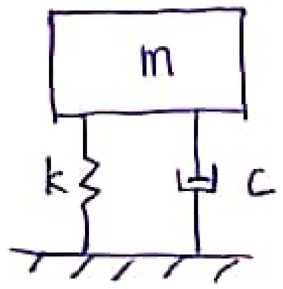
ASSIGNMENT 2

ME 362

INDEX No: 9383417

Q1 Simple mass spring damper model



mass (m) = 150 kg

Stiffness of spring (k) = 1500 N/m

damping coefficient (c) = 200 kg/s

(i) Undamped Natural Frequency, $\omega_n = \sqrt{\frac{k}{m}}$

$$= \sqrt{\frac{1500}{150}}$$

$$\therefore \omega_n = 3.16 \text{ rad/s}$$

(ii) Damping ratio, $\zeta = \frac{c}{2\sqrt{mk}}$

$$= \frac{200}{2\sqrt{150 \times 1500}}$$

$$\zeta = 0.21$$

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iii) Damped natural frequency, $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$= 3.1623 \cdot \sqrt{1-(0.2108)^2}$$

$$= 3.09 \text{ rad/s}$$

iv) Since $\zeta = 0.21$, and $0.21 < 1$

$\therefore$ the system is underdamped

v) The system would oscillate

vi) $X(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$

For this system,

$$m\lambda^2 + c\lambda + k = 0$$

$$\Rightarrow \quad 150\lambda^2 + 200\lambda + 1500 = 0$$

$$\therefore \lambda = -\frac{2}{3} + \frac{\sqrt{86}}{3}i$$

$$\therefore X(t) = Ae^{-\frac{2}{3}t}\sin(3.09t + \phi)$$

$$\dot{x}(t) = -\frac{2}{3}Ae^{-\frac{2}{3}t}\sin(3.09t+\phi) + Ae^{-\frac{2}{3}t}(3.09)\cos(3.09t+\phi)$$

Given $x(0) = -5$mm and $\dot{x}(0) = 10$mm/s

$$x(0) = Ae^{-\frac{2}{3}(0)}\sin(3.09(0) + \phi)$$

$\Rightarrow \quad -5 = A\sin\phi$ —— (1)

Also

$\dot{x}(0) = 10$

$\Rightarrow \quad 10 = -\frac{2}{3}A\sin\phi + 3.09\,A\cos\phi$ —— (2)

Substitute $A\sin\phi = -5$ into (2)

$\Rightarrow \quad 10 = \frac{10}{3} + 3.09\,A\cos\phi$

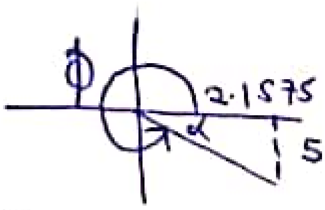
$\Rightarrow \quad 2.1575 = A\cos\phi$ —— (3)

From (1) and (3)

$A = \sqrt{(2.1575)^2 + (-5)^2} = 5.446$

$A\sin\phi = -5$

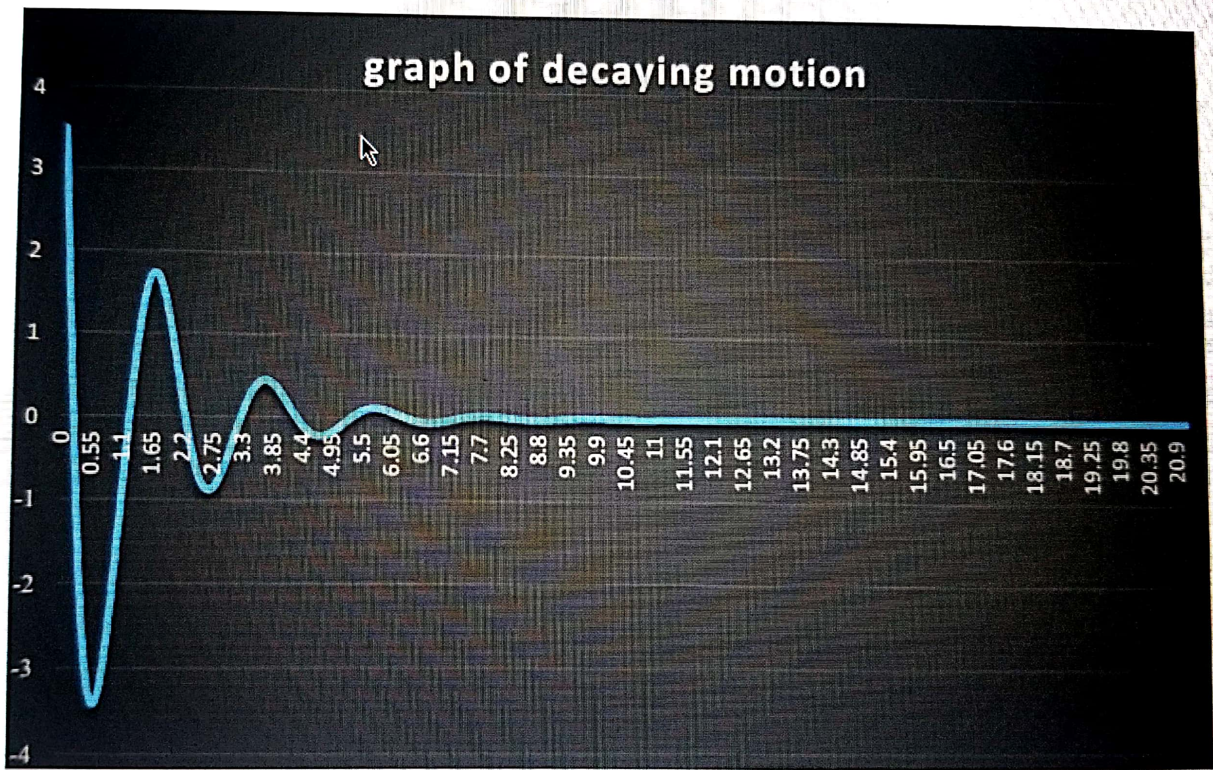
$A\cos\phi = 2.1575$



$\alpha = \tan^{-1}\left(\frac{5}{2.1575}\right) = 66.66^\circ$

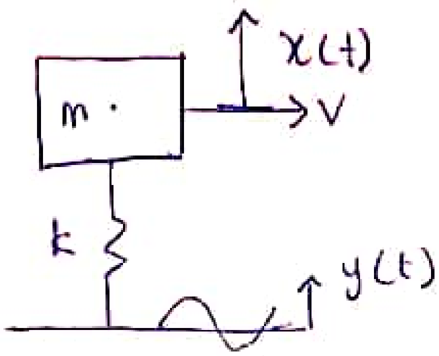
$\therefore \quad \phi = 360 - 66.66^\circ = 293.34^\circ$

$\therefore \quad X(t) = 5.45\, e^{-0.67t} \sin(3.09t + 293.34^\circ)$



the settling time from the graph is 12.1s

Q2



$m = 9\text{tons} = 8164.66\,kg$

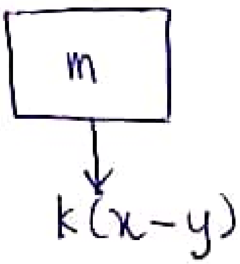
$k = 100 \times 10^3\ N/m$

$\lambda = 0.8m$

$y_o = 15mm = 0.015m$

$V = 80km/h$

Free body diagram



Equation of motion

$$m\ddot{x} + k(x-y) = 0$$

$$m\ddot{x} + kx = ky$$

but $y = Y_o \sin\omega t$

$$\omega = \frac{2\pi V}{\lambda}$$

$$\omega = \frac{2\pi \times 80 \times 1000}{3600 \times 0.8} = 174.533 \text{ rad/s}$$

$$m\ddot{x} + kx = kY_0 \sin\omega t \quad \text{——} (2)$$

The steady state response is given by the particular Solution of eqn 2

let $x_p(t) = X_0 \sin\omega t$

$$\ddot{x}_p = -X_0 \omega^2 \sin\omega t$$

$$\Rightarrow m(-X_0\omega^2 \sin\omega t) + k(X_0 \sin\omega t) = kY_0 \sin\omega t$$

$$\Rightarrow [k - m\omega^2] X_0 \sin\omega t = kY_0 \sin\omega t$$

comparing L.H.S and R.H.S of equation

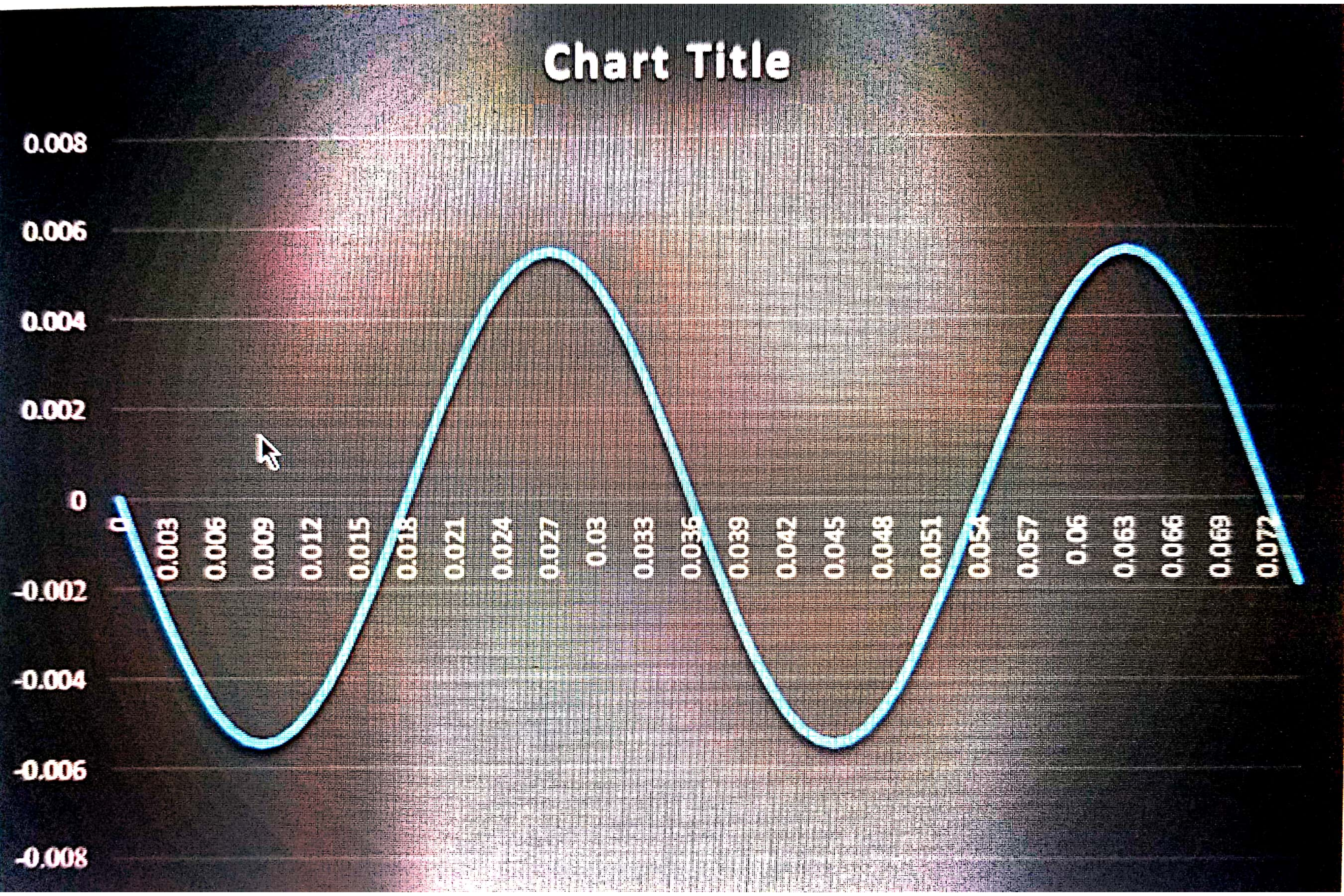
$$\Rightarrow [k - m\omega^2] X_0 = kY_0$$

$$\therefore X_0 = \frac{kY_0}{k - m\omega^2}$$

$$X_0 = \frac{100 \times 10^3 \times 0.015}{(100 \times 10^3) - (8164.66 \times 174.533^2)} = -6.03 \times 10^{-6}$$

$\therefore$ Steady state response

$$x_p(t) = -6.03 \times 10^{-6} \sin(174.53t)$$



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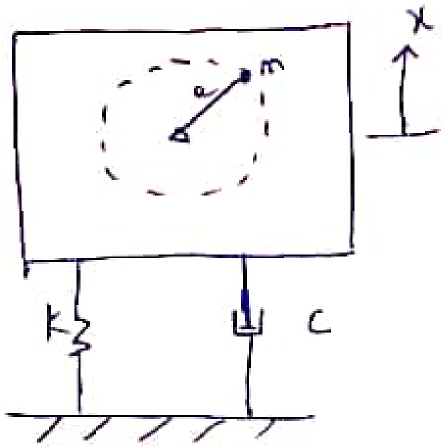
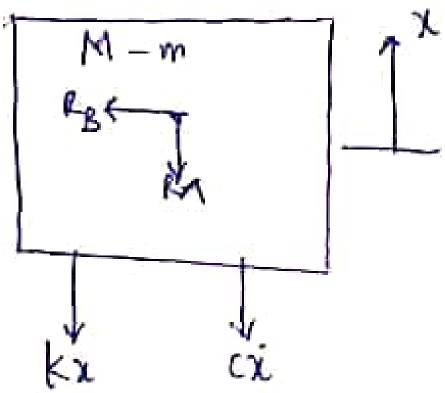
Q3 Model of System

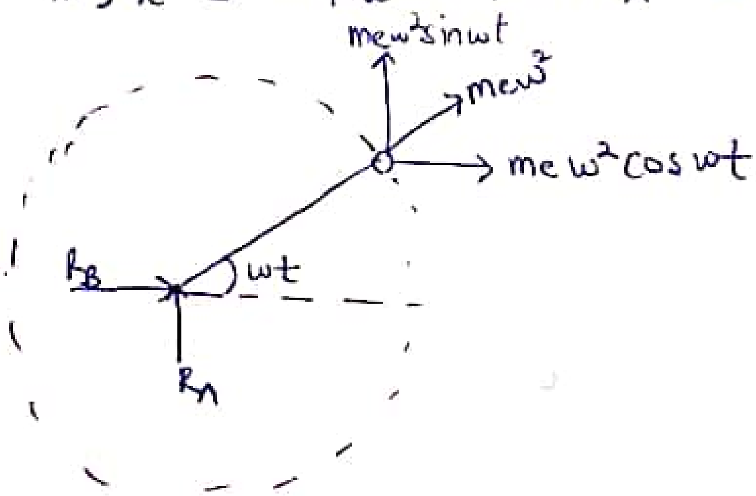


Free body Diagrams



$$\sum F_x = m\ddot{x}$$

$$(M-m)\ddot{x} = -kx - c\dot{x} - R_A \quad \text{——} (1)$$



Equation of motion

$$m\ddot{x} = R_A + me\omega^2 \sin\omega t$$

$$R_A = m\ddot{x} - me\omega^2 \sin\omega t \quad \text{——— (2)}$$

Substituting eqn 2 into eqn 1

$$\Rightarrow (M-m)\ddot{x} = -kx - c\dot{x} - m\ddot{x} + me\omega^2 \sin\omega t$$

$$\therefore \quad M\ddot{x} + c\dot{x} + kx = me\omega^2 \sin\omega t$$

let $X = X_0 \sin(\omega t - \theta)$

$$\dot{X} = \omega X_0 \cos(\omega t - \theta)$$

$$\ddot{X} = -\omega^2 X_0 \sin(\omega t - \theta)$$

$$\Rightarrow M(-\omega^2 X_0 \sin(\omega t - \theta)) + c\omega X_0 \cos(\omega t - \theta) + kX_0 \sin(\omega t - \theta) = me\omega^2 \sin\omega t$$

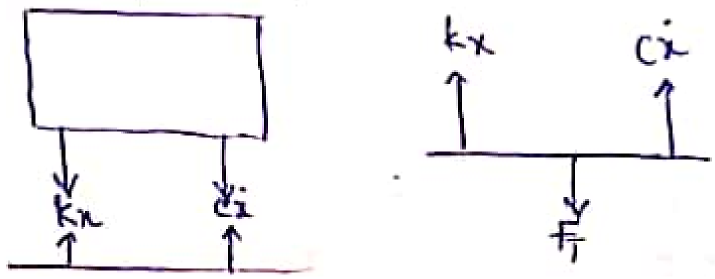
$$X_0\left[(k - M\omega^2)(\sin\omega t - \theta) + c\omega \cos(\omega t - \theta)\right] = me\omega^2 \sin\omega t$$

Taking absolute of both sides

$$\Rightarrow X_0 \sqrt{(k - M\omega^2)^2 + (c\omega)^2} = me\omega^2 \qquad F_0 = me\omega^2$$

$$\therefore \quad X_0 = \frac{me\omega^2}{\sqrt{(k - M\omega^2)^2 + (c\omega)^2}}$$

force transmitted



$$F_T = kx + c\dot{x}$$

$$\therefore F_T = kX_0 \sin(\omega t - \theta) + c\omega X_0 \cos(\omega t - \theta)$$

$$|F_T| = X_0 \sqrt{k^2 + (c\omega)^2} \quad \text{——— (3)}$$

substitute value of X_0 into eqn 3

$$\Rightarrow |F_T| = me\omega^2 \sqrt{\frac{k^2 + (c\omega)^2}{(k - M\omega^2)^2 + (c\omega)^2}}$$

let $r = \frac{\omega}{\omega_n}$, $\zeta = \frac{c}{2m\omega_n}$

$$\therefore |F_T| = me\omega^2 \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}$$

Given $F_T = 0.1F_0$, $F_0 = Me\omega^2$ $\zeta = 0.2$

$$\Rightarrow 0.1\, me\omega^2 = me\omega^2 \sqrt{\frac{1 + (0.4r)^2}{(1 - r^2)^2 + (0.4r)^2}}$$

$$\Rightarrow 0.01 = \frac{1 + 0.16r^2}{1 - 2r^2 + r^4 + 0.16r^2}$$

$$0.01 - 0.0184r^2 + 0.01r^4 = 1 + 0.16r^2$$

$$\Rightarrow 0.01r^4 - 0.1784r^2 - 0.99 = 0$$

let $r^2 = a$

$$\therefore 0.01a^2 - 0.1784a - 0.99 = 0$$

$\therefore \quad q = 22.2829$

$\Rightarrow \quad r^2 = 22.2829$

but $r = \dfrac{w}{w_n}$ $\qquad w = \dfrac{1480 \times 2\pi}{60} = 154.985\,rad/s$

$$\Rightarrow \quad \frac{w^2}{w_n^2} = 22.2829$$

$$\Rightarrow \quad \frac{(154.985)^2}{w_n^2} = 22.2829$$

$$w_n^2 = \frac{(154.985)^2}{22.2829} \qquad \text{but } w_n = \sqrt{\frac{k}{M}}$$

$$\Rightarrow \quad \frac{k}{M} = \frac{(154.985)^2}{22.2829} \qquad M = 40kg$$

$$\therefore \quad k = \frac{(154.985)^2 \times 40}{22.2829}$$

$$= 43118.9\,N/m$$

$$k \doteq 43.119\,kN/m$$

when damping element is removed ($\xi = 0$)

$$\Rightarrow \quad \frac{F_T}{F_0} = \sqrt{\frac{1}{(1-r^2)^2}}$$

$$\frac{F_T}{F_o} = \sqrt{\frac{1}{(1-22.2829)^2}}$$

$\therefore$ Transmissibity $(T) = \frac{F_T}{F_o} = 0.0469$

Since $F_T = 0.0469 F_o$

$\therefore$ When the damping element is removed, the force transmitted to the ground decreases