

Lecture 16 - The pn Junction Diode (II)

EQUIVALENT CIRCUIT MODEL

April 8, 2003

Contents:

1. I-V characteristics (*cont.*)
2. Small-signal equivalent circuit model
3. Carrier charge storage: diffusion capacitance

Reading assignment:

Howe and Sodini, Ch. 6, §§6.4, 6.5, 6.9

Announcements:

Quiz 2: 4/16, 7:30-9:30 PM, Walker (lectures #10-17)
open book, must bring calculator

Key questions

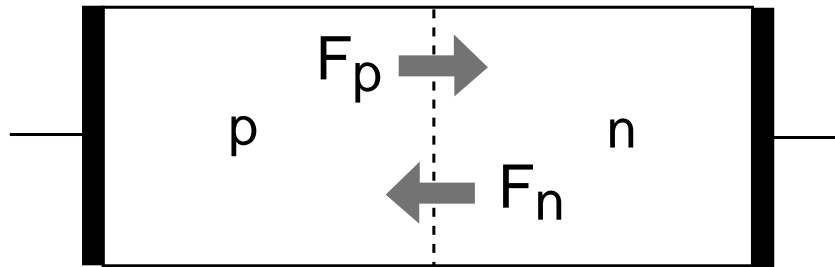
- How does a pn diode look like from a small-signal point of view?
- What are the leading dependences of the small-signal elements?
- In addition to the junction capacitance, are there any other capacitive effects in a pn diode?

1. I-V characteristics (*cont.*)

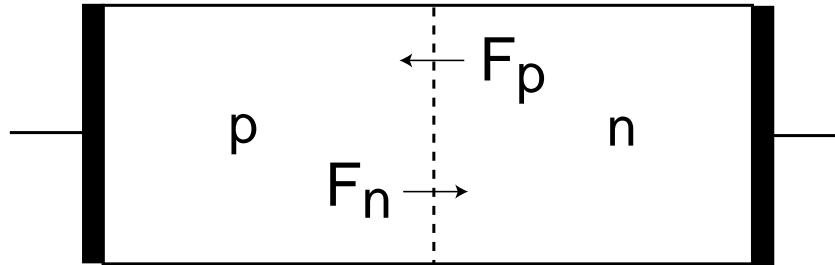
Diode current equation:

$$I = I_o \left(\exp \frac{qV}{kT} - 1 \right)$$

Physics of forward bias:



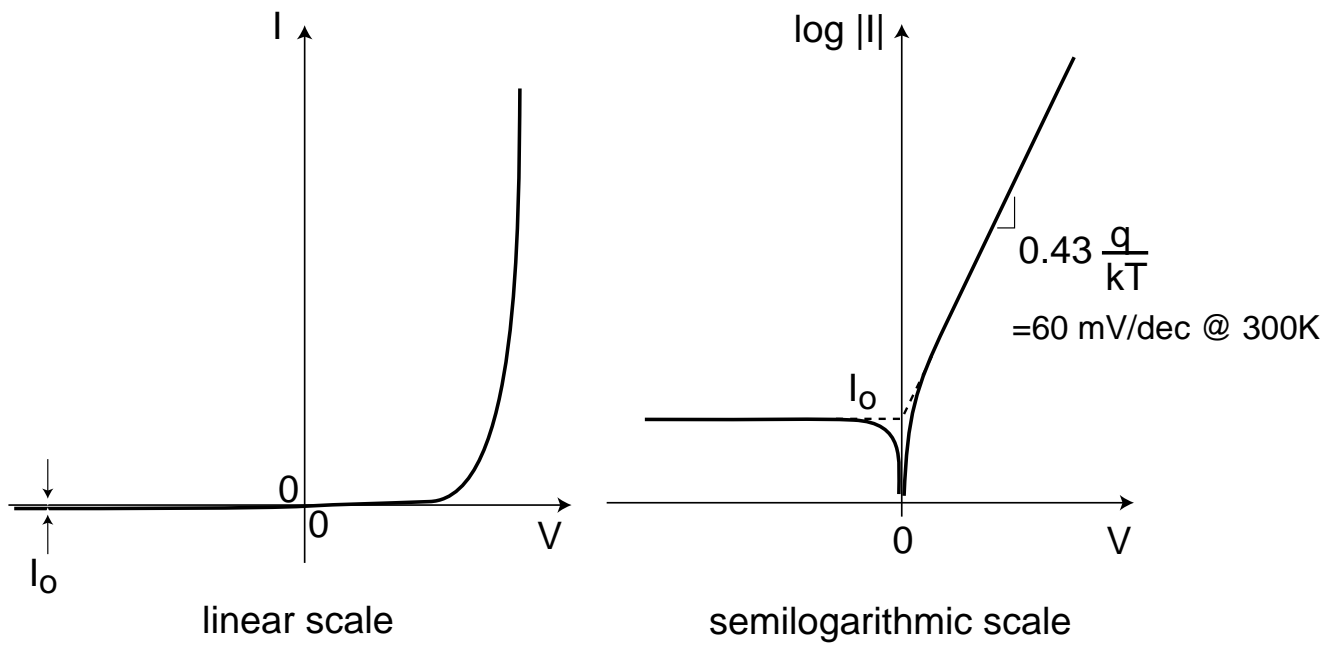
- potential difference across SCR reduced by $V \Rightarrow$ minority carrier injection in QNR's
- minority carrier diffusion through QNR's
- minority carrier recombination at surface of QNR's
- large supply of carriers available for injection
 $\Rightarrow I \propto e^{qV/kT}$



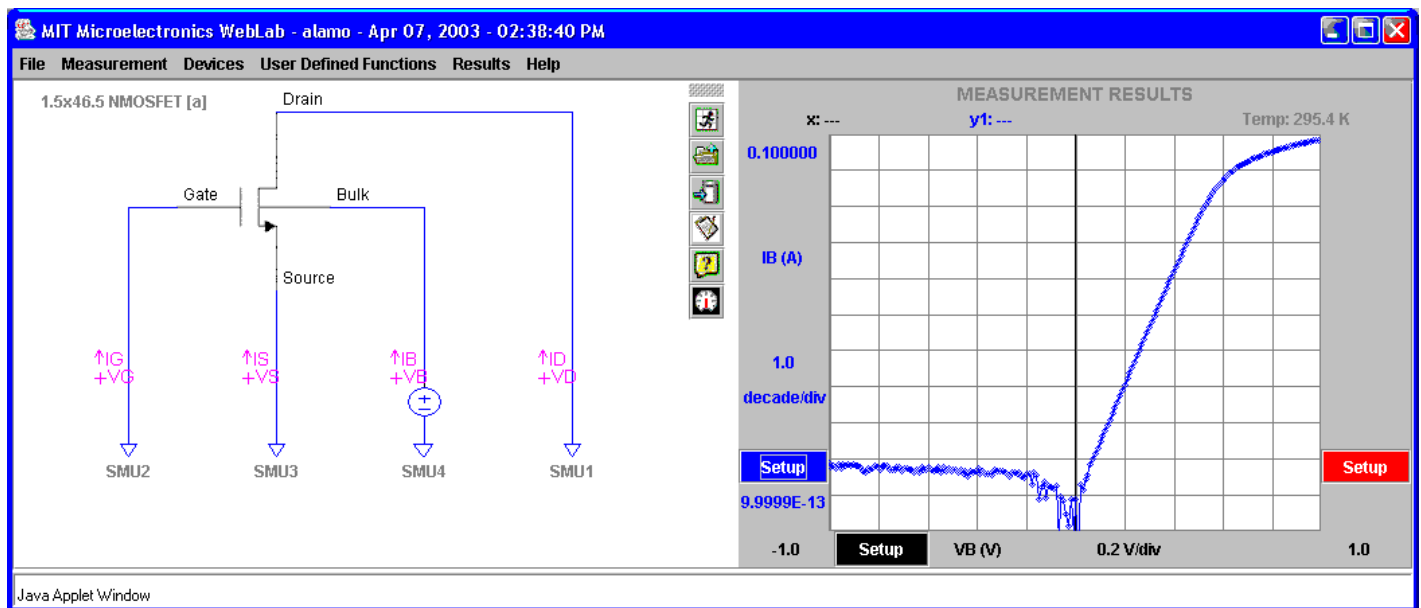
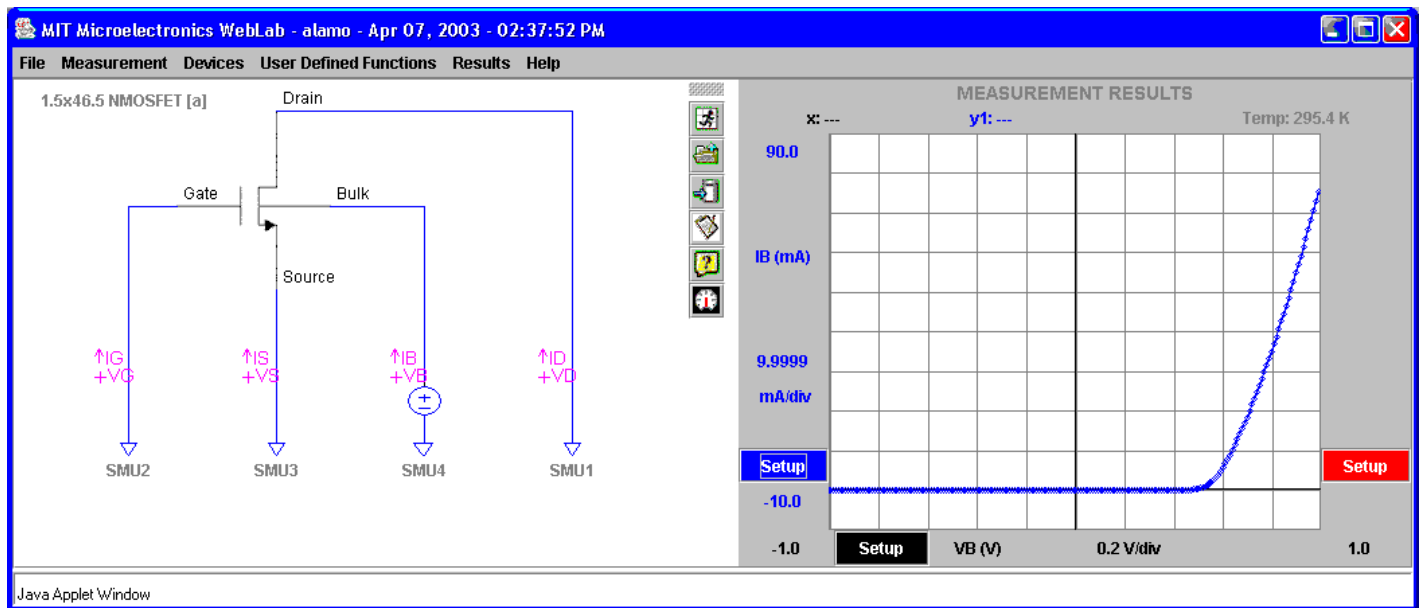
Physics of reverse bias:

- potential difference across SCR increased by V
 \Rightarrow minority carrier extraction from QNR's
- minority carrier diffusion through QNR's
- minority carrier generation at surface of QNR's
- very small supply of carriers available for extraction
 $\Rightarrow I$ saturates to small value

I-V characteristics: $I = I_o(\exp \frac{qV}{kT} - 1)$



Source/drain-body pn diode of NMOSFET:



Key dependences of diode current:

$$I = qAn_i^2 \left(\frac{1}{N_a} \frac{D_n}{W_p - x_p} + \frac{1}{N_d} \frac{D_p}{W_n - x_n} \right) \left(\exp \frac{qV}{kT} - 1 \right)$$

- $I \propto \frac{n_i^2}{N} (\exp \frac{qV}{kT} - 1) \equiv \text{excess minority carrier concentration at edges of SCR}$
 - in forward bias: $I \propto \frac{n_i^2}{N} \exp \frac{qV}{kT}$: the more carrier are injected, the more current flows
 - in reverse bias: $I \propto -\frac{n_i^2}{N}$: the minority carrier concentration drops to negligible values and the current saturates
- $I \propto D$: faster diffusion \Rightarrow more current
- $I \propto \frac{1}{W_{QNR}}$: shorter region to diffuse through \Rightarrow more current
- $I \propto A$: bigger diode \Rightarrow more current

2. Small-signal equivalent circuit model

Examine effect of small signal overlapping bias:

$$I + i = I_o \left[\exp \frac{q(V + v)}{kT} - 1 \right]$$

If v small enough, linearize exponential characteristics:

$$\begin{aligned} I + i &= I_o \left(\exp \frac{qV}{kT} \exp \frac{qv}{kT} - 1 \right) \simeq I_o \left[\exp \frac{qV}{kT} \left(1 + \frac{qv}{kT} \right) - 1 \right] \\ &= I_o \left(\exp \frac{qV}{kT} - 1 \right) + I_o \left(\exp \frac{qV}{kT} \right) \frac{qv}{kT} \end{aligned}$$

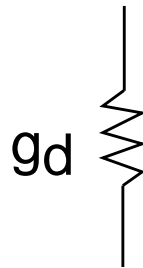
Then:

$$i = \frac{q(I + I_o)}{kT} v$$

From small signal point of view, diode behaves as conductance of value:

$$g_d = \frac{q(I + I_o)}{kT}$$

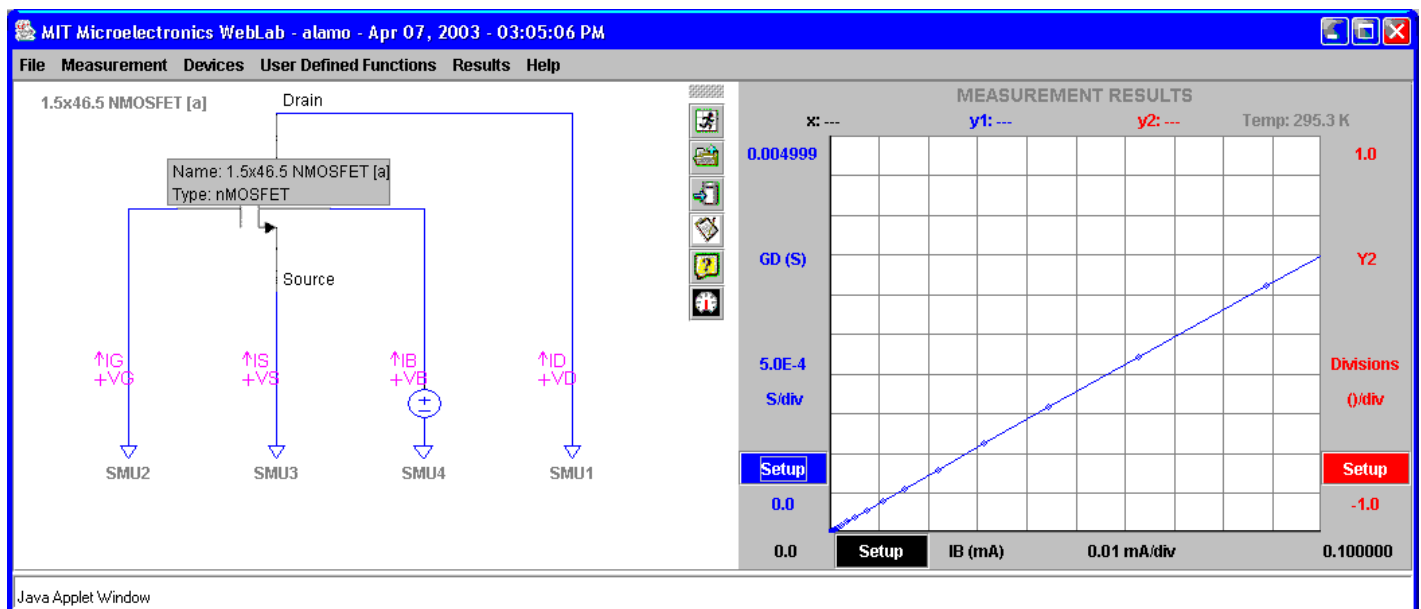
Small-signal equivalent circuit model, so far:



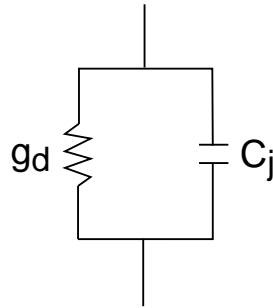
g_d depends on bias. In forward bias:

$$g_d \simeq \frac{qI}{kT}$$

g_d is linear in diode current.



Must add capacitance associated with depletion region:



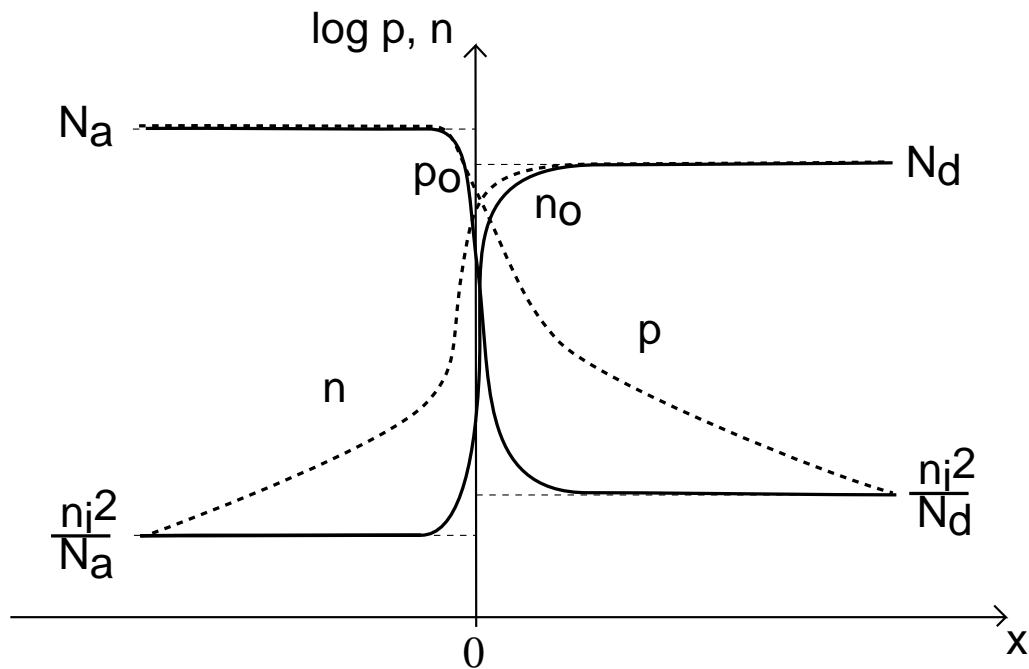
Depletion or junction capacitance:

$$C_j = \frac{C_{j0}}{\sqrt{1 - \frac{V}{\phi_B}}}$$

3. Carrier charge storage: diffusion capacitance

What happens to the majority carriers?

Carrier picture so far:

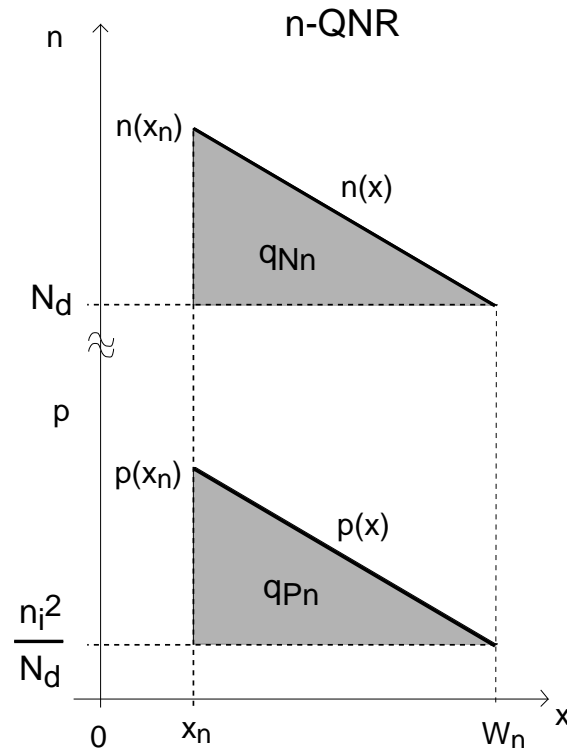


If in QNR minority carrier concentration \uparrow but majority carrier concentration unchanged
 \Rightarrow quasi-neutrality is violated.

Quasi-neutrality demands that at every point in QNR:

excess minority carrier concentration

= excess majority carrier concentration



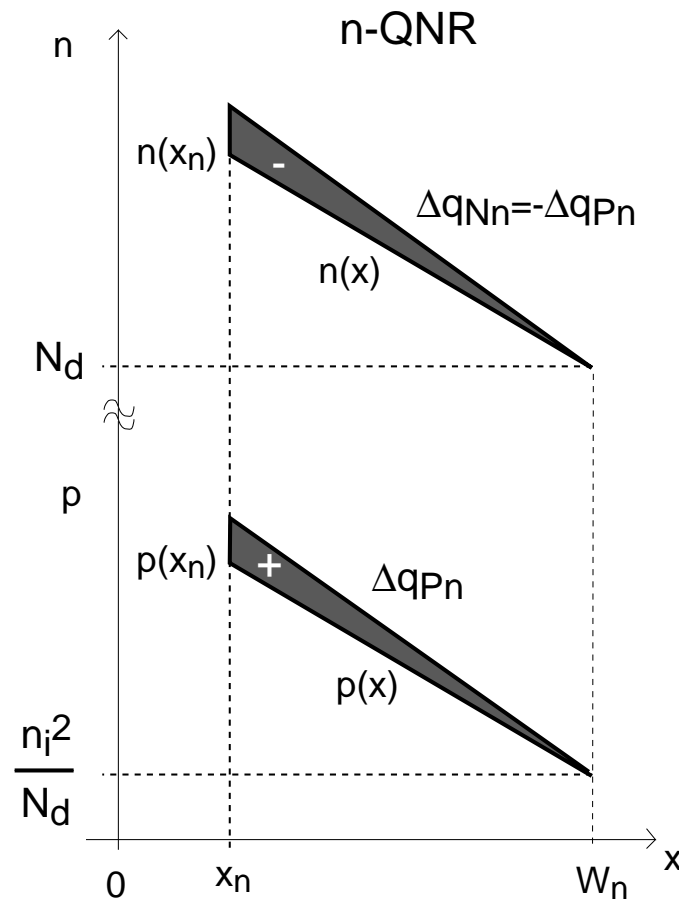
Mathematically:

$$p'(x) = p(x) - p_o \simeq n'(x) = n(x) - n_o$$

Define integrated carrier charge:

$$\begin{aligned} q_{Pn} &= qA \frac{1}{2} p'(x_n) (w_n - x_n) = \\ &= qA \frac{w_n - x_n}{2} \frac{n_i^2}{N_d} \left(\exp \frac{qV}{kT} - 1 \right) = -q_{Nn} \end{aligned}$$

Now examine small increase in V :



Small increase in $V \Rightarrow$ small increase in $q_{Pn} \Rightarrow$ small increase in $|q_{Nn}|$

Behaves as capacitor of capacitance:

$$C_{dn} = \frac{dq_{Pn}}{dV}$$

Can write q_{Pn} in terms of I_p (portion of diode current due to holes in n-QNR):

$$\begin{aligned} q_{Pn} &= \frac{(W_n - x_n)^2}{2D_p} qA \frac{n_i^2}{N_d W_n - x_n} \frac{D_p}{kT} (\exp \frac{qV}{kT} - 1) \\ &= \frac{(W_n - x_n)^2}{2D_p} I_p \end{aligned}$$

Define *transit time* of holes through n-QNR:

$$\tau_{Tp} = \frac{(W_n - x_n)^2}{2D_p}$$

Transit time is *average time for a hole to diffuse through n-QNR* [will discuss in more detail in BJT]

Then:

$$q_{Pn} = \tau_{Tp} I_p$$

and

$$C_{dn} \simeq \frac{q}{kT} \tau_{Tp} I_p$$

Similarly for p-QNR:

$$C_{dp} \simeq \frac{q}{kT} \tau_{Tn} I_n$$

where τ_{Tn} is *transit time* of electrons through p-QNR:

$$\tau_{Tn} = \frac{(W_p - x_p)^2}{2D_n}$$

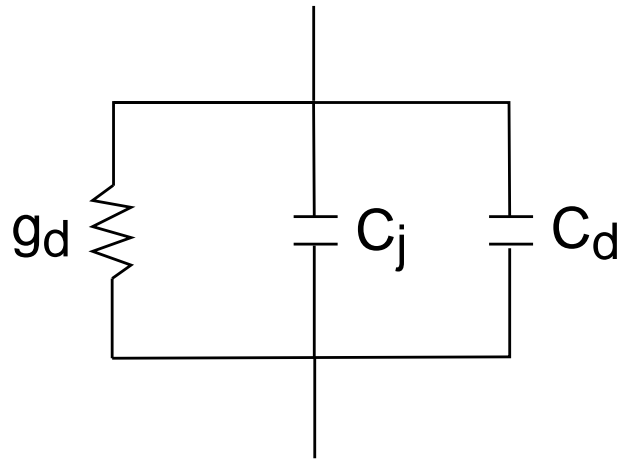
Both capacitors sit in *parallel* \Rightarrow total diffusion capacitance:

$$C_d = C_{dn} + C_{dp} = \frac{q}{kT} (\tau_{Tn} I_n + \tau_{Tp} I_p) = \frac{q}{kT} \tau_T I$$

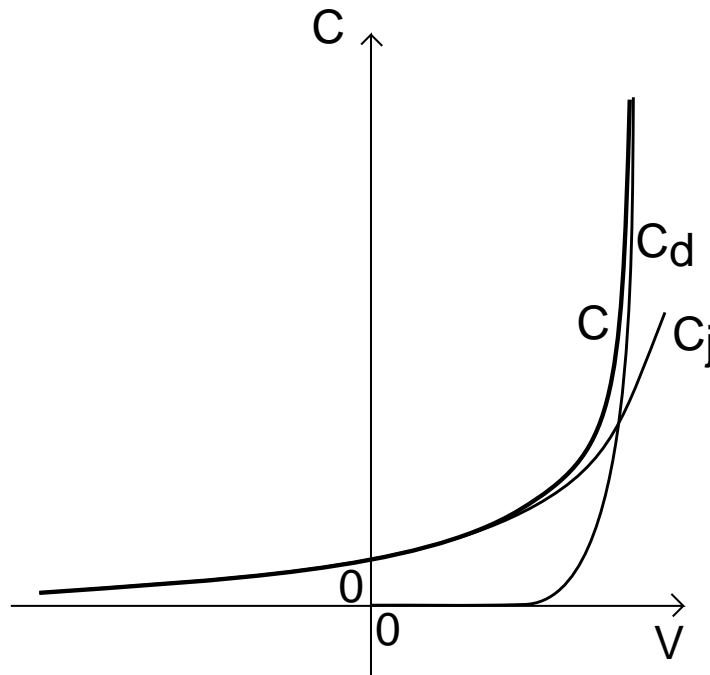
with:

$$\tau_T = \frac{\tau_{Tn} I_n + \tau_{Tp} I_p}{I}$$

Complete small-signal equivalent circuit model for diode:



Bias dependence of C_j and C_d :



- C_j dominates in reverse bias and small forward bias ($\sim 1/\sqrt{\phi_B - V}$)
- C_d dominates in strong forward bias ($\sim e^{qV/kT}$)

Key conclusions

Small-signal behavior of diode:

- *conductance*: associated with current-voltage characteristics

$g_d \sim I$ in forward bias, negligible in reverse bias

- *junction capacitance*: associated with charge modulation in depletion region

$$C_j \sim 1/\sqrt{\phi_B - V}$$

- *diffusion capacitance*: associated with charge storage in QNR's to keep quasi-neutrality

$$C_d \sim e^{qV/kT}$$