1 a. Distinguish between the functions of the governor and the flywheel of an engine [2 marks]

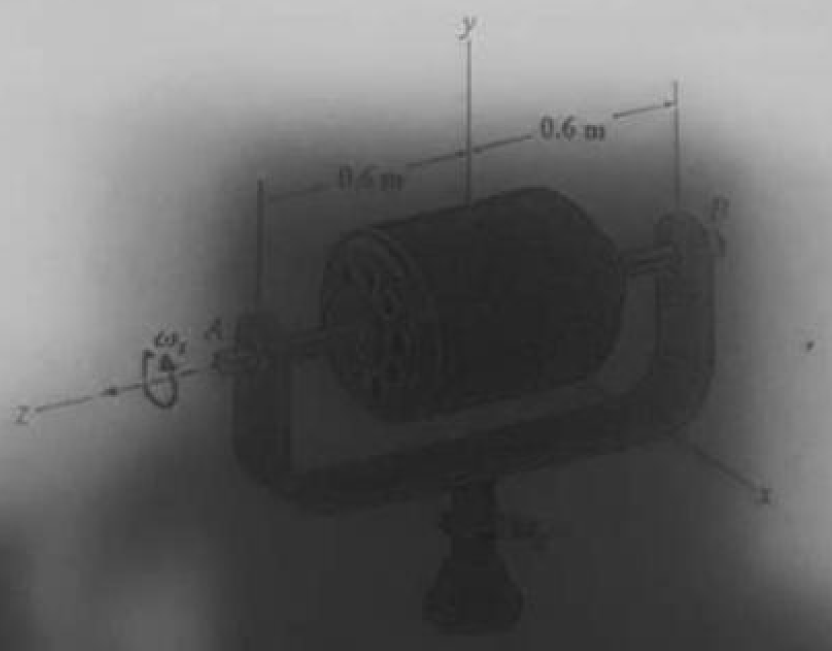
b. The upper and lower arms of a four-fly ball Porter governor are each 550 mm long and pivoted on the axis of rotation, and the mass of each fly ball is 6 kg. When the radius of rotation of the balls is 350 mm, the speeds of governor are, respectively, 160 rpm when the sleeve is moving upward and 150 rpm when the sleeve is moving downward. Find the mass of the sleeve and the friction force acting between the sleeve and the spindle. [10 marks]

c. A four-stroke engine develops 200 kW of power at a mean speed of 75 rpm. The coefficient of energy fluctuation is 0.17 and speed fluctuates within ±2% of the mean speed. Find the moment of inertia of the flywheel that must attached to the crankshaft of the engine in order to maintain the speed within the acceptable range. [6 marks]

2. The motor shown in Figure 1 weighs 60 kg and has a radius of gyration of 0.8 m about the z axis. The shaft of the motor is supported by bearings at A and B, and spins at a constant rate of $\omega_z = 20\,rad/s$, while the frame has an angular velocity of $\omega_y = 0.5\,rad/s$.

i. Determine the gyroscopic couple and its direction on the motor [9 marks]

ii. What are the bearing forces at A and B due to this motion. [9 marks]



KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI
COLLEGE OF ENGINEERING

BSC. MECHANICAL ENGINEERING. ME 361 DYNAMICS OF MACHINERY

END-OF-SEMESTER EXAMINATION. JANUARY 2019

Answer any three questions **Time allowed: 2 Hours**

(24 Points)

Question 1
Kinematics of a rigid body

Find the velocity and acceleration of slider block C and the angular velocity of link BC at the instant shown in Figure 1.1, assuming that the angular acceleration of link $AB = 0$.

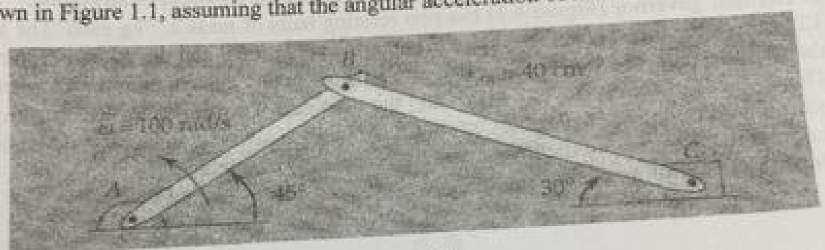
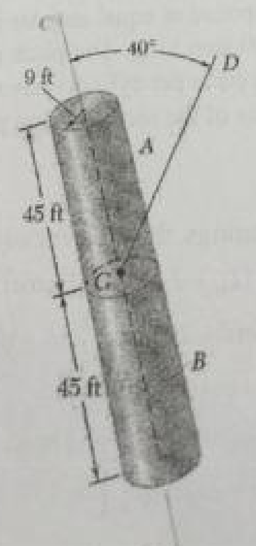


Figure 1.1

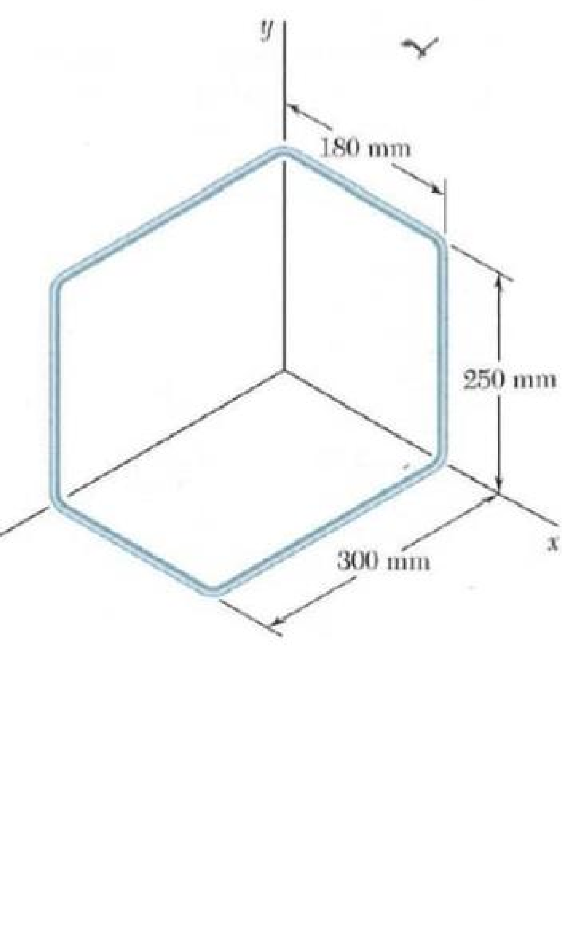
(24 Points)

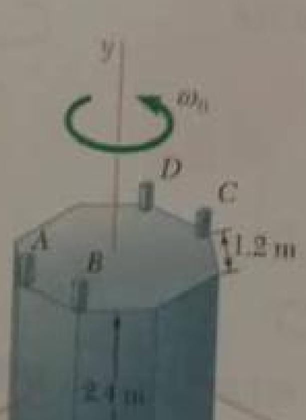
Question 2
Three-dimensional kinetics of a rigid body

A space station consists of two sections A and B of equal masses, which are rigidly connected. Each section is dynamically equivalent to a homogenous cylinder of length 45 ft. and radius 9 ft. Knowing that the station is precessing about the fixed direction GD at the constant rate of 2 rev/h, determine the rate of spin of the station about its axis of symmetry CC'.



The figure shown is formed of 1.5-mm diameter aluminum wire. The density of aluminum is 2800 kg/m3. Determine the mass products of inertia of the wire figure. (Partial Answer: $I_{xy}=3.46x10^{-5}$ *kg-m²*)





18.124 The angular velocity vector of a football which has just been kicked is horizontal, and its axis of symmetry OC is oriented as shown. Knowing that the magnitude of the angular velocity is 200 rpm and that the ratio of the axis and transverse moments of inertia is $I/I' = \frac{1}{3}$ determine (*a*) the orientation of the axis of precession OA, (*b*) the rates of precession and spin.

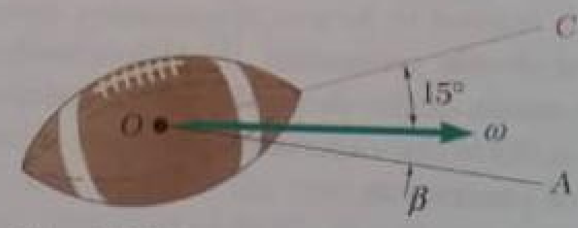


Fig. P18.124

18.125 A 2500-kg satellite is 2.4 m high and has octagonal bases of sides 1.2 m. The coordinate axes shown are the principal centroidal axes of inertia of the satellite, and its radii of gyration are $k_x = k_z =$ 0.90 m and $k_y = 0.98$ m. The satellite is equipped with a main 500-N thruster E and four 20-N

	M (kg)	r (m)	z (m)	mr	mrz (kg m)
1	50	0.15	-.400	7.5	-3
2	50	0.15	0	7.5	0
3	50	0.15	0.400	7.5	3

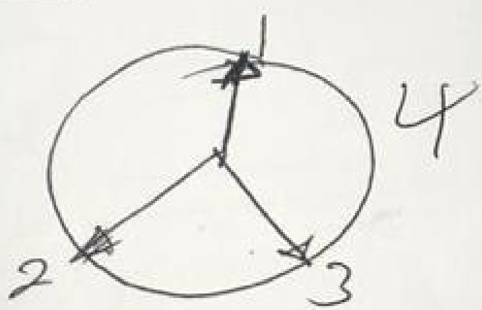
0.3

20.3

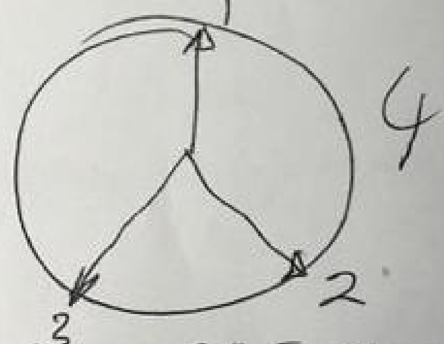
0.3

$l = 0.5m$

Forces



1° Forces

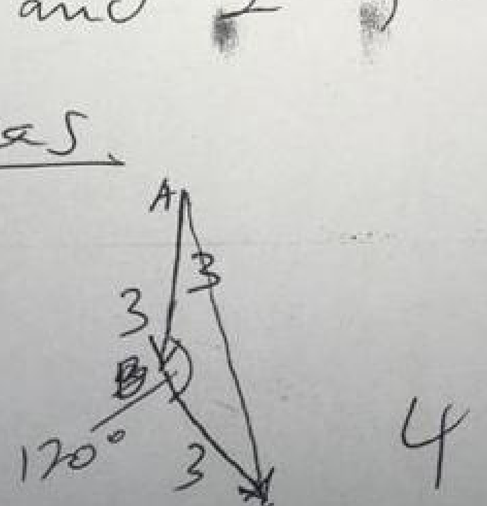


2° Forces.

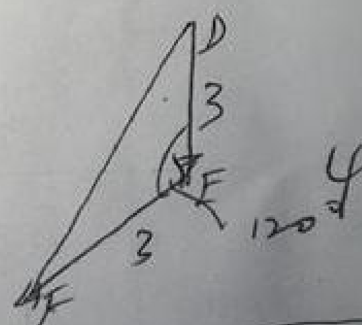
1° and 2° forces balanced.

Couples

1°



2°

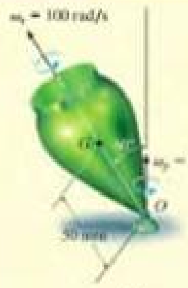


$AC = \sqrt{3^2 + 3^2 - (2)(3)(3)\cos 120} = 1.\,n a 7 i r$

$FD = \sqrt{3^2 + 3^2 - 2(3)(3)\cos 120}$

$= 1.0934 \times \%$

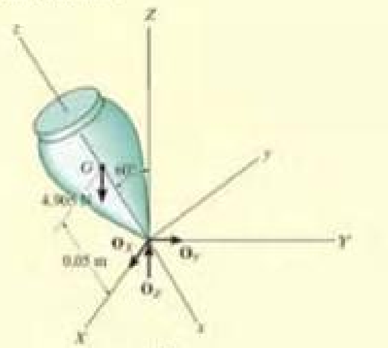
EXAMPLE 21.7



(a)

Fig. 21–20

The top shown in Fig. 21–20*a* has a mass of 0.5 kg and is precessing about the vertical axis at a constant angle of $\theta = 60°$. If it spins with an angular velocity $\omega_s = 100$ rad/s, determine the precession ω_p. Assume that the axial and transverse moments of inertia of the top are $0.45(10^{-3})\ \text{kg}\cdot\text{m}^2$ and $1.20(10^{-3})\ \text{kg}\cdot\text{m}^2$, respectively, measured with respect to the fixed point O.



(b)

SOLUTION

Equation 21–30 will be used for the solution since the motion is *steady precession*. As shown on the free-body diagram, Fig. 21–20*b*, the coordinate axes are established in the usual manner, that is, with the positive z axis in the direction of spin, the positive Z axis in the direction of precession, and the positive x axis in the direction of the moment ΣM_x (refer to Fig. 21–16). Thus,

$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\dot{\phi}\sin\theta(\dot{\phi}\cos\theta + \dot{\psi})$$

$$4.905\ \text{N}(0.05\ \text{m})\sin 60° = -[1.20(10^{-3})\ \text{kg}\cdot\text{m}^2\,\dot{\phi}^2]\sin 60°\cos 60°$$

$$+\,[0.45(10^{-3})\ \text{kg}\cdot\text{m}^2]\dot{\phi}\sin 60°(\dot{\phi}\cos 60° + 100\ \text{rad/s})$$

or

$$\dot{\phi}^2 - 120.0\dot{\phi} + 654.0 = 0 \qquad (1)$$

Solving this quadratic equation for the precession gives

$$\dot{\phi} = 114\ \text{rad/s} \qquad \text{(high precession)} \qquad \textit{Ans.}$$

and

$$\dot{\phi} = 5.72\ \text{rad/s} \qquad \text{(low precession)} \qquad \textit{Ans.}$$

NOTE: In reality, low precession of the top would generally be observed, since high precession would require a larger kinetic energy.

EXAMPLE 21.8

The 1-kg disk shown in Fig. 21–21*a* spins about its axis with a constant angular velocity $\omega_D = 70$ rad/s. The block at B has a mass of 2 kg, and by adjusting its position s one can change the precession of the disk about its supporting pivot at O while the shaft remains horizontal. Determine the position s that will enable the disk to have a constant precession $\omega_p = 0.5$ rad/s about the pivot. Neglect the weight of the

Question 5

Flywheels and governors

a) Distinguish between the functions of the governor and the flywheel of an engine. (2 **points)**

b) An engine runs at 1000 rev/min and a curve of the turning moment plotted on a crank angle base showed the following areas alternately above and below the mean turning-moment line: 700 (above), (480 below), 520, 620, 260, 460, 340, and 420 mm^2. The scales used were 1 mm = 400 N m for turning moment, and 1 mm = 1° for crank angle. Determine the coefficient of fluctuation of speed. The rotating parts are equivalent to a mass of 45 kg at a radius of gyration of 140 mm. (8 **points)**

c) In a Porter governor (Figure 5.1), the upper and lower arms are each inclined at 30° to the vertical when the sleeve is in its lowest position. The points of suspension are each 36 mm from the axis of the spindle. The mass of each rotating ball is 3 kg, and that of the central load on the sleeve is 20 kg. If the movement of the sleeve is 36 mm, find the range of speed of the governor.

(7.5 points)

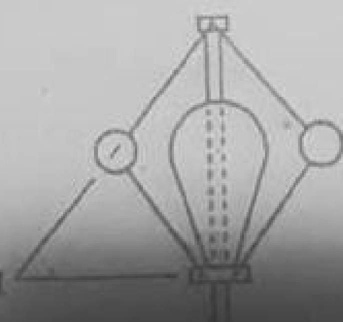


Figure 5.1

we can write:

Q 4 cnt'd

~~es~~ limit

$$(2.7) W_{maxu}^2 (0.1436)(0.14) = \frac{(329.07 + 14)(0.09)}{2}$$

$$N_{maxu} = \underline{\underline{161.04 \text{ rev/min}}}$$

$$= 16.86 \text{ rad/s}$$

$$(2.7) W_{maxl}^2 (0.1436)(0.14) = \frac{(329.07 - 14)(0.09)}{2}$$

$$N_{maxl} = \underline{\underline{154.3 \text{ rev/min}}}$$

$$16.147 \text{ rad/s}$$

lower limit

$$(2.7)(W_{minu}^2)(0.1063)(0.14) = \frac{(161.07 + 14)0.09}{2}$$

$$N_{minu} = \underline{\underline{133.7 \text{ rev/min}}}$$

$$= 14 \text{ rad/s}$$

$$(2.7)(W_{minl}^2)(0.1063)(0.14) = \frac{(161.07 - 14)(0.09)}{2}$$

$$N_{minl} = \underline{\underline{122.5 \text{ rev/min}}}$$

$$= 12.82 \text{ rad/s}$$

1° Couple $= 1.0934 \, kg \, m^2$ G 5 cntd

$= 1.0934 \times \left(\frac{2\pi(400)}{60}\right)^2$

$= \underline{\underline{1{,}918.5 \ N \cdot m}}$

2° Couples

$= 1.0934 \times \left(\frac{0.15}{0.5}\right)\left(\frac{2\pi \times 400}{60}\right)^2$

$= \underline{\underline{575.5 \ N \cdot m}}$, remembering that

$C = Fz = m\omega^2 r z \left(\cos\theta + \frac{r}{l}\cos 2\theta\right)$

KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI
COLLEGE OF ENGINEERING

BSc. Mechanical Engineering, Mid-Semester Examination, November, 2017

ME 361 Dynamics of Machinery

Answer All Questions **Time allowed: 1 Hour**

Question 1 (5 Points)

Rigid body geometry

The component shown in Fig 2. is formed of 1.5-mm diameter aluminium wire. If the density of aluminium is 2800 kg/m^3, determine the moments of inertia I_{xx}, I_{yy}, and I_{zz} of the wire figure.

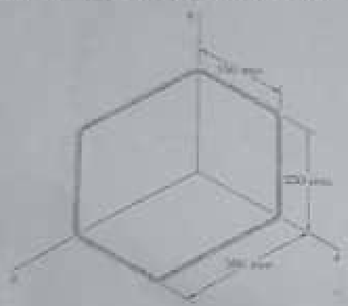


Fig 2.1

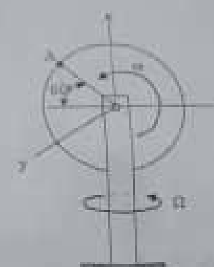


Fig 3.1

Question 2 (5 Points)

Kinematics of a rigid body

A disc of radius 0.4 m is supported by a vertical shaft as shown in Figure Fig 3.1 above. The shaft rotates about its vertical axis with a constant angular velocity $\Omega = 6$ rad/s. The disc rotates with a constant angular velocity $\omega = 8$ rad/s relative to the shaft. Determine the angular velocity and acceleration of the disc.

Question 3 (5 Points)

Kinetics of a rigid body

The angular velocity of an American football (Fig 1.1) which has just been kicked is horizontal, and its axis of symmetry OC is oriented as shown. Knowing that the magnitude of the angular velocity is 200 rpm and that the ratio of the axis and transverse moments of inertia is $I/I' = 1/3$, determine the orientation of the axis of precession OA, (b) the rates of precession and spin.

Fig 1.1

S.M. Sackey

Question 3 **(24 Points)**

Turning-moment diagrams/Flywheels

The turning-moment diagram for an engine, which has been drawn to scales of 1 mm to 50 N m and 1 mm to 1° of rotation of the crankshaft, shows the greatest amount of energy which has to be stored by the flywheel is represented by an area of 2250 mm^2. The flywheel is to run at a mean speed of 240 rev/min with a total speed variation of 2 per cent. If the mass of the flywheel is to be 450 kg, determine suitable dimensions for the rim, the internal diameter being 0.9 of the external diameter. Neglect the inertia of the arms and hub of the wheel. Cast iron has a density of 7.2 Mg/m^3.

Question 4 **(24 Points)**

Flywheels and governors

a) Distinguish between the functions of the governor and the flywheel of an engine. **(8 points)**

b) A Hartnell governor (Figure 4.1) has two rotating balls of mass 2.7 kg each. The ball radius is 125 mm in the mean position when the ball arms are vertical and the speed is 150 rev/min with the sleeve rising. The length of the ball arm is 140 mm and the length of the sleeve arms 90 mm. The stiffness of the spring is 7 kN/m and the total sleeve movement is ±12 mm from the mean position. Allowing for a constant friction force of 14 N acting at the sleeve, determine the speed range of the governor in the lowest and highest sleeve positions.
Neglect the obliquity of the ball arms. **(16 points)**

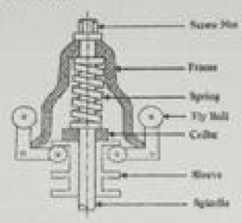


Figure 4.1

Question 5 **(24 Points)**

Balancing

A three-cylinder engine has the cranks spaced at equal angular intervals of 120°. Each crank is 150 mm long and each connecting rod is 500 mm long. The pitch of the cylinders is 400 mm and the speed is 400 rev/min. If the reciprocating parts per cylinder have a mass of 50 kg, find the maximum unbalanced primary and secondary effects of the reciprocating parts.

FORMULA SHEET

With all symbols bearing their usual meanings, the moment equations of motion are:

$$\sum M_x = I_{xx}\ddot{\theta} + (I_{zz} - I_{xx})\dot{\phi}^2 \sin\theta\cos\theta + I_{zz}\dot{\psi}\dot{\phi}\sin\theta$$

$$\sum M_y = I_{xx}(\ddot{\phi}\sin\theta + 2\dot{\phi}\dot{\theta}\cos\theta) - I_{zz}\dot{\theta}(\dot{\phi}\cos\theta + \dot{\psi})$$

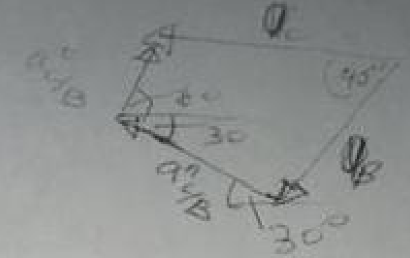
$$\sum M_z = I_{zz}(\ddot{\psi} + \ddot{\phi}\cos\theta - \dot{\phi}\dot{\theta}\sin\theta)$$

here the angles ϕ, θ and ψ are the precession, nutation and spin, respectively. Other equations are:

$$\omega = \omega_1 + \omega_2 + ..\omega_n \qquad \alpha = \alpha_1 + \alpha_2 + \alpha_3 \qquad \dot{u} = (\dot{u})_{rel} + \Omega \times u$$

$$r_B = r_A + r_{B/A} \qquad v_B = v_A + (v_{B/A})_{xyz} + \Omega \times r_{B/A}$$

$$a_B = a_A + (a_{B/A})_{xyz} + \dot{\Omega} \times r_{B/A} + 2\Omega \times (v_{B/A})_{xyz} + \Omega \times (\Omega \times r_{B/A})$$



$$a_C = a_B + \alpha_{BC} \times r_{C/B} + \cancel{(a_{C/B})_{xyz}} + \cancel{2W_{BC} \times (V_{C/B})_{xyz}} + W_{BC} \times (W_{BC} \times r_{C/B})$$

$$= a_B + \alpha_{BC} \times r_{C/B} + W_{BC} \times (W_{BC} \times r_{C/B})$$

$$|a_B| = w^2_{AB} r_{AB} = 0.4 \times 100^2 = 4000 \text{ m/s}^2$$

$$|a^n_{C/B}| = w^2_{BC} r_{C/B} = (57.73)^2 \cdot (0.5657)$$
$$= 1885 \text{ m/s}^2$$

Then

$$-a_c i = -4000\cos 45 i - 4000 \sin 45 j + 1885 \sin 30 j$$
$$-1885 \cos 30 j + \alpha \cos 60 i + \alpha \sin 60 j$$

$$\alpha = (4000 \sin 45 + 1885 \cos 30 - 1885 \sin 30)/\sin 60$$

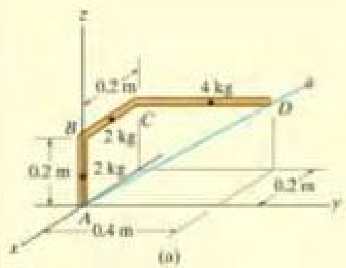
$$-a_c i = -2828.43 i - 2828.43 j - 1632.4 i$$
$$942.5 j - (\alpha k) \times (0.4899 i - 0.2828 j)$$

$$= -2828.43 i - 1632.4 i - 0.2828 \alpha i$$
$$-2828.43 j + 942.5 j - 0.4899 \alpha j$$

$$\alpha = -3{,}849.6, \quad a_c = 3372.15 \text{ m/s}^2$$

axes, respectively (see Appendix C).

EXAMPLE 21.1



Determine the moment of inertia of the bent rod shown in Fig. 21–5*a* about the *Aa* axis. The mass of each of the three segments is given in the figure.

SOLUTION

Before applying Eq. 21–5, it is first necessary to determine the moments and products of inertia of the rod with respect to the x, y, z axes. This is done using the formula for the moment of inertia of a slender rod, $I = \frac{1}{12}ml^2$, and the parallel-axis and parallel-plane theorems, Eqs. 21–3 and 21–4. Dividing the rod into three parts and locating the mass center of each segment, Fig. 21–5*b*, we have

$$I_{xx} = \left[\tfrac{1}{12}(2)(0.2)^2 + 2(0.1)^2\right] + [0 + 2(0.2)^2] + \left[\tfrac{1}{12}(4)(0.4)^2 + 4((0.2)^2 + (0.2)^2)\right] = 0.480\ \text{kg}\cdot\text{m}^2$$

$$I_{yy} = \left[\tfrac{1}{12}(2)(0.2)^2 + 2(0.1)^2\right] + \left[\tfrac{1}{12}(2)(0.2)^2 + 2((-0.1)^2 + (0.2)^2)\right] + [0 + 4((-0.2)^2 + (0.2)^2)] = 0.453\ \text{kg}\cdot\text{m}^2$$

$$I_{zz} = [0 + 0] + \left[\tfrac{1}{12}(2)(0.2)^2 + 2(-0.1)^2\right] + \left[\tfrac{1}{12}(4)(0.4)^2 + 4((-0.2)^2 + (0.2)^2)\right] = 0.400\ \text{kg}\cdot\text{m}^2$$

$$I_{xy} = [0 + 0] + [0 + 0] + [0 + 4(-0.2)(0.2)] = -0.160\ \text{kg}\cdot\text{m}^2$$

$$I_{yz} = [0 + 0] + [0 + 0] + [0 + 4(0.2)(0.2)] = 0.160\ \text{kg}\cdot\text{m}^2$$

$$I_{zx} = [0 + 0] + [0 + 2(0.2)(-0.1)] + [0 + 4(0.2)(-0.2)] = -0.200\ \text{kg}\cdot\text{m}^2$$

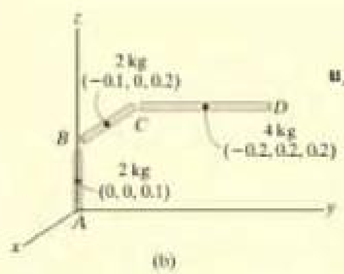


Fig. 21–5

The *Aa* axis is defined by the unit vector

$$\mathbf{u}_{Aa} = \frac{\mathbf{r}_D}{r_D} = \frac{-0.2\mathbf{i} + 0.4\mathbf{j} + 0.2\mathbf{k}}{\sqrt{(-0.2)^2 + (0.4)^2 + (0.2)^2}} = -0.408\mathbf{i} + 0.816\mathbf{j} + 0.408\mathbf{k}$$

Thus,

$$u_x = -0.408 \qquad u_y = 0.816 \qquad u_z = 0.408$$

Substituting these results into Eq. 21–5 yields

$$\begin{aligned} I_{Aa} &= I_{xx}u_x^2 + I_{yy}u_y^2 + I_{zz}u_z^2 - 2I_{xy}u_xu_y - 2I_{yz}u_yu_z - 2I_{zx}u_zu_x \\ &= 0.480(-0.408)^2 + (0.453)(0.816)^2 + 0.400(0.408)^2 \\ &\quad - 2(-0.160)(-0.408)(0.816) - 2(0.160)(0.816)(0.408) \\ &\quad - 2(-0.200)(0.408)(-0.408) \\ &= 0.169\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

Ans.

(b) A generating set is arranged on board a ship with its axis parallel to the longitudinal centre-line of the ship. The revolving parts have a mass of 1400 kg, a radius of gyration of 400 mm and revolve at 720 rev/min. If the ship steams at 36 km/h, round a curve of 180 m radius, find the magnitude and sense of the gyroscopic couple transmitted to the ship. Show with a sketch the effect of the gyroscopic couple on the bow of the ship.

(9 Points)

The equations of motion are:

$$\sum M_x = I_{xx}\ddot{\theta} + (I_{zz} - I_{xx})\dot{\phi}^2 \sin\theta\cos\theta + I_{zz}\dot{\psi}\dot{\phi}\sin\theta$$

$$\sum M_y = I_{xx}\left(\ddot{\phi}\sin\theta + 2\dot{\phi}\dot{\theta}\cos\theta\right) - I_{zz}\dot{\theta}\left(\dot{\phi}\cos\theta + \dot{\psi}\right)$$

$$\sum M_z = I_{zz}\left(\ddot{\psi} + \ddot{\phi}\cos\theta - \dot{\phi}\dot{\theta}\sin\theta\right)$$

where the angles ϕ, θ and ψ refer to the precession, nutation and spin, respectively, other symbols bearing their usual meanings.

Question 3

Balancing of machinery

A five cylinder in-line engine has asimilar reciprocating parts at equal centre distances and the cranks are successively 72^0 apart. Show that the primary and secondary forces balance for all positions of the crank shaft.If each reciprocating mass is 4 kg, each crank is 50 mm and each connecting rod is 175 mm long, and the cylinder centre distances are 100 mm, determine, using diagrams and calculation, the maximum values of (a) the primary and (b) the secondary couples when the speed is 120 rev/min and state the positions of the central crank when these maxima occur.

(17.5 Points)

Question 4

Rigid body geometry

Determine the moment of inertia of the rod shown in Figure 4.1 about the *Aa* axis. The mass of each of the three segments is shown in the figure. **(17.5 Points)**

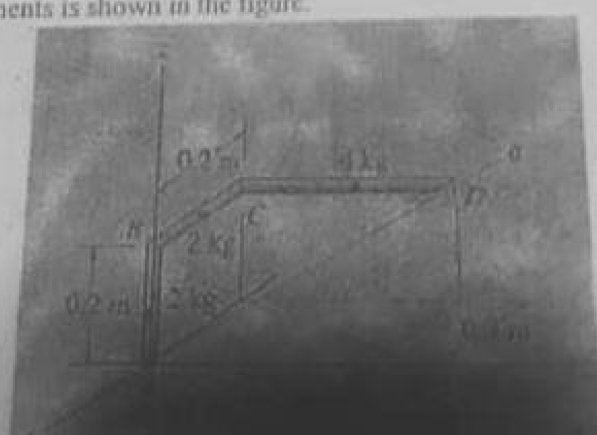
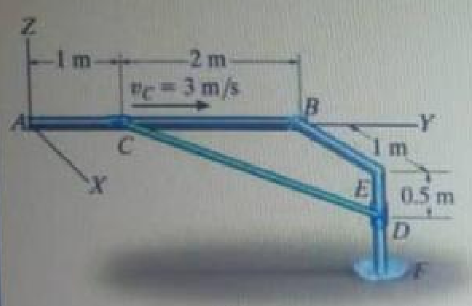


Figure 4.1

EXAMPLE 20.3



(a)

If the collar at C in Fig. 20–10a moves towards B with a speed of 3 m/s, determine the velocity of the collar at D and the angular velocity of the bar at the instant shown. The bar is connected to the collars at its end points by ball-and-socket joints.

SOLUTION

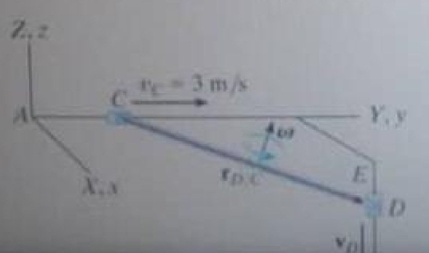
Bar CD is subjected to general motion. Why? The velocity of point D on the bar can be related to the velocity of point C by the equation

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{D/C}$$

The fixed and translating frames of reference are assumed to coincide at the instant considered, Fig. 20–10b. We have

$$\mathbf{v}_D = -v_D\mathbf{k} \qquad \mathbf{v}_C = \{3\mathbf{j}\} \text{ m/s}$$

$$\mathbf{r}_{D/C} = \{1\mathbf{i} + 2\mathbf{j} - 0.5\mathbf{k}\} \text{ m} \qquad \boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

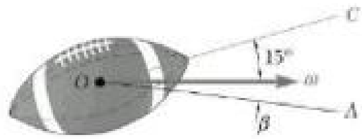


Substituting into the above equation we get

$$-v_D\mathbf{k} = 3\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 1 & 2 & -0.5 \end{vmatrix}$$

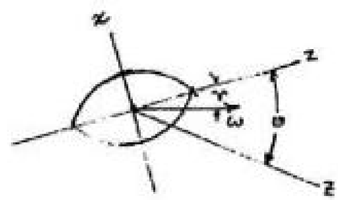
Expanding and equating the respective i, j, k components yields

PROBLEM 18.125



The angular velocity vector of a football which has just been kicked is horizontal, and its axis of symmetry OC is oriented as shown. Knowing that the magnitude of the angular velocity is 200 rpm and that the ratio of the axis and transverse moments of inertia is $I/I' = \frac{1}{3}$, determine (*a*) the orientation of the axis of precession OA, (*b*) the rates of precession and spin.

SOLUTION



$$\tan\gamma = -\frac{\omega_x}{\omega_z}$$

$$\gamma = 15°$$

For steady precession with no force,

$$\tan\theta = \frac{I'}{I}\tan\gamma$$

$$= 3\tan 15°$$

$$\theta = 38.794°$$

(*a*) $$\beta = \theta - \gamma = 38.794 - 15°$$ $\beta = 23.8°$ ◄

(*b*) $$\omega_x = -\dot{\varphi}\sin\theta = -\omega\sin\gamma$$

$$\dot{\varphi} = \frac{\omega\sin\gamma}{\sin\theta} = \frac{(200 \text{ rpm})\sin 15°}{\sin(38.794°)}$$

$$= 82.621 \text{ rpm}$$

precession: $\dot{\varphi} = 82.6$ rpm ◄

$$\omega_z = \dot{\psi} + \dot{\varphi}\cos\theta = \omega\cos\gamma$$

$$\dot{\psi} = \omega\cos\gamma - \dot{\varphi}\cos\theta$$

$$= 200\cos 15° - 82.621\cos 38.794°$$

spin: $\dot{\psi} = 128.8$ rpm ◄

ME 361 REGULAR
SCHEME

Q1. $V_C = V_B + (V_{C/B})_{xyz} + \omega_{BC} \times r_{C/B}$

$V_B = \omega_{AB} R_{AB} = (100)(40) = 4000\ cm/s$

$= 40 m/s.$ 1

$V_B = -40\cos 45 i + 40 \sin 45 j$

$= -28.28 i + 28.28 j$ 1

$BC = \dfrac{AB \sin 45}{\sin 30} = 0.5657\ m$ 1

$r_{C/B} = 0.5657 \cos 30 i - 0.5657 \sin 30 j$

$= 0.4899 i - 0.2828 j$ 2

Then

$-V_c i = -28.28 i + 28.28 j + \omega_{BC} k \times (0.4899 i - 0.2828 j)$

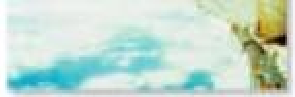
$= -28.28 i + 28.28 j + 0.4899 \omega_z j + 0.2828 \omega_z i$ 2

$\therefore\ \omega_z = -28.28 / 0.4899 = -57.73\ rad/s$

$= \underline{\underline{57.73}}\ rad/s$ 2

Then $-V_c = -28.28 + 0.2828(-57.73)$

$V_c = \underline{\underline{44.61}}\ m/s$ 2



EXAMPLE 21.9

The motion of a football is observed using a slow-motion projector. From the film, the spin of the football is seen to be directed 30° from the horizontal, as shown in Fig. 21–24*a*. Also, the football is precessing about the vertical axis at a rate $\dot{\phi} = 3$ rad/s. If the ratio of the axial to transverse moments of inertia of the football is $\frac{1}{3}$, measured with respect to the center of mass, determine the magnitude of the football's spin and its angular velocity. Neglect the effect of air resistance.

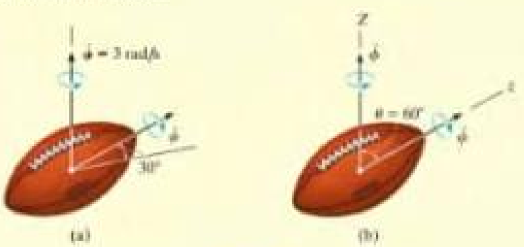


Fig. 21–24

SOLUTION

Since the weight of the football is the only force acting, the motion is torque-free. In the conventional sense, if the z axis is established along the axis of spin and the Z axis along the precession axis, as shown in Fig. 21–24*b*, then the angle $\theta = 60°$. Applying Eq. 21–37, the spin is

$$\dot{\psi} = \frac{I - I_z}{I_z}\dot{\phi}\cos\theta = \frac{I - \frac{1}{3}I}{\frac{1}{3}I}(3)\cos 60°$$

$$= 3 \text{ rad/s} \qquad \textit{Ans.}$$

Using Eqs. 21–34, where $H_G = \dot{\phi}I$ (Eq. 21–36), we have

$$\omega_x = 0$$

$$\omega_y = \frac{H_G \sin\theta}{I} = \frac{3I \sin 60°}{I} = 2.60 \text{ rad/s}$$

$$\omega_z = \frac{H_G \cos\theta}{I_z} = \frac{3I \cos 60°}{\frac{1}{3}I} = 4.50 \text{ rad/s}$$

Thus,

$$\omega = \sqrt{(\omega_x)^2 + (\omega_y)^2 + (\omega_z)^2}$$

$$= \sqrt{(0)^2 + (2.60)^2 + (4.50)^2}$$

$$= 5.20 \text{ rad/s} \qquad \textit{Ans.}$$

PROBLEMS

2

$0 = (I_{zz} - I_{xx})\dot{\phi}^2 \cos\theta + I_{zz}\dot{\psi}\dot{\phi}$

$\therefore \dot{\psi} = \left(\frac{I_{xx}}{I_{zz}} - 1\right)\dot{\phi}\cos\theta$ 3

$\theta = 40°$

2 $\dot{\phi} = 2\,\text{rev/h} = \frac{2}{3600}\,\text{rev/s}$ 2

$= 0.00349\,\text{rad/s}$

$I_{xx} = \frac{m}{12}(3r^2 + h^2) = \left(\frac{3 \times 9^2 + 90^2}{12}\right)m$

$= 695.25m$ 2

$I_{zz} = \frac{mr^2}{2} = \frac{m \times 9^2}{2} = 40.5m$

2

$\therefore \frac{I_{xx}}{I_{zz}} = 17.1667$ 3

$\therefore \dot{\psi} = (17.1667 - 1) \cdot \frac{2}{3600} \times 2\pi \times \cos 40$ 5

$= \underline{\underline{~~0.00688~~\ \text{rad/s}}}$ 3

$= 0.043229\,\text{rad/s}$

$= 0.4128\,\text{rev/min}$

$W_{av} = 25.13$ rad/s

$W_{max} = 25.3813$ rad/s

$W_{min} = 24.8787$ rad/s

ie 2% total variation

The $\Delta E = \frac{I}{2}(W_{max}^2 - W_{min}^2)$ 3

$$2250 \times \frac{50\pi}{180} = \frac{I}{2}(25.3813^2 - 24.8787^2)$$

$$I = 155.4 \text{ kg m}^2$$ 3

ie $$I = 155.4 = \frac{m}{2}(R^2 + r^2)$$ 4 $r = 0.9R$

$$\therefore 1.81R^2 = \frac{2 \times 155.4}{450}$$

$$R^2 = 0.3815$$ 4

$$R = 0.6177 \text{ m}, \quad D = 1.235 \text{ m}$$

Now volume of flywheel is mass/ρ

ie, $$V = \frac{450}{7200} = 0.0625 \text{ m}^3$$

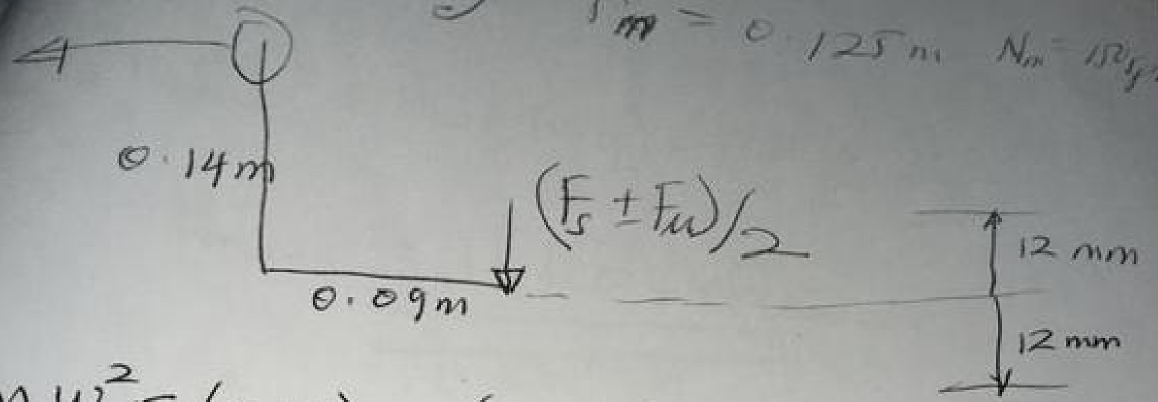
ie $$[\pi R^2 - \pi(0.81)R^2]t = 0.0625$$ 4

or $$R^2 t = 0.1047 \text{ m}^3$$ 4

Substituting for R, we get

$$t = \frac{0.1047}{0.6177^2} = 0.2744 \text{ m}$$ 4

$m = 2.7\,kg$ $\quad r_m = 0.125\,m$ $\quad N_m = 150\,rpm$



$$m\omega_m^2 r_m (0.14) = \frac{(F_s + 14)}{2}\,0.09$$

$$(2.7)\left(\frac{2\pi 150}{60}\right)^2(0.125)(0.14) = \frac{(F_s + 14)}{2}\times 0.09$$

$$F_{sm} = 245.07\,N$$

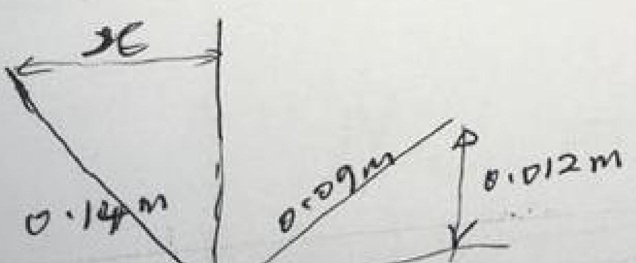
$$\therefore \delta_m = F_{sm}/k = 245.07/7000$$

$$= 35.01\,mm$$

Then $\delta_u = 47.01\,mm$, $\quad F_u = 7\times 47.01\,N = 329.07\,N$

$\delta_l = 23.01\,mm$ $\quad F_l = 7\times 23.01\,N = 161.07\,N$

$$x = \frac{0.12}{0.09}\times 0.14 = 0.0186\,m$$



Then $r_u = 0.1436\,m$

$r_l = 0.1063\,m$