Chapter #2: Operational Amplifiers

from **Microelectronic Circuits** Text

by Sedra and Smith

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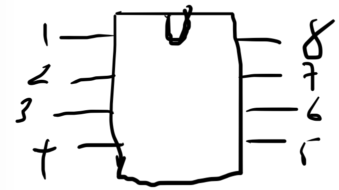
Introduction

- **IN THIS CHAPTER YOU WILL LEARN**
  - The terminal characteristics of the ideal op-amp.
  - How to analyze circuits containing op-amps, resistors, and capacitors.
  - How to use op-amps to design amplifiers having precise characteristics.

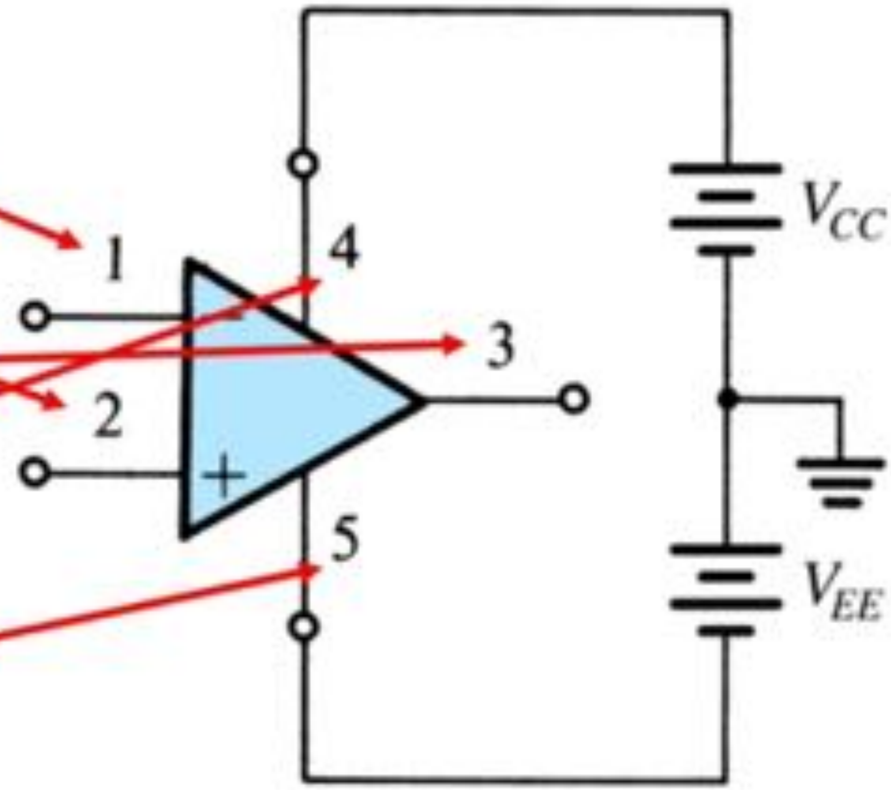
Introduction

- **IN THIS CHAPTER YOU WILL LEARN**
  - How to design more sophisticated op-amp circuits, including summing amplifiers, instrumentation amplifiers, integrators, and differentiators.
  - Important non-ideal characteristics of op-amps and how these limit the performance of basic op-amp circuits.

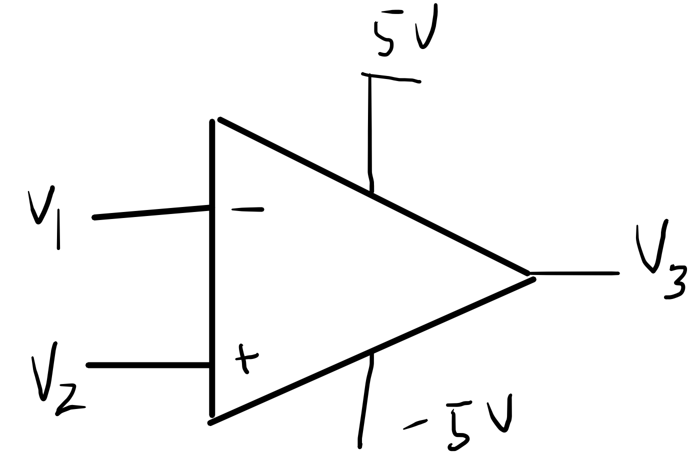
2.1.1. **The Op Amp Terminals**



- **terminal #1**
  - inverting input
- **terminal #2**
  - non-inverting input
- **terminal #3**
  - output
- **terminal #4**
  - positive supply V_{CC}
- **terminal #5**
  - negative supply V_{EE}



2.1.2. Function and Characteristics of Ideal Op Amp



Open loop gain A_o

- ideal gain is defined below

$A_o = 1000$

$$v_3 = A(v_2 - v_1)$$

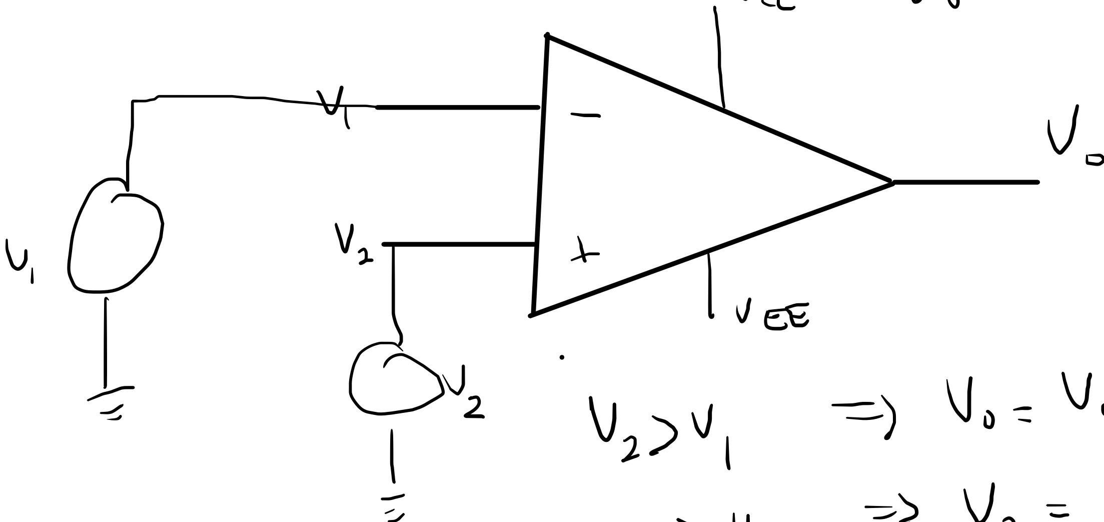
$V_2 - V_1 = 1V$

$V_2 - V_1 = 0.5V$

- ideal input characteristic is infinite impedance
- ideal output characteristic is zero impedance
- differential gain (A) is infinite
- bandwidth gain is constant from dc to high frequencies

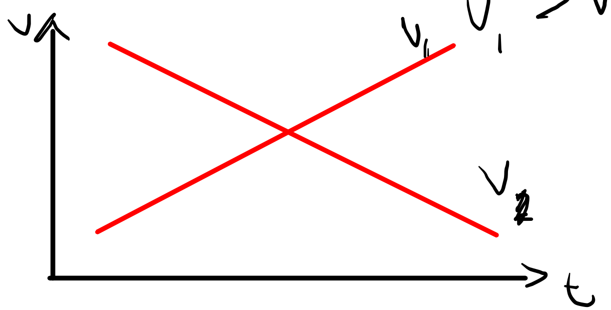
Q: But, is an amplifier with infinite gain of any use?

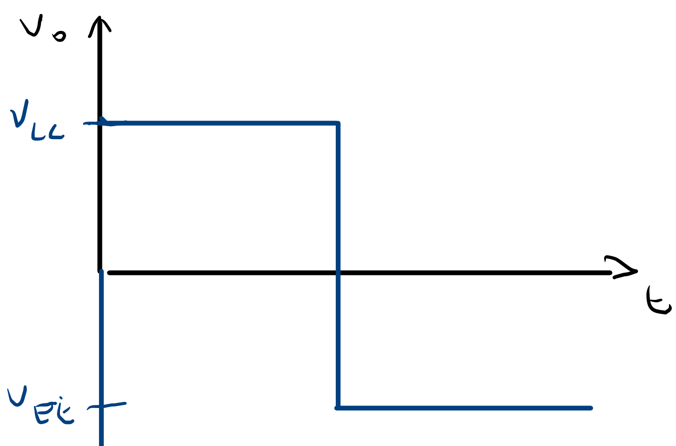
$$V_o = A_o(V_2 - V_1)$$



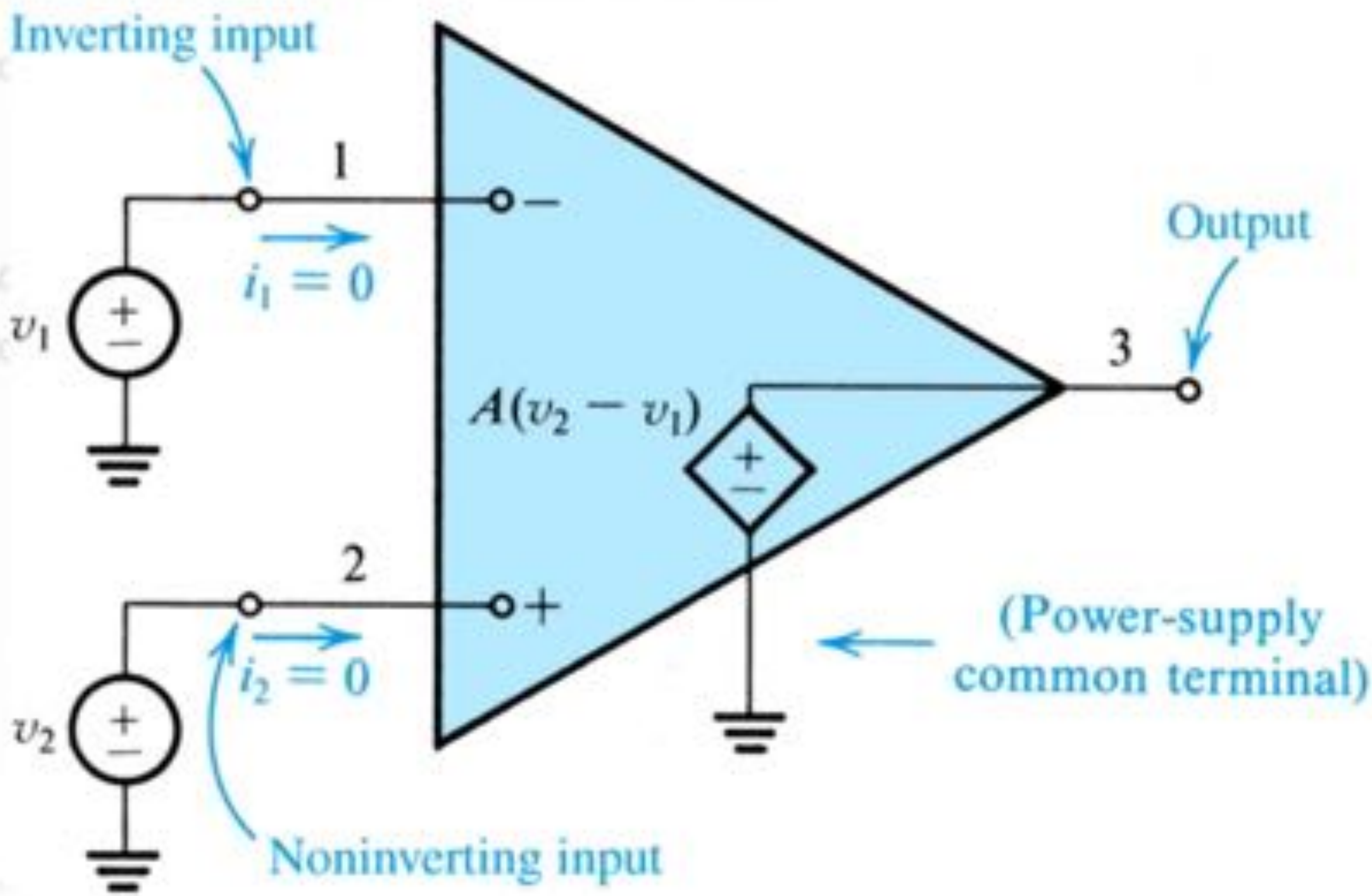
$$V_2 > V_1 \Rightarrow V_o = V_{cc}$$

$$V_1 > V_2 \Rightarrow V_o = V_{EE}$$





2.1.2. Function and Characteristics of Ideal Op Amp



2.1.2. Function and Characteristics of Ideal Op-Amp

- An amplifier's input is composed of two components…
  - **differential input** (v_{dfi}) – is difference between inputs at inverting and non-inverting terminals
  - **common-mode input** (v_{cmi}) – is input present at both inverting and non-inverting terminals

$$v_{in} = (10+1) - (10-1) = \overbrace{(10-10)}^{\text{common-mode input } (v_{cmi})} + \overbrace{(1+1)}^{\text{differential input } (v_{dfi})}$$

2.1.2. Function and Characteristics of Ideal Op-Amp

- Similarly, two components of gain exist…
  - **differential gain** (A) – gain applied to differential input ONLY
  - **common-mode gain** (A_{cm}) – gain applied to common-mode input ONLY

$$v_{out} = \overbrace{(A_{cm}10 + A1)}^{\text{e.g. } v_1 = 10+1} - \overbrace{(A_{cm}10 - A1)}^{\text{e.g. } v_2 = 10-1} = \overbrace{A_{cm}(10-10)}^{\text{common-mode output}} + \overbrace{A(1+1)}^{\text{differential output}}$$

2.1.2. Function and Characteristics of Ideal Op Amp

- **Table 2.1: Characteristics of Ideal Op Amp**
  - infinite input impedance
  - zero output impedance
  - zero common-mode gain ($A_{cm} = 0$)
    - complete common-mode rejection
  - infinite open-loop gain (A = infinity)
  - infinite bandwidth

2.1.3. Differential & Common-Mode Signals

- **Q:** How is common-mode input (v_{cmi}) defined in terms of v_1 and v_2?

$$\overbrace{v_{cmi} = \frac{1}{2}(v_1 + v_2)}^{\text{common-mode input}}$$ but also...

$$\overbrace{v_1 = v_{cmi} - \underbrace{v_{di}}_{\text{diff}}/2}^{\text{inverting input}}$$

$$\underbrace{v_2 = v_{cmi} + v_{di}/2}_{\text{non-inverting input}}$$

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but also...

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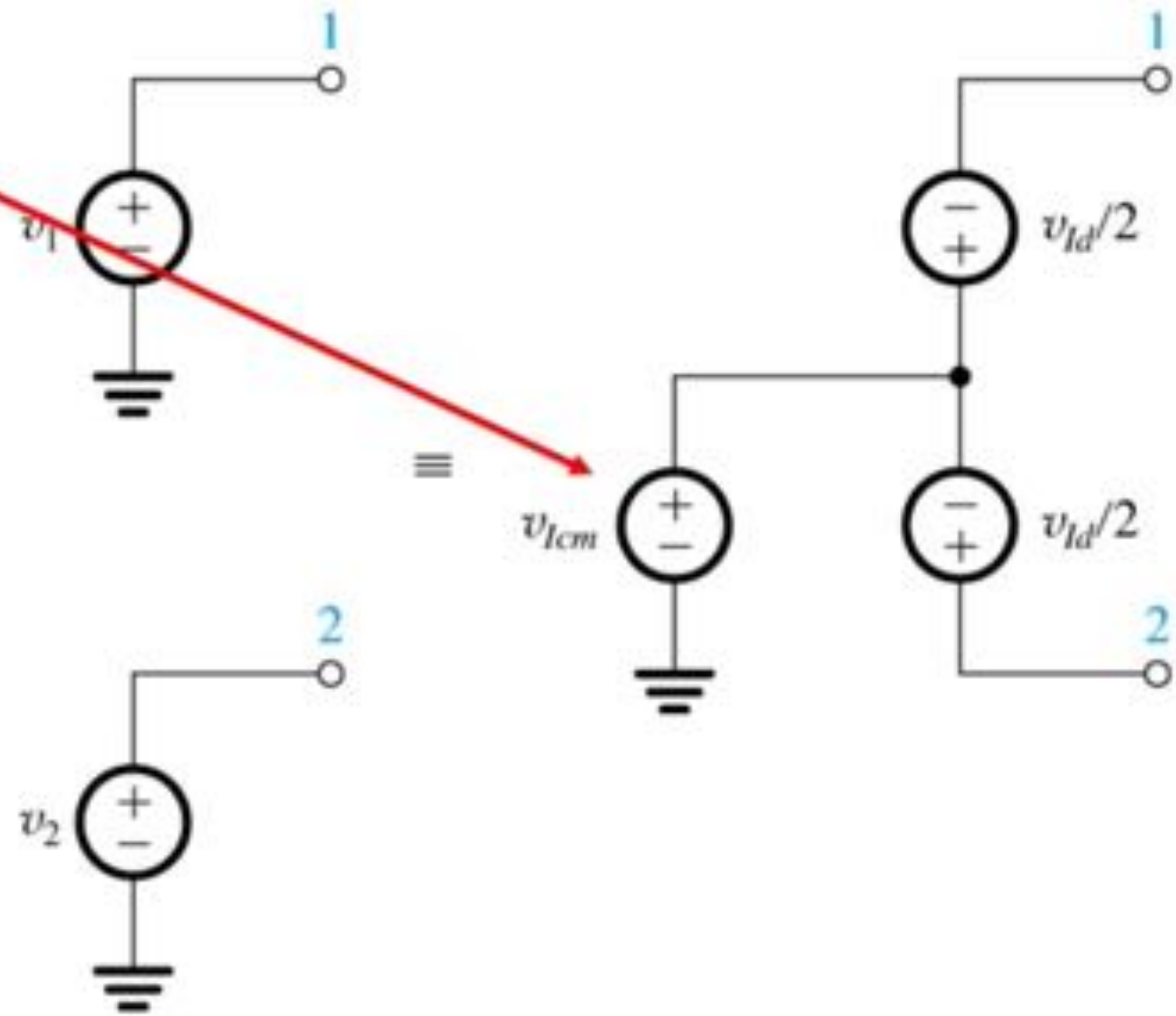
$$\underbrace{v_2 = v_{cmi} + v_{di} / 2}_{\text{non-inverting input}}$$



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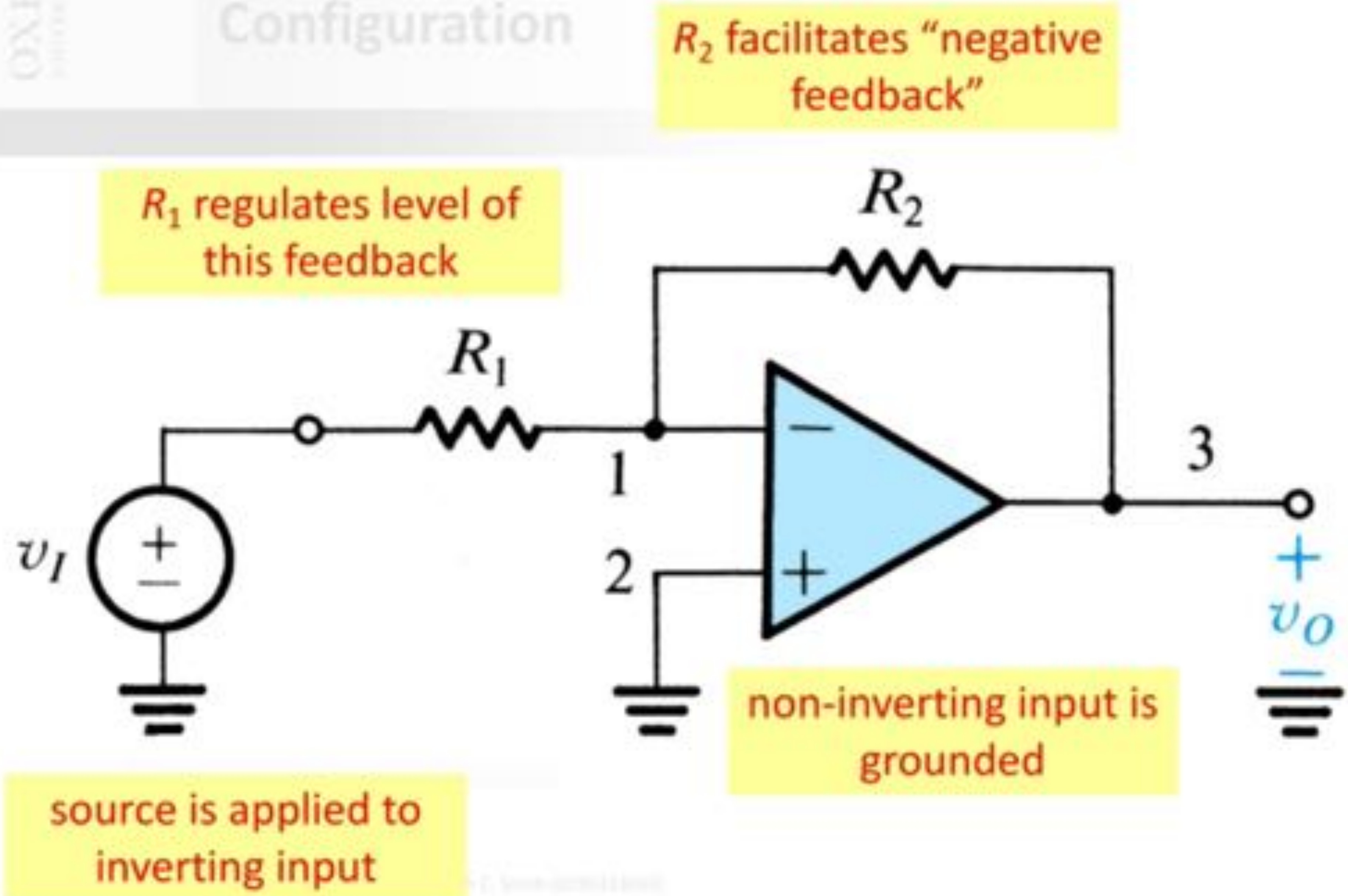
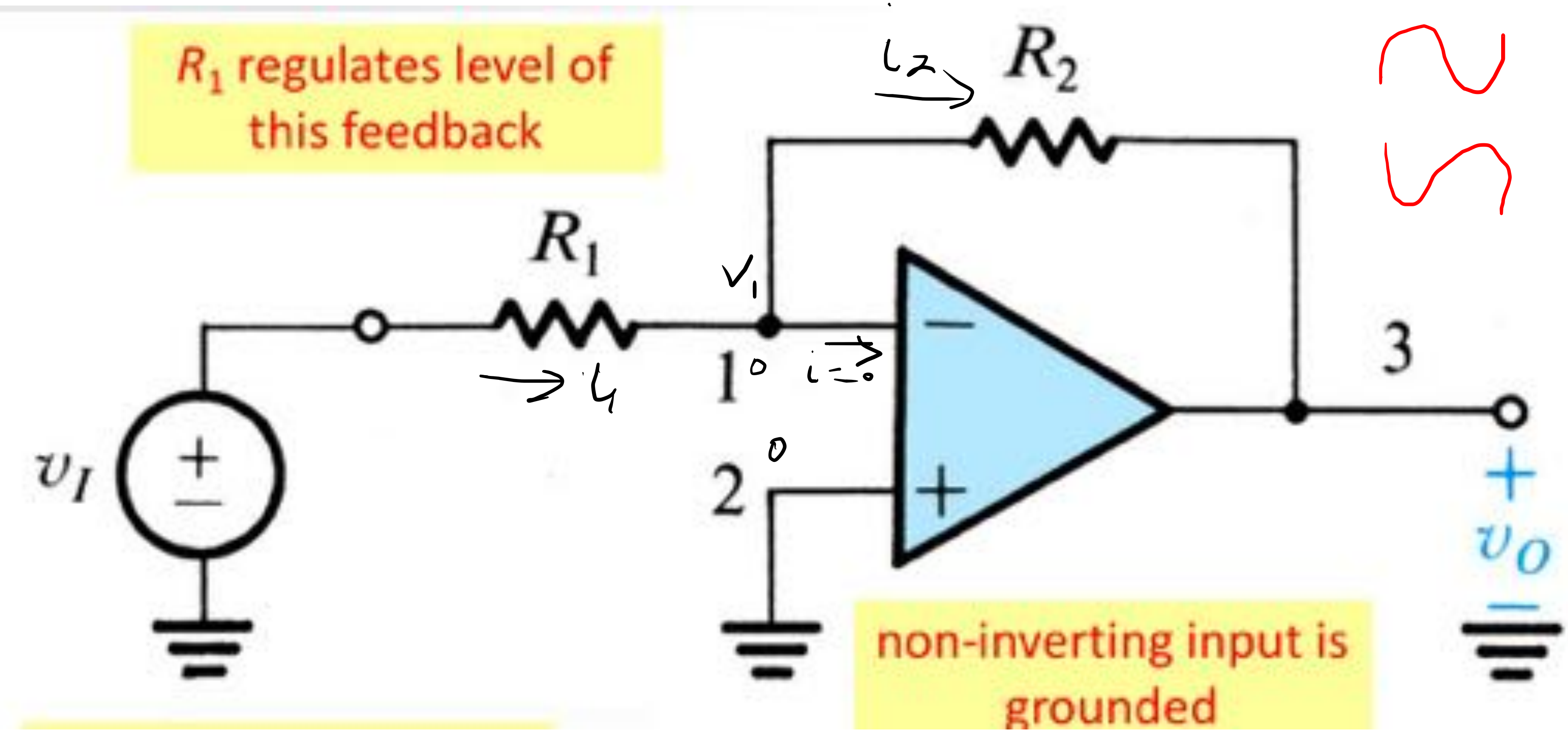
2.2. The Inverting Configuration

- **Q:** What are two basic closed-loop op-amp configurations which employ op-amp and resistors alone?
  - **A:** 1) inverting and 2) non-inverting op amp

Figure 2.5: The inverting closed-loop configuration.





$$i_1 = i_2 \Rightarrow \frac{V_i - V_1}{R_1} = \frac{V_1 - V_o}{R_2}$$

$$V_1 = 0 \qquad \frac{V_i}{R_1} = -\frac{V_o}{R_2} \Rightarrow A_v = \frac{V_o}{V_1} = -\frac{R_2}{R_1}$$

2.2.1. Closed-Loop Gain

$V_0 = A_0(V_2 - V_1)$

$V_2 - V_1 = V_0/A_0 \sim 0$

$\Rightarrow V_2 = V_1$

- **Q:** How does one analyze closed-loop gain for inverting configuration of an ideal op-amp?
  - **step #1:** Begin at the output terminal
  - **step #2:** If v_{Out} is finite, then differential input must equal 0
    - virtual short circuit btw v_1 and v_2
    - virtual ground exists at v_1

$$\overbrace{v_2 - v_1 = \frac{v_{Out}}{\underbrace{A}_{\infty}}}^{\text{because } A \text{ in infinite}} = 0$$

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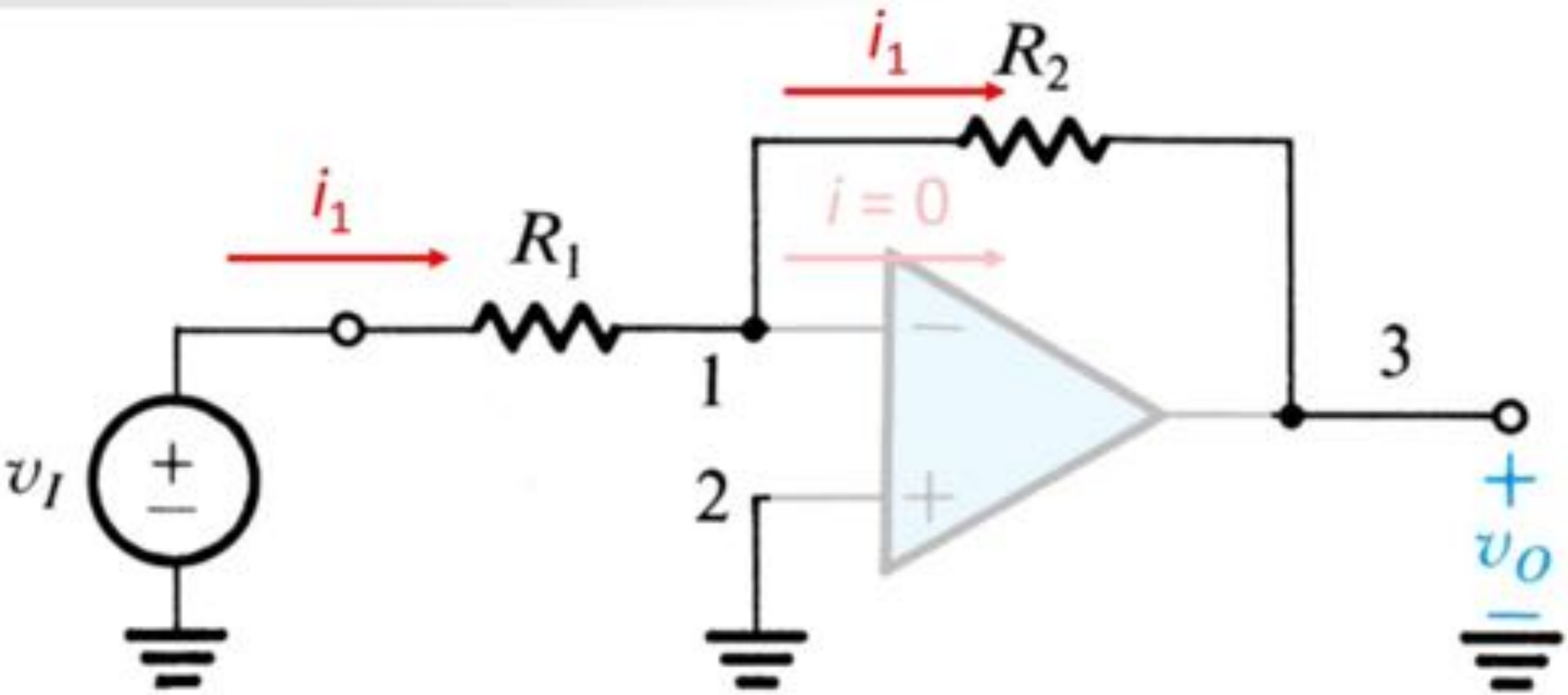
2.2.1. Closed-Loop Gain

- **step #3:** Define current in to inverting input (i_1).
- **step #4:** Determine where this current flows?
  - refer to following slide…

$$i_1 = \frac{(v_{In}) - \overbrace{(v_1)}^{\text{virtual ground}}}{R_1} = \frac{v_{In} - 0}{R_1} = \frac{v_{In}}{R_1}$$

Figure 2.5: The inverting closed-loop configuration.



2.2.1. Closed-Loop Gain

- **step #5:** Define v_{Out} in terms of current flowing across R_2.
- **step #6:** Substitute v_{in} / R_1 for i_1.

$$v_{Out} = \overbrace{(v_1)}^{\text{virtual ground}} - (i_1 R_2) = -i_1 R_2$$

$$\boxed{v_{Out} = -\frac{R_2}{R_1} v_{In}}$$

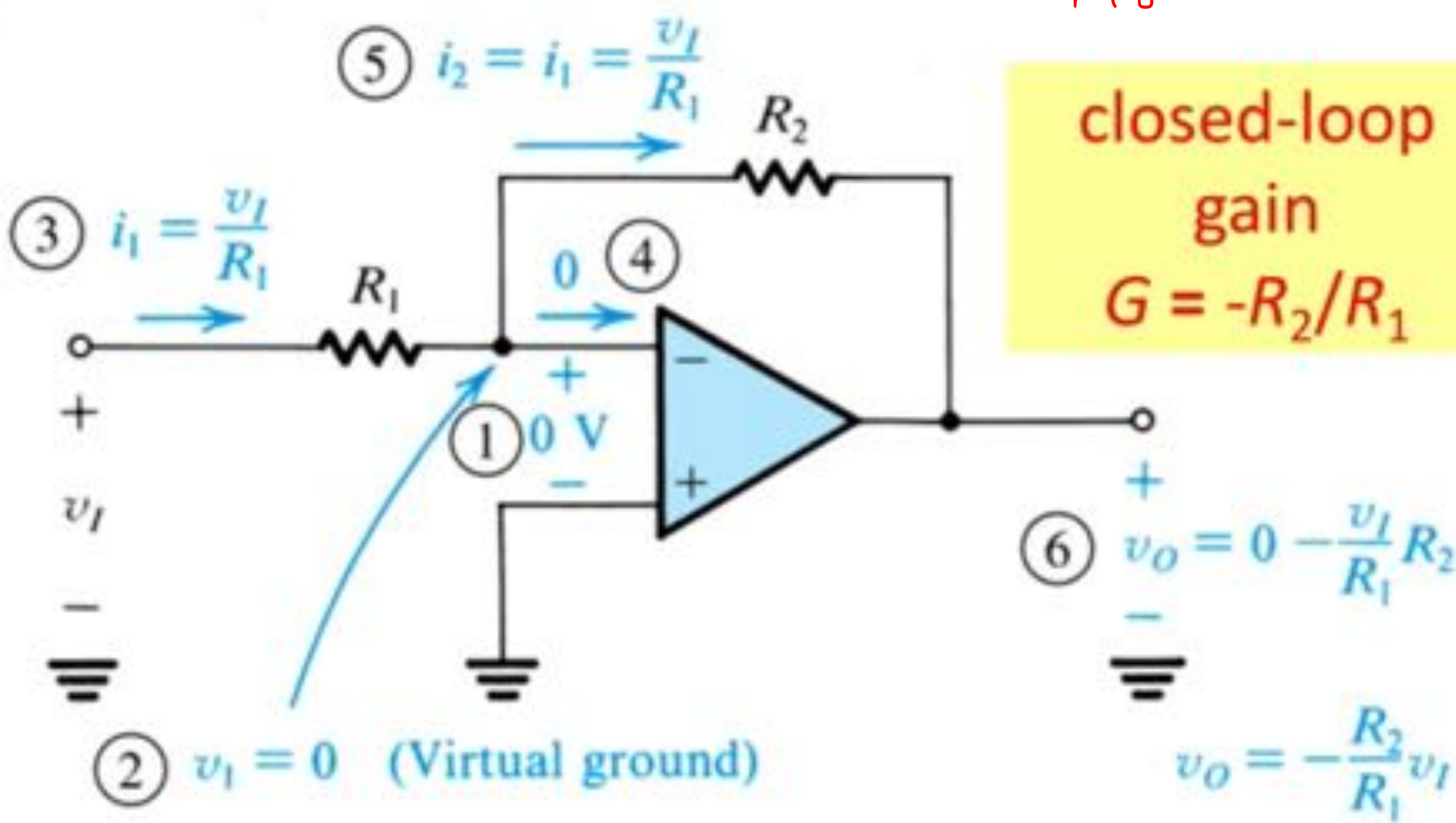
solution

note: this expression is one of the fundamentals of electronics

Figure 2.6: Analysis of the inverting configuration. The circled numbers indicate the order of the analysis steps.

Closed-Loop Gain

$A_0 = \infty$



closed-loop gain

$G = -R_2/R_1$

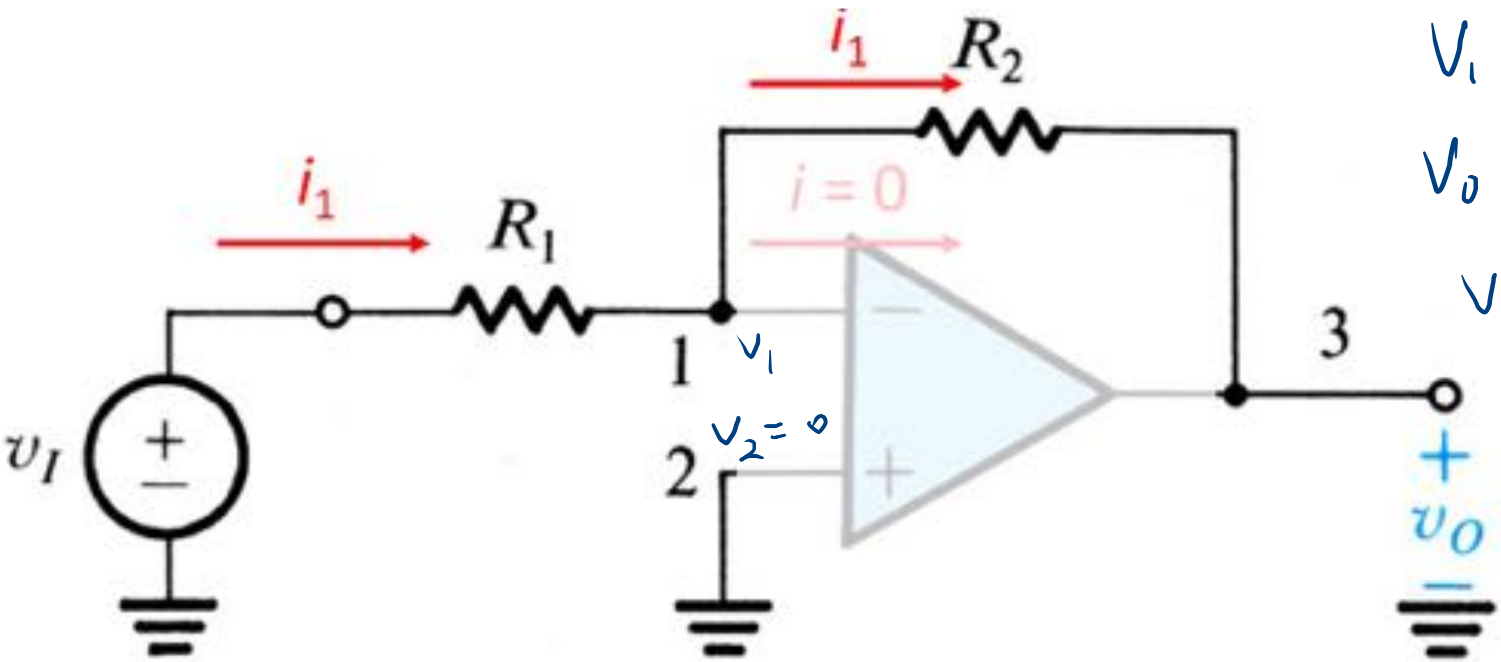
2.2.1. **Effect of Finite Open-Loop Gain**

- **Q:** How does the gain expression change if open loop gain (A) is not assumed to be infinite?
  - **A:** One must employ analysis similar to the previous, result is presented below…

$$G_{A<\infty} = \frac{v_{Out}}{v_{In}} = \underbrace{\frac{-R_2/R_1}{1+\left(\frac{1+(R_2/R_1)}{A}\right)}}_{\text{if } A=\infty \text{ then the previous gain expression is yielded}} \neq \overbrace{-\frac{R_2}{R_1}}^{G_{A=\infty}}$$

non-ideal gain

ideal gain



$A_0 < \infty$

$V_1 \neq V_2$

$V_0 = A_0(V_2 - V_1)$

$V_0 = A_0(-V_1)$

$\Rightarrow V_1 = -V_0/A_0$

$$\frac{V_i - V_1}{R_1} = \frac{V_1 - V_0}{R_2}$$

$$\frac{V_i + V_0/A_0}{R_1} = \frac{-V_0/A_0 - V_0}{R_2}$$

$$\frac{V_i + \frac{V_o}{A_o}}{R_1} = \frac{-\frac{V_o}{A_o} - V_o}{R_2}$$

$$\frac{V_o}{A_o R_1} + \frac{V_o}{A_o R_2} + \frac{V_o}{R_2} = -\frac{V_i}{R_1}$$

$$V_o\left(\frac{R_2 + R_1 + A_o R_1}{A_o R_1 R_2}\right) = -\frac{V_i}{R_1}$$

$$\frac{V_o}{V_i} = \frac{-R_2}{R_1}\left(\frac{A_o R_1}{R_2 + R_1 + A_o R_1}\right)$$

$$\boxed{A_v = \frac{V_o}{V_i} = -\frac{R_2}{R_1}\left[\frac{1}{\frac{1}{A_o}\left(\frac{R_2}{R_1}+1\right)+1}\right]} \rightarrow$$

Closed loop gain

$A_o < \infty$

2.2.1. Effect of Finite Open-Loop Gain

- **Q:** Under what condition can $G = -R_2 / R_1$ be employed over the more complex expression?
  - **A:** If $1 + (R_2/R_1) \ll A$, then simpler expression may be used.

$$\text{if } 1+\frac{R_2}{R_1} \ll A \text{ then } G_{A=\infty} = -\frac{R_2}{R_1} \text{ else } G_{A<\infty} = \frac{-R_2/R_1}{1+\left(\frac{1+(R_2/R_1)}{A}\right)}$$

ideal gain

non-ideal gain

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Example 2.1: Simple Inverting Amplifier

$A_1 \quad \frac{-R_2}{R_1} \qquad \frac{A_2 - A_1}{A_1}$

- **Problem Statement:** Consider an inverting configuration with $R_1 = 1kOhm$ and $R_2 = 100kOhm$.
- **Q(a):** Find the closed-loop gain (G) for the cases below. In each case, determine the percentage error in the magnitude of G relative to the ideal value.
  - cases are $A = 10^3, 10^4, 10^5$...
- **Q(b):** What is the voltage v_1 that appears at the inverting input terminal when $v_{In} = 0.1V$.
- **Q(c):** If the open loop gain (A) changes from $100k$ to $50k$, what is percentage change in gain (G)?

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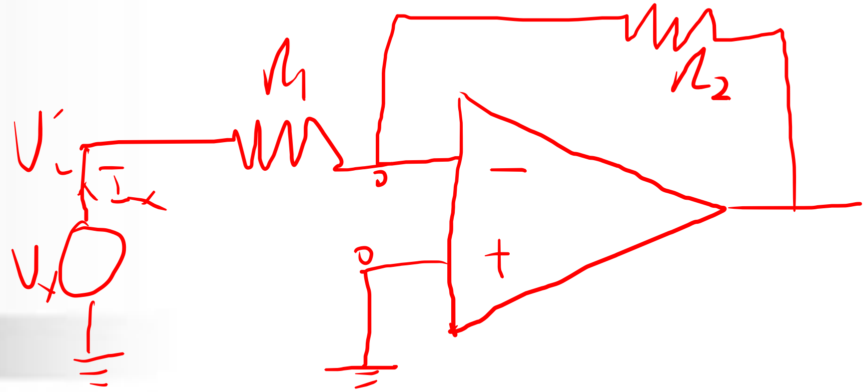
2.2.3. Input and Output Resistances



- **Q:** What is input resistance for inverting op-amp? How is it defined mathematically?
  - **A:** R_1 (refer to math below)
- **Q:** What does this say?
  - **A:** That, for the combination of ideal op-amp and external resistors, input resistance will be finite…

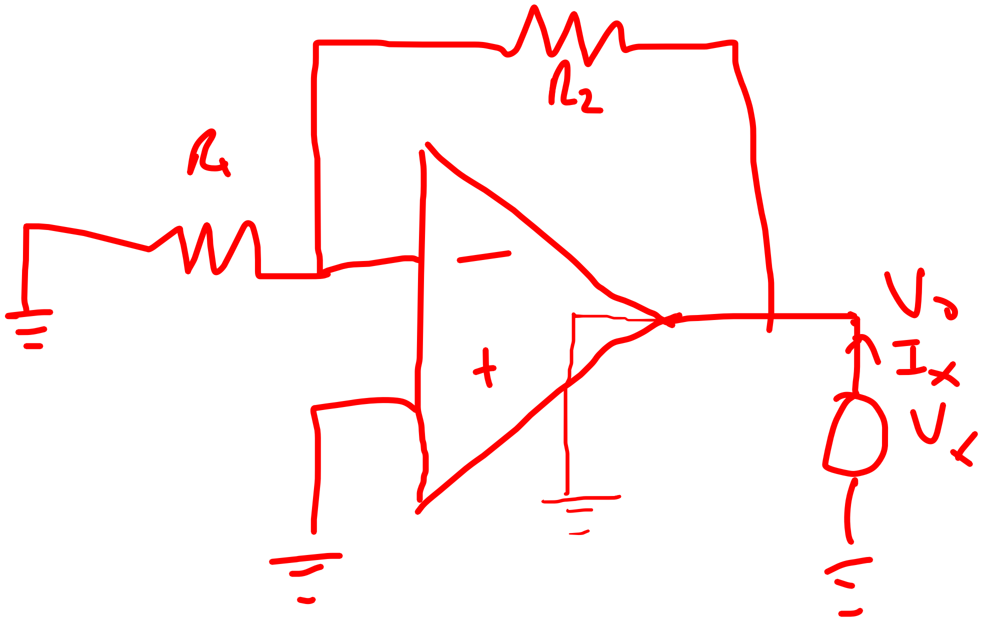
$$R_i = \frac{V_x}{I_x}$$

$$I_x = \frac{V_x - 0}{R_1}$$

$$\Rightarrow R_i = R_1$$

this assumes that ideal op-amp and external resistors are considered "one unit"

$$R_i = \frac{v_{in}}{\underbrace{i_{in}}_{\text{same as } i_1}} = \overbrace{\frac{v_{in}}{(v_{in} - \underbrace{v_1}_{\text{virtual ground} =0})/R_1}}^{\text{action: simplify}} = \overbrace{\frac{v_{in}}{v_{in}/R_1}}^{\text{action: simplify}} = R_1$$



$$R_o = \frac{V_x}{I_x} = 0$$

Example 2.2: Another Inverting Op-Amp

- **Problem Statement:** Consider the circuit below…
- **Q(a):** Derive an expression for the closed-loop gain v_{Out}/v_{In} of this circuit.
- **Q(b):** Use this circuit to design an inverting amplifier with gain of 100 and input resistance of 1*Mohm*.
  - Assume that one cannot use any resistor with resistance larger than 1*Mohm*.
- **Q(c):** Compare your design with that based on traditional inverting configuration.

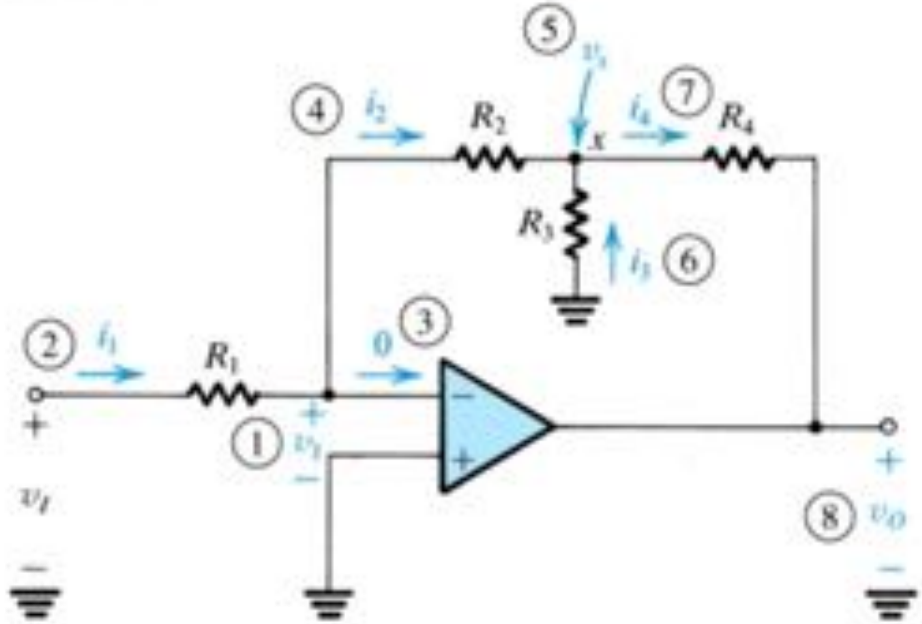
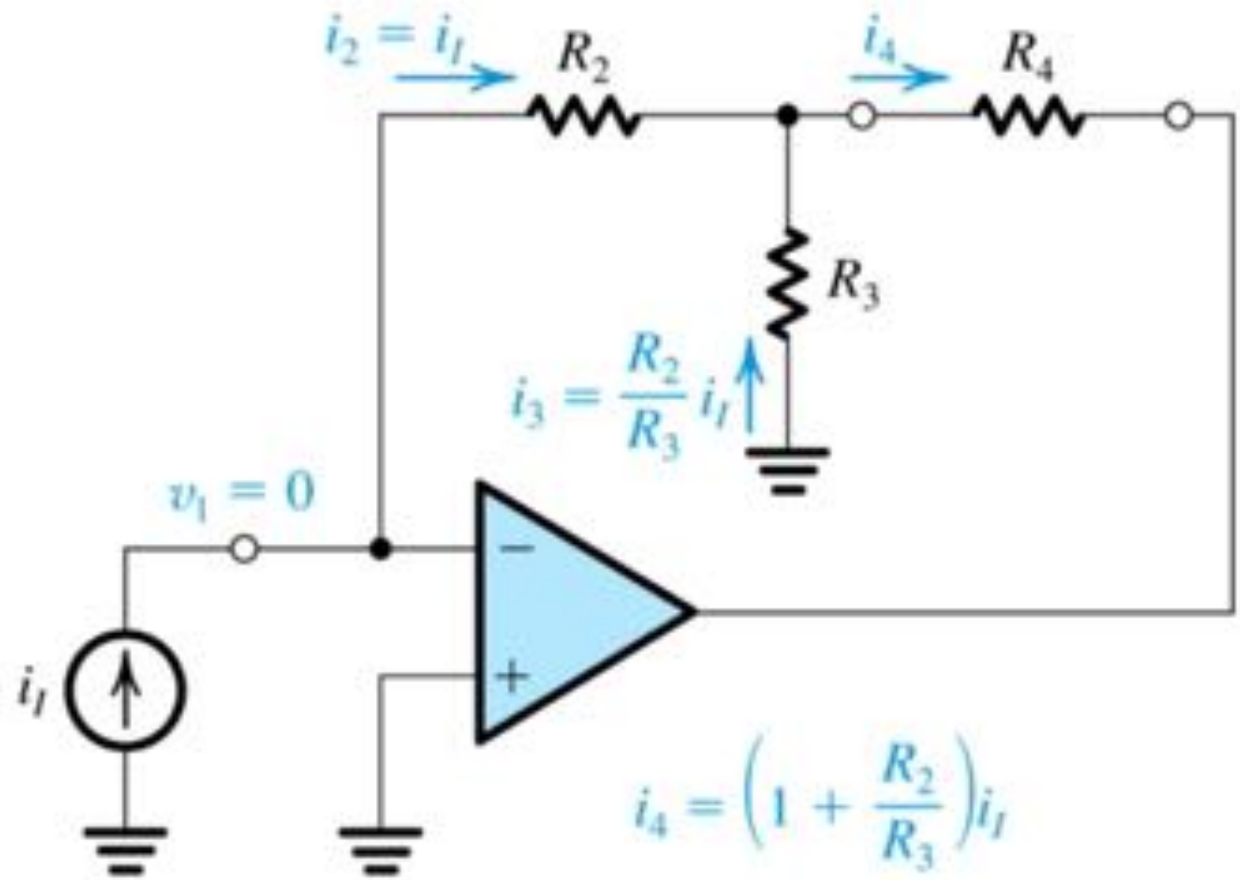


Figure 2.8: Circuit for Example 2.2. The circled numbers indicate the sequence of the steps in the analysis.

Figure 2.9: A current amplifier based on the circuit of Fig. 2.8. The amplifier delivers its output current to R_4. It has a current gain of $(1 + R_2/R_3)$, a zero input resistance, and an infinite output resistance. The load (R_4), however, must be floating (i.e., neither of its two terminals can be connected to ground).

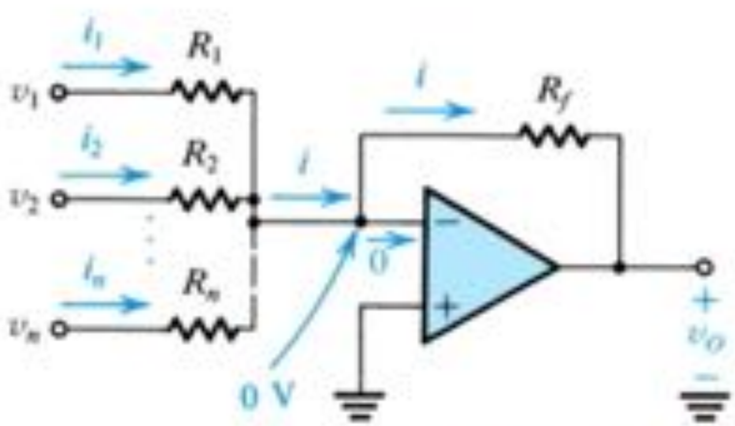


2.2.4. An Important Application – The Weighted Summer

- **weighted summer** - is a closed-loop amplifier configuration which provides an output voltage which is **weighted sum** of the inputs.

$$i_1 + i_2 + \dots i_n = i$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = \frac{-V_o}{R_f}$$



$$v_O = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \cdots + \frac{R_f}{R_n}v_n\right)$$

Figure 2.10: A weighted summer.

$$V_o = -R_f\left(\frac{1}{R_1}V_1 + \frac{1}{R_2}V_2 + \dots + \frac{1}{R_n}V_n\right)$$

$$v_{Out} = -[\ (R_f/R_{In1})v_{In1} + (R_f/R_{In2})v_{In2} + (R_f/R_{In3})v_{In3} + \ldots\ ]$$

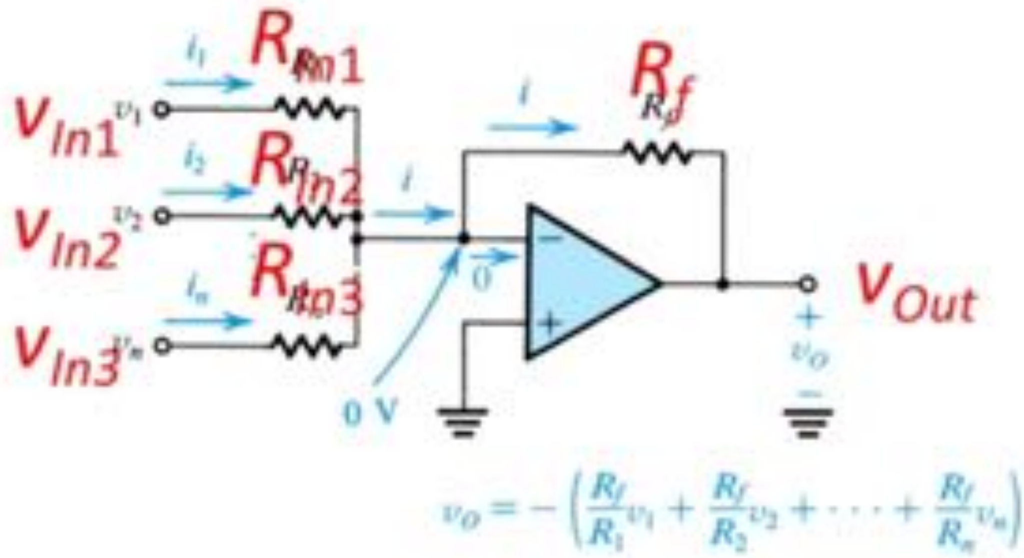
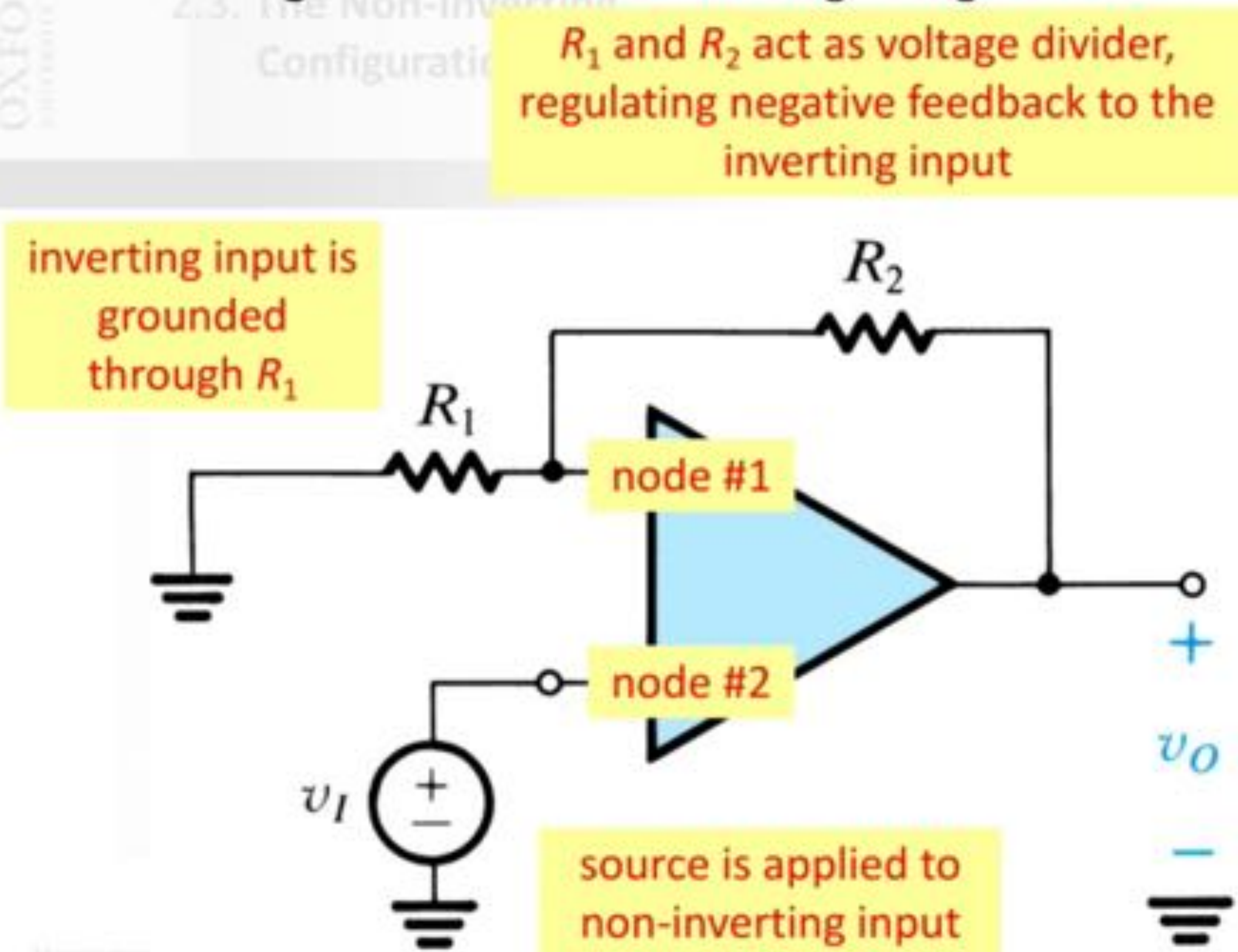


Figure 2.10: A weighted summer.

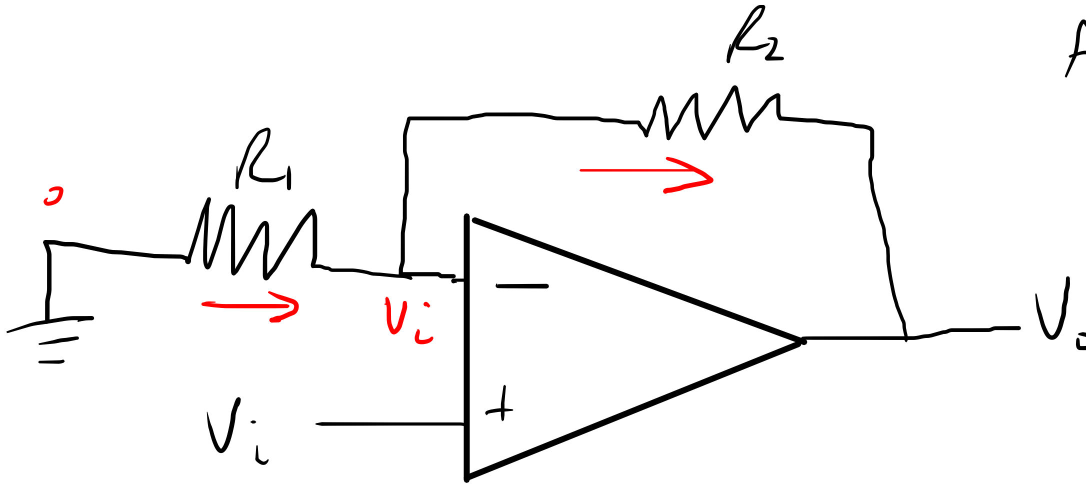
2.3. The Non-Inverting Configuration

- **non-inverting op-amp configuration** – is one which utilizes external resistances (like the previous) to effect voltage gain. However, the polarity / phase of the output is same as input.

Figure 2.12: The non-inverting configuration.



$A_o = \infty$



$$\frac{0 - V_i}{R_1} = \frac{V_i - V_o}{R_2}$$

$$\frac{V_o}{R_2} = V_i\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{V_o}{V_i} = R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\boxed{A_v = \frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}}$$

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Characteristics of Non-Inverting Op-Amp Configuration

ideal gain $\left(A >> 1 + \frac{R_2}{R_1}\right)$: $\overbrace{G_{A=\infty}}^{\frac{v_{Out}}{v_{In}}} = 1 + \frac{R_2}{R_1}$

non-ideal gain: $\underbrace{G_{A<\infty}}_{\frac{v_{Out}}{v_{In}}} = \frac{1 + (R_2 / R_1)}{1 + \frac{1 + (R_2 / R_1)}{A}}$

percent gain error: $pge = 100\left|\frac{1 + (R_2 / R_1)}{A + 1 + (R_2 / R_1)}\right|$

inverting input potential: $v_1 = v_{Out}\left(\frac{R_1}{R_1 + R_2}\right)$

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Configuration and Characteristics of Buffer / Voltage-Follower Op-Amp Configuration

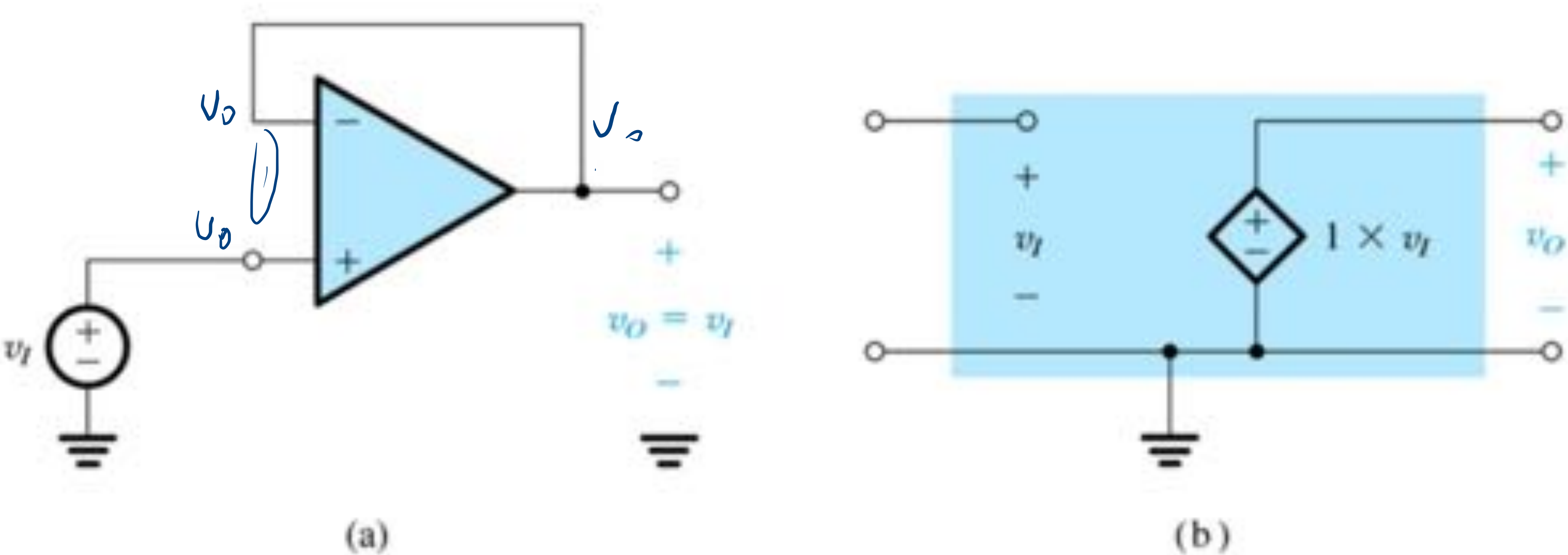


Figure 2.14: (**a**) The unity-gain buffer or follower amplifier. (**b**) Its equivalent circuit model.

Main point? For the buffer amp, output voltage is equal (in both magnitude and phase) to the input source. However, any current supplied to the load is drawn from amplifier supplies (V_{CC}, V_{EE}) and not the input source (v_I).

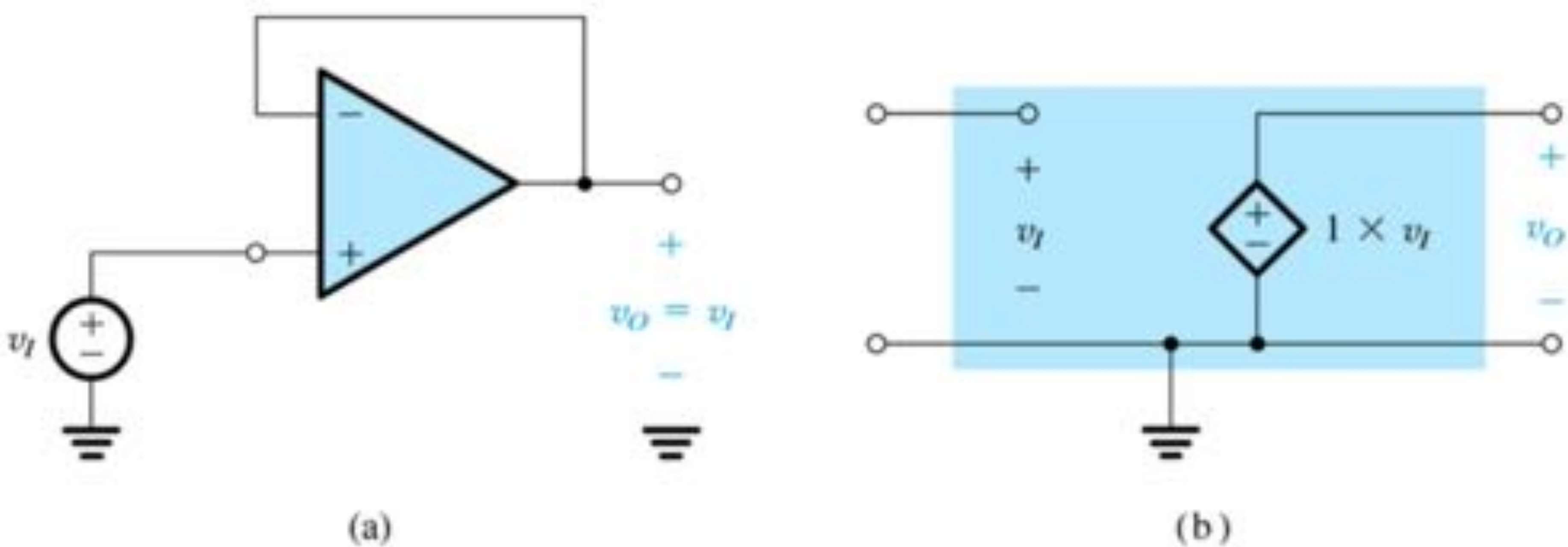


Figure 2.14: (**a**) The unity-gain buffer or follower amplifier. (**b**) Its equivalent circuit model.

2.4. Difference Amplifiers

- **difference amplifier** – is a closed-loop configuration which responds to the difference between two signals applied at its input and ideally rejects signals that are common to the two.
  - Ideally, the amp will amplify only the differential signal (v_{dfi}) and reject completely the common-mode input signal (v_{cmi}). However, a practical circuit will behave as below…

$$v_{Out} = Av_{dfi} + A_{cm}v_{cmi}$$

v_{i1} $\quad v_{dfi} = v_{i2} - v_{i2}$

v_{i2} $\quad v_{cmi} = \dfrac{v_{i1} + v_{i2}}{2}$

$v_{i1} = v_{cm} - \tfrac{1}{2} v_{dfi}$

$v_{i2} = v_{cm} + \tfrac{1}{2} v_{dfi}$

common-mode input

common-mode gain

differential input

differential gain

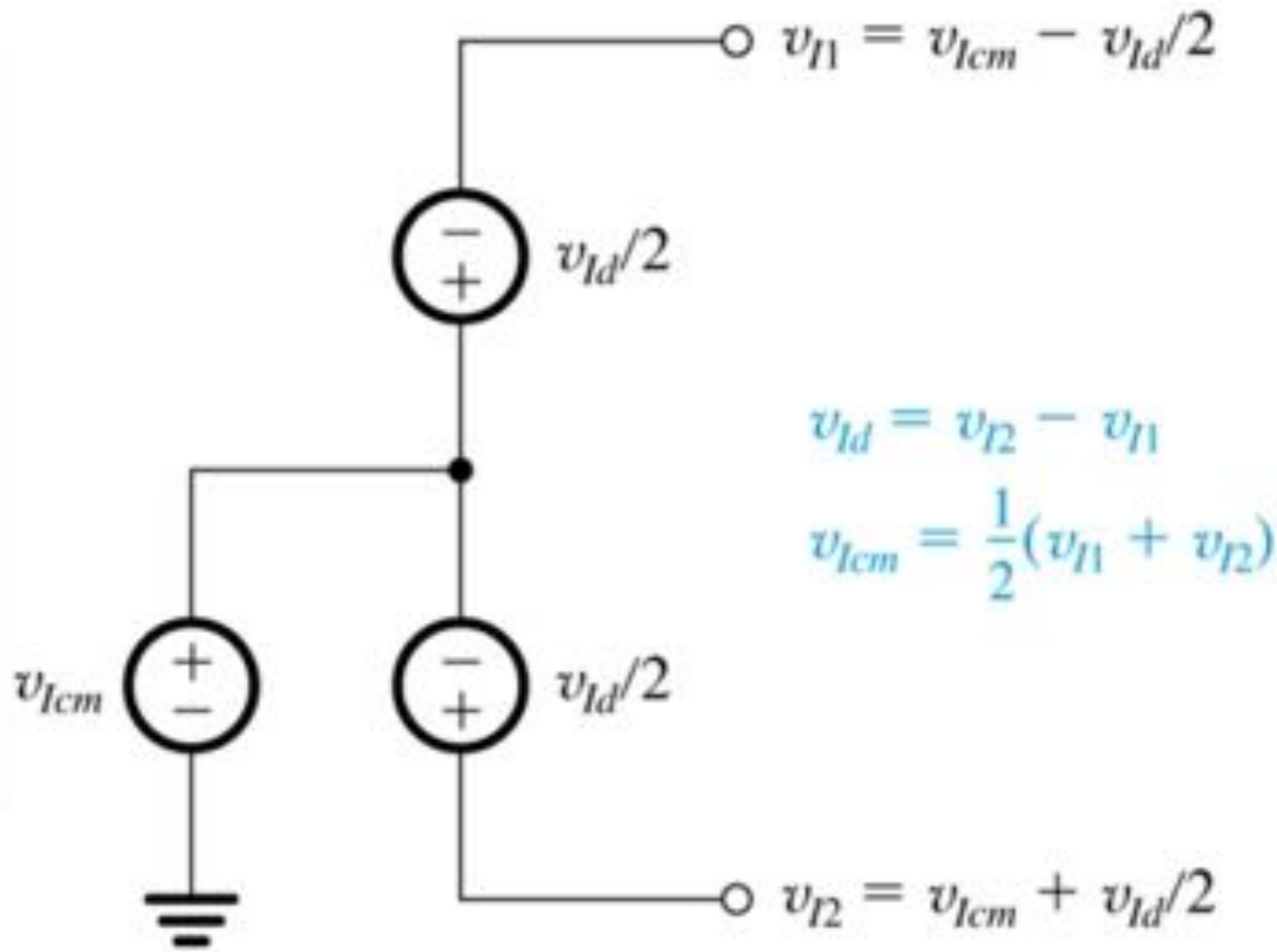
$$v_{Out} = A v_{dfi} + A_{cm} v_{cmi}$$

2.4. Difference Amplifiers

- **common-mode rejection ratio** (CMRR) – is the degree to which a differential amplifier "rejects" the common-mode input.
  - Ideally, CMRR = *infinity*…

$$CMRR = 20\ \mathbf{log}_{10}\left|\frac{A}{A_{Cm}}\right|$$

Figure 2.15: Representing the input signals to a differential amplifier in terms of their differential and common-mode components.



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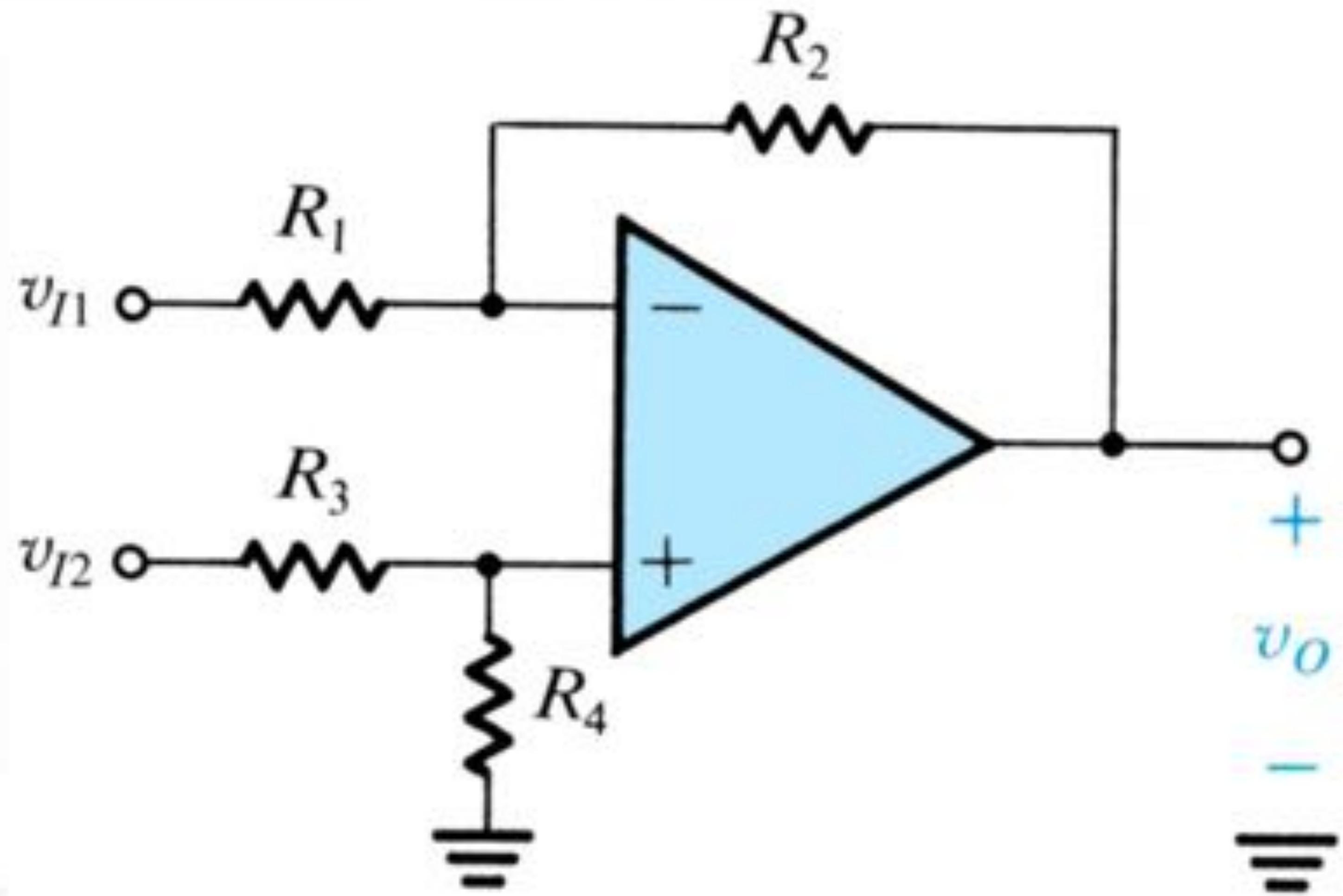
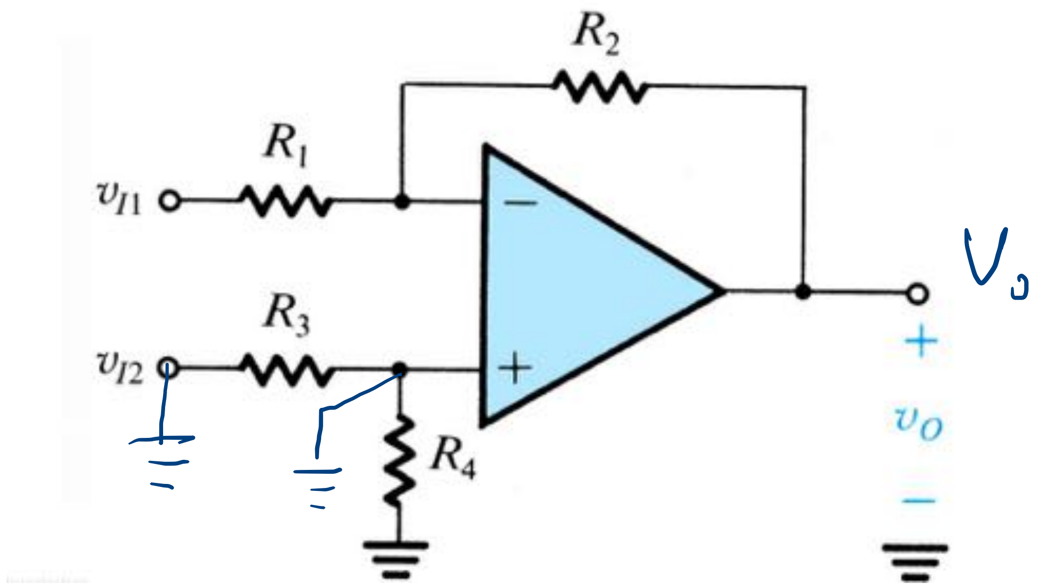
2.4. Difference Amplifiers

$$V_o = A_o(V_2 - V_1)$$

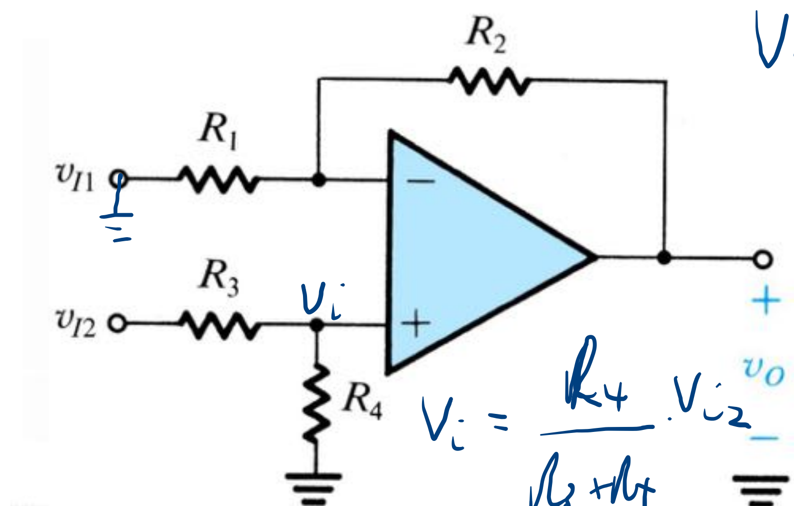
- **Q:** The op amp itself is differential in nature, why cannot it be used by itself?
  - **A:** It has an infinite gain, and therefore cannot be used by itself. One must devise a closed-loop configuration which facilitates this operation.

Figure 2.16: A difference amplifier.





$$V_{o1} = -\frac{R_2}{R_1} V_{i1}$$



$$V_{o2} = \left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right) V_{i2}$$

$$V_i = \frac{R_4}{R_3+R_4} \cdot V_{i2}$$

$$V_o = V_{o1} + V_{o2}$$

$$V_{o2} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2}$$

$$V_o = \left(\left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) - \frac{R_2}{R_1}\right) V_{cmi}$$

$$\boxed{\begin{aligned} R_4 &= R_2 \\ R_3 &= R_1 \end{aligned}}$$

$$V_o = V_{o2} + V_{o1}$$

$$V_o = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2} - \frac{R_2}{R_1} V_{i1}$$

$$V_o = \frac{R_2}{R_1}\left(V_{i2} - V_{i1}\right)$$

$$V_o = A_v\left(V_{i2} - V_{i1}\right) + A_{cm} V_{cmi}$$

$$V_o = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2} - \frac{R_2}{R_1} V_{i1}$$

$$V_o = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)V_{i_2} - \frac{R_2}{R_1}V_{i_1}$$

$$V_o = A_d V_{dfi} + A_{cm} V_{cmi}$$

assume common-mode input $\Rightarrow V_{i_1} = V_{i_2} = V_{cmi}$

$$V_o = \left[\left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) - \frac{R_2}{R_1}\right]V_{cmi}$$

$$A_{cm} = 0 \Rightarrow \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) - \frac{R_2}{R_1} = 0$$

$$\left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) = \frac{R_2}{R_1}$$

2.4.1. A Single Op-Amp Difference Amp

- **Q:** What are the characteristics of the difference amplifier?
  - **A:** Refer to following equations...

$$v_{Out} = \frac{(R_2 + R_1)R_4}{(R_4 + R_3)R_1} v_{In2} - \frac{R_2}{R_1} v_{In1}$$

but if $\begin{Bmatrix} R_1 = R_3 \\ R_2 = R_4 \end{Bmatrix}$ then $v_{Out} = \frac{R_2}{R_1}(v_{In2} - v_{In1})$

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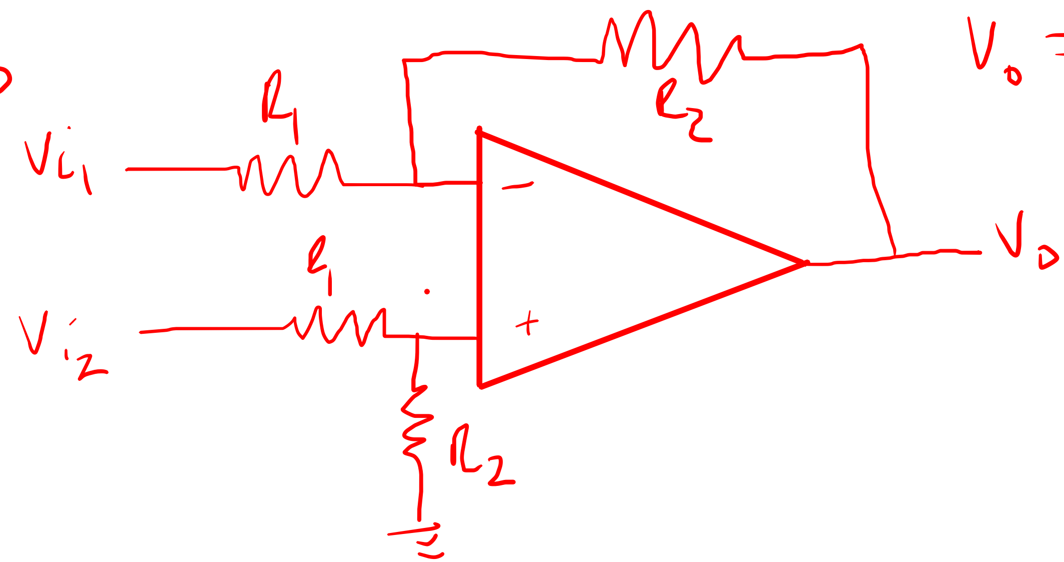
A Shift in Notation

- Before this point…
  - The parameter *A* is used to represent open-loop gain of an op amp.
  - The parameter *G* is used to represent ideal / non-ideal closed-loop gain of an op amp.
- After this point…
  - The parameter *A* is used to represent ideal gain of an op amp in a given closed-loop configuration.
  - The parameter *G* is not used.

Properties of an ideal operational Amp.

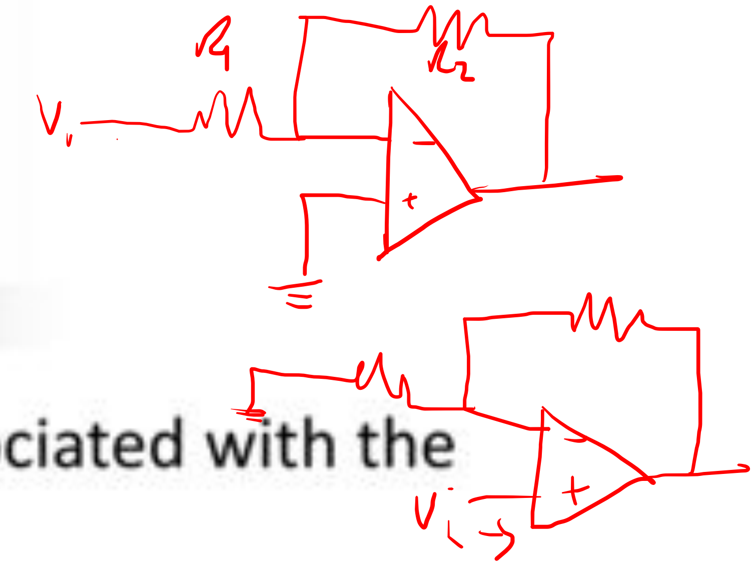
* $A_0 = \infty$ $\Rightarrow$ Virtual ground.
* $R_{in} = \infty$
* $R_0 = 0$
* Bandwidth $= \infty$
* $A_{cm} = 0$ $\Rightarrow$ $CMRR = \infty$

$R_{in} = \infty$



$$V_0 = \frac{R_2}{R_1}\left(V_{i2} - V_{i1}\right)$$

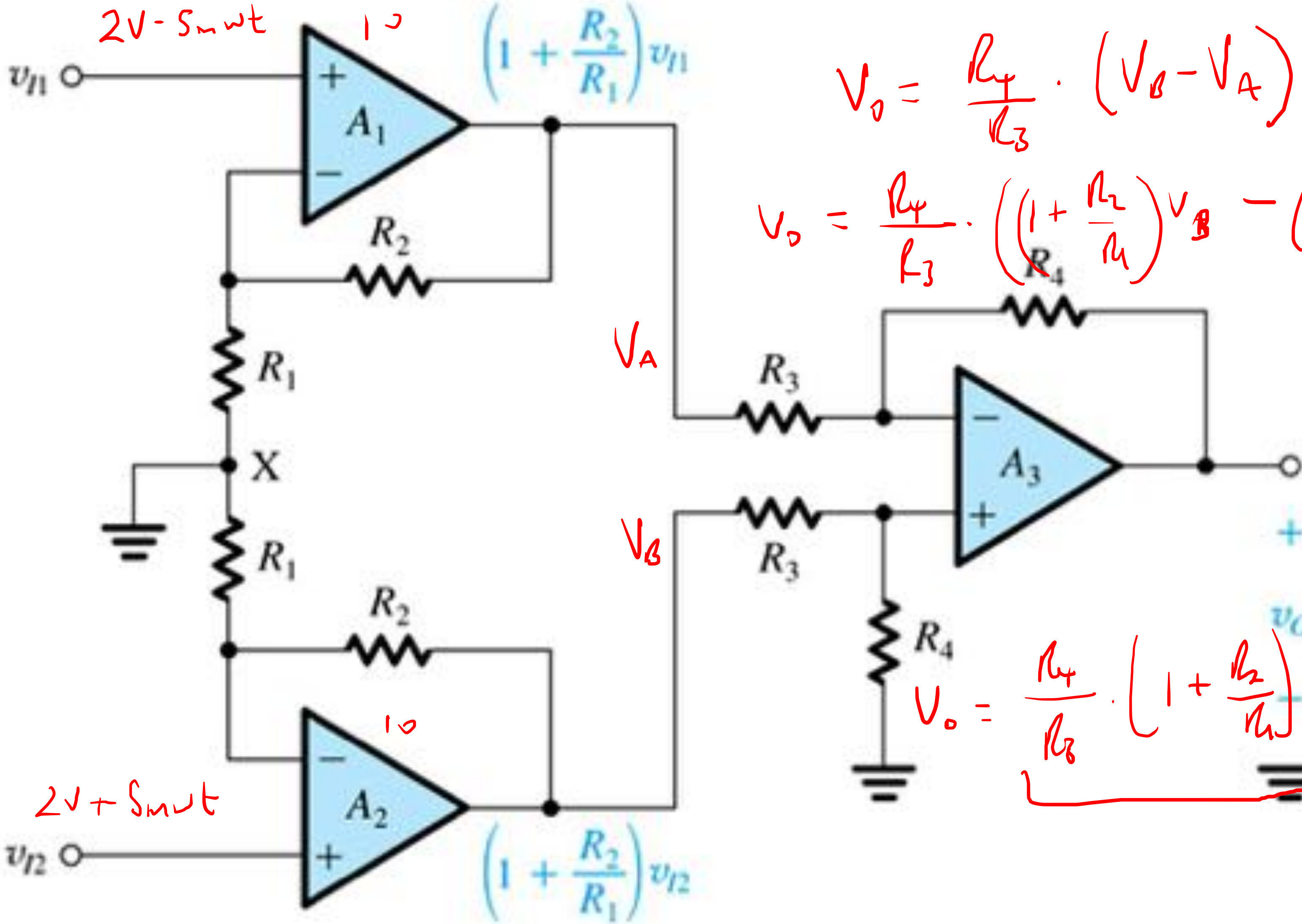
2.4.2. The Instrumentation Amplifier



- **Q:** What is one problem associated with the difference amplifier?
  - **A:** Low input impedance.
- **Q:** And, what does this mean practically?
  - **A:** That source impedance will have an effect on gain.
- **Q:** What is the solution?
  - **A:** Placement of two buffers at the input terminals, amplifiers which transmit the voltage level but draw minimal current.

2.4.2. **The Instrumentation Amplifier**

- **Q:** However, can one get "more" from these amps than simply impedance matching?
  - **A:** Yes, maybe additional voltage gain???

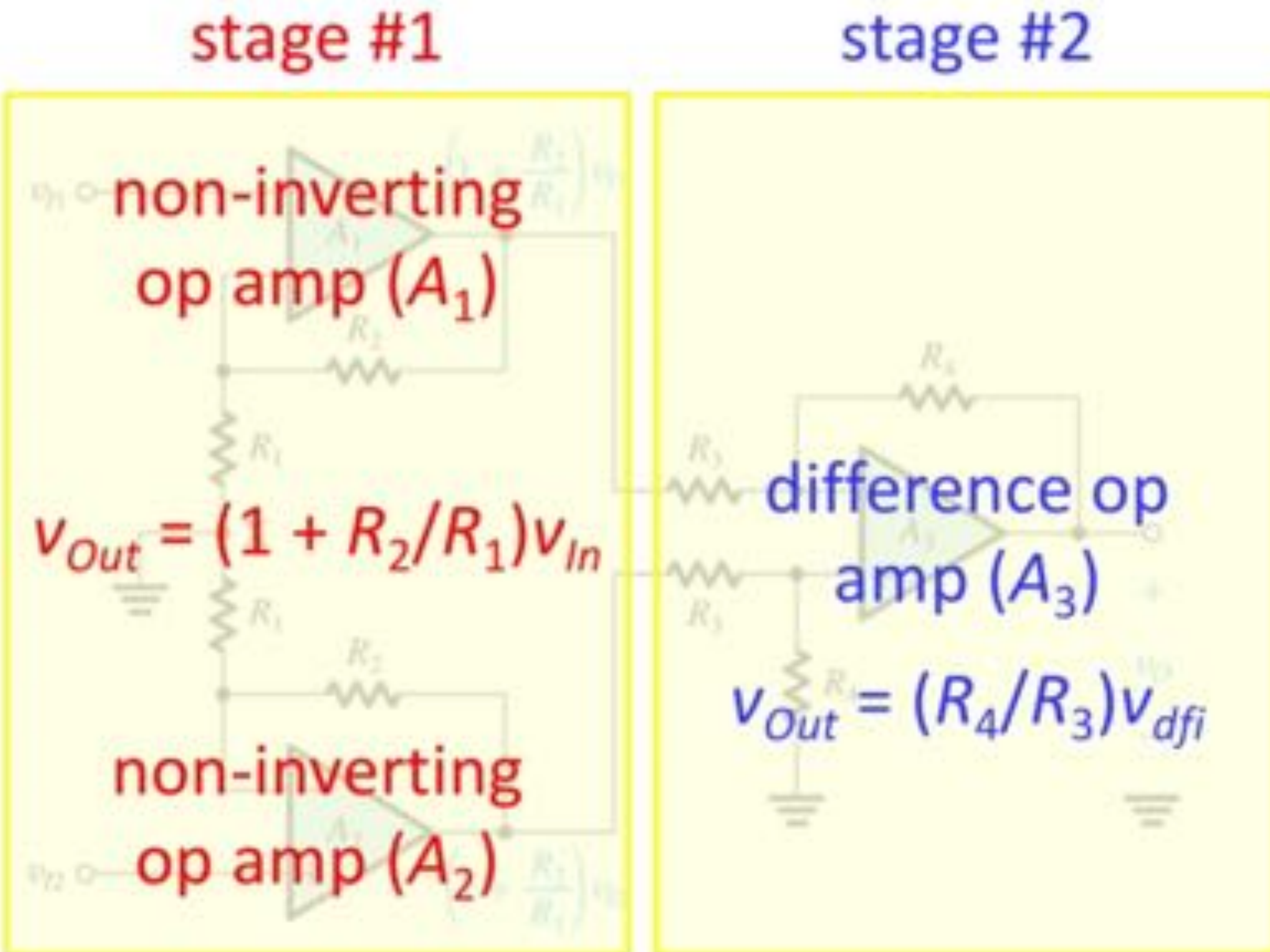


$$V_0 = \frac{R_4}{R_3}\cdot(V_B - V_A)$$

$$V_0 = \frac{R_4}{R_3}\cdot\left(\left(1+\frac{R_2}{R_1}\right)V_B - \left(1+\frac{R_2}{R_1}\right)V_A\right)$$

$$V_0 = \frac{R_4}{R_3}\cdot\left(1+\frac{R_2}{R_1}\right)\cdot(V_B - V_A)$$

Figure 2.20: A popular circuit for an instrumentation amplifier.



2.4.2. The Instrumentation Amplifier

- **Q:** However, can one get "more" from these amps than simply impedance matching?
  - **A:** Yes, maybe additional voltage gain???

$$\overbrace{v_{Out} = \underbrace{\frac{R_4}{R_3}\left(1+\frac{R_2}{R_1}\right)}_{\mathbf{A}_{inst}(\underline{R})} v_{dfi}}^{\text{transfer function for instrumentation amplifier of figure 2.20.}}$$

additional voltage gain

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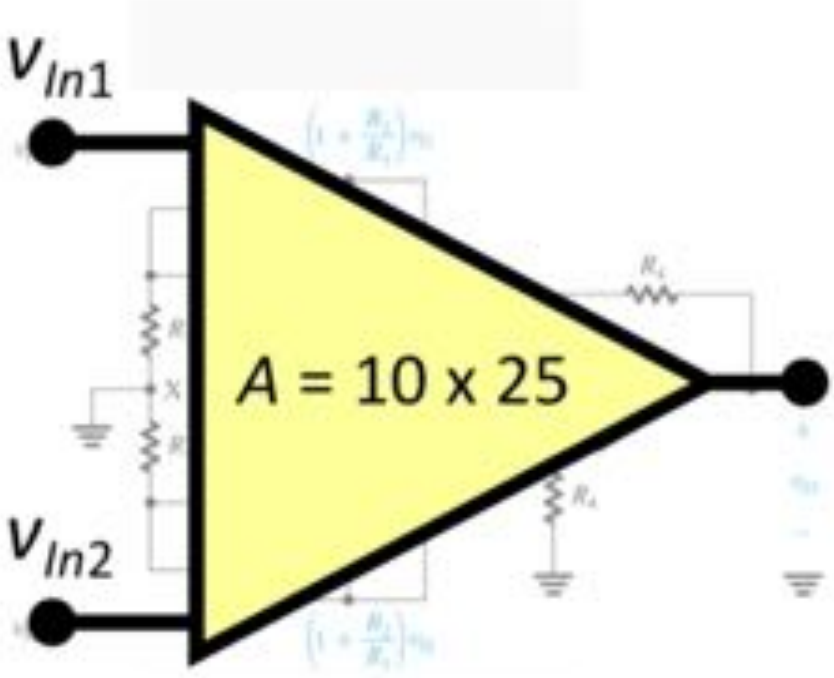
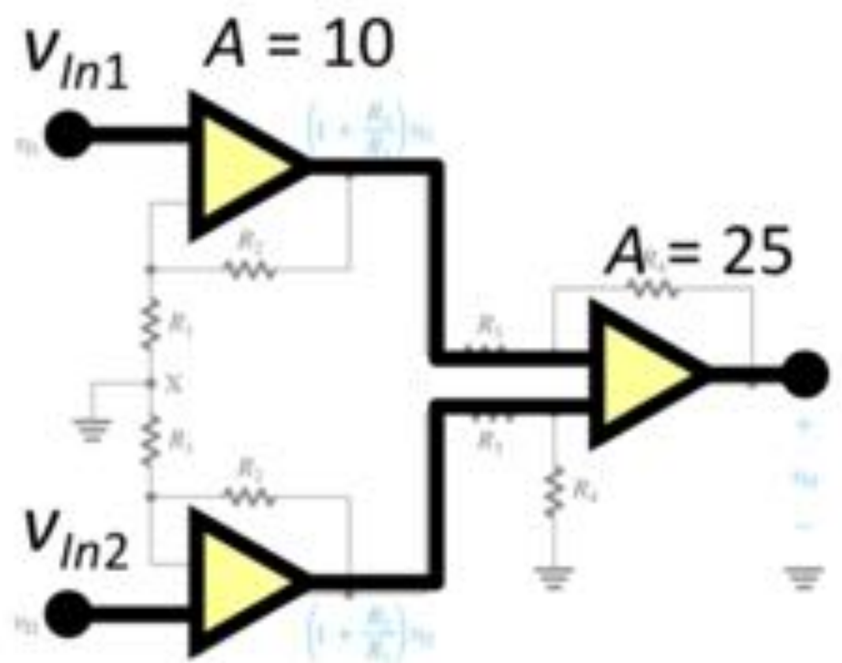
2.4.2. **The Instrumentation Amplifier**

- **advantages** of instrumentation amp
  - very high input resistance
  - high differential gain
  - symmetric gain (assuming that A_1 and A_2 are matched)
- **disadvantages** of instrumentation amp
  - A_{Di} and A_{Cm} are equal in first stage – meaning that the common-mode and differential inputs are amplified with equal gain…

What is problem with $A_{Cm} = A$?



differential gain >> common-mode gain



differential gain = common-mode gain

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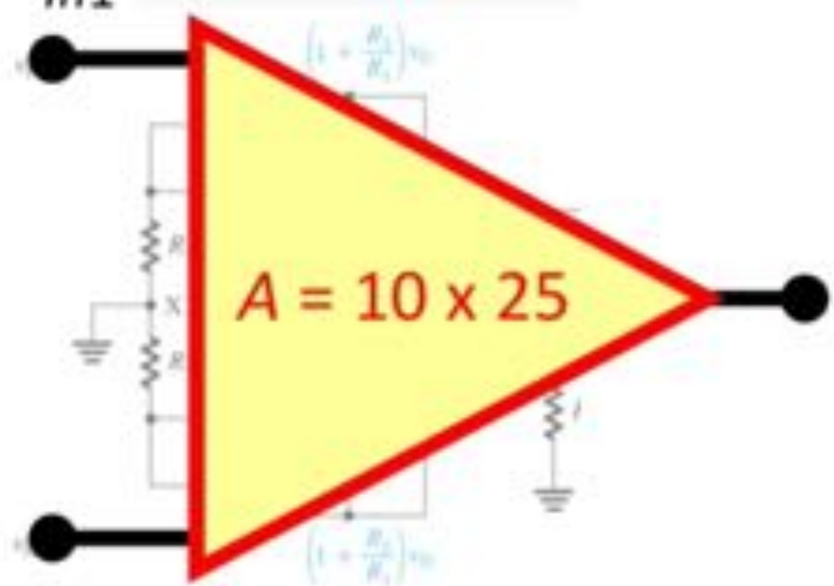
differential gain >> common-mode gain

$v_{In1} = 10.03V$



$v_{In2} = 10.02V$

$v_{Out} = 250 \times (10.03\text{-}10.02)V$

$v_{Out} = 2.5V$ **no problem!!!**

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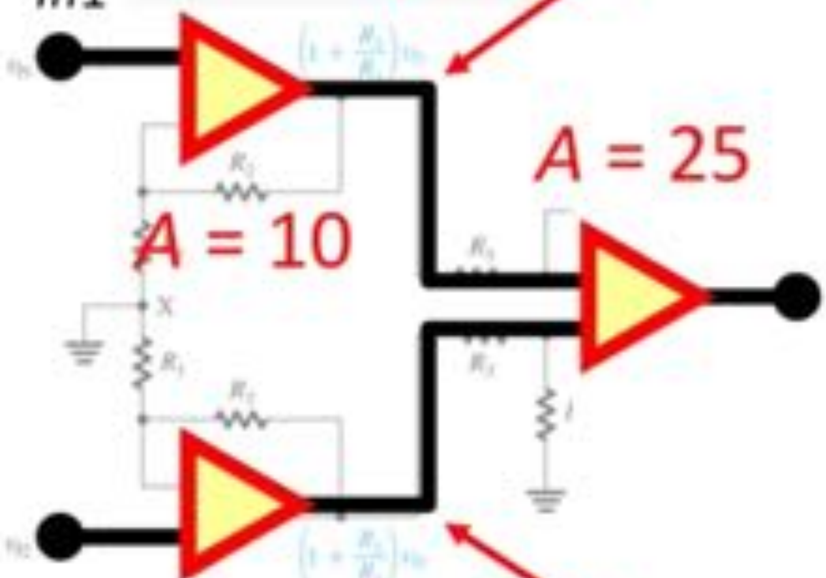
differential gain = common-mode gain

$v_{In1} = 10.03V$

$v_{Out1} = 10 \times 10.03 = 15V$ saturation



$v_{Out} = 25 \times (15\text{-}15)V$

$v_{Out} = 0V$ **problem!!!**

$v_{In2} = 10.02V$

$v_{Out2} = 10 \times 10.02 = 15V$ saturation

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2.4.2. **The Instrumentation Amplifier**

- **advantages** of instrumentation amp
  - very high input resistance
  - high differential gain
  - symmetric gain (assuming that A_1 and A_2 are matched)
- **disadvantages** of instrumentation amp
  - A_{Di} and A_{Cm} are equal in first stage – meaning that the common-mode and differential inputs are amplified with equal gain…
  - need for matching – if two op amps which comprise stage #1 are not perfectly matched, one will see unintended effects

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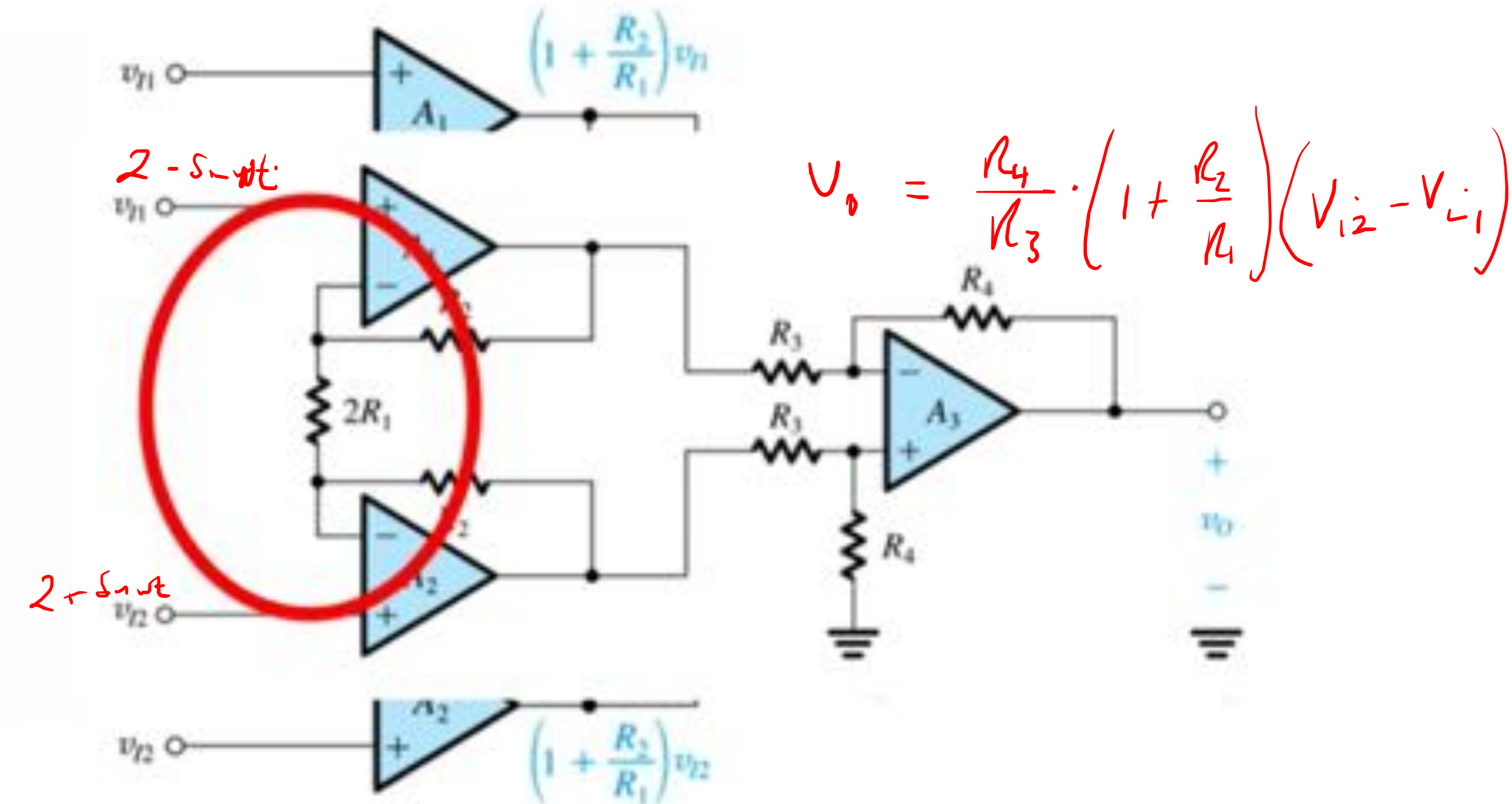
2.4.2. The Instrumentation Amplifier

- **Q:** How can one fix this (alleviate these disadvantages)?
  - **A:** Disconnect the two resistors (R_1) connected to node X from ground, making the configuration "floating" in nature...
  - **A:** Refer to following slide...

Figure 2.20: A popular circuit for an instrumentation amplifier. **(b)** The circuit in **(a)** with the connection between node X and ground removed and the two resistors R_1 and R_1 lumped together. This simple wiring change dramatically improves performance.



$$V_o = \frac{R_4}{R_3} \cdot \left(1 + \frac{R_2}{R_1}\right)\left(V_{i2} - V_{i1}\right)$$

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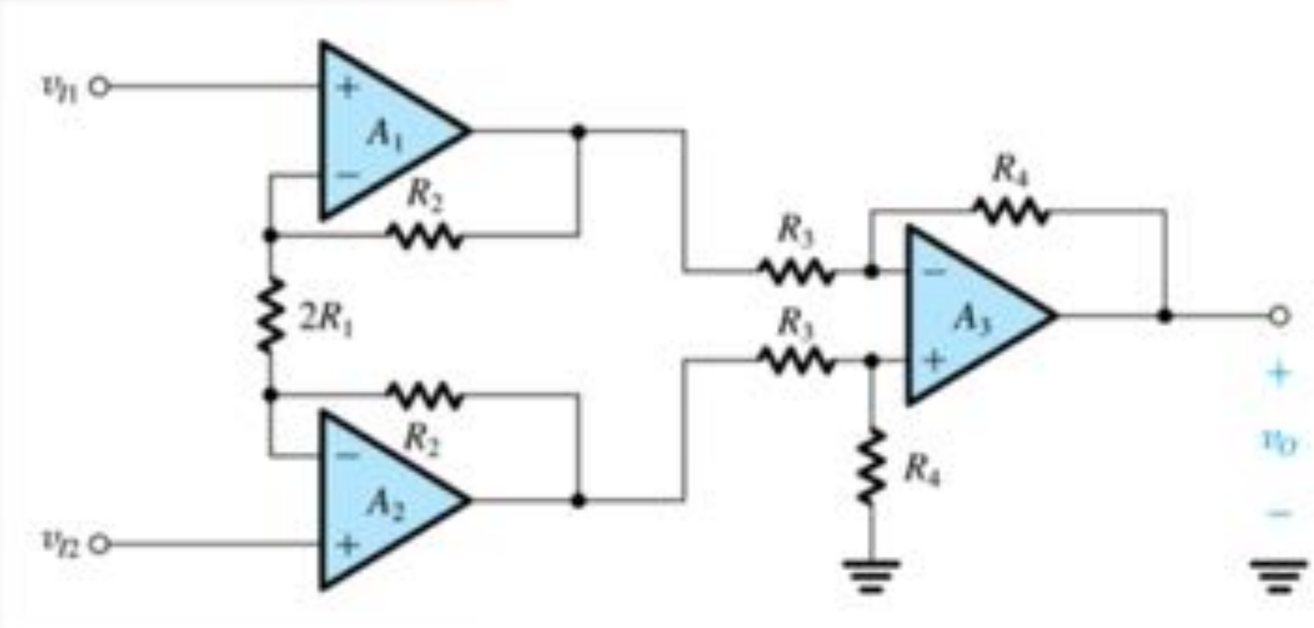
2.4.2. The Instrumentation Amplifier

- **Q:** How can one analyze this circuit?



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2.4.2. The Instrumentation Amplifier

- **step #1:** note that virtual short circuit exists across terminals of op amp A_1 and A_2
- **step #2:** define current flow across the resistor $2R_1$
- **step #3:** define output of A_1 and A_2

$$\overbrace{v_{(+)} - v_{(-)} = 0}^{\text{for both op amp } A_1 \text{ and } A_2}$$

...therefore

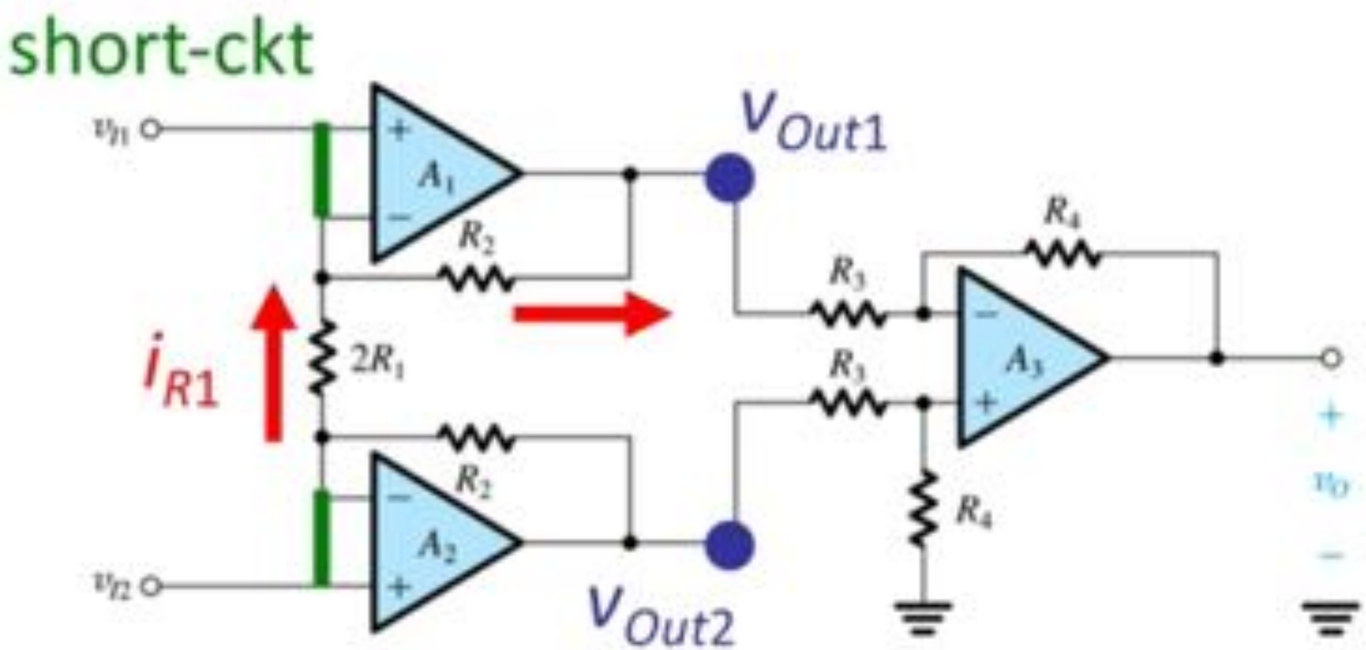
$$v_{(+)} = v_{(-)}$$

$$i_{R1} = \frac{\overbrace{v_{In2} - v_{In1}}^{v_{InDi}}}{2R_1}$$

$$\overbrace{\begin{aligned} v_{Out1} &= v_{In1} - i_{R1}R_2 \end{aligned}}^{\text{because no current will flow into ideal op amp, all of } iR_1 \text{ will flow across } R_2}$$

$$v_{Out2} = v_{In2} + i_{R1}R_2$$

2.4.2. **The Instrumentation Amplifier**



2.4.2. The Instrumentation Amplifier

- **step #4:** Define output of A_1 and A_2 in terms of input alone

$$\overbrace{v_{Out2} - v_{Out1} = \underbrace{\left[v_{In2} + \left(\frac{v_{In2} - v_{In1}}{2R_1} \right) R_2 \right]}_{v_{Out2} = v_{In2} + i_{R1}R_2} - \cdots}^{\textbf{action}\text{: define (from equations above) the differential input } v_{Out2} - v_{Out1} \text{ to stage \#2}}$$

$$\cdots - \underbrace{\left[v_{In1} - \left(\frac{v_{In2} - v_{In1}}{2R_1} \right) R_2 \right]}_{v_{Out1} = v_{In1} - i_{R1}R_2}$$

$$\overbrace{v_{Out2} - v_{Out1} = \underbrace{(v_{In2} - v_{In1})}_{v_{InDi}} + 2\left(\frac{\overbrace{v_{In2} - v_{In1}}^{v_{InDi}}}{2R_1} \right) R_2}^{\textbf{action}\text{: combine terms}}$$

$$v_{Out2} - v_{Out1} = \left(1 + \frac{2R_2}{2R_1} \right) v_{InDi}$$

2.4.2. The Instrumentation Amplifier

- **step #5:** Define output of A_3.
- **step #6:** Define gain of revised instrumentation amplifier.

$$v_{Out} = \frac{R_4}{R_3}\overbrace{(v_{Out2} - v_{Out1})}^{\text{action: define in terms of } v_{dfi}}$$

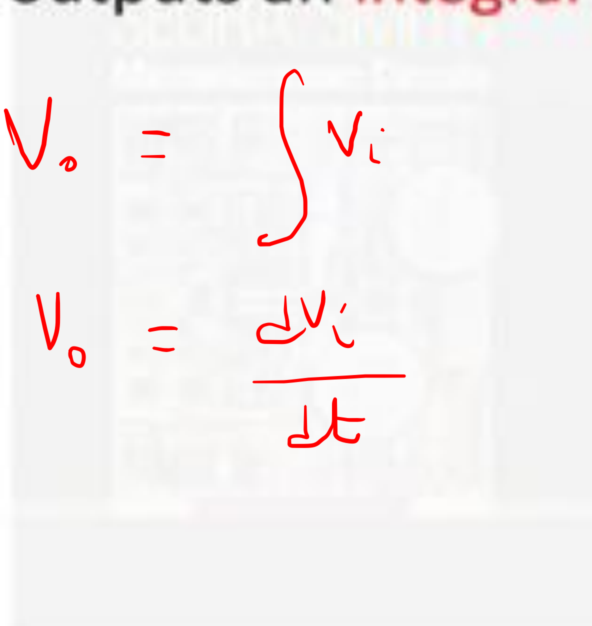
$$v_{Out} = \frac{R_4}{R_3}\left(1 + \frac{2R_2}{2R_1}\right)v_{dfi}$$

$$\boxed{\frac{v_{Out}}{v_{dfi}} = A_{Di} = \frac{R_4}{R_3}\left(1 + \frac{2R_2}{2R_1}\right)}$$

solution

2.5. Integrators and Differentiators

- **integrator / differentiator amplifier** – is one which outputs an integral or derivative of the input signal.



$$V_o = \int V_i$$

$$V_o = \frac{dV_i}{dt}$$

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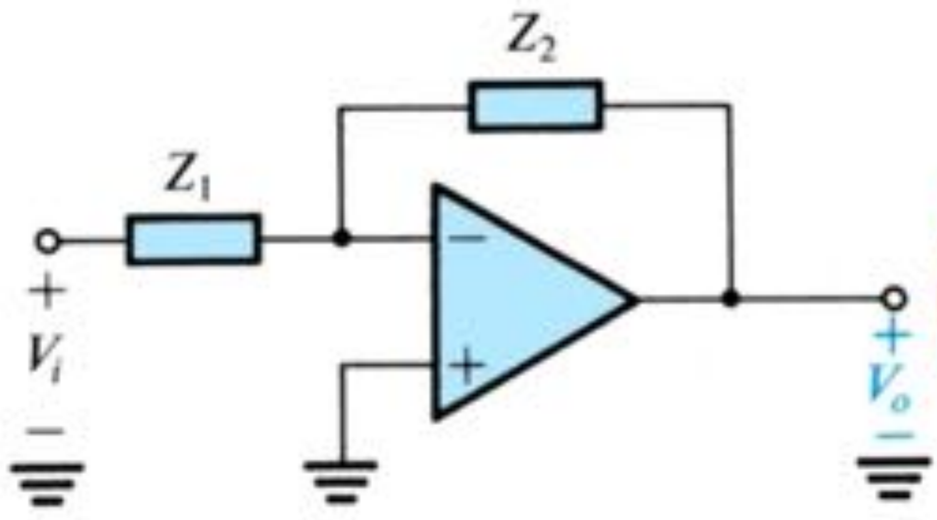
2.5.1. The Inverting Configuration with General Impedances

- **Q:** Does the transfer function for the inverting op amp change if the feedback and input impedances are not purely resistive?
  - **A:** No, not in form…



$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$

Example 2.4: Other Op-Amp Configurations

- Consider the circuit on next slide page.
- **Q(a):** Derive an expression for the transfer function v_{Out} / v_{In}.
- **Q(b):** Show that the transfer function is of a low-pass STC circuit.
- **Q(c):** By expressing the transfer function in standard form of Table 1.2, find the dc-gain and 3dB frequency.

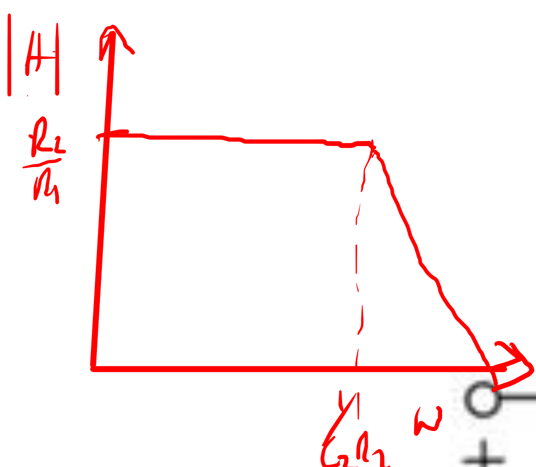
Example 2.4: Other Op-Amp Configurations

$$A_v = \frac{-Z_2}{Z_1} \qquad Z_2 = X_{C2} \parallel R_2$$

$$H(s) = \frac{-R_2}{R_1} \cdot \frac{1}{1+sC_2R_2}$$

$$Z_2 = \frac{\frac{1}{sC_2} \cdot R_2}{\frac{1}{sC_2} + R_2}$$

$$X_{C2} = \frac{1}{j\omega C_2} = \frac{1}{sC_2}$$



$$Z_2 = \frac{R_2}{1+sC_2R_2}$$

$$\omega = \frac{1}{C_2R_2}$$

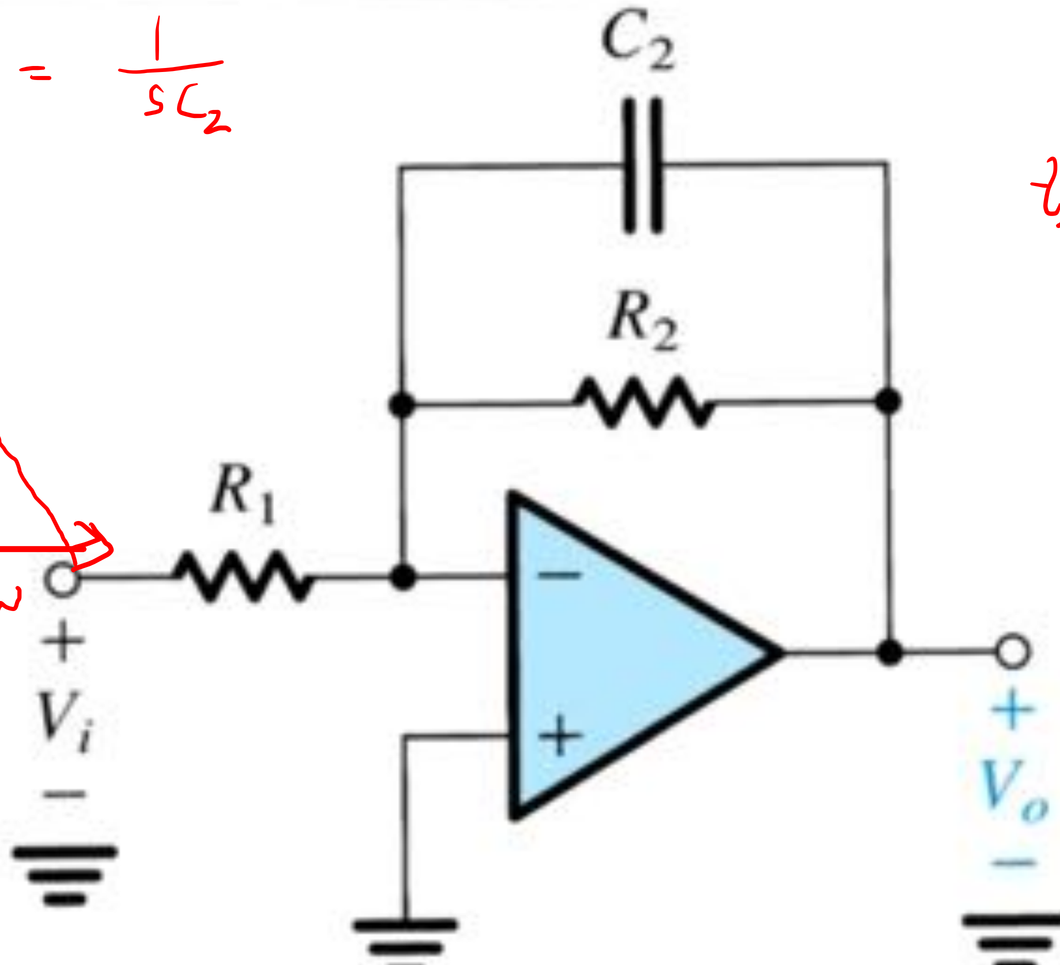


Figure 2.23: Circuit for Example 2.4.

2.5.2. The Inverting Integrator

- **Q:** How can inverting op-amp be adapted to perform integration?
    - **A:** Utilization of capacitor as feedback impedance.

Figure 2.24: (a) The miller or inverting integrator. **(b)** Frequency response of the integrator.

$1/s$

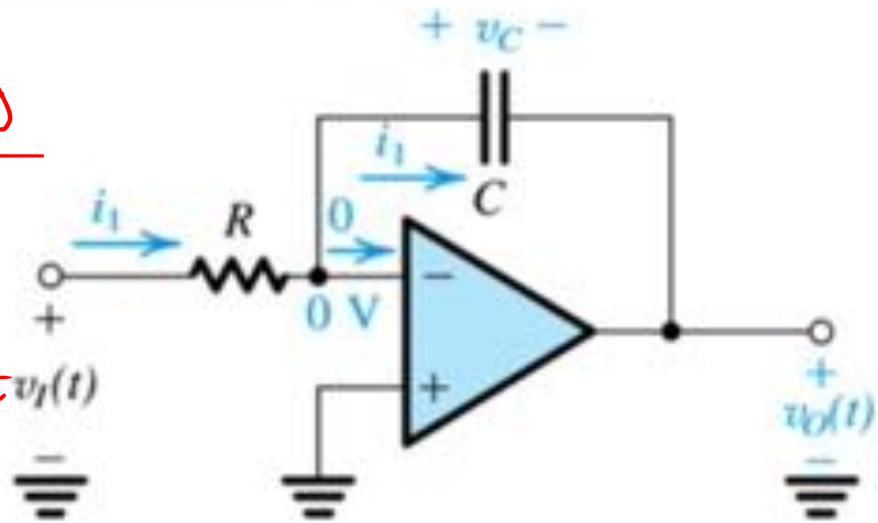
$$H(s) = -\frac{\frac{1}{sC}}{R} = -\frac{1}{sRC}$$

$$i(t) = C\,\frac{dV(t)}{dt}$$

$$\frac{V_i(t)}{R} = C \cdot \frac{-dV_o(t)}{dt}$$

$$\int dV_o(t) = \frac{-1}{RC}\int V_i(t)\,dt$$

$$V_o(t) = -\frac{1}{RC}\int V_i(t)\,dt$$



transient description (dc): $$v_{Out}(t) = -\frac{1}{R_1C_F}\left(\int_{t=0}^{t} v_{In}(t)\,dt\right) - \overbrace{v_{Out}(t_0)}^{\text{initial output voltage}}$$

steady-state description (ac): $$\frac{v_{Out}}{v_{In}} = -\frac{1}{sR_1C_F}$$

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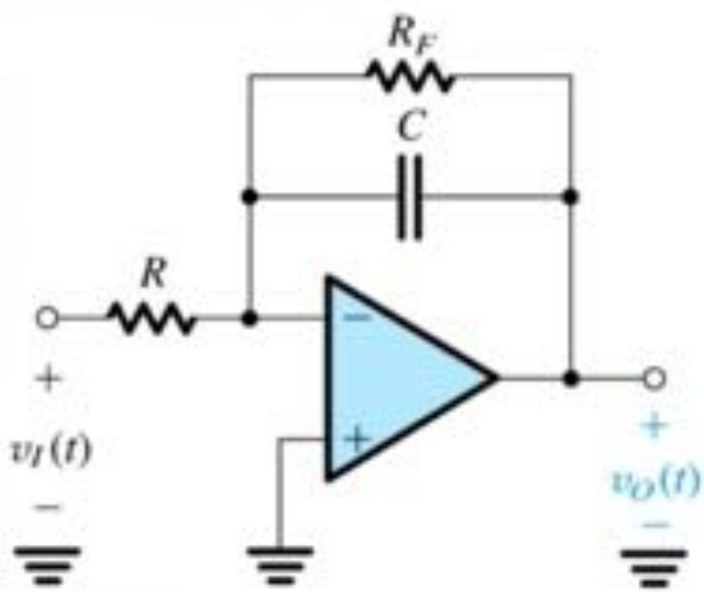
2.5.2. The Inverting Integrator

- **Q:** What is the problem with this configuration (related to dc gain)?
  - **A:** At dc frequency (ω = 0), gain is infinite
  - Gain = 1 / (ω R_1C_F)
- **Q:** Solution?
  - **A:** By placing a very large resistor in parallel with the capacitor, negative feedback is employed to make dc gain "finite."

Figure 2.25: The Miller integrator with a large resistance *RF* connected in parallel with *C* in order to provide negative feedback and hence finite gain at dc.



transient description (dc): depends on input signal???

steady-state description (ac): $\dfrac{v_{Out}}{v_{In}} = -\dfrac{R_F / R_1}{1 + sR_F C_F}$

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Example 2.5: Miller Integrator

- Consider the Miller integrator…
- **Q(a):** Find response of a Miller Integrator to input pulse of 1*V* height and 1*ms* width.
  - $R_1 = 10k\Omega hm$, $C_F = 10nF$
- **Q(b):** If the integrator capacitor is shunted by a 1*MOhm* resistor, how will the response be modified?
  - **note:** the op amp will saturate at +/- 13V

2.5.3. The Op-Amp Differentiator

$$H(s) = -\frac{R}{1/sC}$$

$$H(s) = -sRC$$

- **Q:** How can one adapt integrator to perform differentiation?
  - **A:** Interchange locations of resistors and capacitors.

$$C\frac{dV_i(t)}{dt} = \frac{-V_o(t)}{R}$$

$$V_o = -RC\frac{dV_i(t)}{dt}$$

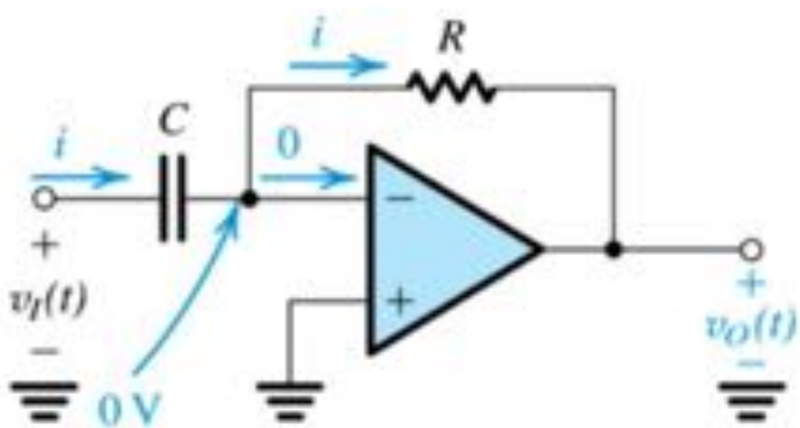

Figure 2.27: A differentiator.

2.5.3. The Op-Amp Differentiator

transient description (dc): $$\mathbf{v}_{Out}(t) = -R_F C_1 \frac{d\mathbf{v}_{In}(t)}{dt}$$

steady-state description (ac): $$\frac{\mathbf{V}_{Out}(s)}{\mathbf{V}_{In}(s)} = -sR_F C_1$$

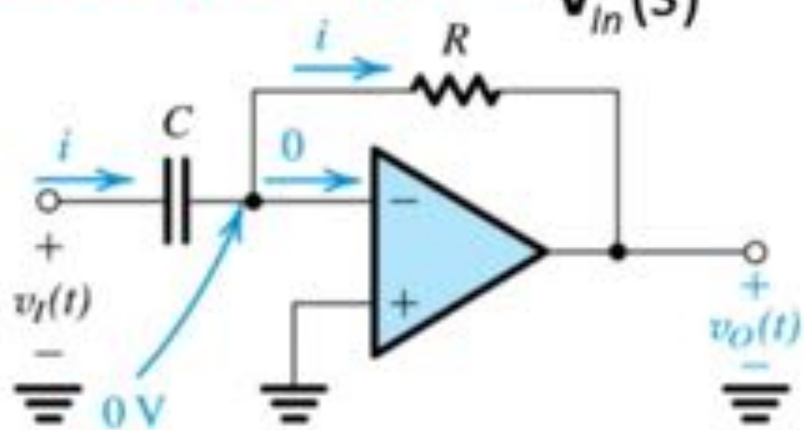

Figure 2.27: A differentiator.

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2.5.3. The Op-Amp Differentiator

- filtering characteristic is high pass filter
- magnitude of transfer function is $|V_{Out} / V_{In}| = \omega R_F C_1$
- phase of transfer function is $\phi = -90^{\circ}$
- differentiator time-constant is frequency at which unity gain occurs and defined as $\omega = 1 / R_F C_1$
- **Q:** What is the problem with differentiator?
  - **A:** Differentiator acts as noise amplifier, exhibiting large changes in output from small (but fast) changes in input. As such, it is rarely used in practice.

2.6. **DC Imperfections**

- **Q:** What will be discussed moving on?
  - **A:** When can one NOT consider an op amp to be ideal, and what effect will that have on operation?

2.6.1. **Offset Voltage**

- **Q:** What is **input offset voltage** (V_{OS})?
  - **A:** An imaginary voltage source in series with the user-supplied input, which effects an op amp output even when $i_{dfi} = 0$.

What will happen when short is applied?

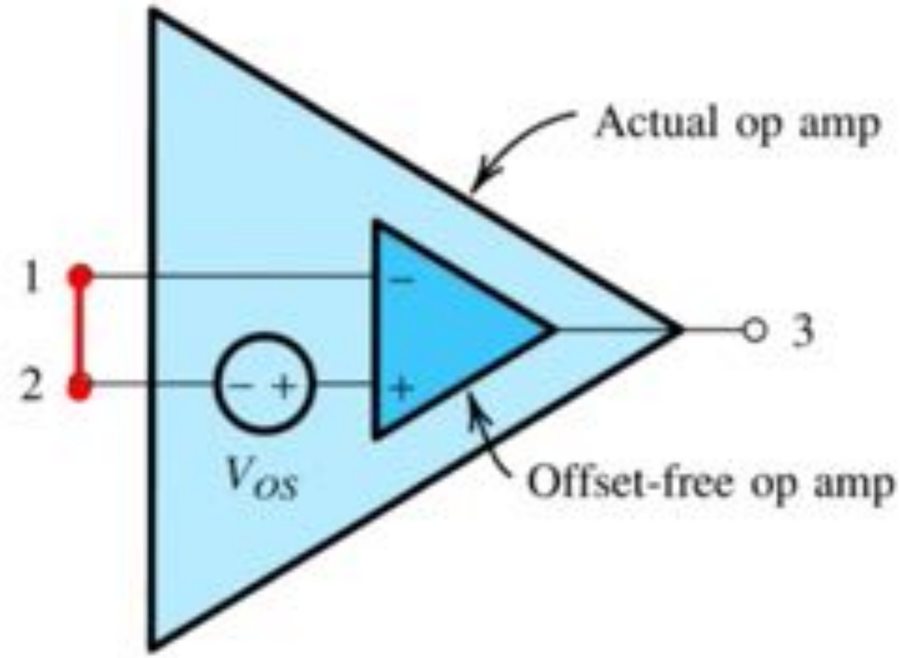

Figure 2.28: circuit model for an op amp with input offset voltage V_{OS}.

2.6.1. **Offset Voltage**

- **Q:** What causes V_{OS}?
  - **A:** Unavoidable mismatches in the differential stage of the op amp. It is impossible to perfectly match all transistors.
- **Q:** Range of magnitude?
  - **A:** 1mV to 5mV

$$\overbrace{V_{dcOut}}^{\text{offset dc output}} = \overbrace{V_{OS}}^{\text{offset voltage}}\left(1+\frac{R_F}{R_1}\right)$$

This relationship between offset voltage (V_{OS}) and offset dc output (V_{OsOut}) applies to both inverting and non-inverting op amp. However, only if one assumes that V_{OS} is present at non-inverting input.

- **Q:** What causes V_{OS}?
  - **A:** Unavoidable mismatches in the differential stage of the op amp. It is impossible to perfectly match all transistors.
- **Q:** Range of magnitude?
  - **A:** $1mV$ to $5mV$

$$\overbrace{V_{dcOut}}^{\text{offset dc output}} = \overbrace{V_{OS}}^{\text{offset voltage}} \left(1 + \frac{R_F}{R_1}\right)$$

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2.6.1. **Offset Voltage**

- **Q:** How can this offset be reduced?
  - **A: offset nulling terminals** – A variable resistor (if properly set) may be used to reduce the asymmetry present and, in turn, reduce offset.
  - **A: capacitive coupling** – A series capacitor placed between the source and op amp may be used to reduce offset, although it will also filter out dc signals.

Figure 2.30: The output dc offset voltage of an op-amp can be trimmed to zero by connecting a potentiometer to the two **offset-nulling terminals**. The wiper of the potentiometer is connected to the negative supply of the op amp.

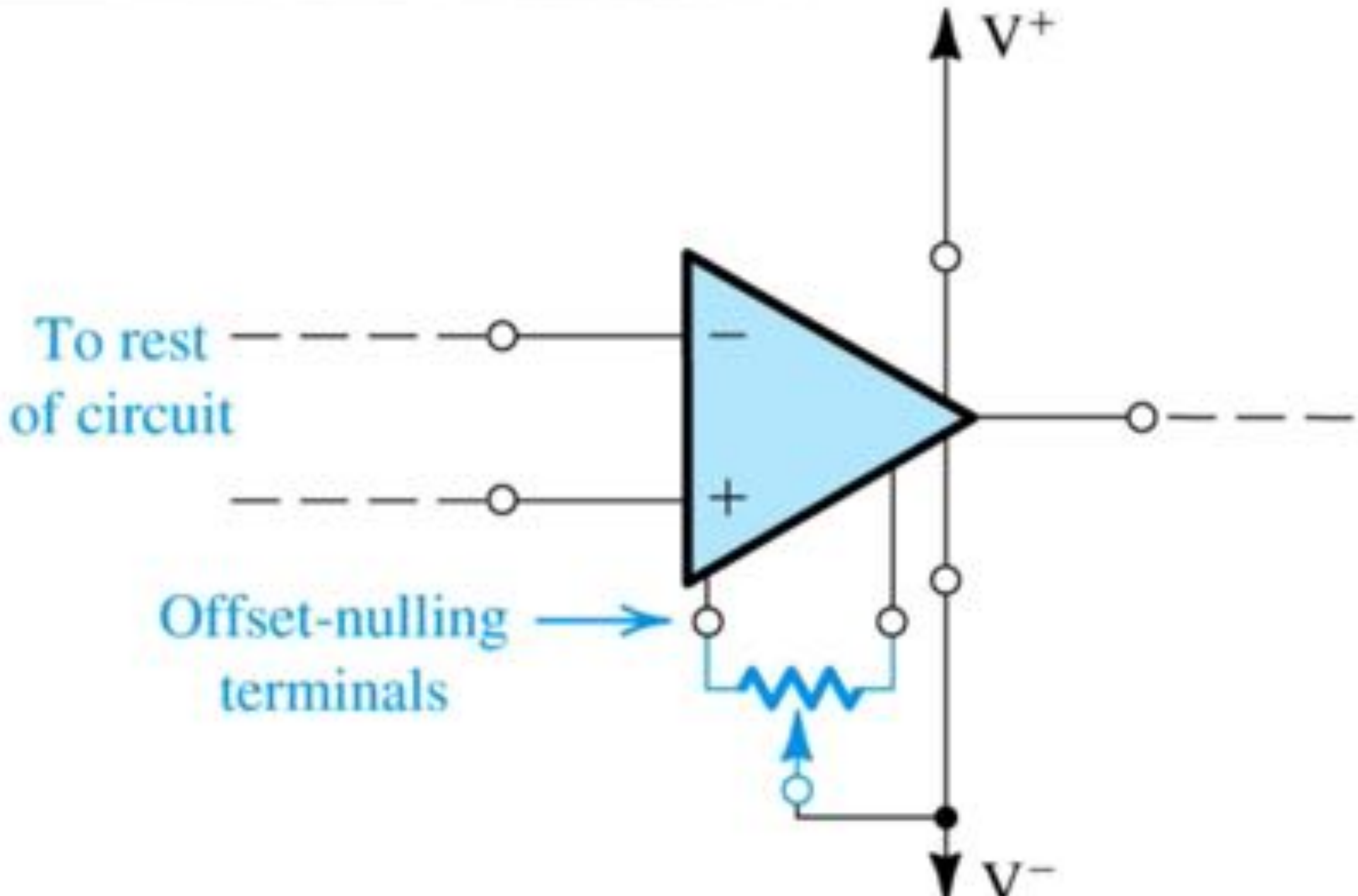

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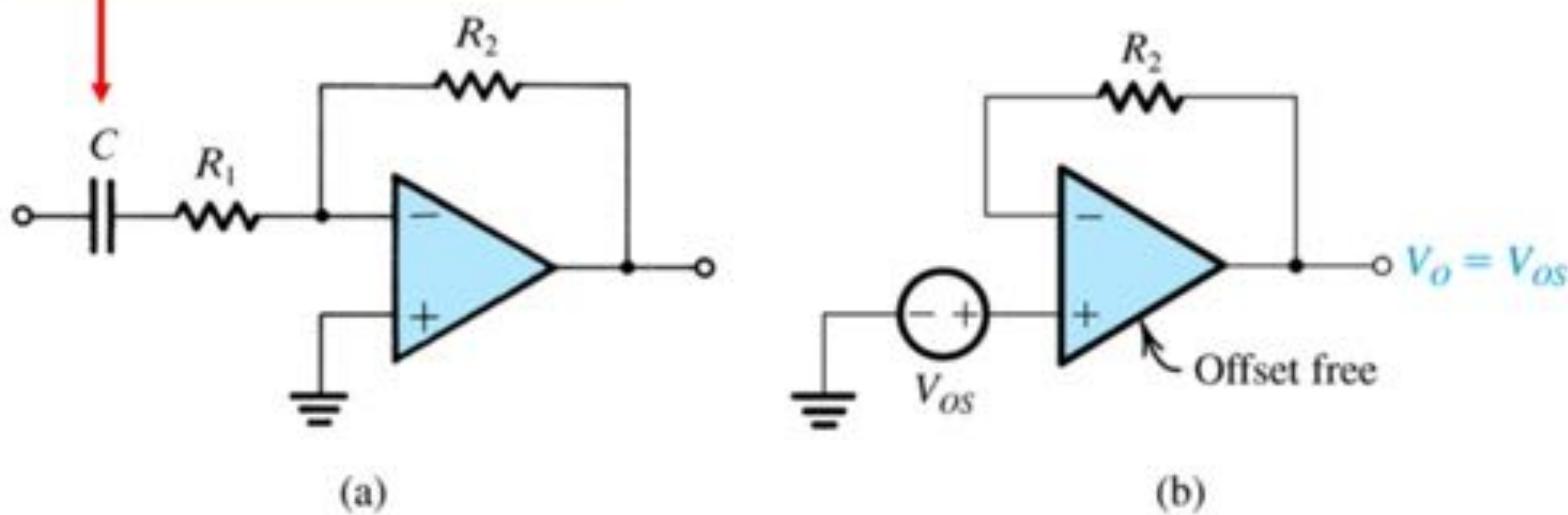
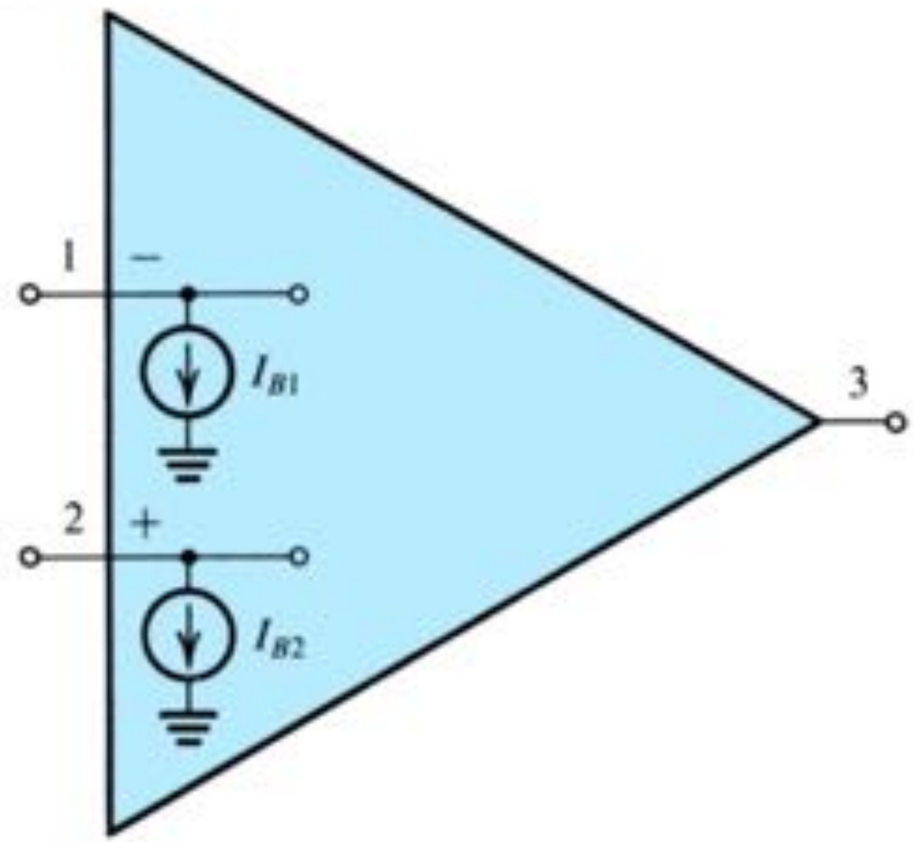
Figure 2.31: (a) A capacitively-coupled inverting amplifier. **(b)** The equivalent circuit for determining its dc output offset voltage V_O.

dc signals cannot pass!



2.6.2. **Input Bias and Offset Currents**

Figure 2.32: The op-amp input bias currents represented by two current sources I_{B1} and I_{B2}.



- **input bias current** - is the dc current which must be supplied to the op-amp inputs for proper operation.
    - Ideally, this current is zero...
- **input offset current** - the difference between bias current at both terminals

input bias current:

$$I_B = \frac{\overbrace{I_{B1} + I_{B2}}^{\text{bias current at terminals \#1 and \#2}}}{2}$$

input offset current:

$$I_{OS} = \overbrace{\left| I_{B1} - I_{B2} \right|}^{\text{difference between bias'}}$$

resulting output voltage:

$$V_{BOut} = I_{B1} R_F$$

2.6.2. Input Bias and Offset Currents

- **Q:** How can this bias be reduced?
  - **A:** Placement of R_3 as additional resistor between non-inverting input and ground.
- **Q:** How is R_3 defined?
  - **A:** Parallel connection of R_F and R_1.

$$R_3 = \underbrace{\frac{R_1 R_F}{R_1 + R_F}}^{\text{resistor placed between non-inverting input and ground } (R_3) \text{ should equal parallel connection of inverting input resistance and feedback}}$$

2.7.1. Frequency Dependence of the Open-Loop Gain

- The differential open-loop gain of an op-amp is not infinite.
- It is finite and decreases with frequency.
- It is high at dc, but falls off quickly starting from 10Hz.

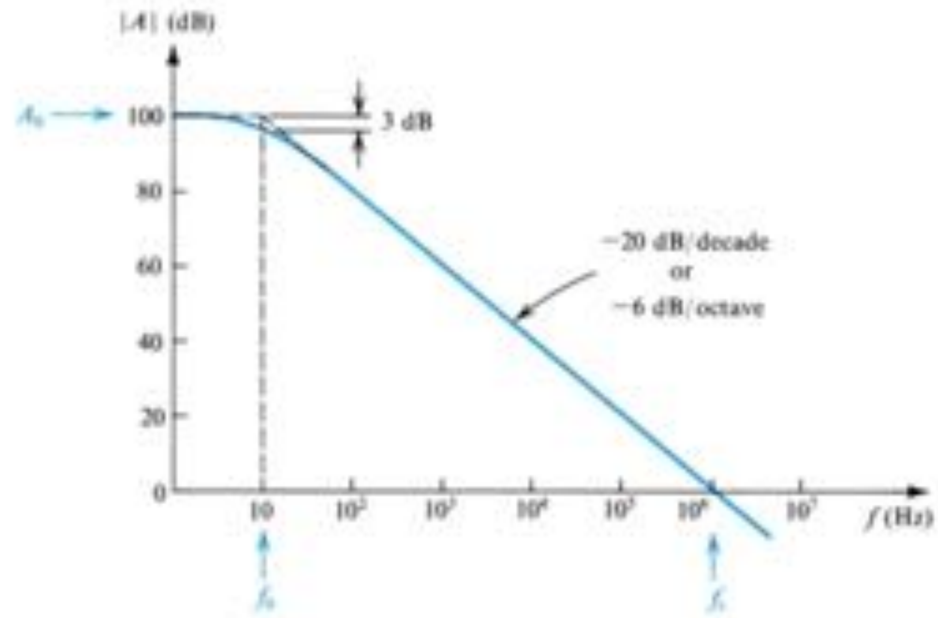


Figure 2.39: Open-loop gain of a typical general-purpose internally compensated op amp.

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2.7.1. **Frequency Dependence of the Open-Loop Gain**

- **internal compensation** – is the presence of internal passive components (caps) which cause op-amp to demonstrate STC low-pass response.
- **frequency compensation** – is the process of modifying the open-loop gain.
  - The goal is to increase stability…

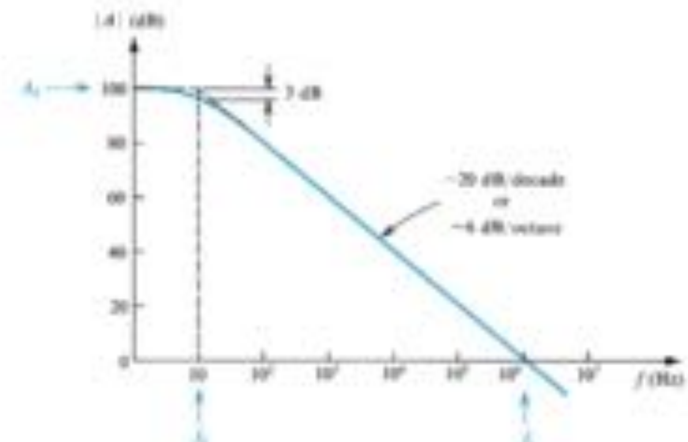

Figure 2.39: Open-loop gain of a typical general-purpose internally compensated op amp.

2.7.1: Frequency Dependence of the Open-Loop Gain

- The gain of an internally compensated op-amp may be expressed as shown below…

transfer function in Laplace domain: $\mathbf{A}(s) = \dfrac{A_0}{1 + s/\omega_b}$

transfer function in frequency domain: $\mathbf{A}(j\omega) = \dfrac{A_0}{1 + j\omega/\omega_b}$

transfer function for high frequencies: $\underbrace{\mathbf{A}(j\omega) \approx \dfrac{A_0\omega_b}{j\omega}}_{\omega_b \text{ is break frequency}}$

magnitude gain for high frequencies: $|\mathbf{A}(j\omega)| \approx \left|\dfrac{A_0\omega_b}{j\omega}\right| \approx \left|\dfrac{\omega_t}{\omega}\right|$

unity gain occurs at ω_t: $\omega_t = A_0\omega_b$

2.7.2. Frequency Response of Closed-Loop Amplifiers

- **Q:** How can we create a more accurate description of closed loop gain for an inverting-type op-amp?
  - **step #1:** Define closed-loop gain of an inverting amplifier with finite open-loop gain (*A*)
  - **step #2:** Insert frequency-dependent description of *A* from last slide
  - **step #3:** Assume $A_0 >> 1 + R_2/R_1$

$$\frac{V_{Out}}{V_{in}} = \frac{-R_2/R_1}{1+(1+R_2/R_1)/\underbrace{(A)}_{\text{open loop gain}}}$$

$$\frac{V_{Out}}{V_{in}} = \frac{-R_2/R_1}{1+\dfrac{1+R_2/R_1}{\underbrace{\left(\dfrac{A_0}{1+s/\omega_0}\right)}_{A\text{ from two slides back}}}} = \frac{-R_2/R_1}{1+\underbrace{\left(\dfrac{1+R_2/R_1}{A_0}\right)(1+s/\omega_b)}_{\textbf{action}\text{: split these terms}}}$$

$$\frac{V_{Out}}{V_{in}} = \frac{-R_2/R_1}{1+\underbrace{\left(\dfrac{1+R_2/R_1}{A_0}\right)}_{\textbf{action}\text{: replace with 0 because } A_0>>1+R_2/R_1}+\underbrace{\dfrac{s}{\omega_b}\left(\dfrac{1+R_2/R_1}{A_0}\right)}_{\textbf{action}\text{: replace with...}}}$$

$$\boxed{\frac{V_{Out}}{V_{in}} = \frac{-R_2/R_1}{1+\dfrac{s(1+R_2/R_1)}{\omega_t}}}$$

solution

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2.7.2. Frequency Response of Closed-Loop Amplifiers

- **Q:** How can we create a more accurate description of closed loop gain for an both inverting and non-inverting type op-amps?

$$\underbrace{\frac{V_{Out}}{V_{In}} = \frac{-R_2/R_1}{1+\dfrac{s(1+R_2/R_1)}{\omega_t}}}^{\text{inverting op amp}} \qquad \overbrace{\frac{V_{Out}}{V_{In}} = \frac{1+R_2/R_1}{1+\dfrac{s(1+R_2/R_1)}{\omega_t}}}^{\text{non-inverting op amp}}$$

2.7.2. Frequency Response of Closed-Loop Amplifiers

- **3*dB* frequency** – is the frequency at which the amplifier gain is attenuated 3*dB* from maximum (aka. dc) value.

$$\omega_{3dB} = \frac{\omega_t}{1 + R_2 / R_1}$$

2.8. **Large-Signal Operation of Op-Amps**

- 2.8.1. **Output Voltage Saturation**
  - If supply is +/- 15*V*, then v_{Out} will saturate around +/- 13*V*.
- 2.8.2. **Output Current Limits**
  - i_{Out} current of op-amp, including that which facilitates feedback, cannot exceed *X*.
  - The book approximates *X* at 20*mA*.

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2.8.3. Slew Rate

- **slew rate** – is maximum rate of change of an op-amp (*V/us*)
- **Q:** How can this be problematic?
  - **A:** If slew rate is less than rate of change of input.

$$\overbrace{SR = \left.\frac{dv_{Out}}{dt}\right|_{\max}}^{\text{slew rate (SR)}}$$

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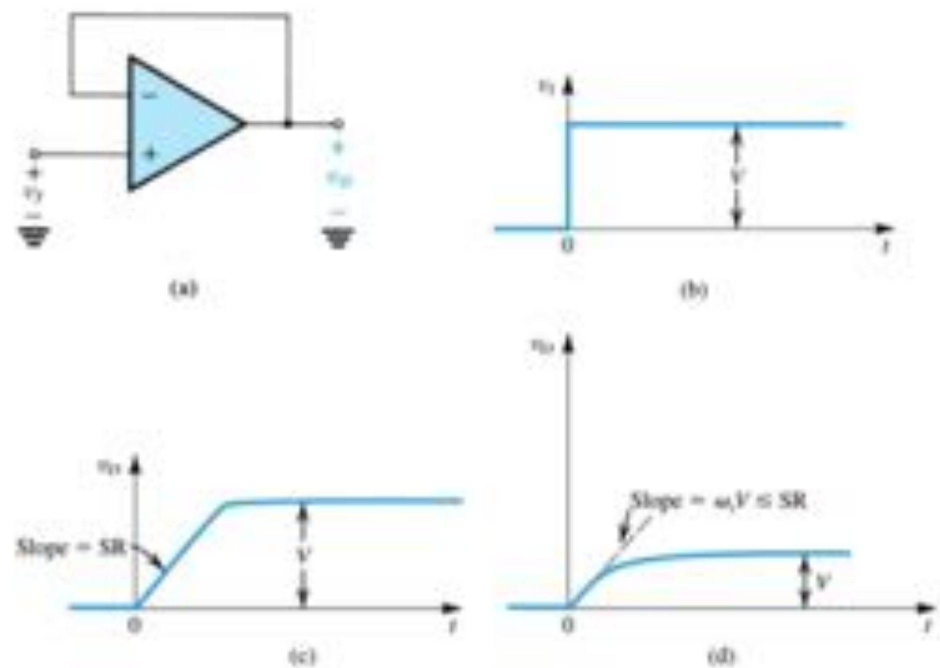
2.8.3. Slew Rate

- **Q:** Why does slewing occur?
  - **A:** In short, the bandwidth of the op-amp is limited – so the output at very high frequencies is attenuated…

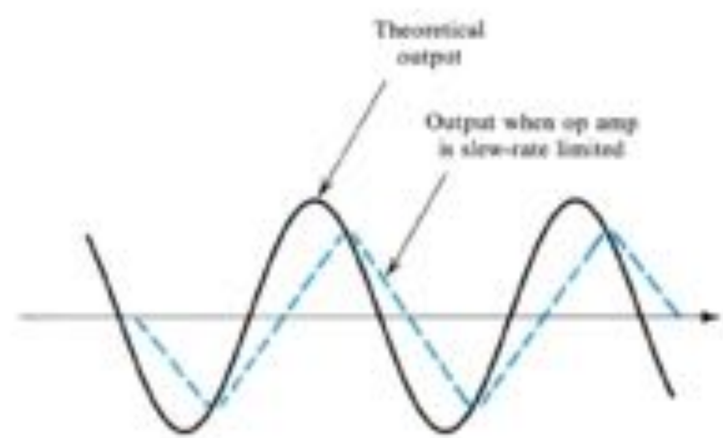


2.8.4. **Full-Power Bandwidth**

- Op-amp slewing will cause nonlinear distortion of sinusoidal waveforms…
  - sine wave
  - rate of change

$$v_{In} = |V_{In}| \sin(\omega t)$$

$$\frac{dv_{In}}{dt} = \omega |V_{In}| \cos(\omega t)$$



2.8.4. Full-Power Bandwidth

- **full-power bandwidth** (f_M) – the maximum frequency at which amplitude of a sinusoidal input and output are equal
- **maximum output voltage** (V_{OutMax}) – is equal to (A^*v_{In})
  - note: an inverse relationship exists between f_M and V_{OutMax}
  - note: beyond ω_M, output may be defined in terms of ω

$$SR = \underbrace{\omega_M}_{\text{FP band.}} \underbrace{V_{OutMax}}_{\text{rated output voltage } = A^*v_{In}}$$

$$\underbrace{f_M = \frac{SR}{2\pi V_{OutMax}}}_{\text{full-power bandwidth}}$$

$$\underbrace{V_{Out} = V_{OutMax} \overbrace{\left(\frac{\omega_M}{\omega}\right)}^{\text{this value cannot be greater than one}}}_{\text{relationship between actual output and maximum}}$$

Conclusion

- The IC op-amp is a versatile circuit building block. It is easy to apply, and the performance of op-amp circuits closely matches theoretical predictions.
- The op-amp terminals are the inverting terminal (1), the non-inverting input terminal (2), the output terminal (3), the positive-supply terminal (4) to be connected to the positive power supply (V_{CC}), and the negative-supply terminal (5) to be connected to the negative supply ($-V_{EE}$).

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Conclusion (2)

- The ideal op-amp responds only to the difference input signal, that is (v_2 - v_1). It yields an output between terminals 3 and ground of $A(v_2 - v_1)$. The open-loop gain (A) is assumed to be infinite. The input resistance (R_{in}) is infinite. The output resistance (R_{out}) is assumed to be zero.
- Negative feedback is applied to an op-amp by connecting a passive component between its output terminal and its inverting (aka. negative) input terminal.

Conclusion (3)

- Negative feedback causes the voltage between the two input terminals to become very small, and ideally zero. Correspondingly, a virtual short is said to exist between the two input terminals. If the positive input terminal is connected to ground, a virtual ground appears on the negative terminal.

Conclusion (4)

- The two most important assumptions in the analysis of op-amp circuits, assuming negative feedback exists, are:
  - the two input terminals of the op-amp are at the same voltage potential.
  - zero current flows into the op-amp input terminals.
- With negative feedback applied and the loop closed, the gain is almost entirely determined by external components: $V_o/V_i = -R_2/R_1$ or $1+R_2/R_1$.

Conclusion (5)

- The non-inverting closed-loop configuration features a very high input resistance. A special case is the unity-gain follower, frequently employed as a buffer amplifier to connect a high-resistance source to a low-resistance load.
- The difference amplifier of Figure 2.16 is designed with $R_4/R_3 = R_2/R_1$, resulting in $v_o = (R_2/R_1)(v_{I2} - v_{I1})$.

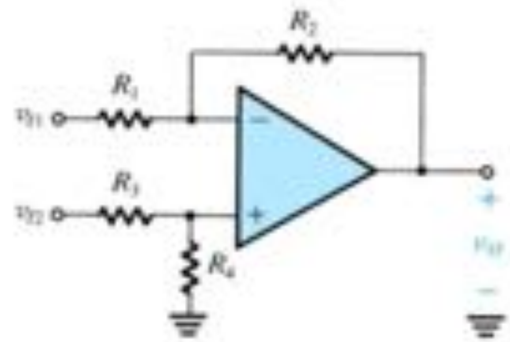

Figure 2.16

Conclusion (6)

- The instrumentation amplifier of Figure 2.20(b) is a very popular circuit. It provides $v_o = (1+R_2/R_1)(R_4/R_3)(v_{I2} - v_{I1})$. It is usually designed with $R_3 = R_4$ and R_1 and R_2 selected to provide the required gain. If an adjustable gain is needed, part of R_1 can be made variable.

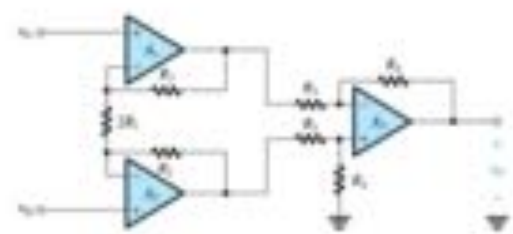

Figure 2.20(b)

Conclusion (7)

- The inverting Miller Amplifier of Figure 2.24 is a popular circuit, frequently employed in analog signal-processing functions such as filters (Chapter 16) and oscillators (Chapter 17).
- The input offset voltage (V_{OS}) is the magnitude of dc voltage that when applied between the op-amp input terminals, with appropriate polarity, reduces the dc offset at the output.

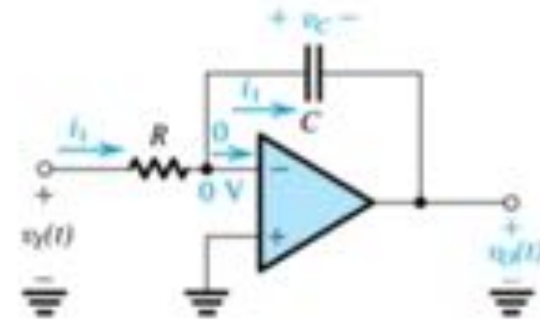

Figure 2.24