Convective Heat Transfer

Assignment 2: Solution

Q. 1 Engine oil at 60 ºC flows over the upper surface of a 5 m long flat plate whose temperature is 20℃ with velocity of 3m/s. Determine the rate of heat transfer per unit width of the entire plate. Properties at mean temperature are: ρ = 876 kg/m^3, Pr = 2962, k = 0.1444 W/m-K and v = 2.485X10^{-4} m^2/s.

A. 1127.2 W
B. 281.8 W
C. 6902.71 W
D. 7839.4 W

Sol. Reynolds number, $Re_L = \frac{VL}{v} = 6.0362 \times 10^4$

For high Prandtl number, $Nu = \frac{hL}{k} = 0.3387\, Re_L{}^{1/2} Pr^{1/3} = 0.3387 \times (6.0362 \times 10^4)^{1/2} \times 2962^{1/3} = 1195.06$

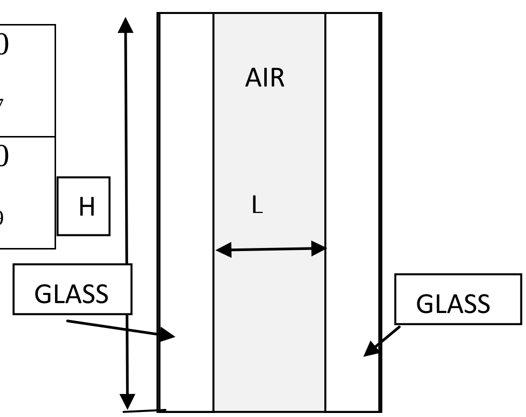
$h = \frac{k}{L} Nu = 34.51\ W/(m^2K)$,

The rate of heat transfer per unit width,

$$\dot{Q} = hA_s(T_\infty - T_s) = 34.51 \times 5 \times 1 \times (60 - 20) = 6902.71\ W$$

Q. 2 The vertical 0.6 m high, 1.5 m wide double-pane window shown in figure, consists of two sheets of glass separated by a 2.5 cm air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 15 ºC and 3 ºC, determine the rate of heat transfer through the window. Properties of air at average temperature are: Pr = 0.716, k = 0.02486 W/m-K and v = 14.28X10^{-6} m^2/s.

$Nu = 0.42 Ra_L^{\frac{1}{4}} Pr^{0.012} \left(\frac{H}{L}\right)^{-0.3}$	10 < H/L < 40 1<Pr< 2x10^4 10^4<Ra$_L$< 10^7
$Nu = 0.046 Ra_L^{1/3}$	10 < H/L < 40 1<Pr< 20 10^6<Ra$_L$< 10^9



A. 21.26 W
B. 50.368 W
C. 13 W
D. 140 W

Sol. Volume Expansion Coefficient,

$$\beta = \frac{1}{T_{avg}} = \frac{1}{282}$$

Rayleigh Number, $Ra_L = \frac{g\beta\,(T_1-T_2)L_c^3}{\nu^2} \times Pr = \frac{9.81\times\left(\frac{1}{282}\right)\times(15-3)\times 0.025^3}{(14.28\times10^{-6})^2} \times 0.716$

$Ra_L = 22902.23$

Nusselt Number, $Nu = 0.42 Ra_L^{\frac{1}{4}} Pr^{0.012}\left(\frac{H}{L}\right)^{-0.3}$

$$Nu = 0.42 \times (22902.23)^{1/4} \times (0.716)^{0.012} \times \left(\frac{0.6}{0.025}\right)^{-0.3} = 1.98$$

$$A_s = H \times W = 0.6 \times 1.5 = 0.9\, m^2$$

$$Q = hA_s(T_1 - T_2) = kNuA_s\frac{T_1 - T_2}{L}$$

$$Q = 0.02486 \times 1.98 \times 0.9 \times \frac{15-3}{0.025} = 21.26\, W$$

Q. 3 A $0.2m \times 0.2m$ vertical plate has a surface temperature that is maintained at 40 ºC. This plate is surrounded by atmospheric air at 20 ºC. Determine the heat flux from the plate surface. Properties at air at given conditions are: Pr = 0.7282, k = 0.02588 W/m-K and ν = $1.608\text{X}10^{-5}$ m^2/s.

A. 17.728 W/m^2
B. 88.64 W/m^2
C. 80.49 W/m^2
D. 167.76 W/m^2

Sol. We know that,

$$\beta = \frac{1}{T_{avg}} = 0.0034$$

Grashof number,

$$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{9.81 \times 0.0034 \times (40-20) \times (0.2)^3}{(1.608\times10^{-5})^2} = 2.003 \times 10^7$$

Nusselt numbers for the natural convection,

$$Nu = 0.6Ra^{1/4}Pr^{1/4} = 0.6 \times (2.003\times10^7 \times 0.7282)^{\frac{1}{4}}(0.7282)^{1/4}$$
$$= 34.2528$$

Convective heat transfer coefficient, $h = \frac{Nu\times k}{L} = 34.2528 \times \frac{0.02588}{0.2} = 4.432\, W/m^2K$

Heat flux , $q'' = h(T_s - T_\infty) = 4.432 \times (40-20) = 88.64\, W/m^2$

Q. 4 If the air at 20 ºC is flowing in parallel over the plate given in Q. 3 with a velocity of 0.4 m/s in upwards direction. What will be the percentage increase in heat flux from the plate surface. Consider $Nu = 0.785Re^{0.5}Pr^{1/3}$ for combined flow assisted convection.

A. 45.42 %
B. 31.23 %
C. 76.83 %
D. 60.15 %

Sol. Reynolds number, $Re_L = \frac{VL}{\nu} = \frac{0.4\times0.2}{1.608\times10^{-5}} = 4975$

→ $Nu = 0.785Re^{0.5}Pr^{1/3} = 0.785 \times 4975^{0.5} \times 0.7282^{1/3} = 49.81$

→ Convective heat transfer coefficient, $h = \frac{Nu\times k}{L} = 49.81 \times \frac{0.02588}{0.2} = 6.445\, W/m^2K$

Heat flux , $q'' = h(T_s - T_\infty) = 6.445 \times (40-20) = 128.9\, W/m^2$

Percentage increase in heat flux = $\frac{128.9-88.64}{88.64} \times 100 = 45.42\%$

Q. 5 Water flows with a velocity of 0.2 m/s over a 75 cm long plate. Free stream temperature is 35℃ and surface temperature is 85℃. Determine the heat transfer coefficient at x = 65 mm and 7.5 mm. Given, non-dimensional temperature gradient at wall, $\frac{\partial\theta}{\partial\eta} = \sqrt{\frac{2\times Pr}{\pi}}$ and local heat transfer coefficient, $h(x) = k\sqrt{\frac{U_\infty}{\nu x}} \times \frac{\partial\theta}{\partial\eta}$.

Properties of water at mean temperature are:

$$k = 0.6507\ W/(m-K)\ ,\text{Pr} = 3.0\ ,\nu = 0.4748 \times 10^{-6} m^2/s$$

A. 6739.23 W/m^{2}K , 2289.2 W/m^{2}K
B. 1583.6 W/m^{2}K, 2687.4 W/m^{2}K
C. 2687.4 W/m^{2}K , 1583.6 W/m^{2}K
D. 2289.2 W/m^{2}K , 6739.23 W/m^{2}K

Sol. Given, $h(x) = k\sqrt{\frac{U_\infty}{\nu x}} \times \frac{\partial\theta}{\partial\eta}$

At x = 65 mm

$$h(x = 65) = 0.6507 \times \sqrt{\frac{0.2}{0.4748 \times 10^{-6} \times 0.065}} \times \sqrt{\frac{2 \times 3.0}{\pi}}$$
$$= 2289.2\ W/m^2K$$

At x = 7.5 mm

$$h(x = 7.5) = 0.6507 \times \sqrt{\frac{0.2}{0.4748 \times 10^{-6} \times 0.0075}} \times \sqrt{\frac{2 \times 3.0}{\pi}}$$
$$= 6739.23\ W/m^2K$$

Q. 6 In Q. 5 if plate is 50 cm wide. Then, calculate the heat transfer rate.

A. 25272.12 W
B. 13474.67 W
C. 50544.25 W
D. 18447.8 W

Sol. Rate of heat transfer, $\dot{q} = \bar{h}A_s(T_s - T_\infty)$

As, we have to calculate the heat transfer rate from the plate surface, therefore, we need to calculate the average heat transfer coefficient over the entire plate surface.

Therefore, $\bar{h} = \frac{1}{L}\int_0^L h(x)\,dx = 2h(L)$

Average heat transfer coefficient, $\bar{h} = 2k\sqrt{\frac{U_\infty}{\nu L}} \times \frac{\partial\theta\,(0)}{\partial\eta}$

$$= 2 \times 0.6507 \times \sqrt{\frac{0.2}{0.4748 \times 10^{-6} \times 0.75}} \times \sqrt{\frac{2 \times 3.0}{\pi}}$$
$$\bar{h} = 1347.846\ W/m^2K$$

Rate of heat transfer, $\dot{q} = 1347.846 \times (0.75 \times .5) \times (85 - 35) = 25272.12\ W$

Q. 7 A wire having a diameter of 0.2 mm is maintained at a constant temperature of 60 °C by an electric current. The wire is exposed to air at 0 °C. Calculate the electric power necessary to maintain the wire temperature if the length is 100 cm. Properties of air at mean film temperature are: $\nu = 15.69 \times 10^{-6}$ m^2/s, k = 0.02624 W/m-K and Pr = 0.708. Take $Nu_d = 0.675Ra_d^{0.058}$ to calculate heat transfer coefficient for cylindrical surface, here, d is the diameter of the wire.

A. 0.89 W
B. 2.79 W
C. 0.836 W
D. 1.395 W

Sol. Film temperature, $T_f = \frac{60+0}{2} = 30\ °C$

$$\beta = \frac{1}{T_f} = \frac{1}{303} = 3.3 \times 10^{-3}$$

Grashof Number, $Gr_d = \frac{g\beta\,(T_s - T_\infty)d^3}{\nu^2} = \frac{9.81\times3.3\times10^{-3}\times(60-0)\times(0.2\times10^{-3})^3}{(15.69\times10^{-6})^2} = 0.0631$

→ Gr Pr = 0.0631X0.708 = 0.04467

→ $Nu_d = 0.675Ra_d^{0.058} = 0.675 \times 0.04467^{0.058} = 0.5636$

→ $h = \frac{Nuk}{d} = \frac{0.5636\times0.02624}{0.2\times10^{-3}} = 73.94\ W/m^2K$

Therefore, required power, $q = hA(T_s - T_\infty) = 73.94 \times \pi \times 0.2 \times 10^{-3} \times 100 \times 10^{-2} \times (60 - 0) = 2.79\ W$

Q. 8 Air at 25 ºC flows over a 0.6 m long panel at 1.8 m/s. The panel is intended to supply 420 W/m^2 to the air. What can be the maximum temperature of the panel? Use correlation $Nu_x = 0.453{Re_x}^{1/2}Pr^{1/3}$. Properties of air are given as:
Pr $= 0.709$, $\nu = 1.784 \times 10^{-5} m^2/s$, $k = 0.0278\ W/mK$

A. 150 ºC
B. 91.2 ºC
C. 87.3 ºC
D. 116.2ºC.

Sol. Trailing edge of the panel will be having maximum temperature.

Therefore, maximum temperature difference, $\Delta T_{max} = \Delta T_{x=L} = \frac{q}{h_{x=L}} = \frac{qL}{kNu_{x=L}}$

$$Re_x = \frac{VL}{\nu} = \frac{1.8 \times 0.6}{1.784 \times 10^{-5}} = 60538$$

$$\Delta T_{max} = \frac{qL}{k \times 0.453 \times {Re_x}^{1/2}Pr^{1/3}} = \frac{420 \times 0.6}{0.0278 \times 0.453 \times (60538)^{\frac{1}{2}} \times (0.709)^{1/3}}$$

$$= 91.2°C$$

Now, maximum temperature, $T_{max} = 25 + 91.2 = 116.2\ °C$