# Power and Torque Transmission -Shaft Design

- Material Selection
- Geometric Layout
- Stress and strength
  - Static strength
  - Fatigue strength
- Sizing
  - Diameters
  - Fillet radii

# Shaft Materials

- Shafts are commonly made from low carbon, CD or HR steel, such as AISI 1020–1050 steels.
- Fatigue properties don't usually benefit much from high alloy content and heat treatment.
- Surface hardening usually only used when the shaft is being used as a bearing surface.

# Shaft Layout

- **Issues to consider for shaft layout**
  - Axial layout of components
  - Supporting axial loads
  - Providing for torque transmission
  - Assembly and Disassembly

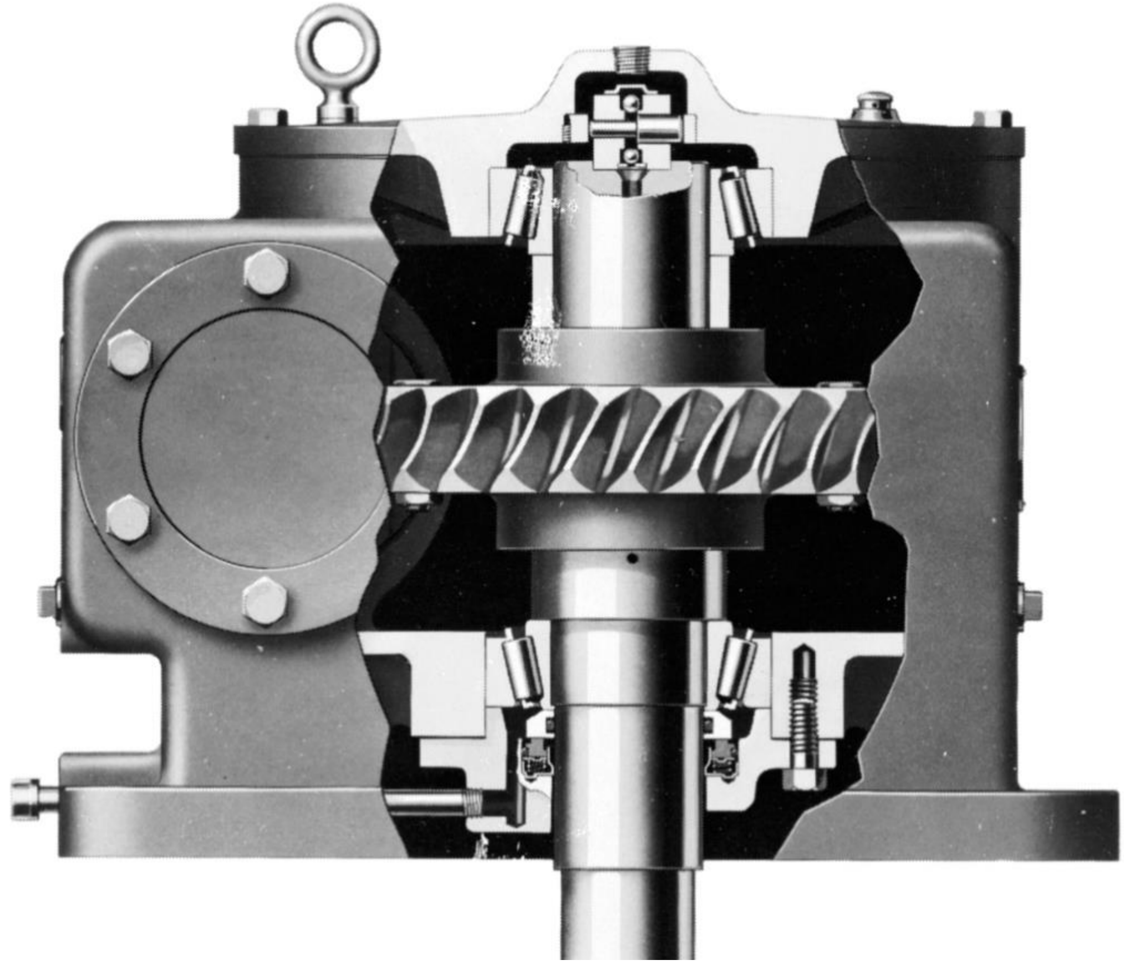


Fig. 7–1

# Axial Layout of Components

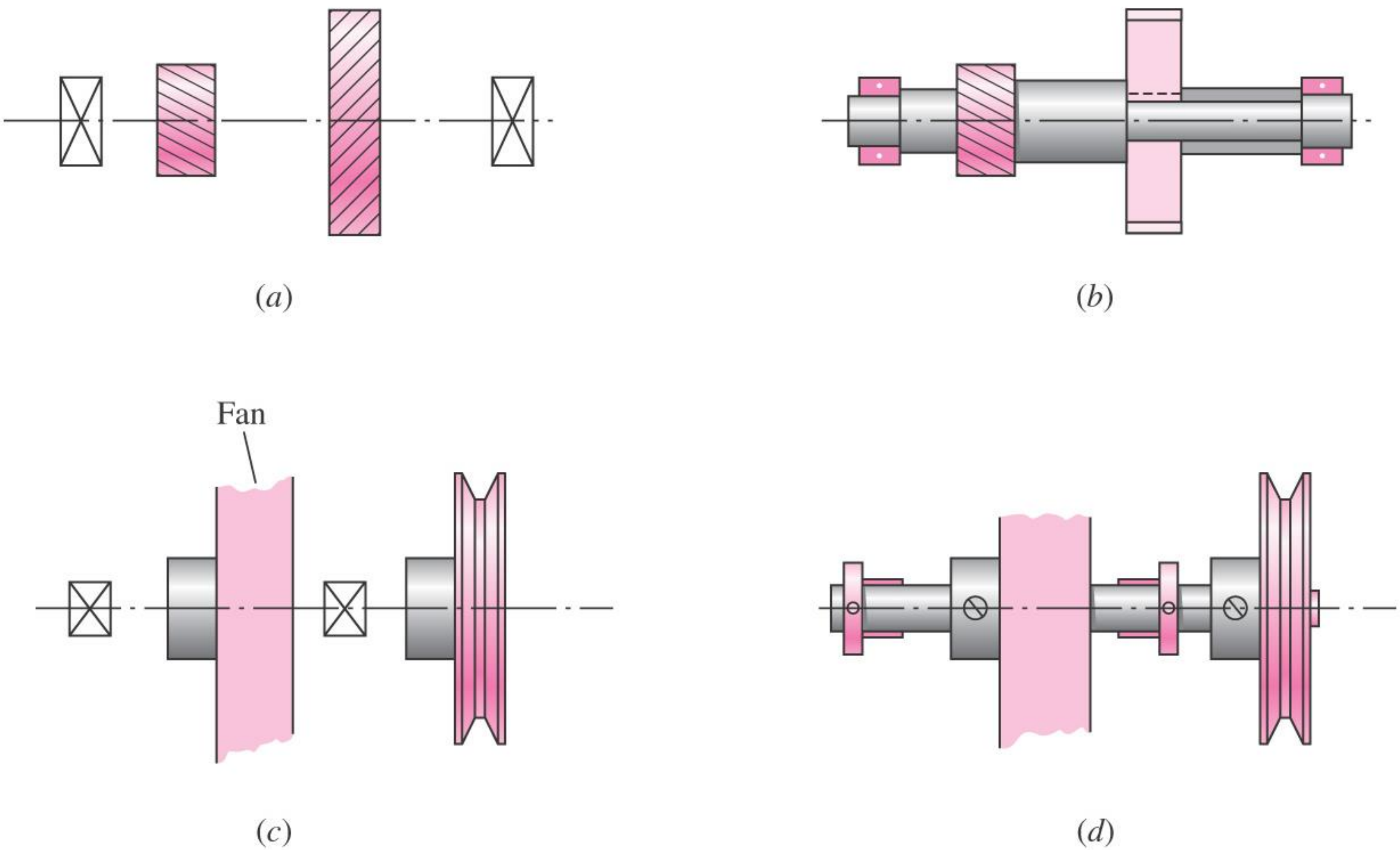



Fig. 7–2

# Providing for Torque Transmission

- Common means of transferring torque to shaft
  - Keys
  - Splines
  - Setscrews
  - Pins
  - Press or shrink fits
  - Tapered fits
- **Keys are one of the most effective**
  - Slip fit of component onto shaft for easy assembly
  - Positive angular orientation of component
  - Can design key to be weakest link to fail in case of overload

# Assembly and Disassembly

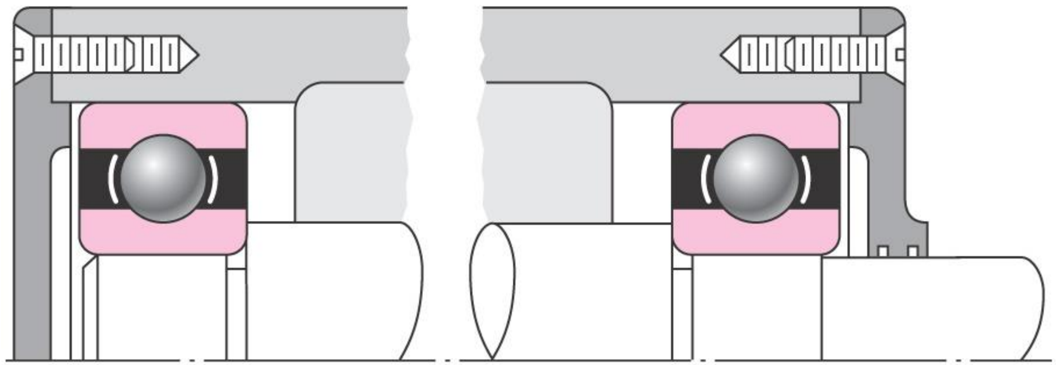


Fig. 7–5

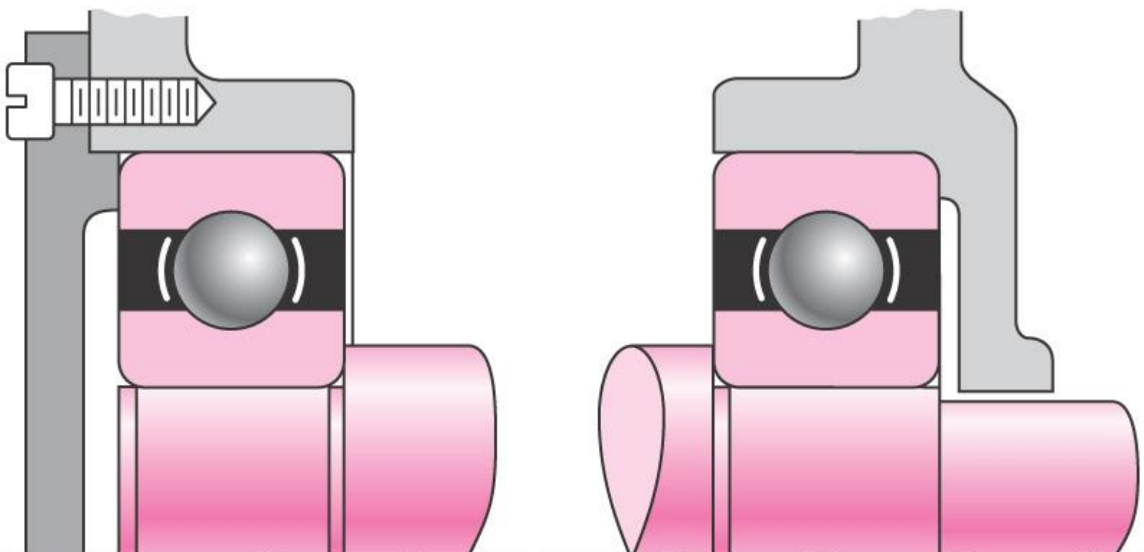


Fig. 7–6

# Assembly and Disassembly

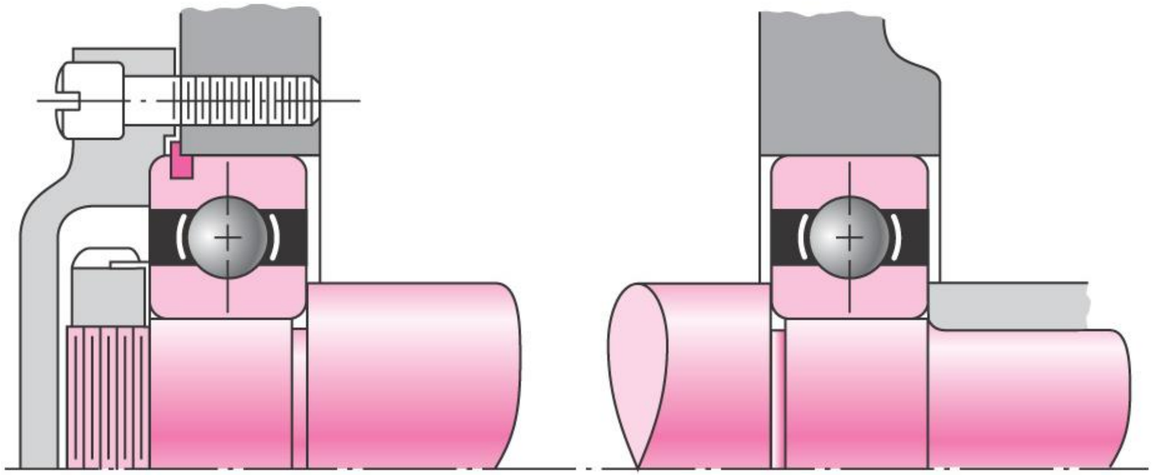


Fig. 7–7

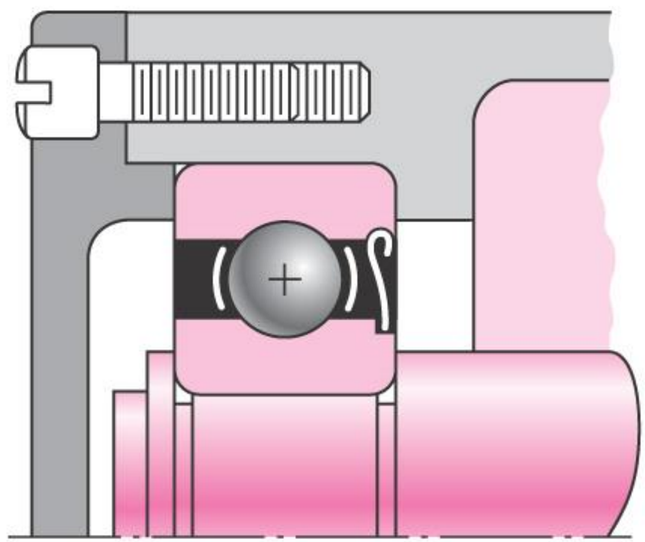


Fig. 7–8

# Shaft Design for Stress

- Stresses are only evaluated at critical locations
- Critical locations are usually
  - Where the bending moment is large
  - Where the torque is present
  - Where stress concentrations exist

# Shaft Stresses

- Standard stress equations can be customized for shafts for convenience
- Axial loads are generally small and constant, so can be ignored
- **Standard alternating and midrange stresses:**

$$\sigma_a = K_f \frac{M_a c}{I} \qquad \sigma_m = K_f \frac{M_m c}{I} \qquad (7\text{–}1)$$

$$\tau_a = K_{fs} \frac{T_a r}{J} \qquad \tau_m = K_{fs} \frac{T_m r}{J} \qquad (7\text{–}2)$$

- **Customized for round solid shafts:**

$$\sigma_a = K_f \frac{32 M_a}{\pi d^3} \qquad \sigma_m = K_f \frac{32 M_m}{\pi d^3} \qquad (7\text{–}3)$$

$$\tau_a = K_{fs} \frac{16 T_a}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16 T_m}{\pi d^3} \qquad (7\text{–}4)$$

# Shaft Stresses

- **Combine stresses into Equivalent von Mises stresses**

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2} \qquad (7\text{–}5)$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2} \qquad (7\text{–}6)$$

# Shaft Stresses

- Substitute von Mises stresses into failure criteria equation. For example, using modified Goodman line,

$$\frac{1}{n} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}$$

$$\frac{1}{n} = \frac{16}{\pi d^3} \left\{ \frac{1}{S_e} \left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_{ut}} \left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2} \right\} \tag{7–7}$$

- **Solving for *d* is convenient for design purposes**

$$d = \left( \frac{16n}{\pi} \left\{ \frac{1}{S_e} \left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_{ut}} \left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2} \right\} \right)^{1/3} \tag{7–8}$$

# Shaft Stresses

- *DE-ASME Elliptic* ***(Recommended for Transmission Shafts)***

$$\frac{1}{n} = \frac{16}{\pi d^3}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}$$

(7–11)

$$d = \left\{\frac{16n}{\pi}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}\right\}^{1/3}$$

(7–12)

- *DE-Soderberg* ***(Conservative Design ONLY)***

$$\frac{1}{n} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_y}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2}\right\}$$

(7–13)

$$d = \left(\frac{16n}{\pi}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_y}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2}\right\}\right)^{1/3}$$

(7–14)

# Shaft Stresses for Rotating Shaft

- **For rotating shaft with steady bending and torsion**
  - Bending stress is completely reversed, since a stress element on the surface cycles from equal tension to compression during each rotation
  - Torsional stress is steady
  - Previous equations simplify with $M_m$ and $T_a$ equal to 0

# Shaft Under Reversed Bending and Constant Torsion

## DE-Elliptic Criterion

Factor of Safety

$$\frac{1}{n} = \frac{16}{\pi d^3}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\cancelto{0}{\left(\frac{K_{fs} T_a}{S_e}\right)^2} + 4\cancelto{0}{\left(\frac{K_f M_m}{S_y}\right)^2} + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2} \tag{7–11}$$

Shaft Diameter

$$d = \left\{\frac{16n}{\pi}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\cancelto{0}{\left(\frac{K_{fs} T_a}{S_e}\right)^2} + 4\cancelto{0}{\left(\frac{K_f M_m}{S_y}\right)^2} + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}\right\}^{1/3} \tag{7–12}$$

# Checking for Yielding in Shafts

- Always necessary to consider static failure, even in fatigue situation
- Soderberg criteria inherently guards against yielding
- ASME-Elliptic criteria takes yielding into account, but is not entirely conservative
- Gerber and modified Goodman criteria require specific check for yielding

# Checking for Yielding in Shafts

- Assume Fatigue Failure and find the shaft diameter, d
- Use von Mises maximum stress to check for yielding using the calculated diameter, *d*

$$\sigma'_{\max} = \left[(\sigma_m + \sigma_a)^2 + 3\,(\tau_m + \tau_a)^2\right]^{1/2}$$

$$= \left[\left(\frac{32K_f\,(M_m + M_a)}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}\,(T_m + T_a)}{\pi d^3}\right)^2\right]^{1/2} \qquad (7\text{–}15)$$

$$n_y = \frac{S_y}{\sigma'_{\max}} \qquad (7\text{–}16)$$

- To safeguard against yielding, n (yield) > n(fatigue)

# Estimating Stress Concentrations

- Stress analysis for shafts is highly dependent on stress concentrations.
- Stress concentrations depend on size specifications, which are not known the first time through a design process.
- Standard shaft elements such as shoulders and keys have standard proportions, making it possible to estimate stress concentrations factors before determining actual sizes.

# Keys and Pins

- Used to secure rotating elements and to transmit torque

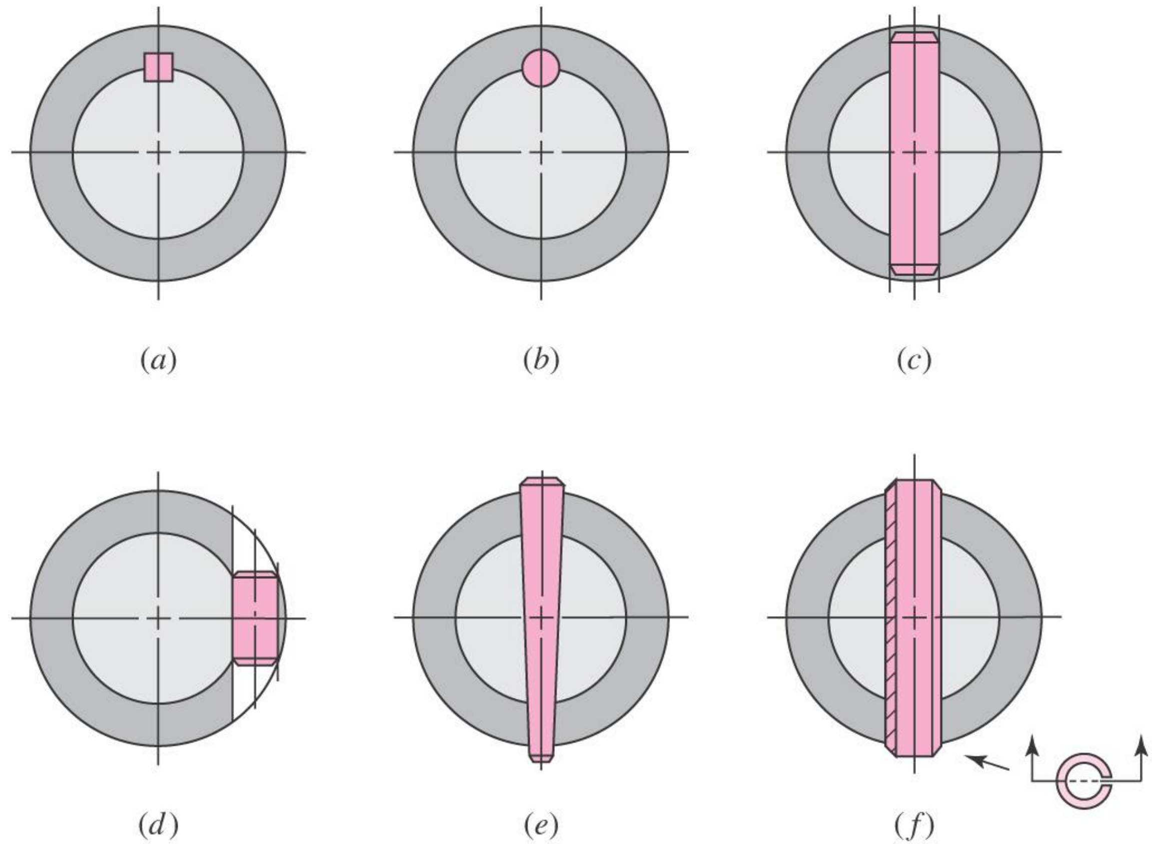



Fig. 7–16

# Stress Concentration Factors for Keys

- For keyseats cut by standard end-mill cutters, with a ratio of $r/d = 0.02$, Peterson's charts give
  - $K_t = 2.14$ for bending
  - $K_t = 2.62$ for torsion without the key in place
  - $K_t = 3.0$ for torsion with the key in place
- The end of the keyseat must be at least $d/10$ from the shoulder fillet to prevent the two stress (keyseat and shoulder) concentrations from combining.

# Retaining Rings

- Retaining rings are often used instead of a shoulder to provide axial positioning

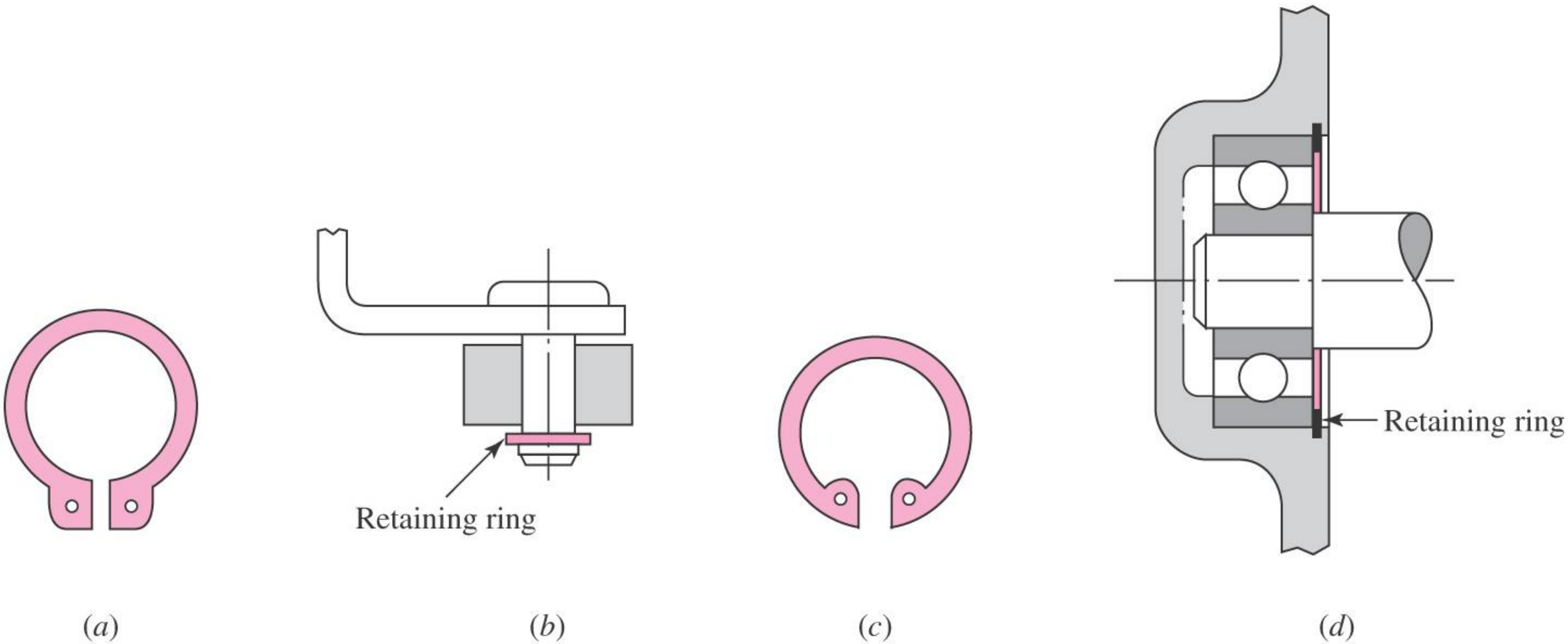



Fig. 7–18

# Retaining Rings

- Retaining ring must seat well in bottom of groove to support axial loads against the sides of the groove.
- This requires sharp radius in bottom of groove.
- Stress concentrations for flat-bottomed grooves are available in Table A–15–16 and A–15–17.
- Typical stress concentration factors are high, around 5 for bending and axial, and 3 for torsion

# Estimating Stress Concentrations

## Table 7–1

First Iteration Estimates for Stress-Concentration Factors $K_t$ and $K_{ts}$.

*Warning:* These factors are only estimates for use when actual dimensions are not yet determined. Do *not* use these once actual dimensions are available.

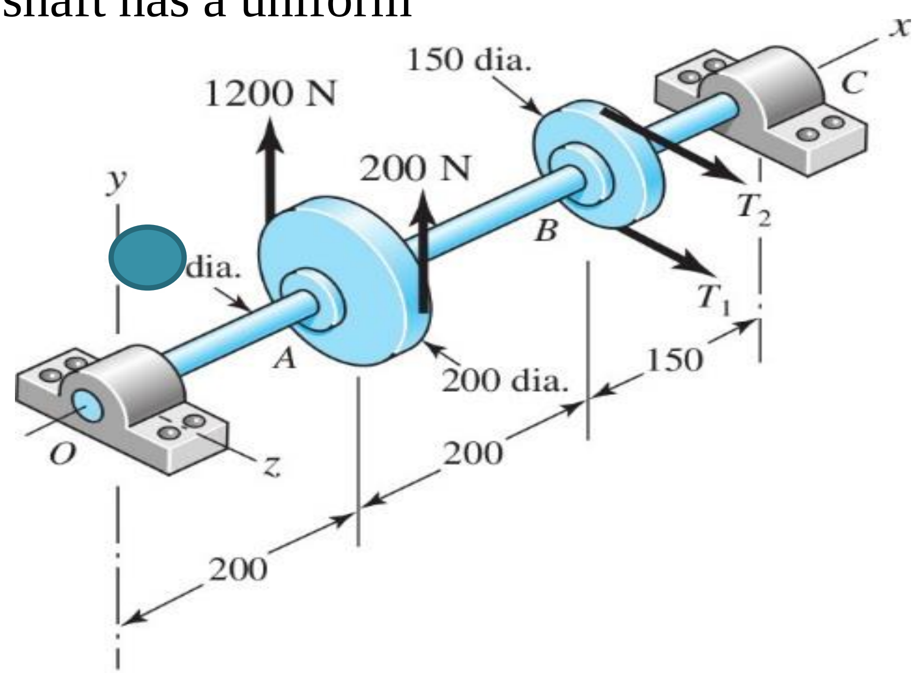
| | Bending | Torsional | Axial |
|---|---|---|---|
| Shoulder fillet—sharp ($r/d = 0.02$) | 2.7 | 2.2 | 3.0 |
| Shoulder fillet—well rounded ($r/d = 0.1$) | 1.7 | 1.5 | 1.9 |
| End-mill keyseat ($r/d = 0.02$) | 2.14 | 3.0 | — |
| Sled runner keyseat | 1.7 | — | — |
| Retaining ring groove | 5.0 | 3.0 | 5.0 |

Missing values in the table are not readily available.

# Axle and Shaft Design Example

Refer to the shaft provided. Pulley at A receives power and transmits it to pulley at B through keys. The torque is steady. The belt tension at the slack side of Pulley B is 15% of the tension at the tight side.

a. Propose a profile for the shaft
b. Specify the necessary diameters and fillet radii for the shaft
c. Specify the dimensions of rectangular keys
d. If there is no rotation and shaft has a uniform diameter, specify the diameter.




Problem 3–70*

Dimensions in millimeters.

# Keys

- Failure of keys is by either direct shear or bearing stress
- Key length is designed to provide desired factor of safety
- Factor of safety should not be excessive, so the inexpensive key is the weak link
- Key length is limited to hub length
- Key length should not exceed 1.5 times shaft diameter to avoid problems from twisting
- Multiple keys may be used to carry greater torque, typically oriented $90^{o}$ from one another
- Stock key material is typically low carbon cold-rolled steel, with dimensions slightly under the nominal dimensions to easily fit end-milled keyway
- A setscrew is sometimes used with a key for axial positioning, and to minimize rotational backlash

# Keys

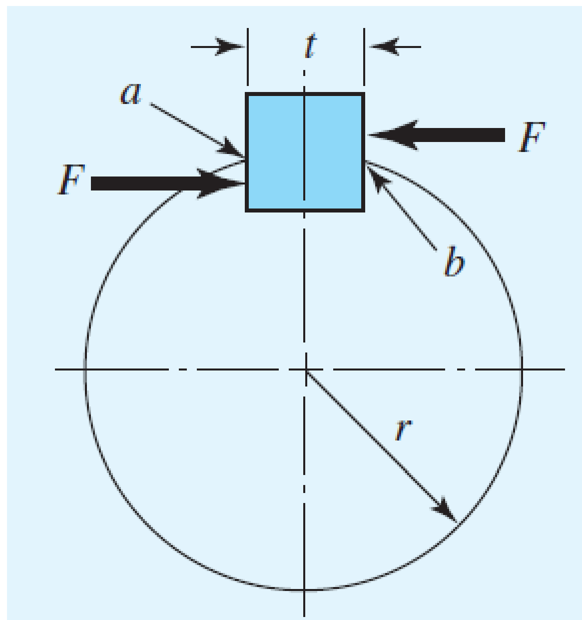
- Keys come in standard square and rectangular sizes
- Shaft diameter determines key size

Table 7–6

| Shaft Diameter | | Key Size | | |
|---|---|---|---|---|
| Over | To (Incl.) | $w$ | $h$ | Keyway Depth |
| $\frac{5}{16}$ | $\frac{7}{16}$ | $\frac{3}{32}$ | $\frac{3}{32}$ | $\frac{3}{64}$ |
| $\frac{7}{16}$ | $\frac{9}{16}$ | $\frac{1}{8}$ | $\frac{3}{32}$ | $\frac{3}{64}$ |
| | | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{16}$ |
| $\frac{9}{16}$ | $\frac{7}{8}$ | $\frac{3}{16}$ | $\frac{1}{8}$ | $\frac{1}{16}$ |
| | | $\frac{3}{16}$ | $\frac{3}{16}$ | $\frac{3}{32}$ |
| $\frac{7}{8}$ | $1\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{3}{16}$ | $\frac{3}{32}$ |
| | | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |
| $1\frac{1}{4}$ | $1\frac{3}{8}$ | $\frac{5}{16}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |
| | | $\frac{5}{16}$ | $\frac{5}{16}$ | $\frac{5}{32}$ |
| $1\frac{3}{8}$ | $1\frac{3}{4}$ | $\frac{3}{8}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |
| | | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{3}{16}$ |
| $1\frac{3}{4}$ | $2\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{8}$ | $\frac{3}{16}$ |
| | | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |
| $2\frac{1}{4}$ | $2\frac{3}{4}$ | $\frac{5}{8}$ | $\frac{7}{16}$ | $\frac{7}{32}$ |
| | | $\frac{5}{8}$ | $\frac{5}{8}$ | $\frac{5}{16}$ |
| $2\frac{3}{4}$ | $3\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |
| | | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{8}$ |

# Example – Key Design

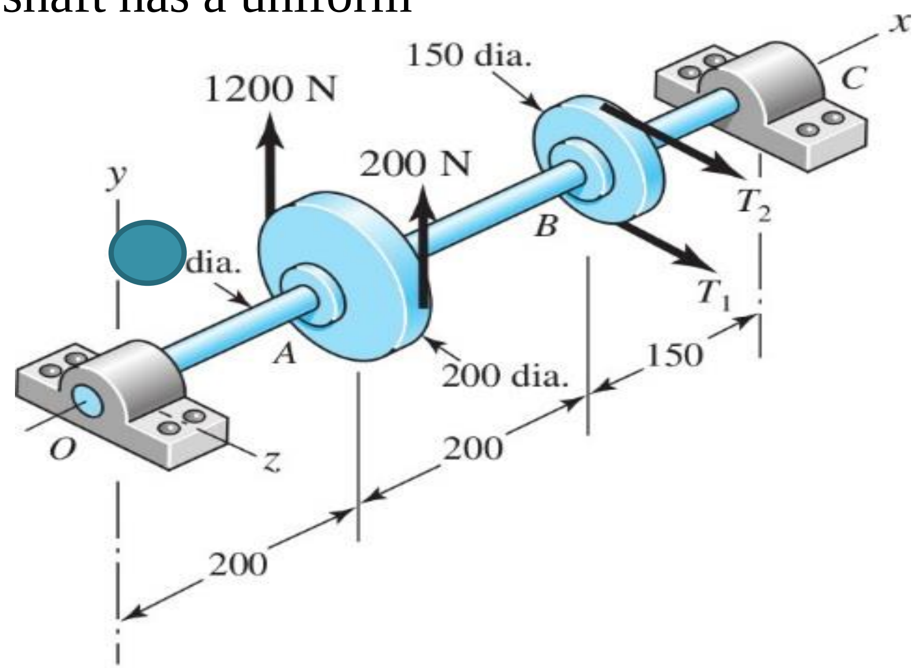
A UNS G1050 steel shaft, heat treated to  minimum yield strength of 525 MPa has a diameter of 36 mm. The shaft rotates at 600 rev/min and transmits 30 kW through a gear. Select a safe key for the Gear.



# Axle and Shaft Design Example

Refer to the shaft provided. Pulley at A receives power and transmits it to pulley at B through keys. The torque is steady. The belt tension at the slack side of Pulley B is 15% of the tension at the tight side.

a. **Propose a profile for the shaft - Done**
b. **Specify the necessary diameters and fillet radii for the shaft - Done**
c. Specify the dimensions of rectangular keys
d. If there is no rotation and shaft has a uniform diameter, specify the diameter.




*Problem 3–70**

Dimensions in millimeters.

# Gib-head Key

- Gib-head key is tapered so that when firmly driven it prevents axial motion
- Head makes removal easy
- Projection of head may be hazardous

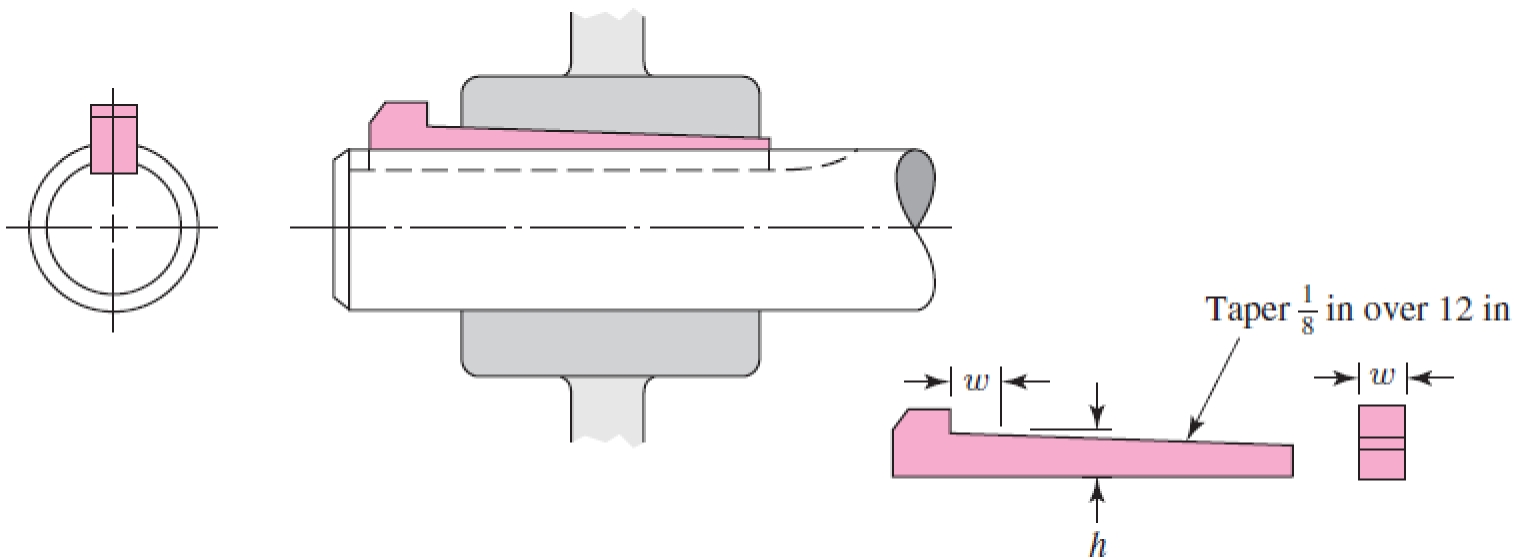



Fig. 7–17 *(a)*

# Woodruff Key

- Woodruff keys have deeper penetration
- Useful for smaller shafts to prevent key from rolling
- When used near a shoulder, the keyway stress concentration interferes less with shoulder than square keyway

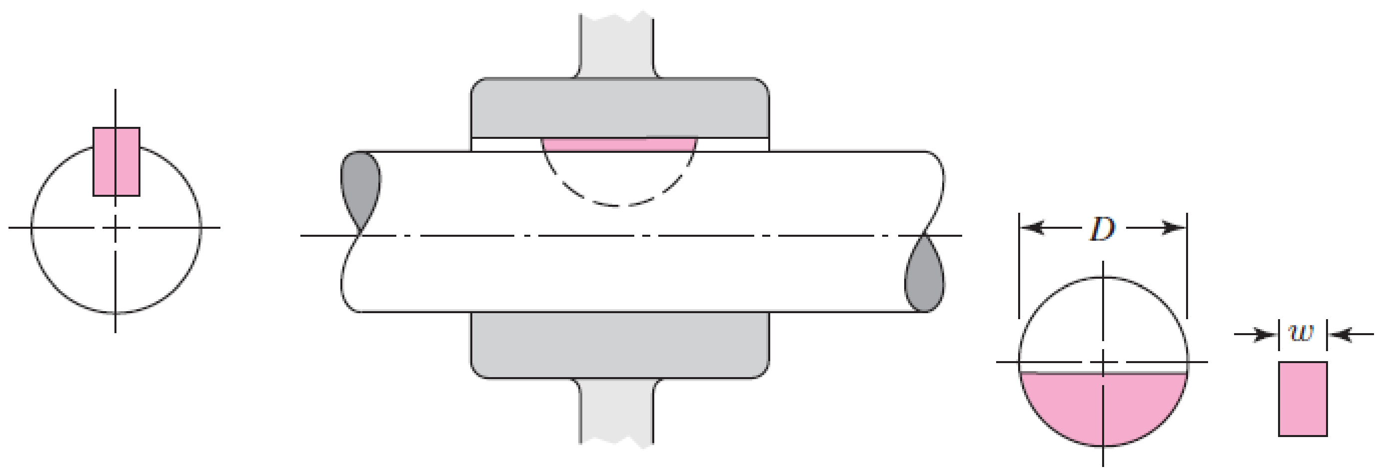



Fig. 7–17 (b)

# Setscrews

- Setscrews resist axial and rotational motion
- They apply a compressive force to create friction
- The tip of the set screw may also provide a slight penetration
- Various tips are available

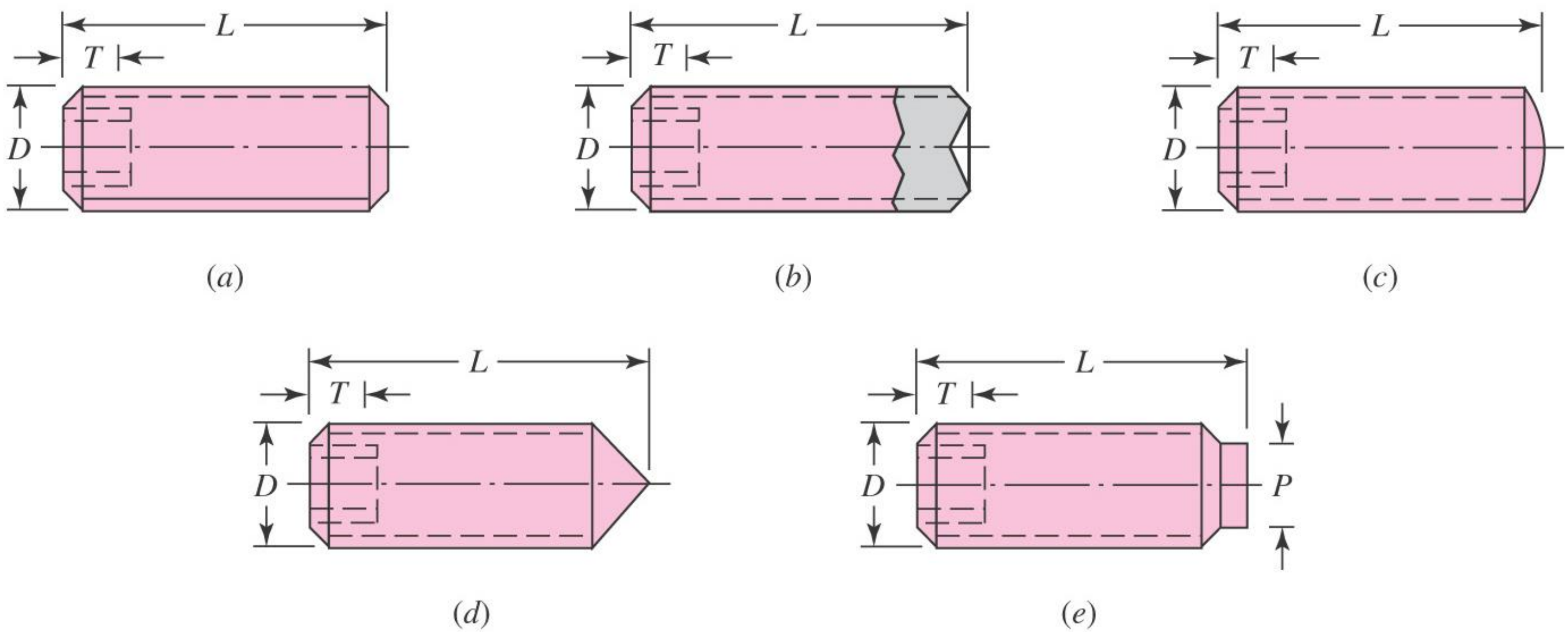



Fig. 7–15

# Reducing Stress Concentration at Shoulder Fillet

- Bearings often require relatively sharp fillet radius at shoulder
- If such a shoulder is the location of the critical stress, some manufacturing techniques are available to reduce the stress concentration
  - (a) Large radius undercut into shoulder
  - (b) Large radius relief groove into back of shoulder
  - (c) Large radius relief groove into small diameter of shaft

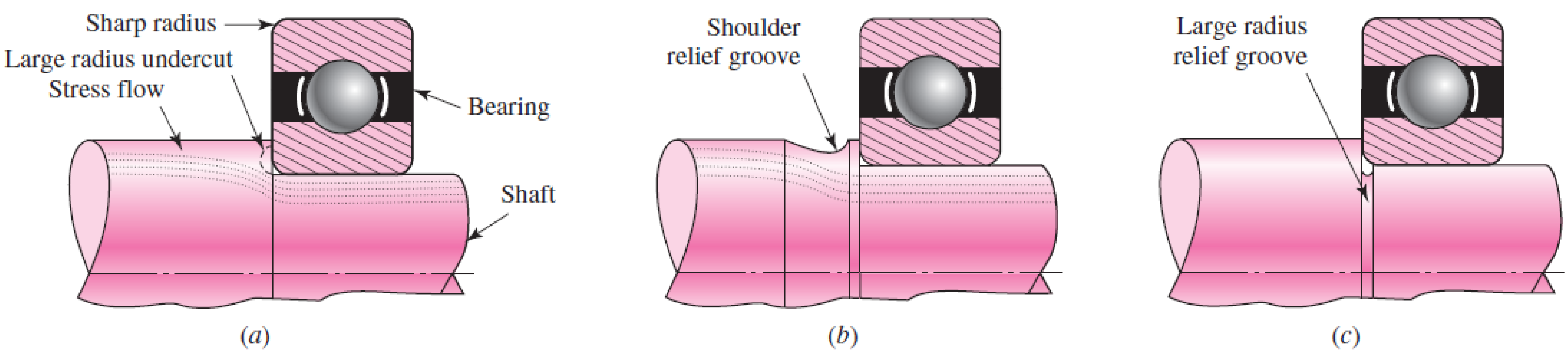



Fig. 7–9

# Deflection Considerations

- Deflection analysis at a single point of interest requires complete geometry information for the entire shaft.
- For this reason, a common approach is to size critical locations for stress, then fill in reasonable size estimates for other locations, then perform deflection analysis.
- Deflection of the shaft, both linear and angular, should be checked at gears and bearings.

# Deflection Considerations

- Deflection analysis is straightforward, but lengthy and tedious to carry out manually.
- Each point of interest requires entirely new deflection analysis.
- Consequently, shaft deflection analysis is almost always done with the assistance of software.
- Options include specialized shaft software, general beam deflection software, and finite element analysis software.

# Critical Speeds for Shafts

- A shaft with mass has a critical speed at which its deflections become unstable.
- Components attached to the shaft have an even lower critical speed than the shaft.
- Designers should ensure that the lowest critical speed is at least twice the operating speed.

# Critical Speeds for Shafts

- For a simply supported shaft of uniform diameter, the first critical speed is

$$\omega_1 = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{m}} = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{gEI}{A\gamma}} \qquad (7\text{–}22)$$

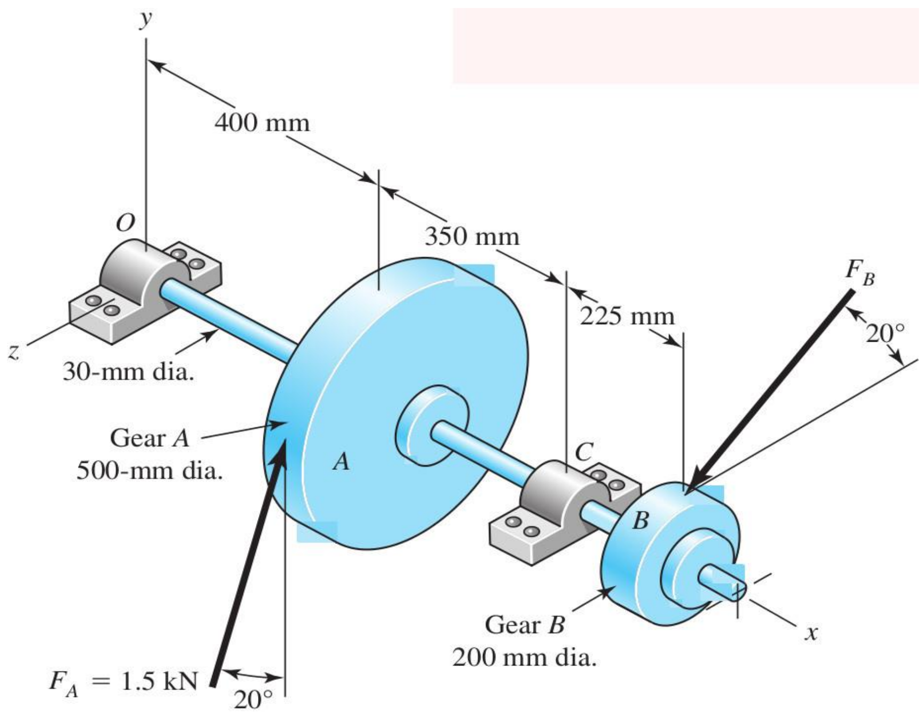
- For an ensemble of attachments, Rayleigh's method for lumped masses gives

$$\omega_1 = \sqrt{\frac{g \sum w_i y_i}{\sum w_i y_i^2}} \qquad (7\text{–}23)$$

# Assignment 1 – Problem 1

**Gear at A receives power and transmits it to gear at B through keys. The torque is steady and shaft is supported by ball bearings at O and C.**

a. **Propose a profile for the shaft**
a. **Specify the necessary diameters and fillet radii for the shaft**
b. **Specify the dimensions of rectangular keys**
c. **Complete the design by showing all dimensions.**

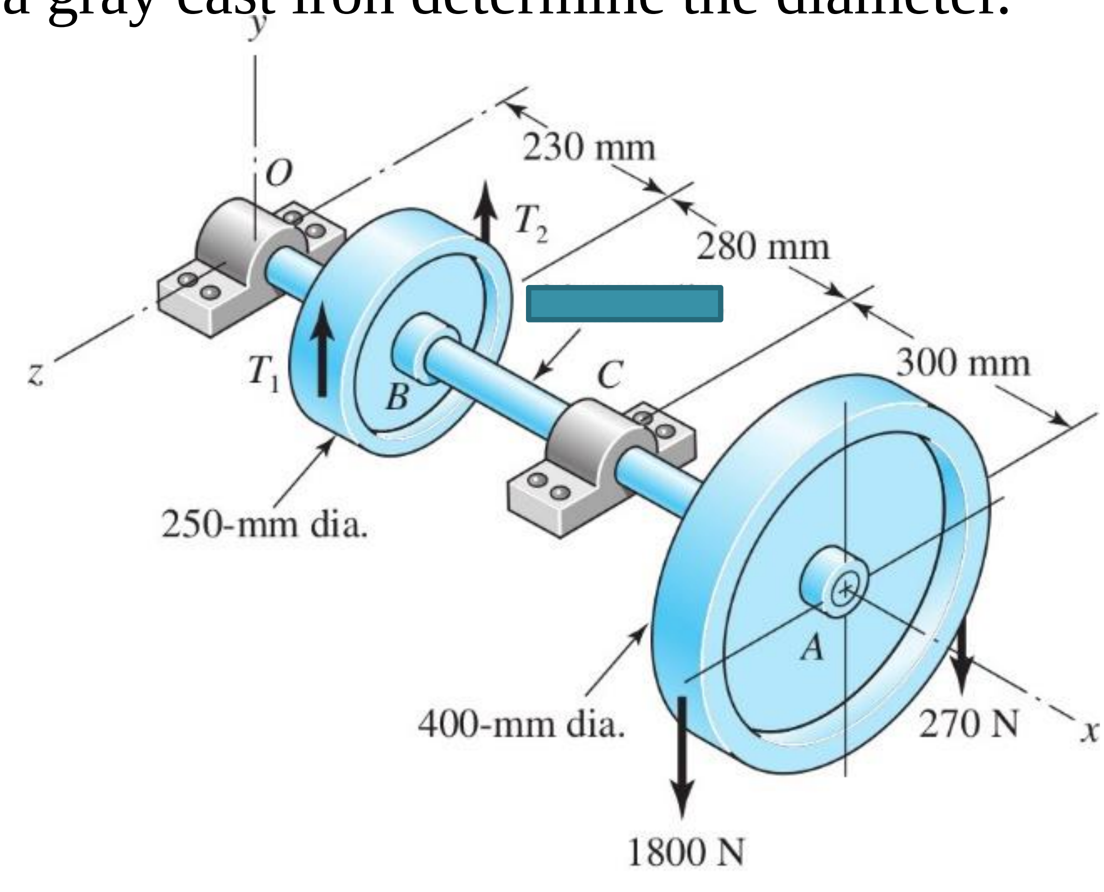



Problem 3–72*

# Assignment 1 – Problem 2

The axle has a uniform diameter and supports two elements as shown. It does not rotate and a factor of safety of 1.5 is desired.

a. Select a plain carbon steel and determine the diameter
b. Select a gray cast iron determine the diameter.




Problem 3–69*