Chapter 11 Question 6.

Solution 1. *We only do part (a) and part (b) for this question:*

We have number of treatment levels $v = 7$, estimated $msE = 30$, significance level $\alpha = 0.05$, block size $k = 5$.

Since we are trying to get a balanced incomplete block design (BIBD), we need:

$$\lambda = \frac{r(k-1)}{v-1} \Longrightarrow \lambda = \frac{4r}{6} = \frac{2}{3}r$$

$$bk = rv \Longrightarrow b = \frac{rv}{k} = \frac{7r}{5}$$

which implies that the replication of each treatment level r must be a mulitple of both 3 and 5, and hence a mulitple of 15.

We have error degree of freedom as:

$$df = bk - b - v + 1 = vr - \frac{vr}{k} - v + 1 = \frac{vr(k-1) - k(v-1)}{k}$$
$$= \frac{28r - 30}{5}$$

With Tukey's method, we can compute the minimum significant difference as:

$$msd = \frac{q_{v,bk-b-v+1}}{\sqrt{2}}\sqrt{msE \cdot \frac{2k}{\lambda v}} = \frac{q_{v,df}}{\sqrt{2}}\sqrt{2msE\left[\frac{(v-1)k}{rv(k-1)}\right]}$$
$$= q_{7,\frac{28r-30}{5},0.05}\sqrt{30 \cdot \frac{30}{28r}} = q_{7,\frac{28r-30}{5},0.05}\sqrt{\frac{225}{7r}}$$

For part (a) and part (b), we need $msd \leq 50$ and $msd \leq 25$ separately, then we can solve from above equation that:

$$r \geq \frac{225}{7} \cdot \frac{\left(q_{7,\frac{28r-30}{5},0.05}\right)^2}{msd^2}$$

Plug in $msd = 50$ and 25 separately, we can get our estimate for r for (a) and (b) separately (also keep in mind that r has to be a multiple of 15):

```
#Ch11Q6 (a) and (b)
msE <-30
v <-7
k <-5
alpha <- 0.05
r <- seq(15, 150, by = 15)
b <- v*r/k
msd <- c(50, 25)
rep <- 0
```

```r
for (i in 1:2){
  for (j in 1:length(r)){
    df <- as.integer((28*r[j] - 30)/5)
    if (r[j] >= 225/7*(qtukey(.05, v, df = df, lower.tail = FALSE))^2/
        (msd[i])^2)
    {
      rep <- r[j]
      break
    }
  }
  cat("For minimal significant difference", msd[i], ", we need at least r = ",
      rep, "\n")
}

## For minimal significant difference 50 , we need at least r =   15
## For minimal significant difference 25 , we need at least r =   15
```

So in both (a) and (b), the number of replication for each treatment level is at least $r = 15$, and in this case we can design a BIBD with the number of blocks $b = \frac{rv}{k} = 21$, and $\lambda = 10$.