Channel Theory

Classifications, Infomorphisms, and Channels

In channel theory, each component of a distributed systems is represented by a *classification* $\mathbf{A} = (tok(\mathbf{A}), typ(\mathbf{A}), \models_{\mathbf{A}})$, consisting of a set of *tokens* $tok(\mathbf{A})$, a set of *types* $typ(\mathbf{A})$ and a *classification relation* $\models_{\mathbf{A}} \subseteq tok(\mathbf{A}) \times typ(\mathbf{A})$ that classifies tokens to types.

The flow of information between components in a distributed system is modelled in channel theory by the way the various classifications that represent the vocabulary and context of each component are connected with each other through *infomorphisms*. An infomorphism $f = \langle f^{\wedge}, f^{\vee} \rangle : \mathbf{A} \rightleftarrows \mathbf{B}$ from classification $\mathbf{A}$ to classification $\mathbf{B}$ is a contravariant pair of functions $f^{\wedge} : typ(\mathbf{A}) \to typ(\mathbf{B})$ and $f^{\vee} : tok(\mathbf{B}) \to tok(\mathbf{A})$ satisfying the following fundamental property, for each type $\alpha \in typ(\mathbf{A})$ and token $b \in tok(\mathbf{B})$:

$$\begin{tikzcd}
\alpha \arrow[r, mapsto, "f^{\wedge}"] \arrow[d, dash, dashed, "\models_{\mathbf{A}}"'] & f^{\wedge}(\alpha) \arrow[d, dash, dashed, "\models_{\mathbf{B}}"] \\
f^{\vee}(b) & b \arrow[l, mapsto, "f^{\vee}"]
\end{tikzcd}$$

$$f^{\vee}(b) \models_{\mathbf{A}} \alpha \quad \text{iff} \quad b \models_{\mathbf{B}} f^{\wedge}(\alpha\rangle$$

A *distributed system* $\mathcal{A}$ consists then of an indexed family $cla(\mathcal{A}) = \{\mathbf{A}_i\}_{i \in I}$ of classifications together with a set $inf(\mathcal{A})$ of infomorphisms all having both domain and codomain in $cla(\mathcal{A})$.

A basic construct of channel theory is that of a *channel*—two classifications $\mathbf{A}$ and $\mathbf{B}$ connected through a core classification $\mathbf{C}$ via two infomorphisms f and g:

$$\begin{tikzcd}
 & typ(\mathbf{C}) \arrow[dd, dash, dashed, "\models_{\mathbf{C}}"] & \\
typ(\mathbf{A}) \arrow[ur, "f^{\wedge}"] \arrow[dd, dash, dashed, "\models_{\mathbf{A}}"'] & & typ(\mathbf{B}) \arrow[ul, "g^{\wedge}"'] \arrow[dd, dash, dashed, "\models_{\mathbf{B}}"] \\
 & tok(\mathbf{C}) \arrow[dl, "f^{\vee}"] \arrow[dr, "g^{\vee}"'] & \\
tok(\mathbf{A}) & & typ(\mathbf{B})
\end{tikzcd}$$

This basic construct captures the information flow between components $\mathbf{A}$ and $\mathbf{B}$. Crucial in Barwise and Seligman's model is that it is the particular tokens that carry information and that information flow crucially involves both types and tokens.

Regular Theories and Local Logics

Channel theory has been developed based on the understanding that information flow results from regularities in a distributed system, and that it is by virtue of regularities among the connections that information of some components of a system carries information of other components. These regularities are implicit in the representation of the systems' components and its connections as classifications and infomorphisms, which can be expressed in a logical fashion. This is done in channel theory with *regular theories* and *local logics*.

A *theory* $T = \langle typ(T), \vdash \rangle$ consists of a set $typ(T)$ of types, and a binary relation $\vdash$ between subsets of $typ(T)$. Pairs $\langle \Gamma, \Delta \rangle$ of subsets of $typ(T)$ are called *sequents*. If $\Gamma \vdash \Delta$, for $\Gamma, \Delta \subseteq typ(T)$, then the sequent $\Gamma \vdash \Delta$ is called a *constraint*. T is *regular* if for all $\alpha \in typ(T)$ and all sets $\Gamma, \Gamma', \Delta, \Delta', \Sigma', \Sigma_0, \Sigma_1$ of types:

1. *Identity:* $\alpha \vdash \alpha$
2. *Weakening:* If $\Gamma \vdash \Delta$, then $\Gamma, \Gamma' \vdash \Delta, \Delta'$
3. *Global Cut:* If $\Gamma, \Sigma_0 \vdash \Delta, \Sigma_1$ for each partition $\langle \Sigma_0, \Sigma_1 \rangle$ of Σ', then $\Gamma \vdash \Delta$.[1]

Regularity arises from the observation that, given any classification of tokens to types, the set of all sequents that are *satisfied* (defined further below) by all tokens always fulfil these three properties. In addition, given a regular theory T we can generate a classification $Cla(T)$ that captures the regularity specified in its constraints. Its tokens are partitions $\langle \Gamma, \Delta \rangle$ of $typ(T)$ that are *not* constraints of T, and types are the types of T, such that $\langle \Gamma, \Delta \rangle \models_{Cla(T)} \alpha$ iff $\alpha \in \Gamma$.[2]

Putting the idea of a classification with that of a regular theory together we get a *local logic* $\mathfrak{L} = \langle tok(\mathfrak{L}), typ(\mathfrak{L}), \models_{\mathfrak{L}}, \vdash_{\mathfrak{L}}, N_{\mathfrak{L}} \rangle$. It consists of a classification $cla(\mathfrak{L}) = \langle tok(\mathfrak{L}), typ(\mathfrak{L}), \models_{\mathfrak{L}} \rangle$, a regular theory $th(\mathfrak{L}) = \langle typ(\mathfrak{L}), \vdash_{\mathfrak{L}} \rangle$ and a subset of $N_{\mathfrak{L}} \subseteq tok(\mathfrak{L})$ of *normal tokens*, which satisfy all the constraints of $th(\mathfrak{L})$; a token $a \in tok(\mathfrak{L})$ satisfies a constraint $\Gamma \vdash \Delta$ of $th(\mathfrak{L})$ if, when a is of all types in Γ, a is of some type in Δ. A local logic $\mathfrak{L}$ is *sound* if $N_{\mathfrak{L}} = tok(\mathfrak{L})$.

The Distributed Logic

The *distributed logic* is the logic that represents the information flow occurring in a distributed system. Assuming a channel

$$\begin{tikzcd} & \mathbf{C} & \\ \mathbf{A}_1 \arrow[ur, "f_1"] & & \mathbf{A}_2 \arrow[ul, "f_2"'] \end{tikzcd}$$

that represents the information flow between $\mathbf{A}_1$ and $\mathbf{A}_2$, the logic we are after is the one we get from *moving* a local logic on the core $\mathbf{C}$ of the channel to the sum of components $\mathbf{A}_1 + \mathbf{A}_2$: The theory will be induced at the core of the channel; this is crucial. The distributed logic is the *inverse image* of the local logic at the core.

Given an infomorphism $f : \mathbf{A} \rightleftarrows \mathbf{B}$ and a local logic $\mathfrak{L}$ on $\mathbf{B}$, the *inverse image* $f^{-1}[\mathfrak{L}]$ of $\mathfrak{L}$ under f is the local logic on $\mathbf{A}$, whose theory is such that

[1] A partition of Σ' is a pair $\langle \Sigma_0, \Sigma_1 \rangle$ of subsets of Σ', such that $\Sigma_0 \cup \Sigma_1 = \Sigma'$ and $\Sigma_0 \cap \Sigma_1 = \emptyset$; Σ_0 and Σ_1 may themselves be empty (hence it is actually a quasi-partition).

[2] These tokens may not seem obvious, but these sequents code the content of the classification table: The left-hand sides of the these sequents indicate which to which types they are classified, while the right-hand sides indicate to which they are not.

$\Gamma \vdash \Delta$ is a constraint of $th(f^{-1}[\mathfrak{L}])$ iff $f\hat{}[\Gamma] \vdash f\hat{}[\Delta]$ is a constraint of $th(\mathfrak{L})$, and whose normal tokens are $N_{f^{-1}[\mathfrak{L}]} = \{a \in tok(\mathbf{A}) \mid a = f\check{}(b) \text{ for some } b \in N_{\mathfrak{L}}\}$. If $f\check{}$ is surjective on tokens and $\mathfrak{L}$ is sound, then $f^{-1}[\mathfrak{L}]$ is sound.

The type and token systems at the core and the classification of tokens to types will determine the local logic at this core. We usually take the *natural logic* as the local logic of the core, which is the local logic $Log(\mathbf{C})$ generated from a classification $\mathbf{C}$, and has as classification $\mathbf{C}$, as regular theory the theory whose constraints are the sequents satisfied by all tokens, and whose tokens are all normal.

Given a channel $\mathcal{C} = \{f_{1,2} : \mathbf{A}_{1,2} \rightleftarrows \mathbf{C}\}$ and a local logic $\mathfrak{L}$ on its core $\mathbf{C}$, the *distributed logic* $DLog_{\mathcal{C}}(\mathfrak{L})$ is the inverse image of $\mathfrak{L}$ under the sum infomorphism $f_1 + f_2 : \mathbf{A}_1 + \mathbf{A}_2 \rightleftarrows \mathbf{C}$. This sum is defined as follows: $\mathbf{A}_1 + \mathbf{A}_2$ has as set of tokens the Cartesian product of $tok(\mathbf{A}_1)$ and $tok(\mathbf{A}_2)$ and as set of types the disjoint union of $typ(\mathbf{A}_1)$ and $typ(\mathbf{A}_2)$, such that for $\alpha \in typ(\mathbf{A}_1)$ and $\beta \in typ(\mathbf{A}_2)$, $\langle a, b\rangle \models_{\mathbf{A}_1+\mathbf{A}_2} \alpha$ iff $a \models_{\mathbf{A}_1} \alpha$, and $\langle a, b\rangle \models_{\mathbf{A}_1+\mathbf{A}_2} \beta$ iff $b \models_{\mathbf{A}_2} \beta$. Given two infomorphisms $f_{1,2} : \mathbf{A}_{1,2} \rightleftarrows \mathbf{C}$, the sum $f_1 + f_2 : \mathbf{A}_1 + \mathbf{A}_2 \rightleftarrows \mathbf{C}$ is defined by $(f_1 + f_2)\hat{}(\alpha) = f_i(\alpha)$ if $\alpha \in \mathbf{A}_i$ and $(f_1 + f_2)\check{}(c) = \langle f\check{}_1(c), f\check{}_2(c)\rangle$, for $c \in tok(\mathbf{C})$.