KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

COLLEGE OF ENGINEERING

BSC MECHANICAL ENGINEERING

TUTORIALS

ME 356: STRENGTH OF MATERIAL II

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Course Writer

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GUIDELINES FOR STUDENTS ASSESSMENT

A. Quiz

1. Three questions will be set from unit 4 to unit 6
2. One question from each unit
3. Each question will come from the Essay Type
4. Answer Two questions

B. Mid Semester

1. Section A: Objectives
   i. Twenty objective questions will be set from unit 1 and unit 2
   ii. Ten questions from each unit
   iii. Six from Objective Type
   iv. Four from Solved Example
   v. Answer ALL

2. Section B Essay
   i. Three questions will be set from unit 1 to unit 3
   ii. One questions from each unit
   iii. All questions are from Solved Example and Essay type
   iv. Answer question in **Unit 3** and any other **One**

C. Final Examination

1. Section A: Objectives
   i. Thirty objective questions will be set from unit 1 to unit 6; except **Unit 3**
   ii. Six questions from each unit
   iii. Four from Objective Type
   iv. Two from Solved Example
   v. Answer ALL

2. Section B Essay
   i. Three questions will be set from unit 1 to unit 3
   ii. One questions from each unit
   iii. All questions are from Solved Example and Essay type
   iv. Question 1: Unit 3 and Unit 6
   v. Question 2: Unit 1 and Unit 2
   vi. Question 3: Unit 4 and Unit 5
   vii. Answer question **1** and any other **One**

Unit 1

DEFLECTION OF BEAMS

Introduction

This unit discusses the various method used to determine the deflection of beams which include the strain energy method, the method of calculus and the Macualay's method

Learning Objectives

After reading this unit you should be able to:

1. Define a deflection of a beam
2. Derive the various equations used to determine the deflection of beams
3. Selection the appropriate boundary conditions for each type of situation
4. Determine the deflection of beam using the three methods

IMPORTANT EQUATIONS

1. Moment: $M = EI\ X\ \dfrac{d^2 y}{dx^2}$

2. Simple supported, central load,

$$EI\,\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16}$$

$$EI.y = \frac{Wx^3}{12} - \frac{Wl^2 x}{16}$$

3. Simply Supported Beam, Eccentric Point Load

$$EI\,\frac{dy}{dx} = \frac{Wax^2}{2l} - \frac{W(x-b)^2}{2} + (EIi_C) - \frac{Wab^2}{2l}$$

$$EI.y = \left\{ \begin{array}{l} \dfrac{Wax^3}{6l} + \dfrac{W(x-b)^3}{6} + (EI.i_C.x) \\ -\dfrac{Wab^2}{2l}x + \dfrac{Wab}{3}(b-a) - (EI.i_C.l) \end{array} \right\}$$

4. Simply Supported Beam with a Uniformly Distributed Load

$$EI\,\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^3}{24}$$

$$EI.y = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^3x}{24}$$

5. Cantilever with a Point Load at the Free End,

$$EI\,\frac{dy}{dx} = -\frac{Wx^2}{2} + \frac{Wl^2}{2} \text{ and}$$

$$EI.y = -\frac{Wx^3}{6} + \frac{Wl^2x}{2} - \frac{Wl^3}{3}$$

6. Cantilever with a Point Load not at the Free End

$$i_C = i_B = -\frac{Wl_1^2}{2EI}\,radians\ ,$$

$$y_C = \frac{Wl_1^3}{3EI} \text{ and}$$

$$y_B = \frac{Wl_1^3}{3EI} + \frac{Wl_1^2}{3EI}(l - l_1)$$

7. Cantilever with a Uniformly Distributed Load

$$EI\,\frac{dy}{dx} = -\frac{wx^3}{6} + \frac{wl^3}{6}$$

$$EI.y = -\frac{wx^4}{24} + \frac{wl^3x}{6} - \frac{wl^4}{8}$$

8. Cantilever Partially Loaded with a Uniformly Distributed Load

$$i_C = i_B = -\frac{wl_1^3}{6EI}\,radians \text{ and}$$

$$y_C = \frac{wl_1^4}{8EI} \text{ and}$$

$$y_B = \frac{wl_1^4}{8EI} + \frac{wl_1^3}{6EI}(l - l_1)$$

9. Simply Supported Beam with a gradually Varying Load

$$EI\,\frac{dy}{dx} = \frac{wlx^2}{12} - \frac{w.x^4}{24l} - \frac{7wl^3}{360}$$

$$EI.y = \frac{wlx^3}{36} - \frac{w.x^5}{120l} - \frac{7wl^3x}{360}$$

10. Cantilever with a gradually Varying Load

$$EI\,\frac{dy}{dx} = -\frac{w.x^4}{24l} + \frac{wl^3}{24}$$

$$EI.y = -\frac{w.x^5}{120l} + \frac{wl^3x}{24} - \frac{wl^4}{30}$$

11. Singularity, Concentrated loads

$$EI\, {d^2y}/{dx^2} = M =$$

$$\left(-W_1x + R[x-a] - W_2[x-b] - W_3[x-c]\right)$$

12. Singularity, Uniformly distributed loads,

$$EI\, {d^2y}/{dx^2} = M = \left(Rx - w[x-a]^2 + w[x-b]^2\right)$$

13. Singularity, Concentrated bending moment,

$$EI\, {d^2y}/{dx^2} = M = \left(Rx - M_0[x-a]^0\right)$$

TYPE A: SOLVED EXAMPLES

Problem 1: *A cantilever 1.5 m long carries a uniformly distributed load over the entire length. Find the deflection at the free end if the slope at the free end is 1.5°.*

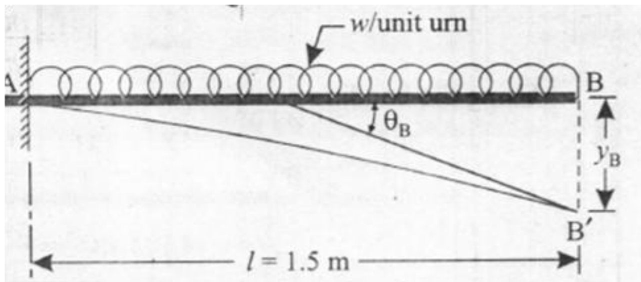


Fig. 1

Solution

Given: Length of the cantilever, l= 1.5 m = 1500 mm; Slope at the free end, = 1.5^0 = 1.5 radian;

Deflection at B

Slope at the free end, $\theta = \dfrac{wl^3}{6EI} \Rightarrow \dfrac{wl^3}{EI} = 6\theta = \dfrac{6\pi(1.5)}{180} = \dfrac{\pi}{20}$

Deflection at the free end, $y_B = \dfrac{wl^4}{8EI} = \left[\dfrac{wl^3}{EI}\right]\left[\dfrac{l}{8}\right] = \left[\dfrac{\pi}{20}\right]\left[\dfrac{1500}{8}\right] = 29.45\ mm$

Problem 2: *A* 2 *meters long cantilever made of steel tube of section 150 mm external diameter and 10 mm thick is loaded as shown in the fig.* 2 *(a). If E = 200 GPa, calculate:*

(i) The2 value of W so that the maximum bending stress is 150 MPa

(ii) The maximum deflection for the loading.

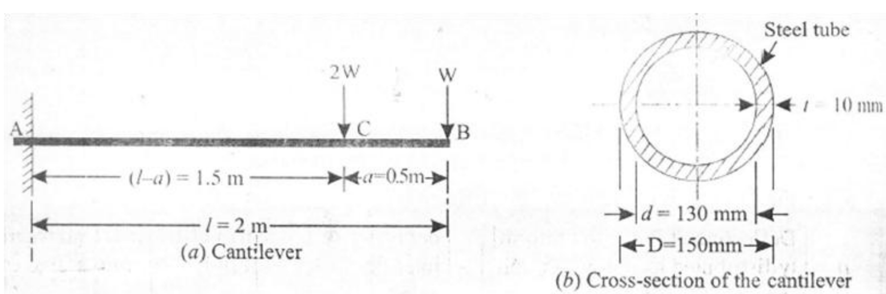


Fig 2

Solution

Given: Length of the cantilever, l = 2 m = 2000 mm; External diameter of the steel tube, D =; 150 mm; Thickness of the tube = 10 mm; Internal diameter of the tube, d = D – 2t = 130 mm; Maximum bending stress, σ_b = 150 MPa = 150 N/mm^2; E = 200 GPa = 200 x10^3 N/mm^2;

Load

Maximum Bending Moment,

$M_{max} = Wl + 2W(l-a) = W(3l-2a) = W[3(2000)-2(500) = 5000W]$

Moment of inertia, $I = \frac{\pi}{66}\left[D^4 - d^4\right] = \frac{\pi}{64}\left[150^4 - 130^4\right] = 10.8x10^6 \ mm^4$

Using the relation, we have $\sigma_b = \frac{My}{I} \Rightarrow 150 = \frac{(5000W)(75)}{10.8x10^6} \Rightarrow W = \frac{(10.8x10^6)(150)}{(5000)(75)} = 4320 \ N$

The maximum deflection

Total deflection at the free end = Deflection at the free end due to the load *W* alone + deflection at the free end due to the load 2W

$$\delta = \frac{Wl^3}{3EI} + \left[\frac{2W(l-a)^3}{3EI} + \frac{2W(l-a)^2}{2EI}.a\right]$$

$$\delta = \frac{4320(2000^3)}{3(200x10^3)(10.8x10^6)} + \left[\frac{2(4320)(1500)^3}{3(200x10^3)(10.8x10^6)} - \frac{2(4320)(1500)^2}{2(200x10^3)(10.8x10^6)}(500)\right] = 12.08 \ mm$$

Hence, maximum deflection = 12.08 mm

Problem 3: *A cantilever of* 3 *metres length and of uniform rectangular cross section 150 mm wide and 300 mm deep is loaded with a 30 kN load at its free end. In addition to this, it carries a uniformly distributed load of* 20 *kN per metre run over its entire length, Calculate:*
(i) The maximum slope and maximum
(ii) The slope and deflection at 2 *metres from the fixed end. Take, E = 210 GNlm²*

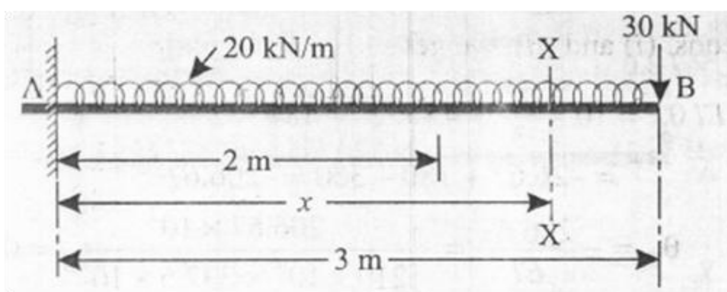


Fig 3

Solution

Given: Length of the cantilever, $l = 3$ m; Cross-section: width, b = 0.15 m; depth, d = 0.3 m; E = 210 GPa = 210 x 10^9 N/m^2;

Moment of inertia, $I = \frac{bd^3}{12} = \frac{0.15(0.3^3)}{12} = 337.5x10^{-6} \ m^4$

Maximum slope

Consider a section XX at a distance *x* from the fixed end,

$$M_x = -30(3-x) - \frac{20(3-x)^2}{2} = -10x^2 + 90x - 180$$

This implies, $EI\frac{d^2y}{dx^2} = M_x = -10x^2 + 90x - 180$

Integrating, we get $EI\frac{dy}{dx} = -\frac{10x^3}{3} + 45x^2 - 180x + C_1$

When, $x = 0; dy/dx = 0 \Rightarrow C_1 = 0$

Hence, $EI\frac{dy}{dx} = -\frac{10x^3}{3} + 45x^2 - 180x$ (i)

Putting x = 3 m, we get, $EI\theta_{max} = -\frac{10(3)^3}{3} + 45(3)^2 - 180(3) = -225$

Hence, $\theta_{max} = \frac{-225}{EI} = \frac{-225}{(200x10^9)(337.5x10^{-6})} = -0.003175\ radian$

Maximum deflection

Integrating eqn. (i) we get $EIy = \frac{10x^4}{12} + 15x^3 - 90x^2 + C_2$

When, x = 0, y = 0 then, C_2 = 0

Hence, $EIy = \frac{10x^4}{12} + 15x^3 - 90x^2$

Putting x = 3 m, we get $EIy = \frac{10(3)^4}{12} + 15(3)^3 - 90(3)^2 = -472.5$

Hence $y = \frac{-472.5}{EI} = \frac{-472.5}{(200x10^9)(337.5x10^{-6})}x10^3\ mm = -6.67\ mm$

Slope at 2 metres from the fixed end
Putting x = 2 m in eqn. (i), we get,

$$EI\theta_{max} = \frac{10(2)^3}{3} + 45(2)^2 - 180(2) = -206$$

Hence, $\theta_{max} = \frac{-2.06.7}{EI} = \frac{-206.7}{(200x10^9)(337.5x10^{-6})} = -0.00292\ radian$

Deflection at 2 metres from the fixed end
Putting x = 2000 in eqn. (ii), we get,

$$EIy = \frac{10(2)^4}{12} + 15(2)^3 - 90(2)^2 = -253.3$$

Hence $y = \frac{-253.3}{EI} = \frac{-253.3}{(200x10^9)(337.5x10^{-6})} x10^3 \ mm = -3.57mm$

Problem 4: *A 250 mm long cantilever of rectangular section 40 mm wide and 30 mm deep carries a uniformly distributed load. Calculate the value of w if the maximum deflection in the cantilever is not to exceed 0.5 mm. 'Take, E = 70 GPa*

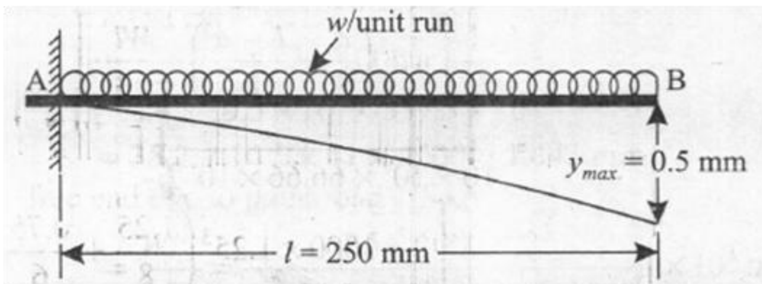


Fig 4

Solution

Given: Length of the cantilever, l = 250 mm; Width, b = 40 mm; depth, d = 30 mm; maximum deflection, y_{max} = 0.5 mm; E = 70 x 10^3 N/mm^2;

Moment of inertia, $I = \frac{bd^3}{12} = \frac{40(30^3)}{12} = 9.0x10^4 \ mm^4$

The value of w

Using the relation, $y_B = \frac{wl^4}{8EI} \Rightarrow w = \frac{8y_B EI}{l^4} = \frac{8(0.5)(70x10^3)(9x10^4)}{250^4} = 6.451 \ N/mm$

Problem 5: *A cantilever* 2 *metres long is of rectangular section 100 mm wide and 200 mm deep. It carries a uniformly distributed load of* 2 *kN per unit metre length for a length of* 1.25 *metres from the fixed end, a point load of 0.8 kN at the free end. Find the lie deflection at the free end. Take, E = 10 GPa*

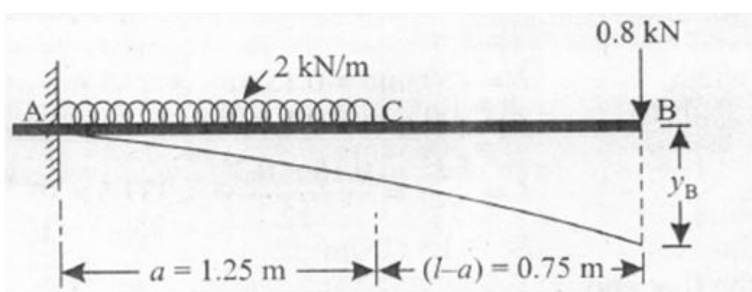


Fig 5

Solution

Given: Length of the cantilever, l = 2 m; Width, b = 0.10 m; depth, d = 0.20 mm; mm; E = 70 x

10^9 N/m^2;

Moment of inertia, $I = \frac{bd^3}{12} = \frac{(0.1)(0.2^3)}{12} = 66.66x10^{-6}\ m^4$

Deflection at the free end

Deflection at the free end *B, (downward)* = Deflection at *B* due to uniformly distributed load + Deflection at *B* due to point load at B

Thus,
$$\delta = \left[\frac{wa^4}{8EI} + \frac{wa^3}{6EI}.(l-a)\right] + \frac{Wl^3}{3EI} = \frac{1}{EI}\left[wa^3\left(\frac{a}{8} + \frac{l-a}{6}\right) + \frac{Wl^3}{3}\right]$$
$$\delta = \frac{10^3}{8(70x10^9)(66.66x10^{-6})}\left[2(1.25^3)\left(\frac{1.25}{8} + \frac{0.75}{6}\right) + 0.8(2^3)\right]x10^{-3} = 4.85\ mm$$

Problem 6: *A* 2 *metres long cantilever of rectangular section 150 mm wide and 300 deep is loaded as shown in the fig.* 6. *Calculate the deflection at the free end. Take, E = 10.5 GNlm^2•*

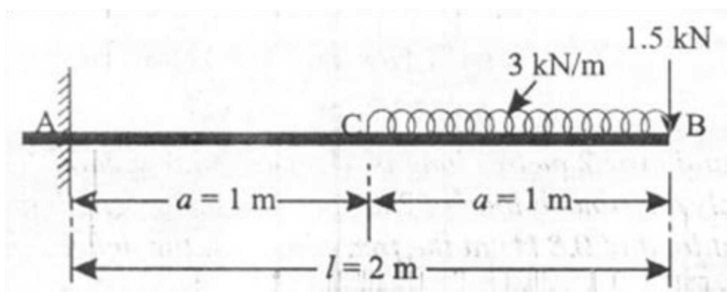


Fig 6

Solution

Given: Length of the cantilever, l = 2000 mm; Width, b = 150 mm; depth, d = 300 mm; mm; E = 10.5 x 10^3 N/mm^2;

Moment of inertia, $I = \frac{bd^3}{12} = \frac{150(300^3)}{12} = 337.5x10^6\ mm^4$

Deflection at the free end

Deflection at B due to uniformly distributed load

$$y_u = \frac{w}{24EI}\left[3l^4 - 4la^3 + a^4\right]$$

$$y_u = \frac{3}{24(10.5x10^3)(337.5x10^6)}\left[3(2000)^4 - 4(2000)(1000)^3 + 1000^4\right] = 1.443\ mm$$

Deflection at B due to point load, $y_p = \frac{Wl^3}{3EI} = \frac{1500(2000)^3}{24(10.5x10^3)(337.5x10^6)} = 1.128\ mm$

Total deflection at the free end *B, (downward)* = Deflection at *B* due to uniformly distributed load + Deflection at *B* due to point load

$y_B = y_u + y_p = 1.443 + 1.128 = 2.57\ mm$

Problem 7: *A girder of uniform section and constant depth is freely supported; over a span of 3metres. If the point load at the mid span is 30 kN and Ixx* 15.614 x 10^6 mm^4, c*alculate:*
(l) The central deflection.
(ii) The slopes at the end of the beam Take: E=200 GPa

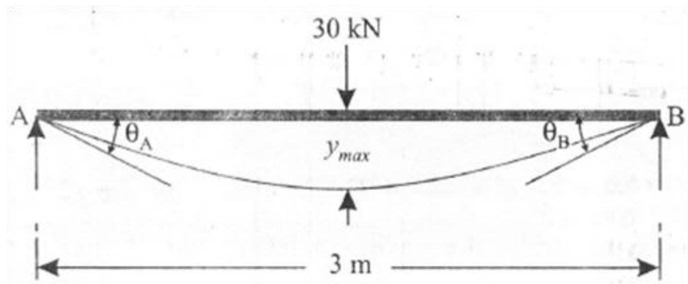


Fig 7

Solution

Given: Length of the span, l = 3 m = 3000 mm; Load at the mid span, W = 30 kN = 30000 N; Moment of inertia, Ixx = 15.614 x 10^6 mm^4; E = 200 x10^3 N/mm^2;

Central deflection

$$y_{\max} = \frac{Wl^3}{48EI} = \frac{30000(3000)^3}{48(200x10^3)(15.614x10^6)} = 5.4\ mm$$

Slopes at the ends of the beam

Slope at end A

$$\theta_A = -\frac{Wl^2}{16EI} = -\frac{30000(3000)^2}{16(200x10^3)(15.614x10^6)} = -0.0054\ radian = -0.309^o$$

Slope at the end B

$$\theta_B = \frac{Wl^2}{16EI} = \frac{30000(3000)^2}{16(200x10^3)(15.614x10^6)} = 0.0054\ radian = 0.309^o$$

Problem 8: *A steel girder of 6 m length acting as a beam carries a uniformly distributed load w N/m runs throughout its length. If I = 30 x 10^6 m^4 and depth 270 mm, calculate:*
(i) The magnitude of w so that the maximum stress developed in the beam section does not exceed 72 *MPa.*
(ii) The slope and deflection (under this load) in the beam at a distance of 1.8 m from one end. Take: E = 200 *GPa^2*

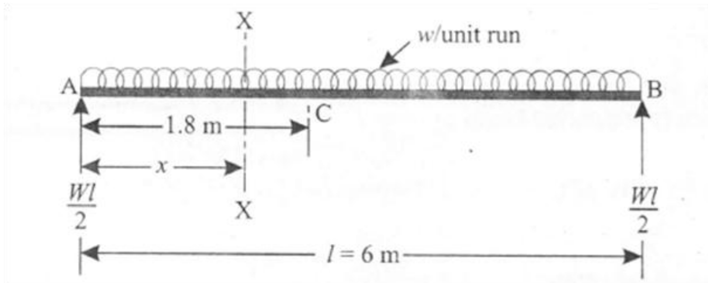


Fig 8

Solution

Given: Length of the span, l = 6 m = 6000 mm; Load at the mid span, Moment of inertia, I = 30 x 10^6 mm^4 = 30 x 10^{-6} m^4; depth, d = 270 mm; σ_b = 72 N/mm^2; E = 200 x10^9 N/mm^2;

Magnitude of w

Maximum bending occurs at the centre of the beam. $M_{\max} = \frac{wl^2}{8} = \frac{w(6000^2)}{8} = 4.5x10^6\, w$

Maximum stress will occur at the extreme layers at a distance of ± *d/2* from the neutral (where *d* is the depth of the section

Now, $\sigma_b = \frac{My}{I} = \frac{(4.5x10^6)(w)(135)}{30x10^6} \Rightarrow w = \frac{(30x10^6)\sigma_b}{(4.5x10^6)(135)} = \frac{(30x10^6)(72)}{(4.5x10^6)(135)} = 3.5\, N/mm$

Considering a section XX at a distance x from the end A, we have

$EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^3}{24}$ and $EIy = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^3x}{24}$

Slope at point C, at x = 1.8 m

$$EI\theta_C = 3.5\left[\frac{(6)(1.8)^2}{4} - \frac{(1.8)^3}{6} - \frac{(6)^3}{24}\right] = -18.17$$

$$\Rightarrow \theta_C = \frac{-17.89}{(200x10^9)(30x10^{-6})} = 0.00303\, rad = -0.173^o$$

Deflection at point C, at x = 1.8 m

$$EIy_C = 3.5\left[\frac{(6)(1.8)^3}{12} - \frac{(1.8)^4}{24} - \frac{(6)^3(1.8)}{24}\right] = -48.779$$

$$\Rightarrow y_C = \frac{-48.779}{(200x10^9)(30x10^{-6})}x10^{-3}\, mm = 8.13\, mm$$

Problem 9: *A uniform beam of length l is simply and symmetrically supported on a span l'. Find the ratio of so that the upwards deflection at each end equals the downward deflection at mid span due to a central point load.*

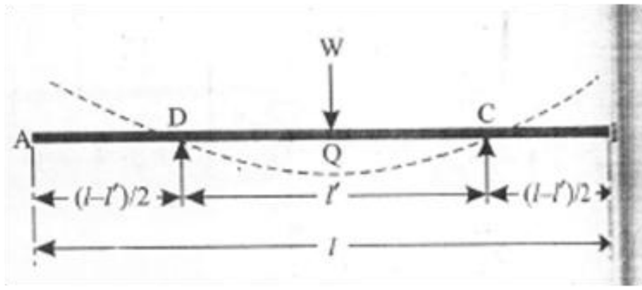


Fig 9

Solution

Given: Length of the beam = *l;* length of the span = *l'*

Ratio

We know that due to a central point load in the case of a simply supported beam of span *1',* W[,2

Slope at the supports $\theta_D = \dfrac{Wl'^2}{16EI}$

Deflection at the centre, $y_Q = \dfrac{Wl'^3}{48EI}$ (i)

Since the angle is very small $y_A = \theta_D x\, AD = \left[\dfrac{Wl'^2}{16EI}\right] x \left[\dfrac{l-l'}{2}\right]$ (ii)

Equation *(i)* and *(ii),* we get. $\left[\dfrac{Wl'^2}{16EI}\right] x \left[\dfrac{l-l'}{2}\right] = \dfrac{Wl'^3}{48EI} \Rightarrow \dfrac{l-l'}{l'} = \dfrac{2}{3}$

Thus, $\dfrac{l}{l'} = \dfrac{2}{3} + 1 = \dfrac{5}{3}$

Problem 10: *A beam AB of length l simply supported at the ends carries a point load W at a distance* ***a*** *from the left end. Find:*

(i) *The deflection under the load.*

(ii) The maximum deflection.

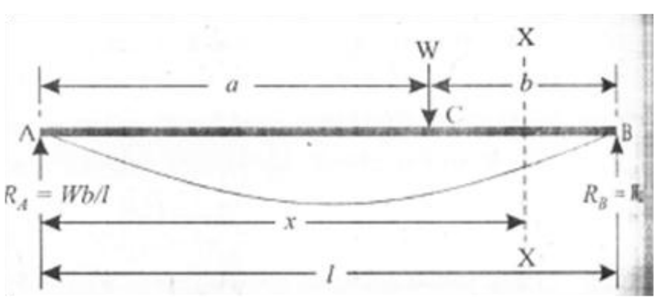


Fig 10

Solution

Given: Fig. 10 shows a beam AB of span *l* carrying a point load *W at* C

Let, *AC* = *a*, *CB* = *b*, and *a* > *b*

To find reactions taking moments about *A*, we get $R_B.l = W.a \Rightarrow R_B = \frac{Wa}{l}$

Also, $R_A + .R_B = W \Rightarrow R_A = W - R_B == W - \frac{Wa}{l} = \frac{W}{l}(l-a) = \frac{Wb}{l}$

The bending moment at any section XX at a distance *x* from the end *A*, following Macaulay method is given by: $M = .EI\frac{d^2y}{dx^2} = \frac{Wb}{l}x - W(x-a)$

Integrating for slope, we get $EI\frac{dy}{dx} = \frac{Wb}{l}\frac{x^2}{2} - W\frac{(x-a)^2}{2} + C_1$

Integrating again for deflection, we get $EIy = \frac{Wb}{l}\frac{x^3}{6} - W\frac{(x-a)^3}{6} + C_1x + C_2$

At A, deflection is zero, *i.e.* when $x = 0$, y = 0 and $C_2 = 0$

At B, deflection is zero, *i.e.* when $x = l$, y = 0,

$$0 = \frac{Wb}{l}\frac{l^3}{6} - W\frac{(l-a)^3}{6} + C_1l \Rightarrow C_1 = -\frac{Wb}{6l}(l^2 - b^2)$$

Hence the *slope and deflection at* any section are given by

Slope equation $EI\frac{dy}{dx} = \frac{Wb}{l}\frac{x^2}{2} - W\frac{(x-a)^2}{2} - \frac{Wb}{6l}(l^2 - b^2)$

Deflection equation $EIy = \frac{Wb}{l}\frac{x^3}{6} - W\frac{(x-a)^3}{6} - \frac{Wbx}{6l}(l^2 - b^2)$

Deflection under the load

To find y_C, putting x = a in the deflection equation, we have

$$EIy_C = \frac{Wba^3}{6l} - \frac{Wab}{6l}(l^2 - b^2) = -\frac{Wab}{6l}\left[l^2 - a^2 - b^2\right]$$

But, $l = a + b$

Therefore, $EIy_C = -\frac{Wab}{6l}\left[(a+b)^2 - a^2 - b^2\right] = -\frac{Wab}{6l}[2ab] = -\frac{Wa^2b^2}{3l} \Rightarrow y_C = -\frac{Wa^2b^2}{3EIl}$

The maximum deflection

The maximum deflection will occur, on the larger segment AC Moreover the slope is zero at the point of maximum deflection. Therefore, equating the slope at a section in A~ to zero we get,

$$0 = \frac{Wb}{l}\frac{x^2}{2} - \frac{Wb}{6l}(l^2 - b^2) \Rightarrow x = \sqrt{\frac{l^2 - b^2}{3}}$$

The maximum deflection can be obtained by putting this value of x in deflection equation

$$EIy_{\max} = \frac{Wb}{6l}\left(\frac{l^2 - b^2}{3}\right)^{3/2} - \frac{Wb}{6l}\left(l^2 - b^2\right)\left(\frac{l^2 - b^2}{3}\right)^{1/2} = -\frac{Wb\left(l^2 - b^2\right)^{3/2}}{\left(9\sqrt{3}\right)l}$$

Hence, $y_{\max} = -\dfrac{Wb\left(l^2 - b^2\right)^{3/2}}{\left(9\sqrt{3}\right)EIl}$

Problem 11: *A beam with a span of* 4.5 *metres carries a point load of 30 kN at 3tmetres from the left support. If for the section, Ixx* = 54.97 x 10^6 *mm^4 and E =1200 GPa, find:*

- *(i) The deflection under the load.*
- *(ii) The position and amount of maximum deflection.*

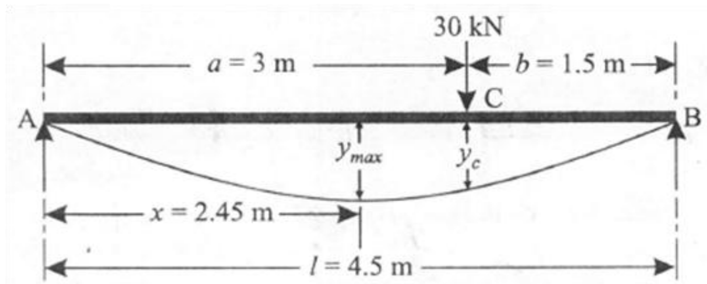


Fig. 11

Solution

Given: The span of the beam, l = 4500 mm; Point load = 30kN = 30000 N; a = 3000 mm; b = 1500 mm; Moment of inertia, **I** = 54·97 x 10^6 mm^4; E = 200 x10^3 N/mm^2;

The deflection under the load

$$y_C = -\frac{Wa^2b^2}{3EIl} = -\frac{\left(30x10^3\right)\left(3000^2\right)\left(1500^2\right)}{3\left(200x10^3\right)\left(54.97x10^6\right)\left(4500\right)} = -4.09\,mm$$

The position (x) and amount of maximum deflection

Position, $x = \sqrt{\dfrac{l^2 - b^2}{3}} = \sqrt{\dfrac{4500^2 - 1500^2}{3}} = 2450\ mm$

Maximum deflection, $y_{\max} = -\dfrac{Wb\left(l^2 - b^2\right)^{3/2}}{\left(9\sqrt{3}\right)EIl} = \dfrac{\left(30x10^3\right)\left(1500\right)\left[4500^2 - 1500^2\right]^{3/2}}{\left(9\sqrt{3}\right)\left(200x10^3\right)\left(54.97x10^6\right)\left(4500\right)} = 4.456\ mm$

Problem 12: *A steel girder of uniform section,* 14 *metres long is simply supported at its ends.* It *carries concentrated loads of 90 kN and* 60 *kN at two points* 3 *metres and* 4.5 *metres from the two ends respectively. Calculate:*

(i) *The deflection of the girder at the points under the two loads.*

(ii) The maximum deflection. Take: 1=64 x *10^8 mm^4 and E = 210 x10^3 N/mm^2.*

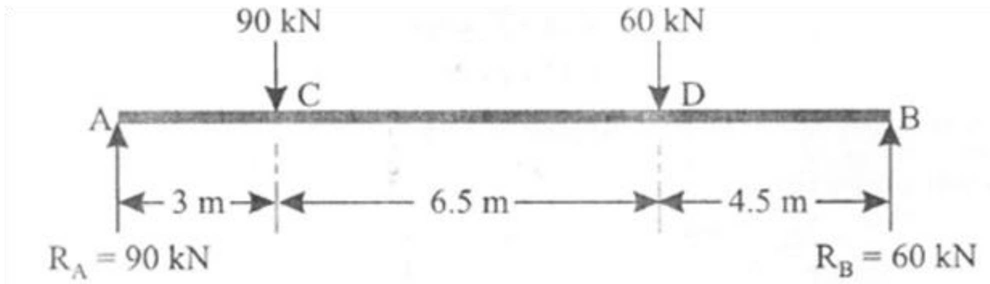


Fig 12

Solution

Given: Span of the steel girder, l= 14 x10^3 mm; Moment of inertia, I= 6.4 x 10^9 mm^4; Young's modulus E = 210 x 10^3 N/mm^2;

Let R_A and R_B be the reactions at the support A and B respectively.

Taking moments about A, we get

$$(14)R_B. = 90(3) + 60(9.5) = 840^3 \Rightarrow R_B = \frac{840}{14} = 60\ kN$$

Also, $R_A + R_B. = 90 + 60 = 150 \Rightarrow R_A = 150 - 60 = 90\ kN$

Consider any section XX at a distance x from the end A, following Macaulay's method, the bending moment is given by $M = EI\frac{d^2y}{dx^2} = 90x - 90(x-3) - 60(x-9.5)$

Integrating for slope, we get $EI\frac{dy}{dx} = 45x^2 - 45(x-3)^2 - 30(x-9.5)^2 + C_1$

Integrating again for deflection, we get

$$EIy = 15x^3 - 15(x-3)^3 - 10(x-9.5)^3 + C_1x + C_2$$

At A, deflection is zero, *i.e.* when x = 0, y = 0 and C_2 = 0

At B, deflection is zero, *i.e.* when x = *14x10*^{3}, y = 0,

$$0 = (15)(14)^3 - (15)(14-3)^3 - (10)(14-9.5)^3 + 14C_1$$

$$\Rightarrow C_1 = \frac{-20.3x10^3}{14} = -1448.84$$

Hence the deflection at any section are given by

$$EIy = 15x^3 - 15(x-3)^3 - 10(x-9.5)^3 + 1448.84$$

Deflection at C

Putting x = 3 m in the deflection equation, we get

$$EIy_C = \left[15(3)^3 - 1448.84(3)\right] = -3945 \Rightarrow y_C = \frac{-3945}{(210x10^6)(6.4x10^{-4})}x10^3 = -2.93\ mm$$

Deflection at D,

Putting $x = 9500$ mm in the deflection equation, we get

$$EIy_C = \left[15(9.5)^3 - 15(9.5-3)^3 - 1448.84(9.5)\right] = -5.03x10^{15} \Rightarrow$$

$$y_C = \frac{-5022.8}{\left(210x10^3\right)\left(6.4x10^9\right)}x10^3 = -3.75\ mm$$

Maximum deflection

Let us assume that the deflection will be maximum at a section between C and *D.* Equating slope at the section to zero, we get $0 = 45x^2 - 45(x-3)^2 - 1448.84$

Thus, $0 = 45x^2 - 45\left(x^2 - 6x + 9\right) - 1448.84 \Rightarrow 270x = 1853.84$

Therefore, $x = \frac{1853.84}{270} = 6.87\ m$

Putting this value of x in the deflection equation, we get

$$EIy_C = \left[15(6.87)^3 - 15(6.870-3)^3 - 14483.84(6.87)\right]x10^3 = -5959.3 \Rightarrow$$

$$y_C = \frac{5959.3}{\left(210x10^6\right)\left(6.4x10^{-4}\right)}x10^3 = -4.44\ mm$$

Problem 13: *A beam AB of* 4 *metres span is simply supported at the ends and is loaded as shown in Fig.* 13. *Determine (i) Deflection at* C,
ii) Maximum deflection and
(iii) Slope at the end A.
Given: E = 200 x *10*^{6} *kN/m*^{2}*, and 1* = 20 x 10^{-6} mm^4

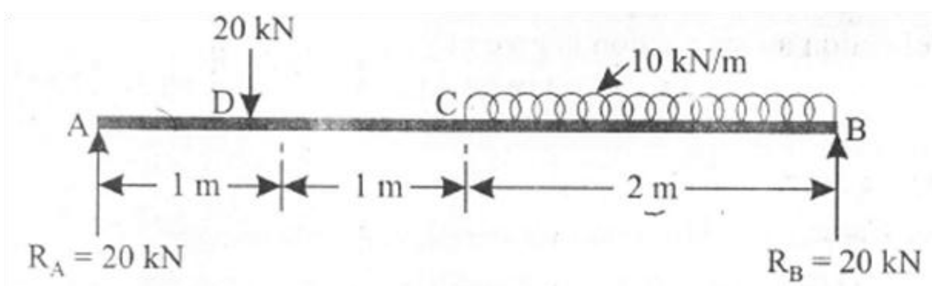


Fig 13

Solution

Given: Span of, the beam, $l = 4000$ mm; *E = 200* x *10*^{3} *N/mm*^{2}*; and I* = 20 x 10^6 mm^4;
Let R_A and R_B be the reactions at the support A and B respectively.

Taking moments about *A*, we get

$$14R_B. = (20)(1) + 10(2)(3) = 80 \Rightarrow R_B = \frac{80}{4} = 20\ kN$$

Also, $R_A + R_B. = 20 + 10(2) = 40 \Rightarrow R_A = 40 - 20 = 20\ kN$

Consider any section XX at a distance *x* from the end A, following Macaulay's method, the bending moment is given by $M = EI \dfrac{d^2y}{dx^2} = 20x - 20(x-1) - 5(x-2)^2$

Integrating for slope, we get $EI \dfrac{dy}{dx} = 10x^2 - 10(x-1)^2 - \dfrac{5}{3}(x-2)^3 + C_1$

Integrating again for deflection, we get

$$EIy = \frac{10}{3}x^3 - \frac{10}{3}(x-1)^3 - \frac{5}{12}(x-2)^4 + C_1x + C_2$$

At A, deflection is zero, *i.e.* when $x = 0$, y = 0 and $C_2 = 0$

At B, deflection is zero, *i.e.* when $x = 4x10^3$, y = 0,

$$0 = \frac{10}{3}(4)^3 - \frac{10}{3}(4-1)^3 - \frac{5}{12}(4-2)^4 + 4C_1 \Rightarrow C_1 = \frac{116.34}{4} = 29.16$$

Hence the slope and deflection at any section are given by

Slope, $EI \dfrac{dy}{dx} = 10x^2 - 10(x-1)^2 - \dfrac{5}{3}(x-2)^3 + 29.16$

Deflection, $EIy = \dfrac{10}{3}x^3 - \dfrac{10}{3}(x-1)^3 - \dfrac{5}{12}(x-2)^4 + 29.16$

Deflection at C

Putting $x = 2000$ mm in the deflection equation, we get

$$EIy_C = \left[\frac{10}{3}(2)^3 - \frac{10x10^3}{3}(2-1)^3 + 29.16(2) = -34.98\right]$$

$$\Rightarrow y_C = \frac{-34.98}{(200x10^6)(20x10^{-6})}x10^3 = -8.74\ mm$$

Maximum deflection

Let us assume that the deflection will be maximum at a section between C and *D.* Equating slope at the section to zero, we get $0 = 10x^2 - 10(x-1)^2 - 29.16$

Thus, $0 = 10x^2 - 10(x^2 - 2x + 1) - 29.16 \Rightarrow 20x = 39.16$

Therefore, $x = \dfrac{39.16}{20} = 1.958\ m$

Putting this value of x in the deflection equation, we get

$$EIy_C = \left[\frac{10}{3}(1.958)^3 - \frac{10}{3}(1.958-1)^3 + 29.16(1.958) = -35\right]$$

$$\Rightarrow y_C = \frac{-35}{(200x10^6)(20x10^{-6})}x10^3 = -8.75\ mm$$

Slope at the end A

Putting $x = 0$ in the slope-equation, we get

$$EI\theta_A = -29.16 \Rightarrow \theta_A = \frac{-29.16}{\left(200x10^6\right)\left(20x10^{-6}\right)} = -0.00729\ radian = -0.417^o$$

Problem 14: *A beam AB of span 8 metres is simply supported at the ends. It carries a uniformly distributed load of 30 kN/m over its entire length and a concentrated. Load of 60 kN at 3 metres from the support A. Determine the maximum deflection in the beam and the location where the deflection occurs. Take: 1 = 80 x 10^{-4} m^4; E = 200 x 10^6 kN/mm^2*

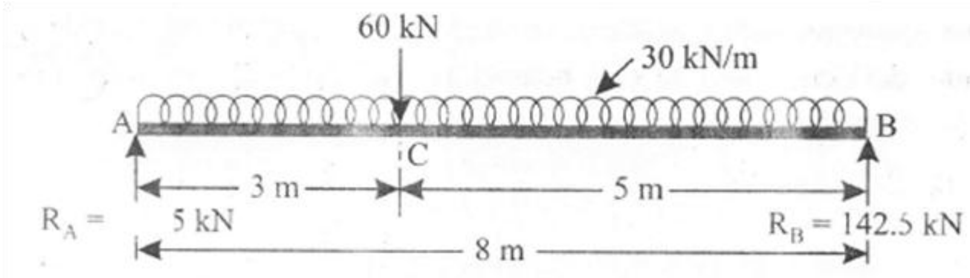


Fig 14

Solution

Given: Length of span of the beam, l= 8000 mm; Moment of inertia, *1* = 80 x 10^{-4} m^4; Young's modulus, E= 200 x 10^6 kN/mm^2

Taking moments about *A*, we get

$$8R_B. = 60(3) + 30(8)(4) = 1140 \Rightarrow R_B = \frac{1140}{4} = 142.5\ kN$$

Also, $R_A + R_B. = 60 + 30(8) = 300 \Rightarrow R_A = 300 - 142.5 = 157.5\ kN$

Consider any section XX at a distance *x* from the end A, following Macaulay's method, the bending moment is given by $M = EI\dfrac{d^2y}{dx^2} = 157.5x - 15x^2 - 60(x-3)$

Integrating for slope, we get $EI\dfrac{dy}{dx} = \dfrac{157.5}{2}x^2 - 5x^3 - 30(x-3)^2 + C_1$

Integrating again for deflection, we get $EIy = \dfrac{157.5}{6}x^3 - \dfrac{5}{4}x^4 - 10(x-3)^3 + C_1x + C_2$

At A, deflection is zero, *i.e.* when $x = 0$, y = 0 and $C_2 = 0$

At B, deflection is zero, *i.e.* when $x = 6x10^3$, y = 0,

$$0 = \frac{157.5}{6}(8)^3 - \frac{5}{4}(8)^4 - 10(8-3)^4 + 8C_1 \Rightarrow C_1 = -\frac{7070}{8} = -883.75$$

Hence the slope and deflection at any section are given by

$$Slope,\ EI\frac{dy}{dx} = \frac{157.5}{2}x^2 - 5x^3 - 30(x-3)^2 - 883.75$$

Deflection, $EIy = \frac{157.5}{6}x^3 - \frac{5}{4}x^4 - 10(x-3)^3 - 883.75x$

Maximum deflection

Let us assume that the deflection will be maximum at a section between C and *D*. Equating slope at the section to zero, we get $0 = 157.5x^2 - 5x^3 - 30(x-3) - 883.75$

Solving for x, x = 3.92 m

Putting this value of *x* in the deflection equation, we get

$$EIy_C = \frac{157.5}{6}(3.92)^3 - \frac{5}{4}(3.92)^4 - 10(3.92-3)^3 - 883.75(3.92) = -2186$$

$$\Rightarrow y_C = \frac{-2186}{(200x10^6)(80x10^{-4})}x10^{-3} = -1.366\ mm$$

TYPE B: OBJECTIVE

CHOOSE THE CORRECT OPTION

1. A simply supported beam carriers a point load at its centre. The slope at its supports is

   (a) $\frac{Wl^2}{16EI}$ (b) $\frac{Wl^3}{16EI}$

   (c) $\frac{Wl^2}{48EI}$ (d) $\frac{Wl^3}{48EI}$

2. A simply supported beam AB of span *(l)* carriers a point load *(W)* at Cat a distance *(a)* from the left end A, such that a< b. The maximum deflection is

   (a) at C

   (b) between A and C

   (c) between C and B

   (d) anywhere between A and B

3. A simply supported beam of span (*l*) is subjected to a uniformly distributed load of *(w)* per unit length over the whole span. The maximum deflections at the centre of the beam is

   (a) $\frac{5Wl^5}{48EI}$ (b) $\frac{5Wl^4}{96EI}$

   (c) $\frac{5Wl^4}{192EI}$ (d) $\frac{5Wl^3}{384EI}$

4. Two simply supported beams of the same span carry the same load. If the first beam carries the total load as a point load at its centre and the other uniformly distributed over the whole span, the ratio of maximum slopes of first beam to the second will be

   (a) 1:1 {b) 1: 1.5

   (c) 1.5:1 *(d)* 2:1

5. Maximum deflection of a cantilever beam of span *I* carrying a point load W at its free end is

   (a) $\frac{Wl^3}{2EI}$ (b) $\frac{Wl^3}{3EI}$

   (c) $\frac{Wl^3}{8EI}$ (d) $\frac{Wl^3}{16EI}$

6. The maximum slope of a cantilever carrying a point load at its; free end is at the
(a) fixed end (b) centre of span
(c) free end *(d)* none of these

A cantilever beam of span l carries a uniformly distributed load w over the entire span. Use the information to answer questions 7 and 8.

7. The maximum slope of the cantilever is.
(a) $\frac{wl^2}{3EI}$ (b) $\frac{wl^2}{4EI}$
(c) $\frac{wl^3}{6EI}$ (d) $\frac{wl^3}{8EI}$

8. Maximum deflection of a cantilever is equal to
(a) $\frac{wl^4}{2EI}$ (b) $\frac{wl^3}{3EI}$
(c) $\frac{wl^4}{8EI}$ (d) $\frac{wl^4}{16EI}$

9. The amount of deflection of a beam subjected to some type of loading depends upon
(a) cross-section
(b) bending moment
(c) either *(a)* and *(b)*
(d) both *(a)* and *(b).*

10. The slope and deflection at a section in a loaded beam can be found out by which of the following methods?
(a) Double integration method
(b) Moment area method
(c) Macaulay's method
(d) Any of the above.

11. The deflection at the free end of a cantilever of length l carrying a point load W *at* its free end is given as:
(a) $\frac{Wl^2}{2EI}$ (b) $\frac{Wl^2}{3EI}$
(c) $\frac{Wl^3}{2EI}$ (d) $\frac{Wl^3}{3EI}$

12. A cantilever of length l is carrying a uniformly distributed load of w per unit run over the whole span. The deflection at the free end is given as:
(a) $\frac{wl^3}{4EI}$ (b) $\frac{wl^2}{4EI}$
(c) $\frac{wl^4}{8EI}$ (d) $\frac{wl^4}{16EI}$

13. A cantilever of length l is carrying a uniformly distributed load of w per unit run for a distance *(a)* from fixed end. The slope at the free end is given as:
(a) $\frac{wa^3}{6EI}$ (b) $\frac{wa^3}{8EI}$
(c) $\frac{wa^3}{12EI}$ (d) $\frac{wa^3}{24EI}$

14. A cantilever AB of length l has a moment M applied at free end. The deflection at the free end B is given as
(a) $\frac{M^2 l}{EI}$ (b) $\frac{Ml^2}{2EI}$
(c) $\frac{Ml}{2EI}$ (d) $\frac{Ml^3}{2EI}$

15. A cantilever AB of length l is carrying a distributed load whose intensity varies uniformly from zero at the free end to w per unit run at the fixed end. The deflection at the free end is given as:
(a) $\frac{wl^2}{30EI}$ (b) $\frac{wl^3}{30EI}$
(c) $\frac{wl^4}{30EI}$ (d) $\frac{wl^5}{30EI}$

16. A cantilever AB of length l is carrying a distributed load whose intensity varies uniformly from zero at the fixed end to w per unit run at the 'free' end. The deflection at free end is given as:
(a) $\frac{wl^3}{48EI}$ (b) $\frac{wl^4}{30EI}$
(c) $\frac{6wl^4}{120EI}$ (d) $\frac{11wl^4}{120EI}$

17. A simply supported beam of span I is carrying a uniformly distributed load of w per unit run over the whole span. The maximum deflection in this case is given as:
(a) $\frac{wl^4}{48EI}$ (b) $\frac{wl^3}{30EI}$
(c) $\frac{5wl^4}{384EI}$ (d) $\frac{wl^4}{384EI}$

18. A simply supported beam of span I is carrying point load W at the mid span. What is the deflection at the centre of the beam?
(a) $\frac{Wl^2}{48EI}$ (b) $\frac{Wl^3}{48EI}$
(c) $\frac{5Wl^3}{384EI}$ (d) $\frac{11Wl^3}{120EI}$

A simply supported beam of span 2.4 m is subjected to a central point load of 15 kN. Take El for the beam as 6 x 10^{10} N-mm^2. Use the information to answer questions 19 and 20.

19. What is the maximum slope at the centre of the beam?
(a) 0.10 rad (b) 0.09 rad
(c) 0.11 rad (d) 0.08 rad

20. Determine the maximum deflection at the centre of the beam.
(a) 74 mm (b) 73 mm
(c) 71 mm (d) 72 mm

21. A beam simply supported at its both ends carries a uniformly distributed load 16 kN/m. If the deflection of the beam at its centre is limited to 2.5 mm; find the span of the beam. Take El for the beam as 9 x 10^{12} N-mm^2. [Ans. 3.22 m]
(a) 3.33 m (b) 3.22 m
(c) 3.44 m (d) 3.55 m

22. A horizontal beam of uniform section and 6 m long is simply supported at its ends. Two vertical concentrated loads of 48 kN and 40 kN act at 1 m and 3 m respectively from the left hand support. Determine the position and magnitude of the maximum deflection, if E = 200 GPa and I = 85 x 10^6 mm^4. [Ans. 16.75 mm] '
(a) 16.75 mm (b) 16.85 mm
(c) 16.65 mm (d) 16.95 mm

23. A composite beam consisting of two timber sections and centrally embedded steel plate, is supported over a span of 4 metres. It carries two concentrated loads of 20 kN each at points 1 m from each support. Find the deflection of the beam under each load. Take flexural rigidity of the beam as 13 x 10^{12} N-mm^2.
(a) 2.03 mm (b) 2.04 mm
(c) 2.02 mm (d) 2.05 mm

A cantilever 2.4 m long carries a point load of 30 kN at its free end. Take flexural rigidity for the cantilever beam as 25 x 10^{12} N-mm^2. Use the information to answer questions 24 and 25

24. Find the slope of the cantilever under the load.
    (a) 0.0025 rad (b) 0.0045 rad
    (c) 0.0055 rad (d) 0.0035 rad

25. Find the deflection of the cantilever under the load.
    (a) 5.6 mm (b) 5.4 mm
    (c) 5.5 mm (d) 5.7 mm

TYPE C: ESSAY

1. What is the relation between slope, deflection and radius of curvature of a simply supported beam?

2. A simply supported beam *AB* of span *l* and stiffness *El* carries a concentrated load *P* at its centre. Find the expression for slope of the beam at the support *A* and deflection of the beam at its centre.

3. Derive a relation for the slope and deflection of a simply supported beam subjected to a uniformly distributed load of w/m length.

4. Explain the procedure for finding out the deflection of a beam of composite section.

5. A cantilever beam of span *l* carries a uniformly distributed load *w* over the entire span. Compute the maximum slope and deflection of a cantilever.

Unit 2

COLUMNS AND STRUTS

Introduction

This unit discusses the derivation and computation of critical load in strut using the various empirical formulas.

Learning Objectives

After reading this unit you should be able to:

1. Define a strut
2. Derive the Euler's critical load equations for the various ended-joints
3. Compute the critical load using the Euler's load formulae
4. Derive the equation of critical load using the other methods
5. Compute the critical load using the other methods

IMPORTANT EQUATIONS

1. Euler's formula $P_E = \pi^2 EI \big/ L_e^2$.
2. Perry-Robertson's Formula,
$$\left[\frac{\sigma_{\max}}{\sigma_d} - 1\right]\left[1 - \frac{\sigma_d}{\sigma_E}\right] = \frac{\delta' y_c}{k^2}$$
3. Rankine Formula, $P_R = \dfrac{A\sigma_c}{1 + a(L_e/k)^2}$
4. Johnson's Formula,
$$P = A\left[\sigma_C - n\left(\frac{L_e}{k}\right)\right]$$
$$P = A\left[\sigma_C - r\left(\frac{L_e}{k}\right)^2\right]$$
5. Indian Standard Code for Columns,
$$\sigma_C = \sigma_C' = \frac{\sigma_y \big/ m}{1 + 0.20\sec\left(\dfrac{L_e}{k}\sqrt{\dfrac{mp_c'}{4E}}\right)}$$
$$\sigma_C = \sigma_C'\left(1.2 - \frac{L_e}{800k}\right)$$
6. Eccentric Loading
$$P = \frac{\sigma_c.\sigma_c}{\left(1 + \dfrac{e.y_e}{k^2}\right)\left[1 + a\left(\dfrac{L_e}{k}\right)^2\right]}$$

$$\sigma = \frac{P}{A} + \frac{M}{Z} = \frac{P}{A} + \frac{1}{Z}.P.e.\sec\left(\frac{L_e}{2}\sqrt{\frac{P}{EI}}\right)$$

TYPE A: SOLVED EXAMPLES

Problem 1: *A solid round bar* 60 *mm in diameter and* 2.5 *m long is used as a strut. One end of the strut is fixed, while its other end* is *hinged. Find the safe compressive load for this strut, using Euler's formula. Assume E = 200 GPa and factor of safety* = 3.

Solution

Given: Diameter of solid round bar, D= 60 mm; Modulus of elasticity, E = 200 GPa = 200 x 10^3 N/mm^2; Factor of safety, F. O. S. = 3; Length of round bar (strut), l= 2.5 m = 2500 mm; End conditions: *One end hinged, other fixed*

Safe compressive load

Effective length, $l_e = \frac{l}{\sqrt{2}} = \frac{2500}{\sqrt{2}} = 1768\ mm$

Moment of inertia, $I = \frac{\pi}{64}D^4 = \frac{\pi}{64}\left(60^4\right) = 636.17x10^3\ mm^4$

Euler's crippling load is given by the relation,

$$P_e = \pi^2 EI \big/ l_e^{\,2} = \frac{\pi^2\left(200x10^3\right)\left(636.17x10^3\right)}{1768^2}x10^{-3}\ kN = 401.74\ kN$$

Problem 2: *In an experimental determination of the buckling load for 1.2 cm diameter mild steel pin-ended struts of various lengths, two of the values obtained were:*

(i) When length = 50 cm the load = 10 kN, and

(ii) When length = 20 cm, the load = 30 kN.

Make the necessary calculations and then state whether either of the above values of loads conforms to the Euler's formula for the critical load. Take E = 200 GNlm^2.

Solution

Given: Diameter, D = 1.2 cm = 12 mm; Length (l_1) = 50 cm = 500 mm; load (P_1) = 10 kN = 10000 N; length (l_2) = 20 cm = 200 mm; load (P_2) =30 kN = 30000 N; Young's modulus of elasticity, *E*= 200 *GPa;* End conditions: *Both ends pin jointed.*

Moment of inertia, $I = \frac{\pi}{64}D^4 = \frac{\pi}{64}\left(12^4\right) = 1017.88\ mm^4$

Effective length, $l_e = l_1 = 500\ mm$

Using the relation, $P_1 = \pi^2 EI \big/ l_e^{\,2} = \frac{\pi^2\left(200x10^3\right)\left(1017.88\right)}{500^2}x10^{-3}\ kN = 8.04\ kN$

Similarly,

Effective length, $l_e = l_2 = 200\ mm$

Using the relation, $P_2 = \pi^2 EI \big/ l_e^{\,2} = \dfrac{\pi^2 (200x10^3)(1017.88)}{200^2} x10^{-3}\ kN = 50.24\ kN$

From the above two calculations, we find that the load in case *(i)* conforms approximately.

***Problem 3*:** *Calculate the safe compressive load on a hollow cast iron column (one end rigidly fixed and the other hinged) of 150 mm external diameter, 100 mm internal diameter and 10 m length. Use Euler's formula with a factor of safety of* 5, *and E* = 95 *GPa.*

Solution
Given: External diameter, *D* = 150 mm; internal diameter, *d* = 100 mm; length of the column, *l* = 10 m = 10000 mm; Factor of safety, F O. S. = 5; *E*= 95 *GPa = 95 x* 10^3 *N/mm*^{2}*;* End conditions: *One end rigidly fixed and the other hinged*

Safe compressive load

Effective length, $l_e = \dfrac{l}{\sqrt{2}} = \dfrac{10^4}{\sqrt{2}} = 7071\, mm$

Moment of inertia, $I = \dfrac{\pi}{64}\left[D^4 - d^4\right] = \dfrac{\pi}{64}\left[(150^4) - (100^4)\right] = 19.94x10^6\ mm^4$

Using the relation, $P_e = \pi^2 EI \big/ l_e^{\,2} = \dfrac{\pi^2 (95x10^3)(19.94x10^6)}{(7071)^2} x10^{-3}\ kN = 373.92\ kN$

Safe load, $P_s = \dfrac{P_e}{FOS} = \dfrac{373.92}{5} = 74.78\ kN$

***Problem·4*:** *A slender pin ended aluminium column* 1.8 *m long and of circular cross-section is to have an outside diameter of 50 mm. Calculate the necessary internal diameter to prevent failure by buckling if the actual load applied is* 13·6 *kN and the critical load applied is twice the actual load. Take E for aluminium as 70 GPa*
2.
Solution
Given: Outside diameter of the column, *D* = 50 mm; length of the column, *l* = 1.8 m = 1800 mm; P_a = 13.6 kN = 13600 N; *E*= 70 *GPa = 70 x* 10^3 *N/mm*^{2}*;* End conditions: *Pin-ended*

Inside diameter of the column

Area, $I = \dfrac{\pi}{4}\left[D^2 - d^2\right] = \dfrac{\pi}{64}\left[(50^4) - (d^2)\right] mm^2$

Moment of Inertia, $I = \dfrac{\pi}{64}\left[D^4 - d^4\right] = \dfrac{\pi}{64}\left[(50^4) - (d^4)\right] mm^4$

Also, critical load $P_c = 2P_a = 2(13.6) = 27.2\ kN$

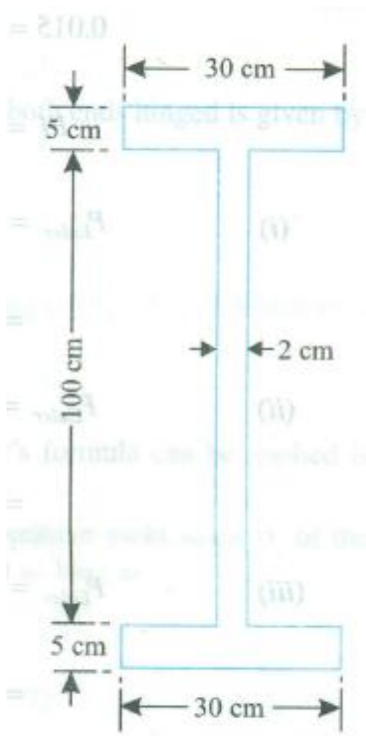
Effective length, $l_e = l = 1800\ mm$

Using the relation,

$$P_e = \pi^2 EI \big/ l_e^2 \Rightarrow 27.2x10^3 = \frac{\pi^2 \left(70x10^3\right) \frac{\pi}{64} \left[\left(50^4\right) - \left(d^4\right)\right]}{(1800)^2} = (0.0104)\left(50^4 - d^4\right)$$

Therefore, $d^4 = 50^4 - 2.62x10^6 = 3.63x10^6 \Rightarrow d = \sqrt[4]{3.63x10^6} = 43.64\ mm$

Problem 5: *A built-up beam shown in Fig. 1 is simply supported at its ends. Compute the length given that when it is subjected to a load of 40 kN per metre length, it deflects by 1 cm. Find out the safe load, if this beam is used as a column with both ends fixed. Assume a factor of safety of 4. Use Euler's formula. E = 210 GPa.*



Solution

Given: Load, w = 40 kN/m = 40 N/mm; E= 210 GPa = 210 x 10^3 N/mm^2; F O S = 4; δ = 1 cm = 10 mm; End conditions: *Fixed ended*

Length of beam

Moment of inertia of section about *x-x* axis,

$$I_{XX} = \frac{1}{12}\left[300\left(1100^3\right) - 280\left(1000^3\right)\right] = 9.94x10^9\ mm^4$$

Using the relation,

$$\delta = \frac{5wl^4}{384EI} \Rightarrow l^4 = \frac{384\delta EI}{5w} = \frac{384(10)\left(210x10^3\right)\left(9.94x10^9\right)}{5\left(40x10^3\right)}$$

Therefore, $l^4 = 4x10^{16} \Rightarrow l = \sqrt[4]{4x10^{16}} = 14.15x10^3\ mm$

Safe load, the beam can carry as a column

Effective length, $l_e = \frac{l}{2} = \frac{14150}{2} = 7074.5\ mm$

Moment of inertia about Y - Y axis

$$I_{YY} = 2\left[\frac{50\left(300^3\right)}{12} + \frac{(1000)\left(20^3\right)}{12}\right] = 225x10^6\ mm^4$$

Using the relation,

$$P_e = \pi^2 EI \big/ l_e^2 = \frac{\pi^2\left(210x10^3\right)\left(225x10^6\right)}{(7074.5)^2} x10^{-3}\ kN = 9330\ kN$$

$$\text{Safe load, } P_s = \frac{P_e}{FOS} = \frac{9330}{4} = .2332.5\ kN$$

Problem·6: *A bar of length 4 m when used as a simply supported beam and subjected to a uniform distributed load of 30 kN/m over the whole span, deflects 15 mm at the centre. Determine the crippling load when it is used as a column with following end conditions:*

(i) Both ends pin-jointed;
(ii) One end fixed and other end hinged;
(iii) Both ends fixed.

Solution

Given: Length of the bar, l=4m = 4000 mm; Uniformly distributed load, w= 30kN/m =30 N/mm; Deflection, δ = 15 mm;

We know that, $\delta = \frac{5wl^4}{384EI} \Rightarrow EI = \frac{5wl^4}{384\delta} = \frac{5(30)(4000)^4}{384(15)} = 6.67x10^{12}$

Both ends pin-jointed;

Effective length, $l_e = l = 4000\ mm$

Using the relation, $P_e = \pi^2 EI \big/ l_e^{\,2} = \frac{\pi^2 (6.67x10^{12})}{(4000)^2} x10^{-3}\ kN = 4114\ kN$

One end fixed and other end hinged;

Effective length, $l_e = \frac{l}{\sqrt{2}} = \frac{4000}{\sqrt{2}} = 2828\ mm$

Using the relation, $P_e = \pi^2 EI \big/ l_e^{\,2} = \frac{\pi^2 (6.67x10^{12})}{(2828)^2} x10^{-3}\ kN = 8231\ kN$

Both ends fixed.

Effective length, $l_e = \frac{l}{2} = \frac{4000}{2} = 2000\ mm$

Using the relation, $P_e = \pi^2 EI \big/ l_e^{\,2} = \frac{\pi^2 (6.67x10^{12})}{(2000)^2} x10^{-3}\ kN = 16457\ kN$

Problem 7: *Calculate the critical load of a strut which* is *made of a bar circular in section of 5 m long and which* is *pin-jointed at both ends. The same bar when freely supported gives mid-span deflection of 10 mm with a load of 80 N at the centre.*

Solution

Given: Length of the bar, l = 5 m = 5000 mm; Uniformly distributed load, P = 80 N; Deflection, δ = 10 mm; End condition, *pin-jointed at both ends*

Effective length, $l_e = l = 5000\ mm$

For deflection, $\delta = \frac{Wl^3}{48EI} \Rightarrow \frac{EI}{l^2} = \frac{Wl}{48\delta}$

We know that $P_e = \pi^2 EI \big/ l_e^{\,2} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 Wl}{48\delta} = \frac{\pi^2 (80)(5000)}{48(10)} x10^{-3}\ kN = 8.22\ kN$

Problem 8: *Calculate the maximum value of the slenderness ratio of a mild steel column which Euler's formula* is *valid. Take σ_c =330 MPa and E = 210GPa*

Solution

The buckling or crippling load P_{cr} for a column with both ends hinged is given by:

$$P_e = \pi^2 EI \Big/ {l_e}^2 = \frac{\pi^2 EI}{l^2} \quad (where\ l_e = l) \qquad \text{(i)}$$

Also, $I = Ak^2 \quad and \quad \sigma_{cr} = \frac{P_{cr}}{A}$ (ii)

Substituting (ii) into (i) and simplifying, we have $\sigma_{cr} = \frac{\pi^2 E}{(l/k)^2}$

Euler's formula for *hinged* ends will apply if $\sigma_{cr} = \frac{\pi^2 E}{(l/k)^2} \le \sigma_c = 330$

Therefore, $\frac{\pi^2 E}{(l/k)^2} \le 330 \Rightarrow \left(\frac{l}{k}\right)^2 \ge \frac{\pi^2 E}{330}$

Thus, $\left(\frac{l}{k}\right)^2 \ge \frac{\pi^2 (210x10^3)}{330} \Rightarrow \frac{l}{k} \ge 80$

Hence for a steel column with *hinged ends*, Euler's formula is *valid for* $\frac{l}{k} \ge 80$

Problem·9: *Determine the ratio of the buckling strengths of two columns of circular cross-section one hollow and other solid when both are made of the same material, have the same length and cross-sectional area and end-conditions. The internal diameter of the hollow column is half of the external diameter.*

Solution

Let D_S = Diameter of the solid column,
D_H = External diameter of the hollow column,
d_H = Internal diameter of the hollow column,
A_S = Area of cross-section of the solid column,
A_H = Area of cross-section of the hollow column,
P_S = Buckling load of the solid column, and
P_H = Buckling load of hollow column.
$As = A_H$ *(Given)*
$d_H = 0.5D_H$

$$\frac{\pi}{4} D_S^2 = \frac{\pi}{4}\left[D_H^2 - d_H^2\right] = \frac{\pi}{4}\left[D_H^2 - (0.5D_H)^2\right] \Rightarrow D_S^2 = 0.75D_H^2 \quad or\ D_s = 0.866D_H$$

Solid column:

Moment of inertia, $I = \frac{\pi}{64} D_S^4 = \frac{\pi}{64}(0.866D_H)^4 = 0.0276D_H^4$

Using the relation, $P_S = \pi^2 EI \Big/ {l_e}^2 = \frac{\pi^2 E(0.0276D_H^4)}{{l_e}^2}$

Hollow column:

Moment of inertia, $I = \frac{\pi}{64}\left[D_H^4 - (0.5D_H)^4\right] = 0.046 D_H^4$

Using the relation, $P_H = \pi^2 EI \big/ {l_e}^2 = \frac{\pi^2 E(0.046 D_H^4)}{{l_e}^2}$

Since end conditions are same in both cases, therefore, l_e will be same. Therefore, the ratio of buckling strengths of the two columns,

$$\frac{P_H}{P_S} = \left[\frac{\pi^2 E(0.046 D_H^4)}{{l_e}^2}\right]\left[\frac{{l_e}^2}{\pi^2 E(0.0276 D_H^4)}\right] = 1.66$$

Problem 10: *A hollow CI column whose outside diameter is 200 mm has a thickness of 20 mm. It is 4.5 m long and is fixed at both ends. Calculate the safe load by Rankine-Gordon formula using a factor of safety of 4. Take σ_c = 550 MPa; a = 1/1600.*

Solution

Given: Outside diameter of the column, D=200 mm; Thickness of the column, t = 20mm; Inside diameter of the column, d = D-2t = 160 mm; Length of the column, l = 4500 mm; Factor of safety, (F. O. S) = 4;

Area of the column, $A = \frac{\pi}{4}\left[D^2 - d^2\right] = \frac{\pi}{4}\left[200^2 - 160^2\right] = 11.3x10^3\ mm^2$

Moment of inertia, $I = \frac{\pi}{64}\left[D^4 - d^4\right] = \frac{\pi}{64}\left[200^4 - 160^4\right] = 46.37x10^6\ mm^4$

k: $k^2 = \frac{I}{A} = \frac{46.37x10^6}{11.3x10^3} = 4103.5$

End conditions: *Both ends fixed,* $l_e = \frac{l}{2} = \frac{4500}{2} = 2250\ mm$

Using the relation, $P_R = \frac{\sigma_C.A}{1 + a.\left({l_e}^2 \big/ k^2\right)} = \frac{550(11.3x10^3)}{1 + \frac{1}{1600}\left(\frac{2250^2}{4103.5}\right)} x10^{-6} MN = 3.51\, MN$

Safe load, $P_s = \frac{P_R}{FOS} = \frac{3.51}{4} = 877\ kN$

Problem·11: *Compare the crippling loads given by Rankine's and Euler's formulae for tubular strut* 2·25 *m long having outer and inner diameters of* 37·5 *mm and* 32·5 *mm loaded through pin-joint at both ends. Take: Yield stress as* 315 *MNlm^2, a =1/7500 and E = 200 GPa. If elastic limit for the material is taken as 200 MPa^2, then for what length of the strut Euler formula cease to apply?*

Solution

Given: Outer diameter of strut, D = 37·5 mm; Inner diameter of the strut, d = 32·5 mm; Length of the strut, l =2250 mm; End condition: *both ends pin-jointed;* Yield stress σ_c = 315 MPa = 315 N/mm^2; Rankine constant, $a = 1/7500$

Comparison of loads

Moment of inertia, $I = \frac{\pi}{64}\left[D^4 - d^4\right] = \frac{\pi}{64}\left[37.5^4 - 32.5^4\right] = 42.3x10^3\ mm^4$

Effective length, $l_e = l = 2250\ mm$

Euler s crippling load, P Euler, $P_e = \pi^2 EI \big/ l_e^{\,2} = \frac{\pi^2 (200x10^3)(42.3x10^3)}{2250^2} x10^{-3}\, kN = 16.5\ kN$

Rankine s crippling load, P Rankine

Area, $A = \frac{\pi}{4}\left[D^2 - d^2\right] = \frac{\pi}{4}\left[37.5^2 - 32.5^2\right] = 275\ mm^2$

k: $k^2 = \frac{I}{A} = \frac{42.3x10^3}{275} = 154$

Using the relation: $P_R = \frac{\sigma_C . A}{1 + a.\left(l_e^{\,2} \big/ k^2\right)} = \frac{550(275)}{1 + \frac{1}{7500}\left(\frac{2250^2}{154}\right)} x10^{-3}\, kN = 16.08\ kN$

Ratio; $\frac{P_e}{P_R} = \frac{16.5}{16.08} = 1.026$

Length of Strut

Now, $\sigma_e = \frac{P_e}{A} = \pi^2 EI \big/ A l_e^{\,2} = \frac{\pi^2 EAk^2}{Al_e^2} \Rightarrow l_e^{\,2} = \frac{\pi^2 Ek^2}{\sigma_e} = \frac{\pi^2 (200x10^3)(154)}{200} = 1.52x10^6$

Hence, $l_e = \sqrt{1.52x10^6} = 1233\ mm$

Problem 12: *A 1·5 m long CI column has a circular cross-section of 5 cm diameter. One end of the column is fixed in direction and position and the other is free. Taking factor of safety as 3, calculate the safe load, using*

(i) *Rankine-Gordon formula; take yield stress 560 MNlm2, and a = 1/1600 for pinned end*

(ii) Euler's formula. Young's modulus for CI = 120 GNlm^2. n

Solution

Given: D = 50 mm; l = 1500 mm; F O S = 3

Area, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}\left[50^2\right] = 1963.5\ mm^2$

Moment of inertia, $I = \frac{\pi}{64} D^4 = \frac{\pi}{64}\left[5^4\right] = 306.8x10^3\ mm^4$

Safe load by Rankine-Gordon formula

Yield stress, σ_c = 560 N/mm^2; Rankine's constant, a = 1/1600;

End conditions: *One end fixed in direction and position and the other free.*
Effective length, $l_e = 2l = 2(1500) = 3000\ mm$

k: $k^2 = \frac{I}{A} = \frac{306.8x10^3}{1963.5} = 156$

Using the relation, $P_R = \frac{\sigma_C . A}{1 + a.\left(l_e^2 / k^2 \right)} = \frac{560(1963.5)}{1 + \frac{1}{1600}\left(\frac{3000^2}{156} \right)} x10^{-3}\, kN = 29.72\ kN$

Safe load, $P_s = \frac{P_R}{FOS} = \frac{29.72}{3} = 9.9\ kN$

Safe load by Euler's formula
Young's modulus, E = 120 x10^3 N/mm^2
Using the relation, $P_e = \pi^2 EI / l_e^2 = \frac{\pi^2 (120x10^3)(306.8x10^3)}{3000^2} x10^{-3}\, kN = 40.4\ kN$

Safe load, $P_s = \frac{P_e}{FOS} = \frac{40.4}{3} = 13.47\ kN$

Problem ·13: *A hollow cylindrical cast iron column is* 4 *m long with both ends fixed Determine the minimum diameter of the column, if it has to carry a safe load of 250kN with of safety of* 5. *Take the internal diameter as 0·8 times the external diameter. Take: a = 1/1600 in Rankine's formula and σ_c = 550 MPa*

Solution
Given: Inner diameter of the strut, d = 0.8D; Length of the strut, l =4000 mm; End condition: *both ends fixed;* Yield stress σ_C = 550 MPa = 550 N/mm^2; Rankine constant, a = 1/1600

Minimum diameter of the column

Area, $A = \frac{\pi}{4}\left[D^2 - d^2\right] = \frac{\pi}{4}\left[D^2 - (0.8D)^2\right] = 0.283D^2$

Moment of inertia, $I = \frac{\pi}{64}\left[D^4 - d^4\right] = \frac{\pi}{64}\left[D^4 - (0.8D)^4\right] = 0.029D^4$

k: $k^2 = \frac{I}{A} = \frac{0.029D^4}{0.28D^2} = 0.102D^2$

Effective length, $l_e = \frac{l}{2} = \frac{4000}{2} = 2000\ mm$

Crippling load, $P_R = P_s\ x\, FOS = 250x5 = 1250\ kN = 1.25x10^6\ N$

Therefore, $P_R = \dfrac{\sigma_C.A}{1+a.\left(l_e^2/k^2\right)} \Rightarrow 1.25x10^6 = \dfrac{550\left(0.283D^2\right)}{1+\dfrac{1}{1600}\left(\dfrac{2000^2}{0.102D^2}\right)} = \dfrac{156D^2}{1+\dfrac{24510}{D^2}}$

Thus, $= \dfrac{D^4}{D^2+24510} = 8013 \Rightarrow D^4 - 8013D^2 - 196.4x10^6 = 0$

Hence, $= D^2 = \dfrac{8013 \pm \sqrt{8013^2 + 4\left(196.4x10^6\right)}}{2} = 18582 \Rightarrow D = \sqrt{18582} = 136.3\, mm$

and $d = 0.8D = 0.8(136.3) = 109\, mm$

Problem·14: *From the following data, determine thickness of cast-iron column:*
Length of column = 6 metres
External diameter = 200 mm
Load = 500 kN
Factor of safety = 6
Assume fixed ends and ultimate compressive stress and constant for hinged ends as 570 MPa and 1/1600 respectively.

Solution

Given: External diameter of the column, D = 200 mm; length, l = 6000 mm; Load P = 500 kN= 500 x 10^3 N; Factor of safety, F O S = 6; σ_c = 570 MPa = 570 N/mm^2; a = 1/1600

Thickness of the column

Area, $A = \dfrac{\pi}{4}\left[D^2 - d^2\right] = 0.785\left[200^2 - d^2\right]$

Moment of inertia, $I = \dfrac{\pi}{64}\left[D^4 - d^4\right] = 0.049\left[200^4 - d^4\right]^4$

k: $k^2 = \dfrac{I}{A} = \dfrac{0.049\left[200^4 - d^4\right]}{0.785\left[200^2 - d^2\right]} = 0.06\left(200^2 + d^2\right)$

Effective length, $l_e = \dfrac{l}{2} = \dfrac{6000}{2} = 3000\, mm$

Crippling load, $P_R = P_s\, x\, FOS = 500x6 = 3000\, kN = 3x10^6\; N$

Therefore, $P_R = \dfrac{\sigma_C.A}{1+a.\left(l_e^2/k^2\right)} \Rightarrow 3x10^6 = \dfrac{570(0.785)\left(200^2 - d^2\right)}{1+\dfrac{1}{1600}\left(\dfrac{3000^2}{0.06\left(200^2+d^2\right)}\right)} = \dfrac{447.45\left(200^2 - d^2\right)}{1+\dfrac{90000}{200^2+d^2}}$

Thus, $= \dfrac{1.6x10^9 - d^4}{d^2 + 13000} = 6700 \Rightarrow d^4 + 6704.6d^2 - 729x10^6 = 0$

Hence, $= d^2 = \dfrac{-6700 \pm \sqrt{6700^2 + 4\left(729x10^6\right)}}{2} = 23857 \Rightarrow D = \sqrt{23857} = 154.5\ mm$

and $t = \dfrac{D-d}{2} = \dfrac{200-154.5}{2} = 22.8\ mm$

Problem 15: *Find the Euler crippling load for a hollow cylindrical cast-iron column 200 mm external diameter,* 25 *mm thick,* 6 *m long and hinged at both ends. E =* 120 *GPa. Compare the load with the crippling load as given by the Rankine formula taking σ_c = 550 MPa^2, and a = 1/1600. What length of column would two formulae give the same crushing load?*

Solution

Given: External diameter, D = 200 mm; Thickness, t = 25 mm; Internal diameter of the column, d = *D* - *2t* = 150 mm; Length of the column, *l* = 6000 mm; End conditions: *Both ends hinged;* σ_c = 550 MPa = 550 N/mm^2; a = 1/1600; E = 120 GPa = 120x10^3 N/mm^2;

Area, $A = \dfrac{\pi}{4}\left[D^2 - d^2\right] = \dfrac{\pi}{4}\left[200^2 - 150^2\right] = 13.7x10^3\ mm^2$

Moment of inertia, $I = \dfrac{\pi}{64}\left[D^4 - d^4\right] = \dfrac{\pi}{64}\left[200^4 - 150^4\right] = 53.7x10^6\ mm^4$

k: $k^2 = \dfrac{I}{A} = \dfrac{53.7x10^6}{13.7x10^3} = 3920\ mm^2$

Effective length, $l_e = l = 6000\ mm$

Euler crushing load

Using the relation, $P_e = \pi^2 EI \big/ l_e^2 = \dfrac{\pi^2\left(120x10^3\right)\left(53.7x10^6\right)}{6000^2} x10^{-3} kN = 1766\ kN$

Rankine crushing load

We know, $P_R = \dfrac{\sigma_C . A}{1 + a.\left(l_e^2 \big/ k^2\right)} = \dfrac{550\left(13.7x10^3\right)}{1 + \dfrac{1}{1600}\left(\dfrac{6000^2}{3920}\right)} x10^{-3} kN = 1116.7\ kN$

$\dfrac{P_e}{P_R} = \dfrac{1766.7}{1116.7} = 1.58$

i.e. Euler load is 1·58 times *Rankine load.*

Effective length

For the same crushing load with both formulae,

$$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EAk^2}{l_e^2} = \frac{\sigma_C . A}{1 + a.\left(l_e^2 / k^2 \right)} \Rightarrow l_e^2 = \frac{\pi^2 Ek^2}{\sigma_c - \pi^2 Ea} = \frac{\pi^2 (120x10^3)(3920)}{550 - \pi^2 (200x10^3)(1/1600)} = -24.3$$

Since $l_e^2 = -24.3$ is negative, then, under given conditions there will be no column to give same crushing load with formulae.

Problem 16: *A short length of tube 3 cm internal and 5 cm external diameter, failed at a compression at a load of 240 kN. When a 2 m length of the same tube was tested as a strut with fixed ends, the load at the failure was 158 kN. Assuming that σ_C in Rankine's formula is given by the failed test and the value of constant 'a' in the same formula what will be the crippling load of this tube if it is used as a strut 3 metres long with one end fixed and the other end hinged?*

Solution

Given: External diameter, D = 50 mm; Thickness, t = 25 mm; Internal diameter of the column, d = 30 mm; Length of the column, l = 2000 mm; W = 240 x 10^3 N; P = 158 x 10^3 N;

Area, $A = \frac{\pi}{4}\left[D^2 - d^2\right] = \frac{\pi}{4}\left[50^2 - 30^2\right] = 1257\ mm^2$

Moment of inertia, $I = \frac{\pi}{64}\left[D^4 - d^4\right] = \frac{\pi}{64}\left[50^4 - 30^4\right] = 267x10^3\ mm^4$

k: $k^2 = \frac{I}{A} = \frac{267x10^3}{1257} = 212.44\ mm^2$

Yield strength

We know, $\sigma_c = \frac{W}{A} = \frac{240000}{1257} = 190.9\ N/mm^2$

Rankine's constant (a) when both ends are fixed

Effective length, $l_e = \frac{l}{2} = \frac{2000}{2} = 1000\ mm$

$$P_R = \frac{\sigma_C . A}{1 + a.\left(l_e^2 / k^2 \right)} \Rightarrow 1.25x10^6 = \frac{190.9(1257)}{1 + a\left(\frac{1000^2}{212.44}\right)} = \frac{240000}{1 + 4707.2a}$$

Therefore, $a = \frac{1}{4707.2}\left[\frac{240000}{158x10^3} - 1\right] = \frac{0.52}{4707.2} = \frac{1}{9070}$

Crippling load (P) when one end is fixed and other hinged

Effective length, $l_e = \frac{l}{\sqrt{2}} = \frac{3000}{\sqrt{2}} = 2121\ mm$

$$P_R = \frac{\sigma_C.A}{1+a.\left(l_e^2 \big/ k^2 \right)} = \frac{191(13.7x10^3)}{1+\frac{1}{9070}\left(\frac{2121^2}{212.44}\right)} x10^{-3} kN = 72\ kN$$

Problem 17: *Find the greatest length for which a mild steel strut of T-shaped cross-section, having an area a of 30 cm² and the least moment of inertia of which is 240 cm², may be used with one end fixed and other entirely free in order to carry a working load of 70 MPa of section, the working load being one-fourth of the crippling load. Rankine constants for mild steel are: a = 1/7500 and σ_c = 330 MPa.*

Solution

Given: Area A = 30 cm² = 30 x 10² mm²; D = 200 mm; least moment of inertia I = 240 cm² = 240 x 10⁴ mm⁴; σ_{cR} = 70 MPa = 70 N/mm² End conditions: *one end fixed and other entirely free;* σ_c = 330 MPa = 330 N/mm²; a = 1/7500

Crippling load

We know, $P_R = \sigma_{cR}.A.FOS = 70x3000x4x10^{-3} kN = 840\ kN$

Rankine's constant (a) when both ends are fixed 0

k: $k^2 = \frac{I}{A} = \frac{240x10^4}{3000} = 800\ mm^2$

$$P_R = \frac{\sigma_C.A}{1+a.\left(l_e^2 \big/ k^2 \right)} \Rightarrow 840x10^3 = \frac{330(3000)}{1+\left(\frac{1}{7500}\right)\left(\frac{l_e^2}{800}\right)} = \frac{990000}{1+ l_e^2 \big/ 6000000} = \frac{5.94x10^{12}}{6x10^6 + l_e^2}$$

Therefore, $l_e^2 = \frac{5.95x10^{12}}{840x10^3} - 6x10^6 = 1.07x10^6 \Rightarrow l_e = \sqrt{1.07x10^6} = 1035\ mm$

We know that for one end fixed and the other free, then, $l_e = 2l \Rightarrow l = \frac{l_e}{2} = \frac{1035}{2} = 517.5\ mm$

Problem18 *From the following data, determine the diameter of the piston rod.*
Diameter of the engine cylinder = 0-3 m
Maximum effective steam pressure in the cylinder = 800 kNlm²
Distance from piston to cross-head centre = 1·5 m.
Factor of safety = 4
Assume σ_c =330 MPa; a = 1/30000 for both ends fixed.

Solution

Given: Diameter of the engine cylinder, D= 0.3 m = 3000 mm; Maximum effective steam pressure in the cylinder = 800 kN/m² = 0.8 N/mm²; Distance from piston to cross-head centre *l* = 1·5 m = 1500 mm; Factor of safety, F O S = 4; σ_c =330 MPa; a = 1/30000; End condition: *both ends fixed*

Diameter of the piston rod

Area, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}\left[300^2\right] = 70.7x10^3\ mm^2$

Moment of inertia, $I = \frac{\pi}{4} D^4 = \frac{\pi}{4}\left[300^4\right] = 397.6x10^6\ mm^4$

Maximum load on the piston, $P_s = p.A. = 0.8\left(70.7x10^3\right)x10^{-3} = 56.55\ kN$

Crippling load, $P_R = P_s.FOS = 56.55x4 = 226.2\ kN$

Effective length, $l_e = l = 1500\ mm$

k: $k^2 = \frac{I}{A} = \frac{(\pi/64)d^4}{(\pi/4)d^2} = \frac{d^2}{16}$

Area, $A = \frac{\pi}{4} d^2$

Using the relation,

$$P_R = \frac{\sigma_C .A}{1 + a.\left(l_e^2 \big/ k^2\right)} \Rightarrow 226.2x10^3 = \frac{330(\pi/4)d^2}{1 + \left(\frac{1}{30000}\right)\left(\frac{1500^2(16)}{d^2}\right)} = \frac{259.2d^2}{1 + \frac{1200}{d^2}} = \frac{259.2d^4}{d^2 + 1200}$$

Therefore, $\frac{226.2x10^3}{259.2} = \frac{d^4}{d^2 + 1200} \Rightarrow d^4 - 873d^2 - 105x10^6$

Then, $d^2 = \frac{873 \pm \sqrt{873^2 + 4\left(105x10^6\right)}}{2} = 1549 \Rightarrow d = \sqrt{1549} = 39.4\ mm$

Problem 19: *From the following data of a column of circular section calculate the stresses on the column section. Also find the maximum eccentricity in order that there may tension anywhere on the section. External diameter = 20 cm Internal diameter = 16 cm Length of the column = 4 m*
Load carried by the column = 200 kN Eccentricity of the load = 2.5 cm (from the axis of the column) End conditions = Both ends fixed Young's modulus = 94 GPa

Solution

Given: External diameter D = 20 cm = 200 mm; Internal diameter d = 16 cm = 160 mm; Length of the column l = 4 m =4000 mm; Load, P = 200 kN = 200 x 10^3 N; Eccentricity of the load, e = 2.5 cm (from the axis of the column); y = 200/2 = 100 mm; End conditions = Both ends fixed Young's modulus, E = 94 GPa = 94 x 10^3 N/mm^2;

Area of the column, $A = \frac{\pi}{4}\left[D^2 - d^2\right] = \frac{\pi}{4}\left[200^2 - 160^2\right] = 11310\ mm^2$

Moment of Inertia, $I = \frac{\pi}{4}\left(D^4 - d^4\right) = \frac{\pi}{4}\left[200^4 - 160^4\right] = 46.37x10^6\ mm^4$

Effective length, $l_e = \frac{l}{2} = \frac{4000}{2} = 2000\ mm$

The angle, $\alpha = \frac{l_e}{2}\sqrt{\frac{P}{EI}} = \frac{2000}{2}\sqrt{\frac{200x10^3}{\left(94x10^3\right)\left(46.37x10^6\right)}} = 0.212\ rad = 12.27^o$

Maximum bending moment; $M = P.e\sec\alpha = \left(200x10^3\right)(25)\sec 12.27 = 5.1x10^6\ Nmm$

Maximum compressive stress, $\sigma_{\max} = \frac{P}{A} + \frac{My}{I} = \frac{200x10^3}{11310} + \frac{\left(5.1x10^6\right)(100)}{46.37x10^6} = 28.7\ N/mm^2$

For no tension (corresponding to the maximum eccentricity)

$$\frac{P}{A} = \frac{M}{I/y} = \frac{P.e\sec\alpha}{\frac{I}{y}} \Rightarrow e = \frac{PI}{AyP\sec\alpha} = \frac{I}{Ay\sec\alpha} = \frac{46.37x10^6}{11310(100)(1.02)} = 40.2\ mm$$

Problem·20: *Fig.* 2 *shows a compound stanchion made up of two channels ISJC 200 weighing 139 N per metre per channel and two* 25 *cm* x 1 *cm plate riveted one to each flange. If the maximum permissible stress is* 70 *MPa, find the maximum eccentricity of a 300 kN load from the* YY-axis *of the column. The load line lies in the vertical plane through the XX-axis. Take: E = 200 GPa, the effective length column being* 3 *metres.*

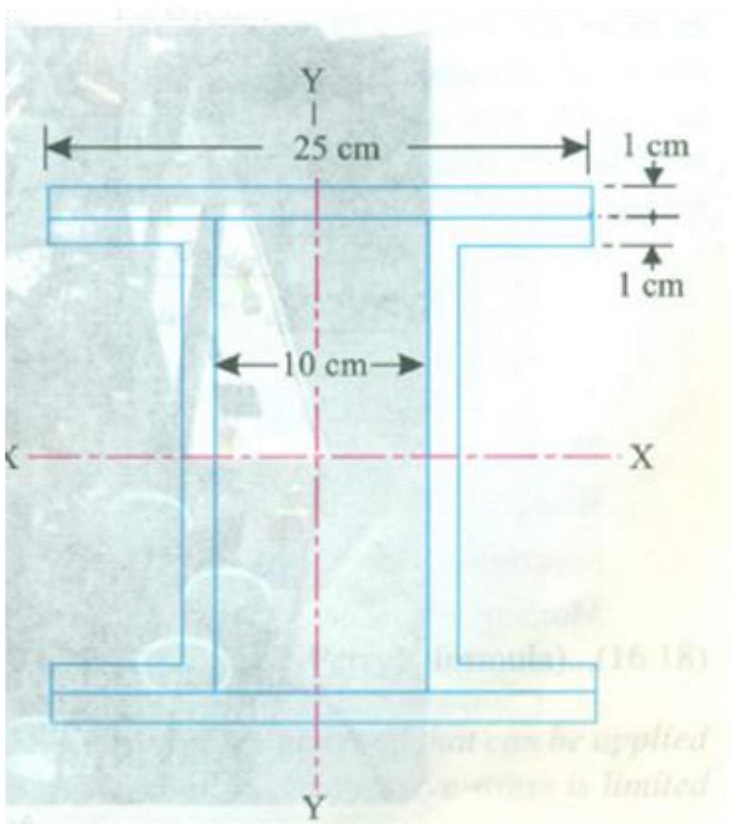


Fig. 2

Solution

Given: Permissible stress, σ_{max} = 70 MPa = 70 N/mm^2; P = 300 kN = 300 x 10^3 N; l_e = 3 m = 3000 mm; E = 200 GPa = 200 x 10^3 N/mm^2

Section properties; Refer to Fig. 2

Area of the section, A = 8554 mm^2

Moment of inertia about YY-axis I_{YY} = 44.923 x 10^6 mm^4

Stress due to the direct load

$$\sigma_d = \frac{P}{A} = \frac{300x10^3}{8554} = 35.07\ N/mm^2$$

Maximum bending stress, $\sigma_b = \sigma_{\max} - \sigma_d = 70 - 35.07 = 34.93\ N/mm^2$

Maximum bending moment, $M_{\max} = \frac{\sigma_{\max} I_{YY}}{x} = \frac{(34.93)(44.923x10^6)}{125} = 12.55x10^6\ Nmm$

Now, $\alpha = \frac{l_e}{2}\sqrt{\frac{P}{EI}} = \frac{3000}{2}\sqrt{\frac{300x10^3}{(200x10^3)(44.923x10^6)}} = 0.274\ rad = 15.7^o$

Then, $M = P.e\sec\alpha \Rightarrow e = \frac{M}{P\sec\alpha} = \frac{12.55x10^6}{(300x10^3)\sec 15.7} = 40.3\ mm$

TYPE B: OBJECTIVES

CHOOSE THE CORRECT OPTION

1. A member of structure or bar which carries an axial compressive load is called
   (a) strut (b) tie
   (c) shaft (d) none of the above

2. When the strut is vertical *i.e.* inclined at 90° to the horizontal is known as
   (a) column (b) pillar
   (c) stanchion (d) any of the above.

3. Which of the following is an example of the strut?
   (a) Piston rods
   (b) Connecting rods
   (c) Side links in forging machines
   (d) All of the above.

4. The ratio of equivalent length of the column to the minimum radius of gyration is called
   (a) Poisson's ratio
   (b) buckling factor
   (c) factor of safety
   (d) none of the above

5. Columns which have length less than 8 times their diameter or slenderness ' ratio less than 32 are called
   (a) short columns
   (b) medium columns
   (c) long columns
   (d) any of the above

6. The ratio between buckling load and safe load is known as

(a) slenderness ratio
(b) buckling factor
(c) factor of safety
(d) none of the above

7. The buckling, in case of a column, takes place about the axis having
   (a) minimum radius of gyration
   (b) maximum radius of gyration
   (c) either of the above
   (d) none of the above

8. The strength of a column depends on which of the following factors ?
   (a) Slenderness ratio
   (b) End conditions
   (c) Both (a) and (b)
   (d) None of the above

9. Slenderness ratio of a column may be defined as the ratio of its length to the
   (a) radius of column
   (b) minimum radius of gyration
   (c) maximum radius of gyration
   (d) none of the above

10. If a slenderness ratio of a column is more than 120 it is termed as
   (a) short column
   (b) medium column
   (c) long column
   (d) none of the above

11. Euler's formula is applicable to
   (a) short columns
   (b) medium columns
   (c) long columns
   (d) none of the above

12. The radius of gyration of a circular column of diameter *d* is
   (a) d/4 (b) d/2
   (c) $d^2/4$ (d) $d^2/16$

13. The ratio of equivalent length of a column having one end fixed arid the other end free, to its length is
   (a) 2 (b) $\sqrt{2}$
   (c) 1/2 (d) $1/\sqrt{2}$

14. The ratio of equivalent length of a column having one end fixed and the other end hinged, to its length is
   (a) 2 (b) $\sqrt{2}$
   (c) 1/2 (d) $1/\sqrt{2}$

15. Rankine formula takes into account which of the following?
   (a) The effect of slenderness ratio
   (b) The initial curvature of the column
   (c) The eccentricity of loading
   (d) The effect of direct compressive stress

16. In the Rankine formula, the material mild steel is
   (a) 1/1200 (b) 1/1600
   (c) 1/7500 (d) 1/5000

17. A beam column is one which carries
   (a) axial load
   (a) transverse loads
   (c) eccentric load
   (d) axial as well as transverse loads

18. The slenderness ratio of a long column is
   (a) 10-20
   (b) 20-30
   (c) 50-60
   (d) above 80

19. A mild steel column of 50 mm diameter is hinged at both of its ends. Find the crippling load for the column, if its length is 2.5 m. Take *E* for the column material as 200 GPa.
   (a) 98.9 Kn (b) 97.9 kN
   (c) 96.9 kN (d) 95.9 kN

20. A hollow cast iron column of 150 mm external diameter and 100 mm internal

diameter is 3.5 m long. If one end of the column is rigidity fixed and the other is free, find the critical load on the column. Assume modulus of elasticity for the column material as 120 GPa
(a) 482 kN (b) 483 kN
(c) 484 kN (d) 485 kN

21. An -section 240 mm x 120 mm x 20 mm is used as 6 m long column with both ends fixed. What is the crippling load for the column? Take Young's modulus for the joist as 200 GPa.
(a) 1293.5 kN (b) 1292.5 kN
(c) 1294.5 kN (d) 1295.5 kN

22. A hollow column of 200 mm external diameter and 160 mm internal diameter is used as a column of 4.5m length. Calculate the Rankine's crippling load when the column is fixed at both ends. Take allowable stress as 350 MPa and Rankine's constant as 1/1600.
(a) 2.53 kN (b) 3.23 kN
(c) 2.23 kN (d) 3.53 kN

23. A hollow cast iron 5 m long column with both ends fixed is required for support a load of 1000 kN. If the external diameter of the column is 250 mm, find its thickness. Take working stress as 80 MPa and Rankine's constant as l/1600.
(a) 29.4 mm (b) 28.4 mm
(c) 27.4 mm (d) 26.4 mm

24. A hollow circular column of 200 mm external diameter and 160 mm internal diameter is 4 m long with both ends fixed. lf the column carries load of 150 kN at an eccentricity of 25 mm, find the extreme stress in the column.
(a) 22.5 MPa
(b) 23.5 MPa
(c) 21.5 MPa
(d) 24.5 MPa

25. An alloy tube 60 mm diameter and 2.8 m length is used as a strut with both ends hinged. If the tube is subjected to an eccentric load equal to 60% of the Euler·s crippling load, find the value of eccentricity. Take yield strength as 320 MPa and modulus of elasticity as 210 GPa.
(a) 12.1 mm
(b) 13.1 mm
(c) 14.1 mm
(d) 15.1 mm

TYPE C: ESSAY

1. Derive a relation for the Euler's crippling load for a column when
   a. it has both ends hinged, and
   b. both ends fixed

2. Explain the term 'slenderness ratio' and describe with mathematical expression, how it limits the use of Euler's formula for crippling load.

3. Obtain a relation for the Rankine's crippling load for columns.

4. Give the Johnson's straight line and parabolic formula for columns.

5. What is Indian Standard Code for columns?

Unit 4

BENDING OF CURVED BARS

Introduction

This unit deals with the derivation and computation of stresses and deflection of beams of small radius of curvature of different cross-section

Learning Objectives

After reading this unit you should be able to:

1. Derive the equations used to compute the stresses in bars of small radius of curvature
2. Calculate the stresses formed in bars of small radius of curvature for particular cross-section
3. Derive the equations used to compute the deflection in beams of small radius of curvature
4. Calculate the deflection in beams of small radius of curvature

IMPORTANT EQUATIONS

1. Bending Stress,

$$\sigma = \frac{M}{AR}\left[1 + \frac{R^2 y}{h^2(R+y)}\right]$$

2. Rectangular Section,

$$h^2 == 2.3\frac{R^3}{D}\log\left(\frac{2R+D}{2R-D}\right) - R^2$$

3. Triangular Section,

$$h^2 = \frac{R^3}{A} x \frac{B}{D}\left[2.3R_2 \log\frac{R_2}{R_1} - D\right] - R^2$$

4. Trapezoidal Section,

$$h^2 = \frac{R^3}{A}\left\{\begin{array}{l} 2.3\log\frac{R_2}{R_1}\left[B_2 + \frac{(B_1 - B_2)R_2}{D}\right] \\ -(B_1 - B_2) \end{array}\right\} - R^2$$

5. Circular Section,

$$h^2 = \frac{d^2}{16} + \frac{1}{8} x \frac{d^4}{16R^2}$$

6. Crane Hooks,

$$\sigma_1 = \frac{W}{A}\left\{1 + \frac{x}{R}\left[1 - \frac{R^2 y_1}{h^2(R - y_1)}\right]\right\}$$

$$\sigma_2 = \frac{W}{A}\left\{1+\frac{x}{R}\left[1-\frac{R^2 y_2}{h^2(R-y_2)}\right]\right\}$$

7. Rings,

$$\sigma_A = \frac{P}{\pi A}\left[1-\frac{R^2}{h^2}x\frac{y_1}{R-y_1}\right]$$

$$\sigma_B = \frac{P}{2A}-\frac{0.182P}{A}\left[1-\frac{R^2}{h^2}x\frac{y_1}{R-y_1}\right]$$

$$\sigma_C = \frac{P}{\pi A}\left[1+\frac{R^2}{h^2}x\frac{y_2}{R+y_2}\right]$$

$$\sigma_D = \frac{P}{2A}-\frac{0.182P}{A}\left[1+\frac{R^2}{h^2}x\frac{y_2}{R+y_2}\right]$$

8. Chain Links, Thus the stress at *A*,

$$\sigma_A = \frac{P}{2A}\left(\frac{l+2R}{l+\pi R}\right)\left[1-\frac{R^2}{h^2}x\frac{y_1}{R-y_1}\right]$$

$$\sigma_B = \frac{P}{2A}-\frac{PR}{2A}\left(\frac{\pi-2}{l+\pi R}\right)\left[1-\frac{R^2}{h^2}x\frac{y_1}{R-y_1}\right]$$

$$\sigma_C = \frac{P}{2A}\left(\frac{l+2R}{l+\pi R}\right)\left[1+\frac{R^2}{h^2}x\frac{y_2}{R+y_2}\right]$$

$$\sigma_D = \frac{P}{2A}-\frac{PR}{2A}\left(\frac{\pi-2}{l+\pi R}\right)\left[1+\frac{R^2}{h^2}x\frac{y_2}{R+y_2}\right]$$

9. Closed Ring,

$$\delta_{UU'} = \frac{PR^3}{EI}\left[\frac{\pi}{4}-\frac{2}{\pi}\left(\frac{R^2}{R^2+h^2}\right)\right]$$

$$\delta_{VV'} = \frac{PR^3}{EI}\left[\frac{1}{2}-\frac{2}{\pi}\left(\frac{R^2}{R^2+h^2}\right)\right]$$

10. Chain Link

$$\delta_{UU'} = \frac{PR^3}{EI}\left[\frac{\pi R}{4}-2C'-R\right]-\frac{RPC'l}{EI}+\frac{Pl}{2AE}$$

$$\delta_{VV'} = -\frac{2PR^2}{EI}\left[C'+\frac{R}{4}\right]-\frac{RPC'l}{EI}$$

TYPE A: SOLVED EXAMPLES

Problem 1: *Fig. 1 shows a frame subjected to a load 2.4 kN. Find:*

(i) *The resultant stresses at points* 1 *and* 2;

(ii) *Position of the neutral axis.*

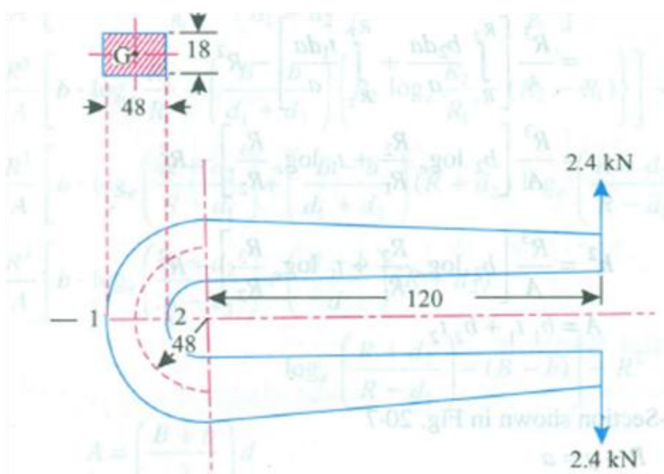


Fig. 1

Solution

Given: R = 48 mm; D = 48 mm; b = 18 mm

Area of section at 1-2, $A = bD. = 18x48. = 864\ mm^2$

Bending Moment, M = (-2.4 x 10^3) x (120 + 48) = - 403·2 x10^3 N.mm

M is taken negative because it tends to decrease the curvature.

From geometry, distance from centre line to extreme fibre, $y = \frac{D}{2}. = \frac{48}{2} = 24\ mm$

Link Radius $h^2 = 2.3\frac{R^3}{D}\log\left[\frac{2R+D}{2R-D}\right]. - R^2 = 2.3x\frac{48^3}{48}\log\left[\frac{2x48+48}{2x48-48}\right] - 48^2 = 227\ mm^2$

(i) Resultant stresses at points 1 and 2

Direct stress, $\sigma_d = \frac{P}{A}. = \frac{2.4x10^3}{864} = 2.77\ N/mm^2$

Bending stress due to M at point 2,

$$\sigma_{2b} = \frac{M}{AR}\left[1 - \frac{R^2.y}{h^2(R-y)}\right]. = \frac{-403x10^3}{864x48}\left[1 - \frac{48^2x24}{227(48-24)}\right]. = 88.95\ N/mm^2$$

Bending moment due to M at point 1,

$$\sigma_{2b} = \frac{M}{AR}\left[1 + \frac{R^2.y}{h^2(R+y)}\right]. = \frac{-403x10^3}{864x48}\left[1 + \frac{48^2x24}{227(48+24)}\right]. = -42.61\ N/mm^2$$

Resultant stress at point 2, $\sigma_2 = \sigma_d + \sigma_{2b} = 2.77 + 88.95 = 91.72\ N/mm^2$

Resultant stress at point 1, $\sigma_1 = \sigma_d + \sigma_{1b} = 2.77 + (-42.61) = -39.84\ N/mm^2$

(ii) Position of the neutral axis

$$y = -\left(\frac{Rh^2}{R^2 + h^2}\right) = -\left(\frac{48x227}{48^2 + 227}\right) = -4.35\ mm$$

Hence, neutral axis is at a radius of **4·35 mm** (Ans.)

Problem 2*: The curved member shown in Fig. 2 has a solid circular cross-section 0.10 m in diameter. If the maximum tensile and compressive stresses in the member are not exceed 150 MPa and 200 MPa respectively, determine the value of load P that can safely be carried by the member.*

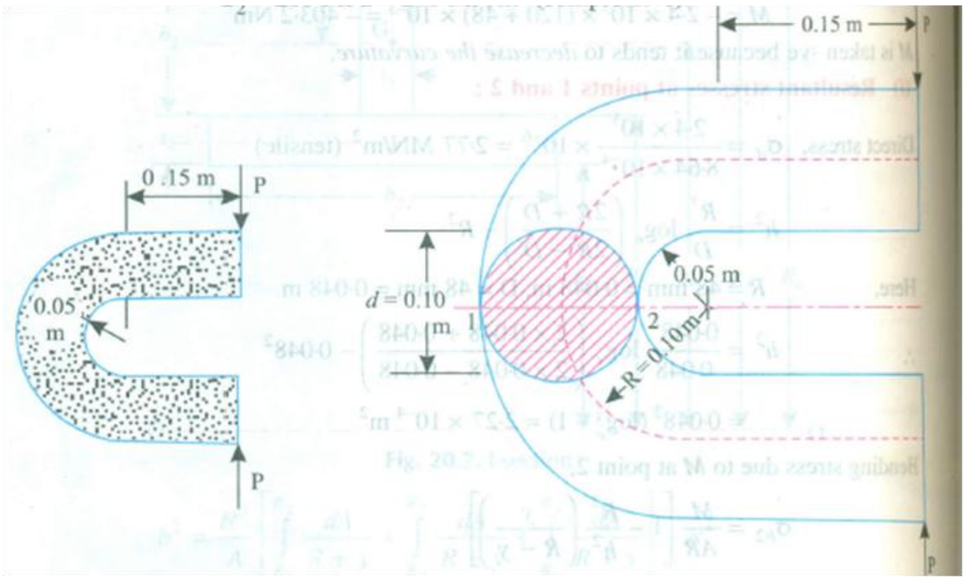


Fig 2

Solution

Given: d = 0·10 m = 100 mm; R = 0·10 m = 100 mm; σ_1 = 150 MPa = 150 N/mm^2 (tensile); σ_2 = 200 MPa = 200 N/mm^2 (compressive)

Area of section $A = \frac{\pi}{4} x d^2 . = \frac{\pi}{4} x 100^2 . = 7.854 x 10^3 \ mm^2$

From geometry, distance from centre line to extreme fibre, $y = \frac{d}{2} . = \frac{100}{2} = 50 \ mm$

Link Radius $h^2 = \frac{d^2}{16} x \frac{1}{128} x \frac{d^4}{R^2} = \frac{100^2}{16} x \frac{1}{128} x \frac{100^4}{100^2} = 703.1 \, mm^2$

(i) Load P

Bending Moment, M = P x (125 + 100) = 250P N.mm

Direct stress, $\sigma_d = \frac{P}{A}. = \frac{P}{7.854x10^3}\ N/mm^2$ (comp)

Bending stress due to M at point 1,

$$\sigma_{1b} = \frac{M}{AR}\left[1+\frac{R^2.y}{h^2(R+y)}\right]. = \frac{250P}{864x48}\left[1+\frac{100^2x50}{703(100+50)}\right].\ N/mm^2 \text{ (tensile)}$$

Bending moment due to M at point 2,

$$\sigma_{2b} = \frac{M}{AR}\left[1-\frac{R^2.y}{h^2(R-y)}\right]. = \frac{250P}{7854x100}\left[1-\frac{100^2x50}{703(100-50)}\right].N/mm^2 \text{ (comp)}$$

Resultant stress at point 1,

$$\sigma_1 = \sigma_d + \sigma_{1b} = -\frac{P}{7854} + \frac{250P}{7854x100}\left[1+\frac{100^2x50}{703(100+50)}\right].\ N/mm^2 = 150 \quad \text{(i)}$$

$$\Rightarrow P = \frac{150}{1.6997x10^{-3}} = 88.25\ kN$$

Resultant stress at point 2,

$$\sigma_2 = \sigma_d + \sigma_{1b} = \frac{P}{7854} + \frac{250P}{7854x100}\left[1-\frac{100^2x50}{703(100-50)}\right].\ N/mm^2 = 200 \quad \text{(ii)}$$

$$\Rightarrow P = \frac{200}{4.336x10^{-3}} = 46.13\ kN$$

Comparing eqn. *(i)* and *(ii)* the safe load P will be lesser of these. Hence, P = **46·13 kN** (Ans.)

Problem 3: *Fig. 3 shows a circular ring of rectangular section, with a slit and subjected load P*

(i) Calculate the magnitude of the force P if the maximum stress along the section 1-2 is not to exceed 225 MNlm²

(ii) Draw the stress distribution along 1-2.

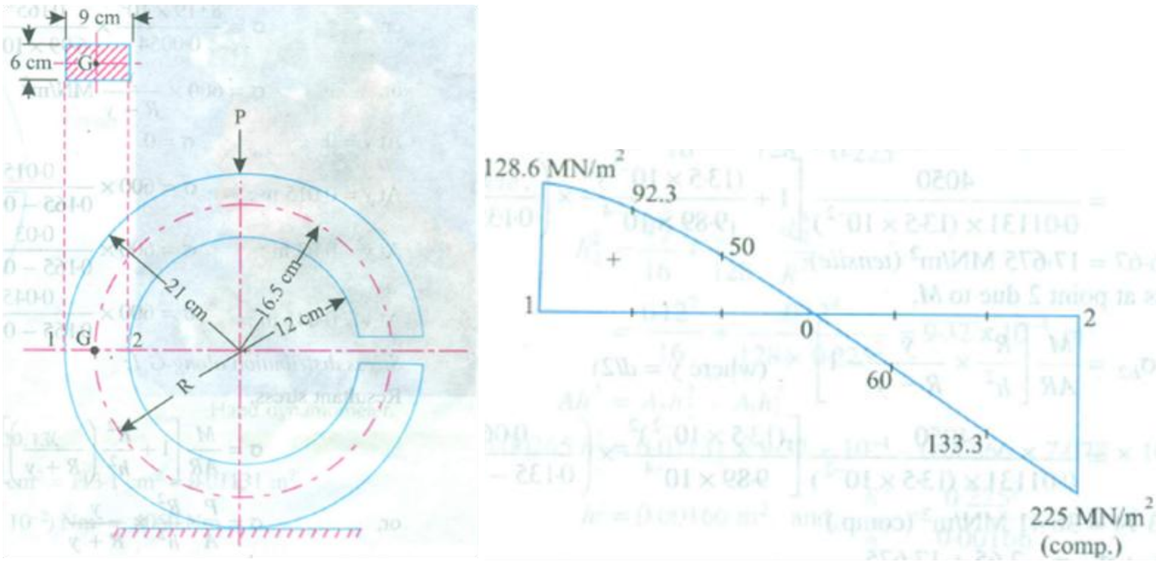


Fig 4

Solution

Given: D = 90 mm; b = 60 mm; R = 165 mm; σ_{max} = 225 MPa = 225 N/mm^2 (compressive)

Area of section $A = bD. = 60x90. = 540\ mm^2$

From geometry, distance from centre line to extreme fibre, $y = \frac{D}{2}. = \frac{90}{2} = 45\ mm$

Link Radius $h^2 = 2.3\frac{R^3}{D}\log\left[\frac{2R+D}{2R-D}\right]. - R^2 = 2.3x\frac{165^3}{90}\log\left[\frac{2x165+90}{2x165-90}\right] - 48^2 = 699\ mm^2$

(i) Load P

Bending Moment, M = P x R= PR N.mm

Direct stress, $\sigma_d = \frac{P}{A}. = \frac{P}{540}\ N/mm^2$ (comp)

Bending moment due to M at point 2,

$$\sigma_{2b}=\frac{P}{A}\left[1-\frac{R^2.y}{h^2(R-y)}\right].=\frac{P}{540}\left[1-\frac{165^2x45}{699(165-45)}\right].N/mm^2$$

Resultant stress at point 2, $\sigma_{\max}=\sigma_d+\sigma_{2b}=-\frac{P}{540}-\frac{P}{540}\left[1-\frac{100^2x50}{703(100-45)}\right].N/mm^2=225$

$\Rightarrow P==83.19\ kN$

o

(ii) Stress distribution along the section 1-2:

Stress distribution along G 2: $\sigma=-\frac{P}{A}\left[1-\frac{R^2.y}{h^2(R-y)}\right].+\frac{P}{A}=\frac{P}{A}x\frac{R^2.y}{h^2(R-y)}=600.\frac{y}{(R-y)}$ (comp)

At y=0; $\sigma=600.\frac{y}{(R-y)}=600x\frac{0}{165-0}=0$

At y=15; $\sigma=600.\frac{y}{(R-y)}=600x\frac{15}{165-15}=60\ N/mm^2$ (comp)

At y=30; $\sigma=600.\frac{y}{(R-y)}=600x\frac{30}{165-30}=133\ N/mm^2$ (comp)

At y=45; $\sigma=600.\frac{y}{(R-y)}=600x\frac{45}{165-45}=225\ N/mm^2$ (comp)

Stress distribution along G 1: $\sigma=\frac{P}{A}\left[1+\frac{R^2.y}{h^2(R+y)}\right].-\frac{P}{A}=\frac{P}{A}x\frac{R^2.y}{h^2(R+y)}=600.\frac{y}{(R+y)}$

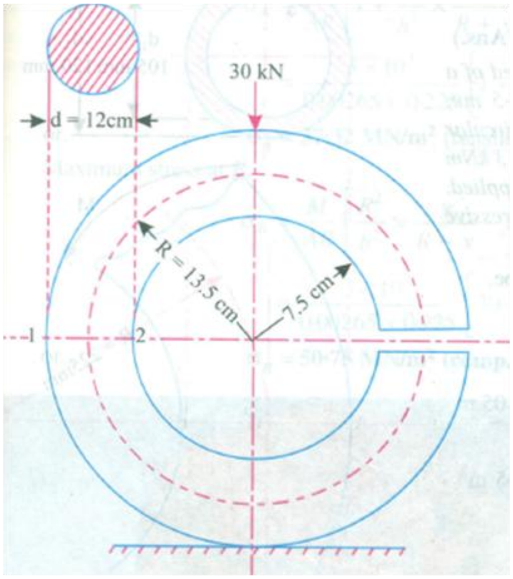
At y = 0; $\sigma=600.\frac{y}{(R+y)}=600x\frac{0}{165+0}=0$

At y = 15; $\sigma=600.\frac{y}{(R+y)}=600x\frac{15}{165+15}=50\ N/mm^2$ (tensile)

At y=30; $\sigma=600.\frac{y}{(R+y)}=600x\frac{30}{165+30}=92.3\ N/mm^2$ (tensile)

At y=45; $\sigma = 600.\frac{y}{(R+y)} = 600x\frac{45}{165+45} = 128.6\ N/mm^2$ (tensile

Problem 4: *Fig. 4 shows a ring carrying a load of 30 kN. Calculate the stresses at 1 and 2.*



Solution

Given: d = 120 mm; R = 135 mm; P = - 30 kN = - 30 x10^3 N

Area of section $A = \frac{\pi}{4} xd^2. = \frac{\pi}{4} x120^2. = 11.31x10^3\ mm^2$

Bending Moment, M = (30 x 10^3) x (135) = 4.05 x10^6 N.mm

From geometry, distance from centre line to extreme fibre, $y = \frac{d}{2}. = \frac{120}{2} = 60\ mm$

Link Radius $h^2 = \frac{d^2}{16} x \frac{1}{128} x \frac{d^4}{R^2} = \frac{120^2}{16} x \frac{1}{128} x \frac{120^4}{120^2} = 989\ mm^2$

Direct stress, $\sigma_d = \frac{P}{A}. = \frac{-30x10^3}{11.31x10^3} = -265\,N/mm^2$

(i) *Stress at Point 1*

Bending stress due to M at point 1,

$$\sigma_{1b} = \frac{M}{AR}\left[1+\frac{R^2.y}{h^2(R+y)}\right]. = \frac{4.05x10^6}{(11.31x10^3)x135}\left[1+\frac{135^2\,x60}{703(135+60)}\right] = 17.67\ N/mm^2$$

Resultant stress at point 1, $\sigma_1 = \sigma_d + \sigma_{1b} = -2.65 + 17.67 = 15.02\ N/mm^2$

(ii) *Stress at Point 2*

Bending moment due to *M* at point 2,

$$\sigma_{2b} = \frac{M}{AR}\left[1-\frac{R^2.y}{h^2(R-y)}\right]. = \frac{4.05x10^6}{(11.31x10^3)x135}\left[1-\frac{135^2\,x60}{703(135-60)}\right] = -36.41\ N/mm^2$$

Resultant stress at point 2, $\sigma_2 = \sigma_d + \sigma_{2b} = -2.65 + (-36.41) = -39.06\ N/mm^2$

Problem·5: *A curved bar is formed of a tube of 120 mm outside diameter and 7·5 mm thickness. The centre line of this beam is a circular arc of radius 225 mm. A bending moment of 3 kNm tending to increase curvature of the bar is applied. Calculate the maximum tensile and compressive stresses set up in the bar.*

Solution

Outside diameter of the tube, *D* = 120 mm; Thickness of the tube = 7·5 mm; Inside diameter of the tube, *D* = 120 - 2 x 7·5 = 105 mm; Bending moment, M = 3kNm

Area of section, $A = \frac{\pi}{4}x(D^2 - d^2). = \frac{\pi}{4}x(120^2 - 105^2.) = 2650\ mm^2$

Area of outer circle, $A = \frac{\pi}{4}xD^2. = \frac{\pi}{4}x120^2 = 11310\ mm^2$

Area of inner circle, $A = \frac{\pi}{4}xd^2. = \frac{\pi}{4}x105^2 = 8660\ mm^2$

From geometry, distance from centre line to extreme fibre, $y = \frac{d}{2}. = \frac{120}{2} = 60\ mm$

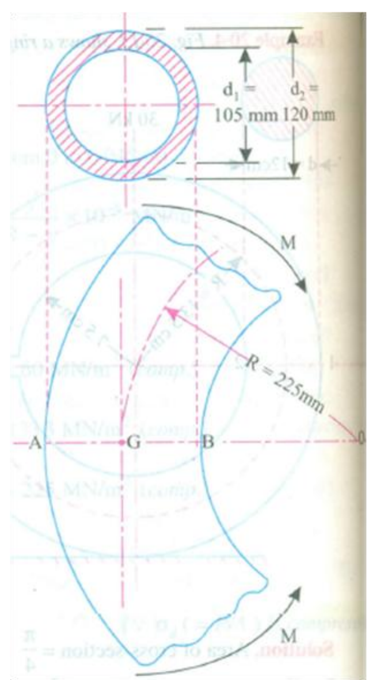


Fig. 5

Link Radius for outer circle, $h_o^{\ 2} = \frac{d^2}{16} x \frac{1}{128} x \frac{d^4}{R^2} = \frac{120^2}{16} x \frac{1}{128} x \frac{120^4}{225^2} = 932\ mm^2$

Link Radius for inner circle, $h_o^{\ 2} = \frac{d^2}{16} x \frac{1}{128} x \frac{d^4}{R^2} = \frac{105^2}{16} x \frac{1}{128} x \frac{105^4}{225^2} = 708\ mm^2$

$$Also,\ Ah^2 = A_o h_o^{\ 2} - A_i h_i^{\ 2} = h^2 = \frac{A_o h_o^{\ 2} - A_i h_i^{\ 2}}{A} = \frac{11310x932 - 8660x708}{265} = 1660\, mm^2$$

(i) Stress at A

$$\sigma_A = \frac{M}{AR}\left[1 + \frac{R^2.y}{h^2(R+y)}\right]. = \frac{3x10^3}{2650x225}\left[1 + \frac{225^2\,x60}{1660(225+60)}\right]. = 37.32\ N/mm^2$$

(ii) Stress at B

$$\sigma_B = \frac{M}{AR}\left[1 - \frac{R^2.y}{h^2(R-y)}\right]. = \frac{3x10^3}{2650x225}\left[1 - \frac{225^2\,x60}{1660(225-60)}\right]. = 50.75\ N/mm^2$$

Problem 6: *Fig. 7 shows a crane hook lifting a load of 150 kN. Determine the compressive and tensile stresses in the critical section of the crane hook.*

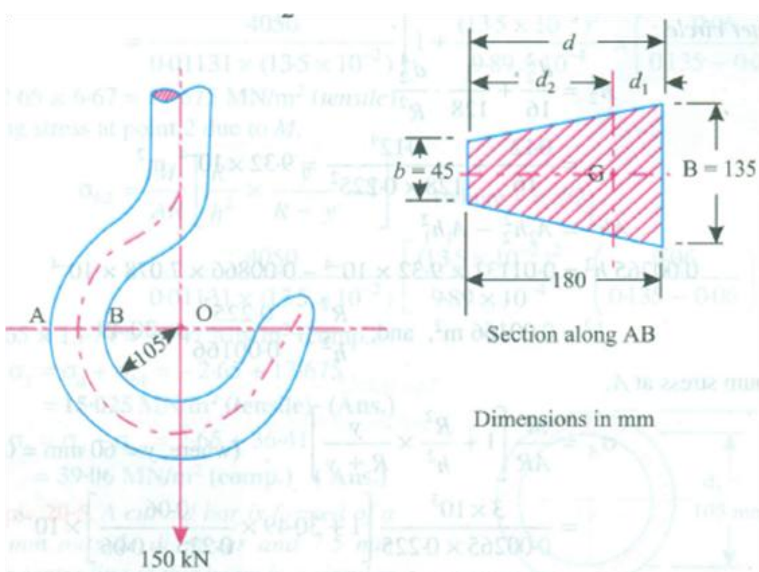


Fig. 7

Solution

Given: Load (P) = 150 kN = 150 x 10^3 N; Outer width (B_2) = 45 mm; Inner width (B_1) = 135 mm and depth *(D)* = 180 mm.

Distance between centre Line and extreme fibre,

$$y_1 = \frac{B_1 + 2B_2}{B_1 + B_2}\left(\frac{D}{3}\right) = \left(\frac{135 + 2x45}{135 + 45}\right)\left(\frac{180}{3}\right) = 75\ mm \text{ and}$$

$$y_2 = D - y_1 = 180 - 75 = 105\ mm$$

The area $A = \frac{D}{2}(B_1 + B_2) = \frac{180}{2}(135 + 45) = 16.2x10^3\ mm^2$

From the geometry of the hook section, we find that

Radius of inner edge, R_1 = 105 mm

Radius of outer edge, R_2 = 105 + 180 = 285 mm

Radius of central line, R = 105 + 75 = 180 mm

Distance between centre of curvature and centre line *(x)* = 105 + 75 = 180 mm;

Link radius for the beam section,

$$h^2 = \frac{R^3}{A}\left\{2.3\log\frac{R_2}{R_1}\left[B_2 + \frac{(B_1 - B_2)R_2}{D}\right] - (B_1 - B_2)\right\} - R^2$$

$$\Rightarrow h^2 = \frac{180^3}{16.2x10^3}\left\{2.3\log\frac{285}{105}\left[45 + \frac{(135 - 45)(285)}{180}\right] - (135 - 45)\right\} - 180^2 = 256\ mm^2$$

Maximum stress at outside edge

At the outside edge, $y = +105\ mm$

The stress is given by

$$\sigma_A = \frac{W}{A}\left\{1 + \frac{x}{R}\left[1 + \frac{R^2 y_2}{h^2(R + y_2)}\right]\right\} = \frac{-150x10^3}{62.5x10^3}\left\{1 + \frac{180}{180}\left[1 + \frac{(180^2)(105)}{(256)(180 + 105)}\right]\right\}$$

$$\Rightarrow \sigma_B = -43.17 = N/mm^2$$

Maximum stress at inside edge

At the inside edge, $y = -75\ mm$

The stress is given by

$$\sigma_B = \frac{W}{A}\left\{1+\frac{x}{R}\left[1-\frac{R^2 y_2}{h^2(R-y_2)}\right]\right\} = \frac{-150x10^3}{62.5x10^3}\left\{1+\frac{180}{180}\left[1-\frac{(180^2)(75)}{(256)(180+75)}\right]\right\}$$

$$\Rightarrow \sigma_B = 83.71 = N/mm^2$$

***Problem 8**: A central horizontal section of hook is a symmetrical trapezium 60 mm deep, the inner width being 60 mm and the outer being 30 mm. Estimate the extreme intensities of stress when the hook carries a load of 30 kN, the load line passing 40 mm from the inside edge of the section and the centre of curvature being in the load line. Also plot the stress distribution across the section.*

From the geometry of the hook section, we find that

Radius of inner edge, $R_1 = 40$ mm

Radius of outer edge, $R_2 = 40 + 60 = 100$ mm

Radius of central line, $R = 40 + 26.7 = 66.7$ mm

Distance between centre of curvature and centre line *(x)* = 40 + 26.7 = 66.7 mm;

Solution

Given: Load (P) = 30 kN = 30 x 10^3 N; Outer width (B_2) = 30 mm; Inner width (B_1) = 60 mm and depth *(D)* = 60 mm.

Distance between centre Line and extreme fibre,

$$y_1 = \frac{B_1+2B_2}{B_1+B_2}\left(\frac{D}{3}\right) = \left(\frac{60+2x30}{60+30}\right)\left(\frac{60}{3}\right) = 26.7\ mm$$

and

$$y_2 = D - y_1 = 600 - 26.67 = 33.3\ mm$$

The area

$$A = \frac{D}{2}(B_1+B_2) = \frac{60}{2}(60+30) = 2700\ mm^2$$

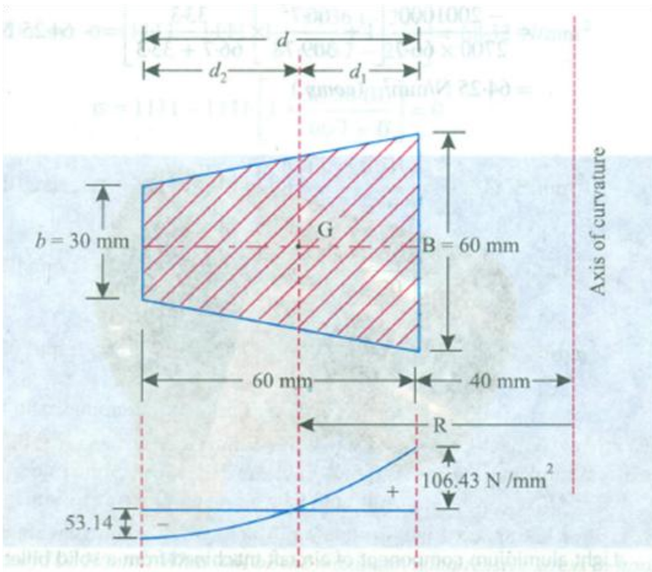


Fig 8

Link radius for the beam section,

$$h^2 = \frac{R^3}{A}\left\{2.3\log\frac{R_2}{R_1}\left[B_2+\frac{(B_1-B_2)R_2}{D}\right]-(B_1-B_2)\right\}-R^2$$

$$\Rightarrow h^2 = \frac{66.7^3}{2700}\left\{2.3\log\frac{100}{40}\left[30+\frac{(60-30)(100)}{60}\right]-(60-30)\right\}-66.7^2 = 309.78\ mm^2$$

Maximum stress at outside edge
At the outside edge, $y = +33.3\,mm$

The stress is given by

$$\sigma_o = \frac{W}{A}\left\{1+\frac{x}{R}\left[1+\frac{R^2 y_2}{h^2(R+y_2)}\right]\right\} = \frac{30x10^3}{2700}\left\{1-\frac{66.7}{66.7}\left[1+\frac{(66.7^2)(33.3)}{(256)(66.7+33.3)}\right]\right\} - 53.14\,N/mm^2$$

Maximum stress at inside edge
At the inside edge, $y = -26.7\ mm$

The stress is given by

$$\sigma_B = \frac{W}{A}\left\{1+\frac{x}{R}\left[1-\frac{R^2 y_2}{h^2(R-y_2)}\right]\right\} = \frac{30x10^3}{2700}\left\{1-\frac{66.7}{66.7}\left[1-\frac{(66.7^2)(26.7)}{(310)(66.7+26.7)}\right]\right\} = 106.43\,N/mm^2$$

(ii) Stress Distribution

At any point distance y from the centroidal axis, the total stress is given by

$$\sigma = \sigma_d + \frac{Wx}{AR}\left[1+\frac{R^2 y_2}{h^2(R+y_2)}\right] = 11.11 + 11.11\left[1+\frac{14.36y_2}{66.7+y}\right]$$

The stress distribution is shown in Fig 8

Problem 9: A *ring is made of round steel bar 30 mm diameter and the mean radius of the ring is 180 mm. Calculate the maximum tensile and compressive stresses in the material of/hl ring if it is subjected to a pull of* 12 *kN.*

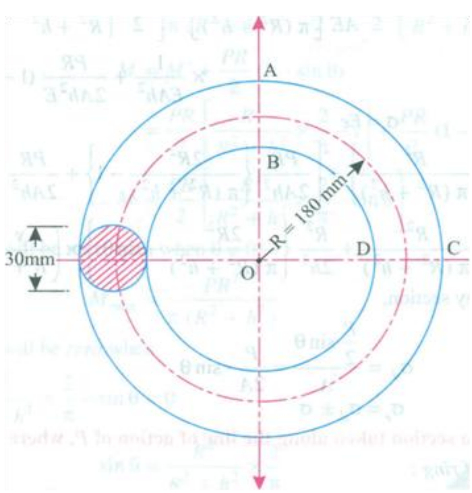


Fig. 9

Solution
Given: Diameter of steel bar *(d)*= 30 mm; Pull *(P)* = 12 kN = 12 x 10^3 N and diameter of the ring *(D)*= 360 mm or radius of ring *(R)* = 180 mm.

We know that area, $A = \frac{\pi}{4} x d^2 = \frac{\pi}{4} x 30^2 = 706.8\ mm^2$

and distance between centre line of the ring and extreme fibre, $y = y_1 = y_2 == 15\ mm$

Link radius for the ring section, $h^2 = \frac{d^2}{16} + \frac{1}{8} x \frac{d^4}{16R^2} = \frac{30^2}{16} + \frac{1}{8} x \frac{30^4}{16x180^2} = 56.4.5\ mm^2$

Stress at A

$$\sigma_A = \frac{P}{\pi A}\left[1 + \frac{R^2}{h^2} x \frac{y_2}{R + y_2}\right] = \frac{12x10^3}{\pi(706.8)}\left[1 + \frac{180^2}{56.4} x \frac{15}{180 + 15}\right] = 243.7\ N/mm^2$$

Stress at B,

$$\sigma_B = \frac{P}{\pi A}\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R - y_1}\right] = \frac{12x10^3}{\pi(706.8)}\left[1 - \frac{180^2}{56.4} x \frac{15}{180 - 15}\right] = -276.3\ N/mm^2$$

Stress at C

$$\sigma_D = \frac{P}{2A} - \frac{0.182P}{A}\left[1 + \frac{R^2}{h^2} x \frac{y_2}{R + y_2}\right]$$

$$= \frac{10^4}{2x100\pi} - \frac{0.182x12x10^3}{706.8}\left[1 + \frac{180^2}{56.4} x \frac{15}{180 + 15}\right] = -122.8\ N/mm^2$$

Stress at D,

$$\sigma_B = \frac{P}{2A} - \frac{0.182P}{A}\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R - y_1}\right]$$

$$= \frac{12x10^3}{2x706.8} - \frac{0.182x12x10^3}{706.8}\left[1 - \frac{180^2}{56.4} x \frac{15}{180 - 15}\right] = 175.48\ N/mm^2$$

Therefore, the maximum tensile stress = 243·7 *MN/m²* *at A and m*aximum compressive stress = 276·3 *MN/m2 at B*

Problem 10: *A chain link (Fig. 10) is made of round steel rod of 15 mm diameter. If R = 45 mm, l = 75 mm and load applied is 1·5 kN determine the maximum compressive stress in the link and tensile stress at the same section.*

Solution

Given: Diameter of steel bar *(d)* = 15 mm; Radius of link *(R)* = 45 mm; Length of straight portion (*l*) = 75 mm and pull *(P)* = 1.5 kN = 1.5 x 10^3 N

We know that area, $A = \frac{\pi}{4} x d^2 = \frac{\pi}{4} x 15^2 = 176.7\ mm^2$

and distance between centre line of the ring and extreme fibre, $y = y_1 = y_2 == 7.5\ mm$

Link radius for the ring section, $h^2 = \frac{d^2}{16} + \frac{1}{8} x \frac{d^4}{16R^2} = \frac{15^2}{16} + \frac{1}{8} x \frac{15^4}{16x45^2} = 14.25\ mm^2$

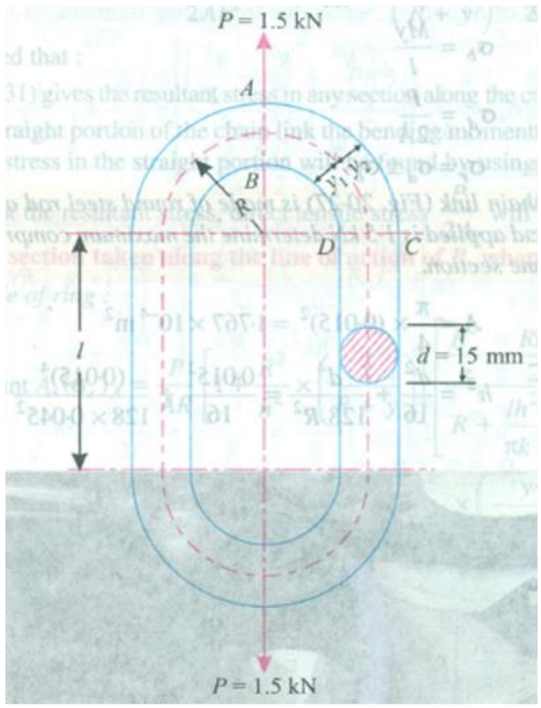


Fig. 10

Stress at A

$$\sigma_A = \frac{P}{2A}\left(\frac{l+2R}{l+\pi R}\right)\left[1 + \frac{R^2}{h^2} x \frac{y_2}{R+y_2}\right]$$

$$= \frac{1.5x10^3}{2x176.7}\left(\frac{75+2x45}{75+\pi x45}\right)\left[1 + \frac{45^2}{14.25} x \frac{7.5}{45+7.5}\right] = 68.74\ N/mm^2$$

Stress at *B*,

$$\sigma_B = \frac{P}{2A}\left(\frac{l+2R}{l+\pi R}\right)\left[1 - \frac{R^2}{h^2} x \frac{y_1}{R-y_1}\right]$$

$$= \frac{1.5x10^3}{2x176.7}\left(\frac{75+2x45}{75+\pi x45}\right)\left[1 - \frac{45^2}{27} x \frac{7.5}{(45-7.5)}\right] = -88.5\ N/mm^2$$

TYPE B: OBJECTIVES

CHOOSE THE CORRECT OPTION

1. The theory of curved beam was postulated by
   (a) Rankline
   (b) Mohr
   (c) Castigliano
   (d) Winkler-Bach

2. In curved beams the distribution of bending stress is
   (a) linear
   (b) parabolic
   (c) uniform
   (d) hyperbolic.

3. The neutral axis in curved beams
   (a) lies at the top of the beam
   (b) lies at the bottom of the beam
   (c) does not coincide with the geometric axis
   (d) coincides with the geometric axis

4. For a crane hook the most suitable section is
   (a) triangular
   (b) trapezoidal
   (c) circular
   (d) rectangular.

5. The nature of stress at the inside surface of a crane hook is
   (a) shear
   (b) tensile
   (c) compressive
   (d) none of the above

6. Which of the following statements is correct with reference to the curved beam theory?
   (a) Shear stress is zero
   (b) Hoop stress is zero
   (c) Radial stress is zero
   (d) Bending stress is zero

7. The maximum stress in a ring under tension occurs
   (a) along the line of action of load
   (b) perpendicular to the line of action of load
   (c) at 45° with the line of action of the load
   (d) none of the above.

8. When fabricating a chain link the joint should come at which of the following locations?
   (a) Parallel to the line of action of the load
   (b) 45° with the line of action of the load
   (c) 60° with the line of action of the load
   (d) Perpendicular to the line of action of the load

9. In a closed ring when a small cut is made at the horizontal diameter the maximum stress will
   (a) decrease
   (b) increase
   (c) remain same
   (d) become infinite.

10. Which of the following assumptions is made in the analysis of curved beams?
   (a) Limit of proportionality is not exceeded
   (b) Radial strain is negligible
   (c) Plane to transverse sections remain plane after bending
   (d) The material considered is isotropic and obeys Hooke's law
   (e) All of the above.

A beam of rectangular section 30 mm x 40 mm has its central line curved to a radius of 60. The beam is subjected to a bending moment of 120 x 10^3 N-mm. Use the information to answer questions 11 and 12

11. Find the greatest tension in the beam
(a) 26 N/mm^2
(b) 13 N/mm^2
(c) 21 N/mm^2
(d) 42 N/mm^2

12. Determine the compression stresses in the beam.
(a) 26 N/mm^2
(b) 13 N/mm^2
(c) 21 N/mm^2
(d) 42 N/mm^2

A crane hook carries a load of 45 kN, the line of load being at a horizontal distance of 40 mm from inner edge of the section, and the centre of curvature coincides with the load line. The horizontal section is trapezium with 50 mm depth, inner width being 60 mm and the outer width being 30 mm. Use the information to answer questions 13 and 14

13. Find the greatest compressive stress in the hook.
(a) 152.8 N/mm^2
(b) 76.4 N/mm^2
(c) 71.6 N/mm^2
(d) 143.2 N/mm^2

14. Compute the greatest tensile stress in the hook.
(a) 152.8 N/mm^2
(b) 76.4 N/mm^2
(c) 71.6 N/mm^2
(d) 143.2 N/mm^2

A chain link is made of 40 mm round steel and is semi-circular at each end, the mean diameter of which is 80 mm. The straight sides of the link are also 80 mm with the link carrying a load of 100 kN. Use the information to answer questions 15 and 16.

15. Estimate the greatest tensile and compressive stresses in the link.
(a) 46.8 N/mm^2
(b) 45.5 N/mm^2
(c) 91.2 N/mm^2
(d) 93.5 N/mm^2

16. Find the greatest compressive stresses in the link.
(a) 46.8 N/mm^2
(b) 45.5 N/mm^2
(c) 91.2 N/mm^2
(d) 93.5 N/mm^2

A steel bar 38 mm in diameter is bent into a curve of mean radius 31·7 mm. If a bending moment of 4·6 Nm tending to increase the curvature, acts on the bar, use the information to answer questions 17and 18.

17. Find the intensity of maximum compressive stress.
(a) 1.6 N/mm^2 (tensile)
(b) 1.6 N/mm^2 (comp.)
(c) 0.56 N/mm^2 (tensile)
(d) 0.56 N/mm^2 (comp.)

18. Compute the intensity of maximum tensile stress.
(a) 1.6 N/mm^2 (tensile)
(b) 1.6 N/mm^2 (comp.)
(c) 0.56 N/mm^2 (tensile)
(d) 0.56 N/mm^2 (comp.)

A bar of circular cross-section is bent in the shape of a horse shoe. The radius of the section is 40 mm and the mean radius is 80 mm. Two equal and opposite forces of 15 kN each are applied so as to straighten the bar. Use the information to answer questions 19 and 20

19. Find the maximum compressive stress.
(a) 39.2 N/mm^2 (tensile)
(b) 99.8 N/mm^2 (tensile
(c) 39.2 N/mm^2 (comp.)

(d) 99.8 N/mm^2 (comp.)

20. Find the maximum tensile stress.
    (a) 39.2 N/mm^2 (tensile)
    (b) 99.8 N/mm^2 (tensile
    (c) 39.2 N/mm^2 (comp.)
    (d) 99.8 N/mm^2 (comp.)

A ring is made of round steel bar 25 mm diameter and the mean radius of the ring is 150 mm. If it is subjected to a pull of 10 kN, use the information to answer questions 21 and 22.

21. Calculate the maximum tensile stresses in the material of the ring
    (a) 292.2 N/mm^2
    (b) 296.8 N/mm^2
    (c) 331.2 N/mm^2
    (d) 336.7 N/mm^2

22. Determine the maximum compressive stress in the material of the ring
    (a) 292.2 N/mm^2
    (b) 296.8 N/mm^2
    (c) 331.2 N/mm^2
    (d) 336.7 N/mm^2

A curved bar of rectangular section 80 nun wide by 100 mm deep in the plane of bending, initially unstressed, is subjected to a bending moment of 3 kNm which tends to straighten the bar. The mean radius of curvature is 200 mm. Use the information to answer questions 23 to 25.

23. Find the position of the neutral axis. [-4.02 mm]
    (a) -4.02 mm
    (b) 0.402 mm
    (c) 4.02 mm
    (d) -0.402 mm

24. Determine the greatest bending stresses at the inner edge. [19·16 MN/m^2
    (a) 19.16 N/mm^2 (tensile)
    (b) 26.93 N/mm^2 (tensile)
    (c) 26.93 N/mm^2 (comp.)
    (d) 19.16 N/mm^2 (comp.)

25. Compute the greatest bending stresses at the inner edge.
    (a) 19.16 N/mm^2 (tensile)
    (b) 26.93 N/mm^2 (tensile)
    (c) 26.93 N/mm^2 (comp.)
    (d) 19.16 N/mm^2 (comp.)

TYPE C: ESSAY

1. Give the assumptions for determining the stresses in the bending of curved bars.
2. Derive an expression for the bending stress on the extreme fibres of a bar
    a. having a small initial curvature, and
    b. having a large initial curvature
3. Obtain the values of link radius for a
    a. triangular section and
    b. trapezoidal section
4. How will you find out the values of maximum tensile and compressive stresses in a crane hook?
5. Obtain from fundamentals the relation for the maximum compressive and tensile stresses in ring.
6. What is link radius

Unit 5

STRESSES AND DEFLECTION OF SPRINGS

Introduction

This unit deals with the derivation and computation of deflection and stress in the various kinds of springs which include carriage or leaf, helical and flat spiral springs

Learning Objectives

After reading this unit you should be able to:

1. Derive the bending stress in semi- and quarter-elliptic leaf spring type
2. Derive the deflection in semi- and quarter-elliptic leaf spring type
3. Derive the equations used to compute the stresses under axial loading and axial torque in helical springs
4. Estimate the diameters of the coils and wire of a helical spring
5. Compute the strain energy and stresses in flat spiral

springs

IMPORTANT EQUATIONS

1. Semi-elliptical leaf spring

$$\sigma = \frac{3Wl}{2nbt^2}$$

$$\delta = \frac{3Wl^3}{8Enbt^3}$$

2. Quarter-elliptical leaf spring

$$\sigma = \frac{6Wl}{nbt^2}$$

$$\delta = \frac{6Wl^3}{Enbt^3}$$

3. Closely-coiled Helical Springs

$$\theta = \frac{Tl}{JC} = \frac{WR.2\pi Rn}{\frac{\pi}{32} x d^4 C} = \frac{64WR^2 n}{Cd^4}$$

$$\delta = R\theta = \frac{64WR^3 n}{Cd^4}$$

$$U = \tfrac{1}{2} W\delta$$

$$s = \frac{W}{\delta} = \frac{Cd^4}{64R^3 n}\ U = \tfrac{1}{2} M.\phi$$

4. Open-coiled Helical Springs

$$\delta = \frac{64WR^3 n \sec\alpha}{d^4}\left[\frac{\cos^2\alpha}{C} + \frac{\sin^2\alpha}{E}\right]$$

5. Flat Spiral Springs

$$U = \tfrac{1}{2} T\theta = 1.25\frac{T^2 l}{2EI}$$

$$\hat{\sigma} = \frac{2T}{Z} = \frac{12T}{bt^2}$$

6. Semi-elliptical leaf spring

$$\sigma = \frac{3Wl}{2nbt^2}$$

$$\delta = \frac{3Wl^3}{8Enbt^3}$$

7. Quarter-elliptical leaf spring

$$\sigma = \frac{6Wl}{nbt^2}$$

$$\delta = \frac{6Wl^3}{Enbt^3}$$

8. Closely-coiled Helical Springs

$$\theta = \frac{Tl}{JC} = \frac{WR.2\pi Rn}{\frac{\pi}{32} x d^4 C} = \frac{64WR^2 n}{Cd^4}$$

$$\delta = R\theta = \frac{64WR^3 n}{Cd^4}$$

$$U = \tfrac{1}{2} W\delta$$

$$s = \frac{W}{\delta} = \frac{Cd^4}{64R^3 n}\ U = \tfrac{1}{2} M.\phi$$

9. Open-coiled Helical Springs

$$\delta = \frac{64WR^3 n \sec\alpha}{d^4}\left[\frac{\cos^2\alpha}{C} + \frac{\sin^2\alpha}{E}\right]$$

10. Flat Spiral Springs

$$U = \tfrac{1}{2} T\theta = 1.25\frac{T^2 l}{2EI}$$

$$\hat{\sigma} = \frac{2T}{Z} = \frac{12T}{bt^2}$$

SOLVED EXAMPLES

Problem 1: *A closely coiled helical spring is to carry a load of 500 N. Its mean coil diameter is to be 10 times that of the wire diameter. Calculate these diameters, if the maximum shear stress in the material of the spring is to be 80 MPa.*

Solution

Given: Load to be carried, W = 500 N; Mean coil diameter, D = 10 d (wire diameter); Shear stress τ = 80 MPa = 80 N/mm^2 and, *i.e.* $D = 126$ mm (Ans.)

Diameters, D and d:

Using the relation $\tau = \frac{16WR}{\pi d^3} = \frac{16W(5d)}{\pi d^3} = \frac{80W}{\pi d^2} \Rightarrow d = \sqrt[3]{\frac{80W}{\tau\pi}} = \sqrt[3]{\frac{80(500)}{80\pi}} = 12.6\ mm$

Also, $D = 10d = 10(12.6) = 126\ mm$

Problem 2: *A helical spring is made of 12 mm diameter steel wire wound on a 120 mm diameter mandrel. If there are 10 active coils, what is spring constant? Take: C = 82 GPa. What force must be applied to the spring to elongate it by 40 mm?*

Solution

Given: Diameter of steel wire, *d*= 12 mm; Diameter of mandrel, *D* = 120 mm; Number of active coils, *n* = 10; Modulus of rigidity C = 82 *GPa* = *82 x10^3 N/mm$^{2;}$* Elongation of the spring, δ = 40 mm;

Spring constant

We know that $k = \frac{W}{\delta} = \frac{CD^4}{64R^3n} = \frac{(82x10^3)(12^4)}{64(60^3)(10)} = 12.3\ N/mm$

Force to be applied to the spring

Again, $k = \frac{W}{\delta} \Rightarrow W = k\delta = 123(4) = 492\ N$

Problem 3:

a) Draw neat illustrative sketches to bring about the difference between helical coil tension spring and helical coil compression spring.

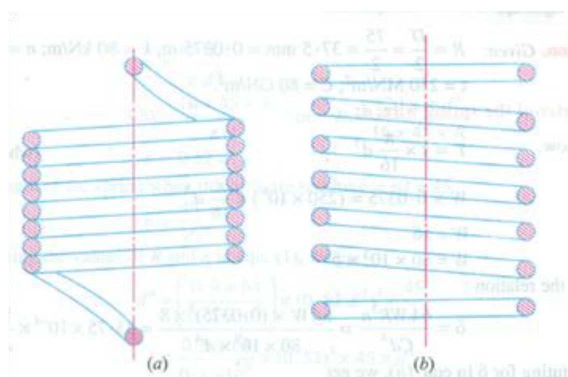
(b) A helical coil spring is made of round steel wire 6·35 mm in diameter. The mean radius of helix is 31· 75 mm, number of complete turns, 12; the spring is close-coiled. If C = 84·36 GPa, find:

(i) The pull required to extend the spring by 25·4 mm, and

(ii) The stress in the wire.

Solution

(a) The helical coil springs consist of a rod or wire wound in the form of the helix as shown fig. which clearly indicates the difference between the helical coil tension and compression springs.



It may be noted that in case of a helical tension spring it is not imperative to provide spacing between coils because helix angle is small while in case of helical compression spring, the helix angle being comparatively more, spacing is provided between the coils.

(b)

Given: R = 31·75 mm; d = 6·35 mm; n = 12; δ = 25-4 mm; C = 84.36 GPa = 84.36 x10^3 N/mm^2

Pull required in extending the spring

Using the relation, $\delta = \frac{64WR^3n}{Cd^4} \Rightarrow W = \frac{Cd^4\delta}{64R^3n} = \frac{(84.36x10^3)(6.35)(25.4)}{64(31.75)^3(12)} = 141.7\ N$

Shear stress in the wire

Using the relation, $\tau = \dfrac{16WR}{\pi d^3} = \dfrac{16(141.7)(31.75)}{\pi (6.35)^3} = 89.5\ N/mm^2$

Problem 4: *A close-coiled helical spring has mean diameter of 75 mm and spring constant of 80 kN/m and has 8 coils. What is the suitable diameter of the spring wire if maximum shear stress is not to exceed 250 MPa? Modulus of rigidity of the spring wire material is 80 GPa. What is the maximum axial load the spring can carry?*

Solution

Given: D = 75 mm; $R = 37\cdot5$ mm; $k = 80$ kN/m = 80 N/mm; $n = 8$; $\tau = 250$ *MPa = 250 N/mm²;* C = 80 *GPa = 80 x 10³ N/mm²*

Diameter of the spring wire

We know, $T = \tau x \dfrac{\pi}{16} d^3$ (i)

But, $T = WR$ (ii)

Also, $W = k\delta$ (iii),

Using the relation, $\delta = \dfrac{64WR^3 n}{Cd^4} = \dfrac{64W(37.5^3)(8)}{(80x10^3)d^4} = 337.5\dfrac{W}{d^4}$ (iv)

Substituting equation *(ii)* into equation *(iii)*

We have $W = k\left[337.5\dfrac{W}{d^4}\right] \Rightarrow d = \sqrt[4]{337.5k} = \sqrt[4]{337.5(80)} = 12.81\ mm$ (v)

Maximum axial load the spring can carry

Combining equations *(i)* and *(ii)* we get $WR = \tau x \dfrac{\pi}{16} d^3 \Rightarrow W = \dfrac{\pi \tau d^3}{16R}$ (vi)

Substituting the value of *d* in *(v)* into eqn. *(vi)*, we get $W = \frac{\pi\tau d^3}{16R} = \frac{\pi(250)(12.81)^3}{16(37.5)} = 2745.2\ N$

Problem 5: *A close-coiled helical spring is to have a stiffness of 900 N/m in compression, with a maximum load of 45 N and a maximum shearing stress of 120 N/mm^2. The solid length of the spring (i.e., coils touching) is* 45 *mm. Find:*

(i) *The wire diameter,*

(ii) The mean coil radius, and

(iii) The number of coils.

Take modulus of rigidity of material of the spring = 0·4 x 10^5 *Nlmm^2*

Solution

Given: k = 900 *N/m* = 0·9 *N/mm; W* = 45 N; τ: = 120 N/mm^2; C = 0·4 x 10^5 *N/mm^2*

The wire diameter

Using the relation, $\delta = \frac{64WR^3n}{Cd^4}$

or, $k = \frac{W}{\delta} = \frac{Cd^4}{64R^3n} \Rightarrow d^4 = \frac{64kR^3n}{C} = \frac{64(900)R^3n}{0.4x10^5}$ (1)

o

Also, $\tau = \frac{16WR}{\pi d^3} \Rightarrow R = \frac{\tau\pi d^3}{16W} = \frac{120\pi d^3}{16(45)} = 0.52d^3$ (2)

Solid length of the spring when the coils are touching

. $nd = 45 \Rightarrow n = \frac{45}{d}$ (3)

Substituting the values of *R* and *n* in eqn. (1), we get

$$d^4 = \frac{64(900)R^3 n}{0.4x10^5} = \frac{64(900)(0.52d^3)^3(45/d)}{0.4x10^5} = \frac{d^8}{109.75}$$

or $d^4 = \frac{d^8}{109.75} \Rightarrow d^4 = 109.75 \quad \therefore d = 3.24\ mm$

The mean coil radius, $R = 0.52d^3 = 0.52(3.24^3) = 17.68\ mm$

The number of coils, $n = \frac{45}{d} = \frac{45}{3.24} = 13.88 \approx 14$

Problem 6: *A safety valve of 76 mm diameter is to blow off at a pressure of 1·12 MPa is held by a close-coiled compression spring of circular steel bar. The mean diameter is152.5 mm and the initial compression is 25·4 mm. Find the diameter of the steel bar and the number of turns necessary if the stress allowed is* 126 *MPa and* C = 79 *GPa.*

Solution

Given: D = 152.5 mm; R = 76.25 mm; δ = 25.4 mm; τ = 126 MPa = 126 N/mm^2; p = 1.12 MPa = 1.12 N/mm^2; C = 79 GPa = 79 x 10^3 N/mm^2

Force required lifting the valve $F = p\frac{\pi}{4}d_v^{\ 2} = \frac{1.12x\pi x76^2}{4} = 5080\ N$

o

We know that $\tau = \frac{16WR}{\pi d^3} \Longrightarrow d = \sqrt[3]{\frac{16WR}{\tau\pi}} = \sqrt[3]{\frac{16(5080)(76.25)}{126\pi}} = 25\ mm$

Also, $\delta = \frac{64WR^3 n}{Cd^4} \Rightarrow n = \frac{\delta Cd^4}{64WR^3} = \frac{(25.4)(79x10^3)(25)^4}{(64)(5080)(76.25)^3} = 5.4 \approx 6$

Problem 7: *A railway wagon weighing 40 kN and moving with a speed of 8 km/hr stopped by a* *buffer of 4 springs whose allowable maximum compression is 150 mm Find out the number of turns in each spring, if the diameter of the spring wire is 14 mm and the diameter of the coil is 80 mm. Assume C = 84 GPa*

Solution

Given: Weight of railway wagon, *W=40kN = 40 x10*^{3} *N;* Speed of the wagon, *v = 8 km/h =2.22 m1s;* δ = 150 mm; d = 14 mm; C = 84 GPa = 84 x 10^3 N/mm^2

K.E. of the wagon $K.E = \frac{1}{2}.\frac{Wv^2}{g} = \frac{(40x10^3)(2.22)^2}{(2)(9.81)} = 10048\ Nm$

Energy absorbed by each spring of the buffer $E = \frac{10048\ Nm}{4} = 2512\ Nm$

Also, energy absorbed = Work done $E = \frac{W\delta}{2} \Rightarrow W = \frac{2E}{\delta} = \frac{2(2512x10^3)}{150} = 33.49\ kN$

Number of coils, *n:*

Using the relation, $\delta = \frac{64WR^3n}{Cd^4} \Rightarrow n = \frac{\delta Cd^4}{64WR^3} = \frac{(150)(84x10^3)(14)^4}{(64)(33490)(40)^3} = 3.53 \approx 4$

o

Problem·8: A *weight of 200 N is dropped on to a helical spring made of* 15 *mm wire freely coiled to a mean diameter of 120 mm with 20 coils. Determine the height of drop if the instantaneous compression is 80 mm. Assume:* C = 84 *GPa*

Solution

Given: Magnitude of falling weight, *W* = 200 *N;* Diameter of wire, *d* = 15 mm; Mean diameter of coils *D* = 120 mm; δ = 80 mm Number of coils, *n =.20;* C = 84 GPa = 84 x 10^3 N/mm^2; Instantaneous compression δ = 80 mm

Height of drop

o

We have $\delta = \dfrac{64WR^3n}{Cd^4} \Rightarrow W = \dfrac{\delta Cd^4}{64R^3n} = \dfrac{(80)(84x10^3)(15)^4}{(64)(60^3)(20)} = 1230\ N$

Also, energy supplied by the impact load = Energy stored

Using the relation, $P(h+\delta) = \frac{1}{2}W\delta \Rightarrow h = \dfrac{W\delta}{2P} - \delta = \dfrac{(1230)(80)}{(2)(200)} - 80 = 166\ mm$

Problem·9: *Determine amount of compression and maximum shear stress produced when a load of 2100 N is dropped axially on a close-coiled helical spring from a height of 240 mm. The spring has 22 coils each of mean diameter 180 mm and wire diameter is* 25 *mm,* C = *84000 N/mm*^{2}.

Solution

Given: Diameter of wire, d = 25 mm; Mean diameter of coil, D = 180 mm; Number of coils, n = 22; Height of fall, h = 240 mm; Falling load, P = 2100 N; Modulus of rigidity, C = 84000 N/mm^2-.

Amount of compression

Let W_e, = Equivalent gradually applied load which shall produce the same effect as produced given falling load of 2100 N

o

Now, work done by the falling load = Work done by W_e

We have $P(h+\delta) = \frac{1}{2}W_e\delta = \dfrac{1}{2}\dfrac{\delta Cd^4}{64R^3n}.\delta \Rightarrow \delta^2 = \dfrac{128P(h+\delta)R^3n}{Cd^4}$

or, $\delta^2 = \dfrac{128(2100)(240+\delta)(90)^3(22)}{(84000)(25)^4} = 131.41\delta + 31539 \Rightarrow \delta^2 - 131.41\delta - 31539 = 0$

or, $\delta^2 - 131.41\delta - 31539 = 0 \Rightarrow \delta = \frac{1}{2}\left[131.41 \pm \sqrt{131.41^2 + 4(31539)}\right] = 255\ mm$

Maximum shear stress

Now substituting the value of 8 in the following relation, we get

$$W_e = \frac{\delta C d^4}{64R^3 n} = \frac{(255)(84x10^3)(25)^4}{(64)(90^3)(22)} = 8151\,N$$

Hence, $\tau = \frac{16W_e R}{\pi d^3} = \frac{16(8151)(90)}{\pi(25)^3} = 239\,N/mm^2$

Problem 10: *An open-coiled helical spring made from wire of circular cross-section is to carry a load of 120 N. If the wire diameter is* 8 *mm, mean coil radius is* 48 *mm, the spring is 30° and the number of turns is* 12, *calculate:*

(i) *Axial deflection;*

(ii) *Angular rotation of free end with respect to the fixed end of the spring. Take: C= 80 GPa; E= 200 GPa.*

Solution

Given: d = 8 mm; R = 48 mm; W = 120 N; α = 30^o; n = 12; C = 80 GPa = 80 x 10^3 N/mm^2; E = 200 GPa = 200 x 10^3 N/mm^2;

Axial deflection

We have

$$\delta = \frac{64WR^3 n \sec\alpha}{d^4}\left[\frac{\cos^2\alpha}{C} + \frac{\sin^2\alpha}{E}\right] = \frac{64(120)(48^3)(12)(\sec 30^o)}{8^4}\left[\frac{(\cos 30^o)^2}{80x10^3} + \frac{(\sin 30^o)^2}{200x10^3}\right] = 34.1\,mm$$

Angular rotation

$$\psi = \frac{64WR^3 n \sin\alpha}{d^4}\left[\frac{1}{C} - \frac{1}{E}\right] = \frac{64(120)(48^3)(12)(\sec 30^o)}{8^4}\left[\frac{1}{80x10^3} + \frac{1}{200x10^3}\right] = 0.648\,rad = 3.71^o$$

Problem 11: *An open-coiled helical spring consists of* 12 *coils, each of mean diameter of 60 mm, the wire forming the coil being* 6 *mm in diameter. Each coil makes an angle of 30° with the plane perpendicular to the axis of the spring.*

(i) *Determine the load required to elongate the spring by* 25 *mm and the bending and shear stresses caused by that load;*

(ii) *Calculate the axial twist that would cause a bending stress of 50 MPa in the coils. Take: E = 200 GPa, and* C = 82 *GPa,*

Solution

Given: d = 6 mm; D = 60 mm; W = 120 N; α = 30°; n = 12; δ = 2 mm; C = 82 GPa = 82 x 10^3 N/mm^2; E = 200 GPa = 200 x 10^3 N/mm^2;

o

Load

we have $\delta = \dfrac{64WR^3 n \sec\alpha}{d^4}\left[\dfrac{\cos^2\alpha}{C} + \dfrac{\sin^2\alpha}{E}\right] = \dfrac{64W(30^3)(12)(\sec 30^o)}{6^4}\left[\dfrac{(\cos 30^o)^2}{82x10^3} + \dfrac{(\sin 30^o)^2}{200x10^3}\right]$

$\Rightarrow \delta =$

Bending moment, $M = WR\sin\alpha = (116)(30)\sin 30^o = 1740\ Nmm$

Now bending stress, $\sigma_b = \dfrac{32M}{\pi d^3} = \dfrac{32(1740)}{\pi(6)^3} = 82.05\ N/mm^2$

Twisting moments about the axis of the spring, $T = WR\cos\alpha = (116)(30)\cos 30^o = 3013\ Nmm$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(3013)}{\pi(6)^3} = 71.04\ N/mm^2$$

Axial twist

Let T' = Axial torque required to cause bending stress of 50 N/mm^2.

Component of axial torque causing bending = T' cos 0.

We have: $\tau = \frac{16T'\cos\alpha}{\pi d^3} \Rightarrow T' = \frac{\tau\pi d^3}{16\cos\alpha} = \frac{(50)(\pi)(6)^4}{16\cos 30^o} = 1220\ Nmm$

Problem 12: *An open-coiled helical spring of wire diameter* 12 *mm, mean coil radius84 mm, helix angle 20° carries an axial load of 480 N. Determine the shear stress and direct stress developed at inner radius of the coil.*

Solution

Given: d = 12 mm; R = 84 mm; W = 480 N; α = 20^o; n = 12;

Shear stress; direct stress

Twisting moment, $T = WR\cos\alpha$.

Bending moment, $M = WR\sin\alpha$.

Torsional shear stress, $\tau_1 = \frac{16T}{\pi d^3} = \frac{16WR\cos\alpha}{\pi d^3} = \frac{16(480)(84)(\cos 20^o)}{\pi(12)^3} = 111.66\ N/mm^2$

Direct shear stress, $\tau_2 = \frac{W}{\frac{\pi}{4}d^2} = \frac{4WR}{\pi d^2} = \frac{4(480)}{\pi(12)^2} = 4.24\ N/mm^2$

:. Total shear stress at the inner coil radius, $\tau = \tau_1 + \tau_2 = 111.66 + 4.24 = 115.9\ N/mm^2$

Direct stress due to bending, $\sigma_b = \frac{32M}{\pi d^3} = \frac{32WR\sin\alpha}{\pi d^3} = \frac{32(480)(84)(\cos 20^o)}{\pi(12)^3} = 81.28\ N/mm^2$

Problem 13: *A composite spring has two closed-coiled springs connected in series; one spring has 12 coils of a mean diameter of* 25 *mm and wire diameter 2.5 mm. Find the wire diameter of the other spring if it has* 15 *coils of mean diameter 40 mm. The stiffness of the composite spring is* 1·5 *kN/m. Determine the greatest load that can be carried by the composite spring and the corresponding extension if the maximum stress is 250 MNlm^2*. C = *80 GNlm^2*.

Solution

Given: D_1 = 25 mm; D_2 = 40 mm; d_1 = 2.5 mm; k = 1.5 kN/m = 1.5 N/mm; τ = 250 MPa = 250 N/mm^2; C = 80 GPa = 80 x10^3 N/mm^2;

In case of spring connected in series, $\delta = \delta_1 + \delta_2 \Rightarrow \frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$

or, $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$ (i)

but, $\frac{1}{k_1} = \frac{Cd_1^{\,4}}{64R_1^{\,3}n} = \frac{(80x10^3)(2.5)^4}{64(12.5)^3(12)} = 2.083\ N/mm$

and $\frac{1}{k_2} = \frac{Cd_2^{\,4}}{64R_2^{\,3}n} = \frac{(80x10^3)d_2^{\,4}}{64(20)^3(15)} = 10.416x10^{-3}d_2^{\,4}$

From eqn. *(i)*, we get $\frac{1}{1.5} = \frac{1}{2.083} + \frac{1}{k_2} \Rightarrow k_2 = 5.39$

From which, $10.41x10^{-3}d_2^{\,4} = 5.39 \Rightarrow d_2 = \sqrt[4]{\frac{5.39}{10.41}x10^3} = 4.74\ mm$

The greatest load that can be carried by the spring will correspond to the smaller

$$\tau = \frac{16WR}{\pi d^3} \Rightarrow W = \frac{\tau\pi d^3}{16R} = \frac{(250)(\pi)(2.5)^3}{\pi(12.5)} = 61.3\ N$$

Total extension, $\delta = \frac{W}{k} = \frac{61.3}{1.5} = 40.8\ mm$

Problem 14: *A helical spring B is placed inside the coils of a second helical spring having the same number of coils and free axial length and of same material. The two springs are compressed by an axial load of 210 N which is shared between them. The mean coil diameters of A and B are 90 mm and 60 mm and the wire diameters are 12 mm and 7 mm respectively. Calculate the load taken and the maximum stress in each spring.*

Solution

Given: D_A = 90 mm; D_B = 60 mm; d_A = 12 mm; d_B = 7 mm; W = 210 N;

Let, W_A = Load shared by spring *A*,

W_B = Load shared by spring *B*,

δ_A = Deflection of spring *A*, and

δ_B = Deflection of spring *B*.

Now, $W_A + W_B = 210\ N$ (Give) ....(i)

Also, $\delta_A = \delta_B;\ \frac{64W_A R_A{}^3 n}{Cd_A{}^4} = \frac{64W_B R_B{}^3 n}{Cd_B{}^4} \Rightarrow \frac{W_A}{W_B} = \left[\frac{R_B}{R_A}\right]^3 \left[\frac{d_A}{d_B}\right]^4 = \left[\frac{30}{45}\right]^3 \left[\frac{12}{7}\right]^4 = 2.559$

or $\frac{W_A}{W_B} = 2.559 \Rightarrow W_A = 2.559W_B$

Substituting this value of W_A in (i) we get $2.559W_B + W_B = 2100 \Rightarrow W_B = \frac{210}{3.559} = 59\ N$

and $W_A = 210 - W_A = 210 - 59 = 151\ N$

Now $\tau_A = \frac{16W_A R_A}{\pi d_A{}^3} = \frac{16(151)(45)}{\pi(12)^3} = 20\ N/mm^2$

Similarly, $\tau_B = \frac{16W_B R_B}{\pi d_B{}^3} = \frac{16(59)(30)}{\pi(7)^3} = 26.28\ N/mm^2$

Problem 15: *A flat spiral spring is* 5 *mm wide, 0·25 mm thick and* 3 *metres long. Assume maximum stress of 1000 MPa to occur at the point of greatest bending moment, calculate:*

(i) *The torque;*

(ii) *The work that can be stored in the spring; and*

(iii) The number of turns required to wind up the spring. Take: E = 200 GPa

Solution

Given: Width of the strip, *b=5mm;* Thickness of the strip, t = 0·25 mm; Length of the strip, l = 3 m = 3 x10^3 mm; Maximum stress, σ_{max} = 1000 MPa = 1000 N/mm^2;

The torque

$$T = \frac{bt^2\sigma_{\max}}{12} = \frac{(5)(0.25)(1000)}{12} = 26\ Nmm$$

The work that can be stored in the spring;

$$U = \frac{\sigma^2{}_{\max}}{24E}\,x volume\ of\ \ spring = \frac{(1000)^2}{24(200x10^3)}x[(5)(0.25)(3000)] = 0.781\ Nm = 0.781\ J$$

Number of turns

We know $\phi = \frac{Tl}{EI} = \frac{(26)(3000)}{(200x10^3)[(5)(0.25)^3/12]} = 59.9\ rad$

Number of turns, $n = \frac{\phi}{2\pi} = \frac{59.9}{2\pi} = 9.533 \approx 10$

Problem 16: *A flat spiral spring is made of a strip* 6 *mm wide, 0·25 mm thick 12 mm long. The torque is applied at the winding spindle and* 9 *complete turns are given. Calculate:*

(i) The torque;

(ii) Maximum stress developed at the point of greatest bending moment;

(iii) The energy stored. *Take: E = 210 GPa*

Solution

Given: Width of the strip, b = 6 mm; Thickness of the strip, t = 0·25 mm; Length of the strip, *l* = 12 m = 12 x10^3 mm; Number of complete turns, n = 9 Young's modulus, E = 210 GPa = 210 x 10^3 N/mm^2;

The torque

Angular rotation, $\phi = 2\pi n = 2\pi(9) = 56.55\ rad$

Also, $\phi = \frac{Tl}{EI} \Rightarrow T = \frac{\phi EI}{l} = \frac{(56.55)(210x10^3)\left[(6)(0.25)^3/12\right]}{12x10^3} = 7.73\ Nmm$

Maximum stress

$$\sigma_{\max} = \frac{12T}{bt^2} = \frac{(12)(7.33)}{(6)(0.25)^2} = 247.4\ N/mm^2$$

The energy stored

$$U = \tfrac{1}{2}T\phi = \tfrac{1}{2}(7.33)(56.55) = 218.56\ Nmm$$

Problem 17: *A carriage spring is to be 600 mm long and made of* 9·5 *mm thick steel plate and* 50 *mm broad. How many plates are required to carry a load of 4·5 kN, without the stress exceeding 230 MPa. What would be central deflection and the initial radius of curvature, if plates straighten under the load? E = 200 GPa*

Solution

Span length, *l* = 600 mm; Thickness of each plate, t = 9·5 mm; Width of each plate, b = 50 mm; Load = W=4·5kN: E= 200 GPa = 200 x 10^3 N/mm^2; σ_b = 230 MPa = 230 N/mm^2;

Number of plates

We know, $M = \frac{Wl}{4N} = \sigma_b . \frac{bt^2}{6} \Rightarrow N = \frac{6Wl}{4\sigma_b bt^2} = \frac{6(4500)(600)}{4(230)(50)(9.5)^2} = 3.9 \approx 4$

Initial radius of curvature

Using the relation $\frac{\sigma_b}{y} = \frac{E}{R_c} \Rightarrow R_c = \frac{Ey}{\sigma_b} = \frac{Et}{2\sigma_b} = \frac{(200x10^3)(9.5)}{2(230)} = 4130\ mm$

Central deflection

We have $\delta = \frac{3Wl^3}{8Enbt^3} = \frac{3(4.5x10^3)(600)^3}{8(200x10^3)(4)(50)(9.5)^3} = 10.6\ mm$

Problem 18: *A laminated steel spring* 1 *m long is to support central load of* 5·8 *kN. If the maximum deflection of spring is not to exceed* 45 *mm and maximum stress should not exceed 300 MPa. Calculate:*

i. *The thickness of the leaves;*
ii. *Their number if each plate is to be* 80 *mm wide. Take: E = 200 GPa*

Solution

Given: l = 1000 mm; b = 80 mm; W = 5·8 kN = 5800 N; δ = 45 mm; σ_b = 300 MPa = 300 N/mm^2;

We know that, $\sigma_b = \frac{3Wl}{2nbt^2} \Rightarrow nt^2 = \frac{3Wl^3}{2b\sigma_b} = \frac{3(5800)(1000)}{2(80)(300)} = 362.5$

Also, $\delta = \frac{3Wl^3}{8Enbt^3} \Rightarrow nt^3 = \frac{3Wl^3}{8Eb\delta} = \frac{3(5800)(1000)^3}{8(200x10^3)(80)(45)} = 3020.8$

Thickness

Dividing *(ii)* by (i), we get $\frac{nt^3}{nt^2} = \frac{3020.8}{362.5} \Rightarrow t = 8.33\ mm$

Number of plates

Substituting the value of t in (i), we have $nt^2 = 362.5 \Rightarrow n = \frac{362.5}{t^2} = \frac{362.5}{8.33^2} = 5.22 \approx 6$

Problem 19: *A leaf spring of semi-elliptic type has 11 plates each 9 cm wide and 1.5 cm thick. The length of spring is 1.5 m. The plates are made of steel having a proof stress (bending) 650 MPa. To what radius should the plates be bent initially? From what height can a load of 600 N fall on to centre of the spring, if maximum stress is to be one-half of the proof stress? E = 200 GPa*

Solution

Given: l = 1500 mm; b = 90 mm; t = 15 mm; W = 600 N; δ = 45 mm; N = 11; σ_b = 650 MPa = 650 N/mm^2; E = 200 GPa = 200 x 10^3 N/mm^2;

Initial Radius

The bending equation is given by,

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R_c} \Rightarrow R_c = \frac{Ey}{\sigma_b} = \frac{Et}{2\sigma_b} = \frac{(200x10^3)(15)}{2(650)} = 2310\ mm$$

Height of drop

The stress in the second case is *half* the proof stress $\sigma_b' = \frac{\sigma_b}{2} = \frac{650}{2} = 325\ N/mm^2$

But $\sigma_b' = \frac{3Wl}{2nbt^2} \Rightarrow W = \frac{2\sigma_b' bnt^2}{3l} = \frac{2(325)(90)(11)(15)^2}{3(1500)} = 32175\ N$

o

We have $\delta = \frac{3Wl^3}{8Enbt^3} = \frac{3(32175)(1500)^3}{8(200x10^3)(11)(90)(15)^3} = 61\ mm$

Also, energy supplied by the impact load = Energy stored

Using the relation, $P(h+\delta)=\frac{1}{2}W\delta \Rightarrow h=\frac{W\delta}{2P}-\delta=\frac{(32175)(61)}{(2)(600)}-61=1574\ mm$

Problem 20: *A quarter elliptical spring has a length of* 50 *cm and consists of plates each 6 cm wide and 0·6 cm thick. Find the least number of plates which can be used if deflection under axially applied load of* 3 *kN is not to exceed* 8 *cm. E = 200 GPa^2.*

Solution

Given: l = 500 mm; b = 60 mm; t = 6 mm; W = 3 kN = 3000 N; δ = 80 mm; E = 200 GPa = 200 x 10^3 N/mm^2;

Least number of plates

Using the relation, $\delta=\frac{6Wl^3}{Enbt^3} \Rightarrow n=\frac{6Wl^3}{\delta Ebt^3}=\frac{3(3000)(500)^3}{(80)(200x10^3)(60)(6)^3}=10.85\approx 11$

Problem 21: *A quarter-elliptical leaf spring has a length of 600 mm and consists of plates each* 50 *mm wide and* 6 *mm thick.*

(i) Determine the least number of plates which can be used if the deflection under a gradually applied load of 1.8 kN is not to exceed 80 mm.

(ii) If the applied load of 1.8 kN, instead of being gradually applied, falls a distance of 6 mm on to the undeflected spring, find the maximum deflection and stress produced. Take: E = 200 GPa

Solution

Given: l = 600 mm; b = 50 mm; t = 6 mm; h = 6 mm; W = 1.8 kN = 1800 N; δ = 80 mm; E = 200 GPa = 200 x 10^3 N/mm^2;

Number of plates

We know, $\delta = \dfrac{6Wl^3}{Enbt^3} \Rightarrow n = \dfrac{6Wl^3}{\delta Ebt^3} = \dfrac{3(1800)(600)^3}{(80)(200x10^3)(50)(6)^3} = 13.5 \approx 14$

Let, W_e = Equivalent gradually applied load which would produce the same caused by the impact load.

We know, $\delta' = \dfrac{6W_e l^3}{Enbt^3} \Rightarrow W_e = \dfrac{\delta' Ebnt^3}{6l^3} = \dfrac{\delta'(200x10^3)(50)(14)(6)^3 n}{6(600)^3} = 23.33\delta'$

Loss of potential energy $PE = W(h+\delta') = 1800(6+\delta')$ (i)

Strain energy absorbed by the spring $SE = \frac{1}{2}W_e\delta' = \frac{1}{2}(23.33\delta')\delta' = 11.667\delta'^2.$ (ii)

Equating (i) and (ii), we get $11.667\delta'^2. = 1800(6+\delta')$

Then $\delta'^2 - 154\delta' - 9257 \Rightarrow \delta' = \frac{1}{2}\left[154 \pm \sqrt{154^2 + 4(9257)}\right] = 159.8\ mm$

Maximum stress

We have $W_e = 23.33\delta' = 23.33(159.8) = 3728\ N$

Using the relation, $\sigma_{\max} = \dfrac{6W_e l}{nbt^2} = \dfrac{6(3728)(600)}{(14)(50)(6)^2} = 532.57\ N/mm^2$

TYPE B: OBJECTIVES

SECTION A: CHOOSE THE CORRECT OPTION

1. If a close coiled helical spring id subjected to load W and the deflection produced is δ, then stiffness of the spring is given by
   (a) W/δ (b) δ/W
   (c) $W.\delta$ (d) $W^2.\delta$

2. Wahl's connection factor (K) is given by the relation
   (a) $K = \dfrac{3S-1}{3S+4} + \dfrac{0.615}{S}$
   (b) $K = \dfrac{4S-1}{4S+4} + \dfrac{0.615}{S}$

(c) $K = \frac{3S-1}{3S+4} + \frac{0.415}{S}$

(d) $K = \frac{4S-1}{4S+4} + \frac{0.415}{S}$

3. The energy stored in a close coiled helical spring when subjected to an axial twist is given by

(a) $\frac{\sigma_b^{\ 2}}{6E} x\ volume\ of\ spring$

(b) $\frac{\sigma_b^{\ 2}}{8E} x\ volume\ of\ spring$

(c) $\frac{\sigma_b^{\ 2}}{4E} x\ volume\ of\ spring$

(d) $\frac{\sigma_b^{\ 2}}{2E} x\ volume\ of\ spring$

4. Two springs of stiffness k_1 and k_2 respectively are connected in series, the stiffness of the composite spring (k) will be given by

(a) $k = k_1 + k_2$ (b) $k = k_1 x k_2$

(c) $k = \frac{k_1 k_2}{k_1 + k_2}$ (d) $k = \frac{k_1 + k_2}{k_1 k_2}$

5. The resilience of a flat spiral spring is given by

(a) $\frac{\sigma_{\max}}{24E}$ (b) $\frac{\sigma^2_{\max}}{24E}$

(c) $\frac{\sigma_{\max}}{12E}$ (d) $\frac{\sigma^2_{\max}}{12E}$

6. In case of a laminated spring the load at which the plates become straight is called

(a) working load (b) safe load

(c) proof load (d) none of the above.

7. ....are called cantilever laminated springs.

(a) semi-elliptical springs

(b) quarter elliptical springs

(c) both (a) and (b)

(d) None of the above

8. In a leaf spring, maximum bending stress developed in the plates is

(a) $\frac{Wl}{nbt^2}$ (b) $\frac{2Wl}{nbt^2}$

(c) $\frac{3Wl}{nbt^2}$ (d) $\frac{3Wl}{2nbt^2}$

9. The maximum deflection at the centre of a leaf spring is

(a) $\frac{\sigma_b l}{Et}$ (b) $\frac{\sigma_b l^2}{2Et}$

(c) $\frac{\sigma_b l^2}{3Et}$ (d) $\frac{\sigma_b l^2}{4Et}$

10. When a closely coiled spring is subjected to an axial load, it is said to be under"

(a) bending (b) shear

(c) torsion (d) all of these

11. The deflection of a closely-coiled helical spring of diameter *(D)* subjected to an axial load is

(a) $\frac{64WR^3n}{Cd^4}$ (b) $\frac{64WR^2n}{Cd^4}$

(c) $\frac{64WRn}{Cd^4}$ (d) $\frac{64WRn^2}{Cd^4}$

12. A laminated spring 1 m long is built in 100 mm x 10 mm plates. If the spring is to carry a load of 10 kN at its centre, determine the number of plates required for the spring. Take allowable bending stress as 150 MPa.

(a) 9 (b) 10

(c) 11 (d) 12

13. A carriage spring 800 mm long is made of 12 plates of 40 mm width. Determine the thickness of the plates, if bending stress is not to exceed 200 MPa and

spring is to carry a load of 6 kN at its centre. Take E as 200 GPa

(a) 7 mm (b) 8 mm
(c) 9 mm (d) 10 mm

14. A carriage spring 800 mm long is made of 12 plates of 40 mm width. Determine the central deflection of the plates, if bending stress is not to exceed 200 MPa and spring is to carry a load of 6 kN at its centre. Take E as 200 GPa.
    (a) 16.5 mm (b) 15.5 mm
    (c) 17 5 mm (d) 14 .5 mm

A laminated spring of the quarter elliptic type 600 mm long is to provide a deflection of 75 mm under an end load of 1960 N and the leaf material is 60 mm wide and 6 mm thick. Use this information to answer questions 15 and 16.

15. Find the number of leaves required
    (a) 12 (b) 14
    (c) 15 (d) 13

16. Find the maximum stress
    (a) 242 N/mm^2 (b) 252 N/mm^2
    (c) 232 N/mm^2 (d) 262 N/mm^2

17. A closely coiled helical spring of mean diameter 140 mm is made up of 12 mm diameter steel wire. Calculate the direct axial load the spring can carry if the maximum stress is not to exceed 100 MPa
    (a) 474 N (b) 484 N
    (c) 494 N (d) 464 N

A closely coiled helical spring is made of 6 mm wire. The maximum shear stress and deflection under a 200 N load is not to exceed 80 MPa and 11 mm respectively. Take C = 84 MPa. Use this information to answer questions 18 and 19.

18. Determine the no. of coils
    (a) 17 (b) 18
    (c) 19 (d) 20

19. Find their mean diameter.
    (a) 35 mm (b) 34 mm
    (c) 33 mm (d) 32 mm

20. A open coil helical spring made of 10 mm diameter wire has 15 coils of 50 mm radius with a 20^o angle of helix. Determine the deflection of the spring when subjected to an axial load of 300 N. Take E = 200 GPa and G= 80 GPa.
    (a) 49.4 mm (b) 48.4 mm
    (c) 47.4 mm (d) 46.4 mm

A leaf spring 1 m long is made up with steel plates with width equal to 6 times its thickness. The spring is subjected to a load of 15 kN when the maximum stress is 100 MPa and deflection not to exceed 16 mm. Use this information to answer questions 21 to 23.

21. Determine the thickness of the leaf spring
    (a) 10.5 mm (b) 11.5 mm
    (c) 12.5 mm (d) 13.5 mm

22. Calculate the width of the springs
    (a) 65 mm (b) 85 mm
    (c) 95 mm (d) 75 mm

23. Find the number of plates
    (a) 12 (b) 13
    (c) 14 (d) 15

TYPE C: ESSAY

1. Derive from first principles, making usual assumptions the formula for the maximum stress and or the central deflection of a leaf spring consisting of *n* leaves and subjected to central load.

2. What are helical springs? Differentiate between a closely coiled helical spring and an coiled helical spring.

3. A closely coiled helical spring with *D* as diameter of the coil and *d* as diameter of the wire subjected to an axial load *W*. Prove that the maximum shear stress produced is equal to $\frac{8WD}{\pi d^3}$

4. Derive an equation for the deflection of an open coiled helical spring.

Unit 6

THIN SHELLS

Introduction

This unit deals with the derivation and computation of axial and tangential stresses and strains in different types of vessels.

Learning Objectives

After reading this unit you should be able to:

1. Derive the equations for axial and tangential stresses in cylindrical and spherical vessels
2. Compute the a axial and tangential stresses using the derived equations
3. Derive the equations for axial and tangential strains in cylindrical and spherical vessels
4. Compute the a axial and tangential strains using the derived equations

IMPORTANT EQUATIONS

1. Stresses in a Thin Cylindrical Shell

$$\sigma_c = \frac{pd}{2t}$$

$$\sigma_c = \frac{pd}{2t\eta}$$

$$\sigma_l = \frac{pd}{4t}$$

$$\sigma_l = \frac{pd}{4t\eta}$$

2. Design of Thin Cylindrical Shells

$$t = \frac{pd}{2\sigma_c}$$

3. Change in Dimensions due to Internal Pressure

$$\delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$\delta l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

4. Change in Volume due to an Internal Pressure
   $\delta V = V(\varepsilon_l + 2\varepsilon_c)$
5. Stresses in Thin Spherical Shells

$$\sigma = \frac{pd}{4t}$$

$$\sigma = \frac{pd}{4t\eta}$$

6. Change in Diameter and Volume due to an Internal Pressure

$$\delta d = \varepsilon . d = \frac{pd^2}{4tE}\left(1 - \frac{1}{m}\right)$$

$$\delta V = \frac{\pi p d^4}{8tE}\left(1 - \frac{1}{m}\right)$$

7. Bulk Modulus

$$K = \frac{-p}{\delta V / V}$$

TYPE A: SOLVED EXAMPLES

Problem 1: *A thin cylindrical shell of diameter 300 mm and wall thickness 6 mm has hemispherical ends. If there is no distortion of the junction under pressure, determine the thickness of the spherical ends. Take Young's modulus of elasticity to be 208 GPa and Poisson ratio 0.3*

Solution

Given: d = 300 mm; t_1 = 6 mm; E = 208 GPa; 1/m = 0.3

$$\frac{1}{m} = 0.3 \Rightarrow m = \frac{1}{0.3} = 3.33$$

6.

Let t_2 be the thickness of hemispherical ends.

For no distortion at the junction

$$\frac{t_2}{t_1} . = \frac{m-1}{2m-1} = \frac{33.3-1}{2x33.3-1} = 0.4117 \Rightarrow t_2 = 0.4117 t_1 = 0.4117 x 6 = 2.47\ mm$$

7.

Problem 2: *Calculate the bursting pressure for cold drawn seamless steel tubing of 60 mm diameter with* 2 *mm wall thickness. The ultimate strength of steel is 380 N/mm^2.*

Solution

Given: d = 60 mm; t = 2 mm; σ_c = 380 N/mm^2

$$\sigma_c = 380 = \frac{pd}{2t}. = \frac{60p}{2x2} = 15p \Rightarrow p = \frac{380}{15} = 25.33\ N/mm^2$$

***Problem 3**: Calculate the thickness of the metal required for a cast-iron main 800 mm in water at a pressure head of 100 m if the maximum permissible tensile stress* is *20 N/mm of water is 10 kNlm^3.*

Solution

Given: d = 800 mm; h = 100 m = 100 x 10^3 mm; σ_c = 20 N/mm^2; w = 10 kN/m^3 = 10 x 10^{-6} N/mm^3;

The pressure, $p = wh = \left(10x10^{-6}\right)x\left(100x10^{3}\right) = 1\ N/mm^2$

$$\sigma_c = 20 = \frac{pd}{2t}. = \frac{60x1}{2t} = \frac{60}{t} \Rightarrow t = \frac{60}{20} = 3\ mm$$

***Problem·4:** A cylindrical water tank of height 25 m, inside diameter 2·2 m, having the vertical axis is open at the top. The tank is made of steel having yield stress of 210 N/mm^2, thickness of steel used when the tank is full of water. Take the efficiency of the circumferential joint =* 70 %; *Factor of safety =* 3.

Solution

Given: d = 2.2 m = 2.2 x 10^3 mm; h = 25 m = 25 x 10^3 mm; σ_t = 210 N/mm^2; w = 10 kN/m^3 = 10 x 10^{-6} N/mm^3; N = 3; η_l = 0.7

The maximum pressure will be at the base; therefore, this pressure must be resisted by steel therefore the thickness must be determined by taking the maximum pressure into consideration. The pressure at the base, $p = wh = \left(10x10^{-6}\right)x\left(25x10^{3}\right) = 0.25\ N/mm^2$

Working stress, $\sigma_c = 20 = \frac{Yield\ \ Stress}{Factor\ \ of\ \ safety}. = \frac{\sigma_t}{N} = \frac{210}{3} = 70\ N/mm^2$

Since the tank is open at the top, there will only be hoop (or circumferential) under any circumstances, should not exceed the working stress.

We know, $\sigma_c = 70 = \frac{pd}{2t\eta_l}. = \frac{(2.2x10^3)x(0.25)}{2x0.7xt} = \frac{392.85}{t} \Rightarrow t = \frac{392.85}{70} = 5.6\ mm$

Problem 5: *A boiler shell is to be made of 15 mm thick plate having tensile 120 N/mm². If the efficiencies of the circumferential and longitudinal joints are 70% and 30% respectively, determine:*

(i) *Maximum permissible diameter of the shell for an internal pressure of 2 N/mm²;*

(ii) *Permissible intensity of internal pressure when the shell diameter is* 1.5 m

Solution

Given: t = 15 mm; σ = 120 N/mm²; η_c = 0.7; η_l = 0.3;

(i) Maximum Permissible Diameter

Let, $d =$ Maximum permissible diameter.

P = 2 N/mm²

Consider circumferential stress as 120 N/mm²,

$$\sigma_c = 120 = \frac{pd}{2t\eta_c}. = \frac{2xd}{2x15x0.7} = 0.095d \Rightarrow d = \frac{120}{0.095} = 1.26x10^3\ mm$$

Consider longitudinal stress as 120 N/mm²,

$$\sigma_l = 120 = \frac{pd}{4t\eta_l}. = \frac{2xd}{4x15x0.3} = 0.111d \Rightarrow d = \frac{120}{0.111} = 1.08x10^3\ mm$$

In order to satisfy both the conditions, ***d* = 1·08 x10³ mm** *(minimum* of the values)

Note: If we provide bigger diameter *(i.e.* 1·26 x10^3 mm) then the longitudinal stress

$$\sigma_l = \frac{pd}{4t\eta_l} . = \frac{2x\left(1.26x10^3\right)}{4x15x0.3} = 140\ N/mm^2$$ be more than the permissible stress (120 N/mm^2*).*

(ii) Permissible Intensity of Internal Pressure

Let, *p = Permissible Intensity of Internal Pressure*

d = 1.5 m = 1.5 x10^3 mm

Consider circumferential stress as 120 N/mm^2,

$$\sigma_c = 120 = \frac{pd}{2t\eta_c} . = \frac{px1.5x10^3}{2x15x0.7} = 71.43p \Rightarrow p = \frac{120}{71.43} = 1.68\ N/mm^2$$

Consider longitudinal stress as 120 N/mm^2,

$$\sigma_l = 120 = \frac{pd}{4t\eta_l} . = \frac{px1.5x10^3}{4x15x0.3} = 83.33p \Rightarrow p = \frac{120}{83.33} = 1.44\ N/mm^2$$

Hence, the permissible intensity of pressure, p **= 1·44** N/mm^2 (minimum of two values)

Problem 6: *A cylindrical air drum is* 2·25 *m in diameter with plates* 1·2 *cm thick. The efficiency of the circumferential and longitudinal joints are respectively* 75% *and* 40%. *If the tensile stress in the plating is to be limited to 120 N/mm*^{2} *find the maximum safe air pressure.*

Solution

Given: d = 2.25 m = 2.25 x10^3 mm; t = 1.2 cm = 12 mm; σ = 120 N/mm^2; η_c = 0.75; η_l = 0.4;

Let, *p = Permissible Intensity of Internal Pressure*

Consider circumferential stress as 120 N/mm^2,

$$\sigma_c = 120 = \frac{pd}{2t\eta_c} . = \frac{px2.25x10^3}{2x12x0.75} = 125p \Rightarrow p = \frac{120}{125} = 0.96\ N/mm^2$$

Consider longitudinal stress as 120 N/mm^2,

$$\sigma_l = 120 = \frac{pd}{4t\eta_l} . = \frac{px2.25x10^3}{4x12x0.4} = 117.2p \Rightarrow p = \frac{120}{117.2} = 1.024\ N/mm^2$$

Hence, the permissible intensity of pressure, p **= 0.96** N/mm^2 (minimum of two values)

8.

Note: If we provide larger pressure (i.e.1024 N/mm^2) then the circumferential stress

$\sigma_c = \frac{pd}{2t\eta_c} . = \frac{1.024x2.25x10^3}{2x12x0.75} = 128\ N/mm^2$, will be more than the permissible stress (120 N/mm^2*)*

9.

Problem 7: *A cylindrical vessel whose ends are closed by means of rigid flange of steel plate 3 mm thick. The internal length and diameter of vessel are 50 cm and 25 cm. Determine the longitudinal and circumferential stresses in the cylindrical shell due to an internal pressure of 3 N/mm^2. Also calculate increase in length, diameter and volume of the vessel. Take: E = 200 GPa and 1/m = 0·3.*

Solution

Given; *l* = 50 cm = 500 mm; t = 3 mm; d = 25 cm = 250 mm; p = 3 N/mm^2; E = 200 GPa = 200 x 10^3 N/mm^2; 1/m = 0.3

(i) Stresses in the cylinder shell

Circumferential or hoop stress, $\sigma_c = \frac{pd}{2t} . = \frac{3x250}{2x3} = 125\ N/mm^2$

Longitudinal stress, $\sigma_l = \frac{pd}{4t} . = \frac{3x250}{4x3} = 62.5\ N/mm^2$

(ii) Change in Dimensions

Let δd Change in diameter

δl Change in length

ε_c Circumferential strain

ε_l Longitudinal strain

Then, $\varepsilon_c = \frac{\sigma_c}{E} - \frac{\sigma_l}{mE} . = \frac{125}{200x10^3} - \frac{0.3x62.5}{200x10^3} . = 5.312x10^{-4}$

Therefore, $\delta d = d\varepsilon_c = 250x(5.312x10^{-4}) = 0.133\ mm$

Similarly, $\varepsilon_l = \frac{\sigma_l}{E} - \frac{\sigma_c}{mE} . = \frac{62.5}{200x10^3} - \frac{0.3x125}{200x10^3} . = 1.25x10^{-4}$

Therefore, $\delta l = l\varepsilon_l = 500x(1.25x10^{-4}) = 0.0.0625\ mm$

Also, $\varepsilon_v = \varepsilon_l + 2\varepsilon_c = [1.25 + (2x5.312)]x10^{-4} = 1.19x10^{-3}$

Therefore, $\delta V = V\varepsilon_v = \frac{\pi}{4} x250^2 x500x(1.19x10^{-3}) = 29.15\ mm^3$

Problem·8: A *built up cylindrical shell of 300 mm diameter,* 3 *m long and 6 mm thick subjected to an internal pressure of 2 GPa. Calculate the change in length, diameter and volume of the cylinder under that pressure if the efficiencies of the circumferential and longitudinal joint are 80% and 50% respectively. Take E = 200 GPa; m =* 3·5

Solution

Given; l = 3 m = 3x10^3 mm; t = 6 mm; d = 300 mm; p = 2 GPa = 2 N/mm^2; E = 200 GPa = 200 x 10^3 N/mm^2; m = 3.5; η_c = 0.8; η_l = 0.5

(i) *Stresses in the cylinder shell*

Circumferential or hoop stress, $\sigma_c = \frac{pd}{2t\eta_c} . = \frac{2x300}{2x6x0.8} = 62.5\ N/mm^2$

Longitudinal stress, $\sigma_l = \frac{pd}{4t\eta_l} . = \frac{2x300}{4x6x0.5} = 50\ N/mm^2$

(ii) *Change in Dimensions*

Let δd Change in diameter

δl Change in length

ε_c Circumferential strain

ε_l Longitudinal strain

Then, $\varepsilon_c = \frac{\sigma_c}{E} - \frac{\sigma_l}{mE}. = \frac{62.5}{200x10^3} - \frac{0.3x50}{200x10^3}. = 2.41x10^{-4}$

Therefore, $\delta d = d\varepsilon_c = 300x\left(2.41x10^{-4}\right) = 0.0723\, mm$

Similarly, $\varepsilon_l = \frac{\sigma_l}{E} - \frac{\sigma_c}{mE}. = \frac{25}{200x10^3} - \frac{0.3x50}{200x10^3}. = 1.16x10^{-4}$

Therefore, $\delta l = l\varepsilon_l = 3x10^3\, x\left(1.61x10^{-4}\right) = 0.483\, mm$

Also, $\varepsilon_v = \varepsilon_l + 2\varepsilon_c = \left[1.61 + (2x2.41)\right]x10^{-5} = 6.43x10^{-4}$

Therefore, $\delta V = V\varepsilon_v = \frac{\pi}{4}x(300)^2 x\left(3x10^3\right)x\left(4.75x10^{-4}\right) = 136.4x10^3\ mm^3$

Problem 9: *A copper cylinder,* 90 *cm long,* 40 *cm external diameter and wall 6 mm has its both ends closed by rigid blank flanges. It is initially full of oil at atmospheric Calculate the additional volume of oil which must be pumped into it in order to raise the 5 MPa above atmospheric pressure. For copper, assume E = 100 GPa and Poisson's = 1/3. Take bulk modulus of oil as* 2·6 *GPa*

Solution

Given: l = 90 cm = 900 mm; d_o = 40 cm = 400 mm; p = 5 MPa = 5 N/mm^2; K_{oil} = 2.6 GPa = 2.6 x 10^3 N/mm^2; E_c = 100 GPa = 100 x 10^3; 1/m = 1/3;

Initial Volume, $V = \frac{\pi}{4}xd^2xl = \frac{\pi}{4}(400 - 2x6)^2 x(900) = 106.4x10^6\ mm^3$

We know that $\frac{\delta V_1}{V} = \frac{pd}{2tE}\left[\frac{5}{2} - \frac{2}{m}\right] = \frac{5x[400 - 2(6)]}{2x6x\left(100x10^3\right)}\left[\frac{5}{2} - \frac{2}{3}\right] = 2.964x10^{-3}$

Therefore, the increase in volume,

$$\delta V = 2.964x10^{-3}V = \left(2.964x10^{-3}\right)x\left(106.4x10^{6}\right) = 315.4x10^{3}\ mm^{3}$$

In addition to this, the liquid will shrink in volume as follows:

$$P = 5 = K\frac{\delta V_2}{V} = \frac{2.6x10^{3}}{106.4x10^{6}}\delta V_2 \Rightarrow \delta V_2 = \frac{5x\left(106.4x10^{6}\right)}{2.6x10^{3}} = 204.67x10^{3}\ mm^{3}$$

Therefore, the net addition of oil which must be pumped into the shell,

$$\delta V = \delta V_1 + \delta V_2 = \left(315x10^{3}\right) + \left(204.67x10^{3}\right) = 520.12x10^{3}\ mm^{3}$$

Problem 10: *A boiler drum consists of a cylindrical portion 4 m long, 1·5 m in diameter and 2.25 cm thick. It is closed by hemispherical ends. In a hydraulic test to 6 MPa, how much water will be pumped in after initial filling at atmospheric pressure? The circumferential strain at the junction of the cylinder and hemisphere may be assumed as same for both. Take = 200 GPa, K (for water) = 2·13 GPa , and 1/m = 0·3*

Solution

Given: l = 4 m = 4 x 10^3 mm; d = 1.5 m = 15 x 10^3 mm; t = 2.25 cm = 22.5 mm; p = 6 MPa = 6 N/mm^2; E = 200 GPa = 200 x 10^3 N/mm^2; 1/m = 0.3; K = 2.13 GPa = 2.13 x 10^3 N/mm^2

For cylindrical portion

Circumferential or hoop stress, $\sigma_c = \frac{pd}{2t}. = \frac{6x\left(1.5x10^{3}\right)}{2x22.5} = 200\ N/mm^{2}$

Longitudinal stress, $\sigma_l = \frac{pd}{4t}. = \frac{6x\left(1.5x10^{3}\right)}{4x22.5} = 100\ N/mm^{2}$

The volume, $V = \frac{\pi}{4}xd^{2}xl = \frac{\pi}{4}x\left(1.5x10^{3}\right)^{2}x\left(4x10^{3}\right) = 7.07x10^{9}\ mm^{3}$

Circumferential or hoop strain, $\varepsilon_c = \frac{\sigma_c}{E}\left(1 - \frac{1}{2m}\right) = \frac{200}{\left(200x10^{3}\right)}\left(1 - \frac{0.3}{2}\right) = 8.5x10^{-4}$

Longitudinal strain, $\varepsilon_l = \frac{\sigma_l}{E}\left(1 - \frac{2}{m}\right) = \frac{100}{\left(200x10^{3}\right)}\left(1 - 0.3x2\right) = 2x10^{-4}$

Change in Volume, $\delta V_1 = 20x10^3 = V(\varepsilon_l + 2\varepsilon_c) = (7.07x10^9)x(10^{-5})(2 + 2x8.5) = 13.43x10^6 \ mm^3$

For Hemispherical ends

The circumferential strain at the junction of the cylinder and hemisphere may be assumed as same for both.

$$\varepsilon = \varepsilon_l = \varepsilon_c \Rightarrow \varepsilon_v = 3\varepsilon_c = 3x(8.5x10^{-3}) = 25.5x10^{-3}$$

The volume, $V = \frac{4\pi}{3}x\left(\frac{d}{2}\right)^3 = \frac{4\pi}{3}x\left(\frac{1.5x10^3}{2}\right)^2 = 1.77x10^9 \ mm^3$

Change in Volume, $\delta V_2 == V\varepsilon_v = (1.77x10^9)x(25.5x10^{-4}) = 4.5x10^6 \ mm^3$

Decrease in Volume due to water pressure,

$$P = 6 = K\frac{\delta V_2}{V} = \frac{2.13x10^3}{106.4x10^6}\delta V_3 \Rightarrow \delta V_3 = \frac{6x(7.07 + 1.77)(10^9)}{2.13x10^3} = 24.9x10^6 \ mm^3$$

The total additional volume of water

$$\delta V = \delta V_1 + \delta V_2 + \delta V_3 = (13.43 + 4.5 + 24.9)(10^6) = 42.83x10^6 \ mm^3$$

Problem 11: *A cast iron cylinder of 200 mm inner diameter and 12·5 mm thick is closed wound with a layer of 4 mm diameter steel wire under a tensile stress of 55 MPa. Determine set up in the cylinder and steel wire if water under a pressure of 3 MPa is admitted in the Take E_c = 100 GPa, E_s = 200 GPa and Poisson's ratio = 0·25*

Solution

Given: d = 200 mm; t = 12.5 mm; d_w = 4 mm; σ_w = 55 MPa = 55 N/mm^2; p = 3 MPa = 3 N/mm^2; E_{ci} = E_C = 100 GPa = 100x10^3 N/mm^2; E_s = E_w = 200 GPa = 200x10^3 N/mm^2; 1/m = 0.25;

Stresses set up in the cylinder and steel wire

Before admitting water into the cylinder:

Tensile force exerted by wire per unit length = Compressive force developed in the cylinder

$$2x\frac{\pi d_w^{\ 2}}{4}x\sigma_w xn = 2xtx1x\sigma_c \text{ but } n = \frac{1}{d_w} \Rightarrow \sigma_c = \frac{\pi d_w}{4t}x\sigma_w$$

Therefore, the initial circumferential compression in cylinder,

$$\sigma_C = \frac{\pi d_w}{4t}x\sigma_w = \frac{\pi x4x55}{4x12.5} = 13.82\ N/mm^2$$

After admitting water into the cylinder:

Due to internal pressure, the longitudinal stress developed in the cylinder,

$$\sigma_l' = \frac{pd}{4t} = \frac{3x200}{4x12.5} = 12\ N/mm^2$$

Total bursting force = Total resisting force per unit length,

$$pxdx1 = \sigma_c' x2tx1 + \sigma_w' x2x\frac{\pi}{4}xd_w^{\ 2}xn \Rightarrow pd = 2t\sigma_c' + \frac{\pi}{2}d_w\sigma_w'$$

$$3x200 = 2x12.5x\sigma_c' + \frac{\pi}{2}x4x\sigma_w' \Rightarrow 25\sigma_c' + 2\pi\sigma_w' = 600 \qquad \text{(i)}$$

Circumferential strain in cylinder = Circumferential strain in wire

$$\frac{\sigma_c'}{E_C} - \frac{\sigma_l'}{mE_C} = \frac{\sigma_w'}{E_w} \Rightarrow \frac{\sigma_c'}{100x10^3} - \frac{12x0.25}{100x10^3} = \frac{\sigma_w'}{200x10^3} \quad \therefore 2\sigma_c' - 6 = \sigma_w'$$

Substituting the value of σ'_w in eqn. (i), we get

10. $25\sigma_c' + 2\pi\sigma_w' = 600 \Rightarrow 25\sigma_c' + 2\pi(2\sigma_c' - 6) = 600$

Or, $(25 + 4\pi)\sigma_c' = 600 + 12\pi \Rightarrow \sigma_c' = \frac{637.7}{37.6} = 17\ N/mm^2$

Therefore, $\sigma'_w = 2\sigma'_c - 6 = 2x17 - 6 = 28\ N/mm^2$

Resultant stress in the cylinder, $\sigma = \sigma'_c - \sigma_c = 17 - 13.8 = 3.2\ N/mm^2$

Resultant stress in the wire, $\sigma = \sigma_w + \sigma'_w = 55 + 28 = 83\ N/mm^2$

Problem 12*: A gun metal tube of 100 mm bore, wall thickness* 2·5 *mm is closely wound externally by a steel wire* 1 *mm diameter. Determine the tension under which the wire must be wound on the tube, if an internal radial pressure of* 3 *MPa is required before the tube is subjected to the tensile stress in the circumferential direction. Take E (For gun metal)* = 102 *GPa, E (For steel)* = 210 *GPa and 1/m* = 0·35 *m*

Solution

Given; d = 100 mm; t = 2·5 mm; d_w = 1 mm; p = 3 MPa = 3 x 10^3 N/mm^2; E (For gun metal) = 102 GPa = 102 x 10^3 N/mm^2; E (For steel) = 210 GPa = 210 x 10^3 N/mm^2; 1/m = 0·35 m
11.

Tension under which wire must be wound

Before subjecting the tube to internal pressure

Tensile force exerted by wire per unit length = Compressive force developed in the

$$2x\frac{\pi d_w{}^2}{4}x\sigma_w x n = 2xtx1x\sigma_c \ but\ n = \frac{1}{d_w} \Rightarrow \sigma_c = \frac{\pi d_w}{4t}x\sigma_w$$

Or $\sigma_C = \frac{\pi d_w}{4t}x\sigma_w = \frac{\pi x1x\sigma_w}{4x2.5} = 0.314\sigma_w$ (i)

After subjecting the tube to internal pressure

Due to internal pressure, the longitudinal stress developed in the cylinder,

$$\sigma'_l = \frac{pd}{4t} = \frac{3x100}{4x2.5} = 30\ N/mm^2$$

Total bursting force = Total resisting force per unit length (for equilibrium)

$$pxdx1 = \sigma'_c x2tx1 + \sigma'_w x2x\frac{\pi}{4}xd_w{}^2xn \Rightarrow pd = 2t\sigma'_c + \frac{\pi}{2}d_w\sigma'_w$$

$$3x100 = 2x2.5x\sigma'_c + \frac{\pi}{2}x1x\sigma'_w \Rightarrow 5\sigma'_c + 1.57\sigma'_w = 300 \qquad \text{(ii)}$$

Circumferential strain in cylinder = Circumferential strain in wire

$$\frac{\sigma'_c}{E_C} - \frac{\sigma'_l}{mE_C} = \frac{\sigma'_w}{E_w} \Rightarrow \frac{\sigma'_c}{102x10^3} - \frac{30x0.35}{102x10^3} = \frac{\sigma'_w}{210x10^3} \quad \therefore 2.059\sigma'_c - 21.62 = \sigma'_w$$

Substituting the value of σ'_w in eqn. (ii), we get

12.

$$5\sigma'_c + 1.57\sigma'_w = 300 \Rightarrow 5\sigma'_c + 1.57(2.059\sigma'_c - 21.62) = 300$$

Or, $8.23\sigma'_c = 334 \Rightarrow \sigma'_c = \frac{334}{8.23} = 40.6\ N/mm^2$

Therefore, $\sigma'_w = 2.059\sigma'_c - 21.62 = 2.059x40.6 - 21.62 = 61.9\ N/mm^2$

But, $\sigma'_c - \sigma_c = 0 \Rightarrow \sigma_c = \sigma'_c = 40.6\ N/mm^2$

Also, from equation (i), $\sigma_C = 40.6 = 0.314\sigma_w \Rightarrow \sigma_w = \frac{40.6}{0.314} = 129.3\ N/mm^2$

Problem·13: *Calculate the increase in volume of a spherical shell 1 m in diameter and 1 cm thick when it is subjected to an internal pressure of 1.6 MPa. Take E = 200 GPa^2, and 1/m = 0.3*

Solution

Given: t = 1 cm = 10 mm; d = 1 m = 10^3 mm; r = 500 mm; p = 1.6 MPa = 1.6 N/mm^2; E = 200 GPa = 200 x 10^3; 1/m = 0.3;

Initial Volume, $V = \frac{4\pi}{3}xr^3 = \frac{4\pi}{3}(500)^3 = 523.7x10^6\ mm^3$

We know that $\varepsilon_v = \frac{3\sigma_c}{E}\left[1 - \frac{1}{m}\right] = \frac{3pd}{4tE}\left[1 - \frac{1}{m}\right] = \frac{3x1.6x10^3}{4x10x(200x10^3)}[1 - 0.3] = 4.2x10^{-4}$

Therefore, the Change in volume,

$$\delta V = \varepsilon_v V = \left(4.2x10^{-4}\right)x\left(523.7x10^{6}\right) = 218.7x10^{3}\ mm^{3}$$

Problem 14: *A thin spherical shell 1 m in diameter with its wall of 1.2 cm thickness filled with a fluid at atmospheric pressure. What intensity of pressure will be developed in it if 175 cm^3 more of fluid is pumped into it? Also, calculate the circumferential stress at that pressure and the increase in diameter. Take: E = 200 GNlm^2, and 1/m = 0·3 m*

Solution

Given: t = 1.2 cm = 12 mm; d = 1 m = 10^3 mm; r = 500 mm; δV = 175 cm^3 = 175 x 10^3 mm^3; E = 200 GPa = 200 x 10^3; 1/m = 0.3;

Initial Volume, $V = \frac{4\pi}{3} xr^{3} = \frac{4\pi}{3}(500)^{3} = 523.7x10^{6}\ mm^{3}$

Increase in Diameter

Volumetric Strain $\varepsilon_v = \frac{\delta V}{V} = \frac{175x10^{3}}{523.7x10^{6}} = 3.34x10^{-4}$

But, $\varepsilon_v = 3\varepsilon_c \Rightarrow \varepsilon_c = \frac{\varepsilon_v}{3} = \frac{3.34x10^{-4}}{3} = 1.11x10^{-4}$

Also, $\varepsilon_c = \frac{\delta d}{d} \Rightarrow \delta d = d\varepsilon_c = \frac{\varepsilon_v}{3} = \left(10^{3}\right)x\left(1.11x10^{-4}\right) = 0.11\,mm$

Circumferential stress

$$\varepsilon_v = 3.34x10^{-4} = \frac{3\sigma_c}{200x10^{3}}\left[1-0.3\right] = 1.05x10^{-5}\sigma_c \Rightarrow \sigma_c = \frac{3.34x10^{-4}}{1.05x10^{-5}} = 31.8\ N/mm^{2}$$

Pressure

$$\sigma_c = 31.8 = \frac{pd}{4t} = \frac{px10^3}{4x12} = 20.83p \Rightarrow p = \frac{31.8}{20.83} = 1.53\ N/mm^2$$

TYPE B: OBJECTIVE

SECTION A: CHOOSE THE CORRECT OPTION

1. Pressure vessels are made of
   (a) non-ferrous materials
   (b) sheet steel
   (c) cast iron
   (d) any of the above.

2. When a thin cylindrical shell is subjected to internal fluid pressure, which of the following stress is developed in its wall?
   (a) Circumferential stress
   (b) Longitudinal stress
   (c) both (a) and (b)
   (d) none of the above

3. Chemical vessels are made of which of the following materials?
   (a) Non-ferrous materials
   (b) Sheet metal
   (c) Cast iron
   (d) Special material.

4. Vessels used for storing fluid under called ...
   (a) cylinders (b) sphere
   (c) shells (d) none of the above

5. A shell with wall thickness small compared with internal diameter is called ...
   (a) thin shell
   (b) thick shell
   (c) either of the above
   (d) none of the above

6. Which of the following are usually as thin cylinders?
   (a) Boilers (b) Tanks
   (c) Steam pipes (d) Water pipes
   (e) All of the above.

7. Thin cylinders are frequently required under pressures up to
   (a) 5 N/mm^2 (b) 15 N/mm^2
   (c) 30 N/mm^2 (d) 250 N/mm^2

8. Longitudinal stresses act... to the longitudinal axis of the shell.
   (a) parallel
   (b) perpendicular
   (c) either of the above
   (d) none of the above

9. In case of seamless shell the change in volume (dV) is given by
   (a) $\sigma_c = \frac{pd}{2t\eta_l}$ (b) $\sigma_c = \frac{pd}{2t\eta_c}$
   (c) $\sigma_c = \frac{pd}{4t\eta_l}$ (d) $\sigma_c = \frac{pd}{4t\eta_l}$

10. In thin shell circumferential stress (σ_c) is given
   (a) $\sigma_c = \frac{pd}{2t\eta_l}$ (b) $\sigma_c = \frac{pd}{2t\eta_c}$
   (c) $\sigma_c = \frac{pd}{4t\eta_l}$ (d) $\sigma_c = \frac{pd}{4t\eta_l}$

11. A thin cylindrical shell of diameter (d), length (l) is subjected to an internal

pressure (p) circumferential stress in the shell is

(a) $\frac{pd}{2t}$ (b) $\frac{pd}{4t}$

(c) $\frac{pd}{6t}$ (d) $\frac{pd}{8t}$

12. In a thin shell, the ratio of longitudinal stress to the circumferential stress is
(a) 1/2 (b) 3/4
(c) 1 (d) 2

13. The design of a thin cylindrical shell is based on
(a) internal pressure
(b) diameter of shell
(c) longitudinal stress
(d) all of these

14. A thin spherical shell of diameter (d) and thickness (t) is subjected to an internal pressure (p). The tensile stress in the shell plates will be

(a) $\frac{pd}{2t}$ (b) $\frac{pd}{4t}$

(c) $\frac{pt}{2d}$ (d) $\frac{pt}{4t}$

A steam boiler of 1.25 m in diameter is subjected to an internal pressure of 1.6 MPa. Take efficiency of the circumferential and longitudinal joints as 75% and 60% respectively. The steam boiler is made up of 20 mm thick plates. Use the information to answer questions 15 and 16.

15. , Calculate the circumferential stress.
(a) 67 MPa
(b) 42 MPa
(c) 21 MPa
(d) 33.5 MPa

16. Compute the longitudinal stress.
(a) 67 MPa
(b) 42 MPa
(c) 21 MPa
(d) 33.5 MPa

A cylindrical vessel 1.8 m long 800 mm in diameter is made up of 8 mm thick plates and it contains fluid under a pressure of 2.5 MPa. Take E = 200 GPa and 1/m = 0.3. Use the information to answer questions 17 to 19

17. Find the changes in length, diameter and volume of the vessel
(a) 0.26 mm
(b) 0.25 mm
(c) 0.24 mm
(d) 0.23 mm

18. Compute the changes in diameter of the vessel
(a) 0.45 mm
(b) 0.41 mm
(c) 0.43 mm
(d) 0.44 mm

19. Determine the changes in volume of the vessel
(a) 1.074 x10^3 mm^3
(b) 1.032 x10^3 mm^3
(c) 2.148 x10^3 mm^3
(d) 2.064 x10^3 mm^3

20. A spherical shell of 1 m diameter is subjected to a pressure of 2.4 MPa. What is the stress induced in the vessel plate, if its thickness is 15 mm?
(a) 35 N/mm^2
(b) 40 N/mm^2
(c) 45 N/mm^2
(d) 50 N/mm^2

21. A spherical shell of 1·5 m diameter has I cm thick wall. Determine the pressure that can increase its volume by 100 cm. Take: E = 200 GPa; 1/m = 0.3.
(a) 0.143 N/mm^2
(b) 0.144 N/mm^2

(c) 0.145 N/mm^2
(d) 0.146 N/mm^2

22. A bronze spherical shell is made of 1·5 cm thick plate. It is subjected to an internal pressure of 1 N/mm^2. If the permissible stress in the bronze is 55 N/mm^2, calculate the diameter of the spherical shell taking the efficiency as 80%. [Ans. 2·64 m
(a) 2.64x 10^3 mm
(b) 2.74 x10^3 mm
(c) 2.84 x 10^3 mm
(d) 2.94 x10^3 mm

23. A vessel in the shape of a thin spherical shell 40 cm radius and 1 cm shell thickness is completely filled with a fluid at atmospheric pressure. Additional fluid is then pumped till the pressure increases by 6 N/mm^2. Find the volume of this additional fluid, given that 1/m = 0.26 and E = 100 GPa for the shell material.
(a) 602.2 x 10^3 mm^3
(b). 603·2 x10^3 mm^3
(c) 604.2 x 10^3 mm^3
(d). 605·2 x10^3 mm^3

TYPE C: ESSAY

1. Distinguish between circumferential stress and longitudinal stress in a cylindrical shell, when subjected to an internal pressure.
2. Show that in the case of a thin cylindrical shell subjected to an internal fluid pressure, the tendency to burst lengthwise is twice as great as in a transverse section.
3. Derive a relation for the changes of diameter and length of a thin cylindrical shell, when subjected to an internal pressure.
4. Distinguish between cylindrical shell and spherical shell.
5. Derive a formula for the hoop stress in a thin spherical shell subjected to an internal pressure,