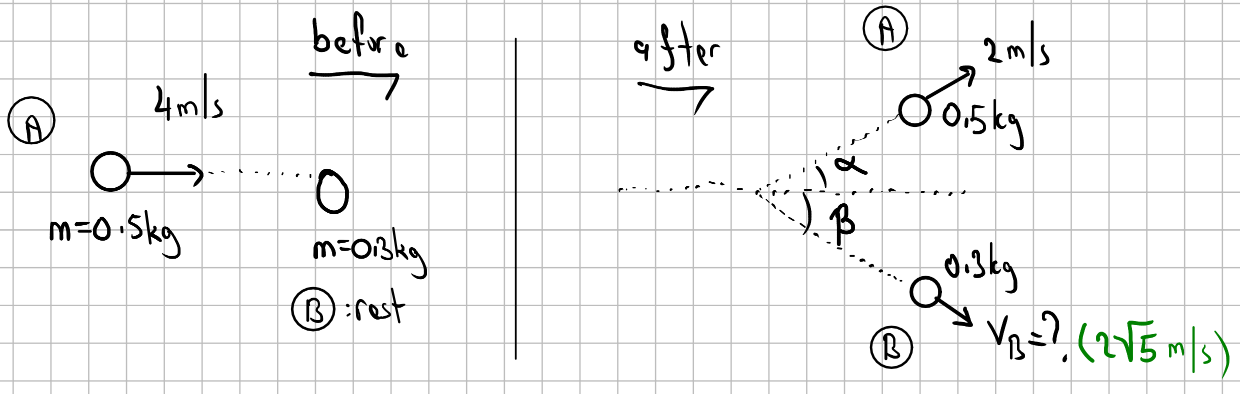
Elastic Coll. | 2D example



* Given that the collision is "elastic"

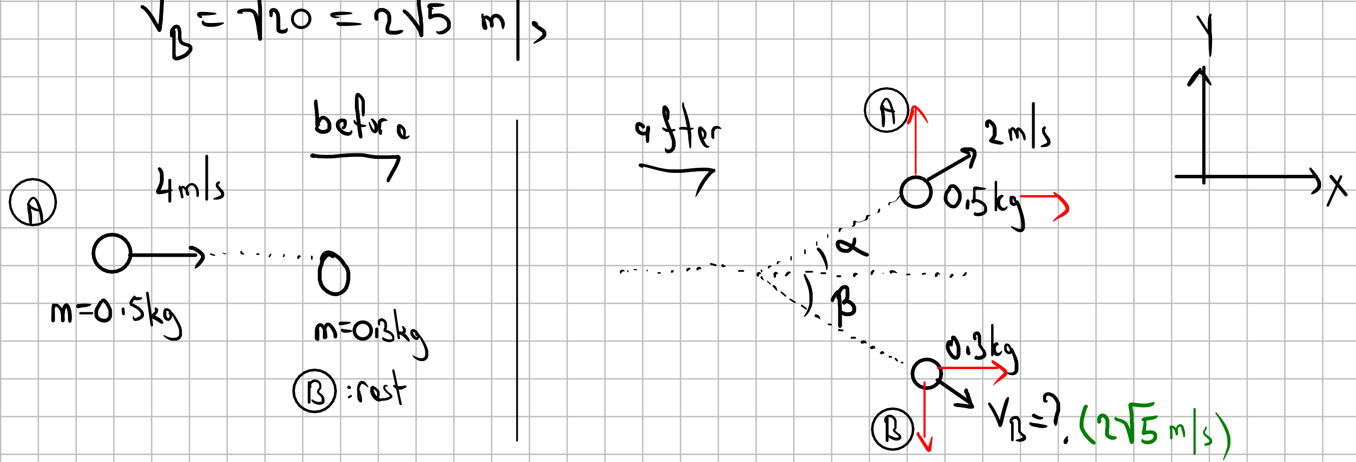
⇒ Find $V_B = ?$ and angles α, β

⊙ Using conservation of energy:

$$K = \frac{1}{2}(0.5)\cdot(4^2) + 0 = \frac{1}{2}(0.5)(2^2) + \frac{1}{2}(0.3)V_B^2$$

$$4 = \cancel{\frac{1}{4}\cdot 4} + \frac{1}{2}\cdot\frac{3}{10}\cdot V_B^2 \rightarrow \cancel{3} = \frac{1}{2}\cdot\frac{\cancel{3}}{10}\cdot V_B^2 \rightarrow V_B^2 = 20$$

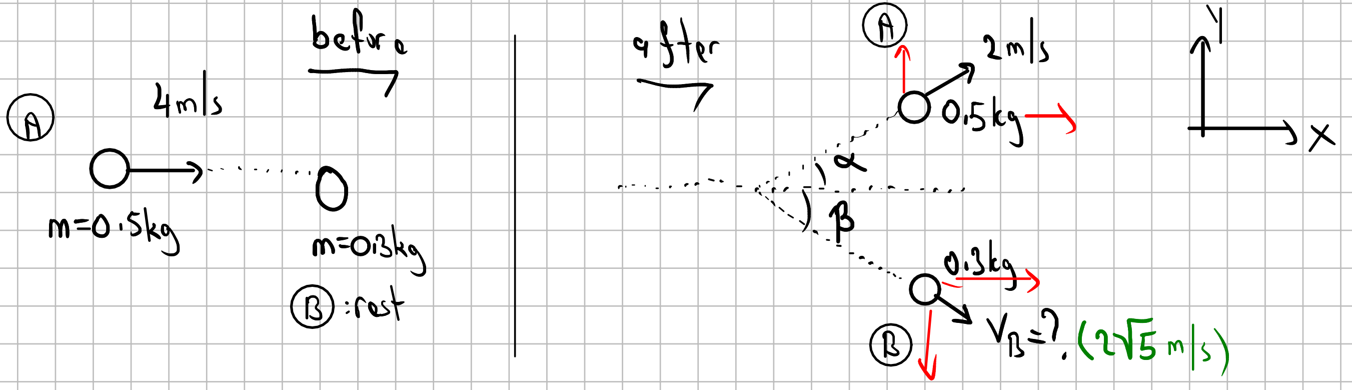
$$V_B = \sqrt{20} = 2\sqrt{5}\ m/s$$



* Conservation of momentum:

$$P_x: \quad (0.5)(4) + 0 = (0.5)(2)\cos\alpha + (0.3)(2\sqrt{5})\cos\beta$$

$$2 = \cos\alpha + \frac{3}{\cancel{10}_5}\cancel{2}\sqrt{5}\cos\beta \rightarrow \boxed{2 = \cos\alpha + \frac{3}{\sqrt{5}}\cos\beta} \quad (1)$$



* Cons. of momentum:

$$p_y: \quad 0 = (0.5)(2)\sin\alpha - (0.3)(2\sqrt{5})\sin\beta$$

$$\Rightarrow \sin\alpha = \frac{3}{\cancel{10}_{5}}\cancel{2}\sqrt{5}\sin\beta \Rightarrow \boxed{\sin\alpha = \frac{3}{\sqrt{5}}\sin\beta} \quad (2)$$

$$\boxed{2 = \cos\alpha + \frac{3}{\sqrt{5}}\cos\beta} \quad (1)$$

$$\boxed{\sin\alpha = \frac{3}{\sqrt{5}}\sin\beta} \quad (2)$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2\beta = 1 - \cos^2\beta$$

$$\cos^2\beta = 1 - \sin^2\beta$$

$$\left(\frac{(2-\cos\alpha)\sqrt{5}}{3} = \cos\beta\right)^2$$

$$\boxed{\cos^2\beta = \frac{5}{9}\left[4 + \cos^2\alpha - 4\cos\alpha\right]}$$

$(2)^2$

$$\sin^2\alpha = \frac{9}{5}\sin^2\beta \Rightarrow \boxed{\frac{5\sin^2\alpha}{9} = \sin^2\beta}$$

$(1)^2$

$$1 - \sin^2\beta = \cos^2\beta = \underbrace{1 - \frac{5}{9}\sin^2\alpha = \frac{5}{9}\left[4 + \cos^2\alpha - 4\cos\alpha\right]}$$

$$9-5\sin^2\alpha = 20+5\cos^2\alpha-20\cos\alpha$$

$$9-5\sin^2\alpha-20-5\cos^2\alpha+20\cos\alpha=0$$

$$-11-5(\underbrace{\sin^2\alpha+\cos^2\alpha}_{1})+20\cos\alpha=0$$

$$-16+20\cos\alpha=0 \Rightarrow \boxed{\cos\alpha=\frac{4}{5}}$$

$$\sin\alpha=\sqrt{1-\cos^2\alpha}$$

$$\sin\alpha=\sqrt{1-\frac{16}{25}}$$

$$\sin\alpha=\frac{3}{5}$$

$$\boxed{\sin\alpha=\frac{3}{\sqrt{5}}\sin\beta}$$

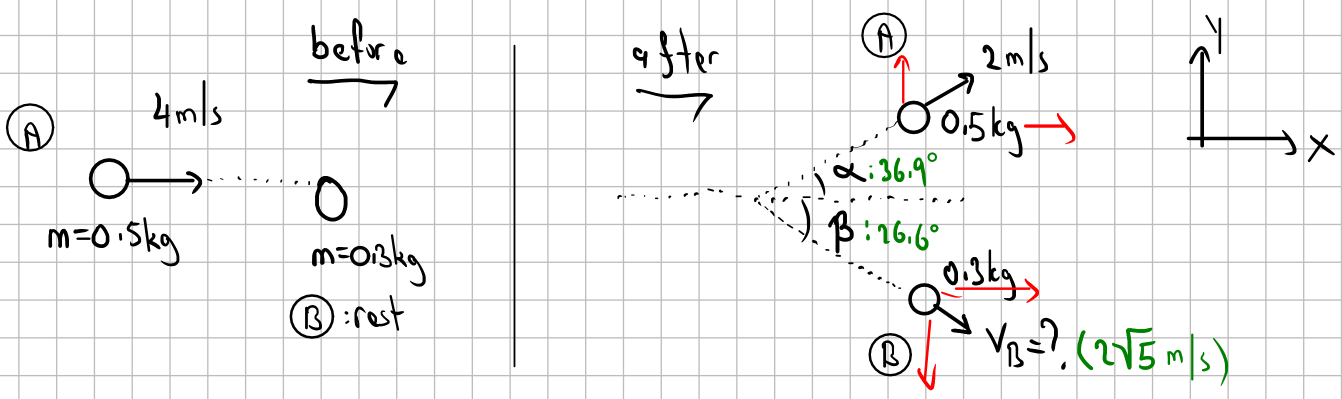
$$\frac{\cancel{3}}{\cancel{5}_{\sqrt{5}}}=\frac{\cancel{3}}{\cancel{\sqrt{5}}}\sin\beta$$

$$\boxed{\frac{1}{\sqrt{5}}=\sin\beta}$$

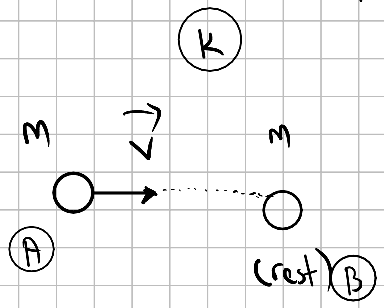
$$\alpha=\cos^{-1}\left(\frac{4}{5}\right) \qquad \alpha\approx 36.9^\circ$$

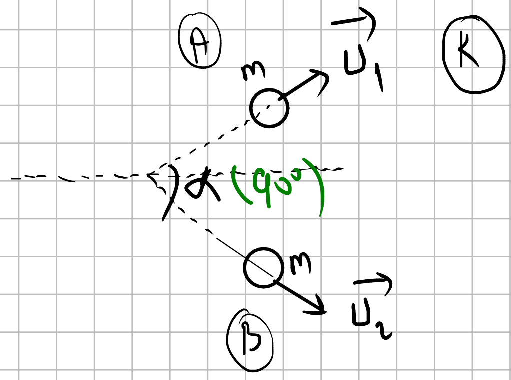
$$\beta=\sin^{-1}(1/\sqrt{5}) \qquad \beta=26.6^\circ$$

$$\boxed{\alpha+\beta\approx 63.5^\circ}$$



Special case: $m_A = m_B = m$ (one of them is at rest)





(elastic collision)

⊛ Conservation of energy:

$$K = \frac{1}{\cancel{2}}\cancel{m}v^2 = \frac{1}{\cancel{2}}\cancel{m}U_1^2 + \frac{1}{\cancel{2}}\cancel{m}U_2^2 \Rightarrow \boxed{v^2 = U_1^2 + U_2^2}$$

$$\vec{p} = \cancel{m}\vec{v} + 0 = \cancel{m}\vec{U_1} + \cancel{m}\vec{U_2} \Rightarrow \boxed{\vec{v} = \vec{U_1} + \vec{U_2}}$$

$$v^2 = \vec{v}\cdot\vec{v} = (\vec{U_1} + \vec{U_2})\cdot(\vec{U_1} + \vec{U_2})$$

$$v^2 = U_1^2 + \vec{U_1}\cdot\vec{U_2} + \vec{U_2}\cdot\vec{U_1} + U_2^2$$

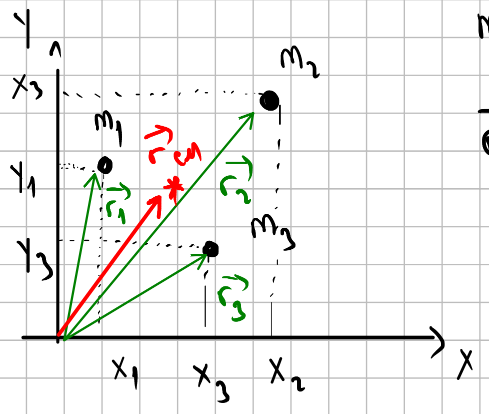
$$v^2 = U_1^2 + U_2^2 + 2\vec{U_1}\cdot\vec{U_2}$$

$$\cancel{U_1^2} + \cancel{U_2^2} = \cancel{U_1^2} + \cancel{U_2^2} + 2\vec{U_1}\cdot\vec{U_2} \rightarrow \vec{U_1}\cdot\vec{U_2} = 0$$

$$\hookrightarrow U_1 U_2 \cos\alpha = 0$$

$$\cos\alpha = 0 \Rightarrow \alpha = 90^\circ$$

Center of Mass



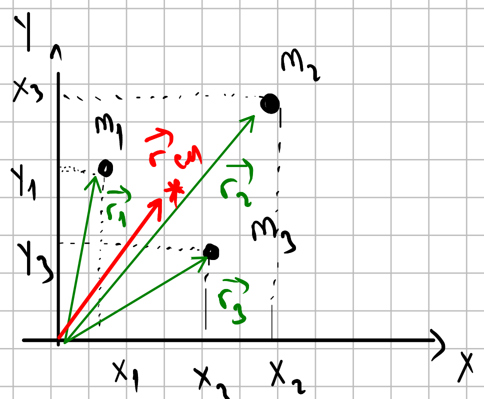
m_1, m_2, m_3 .... : masses

$\vec{r}_1, \vec{r}_2, \vec{r}_3$ ..... : position vectors.

define the center of mass : (C.M)

⇒ C.M is a point as if all the mass $(m_1+m_2+m_3+\cdots)$ is concenctrated at.

↳ C.M is the weighted average of positions (weighted by mass values)



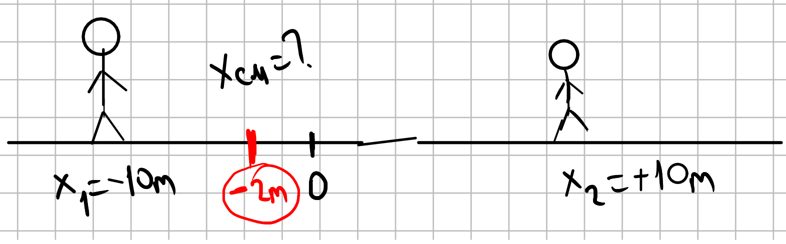
$$\vec{r}_{cM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1+m_2+m_3+\cdots}$$

$m_1 + m_2 + \cdots = M$ (total mass)

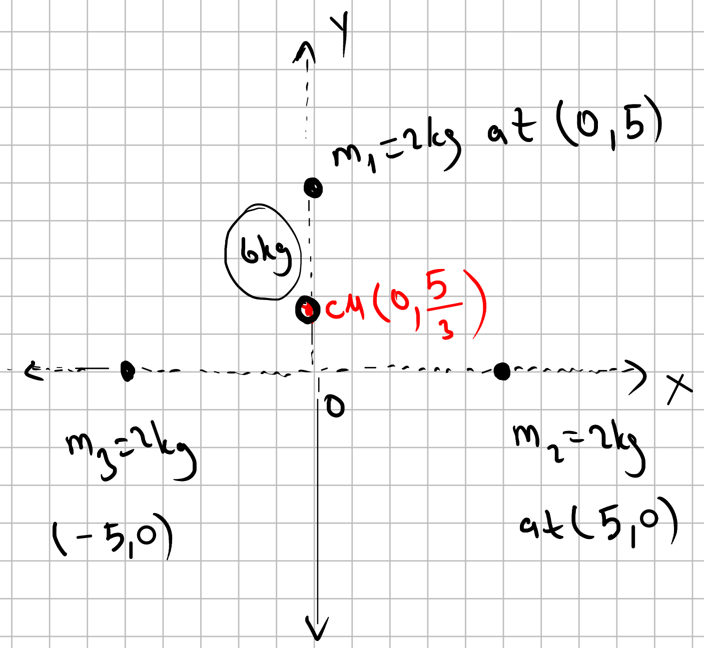
Coordinates of the centre of Mass:

$$x_{cM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{M} \qquad y_{cM} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{M}$$

$m_1 = 90\,kg$ $\qquad$ $m_2 = 60\,kg$



$$x_{cm} = \frac{(90)(-10)+(60)(10)}{150} = \frac{-900+600}{150} \Rightarrow x_{cm} = -2\,m$$



* CM ? in x_{cm}, Y_{cm}

$x_1 = 0 \quad Y_1 = 5$

$x_2 = 5 \quad Y_2 = 0$

$x_3 = -5 \quad Y_3 = 0$

$M = 6\,kg$

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M} = \frac{2\cdot 0 + 2\cdot 5 + 2\cdot(-5)}{6} = 0$$

$$Y_{cm} = \frac{m_1Y_1 + m_2Y_2 + m_3Y_3}{M} = \frac{2(5)+2(0)+2(0)}{6} = \frac{10}{6} = \frac{5}{3}$$

Center of Mass Motion:

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{M} + \cdots \quad (M = m_1 + m_2 + \cdots)$$

$$M\vec{r}_{cm} = (m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots\cdots)$$

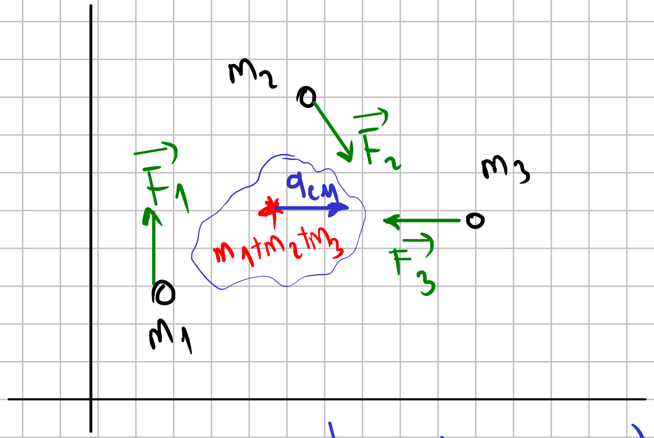
$\{ \vec{F} = m \cdot \vec{a} \}$

$$\boxed{M\vec{V}_{cm} = (m_1\vec{V}_1 + m_2\vec{V}_2 + m_3\vec{V}_3 + \cdots\cdots)}$$

$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d^2\vec{r}}{dt^2}$

$$M\,\vec{a}_{cm} = (m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots\cdots)$$

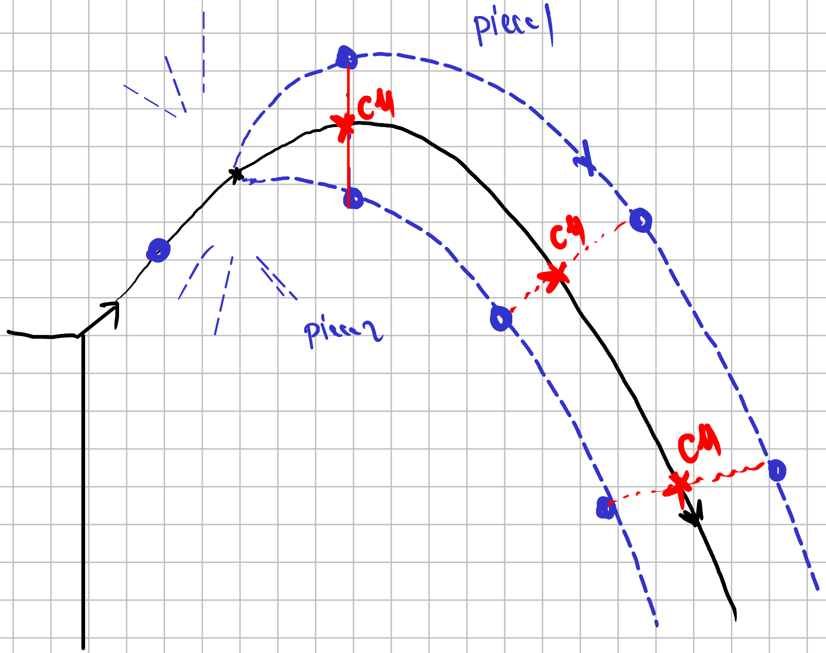
$\vec{V} = \frac{d\vec{r}}{dt}$



$$M\vec{a}_{cm} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

$$\boxed{M\vec{a}_{cm} = \vec{F}_{NET}}$$

* Center of mass behaves like a single object with a total mass $(m_1 + m_2 + \cdots)$ with a net acceleration of a_{cm}



$$\boxed{M\vec{V}_{cm} = (m_1\vec{V}_1 + m_2\vec{V}_2 + m_3\vec{V}_3 + \cdots)}$$