Homework 1

Extended Bridge to CS, Spring 2025

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Question 1:

A. Convert the following numbers to their decimal representation. Show your work.

1. 10011011_2

$$
\begin{aligned}
&= 1 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 0 \cdot 2^5 + 0 \cdot 2^6 + 1 \cdot 2^7 \\
&= 1 + 2 + 0 + 8 + 16 + 0 + 0 + 128 \\
&= 155_{10}
\end{aligned}
$$

2. 456_7

$$
\begin{aligned}
&= 6 \cdot 7^0 + 5 \cdot 7^1 + 4 \cdot 7^2 \\
&= 6 + 35 + 196 \\
&= 237_{10}
\end{aligned}
$$

3. $38A_{16}$

$$
\begin{aligned}
&= 10 \cdot 16^0 + 8 \cdot 16^1 + 3 \cdot 16^2 \\
&= 10 + 128 + 768 \\
&= 906_{10}
\end{aligned}
$$

4. 2214_5

$$
\begin{aligned}
&= 4 \cdot 5^0 + 1 \cdot 5^1 + 2 \cdot 5^2 + 2 \cdot 5^3 \\
&= 4 + 5 + 50 + 250 \\
&= 309_{10}
\end{aligned}
$$

B. Convert the following numbers to their binary representation:

1. $69_{10} = (1000101)_2$

   - $69 \div 2 = 34$ remainder 1
   - $34 \div 2 = 17$ remainder 0
   - $17 \div 2 = 8$ remainder 1
   - $8 \div 2 = 4$ remainder 0
   - $4 \div 2 = 2$ remainder 0
   - $2 \div 2 = 1$ remainder 0
   - $1 \div 2 = 0$ remainder 1

   Reading the remainders from bottom to top gives $(1000101)_2$.

2. $485_{10} = (111100101)_2$

   - $485 \div 2 = 242$ remainder 1
   - $242 \div 2 = 121$ remainder 0
   - $121 \div 2 = 60$ remainder 1
   - $60 \div 2 = 30$ remainder 0
   - $30 \div 2 = 15$ remainder 0
   - $15 \div 2 = 7$ remainder 1
   - $7 \div 2 = 3$ remainder 1
   - $3 \div 2 = 1$ remainder 1
   - $1 \div 2 = 0$ remainder 1

   Reading the remainders from bottom to top gives $(111100101)_2$.

3. $6D1A_{16} = (0110110100011010)_2$

   - $A_{16} = 10_{10} = 1010_2$
   - $1_{16} = 0001_2$
   - $D_{16} = 13_{10} = 1101_2$
   - $6_{16} = 0110_2$

   Therefore, $6D1A_{16} = (0110110100011010)_2$

C. Convert the following numbers to their hexadecimal representation:

1. $1101011_2 = 6B_{16}$

   - $1101011_2 = 01101011_2$
   - $0110_2 = 0 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^2 + 0 \cdot 2^3 = 6_{16}$
   - $1011_2 = 1 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 1 \cdot 2^3 = B_{16}$

   Therefore, $1101011_2 = 6B_{16}$

2. $895_{10} = 37F_{16}$

   - $895 \div 16 = 55$ remainder 15
   - $55 \div 16 = 3$ remainder 7
   - $3 \div 16 = 0$ remainder 3
   - $15_{10} = F_{16}$

   Therefore, $895_{10} = 37F_{16}$

Question 2:

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 = 14303_8$

$$\begin{array}{r} {}^{1}\ \ {}^{111} \\ 7566_8 \\ +\,4515_8 \\ \hline 1\,4303_8 \end{array}$$

2. $10110011_2 + 1101_2 = 11000000_2$

$$\begin{array}{r} {}^{111111} \\ 10110011_2 \\ +\,00001101_2 \\ \hline 11000000_2 \end{array}$$

3. $7A66_{16}$+$45C5_{16}$ $= C02B_{16}$

$$\begin{array}{r} {}^{1\ 1}\phantom{6\ 6_{16}} \\ 7\,A\,6\,6_{16} \\ +\ 4\,5\,C\,5_{16} \\ \hline C\,0\,2\,B_{16} \end{array}$$

4. 3022_5 - 2433_5 $= 34_5$

$$\begin{array}{r} {}^{241} \\ \cancel{3}\cancel{0}\cancel{2}2_5 \\ -\,2433_5 \\ \hline 0034_5 \end{array}$$

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. $124_{10} = (01111100)_{\text{8 bit 2's comp}}$

   - $124 \div 2 = 62$ remainder 0
   - $62 \div 2 = 31$ remainder 0
   - $31 \div 2 = 15$ remainder 1
   - $15 \div 2 = 7$ remainder 1
   - $7 \div 2 = 3$ remainder 1
   - $3 \div 2 = 1$ remainder 1
   - $1 \div 2 = 0$ remainder 1

   Reading the remainders from bottom to top gives $(01111100)_{\text{8 bit 2's comp}}$.

2. $-124_{10} = (10000100)_{\text{8 bit 2's comp}}$

   - $124_{10} = (01111100)_2$
   - Flip the bits gives $(10000011)_2$
   - Add 1 to complement

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 10000011_2 \\ +\,00000001_2 \\ \hline 10000100_2 \end{array}$$

   Therefore, $-124_{10} = (10000100)_{\text{8 bit 2's comp}}$.

3. $109_{10} = (01101101)_{\text{8 bit 2's comp}}$

   - $109 \div 2 = 54$ remainder 1
   - $54 \div 2 = 27$ remainder 0
   - $27 \div 2 = 13$ remainder 1
   - $13 \div 2 = 6$ remainder 1
   - $6 \div 2 = 3$ remainder 0
   - $3 \div 2 = 1$ remainder 1
   - $1 \div 2 = 0$ remainder 1

   Reading the remainders from bottom to top gives $(01101101)_{\text{8 bit 2's comp}}$.

4. $-79_{10} = (10110001)_{\text{8 bit 2's comp}}$

   - $79 \div 2 = 39$ remainder 1
   - $39 \div 2 = 19$ remainder 1

- $19 \div 2 = 9$ remainder 1
- $9 \div 2 = 4$ remainder 1
- $4 \div 2 = 2$ remainder 0
- $2 \div 2 = 1$ remainder 0
- $1 \div 2 = 0$ remainder 1
- Reading the remainders from bottom to top gives $= (01001111)_2$.
- Flip the bits gives $(10110000)_2$
- Add 1 to complement

$$\begin{array}{r} 10110000_2 \\ +\,00000001_2 \\ \hline 10110001_2 \end{array}$$

Therefore, $-79_{10} = (10110001)_{8\text{ bit 2's comp}}$.

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8\text{ bit 2's comp}}$

$$\begin{aligned} &= 0 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 0 \cdot 2^5 + 0 \cdot 2^6 + 0 \cdot 2^7 \\ &= 0 + 2 + 4 + 8 + 16 + 0 + 0 + 0 \\ &= 30_{10} \end{aligned}$$

2. $11100110_{8\text{ bit 2's comp}}$

$$\begin{aligned} &= 0 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^2 + 0 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 + 1 \cdot 2^6 + (-1) \cdot 2^7 \\ &= 0 + 2 + 4 + 0 + 0 + 32 + 64 - 128 \\ &= -26_{10} \end{aligned}$$

3. $00101101_{8\text{ bit 2's comp}}$

$$\begin{aligned} &= 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 + 0 \cdot 2^6 + 0 \cdot 2^7 \\ &= 1 + 0 + 4 + 8 + 0 + 32 + 0 + 0 \\ &= 45_{10} \end{aligned}$$

4. $10011110_{8\text{ bit 2's comp}}$

$$\begin{aligned} &= 0 \cdot 2^0 + 1 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 0 \cdot 2^5 + 0 \cdot 2^6 + (-1) \cdot 2^7 \\ &= 0 + 2 + 4 + 8 + 16 + 0 + 0 - 128 \\ &= -98_{10} \end{aligned}$$

Question 4:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4, sections b, c

   Write a truth table for each expression.

   (b) $\neg(p \vee q)$

p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

   (c) $r \vee (p \wedge \neg q)$

p	q	r	$p \wedge \neg q$	$r \vee (p \wedge \neg q)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

2. Exercise 1.3.4, sections b, d

   Give a truth table for each expression.

   (b) $(p \to q) \to (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

   (d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question 5:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.7, sections b, c

   Consider the following pieces of identification a person might have to apply for a credit card:

   - B: Applicant presents a birth certificate.
   - D: Applicant presents a driver's license.
   - M: Applicant presents a marriage license.

   Write a logical expression for the requirements under the following conditions:

   (b) The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

   $(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$

   (c) Applicant must present either a birth certificate or both a driver's license and a marriage license.

   $B \vee (D \wedge M)$

2. Exercise 1.3.7, sections b – e

   Define the following propositions:

   - s: a person is a senior
   - y: a person is at least 17 years of age
   - p: a person is allowed to park in the school parking lot

   Express each of the following English sentences with a logical expression:

   (b) A person can park in the school parking lot if they are a senior or at least 17 years of age.

   $(s \vee y) \rightarrow p$

   (c) Being 17 years of age is a necessary condition for being able to park in the school parking lot.

   $p \rightarrow y$

   (d) A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

   $p \leftrightarrow (s \wedge y)$

   (e) Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

   $p \rightarrow (s \vee y)$

3. Exercise 1.3.9, sections c, d

   Use the definitions of the variables below to translate each English statement into an equivalent logical expression.

   - y: the applicant is at least 18 years old
   - p: the applicant has parental permission
   - c: the applicant can enroll in the course

   Express each of the following English sentences with a logical expression:

   (c) The applicant can enroll in the course only if the applicant has parental permission.

   $c \to p$

   (d) Having parental permission is a necessary condition for enrolling in the course.

   $c \to p$

Question 6:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.3.6, sections b - d

   Give an English sentence in the form "If. . . then. . . " that is equivalent to each sentence.

   (b) Maintaining a B average is necessary for Joe to be eligible for the honors program.

   If Joe is eligible for the honors program, then he maintains a B average.

   (c) Rajiv can go on the roller coaster only if he is at least four feet tall.

   If Rajiv can go on the roller coaster, then he is at least four feet tall.

   (d) Rajiv can go on the roller coaster if he is at least four feet tall.

   If Rajiv is at least four feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10, sections c – f

   The variable p is true, q is false, and the truth value for variable r is unknown. Indicate whether the truth value of each logical expression is true, false, or unknown.

   (c) $(p \vee r) \leftrightarrow (q \wedge r)$

p	q	r	$p \vee r$	$q \wedge r$	$(p \vee r) \leftrightarrow (q \wedge r)$
T	F	T	T	F	F
T	F	F	T	F	F

   Therefore, $(p \vee r) \leftrightarrow (q \wedge r)$ = false

   (d) $(p \wedge r) \leftrightarrow (q \wedge r)$

p	q	r	$p \wedge r$	$q \wedge r$	$(p \wedge r) \leftrightarrow (q \wedge r)$
T	F	T	T	F	F
T	F	F	F	F	T

   Therefore, $(p \wedge r) \leftrightarrow (q \wedge r)$ = unknown

   (e) $p \rightarrow (r \vee q)$

p	q	r	$r \vee q$	$p \rightarrow (r \vee q)$
T	F	T	T	T
T	F	F	F	F

   Therefore, $p \rightarrow (r \vee q)$ = unknown

   (f) $(p \wedge q) \rightarrow r$

p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow r$
T	F	T	F	T
T	F	F	F	T

   Therefore, $(p \wedge q) \rightarrow r$ = true

Question 7:

Solve Exercise 1.4.5, sections b – d, from the Discrete Math zyBook:

Define the following propositions:

- j: Sally got the job.
- l: Sally was late for her interview
- r: Sally updated her resume.

Express each pair of sentences using logical expressions. Then prove whether the two expressions are logically equivalent.

(b) If Sally did not get the job, then she was late for her interview or did not update her resume.
$\neg j \to (l \vee \neg r)$
If Sally updated her resume and was not late for her interview, then she got the job.
$(r \wedge \neg l) \to j$

j	l	r	$\neg j \to (l \vee \neg r)$	$(r \wedge \neg l) \to j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

Two compound propositions are logically equivalent if they have the same truth value regardless of the truth values of their individual propositions. Therefore, the two expressions are logically equivalent.

(c) If Sally got the job then she was not late for her interview.
$j \to \neg l$
If Sally did not get the job, then she was late for her interview.
$\neg j \to l$

j	l	$j \to \neg l$	$\neg j \to l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

Two compound propositions are logically equivalent if they have the same truth value regardless of the truth values of their individual propositions. Therefore, the two expressions are NOT logically equivalent.

(d) If Sally updated her resume or she was not late for her interview, then she got the job.
$(r \vee \neg l) \rightarrow j$
If Sally got the job, then she updated her resume and was not late for her interview.
$j \rightarrow (r \wedge \neg l)$

j	l	r	$(r \vee \neg l) \rightarrow j$	$j \rightarrow (r \wedge \neg l)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

Two compound propositions are logically equivalent if they have the same truth value regardless of the truth values of their individual propositions. Therefore, the two expressions are NOT logically equivalent.

Question 8:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.5.2, sections c, f, i

   Use the laws of propositional logic to prove the following:

   (c) $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$$\begin{aligned}
(p \to q) \land (p \to r) &\equiv (\neg p \lor q) \land (p \to r) && \text{[Conditional identity]} \\
&\equiv (\neg p \lor q) \land (\neg p \lor r) && \text{[Conditional identity]} \\
&\equiv \neg p \lor (q \land r) && \text{[Distributive law]} \\
&\equiv p \to (q \land r) && \text{[Conditional identity]}
\end{aligned}$$

   (f) $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$$\begin{aligned}
\neg(p \lor (\neg p \land q)) &\equiv \neg((p \lor \neg p) \land (p \lor q)) && \text{[Distributive law]} \\
&\equiv \neg(T \land (p \lor q)) && \text{[Complement law]} \\
&\equiv \neg(p \lor q) && \text{[Identity law]} \\
&\equiv \neg p \land \neg q && \text{[De Morgan's law]}
\end{aligned}$$

   (i) $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

$$\begin{aligned}
(p \land q) \to r &\equiv \neg(p \land q) \lor r && \text{[Conditional identity]} \\
&\equiv (\neg p \lor \neg q) \lor r && \text{[De Morgan's law]} \\
&\equiv r \lor (\neg p \lor \neg q) && \text{[Commutative law]} \\
&\equiv (r \lor \neg p) \lor \neg q && \text{[Associative law]} \\
&\equiv (\neg p \lor r) \lor \neg q && \text{[Commutative law]} \\
&\equiv (\neg p \lor \neg(\neg r)) \lor \neg q && \text{[Double negation law]} \\
&\equiv \neg(p \land \neg r) \lor \neg q && \text{[De Morgan's law]} \\
&\equiv (p \land \neg r) \to \neg q && \text{[Conditional identity]}
\end{aligned}$$

2. Exercise 1.5.3, sections c, d

   Use the laws of propositional logic to prove that each statement is a tautology.

   (c) $\neg r \lor (\neg r \to p)$

$$\begin{aligned}
\neg r \lor (\neg r \to p) &\equiv \neg r \lor (\neg(\neg r) \lor p) && \text{[Conditional identity]} \\
&\equiv \neg r \lor (r \lor p) && \text{[Double negation law]} \\
&\equiv (\neg r \lor r) \lor p && \text{[Associative law]} \\
&\equiv T \lor p && \text{[Complement law]} \\
&\equiv T && \text{[Domination law]}
\end{aligned}$$

   (d) $\neg(p \to q) \to \neg q$

$$
\begin{aligned}
\neg(p \to q) \to \neg q &\equiv \neg(\neg(p \to q)) \vee \neg q && \text{[Conditional identity]} \\
&\equiv (p \to q) \vee \neg q && \text{[Double negation law]} \\
&\equiv (\neg p \vee q) \vee \neg q && \text{[Conditional identity]} \\
&\equiv \neg p \vee (q \vee \neg q) && \text{[Associative law]} \\
&\equiv \neg p \vee T && \text{[Complement law]} \\
&\equiv T && \text{[Domination law]}
\end{aligned}
$$

Table 1: Laws of propositional logic.

Name		
Idempotent laws	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive laws	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws	$p \vee F \equiv p$	$p \wedge T \equiv p$
Domination laws	$p \wedge F \equiv F$	$p \vee T \equiv T$
Double negation law	$\neg(\neg p) \equiv p$	
Complement laws	$p \vee \neg p \equiv T$	$p \wedge \neg p \equiv F$
	$\neg T \equiv F$	$\neg F \equiv T$
De Morgan's laws	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Conditional identities	$p \to q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

Question 9:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.6.3, sections c, d

   Consider the following statements in English. Write a logical expression with the same meaning. The domain is the set of all real numbers.

   (c) There is a number that is equal to its square.

   $\exists x(x = x^2)$

   (d) Every number is less than or equal to its square plus 1.

   $\forall x(x \leq x^2 + 1)$

2. Exercise 1.7.4, sections b - d

   In the following question, the domain is a set of employees who work at a company. Ingrid is one of the employees at the company. Define the following predicates:

   - $S(x)$: x was sick yesterday
   - $W(x)$: x went to work yesterday
   - $V(x)$: x was on vacation yesterday

   (b) Everyone was well and went to work yesterday.

   $\forall x(\neg S(x) \vee W(x))$

   (c) Everyone who was sick yesterday did not go to work.

   $\forall x(S(x) \rightarrow \neg W(x))$

   (d) Yesterday someone was sick and went to work.

   $\exists x(S(x) \wedge W(x))$

Question 10:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.7.9, sections c - i

   The domain for this question is the set {a,b,c,d,e}. The following table gives the value of predicates P, Q, and R for each element in the domain. For example, Q(c) = T because the truth value in the row labeled c and the column Q is T. Using these values, determine whether each quantified expression evaluates to true or false.

	$P(x)$	$Q(x)$	$R(x)$
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

   (c) $\exists x((x = c) \to P(x))$

   True: The expression shows that there exists an x that x = c is false implies $P(x)$ is true. If x = a, b, d or e, then x = c is false and $P(x)$ is true. Therefore, the expression is True.

   (d) $\exists x(Q(x) \land R(x))$

   True: The expression shows that there exists an x that $Q(x) \land R(x)$ is true. If x = e, then $Q(x) \land R(x)$ is true. Therefore, the expression is True.

   (e) $Q(a) \land P(d)$

   True: The table shows that $Q(a)$ and $P(d)$ are true. Therefore, the expression is True.

   (f) $\forall x((x \neq b) \to Q(x))$

   True: The expression shows that if $x \neq b$, then $Q(x)$ is always true for all x. Therefore, the expression is True.

   (g) $\forall x(P(x) \lor R(x))$

   False: The expression shows that $P(x) \lor R(x)$ is always true (or false) for all x. If x = c, then $P(x) \lor R(x)$ is false meanwhile others are true. Therefore, the expression is False.

   (h) $\forall x(R(x) \to P(x))$

   True: The expression shows that $(R(x) \to P(x))$ is always true (or false) for all x. If x = a, b, c, d and e, then $R(x)$ is false (or $P(x)$ is true) and $(R(x) \to P(x))$ is always true for all x. Therefore, the expression is True.

   (i) $\exists x(Q(x) \lor R(x))$

   True: The expression shows that there exists an x that $\exists x(Q(x) \lor R(x))$ is true. If x = a, c, d or e then $\exists x(Q(x) \lor R(x))$ is true. Therefore, the expression is True.

2. Exercise 1.9.2, sections b - i

The tables below show the values of predicates P(x,y), Q(x,y), and S(x,y) for every possible combination of values of the variables x and y. The row number indicates the value for x, and the column number indicates the value for y. The domain for x and y is {1, 2, 3}.

P	1	2	3	Q	1	2	3	S	1	2	3
1	T	F	T	1	F	F	F	1	F	F	F
2	T	F	T	2	T	T	T	2	F	F	F
3	T	T	F	3	T	F	F	3	F	F	F

Indicate whether each of the quantified statements is true or false.

(b) $\exists x \forall y Q(x, y)$

True: When x = 2, $Q(x, y)$ is true for all y.

(c) $\exists y \forall x P(x, y)$

True: When y = 1, $P(x, y)$ is true for all x.

(d) $\exists x \exists y S(x, y)$

False: $S(x, y)$ is always false for all x and y.

(e) $\forall x \exists y Q(x, y)$

False: When x = 1, $Q(x, y)$ is false for all y.

(f) $\forall x \exists y P(x, y)$

True: When x = 1 and y = 1 or 3, x = 2 and y = 1 or 3, or x = 3 and y = 1 or 2, $P(x, y)$ is true for all x.

(g) $\forall x \forall y P(x, y)$

False: When x = 1 and y = 2, x = 2 and y = 2 or x = 3 and y = 3, $P(x, y)$ is false.

(h) $\exists x \exists y Q(x, y)$

True: When x = 2 and y = 1, 2 or 3, and x = 3 and y = 1, $Q(x, y)$ is true.

(i) $\forall x \forall y \neg S(x, y)$

True: $Q(x, y)$ is always false for all x and y, so $\neg S(x, y)$ is always true for all x and y.

Question 11:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.10.4, sections c - g

   Translate each of the following English statements into logical expressions. The domain is the set of all real numbers.

   (c) There are two numbers whose sum is equal to their product.

   $\exists x \exists y(x + y = xy)$

   (d) The ratio of every two positive numbers is also positive.

   $\forall x \forall y(((x > 0) \wedge (y > 0)) \rightarrow (\frac{x}{y} > 0))$

   (e) The reciprocal of every positive number less than one is greater than one.

   $\forall x(((x > 0) \wedge (x < 1)) \rightarrow (\frac{1}{x} > 1))$

   (f) There is no smallest number.

   $\forall x \exists y(y < x)$

   (g) Every number other than 0 has a multiplicative inverse.

   $\forall x \exists y((x \neq 0) \rightarrow (y = \frac{1}{x}))$

2. Exercise 1.10.7, sections c - f

   The domain is a group working on a project at a company. One of the members of the group is named Sam. Define the following predicates.

   - $P(x, y)$: x knows y's phone number. (One may or may not know one's own phone number.)
   - $D(x)$: x missed the deadline.
   - $N(x)$: x is a new employee.

   (c) At least one new employee missed the deadline.

   $\exists x(N(x) \wedge D(x))$

   (d) Sam knows the phone number of everyone who missed the deadline.

   $\forall y(D(y) \rightarrow P(\text{Sam}, y))$

   (e) There is a new employee who knows everyone's phone number.

   $\exists x \forall y(N(x) \wedge P(x, y))$

   (f) Exactly one new employee missed the deadline.

   $\exists x \forall y(((N(x) \wedge D(x)) \wedge (N(y) \wedge (y \neq x))) \rightarrow \neg D(y))$

3. Exercise 1.10.10, sections c – f

   The domain for the first input variable to predicate T is a set of students at a university. The domain for the second input variable to predicate T is the set of math classes offered at that university. The predicate T(x,y) indicates that student x has taken class y. Sam is a student at the university, and Math 101 is one of the courses offered at the university. Give a logical expression for each sentence.

   (c) Every student has taken at least one class other than Math 101.

   $\forall x \exists y (T(x,y) \land (y \neq \text{Math 101}))$

   (d) There is a student who has taken every math class other than Math 101.

   $\exists x \forall y ((y \neq \text{Math 101}) \to T(x,y))$

   (e) Everyone other than Sam has taken at least two different math classes.

   $\forall x \exists y_1 \exists y_2 ((x \neq \text{Sam}) \to (T(x,y_1) \land T(x,y_2) \land (y_1 \neq y_2)))$

   (f) Sam has taken exactly two math classes.

   $\exists y_1 \exists y_2 \exists y_3 ((T(\text{Sam},y_1) \land T(\text{Sam},y_2) \land (y_1 \neq y_2) \land (y_3 \neq y_1) \land (y_3 \neq y_2)) \to \neg T(\text{Sam},y_3))$

Question 12:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.8.2, sections b – e

   In the following question, the domain is a set of patients in a clinical study. Define the following predicates:

   - $P(x)$: x was given the placebo
   - $D(x)$: x was given the medication
   - $M(x)$: had migraines

   Translate each statement into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

   Sample question: Some patient was given the placebo and the medication.

   - $\exists x(P(x) \wedge D(x))$
   - Negation: $\neg\exists x(P(x) \wedge D(x))$
   - Applying De Morgan's law: $\forall x(\neg P(x) \vee \neg D(x))$
   - English: Every patient was either not given the placebo or not given the medication (or both).

   (b) Every patient was given the medication or the placebo or both.

$$\begin{aligned}
& \forall x(D(x) \vee P(x)) \\
\text{Negation:} \quad & \neg\forall x(D(x) \vee P(x)) \\
\text{Applying De Morgan's law:} \quad & \exists x \neg(D(x) \vee P(x)) \\
\text{Applying De Morgan's law:} \quad & \exists x(\neg D(x) \wedge \neg P(x))
\end{aligned}$$

   English: There is a patient who was neither given the medication nor the placebo.

   (c) There is a patient who took the medication and had migraines.

$$\begin{aligned}
& \exists x(D(x) \wedge M(x)) \\
\text{Negation:} \quad & \neg\exists x(D(x) \wedge M(x)) \\
\text{Applying De Morgan's law:} \quad & \forall x \neg(D(x) \wedge M(x)) \\
\text{Applying De Morgan's law:} \quad & \forall x(\neg D(x) \vee \neg M(x))
\end{aligned}$$

   English: Every patient was not given the medication or did not have migraines or both.

   (d) Every patient who took the placebo had migraines. (Hint: Apply the conditional identity, $p \to q \equiv \neg p \vee q$.)

$$
\begin{aligned}
& \forall x(P(x) \to M(x)) \\
\text{Negation:} \quad & \neg\forall x(P(x) \to M(x)) \\
\text{Applying De Morgan's law:} \quad & \exists x \neg(P(x) \to M(x)) \\
\text{Applying conditional identity:} \quad & \exists x \neg(\neg P(x) \vee M(x)) \\
\text{Applying De Morgan's law:} \quad & \exists x(\neg(\neg P(x)) \wedge \neg M(x)) \\
\text{Applying double negation law:} \quad & \exists x(P(x) \wedge \neg M(x))
\end{aligned}
$$

English: There is a patient who was given the placebo and did not have migraines.

(e) There is a patient who had migraines and was given the placebo.

$$
\begin{aligned}
& \exists x(M(x) \wedge P(x)) \\
\text{Negation:} \quad & \neg\exists x(M(x) \wedge P(x)) \\
\text{Applying De Morgan's law:} \quad & \forall x \neg(M(x) \wedge P(x)) \\
\text{Applying De Morgan's law:} \quad & \forall x(\neg M(x) \vee \neg P(x))
\end{aligned}
$$

English: Every patient did not have migraines or was not given the placebo.

2. Exercise 1.9.4, sections c - e

Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.

(c) $\exists x \forall y(P(x,y) \to Q(x,y))$

$$
\begin{aligned}
\neg\exists x \forall y(P(x,y) \to Q(x,y)) &\equiv \forall x \neg\forall y(P(x,y) \to Q(x,y)) && \text{[De Morgan's law]} \\
&\equiv \forall x \exists y \neg(P(x,y) \to Q(x,y)) && \text{[De Morgan's law]} \\
&\equiv \forall x \exists y \neg(\neg P(x,y) \vee Q(x,y)) && \text{[Conditional identity]} \\
&\equiv \forall x \exists y(\neg(\neg P(x,y)) \wedge \neg Q(x,y)) && \text{[De Morgan's law]} \\
&\equiv \forall x \exists y(P(x,y) \wedge \neg Q(x,y)) && \text{[Double negation law]}
\end{aligned}
$$

(d) $\exists x \forall y(P(x,y) \leftrightarrow P(y,x))$

$$
\begin{aligned}
\neg\exists x \forall y(P(x,y) \leftrightarrow P(y,x)) &\equiv \forall x \neg\forall y(P(x,y) \leftrightarrow P(y,x)) && \text{[De Morgan's law]} \\
&\equiv \forall x \exists y \neg(P(x,y) \leftrightarrow P(y,x)) && \text{[De Morgan's law]} \\
&\equiv \forall x \exists y \neg((P(x,y) \to P(y,x)) \wedge (P(y,x) \to P(x,y))) && \text{[Conditional identity]} \\
&\equiv \forall x \exists y(\neg(P(x,y) \to P(y,x)) \vee \neg(P(y,x) \to P(x,y))) && \text{[De Morgan's law]} \\
&\equiv \forall x \exists y(\neg(\neg P(x,y) \vee P(y,x)) \vee \neg(\neg P(y,x) \vee P(x,y))) && \text{[Conditional identity]} \\
&\equiv \forall x \exists y((\neg(\neg P(x,y)) \wedge \neg P(y,x)) \vee (\neg(\neg P(y,x)) \wedge \neg P(x,y))) && \text{[De Morgan's law]} \\
&\equiv \forall x \exists y((P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y))) && \text{[Double negation law]}
\end{aligned}
$$

(e) $\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y)$

$$\begin{aligned}
\neg(\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y)) &\equiv \neg\exists x \exists y P(x,y) \vee \neg\forall x \forall y Q(x,y) && \text{[De Morgan's law]} \\
&\equiv \forall x \neg\exists y P(x,y) \vee \exists x \neg\forall y Q(x,y) && \text{[De Morgan's law]} \\
&\equiv \forall x \forall y \neg P(x,y) \vee \exists x \exists y \neg Q(x,y) && \text{[De Morgan's law]}
\end{aligned}$$