

Tutorial: Modes of Heat Transfer

General

1. What are the mechanisms of energy transfer to a closed system?
2. How is heat transfer distinguished from the other forms of energy transfer?
3. What are the physical mechanisms associated with heat transfer by conduction, convection and radiation?
4. What is the driving potential for heat transfer?
5. Explain the difference between a heat transfer rate and a heat flux. State the unit of each.
6. Define temperature gradient and state its units. What is the relationship of heat flow to a temperature gradient?
7. A 1-kW iron is left on the ironing board with its base exposed to the air. 90% of the heat generated in the iron is dissipated through its base whose surface area is 150 cm^2 , and the remaining 10% through other surfaces. Assuming the heat transfer from the surface to be uniform, determine:
 - (a) The amount of heat dissipated by the iron in 2 hours.
 - (b) The heat flux on the surface of the iron base, in W m^{-2} .

The first law of thermodynamics

1. On a very hot day, a student turns his fan on when he leaves his room in the morning and closed all the doors and windows. When he returns in the evening, will his room be warmer or cooler than the neighbouring rooms? Why?
2. Consider two identical rooms, one with a refrigerator in it and the other without one. If all doors and windows are closed, which of the two rooms will be cooler than the other? Explain.
3. A rigid tank contains 20 lb-mass of air at 50 psia and 80°F . The air is now heated until its pressure is doubled. Determine (a) the volume of the tank and (b) the amount of heat transfer.

Heat conduction

1.
 - (i) Define thermal conductivity, k , and explain its significance in heat transfer.
 - (ii) The unit of k is $\text{W m}^{-1} \text{ K}^{-1}$. Judging from this, can k be defined as "*rate of heat transfer through a material per unit thickness per degree rise in temperature?*" Explain.
 - (iii) The brick wall of a room is 5m long by 6 m wide and of thickness 30 cm. On a very hot day, the temperature inside the air-conditioned room is 18°C while the outside temperature is 33°C . Determine the rate of heat conduction through the homogeneous wall of the room. ($k_{\text{brick}} = 0.72 \text{ W m}^{-1} \text{ K}^{-1}$)
2. A metal is attached to the inside surface of a refrigerator to measure the heat flux across it. The meter registers a heat flux of 30 W m^{-2} across the 5 cm thick door of the refrigerator. Temperature inside the ref is 5°C while its outside surface temperature is 20°C . What is the average thermal conductivity of the refrigerator door over this temperature range?
3. An aluminum pan has a flat bottom with diameter 20 cm and thickness 0.4 cm. Heat is transferred steadily to boiling water in the pan through its bottom at a rate of 800W. If the inner surface of the bottom of the pan is at 105°C , find the temperature of the outer surface of the bottom of the pan. $k_{\text{al}} = 237 \text{ W m}^{-1} \text{ K}^{-1}$.

4. The two surfaces of a plate 2.5 cm thick are maintained at 5°C and 85°C respectively. Heat is transferred through the plate at 500 W m^{-2} . Calculate the k of the plate material.
5. Consider two walls of a house have identical area of cross-section, except in that one is made of 10-cm-thick wood, while the other is made of 25-cm-thick brick. If both walls have the same temperature difference across them: (i) through which wall will the house lose more heat in winter? (ii) determine the wood thickness such that both loose heat at the same rate.

$$k_{\text{wood}} = 0.17 \text{ W m}^{-1} \text{ K}^{-1} \quad k_{\text{brick}} = 0.72 \text{ W m}^{-1} \text{ K}^{-1}$$

Convection

- ✓ 1. (a) What is the difference between natural convection and forced convection?
 (b) State Newton's law of cooling and write its equation from memory. Define heat transfer coefficient and state its units.
- ✓ 2. Hot air at 90°C is blown over a flat surface 3 m by 4 m in dimension, which is at 35°C . The average heat transfer coefficient (h.t.c) over the surface is $50 \text{ W m}^{-2} \text{ }^{\circ}\text{C}^{-1}$. Calculate the rate of heat transfer from air to the plate.
- 3.
- (i) A 5 cm diameter pipe, which is 10 m, long carries hot water to a house. The hot water inside the cylindrical pipe is at 80°C while the surrounding air outside the pipe is at 5°C . This pipe is losing heat to the surrounding by natural convection with the h.t.c. of $25 \text{ W m}^{-2} \text{ }^{\circ}\text{C}^{-1}$. Find the rate of heat loss by the pipe.
 - (ii) Repeat (i) for pipe diameter of (a) 3 cm, (b) 7 cm and (c) 9 cm. Plot the rate of heat loss against container diameter to show how heat loss vary with diameter. Discuss your result.
4. A 60 m long section of a steam pipe whose outer diameter is 10 cm passes through an open space at 10°C . The average temperature of the outer surface of the pipe is 140°C , and the average heat transfer coefficient on that surface is $34 \text{ W m}^{-2} \text{ K}^{-1}$. Determine:
- (a) the rate of heat loss from the steam pipe and
 - (b) the annual cost of this energy loss if steam is generated in a natural gas furnace with an efficiency of 86%, given that the price of natural gas is $\$5 \times 10^{-9}$ per Joule.

Heat transfer by radiation

(Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$)

- ✓ 1. Write Stefan-Boltzman equation and what unit of temperature must be used with the equation?
- (a) (i) Does any of the energy of the sun reach the earth by conduction or convection? Explain.
 (ii) Define emissivity and absorptivity.
 (iii) Define irradiation and state its units.
 - (b) Consider a person whose exposed surface area is 1.7 m^2 , emissivity is 0.7, and surface temperature is 32°C . Determine the rate of heat loss from that person by radiation in a large room having walls at a temperature of (a) 300 K and (b) 280 K.
2. (a) What is a blackbody? How do real bodies differ from blackbodies?
 (b) A metal tube with an emissivity of 0.65 and surface area of 500 cm^2 is at a temperature 300°C in a large enclosure which contains a gas at a temperature of 1000 K. The enclosure contains a non-radiation absorbing environment that can be considered a "black body." Calculate the net rate of thermal radiation from the enclosure to the metal tube.

Mixed Modes of heat transfer

1. A surface at a higher temperature is exposed to cooler surroundings. By what mode(s) is heat transferred from the surface if: (a) it is in perfect contact with another solid at lower temperature? (b) it is exposed to the flow of a cold liquid, (c) it is exposed to the flow of a gas, and (d) it is kept in an evacuated chamber (i.e. vacuum)?
2.
 - (a) After attending a lecture on heat transfer, a second year petrochemical engineering student was walking across the campus on a cold harmattan morning. For each of the following scenarios, which mode of heat transfer (i.e. conduction, convection, radiation) are operative and which mode do you think is dominant? Explain your answer in each case:
 - (i) student feels colder when a sudden gust of wind arises
 - (ii) student sat on a concrete bench and feels even colder, especially in the part of the body that is in contact with the bench
 - (iii) seeing some rubbish being incinerated in the bush, the student moved and stood near the fire and feels warmer.
 - (b) Consider a person standing in a room at 23°C . The expose surface area and the skin temperature of the person are 1.7 m^2 and 32°C , respectively. If the combined emissivity of the skin and clothes is 0.9 and assuming the inner surfaces of the room is at the same temperature as air in the room, determine the total rate of heat transfer from this person.
3. (a) Will radiation heat transfer require the presence of a medium for the transfer to occur? How does this differ from convection or conduction heat transfer?
(b) A 5 cm diameter spherical ball whose surface is maintained at a temperature of 80°C is suspended in the middle of a room. The temperature in the room is 21°C . (i) What are the modes of heat transfer from the ball to the wall of the room. (ii) Determine the total rate of heat transfer from the ball.
Data: $\text{h.t.c.} = 68 \text{ W m}^{-2} \text{ }^{\circ}\text{C}^{-1}$, emissivity of the ball's surface = 0.8
- * 13. (a) Give one example of each of conduction, convection and radiation, from everyday life.
(b) A 1 kW iron is left on the iron board for 2 hours, with its base exposed to the air at 20°C . The heat transfer coefficient between the base surface and the surrounding air is $35 \text{ W m}^{-2} \text{ }^{\circ}\text{C}^{-1}$. If the base has an emissivity of 0.6 and a surface area of 200 cm^2 , determine the temperature of the base of the iron.
14. As your neighbour watches you boil yam on the stove, s/he suggests that if you increase the flame on the stove the yam will cook faster because the water will be hotter and boil quicker. From your understanding of heat transfer, is your neighbour correct or not? Explain your answer.

Tutorial: Analysis of Convection heat transfer

General principle

- a. How does forced convection differ from natural convection?
In which mode of heat transfer is the convection heat transfer coefficient usually higher, natural convection or forced convection? Explain your answer.
- b. What is external forced convection? How does it differ from internal forced convection? Can a heat transfer system involve both internal and external convection at the same time? Give an example.
- c. Consider a piece of hot roasted yam. Will the yam cool faster or slower when we blow the warm air coming from our lungs on it instead of letting it cool naturally in the cooler air in the room? Explain.
- d. (a) Write an expression for the Reynold's number. What is its significance? (b) How is Nusselt number defined? What is its physical significance?
- e. When is heat transfer through a fluid conduction and when is it convection? Which of these modes in a fluid has a higher rate of heat transfer? How does the convection heat transfer coefficient differ from the thermal conductivity of a fluid?
- f. Define incompressible flow and incompressible fluid. Must the flow of a compressible fluid necessarily be treated as compressible?

Forced convection in external flow

1. Consider atmospheric air at 25°C and a velocity of 25 ms^{-1} flowing over both surfaces of a 1m long flat plate that is maintained at 125°C . Determine the rate of heat transfer per unit width from the plate for values of the critical Reynolds number corresponding to (a) 10^5 and (b) 10^6 .
2. An uninsulated steam pipe, 0.5m diameter, is used to transport high temperature steam from one building to another. The pipe surface temperature is 150°C , and is exposed to ambient air at a temperature of -10°C . The air moves in cross flow over the pipe with a velocity of 5 m/s .
(a) What is the heat transfer rate per unit length of pipe? (b) The management is considering insulating the pipe with a 50 mm thick layer of rigid urethane foam ($k = 0.026 \text{ Wm}^{-1}\text{K}^{-1}$). Evaluate the heat transfer rate with the insulation layer and therefore, determine the saving in heat loss.
3. A 25 cm diameter stainless steel ball ($\rho = 8055 \text{ kgm}^{-3}$, $C_p = 480 \text{ J kg}^{-1}\text{C}^{-1}$) is removed from the oven at a uniform temperature of 300°C . The ball is then subjected to the flow of air at 1 atm pressure and 25°C with a velocity of 3 ms^{-1} . The surface temperature of the ball eventually drops to 200°C . Determine the average convection heat transfer coefficient during this cooling process and estimate how long the process will take.
4. Water at 20°C flows over a 20 mm diameter sphere with a velocity of 5 ms^{-1} . The surface of the sphere is at 60°C . What is the rate of heat transfer from the sphere?

Forced convection in internal flow

1. In the final stages of production, a pharmaceutical is sterilized by heating it from 25 to 75°C as it moves at 0.2 m/s through a straight thin-walled stainless steel tube of 12.7-mm diameter. A uniform heat flux is maintained by an electric resistance heater wrapped around the outer surface of the tube. If the tube is 10 m long, what is the required heat flux? Neglecting entrance effects, what is the surface temperature at the tube exit? Fluid properties may be approximated as: density = 1000 kg/m^3 , $cp=4000 \text{ J/kg K}$, viscosity = $2 \times 10^{-3} \text{ Nsm}^{-2}$, $k = 0.48 \text{ W/m K}$, and $Pr = 10$.
2. Air enters a 7-m -long section of a rectangular duct of cross section 15 cm by 20 cm at 50°C at an average velocity of 7 m/s . If the walls of the duct are maintained at 10°C , determine (a) the outlet temperature of the air, (b) the rate of heat transfer from the air. *Answers: (a) 8°C , (b) 3674 W*
3. Water flows steadily at 2 kgs^{-1} through a 40 mm diameter tube that is 4 m long. The water enters at 25°C , and the tube temperature is maintained at 95°C by steam condensing on the exterior surface. Determine the outlet temperature of the water and the rate of heat transfer to the water.

4. Heated air required for a food-drying process is generated by passing ambient air at 20°C through long, circular tubes ($D = 50 \text{ mm}$, $L = 5 \text{ m}$) housed in a steam condenser. Saturated steam at atmospheric pressure condenses on the outer surface of the tubes, maintaining a uniform surface temperature of 100°C . If an air flowrate of 0.01 kg s^{-1} is maintained in each tube, determine the air outlet temperature $T_{m,o}$ and the total heat rate q for the tube.

5. Oil flows at 25°C and an average velocity of 2.5 ms^{-1} through a 35-cm-diameter pipeline. A 250-m-long section of the pipeline passes through very cold waters of a lake at 5°C . Measurements indicate that the surface temperature of the pipe through the lake is approximately 5°C . Ignoring the thermal resistance of the pipe material determine:

- (a) the temperature of the oil when the pipe leaves the lake,
- (b) the rate of heat loss from the oil,
- (c) the pumping power required to overcome the pressure losses and to maintain the flow of the oil in the pipe.

Properties of the oil: Density = 890 kg m^{-3} Specific heat = $1880 \text{ J kg}^{-1} \text{ K}^{-1}$ Prandtl number = 10500

Thermal conductivity = $0.147 \text{ W m}^{-1} \text{ K}^{-1}$ Kinematic viscosity = $0.9 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$

Free (natural) convection

1. Under steady-state operation, the surface temperature of a small 20-W incandescent light bulb is 125°C when the temperature of the room air and walls is 25°C . Approximating the bulb as a sphere 40 mm in diameter with a surface emissivity of 0.8, what is the rate of heat transfer from the surface of the bulb to the surroundings?

Boiling and condensation.

(Note: Properties of ammonia and water may be found in the literature, for example, the steam table).

1.
 - a) Is peak nucleate boiling a stable condition? Explain
 - b) Explain why film boiling is associated with a lower convective heat transfer coefficient.
 - c) When would radiation predominate in boiling?
 - d) What is the difference between drop-wise condensation and film condensation?
 - e) Is film condensation more effective on vertical tubes or on horizontal tubes? Explain
 - f) Explain why pressure should affect convective heat transfer coefficient in boiling.
 2. Determine the heat flux for pool boiling water at 200 kPa , when the surface temperature of the container is 135°C . The surface tension for water at 200 kPa is 45 mN/m . (note: $1 \text{ mN} = 10^{-3} \text{ N}$)
 3. Water is to be condensed at 1 atm pressure and 60°C , in a compact heat exchanger having vertical stainless steel plates.
 - i. Assuming laminar flow, sketch a graph of the condensation boundary layer thickness as a function of distance down the plate, as the condensed water flows down the vertical surface. On your sketch indicate regions of (a) laminar condensate flow and (b) turbulent flow, and use the condensation Reynolds number, Re_{cond} to define the criteria for each flow type.
 - ii. Calculate the heat flux for this condensation.
 4. In a chiller unit, ammonia is to be condensed at a saturation temperature of 50°C by passing the ammonia vapour over vertical plates 100 cm high by 150 cm wide, and at a surface temperature of 20°C . Estimate the average convective heat transfer coefficient and the amount of ammonia condensing from one plate surface. Take h_{fg} of ammonia as $1052.8 \text{ kJ kg}^{-1}$.
 5. Saturated steam at atmospheric pressure condenses on a vertical plate $30 \text{ cm} \times 30 \text{ cm}$. The plate is at a temperature of 98°C . Determine:
 - i. The heat transfer coefficient for the condensation and hence the rate of heat transfer.
 - ii. The rate of condensation of the steam
 - iii. The Reynolds number of condensate flow.
- DATA: properties of water at the film temperature may be obtained from the steam table.
6. A vertical plate 50 cm high and 20 cm wide is to be used to condense saturated steam at 1 atm. If the condensation rate, m is 35 kg hr^{-1} , Determine (i) the rate of heat transfer and (ii) the surface temperature at which the plate is maintained.

Tutorial: Steady-state 1-D heat conduction (no heat generation)

Plane wall

1. Consider steady state conditions for 1-dimensional thermal conduction in a plane wall of thickness $L = 25 \text{ cm}$, $k = 50 \text{ W m}^{-1}\text{K}^{-1}$, with surface temperatures T_1 and T_2 respectively. Assume no energy generation. Determine the heat flux and the unknown quantity for each case and sketch the temperature distribution, indicating the direction of the heat flux:

Case	$T_1 \text{ } ^\circ\text{C}$	$T_2 \text{ } ^\circ\text{C}$	Temperature gradient $dT/dx \text{ (K m}^{-1})$
1	50	-20	
2	-30	-10	
3	70		160
4		40	-80
5		30	200

Show all calculations.

2. The walls of a refrigerator are typically constructed by sandwiching a layer of insulation between sheet metal panels. Consider a wall made from fiberglass insulation and steel panels. If the wall separates refrigerated air from ambient air what is the heat flux?

Data:

Fiberglass: thermal conductivity $k_f = 0.045 \text{ W m}^{-2}\text{K}^{-1}$ and thickness $L_f = 52 \text{ mm}$

Steel panels: thermal conductivity $k_p = 60 \text{ W m}^{-1}\text{K}^{-1}$ and thickness $L_p = 3 \text{ mm}$ each

Temperature of refrigerated air: $T_{\infty,f} = 4^\circ\text{C}$

Ambient air temperature: $T_{\infty,o} = 25^\circ\text{C}$

Natural convection coefficient at the inner and outer surfaces: $h_i = h_o = 5 \text{ W m}^{-2}\text{K}^{-1}$.

Heat Conduction in Cylinders and Spheres

3. A cold aluminum canned drink 12.5 cm high with a diameter of 6 cm, is initially at a uniform temperature of 3°C . If the combined convection/radiation heat transfer coefficient between the can and the surrounding air at 25°C is $10 \text{ W m}^{-2}\text{C}^{-1}$, determine how long it will take for the average temperature of the drink to rise to 10°C . In an effort to slow down the warming of the cold drink, the student puts the can in a perfectly fitting 1-cm-thick cylindrical rubber insulation ($k = 0.13 \text{ m}^{-1}\text{C}^{-1}$), under this condition, how long will it take for the average temperature of the drink to rise to 10°C ? Assume the top of the can is not covered.

4.

- (a) What is an infinitely long cylinder? When is it proper to treat an actual cylinder as being infinitely long, and when is it not?

- (b) Can the thermal resistance concept be used for a solid cylinder or sphere in steady-state operation? Explain.

5. A 5m internal diameter spherical tank made of 1.5-cm thick stainless steel ($k = 15 \text{ W m}^{-1}\text{C}^{-1}$) is used to store iced water at 0°C . The tank is located in a room whose temperature is 30°C . The walls of the room are also at 30°C . The outer surface of the tank is black (emissivity = 1), and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are: $80 \text{ W m}^{-2}\text{C}^{-1}$ and $45 \text{ W m}^{-2}\text{C}^{-1}$, respectively. Determine:

(a) the rate of heat transfer to the iced water in the tank and

(b) the amount of ice at 0°C that melts during a 24-h period.

The heat of fusion of water at atmospheric pressure is $h = 333.7 \text{ kJ kg}^{-1}$.

6. A 50-m-long section of a steam pipe whose outer diameter is 10 cm passes through an open space at an ambient temperature of 15°C . The average temperature of the outer surface of the pipe is measured to be 150°C . If the combined heat transfer coefficient on the outer surface of the pipe is $20 \text{ W m}^{-2}\text{C}^{-1}$, determine:

(i) the rate of heat loss from the steam pipe,

(ii) the annual cost of this energy lost if steam is generated in a natural gas furnace that has an efficiency of 75% and the price of natural gas is \$0.52/therm (1 therm = $105,500 \text{ kJ}$), and

(iii) the thickness of fiberglass insulation ($k = 0.035 \text{ W m}^{-1}\text{C}^{-1}$) needed in order to save 90 percent of the heat lost.

Assume the pipe temperature remains constant at 150°C .

7. A 5-mm-diameter spherical ball at 50°C is covered by a 1-mm-thick plastic insulation ($k = 0.13 \text{ W m}^{-1}\text{C}^{-1}$). The ball is exposed to a medium at 15°C , with a combined convection and radiation heat transfer coefficient of $20 \text{ W m}^{-2}\text{C}^{-1}$. Calculate the critical radius of the insulation for this ball, hence determine if the plastic insulation on the ball will reduce or enhance heat transfer from the ball.

8. A steam pipe of 120-mm outside diameter is covered with a 20-mm-thick layer of calcium silicate insulation ($k = 0.089 \text{ W m}^{-1}\text{C}^{-1}$). The pipe surface temperature is 530°C , and the ambient air and surroundings temperatures are 25°C . The convection and radiation coefficients for the outer surface of the insulation are estimated as 5.5 and $10 \text{ W m}^{-2}\text{C}^{-1}$, respectively. Determine the heat rate per unit length from the pipe (Wm^{-1}) and the outer surface temperature of the insulation.

9. A stainless steel tube ($k = 14 \text{ W m}^{-1}\text{K}^{-1}$) used to transport a chilled pharmaceutical product has an inner diameter of 35 mm and a wall thickness of 2 mm. The pharmaceutical and ambient air are at temperatures of 6°C & 23°C , respectively, while the corresponding inner and outer convection coefficients are $400 \text{ W m}^{-2}\text{K}^{-1}$ and $6 \text{ W m}^{-2}\text{K}^{-1}$ respectively.

- (a) What is the heat transfer rate per unit tube length?
(b) What is the heat transfer rate per unit length if a 10-mm-thick layer of calcium silicate insulation is applied to the outer surface of the tube? ($k_{\text{ins}} = 0.050 \text{ W m}^{-1}\text{K}^{-1}$)

10. The wall of a spherical tank of 1-m diameter contains an exothermic chemical reaction and is at 200°C when the ambient air temperature is 25°C . What thickness of urethane foam is required to reduce the exterior temperature to 40°C , assuming the convection coefficient is $20 \text{ W m}^{-2}\text{C}^{-1}$ for both situations? ($k_{\text{urethane}} = 0.026 \text{ W m}^{-1}\text{K}^{-1}$), what is the percentage reduction in heat rate achieved by using the insulation?

Critical Radius of Insulation

11.

- (i) Explain what you understand as the "critical radius" of insulation. How is it defined for cylindrical layer?
(ii) A pipe is insulated such that the outer radius of the insulation is less than the critical radius. Now the insulation is taken off. Will the rate of heat transfer from the pipe increase or decrease for the same pipe surface temperature?

12.

- (a) A pipe is insulated to reduce the heat loss from it. However, measurements indicate that the rate of heat loss has increased instead of decreasing. Can the measurements be right? Explain.
(b) Consider a pipe at a constant temperature whose radius is greater than the critical radius of insulation. A duty technician recorded that the rate of heat loss from the pipe has increased when some insulation is added to the pipe. Is this record valid?
(c) Consider an insulated pipe exposed to the atmosphere. Will the critical radius of insulation be greater on calm days or on windy days? Why?

Tutorial: Steady state heat conduction with heat generation

1. A nuclear fuel element is of thickness 15 mm. Energy is generated within it at a rate 20 MW m^{-3} and this is removed by cooling fluid flowing on its outside at temperature 200°C and with heat transfer coefficient $h = 10 \text{ kW m}^{-2}\text{K}^{-1}$. The fuel element has $k = 60 \text{ W m}^{-1}\text{K}^{-1}$. What are the highest and the lowest temperature in the fuel element and where are these located?

2. An infinite slab is made of composite materials "A" and "B". Material "A" is on the inside in which heat generation occurs. Material "B" is on the outside and it completely surrounds "A". The thickness of "A" is 10 cm while "B" is 2 cm thick. Heat is generated inside "A" at 1.5 MW m^{-3} . The respective thermal conductivities of the materials are: $k_A = 75 \text{ W m}^{-1}\text{K}^{-1}$ and $k_B = 150 \text{ W m}^{-1}\text{K}^{-1}$. On the outer surface of the composite cooling water flows at constant temperature of 30°C with a heat transfer coefficient of $1 \text{ kW m}^{-2}\text{K}^{-1}$.

- (i) Find (a) Temperature at the centre of the reactor (b) Temperature of the outside surface of the tube
(ii) Sketch the steady state temperature distribution in the system.

3. A long cylindrical rod, diameter 200 mm and thermal conductivity of $0.5 \text{ W m}^{-1}\text{K}^{-1}$, experiences uniform volumetric energy generation of 24 kW m^{-3} . The rod is encapsulated by a circular sleeve with an outer diameter of 400 mm and a thermal conductivity of $4 \text{ W m}^{-1}\text{K}^{-1}$. The outer surface of the sleeve is exposed to cross flow of air at 27°C with a convection coefficient of $25 \text{ W m}^{-2}\text{K}^{-1}$.

- (a) Find the temperature at the interface between the rod and sleeve (b) Temperature on the outer surface.
(c) What is the temperature at the center of the rod?

4. Radioactive wastes ($k = 20 \text{ W m}^{-1}\text{K}^{-1}$) are stored in a spherical stainless steel container ($k = 15 \text{ W m}^{-1}\text{K}^{-1}$) of inner and outer radii $R_i = 0.5 \text{ m}$ and $R_o = 0.6 \text{ m}$, respectively. Energy is generated volumetrically within the wastes at a uniform rate of 10^5 W m^{-3} and outer surface of the container is exposed to a water flow at constant temperature of 25°C and heat transfer coefficient $1 \text{ kW m}^{-2}\text{K}^{-1}$. (a) Evaluate the outer and inner surface temperatures, $T_{s,o}$ and $T_{s,i}$ respectively.

- (b) What is the maximum temperature in the system, and where is its location?

Tutorial: Transient heat conduction

Lumped Thermal Capacitance method

1. (a) What is lumped thermal Capacitance (LTC) system analysis? When is it applicable?
Consider heat transfer between two identical hot solid bodies and the air surrounding them. A fan is cooling the first solid while the second one is allowed to cool naturally. For which solid is the LTC system analysis more likely to be applicable? Why?
2. (a) What is the physical significance of the Biot's number? Is the Biot number more likely to be larger for highly conducting solids or poorly conducting ones? (b) For which solid is the lumped thermal capacitance method more likely to be applicable: an actual apple or a model of an apple of the same size, made of gold? Why?
3. A long metal cylinder of 30 mm diameter, initially at a uniform temperature of 1000 K, is suddenly quenched in a large, constant-temperature oil bath at 350 K. (a) Calculate the time required for the surface of the cylinder to reach 500 K. (b) What is the temperature after 10 mins?
Properties of the cylinder are: $k = 17 \text{ W m}^{-1} \text{K}^{-1}$, $C_p = 160 \text{ J kg}^{-1} \text{K}^{-1}$ and $\rho = 4000 \text{ kg m}^{-3}$, while the convection coefficient is $50 \text{ W m}^{-2} \text{K}^{-1}$.
4. Carbon steel balls 8mm in diameter, are annealed by heating them first to 900°C in a furnace and then allowing them to cool slowly to 100°C in ambient air at 35°C. If the average heat transfer coefficient is 20 Wm⁻²°C⁻¹,
(i) determine how long the annealing process will take,
(ii) If 2500 balls are to be annealed per hour, calculate the total heat transferred from the balls to the ambient air.
Properties of the carbon steel: $\rho = 7833 \text{ kg m}^{-3}$, $k = 40 \text{ W m}^{-1} \text{C}^{-1}$, $C_p = 600 \text{ J kg}^{-1} \text{C}^{-1}$.

5. Stainless steel ball bearings having a diameter of 1.2 cm are to be quenched in water. The balls leave the oven at a uniform temperature of 900°C and are exposed to air at 30°C for a while before they are dropped into the water.

If the temperature of the balls is not to fall below 850°C prior to entering the quenching water and the convection heat transfer coefficient in the air is 125 W m⁻²K⁻¹, determine how long they can stand in the air before being dropped into the water.

Properties of the ball bearing: $\rho = 8085 \text{ kg m}^{-3}$, $k = 15.1 \text{ W m}^{-1} \text{C}^{-1}$, $C_p = 0.480 \text{ kJ kg}^{-1} \text{C}^{-1}$, and thermal diffusivity, $\alpha = 3.91 \times 10^{-6} \text{ m}^2 \text{s}^{-1}$

Transient heat transfer in infinite geometries

1.
 - (i) What is the physical significance of the Fourier number? Will the Fourier number for a specified heat transfer problem double when the time is doubled?
 - (ii) How can the transient temperature charts be used when the surface temperature of geometry is specified instead of the temperature of the surrounding medium and the convection heat transfer coefficient.
 - (iii) A body at an initial temperature of T_i is brought into a medium at a constant temperature of T . How will you determine the maximum possible amount of heat transfer between the body and the surrounding medium?
2. Annealing is a process by which steel is reheated and then cooled to make it less brittle. Consider reheat of a steel plate 100 mm thick, initially at a uniform temperature of 200°C and is to be reheated to a minimum temperature of 550°C. Heating is effected in a gas-fired furnace where products of combustion at a steady temperature of 800°C maintain a convection heat transfer coefficient of 250 Wm⁻² K⁻¹, on both surfaces of the steel plate. Determine how long the plate should be left in the furnace. Thermophysical properties of steel are: $\rho = 7830 \text{ kg m}^{-3}$, $C_p = 550 \text{ J kg}^{-1} \text{K}^{-1}$, $k = 48 \text{ W m}^{-1} \text{K}^{-1}$.
3. A long rod 40 mm in diameter, fabricated from aluminum oxide and initially at a uniform temperature of 800 K, is suddenly cooled by a fluid at 300 K having a heat transfer coefficient of 1600 Wm⁻² K⁻¹. After 35 s, the rod is wrapped in insulation and experiences no heat losses. Determine the temperature of the rod after a long period of time. Properties of Aluminium oxide: $k = 50 \text{ Wm}^{-1} \text{K}^{-1}$, $C_p = 500 \text{ J kg}^{-1} \text{K}^{-1}$ and $\rho = 7800 \text{ kg m}^{-3}$
4. In a process to manufacture glass beads ($k = 1.4 \text{ Wm}^{-1} \text{K}^{-1}$, $\rho = 2200 \text{ kg m}^{-3}$, $C_p = 800 \text{ J kg}^{-1} \text{K}^{-1}$) of 3-mm diameter, the beads are suspended in an upwardly directed airstream that is at a temperature of 15°C and maintains a convection coefficient of $h = 400 \text{ W m}^{-2} \text{K}^{-1}$. If the beads are at an initial temperature $T_i = 477^\circ\text{C}$, how long must they be suspended to achieve a center temperature of 80°C? What is the corresponding surface temperature?
5. An ordinary egg can be approximated as a 5.5-cm-diameter sphere whose properties are roughly $k = 0.6 \text{ W m}^{-1} \text{C}^{-1}$ and $\alpha = 0.14 \times 10^{-6} \text{ m}^2 \text{s}^{-1}$. The egg is initially at a uniform temperature of 8°C and is dropped into boiling water at 97°C. Taking the convection heat transfer coefficient to be $h = 1400 \text{ Wm}^{-2} \text{C}^{-1}$, determine how long it will take for the center of the egg to reach 70°C.

Transient heat transfer in semi-infinite solid

- 6.
- What is a semi-infinite medium? Give examples of solid bodies that can be treated as semi-infinite mediums for heat transfer purposes.
 - A thick wood slab ($k=0.17 \text{ W/m}^\circ\text{C}$ and $\alpha = 1.28 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$) that is initially at a uniform temperature of 25°C is exposed to hot gases at 550°C for a period of 5 minutes. The heat transfer coefficient between the gases and the wood slab is $35 \text{ W m}^{-2} \text{ }^\circ\text{C}^{-1}$. If the ignition temperature of the wood is 450°C , determine if the wood will ignite. Calculate the total heat transferred in 5 minutes.

Transient Heat Transfer in solids of finite length

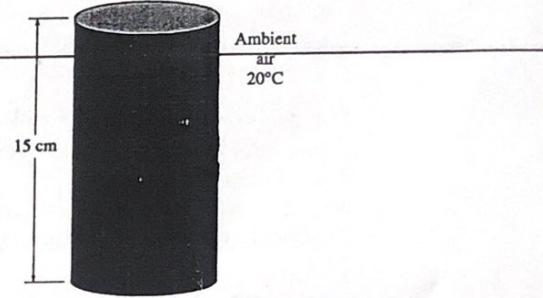
7. (a) A short cylinder, initially at a uniform temperature T_i , has both top and bottom surfaces insulated. The cylinder is subjected to convection heating from its side surface to a medium at temperature T_∞ with a heat transfer coefficient of h . Is the heat transfer in this short cylinder one- or two-dimensional? Explain.

(b) A short brass cylinder 8 cm diameter with a height of 15 cm is initially at a uniform temperature of 150°C . The cylinder is now placed in atmospheric air at 20°C , where heat transfer takes place by convection with a heat transfer coefficient of $h=40 \text{ W m}^{-2} \text{ K}^{-1}$. Calculate:

- the center temperature of the cylinder,
- the center temperature at the top surface of the cylinder, and
- the total heat transfer from the cylinder 15 min after the start of the cooling.

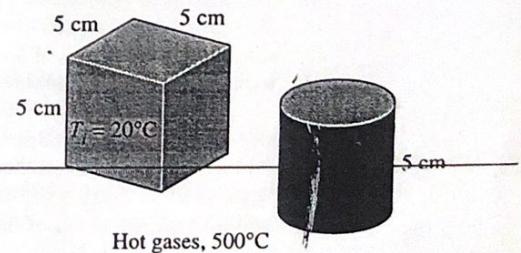
Data:

(density = 8530 kg/m^3 , $C_p = 0.389 \text{ kJ kg}^{-1} \text{ K}^{-1}$, $k = 110 \text{ W m}^{-1} \text{ K}^{-1}$, and $\alpha = 3.39 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$)



Mixed problems on transient heat transfer

8. An apple, diameter 0.06 m at an initial temperature of 30°C , is placed in a refrigerator held at a temperature of 4°C . The convective h.t.c. at the surface of the apple is $57 \text{ W m}^{-2} \text{ K}^{-1}$. Estimate how long it takes for the centre of the apple to reach 5°C . Thermophysical properties of apple are: $C_p = 3888 \text{ J kg}^{-1} \text{ K}^{-1}$, $k = 0.6 \text{ W m}^{-1} \text{ K}^{-1}$ and $\rho = 1010 \text{ kg m}^{-3}$.
9. Canned tomato puree, 0.08 m in diameter and 0.18 m long, is to be sterilized in a steam chamber. The initial temperature of the tomato puree is 60°C , and it has been established that to give the specified shelf-life the canned product must be heated to a centre temperature of 150°C and held at that temperature for 1 minute. Steam is supplied to the chamber at 240°C and the convective heat transfer coefficient at the surface of the can is $15 \text{ W m}^{-2} \text{ K}^{-1}$. Assuming the resistance of the metal can is negligible, calculate:
- the residence time required in the steam chamber,
 - the surface temperature of the tomato puree at the end of the process, and
 - the total heat transferred to sterilise the can.
- Thermophysical properties of tomato puree are: $C_p = 4100 \text{ J kg}^{-1} \text{ K}^{-1}$, $k = 0.64 \text{ W m}^{-1} \text{ K}^{-1}$ and $\rho = 1050 \text{ kg m}^{-3}$.
10. Consider a cubic block whose sides are 5 cm long and a cylindrical block whose height and diameter are also 5 cm. Both blocks are initially at 20°C and are made of granite ($k = 2.5 \text{ W m}^{-1} \text{ K}^{-1}$ and $\alpha = 1.15 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$). Now both blocks are exposed to hot gases at 500°C in a furnace on all of their surfaces with a heat transfer coefficient of $40 \text{ W m}^{-2} \text{ K}^{-1}$. Determine the center temperature of each geometry after 20 minutes.



Further Practice exercise: Heat exchangers

1. Hot oil is used to heat water in a double-pipe heat exchanger. The water flows at a rate of 0.1 kg s^{-1} and is heated from 40°C to 80°C . If the hot oil enters the exchanger at 105°C and leaves at 75°C ,
 - (a) Determine the heat transfer area for
 - (i) co-current flow
 - (ii) counter-current flow
 - (b) Which of the two flow arrangements of part (a) you would recommend? Give reasons.
 - (c) A shell and tube heat exchanger is substituted for the double pipe one. If all other conditions remain constant, calculate the area of shell and tube exchanger that would be required (You may use the chart for correction factor, F_c).

Physical properties of oil:

-
2.
 - (a) Sketch a typical curve to illustrate how heat flux changes with increasing excess temperature in pool boiling. Label the main regions of the curve.
 - (b) A vertical tube is 1 m long and has an outer diameter of 8 cm. The outer surface of the tube is exposed to steam at atmospheric pressure, and it is maintained at a temperature of 50°C by the flow of cooling water through the tube. Calculate:
 - (i) the rate of heat transfer to the coolant
 - (ii) the rate of steam condensation on the outer surface of the tube.

DATA

Properties of steam and water at the operating conditions may be obtained from steam tables.

3. A hot gas at 250°C flows through a horizontal pipe whose internal diameter is 15 cm and outer diameter is 17.5 cm. The pipe is lagged with a layer of 4.5 cm thickness of 85% magnesium insulation and the outside air temperature is 15°C .

- (a) Calculate:
 - (i) the overall heat transfer coefficient
 - (ii) the heat loss per unit area through the pipe wall (heat flux)
 - (iii) the external surface temperature of the lagging.
 - (b) If a layer of dirt develops inside the pipe, explain its effect on the rate of heat transfer.

DATA

- a. Thermal conductivities:

$$(i) \text{ of pipe : } 44.5 \text{ W m}^{-1} \text{ K}^{-1} \quad (ii) \text{ of 85\% magnesium: } 0.058 \text{ W m}^{-1} \text{ K}^{-1}$$

-
- b. Inside film heat transfer coefficient (based on inside area of the pipe): $22.8 \text{ W m}^{-2} \text{ K}^{-1}$
 - c. Combined radiation and convection external heat transfer coefficient: $20 \text{ W m}^{-2} \text{ K}^{-1}$
(based on inside area of the pipe)

4. Water is to be boiled at atmospheric pressure, in a copper tube of 25 mm outside diameter. If the temperature of the copper surface is 210°C , calculate the boiling film coefficient and the heat flux.
5. A vertical tube condenser is to be used to condense vapour flowing at 0.1 kg s^{-1} , which enters at atmospheric pressure. Condenser uses cooling water flowing through 2 m long tubes at an average temperature of 25°C . Assuming the tubes are clean and resistance of the tube wall are negligible, how many tubes will be required?

DATA

For the tubes:	$d_o = 28 \text{ mm}$	$d_i = 24 \text{ mm}$
Condensing temperature of the vapour		$T_{\text{sat}} = 78.4^{\circ}\text{C}$
Latent heat of vaporization of vapour		$h_{fg} = 900 \text{ J g}^{-1}$
Density of condensate		$\rho_l = 700 \text{ kg m}^{-3}$

-
6. A pipeline 50 cm OD carrying hot oil at 30°C , is exposed to an ambient temperature of 20°C . A 5 cm thick layer of special insulation with thermal conductivity $0.007 \text{ W m}^{-1} \text{ k}^{-1}$ surrounds the pipe. The convective heat transfer coefficient on the outside of the insulated pipe is $12 \text{ W m}^{-2} \text{ k}^{-1}$. Determine the heat loss per metre length from the pipe. State any assumptions you make.

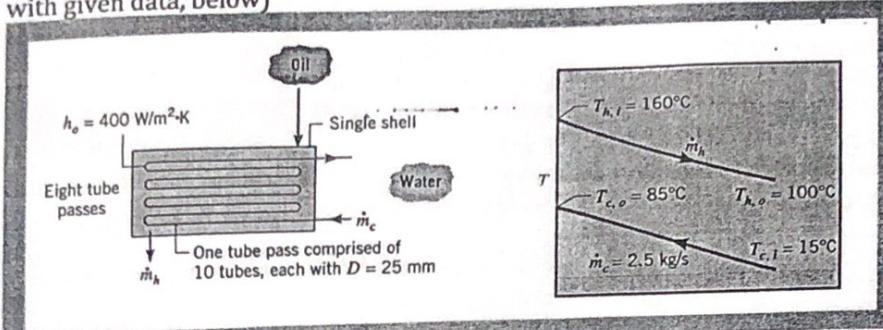
Properties for oil:

7. In a heat exchanger design calculation, the logarithmic mean temperature difference, ΔT_{LM} is used rather than the arithmetic mean temperature difference ΔT . Explain the difference between these two types of mean temperature differences. You may use an appropriate example.

Tutorial: Heat exchanger

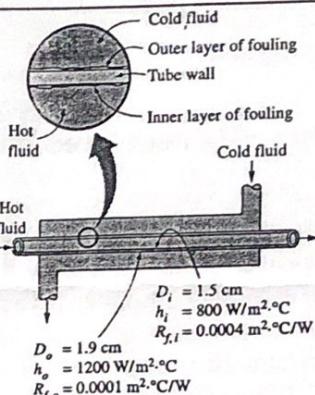
1.

A shell-and-tube heat exchanger must be designed to heat 2.5 kg/s of water from 15 to 85°C. The heating is to be accomplished by passing hot engine oil, which is available at 160°C, through the shell side of the exchanger. The oil is known to provide an average convection coefficient of $h_o = 400 \text{ W/m}^2 \text{ K}$ on the outside of the tubes. Ten tubes pass the water through the shell. Each tube is thin walled, of diameter $D = 25 \text{ mm}$, and makes eight passes through the shell. If the oil leaves the exchanger at 100°C, what is its flow rate? How long must each tube be to accomplish the desired heating? (See the schematic diagram with given data, below)



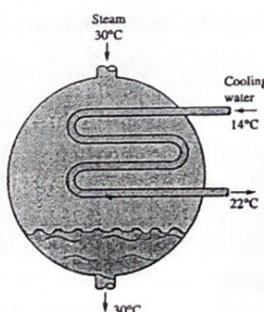
2.

A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel ($k = 15.1 \text{ W/m} \cdot {}^\circ\text{C}$) inner tube of inner diameter $D_i = 1.5 \text{ cm}$ and an outer shell of inner diameter 3.2 cm . The convection heat transfer coefficient is given to be $h_i = 800 \text{ W/m}^2 \cdot {}^\circ\text{C}$ on the inner surface of the tube and $h_o = 1200 \text{ W/m}^2 \cdot {}^\circ\text{C}$ on the outer surface. For a fouling factor of $0.0004 \text{ m}^2 \cdot {}^\circ\text{C/W}$ on the tube side and $0.0001 \text{ m}^2 \cdot {}^\circ\text{C/W}$ on the shell side, determine (a) the thermal resistance of the heat exchanger per unit length and (b) the overall heat transfer coefficients, U_i and U_o based on the inner and outer surface areas of the tube, respectively.



3.

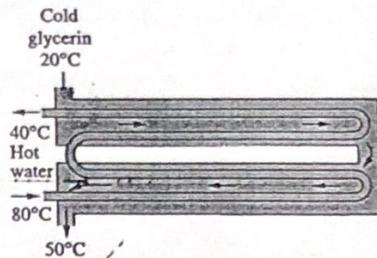
Steam in the condenser of a power plant is to be condensed at a temperature of 30°C with cooling water from a nearby lake, which enters the tubes of the condenser at 14°C and leaves at 22°C, (as shown in the schematic diagram). The surface area of the tubes is 45 m², and the overall heat transfer coefficient is 2100 W/m² · °C. Determine the mass flow rate of the cooling water needed and the rate of condensation of the steam in the condenser.



4. A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m² · °C, determine the length of the heat exchanger required to achieve the desired heating.

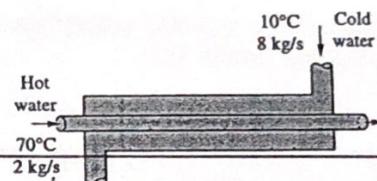
5. Effect of fouling

2-shell passes and 4-tube passes heat exchanger is used to heat glycerin from 20°C to 50°C by hot water, which enters the thin-walled 2-cm-diameter tubes at 80°C and leaves at 40°C (see figure). The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 W/m²·K on the glycerin (shell) side and 160 W/m²·K on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (a) before any fouling occurs and (b) after fouling with a fouling factor of 0.0006 m² · °C / W occurs on the outer surfaces of the tubes.



Schematic diagram for Q5

6. Cold water enters a counter-flow heat exchanger at 10°C at a rate of 8 kg/s, where it is heated by a hot water stream that enters the heat exchanger at 70°C at a rate of 2 kg/s. Assuming the specific heat of water to remain constant at $C_p = 4.18 \text{ kJ/kg} \cdot ^\circ\text{C}$, determine the maximum heat transfer rate and the outlet temperatures of the cold and the hot water streams for this limiting case.

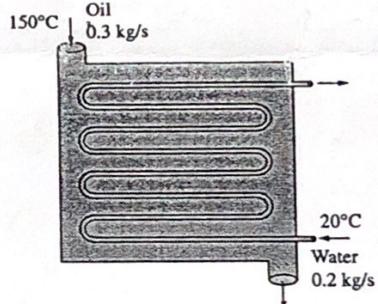


Schematic diagram for Q6

7. Repeat Q4 using the effectiveness – NTU method

8.

Hot oil is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is 310 W/m² · °C. Water flows through the tubes at a rate of 0.2 kg/s, and the oil through the shell at a rate of 0.3 kg/s. The water and the oil enter at temperatures of 20°C and 150°C, respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.



Schematic diagram for Q8

9. In a dairy plant, milk is pasteurized by hot water supplied by a natural gas furnace. The hot water is then discharged to an open floor drain at 80°C at a rate of 15 kg/min. The plant operates 24 h a day and 365 days a year. The furnace has an efficiency of 80 percent, and the cost of the natural gas is \$0.40 per therm (1 therm = 105,500 kJ). The average temperature of the cold water entering the furnace throughout the year is 15°C. The drained hot water cannot be returned to the furnace and re-circulated, because it is contaminated during the process.

In order to save energy, installation of a water-to-water heat exchanger to pre-heat the incoming cold water by the drained hot water is proposed. Assuming that the heat exchanger will recover 75 percent of the available heat in the hot water, determine the heat transfer rating of the heat exchanger that needs to be purchased and suggest a suitable type. Also, determine the amount of money this heat exchanger will save the company per year from natural gas savings.

Worked Example

A shell and tube heat exchanger is used to heat water (in the tube side) from 30°C to 45°C at a mass flow rate of 4 kg s⁻¹. The fluid used for heating (shell side) is water (entering temperature of 90°C) with a mass flow rate of 2 kg s⁻¹. A single shell pass is utilized. The overall heat transfer coefficient (based on inside tube area) is 1390 W/m²·°C.

Tubes are 1.875 cm (inside diameter) and require an average water velocity of 0.375 m/sec. Available unit floor space limits the tube length to 1.75 m.

For this heat exchanger find: (i) the number of passes, (ii) number of tubes per pass, and (iii) tube length.

Solution:

Given: $T_{c,i} = 30^\circ\text{C}$ $T_{c,o} = 45^\circ\text{C}$ $m_c = 4 \text{ kg s}^{-1}$ $u_{l,av} = 0.375 \text{ ms}^{-1}$ $D_l = 1.875 \text{ cm}$
 tube length = 1.75 m
 $T_{h,i} = 90^\circ\text{C}$ $m_h = 2 \text{ kg s}^{-1}$.
 $U = 1390 \text{ W m}^{-2} \text{ °C}^{-1}$ (based on tube side)

Find:

Number of tube passes (ii) number of tubes per pass (iii) tube length

Step 1: From energy balance (i.e. 1st law of thermodynamics) determine the exit temperature of heating fluid:

$T_{\text{mean},c} = 0.5(45+30) = 37.5^\circ\text{C}$. At this temp, $C_p \text{ water} = 4174 \text{ J kg}^{-1} \text{ °C}^{-1}$. Density = 993 kg m⁻³

(assume: same C_p for hot fluid)

$$(mC_p)_c \Delta T_c = (mC_p)_h \Delta T_h \Rightarrow \Delta T_h = \frac{(mC_p)_c \Delta T_c}{(mC_p)_h} = \frac{4 \times 4174 \times (45-30)}{2 \times 4174} = 30^\circ\text{C}$$

$$\therefore \Delta T_h = 90 - T_{h,o} = 30 \Rightarrow \therefore T_{h,c} = 90 - 30 = 60^\circ\text{C}$$

Step 2: Heat Transfer for the system: $q = (mC_p \Delta T)_c = 4 \times 4174 \times 15 = 250.4 \text{ kW}$

Step 3: Determine the L.M.T.D. for an equivalent counter-current flow system:

Step 4: Determine the total surface area required for the counter-current flow system:

$$q = UA \Delta T_{lm} \Rightarrow \therefore A = \frac{q}{U \Delta T_{lm}} = \frac{250.4 \times 1000}{1390 \times 37} = 4.87 \text{ m}^2$$

Step 5: Now determine the cross-sectional area required, using the relationship between mass flow rate and average velocity:

$$\text{mass flow rate} = \rho u A_{\text{section}} \Rightarrow \therefore A_{\text{section}} = \frac{\text{mass flow rate}}{\rho u} = \frac{4}{993 \times 0.37} = 0.0107 \cdot m^2$$

Step 6: Calculate no of tubes required: $n = (\text{required cross section area}) / (\text{cross section area of 1 tube})$

$n = 38.75$ tubes. Therefore, take $n=39$ tubes.

Step 7: Calculate tube length, L :

$$L = \frac{\text{Total surface area}}{\text{number of tubes} \times \text{perimeter of each tube}} = \frac{4.87}{39 \times (\pi D)} = \frac{4.87}{39 \times 0.01875 \times \pi} = 2.12 \text{ m}$$

This length exceeds the length limit of 1.875 m stipulated. In order to get a workable system. we go up to a two-tube pass system.

Calculate the parameters of Figure:

From figure ... we obtain an F value = 0.945.

Thus the actual surface area for heat transfer is:

$$q = FUA\Delta T_{lm} \Rightarrow A = \frac{q}{FUA\Delta T_{lm}} = \frac{4.87}{0.945} = 5.15 \text{ m}^2$$

and the length of each tube is:

$$L = \frac{\text{Total surface area}}{\text{number of tubes} \times \text{perimeter of each tube}} = \frac{5.15}{2 \times 39 \times (\pi D)} = \frac{5.15}{2 \times 39 \times 0.01875 \times \pi} = 2.12 \text{ m}$$

This length is within the stipulated range.

Ans: the final system is a 2-tube passes HX with 39 tubes each of length 1.12 m