## Channel Theory

## Classifications, Infomorphisms, and Channels

In channel theory, each component of a distributed systems is represented by a classification  $\mathbf{A} = (tok(\mathbf{A}), typ(\mathbf{A}), \models_{\mathbf{A}})$ , consisting of a set of tokens  $tok(\mathbf{A})$ , a set of types  $typ(\mathbf{A})$  and a classification relation  $\models_{\mathbf{A}} \subseteq tok(\mathbf{A}) \times typ(\mathbf{A})$  that classifies tokens to types.

The flow of information between components in a distributed system is modelled in channel theory by the way the various classifications that represent the vocabulary and context of each component are connected with each other through *infomorphisms*. An infomorphism  $f = \langle f \hat{\ }, f \rangle : \mathbf{A} \rightleftharpoons \mathbf{B}$  from classification  $\mathbf{A}$  to classification  $\mathbf{B}$  is a contravariant pair of functions  $f \hat{\ } : typ(\mathbf{A}) \rightarrow$  $typ(\mathbf{B})$  and  $f \hat{\ } : tok(\mathbf{B}) \rightarrow tok(\mathbf{A})$  satisfying the following fundamental property, for each type  $\alpha \in typ(\mathbf{A})$  and token  $b \in tok(\mathbf{B})$ :

$$\alpha \vdash \xrightarrow{f^{\hat{}}} f^{\hat{}}(\alpha)$$

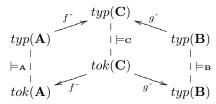
$$\models_{\mathbf{A}} \mid \qquad \mid_{\models_{\mathbf{B}}}$$

$$f^{\hat{}}(b) \xleftarrow{f^{\hat{}}} b$$

$$f(b) \models_{\mathbf{A}} \alpha \text{ iff } b \models_{\mathbf{B}} f(\alpha)$$

A distributed system  $\mathcal{A}$  consists then of an indexed family  $cla(\mathcal{A}) = \{\mathbf{A}_i\}_{i \in I}$  of classifications together with a set  $inf(\mathcal{A})$  of infomorphisms all having both domain and codomain in  $cla(\mathcal{A})$ .

A basic construct of channel theory is that of a *channel*—two classifications **A** and **B** connected through a core classification **C** via two infomorphisms f and g:



This basic construct captures the information flow between components **A** and **B**. Crucial in Barwise and Seligman's model is that it is the particular tokens that carry information and that information flow crucially involves both types and tokens.

## Regular Theories and Local Logics

Channel theory has been developed based on the understanding that information flow results from regularities in a distributed system, and that it is by virtue of regularities among the connections that information of some components of a system carries information of other components. These regularities are implicit in the representation of the systems' components and its connections as classifications and infomorphisms, which can be expressed in a logical fashion. This is done in channel theory with *regular theories* and *local logics*.

A theory  $T = \langle typ(T), \vdash \rangle$  consists of a set typ(T) of types, and a binary relation  $\vdash$  between subsets of typ(T). Pairs  $\langle \Gamma, \Delta \rangle$  of subsets of typ(T) are called sequents. If  $\Gamma \vdash \Delta$ , for  $\Gamma, \Delta \subseteq typ(T)$ , then the sequent  $\Gamma \vdash \Delta$  is called a constraint. T is regular if for all  $\alpha \in typ(T)$  and all sets  $\Gamma, \Gamma', \Delta, \Delta', \Sigma', \Sigma_0, \Sigma_1$  of types:

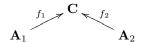
- 1. *Identity:*  $\alpha \vdash \alpha$
- 2. Weakening: If  $\Gamma \vdash \Delta$ , then  $\Gamma, \Gamma' \vdash \Delta, \Delta'$
- 3. Global Cut: If  $\Gamma, \Sigma_0 \vdash \Delta, \Sigma_1$  for each partition  $(\Sigma_0, \Sigma_1)$  of  $\Sigma'$ , then  $\Gamma \vdash \Delta$ .

Regularity arises from the observation that, given any classification of tokens to types, the set of all sequents that are *satisfied* (defined further below) by all tokens always fulfil these three properties. In addition, given a regular theory T we can generate a classification Cla(T) that captures the regularity specified in its constraints. Its tokens are partitions  $\langle \Gamma, \Delta \rangle$  of typ(T) that are *not* constraints of T, and types are the types of T, such that  $\langle \Gamma, \Delta \rangle \models_{Cla(T)} \alpha$  iff  $\alpha \in \Gamma$ .<sup>2</sup>

Putting the idea of a classification with that of a regular theory together we get a local logic  $\mathfrak{L} = \langle tok(\mathfrak{L}), typ(\mathfrak{L}), \models_{\mathfrak{L}}, \vdash_{\mathfrak{L}}, N_{\mathfrak{L}} \rangle$ . It consists of a classification  $cla(\mathfrak{L}) = \langle tok(\mathfrak{L}), typ(\mathfrak{L}), \models_{\mathfrak{L}} \rangle$ , a regular theory  $th(\mathfrak{L}) = \langle typ(\mathfrak{L}), \vdash_{\mathfrak{L}} \rangle$  and a subset of  $N_{\mathfrak{L}} \subseteq tok(\mathfrak{L})$  of normal tokens, which satisfy all the constraints of  $th(\mathfrak{L})$ ; a token  $a \in tok(\mathfrak{L})$  satisfies a constraint  $\Gamma \vdash \Delta$  of  $th(\mathfrak{L})$  if, when a is of all types in  $\Gamma$ , a is of some type in  $\Delta$ . A local logic  $\mathfrak{L}$  is sound if  $N_{\mathfrak{L}} = tok(\mathfrak{L})$ .

## The Distributed Logic

The *distributed logic* is the logic that represents the information flow occurring in a distributed system. Assuming a channel



that represents the information flow between  $\mathbf{A}_1$  and  $\mathbf{A}_2$ , the logic we are after is the one we get from *moving* a local logic on the core  $\mathbf{C}$  of the channel to the sum of components  $\mathbf{A}_1 + \mathbf{A}_2$ : The theory will be induced at the core of the channel; this is crucial. The distributed logic is the *inverse image* of the local logic at the core.

Given an infomorphism  $f : \mathbf{A} \rightleftharpoons \mathbf{B}$  and a local logic  $\mathfrak{L}$  on  $\mathbf{B}$ , the *inverse image*  $f^{-1}[\mathfrak{L}]$  of  $\mathfrak{L}$  under f is the local logic on  $\mathbf{A}$ , whose theory is such that

<sup>&</sup>lt;sup>1</sup>A partition of  $\Sigma'$  is a pair  $\langle \Sigma_0, \Sigma_1 \rangle$  of subsets of  $\Sigma'$ , such that  $\Sigma_0 \cup \Sigma_1 = \Sigma'$  and  $\Sigma_0 \cap \Sigma_1 = \emptyset$ ;  $\Sigma_0$  and  $\Sigma_1$  may themselves be empty (hence it is actually a quasi-partition).

<sup>&</sup>lt;sup>2</sup>These tokens may not seem obvious, but these sequents code the content of the classification table: The left-hand sides of the these sequents indicate which to which types they are classified, while the right-hand sides indicate to which they are not.

 $\Gamma \vdash \Delta$  is a constraint of  $th(f^{-1}[\mathfrak{L}])$  iff  $f\hat{\ } [\Gamma] \vdash f\hat{\ } [\Delta]$  is a constraint of  $th(\mathfrak{L})$ , and whose normal tokens are  $N_{f^{-1}[\mathfrak{L}]} = \{a \in tok(\mathbf{A}) \mid a = f\check{\ } (b) \text{ for some } b \in N_{\mathfrak{L}} \}$ . If  $f\tilde{\ }$  is surjective on tokens and  $\mathfrak{L}$  is sound, then  $f^{-1}[\mathfrak{L}]$  is sound.

The type and token systems at the core and the classification of tokens to types will determine the local logic at this core. We usually take the *natural logic* as the local logic of the core, which is the local logic  $Log(\mathbf{C})$  generated from a classification  $\mathbf{C}$ , and has as classification  $\mathbf{C}$ , as regular theory the theory whose constraints are the sequents satisfied by all tokens, and whose tokens are all normal.

Given a channel  $C = \{f_{1,2} : \mathbf{A}_{1,2} \rightleftarrows \mathbf{C}\}$  and a local logic  $\mathfrak L$  on its core  $\mathbf C$ , the distributed logic  $DLog_{\mathcal C}(\mathfrak L)$  is the inverse image of  $\mathfrak L$  under the sum infomorphism  $f_1 + f_2 : \mathbf A_1 + \mathbf A_2 \rightleftarrows \mathbf C$ . This sum is defined as follows:  $\mathbf A_1 + \mathbf A_2$  has as set of tokens the Cartesian product of  $tok(\mathbf A_1)$  and  $tok(\mathbf A_2)$  and as set of types the disjoint union of  $typ(\mathbf A_1)$  and  $typ(\mathbf A_2)$ , such that for  $\alpha \in typ(\mathbf A_1)$  and  $\beta \in typ(\mathbf A_2)$ ,  $\langle a,b \rangle \models_{\mathbf A_1+\mathbf A_2} \alpha$  iff  $a \models_{\mathbf A_1} \alpha$ , and  $\langle a,b \rangle \models_{\mathbf A_1+\mathbf A_2} \beta$  iff  $b \models_{\mathbf A_2} \beta$ . Given two infomorphisms  $f_{1,2} : \mathbf A_{1,2} \rightleftarrows \mathbf C$ , the sum  $f_1 + f_2 : \mathbf A_1 + \mathbf A_2 \rightleftarrows \mathbf C$  is defined by  $(f_1 + f_2)^{\hat{}}(\alpha) = f_i(\alpha)$  if  $\alpha \in \mathbf A_i$  and  $(f_1 + f_2)^{\hat{}}(c) = \langle f^{\hat{}}_1(c), f^{\hat{}}_2(c) \rangle$ , for  $c \in tok(\mathbf C)$ .