BIOS 880: Bayesian Statistics
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Take-Home Test 1

Instructions: Please complete the following four (4) exam questions. When you finish, write your results with a narrative and provide an attachment of your code and output in an appendix (you may use R to write code, where appropriate). **Do not communicate with anyone about this exam – you must work on it on your own.** The exam is ***due*** by Tuesday, October 9, 2018, 3:00 PM. You may communicate with me only to clarify the question(s). Good luck!

Questions

1. Suppose one observes mutually independent random variables $x_1,\ldots,x_n$, $y_1,\ldots,y_m$, that are respectively random samples from Poisson distribution $x|\theta\sim$Poisson(θ) and $y|\delta\sim$Poisson(δ). The parameters θ and δ are unknown and have prior densities. $\theta|$a,b~Gamma(a,b) and $\delta|$c,d ~Gamma(c,d).
   - (i) Given observations $x_1,\ldots,x_n$, $y_1,\ldots,y_m$. Find the posterior density of θ and δ.
   - (ii) What is the posterior mode for each in part (i)?
   - (iii) Determine the derivative and second derivatives of the log posterior densities in part (i).
   - (iv) Construct the normal approximation based on the second derivative of the log posterior at that mode for both θ and δ.

2. For 347 mothers in the KUDOS clinical trial, 169 of them were in non-supplement group and 178 in supplement group. Consider the number of serious adverse events reported for each mom (infant or mom related) and stored as $x_1,\ldots,x_n$, for non-supplement group and $y_1,\ldots,y_m$, for the supplement group (see data below). Consider modeling these data (data are below) using your answers in question 1.
   (i) Using the posterior densities in question 1 (i), find the posterior probability that θ is less than δ (you may use simulation).
   (ii) Using the normal approximation in question 1 (iv), estimate the posterior probability that θ is less than δ.
   (iii) Are your answers close in (i) and (ii)? Why or why not?

Data for problem 2.

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x=c(0,1,0,0,0,0,1,0,0,0,1,0,0,1,0,0,1,2,0,0,0,0,0,0,0,0,0,0,0,0,1,0,
0,0,0,1,0,0,0,1,0,1,0,0,0,1,1,0,0,1,0,0,0,2,0,0,0,0,0,0,1,1,0,0,0,0,
0,1,1,0,0,1,3,0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,0,2,0,0,1,1,0,0,0,0,0,0,
0,0,0,0,0,1,0,0,0,1,0,0,3,0,1,2,0,0,3,0,0,0,2,0,0,0,1,0,0,0,0,1,0,1,
0,0,0,0,0,0,4,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,
5)

y=c(0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,2,4,0,0,0,0,0,0,0,1,0,0,0,
0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,1,0,
0,0,0,0,1,0,1,1,0,0,0,0,0,0,0,0,0,3,0,0,0,0,0,0,0,1,0,0,0,0,0,0,2,0,
1,0,0,0,0,0,1,0,0,0,1,0,0,1,0,1,0,0,0,1,0,0,0,0,0,0,1,0,1,0,0,0,1,0,
0,0,0,0,0,0,3,0,0,0,0,1,1,0,0,0,1,0,1,0,0,1,0,1,1,0,1,0,0,0,0,0,1,0,
0,2,0,1,0,0,0,1,1,0)
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3. Suppose I have 12 months of accrual data (number of patients enrolled per month). Suppose that there was a study participant marketing recruiting campaign at month four. Also, assume Poisson models before and after a suspected change point. We suspect the change point sometime after four months and it occurs at theta (unknown parameter). The likelihood is

   $f\left(\underset{\sim}{y} \mid \underset{\sim}{\theta}\right) = \prod_{i=1}^{3+\theta} Poisson\left(y_i \mid \lambda_1\right) \prod_{i=4+\theta}^{12} Poisson\left(y_i \mid \lambda_2\right)$. Assume the priors are:

   $\pi\left(\lambda_1\right) \sim Gamma\left(.001,.001\right)$, $\pi\left(\lambda_2\right) \sim Gamma\left(.001,.001\right)$, and $\pi\left(\theta\right) = 1/8$, where $\theta = 1,2,3,\ldots,8$. Use successive substitution sampling (Gibbs) to simulate the posterior distribution of λ_1, λ_2, and θ and report their density plots.

We observe the data as
y=c(7, 7, 15, 11, 7, 9, 16 , 15, 15, 16, 22, 18).

4. Suppose $g(\theta) = \dfrac{\exp(2\theta)}{\left(1+\exp(2\theta)\right)^2}$ where $-\infty < \theta < \infty$. Use two techniques (rejection, weighted bootstrap, or Metropolis) to sample from the distribution $\pi(\theta) = g(\theta)/c$. In both cases draw a graph of this posterior distribution.