# Problem Sheet 4

**Solve any three!**

1. The scalar Yukawa Lagrangian, as discussed in the lecture, is given by

$$\mathcal{L} = \partial^\mu \psi^* \partial_\mu \psi + \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - M^2 \psi^* \psi - \frac{1}{2} m^2 \phi^2 - g\psi^* \psi \phi, \tag{1}$$

where $\psi$ and $\phi$ are, respectively, complex and real scalar fields. With the Feynman rules stated in the lecture, find the leading order scattering amplitude for the processes

   (a) $\psi\phi \to \psi\phi$
   (b) $\psi\psi^* \to \phi\phi$

2. Use the properties (i) $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ and (ii) cyclic property of trace, to do the following:

   (a) Show that the trace of any odd number of $\gamma^\mu$ ($\mu = 0, 1, 2, 3$) is zero.
   (b) Find an expression for $\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma)$ in terms of Minkowski metric.
   (c) Find an expression for $\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\alpha \gamma^\beta)$.
   (d) Can you guess how many additive terms it will have in the expression for the trace of eight $\gamma^\mu$ matrices?

3. The Lagrangian for QED is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - m)\psi - e\bar{\psi}\gamma^\mu A_\mu \psi \tag{2}$$

Using the Feynman rules stated in the lecture, please find the leading order scattering amplitude for the following:

   (a) Compton Scattering: $e^-\gamma \to e^-\gamma$.
   (b) Bhabha Scattering: $e^-e^+ \to e^-e^+$.

4. The Lagrangian for Scalar QED (complex scalar field $\phi$ interacting with field $A^\mu$.) is given by-

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + (D_\mu \phi)^* (D_\mu \phi) - m_\phi^2 \phi^* \phi \tag{3}$$

where $D_\mu = \partial_\mu + ieA_\mu$ is the usual gauge-covariant derivative.

   (a) Compute the Interaction Lagrangian .

(b) Derive the Feynman rules for the vertices and propagators of the above theory.

**Deadline: 19/09/2024**

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