

## Notes for Students – Lesson 14

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Two variables

→ Different design

- How we collect our data
- How we visualize our data

→ Analysis is similar

Two Variables that are probably related

Height and weight

Time spent studying and your grade

Temperature of the weather outside and ankle injuries

Q1.

$x$ Miles on a car	$y$ Value of the car
60,000	\$ 12,000
80,000	\$ 10,000
90,000	\$ 9,000
100,000	\$ 7,500
120,000	\$ 6,000

$X$ : predictor, explanatory,  
independent variable

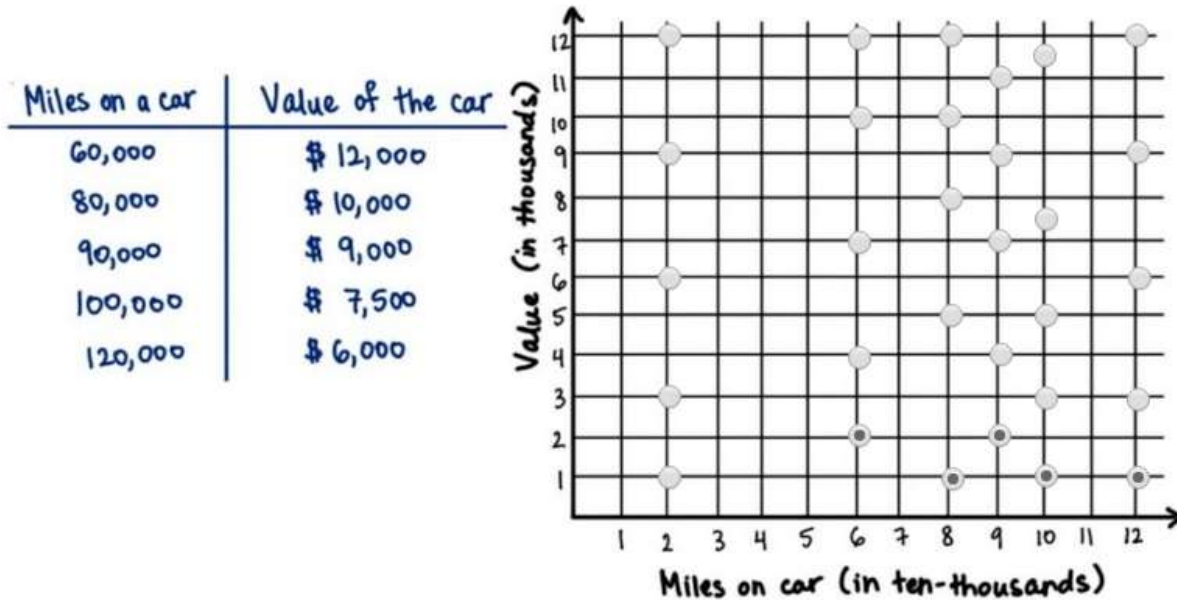
$Y$ : ☐ outcome  
☐ determiner  
☐ response  
☐ stand-alone  
☐ dependent

Q2.

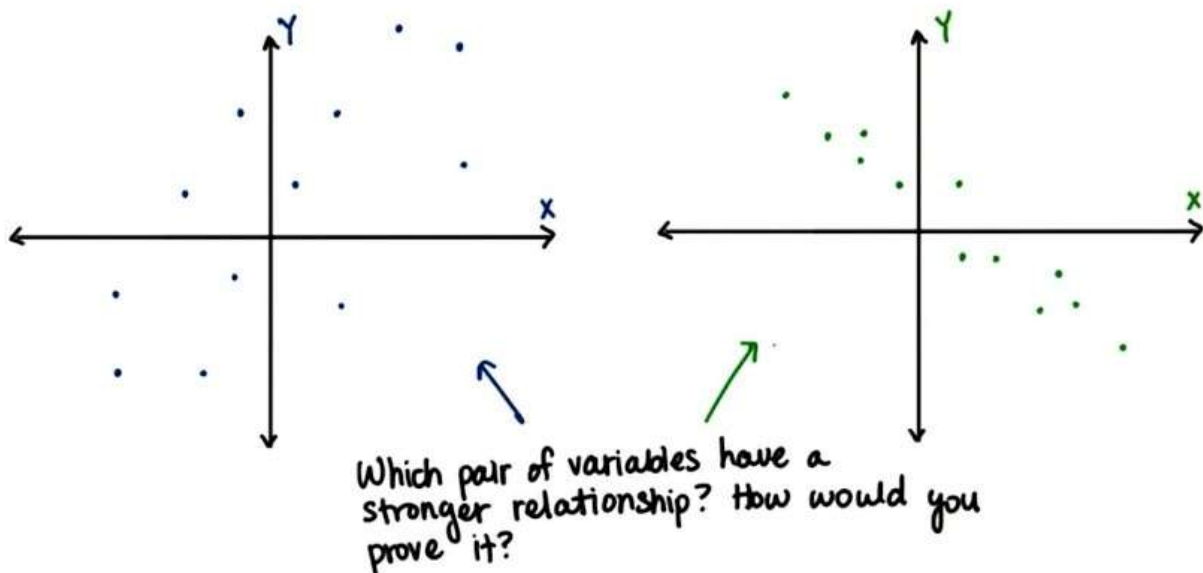
Miles on a car	Value of the car
60,000	\$ 12,000
80,000	\$ 10,000
90,000	\$ 9,000
100,000	\$ 7,500
120,000	\$ 6,000

How can we show that  
these variables have a  
relationship?

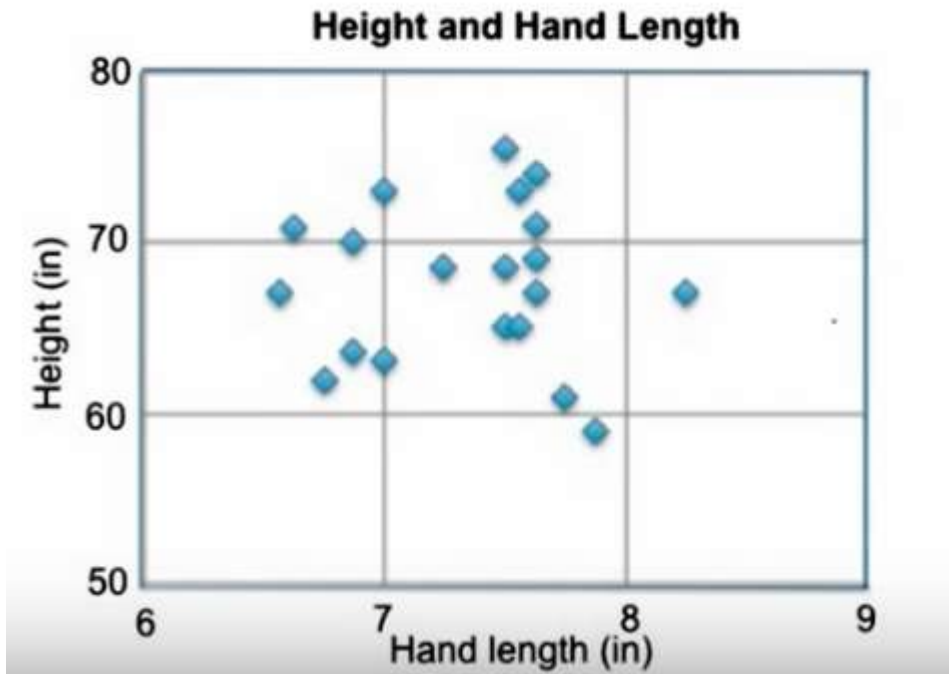
### Q3. Let's draw a scatter plot !!



### Q4.

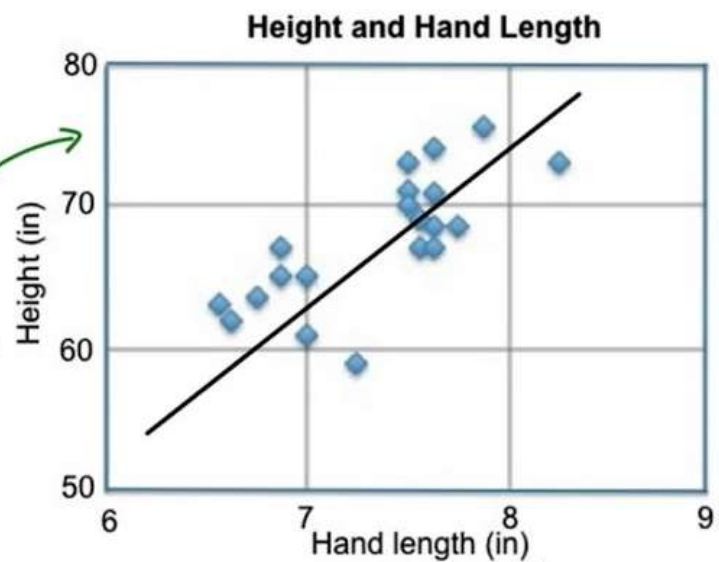


Q5.



A strong relationship has direction.

As  $x$  increases,  $Y$   
o increases  
o decreases

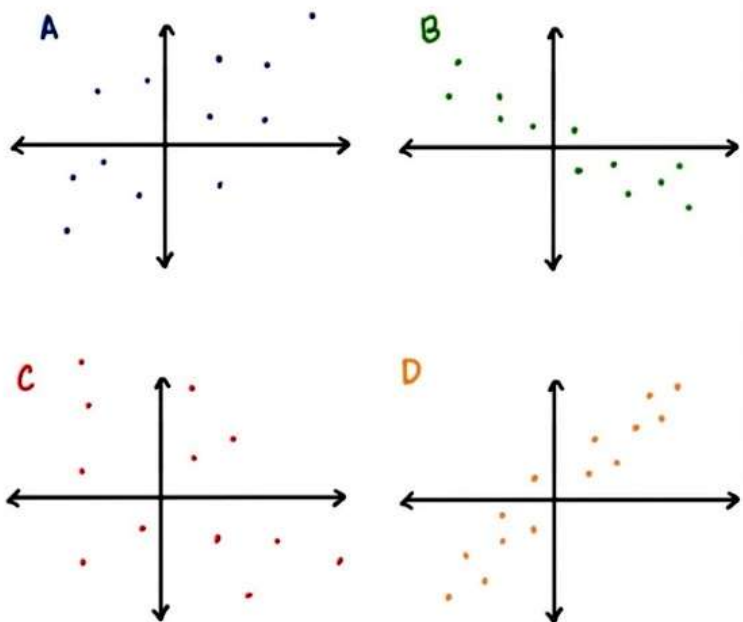


Q6.

We often refer to the relationship between two variables as the correlation. If two variables have a strong relationship, then they have a high correlation. We can also say strong correlation.

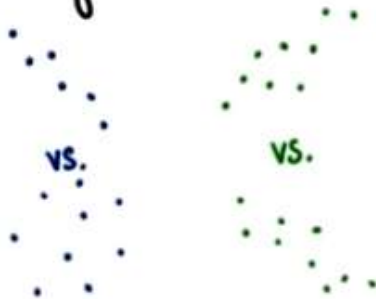
Which of these has a strong positive correlation? Or strong in negative? Which shows a weak and positive correlation and finally which is a weak but negative correlation. Put the appropriate letter in the boxes here on the left.

- \_\_\_ Strong, positive
- \_\_\_ Strong, negative
- \_\_\_ Weak, positive
- \_\_\_ Weak, negative



Correlation Coefficient

## Strength & Direction of Relationships

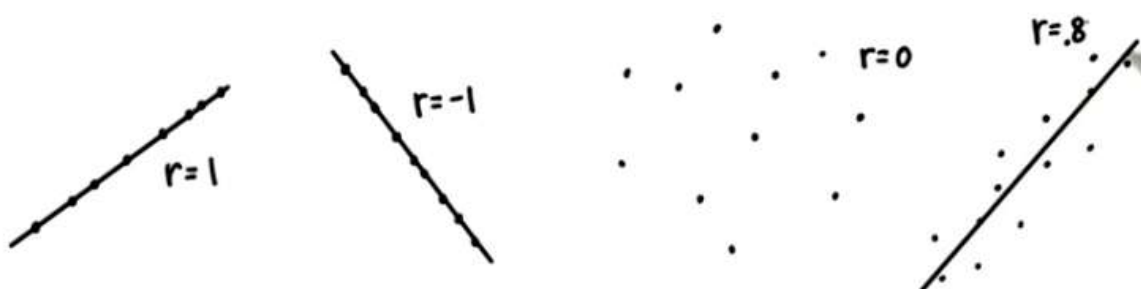


Quantify relationship with **Pearson's r** correlation coefficient ( $r$ ).  
↓

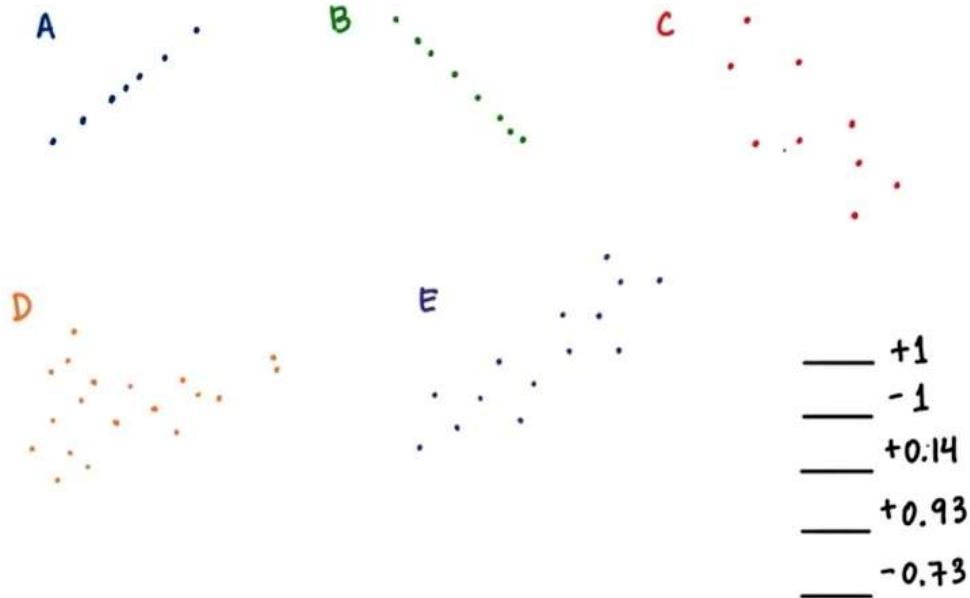
$$r = \frac{\text{cov}(X, Y)}{S_x \cdot S_y} = \frac{\text{cov}_{X,Y}}{S_x \cdot S_y} \quad (\text{not a percentage})$$

$r^2$  = % of the variation in  $Y$  explained by the variation in  $X$ .

$$r = \frac{1}{n-1} \left( \frac{\sum_x \sum_y (x - \bar{x})(y - \bar{y})}{s_x s_y} \right)$$



Q7. Match with r.



Q8.

What are the correlation coefficients?

X = Age in years

Y = Age in months

r =



Q9.

What are the correlation coefficients?

X = Hours awake

Y = Hours asleep

$r =$

Calculating r in Excel

Q10.

=PEARSON(start cell for variable x: end cell for variable x,  
start cell for variable y: end cell for variable y)

Click link below to see poll results.

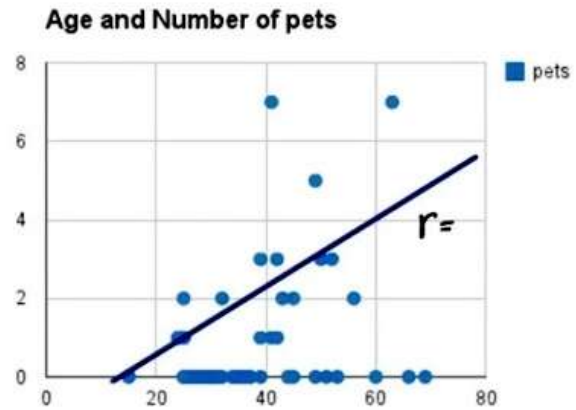
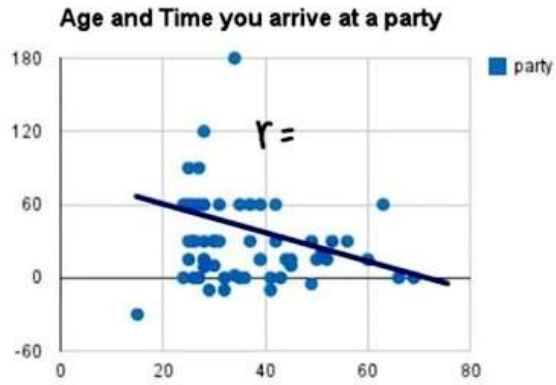
- Age, when you arrive at a party
- Age, number of pets

Create a scatterplot for each. Does it look like there is a relationship?

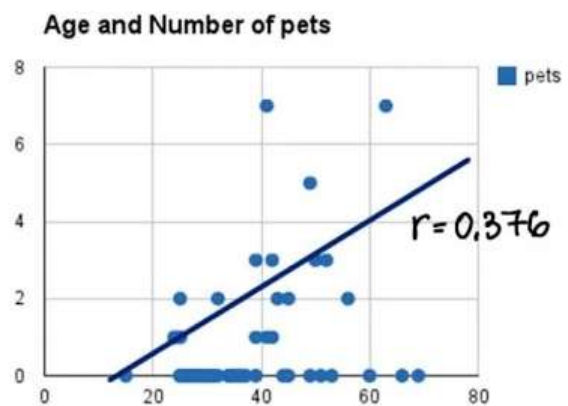
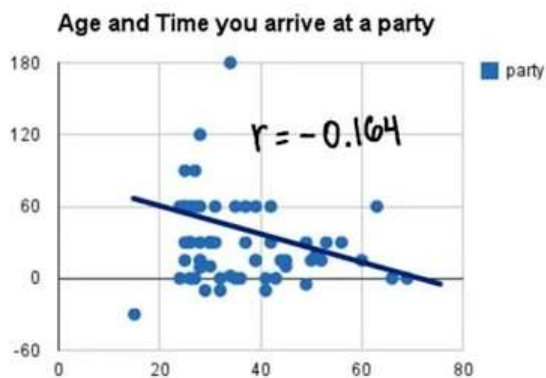
<https://docs.google.com/spreadsheets/d/14Av6vXwfcC166cN6MoGyYg97KD8Tv1mGQyWgTBHr9go/edit?usp=sharing>



Q11.

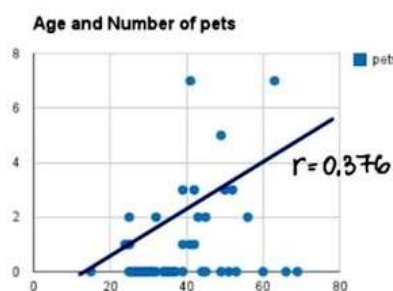
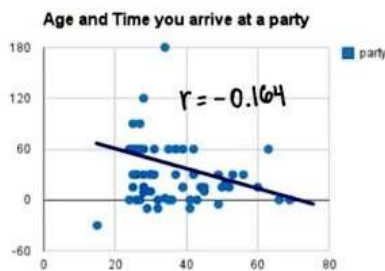


Q12. Which one is stronger ??



## Q13.

Even though two variables might appear to have a relationship if we look at the sample data, this could just be due to chance. Just because our sample data works like it has a relationship, the actual population of people might not or vice versa. Our sample data might not have a relationship but the population actually does. Things like this naturally happen with sampling error as you've seen before. And we are surer as the sample size increases. The question is how much variation due to sampling error is allowable before we decide that our results are not due to chance? Again, we do Hypothesis Testing. Now,  $r$  is the correlation we get from our sample data.  $\rho$  is the true correlation for the population. Given this what do you think the null and alternative hypothesis are?



$\rho$  ( $\rho$ ) = true correlation for population.

### Hypothesis Testing

$$\begin{aligned} H_0: r &= 0 \\ H_A: r &< 0 \\ &r > 0 \\ &r \neq 0 \end{aligned}$$

$$\begin{aligned} H_0: \rho &= 0 \\ H_A: \rho &< 0 \\ &\rho > 0 \\ &\rho \neq 0 \end{aligned}$$

$$\begin{aligned} H_0: \rho &< 0 \\ &\rho > 0 \\ &\rho \neq 0 \\ H_A: \rho &= 0 \end{aligned}$$

$$\begin{aligned} H_0: r &< 0 \\ &r > 0 \\ &r \neq 0 \\ H_A: r &= 0 \end{aligned}$$

## Q14.

Hypothesis testing about the true population correlation is a type of t test. It uses the t distribution. Since we already know all about t tests, we know when to reject or fail to reject the null.

### Example :-

#### Hypothesis Testing

$$H_0: \rho = 0$$

$$H_A: \rho < 0$$

$$\rho > 0$$

$$\rho \neq 0$$

$\rho$  (rho) = true correlation for population.

$$t = \frac{r\sqrt{N-2}}{\sqrt{1-r^2}} \quad df = N-2$$

	X	Y
1		
2		
...		
N		

25 participants ( $N=25$ )  
Non-directional test ( $\alpha=0.05$ )  
 $t=2.71$

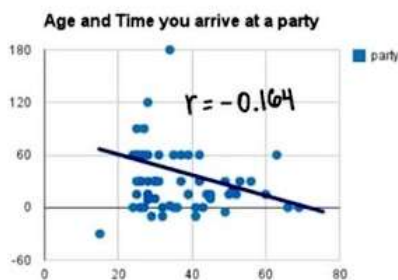
There <sup>is</sup> <sub>is not enough</sub> evidence to reject the null;

there <sup>is</sup> <sub>is not</sub> a significant relationship between x and y.

## Q15.

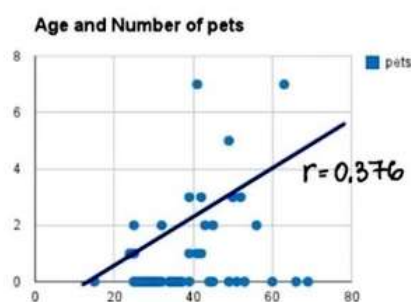
Since we know that the sample statistics often aren't exactly what the population parameters are, we might want a confidence interval for the true correlation. In other words, a range of likely values for the population correlation coefficient rho.

For the correlation between age and time you arrive at a party, the 95% confidence interval went from negative 0.3995, to positive 0.0914. And for age and number of pets, this ranged from 0.1369 to 0.5733. Based on these Confidence Intervals, what could you logically conclude? What would be your statistical decision for each?



95% CI =  
(-0.3995, 0.0914)

- Reject the null
- Fail to reject the null



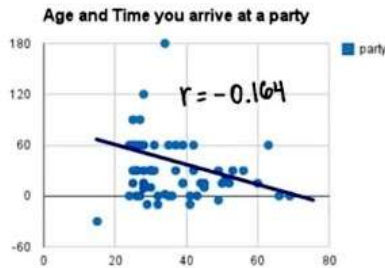
95% CI =  
(0.1369, 0.5733)

- Reject the null
- Fail to reject the null

$$H_0: \rho = 0$$
$$H_A: \rho < 0$$
$$\rho > 0$$
$$\rho \neq 0$$

$\rho$  (rho) = true  
correlation for  
population.

## Q16.



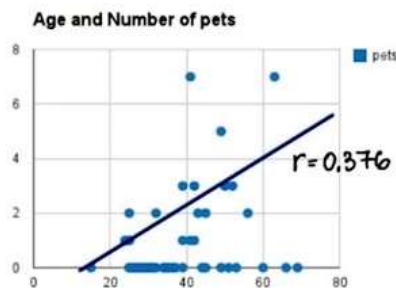
95% CI =

(-0.3995, 0.0914)

Fail to reject the null

$t = -1.2787$   $df = 59$

$p =$



95% CI =

(0.1369, 0.5733)

Reject the null

$t = 3.1146$   $df = 59$

$p =$

$H_0: \rho = 0$

$H_A: \rho < 0$

$\rho > 0$

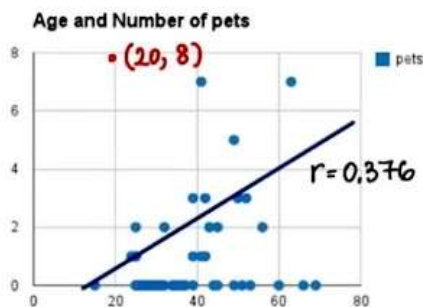
$\rho \neq 0$

$\rho$  (rho) = true correlation for population.

$\alpha = 0.05$

## OUTLIER

## Q17.



95% CI =

(0.1369, 0.5733)

Reject the null

$t = 3.1146$   $df = 59$

$p = 0.0028$

What is the new correlation coefficient when we add the point (20, 8) to the data?

# Dimensionless Technologies Private Limited

Visit us at: [www.dimensionless.in](http://www.dimensionless.in)



- [info@dimensionless.in](mailto:info@dimensionless.in)



- 9923170071, 8108094992

