Vectors

Scalars and Vectors

A **vector** is a quantity that has a **magnitude** and a **direction**. One example of a vector is velocity. The velocity of an object is determined by the magnitude(speed) and direction of travel. Other examples of vectors are force, displacement and acceleration.

A **scalar** is a quantity that has magnitude only. Mass, time and volume are all examples of scalar quantities.

Example 1.

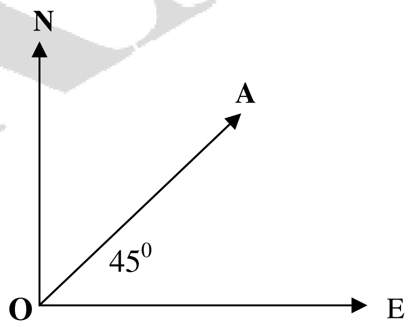
If a car travels from point O to point A, which is 50km. away in a north-easterly direction, then the **displacement** of the car from O is **50km.NE.** The displacement of the car is specified by the distance travelled (50km.) and the direction of travel (NE.) from O.

Displacement is therefore a vector, and the magnitude of the displacement (distance), is a scalar.

On the diagram below the displacement is represented by the directed line segment $\overrightarrow{OA}$.

The length of the line represents the magnitude of the displacement and is written $\left|\overrightarrow{OA}\right|$.

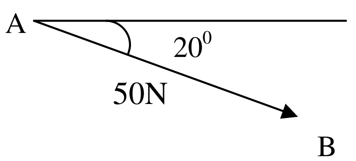
The arrowhead represents the direction of the displacement.



$\left|\overrightarrow{OA}\right|$ =50km.

Example 2.

A force of 50 Newton at an angle of 20^0 to the horizontal downward, is applied to a wheelbarrow. The diagram below shows a vector representing this force.



$\left|\overrightarrow{AB}\right| = 50\text{N}$

Geometric Vectors

We will be considering vectors in three-dimensional space defined by three mutually perpendicular directions.

Definitions and conventions.

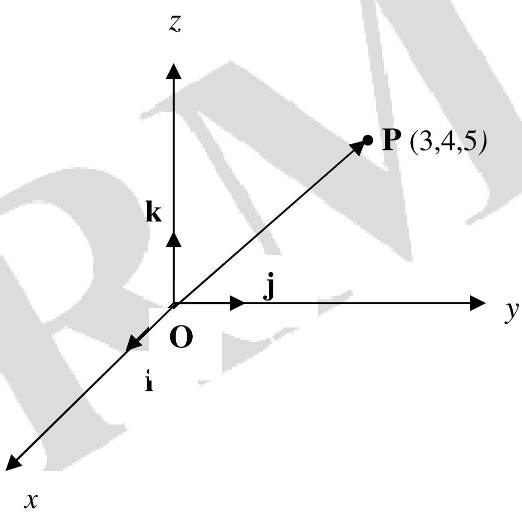
Vectors will be denoted by lower case **bold** letters such as **a, b, c.**

Unit vectors **i, j, k**

Vectors with a magnitude of **one** in the direction of the *x*-axis, *y*-axis and *z*-axis will be denoted by **i, j,** and **k** respectively.

The notation (*a, b, c*) will be used to denote the vector ($a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$) as well as the co-ordinates of a point P (*a, b, c*). The context will determine which meaning is correct.

Example



$$|\mathbf{i}| = |\mathbf{j}| = |\mathbf{k}| = 1$$

In the diagram above the point P has coordinates (3,4,5).

The vector $\overrightarrow{OP}$ is the vector $3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$. This may also be written (3,4,5).

Directed Line Segment.

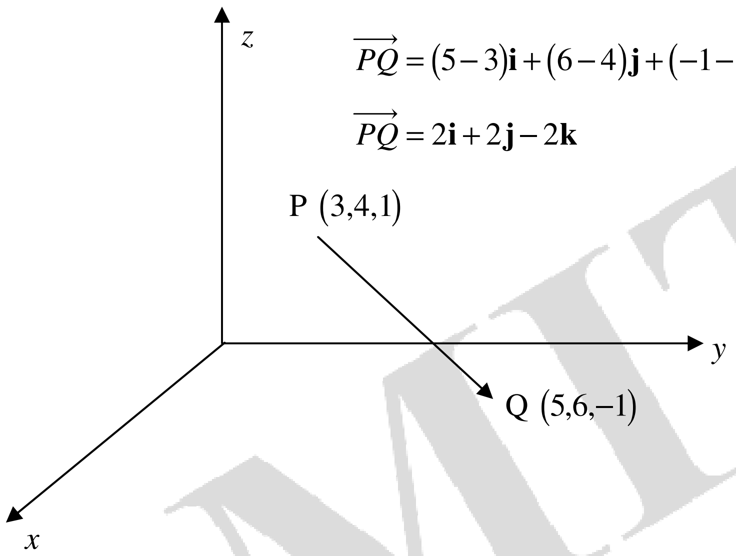
The directed line segment, or geometric vector, $\overrightarrow{PQ}$, from P($x_1,\ y_1,\ z_1$) to Q($x_2,\ y_2,\ z_2$) is found by subtracting the co-ordinates of P (the initial point) from the co-ordinates of Q (the final point).

$$\overrightarrow{PQ} = \left((x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}\right)$$

Example.

$$\overrightarrow{PQ} = (5-3)\mathbf{i} + (6-4)\mathbf{j} + (-1-1)\mathbf{k}$$

$$\overrightarrow{PQ} = 2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$$



The directed line segment $\overrightarrow{PQ}$ is represented by the vector 2**i** + 2**j** – 2**k,** or (2, 2, –2). Any other directed line segment with the **same length and same directon as** $\overrightarrow{PQ}$ is also represented by 2**i** + 2**j** – 2**k** or (2, 2, –2).

The directed line segment $\overrightarrow{QP}$ has the same length as $\overrightarrow{PQ}$ but is in the opposite direction.

$\overrightarrow{QP} = -\overrightarrow{PQ} = -(2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) = -2\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ or (–2,–2, 2)

Position Vector.

The position vector of any point is the directed line segment from the origin O (0,0,0) to the point and is given by the co-ordinates of of the point.

The position vector of P(3, 4, 1) is 3**i** + 4**j** + **k**, or (3, 4, 1).

Exercise 1. Given the points A(3, 0,4) B(–2, 4, 3) and C(1,–5,0), find:

(a) $\overrightarrow{AB}$ (b) $\overrightarrow{AC}$ (c) $\overrightarrow{CB}$

(d) $\overrightarrow{BC}$ (e) $\overrightarrow{CA}$

(f) The position vectors of A, B and C.

Compare your answers 1(b)and 1(e), and 1(c) and 1(d). What do you notice?

Length or Magnitude of a Vector.

The length of a vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ is written $|\mathbf{a}|$ and is evaluated by:

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

For example the length of the vector $2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ equals $\sqrt{2^2 + 2^2 + (-2)^2} = \sqrt{12} = 2\sqrt{3}$

a_1, a_2, a_3 are often referred to as the components of vector **a.**

Unit Vector

A vector with a magnitude of one is called a unit vector. If **a** is any vector then a unit vector parallel to **a** is written $\hat{\mathbf{a}}$ (**a** "hat"). The "hat" symbolises a unit vector.

Vector **a** can then be written $\mathbf{a} = |\mathbf{a}|\hat{\mathbf{a}}$

therefore $\hat{\mathbf{a}} = \dfrac{\mathbf{a}}{|\mathbf{a}|}$

Example

A unit vector parallel to $\mathbf{a} = (1, 2, 3)$

is the vector $\hat{\mathbf{a}} = \dfrac{\mathbf{a}}{|\mathbf{a}|} = \dfrac{(1,\ 2,\ 3)}{\sqrt{1^2 + 2^2 + 3^2}} = \dfrac{1}{\sqrt{14}}(1,\ 2,\ 3)$

Adding and Subtracting Vectors.

Vectors are added or subtracted by

- adding or subtracting their corresponding components
- using the triangle rule
- by using the parallogram rule.

Example

If $\mathbf{a} = (-3, 4, 2)$ and $\mathbf{b} = (-1\ -2, 3)$, find:

(i) $\mathbf{a} + \mathbf{b}$ (ii) $\mathbf{a} - \mathbf{b}$.

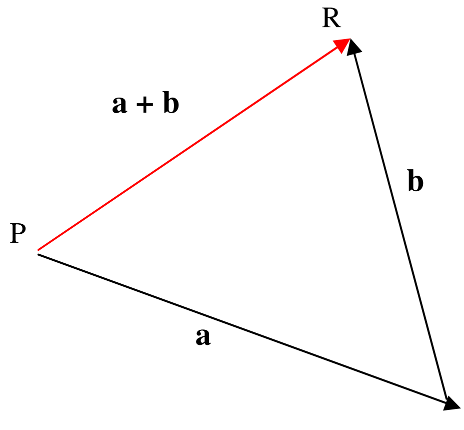
Adding or subtracting components

(i) $\mathbf{a} + \mathbf{b} = (-3, 4, 2) + (-1, -2, 3) = (-4, 2, 5)$

Similarly (ii) $\mathbf{a} - \mathbf{b} = (-3, 4, 2) - (-1, -2, 3) = (-2, 6, -1)$

Triangle Rule

(i) **a + b**



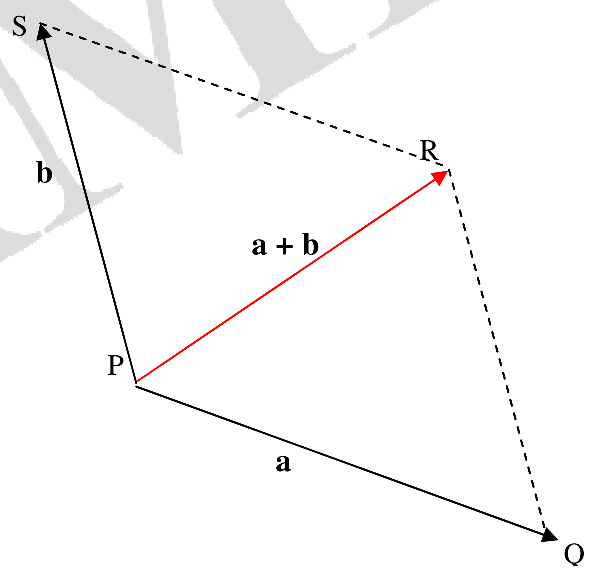
Place the tail of vector **b** at the head of vector **a** (point Q). The directed line segment $\overrightarrow{PR}$ from the tail of vector **a** to the head of vector **b** is the vector **a + b**.

(ii) To subtract **b** from **a,** reverse the direction of **b** to give **–b** then add **a** and −**b.**

a –b = a + (–b)

Parallelogram Rule

(i) **a + b**



a and **b** are placed "tail-to-tail"(point P) and the parallelogram (PQRS) completed. The diagonal PR is the sum **a + b.**

(ii) To find (**a – b**), reverse the direction of **b** to give **–b** then add **a** and **–b.**

<u>Exercise</u> 2.

For vectors **p** (3, 6, 5), **q** (–4, 1, 0) and **r** (1, –3, 5) find:

(a) **p + q** (b) **r + p** (c) **p – q**

Multiplication of a vector by a scalar.

To multiply vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ by a scalar, m, multiply each component of **a** by m.

$$m\mathbf{a} = ma_1\mathbf{i} + ma_2\mathbf{j} + ma_3\mathbf{k}$$

The result is a vector of length $|m|\times|\mathbf{a}|$

If $m > 0$ the resultant vector is in the same direction as **a**

If $m < 0$ the resultant vector is in the opposite direction from **a**.

Two vectors **a** and **b** are said to be parallel if and only if $\mathbf{a} = k\mathbf{b}$ where k is a real constant.

Example 1

$\mathbf{a} = (3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ is multiplied by 7

$7\mathbf{a} = 7(3\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 21\mathbf{i} + 7\mathbf{j} - 14\mathbf{k}.$

The magnitude of **a** is

$$|\mathbf{a}| = \sqrt{3^2 + 1^2 + (-2)^2} = \sqrt{14}$$

$$|7\mathbf{a}| = \sqrt{21^2 + 7^2 + (-14)^2} = \sqrt{686} = 7\sqrt{14} = 7|a|$$

Example 2

Find the value of m so that the vector **a,** (4, m, 8) is parallel to the vector **b**, (–6, 3, 12).

For **a** and **b** to be parallel $\mathbf{a} = k\mathbf{b}$

Therefore $(4, m, 8) = k(-6, 3, -12) = (-6k, 3k, -12k)$

equating "**i**"components

$4 = -6k$

$$k = \frac{-2}{3}$$

equating "**j**"components

$m = 3k$

$$\therefore m = 3\times\frac{-2}{3}$$

$m = -2$

Exercise 3

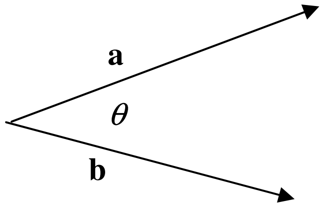
Find the following

(a) $3 \times(\mathbf{i} + 3\mathbf{j} - 5\mathbf{k})$ (b) $8 \times (7\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})$ (c) $-4 \times (\mathbf{j} - 3\mathbf{k})$

Multiplication of a vector by a vector

(1) ***Dot product or scalar product***

The dot product of two vectors **a** (a_1, a_2, a_3) and **b** (b_1, b_2, b_3) is a **scalar**, defined by



$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between **a** and **b**

and $\mathbf{a} \bullet \mathbf{b} = \mathbf{b} \bullet \mathbf{a}$

If **a** *is perpendicular to* **b** *then* $\mathbf{a} \bullet \mathbf{b} = 0$ (*cos*(π/2)= 0).

In particular $\mathbf{i} \bullet \mathbf{j} = \mathbf{j} \bullet \mathbf{k} = \mathbf{k} \bullet \mathbf{i} = 0$

If **a** *is parallel to* **b** *then* $\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}||\mathbf{b}|$ (*cos*(0)=1)

In particular $\mathbf{i} \bullet \mathbf{i} = \mathbf{j} \bullet \mathbf{j} = \mathbf{k} \bullet \mathbf{k} = 1$

Also $(a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) \bullet (b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}) =$

$$(a_1b_1\mathbf{i} \bullet \mathbf{i} + a_1b_2\mathbf{i} \bullet \mathbf{j} + a_1b_3\mathbf{i} \bullet \mathbf{k} + a_2b_1\mathbf{j} \bullet \mathbf{i} + a_2b_2\mathbf{j} \bullet \mathbf{j} + a_2b_3\mathbf{j} \bullet \mathbf{k} + a_3b_1\mathbf{k} \bullet \mathbf{i} + a_3b_2\mathbf{k} \bullet \mathbf{j} + a_3b_3\mathbf{k} \bullet \mathbf{k})$$

Thus $\mathbf{a} \bullet \mathbf{b}$ can be defined by $\mathbf{a} \bullet \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$

Example 1

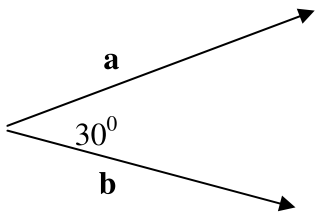
Find the dot product of $(2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$ and $(-\mathbf{i} - 2\mathbf{j} + \mathbf{k})$

$$\begin{aligned}(2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) \bullet (-\mathbf{i} - 2\mathbf{j} + \mathbf{k}) &= 2(-1) + 3(-2) + (4)(1) \\ &= -2 - 6 + 4 \\ &= -4\end{aligned}$$

$$(2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) \bullet (-\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = -4$$

Example 2

Find the scalar product of **a** and **b**, as drawn, below where $|\mathrm{a}| = \sqrt{14}$ and $|\mathrm{b}| = \sqrt{6}$



$$\begin{aligned}\mathbf{a} \bullet \mathbf{b} &= |\mathbf{a}|\;|\mathbf{b}|\cos\theta \\ &= \sqrt{14} \times \sqrt{6} \times \cos 30^0 \\ &= \sqrt{14} \times \sqrt{6} \times \frac{\sqrt{3}}{2} \\ &= 3\sqrt{7}\end{aligned}$$

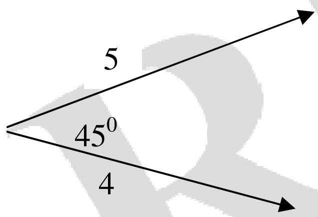
$$\mathbf{a} \bullet \mathbf{b} = 3\sqrt{7}$$

Exercise 4 Find the dot product of the following vectors:

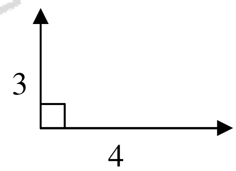
(a) $3\mathbf{i}$ and $5\mathbf{j}$ (b) $2\mathbf{i} + 3\mathbf{k}$ and $7\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ (c) $5\mathbf{k}$ and $\mathbf{j} - 2\mathbf{k}$

(d) (2, 0, 4) and (– 3, 1, 3) (e) (0, 5, 1) and (4, 0, 0)

(f)



(g)



(2) *Cross product or vector product*

The cross product of two vectors **a** and **b** is the vector $\mathbf{a} \times \mathbf{b}$**,** which is perpendicular to **both a and b** and is given by

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

The magnitude of $\mathbf{a} \times \mathbf{b}$ is given by $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$ where θ is the angle between **a** and **b**.

The direction of $\mathbf{a} \times \mathbf{b}$ is that in which your thumb would point if the fingers of your right are curled from **a** to **b**.

In particular $\mathbf{i} \times \mathbf{j} = \mathbf{k}$, $\mathbf{j} \times \mathbf{k} = \mathbf{i}$, $\mathbf{k} \times \mathbf{i} = \mathbf{j}$

$\mathbf{i} \times \mathbf{k} = -\mathbf{j}$ $\mathbf{k} \times \mathbf{j} = -\mathbf{i}$ $\mathbf{j} \times \mathbf{i} = -\mathbf{k}$

If $\mathbf{a}$ *is parallel to* $\mathbf{b}$ *then* $\mathbf{a} \times \mathbf{b} = 0$. ($sin0^0 = 0$)

In particular $\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$

If $\mathbf{a}$ *is perpendicular to* $\mathbf{b}$ *then* $|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|$ ($sin90^0 = 1$)

$\mathbf{a} \times \mathbf{b} = -\,\mathbf{b} \times \mathbf{a}$ (*the cross product is not commutative*.)

Example 1

Find $\mathbf{a} \times \mathbf{b}$ if $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 5\mathbf{j} + 3\mathbf{k}$

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 1 \\ 0 & 5 & 3 \end{vmatrix} = (9-5)\mathbf{i} - (6-0)\mathbf{j} + (10-0)\mathbf{k}$$

$$\mathbf{a}\times\mathbf{b} = 4\mathbf{i} - 6\mathbf{j} + 10\mathbf{k}$$

Example 2

Find $\mathbf{a} \times \mathbf{b}$ if $\mathbf{a} = (2,1,1)$ and $\mathbf{b} = (-2,4,0)$

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 1 \\ -2 & 4 & 0 \end{vmatrix} = (0-4)\mathbf{i} - (0+2)\mathbf{j} + (8+2)\mathbf{k}$$

$$\mathbf{a}\times\mathbf{b} = -4\mathbf{i} - 2\mathbf{j} + 10\mathbf{k}$$

Example 3

Find $\mathbf{a} \times \mathbf{b}$ if $\mathbf{a} = (2,1,1)$ and $\mathbf{b} = (8,4,4)$

Because $\mathbf{a} = 4\mathbf{b}$, $\mathbf{a}$ is parallel to $\mathbf{b}$ therefore $\mathbf{a} \times \mathbf{b} = \mathbf{0}$

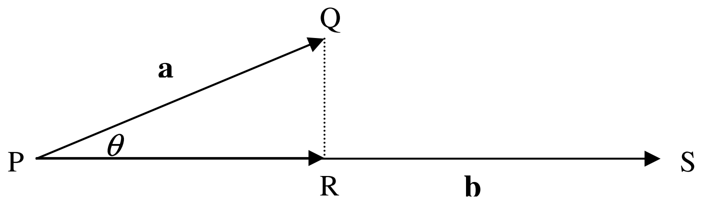
Exercise 5 Find the cross product of the following vectors:

(a) $\mathbf{j} \times \mathbf{k}$ (b) $\mathbf{i} \times 4\mathbf{i}$ (c) $(2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) \times (3\mathbf{j} + 2\mathbf{k})$

(d) $3\mathbf{j} \times 5\mathbf{i}$ (e) $(\mathbf{i} - 3\mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - \mathbf{k})$

Projection of vectors.

Consider the diagram below:



Let $\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{PS} = \mathbf{b}$.

Scalar projection

The **scalar projection** of vector **a** in the direction of vector **b** is the length of the straight line PR or $\left|\overrightarrow{PR}\right|$.

$$\left|\overrightarrow{PR}\right| = |\mathbf{a}|\cos\theta. \qquad \text{Also} \quad \cos\theta = \frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \quad (\text{because } \mathbf{a}\bullet\mathbf{b} = |\mathbf{a}|\,|\mathbf{b}|\cos\theta)$$

Therefore

$$\left|\overrightarrow{PR}\right| = (|\mathbf{a}|)\left(\frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right) = \frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{b}|} = \mathbf{a}\bullet\hat{\mathbf{b}} \quad (\text{cancel } |\mathbf{a}|, \text{ and use } \frac{\mathbf{b}}{|\mathbf{b}|} = \hat{\mathbf{b}})$$

The **scalar projection** of a vector **a** in the direction of vector **b** is given by

$$\frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{b}|} = \mathbf{a}\bullet\hat{\mathbf{b}} \quad \text{or} \quad |\mathbf{a}|\cos\theta$$

Vector projection

The **vector projection** of vector **a** in the direction of vector **b** is a **vector** in the direction of **b** with **a magnitude** equal to the length of the straight line PR or $\left|\overrightarrow{PR}\right|$.

Therefore the vector projection of **a** in the direction of **b** is the scalar projection multiplied by a unit vector in the direction of **b.**

The **vector projection** of vector **a** in the direction of vector **b** is given by

$$(\mathbf{a}\bullet\hat{\mathbf{b}})\,\hat{\mathbf{b}} = \frac{(\mathbf{a}\bullet\mathbf{b})\,\mathbf{b}}{|\mathbf{b}|^2}$$

Angle between two vectors

The angle, θ between two vectors can be found from the definition of the dot product

$$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

therefore $\cos\theta = \dfrac{\mathbf{a} \bullet \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

θ can also be found from $\cos\theta = \dfrac{\mathbf{a} \bullet \hat{\mathbf{b}}}{|\mathbf{a}|}$

Example

Find (a) the scalar projection of vector **a** = (2, 3, 1) in the direction of vector **b** = (5, −2, 2).

(b) the angle between **a** and **b**.

(c) the vector projection of **a** in the direction of **b**.

(a) **Scalar projection**

$$|\mathbf{b}| = \sqrt{25+4+4} = \sqrt{33} \quad \text{therefore} \quad \hat{\mathbf{b}} = \frac{(5,\ -2,\ 2)}{\sqrt{33}}$$

$$\mathbf{a} \bullet \hat{\mathbf{b}} = (2,\ 3,\ 1) \bullet \frac{(5,\ -2,\ 2)}{\sqrt{33}} = \frac{10+(-6)+2}{\sqrt{33}} = \frac{6}{\sqrt{33}}$$

The scalar projection of **a** in the direction of **b** is $\dfrac{6}{\sqrt{33}}$

(b) **Angle between a and b**

The scalar projection of **a** in the direction of **b** is also equal to $|\mathbf{a}|\cos\theta$, where θ is the angle between **a** and **b**.

Therefore $\dfrac{6}{\sqrt{33}} = |\mathbf{a}|\cos\theta$. $\quad |\mathbf{a}| = \sqrt{2^2+3^2+1^2} = \sqrt{14}$

$$\therefore \frac{6}{\sqrt{33}} = \sqrt{14}\cos\theta$$

$$\therefore \cos\theta = \frac{6}{\sqrt{33\times 14}} = 0.2791$$

$$\therefore \theta = 74°$$

The angle between **a** and **b** is 74^0.

(c) **Vector projection**

The vector projection **a** in the direction of **b** equals:

(**scalar projection a in the direction of b**) $\hat{\mathbf{b}}$

$$= \frac{6}{\sqrt{33}} \times \frac{(5,\ -2,\ 2)}{\sqrt{33}} = \frac{6(5,-2,2)}{33}$$

The vector projection of **a** in the direction of **b** is $\dfrac{6(5,-2,2)}{33}$

<u>**Exercise**</u> **6** For the following pairs of vectors find:

(i) the scalar projection of **a** on **b**

(ii) the angle between **a** and **b**

(iii) the vector projection of **a** on **b**

(a) **a** = (2, 3 ,1) **b** = (5, 0, 3)

(b) **a** = (0, 0, 3) **b** = (0, 0, 7)

(c) **a** = (5, 0, 0) **b** = (0, 3, 0)

(d) **a** = (−3, 2, −1) **b** = (2, 1, 2)

Answers

1.(a) $(-5, 4, -1)$ **(b)** $(-2, -5, -4)$ **(c)** $(-3, 9, 3)$

(d) $(3, -9, -3)$ **(e)** $(2, 5, 4)$

(f) $\overrightarrow{OA} = 3\mathbf{i} + 4\mathbf{k}$, $\overrightarrow{OB} = -2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OC} = \mathbf{i} - 5\mathbf{j}$

2. (a) $(-1, 7, 5)$ **(b)** $(4, 3, 10)$ **(c)** $(7, 5, 5)$

3. (a) $3\mathbf{i} + 9\mathbf{j} - 15\mathbf{k}$ **(b)** $56\mathbf{i} + 16\mathbf{j} + 32\mathbf{k}$ **(c)** $-4\mathbf{j} + 12\mathbf{k}$

4. (a) 0 **(b)** 26 **(c)** -10 **(d)** 6 **(e)** 0 **(f)** $10\sqrt{2}$ **(g)** 0

5. (a) $\mathbf{i}$ **(b)** 0 **(c)** $9\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ **(d)** $-15\mathbf{k}$ **(e)** $-\mathbf{i} + 9\mathbf{j} + 7\mathbf{k}$

6.(a)(i) $\frac{13}{\sqrt{34}}$ **(ii)** 53^0 **(iii)** $\frac{13}{34}(5,\ 0,\ 3)$

(b)(i) 3 **(ii)** 0^0 **(iii)** $\frac{3}{7}(0,\ 0,\ 7)$

(c)(i) 0 **(ii)** 90^0 **(iii)** 0

(d)(i) -2 **(ii)** 122^0 **(iii)** $\frac{-2}{3}(2,\ 1,\ 2)$