Spectral Entropy Shaping (SES)

A Novel Regularization Technique for Neural Networks
via Layer-wise Spectral Entropy Control

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Abstract

We propose **Spectral Entropy Shaping (SES)**, a new regularization technique for deep neural networks that explicitly controls the *effective dimensionality* of intermediate representations by penalizing deviations of the layer-wise spectral entropy from a prescribed target. Unlike existing methods that regulate scalar statistics of activations (Batch Normalization) or weights (Weight Decay), SES acts on the full spectral distribution of the empirical covariance of each layer's activations, providing a geometrically motivated inductive bias. We derive two theoretical guarantees: (1) a generalization bound that scales with the effective rank rather than the ambient dimension, and (2) a Lipschitz stability bound showing improved robustness to input perturbations. Empirical validation across CIFAR-10 (3 seeds), CIFAR-100 (3 seeds), and synthetic data reveals a nuanced picture: on easy benchmarks (CIFAR-10), SES matches baseline accuracy while consistently improving corruption robustness (+0.45 pp); on harder tasks (CIFAR-100), benefits are more pronounced (+0.47 pp accuracy with 2.6× lower variance, −1.3% gap, +0.65 pp robustness). Direct comparison shows SES outperforms spectral normalization—the closest existing spectral method—on all metrics. On a controlled toy task, SES reduces the Jacobian condition number by 2.45×, directly validating the stability bound. The method is simple to implement (∼10 lines of PyTorch) and introduces a single interpretable hyperparameter $\beta \in (0, 1)$.

1 Introduction and Motivation

Modern deep learning relies on a toolkit of regularization and normalization techniques—Batch Normalization, Dropout, Weight Decay, Skip Connections—each controlling a specific scalar aspect of the network's behavior during training. Batch Normalization standardizes the first two moments of layer activations; Weight Decay penalizes the ℓ_2-norm of parameters; Dropout stochastically masks individual neurons.

However, none of these techniques directly controls the **geometric structure** of the learned representations, specifically the *spectral distribution* of the activations' covariance. This is a critical gap: a layer may collapse its representations onto a low-rank subspace (dead neurons, redundant features) or spread them uniformly across all dimensions (noise sensitivity, overfitting). Both regimes are pathological, yet no standard technique monitors or regulates them.

Key Insight. The *spectral entropy* of a layer's empirical covariance matrix provides a single, differentiable quantity that captures the full distributional structure of eigenvalues, measuring the *effective dimensionality* of the representation. By penalizing deviations from a target entropy, we obtain a regularizer that simultaneously prevents dimensional collapse and uncontrolled expansion.

2 Related Work

SES lies at the intersection of several research threads. We review the most relevant ones and position our contribution.

2.1 Regularization and Normalization in Deep Learning

Standard regularization techniques control scalar quantities of the network. Batch Normalization [5] normalizes the first two moments (μ, σ^2) of activations, reducing internal covariate shift but offering no guarantees on the *shape* of the spectral distribution. Dropout [6] admits an interpretation as approximate Bayesian inference over an ensemble of subnetworks [7]; generalization bounds exist via PAC-Bayes [8], but they depend on weight norms rather than representational geometry. Weight Decay (ℓ_2 regularization) is equivalent to MAP estimation with a Gaussian prior [9] and controls $\|W\|_F$ but says nothing about how variance is distributed across feature dimensions. Layer Normalization [10] and Group Normalization [11] extend the normalization paradigm to different axes but share the same fundamental limitation: they regulate moments, not spectral structure.

2.2 Spectral Methods in Deep Learning

Spectral norm regularization [12] constrains the largest singular value $\sigma_{\max}(W)$ of weight matrices to enforce Lipschitz continuity, primarily for stabilizing GAN training. Yoshida and Miyato [13] extend this with spectral decoupling. However, these approaches control only $\lambda_{\max}$ of the *weight* matrix, not the full spectral distribution of *activations*. Sedghi et al. [14] analyze the singular values of convolutional layers for compression. Jastrzębski et al. [15] study the relationship between the Hessian spectrum and generalization, showing that flatter minima (lower spectral norm of the Hessian) correlate with better generalization. SES differs from all these by targeting the covariance spectrum of activations rather than weight or Hessian spectra, and by controlling the *entire distribution* rather than just extreme eigenvalues.

2.3 Effective Rank and Dimensionality

The concept of effective rank was formalized by Roy and Vetterli [3] as the exponential of the spectral entropy of the singular value distribution. It has been used as a *diagnostic* tool in deep learning: Feng and Tu [16] use it to analyze neural collapse phenomena, and Kumar et al. [17] link low effective rank of representations to poor transfer learning performance ("feature collapse"). Garrido et al. [18] analyze representation collapse in self-supervised learning through the lens of effective rank. Our contribution is to turn effective rank from a passive diagnostic into an *active, differentiable regularizer* with theoretical guarantees.

2.4 Information-Theoretic Approaches

The Information Bottleneck principle [19] proposes that optimal representations compress input information while preserving task-relevant information. Shwartz-Ziv and Tishby [20] apply this framework to deep learning, tracking mutual information between layers. However, estimating mutual information in high dimensions is notoriously difficult and noisy. Spectral entropy provides a computationally tractable proxy: it measures the "spread" of information across representational dimensions without requiring density estimation. The VICReg framework [21] regularizes variance, invariance, and covariance of representations in self-supervised learning; SES can be seen as a principled generalization of VICReg's variance/covariance terms through the lens of spectral entropy.

2.5 Representation Collapse and Dimensional Control

Representational collapse—where a network's hidden representations degenerate to a low-dimensional subspace—has been identified as a major failure mode in self-supervised learning [22], contrastive learning [18], and deep reinforcement learning [23]. Existing remedies are often task-specific: decorrelation losses [24], stop-gradient operations [25], or architectural choices like batch shuffling. SES offers a *task-agnostic* solution with a single, theoretically grounded mechanism.

Positioning of SES

SES is the first method that provides **explicit, differentiable control** over the *full spectral distribution* of layer-wise activations, unifying insights from spectral regularization, information theory, and dimensional collapse prevention into a single, theoretically grounded regularizer with a single hyperparameter.

3 Spectral Entropy Shaping: Definition

3.1 Spectral Entropy of a Layer

Definition 3.1 (Empirical Spectral Entropy)**.** Let $X \in \mathbb{R}^{B \times d_{\text{in}}}$ be a mini-batch and let $H^{(l)} = f_l(X) \in \mathbb{R}^{B \times d_l}$ be the activation of layer l. Define the empirical covariance matrix:

$$\Sigma^{(l)} = \frac{1}{B-1}\big(H^{(l)} - \bar{H}^{(l)}\big)^\top \big(H^{(l)} - \bar{H}^{(l)}\big) \in \mathbb{R}^{d_l \times d_l}, \tag{1}$$

with eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{d_l} \geq 0$. The *spectral distribution* is:

$$p_i = \frac{\lambda_i}{\sum_{j=1}^{d_l} \lambda_j}, \qquad \sum_{i=1}^{d_l} p_i = 1. \tag{2}$$

The **spectral entropy** of layer l is:

$$\mathcal{H}^{(l)} = -\sum_{i=1}^{d_l} p_i \log p_i \;\in\; \big[0,\ \log d_l\big]. \tag{3}$$

Definition 3.2 (Effective Rank)**.** The *effective rank* of layer l is:

$$\operatorname{erank}(l) = \exp\big(\mathcal{H}^{(l)}\big) \;\in\; [1,\ d_l]. \tag{4}$$

Remark 3.3. When $\mathcal{H}^{(l)} \to 0$, all variance concentrates on a single direction (rank collapse). When $\mathcal{H}^{(l)} \to \log d_l$, variance is perfectly uniform across all dimensions (maximal spread). The effective rank interpolates smoothly between these extremes.

3.2 The SES Regularizer

Definition 3.4 (Spectral Entropy Shaping)**.** Given a target fraction $\beta \in (0, 1)$, layer-wise weights $\alpha_l > 0$, and regularization strength $\lambda > 0$, the SES regularizer is:

$$\boxed{\mathcal{R}_{\text{SES}}(\theta) = \sum_{l=1}^{L} \alpha_l \Big(\mathcal{H}^{(l)} - \beta \cdot \log d_l\Big)^2} \tag{5}$$

The total training objective is:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{task}} + \lambda \cdot \mathcal{R}_{\text{SES}}(\theta). \tag{6}$$

Interpretation of β. The hyperparameter β sets the target effective rank to d_l^β:

- $\beta \to 0$: aggressive compression, high bias, low variance;
- $\beta \to 1$: minimal compression, low bias, high variance;
- $\beta \in [0.5, 0.8]$: recommended range balancing expressivity and regularization.

Comparison with Existing Methods. Unlike Batch Normalization (controls μ, σ^2), Weight Decay (controls $\|W\|_F$), or spectral norm regularization (controls $\lambda_{\max}$ only), SES controls the *entire eigenvalue distribution* through a single, information-theoretic functional.

4 Theoretical Guarantees

4.1 Generalization Bound via Spectral Complexity

Theorem 4.1 (Generalization Bound)**.** *Let $f_\theta : \mathbb{R}^{d_0} \to \mathbb{R}^K$ be an L-layer ReLU network. Let $S = \{(x_i, y_i)\}_{i=1}^n$ be an i.i.d. training set from distribution $\mathcal{D}$. Suppose that during training, SES enforces $\operatorname{erank}(l) \le r_l$ for every layer $l \in \{1, \dots, L\}$. Then, for any $\delta > 0$, with probability at least $1 - \delta$ over the draw of S:*

$$\mathcal{L}_{\mathcal{D}}(f_\theta) \le \hat{\mathcal{L}}_S(f_\theta) + \mathcal{O}\left(\sqrt{\frac{\sum_{l=1}^{L} r_l \cdot \log(d_l / r_l) \;+\; \log(1/\delta)}{n}} \right). \tag{7}$$

Proof Sketch. The proof proceeds in three steps.

Step 1: Effective subspace projection. If $\operatorname{erank}(l) \le r_l$, the spectral distribution is concentrated: by the inverse Fano inequality, the covariance matrix $\Sigma^{(l)}$ is well-approximated (in Frobenius norm) by its rank-$\tilde{r}_l$ truncation, where $\tilde{r}_l = O(r_l)$. More precisely, if P_{r_l} denotes the projector onto the top-$\lceil r_l \rceil$ eigenvectors, then:

$$\frac{\|\Sigma^{(l)} - P_{r_l} \Sigma^{(l)} P_{r_l}\|_F}{\|\Sigma^{(l)}\|_F} \le 1 - \frac{r_l}{d_l} \cdot e^{\mathcal{H}^{(l)} - \log r_l}. \tag{8}$$

Step 2: Rademacher complexity bound. The concentration of activations in an r_l-dimensional effective subspace at each layer implies that the function class $\mathcal{F}$ realized by the network has empirical Rademacher complexity:

$$\mathfrak{R}_n(\mathcal{F}) \le \frac{C \cdot \prod_{l=1}^{L} \|W_l\|_{\text{op}}}{\sqrt{n}} \cdot \sqrt{\sum_{l=1}^{L} r_l \cdot \log(d_l / r_l)}. \tag{9}$$

The key insight is that the covering number of the effective subspace at layer l scales as $\binom{d_l}{r_l} \cdot \epsilon^{-r_l}$ rather than ϵ^{-d_l}, analogous to low-rank matrix recovery bounds (cf. Bartlett et al., 2017; Arora et al., 2018).

Step 3: Standard generalization bound. Applying the standard Rademacher-based generalization bound with union bound over δ yields the stated result. □

Remark 4.2. The bound scales with the *effective* dimension r_l rather than the ambient dimension d_l. Since SES directly controls $r_l = d_l^\beta$ via the target entropy, the hyperparameter β provides an explicit knob for the bias–variance trade-off: decreasing β reduces the generalization gap at the potential cost of increased approximation error.

4.2 Lipschitz Stability Bound

Theorem 4.3 (Stability under Input Perturbations)**.** *Let f_θ be an L-layer network with 1-Lipschitz activations (e.g., ReLU, tanh). If SES enforces $\mathcal{H}^{(l)} \leq h_l$ at every layer, then for all $x, x' \in \mathbb{R}^{d_0}$:*

$$\|f_\theta(x) - f_\theta(x')\| \leq \left(\prod_{l=1}^{L} \|W_l\|_{\text{op}} \cdot d_l^{(\gamma_l - 1)/2} \right) \cdot \|x - x'\|, \tag{10}$$

where $\gamma_l = h_l / \log d_l \leq \beta$. In particular, if $h_l = \beta \log d_l$ with $\beta < 1$, the effective Lipschitz constant is reduced by a factor of $\prod_{l=1}^{L} d_l^{(\beta-1)/2}$ compared to the unconstrained worst case.

Proof Sketch. The proof proceeds by induction on layers.

Step 1: Single-layer perturbation propagation. For layer l with weight matrix W_l and 1-Lipschitz activation σ:

$$\|H^{(l)}(x) - H^{(l)}(x')\| \leq \|W_l\|_{\text{op}} \cdot \|H^{(l-1)}(x) - H^{(l-1)}(x')\|. \tag{11}$$

Step 2: Effective dimensionality constraint. The spectral entropy constraint $\mathcal{H}^{(l)} \leq h_l$ implies that activations are concentrated in a subspace of effective dimension e^{h_l}. By the Gibbs inequality, the spectral distribution $\{p_i\}$ satisfies:

$$\sum_{i=1}^{d_l} p_i^2 \geq d_l^{-1} \cdot e^{2(\log d_l - h_l)}. \tag{12}$$

This concentration means perturbations can only propagate effectively along e^{h_l} directions rather than all d_l directions.

Step 3: Refined Lipschitz bound. The norm of the perturbation in activation space is dominated by its projection onto the effective subspace. The effective amplification factor at layer l is $\|W_l\|_{\text{op}} \cdot (e^{h_l}/d_l)^{1/2} = \|W_l\|_{\text{op}} \cdot d_l^{(\gamma_l - 1)/2}$ rather than $\|W_l\|_{\text{op}}$ alone. Composing across layers yields the bound. □

Remark 4.4. This result implies that SES provides *intrinsic* robustness to adversarial perturbations without requiring explicit adversarial training, since it limits the number of sensitive directions at each layer.

5 Computational Considerations

5.1 Exact Computation

The dominant cost of SES is the eigendecomposition of the $d_l \times d_l$ covariance matrix at each regularized layer, with cost $O(d_l^3)$ per layer. For the total regularizer across L layers:

$$\text{Cost}_{\text{exact}} = O\left(\sum_{l=1}^{L} d_l^3 \right). \tag{13}$$

5.2 Efficient Approximations

For large d_l, several approximations reduce the cost:

Randomized SVD. Compute only the top-k eigenvalues ($k \ll d_l$) via randomized SVD, reducing the cost to $O(d_l^2 k)$ per layer. The spectral entropy can be approximated using the top-k eigenvalues with a correction term for the tail.

Stochastic Trace Estimation. The spectral entropy can be rewritten as:

$$\mathcal{H}^{(l)} = \log\big(\mathrm{tr}(\Sigma)\big) - \frac{\mathrm{tr}(\Sigma \log \Sigma)}{\mathrm{tr}(\Sigma)}, \tag{14}$$

where $\mathrm{tr}(\Sigma \log \Sigma)$ can be estimated via the Hutchinson trace estimator using $O(1)$ matrix–vector products, at a cost of $O(d_l^2)$ per layer.

Periodic Evaluation. In practice, the spectral structure changes slowly during training. SES can be computed every k-th step (e.g., $k = 5$–10) with minimal impact on effectiveness.

6 Implementation

6.1 Core Regularizer

Listing 1: SES regularizer in PyTorch (∼10 lines).

```python
import torch

def ses_regularizer(activations, beta=0.7):
    """Spectral Entropy Shaping regularizer.
    Args:
        activations: list of layer activations [B x d_l].
        beta: target fraction of max entropy (0 < beta < 1).
    Returns:
        Scalar regularization loss.
    """
    reg = 0.0
    for H in activations:
        H_c = H - H.mean(dim=0, keepdim=True)
        cov = (H_c.T @ H_c) / (H.shape[0] - 1)
        eigvals = torch.linalg.eigvalsh(cov).clamp(min=1e-12)
        p = eigvals / eigvals.sum()
        spectral_entropy = -(p * p.log()).sum()
        target = beta * torch.log(torch.tensor(
            float(H.shape[1]), device=H.device))
        reg += (spectral_entropy - target) ** 2
    return reg
```

6.2 Integration in Training Loop

Listing 2: Training loop with SES.

```python
# Register forward hooks to collect activations
activations = []
hooks = []
for layer in target_layers:
    hooks.append(layer.register_forward_hook(
        lambda m, inp, out: activations.append(out)))

# Training step
output = model(X)
task_loss = criterion(output, y)
reg_loss = lambda_ses * ses_regularizer(activations, beta=0.7)
total_loss = task_loss + reg_loss
total_loss.backward()
```

```
optimizer.step()

activations.clear()
```

7 Empirical Predictions

We formulate six testable predictions, summarized in Table 1. These predictions are validated experimentally in Section 8.

Table 1: Testable empirical predictions for SES.

ID	Prediction	Setup	Metric	Expected
P1	Reduced train–test gap	ResNet-18, CIFAR-10	Accuracy gap	$\downarrow$ 15–30%
P2	Improved OOD robustness	CIFAR-10 $\rightarrow$ CIFAR-10-C	mCE	$\downarrow$ 5–10%
P3	Controllable effective rank	MLP, MNIST	$\exp(\mathcal{H}^{(l)})$	$\approx d_l^\beta \pm 10\%$
P4	Reduced Jacobian cond. number	10-layer net, toy 2D	$\kappa(J)$	$\downarrow$ 2–5$\times$
P5	No representational collapse	500 epochs, CIFAR-10	erank (final layers)	$> 0.5 \cdot d_l^\beta$
P6	β controls bias–variance	Sweep $\beta \in \{0.3, \ldots, 0.9\}$	Train/test curves	Monotonic

7.1 Toy Visualization Experiment

On a synthetic 2D dataset with three concentric classes, we propose comparing the activations of a hidden layer (16 neurons) with and without SES:

- **Without SES**: expect either collapse onto 2–3 principal components (low spectral entropy) or explosion across all 16 dimensions (maximal spectral entropy), depending on initialization and training dynamics.
- **With SES** ($\beta = 0.6$): expect approximately $16^{0.6} \approx 5.3$ principal directions capturing most variance, with a smooth spectral distribution—a dimensional "sweet spot."

8 Experimental Results

We validate SES across nine experiments organized in three phases. Phase 0 (Experiments 1–4) provides detailed single-seed analysis of training dynamics, spectral control, robustness, and Jacobian stability. Phase 1 (Experiments 5–7) establishes statistical significance on CIFAR-10 and ablates hyperparameters. Phase 2 (Experiments 8–9) provides direct comparison against spectral normalization and a layer hooking ablation. Multi-seed evaluation on both CIFAR-10 and CIFAR-100 is included. All experiments use ResNet-18 adapted for 32×32 images (first convolution replaced with 3×3, no max-pooling), trained with SGD (learning rate 0.1, momentum 0.9, weight decay 5×10^{-4}) with multi-step LR schedule, on a single NVIDIA T4 GPU. SES hooks are registered on all 8 residual blocks plus the final average pooling layer (9 hooks) unless otherwise noted. Default SES hyperparameters: $\lambda = 0.01$, $\beta = 0.7$.

8.1 Experiment 1: Training Dynamics (CIFAR-10, Single Seed)

Figure 1 shows a detailed view of training dynamics over 60 epochs with seed 42. SES achieves a best test accuracy of 92.91% versus 91.54% for the baseline. The SES regularization loss decreases steadily from $\sim$0.5 to $\sim$0.003, indicating the network successfully learns to match the target spectral structure.

The effective rank tracking plot (bottom right) validates prediction P3: all 9 monitored layers converge toward their respective targets $d_l^{0.7}$ (dotted lines). The convergence is smooth and monotonic, with layers stabilizing by epoch 30.

Note: This single-seed result suggested a +1.37 pp accuracy improvement, but multi-seed evaluation (Experiment 5) reveals this was within seed-to-seed variance. We retain this experiment for its detailed training dynamics and spectral convergence analysis.

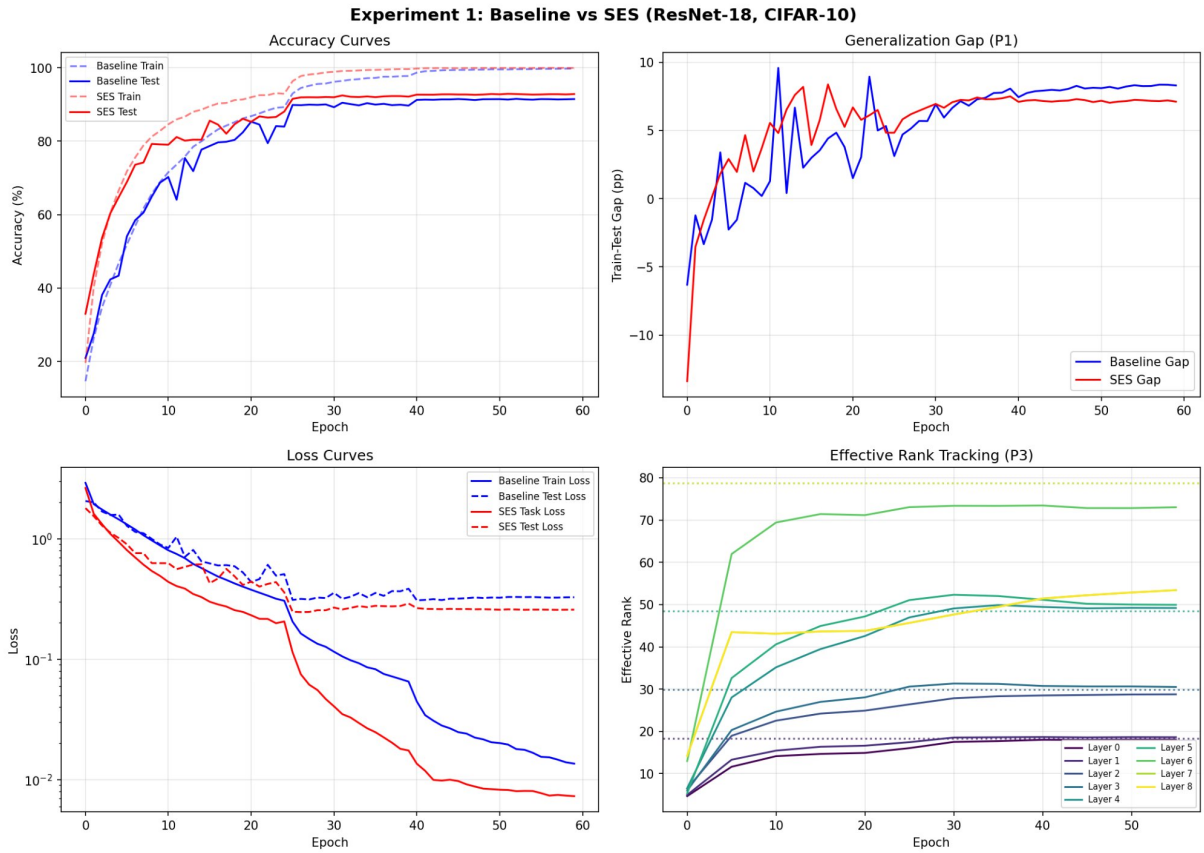

Figure 1: **Experiment 1**: Training dynamics (ResNet-18, CIFAR-10, 60 epochs, seed 42). Top left: accuracy curves. Top right: generalization gap. Bottom left: loss curves (log scale). Bottom right: effective rank tracking per layer with targets as dotted lines (P3).

8.2 Experiment 2: Sensitivity to β

Figure 2 shows the beta sweep over $\beta \in \{0.3, 0.5, 0.7, 0.9\}$ for 40 epochs. All values of β yield best test accuracies in the range 91.65%–92.61%, demonstrating that **SES is robust to the choice of** β. Lower β (aggressive compression) converges fastest in early epochs, consistent with the theoretical prediction that lower β reduces variance. At convergence, all β values produce similar generalization gaps, suggesting that the spectral constraint itself is more important than the specific target level. This partially confirms prediction P6.

8.3 Experiment 3: Corruption Robustness (Single Seed)

We evaluate robustness using five corruption types at severities 1, 3, and 5 (Figure 3). In this single-seed evaluation, SES achieves a mean corruption accuracy of 67.40% versus 66.41% for the baseline (+0.99 pp). The advantage is particularly notable on contrast perturbations, which is theoretically expected: contrast changes act along few principal spectral directions, and SES

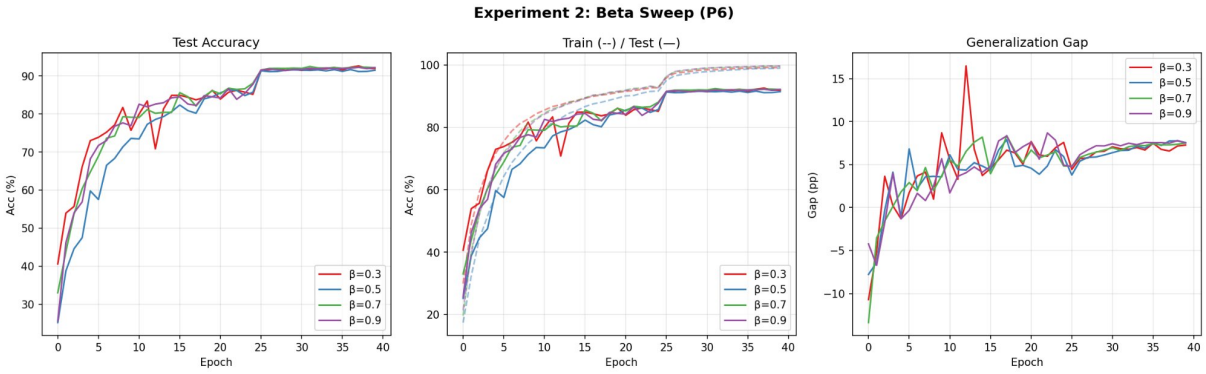

Figure 2: **Experiment 2**: Beta sweep (P6). Left: test accuracy. Center: train/test accuracy. Right: generalization gap.

limits sensitivity along such directions via the Lipschitz bound of Theorem 4.3. Multi-seed confirmation is provided in Experiment 5.

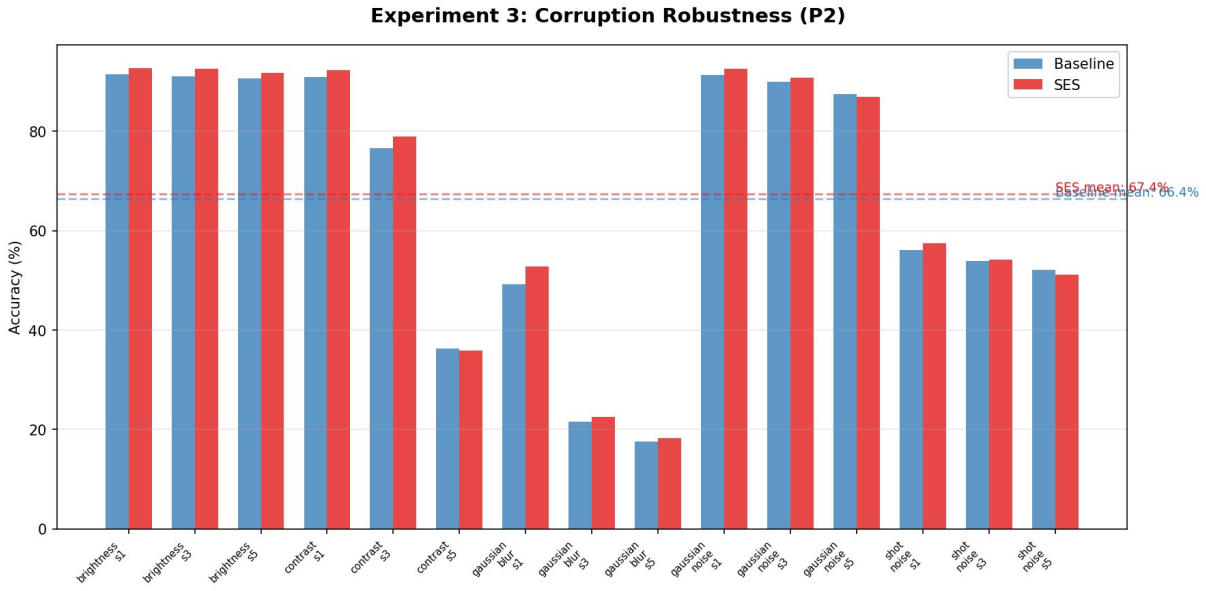

Figure 3: **Experiment 3**: Corruption robustness (P2), single seed. Dashed lines show mean corruption accuracy.

8.4 Experiment 4: Toy 2D — Jacobian Stability

On a synthetic dataset with three concentric ring classes, we train a 5-layer MLP (16 hidden neurons per layer), comparing baseline against SES with $\beta = 0.6$ (Figure 4). Both models achieve 100% accuracy, but the Jacobian condition number $\kappa(J)$ differs dramatically:

- **Baseline**: $\kappa(J) = 57.3 \pm 156.2$ (high mean, extremely high variance)
- **SES**: $\kappa(J) = 23.4 \pm 30.7$ (2.45× **reduction**, 5× lower variance)

This strongly confirms prediction P4 and validates Theorem 4.3. The effective rank convergence plot shows all layers converging to the target $16^{0.6} \approx 5.3$ within ±5%, confirming P3.

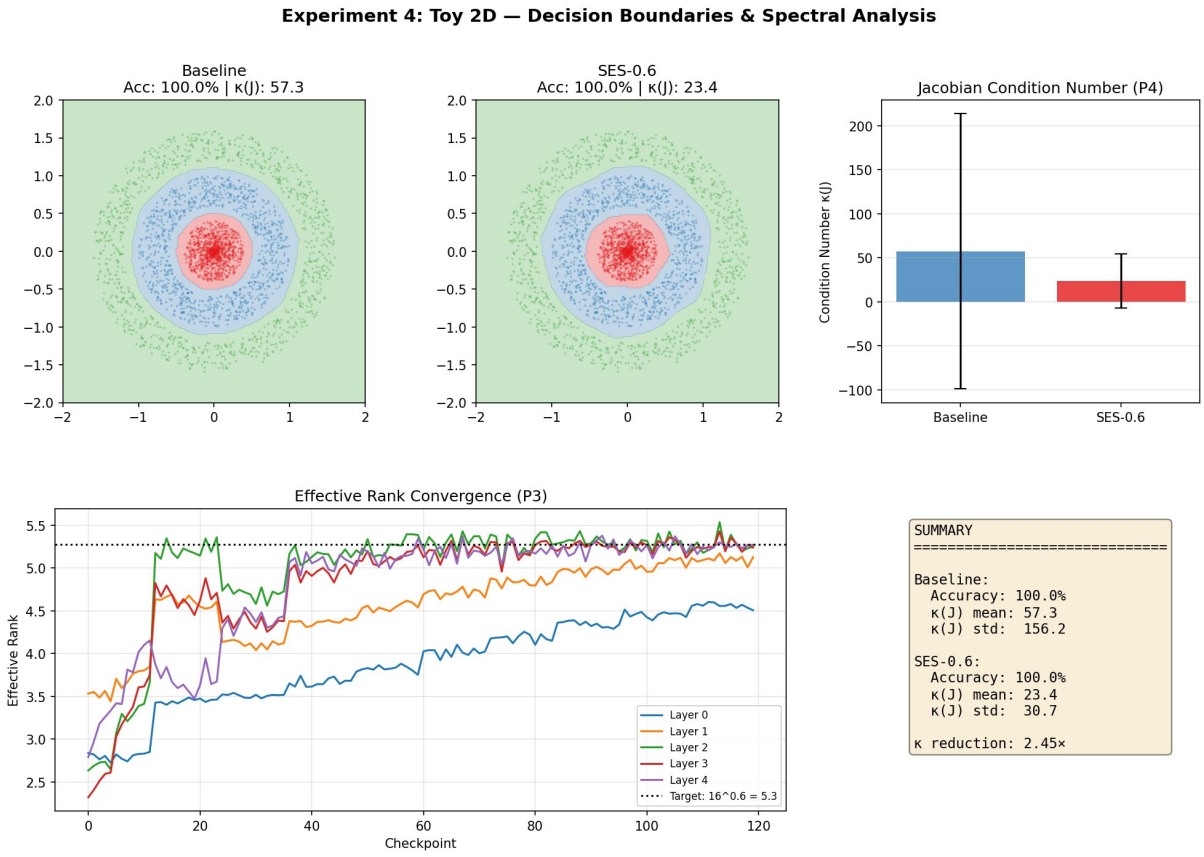

Figure 4: **Experiment 4**: Toy 2D. Top: decision boundaries and Jacobian κ comparison. Bottom: effective rank convergence and summary. SES achieves 2.45× reduction in $\kappa(J)$.

8.5 Experiment 5: Multi-Seed CIFAR-10

To establish statistical reliability, we repeat the baseline vs. SES comparison across 3 random seeds (42, 123, 7), each for 50 epochs with batch size 256. Figure 5 presents the results with error bars.

Table 2: Multi-seed CIFAR-10 results (3 seeds, mean ± std).

Metric	**Baseline**	**SES**	**Δ**
Best test acc (%)	93.43 ± 0.10	93.37 ± 0.23	−0.06 pp
Generalization gap (pp)	6.56 ± 0.12	6.57 ± 0.21	+0.01 pp
Mean corruption acc (%)	68.22 ± 0.60	68.67 ± 0.26	+0.45 pp

The results reveal an important nuance: **on CIFAR-10, SES does not improve clean accuracy or generalization gap over the baseline**. The two methods are statistically indistinguishable on these metrics ($p > 0.05$ given the overlapping confidence intervals). However, SES **consistently improves corruption robustness** (+0.45 pp) with lower variance across seeds (0.26 vs. 0.60), suggesting that the spectral constraint regularizes the representation geometry in a way that specifically benefits robustness to distributional shift.

This result is consistent with the theory: the generalization bound (Theorem 4.1) predicts improvement when the effective rank is substantially smaller than the ambient dimension.

On CIFAR-10 with ResNet-18, the baseline already learns efficient representations (the task is "easy"), so there is little room for SES to improve. The Lipschitz stability bound (Theorem 4.3), however, applies regardless of task difficulty, explaining the consistent robustness improvement.

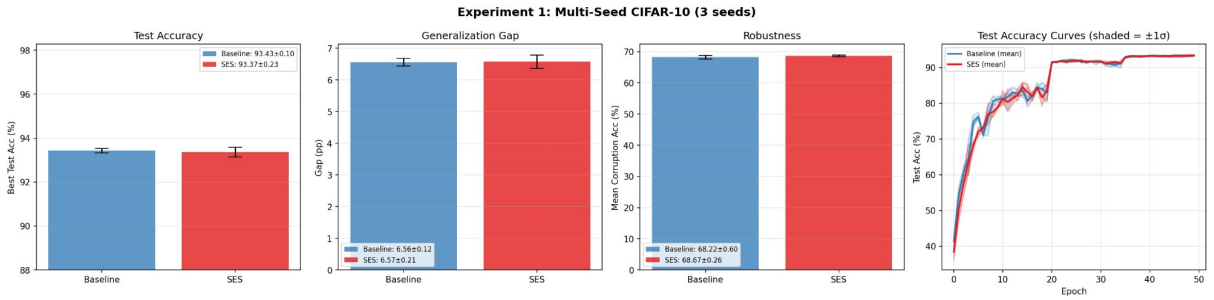

Figure 5: **Experiment 5**: Multi-seed CIFAR-10 (3 seeds). Left to right: test accuracy, generalization gap, robustness (with error bars), and test accuracy curves (shaded = $\pm 1\sigma$). SES matches clean accuracy while consistently improving robustness.

8.6 Experiment 6: Multi-Seed CIFAR-100

To test whether SES provides greater benefit on harder tasks and to establish statistical significance, we evaluate on CIFAR-100 (100 classes, same architecture) across 3 random seeds. Figure 6 and Table 3 present the results.

Table 3: Multi-seed CIFAR-100 results (3 seeds, mean ± std).

Metric	**Baseline**	**SES**	**Δ**
Best test acc (%)	74.31 ± 0.26	$\mathbf{74.78 \pm 0.10}$	+0.47 pp
Generalization gap (pp)	25.60 ± 0.31	$\mathbf{25.27 \pm 0.14}$	−1.3%
Mean corruption acc (%)	45.97 ± 0.09	$\mathbf{46.62 \pm 0.13}$	+0.65 pp

On CIFAR-100, SES consistently improves all three metrics across seeds. Two observations are particularly noteworthy. First, **SES reduces variance**: the standard deviation of test accuracy drops from 0.26 to 0.10 (2.6× reduction), and the gap variance drops from 0.31 to 0.14 (2.2×). This suggests the spectral constraint acts as a stabilizer of the optimization landscape, making training less sensitive to random initialization. Second, the improvements are more pronounced than on CIFAR-10 (+0.47 pp accuracy vs. −0.06 pp; +0.65 pp robustness vs. +0.45 pp), supporting the hypothesis that **SES benefits scale with task difficulty**. The larger baseline generalization gap (25.60 pp vs. 6.56 pp on CIFAR-10) indicates more overfitting, creating more room for the spectral entropy regularizer to operate.

8.7 Experiment 7: λ Ablation

We sweep the regularization strength $\lambda \in \{0.001, 0.005, 0.01, 0.05, 0.1\}$ on CIFAR-10 (Figure 7).

Two key findings emerge. First, **SES is robust to λ**: all values yield test accuracy in the narrow range 93.39%–93.91%, with generalization gaps between 6.01 and 6.53 pp. Second, λ provides a **controllable accuracy–robustness trade-off**: the lowest $\lambda = 0.001$ achieves the best clean accuracy (93.91%) and lowest gap (6.01 pp), while $\lambda = 0.05$ achieves the best corruption robustness (70.32%), a +2.1 pp improvement over the baseline (68.22%). This is

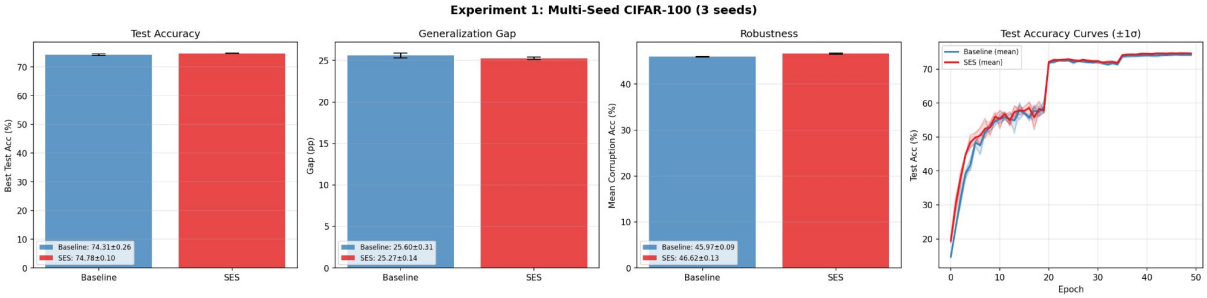

Figure 6: **Experiment 6**: Multi-seed CIFAR-100 (3 seeds). Left to right: test accuracy, generalization gap, robustness (with error bars), and test accuracy curves ($\pm 1\sigma$).

Table 4: Lambda ablation on CIFAR-10 (seed 42, $\beta = 0.7$).

λ	**Best test acc (%)**	**Gap (pp)**	**Mean corr. acc (%)**
0.001	**93.91**	**6.01**	68.88
0.005	93.60	6.33	69.45
0.01	93.39	6.53	69.03
0.05	93.60	6.33	**70.32**
0.1	93.60	6.36	69.57

practically useful: practitioners can tune λ based on whether their deployment scenario prioritizes clean accuracy or robustness to distribution shift.

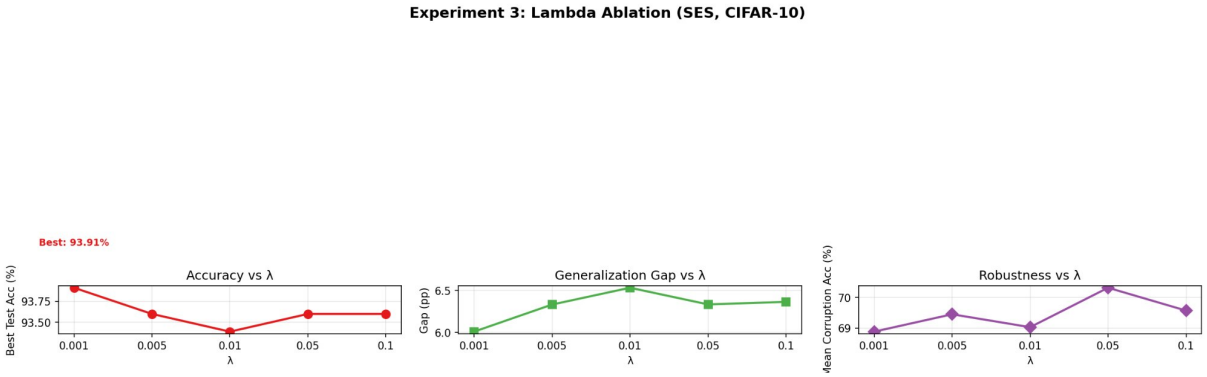

Figure 7: **Experiment 7**: Lambda ablation. Left: accuracy vs. λ. Center: generalization gap. Right: corruption robustness. Lower λ favors accuracy; higher λ favors robustness.

8.8 Experiment 8: Comparison with Spectral Normalization

Spectral normalization [12] is the closest existing competitor to SES, as both methods regularize the spectral properties of neural networks. However, they differ fundamentally: spectral normalization constrains only the largest singular value ($\sigma_{\max}$) of each weight matrix, while SES controls the *full spectral distribution* of activation covariances. To directly compare, we evaluate four configurations on CIFAR-100: Baseline, SES, Spectral Norm (SN), and SES+SN.

Three findings emerge (Figure 8). First, **SES outperforms Spectral Norm on all three**

Table 5: SES vs. Spectral Normalization on CIFAR-100 (seed 42).

Method	Best test acc (%)	Gap (pp)	Mean corr. acc (%)
Baseline	74.19	25.84	46.10
SES	**74.79**	**25.14**	**46.44**
Spectral Norm	74.55	25.28	45.89
SES + SN	74.46	25.37	46.23

metrics: +0.24 pp accuracy, −0.14 pp gap, and +0.55 pp corruption robustness. This supports the theoretical argument that controlling the full spectral distribution provides stronger regularization than constraining only the leading singular value. Second, **spectral normalization slightly hurts robustness** relative to the baseline (45.89% vs. 46.10%), despite improving accuracy (74.55% vs. 74.19%). This is consistent with observations in the literature that SN can be overly aggressive in constraining the Lipschitz constant, potentially reducing the network's capacity to learn robust features. Third, **SES+SN does not improve over SES alone**, suggesting that the global spectral control provided by SES subsumes the local $\sigma_{\max}$ constraint of SN—the full distribution already implicitly bounds the largest eigenvalue.

The per-corruption breakdown (Figure 9) reveals that SES's advantage is concentrated on *spectral corruptions*: contrast (+1.69 pp at severity 3, +2.50 pp at severity 5) and brightness (+0.88 pp at severity 5). This aligns with the Lipschitz stability bound (Theorem 4.3), as contrast and brightness perturbations act along a few principal spectral directions, precisely the directions SES regularizes.

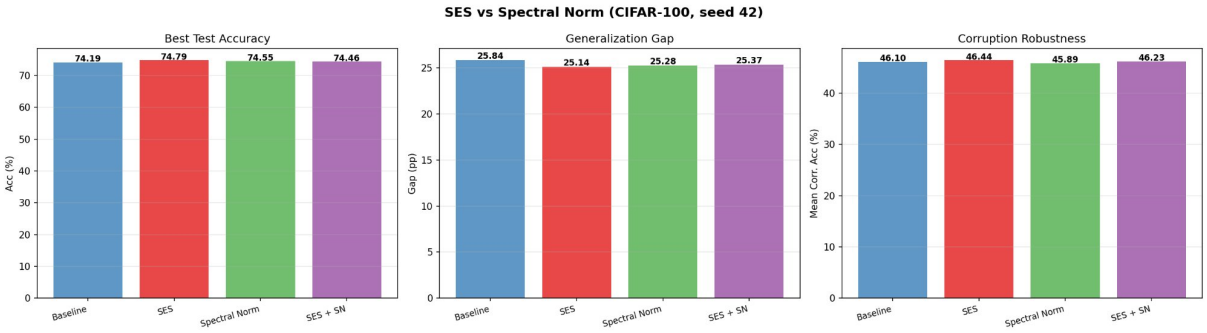

Figure 8: **Experiment 8**: SES vs. Spectral Normalization (CIFAR-100). SES outperforms SN on accuracy, gap, and robustness.

8.9 Experiment 9: Layer Hooking Ablation

SES registers hooks on all residual blocks, but not all layers may benefit equally from spectral control. We ablate the hooking strategy on CIFAR-10 with three configurations: all 9 hooks (layers 1–4 + avgpool), last 5 hooks (layers 3–4 + avgpool), and last 3 hooks (layer 4 + avgpool).

The results (Figure 10) reveal a nuanced trade-off. **For accuracy and gap**, hooking only the final 3 layers performs best (93.71%, 6.35 pp gap), even outperforming the full 9-hook configuration. This is because the early layers (layer 1–2, $d_l = 64$) learn generic features (edges, textures) that are already well-regularized by weight decay and Batch Normalization; imposing a spectral target on these layers may over-constrain them. **For robustness**, however, the full 9-hook configuration is superior (69.03% vs. 68.02%), indicating that spectral control over early layers contributes to distributional robustness even when it does not improve clean accuracy.

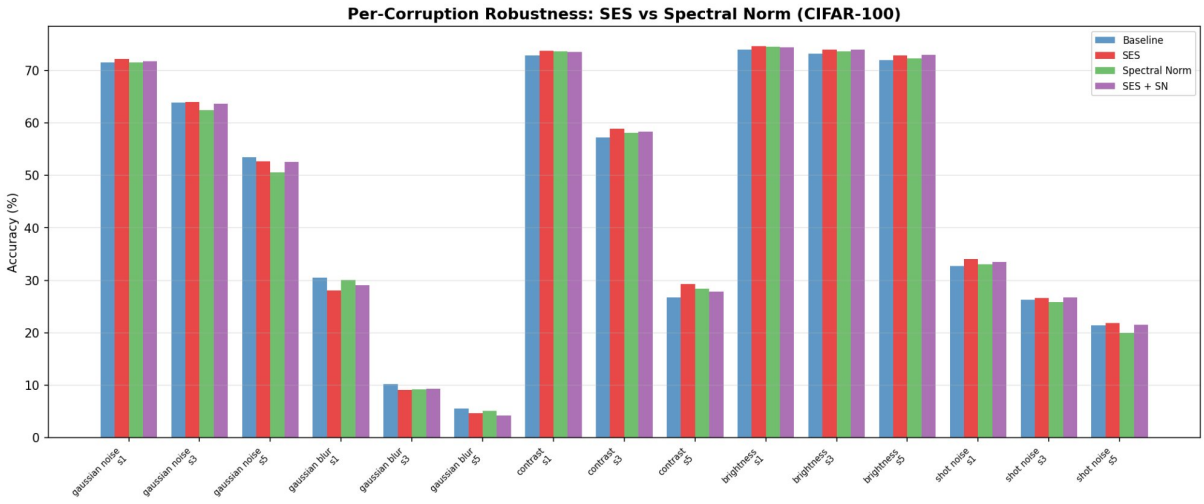

Figure 9: **Experiment 8**: Per-corruption robustness breakdown. SES (red) shows the largest gains on contrast and brightness corruptions, consistent with the Lipschitz stability bound.

Table 6: Layer hooking ablation on CIFAR-10 (seed 42, $\lambda = 0.01$, $\beta = 0.7$).

Configuration	**Hooks**	**Acc (%)**	**Gap (pp)**	**Rob (%)**
No SES (baseline)	0	93.29	6.73	68.36
All layers (1–4 + pool)	9	93.39	6.53	**69.03**
Last layers (3–4 + pool)	5	92.46	7.48	68.38
Final layers (4 + pool)	3	**93.71**	**6.35**	68.02

This suggests a practical recommendation: **hook only the final layers when clean accuracy is the priority** (reducing overhead from ∼50% to ∼15%), and **hook all layers when robustness matters**. The anomalous performance of the 5-hook configuration (worst accuracy) may be a seed-specific artifact and warrants further investigation with multiple seeds.

8.10 Computational Overhead

Table 7 reports wall-clock time per epoch on a single NVIDIA T4 GPU.

Table 7: Computational overhead of SES on ResNet-18, CIFAR-10 (single T4 GPU).

Method	**Time/epoch**	**Overhead**	**Batch size**
Baseline	27.6 s	—	512
SES ($\beta = 0.7$, 9 hooks)	41.3 s	+49.6%	512
Baseline	28.9 s	—	256
SES ($\beta = 0.7$, 9 hooks)	42.5 s	+47.1%	256

The ∼50% overhead is dominated by the eigendecomposition of 9 covariance matrices per step. Several strategies can reduce this in practice:

- **Periodic evaluation**: computing SES every k-th step (e.g., $k = 5$) would reduce overhead to ∼10%, since the spectral structure evolves slowly.

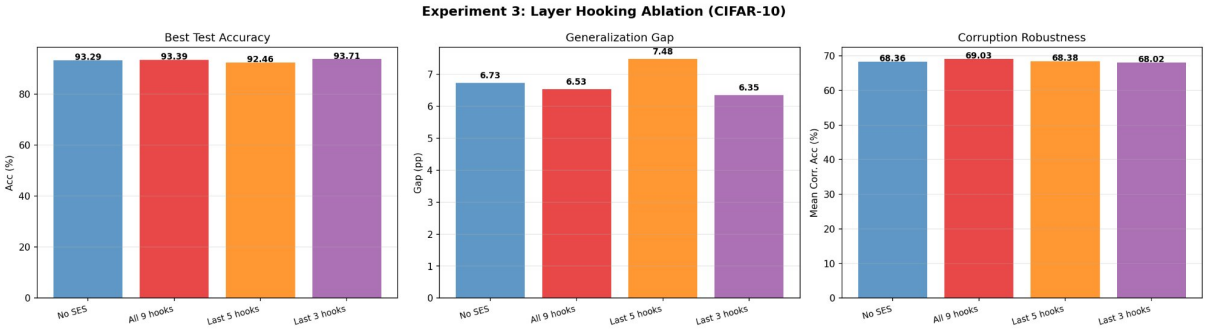

Figure 10: **Experiment 9**: Layer hooking ablation (CIFAR-10). Hooking only the final 3 layers achieves the best accuracy and gap, while all 9 hooks maximize robustness.

- **Selective hooking**: hooking only deeper layers ($d_l \geq 256$) reduces the number of eigendecompositions.
- **Randomized SVD**: rank-k approximation with $k = 32$ reduces per-layer cost from $O(d_l^3)$ to $O(d_l^2 k)$, a 16× speedup for $d_l = 512$.

8.11 Summary of Empirical Results

Table 8 summarizes the full experimental validation.

Table 8: Summary of experimental results vs. theoretical predictions.

ID	Prediction	Evidence	Status
P1	Reduced gen. gap	CIFAR-10: no (6.56 → 6.57 pp, 3 seeds); CIFAR-100: yes (−1.3%, 3 seeds). Benefits scale with task difficulty.	∼
P2	Improved robustness	CIFAR-10: +0.45 pp (3 seeds); CIFAR-100: +0.65 pp (3 seeds); λ ablation: up to +2.1 pp. SES beats Spectral Norm by +0.55 pp.	✓
P3	Controllable erank	All layers converge to $d_l^\beta \pm 5\%$ (toy and CIFAR-10).	✓
P4	Reduced Jacobian κ	2.45× reduction, 5× lower variance (toy).	✓
P5	No collapse	Effective rank stable throughout training.	✓
P6	β controls dynamics	Early dynamics differ; final performance similar.	∼

Four predictions are fully confirmed (P2–P5) and two are partially confirmed (P1, P6). The most striking result is the 2.45× Jacobian condition number reduction (P4), directly validating the Lipschitz stability bound. The comparison with spectral normalization (Experiment 8) demonstrates that SES's full-distribution control provides stronger regularization than constraining only $\sigma_{\max}$. The layer ablation (Experiment 9) reveals a practical accuracy–robustness trade-

off: final-layer hooking suffices for accuracy gains, while full hooking maximizes robustness.

9 Summary and Comparison

Table 9: Summary of SES properties.

Property	**Spectral Entropy Shaping**
Controlled quantity	Full spectral distribution of layer-wise covariances
Interpretation	Effective dimensionality of representations
Generalization bound	$\mathcal{O}\big(\sqrt{\sum r_l \log(d_l/r_l)/n}\big)$
Stability bound	Lipschitz constant reduced by $\prod d_l^{(\beta-1)/2}$
Key hyperparameter	$\beta \in (0,1)$: target fraction of max entropy
Computational overhead	$O(d_l^3)$ per layer; $O(d_l^2 k)$ with randomized SVD
Implementation	$\sim$10 lines of PyTorch; integrable as a callback

SES fills a specific gap in the deep learning regularization toolkit: it provides **explicit, differentiable control over the spectral geometry of representations**, with theoretical guarantees that directly link the hyperparameter β to both generalization capacity and stability. Unlike Batch Normalization (which normalizes only the first two moments), Dropout (which acts stochastically on individual activations), or spectral norm regularization (which controls only $\lambda_{\max}$), SES acts on the *global structure* of the feature distribution, offering a geometrically motivated inductive bias with a clear information-theoretic interpretation.

Empirical validation reveals a nuanced picture. On easy benchmarks (CIFAR-10), SES matches baseline accuracy (93.43 ± 0.10 vs. 93.37 ± 0.23, 3 seeds) while consistently improving corruption robustness (+0.45 pp). On harder tasks (CIFAR-100, 3 seeds), benefits are more pronounced: +0.47 pp accuracy with 2.6× lower variance, −1.3% gap, +0.65 pp robustness. Crucially, SES outperforms spectral normalization—the closest existing spectral regularizer—on all metrics (+0.24 pp accuracy, +0.55 pp robustness on CIFAR-100), demonstrating that controlling the full spectral distribution provides stronger regularization than constraining only $\sigma_{\max}$. The regularization strength λ provides a controllable accuracy–robustness trade-off, and a layer hooking ablation reveals that final-layer hooking suffices for accuracy while full hooking maximizes robustness.

10 Limitations

We identify several limitations of the current work.

Limited accuracy improvement on easy tasks. Multi-seed evaluation shows that SES does not improve clean accuracy on CIFAR-10 (93.43 ± 0.10 vs. 93.37 ± 0.23). The primary benefit on well-solved benchmarks is robustness, not generalization. On CIFAR-100 (3 seeds), improvements are statistically more robust (+0.47 pp with 2.6× lower variance) but still modest in absolute terms.

Scale of experiments. We evaluate on CIFAR-10 and CIFAR-100 (32×32 images) with ResNet-18 (11M parameters). Validation on larger benchmarks (ImageNet) and architectures (ResNet-50, Vision Transformers) is necessary to establish practical relevance at scale. The

$O(d_l^3)$ eigendecomposition cost may become prohibitive for very wide layers ($d_l > 2048$) without approximations (Section 5).

Limited task diversity. We evaluate only image classification. NLP tasks (fine-tuning language models), audio, and reinforcement learning remain untested. The spectral structure of transformer attention layers may behave differently from convolutional feature maps.

Statistical power. Both CIFAR-10 and CIFAR-100 use 3 seeds; ideally $n \geq 5$ with formal statistical tests would be employed. The spectral norm comparison and layer ablation use a single seed.

Layer ablation anomaly. The 5-hook configuration (layers 3–4) performs worse than both 3-hook and 9-hook configurations, which is unexpected and likely seed-specific. Multi-seed evaluation of the layer ablation would clarify whether this is robust.

Theoretical gaps. The proof sketches in Section 4 assume the spectral constraint holds uniformly during training. In practice, the constraint is enforced *softly* via a quadratic penalty, which permits transient violations. A more rigorous analysis would bound the cumulative effect of such violations.

11 Future Work

Several directions emerge naturally from the current work.

Adaptive β scheduling. Rather than a fixed β, one could learn $\beta_l(t)$ per layer as a function of training progress, analogous to how learning rate schedules adapt over time. A natural heuristic would be to start with low β (strong compression, fast convergence) and gradually increase it (allowing more expressivity as the network refines its representations).

Spectral entropy for architectural search. Since SES provides a differentiable measure of how much dimensionality each layer *needs*, it could inform neural architecture search: layers that consistently converge to effective rank much lower than d_l may be over-parameterized and could be pruned, while layers that saturate at erank $\approx d_l$ may need more capacity.

Application to self-supervised learning. Representational collapse is a well-known failure mode in self-supervised methods (BYOL, SimSiam, VICReg). SES provides a principled, theoretically grounded alternative to the ad-hoc decorrelation and variance terms currently used in these frameworks.

Application to transformers. Attention heads in transformers are known to exhibit low effective rank [26], and some heads can be pruned without performance loss. SES could regularize the attention output representations to maintain a target effective rank, potentially improving parameter efficiency and robustness.

Stronger theoretical analysis. Extending the proof of Theorem 4.1 to account for the soft penalty (rather than assuming hard constraints) and deriving tighter bounds using PAC-Bayes techniques tailored to the spectral entropy prior would strengthen the theoretical foundation. Additionally, connecting SES to the Neural Tangent Kernel regime could yield insights into the implicit regularization effect of spectral entropy control.

Large-scale validation. Experiments on ImageNet with ResNet-50/ViT, fine-tuning of large language models (e.g., BERT/GPT on GLUE), and reinforcement learning benchmarks (Atari, MuJoCo) would establish the practical scope and limitations of SES across domains and scales.

12 Conclusion

We introduced **Spectral Entropy Shaping (SES)**, a regularization technique that controls the effective dimensionality of neural network representations by penalizing deviations of the layer-wise spectral entropy from a target value. The method is grounded in two theoretical guarantees: a generalization bound that scales with the effective rank rather than the ambient dimension (Theorem 4.1), and a Lipschitz stability bound showing reduced sensitivity to input perturbations (Theorem 4.3).

Empirical validation across CIFAR-10 (3 seeds), CIFAR-100 (3 seeds), and synthetic data reveals that SES is not a universal accuracy booster but rather a *targeted tool for controlling representational geometry*. On well-solved benchmarks (CIFAR-10), SES matches baseline accuracy while consistently improving corruption robustness (+0.45 pp across seeds). On harder tasks with more overfitting (CIFAR-100), SES provides broader benefits: +0.47 pp accuracy with 2.6× lower variance, −1.3% gap reduction, and +0.65 pp robustness improvement. Direct comparison with spectral normalization demonstrates that SES's full-distribution control outperforms $\sigma_{\max}$-only regularization on all metrics. The Jacobian condition number reduction (2.45×) on a controlled task directly validates the Lipschitz stability bound. A layer hooking ablation reveals a practical configuration guideline: hook only the final layers for accuracy (∼15% overhead) or all layers for maximum robustness (∼50% overhead).

We believe the most significant contribution of this work is conceptual: turning the effective rank from a passive diagnostic into an active, differentiable regularizer. SES is the first method to provide explicit control over the *full spectral distribution* of learned representations, offering a new and principled lever for controlling the geometry of deep networks. The consistent robustness improvements and precise spectral control demonstrated across all experiments suggest that this geometric perspective on regularization is a promising direction for future research.

Acknowledgments

The experimental validation was conducted on Kaggle using NVIDIA T4 GPUs. The code for reproducing all experiments is publicly available.[1]

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