Angular / Rotational eqns of motions:

constant angular velocity : $\theta = \theta_0 + \omega_0 \Delta t$

constant angular acceleration: $\omega = \omega_0 + \alpha \Delta t$

$\theta = \theta_0 + \omega_0 \Delta t + \frac{1}{2} \alpha \Delta t^2$

$\omega^2 - \omega_0^2 = 2 \alpha \Delta\theta$

$x \rightarrow \theta$

$v \rightarrow \omega$

$a \rightarrow \alpha$

$M \rightarrow I$

$F \rightarrow ?$

Rotational kinetic energy:

$\frac{1}{2} I \omega^2 = K_{rot}$ $\quad I = \sum m_i r_i^2$: moment of inertia

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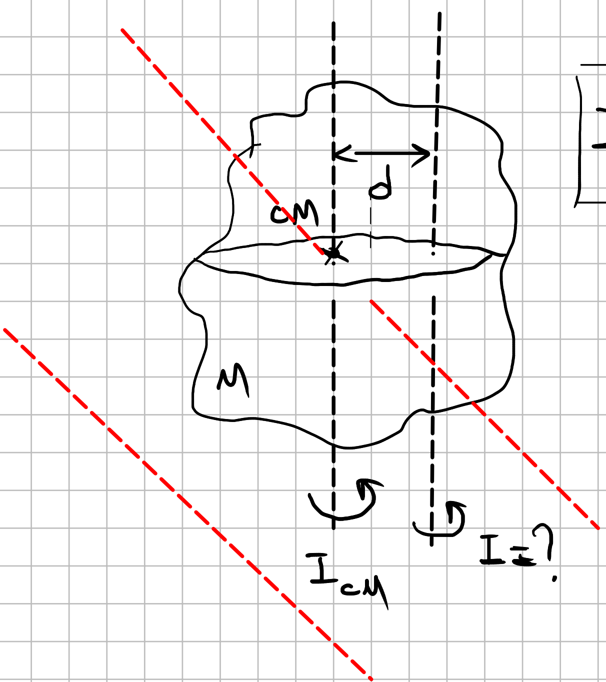
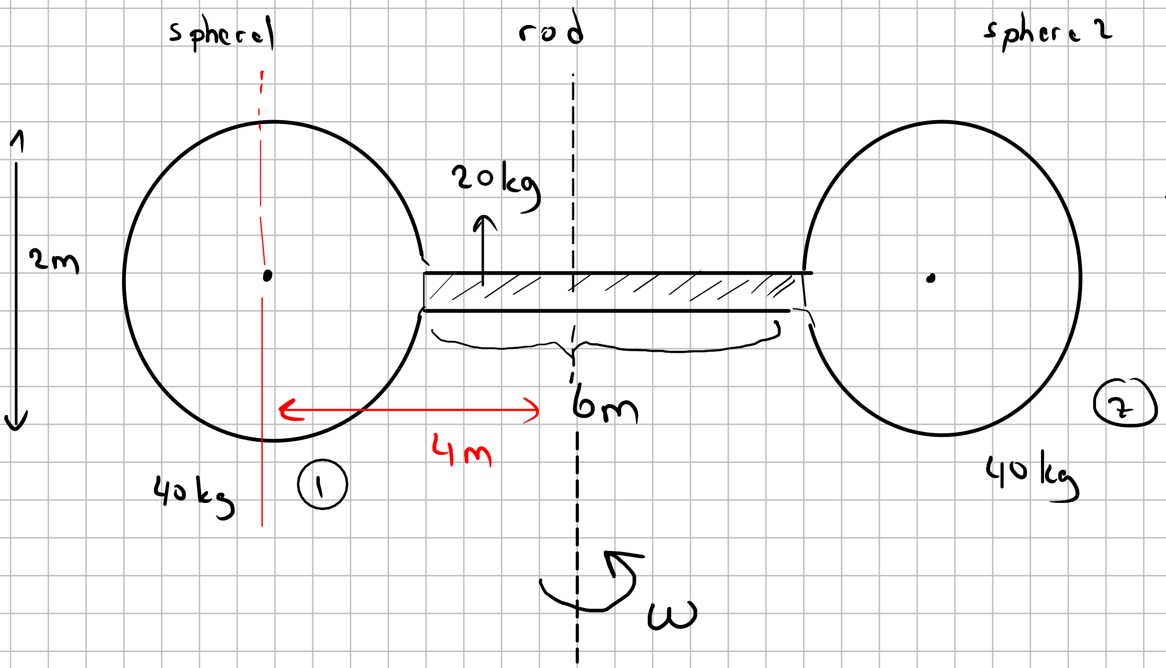
parallel axis theorem:



$$\boxed{I = I_{CM} + Md^2}$$

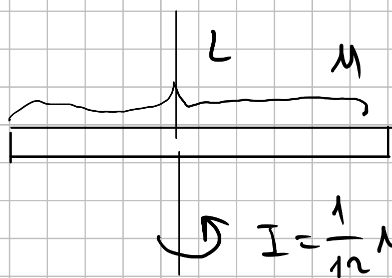
- Two axes must be parallel
- One of the axes must pass through C.M.



$I = ?$

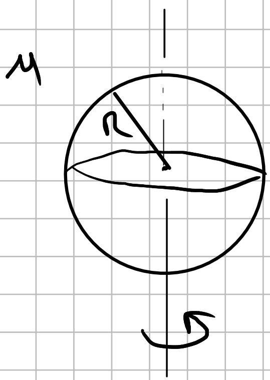
$$I = I_1 + I_2 + I_{rod}$$

Rod:



$$I = \frac{1}{12} M L^2$$

$$I_{rod} = \frac{1}{12} \cdot 20 \cdot 6^2 = \frac{20 \cdot \cancel{36}^{\,3}}{\cancel{12}} = 60\ kgm^2$$



$$I = \frac{2}{5} M R^2$$

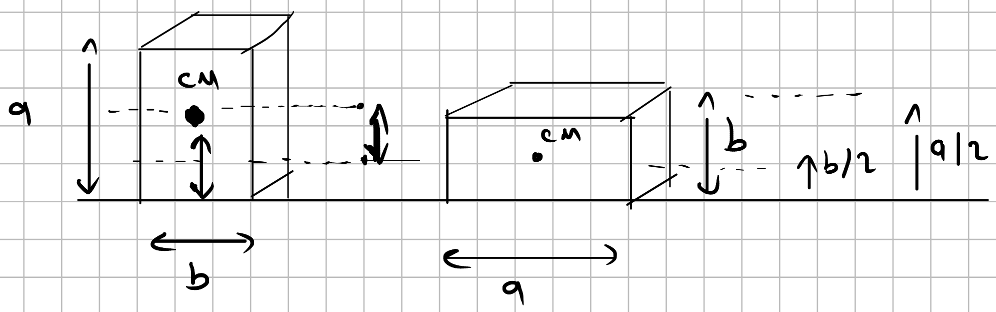
$$\Rightarrow I_1 = \frac{2}{5} M R^2 + M d^2 \quad d = 4m$$

$$\Rightarrow I_1 = \frac{2}{\cancel{5}} \cdot \cancel{40}^{\,8} \cdot 1^2 + 40 \cdot 4^2 = 16 + 640 = 656\ kgm^2$$

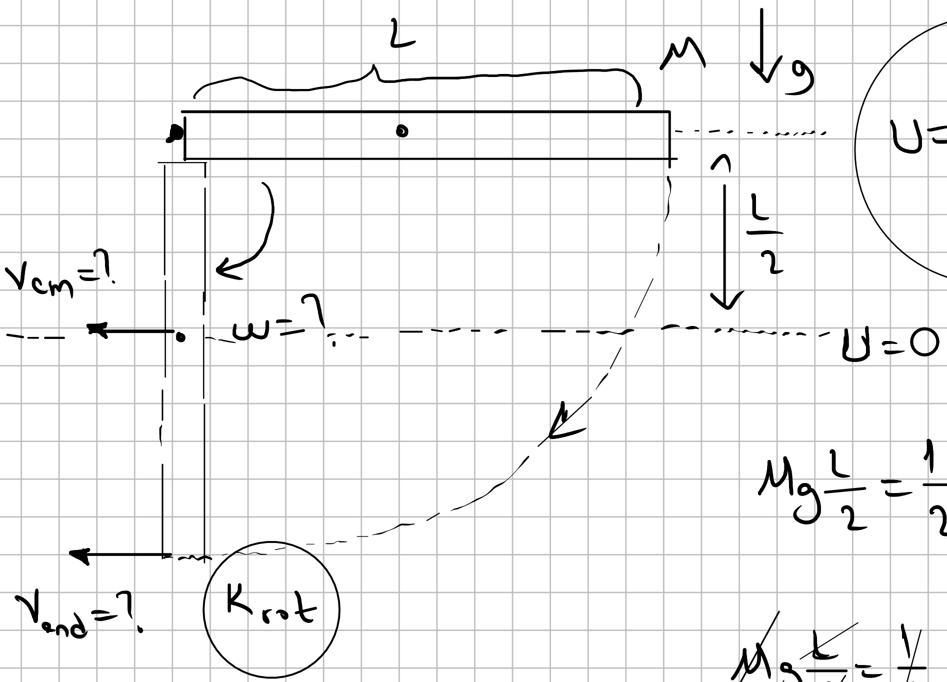
$$I_2 = I_1 = 656\ kgm^2$$

$$\boxed{I = 656 + 656 + 60 = 1372\ kgm^2}$$

Gravitational potential energy of non-point objects



$$\boxed{U = mg\cdot\left(\frac{b-a}{2}\right)}$$



$$U = Mg\frac{L}{2}$$

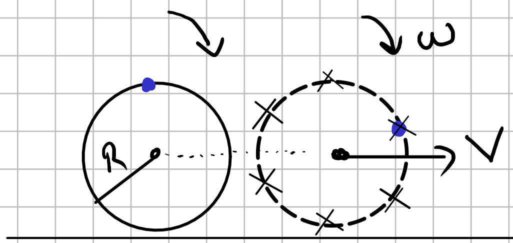
$$Mg\frac{L}{2} = \frac{1}{2}I\omega^2 \qquad I = \frac{1}{3}ML^2$$

$$\cancel{M}g\frac{\cancel{L}}{\cancel{2}} = \frac{1}{\cancel{2}}\frac{1}{3}\cancel{M}L^{\cancel{2}}\cdot\omega^2$$

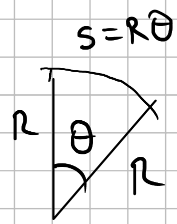
$$\Rightarrow \boxed{\omega^2 = \frac{3g}{L}} \quad \omega = \sqrt{\frac{3g}{L}} \qquad V = \omega R$$

$$V_{end} = \omega\cdot L \Rightarrow \boxed{V_{end} = \sqrt{3gL}} \qquad \boxed{V_{cm} = \sqrt{\frac{3gL}{4}}}$$

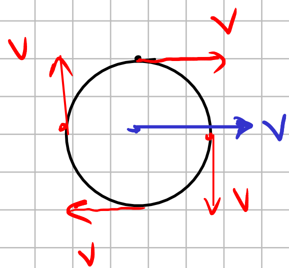
"Rolling without slipping"



Translation + rotation

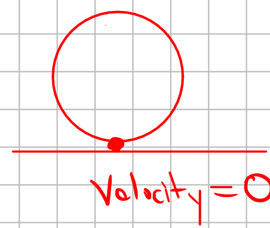


$x = R\theta$ (condition of no-slip)



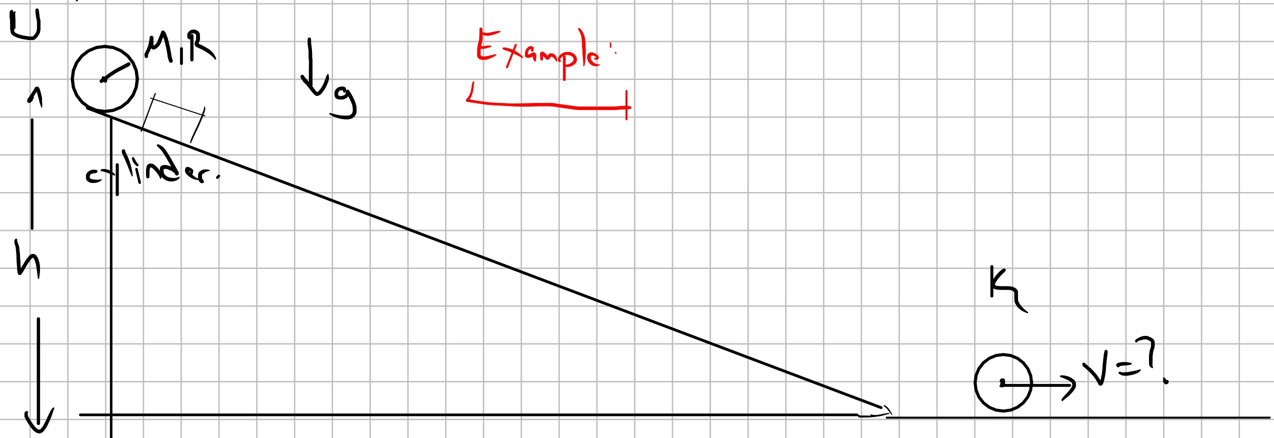
$$V = \frac{dx}{dt} \qquad \omega = \frac{d\theta}{dt}$$

$$\hookrightarrow V = \frac{d(R\theta)}{dt} = R\frac{d\theta}{dt} \Rightarrow \boxed{V = \omega R}$$



$$\boxed{K = \frac{1}{2} I \omega^2 + \frac{1}{2} M V_{cm}^2}$$

Example:



$$U = Mgh = \frac{1}{2} M V^2 + \frac{1}{2} I \omega^2 \qquad V = \omega R \qquad I = \frac{1}{2} M R^2$$

$$\omega = \frac{V}{R}$$

$$\cancel{M} g h = \frac{1}{2} \cancel{M} v^2 + \frac{1}{2} \frac{1}{2} \cancel{M} \cancel{R^2} \cdot \frac{v^2}{\cancel{R^2}}$$

$$gh = \frac{v^2}{2} + \frac{v^2}{4} = \frac{3}{4} v^2 \Rightarrow \boxed{v = \sqrt{\frac{4}{3} gh}}$$

$x \rightarrow \theta$

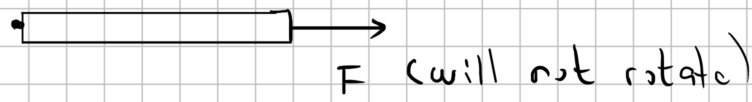
$v \rightarrow \omega$

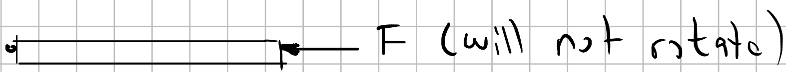
$a \rightarrow \alpha$

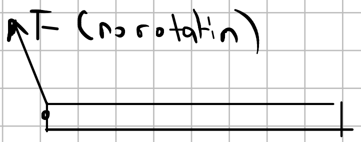
$M \rightarrow I$

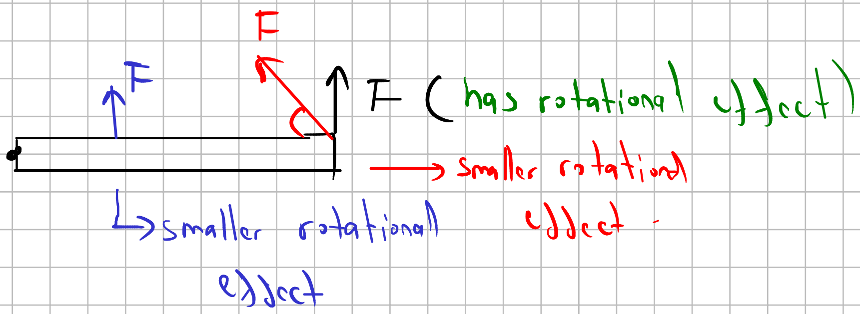
$F \rightarrow \tau$

τ: Torque: ~ force that rotates.

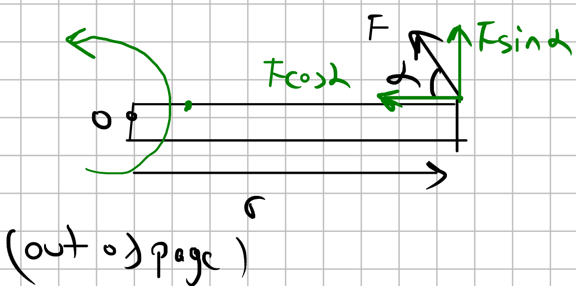








(distance, angle): rotational.

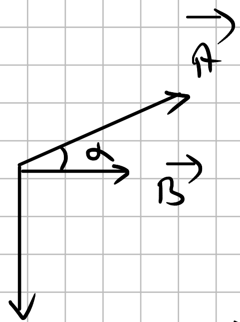


$$\boxed{\tau = F\sin\alpha \cdot r}$$

$\vec{r}, \vec{F}$: vectorial

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

$\vec{\tau} \perp \vec{F}$ $\quad \vec{\tau} \perp \vec{r}$

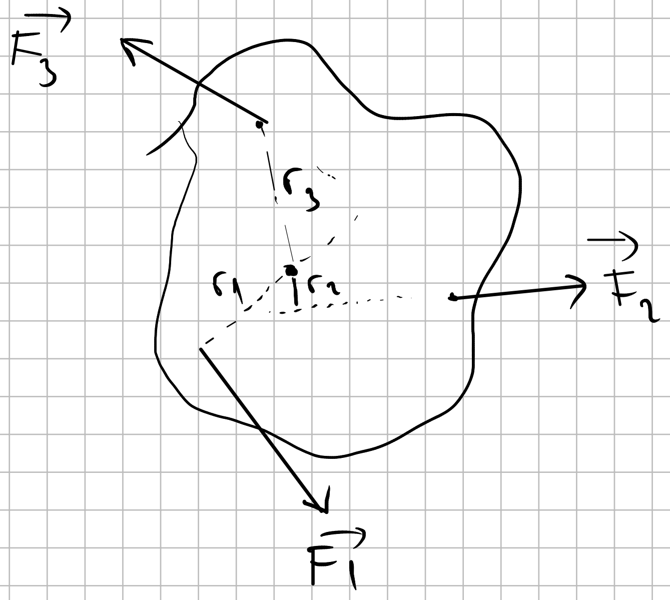


$\vec{A} \times \vec{B}$

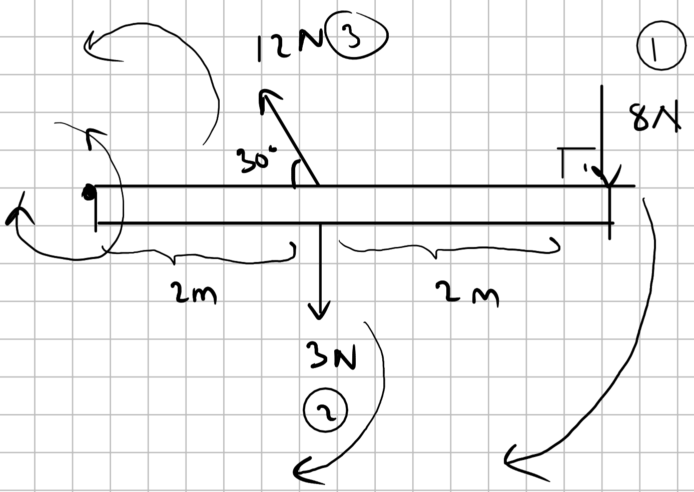
$\vec{A} \times \vec{B}$: $|\vec{A} \times \vec{B}| = AB\sin\alpha$

⊙ out from page : +

⊗ into the page : −



$$\vec{\tau} = \vec{r_1}\times\vec{F_1} + \vec{r_2}\times\vec{F_2} + \dots$$



$$\tau = ? = \tau_1 + \tau_2 + \tau_3$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\tau_1 = r_1 \cdot F_1 \sin\alpha$$

$$\tau_1 = 4\cdot 8\cdot 1 = -32\,Nm$$

$$\tau_2 = 2\cdot 3\cdot 1 = -6\,Nm \qquad \tau_3 = 2\cdot 12\cdot \sin 30^\circ = +12\,Nm$$

$$\tau = -32-6+12 = -26\,Nm \text{ (into page)}$$

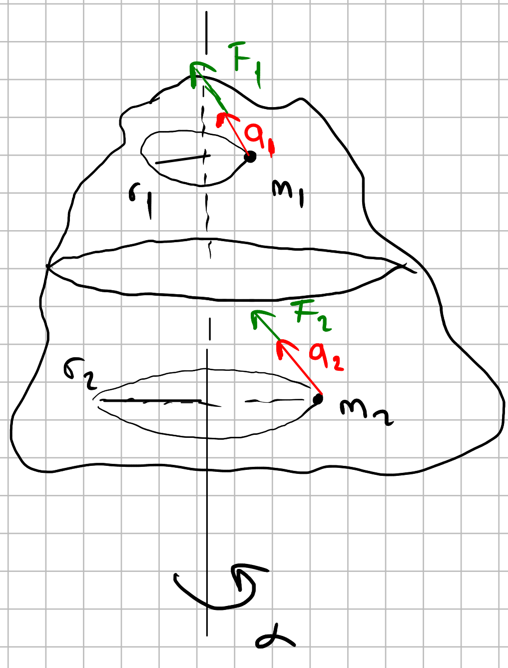
$x \to \theta$

$V \to \omega$

$a \to \alpha$

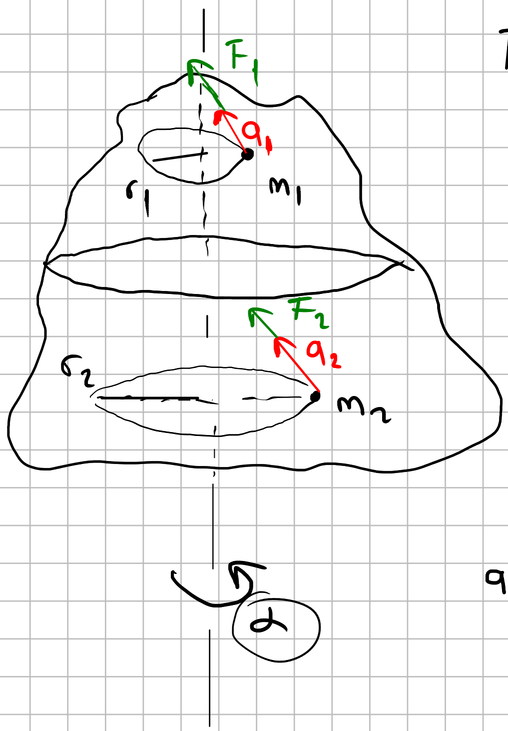
$M \to I$

$F \to \tau$



$$F = m\cdot a$$

$$\boxed{\tau = I\alpha}$$



$$F_1 = m_1 a_1 \qquad a_1 = \alpha \cdot r_1 \qquad F_1 = m_1 r_1 \alpha \quad \times r_1$$

$$F_2 = m_2 a_2 \qquad a_2 = \alpha \cdot r_2 \qquad F_2 = m_2 r_2 \alpha \quad \times r_2$$

$$a = \alpha \cdot r$$

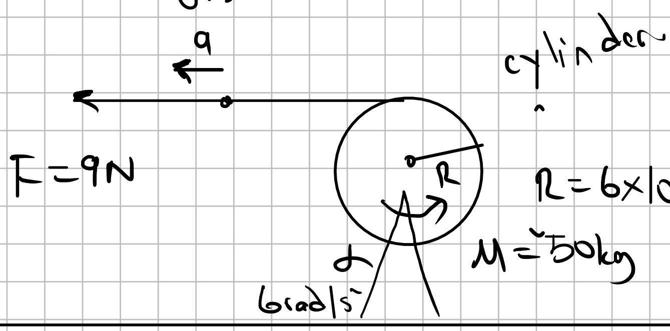
$$F_1 r_1 = m_1 r_1^2 \alpha$$

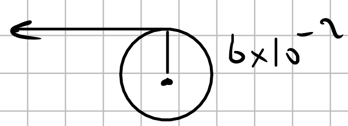
$$F_2 r_2 = m_2 r_2^2 \alpha$$

$+$

$$\tau_1 + \tau_2 + \cdots = \tau = \alpha(\underbrace{m_1 r_1^2 + m_2 r_2^2 + \cdots}_{I})$$

$$\boxed{\tau = I\alpha}$$





$$\tau = F \cdot R$$

$$\tau = 9 \times 6 \times 10^{-2} = 54 \times 10^{-2}\ Nm$$

$$\boxed{\tau = I\alpha} \qquad I = \frac{1}{2} M R^2 = \frac{1}{2} 50 \times 36 \times 10^{-4}$$

$$54 \times 10^{-2} = \frac{1}{2} \cdot 50 \cdot 36 \cdot 10^{-4} \cdot \alpha \qquad \boxed{\alpha = 6\ rad/s^2}$$

$$a = \alpha \cdot R = a = 6 \cdot 6 \times 10^{-2} = \boxed{0.36\ m/s^2}$$

$x \longrightarrow \theta$

$V \longrightarrow \omega$

$a \longrightarrow \alpha$

$M \longrightarrow I$

$F \longrightarrow \tau$

$p \longrightarrow ?\ L$ (angular momentum)

Next : Angular Momentum.