

MATHEMATICS FOR COMPUTATION 2024
PROBLEM SHEET 2

ISSUED 11 MARCH 2024, DUE 22 APRIL 2024

Coursework forms 25% of the assessment for this unit and will be comprised of 2 problem sheets equally weighted. On this sheet, each question is equally weighted.

1.

- (a) Write a linear equation in the form $aX + bY + cZ = d$ defining a plane in \mathbb{R}^3 passing through three points, $(0, 1, 2)$, $(5, -1, -1)$ and $(2, 0, 0)$.
- (b) Is there the unique plane passing through the following three points in \mathbb{R}^3 : $(0, 0, 0)$, $(1, 1, 0)$, $(-2, -2, 0)$? If yes, prove it. Otherwise, how many planes? Justify your answer.

2. Prove that the following functions are continuous, using the ε/δ definition.

(i)

$$f : \mathbb{R} \rightarrow \mathbb{R}$$
$$x \mapsto \frac{1}{2} x^2$$

(ii)

$$f : \mathbb{R} \rightarrow \mathbb{R}$$
$$x \mapsto -2 |x|$$

3. Consider the function f defined by the following expression

$$f(x) = \frac{x^4}{\ln x} - \sin(e^{2x-1}) + e^{1/x} + 2.$$

(Reminder: $\ln x$ stands for $\log_e x$, where $e = 2.71828 \dots$)

What is the largest subset of \mathbb{R} in which f is defined? Calculate the derivative $f'(x)$ of this function.

4. Find the second derivative $f''(x)$ of the function

$$f(x) = \begin{cases} x^3 & \text{if } x \leq 0 \\ -x^3 & \text{if } x > 0 \end{cases}.$$

Does this function have the third derivative? Justify your answers.

5. For each of the following two matrices decide whether or not it has eigenvalues among real numbers. If yes, find all real eigenvalues and corresponding eigenvectors. If not, explain why not.

(i)

$$\begin{pmatrix} 3 & 0 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

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(ii)

$$\begin{pmatrix} 1 & -1 \\ 5 & 0 \end{pmatrix}$$