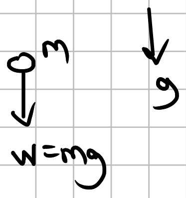
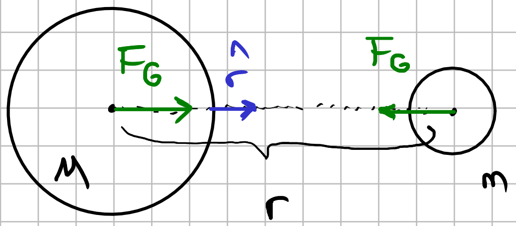
Ch #13 / Gravity:

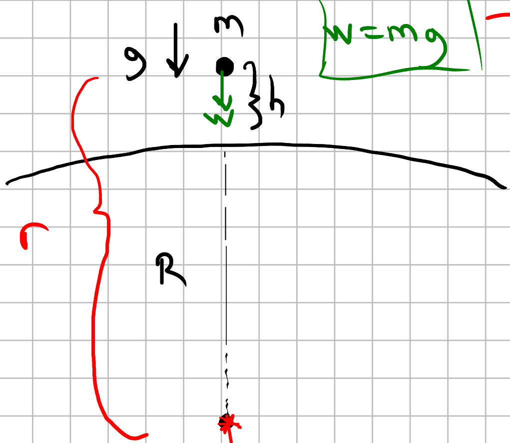


Newton's Law of Gravity:



$$F_G = -G \cdot \frac{Mm}{r^2}$$ (− : attractive force)

$$\vec{F}_G = -G \frac{Mm}{r^2} \hat{r}$$ $\hat{r}$: unit vector $|\hat{r}| = 1$



$\boxed{w = mg}$ → true only when you are very close to the surface.

$$F_G = -G \frac{Mm}{r^2}$$ $r = R + h$

$h \ll R \rightarrow \boxed{r = R}$

weight / g near surface: $W = F_G = \frac{-GM\cancel{m}}{R^2} = \cancel{m} \cdot g$

$$\Rightarrow \quad g = \frac{-GM}{R^2}$$

M : mass of earth

R : radius of earth

G : gravitational constant

$G = 6.67 \times 10^{-11}\ m^3/kg\,s^2$ $\quad M = 5.97 \times 10^{24}\ kg$ $\quad R = 6371\ km$

$= 6371 \times 10^3\ m$

$$g=\frac{-6.67\times10^{-11}\times5.97\times10^{\cancel{24}\,18}}{(6371)^2\times\cancel{10^6}}=-10^7\times\frac{6.67\times5.97}{6371^2}=\underbrace{-9.81\ m/s^2}$$

$g_{moon}=?$

$$g_{moon}=\frac{-G\cdot M_{moon}}{R_{moon}^2}$$

$m_{moon}=7.34\times10^{22}\ kg$

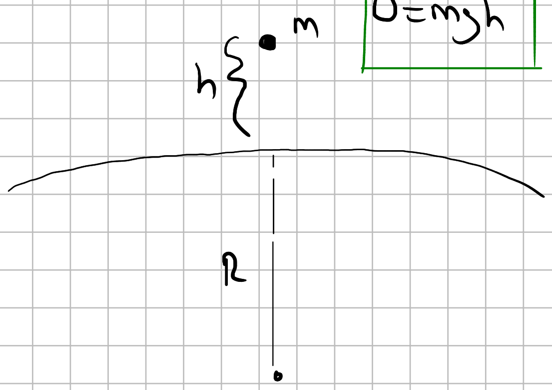
$R_{moon}=1737\ km=1737\times10^3\ m$

$$g_{moon}=\frac{-6.67\times10^{-11}\times7.34\times10^{\cancel{22}\,16}}{(1737)^2\times\cancel{10^6}}=-10^5\times\frac{6.67\times7.34}{1737^2}=\underbrace{-1.62\ m/s^2}$$

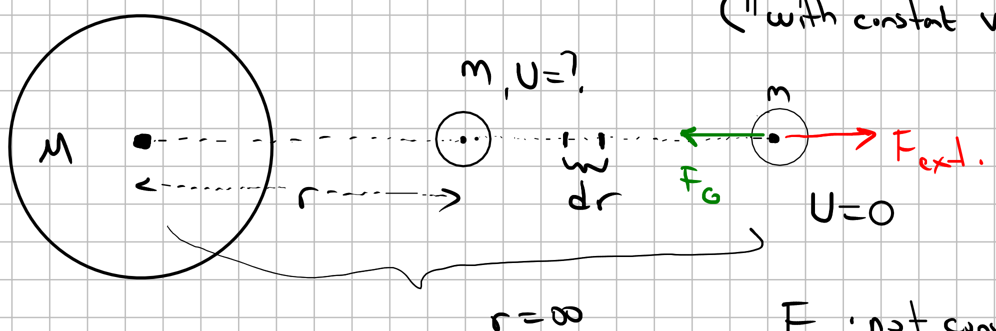
$$\Delta y=V_0\Delta t+\frac{1}{2}g_{moon}\Delta t^2\ \ldots$$

Gravitational potential energy

$\boxed{U=mgh}$ → ONLY when you are very close to the surface.



* potential energy: the amount of necessary work done by an external force when moving the object from one location to another ("with constant velocity")



F_G: not constant

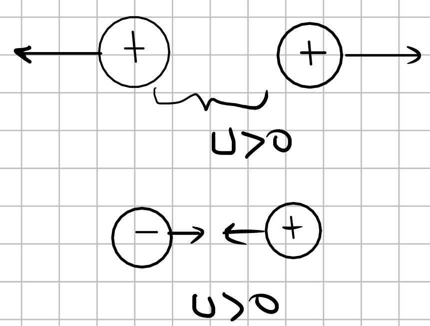
$$U = W_{ext} = \int \underbrace{+G\frac{Mm}{r^2}}_{F} \cdot dr \qquad \left\{ W = \int F \cdot dx \right\} \qquad \left\{ F = -F_G \right\}$$

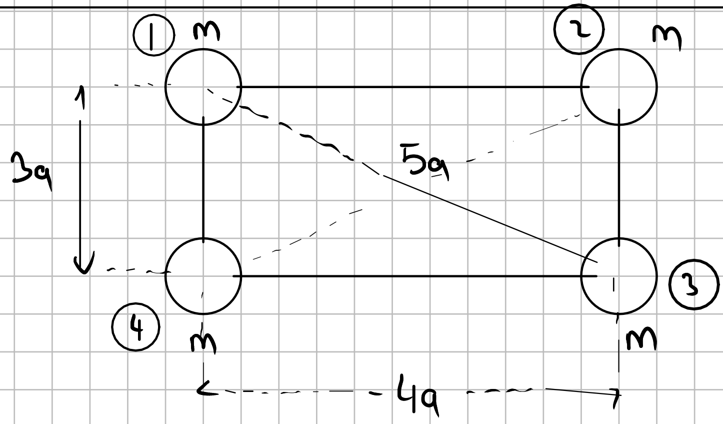
$$U = W_{ext} = +GMm \int_{\infty}^{r} \frac{1}{r^2} \cdot dr \qquad \left\{ \int r^{-2} \cdot dr = \frac{r^{-2+1}}{-2+1} = \frac{-1}{r} \right\}$$

$$\Rightarrow U = +GMm\left(\frac{1}{r}\Big|_{\infty}^{r}\right) = +GMm\left(\underbrace{\cancel{\frac{1}{\infty}}}_{0} - \frac{1}{r}\right)$$

$$\Rightarrow \boxed{U = -G\frac{Mm}{r}} \rightarrow \text{Negative potential}$$

{gravity is attractive}





(G, m, a) given $\longrightarrow U = ?$

$$U = U_{12} + U_{23} + U_{34} + U_{41} + U_{13} + U_{24}$$

$$U_{12} = U_{34} = -G\frac{m^2}{4a}$$

$$U_{41} = U_{23} = -G\frac{m^2}{3a}$$

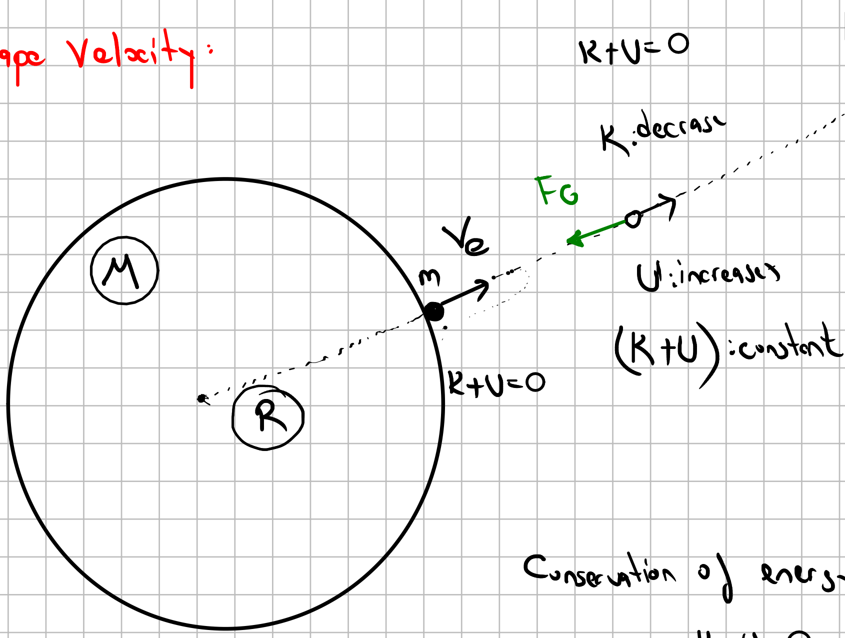
$$U_{13} = U_{24} = -G\frac{m^2}{5a}$$

$+$

$$\Rightarrow = -G\frac{m^2}{a}\left[\underset{15}{\frac{1}{4}} + \underset{20}{\frac{1}{3}} + \underset{12}{\frac{1}{5}}\right] = -G\frac{m^2}{a} \cdot \frac{47}{60}$$

$$\boxed{U = -G\frac{m^2}{a}\left(\frac{47}{30}\right)} \checkmark$$

Escape Velocity:



V_e: is the minimum velocity in such a way that the projectile never comes back.

Conservation of energy $(K+U)$: constant

$K+U=0$

initially $K+U=0$ $\quad K=\frac{1}{2}mV_e^2 \quad U=-G\frac{Mm}{R}$

$$K+U=\frac{1}{2}mV_e^2-G\frac{Mm}{R}=0 \Rightarrow \frac{1}{2}\cancel{m}V_e^2=G\frac{M\cdot \cancel{m}}{R}$$

$$\Rightarrow V_e^2=\frac{2GM}{R} \qquad \Rightarrow V_e=\sqrt{\frac{2GM}{R}}$$

$G=6.67\times10^{-11}\, m^3/kg\,s^2$

$R=6371\,km=6371\times10^3\,m.$

$M=5.97\times10^{24}\,kg$

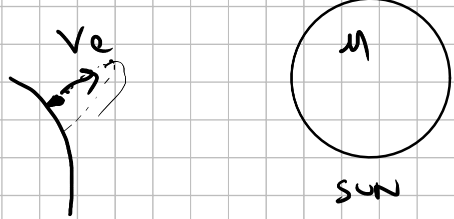
$$V_e^{(earth)}=\left(\frac{2\times6.67\times10^{-11}\times5.97\times10^{\cancel{24}\,21}}{6371\times\cancel{10^3}}\right)^{1/2}$$

$$=\left(10^{10}\times\frac{2\times6.67\times5.97}{6371}\right)^{1/2}=11\,180\ m/s \quad \boxed{11{,}18\ km/s}$$

$V_e^{(mars)}\simeq 5\,km/s$ etc. ...

Black holes:

$V_e >$ speed of light



c: speed of light

$$c=\sqrt{\frac{2GM}{R}}$$

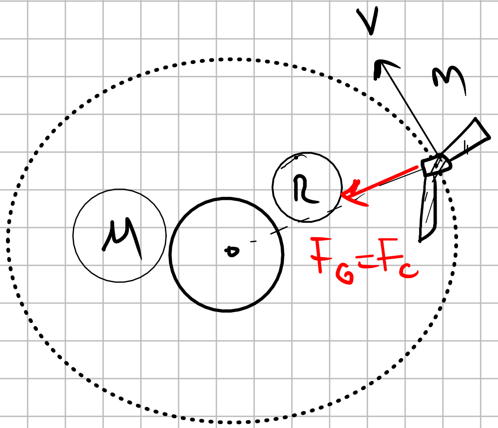
M: mass of sun $= 1{,}98\times10^{30}$ kg

R: radius of the sun $= 696340$ km

$c = 300000$ km/s

$\Rightarrow \underbrace{R}\; = 3$ km   R: schwartschild radius.

Satellite Motion / Keppler's (3rd) Law.



⊛ period $T=?$ in terms of $M,R=?$

F_c: centripetal force.

$$F_G=+G\frac{M\cancel{m}}{R^{\cancel{2}}}=+\cancel{m}\frac{V^2}{\cancel{R}}=F_c$$

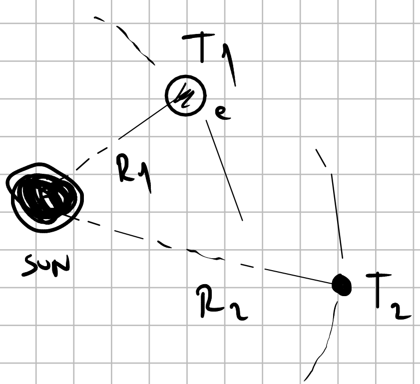
$$\Rightarrow V=\left(G\frac{M}{R}\right)^{1/2}$$

$$V\cdot T=2\pi\cdot R$$

$$T=\frac{2\pi R}{V}=2\pi\left(\frac{R^2\cdot R}{GM}\right)^{1/2}$$

$$T=2\pi\sqrt{\frac{R^3}{MG}}\Rightarrow T^2=4\pi^2\frac{R^3}{MG}\Rightarrow\frac{R^3}{T^2}=\frac{MG}{4\pi^2}\text{ : constant}$$

↳ {Keppler's ? Law}



$$\boxed{\frac{R_1^3}{T_1^2}=\frac{R_2^3}{T_2^2}}$$