INSTITUTE OF DISTANCE LEARNING

KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI, GHANA

ME 362
ME 362 VIBRATIONS I

Lecture 2

RESPONSE TO HARMONIC EXCITATIONS

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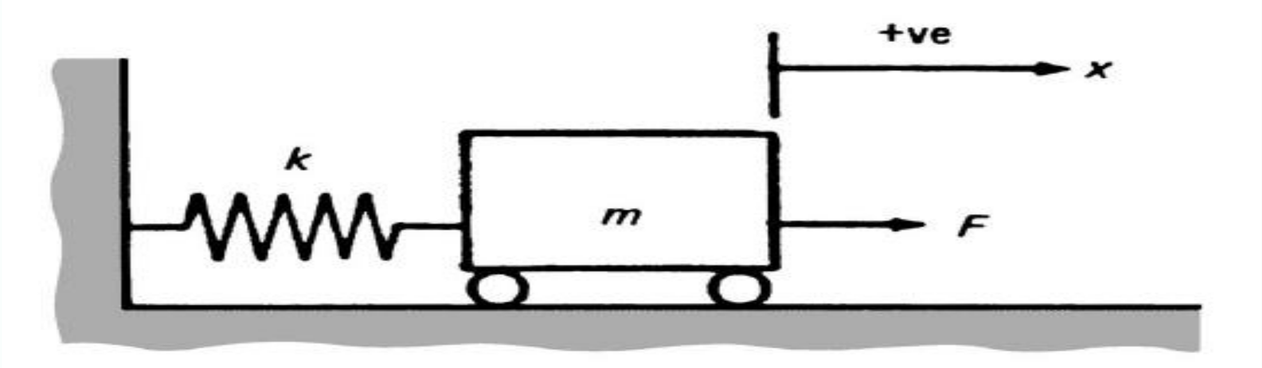
FORCED UNDAMPED VIBRATION



$Let\ F\ be\ a\ sinusoidal\ force,$

$$F = F_o sin\omega t$$

$$m\ddot{x} + kx = F_o sin\omega t$$

$$\ddot{x} + \omega_n{}^2 x = (F_o/m) sin\omega t$$

$$x = x_h + x_p$$

$$x_h = Asin\omega_n t + Bcos\omega_n t$$

$$x_p = X_o sin\omega t$$

$$\dot{x}_p = X_o \omega cos\omega t;\ \ddot{x}_p = -X_o \omega^2 sin\omega t$$

$$-X_o \omega^2 sin\omega t + X_o \omega_n{}^2 sin\omega t = F_o sin\omega t$$

$$X_o = \frac{F_o}{\omega_n{}^2 - \omega^2}$$

$when\ \ \omega = \omega_n;\ X_o \rightarrow \infty$

This *condition is known as resonance and* ***must*** *be avoided.*

The total response then is,

$$x = Asin\omega_n t + Bcos\omega_n t + \frac{F_o}{\omega_n{}^2 - \omega^2} sin\omega t$$

Example 2-1

Compute and plot the response of a sound system subjected to a force of magnitude 23 N, driving frequency of twice the natural frequency and initial conditions given by x_0 = 0 m and v_0= 0.2 m/s. The mass of the system is 10 kg and the spring stiffness is 1000 N/m.

Solution 2

First compute the various coefficients for the response

$$\omega_n = \sqrt{\frac{1000\ \mathrm{N/m}}{10\ \mathrm{kg}}} = 10\ \mathrm{rad/s},\ \omega = 2\omega_n = 20\mathrm{rad/s}$$

$$f_0 = \frac{23}{10} = 2.3\ \mathrm{N/kg}$$

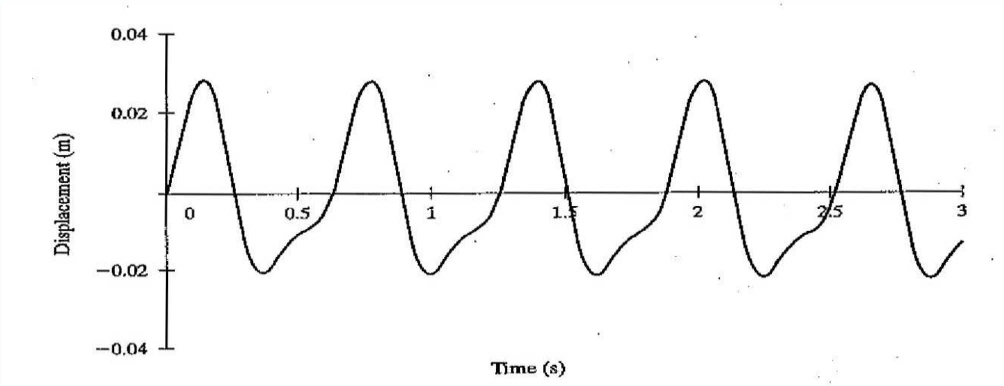
$$\frac{v_0}{\omega_n} = \frac{0.2\ \mathrm{m/s}}{10\ \mathrm{rad/s}} = 0.02\mathrm{m},$$

$$\frac{f_0}{\omega_n^2-\omega^2} = \frac{2.3\ \mathrm{N/kg}}{(10^2-20^2)\mathrm{rad^2/s^2}} = -7.9667\times 10^{-3}\mathrm{m}$$

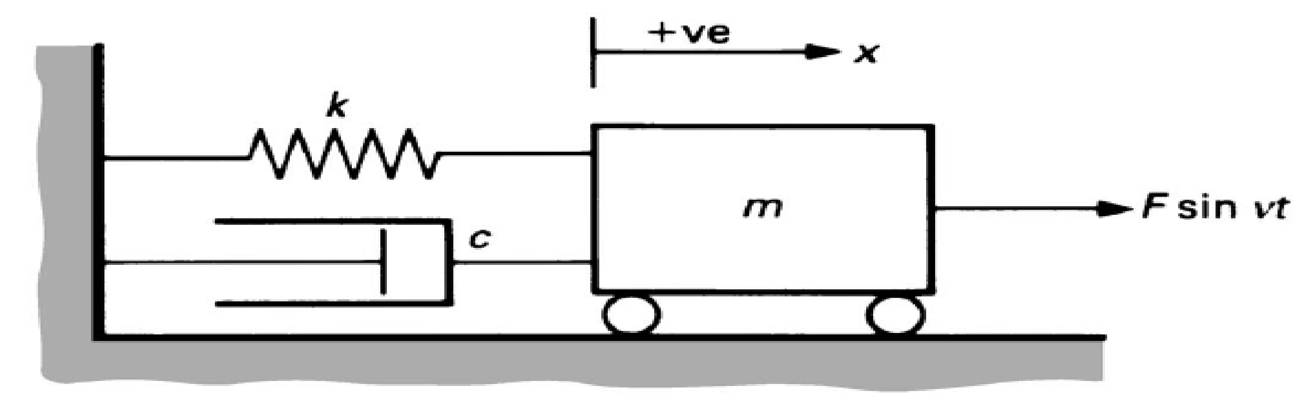
With these values, the response becomes

$$x(t) = 0.02\sin 10t + 7.667\times 10^{-3}(\cos 10t - \cos 20t)\mathrm{m}$$

The plot of the time response is given in the Figure .



FORCED DAMPED SYSTEMS



$$m\ddot{x} + c\dot{x} + kx = F_o sin\nu t$$

$$x = x_h + x_p$$

The homogenous solution, x_h depends on the value of ξ

The particular solution

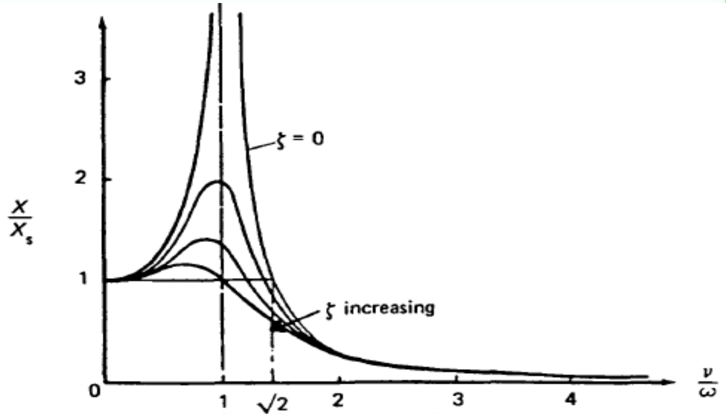
$$x_p = X_o \sin(\omega t - \emptyset)$$

$$\dot{x}_p = X_o \omega cos(\omega t - \emptyset); \quad \ddot{x}_p = -X_o \omega^2 \sin(\omega t - \emptyset)$$

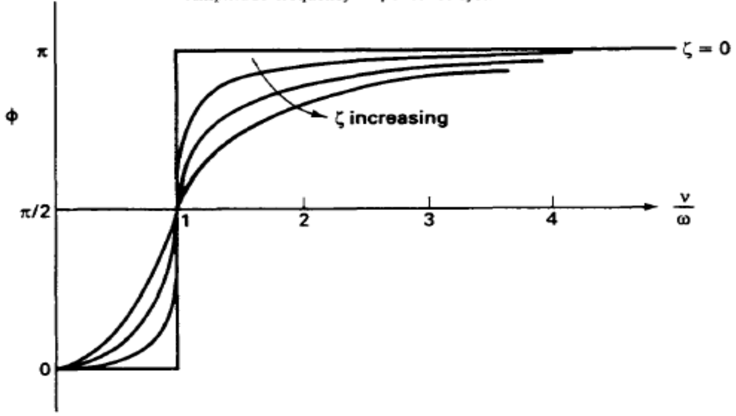
$$X_o = \frac{F_o/m}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\xi\omega_n\omega)^2}} = \frac{\delta}{\sqrt{(1-\tau^2)^2 + (2\xi\tau)^2}};$$

$$\tau = \omega/\omega_n$$

$$\emptyset = \tan^{-1}\left(\frac{2\xi\omega_n\omega}{\omega_n^2 - \omega^2}\right) = \tan^{-1}\frac{(2\xi\tau)}{(1-\tau^2)}$$



Amplitude–frequency response for system



Phase–frequency response for system

Amplitude and Phase response

$$\frac{X_o}{\delta} = \frac{1}{\sqrt{(1-\tau^2)^2 + (2\xi\tau)^2}}$$

$$@\,\tau = 0, \frac{X_o}{\delta} = 1; \;\; \text{as } \tau \to 1 \;\&\; \xi = 0; \; \frac{X_o}{\delta} \to \infty$$

$$\text{as } \tau \to \infty, \frac{X_o}{\delta} \to 0; \text{as } \tau \to 1 \;\&\; \xi = 0; \; \frac{X_o}{\delta} \to \infty$$

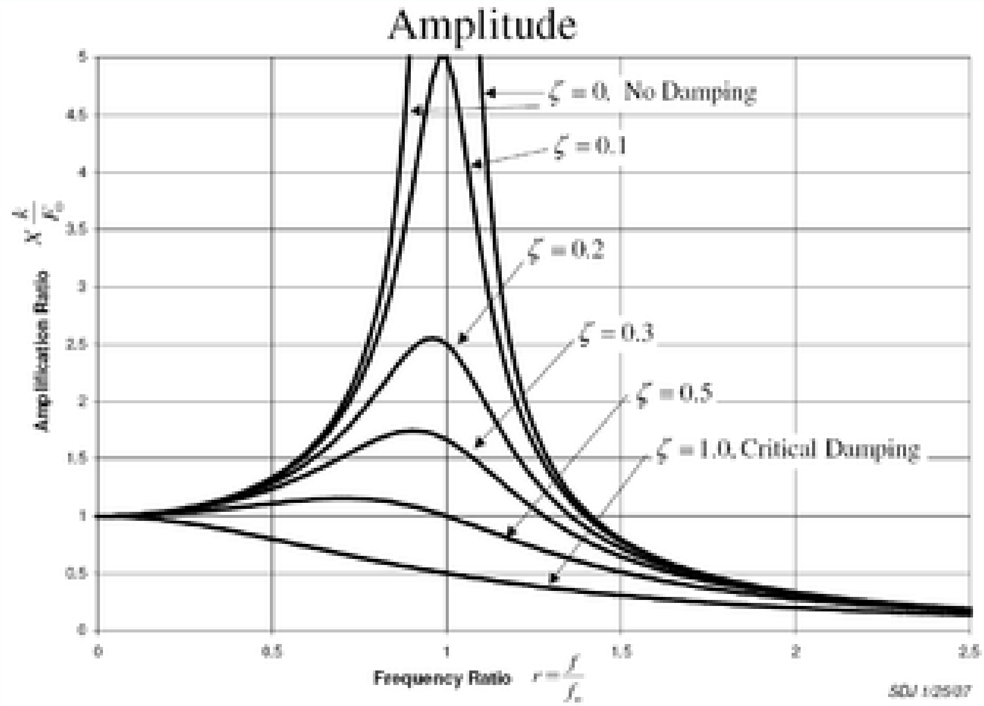
$$@\,\tau = 1, \frac{X_o}{\delta} = \frac{1}{2\xi} = Q @\, \xi = 0 \;\&\; \frac{X_o}{\delta} = 1; \tau = 0 \; or \; \sqrt{2}$$

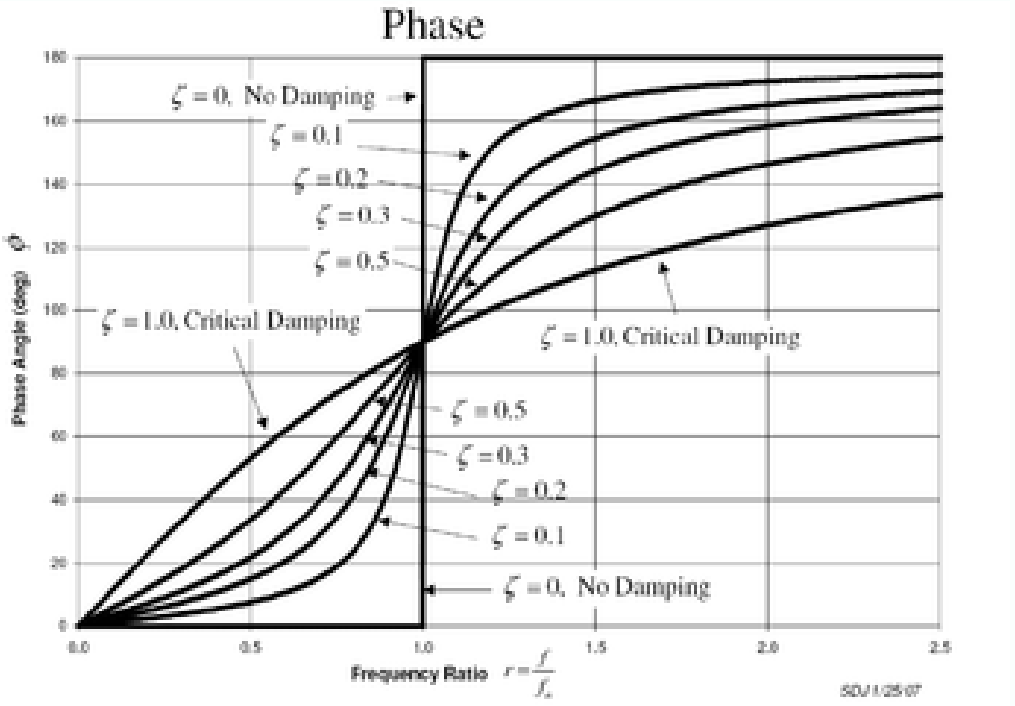
$$\emptyset = \tan^{-1}\frac{(2\xi\tau)}{(1-\tau^2)}$$

$$@\,\tau = 0, \emptyset = 0^o$$

$$@\,\tau = 1, \emptyset = 90^o$$

$$\text{as } \tau \to \infty, \emptyset \to 180^o$$





Example 5

Consider again the simple spring-mass system of consisting of a spring and a bolt. Calculate the value of the steady-state response if the driving frequency is 132 rad/s for f_0 = 10 N/kg. Calculate the change in amplitude if the driving frequency is reduced to 125 rad/s. , the natural frequency and damping ratio are given as 132 rad/s and 0.0085 respectively.

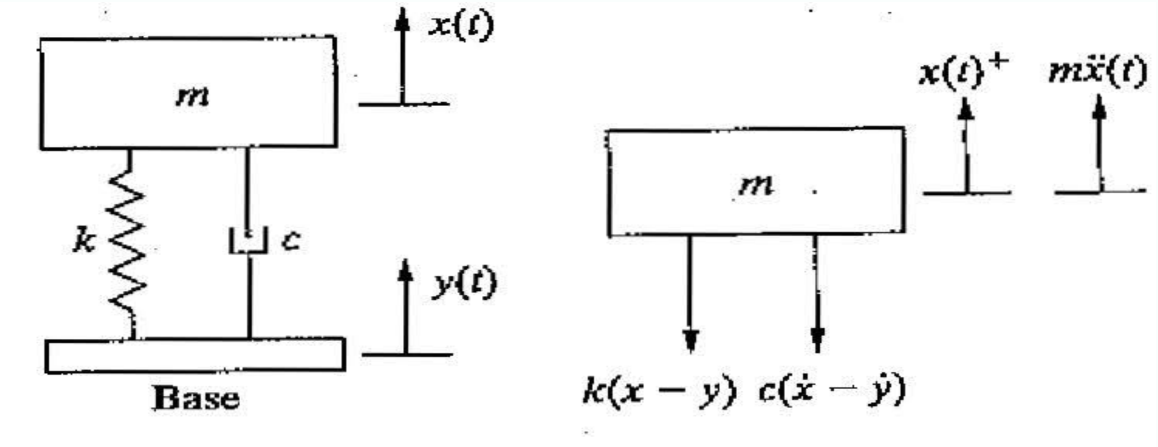
Solution

$$X_o = \frac{F_o/m}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\xi\omega_n\omega)^2}}$$

$$X_o = \frac{10}{\{[(132)^2-(132)^2]^2+[2(0.0085)(132)(132)]^2\}^{1/2}} = \frac{10}{2(0.0085)(132)^2}=0.034$$

If the driving frequency is changed to 125 rad/ s, the amplitude becomes 0.005 m. So a slight change in the driving frequency from near resonance at 132 rad/s to 125 rad/s (5%) causes an order-of-magnitude change in the amplitude of the steady-state response.

Base Excitation



EQUATION OF MOTION

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

$$y(t) = Y\, sin\omega\, t$$

ABSOLUTE DISPLACEMENT, X

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

$$Amplitude\ ratio = T$$

$$T = \frac{X}{Y} = \sqrt{\frac{1 + (2\xi\tau)^2}{(1 - \tau^2)^2 + (2\xi\tau)^2}}$$

RELATIVE DISPLACEMENT, z

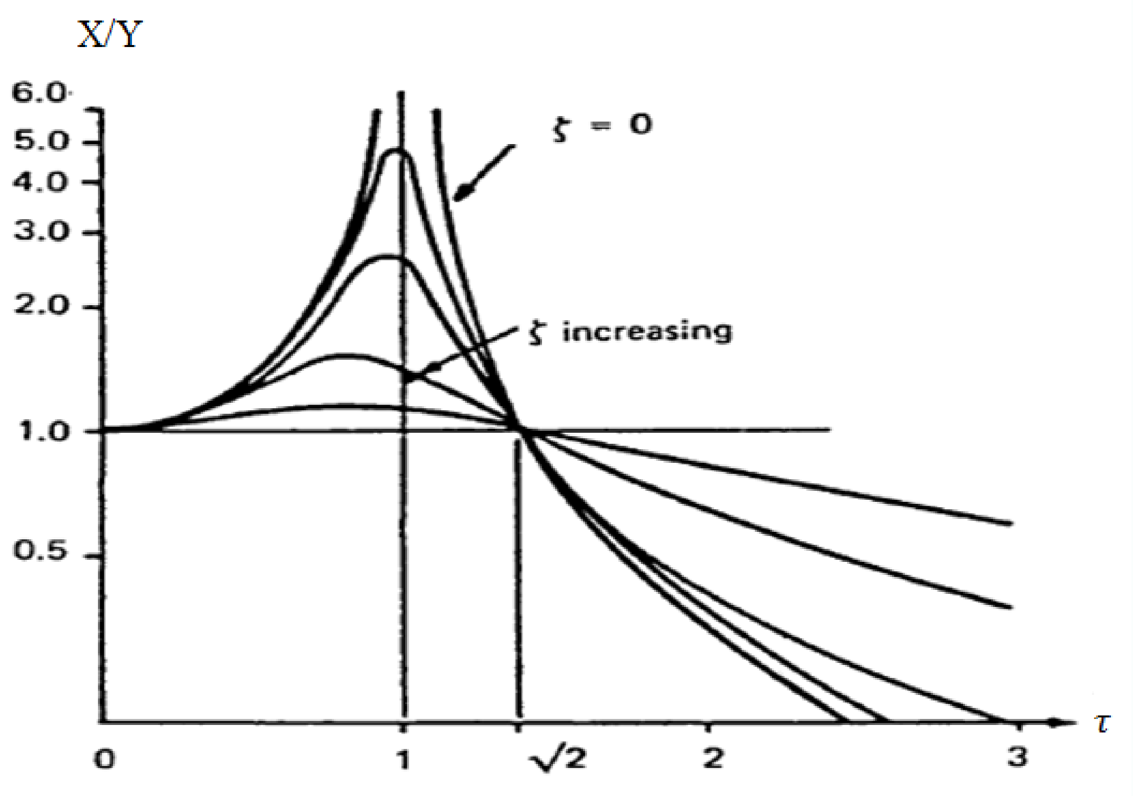
$$let\ z = x - y$$

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y}$$

$$\frac{Z}{Y} = \frac{\tau^2}{\sqrt{(1 - \tau^2)^2 + (2\xi\tau)^2}}$$

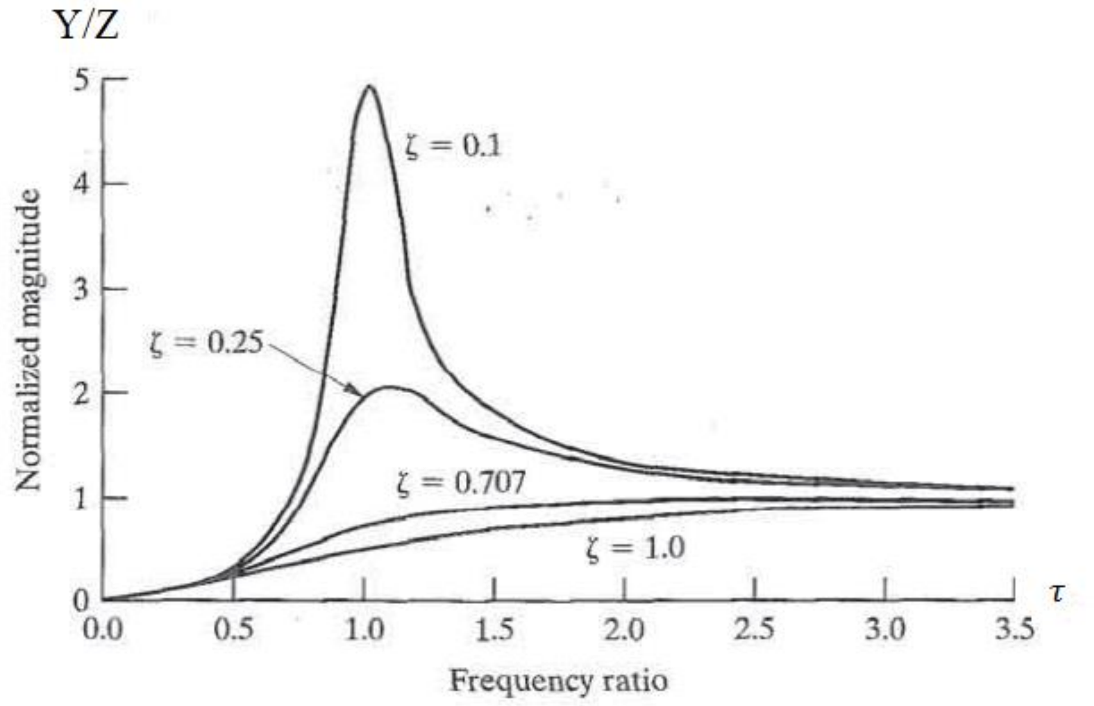
Base Excitation

- ABSOLUTE DISPLACEMENT RESPONSE



Transmissibility-frequency ratio response.

RELATIVE DISPLACEMENT RESPONSE



Example 6

A very common example of base motion is the single-degree-of-freedom model of an automobile driving over a rough road or an airplane taxiing over a rough runway, indicated below. The road is approximated as sinusoidal in cross section providing a base motion displacement of

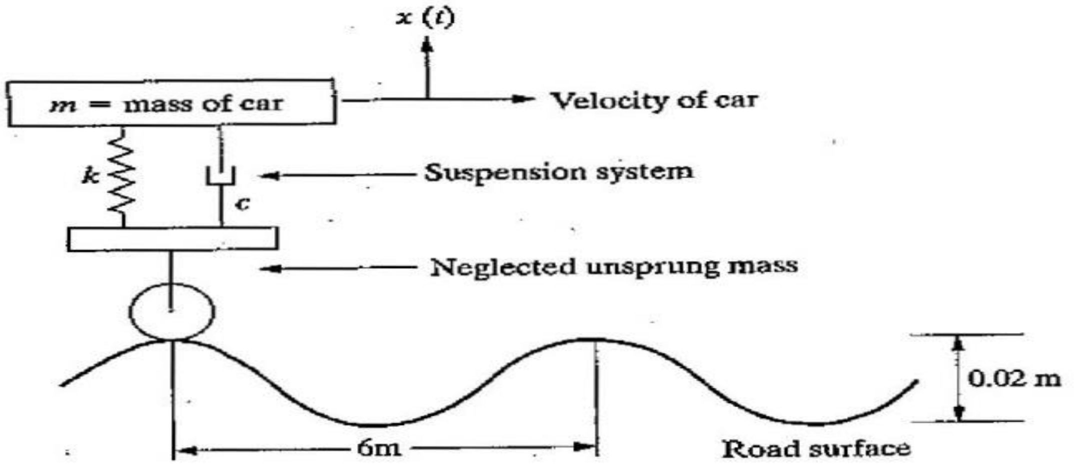
$$y(t) = 0.01 \sin \omega\, t \text{ m}$$

Assume the vehicle is moving with a speed of 20 km/h.

Solution

$$\omega = \frac{2\pi v}{\lambda} = 5.818 \text{ rad/s}$$

where v denotes the vehicle's velocity in m/s.



Solution

$$\omega_n = \sqrt{\frac{4 \times 10^5 \text{ N/m}}{1007 \text{ kg}}} = 19.93 \text{ rad/s}$$

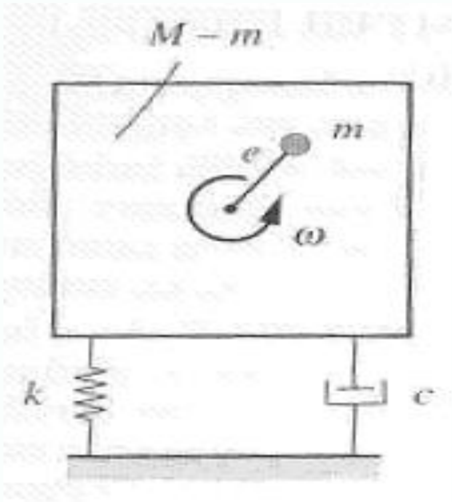
So that τ = 5.818/19.93 = 0.292 and

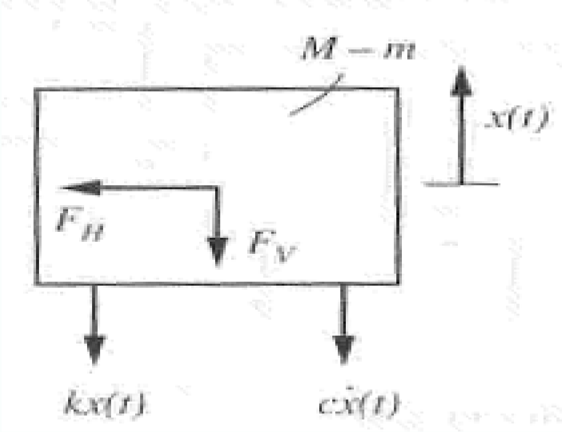
$$\xi = \frac{c}{2\sqrt{km}} = \frac{20{,}000 \text{ N.s/m}}{2(4 \times 10^5 \text{ N/m})(1007 \text{ kg})} = 0.498$$

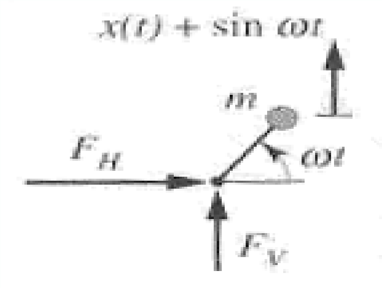
The deflection experienced by the car will be

$$X = (0.01 \text{ m})\sqrt{\frac{1 + [2(0.498)(0.292)]^2}{[1 - (0.292)^2]^2 + [2(0.498)(0.292)]^2}} = 0.0108 \text{ m}$$

Rotating Unbalance







Displacement of Total Mass

$$(\mathrm{M}-\mathrm{m})\ddot{\mathrm{x}}+\mathrm{m}\frac{d^2}{dt^2}(x+esin\omega t)=-\mathrm{c}\dot{\mathrm{x}}-\mathrm{kx}$$

$$\mathrm{M}\ddot{\mathrm{x}}+\mathrm{c}\dot{\mathrm{x}}+\mathrm{kx}=\mathrm{m}\omega^2\,e\,\sin\omega\,\mathrm{t}$$

$$X=\frac{me}{M}\frac{\tau^2}{\sqrt{(1-\tau^2)^2+(2\xi\tau)^2}}$$

Force Transmitted to support

$$F_T=\mathrm{c}\dot{\mathrm{x}}+\mathrm{kx}$$

$$x=X_o\sin(\omega\,\mathrm{t}-\emptyset)$$

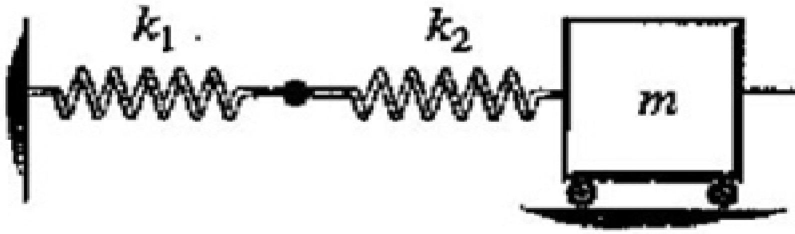
$$F_T=F_o\sqrt{\frac{1+(2\xi\tau)^2}{(1-\tau^2)^2+(2\xi\tau)^2}}$$

$$Transmissibility=T=\frac{F_T}{F_o}$$

$$\emptyset=\tan^{-1}\frac{(2\xi\tau)}{(1-\tau^2)}$$

Assignment 1

Consider the system in the Figure below. Write the equation of motion, and calculate the response assuming that the system is initially at rest for the values k_1, = 1 kN/m, k_2= 5 kN/m, and m = 50 kg.



Plot the response of this system using **excel**

ASSIGNMENT 2

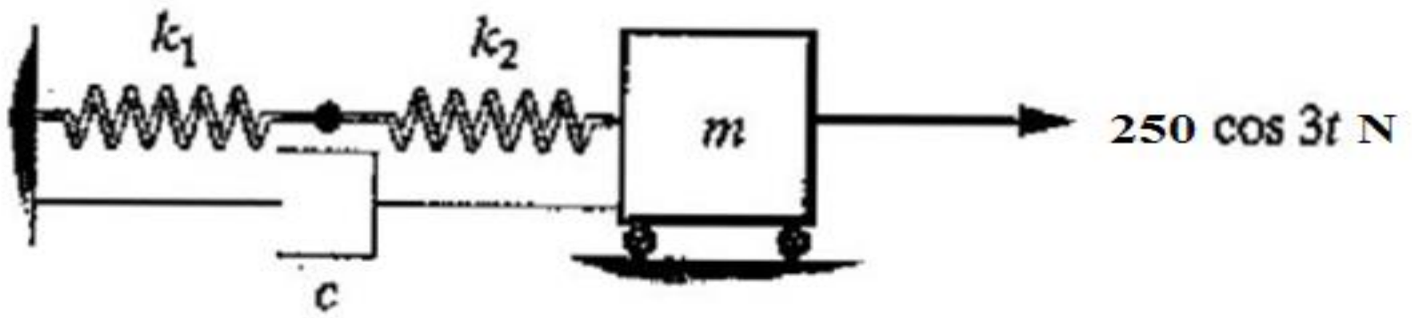
A generator of mass 100 kg is supported on springs which deflect 20 mm under the load. The vibration of the machine on the springs is constrained to be linear and vertical, and a dash-pot fitted in order to reduce the amplitude to one-quarter of its initial value in two complete oscillations.

Find the magnitude of the damping force required at unit speed and compare the frequency of the damped and free vibrations of the system.

Assignment 3

Compute the response of the system below, if the system is initially at rest for the values

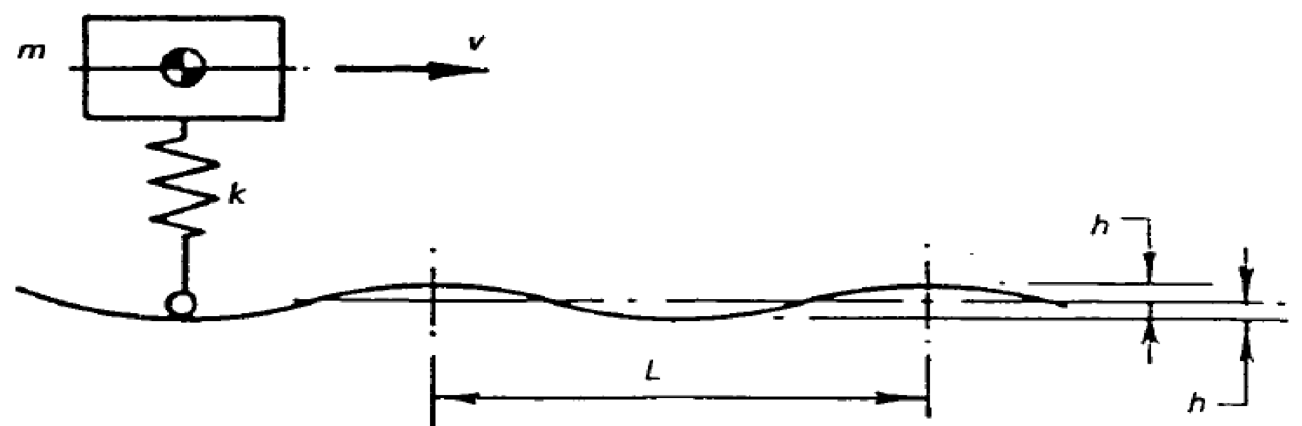
k_1 = 1 k N/m, k_2 = 5 kN/m, c = 200 kg/s, and m = 50 kg.



Plot the response of this system using Microsoft **Excel**

Assignment 4

The figure shows a simple model of a vehicle which is moving with a constant speed *80 km/h* over a surface with a sinusoidal profile. The mass of the vehicle is 9 *tons* and the stiffness of its suspension springs is *100 kN/m*. The wavelength of the surface profile is *0.8 m* and the amplitude of its undulations is *15 mm*. Obtain an equation for the steady state vertical displacement of the vehicle and sketch the response(Use MS Excel).



Assignment 5

A fan is mounted on a spring and viscous damper in parallel so that only linear motion in the vertical direction occurs.

Briefly derive an expression for the force transmitted to the ground through the spring and damper if the fan generates a harmonic disturbing force in the vertical direction.

Given that the fan has a mass of 40 kg and a rotating unbalance of 0.01 kg m, determine the spring stiffness required for 10% force transmission. Take the damping ratio of the system to be 0.2 and the fan speed as 1480 rev/min.

When running under these conditions what effect on the transmission would removal of the damping element have?