

## **Lecture 4 - PN Junction and MOS Electrostatics (I)**

### **SEMICONDUCTOR ELECTROSTATICS IN THERMAL EQUILIBRIUM**

February 13, 2003

#### **Contents:**

1. Non-uniformly doped semiconductor in thermal equilibrium
2. Quasi-neutral situation
3. Relationships between  $\phi(x)$  and equilibrium carrier concentrations (*Boltzmann relations*), "60 mV Rule"

#### **Reading assignment:**

Howe and Sodini, Ch. 3, §§3.1-3.2

#### **Announcements:**

Spare handouts available on the web and outside TA's office (Rm. 24-320).

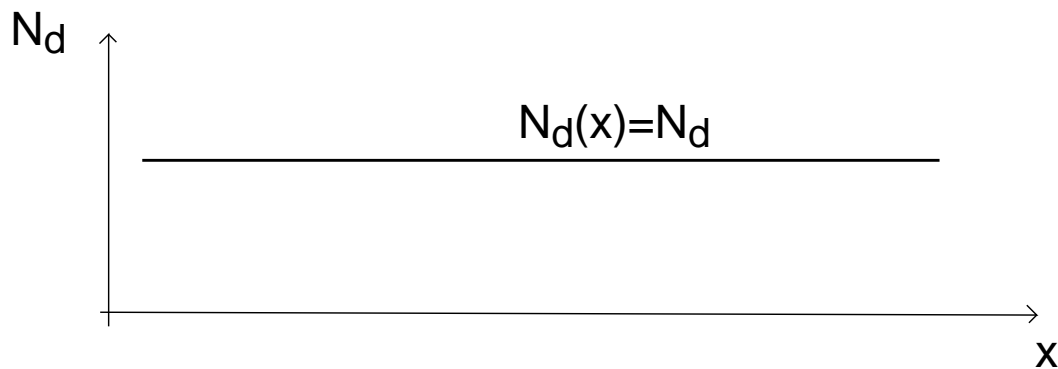
Deadline for all assignments: Friday, 1 PM in 26-310.

## Key questions

- Is it possible to have an electric field inside a semiconductor in thermal equilibrium?
- If there is a doping gradient in a semiconductor, what is the resulting majority carrier concentration in thermal equilibrium?

# 1. Non-uniformly doped semiconductor in thermal equilibrium

Consider first *uniformly doped* n-type Si in thermal equilibrium:



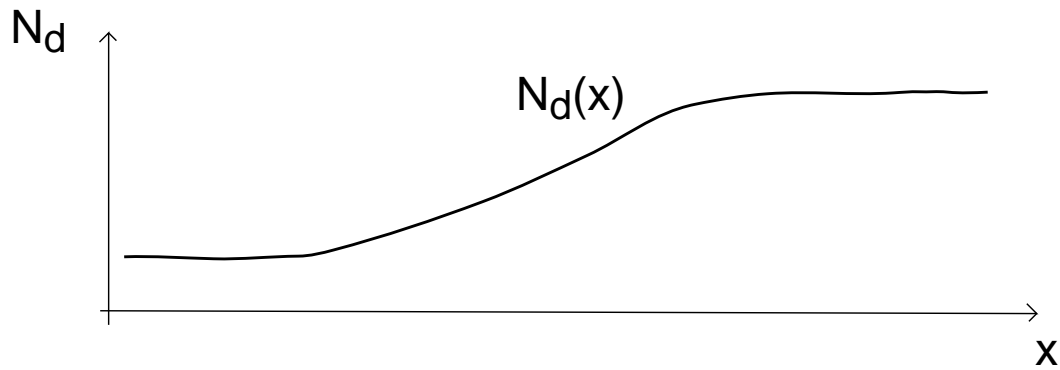
n-type  $\Rightarrow$  lots of electrons, few holes  
 $\Rightarrow$  focus on electrons

$$n_o = N_d \quad \text{independent of } x$$

Volume charge density [ $C/cm^3$ ]:

$$\rho = q(N_d - n_o) = 0$$

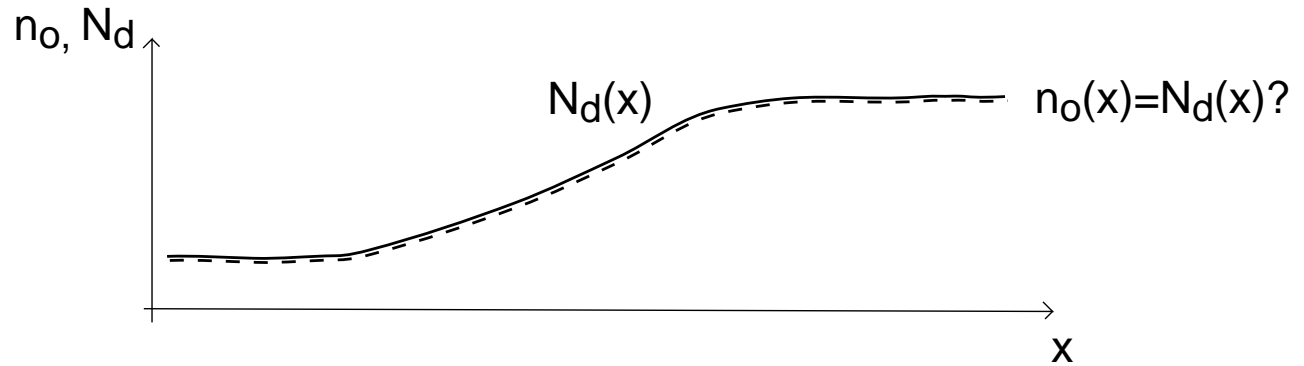
Next, consider piece of n-type Si in thermal equilibrium with *non-uniform dopant distribution*:



What is the resulting electron concentration in thermal equilibrium?

OPTION 1: Every donor gives out one electron  $\Rightarrow$

$$n_o(x) = N_d(x)$$



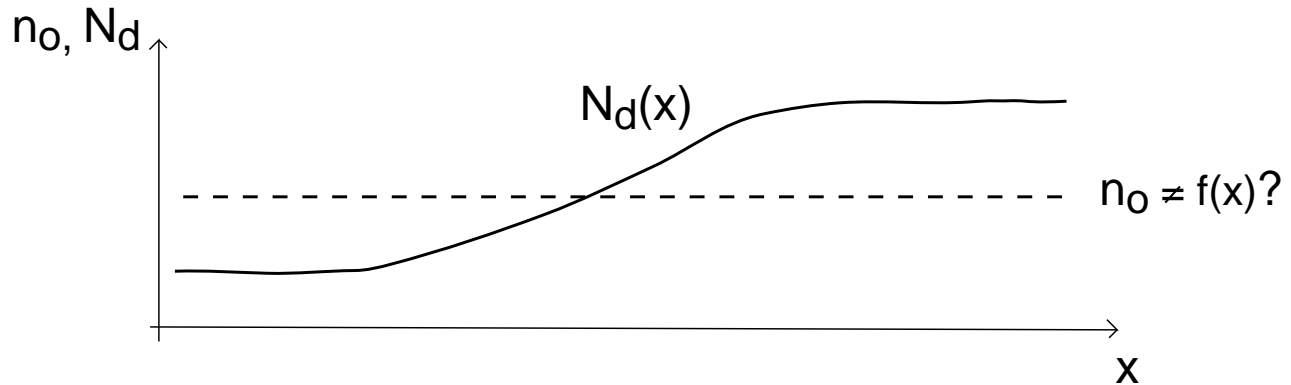
Gradient of electron concentration:

$\Rightarrow$  net electron diffusion

$\Rightarrow$  not thermal equilibrium!

## OPTION 2: Electron concentration uniform in space:

$$n_o = n_{ave} \neq f(x)$$



Think about space charge density:

$$\rho(x) = q[N_d(x) - n_o(x)]$$

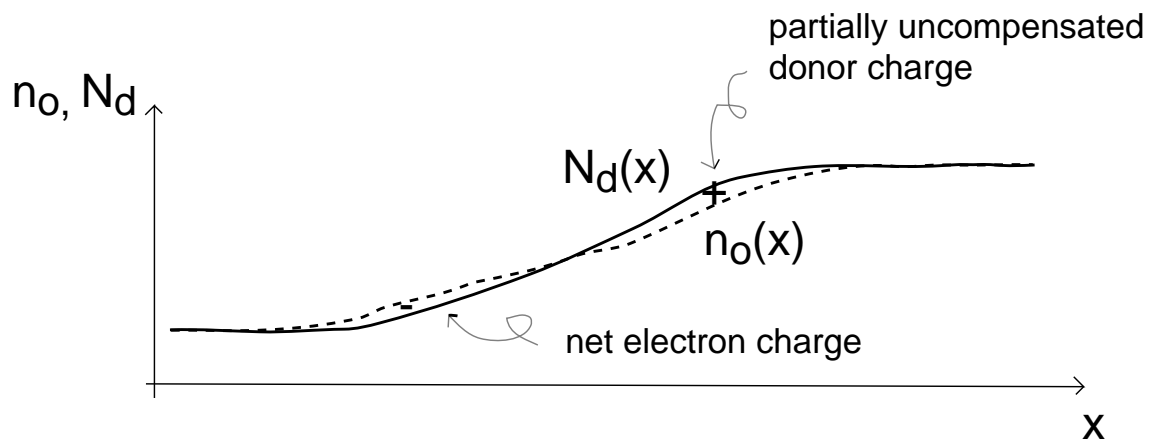
If  $N_d(x) \neq n_o(x) \Rightarrow \rho(x) \neq 0$   
 $\Rightarrow$  electric field  
 $\Rightarrow$  net electron drift  
 $\Rightarrow$  not thermal equilibrium!

OPTION 3: Demand  $J_e = 0$  in thermal equilibrium (and  $J_h = 0$  too) at every  $x \Rightarrow$

Diffusion precisely balances drift:

$$J_e(x) = J_e^{drift}(x) + J_e^{diff}(x) = 0$$

What is  $n_o(x)$  that satisfies this condition?



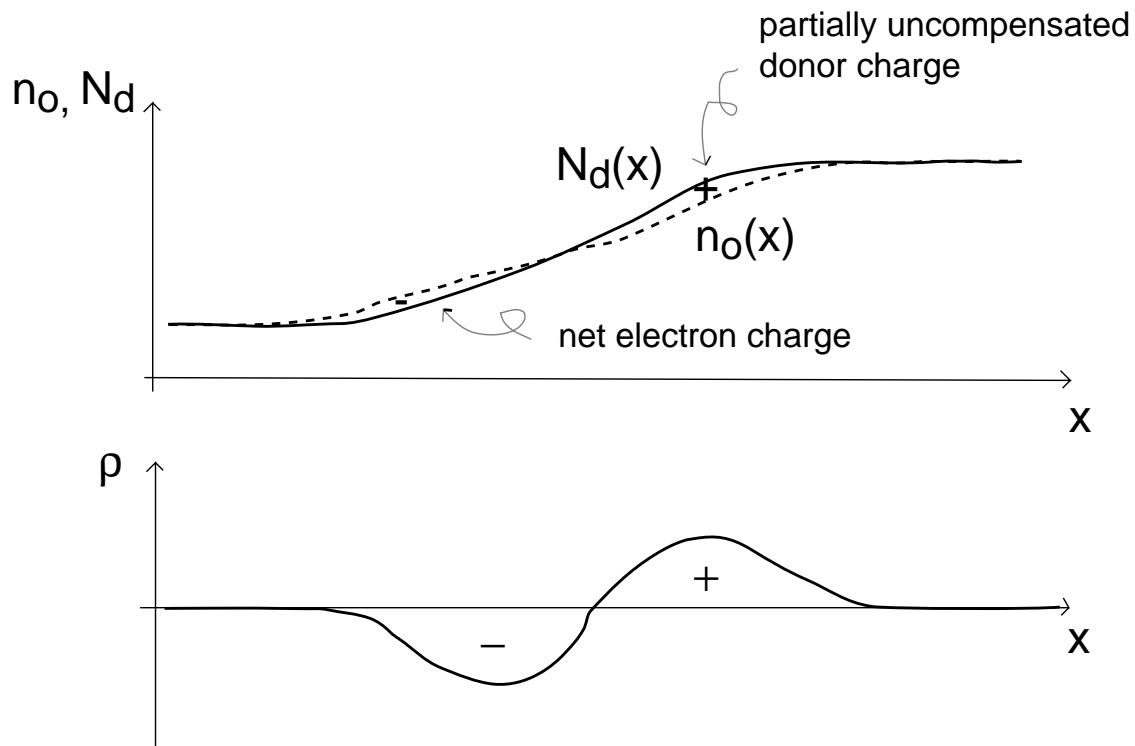
In general, then:

$$n_o(x) \neq N_d(x)$$

What are the implications of this?

- *Space charge density:*

$$\rho(x) = q[N_d(x) - n_o(x)]$$





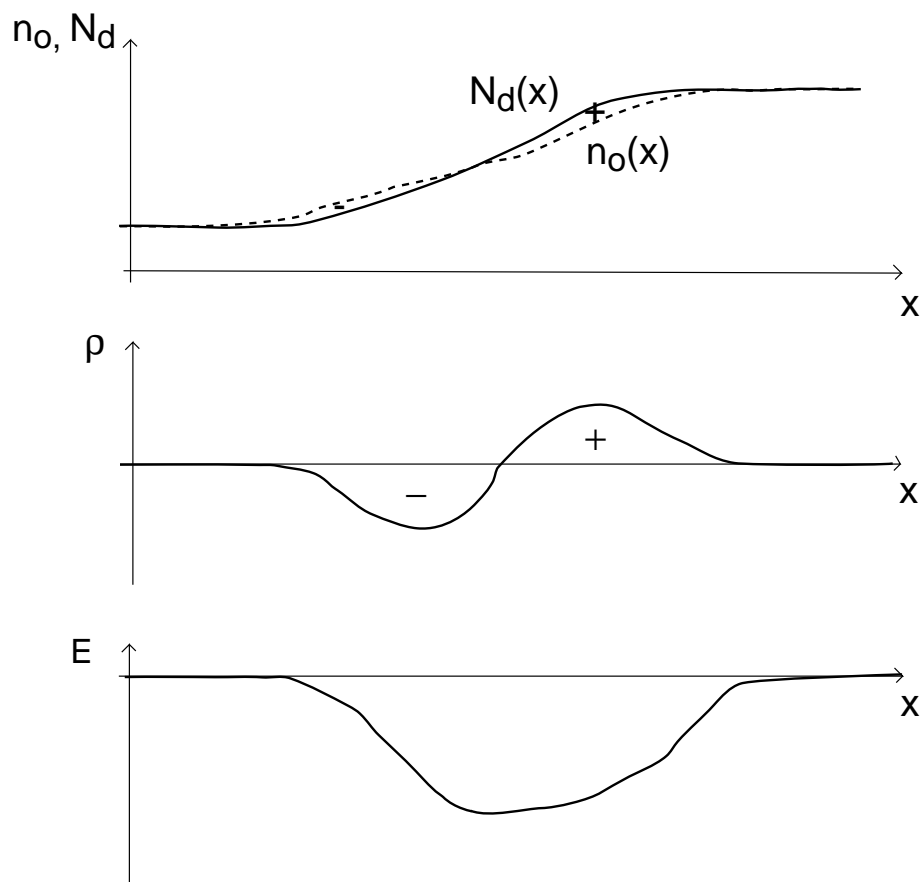
- *Electric field:*

Gauss' equation:

$$\frac{dE}{dx} = \frac{\rho}{\epsilon_s}$$

Integrate from  $x = 0$  to  $x$ :

$$E(x) - E(0) = \frac{1}{\epsilon_s} \int_0^x \rho(x) dx$$



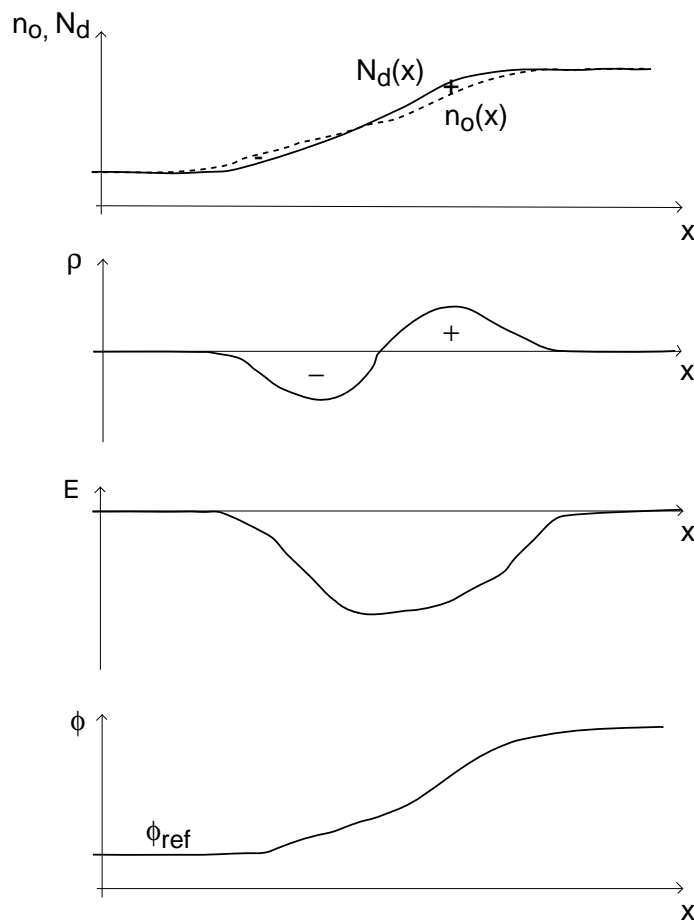
- *Electrostatic potential:*

$$\frac{d\phi}{dx} = -E$$

Integrate from  $x = 0$  to  $x$ :

$$\phi(x) - \phi(0) = - \int_0^x E(x) dx$$

Need to select reference (physics is in potential difference, not in absolute value!); select  $\phi(x = 0) = \phi_{ref}$ :



Given  $N_d(x)$ , want to know  $n_o(x)$ ,  $\rho(x)$ ,  $E(x)$ , and  $\phi(x)$ .

Equations that describe problem:

$$J_e = q\mu_n n_o E + qD_n \frac{dn_o}{dx} = 0$$

$$\frac{dE}{dx} = \frac{q}{\epsilon_s} (N_d - n_o)$$

Express them in terms of  $\phi$ :

$$-q\mu_n n_o \frac{d\phi}{dx} + qD_n \frac{dn_o}{dx} = 0 \quad (1)$$

$$\frac{d^2\phi}{dx^2} = \frac{q}{\epsilon_s} (n_o - N_d) \quad (2)$$

Plug [1] into [2]:

$$\frac{d^2(\ln n_o)}{dx^2} = \frac{q^2}{\epsilon_s kT} (n_o - N_d) \quad (3)$$

One equation with one unknown. Given  $N_d(x)$ , can solve for  $n_o(x)$  and all the rest, but...

... no analytical solution for most situations!

## 2. Quasi-neutral situation

$$\frac{d^2(\ln n_o)}{dx^2} = \frac{q^2}{\epsilon_s kT} (n_o - N_d)$$

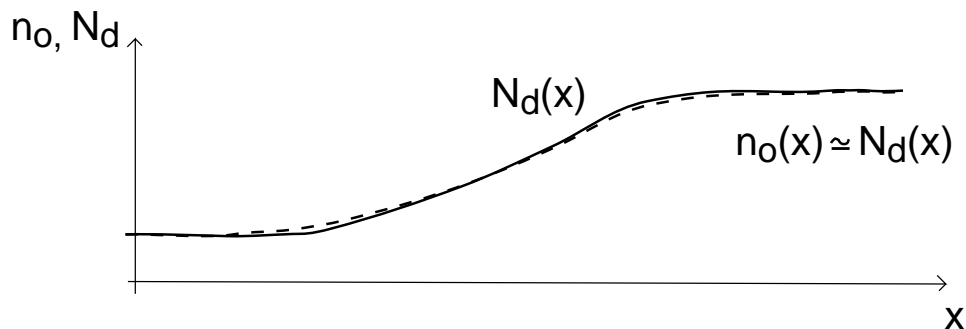
If  $N_d(x)$  changes slowly with  $x$ :

$\Rightarrow n_o(x)$  also changes slowly with  $x$

$\Rightarrow \frac{d^2(\ln n_o)}{dx^2}$  small

$\Rightarrow n_o(x) \simeq N_d(x)$

$n_o(x)$  tracks  $N_d(x)$  well  $\Rightarrow$  minimum space charge  $\Rightarrow$  semiconductor is *quasi-neutral*



Quasi-neutrality good if:

$$\left| \frac{n_o - N_d}{n_o} \right| \ll 1 \quad \text{or} \quad \left| \frac{n_o - N_d}{N_d} \right| \ll 1$$

### 3. Relationships between $\phi(x)$ and equilibrium carrier concentrations (Boltzmann relations)

From [1]:

$$\frac{\mu_n}{D_n} \frac{d\phi}{dx} = \frac{1}{n_o} \frac{dn_o}{dx}$$

Using Einstein relation:

$$\frac{q}{kT} \frac{d\phi}{dx} = \frac{d(\ln n_o)}{dx}$$

Integrate:

$$\frac{q}{kT} (\phi - \phi_{ref}) = \ln n_o - \ln n_o(ref) = \ln \frac{n_o}{n_o(ref)}$$

Then:

$$n_o = n_o(ref) e^{q(\phi - \phi_{ref})/kT}$$

Any reference is good.

In 6.012,  $\phi_{ref} = 0$  at  $n_o(ref) = n_i$ .

Then:

$$n_o = n_i e^{q\phi/kT}$$

If do same with holes (starting with  $J_h = 0$  in thermal equilibrium, or simply using  $n_o p_o = n_i^2$ ):

$$p_o = n_i e^{-q\phi/kT}$$

Can rewrite as:

$$\phi = \frac{kT}{q} \ln \frac{n_o}{n_i}$$

and

$$\phi = -\frac{kT}{q} \ln \frac{p_o}{n_i}$$

## □ ”60 mV” Rule:

At room temperature for Si:

$$\phi = (25 \text{ mV}) \ln \frac{n_o}{n_i} = (25 \text{ mV}) \ln(10) \log \frac{n_o}{n_i}$$

Or

$$\phi \simeq (60 \text{ mV}) \log \frac{n_o}{10^{10}}$$

*For every decade of increase in  $n_o$ ,  $\phi$  increases by 60 mV at 300K.*

### ● EXAMPLE 1:

$$n_o = 10^{18} \text{ cm}^{-3} \Rightarrow \phi = (60 \text{ mV}) \times 8 = 480 \text{ mV}$$

With holes:

$$\phi = -(25 \text{ mV}) \ln \frac{p_o}{n_i} = -(25 \text{ mV}) \ln(10) \log \frac{p_o}{n_i}$$

Or

$$\phi \simeq -(60 \text{ mV}) \log \frac{p_o}{10^{10}}$$

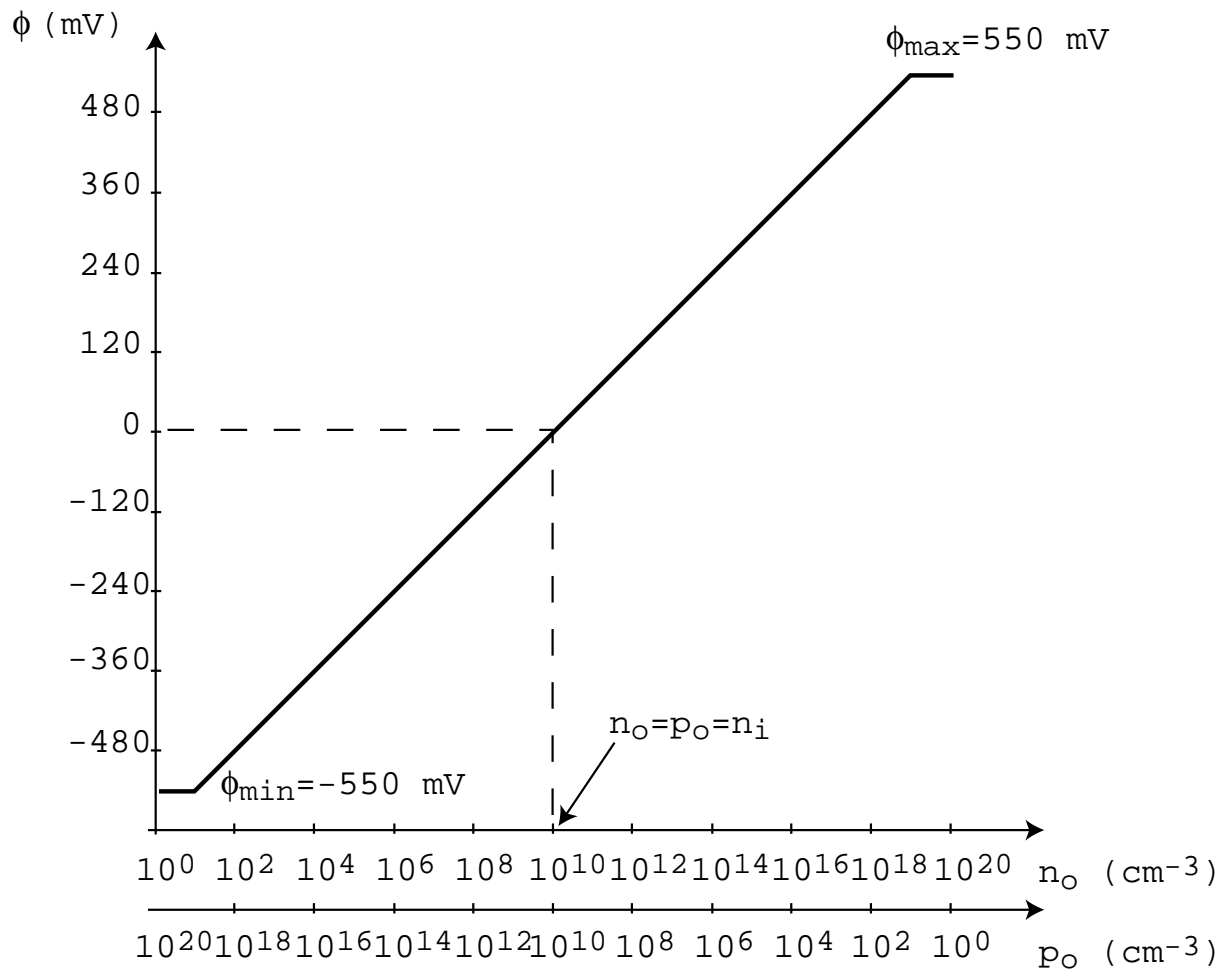
• EXAMPLE 2:

$$n_o = 10^{18} \text{ cm}^{-3} \Rightarrow p_o = 10^2 \text{ cm}^{-3}$$

$$\Rightarrow \phi = -(60 \text{ mV}) \times (-8) = 480 \text{ mV}$$



Relationship between  $\phi$  and  $n_o$  and  $p_o$ :



Note:  $\phi$  cannot exceed  $550$  mV or be smaller than  $-550$  mV (beyond these points, different physics come into play).

● **EXAMPLE 3:** Compute potential difference in thermal equilibrium between region where  $n_o = 10^{17} \text{ cm}^{-3}$  and region where  $p_o = 10^{15} \text{ cm}^{-3}$ :

$$\phi(n_o = 10^{17} \text{ cm}^{-3}) = 60 \times 7 = 420 \text{ mV}$$

$$\phi(p_o = 10^{15} \text{ cm}^{-3}) = -60 \times 5 = -300 \text{ mV}$$

$$\phi(n_o = 10^{17} \text{ cm}^{-3}) - \phi(p_o = 10^{15} \text{ cm}^{-3}) = 720 \text{ mV}$$

● **EXAMPLE 4:** Compute potential difference in thermal equilibrium between region where  $n_o = 10^{20} \text{ cm}^{-3}$  and region where  $p_o = 10^{16} \text{ cm}^{-3}$ :

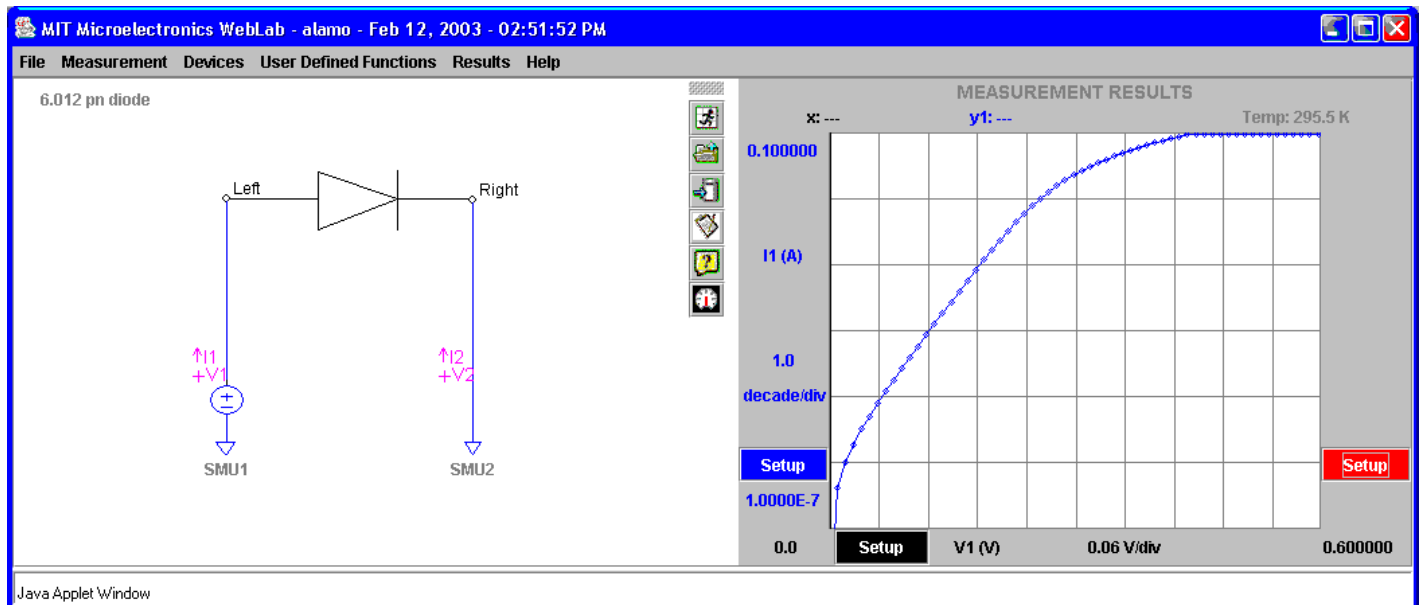
$$\phi(n_o = 10^{20} \text{ cm}^{-3}) = \phi_{max} = 550 \text{ mV}$$

$$\phi(p_o = 10^{16} \text{ cm}^{-3}) = -60 \times 6 = -360 \text{ mV}$$

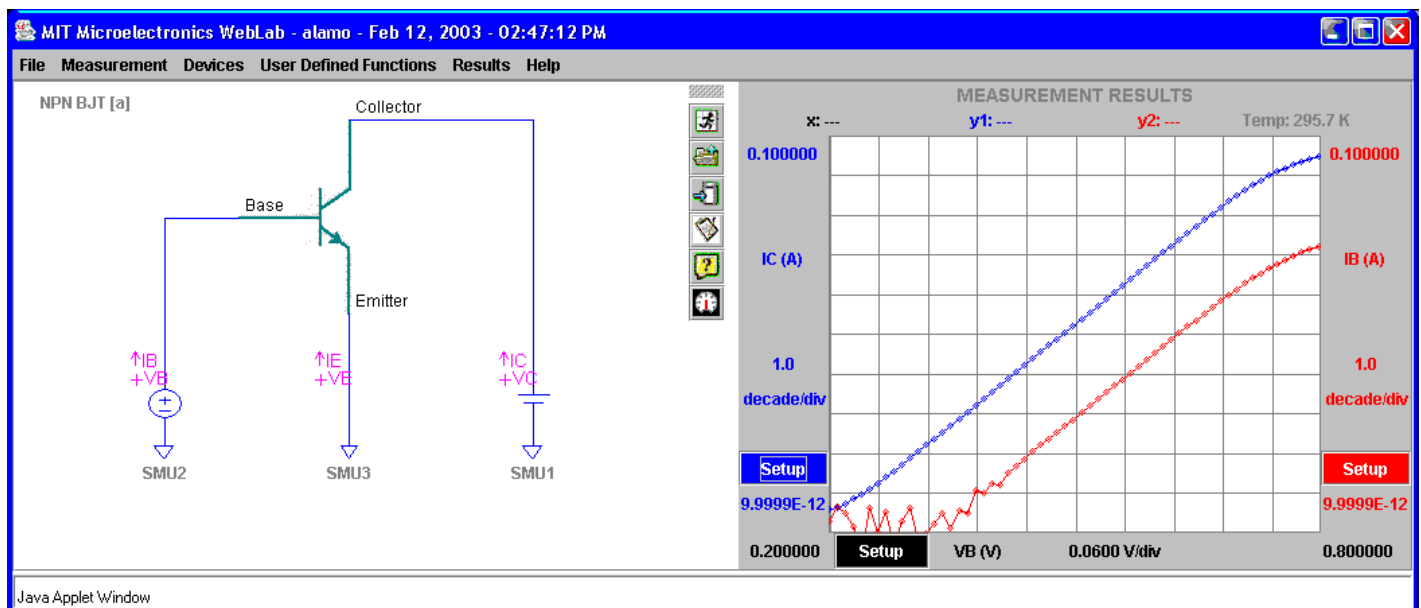
$$\phi(n_o = 10^{20} \text{ cm}^{-3}) - \phi(p_o = 10^{16} \text{ cm}^{-3}) = 910 \text{ mV}$$

Boltzmann relations readily seen in device behavior!

□ pn diode current-voltage characteristics:



□ Bipolar transistor transfer characteristics:



## Key conclusions

- It is possible to have an electric field inside a semiconductor in thermal equilibrium  
 $\Rightarrow$  *non-uniform doping distribution*.
- In a slowly varying doping profile, majority carrier concentration tracks well doping concentration.
- In thermal equilibrium, there are fundamental relationships between  $\phi(x)$  and the equilibrium carrier concentrations  
 $\Rightarrow$  *Boltzmann relations* (or "60 mV Rule").