COLLEGE OF ENGINEERING
Kwame Nkrumah University of Science & Technology

KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

ME 161
BASIC MECHANICS

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INTRODUCTION

- Engineering involves specifying; size, type, shape etc.
- A good understanding of factors to consider when making specifications is a must for every engineer.
- In engineering, a major factor to consider when specifying sizes and materials especially is forces.
- The study of forces and their effects of on bodies at rest or in motion constitutes the field of ***Mechanics***
- Mechanics is the foundation of most engineering sciences and is often an indispensable prerequisite to their study.

COURSE OBJECTIVES

- Upon successful completion of this course, students should be able to:
  - ✓ Understand and apply Newton's laws of motion and other basic theories and laws of Newtonian mechanics to particles and rigid bodies.
  - ✓ Understand and use appropriate units of measurement, and SI unit prefixes.
  - ✓ Understand the basis of force and moments, and draw free body diagrams.
  - ✓ Analyze 2-D and 3-D equilibrium of system of forces for tensions in ropes/cables, forces in links, and support and contact reactions.
  - ✓ Determine centroids and centre of gravity of single and composite bodies.
  - ✓ Find support reactions and internal forces of two-dimensional determinant structures.
  - ✓ Solve static and dynamic problems involving dry friction.
  - ✓ Evaluate mechanical advantage, velocity ratio and efficiency of simple Machines.
  - ✓ Understand and Solve two-dimensional problems involving equation of motion, momentum, impulse and energy.
  - ✓ Solve simple one degree-of-freedom conservative vibration problems.
  - ✓ Solve simple applied mechanics problems involving combinations of items 1 to 10.

OUTLINE

- Fundamental Principles and Concepts
- Forces & Moments
- Equilibrium of Particles and Rigid Bodies
- Structural Analysis
- Friction
- Simple Machines
- Method of Virtual Work
- Basic Dynamics of Particles and Rigid Bodies
- Simple Harmonic Motion

READING MATERIALS

- Main Text:
  - Basic Engineering Mechanics, J. Antonio
- Other Texts:
  - Vector Mechanics for Engineers, Beer *et al.*
  - Engineering Mechanics – Statics and Dynamics, R. C. Hibbler.
  - Engineering Mechanics – Statics and Dynamics, J.L. Meriam and L.G. Kraige.
  - Engineering Mechanics – Statics, Pytel and Kiusalaas.
  - Any book on Vector Mechanics or Engineering Mechanics.

ASSESSMENT

Assignments	10%
Pop Quizzes	5%
Mid-Semester Exam	15%
End of Semester Exam	70%
TOTAL	**100%**

Class Attendance 5 marks

Note: Class attendance will be considered if and only if Continuous Assessment is less than 30

FUNDAMENTAL PRINCIPLES & CONCEPTS

Definition and Branches of Mechanics

Some Basic Concepts in Mechanics

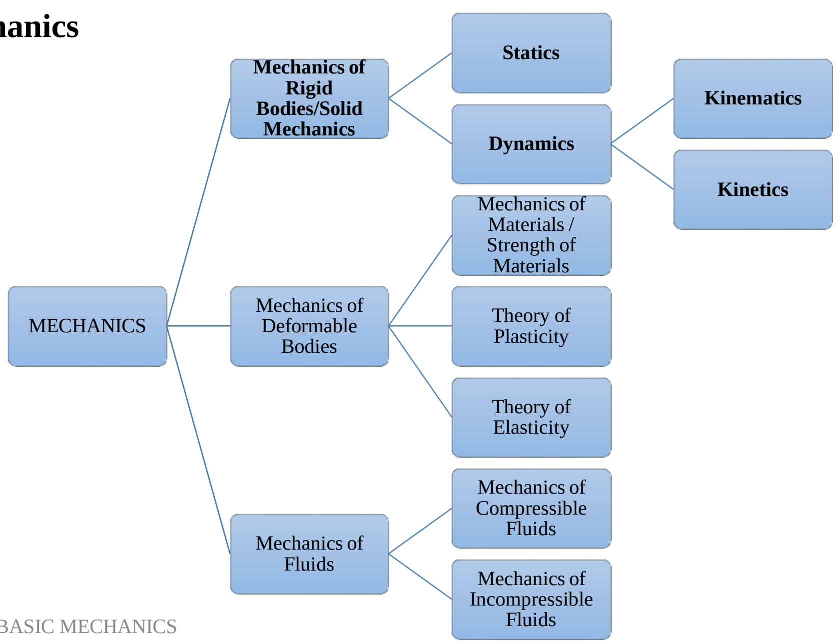
Fundamental Quantities in Mechanics

Units of Measurement and Dimensions

Newton's Laws of Motion and Universal Gravitation

FUNDAMENTAL PRINCIPLES & CONCEPTS

Branches of Mechanics



FUNDAMENTAL CONCEPTS

Some Basic Concepts Mechanics

- Particle

  A very small amount of matter which may be assumed to occupy a single point in space. Idealizing bodies as points simplifies problems since body geometry is not considered.

- Rigid body

  A collection of a large number of particles that remain at a fixed distance from each other, even when under the influence of a load.

FUNDAMENTAL PRINCIPLES & CONCEPTS

Some Basic Concepts Mechanics

- Space

  This is associated with notion of describing a point in terms of co-ordinates measured from a reference point.

- Time

  A measure of the succession of two events or the duration of an event. Of significance in dynamics.

- Mass

  A measure of the inertia of a body, which is its resistance to a change of velocity. The mass of a body affects the gravitational attraction force between it and other bodies.

- Force

- In Newtonian Mechanics, space, time, and mass are absolute concepts, independent of each other. Force, however, is related to the mass of the body and the variation of its velocity with time.

FUNDAMENTAL PRINCIPLES & CONCEPTS

- Quantities
  - Time – second (*s*)
  - Mass – kilogram (*kg*)
  - Length – metre (*m*)
  - Force – Newton (*N*)
- Quantities may also be derived in terms of the basic quantities. The quantities may be scalars or vectors.
- Units and Relations (formulae) are normally in terms of these fundamental quantities, in which case, they must be dimensionally homogenous (all the terms in it have the same dimensions).

FUNDAMENTAL PRINCIPLES & CONCEPTS

Some Quantities and their Corresponding Dimensions and Units

Physical Quantity	**Dimension**	**Common SI Units**
Length	L	m, cm, mm
Area	L^2	m^2, cm^2, mm^2
Angle	1(L/L)	rad, degree
Time	T	s
Linear velocity	L/T or LT^{-1}	m/s or ms^{-1}
Linear acceleration	L/T^2 or LT^{-2}	m/s^2 or ms^{-2}
Angular velocity	1/T or T^{-1}	rad/s
Angular acceleration	$1/T^2$ or T^{-2}	rad/s^2
Mass	M	kg
Force	ML/T^2 or MLT^{-2}	N
Moment of a force	ML^2/T^2 or ML^2T^{-2}	N.m or N-m
Pressure, Stress	M/LT^2 or $ML^{-1}T^{-2}$	Pa, kPa, MPa
Work and Energy	ML^2/T^2 or ML^2T^{-2}	J, kJ
Power	ML^2/T^3 or ML^2T^{-3}	W, kW
Momentum and linear impulse	ML/Tor MLT^{-1}	N.s or N-s

FUNDAMENTAL PRINCIPLES & CONCEPTS

Example on dimensional homogeneity

Determine the dimensions of *I, R, w, M* and *C* in the dimensionally homogeneous equation

$$EIy = Rx^3 - P(x-a)^3 - wx^4 + M'x^2 + C$$

in which x and y are lengths, P is a force, and E is a force per unit area.

FUNDAMENTAL PRINCIPLES & CONCEPTS

Solution

The equation can be written dimensionally as

$$\left(\frac{F}{L^2}\right)(I)(L) = R(L)^3 - (F)(L-a)^3 - (w)(L)^4 + (M')(L)^2 + C$$

Let the dimension for *a* be *L*, then

$$\left(\frac{F}{L^2}\right)(I)(L) = R(L)^3 = (F)(L)^3 = (w)(L)^4 = (M'[)(L)^2 = C$$

The dimensions of each of the unknown quantities are obtained as follows:

$$I = \left(\frac{L}{F}\right)(FL^3) \qquad I = L^4 \qquad M' = \left(\frac{1}{L^2}\right)(FL^3) \qquad M' = FL$$

$$R = \left(\frac{1}{L^3}\right)(FL^3) \qquad R = F \qquad C = FL^3$$

$$w = \left(\frac{1}{L^4}\right)(FL^3) \qquad w = \frac{F}{L}$$

FUNDAMENTAL PRINCIPLES & CONCEPTS

Some Prefixes

Factor by which unit is multiplied	Name of prefix	Symbol of prefix	Example
10^{12}	tera	T	1.23 TJ = 1 230 000 000 000 J
10^{9}	giga	G	4.53 GPa = 4 530 000 000 Pa
10^{6}	mega	M	7.68 MW = 7 680 000 W
10^{3}	kilo	k	5.46 kg = 5 460 g
10^{-2}	centi	c	3.34 cm = 0.0334 m
10^{-3}	milli	m	395 mm = 0.395 m
10^{-6}	micro	μ	65 μm = 0.000 065 m
10^{-9}	nano	n	34 nm = 0.000 000 034 m

FUNDAMENTAL PRINCIPLES & CONCEPTS

Some Laws and Principles

- Newton's Laws of Motion

  - 1st Law – a body will maintain it state of motion (remain at rest or continue to move in a straight line) unless the resultant force on it is not zero.
  - 2nd Law – A body under the influence of a force experiences a proportionate acceleration in the direction of that force.

$$\vec{F} = m\vec{a}$$

  - 3rd Law – Action and Reaction are equal and opposite.

FUNDAMENTAL PRINCIPLES & CONCEPTS

Some Laws and Principles

- Newton's Law of Universal Gravitation
  - Two particles are attracted to each other by a force defined mathematically as;

$$F = G\frac{Mm}{r^2} \qquad W = mg, \quad g = \frac{GM}{R^2}$$

  - g varies from place to place on earth.
- Two or more forces acting on a particle may be replaced by a single force, the resultant.

FORCES & MOMENTS

Characteristics of Forces

Characteristics of Vectors

Resultants of Forces

Moments of Forces

Equivalent Force Systems

FORCES & MOMENTS

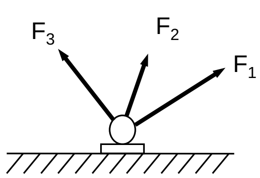
Characteristics of Forces

- Quantities/dimensions are either scalar or vectors.
- All the basic quantities mentioned are scalars with the exception of Forces.
- As such, forces are essentially treated as vectors in Rigid Body Mechanics.
- The external effects of force(s) on a body/particle depend on:
  - The magnitude of the force(s)
  - The direction of the force(s)
  - The line of action of the force(s)
- Two or more Forces acting on a body/particle (A System of forces) may be collinear, parallel, coplanar or concurrent.
- Two Systems of forces are considered equivalent if they produce the same effect on a rigid body.

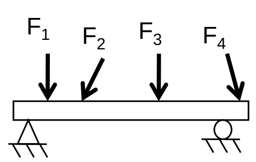
FORCES & MOMENTS

Characteristics of Forces

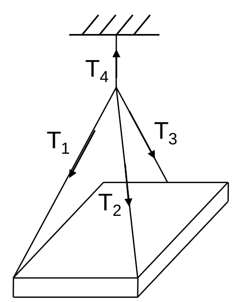
Types of Force Systems



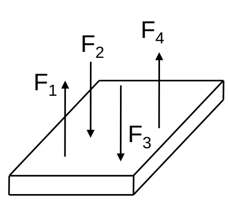
Concurrent Coplanar Force System



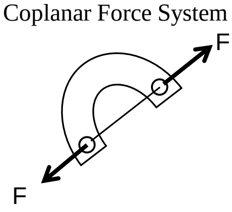
Non-Concurrent Coplanar Force System



Concurrent Spatial Force System



Parallel spatial Force System

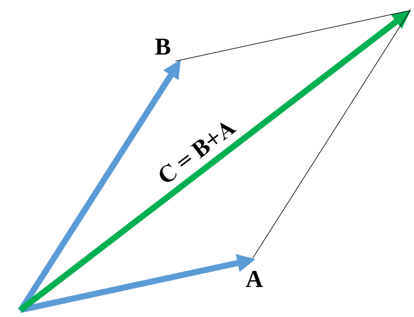


Collinear Force System

FORCES & MOMENTS

Some Characteristics of Vectors

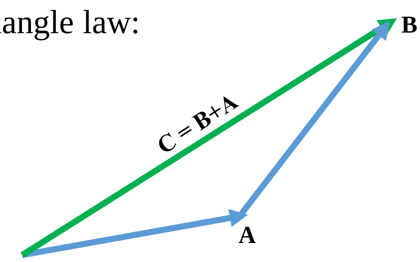
- Vectors may be Fixed or bound, Free or Sliding.
- Vectors are considered equal if they have the same magnitude and direction.
- Scalar multiplication of a vector changes only it's magnitude, unless the scalar is –ve in which case a change in direction is also produced.
- They obey the parallelogram law of addition:



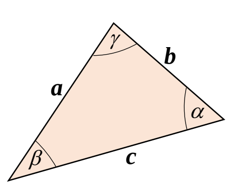
FORCES & MOMENTS

Some Characteristics of Vectors

- They obey the triangle law:



- They obey sine and cosine laws:

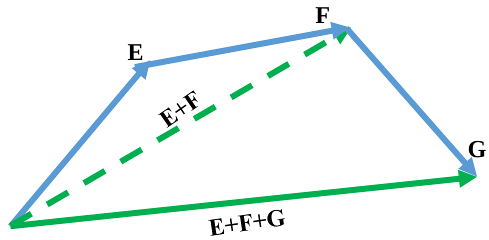


Law of Sines	$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
Law of cosines	$a^2 = b^2 + c^2 - 2bc\cos\alpha$ $b^2 = c^2 + a^2 - 2ca\cos\beta$ $c^2 = a^2 + b^2 - 2ab\cos\gamma$

FORCES & MOMENTS

Some Characteristics of Vectors

- Obey the polygon rule of addition.



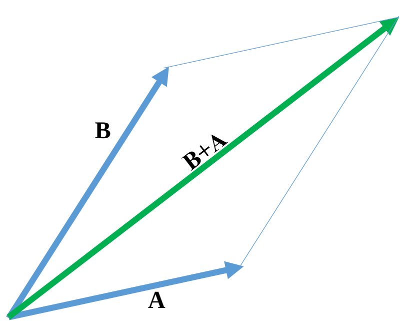
- Successive application of the parallelogram, triangle laws is possible.
- Vector addition is commutative and associative.

$$\vec{P}+\vec{Q}=\vec{Q}+\vec{P}$$
$$\vec{P}+\vec{Q}+\vec{S}=\left(\vec{P}+\vec{Q}\right)+\vec{S}=\vec{P}+\left(\vec{Q}+\vec{S}\right)$$

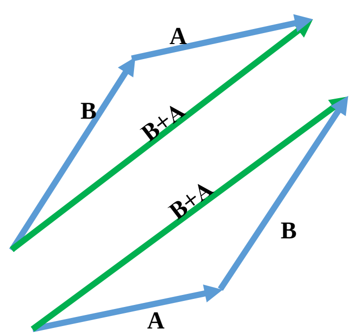
FORCES & MOMENTS

Resultants of Forces

- This is the simplest equivalent force system to which a system of forces can be reduced.
- The resultant is the vector sum of all the individual forces acting on the particle or rigid body.



Parallelogram law



Triangle law

FORCES & MOMENTS

Resultants of Forces

- Resultants may be determined through the Graphical or Trigonometric Approaches.
  - Graphical approach – Parallelogram, Triangle or Polygon rules of vector addition.
  - Trigonometric approach – Sine and Cosine rules.

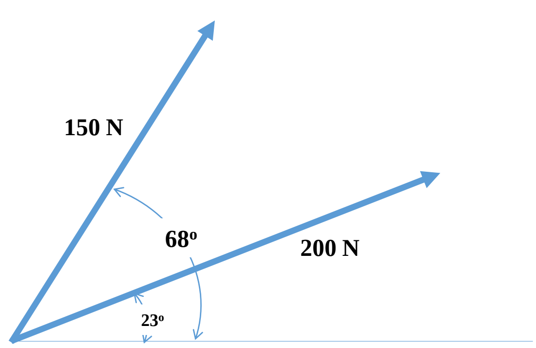
- Force Components approach.
  - Rectangular components approach
  - Unit vector approach
- This approach is more suitable for non-coplanar forces.

FORCES & MOMENTS

Resultants of Forces

Example

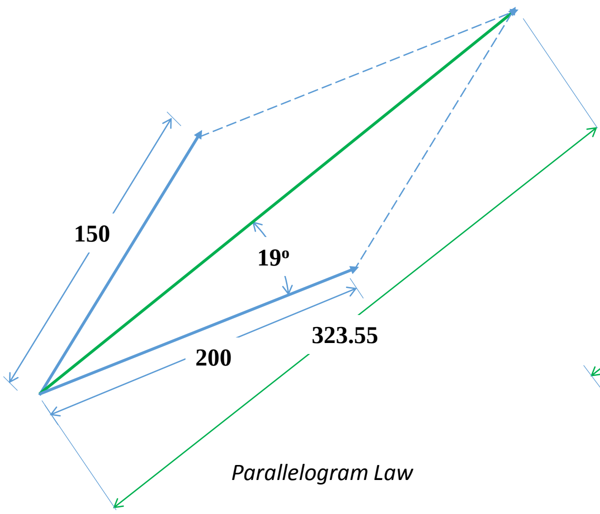
Determine the Resultant of the forces shown below:



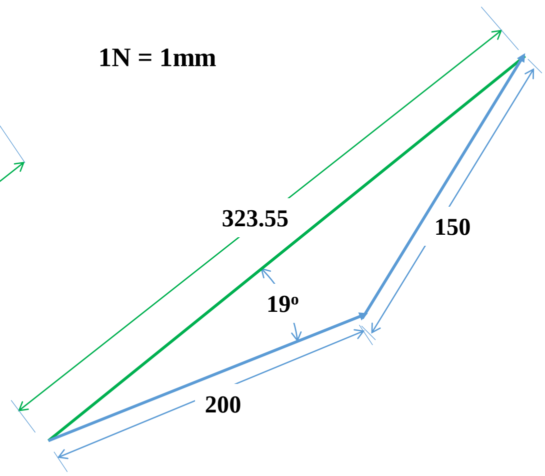
FORCES & MOMENTS
Resultant of Forces

Solution – Graphical Approach

1N = 1mm



Parallelogram Law

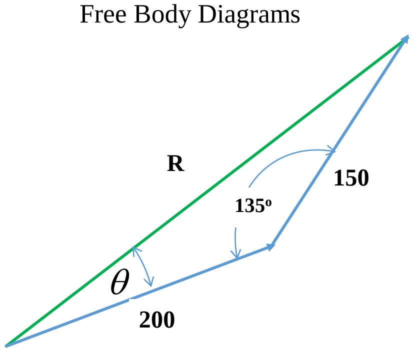


Triangle Law

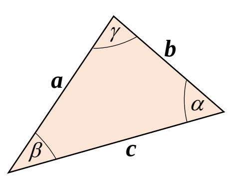
FORCES & MOMENTS
Resultant of Forces

Solution (Continued) - Trigonometric Approach

Free Body Diagrams



Recall



Law of Sines	$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
Law of cosines	$a^2 = b^2 + c^2 - 2bc\cos\alpha$ $b^2 = c^2 + a^2 - 2ca\cos\beta$ $c^2 = a^2 + b^2 - 2ab\cos\gamma$

From the Cosine Rule,

$$R^2 = 200^2 + 150^2 - 2(200)(150)\cos 135^o$$

$$R = 323.9 \text{ N.}$$

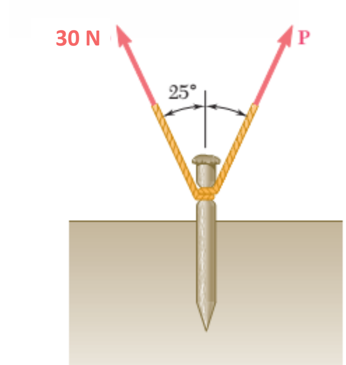
From the Sine Law

$$\frac{323.9 \text{ N}}{\sin 135^o} = \frac{150 \text{ N}}{\sin \theta^o}, \quad \theta = 19.11^o$$

FORCES & MOMENTS
Resultant of Forces

Example

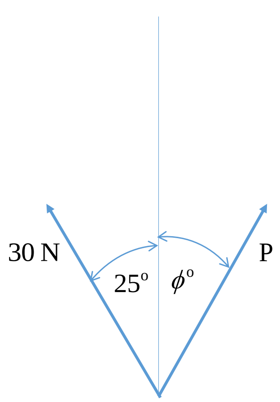
A stake is being pulled out of the ground by means of two ropes as shown below. Knowing the magnitude and direction of the force exerted on one rope, determine the magnitude and direction of the force *P*, that should be exerted on the other rope if the resultant of these two forces is to be a 40 N vertical force. Also determine the angle the 30 N force makes with the unknown force.

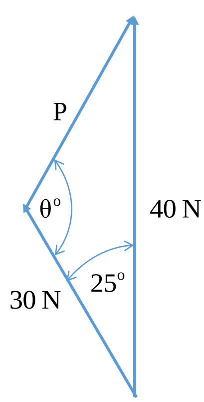


FORCES & MOMENTS
Resultant of Forces

Solution

Free Body Diagram





From the Cosine Rule,

$$P^2 = 40^2 + 30^2 - 2(40)(30)\cos 25^\circ$$

$$P = 18.02 \text{ N.}$$

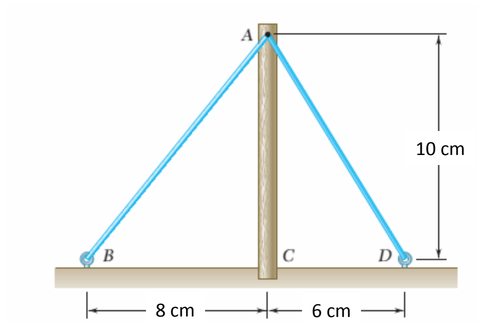
From the Sine Law

$$\frac{18.02 \text{ N}}{\sin 25^\circ} = \frac{40 \text{ N}}{\sin \theta^\circ}, \quad \theta = 69.74^\circ$$

FORCES & MOMENTS
Resultant of Forces

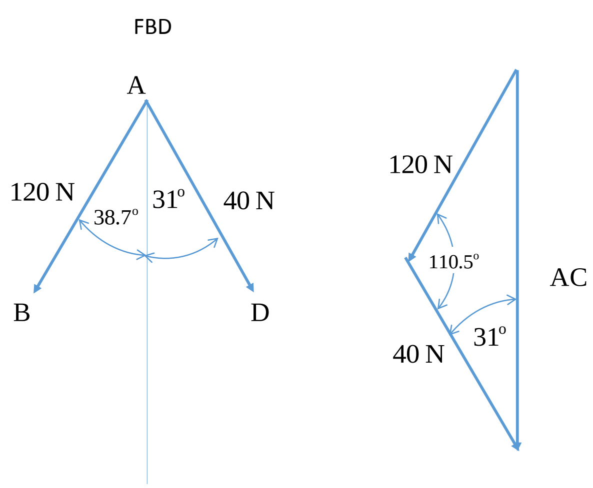
Example

The cable stays AB and AD help support pole AC. Knowing that the tension is 120 N in AB and 40 N in AD, determine the magnitude of the resultant of the forces exerted by the stays at A.



FORCES & MOMENTS
Resultant of Forces

Solution



From the Cosine Rule,

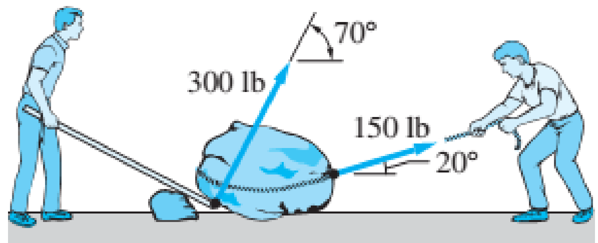
$$AC^2 = 40^2 + 120^2 - 2(40)(120)\cos 110.3^o$$

$$AC = 139.04 \text{ N}.$$

FORCES & MOMENTS
Resultant of Forces

Example

Two men are trying to roll the boulder by applying the forces shown. Determine the magnitude and direction of the force that is equivalent to the two applied forces.

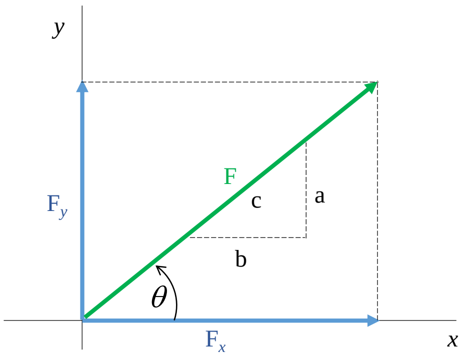


FORCES & MOMENTS
Resultant of Forces - Force Components approach

- This approach requires the forces to be resolved into Rectangular or Cartesian components.
- Like components are then summed to get the components of the resultant force.
- Magnitude and direction of the resultant force can be obtained through appropriate Trigonometry techniques

FORCES & MOMENTS

Resolving Forces in a Plane Into Rectangular Components – Scalar Approach



$$\vec{F} = \vec{F}_x + \vec{F}_y$$

$$\vec{F}_x = F\cos\theta = F\left(\frac{b}{c}\right)$$

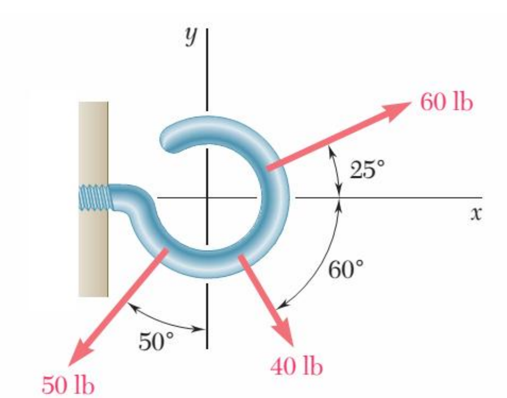
$$\vec{F}_y = F\sin\theta = F\left(\frac{a}{c}\right)$$

$$\theta_x = \cos^{-1} = \left(\frac{F_x}{F}\right) = \cos^{-1} = \left(\frac{b}{c}\right)$$

FORCES & MOMENTS

Example

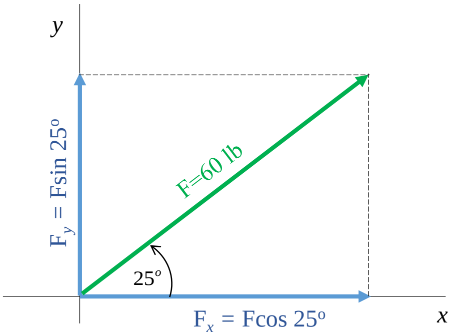
Resolve the forces shown into components.



FORCES & MOMENTS

Solution

For 60 lb force,



For the 60 lb force,

$\vec{F}_x = 60\cos 25^\circ = 54.38\text{ lb}$

$\vec{F}_y = 60\sin 25^\circ = 25.36\text{ lb}$

Similarly,

For the 40 lb force,

$\vec{F}_x = 40\cos 60^\circ = 20\text{ lb}$

$\vec{F}_y = 40\cos 60^\circ = -34.64\text{ lb}$

For the 50 lb force,

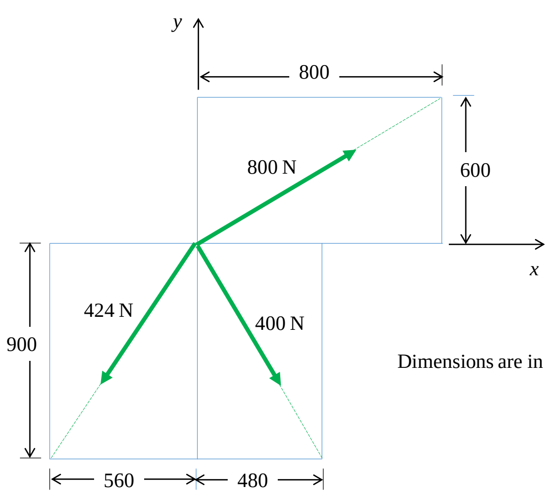
$\vec{F}_x = 50\sin 50^\circ = -38.30\text{ lb}$

$\vec{F}_y = 50\cos 50^\circ = -32.14\text{ lb}$

FORCES & MOMENTS

Example

Resolve the concurrent forces shown into components.



Dimensions are in mm

For 800 N force,

$\vec{F}_{1x} = 639.75\text{ N}$

$\vec{F}_{1y} = 480.34\text{ N}$

For 400 N force,

$\vec{F}_x = 188.41\text{ N}$

$\vec{F}_y = -352.85\text{ N}$

For 424 N force,

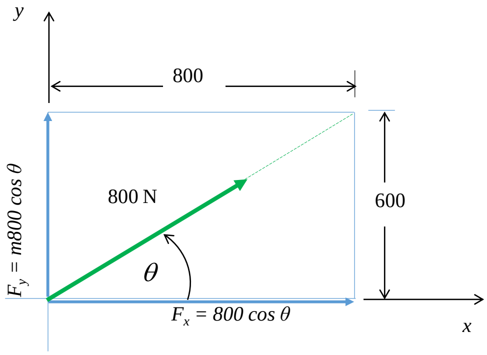
$\vec{F}_x = -224.06\text{ N}$

$\vec{F}_y = -359.96\text{ N}$

FORCES & MOMENTS

Solution

For 800 N force,



$$\tan\theta = \frac{600mm}{800mm} \Rightarrow \theta = 36.9^{o}$$

Therefore,

$\vec{F}_x = 639.75$ N

$\vec{F}_y = 480.34$ N

Similarly,

For 400 N force,

$\vec{F}_x = 188.41$ N

$\vec{F}_y = -352.85$ N

For 424 N force,

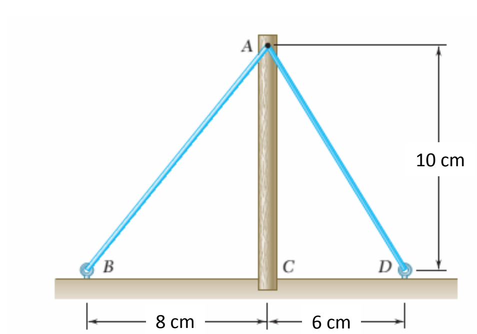
$\vec{F}_x = -224.06$ N

$\vec{F}_y = -359.96$ N

FORCES & MOMENTS

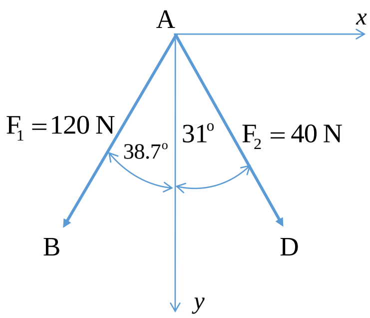
Example

The cable stays AB and AD help support pole AC. Knowing that the tension is 120 N in AB and 40 N in AD, determine the components of the forces in the stays.



FORCES & MOMENTS

Solution



$$\vec{F}_{1x} = F_1 \sin\theta = 40 \sin 31^o = 20.6 \text{ N}$$
$$\vec{F}_{1y} = F_1 \cos\theta = 40 \cos 31^o = -34.29 \text{ N}$$

$$\vec{F}_{2x} = F_2 \sin\theta = 120 \sin 38.7^o = -75.03 \text{ N}$$
$$\vec{F}_{2y} = F_2 \cos\theta = 120 \cos 38.7^o = -93.65 \text{ N}$$

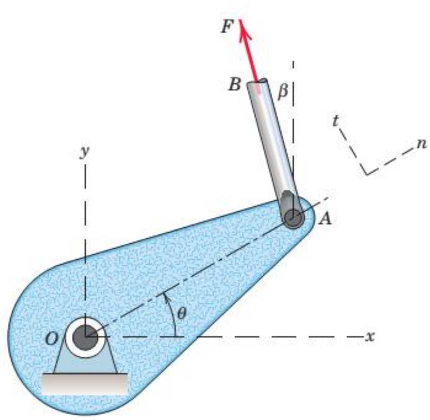
FORCES & MOMENTS

Example

Determine the n and t components of the force **F** which is exerted by the rod AB on the crank OA. Evaluate your general expression for $F = 100\ N$ and

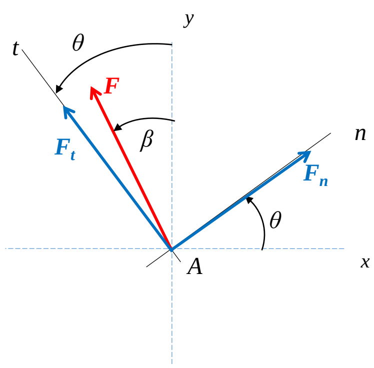
a) $\theta = 30^o$, $\beta = 10^o$

b) $\theta = 15^o$, $\beta = 25^o$



FORCES & MOMENTS

Solution

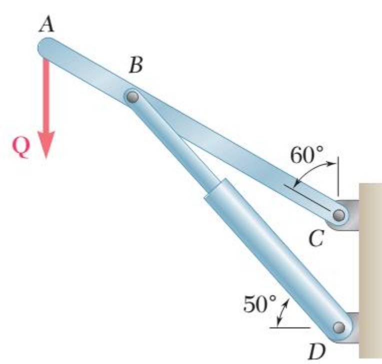


$$\vec{F}_t = F\cos(\theta - \beta)$$
$$\vec{F}_n = F\sin(\theta - \beta)$$

FORCES & MOMENTS

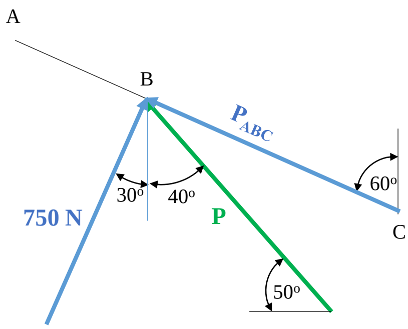
Example

The hydraulic cylinder BD exerts on member ABC, a force P directed along BD. Knowing that P has a 750 N component perpendicular to member ABC, determine the magnitude of the force P and its component parallel to ABC.



FORCES & MOMENTS

Solution



$$P\cos 70^o = 750N$$

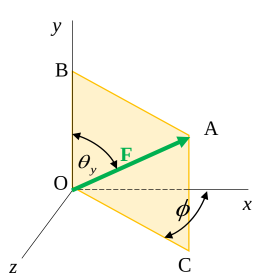
$$\vec{P}_{ABC} = P\sin 70^o$$

FORCES & MOMENTS

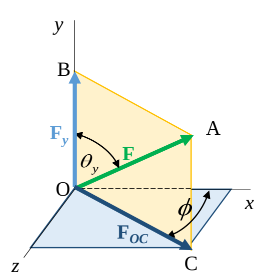
Resolving Forces in Space Into Rectangular Components – Scalar Approach

- The same idea is extended to forces in space. A third component, F_z is introduced.
- For instance;

For instance, the force A is in space (required three components to fully describe it.)



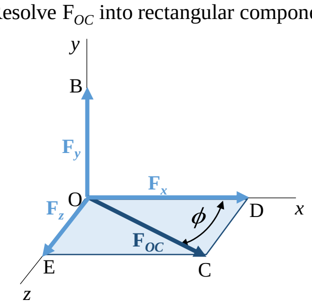
Resolve $\vec{F}$ into vertical (y) and OC components and vertical components.



$$F_y = F\cos\theta_y$$

$$F_{OC} = F\sin\theta_y$$

Resolve F_{OC} into rectangular components

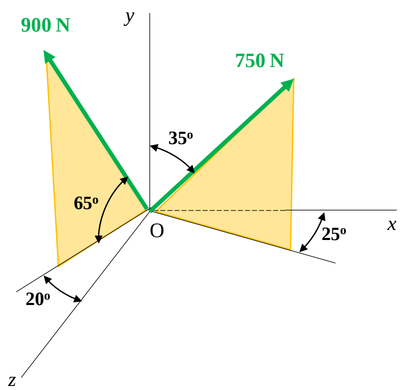


$$F_x = F_{OC}\cos\phi$$
$$= F\sin\theta_y\cos\phi$$
$$F_z = F_{OC}\sin\phi$$
$$= F\sin\theta_y\sin\phi$$

FORCES & MOMENTS

Example

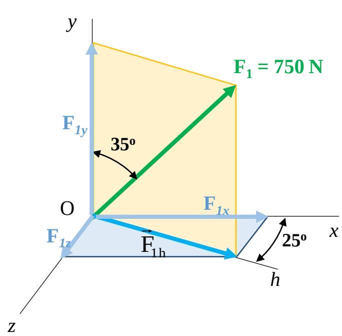
Determine the components of the forces shown.

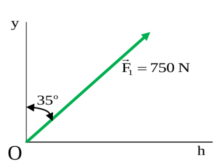


FORCES & MOMENTS

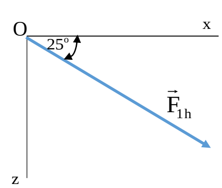
Solution

For the 750 N force,





As viewed in y-h plane



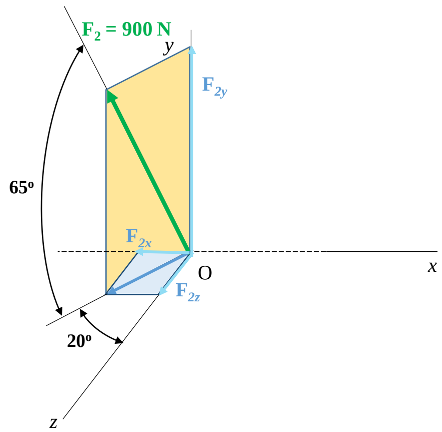
As viewed in x-z plane

$$\vec{F}_{1y} = F_1 \cos\theta = 750\cos 35^\circ = 614.36 \text{ N}$$
$$\vec{F}_{1h} = F_1 \sin\theta = 750\sin 35^\circ = 430.18 \text{ N}$$
$$\vec{F}_{1x} = F_{1h} \cos\theta = 430.18\cos 25^\circ = 389.88 \text{ N}$$
$$\vec{F}_{1z} = F_{1h} \sin\theta = 430.18\sin 25^\circ = 181.8 \text{ N}$$

FORCES & MOMENTS

Solution (Continued)

Applying same to the 900 N force,

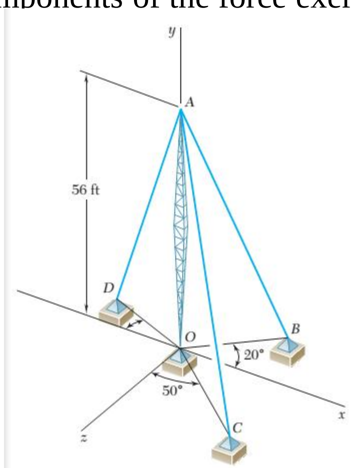


$$\vec{F}_{2y} = F_2 \sin\theta = 900 \sin 65^{\circ} = 815.68 \text{ N}$$
$$\vec{F}_{2h} = F \cos\theta = 900 \cos 65^{\circ} = 380.36 \text{ N}$$
$$\vec{F}_{2x} = F_{2h} \sin\theta = 380.36 \sin 20^{\circ} = -130.09 \text{ N}$$
$$\vec{F}_{2z} = F_{2h} \cos\theta = 380.36 \cos 20^{\circ} = 357.42 \text{ N}$$

FORCES & MOMENTS

Example

Cable AB is 65 ft long, and the tension in that cable is 3900 lb. Determine (a) the x, y, and z components of the force exerted by the cable on the anchor B.



$$F_y = 3360.35N$$
$$F_x = 3045.51N$$
$$F_z = 1420.15N$$