

## **Lecture 5 - PN Junction and MOS Electrostatics (II)**

### **PN JUNCTION IN THERMAL EQUILIBRIUM**

February 20, 2003

#### **Contents:**

1. Introduction to pn junction
2. Electrostatics of pn junction in thermal equilibrium
3. The depletion approximation
4. Contact potentials

#### **Reading assignment:**

Howe and Sodini, Ch. 3, §§3.3-3.4

## Key questions

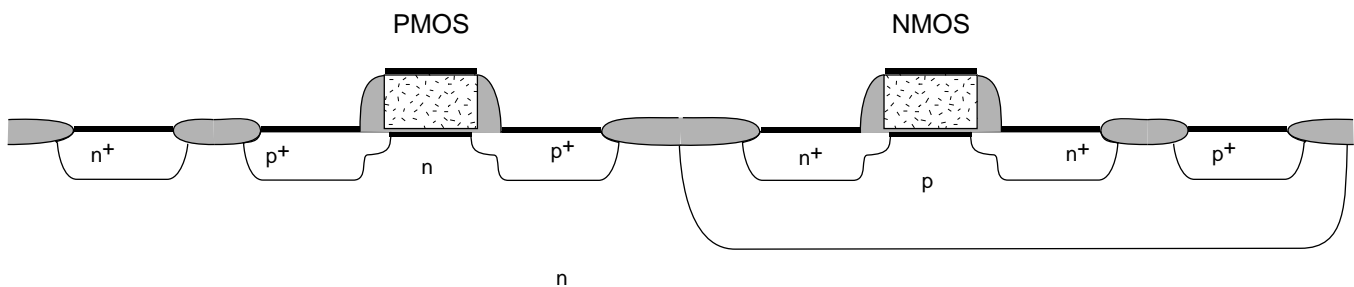
- What happens if the doping distribution in a semiconductor abruptly changes from n-type to p-type?
- Is there a simple description of the electrostatics of a pn junction?

# 1. Introduction to pn junction

- pn junction: p-region and n-region in intimate contact
- Why is the p-n junction worth studying?

It is present in virtually every semiconductor device!

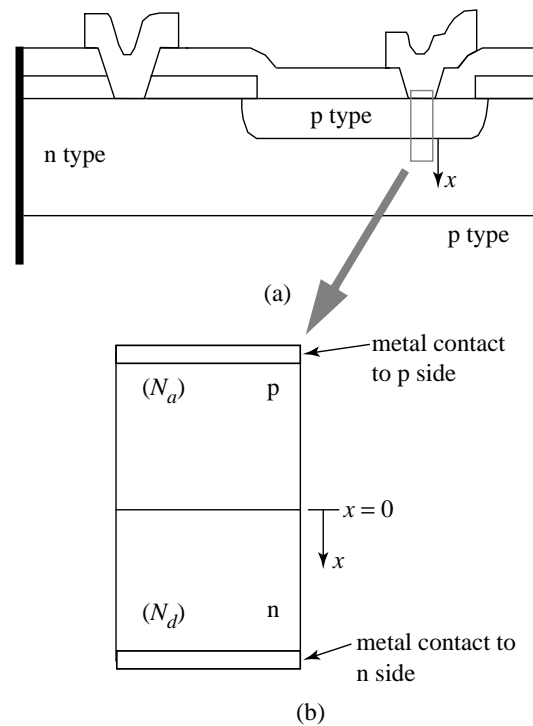
Example: CMOS cross section



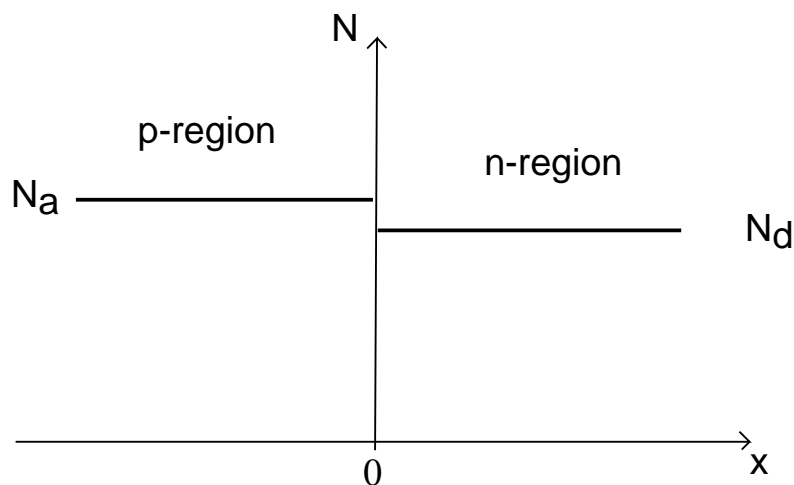
Understanding p-n junction is essential to understanding transistor operation.

## 2. Electrostatics of p-n junction in equilibrium

Focus on intrinsic region:

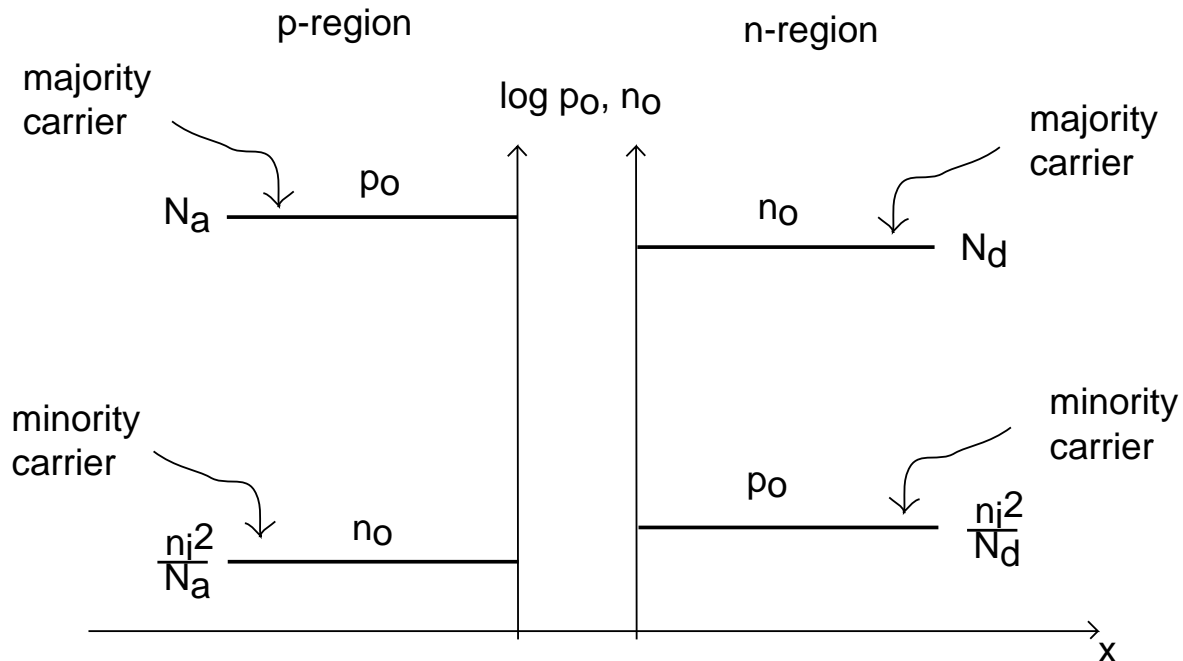


Doping distribution of abrupt p-n junction:



What is the carrier concentration distribution in thermal equilibrium?

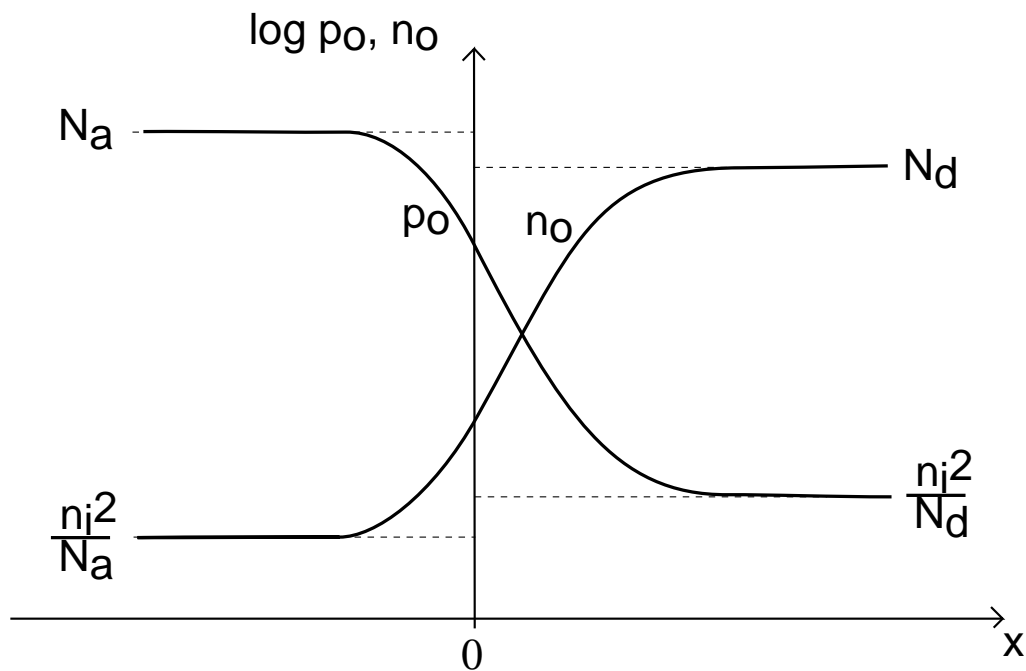
First think of two sides separately:



Now bring them together. What happens?

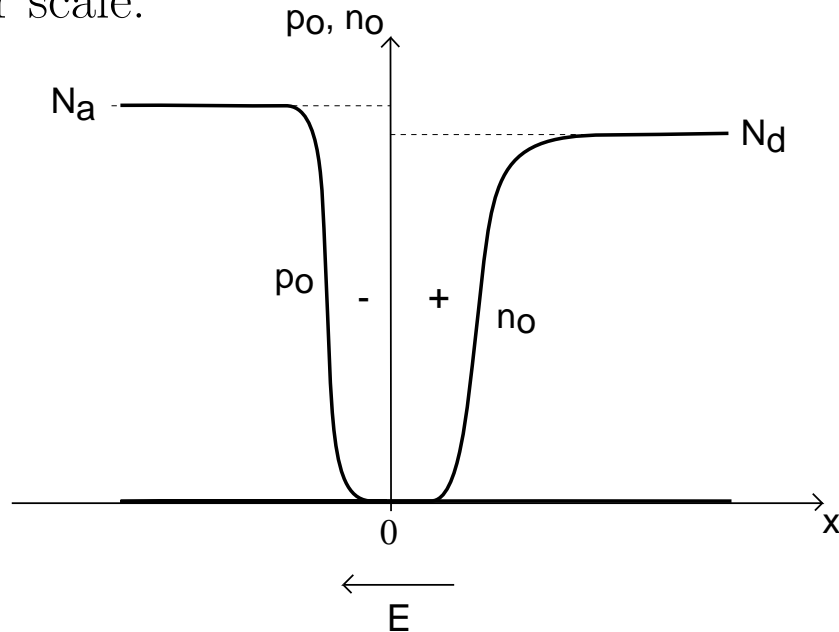
Diffusion of electrons and holes from majority carrier side to minority carrier side until drift balances diffusion.

Resulting carrier profile in thermal equilibrium:



- Far away from metallurgical junction: nothing happens
  - two *quasi-neutral regions*
- Around metallurgical junction: carrier drift must cancel diffusion
  - *space-charge region*

In a linear scale:



Thermal equilibrium: balance between drift and diffusion

$$\begin{array}{c}
 \xrightarrow{J_h^{\text{diff}}} \\
 \xleftarrow{J_h^{\text{drift}}} \\
 \xrightarrow{J_e^{\text{diff}}} \\
 \xleftarrow{J_e^{\text{drift}}}
 \end{array}$$

Can divide semiconductor in three regions:

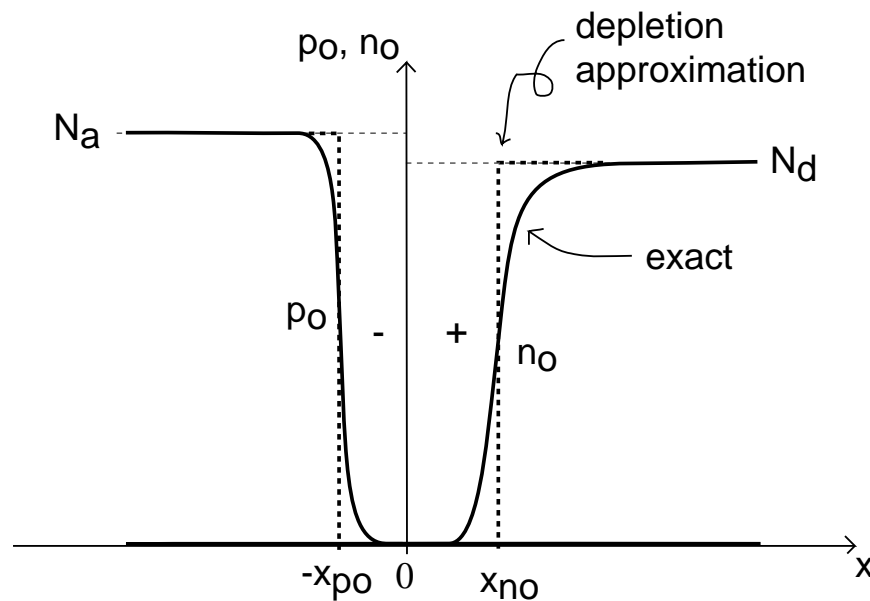
- two quasi-neutral n- and p-regions (QNR's)
- one space charge region (SCR)

Now, want to know  $n_o(x)$ ,  $p_o(x)$ ,  $\rho(x)$ ,  $E(x)$ , and  $\phi(x)$ .

Solve electrostatics using simple, powerful approximation.

### 3. The depletion approximation

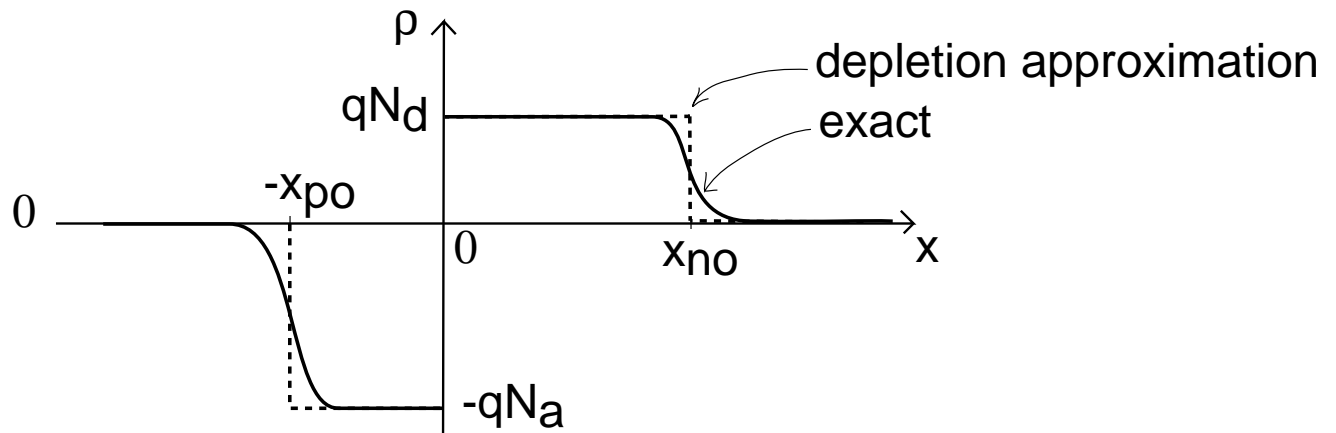
- Assume QNR's perfectly charge neutral
- assume SCR depleted of carriers (*depletion region*)
- transition between SCR and QNR's sharp  
(must calculate where to place  $-x_{po}$  and  $x_{no}$ )



- $x < -x_{po}$   $p_o(x) = N_a, n_o(x) = \frac{n_i^2}{N_a}$
- $-x_{po} < x < 0$   $p_o(x), n_o(x) \ll N_a$
- $0 < x < x_{no}$   $n_o(x), p_o(x) \ll N_d$
- $x_{no} < x$   $n_o(x) = N_d, p_o(x) = \frac{n_i^2}{N_d}$



## ● SPACE CHARGE DENSITY

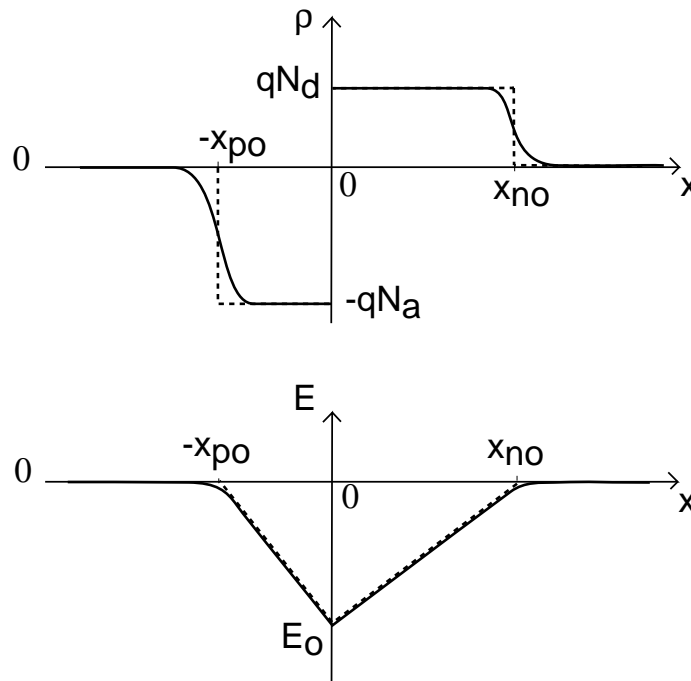


$$\begin{aligned}
 \rho(x) &= 0 & x < -x_{po} \\
 &= -qN_a & -x_{po} < x < 0 \\
 &= qN_d & 0 < x < x_{no} \\
 &= 0 & x_{no} < x
 \end{aligned}$$

## ● ELECTRIC FIELD

Integrate Gauss' equation:

$$E(x_2) - E(x_1) = \frac{1}{\epsilon_s} \int_{x_1}^{x_2} \rho(x) dx$$



$$\bullet \quad x < -x_{po} \quad E(x) = 0$$

$$\bullet \quad -x_{po} < x < 0 \quad E(x) - E(-x_{po}) = \frac{1}{\epsilon_s} \int_{-x_{po}}^x -qN_a dx \\ = \frac{-qN_a}{\epsilon_s} x \Big|_{-x_{po}}^x = \frac{-qN_a}{\epsilon_s} (x + x_{po})$$

$$\bullet \quad 0 < x < x_{no} \quad E(x) = \frac{qN_d}{\epsilon_s} (x - x_{no})$$

$$\bullet \quad x_{no} < x \quad E(x) = 0$$

# ● ELECTROSTATIC POTENTIAL

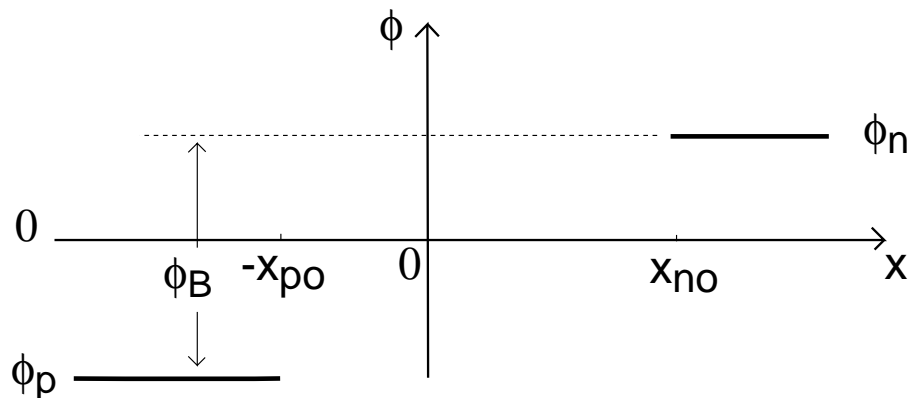
(with  $\phi = 0$  @  $n_o = p_o = n_i$ ):

$$\phi = \frac{kT}{q} \ln \frac{n_o}{n_i} \qquad \phi = -\frac{kT}{q} \ln \frac{p_o}{n_i}$$

In QNR's,  $n_o$ ,  $p_o$  known  $\Rightarrow$  can determine  $\phi$ :

$$\text{in p-QNR: } p_o = N_a \Rightarrow \phi_p = -\frac{kT}{q} \ln \frac{N_a}{n_i}$$

$$\text{in n-QNR: } n_o = N_d \Rightarrow \phi_n = \frac{kT}{q} \ln \frac{N_d}{n_i}$$



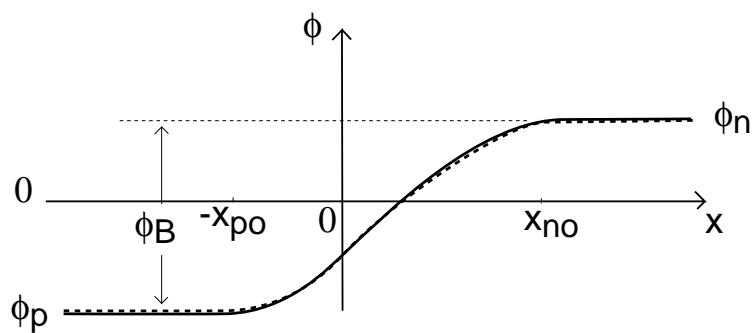
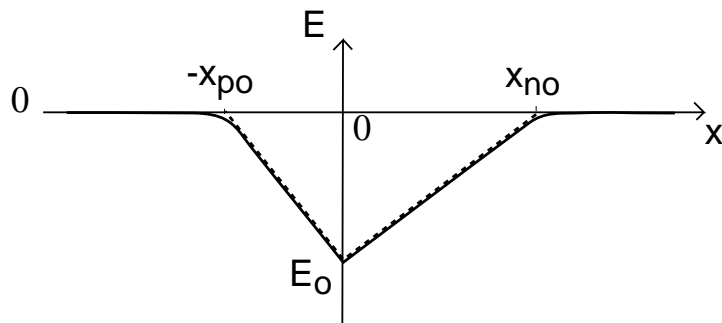
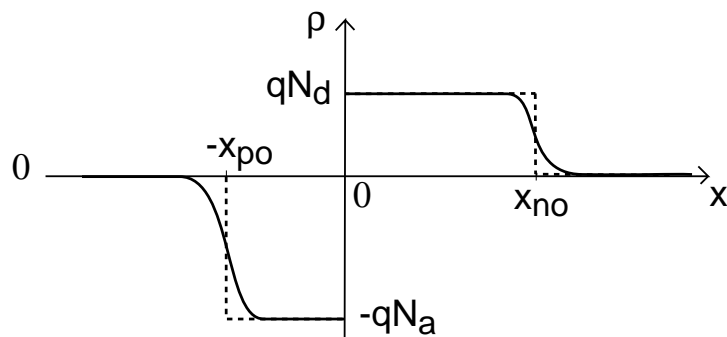
Built-in potential:

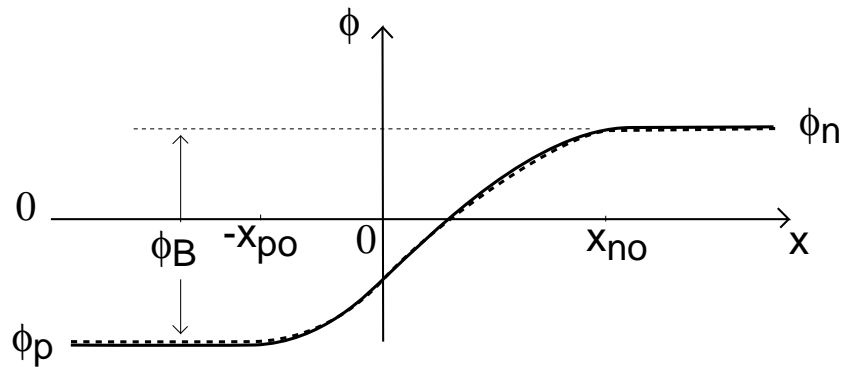
$$\phi_B = \phi_n - \phi_p = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}$$

General expression: did not use depletion approximation.

To get  $\phi(x)$  in between, integrate  $E(x)$ :

$$\phi(x_2) - \phi(x_1) = - \int_{x_1}^{x_2} E(x) dx$$





- $x < -x_{po}$                        $\phi(x) = \phi_p$
- $-x_{po} < x < 0$                $\phi(x) = \phi(-x_{po}) - \int_{-x_{po}}^x \frac{qN_a}{\epsilon_s}(x + x_{po})dx$   
 $= \frac{qN_a}{2\epsilon_s}(x + x_{po})^2$
- $0 < x < x_{no}$                        $\phi(x) = \phi_n - \frac{qN_d}{2\epsilon_s}(x - x_{no})^2$
- $x_{no} < x$                        $\phi(x) = \phi_n$

Almost done...

Still don't know  $x_{no}$  and  $x_{po} \Rightarrow$  need two more equations

1. Require overall charge neutrality:

$$qN_ax_{po} = qN_dx_{no}$$

2. Require  $\phi(x)$  continuous at  $x = 0$ :

$$\phi_p + \frac{qN_a}{2\epsilon_s}x_{po}^2 = \phi_n - \frac{qN_d}{2\epsilon_s}x_{no}^2$$

Two equations with two unknowns. Solution:

$$x_{no} = \sqrt{\frac{2\epsilon_s\phi_B N_a}{q(N_a + N_d)N_d}} \quad x_{po} = \sqrt{\frac{2\epsilon_s\phi_B N_d}{q(N_a + N_d)N_a}}$$

Now problem completely solved.

Other results:

Total width of space charge region:

$$x_{do} = x_{no} + x_{po} = \sqrt{\frac{2\epsilon_s\phi_B(N_a + N_d)}{qN_aN_d}}$$

Field at metallurgical junction:

$$|E_o| = \sqrt{\frac{2q\phi_B N_a N_d}{\epsilon_s(N_a + N_d)}}$$

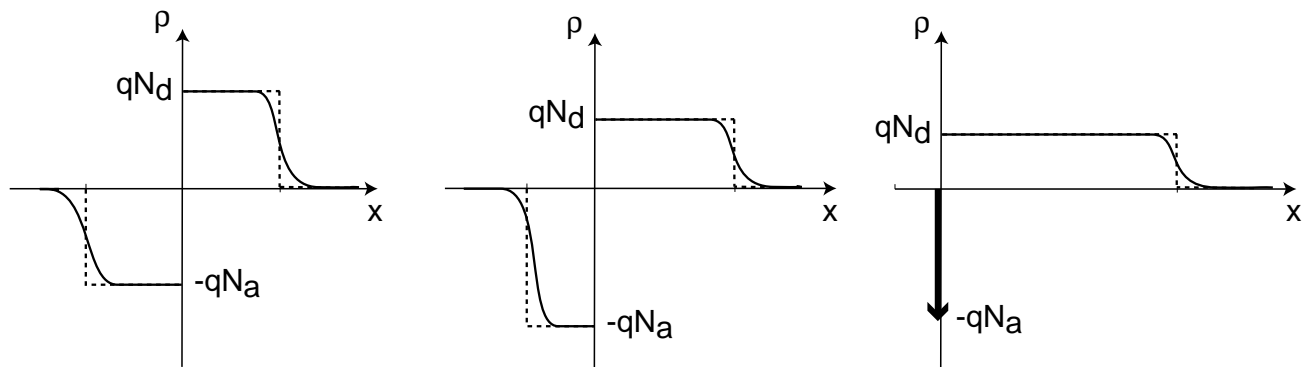
Three cases:

- Symmetric junction:  $N_a = N_d \Rightarrow x_{po} = x_{no}$
- Asymmetric junction:  $N_a > N_d \Rightarrow x_{po} < x_{no}$
- Strongly asymmetric junction:  
*i.e.* p<sup>+</sup>n junction:  $N_a \gg N_d$

$$x_{po} \ll x_{no} \simeq x_{do} \simeq \sqrt{\frac{2\epsilon_s \phi_B}{qN_d}} \propto \frac{1}{\sqrt{N_d}}$$

$$|E_o| \simeq \sqrt{\frac{2q\phi_B N_d}{\epsilon_s}} \propto \sqrt{N_d}$$

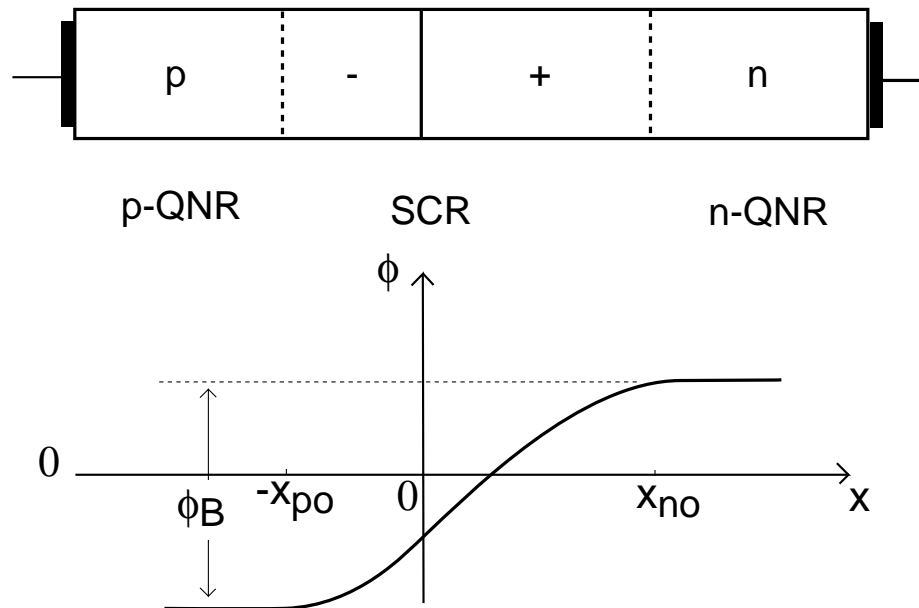
The lowly-doped side controls the electrostatics of the pn junction.





## 4. Contact potentials

Potential distribution in thermal equilibrium so far:



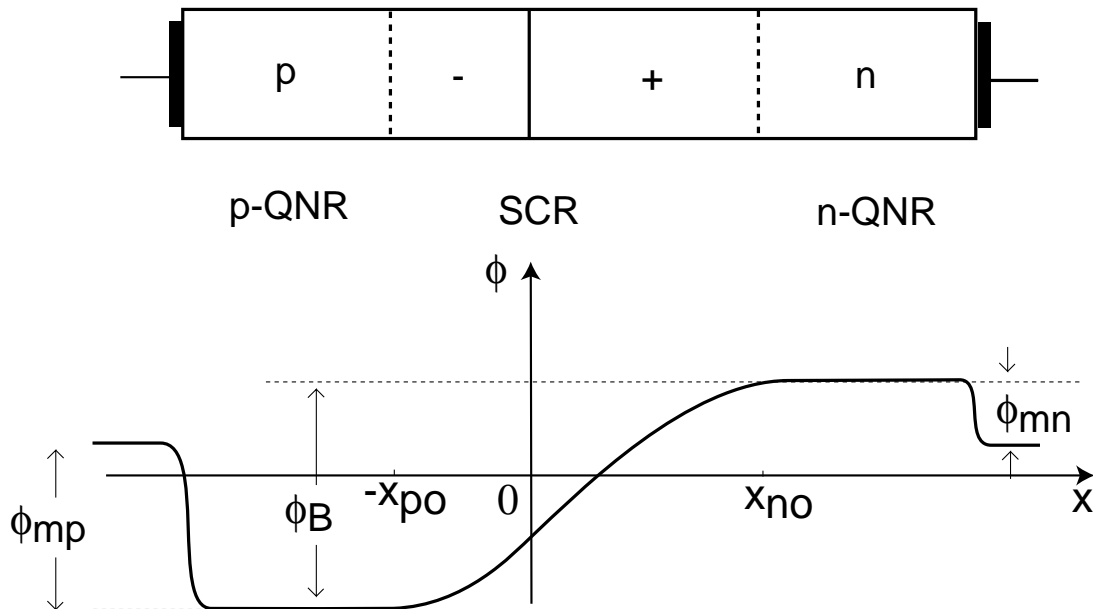
Question 1: *If I apply a voltmeter across diode, do I measure  $\phi_B$ ?*

☐ yes      ☐ no      ☐ it depends

Question 2: *If I short diode terminals, does current flow on outside circuit?*

☐ yes      ☐ no      ☐ sometimes

We are missing *contact potential* at metal-semiconductor contacts:



Metal-semiconductor contacts: junctions of dissimilar materials

$\Rightarrow$  built-in potentials:  $\phi_{mn}$ ,  $\phi_{mp}$

Potential difference across structure must be zero

$\Rightarrow$  cannot measure  $\phi_B$ !

$$\phi_B = \phi_{mn} + \phi_{mp}$$

## Key conclusions

- Electrostatics of pn junction in equilibrium:
  - a *space-charge region*
  - surrounded by two *quasi-neutral regions*
  - ⇒ built-in potential across p-n junction
- To first order, carrier concentrations in space-charge region are much smaller than doping level
  - ⇒ *depletion approximation*.
- Contact potential at metal-semiconductor junctions:
  - ⇒ from contact to contact, there is no potential build-up across pn junction