MATHEMATICS FOR COMPUTATION 2024
PROBLEM SHEET 1

ISSUED 12 FEBRUARY 2024, DUE 11 MARCH 2024

Coursework forms 25% of the assessment for this unit and will be comprised of 2 problem sheets equally weighted. On this sheet, each question is equally weighted.

1. Solve the following system of linear equations using the Cramer's rule.

$$3X - Y = 8$$
$$-2X + Y + Z = 9$$
$$2X - Y + 4Z = -5.$$

The solution should be represented by simple fractions. Show your calculations of necessary determinants.

2. Solve the same system of equations by first inverting its matrix

$$A = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & 4 \end{pmatrix}.$$

Compute the inverse of A using the calculation of the adjugate matrix A^*. The elements of A^{-1} should be represented by simple fractions. Show your working, including all intermediate steps.

3. Given a matrix

$$A = \begin{pmatrix} 1 & 4 & 1 \\ -1 & 2 & 2 \\ 3 & 1 & -1 \end{pmatrix},$$

use Gaussian elimination to compute the determinant $\det(A)$ of A and to solve the system of linear equations $AX = b$, where $X = (X_1, X_2, X_3)^T$ is the vector of unknowns and $b = (1, 0, 1)^T$. The solution should be represented by simple fractions. Show your working.

4. Using Gaussian elimination, compute the rank of the following (4×4)-matrix

$$\begin{pmatrix} -1 & 2 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 3 & 1 & 3 \\ -2 & 4 & 2 & 4 \end{pmatrix}$$

Show your working.

5. Consider the following system of linear equations.

$$3X_1 - 2X_2 + 3X_3 - X_4 = 1$$
$$X_2 + X_4 = 3$$
$$X_1 + X_2 - 2X_3 + 4X_4 = 1$$

Using Gaussian elimination, show that it has at least one solution. Represent the general solution as an affine map from one vector space to another, in matrix/vector form. Find one specific solution. Show your working.