MATH 118 Probability and Statistics

Final Presentation Report

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Introduction

1. Topics
   - Probability of an Event
   - Conditional Probability

2. Abstract
   - First, we will see Terminology of Probability, Event and Probability of an Event so that we can understand Conditional Probability.
   - We will see some of basic examples of the probability.
   - Then, We will study conditional probability.
   - With conditional probability, we will see Multiplication Rule, Addition Rule and Probability Trees.
   - Lastly, we will be finishing with a good example of conditional probability.

Terminology of Probability

Probability

Probability is a measure that is associated with how certain we are of outcomes of a particular experiment or activity.

Experiment

An experiment is a planned operation carried out under controlled conditions.

Sample Space

The sample space of an experiment is the set of all possible outcomes.

Outcome

A result of an experiment is called an outcome.

Event

Event is an outcome or occurrence that has a probability assigned to it.

Probability of an Event

$$Probability = \frac{\text{The number of wanted outcomes}}{\text{The number of possible outcomes}}$$

If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes correspond to event A, then the probability of event A is

$$P(A) = \frac{\mathrm{n}}{\mathrm{N}},$$

$$0 \leq P(A) \leq 1.$$

Example

The names of four directors of a company will be placed in a hat and a 2-member delegation will be selected at random to represent the company at an international meeting. Let A, B, C and D denote the directors of the company. What is the probability that

1. A is selected?
2. A is not selected?

Solution(i)

Firstly, The sample space for this experiment is;

$$S = \{AB, AC, AD, BC, BD, CD\}$$

When we choose A, we must choose one of the remaining 3 directors to go with A. There are;

$$\binom{4}{2} = 6$$

possible combinations.

Solution(i) cont.

Then, the probability that A is selected is;

$$\frac{\binom{1}{1} \times \binom{3}{1}}{\binom{4}{2}} = \frac{3}{6} = \frac{1}{2}$$

Solution(ii)

We found A is selected is $\frac{1}{2}$, then by using **Complement Rule** A is not selected is;

$$1 - \frac{1}{2} = \frac{1}{2}$$

Conditional Probability

Conditional Probability

The conditional probability of an event B is the probability that the event will occur given the knowledge that an event A has already occurred. This probability is written $P(B \mid A)$ notation for the probability of B given A. In the case where events A and B are independent (where event A has no effect on the probability of event B), the conditional probability of event B given event A is simply the probability of event B, that is P(B).

How to express?

If we want to express the probability of one event happening given another one has already happened, we use the " | " symbol to mean "given", and we say;

$P(A \mid B) =$ The probability of A given that we know B has happened.

Definition

If events A and B are not independent, then the probability of the intersection of A and B (the probability that both events occur) is defined by

$$P(A \cap B) = P(A)P(B \mid A)$$

From this definition, the conditional probability $P(B \mid A)$ is easily obtained by dividing by P(A);

$$P(B \mid A) = \frac{P(A \cap B)}{P(B)} \quad \text{when} \quad P(B) > 0.$$

Example

In Gebze Technical University, 60% of the boys play tennis, and 24% of the boys play tennis and football.
What percent of those that play tennis also play football?

Solution

$$P(Tennis \cap Football) = 0.24$$

$$P(Tennis) = 0.6$$

$$P(Football \mid Tennis) = ?$$

$$P(Football \mid Tennis) = \frac{P(Football \cap Tennis)}{P(Tennis)} = \frac{0.24}{0.6} = 0.40 = 40\%$$

Independence

Definition

Events A and B are independent, if information about one does not affect the other. This is;

$$P(A \mid B) = P(A)$$

$$P(B \mid A) = P(B)$$

This is equivalent to, events A and B are independent if and only if

$$P(A \cap B) = P(A)P(B)$$

Therefore, to obtain the probability that two independent events will both occur, we simply find the product of their individual probabilities.

Multiplication Rule

Definition

This rule follows directly from the definition of conditional probability;

$$P(A \cap B) = P(A)P(B \mid A)$$

or

$$P(A \cap B) = P(B)P(A \mid B)$$

Example

What is the probability that two female students will be selected at random to participate in a certain research project, from a class of 7 males and 3 female students?

Solution

First we need to define events; Let;

A = the first student selected is a female
B = the second student selected is a female

$$P(A \cap B) = P(A)P(B \mid A) = \frac{3}{10} \times \frac{2}{9} = \frac{6}{90}$$

$$P(A \cap B) = \frac{1}{15}$$

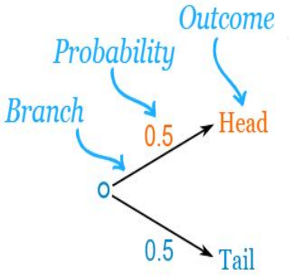
Addition Rule

For any two events A and B;

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Probability Trees

This is a useful device to calculate probabilities when using the probability rules.



- We multiply probabilities along the branches.
- We add probabilities down columns.

Example

Student G wakes up late on average 3 days in every 5 days.

If G wakes up late, the probability G is late for school $= \frac{9}{10}$

If G does not wakes up late, the probability G is late for school $= \frac{3}{10}$

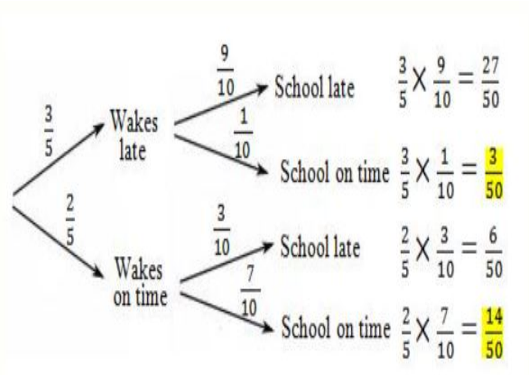
On what percent of days does G get to school on time?

Solution

The probability G wakes up late $= \frac{3}{5}$

The probability G wakes up on time $= \frac{2}{5}$

If we draw Probability Tree we will get;



Solution cont.

Thus, the probability G gets to school on time;

$$P(G) = \frac{3}{50} + \frac{14}{50} = \frac{17}{50} = \frac{34}{100}$$

As a result, Student G gets to school on time 34% of the time.

Example

A Ph.D. graduate has applied for a job with two universities: A and B. The graduate feels that she has a 60% chance of receiving an offer from university A and a 50% chance of receiving an offer from university B. If she receives an offer from university B, she believes that she has an 80% chance of receiving an offer from university A.

1. What is the probability that both universities will make her an offer?
2. What is the probability that at least one university will make her an offer?
3. If she receives an offer from university B, what is the probability that she will not receive an offer from university A?

*This example includes new questions which is not in presentation.

Solution

From question we have;

$$P(A) = 0.6$$

$$P(B) = 0.5$$

$$P(A \mid B) = 0.8$$

1. $P(A \cap B) = ?$

$$P(A \cap B) = P(B)P(A \mid B)$$

$$P(A \cap B) = 0.5 \times 0.8 = 0.4$$

2. $P(A \cup B) = ?$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.6 + 0.5 - 0.4 = 0.7$$

Solution cont.

(iii) If she receives an offer from university B, what is the probability that she will not receive an offer from university A?

Let;

P(D) = receiving offer from university B and not receiving offer from university A.

P(B) = receiving offer from university B

$P(A \cap B)$ = receiving offer from university A and receiving offer from university B

Then P(D) will be;

$$P(D) = P(B) - P(A \cap B)$$

$$P(D) = 0.5 - 0.4 = 0.1$$

Solution cont.

Let;

$P(E) =$ not receiving offer from university A, Then;

$$P(E) = 1 - 0.6 = 0.4$$

Let;

$P(B \mid E) =$ receiving offer from university B | not receiving offer from university A

Then $P(B \mid E)$ will be;

$$P(B \mid E) = \frac{\text{receiving offer from uni. B and not receiving offer from uni. A}}{\text{not receiving offer from university A}}$$

$$P(B \mid E) = \frac{P(D)}{P(E)} = \frac{0.1}{0.4} = 0.25$$

References

Books

- Probability and Statistics for Engineers and Scientists 9th edition
- Head First Statistics 1st Edition

Web Sources

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