Numerical Analysis
Assignment 2

1. Given
$$\begin{aligned} 2x_1 &+ -x_2 + 3x_3 &= 2 \\ -x_1 &+ 2x_2 - x_3 &= 1 \\ 3x_1 &- x_2 + 2x_3 &= 3 \end{aligned}$$
Find the following.

   (a) Under what condition can the Cholesky Factorization Method be used to solve a Linear System?

   (b) LU factorization of the coefficient matrix using Cholesky reduction, hence the solution to the systems of equation.

   (c) from (b) above, Find the inverse of the coefficient matrix.

2. Prove that the Pascal Matrix below is Symmetric Positve Definite.
$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{pmatrix}$$

3. Solve the following set of three linear equations using the SOR iteration method. Use a relaxation parameter $w = 1.25$ and initial guess $x^{(0)} = [1, 1, 1]^T$
$$\begin{aligned} 4x_1 &+ 3x_2 &= 24 \\ 3x_1 &+ 4x_2 - x_3 &= 30 \\ &- x_2 + 4x_3 &= -24 \end{aligned}$$

4. Using $[1, 3, 5]^T$ as the initial guess, find the value of $[x_1, x_2, x_3]^T$ after three iterations in Gauss Seidel method for $\begin{bmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 6 \end{bmatrix}$

5. Solve the following system using the LU-Decomposition Method.
$$\begin{pmatrix} 6 & 2 & 1 & 1 \\ 2 & 4 & 1 & 0 \\ 1 & 1 & 4 & -1 \\ -1 & 0 & -1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 11 \\ 20 \end{pmatrix}$$