Homework 3
Extended Bridge to CS, Spring 2025

Akihito Chinda

April 28, 2025

Question 7:

Solve the following questions from the Discrete Math zyBook:

1. a) Exercise 3.1.1, sections a-g

    Use the definitions for the sets given below to determine whether each statement is true or false:

$$A = \{x \in \mathbb{Z} : x \text{ is an integer multiple of } 3\}$$
$$B = \{x \in \mathbb{Z} : x \text{ is a perfect square}\}$$
$$C = \{4, 5, 9, 10\}$$
$$D = \{2, 4, 11, 14\}$$
$$E = \{3, 6, 9\}$$
$$F = \{4, 6, 16\}$$

An integer x is a perfect square if there is an integer y such that $x = y^2$.

(a) $27 \in A$

True: 3 x 9 = 27

(b) $27 \in B$

False: 27 is not a perfect square

(c) $100 \in B$

True: 10 x 10 = 100

(d) $E \subseteq C$ or $C \subseteq E$

False: E doesn't include all elements from C, nor C doesn't from E.

(e) $E \subseteq A$

True: A includes all elements from E.

(f) $A \subseteq E$

False: E doesn't include all elements from A.

(g) $E \in A$

False: There doesn't exist a set E{3, 6, 9} in A.

2. b) Exercise 3.1.2, sections a-e

   (a) $15 \subset A$

   False: A doesn't include a proper subset 15 that is not a set.

   (b) $\{15\} \subset A$

   True: A includes a proper subset $\{15\}$.

   (c) $\emptyset \subset C$

   True: C includes a proper subset of empty set $\{\}$.

   (d) $D \subseteq D$

   True: D is a subset of itself.

   (e) $\emptyset \in B$

   False: There doesn't exist an empty set $\{\}$ in B.

3. c) Exercise 3.1.5, sections c, d

   Express each set using set builder notation. Then if the set is finite, give its cardinality. Otherwise, indicate that the set is infinite.

   (b) $\{3, 6, 9, 12, \ldots\}$

   $A = \{x \in \mathbb{Z}^+ : x \text{ is an integer multiple of } 3\}$

   Therefore, the set is infinite.

   (d) $\{0, 10, 20, 30, \ldots 1000\}$

   $B = \{x \in \mathbb{N} : 0 \leq x \leq 1000 \text{ and } x \text{ is an integer multiple of } 10\}$

   Therefore, the cardinality of set $|B| = 101$.

4. d) Exercise 3.2.1, sections a-k

   Let $X = \{1, \{1\}, \{1, 2\}, 2, \{3\}, 4\}$. Which statements are true?

   (a) $2 \in X$

   True: There exists an object 2 in X.

   (b) $\{2\} \subseteq X$

   True: $2 \in \{2\} \rightarrow 2 \in X$, therefore $\{2\} \subseteq X$.

   (c) $\{2\} \in X$

   False: There doesn't exist a set $\{2\}$ in X.

   (d) $3 \in X$

   False: There doesn't exist an object 3 in X.

   (e) $\{1, 2\} \in X$

   True: There exists a set $\{1, 2\}$ in X.

   (f) $\{1, 2\} \subseteq X$

2

True: $1 \in \{1, 2\} \to 1 \in X$ and $2 \in \{1, 2\} \to 2 \in X$, therefore $\{1, 2\} \subseteq X$.

(g) $\{2, 4\} \subseteq X$

True: $2 \in \{2, 4\} \to 2 \in X$ and $4 \in \{2, 4\} \to 4 \in X$, therefore $\{2, 4\} \subseteq X$.

(h) $\{2, 4\} \in X$

False: There doesn't exist a set $\{2, 4\}$ in X.

(i) $\{2, 3\} \subseteq X$

False: $2 \in \{2, 3\} \to 2 \in X$, but not $3 \in \{2, 3\} \to 3 \in X$, therefore $\{2, 3\} \nsubseteq X$.

(j) $\{2, 3\} \in X$

False: There doesn't exist a set $\{2, 3\}$ in X.

(k) $|X| = 7$

False: $|X| = 6$, therefore $|X| \neq 7$

Question 8:

Solve Exercise 3.2.4, section b from the Discrete Math zyBook:
(b) Let $A = \{1, 2, 3\}$. What is $\{X \in P(A) : 2 \in X\}$?

$A = \{1, 2, 3\}$, $|A| = 3$
$|P(A)| = 2^3 = 8$
$= \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$

$2 \in \{2\} \rightarrow 2 \in X$, $2 \in \{1, 2\} \rightarrow 2 \in X$, $2 \in \{2, 3\} \rightarrow 2 \in X$ and $2 \in \{1, 2, 3\} \rightarrow 2 \in X$.
Therefore, $X = \{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}$.

Question 9:

Solve the following questions from the Discrete Math zyBook:

1. a) Exercise 3.3.1, sections c-e

   Define the sets A, B, C, and D as follows:

   $$A = \{-3, 0, 1, 4, 17\}$$
   $$B = \{-12, -5, 1, 4, 6\}$$
   $$C = \{x \in \mathbb{Z} : x \text{ is odd }\}$$
   $$D = \{x \in \mathbb{Z} : x \text{ is positive }\}$$

   For each of the following set expressions, if the corresponding set is finite, express the set using roster notation. Otherwise, indicate that the set is infinite.

   (c) $A \cap C$

   $= \{-3, 0, 1, 4, 17\} \cap \{x \in \mathbb{Z} : x \text{ is odd }\}$

   $= \{-3, 1, 17\}$

   (d) $A \cup (B \cap C)$

   $= \{-3, 0, 1, 4, 17\} \cup (\{-12, -5, 1, 4, 6\} \cap \{x \in \mathbb{Z} : x \text{ is odd }\})$

   $= \{-3, 0, 1, 4, 17\} \cup (\{-5, 1\})$

   $= \{-5, -3, 0, 1, 4, 17\}$

   (e) $A \cap B \cap C$

   $= \{-3, 0, 1, 4, 17\} \cap \{-12, -5, 1, 4, 6\} \cap \{x \in \mathbb{Z} : x \text{ is odd }\}$

   $= \{1, 4\}$

   $= \{1\}$

2. b) Exercise 3.3.3, sections a, b, e, f

   Use the following definitions to express each union or intersection given. Roster or set builder notation can be used in your responses, but no set operations. For each definition, $i \in \mathbb{Z}^+$.

   $$A_i = \{i^0, i^1, i^2\} \text{ (Recall that for any number } x, x^0 = 1 \text{ when } x \neq 0)$$
   $$B_i = \left\{x \in \mathbb{R} : -i \leq x \leq \frac{1}{i}\right\}$$
   $$C_i = \left\{x \in \mathbb{R} : -\frac{1}{i} \leq x \leq \frac{1}{i}\right\}$$

   (a) $\displaystyle\bigcap_{i=2}^{5} A_i$

5

$= A_2 \cap A_3 \cap A_4 \cap A_5$

$= \{2^0, 2^1, 2^2\} \cap \{3^0, 3^1, 3^2\} \cap \{4^0, 4^1, 4^2\} \cap \{5^0, 5^1, 5^2\}$

$= \{1, 2, 4\} \cap \{1, 3, 9\} \cap \{1, 4, 16\} \cap \{1, 5, 25\}$

$= \{1\}$

(b) $\bigcup\limits_{i=2}^{5} A_i$

$= A_2 \cup A_3 \cup A_4 \cup A_5$

$= \{2^0, 2^1, 2^2\} \cup \{3^0, 3^1, 3^2\} \cup \{4^0, 4^1, 4^2\} \cup \{5^0, 5^1, 5^2\}$

$= \{1, 2, 4\} \cup \{1, 3, 9\} \cup \{1, 4, 16\} \cup \{1, 5, 25\}$

$= \{1, 2, 3, 4, 5, 9, 16, 25\}$

(e) $\bigcap\limits_{i=1}^{100} C_i$

$= \{x \in \mathbb{R}\} \cap \left\{ -\frac{1}{1} \le x \le \frac{1}{1} \right\} \cap \left\{ -\frac{1}{2} \le x \le \frac{1}{2} \right\} \cap \left\{ -\frac{1}{3} \le x \le \frac{1}{3} \right\} \ldots \cap \left\{ -\frac{1}{100} \le x \le \frac{1}{100} \right\}$

The intersection of all these intervals is the smallest interval among them. Therefore, the intersection of all intervals from $i = 1$ to $i = 100$ is $\{x \in \mathbb{R} : -\frac{1}{100} \le x \le \frac{1}{100}\}$.

(f) $\bigcup\limits_{i=1}^{100} C_i$

$= \{x \in \mathbb{R}\} \cap \left\{ -\frac{1}{1} \le x \le \frac{1}{1} \right\} \cup \left\{ -\frac{1}{2} \le x \le \frac{1}{2} \right\} \cup \left\{ -\frac{1}{3} \le x \le \frac{1}{3} \right\} \ldots \cup \left\{ -\frac{1}{100} \le x \le \frac{1}{100} \right\}$

The union of all these intervals covers the entire range from -1 to 1. Therefore, the union of all intervals from $i = 1$ to $i = 100$ is $\{x \in \mathbb{R} : -\frac{1}{1} \le x \le \frac{1}{1}\}$.

3. Exercise 3.3.4, sections b, d

Use the set definitions $A = \{a, b\}$ and $B = \{b, c\}$ to express each set below. Use roster notation in each solution.

(b) $P(A \cup B)$

$= P(\{a, b\} \cup \{b, c\})$

$= P(\{a, b, c\})$

$= \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$

(d) $P(A) \cup P(B)$

$= P(\{a, b\}) \cup P(\{b, c\})$

$= P(\emptyset, \{a\}, \{b\}, \{a, b\}) \cup P(\emptyset, \{b\}, \{c\}, \{b, c\})$

$= \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$

Question 10:

Solve the following questions from the Discrete Math zyBook:

1. a) Exercise 3.5.1, sections b, c

   The sets A, B, and C are defined as follows:

   $$A = \{\text{tall}, \text{grande}, \text{venti}\}$$
   $$B = \{\text{foam}, \text{no-foam}\}$$
   $$C = \{\text{non-fat}, \text{whole}\}$$

   Use the definitions for A, B, and C to answer the questions. Express the elements using n-tuple notation, not string notation.

   (b) Write an element from the set $B \times A \times C$.

   (foam, tall, non-fat)

   (c) Write the set $B \times C$ using roster notation.

   $\{(\text{foam}, \text{non-fat}), (\text{foam}, \text{whole}), (\text{no-foam}, \text{non-fat}), (\text{no-foam}, \text{whole})\}$

2. b) Exercise 3.5.3, sections b, c, e

   Indicate which of the following statements are true.

   (b) $\mathbb{Z}^2 \subseteq \mathbb{R}^2$

   True: $\mathbb{Z} \in \mathbb{R}$, therefore $\mathbb{Z}^2 \subseteq \mathbb{R}^2$.

   (c) $\mathbb{Z}^2 \cap \mathbb{Z}^3 = \emptyset$

   True: Since $\mathbb{Z}^2$ consists of pairs and $\mathbb{Z}^3$ consists of triples, there are no common elements between these sets. Therefore, the intersection of $\mathbb{Z}^2$ and $\mathbb{Z}^3$ is empty.

   (e) For any three sets, A, B, and C, if $A \subseteq B$, then $A \times C \subseteq B \times C$.

   True: If $A \subseteq B$, then B includes all elements from A. Therefore, $A \times C \subseteq B \times C$.

3. c) Exercise 3.5.6, sections d, e

   Express the following sets using the roster method. Express the elements as strings, not n-tuples.

   (d) $\{xy : \text{ where } x \in \{0\} \cup \{0\}^2 \text{ and } y \in \{1\} \cup \{1\}^2\}$

   $x \in \{0\} \cup \{0\}^2 = \{0, 00\}$

   $y \in \{1\} \cup \{1\}^2 = \{1, 11\}$

   Therefore, $xy = \{01, 011, 001, 0011\}$

   (e) $\{xy : x \in \{aa, ab\} \text{ and } y \in \{a\} \cup \{a\}^2\}$

$x \in \{aa, ab\}$

$y \in \{a\} \cup \{a\}^2 = \{a, aa\}$

Therefore, $xy = \{aaa, aaaa, aba, abaa\}$

4. d) Exercise 3.5.7, sections c, f, g

Use the following set definitions to specify each set in roster notation. Except where noted, express elements of Cartesian products as strings.

$$A = \{a\}$$
$$B = \{b, c\}$$
$$C = \{a, b, d\}$$

(c) $(A \times B) \cup (A \times C)$

$= (\{a\} \times \{b, c\}) \cup (\{a\} \times \{a, b, d\})$

$= (\{ab, ac\}) \cup (\{aa, ab, ad\})$

$= \{ab, ac, aa, ad\}$

(f) $P(A \times B)$

$= P(\{a\} \times \{b, c\})$

$= P(\{ab, ac\})$

$= \{\emptyset, \{ab\}, \{ac\}, \{ab, ac\}\}$

(g) $P(A) \times P(B)$. Use ordered pair notation for elements of the Cartesian product.

$= P(\{a\}) \times P(\{b, c\})$

$= \{\emptyset, \{a\}\} \times \{\emptyset, \{b\}, \{c\}, \{bc\}\}$

$= \{(\emptyset, \emptyset), (\emptyset, \{b\}), (\emptyset, \{c\}), (\emptyset, \{bc\}), (\{a\}, \emptyset), (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{bc\})\}$

Question 11:

Solve the following questions from the Discrete Math zyBook:

1. a) Exercise 3.6.2, sections b, c

   Use the set identities given in the table to prove the following new identities. Label each step in the proof with the set identity used to establish that step.

   (b) $(B \cup A) \cap (\overline{B} \cup A) = A$

$$
\begin{aligned}
(B \cup A) \cap (\overline{B} \cup A) &\equiv (A \cup B) \cap (\overline{A} \cup B) && \text{[Commutative law]} \\
&\equiv A \cup (B \cap \overline{B}) && \text{[Distributed law]} \\
&\equiv A \cup \emptyset && \text{[Complement law]} \\
&\equiv A && \text{[Identity law]}
\end{aligned}
$$

   (c) $\overline{A \cap \overline{B}} = \overline{A} \cup B$

$$
\begin{aligned}
\overline{A \cap \overline{B}} &\equiv (\overline{A} \cup \overline{\overline{B}}) && \text{[De Morgan's law]} \\
&\equiv \overline{A} \cup B && \text{[Double complement law]}
\end{aligned}
$$

2. b) Exercise 3.6.3, sections b, d

   A set equation is not an identity if there are examples for the variables denoting the sets that cause the equation to be false. For example $A \cup B = A \cap B$ is not an identity because if $A = \{1, 2\}$ and $B = \{1\}$, then $A \cup B = \{1, 2\}$ and $A \cap B = \{1\}$, which means that $A \cup B \neq A \cap B$.

   Show that each set equation given below is not a set identity.

   (b) $A - (B \cap A) = A$

   If $A = \{1\}$ and $B = \{1, 2\}$, then $B \cap A = \{1\}$ and $A - (B \cap A) = \emptyset$. Therefore, $A - (B \cap A) \neq A$.

   (d) $(B - A) \cup A = A$

   If $A = \{1\}$ and $B = \{1, 2\}$, then $B - A = \{2\}$ and $(B - A) \cup A = \{1, 2\}$. Therefore, $(B - A) \cup A \neq A$.

3. c) Exercise 3.6.4, sections b, c

   The set subtraction law states that $A - B = A \cap \overline{B}$. Use the set subtraction law and the other set identities given in the table to prove each of the following new identities. Label each step in the proof with the set identity used to establish that step.

   (b) $A \cap (B - A) = \emptyset$

$$A \cap (B - A) \equiv A \cap (B \cap \overline{A}) \quad \text{[By def}^n\text{(Set subtraction law)]}$$
$$\equiv (B \cap \overline{A}) \cap A \quad \text{[Commutative law]}$$
$$\equiv B \cap (\overline{A} \cap A) \quad \text{[Associative law]}$$
$$\equiv B \cap \emptyset \quad \text{[Complement law]}$$
$$\equiv \emptyset \quad \text{[Domination law]}$$

(c) $A \cup (B - A) = A \cup B$

$$A \cup (B - A) \equiv A \cup (B \cap \overline{A}) \quad \text{[By def}^n\text{(Set subtraction law)]}$$
$$\equiv (A \cup B) \cap (A \cup \overline{A}) \quad \text{[Distributed law]}$$
$$\equiv (A \cup B) \cap U \quad \text{[Complement law]}$$
$$\equiv A \cup B \quad \text{[Identity law]}$$

	Name	**Identities**
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \emptyset = A$	$A \cap U = A$
Domination laws	$A \cap \emptyset = \emptyset$	$A \cup U = U$
Double complement law	$\overline{\overline{A}} = A$	
Complement laws	$A \cap \overline{A} = \emptyset$ $\overline{U} = \emptyset$	$A \cup \overline{A} = U$ $\overline{\emptyset} = U$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$