RELATIONAL MAGNITUDES

Relational Magnitudes are invariant vectorial quantities that conserve their value and form under transformations of translation and rotation.

I. Definitions I (Relational Magnitudes)

The relational position ($\mathbf{r}_i$), relational velocity ($\mathbf{v}_i$), and relational acceleration ($\mathbf{a}_i$) of a particle i with respect to an Auxiliary Reference Frame, are given by:

$$\mathbf{r}_i \doteq \vec{r}_i$$

$$\mathbf{v}_i \doteq d(\vec{r}_i)/dt = \vec{v}_i$$

$$\mathbf{a}_i \doteq d^2(\vec{r}_i)/dt^2 = \vec{a}_i$$

Where $\vec{r}_i$, $\vec{v}_i$ and $\vec{a}_i$ are the ordinary vectorial position, velocity, and acceleration of the particle i with respect to the Auxiliary Reference Frame.

Note: The Relational (Vectorial) Magnitudes are always the same as the Ordinary (Vectorial) Magnitudes in the Auxiliary Reference Frame.

II. Definitions II (Relational Magnitudes)

The relational position ($\mathbf{r}_i$), relational velocity ($\mathbf{v}_i$), and relational acceleration ($\mathbf{a}_i$) of a particle i with respect to any Reference Frame S, are given by:

$$\mathbf{r}_i \doteq \vec{r}_i - \vec{R}$$

$$\mathbf{v}_i \doteq (\vec{v}_i - \vec{V}) - \vec{\omega} \times (\vec{r}_i - \vec{R})$$

$$\mathbf{a}_i \doteq (\vec{a}_i - \vec{A}) - 2\vec{\omega} \times (\vec{v}_i - \vec{V}) + \vec{\omega} \times [\, \vec{\omega} \times (\vec{r}_i - \vec{R}) \,] - \vec{\alpha} \times (\vec{r}_i - \vec{R})$$

Where: $\vec{r}_i, \vec{v}_i, \vec{a}_i$ are the ordinary vectorial position, velocity, and acceleration of particle i with respect to the Frame S. $\vec{R}, \vec{V}, \vec{A}$ are the position, velocity, and acceleration of the Auxiliary Frame's origin with respect to S. $\vec{\omega}$ and $\vec{\alpha}$ are the angular velocity and angular acceleration of the Auxiliary Frame with respect to S.

III. Transformations (Invarianza·Relations)

The transformations of relational position, relational velocity and relational acceleration of a particle i between a Reference Frame S and another Reference Frame S', are given by:

$$\mathbf{r}_i \doteq (\vec{r}_i - \vec{R}) = \mathbf{r}'_i$$

$$\mathbf{r}'_i \doteq (\vec{r}'_i - \vec{R}') = \mathbf{r}_i$$

$$\mathbf{v}_i \doteq (\vec{v}_i - \vec{V}) - \vec{\omega} \times (\vec{r}_i - \vec{R}) = \mathbf{v}'_i$$

$$\mathbf{v}'_i \doteq (\vec{v}'_i - \vec{V}') - \vec{\omega}' \times (\vec{r}'_i - \vec{R}') = \mathbf{v}_i$$

$$\mathbf{a}_i \doteq (\vec{a}_i - \vec{A}) - 2\vec{\omega} \times (\vec{v}_i - \vec{V}) + \vec{\omega} \times [\, \vec{\omega} \times (\vec{r}_i - \vec{R}) \,] - \vec{\alpha} \times (\vec{r}_i - \vec{R}) = \mathbf{a}'_i$$

$$\mathbf{a}'_i \doteq (\vec{a}'_i - \vec{A}') - 2\vec{\omega}' \times (\vec{v}'_i - \vec{V}') + \vec{\omega}' \times [\, \vec{\omega}' \times (\vec{r}'_i - \vec{R}') \,] - \vec{\alpha}' \times (\vec{r}'_i - \vec{R}') = \mathbf{a}_i$$

IV. Bibliography

[1] **A. Blatter**, A Reformulation of Classical Mechanics, (2015).(PDF)

[2] **A. Tobla**, A Reformulation of Classical Mechanics, (2024).(PDF)