EQUILIBRIUM OF PARTICLES & RIGID BODIES

Static Equilibrium of particles

Static Equilibrium of rigid bodies

EQUILIBRIUM OF PARTICLES & RIGID BODIES

- A particle or body is said to be in equilibrium if the resultant force and moment acting on it is zero.
- This is necessary condition for Newton's first law.
- The equilibrant of a system of forces acting on a particle can easily be obtained applying the already discussed techniques for finding the resultant of forces on a particle.
- Problems on equilibrium often require equations of equilibrium to obtained and solved.

EQUILIBRIUM OF PARTICLES & RIGID BODIES

- Solving problems on equilibrium involve three main steps;
  - Draw a free body diagram for the problem
  - Obtain the equations of equilibrium for the problem
  - Solve the equations and interpret your results

EQUILIBRIUM OF PARTICLES & RIGID BODIES

Static Equilibrium of a Particle

- For particles,

$$\vec{R} = \sum F = 0$$

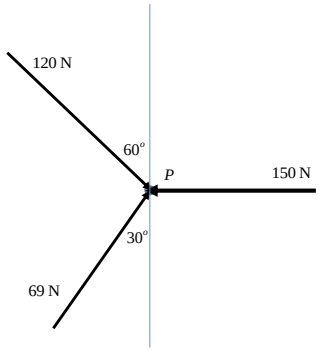
$$\Rightarrow \sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

EQUILIBRIUM OF PARTICLES & RIGID BODIES

Static Equilibrium of a Particle

➢ Example 4.1

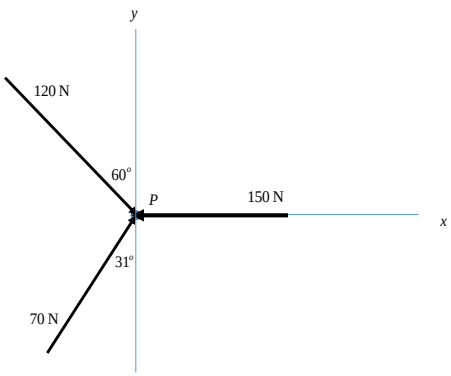
Determine if the particle P is in equilibrium under the influence of the forces shown in the diagram.



EQUILIBRIUM OF PARTICLES & RIGID BODIES

Static Equilibrium of a Particle

➢ Example 4.1 - Solution



Equations of Equilibrium

$$\overset{+}{\rightarrow}\sum F_x = 120\sin 60^o + 70\sin 31^o - 150 = -11.02 \text{ N}$$

$$+\uparrow \sum F_y = -120\cos 60^o + 70\cos 31^o = -0.0017 \text{ N} = 0.00 \text{ N}$$

For equilibrium, $\sum F_x = 0 = \sum F_y$

But $\sum F_x \neq 0$

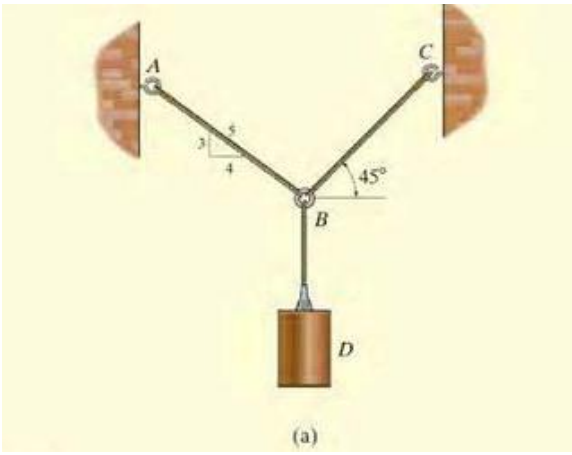
Hence, P is not in equilibrium

EQUILIBRIUM OF PARTICLES & RIGID BODIES

Static Equilibrium of a Particle

➢ Example 4.2

Determine the required tensions in cables BC and BA so they can sustain the 60kg weight.

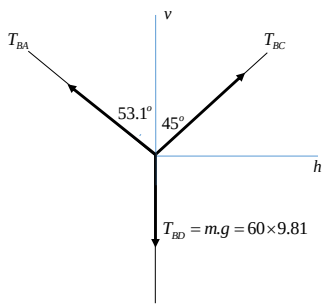


EQUILIBRIUM OF PARTICLES & RIGID BODIES

Static Equilibrium of a Particle

➢ Example 4.2 - Solution

FBD



Equations of Equilibrium

$$\overset{+}{\rightarrow} \sum F_h = 0 : T_{BC} \sin 45^o - T_{BA} \sin 53.1^o = 0 \quad \text{--- (1)}$$

$$+\uparrow \sum F_v = 0 : T_{BC} \cos 45^o + T_{BA} \cos 53.1^o - T_{BD} = 0$$

$$= T_{BC} \cos 45^o + T_{BA} \cos 53.1^o = 588.6 \text{ N} \quad \text{--- (2)}$$

Solving (1) and (2) simultaneously,

$T_{BC} = 475.41$ N

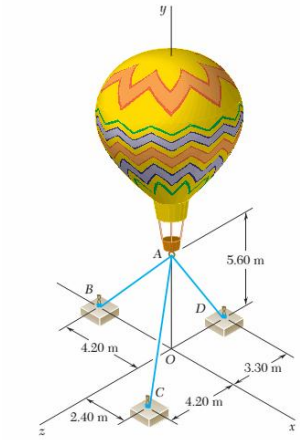
$T_{BA} = 420.43$ N

EQUILIBRIUM OF PARTICLES AND BODIES

Static Equilibrium of a Particle

Example

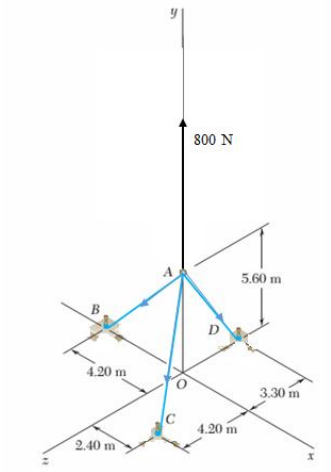
Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800-N vertical force at A, determine the tension in each cable.



EQUILIBRIUM OF PARTICLES AND BODIES

Static Equilibrium of a Particle

Example 4.3 - Solution



Equations of Equilibrium

$$\vec{T}_{AB} = T_{AB}\frac{-4.2i-5.6j}{\sqrt{4.2^2+5.6^2}} = -0.6T_{AB}i - 0.8T_{AB}j$$

$$\vec{T}_{AC} = T_{AC}\frac{2.4i-5.6j+4.2k}{\sqrt{2.4^2+5.6^2+4.2^2}} = 0.37T_{AC}i - 0.87T_{AC}j + 0.65T_{AC}k$$

$$\vec{T}_{AD} = T_{AD}\frac{-5.6j-3.3k}{\sqrt{5.6^2+3.3^2}} = -0.86T_{AD}j - 0.51T_{AD}k$$

$$F_{Ay} = 800\text{ N}j$$

$$\sum F_x = -0.6T_{AB} + 0.37T_{AC} = 0 \quad \text{--- (1)}$$

$$\sum F_y = -0.8T_{AB} - 0.87T_{AC} - 0.86T_{AD} + 800 = 0 \quad \text{--- (2)}$$

$$\sum F_z = 0.65T_{AC} - 0.51T_{AD} = 0 \quad \text{--- (3)}$$

Solving (1), (2) and (3) simultaneously,

$T_{AB} = 200.6$ N

$T_{AC} = 325.3$ N

$T_{AD} = 414.6$ N

EQUILIBRIUM OF PARTICLES AND BODIES

Static Equilibrium of Rigid Bodies

- For rigid bodies, the sum of moments is considered in addition to the sum of forces.

$$\vec{R} = \sum F = 0$$
$$\Rightarrow \sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$
$$\vec{M} = \sum M = 0$$
$$\Rightarrow \sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

- Reaction at supports of rigid bodies must not be ignored in equilibrium analysis.
- A free body diagram is indispensable in solving problems on equilibrium of rigid bodies.
- All support reactions must be accounted on the free body diagram in order for the problem to be solved correctly.