LOCAL AND GLOBAL EXTREMA

1. Find and analyze the critical points of $f(x,y) = x^2 - 2x + y^2 - 4y + 5$

2. Find and analyze any critical points of $f(x,y) = -\sqrt{x^2+y^2}$ Find the local maxima, local minima, or saddle points of the given function.

3. $f(x,y) = \dfrac{x^2}{2} + 3y^3 + 9y^2 - 3xy + 9y - 9x$

4. $f(x,y) = x^3 - 3x + y^3 - 3y$

5. $f(x,y) = (x+y)(xy+1)$

6. $E(x,y) = 1 - \cos x + \dfrac{y^2}{2}$

7. $P(x,y) = 400 - 3x^2 - 4x + 2xy - 5y^2 + 48y$
   Each function in Problems 8-9 has a critical point at (0,0). What sort of critical point is it?

8. $f(x,y) = x^6 + y^6$

9. $h(x,y) = \cos x \cos y$

10. Suppose $f_x = f_y = 0 at (1,3)$ and $f_{xx} > 0, f_{yy} > 0, f_{xy} = 0$.
    What can you conclude about the behavior of the function near the point (1,3)?

11. Find the maximum and minimum values of $x + y$ on the circle $x^2 + y^2 = 4$
    Use Langrange multipliers to find the maximum and minimum values of $f(x,y)$ subject to the given constraints.

12. $f(x,y) = x + y$, $x^2 + y^2 = 1$

13. $f(x,y) = x^2 + y^2$, $x^4 + y^4 = 2$

14. $f(x,y) = 3x - 2y$, $x^2 + 2y^2 = 44$

15. $f(x,y) = xy$, $4x^2 + y^2 = 8$