Introductory
NUCLEAR PHYSICS

PHY 170

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introduction

- Elements constituent
- Isotopes
- Atoms found in nature are either stable or unstable
- Unstable atoms are radioactive and called radionuclides
- Large atomic nuclei, with more than 83 protons and their associated complement of neutrons, are inherently unstable. Uranium and plutonium are examples of such elements.
- Constantly vibrate to attain stability through:
- converting one to the other with the ejection of a beta particle or positron
- the release of additional energy by photon (i.e., gamma ray) emission.

Nuclear Decay

When the unstable nuclear decays, decay products like γ-rays (high energy photons), α-particles (helium nuclei), β– particles (electrons) and β+ particles (positrons) are produced.

Laws Governing Nuclear Decay Reactions

In nuclear decay reactions the following laws are obeyed:

1. Conservation of mass-energy.
2. Conservation of charge.
3. Conservation of linear and angular momenta.
4. Conservation of nucleons.

Law Radioactive Decay

In a typical radioactive decay an initial nucleus (a parent) decays by emitting a particle forming a new nucleus (a daughter). Generally,

$$N(t) = N_0 \mathrm{e}^{-\lambda t} \qquad (1)$$

where N_0 is the unstable parent nucleus,

$N(t)$ is the daughter nucleus,

λ is the decay or disintegration constant and t is time.

Half-Life $(T_{1/2})$

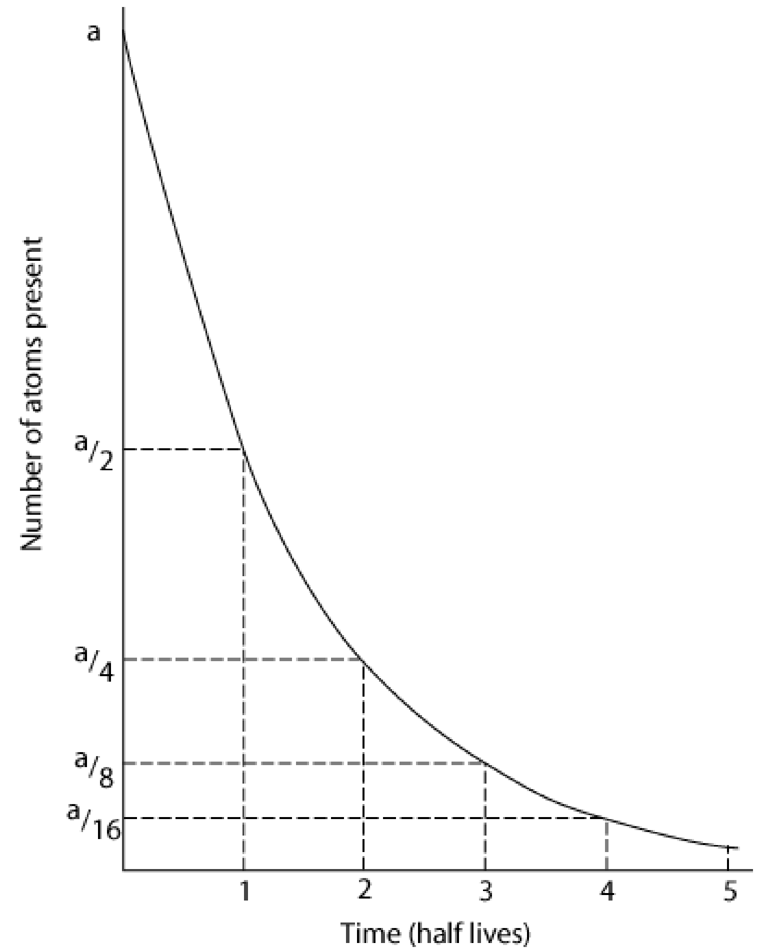
This is defined as the time interval required for the number of parent nuclei present at the beginning to be reduced by a factor of one-half. Thus, using

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \qquad (2)$$

Average of Mean Life time (Tm)

This is defined as

$$T_m = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2}$$



Average of Mean Life time Tm

This is defined as

$$T_m = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} \qquad (3)$$

Derivation

$$T_m = \frac{\int_{N_0}^{0} t\,dN}{\int_{N_0}^{0} dN} = \frac{1}{-N_0}\int_{N_0}^{0} t\,dN \qquad (4)$$

Now taking the first time differential of equation (1) gives

$$dN = -\lambda N_0 e^{-\lambda t} dt \qquad (5)$$

Using (5) in (4) and changing the limits N0,0 to ,0 in ∞ terms of the time variable t gives

$$T_m = \frac{1}{-N_0} \int_0^\infty t\left(-\lambda N_0 e^{-\lambda t} dt\right) = \lambda \int_0^\infty t e^{-\lambda t} dt = \lambda\left(\frac{1}{\lambda^2}\right) = \frac{1}{\lambda}$$

Activity

The activity (i.e. the absolute value of the rate of disintegration) of a nucleus is defined as

$$Activity = \left|\frac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t} = \lambda N$$

Activity is measured in the unit of curie (Ci). 1 Ci = 3.7×10^{10} Bequerel (Bq) or disintegrations per second. 1 Bq = 1 disintegration per second.

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Question

What is the activity of 1g of ${}^{226}_{88}Ra$ whose half life is 1622 year?

The number of atoms of 1 g of radium is given by

$$N = (1g)\left(\frac{1g - mole}{226g}\right)\left(6.025\times10^{23}\ \frac{atoms}{g - mole}\right) = 2.666\times10^{21}$$

The decay constant is related to the half-life by

$$\lambda = \left(\frac{0.693}{T_{1/2}}\right) = \left(\frac{0.693}{1622y}\right)\left(\frac{1y}{365d}\right)\left(\frac{1d}{8.64\times10^{4}s}\right) = 1.355\times10^{-11}s^{-1}$$

The activity is then found from:

Activity = λN=$(1.355 \times 10^{-11}\ s^{-1})(2.666 \times 10^{21}) = 3.612 \times 10^{10}$ disintegrations/s

Gamma Decay

This occurs when a nucleus in an excited energy state makes a transition to a lower energy state and accordingly emits a -ray in the process. If the nucleus makes a transition from a higher energy state Eu to a lower energy state El then

$$E_u - E_l = hv$$

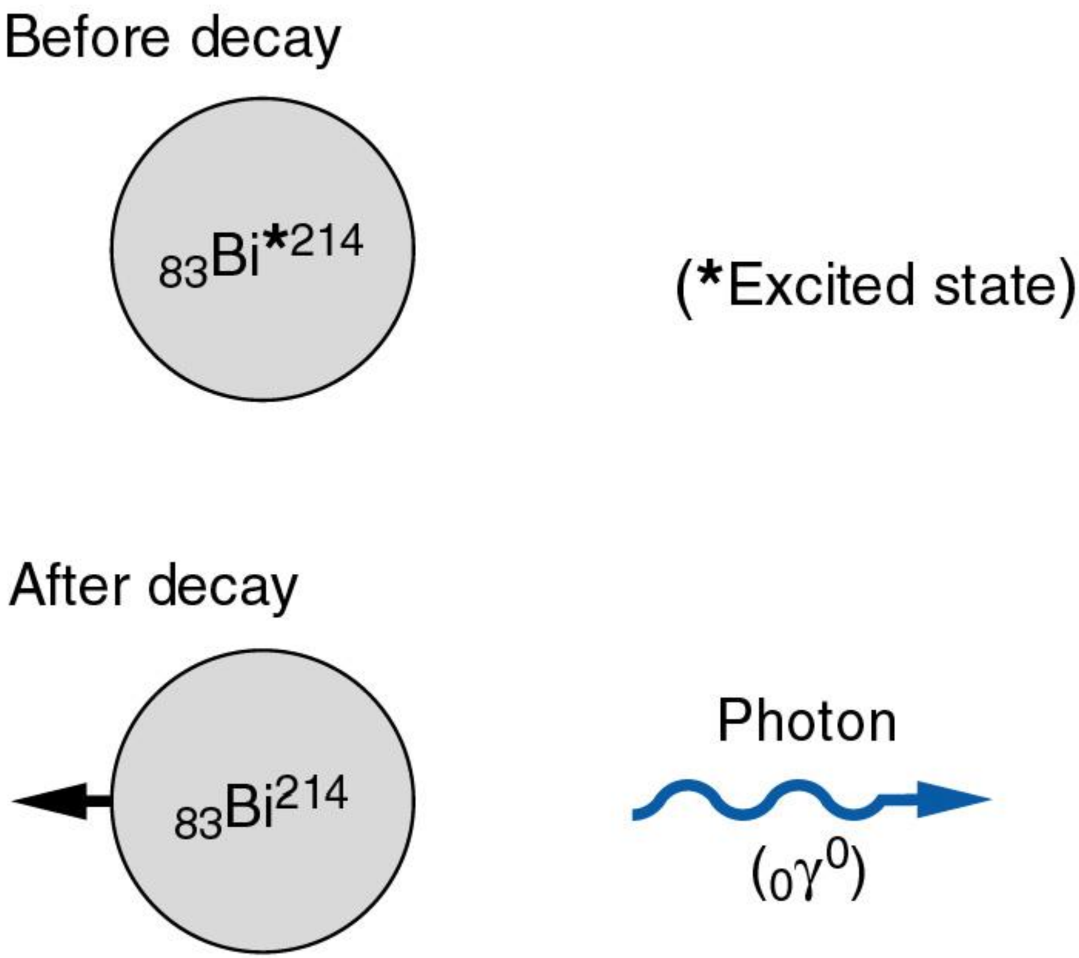
Excited nuclei are called *isomers* and the excited states are referred to as *isomeric states*.

Example

$${}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X - \gamma$$

<u>Gamma decay</u> of bismuth-214.

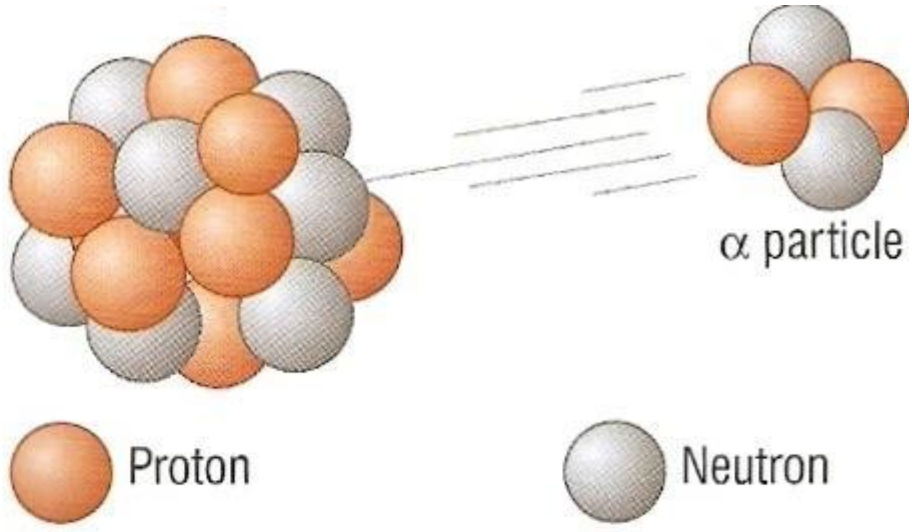
The daughter isotope is a more stable (lower-energy) version of the original bismuth-214.



Alpha Decay

$${}^{4}_{2}\mathbf{He}$$

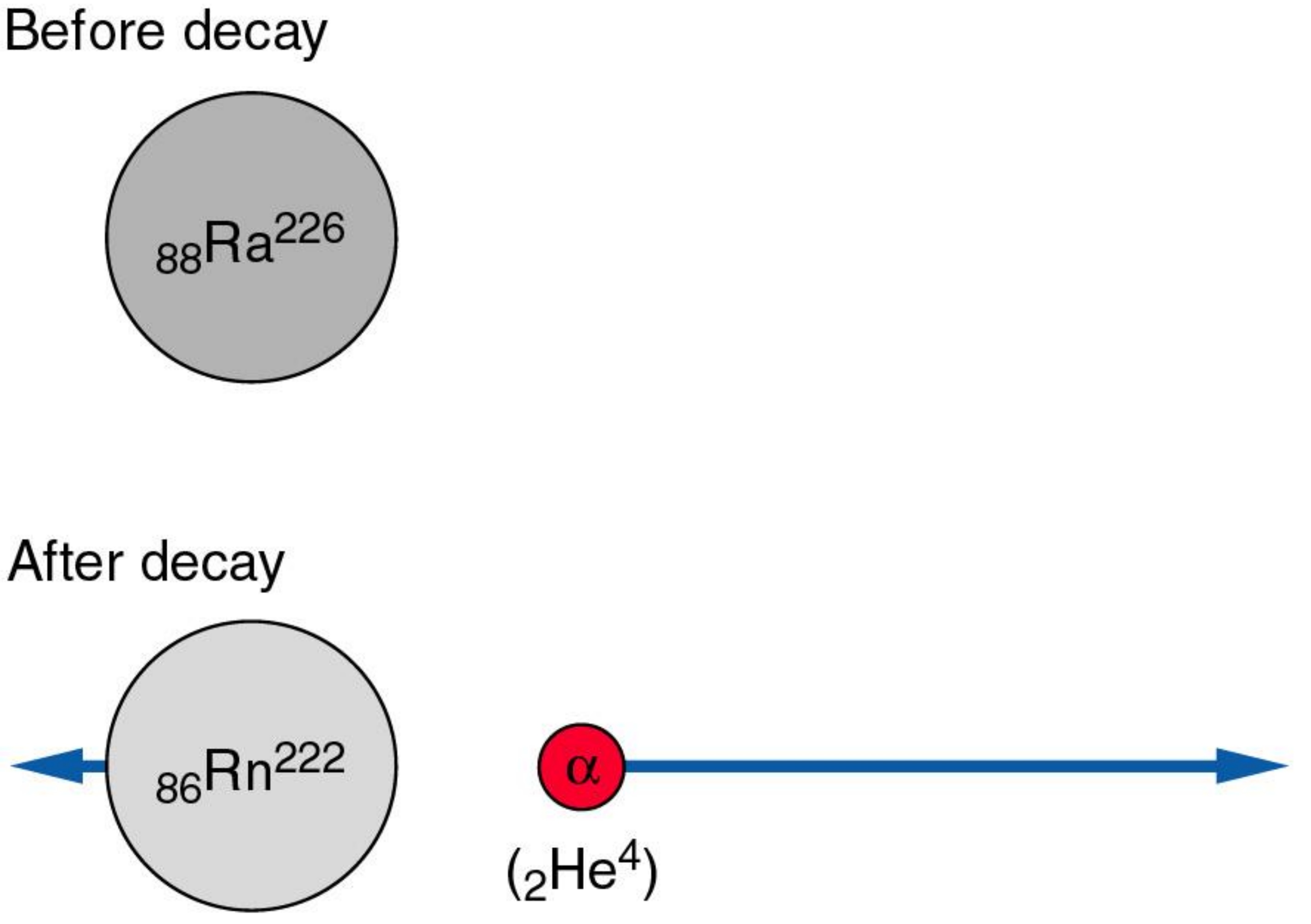
The α-particle is a helium nucleus.
Alpha particles consist of two protons plus two neutrons.
They are emitted by some of the isotopes of the heaviest elements.



$${}^{A}_{Z}Y \rightarrow {}^{A-4}_{Z-1}X + {}^{4}_{2}He$$

Alpha decay of Radium-226.

The daughter isotope is Radon-222.



Example: The decay of Uranium 238

$$ {}^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\alpha $$

Uranium 238 decays to **Thorium 234** plus an **alpha particle**.

Notes:

1. The mass and atomic numbers must balance on each side of the equation: (238 = 234 + 4 AND 92 = 90 +2)

Question

*Show the equation for Plutonium 239 (**Pu**) decaying by alpha emission to Uranium (atomic number 92).*

Ans:

$$^{239}_{94}\mathbf{Pu} \longrightarrow \, ^{235}_{92}\mathbf{U} + \, ^{4}_{2}\boldsymbol{\alpha}$$

If the parent nucleus is initially at rest then, energy conservation implies that

$$M_p c^2 = M_D c^2 + M_\alpha c^2 + K_D + K_\alpha$$

where KD, K_ are the kinetic energies of the daughter nuclei and MP, MD and M_ are the masses of the parent, daughter nuclei and alpha particle respectively. Since the kinetic energy can never be negative, alpha decay occurs if and only if

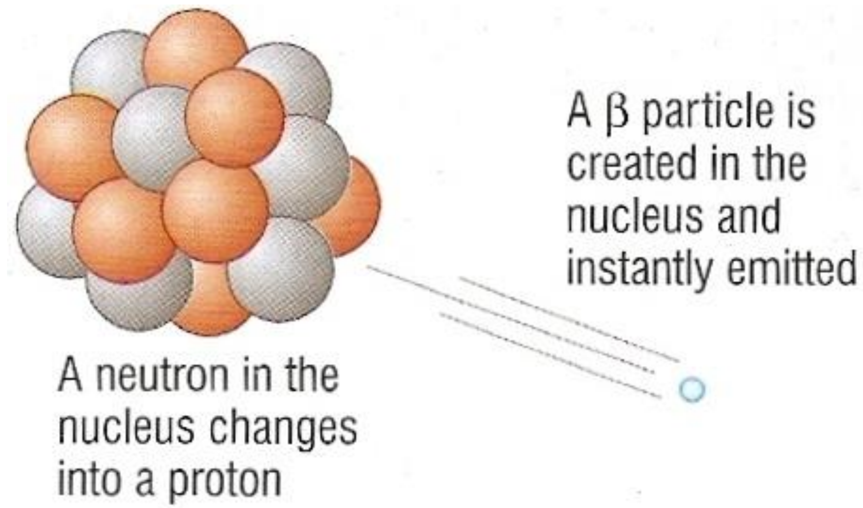
$$M_p \geq M_D + M_\alpha$$

Beta Decay

Beta particles consist of high speed electrons.

They are emitted by isotopes that have too many neutrons.

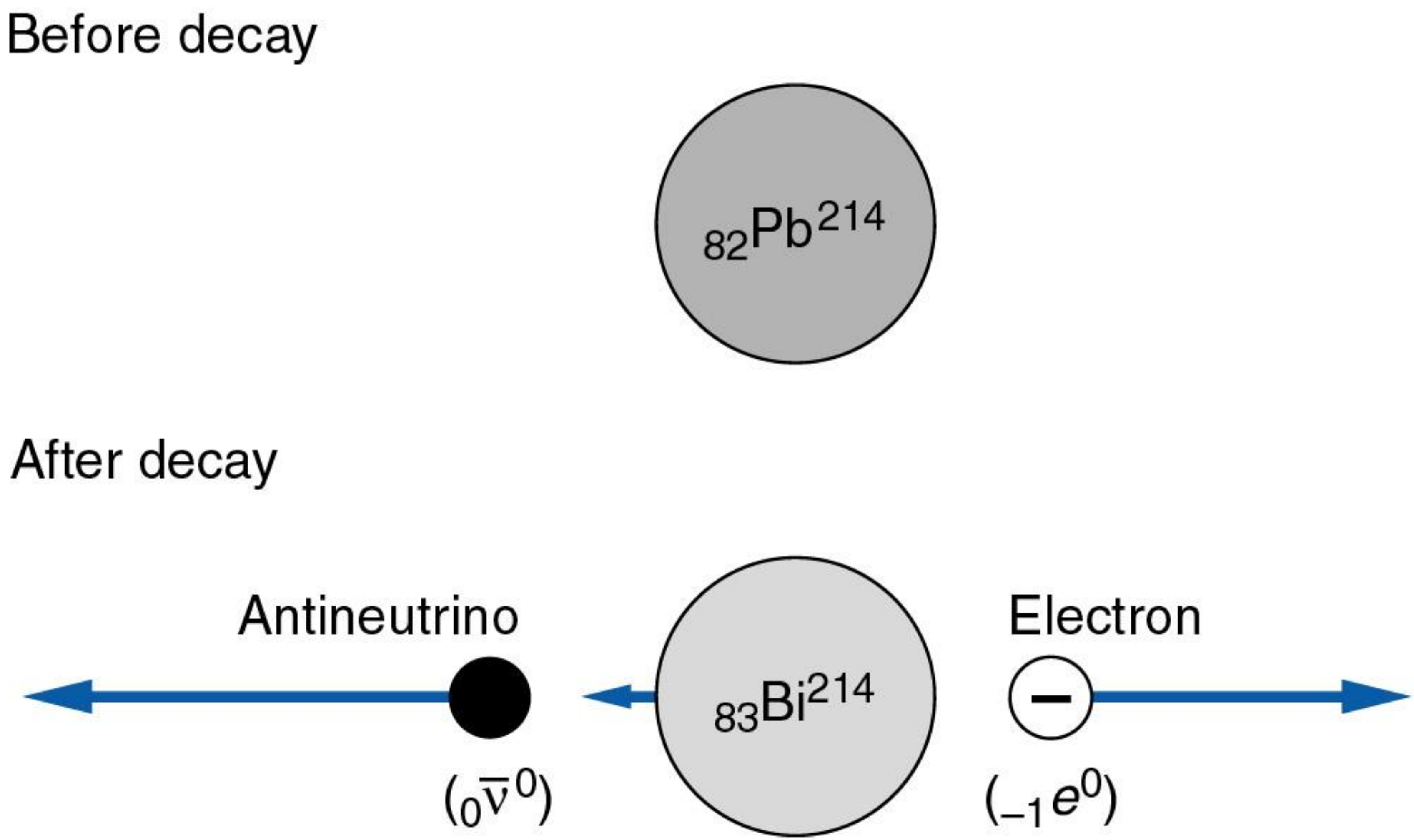
One of these neutrons decays into a proton and an electron. The proton remains in the nucleus but the electron is emitted as the beta particle.



$$n \rightarrow p + e^{-} + \bar{v}$$

<u>Beta decay</u> of Lead-214.

The daughter isotope, Bismuth-214, has a higher atomic number than Lead.



- Beta Decay
  - ${}^{A}Z \rightarrow {}^{A}(Z+1) + e^{-}$ + an anti-neutrino
    - A neutron has converted into a proton, electron and an anti-neutrino.

- Positron Decay
  - ${}^{A}Z \rightarrow {}^{A}(Z-1) + e^{+}$ + a neutrino
    - A proton has converted into a neutron, positron and a neutrino.

- Electron Capture
  - ${}^{A}Z + e^{-} \rightarrow {}^{A}(Z-1)$ + a neutrino
    - A proton and an electron have converted into a neutron and a neutrino.

Example: The decay of Carbon 14

$$^{14}_{6}\mathrm{C} \longrightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}\beta^{-}$$

Carbon 14 decays to **Nitrogen 14** plus a **beta particle**.

Notes:

1. The beta particle, being negatively charged, has an effective atomic number of minus one.

2. The beta particle can also be notated as:

$$^{0}_{-1}\mathrm{e}$$

Question

*Show the equation for Sodium 25 (**Na**), atomic number 11, decaying by beta emission to Magnesium (**Mg**).*

$$^{25}_{11}\mathbf{Na} \longrightarrow {}^{25}_{12}\mathbf{Mg} + {}^{0}_{-1}\boldsymbol{\beta}^{-}$$

Several Decay Processes:

α decay:

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

$$e.g., {}^{210}_{84}Po \rightarrow {}^{206}_{82}Pb + {}^{4}_{2}He$$

β⁻ decay:

$${}^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + e^{-} + \tilde{\nu}$$

$$e.g., {}^{99}_{43}Tc \rightarrow {}^{99}_{44}Rb + e^{-} + \tilde{\nu}$$

β⁺ decay:

$${}^{A}_{Z}X \rightarrow {}_{Z-1}^{A}Y + e^{+} + \nu$$

$$e.g., {}^{12}_{7}N \rightarrow {}^{12}_{6}C + e^{+} + \nu$$

Electron capture:

$${}^{A}_{Z}X + e^{-} \rightarrow {}_{Z-1}^{A}Y + \nu$$

$$e.g., {}^{12}_{7}N + e^{-} \rightarrow {}^{12}_{6}C + \nu$$

γ decay:

$${}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma$$

$$e.g., {}^{99}_{43}Tc^{*} \rightarrow {}^{99}_{43}Tc + \gamma(140\,keV)$$

Problems

1. Derive the decay law

$$N = N_0 e^{-\lambda t}$$

2. What is the activity of one gram of $^{226}_{88}Ra$ whose half life is 1622 years?

3. Over what distance in free space will the intensity of a 5 eV neutron beam be reduced by a factor of one-half? (T ½ = 12.8 min)

Changing Elements

Both alpha and beta decay cause the an isotope to change atomic number and therefore element. Alpha decay also causes a change in mass number.

Decay type	Atomic number	Mass number
alpha		
beta		
gamma		

NUCLEAR FISSION

Fission is a phenomenon by which an unstable nucleus disintegrates into two smaller nuclides of approximately the same order of mass as well as the emission of ionizing radiations or particles with the release of nuclear energy.

There are 2 types of fission that exist:

1. Spontaneous Fission

2. Induced Fission

$$^{235}_{92}U + ^{1}_{0}n \rightarrow \left[^{236}_{92}U\right] \rightarrow ^{A_1}_{z_1}X + ^{A_2}_{z_2}Y + \varepsilon\, ^{1}_{0}n$$

Spontaneous Fission

Some radioisotopes contain nuclei which are highly unstable and decay spontaneously by splitting into 2 smaller nuclei.

Such spontaneous decays are accompanied by the release of neutrons.

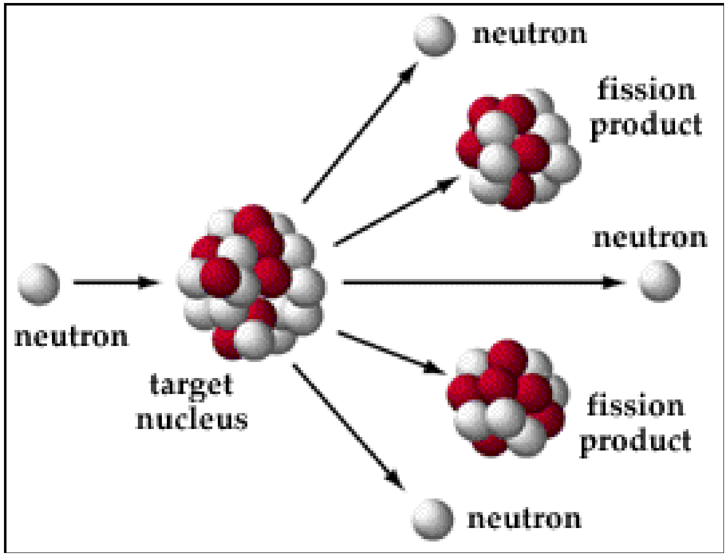
Induced Fission

Nuclear fission can be induced by bombarding atoms with neutrons.

The nuclei of the atoms then split into 2 equal parts.

Induced fission decays are also accompanied by the release of neutrons.

Examples of Fission Reactions



$$^{235}_{92}U(n,3n)^{141}_{56}Ba,^{92}_{36}Kr$$

$$^{118}_{50}Sn(p,4n)^{66}_{31}Ga,^{49}_{20}Ca$$

Equivalence of mass and energy ($E=mc^2$)

1. An atomic particle at rest possess a rest mass energy E_0 given by $$E_0 = m_0 c^2$$ where m_0 is the rest mass and c is the speed of electromagnetic waves in vacuum.

2. A dynamic particle possesses both kinetic energy and rest mass energy, the sum of which is known as dynamic energy E. Mathematically, $$E = mc^2 = E_R + E_K$$ where m is dynamic mass, ER = rest mass energy and E_K = kinetic mass energy

3. Hence, for moving atomic particles,

$$E = m_0 c^2 + E_K$$

4. But for photons

$$E = hv$$

since photons have zero rest masses as they are quanta of electromagnetic radiations moving with the speed of light

(c = 3.0×10^8 m/s).

Mass Defect

It has been experimentally proven that when an atomic nucleus disintegrates into its constituent nucleons, the total mass (i.e. rest mass) of the nucleons is usually more than that of the initial nucleus or nuclide. This difference in mass is known as the mass defect. Thus,

$$\text{Mass Defect} = \sum(\text{Mass of Nucleons}) - \sum(\text{Mass of Nucleus})$$

The <u>*nuclear binding energy*</u> is a consequence of the mass defect. The example below illustrates this point. Consider the reaction

$$X \rightarrow a + b$$

where X s the initial nuclide and a and b are nucleons. Then sum of the masses of nucleons = *ma* + *mb* and mass of *X* is *mx*. But

$$\left(m_a + m_b\right) > m_x$$

$$\Rightarrow$$

$$\text{Mass Defect} = \left(m_a + m_b\right) - m_x \ \text{ or } \ \left|\mathrm{m}_{\mathrm{x}} - \left(m_a + m_b\right)\right|$$

Disintegration Energy(Q)

A particle undergoing a translational motion in space-time continuum must possess both rest mass energy and kinetic energy. In such a case the total energy E_T of the particle is given by

$$E_T = E_0 + E_K$$

Assume that in the reaction : X(a, b)Y, Z all the particle i.e. *a ,b and the nuclides* possess non-zero translational kinetic energy (mass energy). The total energies of the individual nuclides and particles are calculated as follows:

$$\text{X:} \quad E_K^X + m_0^X c^2, \quad \text{a:}\ E_K^a + m_0^a c^2, \quad \text{b:}\ E_K^b + m_0^b c^2, \ \text{Y:}\ E_K^Z + m_0^Z c^2$$

where 'm' is in 'kg' and 'c' is in 'm/s'.

UNITS

- Note that, in usual nuclear calculation, you may encounter atomic masses in either kilogram or unified atomic units.
- Please, make sure you use consistent units:
- $1u = 1.6605402 \times 10^{-27} kg$

Energy from Fission

$$ {}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{138}_{55}\mathrm{Cs} + {}^{96}_{37}\mathrm{Rb} + 2\,{}^{1}_{0}\mathrm{n} $$

Element	Atomic Mass (kg)
${}^{235}_{92}U$	3.9014×10^{-25}
${}^{138}_{55}Cs$	2.2895×10^{-25}
${}^{96}_{37}Rb$	1.5925×10^{-25}
${}^{1}_{0}n$	1.6750×10^{-27}

Worked Eample

For the reaction

$$ {}_{2}^{4}He \rightarrow 2n + 2p $$

$$ m\left({}_{2}^{4}He\right) = 4.00154\,u, \quad m\left(n\right) = 1.0073\,u, $$

$$ m\left(p\right) = 1.0087\,u, $$

The mass defect is computed as follows:

Mass of nucleons = 2(1.0073u) + 2(1.0087u) = 4.032u

Mass defect = 4.032 u - 4.00154 u = 0.03046 u

But $E = mc^2$

Hence, mass defect

$= 0.03046 \times c2$

$= 0.03046 \times 931$ Mev

Question : **Uranium decay energy release.** Calculate the disintegration energy when $^{232}_{92}U$ (mass = 232.037146 u) decays to $^{228}_{90}Th$ (228.028731 u) with the emission of an α particle. (As always, masses are for neutral atoms.)

Solution:

We use conservation of energy as expressed $^{232}_{92}U$ is the parent, $^{228}_{90}Th$ is the daughter.

$$^{232}_{92}U \longrightarrow {}^{228}_{90}Th + {}^{4}_{2}He$$

$$M_P c^2 = M_D c^2 + m_\alpha c^2 + Q,$$

$$Q = M_P c^2 - (M_D + m_\alpha)c^2$$

Since the mass of the $^{4}_{2}He$ is 4.002603 u , the total mass in the final state is

$$228.028731 \text{ u} + 4.002603 \text{ u} = 232.031334 \text{ u}.$$

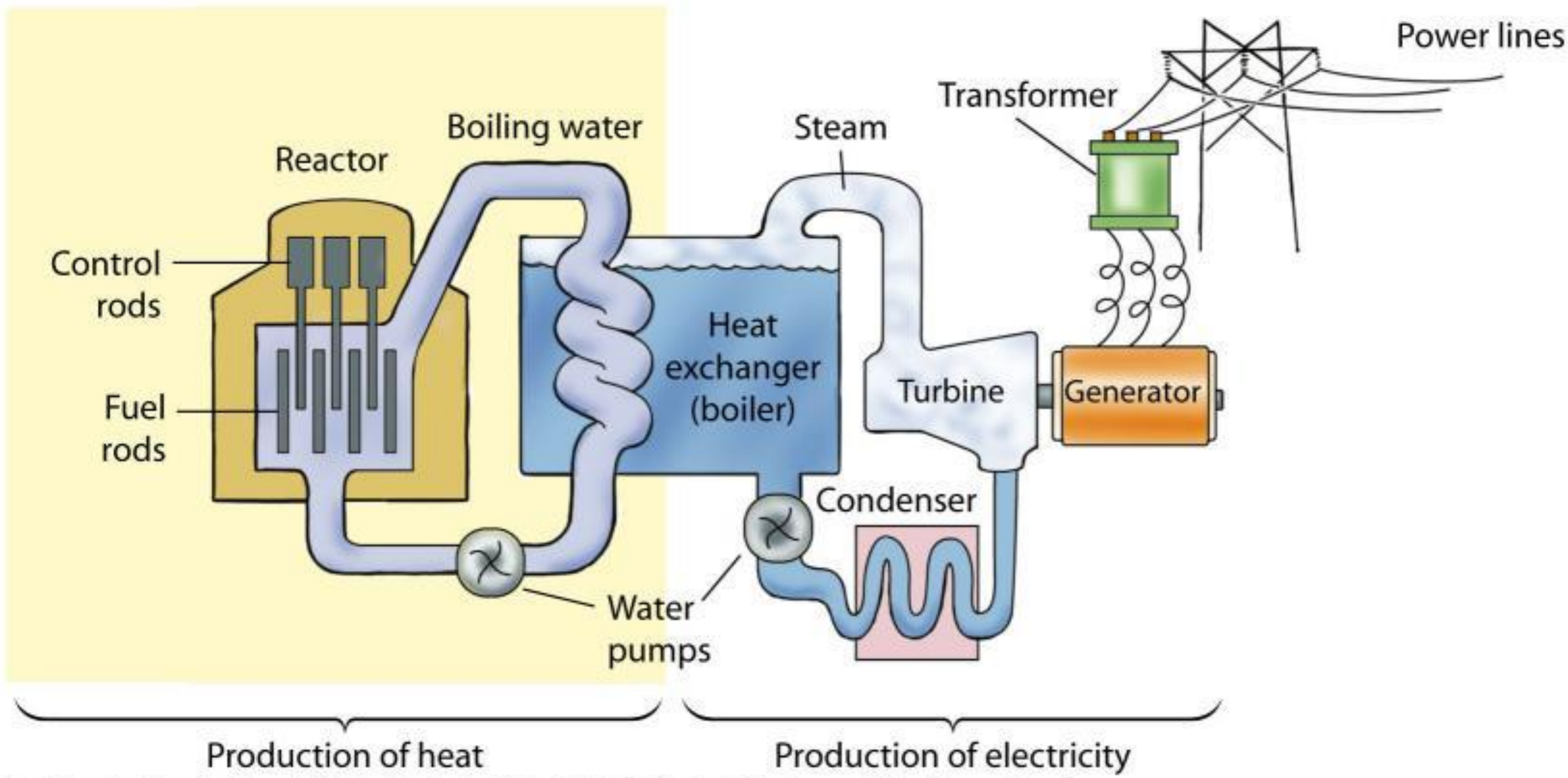
The mass lost when the $^{232}_{92}U$ decays is

$$232.037146 \text{ u} - 232.031334 \text{ u} = 0.005812 \text{ u}.$$

Since $1 \text{ u} = 931.5 \text{ MeV}$, the energy Q released is

$$Q = (0.005812 \text{ u})(931.5 \text{ MeV/u})$$
$$\approx 5.4 \text{ MeV},$$

Energy release in nuclear reaction



Nuclear Reactor Parts

Bombarding Particles of Appropriate Energies

Fuel rods

These contain U235 or Pu239. They become very hot due to nuclear fission.

Control rods

Made of boron, when placed in-between the fuel rods these absorb neutrons and so reduce the rate of fission. Their depth is adjusted to maintain a constant rate of fission.

Moderator

This surrounds the fuel rods and slows neutrons down to make further fission more likely. The moderator can be water or graphite.

Coolant

This transfers the heat energy of the fuel rods to the heat exchanger. Coolant be water, carbon dioxide gas or liquid sodium.

Heat exchanger

Here water is converted into high pressure steam using the heat energy of the coolant.

Reactor core

This is a thick steal vessel designed to withstand the very high pressure and temperature in the core.

Concrete shield

This absorbs the radiation coming from the nuclear reactor.

Nuclear Fusion

Fusion is a nuclear phenomenon by which two small masses of high atomic nuclides under controlled thermo-nuclear conditions aggregate into a composite (single) atomic nuclide with the consequent release of nuclear energy. Fusion is one of the methods by which energy can be obtained from the nucleus. The fusion process is termed as thermo-nuclear process because it requires an initial input of thermal energy of a very great magnitude and consequently requires super-high temperatures (of the order of 10^6 K) for ignition. Nevertheless, the fusion process after it has been triggered produces a large avalanche of nuclear energy. A few examples of fusion reaction are listed below:

The Fusion Process

Energy from Fusion

$$ {}^{2}_{1}H + {}^{3}_{1}H \longrightarrow {}^{4}_{2}He + {}^{1}_{0}n + \text{Energy} $$

Element	Atomic Mass (kg)
${}^{2}_{1}H$	3.345×10^{-27}
${}^{3}_{1}H$	5.008×10^{-27}
${}^{4}_{2}He$	6.647×10^{-27}
${}^{1}_{0}n$	1.6750×10^{-27}

$$ {}_{1}^{3}H + {}_{1}^{1}H \rightarrow {}_{2}^{4}He + \gamma $$ or $$ T\left(p,\gamma\right){}_{2}^{4}He $$

$$ {}_{1}^{2}H + {}_{1}^{2}H \rightarrow {}_{2}^{3}He + {}_{0}^{1}n $$ or $$ D\left(d,n\right){}_{2}^{3}He $$

$$ {}_{3}^{6}Li\left(d,\alpha\right){}_{3}^{4}He $$

$$ {}^{7}Li\left(p,\alpha\right){}^{4}He $$

$$ {}^{7}Li\left(d,n\right){}^{8}Be $$

Theoretical Background for the Computation of Q Value

A particle undergoing a translational motion in space-time continuum must possess both rest mass energy and kinetic energy. In such a case the total energy E_T of the particle is given by

$$E_T = E_0 + E_K$$

General Case:

$$X\left(a,b\right)Y,Z$$

Assume that in the above reaction all the particle i.e. *a,b* and the nuclides possess non-zero translational kinetic energy (mass energy). The total energies of the individual nuclides and particles are calculated as follows:

X: $$E_K^X + m_0^X c^2$$

a: $$E_K^a + m_0^a c^2$$

b: $$E_K^b + m_0^b c^2$$

Z: $$E_K^Z + m_0^Z c^2$$

where m_0 is in kg and *c* is in ms^{-1}.

Conservation of mass in nuclear reactions implies that the energy of the interacting system is equal to the energy of the resultant system. Thus,

$$E_K^X + m_0^X c^2 + E_K^a + m_0^a c^2 = E_K^Y + m_0^Y c^2 + E_K^Z + m_0^Z c^2 + E_K^b + m_0^b c^2$$

$$\Rightarrow$$

$$\left(m_0^Y c^2 + m_0^Z c^2 + m_0^b c^2\right) - \left(m_0^X c^2 + m_0^a c^2\right) = \left(E_K^b + E_K^Y + E_K^Z\right) - \left(E_K^X + E_K^a\right) = Q$$

But 1u (atomic mass unit amu) = 931 MeV

$$\therefore Q = \left[\left(E_K^b + E_K^Y + E_K^Z\right) - \left(E_K^X + E_K^a\right)\right] \times 931 \quad \text{MeV}$$

Energy from Fusion

Calculate the following:

- The mass difference.
- The energy released per fusion.

Energy from Fusion

$$^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + ^{1}_{0}n + \text{Energy}$$

The total mass before fusion (LHS of the equation):

$$\mathbf{3.345 \times 10^{-27} + 5.008 \times 10^{-27} = \underline{8.353 \times 10^{-27}\ kg}}$$

The total mass after fission (RHS of the equation):

$$\mathbf{6.647 \times 10^{-27} + 1.675 \times 10^{-27} = \underline{8.322 \times 10^{-27}\ kg}}$$

Energy from Fusion

m = total mass before fission – total mass after fission

$m = 8.353 \times 10^{-27} - 8.322 \times 10^{-27}$

$m = 3.1 \times 10^{-29}$ kg

Energy from Fusion

$$^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + ^{1}_{0}n + Energy$$

$m = 3.1 \times 10^{-29}$ kg

$c = 3 \times 10^{8}$ ms^{-1}

$E = E$

$E = mc^2$

$E = 3.1 \times 10^{-29} \times (3 \times 10^{8})^2$

$\mathbf{E = 2.79 \times 10^{-12}}$ **J**

The energy released per fusion is 2.79 x 10^{-12} J.

ACCELERATED CHARGES AND BREMSSTRAHLUNG

Radiation from Accelerated, Charged Particles

A charged particle undergoing acceleration radiates photons. A ready example of this is when electrons moving back and forth in antennae produce electromagnetic radiation, such as transmitted by radio stations.

The power in electromagnetic radiation emitted by a particle of charge (q)with an acceleration a is given by Larmor's formula:

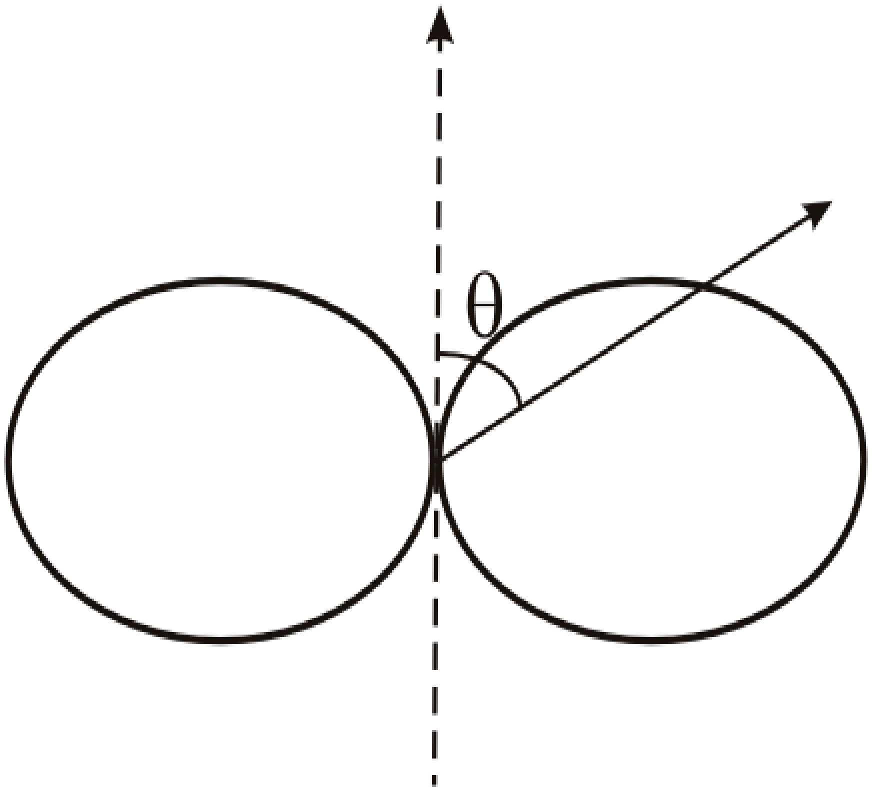
$$P = \frac{2q^2a^2}{3c^2}$$

The radiation has some very interesting properties:

• the emitted power, P, is proportional to the square of the charge(q^2) and the square of the acceleration (a^2).

•the photons are emitted in a characteristic dipolar form.

Maximum emission takes place perpendicular to the direction of the acceleration, and is proportional to $\sin^2\theta$.
($P \propto \sin^2\theta$)



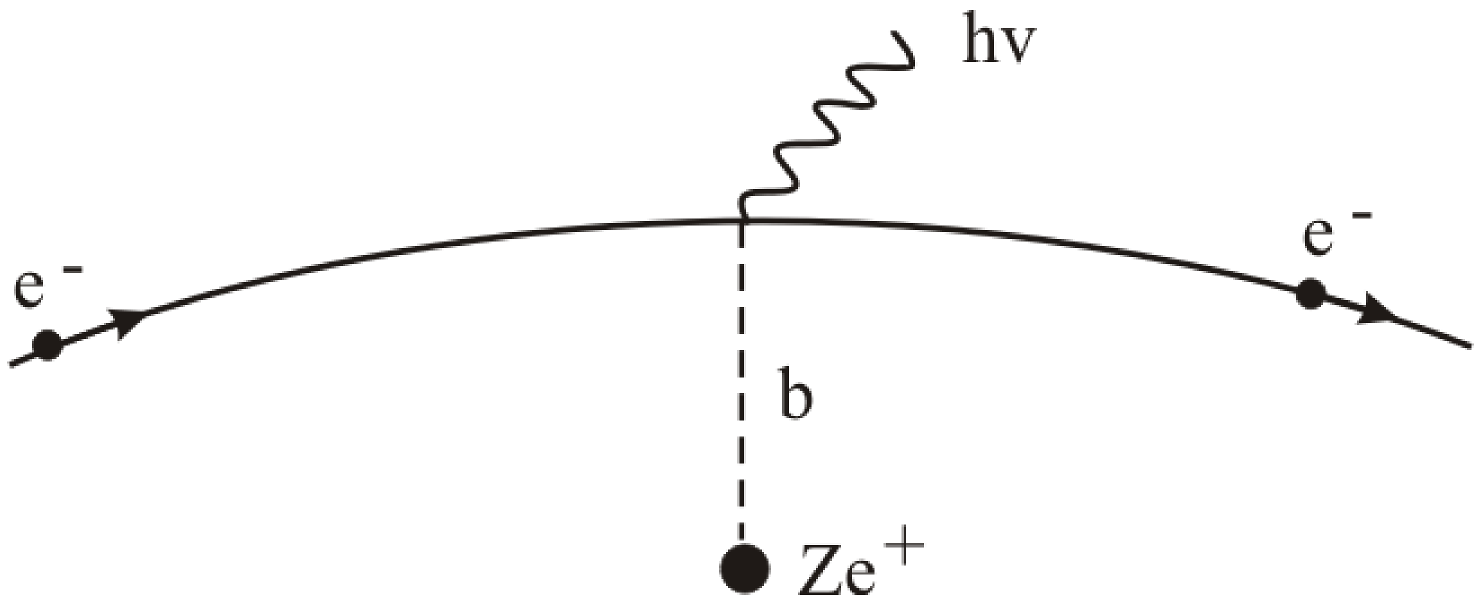
Dipolar emission from an accelerated charge

Bremsstrahlung from an Electron Passing a Charged Particle

Bremsstrahlung, or braking radiation, is emitted when a charged particle moves in an electric field, E. The particle emits energy in the form of electromagnetic radiation, at the expense of its kinetic energy, hence the name ``braking radiation''.

The major astrophysically relevant example of bremsstrahlung, is when anelectron, e^- with velocity v, passes a charge consisting of Z protons, with total charge Ze^+. The impact parameter b of the interaction is the distance of closest approach.

The electron is accelerated during its interaction, and since the acceleration is not uniform it emits photons with a range of wavelengths, i.e. a spectrum.

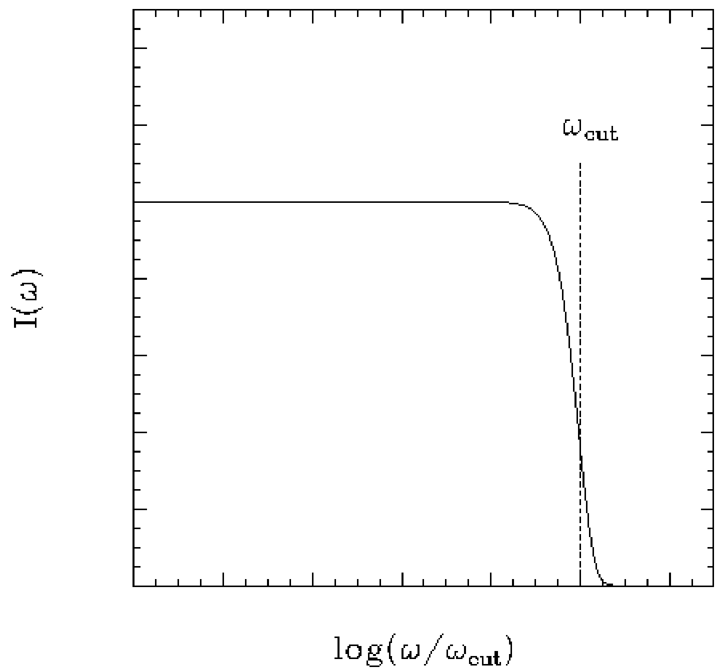


Bremsstrahlung radiation. An electron e- passes an ion with charge Ze+ with an impact parameter, b. Forces acting on the charge during the passage cause the emission of photons, hv.

- The electron is accelerated during its interaction
- If acceleration is not uniform it emits photons with a range of wavelengths, i.e. a spectrum.
- The power emitted can be computed from Larmor's formula
- flat spectrum in frequency with an upper cutoff, ω_{cut}), which is related to the interaction time, $\Delta t = v/b$, or interaction frequency $w = 1/\Delta t = b/v$, is produced.

The intensity in the flat part of the spectrum, where ($\omega < \omega_{cut}$) is given by

$$I = \frac{8Z^2 e^6}{3\pi . c^3 m_e^{\,2} v^2 b^2}$$



Radiocarbon (^{14}C) formation and decay

-formed by interaction of cosmic ray spallation products with stable N gas

$$ {}^{1}_{0}\mathrm{n} + {}^{14}_{7}N \rightarrow {}^{14}_{6}C + {}^{1}_{1}H $$

-radiocarbon subsequently decays by β- decay back to ^{14}N with a half-life of 5730y

$$ {}^{14}_{6}C \rightarrow {}^{14}_{7}N + \beta^{-} + \overline{\nu} + Q $$

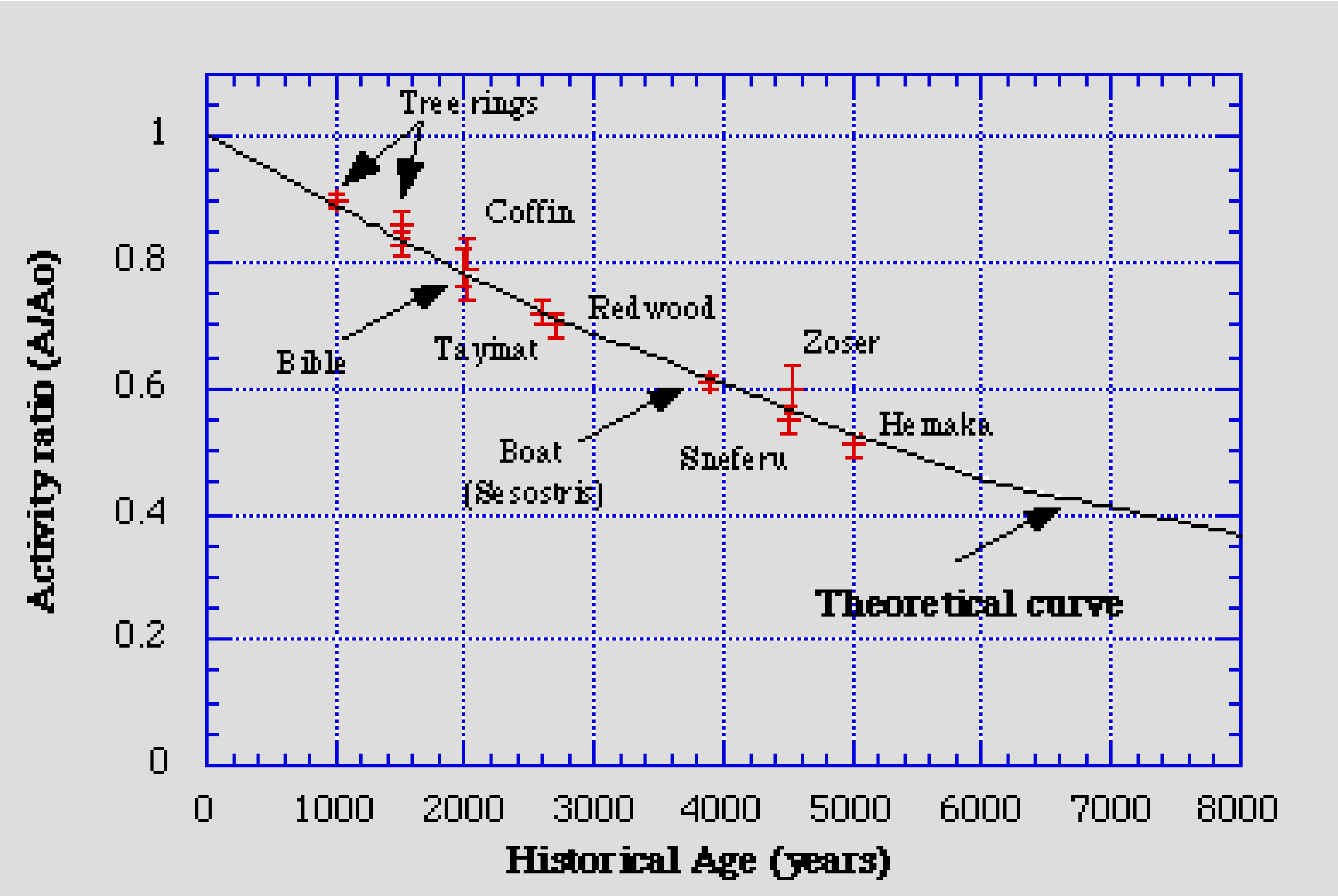
The activity of radiocarbon in the atmosphere represents a balance of its production, its decay,and its uptake by the biosphere, weathering, etc.

Radiocarbon Dating

- As plants uptake C through photosynthesis, they take on the 14C activity of the atmosphere.
- Anything that derives from this C will also have atmospheric ^{14}C activity (including you and I).
- If something stops actively exchanging C (it dies, is buried, etc), that 14C begins to decay.

$$A = A_0 e^{-\lambda t}$$

where present-day, pre-bomb, 14C activity = 13.56dpm/g C



So all you need to know to calculate an age is A_0, which to first order is 13.56dpm/g, BUT

*small variations (several percent) in atmospheric ^{14}C in the past lead to dating errors of up to 20%!

<u>Sources of variability:</u>

1) geomagnetic field strength
2) solar activity
3) carbon cycle changes

Radiocarbon Measurements and Reporting

1) Radiocarbon dates are determined by measuring the ratio of ^{14}C to ^{12}C in a sample, relative to a standard, usually in an accelerator mass spectrometer.

standard = oxalic acid that represents activity of 1890 wood

14C ages are reported as "^{14}C years BP", where BP is 1950

2) <u>Fact</u>: Most living things do not uptake C in atmospheric ratios – i.e. they fractionate carbon, (lighter ^{12}C preferentially used), must correct for this fractionation because it affects the $^{14}C/^{12}C$ ratio

Researchers collect the $^{13}C/^{12}C$ ratio, use it to correct for "missing" ^{14}C

$$\delta^{13}C = \left[\frac{\left({}^{13}C/{}^{12}C\right)_{spl} - \left({}^{13}C/{}^{12}C\right)_{std}}{\left({}^{13}C/{}^{12}C\right)_{std}} - 1 \right] * 1000$$

So the less ^{13}C a sample has, the less ^{14}C it has, and so the uncorrected ^{14}C age will be ________ than the calendar age?

$$A_{corr} = A_{meas}\left[1 - \frac{2(25 + \delta^{13}C_{PDB})}{1000}\right] dpm/g$$

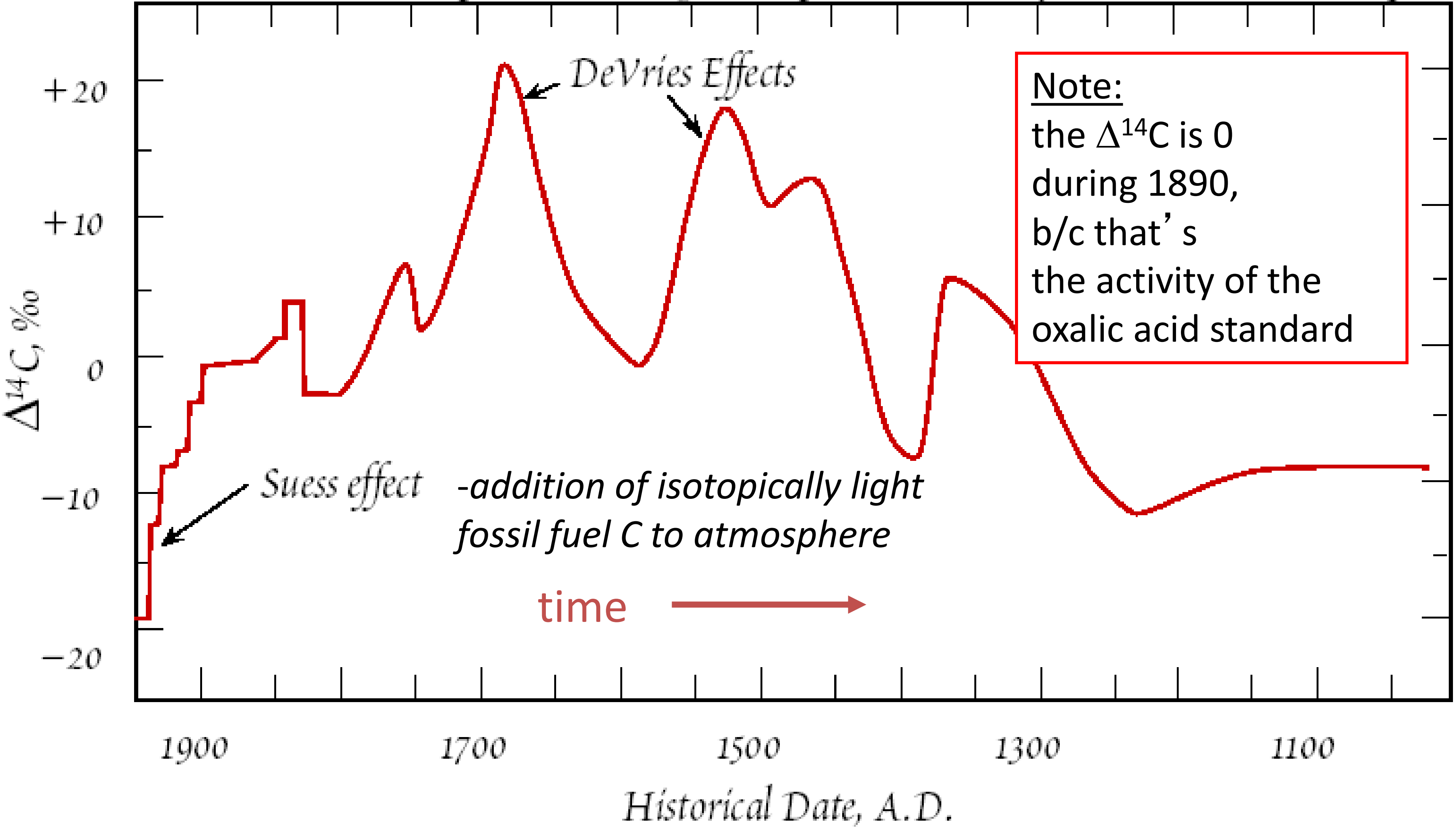
Samples are "normalized" to a $\delta^{13}C_{PDB}$ value of -25‰

3) The final step is to obtain a "calibrated ^{14}C age" using the atmospheric radiocarbon content when the sample grew.

Atmospheric Radiocarbon Variability through Time

<u>Convention</u>:
The atmospheric radiocarbon anomaly with respect to a standard is defined as $D^{14}C$

$$\Delta^{14}C = \left[\frac{\left({}^{14}C / {}^{12}C \right)_{spl}}{\left({}^{14}C / {}^{12}C \right)_{std}} - 1 \right] * 1000$$



VARIATION OF INITIAL SPECIFIC ACTIVITY OF C-14 IN THE PAST