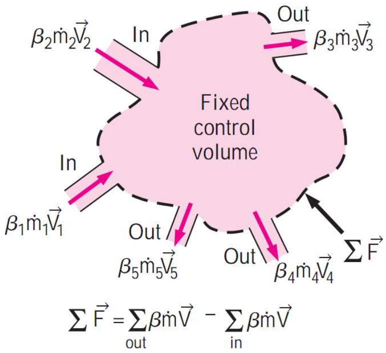
Steady Flow

- If the flow is also steady, the time derivative term vanishes and we are left with



$$\sum\vec{F} = \sum_{out}\beta\dot{m}\vec{V} - \sum_{in}\beta\dot{m}\vec{V}$$

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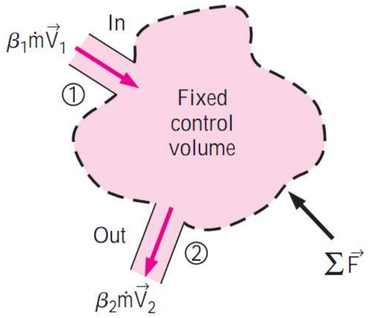
Steady Flow with One Inlet and One Outlet

- Many practical problems involve just one inlet and one outlet.
- The mass flow rate for such single-stream systems remains constant.



$$\Sigma\vec{F} = \dot{m}(\beta_2\vec{V}_2 - \beta_1\vec{V}_1)$$

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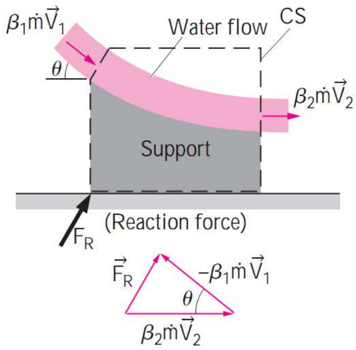
Steady Flow with One Inlet and One Outlet



- Along x-coordinate

$$\sum F_x = \dot{m}(\beta_2 V_{2,x} - \beta_1 V_{1,x})$$

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Flow with No External Forces

- An interesting situation arises when there are no external forces such as weight, pressure, and reaction forces acting on the body in the direction of motion—a common situation for space vehicles and satellites.
- For a control volume with multiple inlets and outlets, we have

$$0 = \frac{d(m\vec{V})_{CV}}{dt} + \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V}$$

Flow with No External Forces

- When the mass m of the control volume remains nearly constant, the first term simply becomes mass times acceleration

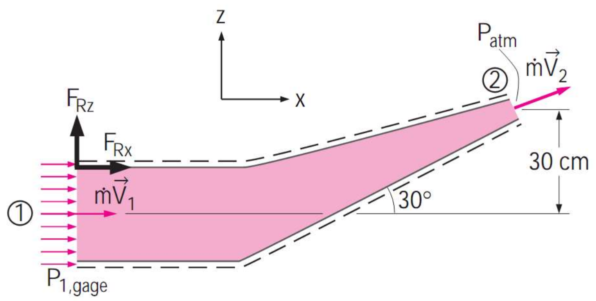
$$\frac{d(m\vec{V})_{CV}}{dt} = m_{CV}\frac{d\vec{V}_{CV}}{dt} = (m\vec{a})_{CV}$$

Therefore, the control volume in this case can be treated as a solid body, with a net force or thrust = 0

$$\vec{F}_{body} = m_{body}\vec{a} = \sum_{in} \beta \dot{m}\vec{V} - \sum_{out} \beta \dot{m}\vec{V}$$

The Force to Hold a Deflector Elbow in Place

A reducing elbow is used to deflect water flow at a rate of 14 kg/s in a horizontal pipe upward 30° while accelerating it. The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 113 cm^2 at the inlet and 7 cm^2 at the outlet. The elevation difference between the centers of the outlet and the inlet is 30 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (a) the gauge pressure at the center of the inlet of the elbow and (b) the anchoring force needed to hold the elbow in place.



SOLUTION A reducing elbow deflects water upward and discharges it to the atmosphere. The pressure at the inlet of the elbow and the force needed to hold the elbow in place are to be determined.

Assumptions **1** The flow is steady, and the frictional effects are negligible. **2** The weight of the elbow and the water in it is negligible. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **4** The flow is turbulent and fully developed at both the inlet and outlet of the control volume, and we take the momentum-flux correction factor to be $\beta = 1.03$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis (*a*) We take the elbow as the control volume and designate the inlet by 1 and the outlet by 2. We also take the *x*- and *z*-coordinates as shown. The continuity equation for this one-inlet, one-outlet, steady-flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 14$ kg/s. Noting that $\dot{m} = \rho AV$, the inlet and outlet velocities of water are

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{14 \text{ kg/s}}{(1000 \text{ kg/m}^3)(0.0113 \text{ m}^2)} = 1.24 \text{ m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{14 \text{ kg/s}}{(1000 \text{ kg/m}^3)(7 \times 10^{-4} \text{ m}^2)} = 20.0 \text{ m/s}$$

We use the Bernoulli equation (Chap. 5) as a first approximation to calculate the pressure. In Chap. 8 we will learn how to account for frictional losses along the walls. Taking the center of the inlet cross section as the reference level ($z_1 = 0$) and noting that $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$P_1 - P_2 = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1\right)$$

$$P_1 - P_{atm} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2) \times \left(\frac{(20\ \text{m/s})^2 - (1.24\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0.3 - 0\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$P_{1,\ gage} = 202.2\ \text{kN/m}^2 = 202.2\ \text{kPa} \quad \text{(gage)}$$

(*b*) The momentum equation for steady one-dimensional flow is

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

We let the x- and z-components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive direction. We also use gage pressure since the atmospheric pressure acts on the entire control surface. Then the momentum equations along the x- and z-axes become

$$F_{Rx} + P_{1,\ gage}A_1 = \beta \dot{m} V_2 \cos\theta - \beta \dot{m} V_1$$

$$F_{Rz} = \beta \dot{m} V_2 \sin\theta$$

Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$F_{Rx} = \beta \dot{m}(V_2 \cos\theta - V_1) - P_{1,\ gage}A_1$$

$$= 1.03(14\ \text{kg/s})[(20 \cos 30° - 1.24)\ \text{m/s}]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$- (202{,}200\ \text{N/m}^2)(0.0113\ \text{m}^2)$$

$$= 232 - 2285 = -2053\ \text{N}$$

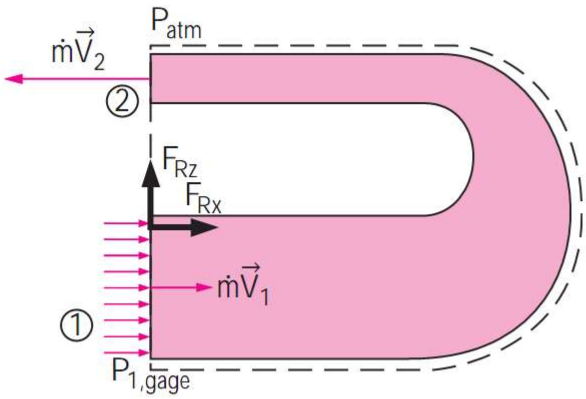
$$F_{Rz} = \beta \dot{m} V_2 \sin\theta = (1.03)(14\ \text{kg/s})(20 \sin 30°\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 144\ \text{N}$$

The negative result for F_{Rx} indicates that the assumed direction is wrong, and it should be reversed. Therefore, F_{Rx} acts in the negative x-direction.

Discussion There is a nonzero pressure distribution along the inside walls of the elbow, but since the control volume is outside the elbow, these pressures do not appear in our analysis. The actual value of $P_{1,\ gage}$ will be higher than that calculated here because of frictional and other irreversible losses in the elbow.

The Force to Hold a Reversing Elbow in Place

The deflector elbow in last example is replaced by a reversing elbow such that the fluid makes a 180° U-turn before it is discharged, as shown in Figure. The elevation difference between the centers of the inlet and the exit sections is still 0.3 m. Determine the anchoring force needed to hold the elbow in place.



SOLUTION The inlet and the outlet velocities and the pressure at the inlet of the elbow remain the same, but the vertical component of the anchoring force at the connection of the elbow to the pipe is zero in this case ($F_{Rz} = 0$) since there is no other force or momentum flux in the vertical direction (we are neglecting the weight of the elbow and the water). The horizontal component of the anchoring force is determined from the momentum equation written in the x-direction. Noting that the outlet velocity is negative since it is in the negative x-direction, we have

$$F_{Rx} + P_{1,\,gage}A_1 = \beta_2\dot{m}(-V_2) - \beta_1\dot{m}V_1 = -\beta\dot{m}(V_2 + V_1)$$

Solving for F_{Rx} and substituting the known values,

$$F_{Rx} = -\beta\dot{m}(V_2 + V_1) - P_{1,\,gage}A_1$$

$$= -(1.03)(14\text{ kg/s})[(20 + 1.24)\text{ m/s}]\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) - (202{,}200\text{ N/m}^2)(0.0113\text{ m}^2)$$

$$= -306 - 2285 = -2591\text{ N}$$

Therefore, the horizontal force on the flange is 2591 N acting in the negative x-direction (the elbow is trying to separate from the pipe). This force is equivalent to the weight of about 260 kg mass, and thus the connectors (such as bolts) used must be strong enough to withstand this force.

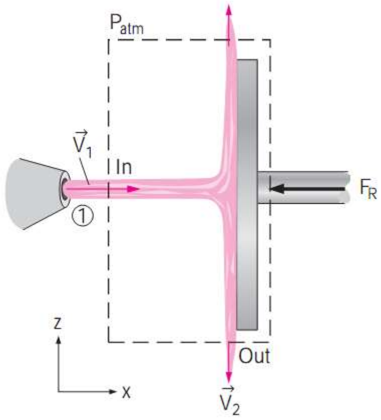
Discussion The reaction force in the x-direction is larger than that of Example 6–2 since the walls turn the water over a much greater angle. If the reversing elbow is replaced by a straight nozzle (like one used by firefighters) such that water is discharged in the positive x-direction, the momentum equation in the x-direction becomes

$$F_{Rx} + P_{1,\,gage}A_1 = \beta\dot{m}V_2 - \beta\dot{m}V_1 \quad\rightarrow\quad F_{Rx} = \beta\dot{m}(V_2 - V_1) - P_{1,\,gage}A_1$$

since both V_1 and V_2 are in the positive x-direction. This shows the importance of using the correct sign (positive if in the positive direction and negative if in the opposite direction) for velocities and forces.

Water Jet Striking a Stationary Plate

Water is accelerated by a nozzle to an average speed of 20 m/s, and strikes a stationary vertical plate at a rate of 10 kg/s with a normal velocity of 20 m/s. After the strike, the water stream splatters off in all directions in the plane of the plate. Determine the force needed to prevent the plate from moving horizontally due to the water stream.



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SOLUTION A water jet strikes a vertical stationary plate normally. The force needed to hold the plate in place is to be determined.

Assumptions **1** The flow of water at nozzle outlet is steady. **2** The water splatters in directions normal to the approach direction of the water jet. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water leaving the control volume is atmospheric pressure, which is disregarded since it acts on the entire system. **4** The vertical forces and momentum fluxes are not considered since they have no

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effect on the horizontal reaction force. **5** The effect of the momentum-flux correction factor is negligible, and thus $\beta \cong 1$.

Analysis We draw the control volume for this problem such that it contains the entire plate and cuts through the water jet and the support bar normally. The momentum equation for steady one-dimensional flow is given as

$$\sum \vec{F} = \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V}$$

Writing it for this problem along the *x*-direction (without forgetting the negative sign for forces and velocities in the negative *x*-direction) and noting that $V_{1,x} = V_1$ and $V_{2,x} = 0$ gives

$$-F_R = 0 - \beta \dot{m}\vec{V}_1$$

Substituting the given values,

$$F_R = \beta \dot{m}\vec{V}_1 = (1)(10 \text{ kg/s})(20 \text{ m/s})\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 200 \text{ N}$$

Therefore, the support must apply a 200-N horizontal force (equivalent to the weight of about a 20-kg mass) in the negative *x*-direction (the opposite direction of the water jet) to hold the plate in place.

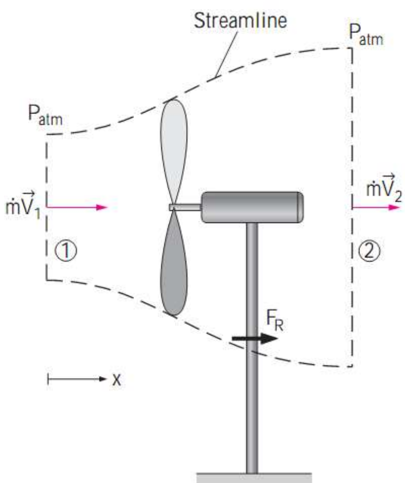
Discussion The plate absorbs the full brunt of the momentum of the water jet since the *x*-direction momentum at the outlet of the control volume is zero. If the control volume were drawn instead along the interface between the water and the plate, there would be additional (unknown) pressure forces in the analysis. By cutting the control volume through the support, we avoid having to deal with this additional complexity. This is an example of a "wise" choice of control volume.

Power Generation and Wind Loading of a Wind Turbine

A wind generator with a 30-ft-diameter blade span has a cut-in wind speed (minimum speed for power generation) of 7 mph, at which velocity the turbine generates 0.4 kW of electric power.

Determine (a) the efficiency of the wind turbine-generator unit and (b) the horizontal force exerted by the wind on the supporting mast of the wind turbine.

What is the effect of doubling the wind velocity to 14 mph on power generation and the force exerted? Assume the efficiency remains the same, and take the density of air to be 0.076 lbm/ft^3



SOLUTION The power generation and loading of a wind turbine are to be analyzed. The efficiency and the force exerted on the mast are to be determined, and the effects of doubling the wind velocity are to be investigated.

Assumptions **1** The wind flow is steady and incompressible. **2** The efficiency of the turbine–generator is independent of wind speed. **3** The frictional effects are negligible, and thus none of the incoming kinetic energy is converted to thermal energy. **4** The average velocity of air through the wind turbine is the same as the wind velocity (actually, it is considerably less—see the discussion that follows the example). **5** The wind flow is uniform and thus the momentum-flux correction factor is $\beta \cong 1$.

Properties The density of air is given to be 0.076 lbm/ft³.

Analysis Kinetic energy is a mechanical form of energy, and thus it can be converted to work entirely. Therefore, the power potential of the wind is proportional to its kinetic energy, which is $V^2/2$ per unit mass, and thus the maximum power is $\dot{m}V^2/2$ for a given mass flow rate:

$$V_1 = (7 \text{ mph})\left(\frac{1.4667 \text{ ft/s}}{1 \text{ mph}}\right) = 10.27 \text{ ft/s}$$

$$\dot{m} = \rho_1 V_1 A_1 = \rho_1 V_1 \frac{\pi D^2}{4} = (0.076 \text{ lbm/ft}^3)(10.27 \text{ ft/s}) \frac{\pi (30 \text{ ft})^2}{4} = 551.7 \text{ lbm/s}$$

$$\dot{W}_{max} = \dot{m}ke_1 = \dot{m}\frac{V_1^2}{2}$$

$$= (551.7 \text{ lbm/s}) \frac{(10.27 \text{ ft/s})^2}{2} \left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right)\left(\frac{1 \text{ kW}}{737.56 \text{ lbf} \cdot \text{ft/s}}\right)$$

$$= 1.225 \text{ kW}$$

Therefore, the available power to the wind turbine is 1.225 kW at the wind velocity of 7 mph. Then the turbine–generator efficiency becomes

$$\eta_{\text{wind turbine}} = \frac{\dot{W}_{act}}{\dot{W}_{max}} = \frac{0.4 \text{ kW}}{1.225 \text{ kW}} = 0.327 \qquad (\text{or } 32.7\%)$$

(*b*) The frictional effects are assumed to be negligible, and thus the portion of incoming kinetic energy not converted to electric power leaves the wind turbine as outgoing kinetic energy. Noting that the mass flow rate remains constant, the exit velocity is determined to be

$$\dot{m}ke_2 = \dot{m}ke_1(1 - \eta_{\text{wind turbine}}) \quad \rightarrow \quad \dot{m}\frac{V_2^2}{2} = \dot{m}\frac{V_1^2}{2}(1 - \eta_{\text{wind turbine}})$$

or

$$V_2 = V_1\sqrt{1 - \eta_{\text{wind turbine}}} = (10.27 \text{ ft/s})\sqrt{1 - 0.327} = 8.43 \text{ ft/s}$$

We draw a control volume around the wind turbine such that the wind is normal to the control surface at the inlet and the outlet and the entire control surface is at atmospheric pressure. The momentum equation for steady one-dimensional flow is given as

$$\sum \vec{F} = \sum_{\text{out}} \beta\dot{m}\vec{V} - \sum_{\text{in}} \beta\dot{m}\vec{V}$$

Writing it along the *x*-direction and noting that $\beta = 1$, $V_{1,x} = V_1$, and $V_{2,x} = V_2$ give

$$F_R = \dot{m}V_2 - \dot{m}V_1 = \dot{m}(V_2 - V_1)$$

Substituting the known values gives

$$F_R = \dot{m}(V_2 - V_1) = (551.7 \text{ lbm/s})(8.43 - 10.27 \text{ ft/s})\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right)$$
$$= -31.5 \text{ lbf}$$

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The negative sign indicates that the reaction force acts in the negative *x*-direction, as expected. Then the force exerted by the wind on the mast becomes $F_{mast} = -F_R =$ **31.5 lbf.**

The power generated is proportional to V^3 since the mass flow rate is proportional to V and the kinetic energy to V^2. Therefore, doubling the wind velocity to 14 mph will increase the power generation by a factor of $2^3 = 8$ to $0.4 \times 8 = 3.2$ kW. The force exerted by the wind on the support mast is proportional to V^2. Therefore, doubling the wind velocity to 14 mph will increase the wind force by a factor of $2^2 = 4$ to $31.5 \times 4 = 126$ lbf.

Discussion To gain more insight into the operation of devices with propellers or turbines such as helicopters, wind turbines, hydraulic turbines, and turbofan engines, we reconsider the wind turbine and draw two streamlines, as shown in Fig. 6–24. (In the case of power-consuming devices such as a fan and a helicopter, the streamlines converge rather than diverge since the exit velocity will be higher and thus the exit area will be lower.) The upper and lower streamlines can be considered to form an "imaginary duct" for the flow of air through the turbine. Sections 1 and 2 are sufficiently far from the turbine so that $P_1 = P_2 = P_{atm}$. The momentum equation for this large control volume between sections 1 and 2 was obtained to be

$$F_R = \dot{m}(V_2 - V_1) \qquad \textbf{(1)}$$

The smaller control volume between sections 3 and 4 encloses the turbine, and $A_3 = A_4 = A$ and $V_3 = V_4$ since it is so slim. The turbine is a device that causes a pressure change, and thus the pressures P_3 and P_4 are different. The momentum equation applied to the smaller control volume gives

$$F_R + P_3A - P_4A = 0 \quad \rightarrow \quad F_R = (P_4 - P_3)A \qquad \textbf{(2)}$$

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The Bernoulli equation is not applicable between sections 1 and 2 since the path crosses a turbine, but it is applicable separately between sections 1 and 3 and sections 4 and 2:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + z_3 \quad \text{and} \quad \frac{P_4}{\rho g} + \frac{V_4^2}{2g} + z_4 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Adding these two equations and noting that $z_1 = z_2 = z_3 = z_4$, $V_3 = V_4$, and $P_1 = P_2 = P_{atm}$ gives

$$\frac{V_2^2 - V_1^2}{2} = \frac{P_4 - P_3}{\rho} \qquad \textbf{(3)}$$

Substituting $\dot{m} = \rho A V_3$ into Eq. 1 and then combining it with Eqs. 2 and 3 gives

$$V_3 = \frac{V_1 + V_2}{2} \qquad \textbf{(4)}$$

Thus we conclude that *the average velocity of a fluid through a turbine is the arithmetic average of the upstream and downstream velocities.* Of course, the validity of this result is limited by the applicability of the Bernoulli equation.

Now back to the wind turbine. The velocity through the turbine can be expressed as $V_3 = V_1(1 - a)$, where $a < 1$ since $V_3 < V_1$. Combining this expression with Eq. 4 gives $V_2 = V_1(1 - 2a)$. Also, the mass flow rate through the turbine becomes $\dot{m} = \rho A V_3 = \rho A V_1(1 - a)$. When the frictional

effects and losses are neglected, the power generated by a wind turbine is simply the difference between the incoming and the outgoing kinetic energies:

$$\dot{W} = \dot{m}(ke_1 - ke_2) = \frac{\dot{m}(V_1^2 - V_2^2)}{2} = \frac{\rho A V_1(1 - a)[V_1^2 - V_1^2(1 - 2a)^2]}{2}$$

$$= 2\rho A V_1^3 a(1 - a)^2$$

Dividing this by the available power of the wind $\dot{W}_{max} = \dot{m}V_1^2/2$ gives the efficiency of the wind turbine in terms of a,

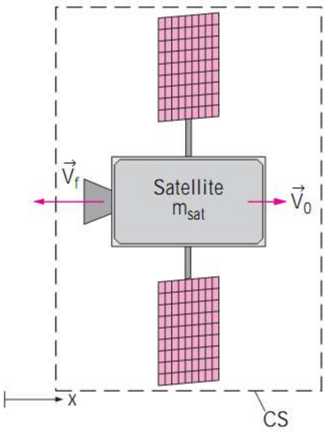
$$\eta_{\text{wind turbine}} = \frac{\dot{W}}{\dot{W}_{max}} = \frac{2\rho A V_1^3 a(1 - a)^2}{(\rho A V_1)V_1^2/2}$$

The value of a that maximizes the efficiency is determined by setting the derivative of $\eta_{\text{wind turbine}}$ with respect to a equal to zero and solving for a. It gives $a = 1/3$. Substituting this value into the efficiency relation just presented gives $\eta_{\text{wind turbine}} = 16/27 = 0.593$, which is the upper limit for the efficiency of wind turbines and propellers. This is known as the **Betz limit.** The efficiency of actual wind turbines is about half of this ideal value.

Repositioning of a Satellite

An orbiting satellite has a mass of m_{sat} = 5000 kg and is traveling at a constant velocity of V_0. To alter its orbit, an attached rocket discharges m_f = 100 kg of gases from the reaction of solid fuel at a velocity V_f = 3000 m/s relative to the satellite in a direction opposite to V_0. The fuel discharge rate is constant for 2s.

Determine (a) the acceleration of the satellite during this 2-s period, (b) the change of velocity of the satellite during this time period, and (c) the thrust exerted on the satellite.



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SOLUTION The rocket of a satellite is fired in the opposite direction to motion. The acceleration, the velocity change, and the thrust are to be determined.

Assumptions **1** The flow of combustion gases is steady and one-dimensional during the firing period. **2** There are no external forces acting on the satellite, and the effect of the pressure force at the nozzle exit is negligible. **3** The mass of discharged fuel is negligible relative to the mass of the satellite, and thus the satellite may be treated as a solid body with a constant mass. **4** The nozzle is well-designed such that the effect of the momentum-flux correction factor is negligible, and thus $\beta \cong 1$.

Analysis (*a*) We choose a reference frame in which the control volume moves with the satellite. Then the velocities of fluid streams become simply their velocities relative to the moving body. We take the direction of motion of the satellite as the positive direction along the *x*-axis. There are no external forces acting on the satellite and its mass is nearly constant. Therefore, the satellite can be treated as a solid body with constant mass, and the momentum equation in this case is simply Eq. 6–28,

$$0 = \frac{d(m\vec{V})_{CV}}{dt} + \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V} \quad \rightarrow \quad m_{sat}\frac{d\vec{V}_{sat}}{dt} = -\dot{m}_f\vec{V}_f$$

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Noting that the motion is on a straight line and the discharged gases move in the negative *x*-direction, we can write the momentum equation using magnitudes as

$$m_{sat}\frac{dV_{sat}}{dt} = \dot{m}_f V_f \quad \rightarrow \quad \frac{dV_{sat}}{dt} = \frac{\dot{m}_f}{m_{sat}} V_f = \frac{m_f/\Delta t}{m_{sat}} V_f$$

Substituting, the acceleration of the satellite during the first 2 s is determined to be

$$a_{sat} = \frac{dV_{sat}}{dt} = \frac{m_f/\Delta t}{m_{sat}} V_f = \frac{(100\text{ kg})/(2\text{ s})}{5000\text{ kg}}(3000\text{ m/s}) = \mathbf{30\ m/s^2}$$

(*b*) Knowing acceleration, which is constant, the velocity change of the satellite during the first 2 s is determined from the definition of acceleration $a_{sat} = dV_{sat}/dt$ to be

$$dV_{sat} = a_{sat}\,dt \quad \rightarrow \quad \Delta V_{sat} = a_{sat}\,\Delta t = (30\text{ m/s}^2)(2\text{ s}) = \mathbf{60\ m/s}$$

(*c*) The thrust exerted on the satellite is, from Eq. 6–29,

$$F_{sat} = 0 - \dot{m}_f(-V_f) = -(100/2\text{ kg/s})(-3000\text{ m/s})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{150\ kN}$$

Discussion Note that if this satellite were attached somewhere, it would exert a force of 150 kN (equivalent to the weight of 15 tons of mass) to its support. This can be verified by taking the satellite as the system and applying the momentum equation.

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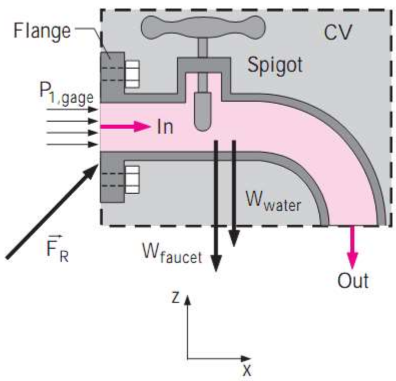
Net Force on a Flange

Water flows at a rate of 18.5 gal/min through a flanged faucet with a partially closed gate valve spigot. The inner diameter of the pipe at the location of the flange is 0.780 in (= 0.0650 ft), and the pressure at that location is measured to be 13.0 psig. The total weight of the faucet assembly plus the water within it is 12.8 lbf. Calculate the net force on the flange.



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SOLUTION Water flow through a flanged faucet is considered. The net force acting on the flange is to be calculated.

Assumptions **1** The flow is steady and incompressible. **2** The flow at the inlet and at the outlet is turbulent and fully developed so that the momentum-flux correction factor is about 1.03. **3** The pipe diameter at the outlet of the faucet is the same as that at the flange.

Properties The density of water at room temperature is 62.3 lbm/ft^3.

Analysis We choose the faucet and its immediate surroundings as the control volume, as shown in Fig. 6–26 along with all the forces acting on it. These forces include the weight of the water and the weight of the faucet assembly, the gage pressure force at the inlet to the control volume, and the net force of the flange on the control volume, which we call $\vec{F}_R$. We use gage pressure for convenience since the gage pressure on the rest of the control surface is zero (atmospheric pressure). Note that the pressure through the outlet of the control volume is also atmospheric since we are assuming incompressible flow; hence, the gage pressure is also zero through the outlet.

We now apply the control volume conservation laws. Conservation of mass is trivial here since there is only one inlet and one outlet; namely, the mass flow rate into the control volume is equal to the mass flow rate out of the control volume. Also, the outflow and inflow average velocities are identical since the inner diameter is constant and the water is incompressible, and are determined to be

$$V_2 = V_1 = V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{18.5 \text{ gal/min}}{\pi (0.065 \text{ ft})^2/4}\left(\frac{0.1337 \text{ ft}^3}{1 \text{ gal}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 12.42 \text{ ft/s}$$

Also,

$$\dot{m} = \rho \dot{V} = (62.3 \text{ lbm/ft}^3)(18.5 \text{ gal/min})\left(\frac{0.1337 \text{ ft}^3}{1 \text{ gal}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 2.568 \text{ lbm/s}$$

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$$V_2 = V_1 = V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{18.5 \text{ gal/min}}{\pi (0.065 \text{ ft})^2/4}\left(\frac{0.1337 \text{ ft}^3}{1 \text{ gal}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 12.42 \text{ ft/s}$$

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Next we apply the momentum equation for steady flow,

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

We let the *x*- and *z*-components of the force acting on the flange be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. The magnitude of the velocity in the *x*-direction is $+V_1$ at the inlet, but zero at the outlet. The magnitude of the velocity in the *z*-direction is zero at the inlet, but $-V_2$ at the outlet. Also, the weight of the faucet assembly and the water within it acts in the $-z$-direction as a body force. No pressure or viscous forces act on the chosen control volume in the *z*-direction.

The momentum equations along the *x*- and *z*-directions become

$$F_{Rx} + P_{1,\text{ gage}}A_1 = 0 - \dot{m}(+V_1)$$

$$F_{Rz} - W_{\text{faucet}} - W_{\text{water}} = \dot{m}(-V_2) - 0$$

100

Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$F_{Rx} = -\dot{m}V_1 - P_{1,\,gage}A_1$$

$$= -(2.568\ \text{lbm/s})(12.42\ \text{ft/s})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) - (13\ \text{lbf/in}^2)\,\frac{\pi(0.780\ \text{in})^2}{4}$$

$$= -7.20\ \text{lbf}$$

$$F_{Rz} = -\dot{m}V_2 + W_{\text{faucet+water}}$$

$$= -(2.568\ \text{lbm/s})(12.42\ \text{ft/s})\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right) + 12.8\ \text{lbf} = 11.8\ \text{lbf}$$

Then the net force of the flange on the control volume can be expressed in vector form as

$$\vec{F}_R = F_{Rx}\vec{i} + F_{Rz}\vec{k} = -7.20\vec{i} + 11.8\vec{k}\quad \text{lbf}$$

10
1

From Newton's third law, the force the faucet assembly exerts on the flange is the negative of $\vec{F}_R$,

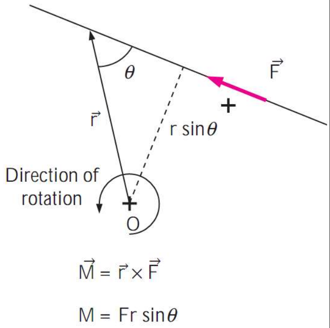
$$\vec{F}_{\text{faucet on flange}} = -\vec{F}_R = 7.20\vec{i} - 11.8\vec{k}\quad \text{lbf}$$

Discussion The faucet assembly pulls to the right and down; this agrees with our intuition. Namely, the water exerts a high pressure at the inlet, but the outlet pressure is atmospheric. In addition, the momentum of the water at the inlet in the *x*-direction is lost in the turn, causing an additional force to the right on the pipe walls. The faucet assembly weighs much more than the momentum effect of the water, so we expect the force to be downward. Note that labeling forces such as "faucet on flange" clarifies the direction of the force.

10
2

ANGULAR MOMENTUM EQUATION

- Many engineering problems involve the moment of the linear momentum of flow streams, and the rotational effects caused by them.
- Such problems are best analyzed by the angular momentum equation, also called the moment of momentum equation.
- An important class of fluid devices, called turbomachines, which include centrifugal pumps, turbines, and fans, is analyzed by the angular momentum equation.
- The moment of a force F about a point O is the vector (or cross) product



$$\vec{M} = \vec{r} \times \vec{F}$$

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ANGULAR MOMENTUM EQUATION

- Replacing the vector F by the momentum vector mV gives the moment of momentum, about a point O as

$$\vec{M} = \vec{r} \times \vec{F}$$

$$\vec{H} = \vec{r} \times m\vec{V}$$

$$\vec{H}_{sys} = \int_{sys} (\vec{r} \times \vec{V})\rho \, dV$$

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ANGULAR MOMENTUM EQUATION

- Rate of change of moment of momentum:

$$\frac{d\vec{H}_{sys}}{dt} = \frac{d}{dt}\int_{sys} (\vec{r} \times \vec{V})\rho \, dV$$

$$\sum \vec{M} = \frac{d\vec{H}_{sys}}{dt}$$

ANGULAR MOMENTUM EQUATION

- By the Reynolds transport theorem.

$$\frac{dH_{sys}}{dt} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V})\rho \, dV + \int_{CS} (\vec{r} \times \vec{V})\rho(\vec{V}_r \cdot \vec{n}) \, dA$$

$$\sum \vec{M} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V})\rho \, dV + \int_{CS} (\vec{r} \times \vec{V})\rho(\vec{V}_r \cdot \vec{n}) \, dA$$

$$\left(\begin{array}{c}\text{The sum of all}\\ \text{external moments}\\ \text{acting on a CV}\end{array}\right) = \left(\begin{array}{c}\text{The time rate of change}\\ \text{of the angular momentum}\\ \text{of the contents of the CV}\end{array}\right) + \left(\begin{array}{c}\text{The net flow rate of}\\ \text{angular momentum}\\ \text{out of the control}\\ \text{surface by mass flow}\end{array}\right)$$

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ANGULAR MOMENTUM EQUATION

For a fixed CV

$$\sum \vec{M} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V})\rho\, dV + \int_{CS} (\vec{r} \times \vec{V})\rho(\vec{V} \cdot \vec{n})\, dA$$

Special Cases

- During steady flow, the amount of angular momentum within the control volume remains constant,
- Thus the time rate of change of angular momentum of the contents of the control volume is zero.
- Then

$$\sum \vec{M} = \int_{CS} (\vec{r} \times \vec{V})\rho(\vec{V}_r \cdot \vec{n})\, dA$$

ANGULAR MOMENTUM EQUATION

- In many practical applications, the fluid crosses the boundaries of the control volume at a certain number of inlets and outlets, and it is convenient to replace the area integral by an algebraic expression written in terms of the average properties over the cross-sectional areas where the fluid enters or leaves the control volume.

$$\sum \vec{M} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V})\rho \, dV + \sum_{out} \vec{r} \times \dot{m}\vec{V} - \sum_{in} \vec{r} \times \dot{m}\vec{V}$$

Steady flow: $$\sum \vec{M} = \sum_{out} \vec{r} \times \dot{m}\vec{V} - \sum_{in} \vec{r} \times \dot{m}\vec{V}$$

The net torque acting on the control volume during steady flow is equal to the difference between the outgoing and incoming angular momentum flow rates.

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ANGULAR MOMENTUM EQUATION

- In many problems, all the significant forces and momentum flows are in the same plane, and thus all give rise to moments in the same plane and about the same axis.
- For such cases, equation can be expressed in scalar form as

$$\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$$

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Flow with No External Moments

- When there are no external moments applied, the angular momentum equation reduces to

$$0 = \frac{d\vec{H}_{CV}}{dt} + \sum_{out} \vec{r} \times \dot{m}\vec{V} - \sum_{in} \vec{r} \times \dot{m}\vec{V}$$

Then the moment of inertia I of the control volume remains constant, the first term of the last equation simply becomes moment of inertia times angular acceleration, $I\alpha$.
Therefore, the control volume in this case can be treated as a solid body, with a net torque of

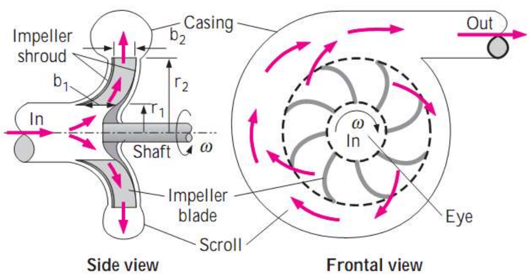
$$\vec{M}_{body} = I_{body}\,\vec{\alpha} = \sum_{in} (\vec{r} \times \dot{m}\vec{V}) - \sum_{out} (\vec{r} \times \dot{m}\vec{V})$$

This approach can be used to determine the angular acceleration of space vehicles and aircraft when a rocket is fired in a direction different than the direction of motion.

11 1

Radial-Flow Devices

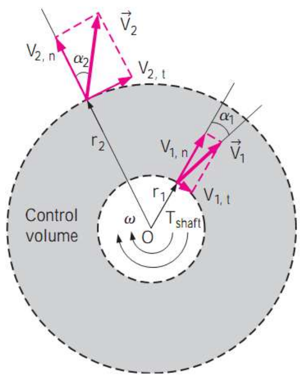
- Many rotary-flow devices such as centrifugal pumps and fans involve flow in the radial direction normal to the axis of rotation and are called radial-flow devices.
- In a centrifugal pump, for example, the fluid enters the device in the axial direction through the eye of the impeller, turns outward as it flows through the passages between the blades of the impeller, collects in the scroll, and is discharged in the tangential direction.



11 2

Radial-Flow Devices

- To analyze the centrifugal pump, we choose the annular region that encloses the impeller section as the control volume.
- The average flow velocity, in general, has normal and tangential components at both the inlet and the outlet of the impeller section.
- Also, when the shaft rotates at an angular velocity of ω, the impeller blades have a tangential velocity of ωr_1 at the inlet and ωr_2 at the outlet.
- For steady incompressible flow, the conservation of mass equation can be written as



$$\dot{V}_1 = \dot{V}_2 = \dot{V} \quad \rightarrow \quad (2\pi r_1 b_1)V_{1,n} = (2\pi r_2 b_2)V_{2,n}$$

3

Radial-Flow Devices

- the average normal components $V_{1,n}$ and $V_{2,n}$ of absolute velocity can be expressed in terms of the volumetric flow rate V as

$$V_{1,n} = \frac{\dot{V}}{2\pi r_1 b_1} \quad \text{and} \quad V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2}$$

The normal velocity components $V_{1,n}$ and $V_{2,n}$ as well as pressure acting on the inner and outer circumferential areas pass through the shaft center, and thus they do not contribute to torque about the origin. Then only the tangential velocity components contribute to torque, and the application of the angular momentum equation

$$\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V \qquad T_{shaft} = \dot{m}(r_2V_{2,t} - r_1V_{1,t})$$

Radial-Flow Devices

$$T_{shaft} = \dot{m}(r_2V_{2,\,t} - r_1V_{1,\,t})$$

- Is known as Euler's turbine formula.
- When the angles α_1 and α_2 between the direction of absolute flow velocities and the radial direction are known, it becomes

$$T_{shaft} = \dot{m}(r_2V_2 \sin \alpha_2 - r_1V_1 \sin \alpha_1)$$

In the idealized case of the tangential fluid velocity being equal to the blade angular velocity both at the inlet and the exit, we have $V_{1,\,t} = \omega r_1$ and $V_{2,\,t} = \omega r_2$, and the torque becomes

$$T_{shaft,\ ideal} = \dot{m}\omega(r_2^2 - r_1^2)$$

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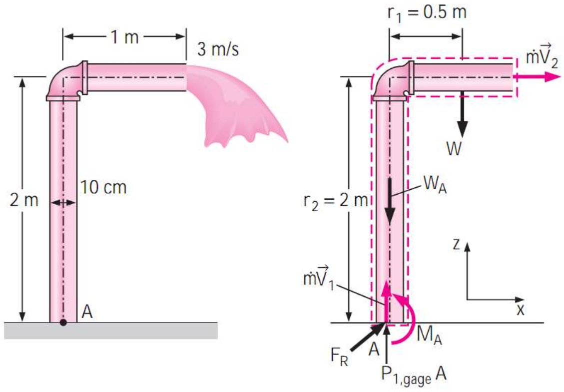
Bending Moment Acting at the Base of a Water Pipe

Underground water is pumped to a sufficient height through a 10-cm-diameter pipe that consists of a 2-m-long vertical and 1-m-long horizontal section, as shown in Figure. Water discharges to atmospheric air at an average velocity of 3 m/s, and the mass of the horizontal pipe section when filled with water is 12 kg per meter length. The pipe is anchored on the ground by a concrete base.
Determine the bending moment acting at the base of the pipe (point A) and the required length of the horizontal section that would make the moment at point A zero.



SOLUTION Water is pumped through a piping section. The moment acting at the base and the required length of the horizontal section to make this moment zero is to be determined.

Assumptions **1** The flow is steady. **2** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **3** The pipe diameter is small compared to the moment arm, and thus we use average values of radius and velocity at the outlet.

Properties We take the density of water to be 1000 kg/m^3.

Analysis We take the entire L-shaped pipe as the control volume, and designate the inlet by 1 and the outlet by 2. We also take the *x*- and *z*-coordinates as shown. The control volume and the reference frame are fixed.

The conservation of mass equation for this one-inlet, one-outlet, steady-flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and $V_1 = V_2 = V$ since A_c = constant. The mass flow rate and the weight of the horizontal section of the pipe are

$$\dot{m} = \rho A_c V = (1000\ \text{kg/m}^3)[\pi(0.10\ \text{m})^2/4](3\ \text{m/s}) = 23.56\ \text{kg/s}$$

$$W = mg = (12\ \text{kg/m})(1\ \text{m})(9.81\ \text{m/s}^2)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 118\ \text{N}$$

To determine the moment acting on the pipe at point *A*, we need to take the moment of all forces and momentum flows about that point. This is a steady-flow problem, and all forces and momentum flows are in the same plane. Therefore, the angular momentum equation in this case can be expressed as

$$\sum M = \sum_{\text{out}} r\dot{m}V - \sum_{\text{in}} r\dot{m}V$$

where *r* is the average moment arm, V is the average speed, all moments in the counterclockwise direction are positive, and all moments in the clockwise direction are negative.

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The free-body diagram of the L-shaped pipe is given in Fig. 6–37. Noting that the moments of all forces and momentum flows passing through point *A* are zero, the only force that yields a moment about point *A* is the weight *W* of the horizontal pipe section, and the only momentum flow that yields a moment is the outlet stream (both are negative since both moments are in the clockwise direction). Then the angular momentum equation about point *A* becomes

$$M_A - r_1 W = -r_2 \dot{m} V_2$$

Solving for M_A and substituting give

$$M_A = r_1 W - r_2 \dot{m} V_2$$

$$= (0.5\ \text{m})(118\ \text{N}) - (2\ \text{m})(23.56\ \text{kg/s})(3\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= -82.5\ \text{N}\cdot\text{m}$$

The negative sign indicates that the assumed direction for M_A is wrong and should be reversed. Therefore, a moment of 82.5 N · m acts at the stem of the pipe in the clockwise direction. That is, the concrete base must apply a 82.5 N · m moment on the pipe stem in the clockwise direction to counteract the excess moment caused by the exit stream.

The weight of the horizontal pipe is $w = W/L$ = 118 N per m length. Therefore, the weight for a length of *L* m is *Lw* with a moment arm of $r_1 = L/2$. Setting $M_A = 0$ and substituting, the length *L* of the horizontal pipe that will cause the moment at the pipe stem to vanish is determined to be

$$0 = r_1 W - r_2 \dot{m} V_2 \quad \rightarrow \quad 0 = (L/2)Lw - r_2 \dot{m} V_2$$

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or

$$L = \sqrt{\frac{2r_2\dot{m}V_2}{w}} = \sqrt{\frac{2 \times 141.4\ \text{N}\cdot\text{m}}{118\ \text{N/m}}} = 2.40\ \text{m}$$

Discussion Note that the pipe weight and the momentum of the exit stream cause opposing moments at point *A*. This example shows the importance of accounting for the moments of momentums of flow streams when performing a dynamic analysis and evaluating the stresses in pipe materials at critical cross sections.

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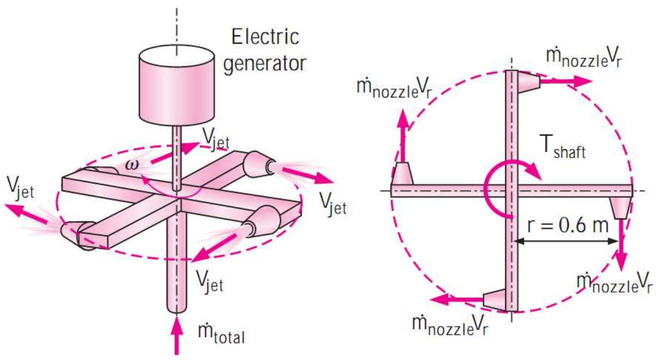
Power Generation from a Sprinkler System

A large lawn sprinkler with four identical arms is to be converted into a turbine to generate electric power by attaching a generator to its rotating head, as shown in Figure. Water enters the sprinkler from the base along the axis of rotation at a rate of 20 L/s and leaves the nozzles in the tangential direction. The sprinkler rotates at a rate of 300 rpm in a horizontal plane. The diameter of each jet is 1 cm, and the normal distance between the axis of rotation and the center of each nozzle is 0.6 m. Estimate the electric power produced.



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SOLUTION A four-armed sprinkler is used to generate electric power. For a specified flow rate and rotational speed, the power produced is to be determined.

Assumptions **1** The flow is cyclically steady (i.e., steady from a frame of reference rotating with the sprinkler head). **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle exit is zero. **3** Generator losses and air drag of rotating components are neglected. **4** The nozzle diameter is small compared to the moment arm, and thus we use average values of radius and velocity at the outlet.

Properties We take the density of water to be 1000 kg/m³ = 1 kg/L.

Analysis We take the disk that encloses the sprinkler arms as the control volume, which is a stationary control volume.

The conservation of mass equation for this steady-flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}_{total}$. Noting that the four nozzles are identical, we have $\dot{m}_{nozzle} = \dot{m}_{total}/4$ or $\dot{V}_{nozzle} = \dot{V}_{total}/4$ since the density of water is constant. The average jet exit velocity relative to the nozzle is

$$V_{jet} = \frac{\dot{V}_{nozzle}}{A_{jet}} = \frac{5 \text{ L/s}}{[\pi(0.01 \text{ m})^2/4]}\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right) = 63.66 \text{ m/s}$$

12
1

The angular and tangential velocities of the nozzles are

$$\omega = 2\pi\dot{n} = 2\pi(300 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 31.42 \text{ rad/s}$$

$$V_{nozzle} = r\omega = (0.6 \text{ m})(31.42 \text{ rad/s}) = 18.85 \text{ m/s}$$

That is, the water in the nozzle is also moving at a velocity of 18.85 m/s in the opposite direction when it is discharged. Then the average velocity of the water jet relative to the control volume (or relative to a fixed location on earth) becomes

$$V_r = V_{jet} - V_{nozzle} = 63.66 - 18.85 = 44.81 \text{ m/s}$$

Noting that this is a cyclically steady-flow problem, and all forces and momentum flows are in the same plane, the angular momentum equation can be approximated as $\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$, where r is the moment arm, all moments in the counterclockwise direction are positive, and all moments in the clockwise direction are negative.

The free-body diagram of the disk that contains the sprinkler arms is given in Fig. 6–38. Note that the moments of all forces and momentum flows passing through the axis of rotation are zero. The momentum flows via the water jets leaving the nozzles yield a moment in the clockwise direction and the effect of the generator on the control volume is a moment also in the clockwise direction (thus both are negative). Then the angular momentum equation about the axis of rotation becomes

$$-T_{shaft} = -4r\dot{m}_{nozzle}V_r \qquad \text{or} \qquad T_{shaft} = r\dot{m}_{total}V_r$$

12
2

Substituting, the torque transmitted through the shaft is determined to be

$$T_{shaft} = r\dot{m}_{total}V_r = (0.6 \text{ m})(20 \text{ kg/s})(44.81 \text{ m/s})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = 537.7 \text{ N}\cdot\text{m}$$

since $\dot{m}_{total} = \rho\dot{V}_{total} = (1 \text{ kg/L})(20 \text{ L/s}) = 20 \text{ kg/s}$.

Then the power generated becomes

$$\dot{W} = 2\pi\dot{n}T_{shaft} = \omega T_{shaft} = (31.42 \text{ rad/s})(537.7 \text{ N}\cdot\text{m})\left(\frac{1 \text{ kW}}{1000 \text{ N}\cdot\text{m/s}}\right) = \mathbf{16.9 \text{ kW}}$$

Therefore, this sprinkler-type turbine has the potential to produce 16.9 kW of power.

Discussion To put the result obtained in perspective, we consider two limiting cases. In the first limiting case, the sprinkler is stuck and thus the angular velocity is zero. The torque developed will be maximum in this case since $V_{nozzle} = 0$ and thus $V_r = V_{jet} = 63.66$ m/s, giving $T_{shaft,\ max} = 764$ N · m. But the power generated will be zero since the shaft does not rotate.

In the second limiting case, the shaft is disconnected from the generator (and thus both the torque and power generation are zero) and the shaft accelerates until it reaches an equilibrium velocity. Setting $T_{shaft} = 0$ in the angular momentum equation gives $V_r = 0$ and thus $V_{jet} = V_{nozzle} = 63.66$ m/s. The corresponding angular speed of the sprinkler is

$$\dot{n} = \frac{\omega}{2\pi} = \frac{V_{nozzle}}{2\pi r} = \frac{63.66 \text{ m/s}}{2\pi(0.6 \text{ m})}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 1013 \text{ rpm}$$

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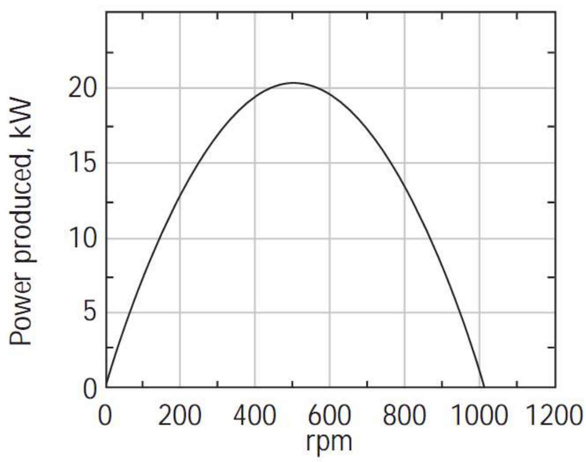
At this rpm, the velocity of the jet will be zero relative to an observer on earth (or relative to the fixed disk-shaped control volume selected).

The variation of power produced with angular speed is plotted in Fig. 6–39. Note that the power produced increases with increasing rpm, reaches a maximum (at about 500 rpm in this case), and then decreases. The actual power produced will be less than this due to generator inefficiency (Chap. 5).



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