Assignment 2

March 30, 2017

1. (a)
   - $f_x = 3y^2e^{3x}$
   - $f_y = 2ye^{3x}$

   (b)
   - $$\begin{aligned} Z_x &= 5(3xy+2x)^4(3y+2), \\ &= 5(3y+2)(3xy+2x)^4. \end{aligned}$$
   - $$\begin{aligned} Z_y &= 5(3xy+2x)^4 \times 3x, \\ &= 15x(3xy+2x)^4. \end{aligned}$$

   (c)
   - $$\begin{aligned} g_x &= e^{x+3y}\sin(xy) + e^{x+3y}y\cos(xy), \\ &= e^{x+3y}\sin(xy) + ye^{x+3y}\cos(xy), \\ &= e^{x+3y}[\sin(xy) + ycos(xy)]. \end{aligned}$$
   - $$\begin{aligned} g_y &= 3e^{x+3y}\sin(xy) + e^{x+3y}\cos(xy)x, \\ &= 3e^{x+3y}\sin(xy) + xe^{x+3y}\cos(xy), \\ &= e^{x+3y}[3\sin(xy) + xcos(xy)]. \end{aligned}$$

2.
$$F(x,y) = \frac{x^2}{y+1} = x^2(y+1)^{-1},$$
$$F_y = -x^2(y+1)^{-2}(1) = -\frac{x^2}{(y+1)^2}.$$
$$F_y(3,2) = -\frac{9}{9} = -1.$$

3.

$$
\begin{aligned}
Z_y &= [7(3x^2)y^6 - 2y](15xy-8)^{-1} + (3x^2y^7 - y^2) \times -(15xy-8)^{-2} \times 15x, \\
&= \frac{21x^2y^6 - 2y}{15xy-8} - \frac{3x^2y^7 - y^2(15x)}{(15xy-8)^2}, \\
&= \frac{21x^2y^6 - 2y}{15xy-8} - \frac{45x^3y^7 - 15xy^2}{(15xy-8)^2}.
\end{aligned}
$$

4.

$$
\begin{aligned}
Z_x &= -(2x^2ay)^{-2}(4xay) + 15x^4abcy^{-1}, \\
&= -\frac{4xay}{(2x^2ay)^2} + \frac{15x^4abc}{y}, \\
&= -\frac{4xay}{(2x^2ay)^2} + \frac{15x^4abc}{y}, \\
&= -\frac{1}{x^3ay} + \frac{15x^4abc}{y}, \\
&= -\frac{1}{y}[\frac{1}{x^3a} - 15x^4abc].
\end{aligned}
$$

5.

$$
\begin{aligned}
\frac{\partial}{\partial\lambda}\left(\frac{x^2y\lambda - 3\lambda^5}{\sqrt{\lambda^2 - 3\lambda + 5}}\right) &= \frac{\partial}{\partial\lambda}\left(x^2y\lambda - 3\lambda^5\right)\left(\lambda^2 - 3\lambda + 5\right)^{-\frac{1}{2}}, \\
&= (x^2y - 15\lambda^4)(\lambda^2 - 3\lambda + 5)^{-\frac{1}{2}} + \\
&(x^2y\lambda - 3\lambda^5)\left[-\frac{1}{2}(\lambda^2 - 3\lambda + 5)^{-\frac{3}{2}}(2\lambda - 3)\right], \\
&= \frac{(x^2y - 15\lambda^4)}{\sqrt{\lambda^2 - 3\lambda + 5}} - \frac{(x^2y\lambda - 3\lambda^5)(2\lambda - 3)}{2(\sqrt{(\lambda^2 - 3\lambda + 5)^3}}, \\
&= \frac{(2x^2y - 30\lambda^4)(\lambda^2 - 3\lambda + 5) - [(x^2y\lambda - 3\lambda^5)(2\lambda - 3)]}{2(\sqrt{\lambda^2 - 3\lambda + 5})^3}.
\end{aligned}
$$

6.

$$
\begin{aligned}
\frac{\partial}{\partial w}(\sqrt{2\pi xyw - 13x^7y^3v}) &= \frac{1}{2}(2\pi xyw - 13x^7y^3v)^{-\frac{1}{2}}(2\pi xy), \\
&= \frac{2\pi xy}{2\sqrt{2\pi xyw - 13x^7y^3v}}, \\
&= \frac{\pi xy}{\sqrt{2\pi xyw - 13x^7y^3v}}.
\end{aligned}
$$

7.

$$\alpha = \frac{e^{x\beta} - 3}{2y\beta + 5} = (e^{x\beta} - 3)(2y\beta + 5)^{-1},$$
$$\frac{\partial \alpha}{\partial \beta} = \frac{xe^{x\beta-3}}{(2y\beta+5)} - \frac{(e^{x\beta-3})2y}{(2y\beta+5)^2},$$
$$= \frac{xe^{x\beta-3}(2y\beta+5) - (e^{x\beta-3})2y}{(2y\beta+5)^2},$$
$$= \frac{(2xy\beta + 5x - 2y)e^{x\beta+3}}{(2y\beta+5)^2}.$$

8.

$$\frac{\partial}{\partial w}[(x^2yw - xy^3w^7)(w-1)^{-1}]^{-\frac{7}{2}}$$
$$= -\frac{7}{2}[(x^2yw - xy^3w^7)(w-1)^{-1}]^{-\frac{9}{2}}[(x^2y - 7xy^3w^6)(w-1)^{-1} +$$
$$(x^2yw - xy^3w^7)[-(w-1)^{-2}]],$$
$$= -\frac{7}{2}\left[(x^2yw - xy^3w^7)(w-1)^{-1}\right]^{-\frac{9}{2}}\left[\frac{x^2y - 7xy^3w^6}{w-1} - \frac{(x^2yw - xy^3w^7)}{(w-1)^2}\right].$$

9.

- $Z = e^y + x + x^2 + 6$ $\qquad p(1, 0, 9)$

$$Z = Z(a,b,c) + Z_x(a,b,c)(x-a) + Z_y(a,b,c)(y-b) + Z_z(a,b,c)(z-c)$$
$$Z_x(a,b,c) = Z_x(1,0,9) = 3, \;\; Z_y(1,0,9) = 1, Z_z(1,0,9) = 0, \;\; Z(1,0,9) = 9$$
$$\implies Z = 9 + 3(x-1) + 1(y-0) = 6 + 3x + y.$$

- $Z = \frac{1}{2}(x^2 + 4y^2)$ $\qquad p(2, 1, 4)$

$$Z = Z(a,b,c) + Z_x(a,b,c)(x-a) + Z_y(a,b,c)(y-b) + Z_z(a,b,c)(z-c)$$
$$Z_x(a,b,c) = Z_x(2,1,4) = 2, \;\; Z_y(2,1,4) = 4, Z_z(2,1,4) = 0, \;\; Z(2,1,4) = 4$$
$$\implies Z = 4 + 2(x-2) + 4(y-1) = 2x + 4y - 4.$$

10.

$$dh = e^{-3t}\cos(x+5t)dx + [-3e^{-3t}\sin(x+5t) + 5e^{-3t}\cos(x+5t)]dt,$$
$$dh = e^{-3t}\cos(x+5t)dx - [3e^{-3t}\sin(x+5t) - 5e^{-3t}\cos(x+5t)]dt.$$

11.

$$dg = 2x\sin(2t)dx + 2x^2\cos(2t)dt,$$
$$\text{At } \; p(2, \frac{\pi}{4}); dg = 4\sin(\frac{\pi}{2})dx + 8\cos(\frac{\pi}{2})dt,$$
$$dg = 4dx + 0dt = 4dx.$$

12. •

$$
\begin{aligned}
\frac{dz}{du} &= \frac{dz}{dx}\frac{dx}{du} + \frac{dz}{dy}\frac{dy}{du},\\
\frac{dz}{du} &= (e^{-y} - ye^{-x})\sin v + (-xe^{-y} + e^{-x}) - vsinu,\\
&= (e^{-y} - ye^{-x})\sin v + xv\sin ue^{-y} - ve^{-x}\sin u,\\
&= (e^{-vcosu} - v\cos ue^{-usinv})\sin v + [u\sin ve^{-v\cos u} - e^{-u\sin v}]v\sin u.
\end{aligned}
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\begin{aligned}
\frac{dz}{dv} &= \frac{dz}{dx}\frac{dx}{dv} + \frac{dz}{dy}\frac{dy}{du},\\
&= (e^{-y} - ye^{-x})u\cos v + [(-xe^{-y} + e^{-x})(\cos u)],\\
&= (e^{-y} - ye^{-x})u\cos v - xe^{-y}\cos u + e^{-x}\cos u,\\
&= (e^{-v\cos u} - v\cos ue^{-u\sin v})u\cos v - (u\sin ve^{-v\cos u} - e^{-u\sin v})\cos u.
\end{aligned}
$$

13.

$$
\begin{aligned}
\frac{dz}{dt} &= \frac{dz}{dx}\frac{dx}{dt} + \frac{dz}{dy}\frac{dy}{dt},\\
&= \frac{1}{y}\cos\left(\frac{x}{y}\right) \times 2 + \left[-\frac{x}{y^2}\cos\left(\frac{x}{y}\right) \times (-2t)\right],\\
&= \frac{2}{y}\cos\left(\frac{x}{y}\right) + \frac{2xt}{y^2}\cos\left(\frac{x}{y}\right),\\
&= \left[\frac{2}{1-t^2} + \frac{2(2t)t}{(1-t^2)^2}\right]\cos\left(\frac{2t}{1-t^2}\right),\\
&= \left[\frac{2}{1-t^2} + \frac{4t^2}{(1-t^2)^2}\right]\cos\left(\frac{2t}{1-t^2}\right).
\end{aligned}
$$

14.

$$\begin{aligned}
f_x &= 2x\cos(x^2+y^2). \\
f_{xx} &= 2\cos(x^2+y^2) + 2x[-2x(\sin(x^2+y^2))], \\
&= 2\cos(x^2+y^2) - 4x^2\sin(x^2+y^2). \\
f_{xy} &= 2y[-2x\sin(x^2+y^2)], \\
&= -4xy\sin(x^2+y^2). \\
f_y &= 2y\cos(x^2+y^2). \\
f_{yy} &= 2\cos(x^2+y^2) + 2y[-2y\sin(x^2+y^2)], \\
&= 2\cos(x^2+y^2) - 4y^2\sin(x^2+y^2) \\
f_{yx} &= -4xy\sin(x^2+y^2). \\
\therefore f_{xy} &= f_{yx}.
\end{aligned}$$

15.

$$\begin{aligned}
f_x &= \frac{1}{y}\cos\left(\frac{x}{y}\right). \\
f_{xx} &= -\frac{1}{y^2}\sin\left(\frac{x}{y}\right). \\
f_{xy} &= -\frac{1}{y^2}\cos\left(\frac{x}{y}\right) + \frac{1}{y}\left(-\frac{x}{y^2}\times -\sin\left(\frac{x}{y}\right)\right), \\
&= -\frac{1}{y^2}\cos\left(\frac{x}{y}\right) + \frac{x}{y^3}\sin\left(\frac{x}{y}\right). \\
f_y &= -\frac{x}{y^2}\cos\left(\frac{x}{y}\right). \\
f_{yy} &= \frac{2x}{y^3}\cos\left(\frac{x}{y}\right) + \frac{x}{y^2}\left[-\frac{x}{y^2}\sin\left(\frac{x}{y}\right)\right], \\
&= \frac{2x}{y^3}\cos\left(\frac{x}{y}\right) - \frac{x^2}{y^4}\sin\left(\frac{x}{y}\right). \\
f_{yx} &= -\frac{1}{y^2}\cos\left(\frac{x}{y}\right) - \frac{x}{y^2}\left[\frac{1}{y}\times -\sin\left(\frac{x}{y}\right)\right], \\
&= -\frac{1}{y^2}\cos\left(\frac{x}{y}\right) + \frac{x}{y^3}\sin\left(\frac{x}{y}\right). \\
\therefore f_{xy} &= f_{yx}.
\end{aligned}$$