



1 Solution to the simple siphon problem

Problem: Find a) The velocity v of the fluid in the tube and b) the pressure at the top of the tube.

Begin by applying Bernoulli's theorem to the top of the fluid in the left container AND to the bottom right hand side of the tube. The pressure at the bottom left is just $P_0 + \rho g z_2$, so at the point where fluid is entering the second tank:

$$P_0 + \rho g z_1 + \frac{1}{2} \rho v_1^2 = P_0 + \rho g z_2 + \frac{1}{2} \rho v^2 \quad (1)$$

and (solving for v , cancel P_0 , assume $v_1 \ll v$, reorganize and):

$$\frac{1}{2} \rho v^2 = \rho g (z_1 - z_2) \rightarrow v \approx \sqrt{2gd} \quad (2)$$

which solves part a).

To find the pressure P_H at the top of the tube (at height H) we have to apply B's T to the top of the left container (again) but also to the fluid in the tube at height H :

$$P_0 + \rho g z_1 + \frac{1}{2} \rho v_1^2 = P_H + \rho g H + \frac{1}{2} \rho v^2 \quad (3)$$

Rearranging and using the fact that $\frac{1}{2} \rho v^2 = \rho g (z_1 - z_2)$ and still assuming $v_1 \ll v$:

$$\begin{aligned} P_H &\approx P_0 - \rho g H + \rho g z_1 - \frac{1}{2} \rho v^2 \\ &\approx P_0 - \rho g H + \rho g z_1 - \rho g (z_1 - z_2) \\ &\approx P_0 - \rho g H + \rho g z_2 = P_0 - \rho g (H - z_2) \end{aligned} \quad (4)$$

which shows that the pressure at the top is the pressure at the bottom right, reduced $\rho g H$ by the height H . This is *precisely consistent* with our assumption

that the pressure at the right hand mouth of the tube is $P_0 + \rho g z_2$. It also follows trivially that the pressure at the *left* hand mouth of the tube is $P_0 + \rho g z_2$.

This latter result makes *sense*! The (essentially static) pressure just outside the tube at the bottom of the left hand tank is $P_0 + \rho g z_1$, and the pressure *difference* of $\rho g(z_1 - z_2)$ is what does the work of accelerating the fluid up to speed v as it enters the tube!

One interesting (and correct!) consequence of this is that there is a limit to the height over which one can siphon a fluid. The pressure in the tube cannot be negative, as (before) it reaches 0 (a vacuum) the fluid will chemically "come apart" and fall down the two columns as air pressure no longer suffices to hold it up in the tube and the picture will become that of two barometers with a shared vacuum space. We see that

$$P_{\min} \approx 0 = P_0 - \rho g(H_{\max} - z_2) \quad (5)$$

or when the difference of H and z_2 (the height of fluid in the *lower* of the two vessels) is equal to $P_0/\rho g$ (about 10 meters for water) the siphon will "break".