LadderLeak: Breaking ECDSA With Less Than One Bit Of Nonce Leakage

To appear in CCS '20
ePrint: 2020/615

Diego F. Aranha[1] Felipe R. Novaes[2] Akira Takahashi[1] Mehdi Tibouchi[3] Yuval Yarom[4]

[1]Dept. of CS and DIGIT, Aarhus University, Denmark

[2]University of Campinas, Brazil

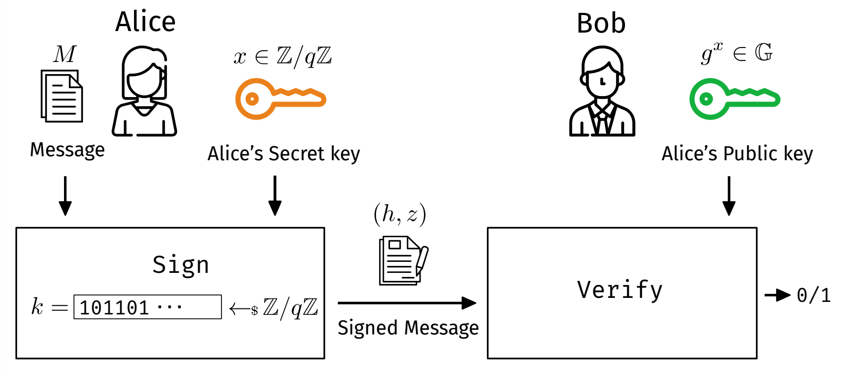
[3]NTT Corporation, Japan

[4]University of Adelaide and Data61, Australia

Attacks on ECDSA “nonce”

- ECDSA/Schnorr: Most popular signature schemes relying on the hardness of the (EC)DLP
- Signing operation involves **secret** randomness $k \in \mathbb{Z}_q$, sometimes called “**nonce**”
- Long history of research on the attacks against $k \ldots$

Randomness in ECDSA/Schnorr-type Schemes

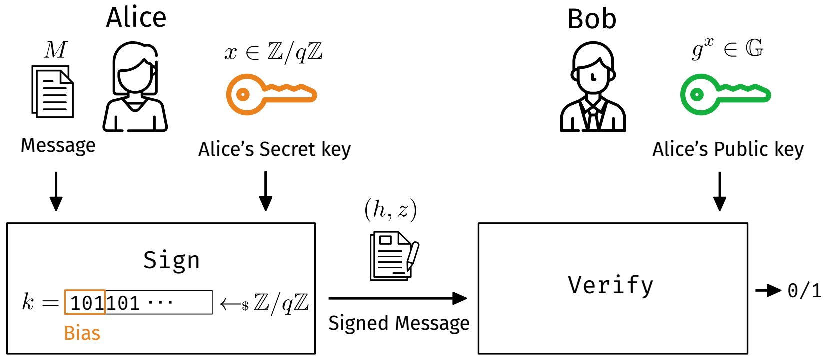


- k is a uniformly random value satisfying

$$k \equiv \underbrace{z}_{\text{public}} + \underbrace{h}_{\text{public}} \cdot x \mod q.$$

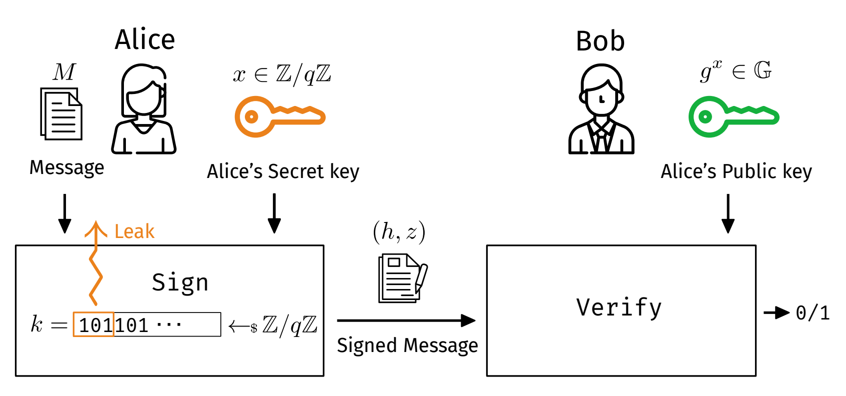
- k should **NEVER** be reused/exposed as $x = (z - z')/(h' - h) \mod q$

Risk of Biased/Leaky Randomness



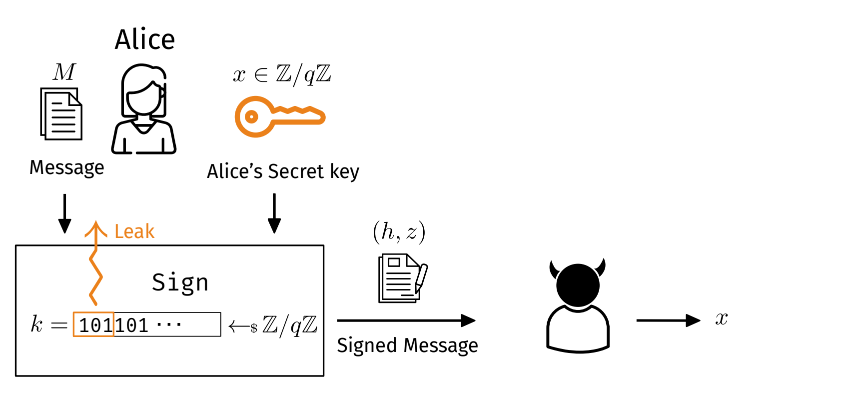
- What if k is slightly biased ?
- Secret key x is recovered by solving the **hidden number problem (HNP)**

Risk of Biased/Leaky Randomness



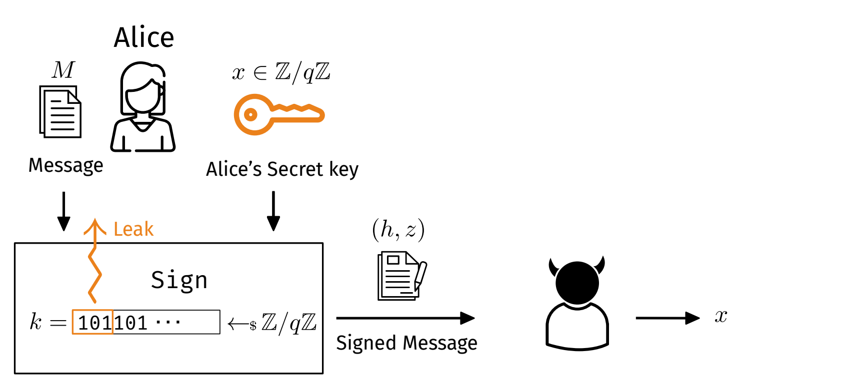
- What if k is slightly biased or partially leaked?
- Secret key x is recovered by solving the **hidden number problem (HNP)**

Risk of Biased/Leaky Randomness



- What if k is slightly biased or partially leaked? $\leadsto$ Attack!
- Secret key x is recovered by solving the **hidden number problem (HNP)**

Risk of Biased/Leaky Randomness



- What if k is slightly biased or partially leaked? $\rightsquigarrow$ Attack!
- Secret key x is recovered by solving the **hidden number problem (HNP)**

Randomness Failure in the Real World

- Poorly designed/implemented RNGs
- Predictable seed (`srand(time(0))`)
- VM resets $\rightsquigarrow$ same snapshot will end up with the same seed
- Side-channel leakage
- and many more...


BBC Sign in News Sport Weather Shop Earth Travel

NEWS

Home UK World Business Politics Tech Science Health Family & Education

Technology

iPhone hacker publishes secret Sony PlayStation 3 key

By Jonathan Fildes
Technology reporter, BBC News

6 January 2011

Share

The PlayStation 3's security has been broken by hackers, potentially allowing anyone to run any software - including pirated games - on the console.

A collective of hackers recently showed off a method that could force the system to reveal secret keys used to load

BBC news. 2011. https://www.bbc.com/news/technology-12116051

Randomness Failure in the Real World

- Poorly designed/implemented RNGs
- Predictable seed (`srand(time(0))`)
- VM resets $\rightsquigarrow$ same snapshot will end up with the same seed
- Side-channel leakage
- and many more. . .


BBC | Sign in | News | Sport | Weather | Shop | Earth | Travel | Mo

NEWS

Home | UK | World | Business | Politics | Tech | Science | Health | Family & Education | E

Technology

iPhone hacker publishes secret Sony PlayStation 3 key

By Jonathan Fildes
Technology reporter, BBC News

6 January 2011

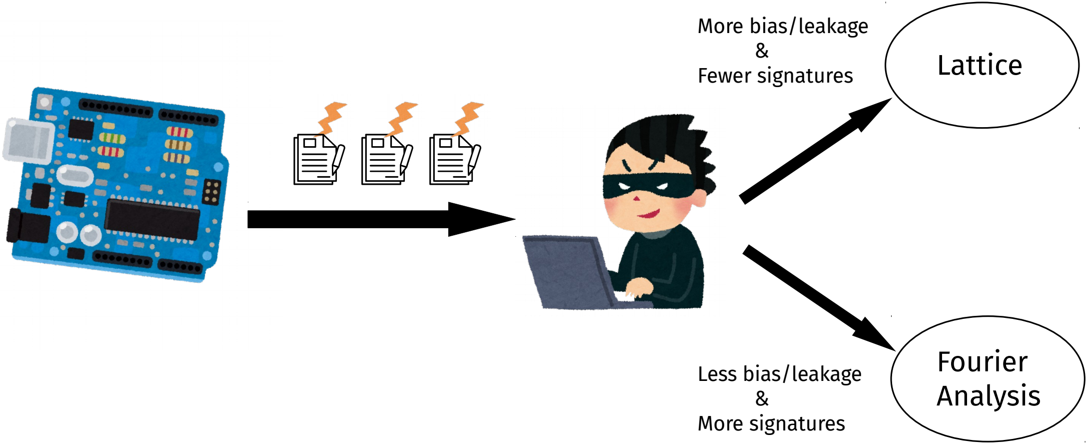
Share

The PlayStation 3's security has been broken by hackers, potentially allowing anyone to run any software - including pirated games - on the console.

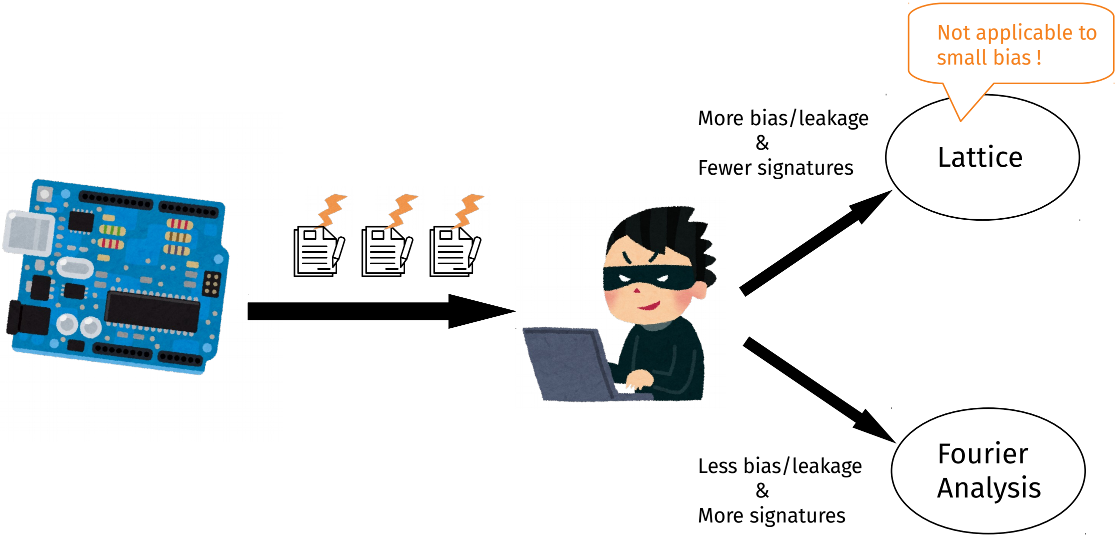
A collective of hackers recently showed off a method that could force the system to reveal secret keys used to load

BBC news. 2011. https://www.bbc.com/news/technology-12116051

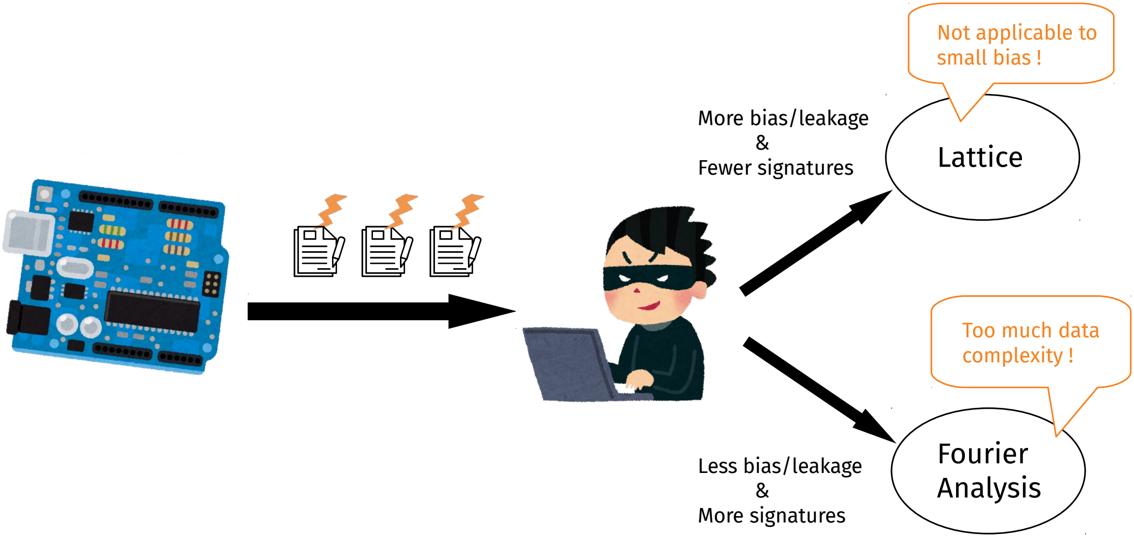
How to solve the HNP



How to solve the HNP



How to solve the HNP



Questions

- Can we reduce the data complexity of Fourier analysis-based attack?
- Can we attack even **less than 1-bit of nonce leakage** (= MSB is only leaked with prob. < 1)?
- Can we obtain such a small leakage from practical ECDSA implementations?

YES!

Questions

- Can we reduce the data complexity of Fourier analysis-based attack?
- Can we attack even **less than 1-bit of nonce leakage** (= MSB is only leaked with prob. < 1)?
- Can we obtain such a small leakage from practical ECDSA implementations?

YES!

Questions

- Can we reduce the data complexity of Fourier analysis-based attack?
- Can we attack even **less than 1-bit of nonce leakage** (= MSB is only leaked with prob. < 1)?
- Can we obtain such a small leakage from practical ECDSA implementations?

YES!

Questions

- Can we reduce the data complexity of Fourier analysis-based attack?
- Can we attack even **less than 1-bit of nonce leakage** (= MSB is only leaked with prob. < 1)?
- Can we obtain such a small leakage from practical ECDSA implementations?

YES!

Summary of results

1. Novel class of cache attacks against the Montgomery ladder scalar multiplication in OpenSSL `1.0.2u` and `1.1.0l`, and RELIC 0.4.0.
   - **Affected curves:** NIST P-192, P-224, P-256 (not by default in OpenSSL), P-384, P-521, B-283, K-283, K-409, B-571, `sect163r1`, `secp192k1`, `secp256k1`
   - **Affected products (?):** VMWare Photon, Chef, Wickr
2. Theoretical improvements to Fourier analysis-based attack on the HNP (originally by Bleichenbacher)
   - Significantly reduced the required input data
   - Analysis in the presence of erroneous leakage information
3. Implemented a full secret key recovery attack against OpenSSL ECDSA over `sect163r1` and NIST P-192.

Summary of results

1. Novel class of cache attacks against the Montgomery ladder scalar multiplication in OpenSSL `1.0.2u` and `1.1.0l`, and RELIC 0.4.0.
    - **Affected curves:** NIST P-192, P-224, P-256 (not by default in OpenSSL), P-384, P-521, B-283, K-283, K-409, B-571, `sect163r1`, `secp192k1`, `secp256k1`
    - **Affected products (?):** VMWare Photon, Chef, Wickr
2. Theoretical improvements to Fourier analysis-based attack on the HNP (originally by Bleichenbacher)
    - Significantly reduced the required input data
    - Analysis in the presence of erroneous leakage information
3. Implemented a full secret key recovery attack against OpenSSL ECDSA over `sect163r1` and NIST P-192.

Summary of results

1. Novel class of cache attacks against the Montgomery ladder scalar multiplication in OpenSSL `1.0.2u` and `1.1.0l`, and RELIC 0.4.0.
   - **Affected curves:** NIST P-192, P-224, P-256 (not by default in OpenSSL), P-384, P-521, B-283, K-283, K-409, B-571, `sect163r1`, `secp192k1`, `secp256k1`
   - **Affected products (?):** VMWare Photon, Chef, Wickr
2. Theoretical improvements to Fourier analysis-based attack on the HNP (originally by Bleichenbacher)
   - Significantly reduced the required input data
   - Analysis in the presence of erroneous leakage information
3. Implemented a full secret key recovery attack against OpenSSL ECDSA over `sect163r1` and NIST P-192.

New attack records for the HNP!

Comparison with the previous records of solutions to HNP: Fourier analysis vs Lattice

	< 1	1	2	3	4
384-bit	—	—	—	[CABuH+19]	[DHMP13]
256-bit	—	—	[TTA18]	[TTA18],[AGB20]	[PGB17, DDE+18, Rya18] [Rya19, MSEH19, WSBS20]
192-bit	This work	This work	—	—	—
160-bit	This work	This work (less data), [AFG+14, Ble05]	[LN13]	[NS02]	—

- Require fewer input signatures to attack 160-bit HNP with 1-bit leak!
- First attack records for 192-bit HNP with (less than) 1-bit leak!

How to acquire ECDSA nonce

ECDSA signing

Scalar multiplication is critical for performance/security of ECC.

Algorithm ECDSA signature generation

Input: $sk \in \mathbb{Z}_q$, $\texttt{msg} \in \{0,1\}^*$

Output: A valid signature (r, s)

1: $k \leftarrow_\$ \mathbb{Z}_q^*$
2: $R = (r_x, r_y) \leftarrow [k]P$
3: $r \leftarrow r_x \mod q$
4: $s \leftarrow (H(\texttt{msg}) + r \cdot sk)/k \mod q$
5: **return** (r, s)

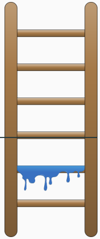
Critical: $[k]P$ should be **constant time** to avoid timing leakage about k.

LadderLeak: Tiny timing leakage from the Montgomery ladder

Algorithm Montgomery ladder

Input: $P = (x, y)$, $k = (1, k_{t-2}, \ldots, k_1, k_0)$
Output: $Q = [k]P$

$k' \leftarrow$ Select $(k + q, k + 2q)$
$R_0 \leftarrow P$, $R_1 \leftarrow [2]P$
for $i \leftarrow \lg(q) - 1$ **downto** 0 **do**
    Swap (R_0, R_1) if $k'_i = 0$
    $R_0 \leftarrow R_0 \oplus R_1$; $R_1 \leftarrow 2R_1$
    Swap (R_0, R_1) if $k'_i = 0$
end for
return $Q = R_0$



Conditions for the attack to work:

- Accumulators (R_0, R_1) are in **projective coordinates**, but initialized with the base point in **affine coordinates**.
- Group order is $2^n - \delta$
- Group law is non-constant time wrt handling Z coordinates $\rightsquigarrow$ **Weierstrass model**

Experiments were carried out with Flush+Reload cache attack technique

$\rightsquigarrow$ MSB of k was detected with $> 99\,\%$ accuracy.

Software countermeasures & coordinated disclosure

There are **at least** three possible fixes:

1. **Randomize** Z coordinates at the beginning of scalar multiplication.
2. Implement group law in constant time, for example using **complete addition formulas** (no branches).
3. Implement ladder over co-Z arithmetic to **not handle** Z directly.

Coordinated disclosure: reported in December 2019 (before EOL of OpenSSL `1.0.2`), fixed in April 2020 with the first countermeasure.

How to exploit ECDSA nonce bias

Bleichenbacher's Attack: High-level Overview

- Step 1. Quantify the modular bias of randomness $k \leftarrow K$
  - $\text{Bias}_q(K) \approx 0$ if k is uniform in $\mathbb{Z}_q$
  - $\text{Bias}_q(K) \approx 1$ if k is biased in $\mathbb{Z}_q$
  - Contribution-1 Analyzed the behavior $\text{Bias}_q(K)$ when k's MSB is biased with probability < 1!
- Step 2. Find a candidate secret key which leads to the peak of $\text{Bias}_q(K)$ (by computing FFT)
- Critical intermediate step: collision search of integers h
  - Detect the bias peak correctly and efficiently
  - Contribution-2 Established **unified time-memory-data tradeoffs** by applying $\mathcal{K}$-list sum algorithm for the GBP!

Bleichenbacher's Attack: High-level Overview

- Step 1. Quantify the modular bias of randomness $k \leftarrow K$
  - $\mathrm{Bias}_q(K) \approx 0$ if k is uniform in $\mathbb{Z}_q$
  - $\mathrm{Bias}_q(K) \approx 1$ if k is biased in $\mathbb{Z}_q$
  - Contribution-1 Analyzed the behavior $\mathrm{Bias}_q(K)$ when k's MSB is biased with probability < 1!
- Step 2. Find a candidate secret key which leads to the peak of $\mathrm{Bias}_q(K)$ (by computing FFT)
- Critical intermediate step: collision search of integers h
  - Detect the bias peak correctly and efficiently
  - Contribution-2 Established **unified time-memory-data tradeoffs** by applying $\mathcal{K}$-list sum algorithm for the GBP!

Bleichenbacher's Attack: High-level Overview

- Step 1. Quantify the modular bias of randomness $k \leftarrow K$
  - $\text{Bias}_q(K) \approx 0$ if k is uniform in $\mathbb{Z}_q$
  - $\text{Bias}_q(K) \approx 1$ if k is biased in $\mathbb{Z}_q$
  - Contribution-1 Analyzed the behavior $\text{Bias}_q(K)$ when k's MSB is biased with probability < 1!
- Step 2. Find a candidate secret key which leads to the peak of $\text{Bias}_q(K)$ (by computing FFT)
- Critical intermediate step: collision search of integers h
  - Detect the bias peak correctly and efficiently
  - Contribution-2 Established **unified time-memory-data tradeoffs** by applying $\mathcal{K}$-list sum algorithm for the GBP!

Tradeoff Graphs for 1-bit Bias

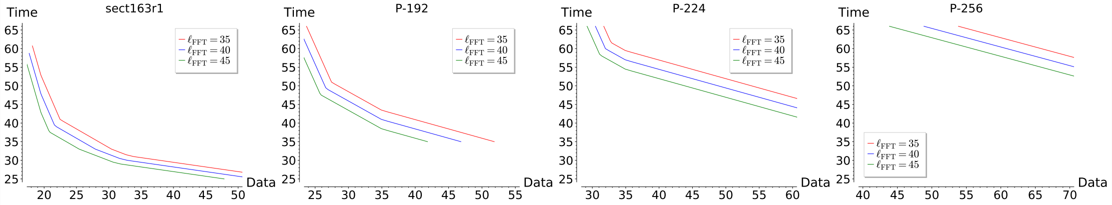


Figure 1: Time–Data tradeoffs when memory is fixed to 2^{35}.

* Optimized data complexity by solving the linear programming problem
* Paper has various tradeoff graphs and improved complexity estimates for 2-3 bits bias

Experimental Results on Full Key Recovery

Target	Facility	Error rate	Input	Output	Thread (Collision)	Time (Collision)	RAM (Collision)	L_{FFT}	Recovered MSBs
NIST P-192	AWS EC2	0	2^{29}	2^{29}	96×24	113h	492GB	2^{38}	39
NIST P-192	AWS EC2	1%	2^{35}	2^{30}	96×24	52h	492GB	2^{37}	39
`sect163r1`	Cluster	0	2^{23}	2^{27}	16×16	7h	80GB	2^{35}	36
`sect163r1`	Workstation	2.7%	2^{24}	2^{29}	48	42h	250GB	2^{34}	35

- Attack on **P-192** is made possible by our highly optimized parallel implementation.
- Attack on `sect163r1` is even feasible with a laptop.
- Recovering remaining bits is much cheaper in Bleichenbacher's framework.
- Attacks on P-224 with 1-bit bias or P-256 with 2-bit bias are also tractable.

Main takeaways

- Securely implementing brittle cryptographic algorithms is still **hard**.
- **Don't** underestimate even less than 1-bit of nonce leakage!
- Interesting connection between the HNP and GBP (from symmetric key crypto)
- Open questions:
  - More list sum algorithms and tradeoffs?
  - Improvements to FFT computation?
  - Other sources of small leakage?

Thank you! & Questions?

More details at `https://ia.cr/2020/615`

Main takeaways

- Securely implementing brittle cryptographic algorithms is still **hard**.
- **Don't** underestimate even less than 1-bit of nonce leakage!
- Interesting connection between the HNP and GBP (from symmetric key crypto)
- Open questions:
  - More list sum algorithms and tradeoffs?
  - Improvements to FFT computation?
  - Other sources of small leakage?

Thank you! & Questions?

More details at `https://ia.cr/2020/615`

Main takeaways

- Securely implementing brittle cryptographic algorithms is still **hard**.
- **Don't** underestimate even less than 1-bit of nonce leakage!
- Interesting connection between the HNP and GBP (from symmetric key crypto)
- Open questions:
  - More list sum algorithms and tradeoffs?
  - Improvements to FFT computation?
  - Other sources of small leakage?

Thank you! & Questions?

More details at `https://ia.cr/2020/615`

Main takeaways

- Securely implementing brittle cryptographic algorithms is still **hard**.
- **Don't** underestimate even less than 1-bit of nonce leakage!
- Interesting connection between the HNP and GBP (from symmetric key crypto)
- Open questions:
  - More list sum algorithms and tradeoffs?
  - Improvements to FFT computation?
  - Other sources of small leakage?

Thank you! & Questions?

More details at `https://ia.cr/2020/615`

Main takeaways

- Securely implementing brittle cryptographic algorithms is still **hard**.
- **Don't** underestimate even less than 1-bit of nonce leakage!
- Interesting connection between the HNP and GBP (from symmetric key crypto)
- Open questions:
  - More list sum algorithms and tradeoffs?
  - Improvements to FFT computation?
  - Other sources of small leakage?

Thank you! & Questions?

More details at `https://ia.cr/2020/615`

- Diego F. Aranha, Pierre-Alain Fouque, Benoît Gérard, Jean-Gabriel Kammerer, Mehdi Tibouchi, and Jean-Christophe Zapalowicz.
  GLV/GLS decomposition, power analysis, and attacks on ECDSA signatures with single-bit nonce bias.
  In Palash Sarkar and Tetsu Iwata, editors, *ASIACRYPT 2014, Part I*, volume 8873 of *LNCS*, pages 262–281. Springer, Heidelberg, December 2014.
- Alejandro Cabrera Aldaya, Cesar Pereida García, and Billy Bob Brumley.
  From a to z: Projective coordinates leakage in the wild.
  Cryptology ePrint Archive, Report 2020/432, 2020.
  `https://eprint.iacr.org/2020/432`.

Daniel Bleichenbacher.
Experiments with DSA.
Rump session at CRYPTO 2005, 2005.
Available from https://www.iacr.org/conferences/crypto2005/r/3.pdf.

Dan Boneh and Ramarathnam Venkatesan.
Hardness of computing the most significant bits of secret keys in Diffie-Hellman and related schemes.
In Neal Koblitz, editor, *CRYPTO'96*, volume 1109 of *LNCS*, pages 129–142. Springer, Heidelberg, August 1996.

Alejandro Cabrera Aldaya, Billy Bob Brumley, Sohaib ul Hassan, Cesar Pereida García, and Nicola Tuveri.
Port contention for fun and profit.
In *2019 IEEE Symposium on Security and Privacy*, pages 870–887. IEEE Computer Society Press, May 2019.

Fergus Dall, Gabrielle De Micheli, Thomas Eisenbarth, Daniel Genkin, Nadia Heninger, Ahmad Moghimi, and Yuval Yarom.
CacheQuote: Efficiently recovering long-term secrets of SGX EPID via cache attacks.
IACR TCHES, 2018(2):171–191, 2018.
https://tches.iacr.org/index.php/TCHES/article/view/879.

Elke De Mulder, Michael Hutter, Mark E. Marson, and Peter Pearson.
Using Bleichenbacher's solution to the hidden number problem to attack nonce leaks in 384-bit ECDSA.
In Guido Bertoni and Jean-Sébastien Coron, editors, *CHES 2013*, volume 8086 of *LNCS*, pages 435–452. Springer, Heidelberg, August 2013.

Itai Dinur.
An algorithmic framework for the generalized birthday problem.
Des. Codes Cryptogr., 87(8):1897–1926, 2019.

Mingjie Liu and Phong Q. Nguyen.
Solving BDD by enumeration: An update.
In Ed Dawson, editor, *CT-RSA 2013*, volume 7779 of *LNCS*, pages 293–309. Springer, Heidelberg, February / March 2013.

References v

Daniel Moghimi, Berk Sunar, Thomas Eisenbarth, and Nadia Heninger.
TPM-FAIL: TPM meets timing and lattice attacks.
CoRR, abs/1911.05673, 2019.
To appear at USENIX Security 2020.

Phong Q. Nguyen and Igor Shparlinski.
The insecurity of the digital signature algorithm with partially known nonces.
Journal of Cryptology, 15(3):151–176, June 2002.

Cesar Pereida García and Billy Bob Brumley.
Constant-time callees with variable-time callers.
In Engin Kirda and Thomas Ristenpart, editors, *USENIX Security 2017*, pages 83–98. USENIX Association, August 2017.

Keegan Ryan.
Return of the hidden number problem.
IACR TCHES, 2019(1):146–168, 2018.
https://tches.iacr.org/index.php/TCHES/article/view/7337.

Keegan Ryan.
Hardware-backed heist: Extracting ECDSA keys from qualcomm's TrustZone.
In Lorenzo Cavallaro, Johannes Kinder, XiaoFeng Wang, and Jonathan Katz, editors, *ACM CCS 2019*, pages 181–194. ACM Press, November 2019.

Akira Takahashi, Mehdi Tibouchi, and Masayuki Abe.
New Bleichenbacher records: Fault attacks on qDSA signatures.
IACR TCHES, 2018(3):331–371, 2018.
https://tches.iacr.org/index.php/TCHES/article/view/7278.

Samuel Weiser, David Schrammel, Lukas Bodner, and Raphael Spreitzer.
Big numbers - big troubles: Systematically analyzing nonce leakage in (ec)dsa implementations.
In *USENIX Security 2020)*, Boston, MA, August 2020. USENIX Association.