CHAPTER II

PROBLEM 11.1

11.1 Determine the modulus of resilience for each of the following grades of structural steel:

(*a*) ASTM	A709 Grade 50:	$\sigma_Y = 50$ ksi
(*b*) ASTM	A913 Grade 65:	$\sigma_Y = 65$ ksi
(*c*) ASTM	A709 Grade 100:	$\sigma_Y = 100$ ksi

SOLUTION

Structural steel $\quad E = 29 \times 10^6$ psi for all three steels given.

(a) $\sigma_Y = 50$ ksi $= 50 \times 10^3$ psi

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(50 \times 10^3)^2}{(2)(29 \times 10^6)} = 43.1 \text{ in}\cdot\text{lb/in}^3$$

(b) $\sigma_Y = 65$ ksi $= 65 \times 10^3$ psi

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(65 \times 10^6)^2}{(2)(29 \times 10^6)} = 72.8 \text{ in}\cdot\text{lb/in}^3$$

(c) $\sigma_Y = 100$ ksi $= 100 \times 10^3$ psi

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(100 \times 10^3)^2}{(2)(29 \times 10^6)} = 172.4 \text{ in}\cdot\text{lb/in}^3$$

PROBLEM 11.2

11.2 Determine the modulus of resilience for each of the following aluminum alloys:

(*a*) 1100-H14:	$E = 70$ GPa,	$\sigma_Y = 55$ MPa
(*b*) 2014-T6:	$E = 72$ GPa	$\sigma_Y = 220$ MPa
(*c*) 6061-Y6:	$E = 69$ GPa	$\sigma_Y = 140$ MPa

SOLUTION

Aluminum alloys

(a) $E = 70 \times 10^9$ Pa, $\sigma_Y = 55 \times 10^6$ Pa

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(55 \times 10^6)^2}{(2)(70 \times 10^9)} = 21.6 \times 10^3 \text{ N}\cdot\text{m/m}^2 = 21.6 \text{ kJ/m}^3$$

(b) $E = 72 \times 10^9$ Pa, $\sigma_Y = 220 \times 10^6$ Pa

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(220 \times 10^6)^2}{(2)(72 \times 10^9)} = 336 \times 10^3 \text{ N}\cdot\text{m/m}^2 = 336 \text{ kJ/m}^3$$

(c) $E = 69 \times 10^9$ Pa, $\sigma_Y = 140 \times 10^6$ Pa

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(140 \times 10^6)^2}{(2)(69 \times 10^9)} = 142.0 \times 10^3 \text{ N}\cdot\text{m/m}^3 = 142.0 \text{ kJ/m}^3$$

PROBLEM 11.3

11.3 Determine the modulus of resilience for each of the following metals:
(*a*) Stainless steel AISI 302 (annealed): $E = 190$ GPa, $\sigma_Y = 260$ MPa
(*b*) Stainless steel AISI 302 (cold-rolled): $E = 190$ GPa $\sigma_Y = 520$ MPa
(*c*) Malleable cast iron: $E = 165$ GPa $\sigma_Y = 230$ MPa

SOLUTION

(a) $E = 190 \times 10^9$ Pa, $\sigma_Y = 260 \times 10^6$ Pa

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(260\times10^6)^2}{(2)(190\times10^9)} = 177.9\times10^3 \text{ N}\cdot\text{m/m}^3 = 177.9 \text{ kJ/m}^3$$

(b) $E = 190\times10^9$ Pa, $\sigma_Y = 520\times10^6$ Pa

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(520\times10^6)^2}{(2)(190\times10^9)} = 712\times10^3 \text{ N}\cdot\text{m/m}^3 = 712 \text{ kJ/m}^3$$

(c) $E = 165\times10^9$ Pa, $\sigma_Y = 230\times10^6$ Pa

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(230\times10^6)^2}{(2)(165\times10^9)} = 160.3\times10^3 \text{ N}\cdot\text{m/m}^3 = 160.3 \text{ kJ/m}^3$$

PROBLEM 11.4

11.4 Determine the modulus of resilience for each of the following alloys:
(*a*) Titanium : $E = 16.5 \times 10^6$ psi: $\sigma_Y = 120$ ksi
(*b*) Magnesium $E = 6.5 \times 10^6$ psi: $\sigma_Y = 29$ ksi
(*c*) Cupronickel (annealed) : $E = 20 \times 10^6$ psi: $\sigma_Y = 16$ ksi

SOLUTION

(a) $E = 16.5\times10^6$ psi, $\sigma_Y = 120\times10^3$ psi

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(120\times10^3)^2}{(2)(16.5\times10^6)} = 436 \text{ in}\cdot\text{lb/in}^3$$

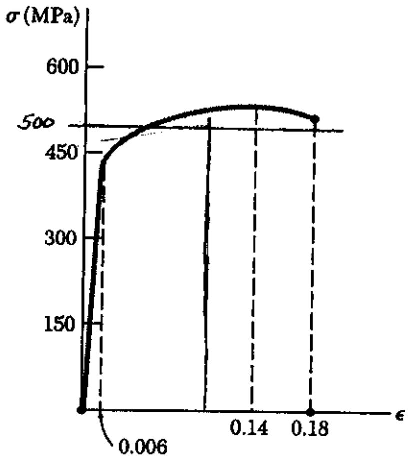
(b) $E = 6.5\times10^6$ psi, $\sigma_Y = 29\times10^3$ psi

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(29\times10^3)^2}{(2)(6.5\times10^6)} = 64.7 \text{ in}\cdot\text{lb/in}^3$$

(c) $E = 20\times10^6$ psi, $\sigma_Y = 16\times10^3$ psi

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(16\times10^3)^2}{(2)(20\times10^6)} = 6.40 \text{ in}\cdot\text{lb/in}^3$$

PROBLEM 11.5



11.5 The stress-strain diagram shown has been drawn from data obtained during a tensile test of an aluminum alloy. Using $E = 72$ GPa, (*a*) determine the modulus of resilience of the alloy, (*b*) the modulus of toughness of the alloy.

SOLUTION

(a) $\sigma_Y = E\varepsilon_Y$

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{1}{2}E\varepsilon_Y^2 = \frac{1}{2}(72\times10^9)(0.006)^2$$

$$= 1296\times10^3 \ \text{N}\cdot\text{m/m}^3 = 1296 \ \text{kJ/m}^3$$

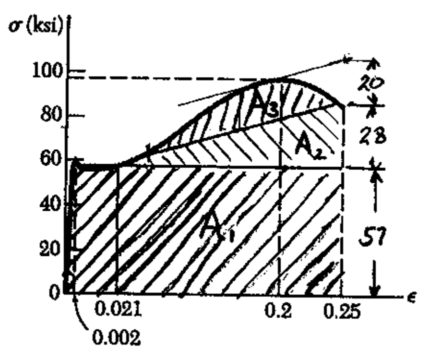
(b) Modulus of toughness = total area under the stress-strain curve

The average ordinate of the stress-strain curve is 500 MPa = 500×10^6 N/m²

The area under the curve is $A = (500\times10^6)(0.18) = 90\times10^6$ N/m²

modulus of toughness $= 90\times10^6 \ \text{J/m}^3 = 90 \ \text{MJ/m}^3$

PROBLEM 11.6



11.6 The stress-strain diagram shown has been drawn from data obtained during the tensile test of a specimen of structural steel. Using $E = 29\times10^6$ psi, (*a*) determine the modulus of resilience of the steel, (*b*) determine the modulus of toughness of the steel.

SOLUTION

(a) $\sigma_Y = E\varepsilon_Y$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{1}{2}E\varepsilon_Y^2 = \frac{1}{2}(29\times10^6)(0.002)^2$$

$$= 58.0 \ \text{in}\cdot\text{lb/in}^3$$

(b) Modulus of toughness = total area under the stress-strain curve

$A_1 = (57)(0.25-0.002) = 14.14$ kips/in² $= 14.14$ in·kip/in³

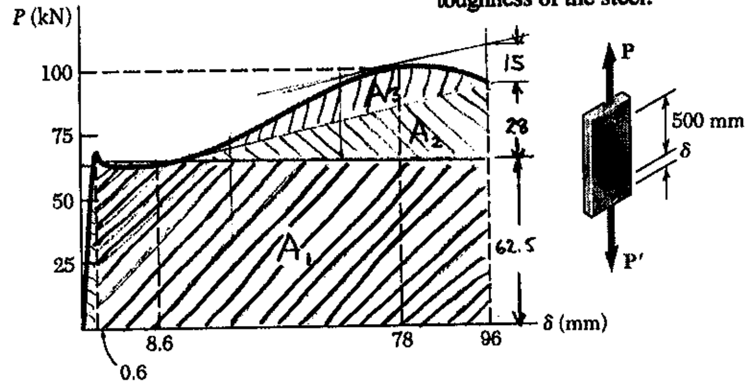
$A_2 = \frac{1}{2}(28)(0.25-0.021) = 3.21$ kips/in² $= 3.21$ in·kip/in³

$A_3 = \frac{2}{3}(20)(0.25-0.075) = 2.33$ kips/in² $= 2.33$ in·kip/in³

modulus of toughness $= U_Y + A_1 + A_2 + A_3 \approx 20$ in kip/in³

PROBLEM 11.7

11.7 The load-deformation diagram shown has been drawn from data obtained during the tensile test of a specimen of structural steel. Knowing that the cross-sectional area of the specimen is 250 mm² and that the deformation was measured using a 500-mm gage length, determine (*a*) the modulus of resilience of the steel, (*b*) the modulus of toughness of the steel.



SOLUTION

Assuming that yielding occurs at $P = 62.5$ kN and $\delta = 0.6$ mm

$$U_Y = \tfrac{1}{2}(62.5\times10^3)(0.6\times10^{-3})$$
$$= 18.75 \text{ N}\cdot\text{m}$$
$$= 18.75 \text{ J}$$

Volume of stressed material $\quad V = AL = (250)(500) = 125\times10^3 \text{ mm}^3$
$$= 125\times10^{-6} \text{ m}^3$$

$$u_Y = \frac{U_Y}{V} = \frac{18.75}{125\times10^{-6}} = 150\times10^3 = 150 \text{ kJ/m}^3$$ ◄

$$A_1 = (62.5\times10^6)(96\times10^{-3}) = 6\times10^3 \text{ N}\cdot\text{m} = 6\times10^3 \text{ J}$$

$$A_2 = \tfrac{1}{2}(28\times10^6)(96-8.6)\times10^{-3} = 1.22\times10^3 \text{ N}\cdot\text{m} = 1.22\times10^3 \text{ J}$$

$$A_3 = \tfrac{2}{3}(15\times10^6)(61\times10^{-3}) = 0.61\times10^3 \text{ N}\cdot\text{m} = 0.61\times10^3 \text{ J}$$

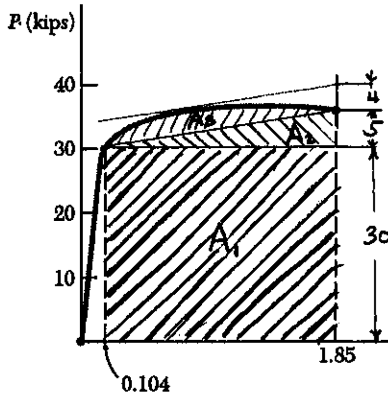
Total energy $\quad U = U_Y + A_1 + A_2 + A_3 = 7.85\times10^3$ J

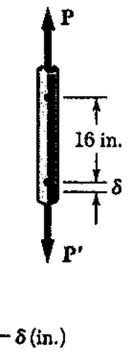
modulus of toughness $= \dfrac{U}{V} = \dfrac{7.85\times10^3}{125\times10^{-6}} = 63\times10^6 \text{ J/m}^3$

$$= 63 \text{ MJ/m}^3$$ ◄

PROBLEM 11.8

11.8 The load-deformation diagram shown has been drawn from data obtained during the tensile test of a 0.75-in.-diameter rod of an aluminum alloy. Knowing that .the deformation was measured using a 16-in. gage length, determine (*a*) the modulus of resilience of the alloy, (*b*) modulus of toughness of the alloy.





SOLUTION

Volume of stressed material involved in the measurement.

$$V = \frac{\pi}{4} d^2 L$$

$$= \frac{\pi}{4}(0.75)^2(16) = 7.0686 \text{ in}^3$$

(a) Modulus of resilience.

$$P_Y = 30 \text{ kips}, \quad \delta_Y = 0.104 \text{ in}$$

$$U_Y = \frac{1}{2} P_Y \delta_Y = \frac{1}{2}(30)(0.104) = 1.56 \text{ in} \cdot \text{kip} = 1560 \text{ in} \cdot \text{lb}.$$

modulus of resilience $\quad u_Y = \frac{U_Y}{V} = \frac{1560}{7.0686} = 221 \text{ in} \cdot \text{lb/in}^3$ ◄

(b) modulus of toughness

$$A_1 = (30)(1.85 - 0.104) = 52.38 \text{ kip} \cdot \text{in} = 52380 \text{ in} \cdot \text{lb/in}^3$$

$$A_2 = \tfrac{1}{2}(5)(1.85 - 0.104) = 4.365 \text{ kip} \cdot \text{in} = 4365 \text{ in lb/in}^3$$

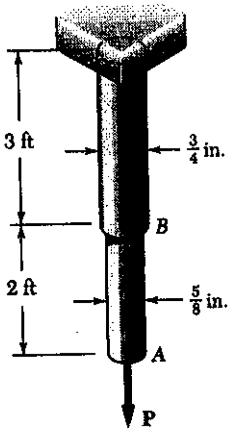
$$A_3 = \tfrac{2}{3}(4)(1.85 - 0.104) = 4.656 \text{ kip} \cdot \text{in} = 4656 \text{ in lb/in}$$

$$U = U_Y + A_1 + A_2 + A_3 = 62961 \text{ in} \cdot \text{lb/in}^3$$

modulus of toughness $\quad \frac{U}{V} = \frac{62961}{7.0686} = 8900 \text{ in} \cdot \text{lb/in}^3$ ◄

PROBLEM 11.9

11.9 Using $E = 29 \times 10^6$ psi, determine (*a*) the strain energy of the steel rod *ABC* when $P = 8$ kips, (*b*) the corresponding strain energy density in portions *AB* and *BC* of the rod.



SOLUTION

$P = 8$ kips, $E = 29 \times 10^3$ ksi

$A = \frac{\pi}{4} d^2$, $V = AL$, $\sigma = \frac{P}{A}$, $u = \frac{\sigma^2}{2E}$

$U = uV$

Portion	d in.	L in	A in^2	V in^3	σ ksi	u in·kip/in^3	U in·kip
AB	0.625	24	0.3608	7.363	26.08	11.72×10^{-3}	86.32×10^{-3}
BC	0.75	36	0.4418	15.904	18.11	5.65×10^{-3}	89.92×10^{-3}
Σ							176.24×10^{-3}

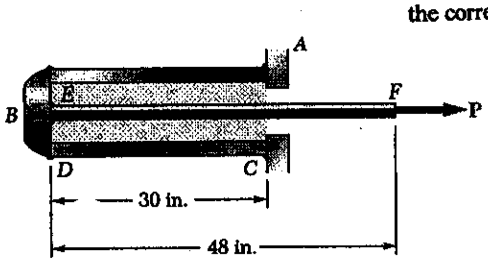
(a) $U = 176.2 \; 10^{-3}$ in·kip $= 176.2$ in·lb ◄

(b) In AB $\quad u = 11.72 \times 10^{-3}$ in·kip/in^3 $= 11.72$ in·lb/in^3 ◄

In BC $\quad u = 5.65 \times 10^{-3}$ in·kip/in^3 $= 5.65$ in·lb/in^3 ◄

PROBLEM 11.10

11.10 A 30-in. length of aluminum pipe of cross-sectional area 1.85 in^2 is welded to a fixed support *A* and to a rigid cap *B*. The steel rod *EF*, of 0.75-in. diameter, is welded to cap *B*. Knowing that the modulus of elasticity is 29×10^6 psi for steel and 10.6×10^6 for aluminum, determine (*a*) the total strain energy of the system when $P = 10$ kips, (*b*) the corresponding strain-energy density in the pipe *CD* and in the rod *EF*.



SOLUTION

For EF: $A = \frac{\pi}{4} d^2 = 0.4418 \; in^2$

CD: $U_{CD} = \dfrac{P^2 L}{2EA} = \dfrac{(10 \times 10^3)^2 (30)}{(2)(10.6 \times 10^6)(1.85)} = 76.49$ in·lb

EF: $U_{EF} = \dfrac{P^2 L}{2EA} = \dfrac{(10 \times 10^3)^2 (48)}{(2)(29 \times 10^6)(0.4418)} = 187.33$ in·lb

Total: $U = U_{CD} + U_{EF} = 264$ in·lb ◄

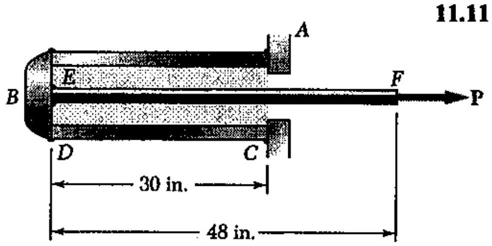
CD: $\sigma = -\dfrac{10000}{1.85} = -5405$ psi, $u = \dfrac{\sigma^2}{2E} = \dfrac{(-5405)^2}{(2)(10.6 \times 10^6)} = 1.378$ in·lb/in^3 ◄

EF: $\sigma = \dfrac{10000}{0.4418} = 22635$ psi, $u = \dfrac{\sigma^2}{2E} = \dfrac{22635^2}{(2)(29 \times 10^6)} = 8.83$ in·lb/in^3 ◄

PROBLEM 11.11

11.10 A 30-in. length of aluminum pipe of cross-sectional area 1.85 in² is welded to a fixed support *A* and to a rigid cap *B*. The steel rod *EF*, of 0.75-in. diameter, is welded to cap *B*. Knowing that the modulus of elasticity is 29×10^6 psi for steel and 10.6×10^6 for aluminum, determine (*a*) the total strain energy of the system when $P = 10$ kips, (*b*) the corresponding strain-energy density in the pipe *CD* and in the rod *EF*.

11.11 Solve Prob. 11.10, when $P = 8$ kips.



SOLUTION

For EF: $A = \frac{\pi}{4} d^2 = 0.4418 \text{ in}^2$

CD: $U_{CD} = \dfrac{P^2 L}{2EA} = \dfrac{(-8000)^2 (30)}{(2)(10.6 \times 10^6)(1.85)} = 48.95 \text{ in}\cdot\text{lb}$

EF: $U_{EF} = \dfrac{P^2 L}{2EA} = \dfrac{(8000)^2 (48)}{(2)(29 \times 10^6)(0.4418)} = 119.89 \text{ in}\cdot\text{lb}$

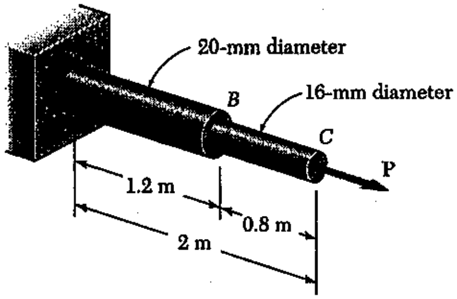
Total $U = U_{CD} + U_{EF} = 168.8 \text{ in}\cdot\text{lb}$ ◄

CD: $\sigma = -\dfrac{8000}{1.85} = -4324 \text{ psi}, \quad u = \dfrac{\sigma^2}{2E} = \dfrac{(-4324)^2}{(2)(10.6 \times 10^6)} = 0.882 \text{ in}\cdot\text{lb/in}^3$ ◄

EF: $\sigma = \dfrac{8000}{0.4418} = 18108 \text{ psi}, \quad u = \dfrac{\sigma^2}{2E} = \dfrac{(18108)^2}{(2)(29 \times 10^6)} = 5.65 \text{ in}\cdot\text{lb/in}^3$ ◄

PROBLEM 11.12

11.12 Using $E = 200$ GPa, determine (*a*) the strain energy of the steel rod *ABC* when $P = 25$ kN, (*b*) the corresponding strain-energy density of portions *AB* and *BC* of the rod.



SOLUTION

$$A_{AB} = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16\times10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2 = 201.06\times10^{-6} \text{ m}^2$$

$$P = 25\times10^3 \text{ N}$$

$$U = \Sigma \frac{P^2L}{2EA}$$

$$= \frac{(25\times10^3)^2(1.2)}{(2)(200\times10^9)(314.16\times10^{-6})} + \frac{(25\times10^3)^2(0.8)}{(2)(200\times10^9)(201.06\times10^{-6})}$$

(a) $U = 5.968 + 6.213 = 12.18 \text{ N-m} = 12.18 \text{ J}$

(b) $\sigma_{AB} = \frac{P}{A_{AB}} = \frac{25\times10^3}{314.16\times10^{-6}} = 79.58\times10^6 \text{ Pa}$

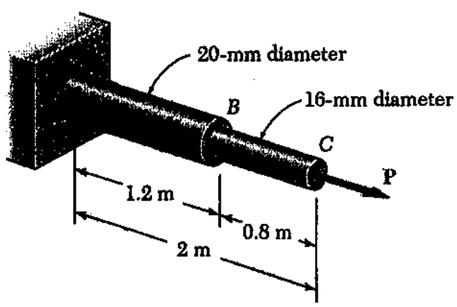
$$u_{AB} = \frac{\sigma_{AB}^2}{2E} = \frac{(79.58\times10^6)^2}{(2)(200\times10^9)} = 15.83\times10^3 = 15.83 \text{ kJ/m}^3$$

$$\sigma_{BC} = \frac{P}{A_{AB}} = \frac{25\times10^3}{201.16\times10^{-6}} = 124.28\times10^6 \text{ Pa}$$

$$u_{BC} = \frac{\sigma_{BC}^2}{2E} = \frac{(124.28\times10^6)^2}{(2)(200\times10^9)} = 38.6\times10^3 = 38.6 \text{ kJ/m}^3$$

PROBLEM 11.13

11.13 The steel rod *ABC* is made of a steel for which the yield strength is $\sigma_Y = 250$ MPa and the modulus of elasticity is $E = 200$ GPa. Determine, for the loading shown, the maximum strain energy that can be acquired by the rod without causing any permanent deformation.



SOLUTION

$$A_{AB} = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16\times10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2 = 201.06\times10^{-6} \text{ m}^2$$

$$P = \sigma_Y A_{min} = (250\times10^6)(201.06\times10^{-6})$$

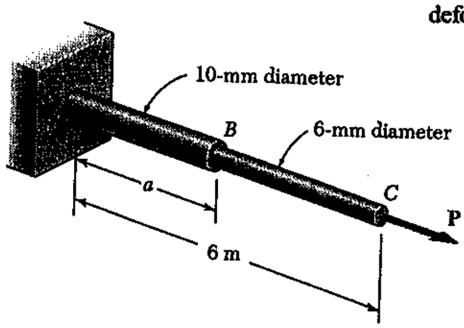
$$= 50.265\times10^3 \text{ Pa}$$

$$U = \Sigma \frac{P^2L}{2EA} = \frac{(50265)^2(1.2)}{(2)(200\times10^9)(314.16\times10^{-6})} + \frac{(50265)^2(0.8)}{(2)(200\times10^9)(201.06\times10^{-6})}$$

$$= 24.13 + 25.13 = 49.3 \text{ J}$$

PROBLEM 11.14

11.14 The steel rods *AB* and *BC* are made of a steel for which the yield strength is $\sigma_Y = 300$ MPa and the modulus of elasticity is $E = 200$ GPa. Determine the maximum strain energy that can be acquired by the assembly without causing any permanent deformation when the length *a* of rod *AB* is (*a*) 2 m, (*b*) 4m.



SOLUTION

$$A_{AB} = \frac{\pi}{4}(10)^2 = 78.54 \text{ mm}^2 = 78.54\times10^{-6} \text{ m}^2$$
$$A_{BC} = \frac{\pi}{4}(6)^2 = 28.274 \text{ mm}^2 = 28.274\times10^{-6} \text{ m}^2$$
$$P = \sigma_Y A_{min} = (300\times10^6)(28.274\times10^{-6}) = 8.4822\times10^3 \text{ N}$$

$$U = \Sigma \frac{P^2 L}{2EA}$$

(a) $a = 2\text{m}$ $\quad L-a = 6-2 = 4\text{m}$

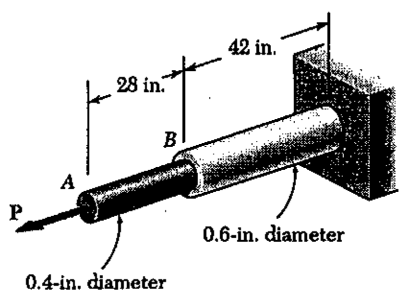
$$U = \frac{(8.4822\times10^3)^2(2)}{(2)(200\times10^9)(78.54\times10^{-6})} + \frac{(8.4822\times10^3)^2(4)}{(2)(200\times10^9)(28.274\times10^{-6})}$$
$$= 4.5803 + 25.4466 = 30.0 \text{ N}\cdot\text{m} = 30.0 \text{ J}$$

(b) $a = 4\text{m}$ $\quad L-a = 6-4 = 2\text{m}$

$$U = \frac{(8.4822\times10^3)^2(4)}{(2)(200\times10^9)(78.54\times10^{-6})} + \frac{(8.4822\times10^3)^2(2)}{(2)(200\times10^9)(28.274\times10^{-6})}$$
$$= 9.1606 + 12.7233 = 21.9 \text{ N}\cdot\text{m} = 21.9 \text{ J}$$

PROBLEM 11.15

11.15 Rod *AB* is made of a steel for which the yield strength is $\sigma_Y = 65$ ksi and the modulus of elasticity is $E = 29\times10^6$ psi; rod *BC* is made of an aluminum alloy for which $\sigma_Y = 40$ ksi and $E = 10.6\times10^6$psi. Determine the maximum strain energy that can be acquired by the composite rod *ABC* without causing permanent deformation.



SOLUTION

$$A_{AB} = \frac{\pi}{4}(0.4)^2 = 0.12566 \text{ in}^2 \qquad E = 29000 \text{ ksi}$$
$$A_{BC} = \frac{\pi}{4}(0.6)^2 = 0.28274 \text{ in}^2 \qquad E = 10600 \text{ ksi}$$

$P_{all} = \sigma_Y A$ for each part

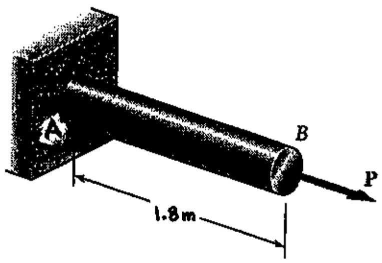
AB: $P_{all} = (65)(0.12566) = 8.1679$ kips

BC: $P_{all} = (40)(0.28274) = 11.3096$ kips

Use smaller value $\quad P = 8.1679$ kips

$$U = \Sigma\frac{P^2L}{2EA} = \frac{(8.1679)^2(28)}{(2)(29000)(0.12566)} + \frac{(8.1679)^2(42)}{(2)(10600)(0.28274)}$$
$$= 256.3\times10^{-3} + 467.5\times10^{-3} = 724\times10^{-3} \text{ in}\cdot\text{kip} = 724 \text{ in}\cdot\text{lb}$$

PROBLEM 11.16



11.16 Rod *AB* is made of a steel for which the yield strength is $\sigma_Y = 300$ MPa and the modulus of elasticity is $E = 200$ GPa. Knowing that a strain energy of 10 J must be acquired by the rod when the axial load **P** is applied, determine the diameter of the rod for which the factor of safety with respect to permanent deformation is six.

SOLUTION

For factor of safety of six on the energy

$$U_Y = (6)(10) = 60 \text{ J}$$

$$u_Y = \frac{\sigma_Y^2}{2E} = \frac{(300\times10^6)^2}{(2)(200\times10^9)} = 225\times10^3 \text{ J/m}^3$$

$U_Y = A L u_Y$

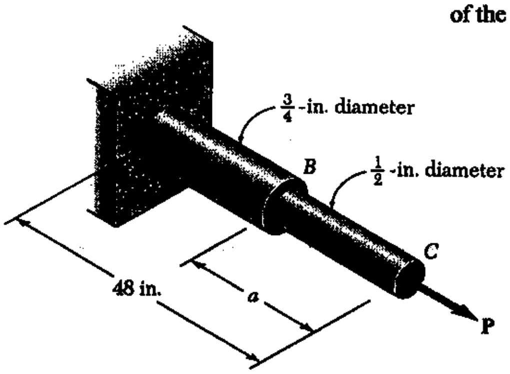
$A = \frac{\pi}{4} d^2$

$$A = \frac{U_Y}{L u_Y} = \frac{60}{(1.8)(225\times10^3)} = 148.148\times10^{-6} \text{ m}^2$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(148.148\times10^{-6})}{\pi}} = 13.73\times10^{-3} \text{ m}$$

$$= 13.73 \text{ mm}$$

PROBLEM 11.17



11.17 The rod *ABC* is made of a steel for which the yield strength is $\sigma_Y = 65$ ksi and the modulus of elasticity is $E = 29\times10^6$ psi. Knowing that a strain energy of 90 in · lb must be acquired by the rod as the axial load **P** is applied, determine the factor of safety of the rod with respect to permanent deformation when $a = 18$ in.

SOLUTION

$$A_{AB} = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$A_{BC} = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$$

$$P_Y = \sigma_Y A_{min} = (65000)(0.19635) = 12763 \text{ lb.}$$

$$U_Y = \Sigma \frac{P_Y^2 L}{2EI}$$

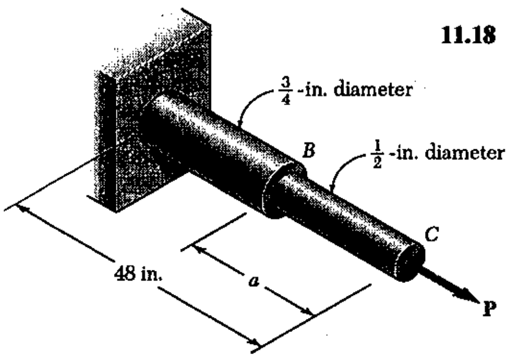
$$U_Y = \frac{(12763)^2(48-18)}{(2)(29\times10^6)(0.4418)} + \frac{(12763)^2(18)}{(2)(29\times10^6)(0.19635)} = 448 \text{ in.lb.}$$

$$F.S. = \frac{U_Y}{U_{design}} = \frac{448}{90} = 4.98$$

PROBLEM 11.18

11.18 The rod ABC is made of a steel for which the yield strength is $\sigma_Y = 65$ ksi and the modulus of elasticity is $E = 29 \times 10^6$ psi. Knowing that a strain energy of 90 in · lb must be acquired by the rod as the axial load **P** is applied, determine the factor of safety of the rod with respect to permanent deformation when $a = 18$ in.

11.18 Solve Prob. 11.17, assuming that $a = 30$ in.



SOLUTION

$$A_{AB} = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$A_{BC} = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$$

$$P_Y = \sigma_Y A_{min} = (65000)(0.19635) = 12763 \text{ lb.}$$

$$U_Y = \sum \frac{P_Y^2 L}{2EA} = \frac{(12763)^2(48-30)}{(2)(29\times10^6)(0.4418)} + \frac{(12763)^2(30)}{(2)(29\times10^6)(0.19635)} = 543.5 \text{ in}\cdot\text{lb}$$

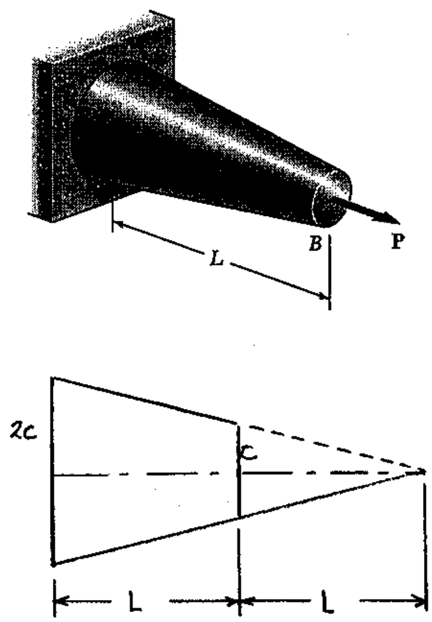
$$F.S. = \frac{U_Y}{U_{design}} = \frac{543.5}{90} = 6.04$$

PROBLEM 11.19

11.19 Show by integration that the strain energy of the tapered rod AB is

$$U = \frac{1}{4}\frac{P^2 L}{EA_{min}}$$

where A_{min} is the cross-sectional area at end B.



SOLUTION

radius $\quad r = \frac{cx}{L} \qquad A_{min} = \pi c^2$

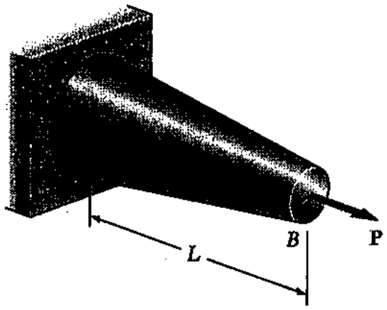
$$A = \pi r^2 = \frac{\pi c^2}{L^2} x^2$$

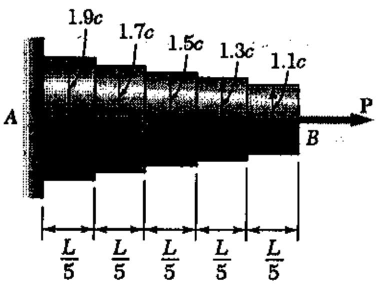
$$U = \int_L^{2L} \frac{P^2 dx}{2EA} = \frac{P^2}{2E}\int_L^{2L} \frac{L^2}{\pi c^2}\frac{dx}{x^2}$$

$$= \frac{P^2 L^2}{2E\pi c^2}\left(-\frac{1}{x}\right)\Big|_L^{2L}$$

$$= \frac{P^2 L^2}{2EA_{min}}\left(-\frac{1}{2L}+\frac{1}{L}\right) = \frac{P^2 L^2}{4EA_{min}}$$

PROBLEM 11.20





11.19 Show by integration that the strain energy of the tapered rod AB is

$$U = \frac{1}{4}\frac{P^2 L}{EA_{min}}$$

where A_{min} is the cross-sectional area at end B.

11.20 Solve Prob. 11.19, using the stepped rod shown as an approximation of the tapered rod. What is the percentage error in the answer obtained?

SOLUTION

$$A_i = \pi r_i^2 \qquad A_{min} = \pi c^2$$

$$U = \sum \frac{P^2 l_i}{2EA_i} = \frac{P^2(L/5)}{2E}\sum\frac{1}{A_i}$$

$$= \frac{P^2 L}{10\pi E}\sum\frac{1}{r_i^2}$$

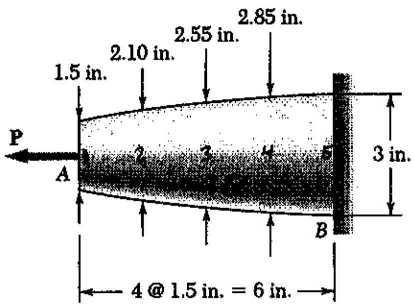
$$= \frac{P^2 L}{10\pi E}\left\{\frac{1}{(1.9c)^2} + \frac{1}{(1.7c)^2} + \frac{1}{(1.5c)^2} + \frac{1}{(1.3c)^2} + \frac{1}{(1.1c)^2}\right\}$$

$$= \frac{P^2 L}{10E(\pi c^2)}\left\{2.4856\right\} = 0.24856\frac{P^2 L}{EA_{min}}$$

$$\%\ \text{error} = \frac{0.24856 - 0.25}{0.25}\times 100\% = -0.575\ \%$$

PROBLEM 11.21

11.21 Using $E = 10.6 \times 10^6$ psi, determine by approximate means the maximum strain energy that can be acquired by the aluminum rod shown if the allowable normal stress is $\sigma_{all} = 22$ ksi.



SOLUTION

$$A_{min} = \frac{\pi}{4}(1.5)^2 = 1.7671 \text{ in}^2$$

$$\sigma_{all} = 22000 \text{ psi}$$

$$P_{all} = \sigma_{all} A_{min} = 38877 \text{ lb.}$$

$$U = \int \frac{P^2 dx}{2EA} = \frac{P^2}{2E}\int \frac{dx}{\frac{\pi}{4}d^2} = \frac{2P^2}{\pi E}\int \frac{dx}{d^2}$$

Use Simpson's rule to compute the integral $\qquad h = 0.15$ in

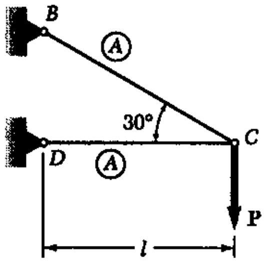
Section	d (in)	$1/d^2$ (in^{-2})	multiplier	$m(1/d^2)$ (in^{-2})
1	1.50	0.4444	1	0.4444
2	2.10	0.22675	4	0.9070
3	2.55	0.15379	2	0.3076
4	2.85	0.12311	4	0.4924
5	3.00	0.11111	1	0.1111
Σ				2.2625

$$\int \frac{dx}{d^2} = \frac{h}{3}\sum m\left(\frac{1}{d^2}\right) = \frac{1.5}{3}(2.2625) = 1.13125 \text{ in}^{-1}$$

$$U = \frac{(2)(38877)^2(1.13125)}{\pi(10.6\times 10^6)} = 102.7 \text{ in}\cdot\text{lb.}$$

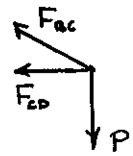
PROBLEM 11.22

11.22 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load **P** is applied.



SOLUTION

Joint C



$+\uparrow \Sigma F_y = 0$

$\frac{1}{2}F_{BC} - P = 0 \qquad F_{BC} = 2P$

$+\rightarrow \Sigma F_x = 0$

$-F_{CD} - \frac{\sqrt{3}}{2}F_{CD} = 0 \qquad F_{BC} = -\sqrt{3}\,P$

$$U = \Sigma \frac{F^2L}{2EA} = \frac{1}{2E}\Sigma\frac{F^2L}{A}$$

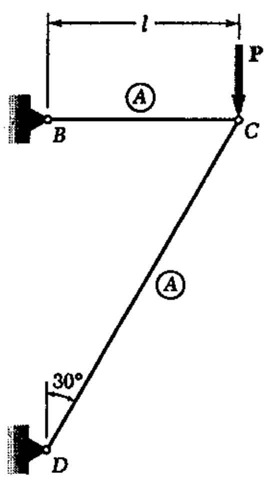
Member	F	L	A	F^2L/A
BC	$2P$	$\frac{2}{\sqrt{3}}l$	A	$\frac{8}{\sqrt{3}}P^2l/A$
CD	$-\sqrt{3}P$	l	A	$3\,P^2l/A$
Σ				$7.62P^2l/A$

$$U = \frac{1}{2E}\left(7.62\frac{P^2l}{A}\right)$$

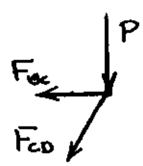
$$= 3.81\frac{P^2l}{EA} \blacktriangleleft$$

PROBLEM 11.23

11.23 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load **P** is applied.



SOLUTION



Joint C

$+\uparrow \Sigma F_y = 0$

$-\frac{\sqrt{3}}{2}F_{CD} - P = 0 \qquad F_{CD} = -\frac{2}{\sqrt{3}}P$

$+\rightarrow \Sigma F_x = 0$

$-F_{BC} - \frac{1}{2}F_{CD} = 0 \qquad F_{BC} = \frac{1}{\sqrt{3}}P$

$$U = \Sigma\frac{F^2L}{2EA} = \frac{1}{2E}\Sigma\frac{F^2L}{A}$$

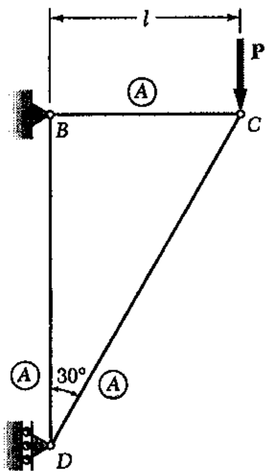
Member	F	L	A	F^2L/A
BC	$\frac{1}{\sqrt{3}}P$	l	A	$\frac{1}{3}P^2l/A$
CD	$-\frac{2}{\sqrt{3}}P$	$2l$	A	$\frac{8}{3}P^2l/A$
Σ				$3P^2l/A$

$$U = \frac{1}{2E}\left(3\frac{P^2l}{A}\right)$$

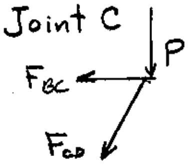
$$= 1.5\frac{P^2l}{EA} \blacktriangleleft$$

PROBLEM 11.24

11.24 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load **P** is applied.



Joint C



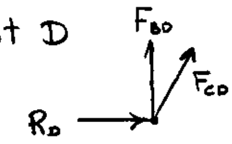
$+\uparrow \Sigma F_y = 0 \qquad -\frac{\sqrt{3}}{2}F_{CD} - P = 0$

$F_{CD} = -\frac{2}{\sqrt{3}}P$

$\xrightarrow{+} \Sigma F_x = 0 \qquad -F_{BC} - \frac{1}{2}F_{CD} = 0$

$F_{BC} = \frac{1}{\sqrt{3}}P$

Joint D



$+\uparrow \Sigma F_y = 0$

$F_{BD} + \frac{\sqrt{3}}{2}F_{CD} = 0$

$F_{BD} = P$

Member	F	L	A	F^2L/A
BC	$\frac{1}{\sqrt{3}}P$	l	A	$\frac{1}{3}P^2l/A$
CD	$-\frac{2}{\sqrt{3}}P$	$2l$	A	$\frac{8}{3}P^2l/A$
BD	P	$\sqrt{3}l$	A	$\sqrt{3}P^2l/A$
Σ				$4.732\,P^2l/A$

$U = \Sigma \frac{1}{2}\frac{F^2L}{EA}$

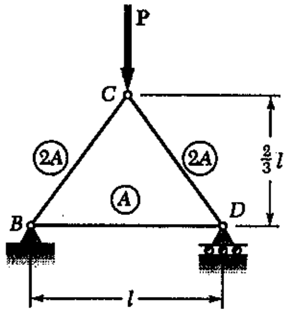
$= \frac{1}{2E}\Sigma\frac{F^2L}{A}$

$= \frac{1}{2E}\left(4.732\frac{P^2l}{A}\right)$

$= 2.37\frac{P^2l}{EA}$

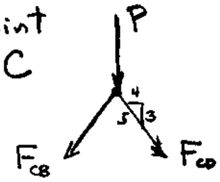
PROBLEM 11.25

11.25 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load **P** is applied



SOLUTION

Joint C



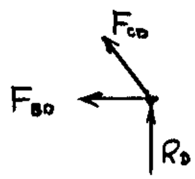
$$\xrightarrow{+} \Sigma F_x = 0 \quad \tfrac{3}{5}F_{CD} - \tfrac{3}{5}F_{CB} = 0$$

$$F_{CB} = F_{CD}$$

$$+\uparrow \Sigma F_y = 0 \quad -P - 2\cdot\tfrac{4}{5}F_{CD} = 0$$

$$F_{CB} = F_{CD} = -\tfrac{5}{8}P$$

Joint D



$$\xrightarrow{+} \Sigma F_x = 0$$

$$-F_{BD} - \tfrac{3}{5}F_{CD} = 0$$

$$F_{BD} = -\tfrac{3}{5}\cdot\tfrac{5}{8}P = \tfrac{3}{8}P$$

$$U = \Sigma \frac{F^2L}{2EA} = \frac{1}{2E}\Sigma\frac{F^2L}{A}$$

Member	F	L	A	F^2L/A
CB	$-\frac{5}{8}P$	$\frac{5}{6}l$	2A	$\frac{125}{768}P^2l/A$
CD	$-\frac{5}{8}P$	$\frac{5}{6}l$	2A	$\frac{125}{768}P^2l/A$
BD	$\frac{3}{8}P$	l	A	$\frac{9}{64}P^2l/A$
Σ				$\frac{179}{384}P^2l/A$

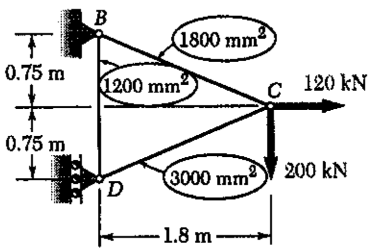
$$= \frac{1}{2E}\left(\frac{179}{384}\frac{P^2l}{A}\right)$$

$$= \frac{179}{768}\frac{P^2l}{EA}$$

$$= 0.233\frac{P^2l}{EA}$$

PROBLEM 11.26

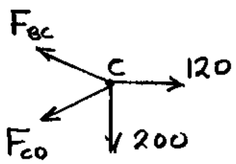
11.26 In the truss shown, all members are made of aluminum and have the uniform cross-sectional area indicated. Using $E = 72$ GPa, determine the strain energy of the truss for the loading shown..



SOLUTION

$$l_{BC} = l_{CD} = \sqrt{1.8^2 + 0.75^2} = 1.95 \text{ m}$$

Joint C



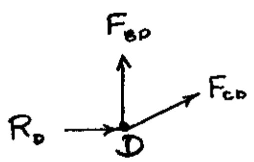
$$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad -\frac{1.8}{1.95} F_{BC} - \frac{1.8}{1.95} F_{CD} + 120 = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0 \quad \frac{0.75}{1.95} F_{BC} - \frac{0.75}{1.95} F_{CD} - 200 = 0 \qquad (2)$$

Solving (1) and (2) simultaneously.

$$F_{BC} = 325 \text{ kN} \qquad F_{CD} = -195 \text{ kN}$$

Joint D



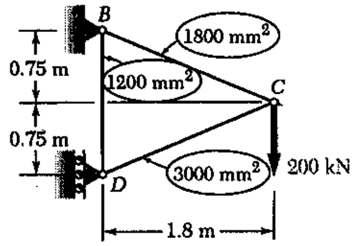
$$+\uparrow \Sigma F_y = 0 \qquad F_{BD} + \frac{0.75}{1.95} F_{CD} = 0 \qquad F_{BD} = 75 \text{ kN}$$

$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2E} \Sigma \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	A (10^{-6} m²)	F^2L/A (N²/m)
BC	325	1.95	1800	114.43×10^{12}
BD	75	1.5	1200	7.03×10^{12}
CD	−195	1.95	3000	24.72×10^{12}
Σ				146.18×10^{12}

$$U = \frac{146.18 \times 10^{12}}{(2)(72 \times 10^9)} = 1.015 \times 10^3 \text{ N·m} = 1015 \text{ J}$$

PROBLEM 11.27



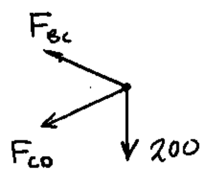
11.27 In the truss shown, all members are made of aluminum and have the uniform cross-sectional area indicated. Using $E = 72$ GPa, determine the strain energy of the truss for the loading shown..

11.27 Solve Prob. 11.26, assuming that the 120-kN load is removed.

SOLUTION

$$\ell_{BC} = \ell_{CD} = \sqrt{1.8^2 + 0.75^2} = 1.95 \text{ m}$$

Joint C



$$+\rightarrow \Sigma F_x = 0 \qquad -\frac{1.8}{1.95} F_{BC} - \frac{1.8}{1.95} F_{CD} = 0$$

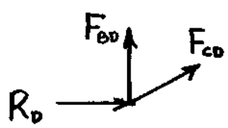
$$+\uparrow \Sigma F_y = 0 \qquad \frac{0.75}{1.95} F_{BC} - \frac{0.75}{1.95} F_{CD} - 200 = 0$$

Solving (1) and (2) simultaneously.

$$F_{BC} = 260 \text{ kN} \qquad F_{CD} = -260 \text{ kN}$$

Joint D

$$+\uparrow \Sigma F_y = 0 \qquad F_{BD} + \frac{0.75}{1.95} F_{CD} = 0 \qquad F_{BD} = 100 \text{ kN}$$



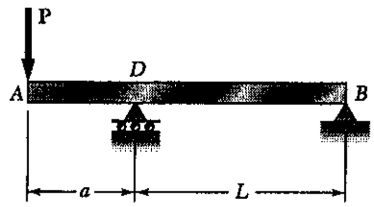
$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2E} \Sigma \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	A (10^{-6} m²)	F^2L/A (N²/m)
BC	260	1.95	1800	73.23×10^{12}
BD	100	1.5	1200	12.50×10^{12}
CD	−260	1.95	3000	43.94×10^{12}
Σ				129.67×10^{12}

$$U = \frac{129.67 \times 10^{12}}{(2)(72 \times 10^9)} = 900 \text{ N}\cdot\text{m} = 900 \text{ J}$$

PROBLEM 11.28

11.28 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.

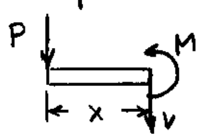


SOLUTION

$$+\circlearrowleft \Sigma M_D = 0$$

$$aP + LR_B = 0 \qquad R_B = -\frac{aP}{L} = \frac{aP}{L}\downarrow$$

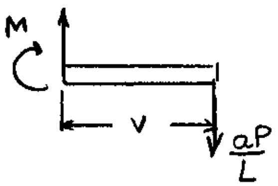
Over portion AD : $M = -Px$



$$U_{AD} = \int_0^a \frac{M^2}{2EI}dx = \frac{1}{2EI}\int_0^a P^2x^2dx = \frac{P^2}{2EI}\frac{x^3}{3}\Big|_0^a$$

$$= \frac{P^2a^3}{6EI}$$

Over portion DB : $M = -\frac{aP}{L}v$



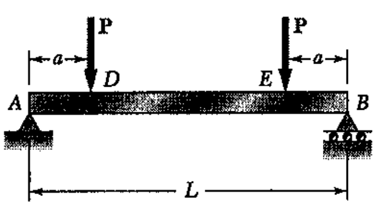
$$U_{DB} = \int_0^L \frac{M^2}{2EI}dv = \frac{1}{2EI}\int_0^L \frac{a^2P^2}{L^2}v^2dv$$

$$= \frac{P^2a^2}{2EIL^2}\int_0^L v^2dv = \frac{P^2a^2}{2EIL^2}\frac{v^3}{3}\Big|_0^L = \frac{P^2a^2L}{6EI}$$

Total $\quad U = U_{AD} + U_{DB} = \frac{P^2a^2}{6EI}(a+L)$ ◄

PROBLEM 11.29

11.29 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



SOLUTION

Symmetric beam and loading $\quad R_A = R_B$.

$$+\uparrow \Sigma F_y = 0 \qquad R_A + R_B - 2P = 0 \qquad R_A = R_B = P$$

Over portion AD : $M = R_A x = Px$

$$U_{AD} = \int_0^a \frac{M^2}{2EI}dx = \frac{P^2}{2EI}\int_0^a x^2dx = \frac{P^2}{2EI}\frac{x^3}{3}\Big|_0^a = \frac{P^2a^3}{6EI}$$

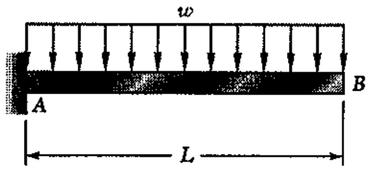
Over portion DE : $M = Pa \qquad U_{DE} = \frac{P^2a^2(L-2a)}{2EI}$

Over portion EB : By symmetry $\quad U_{EB} = U_{AD} = \frac{P^2a^3}{6EI}$

Total $\quad U = U_{AD} + U_{DE} + U_{EB} = \frac{P^2a^2}{6EI}(3L-4a)$ ◄

PROBLEM 11.30

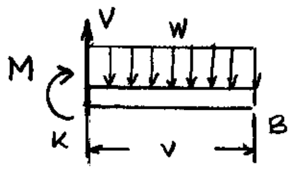
11.30 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam *AB* for the loading shown.



SOLUTION

$$+\circlearrowleft \Sigma M_K = 0 \qquad -M - (wv)\left(\frac{v}{2}\right) = 0$$

$$M = -\frac{1}{2}wv^2$$



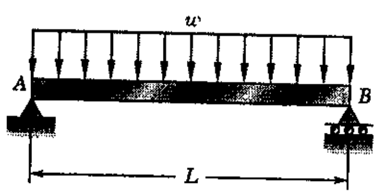
$$U = \int_0^L \frac{M^2}{2EI}\,dv = \frac{1}{2EI}\int_0^L \left(\frac{1}{2}wv^2\right)^2 dv$$

$$= \frac{w^2}{8EI}\int_0^L v^4\,dv = \frac{w^2}{8EI}\,\frac{v^5}{5}\Big|_0^L$$

$$= \frac{w^2L^5}{40EI}$$

PROBLEM 11.31

11.31 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam *AB* for the loading shown.



SOLUTION

$$+\circlearrowleft \Sigma M_B = 0 \qquad -R_A L + (wL)\left(\frac{L}{2}\right) = 0 \qquad R_A = \frac{wL}{2}$$

Bending moment $\quad M = R_A x - \frac{1}{2}wx^2$

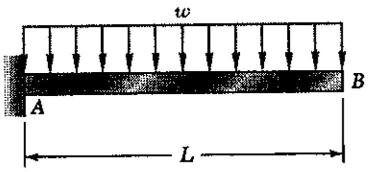
$$= \frac{w}{2}(Lx - x^2)$$

$$U = \int_0^L \frac{M^2}{2EI}\,dx = \frac{w^2}{8EI}\int_0^L (Lx - x^2)^2\,dx$$

$$= \frac{w^2}{8EI}\int_0^L (L^2x^2 - 2Lx^3 + x^4)\,dx = \frac{w^2}{8EI}\left[\frac{L^2x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5}\right]\Big|_0^L$$

$$= \frac{w^2L^5}{8EI}\left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right] = \frac{w^2L^5}{240EI}$$

PROBLEM 11.32

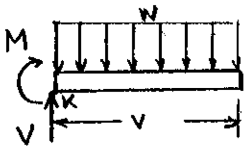


11.32 Assuming that the prismatic beam AB has a rectangular cross section, show that for the given loading the maximum value of the strain energy-density in the beam is

$$u_{max} = 15\frac{U}{V}$$

where U is the strain energy of the beam and V is its volume.

SOLUTION



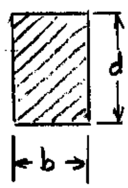
$$+\circlearrowleft \Sigma M_K = 0 \qquad -M - (wv)\frac{v}{2} = 0$$

$$M = -\frac{1}{2}wv^2$$

$$U = \int_0^L \frac{M^2}{2EI}\,dv = \frac{1}{2EI}\int_0^L \left(\frac{1}{2}wv^2\right)^2 dv = \frac{w^2}{8EI}\int_0^L v^4\,dv = \frac{w^2}{8EI}\,\frac{x^5}{5}\Big|_0^L = \frac{w^2L^5}{40\,EI}$$

$$M_{max} = \frac{1}{2}wL^2 \qquad \sigma_{max} = \frac{M_{max}\,c}{I}$$

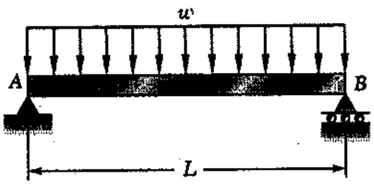
$$u_{max} = \frac{\sigma_{max}^2}{2E} = \frac{M_{max}^2\,c^2}{2EI^2} = \frac{\frac{1}{4}w^2L^4c^2}{2EI^2} = \frac{w^2L^4c^2}{8EI^2}$$



$$\frac{U}{u_{max}} = \frac{LI}{5c^2} = \frac{L\left(\frac{1}{12}bd^3\right)}{5\left(\frac{d}{2}\right)^2} = \frac{1}{15}Lbd = \frac{1}{15}V$$

$$u_{max} = 15\frac{U}{V}$$

PROBLEM 11.33



11.33 Assuming that the prismatic beam AB has a rectangular cross section, show that for the given loading the maximum value of the strain energy-density in the beam is

$$u_{max} = \frac{45}{8}\frac{U}{V}$$

where U is the strain energy of the beam and V is its volume.

SOLUTION

$$+\circlearrowleft M_B = 0 \qquad -R_A L + (wL)\frac{L}{2} = 0 \qquad R_A = \frac{1}{2}wL$$

$$M = R_A x - \frac{1}{2}wL^2 = \frac{1}{2}w(Lx - x^2)$$

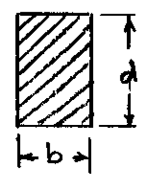
$$U = \int_0^L \frac{M^2}{2EI}dx = \frac{w^2}{8EI}\int_0^L (L^2x^2 - 2Lx^3 + x^4)dx = \frac{w^2}{8EI}\left[\frac{L^2x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5}\right]\Bigg|_0^L$$

$$= \frac{w^2L^5}{8EI}\left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right] = \frac{w^2L^5}{240EI}$$

$$M_{max} = \frac{1}{2}w\left[L\cdot\frac{L}{2} - \left(\frac{L}{2}\right)^2\right] = \frac{1}{8}wL^2 \qquad \sigma_{max} = \frac{M_{max}c}{I} = \frac{wL^2c}{8I}$$

$$u_{max} = \frac{\sigma_{max}^2}{2E} = \frac{w^2L^4c^2}{128EI^2}$$

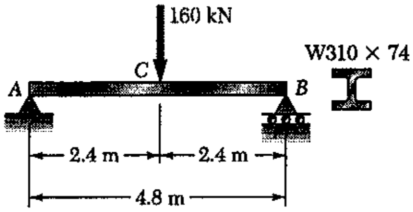
$$\frac{U}{u_{max}} = \frac{8LI}{15c^2} = \frac{8L\left(\frac{1}{12}bd^3\right)}{15\left(\frac{d}{2}\right)^2} = \frac{8}{45}Lbd = \frac{8}{45}V$$



$$u_{max} = \frac{45}{8}\frac{U}{V}$$

PROBLEM 11.34

11.34 Using $E = 200$ GPa, determine the strain energy due to bending for the steel beam and loading shown.

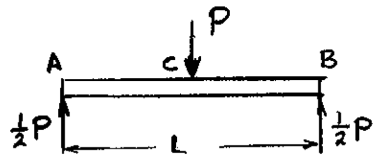


SOLUTION

Over portion AC $\qquad M = \frac{1}{2}Px$

$$U_{AC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dx = \frac{P^2}{8EI}\int_0^{\frac{L}{2}} x^2\,dx$$

$$= \frac{P^2}{8EI}\,\frac{x^3}{3}\Big|_0^{\frac{L}{2}} = \frac{P^2L^3}{192\,EI}$$



By symmetry $\quad U_{CB} = U_{AC} = \frac{P^2L^3}{192\,EI}$

Total: $\quad U = U_{AC} + U_{CB} = \frac{P^2L^3}{96\,EI}$

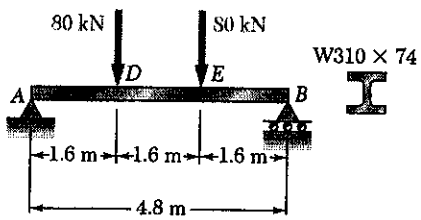
Data: $P = 160\times10^3$ N, $L = 4.8$ m, $E = 200\times10^9$ Pa

$I = 165\times10^6\ \text{mm}^4 = 165\times10^{-6}\ \text{m}^4$

$$U = \frac{(160\times10^3)^2(4.8)^3}{(96)(200\times10^9)(165\times10^{-6})} = 894\ \text{N}\cdot\text{m} = 894\ \text{J}$$

PROBLEM 11.35

11.35 Using $E = 200$ GPa, determine the strain energy due to bending for the steel beam and loading shown.

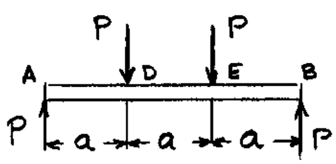


SOLUTION

Over portion AD: $\qquad M = Px$

$$U_{AD} = \int_0^a \frac{M^2}{2EI}\,dx = \frac{1}{2EI}\int_0^a (Px)^2\,dx$$

$$= \frac{P^2}{2EI}\,\frac{x^3}{3}\Big|_0^a = \frac{P^2a^3}{6\,EI}$$



Over portion DE: $\qquad M = Pa$

$$U_{DE} = \frac{(Pa)^2 a}{2EI} = \frac{P^2a^3}{2EI}$$

By symmetry $\quad U_{EB} = U_{AD} = \frac{P^2a^3}{6\,EI}$

$$U = U_{AD} + U_{DE} + U_{EB} = \frac{5}{6}\,\frac{P^2a^3}{EI}$$

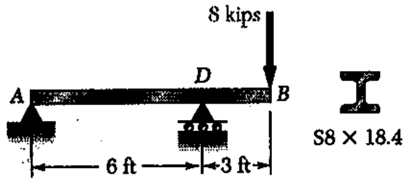
Data: $P = 80\times10^3$ N, $a = 1.6$ m, $E = 200\times10^9$ Pa

$I = 165\times10^6\ \text{mm}^4 = 165\times10^{-6}\ \text{m}^4$

$$U = \frac{5}{6}\,\frac{(80\times10^3)^2(1.6)^3}{(200\times10^9)(165\times10^{-6})} = 662\ \text{N}\cdot\text{m} = 662\ \text{J}$$

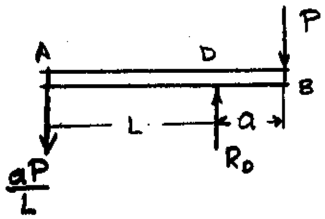
PROBLEM 11.36

11.36 Using $E = 29 \times 10^6$ psi, determine the strain energy due to bending for the steel beam and loading shown.

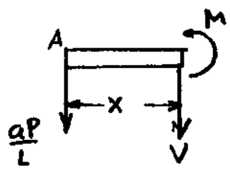


SOLUTION

$\circlearrowright \Sigma M_D = 0 \qquad -R_A L - aP = 0 \qquad R_A = \frac{aP}{L} \downarrow$



Over portion AD $\qquad M = -\frac{aP}{L}x$

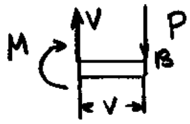


$$U_{AD} = \int_0^L \frac{M^2}{2EI}dx = \frac{1}{2EI}\int_0^L \left(\frac{aP}{L}x\right)^2 dx$$

$$= \frac{P^2a^2}{2EIL^2}\int_0^L x^2 dx$$

$$= \frac{P^2a^2L}{6EI}$$

Over portion DB $\qquad M = -Pv$



$$U_{DB} = \int_0^a \frac{M^2}{2EI}dv = \frac{1}{2EI}\int_0^a P^2x^2dx = \frac{P^2a^3}{6EI}$$

Total: $\quad U = U_{AD} + U_{DB} = \frac{P^2a^2}{6EI}(a+L)$

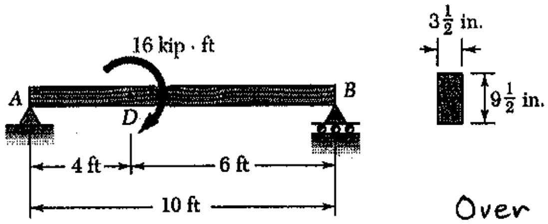
Data: $P = 8000$ lb., $L = 6$ ft. $= 72$ in, $a = 3$ ft $= 36$ in, $E = 29 \times 10^6$ psi

$I = 57.6 \text{ in}^4$

$$U = \frac{(8000)^2(36)^2(72+36)}{(6)(29\times10^6)(57.6)} = 894 \text{ in} \cdot \text{lb.}$$

PROBLEM 11.37

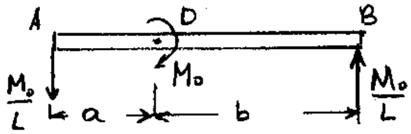
11.37 Using $E = 1.8 \times 10^6$ psi, determine the strain energy due to bending for the timber beam and loading shown.



SOLUTION

$$\circlearrowright \Sigma M_A = 0 \qquad -M_o + R_B L = 0 \qquad R_B = \frac{M_o}{L} \uparrow$$

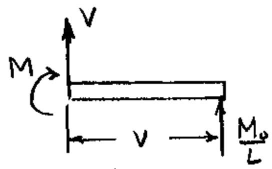
$$R_A = \frac{M_o}{L} \downarrow$$



Over portion AD $\quad M = \frac{M_o x}{L}$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{M_o^2}{2EIL^2} \int_0^a x^2 dx$$

$$= \frac{M_o^2 a^3}{6EIL^2}$$



Over portion DB $\quad M = \frac{M_o}{L} v$

$$U_{DB} = \int_0^b \frac{M^2}{2EI} dx = \frac{M_o^2}{2EIL^2} \int_0^b x^2 dx = \frac{M_o^2 b^3}{6EIL^2}$$

Total $\quad U = U_{AD} + U_{DB} = \dfrac{M_o^2 (a^3 + b^3)}{6EIL^2}$

Data: $\quad M_o = 16$ kip·ft, $\quad a = 4$ ft, $\quad b = 6$ ft, $\quad L = 10$ ft

$E = 1.8 \times 10^3$ ksi

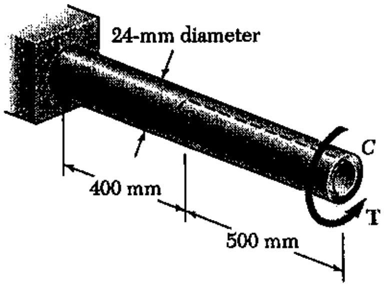
$I = \frac{1}{12}(3\frac{1}{2})(9\frac{1}{2})^3 = 250.07$ in^4

$EI = (1.8 \times 10^3)(250.07) = 450.13 \times 10^3$ kip·in^2 $= 3126$ kip·ft^2

$$U = \frac{(16)^2(4^3 + 6^3)}{(6)(3126)(10)^2} = 38.2 \times 10^{-3} \text{ kip·ft} = 38.2 \text{ ft·lb}$$

$$= 458 \text{ in·lb}$$

PROBLEM 11.38



11.38 Rod *AC* is made of aluminum (G = 73 GPa) and is subjected to a torque **T** applied at end *C*. Knowing that portion *BC* of the rod is hollow and has an inside diameter of 16 mm, determine the strain energy of the rod for a maximum shearing stress of 120 MPa.

SOLUTION

$c_o = \frac{d_o}{2} = 12 \text{ mm}, \quad c_i = \frac{d_i}{2} = 8 \text{ mm}$

$J_{AB} = \frac{\pi}{2} c_o^4 = \frac{\pi}{2}(12)^4 = 32.572 \times 10^3 \text{ mm}^4 = 32.572 \times 10^{-9} \text{ m}^4$

$J_{BC} = \frac{\pi}{2}(c_o^4 - c_i^4) = \frac{\pi}{2}(12^4 - 8^4) = 26.138 \times 10^3 \text{ mm}^4$

$= 26.138 \times 10^{-9} \text{ m}^4$

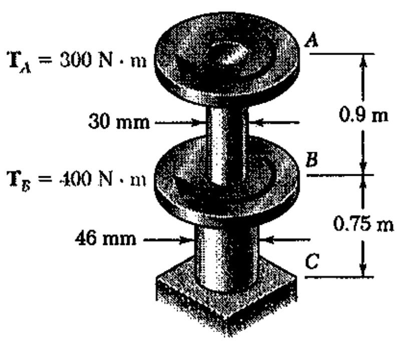
$\tau_{all} = \frac{Tc}{J_{min}} \qquad T = \frac{J_{min}\tau_{all}}{c} = \frac{(26.138 \times 10^{-9})(120 \times 10^6)}{12 \times 10^{-3}} = 261.38 \text{ N}\cdot\text{m}$

$U_{AB} = \frac{T^2 L_{AB}}{2GJ_{AB}} = \frac{(261.38)^2(400 \times 10^{-3})}{(2)(73 \times 10^9)(32.572 \times 10^{-9})} = 5.747 \text{ J}$

$U_{BC} = \frac{T^2 L_{BC}}{2GJ_{BC}} = \frac{(261.38)^2(500 \times 10^{-3})}{(2)(73 \times 10^9)(26.138 \times 10^{-9})} = 8.951 \text{ J}$

Total $U = U_{AB} + U_{BC} = 14.70 \text{ J}$ ◄

PROBLEM 11.39



11.39 In the assembly shown torques $\mathbf{T}_A$ and $\mathbf{T}_B$ are exerted on disks *A* and *B* respectively. Knowing that both shafts are solid and made of aluminum (G = 73 GPa), determine the total energy acquired by the assembly. .

SOLUTION

Over portion AB

$T_{AB} = T_A = 300 \text{ N}\cdot\text{m}$

$J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2}\left(\frac{30}{2}\right)^4 = 79.52 \times 10^3 \text{ mm}^4 = 79.52 \times 10^{-9} \text{ m}^4$

$L_{AB} = 0.9 \text{ m}$

$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} = \frac{(300)^2(0.9)}{(2)(73 \times 10^9)(79.52 \times 10^{-9})}$

$= 6.977 \text{ J}$

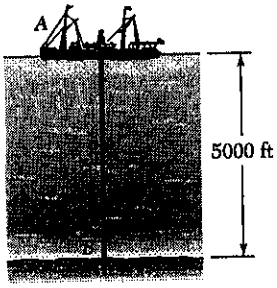
Over portion BC: $T_{BC} = T_A + T_B = 300 + 400 = 700 \text{ N}\cdot\text{m}, \quad L_{BC} = 0.75 \text{ m}$

$J_{BC} = \frac{\pi}{2}\left(\frac{46}{2}\right)^4 = 439.57 \times 10^3 \text{ mm}^4 = 439.57 \times 10^{-9} \text{ m}^4$

$U_{BC} = \frac{T_{BC}^2 L_{BC}}{2GJ_{BC}} = \frac{(700)^2(0.75)}{(2)(73 \times 10^9)(439.57 \times 10^{-9})} = 5.726 \text{ J}$

Total $U = U_{AB} + U_{BC} = 6.977 + 5.726 = 12.70 \text{ J}$ ◄

PROBLEM 11.40



11.40 The ship at A has just started to drill for oil on the ocean floor at a depth of 5000 ft. The steel drill pipe has an outside diameter of 8 in. and a uniform wall thickness of 0.5 in. Knowing that the top of the drill pipe rotates through two complete revolutions before the drill bit at B starts to operate and using $G = 11.2 \times 10^6$ psi, determine the maximum strain energy acquired by the drill pipe.

SOLUTION

$$\varphi = (2)(2\pi) = 4\pi \text{ rad}$$

$$L = 5000 \text{ ft} = 60\times10^3 \text{ in}$$

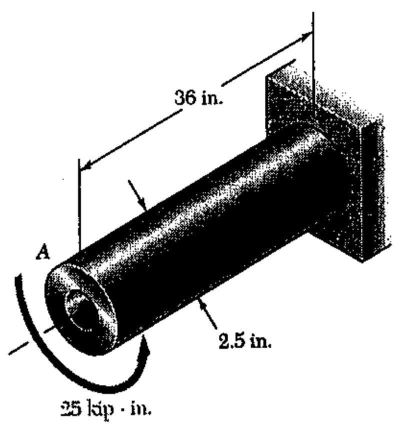
$$c_o = \frac{d_o}{2} = 4 \text{ in.} \qquad c_i = c_o - t = 3.5 \text{ in.}$$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4) = 166.406 \text{ in}^4$$

$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L} \qquad U = \frac{T^2L}{2GJ} = \left(\frac{GJ\varphi}{L}\right)^2\frac{L}{2GJ} = \frac{GJ\varphi^2}{2L}$$

$$U = \frac{(11.2\times10^6)(166.406)(4\pi)^2}{(2)(60\times10^3)} = 2.45\times10^6 \text{ in}\cdot\text{lb}$$

PROBLEM 11.41



11.41. The design specifications for the steel shaft AB require that the shaft acquire a strain energy of 300 in·lb as the 25-kip·in. torque is applied. Using $G = 11.2 \times 10^6$ psi, determine (*a*) the largest inside diameter of the shaft that can be used, (*b*) the corresponding maximum shearing stress in the shaft.

SOLUTION

$$U = 300 \text{ in}\cdot\text{lb} \qquad T = 25 \text{ kip}\cdot\text{in} = 25\times10^3 \text{ lb}\cdot\text{in}$$

$$L = 36 \text{ in.}$$

$$U = \frac{T^2L}{2GJ}$$

$$J = \frac{T^2L}{2GU} = \frac{(25\times10^3)^2(36)}{(2)(11.2\times10^6)(300)} = 3.3482 \text{ in}^4$$

$$\text{But} \quad J = \frac{\pi}{2}\left[\left(\frac{d_o}{2}\right)^4 - \left(\frac{d_i}{2}\right)^4\right] = \frac{\pi}{32}(d_o^4 - d_i^4)$$

$$d_i^4 = d_o^4 - \frac{32}{\pi}J = 2.5^4 - \frac{32}{\pi}(3.3482) = 4.95787 \text{ in}^4$$

$$d_i = 1.492 \text{ in.}$$

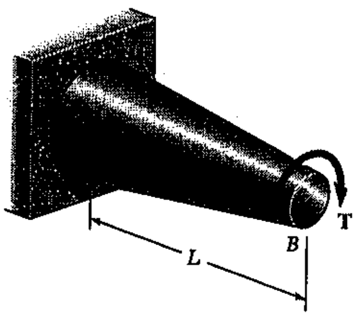
$$\tau = \frac{Tc_o}{J} = \frac{(25\times10^3)(1.25)}{3.3482} = 9.33\times10^3 \text{ psi} = 9.33 \text{ ksi}$$

PROBLEM 11.42

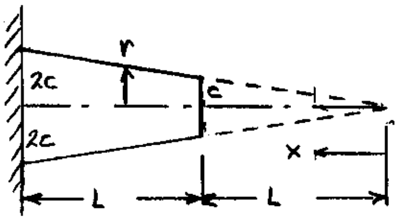
11.42 Show by integration that the strain energy in the tapered rod AB is

$$U = \frac{7}{48}\frac{T^2 L}{GJ_{min}}$$

where J_{min} is the polar moment of inertia of the rod at end B.



SOLUTION



$$r = \frac{cx}{L}$$

$$J = \frac{\pi}{2} r^4 = \frac{\pi}{2}\frac{c^4}{L^4} x^4, \quad J_{min} = \frac{\pi}{2} c^4$$

$$U = \int_L^{2L} \frac{T^2\,dx}{2GJ} = \int_L^{2L} \frac{T^2\,dx}{2G\left(\frac{\pi}{2}\frac{c^4}{L^4}x^4\right)}$$

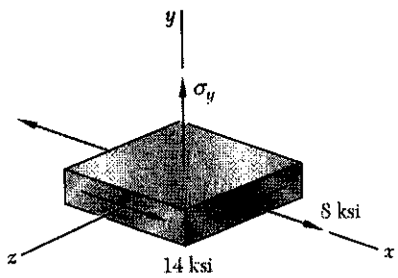
$$= \frac{T^2 L^4}{2GJ_{min}} \int_L^{2L} \frac{dx}{x^4}$$

$$= \frac{T^2 L^4}{2GJ_{min}} \left(-\frac{1}{3x^3}\right)\Bigg|_L^{2L}$$

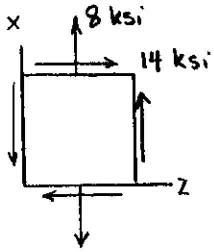
$$U = \frac{T^2 L^2}{2GJ_{min}} \left(-\frac{1}{3(2L)^3} + \frac{1}{3L^3}\right) = \frac{7}{48}\frac{T^2 L}{GJ_{min}}$$

PROBLEM 11.43

11.43 The state of stress shown occurs in a machine component made of a grade of steel for which $\sigma_Y = 65$ ksi. Using the maximum-distortion-energy criterion, determine the range of values of σ_y for which the factor of safety associated with the yield strength is equal to or larger than 2.2.



SOLUTION



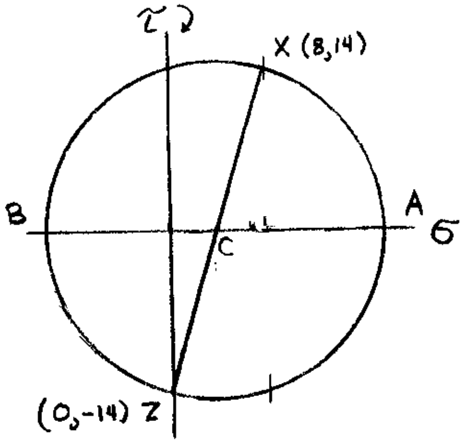
$$\sigma_{ave} = \tfrac{1}{2}(0 + 8) = 4 \text{ ksi}$$

$$\frac{\sigma_x - \sigma_z}{2} = \frac{8-0}{2} = 4 \text{ ksi}$$

$$\tau_{xz} = 14 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= \sqrt{4^2 + 14^2} = 14.56 \text{ ksi}$$



$$\sigma_a = \sigma_{ave} + R = 18.56 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = -10.56 \text{ ksi}$$

$$\sigma_c = \sigma_y$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_Y}{F.S.}\right)^2$$

$$(18.56 + 10.56)^2 + (-10.56 - \sigma_y)^2 + (\sigma_y - 18.56)^2 = 2\left(\frac{65}{2.2}\right)^2$$

$$847.97 + (111.51 + 21.12\,\sigma_y + \sigma_y^2) + (\sigma_y^2 - 37.12\,\sigma_y + 344.47 = 1745.87$$

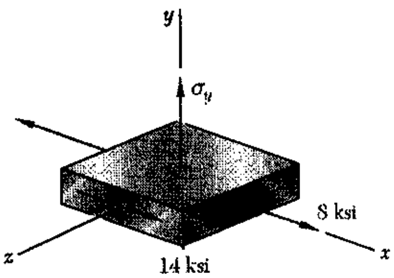
$$2\sigma_y^2 - 16\,\sigma_y - 441.92 = 0$$

$$\sigma_y = \frac{16 \pm \sqrt{16^2 + (4)(2)(441.92)}}{(2)(2)} = 4 \pm 15.39$$

$$\sigma_y = 19.39 \text{ ksi}, \quad -11.39 \text{ ksi}$$

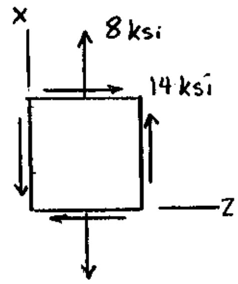
$$-11.39 \text{ ksi} \leq \sigma_y \leq 19.39 \text{ ksi}$$

PROBLEM 11.44



11.44 The state of stress shown occurs in a machine component made of a grade of steel for which $\sigma_Y = 65$ ksi. Using the maximum-distortion-energy criterion, determine the factor of safety associated with the yield strength when (*a*) $\sigma_y = +16$ ksi, (*b*) $\sigma_y = -16$ ksi,

SOLUTION

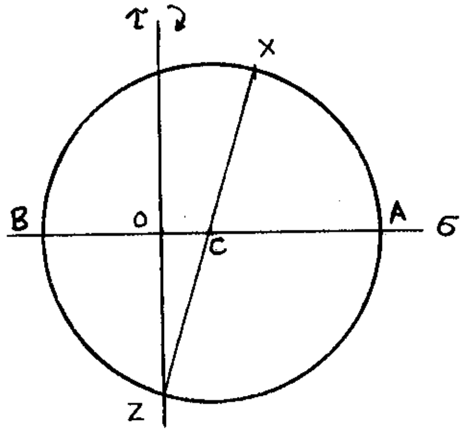


$$\sigma_{ave} = \tfrac{1}{2}(0 + 8) = 4 \text{ ksi}$$

$$\frac{\sigma_x - \sigma_z}{2} = \frac{8-0}{2} = 4 \text{ ksi}$$

$$\tau_{xz} = 14 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \sqrt{4^2 + 14^2} = 14.56 \text{ ksi}$$



$$\sigma_a = \sigma_{ave} + R = 18.56$$

$$\sigma_b = \sigma_{ave} - R = -10.56$$

$$\sigma_c = \sigma_y$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_Y}{F.S.}\right)^2$$

(a) $\sigma_c = \sigma_y = 16$ ksi

$$(18.56 + 10.56)^2 + (-10.56 - 16)^2 + (16 - 18.56)^2 = 2\left(\frac{65}{F.S.}\right)^2$$

$$847.97 + 705.43 + 6.55 = \frac{8450}{(F.S.)^2}$$

F.S. = 2.33

(b) $\sigma_c = \sigma_y = -16$ ksi

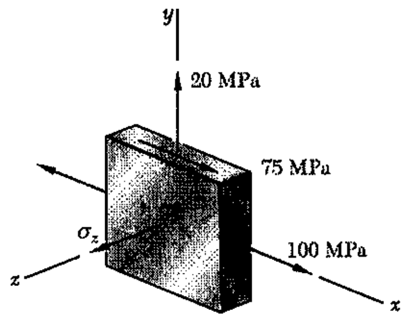
$$(18.56 + 10.56)^2 + (-10.56 + 16)^2 + (-16 - 18.56)^2 = 2\left(\frac{65}{F.S.}\right)^2$$

$$847.97 + 29.59 + 1194.39 = \frac{8450}{(F.S.)^2}$$

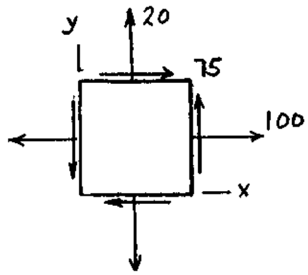
F.S. = 2.02

PROBLEM 11.45

11.45 The state of stress shown occurs in a machine component made of a brass for which $\sigma_Y = 160$ MPa. Using the maximum-distortion-energy criterion, determine whether yield occurs when (*a*) $\sigma_z = +45$ MPa, (*b*) $\sigma_z = -45$ MPa.



SOLUTION



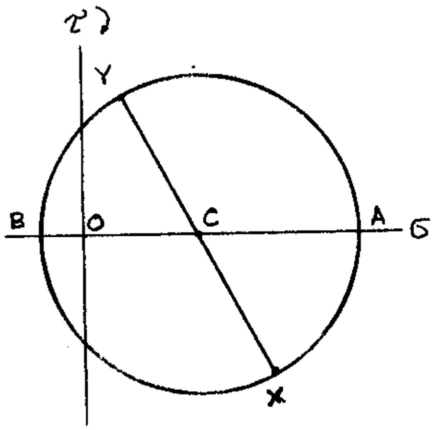
$\sigma_{ave} = \frac{1}{2}(100 + 20) = 60$ MPa

$\frac{\sigma_x - \sigma_y}{2} = \frac{100 - 20}{2} = 40$ MPa

$\tau_{xy} = 75$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{40^2 + 75^2} = 85$ MPa



$\sigma_a = \sigma_{ave} + R = 145$ MPa

$\sigma_b = \sigma_{ave} - R = -25$ MPa

$\sigma_c = \sigma_z$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 \overset{?}{\lessgtr} 2\sigma_Y^{2}$$

(a) $\sigma_c = \sigma_z = +45$ MPa

$(143 + 25)^2 + (-25 - 45)^2 + (45 - 145)^2 \overset{?}{<} 2(160)^2 = 51200$

$28900 + 4900 + 10000 = 43800 < 51200$ (No yield)

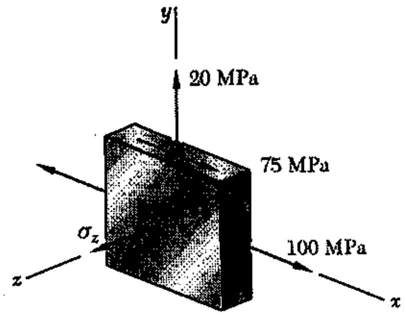
(b) $\sigma_c = \sigma_z = -45$ MPa

$(145 + 25)^2 + (-25 + 45)^2 + (-45 - 145)^2 \overset{?}{<} 51200$

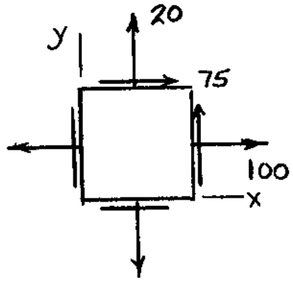
$28900 + 400 + 36100 = 65400 > 51200$ (Yield occurs)

PROBLEM 11.46

11.46 The state of stress shown occurs in a machine component made of a brass for which $\sigma_Y = 160$ MPa. Using the maximum-distortion-energy criterion, determine the range of values of σ_z for which yield does not occur.



SOLUTION



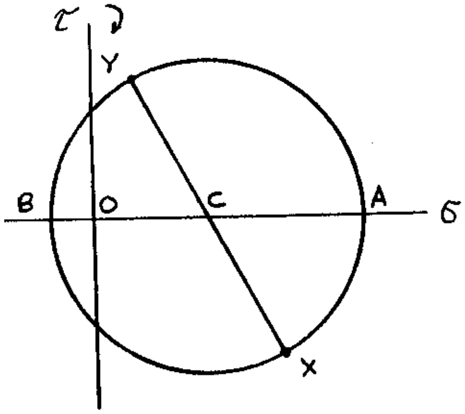
$$\sigma_{ave} = \tfrac{1}{2}(100 + 20) = 60 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{100 - 20}{2} = 40 \text{ MPa}$$

$$\tau_{xy} = 75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$



$$\sigma_a = \sigma_{ave} + R = 145 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -25 \text{ MPa}$$

$$\sigma_c = \sigma_z$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\sigma_Y^2$$

$$(145 + 25)^2 + (-25 - \sigma_z)^2 + (\sigma_z - 145)^2 = (2)(160)^2$$

$$28900 + (625 + 50\sigma_z + \sigma_z^2) + (\sigma_z^2 - 290\sigma_z + 21025 = 51200$$

$$2\sigma_z^2 - 240\sigma_z - 650 = 0$$

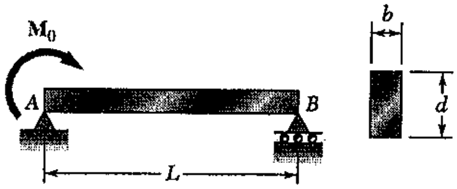
$$\sigma_z = \frac{240 \pm \sqrt{240^2 + (4)(2)(650)}}{(2)(2)} = 60 \pm 62.65$$

$$\sigma_z = 122.65 \text{ MPa}, \quad -2.65 \text{ MPa}$$

$$-2.65 \text{ MPa} < \sigma_z < 122.65 \text{ MPa}$$

PROBLEM 11.47

11.47 Determine the strain energy of the prismatic beam AB, taking into account the effect of both normal and shearing stresses.

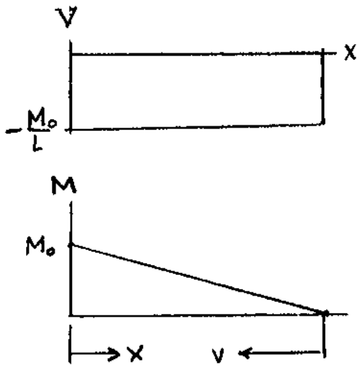


SOLUTION

Reactions $\quad R_A = \frac{M_0}{L}\downarrow, \quad R_B = \frac{M_0}{L}\uparrow$

Shear: $\quad V = -\frac{M_0}{L}$

Bending moment: $\quad M = \frac{M_0}{L}v$



For bending

$$U_1 = \int_0^L \frac{M^2}{2EI}\,dv = \frac{M_0^2}{2EIL^2}\int_0^L v^2\,dv = \frac{M_0 L^3}{6EIL^2}$$

$$= \frac{M_0^2 L}{6EI}$$

For shear

$$\tau_{xy} = \frac{3}{2}\frac{V}{A}\left(1 - \frac{y^2}{c^2}\right) \qquad c = \tfrac{1}{2}d$$

$$u = \frac{\tau_{xy}^2}{2G} = \frac{9V^2}{8GA^2}\left(1 - \frac{y^2}{c^2}\right)^2 = \frac{9M_0^2}{8G(bd)^2L^2}\left(1 - 2\frac{y^2}{c^2} + \frac{y^4}{c^4}\right)$$

$$U_2 = \int u\,d\mathcal{V} = \int_0^L\int_{-c}^{c} u\; b\,dy\,dx = \frac{9M_0^2 b}{8Gb^2d^2L^2}\int_0^L\int_{-c}^{c}\left(1 - 2\frac{y^2}{c^2} + \frac{y^4}{c^4}\right)dy\,dx$$

$$= \frac{9M_0^2}{8Gbd^2L^2}\int_0^L\left(y - \frac{2}{3}\frac{y^3}{c^2} + \frac{1}{5}\frac{y^5}{c^4}\right)\Big|_{-c}^{c}dx = \frac{9M_0^2}{8Gbd^2L^2}\int_0^L\left(2c - \frac{4}{3}c + \frac{2}{5}c\right)dx$$

$$\frac{9M_0^2}{8Gbd^2L^2}\left(\frac{16}{15}c\right)L = \frac{6}{5}\frac{M_0^2 c}{Gbd^2L} = \frac{3}{5}\frac{M_0^2}{GbdL}$$

Total $\quad U = U_1 + U_2 = \dfrac{M_0^2 L}{6EI} + \dfrac{3}{5}\dfrac{M_0^2}{GbdL}$

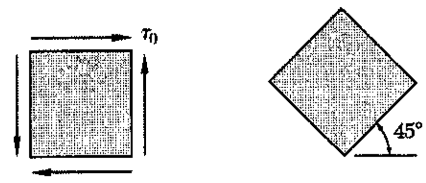
With $\quad I = \frac{1}{12}bd^3$

$$U = \frac{2M_0^2L}{Ebd^3} + \frac{3}{5}\frac{M_0^2}{GbdL} = \frac{2M_0^2L}{Ebd^3}\left\{1 + \frac{3}{10}\cdot\frac{E}{G}\frac{d^2}{L^2}\right\}$$

PROBLEM 11.48

11.48 For the state of stress shown in Fig. *a*, determine the stresses in an element oriented as shown in Fig. *b*. Compare the strain energy density in the given state first by using Fig. *a* and then by using Fig. *b*. Equating the two results obtained, show that

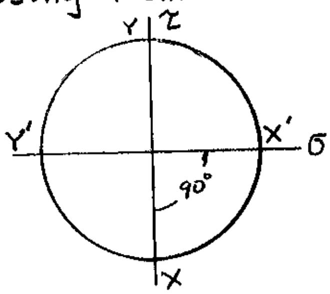
$$G = \frac{E}{2(1+\upsilon)}$$

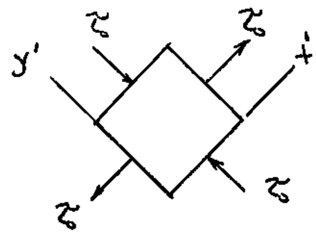


(*a*) (*b*)

SOLUTION

Using Mohr's circle





(a) $\sigma_x = 0, \quad \sigma_y = 0, \quad \tau_{xy} = \tau_0$

$$u = \frac{1}{2E}\left(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y\right) + \frac{1}{2G}\tau_{xy}^2 = \frac{\tau_0^2}{2G}$$

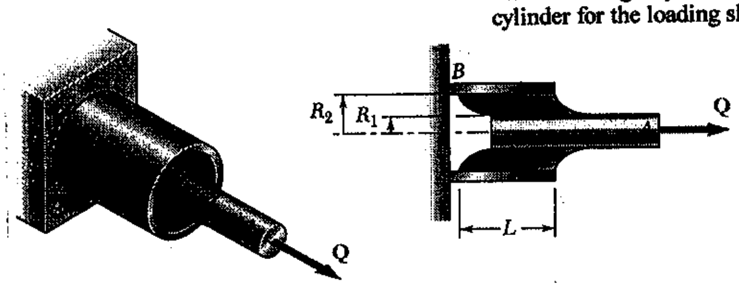
(b) $\sigma_{x'} = \tau_0, \quad \sigma_{y'} = -\tau_0, \quad \tau_{x'y'} = 0$

$$u = \frac{1}{2E}\left(\sigma_{x'}^2 + \sigma_{y'}^2 - 2\nu\sigma_{x'}\sigma_{y'}\right) + \frac{1}{2G}\tau_{x'y'}^2 = \frac{(2+2\nu)\tau_0^2}{2E}$$

Equate $\dfrac{\tau_0^2}{2G} = \dfrac{(2+2\nu)\tau_0^2}{2E}$ $\qquad G = \dfrac{E}{2(1+\nu)}$

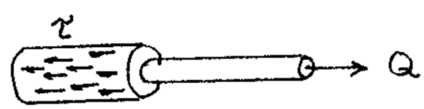
PROBLEM 11.49

***11.49** A vibration isolation support is made by bonding a rod *A*, of radius R_1, and a tube *B*, of inner radius R_2 to a hollow rubber cylinder. Denoting by *G* the modulus of rigidity of the rubber, determine the strain energy of the hollow rubber cylinder for the loading shown.



(*a*) (*b*)

SOLUTION



$\xrightarrow{+} \Sigma F_x = 0 \qquad -\tau(2\pi r L) + Q = 0$

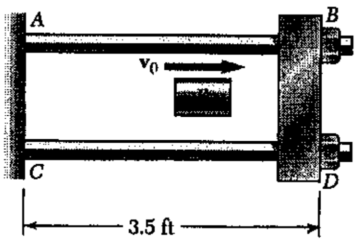
$$\tau = \frac{Q}{2\pi r L}$$

$$u = \frac{\tau^2}{2G} = \frac{Q^2}{8\pi^2 r^2 L^2 G}$$

$$U = \int u\, dV = \frac{Q^2}{8\pi^2 G L^2}\int \frac{dV}{r^2} = \frac{Q^2}{8\pi^2 G L^2}\int_0^L\int_{R_1}^{R_2} \frac{2\pi r\, dr}{r^2}\, dx$$

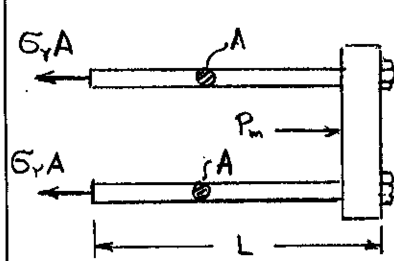
$$= \frac{Q^2}{4\pi G L^2}\int_0^L\int_{R_1}^{R_2}\frac{dr}{r}\,dx = \frac{Q^2}{4\pi G L^2}\int_0^L \left(\ln r\Big|_{R_1}^{R_2}\right) dx = \frac{Q^2}{4\pi G L}\ln\frac{R_2}{R_1}$$

PROBLEM 11.50



11.50 The cylindrical block E has a speed $v_0 = 16$ ft/s when it strikes squarely the yoke BD that is attached to the $\frac{7}{8}$-in.-diameter rods AB and CD. Knowing that the rods are made of a steel for which $\sigma_Y = 50$ ksi and $E = 29 \times 10^6$ psi, determine the weight of the block E for which the factor of safety is five with respect to permanent deformation of the rods.

SOLUTION



At the onset of yielding the force in each rod is

$$F = \sigma_Y A$$

Corresponding strain energy.

$$U_{AB} = \frac{F_{AB}^2 L_{AB}}{2EA_{AB}} = \frac{\sigma_Y^2 A^2 L}{2EA} = \frac{\sigma_Y^2 AL}{2E}$$

$$U_{CD} = \text{same} = \frac{\sigma_Y^2 AL}{2E}$$

$$U_m = U_{AB} + U_{CD} = \frac{\sigma_Y^2 AL}{E}$$

$$U_m = \left(\tfrac{1}{2} m v_0^2\right)(F.S.) = \left(\tfrac{1}{2}\frac{W}{g} v_0^2\right)(F.S.)$$

Solving for W: $\quad W = \dfrac{2g U_m}{v_0^2 (F.S.)} = \dfrac{2g\,\sigma_Y^2 AL}{v_0^2 (F.S.)\,E}$

Data: $g = 32.17 \text{ ft/sec}^2 = 386 \text{ in/sec}^2$, $\sigma_Y = 50 \times 10^3$ psi,

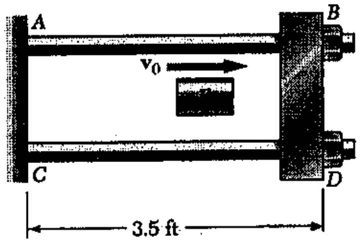
$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2 \qquad E = 29 \times 10^6$ psi

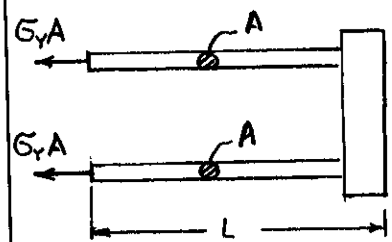
$L = 3.5 \text{ ft} = 42 \text{ in} \qquad F.S. = 5$

$v_0 = 16 \text{ ft/sec} = 192 \text{ in/sec}$

$$W = \frac{(2)(386)(50 \times 10^3)^2 (0.60132)(42)}{(192)^2 (5)(29 \times 10^6)} = 9.12 \text{ lb.}$$

PROBLEM 11.51





11.51 The 18-lb cylindrical block E has a horizontal velocity $\mathbf{v}_0$ when it strikes squarely the yoke BD that is attached to the $\frac{7}{8}$-in.-diameter rods AB and CD. Knowing that the rods are made of a steel for which $\sigma_Y = 50$ ksi and $E = 29 \times 10^6$ psi, determine the maximum allowable speed v_0 if the rods are not to be permanently deformed.

SOLUTION

At the onset of yielding the force in each rod is

$$F = \sigma_Y A$$

Corresponding strain energy

$$U_{AB} = \frac{F_{AB}^2 L_{AB}}{2EA_{AB}} = \frac{\sigma_Y^2 A^2 L}{2EA} = \frac{\sigma_Y^2 AL}{2E}$$

$$U_{CD} = \text{same} = \frac{\sigma_Y^2 AL}{2E}$$

$$\text{Total} \quad U_m = U_{AB} + U_{CD} = \frac{\sigma_Y^2 AL}{E}$$

$$U_m = \frac{1}{2} m v_0^2 = \frac{1}{2}\frac{W}{g} v_0^2$$

Solving for v_0^2 $\qquad v_0^2 = \frac{2gU_m}{W} = \frac{2g\sigma_Y^2 AL}{EW}$

$$v_0 = \sqrt{\frac{2g\sigma_Y^2 AL}{EW}}$$

Data: $g = 32.17 \text{ ft/sec}^2 = 386 \text{ in/sec}^2$, $\qquad \sigma_Y = 50 \times 10^3$ psi

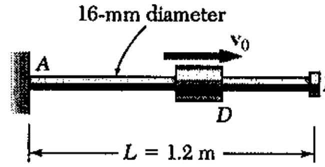
$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$, $\qquad E = 29 \times 10^6$ psi

$L = 3.5 \text{ ft} = 42 \text{ in.}$ $\qquad W = 18$ lb.

$$v_0 = \sqrt{\frac{(2)(386)(50\times10^3)^2(0.60132)(42)}{(29\times10^6)(18)}} = 305.6 \text{ in/sec}$$

$$= 25.5 \text{ ft./sec}$$

PROBLEM 11.52



11.52 The uniform rod AB is made of a brass for which $\sigma_Y = 125$ MPa and $E = 105$ GPa. Collar D moves along the rod and has a speed $v_0 = 3$ m/s as it strikes a small plate attached to end B of the rod. Using a factor of safety of four, determine the largest allowable mass of the collar if the rod is not to be permanently deformed.

SOLUTION

At onset of yielding $P_m = \sigma_Y A$

$\sigma_Y = 125 \times 10^6$ Pa

$A = \frac{\pi}{4}(16)^2 = 201.06\ \text{mm}^2 = 201.06 \times 10^{-6}\ \text{m}^2$

$P_m = 25133$ N

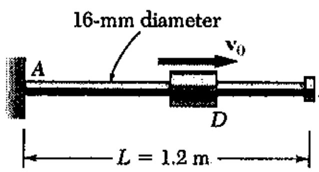
Corresponding strain energy $\quad U_m = \dfrac{P_m^2 L}{2EA} = \dfrac{(25133)^2(1.2)}{(2)(105 \times 10^9)(201.06 \times 10^{-6})}$

$= 17.953$ J

Kinetic energy times safety factor $= \frac{1}{2} m v_0^2 (\text{F.S.}) = 2 m v_0^2$

$2 m v_0^2 = U_m, \qquad m = \dfrac{U_m}{2 v_0^2} = \dfrac{17.953}{(2)(3)^2} = 0.997$ kg. ◄

PROBLEM 11.53



11.52 The uniform rod AB is made of a brass for which $\sigma_Y = 125$ MPa and $E = 105$ GPa. Collar D moves along the rod and has a speed $v_0 = 3$ m/s as it strikes a small plate attached to end B of the rod. Using a factor of safety of four, determine the largest allowable mass of the collar if the rod is not to be permanently deformed

11.53 Solve Prob. 11.52, assuming that the length of the brass rod is increased from 1.2 m to 2.4 m.

SOLUTION

At onset of yielding $\quad P_m = \sigma_Y A \qquad \sigma_Y = 125 \times 10^6$ Pa

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(16) = 201.06\ \text{mm}^2 = 201.06 \times 10^{-6}\ \text{m}^2$

$P_m = 25133$ N

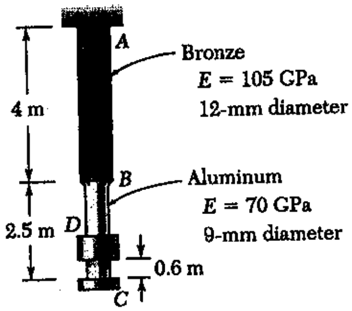
Corresponding strain energy $\quad U_m = \dfrac{P_m^2 L}{2EA} = \dfrac{(25133)^2 (2.4)}{(2)(105 \times 10^9)(201.06 \times 10^{-6})}$

$= 35.906$ J

Kinetic energy time safety factor $= \frac{1}{2} m v_0^2 (4) = 2 m v_0^2$

$2 m v_0^2 = U_m \qquad m = \dfrac{U_m}{2 v_0^2} = \dfrac{35.906}{(2)(3)^2} = 1.995$ kg. ◄

PROBLEM 11.54



11.54 Collar D is released from rest in the position shown and is stopped by a small plate attached at end C of the vertical rod ABC. Determine the mass of the collar for which the maximum normal stress in portion BC is 125 MPa.

SOLUTION

Portion BC: $\sigma_m = 125 \times 10^6$ Pa

$A_{BC} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$

$P_m = \sigma_m A_{BC} = 7952 \text{ N}$

Corresponding strain energy

$$U_{BC} = \frac{P_m^2 L_{BC}}{2E_{BC} A_{BC}} = \frac{(7952)^2(2.5)}{(2)(70 \times 10^9)(63.617 \times 10^{-6})} = 17.750 \text{ J}$$

$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.907 \text{ mm}^2 = 113.907 \times 10^{-6} \text{ m}^2$$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2E_{AB} A_{AB}} = \frac{(7952)^2(4)}{(2)(105 \times 10^9)(113.907 \times 10^{-6})} = 10.574 \text{ J}$$

$$U_m = U_{BC} + U_{AB} = 28.324 \text{ J}$$

Corresponding elongation Δ_m $\qquad \frac{1}{2} P_m \Delta_m = U_m$

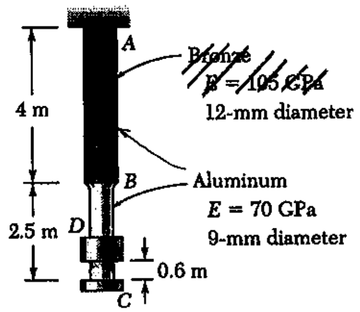
$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(28.324)}{7952} = 7.12 \times 10^{-3} \text{ m}$$

Falling distance $\quad h = 0.6 + 7.12 \times 10^{-3} = 0.60712$ m.

Work of weight $= U_m \qquad Wh = mgh = U_m$

$$m = \frac{U_m}{gh} = \frac{28.324}{(9.81)(0.60712)} = 4.76 \text{ kg}$$

PROBLEM 11.55



11.54 Collar *D* is released from rest in the position shown and is stopped by a small plate attached at end *C* of the vertical rod *ABC*. Determine the mass of the collar for which the maximum normal stress in portion *BC* is 125 MPa.

11.55 Solve Prob. 11.54, assuming that both portions of rod *ABC* are made of aluminum.

SOLUTION

Portion BC: $\sigma_m = 125 \times 10^6$ Pa

$A_{BC} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$

$P_m = \sigma_m A_{BC} = 7952$ N

Corresponding strain energy

$$U_{BC} = \frac{P_m^2 L_{BC}}{2EA_{BC}} = \frac{(7952)^2(2.5)}{(2)(70\times10^9)(63.617\times10^{-6})} = 17.750 \text{ J}$$

$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.907 \text{ mm}^2 = 113.907\times10^{-6} \text{ m}^2$$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2EA_{AB}} = \frac{(7952)^2(4)}{(2)(70\times10^9)(113.907\times10^{-6})} = 15.861 \text{ J}$$

Total $\quad U_m = U_{BC} + U_{AB} = 33.611$ J

Corresponding elongation $\Delta_m \qquad \frac{1}{2}P_m\Delta_m = U_m$

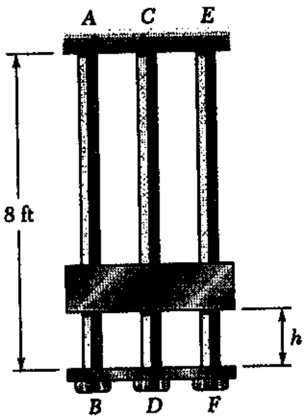
$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(33.611)}{7952} = 8.45\times10^{-3} \text{ m}$$

Falling distance $\qquad h = 0.6 + \Delta_m = 0.60845$ m

Work of weight $= U_m \qquad Wh = mgh = U_m$

$$m = \frac{U_m}{gh} = \frac{33.611}{(9.81)(0.60845)} = 5.63 \text{ kg}$$

PROBLEM 11.56



11.56 The 100-lb collar G is released from rest in the position shown and is stopped by plate BDF that is attached to the $\frac{7}{8}$-in.-diameter steel rod CD and to the $\frac{5}{8}$-in.-diameter steel rods AB and EF. Knowing that for the grade of steel used $\sigma_{all} = 24$ ksi and $E = 29 \times 10^6$ psi, determine the largest allowable distance h.

SOLUTION

Let Δ_m be the elongation

$$\Delta_m = \frac{\sigma_{AB} L}{E} = \frac{\sigma_{CD} L}{E} = \frac{\sigma_{EF} L}{E}$$

$$\sigma_{AB} = \sigma_{CD} = \sigma_{EF} = 24 \times 10^3 \text{ psi}$$

$$L = 8 \text{ ft} = 96 \text{ in}$$

$$\Delta_m = \frac{(24 \times 10^3)(96)}{29 \times 10^6} = 79.448 \times 10^{-3} \text{ in.}$$

For each rod $U = \dfrac{F_m^2 L}{2EA} = \dfrac{(EA\Delta_m/L)^2 L}{2EA} = \dfrac{EA\,\Delta_m^2}{2L}$

Rod CD: $A_{CD} = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$

$$U_{CD} = \frac{(29 \times 10^6)(0.60132)(79.448 \times 10^{-3})^2}{(2)(96)} = 573.28 \text{ in}\cdot\text{lb.}$$

Rods AB and EF: $A_{AB} = A_{EF} = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.30680 \text{ in}^2$

$$U_{AB} = U_{EF} = \frac{(29 \times 10^6)(0.30680)(79.448 \times 10)^2}{(2)(96)} = 292.49 \text{ in}\cdot\text{lb}$$

Total $U_m = U_{AB} + U_{CD} + U_{EF} = 1158.27 \text{ in}\cdot\text{lb}$

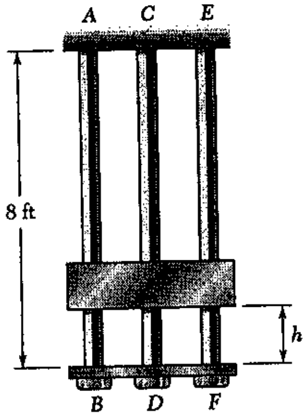
Falling distance is $h + \Delta_m$, $W = 100$ lb

$$W(h + \Delta_m) = U_m$$

$$h + \Delta_m = \frac{U_m}{W} = \frac{1158.27}{100} = 11.583 \text{ in.}$$

$$h = 11.583 - 79.448 \times 10^{-3} = 11.50 \text{ in.}$$

PROBLEM 11.57



11.56 The 100-lb collar G is released from rest in the position shown and is stopped by plate BDF that is attached to the $\frac{7}{8}$-in.-diameter steel rod CD and to the $\frac{5}{8}$-in.-diameter steel rods AB and EF. Knowing that for the grade of steel used $\sigma_{all} = 24$ ksi and $E = 29 \times 10^6$ psi, determine the largest allowable distance h.

11.57 Solve Prob. 11.56, assuming that the $\frac{7}{8}$-in.-diameter steel rod CD is replaced by a $\frac{7}{8}$-in.-diameter rod made of a grade of aluminum for which $\sigma_{all} = 20$ ksi and $E = 10.6 \times 10^6$ psi.

SOLUTION

Let Δ_m be the elongation. $\quad L = 8\text{ ft} = 96\text{ in}$

$$\Delta_m = \frac{\sigma_{AB} L}{E_{AB}} = \frac{\sigma_{CD} L}{E_{CD}} = \frac{\sigma_{EF} L}{E_{EF}}$$

If $\sigma_{AB} = 24 \times 10^3$ psi, $\quad \Delta_m = \dfrac{(24\times10^3)(96)}{29\times10^6} = 79.448\times10^{-3}$ in.

If $\sigma_{CD} = 20\times10^3$ psi, $\quad \Delta_m = \dfrac{(20\times10^3)(96)}{10.6\times10^6} = 181.13\times10^{-3}$ in.

Smaller value governs $\quad \Delta_m = 79.448\times10^{-3}$ in.

For each rod $\quad U = \dfrac{F^2 L}{2EA} = \dfrac{(EA\Delta_m/L)^2 L}{2EA} = \dfrac{EA\,\Delta_m^2}{2L}$

Rod CD: $\quad A_{CD} = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132\text{ in}^2, \quad E_{CD} = 10.6\times10^6$ psi

$$U_{CD} = \frac{(10.6\times10^6)(0.60132)(79.448\times10^{-3})^2}{(2)(96)} = 209.54\text{ in-lb}$$

Rods AB and EF: $\quad A_{AB} = A_{EF} = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.30680\text{ in}^2$

$$U_{AB} = U_{EF} = \frac{(29\times10^6)(0.30680)(79.448\times10^{-3})^2}{(2)(96)} = 292.49\text{ in-lb}$$

Total $\quad U_m = U_{AB} + U_{CD} + U_{EF} = 794.52$ in-lb.

Falling distance is $h + \Delta_m \qquad W = 100$ lb.

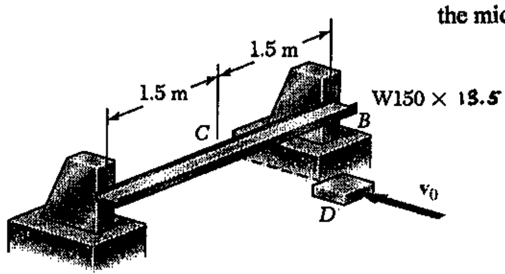
$$W(h + \Delta_m) = U_m$$

$$h + \Delta_m = \frac{U_m}{W} = \frac{794.52}{100} = 7.9452\text{ in}$$

$$h = 7.9452 - 79.448\times10^{-3} = 7.87\text{ in.}$$

PROBLEM 11.58

11.58 The steel beam AB is struck squarely at its midpoint C by a 45-kg block moving horizontally with a speed $v_0 = 2$ m/s. Using $E = 200$ GPa, determine (*a*) the equivalent static load, (*b*) the maximum normal stress in the beam, (*c*) the maximum deflection of the midpoint C of the beam.



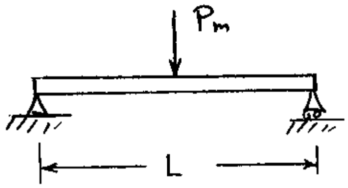
SOLUTION

From Appendix C, for W 150 × 13.5

$$I_x = 6.87 \times 10^6 \text{ mm}^4 = 6.87 \times 10^6 \text{ m}^4$$

$$S_x = 91.6 \times 10^3 \text{ mm}^3 = 91.6 \times 10^{-6} \text{ m}^3$$

Kinetic energy $T = \frac{1}{2} m V_0^2 = \frac{1}{2}(45)(2)^2 = 90 \text{ J}$



From Appendix D, Case 4

$$|y_m| = \frac{PL^3}{48EI}, \qquad M_{max} = \frac{PL}{4}$$

$$U = \frac{1}{2} P_m |y_m| = \frac{P_m^2 L^3}{96EI} = T$$

(a) $$P_m = \sqrt{\frac{96\,EIT}{L^3}} = \sqrt{\frac{(96)(200\times10^9)(6.87\times10^{-6})(90)}{(3.0)^3}} = 20.968\times10^3 \text{ N}$$

$$= 21.0 \text{ kN}$$

(b) $$\sigma_m = \frac{M_{max}}{S} = \frac{P_m L}{4S} = \frac{(20.968\times10^3)(3.0)}{(4)(91.6\times10^{-6})} = 171.7\times10^6 \text{ Pa}$$

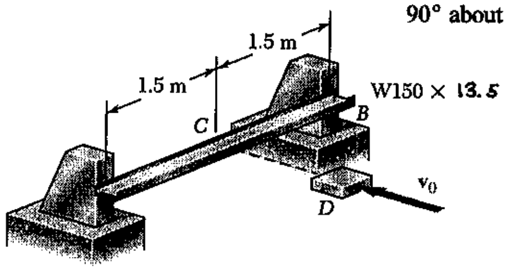
$$= 171.7 \text{ MPa}$$

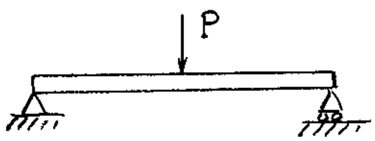
(c) $$|y_m| = \frac{2U}{P_m} = \frac{(2)(90)}{20.968\times10^3} = 8.58\times10^{-3} \text{ m} = 8.58 \text{ mm}$$

PROBLEM 11.59

11.58 The steel beam AB is struck squarely at its midpoint C by a 45-kg block moving horizontally with a speed $v_0 = 2$ m/s. Using $E = 200$ GPa, determine (*a*) the equivalent static load, (*b*) the maximum normal stress in the beam, (*c*) the maximum deflection of the midpoint C of the beam.

11.59 Solve Prob. 11.58, assuming that the W150 × 13.5 rolled-steel beam is rotated by 90° about its longitudinal axis so that its web is vertical.





SOLUTION

From Appendix C, for W 150 × 13.5

$$I_y = 0.918 \times 10^6\ \text{mm}^4 = 0.918 \times 10^{-6}\ \text{m}^4$$

$$S_y = 18.4 \times 10^3\ \text{mm}^3 = 18.4 \times 10^{-6}\ \text{m}^3$$

Kinetic energy $T = \frac{1}{2} m v_0^2 = \frac{1}{2}(45)(2)^2 = 90\ \text{J}$

From Appendix D, Case 4

$$|y_m| = \frac{PL^3}{48EI} \qquad M_{max} = \frac{PL}{4}$$

$$U = \frac{1}{2} P_m |y_m| = \frac{P_m^2 L^3}{96EI} = T$$

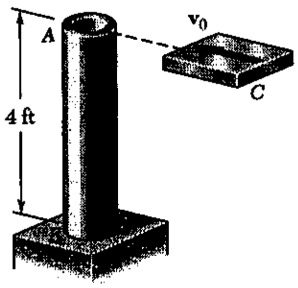
(a) $$P_m = \sqrt{\frac{96EIT}{L^3}} = \sqrt{\frac{(96)(200\times10^9)(0.918\times10^{-6})(90)}{(3.0)^3}} = 7.665\times10^3\ \text{N} = 7.67\ \text{kN}$$

(b) $$\sigma_m = \frac{M_{max}}{S} = \frac{P_m L}{4S} = \frac{(7.665\times10^3)(3.0)}{(4)(18.4\times10^{-6})} = 312\times10^6\ \text{Pa} = 312\ \text{MPa}$$

(c) $$|y_m| = \frac{2U}{P_m} = \frac{(2)(90)}{7.665\times10^3} = 23.5\times10^{-3}\ \text{m} = 23.5\ \text{mm}$$

PROBLEM 11.60

11.60 The post AB consists of a steel pipe of 3.5-in outer diameter and 0.3-in. wall thickness. A 15-lb block C moving horizontally with a velocity $\mathbf{v}_0$ hits the post squarely at A. Using $E = 29 \times 10^6$ psi, determine the largest speed v_0 for which the maximum normal stress in the pipe does not exceed 24 ksi.



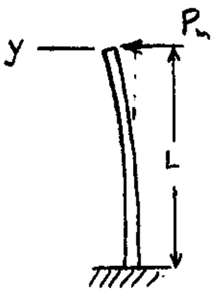
SOLUTION

$$c_o = \tfrac{1}{2} d_o = \tfrac{1}{2}(3.5) = 1.75 \text{ in.}, \quad c_i = c_o - t = 1.75 - 0.3 = 1.45 \text{ in.}$$

$$I = \tfrac{\pi}{4}(c_o^4 - c_i^4) = 3.8943 \text{ in}^4 \qquad \sigma_m = 24000 \text{ psi}$$

$$\sigma_m = \frac{M_m c}{I}, \quad M_m = \frac{I\sigma_m}{c} = \frac{(3.8943)(24000)}{1.75} = 53407 \text{ lb}\cdot\text{in}$$

$$P_m = \frac{M_m}{L} = \frac{53407}{48} = 1112.66 \text{ lb.}$$



By Appendix D, Case 1

$$y_m = \frac{P_m L^3}{3EI} = \frac{(1112.66)(48)^3}{(3)(29\times10^6)(3.8943)} = 0.36319 \text{ in}$$

$$U_m = \tfrac{1}{2} P_m y_m = \tfrac{1}{2}(1112.66)(0.36319) = 202.05 \text{ in}\cdot\text{lb.}$$

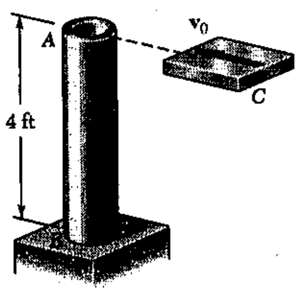
$$\frac{1}{2}\frac{W}{g} v_o^2 = U_m \qquad v_o^2 = \frac{2gU_m}{W} = \frac{(2)(386)(202.05)}{15} = 10399 \text{ in}^2/\text{sec}^2$$

$$v_o = 102.0 \text{ in/sec} = 8.50 \text{ ft/sec}$$

PROBLEM 11.61

11.60 The post AB consists of a steel pipe of 3.5-in outer diameter and 0.3-in. wall thickness. A 15-lb block C moving horizontally with a velocity $\mathbf{v}_0$ hits the post squarely at A. Using $E = 29 \times 10^6$ psi, determine the largest speed v_0 for which the maximum normal stress in the pipe does not exceed 24 ksi.

11.61 Solve Prob 11.60, assuming that the post AB consists of a solid steel rod of 3.5-in outer diameter.



SOLUTION

$$c = \tfrac{1}{2} d = 1.75 \text{ in} \qquad I = \tfrac{\pi}{4} c^4 = 7.3662 \text{ in}^4$$

$$\sigma_m = 24000 \text{ psi} \qquad L = 4 \text{ ft} = 48 \text{ in}$$

$$\sigma_m = \frac{M_m c}{I}, \quad M_m = \frac{I\sigma_m}{c} = \frac{(7.3662)(24000)}{1.75} = 101022 \text{ lb}\cdot\text{in}$$

$$P_m = \frac{M_m}{L} = 2104.6 \text{ lb.}$$

By Appendix D, Case 1

$$y_m = \frac{P_m L^3}{3EI} = \frac{(2104.6)(48)^3}{(3)(29\times10^6)(7.3662)} = 0.36319 \text{ in}$$

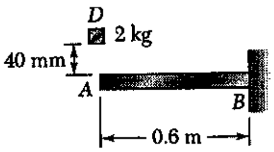
$$U_m = \tfrac{1}{2} P_m y_m = \tfrac{1}{2}(2104.6)(0.36319) = 382.19 \text{ in}\cdot\text{lb.}$$

$$\frac{1}{2}\frac{W}{g} v_o^2 = U_m, \quad v_o^2 = \frac{2gU_m}{W} = \frac{(2)(386)(382.19)}{15} = 19670 \text{ in}^2/\text{sec}^2$$

$$v_o = 140.25 \text{ in/sec} = 11.69 \text{ ft/sec}$$

PROBLEM 11.62

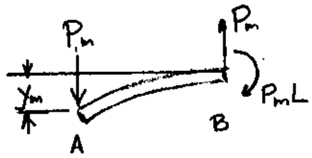
11.62 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (*a*) the maximum deflection of end A, (*b*) the maximum bending moment in the rod, (*c*) the maximum normal stress in the rod.



SOLUTION

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{16}{2}\right)^4 = 3.2170\times10^3\ \text{mm}^4 = 3.2170\times10^{-9}\ \text{m}^4$$

$$c = \frac{d}{2} = 8\ \text{mm} = 8\times10^{-3}\ \text{m} \qquad L_{AB} = 0.6\ \text{m}$$



Appendix D, Case 1

$$y_m = \frac{P_m L_{AB}^3}{3EI} \qquad M_m = P_m L_{AB}$$

$$P_m = \frac{3EI}{L_{AB}^3}\, y_m = \frac{(3)(200\times10^9)(3.217\times10^{-9})}{(0.6)^3} = 8.9361\times10^3\ y_m$$

$$U_m = \frac{1}{2}P_m y_m = \frac{1}{2}(8.9361\times10^3)\, y_m^2 = 4.4681\times10^3\ y_m^2$$

Work of dropped weight $\quad mg(h + y_m) = (2)(9.81)(0.040 + y_m)$

$$= 0.7848 + 19.62\, y_m$$

Equating work and energy

$$0.7848 + 19.62\, y_m = 4.4681\times10^3\ y_m^2$$

$$y_m^2 - 4.3911\times10^{-3}\, y_m - 175.645\times10^{-6} = 0$$

(a) $$y_m = \frac{1}{2}\left\{4.3911\times10^{-3} + \sqrt{(4.3911\times10^{-3})^2 + (4)(175.645\times10^{-6})}\right\}$$

$$= 15.629\times10^{-3}\ \text{m} = 15.63\ \text{mm}$$

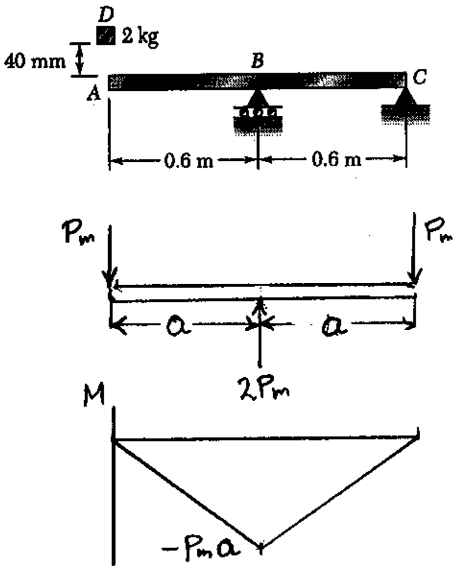
$$P_m = (8.9361\times10^3)(15.629\times10^{-3}) = 139.66\ \text{N}$$

(b) $$M_m = -P_m L_{AB} = -(139.66)(0.6) = -83.8\ \text{N}\cdot\text{m}$$

(c) $$\sigma_m = \frac{|M_m|c}{I} = \frac{(83.8)(8\times10^{-3})}{3.2170\times10^{-9}} = 208\times10^6\ \text{Pa} = 208\ \text{MPa}$$

PROBLEM 11.63

11.63 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (*a*) the maximum deflection of end A, (*b*) the maximum bending moment in the rod, (*c*) the maximum normal stress in the rod.



SOLUTION

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{16}{2}\right)^4 = 3.2170\times10^3\ mm^4 = 3.2170\times10^{-9}\ m^4$$

$$c = \frac{d}{2} = 8\ mm = 8\times10^{-3}\ m \qquad a = 0.6\ m$$

Over AB $\qquad M = -P_m x \qquad M_m = -P_m a$

$$U_{AB} = \int_0^a \frac{P_m^2 x^2}{2EI}dx = \frac{P_m^2 a^3}{6EI}$$

$$= \frac{(0.6)^3}{(6)(200\times10^9)(3.2170\times10^{-9})} P_m^2$$

$$= 55.953\times10^{-6}\ P_m^2$$

By symmetry of bending moment diagram

$$U_{BC} = U_{AB} = 55.953\times10^{-6}\ P_m^2$$

$$U_m = U_{AB} + U_{BC} = 111.906\times10^{-6}\ P_m^2$$

$$\frac{1}{2}P_m y_m = U_m = 111.906\times10^{-6}\ P_m^2 \qquad P_m = 4.4681\times10^3\ y_m$$

$$U_m = \frac{1}{2}P_m y_m = 2.2340\times10^3\ y_m^2$$

Work of dropped weight $\qquad mg(h+y_m) = (2)(9.81)(0.040 + y_m)$

$$= 0.7848 + 19.62\ y_m$$

Equating work and energy

$$0.7848 + 19.62\ y_m = 2.2340\times10^3\ y_m^2$$

$$y_m^2 - 8.7825\times10^{-3}\ y_m - 351.298\times10^{-6} = 0$$

(a) $\quad y_m = \frac{1}{2}\left\{8.7825\times10^{-3} + \sqrt{(8.7282\times10^{-3})^2 + (4)(351.298\times10^{-6})}\right\}$

$$= 23.636\times10^{-3}\ m = 23.6\ mm$$

$$P_m = (4.4681\times10^3)(23.636\times10^{-6}) = 105.61\ N$$

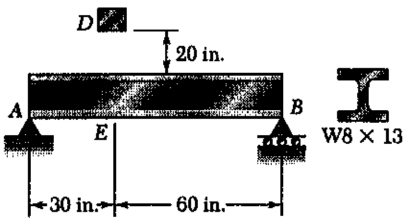
(b) $\quad M_m = -(105.61)(0.6) = -64.4\ N\cdot m$

(c) $\quad \sigma_m = \frac{|M_m|c}{I} = \frac{(64.4)(8\times10^{-3})}{3.2170\times10^{-9}} = 157.6\times10^6\ Pa$

$$= 157.6\ MPa$$

PROBLEM 11.64

11.64 The 50-lb block D is dropped from a height of 20 in. onto the steel beam AB. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the maximum deflection at point E, (*b*) the maximum normal stress in the beam.



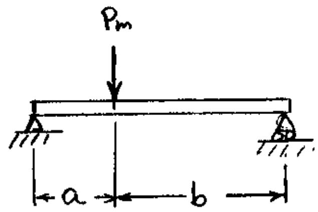
SOLUTION

$I_x = 39.6 \text{ in}^4 \quad , \quad S_x = 9.91 \text{ in}^3$

Appendix D, Case 5

$$y_E = \frac{P_m a^2 b^2}{3EIL} = \frac{(30)^2(60)^2 P_m}{(3)(29\times10^6)(39.6)(90)}$$

$$= 10.4493\times10^{-6}\, P_m$$



$$P_m = 95700\, y_E$$

$$U_m = \tfrac{1}{2} P_m y_E = 47850\, y_E^2$$

Work of falling weight $\quad W(h + y_E) = 50(20 + y_E) = 1000 + 50\, y_E$

Equating work and energy: $\quad 1000 + 50\, y_E = 47850\, y_E^2$

$$y_E^2 - 1.04493\times10^{-3} - 20.899\times10^{-3} = 0$$

(a) $\quad y_E = \frac{1}{2}\left\{ 1.04493\times10^{-3} + \sqrt{(1.04493\times10^{-6})^2 + (4)(20.899\times10^{-3})} \right\}$

$$= 0.1451 \text{ in}$$

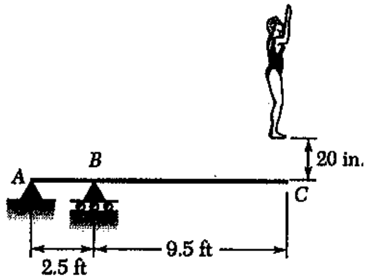
$$P_m = (95700)(0.1451) = 13885 \text{ lb.}$$

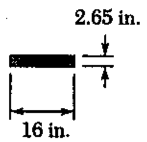
$$M_m = \frac{P_m ab}{L} = \frac{(13885)(30)(60)}{90} = 277.7\times10^3 \text{ lb}\cdot\text{in.}$$

(b) $\quad \sigma_m = \dfrac{M_m}{S_x} = \dfrac{277.7\times10^3}{9.91} = 28.0\times10^3 \text{ psi} = 28.0 \text{ ksi}$

PROBLEM 11.65

11.65 A 160-lb diver jumps from a height of 20 in. onto end *C* of a diving board having the uniform cross section shown. Assuming that the diver's legs remain rigid and using $E = 1.8 \times 10^6$ psi, determine (*a*) the maximum deflection at point *C*, (*b*) the maximum normal stress in the board, (*c*) the equivalent static load.



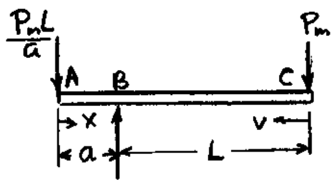


SOLUTION

$I = \frac{1}{12}(16)(2.65)^3 = 24.813 \text{ in}^4$

$L = 9.5 \text{ ft.} = 114 \text{ in}, \quad a = 2.5 \text{ ft} = 30 \text{ in}$

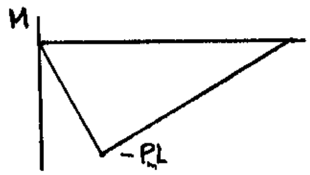
$c = \frac{1}{2}(2.65) = 1.325 \text{ in.}$



Over portion AB $\quad M = -\frac{P_mL}{a}x$

$$U_{AB} = \int_0^a \frac{M^2}{2EI}dx = \frac{P_m^2L^2}{2EIa^2}\int_0^a x^2\,dx = \frac{P_m^2L^2a}{6EI}$$

Over portion BC $\quad M = -P_m v$



$$U_{BC} = \int_0^L \frac{M^2}{2EI}dv = \frac{P_m^2}{2EI}\int_0^L v^2\,dv = \frac{P_m^2L^3}{6EI}$$

Total $\quad U = U_{AB} + U_{BC} = \frac{P_m^2L^2(a+L)}{6EI}$

$\frac{1}{2}P_m y_m = U_m \qquad y_m = \frac{2U_m}{P_m} = \frac{P_mL^2(a+L)}{3EI}$

$$P_m = \frac{3EI}{L^2(a+L)}y_m = \frac{(3)(1.8\times10^6)(24.813)}{(114)^2(114+30)}y_m = 71.598\ y_m$$

$U_m = \frac{1}{2}P_m y_m = 35.799\ y_m^2$

Work of weight $= W(h+y_m) = (160)(20+y_m) = 3200 + 160\,y_m$

Equating $\quad 3200 + 160\,y_m = 35.799\,y_m^2$

$y_m^2 - 4.4694\,y_m - 89.388 = 0$

(a) $\quad y_m = \frac{1}{2}\left\{4.4694 + \sqrt{4.4694^2 + (4)(89.388)}\right\} = 11.95 \text{ in}$ ◄

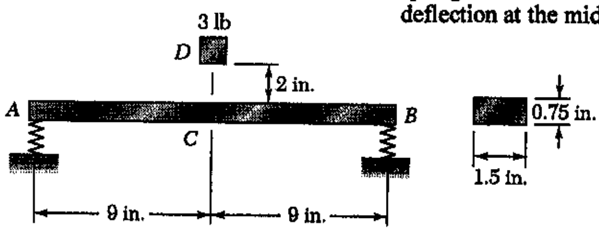
(c) $\quad P_m = (71.598)(11.95) = 856 \text{ lb}$ ◄

$M_m = -(856)(114) = 97535 \text{ lb}\cdot\text{in}$

(b) $\quad \sigma_m = \frac{|M_m|c}{I} = \frac{(97535)(1.325)}{24.813} = 5210 \text{ psi}$ ◄

PROBLEM 11.66

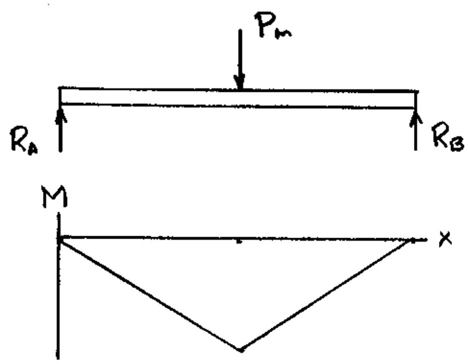
11.66 The 3-lb block D is released from rest in the position shown and strikes a steel bar AB having the uniform cross section shown. The bar is supported at each end by springs of constant 20 kips/in. Using $E = 29 \times 10^6$ psi, determine the maximum deflection at the midpoint of the bar.



SOLUTION

$k = 20$ kips/in $= 20 \times 10^3$ lb/in

$R_A = R_B = \frac{1}{2} P_m$



For spring A, $U_A = \frac{1}{2} R_A y_A = \frac{1}{2}\frac{R_A^2}{k} = \frac{1}{8}\frac{P_m^2}{k}$

For spring B, $U_B = \frac{1}{2} R_B y_B = \frac{1}{2}\frac{R_B^2}{k} = \frac{1}{8}\frac{P_m^2}{k}$

Portion AC of beam ACB $\quad M = \frac{1}{2} P_m x$

$$U_{AC} = \int_0^{L_{AC}} \frac{M^2}{2EI}\,dx = \frac{P_m^2}{8EI}\int_0^{L_{AC}} x^2\,dx = \frac{P_m^2 L_{AC}^3}{24\,EI}$$

Portion CB of beam

By symmetry $U_{CB} = U_{AC} = \frac{P_m^2 L_{AC}^3}{24EI}$

Total $\quad U = U_A + U_B + U_{AC} + U_{CB} = \frac{P_m^2}{4k} + \frac{P_m^2 L_{AC}^3}{12EI}$

$$I = \frac{1}{12} bd^3 = \frac{1}{12}(1.5)(0.75)^3 = 52.734 \times 10^{-3}\ \text{in}^4$$

$$U = \left\{\frac{1}{(4)(20\times10^3)} + \frac{(9)^3}{(12)(29\times10^6)(52.734\times10^{-3})}\right\}P_m^2 = 52.224\times10^{-6}\ P_m^2 = \frac{1}{2}P_m y_m$$

$$y_m = \frac{2U}{P_m} = 104.448\times10^{-6}\ P_m \qquad P_m = 9.5741\times10^3\ y_m$$

$$U = (52.224\times10^{-6})(9.5741\times10^3)^2 y_m^2 = 4.7871 \times 10^3\ y_m^2$$

Work of falling weight $\quad W(h + y_m) = (3)(2 + y_m) = 6 + 3y_m$

Equating $\quad 6 + 3y_m = 4.7871 \times 10^3\ y_m^2$

$$y_m^2 - 626.69\times10^{-6}\ y_m - 1.25338\times10^{-3} = 0$$

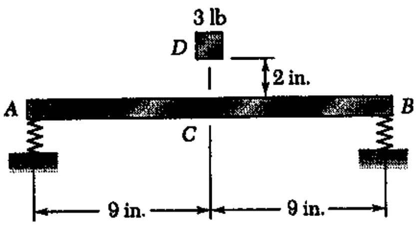
$$y_m = \frac{1}{2}\left\{626.69\times10^{-6} + \sqrt{(626.69\times10^{-6})^2 + (4)(1.25338\times10^{-3})}\right\}$$

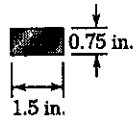
$$= 35.7\times10^{-3}\ \text{in.} = 0.0357\ \text{in.}$$

PROBLEM 11.67

11.67 The 3-lb block *D* is released from rest in the position shown and strikes a steel bar *AB* having the uniform cross section shown. The bar is supported at each end by springs of constant 20 kips/in. Using $E = 20 \times 10^6$ psi, determine the maximum deflection at the midpoint of the bar.

11.67 Solve Prob 11.66, assuming that the constant of each spring is 40 kips/in.

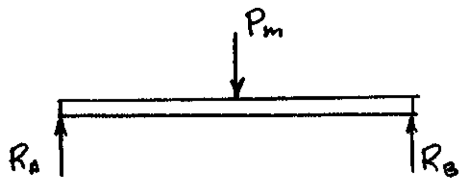




SOLUTION

$k = 40$ kips/in $= 40 \times 10^3$ lb/in.

$R_A = R_B = \frac{1}{2} P_m$



For spring A, $U_A = \frac{1}{2} R_A y_A = \frac{1}{2} \frac{R_A^2}{k} = \frac{1}{8} \frac{P_m^2}{k}$

For spring B, $U_B = \frac{1}{2} R_B y_B = \frac{1}{2} \frac{R_B^2}{k} = \frac{1}{8} \frac{P_m^2}{k}$

Portion AC of beam ACB $\qquad M = \frac{1}{2} P_m x$

$$U_{AC} = \int_0^{L_{AC}} \frac{M^2}{2EI} dx = \frac{P_m^2}{8EI} \int_0^{L_{AC}} x^2 dx = \frac{P_m^2 L_{AC}^2}{24 EI}$$

Portion CB of beam

By symmetry $U_{CB} = U_{AC} = \dfrac{P_m^2 L_{AC}^2}{24 EI}$

Total $U = U_A + U_B + U_{AC} + U_{CB} = \dfrac{P_m^2}{4k} + \dfrac{P_m^2 L_{AC}^3}{12 EI}$

$I = \frac{1}{12} b d^3 = \frac{1}{12}(1.5)(0.75)^3 = 52.734 \times 10^{-3}$ in^4

$$U = \left\{ \frac{1}{(4)(40 \times 10^3)} + \frac{(9)^3}{(12)(29 \times 10^6)(52.734 \times 10^{-3})} \right\} P_m^2 = 45.974 \times 10^{-6} P_m^2 = \frac{1}{2} P_m y_m$$

$y_m = \dfrac{2U}{P_m} = 91.949 \times 10^{-6} P_m \qquad P_m = 10.8756 \times 10^3 y_m$

$U = (45.974 \times 10^{-6})(10.8756 \times 10^3)^2 y_m^2 = 5.4378 \times 10^3 y_m^2$

Work of falling weight $\qquad W(h + y_m) = (3)(2 + y_m) = 6 + 3y_m$

Equating $\qquad 6 + 3y_m = 5.4378 \times 10^3 y_m^2$

$$y_m^2 - 551.70 \times 10^{-6} y_m - 1.1034 \times 10^{-3} = 0$$

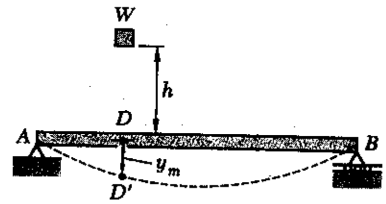
$$y_m = \frac{1}{2}\left\{ 551.70 \times 10^{-6} + \sqrt{(551.70 \times 10^{-3})^2 + (4)(1.1034 \times 10^{-3})} \right\}$$

$$= 33.5 \times 10^{-3} \text{ in.} = 0.0335 \text{ in.}$$

PROBLEM 11.68

11.68 A block of weight W is placed in contact with a beam at some given point D and released. Show that the resulting maximum deflection at point D is twice as large as the deflection due to a static weight W applied at D.

SOLUTION



Consider dropping the weight from a height h above the beam. The work done by the weight is

$\text{Work} = W(h + y_m)$

Strain energy $\quad U = \frac{1}{2} P_m y_m = \frac{1}{2} k y_m^2$

where k is the spring constant of the beam for loading at point D.

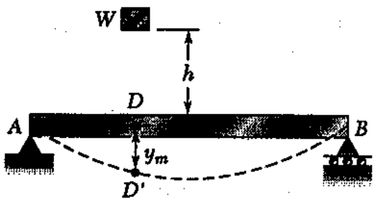
Equating work and energy $\quad W(h + y_m) = \frac{1}{2} k y_m^2$

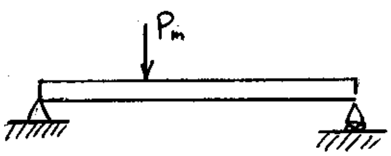
Setting $h = 0$, $\quad W y_m = \frac{1}{2} k y_m^2, \quad y_m = \frac{2W}{k}$

The static deflection at point D due to weight applied at D is

$\delta_{st} = \frac{W}{k} \qquad$ Thus $\qquad y_m = 2\delta_{st}$

PROBLEM 11.69





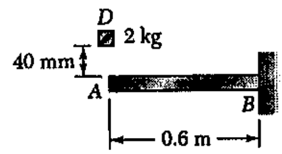
11.69 A block of weight W is dropped from a height h onto the horizontal beam AB and hits it at point D. (*a*) Show that the maximum deflection y_m at point D can be expressed as

$$y_m = y_{st}\left(1 + \sqrt{1 + \frac{2h}{y_{st}}}\right)$$

where y_{st} represents the deflection at D caused by a static load W applied at that point and where the quantity in parentheses is referred to as the *impact factor*. (*b*) Compute the impact factor for the beam and impact factor of Prob. 11.62.

11.62 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (*a*) the maximum deflection of end A, (*b*) the maximum bending moment in the rod, (*c*) the maximum normal stress in the rod.

SOLUTION



Work of falling weight Work $= W(h + y_m)$

Strain energy $U = \frac{1}{2} P y_m = \frac{1}{2} k y_m^2$

where k is the spring constant for a load applied at point D.

Equating work and energy

$$W(h + y_m) = \frac{1}{2} k y_m^2$$

$$y_m^2 - \frac{2W}{k} y_m - \frac{2W}{k} h = 0$$

$$y_m^2 - 2 y_{st} y_m - 2 y_{st} h = 0 \qquad \text{where } y_{st} = \frac{W}{k}$$

$$y_m = \frac{2 y_{st} + \sqrt{4 y_{st}^2 + 8 y_{st} h}}{2} = y_{st}\left(1 + \sqrt{1 + \frac{2h}{y_{st}}}\right)$$

For Prob. 11.62 $W = mg = (2)(9.81) = 19.62$ N

$E = 200 \times 10^9$ Pa $I = \frac{\pi}{4}\left(\frac{16}{2}\right)^4 = 3.217 \times 10^3 \text{ mm}^4 = 3.217 \times 10^{-9} \text{ m}^4$

$L = 0.6$ m $h = 40 \text{ mm} = 40 \times 10^{-3}$ m

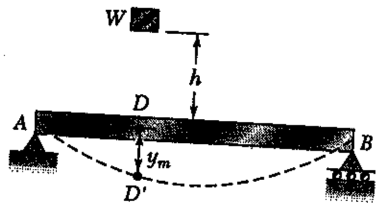
Using Appendix D Case 1 $y_{st} = \frac{WL^3}{3EI}$

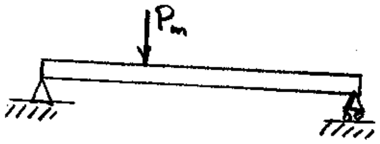
$$y_{st} = \frac{(19.62)(0.6)^3}{(3)(200 \times 10^9)(3.217 \times 10^{-9})} = 2.196 \times 10^{-3} \text{ m}$$

$$\frac{2h}{y_{st}} = \frac{(2)(40 \times 10^{-3})}{2.196 \times 10^{-3}} = 36.44$$

$$\text{impact factor} = 1 + \sqrt{1 + 36.44} = 7.12$$

PROBLEM 11.70

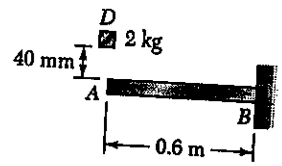




11.70 A block of weight W is dropped from a height h onto the horizontal beam AB and hits it at point D. (a) Denoting by y_m the exact value of the maximum deflection at D and by y_m' the value obtained by neglecting the effect of this deflection on the change in potential energy of the block, show that the absolute value of the relative error is $(y_m - y_m')/y_m$ never exceeds $y_m'/2h$. (b) Check the result obtained in part a by solving part a of Prob. 11.62 without taking y_m' into account when determining the change in potential energy of the load, and comparing the answer obtained in this way with the exact answer to that problem.

11.62 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (a) the maximum deflection of end A, (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.

SOLUTION



$U = \frac{1}{2} P_m y_m = \frac{1}{2} k y_m^2$ where k is the spring constant for a load at point D.

Work of falling weight: exact: Work $= W(h + y_m)$

approximate: Work $\approx Wh$

Equating work and energy: $\frac{1}{2} k y_m^2 = W(h + y_m)$ (1) exact

$\frac{1}{2} k y_m'^2 = W h$ (2) approximate

where y_m' is the approximate value for y_m

Subtracting $\frac{1}{2} k (y_m^2 - y_m'^2) = W y_m$

$$y_m^2 - \tilde{y}_m^2 = (y_m - y_m')(y_m + y_m') = \frac{2W}{k} y_m$$

Relative error $\frac{y_m - y_m'}{y_m} = \frac{2W}{k(y_m + \tilde{y}_m)}$

But $\frac{2W}{k} = \frac{y_m'^2}{h}$ from equation (2)

(a) Relative error $= \frac{y_m - y_m'}{y_m} = \frac{y_m'^2}{h(y_m + y_m')} < \frac{y_m'}{2h}$

(b) From the solution to Prob. 11.62 $y_m = 15.63$ mm

Approximate solution: $W = mg = (2)(9.81) = 19.62$ N

$E = 200 \times 10^9$ Pa $\quad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{16}{2}\right)^4 = 3.217 \times 10^3 \text{ mm}^4 = 3.217 \times 10^{-9} \text{ m}^4$

$L = 0.6$ m, $\quad h = 40$ mm $= 40 \times 10^{-3}$ m

$$k = \frac{3EI}{L^3} = \frac{(3)(200 \times 10^9)(3.217 \times 10^{-9})}{(0.6)^3} = 8.936 \times 10^3 \text{ N/m}$$

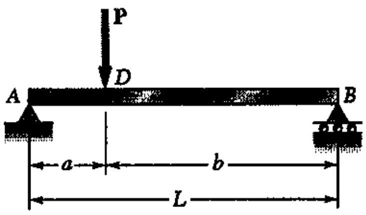
$$y_m'^2 = \frac{2Wh}{k} = \frac{(2)(19.62)(40 \times 10^{-3})}{8.936 \times 10^3} = 175.65 \times 10^{-6} \text{ m}^2$$

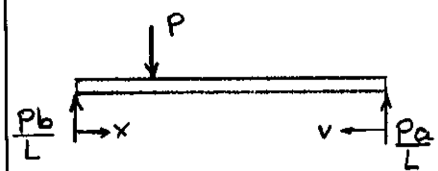
$y_m' = 13.25 \times 10^{-3}$ m $= 13.25$ mm

relative error $= \frac{15.63 - 13.25}{15.63} = 0.152$ ◄ $\quad \frac{y_m'}{2h} = 0.166$ ◄

PROBLEM 11.71

11.71 Using the method of work-energy, determine the deflection at point D caused by the load **P**





SOLUTION

Reactions: $R_A = \frac{Pb}{L}$, $R_B = \frac{Pa}{L}$

Over AD $\quad M = R_A x = \frac{Pbx}{L}$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2 b^2}{2EIL^2} \int_0^a x^2 dx$$

$$= \frac{P^2 b^2 a^3}{6EIL^2}$$

Over DB $\quad M = R_B v = \frac{Pav}{L}$

$$U_{DB} = \int_0^b \frac{M^2}{2EI} dv = \frac{P^2 a^2}{2EIL^2} \int_0^b v^2 dv$$

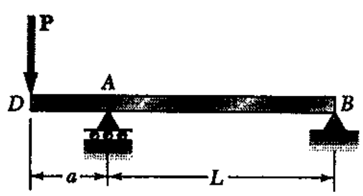
$$= \frac{P^2 a^2 b^3}{6EIL^2}$$

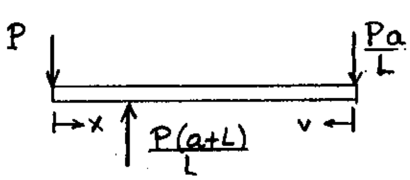
Total $\quad U = U_{AB} + U_{BC} = \frac{P^2 a^2 b^2 (a+b)}{6EIL^2} = \frac{P^2 a^2 b^2}{6EIL}$

$\frac{1}{2} P \delta_D = U \qquad \delta_D = \frac{2U}{P} = \frac{Pa^2 b^2}{3EIL} \downarrow$

PROBLEM 11.72

11.72 Using the method of work-energy, determine the deflection at point D caused by the load **P**





SOLUTION

$\circlearrowleft \Sigma M_A = 0 \qquad Pa + R_B L = 0 \qquad R_B = -\frac{Pa}{L}$

Over portion DA $\quad M = -Px$

$$U_{DA} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^a x^2 dx = \frac{P^2 a^3}{6EI}$$

Over portion AB $\quad M = -\frac{Pav}{L}$

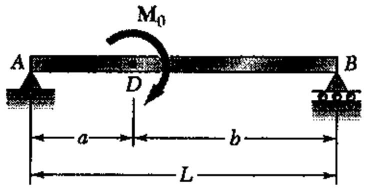
$$U_{AB} = \int_0^L \frac{M^2}{2EI} dv = \frac{P^2 a^2}{2EIL^2} \int_0^L v^2 dv = \frac{Pa^2 L}{6EI}$$

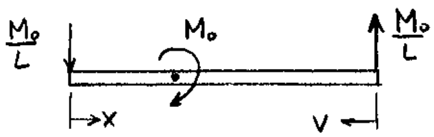
Total $\quad U = U_{DA} + U_{AB} = \frac{P^2 a^2 (a+L)}{6EI}$

$\frac{1}{2} P \delta_D = U \qquad \delta_D = \frac{2U}{P} = \frac{Pa^2 (a+L)}{3EI} \downarrow$

PROBLEM 11.73

11.73 Using the method of work-energy, determine the slope at point D caused by the couple $\mathbf{M}_0$.





SOLUTION

Reactions $\quad R_A = \frac{M_o}{L}\downarrow \qquad R_B = \frac{M_o}{L}\uparrow$

Over portion AD $\quad M = -\frac{M_o x}{L}$

$$U_{AD} = \int_0^a \frac{M^2}{2EI}dx = \frac{M_o^2}{2EIL^2}\int_0^a x^2 dx$$

$$= \frac{M_o a^3}{6EIL^2}$$

Over portion DB $\quad M = \frac{M_o v}{L}$

$$U_{DB} = \int_0^b \frac{M^2}{2EI}dv = \frac{M_o^2}{2EIL^2}\int_0^b v^2 dv$$

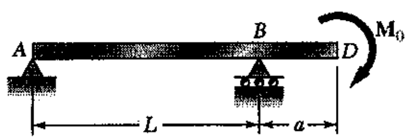
$$= \frac{M_o^2 b^3}{6EIL^2}$$

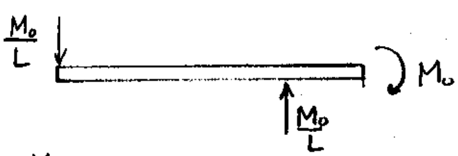
Total $\quad U = U_{AD} + U_{DB} = \frac{M_o^2(a^3+b^3)}{6EIL^2}$

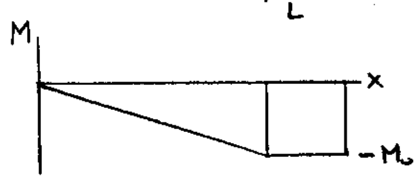
$\frac{1}{2}M_o\theta_D = U \qquad \theta_D = \frac{2U}{M_o} = \frac{M_o(a^3+b^3)}{3EIL^2}$ ◀

PROBLEM 11.74

11.74 Using the method of work-energy, determine the slope at point D caused by the couple $\mathbf{M}_0$.







SOLUTION

Reactions $\quad R_A = \frac{M_o}{L}\downarrow \qquad R_B = \frac{M_o}{L}\uparrow$

Over portion AB $\quad M = -\frac{M_o x}{L}$

$$U_{AB} = \int_0^L \frac{M^2}{2EI}dx = \frac{M_o^2}{2EIL^2}\int_0^L x^2 dx$$

$$= \frac{M_o^2 L}{6EI}$$

Over portion BD $\quad M = -M_o$

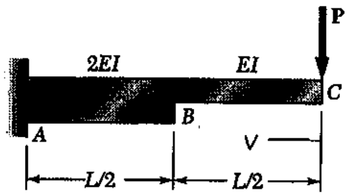
$$U_{BD} = \frac{M_o^2 a}{2EI}$$

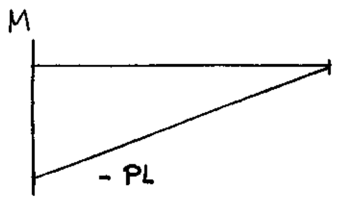
Total $\quad U = U_{AB} + U_{BD} = \frac{M_o^2(L+3a)}{6EI}$

$\frac{1}{2}M_o\theta_D = U \qquad \theta_D = \frac{2U}{M_o} = \frac{M_o(L+3a)}{3EI}$ ◀

PROBLEM 11.75

11.75 Using the method of work and energy, determine the deflection at point C caused by the load **P**.





SOLUTION

Bending moment $\quad M = -Pv$

Over AB

$$U_{AB} = \int_{\frac{L}{2}}^{L} \frac{M^2}{4EI}\,dv = \frac{P^2}{4EI}\int_{\frac{L}{2}}^{L} v^2\,dv$$

$$= \frac{P^2}{12EI}\left[L^3 - \left(\tfrac{L}{2}\right)^3\right] = \frac{7}{96}\frac{P^2L^3}{EI}$$

Over BC $\quad U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dv = \frac{P^2}{2EI}\int_0^{\frac{L}{2}} v^3\,dv$

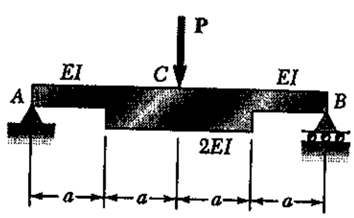
$$= \frac{1}{48}\frac{P^2L^3}{EI}$$

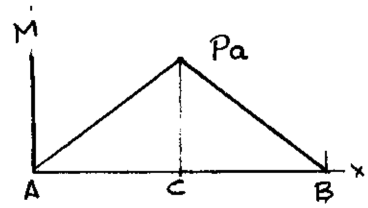
Total $\quad U = U_{AB} + U_{BC} = \frac{3}{32}\frac{P^2L^3}{EI}$

$\frac{1}{2}P\delta_c = 0 \qquad \delta_c = \frac{2U}{P} = \frac{3}{16}\frac{PL^3}{EI}\downarrow$

PROBLEM 11.76

11.76 Using the method of work and energy, determine the deflection at point C caused by the load **P**.





SOLUTION

Symmetric beam and loading $\quad R_A = R_B = \frac{1}{2}P$

From A to C $\qquad M = R_A x = \frac{1}{2}Px$

$$U_{AC} = \int_0^{a} \frac{M^2}{2EI}\,dx + \int_a^{2a}\frac{M^2}{4EI}\,dx$$

$$= \frac{P^2}{8EI}\int_0^a x^2\,dx + \frac{P^2}{16EI}\int_a^{2a} x^2\,dx$$

$$\frac{P^2a^3}{24EI} + \frac{P^2}{48EI}\left[(2a)^3 - a^3\right] = \frac{3}{16}\frac{P^2a^3}{EI}$$

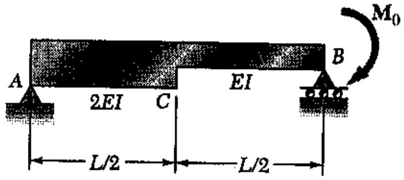
By symmetry $\quad U_{CB} = U_{AB} = \frac{3}{16}\frac{P^2a^3}{EI}$

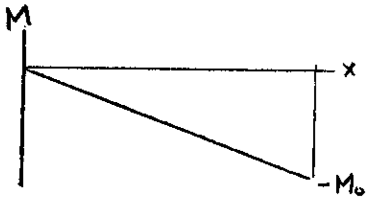
Total $\quad U = U_{AB} + U_{BC} = \frac{3}{8}\frac{P^2a^3}{EI}$

$\frac{1}{2}P\delta_c = U \qquad \delta_c = \frac{2U}{P} = \frac{3}{4}\frac{Pa^3}{EI}$

PROBLEM 11.77

11.77 Using the method of work and energy, determine the slope at point *B* caused by the couple $\mathbf{M}_0$.





SOLUTION

$$\circlearrowleft \Sigma M_B = 0 \qquad -R_A L - M_0 = 0 \qquad R_A = -\frac{M_0}{L}$$

$$M = R_A x = -\frac{M_0}{L} x$$

Over portion AC $\quad U_{AC} = \int_0^{\frac{L}{2}} \frac{M^2}{2(2EI)} dx$

$$U_{AC} = \frac{M_0^2}{4EIL^2} \int_0^{\frac{L}{2}} x^2 dx = \frac{1}{96} \frac{M_0^2 L}{EI}$$

Over portion CB $\quad U_{CB} = \int_{\frac{L}{2}}^{L} \frac{M^2}{2EI} dx$

$$U_{CB} = \frac{M_0^2}{2EIL^2} \int_{\frac{L}{2}}^{L} x^2 dx = \frac{M_0^2}{6EIL^2}\left[L^3 - \left(\frac{L}{2}\right)^3\right]$$

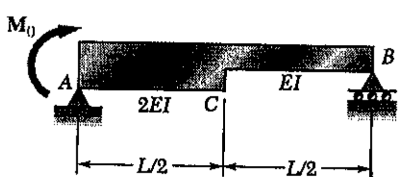
$$= \frac{7}{48} \frac{M_0^2 L}{EI}$$

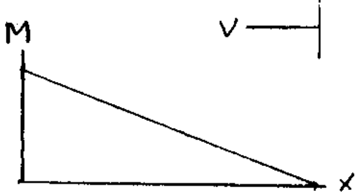
Total $\quad U = U_{AC} + U_{CB} = \frac{5}{32} \frac{M_0^2 L}{EI}$

$$\frac{1}{2} M_0 \theta_B = U \qquad \theta_B = \frac{2U}{M_0} = \frac{5}{16} \frac{M_0 L}{EI}$$

PROBLEM 11.78

11.78 Using the method of work-energy, determine the slope at point *A* caused by the couple $\mathbf{M}_0$.





SOLUTION

$$R_B = \frac{M_0}{L}$$

$$M = R_B v = \frac{M_0}{L} v$$

Over AC $\quad U_{AC} = \int_{\frac{L}{2}}^{L} \frac{M^2}{2(2EI)} dv$

$$U_{AC} = \frac{M_0^2}{4EIL^2} \int_{\frac{L}{2}}^{L} v^2 dv = \frac{M_0^2}{12EIL^2}\left[L^3 - \left(\frac{L}{2}\right)^3\right]$$

$$= \frac{7}{96} \frac{M_0^2 L}{EI}$$

Over CB $\quad U_{CB} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv$

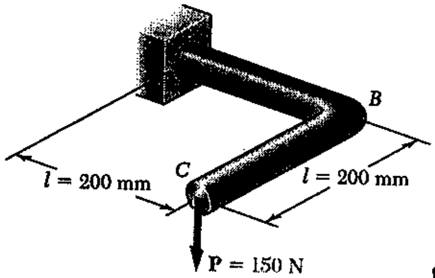
$$U_{CB} = \frac{M_0^2}{2EIL^2} \int_0^{\frac{L}{2}} v^2 dv = \frac{1}{48} \frac{M_0^2 L}{EI}$$

Total $\quad U = U_{AC} + U_{CB} = \frac{3}{32} \frac{M_0^2 L}{EI}$

$$\frac{1}{2} M_0 \theta_A = U \qquad \theta_A = \frac{2U}{M_0} = \frac{3}{16} \frac{M_0 L}{EI}$$

PROBLEM 11.79

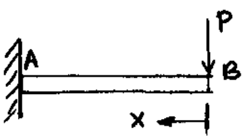
11.79 The 12-mm-diameter steel rod *ABC* has been bent into the shape shown. Knowing that $E = 200$ GPa and $G = 77.2$ GPa, determine the deflection of end *C* caused by the 150-N force.



SOLUTION

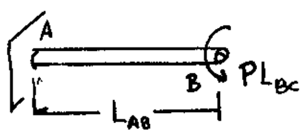
$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{12}{2}\right)^4 = 2.0358\times10^3 \text{ mm}^4$$
$$= 2.0358\times10^{-9} \text{ m}^4$$

$$I = \frac{1}{2}J = 1.0179\times10^{-9} \text{ m}^4$$



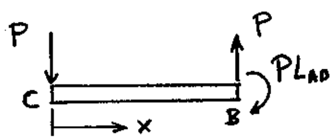
Portion AB: bending $M = -Px$

$$U_{AB,b} = \int_0^{L_{AB}} \frac{M^2}{2EI}dx = \frac{P^2}{2EI}\int_0^{L_{AB}} x^2\,dx$$
$$= \frac{P^2 L_{AB}^3}{6EI} = \frac{(150)^2(200\times10^{-3})^3}{(6)(200\times10^9)(1.0179\times10^{-9})}$$
$$= 0.14736 \text{ J}$$



torsion $T = PL_{BC}$

$$U_{AB,t} = \frac{T^2 L_{AB}}{2GJ} = \frac{P^2 L_{BC}^2 L_{AB}}{2GJ}$$
$$= \frac{(150)^2(200\times10^{-3})^2(200\times10^{-3})}{(2)(77.2\times10^9)(2.0358\times10^{-9})}$$
$$= 0.57265 \text{ J}$$



Portion BC: $M = -Px$

$$U_{BC} = \int_0^{L_{BC}} \frac{M^2}{2EI}dx = \frac{P^2}{3EI}\int_0^{L_{BC}} x^2 dx = \frac{P^2 L_{BC}^3}{6EI}$$
$$= \frac{(150)^2(200\times10^{-3})^3}{(6)(200\times10^9)(1.0179\times10^9)} = 0.14736 \text{ J}$$

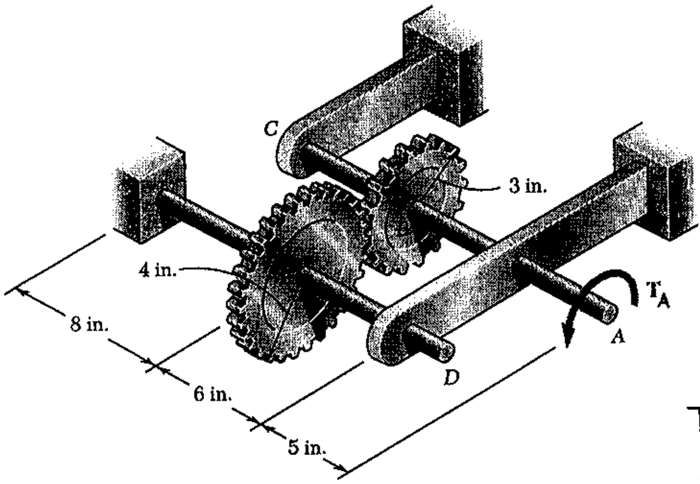
Total: $U = U_{AB,b} + U_{AB,t} + U_{BC} = 0.86737 \text{ J}$

Work-energy $\frac{1}{2}PS = U \qquad S = \frac{2U}{P} = \frac{(2)(0.86737)}{150}$

$$= 11.57\times10^{-3} \text{ m} = 11.57 \text{ mm} \downarrow$$

PROBLEM 11.80

11.80 Two steel shafts, each of 0.75-in. diameter, are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi and that shaft DF is fixed at F, determine the angle through which end A rotates when a 750-lb·in. torque is applied at A. (Ignore the strain energy due to the bending of the shafts)



SOLUTION

Work-energy equation

$$\frac{1}{2} T_A \varphi_A = U$$

$$\varphi_A = \frac{2U}{T_A}$$

Portion AB of shaft ABC:

$$T_{AB} = T_A = 750 \text{ lb}\cdot\text{in}$$

$$L_{AB} = 5 + 6 = 11 \text{ in}$$

$$J_{AB} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{2}\left(\frac{0.75}{2}\right)^4 = 31.063 \times 10^{-3} \text{ in}^4$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} = \frac{(750)^2(11)}{(2)(11.2\times10^6)(31.063\times10^{-3})} = 8.892 \text{ in}\cdot\text{lb}$$

Portion BC of shaft ABC: $U_{BC} = 0$

Gear B $\quad F_{BE} = \frac{T_B}{r_B} = \frac{T_{AB}}{r_B} = \frac{750}{3} = 250 \text{ lb}$

Gear E $\quad T_E = r_E F_{BE} = (4)(250) = 1000 \text{ lb}\cdot\text{in}$

Portion DE of shaft DEF. $\quad U_{DE} = 0$

Portion EF of shaft DEF; $\quad T_{EF} = T_E = 1000 \text{ lb}\cdot\text{in}$

$L_{EF} = 8 \text{ in} \qquad J_{EF} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = 31.063 \times 10^{-3} \text{ in}^4$

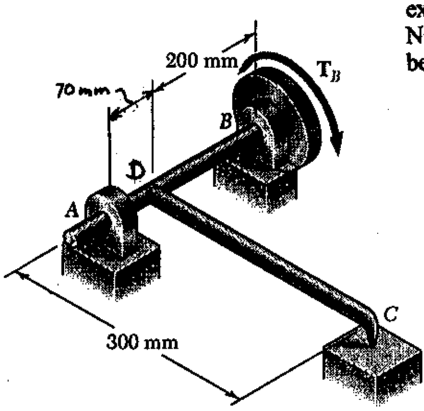
$$U_{EF} = \frac{T_{EF}^2 L_{EF}}{2GJ_{EF}} = \frac{(1000)^2(8)}{(2)(11.2\times10^6)(31.063\times10^{-3})} = 11.497 \text{ lb}\cdot\text{in}$$

Total: $\quad U = U_{AB} + U_{BC} + U_{DE} + U_{EF} = 20.389 \text{ in}\cdot\text{lb}$

$$\varphi_A = \frac{2U}{T_A} = \frac{(2)(20.389)}{750} = 54.4\times10^{-3} \text{ rad} = 3.12^\circ$$

PROBLEM 11.81

11.81 The 20-mm-diameter steel rod *CD* is welded to the 20-mm-diameter steel shaft *AB* as shown. End *C* of rod *CD* is touching the rigid surface shown when a couple $\mathbf{T}_B$ is applied to a disk attached to shaft *AB*. Knowing that the bearings are self aligning and exert no couples on the shaft, determine the angle of rotation of the disk when $T_B = 400$ N·m. Use $E = 200$ GPa and $G = 77.2$ GPa. (Consider the strain energy due to both bending and twisting in shaft *AB* and to bending in arm *CD*.)



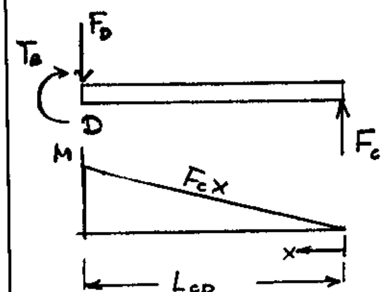
SOLUTION

$$\circlearrowleft \Sigma M_{AB} = 0 \qquad r_{CD} F_C = T_B \qquad F_C = \frac{T_B}{r_{CD}}$$

$$F_C = \frac{400}{300 \times 10^{-3}} = 1333.3 \text{ N} \qquad F_D = 1333.3 \text{ N}$$

Bending of rod CD:

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{20}{2}\right)^4 = 7.854 \times 10^3 \text{ mm}^4 = 7.854 \times 10^{-9} \text{ m}^4$$



$$M = F_C x$$

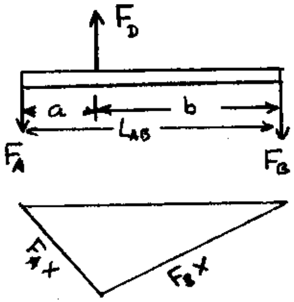
$$U = \int_0^{L_{CD}} \frac{(F_C x)^2}{2EI} = \frac{F_C^2 L_{CD}^3}{6EI}$$

$$= \frac{(1333.3)^2 (300 \times 10^{-3})^3}{(6)(200 \times 10^9)(7.854 \times 10^{-9})} = 5.093 \text{ J}$$

Bending of shaft ADB

$$\circlearrowleft \Sigma M_B = 0 \qquad -F_A L_{AB} + F_D b = 0 \qquad F_A = \frac{F_D b}{L_{AB}}$$

$$\circlearrowleft \Sigma M_A = 0 \qquad +F_A L_{AB} - F_D b = 0 \qquad F_A = \frac{F_D a}{L_{AB}}$$



$$U = \frac{1}{2EI}\left\{\int_0^a \left(\frac{F_D b}{L_{AB}}\right)^2 dx + \int_0^b \left(\frac{F_D a}{L_{AB}}\right)^2 dx\right\} = \frac{F_D^2 a^2 b^2}{6EI L_{AB}}$$

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = 7.854 \times 10^3 \text{ mm}^4 = 7.854 \times 10^{-9} \text{ m}^4$$

$$L_{AB} = (270 \times 10^{-3}) \text{ m}$$

$$U = \frac{(1333.3)^2 (70 \times 10^{-3})^2 (200 \times 10^{-3})^2}{(6)(200 \times 10^9)(7.854 \times 10^{-9})(270 \times 10^{-3})} = 0.137 \text{ J}$$

Torsion: Only portion DB carries torque. $J = 2I = 15.708 \times 10^{-9} \text{ m}^4$

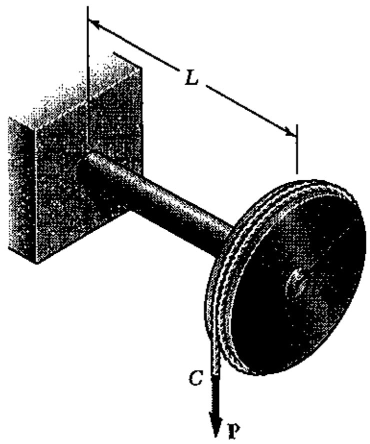
$$U = \frac{T_B^2 L_{DB}}{2GJ} = \frac{(400)^2 (200 \times 10^{-3})}{(2)(77.2 \times 10^9)(15.708 \times 10^{-9})} = 13.194 \text{ J}$$

Total: $U = 5.093 + 0.137 + 13.194 = 18.424$ J

$$\frac{1}{2} T_B \varphi_B = U$$

$$\varphi_B = \frac{2U}{T_B} = \frac{(2)(18.424)}{400} = 92.1 \times 10^{-3} \text{ rad}$$

PROBLEM 11.82



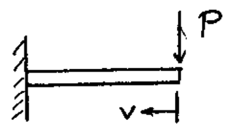
11.82 A disk of radius a has been welded to end B of the solid steel shaft AB.. A cable is then wrapped around the disk and a vertical force **P** is applied to end C of the cable. Knowing that the radius of the shaft is r and neglecting the deformations of the disk and of the cable, show that the deflection of point C caused by the application of **P** is

$$\delta_C = \frac{PL^3}{3EI}\left(1 + 1.5\frac{Ea^2}{GL^2}\right)$$

SOLUTION

Torsion: $T = Pa$

$$U_t = \frac{T^2L}{2GJ} = \frac{P^2a^2L}{2GJ}$$



Bending: $M = Pv$

$$U_b = \int_0^L \frac{M^2\,dv}{2EI} = \int_0 \frac{P^2v^2\,dv}{2EI}$$

$$= \frac{P^2L^3}{6EI}$$

Total $\quad U = \dfrac{P^2a^2L}{2GJ} + \dfrac{P^2L^3}{6EI} = \dfrac{1}{2}P\delta_c$

$$\delta_c = \frac{Pa^2L}{GJ} + \frac{PL^3}{3EI} = \frac{PL^3}{3EI}\left(1 + \frac{3EIa^2}{GJL^2}\right)$$

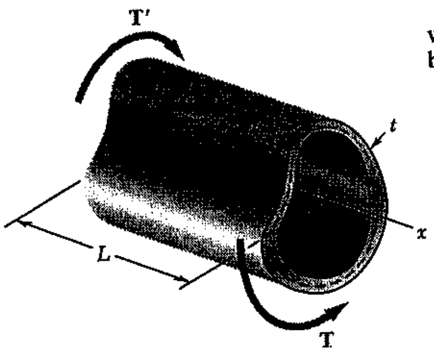
Since $J = 2I \qquad \delta_c = \dfrac{PL^3}{3EI}\left(1 + \dfrac{3}{2}\dfrac{Ea^2}{GL^2}\right)$

PROBLEM 11.83

11.83 The thin-walled hollow cylindrical member AB has a noncircular cross section of nonuniform thickness. Using the expression given in Eq. (3.53) of Sec. 3.13, and the expression for the strain-energy density given in Eq. (11.19) of Sec. 11.4, show that the angle of twist of member AB is

$$\phi = \frac{TL}{4A^2G}\oint\frac{ds}{t}$$

where ds is an element of the centerline of wall cross section and A is the area enclosed by that centerline.



SOLUTION

From equation (3.53) $\quad \tau = \frac{T}{2tA}$

Strain energy density

$$u = \frac{\tau^2}{2G} = \frac{T^2}{8Gt^2A^2}$$

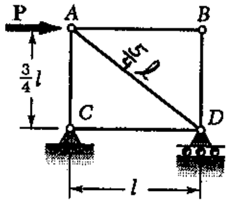
$$U = \int_0^L \oint u\, t\, ds\, dx$$

$$= \int_0^L \frac{T^2}{8GA^2}\oint\frac{ds}{t}\, dx = \frac{T^2L}{8GA^2}\oint\frac{ds}{t}$$

Work of torque $= \frac{1}{2}T\varphi = \frac{T^2L}{8GA^2}\oint\frac{ds}{t} \qquad \varphi = \frac{TL}{4GA^2}\oint\frac{ds}{t}$

PROBLEM 11.84

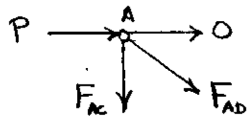
11.84 Each member of the truss shown has a uniform cross-sectional area A. Using the method of work and energy, determine the horizontal deflection of the point of application of the load **P**.



SOLUTION

Members AB and BD are zero force members.

Joint A



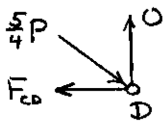
$+\rightarrow \Sigma F_x = 0$

$\frac{4}{5} F_{AD} + P = 0 \qquad F_{AD} = -\frac{5}{4}P$

$+\uparrow \Sigma F_y = 0$

$-F_{AC} - \frac{3}{5} F_{AD} = 0 \qquad F_{AC} = \frac{3}{4}P$

Joint D $\quad +\rightarrow \Sigma F_x = 0$



$\frac{4}{5} \cdot \frac{5}{4} P - F_{CD} = 0$

$F_{CD} = P$

Member	F	L	F^2L
AB	0	ℓ	0
BD	0	$\frac{3}{4}\ell$	0
AD	$-\frac{5}{4}P$	$\frac{5}{4}\ell$	$\frac{125}{64}P^2\ell$
CD	P	ℓ	$P^2\ell$
AC	$\frac{3}{4}P$	$\frac{3}{4}L$	$\frac{27}{64}P^2\ell$
Σ			$\frac{27}{8}P^2\ell$

$$U = \Sigma \frac{F^2L}{2EA} = \frac{1}{2EA}\Sigma F^2 L$$

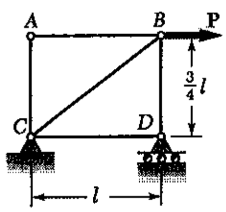
$$= \frac{27}{16}\frac{P^2\ell}{EA}$$

Work of $P = \frac{1}{2}P\Delta = U$

$$\Delta = \frac{2U}{P} = \frac{27}{8}\frac{P\ell}{EA} = 3.375\frac{P\ell}{EA} \blacktriangleleft$$

PROBLEM 11.85

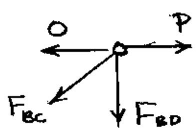
11.85 Each member of the truss shown has a uniform cross-sectional area A. Using the method of work and energy, determine the horizontal deflection of the point of application of the load **P**.



SOLUTION

Members AB, AC, and CD are zero force members.

Joint B



$+\rightarrow \Sigma F_x = 0$

$P - \frac{4}{5}F_{BC} = 0 \qquad F_{BC} = \frac{5}{4}P$

$+\uparrow \Sigma F_y = 0$

$-F_{BD} - \frac{3}{5}F_{BC} = 0 \qquad F_{BD} = -\frac{3}{4}P$

$$U = \Sigma \frac{F^2L}{2EA} = \frac{1}{2EA}\Sigma F^2L$$

$$= \frac{19}{16}\frac{P^2\ell}{EA}$$

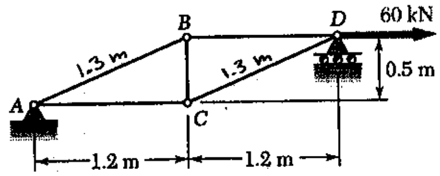
Member	F	L	F^2L
AB	0	ℓ	0
AC	0	$\frac{3}{4}\ell$	0
CD	0	ℓ	0
BC	$\frac{5}{4}P$	$\frac{5}{4}\ell$	$\frac{125}{64}P^2\ell$
BD	$-\frac{3}{4}P$	$\frac{3}{4}\ell$	$\frac{27}{64}P^2\ell$
Σ			$\frac{19}{8}P^2\ell$

Work of $P = \frac{1}{2}P\Delta = U$

$$\Delta = \frac{2U}{P} = \frac{19}{8}\frac{P\ell}{EA} = 2.375\frac{P\ell}{EA} \blacktriangleleft$$

PROBLEM 11.86

11.86 Each member of the truss shown is made of steel; the cross-sectional area of member BC is 800 mm² and for all other members the cross-sectional area is 400 mm². Using $E = 200$ GPa, determine the deflection of point D caused by the 60-kN load shown.

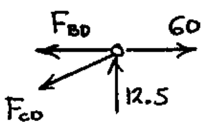


SOLUTION

Entire truss $\circlearrowright \Sigma M_A = 0$

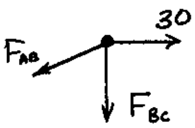
$2.4\, R_D - (0.5)(60) = 0 \qquad R_D = 12.5$ kN

Joint D $\quad +\uparrow \Sigma F_y = 0 \qquad 12.5 - \frac{0.5}{1.3} F_{CD} = 0 \qquad F_{CD} = 32.5$ kN



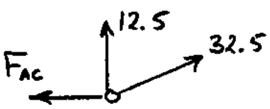
$\xrightarrow{+} \Sigma F_x = 0 \qquad 60 - F_{BD} - \frac{1.2}{1.3} F_{CD} = 0 \qquad F_{BD} = 30$ kN

Joint B $\quad \xrightarrow{+} \Sigma F_x = 0 \qquad 30 - \frac{1.2}{1.3} F_{AB} = 0 \qquad F_{AB} = 32.5$ kN



$+\uparrow \Sigma F_y = 0 \qquad -\frac{0.5}{1.3} F_{AB} + F_{BC} = 0 \qquad F_{BC} = 12.5$ kN

Joint C $\quad \xrightarrow{+} \Sigma F_x = 0 \qquad -F_{AC} + \frac{1.2}{1.3}(32.5) = 0 \qquad F_{AC} = 30$ kN



$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2E} \Sigma \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	A (10^{-6} m²)	F^2L/A (N²/m)
CD	32.5	1.3	400	3.4328×10^{12}
BD	30	1.2	400	2.7×10^{12}
AB	32.5	1.3	400	3.4328×10^{12}
BC	12.5	0.5	800	0.0977×10^{12}
AC	30	1.2	400	2.7×10^{12}
				12.3633×10^{12}

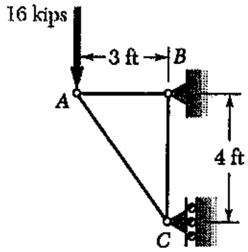
$$U = \frac{12.3633 \times 10^{12}}{(2)(200 \times 10^9)} = 30.908 \text{ J}$$

$$\tfrac{1}{2} P \Delta = U \qquad \Delta = \frac{2U}{P} = \frac{(2)(30.908)}{60 \times 10^3} = 1.030 \times 10^{-3} \text{ m}$$

$$= 1.030 \text{ mm} \rightarrow$$

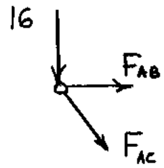
PROBLEM 11.87

11.87 Each member of the truss shown is made of steel and has a uniform cross-sectional area of 3 in². Using $E = 29\times 10^6$ psi, determine the vertical deflection of the point of application of joint A caused by the 16-kip load.



SOLUTION

Joint A



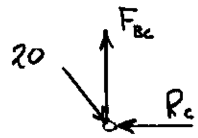
$+\uparrow \Sigma F_y = 0$

$-16 - \frac{4}{5}F_{AC} = 0 \qquad F_{AC} = -20$ kips

$+\rightarrow \Sigma F_x = 0$

$\frac{3}{5}F_{AC} + F_{AB} = 0 \qquad F_{AB} = 12$ kips

Joint C $\qquad +\uparrow \Sigma F_y = 0 \qquad F_B - \frac{4}{5}(20) = 0 \qquad F_{BC} = 16$ kips



$$U = \Sigma \frac{F^2L}{2EA} = \frac{1}{2EA}\Sigma F^2L$$

$E = 29\times 10^3$ ksi

$A = 3$ in²

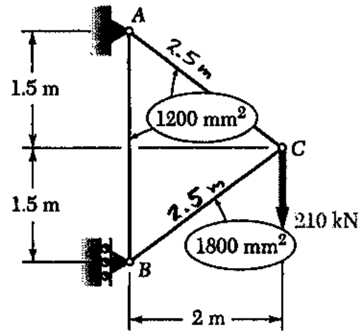
Member	F (kips)	L (in)	F^2L (kip²·in)
AB	12	36	5184
AC	-20	60	24000
BC	16	48	12288
			41472

$$U = \frac{41472}{(2)(29\times10^3)(3)} = 0.23834 \text{ kip}\cdot\text{in}$$

$\frac{1}{2}P\Delta = U \qquad \Delta = \frac{2U}{P} = \frac{(2)(0.23834)}{16} = 0.0298$ in. ↓ ◄

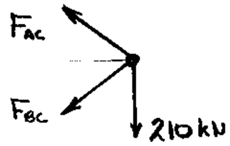
PROBLEM 11.88

11.88 Members of the truss shown are made of steel and have the cross-sectional areas shown. Using $E = 200$ GPa, determine the vertical deflection of joint C caused by the application of the 210-kN load.



SOLUTION

Joint C



$\overset{+}{\rightarrow} \Sigma F_x = 0$

$-\frac{4}{5} F_{AC} - \frac{4}{5} F_{BC} = 0$

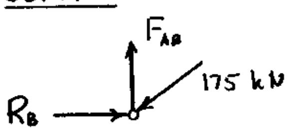
$+\uparrow \Sigma F_y = 0$

$\frac{3}{5} F_{AC} - \frac{3}{5} F_{BC} - 210 = 0$

Solving simultaneously

$F_{AC} = 175$ kN $\qquad F_{BC} = -175$ kN

Joint B



$+\uparrow \Sigma F_y = 0$

$F_{AB} - \left(\frac{3}{5}\right)(175) = 0$

$F_{AB} = 105$ kN

$U_m = \Sigma \frac{F^2 L}{2EA}$

Member	F (kN)	L (m)	A (10^{-6} m²)	F^2L/A (N²/m)
AB	105	3.0	1200	27.5625×10^{12}
AC	175	2.5	1200	63.8021×10^{12}
BC	−175	2.5	1800	42.5347×10^{12}
				133.8993×10^{12}

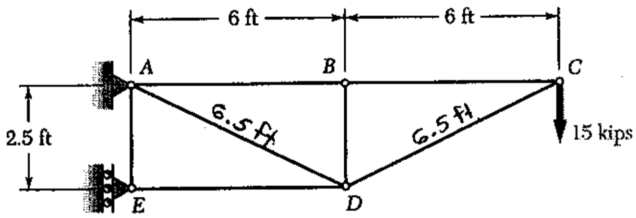
$$U_m = \frac{1}{2E} \Sigma \frac{F^2 L}{A} = \frac{133.8993 \times 10^{12}}{(2)(200 \times 10^9)} = 334.75 \text{ J}$$

$$\frac{1}{2} P_m \Delta_m = U_m$$

$$\Delta_m = \frac{2 U_m}{P_m} = \frac{(2)(334.75)}{210 \times 10^3} = 3.19 \times 10^{-3} \text{ m} = 3.19 \text{ mm}$$

PROBLEM 11.89

11.89 Each member of the truss shown is made of steel and has a uniform cross-sectional area of 5 in². Using $E = 29\times 10^6$ psi, determine the vertical deflection of the point of application of joint C caused by the 15-kip load.



SOLUTION

Members BD and AE are zero force members.

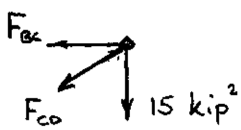
For entire truss $+\circlearrowleft M_A = 0$

$2.5\,R_D - (12)(15) = 0$

$R_D = 72$ kips

For equilibrium of joint E $\quad F_{ED} = -R_D = -72$ kips

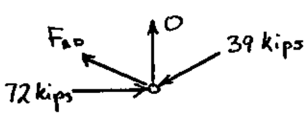
<u>Joint C</u> $\quad +\uparrow \Sigma F_y = 0 \qquad\qquad \xrightarrow{+} \Sigma F_x = 0$



$-\frac{2.5}{6.5}F_{CD} - 15 = 0 \qquad -\frac{6}{6.5}F_{CD} - F_{BC} = 0$

$F_{CD} = -39$ kips $\qquad F_{BC} = 36$ kips.

<u>Joint D</u> $\quad \xrightarrow{+} \Sigma F_x = 0$



$72 - \frac{6}{6.5}(F_{AD} + 39) = 0$

$F_{AD} = 39$ kips

<u>Joint B</u> $\quad \Sigma F_x = 0$

$-F_{AB} + F_{BC} = 0$

$F_{AB} = 36$ kips

Strain energy $\quad U_m = \Sigma \frac{F^2L}{2EA} = \frac{1}{2EA}\Sigma F^2L$

Member	F (kips)	L (in)	F^2L (kip²·in)
AB	36	72	93312
BC	36	72	93312
CD	-39	78	118638
DE	-72	72	373248
BD	0	30	0
AE	0	30	0
AD	39	78	118638
Σ			797148

Data: $E = 29\times 10^3$ ksi

$A = 5$ in²

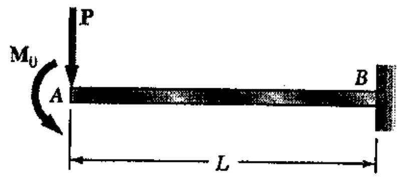
$U_m = \frac{797148}{(2)(29\times 10^3)(5)}$

$= 2.7488$ kip·in

$\frac{1}{2}P_m\Delta_m = U \qquad \Delta_m = \frac{2U_m}{P_m} = \frac{(2)(2.7488)}{15} = 0.366$ in. $\downarrow$

PROBLEM 11.90

11.90 Using the information provided in Appendix D, compute the work of the loads as they are applied to the beam (*a*) if the load P is applied first, (*b*) if the couple $\mathbf{M}_0$ is applied first.



SOLUTION

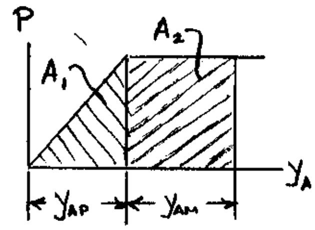
From Appendix D, Case 1

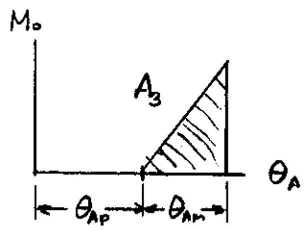
$$y_{AP} = \frac{PL^3}{3EI} \qquad \theta_{AP} = \frac{PL^2}{2EI}$$

From Appendix D, Case 3

$$y_{AM} = \frac{M_0 L^2}{2EI} \qquad \theta_{AM} = \frac{M_0 L}{EI}$$

(a) First P, then M_0.



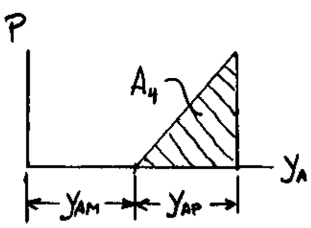


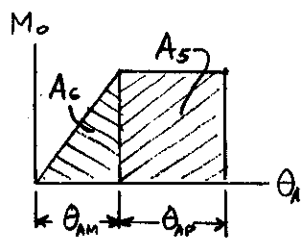
$$U = A_1 + A_2 + A_3$$

$$= \tfrac{1}{2}P y_{AP} + P y_{AM} + \tfrac{1}{2} M_0 \theta_{AM}$$

$$= \frac{P^2L^3}{6EI} + \frac{PM_0L^2}{2EI} + \frac{M_0^2 L}{2EI} \blacktriangleleft$$

(b) First M_0, then P





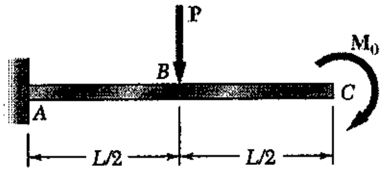
$$U = A_4 + A_5 + A_6$$

$$= \tfrac{1}{2}P y_{AP} + M_0 \theta_{AP} + \tfrac{1}{2} M_0 \theta_{AM}$$

$$= \frac{P^2L^3}{6EI} + \frac{M_0PL^2}{2EI} + \frac{M_0^2 L}{2EI} \blacktriangleleft$$

PROBLEM 11.91

11.91 Using the information provided in Appendix D, compute the work of the loads as they are applied to the beam (*a*) if the load P is applied first, (*b*) if the couple M_0 is applied first.



SOLUTION

Appendix D Cases 1 and 3

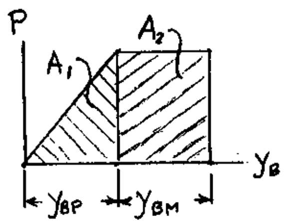
$$y_{BP} = \frac{P(L/2)^3}{3EI} = \frac{PL^3}{24EI}$$

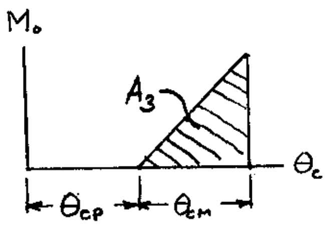
$$\theta_{CP} = \frac{P(L/2)^2}{2EI} = \frac{PL^2}{8EI}$$

$$y_{BM} = \frac{M_0(L/2)^2}{2EI} = \frac{M_0 L^2}{8EI}$$

$$\theta_{BM} = \frac{M_0 L}{EI}$$

(a) First P, then M_0



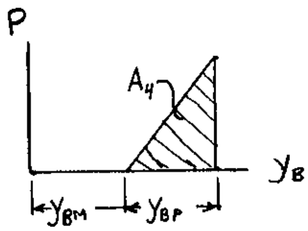


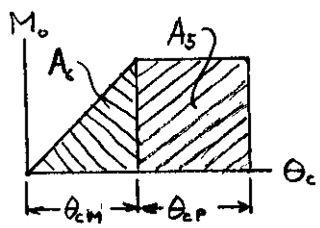
$$U = A_1 + A_2 + A_3$$

$$= \tfrac{1}{2} P y_{BP} + P y_{BM} + \tfrac{1}{2} M_0 \theta_{CM}$$

$$= \frac{P^2 L^3}{48EI} + \frac{PM_0 L^2}{8EI} + \frac{M_0^2 L}{2EI} \quad \blacktriangleleft$$

(b) First M_0, then P



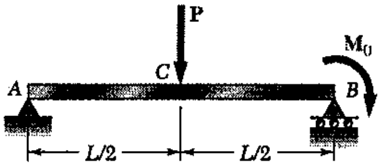


$$U = A_4 + A_5 + A_6$$

$$= \tfrac{1}{2} P y_{BP} + M_0 \theta_{CP} + \tfrac{1}{2} M_0 \theta_{CM}$$

$$= \frac{P^2 L^3}{48EI} + \frac{M_0 P L^2}{8EI} + \frac{M_0^2 L}{2EI} \quad \blacktriangleleft$$

PROBLEM 11.92



11.92 Using the information provided in Appendix D, compute the work of the loads as they are applied to the beam (*a*) if the load P is applied first, (*b*) if the couple $\mathbf{M}_0$ is applied first.

SOLUTION

From Appendix D, Case 4

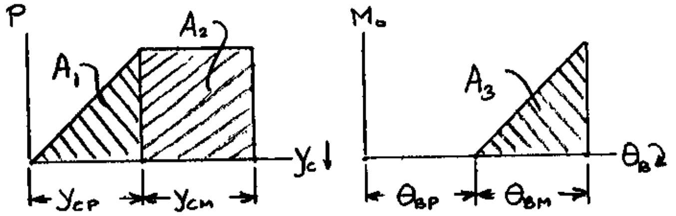
$$\downarrow y_C = \frac{PL^3}{48EI} \qquad \circlearrowleft \theta_B = -\frac{PL^2}{16EI}$$

From Appendix D, Case 7

$$\downarrow y_C = \frac{M_0}{6EIL}\left(\left(L/2\right)^3 - L^2(L/2)\right) = -\frac{M_0L^2}{16EI}$$

$$\circlearrowright \theta_B = \frac{M_0 L}{3EI}$$

(a) First P, then M_0

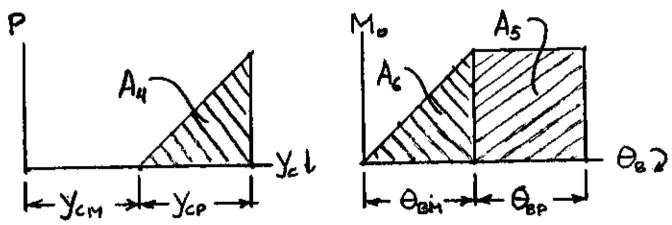


$$U = A_1 + A_2 + A_3$$

$$= \tfrac{1}{2}Py_{CP} + Py_{CM} + \tfrac{1}{2}M_0\theta_{BM}$$

$$= \frac{P^2L^3}{96EI} - \frac{PM_0L^2}{16EI} + \frac{M_0^2L}{6EI}$$

(b) First M_0, then P



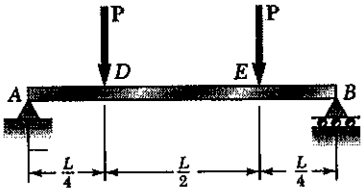
$$U = A_4 + A_5 + A_6$$

$$= \tfrac{1}{2}Py_{CP} + M_0\theta_{BP} + \tfrac{1}{2}M_0\theta_{BM}$$

$$= \frac{P^2L^3}{96EI} - \frac{M_0PL^2}{16EI} + \frac{M_0^2L}{6EI}$$

PROBLEM 11.93

11.93 For the beam and loading shown, (*a*) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (*b*) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part *a*.



SOLUTION

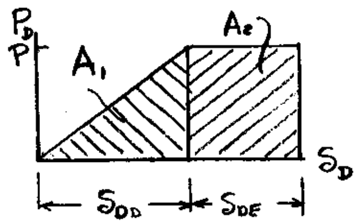
(a) Label the forces P_D and P_E.

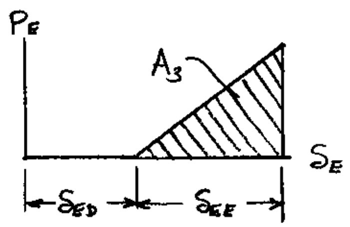
Using Appendix D Case 5

$$\delta_{EE} = \frac{P_E a^2 b^2}{3EIL} = \frac{P_E\left(\frac{3L}{4}\right)^2\left(\frac{L}{4}\right)^2}{3EIL} = \frac{3}{256}\frac{P_E L^3}{EI}$$

$$\delta_{DE} = \frac{P_E b}{6EIL}\left[(L^2 - b^2)x - x^3\right] = \frac{P_E\left(\frac{L}{4}\right)}{6EIL}\left[\left(L^2 - \left(\frac{L}{4}\right)^2\right)\left(\frac{L}{4}\right) - \left(\frac{L}{4}\right)^3\right] = \frac{7}{768}\frac{P_E L^3}{EI}$$

Likewise $\delta_{DD} = \frac{3}{256}\frac{P_D L^3}{EI}$ and $\delta_{ED} = \frac{7}{768}\frac{P_D L^3}{EI}$





Let P_D be applied first.

$$U = A_1 + A_2 + A_3$$

$$U = \tfrac{1}{2}P_D\delta_{DD} + P_D\delta_{DE} + \tfrac{1}{2}P_E\delta_{EE} = \frac{3}{512}\frac{P_D^2L^3}{EI} + \frac{7}{768}\frac{P_DP_EL^3}{EI} + \frac{3}{512}\frac{P_E^2L^3}{EI}$$

With $P_D = P_E = P$ $\qquad U = \frac{1}{48}\frac{P^2L^3}{EI}$ ◄

(b) Reactions $R_A = R_B = P$

Over portion AD $\quad 0 < x < \frac{L}{4} \qquad M = Px$

$$U_{AD} = \int_0^{\frac{L}{4}} \frac{M^2}{2EI}\,dx = \frac{P^2}{2EI}\int_0^{\frac{L}{4}} x^2\,dx = \frac{P^2}{2EI}\cdot\frac{1}{3}\left(\frac{L}{4}\right)^3 = \frac{1}{384}\frac{P^2L^3}{EI}$$

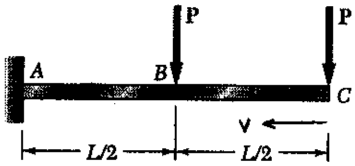
Over portion DE $\qquad M = \frac{PL}{4} \qquad U_{DE} = \frac{M^2\left(\frac{L}{2}\right)}{2EI} + \frac{P^2L^2}{2EI}\cdot\frac{1}{16}\cdot\frac{L}{2} = \frac{P^2L^3}{64EI}$

Over portion EB: $\qquad$ By symmetry $\quad U_{EB} = U_{AD} = \frac{1}{384}\frac{P^2L^3}{EI}$

Total $\quad U = U_{AB} + U_{DE} + U_{EB} = \left(\frac{1}{384} + \frac{1}{64} + \frac{1}{384}\right)\frac{P^2L^3}{EI} = \frac{1}{48}\frac{P^2L^3}{EI}$ ◄

PROBLEM 11.94

11.94 For the beam and loading shown, (*a*) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (*b*) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part *a*.



SOLUTION

(a) Label the forces P_B and P_C

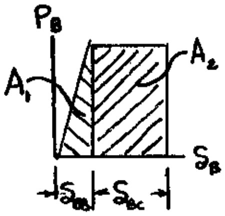
Using Appendix D Case 1

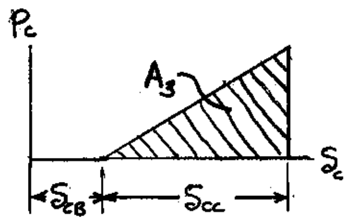
$$\delta_{BB} = \frac{P_B(L/2)^3}{3EI} = \frac{1}{24}\frac{P_BL^3}{EI}$$

$$\delta_{CB} = \delta_{BB} + \frac{L}{2}\theta_B = \frac{1}{24}\frac{P_BL^3}{3EI} + \frac{L}{2}\frac{P_B(L/2)^2}{2EI} = \left(\frac{1}{24}+\frac{1}{16}\right)\frac{P_BL^3}{EI} = \frac{5}{48}\frac{P_BL^3}{EI}$$

$$\delta_{CC} = \frac{1}{3}\frac{P_CL^3}{EI}$$

$$\delta_{BC} = \frac{P_C}{6EI}(3Lx^2 - x^3) = \frac{P_C}{6EI}\left(3L\left(\frac{L}{2}\right)^2 - \left(\frac{L}{2}\right)^3\right) = \frac{5}{48}\frac{P_CL^3}{EI}$$





Apply P_B first, then P_C

$U = A_1 + A_2 + A_3$

$$U = \frac{1}{2}P_B\delta_{BB} + P_B\delta_{BC} + \frac{1}{2}P_C\delta_{CC} = \frac{1}{48}\frac{P_BL^3}{EI} + \frac{5}{48}\frac{P_BP_CL^3}{EI} + \frac{1}{6}\frac{P_C^2L^3}{EI}$$

With $P_B = P_C = P$ $\quad U = \left(\frac{1}{48} + \frac{5}{48} + \frac{1}{6}\right)\frac{P^2L^3}{EI} = \frac{7}{24}\frac{P^2L^3}{EI}$ ◄

Over AB $\quad M = Pv + P\left(v - \frac{L}{2}\right) = P\left(2v - \frac{L}{2}\right)$

$$U_{AB} = \int_{\frac{L}{2}}^{L}\frac{M^2}{2EI}dv = \frac{P^2}{2EI}\int_{\frac{L}{2}}^{L}\left(4v^2 - 2Lv + \frac{1}{4}L^2\right)dv$$

$$= \frac{P^2}{2EI}\left\{\frac{4}{3}\left[L^3 - \left(\frac{L}{2}\right)^3\right] - 2L\cdot\frac{1}{2}\left[L^2 - \left(\frac{L}{2}\right)^2\right] + \frac{1}{4}L^2\left[L - \frac{L}{2}\right]\right\}$$

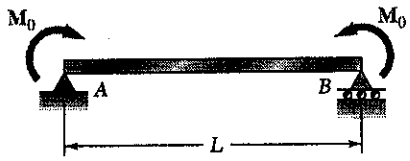
$$= \frac{P^2}{2EI}\left\{\frac{7}{6}L^3 - \frac{3}{4}L^3 + \frac{1}{8}L^3\right\} = \frac{13}{48}\frac{P^2L^3}{EI}$$

Over BC $\quad M = Pv \quad U_{BC} = \int_0^{\frac{L}{2}}\frac{M^2}{2EI}dv = \frac{P^2}{2EI}\int_0^{\frac{L}{2}}v^2dv = \frac{P^2}{2EI}\cdot\frac{1}{3}\left(\frac{L}{2}\right)^3$

$$= \frac{P^2L^3}{48EI}$$

Total $\quad U = U_{AB} + U_{BC} = \left(\frac{13}{48} + \frac{1}{48}\right)\frac{P^2L^3}{EI} = \frac{7}{24}\frac{P^2L^3}{EI}$ ◄

PROBLEM 11.95



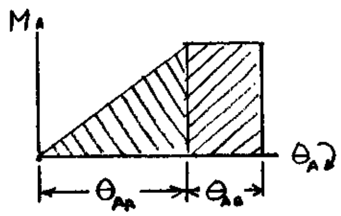
11.95 For the beam and loading shown, (*a*) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (*b*) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part *a*.

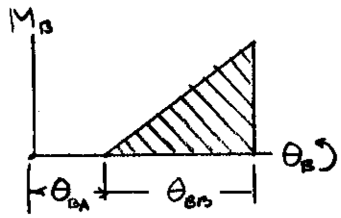
SOLUTION

(a) Label the couples M_A and M_B

Using Appendix D, Case 7

$$\theta_{AA} = \frac{M_A L}{3EI} \qquad \theta_{BA} = \frac{M_A L}{6EI} \qquad \theta_{BB} = \frac{M_B L}{3EI} \qquad \theta_{AB} = \frac{M_B L}{6EI}$$





Apply M_A first, then M_B.

$$U = A_1 + A_2 + A_3$$

$$U = \tfrac{1}{2} M_A \theta_{AA} + M_A \theta_{AB} + \tfrac{1}{2} M_B \theta_{BB} = \frac{1}{6}\frac{M_A^2 L}{EI} + \frac{1}{6}\frac{M_A M_B L}{EI} + \frac{1}{6}\frac{M_B^2 L}{EI}$$

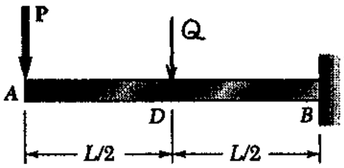
With $M_A = M_B = M_0$ $\qquad U = \dfrac{1}{2}\dfrac{M_0^2 L}{EI}$ ◄

(b) Bending moment $\qquad M = M_0$

$$U = \int_0^L \frac{M^2}{2EI}\,dx = \frac{M_0^2 L}{2EI}$$ ◄

PROBLEM 11.96

11.96 For the prismatic beam shown, determine the deflection at point D.



SOLUTION

Add force Q at point D.

$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\delta_D = \frac{\partial U}{\partial Q} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial Q}\,dx = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial Q}\,dx$$

Over portion AD $\quad 0 < x < \frac{L}{2} \quad M = -Px, \quad \frac{\partial M}{\partial Q} = 0$

Over portion DB $\quad \frac{L}{2} < x < L \quad M = -Px - Q\left(x - \frac{L}{2}\right), \quad \frac{\partial M}{\partial Q} = -\left(x - \frac{L}{2}\right)$

Set $Q = 0$

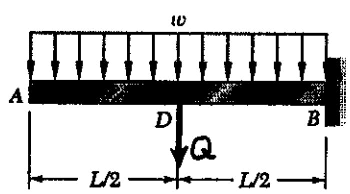
$$\delta_D = \frac{1}{EI}\int_0^{\frac{L}{2}}(-Px)(0)\,dx + \frac{1}{EI}\int_{\frac{L}{2}}^{L}(-Px)\left[-\left(x-\frac{L}{2}\right)\right]dx$$

$$= \frac{P}{EI}\int_{\frac{L}{2}}^{L}\left(x^2 - \frac{L}{2}x\right)dx = \frac{P}{EI}\left\{\frac{1}{3}L^3 - \frac{1}{3}\left(\frac{L}{2}\right)^3 - \left(\frac{L}{2}\right)\frac{1}{2}L^2 + \frac{L}{2}\,\frac{1}{2}\left(\frac{L}{2}\right)^2\right\}$$

$$= \left(\frac{1}{3} - \frac{1}{24} - \frac{1}{4} + \frac{1}{16}\right)\frac{PL^3}{EI} = \frac{5}{48}\frac{PL^3}{EI} \blacktriangleleft$$

PROBLEM 11.97

11.97 For the prismatic beam shown, determine the deflection at point D.



SOLUTION

Add force Q at point D.

$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\delta_D = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial Q}\,dx$$

Over portion AD $\quad 0 < x < \frac{L}{2} \quad M = -\frac{1}{2}wx^2 \quad \frac{\partial M}{\partial Q} = 0$

Over portion DB $\quad \frac{L}{2} < x < L \quad M = -\frac{1}{2}wx^2 - Q\left(x - \frac{L}{2}\right) \quad \frac{\partial M}{\partial Q} = -\left(x - \frac{L}{2}\right)$

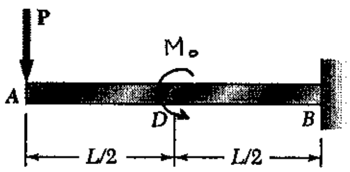
Set $Q = 0$

$$\delta_D = \frac{1}{EI}\int_0^{\frac{L}{2}}\left(-\frac{1}{2}wx^2\right)(0)\,dx + \frac{1}{EI}\int_{\frac{L}{2}}^{L}\left(-\frac{1}{2}wx^2\right)\left[-\left(x-\frac{L}{2}\right)\right]dx$$

$$= \frac{w}{2EI}\int_{\frac{L}{2}}^{L}\left(x^3 - \frac{L}{2}x^2\right)dx = \frac{w}{2EI}\left\{\frac{1}{4}L^4 - \frac{1}{4}\left(\frac{L}{2}\right)^4 - \left(\frac{L}{2}\right)\frac{1}{3}L^3 + \left(\frac{L}{2}\right)\frac{1}{3}\left(\frac{L}{2}\right)^3\right\}$$

$$= \frac{1}{2}\left(\frac{1}{4} - \frac{1}{64} - \frac{1}{6} + \frac{1}{48}\right)\frac{wL^4}{EI} = \frac{17}{384}\frac{wL^4}{EI} = 0.04427\frac{wL^4}{EI} \blacktriangleleft$$

PROBLEM 11.98



11.98 For the prismatic beam shown, determine the slope at point *D*.

SOLUTION

Add couple M_o at point D.

$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\circlearrowleft \theta_D = \frac{\partial U}{\partial M_o} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_o}\,dx = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial M_o}\,dx$$

Over portion AD $\quad 0 < x < \frac{L}{2} \qquad M = -Px \qquad \frac{\partial M}{\partial M_o} = 0$

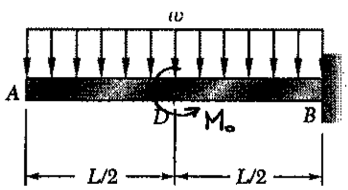
Over portion DB $\quad \frac{L}{2} < x < L \qquad M = -Px - M_o \qquad \frac{\partial M}{\partial M_o} = -1$

Set $M_o = 0$. $\quad \theta_D = \frac{1}{EI}\int_{\frac{L}{2}}^{L}(-Px)(0)\,dx + \frac{1}{EI}\int_{\frac{L}{2}}^{L}(-Px)(-1)\,dx$

$$= \frac{P}{EI}\int_{\frac{L}{2}}^{L} x\,dx = \frac{P}{EI}\left[\frac{1}{2}L^2 - \frac{1}{2}\left(\frac{L}{2}\right)^2\right]$$

$$= \left(\frac{1}{2} - \frac{1}{8}\right)\frac{PL^2}{EI} = \frac{3}{8}\frac{PL^2}{EI} \measuredangle$$

PROBLEM 11.99



11.99 For the prismatic beam shown, determine the slope at point *D*.

SOLUTION

Add couple M_o at point D.

$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\circlearrowleft \theta_D = \frac{\partial U}{\partial M_o} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_o}\,dx = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial M_o}\,dx$$

Over portion AD $\quad 0 < x < \frac{L}{2} \qquad M = -\frac{1}{2}wx^2 \qquad \frac{\partial M}{\partial M_o} = 0$

Over portion DB $\quad \frac{L}{2} < x < L \qquad M = -\frac{1}{2}wx^2 - M_o \qquad \frac{\partial M}{\partial M_o} = -1$

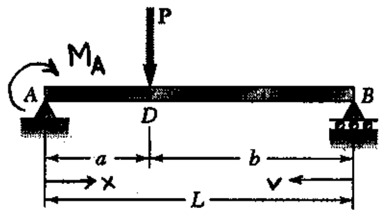
Set $M_o = 0$. $\quad \theta_D = \frac{1}{EI}\int_{\frac{L}{2}}^{L}\left(-\frac{1}{2}wx^2\right)(0)\,dx + \frac{1}{EI}\int_{\frac{L}{2}}^{L}\left(-\frac{1}{2}wx^2\right)(-1)\,dx$

$$= \frac{w}{2EI}\int_{\frac{L}{2}}^{L} x^2\,dx = \frac{w}{2EI}\left[\frac{1}{3}L^3 - \frac{1}{3}\left(\frac{L}{2}\right)^3\right]$$

$$\frac{1}{6}\left(1 - \frac{1}{8}\right)\frac{wL^3}{EI} = \frac{7}{48}\frac{wL^3}{EI} \measuredangle$$

PROBLEM 11.100

11.100 and *11.101* For the prismatic beam shown, determine the slope at point A.



SOLUTION

Add couple M_A at point A.

Reactions: $R_A = \frac{Pb}{L} - \frac{M_A}{L}, \quad R_B = \frac{Pa}{L} + \frac{M_A}{L}$

$$U = \int_0^L \frac{M^2}{2EI}\,dx = \frac{1}{2EI}\int_0^a M^2\,dx + \frac{1}{2EI}\int_0^b M^2\,dv$$

$$\curvearrowleft \theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI}\int_0^a M\frac{\partial M}{\partial M_A}\,dx + \frac{1}{EI}\int_0^b M\frac{\partial M}{\partial M_A}\,dv$$

Over portion AD $(0<x<a)$ $\quad M = M_A + R_A x = M_A\left(1-\frac{x}{L}\right) + \frac{Pbx}{L}, \quad \frac{\partial M}{\partial M_A} = 1 - \frac{x}{L}$

Over portion DB $(0<v<b)$ $\quad M = R_B v = \frac{Pav}{L} + \frac{M_A v}{L}, \quad \frac{\partial M}{\partial M_A} = \frac{v}{L}$

Set $M_A = 0$ $\quad \theta_A = \frac{1}{EI}\int_0^a \left(\frac{Pbx}{L}\right)\left(1-\frac{x}{L}\right)dx + \frac{1}{EI}\int_0^b \frac{Pav}{L}\frac{v}{L}\,dv$

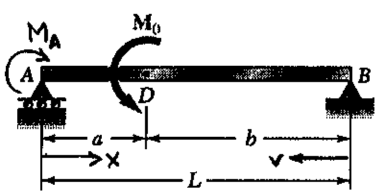
$$= \frac{P}{EIL^2}\left(\tfrac{1}{2}bLa^2 - \tfrac{1}{3}ba^3 + \tfrac{1}{3}ab^3\right)$$

$$= \frac{Pab}{6EIL^2}\left(3La - 2a^2 + 2b^2\right)$$

PROBLEM 11.101

11.100 and *11.101* For the prismatic beam shown, determine the slope at point A.

SOLUTION



Add couple M_A at point a

Reactions: Positive if upward

$R_A = \frac{M_0 - M_A}{L}, \quad R_B = \frac{M_A - M_0}{L}$

$$U = \int_0^L \frac{M^2}{2EI}\,dx = \frac{1}{2EI}\int_0^a M^2\,dx + \frac{1}{2EI}\int_0^b M^2\,dv$$

$$\curvearrowleft \theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI}\int_0^a M\frac{\partial M}{\partial M_A}\,dx + \frac{1}{EI}\int_0^b M\frac{\partial M}{\partial M_A}\,dv$$

Over portion AD $(0<x<a)$ $\quad M = M_A + R_A x = M_A\left(1-\frac{x}{L}\right) + \frac{M_0 x}{L}, \quad \frac{\partial M}{\partial M_A} = 1 - \frac{x}{L}$

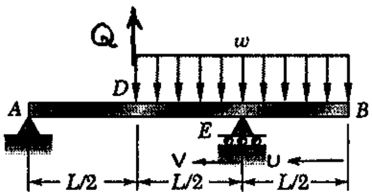
Over portion DB $(0<v<b)$ $\quad M = R_B v = \frac{(M_A - M_0)v}{L} \quad \frac{\partial M}{\partial M_A} = \frac{v}{L}$

Set $M_A = 0$ $\quad \theta_A = \frac{1}{EI}\int_0^a \left(\frac{M_0 x}{L}\right)\left(1-\frac{x}{L}\right)dx + \frac{1}{EI}\int_0^b \left(\frac{M_0 v}{L}\right)\left(-\frac{v}{L}\right)dv$

$$= \frac{M_0}{EIL^2}\left(\tfrac{1}{2}La^2 - \tfrac{1}{3}a^3 - \tfrac{1}{3}b^3\right)$$

$$= \frac{M_0}{6EIL^2}\left(3La^2 - 2a^3 - 2b^3\right)$$

PROBLEM 11.102



11.102 For the prismatic beam shown, determine the deflection at point D.

SOLUTION

Add force Q at point D.

Reactions: $R_A = -\frac{1}{2}Q$, $\quad R_B = wL - \frac{1}{2}Q$

$U = U_{AD} + U_{DE} + U_{EB}$; $\qquad \delta_D = \dfrac{\partial U}{\partial Q}$

Over portion AD: with $Q = 0$ $\quad M = 0 \quad \dfrac{\partial U_{AD}}{\partial Q} = 0$

Over portion DE: $M = R_B v - \frac{1}{2}w\left(v + \frac{L}{2}\right)^2 = wLv - \frac{1}{2}w\left(v+\frac{L}{2}\right)^2 - \frac{1}{2}Qv$

$$\frac{\partial M}{\partial Q} = -\frac{1}{2}v \qquad U_{DE} = \frac{1}{2EI}\int_0^{\frac{L}{2}} M^2\,dv \qquad \text{Set } Q = 0$$

$$\frac{\partial U_{DE}}{\partial Q} = \frac{1}{EI}\int_0^{\frac{L}{2}} M\frac{\partial M}{\partial Q}\,dv = \frac{1}{EI}\int_0^{\frac{L}{2}}\left[wLv - \frac{1}{2}w\left(v+\frac{L}{2}\right)^2\right]\left(-\frac{1}{2}v\right)dv$$

$$= \frac{w}{2EI}\int_0^{\frac{L}{2}}\left[-Lv^2 + \frac{1}{2}\left(v^3 + Lv^2 + \frac{1}{4}L^2 v\right)\right]dv$$

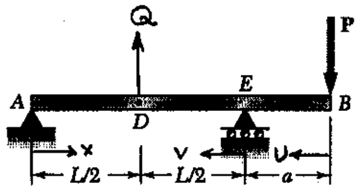
$$= \frac{w}{2EI}\left[-L\cdot\frac{1}{3}\left(\frac{L}{2}\right)^3 + \frac{1}{2}\left(\frac{1}{4}\left(\frac{L}{2}\right)^4 + L\cdot\frac{1}{3}\left(\frac{L}{2}\right)^3 + \frac{1}{4}L^2\frac{1}{2}\left(\frac{L}{2}\right)^2\right)\right]$$

$$= \frac{1}{2}\left(-\frac{1}{24} + \frac{1}{128} + \frac{1}{48} + \frac{1}{64}\right)\frac{wL^4}{EI} = \frac{1}{768}\frac{wL^4}{EI}$$

Over portion EB: $M = -\frac{1}{2}wu^2 \qquad \dfrac{\partial M}{\partial Q} = 0 \qquad \dfrac{\partial U_{EB}}{\partial Q} = 0$

$$\delta_D = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q} = 0 + \frac{1}{768}\frac{wL^4}{EI} + 0 = \frac{1}{768}\frac{wL^4}{EI}$$

PROBLEM 11.103



11.103 For the prismatic beam shown, determine the deflection at point D.

SOLUTION

Add force Q at point D.

Reactions $R_A = -\frac{Pa}{L} - \frac{1}{2}Q$, $R_E = \frac{P(a+L)}{L} - \frac{1}{2}Q$

$U = U_{AD} + U_{DE} + U_{EB}$; $\delta_D = \frac{\partial U}{\partial Q}$

Over portion AD: $U_{AD} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}dx$, $M = R_A x = -\frac{Pa}{L}x - \frac{1}{2}Qx$, $\frac{\partial M}{\partial Q} = -\frac{1}{2}x$

Set $Q = 0$. $\frac{\partial U_{AD}}{\partial Q} = \frac{1}{EI}\int_0^{\frac{L}{2}}\left(-\frac{Pa}{L}x\right)\left(-\frac{1}{2}x\right)dx = \frac{Pa}{2EIL}\int_0^{\frac{L}{2}} x^2 dx$

$$= \frac{Pa}{2EIL}\,\frac{1}{3}\left(\frac{L}{2}\right)^3 = \frac{PaL^2}{48\,EI}$$

Over portion DE: $U_{DE} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}dv$ $\qquad \frac{\partial M}{\partial Q} = -\frac{1}{2}v$

$M = R_E v - P(a+v) = \frac{P(a+L)}{L}v - \frac{1}{2}Qv - P(a+v) = \frac{Pa}{L}v - Pa - \frac{1}{2}Qv$

Set $Q = 0$. $\frac{\partial U_{DE}}{\partial Q} = \frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{Pa}{L}v - Pa\right)\left(-\frac{1}{2}v\right) = \frac{Pa}{2EIL}\int_0^{\frac{L}{2}}(-v^2 + Lv)\,dv$

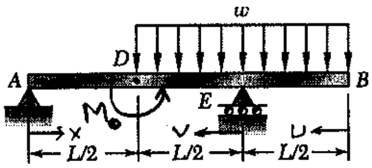
$$= \frac{Pa}{2EIL}\left[-\frac{1}{3}\left(\frac{L}{2}\right)^3 + (L)\frac{1}{2}\left(\frac{L}{2}\right)^2\right] = \frac{Pa}{2EIL}\left[-\frac{L^3}{24} + \frac{L^3}{8}\right]$$

$$= \frac{1}{24}\frac{PaL^2}{EI}$$

Over portion EB $\qquad M = -Pu \qquad \frac{\partial M}{\partial Q} = 0 \qquad \frac{\partial U_{EB}}{\partial Q} = 0$

$$\delta_D = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q} = \frac{PaL^2}{48EI} + \frac{PaL^2}{24EI} + 0 = \frac{1}{16}\frac{PaL^2}{EI}\uparrow$$

PROBLEM 11.104



11.104 For the prismatic beam shown, determine the slope at point *D*.

SOLUTION

Add couple M_o at point D.

Reactions: $R_A = \frac{M_o}{L}$, $\quad R_E = wL - \frac{M_o}{L}$

$U = U_{AD} + U_{DE} + U_{EB}$, $\qquad \theta_D = \frac{\partial U}{\partial M_o}$

Over portion AD: $\quad M = \frac{M_o}{L}x = 0$ with $M_o = 0 \qquad \frac{\partial U_{AD}}{\partial M_o} = 0$

Over portion DE: $\quad M = R_B v - \frac{1}{2}w(v+\frac{L}{2})^2 = wLv - \frac{1}{2}w(v+\frac{L}{2})^2 - \frac{M_o}{L}v$

$\frac{\partial M}{\partial M_o} = -\frac{1}{L}v, \qquad U_{DE} = \frac{1}{2EI}\int_0^{\frac{L}{2}} M^2\,dv \qquad$ Set $M_o = 0$

$$\frac{\partial U_{DE}}{\partial M_o} = \frac{1}{EI}\int_0^{\frac{L}{2}} M\frac{\partial M}{\partial M_o}dv = \frac{1}{EI}\int_0^{\frac{L}{2}}\left[wLv - \frac{1}{2}w(v+\frac{L}{2})^2\right](-\frac{1}{L}v)\,dv$$

$$= \frac{w}{EIL}\int_0^{\frac{L}{2}}\left[-Lv^2 + \frac{1}{2}(v^3 + Lv^2 + \frac{1}{4}L^2v)\right]dv$$

$$= \frac{w}{EIL}\left[-L\cdot\frac{1}{3}(\frac{L}{2})^3 + \frac{1}{2}\left(\frac{1}{4}(\frac{L}{2})^4 + L\cdot\frac{1}{3}(\frac{L}{2})^3 + \frac{1}{4}L^2\frac{1}{2}(\frac{L}{2})^2\right)\right]$$

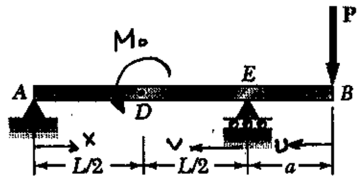
$$= \left(-\frac{1}{24} + \frac{1}{128} + \frac{1}{48} + \frac{1}{64}\right)\frac{wL^3}{EI} = \frac{1}{384}\frac{wL^3}{EI}$$

Over portion EB: $\quad M = -\frac{1}{2}wu^2 \qquad \frac{\partial M}{\partial M_o} = 0 \qquad \frac{\partial U_{EB}}{\partial M_o} = 0$

$$\theta_D = \frac{\partial U_{AD}}{\partial M_o} + \frac{\partial U_{DE}}{\partial M_o} + \frac{\partial U_{EB}}{\partial M_o} = 0 + \frac{1}{384}\frac{wL^3}{EI} + 0 = \frac{1}{384}\frac{wL^3}{EI}$$

PROBLEM 11.105

11.105 For the prismatic beam shown, determine the slope at point D.



SOLUTION

Add couple M_o at point D.

Reactions: $R_A = -\frac{Pa}{L} + \frac{M_o}{L}, \quad R_B = \frac{P(a+L)}{L} - \frac{M_o}{L}$

$U = U_{AD} + U_{DE} + U_{EB} \qquad \theta_D = \frac{\partial U}{\partial M_o}$

Over portion AD: $U_{AD} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dx, \quad M = R_A x = -\frac{Pa}{L}x + \frac{M_o}{L}x, \quad \frac{\partial M}{\partial M_o} = \frac{1}{L}x$

Set $M_o = 0$, $\frac{\partial U_{AD}}{\partial M_o} = \frac{1}{EI}\int_0^{\frac{L}{2}} \left(-\frac{Pa}{L}x\right)\left(\frac{1}{L}x\right) = -\frac{Pa}{EIL^2}\int_0^{\frac{L}{2}} x^2\,dx$

$= -\frac{Pa}{EIL^2}\,\frac{1}{3}\left(\frac{L}{2}\right)^3 = -\frac{PaL}{24EI}$

Over portion DE: $U_{DE} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dv \qquad \frac{\partial M}{\partial M_o} = -\frac{1}{L}v$

$M = R_E v - P(a+v) = \frac{P(a+L)}{L}v - \frac{M_o}{L}v - P(a+v) = \frac{Pa}{L}v - Pa - \frac{M_o}{L}v$

Set $M_o = 0$, $\frac{\partial U_{DE}}{\partial M_o} = \frac{1}{EI}\int_0^{\frac{L}{2}} \left(\frac{Pa}{L}v - Pa\right)\left(-\frac{1}{L}v\right)dv = -\frac{Pa}{EIL^2}\int_0^{\frac{L}{2}}(v^2 - Lv)\,dv$

$= -\frac{Pa}{EIL^2}\left[\frac{1}{3}\left(\frac{L}{2}\right)^3 - L\cdot\frac{1}{2}\left(\frac{L}{2}\right)^2\right] = -\frac{Pa}{EIL^2}\left[\frac{1}{24}L^3 - \frac{1}{8}L^3\right]$

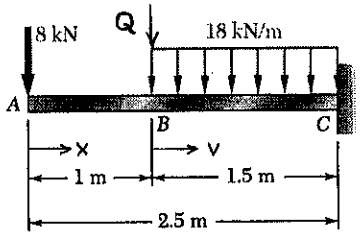
$= \frac{1}{12}\,\frac{PaL}{EI}$

Over portion EB: $M = -Pu \qquad \frac{\partial M}{\partial M_o} = 0 \qquad \frac{\partial U_{EB}}{\partial M_o} = 0$

Total $\theta_D = \frac{\partial U_{AD}}{\partial M_o} + \frac{\partial U_{DE}}{\partial M_o} + \frac{\partial U_{EB}}{\partial M_o} = -\frac{1}{24}\frac{PaL}{EI} + \frac{1}{12}\frac{PaL}{EI} + 0 = \frac{1}{24}\frac{PaL}{EI}$

PROBLEM 11.106

11.106 For the beam and loading shown, determine the deflection at point *B*. Use *E* = 200 GPa.



W250 × 22.3

SOLUTION

Add force Q at point B.

Units: Forces in kN, lengths in m.

Over AB $\quad M = -8x \qquad \frac{\partial M}{\partial Q} = 0$

Over BC $\quad M = -8(v+1) - \frac{1}{2}(18)v^2 - Qv \qquad \frac{\partial M}{\partial Q} = -v$

$E = 200 \times 10^9$ Pa, $\quad I = 28.9 \times 10^6\ \text{mm}^4 = 28.9 \times 10^{-6}\ \text{m}^4$

$EI = (200 \times 10^9)(28.9 \times 10^{-6}) = 5.78 \times 10^6\ \text{N}\cdot\text{m}^2 = 5780\ \text{kN}\cdot\text{m}^2$

$$U = \int_0^1 \frac{M^2}{2EI}dx + \int_0^{1.5} \frac{M^2}{2EI}dv$$

$$\delta_B = \frac{\partial U}{\partial Q} = \frac{1}{EI}\left\{\int_0^1 M\frac{\partial M}{\partial Q}dx + \int_0^{1.5} M\frac{\partial M}{\partial Q}dv\right\}$$

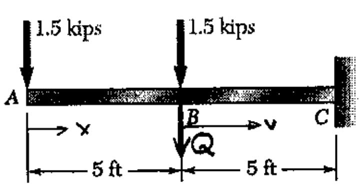
$$= \frac{1}{EI}\left\{0 + \int_0^{1.5}\left[-8(v+1) - \tfrac{1}{2}(18)v^2\right](-v)\,dv\right\} = \frac{1}{EI}\int_0^{1.5}(9v^3 + 8v^2 + 8v)\,dv$$

$$= \frac{1}{EI}\left\{\tfrac{9}{4}(1.5)^4 + \tfrac{8}{3}(1.5)^3 + \tfrac{8}{2}(1.5)^2\right\} = \frac{29.391}{EI} = \frac{29.391}{5780}$$

$$= 5.08 \times 10^{-3}\ \text{m} = 5.08\ \text{mm} \downarrow$$

PROBLEM 11.107

11.107 For the beam and loading shown, determine the deflection at point *B*. Use *E* = 29 × 10³ ksi.



W8 × 13

SOLUTION

Add force Q at point B

Units: forces in kips, lengths in ft.

$E = 29 \times 10^3$ ksi $\qquad I = 39.6\ \text{in}^4$

$EI = (29 \times 10^3)(39.6) = 1.148 \times 10^6\ \text{kip}\cdot\text{in}^2 = 7975\ \text{kip}\cdot\text{ft}^2$

$$U = \int_0^5 \frac{M^2}{2EI}dx + \int_0^5 \frac{M^2}{2EI}dv \qquad \delta_B = \frac{\partial U}{\partial Q} = \frac{1}{EI}\left\{\int_0^5 M\frac{\partial M}{\partial Q}dx + \int_0^5 M\frac{\partial M}{\partial Q}dv\right\}$$

Over AB: $\quad M = -1.5x, \quad \frac{\partial M}{\partial Q} = 0 \qquad \int_0^5 M\frac{\partial M}{\partial Q}dx = 0$

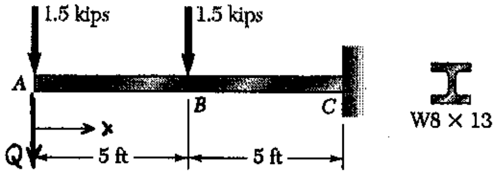
Over BC: $\quad M = -1.5(v+5) - 1.5v - Qv = -3v - 7.5 - Qv, \qquad \frac{\partial M}{\partial Q} = -v$

$$\int_0^5 M\frac{\partial M}{\partial Q}dv = \int_0^5 (3v^2 + 7.5v)\,dv = (3)(\tfrac{1}{3})(5)^3 + (7.5)(\tfrac{1}{2})(5)^2 = 218.75$$

$$\delta_B = \frac{1}{EI}\left\{0 + 143.75\right\} = \frac{218.75}{7975} = 27.43 \times 10^{-3}\ \text{ft} = 0.329\ \text{in.} \downarrow$$

PROBLEM 11.108

11.108 For the beam and loading shown, determine the deflection at point A. Use $E = 29 \times 10^3$ ksi.



SOLUTION

Add force Q at point A.

Units: forces in kips, lengths in ft.

$E = 29 \times 10^3$ ksi, $I = 39.6$ in^4

$EI = (29 \times 10^3)(39.6) = 1.148 \times 10^6$ kip·in^2 $= 7975$ kip·ft^2

$U = \int_0^{10} \frac{M^2}{2EI} dx \qquad \delta_A = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int_0^{10} M \frac{\partial M}{\partial Q} dx$

Over portion AB $\quad 0 < x < 5, \quad M = -1.5x - Qx \quad \frac{\partial M}{\partial Q} = -x$

$$\int_0^5 M \frac{\partial M}{\partial Q} dx = \int_0^5 (1.5x)(x) dx = 1.5 \int_0^5 x^2 dx = (1.5)(\tfrac{1}{3})(5)^3 = 62.5$$

Over portion BC $\quad 5 < x < 10 \quad M = -1.5x - 1.5(x-5) - Qx$

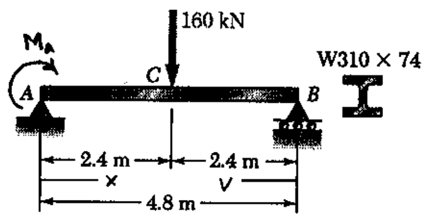
$M = -3x + 7.5 - Qx \qquad \frac{\partial M}{\partial Q} = -x$

$$\int_5^{10} M \frac{\partial M}{\partial Q} dx = \int_5^{10} (3x^2 - 7.5x) dx = (3)(\tfrac{1}{3})(10^3 - 5^3) - (7.5)(\tfrac{1}{2})(10^2 - 5^2)$$
$$= 593.75$$

$$\delta_A = \frac{1}{EI}\{62.5 + 593.75\} = \frac{656.29}{7975} = 82.29 \times 10^{-3} \text{ ft} = 0.987 \text{ in.} \downarrow$$

PROBLEM 11.109

11.109 For the beam and loading shown, determine the slope at end A.. Use $E = 200$ GPa



SOLUTION

Add couple M_A at point A.

Units: forces in kN, lengths in m.

$E = 200 \times 10^9$ Pa, $I = 165 \times 10^6\ \text{mm}^4 = 165 \times 10^{-6}\ \text{m}^4$

$EI = (200 \times 10^9)(165 \times 10^{-6}) = 33 \times 10^6\ \text{N·m}^2 = 33000\ \text{kN·m}^2$

Reactions: $R_A = 80 - \frac{M_A}{4.8}$ $\qquad R_B = 80 + \frac{M_A}{L}$

$$U = U_{AB} + U_{BC} = \int_0^{2.4} \frac{M^2}{2EI}dx + \int_0^{2.4} \frac{M^2}{2EI}dv \qquad \theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AB}}{\partial M_A} + \frac{\partial U_{BC}}{\partial M_A}$$

Over AB: $\quad M = M_A + R_A x = M_A + 80x - \frac{M_A}{4.8}x \qquad \frac{\partial M}{\partial M_A} = \left(1 - \frac{x}{4.8}\right)$

Set $M_A = 0$ $\quad \frac{\partial U_{AB}}{\partial M_A} = \frac{1}{EI}\int_0^{2.4}(80x)\left(1 - \frac{x}{4.8}\right)dx = \frac{1}{EI}\int_0^{2.4}(80x - 16.6667x^2)dx$

$$= \frac{1}{EI}\left\{(80)\left(\tfrac{1}{2}\right)(2.4)^2 - (16.6667)\left(\tfrac{1}{3}\right)(2.4)^3\right\} = \frac{153.6}{EI}$$

Over BC: $\quad M = R_B v = 80v + \frac{M_A}{4.8}v, \qquad \frac{\partial M}{\partial M_A} = \frac{1}{4.8}v$

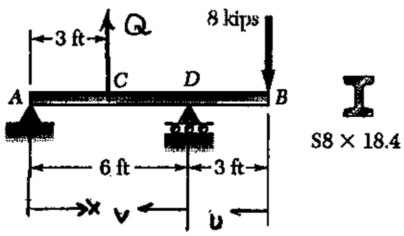
Set $M_A = 0$ $\quad \frac{\partial U_{BC}}{\partial M_A} = \frac{1}{EI}\int_0^{2.4}(80v)\left(\frac{1}{4.8}v\right)dv = \frac{16.6667}{EI}\int_0^{2.4} v^2\,dv$

$$= \frac{(16.6667)(2.4)^3}{3EI} = \frac{76.8}{EI}$$

$$\theta_A = \frac{1}{EI}\{153.6 + 76.8\} = \frac{230.4}{33000} = 6.98 \times 10^{-3}\ \text{rad.}$$

PROBLEM 11.110

11.110 For the beam and loading shown, determine the deflection at point *C*. Use $E = 29 \times 10^3$ ksi.



SOLUTION

Units: Forces in kip, lengths in ft.

$E = 29 \times 10^3$ ksi $\quad I = 57.6$ in^4

$EI = (29\times10^3)(57.6) = 1.6704\times10^6$ kip·in^2 = 11600 kip·ft^2

Add dummy force Q at point C. Reactions $R_A = 4 + \frac{1}{2}Q \downarrow$, $R_D = 12 - \frac{1}{2}Q \uparrow$

$$U = U_{AC} + U_{CD} + U_{DB} \qquad \delta_C = \frac{\partial U}{\partial Q} = \frac{\partial U_{AC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q} + \frac{\partial U_{DB}}{\partial Q}$$

Over AC $\quad 0 < x < 3 \qquad M = -(4 + \frac{1}{2}Q)x \qquad \frac{\partial M}{\partial Q} = -\frac{1}{2}x \qquad$ Set $Q = 0$.

$$\frac{\partial U_{AC}}{\partial Q} = \frac{1}{EI}\int_0^3 (4x)(\tfrac{1}{2}x)\,dx = \frac{2}{EI}\int_0^3 x^2\,dx = \frac{(2)(3)^3}{3EI} = \frac{18}{EI}$$

Over CD $\quad 0 < v < 3 \qquad M = R_B v - 8(v+3) = 12v - \frac{1}{2}Q - 8v - 24 = 4v - 24 - \frac{1}{2}Qv$

$$\frac{\partial M}{\partial Q} = -\frac{1}{2}v \qquad \text{Set } Q = 0$$

$$\frac{\partial U_{CD}}{\partial Q} = \frac{1}{EI}\int_0^3 (24 - 4v)(\tfrac{1}{2}v)\,dv = \frac{1}{EI}\int_0^3 (12v - 2v^2)\,dv = \frac{1}{EI}\left\{(12)\frac{(3)^2}{2} - (2)\frac{(3)^3}{3}\right\}$$

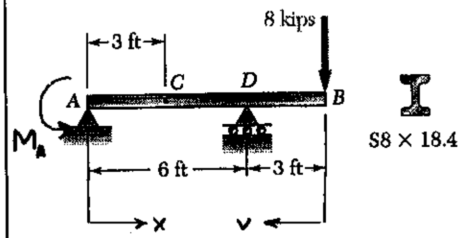
$$= \frac{36}{EI}$$

Over DB $\quad 0 < u < 3 \qquad M = -8u \qquad \frac{\partial M}{\partial Q} = 0 \qquad \frac{\partial U_{DB}}{\partial Q} = 0$

$$\delta_C = \frac{18}{EI} + \frac{36}{EI} + 0 = \frac{54}{11600} = 4.655\times10^{-3}\text{ ft} = 0.0559\text{ in.}\uparrow$$

PROBLEM 11.111

11.111 For the beam and loading shown, determine the slope at end *A*. Use $E = 29 \times 10^3$ ksi.



SOLUTION

Units: Forces in kips, lengths in ft.

$E = 29 \times 10^3$ ksi, $\quad I = 57.6$ in^4

$EI = (29\times10^3)(57.6) = 1.6704\times10^6$ kip·in^2 $= 11600$ kip·ft^2

Add dummy couple M_A at end A. Reactions: $R_A = -4 + \frac{M_A}{6}$, $R_B = 12 - \frac{M_A}{6}$

$$U = U_{AD} + U_{DB} \qquad \theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AD}}{\partial M_A} + \frac{\partial U_{DB}}{\partial M_A}$$

Over AD $\quad 0 < x < 6 \qquad M = -M_A + R_A x = -M_A - 4x + \frac{M_A}{6}x$

$$\frac{\partial M}{\partial M_A} = -\left(1 - \frac{x}{6}\right) \qquad \text{Set } M_A = 0.$$

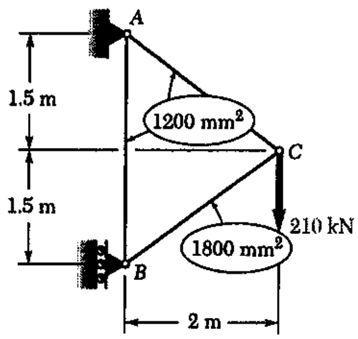
$$\frac{\partial U_{AD}}{\partial M_A} = \frac{1}{EI}\int_0^6 (4x)\left(1-\frac{x}{6}\right)dx = \frac{1}{EI}\int_0^6 \left(4x - \frac{2}{3}x^2\right)dx = \frac{1}{EI}\left\{(4)\frac{6^2}{2} - \frac{2}{3}\,\frac{6^3}{3}\right\}$$

$$= \frac{24}{EI}$$

Over DB $\quad 0 < v < 3 \qquad M = -8v \qquad \frac{\partial M}{\partial M_A} = 0 \qquad \frac{\partial U_{DB}}{\partial M_A} = 0$

$$\theta_A = \frac{24}{EI} + 0 = \frac{24}{11600} = 2.07\times10^{-3} \text{ rad}$$

PROBLEM 11.112



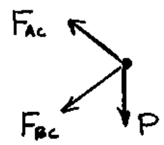
11.112 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200$ GPa, determine the vertical deflection of joint C.

SOLUTION

Call the vertical load P. The vertical deflection of joint C is δ_P

$$\delta_P = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P}\sum \frac{F^2 L}{2EA} = \frac{1}{E}\sum \frac{FL}{A}\frac{\partial F}{\partial P}$$

Joint C: $\xrightarrow{+} \Sigma F_x = 0 \qquad -\frac{4}{5}F_{AC} - \frac{4}{5}F_{BC} = 0$

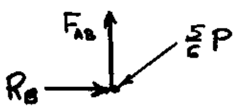


$+\uparrow \Sigma F_y \qquad \frac{3}{5}F_{AC} + \frac{3}{5}F_{BC} - P = 0$

Solving simultaneously

$F_{AC} = \frac{5}{6}P \qquad F_{BC} = -\frac{5}{6}P$

Joint B $\quad +\uparrow \Sigma F_y = 0$



$F_{AB} - \frac{3}{5}\cdot\frac{5}{6}P = 0$

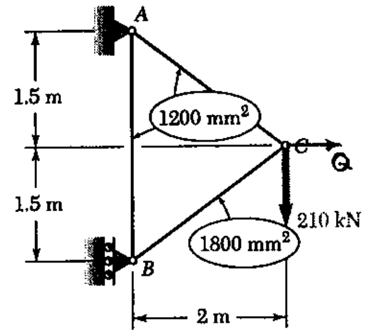
$F_{AB} = \frac{1}{2}P$

Member	F	$\partial F/\partial P$	L (m)	A (10^{-6} m²)	$F(\partial F/\partial P)L/A$
AB	$\frac{1}{2}P$	$\frac{1}{2}$	3	1200	625 P
AC	$\frac{5}{6}P$	$\frac{5}{6}$	2.5	1200	1446.76 P
BC	$-\frac{5}{6}P$	$-\frac{5}{6}$	2.5	1800	964.51 P
Σ					3036.27 P

$$\delta_P = \frac{1}{E}(3036.27\,P) = \frac{(3036.27)(210\times10^3)}{200\times10^9} = 3.19\times10^{-3}\text{ m}$$

$$= 3.19 \text{ mm} \downarrow$$

PROBLEM 11.113



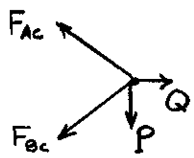
11.113 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200$ GPa, determine the horizontal deflection of joint C.

SOLUTION

Call the vertical force P. Add a dummy horizontal force Q (positive →) at joint C. The horizontal deflection of joint C is

$$\delta_Q = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q}\sum \frac{F^2 L}{2EI} = \frac{1}{E}\sum \frac{FL}{A}\frac{\partial F}{\partial Q}$$

Joint C $\quad \xrightarrow{+} \Sigma F_x = 0$



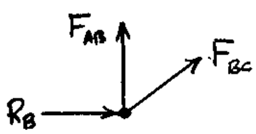
$$-\frac{4}{5}F_{AC} - \frac{4}{5}F_{BC} + Q = 0$$

$+\uparrow \Sigma F_y = 0$

$$\frac{3}{5}F_{AC} - \frac{3}{5}F_{BC} - P = 0$$

Solving simultaneously $\quad F_{AC} = \frac{5}{6}P + \frac{5}{8}Q \qquad F_{BC} = -\frac{5}{6}P + \frac{5}{8}Q$

Joint B $\quad +\uparrow \Sigma F_y = 0$



$$F_{AB} + \frac{3}{5}F_{BC} = 0$$

$$F_{AB} = -\frac{3}{5}F_{BC} = \frac{1}{2}P - \frac{3}{8}Q$$

Member	F	$\partial F/\partial Q$	L (m)	A (10^{-6} m²)	with Q = 0 F ($\partial F/\partial Q$) L / A
AB	$\frac{1}{2}P - \frac{3}{8}Q$	$-\frac{3}{8}$	3	1200	-468.75 P
AC	$\frac{5}{6}P + \frac{5}{8}Q$	$+\frac{5}{8}$	2.5	1200	1085.07 P
BC	$-\frac{5}{6}P + \frac{5}{8}Q$	$+\frac{5}{8}$	2.5	1800	-723.38 P
					-107.06 P

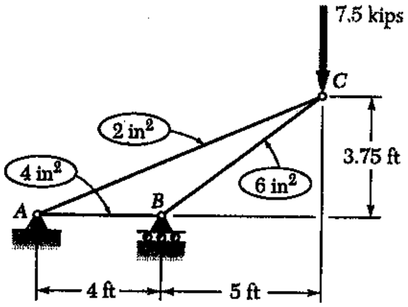
$$\delta_Q = \frac{1}{E}(-107.06\,P) = -\frac{(107.06)(210\times10^3)}{200\times10^9} = -0.1124\times10^{-3}\text{ m}$$

$$= 0.1124 \text{ mm} \leftarrow$$

PROBLEM 11.114

11.114 and 11.115 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 29 \times 10^6$ psi, determine the deflection indicated.

11.114 Vertical deflection of joint *C*.



SOLUTION

Call the vertical load P. The vertical deflection of joint C is δ_P

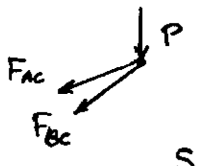
$$\delta_P = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P}\sum \frac{F^2 L}{2EA} = \frac{1}{E}\sum \frac{FL}{A}\frac{\partial F}{\partial P}$$

Geometry $\overline{AC} = \sqrt{9^2 + 3.75^2} = 9.75 \text{ ft} = 117 \text{ in.}$

$\overline{BC} = \sqrt{5^2 + 3.75^2} = 6.25 \text{ ft} = 75 \text{ in}$

$4 \text{ ft} = 48 \text{ in.}, \quad 5 \text{ ft} = 60 \text{ in}, \quad 3.75 \text{ ft} = 45 \text{ in.}$

Joint C

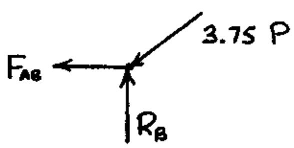


$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad -\frac{108}{117}F_{AC} - \frac{60}{75}F_{BC} = 0$

$+\uparrow \Sigma F_y = 0 \qquad -\frac{45}{117}F_{AC} - \frac{45}{75}F_{BC} - P = 0$

Solving simultaneously $F_{AC} = 3.25\,P, \quad F_{BC} = -3.75\,P$

Joint B



$\overset{+}{\rightarrow} \Sigma F_{AB} = 0 \qquad -F_{AB} - \frac{60}{75}F_{AC} = 0$

$F_{AB} = -3.00\,P$

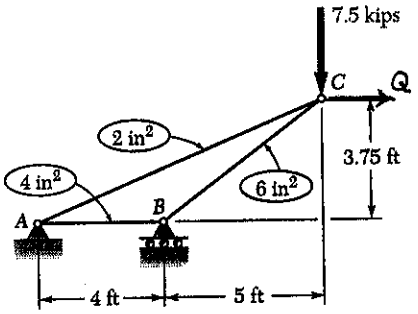
Member	F	$\partial F/\partial P$	L (in)	A (in²)	$F(\partial F/\partial P)L/A$
AB	−3.00 P	−3.00	48	4	108.00 P
AC	3.25 P	3.25	117	2	617.91 P
BC	−3.75 P	−3.75	75	6	175.78 P
Σ					901.69 P

$$\delta_P = \frac{901.69\,P}{E} = \frac{(901.69)(7.5 \times 10^3)}{29 \times 10^6} = 0.233 \text{ in.} \downarrow$$

PROBLEM 11.115

11.114 and 11.115 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 29 \times 10^6$ psi, determine the deflection indicated.

11.115 Horizontal deflection of joint *C*.



SOLUTION

Call the vertical load P. Add horizontal dummy load Q at joint C. The horizontal deflection of joint C is

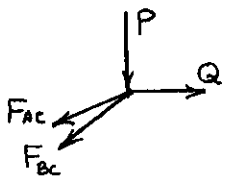
$$\delta_Q = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q}\sum \frac{F^2 L}{2EA} = \frac{1}{E}\sum \frac{FL}{A}\frac{\partial F}{\partial Q}$$

Geometry $\overline{AC} = \sqrt{9^2 + 3.75^2} = 9.75 \text{ ft} = 117 \text{ in}$

$\overline{BC} = \sqrt{5^2 + 3.75^2} = 6.25 \text{ ft} = 75 \text{ in}$

$4 \text{ ft} = 48 \text{ in}, \quad 5 \text{ ft} = 60 \text{ in}, \quad 3.75 \text{ ft} = 45 \text{ in}$

Joint C



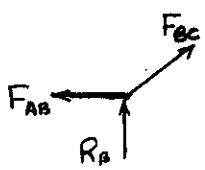
$+\rightarrow \Sigma F_x = 0 \qquad -\frac{108}{117}F_{AC} - \frac{60}{75}F_{BC} + Q = 0$

$+\uparrow \Sigma F_y = 0 \qquad -\frac{45}{117}F_{AC} - \frac{45}{75}F_{BC} - P = 0$

Solving simultaneously $\qquad F_{AC} = 3.25\,P + 2.4375\,Q$

$F_{BC} = -3.75\,P - 1.5625\,Q$

Joint B



$+\rightarrow \Sigma F_x = 0 \qquad \frac{4}{5}F_{AC} - F_{AB} = 0$

$F_{AB} = \frac{4}{5}F_{BC} = -3.00\,P - 1.25\,Q$

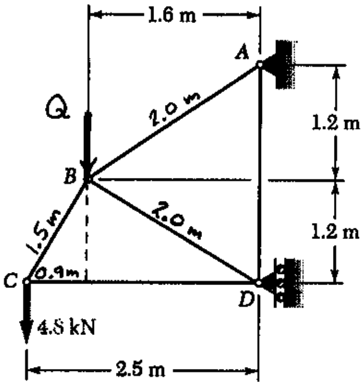
Member	F	$\partial F/\partial Q$	L (in.)	A (in²)	Q = 0 $F(\partial F/\partial Q)L/A$
AB	$-3.00\,P - 1.25\,Q$	-1.25	48	4	45.00 P
AC	$3.25\,P + 2.4375\,Q$	2.4375	117	2	463.43 P
BC	$-3.75\,P - 1.5625\,Q$	-1.5625	75	6	73.24 P
Σ					581.67 P

$$\delta_Q = \frac{581.67\,P}{E} = \frac{(581.67)(7.5\times 10^3)}{29\times 10^6} = 0.1504 \text{ in.} \rightarrow$$

PROBLEM 11.116

11.116 and 11.117 Each member of the truss shown is made of steel and has a cross-sectional area of 500 mm². Using $E = 200$ GPa, determine the deflection indicated.

11.116 Vertical deflection of joint B.



SOLUTION

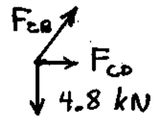
Find the length of each member as shown.

Add dummy vertical force Q at joint B.

$$\delta_B = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q}\sum \frac{F^2 L}{2EA} = \frac{1}{EA}\sum F\frac{\partial F}{\partial Q}L$$

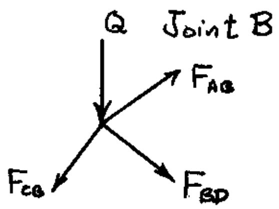
Joint C $\quad +\uparrow \Sigma F_y = 0 \quad \frac{4}{5}F_{CB} - 4.8 = 0$

$F_{CB} = 6.0$ kN



$+\rightarrow \Sigma F_x = 0 \quad \frac{3}{5}F_{CB} + F_{CD} = 0$

$F_{CD} = -3.6$ kN



$+\rightarrow \Sigma F_x = 0 \quad \frac{4}{5}F_{AB} + \frac{4}{5}F_{BD} - 3.6 = 0$

$+\uparrow \Sigma F_y = 0 \quad \frac{3}{5}F_{AB} - \frac{3}{5}F_{BD} - 4.8 - Q = 0$

Solving simultaneously $\quad F_{AB} = 6.25 + 0.8333\,Q$ kN

$F_{BD} = -1.75 - 0.8333\,Q$ kN

Joint D

$+\uparrow \Sigma F_y = 0 \quad \frac{3}{5}F_{BD} + F_{AD} = 0$

$F_{AD} = -\frac{3}{5}F_{BD} = 1.05 + 0.5\,Q$

Member	F (10^3 N)	$\partial F/\partial Q$	L (m)	with Q = 0 $F(\partial F/\partial Q)L$ (10^3 N·m)
AB	$6.25 + 0.8333\,Q$	0.8333	2.0	10.4167
AD	$1.05 + 0.5Q$	0.5	2.4	1.26
BD	$-1.75 - 0.8333\,Q$	-0.8333	2.0	2.9167
BC	6.0	0	1.5	0
CD	-3.6	0	2.5	0
Σ				14.593

$$\delta_B = \frac{1}{EA}\sum F(\partial F/\partial Q)L = \frac{14.593\times 10^3}{(200\times 10^9)(500\times 10^{-6})} = 145.9\times 10^{-6}\ \text{m}$$

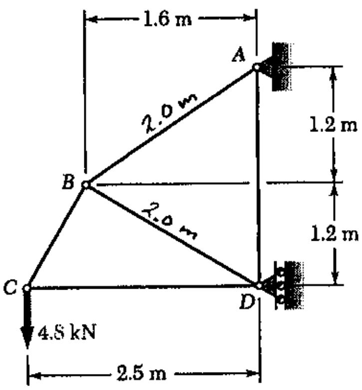
$$= 0.1459\ \text{mm}\downarrow$$

PROBLEM 11.117

11.116 and 11.117 Each member of the truss shown is made of steel and has a cross-sectional area of 500 mm². Using $E = 200$ GPa, determine the deflection indicated.

11.117 Horizontal deflection of joint B.

SOLUTION

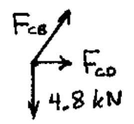


Find the length of each member as shown.

Add dummy horizontal force Q at joint B.

$$\delta_B = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q}\sum \frac{F^2 L}{2EA} = \frac{1}{EA}\sum F\frac{\partial F}{\partial Q}L$$

Joint C



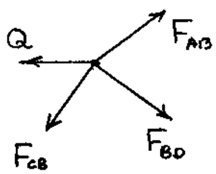
$+\uparrow \Sigma F_y = 0 \qquad \frac{4}{5}F_{CB} - 4.8 = 0$

$F_{CB} = 6.0$ kN

$+\rightarrow \Sigma F_x = 0 \qquad \frac{3}{5}F_{CB} + F_{CD} = 0$

$F_{CD} = -3.6$ kN

Joint B



$+\rightarrow \quad \Sigma F_x = 0 \qquad \frac{4}{5}F_{AB} + \frac{4}{5}F_{BD} - 3.6 - Q = 0$

$+\uparrow \quad \Sigma F_y = 0 \qquad \frac{3}{5}F_{AB} - \frac{3}{5}F_{BD} - 4.8 = 0$

Solving simultaneously $\qquad F_{AB} = 6.25 + 0.625Q$ kN

$F_{BD} = -1.75 + 0.625Q$ kN

Joint D

$+\uparrow \Sigma F_y = 0 \qquad \frac{3}{5}F_{BD} + F_{AD} = 0$

$F_{AD} = -\frac{3}{5}F_{BD} = 1.05 - 0.375Q$

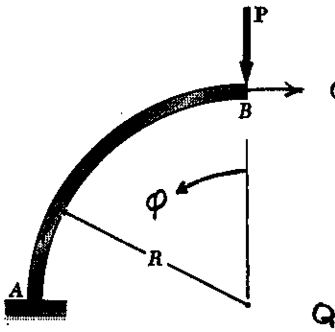
Member	F 10^3 N	$\partial F/\partial Q$	L (m)	$F(\partial F/\partial Q)L$ (10^3 N·m)
AB	6.25 + 0.625Q	0.625	2.0	7.8125
AD	1.05 + 0.375Q	−0.375	2.4	−0.9450
BD	−1.75 + 0.625Q	0.625	2.0	−2.1875
BC	6.0	0	1.5	0
CD	−3.6	0	2.5	0
Σ				4.680

$$\delta_B = \frac{1}{EA}\Sigma F(\partial F/\partial Q)L = \frac{4.680\times 10^3}{(200\times 10^9)(500\times 10^{-6})} = 46.8\times 10^{-6}\text{ m}$$

$$= 0.0468 \text{ mm} \leftarrow$$

PROBLEM 11.118

***11.118** For the uniform rod and loading shown and using Castigliano's theorem, determine (*a*) the horizontal deflection of point *B*, (*b*) the vertical deflection of point *B*.

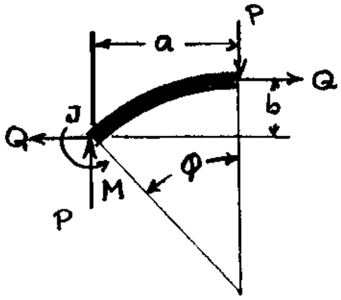


SOLUTION

Add dummy load Q at point B.

Use polar coordinate φ

$$U = \int_0^{\frac{\pi}{2}} \frac{M^2}{2EI} R\, d\varphi$$



Bending moment

$$+\circlearrowleft \Sigma M_J = 0 \qquad M - Pa - Qb = 0$$

$$M = Pa + Qb$$
$$= PR\sin\varphi + QR(1-\cos\varphi)$$

$$\frac{\partial M}{\partial P} = R\sin\varphi \qquad \frac{\partial M}{\partial Q} = R(1-\cos\varphi) \qquad \text{Set } Q = 0$$

(a) $$\delta_Q = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int_0^{\frac{\pi}{2}} M\frac{\partial M}{\partial Q} R\, d\varphi = \frac{1}{EI}\int_0^{\frac{\pi}{2}} PR\sin\varphi\, R(1-\cos\varphi)\, R\, d\varphi$$

$$= \frac{PR^3}{EI}\int_0^{\frac{\pi}{2}} (\sin\varphi - \sin\varphi\cos\varphi)\, d\varphi = \frac{PR^3}{EI}\left(-\cos\varphi - \tfrac{1}{2}\sin^2\varphi\right)\Big|_0^{\frac{\pi}{2}}$$

$$= \frac{PR^3}{EI}\left(-\cos\tfrac{\pi}{2} + \cos 0 - \tfrac{1}{2}\sin^2\tfrac{\pi}{2} + \tfrac{1}{2}\sin^2 0\right)$$

$$= \frac{PR^3}{EI}\left(0 + 1 - \tfrac{1}{2} + 0\right) = \frac{1}{2}\frac{PR^3}{EI}$$ ◄

(b) $$\delta_P = \frac{\partial U}{\partial P} = \frac{1}{EI}\int_0^{\frac{\pi}{2}} M\frac{\partial M}{\partial P} R\, d\varphi = \frac{1}{EI}\int_0^{\frac{\pi}{2}} PR\sin\varphi\, R\sin\varphi\, R\, d\varphi$$

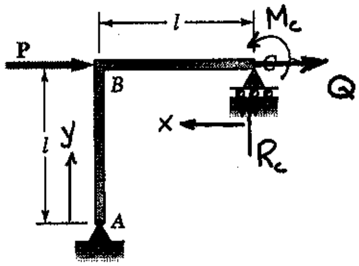
$$= \frac{PR^3}{EI}\int_0^{\frac{\pi}{2}} \sin^2\varphi\, d\varphi = \frac{PR^3}{EI}\int_0^{\frac{\pi}{2}} \tfrac{1}{2}(1-\cos 2\varphi)\, d\varphi$$

$$= \frac{PR^3}{EI}\left(\tfrac{1}{2}\varphi - \tfrac{1}{4}\sin 2\varphi\right)\Big|_0^{\frac{\pi}{2}} = \frac{PR^3}{EI}\left(\tfrac{1}{2}\cdot\tfrac{\pi}{2} - \tfrac{1}{2}\cdot 0 - \tfrac{1}{4}\sin\pi + \tfrac{1}{4}\cdot\sin 0\right)$$

$$= \frac{PR^3}{EI}\left(\tfrac{\pi}{4} - 0 - 0 + 0\right) = \frac{\pi}{4}\frac{PR^3}{EI}$$ ◄

PROBLEM 11.119

11.119 Two rods *AB* and *BC* of the same flexural rigidity *EI* are welded together at *B*. For the loading shown, determine (*a*) the deflection of point *C*, (*b*) the slope of member *BC* at point C.



SOLUTION

Add dummy force Q and dummy couple M_c at C.

$\circlearrowleft \Sigma M_A = 0 \qquad R_c \ell + M_c + (P+Q)\ell = 0$

$$R_c = P + Q + \frac{M_c}{\ell}$$

$\xrightarrow{+} \Sigma F_x = 0 \qquad P + Q + R_{Ax} = 0 \qquad R_{Ax} = P + Q \leftarrow$

Member AB: $\quad M = R_{Ax} y = (P+Q)y, \quad \frac{\partial M}{\partial Q} = y, \quad \frac{\partial M}{\partial M_c} = 0$

$U_{AB} = \int_0^\ell \frac{M^2}{2EI}\,dy \qquad$ Set $Q = 0$ and $M_c = 0$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI}\int_0^\ell M \frac{\partial M}{\partial Q}\,dy = \frac{1}{EI}\int_0^\ell (Py)(y)\,dy = \frac{1}{3}\frac{P\ell^3}{EI}$$

$$\frac{\partial U_{AB}}{\partial M_c} = \frac{1}{EI}\int_0^\ell M \frac{\partial M}{\partial M_A}\,dx = 0$$

Member BC: $\quad M = M_c + R_c x = M_c + (P + Q + \frac{M_c}{\ell})x$

$$\frac{\partial M}{\partial Q} = x, \qquad \frac{\partial M}{\partial M_c} = 1 - \frac{x}{\ell}$$

$U_{BC} = \int_0^\ell \frac{M^2}{2EI}\,dx \qquad$ Set $Q = 0$ and $M_c = 0$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI}\int_0^\ell M \frac{\partial M}{\partial Q}\,dx = \frac{1}{EI}\int_0^\ell (Px)x\,dx = \frac{1}{3}\frac{P\ell^3}{EI}$$

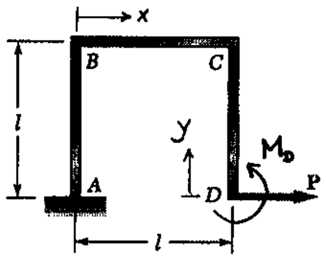
$$\frac{\partial U}{\partial M_A} = \frac{1}{EI}\int_0^\ell M \frac{\partial M}{\partial M_A}\,dx = \frac{1}{EI}\int_0^\ell (Px)(1 - \frac{x}{\ell})\,dx = \frac{P}{EI}\int_0^\ell (x - \frac{x^2}{\ell})\,dx$$

$$= \frac{P}{EI}\left(\frac{1}{2}\ell^2 - \frac{1}{3}\ell^2\right) = \frac{1}{6}\frac{P\ell^2}{EI}$$

(a) Deflection at C $\qquad \delta_c = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{2}{3}\frac{P\ell^3}{EI} \rightarrow$ ◄

(b) Slope at C $\qquad \circlearrowleft \theta_c = \frac{\partial U_{AB}}{\partial M_A} + \frac{\partial U_{BC}}{\partial M_c} = \frac{1}{6}\frac{P\ell^2}{EI} \circlearrowleft$ ◄

PROBLEM 11.120



11.120 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (*a*) the horizontal deflection of point D, (*b*) the slope at point D.

SOLUTION

Add dummy couple M_D at point D.

Reactions at A: $R_{Ay} = 0$, $R_{Ax} = P \leftarrow$, $M_A = M_o$ ↺

Member AB: $M = M_A + R_A y = M_D + Py$ $\qquad \frac{\partial M}{\partial P} = y$, $\frac{\partial M}{\partial M_D} = 1$

$U_{AB} = \int_0^l \frac{M^2}{2EI} dy$ $\qquad$ Set $M_D = 0$

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial P} dy = \frac{1}{EI}\int_0^l (Py) y \, dy = \frac{Pl^3}{3EI}$$

$$\frac{\partial U_{AB}}{\partial M_o} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial M_o} dy = \frac{1}{EI}\int_0^l (Py)(1) dy = \frac{Pl^2}{2EI}$$

Member BC $\quad M = M_A + R_A l = M_D + Pl$ $\qquad \frac{\partial M}{\partial P} = l$, $\frac{\partial M}{\partial M_D} = 1$

$U_{BC} = \int_0^l \frac{M^2}{2EI} dx$ $\qquad$ Set $M_D = 0$

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial P} dx = \frac{1}{EI}\int_0^l (Pl)(l) dx = \frac{Pl^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial M_D} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial M_D} dx = \frac{1}{EI}\int_0^l (Pl)(1) dx = \frac{Pl^2}{EI}$$

Member CD $\quad M = M_D + Py$ $\qquad \frac{\partial M}{\partial P} = y \qquad \frac{\partial M}{\partial M_D} = 1$

$U_{CD} = \int_0^l \frac{M^2}{2EI} dy$ $\qquad$ Set $M_D = 0$

$$\frac{\partial U_{CD}}{\partial P} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial P} dy = \frac{1}{EI}\int_0^l (Py)(y) dy = \frac{Pl^3}{3EI}$$

$$\frac{\partial U}{\partial M_D} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial M_D} dy = \frac{1}{EI}\int_0^l (Py)(1) dy = \frac{Pl^2}{2EI}$$

(a) horizontal deflection of point D.

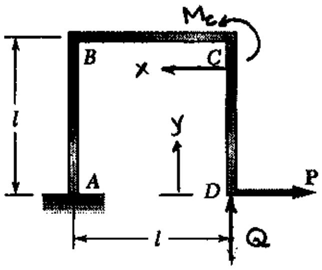
$$\delta_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} + \frac{\partial U_{CD}}{\partial P} = \left(\frac{1}{3} + 1 + \frac{1}{3}\right)\frac{Pl^3}{EI} = \frac{5}{3}\frac{Pl^3}{EI} \rightarrow$$ ◄

(b) slope at point D

$$\theta_D = \frac{\partial U_{AB}}{\partial M_D} + \frac{\partial U_{BC}}{\partial M_o} + \frac{\partial U_{CD}}{\partial M_D} = \left(\frac{1}{2} + 1 + \frac{1}{2}\right)\frac{Pl^2}{EI} = 2\frac{Pl^2}{EI} \circlearrowright$$ ◄

PROBLEM 11.121

11.121 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (*a*) the vertical deflection of point D, (*b*) the slope of BC at point C.



SOLUTION

Add dummy force Q at point D and dummy couple M_c at point C.

Reactions at A: $R_{Ax} = P \leftarrow$, $R_{Ay} = Q \downarrow$

$M_A = Ql + M_c$ ↻

Member AB: $M = M_A + R_{Ax}y = Ql + M_c + Py$, $\frac{\partial M}{\partial Q} = l$, $\frac{\partial M}{\partial M_c} = 1$

$U_{AB} = \int_0^l \frac{M^2}{2EI}dy$ Set $Q = 0$ and $M_c = 0$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial Q}dy = \frac{1}{EI}\int_0^l (Py)(l)\,dy = \frac{Pl^3}{2EI}$$

$$\frac{\partial U_{AB}}{\partial M_c} = \frac{1}{EI}\int_0^l M\frac{dM}{\partial M_c}dy = \frac{1}{EI}\int_0^l (Py)(1)\,dy = \frac{Pl^2}{2EI}$$

Member BC $M = M_c + Pl + Qx$ $\frac{\partial M}{\partial Q} = x$ $\frac{\partial M}{\partial M_c} = 1$

$U_{BC} = \frac{1}{EI}\int_0^l \frac{M^2}{2EI}dx$ Set $Q = 0$ and $M_c = 0$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial Q}dx = \frac{1}{EI}\int_0^l (Pl)(x)\,dx = \frac{Pl^3}{2EI}$$

$$\frac{\partial U_{BC}}{\partial M_c} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial M_c}dx = \frac{1}{EI}\int_0^l (Pl)(1)\,dx = \frac{Pl^2}{EI}$$

Member CD $M = Py$ $\frac{\partial M}{\partial Q} = 0$ $\frac{\partial M}{\partial M_c} = 0$

$\frac{\partial U_{CD}}{\partial Q} = 0$ $\frac{\partial U_{CD}}{\partial M_c} = 0$

(a) vertical deflection of point D

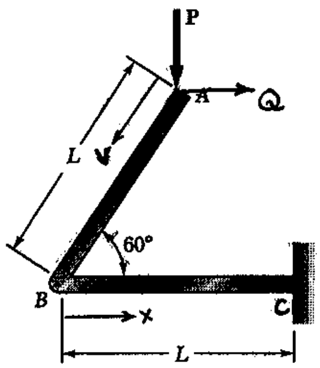
$$\delta_Q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q} = \left(\tfrac{1}{2} + \tfrac{1}{2} + 0\right)\frac{Pl^3}{EI} = \frac{Pl^3}{EI} \uparrow$$ ◄

(b) slope of BC at C

$$\theta_c = \frac{\partial U_{AB}}{\partial M_c} + \frac{\partial U_{BC}}{\partial M_c} + \frac{\partial U_{CD}}{\partial M_c} = \left(\tfrac{1}{2} + 1 + 0\right)\frac{Pl^2}{EI} = \frac{3}{2}\frac{Pl^2}{EI}$$ ◄

PROBLEM 11.122

11.122 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (*a*) the vertical deflection of point A, (*b*) the horizontal deflection of point A.



SOLUTION

Add dummy horizontal force Q at point A.

Over AB $\quad M = \frac{1}{2}Pv + \frac{\sqrt{3}}{2}Qv$

$$\frac{\partial M}{\partial P} = \frac{1}{2}v \qquad \frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2}v$$

$$U_{AB} = \int_0^L \frac{M^2}{2EI}\,dx \qquad\qquad \text{Set } Q = 0$$

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial P}\,dv = \frac{1}{EI}\int_0^L \left(\frac{1}{2}Pv\right)\left(\frac{1}{2}v\right)dv$$

$$= \frac{1}{12}\frac{PL^3}{EI}$$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial Q}\,dv = \frac{1}{EI}\int_0^L \left(\frac{1}{2}Pv\right)\frac{\sqrt{3}}{2}\,dv$$

$$= \frac{\sqrt{3}}{12}\frac{PL^3}{EI}$$

Over BC $\quad M = -P\left(x - \frac{L}{2}\right) + \frac{\sqrt{3}}{2}QL, \qquad \frac{\partial M}{\partial P} = -\left(x - \frac{L}{2}\right), \quad \frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2}L$

$$U_{BC} = \int_0^L \frac{M^2}{2EI}\,dx \qquad\qquad \text{Set } Q = 0$$

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial P}\,dx = \frac{1}{EI}\int_0^L P\left(x-\frac{L}{2}\right)^2 dx = \frac{P}{3EI}\left(x-\frac{L}{2}\right)^3\Big|_0^L = \frac{1}{12}\frac{PL^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial Q}\,dx = \frac{-1}{EI}\int_0^L P\left(x-\frac{L}{2}\right)\left(\frac{\sqrt{3}}{2}\right)L\,dx = -\frac{\sqrt{3}P}{4EI}\left(x-\frac{L}{2}\right)^2\Big|_0^L = 0$$

(a) vertical deflection of point A

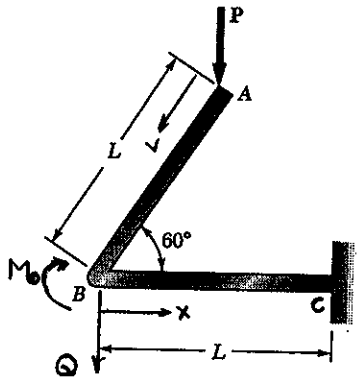
$$\delta_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} = \frac{1}{6}\frac{PL^3}{EI}\ \downarrow$$

(b) horizontal deflection of point A

$$\delta_Q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{\sqrt{3}}{12}\frac{PL^3}{EI} = 0.1443\frac{PL^3}{EI}\ \rightarrow$$

PROBLEM 11.123

11.123 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (*a*) the vertical deflection of point B, (*b*) the slope of BC at point B.



SOLUTION

Add dummy vertical Q and dummy couple M_o at B.

Over AB $\quad M = \frac{1}{2}Pv, \quad \frac{\partial M}{\partial Q} = 0, \quad \frac{\partial M}{\partial M_o} = 0$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dv$$

$$\frac{\partial U_{AB}}{\partial Q} = 0 \qquad \frac{\partial U_{AB}}{\partial M_o} = 0$$

Over BC $\quad M = -P(x - \frac{L}{2}) - Qx + M_o \qquad \frac{\partial M}{\partial Q} = -x, \quad \frac{\partial M}{\partial M_o} = 1$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dx \qquad \text{Set } Q = 0 \text{ and } M_o = 0$$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI}\int_0^L M \frac{\partial M}{\partial Q} dx = \frac{1}{EI}\int_0^L P(x - \frac{L}{2})x\, dx = \frac{P}{EI}\left[\frac{L^3}{3} - \left(\frac{L}{2}\right)\frac{L^2}{2}\right]$$

$$= \frac{1}{12}\frac{PL^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial M_o} = \frac{1}{EI}\int_0^L M \frac{\partial M}{\partial M_o} dx = \frac{-1}{EI}\int_0^L P(x - \frac{L}{2}) dx = 0$$

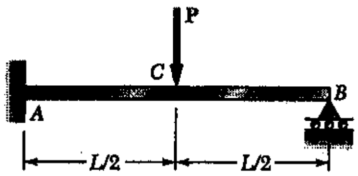
(a) vertical deflection of point B

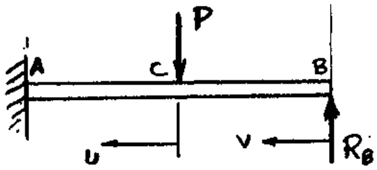
$$\delta_Q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{1}{12}\frac{PL^3}{EI} \downarrow$$

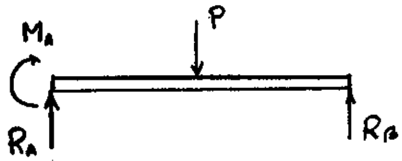
(b) slope of BC at point B

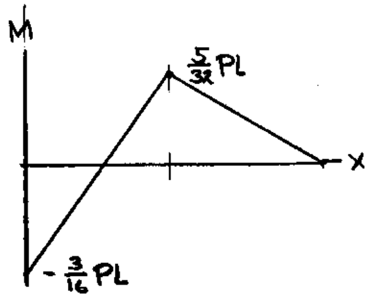
$$\theta_B = \frac{\partial U_{AB}}{\partial M_o} + \frac{\partial U_{BC}}{\partial M_o} = 0$$

PROBLEM 11.124









11.124 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION

Remove support B and add reaction R_B as a load.

$$U = U_{AC} + U_{CB} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,du + \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dv$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{\partial U_{AB}}{\partial R_B} + \frac{\partial U_{CB}}{\partial R_B} = 0$$

Over AC: $M = R_B(u + \frac{L}{2}) - Pu$, $\frac{\partial M}{\partial R_B} = (u + \frac{L}{2})$

$$\frac{\partial U_{AB}}{\partial R_B} = \frac{1}{EI}\int_0^{\frac{L}{2}} \left[R_B(u+\tfrac{L}{2}) - Pu\right](u+\tfrac{L}{2})\,du$$

$$= \frac{R_B}{EI}\int_0^{\frac{L}{2}} (u+\tfrac{L}{2})^2 du - \frac{P}{EI}\int_0^{\frac{L}{2}} u\,(u+\tfrac{L}{2})\,du$$

$$= \frac{R_B}{3EI}\left[L^3 - (\tfrac{L}{2})^3\right] - \frac{P}{EI}\left[\tfrac{1}{3}(\tfrac{L}{2})^3 + \tfrac{L}{2}\cdot\tfrac{1}{2}(\tfrac{L}{2})^2\right]$$

$$\frac{7}{24}\frac{R_B L^3}{EI} - \frac{5}{48}\frac{PL^3}{EI}$$

Over CB: $M = R_B v$ $\quad \frac{\partial M}{\partial R_B} = v$

$$\frac{\partial U_{CB}}{\partial R_B} = \frac{1}{EI}\int_0^{\frac{L}{2}} (R_B v)v\,dv = \frac{R_B}{3EI}\left(\frac{L}{2}\right)^3 = \frac{1}{24}\frac{R_B L^3}{EI}$$

$$y_B = \left(\frac{7}{24} + \frac{1}{24}\right)\frac{R_B L^3}{EI} - \frac{5}{48}\frac{PL^3}{EI} = 0$$

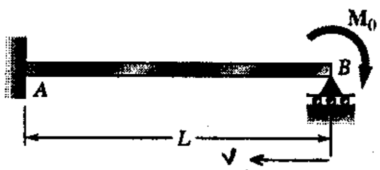
$$R_B = \frac{5}{16}P \uparrow$$

$$M_C = R_B\frac{L}{2} = \frac{5}{32}PL$$

$$M_A = R_B L - P\frac{L}{2} = \left(\frac{5}{16} - \frac{1}{2}\right)PL = -\frac{3}{16}PL$$

PROBLEM 11.125

11.125 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

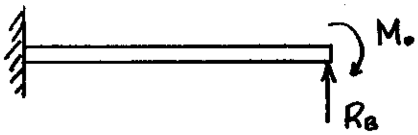


SOLUTION

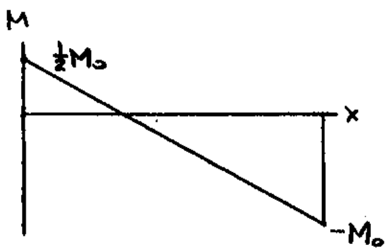
Remove support B and add reaction R_B as a load.

$$U = \int_0^L \frac{M^2}{2EI}\,dv$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial R_B}\,dv = 0$$



$$M = R_B v - M_0 \qquad \frac{\partial M}{\partial R_B} = v$$



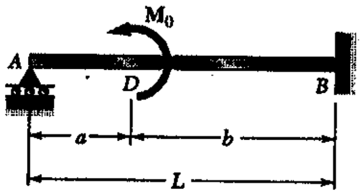
$$y_B = \frac{1}{EI}\int_0^L (R_B v - M_0)\,v\,dv$$

$$= \frac{R_B}{EI}\int_0^L v^2 dv - \frac{M_0}{EI}\int_0^L v\,dv$$

$$= \frac{R_B L^3}{3EI} - \frac{M_0 L^2}{2EI} = 0 \qquad R_B = \frac{3}{2}\frac{M_0}{L}\uparrow$$

$$M_A = R_B - M_0 = \frac{3}{2}M_0 - M_0 = \frac{1}{2}M_0$$

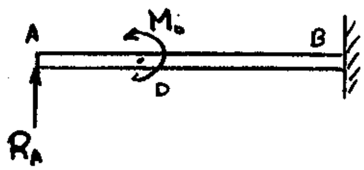
PROBLEM 11.126



11.126 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION

Remove support A and add reaction R_A as a load.



$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial R_A}\,dx = 0$$

Portion AD $\quad 0 < x < a \qquad M = R_A x \qquad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AD}}{\partial R_A} = \frac{1}{EI}\int_0^a (R_A x)(x)\,dx = \frac{R_A a^3}{3EI}$$

Portion DB $\quad (a < x < L) \qquad M = R_A x - M_0 \qquad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI}\int_a^L (R_A x - M_0)(x)\,dx = \frac{1}{EI}\left\{\frac{1}{3}R_A(L^3 - a^3) - \frac{1}{2}M_0(L^2 - a^2)\right\}$$

$$\delta_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI}\left\{R_A\left(\frac{1}{3}a^3 + \frac{1}{3}L^3 - \frac{1}{3}a^3\right) - \frac{1}{2}M_0(L^2 - a^2)\right\} = 0$$

$$R_A = \frac{3}{2}\frac{M_0(L^2 - a^2)}{L^3} = \frac{3}{2}\frac{M_0 b(L+a)}{L^3} \uparrow \quad ◄$$

$$M_A = 0 \quad ◄$$

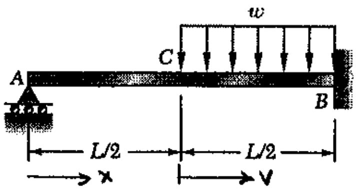
$$M_{D-} = R_A a = \frac{3}{2}\frac{M_0 ab(L+a)}{L^3} \quad ◄$$

$$M_{D+} = M_{D-} + M_0 = \frac{3}{2}\frac{M_0 ab(L+a)}{L^3} - M_0 \quad ◄$$

$$M_B = R_A L - M_0 = \frac{3}{2}\frac{M_0 b(L+a)}{L^2} - M_0 \quad ◄$$

PROBLEM 11.127

11.127 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

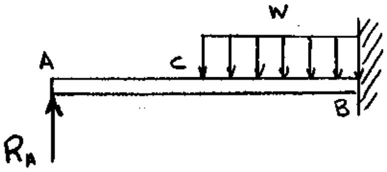


SOLUTION

Remove support A and add reaction R_A as a load.

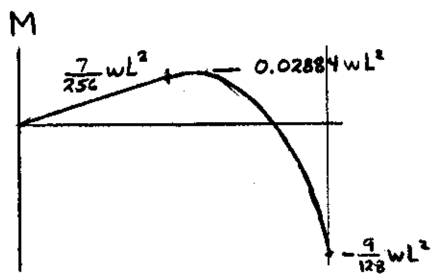
$$U = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dx + \int_0^{\frac{L}{2}} \frac{M^2}{2EI}\,dv$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI}\int_0^{\frac{L}{2}} M\frac{\partial M}{\partial R_A}\,dx + \frac{1}{EI}\int_0^{\frac{L}{2}} M\frac{\partial M}{\partial R_A}\,dv = 0$$



Portion AC: $0 < x < \frac{L}{2}$ $\quad M = R_A x \quad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AC}}{\partial R_A} = \frac{1}{EI}\int_0^{\frac{L}{2}} (R_A x)(x)\,dx = \frac{R_A L^3}{24EI}$$



Portion CB $\quad 0 < v < \frac{L}{2}$

$$M = R_A\left(v + \frac{L}{2}\right) - \frac{1}{2}wv^2 \qquad \frac{\partial M}{\partial R_A} = \left(v + \frac{L}{2}\right)$$

$$\frac{\partial U_{CB}}{\partial R_A} = \frac{1}{EI}\int_0^{L}\left[R_A\left(v+\frac{L}{2}\right) - \frac{1}{2}wv^2\right]\left(v+\frac{L}{2}\right)dv$$

$$= \frac{1}{EI}\left\{R_A\int_0^{\frac{L}{2}}\left(v+\frac{L}{2}\right)^2 dv - \frac{1}{2}w\int_0^{\frac{L}{2}}\left(v^3 + \frac{L}{2}v^2\right)dv\right\}$$

$$= \frac{R_A}{EI}\left[\frac{1}{3}L^3 - \frac{1}{3}\left(\frac{L}{2}\right)^3\right] - \frac{w}{2EI}\left[\frac{1}{4}\left(\frac{L}{2}\right)^4 + \frac{L}{2}\frac{1}{3}\left(\frac{L}{2}\right)^3\right]$$

$$= \left(\frac{1}{3} - \frac{1}{24}\right)\frac{R_A L^3}{EI} - \frac{7}{384}\frac{wL^4}{EI}$$

$$\delta_A = \frac{\partial U_{AC}}{\partial R_A} + \frac{\partial U_{CB}}{\partial R_A} = \frac{1}{3}\frac{R_A L^3}{EI} - \frac{7}{384}\frac{wL^4}{EI} = 0$$

$$R_A = \frac{7}{128}wL \uparrow$$

Over AC $\quad M = \frac{7}{128}wLx$

$$M_C = \frac{7}{256}wL^2 = 0.02734\,wL^2$$

Over CB $\quad M = \frac{7}{128}wL\left(v + \frac{L}{2}\right) - \frac{1}{2}wv^2$

$$M_B = \frac{7}{128}wL^2 - \frac{1}{2}w\left(\frac{L}{2}\right)^2 = -\frac{9}{128}wL^2$$

$$= -0.07031\,wL^2$$

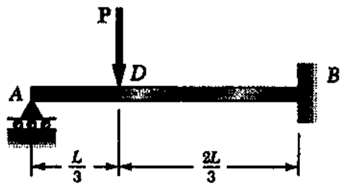
$$\frac{dM}{dv} = \frac{7}{128}wL - wv_m = 0 \qquad v_m = \frac{7}{128}L$$

$$M_m = \frac{7}{128}wL\left(\frac{7}{128}L + \frac{L}{2}\right) - \frac{1}{2}w\left(\frac{7}{128}L\right)^2$$

$$= \frac{945}{32768}wL^2 = 0.02884\,wL^2$$

PROBLEM 11.128

11.128 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

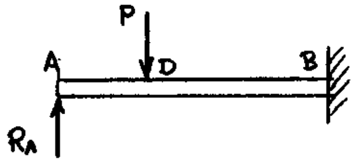


SOLUTION

Remove support A and add reaction R_A as a load.

$$U = \int_0^L \frac{M^2}{2EI}\,dx$$

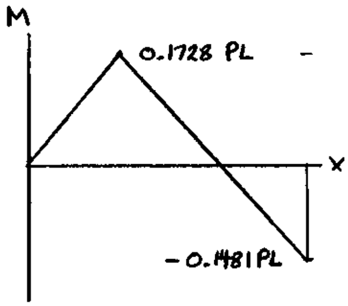
$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial R_A}\,dx$$



Portion AD $\quad 0 < x < \frac{L}{3} \qquad M = R_A x \qquad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AD}}{\partial R_A} = \frac{1}{EI}\int_0^{\frac{L}{3}} M\frac{\partial M}{\partial R_A}\,dx = \frac{1}{EI}\int_0^{\frac{L}{3}}(R_A x)(x)\,dx$$

$$= \frac{R_A}{3EI}\left(\frac{L}{3}\right)^3 = \frac{1}{81}\frac{R_A L^3}{EI}$$



Portion DB $\quad \frac{L}{3} < x < L \qquad M = R_A x - P\left(x - \frac{L}{3}\right)$

$$\frac{\partial M}{\partial R_A} = x$$

$$\frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI}\int_{\frac{L}{3}}^{L} M\frac{\partial M}{\partial R_A}\,dx = \frac{1}{EI}\int_{\frac{L}{3}}^{L}\left[R_A - P\left(x - \frac{L}{3}\right)\right]x\,dx$$

$$= \frac{R_A}{EI}\int_{\frac{L}{3}}^{L} x^2\,dx - \frac{P}{EI}\int_{\frac{L}{3}}^{L}\left(x^2 - \frac{L}{3}x\right)dx$$

$$= \frac{R_A}{3EI}\left[L^3 - \left(\frac{L}{3}\right)^3\right] - \frac{P}{EI}\left[\frac{1}{3}\left(L^3 - \left(\frac{L}{3}\right)^3\right) - \frac{L}{6}\left(L^2 - \left(\frac{L}{3}\right)^2\right)\right]$$

$$= \left(\frac{1}{3} - \frac{1}{81}\right)\frac{R_A L^3}{EI} - \left(\frac{1}{3} - \frac{1}{81} - \frac{1}{6} + \frac{1}{54}\right)\frac{PL^3}{EI}$$

$$\delta_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \left(\frac{1}{81} + \frac{1}{3} - \frac{1}{81}\right)\frac{R_A L^3}{EI} - \frac{14}{81}\frac{PL^3}{EI} = \frac{1}{3}\frac{R_A L^3}{EI} - \frac{14}{81}\frac{PL^3}{EI} = 0$$

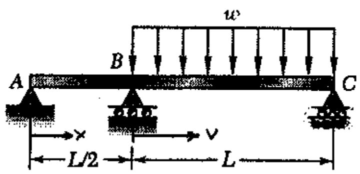
$$R_A = \frac{14}{27}P$$

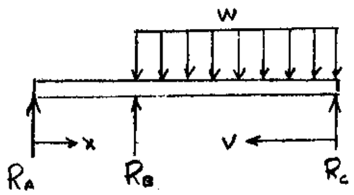
Bending moments $\quad M_D = R_A\left(\frac{L}{3}\right) = \frac{14}{81}PL = 0.1728\,PL$

$$M_B = R_A L - P\left(\frac{2L}{3}\right) = -\frac{4}{27}PL = -0.1481\,PL$$

PROBLEM 11.129

11.129 For the uniform beam and loading shown, determine the reaction at each support.





SOLUTION

Remove support A and add reaction R_A as a load.

$$\Sigma M_B = 0 \quad -R_A\frac{L}{2} - \frac{1}{2}wL^2 + R_C L = 0$$

$$R_C = \frac{1}{2}R_A + \frac{1}{2}wL$$

$$U = U_{AB} + U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI}dx + \int_0^L \frac{M^2}{2EI}dv$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = 0$$

Portion AB: $\quad M = R_A x, \quad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AB}}{\partial R_A} = \frac{1}{EI}\int_0^{\frac{L}{2}} M\frac{\partial M}{\partial R_A}dx = \frac{1}{EI}\int_0^{\frac{L}{2}}(R_A x)(x)dx = \frac{R_A}{3EI}\left(\frac{L}{2}\right)^3 = \frac{1}{24}\frac{R_A L^3}{EI}$$

Portion BC: $\quad M = R_C v - \frac{1}{2}wv^2 = \frac{1}{2}R_A v + \frac{1}{2}wLv - \frac{1}{2}wLv^2$

$$\frac{\partial M}{\partial R_A} = \frac{1}{2}v$$

$$\frac{\partial U_{BC}}{\partial R_A} = \frac{1}{EI}\int_0^L\left[\frac{1}{2}R_A v + \frac{1}{2}w(Lv - v^2)\right]\left(\frac{1}{2}v\right)dv = \frac{1}{4EI}\int_0^L\left[R_A v^2 + w(Lv^2 - v^3)\right]dv$$

$$= \frac{1}{4EI}\left[R_A\frac{L^3}{3} + w\left(\frac{L^4}{3} - \frac{L^4}{4}\right)\right] = \frac{R_A L^3}{12EI} + \frac{wL^4}{48EI}$$

$$\delta_A = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = \left(\frac{1}{24} + \frac{1}{12}\right)\frac{R_A L^3}{EI} + \frac{wL^4}{48EI} = 0$$

$$R_A = -\frac{1}{6}wL = \frac{1}{6}wL\downarrow$$

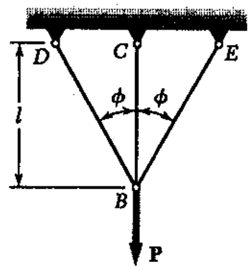
$$R_C = \frac{1}{2}\left(-\frac{1}{6}wL\right) + \frac{1}{2}wL = \frac{5}{12}wL\uparrow$$

$+\uparrow\Sigma F_y = 0 \quad R_A + R_B + R_C - wL = 0$

$-\frac{1}{6}wL + R_B + \frac{5}{12}wL - wL = 0 \qquad R_B = \frac{3}{4}wL$

PROBLEM 11.130

11.130 Three members of the same material and same cross-sectional area are used to support the load **P**. Determine the force in member *BC*.



SOLUTION

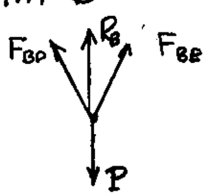
Detach member BC at support C.

Add reaction R_c as a load

$$U = \sum \frac{F^2 L}{2EA} \qquad y_c = \frac{\partial U}{\partial R_c} = \sum \frac{FL}{EA}\frac{\partial F}{\partial R_c} = 0$$

Joint C , $F_{BC} = R_c$

Joint B



$\xrightarrow{+} \sum F_x = 0 \qquad F_{BE} \sin\varphi - F_{BD} \sin\varphi = 0 \qquad F_{BE} = F_{BD}$

$+\uparrow \sum F_y = 0 \qquad F_{BD} \cos\varphi + F_{BE} \cos\varphi + R_B - P$

$$F_{BD} = F_{BE} = \frac{P - R_B}{2\cos\varphi}$$

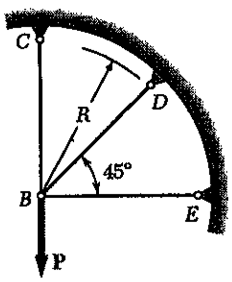
Member	F	$\partial F/\partial R_B$	L	$(FL/EA)\, \partial F/\partial R_B$
BD	$(P - R_B)/2\cos\varphi$	$-1/2\cos\varphi$	$\ell/\cos\varphi$	$(R_B - P)\ell/4EA\cos^3\varphi$
BE	$(P - R_B)/2\cos\varphi$	$-1/2\cos\varphi$	$\ell/\cos\varphi$	$(R_B - P)\ell/4EA\cos^3\varphi$
BC	R_B	1	ℓ	$R_B\ell/EA$

$$y_B = -P\ell/2EA\cos^3\varphi + R_B\ell/2EA\cos^3\varphi + R_B\ell/EA = 0$$

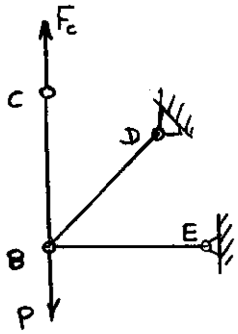
$$R_B = \frac{P}{1 + 2\cos^3\varphi} \qquad\qquad F_{BC} = R_B = \frac{P}{1 + 2\cos^3\varphi}$$

PROBLEM 11.131

11.131 Three members of the same material and same cross-sectional area are used to support the load **P**. Determine the force in member *BC*.

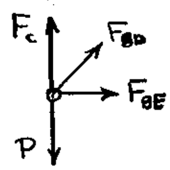


SOLUTION



Detach member BC from its support at point C. Add reaction F_c as a load.

Joint B. $+\uparrow \Sigma F_y = 0$



$$\frac{\sqrt{2}}{2} F_{BD} + F_c - P = 0$$

$$F_{BD} = \sqrt{2}\,P - \sqrt{2}\,F_c$$

$\xrightarrow{+} \Sigma F_x = 0$

$$\frac{\sqrt{2}}{2} F_{BD} + F_{BE} = 0$$

$$F_{BE} = -P + F_c$$

$$U = \Sigma \frac{F^2 R}{2EA} = \frac{R}{2EA} \Sigma F^2$$

$$\delta_c = \frac{\partial U}{\partial F_c} = \frac{R}{EA} \Sigma F \frac{\partial F}{\partial F_c} = 0$$

Member	F	$\partial F / \partial F_c$	$F(\partial F / \partial F_c)$
BC	F_c	1	F_c
BD	$\sqrt{2}P - \sqrt{2}F_c$	$-\sqrt{2}$	$-2P + 2F_c$
BE	$-P + F_c$	1	$-P + F_c$
Σ			$-3P + 4F_c$

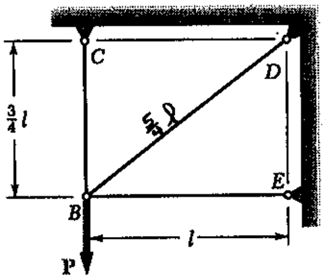
$$\delta_c = \frac{R}{EA}(-3P + 4F_c) = 0$$

$$F_c = \frac{3}{4}P$$

$$F_{BC} = F_c = \frac{3}{4}P$$

PROBLEM 11.132

11.132 Three members of the same material and same cross-sectional area are used to support the load **P**. Determine the force in member *BC*.



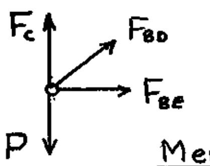
SOLUTION

Detach member BC from support C. Add reaction F_c as a load.

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$\delta_c = \frac{\partial U}{\partial F_c} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_c} L$$

Joint B



$+\uparrow \sum F_y = 0 \qquad F_c - P + \frac{3}{5} F_{BD} = 0 \qquad F_{BD} = \frac{5}{3}P - \frac{5}{3}F_c$

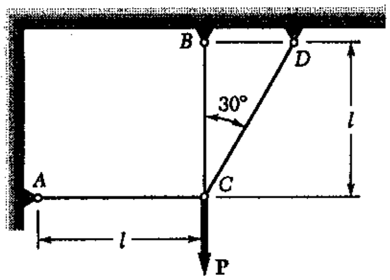
$\underrightarrow{+} \sum F_x = 0 \qquad F_{BE} + \frac{4}{5} F_{BD} = 0 \qquad F_{BE} = -\frac{4}{3}P + \frac{4}{3}F_c$

Member	F	$\partial F/\partial F_c$	L	$F(\partial F/\partial F_c)L$
BC	F_c	1	$\frac{3}{4}\ell$	$\frac{3}{4}F_c\ell$
BD	$\frac{5}{3}P - \frac{5}{3}F_c$	$-\frac{5}{3}$	$\frac{5}{4}\ell$	$-\frac{125}{36}P\ell + \frac{125}{36}F_c\ell$
BE	$-\frac{4}{3}P + \frac{4}{3}F_c$	$\frac{4}{3}$	ℓ	$-\frac{16}{9}P\ell + \frac{16}{9}F_c\ell$
Σ				$-\frac{21}{4}P\ell + 6F_c\ell$

$$\delta_c = \frac{1}{EA}\left(-\frac{21}{4}P\ell + 6F_c\ell\right) = 0 \qquad F_c = \frac{7}{8}P \qquad F_{BC} = F_c = \frac{7}{8}P$$

PROBLEM 11.133

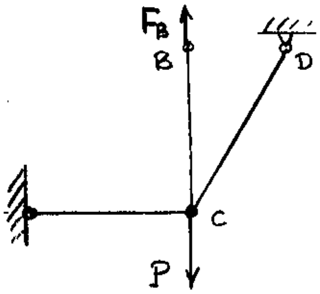
11.133 Three members of the same material and same cross-sectional area are used to support the load **P**. Determine the force in member *BC*.



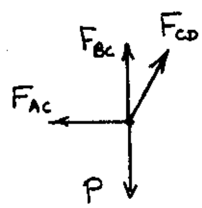
SOLUTION

Cut member BC at end B and replace member force F_{BC} by load F_B acting on member BC at B.

$$S_B = \frac{\partial U}{\partial F_B} = \frac{\partial}{\partial F_B}\sum \frac{F^2 L}{EA} = \frac{1}{EA}\sum F \frac{\partial F}{\partial F_B} L$$



Joint C $\quad +\uparrow \sum F_y = 0 \quad \frac{\sqrt{3}}{2} F_{CD} + F_{BC} - P = 0$



$$F_{CD} = \frac{2}{\sqrt{3}} P - \frac{2}{\sqrt{3}} F_B$$

$\xrightarrow{+} \sum F_x = 0 \quad F_{AC} - \frac{1}{2} F_{CD} = 0$

$$F_{AC} = \frac{1}{\sqrt{3}} P - \frac{1}{\sqrt{3}} F_B$$

Member	F	$\partial F/\partial F_B$	L	$F(\partial F/\partial F_B)L$
AC	F_B	1	l	$F_B l$
BC	$\frac{1}{\sqrt{3}}P - \frac{1}{\sqrt{3}}F_B$	$-\frac{1}{\sqrt{3}}$	l	$-\frac{1}{3}Pl + \frac{1}{3}F_B l$
CD	$\frac{2}{\sqrt{3}}P - \frac{2}{\sqrt{3}}F_B$	$-\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}l$	$-\frac{8}{\sqrt{3}}Pl + \frac{8}{\sqrt{3}}F_B l$
Σ				$-\left(\frac{1}{3}+\frac{8}{\sqrt{3}}\right)Pl + \left(\frac{4}{3}+\frac{8}{\sqrt{3}}\right)F_B l$

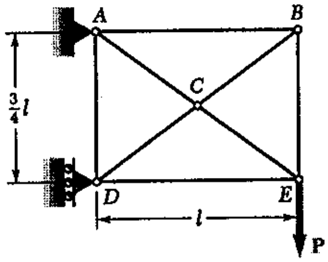
$$S_B = -\left(\frac{1}{3}+\frac{8}{\sqrt{3}}\right)\frac{Pl}{EA} + \left(\frac{4}{3}+\frac{8}{\sqrt{3}}\right)\frac{F_B l}{EA}$$

$$F_B = \frac{\frac{1}{3}+\frac{8}{\sqrt{3}}}{\frac{4}{3}+\frac{8}{\sqrt{3}}} P = \frac{8+\sqrt{3}}{8+4\sqrt{3}} P = 0.652\ P$$

$$F_{BC} = F_B = 0.652\ P$$

PROBLEM 11.134

11.134 Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member *AB*.

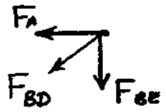


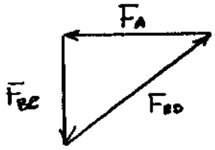
SOLUTION

Cut member AB at end A and replace member force F_{AB} by load $F_A \leftarrow$ acting on member AB at end A.

$$S_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A}\sum \frac{F^2 L}{2EA} = \frac{1}{EA}\sum F\frac{\partial F}{\partial F_A}L = 0$$

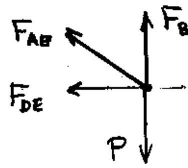
Joint B





$F_{BD} = -\frac{5}{4}F_A \qquad F_{BE} = \frac{3}{4}F_A$

Joint E



$+\uparrow \Sigma F_y = 0$

$F_{BE} - P + \frac{3}{5}F_{AE}$

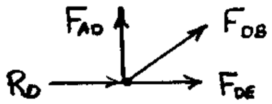
$F_{AE} = \frac{5}{3}P - \frac{5}{3}F_{BE}$

$= \frac{5}{3}P - \frac{5}{4}F_A$

$\xrightarrow{+} \Sigma F_x = 0 \qquad -\frac{4}{5}F_{AE} - F_{DE} = 0$

$F_{DE} = -\frac{4}{5}F_{AE} = -\frac{4}{3}P + F_A$

Joint D



$+\uparrow \Sigma F_y = 0 \qquad F_{AD} + \frac{3}{5}F_{DB} = 0$

$F_{AD} = -\frac{3}{5}F_{DB} = -\frac{3}{4}F_A$

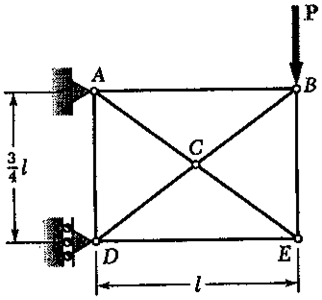
Member	F	$\partial F/\partial F_A$	L	$F(\partial F/\partial F_A)L$
AB	F_A	1	l	$F_A l$
AD	$-\frac{3}{4}F_A$	$-\frac{3}{4}$	$\frac{3}{4}l$	$\frac{27}{64}F_A l$
AE	$\frac{5}{3}P - \frac{5}{4}F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$-\frac{125}{48}Pl + \frac{125}{64}F_A l$
BD	$-\frac{5}{4}F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$\frac{125}{64}F_A l$
BE	$\frac{3}{4}F_A$	$\frac{3}{4}$	$\frac{3}{4}l$	$\frac{27}{64}F_A l$
DE	$-\frac{4}{3}P + F_A$	1	l	$-\frac{4}{3}Pl + F_A l$
Σ				$-\frac{63}{16}Pl + \frac{27}{4}F_A l$

$$S_A = \frac{1}{EA}\left(-\frac{63}{16}Pl + \frac{27}{4}F_A l\right) = 0 \qquad F_A = \frac{7}{12}P$$

$$F_{AB} = F_A = \frac{7}{12}P = 0.583\,P$$

PROBLEM 11.135

11.135 Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member *AB*.

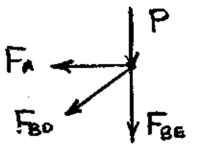


SOLUTION

Cut member AB at end A and replace member force F_{AB} by load $F_A \leftarrow$ acting on member AB at end A.

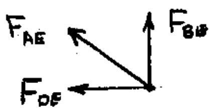
$$\delta_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A}\sum \frac{F^2 L}{EA} = \frac{1}{EA}\sum F\frac{\partial F}{\partial F_A}L = 0$$

Joint B $\quad \overset{+}{\rightarrow}\Sigma F_x = 0 \quad -F_A - \frac{4}{5}F_{BD} = 0 \quad F_{BD} = -\frac{5}{4}F_A$



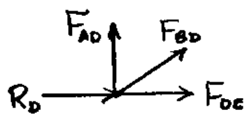
$+\uparrow\Sigma F_y = 0 \quad -P - F_{BE} - \frac{3}{5}F_{BD} = 0 \quad F_{BE} = -P + \frac{3}{4}F_A$

Joint E $\quad +\uparrow\Sigma F_y = 0 \quad F_{BE} + \frac{3}{5}F_{AE} = 0 \quad F_{AE} = \frac{5}{3}P - \frac{5}{4}F_A$



$\overset{+}{\rightarrow}\Sigma F_x = 0 \quad -\frac{4}{5}F_{AE} - F_{DE} = 0 \quad F_{DE} = -\frac{4}{3}P + F_A$

Joint D



$+\uparrow\Sigma F_y = 0 \quad F_{AD} + \frac{3}{5}F_{BD} = 0$

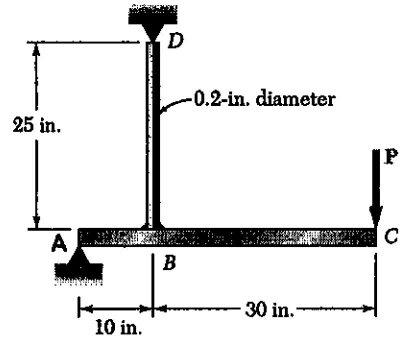
$F_{AD} = \frac{3}{4}F_A$

Member	F	$\partial F/\partial F_A$	L	$F(\partial F/\partial F_A)L$
AB	F_A	1	l	$F_A l$
AD	$\frac{3}{4}F_A$	$\frac{3}{4}$	$\frac{3}{4}l$	$\frac{27}{64}F_A l$
AE	$\frac{5}{3}P - \frac{5}{4}F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$-\frac{125}{48}Pl + \frac{125}{64}F_A l$
BD	$-\frac{5}{4}F_A$	$-\frac{5}{4}$	$\frac{5}{4}l$	$\frac{125}{64}F_A l$
BE	$-P + \frac{3}{4}F_A$	$\frac{3}{4}$	$\frac{3}{4}l$	$-\frac{9}{16}Pl + \frac{27}{64}F_A l$
DE	$-\frac{4}{3}P + F_A$	1	l	$-\frac{4}{3}Pl + F_A l$
Σ				$-\frac{9}{2}Pl + \frac{27}{4}F_A l$

$$\delta_A = \frac{1}{EA}\left(-\frac{9}{2}Pl + \frac{27}{4}F_A l\right) = 0 \qquad F_A = \frac{2}{3}P$$

$$F_{AB} = F_A = \frac{2}{3}P = 0.667P \quad \blacktriangleleft$$

PROBLEM 11.136



11.136 The steel bar *ABC* has a square cross section of side 0.75 in. and is subjected to a 50-lb load **P**. Using $E = 29 \times 10^6$, determine the deflection of point *C*.

SOLUTION

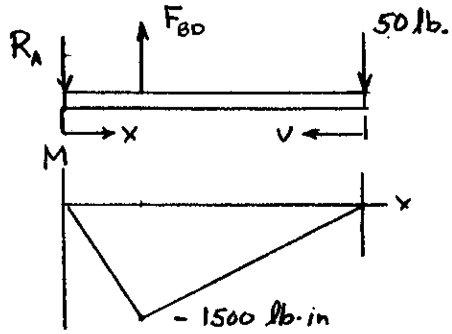
Assume member BD is a two-force member.

$\Sigma M_A = 0 \qquad 10\,F_{BD} - (40)(50) = 0 \qquad F_{BD} = 200 \text{ lb.}$

$A_{BD} = \frac{\pi}{4}(0.2)^2 = 31.416 \times 10^{-3} \text{ in}^2$

$U_{BD} = \dfrac{F_{BD}^2 L_{BD}}{2EA} = \dfrac{(200)^2(25)}{(2)(29\times10^6)(31.416\times10^{-3})}$

$= 0.5488 \text{ in-lb.}$



Member ABC

$I = \frac{1}{12}(0.75)(0.75)^3 = 26.367 \times 10^{-3} \text{ in}^4$

Portion AB $\qquad M = -1500\frac{x}{10} = -150x$

$U_{AB} = \int_0^{10} \dfrac{M^2}{2EI}\,dx = \dfrac{150^2}{2EI}\int_0^{10} x^2\,dx$

$= \dfrac{(150)^2(10^3)}{(2)(29\times10^6)(26.367\times10^{-3})(3)} = 4.904 \text{ in-lb.}$

Portion BC: $\quad M = -50v \qquad U_{BC} = \int_0^{30}\dfrac{M^2}{2EI}\,dv = \dfrac{50^2}{2EI}\int_0^{30} v^3\,dv$

$= \dfrac{(50)^2(30)^3}{(2)(29\times10^6)(26.367\times10^{-3})(3)} = 14.713 \text{ in-lb.}$

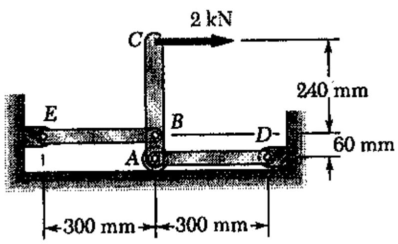
Total $\quad U = U_{BD} + U_{AB} + U_{BD} = 20.166 \text{ in-lb.}$

$\frac{1}{2}P\delta_C = U$

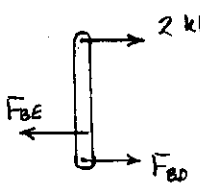
$\delta_C = \dfrac{2U}{P} = \dfrac{(2)(20.166)}{50} = 0.807 \text{ in.} \downarrow$ ◄

PROBLEM 11.137

11.137 The steel bars *BE* and *AD* have each a 5 × 15-mm cross section. Assuming that lever *ABC* is rigid and using $E = 200$ GPa, determine the deflection of point *C*.



SOLUTION



$\circlearrowleft \Sigma M_A = 0 \qquad 60 F_{BE} - (300)(2) = 0$

$F_{BE} = 10$ kN.

$\circlearrowleft \Sigma M_B = 0 \qquad 60 F_{AD} - (240)(2) = 0$

$F_{AD} = 8$ kN

For bars BE and AD $\qquad A = 5 \times 15 = 75 \text{ mm}^2 = 75 \times 10^{-6} \text{ m}^2$

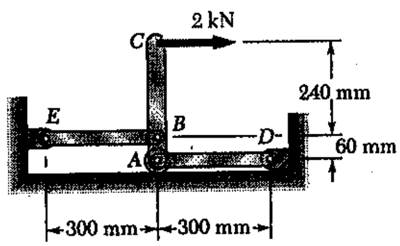
$$U = \frac{F_{BE}^2 L_{BE}}{2EA} + \frac{F_{AD}^2 L_{AD}}{2EA} = \frac{(10 \times 10^3)^2 (300 \times 10^{-3})}{(2)(200 \times 10^9)(75 \times 10^{-6})} + \frac{(8 \times 10^3)^2 (300 \times 10^{-3})}{(2)(200 \times 10^9)(75 \times 10^{-6})}$$

$$= 1.0000 + 0.6400 = 1.6400 \text{ N}\cdot\text{m}^2$$

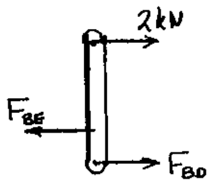
$$\frac{1}{2} P \delta_c = U \qquad \delta_c = \frac{2U}{P} = \frac{(2)(1.6400)}{2 \times 10^3} = 1.64 \times 10^{-3} \text{ m} = 1.64 \text{ mm} \rightarrow$$

PROBLEM 11.138

11.138 The steel bars *BE* and *AD* have each a 5 × 15-mm cross section and the steel lever *ABC* has a square cross section of side 25 mm. Using $E = 200$ GPa, determine the deflection of point *C*.



SOLUTION



$+\circlearrowleft \Sigma M_A = 0 \qquad 60F_{BE} - (300)(2) = 0$

$F_{BE} = 10 \text{ kN}$

$+\circlearrowleft \Sigma M_B = 0 \qquad 60F - (240)(2) = 0$

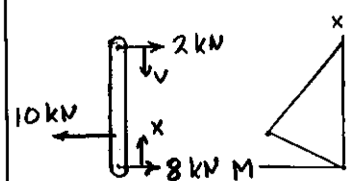
$F = 8 \text{ kN}$

For bars BE and AD $\qquad A = 5 \times 15 = 75 \text{ mm}^2 = 75 \times 10^{-6} \text{ m}^2$

$$U_{BE} = \frac{F_{BE}^2 L_{BE}}{2EA} = \frac{(10)^2(300 \times 10^{-3})}{(2)(200 \times 10^9)(75 \times 10^{-6})} = 1.0000 \text{ J}$$

$$U_{AD} = \frac{F_{AD}^2 L_{AD}}{2EA} = \frac{(8)^2(300 \times 10^{-3})}{(2)(200 \times 10^9)(75 \times 10^{-6})} = 0.6400 \text{ J}$$

Beam ABC: $\quad I = \frac{1}{12}(25)(25)^3 = 32.552 \times 10^3 \text{ mm}^4 = 32.552 \times 10^{-9} \text{ m}^4$



Bending moment at B

$$M_B = (2 \times 10^3)(240 \times 10^{-3}) = 480 \text{ N·m}$$

Portion AB: $\quad M = \frac{480}{L_{AB}} x$

$$U_{AB} = \int_0^{L_{AB}} \frac{M^2}{2EI} = \frac{(480)^2}{2EI L_{AB}^2} \int_0^{L_{AB}} x^2\,dx$$

$$= \frac{(480^2) L_{AB}^3}{6EI L_{AB}^2} = \frac{(480)^2 L_{AB}}{6EI}$$

$$= \frac{(480)^2(60 \times 10^{-3})}{(6)(200 \times 10^9)(32.552 \times 10^{-9})} = 0.3539 \text{ J}$$

Portion BC: $\quad M = \frac{480}{L_{BC}} v$

$$U_{BC} = \int_0^{L_{BC}} \frac{M^2}{2EI} dv = \frac{(480)^2}{2EI L_{BC}^2} \int_0^{L_{BC}} v^2\,dv = \frac{(480)^2 L_{BC}^3}{6EI L_{BC}^2} = \frac{(480)^2 L_{BC}}{6EI}$$

$$= \frac{(480)^2(240 \times 10^{-3})}{(6)(200 \times 10^9)(32.552 \times 10^{-9})} = 1.4156 \text{ J}$$

Total $\quad U = U_{BE} + U_{AD} + U_{AB} + U_{BC}$

$$= 1.0000 + 0.6400 + 0.3539 + 1.4156 = 3.4095 \text{ J}$$

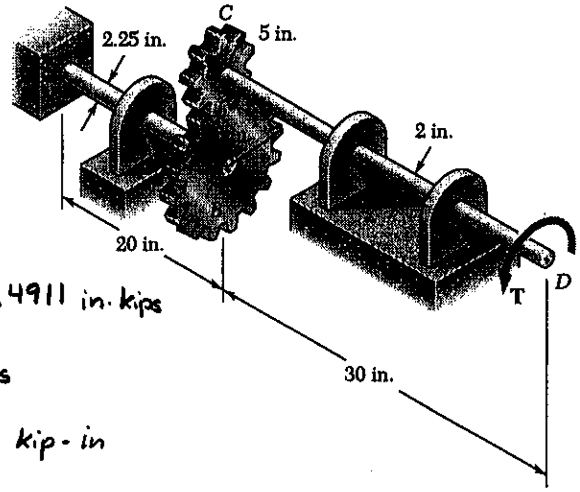
$\frac{1}{2} P \delta_c = U \qquad \delta_c = \frac{2U}{P} = \frac{(2)(3.4095)}{2 \times 10^3} = 3.41 \times 10^{-3} \text{ m}$

$= 3.41 \text{ mm} \rightarrow$

PROBLEM 11.139

11.139 Two solid steel shafts are connected by the gears shown. Using $G = 11.2 \times 10^6$ psi, determine the strain energy in each shaft when a 24 kip·in. torque is applied at D. (Ignore the strain energy due to bending of the shafts.)

SOLUTION



Shaft CD: $T_{CD} = T_C = 24$ kip·in

$J_{CD} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{2}\left(\frac{2.0}{2}\right)^4 = 1.5708 \text{ in}^4$

$L_{CD} = 30$ in., $G = 11.2 \times 10^6$ psi $= 11.2 \times 10^3$ ksi

$U_{CD} = \frac{T_{CD}^2 L_{CD}}{2 G J_{CD}} = \frac{(24)^2(30)}{(2)(11.2\times10^3)(1.5708)} = 0.4911$ in. kips

Gear C $\quad F_{CB} = \frac{T_C}{r_C} = \frac{T_{CD}}{r_C} = \frac{24}{5} = 4.8$ kips

Gear B $\quad T_B = r_B F_{CB} = (8)(4.8) = 38.4$ kip·in

Shaft AB $\quad T_{AB} = T_B = 38.4$ kip·in $\qquad L_{AB} = 20$ in

$J_{AB} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{2}\left(\frac{2.25}{2}\right)^4 = 2.5161 \text{ in}^4$

$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2 G J_{AB}} = \frac{(38.4)^2(20)}{(2)(11.2\times10^3)(2.5161)} = 0.5233$ in. kips

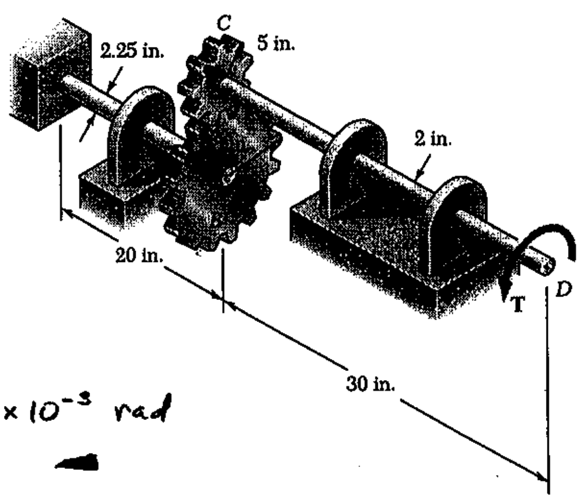
Total $\quad U = U_{AB} + U_{CD} = 0.5233 + 0.4911 = 1.0144$ in. kips.

PROBLEM 11.140

11.140 Two solid steel shafts are connected by the gears shown. Using $G = 11.2 \times 10^6$ psi, determine the angle through which end D rotates when $T = 24$ kip·in.

(Ignore the strain energy of bending of the shafts.)

SOLUTION



From Prob. 11.139

$U = 1.0144$ in. lb.

$\frac{1}{2} T_D \varphi_D = U$

$\varphi_D = \frac{2U}{T_D} = \frac{(2)(1.0144)}{24} = 84.5 \times 10^{-3}$ rad

$= 4.84°$

PROBLEM 11.141

11.141 (*a*) Determine the modulus of resilience of a grade of structural steel for which $\sigma_Y = 300$ MPa and $E = 200$ GPa. (*b*) Determine the required yield strength of an aluminum alloy for which $E = 72$ GPa if the modulus of resilience of the alloy is to be the same as that of the structural steel.

SOLUTION

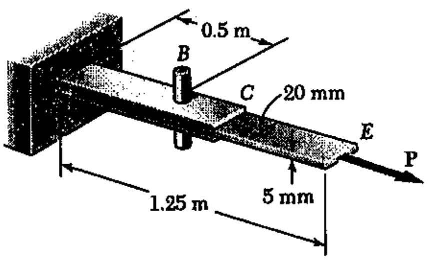
(a) $E = 200 \times 10^9$ Pa, $\sigma_Y = 300 \times 10^6$ Pa

$$u_Y = \frac{\sigma_{YS}^2}{2E_s} = \frac{(300\times10^6)^2}{(2)(200\times10^9)} = 225\times10^3\ \mathrm{N\cdot m/m^3} = 225\ \mathrm{kJ/m^3}$$

(b) $$\sigma_{Ya} = \sqrt{2E_a u_{Ya}} = \sqrt{(2)(72\times10^9)(225\times10^3)} = 180\times10^6\ \mathrm{Pa} = 180\ \mathrm{MPa}$$

PROBLEM 11.142

11.142 A single 6-mm-diameter steel pin *B* is used to connect the steel strip *DE* to two aluminum strips, each of 20-mm width and 5-mm thickness. The modulus of elasticity is 200 GPa for the steel and 70 GPa for the aluminum. Knowing that for the pin at *B* the allowable shearing stress is $\tau_{all} = 85$ MPa, determine, for the loading shown, the maximum strain energy that can be acquired by the assembled strips.



SOLUTION

$$A_{pin} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(6)^2 = 28.274\ \mathrm{mm}^2 = 28.274\times10^{-6}\ \mathrm{m}^2$$

$$\tau_{all} = 85\times10^6\ \mathrm{Pa}$$

Double shear $P = 2A\tau = (2)(28.274\times10^{-6})(85\times10^6)$
$= 4.8066\times10^3$ N

For strips AB, DB, BE $\quad A = (20)(5) = 100\ \mathrm{mm}^2 = 100\times10^{-6}\ \mathrm{m}^2$

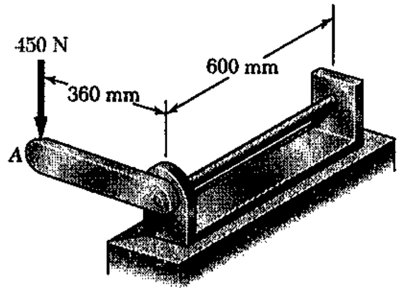
$F_{AB} = F_{DB} = \frac{1}{2}P = 2.4033\times10^3$ N

$$U_{AB} = U_{DB} = \frac{F_{AB}^2 L_{AB}}{2E_a A_{AB}} = \frac{(2.4033)^2(0.5)}{(2)(70\times10^9)(100\times10^{-6})} = 206.3\times10^{-3}\ \mathrm{J}$$

$$U_{BE} = \frac{F_{BE}^2 L_{BE}}{2E_s A_{BE}} = \frac{(4.8066\times10^3)^2(1.25-0.5)}{(2)(200\times10^9)(100\times10^{-6})} = 433.2\times10^{-3}\ \mathrm{J}$$

Total: $U = U_{AB} + U_{DB} + U_{BE} = 846\times10^{-3}\ \mathrm{J} = 0.846\ \mathrm{J}$

PROBLEM 11.143



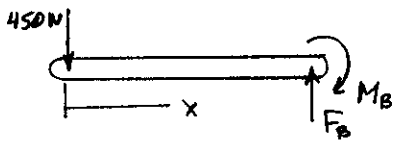
11.143 The 18-mm-diameter steel rod BC is attached to the lever AB and to the fixed support C. The uniform steel lever AB is 9 mm wide and 24 mm deep. Using $E = 200$ GPa, $G = 77$ GPa, and the method of work and energy, determine the deflection of point A

SOLUTION

Member AB

$$I = \frac{1}{12}(9)(24)^3 = 10.368 \times 10^3 \text{ mm}^4 = 10.368 \times 10^{-9} \text{ m}^4$$

$$E = 200 \times 10^9$$



$$M = 450\, x \qquad M_B = 162 \text{ N}\cdot\text{m}$$

$$U_{AB} = \int_0^{L_{AB}} \frac{M^2}{2EI} dx = \frac{(450)^2}{2EI} \int_0^{L_{AB}} x^2 dx$$

$$= \frac{(450)^2 L_{AB}^3}{6EI} = \frac{(450)^2 (360 \times 10^{-3})^3}{(6)(200 \times 10^9)(10.368 \times 10^{-9})}$$

$$= 0.75938 \text{ J}$$

Member BC $\quad T = M_B = 162 \text{ N}\cdot\text{m} \qquad L = 600 \times 10^{-3} \text{ m}$

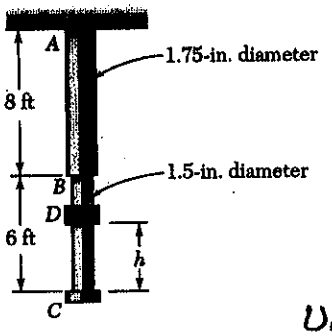
$$J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{2}\left(\frac{18}{2}\right)^4 = 10.306 \times 10^3 \text{ mm}^4 = 10.306 \times 10^{-9} \text{ m}^4$$

$$U_{BC} = \frac{T^2 L}{2GJ} = \frac{(162)^2 (600 \times 10^{-3})}{(2)(77 \times 10^9)(10.306 \times 10^{-9})} = 9.9213 \text{ J}$$

Total $\quad U = U_{AB} + U_{BC} = 10.681 \text{ J}$

$$\frac{1}{2} P \delta_A = U \qquad \delta_A = \frac{2U}{P} = \frac{(2)(10.681)}{450} = 47.5 \times 10^{-3} \text{ m} = 47.5 \text{ mm} \downarrow$$

PROBLEM 11.144



11.144 The 75-lb collar D is released from rest in the position shown and is stopped by a plate attached at end C of the vertical rod ABC. Knowing that $E = 29 \times 10^6$ psi for both portions of the rod, determine the distance h for which the maximum stress in the rod is 36 ksi.

SOLUTION

Portion BC: $A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$

$\sigma_{BC} = 30000$ psi $\qquad L_{BC} = 6 \text{ ft} = 72 \text{ in.}$

Force at C $\quad P = \sigma_{BC} A_{BC} = 53014$ lb.

$$U_{BC} = \frac{P^2 L_{BC}}{2 E A_{BC}} = \frac{(53014)^2 (72)}{(2)(29\times10^6)(1.76715)} = 1974.3 \text{ in}\cdot\text{lb.}$$

Portion AB: $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.75)^2 = 2.40528 \text{ in}^2. \qquad L_{AB} = 8 \text{ ft} = 96 \text{ in.}$

$$U_{AB} = \frac{P^2 L_{AB}}{2 E A_{AB}} = \frac{(53014)^2 (96)}{(2)(29\times10^6)(2.40528)} = 1934.0 \text{ in}\cdot\text{lb}$$

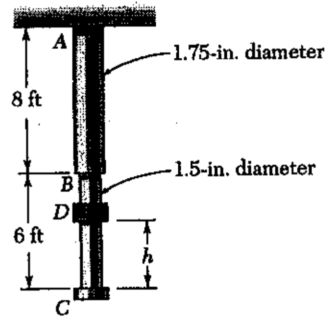
Total $\quad U = U_{AB} + U_{BC} = 3908.3 \text{ in}\cdot\text{lb}$

$\frac{1}{2} P \delta_c = U \qquad \delta_c = \frac{2U}{P} = \frac{(2)(3908.3)}{53014} = 0.14744 \text{ in.}$

$W(h + \delta_c) = U \qquad h = \frac{U}{W} - \delta_c = \frac{3908.3}{75} - 0.14744 = 52.0 \text{ in.}$

PROBLEM 11.145

11.145 The 75-lb collar D is released from rest when $h = 20$ in. and is stopped by a plate attached at end C of the vertical rod ABC. Knowing that $E = 29 \times 10^6$ psi for both portions of the rod, determine (*a*) the maximum deflection of end C, (*b*) the equivalent static load, (*c*) the maximum stress that occurs in the rod.



SOLUTION

Let P_m be the equivalent static load in lb.

Portion AB: $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.75)^2 = 2.40528 \text{ in}^2$

$L_{AB} = 8 \text{ ft} = 96 \text{ in}$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2EA} = \frac{P_m^2 (96)}{(2)(29\times10^6)(2.40528)} = 688.14\times10^{-9} P_m^2$$

Portion BC: $A_{BC} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$ $\quad L_{BC} = 6 \text{ ft} = 72 \text{ in}$

$$U_{BC} = \frac{P_m^2 L_{BC}}{2EA_{BC}} = \frac{P_m^2 (72)}{(2)(29\times10^6)(1.76715)} = 702.48\times10^{-9} P_m^2$$

Total: $U = U_{AB} + U_{BC} = 1.39062\times10^{-6} P_m^2$

$\frac{1}{2} P_m \delta_m = U, \quad \delta_m = \frac{2U}{P_m} = 2.78124\ 10^{-6} P_m; \quad P_m = 359.552\times10^3\ \delta_m$

$U = \frac{1}{2} P_m \delta_m = 179.776\times10^3\ \delta_m^2$

Work of falling weight $\quad W(h + \delta_m) = 75(20 + \delta_m) = 1500 + 75\,\delta_m$

Equating work and energy $\quad 1500 + 75\,\delta_m = 179.776\times10^3\ \delta_m^2$

$$\delta_m^2 - 417.185\times10^{-6}\ \delta_m - 8.3437\times10^{-3} = 0$$

(a) $\delta_m = \frac{1}{2}\left\{ 417.185\times10^{-6} + \sqrt{(417.185\times10^{-6})^2 + (4)(8.3437\times10^{-3})} \right\}$

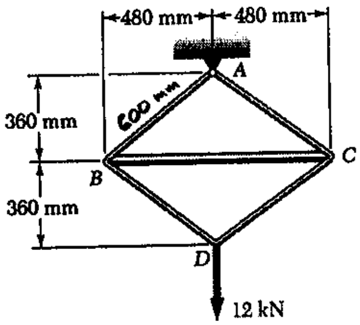
$= 0.091553$ in. $\qquad \delta_m = 0.0916$ in. ◄

(b) $P_m = (359.552\times10^3)(0.091553) = 32\,917$ lb $\qquad P_m = 32900$ lb. ◄

$$\sigma_m = \frac{P_m}{A_{min}} = \frac{32917}{1.76715} = 18\,630 \text{ psi} = 18.63 \text{ ksi}$$ ◄

PROBLEM 11.146

11.146 The steel rod *BC* has a 24-mm diameter and the steel cable *ABDCA* has a 12-mm diameter. Using $E = 200$ GPa, determine the deflection of point *D* caused by the 12-kN load.



SOLUTION

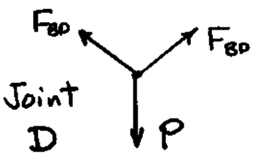
Owing to symmetry $F_{AB} = F_{BD} = F_{DC} = F_{CA}$

$U_{AB} = U_{BD} = U_{DC} = U_{CA}$

$$U = 4U_{BD} + U_{BC} = 4\frac{F_{BD}^2 L_{BD}}{2EA_{BD}} + \frac{F_{BC}^2 L_{BC}}{2EA_{BC}}$$

Let P be the load at D

$$\delta_D = \frac{\partial U}{\partial P} = 4\frac{F_{BD}L_{BD}}{EA_{BD}}\frac{\partial F_{BD}}{\partial P} + \frac{F_{BC}L_{BC}}{EA_{BC}}\frac{\partial F_{BD}}{\partial P}$$

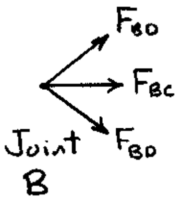


$+\uparrow \Sigma F_y = 0$

$2\,\frac{3}{5}F_{BD} - P = 0$

$F_{BD} = \frac{5}{6}P$

$\frac{\partial F_{BD}}{\partial P} = \frac{5}{6}$



$\xrightarrow{+} \Sigma F_x = 0$

$F_{BC} + (2)\frac{4}{5}F_{BD} = 0$

$F_{BC} = -\frac{8}{5}F_{BD} = -\frac{4}{3}P$

$\frac{\partial F_{BC}}{\partial P} = -\frac{4}{3}$

$$\delta_D = 4\left(\frac{5}{6}\right)^2 \frac{PL_{BD}}{EA_{BD}} + \left(\frac{4}{3}\right)^2 \frac{PL_{BC}}{EA_{BC}} = \frac{P}{E}\left\{\frac{25}{9}\frac{L_{BD}}{A_{BD}} + \frac{16}{9}\frac{L_{BC}}{A_{BC}}\right\}$$

Data: $P = 12\times10^3$ N $\qquad E = 200\times10^9$ Pa

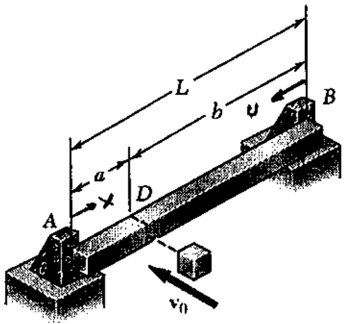
$L_{BD} = 600\times10^{-3}$ m $\qquad A_{BD} = \frac{\pi}{4}(12)^2 = 113.097 \text{ mm}^2 = 113.097\times10^{-6} \text{ m}^2$

$L_{BC} = 960\times10^{-3}$ m $\qquad A_{BC} = \frac{\pi}{4}(24)^2 = 452.39 \text{ mm}^2 = 452.39\times10^{-6} \text{ m}^2$

$$\delta_D = \frac{12\times10^3}{200\times10^9}\left\{\frac{25}{9}\cdot\frac{600\times10^{-3}}{113.097\times10^{-6}} + \frac{16}{9}\,\frac{960\times10^{-3}}{452.39\times10^{-6}}\right\} = 1.111\times10^{-3} \text{ m}$$

$$= 1.111 \text{ mm} \downarrow$$

PROBLEM 11.147



11.147 The simply supported beam AB is struck squarely at D by a block of mass m moving horizontally with a velocity $\mathbf{v}_0$. Show that the resulting maximum normal stress σ_m in the beam due to bending is independent of the location of point D

SOLUTION

Let P_m be the equivalent static load at point D

Reactions: $R_A = \dfrac{P_m b}{L}$, $R_B = \dfrac{P_m a}{L}$

Maximum bending moment $M_m = R_A a = \dfrac{P_m ab}{L}$

Portion AD $\quad U_{AD} = \displaystyle\int_0^a \frac{M^2}{2EI}dx = \int_0^a \frac{(R_A x)^2}{2EI}dx = \frac{P_m^2 b^2}{2EIL^2}\int_0^a x^2 dx = \frac{P_m^2 b^2 a^3}{6EIL^2}$

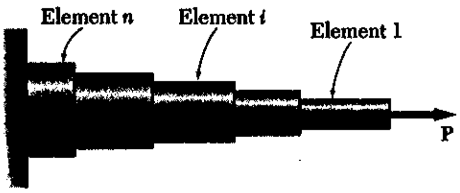
Portion DB $\quad U_{DB} = \displaystyle\int_0^b \frac{M^2}{2EI}dv = \int_0^b \frac{(R_B v)^2}{2EI}dv = \frac{P_m^2 a^2}{2EIL^2}\int_0^b v^2 dv = \frac{P_m^2 a^2 b^3}{6EIL^2}$

Total $\quad U = \dfrac{P_m^2 a^2 b^2 (a+b)}{6EIL^2} = \dfrac{P_m^2 a^2 b^2}{6EIL} = \dfrac{M_m^2 L}{6EI}$

$\frac{1}{2} m v_0^2 = U = \dfrac{M_m^2 L}{6EI} \qquad M_m = \sqrt{\dfrac{3EImv_0^2}{L}}$

Stress $\quad \sigma_m = \dfrac{M_m c}{I} = \sqrt{\dfrac{3Emv_0^2 c^2}{IL}}$, which is independent of a or b.

PROBLEM 11.C1



11.C1 A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to a load **P** applied at its free end. The length of element i is denoted by L_i and its diameter by d_i. (*a*) Denoting by E the modulus of elasticity of the material used in the rod, write a computer program that can be used to determine the strain energy acquired by the rod and the deformation measured at the free end. (*b*) Use this program to determine the strain energy and deformation of the rods of Probs. 11.9 and 11.12.

SOLUTION

ENTER: P AND E

FOR EACH ELEMENT

ENTER A_i AND D_i

COMPUTE: NORMAL STRESS: $\sigma_i = \dfrac{P}{A_i}$

STRAIN ENERGY: $U_i = \dfrac{P^2 L_i}{2 A_i E}$

STRAIN ENERGY DENSITY: $u = \dfrac{\sigma_i^2}{2E}$

TOTAL STRAIN ENERGY

UPDATE THROUGH n ELEMENTS

$U = U + U_i$

TOTAL DEFORMATION

$\frac{1}{2} P \Delta = U \quad : \quad \Delta = \dfrac{2U}{P}$

PROGRAM OUTPUT

Problem 11.9

Axial load = 8.000 kips Modulus of elasticity = 29 x 10^6 psi

Element	Length in.	delta L in.	Stress ksi	Strain Energy in·lb	Strain Energy Density lb·in./in.^3
1	24.000	0.022	26.08	86.32	11.72
2	36.000	0.022	18.11	89.92	5.65

Total Strain Energy = 176.24 in·lb
Total Deformation = 0.0441 in.

Problem 11.12

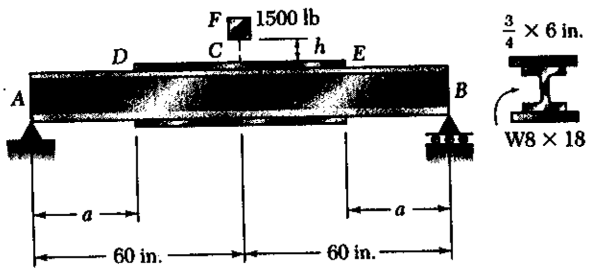
Axial load = 25.000 kN Modulus of elasticity = 200 GPa

Element	Length m	delta L mm	Stress MPa	Strain Energy J	Strain Energy Density kJ/m^3
1	0.80	0.497	124.34	6.22	38.65
2	1.20	0.477	79.58	5.97	15.83

Total Strain Energy = 12.1853 J
Total Deformation = 0.9748 mm

PROBLEM 11.C2

11.C2 Two 0.75 × 6-in. cover plates are welded to a W8 × 18 rolled-steel beam as shown. The 1500-lb block is to be dropped from a height $h = 2$ in. onto the beam. (*a*) Write a computer program to calculate the maximum normal stress on transverse sections just to the left of *D* and at the center of the beam for values of *a* from 0 to 60 in., using 5-in. increments. (*b*) From the values considered in part *a*, select the distance *a* for which the maximum normal stress is as small as possible. Use $E = 29 \times 10^6$ psi.



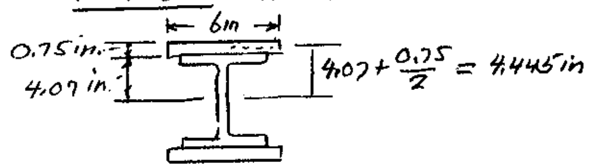
SOLUTION

COMPUTE AND ENTER MOMENTS OF INERTIA AND SECTION MODULI

FOR AD AND EB: W8×18

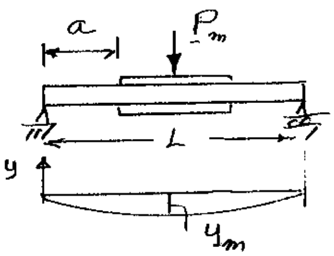
$I_1 = 61.9 \text{ in}^4 \qquad S_1 = 15.2 \text{ in}^3$ ◁

FOR DCE: W8×18 PLUS COVER PLATES



$$I_2 = 61.9 + 2(6 \times 0.75)(4.445)^2 = 239.72 \text{ in}^4$$ ◁

$$S_2 = \frac{I_2}{(4.07 + 0.75)} = \frac{239.72}{4.82} = 49.7 \text{ in}^3$$ ◁



$y_m = P_m \alpha$ WHERE α = INFLUENCE COEFFICIENT

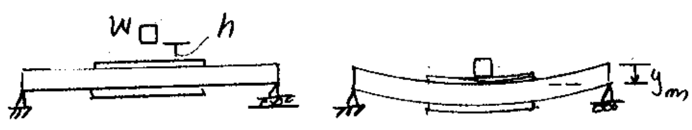
SEE NEXT PAGE FOR DETERMINATION OF α

P_m = EQUIVALENT STATIC LOAD

$$U_2 = \frac{1}{2} P_m y_m = \frac{1}{2} \frac{y_m^2}{\alpha}$$

WORK DONE BY W IS $W(h + y_m)$

$$\frac{1}{2} \frac{y_m^2}{\alpha} = Wh + W y_m$$



OR: $y_m^2 - 2W\alpha y_m - 2Wh\alpha$ (A)

PROGRAM SOLUTION OF (A) FOR y_m

ENTER $L = 120$ in., $h = 2$ in., $W = 1500$ lb, $E = 29 \times 10^6$ psi

FOR $a = 0$ TO 60 in, STEP 5 in.:

SOLVE (A) FOR y_m, $P_m = y_m / \alpha$; $y_{ST} = W\alpha$

$\sigma_D = \sigma_1 = \frac{1}{2} P_m a / S_1$; $\sigma_C = \sigma_2 = \frac{1}{4} P_m L / S_2$

PRINT: a, y_{ST}, y_m, P_m, σ_1, σ_2, AND $(\sigma_1 - \sigma_2)$

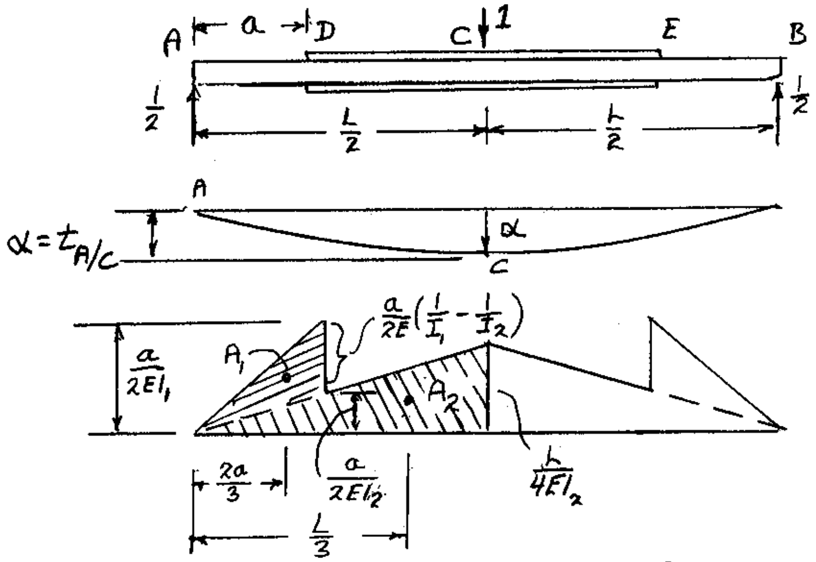
REPEAT WITH SMALLER INTERVALS TO FIND a FOR $(\sigma_1 - \sigma_2) = 0$

THIS IS THE DISTANCE a FOR σ_{max} AS SMALL AS POSSIBLE

CONTINUED

PROBLEM 11.C2 - CONTINUED

DETERMINATION OF α: α IS DEFLECTION AT C FOR A UNIT LOAD AT C.



$$\alpha = t_{A/C} = A_1\left(\frac{2a}{3}\right) + A_2\left(\frac{L}{3}\right) = \left[\frac{1}{2}\frac{a}{E}\left(\frac{1}{I_1}-\frac{1}{I_2}\right)\frac{a}{2}\right]\frac{2a}{3} + \left[\frac{1}{2}\cdot\frac{L}{4EI_2}\cdot\frac{L}{2}\right]\frac{L}{3}$$

$$\alpha = \left[\left(\frac{1}{I_1}-\frac{1}{I_2}\right)a^3 + \frac{1}{8I_2}L^3\right]\Big/6E \quad \triangleleft$$

PROGRAM OUTPUT

Beam = W 8x18 with two 6 by 0.75-in. cover plates
h = 2 in. W = 1500 lb L = 120 in.

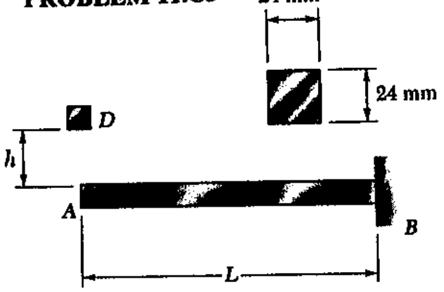
a in.	ystat in.	ymax in.	Pmax lb	σ1 ksi	σ2 ksi	σ1 - σ2 ksi
0.00	0.00777	0.1842	35572	0.00	21.46	-21.46
5.00	0.00778	0.1844	35544	5.85	21.44	-15.59
10.00	0.00787	0.1855	35348	11.63	21.32	-9.69
15.00	0.00812	0.1885	34834	17.19	21.01	-3.82
20.00	0.00859	0.1942	33896	22.30	20.45	1.85
25.00	0.00938	0.2033	32509	26.73	19.61	7.13
30.00	0.01056	0.2163	30736	30.33	18.54	11.79
35.00	0.01220	0.2334	28706	33.05	17.32	15.73
40.00	0.01438	0.2546	26563	34.95	16.02	18.93
45.00	0.01718	0.2799	24436	36.17	14.74	21.43
50.00	0.02068	0.3090	22415	36.87	13.52	23.35
55.00	0.02496	0.3419	20550	37.18	12.40	24.78
60.00	0.03008	0.3783	18862	37.23	11.38	25.85

Use smaller increments to seek the smallest maximum normal stress

18.33	0.00840	0.1919	34259	20.657	20.665	-0.01
18.34	0.00840	0.1920	34257	20.667	20.664	0.00
18.35	0.00841	0.1920	34255	20.677	20.663	0.01

Max stress small as possible for a = 18.34in.
Smallest max stress = 20.67 ksi

PROBLEM 11.C3



11.C3 The 16-kg block D is dropped from a height h onto the free end of the steel bar AB. For the steel used $\sigma_{all} = 120$ MPa and $E = 200$ GPa. (*a*) Write a computer program to calculate the maximum allowable height h for values of the length L from 100 mm to 1.2 m, using 100-mm increments. (*b*) From the values considered in part *a*, select the length corresponding to the largest allowable height.

SOLUTION

ENTER $\sigma_{all} = 120\,MPa$, $E = 200\,GPa$, $d = 0.024\,m$

$m = 16\,kg$, $g = 9.81\,m/s^2$

$I = d^4/12$ $\quad S = \frac{I}{c} = \frac{I}{d/2} = d^3/6$

FOR $L = 100\,m$ TO $1200\,m$ STEP $100\,mm$

$L = L/1000$

$y_{st} = mg\,L^3/3EI$

$M_{max} = \sigma_{all}\,S$

$P_{max} = M_{max}/L$

$y_{max} = P_{max}\,L^3/3EI$

FROM PROB. 11.69, page 705

$y_m = y_{st}\left[1+\sqrt{1+\frac{2h}{y_{st}}}\right]$ SOLVE FOR $\longrightarrow$ $h = \left[\left(\frac{y_{max}}{y_{st}} - 1\right)^2 - 1\right]\frac{y_{st}}{2}$

PRINT: $L, y_{st}, y_{max}, P_{max}, M_{max}, h$

RETURN

PROGRAM OUTPUT

Problem 11.C3

m = 16.0 kg d = 24 mm σ = 120 MPa G = 200 GPa

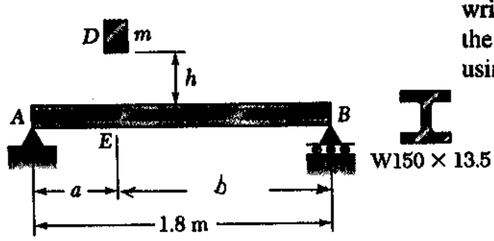
L mm	ystat mm	ymax mm	Pmax N	Mmax N·m	h mm
100	0.00946	0.167	2764.8	276.48	1.301
200	0.07569	0.667	1382.4	276.48	2.269
300	0.25547	1.500	921.6	276.48	2.904
400	0.60556	2.667	691.2	276.48	3.205
500	1.18273	4.167	553.0	276.48	3.173
600	2.04375	6.000	460.8	276.48	2.807
700	3.24540	8.167	395.0	276.48	2.109
800	4.84445	10.667	345.6	276.48	1.076
900	6.89766	13.500	307.2	276.48	-0.289
1000	9.46181	16.667	276.5	276.48	-1.988
1100	12.59367	20.167	251.3	276.48	-4.020
1200	16.35000	24.000	230.4	276.48	-6.385

Use smaller increments to seek the largest height h

435	0.77883	3.154	635.6	276.48	3.2316
440	0.80599	3.227	628.4	276.48	3.2320
445	0.83378	3.300	621.3	276.48	3.2317

PROBLEM 11.C4

11.C4 The block D of mass $m = 8$ kg is dropped from a height $h = 750$ mm onto the rolled-steel beam AB. Knowing that $E = 200$ GPa, write a computer program to calculate the maximum deflection of point E and the maximum normal stress in the beam for values of a from 100 to 900 mm, using 100-mm increments.



SOLUTION

ENTER: $L = 1.8\,m$, $E = 200\,GPa$, $h = 0.75\,m$
$m = 8\,kg$, $g = 9.81\,m/s^2$
$I = 6.87 \times 10^{-6}\,m^4$
$S = 91.6 \times 10^{-6}\,m^4$

FOR a = 100 mm TO 900 mm STEP 100 mm
$a = a/1000$
$b = L - a$

SEE PROB. 11.71, page 705 → $y_{st} = mga^2b^2/3EIL$

INFLUENCE COEFFICIENT FOR Δ_E FOR UNIT LOAD AT E → $\alpha = a^2b^2/3EIL$

SEE PROB. 11.69, page 705 → $y_m = y_{st}\left[1 + \sqrt{1 + \dfrac{2h}{y_{st}}}\right]$

$P_{max} = y_m/\alpha$

$M_{max} = P_{max}\, a\, b/L$

$\sigma_{max} = M_{max}/S$

PRINT: a, y_{st}, y_m, P_{max}, σ_{max}

RETURN

Problem 11.C4

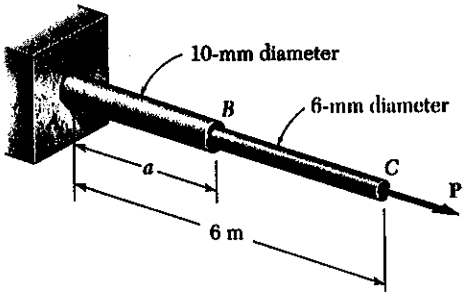
Beam: W 150 x 13.5
I = 6.87 x10^-6 m^4   S = 91.6x10^-6 m^3

L = 1.8 m   h = 750 mm   m = 8 kg   g = 9.81 m/s^2

a mm	ystat mm	ymax mm	Pmax N	σ_{max} MPa
100	0.0003	0.6775	173.93	179.33
200	0.0011	1.2757	92.43	179.40
300	0.0021	1.7946	65.75	179.46
400	0.0033	2.2339	52.85	179.51
500	0.0045	2.5936	45.55	179.55
600	0.0055	2.8734	41.13	179.59
700	0.0063	3.0734	38.46	179.61
800	0.0068	3.1934	37.02	179.63
900	0.0069	3.2334	36.56	179.63

NOTE: THE SMALL VARIATION IN σ_{max}... THIS IS DUE TO THE ENERGY ACQUIRED BY THE MASS AS IT FALLS THROUGH y_{max}.
SEE PROB. 11.147, page 731, FOR A CASE WHERE ENERGY DELIVERED IS CONSTANT AND σ_{max} IS ALSO CONSTANT.

PROBLEM 11.C5



11.C5 The steel rods *AB* and *BC* are made of a steel for which $\sigma_Y = 300$ MPa and $E = 200$ GPa. (*a*) Write a computer program to calculate, for values of *a* from 0 to 6 m, using 1-m increments, the maximum strain energy that can be acquired by the assembly without causing any permanent deformation. (*b*) For each value of *a* considered, calculate the diameter of a uniform rod of length 6 m and of the same mass as the original assembly, and the maximum strain energy that could be acquired by this uniform rod without causing permanent deformation.

SOLUTION

ENTER: $\sigma_Y = 300\,MPa$, $E = 200\,GPa$, $L = 6\,m$

$AREA_{AB} = \frac{\pi}{4}(0.010\,m)^2$, $AREA_{BC} = \frac{\pi}{4}(0.006\,m)^2$

$P_m = \sigma_Y\,AREA_{BC}$

FOR $a = 0$ TO $6\,m$ STEP $1\,m$

$$U = \frac{P_m^2}{2E}\left(\frac{a}{AREA_{AB}} + \frac{L-a}{AREA_{BC}}\right)$$

FOR UNIFORM ROD OF SAME VOLUME

$$VOL = a\,(AREA_{AB}) + (L-a)(AREA_{BC})$$

$$d = \sqrt{\frac{4\,VOL}{\pi\,L}}$$

$$AREA_{NEW} = \frac{\pi}{4}d^2$$

$$P_{NEW} = \sigma_Y\,(AREA_{NEW})$$

$$U_{NEW} = \frac{P_{NEW}^2\,L}{2E(AREA_{NEW})}$$

PRINT a, U, VOL, d, P_{NEW}, U_{NEW}

RETURN

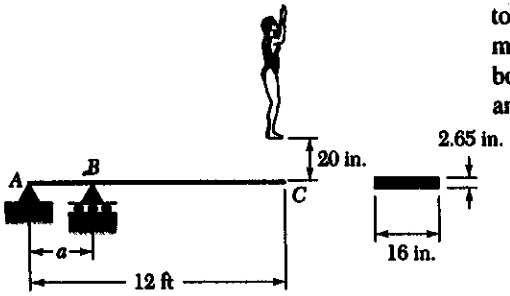
PROGRAM OUTPUT

Problem 11C5

sigmaY = 300 MPa, Pm = 8482 N, L = 6 m, E = 200 GPA

a m	U J	Vol m^3	d mm	New P N	newU J
0.00	38.17	169.65	6.00	8482.30	38.17
1.00	34.10	219.91	6.83	10995.58	49.48
2.00	30.03	270.18	7.57	13508.85	60.79
3.00	25.96	320.44	8.25	16022.12	72.10
4.00	21.88	370.71	8.87	18535.40	83.41
5.00	17.81	420.97	9.45	21048.67	94.72
6.00	13.74	471.24	10.00	23561.95	106.03

PROBLEM 11.C6



11.C6 A 160-lb diver jumps from a height of 20 in. onto end C of a diving board having the uniform cross section shown. Write a computer program to calculate for values of a from 10 to 50 in., using 10-in. increments, (*a*) the maximum deflection of point C, (*b*) the maximum bending moment in the board, (*c*) the equivalent static load. Assume that the diver's legs remain rigid and use $E = 1.8 \times 10^6$ psi.

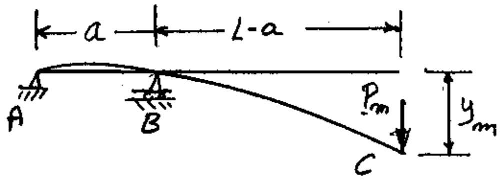
SOLUTION

ENTER: $L = 12\text{ ft}$, $h = 20\text{ in.}$, $W = 160\text{ lb}$

$E = 1.8 \times 10^6\text{ psi}$

$I = (16\text{ in.})(2.65\text{ in})^3/12$

$S = (16\text{ in.})(2.65\text{ in})^2/6$

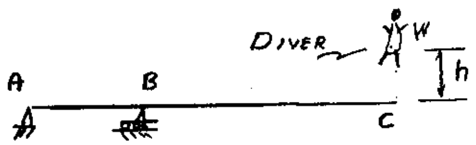


$y_m = P_m\,\alpha$ WHERE α = INFLUENCE COEFFICIENT

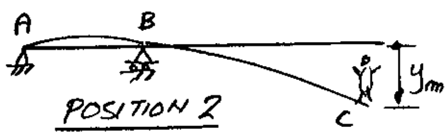
SEE BELOW FOR DETERMINATION OF α

WHERE P_m = EQUIVALENT STATIC LOAD

$$U_2 = \frac{1}{2}P_m y_m = \frac{1}{2}\frac{y_m^2}{\alpha}$$



POSITION 1



POSITION 2

$$\text{WORK} = W(h + y_m)$$

$$\text{WORK} = U_2$$

$$W(h + y_m) = \frac{1}{2}\frac{y_m^2}{\alpha} \qquad A$$

PROGRAM SOLUTION OF A FOR y_m. ENTER α

FOR $a = 10\text{ in.}$ TO 50 in. STEP 10 in.

SOLVE A FOR y_m, $P_m = y_m/\alpha$

$M_{max} = M_B = P_m(L - a)$

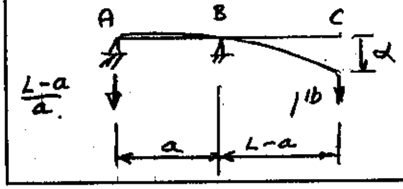
$\sigma = M_{max}/S$

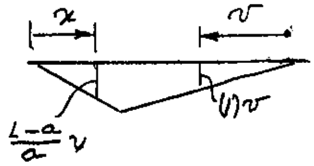
PRINT a, y_m, P_m, M_m, σ

PROGRAM OUTPUT

a in.	ym in.	Pm lb	Max M kip·in.	sigma psi
10	14.622	757.7	101.532	5422
20	13.262	802.6	99.519	5314
30	11.950	855.6	97.536	5208
40	10.683	919.1	95.583	5104
50	9.462	996.4	93.661	5001

DETERMINATION OF INFLUENCE COEFFICIENT α





M-DIAGRAM

$$U = \frac{1}{2}(1^{lb})\alpha = \Sigma\int \frac{M^2}{2EI}\,dx$$

$$\frac{\alpha}{2} = \frac{1}{2EI}\left[\int_0^a \left(\frac{L-a}{a}\right)^2 x^2\,dx + \int_0^{L-a} v^2\,dv\right]$$

$$\alpha = \frac{1}{EI}\left[\frac{(L-a)^2}{a^2}\frac{a^3}{3} + \frac{(L-a)^3}{3}\right]$$

$$\alpha = \frac{1}{3EI}\left[(L-a)^2 a + (L-a)^3\right] \quad \triangleleft$$