

Bitwise Manipulation

Operators:

1. AND

If any is false then entire is false. If both are true then it is true.

a	b	a & b
0	0	0
0	1	0
1	0	0
1	1	1

Q: Given a number N find if it's even.

Observation:

- When you & 1 with any number the digits remain the same.

Ex:

A handwritten example of a bitwise AND operation. The first number is 110010100 (decimal 244) and the second number is 1111111 (decimal 127). The result of the AND operation is 110010100 (decimal 244). The numbers are written in binary, and the result is shown below a horizontal line.

$$\begin{array}{r} 110010100 \\ 1111111 \\ \hline 110010100 \end{array}$$

2. OR

If any one is true, then entire is true.

a	b	a b
0	0	0
0	1	1
1	0	1
1	1	1

3. XOR (^) (Aka Exclusive OR)

If and only if. (Only 1 should be true)

a	b	a ^ b
0	0	0
0	1	1
1	0	1
1	1	0

Observation:

- When you ^ 1 with any number we get the compliment of that number.

$$a \wedge 1 = \bar{a}$$

$$0 \wedge 1 = 0$$

$$1 \wedge 1 = 1$$

- When you $\wedge 0$ with any number we get the number itself.

$$a \wedge 0 = a$$

$$a \wedge 0 = a$$

- When you \wedge number with itself we get 0.

$$a \wedge a = 0$$

$$a \wedge a = 0$$

4. Compliment (~)

Opposite of the number.

$$a = 10110$$

$$\overline{a} = 01001$$

Number Systems:

1. Decimal Number $\rightarrow 0, 1, 2, \dots, 9$ (Base 10)

$$(357)_{10}$$

$$(10)_{10}$$

2. Binary Number $\rightarrow 0 \text{ \& } 1$ (Base 2)

$$(10)_{10} = (1010)_2$$

3. Octal \rightarrow 0, 1, 2, 3, ..., 7 (Base 8)

Octal 0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, ...

$$(9)_{10} = (11)_8$$

4. Hexadecimal \rightarrow 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F (Base 16)

$$(10)_{10} = (A)_{16} \quad (12)_{10} = (C)_{16}$$

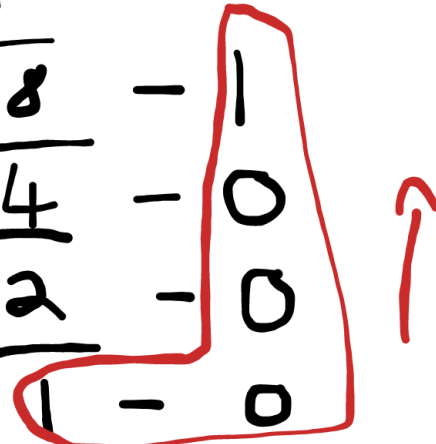
Conversion of Number Systems

- Convert Decimal to any base b

Convert $(17)_{10}$ to base 2

Keep dividing by base, take remainders, write it in opposite.

2	17		
2	8	-	1
2	4	-	0
2	2	-	0
	1	-	0



$$= (10001)_2 = (17)_{10}$$

Convert $(17)_{10}$ to base 8

Keep dividing by base, take remainders, write it in opposite.

$$\begin{array}{r} 8 \overline{) 17} \\ \underline{16} \\ 1 \end{array}$$

The remainder 1 is circled in red in the original image.

$$= (21)_8 = (17)_{10}$$

- Convert any base b to Decimal

$$(1001)_2 = ()_{10} ?$$

Steps:

Multiply and add the power of base with the digits.

$$= 1 * 2^4 + 0 * 2^3 + 0 * 2^2 + 0 * 2^1 + 1 * 2^0$$

$$= 16 + 0 + 0 + 0 + 1 = (17)_{10}$$

$$(21)_8 = ()_{10} ?$$

Steps:

Multiply and add the power of base with the digits.

$$= 2 * 8^1 + 1 * 8^0$$

$$= 16 + 1 = (17)_{10}$$

- If we are given to Convert Base 2 to Base 8, then first convert Base 2 to decimal and then to Base 8.

Operators Continuation

5. Left Shift Operator (<<)

$$(10)_{10} = (1010)_2$$

$$10 \ll 1$$

Steps:

First Convert it into Binary



Add extra 0 when it is not enough

$$= 10100$$

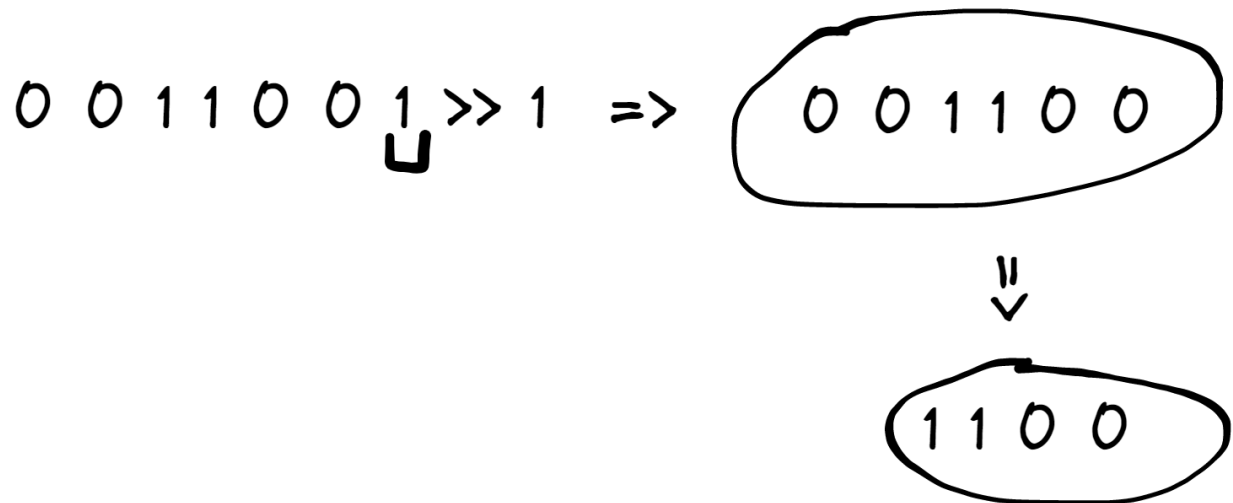
$$= 1 * 2^4 + 0 * 2^3 + 1 * 2^2 + 0 * 2^1 + 0 * 2^0$$

$$= 16 + 4 = 20$$

$$a \ll 1 = 2a$$

$$a \ll b = a * 2^b$$

6. Right Shift Operator (>>)



$$a \gg 1 = a / 2$$

$$a \gg b = a / 2^b$$

Questions

Q1: Given a number n find if it is odd or even.

Ans: Check if the last bit (LSB) is 1 or 0.

If it is 1 then its odd, else even

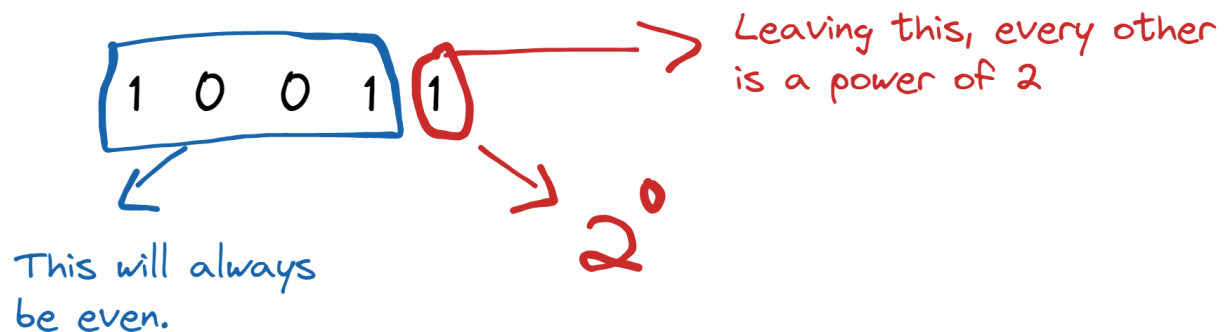
LSB: Least Significant Bit.

```
static boolean isOdd(int num) {  
    return (num & 1) == 1;  
}
```

Q: Given a number n find if it is odd or even.

Point: Every no. is calculated in binary form internally

$$\begin{array}{r} 12 + 7 \Rightarrow \\ (19)_{10} = (10011)_2 \end{array} \qquad \begin{array}{r} 1100 \\ + 0111 \\ \hline 10011 \end{array}$$



Hence: If 2^0 place == 1 \Rightarrow The number is odd
== 0 \Rightarrow The number is even.

Q2:

Given a non-empty array of integers nums, every element appears twice except for one. Find that single one.

Ans:

We know that any number xor with itself we get zero.

And we also know that any number xor with 0 will be the number itself.

So here we will xor the entire array and the resultant will be the single element.

```
public static int singleNumber(int[] nums) {  
    int xor = 0;  
    for (int num : nums) {
```

```

        xor ^= num;
    }
    return xor;
}

```

Q3. Find the ith bit of a number.

Ans: AND that particular it with 1

Example: Find the 5th bit for: 10110110

```

    1 0 1 1 0 1 1 0
&  0 0 0 1 0 0 0 0  --> This is called a mask.
-----
    0 0 0 1 0 0 0 0

```

We need a mask with n - 1 zeros.

To create a mask we can use left shift with n - 1

1 << (n - 1)

1 << 4 => 10000

```
return num & (1 << ( i - 1 ));
```

Q4: Set the ith bit -> Turn it to 1

-> 0 --> 1

-> 1 --> 1

Ans: OR that particular ith with 1

Example: Set the 4th bit for: 1010110

```

    1 0 1 0 1 1 0
||  0 0 0 1 0 0 0  --> Mask
-----
    1 0 1 1 1 1 0

```

We need a mask with n - 1 zeros.

To create a mask we can use left shift with n - 1

1 << (n - 1)

1 << 3 => 0001000

```
return num | (1 << ( i - 1 ));
```

```
static int setTheIthBit(int num, int i) {
    return num | (1 << ( i - 1 ));
}

```

```

Q5: Reset the ith bit -> Turn it to 0
-> 0 --> 0
-> 1 --> 0
Ans: AND everything with 1 except that particular ith with 0
Example: Reset the 5th bit for: 1010110
      1 0 1 0 1 1 0
&    1 1 0 1 1 1 1  --> Mask
-----
      1 0 0 0 1 1 0

```

To create a mask we can use left shift with $n - 1$

```

1 << (n - 1)
1 << 3 ==> 001000
Then take the complement of it, which is: 110111
Mask: ~(1 << (n - 1))

```

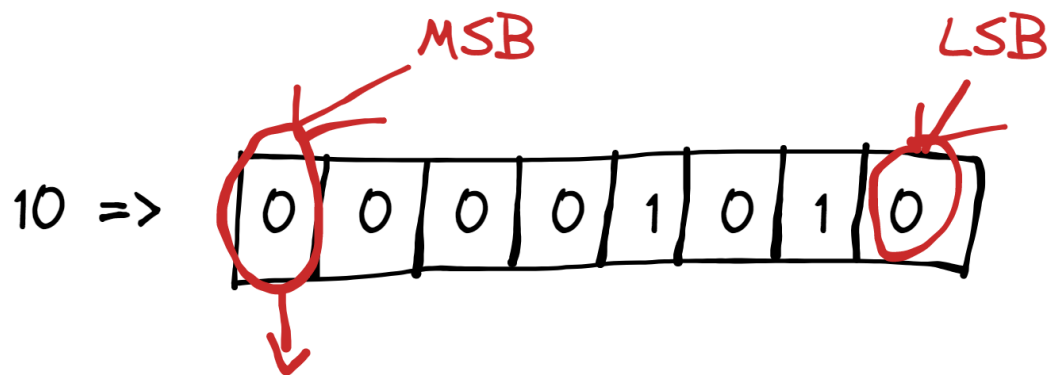
```

static int setTheIthBit(int num, int i) {
    return num & ~(1 << ( i - 1 ));
}

```

Negative of a number in binary form

1 byte = 8 bits



Tells us if number is positive or negative

1 -> -ve
0 -> +ve

How to find negative of a number:-

Steps:

1. Take complement of the number.
2. Add 1 to it.

} 2's Complement

$(10)_{10} = (00001010)_2$

(1) 1 1 1 1 0 1 0 1

(2) 1 1 1 1 0 1 0 1
 + 1

$(11110110)_2 \Rightarrow (-10)_{10}$

Why two?

1 0 1 0 1 0 1 1
 discarded

Range of Numbers:-

Range of numbers.

(1) 1 byte:

sign of number

0 1 0 1 0 1 0 1

128

Total = 2 * 2 * 2 * ... 8 times
 = $2^8 = 256$

actual no is stored in bits = $n-1$ \rightarrow total bits

1 byte : 7 bits

Total 1 can make from 7 bits
 $= 2^7 = \underline{128}$

— 128 to 127
 \rightarrow 0 is here as well

① \Rightarrow

0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1

②

1	1	1	1	1	1	1	1
+							1

$(10000000) \Rightarrow$ 9 bits
 will be discarded. \rightarrow $-0 = 0$

Range formula: for n bits

$-2^{n-1} \text{ to } 2^{n-1} - 1$