Dimensionless Technologies Private Limited
Visit us at: www.dimensionless.in
info@dimensionless.in
9923170071, 8108094992

Notes For Students - Lesson 10
T – test [One Sample and Dependent Sample]

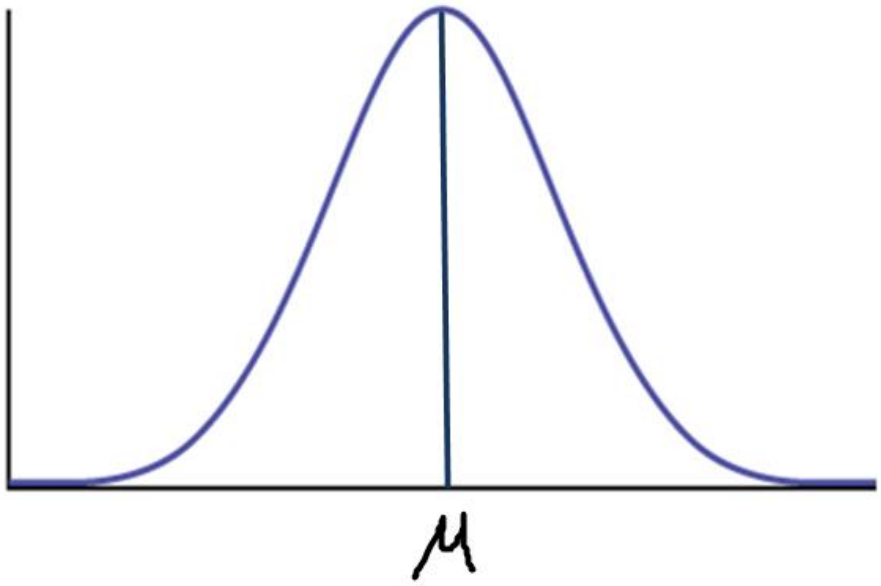
- We have studied the use of z test to analyse if the population mean is significantly different from what it is believed to be or not.

- We studied how z test can be used when n is large and population standard deviation is not known.

However, there may be cases when n is not sufficiently large for the z statistic to approximately follow standard normal distribution.

When n is small and population standard deviation is not known, we use t statistic for our hypothesis testing.

Dimensionless Technologies Private Limited
Visit us at: www.dimensionless.in
info@dimensionless.in
9923170071, 8108094992



t distribution

z distribution

When population s.d. is known, $\bar{X}$ would follow standard normal distribution.

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

When population s.d. is not known, but n > 30, so we use sample s.d.

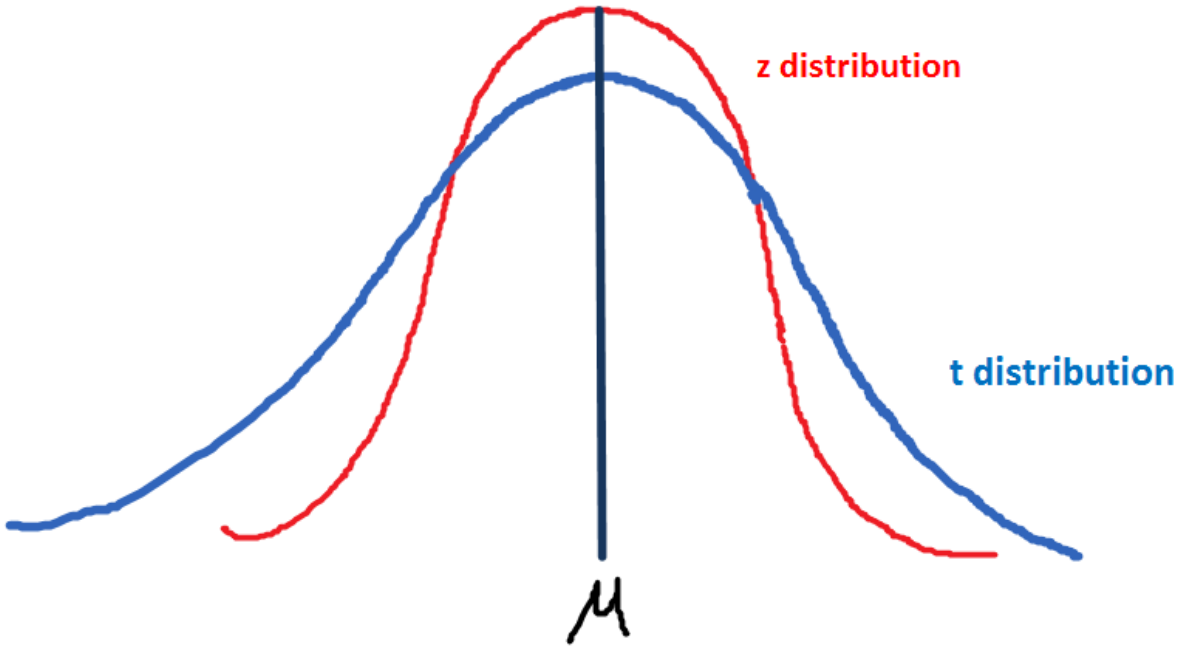
T distribution

When population s.d. is not known and n is small, $\bar{X}$ would follow t distribution (we use sample s.d.)

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

Dimensionless Technologies Private Limited
Visit us at: www.dimensionless.in
info@dimensionless.in
9923170071, 8108094992

Q. What happens to t distribution if n increases?



As n increases, t distribution approaches standard normal distribution.

Q. As t distribution approaches z distribution, the standard error

Decreases

Increases

Dimensionless Technologies Private Limited
Visit us at: www.dimensionless.in
info@dimensionless.in
9923170071, 8108094992

Degrees Of Freedom

A college had organized on week fest. Each day, a chief guest was invited for an event. Five chief guests were invited for each day. It is decided to allot each guest a given day. What will be the degree of freedom in this scenario?

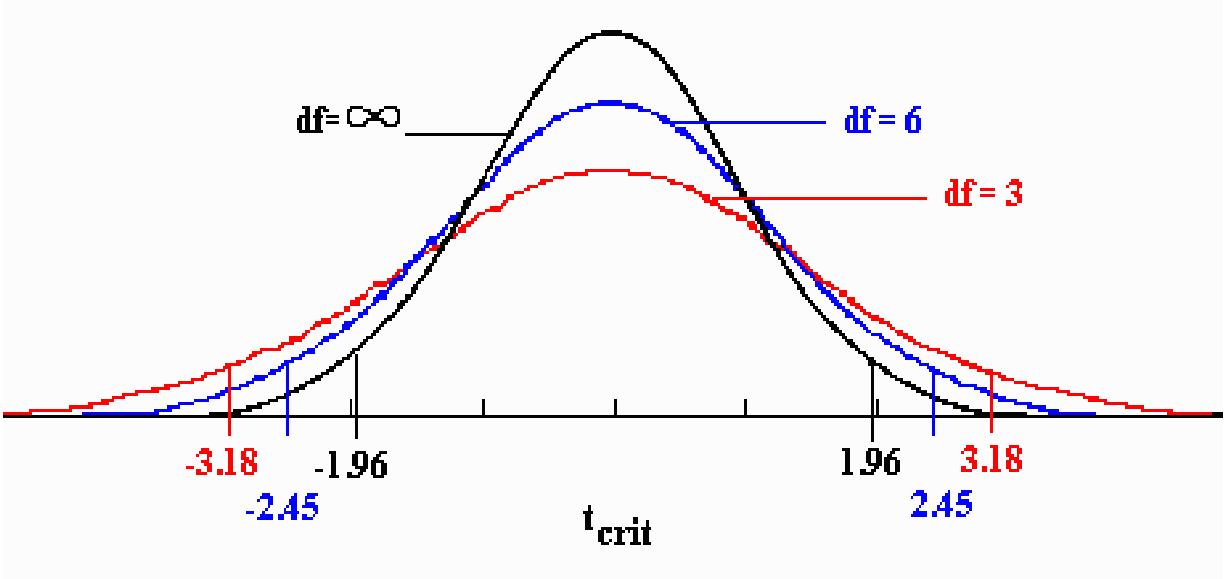
Q. The mean of 10 numbers is 7. How many numbers can you choose independently?

9

Degrees of Freedom for a sample size of n

= **n – 1**

Dimensionless Technologies Private Limited
Visit us at: www.dimensionless.in
info@dimensionless.in
9923170071, 8108094992



Q. In a population, the average IQ = 120. Doctors have invented a medication to increase intelligence. They tested it on a sample of 40 people and found the average IQ to be 150 and standard deviation 30.

Did the medication affect the people or was it just a matter of chance?

H_o : $\mu = 120$

H_a : $\mu > 120$

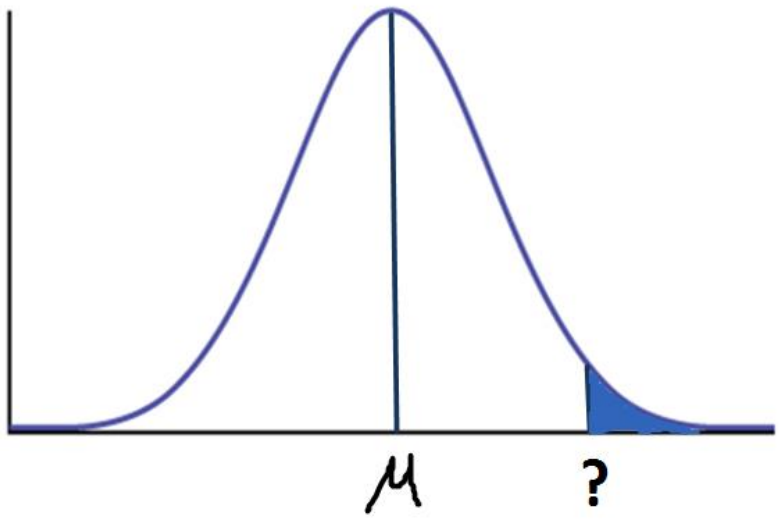
Q. What is the value of the t statistic?

Dimensionless Technologies Private Limited
Visit us at: www.dimensionless.in
info@dimensionless.in
9923170071, 8108094992

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

6.32

Q. What is the t critical value at 5% significance level?

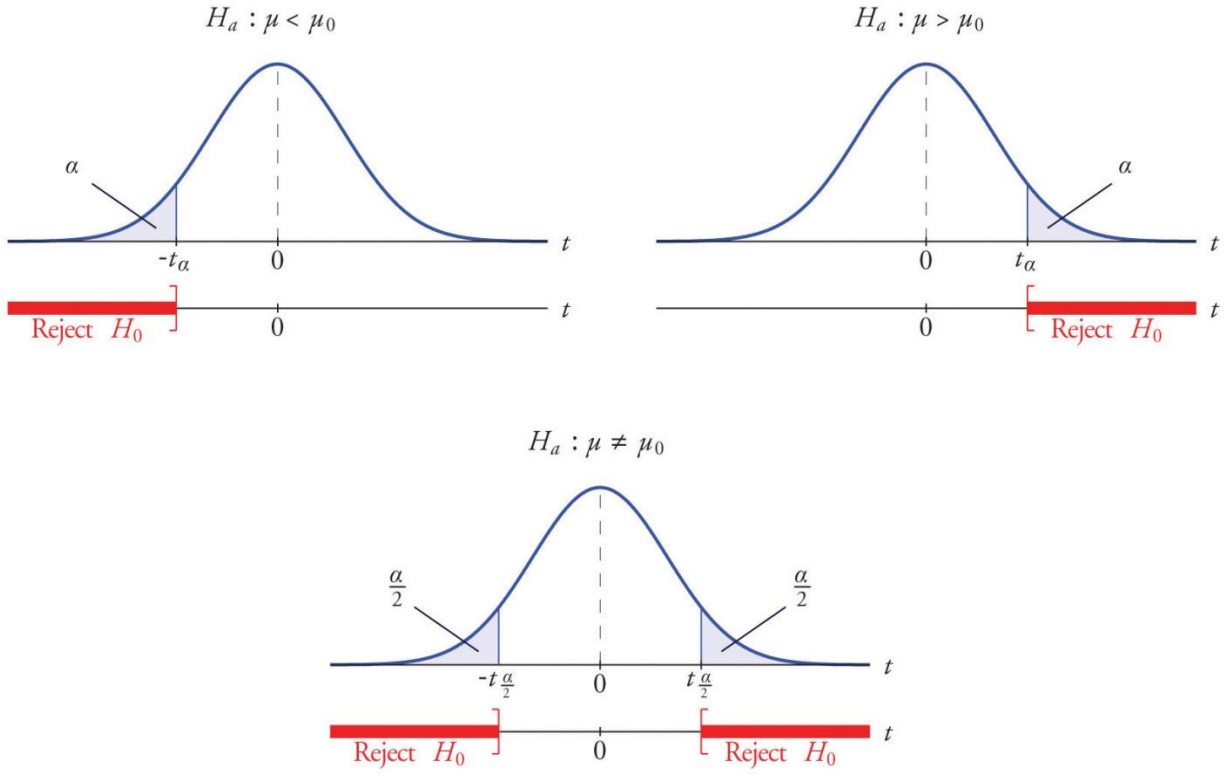


Dimensionless Technologies Private Limited
Visit us at: www.dimensionless.in
info@dimensionless.in
9923170071, 8108094992

Q. Would you reject or retain the null hypothesis?

Reject

Do Not Reject



Let's see how to perform t test on Excel

Use TINV to calculate t critical value

Calculate t value in Excel according to the formula

Use TDIST to calculate P-value

Dimensionless Technologies Private Limited
Visit us at: www.dimensionless.in
info@dimensionless.in
9923170071, 8108094992

Q. Calculate the 95% confidence interval for the true value of μ.

C.I. for the true value of μ for a given significance level is given by

$$\bar{X} \pm t^* \frac{s}{\sqrt{n}}$$

(141.4 , 158.59)

Q. Find the t critical value for the following levels of α.

a) 0.05 , two-tail

b) 0.10 , two tail

c) 0.01 , one tail

Q. What will be the α value for the following t values?

a) 0.896, n = 8

b) 2.145, n = 15

Q. Larger the t value, _____ would be the level of significance?

Smaller

Larger

Dimensionless Technologies Private Limited
Visit us at: www.dimensionless.in
info@dimensionless.in
9923170071, 8108094992

Q. 1000 customers on an average visit a store in a week. The store manager carried out some promotional advertisements. The following number of customers was noted to have visited the store in various weeks.

1500

1200

900

700

1400

2000

1600

700

1100

1000

Q. What is $\overline{X}$?

Q. What will be the null and alternate hypothesis?

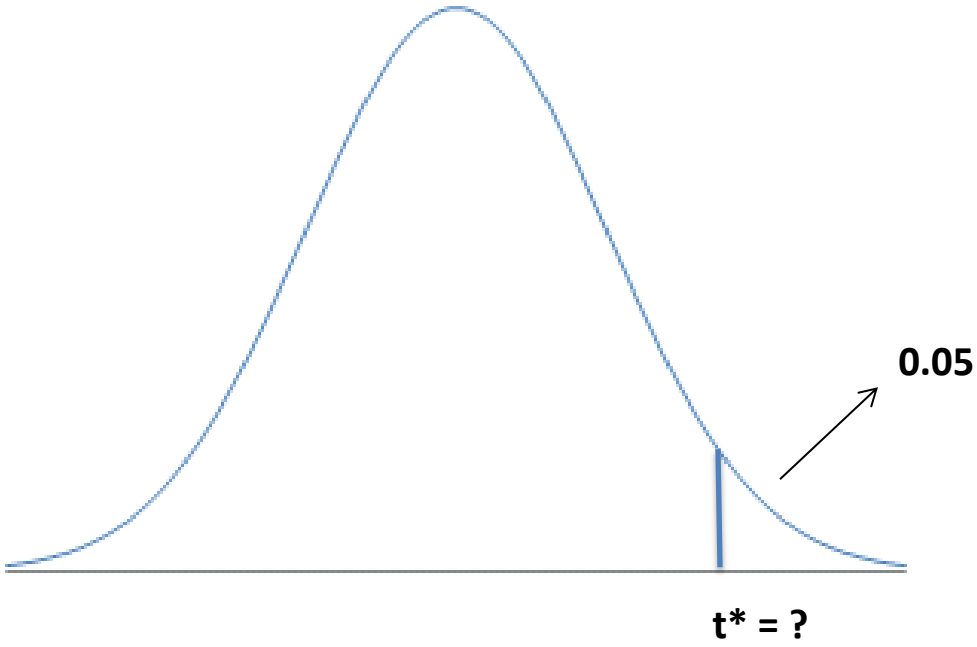
H_o : ?

H_a : ?

Dimensionless Technologies Private Limited
Visit us at: www.dimensionless.in
info@dimensionless.in
9923170071, 8108094992

Q. What is the t statistic value?

Q. What is the t critical value for a significance level of 0.05?



Q. Would you retain or reject the null hypothesis?

Q. What conclusion can be drawn from this observation?

Q. Would your conclusion be the same if the significance level was 99%?

Q. Calculate the 95% C.I. for the true value of μ.

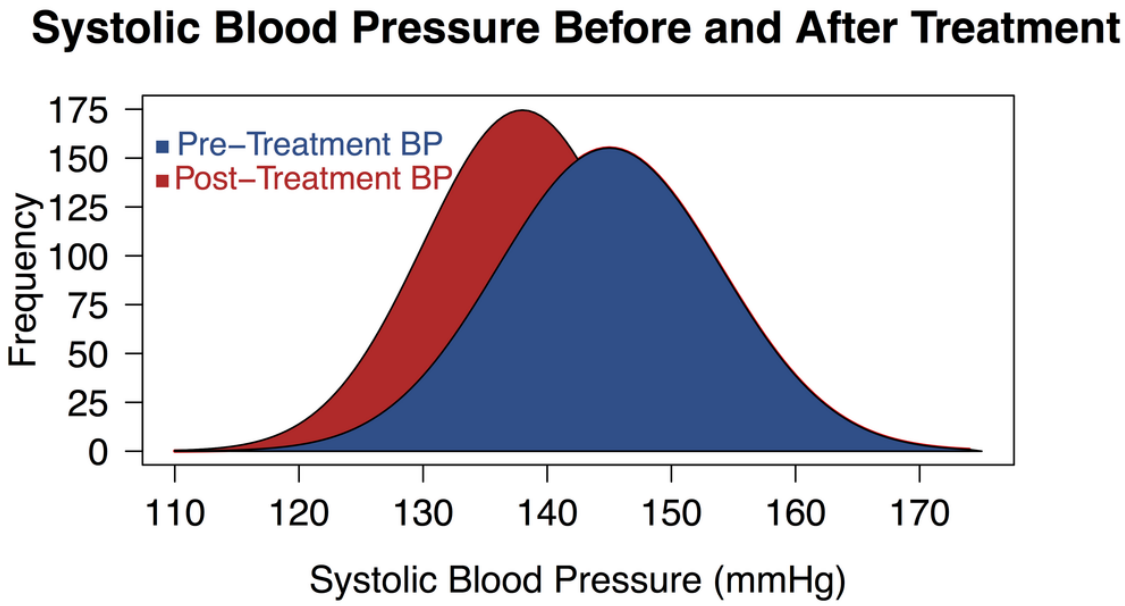
Dimensionless Technologies Private Limited
Visit us at: www.dimensionless.in
info@dimensionless.in
9923170071, 8108094992

Paired T test / Dependent Sample t test

- **A single subject is used**
- **It is measured twice, pre test and post test**
- **It is used in case of a longitudinal study (Growth over time)**

Advantages of Dependent Sample t test

- It uses fewer subjects
- It is cost effective
- It is less time consuming
- It is less expensive



Dimensionless Technologies Private Limited
Visit us at: www.dimensionless.in
info@dimensionless.in
9923170071, 8108094992

Q. The time taken (in seconds) by horses to run over a set distance was noted before and after feeding them high fat feed.

Before	After	Difference
50	47	3
40	42	-2
46	39	7
32	25	7
25	24	1
38	22	16
47	40	7
35	42	-7
47	36	11
30	24	6

Q. What will be the null hypothesis to test is there is a significant difference in the average time taken by horses before and after having high fat feed?

H_o : $\mu_1 - \mu_2$

Q. What will be the alternate hypothesis?

H_a : $\mu_1 \neq \mu_2$

H_a : $\mu_1 > \mu_2$

H_a : $\mu_1 < \mu_2$

Q. What is the t statistic value?

$$t = \frac{\overline{D}}{s/\sqrt{n}}$$

(Here s is the sample standard deviation of the sample of differences in time)

How do you think $\overline{D}$ would be calculated?

1. $\overline{X}_A - \overline{X}_B$ 2. $\overline{X}_A - \overline{X}_B$ 3. Both are correct

t = 2.504

Q. What is the t critical value for a significance level of 0.05?

To calculate Paired t-test in Excel, choose t-test : Paired Two Sample for Means in Data Analysis toolbox.

1.812

Q. Would you reject or retain the null hypothesis?

Reject

Do Not Reject

Calculate the 95% C.I. for the true mean of difference.

$$\overline{X}_D \pm t^* \frac{s_D}{\sqrt{n}}$$

Q. A study wants to check if the burning of firecrackers significantly increase pollution or not. The level of PM 2.5 present in the air was recorded before and after the festival of Diwali at various locations in a city.

Before	After
100	200

300	**1000**
50	**200**
500	**800**
800	**2000**
200	**400**
100	**80**
600	**700**
450	**300**
290	**500**

Q. Do you think there is a significant difference in air quality?

t-Test: Paired Two Sample for Means

	Variable 1	*Variable 2*
Mean	339	618
Variance	59987.78	321906.7
Observations	10	10
Pearson Correlation	0.817953	
Hypothesized Mean Difference	0	
Df	9	
t Stat	-2.24413	
P(T<=t) one-tail	0.025747	
t Critical one-tail	1.833113	
P(T<=t) two-tail	0.051495	
t Critical two-tail	2.262157	