# BIOS 830: Homework 4

Due on April 16, 2015

April 2, 2015

<u>**Instructions**</u>: Students are encouraged to work together on the assigned problems. However, each student is expected to *independently* write up the assigned problems. Since the questions that follow involve a combination of analysis, computing, and theory, students are welcome to hand in a combination of hand written solutions and computer output. For questions that involve computing and analysis, please provide the code (R/SAS) in a clean and readable document; points will be deducted for "messy code". **Assignments are to be turned in at the beginning of lecture on the due date above. Any assignments not turned in at this time will be considered late.**

**Question 1: Variance of least squares estimates -** Suppose we have a simple linear regression model:

$$Y_{xt} = \beta_0 + \beta_1 X + \epsilon_{xt}, \qquad \epsilon_{xt} \overset{iid}{\sim} N(0, \sigma^2)$$

with least squares estimates for $\beta_0$ and $\beta_1$ given by $\widehat{\beta}_0$ and $\widehat{\beta}_1$, respectively. Answer the following questions:

(a) Derive $var(\widehat{\beta}_1)$.

(b) Derive $var(\widehat{\beta}_0)$.

(c) We can use the regression line to estimate the expected mean response $E[Y_{xt}]$ at any particular value of $x$, say $x_a$; that is,

$$\widehat{E}[Y_{x_a t}] = \widehat{y}_{x_a t} = \widehat{\beta}_0 + \widehat{\beta}_1 x_a$$

Show that $var(\widehat{Y}_{x_a t}) = \sigma^2 \left( \frac{1}{N} + \frac{(x_a - \bar{x}_{..})^2}{ss_{xx}} \right)$.

Hints: For part (c), recall that for random variables $X$ and $Y$:

- $var(X + Y) = var(X) + var(Y) + 2cov(X, Y)$.
- $cov(X - Y, X + Y) = cov(X, X) - 2cov(X, Y) + cov(Y, Y)$.
- $cov(X, X) = var(X)$

In addition to the above questions, please do the following questions from Dean & Voss

| Chapter | Question(s) |
|---|---|
| 8 | 6 |
| 9 | 2, 4 |
| 10 | 4, 10 |