

Sorting Algorithms I

Basic Sorts

Why do we need to know sorting?

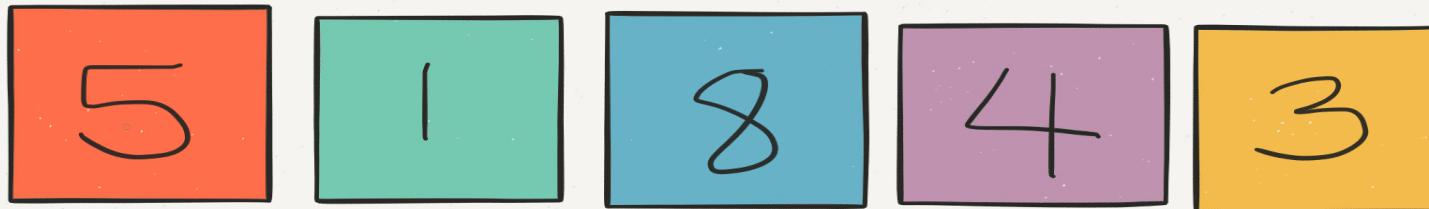
- Fundamental routine
- Many applications use them
- Many sorting algorithms choices
- Consider constraints & tradeoffs
 - dataset type and size
 - stability requirements
 - ease of understanding
 - time and space complexity

Basic Sorting Algorithms

- Bubble Sort
- Selection Sort
- Insertion Sort

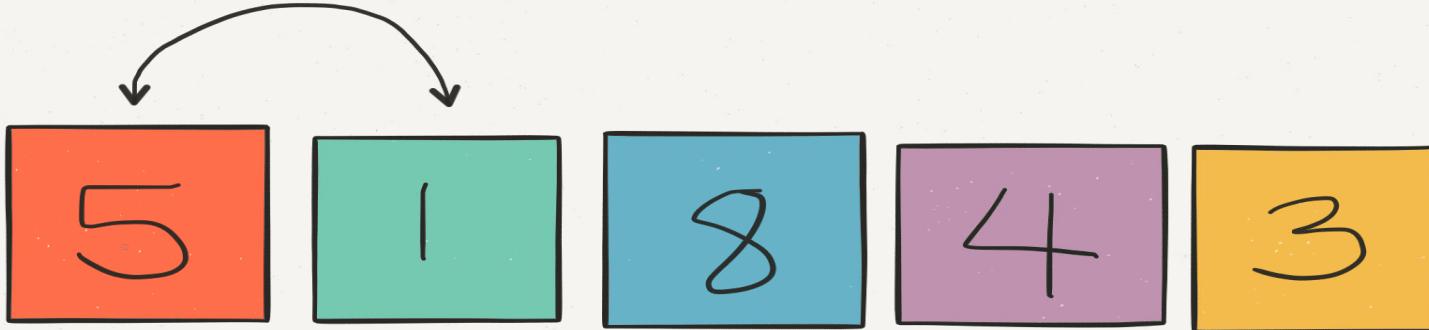
Purpose?

- Easy to understand
- Good starting point
- Useful for small inputs ($n < 10$)



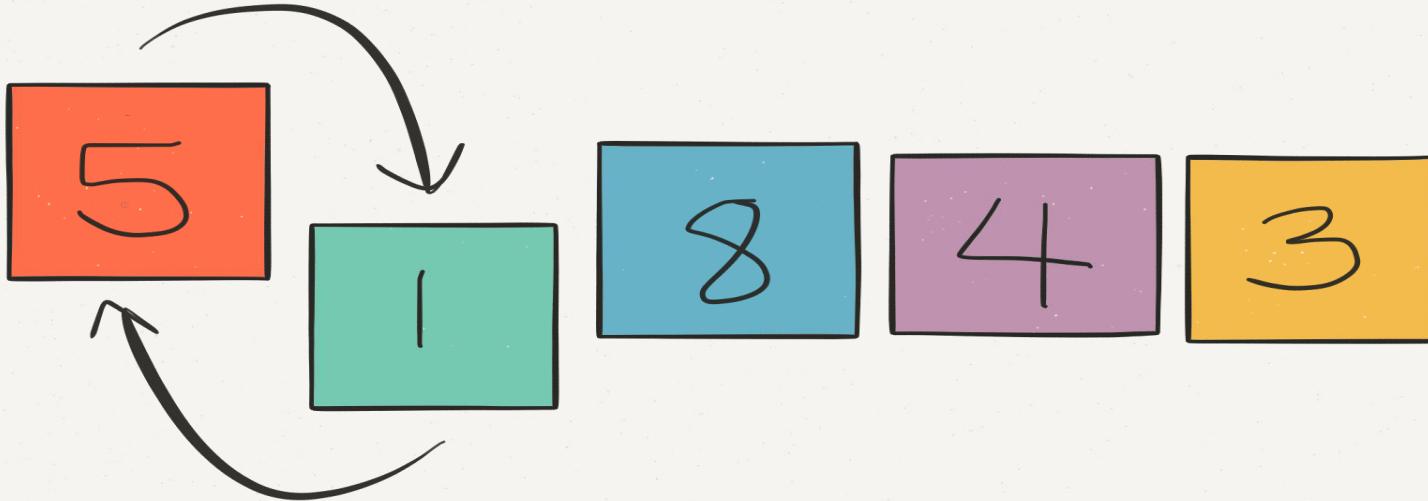
Bubble



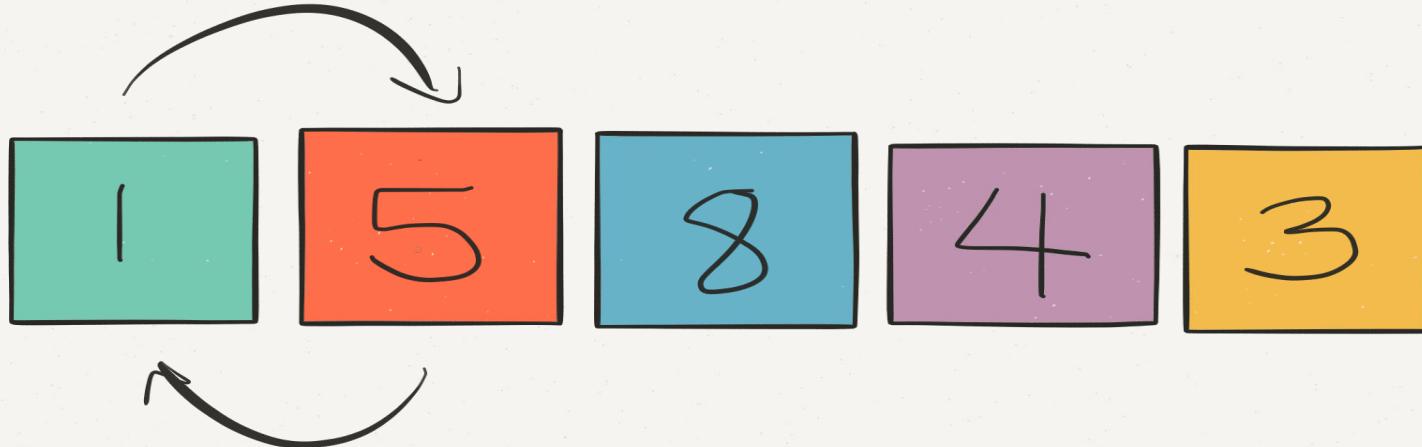


5 > 1

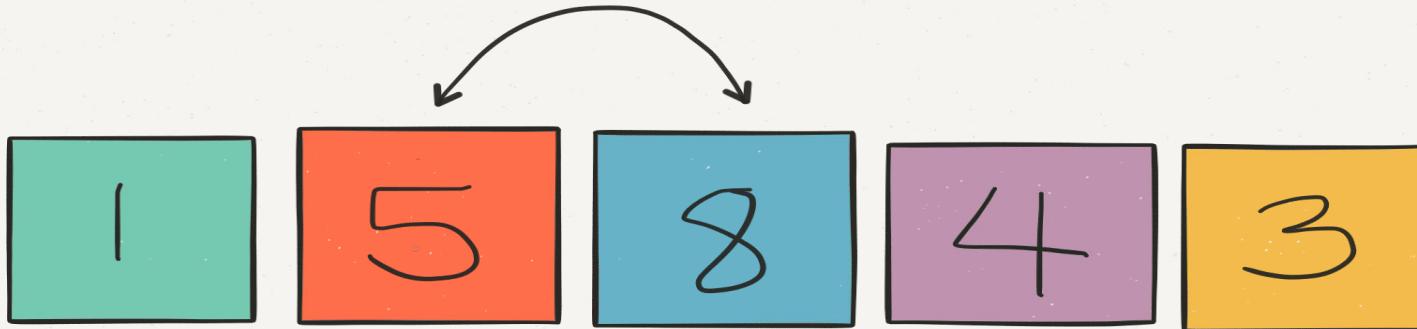
SWAP?


$$5 > 1$$

SWAP : TRUE



5 > 1


$$5 < 8$$

1

5

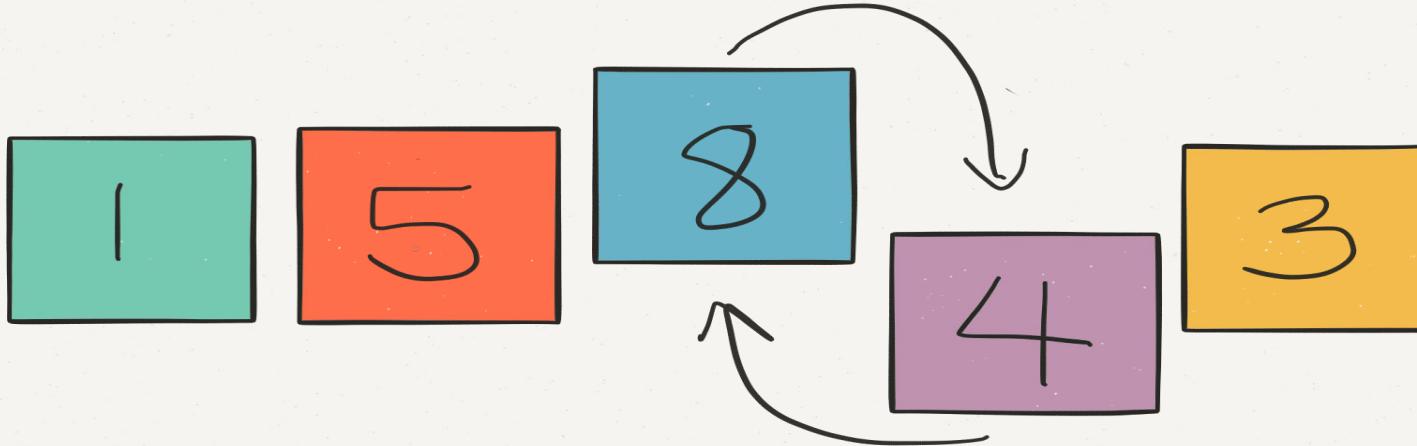
8

4

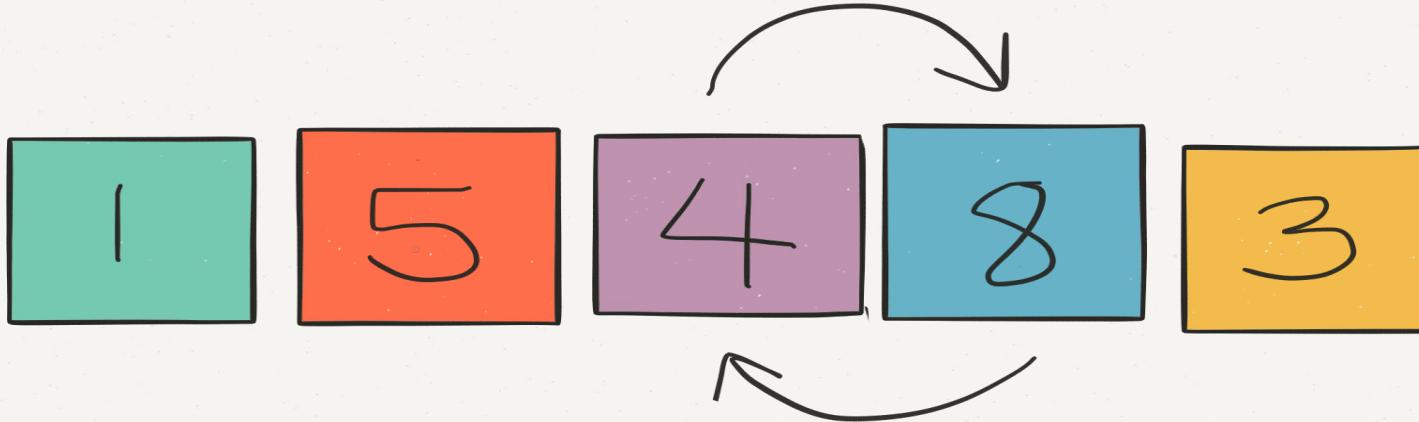
3



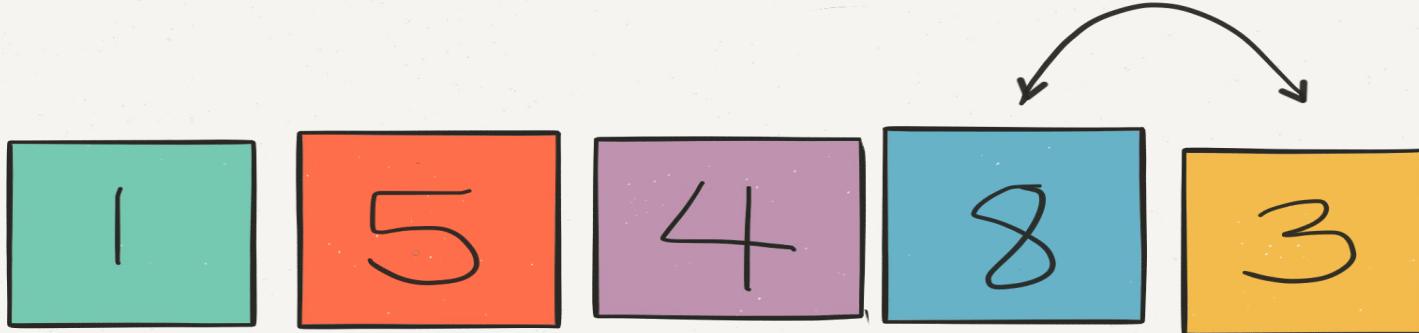
$$8 > 4$$



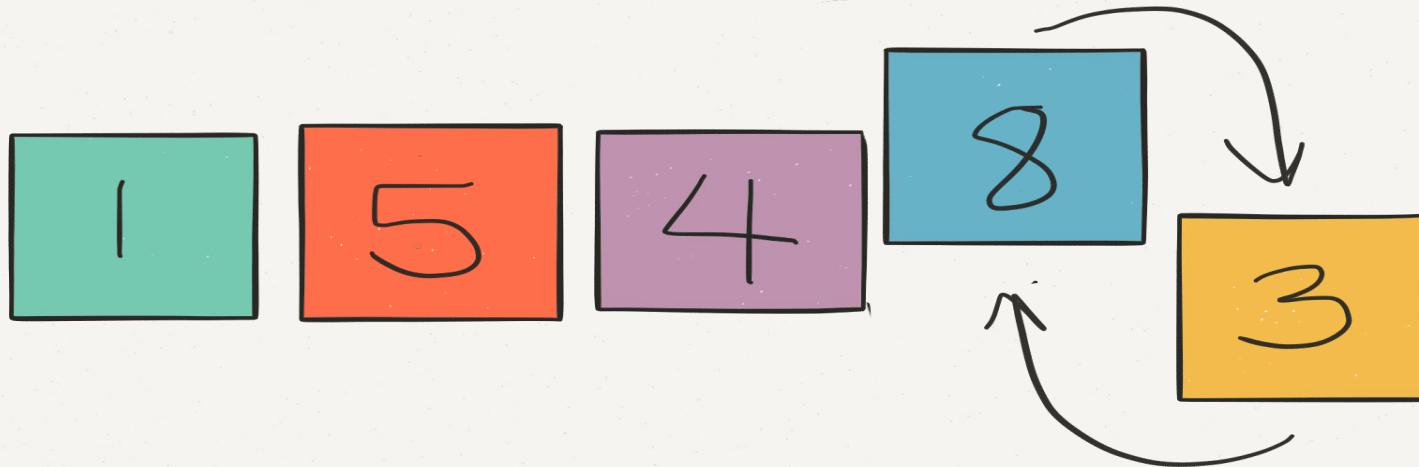
$$8 > 4$$

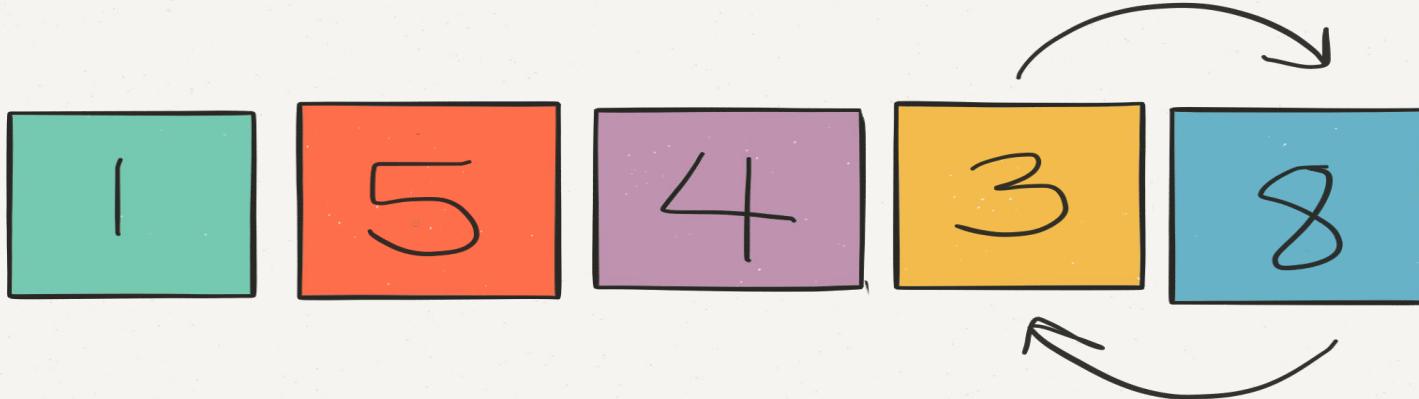


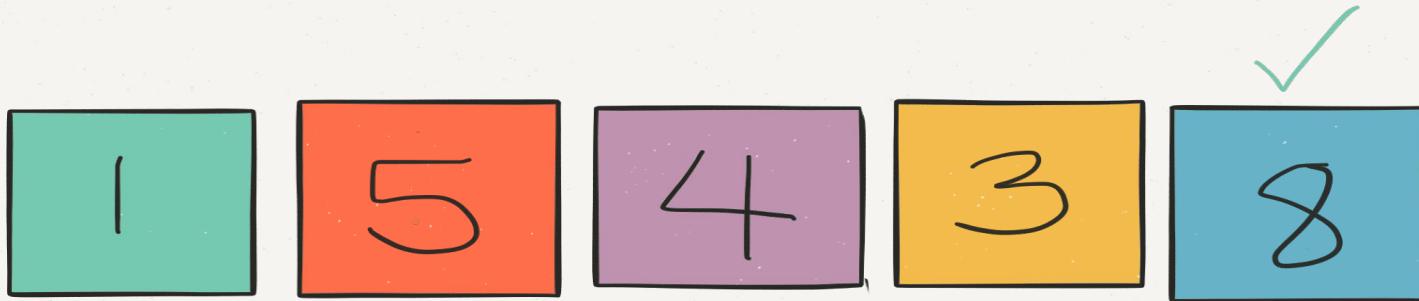
$$8 > 4$$



8 > 3

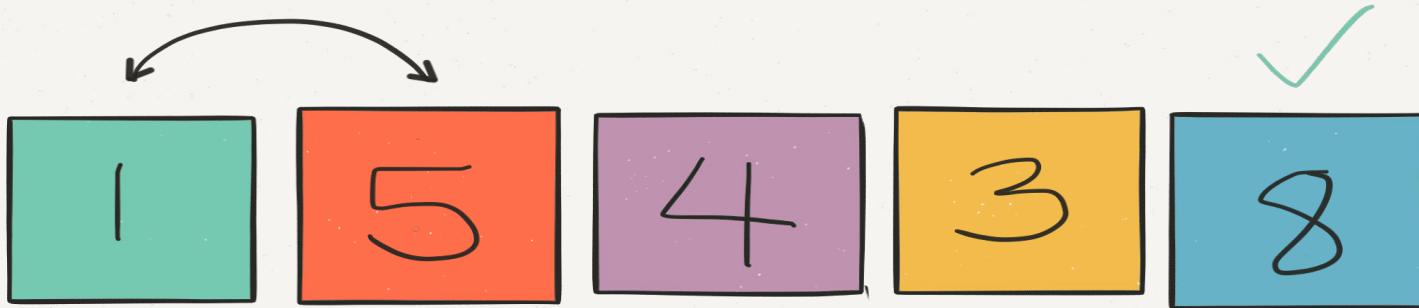

$$8 > 3$$


$$8 > 3$$



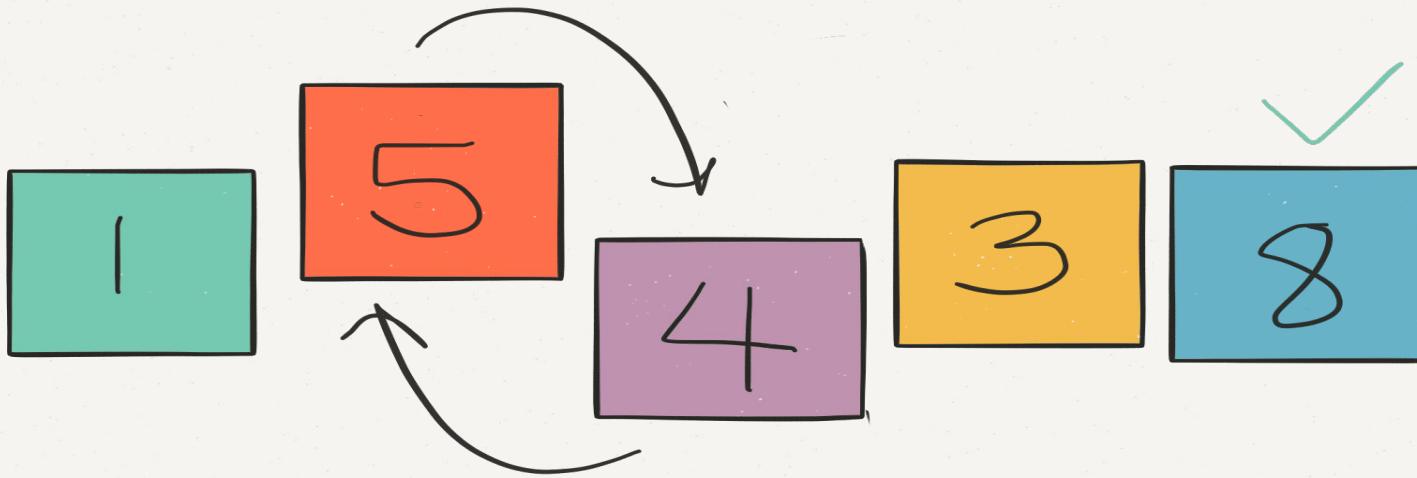
SWAP : TRUE

REPEAT



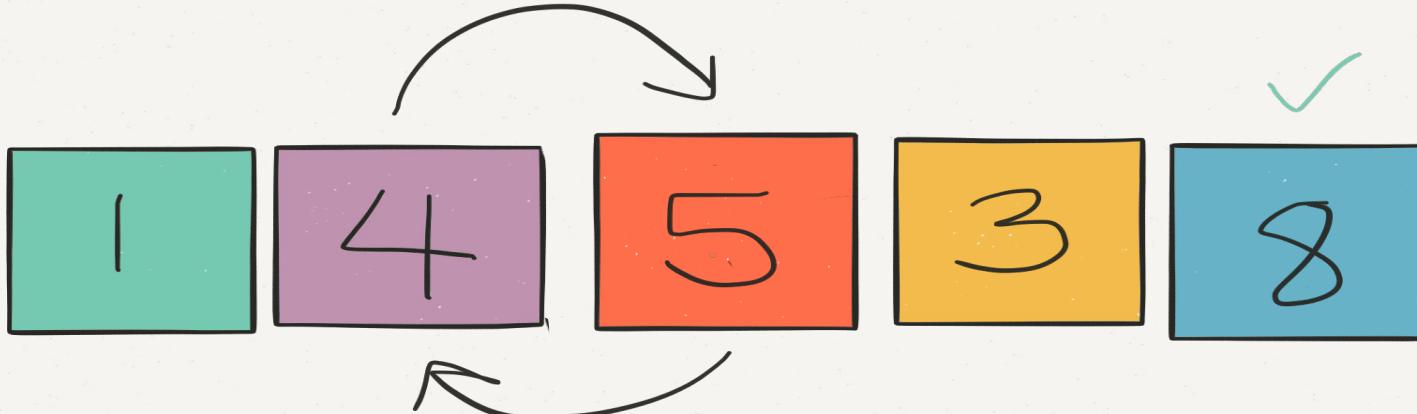
$$1 < 5$$

~~SWAP~~ : ?



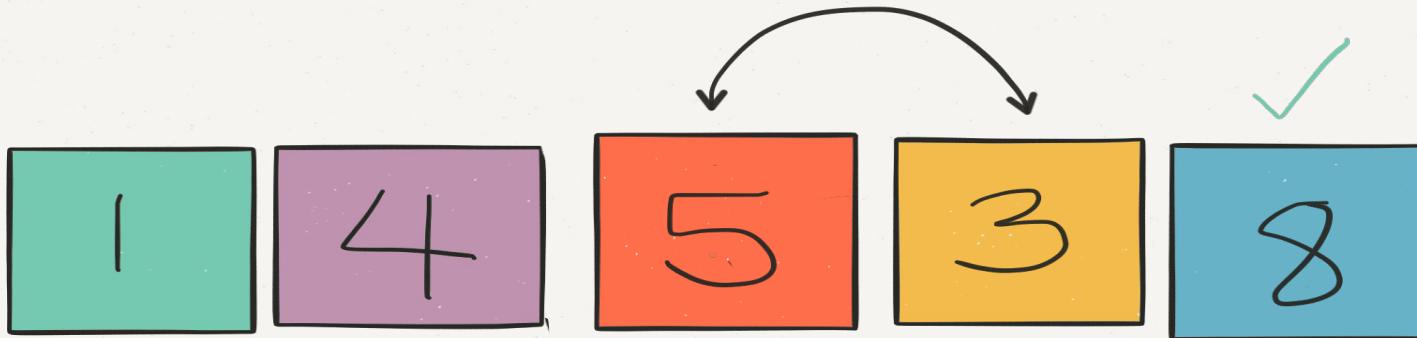
$$5 > 4$$

SWAP : TRUE



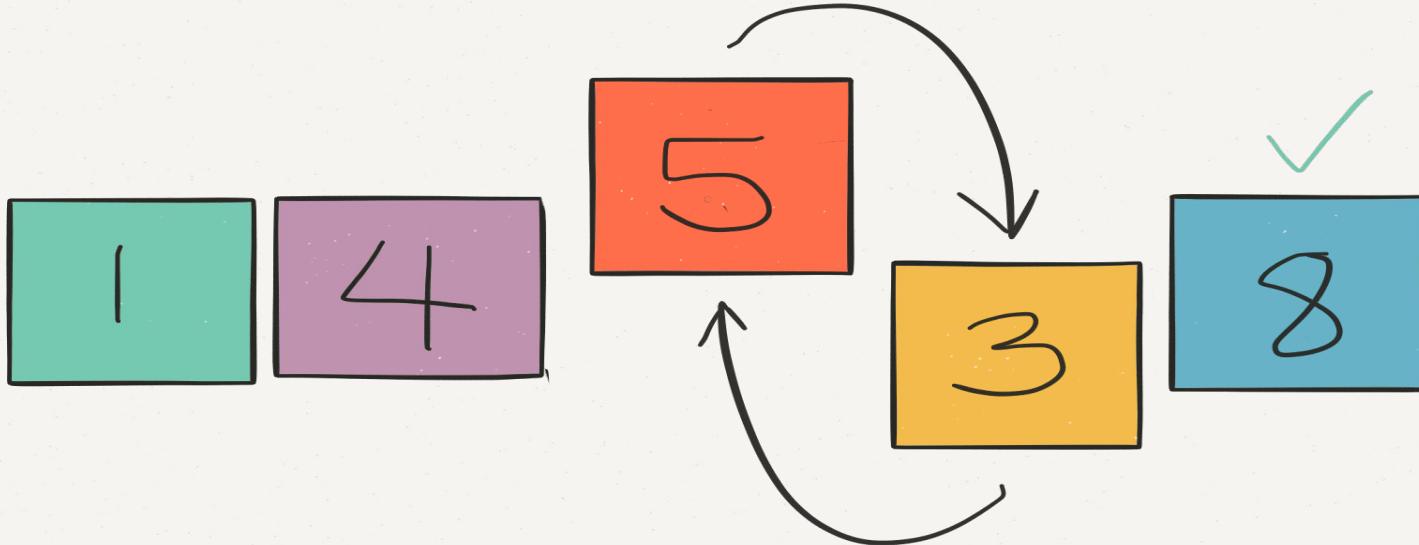
$$5 > 4$$

SWAP : TRUE

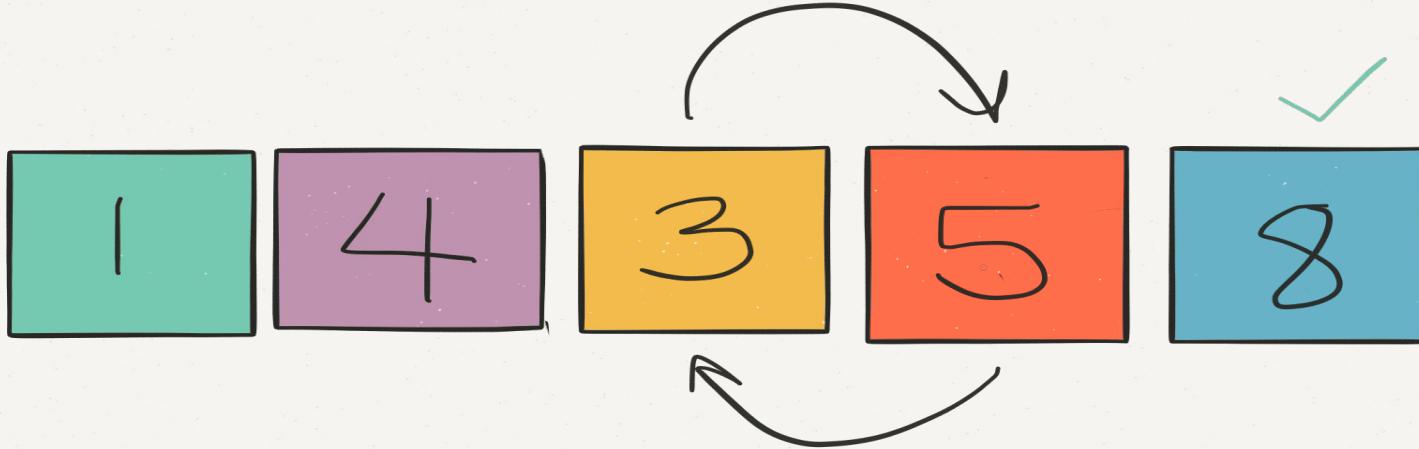


$$5 > 3$$

SWAP : TRUE



~~SWAP~~ : TRUE



SWAP : TRUE

1

4

3

5

8



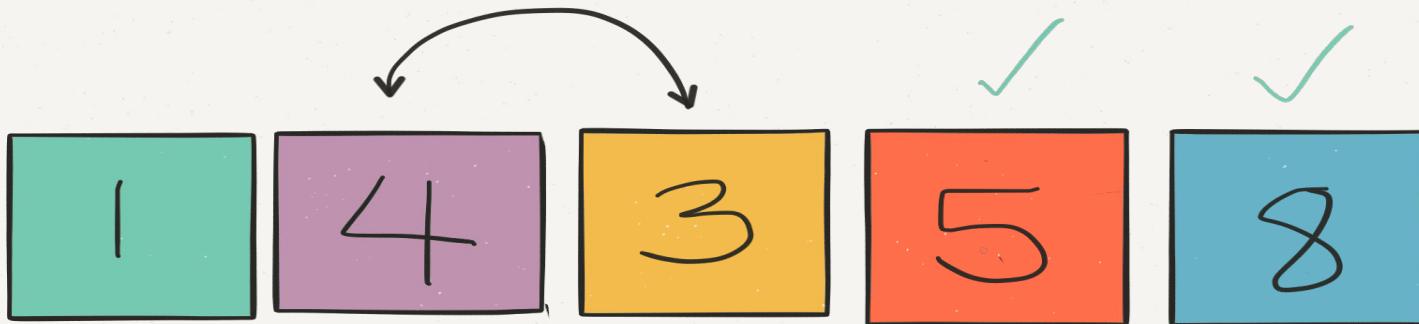
REPEAT!

~~SWAP~~ : TRUE



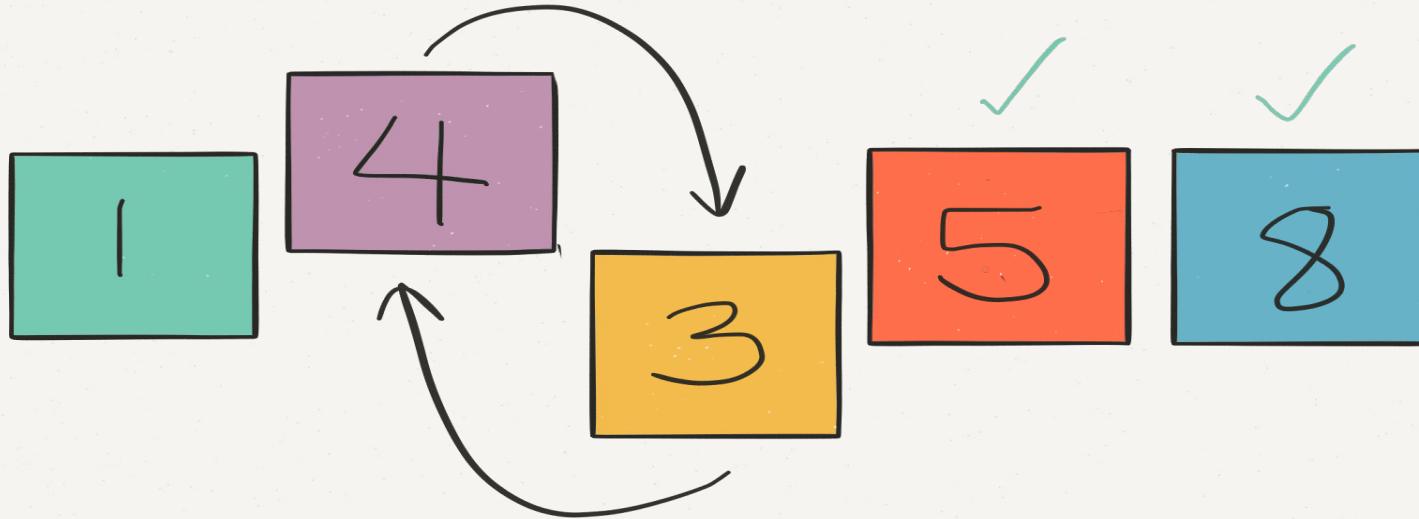
$$1 < 4$$

~~SWAP~~ : ?

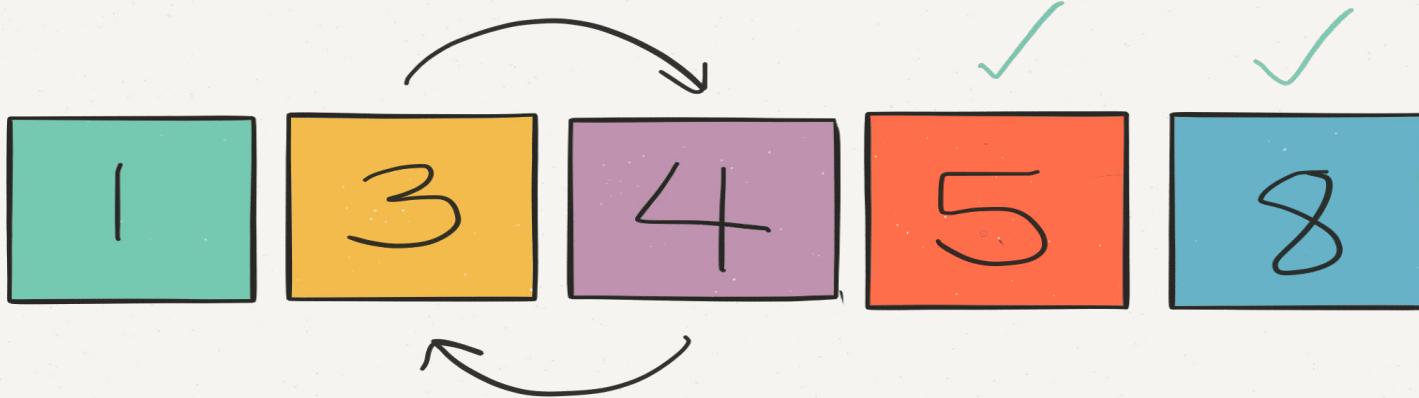


473

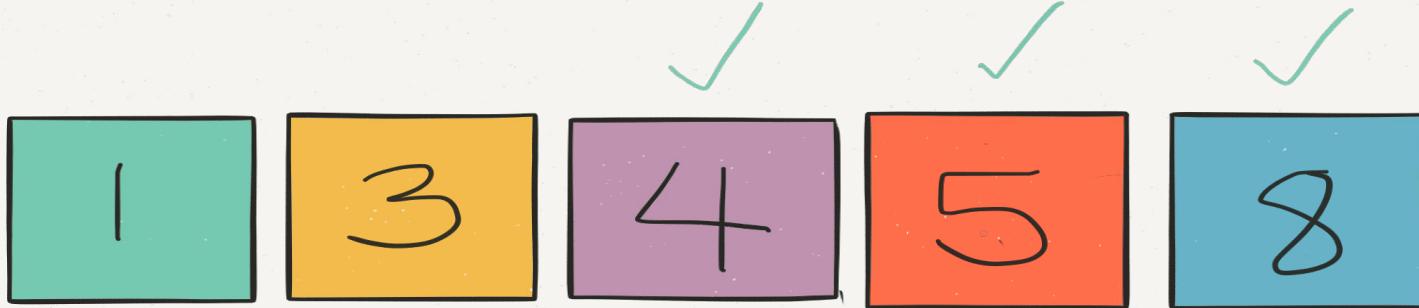
~~SWAP~~: TRUE



SWAP : TRUE

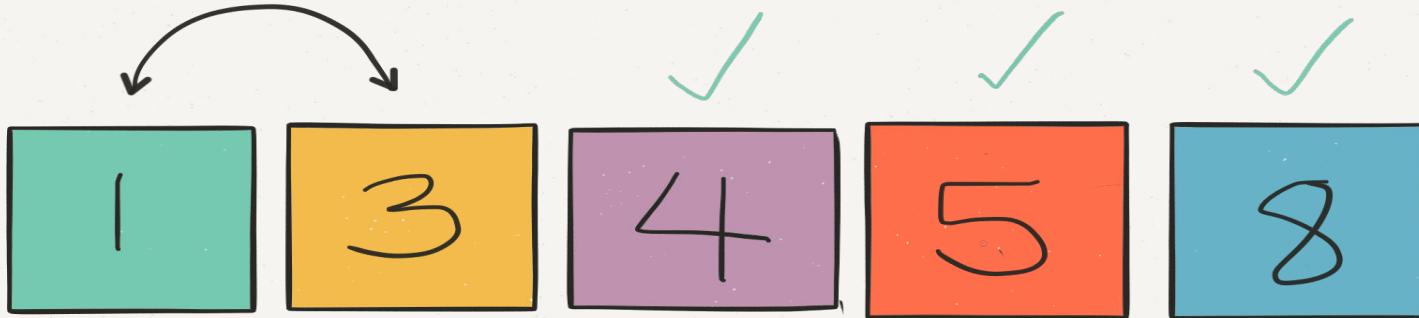


~~SWAP~~: TRUE



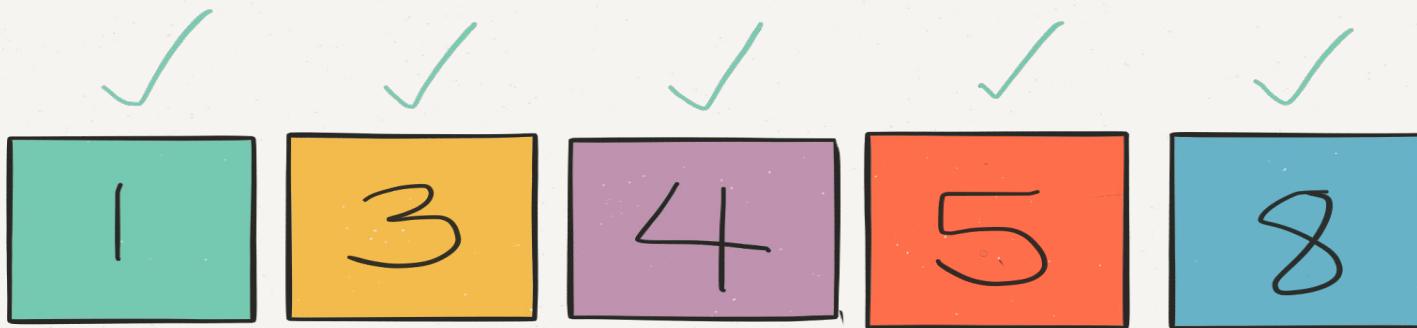
REPE~~X~~T!

SWAP: TRUE



$$1 < 3$$

~~SWAP~~ : ?



DONE!

SWAP : FALSE

Bubble Sort

```
def bubble(arr):
```

Bubble Sort

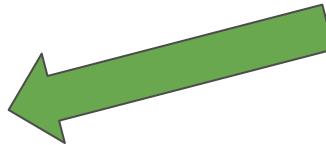
loop backwards

```
def bubble(arr):  
    for end in range(len(arr)-1, -1, -1):
```



Bubble Sort

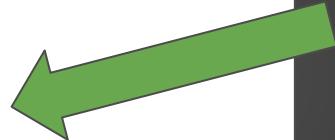
```
def bubble(arr):  
    for end in range(len(arr)-1, -1, -1):  
        for i in range(0, end):
```



loop from 0 to “end”

Bubble Sort

```
def bubble(arr):  
    for end in range(len(arr)-1, -1, -1):  
  
        for i in range(0, end):  
            if arr[i] > arr[i+1]:  
                arr[i], arr[i+1] = arr[i+1], arr[i]
```



perform a swap if
current and next is
out of order

Bubble Sort

```
def bubble(arr):  
    for end in range(len(arr)-1, -1, -1):  
  
        for i in range(0, end):  
            if arr[i] > arr[i+1]:  
                arr[i], arr[i+1] = arr[i+1], arr[i]  
  
    return arr
```

return sorted array

Bubble Sort

```
def bubble(arr):  
    for end in range(len(arr)-1, -1, -1):  
        swap = False  
        for i in range(0, end):  
            if arr[i] > arr[i+1]:  
                arr[i], arr[i+1] = arr[i+1], arr[i]  
  
    return arr
```



add flag to see if
there was a swap
within that loop

Bubble Sort

```
def bubble(arr):  
    for end in range(len(arr)-1, -1, -1):  
        swap = False  
        for i in range(0, end):  
            if arr[i] > arr[i+1]:  
                arr[i], arr[i+1] = arr[i+1], arr[i]  
                swap = True  
  
    return arr
```

if there is a swap,
change to ‘True’



Bubble Sort

```
def bubble(arr):  
    for end in range(len(arr)-1, -1, -1):  
        swap = False  
        for i in range(0, end):  
            if arr[i] > arr[i+1]:  
                arr[i], arr[i+1] = arr[i+1], arr[i]  
                swap = True  
        if swap == False:  
            break  
    return arr
```



if no swaps were made in one outer loop, array is sorted

Bubble Sort

```
def bubble(arr):
    for end in range(len(arr)-1,-1,-1):
        swap = False
        for i in range(0, end):
            if arr[i] > arr[i+1]:
                arr[i], arr[i+1] = arr[i+1], arr[i]
                swap = True
            if swap == False:
                break
    return arr
```

Bubble Sort - Time Complexity

```
def bubble(arr):
    for end in range(len(arr)-1,-1,-1):
        swap = False
        for i in range(0, end):
            if arr[i] > arr[i+1]:
                arr[i], arr[i+1] = arr[i+1], arr[i]
                swap = True
            if swap == False:
                break
    return arr
```

Bubble Sort - Time Complexity

```
def bubble(arr):
    for end in range(len(arr)-1, -1, -1): # n
        swap = False
        for i in range(0, end):
            if arr[i] > arr[i+1]:
                arr[i], arr[i+1] = arr[i+1], arr[i]
                swap = True
            if swap == False:
                break
    return arr
```

Bubble Sort - Time Complexity

```
def bubble(arr):
    for end in range(len(arr)-1, -1, -1):
        swap = False
        for i in range(0, end):
            if arr[i] > arr[i+1]:
                arr[i], arr[i+1] = arr[i+1], arr[i]
                swap = True
            if swap == False:
                break
    return arr
```

n
1

Bubble Sort - Time Complexity

```
def bubble(arr):
    for end in range(len(arr)-1, -1, -1):
        swap = False
        for i in range(0, end):
            if arr[i] > arr[i+1]:
                arr[i], arr[i+1] = arr[i+1], arr[i]
                swap = True
            if swap == False:
                break
    return arr
```

n
1
n/2

Bubble Sort - Time Complexity

```
def bubble(arr):  
    for end in range(len(arr)-1, -1, -1):  
        swap = False  
        for i in range(0, end):  
            if arr[i] > arr[i+1]:  
                arr[i], arr[i+1] = arr[i+1], arr[i]  
                swap = True  
            if swap == False:  
                break  
    return arr
```

n
1
$n/2$
1
1
1

Bubble Sort - Time Complexity

```
def bubble(arr):  
    for end in range(len(arr)-1, -1, -1):  
        swap = False  
        for i in range(0, end):  
            if arr[i] > arr[i+1]:  
                arr[i], arr[i+1] = arr[i+1], arr[i]  
                swap = True  
            if swap == False:  
                break  
    return arr
```

n
1
$n/2$
1
1
1
1
1
1

Bubble Sort - Time Complexity

```
def bubble(arr):  
    for end in range(len(arr)-1, -1, -1):  
        swap = False  
        for i in range(0, end):  
            if arr[i] > arr[i+1]:  
                arr[i], arr[i+1] = arr[i+1], arr[i]  
                swap = True  
        if swap == False:  
            break  
    return arr
```

n
1
$n/2$
1
1
1
1
1
1
1

Bubble Sort - Time Complexity

```
def bubble(arr):  
    for end in range(len(arr)-1, -1, -1):  
        swap = False  
        for i in range(0, end):  
            if arr[i] > arr[i+1]:  
                arr[i], arr[i+1] = arr[i+1], arr[i]  
                swap = True  
            if swap == False:  
                break  
    return arr
```

n
1
$n/2$
3

1
1
1

Bubble Sort - Time Complexity

```
def bubble(arr):  
    for end in range(len(arr)-1, -1, -1):  
        swap = False  
        for i in range(0, end):  
            if arr[i] > arr[i+1]:  
                arr[i], arr[i+1] = arr[i+1], arr[i]  
                swap = True  
            if swap == False:  
                break  
    return arr
```

n
1
$\frac{3}{2}n$

1
1
1

Bubble Sort - Time Complexity

```
def bubble(arr):  
    for end in range(len(arr)-1, -1, -1):  
        swap = False  
        for i in range(0, end):  
            if arr[i] > arr[i+1]:  
                arr[i], arr[i+1] = arr[i+1], arr[i]  
                swap = True  
            if swap == False:  
                break  
    return arr
```

n
$3/2n + 3$

1

Bubble Sort - Time Complexity

```
def bubble(arr):
    for end in range(len(arr)-1, -1, -1):
        swap = False
        for i in range(0, end):
            if arr[i] > arr[i+1]:
                arr[i], arr[i+1] = arr[i+1], arr[i]
                swap = True
            if swap == False:
                break
    return arr
```

$3/2n^2 + 3n$

1

Bubble Sort - Time Complexity

```
def bubble(arr):
    for end in range(len(arr)-1, -1, -1):
        swap = False
        for i in range(0, end):
            if arr[i] > arr[i+1]:
                arr[i], arr[i+1] = arr[i+1], arr[i]
                swap = True
            if swap == False:
                break
    return arr
```

$3/2n^2 + 3n + 1$

Bubble Sort - Time Complexity

```
def bubble(arr):
    for end in range(len(arr)-1, -1, -1):
        swap = False
        for i in range(0, end):
            if arr[i] > arr[i+1]:
                arr[i], arr[i+1] = arr[i+1], arr[i]
                swap = True
            if swap == False:
                break
    return arr
```

~~$\frac{3}{2}n^2 + 3n + 1$~~

drop coefficients
and lower order
terms

Bubble Sort - Time Complexity

```
def bubble(arr):
    for end in range(len(arr)-1, -1, -1):
        swap = False
        for i in range(0, end):
            if arr[i] > arr[i+1]:
                arr[i], arr[i+1] = arr[i+1], arr[i]
                swap = True
            if swap == False:
                break
    return arr
```

Quadratic $O(n^2)$

Time Complexity

- Worse case
 - $O(n^2)$
- Average Case
 - $O(n^2)$
- Best Case
 - $\Omega(n)$



Advantages/Disadvantage

- Simple to understand
- Only linear time if sorted
- Constant auxiliary space
- Stable
- Impractical for virtually all applications
 - Insertion sort is faster

Stable vs Non-Stable

- Stable sorts leave equal values in original order.
- Non-stable sorts can not guarantee original order of equal values

High scores

24

JOHN

55

KATE

51

JANE

34

CARL

55

ROB

High scores - sorted

55

KATE

55

ROB

51

JANE

34

CARL

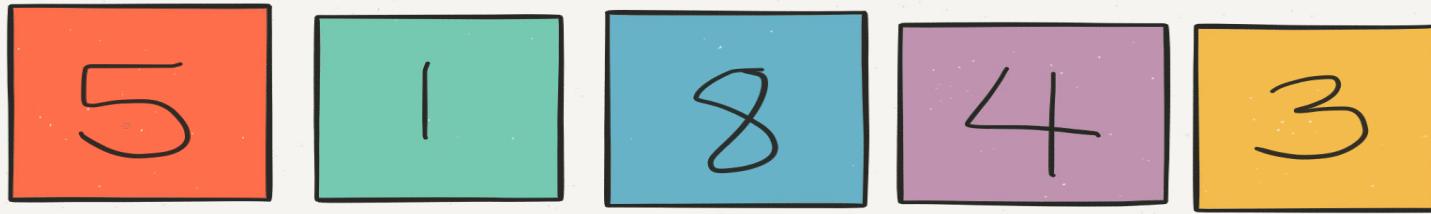
24

JOHN

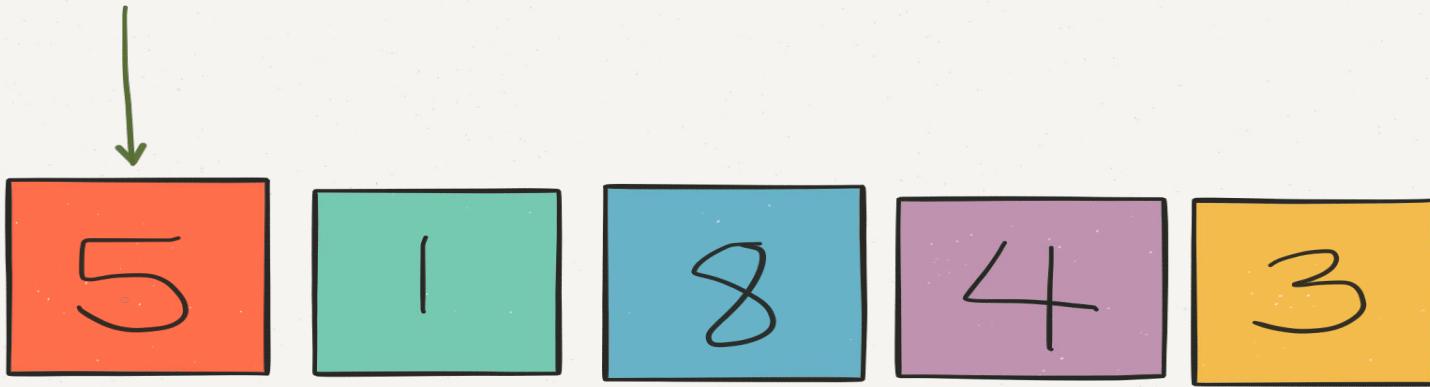
When would we need a stable sort?

- Tiebreakers
 - Last, First Name
- Maintain order
 - top scores

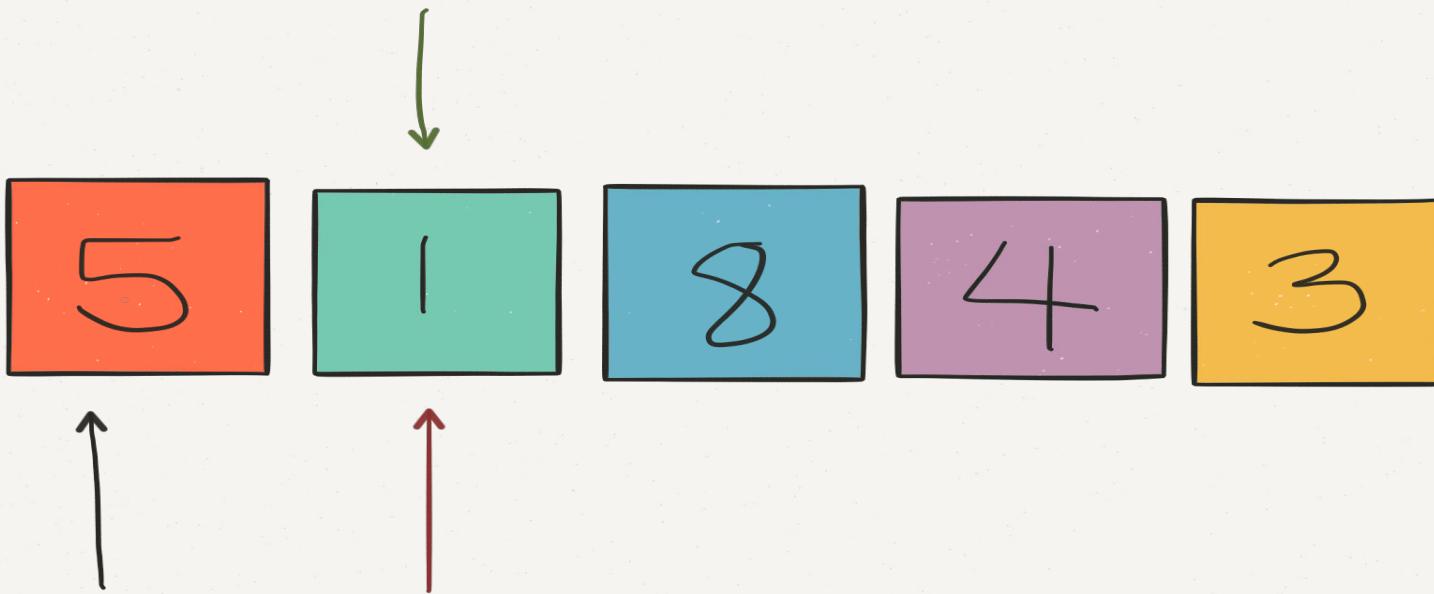
1	Kate : 55 pts
2	Rob : 55 pts
3	Jane : 51 pts



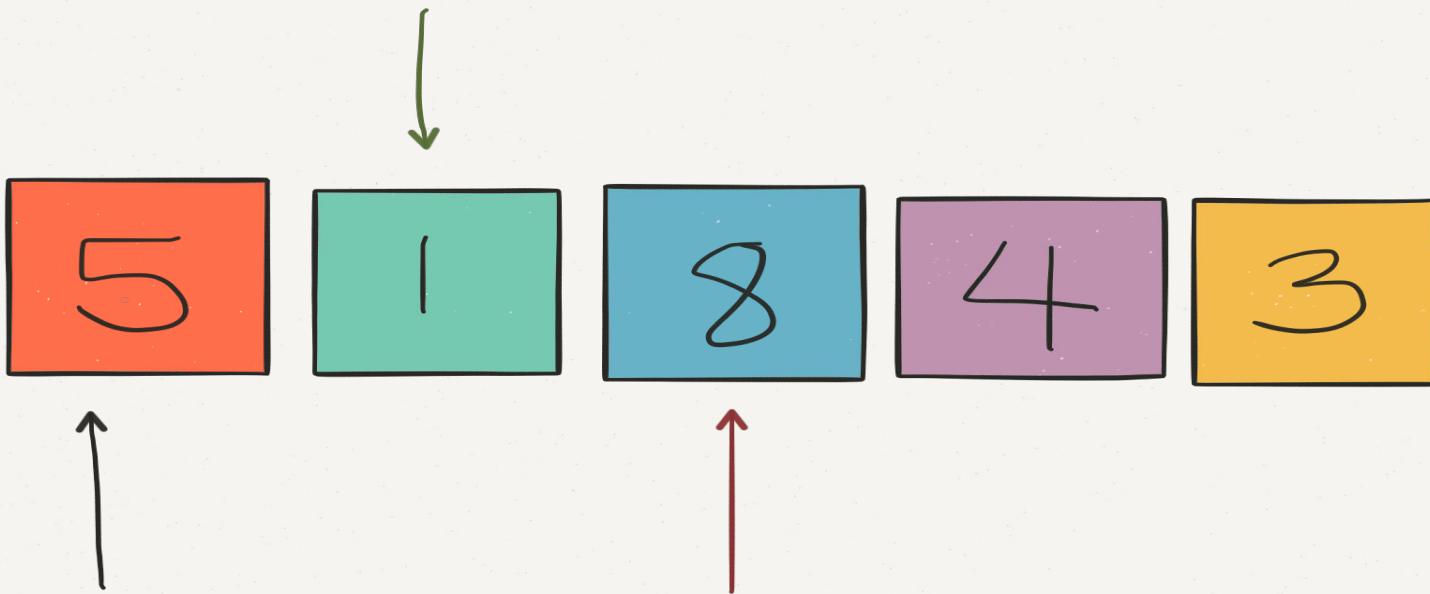
SELECTION



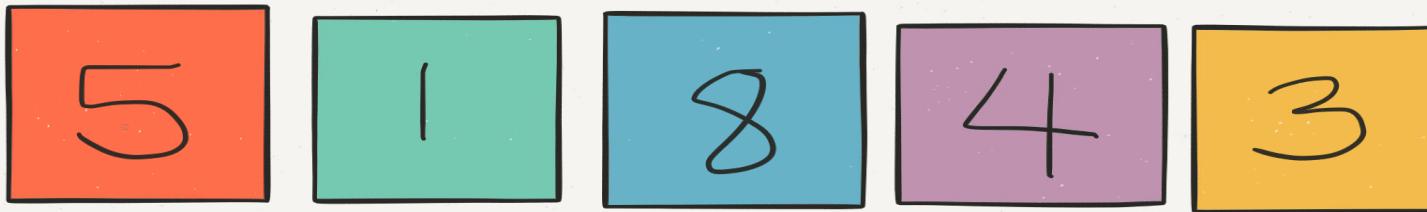
$$\min = 5$$



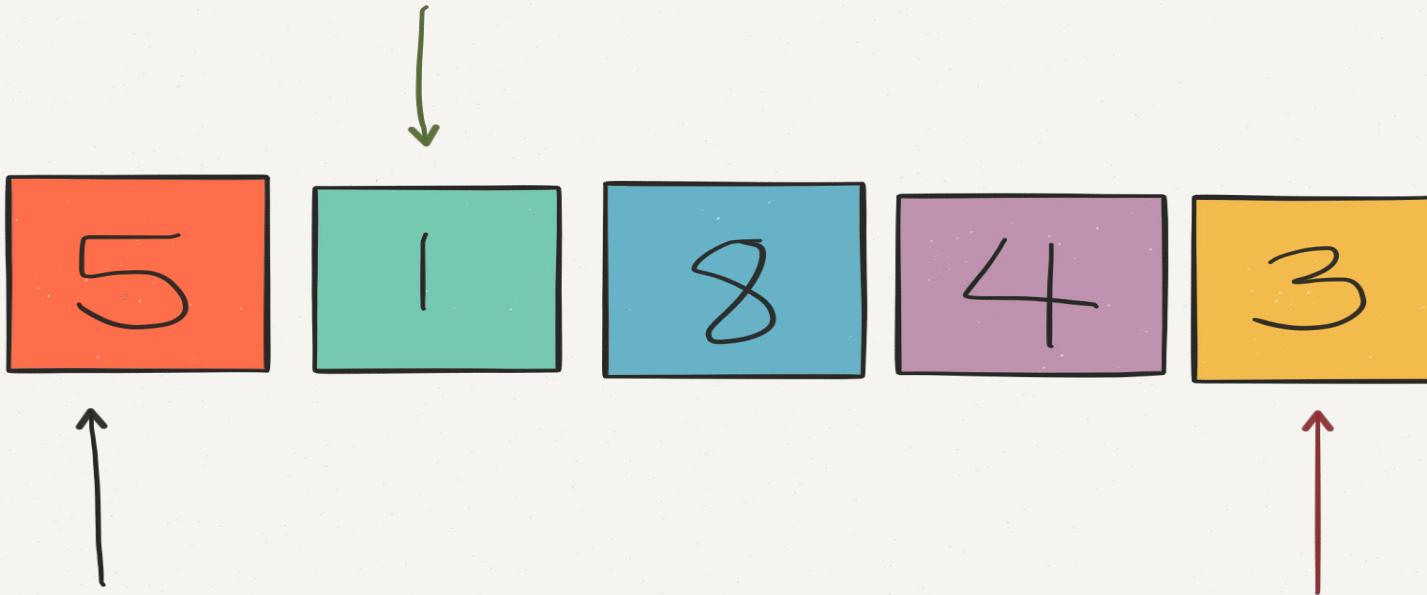
$$\min = 1$$



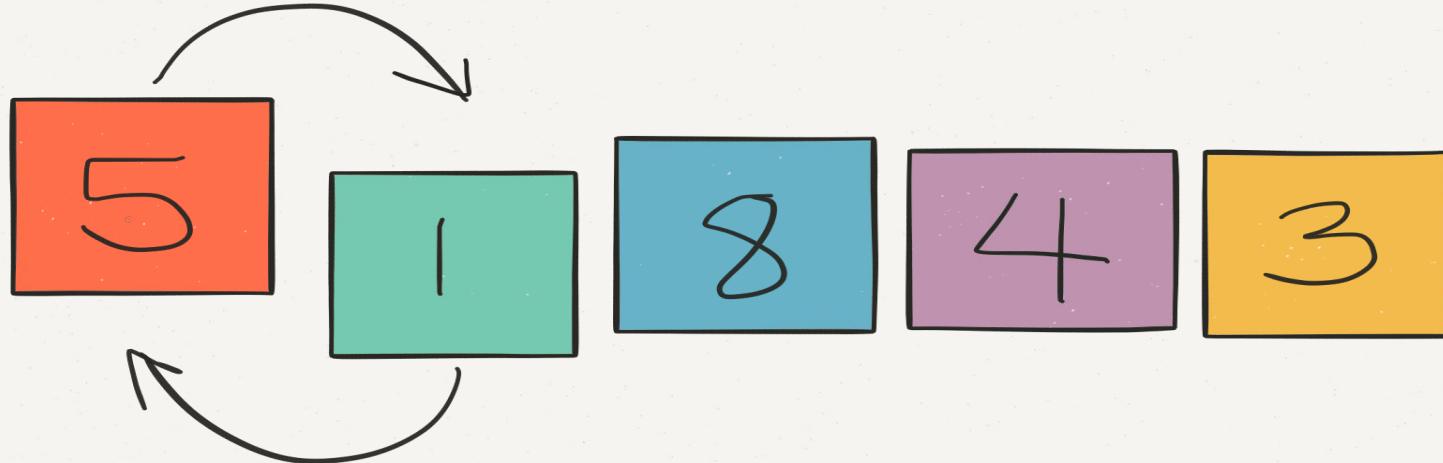
$$\min = 1$$



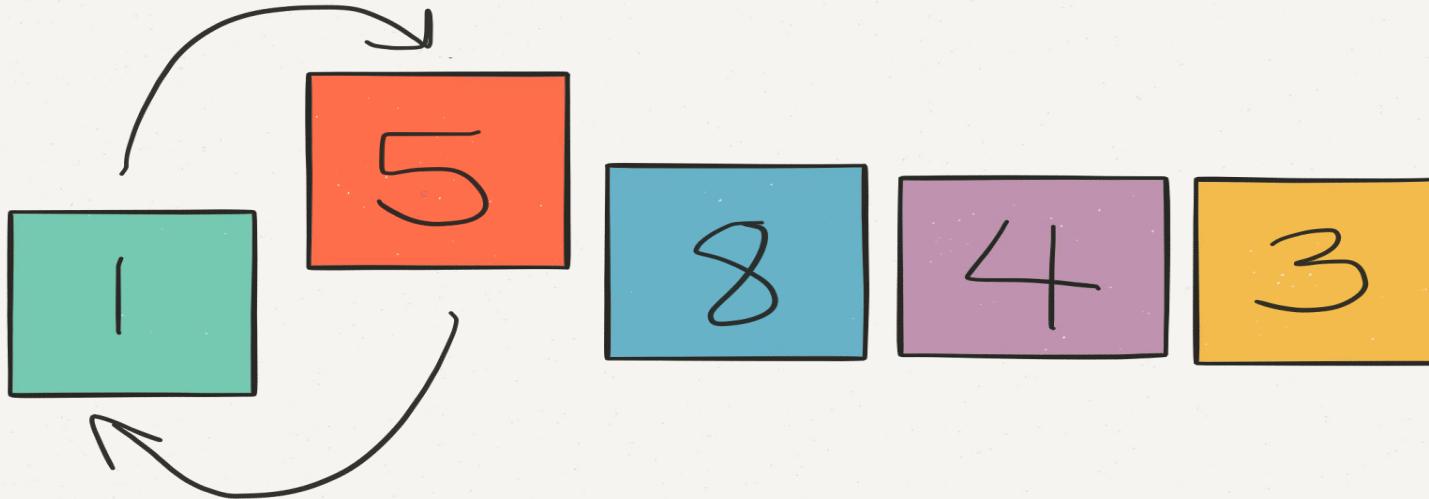
MIN = 1



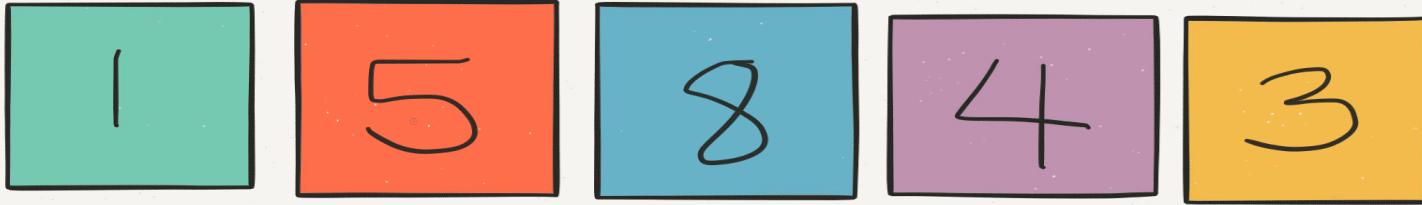
$$\min = 1$$



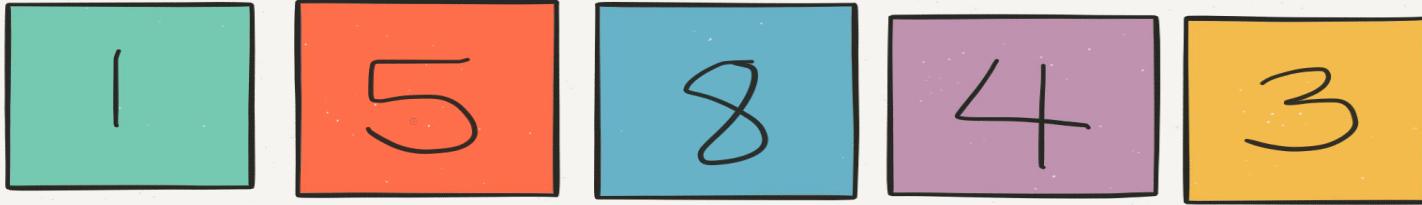
$$\min = 1$$



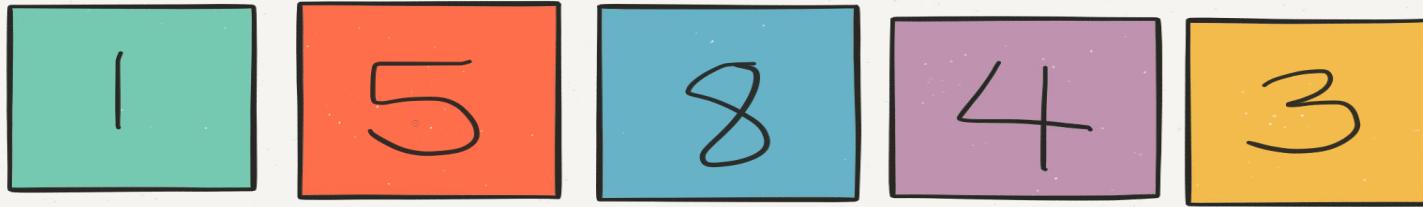
$$\min = 1$$



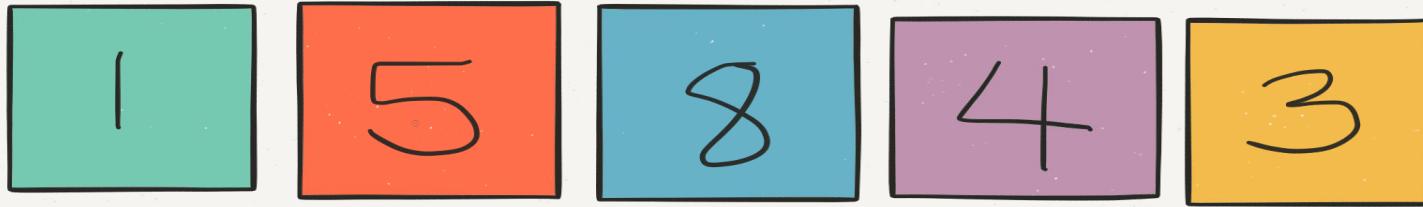
$\min = 5$



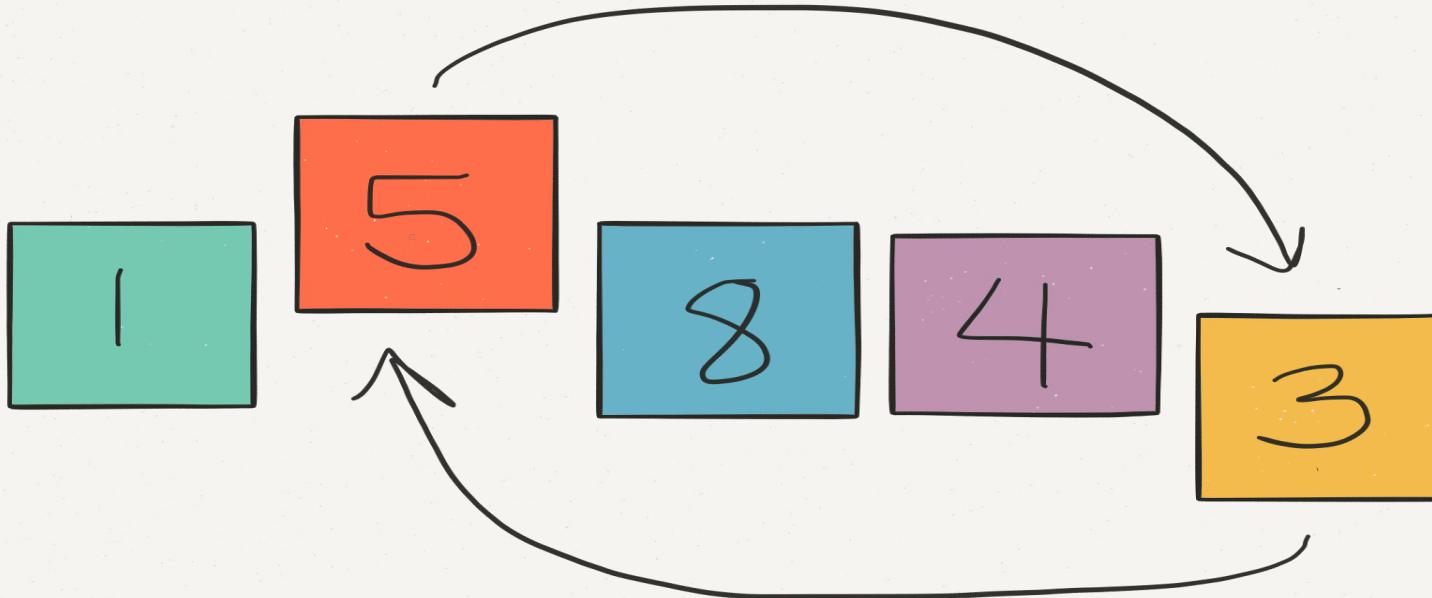
$\min = 5$



min = 4



$\min = 3$



$$\min = 3$$

1

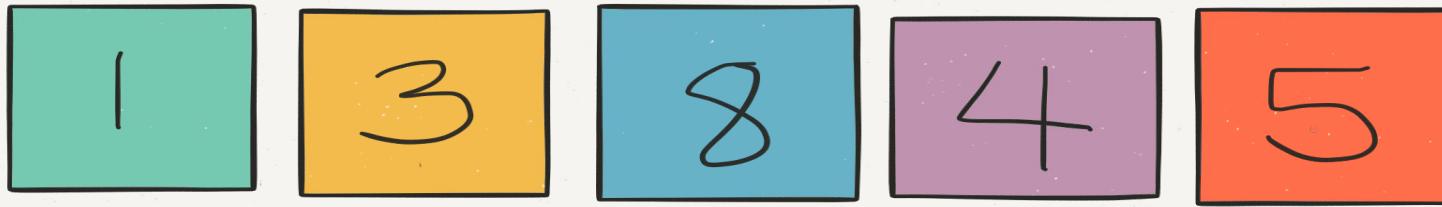
3

8

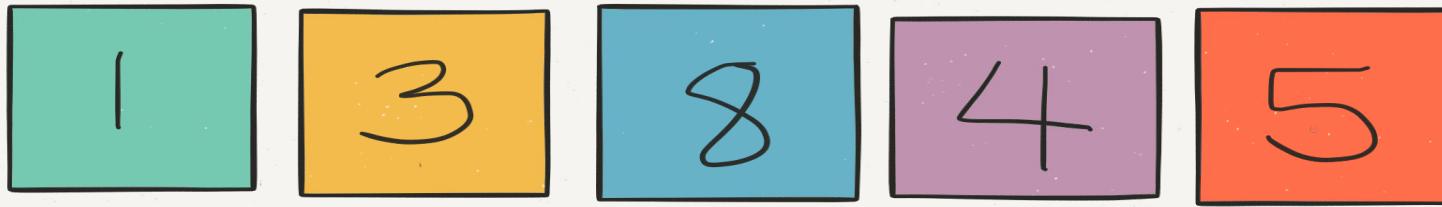
4

5

min = 3



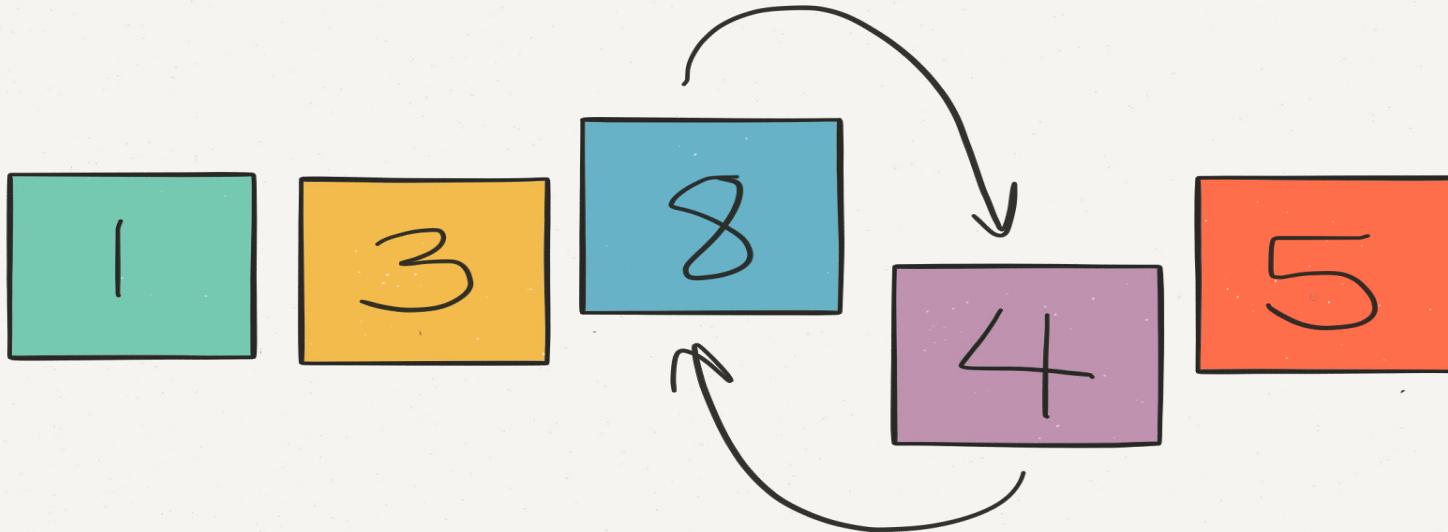
$$\min = 8$$



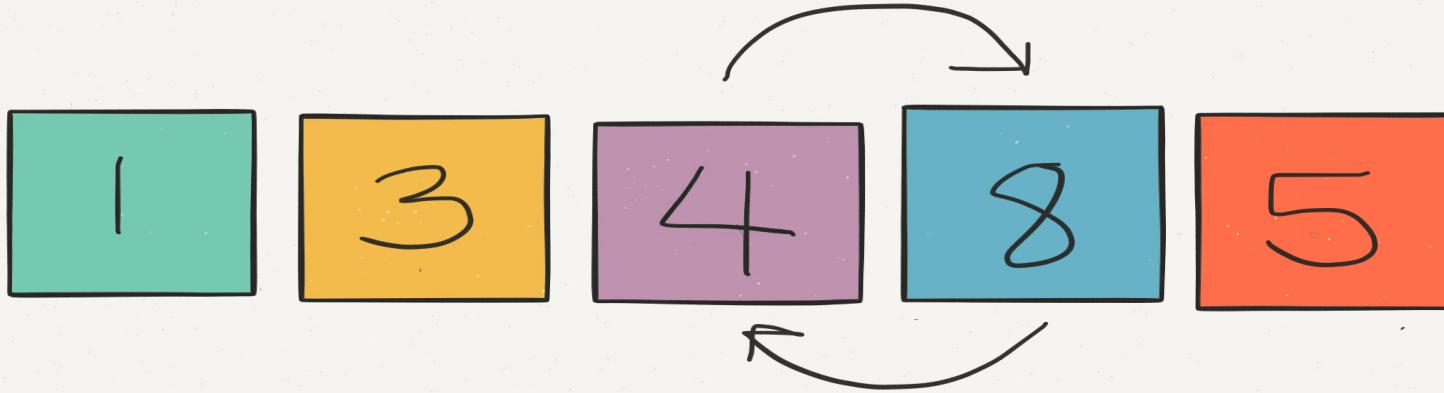
$\min = 4$



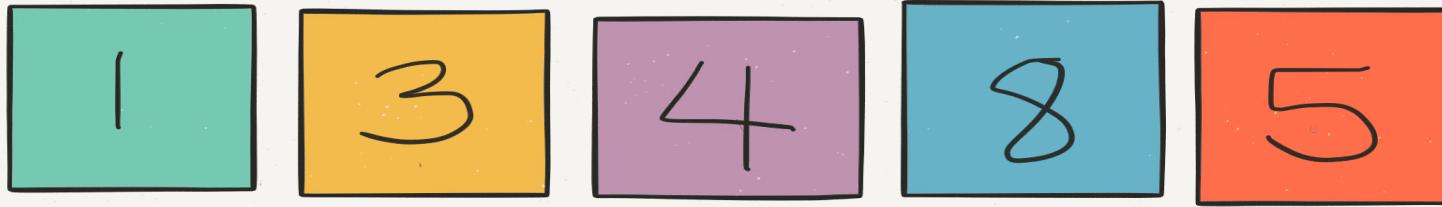
$$\min = 4$$



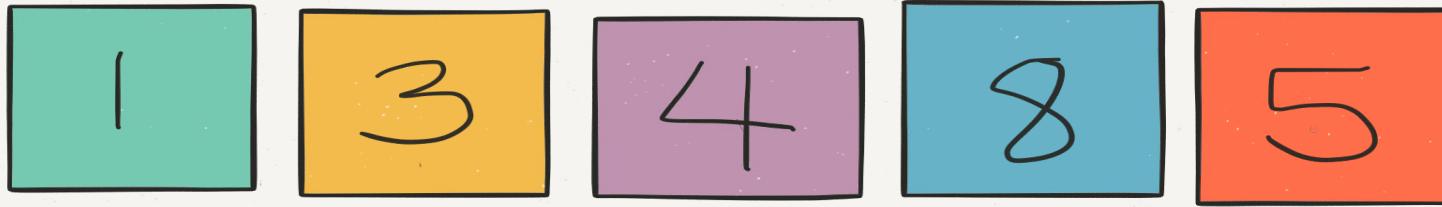
$$\min = 4$$



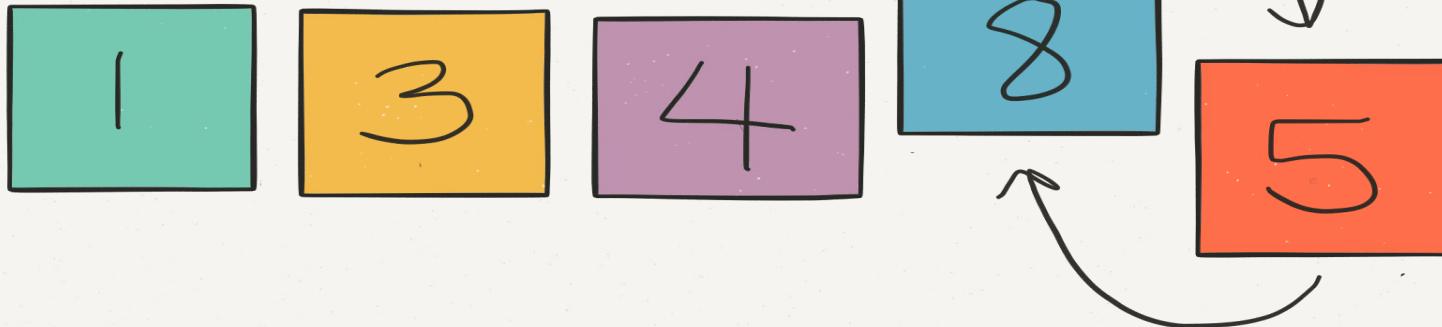
$$\min = 4$$



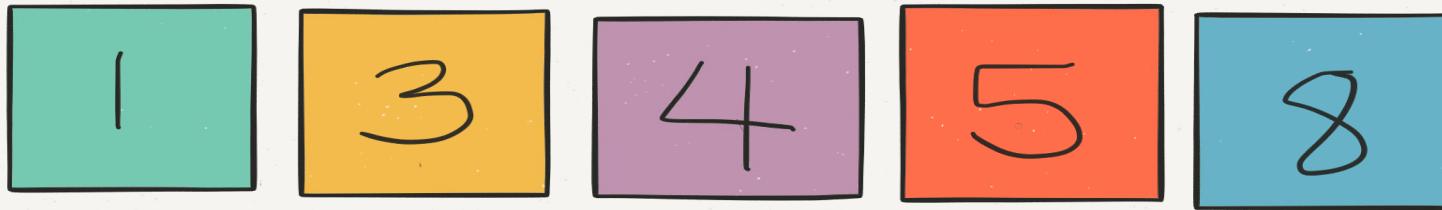
$\min = 8$



$\min = 5$



$\min = 5$



$\min = 5$

1

3

4

5

8

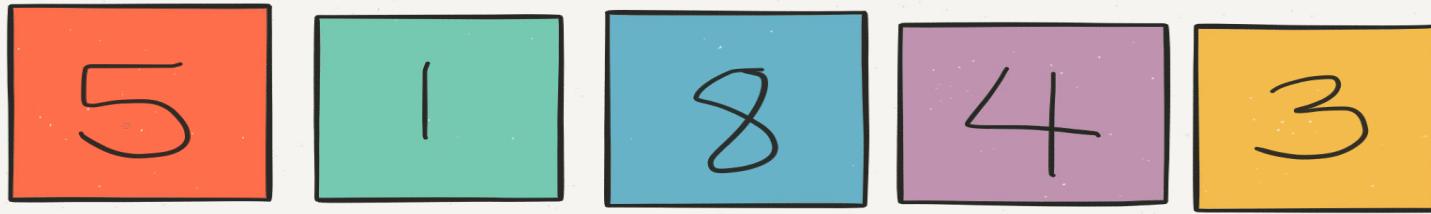
Time Complexity

- Worse case
 - $O(n^2)$
- Average Case
 - $O(n^2)$
- Best Case
 - $\Omega(n^2)$

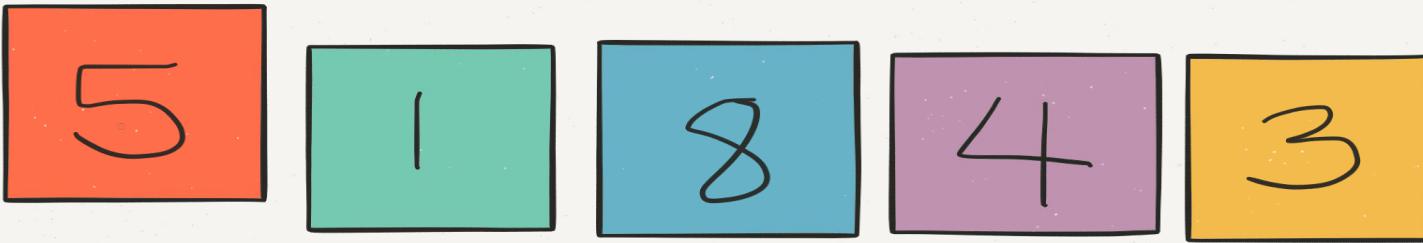


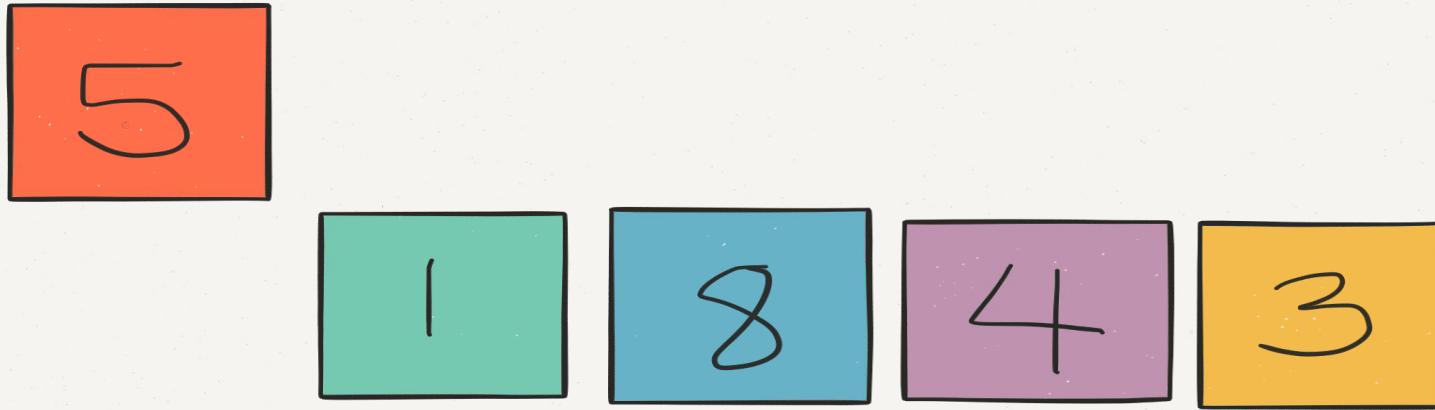
Advantages/Disadvantage

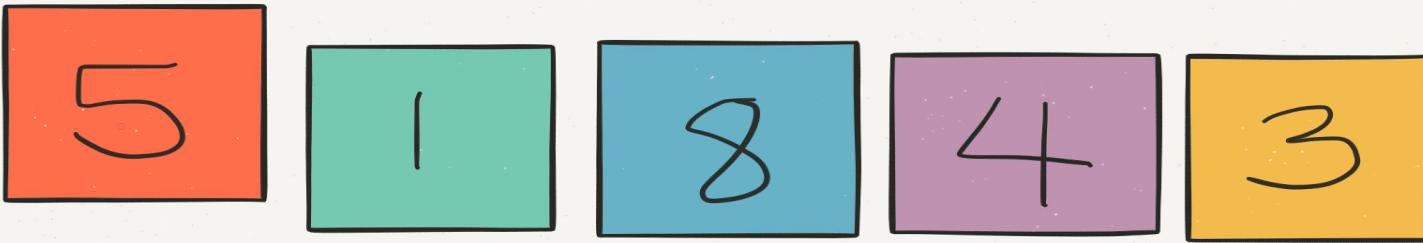
- Simple to understand
- Constant auxiliary space
- Quadratic time at best
- Not stable



INSERTION







5

1

8

4

3



5

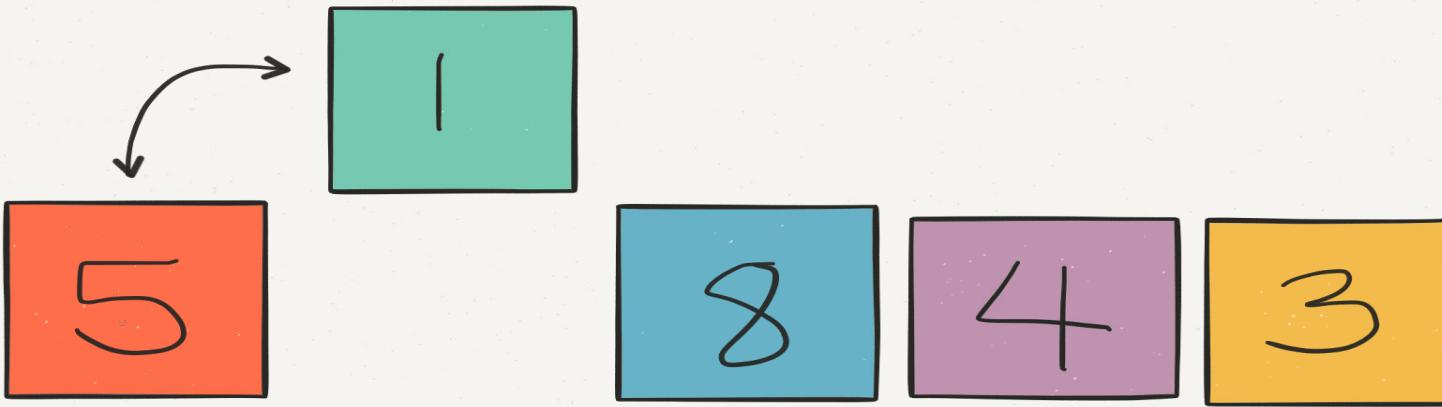
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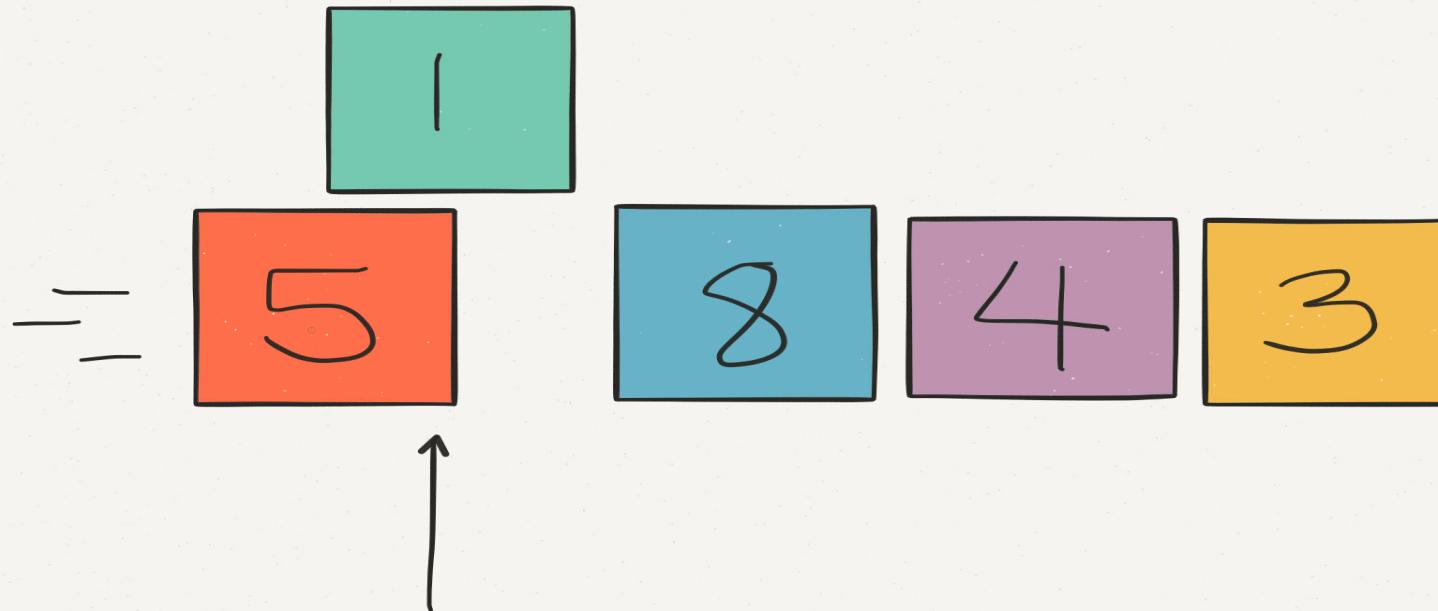
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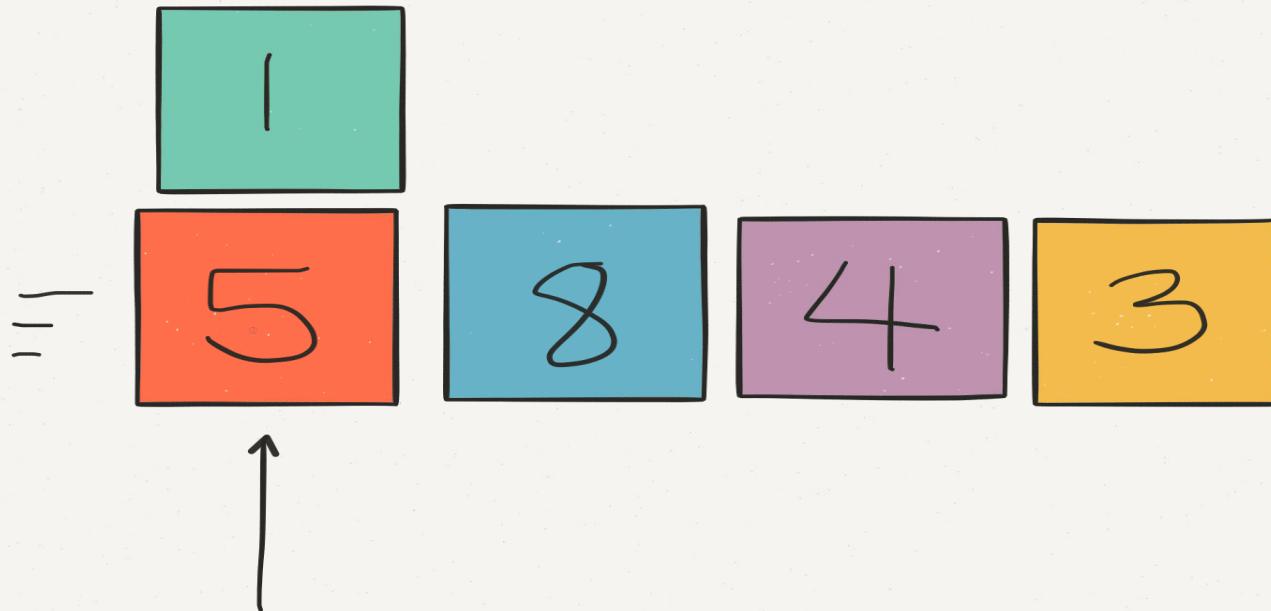
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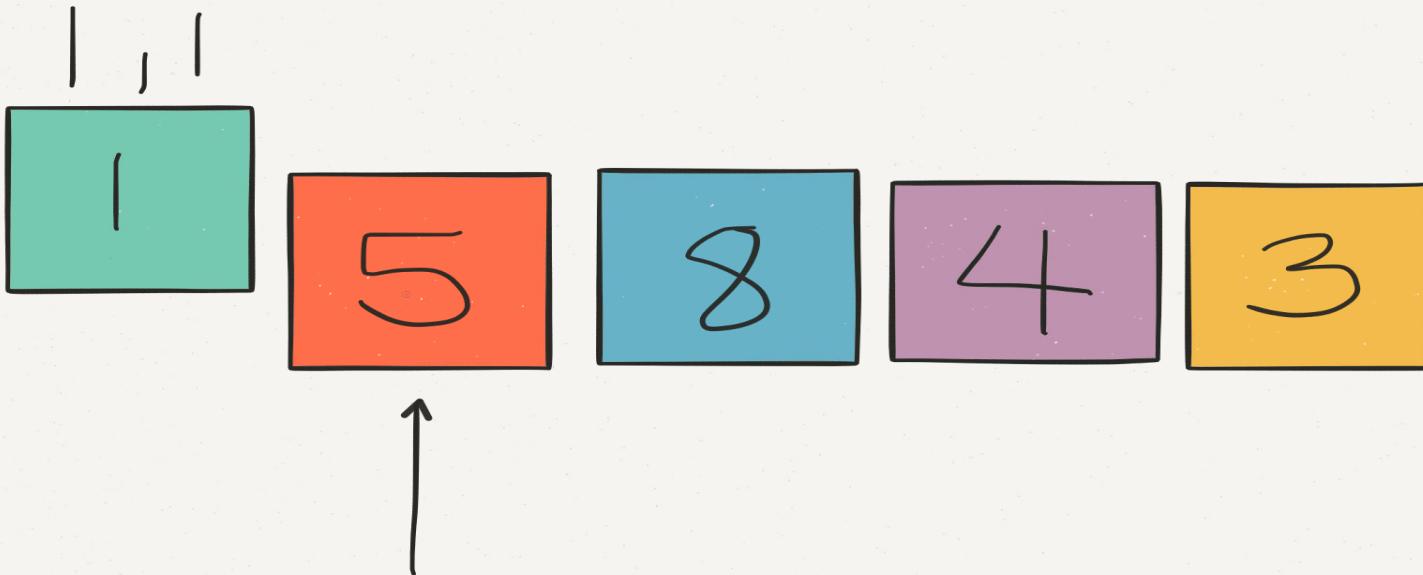
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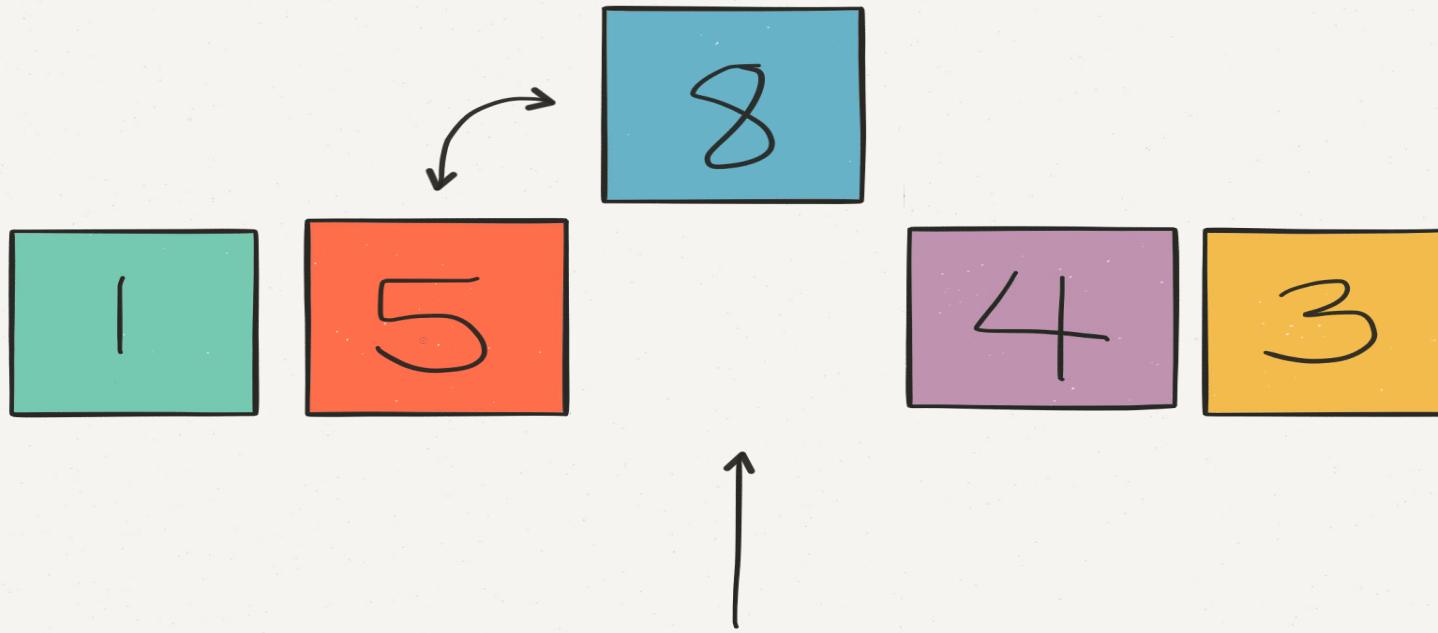
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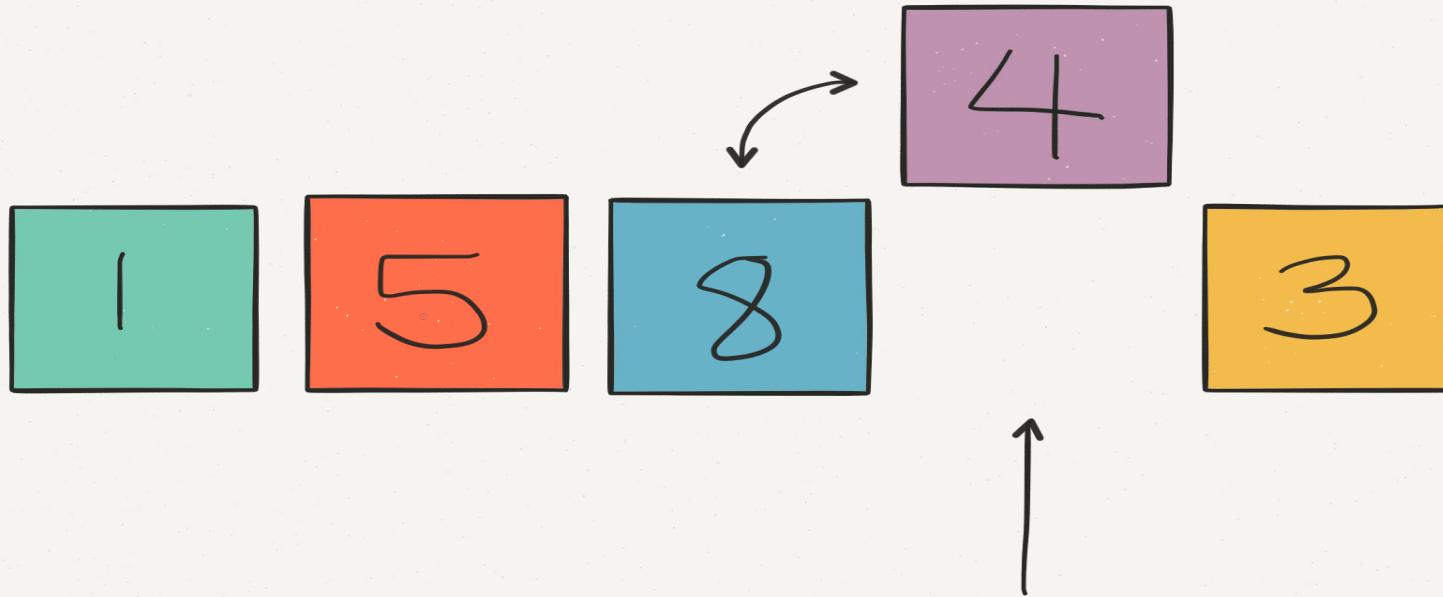
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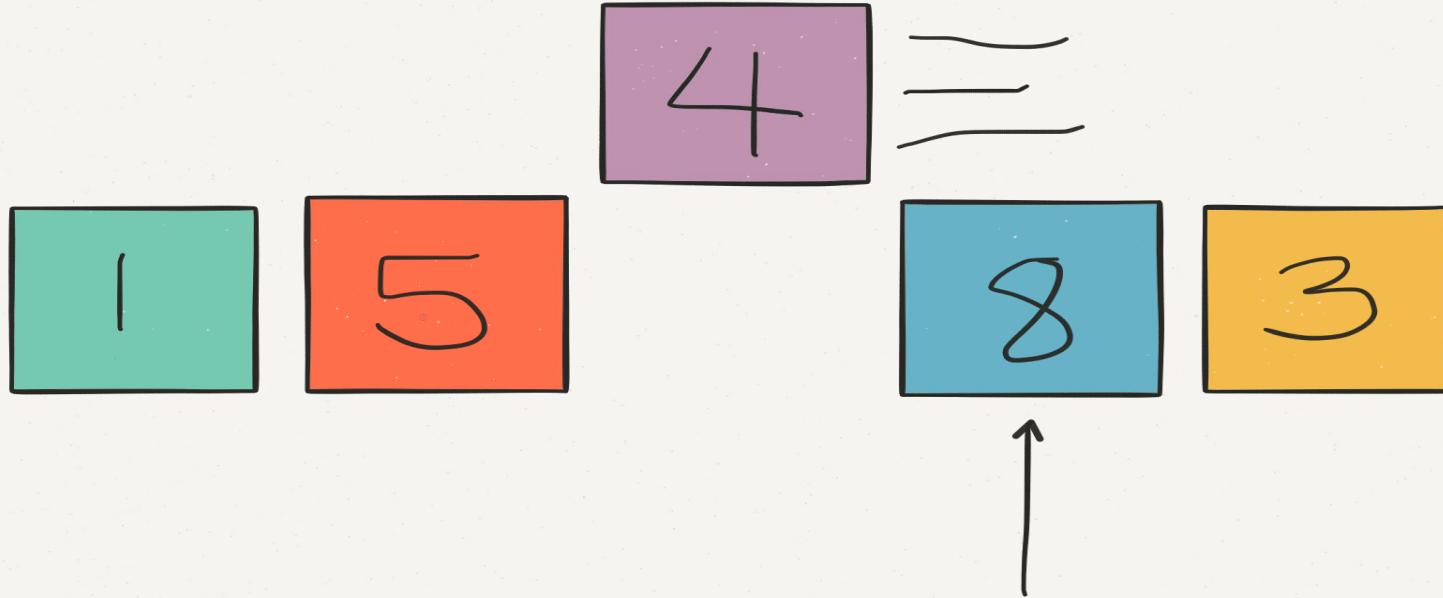
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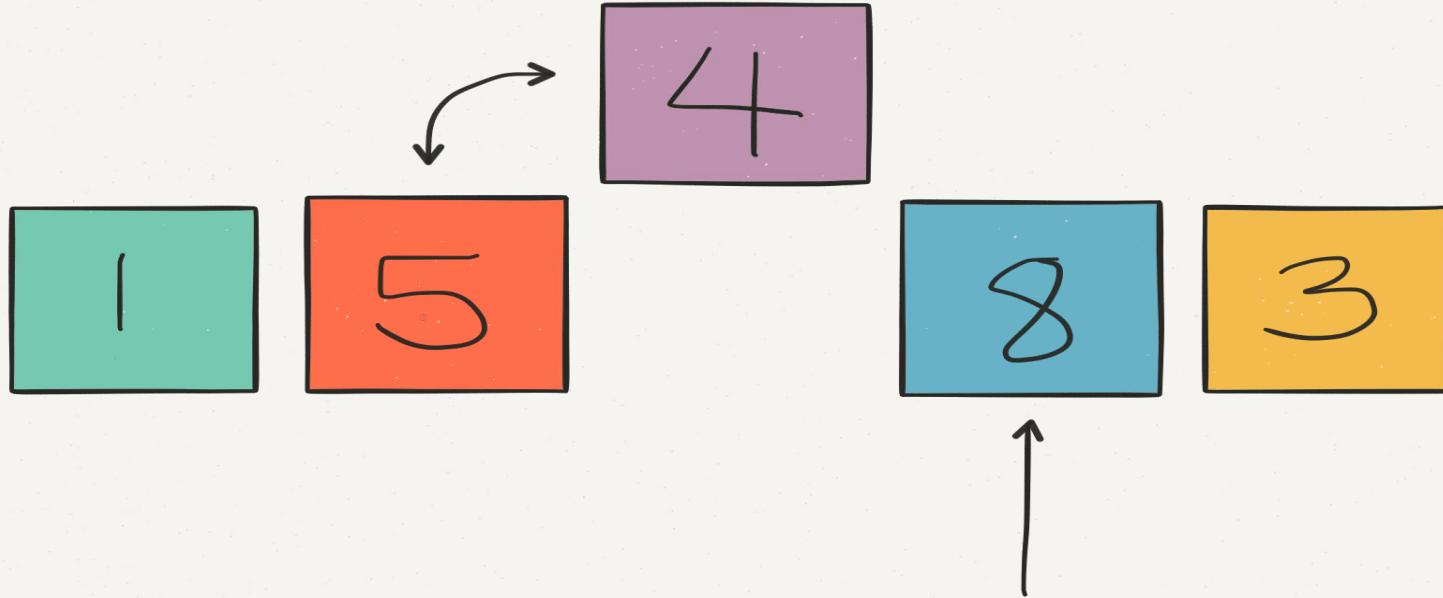
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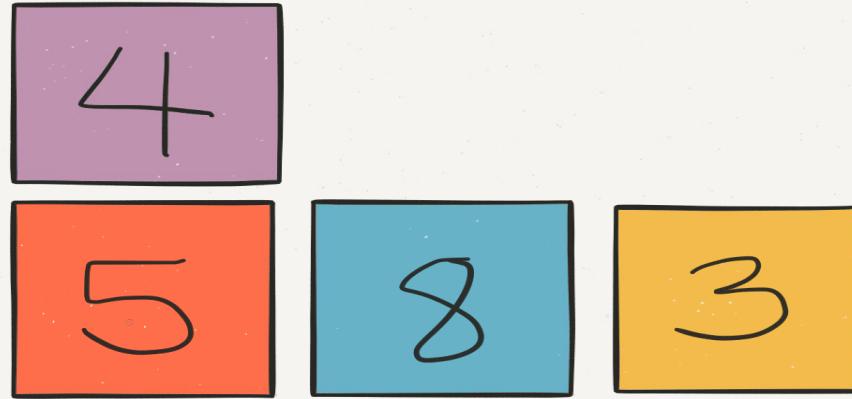


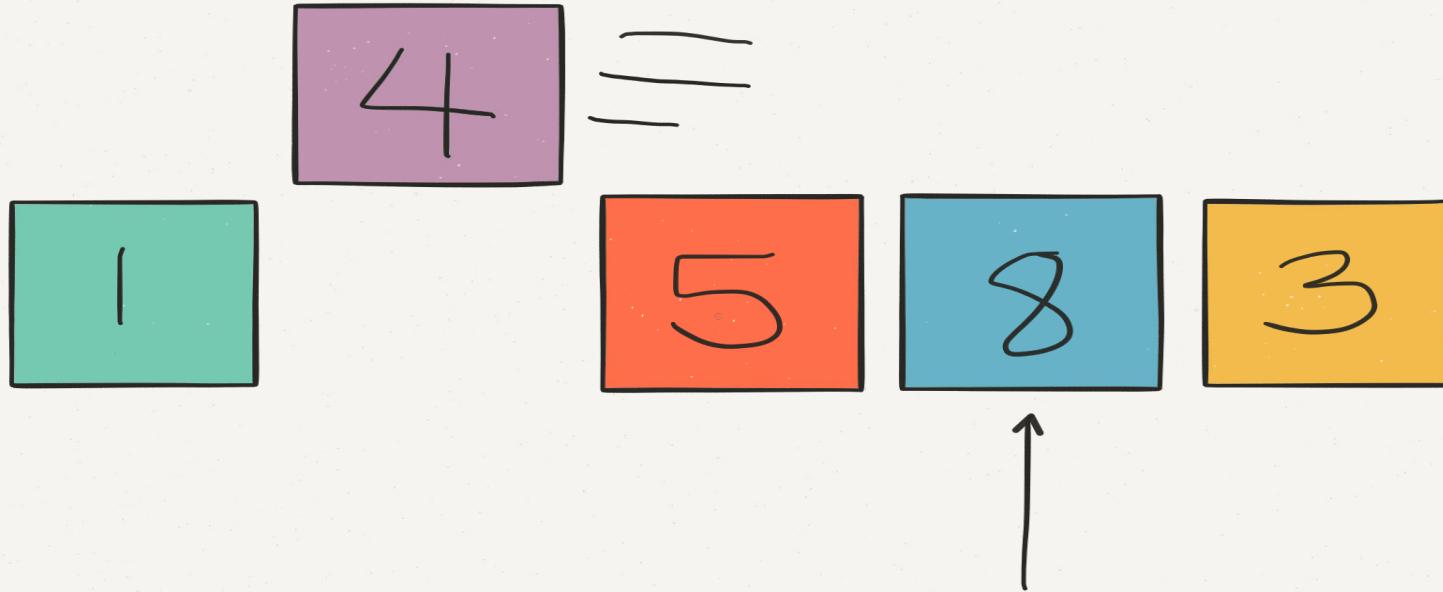


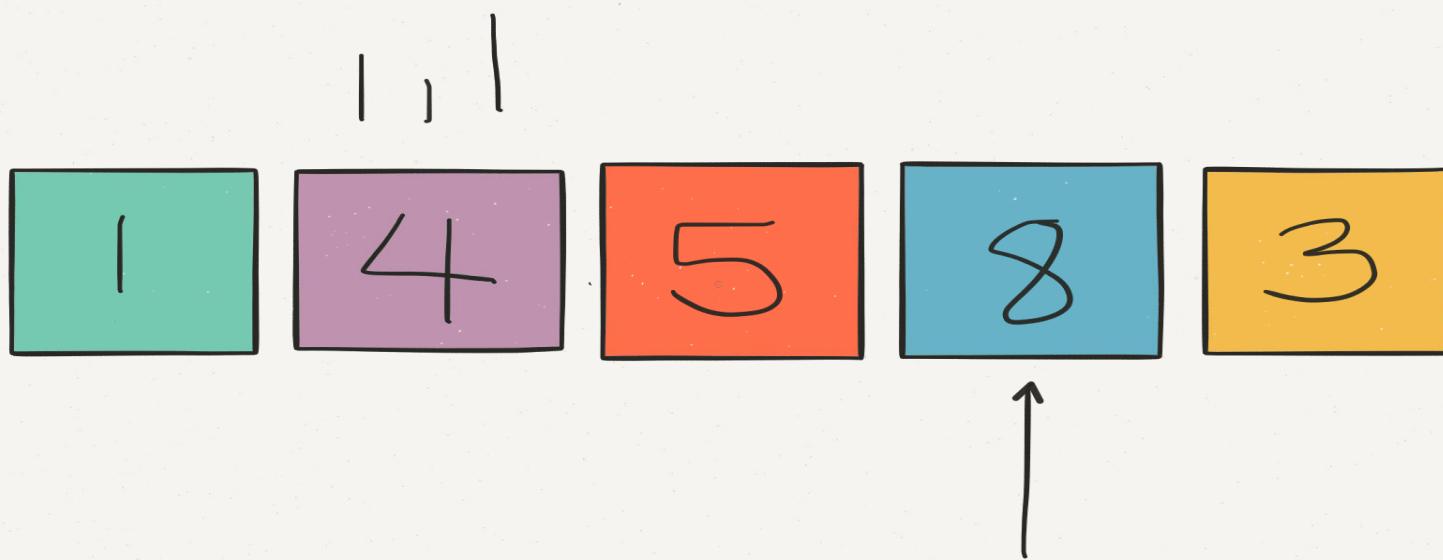


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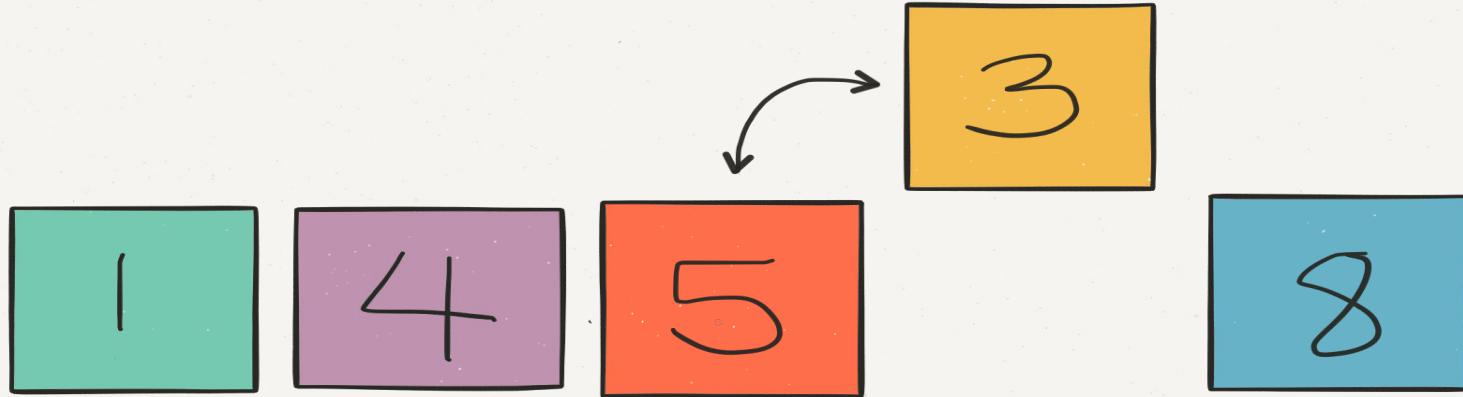
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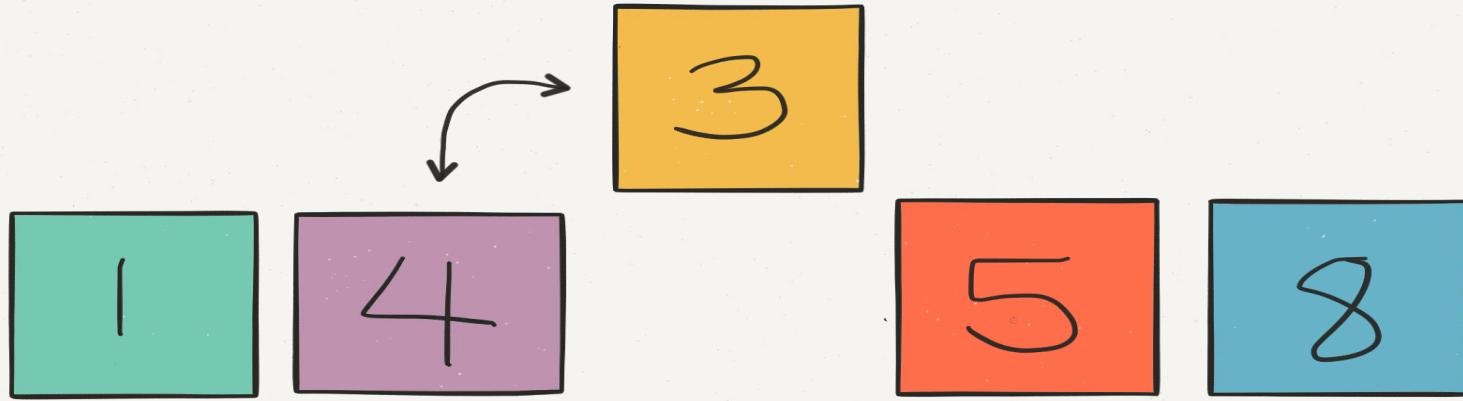
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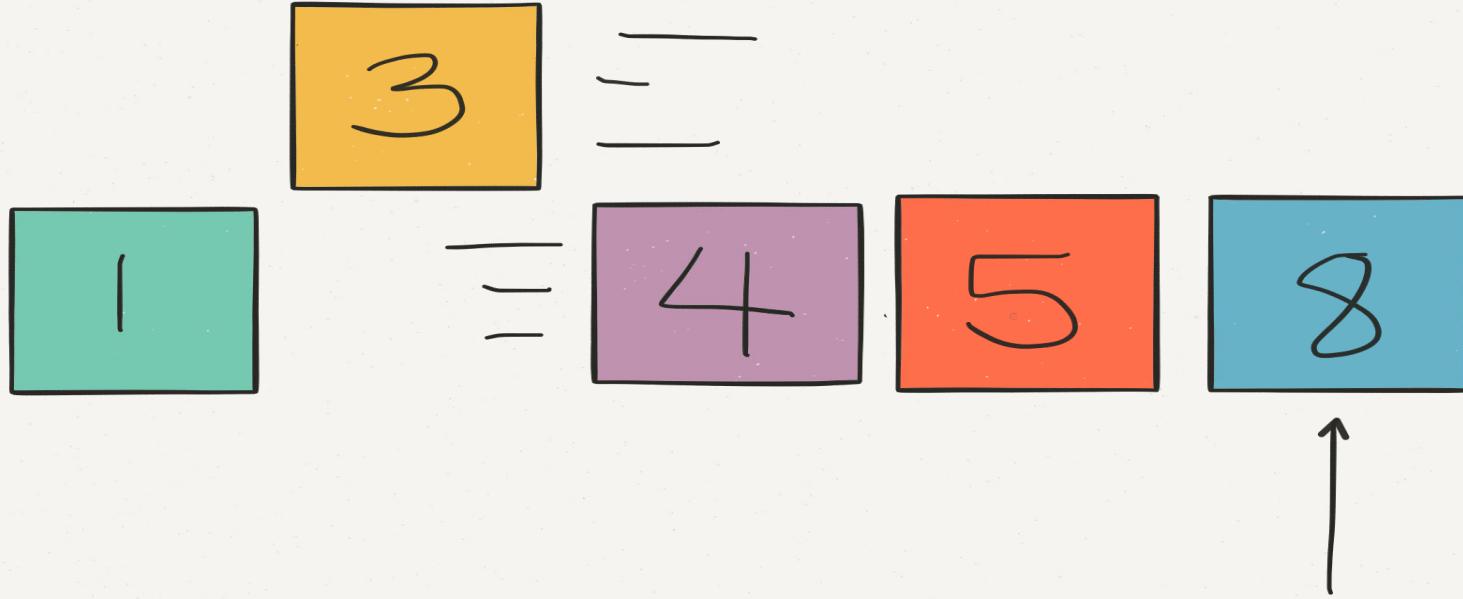
$$\begin{array}{r} 3 \\ = \\ 5 \end{array}$$

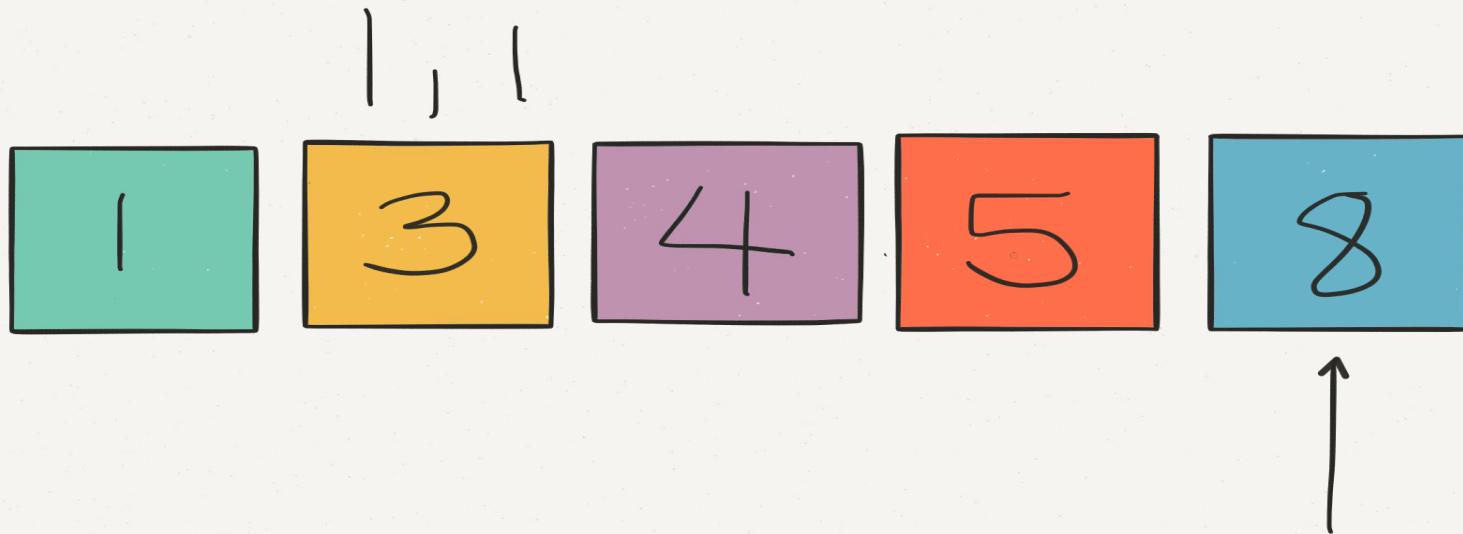
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Time Complexity

- Worse case
 - $O(n^2)$
- Average Case
 - $O(n^2)$
- Best Case
 - $\Omega(n)$



Advantages

- Simple to understand
- Constant auxiliary space
- Average twice as fast as bubble & selection
- Stable sort
- Streamable
- Fast for nearly sorted data $O(kn)$
- Linear time for sorted arrays

Which basic sort should I use?

- Insertion sort
 - For $n < 10$
 - When the dataset is almost sorted
 - Want to maintain simplicity

Resources

- Khan Academy Algorithms Course
 - Javascript
 - <https://www.khanacademy.org/computing/computer-science/algorithms/sorting-algorithms/a/Sorting>
 - <https://www.khanacademy.org/computing/computer-science/algorithms/insertion-sort/a/Insertion-Sort>
- Visualgo.net
 - <http://visualgo.net/Sorting.html>
- USF Computer Science
 - <https://www.cs.usfca.edu/~galles/visualization/ComparisonSort.html>