APPLIED ELECTRICITY (EE 151)

Compiled by:

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Taught by:

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EDUCATIONAL AIM

“To introduce students to the physical principles underpinning Electrical and Electronic Engineering, including tools required to analyze electric and magnetic circuits.”

LEARNING OUTCOMES

a. Knowledge and understanding

- Understand Kirchhoff’s laws, Norton and Thévenin equivalent circuits and be able to apply them to simple circuits
- Understand the superposition principle and be able to apply them to simple circuits
- Understand the concept of phasors and be able to apply them to simple AC circuits

b. Intellectual skills

- Be able to reduce complex circuits to a simple form
- Perform calculations on three-phase circuits
- Perform analysis on linear and non-linear magnetic circuits

c. Professional practical skills

- Students should be able to apply appropriate method to analyze various circuits/systems

METHODOLOGY

- Classroom lectures

MATERIAL REQUIRED

- Powerpoint presentation:**https://goo.gl/W9rKXm**
- Text book : FUNDAMENTALS OF ELECTRIC AND MAGNETIC CIRCUITS by P. Y. Okyere and E. A. Frimpong

COURSE OUTLINE

Unit 1: Circuits and Network Theorems – Kirchhoff's laws, Thevenin's Theorem, Norton's Theorem, Superposition Theorem, Reciprocity Theorem and Delta- Star Transformation.

Unit 2: Alternating current circuits – Determination of Average and RMS values, Harmonics, Phasors, impedance, current and power in ac circuits

Unit 3: Three-phase circuits – Connection of three-phase windings, three phase loads, power in three-phase circuits, solving three-phase circuit problems

Unit 4: Magnetic circuits – Components and terminologies, solving magnetic circuit problems

CLASSROOM NORMS

1. No phone usage
2. No eating
3. No noise-making
4. No lateness
5. No intimidation
6. No sleeping

- Students who fail to abide by these norms will be asked to leave the classroom

ASSESSMENT

1. Quizzes
2. Laboratory work
3. Mid-semester examination

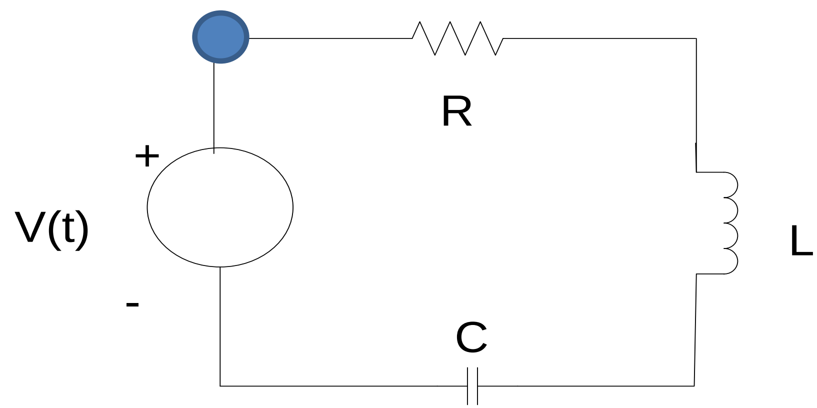
$\left.\right\}$ **30%**

4. End of semester examination 70%

UNIT 1: CIRCUIT AND NETWORK THEOREMS

- Definition of a circuit

An interconnection of elements forming a closed path along which current can flow.



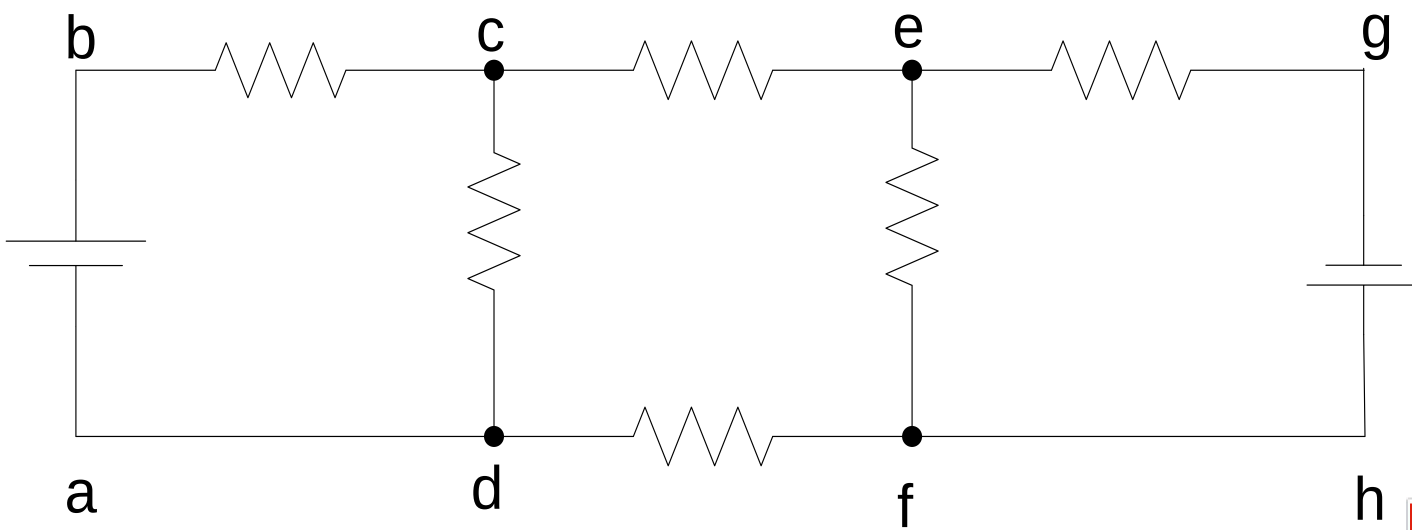
- Elements of an electric circuit

Active elements: Energy producing elements eg. Batteries, Generators, Solar cells, Transistor models

Passive elements: Energy using elements eg. Resistors, inductors, capacitors

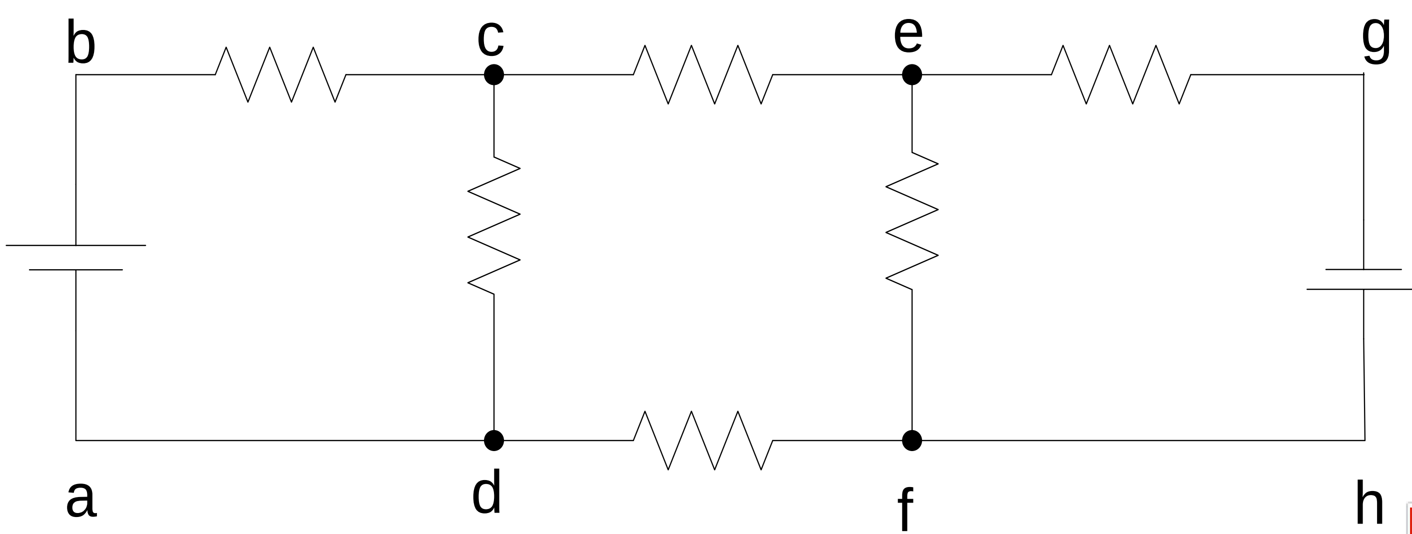
CIRCUIT TERMINOLOGIES

- ❖ Node (Junction) –A point where currents split or come together [points c, d, e and f]
- ❖ Path – Any connection where current flows [eg bc, be, fa]



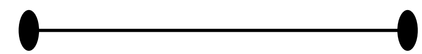
CIRCUIT TERMINOLOGIES

- **Branch** – A connection (path) between two nodes [eg. cd, cbad, df]
- **Loop/Mesh** – a closed path of a circuit [eg. cghdc]



CIRCUIT TERMINOLOGIES

❖ **Short-circuit**– A branch of theoretically zero resistance. It diverts to itself all currents that would have flown in adjacent branches (branches hooked to the same node) except branches with sources.



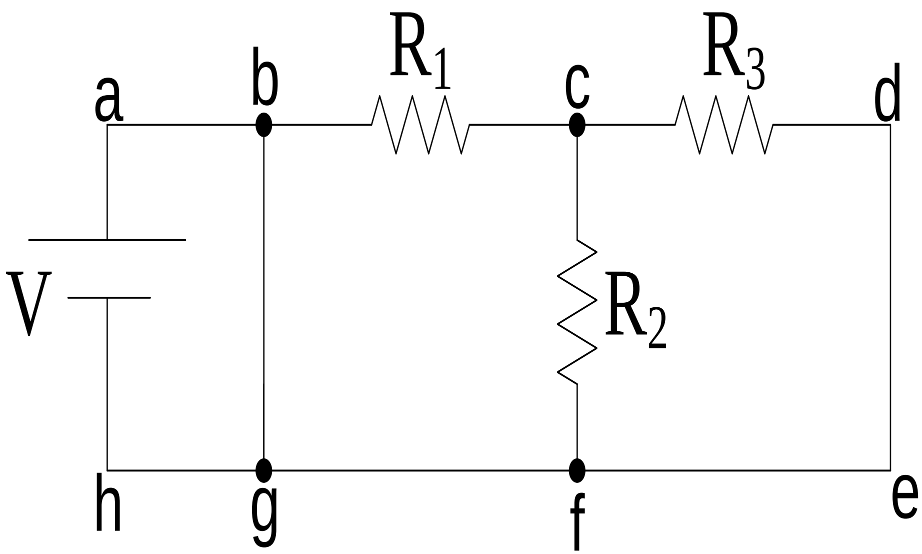


Fig. 1

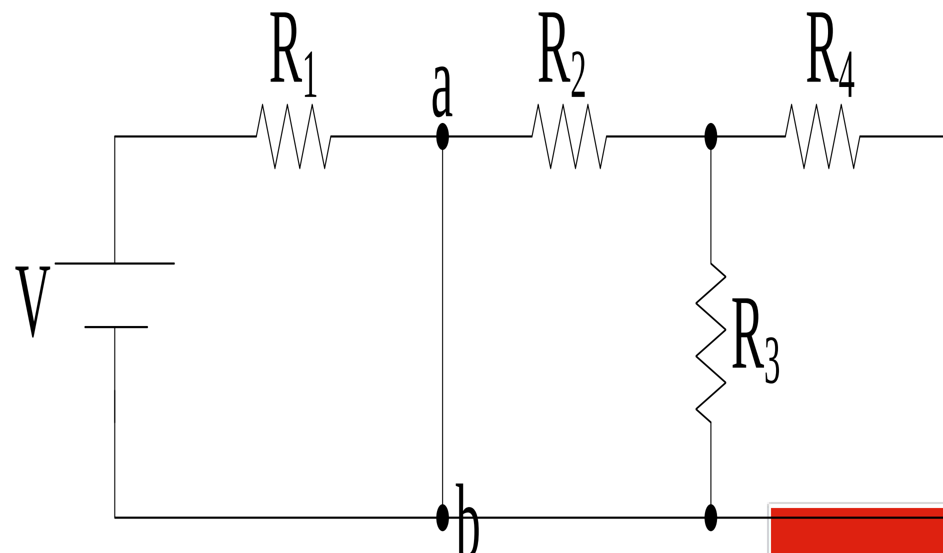


Fig. 2

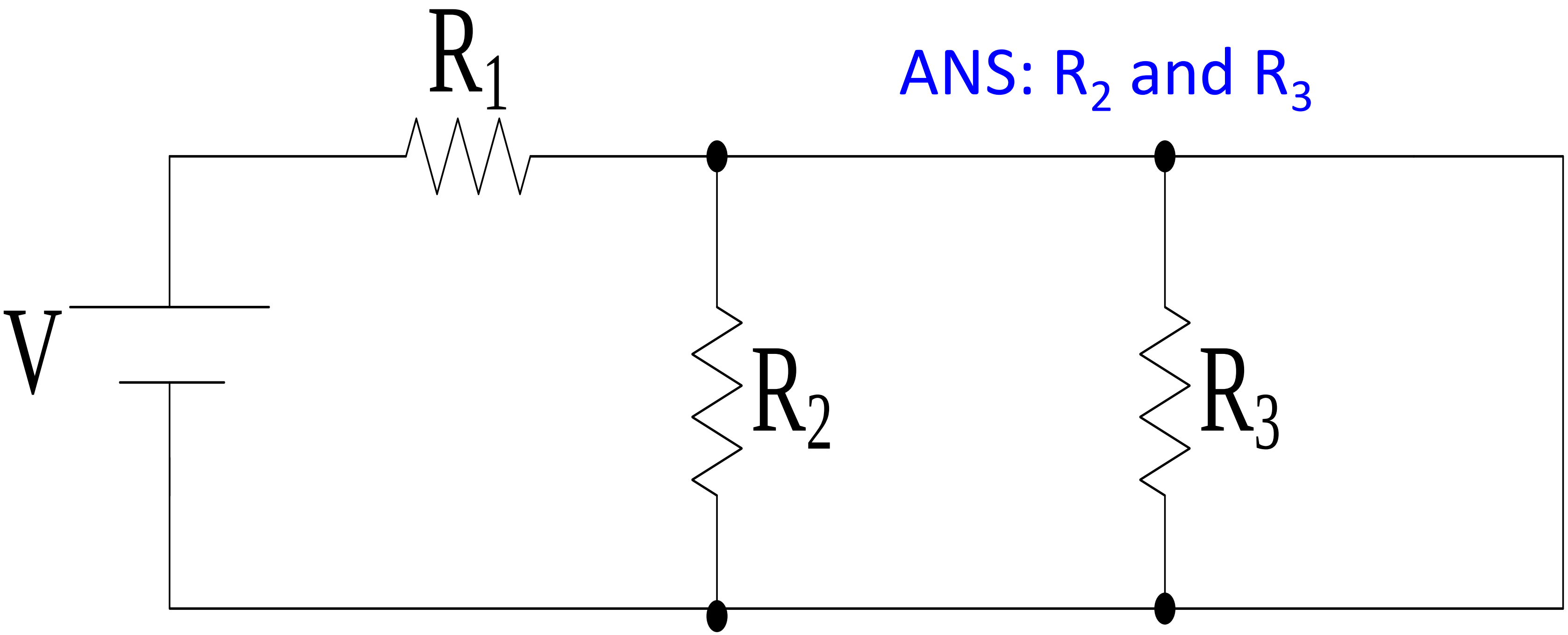
CIRCUIT TERMINOLOGIES

❖ Short-circuit *cont.*

Self assessment

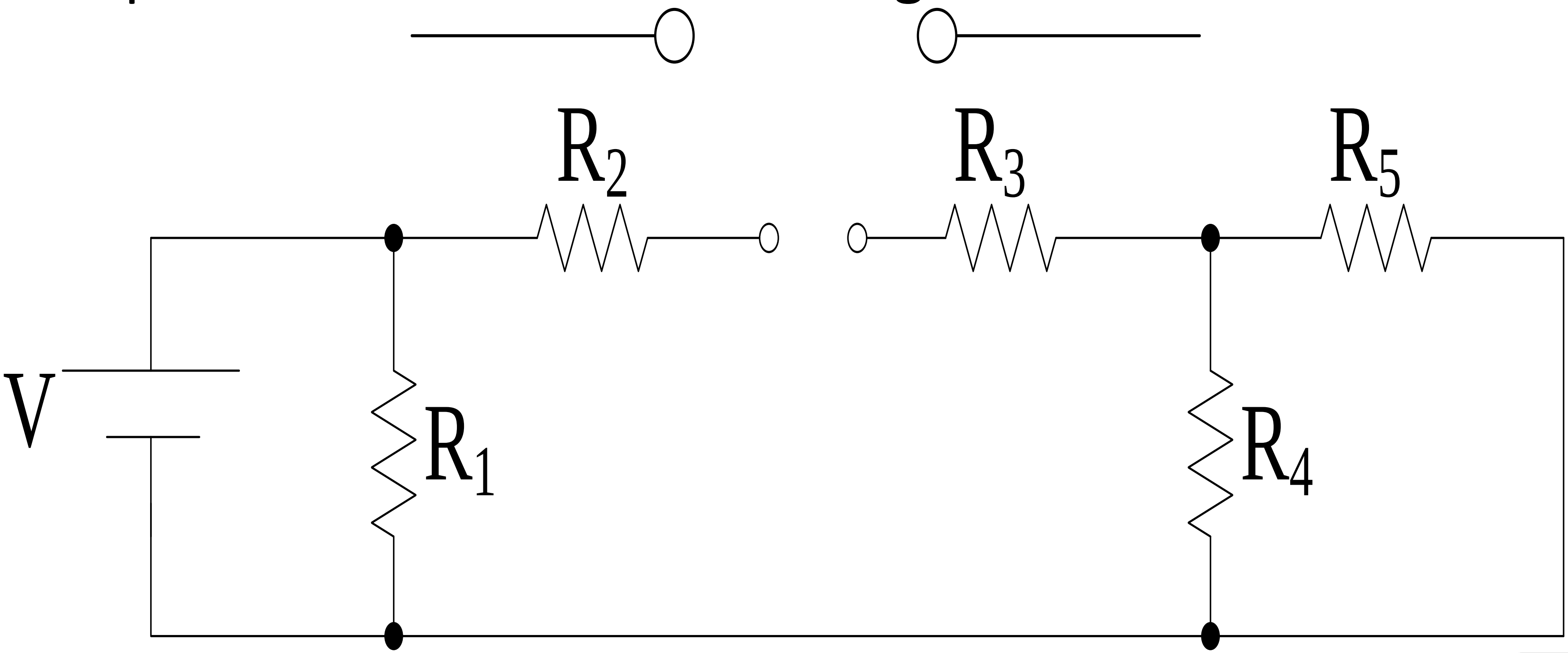
Which of the resistors in the circuit below have been short-circuited?

ANS: R_2 and R_3

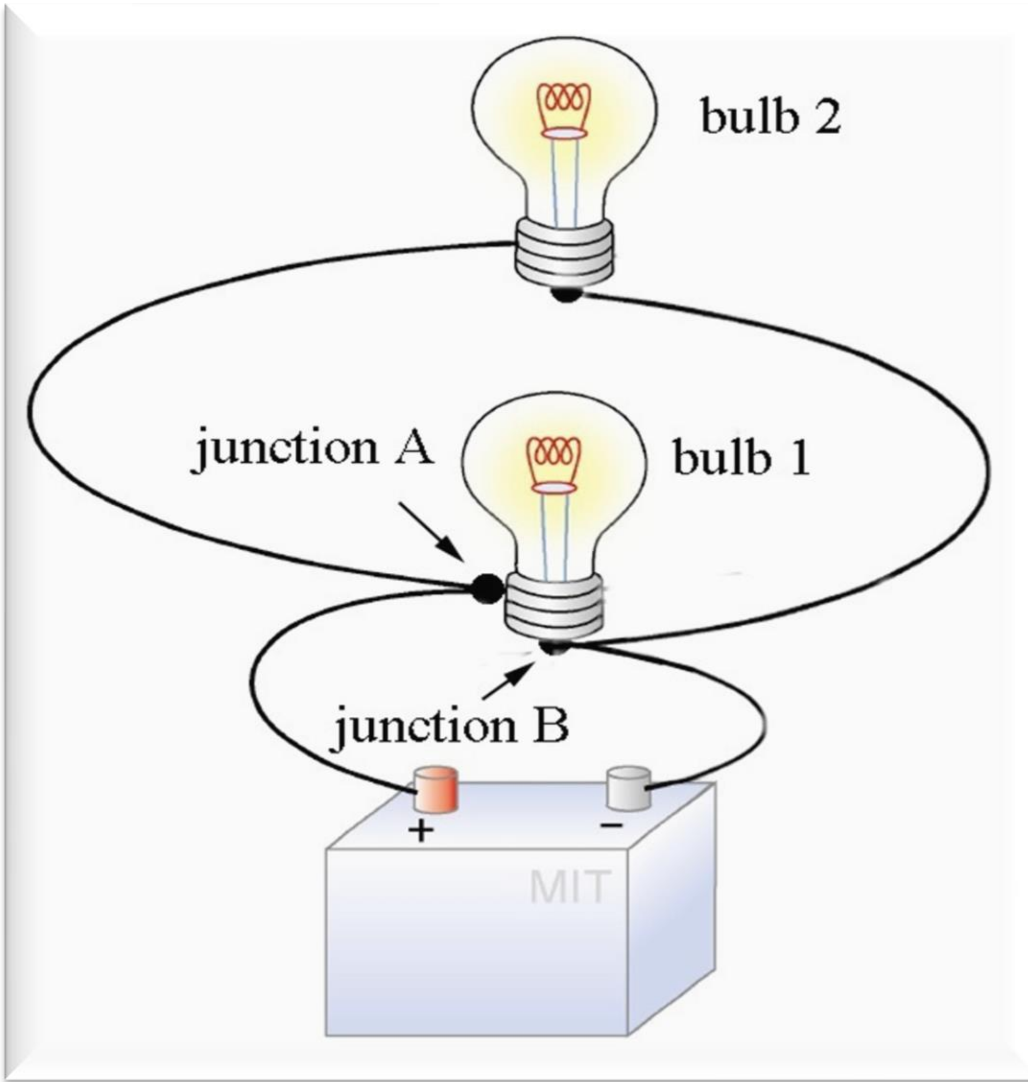


CIRCUIT TERMINOLOGIES

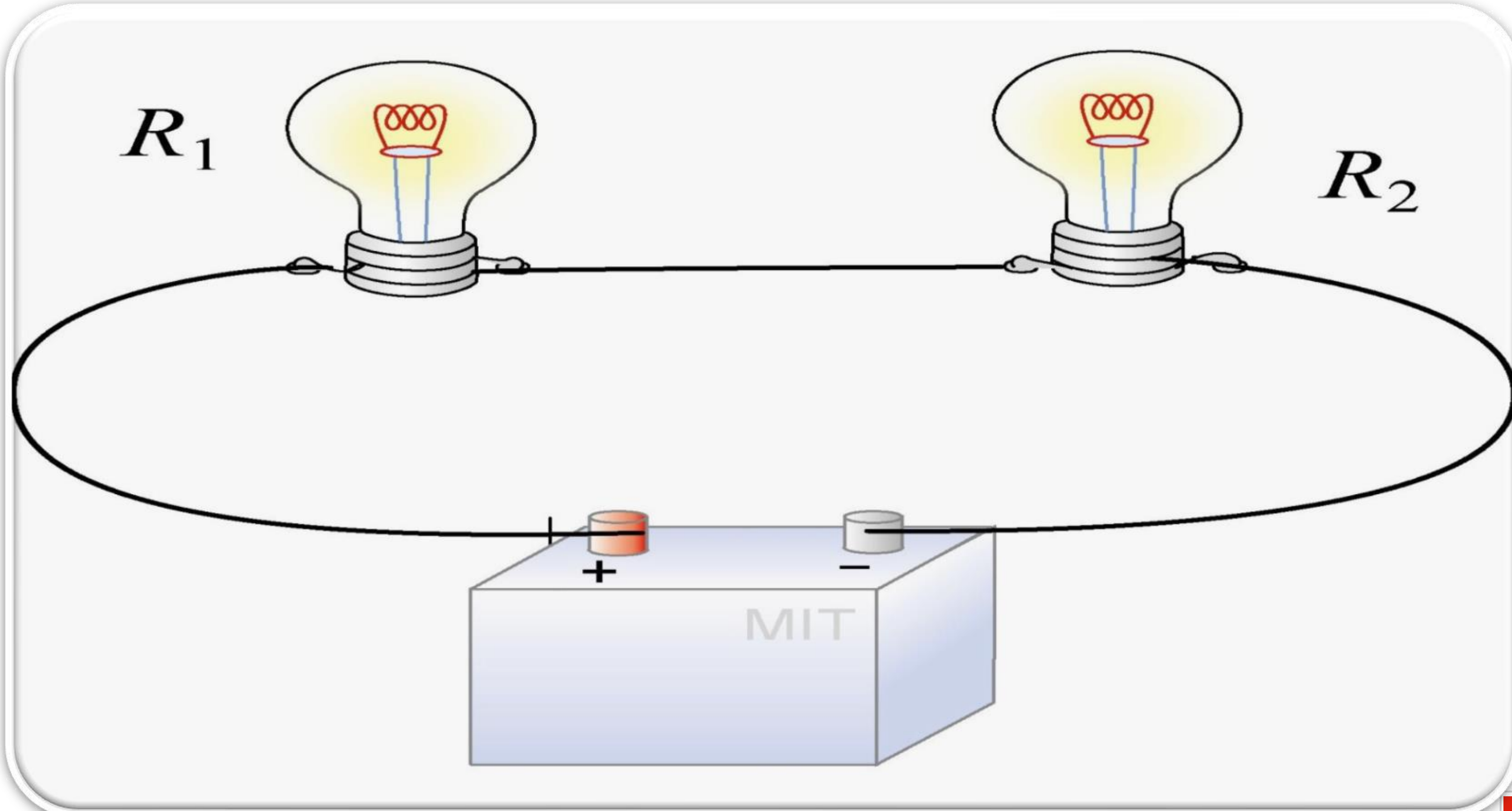
❖ Open circuit – A branch of theoretically infinite resistance. It prevents current from flowing in its branch.



RESISTORS IN PARALLEL



RESISTORS IN SERIES



RESISTORS IN SERIES

Resistors are in series when the same current flows through them. There is NO JUNCTION between them.

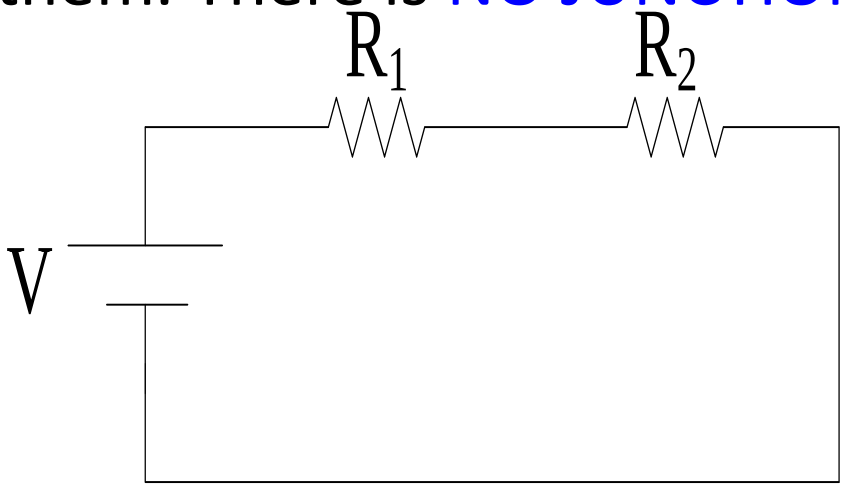


Fig. 1

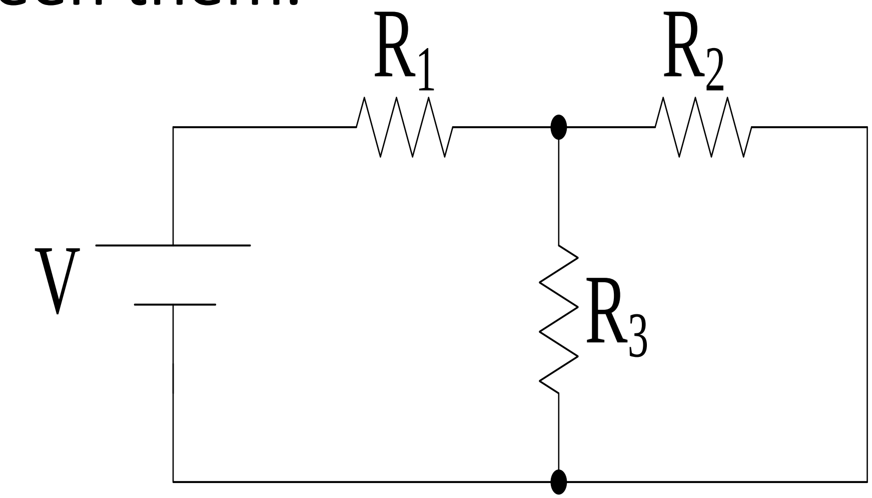


Fig. 2

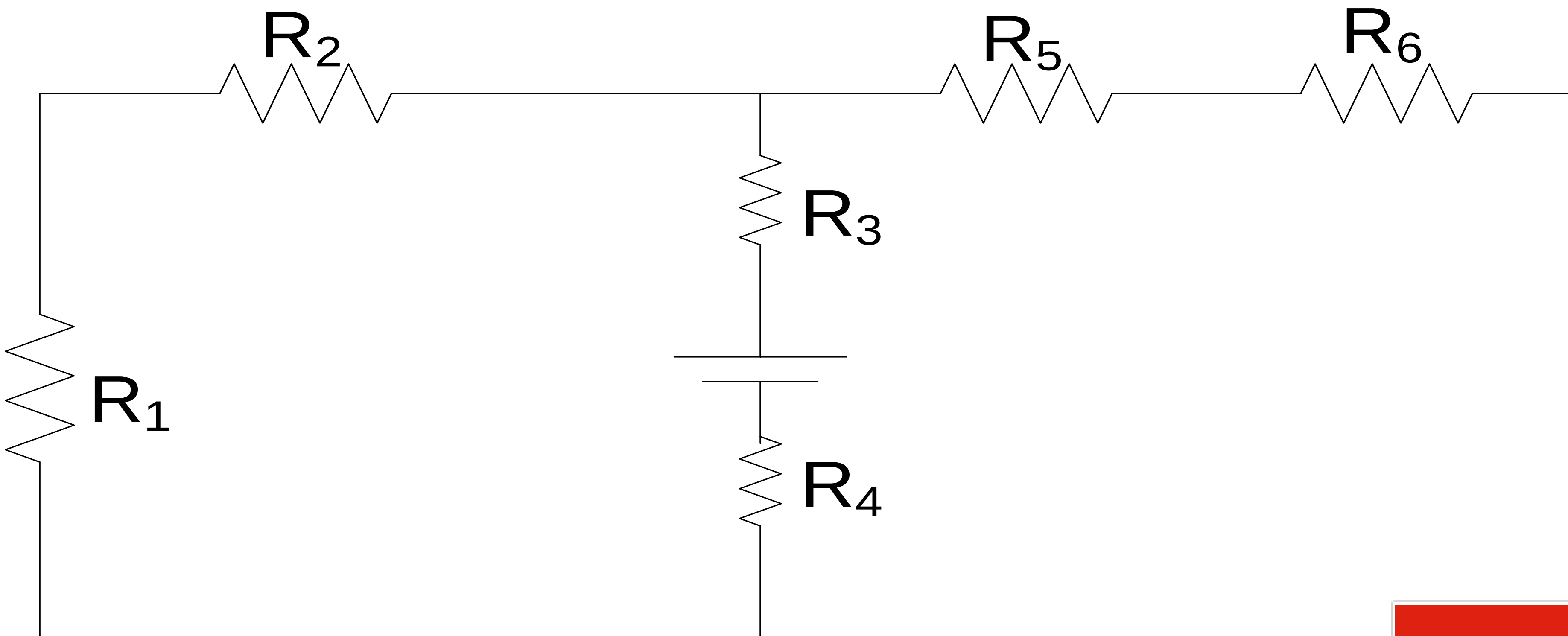
In Fig. 1 : R_1 and R_2 are in series

In Fig. 2: None of the resistors are in series

RESISTORS IN SERIES

- Self assessment 1

Which of the following resistors are in series?



ANS: R_1&R_2, R_3 & R_4 and R_5 & R_6

RESISTORS IN SERIES

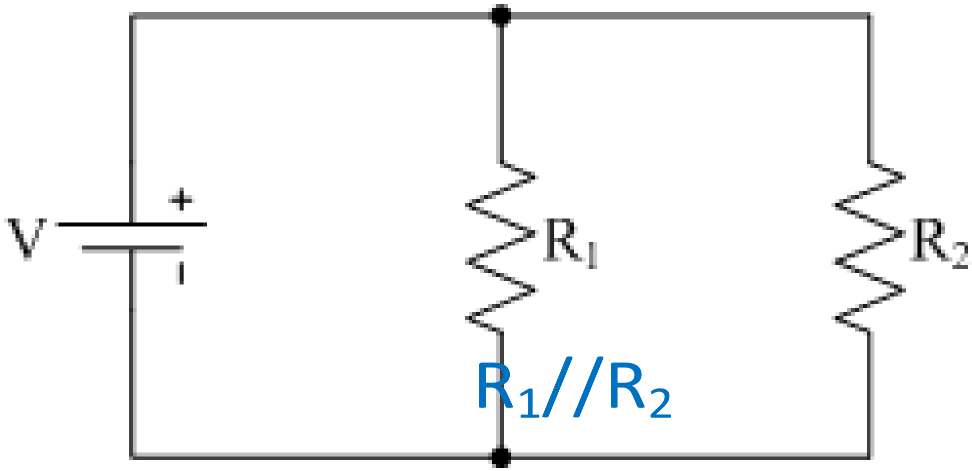
- Total (effective) resistance of series resistors

The total resistance R_T for resistors R_1, R_2, R_3, ….., R_N which are in series is given by:

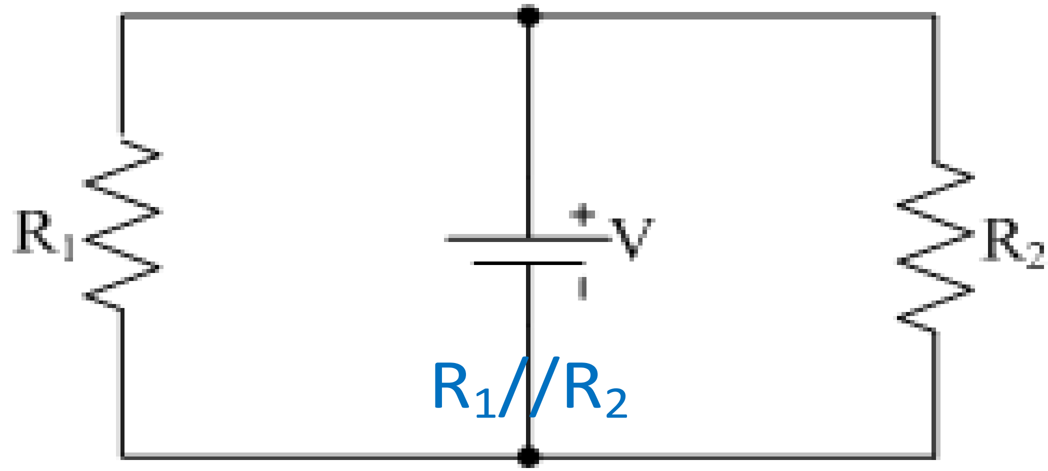
$$R_T = R_1 + R_2 + \ldots + R_N$$

RESISTORS IN PARALLEL

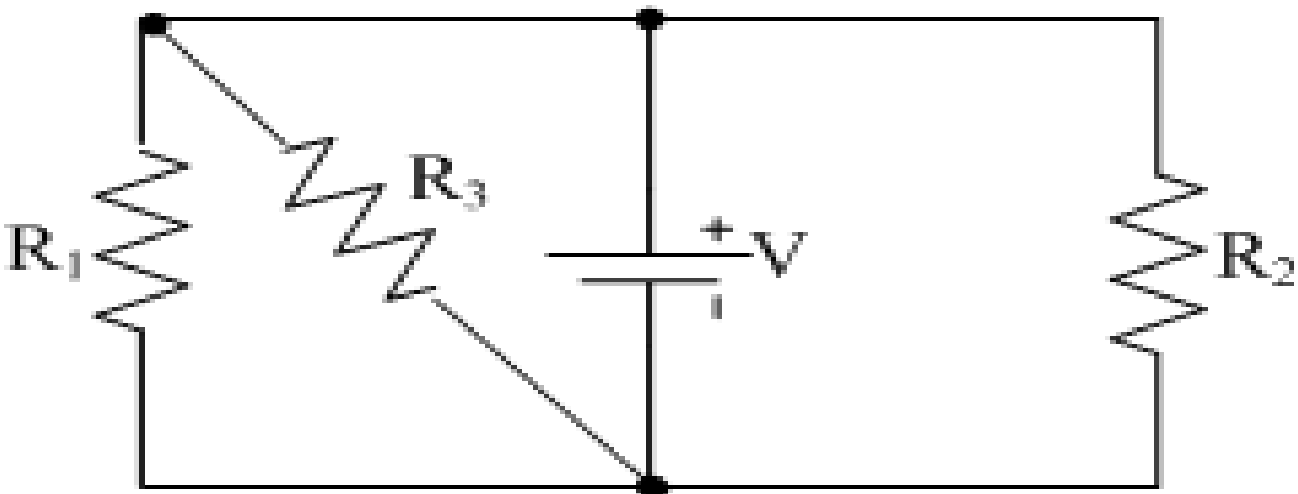
Resistors are said to be in parallel when the voltage across them is the same.



$R_1//R_2$



$R_1//R_2$



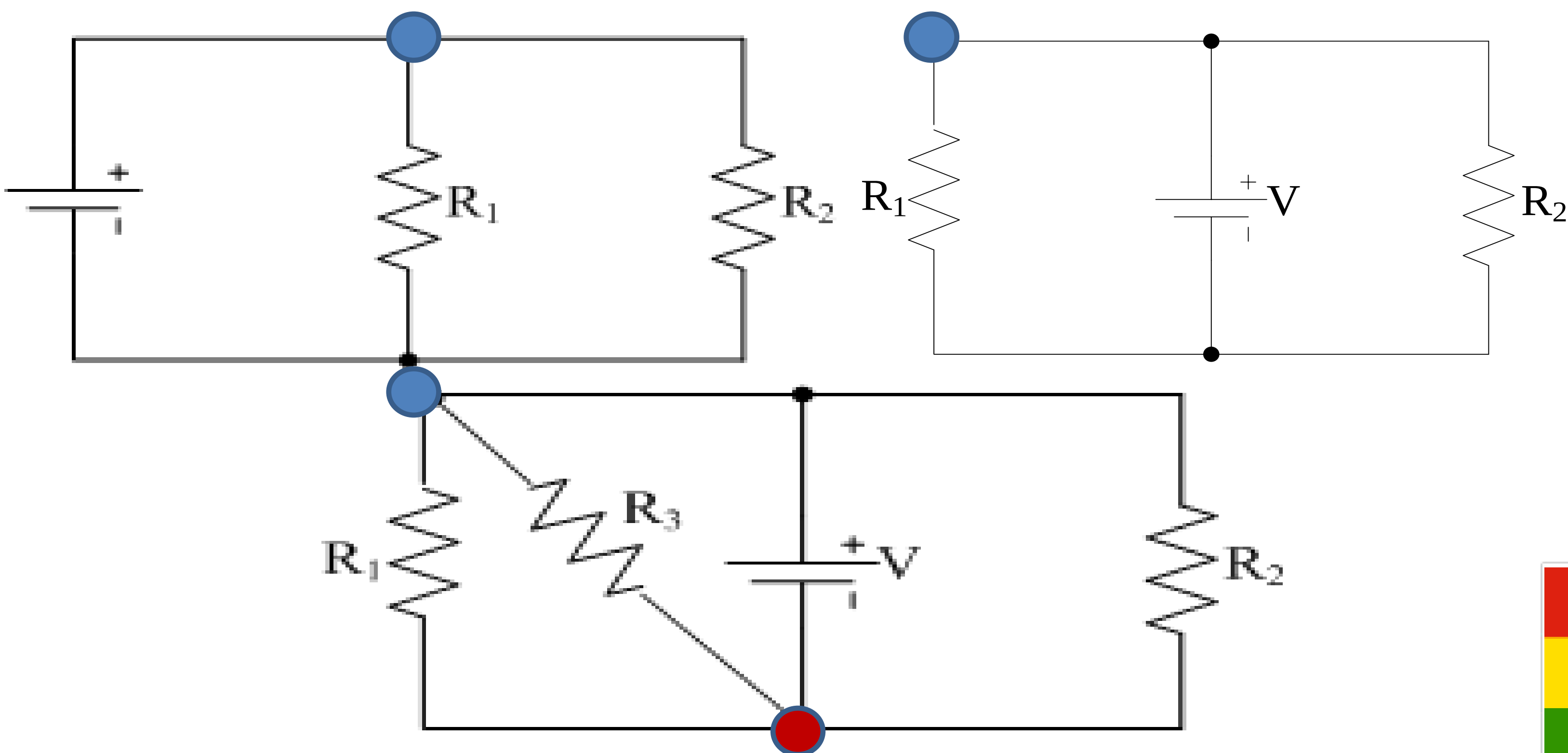
$R_1//R_2$

$R_1//R_3$

$R_3//R_2$

RESISTORS IN PARALLEL

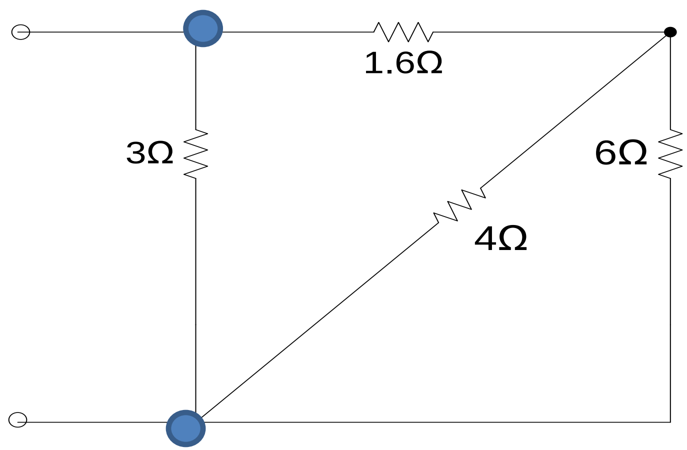
Colloquially, TWO resistors are in parallel if it is possible to traverse them without passing through another element.



RESISTORS IN PARALLEL

Self assessment

Which of the resistors in the circuit below are in parallel?



ANS: 4//6

RESISTORS IN PARALLEL

Total resistance

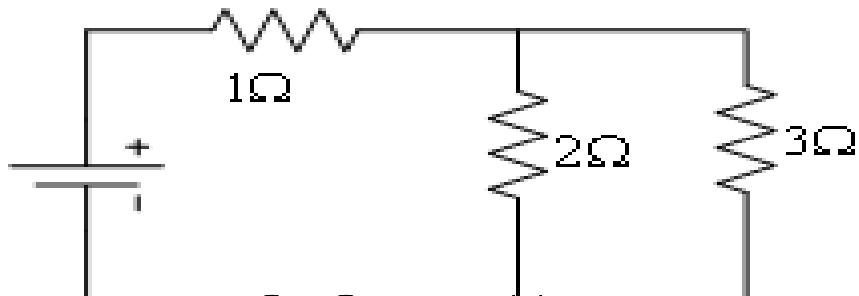
When resistors R_1 and R_2 are in parallel, the total resistance R_T is given by:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \quad R_T = \frac{R_1 R_2}{R_1 + R_2}$$

EFFECTIVE RESISTANCE OF A CIRCUIT

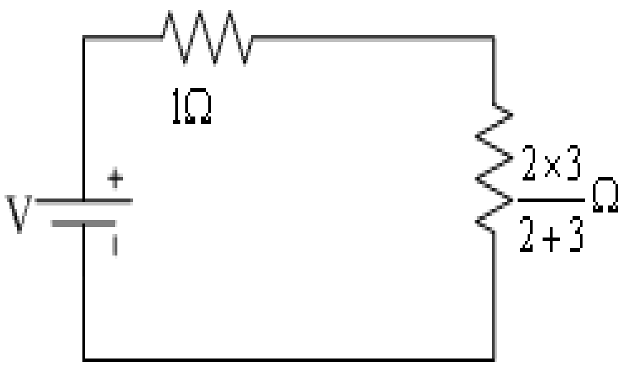
Effective circuit resistance is found by identifying and putting together series and or parallel resistors

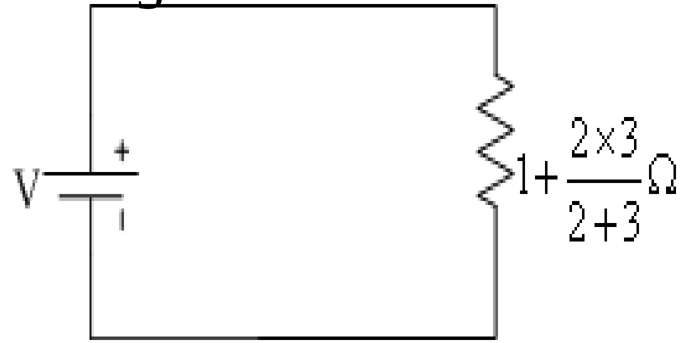
Eg 1. Find the total resistance of the circuit below.



❖Solution

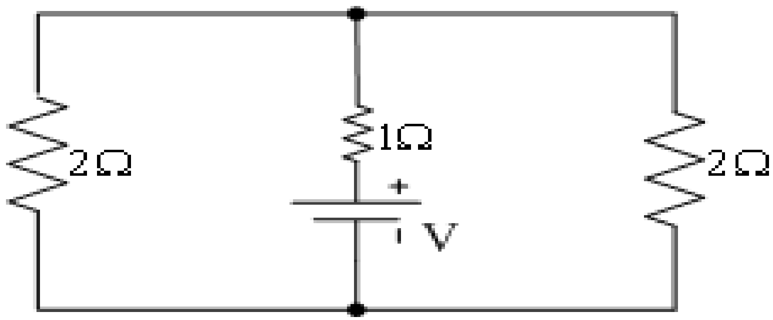
$$R_T = (2 // 3) + 1 = \frac{2\times 3}{2+3} + 1 = \frac{11}{5}\Omega$$



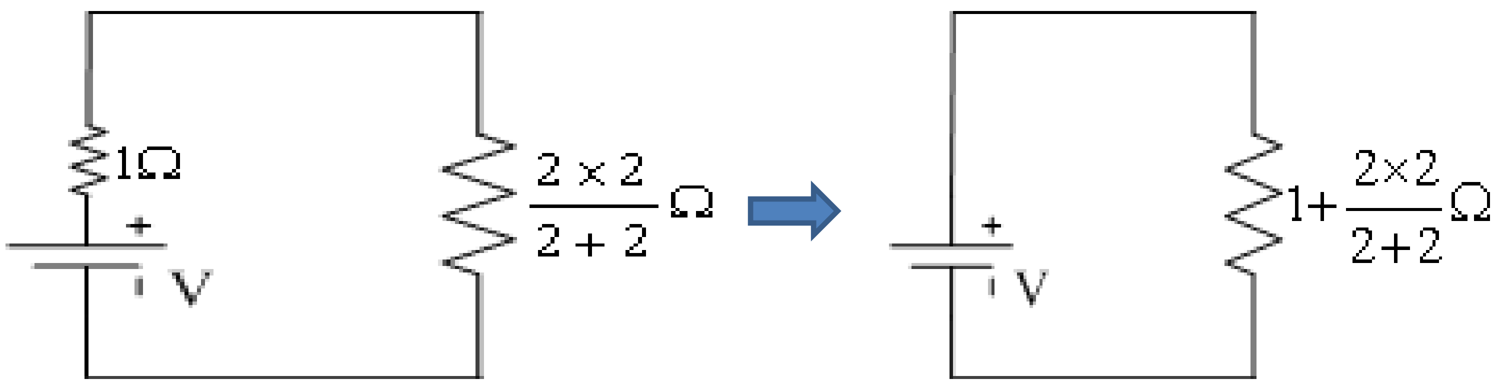


EFFECTIVE RESISTANCE OF A CIRCUIT

Eg. 2. Find the total resistance of the circuit below.



❖Solution

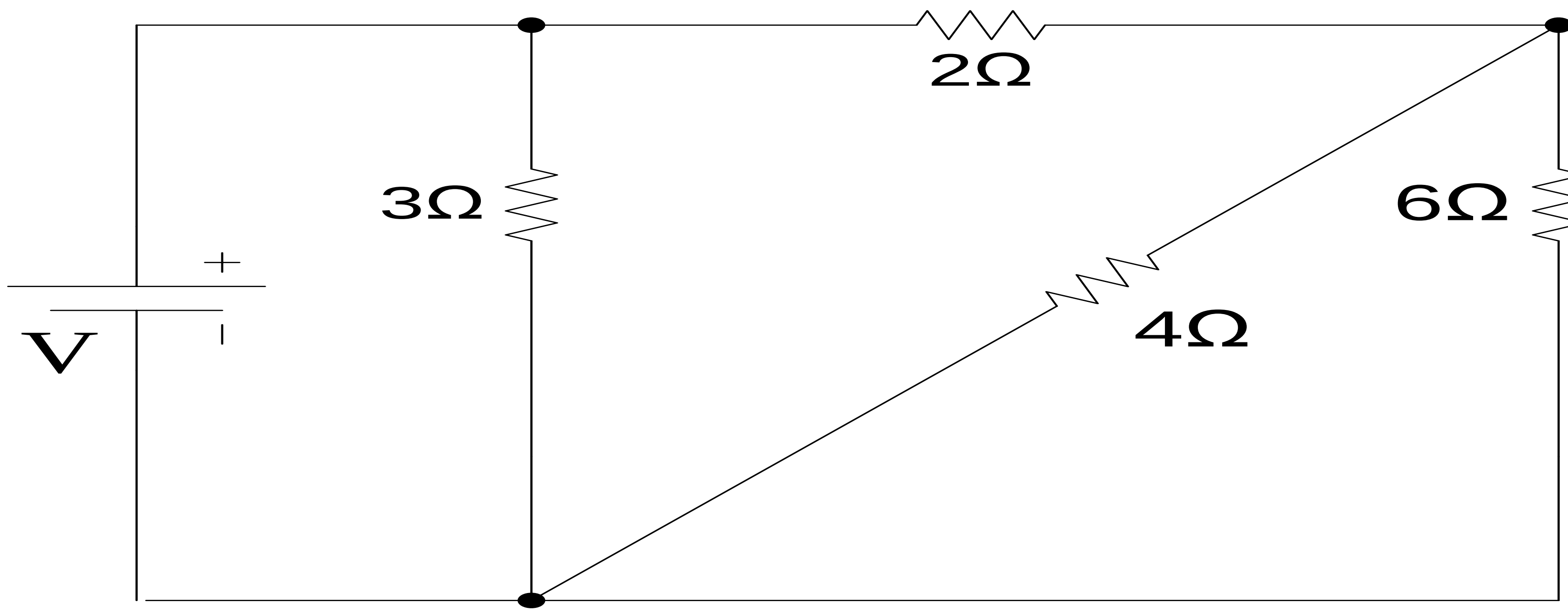


$$R_T = (2 // 2) + 1 = \frac{2 \times 2}{2+2} + 1 = 2\Omega$$

EFFECTIVE RESISTANCE OF A CIRCUIT

Self Assessment 1

Find the total resistance of the circuit below.



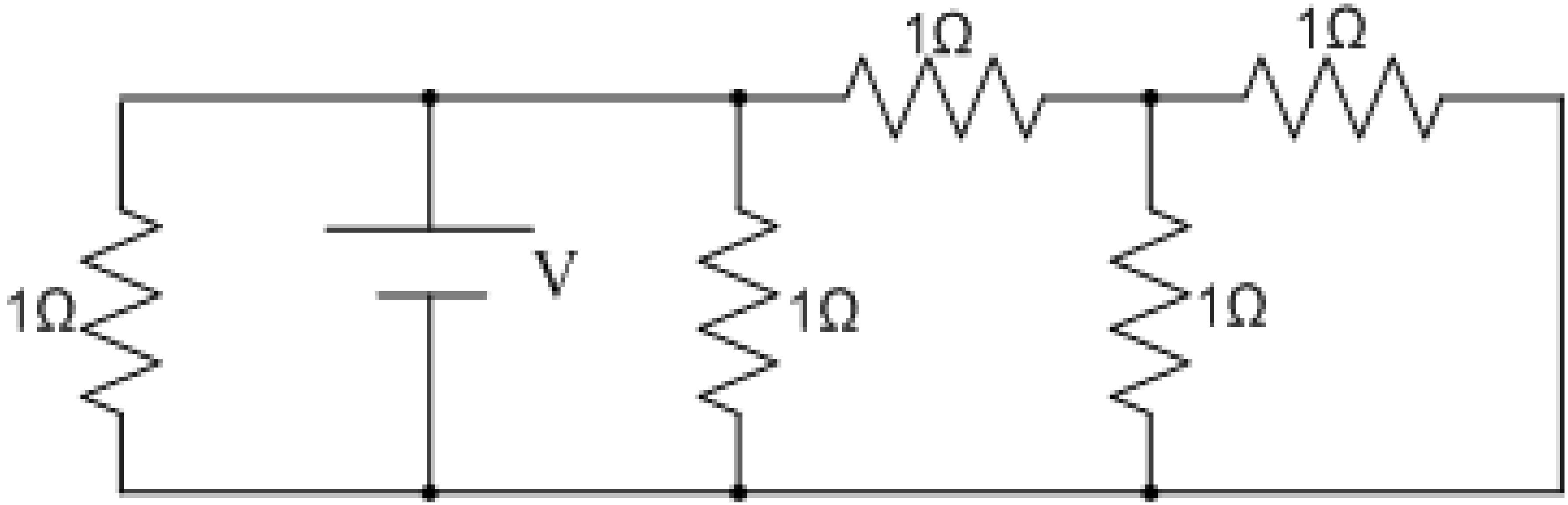
Answer

$$R_T = [(4//6)+2]//3 = \left[\frac{4\times 6}{4+6}+2\right]//3 = \frac{22}{5}//3 = \frac{66}{37}\Omega$$

EFFECTIVE RESISTANCE OF A CIRCUIT

Self Assessment 2

Find the total resistance of the circuit below.



Answer

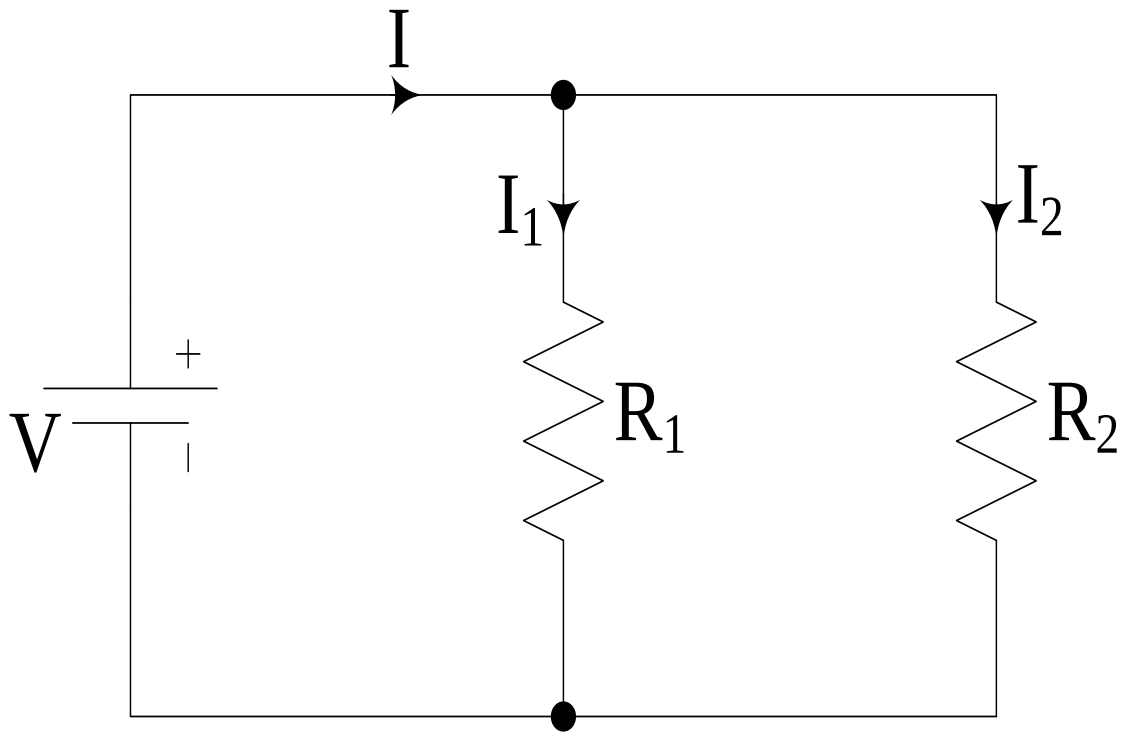
$$R_T = [(1//1)+1]//1//1 = \left[\frac{1}{2}+1\right]//1//1 = \frac{3}{2}//\frac{1}{2} = \frac{3}{8}\Omega$$

CURRENT DIVISION RULE

The current division rule is applied to share current between parallel branches. Consider the circuits below

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

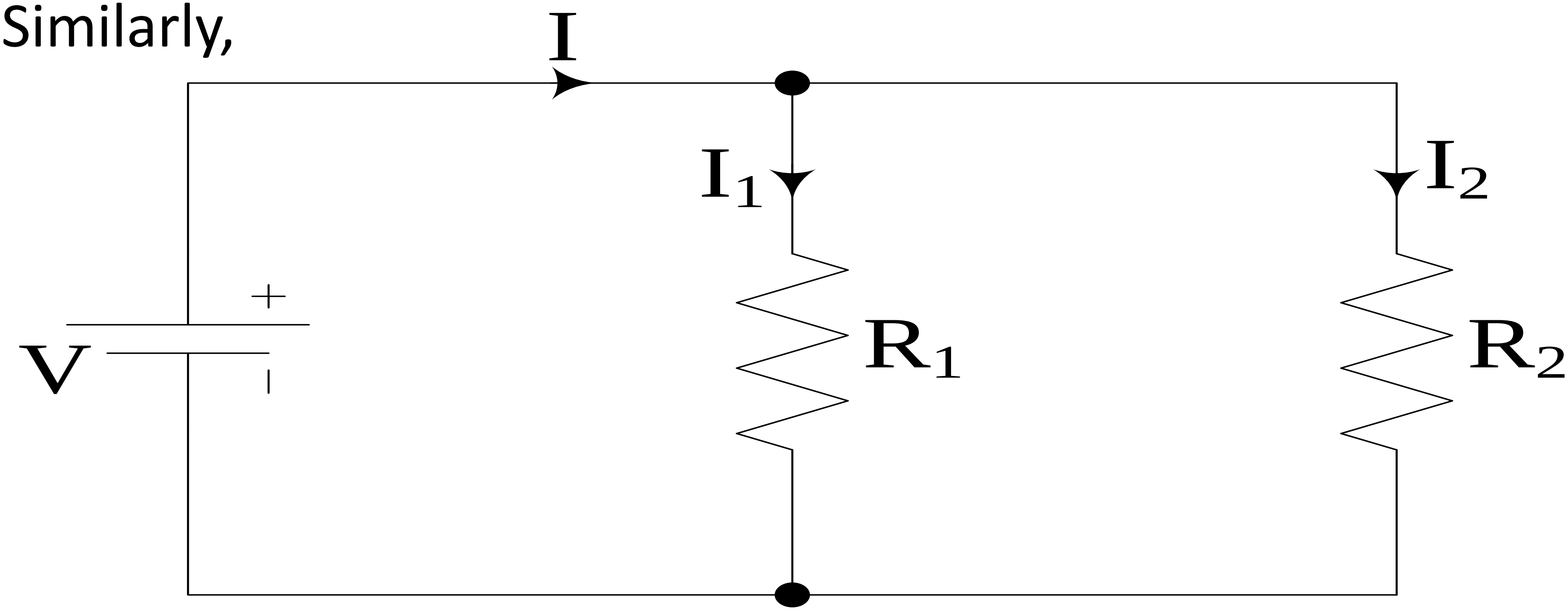
$$V = IR_T = I\frac{R_1 R_2}{R_1 + R_2}$$



$$I_1 = \frac{V}{R_1} = \frac{I\dfrac{R_1 R_2}{R_1 + R_2}}{R_1} = I\frac{R_1 R_2}{R_1 + R_2} \times \frac{1}{R_1} = I\frac{R_2}{R_1 + R_2}$$

CURRENT DIVISION RULE

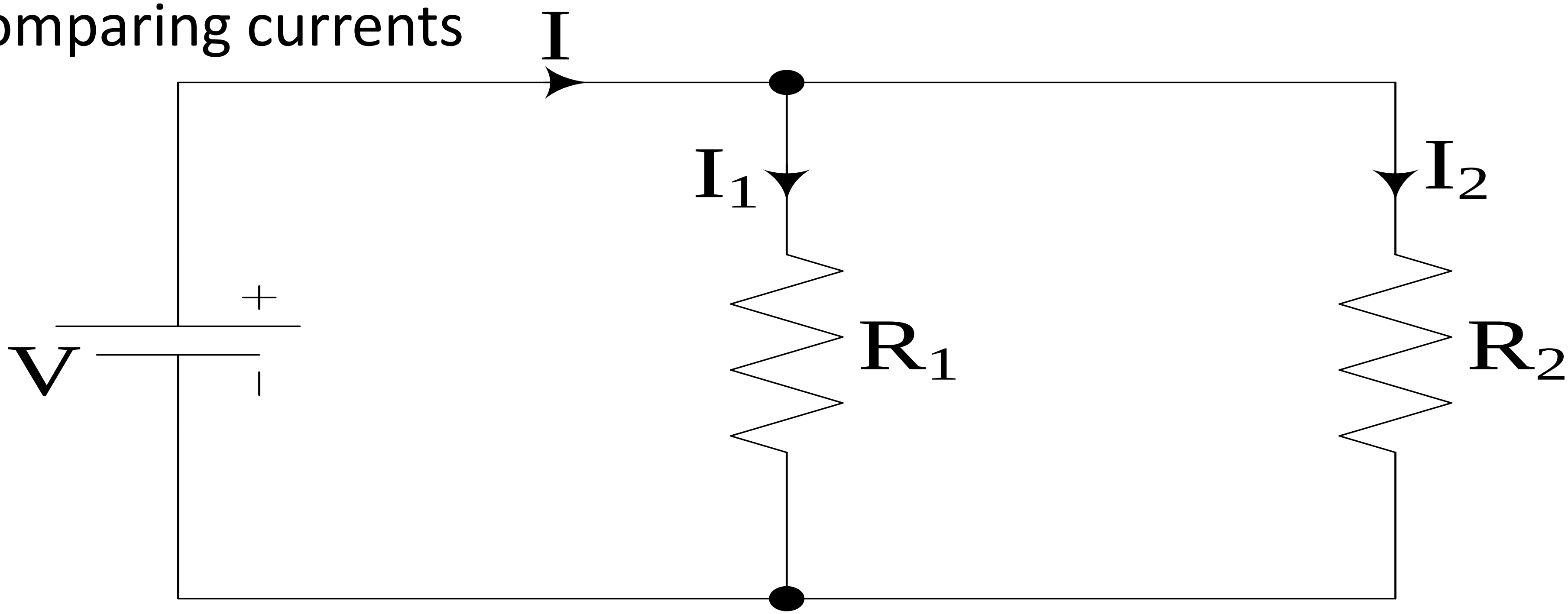
Similarly,



$$I_2 = I \frac{R_1}{R_1 + R_2}$$

CURRENT DIVISION RULE

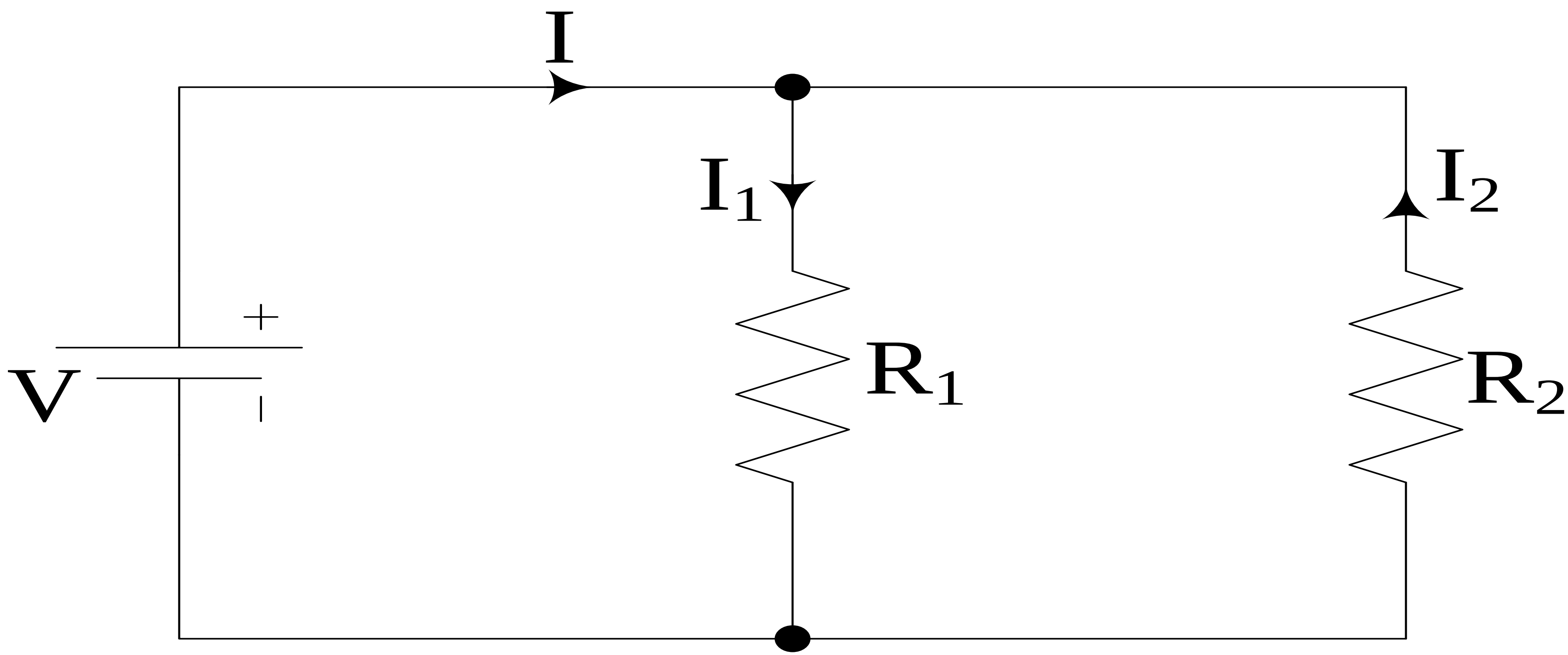
Comparing currents



$$I_1 = I\frac{R_2}{R_1 + R_2} \qquad I_2 = I\frac{R_1}{R_1 + R_2}$$

CURRENT DIVISION RULE

Consider the figure below,

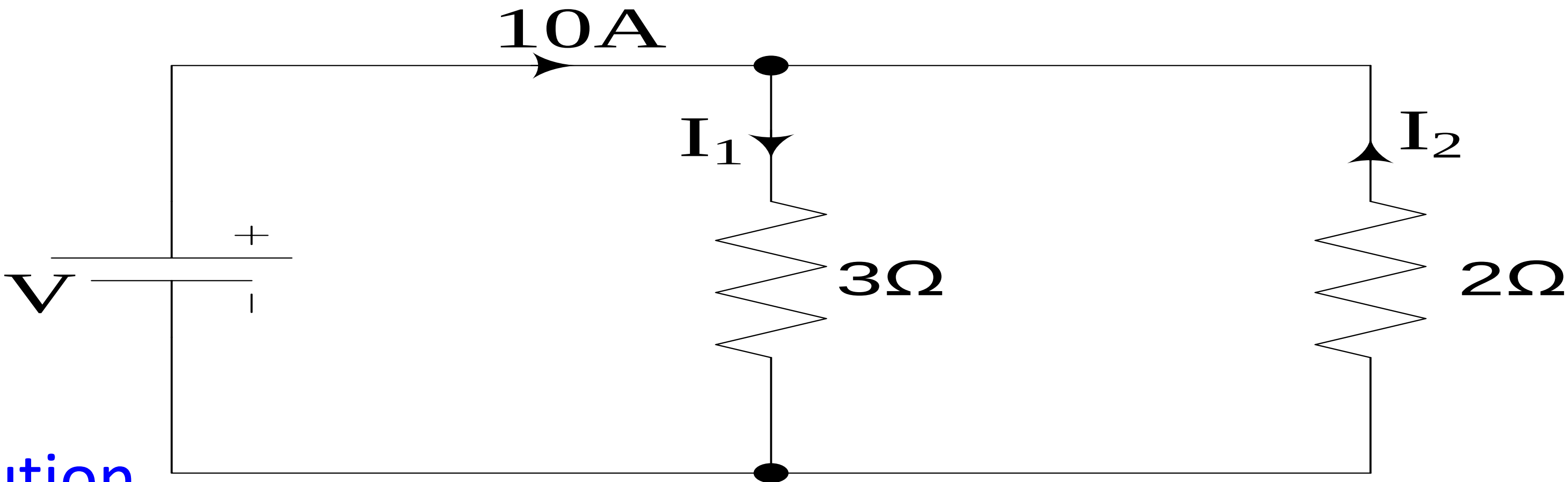


$$I_1 = \frac{R_2}{R_1 + R_2} I \qquad I_2 = -\frac{R_1}{R_1 + R_2} I$$

CURRENT DIVISION RULE

Example 1

Find the values of I_1 and I_2 in the circuit below.



Solution

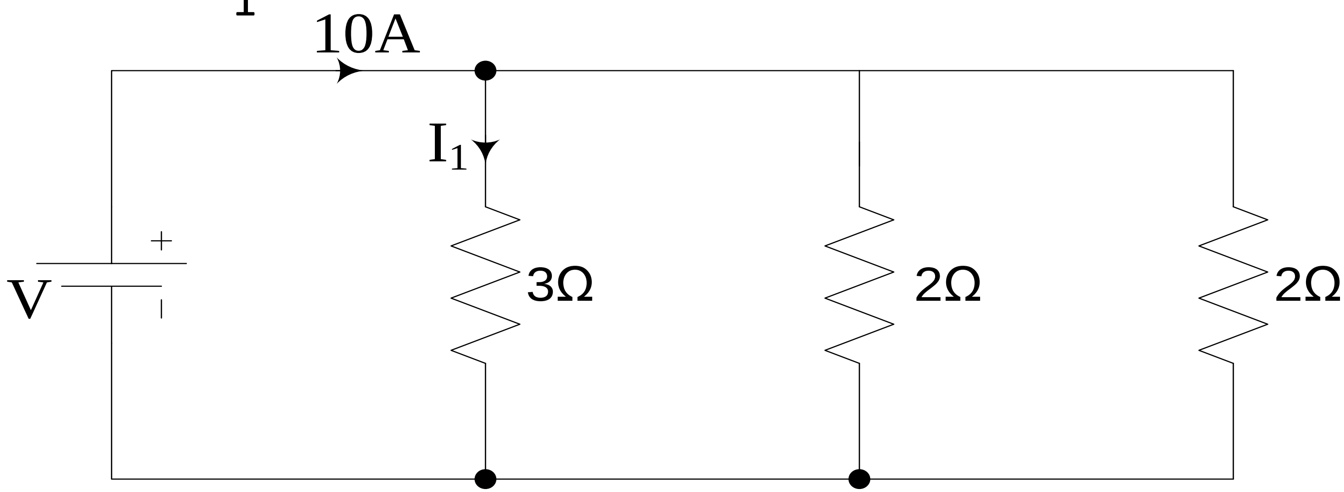
$$I_1 = \frac{R_2}{R_1 + R_2} I = \frac{2}{2+3} \times 10 = 4A$$

$$I_2 = -\frac{R_1}{R_1 + R_2} I = -\frac{3}{2+3} \times 10 = -6A$$

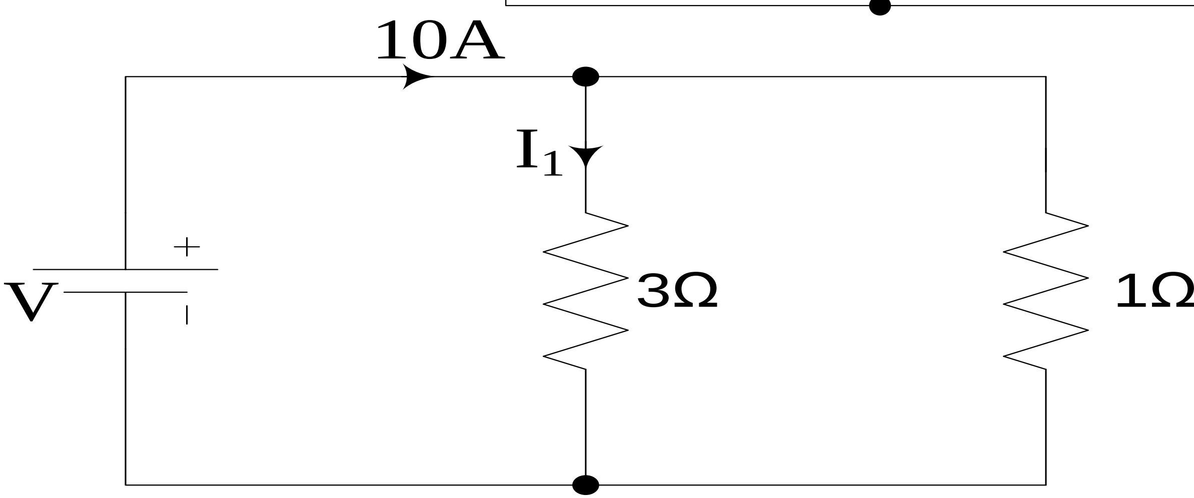
CURRENT DIVISION RULE

Example 2

Find the value of I_1 in the circuit below.



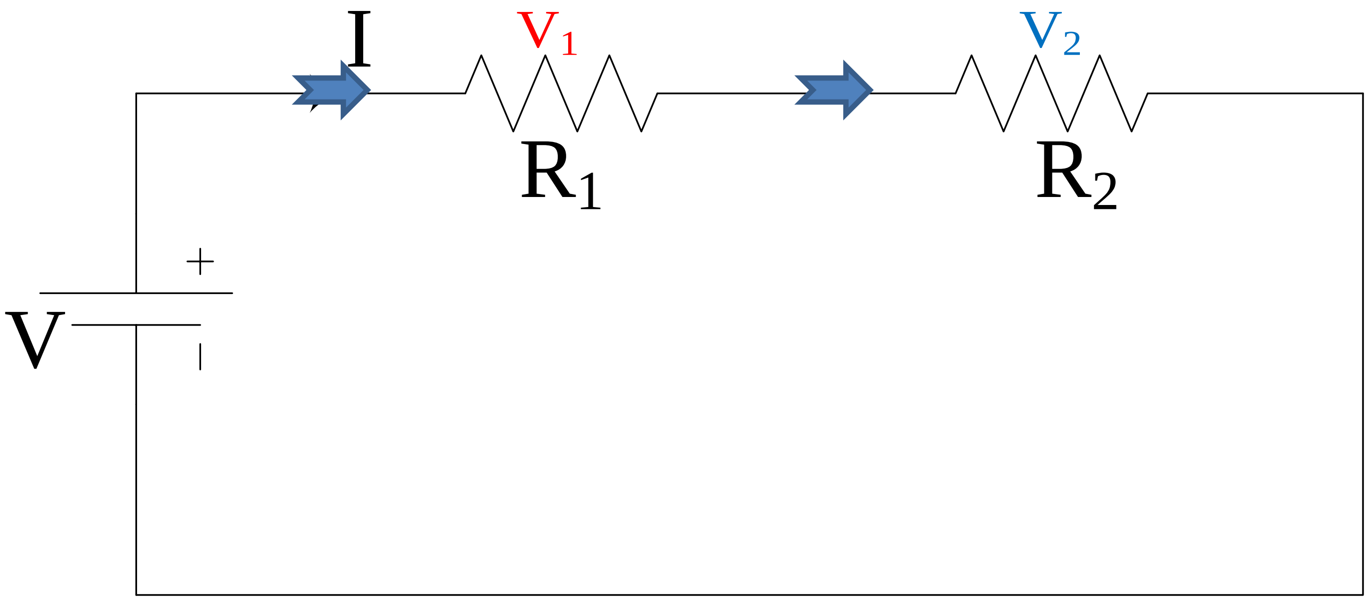
Solution



$$I_1 = \frac{1}{1+3} \times 10 = 2.5A$$

VOLTAGE DROP

- Any time a voltage drives current through a resistor, some of the voltage drops across the resistor.
- The magnitude of the drop is the product of the resistance and current

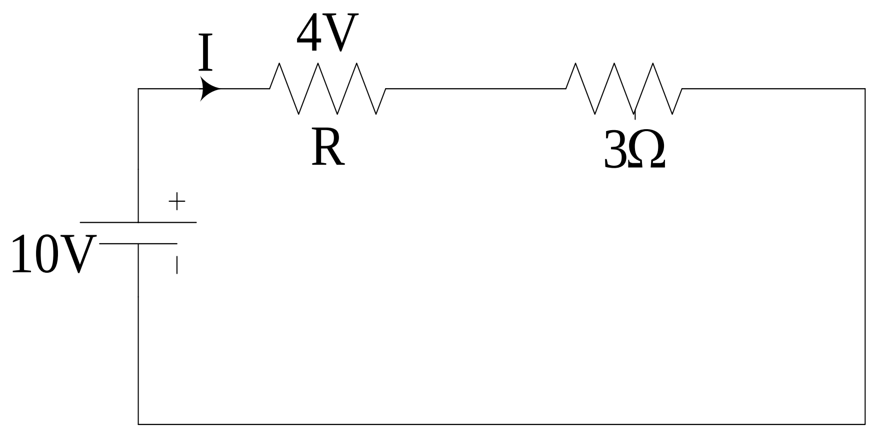


$$V_2 = V - V_1$$

VOLTAGE DROP

Example

Find the values of I and R in the circuit below.



Solution

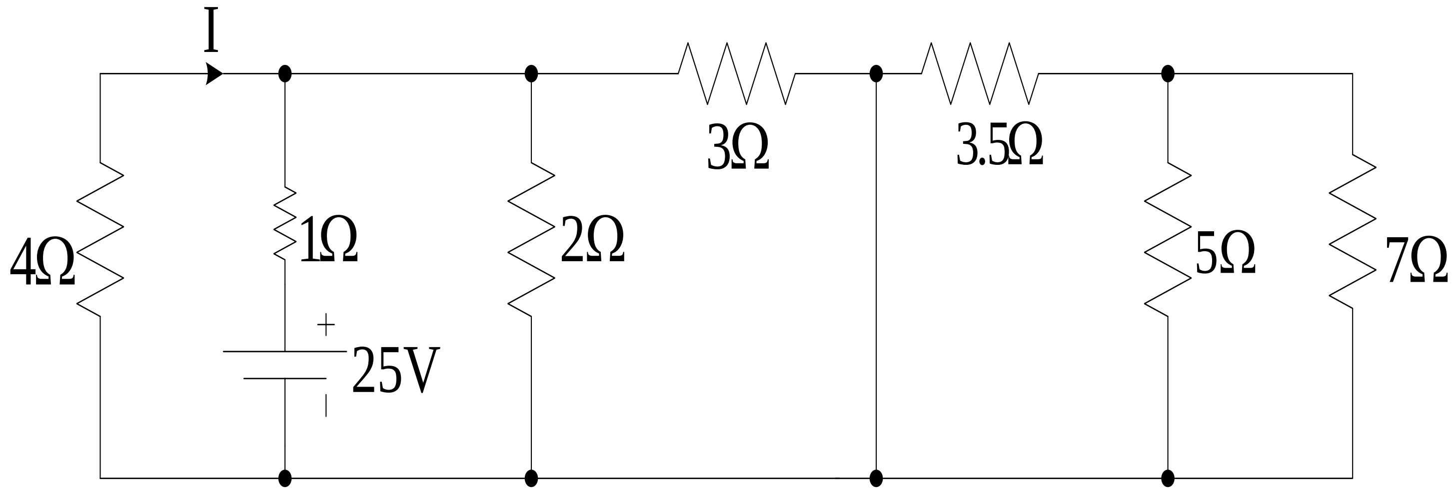
Voltage across 3Ω resistor = 10 – 4 = 6V

Current in 3Ω resistor = I = 6/3 = 2A

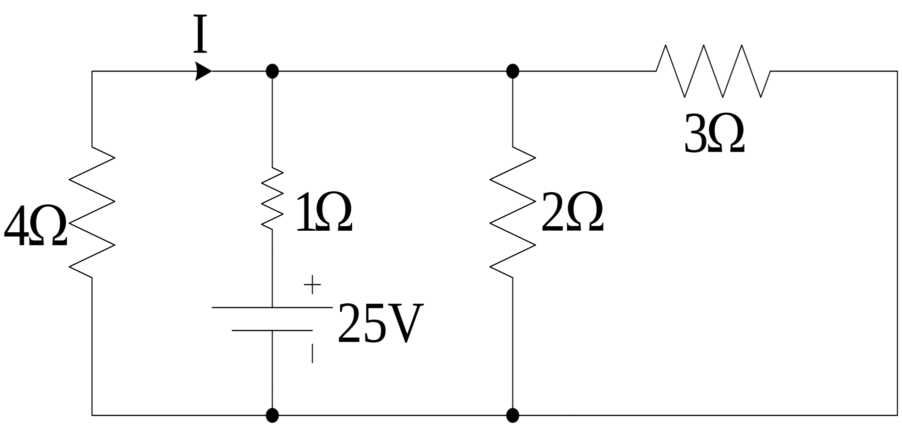
Resistance R = 4V/I = 4/2 = 2Ω

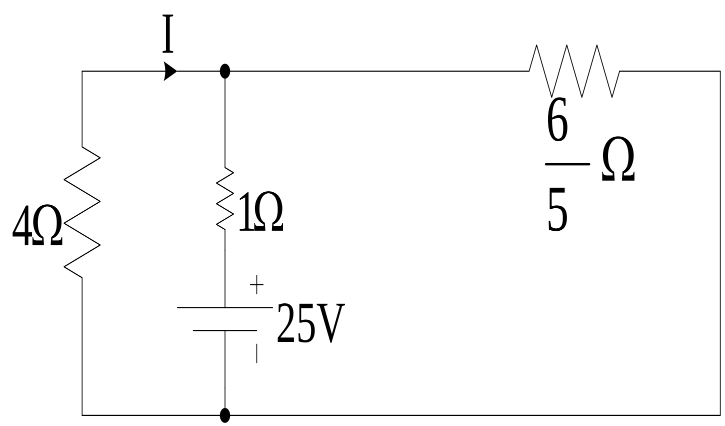
REVISION EXERCISE

Find the value of I in the circuit below.

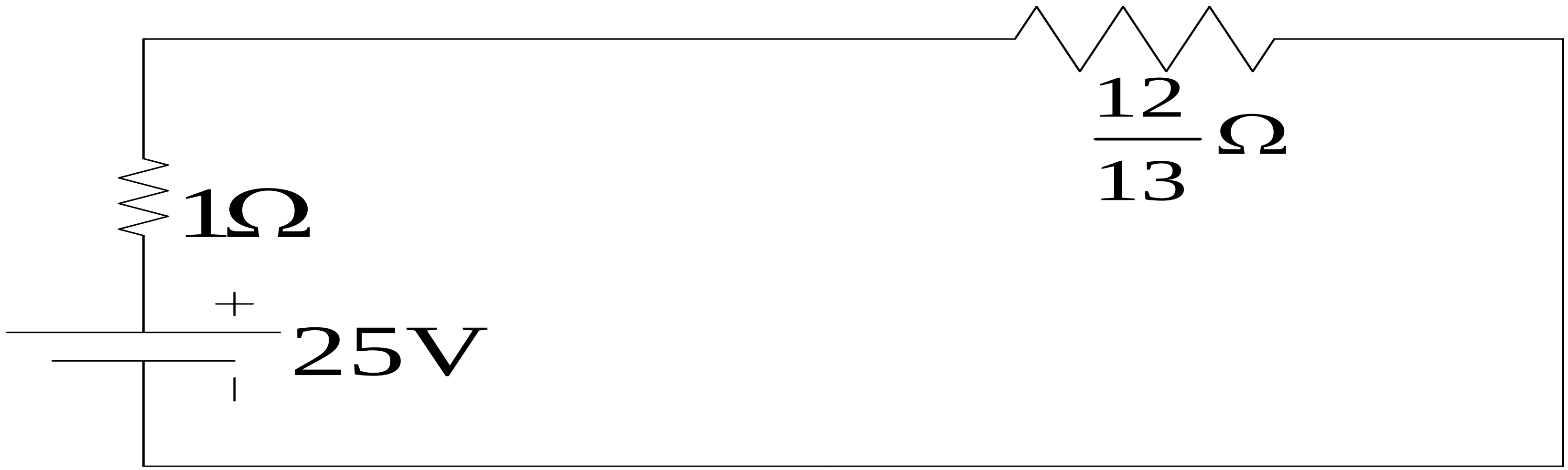


Solution



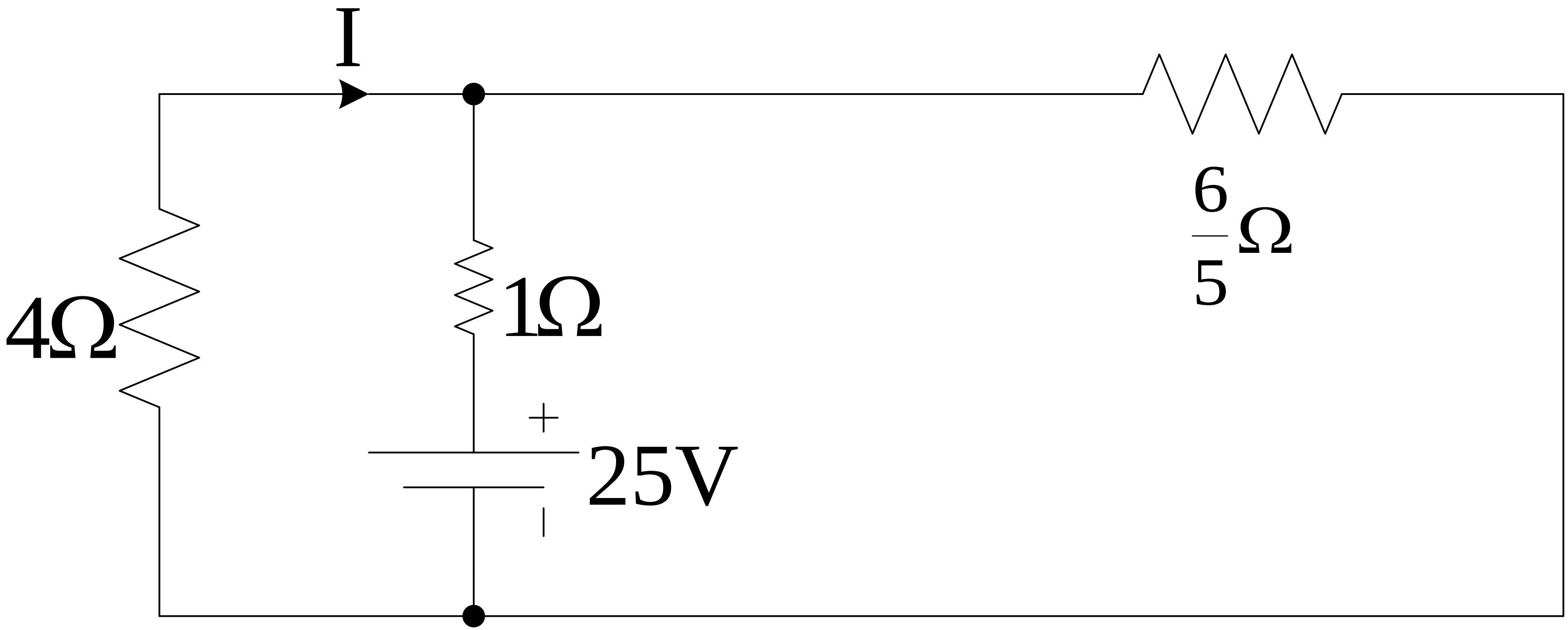


REVISION EXERCISE



$$R_T = \frac{25}{13}\Omega \quad I_T = \frac{V}{R_T} = \frac{25}{25/13} = 13A$$

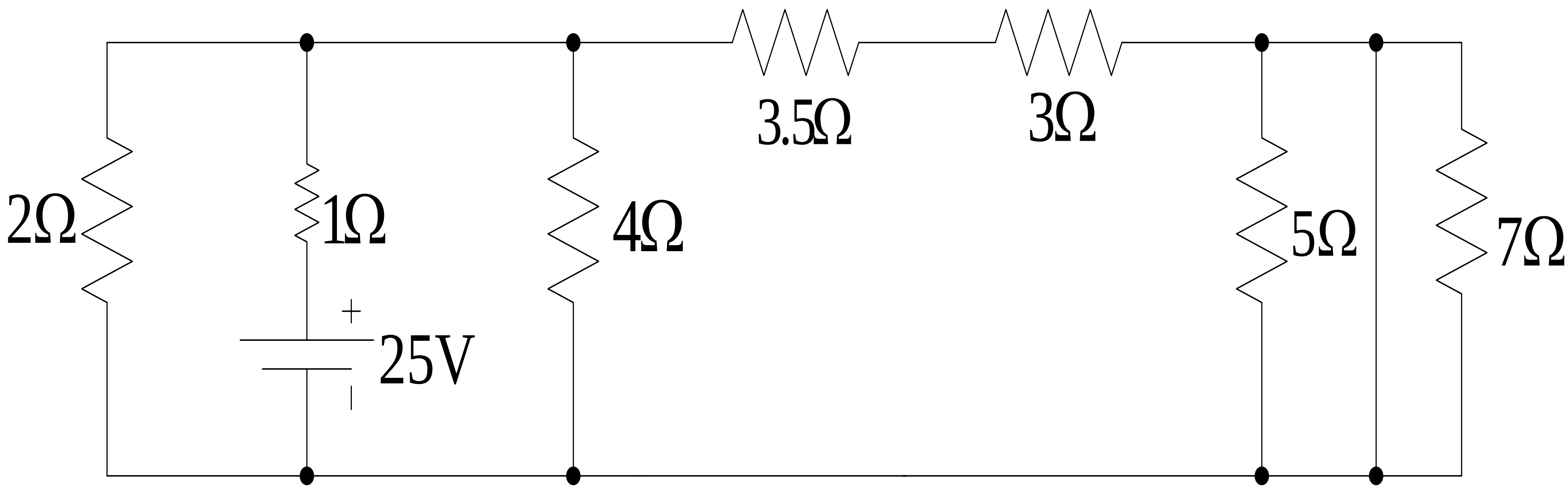
REVISION EXERCISE



$$I = -\frac{\frac{6}{5}}{\frac{6}{5}+4}\times 13 = -3A$$

Group Assignment 1

Find the value of the current in all resistors of the circuit below using total resistance and voltage drop principles. DO NOT use current division rule.



Submission date: God willing a week today

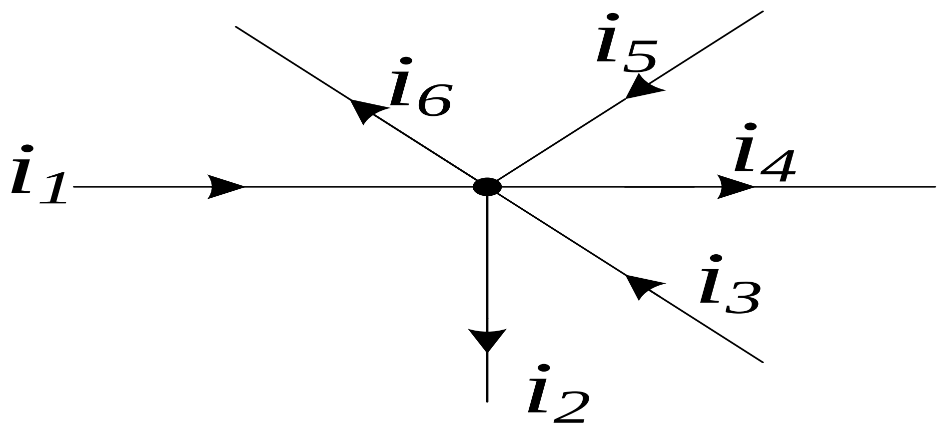
Submission time: Before lecture starts

Where to submit: Electrical Engineering office

KIRCHHOFF'S CURRENT LAW(KCL)

❖ The Law

The sum of currents entering a node equals the sum of currents leaving the node.



Sum of currents entering ➡ $i_1 + i_3 + i_5$

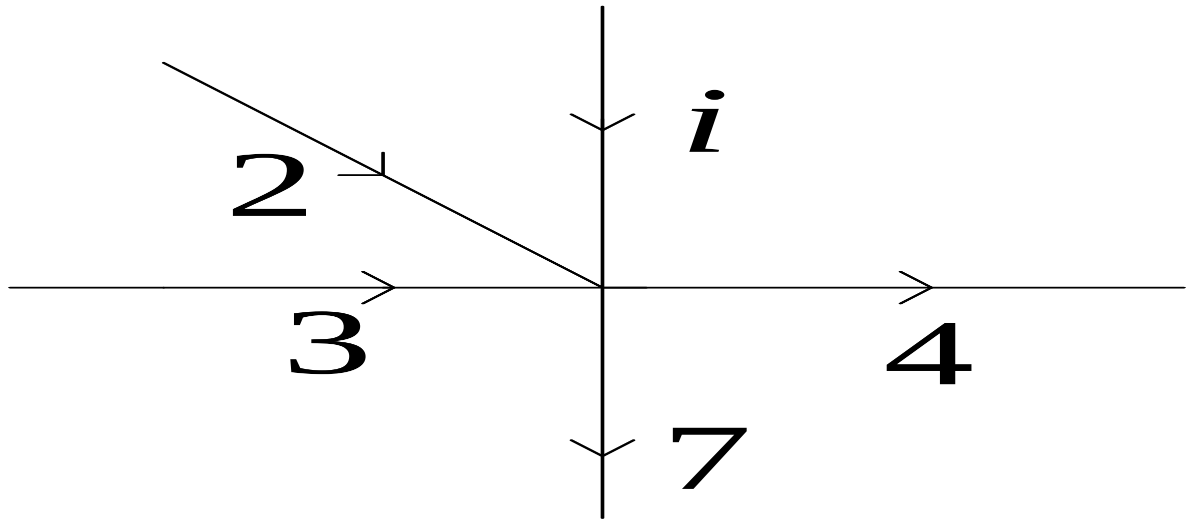
Sum of currents Leaving ➡ $i_2 + i_4 + i_6$

Applying KCL ➡ $i_1 + i_3 + i_5 = i_2 + i_4 + i_6$

KIRCHHOFF'S CURRENT LAW(KCL)

❖ Example

Find the value of i in the figure below.



Solution

$$i + 2 + 3 = 4 + 7$$

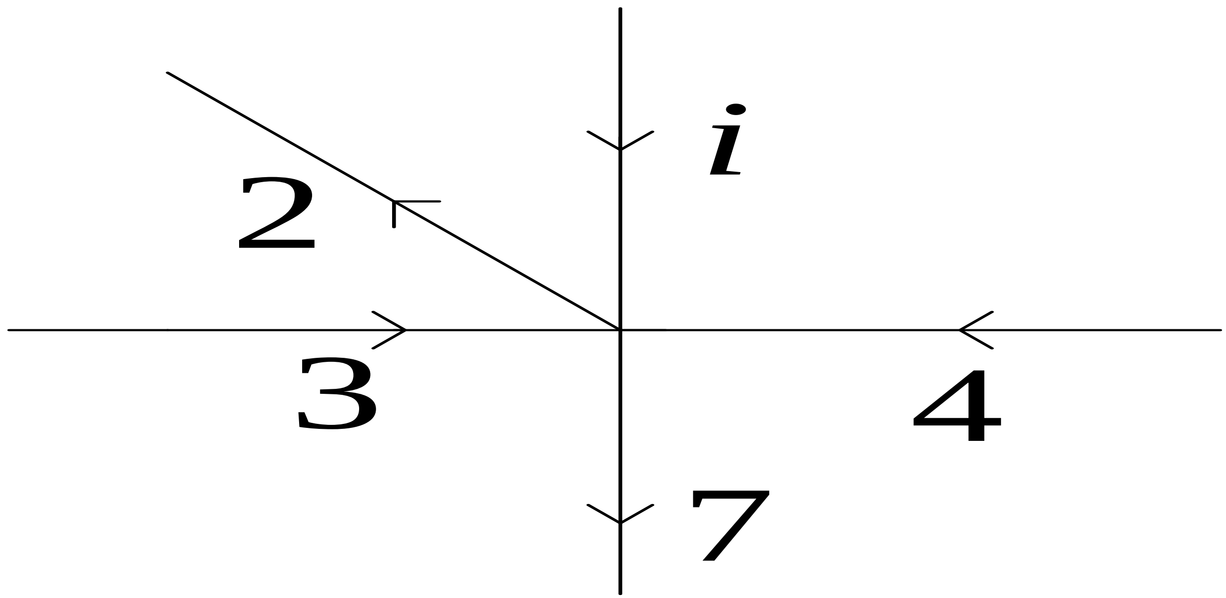
$$i + 5 = 11$$

$$i = 6$$

KIRCHHOFF'S CURRENT LAW(KCL)

❖ Self assessment

Find the value of i in the figure below.

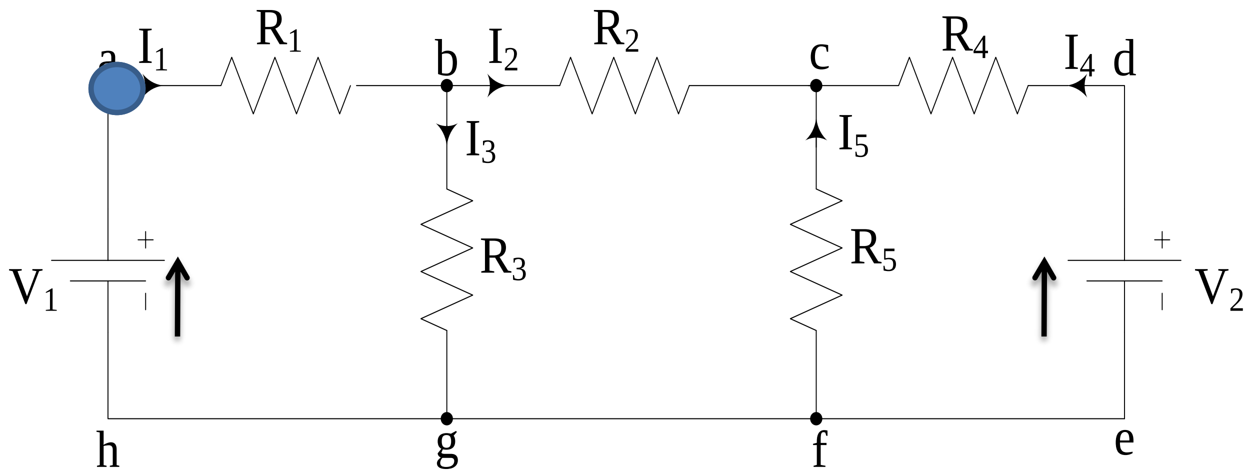


ANS $$i = 2$$

KIRCHHOFF'S VOLTAGE LAW(KVL)

❖ The law

The algebraic sum of the voltages in a loop (closed path) equals zero. Alternatively, in a loop, the algebraic sum of voltage sources equals the algebraic sum of voltage drops.



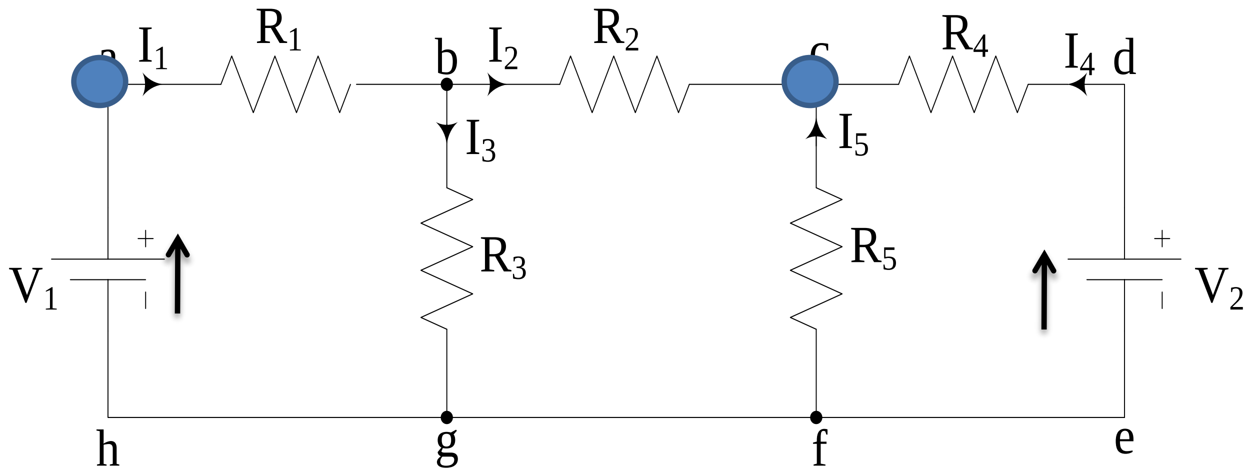
Loop abgha

$$V_1 = I_1R_1 + I_3R_3$$

Loop adeha

$$V_1 - V_2 = I_1R_1 + I_2R_2 - I_4R_4$$

KIRCHHOFF'S VOLTAGE LAW(KVL)



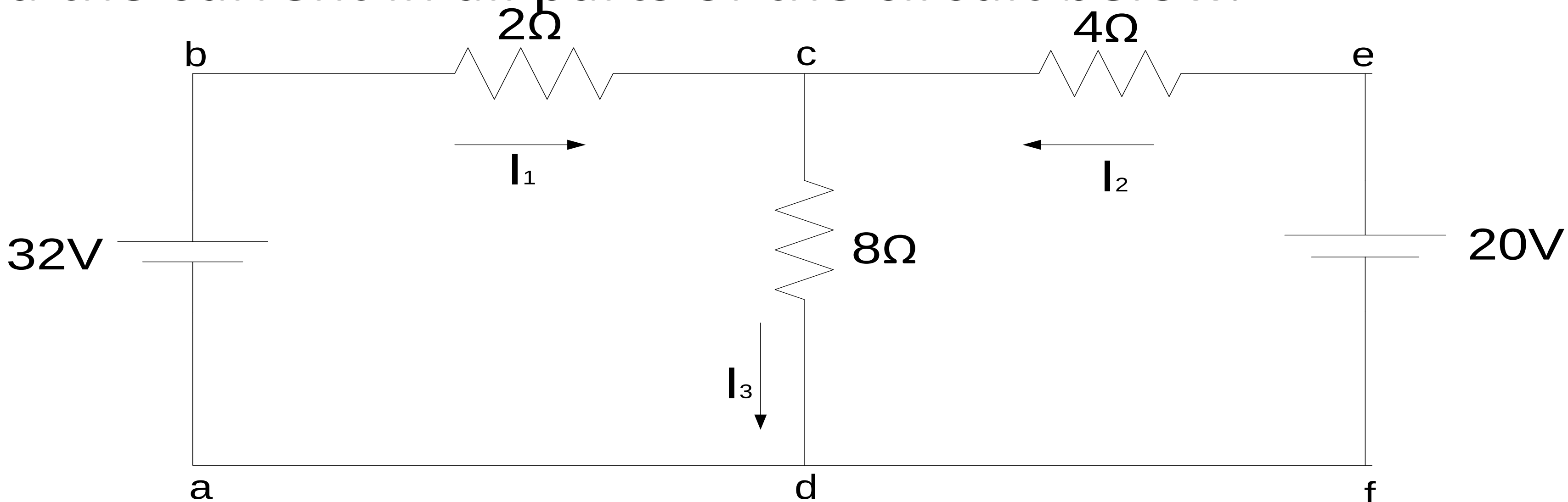
Loop cbgfc $\quad 0 = -I_2R_2 + I_3R_3 + I_5R_5$

Loop acfha $\quad V_1 = I_1R_1 + I_2R_2 - I_5R_5$

KIRCHHOFF'S VOLTAGE LAW(KVL)

Example 1

Find the current in all parts of the circuit below.



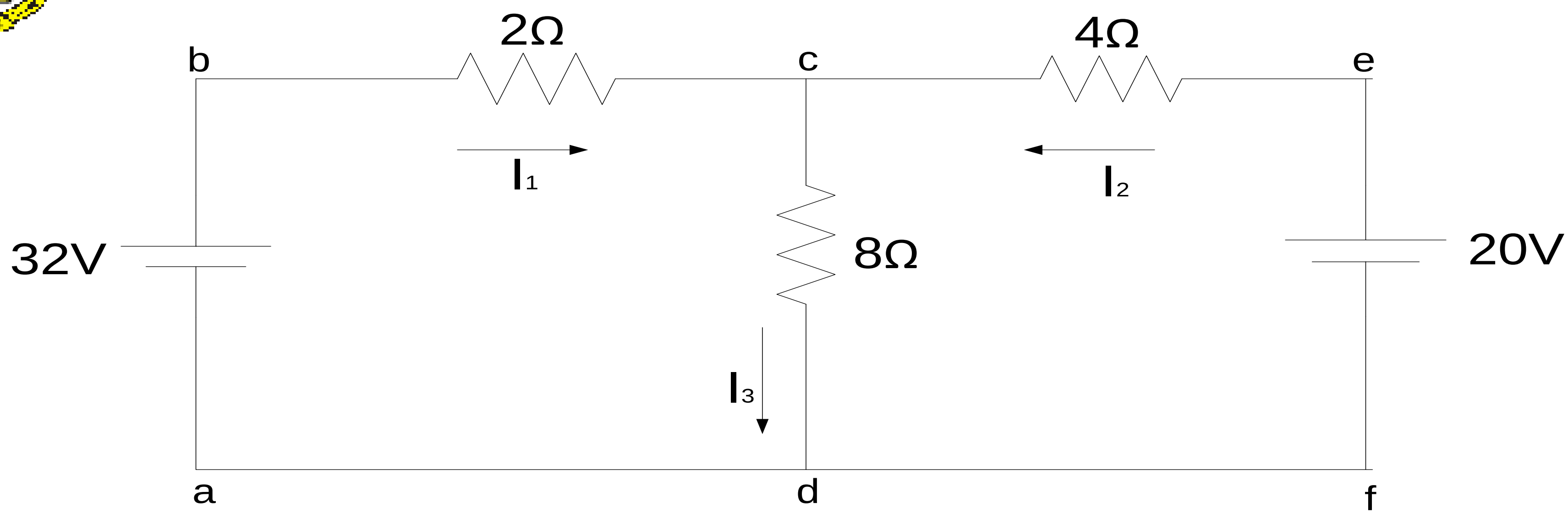
- Applying KVL to loop bcdab $\quad 32 - 2I_1 - 8I_3 = 0$

$$\Rightarrow \quad 32 = 2I_1 + 8I_3 \qquad (1)$$

- Applying KVL to loop ecdfe $\quad 20 - 4I_2 - 8I_3 = 0$

$$\Rightarrow \quad 20 = 4I_2 + 8I_3 \qquad (2)$$

KIRCHHOFF'S VOLTAGE LAW(KVL)



❖Applying KCL to node c: $$I_3 = I_1 + I_2 \qquad (3)$$

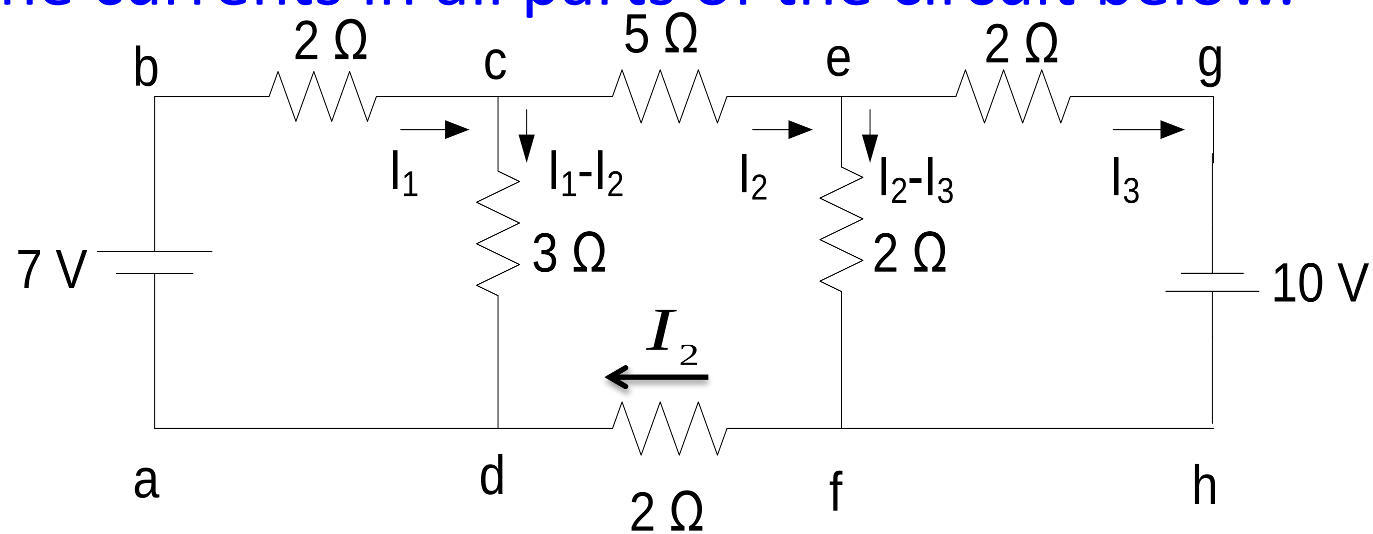
Solving the equations simultaneously yields

$I_1 = 4A, \quad I_2 = -1A \quad \text{and} \quad I_3 = 3A$

KIRCHHOFF'S VOLTAGE LAW(KVL)

Example 2

Find the currents in all parts of the circuit below.



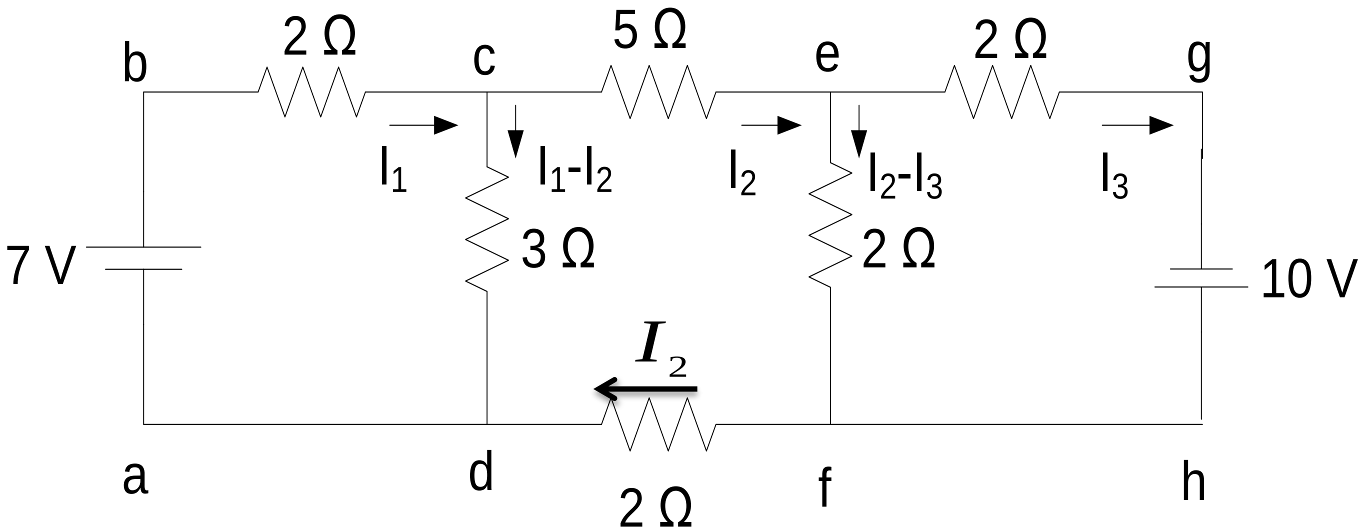
Solution

Apply KVL to loop cefdc

$$5I_2 + 2(I_2 - I_3) + 2I_2 - 3(I_1 - I_2) = 0$$

KIRCHHOFF'S VOLTAGE LAW(KVL)

$$\Rightarrow \quad 0 = -3I_1 + 12I_2 - 2I_3 \quad (1)$$



Apply KVL to loop abcda: $7 = 2I_1 + 3(I_1 - I_2)$

$$\Rightarrow 7 = 5I_1 - 3I_2 \quad (2)$$

Apply KVL to loop ghfeg: $10 = -2(I_2 - I_3) + 2I_3$

$$\Rightarrow 10 = -2I_2 + 4I_3 \quad (3)$$

KIRCHHOFF'S VOLTAGE LAW(KVL)

Solving the three equations:

$$0 = -3I_1 + 12I_2 - 2I_3 \qquad (1)$$

$$7 = 5I_1 - 3I_2 \qquad (2)$$

$$10 = -2I_2 + 4I_3 \qquad (3)$$

Simultaneously,

$$I_1 = 2.0\text{A}, \quad I_2 = 1.0\text{A} \quad \text{and } I_3 = 3.0\text{A}$$

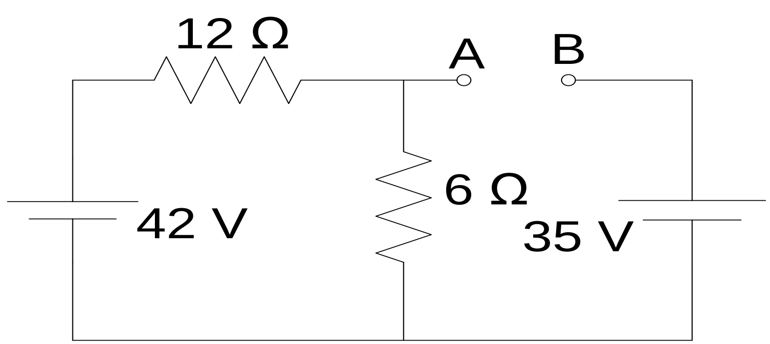
Thevenin's Theorem

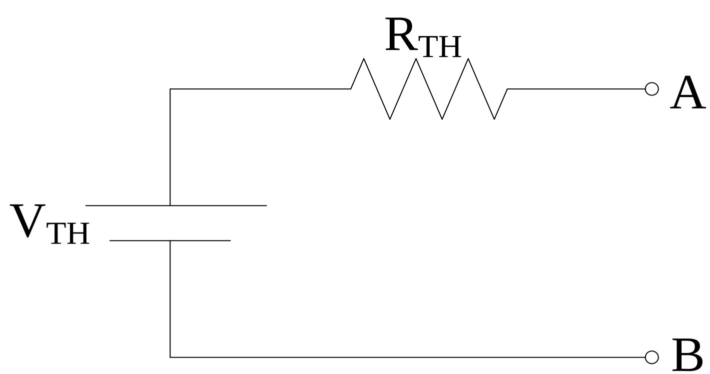
Theorem:

Any linear circuit connected between two terminals can be replaced by a Thevenin's voltage(V_{TH}) in series with a Thevenin's resistance (R_{TH}).

V_{TH} is the open-circuit voltage across the two terminals

R_{TH} is the resistance seen from the two terminals when all sources have been deactivated

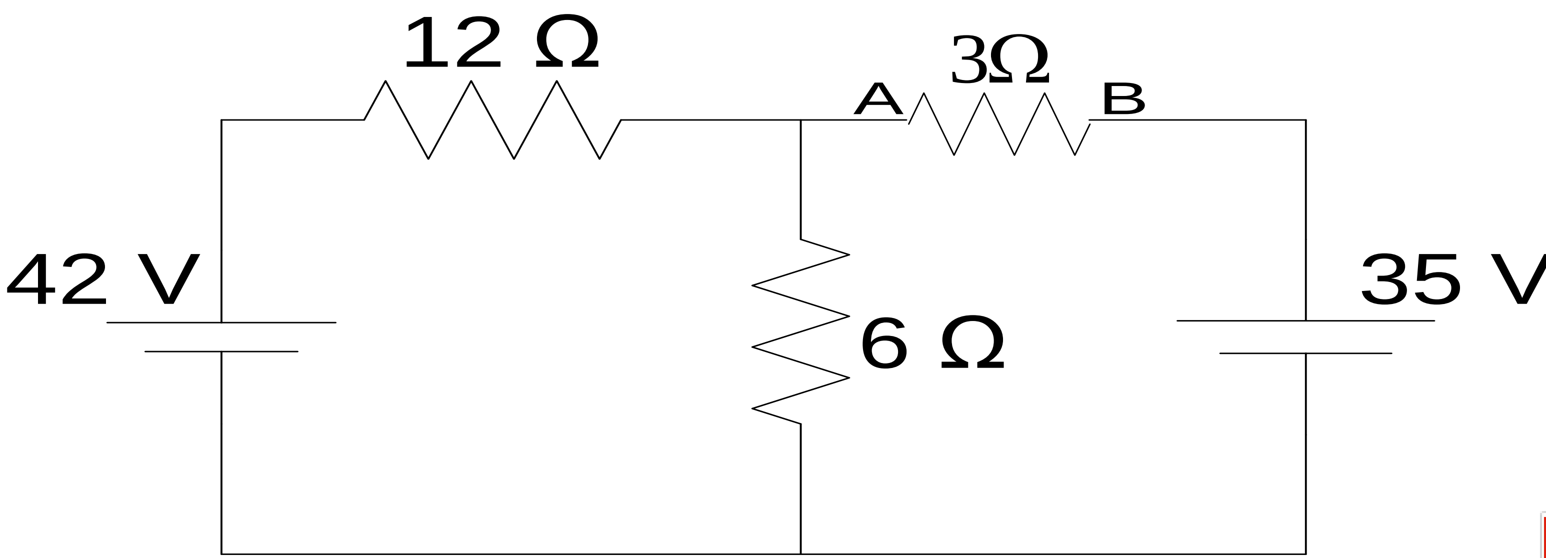




Thevenin's Theorem

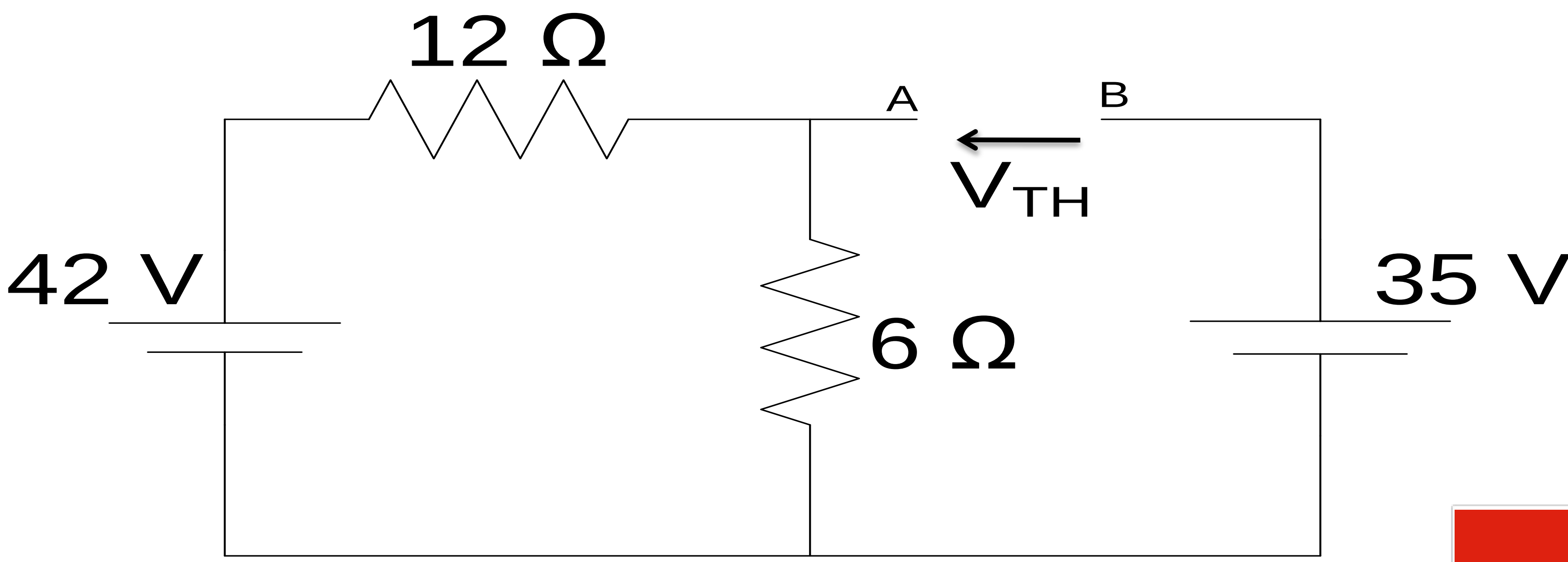
To find the current through a resistor in a circuit, the following steps are taken:

1. **Remove the resistor from the circuit and mark the two terminals.**



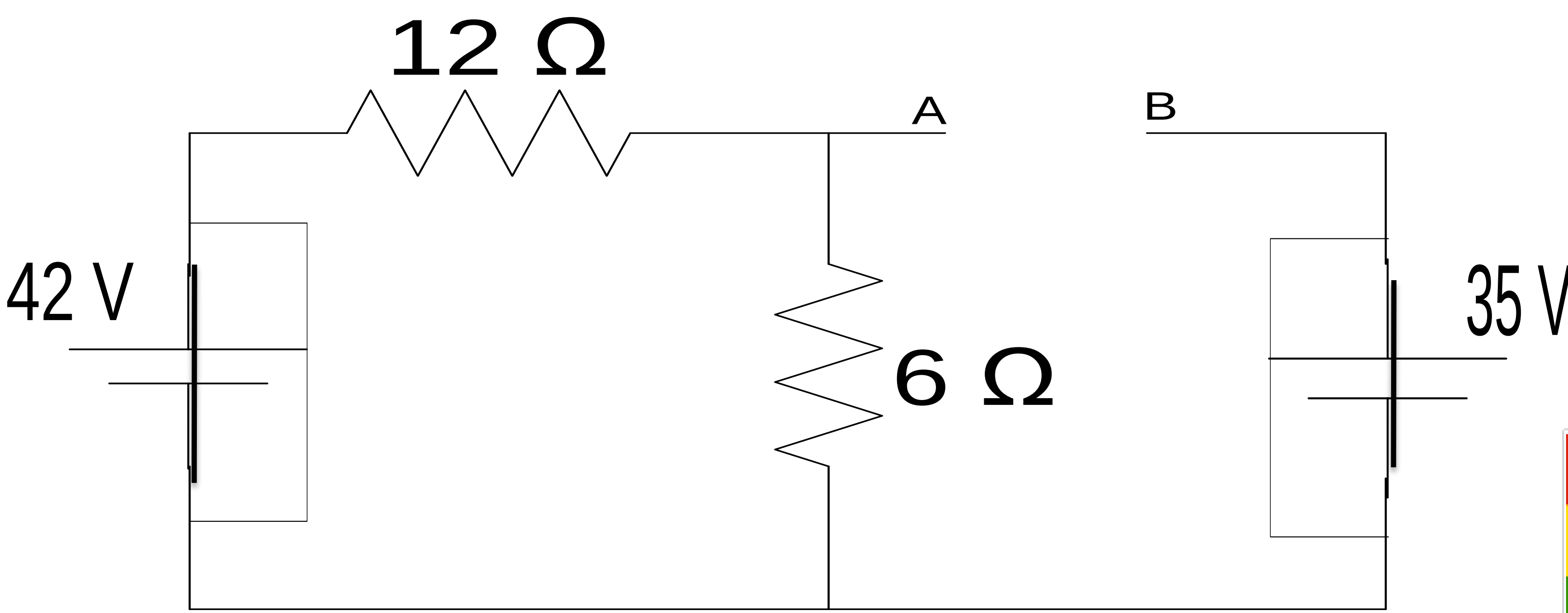
Thevenin's Theorem

2. **Find the open-circuit voltage (V_{TH}) across the two terminals by applying KVL. Treat V_{TH} as a source.**



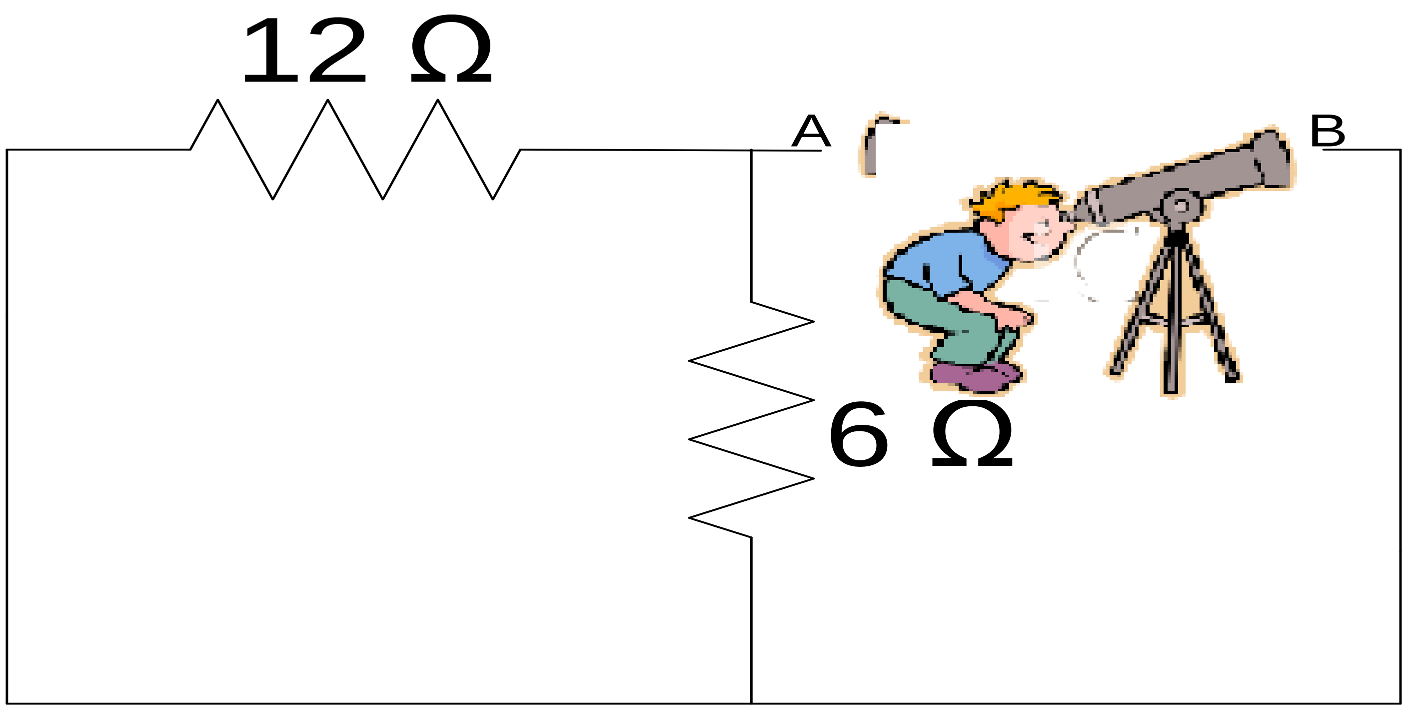
Thevenin's Theorem

3. **Recall the circuit created before step 2 and deactivate all sources. Short-circuit voltage sources and Open-circuit current sources.**



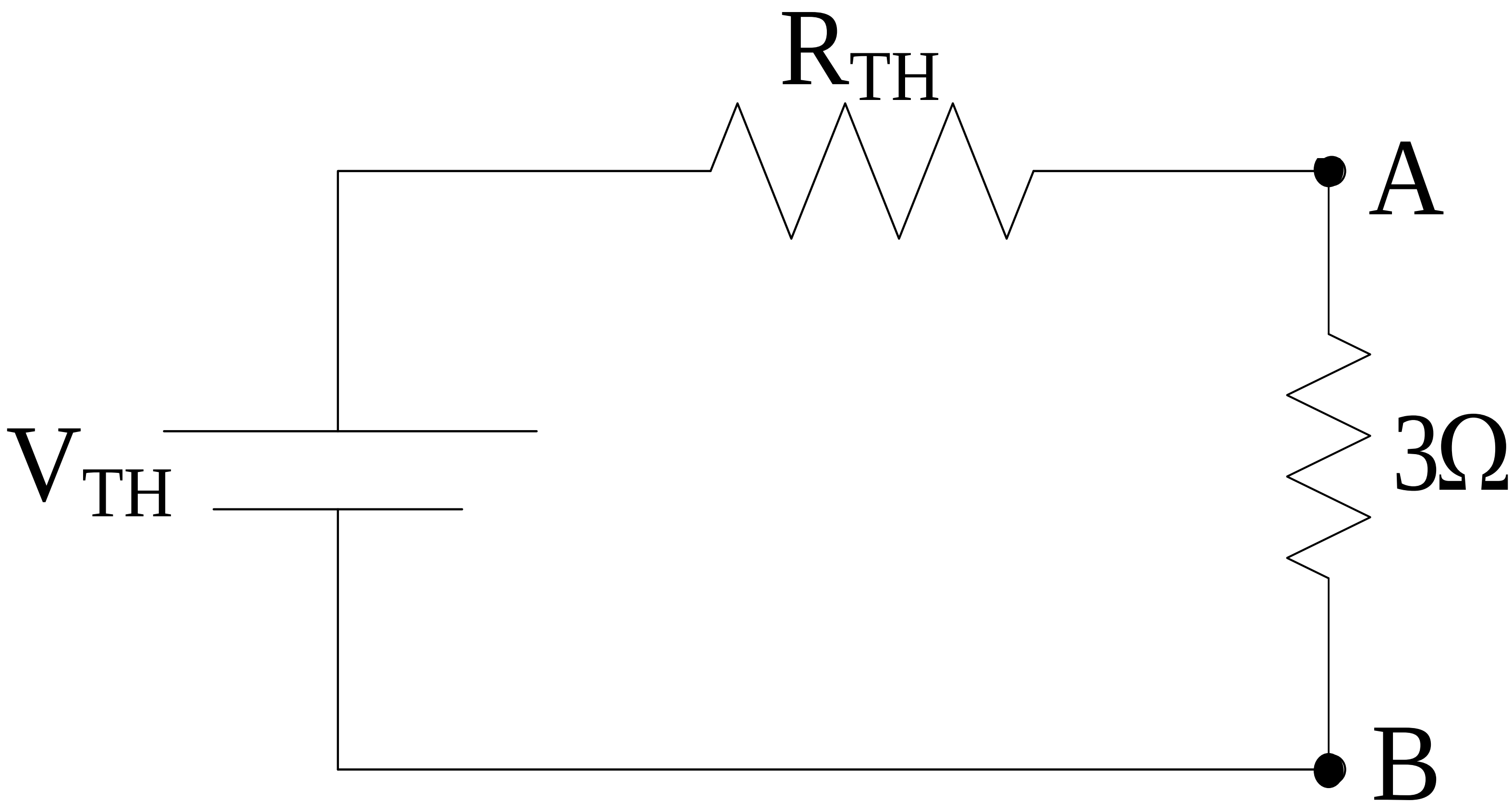
THEVENIN’S THEOREM

4. **Find the total resistance of the circuit resulting from step 3 as seen from the two terminals**



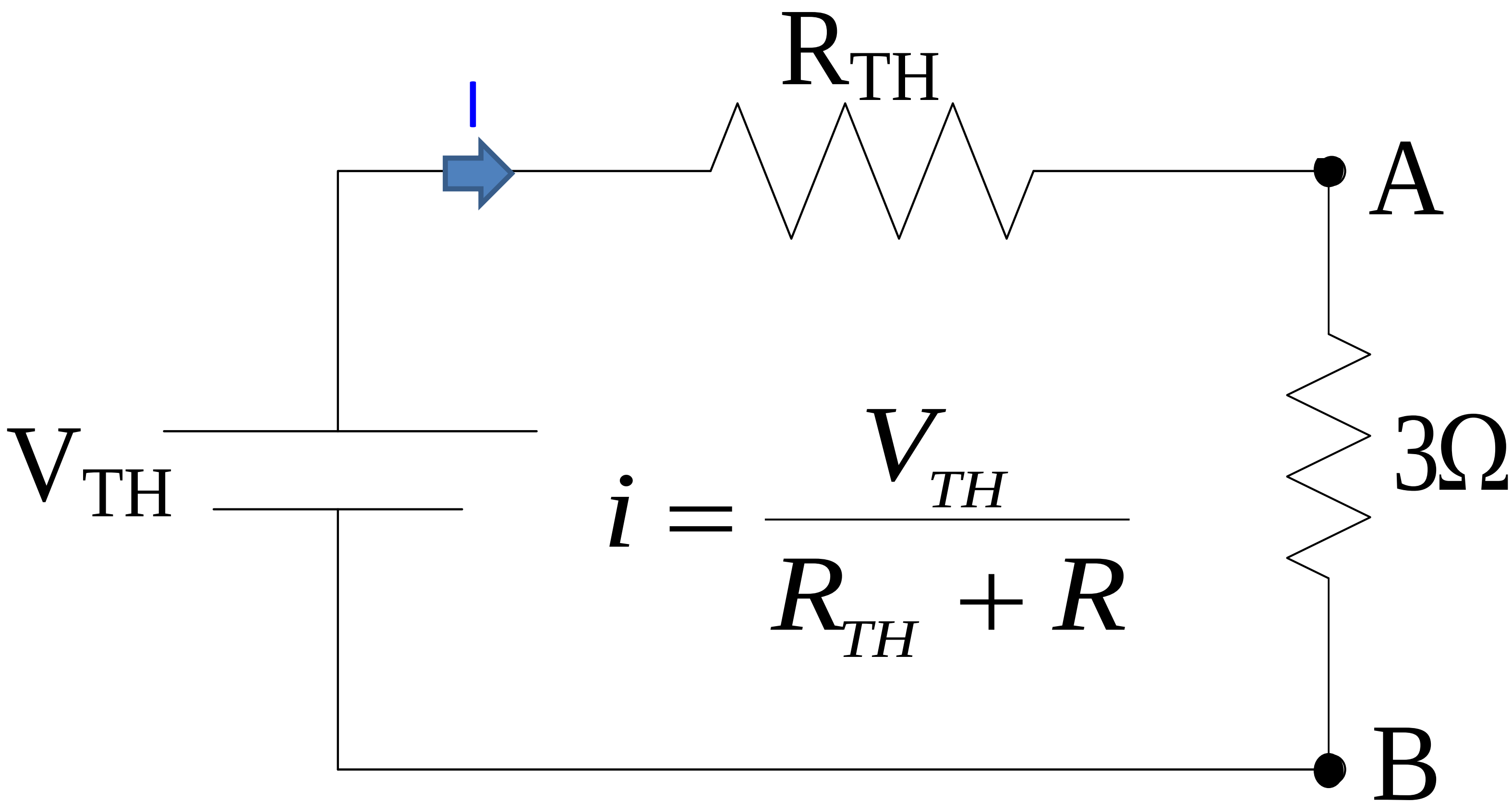
THEVENIN'S THEOREM

5. **Reproduce the Thevenin's equivalent circuit and connect the resistor whose current is to be found.**



THEVENIN'S THEOREM

6. **Calculate the current in the circuit in step 5. This is the current being sought.**

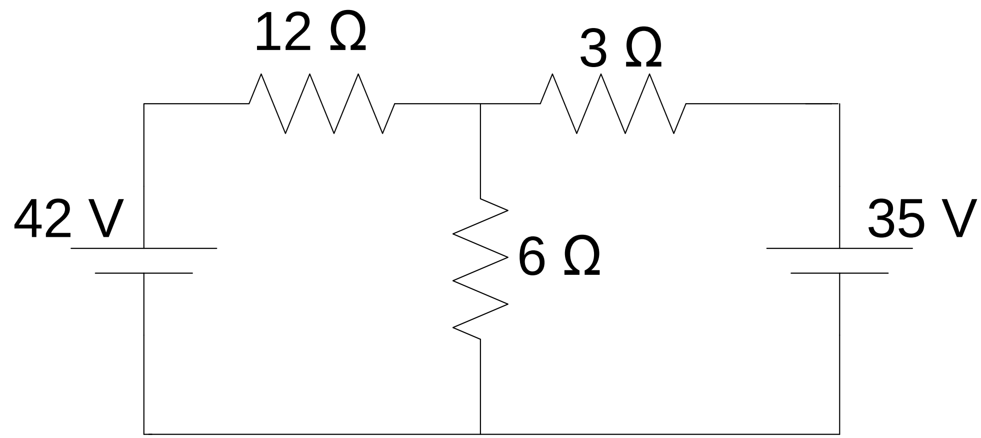


$$i = \frac{V_{TH}}{R_{TH} + R}$$

THEVENIN'S THEOREM

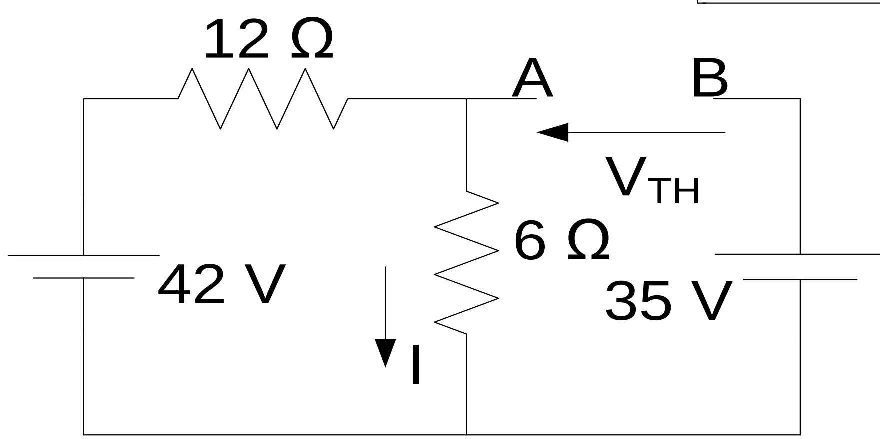
Example 1

Using Thevenin's theorem, determine the current in the 3-Ω resistor of the circuit below.

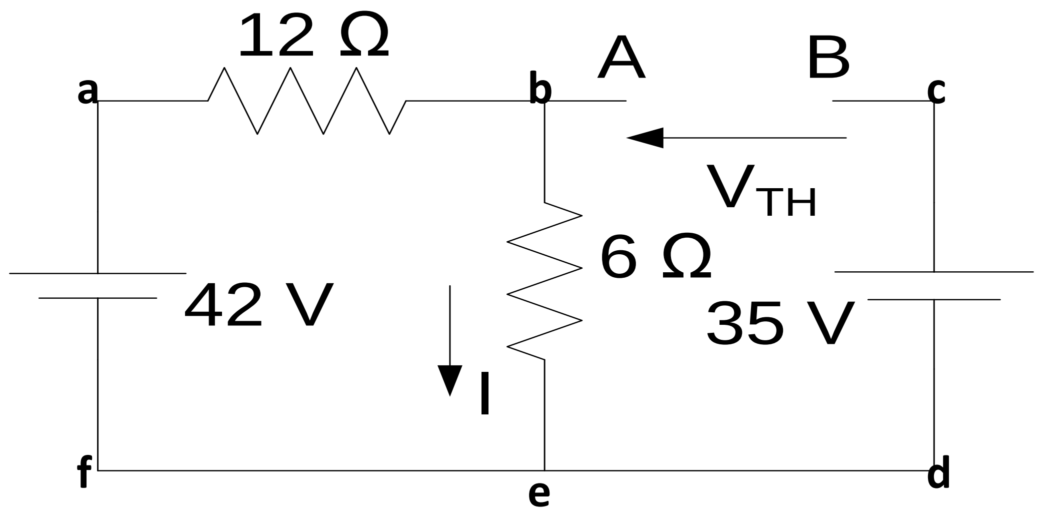


Solution

Steps 1 & 2



THEVENIN'S THEOREM



Applying KVL to loop dcbed: $$35 + V_{TH} = 6I \qquad (1)$$

Applying KVL to loop fabef: $$42 = (12+6)I$$

$$\Rightarrow \quad I = \frac{7}{3}A$$

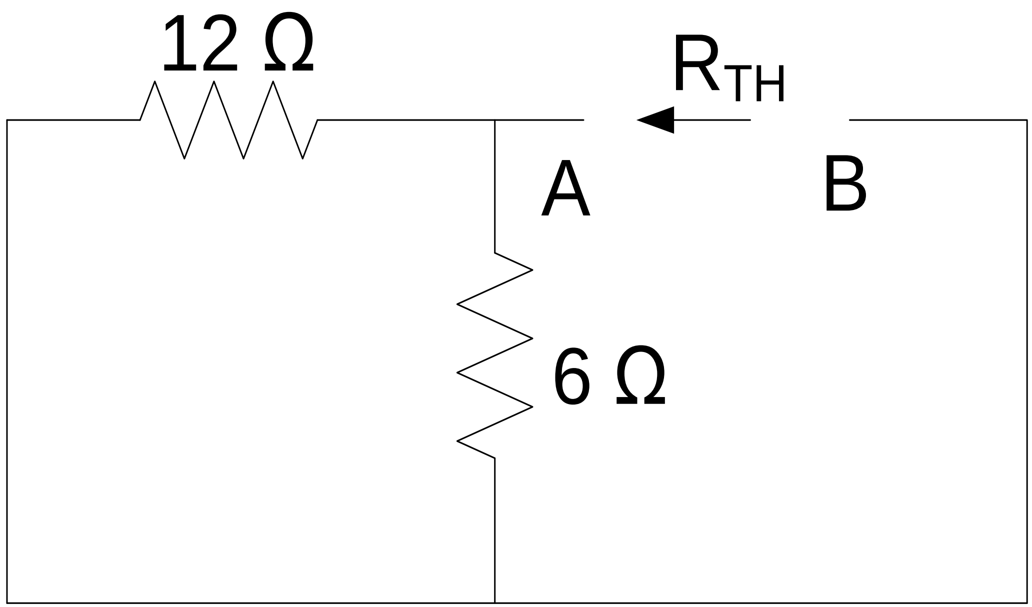
THEVENIN'S THEOREM

Substituting for I in equation 1:

$$35 + V_{TH} = 6\left(\frac{7}{3}\right)$$

$$V_{TH} = -21V$$

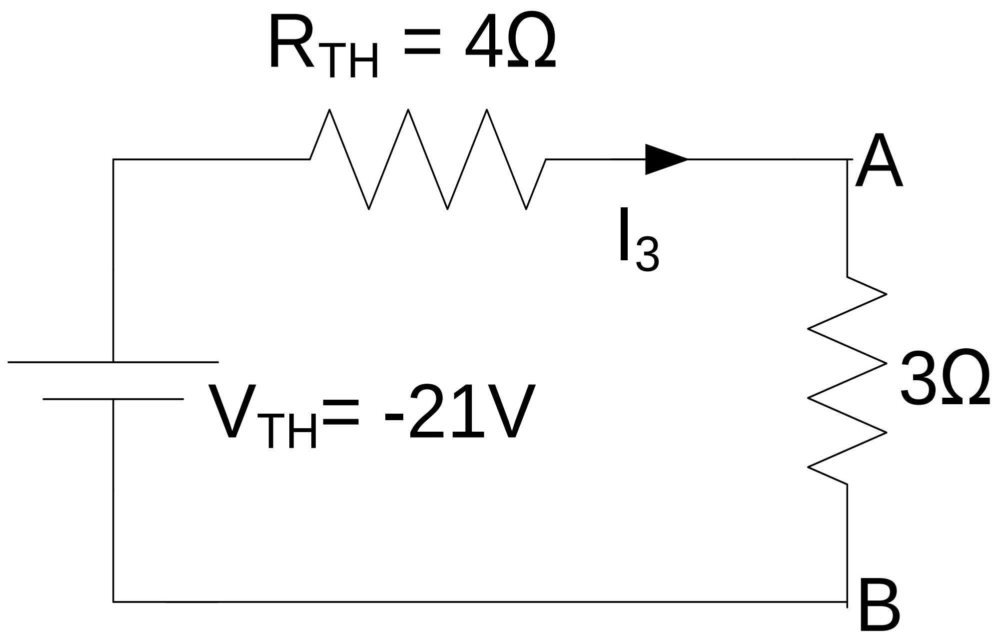
Steps 3 & 4



$$R_{TH} = 12 // 6 = \frac{12 \times 6}{12 + 6} = 4\Omega$$

THEVENIN'S THEOREM

Steps 5 & 6

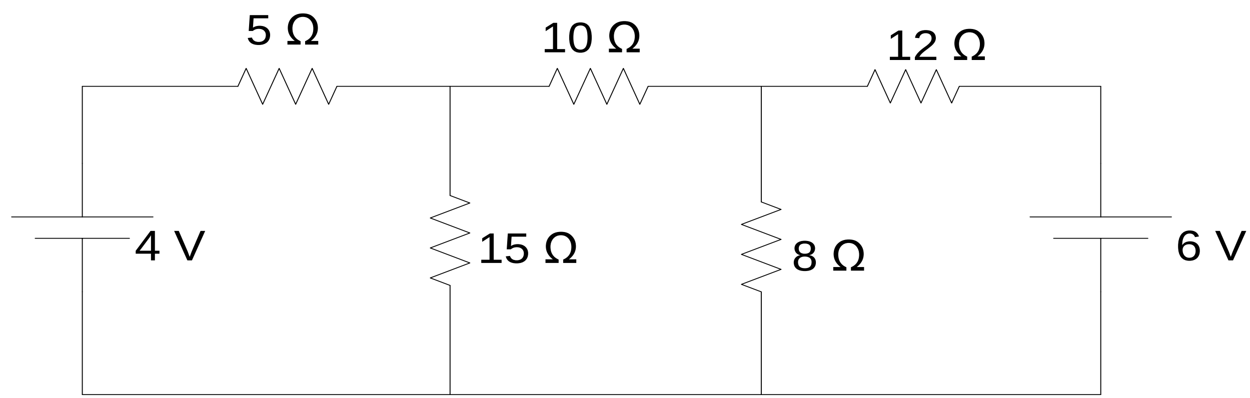


$$I_3 = \frac{V_{TH}}{R_{TH} + 3} = -\frac{21}{4+3} = -3\,A$$

THEVENIN'S THEOREM

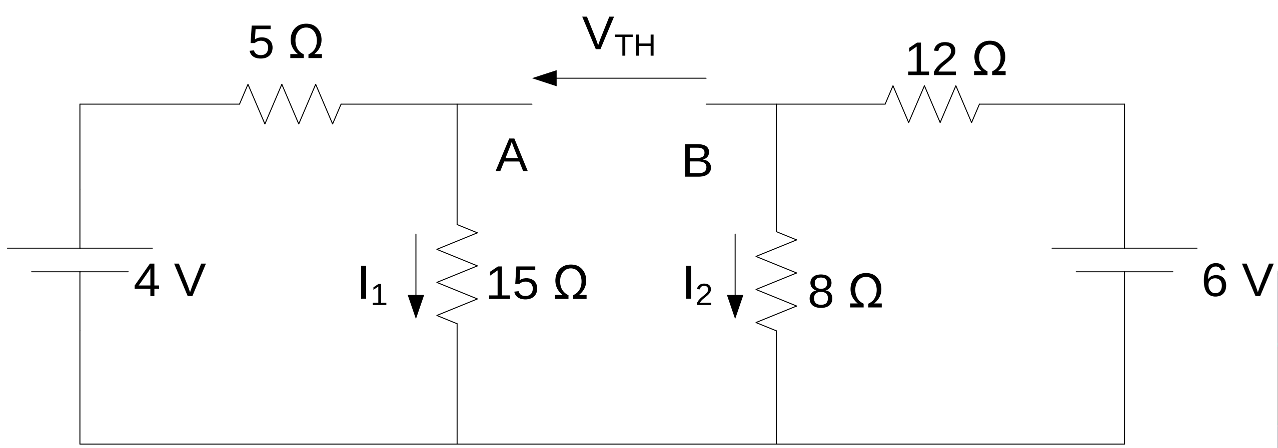
Example 2

Find the current in the 10-Ω resistor of the circuit below using Thevenin's theorem.

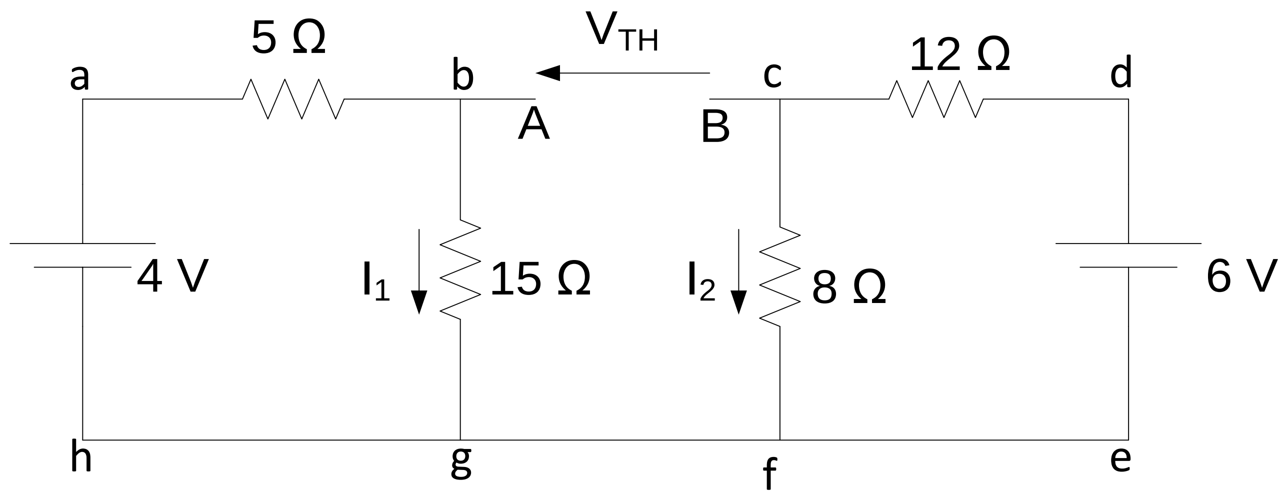


Solution

Steps 1 & 2



THEVENIN'S THEOREM



Applying KVL to loop cbgfc: $V_{TH} = 15I_1 - 8I_2$ (1)

Applying KVL to loop abgha: $4 = (5+15)I_1 \Rightarrow I_1 = \frac{1}{5}A$

Applying KVL to loop dcfed: $6 = (12+8)I_2$

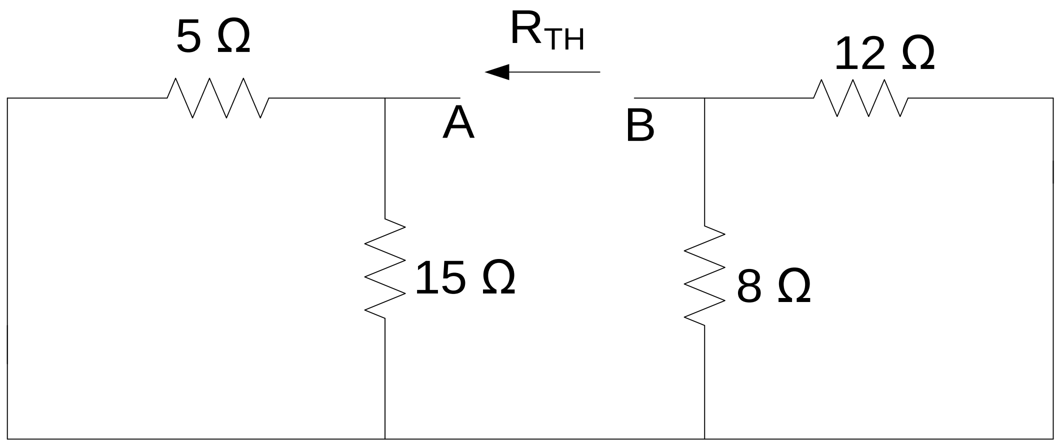
$\Rightarrow I_2 = \frac{3}{10}A$

THEVENIN'S THEOREM

Substituting for I_1 and I_2 in equation 1 yields:

$$V_{TH} = 15\left(\frac{1}{5}\right) - 8\left(\frac{3}{10}\right) = \frac{3}{5}V$$

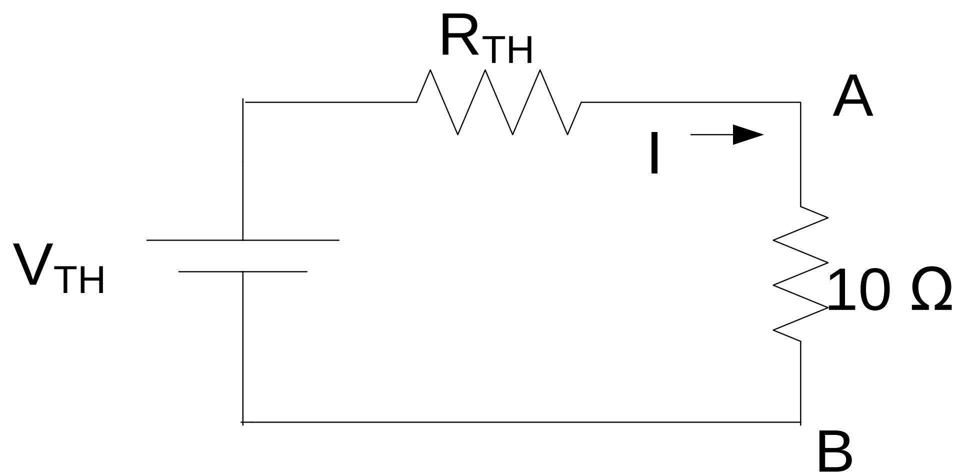
Steps 3 & 4



$$R_{TH} = (5//15) + (12//8) = \frac{171}{20}\Omega$$

THEVENIN'S THEOREM

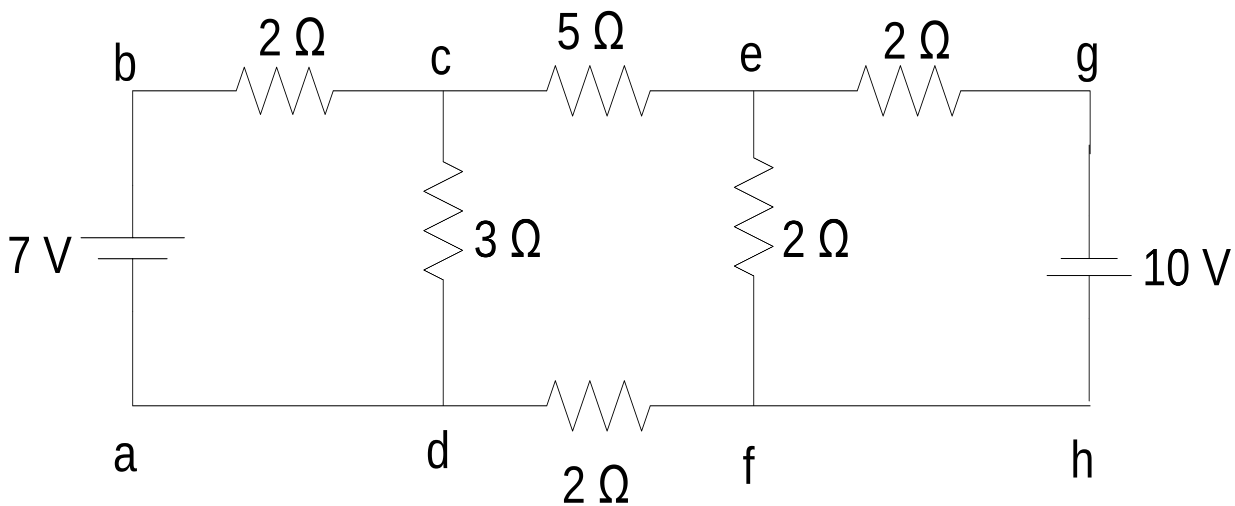
Steps 5 & 6



$$I = \frac{V_{TH}}{R_{TH} + 10} = \frac{3}{5\left(\frac{171}{20} + 10\right)} = \frac{3 \times 20}{5 \times 371} = 0.032\ \text{A}$$

Group Assignment 2

Use Thevenin’s theorem to find the current in the 5Ω resistor of the circuit below.



Submission date: God willing a week today

Submission time: Before lecture starts

Where to submit: Electrical Engineering office

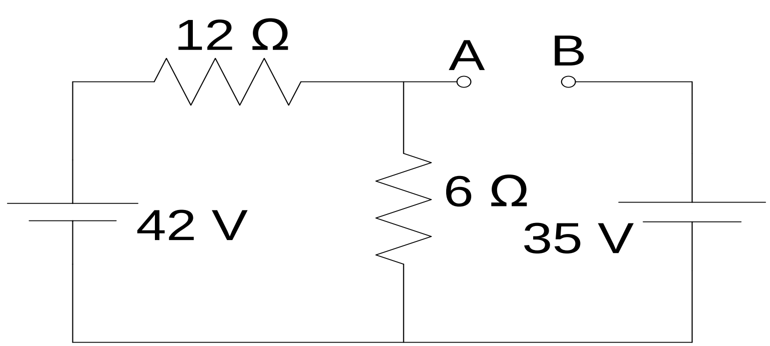
NORTON'S Theorem

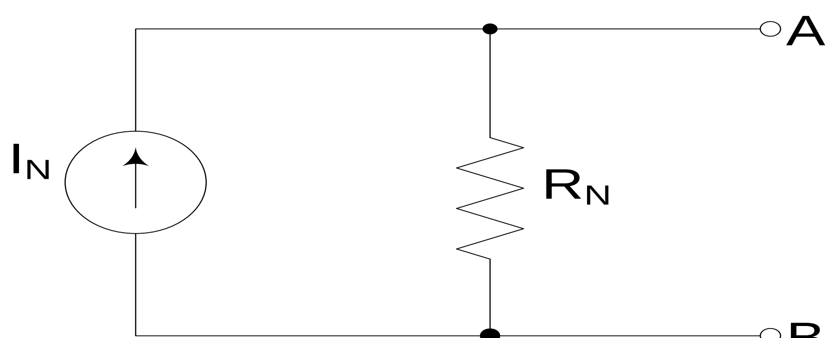
Theorem:

Any linear circuit connected between two terminals can be replaced by a Norton's current(I_N) in parallel with a Norton's resistance (R_N).

I_N is the short-circuit current between the two terminals

R_N is the resistance seen from the two terminals when all sources have been deactivated ($R_N = R_{TH}$)

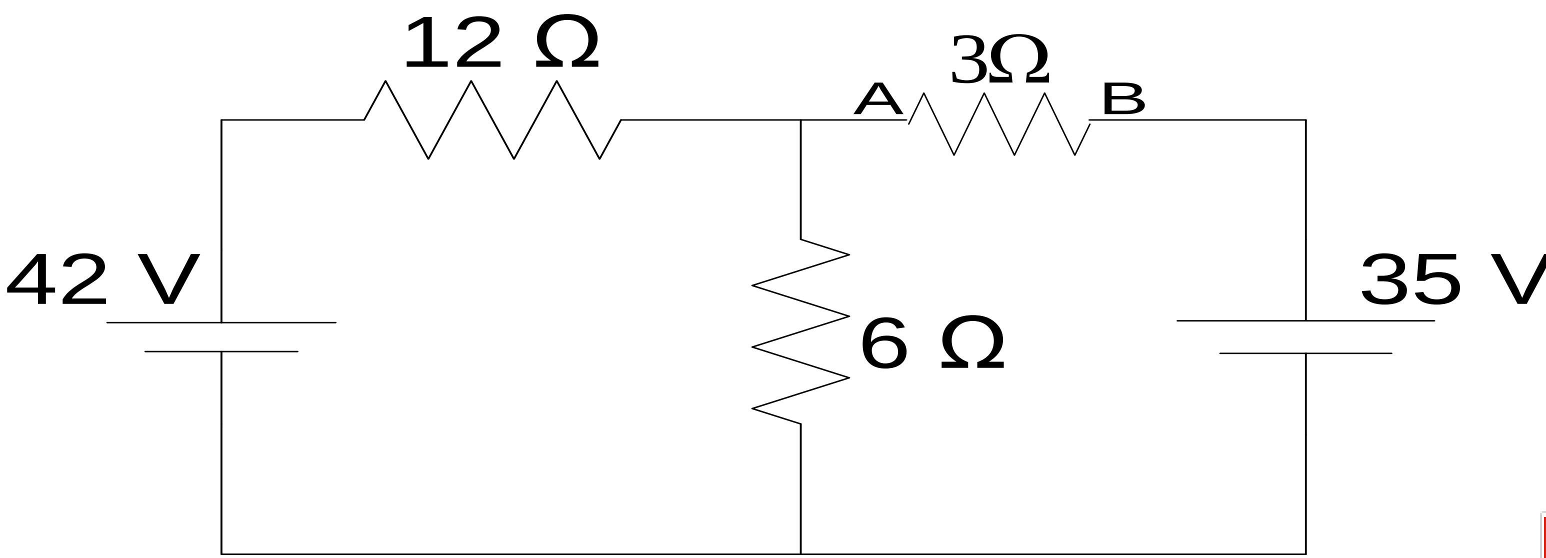




NORTON'S Theorem

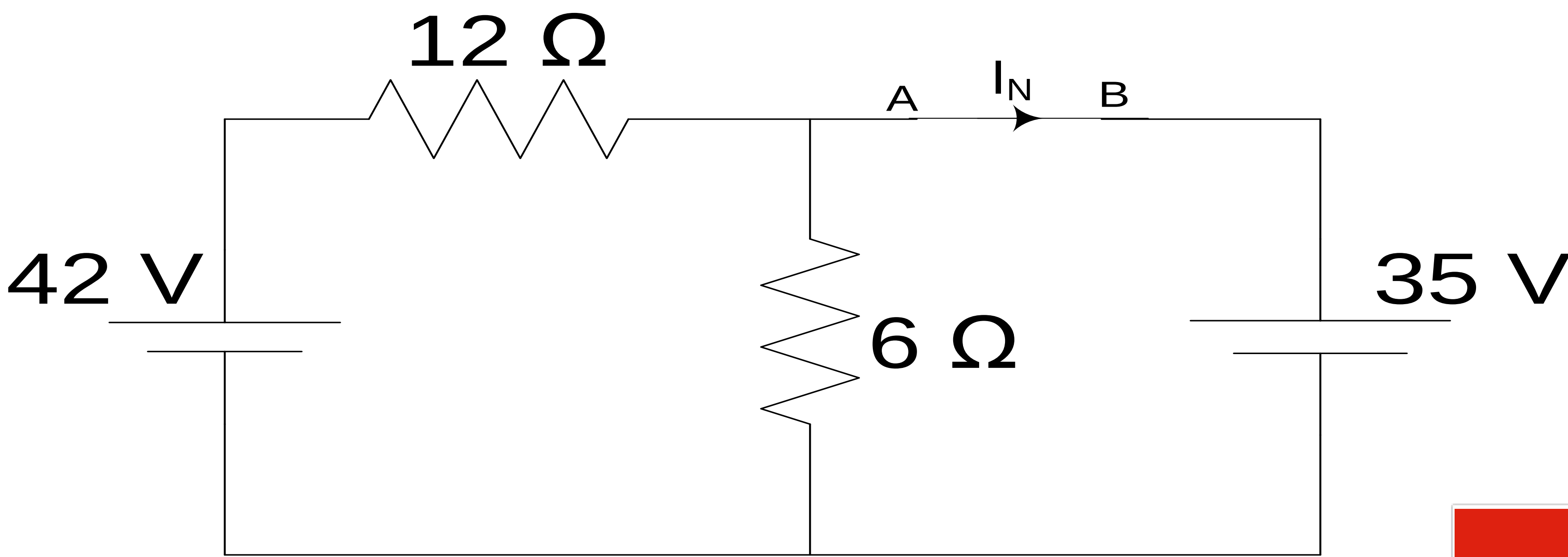
To find the current through a resistor in a circuit, the following steps are taken:

1. **Remove the resistor from the circuit and mark the two terminals.**



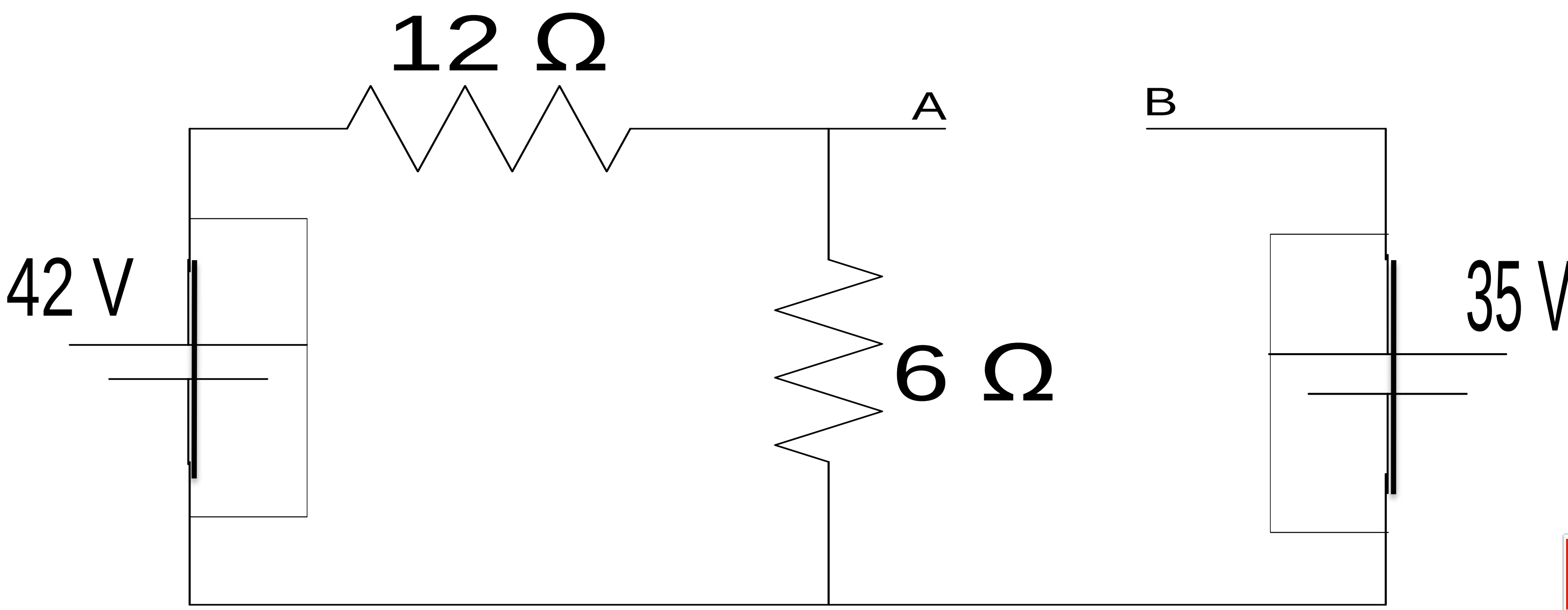
NORTON'S Theorem

2. **Find the short-circuit current (I_N) through the two terminals by applying KVL.**



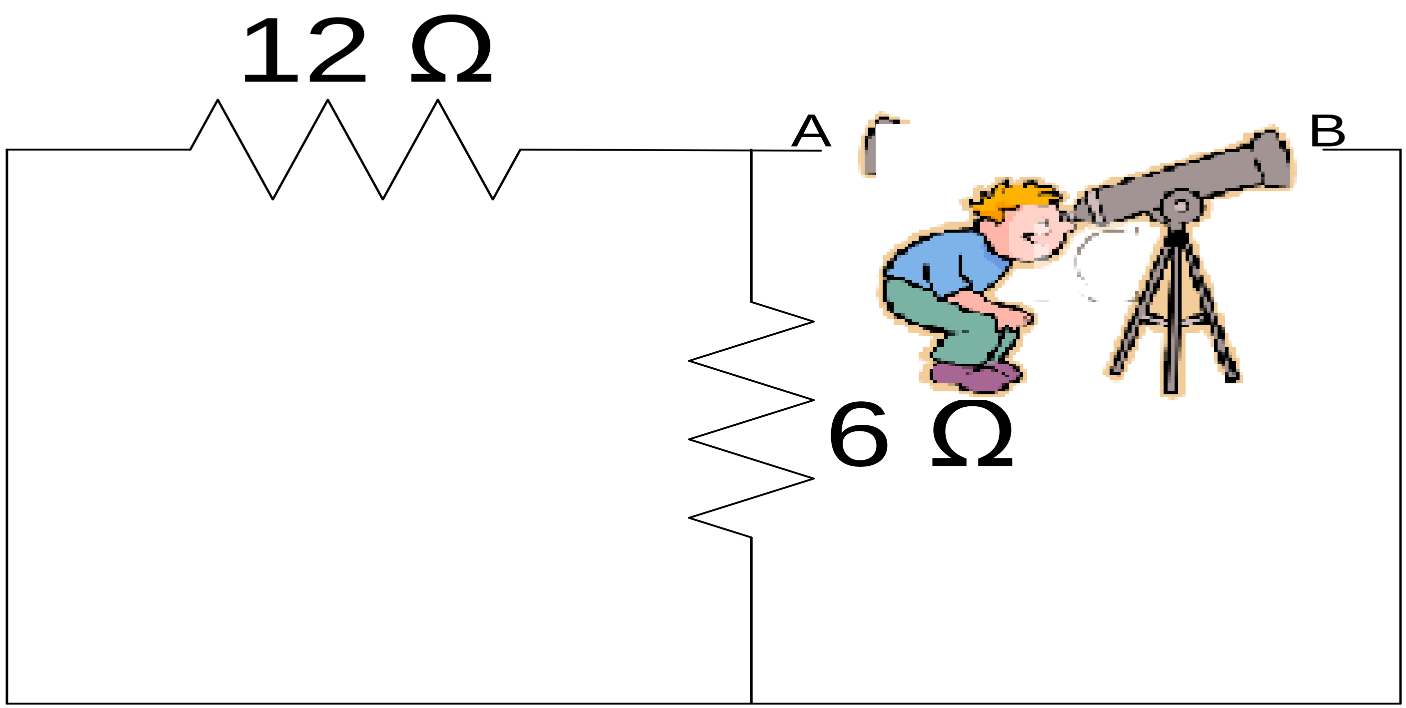
NORTON'S Theorem

3. **Recall the circuit created before step 2 and deactivate all sources. Short-circuit voltage sources and Open-circuit current sources.**



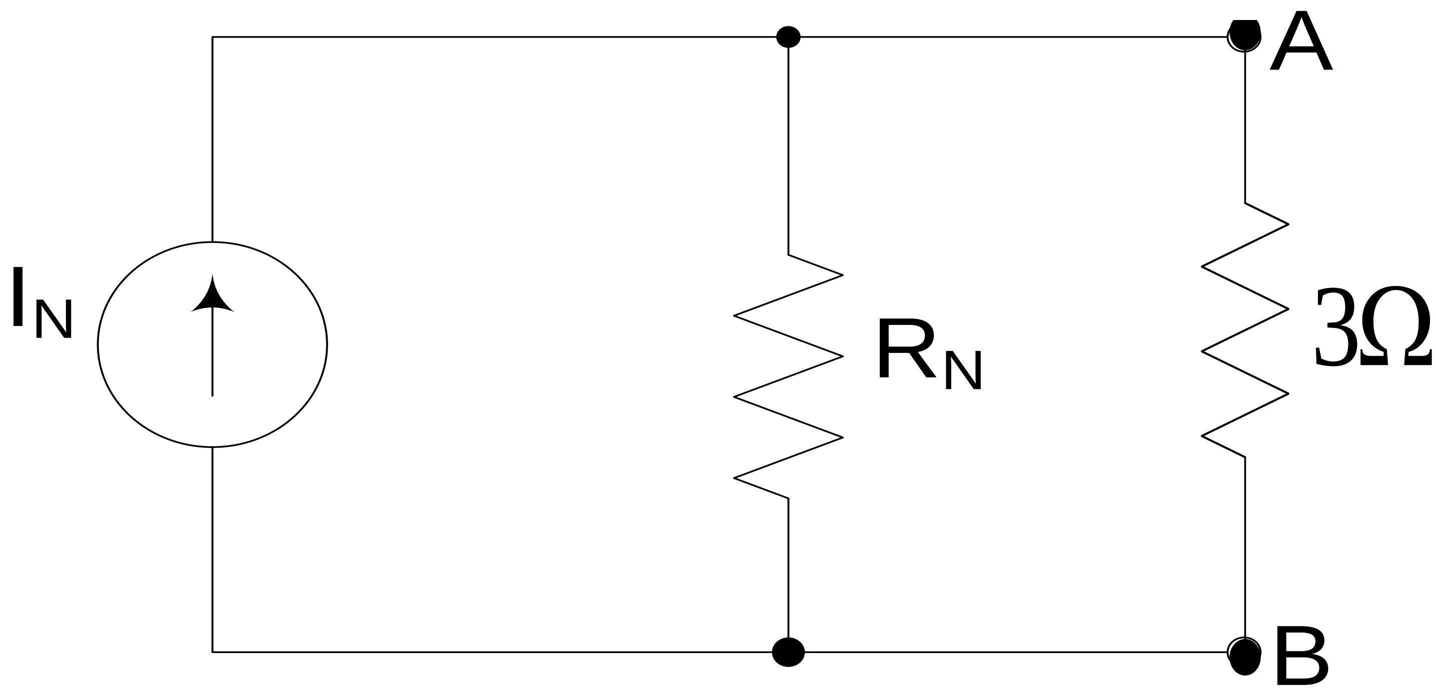
NORTON'S THEOREM

4. **Find the total resistance of the circuit resulting from step 3 as seen from the two terminals**



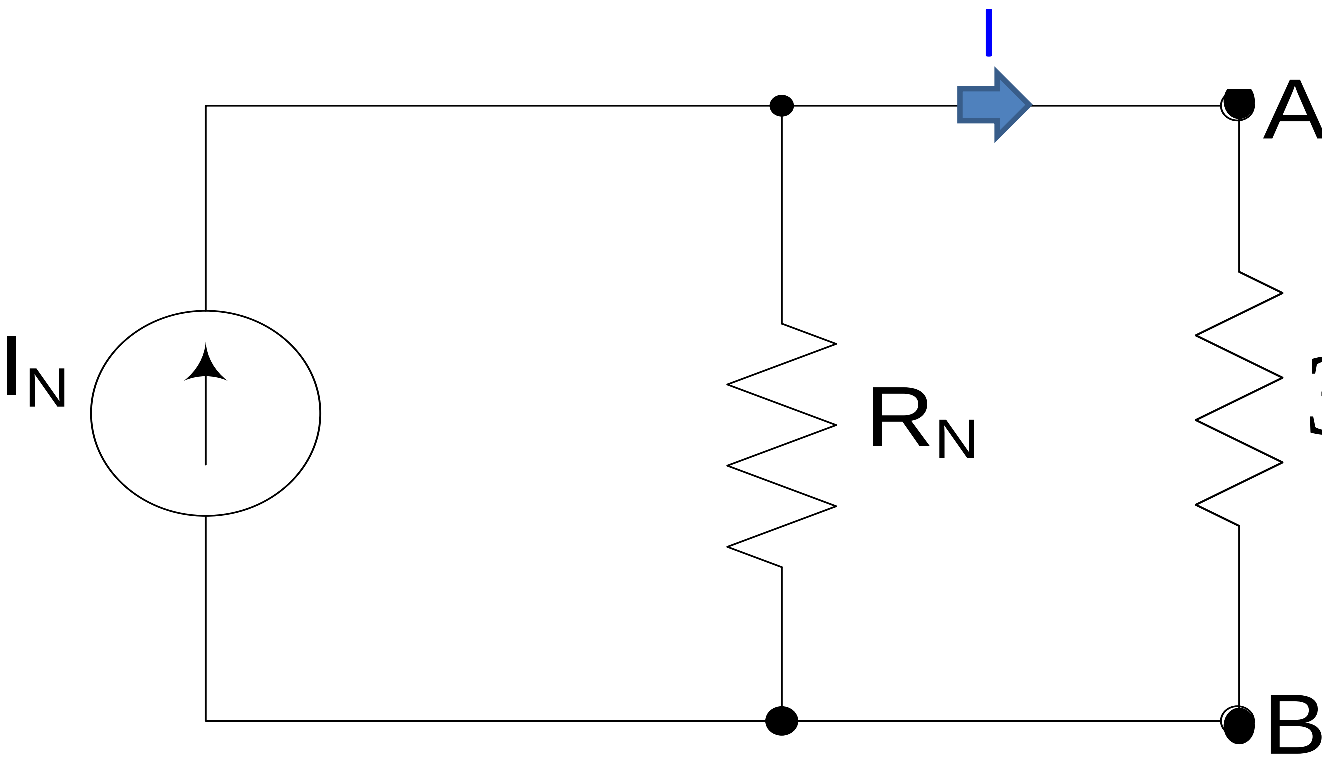
NORTON'S THEOREM

5. **Reproduce the Norton's equivalent circuit and connect the resistor whose current is to be found.**



NORTON'S THEOREM

6. **Calculate the current in the circuit in step 5. This is the current being sought for.**

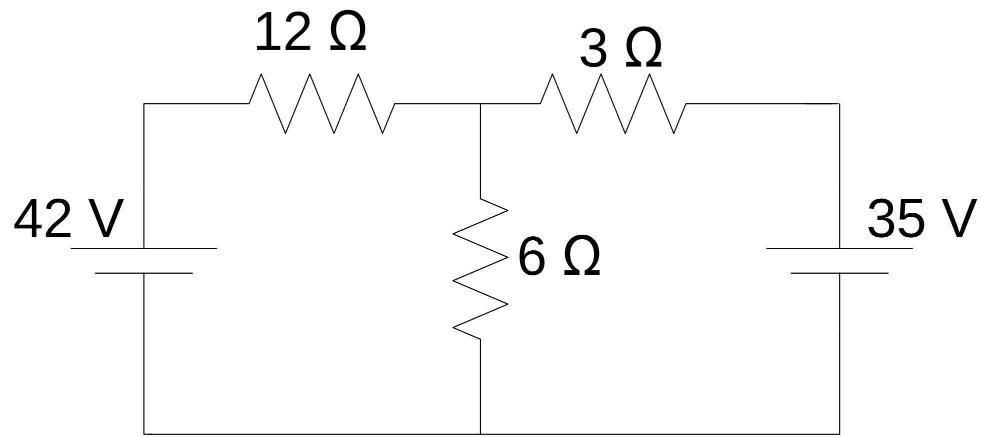


$$i = \frac{R_N}{R_N + 3} \times I_N$$

NORTON'S THEOREM

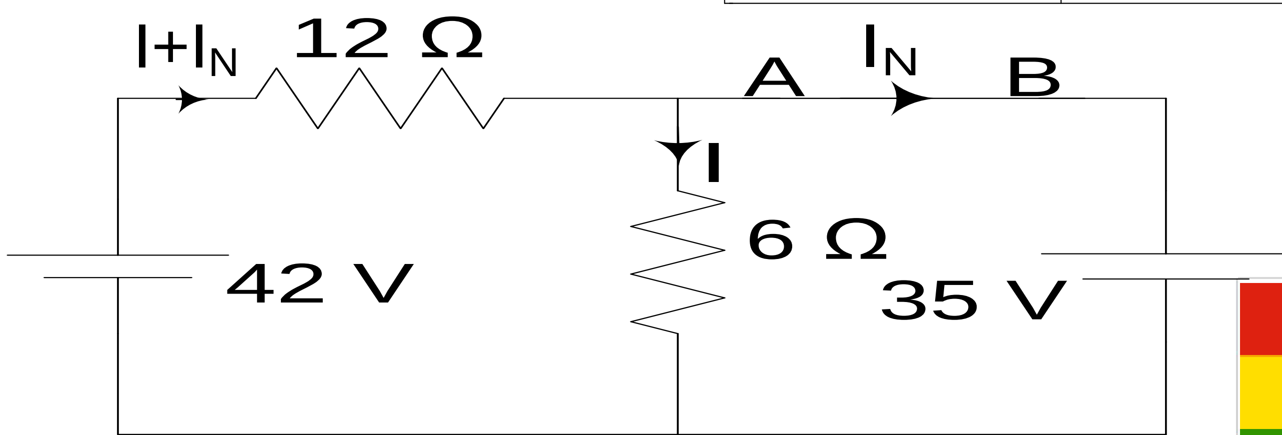
Example 1

Using Norton's theorem, determine the current in the 3-Ω resistor of the circuit below.

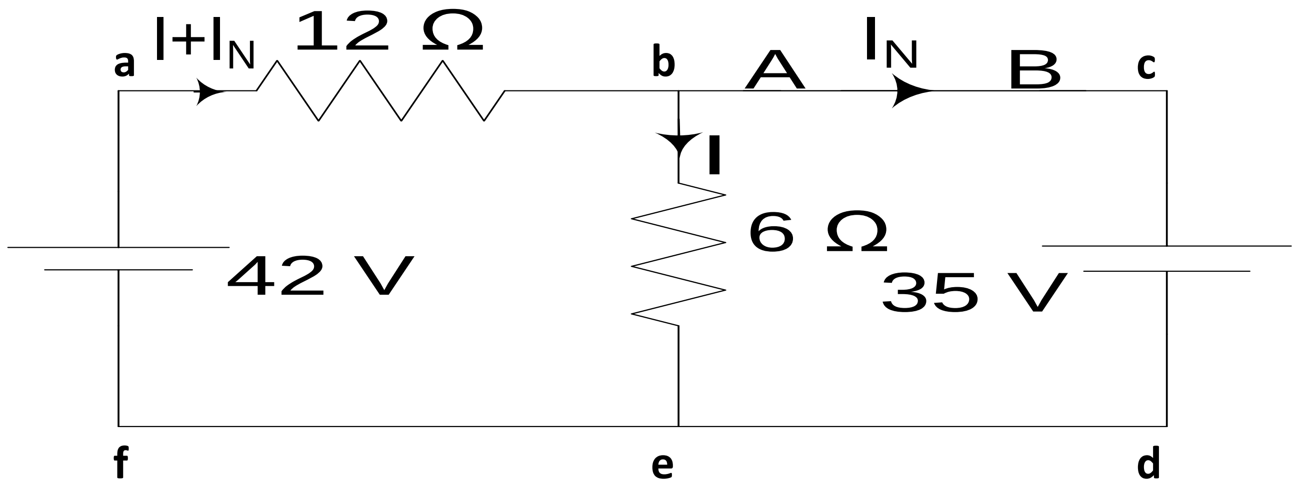


Solution

Steps 1 & 2



NORTON'S THEOREM



Applying KVL to loop abefa: $42 = 12(I + I_N) + 6I$

$$42 = 18I + 12I_N \qquad \textbf{(1)}$$

Applying KVL to loop cbedc: $35 = 6I$

$$\Rightarrow I = \frac{35}{6}A$$

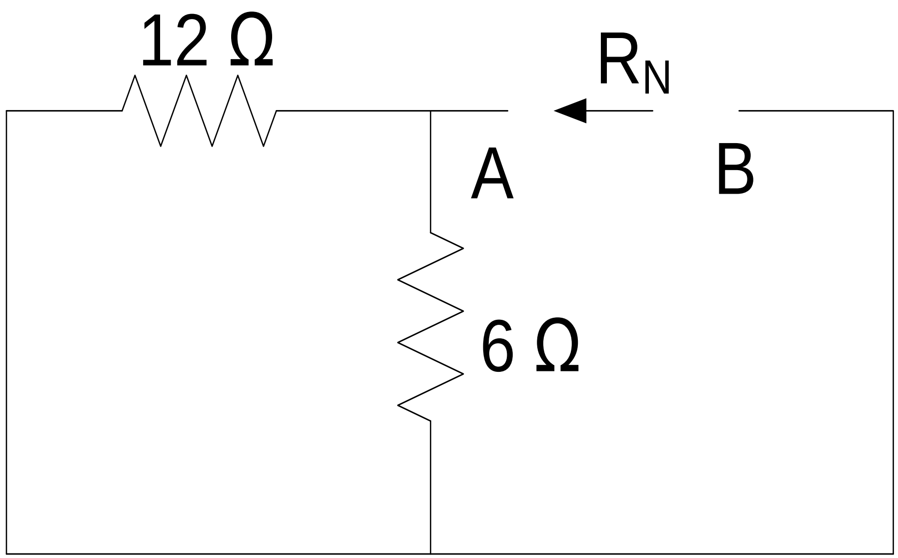
NORTON'S THEOREM

Substituting for I in equation 1:

$$42 = 18\left(\frac{35}{6}\right) + 12I_N$$

$$I_N = \frac{-21}{4}A$$

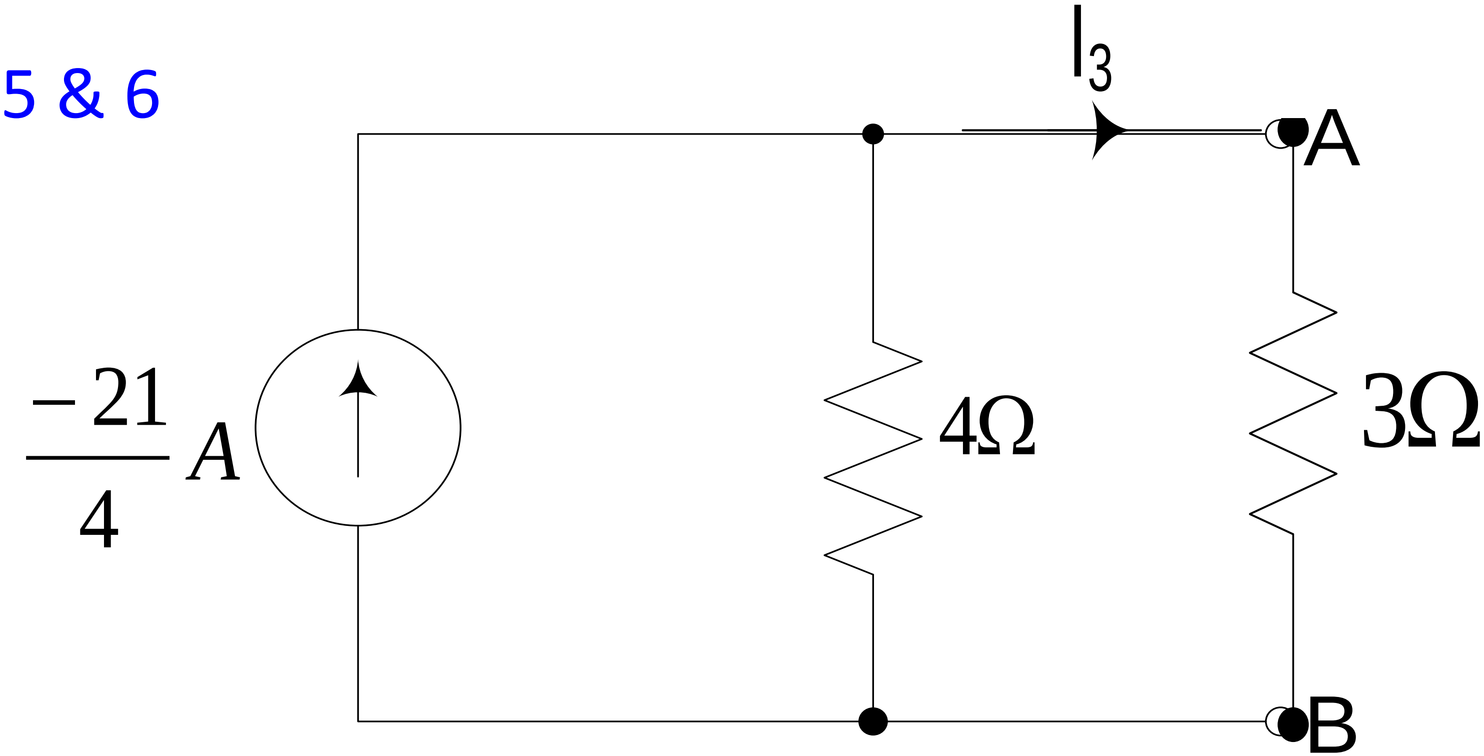
Steps 3 & 4



$$R_N = 12 // 6 = \frac{12 \times 6}{12 + 6} = 4\Omega$$

NORTON'S THEOREM

Steps 5 & 6

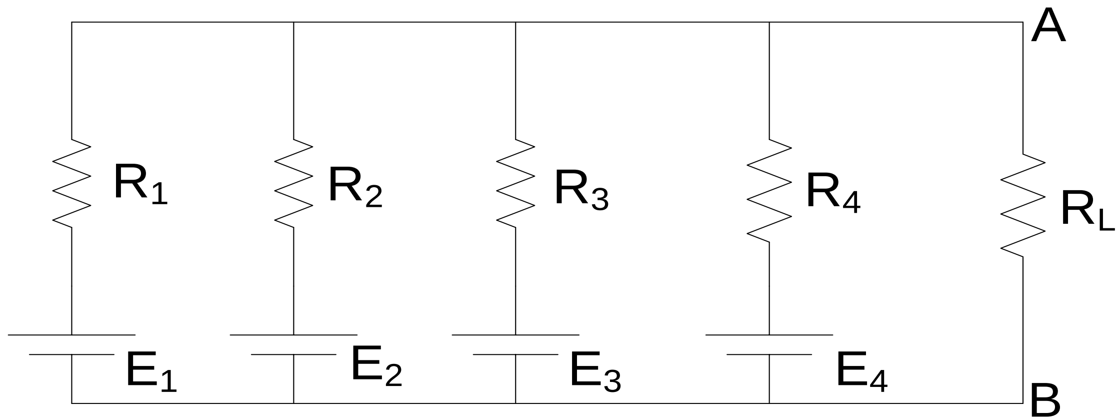


$$I_3 = \frac{4}{4+3} \times \frac{-21}{4} = -3A$$

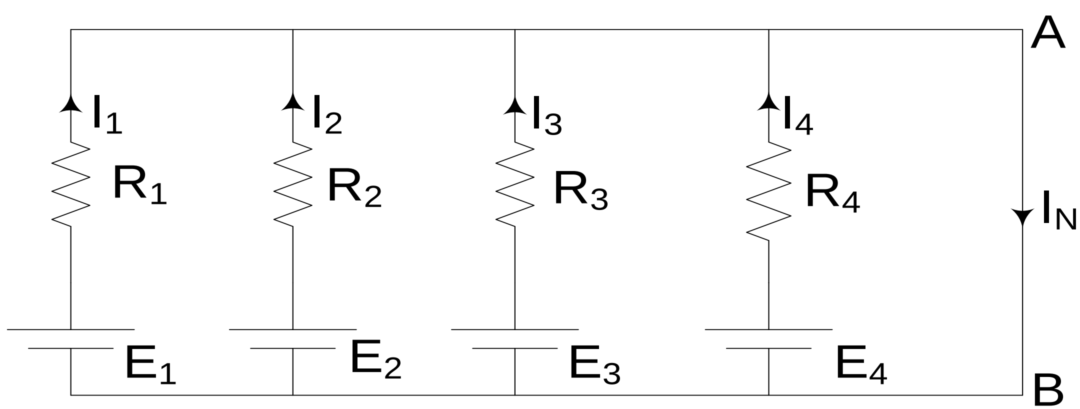
NORTON'S THEOREM

Example 2

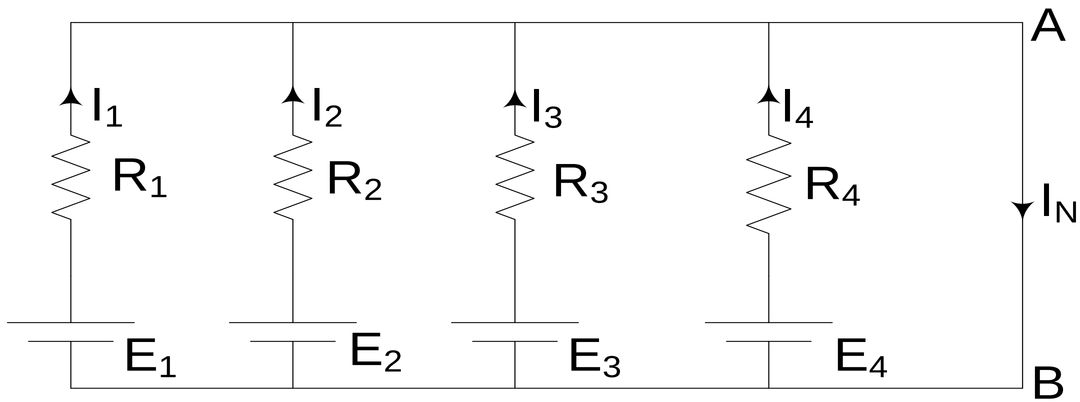
Determine the current in the load resistor R_L



Solution



NORTON'S THEOREM



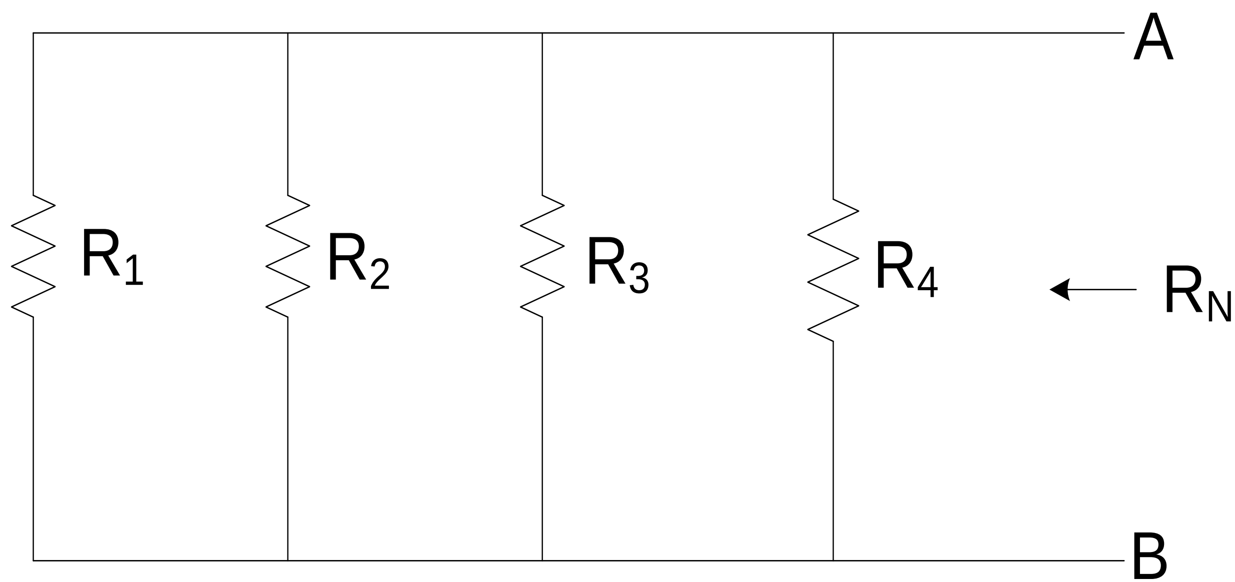
Solution

Applying KCL

$$I_N = I_1 + I_2 + I_3 + I_4$$
$$= \frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3} + \frac{E_4}{R_4}$$

NORTON'S THEOREM

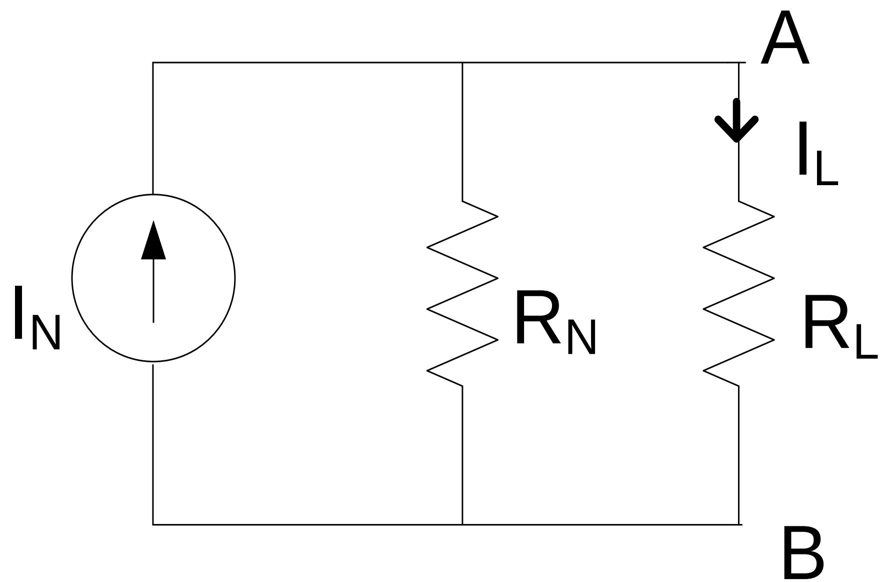
Finding R_N



$$\frac{1}{R_N} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

NORTON'S THEOREM

Finding I_L



$$I_L = \frac{R_N}{R_N + R_L} \times I_N$$

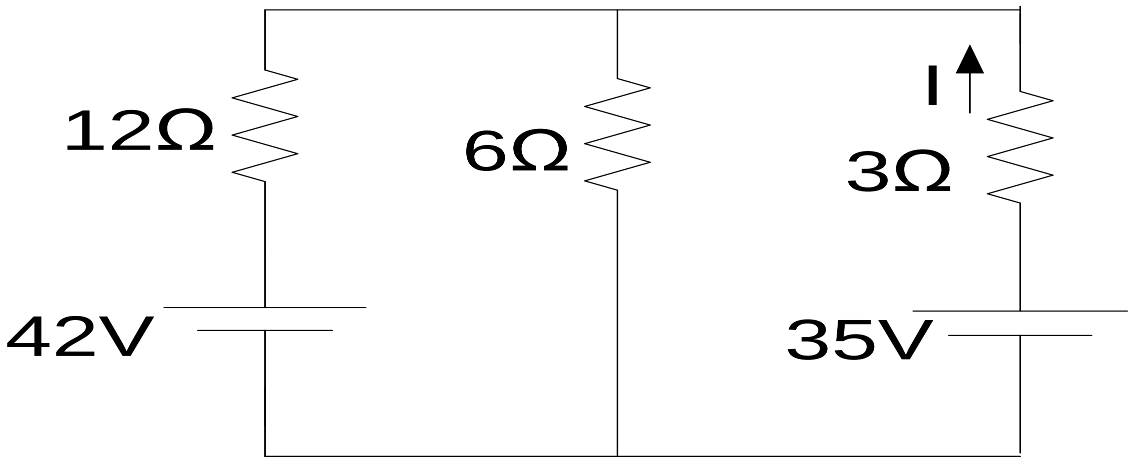
SUPERPOSITION THEOREM

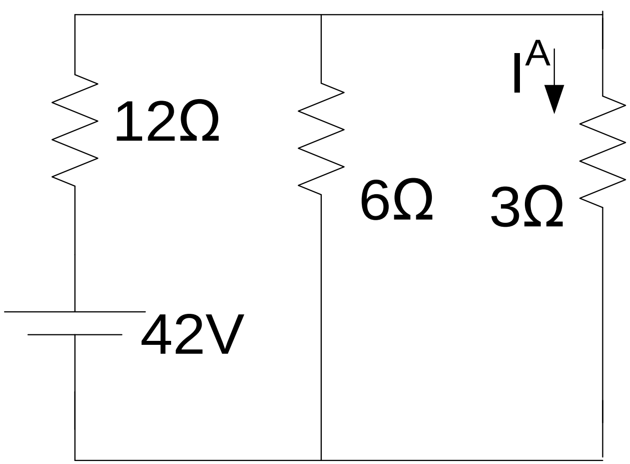
The Theorem

The current through(or the voltage across) any element in a multiple-source linear circuit can be found by taking the algebraic sum of the current through(or the voltage across) that element due to each individual source acting alone.

SUPERPOSITION THEOREM

The Theorem





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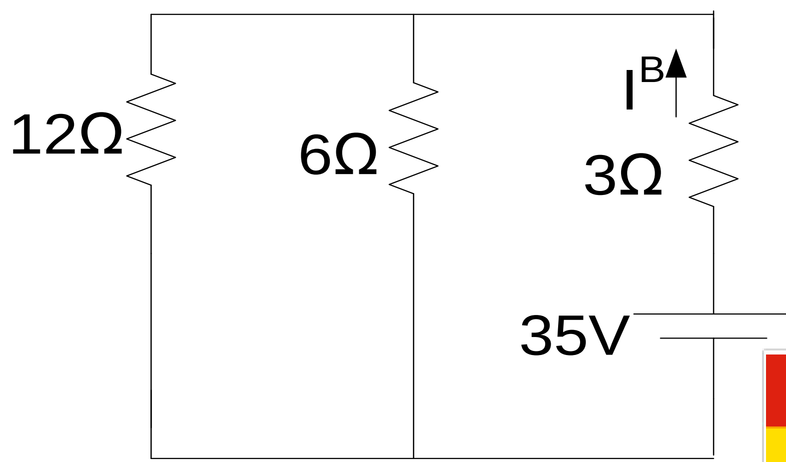
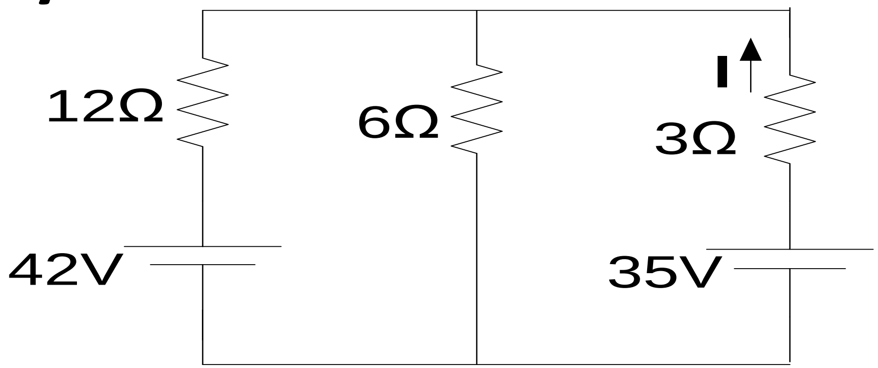


SUPERPOSITION THEOREM

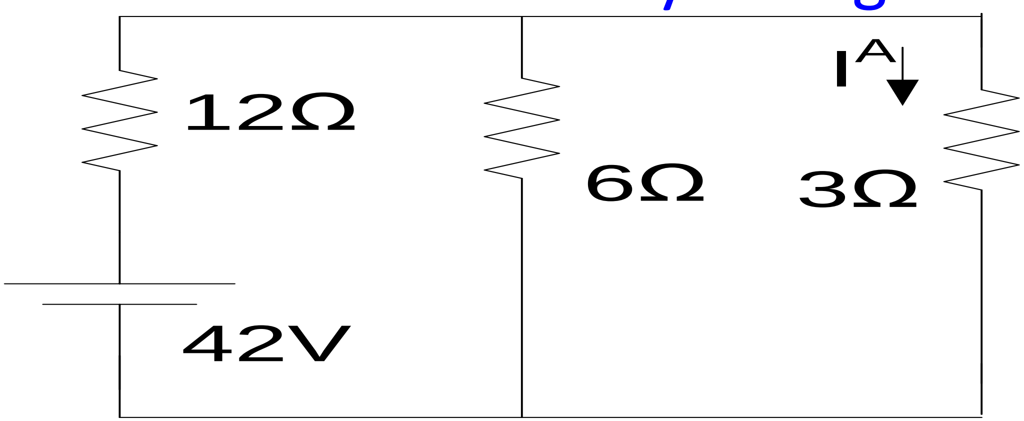
Example 1

Use superposition theorem to find the current supplied by the 35V battery of the circuit below.



Solution

With the 42V battery acting alone,



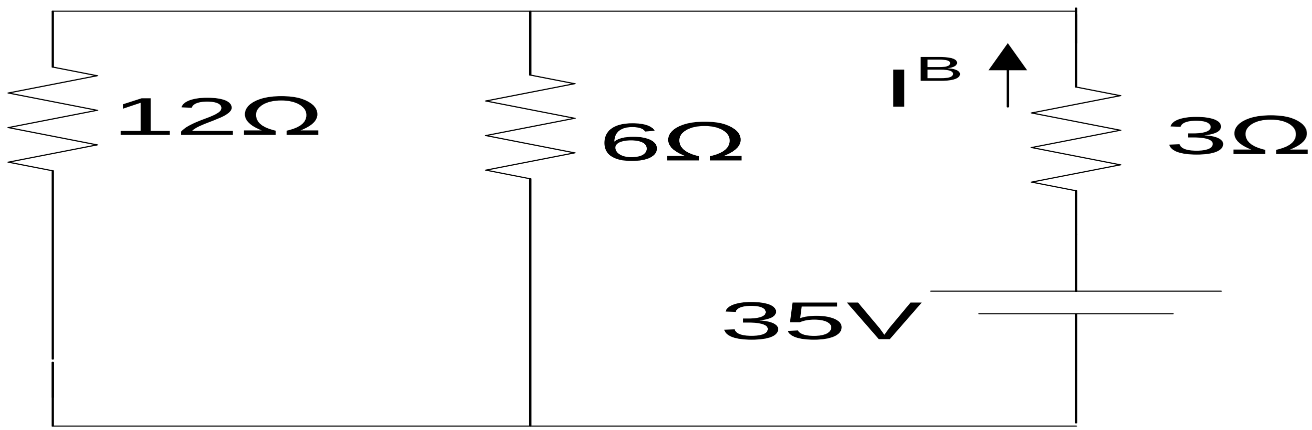
$$R_T = (3//6) + 12 \quad = 14\Omega$$

$$I_T = \frac{42}{14} = 3A$$

$$I_A = \frac{6}{6+3} \times 3 = 2A$$

SUPERPOSITION THEOREM

With the 35V battery acting alone,



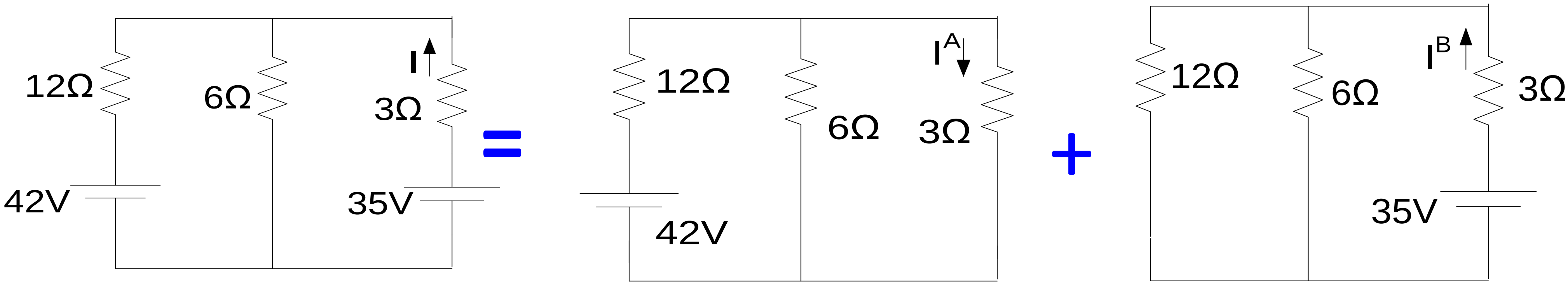
$$R_T = (12//6) + 3 = 7\Omega$$

$$I_T = \frac{35}{7} = 5A$$

$$I_B = I_T = 5A$$

SUPERPOSITION THEOREM

With both batteries acting,

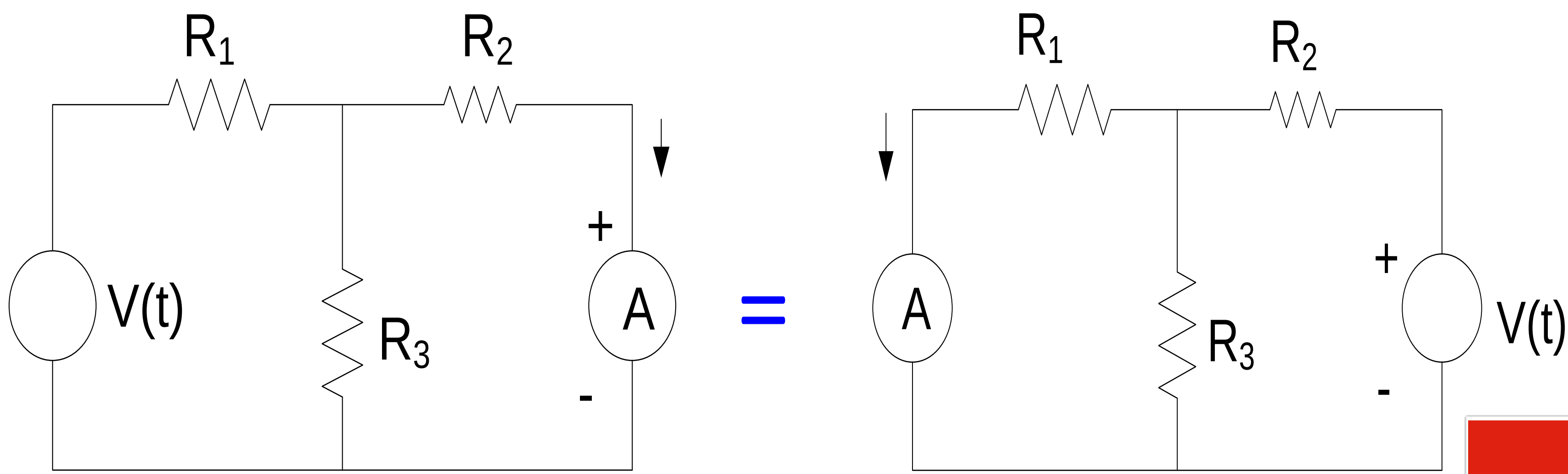


$$I = I_B - I_A$$
$$= 5 - 2 = 3A$$

RECIPROCITY THEOREM

The Theorem

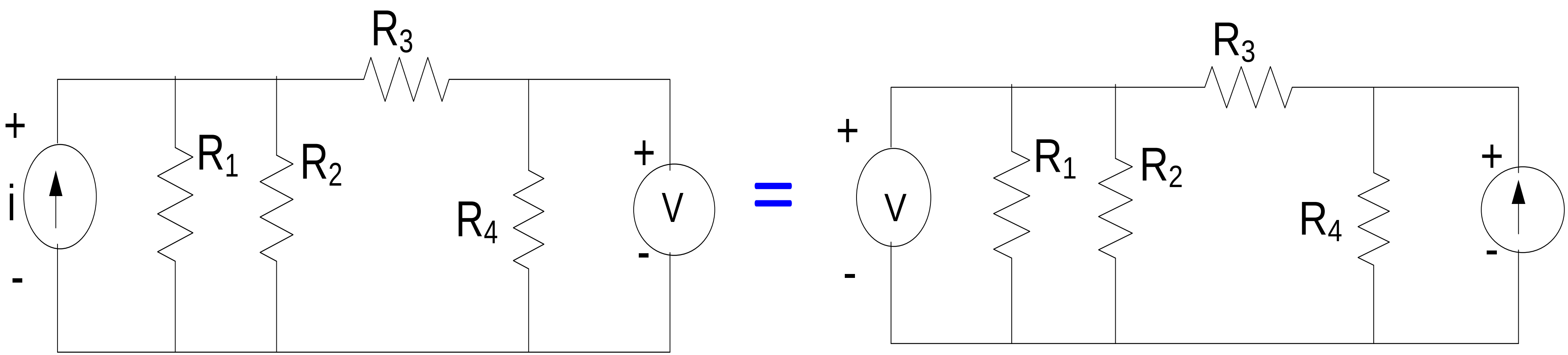
An ideal ammeter and ideal voltage source when inserted in two different branches of a linear network can be interchanged without changing the reading of the ammeter



RECIPROCITY THEOREM

Similarly,

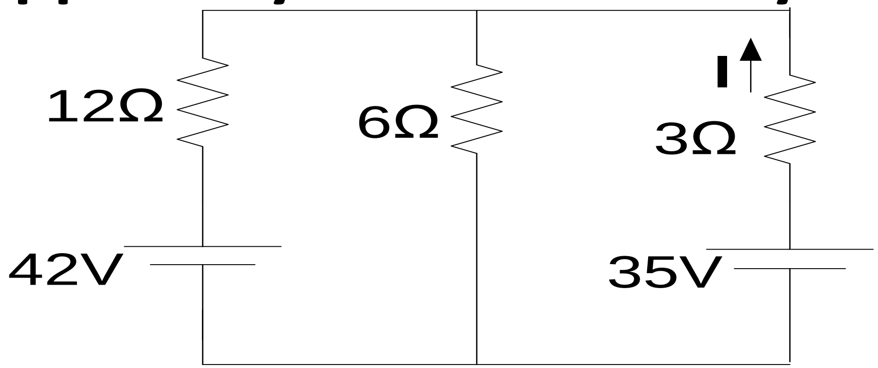
An ideal voltmeter and ideal current source when connected across two different branches of a network can be interchanged without changing the reading of the voltmeter.



RECIPROCITY THEOREM

Example 1

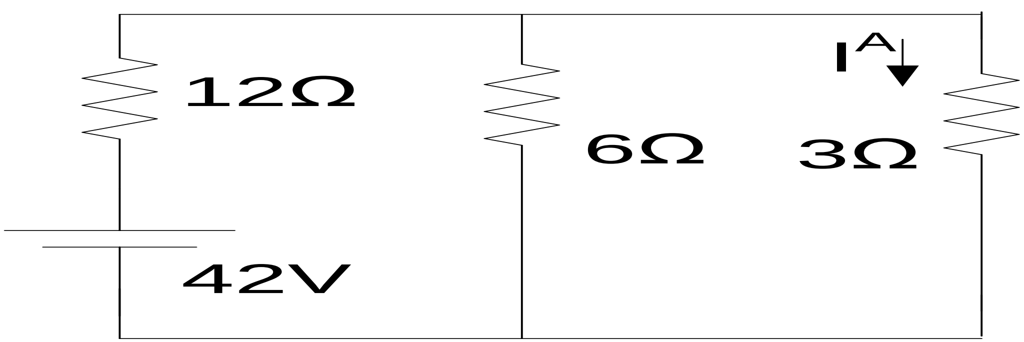
Jointly use superposition and reciprocity theorems to find the current supplied by the 35V battery of the circuit below.



Solution

With the 42V battery acting alone,

$$R_T = (3//6) + 12 = 14\Omega$$

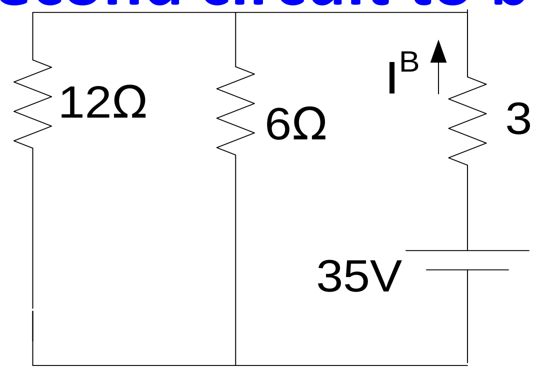


$$I_T = \frac{42}{14} = 3A$$

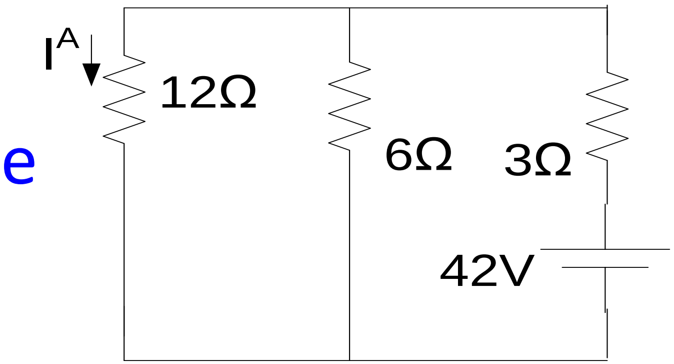
$$I_A = \frac{6}{6+3} \times 3 = 2A$$

RECIPROCITY THEOREM

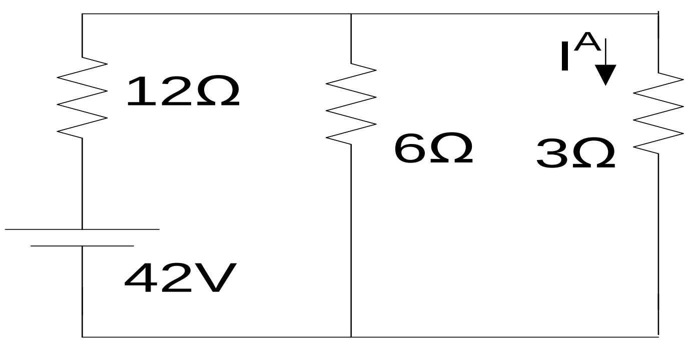
The second circuit to be solved is:



is comparable to

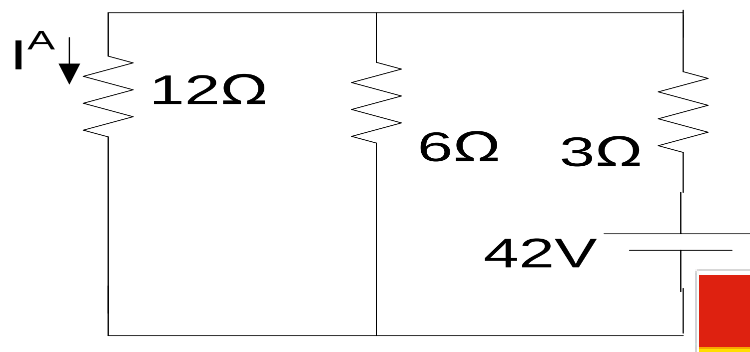


Applying the reciprocity theorem



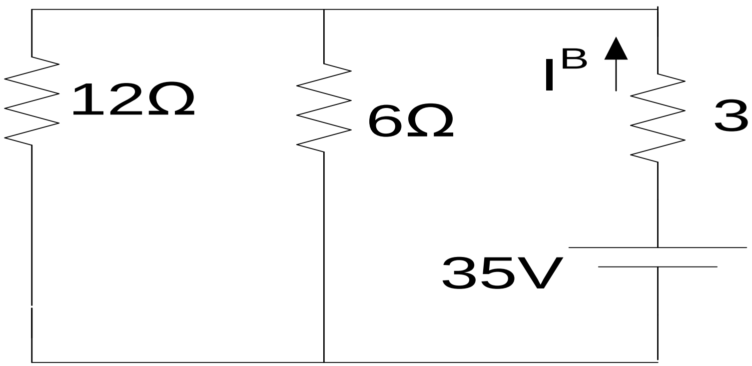
A

=

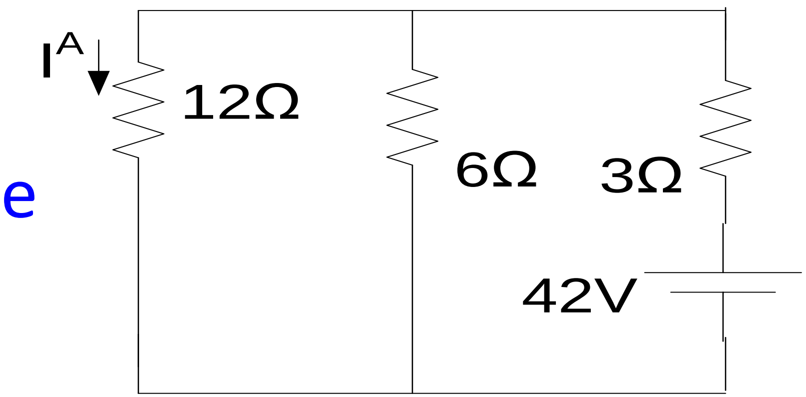


B

RECIPROCITY THEOREM



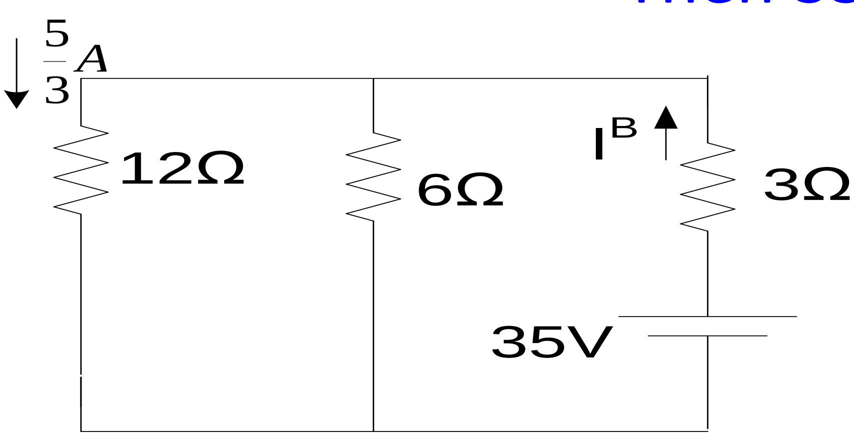
is comparable to circuit B



Applying proportion,

If 42V = I^A =2A,

Then 35V =(35*2)/42=5/3A



Applying KVL,

$$35 = 3I^B + 12 \times \frac{5}{3}$$

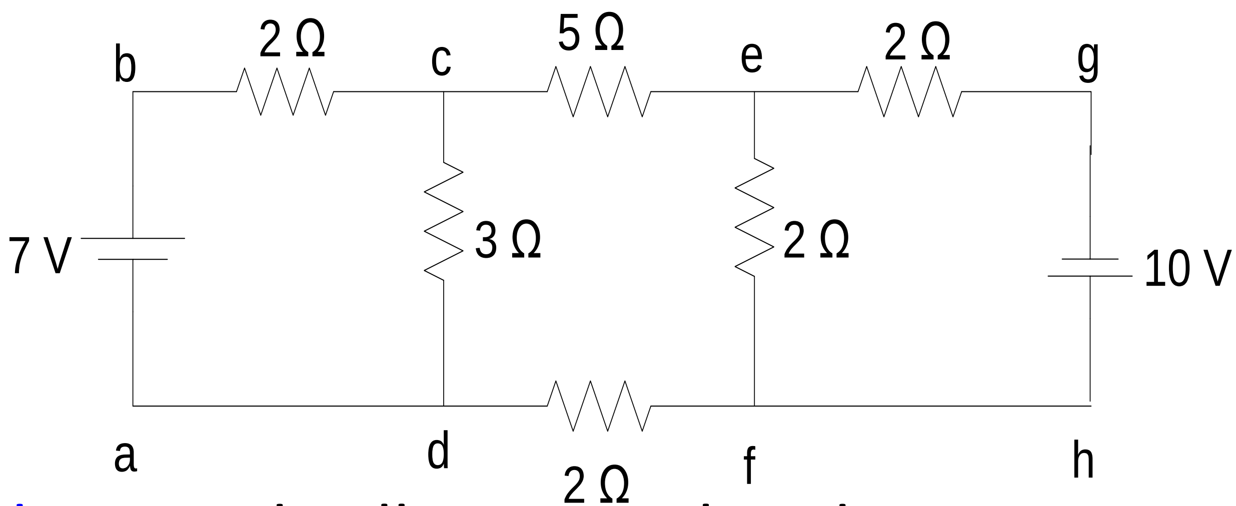
$$\Rightarrow I^B = 5A$$

RECIPROCITY THEOREM

$$I = I_B - I_A$$
$$= 5 - 2 = 3A$$

Group Assignment 3

Use Norton's theorem to find the current in the 2Ω resistor connected between *e* and *f* in the circuit below.



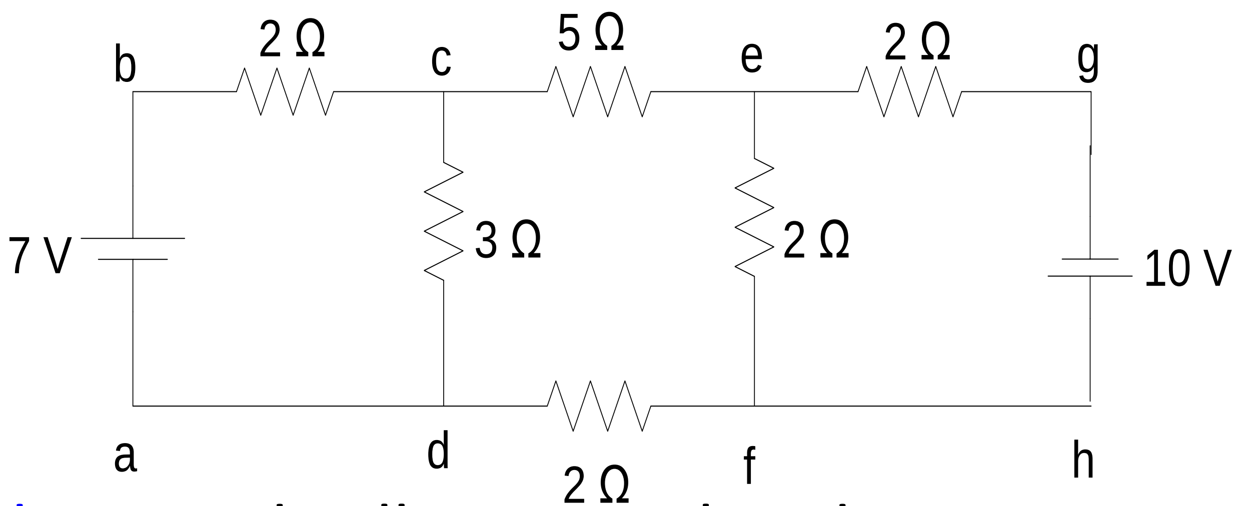
Submission date: God willing a week today

Submission time: Before lecture starts

Where to submit: Electrical Engineering office

Group Assignment 3

Use Norton's theorem to find the current in the 2Ω resistor connected between *e* and *f* in the circuit below.



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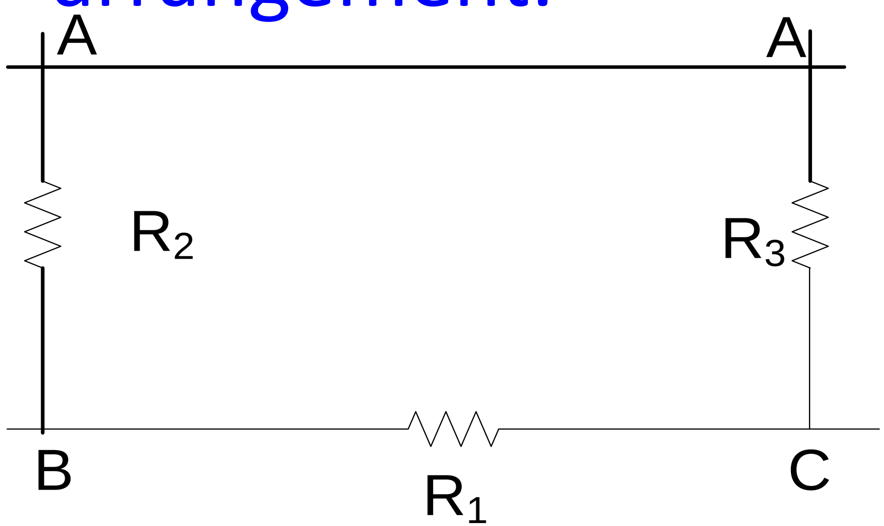
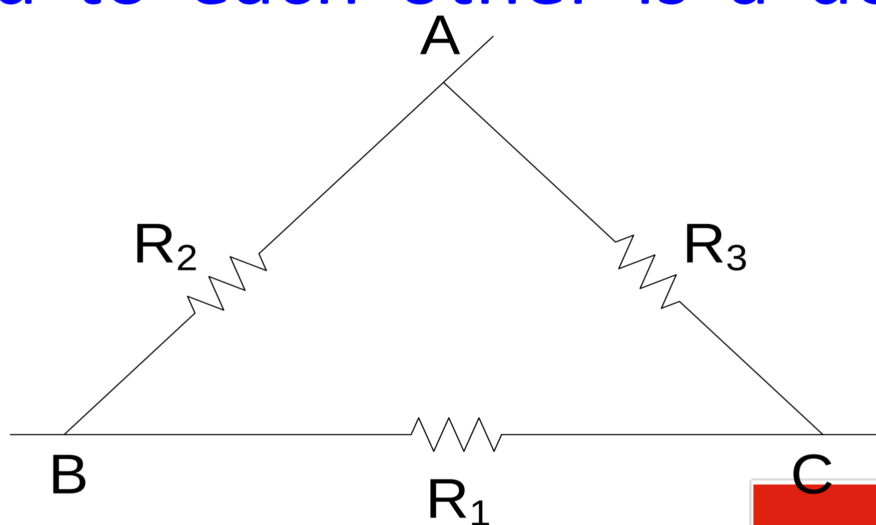
DELTA-STAR TRANSFORMATION

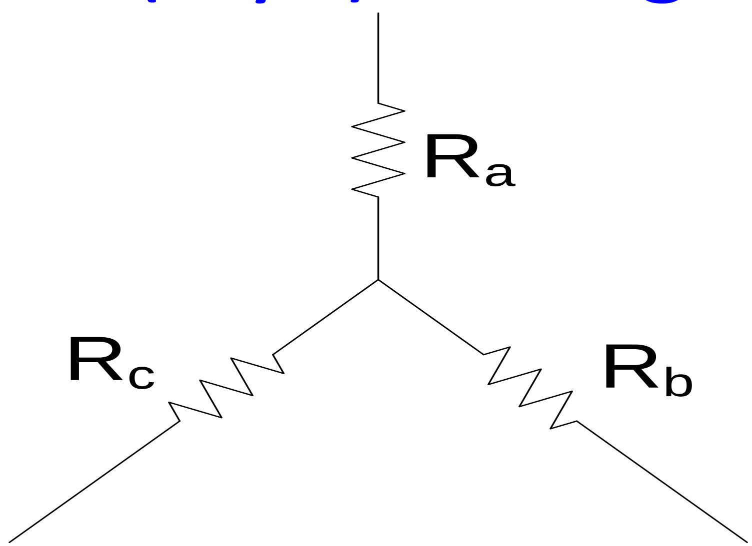
- The transformation is employed in situations where neither series nor parallel arrangements can be identified.
- An arrangement of three(3) resistors where the resistors are connected to each other is a delta arrangement.

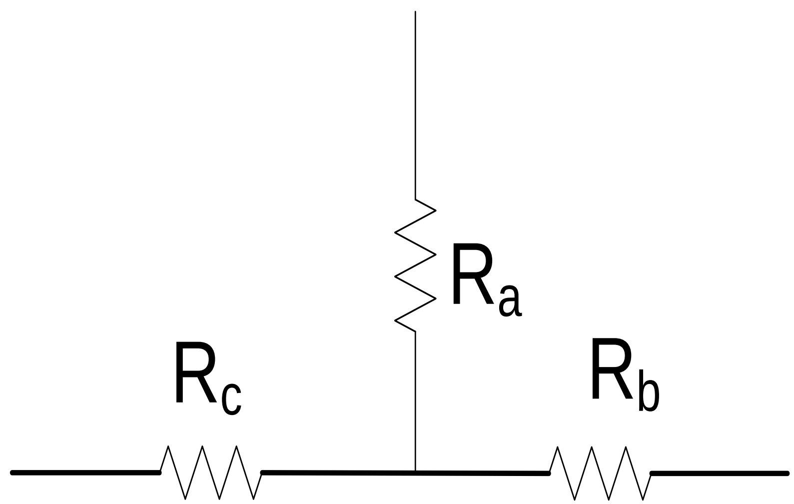




DELTA-STAR TRANSFORMATION

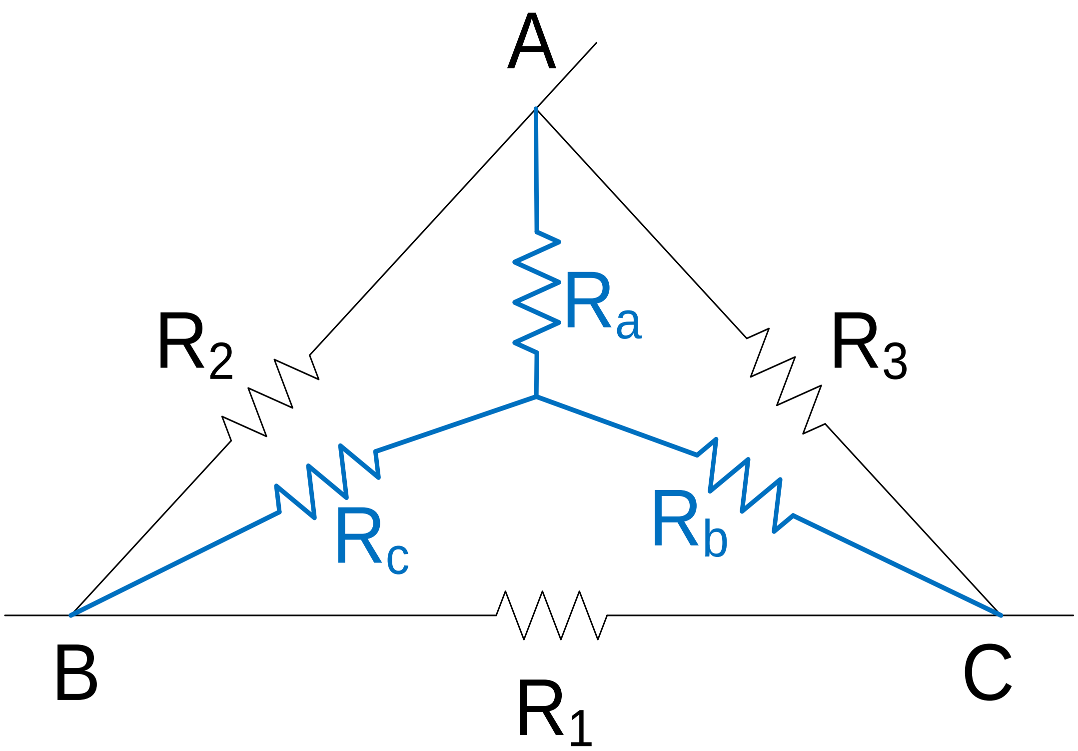
- An arrangement of three(3) resistors where all resistors have a common point of connection through one terminal is a star(wye) arrangement.





DELTA-STAR TRANSFORMATION

- A delta arrangement can be changed to star and vice versa using the following relations:

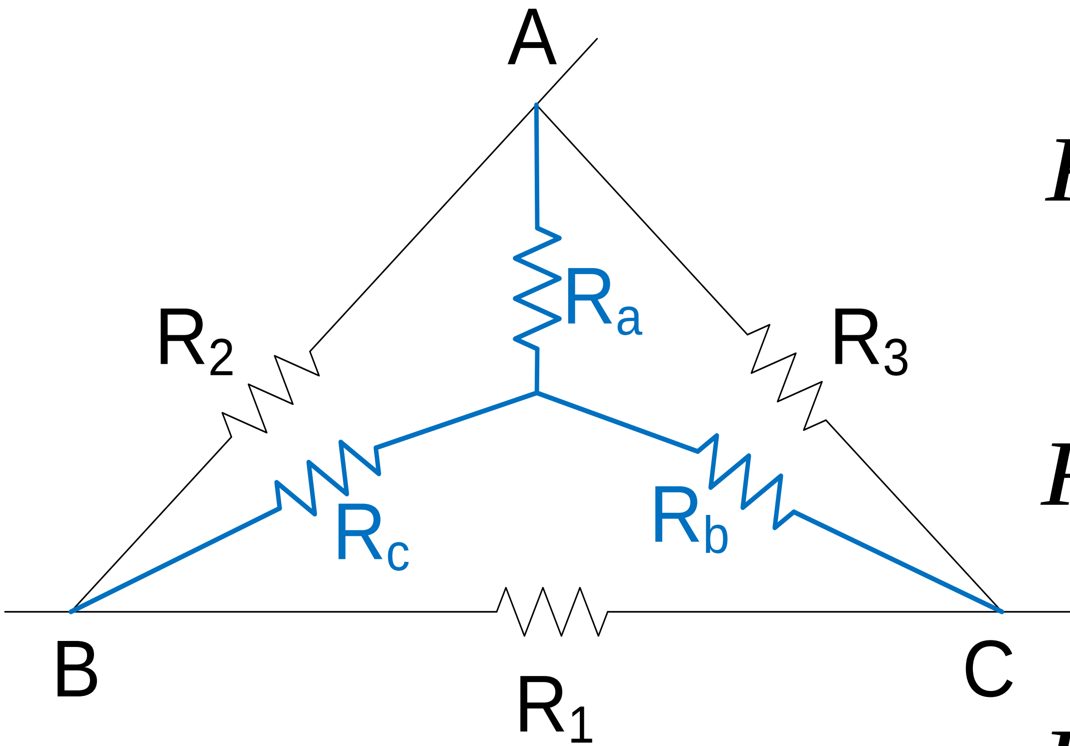


$$R_a = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

DELTA-STAR TRANSFORMATION



$$R_1 = R_b + R_c + \frac{R_b R_c}{R_a}$$

$$R_2 = R_a + R_c + \frac{R_a R_c}{R_b}$$

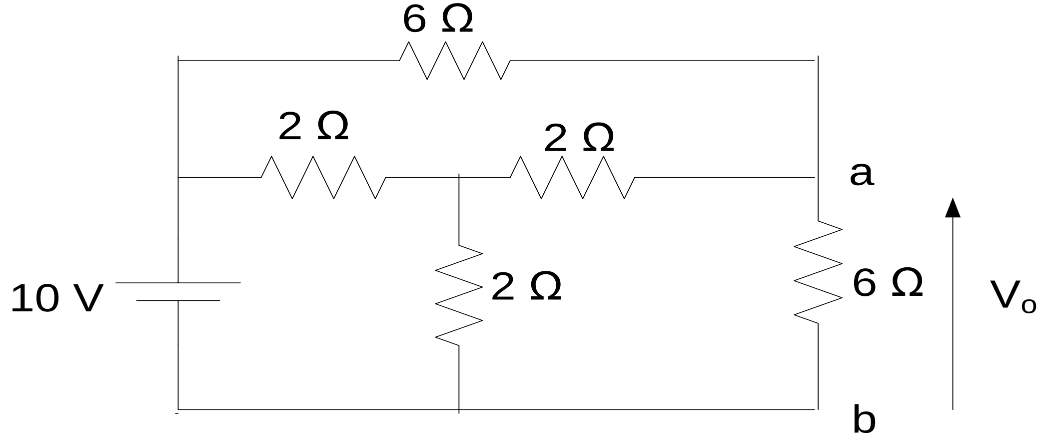
$$R_3 = R_a + R_b + \frac{R_a R_b}{R_c}$$

When all values are the same, delta values are 3 times star values

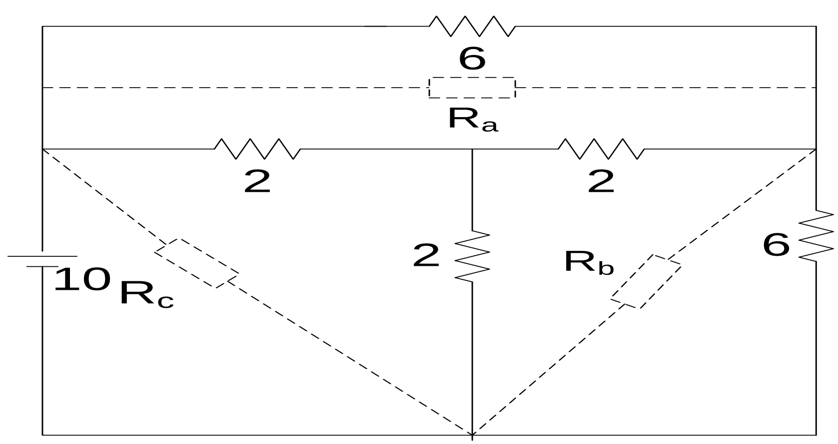
DELTA-STAR TRANSFORMATION

Example 1

Determine the voltage V_0 across the 6Ω resistor of the circuit below

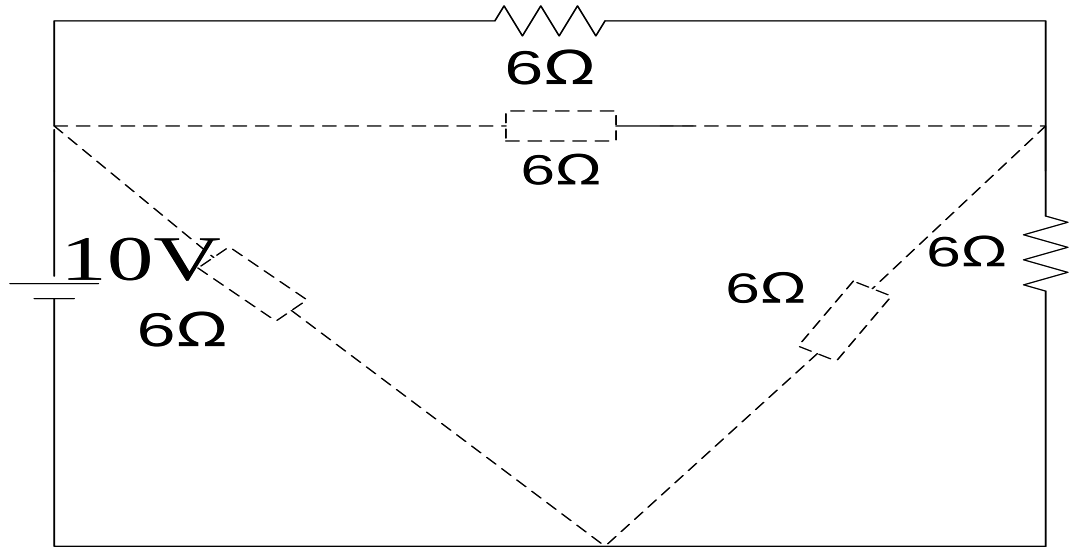


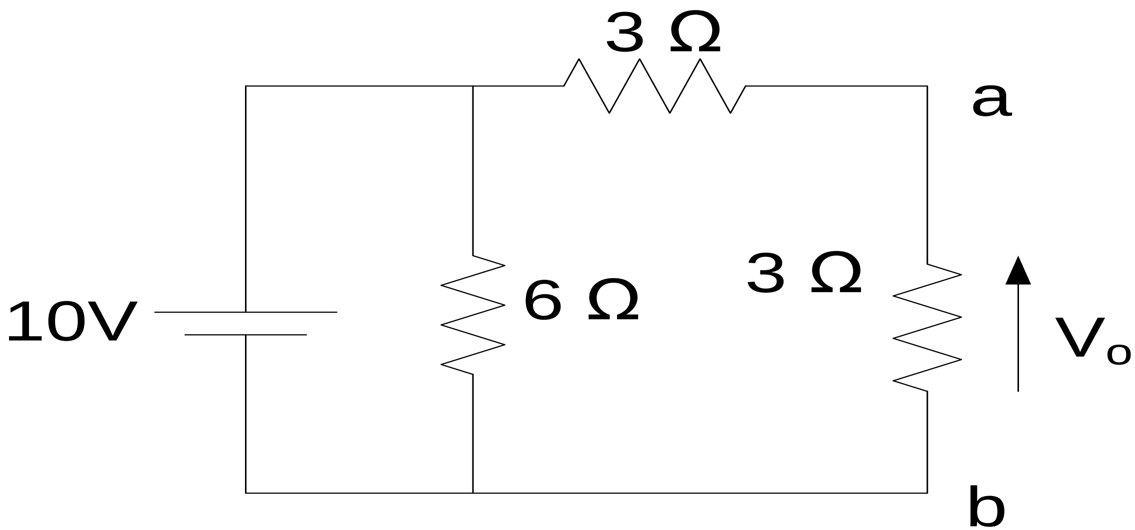
Solution



$$R_a = R_b = R_c = 2 \times 3 = 6\Omega$$

DELTA-STAR TRANSFORMATION

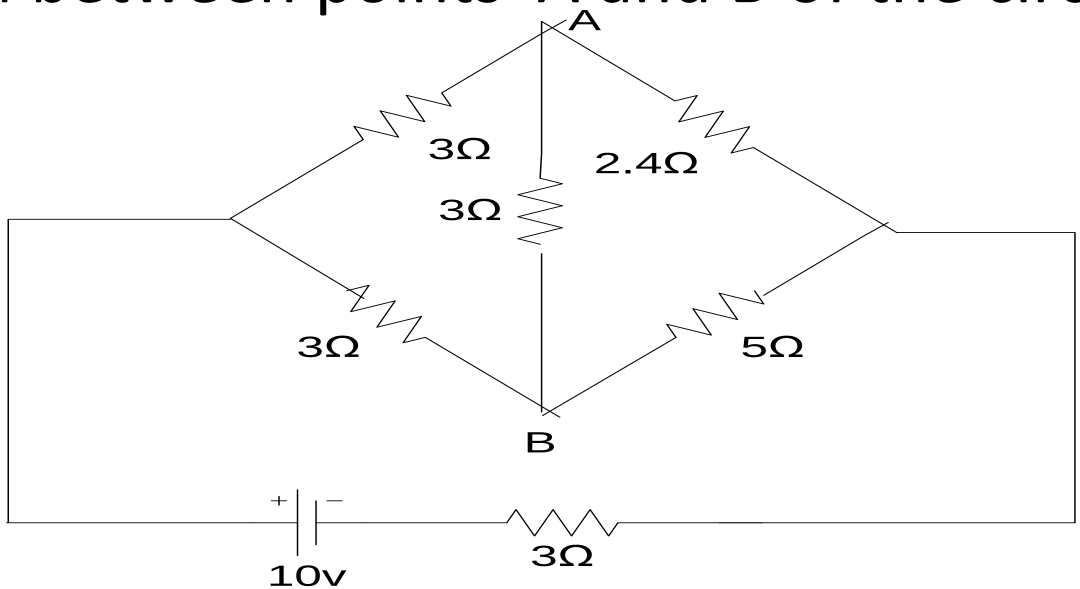




$$V_0 = \frac{3}{3+3} \times 10V = 5V$$

Group Assignment 4

Use Norton's theorem to find the current in the 3Ω resistor connected between points A and B of the circuit below.



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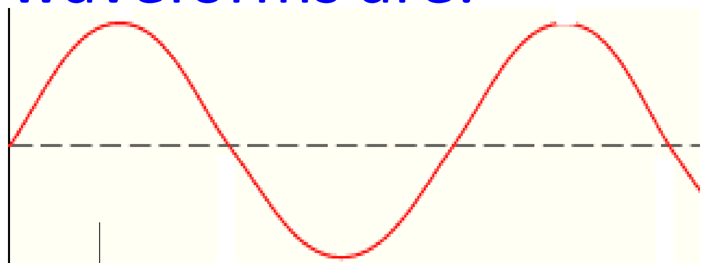
Submission time: Before lecture starts

Where to submit: Electrical Engineering office

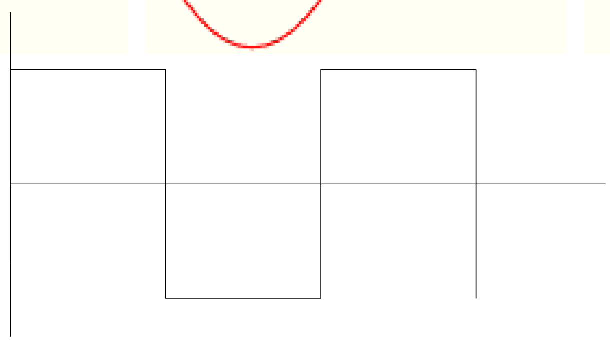
ALTERNATING CURRENT CIRCUITS

- Alternating current (AC) circuits are circuits with currents and voltages which are time-varying

- Examples of AC waveforms are:
  - Sine wave



  - Square wave

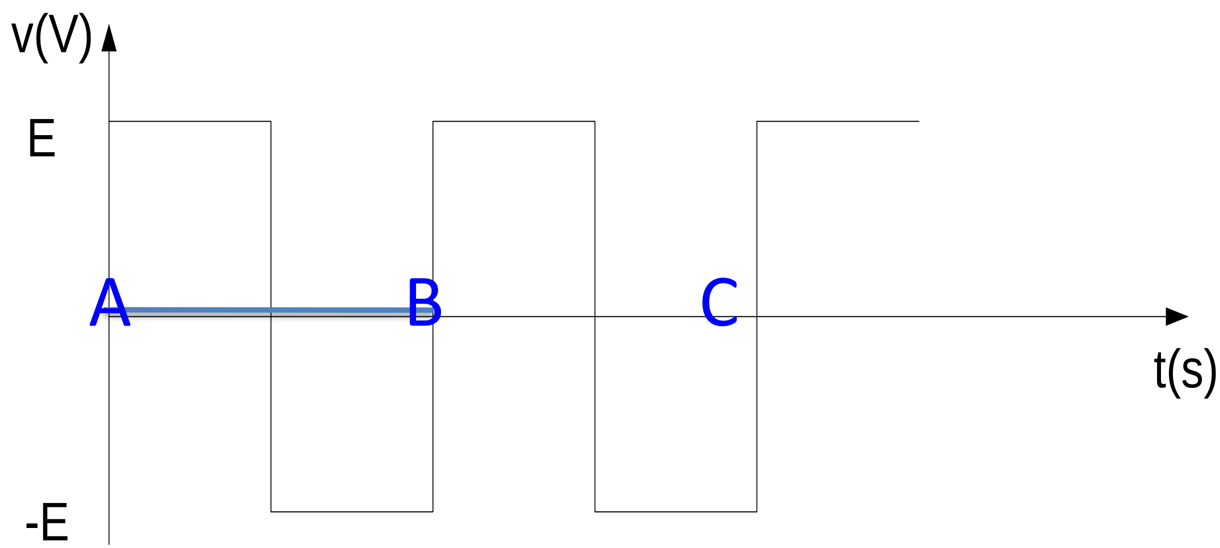


  - Triangular wave

TERMINOLOGIES IN AC CIRCUITS

- **Amplitude (peak):** The maximum deviation of the function from its center position
- **Cycle:** A repeating portion of a function (wave).
- **Period (T):** The duration of a cycle
- **Frequency(f):** The inverse of period.

$$f = \frac{1}{T}$$



AVERAGE VALUE

- **Average value:** The average value of a periodic function is its dc value.

If $$i = f(t)$$

Then $$I_{av} = \frac{1}{T}\int_0^T f(t)dt = \frac{area[f(t)]}{T}$$

AVERAGE VALUE

The following steps are followed when finding average values of waveforms:

1. **Identify a cycle of the wave**

2. **Note the period**

3. **Find the area of the cycle**

4. **Divide the area by the period**

ROOT MEAN SQUARE VALUE

The Root Mean Square (RMS) or Effective value of an alternating quantity is the value of a direct current which when flowing through a given resistance for a given time produces the same heat as produced by the alternating current when flowing through the same resistance.

The RMS value of an alternating current $i = f(t)$ is:

$$I_{rms} = \left\{ \frac{1}{T} \int_0^T [f(t)]^2 dt \right\}^{1/2} = \sqrt{\frac{area[(f(t))^2]}{T}}$$

ROOT MEAN SQUARE VALUE

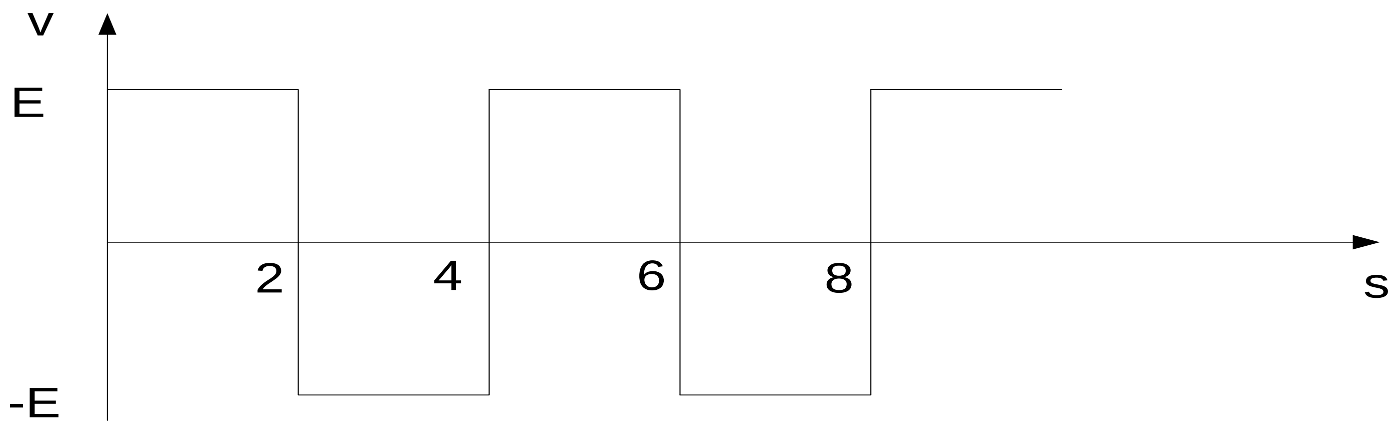
The following steps are taken when finding the RMS value of a waveform :

1. **Identify a cycle of the waveform**
2. **Note the period**
3. **Square the cycle**
4. **Find the area under the squared cycle**
5. **Divide the area by the period**
6. **Take the square root of the result**

ROOT MEAN SQUARE VALUE

Example 1

Find the average and rms values of the waveform below.



Solution

Average Value

- Cycle spans from 0 to 4
- Period= 4s
- Area of cycle $=(2\times E)+(2\times -E)$

ROOT MEAN SQUARE VALUE

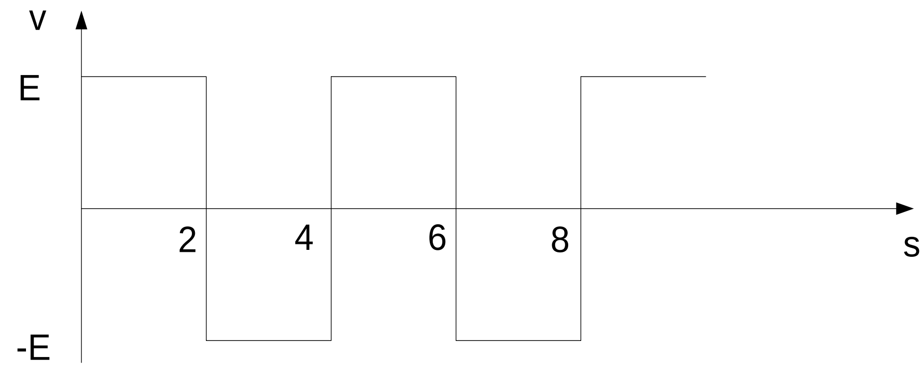
$$= 0$$

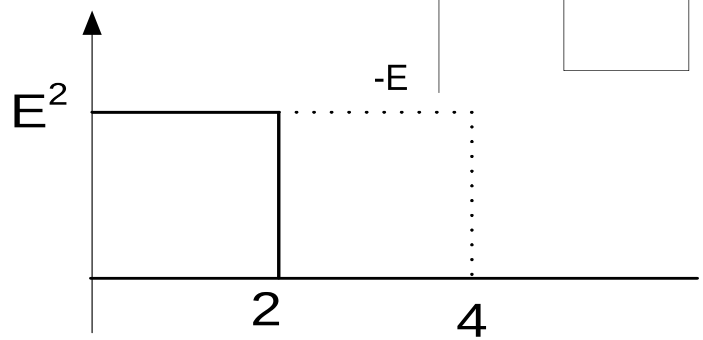
$$\therefore V_{avg} = \frac{Area}{period} = \frac{0}{4} = 0V$$

ROOT MEAN SQUARE VALUE

RMS value

- ❑Cycle spans 0 to 4
- ❑Period = 4s
- ❑Squared cycle





❑Area covered by squared cycle $= 4 \times E^2 = 4E^2$

❑Division of area by period $= \dfrac{4E^2}{4} = E^2$

ROOT MEAN SQUARE VALUE

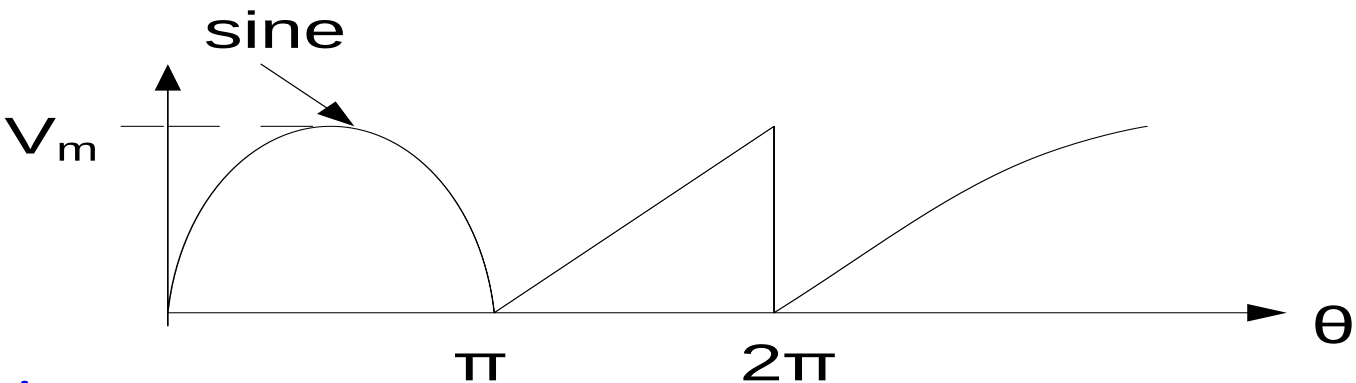
- Taking square root

$$V_{rms} = \sqrt{E^2} = E$$

ROOT MEAN SQUARE VALUE

Example 2

Calculate the effective value of the voltage below.



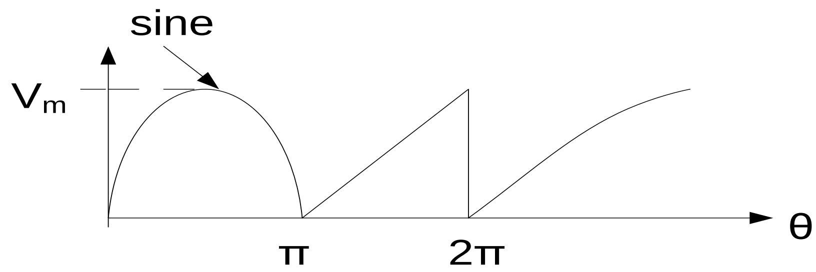
Solution

- Cycle spans from 0 to 2π
- Area of sine part

$$A_s = \int_0^{\pi} V_m{}^2 \sin^2 \theta d\theta = \int_0^{\pi} \frac{V_m{}^2 (1-\cos 2\theta)}{2} d\theta$$

$$= \frac{V_m{}^2}{2}\pi$$

ROOT MEAN SQUARE VALUE



Area of triangular part

$$A_t = \int_0^{\pi} \left(\frac{V_m}{\pi} \right)^2 \theta^2 d\theta = \left[\frac{V_m{}^2}{\pi^2} \cdot \frac{\theta^3}{3} \right]_0^{\pi} = \frac{V_m^2 \pi}{3}$$

Total area: $= \dfrac{V_m^2 \pi}{2} + \dfrac{V_m^2 \pi}{3} = \dfrac{5}{6} V_m^2 \pi$

Mean $= \dfrac{\frac{5}{6} V_m{}^2 \pi}{2\pi} = \dfrac{5}{12} V_m^2$

ROOT MEAN SQUARE VALUE

RMS value $$= \sqrt{\frac{5}{12} V_m^2} = V_m \sqrt{\frac{5}{12}}$$

Note:

The area of a squared right-angled triangular wave is

$$= \frac{bh^2}{3}$$

SINUSOIDAL VOLTAGES AND CURRENT

Voltages and currents of commercial ac generators have the following expressions:

$$v = V_m \sin \omega t \quad \text{or} \quad v = V_m \sin 2\pi f t$$

V_m is the peak voltage

f is the frequency in Hz

ω is the angular frequency in radian per second. It specifies how many oscillations occur in a unit time interval

SINUSOIDAL VOLTAGES AND CURRENT

$$i = I_m \sin \omega t$$ or $$i = I_m \sin 2\pi f t$$

I_m is the peak current

f is the frequency in Hz

ω Is the angular frequency in radian per second. It specifies how many oscillations occur in a unit time interval

RMS VALUE OF SINUSOIDAL QUANTITIES

The RMS value of a sinusoidal voltage is given by:

$$v = V_m \sin 2\pi f t$$

$$V = \left[\frac{1}{T}\int_0^T V_m^2 \sin^2 2\pi f t dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T \frac{V_m^2}{2}(1 - \cos 4\pi f t) dt\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2}\frac{T}{T}\right]^{1/2} \boxed{= \frac{V_m}{\sqrt{2}}}$$

RMS VALUE OF SINUSOIDAL QUANTITIES

Similarly,

The RMS value of a sinusoidal current is given by:

$$i = I_m \sin 2\pi f t$$

$$\boxed{I = \frac{I_m}{\sqrt{2}}}$$

RMS VALUE OF SINUSOIDAL QUANTITIES

Example 1

Find the rms values of the following quantities:

(a) $$i = 10\sqrt{2}\sin 100\pi t$$

(b) $$v = 20\sin 100\pi t$$

Solution

(a) $$I = \frac{10\sqrt{2}}{\sqrt{2}} = 10$$

(b) $$V = \frac{20}{\sqrt{2}} = 14.14$$

HARMONICS

Non-sinusoidal periodic voltages and currents can be expressed as the sum of sine waves in which the lowest frequency is f and all other frequencies are integral multiples of f.

For example, a square wave $v(t)$ of amplitude E can be expressed as:

$$v(t) = \frac{4E}{\pi}\left[\sin 2\pi ft + \frac{1}{3}\sin 6\pi ft + \frac{1}{5}\sin 10\pi ft + ---\right]$$

HARMONICS

- Any quantity which contains multiple frequencies is a harmonic quantity.
- The frequency of which others have been expressed as multiples of is the fundamental frequency.
- An odd multiple of the fundamental is an odd harmonic.
- An even multiple of the fundamental is an even harmonic.

RMS VALUE OF A HARMONIC QUANTITY

The effective value of a harmonic quantity is obtained by:

- First obtaining the square of the rms value of each term
- Adding the obtained squared rms values
- Taking the square root of the sum

$$v(t) = a_o + a_1 \sin(\omega t + \phi_1) + a_2 \sin(2\omega t + \phi_2) + a_3 \sin(3\omega t + \phi_3) + \ldots\ldots$$

$$V = \sqrt{a_o{}^2 + \left(\frac{a_1}{\sqrt{2}}\right)^2 + \left(\frac{a_2}{\sqrt{2}}\right)^2 + \left(\frac{a_3}{\sqrt{2}}\right)^2 + \ldots\ldots\ldots}$$

RMS VALUE OF A HARMONIC QUANTITY

Example

Find the RMS value of the current

$$i(t) = 2 + 5\sin wt + 3\sqrt{2}\sin\left(3wt + 30^0\right)$$

Solution

$$I = \sqrt{2^2 + \left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{3\sqrt{2}}{\sqrt{2}}\right)^2}$$

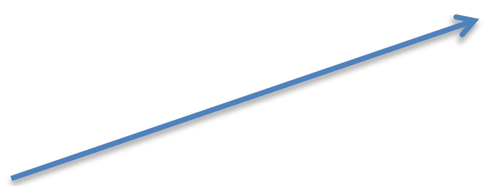
$$= \mathbf{5.05}$$

PHASORS

- Phasors are used to represent sinusoidal quantities to avoid drawing the sine waves.

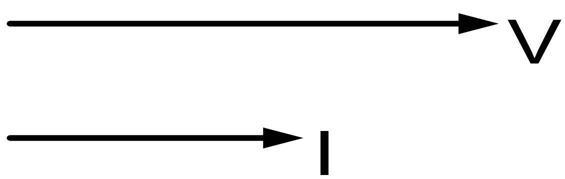
- A phasor is a straight line whose length is proportional to the rms voltage or current it represents.

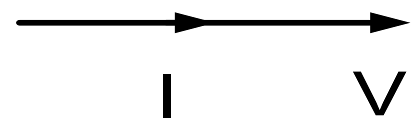
- To show the phase angle or phase displacement between voltages and currents, the phasors bear an arrow.



PHASORS

❖ Two phasors are said to be in phase when they point in the same direction. The phase angle between them is then zero.

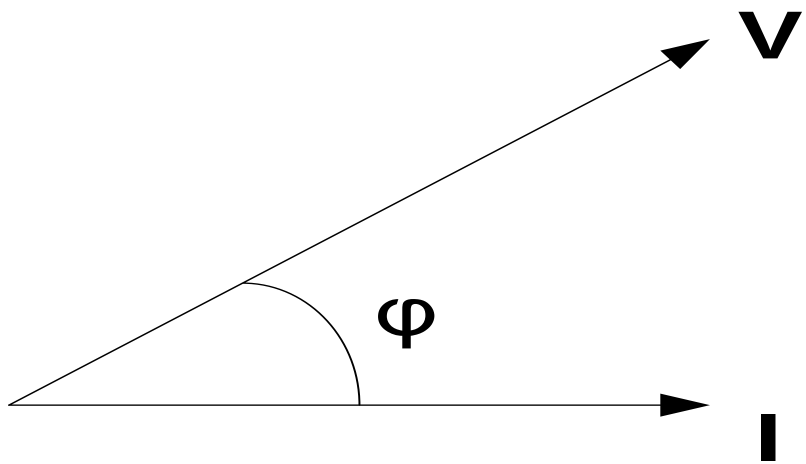




PHASORS

- Two phasors are said to be out of phase when they point in different directions.

- The phase angle between them is the angle through which one of them has to be rotated to make it point in the same direction as the other.



PHASOR DIAGRAMS

- It is used to show at a glance the magnitude and phase relations among the various quantities within a network. This is often helpful in the analysis of the network.

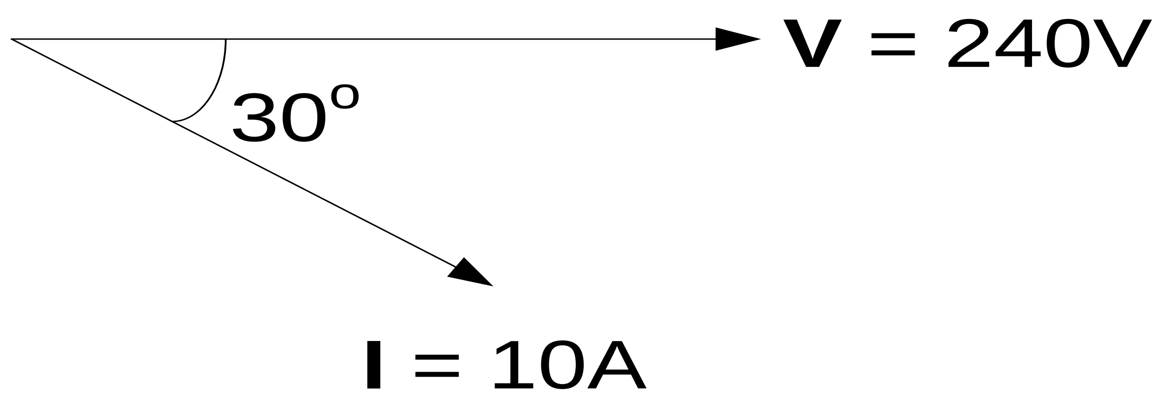
- Example

  A 50 Hz source having rms voltage of 240 V delivers a rms current of 10 A to a circuit. The current lags the voltage by 30°. (a) Draw the phasor diagram for the circuit. (b) Express the voltage and current as functions of time.

PHASOR DIAGRAMS

❖ **Solution**

(a) Take V as the reference



(b)

$$v(t) = 240\sqrt{2}\sin 100\pi t$$

$$i(t) = 10\sqrt{2}\sin(100\pi t - 30^\circ)$$

ADDITION AND SUBTRACTION OF SINUSOIDAL QUANTITIES

- The sum of sinusoidal quantities is obtained by taking the vector sum of their phasors.
- The difference of sinusoidal quantities is obtained by first reversing the subtracted quantity and adding it as a vector to the other phasors.
- A sinusoidal quantity is reversed by adding 180^0 to its angle
- Only sinusoidal quantities of the SAME FREQUENCY can be added or subtracted.

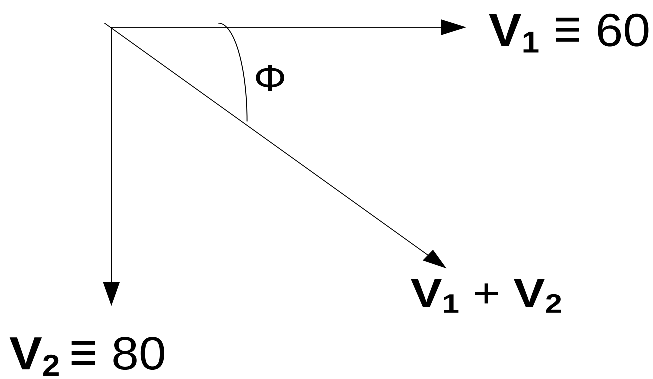
ADDITION AND SUBTRACTION OF SINUSOIDAL QUANTITIES

- **Example 1**

Let $v_1(t) = 60 \sin \omega t$ and $v_2(t) = 80 \sin(\omega t - 90°)$. Determine (a) $v_1 + v_2$ and (b) $v_1 - v_2$

- **Solution**

(a) Phasor diagram



$$\left|V_1 + V_2\right| = \sqrt{60^2 + 80^2} = 100$$

$$\phi = \tan^{-1}\left(\frac{80}{60}\right) = 53^0$$

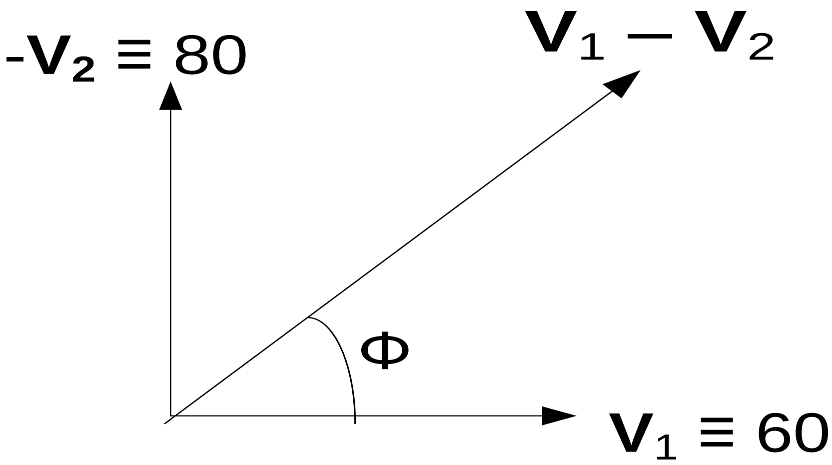
ADDITION AND SUBTRACTION OF SINUSOIDAL QUANTITIES

$$\therefore v_1 + v_2 = 100\left(\sin \omega t - 53^0\right)$$

(b) $$v_1 - v_2 = v_1 + \left(-v_2\right)$$
$$= 60\sin \omega t + 80\sin\left(\omega t - 90^0 + 180^0\right)$$
$$= 60\sin \omega t + 80\sin\left(\omega t + 90^0\right)$$

Phasor diagram

ADDITION AND SUBTRACTION OF SINUSOIDAL QUANTITIES



$$\left|V_1 - V_2\right| = \sqrt{60^2 + 80^2} = 100$$

$$\phi = \tan^{-1}\left(\frac{80}{60}\right) = 53^0$$

$$\therefore v_1 - v_2 = 100\, sin\left(\omega t + 53^0\right)$$

ADDITION AND SUBTRACTION OF SINUSOIDAL QUANTITIES

❖ Example 2

Four conductors meet at a junction. The following relationship exists between them. $i_4 = i_1 + i_2 + i_3$

Find the value of i_4 given that

$$i_1 = 5\sin\omega t$$

$$i_2 = 8\sin\left(\omega t + \tfrac{\pi}{3}\right) + 5\sin 3\omega t$$

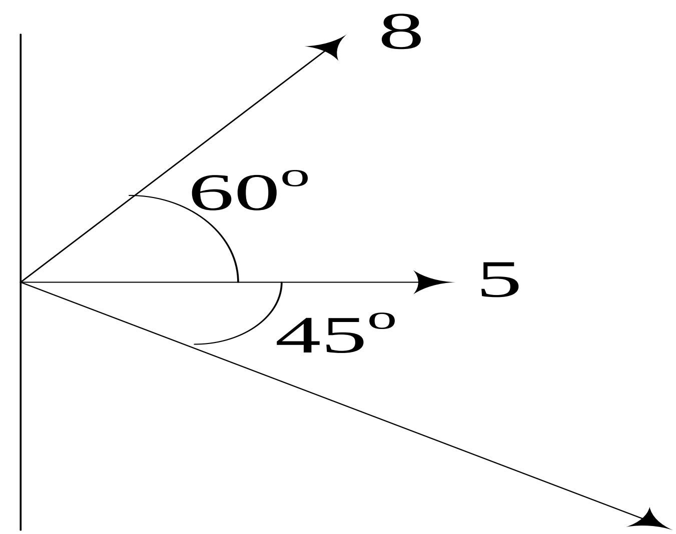
$$i_3 = 15\sin\left(\omega t - \tfrac{\pi}{4}\right) + 8\sin\left(3\omega t + \tfrac{\pi}{3}\right)$$

ADDITION AND SUBTRACTION OF SINUSOIDAL QUANTITIES

❖ Solution

(a) There are two different frequencies. They must therefore be added separately.

<u>Addition of ω part.</u>



X-component

$$= 8\cos 60^{o} + 5 + 15\cos 45^{0}$$

$$= 19.607$$

Y-component

$$= 8\sin 60^{o} + 0 - 15\sin 45^{0}$$

$$= -3.678$$

ADDITION AND SUBTRACTION OF SINUSOIDAL QUANTITIES

Amplitude

$$= \sqrt{(19.607)^2 + (-3.678)^2}$$

$$= 19.949$$

Angle

$$= \tan^{-1}\left(\frac{-3.678}{19.607}\right) = -10.62^0$$

Therefore, ω **part of** i_4 **is**

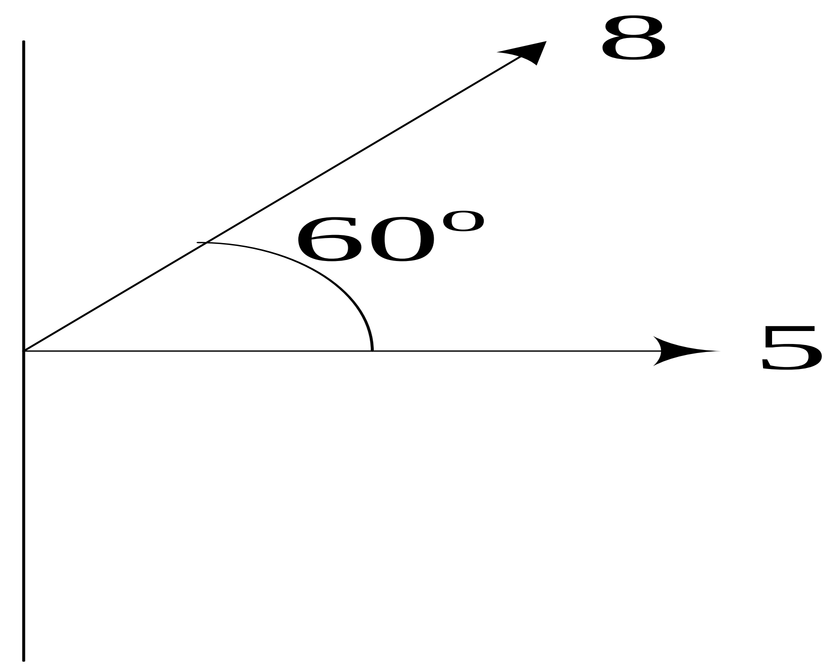
$$19.949 \sin\left(\omega t - 10.62^0\right)$$

ADDITION AND SUBTRACTION OF SINUSOIDAL QUANTITIES

Addition of 3ω part.

X-component

$$= 8\cos 60^0 + 5$$
$$= 9$$



Y-component

$$= 8\sin 60^0 + 0$$
$$= 6.928$$

Amplitude

$$= \sqrt{9^2 + 6.928^2}$$
$$= 11.358$$

ADDITION AND SUBTRACTION OF SINUSOIDAL QUANTITIES

Angle

$$= \tan^{-1}\left(\frac{6.928}{9}\right) = 37.59^0$$

Therefore, 3ω part of i_4 is

$$11.358\sin\left(3\omega t + 37.59^0\right)$$

Hence $$i_4 = 19.949\sin\left(\omega t - 10.62^0\right) + 11.358\sin\left(3\omega t + 37.59^0\right)$$

GROUP ASSIGNMENT 5

Four circuit elements are connected in series across a sinusoidal alternating voltage given by $e = 110\sin\left(\omega t + 30^0\right)$. The instantaneous voltage across three of the elements are given by

$$v_1 = 30\sin\omega t,\ v_2 = 60\sin\left(\omega t + 60^0\right) \text{and } v_3 = 30\sin\left(\omega t - 30^0\right)$$

(a) Determine the expression for the fourth voltage in the form

$$v_4 = A\sin(\omega t + \beta)$$

(b) What is the r.m.s. value of v_4?

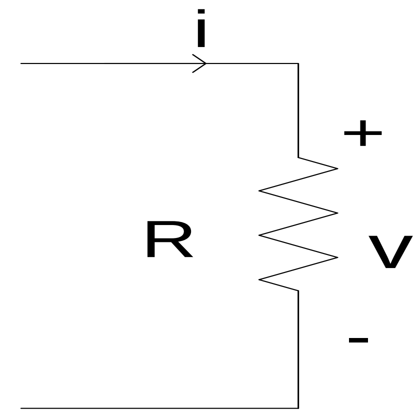
Use phasor diagrams. DO NOT use complex number approach

IMPEDANCE (Z)

- The opposition to current flow in ac circuits owing to the presence of combinations of resistive, inductive and capacitive elements.
- Opposition due to inductance (L) is called inductive reactance(X_L).
- Opposition due to capacitance is called capacitive reactance(X_C).

IMPEDANCE (Z)

❖ Phase relationship between the current and voltage in a resistor



$$i = \frac{v}{R}$$

Let $v = V_m \sin \omega t$

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$$

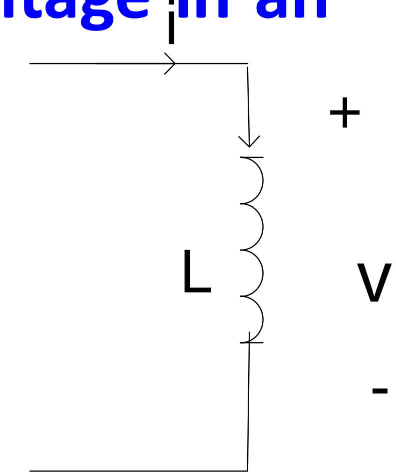
It is noted that the voltage across and the current through a resistor are in phase.

IMPEDANCE (Z)

- Phase relationship between the current and voltage in an inductor

$$v = L\frac{di}{dt}$$

$$V_L = I_L X_L$$



Let $i = I_m \sin\omega t$

$$v = L \times \omega I_m \cos\omega t = \omega L I_m \sin\left(\omega t + 90^0\right)$$

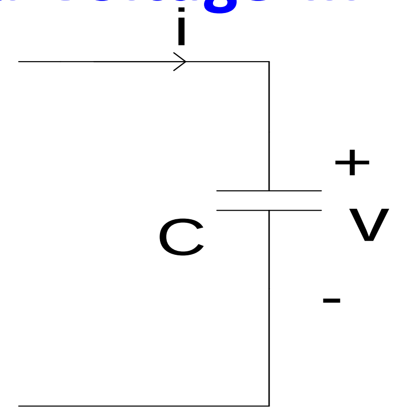
$$= V_m \sin\left(\omega t + 90^0\right)$$

It is noted that the current through an inductor lags the voltage by 90^0.

$$X_L = \omega L$$

IMPEDANCE (Z)

- Phase relationship between the current and voltage in a capacitor



$$i = c\frac{dv}{dt} \qquad V_C = I_C X_C$$

Let $v = V_m \sin \omega t$

$$i = C \times \omega V_m \cos \omega t = \omega C V_m \sin\left(\omega t + 90^0\right)$$

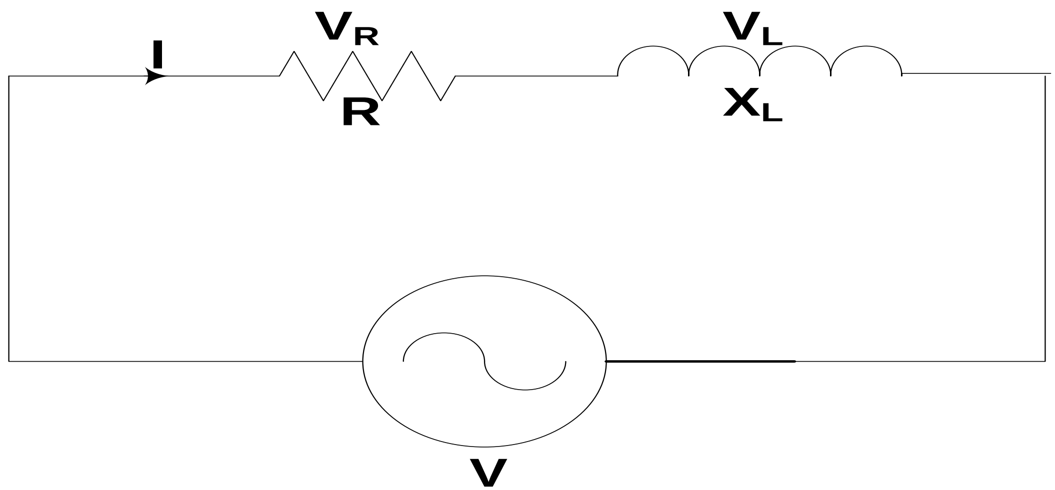
$$= I_m\left(\sin \omega t + 90^0\right)$$

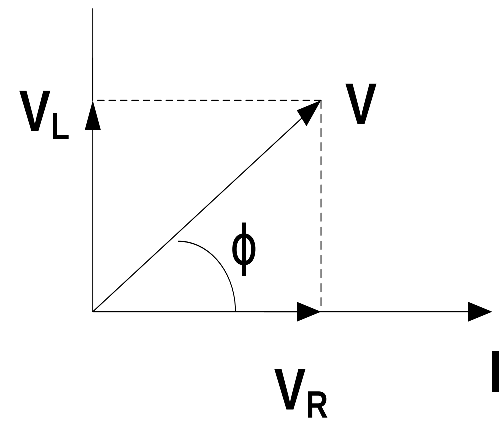
It is noted that the current through a capacitor leads the voltage by 90^0.

$$X_c = {1}/{\omega c}$$

IMPEDANCE (Z)

❖ Series circuit containing R and L





$$\overline{V} = \overline{V_R} + \overline{V_L}$$

$$V^2 = V_R^{\,2} + V_L^{\,2}$$

$$(IZ)^2 = (IR)^2 + (IX_L)^2$$

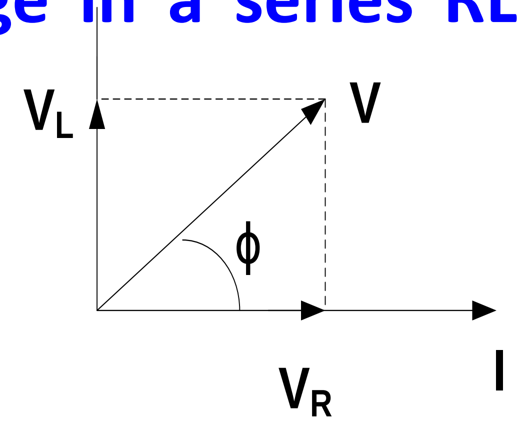
$$Z^2 = R^2 + X_L^{\,2}$$

$$Z = \sqrt{R^2 + X_L^{\,2}}$$

IMPEDANCE (Z)

- Phase angle between current and voltage in a series RL circuit

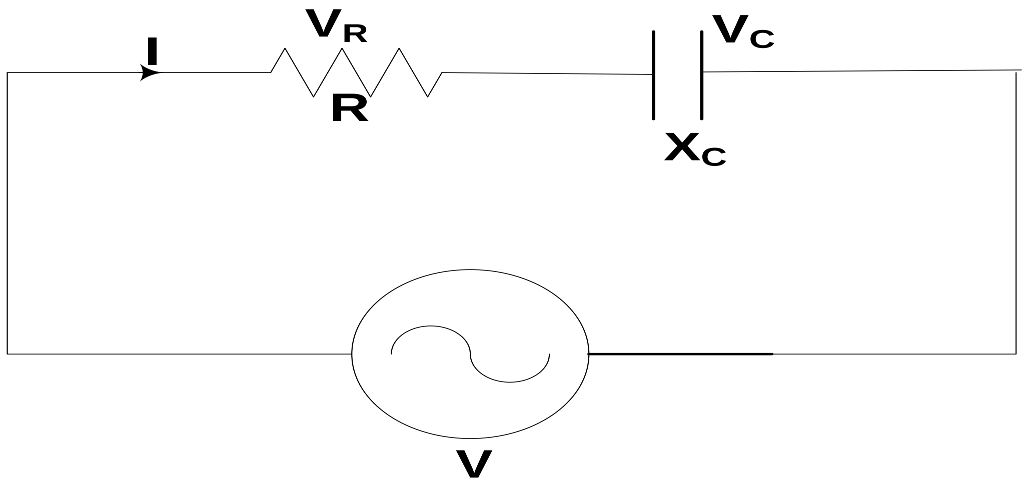
$$\phi = tan^{-1}\left(\frac{X_L}{R}\right)$$

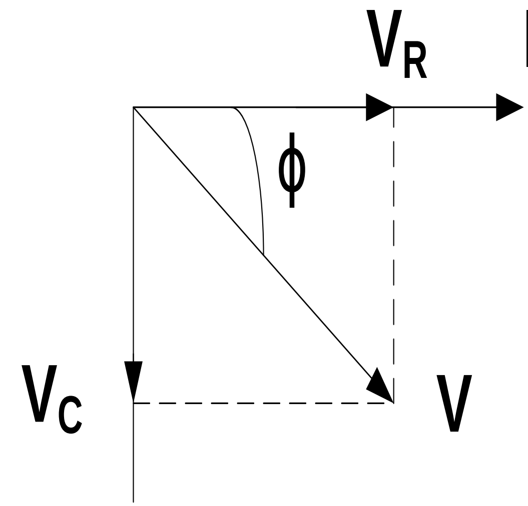


- The current in a series RL circuit lags the voltage but not by 90^0

IMPEDANCE (Z)

❖ Series circuit containing R and C





$$\overline{V} = \overline{V_R} + \overline{V_C}$$

$$V^2 = V_R^{\,2} + V_C^{\,2}$$

$$(IZ)^2 = (IR)^2 + (IX_C)^2$$

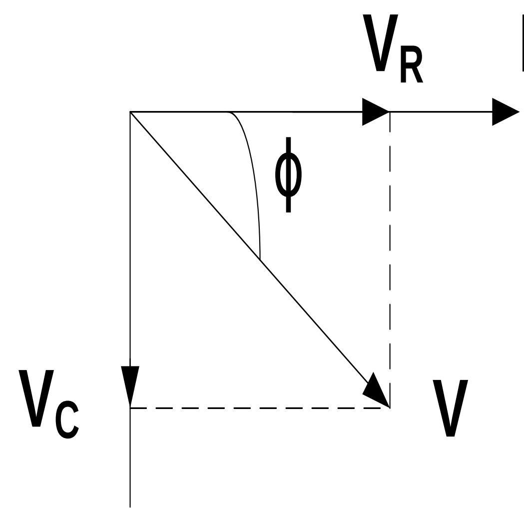
$$Z^2 = R^2 + X_C^{\,2}$$

$$Z = \sqrt{R^2 + X_C^{\,2}}$$

IMPEDANCE (Z)

- Phase angle between current and voltage in a series RC circuit

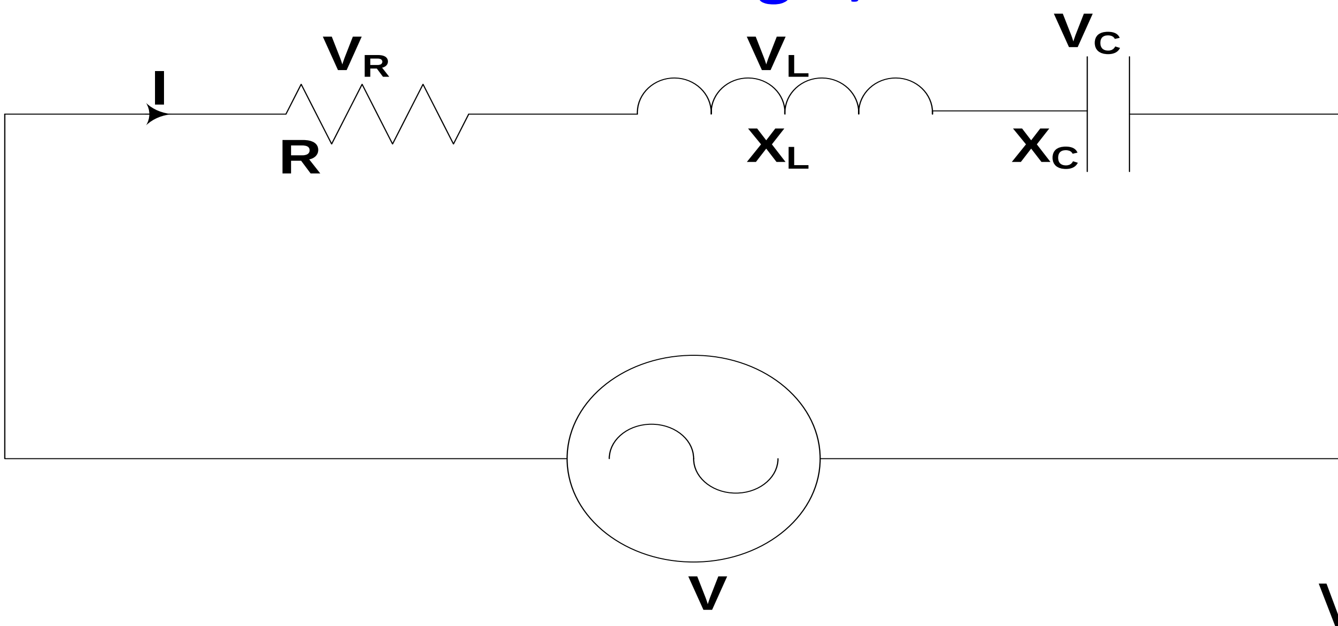
$$\phi = tan^{-1}\left(\frac{X_C}{R}\right)$$

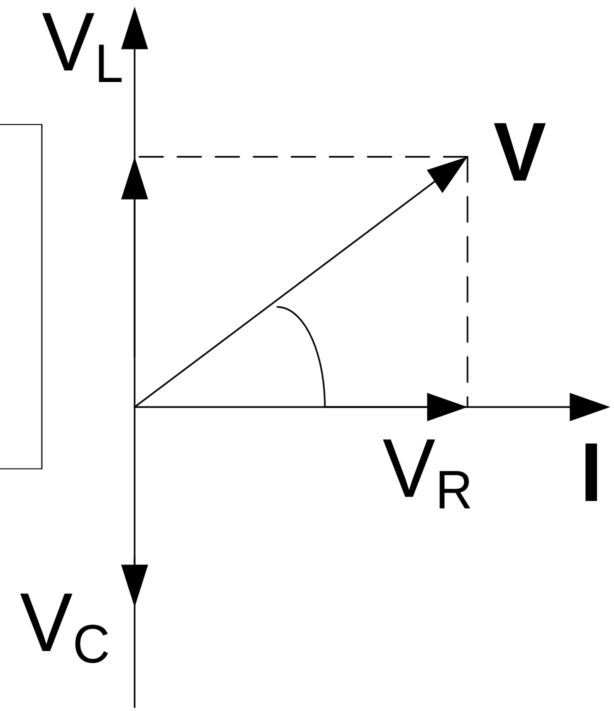


- The current in a series RC circuit leads the voltage but not by 90^0

IMPEDANCE (Z)

❖ Series circuit containing R, L and C





$$\overline{V} = \overline{V}_R + \overline{V}_L + \overline{V}_c$$

$$V^2 = V_R^{\,2} + \left(V_L - V_C\right)^2$$

IMPEDANCE (Z)

$$V^2 = V_R^{\,2} + (V_L - V_C)^2$$

$$= (IR)^2 + (IX_L - IX_c)^2$$

$$V = \sqrt{(IR)^2 + (IX_L - IX_c)^2}$$

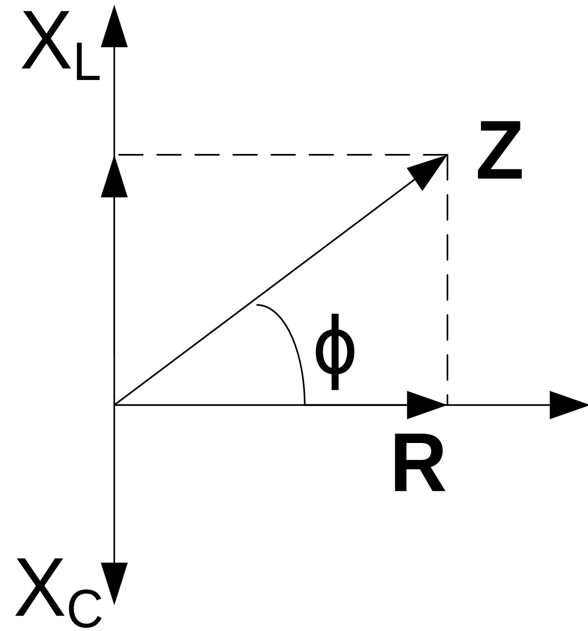
$$IZ = I\sqrt{R^2 + (X_L - X_c)^2}$$

$$\Rightarrow Z = \sqrt{R^2 + (X_L - X_c)^2}$$

IMPEDANCE (Z)

- Phase angle between current and voltage in a series RLC circuit

$$\phi = tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

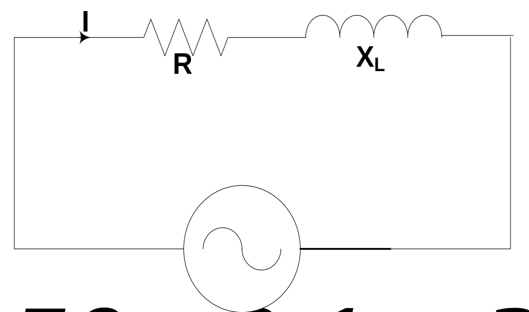


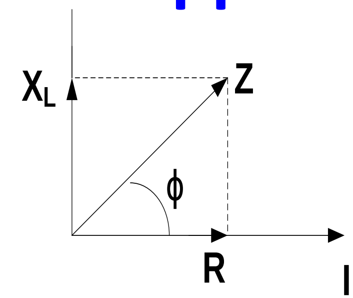
- Current in a series RLC circuit may lead or lag the voltage depending on the relative values of X_L and X_C

IMPEDANCE (Z)

❖ **Example 1**

A coil has R=12Ω and L=0.1H. It is connected across a 100V, 50Hz supply. Calculate (a) the reactance and impedance of the coil (b) the current and (c) the phase difference or angle between the current and the applied voltage.





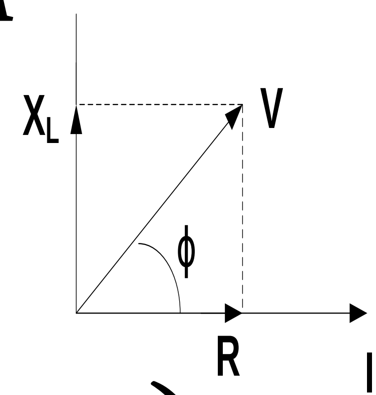
❖ **Solution**

(a) $$X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.416\,\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + 31.416^2}$$
$$= 33.630\,\Omega$$

IMPEDANCE (Z)

(b) $$I = \frac{V}{Z} = \frac{100}{33.630} = 2.974\,A$$



(c) $$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{31.416}{12}\right)$$

$$= 69.09^0$$

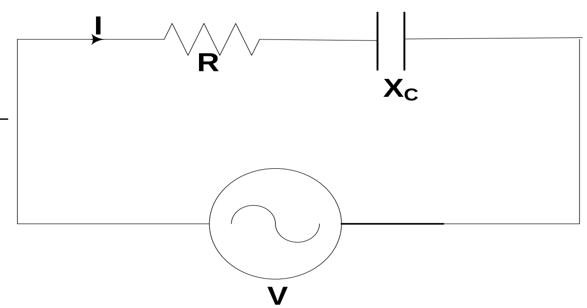
IMPEDANCE (Z)

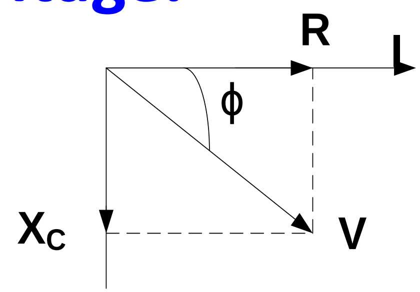
❖ Example 2

A metal filament lamp, rated at 750W, 100V is to be connected in series with a capacitor across a 230V, 50Hz supply. Calculate (a) the capacitance required and (b) the phase angle between the current and supply voltage.

❖ Solution

$$X_C = \frac{V_C}{I_C}$$





(a) $V^2 = V_R^2 + V_C^2$

$$V_C = \sqrt{V^2 - V_R^2} = \sqrt{230^2 - 100^2}$$

$$= 207.123V$$

IMPEDANCE (Z)

(a) $$I_R = I_C = I = \frac{P_R}{V_R} = \frac{750}{100} = 7.5\,A$$

$$\therefore X_C = \frac{V_C}{I_C} = \frac{207.123}{7.5} = 27.616\,\Omega$$

Hence, $$c = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 27.616}$$

$$= 115\,\mu F$$

IMPEDANCE (Z)

(b) $$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{V_C}{V_R}\right)$$

$$= \tan^{-1}\left(\frac{207.123}{100}\right)$$

$$= 64.23^0$$

POWER IN AC CIRCUITS

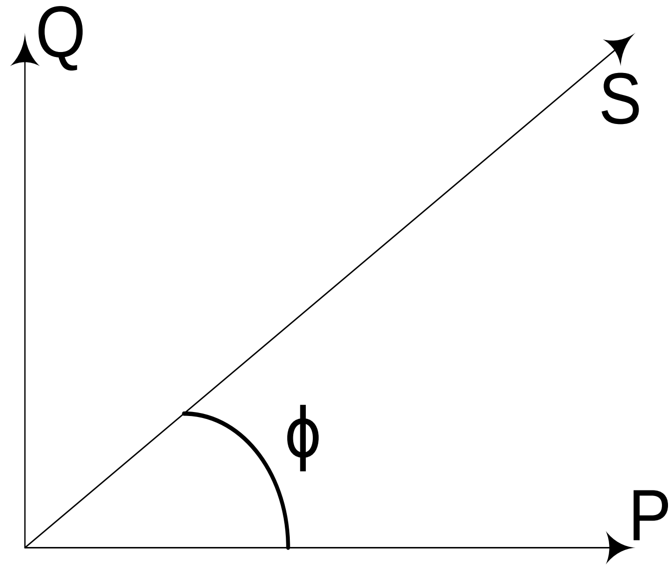
❖ There are three kinds of power in ac circuits

1. Apparent Power (S) which is measured in Volt-amperes (VA)
2. Active Power (P) which is measured in Watts (W).

   Active Power is also called Actual Power, Useful Power, True Power, Real Power or simply, Power
3. Reactive Power (Q) which is measured in Volt-amperes reactive (VAR)

POWER IN AC CIRCUITS

❖ The following relationships exist between S, P and Q



$$S = VI$$

$$P = VI\cos\phi$$

$$Q = VI\sin\phi$$

$$S^2 = P^2 + Q^2$$

$$P = S\cos\phi$$

$$Q = S\sin\phi$$

$\cos\phi$ is called power factor (*pf*)

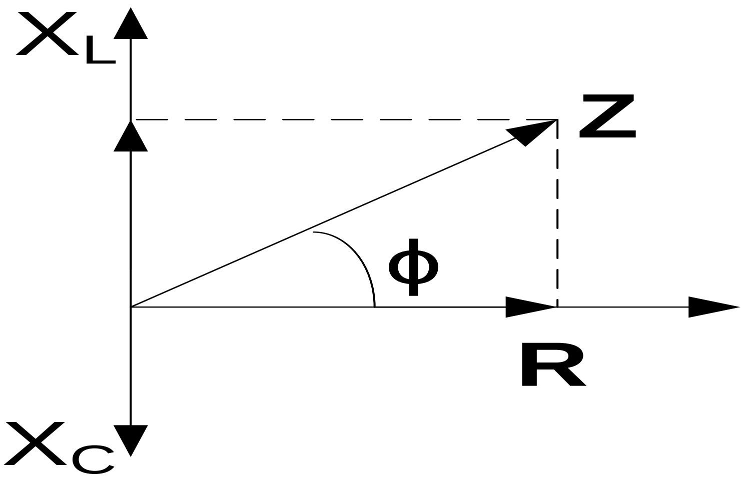
$$pf = \frac{P}{S}$$

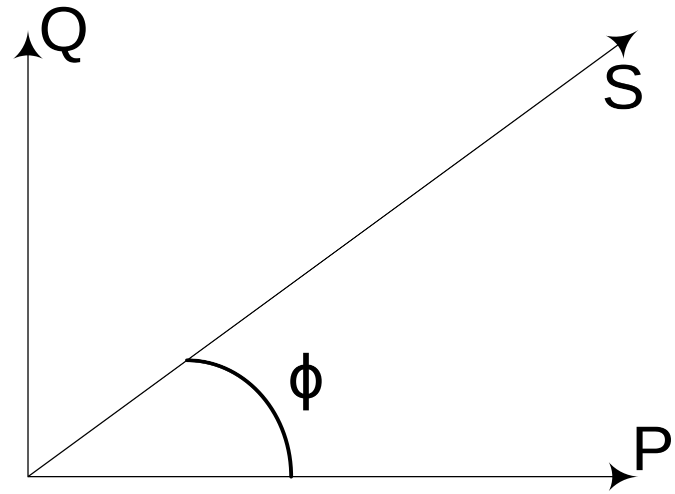
POWER IN AC CIRCUITS

- Power factor may be said to be lagging or leading.
- Power factor is lagging when current lags voltage
- Power factor is leading when current leads voltage

POWER IN AC CIRCUITS

❖ **Relationships between the three passive elements , P and Q.**





1. **Resistors consume only P**
2. **Inductors consume only Q**
3. **Capacitors do not consume P and Q. They rather supply Q or reduce the consumption of Q.**

POWER IN AC CIRCUITS

❖ **Example 1**

A single-phase motor connected to a 400-V, 50-Hz supply is developing 10 kW with efficiency of 84 per cent and a power factor of 0.7 lagging. Calculate (a) the input kVA (b) the active and reactive components of the current and (c) the reactive kVA.

❖ **Solution**

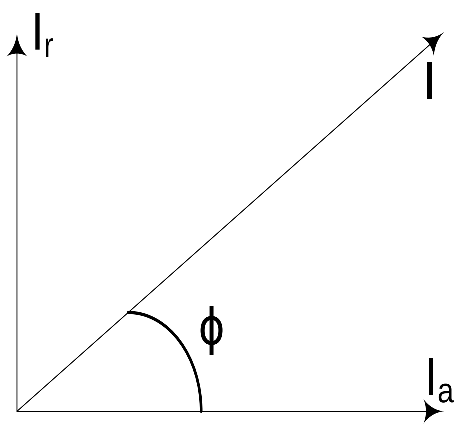
(a) $$P_{in} = \frac{P_{out}}{\eta} = \frac{10}{0.84} = 11.905kW$$

POWER IN AC CIRCUITS

$$S = \frac{P_{in}}{pf} = \frac{11.905}{0.7} = 17.007\,kVA$$

(b) $$S = VI \Rightarrow I = \frac{S}{V} = \frac{17.007 \times 10^3}{400}$$

$$= 42.518\,A$$



$$I_a = I\cos\phi = 42.518 \times 0.7$$

$$= 29.766\,A$$

$$I_r = \sqrt{I^2 - I_a{}^2}$$

$$= 30.361\,A$$

POWER IN AC CIRCUITS

(b) $$Q = VI\sin\phi = VI_r \quad = 400 \times 30.361$$
$$= 12.144kVAR$$

POWER IN AC CIRCUITS

❖ **Example 2**

An emf whose instantaneous value is given by 283sin(314t + π/4)V is applied to an inductive circuit and the current in the circuit is 5.66sin(314t – π/6)A. Determine (a) the frequency of the emf (b) the R and L (c) the power absorbed.

❖ **Solution**

(a)

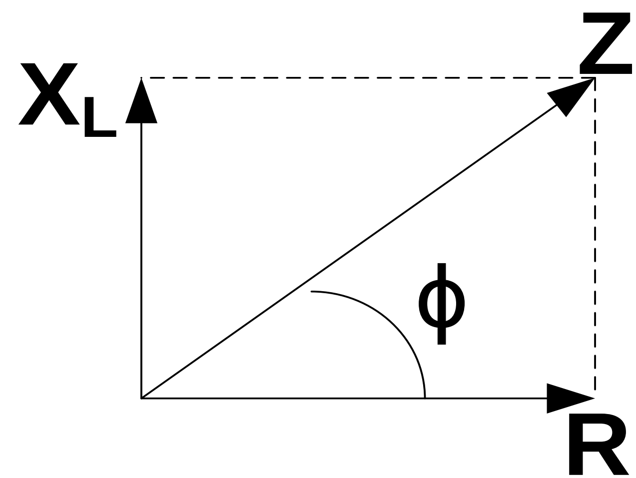
$$2\pi f = 314 \Rightarrow f = \frac{314}{2\pi} = 50\,Hz$$

(b)

$$Z = \frac{V}{I} = \frac{283}{\sqrt{2}} \div \frac{5.66}{\sqrt{2}} = 50\,\Omega$$

POWER IN AC CIRCUITS

(b) $$\phi = \frac{\pi}{4} - -\frac{\pi}{6} = \frac{10}{24}\pi = 75^0$$



$$R = Z\cos\phi = 50\cos 75^0$$
$$= 12.941\Omega$$

$$X_L = Z\sin\phi = 50\sin 75^0 = 48.296\Omega$$

$$X_L = 2\pi fL \Rightarrow L = \frac{X_L}{2\pi f} = \frac{48.296}{2\pi \times 50}$$

$$= 0.154\,H$$

POWER IN AC CIRCUITS

(c) $$P = VI\cos\phi = \frac{283}{\sqrt{2}} \times \frac{5.66}{\sqrt{2}} \cos 75^0$$

$$= 207.286W$$

USING COMPLEX NUMBERS TO SOLVE AC CIRCUIT PROBLEMS

- The ability to make a vector quantity appear as a scalar quantity using complex numbers is utilized in the analysis of ac circuits.
- All the mathematical manipulations in complex algebra hold when employing complex numbers in analyzing ac circuits.
- The operator ‘i’ is replaced with ‘j’ in other to avoid confusing it with current.

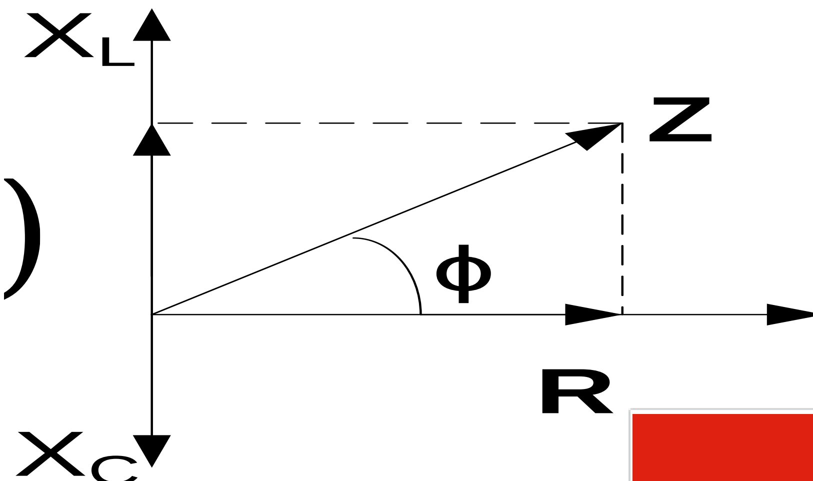
USING COMPLEX NUMBERS TO SOLVE AC CIRCUIT PROBLEMS

❖ The three passive elements are represented as follows:

1. R as a complex number is R
2. X_L as a complex number is jX_L
3. X_C as a complex number is $-jX_C$

4. Series impedance

$$Z = R + j\left(X_L - X_C\right)$$



USING COMPLEX NUMBERS TO SOLVE AC CIRCUIT PROBLEMS

- **Example 1**

Express in rectangular and polar notations, the impedance of each of the following circuits at a frequency of 50 Hz: (a) a resistance of 20 Ω (b) a resistance of 20 Ω in series with an inductance of 0.1 H (c) a resistance of 50 Ω in series with a capacitance of 40 μF.

- **Solution**

(a) $Z = 20 + j0 = 20\angle 0^0$

(b) $X_L = 2\pi fL = 2\pi \times 50 \times 0.1 = 31.416\Omega$

$$\therefore Z = 20 + j31.416$$
$$= 37.242\angle 57.52^0$$

USING COMPLEX NUMBERS TO SOLVE AC CIRCUIT PROBLEMS

(c) $$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 40 \times 10^{-6}}$$

$$= 79.577\,\Omega$$

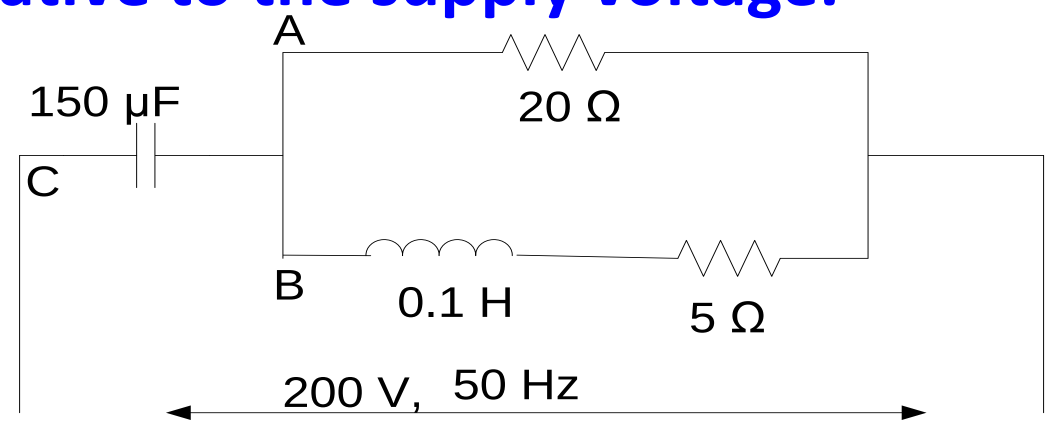
$$\therefore Z = 50 - j79.577$$

$$= 93.981\angle -57.86^0$$

USING COMPLEX NUMBERS TO SOLVE AC CIRCUIT PROBLEMS

❖Example 2

A circuit is arranged as indicated in the figure below, the values being as shown. Calculate the value of the current in each branch and its phase relative to the supply voltage.



❖Solution

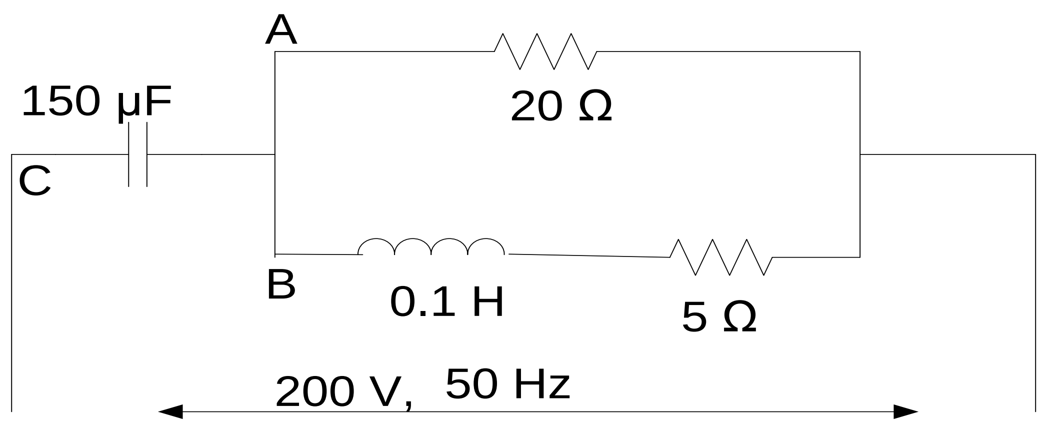
$$Z_A = 20 + j0$$

$$Z_B = R + jX_L = 5 + j31.4$$

$$Z_C = -jX_C = -j21.2\Omega$$

$$Z_{AB} = Z_A // Z_B = 15.84\angle 29.48^0$$

USING COMPLEX NUMBERS TO SOLVE AC CIRCUIT PROBLEMS



$$= 13.78 + j7.8$$

$$Z_T = Z_C + Z_{AB}$$

$$= -j21.2 + (13.78 + j7.8)$$

$$= 19.22\angle -44.2^0$$

Choosing the voltage as the reference phasor,

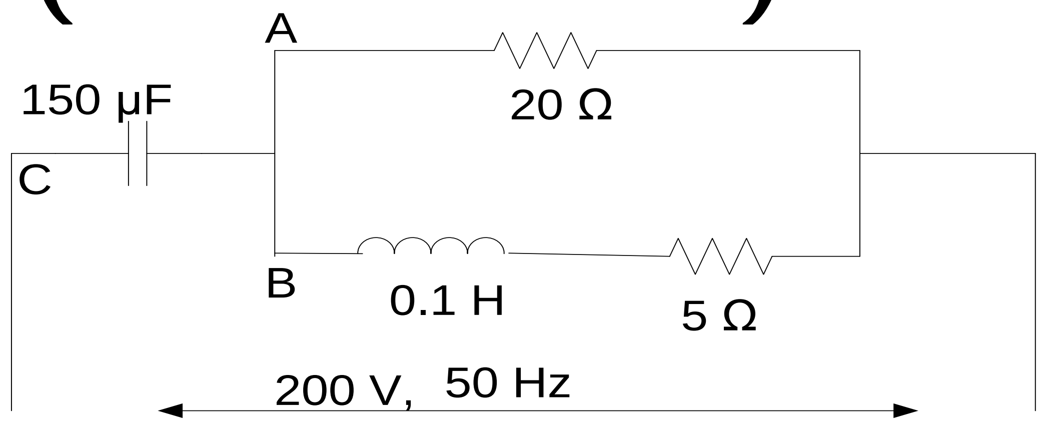
$$I_C = I_T = \frac{V}{Z_T} = \frac{200\angle 0^0}{19.22\angle -44.2^0}$$

$$= 10.4\angle 44.2^0$$

USING COMPLEX NUMBERS TO SOLVE AC CIRCUIT PROBLEMS

Current leads supply voltage by 44.2^0

$$V_{AB} = IZ_{AB}$$
$$= \left(10.4\angle 44.2^0\right)\times\left(15.84\angle 29.48^0\right)$$
$$= 164.8\angle 73.68^0$$



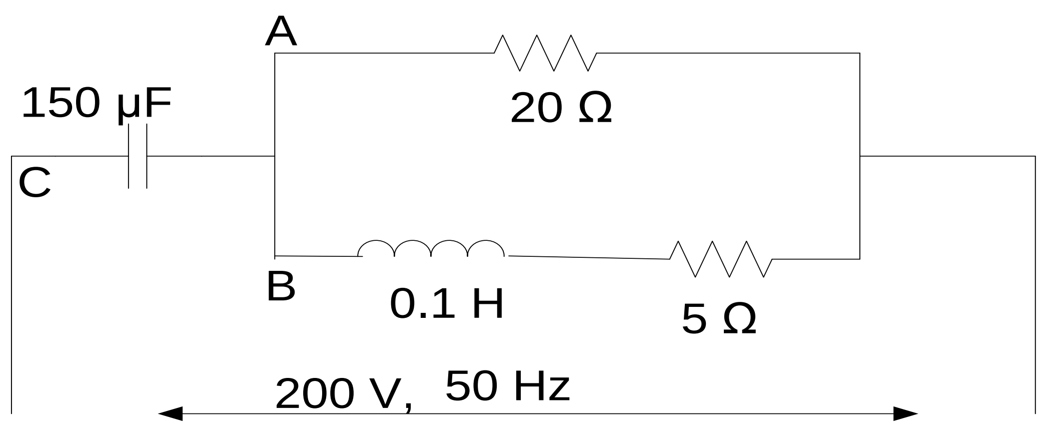
$$I_A = \frac{V_{AB}}{Z_A}$$
$$= \frac{164.8\angle 73.68^0}{20}$$
$$= 8.24\angle 73.68^0$$

Current leads supply voltage by 73.68^0

USING COMPLEX NUMBERS TO SOLVE AC CIRCUIT PROBLEMS

$$I_B = \frac{V_{AB}}{Z_B} = \frac{164.8\angle 73.68^0}{31.79\angle 80.95^0} = 5.18\angle -7.27^0$$

Current lags supply voltage by 7.27^0



USING COMPLEX NUMBERS TO SOLVE AC CIRCUIT PROBLEMS

$$\begin{aligned} S &= VI^{*} \\ &= P + jQ \end{aligned}$$

- Q is positive when the current lags the voltage
- Q is negative when the current leads the voltage

CALCULATION OF COMPLEX POWER

❖ Example 1

The potential difference across and the current in a circuit are represented by 100 + j200 v and 10 + j5 a respectively. Calculate the power and reactive voltamperes (or vars).

❖ Solution

$$S = VI^{*} = (100 + j200)(10 + j5)^{*}$$
$$= (100 + j200)(10 - j5)$$
$$= 2000 + j1500$$

$$P = 2000W \qquad Q = 1500VAR$$

CALCULATION OF COMPLEX POWER

- **Example 2**

A small installation consists of the following loads connected in parallel across a single-phase 240V, 50Hz supply:

(a) a fan motor taking an input of 1.5kVA at 0.75pf lag,

(b) a 1000W radiator operating at unity power factor

(c) a number of fluorescent lamps taking a total load of 1.2kVA at 0.95pf lagging

Find the total current, kW, kVA and power factor of the load.

CALCULATION OF COMPLEX POWER

❖ Solution

The problem is solved by obtaining the active and reactive component of each load and then summing all reactive components and all active components

Load (kVA)	cosϕ	P(kW) = Scosϕ	sinϕ	Q(kVAR) = Ssinϕ
(a) 1.5	0.75	1.125	0.66	0.99
(b) 1.0	1.0	1.0	0	0
(c) 1.2	0.95	1.14	0.312	0.374
TOTAL		3.265		1.364

CALCULATION OF COMPLEX POWER

$$\therefore \textit{Total kW} = 3.265$$

$$\textit{Total kVA} = \sqrt{P^2 + Q^2}$$

$$= \sqrt{3.265^2 + 1.364^2}$$

$$= 3.54$$

$$I = \frac{S}{V} = \frac{3.54 \times 10^3}{240} = 14.8\,A$$

$$pf = \frac{P}{S} = \frac{3.265}{3.54} = 0.923\ \textit{lagging}$$

CALCULATION OF COMPLEX POWER

❖ NOTE

In the problem above, if one load had a leading power factor, then its Q would have a negative sign and thus would have subtracted from the others.

Group Assignment 6

A small installation consists of the following loads connected in parallel across a single-phase 240V, 50Hz supply:

(a) a fan motor having an input of 2kVA at 0.75pf lag,

(b) a 1.5kW radiator operating at unity power factor,

(c) a 1kVA load operating 0.85 leading power factor.

Find the total current, kW, kVA and power factor of the load.

Submission date: God willing a week today

Submission time: Before lecture starts

Where to submit: Electrical Engineering office

THREE-PHASE CIRCUITS

- A single-phase generator produces a single sinusoidal voltage.
- A 3-phase generator on the other hand produces three equal voltages which are out of phase with one another by 120^{o}.
- The three voltages are generated in three separate windings arranged in a special way in the machine.

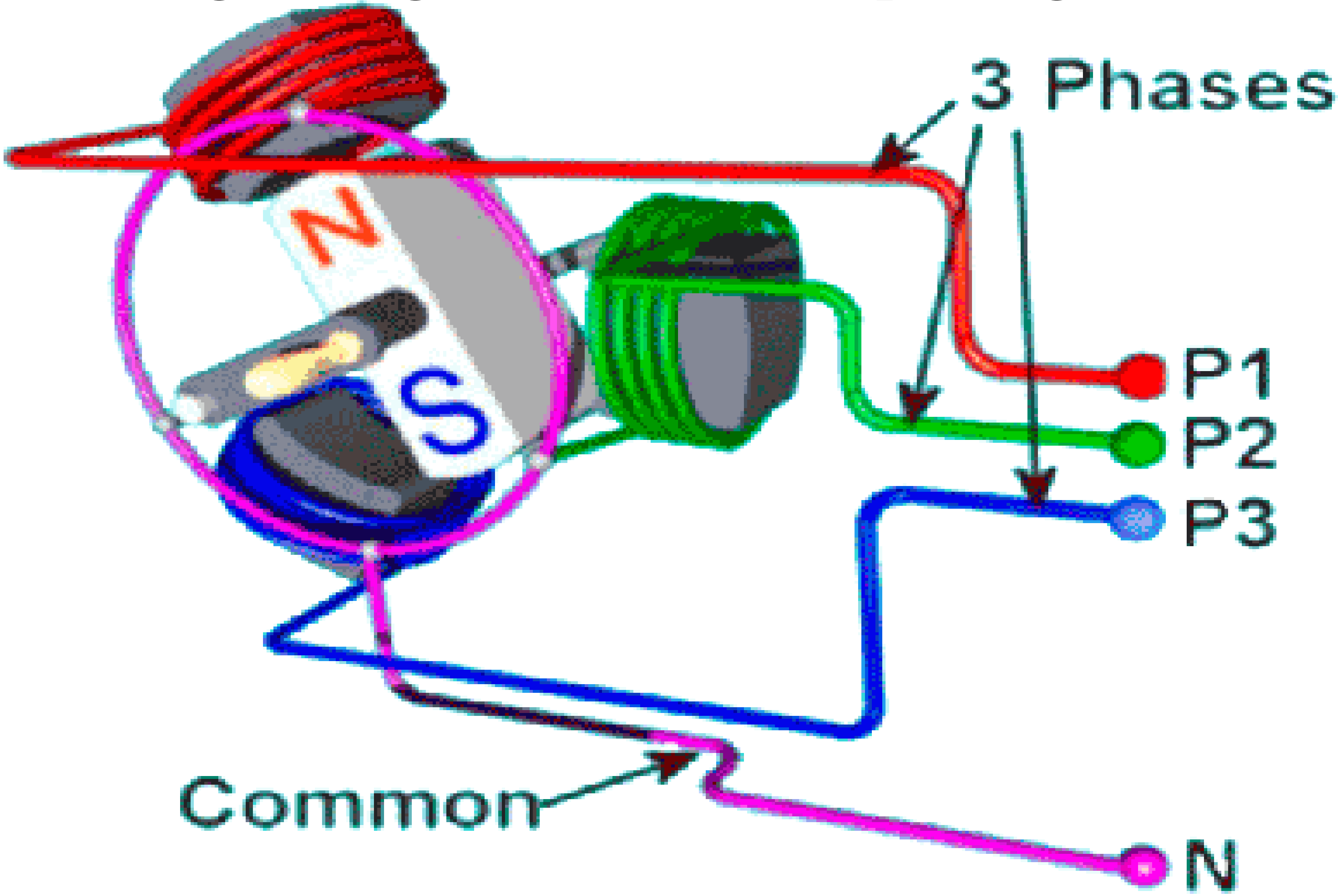
THREE-PHASE CIRCUITS

- A 3-phase system is a power supply system consisting of three voltages which are 120^{o} out of phase with one another.
- Three-phase systems have the following advantages over single phase systems
  - Three-phase motors, generators and transformers are simpler, cheaper and more efficient
  - Three-phase transmission lines can deliver more power for a given weight and cost

THREE-PHASE CIRCUITS

- The voltage regulation of three-phase transmission lines is inherently better
- A 1-phase supply can be obtained from a 3-phase one

Winding arrangement of a three-phase generator



A three-phase transmission line

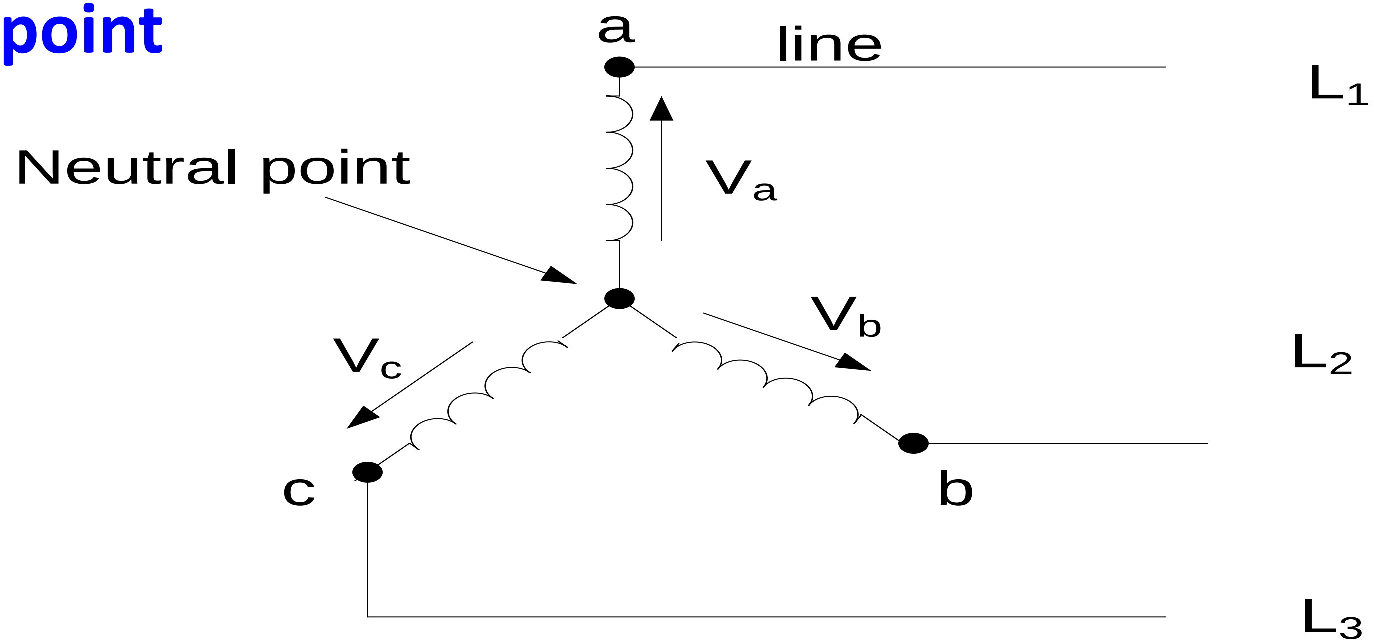
11kV Distribution Feeder

A distribution line

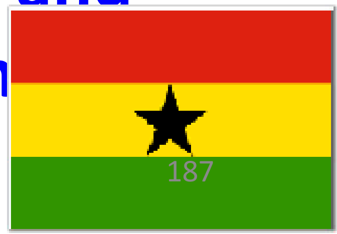
THREE-PHASE CIRCUITS

❖ The two main connections of three-phase windings

1. A star arrangement where all winding have a common point

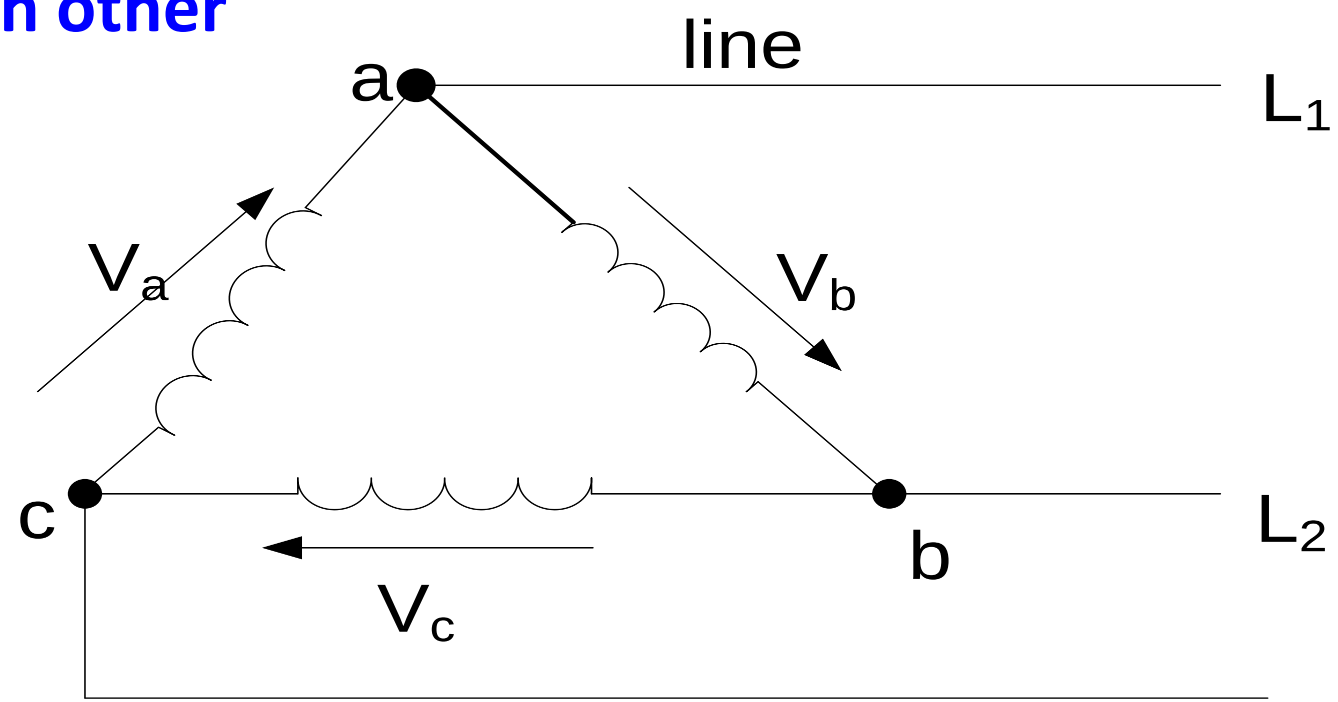


❑ Letters a, b and c , colours red (R), yellow (Y) and blue (B) or numbers 1, 2 and 3 are used to nan
windings



THREE-PHASE CIRCUITS

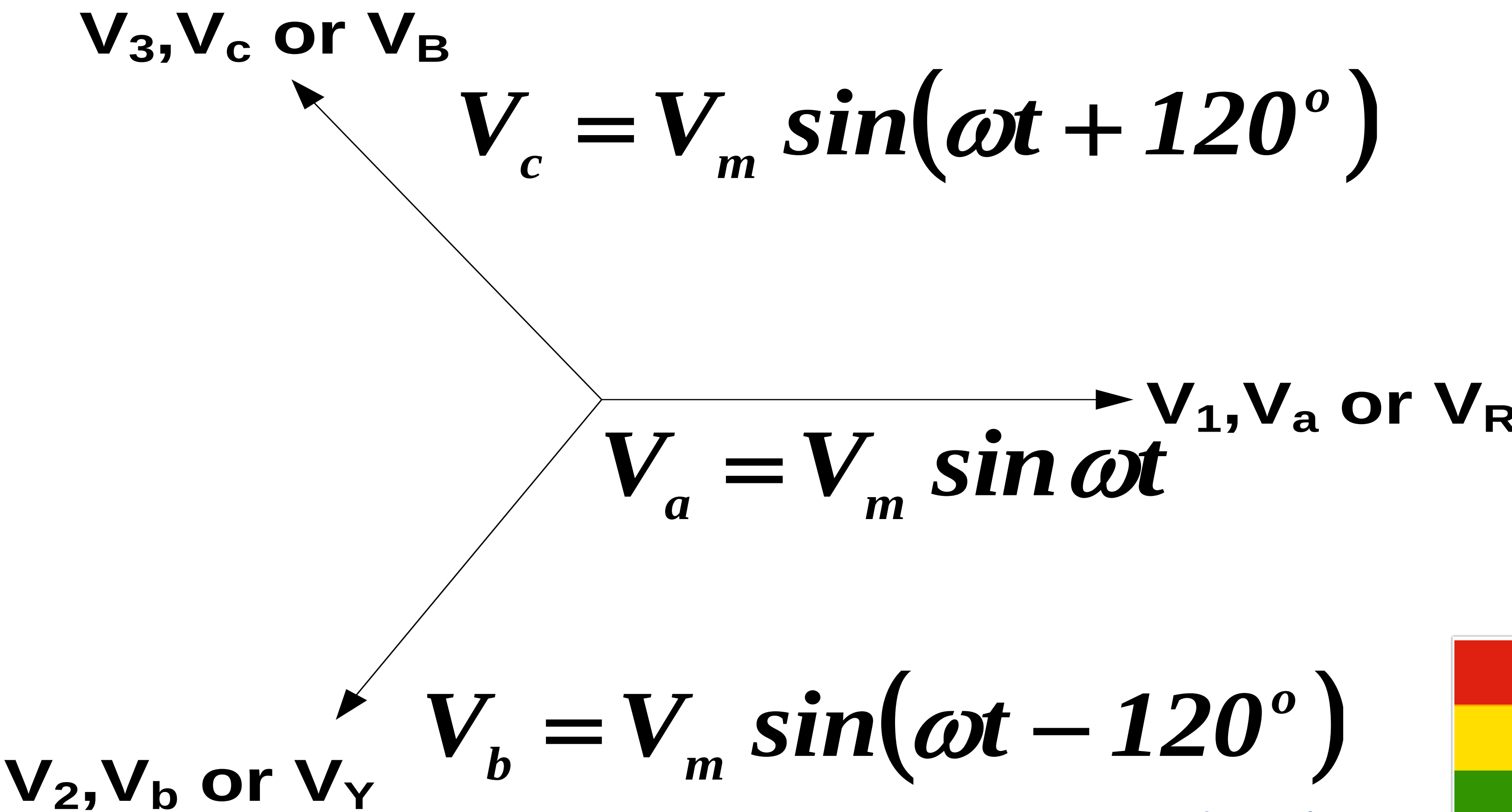
2. A delta arrangement where all winding are connected to each other



- Letters a, b and c , colours red (R), yellow(Y) and blue (B) or numbers 1, 2 and 3 are used to name th windings

THREE-PHASE CIRCUITS

❖ The phasor diagram for the three-voltages (in star or delta) is indicated below.



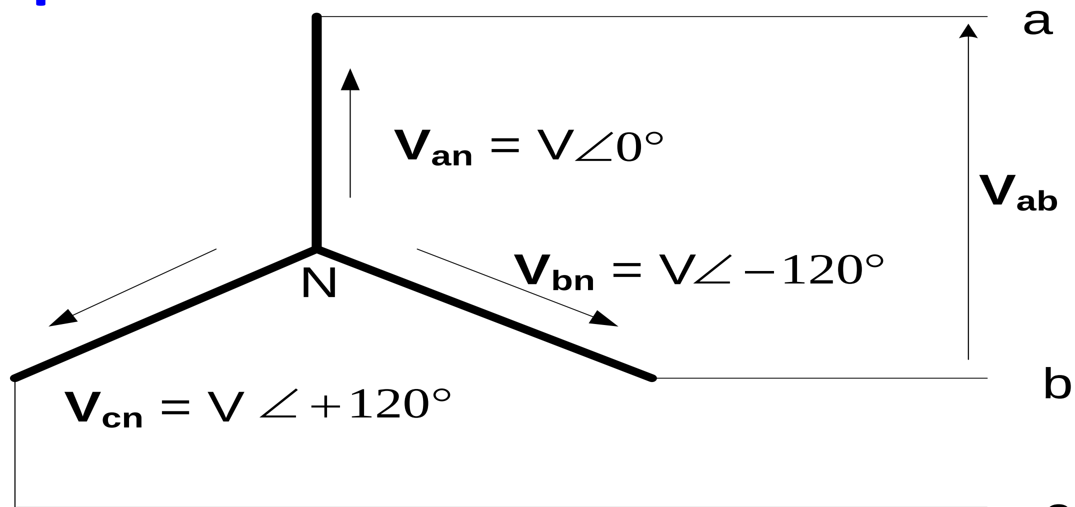
$$V_c = V_m \sin\left(\omega t + 120^o\right)$$

$$V_a = V_m \sin \omega t$$

$$V_b = V_m \sin\left(\omega t - 120^o\right)$$

THREE-PHASE CIRCUITS

- **Line and phase voltages**
  - The voltage from one line to another is called a line-to-line voltage or simple a line voltage
  - The voltage across each winding is a phase voltage
  - On a phasor diagram, a line voltage is drawn from the end of one phase to another in the anti clockwise direction



THREE-PHASE CIRCUITS

- Relationship between line and phase voltages for a star connection

$$
\begin{aligned}
V_{ab} &= V_{an} - V_{bn} \\
&= V\angle 0^o - V\angle -120^0 \\
&= V\left(1 - 1\angle -120^0\right) \\
&= V\left(1 - \cos(-120) - j\sin(-120)\right) \\
&= V\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V\angle 30^0
\end{aligned}
$$

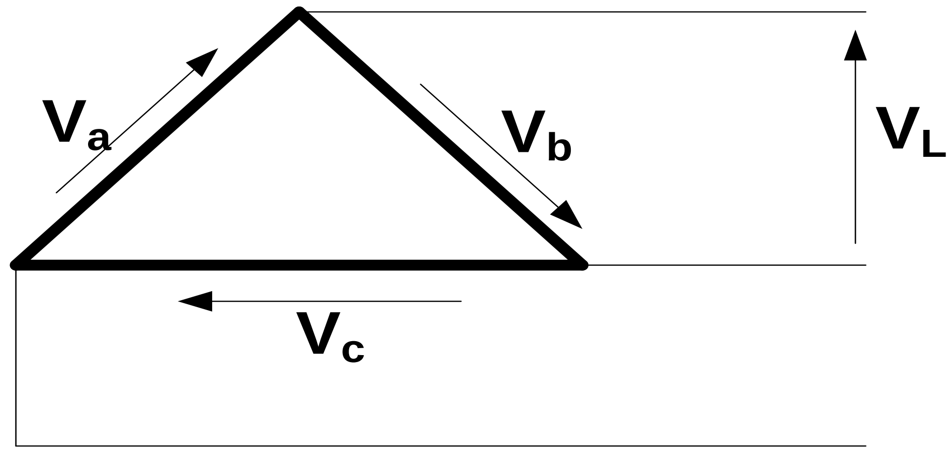
Hence, for a star connection, the line voltage is $\sqrt{3}$ times the phase voltage.

$$V_L = \sqrt{3}V_p$$

THREE-PHASE CIRCUITS

❖ **Relationship between Line and phase voltages for a delta connection**

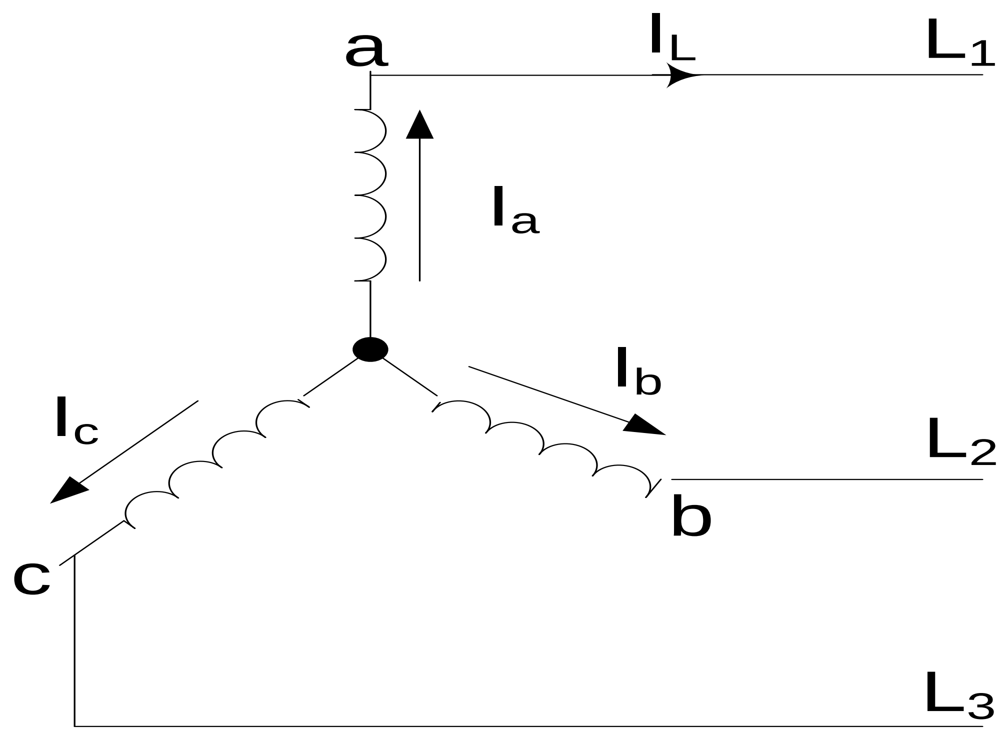
$$V_L = V_p$$



THREE-PHASE CIRCUITS

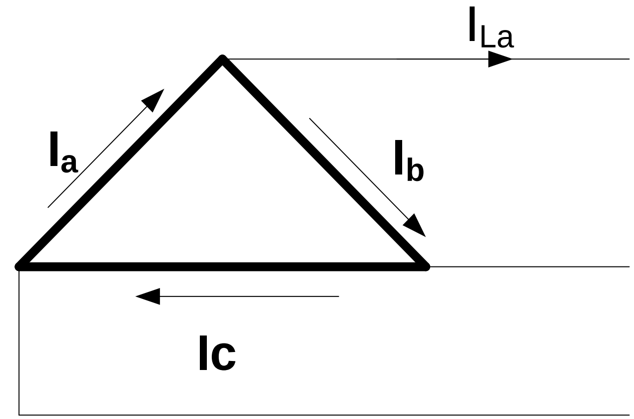
❖ **Relationship between Line and phase current for a star connection**

$$I_L = I_p$$



THREE-PHASE CIRCUITS

❖ Relationship between Line and phase currents for a delta connection



$$I_{La} = I_a - I_b$$
$$= I\angle 0^o - I\angle -120^0$$
$$= I\left(1 - 1\angle -120^0\right)$$
$$= I\left(1 - \cos(-120) - j\sin(-120)\right)$$
$$= I\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}I\angle 30^0$$

Hence, for a delta connection, the line current is $\sqrt{3}$ times the phase current

$$I_L = \sqrt{3}I_p$$

THREE-PHASE CIRCUITS

- Analysis of three-phase balanced circuits
  - A balanced three-phase circuit is that in which identical loads are connected in each phase.
  - The currents that flow in a balanced three-phase system are equal in magnitude and also 120° out of phase.
  - A balanced three-phase circuit is analysed by considering just one phase
  - When finding total power, the per phase power is multiplied by three
  - 1-phase power factor is the same as 3-phase

THREE-PHASE CIRCUITS

❖ **Example 1**

Three identical resistors are connected in star across a 3-phase, 415-V supply. If each resistor has a resistance of 50 ohms, calculate (a) the voltage across each resistor (b) the current in each resistor (c) the total power supplied to the load

❖ **Solution**

(a) $$V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240$$

(b) $$I_p = \frac{V_p}{R} = \frac{240}{50} = 4.8A$$

THREE-PHASE CIRCUITS

(c) $P_p = V_p I_p = 240 \times 4.8 = 1152W$

$\therefore P_T = 3 \times P_p = 3 \times 1152 = 3456W$

THREE-PHASE CIRCUITS

- **Example 2**

  Three identical impedances are connected in delta across a 3-phase, 415-V supply. If the line current is 10 A, calculate (a) the current in each impedance (b) the value of each impedance.

- **Solution**

- (a) $$I_p = \frac{I_L}{\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.78\,A$$

- (b) $$Z_p = \frac{V_p}{I_p} = \frac{415}{5.78} = 71.80\,\Omega$$

THREE-PHASE CIRCUITS

- **Example 3**

  A 3-phase, 450-V system supplies a balanced delta-connected load of 12 kW at 0.8 power factor lagging. Calculate (a) the phase currents (b) the line currents and (c) the effective impedance per phase.

- **Solution**

- (a) $P_p = V_p I_p \cos\theta$

$$\Rightarrow I_p = \frac{P_p}{V_p \cos\theta} = \frac{\frac{12}{3}\times 10^3}{450\times 0.8} = 11.1A$$

THREE-PHASE CIRCUITS

(b) $$I_L = \sqrt{3} I_p = \sqrt{3} \times 11.1 = 19.2 A$$

(c) $$Z_p = \frac{V_p}{I_p}$$

$$= \frac{450\angle 0^o}{11.1\angle -\cos^{-1}(0.8)} = 40.5\angle 36.9^0$$

THREE-PHASE CIRCUITS

❖ **Power in three-phase circuits**

- The total apparent power of a balanced three-phase circuit (star or delta) is given by:

$$S = \sqrt{3}V_L I_L$$

- The total active power of a balanced three-phase circuit (star or delta) is given by:

$$P = \sqrt{3}V_L I_L \cos\theta$$

- The total reactive power of a balanced three-phase circuit (star or delta) is given by:

$$Q = \sqrt{3}V_L I_L \sin\theta$$

THREE-PHASE CIRCUITS

- Analysis of parallel balanced three-phase circuit problems
  - When the parallel loads are not of the same kind(say one is delta and the other is star), then they must be changed either all to star or all to delta.
  - When all are in star, the circuit is analyzed by taking one phase of each and applying the star characteristics.
  - When all are in delta, the circuit is analyzed by taking one phase of each and applying the delta characteristics.

THREE-PHASE CIRCUITS

- Parallel circuits are better analysed by reverting to complex numbers.

- **Example 1**

A Y-connected impedance $Z_1 = 20.0 + j37.75\ \Omega$ per phase is connected in parallel with Δ-connected impedance $Z_2 = 30.0 - j159.3\ \Omega$ per phase. For an impressed 3-phase voltage of 398 V line to line, compute the line current, power factor and the power taken by the parallel combination.

- **Solution**

The circuit is solved by making all star.

THREE-PHASE CIRCUITS

Y-connected impedance Z_1 = 20.0 + j37.75 Ω

Δ-connected impedance Z_2 = 30.0 – j159.3 Ω

Changing the delta to star

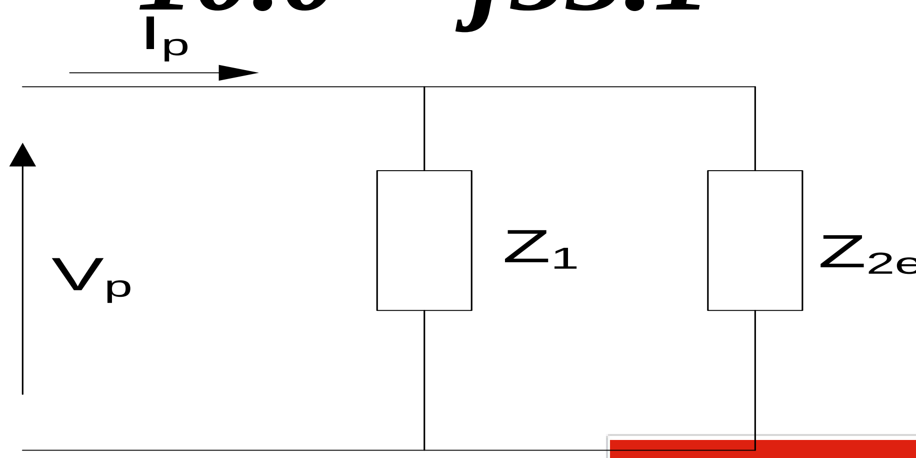
$$Z_{2e} = \frac{30.0 - j159.3}{3}$$

$$= 10.0 - j53.1$$

$$V_p = \frac{398}{\sqrt{3}} = 230V$$

$$I_p = \frac{V_p}{Z_1} + \frac{V_p}{Z_{2e}}$$

$$= 3.37\angle -9.9^0$$



THREE-PHASE CIRCUITS

$$I_L = I_p = 3.37\,A$$

$$pf = \cos\theta = \cos 9.9^0 = 0.985\, lagging$$

$$P = \sqrt{3}V_L I_L \cos\theta$$

$$= \sqrt{3} \times 398 \times 3.37 \times 0.985$$

$$= 2.29kW$$

THREE-PHASE CIRCUITS

❖ Example 2

A manufacturing plant draws 415 kVA from a 2400 V, 3-phase line. If the plant power factor is 0.875 lagging, calculate (a) the impedance of the plant per phase (b) the phase angle between the phase voltage and phase current.

❖ Solution

The load is considered to be star.

(a)

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{2400}{\sqrt{3}} = 1390V$$

THREE-PHASE CIRCUITS

$$S = \sqrt{3}V_L I_L \Rightarrow I_L = \frac{S}{\sqrt{3}V_L}$$

$$\therefore I_p = I_L = \frac{415000}{\sqrt{3 \times 2400}} = 100\,A$$

$$Z_p = \frac{V_p}{I_p} = \frac{1390}{100} = 13.9\Omega$$

$$\theta = \cos^{-1}(0.875) = 29^0$$

THREE-PHASE CIRCUITS

❖ **Example 3**

A star-connected motor is connected to a 4000 V, 3-phase, 50 Hz line. The motor produces an output power of 2681 kW at efficiency of 93 % and a power factor of 0.9 lagging. Calculate (a) the active power absorbed by the motor, (b) the reactive power absorbed by the motor, (c) the apparent power supplied by the transmission line and (d) the motor line current.

❖ **Solution**

(a) $$P_{in} = \frac{P_{out}}{\eta} = \frac{2681}{0.93} = 2883kW$$

THREE-PHASE CIRCUITS

(a) $$P_{in} = \frac{P_{out}}{\eta} = \frac{2681}{0.93} = 2883kW$$

(b) $$S = \frac{P_{in}}{pf} = \frac{2883}{0.9} = 3203kVA$$

$$Q = \sqrt{S^2 - P^2}$$

$$= \sqrt{3203^2 - 2883^2} = 1395kVAR$$

(c) $$S = 3203kVA$$

THREE-PHASE CIRCUITS

(d) $$I_L = \frac{S}{\sqrt{3}V_L} = \frac{3203 \times 10^3}{\sqrt{3} \times 4000} = 462\,A$$

THREE-PHASE CIRCUITS

(a)Three-phase four-wire feeding unbalanced star-connected loads

- Three-phase unbalanced load does not have the same impedance in all the three phases.
- The neutral points of the source and the load are the same, so the voltage across each phase of the load is the phase voltage of the source
- Three-phase four wire is usually used for low-voltage power distribution
- The neutral carries current if the load is unbalan

$$I_N = I_{pa} + I_{pb} + I_{pc}$$

THREE-PHASE CIRCUITS

- ❑ Each phase is analyzed separately when the load is unbalanced

❖ **Example 1**

A three-phase four-wire system has the following data: Supply voltage is 415 V, $Z_1 = 8 + j0\ \Omega$, $Z_2 = 0 - j8\ \Omega$ and $Z_3 = 0 + j8\ \Omega$. Determine the load currents and the current in the neutral.

❖ **Solution**

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240V$$

THREE-PHASE CIRCUITS

$$I_{p1} = \frac{V_p}{Z_1} = \frac{240\angle 0^0}{8 + j0} = 30\angle 0^0$$

$$I_{p2} = \frac{V_p}{Z_2} = \frac{240\angle -120^0}{0 - j8} = 30\angle -30^0$$

$$I_{p3} = \frac{V_p}{Z_3} = \frac{240\angle 120^0}{0 + j8} = 30\angle 30^0$$

$$I_N = I_{p1} + I_{p2} + I_{p3}$$

$$I_N = 30\angle 0^0 + 30\angle -30^0 + 30\angle 30^0$$

THREE-PHASE CIRCUITS

$$I_N = 82A$$

THREE-PHASE CIRCUITS

❖ Example 2

In a three-phase four-wire system, the line voltage is 400V and resistive loads of 10 kW, 8 kW and 5 kW are connected between the three lines and neutral. Calculate (a) the current in each line and (b) the current in the neutral conductor.

❖ Solution

(a) $$V_p = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231V$$

THREE-PHASE CIRCUITS

Let I_a, I_b and I_c be the currents drawn by the 10kW, 8kW and 5kW loads respectively.

$$I_a = \frac{P_a}{V_p \cos\theta} = \frac{10 \times 10^3}{231 \times 1} = 43.3\,A$$

$$I_b = \frac{P_b}{V_p} = \frac{8 \times 10^3}{231} = 34.6\,A$$

$$I_c = \frac{P_c}{V_p} = \frac{5 \times 10^3}{231} = 21.65\,A$$

THREE-PHASE CIRCUITS

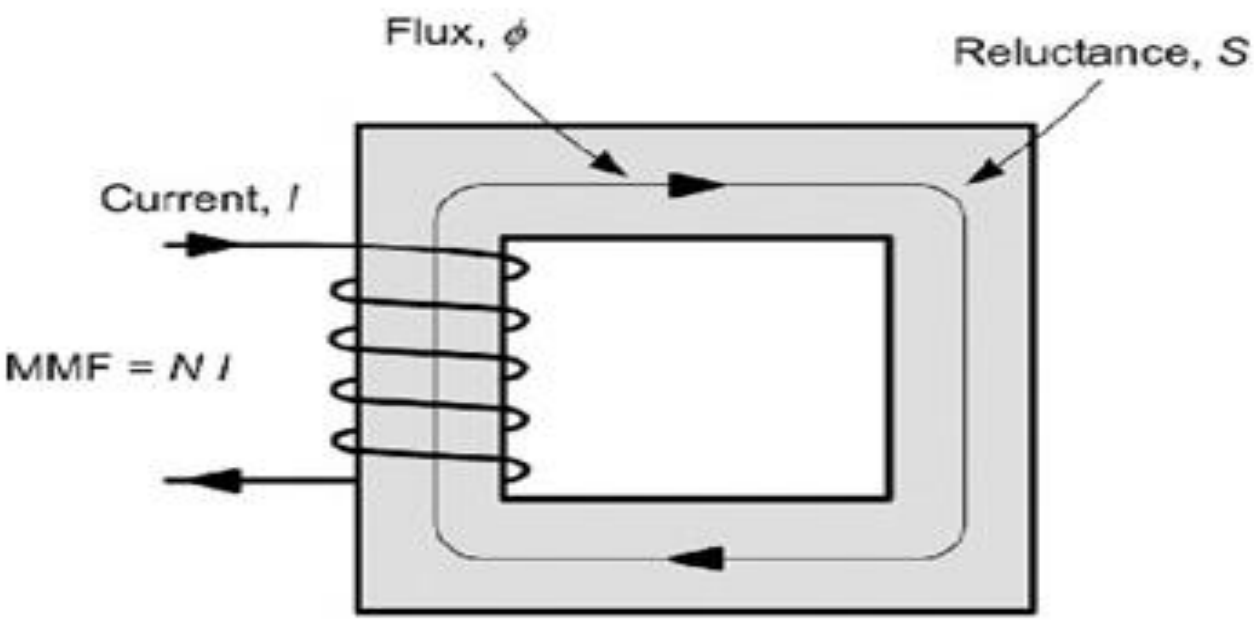
$$I_N = I_a + I_b + I_c$$

$$= 43.3\angle 0^0 + 34.6\angle -120^0 + 21.65\angle 120^0$$

$$= 18.9A$$

MAGNETIC CIRCUITS

- A magnetic circuit is a closed path followed by any group of lines of magnetic flux.
- Magnetic circuits are created with coils and ferromagnetic (iron, cobalt, nickel, etc) or permanent magnetic materials.



MAGNETIC CIRCUITS

❖ Terminologies in magnetic circuits

❑ Flux(ϕ)

It is a measure of the amount of magnetic field passing through a given surface. The SI unit of flux is Weber(Wb).

❑ Flux density (B)

It is the flux per unit area. It is the flux divided by the cross-sectional area. The SI unit is Weber per square meter (Wb/m^2) or Tesla (T)

$$B = \frac{\phi}{A}$$

MAGNETIC CIRCUITS

- Terminologies in magnetic circuits
  - Magnetomotive force (F)

    It is the source which sets up the flux flowing around a magnetic circuit. The unit is Amperes (A) or Ampere-turns(AT). It is the product of current in coil and number of turns of coil

$$\boldsymbol{F = NI}$$

MAGNETIC CIRCUITS

- Reluctance(S or $\mathfrak{R}$)

  It is likened to resistance. It is the opposition offered by a material to the flow of magnetic flux.

$$S = \frac{L}{\mu_o \mu_r A}$$

L is the length of the magnetic path in meters (m)

A is the cross sectional area in square meter (m^2)

The unit of reluctance is ampere-turns per weber (AT/Wb)

$$F = \phi S$$

MAGNETIC CIRCUITS

- Terminologies in magnetic circuits
  - Magnetic field intensity (H)

    It is the mmf per unit length. The unit is Amperes per meter (A/m)or Ampere-Turns per meter (AT/m)

$$H = \frac{F}{L}$$

MAGNETIC CIRCUITS

❖ B-H curves

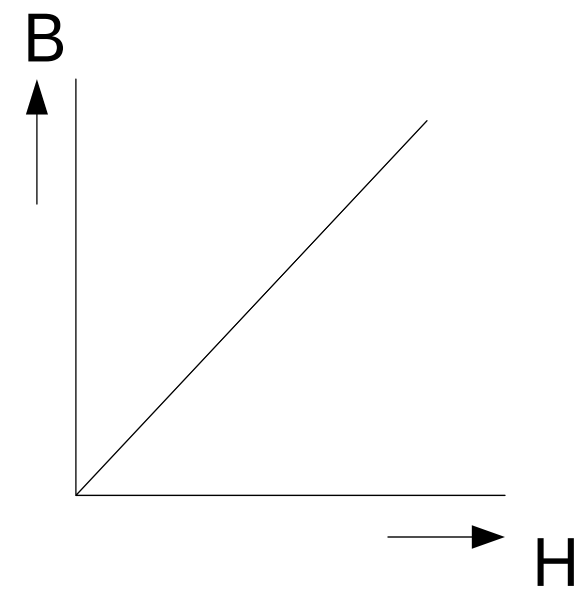
They are curves which show the relation between magnetic flux density and magnetic field intensity for various materials

❑ The B-H curve of vacuum is as shown

$$B = \mu_o H$$

μ_o is a constant called magnetic constant or permeability of free space

$$\mu_o = 4\pi \times 10^{-7}\ H/m$$



MAGNETIC CIRCUITS

- B-H curve of non-magnetic materials

  The B-H curves are almost identical to that of vacuum.

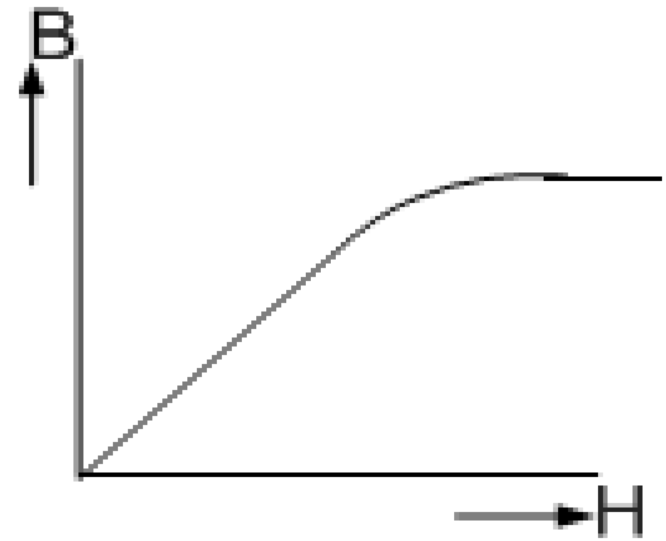
$$B = \mu_o H$$

MAGNETIC CIRCUITS

- B-H curve of magnetic materials

The B-H curves have linear, non-linear and saturation regions

$$B = \mu_o \mu_r H$$



μ_r is a the relative permeability of the magnetic material.

The above formula only works for the linear region.

MAGNETIC CIRCUITS

- **Analysis of linear magnetic circuits**

  Magnetic circuits are analyzed by drawing an equivalent electric circuit and applying the laws used to analyze electric circuits. The following similarities exit.

  (a) Flux is similar to current

  (b) Magnetomotive force is similar to electromotive force

  (c) Reluctance is similar to resistance

MAGNETIC CIRCUITS

❖Analysis of non-linear magnetic circuits

Not all the laws of electric circuits can be applied to magnetic circuits. Kirchoff’s laws are applicable and when applied to magnetic circuits are stated as follows:

First law: The total magnetic flux towards a junction is equal to the total magnetic flux away from that junction.

Second law: In any closed magnetic circuit, the algebraic sum of the product of magnetic field strength and the length of each part of the circuit is equal to the resultant magnetomotive force.

MAGNETIC CIRCUITS

❖Example 1

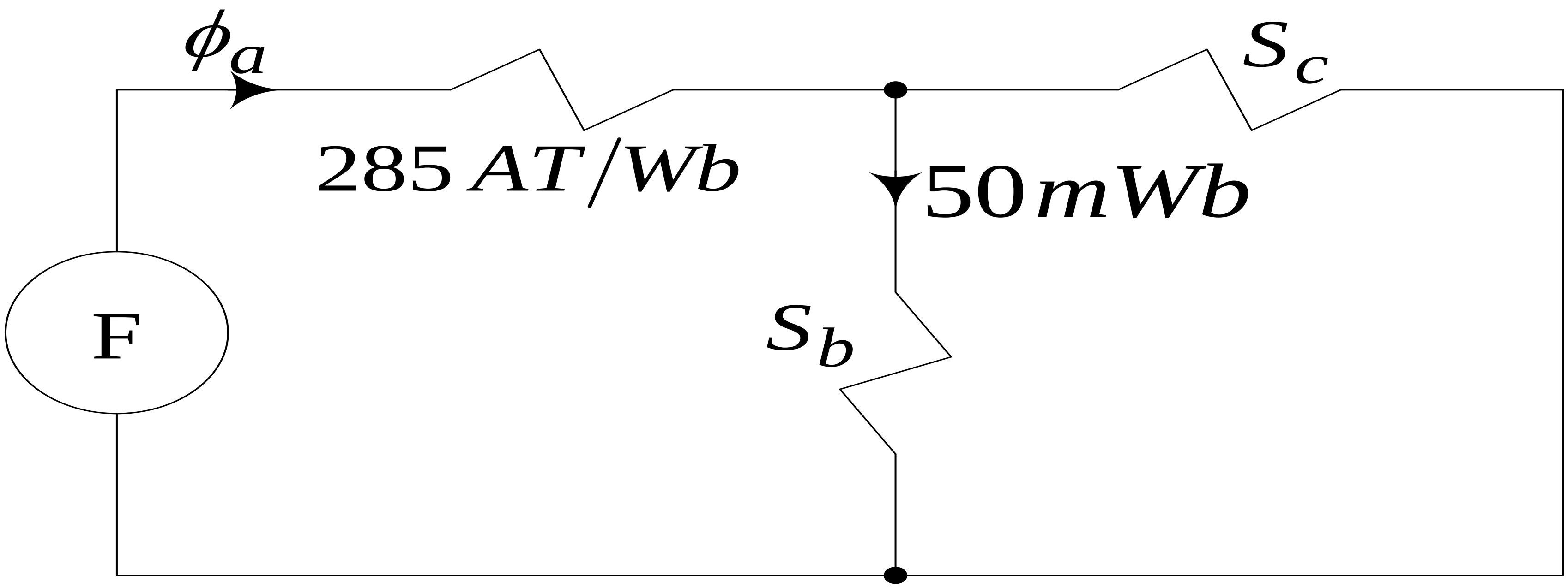
A 600-turn coil carrying a current of 0.2A sets up a flux of 200mWb in a magnetic circuit. (i) What is the mmf in amperes developed in the circuit?

(ii) Determine the total reluctance of the circuit.

MAGNETIC CIRCUITS

❖ **Example 2**

The figure below shows the equivalent electric circuit of a three-part magnetic circuit energised by a 100-turn coil carrying a current of 1A. The mmf drop across part 'b' is 40AT



MAGNETIC CIRCUITS

(a) Calculate the flux in part c.

(b) Determine the reluctance of parts b and c.

(c) Obtain the flux density in part b if the cross sectional area of the material used is $0.05 m^2$.

LINEAR MAGNETIC CIRCUIT PROBLEMS

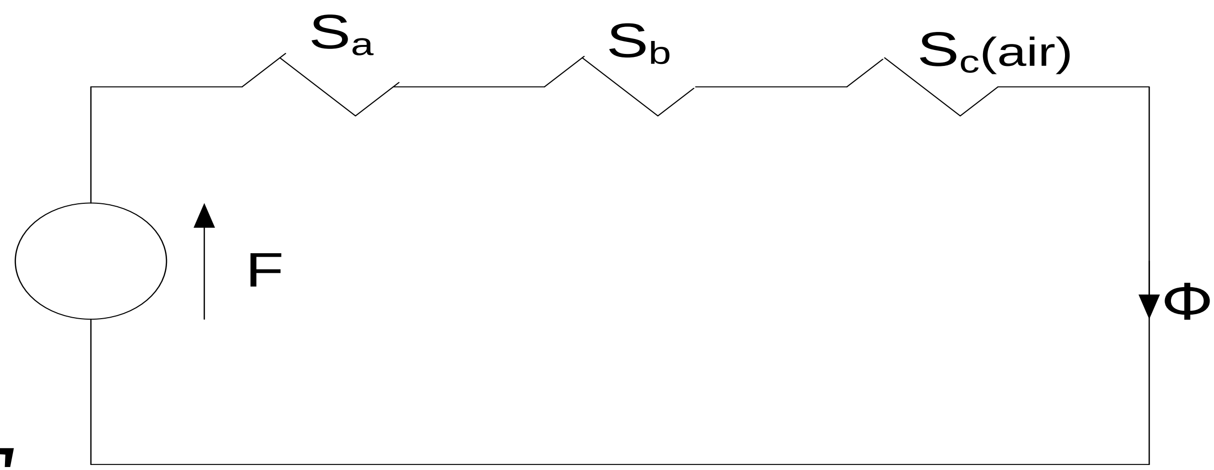
- **Example 1**

A magnetic circuit comprises three parts in series each of uniform cross section. They are (a) a length of 80 mm and cross sectional area 50mm^2 (b) a length of 60mm and cross sectional area 90 mm^2 (c) an air gap of length 0.5 mm and cross sectional area 150mm^2 . A coil of 4000 turns is wound on part (b) and the flux density in the air gap is 0.3 T. Assuming that all the flux passes through the given circuit and that the relative permeability of section (a) and (b) is μ_r = 3000, estimate the coil current to produce such a flux density.

MAGNETIC CIRCUITS

❖Solution

The electric analogue circuit is shown below.



$$F = NI$$

$$\Rightarrow I = \frac{F}{N}$$

$$F = F_a + F_b + F_c$$

$$F_a = \phi S_a = B_{air} A_{air} \times \frac{L_a}{\mu_a \mu_r A} = 19.1 AT$$

MAGNETIC CIRCUITS

$$F_b = \phi S_b = \phi \frac{L_b}{\mu_a \mu_r A} = 7.96AT$$

$$F_c = \phi S_c = \phi \frac{L_c}{\mu_a \mu_r A} = 119.3AT$$

$$F = F_a + F_b + F_c = 146.36AT$$

$$\therefore I = \frac{F}{N} = \frac{146.36}{4000} = 36.59mA$$

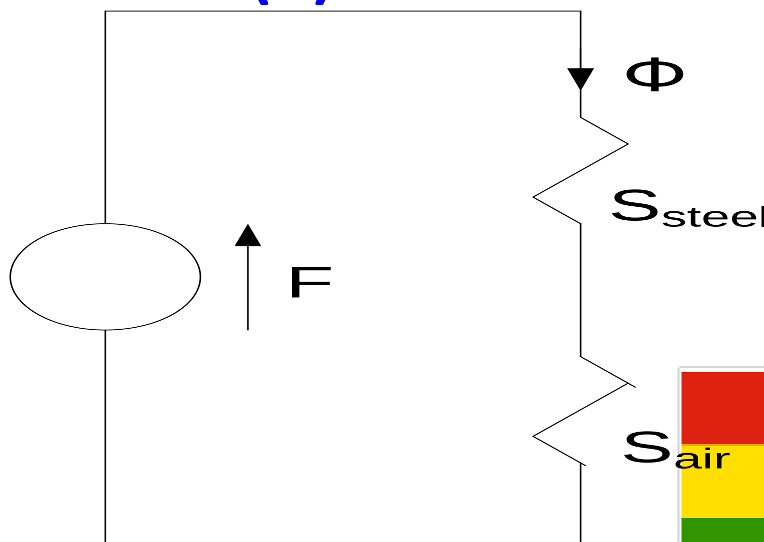
MAGNETIC CIRCUITS

❖Example 2

A steel ring of cross-sectional area $50mm^2$ has an air gap of 2mm and of the same cross-sectional area as shown in Fig. 4.5.a. The coil shown has 2000 turns with current 10 A. If the mean radius of the steel ring is 5cm and $\mu_r = 800$, calculate (a) the total reluctance of the circuit (b) the flux Φ in the ring.

❖Solution

(a) The equivalent electric circuit is as shown



MAGNETIC CIRCUITS

$$S_s = \frac{L_s}{\mu_0 \mu_r A_s} = \frac{2\pi \times 5 \times 10^{-2}}{4\pi \times 10^{-7} \times 800 \times 50 \times 10^{-6}}$$

$$= \frac{10^8}{16} A/Wb$$

$$S_a = \frac{L_a}{\mu_0 A_a} = \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 50 \times 10^{-6}}$$

$$= \frac{10^8}{\pi} A/Wb$$

MAGNETIC CIRCUITS

Total reluctance $$S = \frac{10^8}{16} + \frac{10^8}{\pi}$$

$$= 0.381 \times 10^8 \ A/Wb$$

(b) $$F = \phi S$$

$$\Rightarrow \phi = \frac{F}{S} = \frac{NI}{S} = \frac{2000 \times 10}{0.381 \times 10^8}$$

$$= 5.25 \times 10^{-4} \ Wb$$

MAGNETIC CIRCUITS

❖Example 3

It is desired to produce a magnetic field of 1 Wb/m^2 in the air gap of the electromagnet shown in Example 2. The cross section of the iron is 1 cm^2, the mean length $L_i + L_g = 10$ cm, gap length $L_g = 5$ mm, and from the B-H curve of the iron μ_r was found to be 500 at B = 1 Wb/m^2. Calculate the m.m.f. required.

❖Solution

$$F = \phi(S_i + S_g) = \phi\left(\frac{L_i}{\mu_0 \mu_r A_i} + \frac{L_g}{\mu_0 A_g}\right)$$

$$A_i = A_g = A$$

MAGNETIC CIRCUITS

$$\Rightarrow F = \frac{\phi}{\mu_0 A}\left(\frac{L_i}{\mu_r} + L_g\right)$$

$$= \frac{1}{4\pi \times 10^{-7}}\left(\frac{9.5 \times 10^{-2}}{500} + 5 \times 10^{-3}\right)$$

$$= 4130\,AT$$

MAGNETIC CIRCUITS

❖Example4

A coil of 1500 turns is wound on a circular wooden former which has a mean circumference of 30cm and a cross-sectional area of 4cm^2. Calculate (a) the flux density in the ring when the coil carries a current of 0.4 A (b) the flux in the ring in webers.

When the wooden former was replaced with a steel former of the same dimensions, the total flux became 600µWb for a current of 0.4 A. Calculate the relative permeability of the steel and the reluctance of the magnetic circuit at this flux density.

MAGNETIC CIRCUITS

❖Solution

A coil of 1500 turns is wound on a circular wooden

(a) $$H = \frac{F}{L} = \frac{NI}{L} = \frac{0.4 \times 1500}{0.3} = 2000\,AT/m$$

$$B = \mu_0 H = 4\pi \times 10^{-7} \times 2000$$
$$= 2.513\,mWb/m^2$$

$$\phi = BA = 2.513 \times 10^{-3} \times 4 \times 10^{-4}$$
$$= 1.005\,\mu Wb$$

MAGNETIC CIRCUITS

With the magnetic former

$$H = 2000\,AT/Wb \qquad \phi = 600\,\mu Wb$$

$$\therefore B = \frac{\phi}{A} = \frac{600 \times 10^{-6}}{4 \times 10^{-4}} = 1.5\,Wb/m^2$$

$$\mu_r = \frac{B}{\mu_0 H} = \frac{1.5}{4\pi \times 10^{-7} \times 2000} = 597$$

$$S = \frac{F}{\phi} = \frac{NI}{\phi} = \frac{1500 \times 0.4}{600 \times 10^{-4}} = 10^4\,AT/Wb$$

THE END

I have enjoyed teaching you.

I wish you the very best in the years ahead.

May the ALMIGHTY GOD be with you.