EQUILIBRIUM OF PARTICLES AND BODIES

Static Equilibrium of Rigid Bodies

- For rigid bodies, the sum of moments is considered in addition to the sum of forces.

$$\vec{R} = \sum F = 0$$
$$\Rightarrow \sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$
$$\vec{M} = \sum M = 0$$
$$\Rightarrow \sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

- Reaction at supports of rigid bodies must not be ignored in equilibrium analysis.
- A free body diagram is indispensable in solving problems on equilibrium of rigid bodies.
- All support reactions must be accounted on the free body diagram in order for the problem to be solved correctly.

EQUILIBRIUM OF PARTICLES AND BODIES

Statically Determinate and Statistically Indeterminate Reactions

- Statically Determinate Problems
  - The number of unknowns in the FBD equals the number of equations obtained.

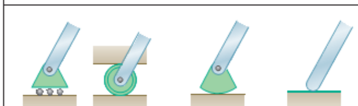
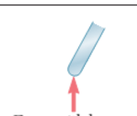
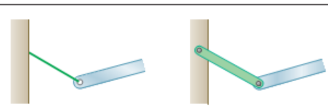
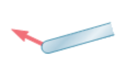
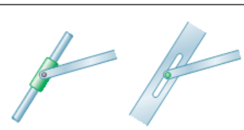
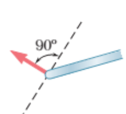
- Statistically Indeterminate Problems
  - The number of unknowns in the FBD exceeds the number of equations obtained. Solving of such problems is beyond the scope of this course.

EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

- Drawing a free body diagram
  - Select the extent of the free-body and detach it from the ground and all other bodies.
  - Indicate point of application, magnitude, and direction of external forces, including the rigid body weight.
  - Indicate point of application and assumed direction of unknown applied forces. These usually consist of reactions through which the ground and other bodies oppose the possible motion of the rigid body.
  - Include the dimensions necessary to compute the moments of the forces.

EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

- Reactions at Supports and Connections for Two-Dimensional Structures

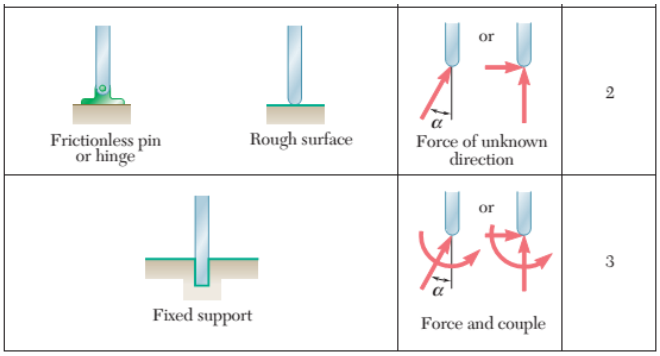
Support or Connection	Reaction	Number of Unknowns
 Rollers, Rocker, Frictionless surface	 Force with known line of action	1
 Short cable, Short link	 Force with known line of action	1
 Collar on frictionless rod, Frictionless pin in slot	 Force with known line of action	1

- Reactions equivalent to a force with known line of action.

Source:
Vector Mechanics for Engineers, Beer *et al.*

EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

➢Reactions at Supports and Connections for Two-Dimensional Structures



- Reactions equivalent to a force of unknown direction and magnitude.

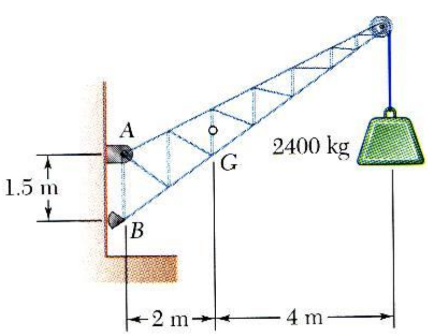
- Reactions equivalent to a force of unknown direction and magnitude and a couple.of unknown magnitude

Source:
Vector Mechanics for Engineers, Beer *et al.*

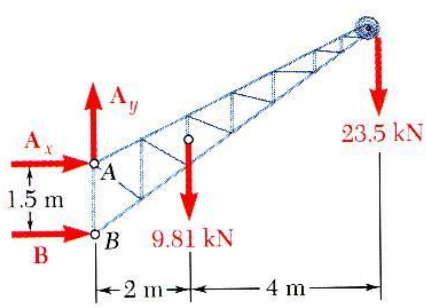
EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

➢Drawing a free body diagram

Problem diagram



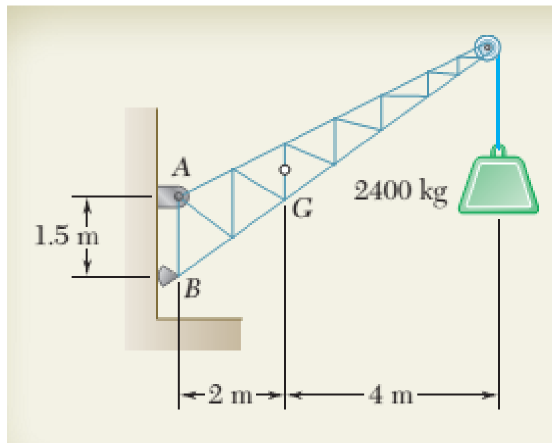
Free-body diagram



EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

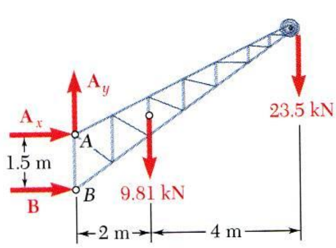
➢Example 4.4

A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G. Determine the components of the reactions at A and B.



EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

➢Solution



At A,

$$\overset{+}{\rightarrow}\sum F_x = 0: \quad A_x + B = 0$$

$$+\uparrow\sum F_y = 0: \quad A_y - 9.81\,\text{kN} - 23.5\,\text{kN} = 0$$

$$A_y = +33.3\,\text{kN}$$

Taking moments about A,

$$\circlearrowleft + \sum M_A = 0: \quad +B(1.5\text{m}) - 9.81\,\text{kN}(2\text{m}) - 23.5\,\text{kN}(6\text{m}) = 0$$

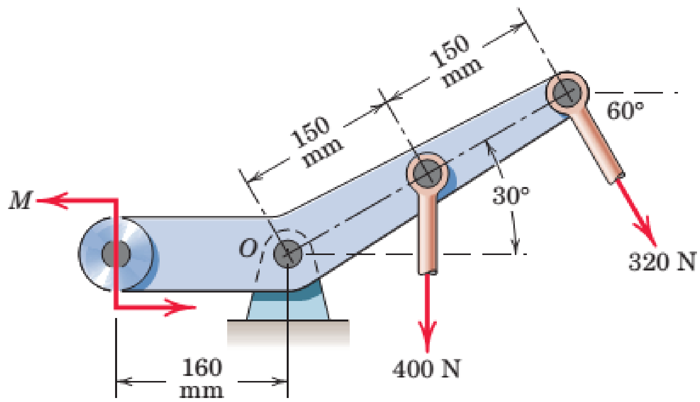
$$B = +107.1\,\text{kN}$$

$$\therefore A_x = -107.1\text{kN}$$

EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

Example

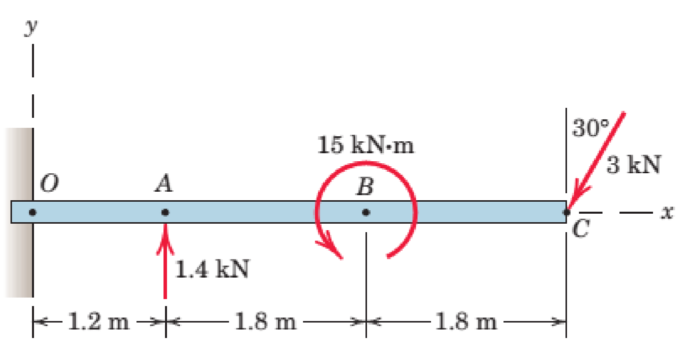
Determine *M* if the link is in equilibrium.



EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

Example

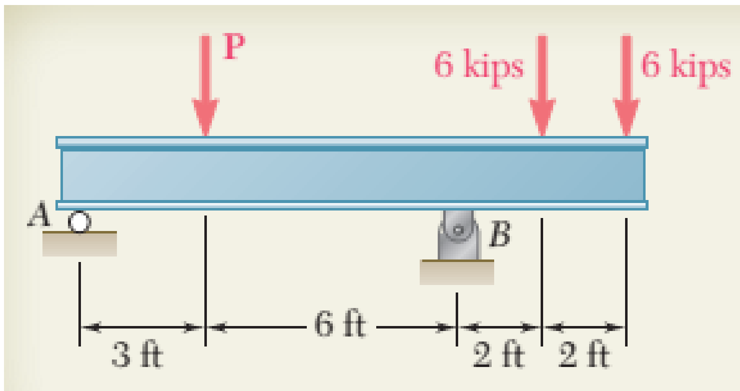
The 500 kg uniform beam is subjected to the three external loads shown. Compute the reactions at the support point *O*.



EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

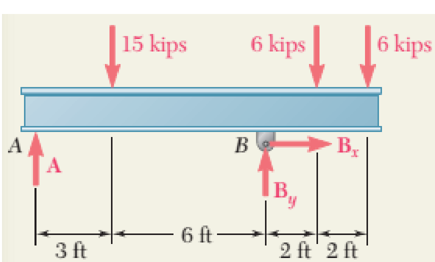
➢Example

Three loads are applied to a beam as shown. The beam is supported by a roller at A and by a pin at B. Neglecting the weight of the beam, determine the reactions at A and B when P is 15 kips.



EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

➢Solution



$$\overset{+}{\rightarrow}\sum F_x = 0: \quad B_x = 0$$

$$+\uparrow\sum F_y = 0: \quad A - 15\text{ kips} + B_y - 6\text{ kips} - 6\text{ kips} = 0$$

$$A + B_y = 27\text{ kips}$$

Taking moments about A (clockwise to be + ve),

$$\circlearrowright+\sum M_A = 0: \quad (15\text{ kips})(3\text{ ft}) + (6\text{ kips})(11\text{ ft}) - (B_y)(9\text{ ft}) + (6\text{ kips})(13\text{ ft})$$

$$B_y = -21.0\text{ kips} = 21.0\text{ kips (acting upwards)}$$

$$\circlearrowright+\sum M_B = 0: \quad (A)(9\text{ ft}) + (-15\text{ kips})(6\text{ ft}) + (6)(2\text{ ft}) + (6\text{ kips})(4\text{ ft})$$

$$\text{A} = -6\text{ kips} = (6\text{ kips upwards})$$

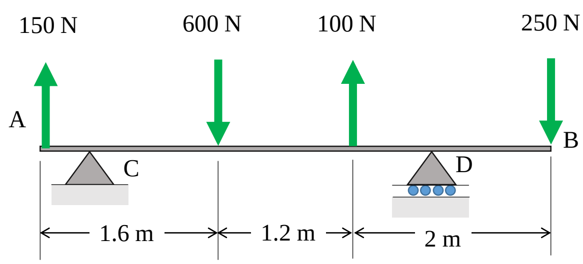
OR

$$A = 27 - B_y = 27\text{ kips} - 21.0\text{ kips} = 6\text{ kips.}$$

EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

Example

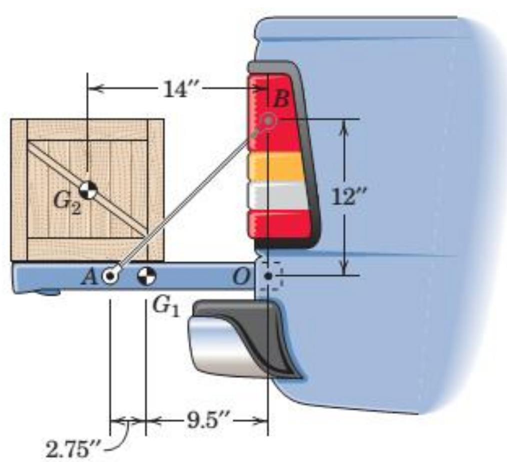
For the beam determine the support reactions at C and D.



EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

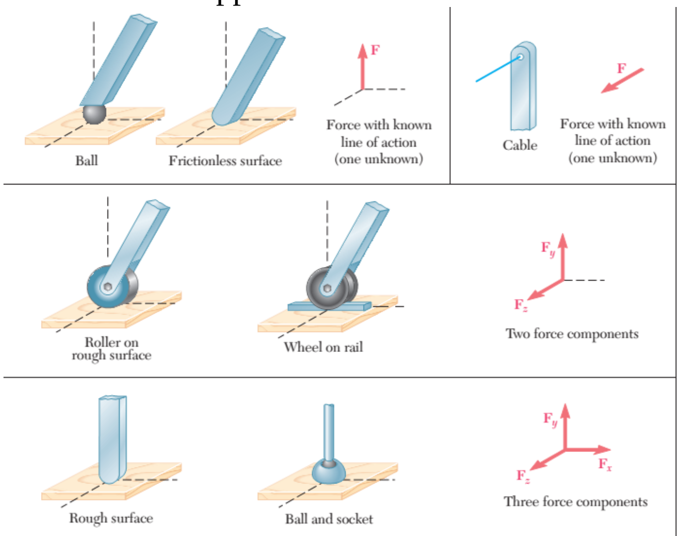
Example

A 150 kg crate rests on the 100 kg pickup tailgate as shown. Calculate the tension T in each of the two restraining cables, one of which is shown. The centres of gravity are at G_1 and G_2. the crate is located midway between the two cables.



EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

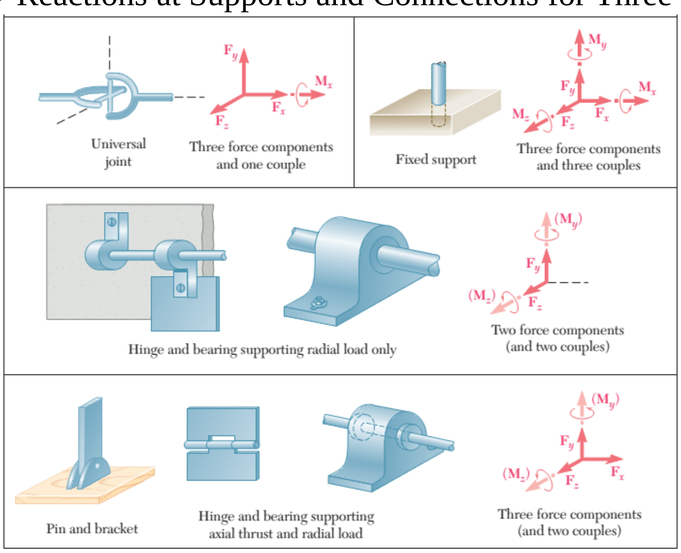
➢Reactions at Supports and Connections for Three-Dimensional Structures



Source:
Vector Mechanics for Engineers, Beer *et al.*

EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

➢Reactions at Supports and Connections for Three-Dimensional Structures



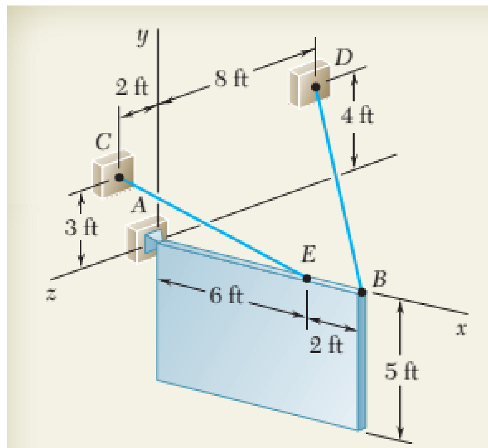
Source:
Vector Mechanics for Engineers, Beer *et al.*

EQUILIBRIUM OF PARTICLES AND BODIES

Static Equilibrium of Rigid Bodies

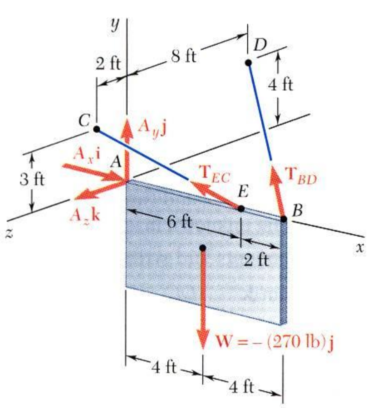
➢Example 4.6

A 5 38-ft sign of uniform density weighs 270 lb and is supported by a ball-and-socket joint at A and by two cables. Determine the tension in each cable and the reaction at A.



EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

➢Example 4.6



$$\vec{T}_{BD} = -\frac{2}{3}T_{BD}i + \frac{1}{3}T_{BD}j - \frac{2}{3}T_{BD}k$$

$$\vec{T}_{EC} = -\frac{6}{7}T_{EC}i + \frac{3}{7}T_{EC}j - \frac{2}{7}T_{EC}k$$

$$W = -270\ \text{lb}j$$

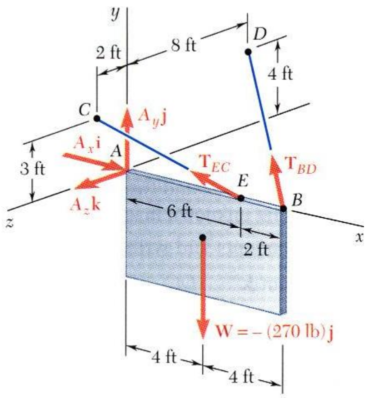
$$\sum F_x = \sum F_i = 0: \quad A_x - \frac{2}{3}T_{BD} - \frac{6}{7}T_{EC} = 0 \qquad \text{---}(1)$$

$$\sum F_y = \sum F_j = 0: \quad A_y + \frac{1}{3}T_{BD} + \frac{3}{7}T_{EC} - 270\ \text{lb} = 0 \qquad \text{---}(2)$$

$$\sum F_z = \sum F_k = 0 : A_z - \frac{2}{3}T_{BD} - \frac{2}{7}T_{EC} = 0 \qquad \text{---}(3)$$

EQUILIBRIUM OF PARTICLES AND BODIES
Static Equilibrium of Rigid Bodies

➢Example 4.6



Taking moments about A,

$$\sum M_B = 0 : (\vec{r}_B)(\vec{T}_{BD}) + \vec{r}_E(\vec{T}_{EC}) + \vec{r}_W(W)$$

$$= (8\text{ ft})i\left(-\frac{2}{3}T_{BD}i + \frac{1}{3}T_{BD}j - \frac{2}{3}T_{BD}k\right) + (6\text{ ft})i\left(-\frac{6}{7}T_{EC}i + \frac{3}{7}T_{EC}j - \frac{2}{7}T_{EC}k\right) + (4\text{ ft})i(-270\text{ lb}j)$$

$$(2.667T_{BD} + 2.571T_{EC} - 1080\text{ lb})k + (5.333T_{BD} - 1.714T_{EC})j = 0$$

$$\therefore\ 2.667T_{BD} + 2.571T_{EC} - 1080\text{ lb} = 0 \qquad ---(4)$$

$$5.333T_{BD} - 1.714T_{EC} = 0 \qquad ---(5)$$

Solvingb (1) to (5) simultaneously,

$T_{BD} = 101.3\text{ lb}$ $\quad T_{EC} = 315\text{ lb}$

$A_x = 338\text{ lb } i$ $\quad A_y = 101.2\text{ lb}j$ $\quad A_z = -22.5\text{ lb } k$