Question 2.

Solution 1. *For part (a):*

We can first compute $E[SSA]$. Notice that

$$ssA = \sum_i \frac{y_{i\cdot\cdot}^2}{bn} - \frac{y_{\cdots}^2}{abn}$$

So

$$\begin{aligned}
E[SSA] &= E\Big[\sum_i \frac{Y_{i\cdot\cdot}^2}{b\cdot n} - \frac{Y_{\cdots}^2}{abn}\Big] \\
&= \frac{1}{bn}\sum_i E\Big[Y_{i\cdot\cdot}^2\Big] - \frac{1}{abn}E\Big[Y_{\cdot\cdot}^2\Big] \\
&= \frac{1}{bn}\sum_i \Big\{\Big(E[Y_{i\cdot\cdot}]\Big)^2 + Var(Y_{i\cdot\cdot})\Big\} - \frac{1}{abn}\Big\{\Big(E[Y_{\cdots}]\Big)^2 + Var(Y_{\cdots})\Big\} \\
&= \frac{1}{bn}\sum_i \Big\{\Big(E[\sum_j\sum_t Y_{ijt}]\Big)^2 + bn\sigma^2\Big\} - \frac{1}{abn}\Big\{\Big(\sum_i\sum_j\sum_t Y_{ijt}\Big)^2 + abn\sigma^2\Big\}
\end{aligned}$$

Notice that the term for σ^2 here is:

$$\begin{aligned}
&\frac{1}{bn}\sum_i bn\sigma^2 - \frac{1}{abn}\cdot(abn)\sigma^2 \\
&= \sum_i \sigma^2 - \sigma^2 \\
&= (a-1)\sigma^2
\end{aligned}$$

so after we divide $a-1$ for MSA we will get σ^2.

Meanwhile, the complete model is:

$$Y_{ijt} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijt}$$

with restriction:

$$\sum_i \alpha_i = \sum_j \beta_j = \sum_i(\alpha\beta)_{ij} = \sum_j(\alpha\beta)_{ij} = 0$$

the last two restriction is given in question 1 and lecture notes,
but that does not make sense since if so, wouldn't $(\alpha\beta)_{i\cdot} = (\alpha\beta)_{\cdot j} = 0$?

and then theoretically, with the right restrictionm, we plug the model back into the equation above, we get what we need.

(I need to discuss with you the algebra here).

For part (b):

When all the main-effect parameters for factor A are all equal, we get

$$\alpha_i = \alpha, 1 \leq i \leq a$$

So

$$\bar{\alpha} = \alpha$$

and hence

$$E[MSA] = \sigma^2 + \frac{bn}{a-1} \sum_i \cancel{(\alpha - \alpha)^2} = \sigma^2$$