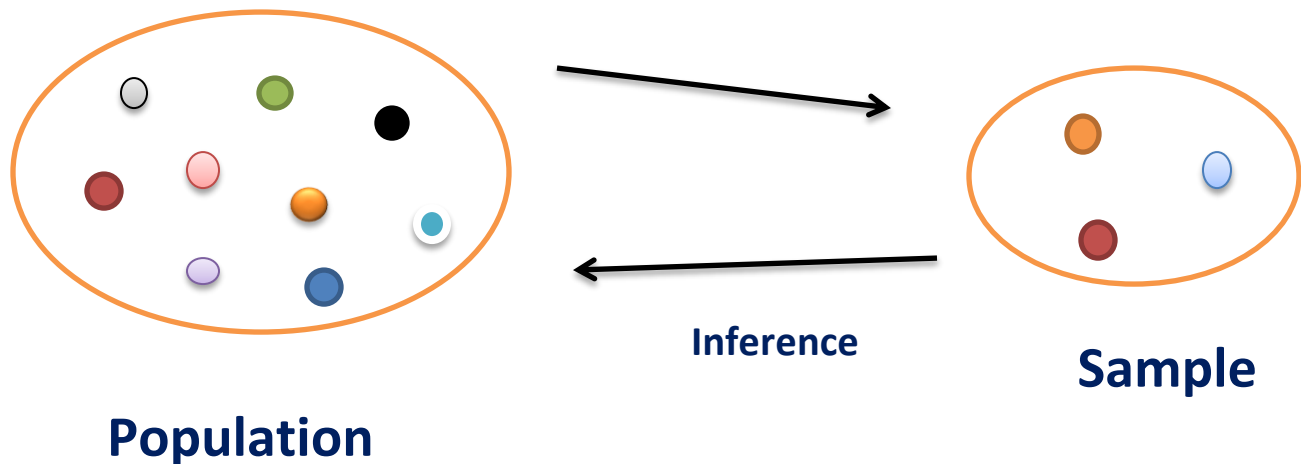


Notes for Students - Lesson 8

Estimation

Statistical Inference is of two types: -**Estimation**
-**Hypothesis Testing**



An Industrial manager is concerned with buying a keyboard table to achieve high productivity & comfort for his typists. He studies the preferred height for experimental keyboard table. Sample of 31 typists was selected & preferred table height was determined for each one.

$$\bar{X} = 80 \text{ cm}$$

$$\sigma = 2 \text{ cm}$$

Q. What will be the point estimate for the keyboard table height?

(A point estimate is a single number that can be regarded as a sensible value for the population parameter)

Q. Can this value be considered as a precise or reliable estimate for the population preferred table height?

a) **Yes**

b) **No**

Due to sampling variability, it is highly impossible that \bar{X} will be equal to μ . Therefore, an alternative to reporting a single sensible value for the parameter being estimated is to calculate and report an entire interval of plausible values i.e. an interval estimate or confidence interval (C.I.).

$$P(-1.96) < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < P(1.96) = 0.95$$

$$-1.96 \frac{\sigma}{\sqrt{n}} < \bar{X} - \mu < 1.96 \frac{\sigma}{\sqrt{n}}$$

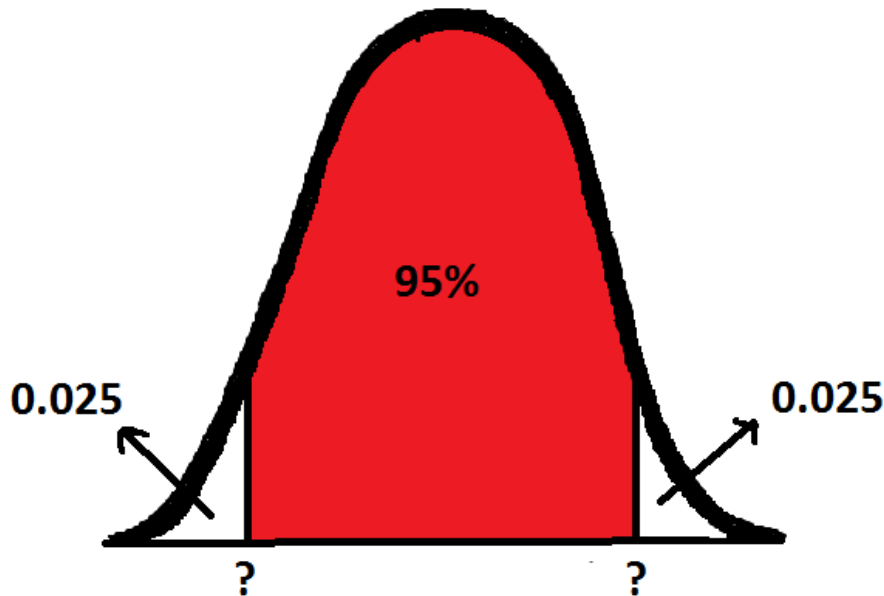
$$\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}$$

$$\bar{X} + 1.96 \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} - 1.96 \frac{\sigma}{\sqrt{n}}$$

$$\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}$$

is the required confidence interval.

Revisiting the 95% significance level



What are the Z values that bound 95% of area within the mean of the distribution?

1.96 and -1.96

Q. The above keyboard table height is known to follow Normal Distribution with $\sigma = 2$ cm. Calculate 95% confidence interval for average height of keyboard.

$$80 - 1.96 * \frac{2}{\sqrt{31}}, 80 + 1.96 * \frac{2}{\sqrt{31}}$$

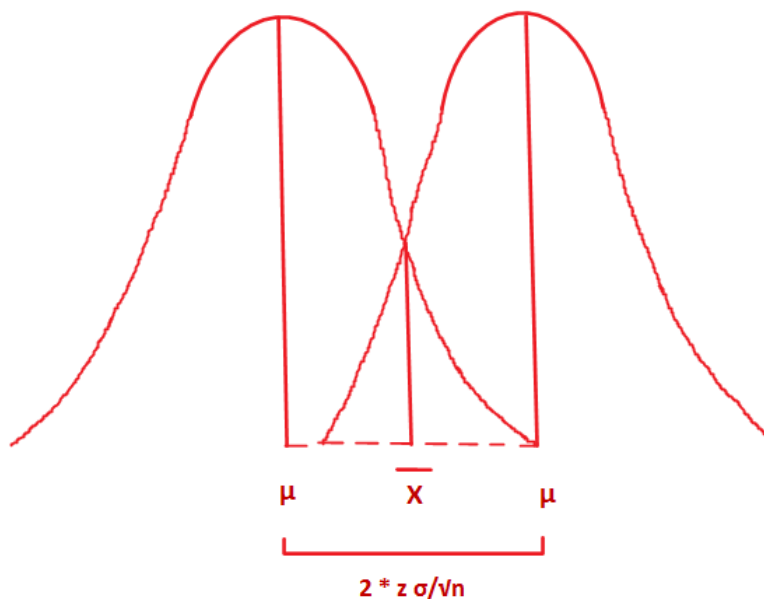
(79.3 , 80.7)

Q. What conclusion can be drawn from this observation?

In the long run, 95 out of 100 times, this confidence interval with contain the true avg. height of keyboard.

Q. Calculate a 99% C.I. for the average keyboard table height.

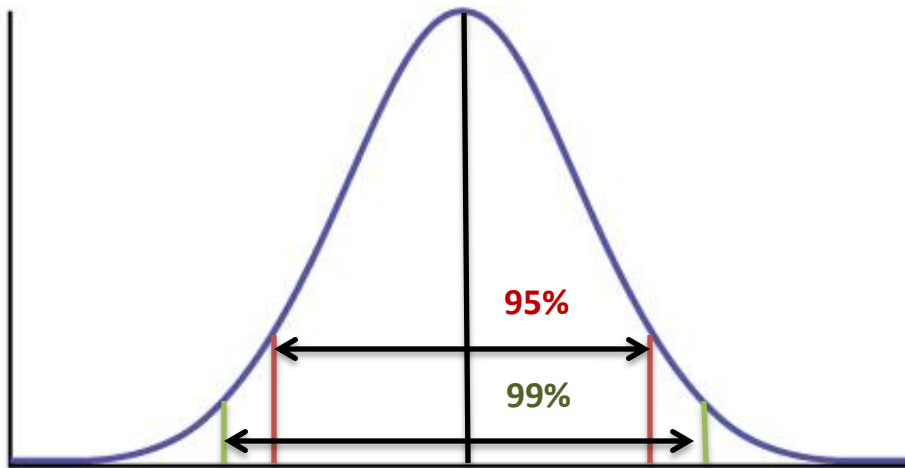
(79.07 , 80.92)



A 100 (1- α) % C.I. for mean of a normal population when standard deviation is known is given by

$$\bar{X} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Q. Why should one settle for 95% C. I. when a 99% C I. is achievable?



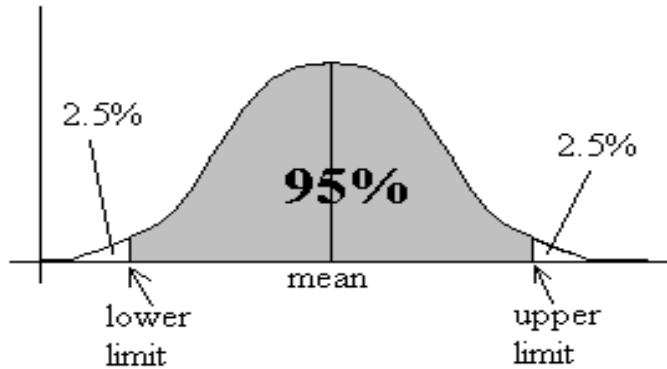
Since a high Confidence level interval would have a wider interval thus making less precision.

$$2 * Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

A highly reliable interval would be less precise while a relatively less reliable interval would have a greater precision.

Q. What is the width of the above 95% confidence interval?

1.40



Q. The manager wants to assure that the width of the 95% confidence interval be at most 1.2. How could he achieve this?

By fixing the size of n such that the width is at most 1.2

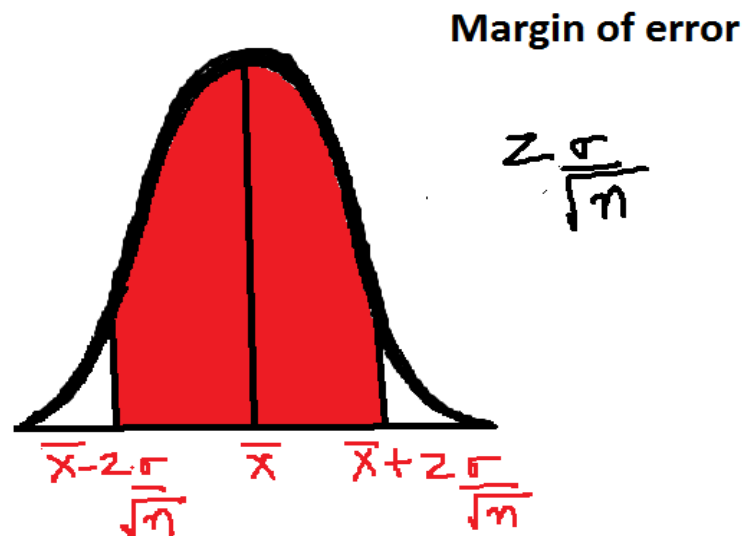
Q. What should be the size of n such that the width is at most 1.2?

$$n = \left(2 * z_{\alpha/2} \frac{\sqrt{\sigma}}{\sqrt{w}} \right)^2$$

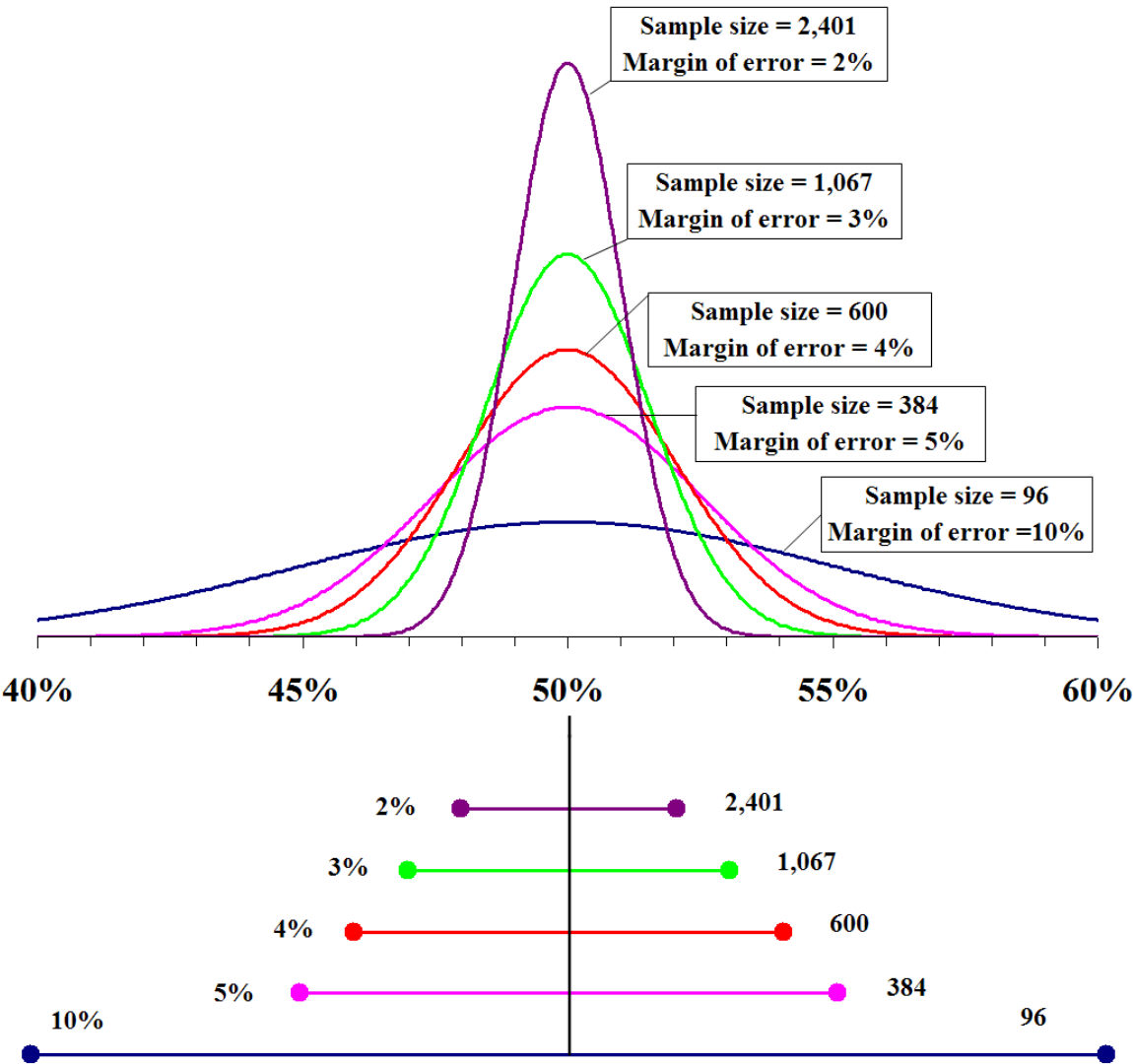
So, a sample of 43 typists needs to be selected.

Q. Which of the following statements is correct?

- a) C.I. are based on population data & give an estimated value for a parameter
- b) C.I. are based on sample data & give a range of plausible values for a parameter
- c) C.I. are based on population data & give a range of plausible values for a parameter
- d) C.I. are based on sample data & give an estimated value for a parameter

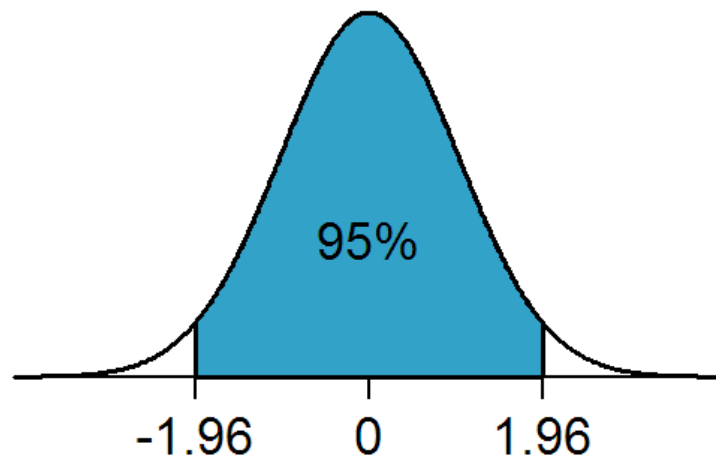


Margin of Error is the range of values that lie above or below the sample statistics.



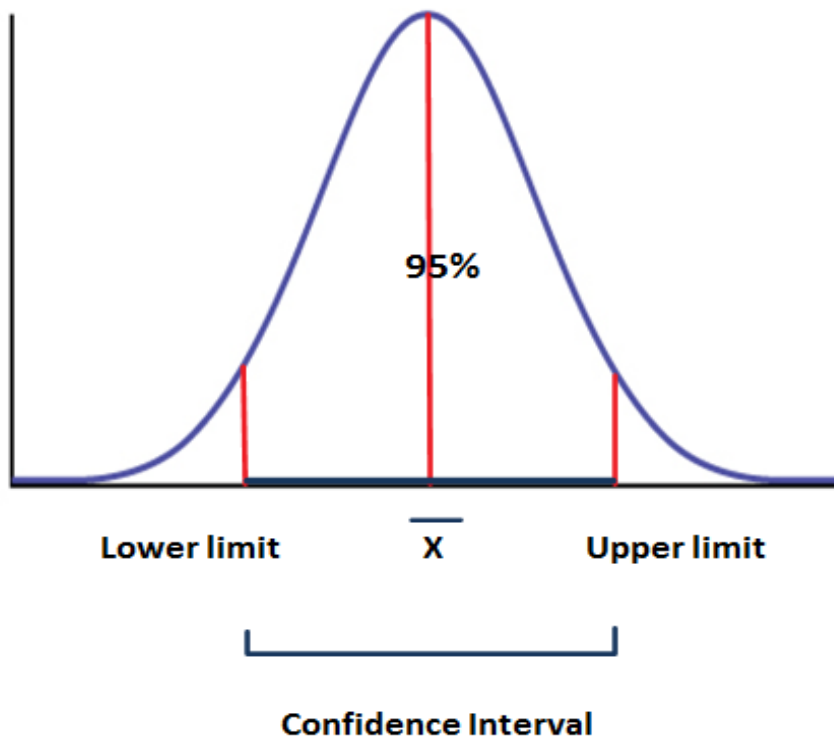
A study reported that for a sample of 50 kitchens with gas cooking applications monitored during a one week period, the sample mean CO₂ level (ppm) was 654, and sample standard deviation was 164 (ppm).

Q. Calculate and interpret 95% C.I. for true average CO₂ level in the population of all homes from which sample was selected.



Q. Will the range of C.I. increase or decrease if we increase the sample size to 100?

Decrease



Q. What is the z score value for a confidence level of 99.7%?

3

Q. What is the confidence level for the interval

$$\bar{X} \pm 2.81 * \frac{\sigma}{\sqrt{n}}$$

99.5%

Q. Response time to a particular editing command in a computer is normally distributed with $\sigma = 25$ millisec.

A new operating system has been installed and we want to estimate the true average response time μ for new system.

The new system is assumed to be normally distributed with $\sigma = 25$, what sample size is necessary to ensure that the resulting 95% C.I. has a width of at most 10?

96

Q. A study reports that the average monthly cost of living for a four person family in Delhi (without rent) is 83000.

While a sample of some families residing in Saket area of Delhi was taken and the average cost of living for that sample was found to be 200000.

What will be the point estimate for average cost of living for a four person family in Saket?

200000

Q. Would the population mean of average cost of living in Saket be Rs. 200000?

No