IE 440 Fall 2019 - Nonlinear Models in Operations Research

Homework # 2

due November 4^{th}, Monday, 17:00

You have the following location-allocation problem which is known as Multi-Facility Weber Problem:

Suppose you have n customers, whose coordinates and demands will be given to you. You are to locate m facilities among those customers so as to satisfy all the demand. The facilities have infinite capacity; that means once you assign a facility to a certain customer, it is able to satisfy the customer's demand fully. You will also be given the cost of transporting one unit of product from a certain facility to a certain customer per unit distance. Your objective is to find the optimum facility locations that would minimize the total cost.

You will be given a sample set of parameters in text files:

h_j : the demand of customer j ("demand-GroupID.txt")
c_{ij} : the cost of transporting 1 unit of product from facility i to customer j per unit distance ("cost-GroupID.txt")
a_j : (a_{1j}, a_{2j}) the coordinates of customer j ("coordinate-GroupID.txt")

Notice $C_{ij} = h_j \cdot c_{ij}$ gives you the total cost of assigning facility i to customer j per unit distance.

Number of customers (n) **and number of facilities** (m) **are not fixed. You should get this information by interpreting the given data.**

Your decision variables will be

x_i : (x_{1i}, x_{2i}) the coordinates of facility i
y_{ij} : binary variable which takes the value 1 if customer j is assigned to facility i and 0 otherwise.

Then, your objective function becomes:

$z = \sum_i \sum_j y_{ij} \cdot C_{ij} \cdot d(x_i, a_j)$, where $d(x_i, a_j)$ is the distance between facility i and customer j.

1. Assume $d(x_i, a_j)$ is squared Euclidean distance and solve single facility problem (You can select any one of the facilities, but please indicate in the report).
2. Assume $d(x_i, a_j)$ is Euclidean distance and solve single facility problem by applying Weiszfeld's Algorithm (Please use same facility that you have selected in the first part. Also, set k_{th} coordinate of the facility to $x_k = \frac{\sum_j h_j \cdot c_{ij} \cdot a_{kj}}{\sum_j h_j \cdot c_{ij}}$ initially, here i is the index of the facility that you have selected).
3. Assume $d(x_i, a_j)$ is squared Euclidean distance
   3.1. Randomly allocate customers to facilities initially
   3.2. Apply Alternative Location-Allocation (ALA) Heuristic
   3.3. Report the average and the best results for 1000 trials
4. Assume $d(x_i, a_j)$ is Euclidean distance
   4.1. Initialize your model as described above
   4.2. Apply ALA Heuristic (using Weiszfeld's Algorithm to solve single facility location problems)
   4.3. Report the average and the best results for 1000 trials

(During iterations in Part 3 and 4 do not forget to consider c_{ij} values when assigning customers to facilities. Also in application of Weiszfeld's Algorithm, to avoid the division by 0, you can add ϵ term to denominators, you can choose your own ϵ).

Your program should give the output of the facilities' final locations and their assigned customers, and the objective function value. Your report should include the output image, any comments you have on the heuristic (on initialization rules, or allocation rules etc.) and your code.

You must use the attached example data files specified for your groups.

The report and the source code are to be named as HW2-GroupID.