Inventory Control Systems

Inventory Systems

Inventory: the stock of any item or resource used in an organisation.

Includes:

- ✓Raw materials
- ✓Component parts
- ✓Work-in-progress
- ✓Finished goods.

An inventory system: the set of policies and controls that monitor levels of inventory and determines what levels should be maintained, when stock should be replenished, and how large orders should be.

Motivation for Holding Inventory

- Economies of Scale/set-up costs
- Uncertainties: internal and external
  - ✓Variation in product demand
  - ✓Variation in raw material delivery time
  - ✓Variation in production process
- Quantity discount
- Smoothing (seasonal demand/limited production capacity)

Characteristics of Inventory System

- Demand
  - ✓Constant or variable
  - ✓Deterministic or stochastic
- Lead time
  - ✓External order: time between placement of an order until arrival of goods
  - ✓Internal production: amount of time required to produce a batch of items
  - ✓Lead time can be deterministic or random
- Replenishment
  - ✓How does the order arrive? Uniform over time, instantaneous batches.

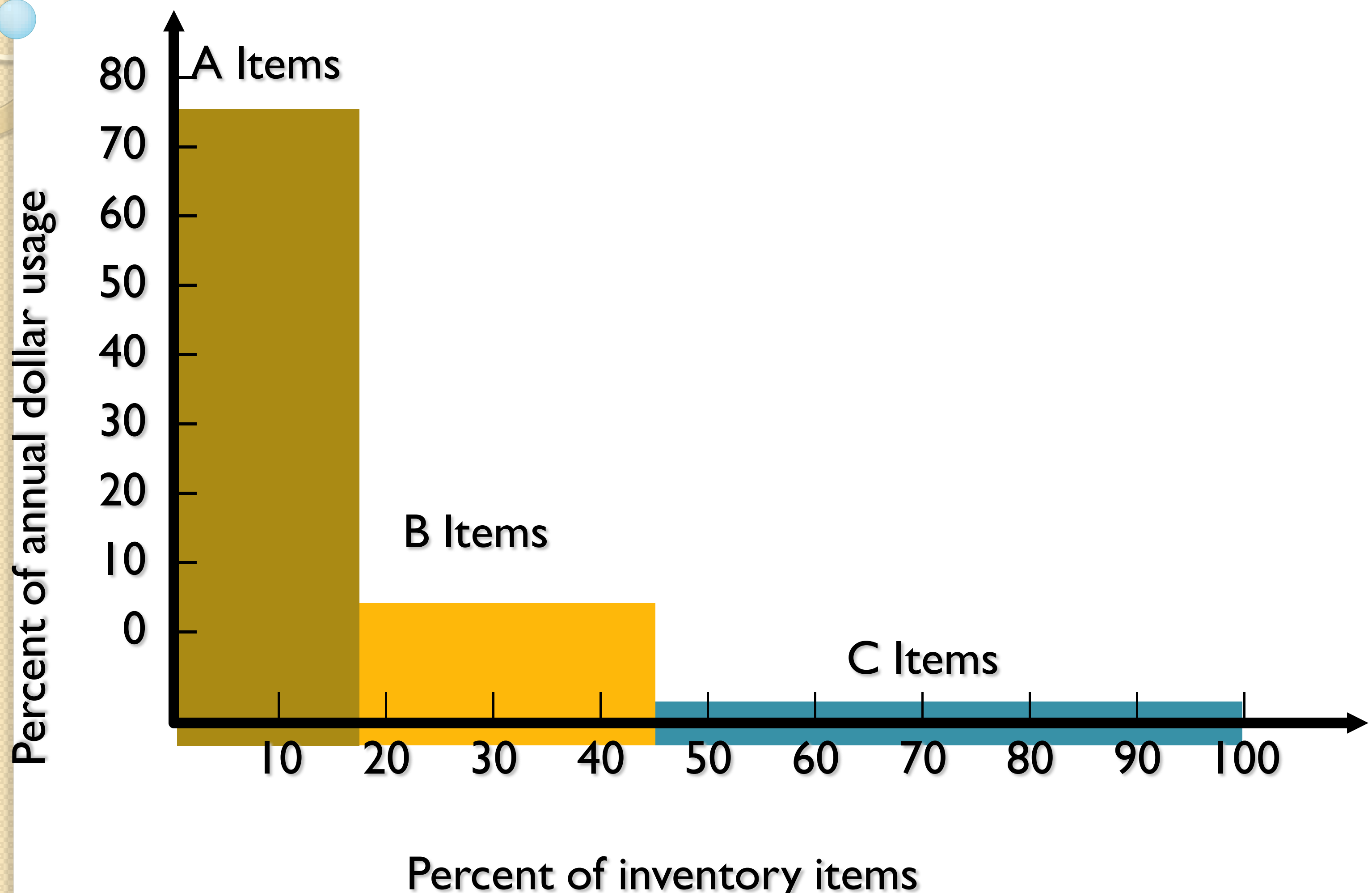
Characteristics of Inventory Systems

- Review Time
  - ✓ Continuous review: inventory level is known at all times
  - ✓ Periodic review: inventory level is known only at discrete points in times (e.g. in a month)
- Excess Demand
  - Back ordering: excess is satisfied in the future
  - Lost sales: excess demand is lost
- Changing inventory
  - Inventory may change over time: limited shelf life (perishable goods - food), obsolescence (e.g. automotive parts)

Inventory Management

- Has two main components:
    1. How inventory items can be classified
    2. How accurate inventory records can be maintained.
- Inventory Classification using ABC Analysis
    - Divides inventory into three classes based on annual dollar volume:
        - Class A - high annual dollar volume
        - Class B - medium annual dollar volume
        - Class C - low annual dollar volume
- Used to establish policies that focus on the few critical parts and not the many trivial ones.

ABC Analysis -1



ABC Analysis -2

- The annual consumption value is calculated with the formula:

  (Annual demand) x (item cost per unit)

- Through this categorization, the supply manager can identify inventory hot spots, and separate them from the rest of the items, especially those that are numerous but not that profitable.

Steps for the classification of items:

1. Find out the unit cost and the usage of each material over a given period;
2. Multiply the unit cost by the estimated annual usage to obtain the net value;
3. List out all the items and arrange them in the descending value (Annual Value);
4. Accumulate value and add up number of items and calculate percentage on total inventory in value and in number.

	Percentage of items	**Percentage value of annual usage**	
Class A items	About 20%	About 80%	Close day to day control
Class B items	About 30%	About 15%	Regular review
Class C items	About 50%	About 5%	Infrequent review

ABC Analysis- An Example

Calculate the total spending per year

Item number	Unit cost	Annual demand	Total cost per year
101	5	48,000	240,000
102	11	2,000	22,000
103	15	300	4,500
104	8	800	6,400
105	7	4,800	33,600
106	16	1,200	19,200
107	20	18,000	360,000
108	4	300	1,200
109	9	5,000	45,000
110	12	500	6,000
Total usage			737,900

Total cost per year: Unit cost * annual demand

Example Contd.

Calculate the usage of item in total usage

Item number	Unit cost	Annual demand	Total cost per year	Usage as a % of total usage
101	5	48,000	240,000	32,5%
102	11	2,000	22,000	3%
103	15	300	4,500	0,6%
104	8	800	6,400	0,9%
105	7	4,800	33,600	4,6%
106	16	1,200	19,200	2,6%
107	20	18,000	360,000	48,8%
108	4	300	1,200	0,2%
109	9	5,000	45,000	6,1%
110	12	500	6,000	0,8%
Total usage			737,900	100%

Calculate the usage of item in total usage

Step 3: Sort the items by usage

Item number	Unit cost	Annual demand	Total cost per year	Usage as a % of total usage	Cumulative % of total
107	20	18,000	360,000	48,8%	48,8%
101	5	48,000	240,000	32,5%	81,3%
109	9	5,000	45,000	6,1%	87,4%
105	7	4,800	33,600	4,6%	92%
102	11	2,000	22,000	3,0%	94,9%
106	16	1,200	19,200	2,6%	97,5%
104	8	800	6,400	0,9%	98,4%
110	12	500	6,000	0,8%	99,2%
103	15	300	4,500	0,6%	99,8%
108	4	300	1,200	0,2%	100%
Total usage			737,900	100%	

Step 4: Results of calculation

Cathegory	Items	Percentage of items	Percentage usage (%)	Action
Class A	107, 101	20%	81,6%	Close control
Class B	109, 105, 102, 106	40%	16,2%	Regular review
Class C	104, 110, 103, 108	40%	2,5%	Infrequent review

Example

Item number	Annual quantity used	Unit value
1	75	80
2	150,000	0,9
3	500	3,0
4	18,000	0,20
5	3,000	0,30
6	20,000	0,10
7	10,000	2

Step 1

Item number	Annual quantity used	Unit value	Usage per year
1	75	80	6,000
2	150,000	0,9	135,000
3	500	3,0	1,500
4	18,000	0,20	3,600
5	3,000	0,30	900
6	20,000	0,10	2,000
7	10,000	2	20,000
Total usage			169,000

Step 2

Item number	Annual quantity used	Unit value	Usage per year	Percentage in total usage (%)
1	75	80	6,000	3,51%
2	150,000	0,9	135,000	79,8%
3	500	3,0	1,500	0,87%
4	18,000	0,20	3,600	2,1%
5	3,000	0,30	900	0,53%
6	20,000	0,10	2,000	1,18%
7	10,000	2	20,000	11,8%
Total usage			169,000	

Step 3

Item number	Cumulative % of items	Annual quantity used	Unit value	Usage per year	Percentage in total usage (%)	Cumulative % of total
2	14%	150,000	0,9	135,000	79,8%	79,8%
7	29%	10,000	2	20,000	11,8%	91,6%
1	42%	75	80	6,000	3,51%	95,11%
4	56%	18,000	0,20	3,600	2,1%	97,21%
6	71%	20,000	0,10	2,000	1,18%	98,39%
3	84%	500	3,0	1,500	0,87%	99,46%
5	100%	3,000	0,30	900	0,53%	100%
Total usage				169,000		

Step 4

Cathegory	Items	Percentage of items	Percentage of usage (%)	Action
Class A items	2	15%	79,8%	Close control
Class B items	7, 1	30%	15,31%	Regular review
Class C items	3, 4, 5, 6	55%	4,89%	Infrequent review

ABC Analysis -2

- Other criteria than annual dollar volume may be used:
  - Anticipated engineering changes
  - Delivery problems
  - Quality problems
  - High unit cost

Inventory Control – Some Definitions

Order Cycle: The time period between placement of two successive orders

Time Horizon: The period over which the inventory level will be controlled.

Lead Time: The time between ordering a replenishment of an item and actually receiving the item into inventory.

Reorder Level: The level between maximum and minimum stock levels at which purchasing/manufacturing activities must initiate actions for replenishment.

Reorder Quantity: The quantity of replacement order. In certain cases, it is the economic order quantity.

Inventory Costs - 1

- Inventory is one of the most expensive assets of many companies representing as much as 50% of total invested capital.
- Operations managers must balance inventory investment and customer service.

Optimisation criterion in models is, therefore, COST MINIMISATION

Inventory Costs -2

Four types of inventory costs are associated with keeping inventory:

1. **Purchase (production cost)** :The cost of purchasing (or producing) a unit of an item.
2. **Ordering (set-up cost):** Cost of arranging specific equipment setups or someone placing an order. Includes paper work, telephone calls, postage, cost of transportation of items ordered, inspection of goods, processing payment etc
   - Fixed and variable part (x units)
   - $S(x) = K + Px$

Inventory Costs -3

3. Holding (carrying costs) – H

- Costs for storage, handling tax/insurance, breakage, deterioration, obsolescence, opportunity cost of alternative investments (e.g. cost of capital)
- H=I * P (where I is annual interest rate and P is the cost of holding one unit per unit time)
- Dimension- (unit of currency) per unit per year

4. Shortage or stock-out costs:

- This cost includes the lost of potential profit that otherwise would have accrued through sales of items demanded, loss of good will, and permanent loss of customers etc.

Overview of Inventory Policies

Two basic questions:

1. When should we reorder (or produce)?
2. How much should we produce or order?

Typical answers to Q1:

- When inventory position is equal (or below) a level s
- Every T time units

Typical answers to Q2:

- Order or produce Q units
- Order or produce such that inventory position becomes S

Overview of Inventory Policies

To answer the two important questions posed :

-when to order,

-how much to order,

we **use three independent demand models**:

1. Economic Order Quantity(EOQ) Model
2. Production Order Quantity(POQ) Model
3. Quantity Discount Model

Independent Vs. Dependent Demand

Inventory control models assume that demand for an item is either:

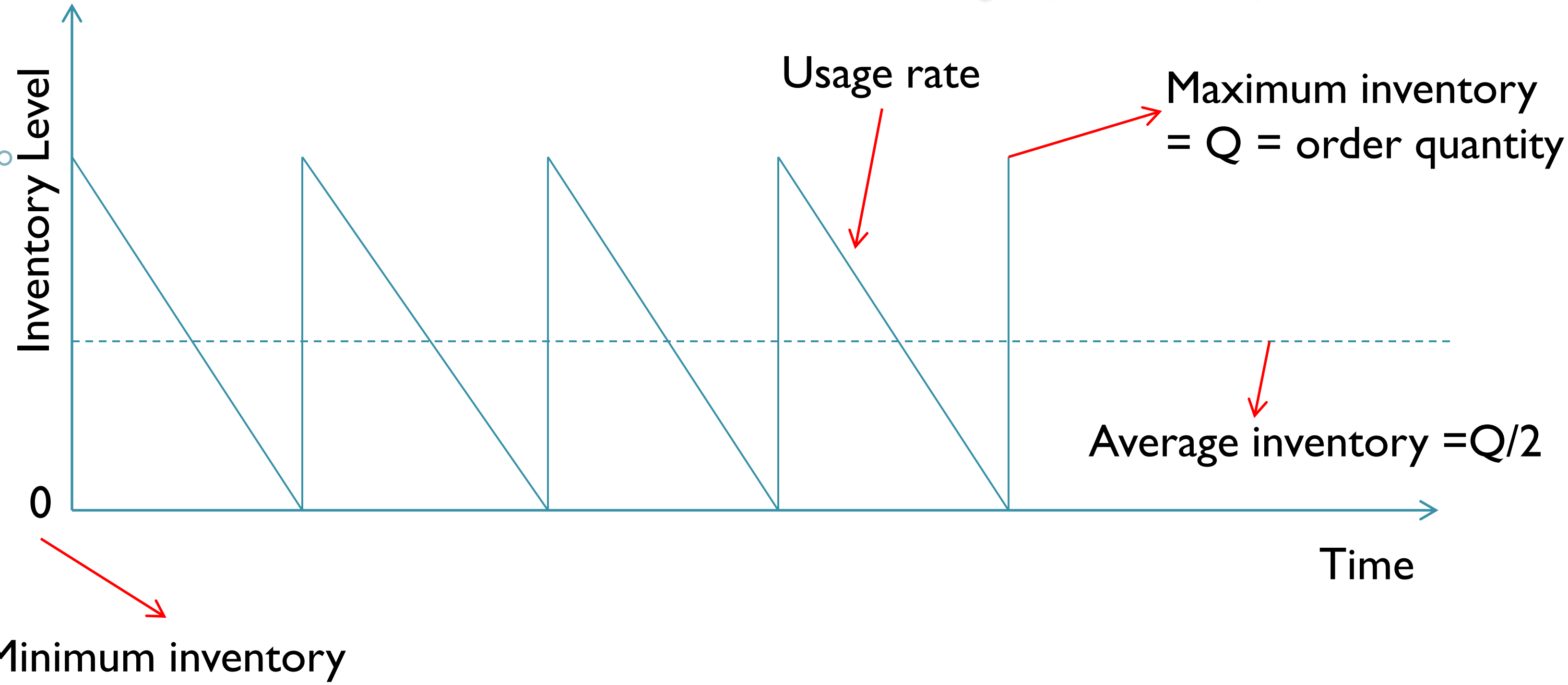
- Independent of or
- Dependent on other items.
- E.g. demand for laptops is independent of demand for mobile phones, however, many inventory problems are interrelated.
- E.g. demand for lawn mower wheels and spark plugs are dependent on the demand for lawn mowers.
    - Four wheels and a spark plug are required for each lawn mower.
    - The relationship between the items is known. Therefore scheduling is done based on the demand of the final item.
    - Our discussion on inventory focuses on using independent demand.

Economic Order Quantity(EOQ) Model

- Oldest, common and most fundamental technique
- A number of organisations still use it
- Relatively easy to use.
- EOQ makes a number of assumptions:

1. Demand is known and constant
2. Lead time,(the time between placing an order and the receipt of the order) is constant.
3. Receipt of inventory is instantaneous (one batch, one time)
4. Quantity discounts are not possible,
5. The only variable costs are the costs of setting up or placing order and the cost of holding or storing inventory over time,
6. Stock outs (shortages) are not allowed.

Economic Order Quantity(EOQ) Model



Average inventory = $\dfrac{\text{maximum inventory} - \text{minimum inventory}}{2}$

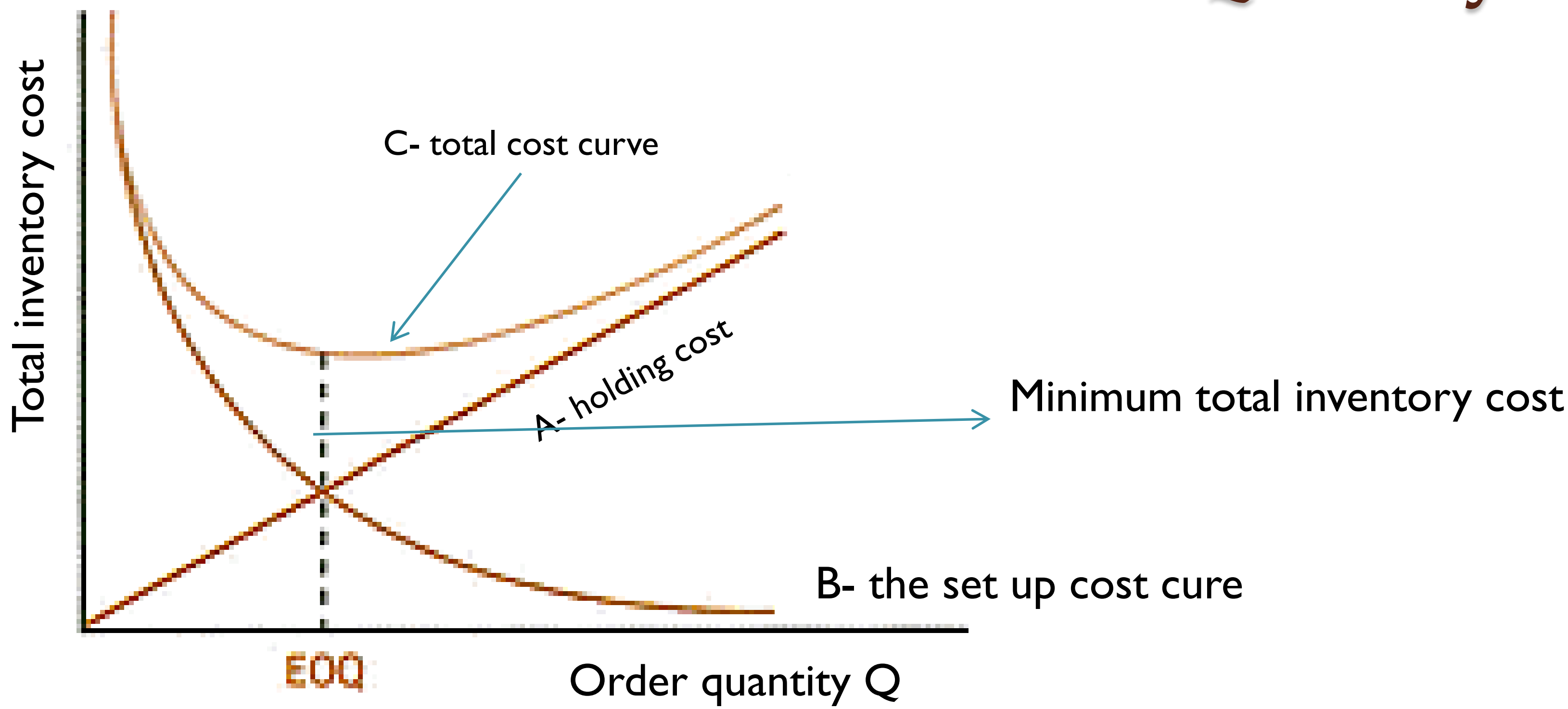
Note: The inventory cycles repeats itself over and over again

Economic Order Quantity(EOQ) Model

Purpose of most inventory models: **minimise the total costs.**

- With the assumptions that underlie the EOQ model, the costs involved are the set up costs and the holding cost. All other costs such as the cost of inventory itself are constant.
- Therefore, if we minimise the sum of setup and holding costs, we will minimise overall inventory cost.
- The optimal order size is the size that will minimise overall costs .
- The optimal order size is Q^*

Total Cost as a Function of Order Quantity



- Note the following:
  - Include only costs affected by order quantity
  - holding cost increases as quantities on-hand increase
  - Ordering costs decrease as quantities on-hand increase
  - Add the holding and ordering costs to determine the annual total cost

EOQ-Determining the Optimal Order Size Q*

Variables involved:

Q = Number of pieces per order

Q* = Optimum number of pieces per order (EOQ)

D = Annual demand in units for the inventory item

S = Set up or ordering cost for each order

H = Holding or carrying cost per unit per year

Derivation of Q*

1. Annual setup cost = (No. of orders placed/yr) * (Setup or order cost/order)

$$= \left(\frac{\text{Annual demand}}{\text{Number of units in each order}}\right)\left(\begin{matrix}\text{Setup or order}\\ \text{cost per order}\end{matrix}\right)$$

$$= \left[\frac{D}{Q}\right] (S)$$

2. Annual holding costs

$$= \left(\frac{\text{Order quantity}}{2}\right) (\text{Holding cost per unit per year}) \quad = \left[\frac{Q}{2}\right] (H)$$

Optimal order quantity is found when annual setup cost = annual holding cost

i.e.

$$\frac{D}{Q} S = \frac{Q}{2} H$$

4. To find the optimal order quantity Q* make Q the subject of the equation.

$$2DS = Q^2H$$

$$Q^2 = 2DS/H$$

$$Q^* = \sqrt{2DS/H}$$

Derivation of Q*

With Q* determined, the expected number of orders = N can be calculated as :

$$N = \frac{\text{Annual Demand}}{\text{Order quantity}} = \frac{D}{Q^*}$$

- Expected time between order = T can also be determined as :
- $$T = \frac{\text{Number of working days/year}}{N}$$

EOQ: An Example

Sharp, Inc., a company that markets painless hypodermic needles to hospitals, would like to reduce its inventory cost by determining the optimal number of hypodermic needles to obtain per order. The annual demand is 1,000 units; the set up or ordering cost is $10 per order; and the holding cost per unit per year is $.50. Sharp, Inc. has 250 working days in a year

Using these figures, calculate the optimal number of units per order. What is the expected number of orders and the expected time between orders?

What is the total annual inventory cost?

Solution

1. Q* = 200 units
2. Expected number of orders N = 5 orders
3. Expected time between orders T = 50 days between orders
4. Total annual inventory cost = $ 100

NB: often the total annual inventory cost expression is written to include the actual cost of the material purchased. The total actual cost will be D * P,

Where D is annual demand and P is price per unit of material

Reorder Point

When to order?

Simple inventory model assume that receipt of inventory is instantaneous. That also means that a firm will wait for its inventory to get to level 0 before placing an order, and that the items will be received immediately. However, the time between placing an order and receiving it (known as lead time or delivery time) can take up to few hours to many months.

As a result, when to order decisions are usually expressed in terms of reorder point.

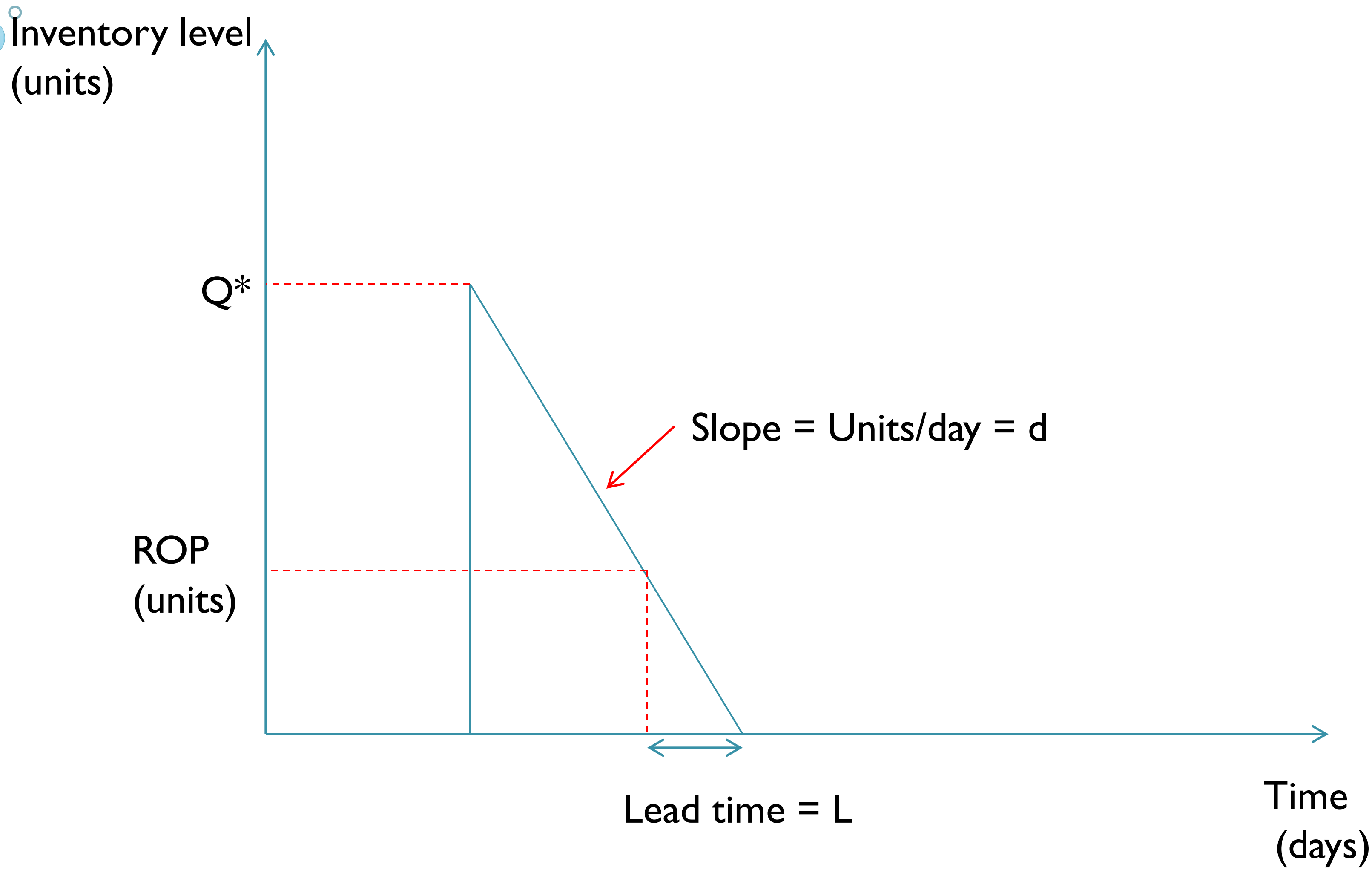
The reorder point = (Demand per day)*(Lead time for a new order in days)

$$ROP = d * L$$

The calculation of the reorder point assumes that demand is uniform and constant. When this is not the case, extra stock (called safety stock), should be added.

$$\text{Demand per day } d = \frac{D}{\text{Number of working days in a year}}$$

Reorder Point Curve



Reorder Point: An Example

Electronic Assembler, Inc. has a demand for TX512 semiconductors of 8,000 per year. The firm operates a 200-day working year. On the average, delivery of an order takes 3 working days. Calculate the reorder point.

Solution

Demand per day d = $\dfrac{D}{\text{Number of working days in a year}}$

$= \dfrac{8000}{200} = 40$ units/day

Reorder point =d *L = 40 units/day * 3 days

= 120 units

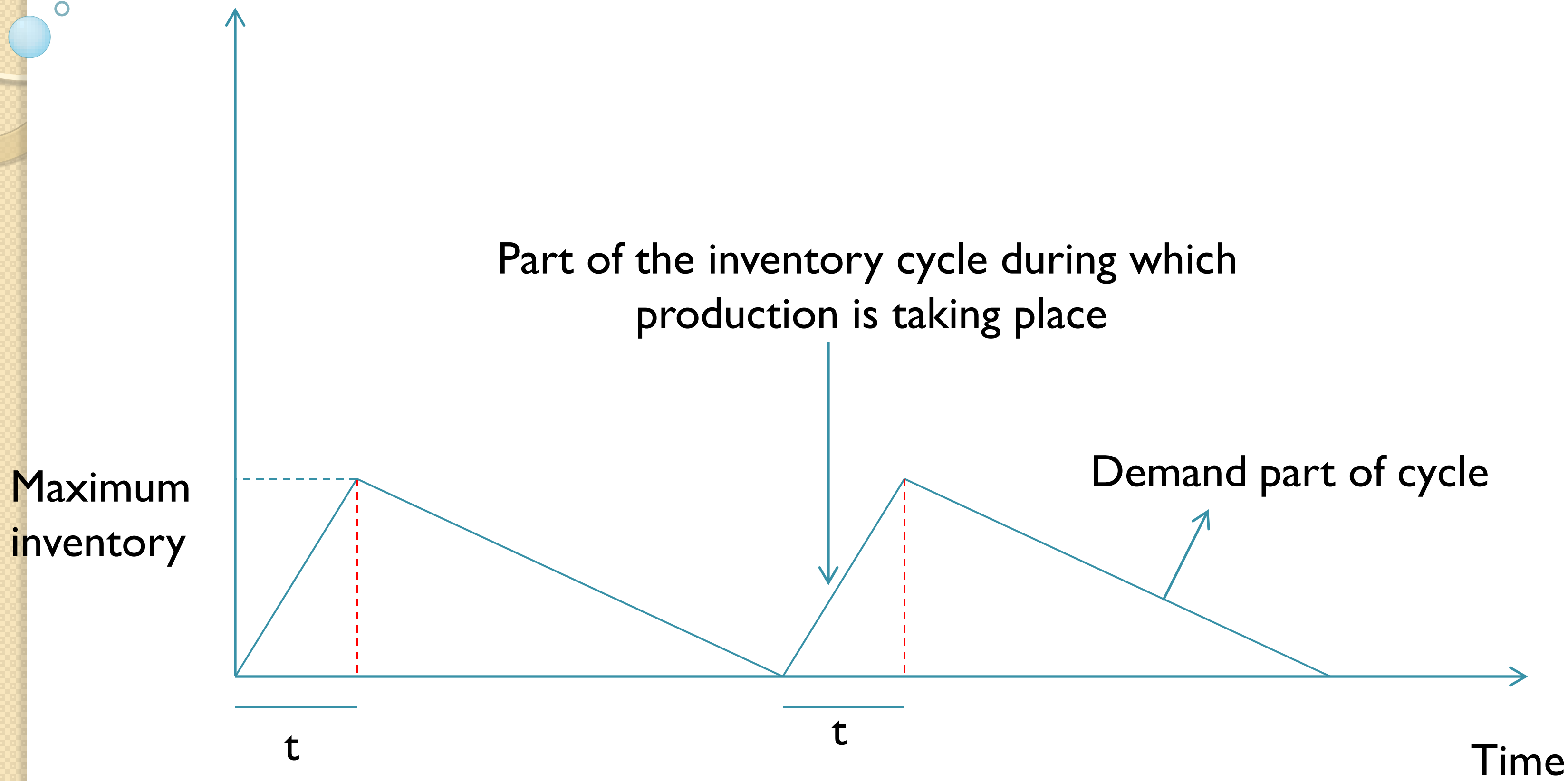
Hence, when the inventory stock drops to 120 units, an order should be placed. The order will arrive three days later, just as the firm's stock is depleted.

Production Order Quantity Model - POQ

In the EOQ model, we assumed that entire inventory order was received at one time. However, there are times when a firm may receive its inventory over a period of time.

- The EOQ model can, therefore, not be used.
- The POQ model is more appropriate
  - Assumption:
  - Inventory continuously flows or builds up over a period of time after an order is placed, or when units are produced.
- This model takes into account daily production rate and daily demand rate.
- Model is more suitable for production and hence its name- Production Order Quantity

Changes in Inventory Levels Over Time for POQ



Production Order Quantity : Assumptions

- EOQ makes a number of assumptions:

1. Demand is known and constant
2. Lead time,(the time between placing an order and the receipt of the order) is constant.
3. Inventory continuously flows or builds up over time.
4. Quantity discounts are not possible,
5. The only variable costs are the costs of setting up or placing and the cost of holding or storing inventory over time,
6. Stock outs (shortages) are not allowed.

Deriving Equation for Optimal Order ,Q*

We derive the POQ model by setting set up costs to be equal to holding cost and solving for Q*.

Symbols:

Q = Number of pieces per order

H =Holding cost per unit per year

p = Daily production rate

d= Daily demand rate, or usage rate

t= Length of the production run in days

Deriving Equation for Optimal Order ,Q*

Annual inventory holding cost = (Average inventory level) * (cost of holding one unit per year)

= (Average inventory level) * H

2. Average inventory level $= \dfrac{\text{Maximum inventory}}{2}$

3. Maximum inventory level = (Total produced during production run) – (Holding cost per unit per year)

= pt – dt

But Q= total produced = pt, →t=Q/p,

$$\left(\begin{array}{c}\text{Maximum}\\ \text{inventory level}\end{array}\right) = p\left(\frac{Q}{p}\right) - d\left(\frac{Q}{p}\right) = Q\left(1 - \frac{d}{p}\right)$$

Annual inventory holding cost $= \dfrac{\text{maximum inventory level}}{2} * H$

$= H\,\dfrac{Q\,(1- d/p)}{2}$

Deriving Equation for Optimal Order ,Q*

The optimal number of pieces Q* is calculated by setting set up cost to be equal to holding cost.

$$\frac{D}{Q} * S = \frac{H\,Q\,(1- d/p)}{2}$$

By making Q the subject equation, we get Q* as

$$Q * = \sqrt{\frac{2DS}{H\left(1-\left(\frac{d}{p}\right)\right.}}$$

This equation yield the optimum order or production quantity when inventory is consumed as it is produced

Production Order Quantity: An Example

Using our earlier example from Sharp Inc. with the following additional variables provided, calculate the optimal order number of units per order.

We have:

Annual demand = D = 1000 units

Set up cost = S = $10

Holding costs = H = $ 0.50 per unit per year

Daily production rate = p = 8 units daily

Daily demand rate = d = 6 times daily

$$Q* = \sqrt{\frac{2DS}{H\left(1 - \left(\frac{d}{p}\right)\right)}}$$

Q^*_p = 400 units

Quantity Discount Model

- Reduced prices are often available when larger quantities are purchased
- Trade-off is between reduced product cost and increased holding cost

Total cost = Setup cost + Holding cost + Product cost

$$TC = \frac{D}{Q}S + \frac{QH}{2} + PD$$

Quantity Discount Model

Our main objective again is to determine the quantity that will minimises total annual inventory cost.

Because there are several discounts, we use four steps:

1. For each discount, calculate Q* using

$$\boxed{Q^* = \sqrt{\frac{2DS}{IP}}}$$

2. If Q* for a discount doesn't qualify, choose the smallest possible order size to get the discount
3. Compute the total cost for each Q* or adjusted value from Step 2
4. Select the Q* that gives the lowest total cost

Quantity Discount: An Example

Discount Number	Unit Price	Order Quantity	Annual Product Cost	Annual Ordering Cost	Annual Holding Cost	Total
1	$5.00	700	$25,000	$350	$350	$25,700
2	$4.80	1,000	$24,000	$245	$480	$24,725
3	$4.75	2,000	$23.750	$122.50	$950	$24,822.50

D is 5000 race cards, inventory carrying charge as a percentage of cost , I, is 20%, ordering cost per order is $49.

What order quantity will minimise the total inventory cost?

Solution

- $Q_1^* = \sqrt{\dfrac{2(5{,}000)(49)}{(.2)(5.00)}}$ = 700 cars order

$Q_2^* = \sqrt{\dfrac{2(5{,}000)(49)}{(.2)(4.80)}}$ = 714 cars order
adjusted to 1000

$Q_3^* = \sqrt{\dfrac{2(5{,}000)(49)}{(.2)(4.75)}}$ = 718 cars order
adjusted to 2000

Test each Q* in the total cost equation to see which one gives the least cost

$$TC = \frac{D}{Q^*}S + \frac{QH}{2} + PD$$

Quantity Discount: An Example

Discount Number	Unit Price	Order Quantity	Annual Product Cost	Annual Ordering Cost	Annual Holding Cost	Total
1	$5.00	700	$25,000	$350	$350	$25,700
2	$4.80	1,000	$24,000	$245	$480	$24,725
3	$4.75	2,000	$23.750	$122.50	$950	$24,822.50