# Homework 6

## Extended Bridge to CS, Spring 2025

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**Question 5**:

Use the definition of $\Theta$ in order to show the following:

1. a. $5n^3 + 2n^2 + 3n = \Theta(n^3)$

   Proof: For $f(n) = \Theta(g(n))$, we take $c_1$, $c_2$ and $n_0$ such that for all n $\geq n_0$.

   $$c_2 \cdot n^3 \leq 5n^3 + 2n^2 + 3n \leq c_1 \cdot n^3$$

   If we take $c_1 = 10$, $c_2 = 5$ and $n_0 = 1$, then for all $n \geq n_0$ we have:

   $$5n^3 \leq 5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3 \leq 10n^3$$
   $$\therefore 5n^3 \leq 5n^3 + 2n^2 + 3n \leq 10n^3$$

   for all n $\geq 1$. Therefore:

   $$5n^3 + 2n^2 + 3n = \Theta(n^3)$$

2. b. $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

   Proof: For $f(n) = \Theta(g(n))$, we take $c_1$, $c_2$ and $n_0$ such that for all n $\geq n_0$.

   $$c_2 \cdot n \leq \sqrt{7n^2 + 2n - 8} \leq c_1 \cdot n$$
   $$\therefore (c_2 \cdot n)^2 \leq 7n^2 + 2n - 8 \leq (c_1 \cdot n)^2$$

   $$2n - 8 \geq 0$$
   $$2n \geq 8$$
   $$n \geq 4$$

   If we take $c_1 = 3$, $c_2 = \sqrt{7}$ and $n_0 = 4$, then for all $n \geq n_0$ we have:

   $$7n^2 \leq 7n^2 + 2n - 8 \leq 7n^2 + 2n^2 \leq 9n^2$$
   $$\therefore 7n^2 \leq 7n^2 + 2n - 8 \leq 9n^2$$
   $$\therefore \sqrt{7}n \leq \sqrt{7n^2 + 2n - 8} \leq 3n$$

   for all n $\geq 4$. Therefore:

   $$\sqrt{7n^2 + 2n - 8} = \Theta(n)$$