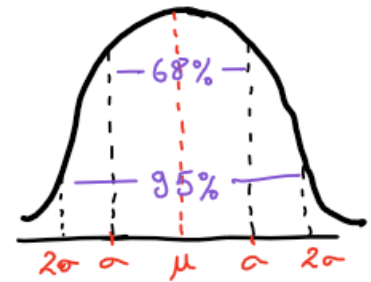


LESSON 7 : SAMPLING DISTRIBUTION

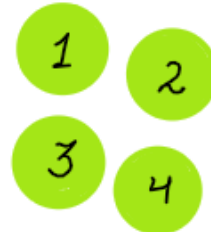
Till now we have learnt that by knowing the MEAN AND STANDARD DEVIATION in normal distribution we can compare any value in that population to rest of the population by determining % less than or greater than that.



BUT WHAT ABOUT A SAMPLE ? IN THIS LESSON WE WILL LEARN HOW TO COMPARE SAMPLES FROM POPULATION.

Q2.

WE HAVE A BAG WITH 4 BALLS NUMBERED 1-4. WE HAVE TO PICK TWO BALLS WITH REPLACEMENT.



PART-I

WHAT IS THE MEAN OF THE POPULATION (1,2,3,4) ?

ANS:

$$\text{MEAN} = \frac{1+2+3+4}{4} = 2.5$$

$$\mu = 2.5$$

PART-II

HOW MANY TOTAL POSSIBILITIES (i.e SAMPLE OF SIZE 2) CAN WE SELECT FROM THIS POPULATION ?



ANS: 16 Samples

1,1	2,1	3,1	4,1
1,2	2,2	3,2	4,2
1,3	2,3	3,3	4,3
1,4	2,4	3,4	4,4

PART-III FIND THE MEAN OF EACH SAMPLE?

ANS:

1,1	$\bar{x}_1 = 1$	2,1	$\bar{x}_5 = 1.5$	3,1	$\bar{x}_9 = 2$	4,1	$\bar{x}_{13} = 2.5$
1,2	$\bar{x}_2 = 1.5$	2,2	$\bar{x}_6 = 2$	3,2	$\bar{x}_{10} = 2.5$	4,2	$\bar{x}_{14} = 3$
1,3	$\bar{x}_3 = 2$	2,3	$\bar{x}_7 = 2.5$	3,3	$\bar{x}_{11} = 3$	4,3	$\bar{x}_{15} = 3.5$
1,4	$\bar{x}_4 = 2.5$	2,4	$\bar{x}_8 = 3$	3,4	$\bar{x}_{12} = 3.5$	4,4	$\bar{x}_{16} = 4$

PART-IV WHAT'S THE MEAN OF THE SAMPLE MEANS?

1,1	$\bar{x}_1 = 1$	2,1	$\bar{x}_5 = 1.5$	3,1	$\bar{x}_9 = 2$	4,1	$\bar{x}_{13} = 2.5$
1,2	$\bar{x}_2 = 1.5$	2,2	$\bar{x}_6 = 2$	3,2	$\bar{x}_{10} = 2.5$	4,2	$\bar{x}_{14} = 3$
1,3	$\bar{x}_3 = 2$	2,3	$\bar{x}_7 = 2.5$	3,3	$\bar{x}_{11} = 3$	4,3	$\bar{x}_{15} = 3.5$
1,4	$\bar{x}_4 = 2.5$	2,4	$\bar{x}_8 = 3$	3,4	$\bar{x}_{12} = 3.5$	4,4	$\bar{x}_{16} = 4$

ANS:

$$\text{MEAN OF SAMPLE MEANS} = \frac{\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \dots + \bar{x}_{16}}{16}$$

(CALCULATE USING EXCEL)

$$\text{MEAN OF SAMPLE MEANS} = 2.5$$

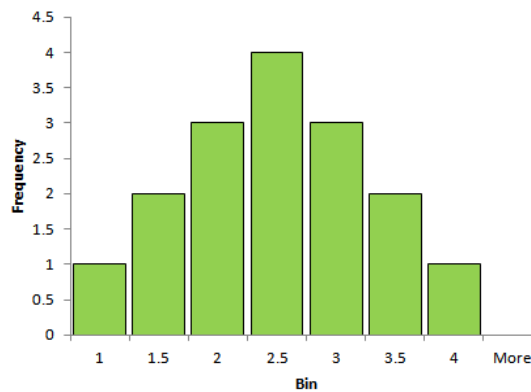
THUS, WE CAN SEE THAT

$$\text{POPULATION MEAN } (\mu) = \text{MEAN OF SAMPLE MEANS } (M)$$

PART - V

CREATE A HISTOGRAM OF SAMPLE MEANS

ANS: Use excel to draw histogram.



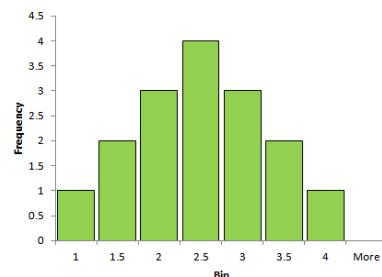
PART - VI

WHAT'S THE SHAPE OF THE DISTRIBUTION ?

- ☐ UNIFORM
- ☐ BIMODAL



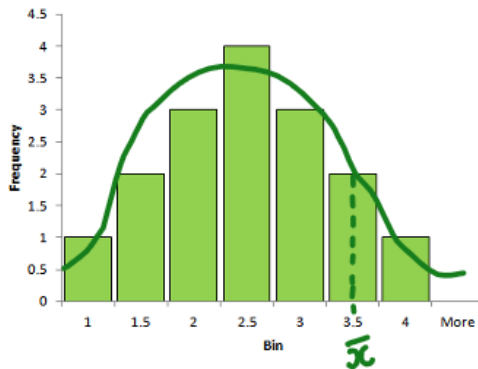
- ☒ NORMAL
- ☐ SKEWED



ANS: So what we have created here is the distribution of sample means which is normal and is known as SAMPLING DISTRIBUTION.

PART-VII

HOW CAN WE COMPARE THE MEAN OF A SINGLE SAMPLE ($\bar{x}=3.5$) WITH OTHER SAMPLES IN THE DISTRIBUTION?



○ TOTAL NO. IN THE POPULATION

✓ ○ STANDARD DEVIATION OF THE DISTRIBUTION OF SAMPLE MEANS

○ TOTAL NO. OF POSSIBLE SAMPLES

ANS: As you studied earlier in the case of the population distribution we can compare one value with the rest of the population by calculating the Z-score. Similarly if we have to compare one sample with the rest of the samples we have to calculate the Z-score, and for that you should know the standard deviation thus our answer.

PART-VIII

FIND THE STANDARD DEVIATION OF SAMPLING DISTRIBUTION AND POPULATION?

ANS:

CALCULATE USING EXCEL :

$$\sigma = 1.118$$

$$S.E = 0.79$$

(S.D. OF SAMPLE MEANS)

STANDARD DEVIATION OF SAMPLING DISTRIBUTION IS CALLED STANDARD ERROR AND IS REPRESENTED BY S.E.

PART - IX

CALCULATE THE RATIO OF $\frac{\sigma}{S.E.}$?

WHAT IS THIS NUMBER ? DOES THE OUTPUT LOOKS FAMILIAR ?

ANS:

$$\frac{\sigma}{S.E.} \text{ (CALCULATE USING SPREADSHEET)} = 1.414$$

THE OUTPUT IS $\sqrt{2}$. AND 2 WAS OUR SAMPLE SIZE .

$$\text{THEREFORE, } \frac{\sigma}{S.E.} = \sqrt{n}$$

PART - X

WHAT IS STANDARD ERROR (S.E.) ?

σ

σ^2/n

\sqrt{n}

✓ # σ/\sqrt{n}

ANS:

$$\text{STANDARD ERROR (S.E.)} = \frac{\sigma}{\sqrt{n}}$$

Q3.

RUN THE APPLET GIVEN IN THE LINK AND ANSWER THE QUESTIONS GIVEN BELOW

<http://www.math.uah.edu/stat/apps/DiceExperiment.html>

PART-I

WHAT HAPPENS WHEN YOU ROLL ONE DICE AT LEAST 100 TIMES . THE DISTRIBUTION IS :

• NORMAL

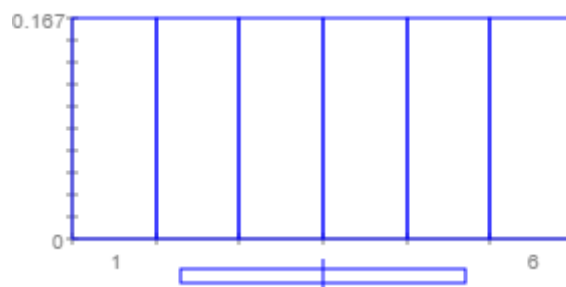
• SKEWED



• UNIFORM

• BIMODAL

ANS: When we roll die once the possibility of any outcome from 1 to 6 is $1/6$, therefore when we roll die many times we get similar no. of all the outcomes so we get a uniform distribution.



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PART-II

WHAT KIND OF DISTRIBUTION WE GET WHEN WE ROLL TWO DICE AT LEAST 100 TIMES AND TAKE THE AVERAGE?



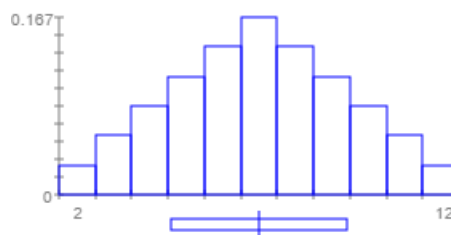
NORMAL

• UNIFORM

• SKEWED

• BIMODAL

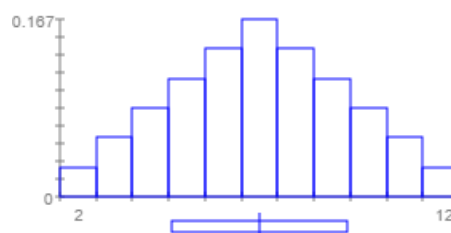
ANS: If we roll two dices [1,2 ; 1,3 ; 2,3 etc.] 100 times and every time we take an average of the outcome and plot the distribution it becomes NORMAL. Because there are more chances to get outcome around 3 and 3.5 as we have seen in example of 4 balls in a bag.



PART-III

WHAT ARE THE MEAN AND S.D. OF THIS SAMPLING DISTRIBUTION WITH $n = 2$?

|| HINT : USE CENTRAL LIMIT THEOREM ||



ANS:

AS WE HAVE STUDIED EARLIER THAT MEAN OF THE SAMPLING DISTRIBUTION IS SAME AS THAT OF POPULATION MEAN

⇒

$$\begin{array}{cc} M = \mu \\ \text{(MEAN OF SAMPLING DISTRIBUTION)} & \text{(POPULATION MEAN)} \end{array}$$

POPULATION
(1, 2, 3, 4, 5, 6)

So,

$$M = \frac{1+2+3+4+5+6}{6}$$

$$M = 3.5$$

⇒

$$\text{S.D FOR SAMPLING DISTRIBUTION} = \text{S.E} = \frac{\sigma}{\sqrt{n}} = \frac{1.7078}{\sqrt{2}}$$

$$\text{S.E.} = 1.2076$$

PART - IV

Will the distribution of means of samples of size 5 be skinnier or wider than the distribution of means taken from samples of size 2?

◦ SKINNIER

◦ WIDER.

ANS: The distribution is going to be skinnier, because as the sample size increases standard error is going to decrease and there will be less variation in data.


$$\text{S.E} = \frac{\sigma}{\sqrt{n}}$$

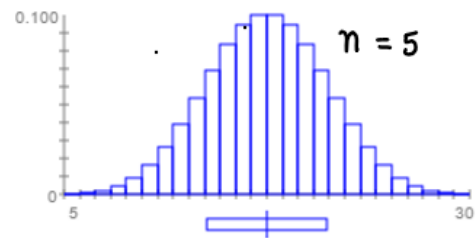
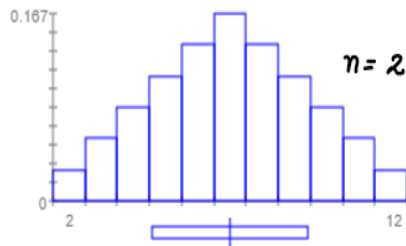
We can compare both the distribution on the applet. We get the following distributions:

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CENTRAL LIMIT THEOREM

THE THEOREM STATES THAT, IF WE COLLECT LARGE NO. OF SAMPLES OF PARTICULAR SIZE FROM A POPULATION AND CREATE A DISTRIBUTION OF SAMPLE MEANS THEN

→ MEAN OF THE SAMPLING DISTRIBUTION (μ) = POPULATION MEAN (μ)

→ STANDARD DEVIATION OF THE SAMPLING DISTRIBUTION (S.E.) = $\frac{\sigma}{\sqrt{n}}$ (S.D. OF POPULATION)

→ THE SHAPE OF THE SAMPLING DISTRIBUTION IS ALWAYS NORMAL IRRESPECTIVE OF THE SHAPE OF THE POPULATION DISTRIBUTION.

DEMONSTRATION OF CENTRAL LIMIT THEOREM :→

http://onlinestatbook.com/stat_sim/sampling_dist/index.html

Obesity

Q4. THE SPREADSHEET ACCOUNTS THE DATA OF WEIGHT OF 10,000 PEOPLE

<https://goo.gl/WmA5zv>



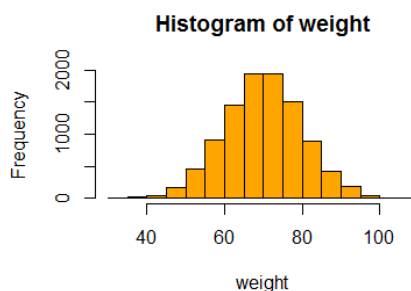
PART-I CALCULATE THE MEAN AND STANDARD DEVIATION OF THE POPULATION ?

ANS:

$$\text{MEAN } (\mu) = 70$$


$$\text{S.D } (\sigma) = 10$$


PART-II IF WE TAKE ALL SAMPLES OF SIZE 25 AND PLOT A DISTRIBUTION OF THEIR MEANS. WHAT WOULD BE THE MEAN AND STANDARD DEVIATION OF THIS DISTRIBUTION ?



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ANS: As studied earlier the mean of the sampling distribution is same as that of population mean.

$$\text{MEAN OF THE SAMPLING DISTRIBUTION } (\mu) = \text{POPULATION MEAN } (\mu)$$

i.e.

$$\mu = 70$$

$$\begin{array}{l} \text{S.E.} \\ \text{(STANDARD DEVIATION} \\ \text{OF THE DISTRIBUTION)} \end{array} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2$$

$$\text{S.E.} = 2$$