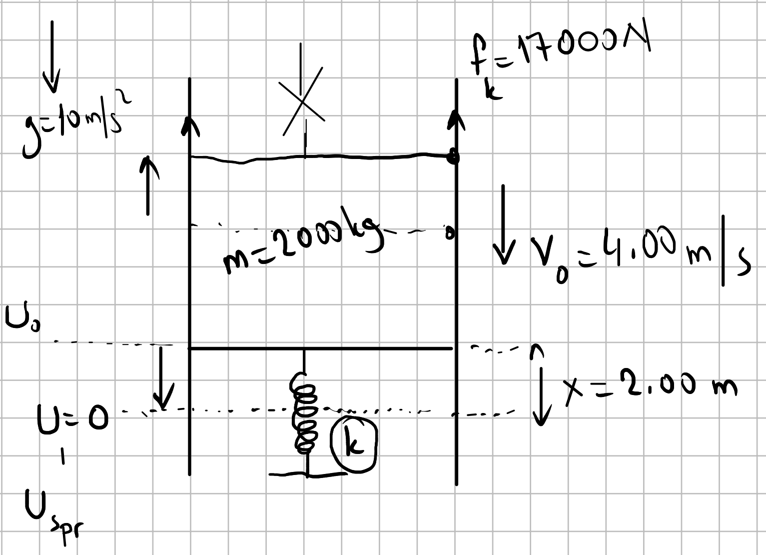
Energy Conservation:

$\Delta E = 0 \qquad E = K + U$

$$\boxed{\Delta E = W_{OTHER}}$$

W_{OTHER} : work done by other (non-conservative or external) forces.

Example:



* find the spring constant "k" (N/m)

Before	After
$K_0 + U_0 = E_0$	$U_{spring} = E_1$

$W_{friction} \neq 0$

$$E_0 = \frac{1}{2}(2000)\cdot 4^2 + (2000)(10)(2) = 16000 + 40000 = 56000 \text{ Joules (J)}$$

$$W_{friction} = -17000(2) = -34000 \text{ Joules}$$

$$E_1 = U_{spring} = \frac{1}{2}kx^2 = (2\cdot k) \text{ : Joules.}$$

$$\Delta E = 2k - 56000 = -34000 \text{ Joules}$$

$$2k = (56-34)\cdot 1000 \Rightarrow \boxed{k = 11000\,N/m}$$

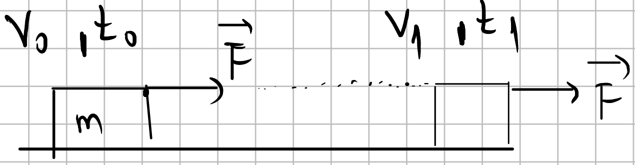
(*) How to take non-conservative / other forces into account?

$\Delta E = W_{OTHER}$

Ch #8 : Impulse momentum :

$$\vec{F} = m\vec{a} \quad , \quad a = \frac{dv}{dt} \quad V = \frac{dx}{dt}$$

$$\left.\begin{aligned} F_x &= m a_x \\ F_y &= m a_y \\ F_z &= m a_z \end{aligned}\right\}$$



$$F = m \cdot \frac{dv}{dt} \Rightarrow F \cdot dt = m \cdot dV$$

$$\Rightarrow \int_{t_0}^{t_1} F dt = \int_{V_0}^{V_1} m dV$$

$$\Rightarrow F \int_{t_0}^{t_1} dt = m \int_{V_0}^{V_1} dV \Rightarrow F(t_1 - t_0) = m(V_1 - V_0)$$

$$\Rightarrow \underbrace{F \cdot \Delta t}_{\text{impulse}} = \underbrace{mV_1}_{\text{momentum}} - \underbrace{mV_0}_{\text{momentum}}$$

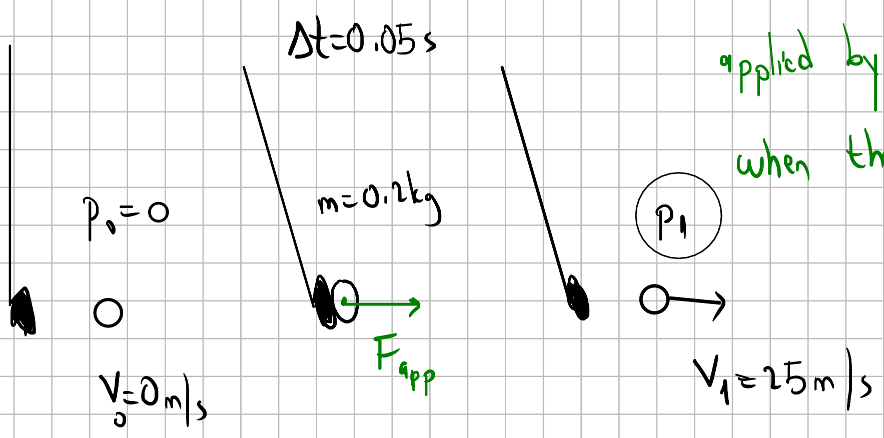
Define: $\vec{J} = \vec{F} \cdot \Delta t$ (impulse)

Define: $\vec{p} = m \cdot \vec{V}$; $kg \frac{m}{s}$

$$\boxed{\vec{J} = \Delta \vec{p}}$$

$\vec{J}$, $\vec{p}$ are vectorial quantities with S.I unit of kg m/s

Example:



* Calculate the force applied by golf stick when they're in contact?

$$F\Delta t = J = p_1 - p_0 = \Delta p$$

$$p_1 = (0.2)(25) = 5\ kgm/s$$

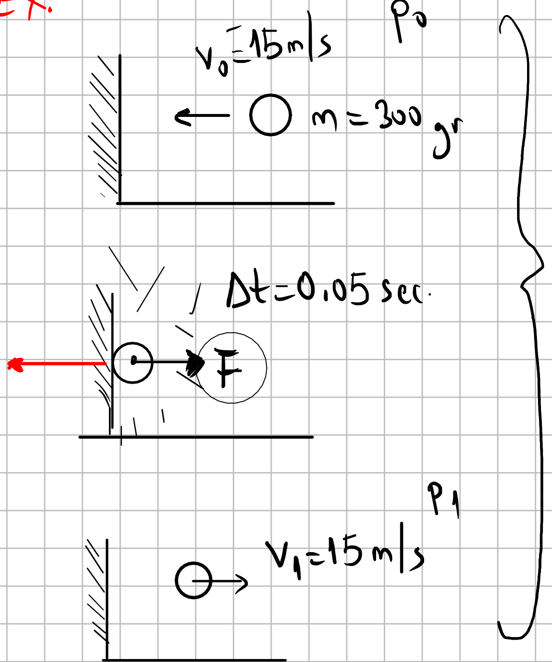
$$p_0 = 0$$

$$\Delta p = p_1 - p_0 = 5\ kgm/s$$

$$J = F_{app} \cdot (0.05) = 5 \Rightarrow \boxed{F_{app} = 100N}$$

$$-x \longleftrightarrow +x$$

Ex:



Find the average force F applied by the wall in "Δt".

$$p_0 = (0.3)(-15) = -4.5\ kgm/s$$

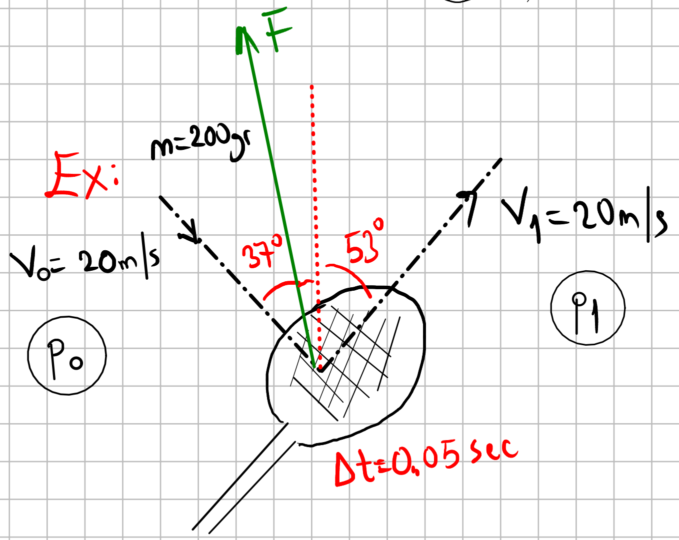
$$p_1 = (0.3)(15) = 4.5\ kgm/s$$

$$J = F\Delta t = \Delta p \qquad \Delta p = (4.5) - (4.5) = 9\ kgm/s$$

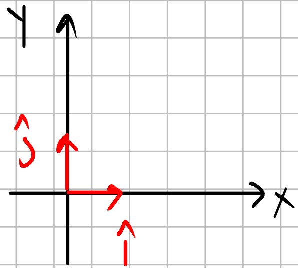
$$\Rightarrow \quad F\cdot(0.05) = 9 \quad \Rightarrow \quad \boxed{F = 180\,N}$$

$$\vec{p} = m\vec{V}$$

Ex:



* Given initial and final velocities, calculate the average force applied by the racket. ($\vec{F}$ in unit vector notation)



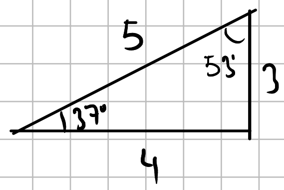
$$\vec{J} = \vec{F}\cdot\Delta t = \Delta\vec{p} = \underbrace{\vec{p}_1}_{} - \underbrace{\vec{p}_0}_{}$$

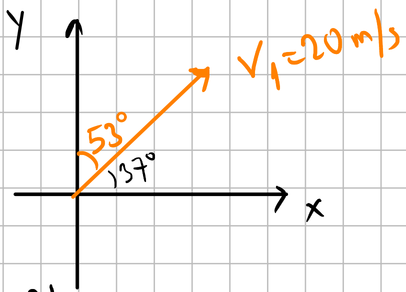
$\cos 37° = 0.8$

$\sin 37° = 0.6$

$\cos 53° = 0.6$

$\sin 53° = 0.8$

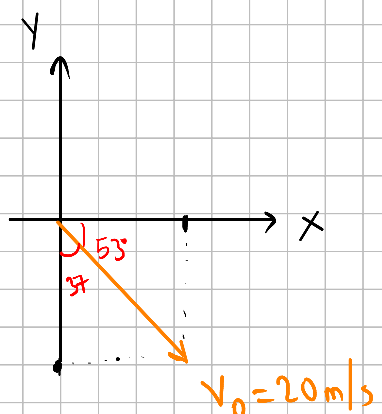




m=0.2kg

$$\vec{V}_1 = 16\hat{i} + 12\hat{j} \; : m/s \qquad \vec{p}_1 = m\vec{V}_1 = 0.2(16\hat{i} + 12\hat{j})$$

$$\vec{p}_1 = [3.2\hat{i} + 2.4\hat{j}] \; kgm/s$$



$$\vec{V}_0 = 12\hat{i} - 16\hat{j} \; : m/s$$

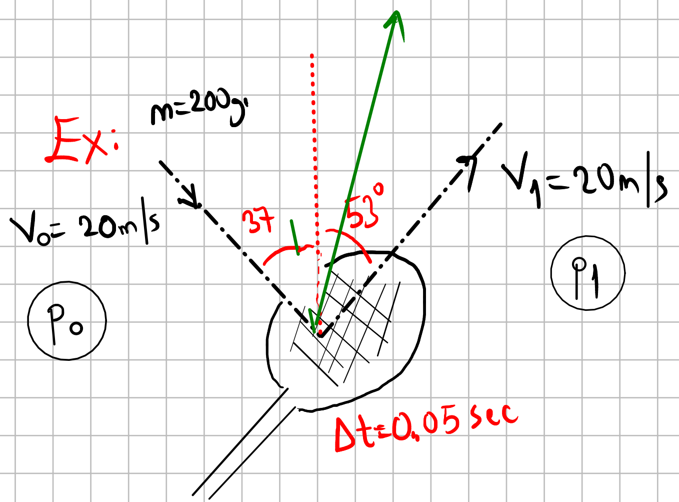
$$\vec{p}_0 = (0.2)[12\hat{i} - 16\hat{j}] = 2.4\hat{i} - 3.2\hat{j} \; : kgm/s$$

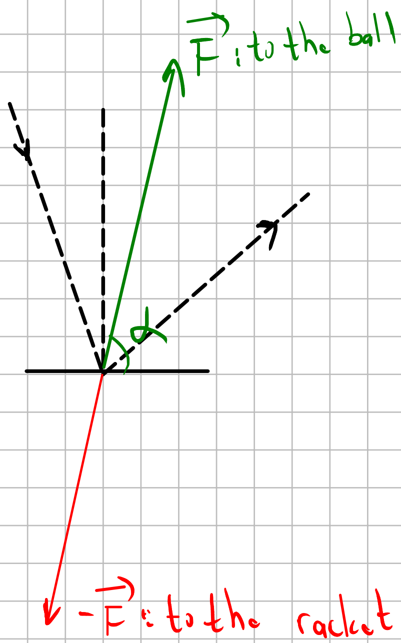
$$\Delta\vec{p} = \vec{p}_1 - \vec{p}_0 = (3.2\hat{i} + 2.4\hat{j}) - (2.4\hat{i} - 3.2\hat{j}) \; : kgm/s$$

$\vec{\Delta p} = 0.8\hat{i} + 5.6\hat{j}$; kgm/s

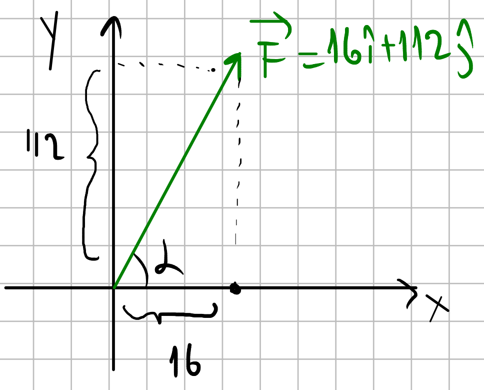
$$\vec{J} = \vec{\Delta p} = \vec{F}\cdot\Delta t \Rightarrow \vec{F} = \frac{\vec{\Delta p}}{\Delta t} \quad (\Delta t = 0.05 \text{ sec})$$

$$\vec{F} = \frac{(0.8\hat{i} + 5.6\hat{j})}{0.05} \Rightarrow \vec{F} = [16\hat{i} + 112\hat{j}] : \text{Newton.}$$





calculate the angle of the force that is applied to the tennis ball (α = ? with horizontal)



$$\tan\alpha = \frac{112}{16} = 7 \qquad \alpha = \tan^{-1}(7)$$

$$= \arctan(7)$$

$$= \text{atan}(7)$$

$\vdots$

$$\Rightarrow \alpha \approx 82^\circ$$

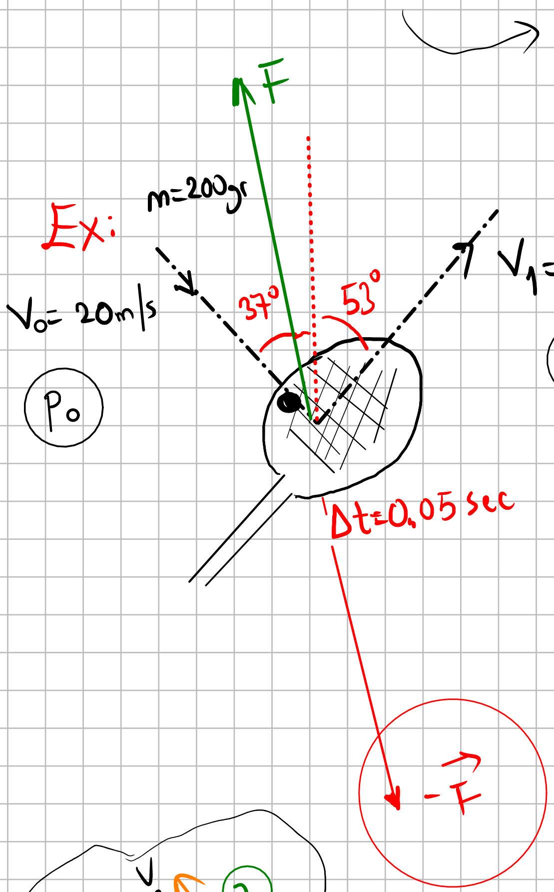
$$\vec{J} = \vec{F}\cdot\Delta t = \vec{\Delta p} \qquad \vec{p} = m\vec{V}$$

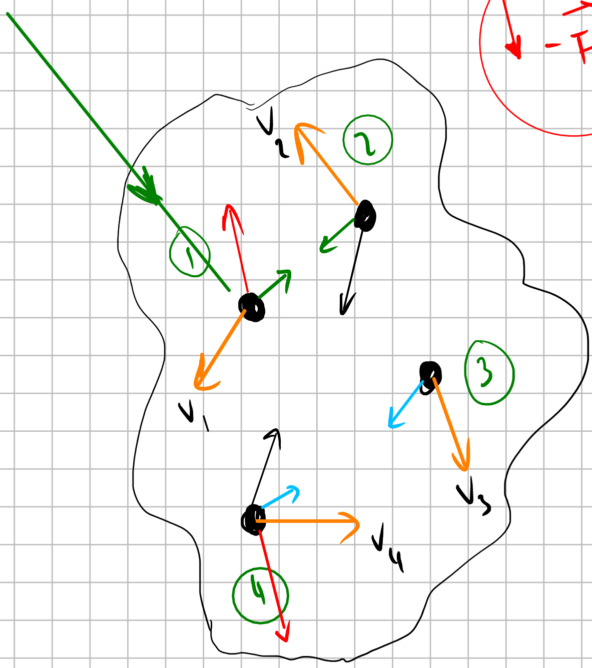
$\vec{J} = \vec{\Delta p}$: when there are no external forces:

$\Rightarrow \vec{J} = 0 \Rightarrow \vec{\Delta p} = 0 \quad \vec{p}_{final} = \vec{p}_{initial}$

$\Rightarrow$ Momentum is conserved

Ex:

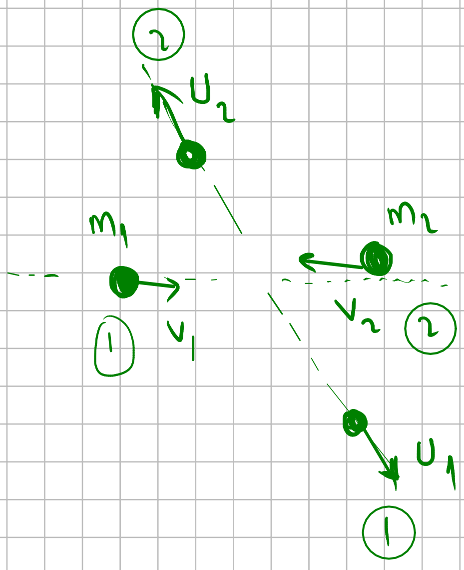




$\Rightarrow \Sigma \vec{F} = 0 \quad \Rightarrow \vec{J} = 0$

$\vec{\Delta p} = 0$: total momentum is conserved

$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots\ldots$: conserved



$$\vec{p}_{in} = m_1 \vec{V_1} + m_2 \vec{V_2}$$

$$\vec{p}_{final} = m_1 \vec{U_1} + m_2 \vec{U_2}$$

$$m_1 \vec{V_1} + m_2 \vec{V_2} = m_1 \vec{U_1} + m_2 \vec{U_2}$$