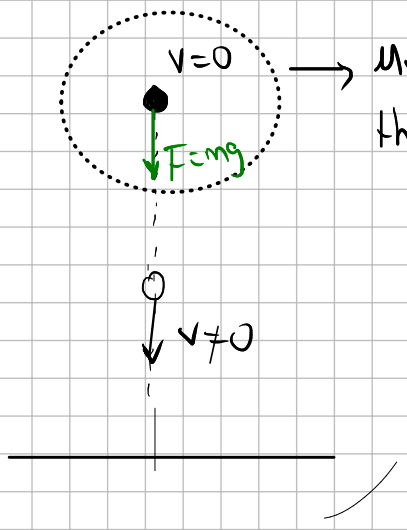
Impulse & Momentum:

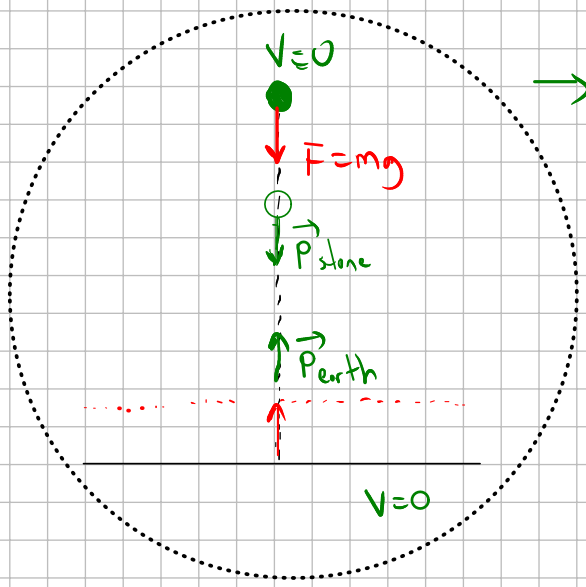
$\vec{J} = \vec{F}\Delta t$ $\quad \boxed{\vec{J} = \Delta\vec{p}} \quad (\Delta\vec{p} = \vec{p}_{final} - \vec{p}_{initial}) \quad (\vec{p} = m\vec{v})$

Consider no net force applies on a **system** $\Rightarrow \Delta\vec{p} = 0 \Rightarrow \vec{p}_{final} = \vec{p}_{initial}$: $\vec{p}$ is conserved

A System: can be only 1 object, or a set of multiple objects....



→ My system is the stone only → It's momentum is NOT conserved $\xrightarrow{?}$ a net force is applied on it

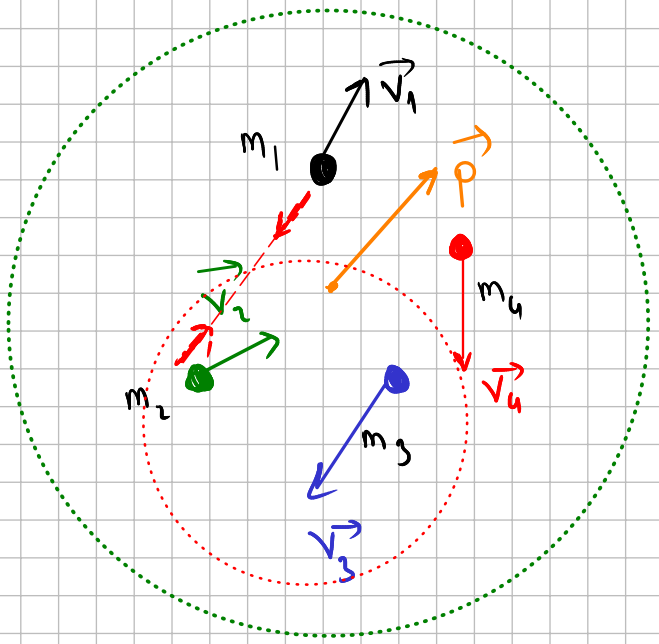


→ My system is earth + stone

⇒ No net force is acting on the system

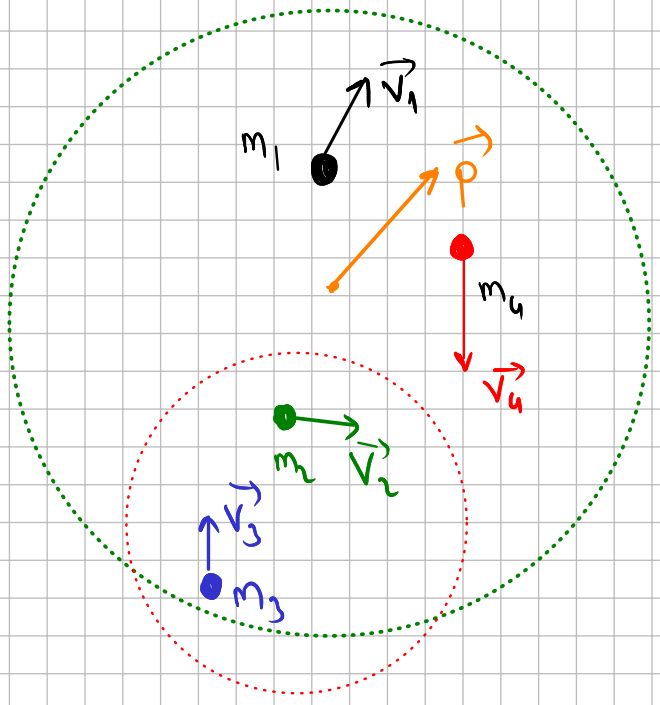
⇒ Momentum of the system is conserved.

Total Momentum:



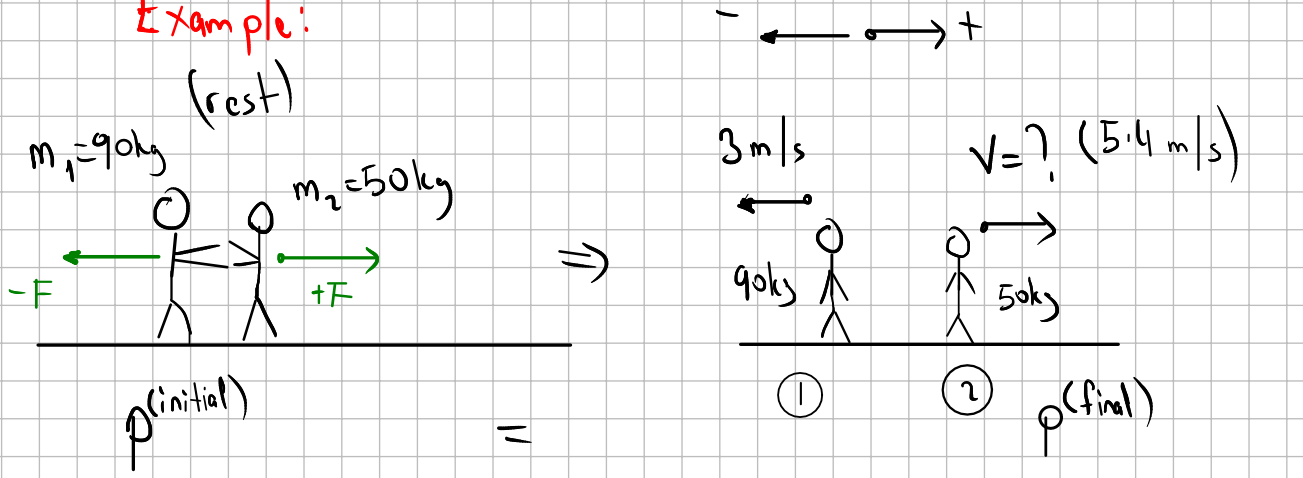
$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4 + \cdots$$

$$= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4 + \cdots$$



$\vec{p}$: will stay the same

Example:



$p^{(initial)} = p_1 + p_2$

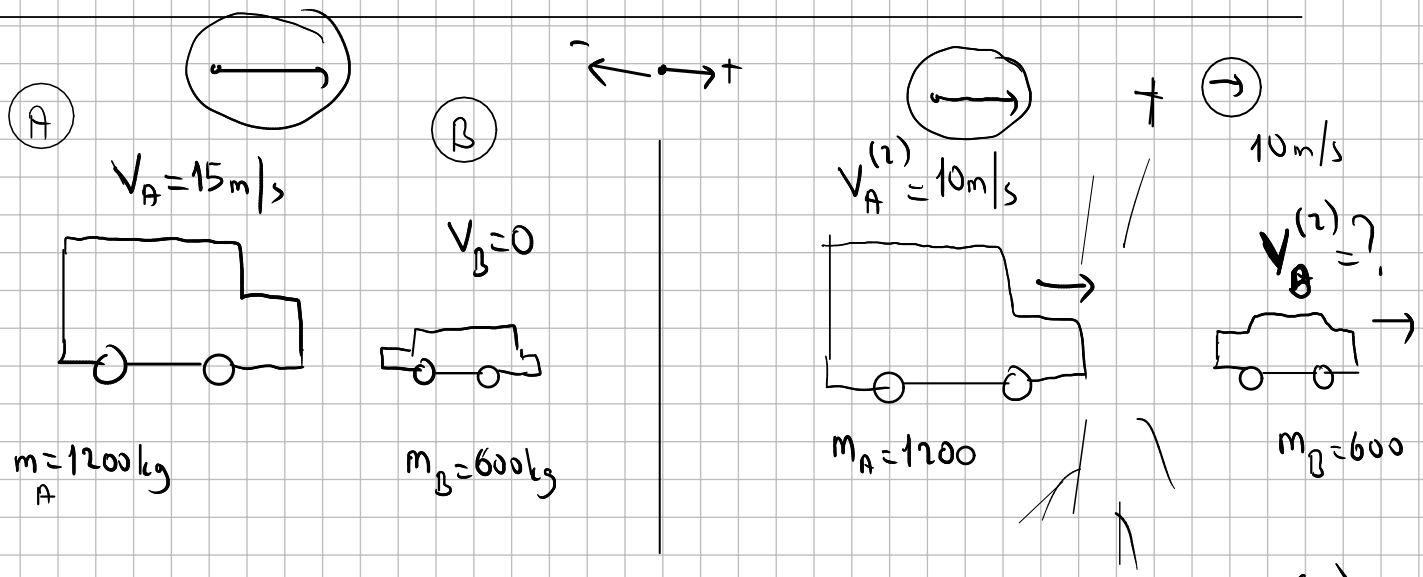
$= 90.0 + 50.0$

$p^{(initial)} = 0$

$p^{(final)} = p_1 + p_2$

$= 90(-3) + 50(V)$

$\Rightarrow \quad -270 + 50V = 0 \Rightarrow \boxed{V = 5.4 \text{ m/s}}$



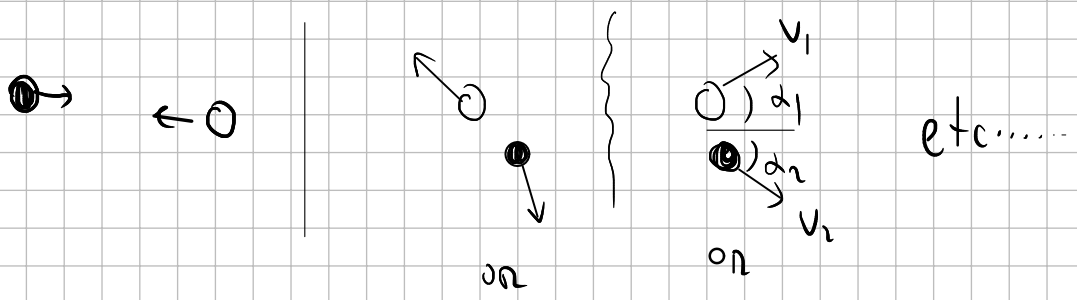
$p_{initial} = (1200)(15) + 0$

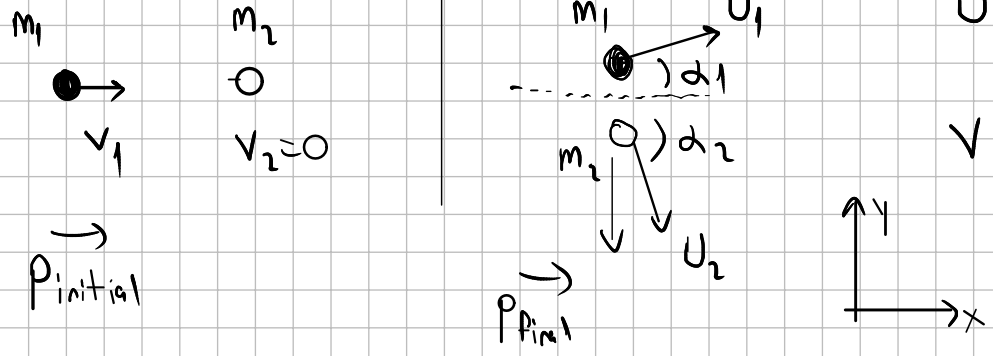
$= 18000 \text{ kg m/s}$

$p_{final} = (1200)\cdot(10) + (600)\cdot V_B^{(2)}$

$18000 = 12000 + 600 \cdot V_B^{(2)} \Rightarrow 6000 = 600 \cdot V_B^{(2)} \Rightarrow \boxed{V_B^{(2)} = 10 \text{ m/s}}$

TWO DIMENSIONAL COLLISION PROBLEMS





U: after collision

V: before collision.

$\vec{p}_{initial}$ $\vec{p}_{final}$

Before After.

$p_{initial_x} = p_{final_x}$

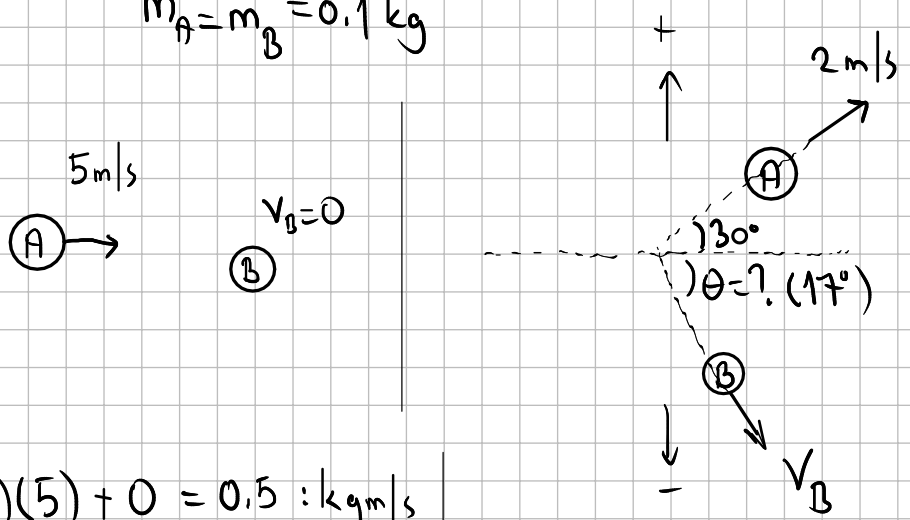
$p_{initial_y} = p_{final_y}$

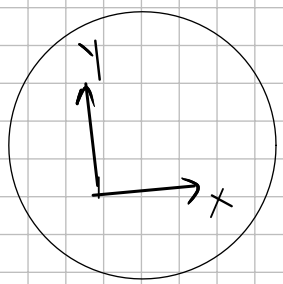
m_1
m_2
V_1
U_1
U_2
α_1
α_2

$x: m_1 V_1 = p_x$	$x: m_1 U_1 \cdot \cos\alpha_1 + m_2 \cdot U_2 \cos\alpha_2$
$y: 0 = p_y$	$y: m_1 U_1 \sin\alpha_1 - m_2 U_2 \sin\alpha_1$

$\downarrow$

$m_A = m_B = 0.1\ kg$





$V_B=?$

$\theta=?$

$p_x = (0.1)(5) + 0 = 0.5\ kgm/s$

$p_y = 0$

$p_x = (0.1)(2\cos 30) + (0.1)(V_B \cos\theta)$

$p_y = (0.1)(2 \cdot \sin 30) - (0.1)(V_B \sin\theta)$

$0.5 = 0.2\cos 30 + 0.1 V_B \cos\theta$ (1)

$\boxed{0 = 0.2\sin 30 - 0.1 V_B \sin\theta}$ (2)

$\sin 30^\circ = 0.5$

$\cos 30^\circ = \sqrt{3}/2$

$0.2\sin 30 = 0.1 V_B \sin\theta \Rightarrow \boxed{V_B = \frac{2\sin 30}{\sin\theta}} \Rightarrow V_B = \frac{1}{\sin\theta}$

$0.5 = 0.2\cos 30 + 0.1 \cdot \frac{1}{\sin\theta} \cdot \cos\theta$

$5 = 2\cos 30 + \cot\theta \Rightarrow \cot\theta = 5 - 2\cos 30 = (5 - \sqrt{3})$

$\cot\theta = 3.27 \quad \tan\theta = \frac{1}{3.27} \quad \theta = \tan^{-1}\left(\frac{1}{3.27}\right) \approx 17^\circ$

$V_B = \frac{1}{\sin(17^\circ)} \approx \frac{1}{0.29} \approx \boxed{3.42\ m/s}$

Types of Collisions:

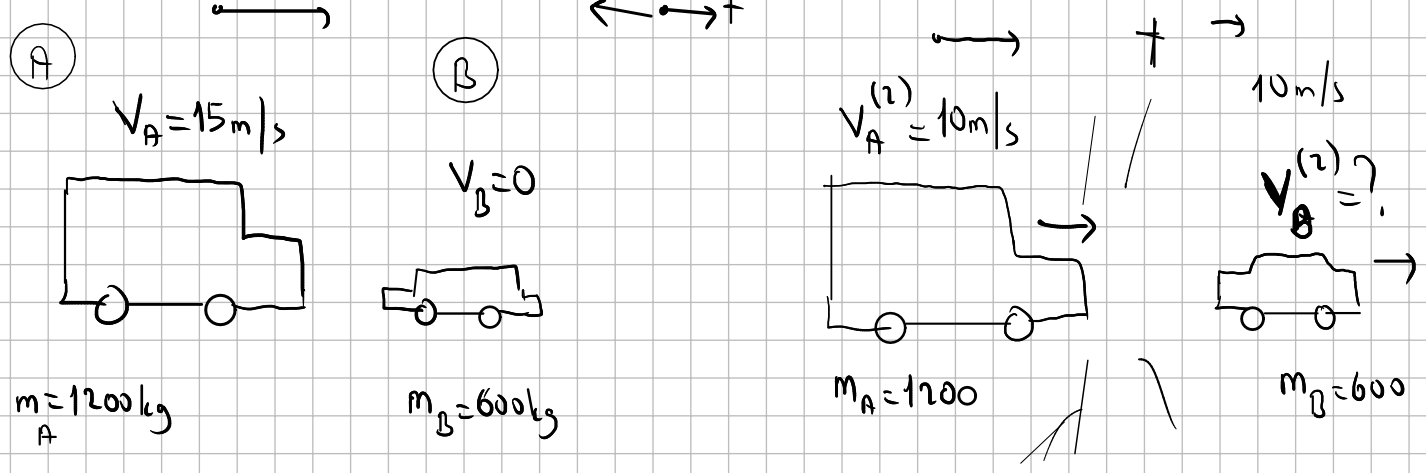
1) Elastic Collision: Mechanical energy is conserved.

2) Inelastic Collision: Mechanical energy is NOT conserved.

↓

perfectly inelastic collision: Maximum loss of energy.

(when objects stick together and move as one after collision)



Now calculate energy loss:

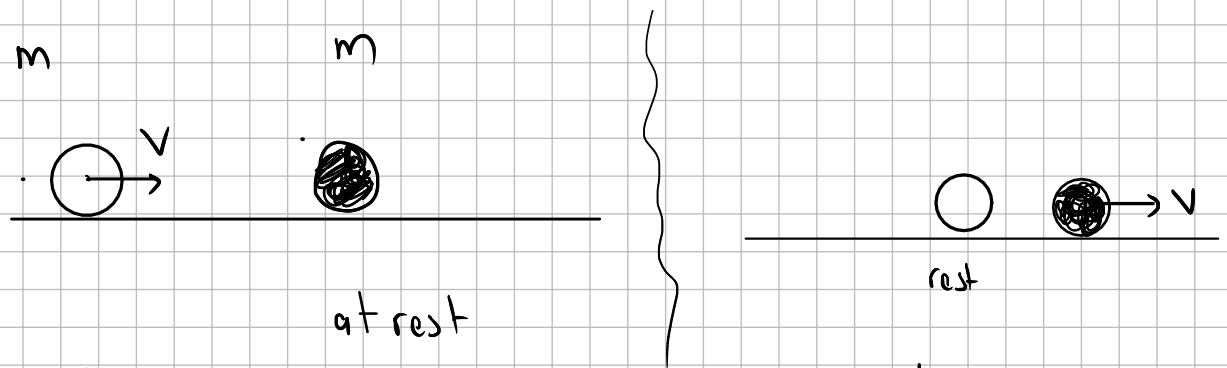
$$E_1 = \frac{1}{2}(1200)\cdot 15^2$$

135000 Joules

$$E_2 = \frac{1}{2}(1200)\cdot 10^2 + \frac{1}{2}(600)\cdot 10^2$$

$60000 + 30000 = 90000\,J$

$$\Delta E = -40000 \text{ Joules}$$



$$E_1 = \frac{1}{2}mV^2$$

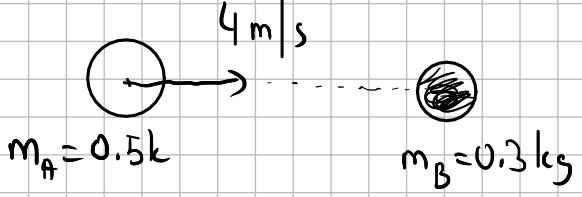
$p = mV$

$$E_2 = \frac{1}{2}mV^2$$

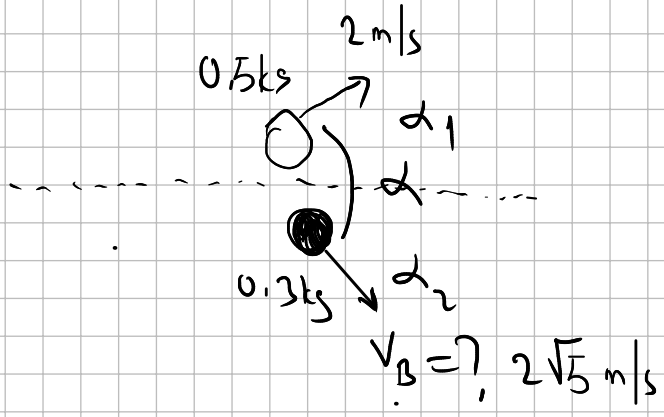
$p = mV$

$\alpha = \alpha_1 + \alpha_2$

(1)



(2)



* Given that the collision is elastic

$V_B = ?$ $\quad \alpha = ?$

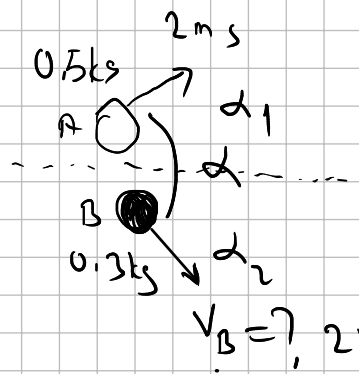
$$E_1 = E_2 \Rightarrow \frac{1}{2}(0.5)\cdot 16 = 4J = E_1$$

$$E_2 = \frac{1}{2}(0.5)\cdot 4 + \frac{1}{2}(0.3)\cdot V_B^2 = 1 + \frac{0.3}{2} V_B^2 = 4$$

$$\Rightarrow \frac{0.3}{2} V_B^2 = 3 \Rightarrow V_B^2 = 20 \qquad V_B = 2\sqrt{5}\ m/s$$

↑ conservation of energy

↓ conservation of momentum.



$P_y = 0$ before/after.

$$(0.5)(2)\cdot \sin\alpha_1 = (0.3)(2\sqrt{5})\cdot \sin\alpha_2$$

$$\sin\alpha_1 = \frac{3}{10}\cdot\sqrt{20}\,\sin\alpha_2 = \sqrt{\frac{9\cdot 20}{100}}\cdot\sin\alpha_2 = \boxed{\frac{3}{\sqrt{5}}\sin\alpha_2 = \sin\alpha_1}$$

$$p_x = (0.5)(4) = 2 = (0.5)(2)\cos\alpha_1 + (0.3)(2\sqrt{5})\cos\alpha_2$$

$$2 = \cos\alpha_1 + \sqrt{\frac{9}{100}\cdot\frac{4.5}{5^2}}\cos\alpha_2 = \boxed{\cos\alpha_1 + \frac{3}{\sqrt{5}}\cos\alpha_2}$$

$$\boxed{\frac{3}{\sqrt{5}}\sin\alpha_2 = \sin\alpha_1}$$

$$\boxed{\cos\alpha_1 + \frac{3}{\sqrt{5}}\cos\alpha_2}$$

$\rightarrow \alpha_1 = ?, \quad \alpha_2 = ?, \quad (\alpha_1 + \alpha_2 = ?)$

$$\cos^2 x + \sin^2 x = 1$$