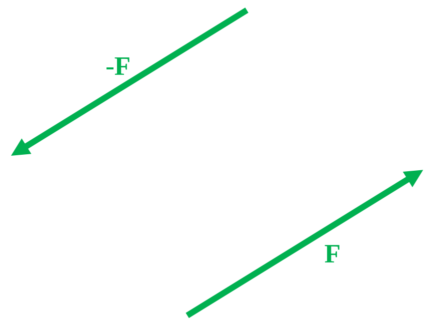
FORCES & MOMENTS
Couples

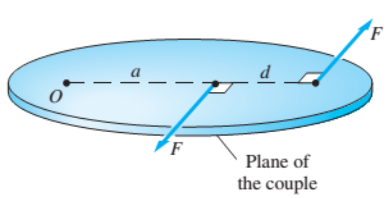
- This refers to two parallel, non-collinear forces that are equal in magnitude and opposite in direction.



- It is a free vector that can be applied anywhere

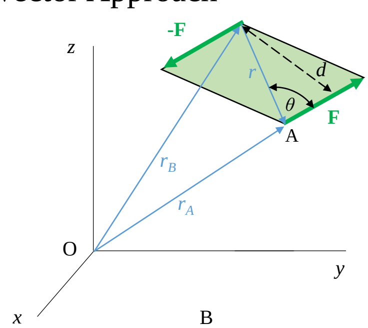
FORCES & MOMENTS
Calculating the Moment of a Couple

- Scalar approach



$$\circlearrowleft + \quad M_O = F(a+d) - F(a) = Fd$$

- Vector Approach



$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times \left(-\vec{F}\right)$$
$$= \left(\vec{r}_A - \vec{r}_B\right) \times \vec{F}$$
$$= \vec{r} \times \vec{F}$$
$$M = rF\sin\theta = Fd$$

FORCES & MOMENTS
Some Properties of Couples

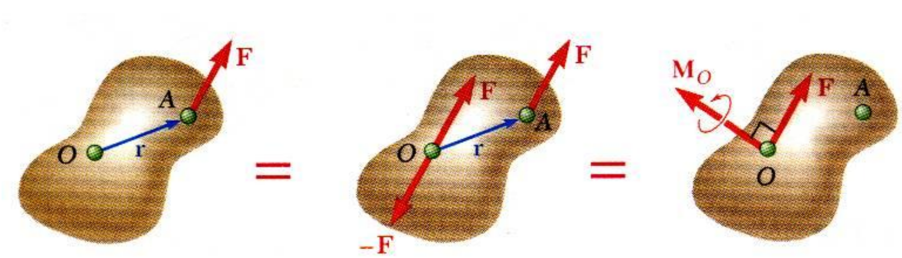
- Two couples are considered equivalent if
  - their moments is of the same magnitude
  - They lie in the same plane
  - Tend to cause rotation in the same direction

- Couples are vectors

- They obey Varignon's Theorem

EQUIVALENT FORCE SYSTEMS
Shifting the line of Action of a Force

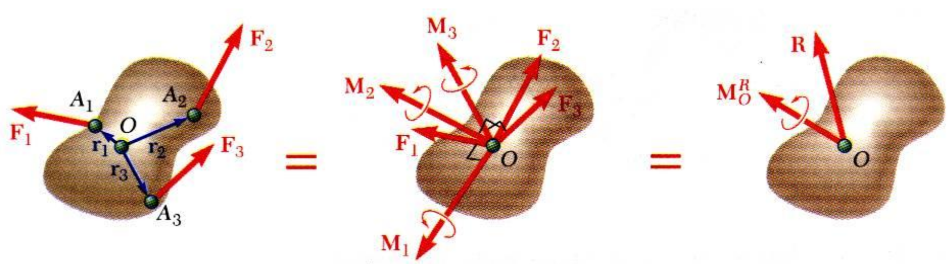
- This can be done by replacing the force with a force-couple system that acts at the desired point.
- The couple is given by the product of the force and the perpendicular distance between the old and new line of actions.



EQUIVALENT FORCE SYSTEMS
Reduction of Several forces into a force couple-system

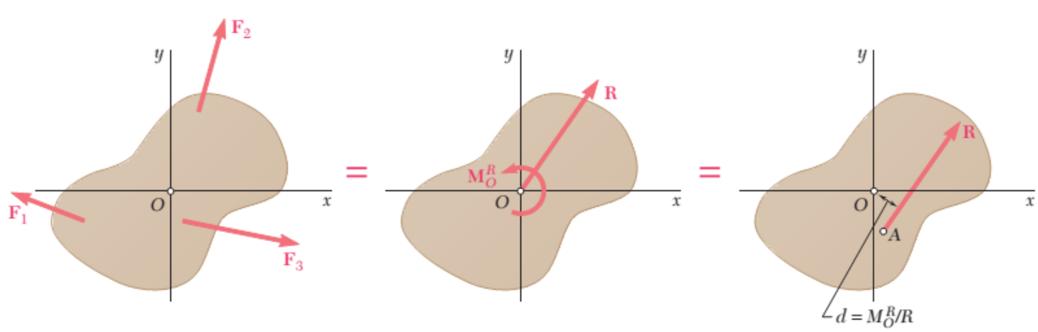
- Where a system of forces act on a body, they can be reduced to a several force-couple systems acting at a desired point.
- The force couple systems can be combined into a resultant force-couple.

$$\vec{R} = \sum \vec{F} \qquad \vec{M}_O^R = \sum \left(\vec{r} \times \vec{F} \right)$$



EQUIVALENT FORCE SYSTEMS
Reduction of Several forces into a force couple-system

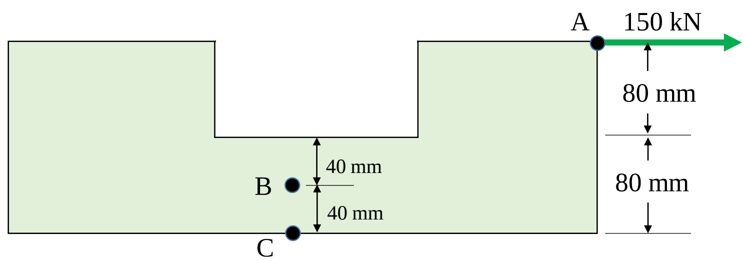
- A system of forces act on a body, they can be reduced to a force-couple system acting at a desired point.



EQUIVALENT FORCE SYSTEMS
Reduction of Several forces into a force couple-system

Example

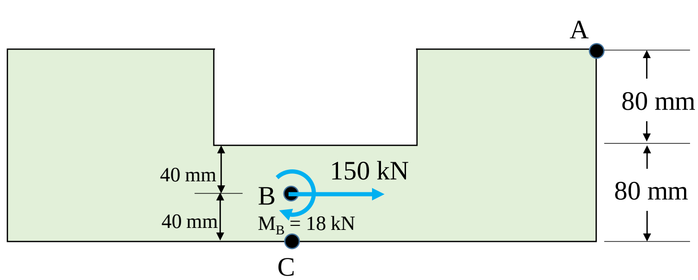
For the machine part shown in the Figure below, replace the applied load of 150 kN acting at point A by an equivalent force-couple system with the force acting at point B.



EQUIVALENT FORCE SYSTEMS
Reduction of Several forces into a force couple-system

Solution

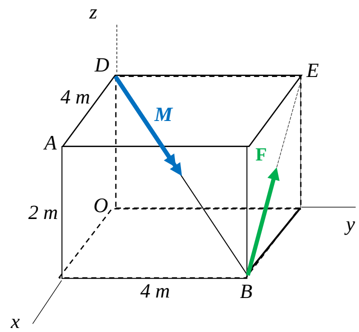
$$\circlearrowleft + \quad M_B = -150(0.08\,\text{m} + 0.04\,\text{m}) = \text{-}18\,\text{kN}$$



EQUIVALENT FORCE SYSTEMS
Reduction of Several forces into a force couple-system

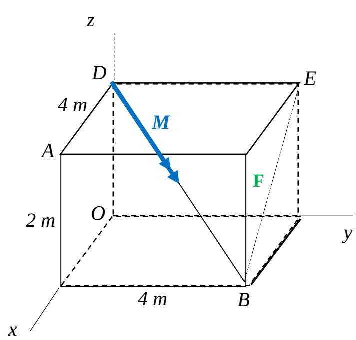
Example

Replace the force-couple system shown in Fig. (a) with an equivalent force-couple system, with the force acting at point A, given that F=100 N and M =120 N.m.



EQUIVALENT FORCE SYSTEMS
Reduction of Several forces into a force couple-system

Solution



$$\vec{F} = 100\lambda_{BE} = 100\left(\frac{-4i+2k}{\sqrt{\left((-4)^2+2^2\right)}}\right) = -89.44i + 44.72k$$

$$\vec{r}_{AB} = 4j - 2k$$

$$\vec{M}_A = \vec{r}_{AB} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 4 & -2 \\ -89.44 & 0 & 44.72 \end{vmatrix}$$

$$= (178.9i + 178.9j + 357.8k)\,\text{N.m}$$

$$\vec{M} = 120\lambda_{DB} = 120\left(\frac{4i+4j-2k}{\sqrt{\left(4^2+4^2+(-2)^2\right)}}\right)$$

$$= (80i + 80j - 40k)\,\text{N.m}$$

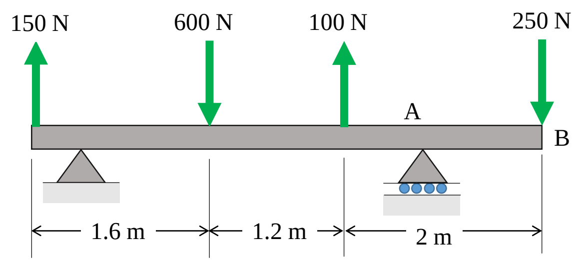
$$\text{M}_{RA} = \vec{M}_A + \vec{M} = (258.9i + 258.9j + 317.8k)\,\text{N.m}$$

$$\text{M}_{RA} = 484.8\ \text{N.m}$$

EQUIVALENT FORCE SYSTEMS
Reduction of Several forces into a force couple-system

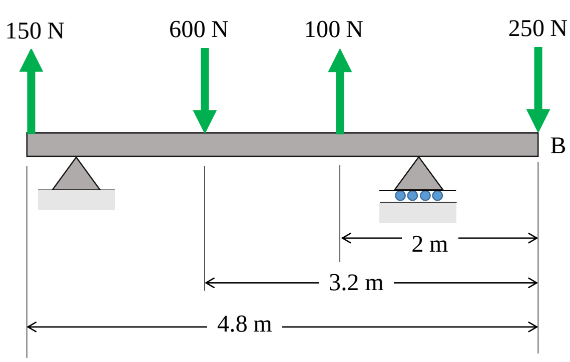
Example

For the beam, reduce the system of forces shown to (a) an equivalent force-couple system at A, (b) an equivalent force couple system at B. (Ignore the support reactions)



EQUIVALENT FORCE SYSTEMS
Reduction of Several forces into a force couple-system

Example



The Resultant force will be

$$
\begin{aligned}
+\uparrow R &= \sum F \\
&= (150\ \text{N}) - (600\ \text{N}) + (100\ \text{N}) - (250\ \text{N}) \\
&= (-600\ \text{N})
\end{aligned}
$$

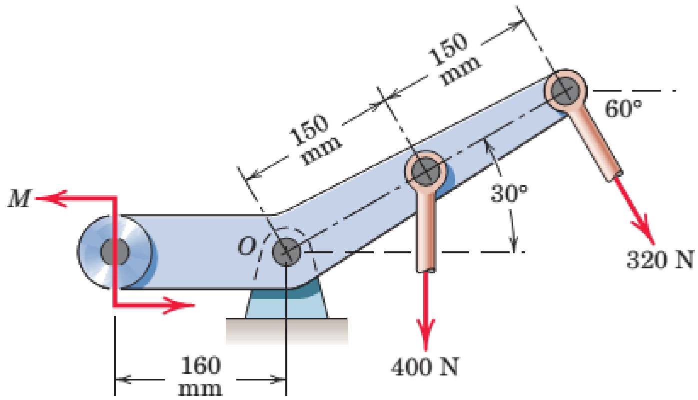
The Resultant Moment

$$
\begin{aligned}
\circlearrowleft + \quad \vec{M}_B &= \sum (r \times F) \\
&= (250N \times 0m) - (100N \times 2m) + (600N \times 3.2m) - (150N \times 4.8m) \\
&= 1000Nm
\end{aligned}
$$

EQUIVALENT FORCE SYSTEMS
Reduction of Several forces into a force couple-system

Example

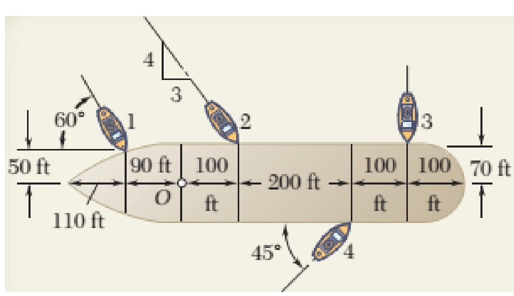
If the resultant of the two forces and couple *M* passes through point *O*, determine *M*.



EQUIVALENT FORCE SYSTEMS
Reduction of Several forces into a force couple-system

Example

Four tugboats are used to bring an ocean liner to its pier. Each tugboat exerts a 5000-lb force in the direction shown. Determine the equivalent force-couple system at the foremast O. Also determine the angle the resultant force makes with the horizontal as well as the direction of rotation of the moment.



$$R = 13.33\,\text{lb},\ \theta_i = 47.3^{O}$$

$$M_{RO} = 1035\,\text{lb.in},\quad \text{Clockwise}$$

CENTROIDS

Centroid of an Area using the moments of area approach

~~*Centroid of an Area using with the integration approach*~~

~~*An Introduction Moments of Inertia.*~~

CENTROIDS

- It is sometimes necessary in mechanics problems to determine the central point of bodies.
- This central point is defined as that point a physical quantity under consideration may be assumed to be centered.
- The central point may have different terminologies for different physical quantities.

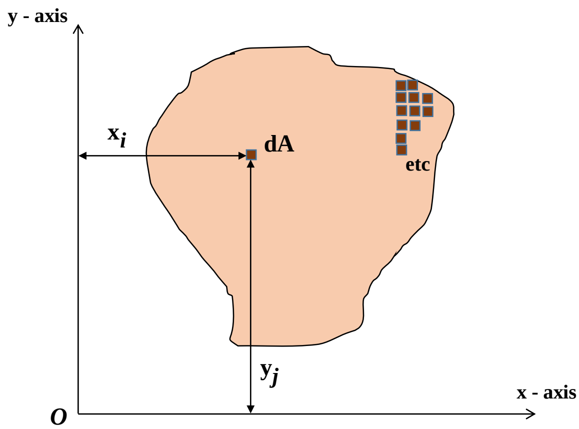
Terminology	**Physical Entity**
Centroid	Length of a curve
Centroid	Area of a surface
Centroid	Volume of a body
Centre of a mass	Mass of a body
Centre of gravity	Gravitational force on a body

- All the terms mentioned above can be determined using the summation approach, which will be discussed. An integration approach can also be used.

CENTROIDS

Determining the Centroid of a Plane Area

- Using the summation approach,
  - We assume the area comprises several smaller elemental areas.
  - We then sum moments of the elemental areas about axes of the origin (1st moments) and divide this by the total area to get the centroid of the area.



$$\bar{x} = \frac{\textit{First Moment of Area about y-axis}, Q_y}{\textit{Total Area}} = \frac{\sum x_i dA_i}{\sum A_i}$$

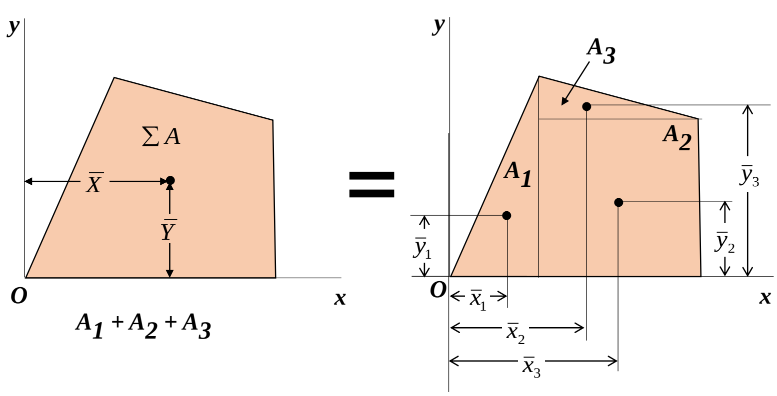
$$\bar{y} = \frac{\textit{First Moment of Area about x-axis}, Q_x}{\textit{Total Area}} = \frac{\sum y_i dA_i}{\sum A_i}$$

Centroid is $(\bar{X}, \bar{Y})$

CENTROIDS

Determining the Centroid of a Plane Area

- The same idea is used in determining the centroids of composite areas.



$$\bar{X} = \frac{\textit{First Moment of each Area about y-axis}, Q_y}{\textit{Total Area}}$$

$$= \frac{\sum \bar{x}_i dA_i}{\sum A_i} = \frac{(\bar{x}_1 A_1) + (\bar{x}_2 A_2) + (\bar{x}_3 A_3)}{A_1 + A_2 + A_3}$$

$$\bar{Y} = \frac{\textit{First Moment of each Area about x-axis}, Q_x}{\textit{Total Area}}$$

$$= \frac{\sum \bar{y}_i dA_i}{\sum A_i} = \frac{(\bar{y}_1 A_1) + (\bar{y}_2 A_2) + (\bar{y}_3 A_3)}{A_1 + A_2 + A_3}$$

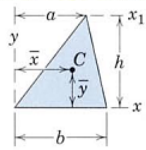
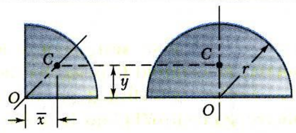
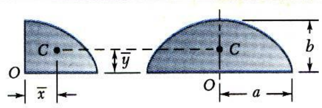
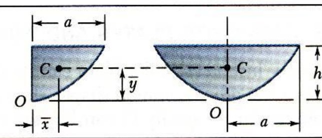
Centroid is $(\bar{X}, \bar{Y})$

Note:

THE ELEMENTAL AREA CENTROID VALUES MAY BE NEGATIVE OR POSITIVE DEPENDING ON THE LOCATION OF THE ORIGIN OF THE COMPOSITE AREA BEING CONSIDERED.

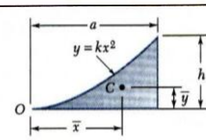
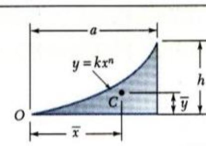
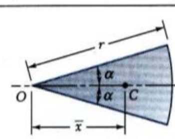
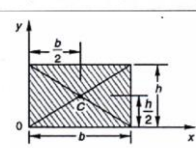
CENTROIDS

Centroids of Common Shapes

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area		$\frac{a+b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$

CENTROIDS

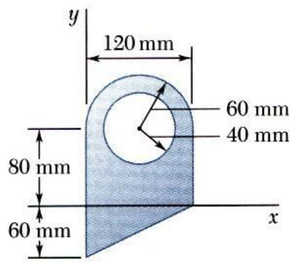
Centroids of Common Shapes

Shape		$\bar{x}$	$\bar{y}$	Area
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2
Rectangle		$b/2$	$h/2$	bh

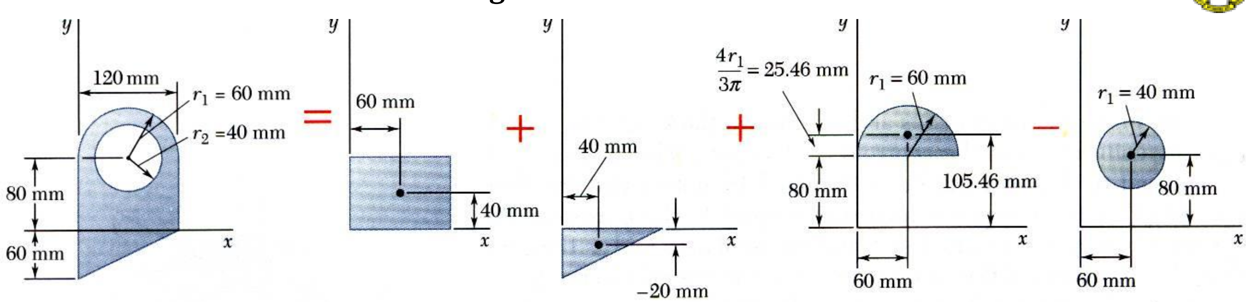
CENTROIDS
Determining the Centroid of a Plane Area

Example

For the plane area shown, determine the first moments with respect to the x and y axes and the location of the centroid.



CENTROIDS
Determining the Centroid of a Plane Area



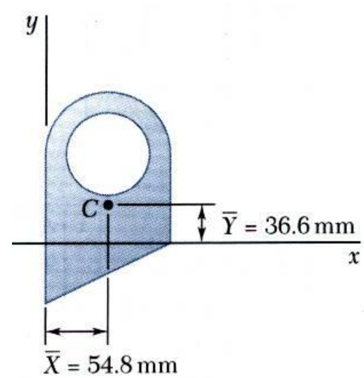
Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	-20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma \bar{x}A = +757.7 \times 10^3$	$\Sigma \bar{y}A = +506.2 \times 10^3$

CENTROIDS

Determining the Centroid of a Plane Area

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{+757.7\times10^3\,\text{mm}^3}{13.828\times10^3\,\text{mm}^2}$$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{+506.2\times10^3\,\text{mm}^3}{13.828\times10^3\,\text{mm}^2}$$

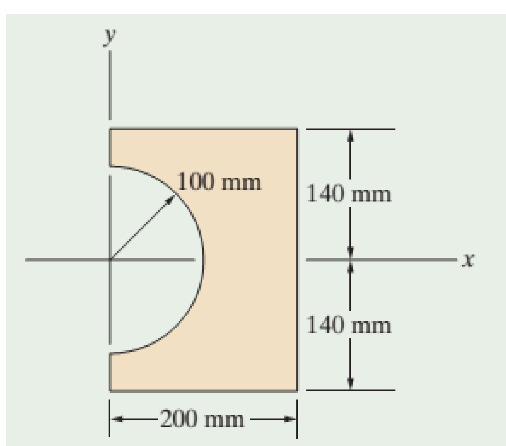


NOTE: The same principles are used in determining the centroids of curves and volumes as well as the centers of mass and gravity of bodies.

CENTROIDS

➢Example

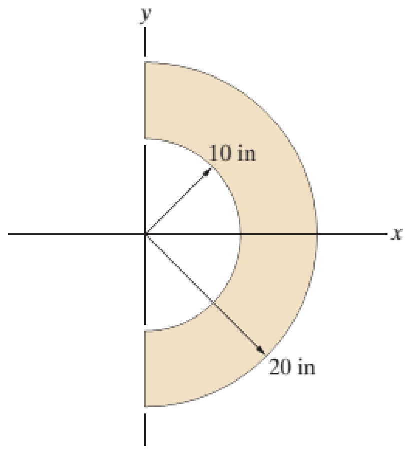
Determine the co-ordinates centroid of the area shown.



CENTROIDS

➢Example

Determine the co-ordinates centroid of the area shown.



CENTROIDS

➢Example

Determine the co-ordinates centroid of the area shown.