

Ch7 Question 8.

Solution 1. for part (a):

Since there is only interaction between A and B, the model is

$$Y_{ijkt} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + \epsilon_{ijkt}, \text{ where } \epsilon_{ijkt} \sim N(0, \sigma^2), i.i.d.$$

for part (b):

the following contrasts may be of interest:

1. effect of contrast of factor A: $\bar{\tau}_{1..} - \bar{\tau}_{2..}$.
2. effect of contrast of factor B: $\bar{\tau}_{.1.} - \bar{\tau}_{.2.}$.
3. effect of contrast of factor C: $\bar{\tau}_{..1} - \bar{\tau}_{..2}$.
4. effect of interaction contrast of factor A and B: $\bar{\tau}_{11.} - \bar{\tau}_{12.} - \bar{\tau}_{21.} + \bar{\tau}_{22.}$.

for part (c):

we have the code:

```
data PaperTowelStrength;
input TreatCombo $ amount $ brand $ type $ strength order @@;
datalines;
111 1 1 1 3279.0 3 111 1 1 1 4330.7 15 111 1 1 1 3843.7 16
112 1 1 2 3260.8 11 112 1 1 2 3134.2 20 112 1 1 2 3206.7 22
121 1 2 1 2889.6 5 121 1 2 1 3019.5 6 121 1 2 1 2451.5 21
122 1 2 2 2323.0 1 122 1 2 2 2603.6 2 122 1 2 2 2893.8 14
211 2 1 1 2964.5 4 211 2 1 1 4067.8 10 211 2 1 1 3327.0 18
212 2 1 2 3114.2 12 212 2 1 2 3009.3 13 212 2 1 2 3242.0 19
221 2 2 1 2883.4 9 221 2 2 1 2581.4 23 221 2 2 1 2385.9 24
222 2 2 2 2142.3 7 222 2 2 2 2364.9 8 222 2 2 2 2189.9 17
;
run;

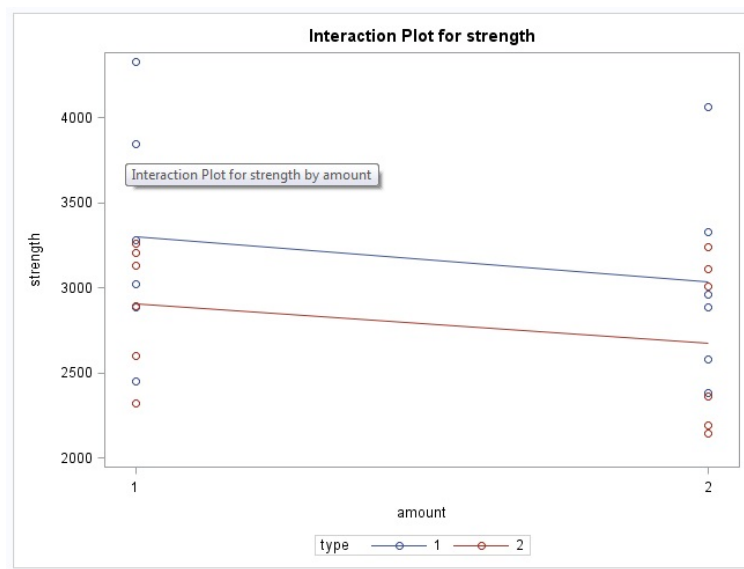
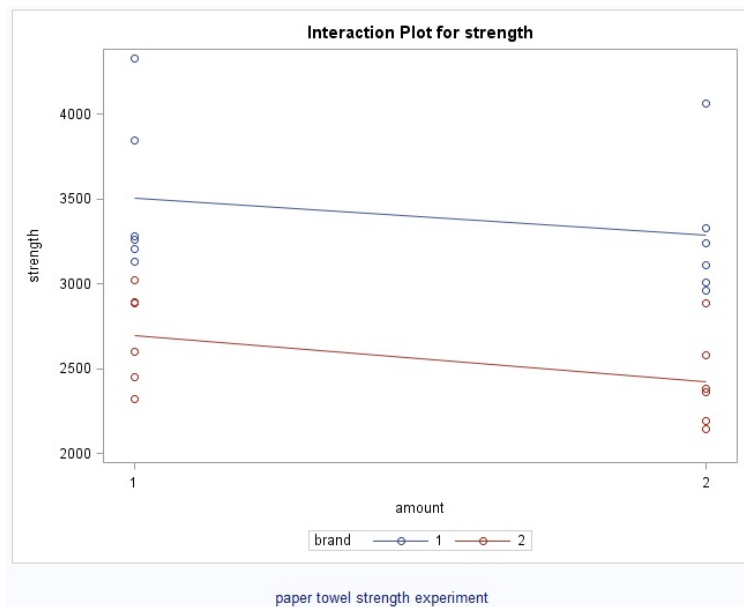
proc glm data = papertowelstrength;
class amount brand;
model strength = amount brand amount*brand;
run;

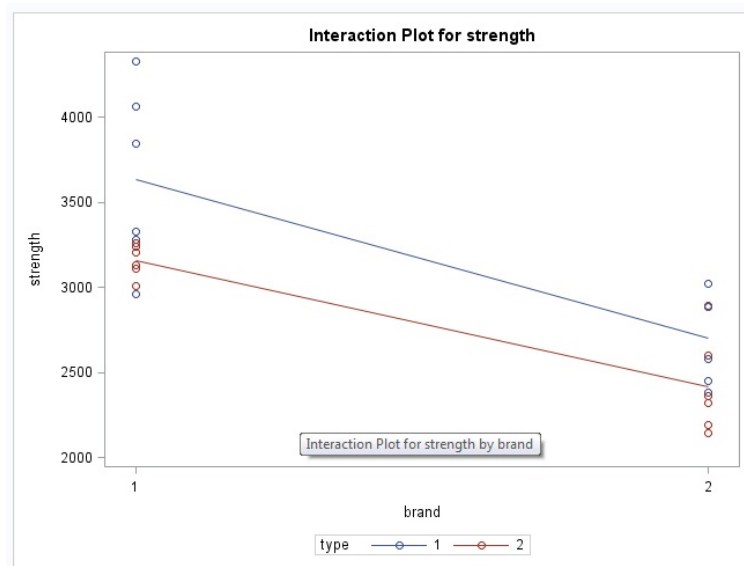
proc glm data = papertowelstrength;
class amount type;
model strength = amount type amount*type;
run;

proc glm data = papertowelstrength;
class brand type;
```

```
model strength = brand type brand*type;  
run;
```

So for interaction between liquid amount and brand:





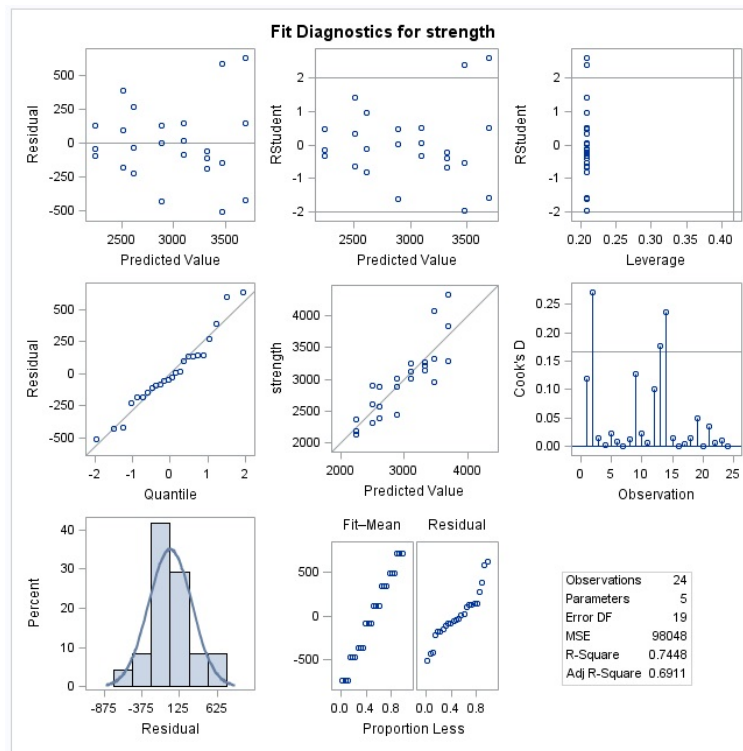
It seems that all lines are near to parallel so we conclude that there is no pairwise interaction.

For part (d):

we run the reg procedure with SAS:

```
proc reg data=papertowelstrength;  
model strength = amount brand type AB;  
run;
```

and we got the following model diagnostic sketches:



residual and studentized residuals against predicted value give supportive information for equal variance assumption. QQ plot and histogram shows supportive signal for normal assumption.

We should really sketch residuals against input order to check independence, but I am very sorry that this time I did not manage to finish in time, same as the rest two sub problems.