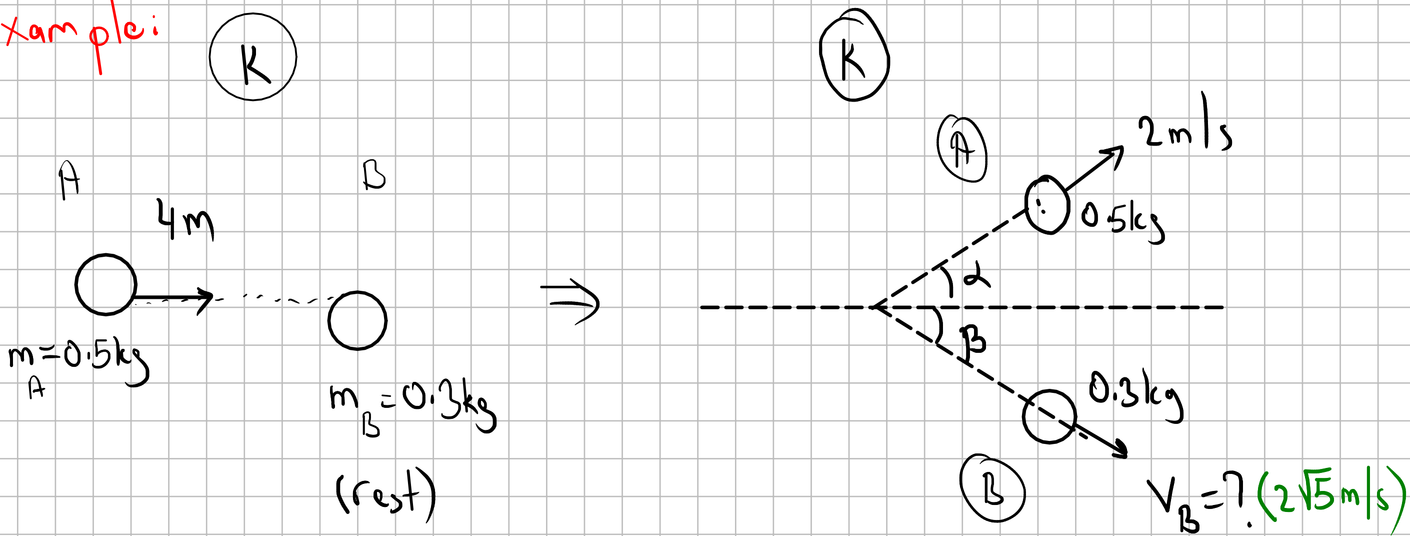
Collisions / conservation of momentum

* inelastic collisions :

* elastic collisions :

Example:



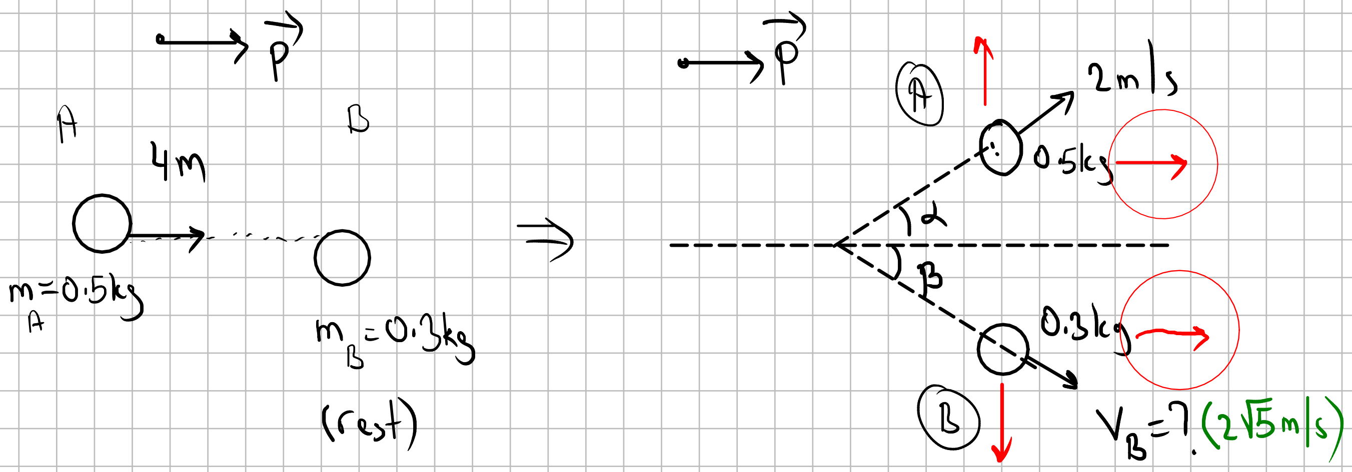
* Given that the collision is "elastic": $V_B = ?$ $\alpha = ?$ $\beta = ?$

⊛ Conservation of energy:

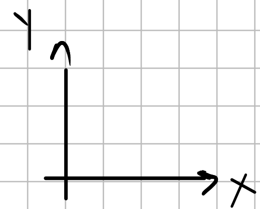
$$K = \frac{1}{2}(0.5)4^2 + 0 = \frac{1}{2}(0.5)2^2 + \frac{1}{2}(0.3)V_B^2$$

$$K = 4 = 1 + \frac{1}{2}\frac{3}{10}V_B^2 \rightarrow \cancel{3} = \frac{1}{\cancel{2}}\frac{\cancel{3}}{10}V_B^2 \rightarrow V_B^2 = 20 \rightarrow V_B = 2\sqrt{5}$$

(m/s)



* Now apply conservation of momentum



P_x: $(0.5)(4)+0 = 2$ kg m/s

$$\Rightarrow (0.5)(2)\cos\alpha + (0.3)(2\sqrt{5})\cdot\cos\beta = 2$$

$$\Rightarrow \cos\alpha + \frac{3}{\cancel{10}_{\sqrt{5}}}\cancel{2}\cancel{\sqrt{5}}\cos\beta \Rightarrow \boxed{\cos\alpha + \frac{3}{\sqrt{5}}\cos\beta = 2} \quad (1)$$

P_y: $0 = (0.5)(2)\sin\alpha - (0.3)(2\sqrt{5})\sin\beta$

$$\sin\alpha = \frac{3}{\cancel{10}_{\sqrt{5}}}\cdot\cancel{2}\cancel{\sqrt{5}}\sin\beta \rightarrow \boxed{\sin\alpha = \frac{3}{\sqrt{5}}\sin\beta} \quad (2)$$

$\{\sin^2 x + \cos^2 x = 1\}$

$(1)^2$

from (1): $\cos\beta = (2-\cos\alpha)\frac{\sqrt{5}}{3} \Rightarrow \boxed{\cos^2\beta = \frac{5}{9}\left[4+\cos^2\alpha - 4\cos\alpha\right]}$

$\sin^2\beta = 1 - \cos^2\beta$

$$\boxed{\sin^2\alpha = \frac{9}{5}\sin^2\beta} \quad (2)^2$$

$\hookrightarrow \sin^2\alpha = \frac{9}{5}\left[1-\cos^2\beta\right]$

$\sin^2\alpha = \frac{9}{5}\left[1 - \frac{5}{9}\left[4+\cos^2\alpha - 4\cos\alpha\right]\right]$

$\sin^2\alpha = \frac{9}{5} - \left[4+\cos^2\alpha - 4\cos\alpha\right]$

$5\sin^2\alpha = 9 - 20 - 5\cos^2\alpha + 20\cos\alpha$

$5\sin^2\alpha \underbrace{-9+20}_{+11} + 5\cos^2\alpha - 20\cos\alpha = 0$

$\underbrace{5(\sin^2\alpha + \cos^2)}_{5} + 11 - 20\cos\alpha = 0 \Rightarrow 16 - 20\cos\alpha = 0$

$\cos\alpha = \frac{16}{20} = \frac{4}{5}$

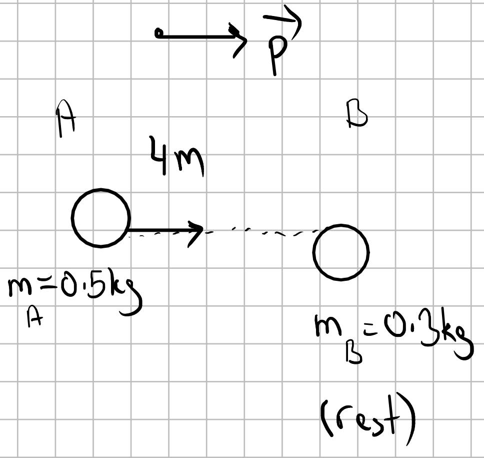
$\boxed{\cos\alpha = \frac{4}{5}}$ $\star$ $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, $\arccos\left(\frac{4}{5}\right)$ $\alpha = 36.9$

$\boxed{\cos\alpha + \frac{3}{\sqrt{5}}\cos\beta = 2} \rightarrow \frac{4}{5} + \frac{3}{\sqrt{5}}\cos\beta = 2$ $\beta = 26.6$

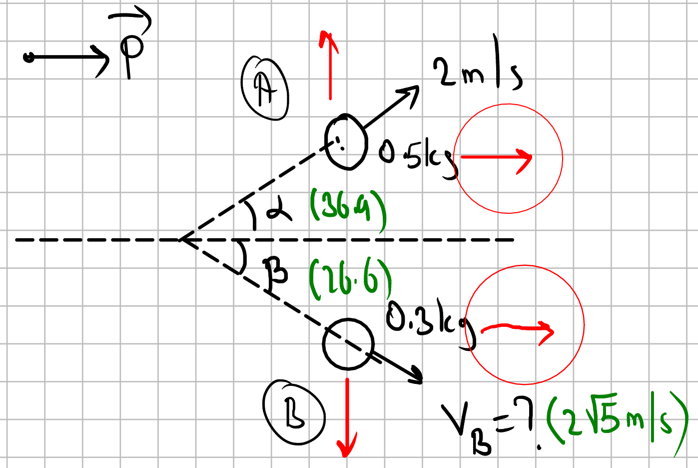
$\rightarrow \frac{\cancel{6}^{\,2}}{\cancel{5}_{\sqrt{5}}} = \frac{\cancel{3}}{\cancel{\sqrt{5}}}\cos\beta \rightarrow \boxed{\cos\beta = \frac{2}{\sqrt{5}}}$ $\star$ $\beta = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$

$\arccos\left(\frac{2}{\sqrt{5}}\right)$

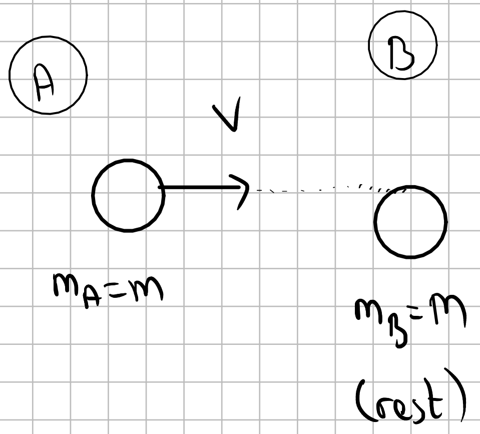
$\boxed{\alpha + \beta = 63.5^\circ}$



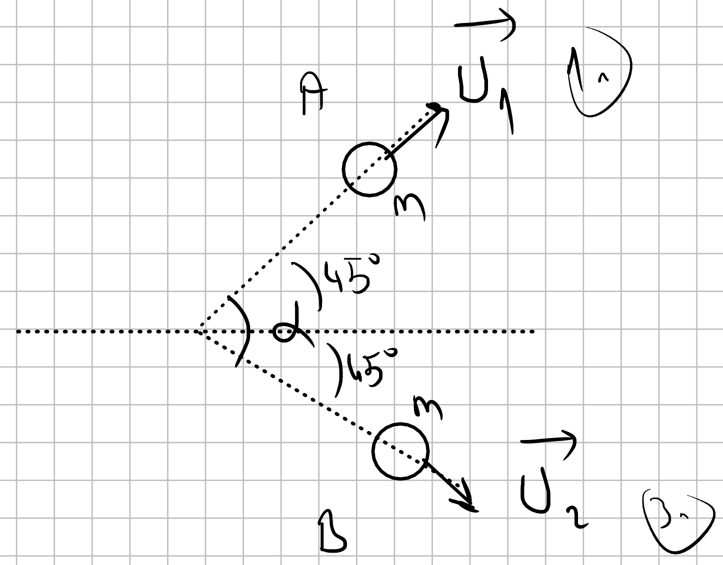
$\Rightarrow$



Special case: elastic



$\Rightarrow$



$$\frac{1}{\cancel{2}}\cancel{m}V^2 = \frac{1}{\cancel{2}}\cancel{m}U_1^2 + \frac{1}{\cancel{2}}\cancel{m}U_2^2$$

$$\boxed{V^2 = U_1^2 + U_2^2}$$

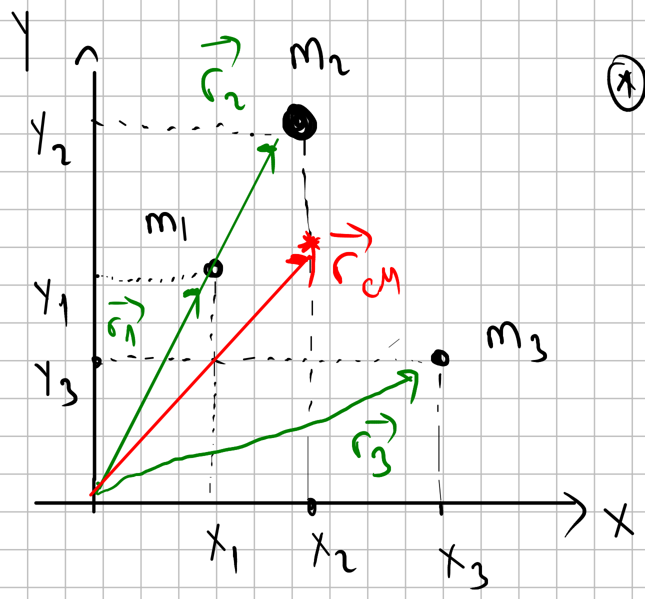
$$\cancel{m}\vec{V} = \cancel{m}\vec{U_1} + \cancel{m}\vec{U_2} \Rightarrow \vec{V} = \vec{U_1} + \vec{U_2}$$

$$\rightarrow \vec{V}\cdot\vec{V} = V^2 = (\vec{U_1} + \vec{U_2})\cdot(\vec{U_1} + \vec{U_2})$$

$$\underbrace{V^2 = U_1^2 + U_2^2}\ + 2\vec{U_1}\cdot\vec{U_2} \rightarrow \vec{U_1}\cdot\vec{U_2} = 0$$

$$U_1 U_2 \cos\alpha = 0 \quad \alpha = 90°$$

Center of Mass:

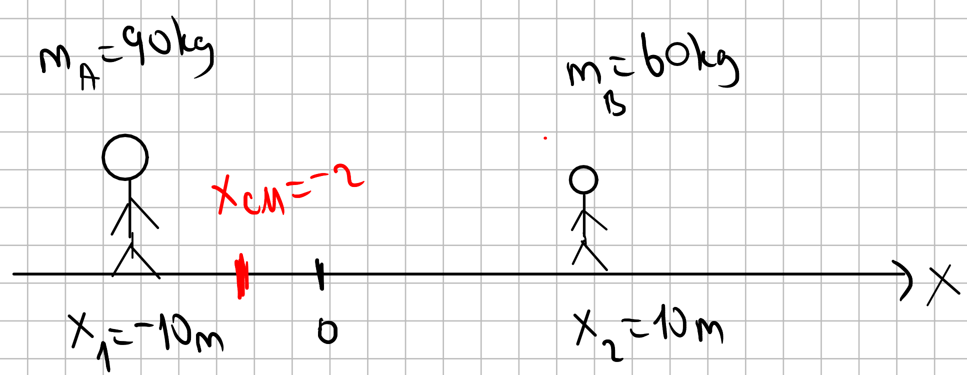


(*) Centre of mass :

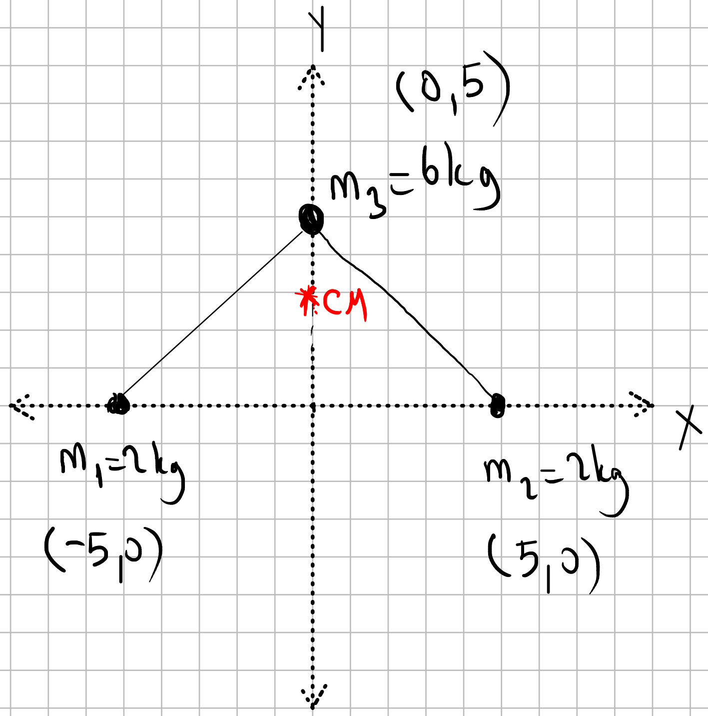
a point where all the mass can be thought to be concentrated at.

$$\vec{r}_{cM} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3} + \cdots}{m_1+m_2+m_3+\cdots\cdots = M}$$

$$X_{cM} = \frac{m_1X_1+m_2X_2+\cdots\cdots}{M} \qquad Y_{cM} = \frac{m_1Y_1+mY_2+\cdots\cdots}{M}$$



$$X_{cM} = \frac{90(-10)+(60)(10)}{150} = \frac{-900+600}{150} = \frac{-300}{150} = -2\ m$$



$x_1 = -5 \quad y_1 = 0$

$x_2 = 5 \quad y_2 = 0$

$x_3 = 0 \quad y_3 = 5$

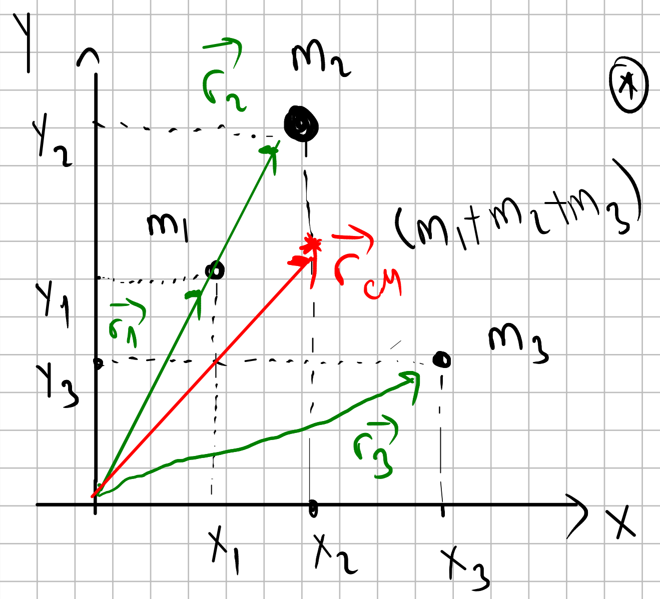
$$x_{cm} = \frac{2(-5)+2(5)+6(0)}{10} = 0$$

$$\vec{r}_{cm} = (0,3)$$

$$y_{cm} = \frac{2(0)+2(0)+6(5)}{10} = 3$$

Centre of Mass Motion

$$\vec{r}_{cM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1+m_2+m_3+\cdots = M}$$



$$\left(\vec{v} = \frac{d\vec{r}}{dt}\right)$$

$$M\vec{r}_{cM} = m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots$$

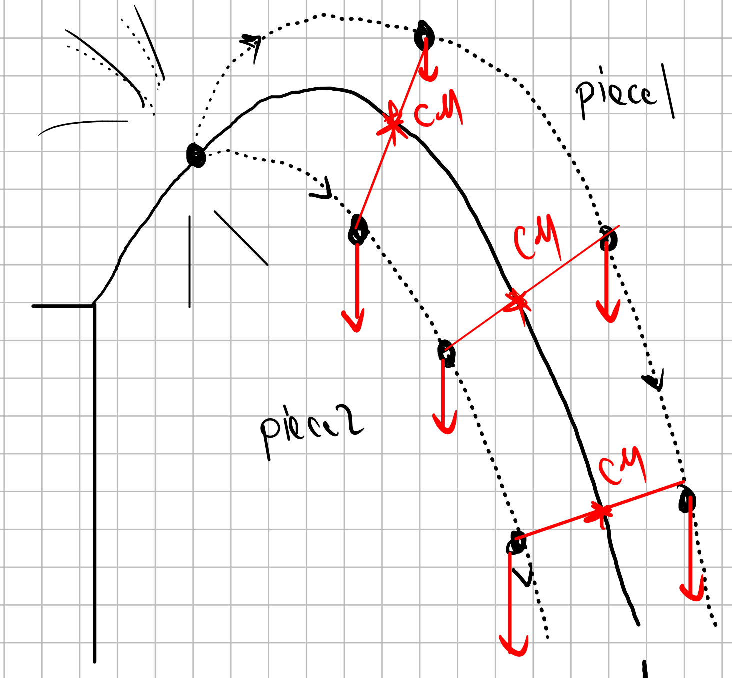
$$\left(\frac{d\vec{v}}{dt} = \vec{a}\right)$$

$$\boxed{M\vec{v}_{cM} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots}$$

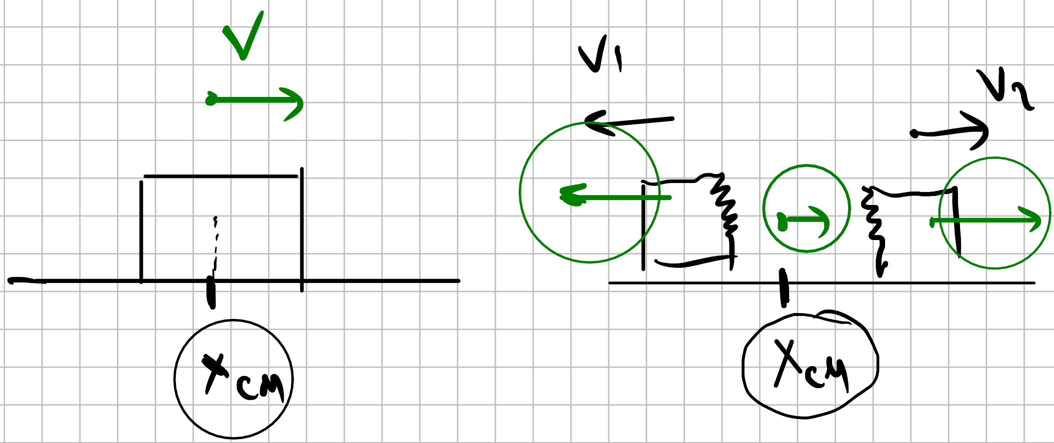
$$M\vec{a}_{cM} = m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots$$

$$M\vec{a}_{cM} = \vec{F}_1 + \vec{F}_2 + \cdots$$

$$\Rightarrow \boxed{\vec{F}_{NET} = M\vec{a}_{cM}}$$



↳ trajectory of center of mass

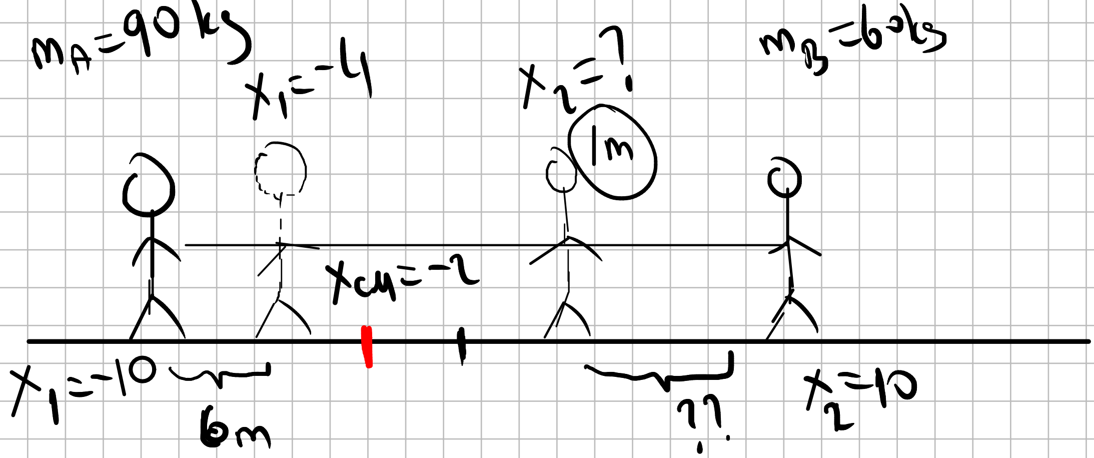


$$\boxed{M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + \dots}$$

$$M\vec{v}_{cm} = \underline{\vec{p}_1 + \vec{p}_2 + \vec{p}_3} + \dots$$

(*) → if no net force is acting

⇒ center of mass momentum is conserved



if person A moves by 6m (right)

how much B has moved? (left)

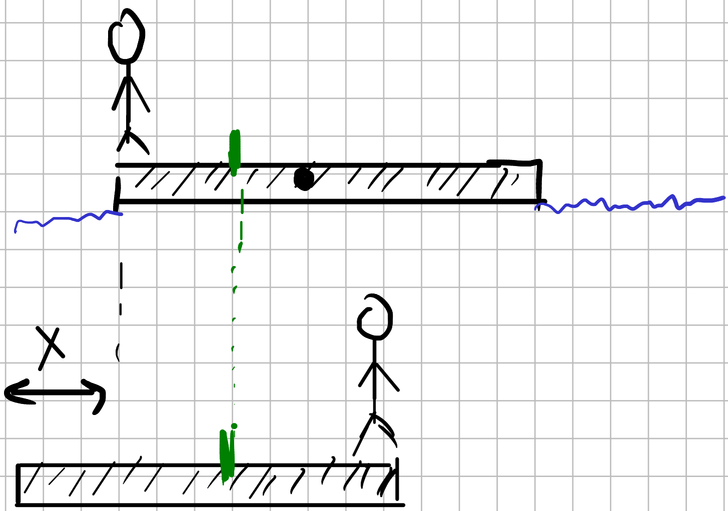
$$x_{cm} = -2 = \frac{90(-4) + 60(x_2)}{150}$$

$$-300 = -360 + 60x_2$$

$$60x_2 = 60 \qquad x_2 = 1\,m$$

$\Rightarrow$ B moves by 9 meters

(when A moves by 6 meters)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