EXAMPLES AND RESEARCH IN THE GALERKIN FINITE ELEMENT METHOD: FROM A VIEWPOINT OF APPLICATION TO ADSORPTION-CHILLERS

A Literature Review submitted by

Dada, Akindolu Oluwakanyinsola

(Course: MSc in Advanced Mechanical Engineering)

(Tutor: Dr Michael Tierney)

To

Department of Mechanical Engineering

University of Bristol

2010

AUTHOR'S DECLARATION

I declare that the work in this literature review was carried out in accordance with the requirements of the University's Regulations and Code of Practice for Taught Postgraduate Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, this work is my own work. Work done in collaboration with, or with the assistance of others, is indicated as such. I have identified all material in this dissertation which is not my own work through appropriate referencing and acknowledgement. Where I have quoted from the work of others, I have included the source in the references/bibliography. Any views expressed in the literature are those of the author.

SIGNED: ………………………………………………………………. DATE: ……………..

(Signature of student/candidate)

EXECUTIVE SUMMARY

Solar refrigeration is a method of obtaining cooling by using energy from the sun; this is a very attractive method of obtaining cooling for the following reasons: The cooling requirements and the amount of solar energy available peaks at about the same time of the day. The parts with the most cooling needs also have the greatest amount of solar energy available. It can be operated independent of the electricity grid. It reduces dependence on oil, thereby reducing the level of CO_2 emissions, hence it can be said to be environmentally benign.

As promising as this technology looks the COP of most solar refrigerators is around 1, as opposed to the vapour compression refrigerators which could be as high as 3. One of the major factors limiting the COP of solar refrigerators is the heat and mass transfer in the system. If this can be enhanced it can result in the reduction of the cycle time and also increase the refrigerant loading. This would result in a significant improvement in the performance of the system.

The heat transfer in the fins and adsorber of the refrigerator is not easy to model by empirical methods because it has to take into account factors like, thermal contact resistance, porosity of the adsorber, internal heat generation in the adsorber due to change of phase of the refrigerant, and some other factors like anisotropy and temperature dependency of properties such as thermal conductivity. It was observed that the finite element method provided a better approach to modeling and analyzing such problems, as such this would be employed in the research. Consequently two Matlab programs were developed to solve 1-dimensional heat conduction problems and two-dimensional rectangular heat conduction problems using the Galerkin finite element method.

TABLE OF CONTENTS

LIST OF FIGURES

1. INTRODUCTION

Solar refrigeration is a method of obtaining cooling by using energy from the sun; this is a very attractive method of obtaining cooling for the following reasons,

- The cooling requirements and the amount of solar energy available peaks at about the same time of the day.
- The parts with the most cooling needs also have the greatest amount of solar energy available.
- It can be operated independent of the electricity grid, this is especially useful in keeping vaccines and blood supplies in rural areas that do not have grid connected electricity.
- It reduces dependence on oil, thereby reducing the level of CO_2 emissions, hence it can be said to be environmentally benign.

Since the 1970s solar refrigeration received great interests. (Kim and Infante Ferreira 2008) There has been sustained interest in its use, especially in the area of medicine, because it can be used for keeping vaccines cold in rural areas without electricity.

As promising as this technology looks it is still in the developmental stages as it has not been possible to match or come close to the COP of the conventional vapour compression refrigeration systems. Most solar refrigeration systems (apart from the ones employing photovoltaic solar panels and vapour compression cycle) have a COP below 1, while the COP of the vapour compression cycles can be as high as3(Klein and Reindl 2005).

But it is needful to say here that the COP of the solar refrigerator is taken in reference to the solar heat input, while that of vapour compression refrigerators is take in reference to the mechanical work input. If the whole process involved in producing the mechanical work and the inefficiencies involved are to be considered, one would be right to conclude that the actual COP of the vapour compression cycle is actually lower than what is generally quoted.

1.1. TYPES OF SOLAR REFRIGERATION SYSTEMS

Solar refrigeration systems can be of the following types;

- Photovoltaic operated refrigeration cycle: this is one in which a PV array is used to produce the energy used in running the refrigeration system; the system would normally use a conventional vapour compression cycle.
- Solar Mechanical Refrigeration: this is similar to the PV operated, apart from the fact that heat from the sun is used to run a heat engine which produces mechanical work to run the refrigeration system, similar to the PV operated type, it also uses a standard vapour compression cycle.
- Absorption Refrigeration: this uses the heat energy from the sun as the major source of energy to drive the refrigeration cycle, it usually consists of an adsorbate-refrigerant pair, and the adsorbate in this case is usually a liquid.
- Adsorption Refrigeration: similar to the Absorption type, but the adsorbate is a solid instead of a liquid.

1.2. THE ADSORPTION REFRIGERATION SYSTEM

"Adsorption refrigerators were first used in the early 1900s as reported by Plank and Kuprianoff" (Critoph 1989). According to (Sumathy, Yeung et al. 2003),(Wang and Oliveira 2006) and (Spahis, Addoun et al. 2007) most adsorption refrigeration systems make use of one of the following working pairs:

- Activated Carbon – Methanol
- Zeolite – Water
- Silica gel – Water
- Calcium Chloride - Ammonia
- Activated Carbon – Ammonia

A lot of research has been done on the methods of improving the performance of Adsorption refrigerators, and this has been in different areas like, selection of the best working pair, improving the heat and mass transfer in the system, use of cascaded systems in a bid to achieve continuous refrigeration.

1.2.1. SELECTION OF BEST ADSORBENT-REFRIGERANT WORKING PAIR

" Critoph had studied the performance limitations of adsorption cycles with different adsorbates, for solar cooling and concluded that, in general, activated carbon-methanol combination was preferable for solar cooling, giving the best COP achievable in a single stage cycle." (Sumathy, Yeung et al. 2003). Selection of appropriate working pair could also take into account other factors apart from the COP of the system, things like cost and availability could also be of paramount importance. If a refrigerator would be used in a rural area a working pair with water as the refrigerant may be preferable as this would be readily available, so a leakage in the system does not render it completely useless, even after the leakage is detected and fixed.

1.2.2. HEAT AND MASS TRANSFER ENHANCEMENT

The heat and mass transfer in the system is an important factor affecting the performance of the system. "Improving the heat and mass transfer performance of the absorber to speed up the adsorption/desorption process is key to improve the efficiency of the adsorption refrigeration. The poor heat and mass transfer in the adsorption refrigeration system is the bottleneck to prevent adsorption system from widespread utilization. It will also be the focus of adsorption refrigeration technology research for a long time." As previously reported (Wang and Oliveira 2006). Heat and mass transfer enhancement has been an area of current research and a lot of research has been done in this area.

The effect of contact resistance of an adsorber block on the system performance was investigated by (Zhu and Wang 2002). They considered the effect of increased contact pressure and the introduction of a material with superior thermal conductivity to the contact surface. They concluded that increasing the contact pressure reduced the contact resistance up to a level after which further increase in contact pressure no longer had an effect on the contact resistance. They also concluded that the application of a high thermal conductivity adhesive had the effect of lowering the contact resistance.

Heat and mass transfer in the adsorbent bed of a solar adsorption cooling system with glass tube insulation was studied by (Sumathy and Dai 2003), and heat and mass transfer in porous spherical pellets of $CaCl_2$ was studied by (Enibe and Iloeje 2000).

Considering the potential of solar refrigeration and the tremendous impact it could have on reducing the worlds carbon footprint and also the life saving impact it could have on rural communities due to it's application in the health sector. It is needful to say that this is an important area of research and would continue to be an area of research interest. Since most of the research previously carried out in this area relates to the adsorbent and their connection to the metal part, there is little about heat transfer in the metal itself, hence this research work would be directed towards optimizing the heat transfer in the metal, and this would be done mainly by the application of the finite element method.

2. THE FINITE ELEMENT METHOD

This is a method employed in solving problems that are modeled by differential equations. The solution method involves discretizing the domain being considered into finite elements (hence the name Finite Element Method), solving the problem over each element and then assembling the solution for the individual elements to give the solution of the problem over the entire domain.

According to (Lewis, Nithiarasu et al. 2004) and (Stasa 1985)the finite element analysis can be broken down into the following basic steps.

- Discretization of the continuum.
- Selection of interpolation or shape functions.
- Formulation of element equations.
- Assembly of the element equations to obtain a system of simultaneous equations.
- Solution of the system of equations.
- Calculation of secondary quantities.

2.1. DISCRETIZATION

Discretization: this involves breaking the body into discrete elements, numbering the elements and nodes, and specifying the nodes attached to each element and the nodal coordinates. According to (Stasa 1985) Discretization results in the specification of the finite element mesh and involves two distinct but related tasks: nodal definitions and element definitions.

Discretization is closely linked to or determines the type of interpolation function to be used.

Depending on the dimension of the body (i.e. 1, 2 or 3-dimensional) different types of elements can be used in discretizing it. According to (Zienkiewicz and Taylor 2000) some element types which can be used in discretizing bodies are given below:

2.1.1. DISCRETIZATION OF 1-DIMENSIONAL BODIES

This typically involves the use of one dimensional lines to discretize the body, i.e. the body is broken into shorter lines with nodes at the start and end points of each element.

The interpolation polynomial used for the elements could be of order 1 (linear), 2 (quadratic) , 3 (cubic) and so on.

2.1.2. DISCRETIZATION OF 2-DIMENSIONAL BODIES

This could be achieved by making use one of the following

- Triangular element (a three sided element)
- Rectangular element (a four sided element)

2.1.3. DISCRETIZATION OF 3-DIMENSIONAL BODIES

This makes use of one of the following element

- Rectangular prisms
- Tetrahedral elements
- Triangular prisms

For all the elements mentioned above the order of the interpolation polynomial used depends on the number of nodes the element has on each edge (or side). At times an extra node is added to the centre of the element to maintain the completeness of the interpolation polynomial.

2.1.4. RESTRICTIONS ON INTERPOLATION FUNCTIONS

- COMPATIBILITY: For C^o-continuous problems the parameter function itself (not the derivative) must be continuous along the boundaries of the element. For C^1 continuous problems the parameter function and its first derivative must be continuous and not necessarily zero along the boundaries of the element. This is generalized as, in C^n-continuous problems the parameter function and its first n derivatives are continuous and not necessarily zero along the boundaries of the element.(Stasa 1985)
- COMPLETENESS: For C^n-continuous problems, the parameter function must be capable of yielding a constant value of field variable as well as a constant partial derivatives of up to order n+1 as the element size decreases to a point.(Stasa 1985)

A C^{n-1}-continuous problem is one whose weak formulation contains at most only *nth* order derivatives.(Stasa 1985)

Discretization can be uniform or variable depending on the problem. Non-uniform meshes make it possible to obtain a more accurate result while using fewer elements than required in a uniform mesh.

According to (Segerlind 1984)there are some rules that need to be followed while discretizing

a) Place nodes closer where the unknown parameter changes rapidly and further apart where the unknown is relatively constant
b) Place a node wherever there is a stepped change in the value of the coefficient.
c) Place a node wherever the numerical value of the field variable is needed.

2.2. SELECTION OF SHAPE AND INTERPOLATION FUNCTIONS

Selection of interpolation or shape function, the interpolation defines how the field property varies over the element, this could be a linear, quadratic or cubic variation. The shape functions of the elements are usually determined from the interpolation function.

The choice of interpolation or shape function used could have great effect on the accuracy of the result for the same number of elements; higher order elements (i.e. quadratic or cubic elements) require fewer elements to accurately model an irregularly shaped body. This is because of their ability to have curved edges as opposed to the straight edges in linear elements.

Shape function for a linear triangular element is developed below, for the element shown in Fig 2.1 below.

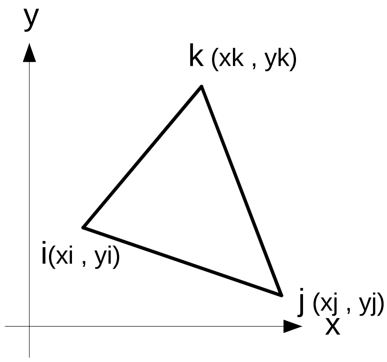

Figure 2.1 A triangular element represented in the global coordinate system

For the linear triangular element an interpolation polynomial

$$T(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y \quad (2.1)$$

The constants $\alpha_1, \alpha_2, \alpha_3$ can be determined by substituting the coordinates of the vertices of the triangle, when this is done it gives a system of three equations.

$$T_i = \alpha_1 + \alpha_2 x_i + \alpha_3 y_i \quad (2.2a)$$

$$T_j = \alpha_1 + \alpha_2 x_j + \alpha_3 y_j \quad (2.2b)$$

$$T_k = \alpha_1 + \alpha_2 x_k + \alpha_3 y_k \quad (2.2c)$$

These equations can then be solved simultaneously to determine the values of $\alpha_1, \alpha_2, \alpha_3$

$$\alpha_1 = \frac{1}{2A}\left[(x_j y_k - x_k y_j)T_i + (x_k y_i - x_i y_k)T_j + (x_i y_j - x_j y_i)T_k\right] \quad (2.3a)$$

$$\alpha_2 = \frac{1}{2A}\left[(y_j - y_k)T_i + (y_k - y_i)T_j + (y_i - y_j)T_k\right] \quad (2.3b)$$

$$\alpha_2 = \frac{1}{2A}\left[(x_k - x_j)T_i + (x_i - x_k)T_j + (x_j - x_i)T_k\right] \quad (2.3c)$$

The shape function is then determined by substituting the values of $\alpha_1, \alpha_2, \alpha_3$ back into equation (2.1) to determine the shape function and the temperature function over the element. The temperature function can be used to determine the temperature at any given point (x,y) in the element if the coordinates and temperature of the vertices are known.

$$T = N_i T_i + N_j T_j + N_k T_k \quad (2.4)$$

$$N_i = \frac{1}{2A}(a_i + b_i x + c_i y) \qquad a_i = x_j y_k - x_k y_j \qquad b_i = y_j - y_k \qquad c_i = x_k - x_j \quad (2.4a)$$

$$N_j = \frac{1}{2A}(a_j + b_j x + c_j y) \qquad a_j = x_k y_i - x_i y_k \qquad b_j = y_k - y_i \qquad c_j = x_i - x_k \quad (2.4b)$$

$$N_k = \frac{1}{2A}(a_k + b_k x + c_k y) \qquad a_k = x_i y_j - x_j y_i \qquad b_k = y_i - y_j \qquad c_k = x_j - x_i \quad (2.4c)$$

Where N_i, N_j , $N_k$are the shape functions at the given point (x,y). The gradient of temperature can also be obtained within the element, by obtaining the partial derivative of T with respect to x and y.

$$g = \begin{bmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & \frac{\partial N_j}{\partial x} & \frac{\partial N_k}{\partial x} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_j}{\partial y} & \frac{\partial N_k}{\partial z} \\ \frac{\partial N_i}{\partial z} & \frac{\partial N_j}{\partial z} & \frac{\partial N_k}{\partial z} \end{bmatrix} \begin{bmatrix} T_i \\ T_j \\ T_k \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} b_i & b_j & b_k \\ c_i & c_j & c_k \end{bmatrix} \begin{bmatrix} T_i \\ T_j \\ T_k \end{bmatrix} = [B]\{T\} \qquad (2.5)$$

2.2.1. ISOPARAMETRIC ELEMENTS AND MAPPING

This is a method for creating curved elements by mapping them onto regular elements, the shape function of the regular parent element is determined in the local s-t coordinate system and them transformed using a coordinate transformation function which is formed using the local shape funcitons of the parent element.

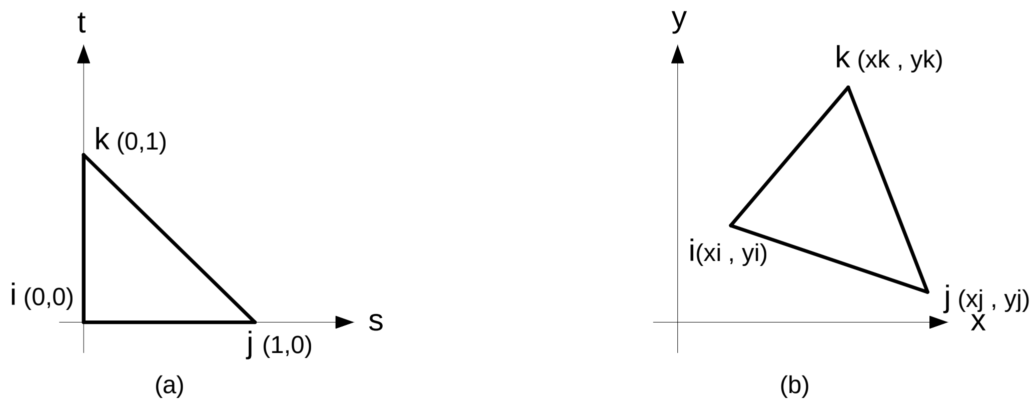

Figure 2.2 (a) Triangular element in local coordinate (b) Triangular element in global coordinate

From Figure 2.2(b). The triangular element in the global coordinate (which is shown as a straight sided triangular element, could also have curved sides) is mapped to the local coordinate using the coordinate transformation given below.

$$x = N_i x_i + N_j x_j + N_k x_k \qquad and \qquad y = N_i y_i + N_j y_j + N_k y_k \qquad (2.6)$$

Where N_i , N_j , N_k are the shape functions of the element in the local coordinate system. For the element shown above the shape functions are given in the local coordinate system by:

$$N_i = 1 - s - t\,, \qquad N_j = s\,, \qquad N_k = t \qquad (2.7)$$

2.3. FORMULATION OF ELEMENT EQUATIONS

Formulation of element equations, basically involves the determination of the element stiffness matrix and the forcing function for each individual element.

$$[K^{(e)}]\,[T] = [f^{(e)}] \qquad (2.8)$$

Where $[K^{(e)}]$ is the element stiffness matrix.
$[T]$ Is the vector of element nodal temperatures.
$[f^{(e)}]$ Is the element forcing vector

The element equations can be determined by several methods, some of these are the Variational Method, the Weighted Residuals Method: this could be the Collocation, Sub-domain, Galerkin or Least Squares depending on the choice of the weighting function used. The element stiffness matrix and forcing vector are functions of the material properties like, thermal conductivity, volumetric heat generation (as this could be a function of the materials resistivity in ohmic-resistance heat generation). Hence when formulating the element equations several things have to be taken into consideration like: the dependence of these element properties on temperature, and direction (i.e. is the material anisotropic or inhomogeneous). It is sometimes desirable to have materials whose properties vary over the body, or in different directions in the body. This is usually the case for composite materials. Most composites exhibit some degree of anisotropy, and this must be considered in the finite element model, in order to obtain accurate results. If this condition is not modeled properly, the solution of the model would still converge, but the model would not be an actual representation of the physical system, so also the result obtained would not be.

2.4. ASSEMBLY OF ELEMENT EQUATIONS

Assembly of the equations involves the formation of the global stiffness matrix, global vector of nodal temperatures and the global forcing vector, this simply involves inputting the contribution of each element at each node into the appropriate position in the global stiffness matrix and forcing vector.

$$[K][T] = [f] \tag{2.9}$$

Where [K] Is the global stiffness matrix.

$[T]$ Is the vector of global nodal temperatures.

$[f]$ Is the global forcing vector

2.5. SOLUTION OF SYSTEM OF EQUATIONS

Solution of the system of equations, this carried out to determine the nodal temperatures, for all the nodes in the body. It should be ensured that the finite element solution is accurate and approximates the physical system being modeled as close as possible. One of the methods of doing this is, to ensure that the solution converges. According to (Tong and Rossettos 1977) convergence of the finite element solution to the exact solution can be achieved by two means, the first involves increasing the degrees of freedom per element with the element size fixed, or by dividing the domain into smaller and smaller elements while keeping the degrees of freedom per element fixed. According to them the former procedure is not normally used because increasing the number of degrees of freedom of an element means reformulating the local approximate solution, which can be tedious and uneconomical. In the latter procedure we only have to subdivide further the domain being considered, which is relatively straight forward.

It has been shown by different researchers that the finite element method can be used to accurately model and predict the thermal behavior of systems, and that the results obtained converges to the analytical solution, and in most cases correlates to the experimental results. (Fotiadis, Boekholt et al. 1990) considered the effects of manipulation of standard CVD (Carbon Vapour Deposition) reactor operating conditions (flow rate, pressure, and carrier gas) on flow and heat transfer characteristics, these effects were considered along with changes in susceptor tilt angle and orientation of the reactor relative to the direction of gravity. Gas phase temperatures were measured by spontaneous Raman scattering and smoke trace experiments were used to visualize flow structures. They reported that excellent agreement was obtained between measured temperatures and those predicted by a finite element analysis of a two-dimensional model.

Despite this it is needful to say that both the analytical and numerical methods are in the best case close approximations of what obtains in the real world as it is impossible to produce a perfect theoretical model of a real world system, but for most practical cases this approximation would suffice.

2.6. CALCULATION OF SECONDARY QUANTITIES

Secondary quantities like, heat flux or a temperature at a point other than a node can be calculated using the interpolation function and the nodal temperatures of the element in which that point is located.

2.7. THE GALERKIN METHOD

This is a method employed in the solution of partial differential equations and it is credited to the Russian Mathematician/Engineer Boris Galerkin (Mottram and Shaw 1996).

According to (Lewis, Nithiarasu et al. 2004) the Galerkin method is applied to heat conduction as shown below. A test function $\bar{T}$ is substituted for T in the function L(T).

$$L(T) = 0 \quad in\ \Omega \tag{2.10}$$

$$T \approx \bar{T} = \sum_{i=1}^{n} a_i N_i(x) \tag{2.11}$$

If this test function is substituted into the governing equation the result is no longer equal to zero, but now equal to a residual R

$$L\,(\bar{T}) = R \neq 0$$

The weighted residual method, requires that;

$$\int_\Omega w_i\, R\, d\Omega = \int_\Omega w_i\, L\,(\bar{T})\ d\Omega = 0 \tag{2.12}$$

This can be applied to the heat conduction problem whose general equation and boundary conditions are shown below:

$$L(\bar{T}) = \frac{\partial}{\partial x}\left(k_x \frac{\partial \bar{T}}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial \bar{T}}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial \bar{T}}{\partial z}\right) + G = 0 \tag{2.13}$$

The boundary conditions are given by:

$$\begin{aligned} &T - T_a = 0 && \text{On surface } S_1 \\ &k_x \frac{\partial \bar{T}}{\partial x} + k_y \frac{\partial \bar{T}}{\partial y} + k_z \frac{\partial \bar{T}}{\partial z} + q = 0 && \text{On surface } S_2 \\ &k_x \frac{\partial \bar{T}}{\partial x} + k_y \frac{\partial \bar{T}}{\partial y} + k_z \frac{\partial \bar{T}}{\partial z} + h(T - T_a) = 0 && \text{On surface } S_3 \end{aligned} \tag{2.14}$$

The general equation can be substituted into the weighted residual formulation of equation (2.12) to give equation (2.15). The boundary conditions above could be referred to as boundary conditions of the first, second and third kind respectively (Ozisik 1968)

$$\int_\Omega w_k \left[\frac{\partial}{\partial x}\left(k_x \frac{\partial \bar{T}}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial \bar{T}}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial \bar{T}}{\partial z}\right)\right] d\Omega + \int_\Omega w_k G\, d\Omega = 0 \tag{2.15}$$

For the Galerkin method of weighted residuals the weight function is the shape function; that is:

$$w_k = N_k$$

Substituting the weight function into equation (2.15) and then integrating by parts by applying the Green's Lemma theory would give the equation below.

$$-\int_{\Omega} \left(k_x \frac{\partial N_k}{\partial x}\frac{\partial N_m}{\partial x} + k_y \frac{\partial N_k}{\partial y}\frac{\partial N_m}{\partial y} + k_z \frac{\partial N_k}{\partial z}\frac{\partial N_m}{\partial z}\right)(\bar{T}_m)\ d\Omega$$

$$+ \int_{\Omega} G\, N_k\, d\Omega + \int_{s} \left(N_k\, k_x \frac{\partial \bar{T}}{\partial x}\tilde{l} + N_k\, k_y \frac{\partial \bar{T}}{\partial y}\tilde{m} + N_k\, k_z \frac{\partial \bar{T}}{\partial z}\tilde{n}\right) ds = 0 \qquad (2.16)$$

The last integral term in the equation above is a surface integral and it is decomposed into the different boundary conditions except the temperature boundary condition which is accounted for after the global stiffness matrix and forcing vectors of the system is formed.

$$-\int_{\Omega} \left(k_x \frac{\partial N_k}{\partial x}\frac{\partial N_m}{\partial x} + k_y \frac{\partial N_k}{\partial y}\frac{\partial N_m}{\partial y} + k_z \frac{\partial N_k}{\partial z}\frac{\partial N_m}{\partial z}\right)(\bar{T}_m)\ d\Omega$$

$$+ \int_{\Omega} G\, N_k\, d\Omega - \int_{s2} N_k\, q\, ds - \int_{s3} h\, N_k N_m\, (\bar{T}_m)\, ds + \int_{s3} h\, T_a\, N_k\ ds = 0 \qquad (2.17)$$

The subscript "m" in the equation stands for the nodes of the element. The equation above can be rearranged to give a form which can then be reduced to the matrix form easily.

$$\int_{\Omega} \left(k_x \frac{\partial N_k}{\partial x}\frac{\partial N_m}{\partial x} + k_y \frac{\partial N_k}{\partial y}\frac{\partial N_m}{\partial y} + k_z \frac{\partial N_k}{\partial z}\frac{\partial N_m}{\partial z}\right)(\bar{T}_m)\ d\Omega$$

$$+ \int_{s3} h\, N_k N_m\, (\bar{T}_m)\, ds = \int_{\Omega} G\, N_k\, d\Omega - \int_{s2} N_k\, q\, ds + \int_{s3} h\, T_a\, N_k\ ds \qquad (2.18)$$

Hence we can write

$$[K]\{\bar{T}\} = \{f\} \qquad (2.19)$$

Where: $$[K] = \int_{\Omega} [B]^T\, [D][B]\, d\Omega + \int_{S3} h\, [N]^T [N]\, ds \qquad (2.20)$$

$$\{f\} = \int_{\Omega} G\, [N]^T\, d\Omega - \int_{s2} q\, [N]^T\ ds + \int_{s3} h\, T_a [N]^T\, ds \qquad (2.21)$$

$[N] = [N_1 N_2 N_3\ .\ .\ .\ N_n]$ depending on the dimension of the element and the degree of the interpolation function.

$$[D] = \begin{bmatrix} k_{xx} & 0 & 0 \\ 0 & k_{yy} & 0 \\ 0 & 0 & k_{zz} \end{bmatrix} \qquad [B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} \cdots\cdots & \frac{\partial N_n}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} \cdots\cdots & \frac{\partial N_n}{\partial y} \\ \frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} \cdots\cdots & \frac{\partial N_n}{\partial z} \end{bmatrix}$$

The gradient vector which gives the temperature gradient of in the domain (or body being considered) is defined as:

$$\{g\} = \begin{Bmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{Bmatrix} = [B]\{T\} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} \cdots\cdots & \frac{\partial N_n}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} \cdots\cdots & \frac{\partial N_n}{\partial y} \\ \frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} \cdots\cdots & \frac{\partial N_n}{\partial z} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{Bmatrix} \qquad (2.22)$$

Once the values of K and F are determined for the elements these can be assembled to form the global stiffness matrix and forcing vector.

3. APPLICATION OF THE FINITE ELEMENT METHOD TO SIMPLE HEAT TRANSFER PROBLEMS

The finite element method can be used to analyze and solve heat transfer problems ranging from the relatively simple ones to the more complex ones for which no analytical solution may be possible. This method is applied to solve some simple one-dimensional and two-dimensional heat conduction problems.

Two Matlab programs were developed to determine the nodal temperatures of simple one-dimensional and two-dimensional conduction problems. A meshing function was developed to generate a structured triangular mesh for the two-dimensional conduction program.

3.1. ONE-DIMENSIONAL HEAT TRANSFER PROBLEMS

1. The temperature distribution in a plane wall is required. The wall is of thickness 60mm and has an internal heat source of $0.3MW/m^3$ and thermal conductivity of $21W/m^oC$. The left hand of the face is insulated and the right hand face is subjected to a convection environment of 93^oC with a surface heat transfer coefficient of $570W/m^{2\,o}C$.(Lewis, Nithiarasu et al. 2004)

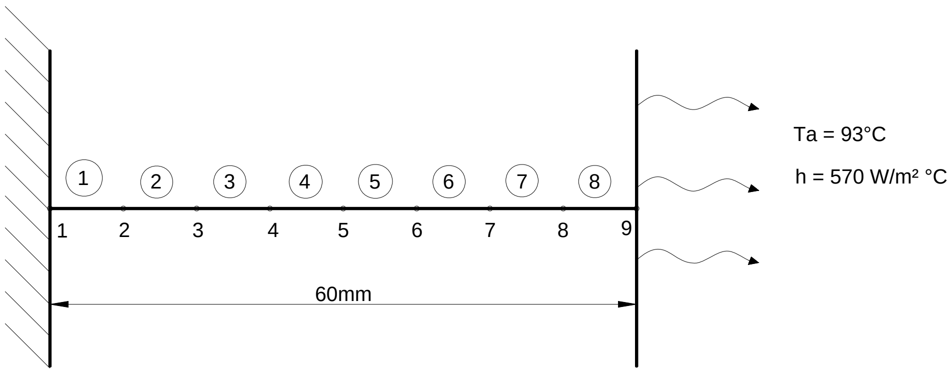

Figure 3.1 Heat transfer in a plane wall

Assumption: the wall is assumed to be infinitely large, hence a one dimensional heat transfer is assumed.

The body is divided into 8 linear elements as shown above.

The shape function for a 1-D linear element is given by $N_i = 1 - \frac{x}{l}$ and $N_j = \frac{x}{l}$

Elements 1, 2, . . . , 7 are similar therefore they would have the same element stiffness matrix and forcing vector, since element 8 is different from the other elements because the end node is exposed to convection, therefore it would have a different element stiffness matrix and forcing vector.

$$[K]^1 = [B]^T[D][B]\,Al$$

Because we are considering a 1-D element the matrix [B] only has derivatives of shape function with respect to x.

$$[B] = \begin{bmatrix} \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} \end{bmatrix} = \begin{bmatrix} \frac{-1}{l} \frac{1}{l} \end{bmatrix} = \frac{1}{l}\,[-1 \;\; 1]$$

$$[D] = [K_x] = K_x$$

$$[K]^1 = \frac{1}{l}\begin{bmatrix} -1 \\ 1 \end{bmatrix} \times K_x \times \frac{1}{l}[-1 \quad 1] \times A \times l = \frac{A\,K_x}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2800 & -2800 \\ -2800 & 2800 \end{bmatrix}$$

$[K]^8 = [B]^T[D][B]\,Al + hA\,[N]^T[N]$ For this element the shape function $N_i = 0 \; and \; N_j = 1$

$$\therefore\; [K]^8 = \frac{A\,K_x}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + hA\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2800 & -2800 \\ -2800 & 3370 \end{bmatrix}$$

$\{F\}^1 = GAl\,[N]^T$ For this element the shape functions $N_i = \frac{1}{2} \; and \; N_j = \frac{1}{2}$ because the heat generation is uniform over the whole element.

$$\therefore\; \{F\}^1 = GAl\begin{bmatrix} \frac{1}{2} \frac{1}{2} \end{bmatrix}^T = \frac{GAl}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1125 \\ 1125 \end{bmatrix}$$

$\{F\}^8 = GAl\,[N]^T + hAT_a[N]^T$

The shape function used with convection, that is, the second term is $N_i = 0 \; and \; N_j = 1$ because only the end node is exposed to convection.

$$\{F\}^8 = GAl\begin{bmatrix} \frac{1}{2} \frac{1}{2} \end{bmatrix}^T + hAT_a[0 \;\; 1]^T = \frac{GAl}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + hAT_a\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1125 \\ 54135 \end{bmatrix}$$

The 8 element stiffness matrices and forcing vectors can be assembled into the global stiffness matrix and forcing vector.

$$\begin{bmatrix} 2800 & -2800 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2800 & 5600 & -2800 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2800 & 5600 & -2800 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2800 & 5600 & -2800 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2800 & 5600 & -2800 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2800 & 5600 & -2800 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2800 & 5600 & -2800 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2800 & 5600 & -2800 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2800 & 3370 \end{bmatrix}\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix} = \begin{bmatrix} 1125 \\ 2250 \\ 2250 \\ 2250 \\ 2250 \\ 2250 \\ 2250 \\ 2250 \\ 54135 \end{bmatrix}$$

When this was solved by matrix inversion using Matlab, the following nodal temperatures were obtained. This problem was solved analytically, using the solution of the one dimensional conduction equation: $\frac{d^2T}{dx^2} + \frac{G}{k} = 0$, this gives the solution $T = \frac{G}{2k}(L^2 - x^2) + \left(\frac{GL}{h} + T_a\right)$ with the appropriate boundary conditions applied (Lewis, Nithiarasu et al. 2004). The result obtained is shown on the right of the result obtained by employing the finite element method.

$$\{T\} = \begin{bmatrix} 150.2932 \\ 149.8914 \\ 148.6861 \\ 146.6772 \\ 143.8647 \\ 140.2486 \\ 135.8289 \\ 130.6057 \\ 124.5789 \end{bmatrix}$$

$$\{T\} = \begin{bmatrix} 150.2932 \\ 149.8914 \\ 148.6861 \\ 146.6772 \\ 143.8647 \\ 140.2486 \\ 135.8289 \\ 130.6057 \\ 124.5789 \end{bmatrix}$$

When this problem was solved using the pogram whose flowchart is shown in Figure A.1 a similar result was obtained, as that obtained by the finite element method, above.

2. The temperature distribution in a plane wall is required. The wall is of thickness 60mm and has an internal heat source of 0.3MW/m^3 and thermal conductivity of 21W/m^{o}C . The left hand face and the right hand face are kept at temperatures of 10^{o}C and 20^{o}C respectively.

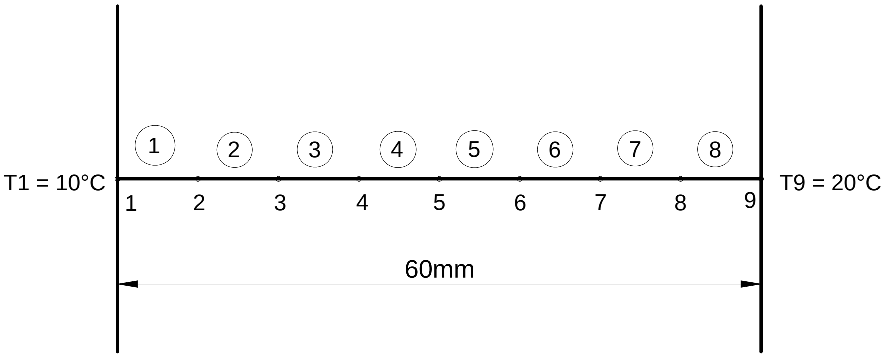

Figure 3.2 Heat transfer in a plane wall with temperature at both sides fixed

The solution method is similar to that above except for the fact that all the elements have the same stiffness matrices and forcing vectors. The temperature condition is applied after the element stiffness matrices and forcing vectors are assembled into the global stiffness matrix and forcing vector respectively.

The element stiffness matrix and forcing vector for all the elements is given below:

$$[K]^e = \begin{bmatrix} 2800 & -2800 \\ -2800 & 2800 \end{bmatrix}$$

$$\{F\}^e = \begin{bmatrix} 1125 \\ 1125 \end{bmatrix}$$

Assembling this gives the global stiffness matrix and element forcing vector below.

$$\begin{bmatrix} 2800 & -2800 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2800 & 5600 & -2800 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2800 & 5600 & -2800 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2800 & 5600 & -2800 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2800 & 5600 & -2800 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2800 & 5600 & -2800 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2800 & 5600 & -2800 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2800 & 5600 & -2800 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2800 & 2800 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix} = \begin{bmatrix} 1125 \\ 2250 \\ 2250 \\ 2250 \\ 2250 \\ 2250 \\ 2250 \\ 2250 \\ 1125 \end{bmatrix}$$

When the temperature condition was applied to the system, the stiffness matrix, and forcing vector became,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5600 & -2800 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2800 & 5600 & -2800 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2800 & 5600 & -2800 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2800 & 5600 & -2800 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2800 & 5600 & -2800 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2800 & 5600 & -2800 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2800 & 5600 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix} = \begin{bmatrix} 1125 + (2800 \times 10) \\ 2250 \\ 2250 \\ 2250 \\ 2250 \\ 2250 \\ 2250 \\ 2250 \\ 1125 + (2800 \times 20) \end{bmatrix}$$

When this was solved by matrix inversion the following nodal temperatures were obtained. This problem was solved analytically, using the solution of the one dimensional conduction equation: $\frac{d^2T}{dx^2} + \frac{G}{k} = 0$, this gives the solution $T = \frac{-G}{2k}x^2 + \left(\frac{10}{L} + \frac{GL}{2k}\right)x + 10$ with the appropriate boundary conditions applied. The result obtained is shown on the right of the result obtained by employing the finite element method.

$$\{T\} = \begin{bmatrix} 10.0000 \\ 14.0625 \\ 17.3214 \\ 19.7768 \\ 21.4286 \\ 22.2768 \\ 22.3214 \\ 21.5625 \\ 20.0000 \end{bmatrix} \qquad \{T\} = \begin{bmatrix} 10.0000 \\ 14.0625 \\ 17.3214 \\ 19.7768 \\ 21.4286 \\ 22.2768 \\ 22.3214 \\ 21.5625 \\ 20.0000 \end{bmatrix}$$

When this problem is solved using the pogram whose flowchart is shown in Figure A.1 a similar result was obtained, as that obtained by the finite element method, above.

From the two cases above it is observed that the result of the FEA method is in close approximation to the theoretical one.

3.2. TWO-DIMENSIONAL PROBLEMS

1. A square plate of unit thickness and size of 1m as shown in the Figure (3.3) is subjected to isothermal boundary conditions of 100°C on all sides except the top side, which is subjected to 500°C. If the thermal conductivity of the material is constant and equal to 10W/m°C, determine the temperature distribution using linear triangular finite elements. (Lewis, Nithiarasu et al. 2004)

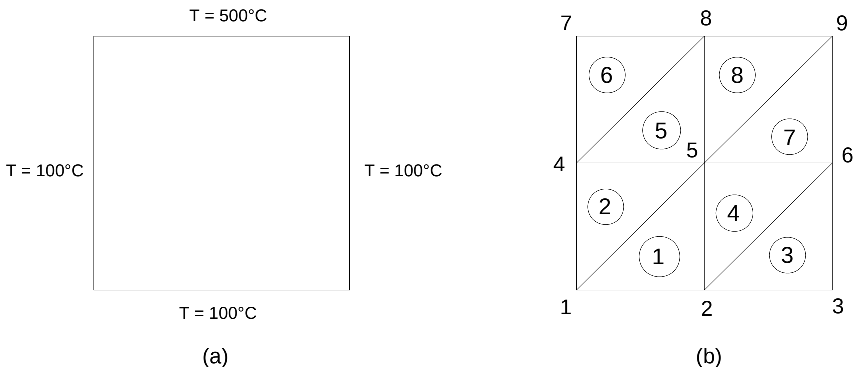

Figure 3.3 2-Dimensional heat transfer in a square body with temperature on all sides fixed

To solve, the square domain is divided into 8 linear triangular elements, the element stiffness matrix and forcing vector are calculated, these are assembled into the global stiffness matrix and forcing function, the temperature conditions are applied and the resulting system of equations solved by matrix inversion using Matlab.

Applying equation (2.20) and (2.21) and using the shape functions as given by equation (2.4a), (2.4b) and (2.4c). The element stiffness matrices for the elements are determined. It is observed that the element stiffness matrices for elements 1, 2, 5 and 7 are equal, and those for elements 2, 4, 6 and 8 are also equal.

$$[K]^1 = \begin{bmatrix} 5 & -5 & 0 \\ -5 & 10 & -5 \\ 0 & -5 & 5 \end{bmatrix} \quad \{f\}^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad [K]^2 = \begin{bmatrix} 5 & -5 & 0 \\ -5 & 10 & -5 \\ 0 & -5 & 5 \end{bmatrix} \quad \{f\}^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Assembling the element stiffness matrices and forcing vectors gives

$$K = \begin{bmatrix} 10 & -5 & 0 & -5 & 0 & 0 & 0 & 0 & 0 \\ -5 & 20 & -5 & 0 & -10 & 0 & 0 & 0 & 0 \\ 0 & -5 & 10 & 0 & 0 & -5 & 0 & 0 & 0 \\ -5 & 0 & 0 & 20 & -10 & 0 & -5 & 0 & 0 \\ 0 & -10 & 0 & -10 & 40 & -10 & 0 & -10 & 0 \\ 0 & 0 & -5 & 0 & -10 & 20 & 0 & 0 & -5 \\ 0 & 0 & 0 & -5 & 0 & 0 & 10 & -5 & 0 \\ 0 & 0 & 0 & 0 & -10 & 0 & -5 & 20 & -5 \\ 0 & 0 & 0 & 0 & 0 & -5 & 0 & -5 & 10 \end{bmatrix} \qquad \{f\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

When the temperature condition is applied the system of equation yields the one below:

$$[K]\{T\} = \{F\} = \begin{bmatrix} 1000\ 0\ 0000 \\ 0100\ 0\ 0000 \\ 0010\ 0\ 0000 \\ 0001\ 0\ 0000 \\ 0000400000 \\ 0000\ 0\ 1000 \\ 0000\ 0\ 0100 \\ 0000\ 0\ 0010 \\ 0000\ 0\ 0001 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 8000 \\ 100 \\ 500 \\ 500 \\ 500 \end{bmatrix}$$

When this is solved it yields the nodal temperatures below. According to (Lewis, Nithiarasu et al. 2004) this problem can be solved by using the formula below:

$T(x,y) = (T_{top} - T_{side})\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}+1}{n}\sin\left(\frac{n\pi x}{w}\right)\frac{\sinh\left(\frac{n\pi y}{w}\right)}{\sinh\left(\frac{n\pi H}{w}\right)} + T_{side}$ The analytical result obtained by using this formula is shown on the right, of the result obtained by finite element method.

$$\{T\} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 200 \\ 100 \\ 500 \\ 500 \\ 500 \end{bmatrix} \qquad \{T\} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 200.11 \\ 100 \\ 500 \\ 500 \\ 500 \end{bmatrix}$$

When this problem is solved using the pogram whose flowchart is shown in Figure A.2 a similar result was obtained, as that obtained by the finite element method, above.

2. Consider the square plate shown below, the left face is maintained at 100°C and the top face at 500°C, while the other two faces are exposed to an environment at 100°C. The coefficient of convection is 10W/m^2 °C and the thermal conductivity is 10W/m °C. The block is 1m square, compute the temperature at nodes 2, 3, 4, 6, 7, 8, 10, 11 and 12 indicated in the figure.

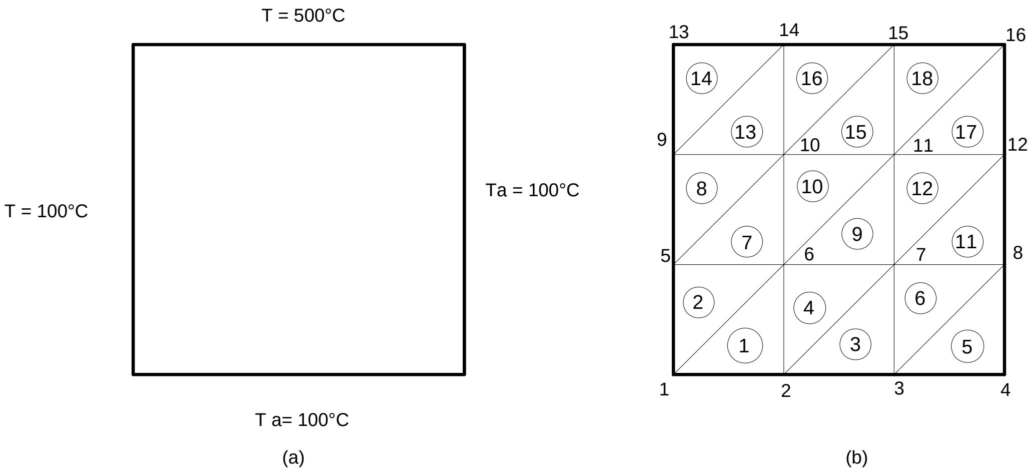

Figure 3.4 2-Dimensional heat transfer in a square body with temperature on two sides fixed, and convection on the other two sides

The problem is solved by the finite element analysis and the result is compared to the finite differences solution as given by (Holman 2002)

The problem is solved using the Matlab program whose flowchart is shown in Figure A.2 and the following results are obtained. The result obtained by the finite differences method according to (Holman 2002) is shown on the right.

$$\{T\} = \begin{bmatrix} T_2 \\ T_3 \\ T_4 \\ T_6 \\ T_7 \\ T_8 \\ T_{10} \\ T_{11} \\ T_{12} \end{bmatrix} = \begin{bmatrix} 158.4089 \\ 184.6395 \\ 172.2828 \\ 192.1800 \\ 230.0027 \\ 214.1380 \\ 280.3083 \\ 329.0531 \\ 305.9015 \end{bmatrix} \qquad \{T\} = \begin{bmatrix} T_2 \\ T_3 \\ T_4 \\ T_6 \\ T_7 \\ T_8 \\ T_{10} \\ T_{11} \\ T_{12} \end{bmatrix} = \begin{bmatrix} 157.70 \\ 184.71 \\ 175.62 \\ 192.38 \\ 231.15 \\ 217.19 \\ 280.67 \\ 330.30 \\ 309.38 \end{bmatrix}$$

3. It is required to determine the temperature at nodes 1 and 2 in the square body shown in Figure 3.5 below. The top side has a fixed temperature of 500°C, the left side has a constant heat flux of 10W/m^2, the bottom face is exposed to convection and the ambient temperature at the bottom face is 100°C, while the right face is exposed to convection but at an ambient temperature of 10°C.

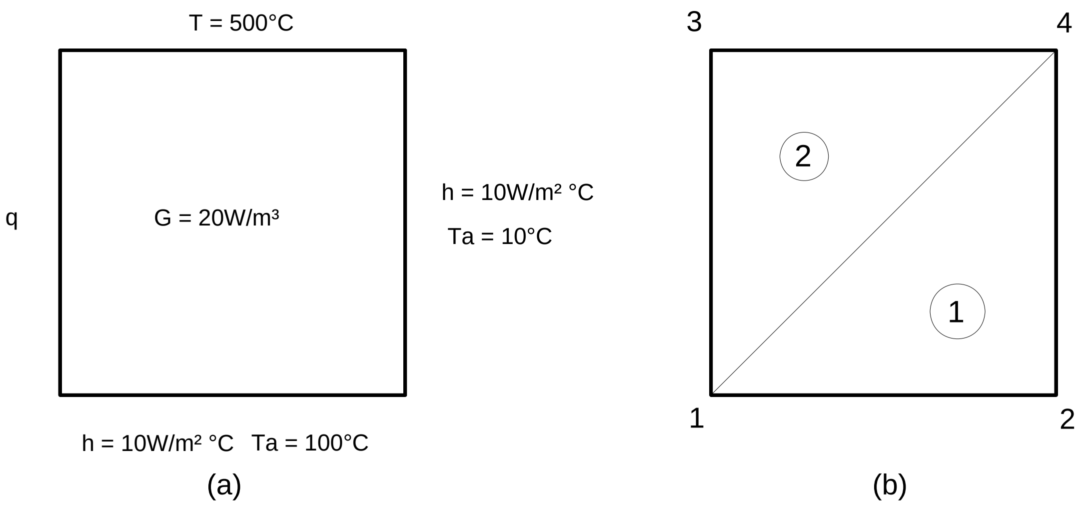

Figure 3.5 2-Dimensional heat transfer in a square body with temperature on one side fixed, convection on one side and heat flux on the last side

When equation (2.20) and (2.21) applied to the problem and the shape functions as given by equations (2.4a), (2.4b) and (2.4c).

$$K^1 = \begin{bmatrix} 8.3333 & -3.3333 & 0 \\ -3.3333 & 16.6667 & -3.3333 \\ 0 & -3.3333 & 8.3333 \end{bmatrix} \qquad F^1 = \begin{bmatrix} 503.3333 \\ 553.3333 \\ 53.3333 \end{bmatrix}$$

$$K^2 = \begin{bmatrix} 5 & -5 & 0 \\ -5 & 10 & -5 \\ 0 & -5 & 5 \end{bmatrix} \qquad F^2 = \begin{bmatrix} 3.3333 \\ -1.6667 \\ -1.6667 \end{bmatrix}$$

The element stiffness matrices and forcing vectors above are assembled to give the global stiffness matrix and forcing vector below

$$K = \begin{bmatrix} 13.3333 & -3.3333 & -5 & 0 \\ -3.3333 & 16.6667 & 0 & -3.3333 \\ -5 & 0 & 10 & -5 \\ 0 & -3.3333 & -5 & 13.3333 \end{bmatrix} \quad F = \begin{bmatrix} 506.6667 \\ 553.3333 \\ 3.3333 \\ 56.6667 \end{bmatrix}$$

When the temperature condition at top face (i.e. node 3 and 4) is applied to the system the global stiffness matrix and forcing vector becomes

$$[K]\{T\} = \{F\} = \begin{bmatrix} 13.3333 & -3.3333 & 0 & 0 \\ -3.3333 & 16.6667 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 3006.6667 \\ 2220 \\ 500 \\ 500 \end{bmatrix}$$

When the system of equations above is solved by matrix inversion using matlab, the following nodal temperatures were obtained.

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 272.0263 \\ 187.6053 \\ 500 \\ 500 \end{bmatrix}$$

When this problem is solved using the program whose flowchart is shown in Figure A.2 a similar result was obtained, as that obtained by the finite element method, above.

4. FINITE ELEMENT IN HEAT TRANSFER RESEARCH

The finite element method has been used extensively in heat transfer research, and it has been proven to be a viable method of analyzing heat transfer problems. In most cases it has been found that the result obtained shows a strong correlation to the analytical and experimental results. In instances where there have been discrepancies, it has been observed that the problem was not the method in itself, but the application.

4.1. USE OF FINITE ELEMENT METHOD IN HEAT TRANSFER RESEARCH IN COMPOSITES

(Carson, Lovatt et al. 2003) studied the influence of material structure on the effective thermal conductivity of porous materials using the finite element method. Although this research showed good correlation between numerical and analytical results, it did not indicate the correlation to experimental results, so one would be forced to ask, if correlation between numerical and analytical results indicate correlation with the real world situation? Are the errors and inconsistencies in the analytical just being propagated to the numerical? If the analytical result has been proven to be well correlated to the real situation why then do we need to develop a numerical solution? The numerical method should be employed when the analytical method is not sufficient, or the problem is too complex to solve with the analytical method.

(Bakker 1997) also used the finite element method to compute the influence of complex porosity and inclusion structures on the thermal and electrical conductivity of the material, the study considered the influence of shape, orientation and distribution of the dispersed phase. (Ramani and Vaidyanathan 1995) also carried out a similar research in which they studied the effects of microstructure of the composite, particle shape, formation of conductive chains at high volume fraction, filler aspect ratio and the interfacial thermal resistance on the effective thermal conductivity of a composite.

From the research by (Bakker 1997) and (Ramani and Vaidyanathan 1995) it can be concluded that empirical relations do not consider all the properties, but generally make some simplifications and assumptions which makes it easy and possible to formulate the relationships and these simplifications affect the results obtained in situations where the simplifications do not hold. An instance of this is in the Series, Parallel, and Maxwell-Eucken models for determining the effective thermal conductivity of a composite material, these models only take into account the volume fraction and thermal conductivity of the matrix and inclusion. It does not consider the spatial distribution which by itself alone can cause the effective thermal conductivity to vary by about 82% when the high thermal conductivity inclusions form conductive chains in the material, as opposed to when they form an agglomeration at the centre, as shown in the research by (Ramani and Vaidyanathan 1995).

Hence it can be inferred that the FEM method is a good way of carrying out heat transfer analysis, in composites as properties which might be difficult to model into the empirical models can be easily included in the finite element model. This is especially useful when it is combined with experimental analysis, which is used to validate the result obtained from FEM.

To appropriately model a non-homogeneous material (Kim and Paulino 2002) considered using Isoparametric Graded Finite elements in which the material properties were modeled using interpolation function whose basis was the shape function of the element.

4.2. USE OF FINITE ELEMENT METHOD IN HEAT TRANSFER RESEARCH OF TEMPERATURE DEPENDENT PROPERTIES

The thermal conductivity of some materials vary with temperature, when this effect occurs in a system it leads to nonlinearities (especially systems with transient heat transfer), and this needs to be appropriately modeled if the effect is much. Since the temperature within a system varies from one point to the other, the thermal conductivity also changes from one point to the other.

In transient systems, this effect of temperature dependent properties becomes more pronounced as the materials properties varies at different locations and also varies with time as the temperature at every point changes. This effect was studied by (Aguirre-Ramirez and Oden 1973).

4.3. USE OF FINITE ELEMENT METHOD IN HEAT TRANSFER RESEARCH OF BODIES WITH INTERNAL HEAT GENERATION

Several systems do possess a volumetric heat generation, this could occur due to ohmic-resistance heat generation in the body, radioactive decay leading to heat generation. At times a body can possess a negative heat generation term (i.e. a heat sink), this can occur in instances where a phase change occurs from solid to liquid or liquid to gas, whereas phase changes in the reverse direction are heat sources. According to (Comini, Del Guidice et al. 1974; Voller, Cross et al. 1987) latent heat effects due to phase change in a system can be modeled by incorporating it as heat capacity variations in the system or as a heat generation term in the conduction equation. (Morgan, Lewis et al. 1978) proposed a simplified and improved method for handling phase change in transient non-linear heat conduction problems. According to them better results can be obtained by direct evaluation of the heat capacity from the rate of change of enthalpy with respect to temperature. This approach removed the need for an averaging process which is not only more complex but can fail in some circumstances. The method employed by Comini et.al involved working with the with the enthalpy, H, which is the integral of the heat capacity with respect to temperature and which is a smooth function even in the phase change zone, from which they obtained the heat capacity by an averaging process.

5. DISCUSSION

From the literature survey it is evident that the applications of solar refrigeration has immense benefits, the only bottle neck being the poor efficiency and COP. As it has been shown by various researches carried out in this area that this is due to some of the following reasons;

- Wrong working pair selection
- Intermittency of cycle
- Long cycle duration
- Poor heat and mass transfer in the system

A lot of research is currently being carried out on most of these areas, the area which is of interest to this research work is in the heat and mass transfer occurring in the adsorber, between the fin and the adsorber, and in the fin itself. It has been shown that optimizing the heat transfer leads to reduction in cycle time and also increases the loading of the adsorber. From the research carried out by (Cornford, Cornaby et al. 2010) it can be inferred that the most important requirement in optimizing the heat transfer is to get as much heat as possible to the adsorber in the shortest time possible. Hence significant improvements in performance can be achieved by optimizing this alone.

As it has been shown by previous research that the heat transfer in composites and at contact boundaries cannot be modeled easily using empirical method, but this can be done relatively easily using the finite element method, as this makes it possible to account for all variations in the structure of the material which might not be possible using empirical means. We are also aware of the ever increasing computing power which we have access to, this has particularly made the finite element method attractive as complex structures can be modeled in great detail without much concern about computing costs. It is also possible to model all complex non-linearities that might exist in a system using the finite element method, thereby enabling the generation of a model which is a true representation of the system in question.

Hence the finite element method would be employed in this research to optimize the heat transfer in the fins of the solar refrigerator.

APPENDIX

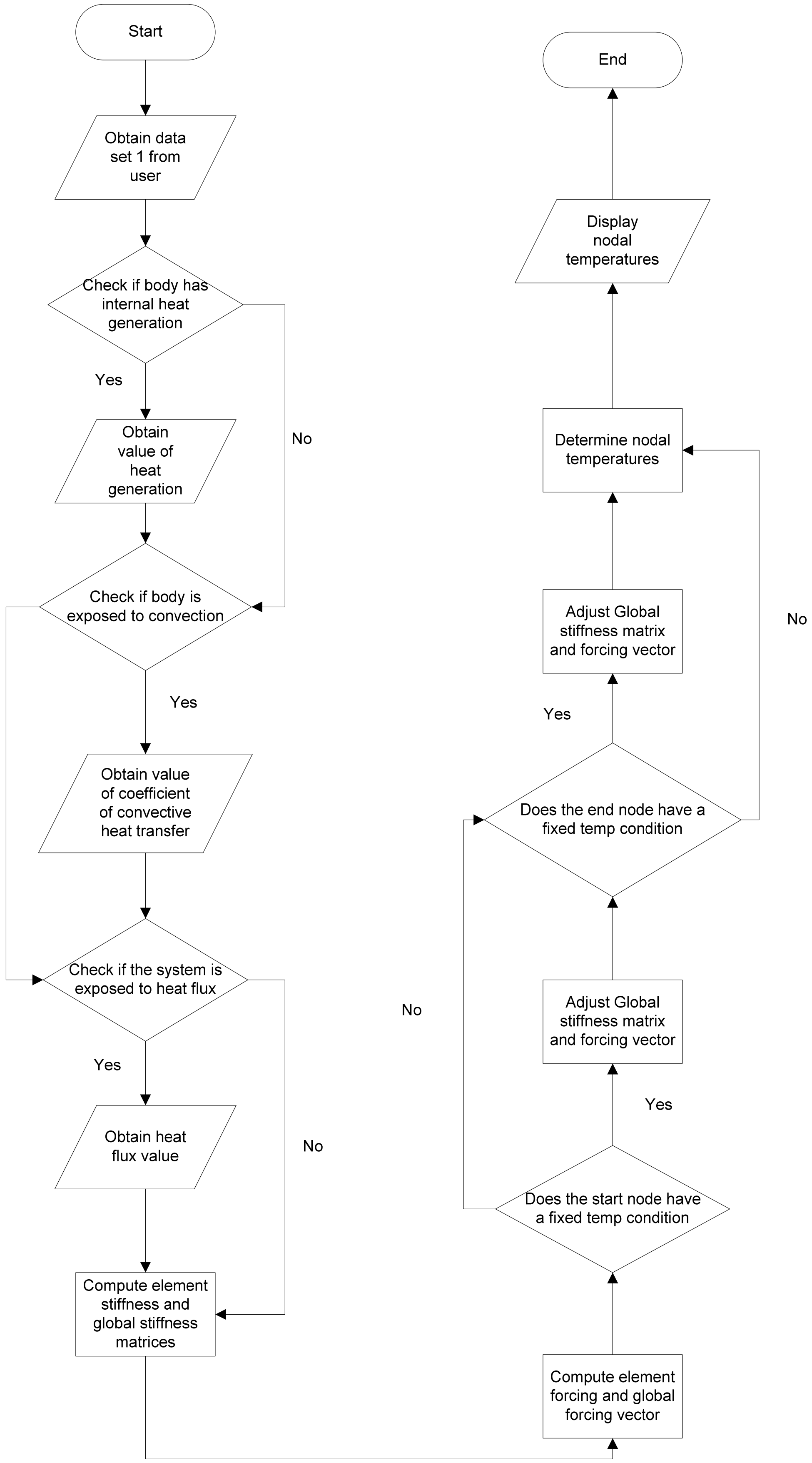

Figure A.1 Flowchart for 1-Dimensional heat conduction program

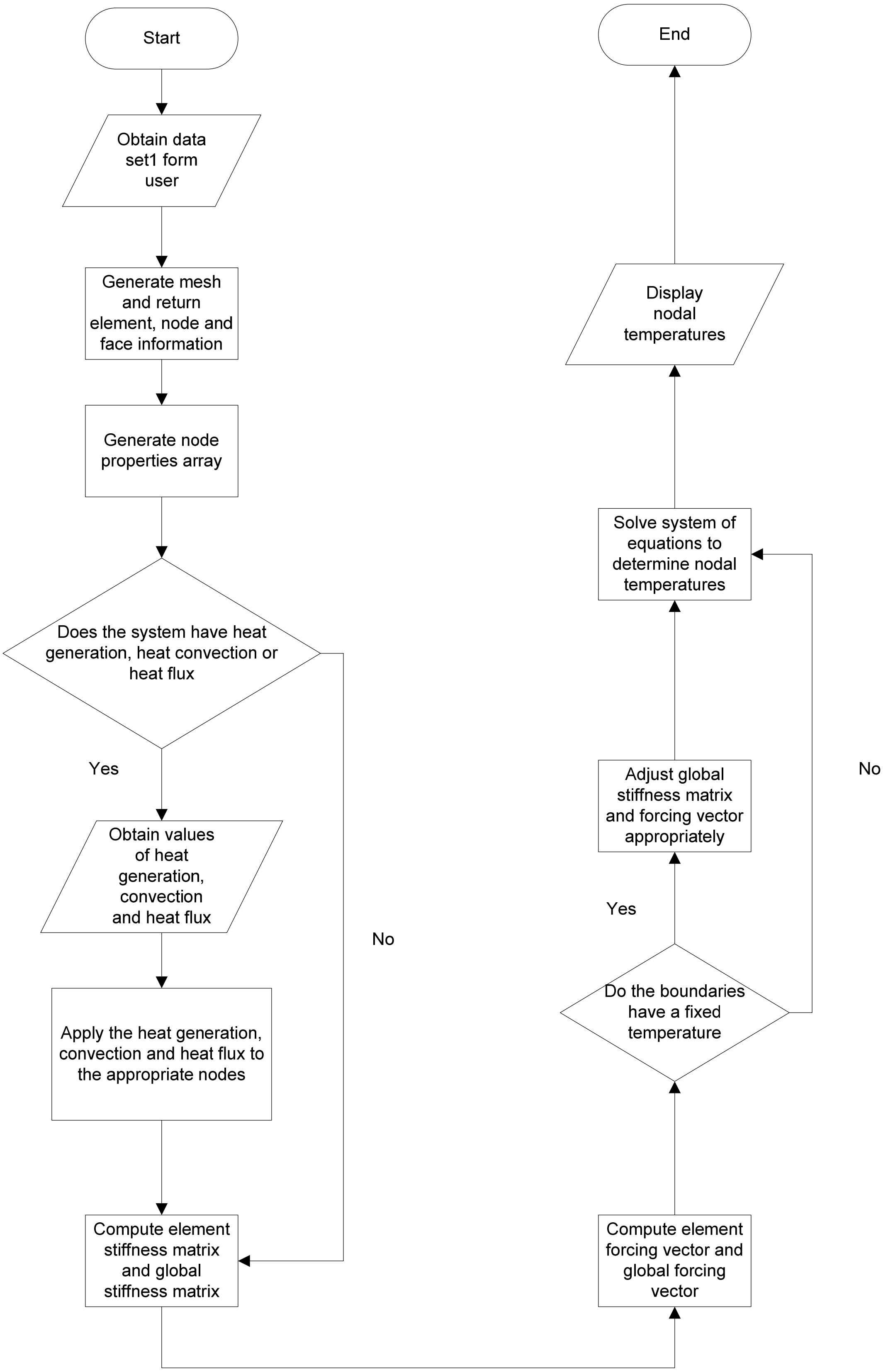

Figure A.2 Flowchart for 2-Dimensional heat conduction program

LIST OF REFERENCES

Aguirre-Ramirez, G. and J. T. Oden (1973). "Finite element technique applied to heat conduction in solids with temperature dependent thermal conductivity." International Journal for Numerical Methods in Engineering **7**(3): 345-355.

Bakker, K. (1997). "Using the finite element method to compute the influence of complex porosity and inclusion structures on the thermal and electrical conductivity." International Journal of Heat and Mass Transfer **40**(15): 3503-3511.

Carson, J. K., S. J. Lovatt, et al. (2003). "An analysis of the influence of material structure on the effective thermal conductivity of theoretical porous materials using finite element simulations." International Journal of Refrigeration **26**(8): 873-880.

Comini, G., S. Del Guidice, et al. (1974). "Finite element solution of non-linear heat conduction problems with special reference to phase change." International Journal for Numerical Methods in Engineering **8**(3): 613-624.

Cornford, J., J. Cornaby, et al. (2010). Low cost solar coolers for food refrigeration in the developing world. Department of Mechanical Engineering. Bristol, University of Bristol. **MEng**.

Critoph, R. E. (1989). "Activated carbon adsorption cycles for refrigeration and heat pumping." Carbon **27**(1): 63-70.

Enibe, S. O. and O. C. Iloeje (2000). "Heat and mass transfer in porous spherical pellets of CaCl2 for solar refrigeration." Renewable Energy **20**(3): 305-324.

Fotiadis, D. I., M. Boekholt, et al. (1990). "Flow and heat transfer in CVD reactors: Comparison of Raman temperature measurements and finite element model predictions." Journal of Crystal Growth **100**(3): 577-599.

Holman, J. P. (2002). Heat transfer, McGraw-Hill.

Kim, D. S. and C. A. Infante Ferreira (2008). "Solar refrigeration options - a state-of-the-art review." International Journal of Refrigeration **31**(1): 3-15.

Kim, J.-H. and G. H. Paulino (2002). "Isoparametric Graded Finite Elements for Nonhomogeneous Isotropic and Orthotropic Materials." Journal of Applied Mechanics **69**(4): 502-514.

Klein, S. A. and D. T. Reindl (2005). "Solar Refigeration." ASHRAE Journal **47**(9): S26 - S30.

Lewis, R. W., P. Nithiarasu, et al. (2004). Fundamentals of the Finite Element Method for Heat and Fluid Flow, John Wiley & Sons, Ltd.

Morgan, K., R. W. Lewis, et al. (1978). "An improved algrorithm for heat conduction problems with phase change." International Journal for Numerical Methods in Engineering **12**(7): 1191-1195.

Mottram, J. T. and C. T. Shaw (1996). Using finite elements in mechanical design, McGraw-Hill.

Ozisik, M. N. (1968). Boundary value problems of heat conduction. [S.l.], Dover Pubns , Inc.

Ramani, K. and A. Vaidyanathan (1995). "Finite Element Analysis of Effective Thermal Conductivity of Filled Polymeric Composites." Journal of Composite Materials **29**(13): 1725-1740.

Segerlind, L. J. (1984). Applied Finite Element Analysis, John Wiley & Sons, Inc.

Spahis, N., A. Addoun, et al. (2007). "Study on solar adsorption refrigeration cycle utilizing activated carbon prepared from olive stones." Revue des Energies Renouvelables **10**(3): 415-420.

Stasa, F. L. (1985). Applied Finite Element Analysis for Engineers. Japan, CBS Publishing.

Sumathy, K. and Y. J. Dai (2003). "Heat and mass transfer in the adsorbent of a solar adsorption cooling system with glass tube insulation." Energy **28**: 1511-1527.

Sumathy, K., K. H. Yeung, et al. (2003). "Technology development in the solar adsorption refrigeration systems." Progress in Energy and Combustion Science **29**(4): 301-327.

Tong, P. and J. N. Rossettos (1977). Finite Element Method, The MIT Press.

Voller, V. R., M. Cross, et al. (1987). "An enthalpy method for convection/diffusion phase change." International Journal for Numerical Methods in Engineering **24**(1): 271-284.

Wang, R. Z. and R. G. Oliveira (2006). "Adsorption refrigeration--An efficient way to make good use of waste heat and solar energy." Progress in Energy and Combustion Science **32**(4): 424-458.

Zhu, D. and S. Wang (2002). "Experimental investigation of contact resistance in adsorber of solar adsorption refrigeration." Solar Energy **73**(3): 177-185.

Zienkiewicz, O. C. and R. L. Taylor (2000). The finite element method, Butterworth Heinemann.