8

Friction

The tractive force that a railroad locomotive can develop depends upon the frictional resistance between the drive wheels and the rails. When the potential exists for wheel slip to occur, such as when a train travels upgrade over wet rails, sand is deposited on top of the railhead to increase this friction.

Introduction

Objectives

- **Examine** the laws of dry friction and the associated coefficients and angles of friction.
- **Consider** the equilibrium of rigid bodies where dry friction at contact surfaces is modeled.
- **Apply** the laws of friction to analyze problems involving wedges and square-threaded screws.
- **Study** engineering applications of the laws of friction, such as in modeling axle, disk, wheel, and belt friction.

Introduction

In the previous chapters, we assumed that surfaces in contact are either *frictionless* or *rough*. If they are frictionless, the force each surface exerts on the other is normal to the surfaces, and the two surfaces can move freely with respect to each other. If they are rough, tangential forces can develop that prevent the motion of one surface with respect to the other.

This view is a simplified one. Actually, no perfectly frictionless surface exists. When two surfaces are in contact, tangential forces, called **friction forces**, always develop if you attempt to move one surface with respect to the other. However, these friction forces are limited in magnitude and do not prevent motion if you apply sufficiently large forces. Thus, the distinction between frictionless and rough surfaces is a matter of degree. You will see this more clearly in this chapter, which is devoted to the study of friction and its applications to common engineering situations.

There are two types of friction: **dry friction**, sometimes called *Coulomb friction*, and **fluid friction** or *viscosity*. Fluid friction develops

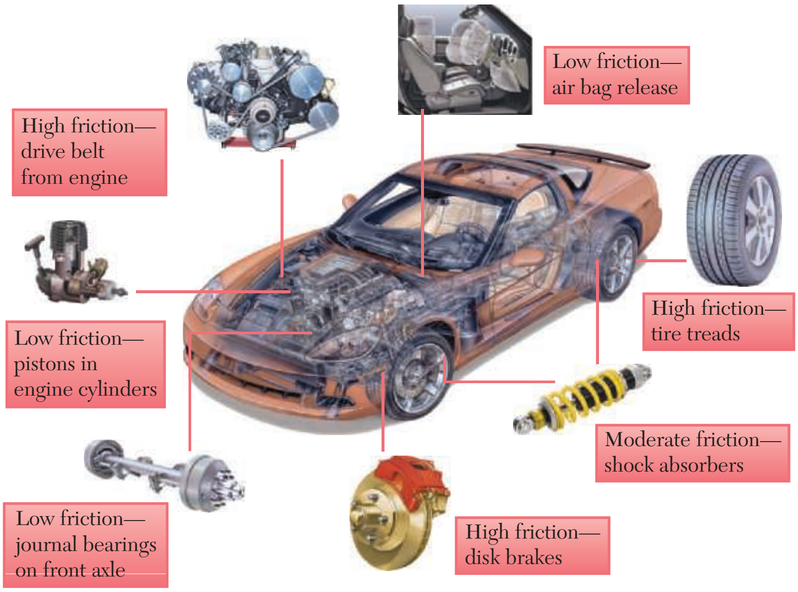


Photo 8.1 Examples of friction in an automobile. Depending upon the application, the degree of friction is controlled by design engineers.

between layers of fluid moving at different velocities. This is of great importance in analyzing problems involving the flow of fluids through pipes and orifices or dealing with bodies immersed in moving fluids. It is also basic for the analysis of the motion of *lubricated mechanisms*. Such problems are considered in texts on fluid mechanics. The present study is limited to dry friction, i.e., to situations involving rigid bodies that are in contact along *unlubricated* surfaces.

In the first section of this chapter, we examine the equilibrium of various rigid bodies and structures, assuming dry friction at the surfaces of contact. Afterward, we consider several specific engineering applications where dry friction plays an important role: wedges, square-threaded screws, journal bearings, thrust bearings, rolling resistance, and belt friction.

8.1 THE LAWS OF DRY FRICTION

We can illustrate the laws of dry friction by the following experiment. Place a block of weight **W** on a horizontal plane surface (Fig. 8.1*a*). The forces acting on the block are its weight **W** and the reaction of the surface. Since the weight has no horizontal component, the reaction of the surface also has no horizontal component; the reaction is therefore *normal* to the surface and is represented by **N** in Fig. 8.1*a*. Now suppose that you apply a horizontal force **P** to the block (Fig. 8.1*b*). If **P** is small, the block does not move; some other horizontal force must therefore exist, which balances **P**. This other force is the **static-friction force F**, which is actually the resultant of a great number of forces acting over the entire surface of contact between the block and the plane. The nature of these forces is not known exactly, but we generally assume that these forces are due to the irregularities of the surfaces in contact and, to a certain extent, to molecular attraction.

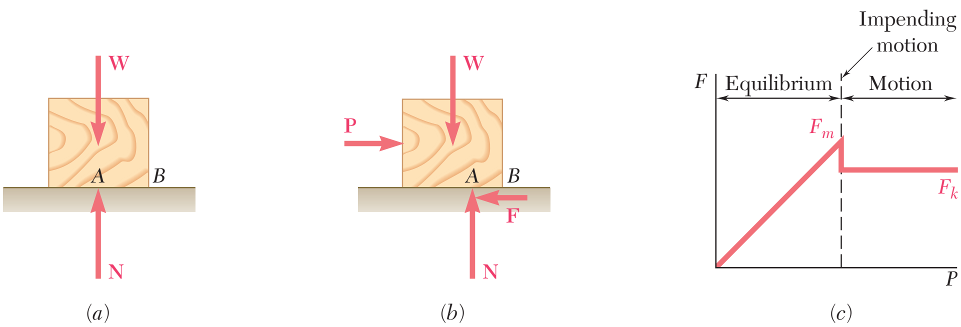


Fig. 8.1 (*a*) Block on a horizontal plane, friction force is zero; (*b*) a horizontally applied force **P** produces an opposing friction force **F**; (*c*) graph of **F** with increasing **P**.

If you increase the force **P**, the friction force **F** also increases, continuing to oppose **P**, until its magnitude reaches a certain *maximum value* F_m (Fig. 8.1*c*). If **P** is further increased, the friction force cannot balance it anymore, and the block starts sliding. As soon as the block has started in motion, the magnitude of **F** drops from F_m to a lower value F_k. This happens because less interpenetration occurs between the irregularities of the surfaces in contact when these surfaces move with respect to each other. From then on, the block keeps sliding with increasing velocity while the friction force, denoted by $\mathbf{F}_k$ and called the **kinetic-friction force**, remains approximately constant.

Note that, as the magnitude F of the friction force increases from 0 to F_m, the point of application A of the resultant $\mathbf{N}$ of the normal forces of contact moves to the right. In this way, the couples formed by $\mathbf{P}$ and $\mathbf{F}$ and by $\mathbf{W}$ and $\mathbf{N}$, respectively, remain balanced. If $\mathbf{N}$ reaches B before F reaches its maximum value F_m, the block starts to tip about B before it can start sliding (see Sample Prob. 8.4).

8.1A Coefficients of Friction

Experimental evidence shows that the maximum value F_m of the static-friction force is proportional to the normal component N of the reaction of the surface. We have

Static friction

$$F_m = \mu_s N \tag{8.1}$$

where μ_s is a constant called the **coefficient of static friction**. Similarly, we can express the magnitude F_k of the kinetic-friction force in the form

Kinetic friction

$$F_k = \mu_k N \tag{8.2}$$

where μ_k is a constant called the **coefficient of kinetic friction**. The coefficients of friction μ_s and μ_k do not depend upon the area of the surfaces in contact. Both coefficients, however, depend strongly on the *nature* of the surfaces in contact. Since they also depend upon the exact condition of the surfaces, their value is seldom known with an accuracy greater than 5%. Approximate values of coefficients of static friction for various combinations of dry surfaces are given in Table 8.1. The corresponding values of the coefficient of kinetic friction are about 25% smaller. Since coefficients of friction are dimensionless quantities, the values given in Table 8.1 can be used with both SI units and U.S. customary units.

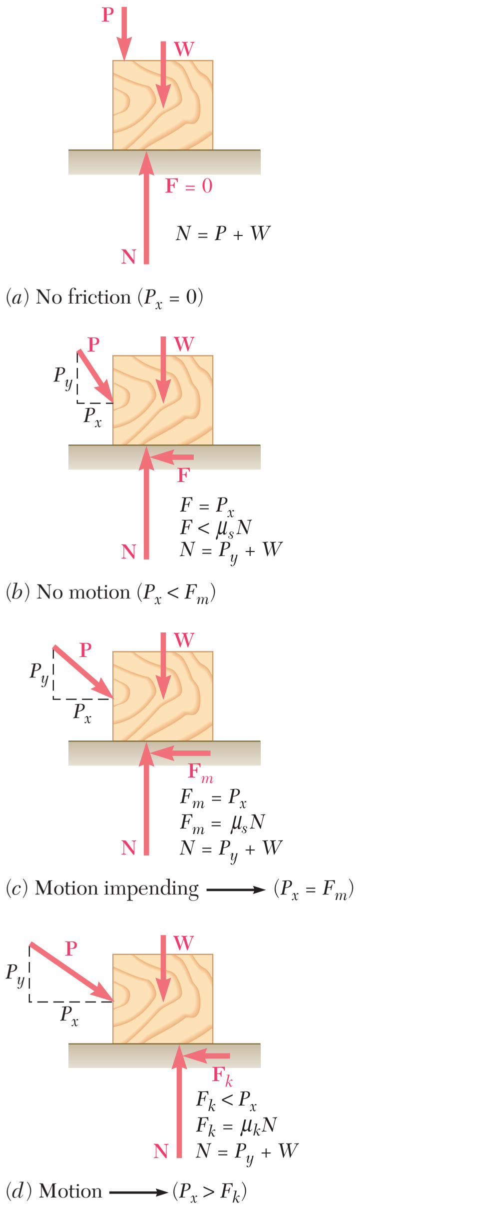


Fig. 8.2 (*a*) Applied force is vertical, friction force is zero; (*b*) horizontal component of applied force is less than F_m, no motion occurs; (*c*) horizontal component of applied force equals F_m, motion is impending; (*d*) horizontal component of applied force is greater than F_k, forces are unbalanced and motion continues.

Table 8.1 Approximate Values of Coefficient of Static Friction for Dry Surfaces

Metal on metal	0.15–0.60
Metal on wood	0.20–0.60
Metal on stone	0.30–0.70
Metal on leather	0.30–0.60
Wood on wood	0.25–0.50
Wood on leather	0.25–0.50
Stone on stone	0.40–0.70
Earth on earth	0.20–1.00
Rubber on concrete	0.60–0.90

From this discussion, it appears that four different situations can occur when a rigid body is in contact with a horizontal surface:

1. The forces applied to the body do not tend to move it along the surface of contact; there is no friction force (Fig. 8.2*a*).
2. The applied forces tend to move the body along the surface of contact but are not large enough to set it in motion. We can find the static-friction force $\mathbf{F}$ that has developed by solving the equations of equilibrium for the body. Since there is no evidence that $\mathbf{F}$ has reached its maximum

value, the equation $F_m = \mu_s N$ *cannot be used* to determine the friction force (Fig. 8.2*b*).

3. The applied forces are such that the body is just about to slide. We say that *motion is impending*. The friction force **F** has reached its maximum value F_m and, together with the normal force **N**, balances the applied forces. Both the equations of equilibrium and the equation $F_m = \mu_s N$ *can be used*. Note that the friction force has a sense opposite to the sense of impending motion (Fig. 8.2*c*).
4. The body is sliding under the action of the applied forces, and the equations of equilibrium no longer apply. However, **F** is now equal to $\mathbf{F}_k$, and we can use the equation $F_k = \mu_k N$. The sense of $\mathbf{F}_k$ is opposite to the sense of motion (Fig. 8.2*d*).

8.1B Angles of Friction

It is sometimes convenient to replace the normal force **N** and the friction force **F** by their resultant **R**. Let's see what happens when we do that.

Consider again a block of weight **W** resting on a horizontal plane surface. If no horizontal force is applied to the block, the resultant **R** reduces to the normal force **N** (Fig. 8.3*a*). However, if the applied force **P** has a horizontal component $\mathbf{P}_x$ that tends to move the block, force **R** has a horizontal component **F** and, thus, forms an angle ϕ with the normal to the surface (Fig. 8.3*b*). If you increase $\mathbf{P}_x$ until motion becomes impending, the angle between **R** and the vertical grows and reaches a maximum value (Fig. 8.3*c*). This value is called the **angle of static friction** and is denoted by ϕ_s. From the geometry of Fig. 8.3*c*, we note that

Angle of static friction

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\boxed{\tan\phi_s = \mu_s} \qquad \textbf{(8.3)}$$

If motion actually takes place, the magnitude of the friction force drops to F_k; similarly, the angle between **R** and **N** drops to a lower value ϕ_k, which is called the **angle of kinetic friction** (Fig. 8.3*d*). From the geometry of Fig. 8.3*d*, we have

Angle of kinetic friction

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\boxed{\tan\phi_k = \mu_k} \qquad \textbf{(8.4)}$$

Another example shows how the angle of friction can be used to advantage in the analysis of certain types of problems. Consider a block resting on a board and subjected to no other force than its weight **W** and the reaction **R** of the board. The board can be given any desired inclination. If the board is horizontal, the force **R** exerted by the board on the block is perpendicular to the board and balances the weight **W** (Fig. 8.4*a*). If the board is given a small angle of inclination θ, force **R** deviates from the perpendicular to the board by angle θ and continues to balance **W** (Fig. 8.4*b*). The reaction **R** now has a normal component **N** with a magnitude of $N = W\cos\theta$ and a tangential component **F** with a magnitude of $F = W\sin\theta$.

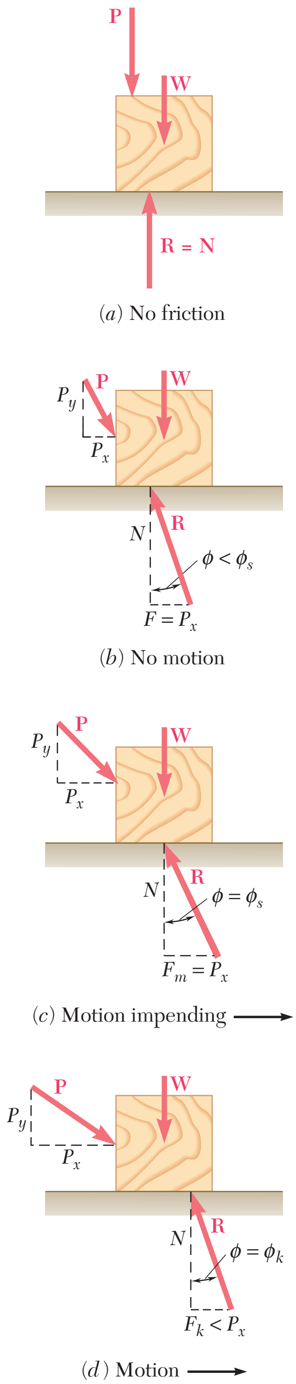


Fig. 8.3 (*a*) Applied force is vertical, friction force is zero; (*b*) applied force is at an angle, its horizontal component balanced by the horizontal component of the surface resultant; (*c*) impending motion, the horizontal component of the applied force equals the maximum horizontal component of the resultant; (*d*) motion, the horizontal component of the resultant is less than the horizontal component of the applied force.

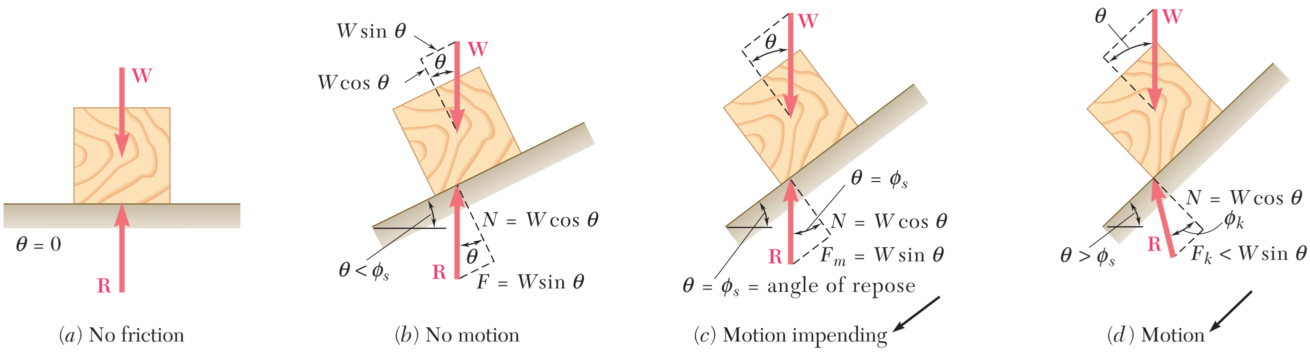


Fig. 8.4 (*a*) Block on horizontal board, friction force is zero; (*b*) board's angle of inclination is less than angle of static friction, no motion; (*c*) board's angle of inclination equals angle of friction, motion is impending; (*d*) angle of inclination is greater than angle of friction, forces are unbalanced and motion occurs.

If we keep increasing the angle of inclination, motion soon becomes impending. At that time, the angle between **R** and the normal reaches its maximum value $\theta = \phi_s$ (Fig. 8.4*c*). The value of the angle of inclination corresponding to impending motion is called the **angle of repose**. Clearly, the angle of repose is equal to the angle of static friction ϕ_s. If we further increase the angle of inclination θ, motion starts and the angle between **R** and the normal drops to the lower value ϕ_k (Fig. 8.4*d*). The reaction **R** is not vertical anymore, and the forces acting on the block are unbalanced.

Photo 8.2 The coefficient of static friction between a crate and the inclined conveyer belt must be sufficiently large to enable the crate to be transported without slipping.

8.1C Problems Involving Dry Friction

Many engineering applications involve dry friction. Some are simple situations, such as variations on the block sliding on a plane just described. Others involve more complicated situations, as in Sample Prob. 8.3. Many problems deal with the stability of rigid bodies in accelerated motion and will be studied in dynamics. Also, several common machines and mechanisms can be analyzed by applying the laws of dry friction, including wedges, screws, journal and thrust bearings, and belt transmissions. We will study these applications in the following sections.

The methods used to solve problems involving dry friction are the same that we used in the preceding chapters. If a problem involves only a motion of translation with no possible rotation, we can usually treat the body under consideration as a particle and use the methods of Chap. 2. If the problem involves a possible rotation, we must treat the body as a rigid body and use the methods of Chap. 4. If the structure considered is made of several parts, we must apply the principle of action and reaction, as we did in Chap. 6.

If the body being considered is acted upon by more than three forces (including the reactions at the surfaces of contact), the reaction at each surface is represented by its components **N** and **F**, and we solve the problem using the equations of equilibrium. If only three forces act on the body under consideration, it may be more convenient to represent each reaction by the single force **R** and solve the problem by using a force triangle.

Most problems involving friction fall into one of the following three groups.

1. All applied forces are given, and we know the coefficients of friction; we are to determine whether the body being considered remains at rest or slides. The friction force **F** *required to maintain equilibrium* is

unknown (its magnitude is *not* equal to $\mu_s N$) and needs to be determined, together with the normal force **N**, by drawing a free-body diagram and solving the equations of equilibrium (Fig. 8.5*a*). We then compare the value found for the magnitude F of the friction force with the maximum value $F_m = \mu_s N$. If F is smaller than or equal to F_m, the body remains at rest. If the value found for F is larger than F_m, equilibrium cannot be maintained and motion takes place; the actual magnitude of the friction force is then $F_k = \mu_k N$.

2. All applied forces are given, and we know the motion is impending; we are to determine the value of the coefficient of static friction. Here again, we determine the friction force and the normal force by drawing a free-body diagram and solving the equations of equilibrium (Fig. 8.5*b*). Since we know that the value found for F is the maximum value F_m, we determine the coefficient of friction by solving the equation $F_m = \mu_s N$.
3. The coefficient of static friction is given, and we know that the motion is impending in a given direction; we are to determine the magnitude or the direction of one of the applied forces. The friction force should be shown in the free-body diagram with a *sense opposite to that of the impending motion* and with a magnitude $F_m = \mu_s N$ (Fig. 8.5*c*). We can then write the equations of equilibrium and determine the desired force.

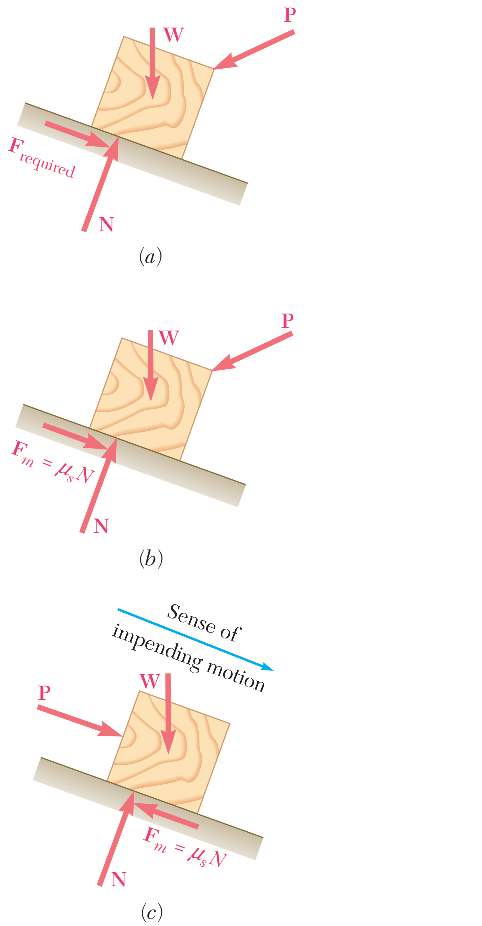


Fig. 8.5 Three types of friction problems: (*a*) given the forces and coefficient of friction, will the block slide or stay? (*b*) given the forces and that motion is pending, determine the coefficient of friction; (*c*) given the coefficient of friction and that motion is impending, determine the applied force.

As noted previously, when only three forces are involved, it may be more convenient to represent the reaction of the surface by a single force **R** and to solve the problem by drawing a force triangle. Such a solution is used in Sample Prob. 8.2.

When two bodies A and B are in contact (Fig. 8.6*a*), the forces of friction exerted, respectively, by A on B and by B on A are equal and opposite (Newton's third law). In drawing the free-body diagram of one of these bodies, it is important to include the appropriate friction force with its correct sense. Observe the following rule: *The sense of the friction force acting on A is opposite to that of the motion (or impending motion) of A as observed from B* (Fig. 8.6*b*). (It is therefore the same as the motion of B as observed from A.) The sense of the friction force acting on B is determined in a similar way (Fig. 8.6*c*). Note that the motion of A as observed from B is a *relative motion*. For example, if body A is fixed and body B moves, body A has a relative motion with respect to B. Also, if both B and A are moving down but B is moving faster than A, then body A is observed, from B, to be moving up.

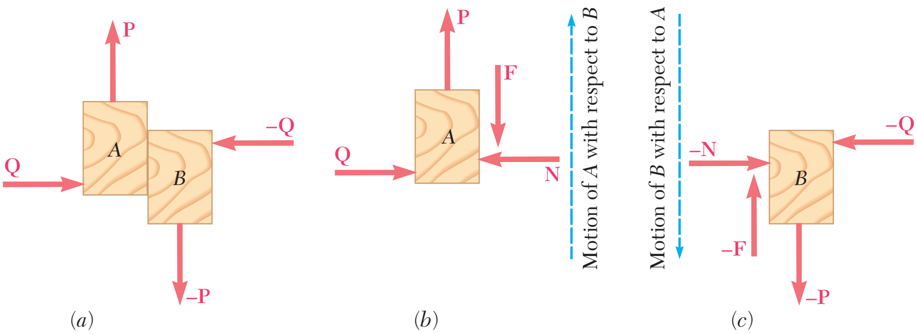
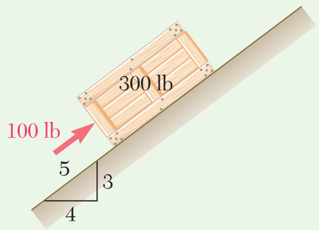


Fig. 8.6 (*a*) Two blocks held in contact by forces; (*b*) free-body diagram for block *A*, including direction of friction force; (*c*) free-body diagram for block *B*, including direction of friction force.



Sample Problem 8.1

A 100-lb force acts as shown on a 300-lb crate placed on an inclined plane. The coefficients of friction between the crate and the plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine whether the crate is in equilibrium, and find the value of the friction force.

STRATEGY: This is a friction problem of the first type: You know the forces and the friction coefficients and want to determine if the crate moves. You also want to find the friction force.

MODELING and ANALYSIS

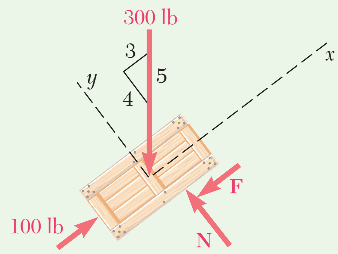


Fig. 1 Free-body diagram of crate showing assumed direction of friction force.

Force Required for Equilibrium. First determine the value of the friction force *required to maintain equilibrium*. Assuming that **F** is directed down and to the left, draw the free-body diagram of the crate (Fig. 1) and solve the equilibrium equations:

$$+\nearrow \Sigma F_x = 0: \qquad 100 \text{ lb} - \tfrac{3}{5}(300 \text{ lb}) - F = 0$$
$$F = -80 \text{ lb} \qquad \mathbf{F} = 80 \text{ lb} \nearrow$$

$$+\nwarrow \Sigma F_y = 0: \qquad N - \tfrac{4}{5}(300 \text{ lb}) = 0$$
$$N = +240 \text{ lb} \qquad \mathbf{N} = 240 \text{ lb} \nwarrow$$

The force **F** required to maintain equilibrium is an 80-lb force directed up and to the right; the tendency of the crate is thus to move down the plane.

Maximum Friction Force. The magnitude of the maximum friction force that may be developed between the crate and the plane is

$$F_m = \mu_s N \qquad F_m = 0.25(240 \text{ lb}) = 60 \text{ lb}$$

Since the value of the force required to maintain equilibrium (80 lb) is larger than the maximum value that may be obtained (60 lb), equilibrium is not maintained and *the crate will slide down the plane*.

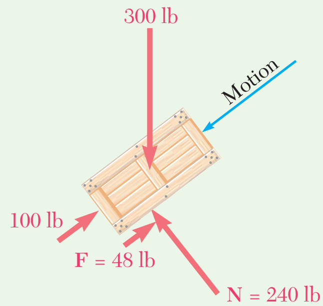


Fig. 2 Free-body diagram of crate showing actual friction force.

Actual Value of Friction Force. The magnitude of the actual friction force is

$$F_{\text{actual}} = F_k = \mu_k N = 0.20(240 \text{ lb}) = 48 \text{ lb}$$

The sense of this force is opposite to the sense of motion; the force is thus directed up and to the right (Fig. 2):

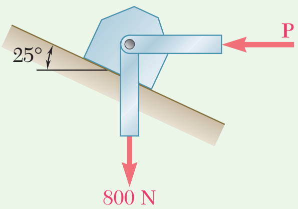
$$\mathbf{F}_{\text{actual}} = 48 \text{ lb} \nearrow \blacktriangleleft$$

Note that the forces acting on the crate are not balanced. Their resultant is

$$\tfrac{3}{5}(300 \text{ lb}) - 100 \text{ lb} - 48 \text{ lb} = 32 \text{ lb} \swarrow$$

REFLECT and THINK: This is a typical friction problem of the first type. Note that you used the coefficient of static friction to determine if the crate moves, but once you found that it does move, you needed the coefficient of kinetic friction to determine the friction force.

Sample Problem 8.2



A support block is acted upon by two forces as shown. Knowing that the coefficients of friction between the block and the incline are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the force **P** required to (*a*) start the block moving up the incline; (*b*) keep it moving up; (*c*) prevent it from sliding down.

STRATEGY: This problem involves practical variations of the third type of friction problem. You can approach the solutions through the concept of the angles of friction.

MODELING:

Free-Body Diagram. For each part of the problem, draw a free-body diagram of the block and a force triangle including the 800-N vertical force, the horizontal force **P**, and the force **R** exerted on the block by the incline. You must determine the direction of **R** in each separate case. Note that, since **P** is perpendicular to the 800-N force, the force triangle is a right triangle, which easily can be solved for **P**. In most other problems, however, the force triangle will be an oblique triangle and should be solved by applying the law of sines.

ANALYSIS:

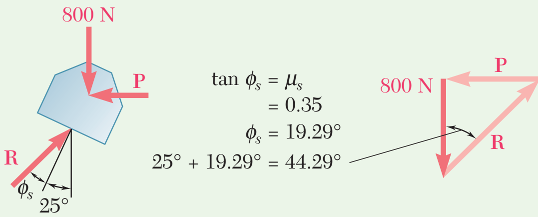


Fig. 1 Free-body diagram of block and its force triangle—motion impending up incline.

a. Force *P* to Start Block Moving Up. In this case, motion is impending up the incline, so the resultant is directed at the angle of static friction (Fig. 1). Note that the resultant is oriented to the left of the normal such that its friction component (not shown) is directed opposite the direction of impending motion.

$$P = (800\text{ N}) \tan 44.29° \qquad \mathbf{P} = 780\text{ N}\leftarrow \quad ◀$$

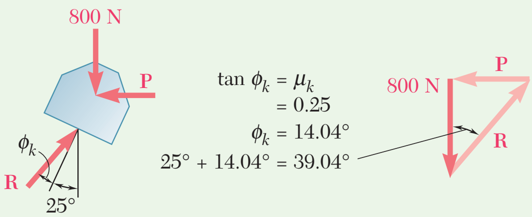


Fig. 2 Free-body diagram of block and its force triangle—motion continuing up incline.

b. Force *P* to Keep Block Moving Up. Motion is continuing, so the resultant is directed at the angle of kinetic friction (Fig. 2). Again, the resultant is oriented to the left of the normal such that its friction component is directed opposite the direction of motion.

$$P = (800\text{ N}) \tan 39.04° \qquad \mathbf{P} = 649\text{ N}\leftarrow \quad ◀$$

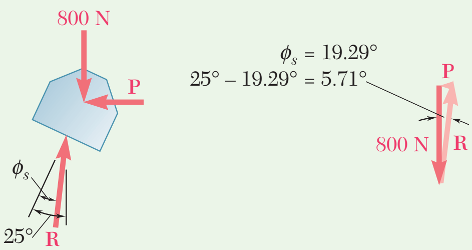


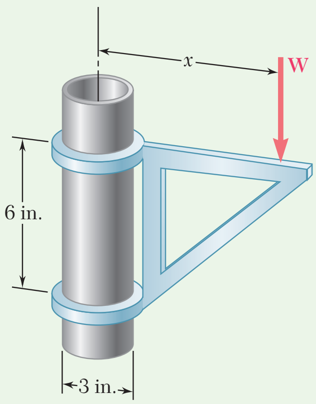
Fig. 3 Free-body diagram of block and its force triangle—motion prevented down the slope.

c. Force *P* to Prevent Block from Sliding Down. Here, motion is impending down the incline, so the resultant is directed at the angle of static friction (Fig. 3). Note that the resultant is oriented to the right of the normal such that its friction component is directed opposite the direction of impending motion.

$$P = (800\text{ N}) \tan 5.71° \qquad \mathbf{P} = 80.0\text{ N}\leftarrow \quad ◀$$

REFLECT and THINK: As expected, considerably more force is required to begin moving the block up the slope than is necessary to restrain it from sliding down the slope.

Sample Problem 8.3



The movable bracket shown may be placed at any height on the 3-in.-diameter pipe. If the coefficient of static friction between the pipe and bracket is 0.25, determine the minimum distance x at which the load **W** can be supported. Neglect the weight of the bracket.

STRATEGY: In this variation of the third type of friction problem, you know the coefficient of static friction and that motion is impending. Since the problem involves consideration of resistance to rotation, you should apply both moment equilibrium and force equilibrium.

MODELING:

Free-Body Diagram. Draw the free-body diagram of the bracket (Fig. 1). When **W** is placed at the minimum distance x from the axis of the pipe, the bracket is just about to slip, and the forces of friction at A and B have reached their maximum values:

$$F_A = \mu_s N_A = 0.25\ N_A$$
$$F_B = \mu_s N_B = 0.25\ N_B$$

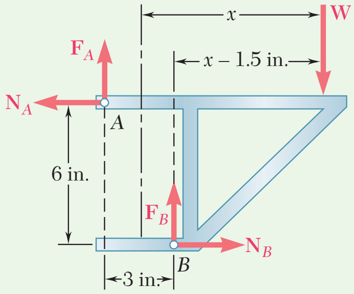


Fig. 1 Free-body diagram of bracket.

ANALYSIS:

Equilibrium Equations.

$\xrightarrow{+}\Sigma F_x = 0$: $\quad N_B - N_A = 0$

$$N_B = N_A$$

$+\uparrow\Sigma F_y = 0$: $\quad F_A + F_B - W = 0$

$$0.25N_A + 0.25N_B = W$$

Since N_B is equal to N_A,

$$0.50N_A = W$$
$$N_A = 2W$$

$+\circlearrowleft\Sigma M_B = 0$: $\quad N_A(6\text{ in.}) - F_A(3\text{ in.}) - W(x - 1.5\text{ in.}) = 0$

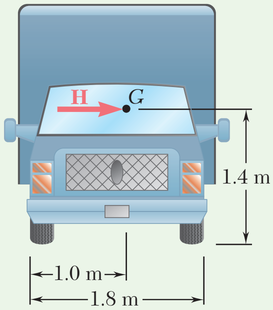
$$6N_A - 3(0.25N_A) - Wx + 1.5W = 0$$
$$6(2W) - 0.75(2W) - Wx + 1.5W = 0$$

Dividing through by W and solving for x, you have

$$x = 12\text{ in.}$$

REFLECT and THINK: In a problem like this, you may not figure out how to approach the solution until you draw the free-body diagram and examine what information you are given and what you need to find. In this case, since you are asked to find a distance, the need to evaluate moment equilibrium should be clear.

Sample Problem 8.4



An 8400-kg truck is traveling on a level horizontal curve, resulting in an effective lateral force **H** (applied at the center of gravity G of the truck). Treating the truck as a rigid system with the center of gravity shown, and knowing that the distance between the outer edges of the tires is 1.8 m, determine (*a*) the maximum force **H** before tipping of the truck occurs, (*b*) the minimum coefficient of static friction between the tires and roadway such that slipping does not occur before tipping.

STRATEGY: For the direction of **H** shown, the truck would tip about the outer edge of the right tire. At the verge of tip, the normal force and friction force are zero at the left tire, and the normal force at the right tire is at the outer edge. You can apply equilibrium to determine the value of **H** necessary for tip and the required friction force such that slipping does not occur.

MODELING: Draw the free-body diagram of the truck (Fig. 1), which reflects impending tip about point B. Obtain the weight of the truck by multiplying its mass of 8400 kg by $g = 9.81\ \text{m/s}^2$; that is, $W = 82\ 400\ \text{N}$ or 82.4 kN.

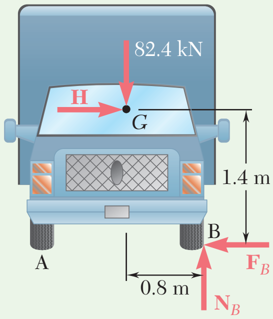


Fig. 1 Free-body diagram of truck.

ANALYSIS:

Free Body: Truck (Fig. 1).

$+\circlearrowleft \Sigma M_B = 0$: $\quad (82.4\ \text{kN})(0.8\ \text{m}) - H(1.4\ \text{m}) = 0$

$H = +47.1\ \text{kN} \qquad H = 47.1\ \text{kN} \rightarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $\quad 47.1\ \text{kN} - F_B = 0$

$F_B = +47.1\ \text{kN}$

$+\uparrow \Sigma F_y = 0$: $\quad N_B - 82.4\ \text{kN} = 0$

$N_B = +82.4\ \text{kN}$

Minimum Coefficient of Static Friction. The magnitude of the maximum friction force that can be developed is

$$F_m = \mu_s N_B = \mu_s\ (82.4\ \text{kN})$$

Setting this equal to the friction force required, $F_B = 47.1$ kN, gives

$$\mu_s\ (82.4\ \text{kN}) = 47.1\ \text{kN} \qquad \mu_s = 0.572 \text{ ◀}$$

REFLECT and THINK: Recall from physics that **H** represents the force due to the centripetal acceleration of the truck (of mass m), and its magnitude is

$$H = m(v^2/\rho)$$

where

v = velocity of the truck
ρ = radius of curvature

In this problem, if the truck were traveling around a curve of 100-m radius (measured to G), the velocity at which it would begin to tip would be 23.7 m/s (or 85.2 km/h). You will learn more about this aspect in your study of dynamics.