ME 161/162 BASIC MECHANICS

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Unit Contents

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Recommended Text Books

Main Textbook

Basic Engineering Mechanics by J. Antonio

Other Textbooks

1. Basic Engineering Mechanics by J. Antonio (Main Handout, (**Statics & Dynamics**)
2. Engineering Mechanics Statics by A. Pytel & J. Kiusalaas (**Statics only**)
3. Vector Mechanics for Engineers: Statics and Dynamics by F. P. Beer & E. R. Johnston (**Statics & Dynamics)**.
4. Statics and Mechanics of Materials: An Integrated Approach by W. F. Riley, et al. (**Statics only**).
5. Vector Mechanics for Engineers: Dynamics by F. P. Beer, et al. (**Dynamics only)**.
6. Principles of Dynamics by R. C. Hibbeler (**Dynamics only**)

Course Contents

FUNDAMENTALS

- Fundamental Concepts
  - Definition of Mechanics
  - Idealization of Mechanics
  - Systems of Units
  - Newton's Laws of Motion
  - Newton's Laws of Universal Gravitation
- Forces and Moments
  - Force Systems and Characteristics of Forces
  - Resultant of Concurrent Coplanar Forces
  - Resolution of Forces- 2D & 3D
  - Equilibrium of a Particle
  - Equilibrium of Rigid Bodies in 2D and 3D

STATICS

- Structural Analysis
  - Simple Trusses
    - Method of Joints
    - Method of Sections
  - Frames
  - Machines
- Dry Friction
  - Definitions and laws of dry friction
  - Problems involving dry friction
- Simple Machines
  - Mechanical Advantage, Velocity Ratio & Efficiency
  - Types of Simple Machines

Course Contents

STATICS

- Method of Virtual Work
  - The Principle of Virtual Work
  - Application of Virtual Work to Multi-degree-of-freedom systems
  - Application of Virtual Work to Completely Constrained systems
  - The Principle of Minimum Potential Energy

DYNAMICS

- Basic Dynamics of Particles
  - Rectilinear, curvilinear & rotational motions
  - Types of Motion: Continuous and Erratic Motions, Projectiles, Dependent Motions and Relative Motions
  - Equation of Motion
  - Work, Energy & Power
  - Momentum & Impulse of particles

- Basic Dynamics of Rigid Bodies
  - Translation of a rigid body
  - Rotation of a rigid body about a fixed axis
  - Moment of inertia
  - General plane motion of a rigid body
  - Potential & Kinetic energies of a rigid body
  - Momentum of a rigid body
  - Momentum & Impulse of Rigid bodies

- Simple Harmonic Motion (SHM)
  - Equation for SHM
  - Examples of SHM
  - Equivalent stiffness of combinations of springs
  - Energy method for conservative systems
  - Allowance for mass of spring

Course Objectives

Upon successful completion of this course, students should be able to:

1. Understand and apply Newton's laws of motion and other basic theories and laws of Newtonian mechanics to particles and rigid bodies.
2. Understand and use appropriate units of measurement, and SI unit prefixes.
3. Understand the basis of force and moments, and draw free body diagrams.
4. Analyze 2-D and 3-D equilibrium of system of forces for tensions in ropes/cables, forces in links, and support and contact reactions.
5. Determine centroids and centre of gravity of single and composite bodies.
6. Find support reactions and internal forces of two-dimensional determinant structures.
7. Solve static and dynamic problems involving dry friction.
8. Evaluate mechanical advantage, velocity ratio and efficiency of simple Machines.
9. Understand and Solve two-dimensional problems involving equation of motion, momentum, impulse and energy.
10. Solve simple one degree-of-freedom conservative vibration problems.
11. Solve simple applied mechanics problems involving combinations of items 1 to 10.

Assessment

- Continuous Assessment (30% of Total)
  1. Mid-Semester Exams (1/3 of Conti. Assess.)
  2. Homework/Assignment (1/3 of Conti. Assess.)
  3. Quiz (1/6 of Conti. Assess.)
  4. Attendance (1/6 of Conti. Assess.)
  5. Contribution to Class Discussions (Bonus, max 10%, only included if sum of items 1 to 4 is less than 30%)
- End of Semester Exams (70% of Total)
  - Multiple Choice/Fill-in Spaces (1/2 of End of Semester Exams)
  - Statics (1/4 of End of Semester Exams)
  - Dynamics (1/4 of End of Semester Exams)

Note

- You will NOT be allowed to write the End of Semester Exams if you miss at least four lectures without permission.
- All homework /assignments are due exactly a week after the assigned day. No excuse will be tolerated.

Unit 1

FUNDAMENTAL CONCEPTS

Mechanics

- *Mechanics* is a branch of physics that deals with the state of rest or motion of bodies under the action of forces.
- Categories of Mechanics:
  - Mechanics of Rigid bodies
    - *Statics-concerns with the equilibrium of bodies under the action of balanced forces*
    - ***Dynamics***-deals with motions of bodies under action of unbalanced forces. It is divided into
      - ***Kinematics-*** it deals with motion of bodies without referring to the forces that cause the motion.
      - ***Kinetics-*** it relates motion of bodies to the forces which cause the motion
  - Mechanics of Materials or Strength of Materials
    - Theory of elasticity
    - Theory of plasticity
  - Fluid Mechanics
    - Mechanics of Compressible fluids
    - Mechanics of incompressible fluids

Scope of Basic Mechanics

- ME 161/162 Basic mechanics is limited to Newtonian mechanics of particles and rigid-bodies. Other branches of mechanics which are not newtonian mechanics include:

- Relativistic mechanics, which addresses phenomena that occur at a cosmic scale (velocities approaching the speed of light, and strong gravitational fields, etc)

- Quantum mechanics, which is concerned with particles at on the atomic or subatomic scale.

Particle and Rigid Body

- A particle is a body (or an object) which is assumed to be so small that it may be regarded as geometric point. Thus, a particle has mass but its size is (assumed to be) negligible.

- When a body is idealised as a particle, the principles of mechanics reduce to a simplified form, since the geometry of the body will not be concerned in the analysis of the problem.

- All the forces acting on a particle are assumed to be applied at the same point, that is the forces are assumed to be concurrent.

- A rigid body is a collection of particles connected together in such a way that the distance between each pair of particles remains constant under all circumstances. That is, the size and the shape of rigid bodies (are assumed to) remain constant at all times.

- This is an ideal situation since all real bodies change in shape and/or size to some extent under a system of forces.

Scalar and Vector Quantities

- A scalar quantity can be described completely by magnitude. e.g. length, area, volume, mass, time, etc.

- A vector quantity is described by its magnitude, direction, line-of-action and sometimes point of application. E.g. force, moment, velocity, acceleration, momentum, etc.

- Note that some vector quantities are completely described by only magnitude and direction. E.g velocity, acceleration, momentum. But, it is inadequate to describe a force by only magnitude and direction. Similarly, moment has no line-of-action and is completely described by magnitude, direction and point of application.

Newton's Laws of Motion

The entire subject of rigid-body mechanics is formulated on the basis of the Newton's three laws of motion, which are:

1st Law: A particle at rest or moving in a straight line with constant velocity will remain in this state except compelled by an external force to act otherwise.

2nd Law: The rate of change of momentum is proportional to the applied external force and it occurs in the direction of the force.

3rd Law: For every force acting on a particle, the particle exerts an equal, opposite and collinear reactive force.

Significances and Deductions from Newton's Laws

❑ From the 1st Law

- For a particle to change its state of rest or motion (i.e to accelerate), it must be subjected to an external force. Only force can change the state of rest or motion of a body.

- However, if the particle is at rest or is moving in a straight line with constant velocity, the external forces acting on it, if any, must be balanced. That is the sum of the forces equals zero.

❑ From the 2nd Law

$$F = k\frac{d}{dt}(mv) = km\frac{d}{dt}(v) + kv\frac{d}{dt}(m)$$

If mass m = constant (for rigid bodies),

$$\frac{d}{dt}(m) = 0 \qquad \Rightarrow F = km\frac{d}{dt}(v)$$

The unit of force in SI unit, Newton, is defined such that k = unity = 1, and dv/dt = a , acceleration

$$F = km\frac{d}{dt}(v) = kma$$

$$F = ma$$

❑ From 3rd Law

- Force is due to interaction between two or more different bodies.

Relation between Newton's 1^{st} and 2^{nd} Laws

- From the 2^{nd} Law (Rigid body), $F = ma$
- If force $F = 0$ then

  either $m = 0$ or $a = 0$

  Since $m \neq 0$ for any matter, $a = 0$
- If $a=0$, then

  either $v = 0$: the body is at rest

  or $dv/dt = 0$: the body is moving in a straight line with a constant velocity.

  Note that dv = change in velocity, which is equal to sum of change in magnitude and direction. If $dv/dt = 0$, then both the magnitude and direction of the velocity are not changing.

  Hence: (a) constant velocity means no change in magnitude

  (b) straight line means no change in direction
- Hence, the 1^{st} Law is special case of the 2^{nd} Law.

Basic Definitions

- **Space** is a geometric region in which the physical events of interest in mechanics occur and it is given in terms of three coordinates measured from a reference point or origin.
- **Length** is used to specify the position of a point in space or size of a body
- **Time** is in an interval between two events or duration of an event
- **Matter** is anything that occupies space.
- **Inertia** is a property that cause a body to resist motion.
- **Mass** is a measure of inertia.
- **Force** is an action of a body upon another body.
- In Newtonian Mechanics, space (or length), time, and mass are absolute concepts, independent of each other. Force, however, is dependent of the other three. The force acting on a body is related to the mass of the body and the variation of its velocity with time.

Example 1-1

A body of mass 50 kg is acted upon by external force whose magnitude is 100 N. What is the acceleration of the body?

$$Solution$$

$$Mass = m = 50kg;\ Force = F = 100N$$

$$Acceleration = a = ?$$

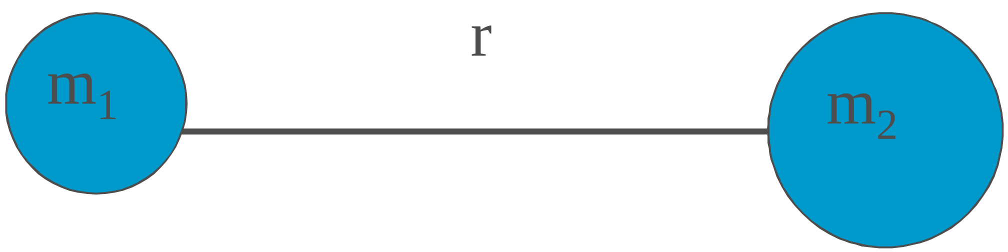
$$From: \quad F = ma$$

$$\Rightarrow\ 100N = 50kg \times a$$

$$\therefore a = \frac{100N}{50kg} = 2 \,{}^{m}\!/\!{}_{s^2}$$

Law of Universal Gravitation (by Kepler)

For any two bodies separated by a distance r, the force of interaction between them is proportional to the product of their masses and inversely proportion to square of the separation distance r.



Mathematically :

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.673(10^{-11})\, {m^3}/{kg.s}\, [SI\ unit]$$

Mass and Weight

- Mass (m) of a body is the quantity of matter in the body and it is independent of geographical location and surroundings in which the body is placed.

- Weight (W) is the product of mass and acceleration due to gravity. Thus, weight depends on the geographical location or/and position of the body relative to some other bodies.

$$Thus\ on\ earth's\ surface$$

$$W = G\frac{m_e m}{{r_e}^2} = mg;$$

$$m_e = mass\ of\ earth$$

$$r_e = radius\ of\ earth$$

$$Therefore,\ \ g = G\frac{m_e}{{r_e}^2}$$

Example 1-2

Calculate the weight W of a body of mass 675 kg at a location on Earth where $g = 9.81\ m/s^2$.

Solution

Mass $m = 675$ kg; $g = 9.81\ m/s^2$

$$\text{Weight } W = mg$$
$$= 675\ kg \times 9.81 m/s^2$$
$$= 6.62 \times 10^3\ N$$

Units of Measurement

1. The building blocks of mechanics are the physical quantities used to express the laws of mechanics.
2. Some of these quantities are mass, length, force and time.
3. Physical quantities are often divided into fundamental quantities and derived quantities.
4. Fundamental quantities cannot be defined in terms of other physical quantities. E.g time, length, mass
5. Derived quantities are those whose defining operations are based on measurement of other physical quantities. E.g. area, volume, velocity, acceleration.
6. In 1960, the Eleventh General Conference on Weights and Measures adopted a system of units of measurement based on metre, kilogram and second (abreviated as MKS) as the international standard . This international standard is known as Systèm International d'Unitès (International System of Units), for which the abbreviation is SI in all languages.
7. The SI units adopted by the conference includes three classes of units: (1) base units, (2) supplementary units, and (3) derived units. The supplementary units may be regarded as either base or derived units.

Units of Measurement

8. Derived units are expressed algebraically in terms of base units and/or supplementary units.
9. Symbols of derived units are obtained by means of the mathematical operations of multiplication and division. Eg. SI unit of velocity is metre per second (m/s).
10. Some derived SI units have special names and symbols eg. SI unit of force (which is derived from mass, time and length) is newton.
11. In SI units, all symbols bearing people's name begins with a capital letter. Eg N for newton.
12. Prefixes are used to form names and symbols for decimal multiples and submultiples of SI units. The multiple should be chosen so that numerical values of the quantity will be between 0.1 and 1000.
13. Only one prefix should be used in forming a multiple of a unit.
14. A prefix cannot be denominator.

Base SI Units and their Symbols

Base Units and Their Symbols

Quantity	Name of SI Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Amount of Substance	mole	mol
Luminous intensity	candela	cd

Supplementary Units and Their Symbols

Quantity	Name of SI Unit	Symbol
Plane Angle	radian	rad
Solid Angle	steradian	sr

Derived SI Units

Quantity	Derived SI Unit	Symbol	Special Name
Area	square meter	m^2	
Volume	cubic meter	m^3	
Linear Velocity	meter per second	m/s	
Linear Acceleration	meter per second squared	m/s^2	
Frequency	(cycle) per second	Hz	Hertz
Density	kilogram per cubic meter	Kg/m^3	
Force	Kilogram meter per second squared	N	Newton
Pressure and Stress	Newton per meter squared	Pa	Pascal
Work and Energy	Newton.meter	J	Joule
Power	Joule per second	W	Watt
Moment of Force	Newton-meter	N.m	

Some Approved Prefixes of SI Units

Factor by which unit is multiplied	Name of prefix	Symbol of prefix	Example
10^{12}	tera	T	1.23 TJ = 1 230 000 000 000 J
10^{9}	giga	G	4.53 GPa = 4 530 000 000 Pa
10^{6}	mega	M	7.68 MW = 7 680 000 W
10^{3}	kilo	k	5.46 kg = 5 460 g
10^{-2}	centi	c	3.34 cm = 0.0334 m
10^{-3}	milli	m	395 mm = 0.395 m
10^{-6}	micro	µ	65 µm = 0.000 065 m
10^{-9}	nano	n	34 nm = 0.000 000 034 m

Note: Always use an appropriate prefix to state numerical value of a quantity between 0.1 and 1000.

Example 1-3

Two balls of masses 5 kg and 10 kg are separated by a distance of 1 m. Calculate the force of attraction between them.

Solution

$$F = G\frac{m_1 m_2}{d^2} = 6.673 \text{ x } 10^{-11} \frac{5 \text{ x } 10}{1^2}$$

$$F = 3.3 \text{ x } 10^{-9} \text{ N}$$

F= 3.3 nN

Dimensional Considerations and Homogeneity

- All physical quantities encountered in mechanics can be expressed dimensionally in terms of the three fundamental quantities: mass, length and time, denoted respectively by M, L and T.
- Dimensions of derived quantities are derived from definitions or physical laws. E.g Dimension of velocity is L/T, which follows from the the definition of velocity: rate of change of position with time.
- An equation is said to be dimensionally homogeneous if the form of the equation does not depend on the units of measurement.

Dimensions of Physical Quantities of Mechanics

Physical Quantity	**Dimension**	**Common SI Units**
Length	L	m, cm, mm
Area	L^2	m^2, cm^2, mm^2
Angle	1(L/L)	rad, degree
Time	T	s
Linear velocity	L/T or LT^{-1}	m/s or ms^{-1}
Linear acceleration	L/T^2 or LT^{-2}	m/s^2 or ms^{-2}
Angular velocity	1/T or T^{-1}	rad/s
Angular acceleration	$1/T^2$ or T^{-2}	rad/s^2
Mass	M	kg
Force	ML/T^2 or MLT^{-2}	N
Moment of a force	ML^2/T^2 or ML^2T^{-2}	N.m or N-m
Pressure, Stress	M/LT^2 or $ML^{-1}T^{-2}$	Pa, kPa, MPa
Work and Energy	ML^2/T^2 or ML^2T^{-2}	J, kJ
Power	ML^2/T^3 or ML^2T^{-3}	W, kW
Momentum and linear impulse	ML/Tor MLT^{-1}	N.s or N-s

Example 1-4

Determine the dimensions of I, R, w, M and C in the dimensionally homogeneous equation

$$EIy = Rx^3 - P(x-a)^3 - wx^4 + Mx^2 + C$$

in which x and y are lengths, P is a force, and E is a force per unit area.

Solution

The equation can be written dimensionally as

$$\left(\frac{F}{L^2}\right)(I)(L) = R(L)^3 - (F)(L-a)^3 - (w)(L)^4 + (M)(L)^2 + C$$

For the above equation to be homogeneous, a must be a length; hence

$$\left(\frac{F}{L^2}\right)(I)(L) = R(L)^3 = (F)(L)^3 = (w)(L)^4 = (M)(L)^2 = C$$

The dimensions of each of the unknown quantities are obtained as follows:

$$I = \left(\frac{L}{F}\right)(FL^3) \qquad I = L^4$$

$$R = \left(\frac{1}{L^3}\right)(FL^3) \qquad R = F$$

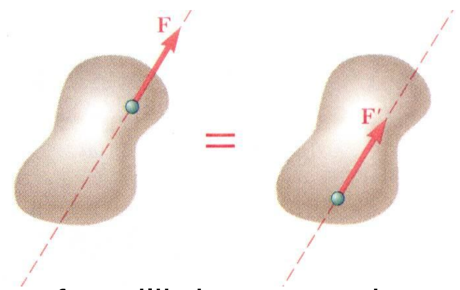
$$w = \left(\frac{1}{L^4}\right)(FL^3) \qquad w = \frac{F}{L}$$

$$M = \left(\frac{1}{L^2}\right)(FL^3) \qquad M = FL$$

$$C = FL^3$$

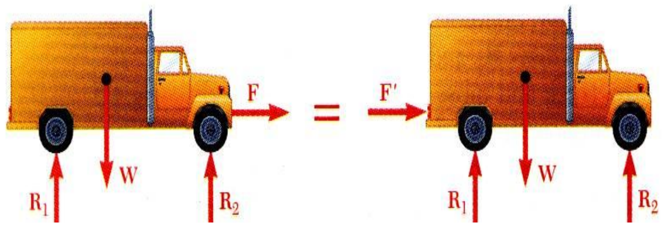
Principle of Transmissibility

The *principle of transmissibility* states that the point of application a force on a rigid body may be placed anywhere along its line of action without changing the conditions of equilibrium or motion of the body



Conditions of equilibrium or motion are not affected by *transmitting* a force along its line of action. NOTE: **F** and **F**' are equivalent forces.

Moving the point of application of the force **F** to the rear bumper does not affect the motion or the other forces acting on the truck.



Principle of transmissibility may not always apply in determining internal forces and deformations.