

Notes for Students – Lesson 10

t- tests

t- distribution

Q1.

Samples

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

What happens as n increases?

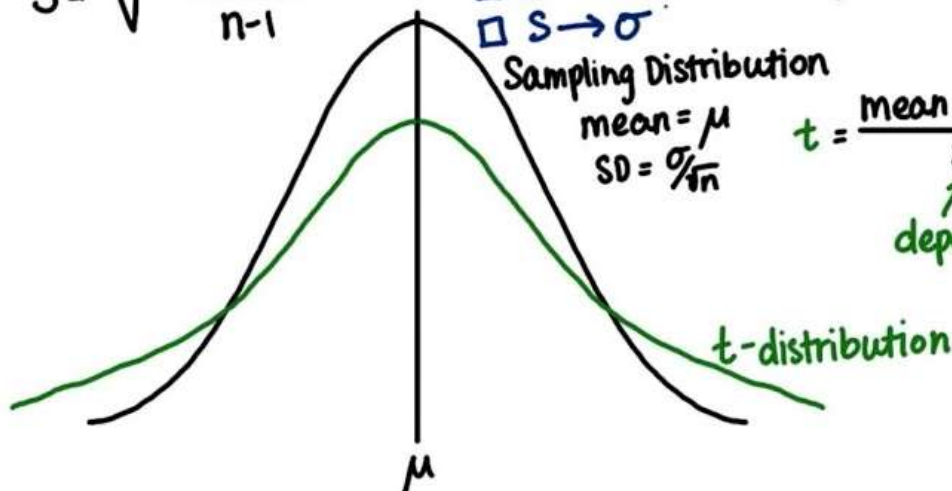
- ☐ The standard error increases
- ☐ The t-distribution approaches a normal distribution
- ☐ The t-distribution gets skinnier tails
- ☐ $S \rightarrow \sigma$

Sampling Distribution

$$\text{mean} = \mu$$
$$SD = \frac{\sigma}{\sqrt{n}}$$

$$t = \frac{\text{mean difference}}{SE}$$

↑
depends on s



Q2.

Degrees of Freedom

Homework assignments

- writing (1 hour)
- statistics (1 hour)
- psychology (1 hour)

Hour-long time slots

1	2	3
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Q3.

Degrees of Freedom

You have to choose n numbers.

How many degrees of freedom?

Q4.

Degrees of Freedom

You have n numbers that must sum to 10.

$$X_1 + X_2 + \dots + X_n = 10$$

How many degrees of freedom?

Q5.

Degrees of Freedom

9 9 9

How many degrees of freedom?

t- table

Table entry for p
and C is the point
 t^* with probability
 p lying above it
and probability C
lying between
 $-t^*$ and t^* .

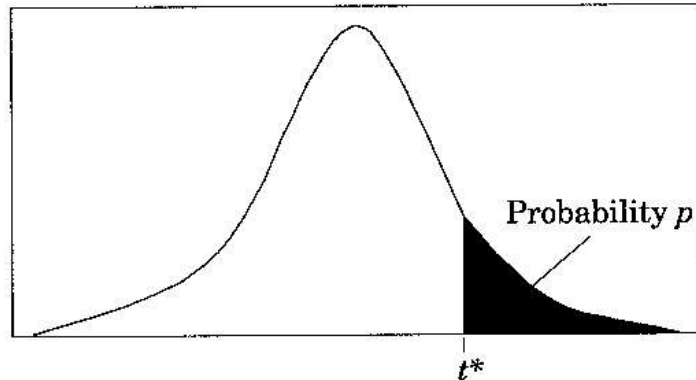


Table B *t* distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Q6.

t-table

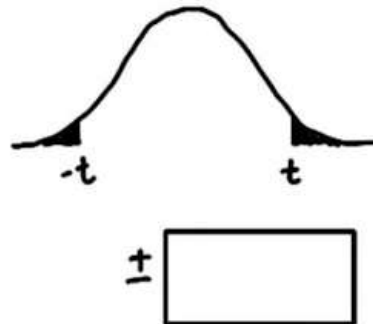
What's the t-critical value for a one-tailed alpha level of 0.05 with 12 degrees of freedom?



Q7.

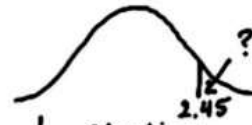
t-table

You have a sample of size 30. What are the t-critical values for a two-tailed test with $\alpha = 0.05$?



Q8.

t-table



Your sample is size 24 and you get a t-statistic of 2.45.
The area to the right of the t-statistic is between
_____ and _____?

(Enter values as proportions exactly as shown in the t-table,
as precise as possible.)

Q9.

s

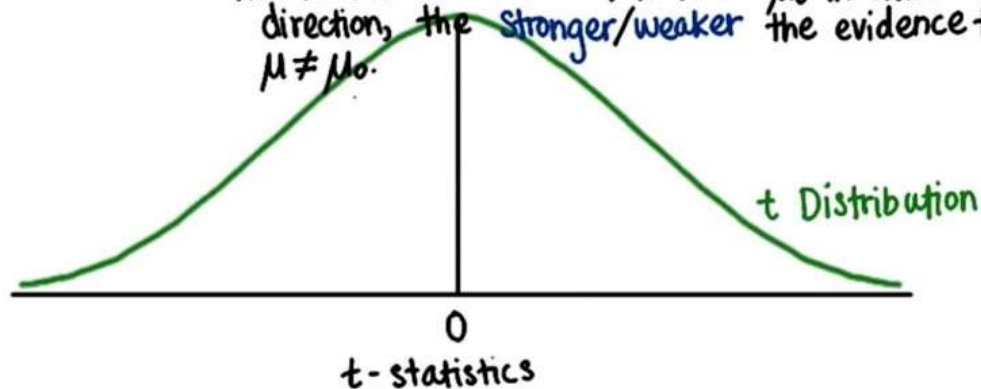
$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

\bar{x} is from population
with mean μ and
SD σ .

The larger/smaller the value of \bar{x} , the stronger
the evidence that $\mu > \mu_0$.

The larger/smaller the value of \bar{x} , the stronger the
evidence that $\mu < \mu_0$.

The further the value of \bar{x} from μ_0 in either
direction, the stronger/weaker the evidence that
 $\mu \neq \mu_0$.



One sample t - test

Q10.

One-Sample t-test $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

What will increase the t-statistic?

- ☐ A larger difference between \bar{x} and μ_0
- ☐ Larger s
- ☐ Larger n
- ☐ Larger standard error

Beak Width Of Finches

Scientists have looked at finches to make hypotheses about evolution. By mapping a trait over time, like beak width. We can tell if the environment is selecting for a particular size of beak width. Maybe it helps the finch survive. But sometimes there are random fluctuations in beak width due to variation in the population. Some finches just have bigger beak widths than others. Therefore to determine whether there is a significant change in beak width within the population over time. We need to use statistics.

Q11.

Map a trait (beak width)

Avg. beak width = 6.07 mm

Do finches today have different-sized beak widths than before?

$H_0: \mu = 6.07$

$H_a: \mu < 6.07$

$\mu > 6.07$

$\mu \neq 6.07$



Q12.

<https://docs.google.com/spreadsheets/d/1SVzT8Jf3YrbZxo2yZgXFz6YgfqQ2-DhOicpmp6puo1s/edit?usp=sharing>

Part 1

Sample size _____
Degrees of freedom _____

Part 2

Find ?

$\bar{X} =$

$S =$

Part 3

Find

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} =$$

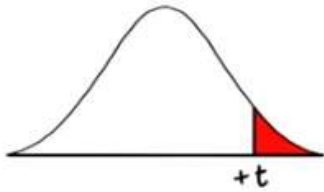
Part 4

So do we

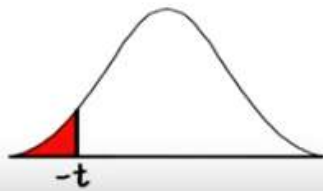
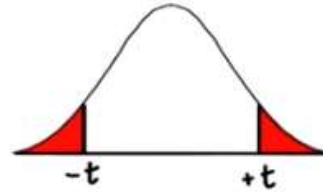
- o Reject H_0
- o Fail to reject H_0

P-Value

One-tailed test



Two-tailed test



Reject the null when the p-value is less than the α level.

Q13.

Part 1

Sample

5

19

11

23

12

7

3

21

$$H_0: \mu = 10$$

$$H_A: \mu \neq 10$$

$$\alpha = 0.05$$

$$t = \frac{\bar{x} - 10}{(s/\sqrt{n})} =$$

Part 2

Sample

5
19
11
23
12
7
3
21

$$\bar{x} = 12.625$$

$$s = 7.596$$

$$H_0: \mu = 10$$

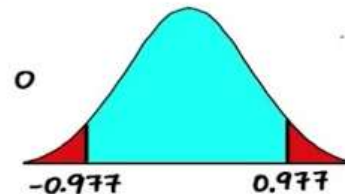
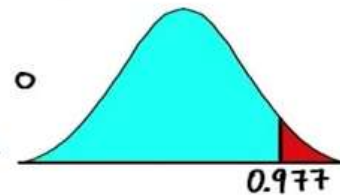
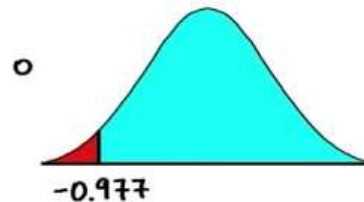
$$H_A: \mu \neq 10$$

$$\alpha = 0.05$$

$$t = 0.977$$

For a 2 tailed test, which image represents the P-value where the P value is the red shaded region.

P-value



Part 3

Graphpad (Link below)

→ "Statistical distributions and interpreting p-values"

The two-tailed p-value equals _____.

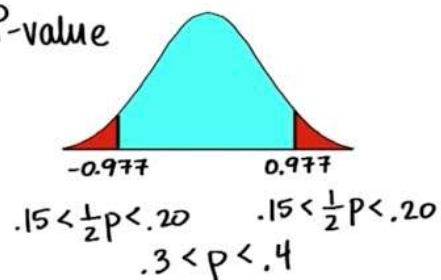
This ☐ is statistically significant
☐ is not statistically significant

Because ☐ $p < \alpha$
☐ $p > \alpha$
☐ $p \neq \alpha$
☐ $p < 1$

Therefore,

☐ We fail to reject the null
☐ We reject the null

P-value



<http://www.graphpad.com/quickcalcs/>

Q14.



This is Santa Clara County in California. And let's say that the mean rent for apartments is 1,830 for all types of units. A certain rental company called Rental California owns and rents a lot of apartments. And they want to know if the amount they're renting it for is significantly different than this population mean. So they examine a random sample of their own units of size 25. They find that on average, they rent each unit out for 1,700. And the sample standard deviation was \$200.

Part 1

Mean rent = \$1830

Rental California examines a random sample ($n=25$) of their units.

$M = \$1700$

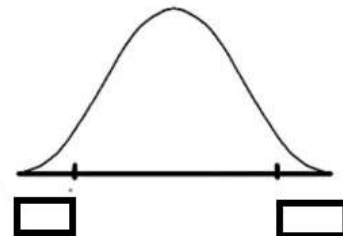
$S = \$200$

$H_0: \mu = 1830$

$H_a: \mu \neq 1830$

$\alpha = 0.05$

Conduct a one sample t-test for them. Here are the null and alternative hypothesis, and remember, they're concerned with whether or not it's significantly different. Even though they found that it's less, this will still be the alternative hypothesis. And they also want an alpha level of 0.05. First of all, find a t critical values for a two-tailed test at alpha equals 0.05.



Part 2. Find t-statistics

Mean rent = \$1830

Rental California examines a random sample ($n=25$) of their units.

$M = \$1700$

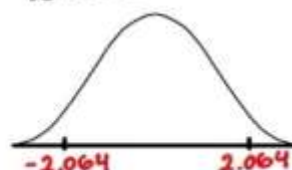
$S = \$200$

$H_0: \mu = 1830$

$H_a: \mu \neq 1830$

$\alpha = 0.05$

$$t = \frac{M - \mu_0}{S / \sqrt{n}} =$$

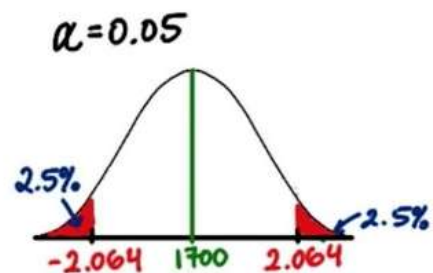


Part 3. Which one is true?

- $H_0: \mu = 1830$
- $H_a: \mu \neq 1830$

Part 4. Calculate a 95% confidence interval

Confidence Interval
1 standard error = $\frac{s}{\sqrt{n}}$
(_____ , _____)



Part 5.

Margin of error =

- $t \cdot s$
- $t \cdot \sqrt{n}$
- s/\sqrt{n}
- $t \cdot s/\sqrt{n}$

Part 6 :- What happens if we increase the sample size to 100? What's our new margin of error for a 95% confidence interval?

Margin of error = $t \cdot s/\sqrt{n}$

If $n=100$, the margin of error is
(95% CI)

Dependent t-test for paired samples

Dependent t-test for paired samples

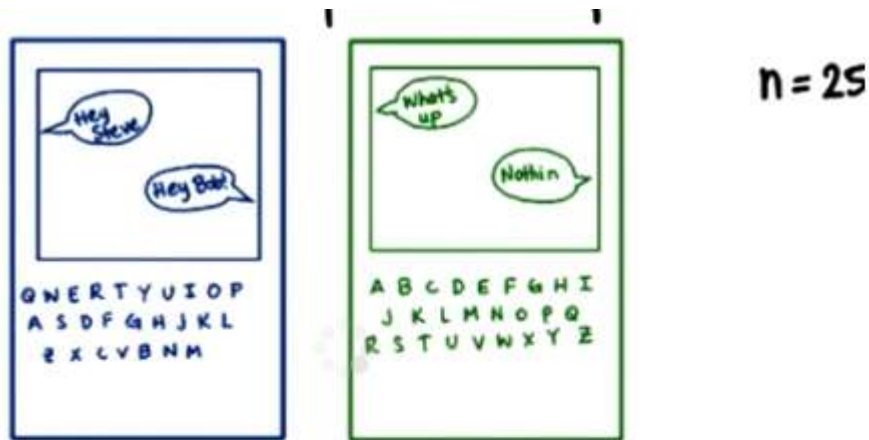
Within-subject designs

- Two conditions
- Pre-test, post-test
- Growth over time (longitudinal study)

x_i	y_i	$D_i = x_i - y_i$
x_1	y_1	$D_1 = x_1 - y_1$
x_2	y_2	$D_2 = x_2 - y_2$
\vdots	\vdots	\vdots

} Follow same procedure as before, using D_i .

Q15.



These are two cell phones with different keyboard configurations. Let's say a researcher is interested in the effects of these cell phone configurations on errors made in typing text messages. To study this, 25 participants used each keyboard type. There is QWERTY and an alphabetical keyboard. And they typed a standardized 20 word text message within 30 seconds. The number of errors for each person using each type of keyboard was recorded. These participants were randomly assigned to which type they used first.

Part 1

In the Google spreadsheet below, you'll find the data for the number of errors each person made using each type of keyboard.

Find the mean number of errors for the QWERTY keyboard and for the alphabetical keyboard.

<https://docs.google.com/spreadsheets/d/1LMvkF2yHQf38-3R3L51dZGjLvU13cyfkjdpq-wH70/edit?usp=sharing>

Part 2.

We want to see if there's a significant difference between the number of errors made on each keyboard. Our null hypothesis is that there is no significant difference. The population of errors from this keyboard, if it were extended to everybody, not just the sample, equals the population of errors using this keyboard. And our alternative hypothesis, is that they're not equal.

Dependent t-test for paired samples



$$\bar{x}_Q = 5.08$$



$$\bar{x}_A = 7.8$$

$$n = 25$$

$$H_0: \mu_Q = \mu_A$$

$$H_A: \mu_Q \neq \mu_A$$

Part 2 - a

What is our point estimate for $\mu_Q - \mu_A$?

Part 2-b

What is s ? (SE of differences)

Part 2-c

Dependent t-test for paired samples

$$t = \frac{\mu_Q - \mu_A}{S/\sqrt{n}} = \frac{-2.72}{S/\sqrt{n}}$$

=



$$\bar{x}_Q = 5.08$$



$$\bar{x}_A = 7.8$$

$$n = 25$$

$$H_0: \mu_Q = \mu_A$$

$$H_A: \mu_Q \neq \mu_A$$

$$S = 3.69$$

Part 2-d

Dependent t-test for paired samples

$$t = \frac{\mu_Q - \mu_A}{S/\sqrt{n}} = \frac{-2.72}{(3.69/\sqrt{25})}$$

$$= -3.69$$



$$\bar{x}_Q = 5.08$$



$$\bar{x}_A = 7.8$$

$$n = 25$$

$$H_0: \mu_Q = \mu_A$$

$$H_A: \mu_Q \neq \mu_A$$

$$S = 3.69$$

FIND T-CRITICAL ?



Part 2-e

Dependent t-test for paired samples

$$t = \frac{\mu_Q - \mu_A}{S/\sqrt{n}} = \frac{-2.72}{(3.69/\sqrt{25})}$$

$$= -3.69$$

• Fail to reject H_0

• Reject H_0



$\bar{x}_Q = 5.08$



$\bar{x}_A = 7.8$

$S = 3.69$

$n = 25$

$H_0: \mu_Q = \mu_A$

$H_A: \mu_Q \neq \mu_A$



Part 3.

Dependent t-test for paired samples

$$t = \frac{\mu_Q - \mu_A}{S/\sqrt{n}} = \frac{-2.72}{(3.69/\sqrt{25})}$$

$$= -3.69$$



$\bar{x}_Q = 5.08$



$\bar{x}_A = 7.8$

$S = 3.69$

$n = 25$

$H_0: \mu_Q = \mu_A$

$H_A: \mu_Q \neq \mu_A$

$$CI: \mu_D \pm t_{critical} \left(\frac{S_D}{\sqrt{n}} \right) = (\quad , \quad)$$

Dependent Samples

Repeated measures design (e.g. errors on two types of keyboards)

$$H_0: \mu_1 = \mu_2$$

Longitudinal

$$H_0: \mu_{\text{time 1}} = \mu_{\text{time 2}}$$

Pretest posttest

$$H_0: \mu_{\text{pre}} = \mu_{\text{post}}$$