Lecture 3

Failures Resulting from Static Loading

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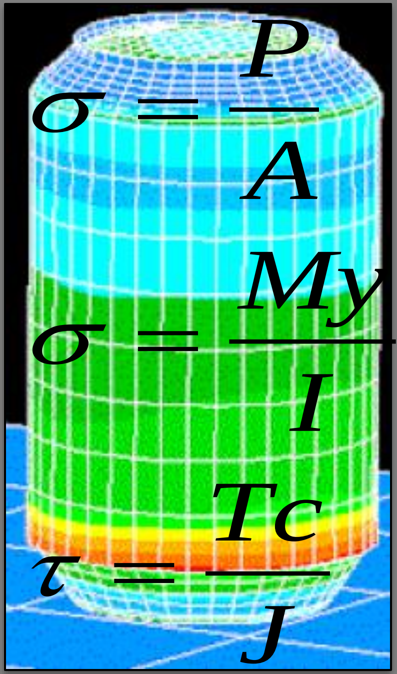


$$\sigma = \frac{P}{A}$$

$$\sigma = \frac{My}{I}$$

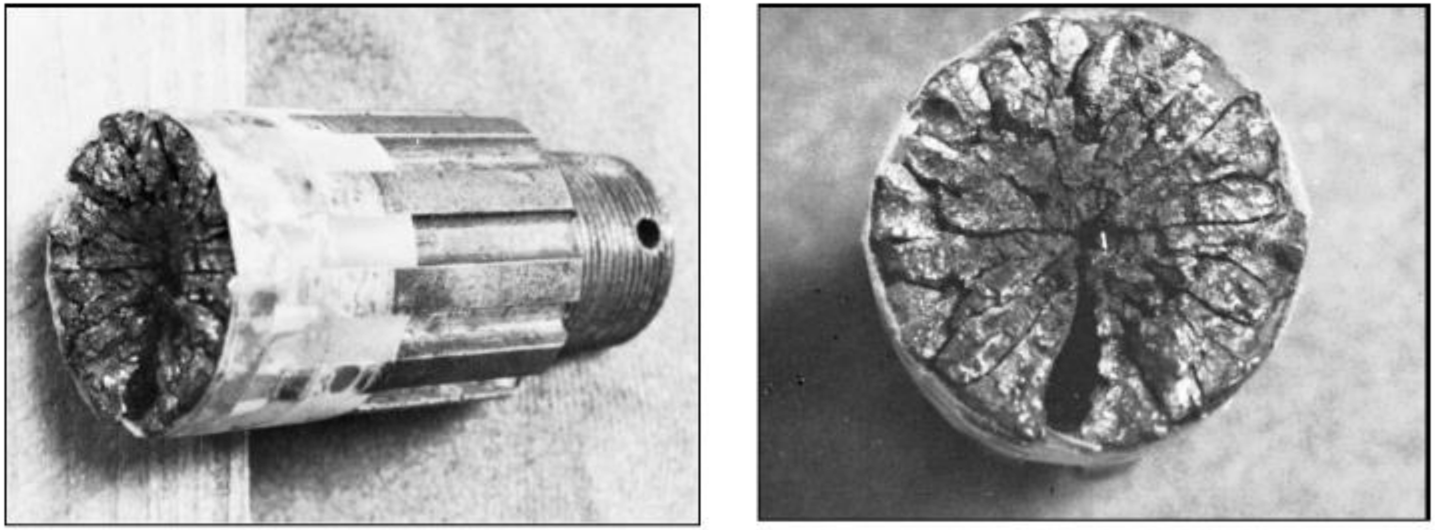
$$\tau = \frac{Tc}{J}$$

ME 274: DESIGN PROJECT II

Lecture Outline

- Introduction-Failure Examples
- Required Background-Stress Analysis
- Required Background- Design Philosophy
- Basic Design Concepts
- Ductile Versus Brittle Behavior
- Fundamental Design Equation for Ductile Failure
  - Maximum Shear Stress Theory (MSS)
  - Distortion Energy (DE) Failure Theory
  - Von-Mises Theory (VMT)
  - Ductile Coulomb Mohr Theory (DCMT)
  - Shear Strength Predictions
- Design Principle for Brittle Materials
  - Maximum Normal Stress Theory (MNST)
  - Brittle Coulomb Mohr Theory (BCMT)
  - Modified Mohr – 1 Theory (MM1T)
- Selection of Failure Criteria in Flowchart Form

Introduction-Failure Examples



(*a*) (*b*)

Fig. 5–1

- Failure of truck driveshaft spline due to corrosion fatigue

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Introduction-Failure Examples

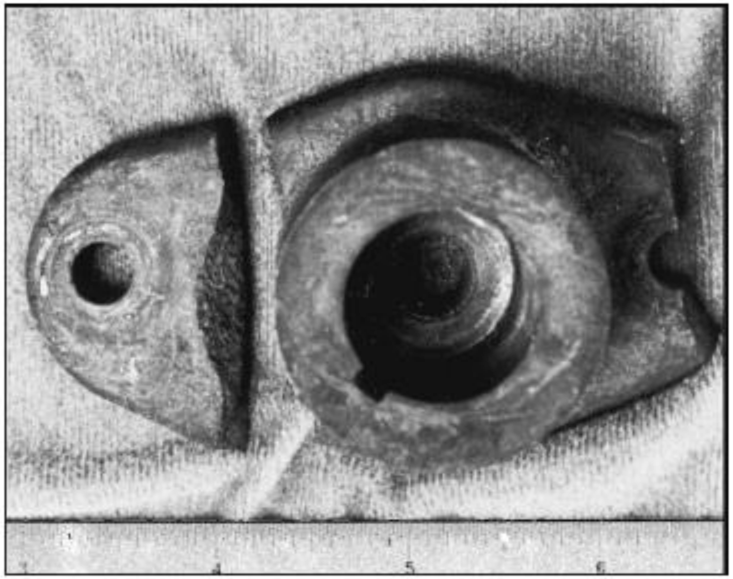

Fig. 5–2

- Impact failure of a lawn-mower blade driver hub.
- The blade impacted a surveying pipe marker.

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Introduction-Failure Examples

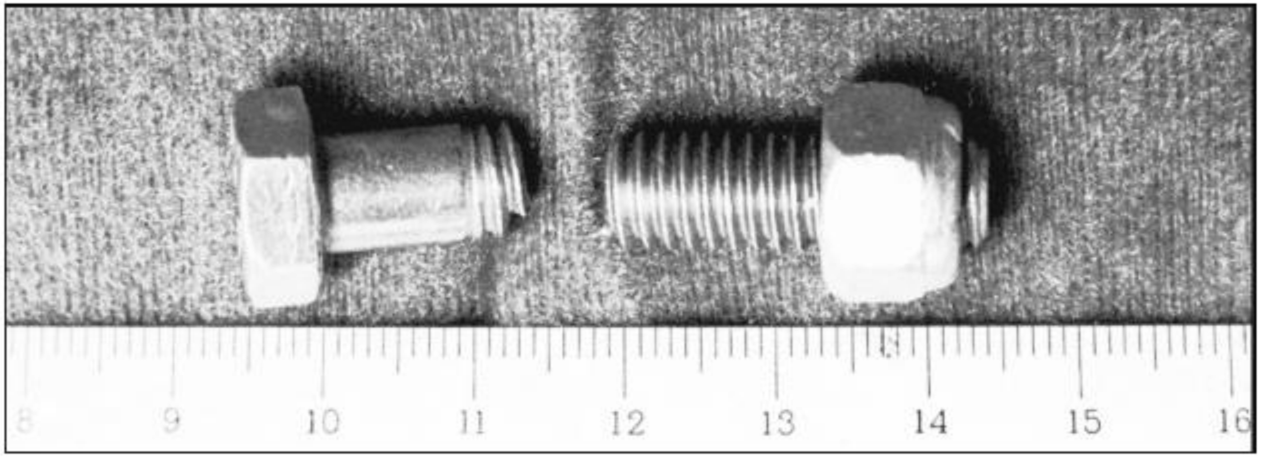

Fig. 5–3

- Failure of an overhead-pulley retaining bolt on a weightlifting machine.
- A manufacturing error caused a gap that forced the bolt to take the entire moment load.

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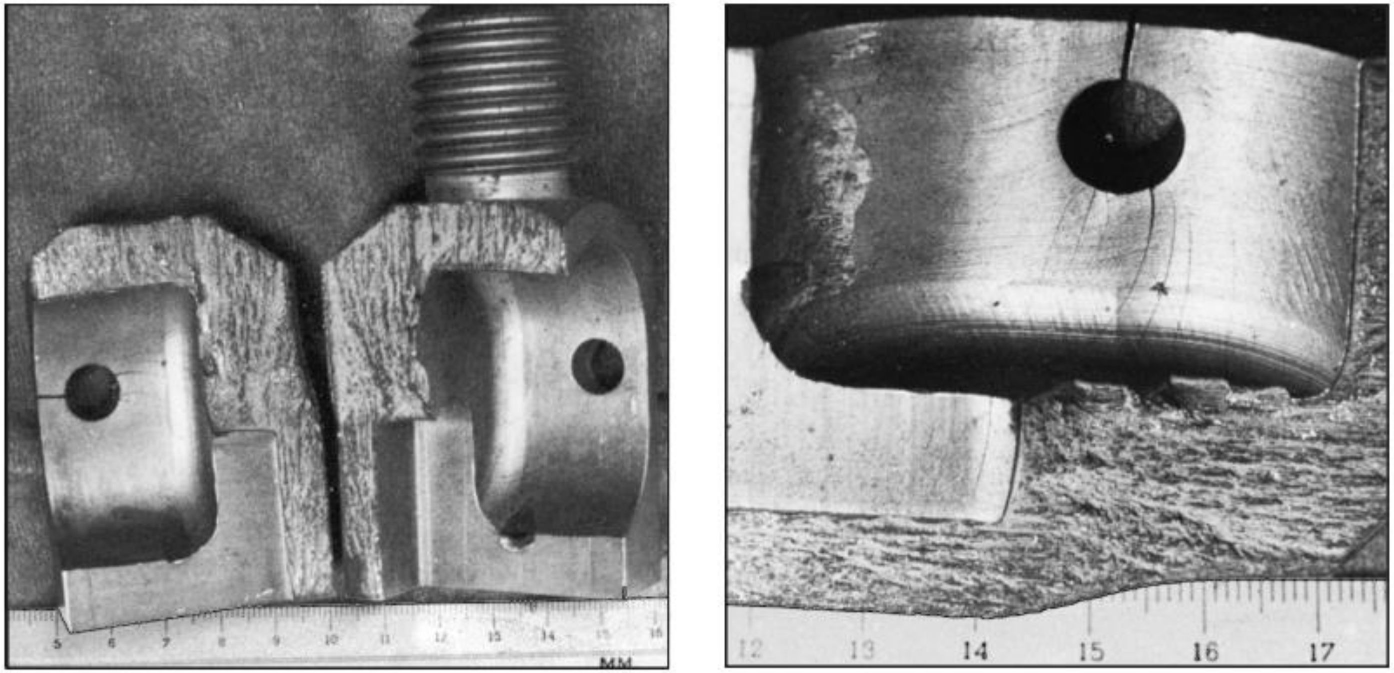
Introduction-Failure Examples



(a) (b)

Fig. 5–4

- Chain test fixture that failed in one cycle.
- To alleviate complaints of excessive wear, the manufacturer decided to case-harden the material
- (a) Two halves showing brittle fracture initiated by stress concentration
- (b) Enlarged view showing cracks induced by stress concentration at the support-pin holes

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Introduction-Failure Examples

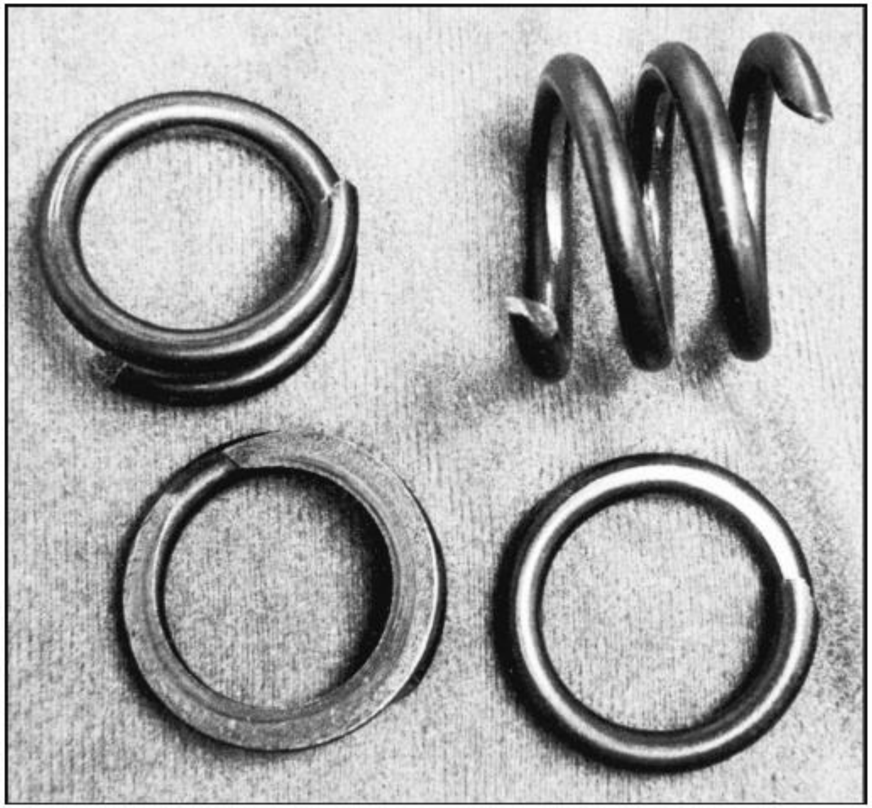

Fig. 5–5

- Valve-spring failure caused by spring surge in an oversped engine.
- The fractures exhibit the classic 45 degree shear failure

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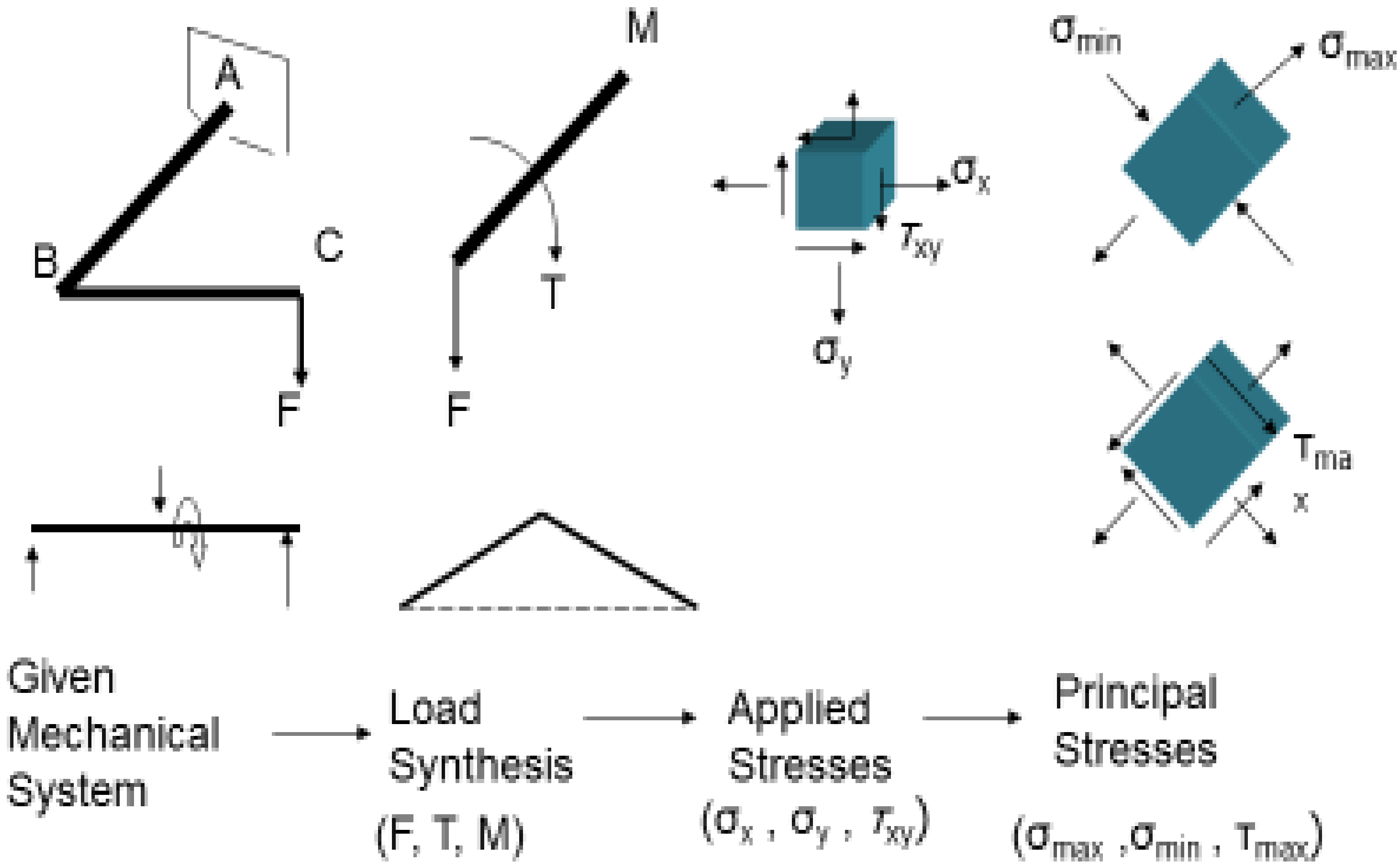
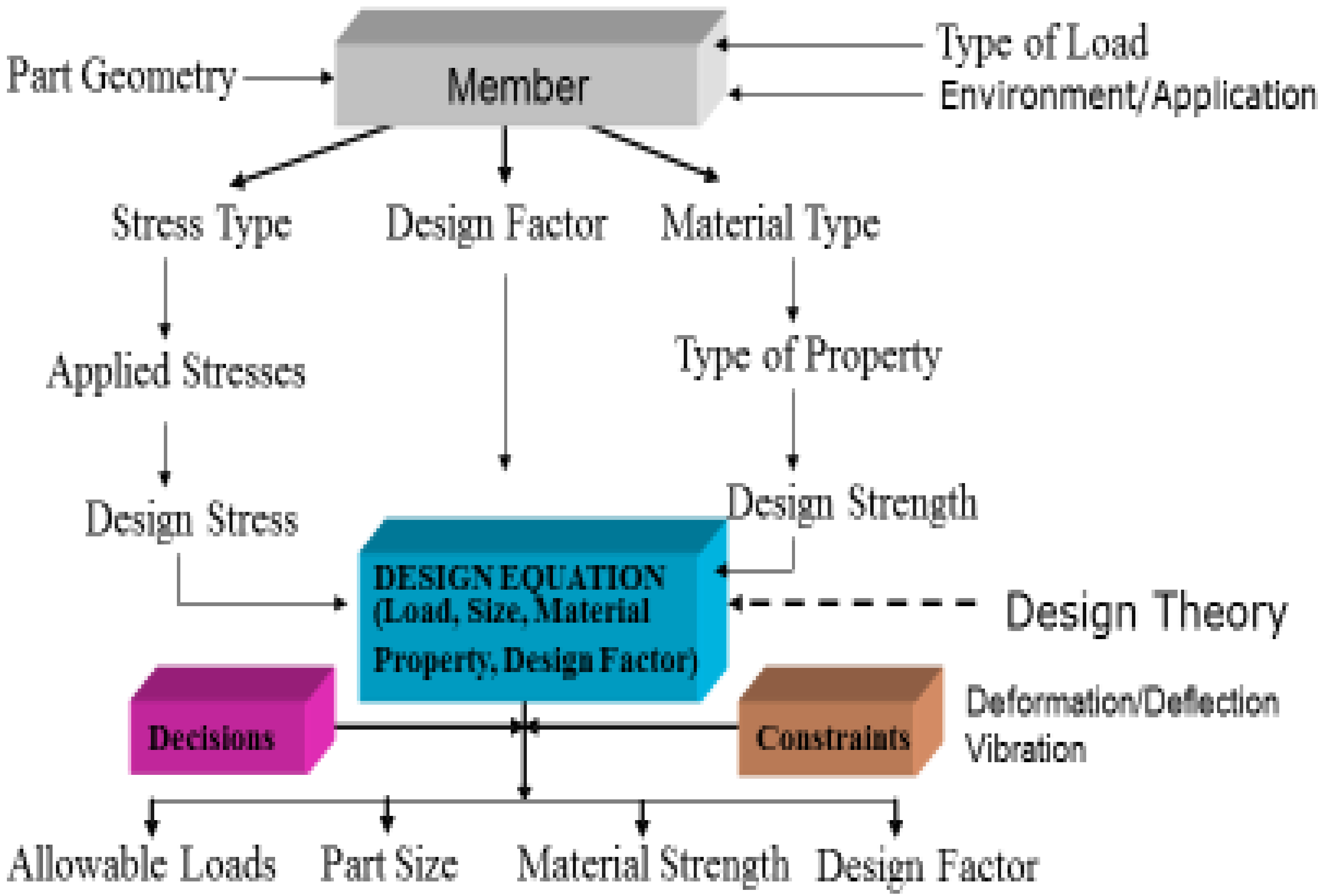
Required Background-Stress Analysis



Required Background- Design Philosophy

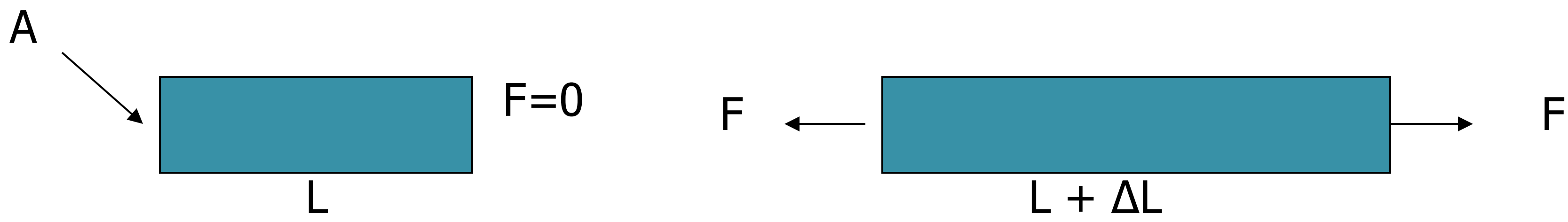


Basic Design Concepts

- Determination of a feasible part configuration (geometry), safe sizes (dimensions), and suitable material.
- *Stress* is applied to or induced in a part through loading conditions.
- *Strength* is an intrinsic property of a material. A part with no load possesses its strength.
- A part is said to have failed if it is stressed to a value that exceeds its strength.
- Failure means that the ***design stress*** in the machine member has reached the ***design strength*** of the material.

Ductile Versus Brittle Behavior

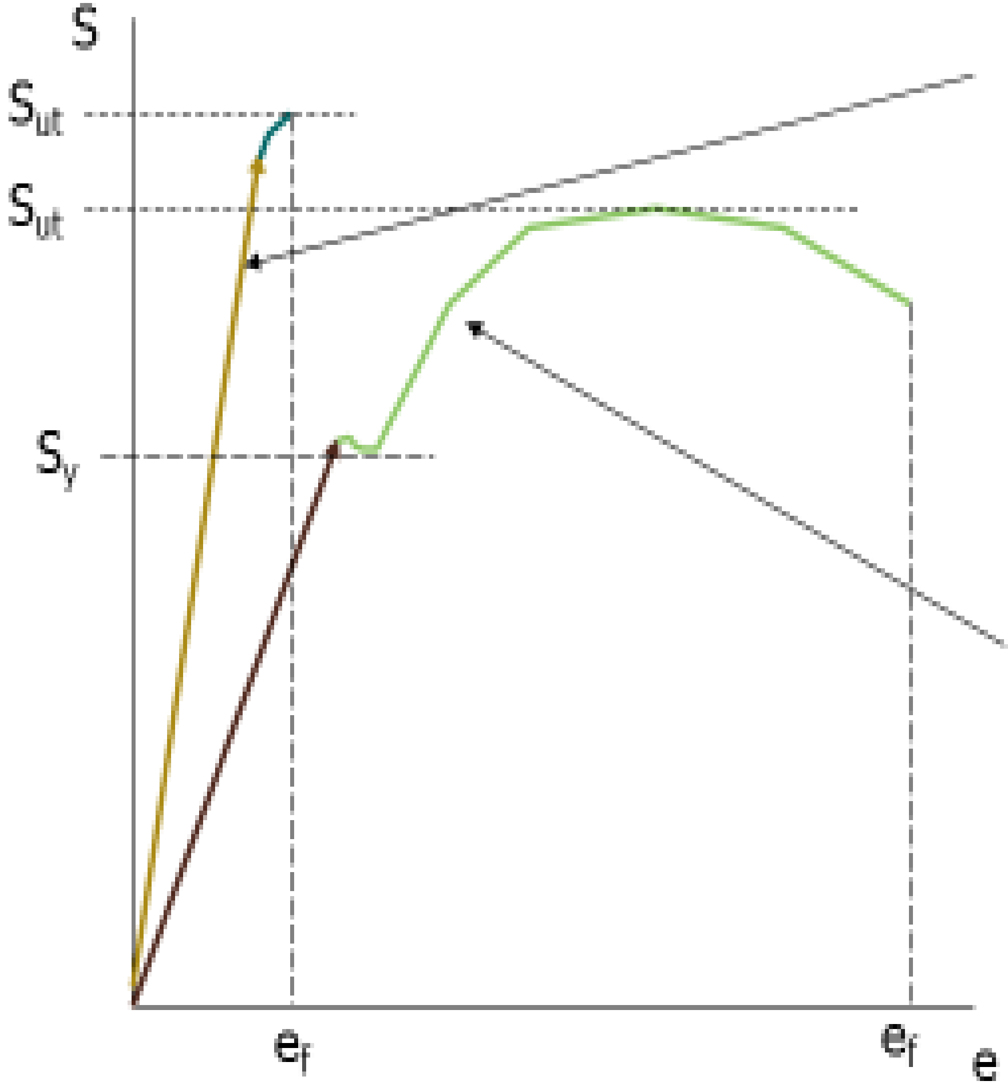
- Consider a test specimen length L and cross-section area, A
- subjected of to a tensile direct normal loading, F



Engineering (Nominal) Stress, $S = F/A$ (Ignores the change in the cross-section Area)

Engineering (Nominal) Strain, $e = \Delta L/L$

Ductile Versus Brittle Behavior



Brittle Behavior
No well defined yield point
Final Strain, $e_f < 0.05$
Ultimate Tensile Strength, S_{ut}
Ultimate Compressive Strength, $S_{uc} = (3.5 \text{ to } 4)\ S_{ut}$

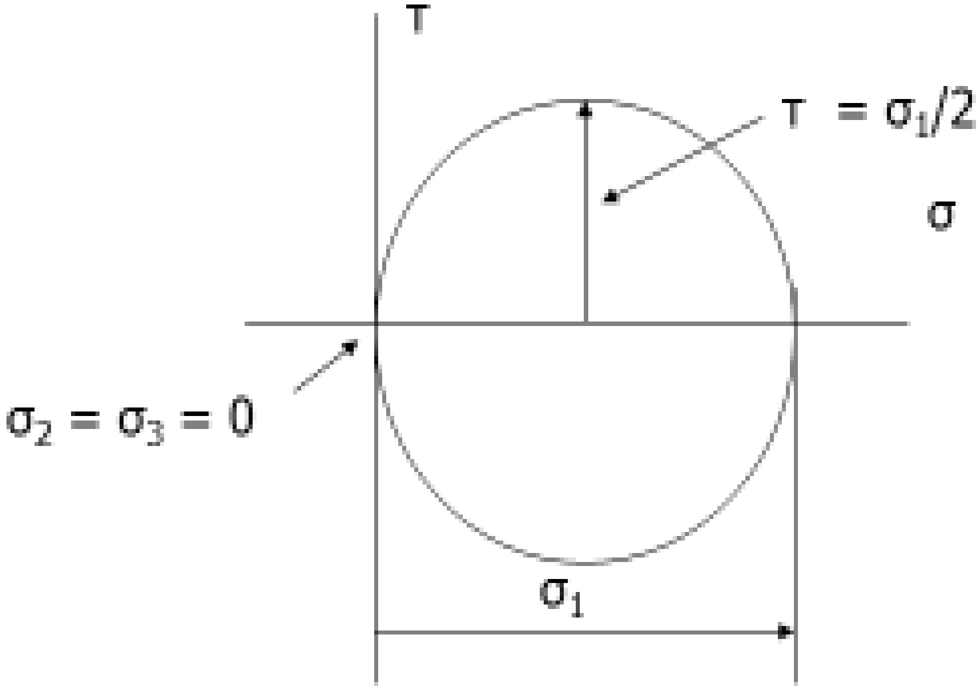
Ductile Behavior
Well defined yield point, S_y
Final Strain, $e_f \geq 0.05$
Ultimate Tensile Strength, S_{ut}
Ultimate Compressive Strength, $S_{uc} = S_{ut}$

Fundamental Design Equation for Ductile Failure

- n = design factor or factor of safety (where n>1)
- Design stress = τ_{max} (maximum shear stress)
- Design strength = S_{ys} (yield strength in shear)
- At the verge of failure: $\tau_{max} = S_{ys}$
- The part is safe when: $\tau_{max} < S_{ys}$
- $\boxed{n = S_{ys} / \tau_{max}}$ (Fundamental Design Equation)

Maximum Shear Stress Theory (MSS)

- Based on Uniaxial Tensile Strength Test



Maximum Shear Stress Theory (MSS)

- Theory:
  - Yielding begins when the *maximum shear stress* in a stress element exceeds the maximum shear stress in a tension test specimen of the same material when that specimen begins to yield.
  - For a tension test specimen, the maximum shear stress is $\sigma_1 /2$.
  - At yielding, when $\sigma_1 = S_y$, the maximum shear stress is $S_y /2$.

- Could restate the theory as follows:

- Theory:
  - Yielding begins when the *maximum shear stress* in a stress element exceeds $S_y/2$.

Maximum Shear Stress Theory (MSS)

- Ordering the principal stresses such that $\sigma_1 \geq \sigma_2 \geq \sigma_3$,

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2} \qquad \text{or} \qquad \sigma_1 - \sigma_3 \geq S_y$$

- Incorporating a factor of safety n

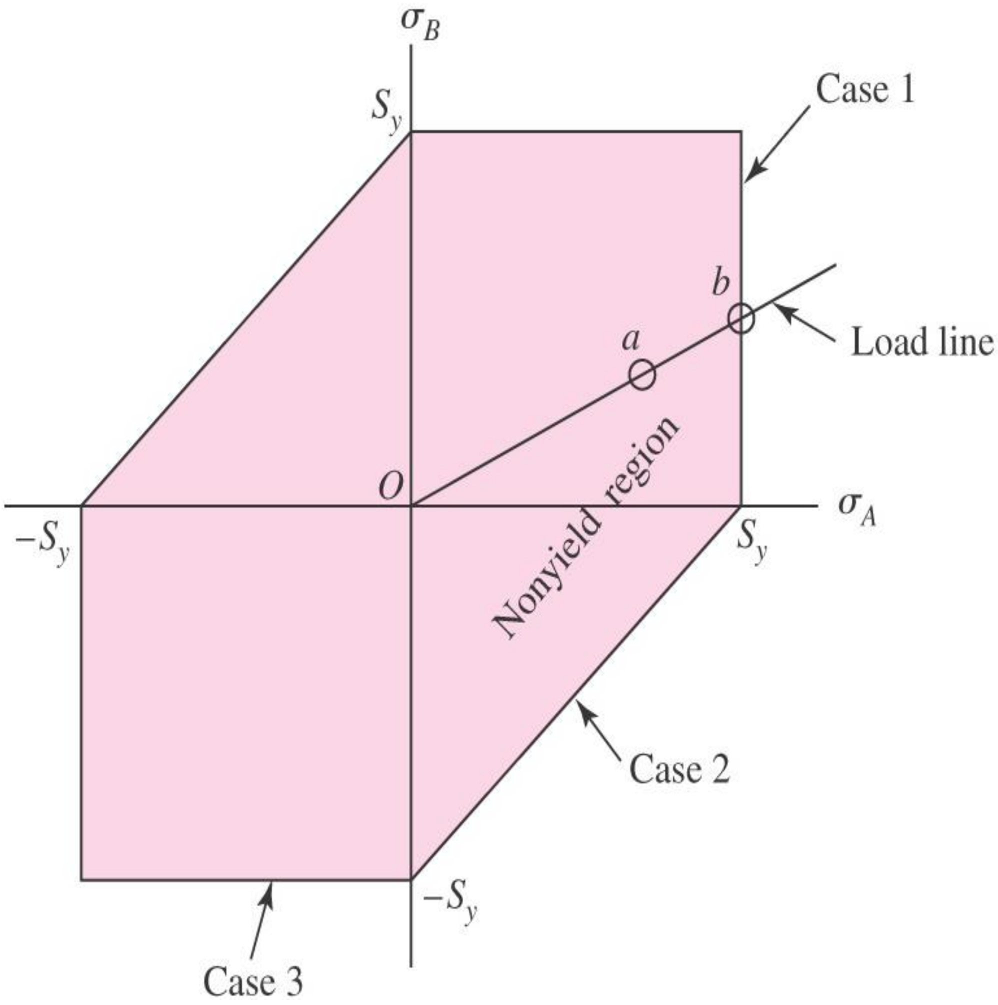
$$\tau_{\max} = \frac{S_y}{2n} \qquad \text{or} \qquad \sigma_1 - \sigma_3 = \frac{S_y}{n}$$

- Or solving for factor of safety

$$n = \frac{S_y / 2}{\tau_{\max}}$$

Maximum Shear Stress Theory (MSS)

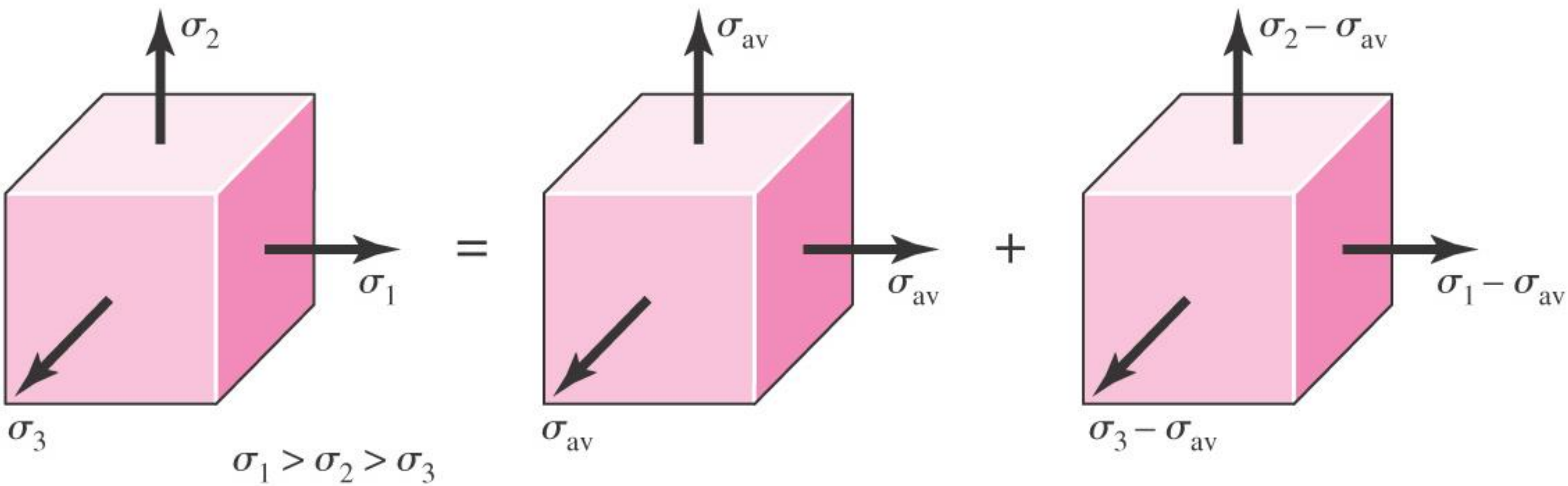
- Let σ_1 and σ_2 represent the two non-zero principal stresses, such that $\sigma_1 \geq \sigma_2$, there are three cases to consider
  - Case 1: $\sigma_1 \geq \sigma_2 \geq 0$
  - Case 2: $\sigma_1 \geq 0 \geq \sigma_2$
  - Case 3: $0 \geq \sigma_1 \geq \sigma_2$



Distortion Energy (DE) Failure Theory

- Originated from observation that ductile materials stressed hydrostatically (equal principal stresses) exhibited yield strengths greatly in excess of expected values.

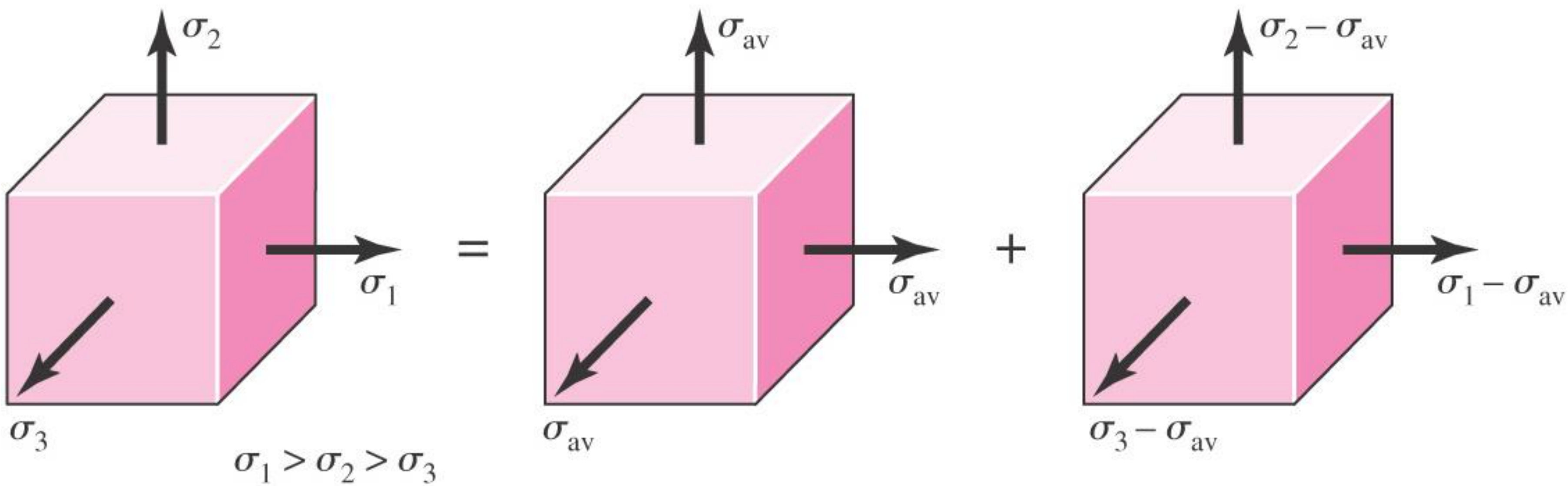
- Theorizes that if strain energy is divided into hydrostatic volume changing energy and angular distortion energy, the yielding is primarily affected by the distortion energy.



(*a*) Triaxial stresses (*b*) Hydrostatic component (*c*) Distortional component

Distortion Energy (DE) Failure Theory

- Theory:
  - Yielding occurs when the *distortion strain energy* per unit volume reaches the distortion strain energy per unit volume for yield in simple tension or compression of the same material.



(*a*) Triaxial stresses (*b*) Hydrostatic component (*c*) Distortional component

Distortion Energy (DE) Failure Theory

- Hydrostatic stress is average of principal stresses

$$\sigma_{av} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

- Strain energy per unit volume, $u = \frac{1}{2}[\epsilon_1\sigma_1 + \epsilon_2\sigma_2 + \epsilon_3\sigma_3]$

- But

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

- Therefore the strain energy equation is

$$u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

Distortion Energy (DE) Failure Theory

- Strain energy for producing only volume change is obtained by substituting σ_{av} for σ_1, σ_2, and σ_3

$$u_v = \frac{3\sigma_{av}^2}{2E}(1-2\nu)$$

- Strain energy for producing only volume change is obtained by substituting σ_1, σ_2, and σ_3 for σ_{av}

$$u_v = \frac{1-2\nu}{6E}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_3\sigma_1)$$

- Obtain distortion energy by subtracting volume changing energy, from total strain energy

$$u_d = u - u_v = \frac{1+\nu}{3E}\left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right] \quad \textbf{Eq. i}$$

Deriving the Distortion Energy

- Tension test specimen at yield has $\sigma_1 = S_y$ and $\sigma_2 = \sigma_3 = 0$

- Applying to U_d, distortion energy for tension test specimen is

$$u_d = \frac{1+\nu}{3E} S_y^2 \qquad \textbf{Eq. ii}$$

- DE theory predicts failure when distortion energy, Eq. (i), exceeds distortion energy of tension test specimen, Eq. (ii)

$$\left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2} \geq S_y \qquad \textbf{Eq. iii}$$

Distortion Energy (DE) Failure Theory

- Left hand side of Eq. iii is defined as *von Mises stress*

$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2}$$

- For plane stress, simplifies to

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$$

- In terms of *xyz* components, in three dimensions

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\right]^{1/2}$$

Distortion Energy (DE) Failure Theory

- In terms of *xyz* components, for plane stress

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$

- Distortion Energy failure theory simply compares von Mises stress to yield strength. $\sigma' \geq S_y$

- Introducing a design factor, $\sigma' = \dfrac{S_y}{n}$

- Expressing as factor of safety, $n = \dfrac{S_y}{\sigma'}$

Von-Mises Theory (VMT)

- Von-Mises Theory is derived from the maximum distortion energy theory which takes all three principal stresses into consideration.

- It is less conservative than the MSST.

- It is desirable to find Sys from a pure shear test.

- If we did this we would have the following stress condition:

$$\sigma_2 = -\sigma_1 \text{ , } \sigma_3 = 0 \text{ and } \sigma_1 = \tau max$$

Von-Mises Theory (VMT)

- Substituting this in von-Mises stress we obtain

$$\sigma_e = \sqrt{3}.\tau_{\max}$$

- The MDET then becomes

$$n = S_{yt} \Big/ \sqrt{3}\,\tau_{\max} \quad \therefore n = 0.577\, S_{yt} \Big/ \tau_{\max}$$

Ductile Coulomb Mohr Theory (DCMT)

- The above theories are good for most ductile materials where the tensile yield strength (**S_{yt}**) and compressive yield strength (**S_{yc}**) are equal.

If $S_{yt} \neq S_{yc}$.

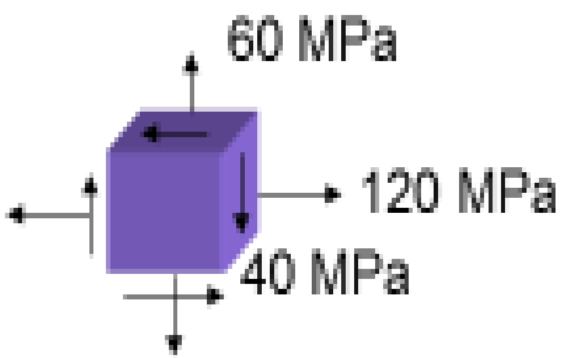
Then the DCMT suggests that $S_{ys} = S_{yt} S_{yc} / \left(S_{yt} + S_{yc} \right)$.

The design equation is $n = S_{ys} / \tau_{\max}$

Example 3-1

The stress element is obtained from the critical section of a machine member made of plain carbon steel. The part is to be designed with a factor of safety of 2.5. Specify a material for the part using:

a. Tresca theory (maximum shear stress theory)
b. Von-Mises theory, and
c. Maximum distortion



Solution

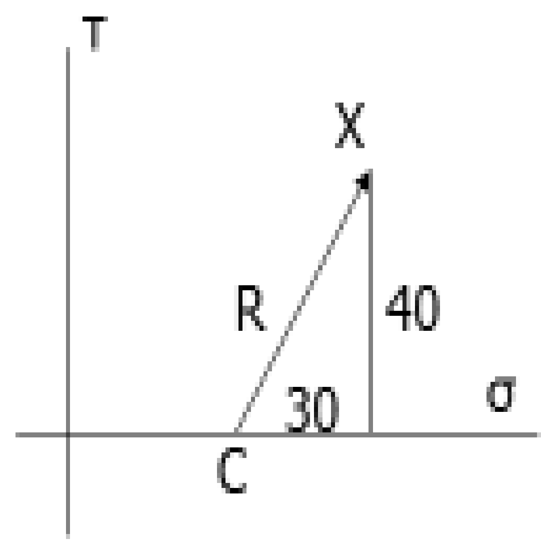
1. Stress Analysis

$\sigma_{avg} = 90$ MPa

Center, C (90,0)

Circumf. X (120,40)

R = 50 MPa



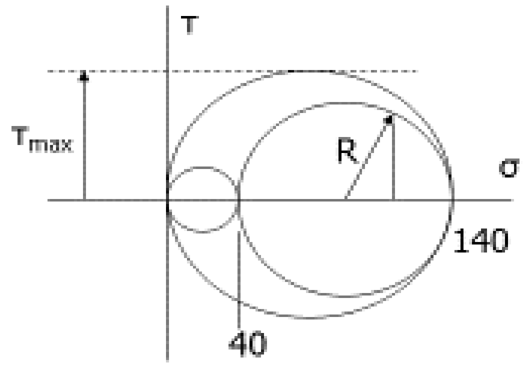
Example 3-1 (Continue)

<u>Principal Stresses:</u>

$\sigma_1 = \sigma_{avg} + R = 90 + 50 = 140$ MPa

$\sigma_1 = \sigma_{avg} - R = 90 - 50 = 40$ MPa

$\tau_{max} = \sigma_1 /2 = 70$ MPa



2. Design Equations and Solution

i. Tresca Failure Theory: $n = 0.5S_y/\tau_{max}$

Design Equation: $2.5 = 0.5S_y/70$

Solution: $S_y = 350$ MPa

ii. Von-Mises Failure Theory: $n = 0.577S_y/\tau_{max}$

Design Equation: $2.5 = 0.577S_y/70$

Solution: $S_y = 303$ MPa

Example 3-1 (Continue)

iii. Maximum Distortion Energy Theory (MDET)

Design Theory: $n = S_y / \sigma_e$

von-Mises stress: $\sigma_e = [\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2]^{1/2}$

$= [140^2 – (140)(40) + 40^2]^{½}$

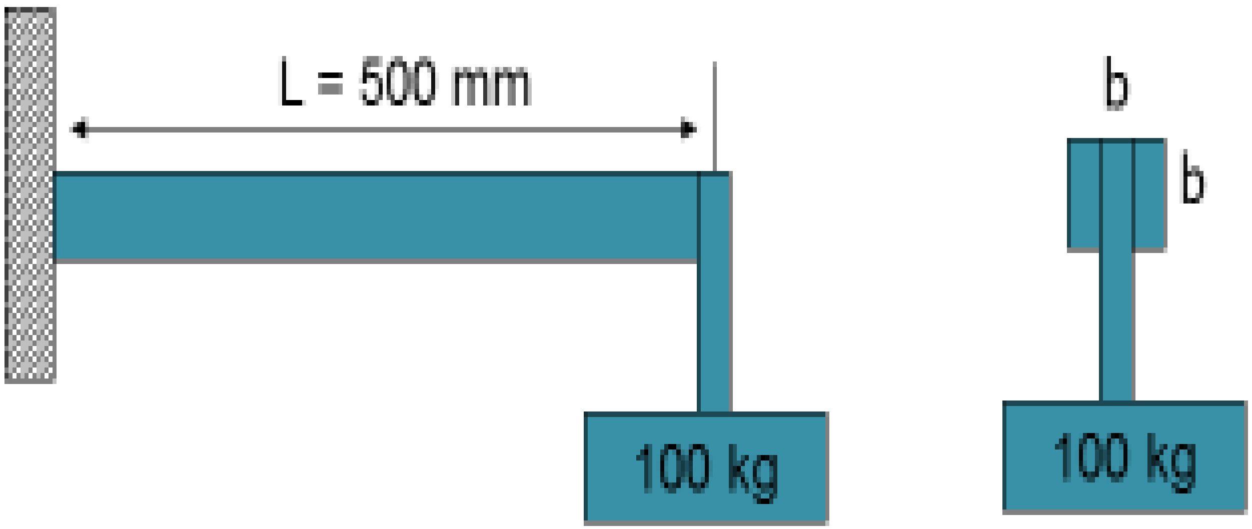
$= 125$ MPa

Design Equation: $2.5 = S_y/125$

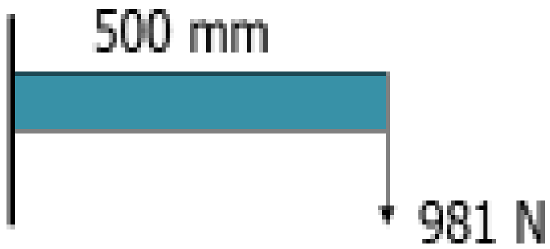
Solution: $S_y = 313$ MPa

Example 3-2

A steel rod 500 mm long and square cross-section supports a 100 kg load as shown. Complete the design of the rod by specifying the type of steel and a reasonable factor of safety.



Example 3-2 (Continue)



A. Load Analysis

$F = 100 \times 9.81$ N

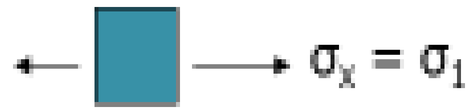
$M = M_{max} = 981 \times 500 = 490.5 \times 10^3$ N.mm

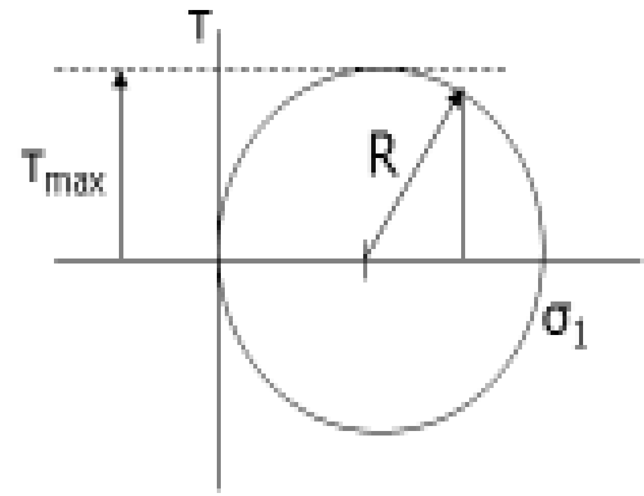
B. Stress Analysis

$\sigma_x = M/Z$ (bending)

$Z = bh^2/6 = b^3/6$ mm^3

$\sigma_x = 6M/b^3 = 2943 \times 10^3/b^3$ MPa





$\tau_{max} = \sigma_1/2 = 1472 \times 10^3/b^3$

Example 3-2 (Continue)

C. Material Type: Steel → Exact Specification is Unknown
→ Ductile Material

D. Von-Mises Design Theory: $n = 0.577S_y/\tau_{max}$

Design Equation: $n = 0.577S_y/[1472 \times 10^3/b^3]$

$n = 0.577b^3S_y/(1472 \times 10^3)$

E. Solution: Select: AISI 1040 CD Steel, $S_y = 490$ MPa
Assume: Normal Operating Conditions, $n = 2.0$
Calculate: $b = 21.84$ mm
Specify: $b = 22$ mm (standard size)
Final: $n = 2.04$ (close to desired value-accept)

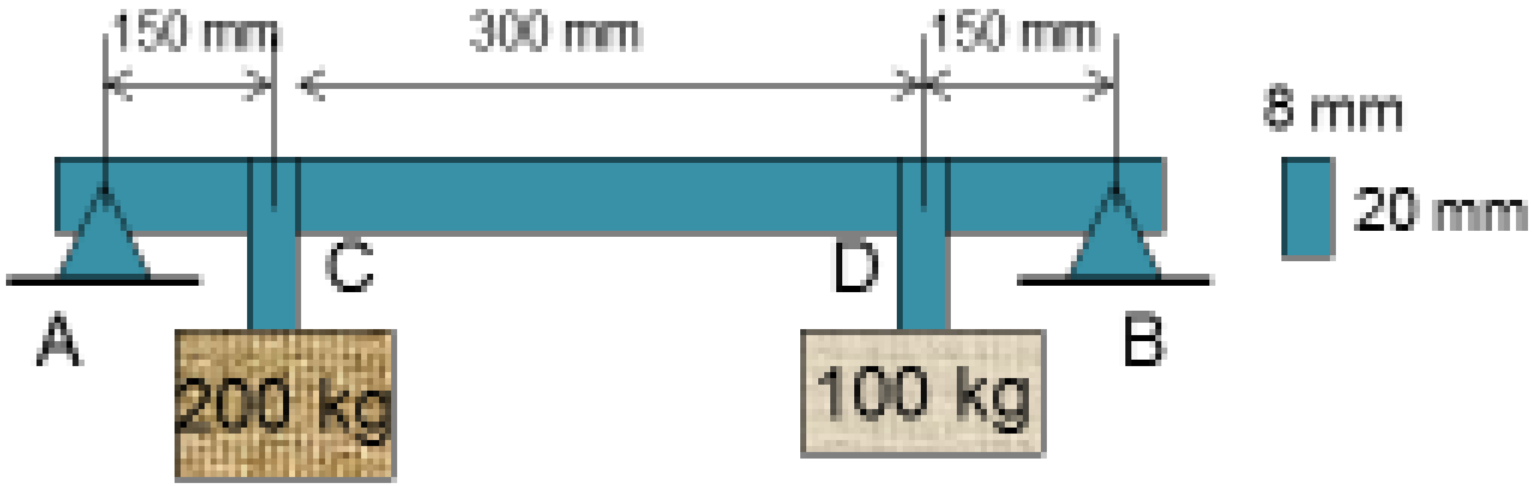
Example 3-3

Consider a simply-supported rectangular beam AB. The beam is designed for hanging loads at C and D. For the loads shown, find the factor of safety if the material is AISI CD 1040 steel.

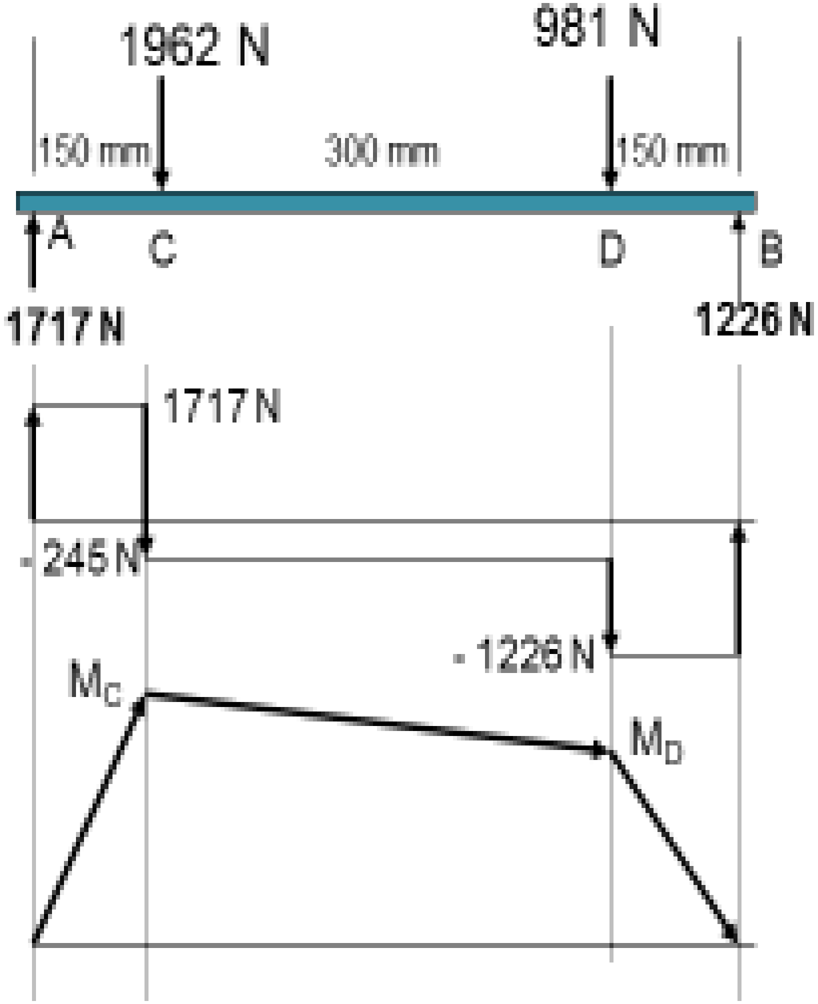
Tensile strength = 590 MPa

Yield Strength = 490 MPa

Elongation = 12%



Example 3-3 (Continue)



A. Load Analysis

Reactions: $R_A = 1717$ N

$R_B = 1226$ N

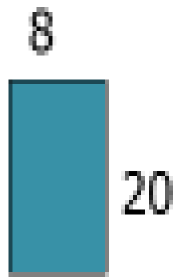
Bending Moments

$M_c = 1717 \times 150 = 257.55 \times 10^3$ N.mm

$M_D = M_C - (245 \times 300)$

$= 184.05 \times 10^3$ N.mm

B. Stress Analysis

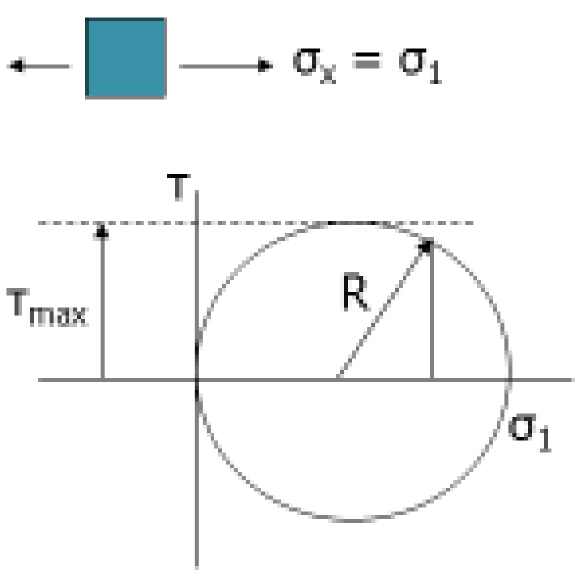


Thus $\sigma_x = 6M_c / bh^2$

$= 483$ MPa

$\tau_{xy} = 0$

Example 3-3 (Continue)



$\tau_{max} = \sigma_1 /2 = 241.5$ MPa

C. Material:
Elongation, $\varepsilon = 12\% > 5\%$
Thus failure mode will be ductile

D. von-Mises Design Theory:
$n = 0.577S_y/\tau_{max}$

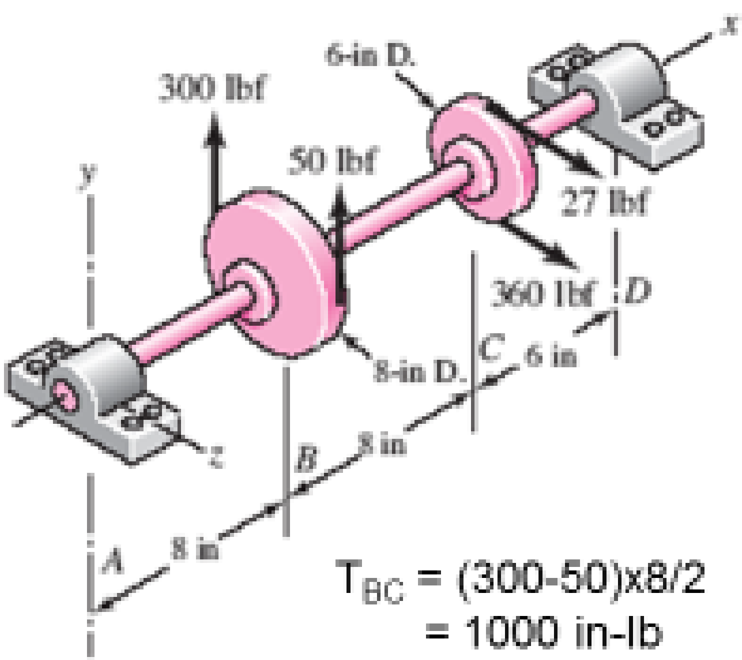
Design Equation: $n = 0.577S_y/241.5$

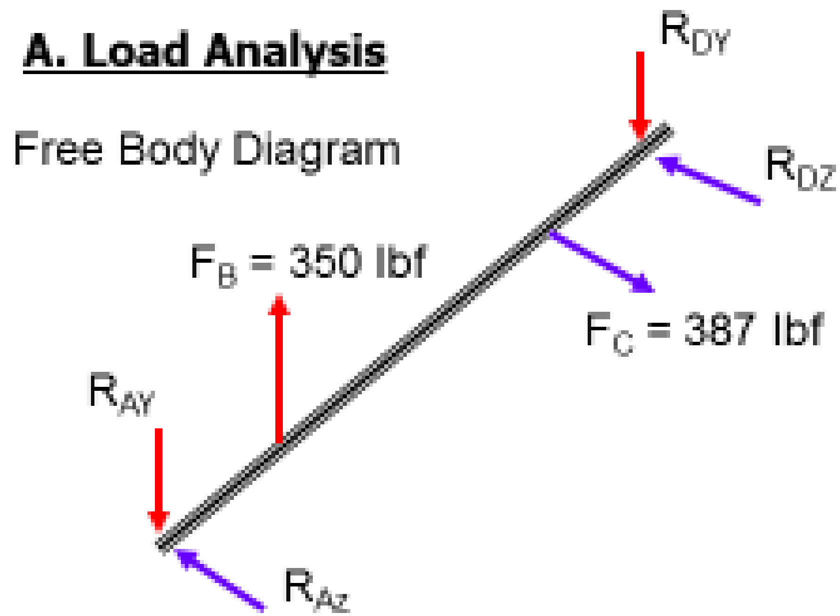
E. Solution: $n = 0.577 \times 490/241.5$
$= 1.17$

Comment: The design is barely safe
Consider increasing the size of the bar

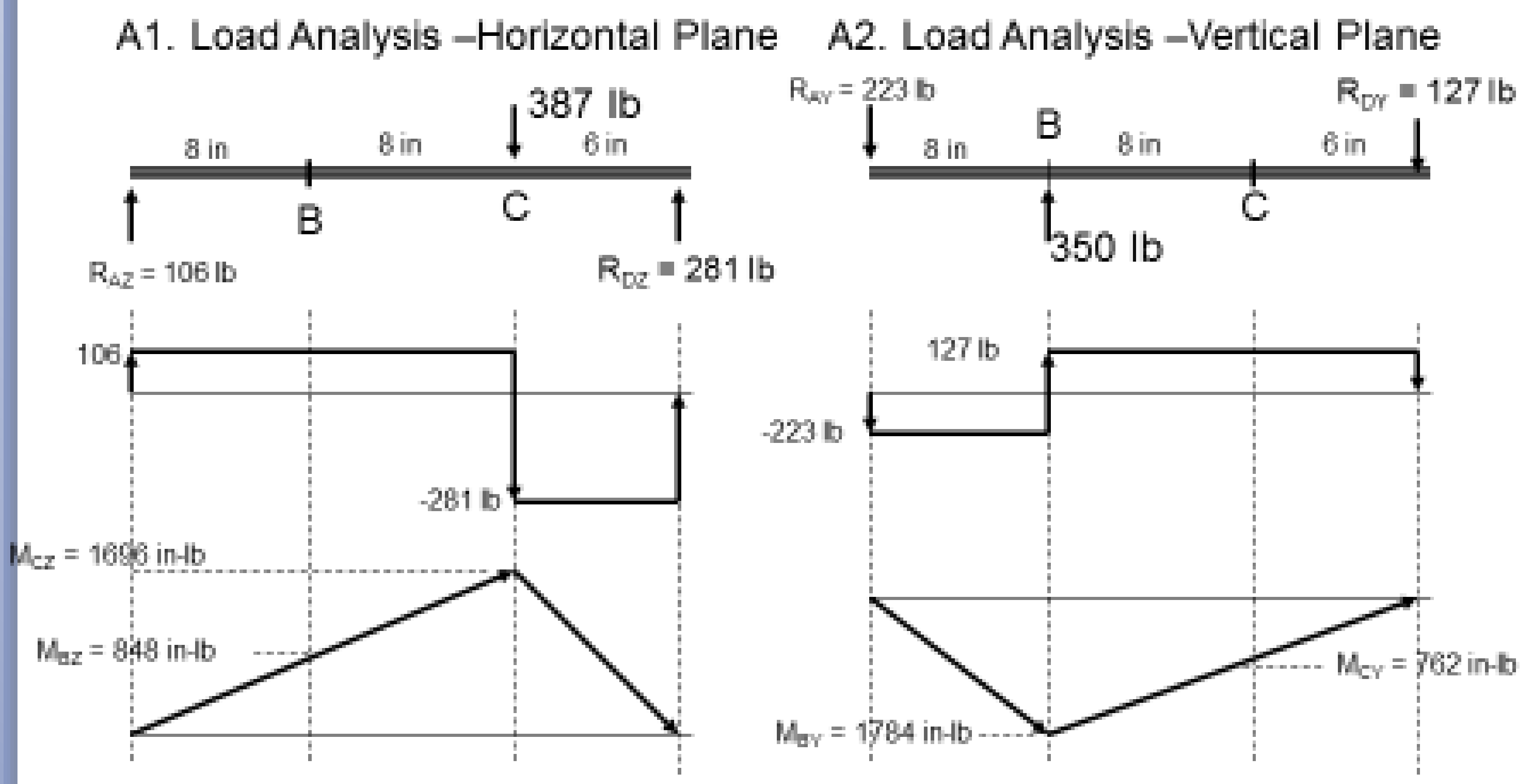
Example 3-4

The solid cylindrical steel non-rotating shaft (axle) AB supports of two pulleys as shown. A design factor of 2.5 is desired. Provide missing information for the shaft.



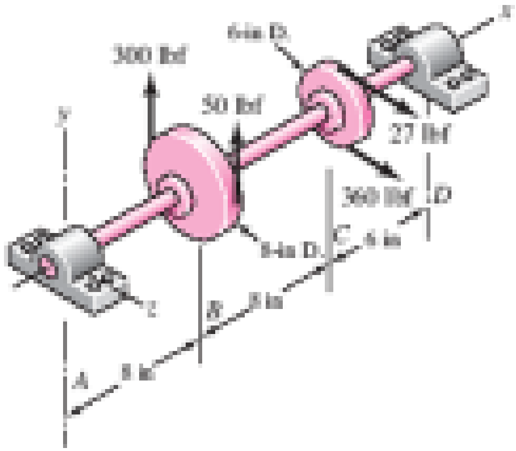


Example 3-4 (Continue)



Example 3-4 (Continue)

$M_B = (848^2+1784^2)^{½} = 1975$ in-lb
$M_c = (1696^2+762^2)^{½} = 1859$ in-lb



Torque Diagram

B. Stress Analysis

Critical section is at B

Normal Stress at Point B:

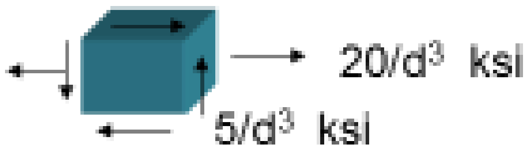
$$\begin{aligned}\sigma_x &= M_B / Z \\ &= 32 \times M_B / \pi d^3 \\ &= 32 \times 1975/\pi(d)^3 \\ &= 20{,}110/d^3 \ \text{lb/in}^2 \\ &= 20/d^3 \ \text{ksi}\end{aligned}$$

Shear Stress at B:

$$\begin{aligned}T_{xy} &= 16T/\pi d^3 \\ &= 16 \times 1000/ \pi(d)^3 \\ &= 5{,}093/d^3 \ \text{lb/in}^2 \\ &= 5/d^3 \ \text{ksi}\end{aligned}$$

Example 3-4 (Continue)

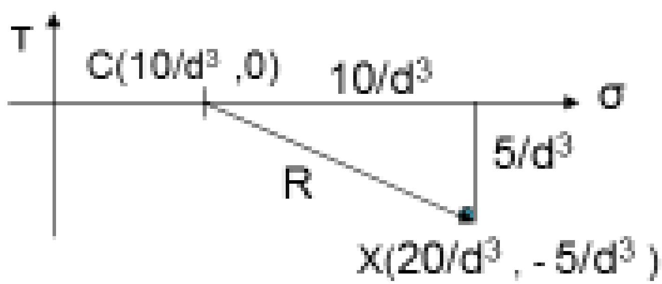
Stress Element at B:



Average stress, $\sigma_{avg} = ½[20/d^3 + 0]$
$= 10/d^3$ ksi

Center Coordinate, C $(10/d^3, 0)$

Circumferential Point, X $(20/d^3, -5/d^3)$



$R = 11.2/d^3$ ksi

Max. Normal Stress, $\sigma_1 = \sigma_{avg} + R$
$= 21.2/d^3$ ksi

Min. Normal Stress, $\sigma_2 = \sigma_{avg} - R$
$= -1.2/d^3$ ksi

Max. Shear Stress, $\tau_{max} = 11.2/d^3$ ksi

B. Design Theory & Equation

von-Mises : $n = 0.577S_y/\tau_{max}$

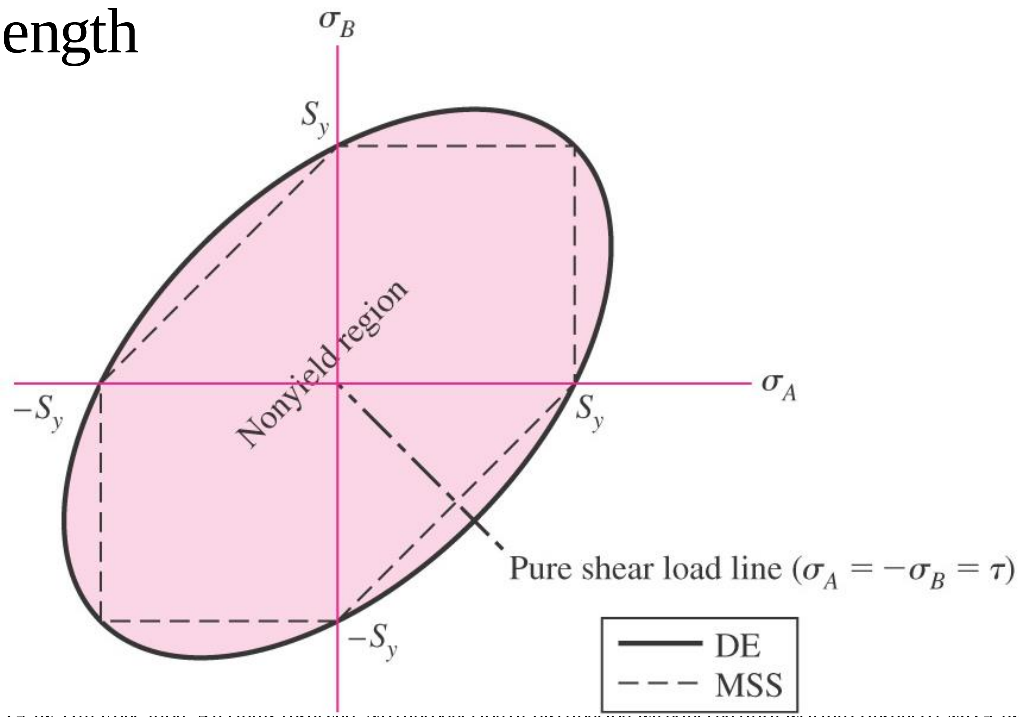
Design Equation: $2.5 = 0.577d^3S_y/11.2$

C Solution:

Select material, read S_y
Calculate d
Standardize d

Shear Strength Predictions

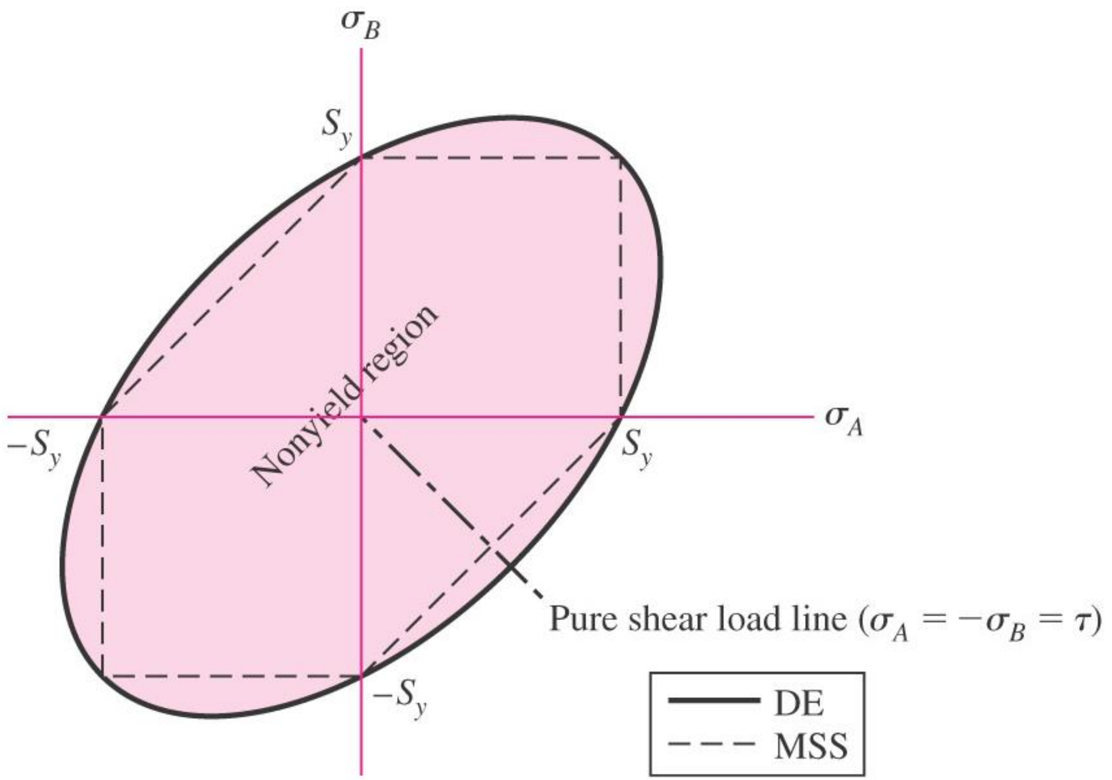
- For pure shear loading, Mohr's circle shows that $\sigma_A = -\sigma_B = \tau$
- Plotting this equation on principal stress axes gives load line for pure shear case
- Intersection of pure shear load line with failure curve indicates shear strength has been reached
- Each failure theory predicts shear strength to be some fraction of normal strength



Shear Strength Predictions

- For MSS theory, intersecting pure shear load line with failure line results in

$$S_{sy} = 0.5 S_y$$



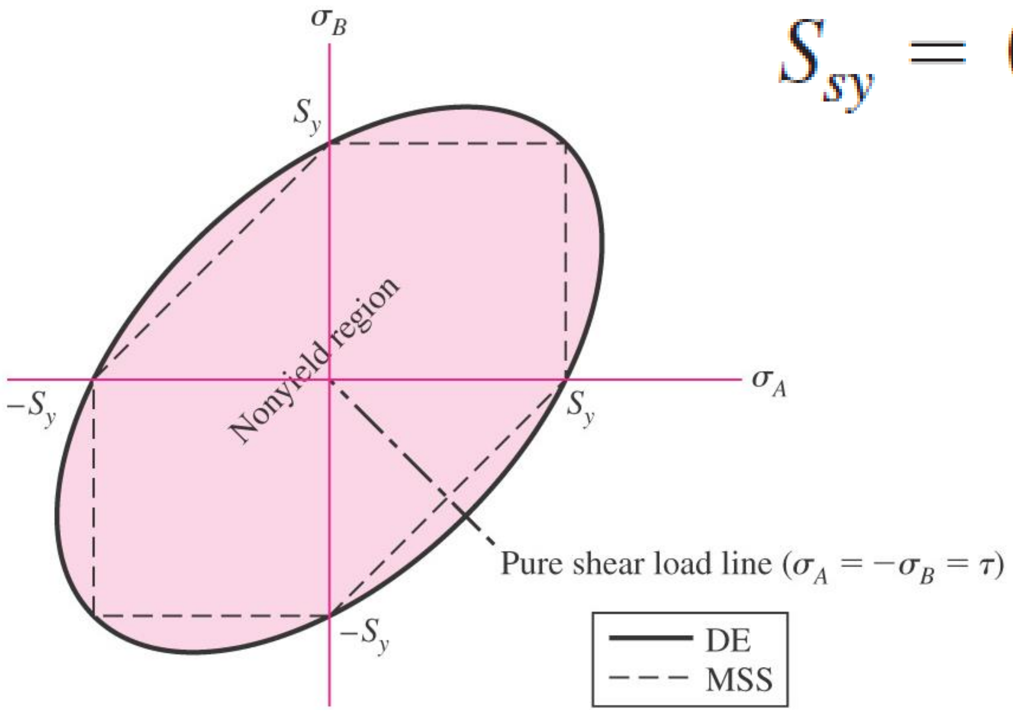
Shear Strength Predictions

- For DE theory, intersection pure shear load line with failure curve gives

$$(3\tau_{xy}^2)^{1/2} = S_y \qquad \text{or} \qquad \tau_{xy} = \frac{S_y}{\sqrt{3}} = 0.577 S_y$$

- Therefore, DE theory predicts shear strength as

$$S_{sy} = 0.577 S_y$$



Example 3–5

A hot-rolled steel has a yield strength of $S_{yt} = S_{yc} = 100$ kpsi and a true strain at fracture of $\varepsilon_f = 0.55$. Estimate the factor of safety for the following principal stress states:

(*a*) $\sigma_x = 70$ kpsi, $\sigma_y = 70$ kpsi, $\tau_{xy} = 0$ kpsi
(*b*) $\sigma_x = 60$ kpsi, $\sigma_y = 40$ kpsi, $\tau_{xy} = -15$ kpsi
(*c*) $\sigma_x = 0$ kpsi, $\sigma_y = 40$ kpsi, $\tau_{xy} = 45$ kpsi
(*d*) $\sigma_x = -40$ kpsi, $\sigma_y = -60$ kpsi, $\tau_{xy} = 15$ kpsi
(*e*) $\sigma_1 = 30$ kpsi, $\sigma_2 = 30$ kpsi, $\sigma_3 = 30$ kpsi

Solution

Since $\varepsilon_f > 0.05$ and S_{yt} and S_{yc} are equal, the material is ductile and both the distortion-energy (DE) theory and maximum-shear-stress (MSS) theory apply. Both will be used for comparison. Note that cases *a* to *d* are plane stress states.

Example 3–5 (continued)

(*a*) Since there is no shear stress on this stress element, the normal stresses are equal to the principal stresses. The ordered principal stresses are $\sigma_A = \sigma_1 = 70$, $\sigma_B = \sigma_2 = 70$, $\sigma_3 = 0$ kpsi.

DE From Eq. (5–13),

$$\sigma' = [70^2 - 70(70) + 70^2]^{1/2} = 70 \text{ kpsi}$$

From Eq. (5–19),

$$n = \frac{S_y}{\sigma'} = \frac{100}{70} = 1.43 \qquad \text{Answer}$$

MSS Noting that the two nonzero principal stresses are equal, τ_{max} will be from the largest Mohr's circle, which will incorporate the third principal stress at zero. From Eq. (3–16),

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{70 - 0}{2} = 35 \text{ kpsi}$$

From Eq. (5–3),

$$n = \frac{S_y/2}{\tau_{max}} = \frac{100/2}{35} = 1.43 \qquad \text{Answer}$$

Example 3–5 (continued)

(*b*) From Eq. (3–13), the nonzero principal stresses are

$$\sigma_A, \sigma_B = \frac{60 + 40}{2} \pm \sqrt{\left(\frac{60 - 40}{2}\right)^2 + (-15)^2} = 68.0, 32.0 \text{ kpsi}$$

The ordered principal stresses are $\sigma_A = \sigma_1 = 68.0$, $\sigma_B = \sigma_2 = 32.0$, $\sigma_3 = 0$ kpsi.

DE $\qquad \sigma' = [68^2 - 68(32) + 32^2]^{1/2} = 59.0 \text{ kpsi}$

$$n = \frac{S_y}{\sigma'} = \frac{100}{59.0} = 1.70$$ Answer

MSS Noting that the two nonzero principal stresses are both positive, τ_{max} will be from the largest Mohr's circle which will incorporate the third principle stress at zero. From Eq. (3–16),

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{68.0 - 0}{2} = 34.0 \text{ kpsi}$$

$$n = \frac{S_y/2}{\tau_{max}} = \frac{100/2}{34.0} = 1.47$$ Answer

Example 3–5 (continued)

(c) This time, we shall obtain the factors of safety directly from the xy components of stress.

DE From Eq. (5–15),

$$\sigma' = (\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2} = [(40^2 + 3(45)^2]^{1/2} = 87.6 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{100}{87.6} = 1.14 \qquad \text{Answer}$$

MSS Taking care to note from a quick sketch of Mohr's circle that one nonzero principal stress will be positive while the other one will be negative, $\tau_{\max}$ can be obtained from the extreme-value shear stress given by Eq. (3–14) without finding the principal stresses.

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0 - 40}{2}\right)^2 + 45^2} = 49.2 \text{ kpsi}$$

$$n = \frac{S_y/2}{\tau_{\max}} = \frac{100/2}{49.2} = 1.02 \qquad \text{Answer}$$

For graphical comparison purposes later in this problem, the nonzero principal stresses can be obtained from Eq. (3–13) to be 69.2 kpsi and −29.2 kpsi.

Example 3–5 (continued)

(*d*) From Eq. (3–13), the nonzero principal stresses are

$$\sigma_A, \sigma_B = \frac{-40 + (-60)}{2} \pm \sqrt{\left(\frac{-40 - (-60)}{2}\right)^2 + (15)^2} = -32.0, -68.0 \text{ kpsi}$$

The ordered principal stresses are $\sigma_1 = 0$, $\sigma_A = \sigma_2 = -32.0$, $\sigma_B = \sigma_3 = -68.0$ kpsi.

DE $$\sigma' = \left[(-32)^2 - (-32)(-68) + (-68)^2\right]^{1/2} = 59.0 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{100}{59.0} = 1.70$$ Answer

MSS From Eq. (3–16),

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{0 - (-68.0)}{2} = 34.0 \text{ kpsi}$$

$$n = \frac{S_y/2}{\tau_{\max}} = \frac{100/2}{34.0} = 1.47$$ Answer

Example 3–5 (continued)

(e) The ordered principal stresses are $\sigma_1 = 30$, $\sigma_2 = 30$, $\sigma_3 = 30$ kpsi

DE From Eq. (5–12),

$$\sigma' = \left[\frac{(30-30)^2 + (30-30)^2 + (30-30)^2}{2}\right]^{1/2} = 0 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{100}{0} \to \infty$$ Answer

MSS From Eq. (5–3),

$$n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{100}{30-30} \to \infty$$ Answer

Example 3–5 (continued)

A tabular summary of the factors of safety is included for comparisons.

	(a)	(b)	(c)	(d)	(e)
DE	1.43	1.70	1.14	1.70	∞
MSS	1.43	1.47	1.02	1.47	∞

Since the MSS theory is on or within the boundary of the DE theory, it will always predict a factor of safety equal to or less than the DE theory, as can be seen in the table.

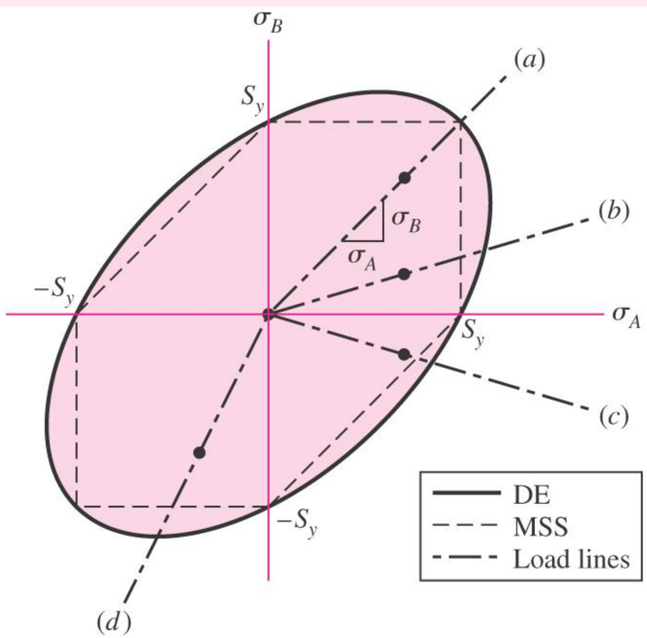


Fig. 5–11

Example 3–5 (continued)

For each case, except case (*e*), the coordinates and load lines in the σ_A, σ_B plane are shown in Fig. 5–11. Case (*e*) is not plane stress. Note that the load line for case (*a*) is the only plane stress case given in which the two theories agree, thus giving the same factor of safety.

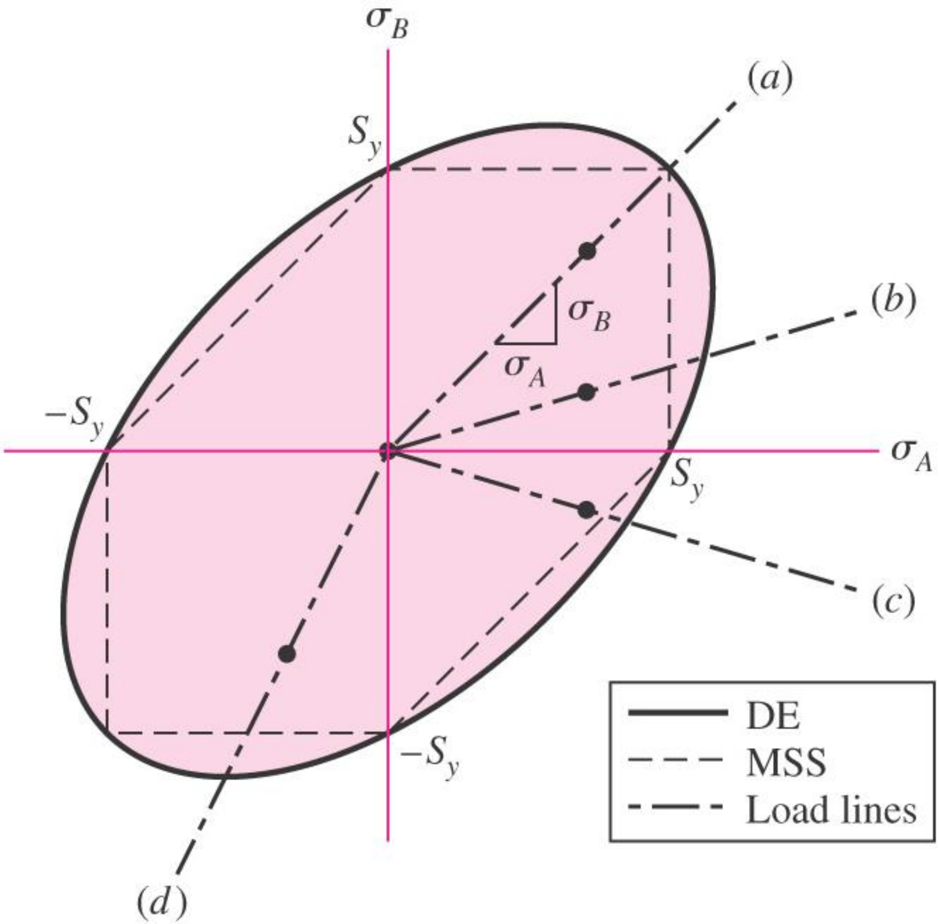


Fig. 5−11

Design Principle for Brittle Materials

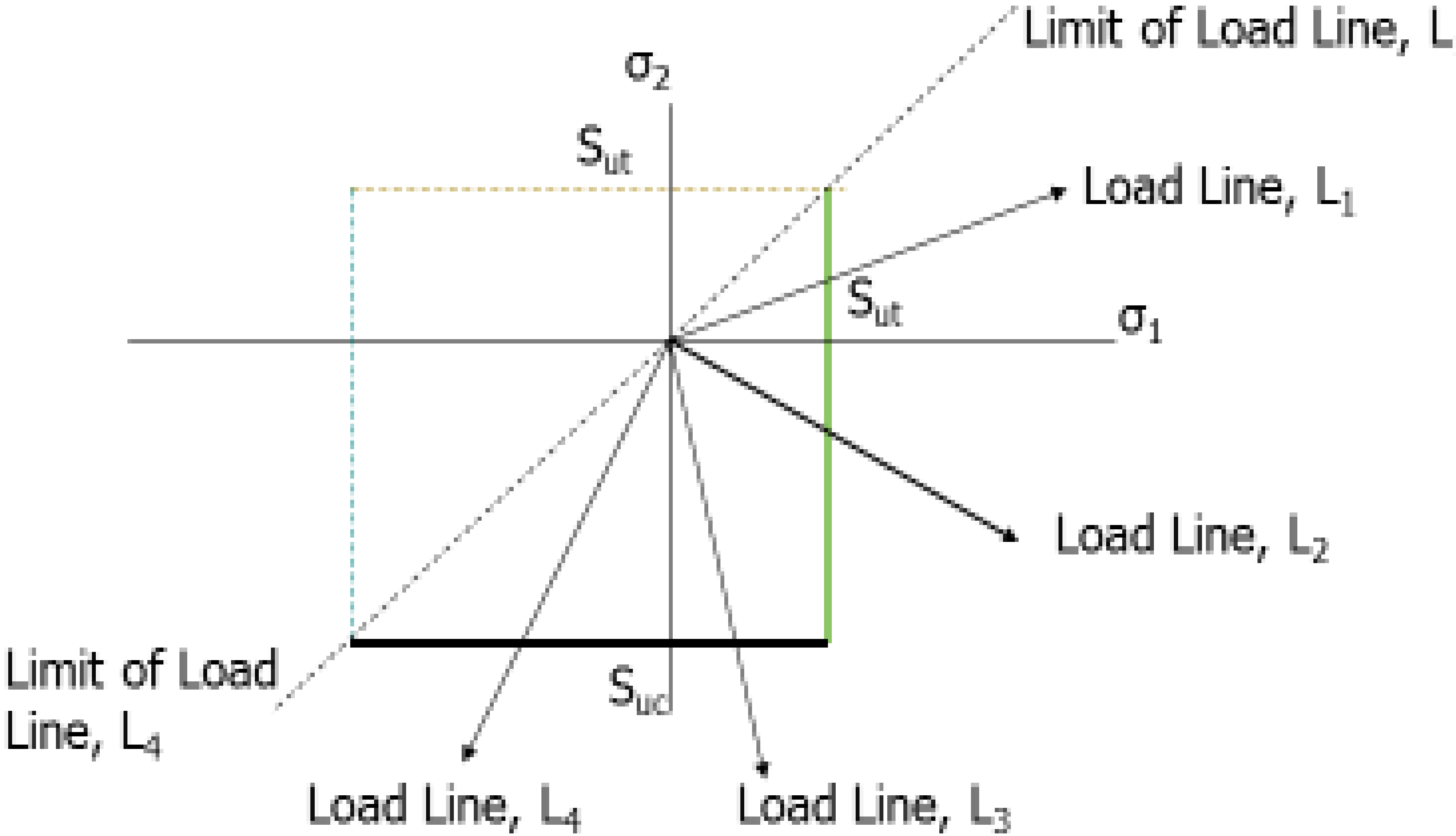
- Brittle failure is due to excessive principal normal stresses σ_1 and σ_2.

- Prevailing material properties are the ultimate tensile strength (S_{ut}) and the ultimate compressive strength (S_{uc}).

- Combination of σ_1 and σ_2 determine the failure characteristics and hence the appropriate design strength.

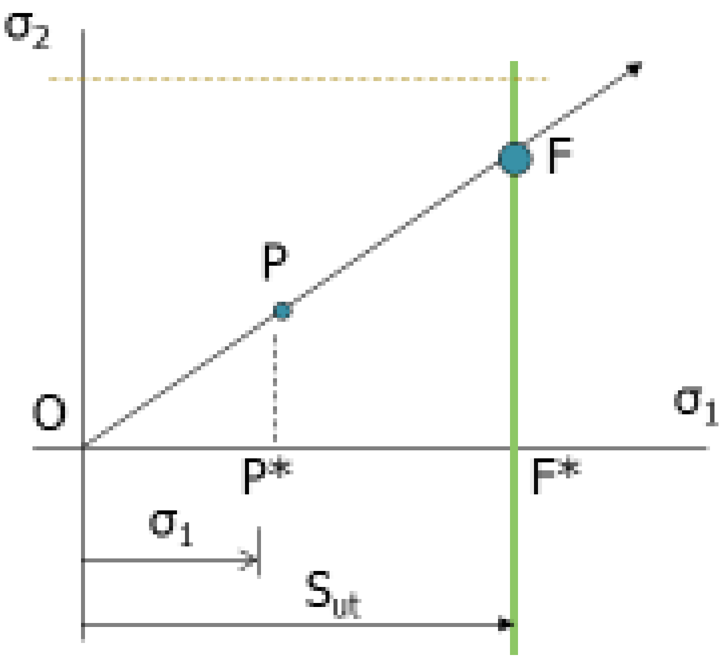
MNST-Effective Failure Boundaries and Load Lines

Note: $\sigma_1 > \sigma_2$ and $\sigma_3 = 0$



MNST Design Equation – 1st Quadrant (L_1)

1. Find σ_1 and σ_2
2. Locate Operating Point P (σ_1, σ_2)
3. Draw Load Line from O thru P to F



Factor of safety, n = OF/OP

Using similar triangles:

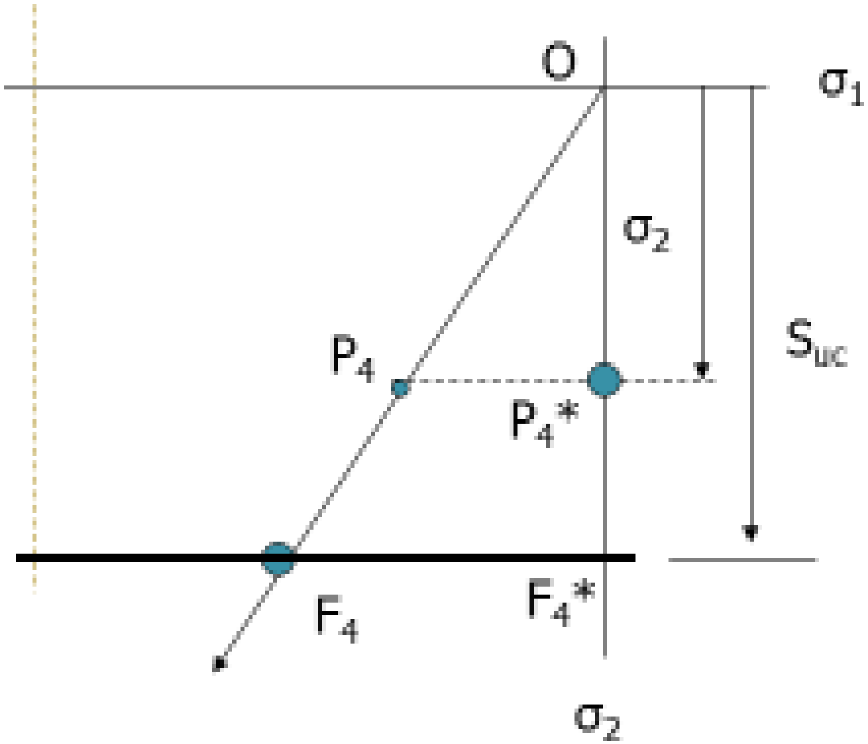
n = OF*/OP*

Design Equation for L_1:

$$\boxed{n = S_{ut}/\sigma_1}$$

MNST Design Equation – 3rd Quadrant (L_4)

1. Find σ_1 and σ_2
2. Locate Operating Point P_4 (σ_1, σ_2)
3. Draw Load Line from O thru P_4 to F_4



Factor of safety, $n = OF_4 / OP_4$

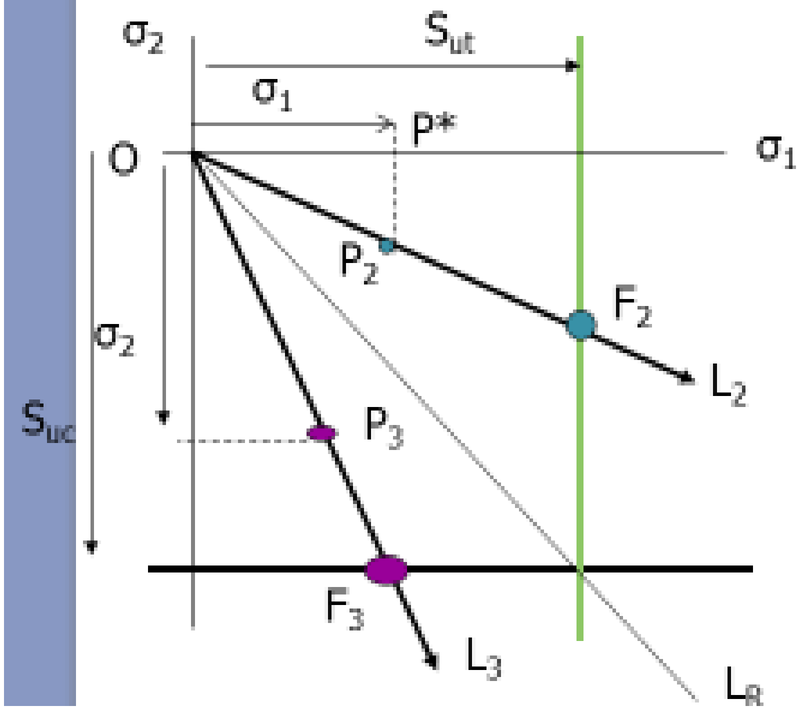
And $n = OF^*/OP^*$

Design Equation for L_4:

$$\boxed{n = S_{uc} / \sigma_2}$$

MSNT Design Equation – 4^{th} Quadrant (L_2 and L_3)

1. Find σ_1 and σ_2
2. Locate Operating Point P (σ_1, σ_2)
3. Draw Load Line from O thru P to F



4. Determine slope of reference line, L_R
   $r_c = S_{uc}/S_{ut}$

5. Find magnitude of slope of Load Line
   $r_L = | \sigma_2 / \sigma_1 |$

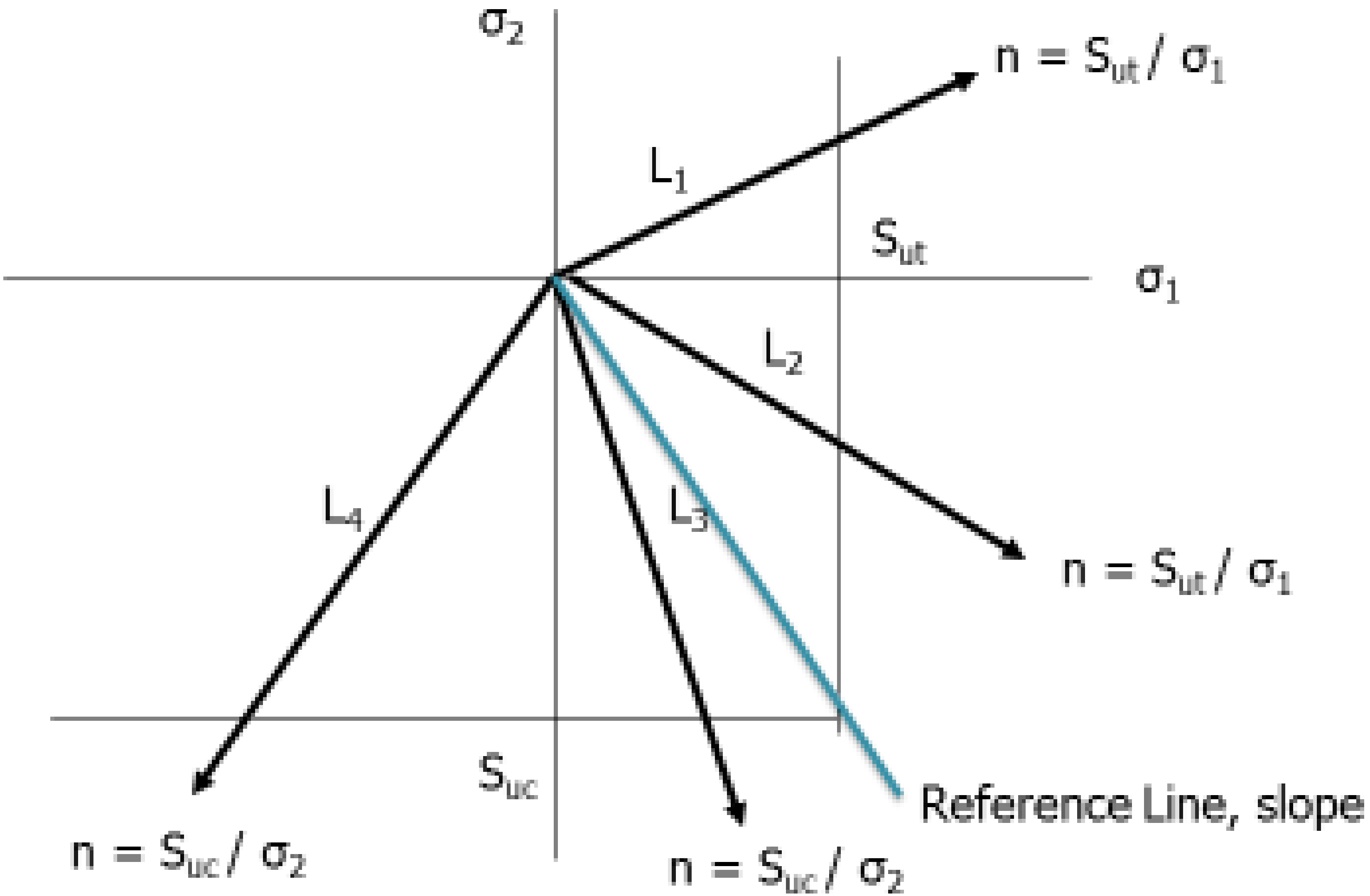
6. If $r_L < r_c$ Then Load Line is L_2
   $n = OF_2/OP_2$

And $$\boxed{n = S_{ut}/\sigma_1}$$

7. If $r_L > r_c$ Then Load Line is L_3,
   $n = OF_3/OP_3$

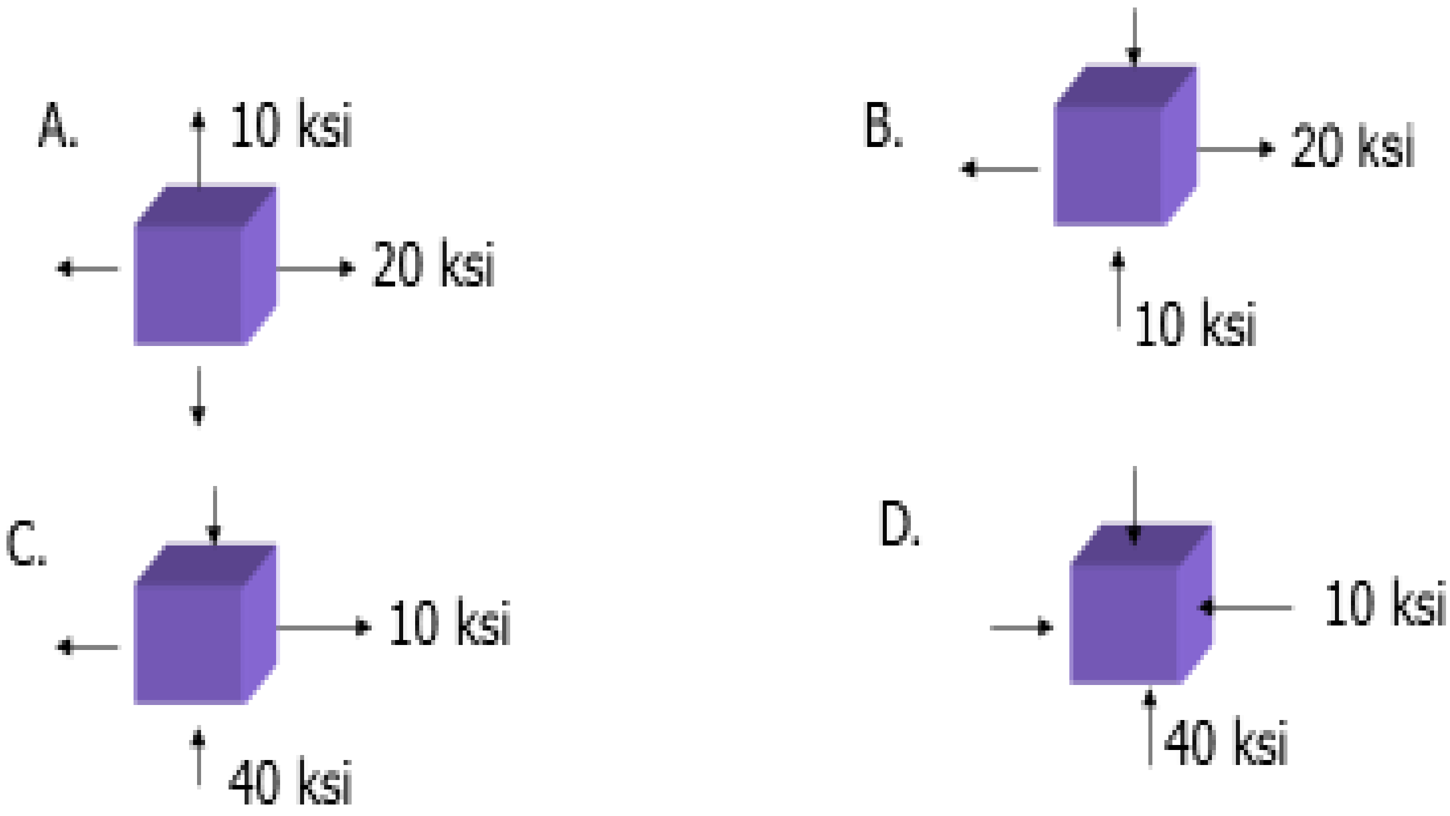
And $$\boxed{n = S_{uc}/\sigma_2}$$

MNST Design Equations



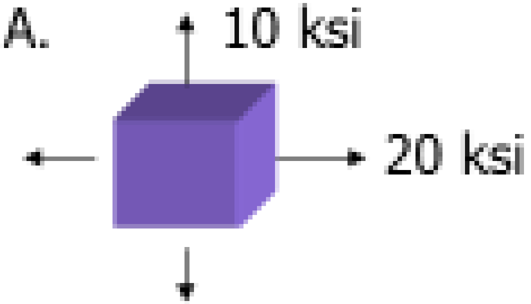
Example 3-6

Each stress element is obtained from the critical section of machine members made of ASTM Class 40 Gray Cast Iron. Find the factor of safety for each situation using MNST.



Example 3-6A

A.



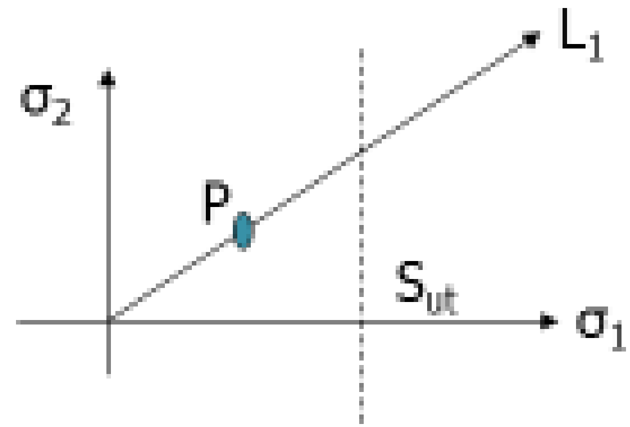
1. Principal Normal Stresses

$\sigma_1 = 20$ ksi

$\sigma_2 = 10$ ksi

2. Operating Point (20,10)

3. Load Line



4. Design Equation/Solution

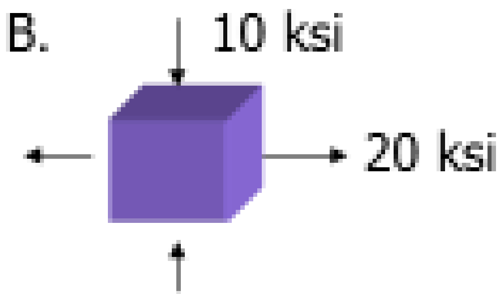
For Load Line, L_1

$n = S_{ut}/\sigma_1$

$n = 42.5/20$

$n = 2.12$

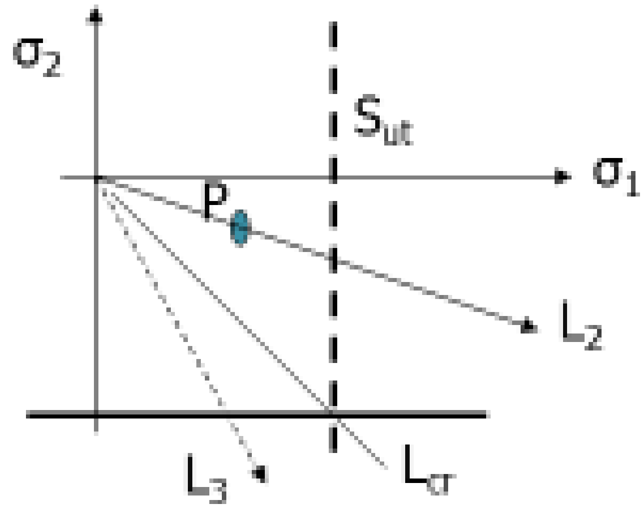
Example 3-6B



1. Principal Normal Stresses
   σ_1 = 20 ksi
   σ_2 = - 10 ksi

2. Operating Point (20, -10)
   4th Quadrant
   Slope of Load Line, $r_L = |\sigma_2/\sigma_1|$
   $= |-10/20| = 0.5$
   Slope of critical line, $r_c = S_{uc}/S_{ut}$
   $= 140/42.5$
   $= 3.3$

3. Load Line

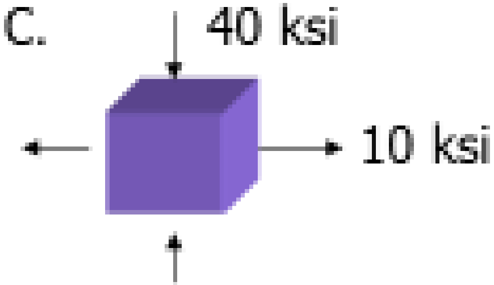


4. Design Equation/Solution

   For Load Line, L_2
   $n = S_{ut}/\sigma_1$
   $n = 42.5/20$
   $n = 2.12$

Example 3-6C

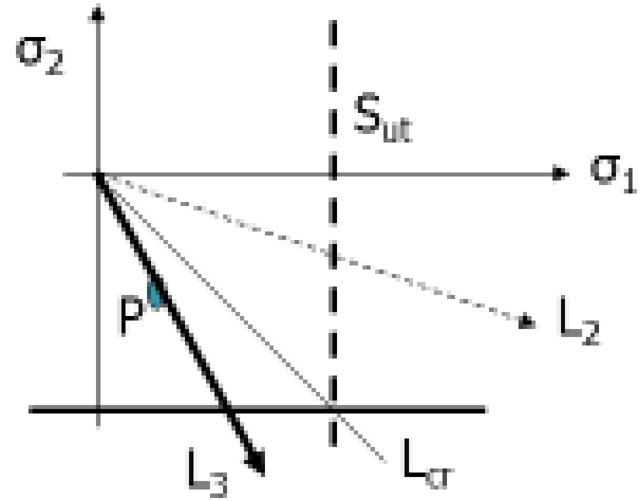
C.



1. Principal Normal Stresses
   σ_1 = 10 ksi
   σ_2 = - 40 ksi

2. Operating Point (10, -40)
   4th Quadrant
   Slope of Load Line, $r_L = |\sigma_2/\sigma_1|$
   $= |-40/10| = 4.0$
   Slope of critical line, $r_c = S_{uc}/S_{ut}$
   $= 140/42.5$
   $= 3.3$

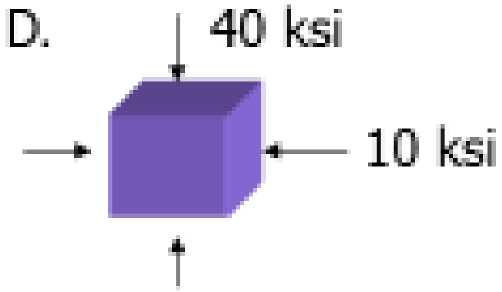
3. Load Line



4. Design Equation/Solution

   For Load Line, L_3
   $n = S_{uc}/\sigma_2$
   $n = 140/40$
   $n = 3.5$

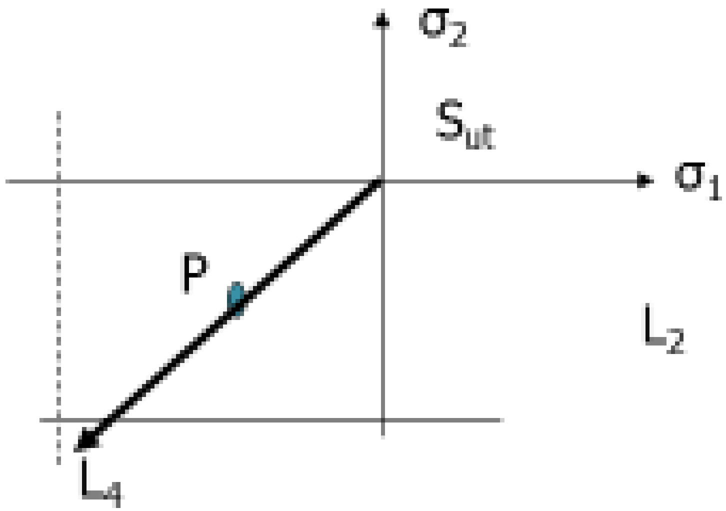
Example 3-6D



1. Principal Normal Stresses
   σ_1 = -10 ksi
   σ_2 = - 40 ksi

2. Operating Point (-10, -40)
   3rd Quadrant

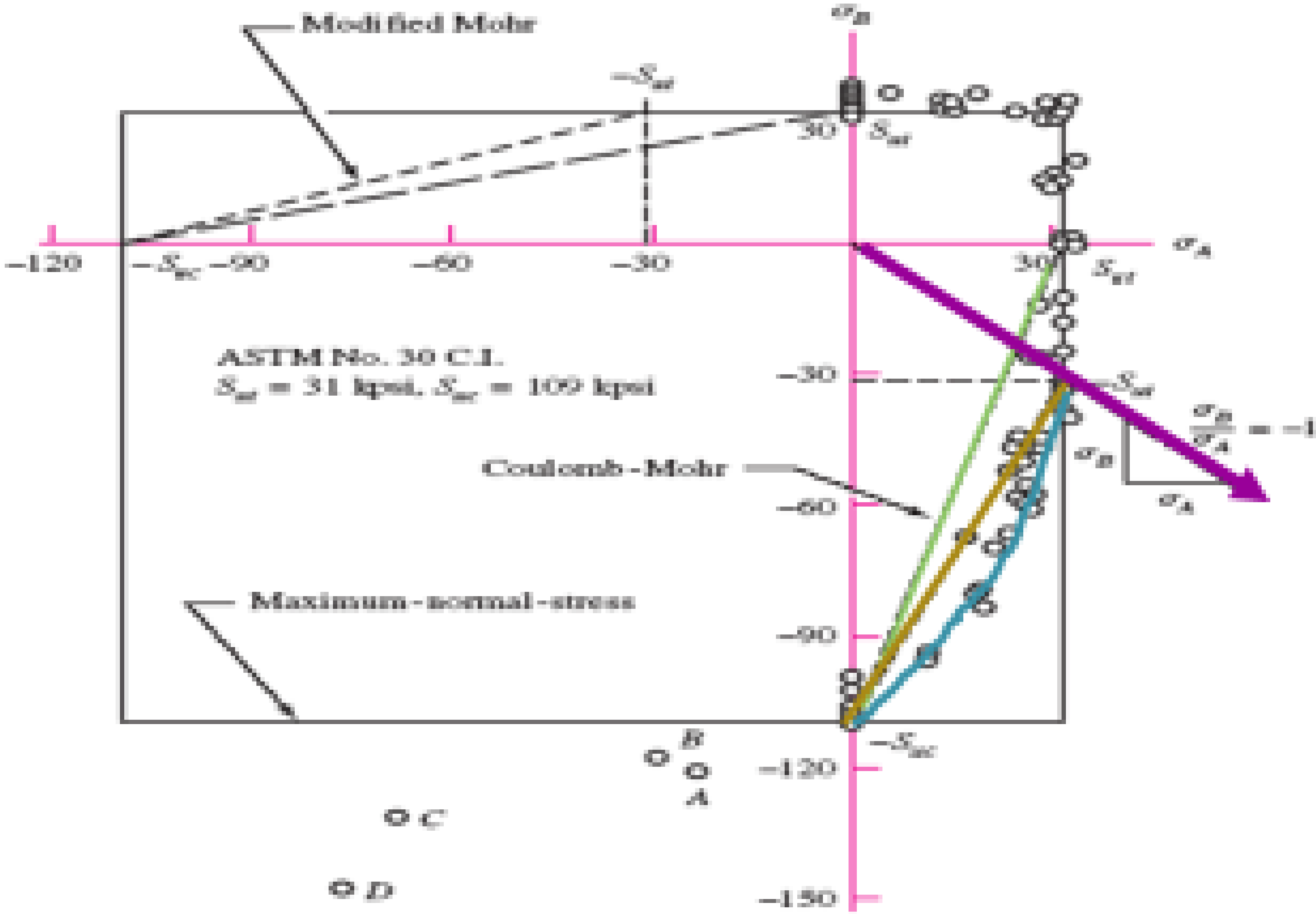
3. Load Line



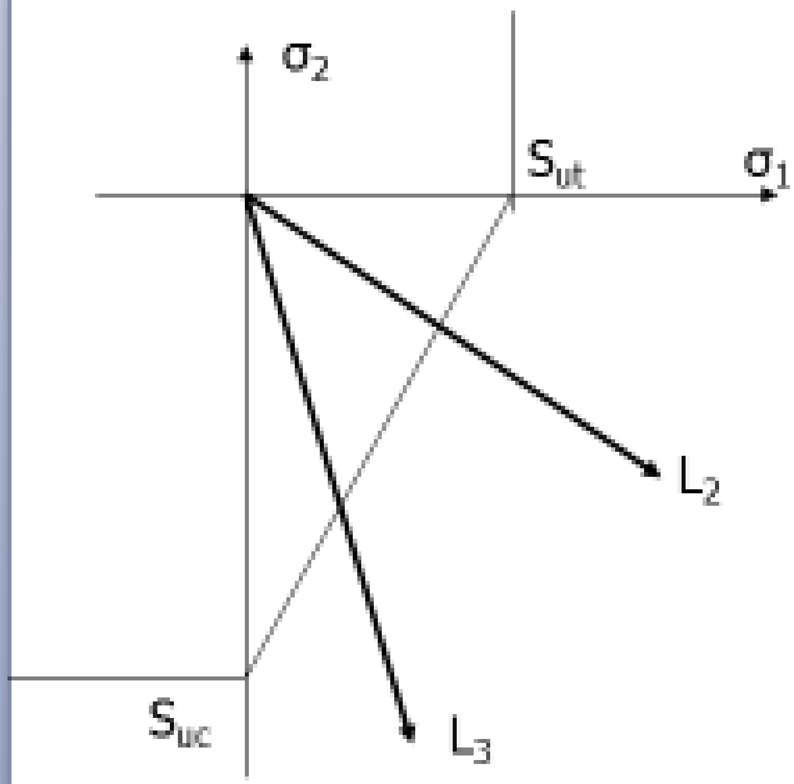
4. Design Equation/Solution

   For Load Line, L_4
   $n = S_{uc}/\sigma_2$
   $n = 140/40$
   $n = 3.5$

Experimental Verification of MNST Failure Boundaries



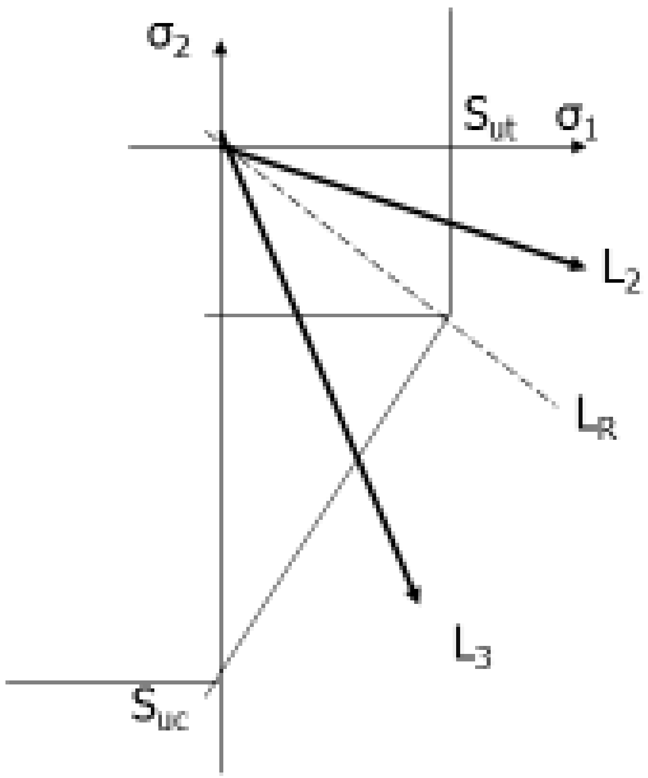
Brittle Coulomb Mohr Theory (BCMT) – 4th Quadrant



Design Equation for L_2 and L_3:

$$\boxed{1/n = \sigma_1/S_{ut} - \sigma_2/S_{uc}}$$

Modified Mohr – 1 Theory (MM1T) : 4th Quadrant



1. Magnitude of slope of reference line, L_R

   $r_c = 1$

2. Find magnitude of slope of Load Line

   $r_L = | \sigma_2 / \sigma_1 |$

3. If $r_L \leq 1$, Then Load Line is L_2

$$\boxed{n = S_{ut}/\sigma_1}$$

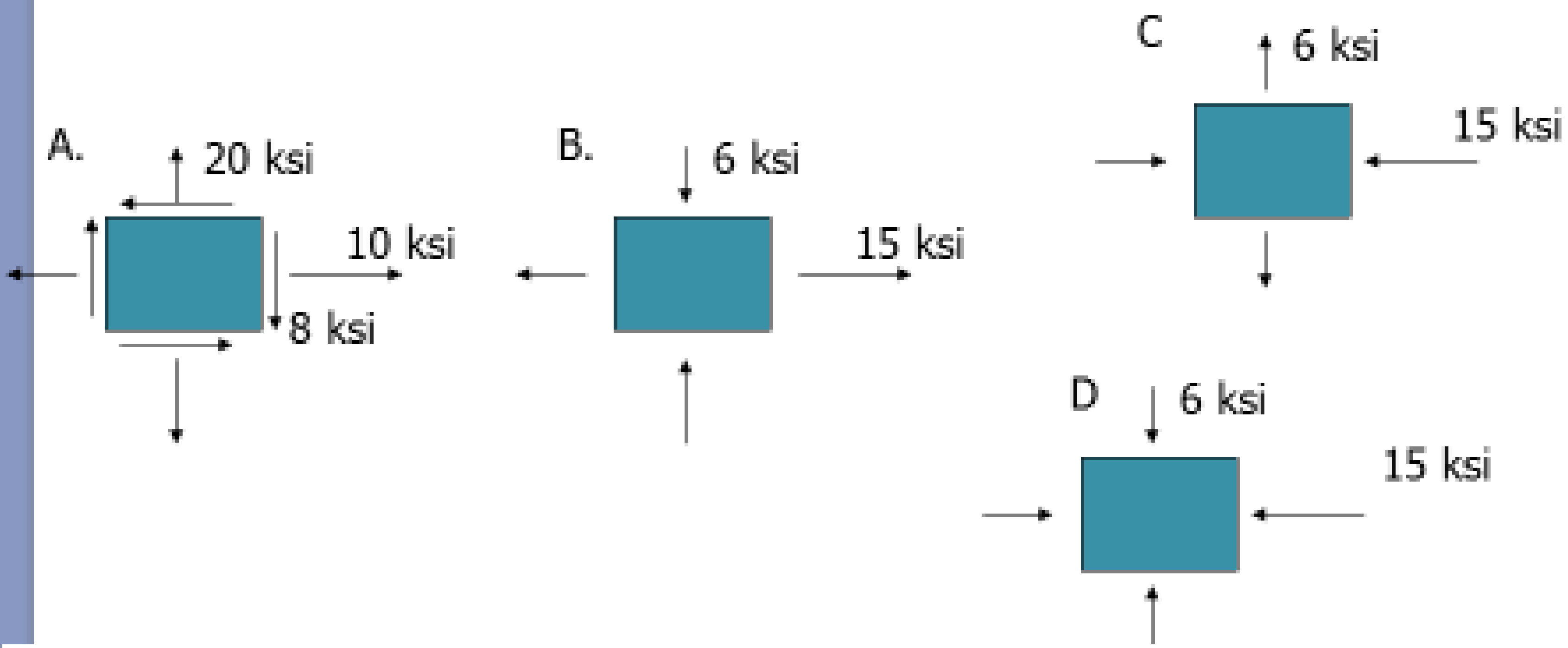
4. If $r_L > 1$, Then Load Line is L_3

$$\boxed{1/n = [(S_{uc} - S_{ut})\sigma_1/S_{uc}\, S_{ut}] - \sigma_2/S_{uc}}$$

Example 3-7

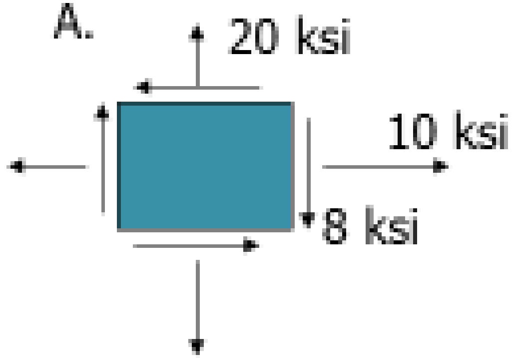
Each stress element is obtained from the critical section of machine members made of ASTM Class 40 Gray Cast Iron. Find the factor of safety for each situation using:

a. Brittle Coulomb-Mohr Theory
b. Modified Mohr-1 Theory



Example 3-7A

A.



Stress Analysis:

Applied Stresses are:

$\sigma_x = 10$ ksi $\sigma_y = 20$ ksi

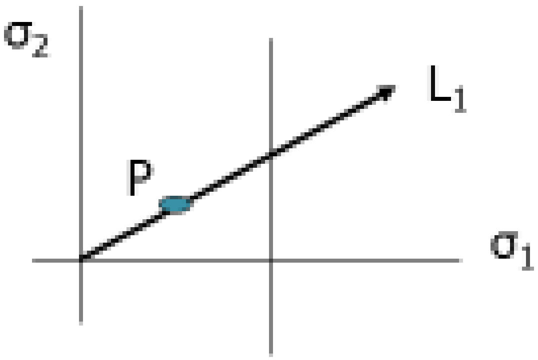
$\tau_{xy} = 8$ ksi

The Principal Stresses are:

$\sigma_1 = 24.4$ ksi

$\sigma_2 = 5.6$ ksi

i. BCMT for P(24.4, 5.6)



Design Equation for L_1 : $n = S_{ut} / \sigma_1$

Solution: $n = 42.5/24.4$

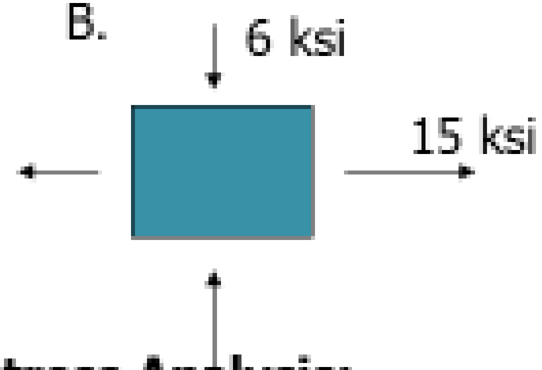
$n = 1.74$

ii. Modified Mohr-1 Theory for P(24.4, 5.6)

L_1 → SAME SOLUTION FOR ALL THEORIES

Example 3-7B

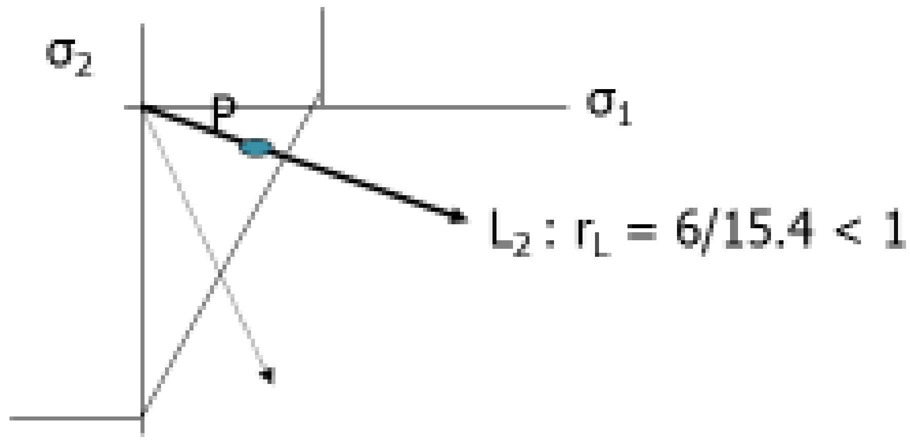
B.



Stress Analysis:
Applied Stresses are Principal Stresses:

$\sigma_x = \sigma_1 = 15$ ksi
$\sigma_y = \sigma_2 = -6$ ksi

i. BCMT for P(15, - 6)



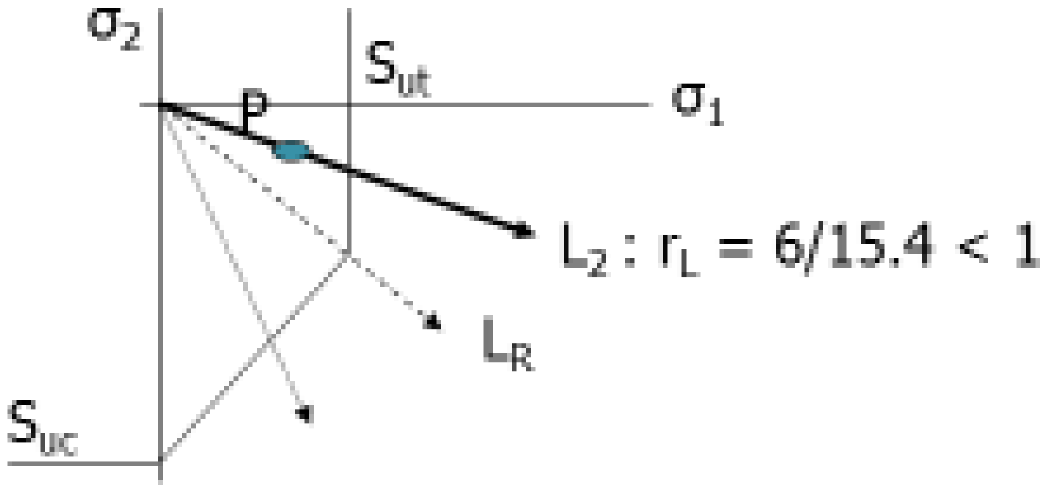
Design Equation: $1/n = \sigma_1/S_{ut} - \sigma_2/S_{uc}$

Solution: $1/n = 15/42.5 - (-6)/140$

$n = 2.53$

Example 3-7B (Continue)

ii. Modified Mohr-1 Theory for P(15, -6)

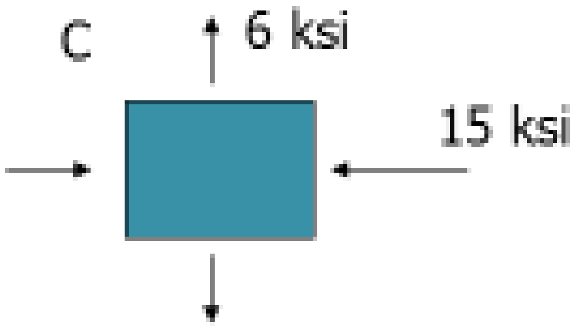


Design Equation: $n = S_{ut}/\sigma_1$

Solution: $n = 42.5/15$

$n = 2.83$

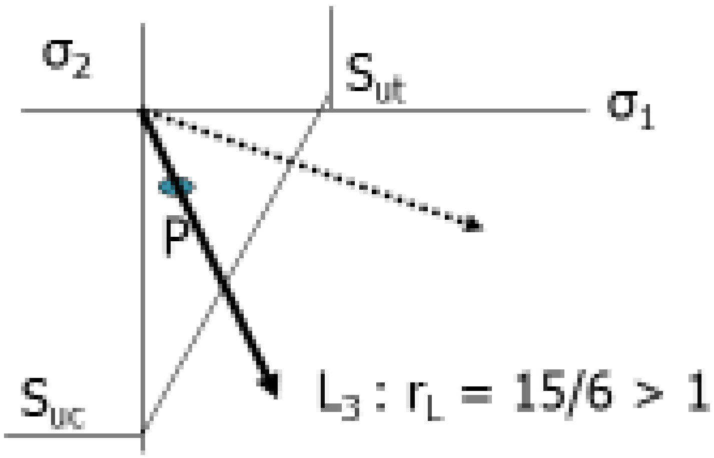
Example 3-7C



Stress Analysis:
Applied Stresses are
Principal Stresses:

$\sigma_Y = \sigma_1 = \quad 6$ ksi
$\sigma_X = \sigma_2 = -15$ ksi

i. BCMT for P(6, - 15)



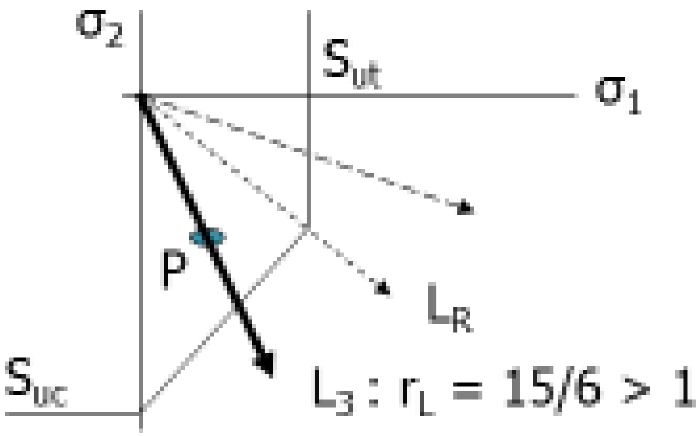
Design Equation: $1/n = \sigma_1/S_{ut} - \sigma_2/S_{uc}$

Solution: $1/n = 6/42.5 - (-15)/140$

$n = 4.03$

Example 3-7C (Continue)

ii. Modified Mohr-1 Theory for P(6, -15)

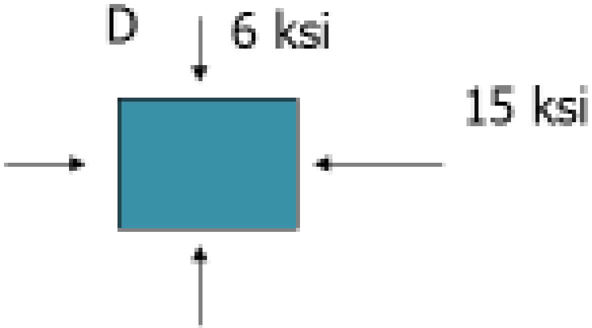


Design Equation for L_3: $1/n = [(S_{uc} - S_{ut})\sigma_1/S_{uc}\, S_{ut}] - \sigma_2/S_{uc}$

Solution: $1/n = [(140 - 42.5)\, 6/(140)(42.5)] - (-15)/140$

$n = 4.87$

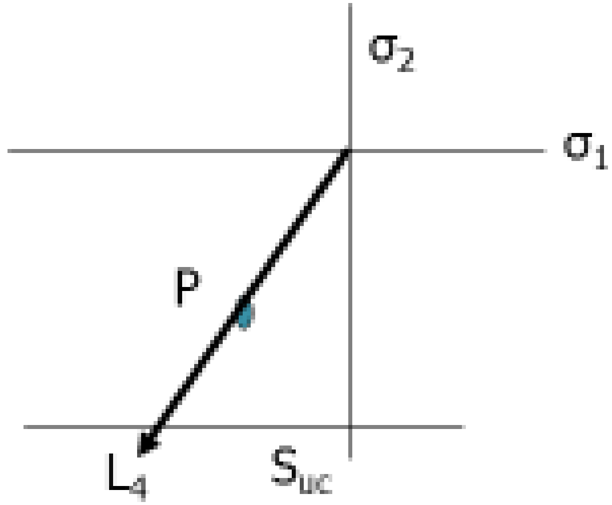
Example 3-7D



Stress Analysis:
Applied Stresses are Principal Stresses:

$\sigma_Y = \sigma_1 = -6$ ksi
$\sigma_X = \sigma_2 = -15$ ksi

i. BCMT for P(-6, - 15)



Design Equation for L_4 : $n = S_{uc} / \sigma_2$

Solution: $n = 140/15$

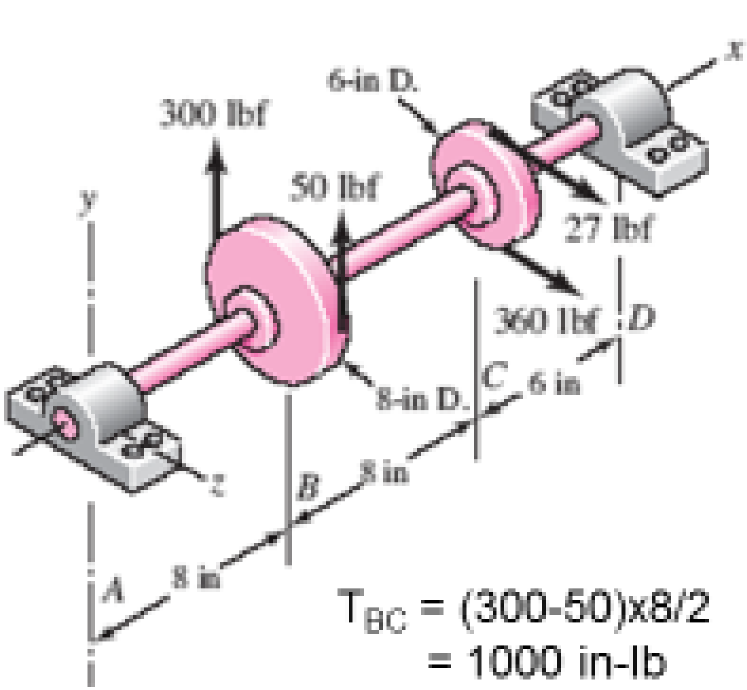
$n = 9.33$ (Over-design)

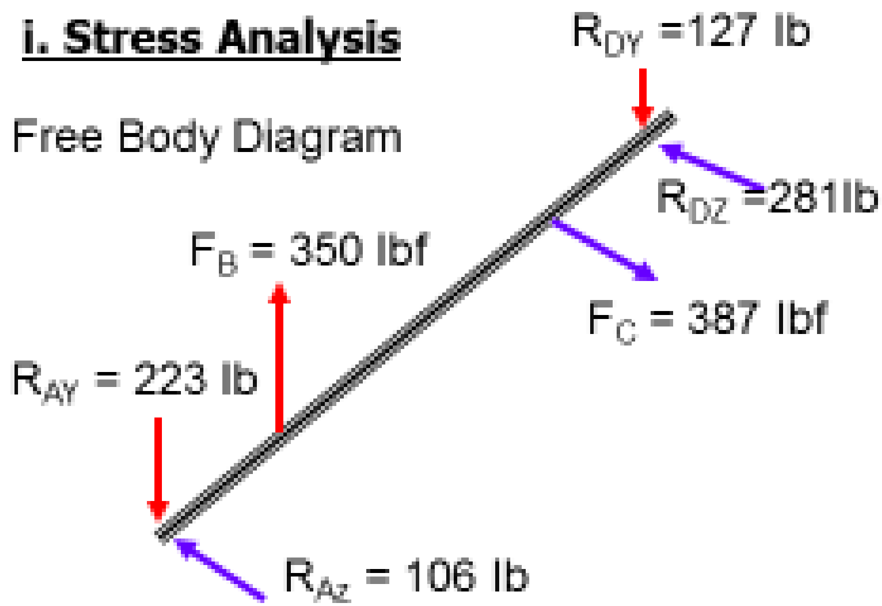
ii. Modified Mohr-1 Theory for P(-6, -15)

$L_4 \rightarrow$ SAME SOLUTION FOR ALL THEORIES

Example 3-8

The solid cylindrical shaft AB supports of two pulleys as shown. The material is cast iron. A design factor of 2.5 is desired. Assume steady conditions. Provide missing information to complete the design of the shaft.



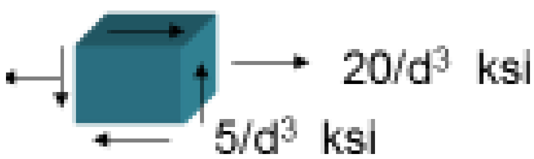


Example 3-8 (Continue)

From Example:

$M_B = 1975$ in-lb
$M_C = 1859$ in-lb

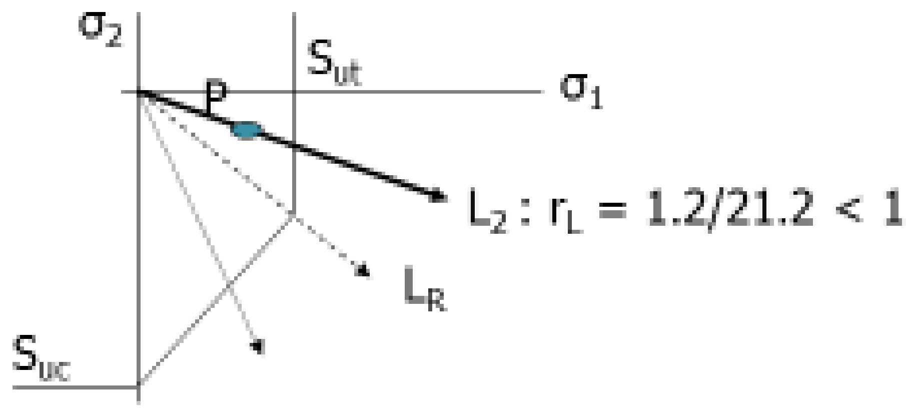
Stress Element at B:



Principal Stresses are:

$\sigma_1 = 21.2/d^3$ ksi
$\sigma_2 = -1.2/d^3$ ksi

ii. Modified Mohr-1 Theory for P(21.2, -1.2)



Design Equation for L_2: $n = S_{ut}/\sigma_1$

$$2.5 = S_{ut}\, d^3 / 21.2$$

Select ASTM Class 25 Gray Cast Iron, $S_{ut} = 31$ ksi

$$d = 1.196 \text{ in}$$

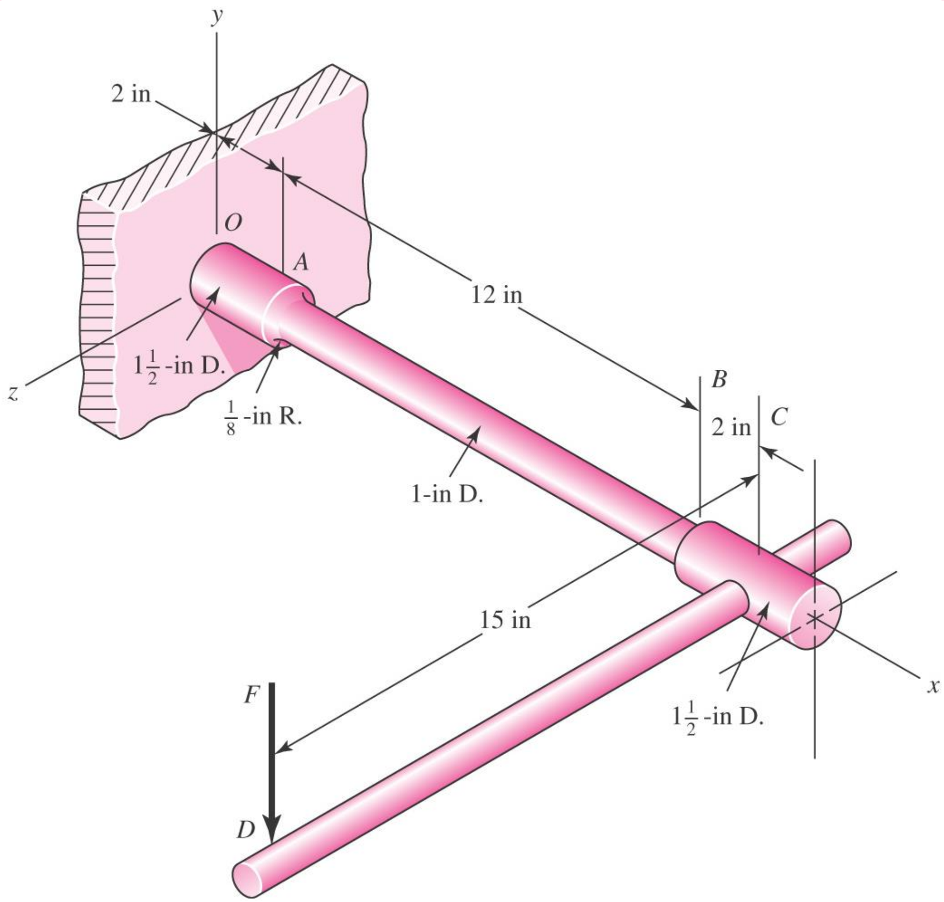
Specify $d = 1.20$ in

Example 3−9

Consider the wrench in Ex. 5–3, Fig. 5–16, as made of cast iron, machined to dimension. The force F required to fracture this part can be regarded as the strength of the component part. If the material is ASTM grade 30 cast iron, find the force F with

(a) Coulomb-Mohr failure model.

(b) Modified Mohr failure model.



Example 3−9 (continued)

We assume that the lever *DC* is strong enough, and not part of the problem. Since grade 30 cast iron is a brittle material *and* cast iron, the stress-concentration factors K_t and K_{ts} are set to unity. From Table A–24, the tensile ultimate strength is 31 kpsi and the compressive ultimate strength is 109 kpsi. The stress element at *A* on the top surface will be subjected to a tensile bending stress and a torsional stress. This location, on the 1-in-diameter section fillet, is the weakest location, and it governs the strength of the assembly. The normal stress σ_x and the shear stress at *A* are given by

$$\sigma_x = K_t \frac{M}{I/c} = K_t \frac{32M}{\pi d^3} = (1)\frac{32(14F)}{\pi(1)^3} = 142.6F$$

$$\tau_{xy} = K_{ts} \frac{Tr}{J} = K_{ts} \frac{16T}{\pi d^3} = (1)\frac{16(15F)}{\pi(1)^3} = 76.4F$$

From Eq. (3–13) the nonzero principal stresses σ_A and σ_B are

$$\sigma_A, \sigma_B = \frac{142.6F + 0}{2} \pm \sqrt{\left(\frac{142.6F - 0}{2}\right)^2 + (76.4F)^2} = 175.8F, -33.2F$$

This puts us in the fourth-quadrant of the σ_A, σ_B plane.

Example 3−9 (continued)

(*a*) For BCM, Eq. (5–31*b*) applies with $n = 1$ for failure.

$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{175.8F}{31(10^3)} - \frac{(-33.2F)}{109(10^3)} = 1$$

Solving for F yields

$$F = 167 \text{ lbf}$$ Answer

(*b*) For MM, the slope of the load line is $|\sigma_B/\sigma_A| = 33.2/175.8 = 0.189 < 1$. Obviously, Eq. (5–32*a*) applies.

$$\frac{\sigma_A}{S_{ut}} = \frac{175.8F}{31(10^3)} = 1$$

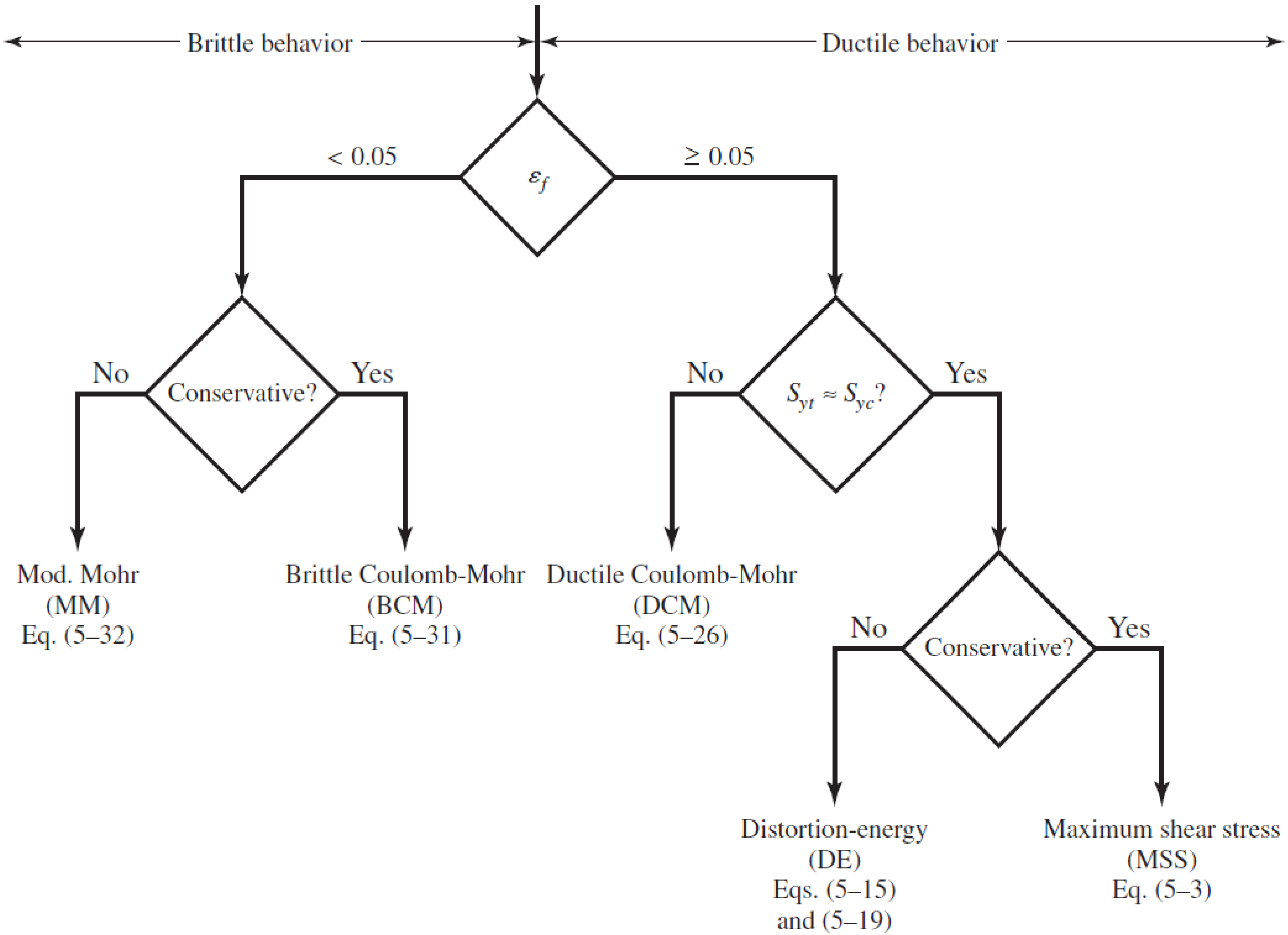
$$F = 176 \text{ lbf}$$ Answer

As one would expect from inspection of Fig. 5–19, Coulomb-Mohr is more conservative.

Selection of Failure Criteria

- First determine ductile vs. brittle
- For ductile
  - MSS is conservative, often used for design where higher reliability is desired
  - DE is typical, often used for analysis where agreement with experimental data is desired
  - If tensile and compressive strengths differ, use Ductile Coulomb-Mohr
- For brittle
  - Mohr theory is best, but difficult to use
  - Brittle Coulomb-Mohr is very conservative in 4^{th} quadrant
  - Modified Mohr is still slightly conservative in 4^{th} quadrant, but closer to typical

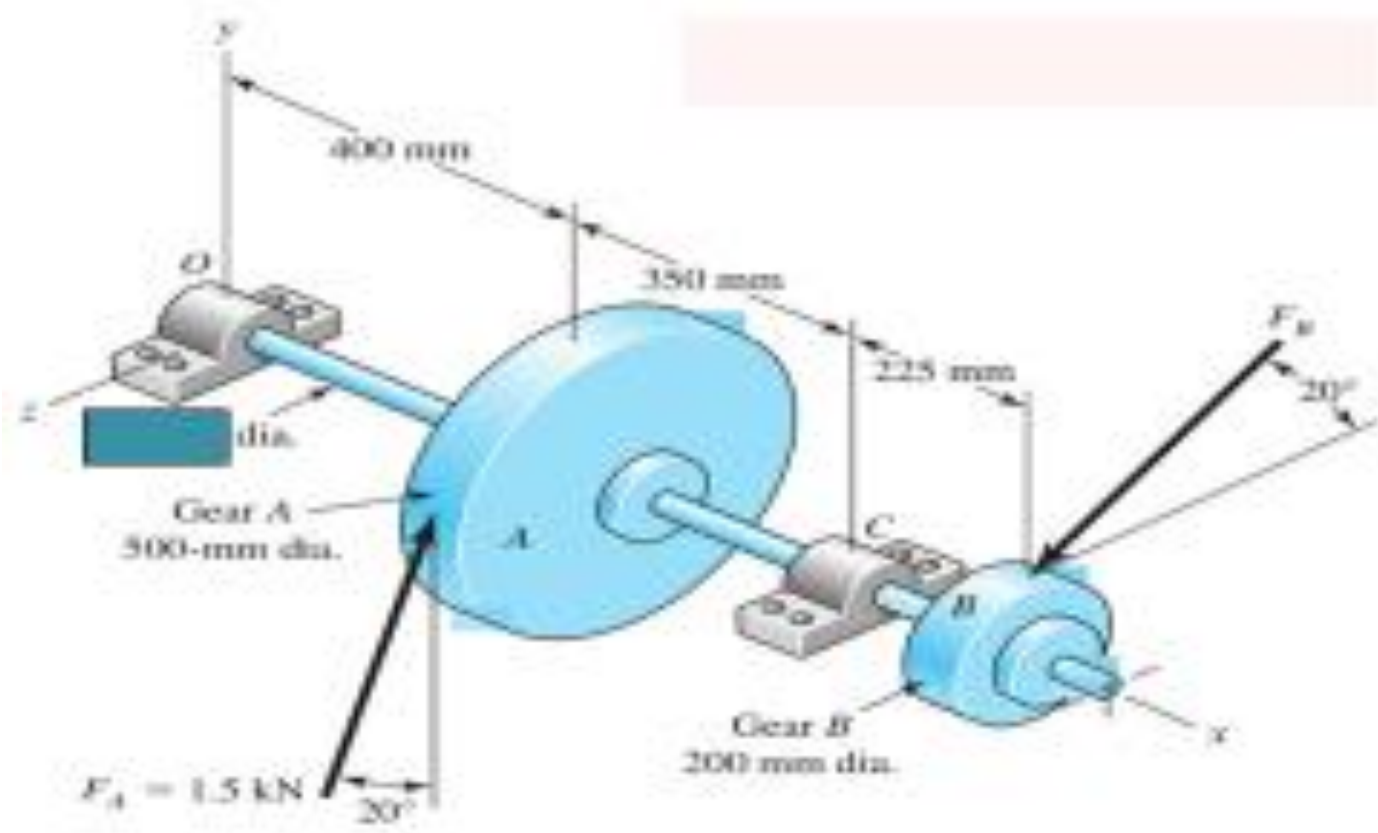
Selection of Failure Criteria in Flowchart Form



Assignment 3- Design of Axle

The axle shown above is to be designed to have the same diameter. Using the design process, provide complete specification for:

a. steel axle and
b. for cast iron axle



Problem 3–72*