Recap:

Let us recap ourselves:

- Started with free theory; write Lagrangian.
- Promote the fields & conjugate mometa into operators via a & $a^\dagger$.
- Introduced interaction term & interaction picture.
- Evolve in the interaction picture.
- Time ordering comes naturally.
- Time ordering has a relation between normal ordering.

$$T(\varphi_1 \varphi_2 \cdots \varphi_n) = N(\varphi_1 \cdots \varphi_n) + \text{all possible contraction.}$$

$$\varphi_1 \cdots \varphi_i \cdots \varphi_j \cdots \varphi_n = \Delta_F(x_i - x_j)\,\underbrace{\varphi_1 \cdots \varphi_n}_{\text{without } \varphi_i \,\&\, \varphi_j}$$

Example:

$$\mathcal{L} = \partial_\mu \psi^* \partial^\mu \psi + \partial_\mu \varphi \partial^\mu \varphi - M^2 \psi^* \psi - \frac{1}{2} m^2 \varphi^2 - g \psi^* \psi \varphi.$$

$\psi, \psi^* \to$ complex scalars.

$$\psi_I(x) = \int \frac{d^3\vec{p}}{(2\pi)^3} \left(b_{\vec{p}}\, e^{-i p \cdot x} + c_{\vec{p}}^\dagger\, e^{i p \cdot x} \right) \frac{1}{\sqrt{2E_{\vec{p}}}}$$

$$\psi_I^\dagger(x) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left(c_{\vec{p}}\, e^{-i p \cdot x} + b_{\vec{p}}^\dagger\, e^{i p \cdot x} \right)$$

$$\varphi_I(x) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left(a_{\vec{p}}\, e^{-i p \cdot x} + a_{\vec{p}}^\dagger\, e^{i p \cdot x} \right)$$

$$\cancel{\langle f|U(t_+,t_-)|i\rangle \;,\; U(t_+,t_-) = T\left(\exp\left(-i\int_{t_-}^{t_+} dt'\, H_I(t')\right)\right)}$$

$$\cancel{= \langle f|\mathbb{1} - ig\, T\left(\int_{t_-}^{t_+} dt'\, \psi^* \psi\, \varphi\right) + \frac{(-ig)^2}{2}\int_{t_-}^{t_+}}$$

$$\langle f|U(t_+,t_-)|i\rangle$$

where $U(t_+,t_-) = T\left(\exp\left(-i\int_{t_-}^{t_+} dt'\, H_I(t')\right)\right)$

$= T\left(\exp\left(-i\int_{t_-}^{t_+} dt' \int d^3x\, \mathcal{H}_I(x)\right)\right)$

$$= \langle f|\mathbb{1}|i\rangle + \langle f|(-ig)\, T\left(\int d^4x\, \psi_I^*(x)\psi_I(x)\,\varphi_I(x)\right)|i\rangle$$

$$+ \frac{(-ig)^2}{2}\langle f|T\left(\int d^4x_1\, d^4x_2\, \psi_I^*(x_1)\psi_I(x_1)\,\varphi_I(x_1)\,\psi_I^*(x_2)\psi_I(x_2)\,\varphi(x_2)\right)|i\rangle$$

Let us look at the first term:

$$\langle f|i\rangle$$

$$= \langle \vec{p}_1{}', \vec{p}_2{}' | \vec{p}_1, \vec{p}_2\rangle$$

$$= \langle 0| b_{\vec{p}_1{}'}\, b_{\vec{p}_2{}'}\, b^\dagger_{\vec{p}_1}\, b^\dagger_{\vec{p}_2}|0\rangle \times \underbrace{\sqrt{2E_{\vec{p}_1{}'}}\,\sqrt{2E_{\vec{p}_2{}'}}\,\sqrt{2E_{\vec{p}_1}}\,\sqrt{2E_{\vec{p}_2}}}_{\#}$$

$$= \#\Big[\langle 0| b_{\vec{p}_1{}'}\,(b^\dagger_{\vec{p}_1}\, b_{\vec{p}_2{}'})\, b^\dagger_{\vec{p}_2}|0\rangle + (2\pi)^3\delta^{(3)}(\vec{p}_1 - \vec{p}_2{}')\, \langle 0| b_{\vec{p}_1{}'}\, b^\dagger_{\vec{p}_2}|0\rangle\Big]$$

$$= \#\Big[\underbrace{\langle 0| b^\dagger_{\vec{p}_1}\, b_{\vec{p}_1{}'}\, b_{\vec{p}_2{}'}\, b^\dagger_{\vec{p}_2}|0\rangle}_{0} + (2\pi)^3\delta^{(3)}(\vec{p}_1 - \vec{p}_1{}')\,\langle 0| b_{\vec{p}_2{}'}\, b^\dagger_{\vec{p}_2}|0\rangle$$

$$+ (2\pi)^3\delta^{(3)}(\vec{p}_1 - \vec{p}_2{}')\,\langle 0| b_{\vec{p}_1{}'}\, \underbrace{b^\dagger_{\vec{p}_2}|0\rangle}_{\frac{|\vec{p}_2\rangle}{\sqrt{2E_{\vec{p}_2}}}}\Big]$$

$$= \sqrt{2E_{\vec{p}_1}}\,\sqrt{2E_{\vec{p}_1{}'}}\,\langle p_2'|p_2\rangle (2\pi)^3\delta^{(3)}(\vec{p}_1 - \vec{p}_1{}')$$

$$+ \sqrt{2E_{\vec{p}_1}}\,\sqrt{2E_{\vec{p}_2{}'}}\,(2\pi)^3\delta^{(3)}(\vec{p}_1 - \vec{p}_2{}')\,\langle p_1'|p_2\rangle$$

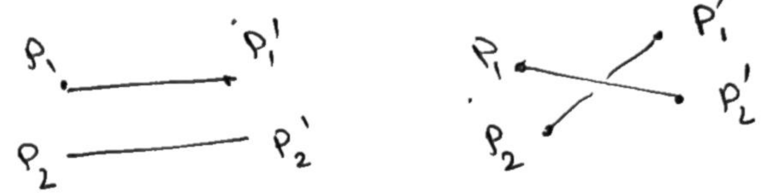
$$= \langle p_2'|p_2\rangle\langle p_1|p_1'\rangle + \langle p_1'|p_2\rangle\langle p_1|p_2\rangle.$$

2 It basically says

$$p_1 \to p_1' \quad \& \quad p_2 \to p_2'$$

$$p_1 \to p_2' \quad \& \quad p_2 \to p_1'$$

In other words, they ~~are~~ have evolved without interacting.



~~1st~~ 2nd term

$$\int d^4x \, -ig \langle f | T(\cancel{\phi^*} \psi^*(x) \psi(x) \varphi(x)) | i \rangle$$

$$\langle 0 | b_{\vec{p}_1'} b_{\vec{p}_2'} (\cdots ( ) a_{\vec{q}} + ( ) a^\dagger_{\vec{q}}) b^\dagger_{\vec{p}_1} b^\dagger_{\vec{p}_2} | 0 \rangle$$

annihilates

annihilates.

2nd term is zero.

hird term

$$\frac{(-ig)^2}{2}\langle f|\, T\int d^4x_1\, d^4x_2\, \psi_I^*(x_1)\psi_I(x_1)\varphi(x_1)\,\psi_I^*(x_2)\psi_I(x_2)\varphi(x_2)\,|i\rangle .$$

~~by this with or~~

$$= \frac{(-ig)^2}{2}\langle f|\, N(\psi\psi\varphi\psi\psi\varphi + \text{contractions})|i\rangle .$$

Let us notice one property:

$$\langle f|\, b^\dagger_{q_1} b^\dagger_{q_2} b^\dagger_{q_3}\cdots b_{q_n}|i\rangle$$

$$= \langle 0|\, b_{\vec p_1'}\, \underbrace{b_{\vec p_2'}\, b^\dagger_{\vec q_1}}\, b^\dagger_{\vec q_2}\, b^\dagger_{\vec q_3}\cdots b_{\vec q_n}|i\rangle$$

$$= \langle 0|\, b_{\vec p_1'}\left(b^\dagger_{q_1} b_{\vec p_2'} + (2\pi)^3\delta^{(3)}(\vec q_1-\vec p_2')\right) b^\dagger_{\vec q_2}\cdots b_{\vec q_n}|i\rangle .$$

$$= \langle 0|\left(\cancel{b^\dagger_{q_1} b_{\vec p_1'}}^{\,0} + (2\pi)^3\delta^{(3)}(q_1-\vec p_1')\right) b_{\vec p_2'}\, b^\dagger_{\vec q_2}\, b^\dagger_{\vec q_3}\cdots b_{\vec q_n}|i\rangle$$

$$+ \langle 0|(2\pi)^3\delta^{(3)}(\vec q_1-\vec p_2')\, b_{\vec p_1'}\, b^\dagger_{\vec q_2}\, b^\dagger_{\vec q_3}\cdots b_{\vec q_n}|i\rangle .$$

$$= \langle 0|(2\pi)^3\delta^{(3)}(\vec q_1-\vec p_1')(2\pi)^3\delta^{(3)}(\vec q_2-\vec p_2')\, b^\dagger_{\vec q_3}\cdots b_{\vec q_n}|i\rangle$$

$$+ \langle 0|(2\pi)^3\delta^{(3)}(\vec q_1-\vec p_2')(2\pi)^3\delta^{(3)}(\vec p_1'-\vec q_2)\, b_{\vec q_3}\cdots b_{\vec q_n}|i\rangle$$

for this process

Only term in the operator with exactly 2 $b^\dagger$ survive.

" " " " 2 b "

Only " "

φ term has a & $a^\dagger$ commute with all b's & $b^\dagger$'s

So, vanishes.

So, only non-vanishing terms come from.

$$N(\psi\psi\psi\psi)\,\overbrace{\varphi\,\varphi} .$$

$$\boxed{\frac{(-ig)^2}{2}\langle f|\, N(\psi^*(x)\psi(x)\psi^*(y)\psi(y))|i\rangle\, \Delta_F^{\varphi}(x-y).}$$

(4) $N\left(\psi^{*}(x_1)\,\psi(x_1)\,\psi^{*}(x_2)\,\psi(x_2)\right)$

$$\psi_I = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec p}}}\, b_{\vec p}\, e^{\cdots} + \cdots$$

$$\psi_I^{\dagger} = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec p}}}\, b^{\dagger}_{\vec p}\, e^{ip\cdot\cdots}$$

$$= \int \left(\prod_{i=1}^{4}\frac{d^3q_i}{(2\pi)^3}\frac{1}{\sqrt{2E_{q_i}}}\right)\left[b^{\dagger}_{\vec q_1}\, b^{\dagger}_{\vec q_3}\, b_{\vec q_2}\, b_{\vec q_4}\right]$$

$$d^4x_1\, d^4x_2 \quad e^{iq_1\cdot x_1}\, e^{-iq_2\cdot x_1}\, e^{+iq_3\cdot x_2}\, e^{-iq_4\cdot x_2}$$

~~Here we define another useful contraction with state.~~

$$\langle 0|\, b_{\vec p_1'}\, b_{\vec p_2'}\, b^{\dagger}_{\vec q_1}\, b^{\dagger}_{\vec q_3}\, b_{\vec q_2}\, b_{\vec q_4}\, b^{\dagger}_{\vec p_1}\, b^{\dagger}_{\vec p_2}\,|0\rangle .$$

$$= \langle 0|\Big[(2\pi)^3\,\delta^{(3)}(\vec p_1'-\vec q_1)\,(2\pi)^3\delta^{(3)}(\vec p_2'-\vec q_3) + (2\pi)^3\,\delta^{(3)}(\vec p_1'-\vec q_3)\,(2\pi)^3\,\delta^{(3)}(\vec p_2'-\vec q_1)\Big]$$

$$\times \left[\delta(q_2-p_1)\,\delta(q_4-p_2) + \delta(q_2,p_2)\,\delta(q_4,p_1)\right]|0\rangle .$$

Putting everything together:

$$\frac{(-ig)^2}{2}\int d^4x_1\, d^4x_2\;\; \Delta_F^{\varphi}(x_1-x_2)\left(e^{ip_1'\cdot x_1+p_2'\cdot x_2} + e^{ip_1'\cdot x_2+ip_2'\cdot x_2}\right)\left(e^{-ip_1\cdot x_1-ip_2\cdot x_2} + e^{-ip_1\cdot x_2-ip_2\cdot x_1}\right)$$

$$= \frac{(-ig)^2}{2}\int d^4x_1\, d^4x_2\;\; \Delta_F^{\varphi}(x_1-x_2)\left[e^{ix_1\cdot(p_1'-p_1)+ix_2(p_2'-p_2)} + e^{ix_1\cdot(p_2'-p_1)+ix_2(p_1'-p_2)} + (x_1\leftrightarrow x_2)\right]$$

$$\Delta_F^{\varphi}(x_1-x_2) = \int\frac{d^4k}{(2\pi)^3}\;\frac{i\,e^{k\cdot(x_1-x_2)}}{k^2-m^2+i\epsilon}.$$

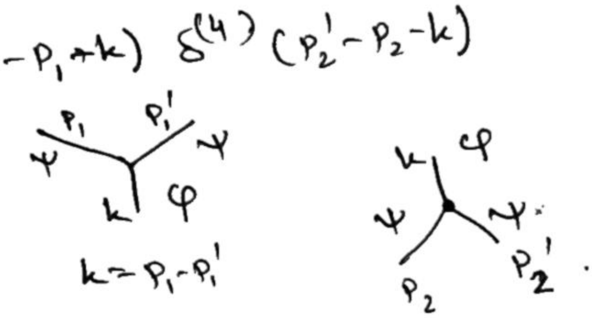
integrate over the x's

$$= (-ig)^2\int\frac{d^4k}{(2\pi)^4}\;\frac{i(2\pi)^8}{k^2-m^2+i\epsilon}\Big[\delta^{(4)}(p_1'-p_1+k)\,\delta^{(4)}(p_2'-p_2-k) + \delta^{(4)}(p_2'-p_1+k)\,\delta^{(4)}(p_1'-p_2-k)\Big].$$

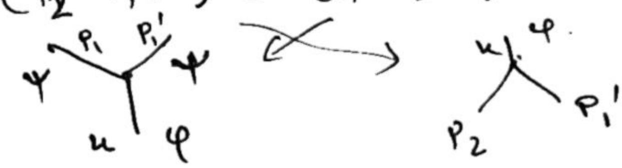
$$= i(-ig)^2\left[\frac{1}{(p_1-p_1')^2-m^2+i\epsilon}+\frac{1}{(p_1-p_2')^2-m^2+i\epsilon}\right]$$

$$(2\pi)^4\,\delta^{(4)}(p_1+p_2-p_1'-p_2').$$

$$\delta^{(4)}(p_1'-p_1+k)\;\delta^{(4)}(p_2'-p_2-k)$$



$$\delta^{(4)}(p_2'-p_1+k)\;\delta^{(4)}(p_1'-p_2-k)$$



The ~~various~~ terms in the perturbative expansion can be represented diagramatically

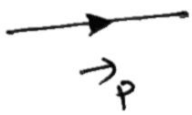
- Draw an external line for each particle in the ~~internal &~~ initial & final state. ($|i\rangle$ & $|f\rangle$).
- with momenta $p_1, p_2 \ldots p_n$.
- Draw vertex corresponding each interation term. $(-ig)$.
- Draw propagator $\xrightarrow{p} = \dfrac{i}{p^2-m^2+i\epsilon}$.
- Impose momentum conservation at each vertex
- Integrate over each undetermined momentum. $\displaystyle\int\frac{d^4k}{(2\pi)^4}$.

QED

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi - e A_\mu \bar{\psi}\gamma^\mu \psi.$$

Propagators.

$$\frac{i\,\eta_{\mu\nu}}{q^2 + i\epsilon}$$



$$\frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$$

Vertex:

$-ie\gamma^\mu.$

External lines:

incoming photons: $\epsilon_\mu(\vec{p})$

outgoing ": $\epsilon^*_\mu(\vec{p})$

incoming fermion: $u^s(p)$

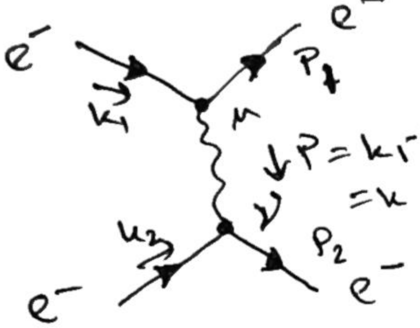
outgoing ": $\bar{u}^s(p)$

incoming ~~fer~~ anti-fermion $\bar{v}^s(p)$

outgoing " $v^s(p)$.

Example:

1. $e^- e^- \longrightarrow e^- e^-$



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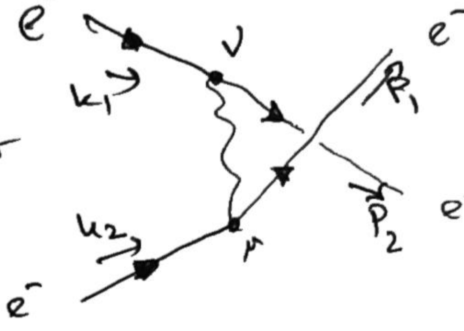

$i\mathcal{M}_1$ $\qquad\qquad$ $i\mathcal{M}_2$.

$$i\mathcal{M} = i\mathcal{M}_1 + i\mathcal{M}_2 .$$

$$i\mathcal{M}_1 = \bar{u}(p_1)(-ie\gamma^\mu)u(k_1)\left[-\frac{i\eta_{\mu\nu}}{p^2+i\epsilon}\right]$$

$$\bar{u}(p_2)(-ie\gamma^\nu)u(k_2)$$

$$= \frac{ie^2}{p^2+i\epsilon}\,\bar{u}(p_1)\gamma^\mu u(k_1)\times\bar{u}(p_2)\gamma_\mu u(k_2)$$

$$-i\mathcal{M}_1^\dagger = \frac{-ie^2}{p^2}\left[\bar{u}(p_1)\gamma^{\mu'}u(k_1)\right]^\dagger\times\left[\bar{u}(p_2)\gamma_{\mu'}u(k_2)\right]^\dagger$$

$$\left[\bar{u}(p_1)\gamma^{\mu'}u(k_1)\right]^\dagger$$

$$= u(k_1)^\dagger\gamma^{\mu'\dagger}\gamma^{0\dagger}u(p_1)$$

$$= u(k_1)^\dagger\gamma^0\gamma^{\mu'}\gamma^0\gamma^0 u(p_1)$$

$$= \bar{u}(k_1)\gamma^{\mu'}u(p_1)$$

$$\left[\begin{array}{l} \gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0 \\ \bar{u} = u^\dagger\gamma^0 \\ \gamma^{0\dagger} = \gamma^0 \end{array}\right.$$

$$-i\mathcal{M}_1^\dagger = \frac{-ie^2}{p^2}\,\bar{u}(k_1)\gamma^{\mu'}u(p_1)\times\bar{u}(k_2)\gamma_{\mu'}u(p_2)$$

$e^-e^- \to e^-e^-$

$$|i\mathcal{M}|^2 = |i\mathcal{M}_1 + i\mathcal{M}_2|^2$$
$$= (-i\mathcal{M}_1^\dagger)(i\mathcal{M}_1) + (-i\mathcal{M}_2^\dagger)(i\mathcal{M}_2) + 2\mathrm{Re}(i\mathcal{M}_1^\dagger(i\mathcal{M}_2))$$

$$|\mathcal{M}_1|^2 = \frac{e^4}{p^4}\left[\bar{u}(k_1)\gamma^{\mu'} u(p_1)\,\bar{u}(p_1)\gamma^{\mu} u(k_1)\right]$$
$$\times\left[\bar{u}(k_2)\gamma_{\mu'} u(p_2)\,\bar{u}(p_2)\gamma_{\mu} u(k_2)\right]$$

Let us write

$$\bar{w}_\gamma(\mu, p_1, k_1)\; u_\gamma(k_1)$$
$$\;\;\shortparallel\qquad\qquad \shortparallel$$
$$\langle w|u\rangle.$$

$$\langle w|u\rangle$$
$$= \sum_i \langle w|e_i\rangle\langle e_i|u\rangle$$
$$= \sum_{i,j} \delta_{ij}\langle w|e_i\rangle\langle e_j|u\rangle$$
$$= \sum_{i,j} \delta_{ij}\langle e_j|u\rangle\langle w|e_i\rangle$$
$$= \sum_{i,j} \delta_{ij}\langle e_j|\underbrace{(|u\rangle\langle w|)}_{\text{operator}}|e_i\rangle$$
$$\qquad\qquad (|u\rangle\langle w|)_{ij}$$
$$= \sum_{i,j}\delta_{ij}(|u\rangle\langle w|)_{ij} = \mathrm{Tr}(|u\rangle\langle w|)$$

$$|\mathcal{M}_1|^2 = \frac{e^4}{p^4}\,\mathrm{Tr}\left[u(k_1)\bar{u}(k_1)\gamma^{\mu'} u(p_1)\bar{u}(p_1)\gamma^{\mu}\right]$$
$$\times\,\mathrm{Tr}\left[\cancel{\bar{u}(k_2)\gamma_{\mu'}}\; u(k_2)\bar{u}(k_2)\gamma_{\mu'} u(p_2)\bar{u}(p_2)\gamma_\mu\right]$$

Let us now remember the completeness relations.

$$\sum_{\substack{\cancel{\text{polarization}}\\ \text{helicities}}} u(k_1)\,\bar{u}(k_1) = \not{k}_1 + m$$

$e^-e^- \to e^-e^-$

$$|\mathcal{M}_1|^2 = \frac{e^4}{p^4}\mathrm{Tr}\left[(\not{k}_1+m)\,\gamma^{\mu'}(\not{p}_1+m)\,\gamma^{\mu}\right]$$
$$\times \mathrm{Tr}\left[(\not{k}_2+m)\,\gamma_{\mu'}(\not{p}_2+m)\,\gamma_\mu\right]$$

$$\boxed{\cancel{=\frac{e^4}{p^4}\mathrm{Tr}}}$$

$$= \frac{e^4}{p^4}\left[\mathrm{Tr}[\not{k}_1\gamma^{\mu'}\not{p}_1\gamma^{\mu}] + m^2\,\mathrm{Tr}[\gamma^{\mu'}\gamma^{\mu}]\right]$$
$$\times\left(\mathrm{Tr}(\not{k}_2\gamma_{\mu'}\not{p}_2\gamma_\mu) + m^2\,\mathrm{Tr}[\gamma_{\mu'}\gamma_\mu]\right)$$

$\mathrm{Tr}[\text{odd } \gamma \text{ matrices}] = 0.$

$\mathrm{Tr}[\gamma^\mu\gamma^\nu] = 4\eta^{\mu\nu}.$

$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta]$
$= 4(\eta^{\mu\nu}\eta^{\alpha\beta} - \eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\beta})$

$$= \frac{e^4}{p^4}\left[4(k_1^{\mu'}p_1^{\mu} - k_1\cdot p_1\,\eta^{\mu'\mu} + k_1^{\mu}p_1^{\mu'}) + 4m^2\eta^{\mu'\mu}\right]$$
$$\times\left[4(k_{2\mu'}p_{2\mu} - k_2\cdot p_2\,\eta_{\mu'\mu} + k_{2\mu}p_{2\mu'}) + 4m^2\eta_{\mu'\mu}\right]$$

$$= \frac{16e^4}{p^4}\Big[k_1\cdot k_2\, p_1\cdot p_2 - \cancel{k_2\cdot p_2}\, k_1\cdot p_1 + k_1\cdot p_2\, k_2\cdot p_1 + k_1\cdot p_1\, m^2$$
$$- k_1\cdot p_1\, \cancel{k_2}\cdot p_2 + 4\, \cancel{k_2\cdot p_2}\, k_1\cdot p_1 - k_1\cdot p_1\,\cancel{k_2}\cdot p_2 - 4m^2 k_1\cdot p_1$$
$$+ k_1\cdot p_2\, k_2\cdot p_1 - k_1\cdot p_1\,\cancel{k_2}\cdot p_2 + k_1\cdot k_2\, p_1\cdot p_2 + m^2 k_1\cdot p_1$$
$$+ m^2 k_2\cdot p_2 - 4k_2\cdot p_2\, m^2 + m^2 k_2\cdot p_2 + m^4\, 4\Big]$$

$$s = (k_1+k_2)^2 = k_1^2 + k_2^2 + 2k_1\cdot k_2 = s. \Rightarrow k_1\cdot k_2 = \frac{s-2m^2}{2} = p_1\cdot p_2.$$

$$t = p^2 = (k_1-p_1)^2 = k_1^2 + p_1^2 - 2k_1\cdot p_1 = t \Rightarrow k_1\cdot p_1 = \frac{2m^2-t}{2}.$$

$$u = (k_1-p_2)^2 = k_1^2 + p_2^2 - 2k_1\cdot p_2 = u \Rightarrow k_1\cdot p_2 = \frac{2m^2-u}{2}.$$

$$= \frac{16e^4}{p^4}\left[\left(\frac{s-2m^2}{2}\right)^2 + \left(\frac{2m^2-u}{2}\right)^2 - 2m^2\left(\frac{2m^2-t}{2}\right) - 2m^2\left(\frac{2m^2-u}{2}\right) + 4m^4\right]$$

$$= \frac{16e^2}{t^2}\left[\left(\frac{s-2m^2}{2}\right)^2 + \left(\frac{2m^2-u}{2}\right)^2 + m^2(t+u+4m^2)\right]$$

$$|M_2|^2 = |M_1|^2 (p_1 \leftrightarrow p_2) \Rightarrow t \leftrightarrow u$$

$$= \frac{16e^2}{u^2 \cancel{p^4}} \left[\frac{s-2m^2}{2} + \left(\frac{2m^2-t}{2}\right)^2 + m^2(t+u+4m^2)\right]$$

$\cancel{M_1^\dagger M_2 =}$

$$iM_2 = \bar{u}(p_1)(-ie\gamma^\mu)\, u(k_2) \left(\frac{-i\eta_{\mu\nu}}{p'^2}\right) \bar{u}(p_2)(-ie\gamma^\nu)\, u(k_1)$$

$$= \frac{+ie^2}{u}\, \bar{u}(p_1)\gamma^\mu u(k_2)\, \bar{u}(p_2)\gamma_\mu u(k_1).$$

$$M_1^\dagger M_2 = \frac{e^2}{\cancel{p}ut}\, \bar{u}(k_1)\gamma^{\mu'} u(p_1)\, \bar{u}(p_1)\gamma^\mu u(k_2)\, \bar{u}(k_2)\gamma_{\mu'} u(p_2)\, \bar{u}(p_2)\gamma_\mu u(k_1)$$

$$= \frac{e^2}{ut} \mathrm{Tr}\left[(\not{k}_1+m)\gamma^{\mu'}(\not{p}_1+m)\gamma^\mu(\not{k}_2+m)\gamma_{\mu'}(\not{p}_2+m)\gamma_\mu\right]$$

<u>Cross-section</u> If two beams of particles collide, they scattering & the ~~fraction~~ total number of scattering events

$N = F\sigma$

F = flux per unit area per unit time.

Probability $P \sim \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle \langle i|i\rangle}$

$$d\sigma = \frac{1}{2E_1\, 2E_2\, |\vec{v}_1 - \vec{v}_2|} |M_{fi}|^2 \int \frac{d^3\vec{p}_i}{2E_{\vec{p}_i}}$$

↳ all final state particles.