INFO-F-409
Learning dynamics

Learning, evolutionary game theory and the evolution of co-operation

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Summary

- What? Why?
- Rational choice
- Strategic games
- Nash Equilibrium
- Best
- Dominance
- Mixed strategies
- Mixed-strategy Nash Equilibria
- Support finding
- Lemke-Howson algorithm
- Extensive-form games
- sub-game perfect equilibrium
- Simultaneous moves
- Chance moves
- Bayesian games
- Assignment 1

2

The formation of agents' beliefs

Now that we can determine the Nash and sub-game perfect equilibria ...

How can we reach them?

Which equilibrium preferred ?

3

The formation of agents' beliefs

Can we expect that the equilibrium will be reached ?

Players could chose their action from an introspective analysis of the game : removing dominated strategies

Learning the beliefs about the other player in response of the information she receives :

1. Best response dynamics
2. Fictitious play
3. Stimulus-response or reinforcement learning
4. Evolutionary or cultural dynamics

4

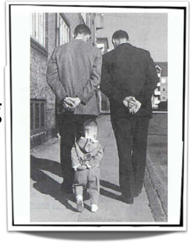
Levels of learning

innate

reflex actions

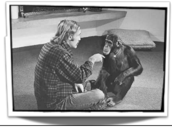
imprinting (specific and irreversible)

Conditioning



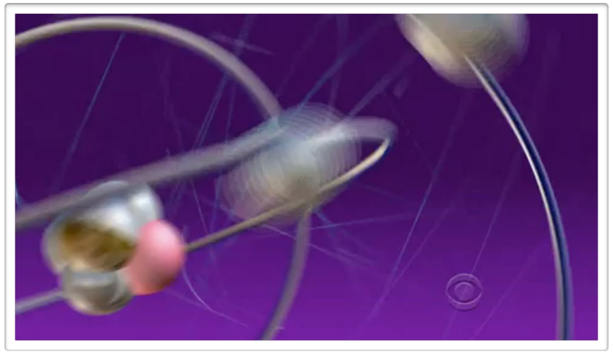
Observational and imitative learning

Teaching



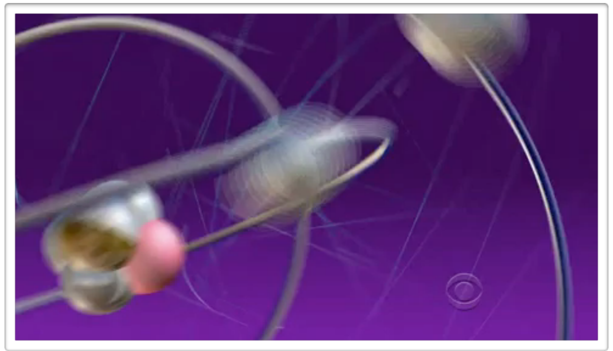
learned

Conditioning



Scene from the Big Bang Theory (S03E03, The Gothowitz Deviation)

Conditioning



Scene from the Big Bang Theory (S03E03, The Gothowitz Deviation)

Best-response dynamics

In the first period, choose a best response to an arbitrary deterministic belief about the other players' actions

In every period after the first, choose the best response to the other players' actions in the previous round

An action profile that remains the same over time is a pure Nash equilibrium of the game

Best-response dynamics

	Bach	Strav.
Bach	1 2	0 0
Strav.	0 0	2 1

Depending on the prior beliefs these dynamics may not converge

Take for instance the Battle of the sexes, which has 3 equilibria ((1,0),(1,0)), ((0,1),(0,1)) and ((2/3,1/3),(1/3, 2/3))

8-1

Best-response dynamics

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BELIEF	A plays	B plays
prior	B	B
1	B	B
2	B	B
...	...	...

8-2

Best-response dynamics

	Bach	Strav.
Bach	1 2	0 0
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Take for instance the Battle of the sexes, which has 3 equilibria ((1,0),(1,0)), ((0,1),(0,1)) and ((2/3,1/3),(1/3, 2/3))

BELIEF	A plays	B plays
prior	B	B
1	B	B
2	B	B
...	...	...

BELIEF	A plays	B plays
prior	S	S
1	S	S
2	S	S
...	...	...

8-3

Best-response dynamics

	Bach	Strav.
Bach	1 2	0 0
Strav.	0 0	2 1

Depending on the prior beliefs these dynamics may not converge

Take for instance the Battle of the sexes, which has 3 equilibria ((1,0),(1,0)), ((0,1),(0,1)) and ((2/3,1/3),(1/3, 2/3))

BELIEF	A plays	B plays
prior	B	B
1	B	B
2	B	B
...	...	...

BELIEF	A plays	B plays
prior	S	S
1	S	S
2	S	S
...	...	...

BELIEF	A plays	B plays
prior	S	B
1	B	S
2	S	B
...	...	...

8-4

Fictitious play

Every agent starts with an arbitrary probabilistic belief about the other players actions.

In the first round she chooses a BR to this prior probabilistic belief and observes the other player's actions, say A.

she changes here belief so that A gets probability 1

In the second round, she produces a best response to this belief and observes the other player's action, say B

she changes here belief to one that assigns 1/2 to action A and 1/2 to action B

In the third round ...

Fictitious play

Consider again the Battle of the sexes:

		BELIEF			
		A plays		B plays	
prior		(1,0)		(0,1)	
1	S	(1,1)	B	(1,1)	TOTAL = 2
2	S	(1,2)	S	(1,2)	TOTAL = 3
3	S	(1,3)	S	(1,3)	TOTAL = 4
4	S	(2,3)	B	(1,4)	TOTAL = 5
5	S	(2,4)	S	(1,5)	TOTAL = 6
6	S	(2,5)	S	(1,6)	TOTAL = 7
7	...	...	...	...	

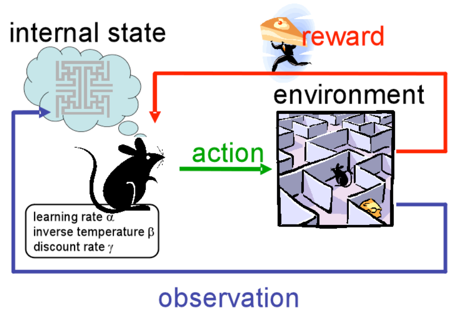
	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

Fictitious play

So in any period, the agent adopts the belief that her opponent is using a mixed strategy in which the probability of each action is proportional to the frequency with which her opponent has chosen that action in the previous rounds

The process converges to a mixed strategy Nash equilibrium from initial beliefs

Stimulus-response learning



Roth-Erev learning, Bush-Mosteller, ...

Stimulus-response learning

Stochastic dynamic models of individual behavior ...

Bush, R. R., & Mosteller, F. (1951). **A mathematical model for simple learning.** Psychological review, 58(5), 313–323.

Roth, A. E., & Erev, I. (1995). **Learning in extensive-form games: Experimental data and simple dynamic models in the intermediate term.** Games and Economic Behavior, 8(1), 164–212.

Erev, I., & Roth, A. E. (1998). **Predicting how people play games: reinforcement learning in experimental games with unique, mixed strategy equilibria.** The American Economic Review, 88(4), 848–881.

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Stimulus-response learning

Take for instance the model proposed by Roth and Erev (1995)

A player is defined by:

A **propensity score** $q_{Ak}(t)$, which expresses the propensity of player A to play action k at time t

A **probability function** $p_{Ak}(t)= q_{Ak}(t)/\sum_j q_{Aj}(t)$, which expresses the probability of A to play action k at time t

An **update function** $q_{Ak}(t+1)= q_{Ak}(t) + x$, where x is the payoff from the interaction. The other actions $q_{Aj}(t)$ remain the same.

Hence actions with a higher probability are more likely to be played (**Law of effect**)

Aim was to design a model that fits psychological literature

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Stimulus-response learning

Consider again the Battle of the sexes:

	Bach	Strav.
Bach	1 / 2	0 / 0
Strav.	0 / 0	2 / 1

player 1	$q_{1k}(t)$	$p_{1k}(t)$
Bach	*1*	*1/2*
Stravinsky	*1*	*1/2*

player 2	$q_{2k}(t)$	$p_{2k}(t)$
Bach	*1*	*1/2*
Stravinsky	*1*	*1/2*

15-1

Stimulus-response learning

Consider again the Battle of the sexes:

	Bach	Strav.
Bach	1 / 2	0 / 0
Strav.	0 / 0	2 / 1

player 1	$q_{1k}(t)$	$p_{1k}(t)$
Bach	*1*	*1/2*
Stravinsky	*1*	*1/2*

B

B

player 2	$q_{2k}(t)$	$p_{2k}(t)$
Bach	*1*	*1/2*
Stravinsky	*1*	*1/2*

15-2

Stimulus-response learning

	Bach	Strav.
Bach	1, 2	0, 0
Strav.	0, 0	2, 1

Consider again the Battle of the sexes:

player 1	$q_{1k}(t)$	$p_{1k}(t)$
Bach	*1*	*1/2*
Stravinsky	*1*	*1/2*

B

2

B

1

player 2	$q_{2k}(t)$	$p_{2k}(t)$
Bach	*1*	*1/2*
Stravinsky	*1*	*1/2*

15-3

Stimulus-response learning

	Bach	Strav.
Bach	1, 2	0, 0
Strav.	0, 0	2, 1

Consider again the Battle of the sexes:

player 1	$q_{1k}(t)$	$p_{1k}(t)$
Bach	*3*	*3/4*
Stravinsky	*1*	*1/4*

player 2	$q_{2k}(t)$	$p_{2k}(t)$
Bach	*2*	*2/3*
Stravinsky	*1*	*1/3*

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Stimulus-response learning

	Bach	Strav.
Bach	1, 2	0, 0
Strav.	0, 0	2, 1

Consider again the Battle of the sexes:

player 1	$q_{1k}(t)$	$p_{1k}(t)$
Bach	*3*	*3/4*
Stravinsky	*1*	*1/4*

player 2	$q_{2k}(t)$	$p_{2k}(t)$
Bach	*2*	*2/3*
Stravinsky	*1*	*1/3*

17-1

Stimulus-response learning

	Bach	Strav.
Bach	1, 2	0, 0
Strav.	0, 0	2, 1

Consider again the Battle of the sexes:

player 1	$q_{1k}(t)$	$p_{1k}(t)$
Bach	*3*	*3/4*
Stravinsky	*1*	*1/4*

B

S

player 2	$q_{2k}(t)$	$p_{2k}(t)$
Bach	*2*	*2/3*
Stravinsky	*1*	*1/3*

17-2

Stimulus-response learning

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

Consider again the Battle of the sexes:

player 1	$q_{1k}(t)$	$p_{1k}(t)$
Bach	*3*	*3/4*
Stravinsky	*1*	*1/4*

B S

0 0

player 2	$q_{2k}(t)$	$p_{2k}(t)$
Bach	*2*	*2/3*
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17-3

Stimulus-response learning

	Bach	Strav.
Bach	2, 1	0, 0
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Consider again the Battle of the sexes:

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Stravinsky	*1*	*1/4*

player 2	$q_{2k}(t)$	$p_{2k}(t)$
Bach	*2*	*2/3*
Stravinsky	*1*	*1/3*

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Stimulus-response learning

	Bach	Strav.
Bach	2, 1	0, 0
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Consider again the Battle of the sexes:

player 1	$q_{1k}(t)$	$p_{1k}(t)$
Bach	*3*	*3/4*
Stravinsky	*1*	*1/4*

S S

player 2	$q_{2k}(t)$	$p_{2k}(t)$
Bach	*2*	*2/3*
Stravinsky	*1*	*1/3*

19-1

Stimulus-response learning

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

Consider again the Battle of the sexes:

player 1	$q_{1k}(t)$	$p_{1k}(t)$
Bach	*3*	*3/4*
Stravinsky	*1*	*1/4*

S S

1 2

player 2	$q_{2k}(t)$	$p_{2k}(t)$
Bach	*2*	*2/3*
Stravinsky	*1*	*1/3*

19-2

Stimulus-response learning

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

Consider again the Battle of the sexes:

player 1	$q_{1k}(t)$	$p_{1k}(t)$
Bach	*3*	*3/5*
Stravinsky	*2*	*2/5*

player 2	$q_{2k}(t)$	$p_{2k}(t)$
Bach	*2*	*2/5*
Stravinsky	*3*	*3/5*

Continue until convergence

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Stimulus-response learning

Three extensions were introduced into this model:

A **cutoff parameter** μ which ensure that $q_{Ak}(t)$ and $p_{Ak}(t)$ can become zero in finite time : when $p_{Ak}(t) \leq \mu$, $q_{Ak}(t)=p_{Ak}(t)=0$

An **error/exploration parameter** ε which prevents a probability $p_{Ak}(t)$ can become zero if it is close to a successful strategy: $q_{Ak}(t+1)=q_{Ak}(t)+(1-\varepsilon)x$ for the successful strategy and $q_{Aj}(t+1)=q_{Aj}(t)+\varepsilon x$ for *the adjacent strategies*

An **forgetting parameter** φ which gradually reduces the importance of each propensity $q_{Ak}(t)$ over time by multiplying each propensity by $(1-\varphi)$.

More details on reinforcement learning by Prof. Nowé

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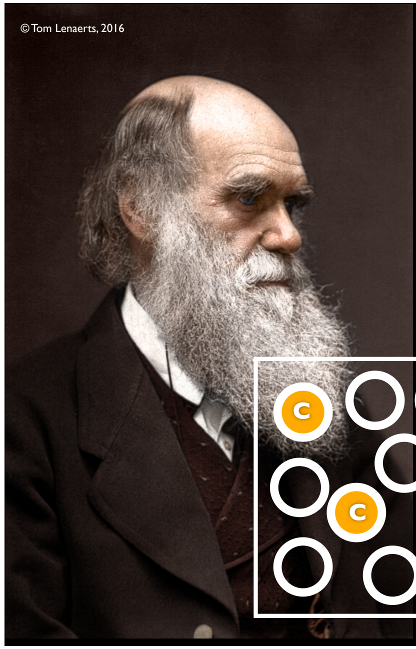
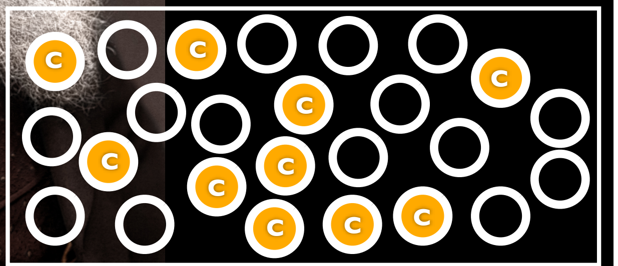


An evolutionary perspective ….

Non-rational players: **preferences = actions**

Darwinian competition in **populations**

Frequency-dependence



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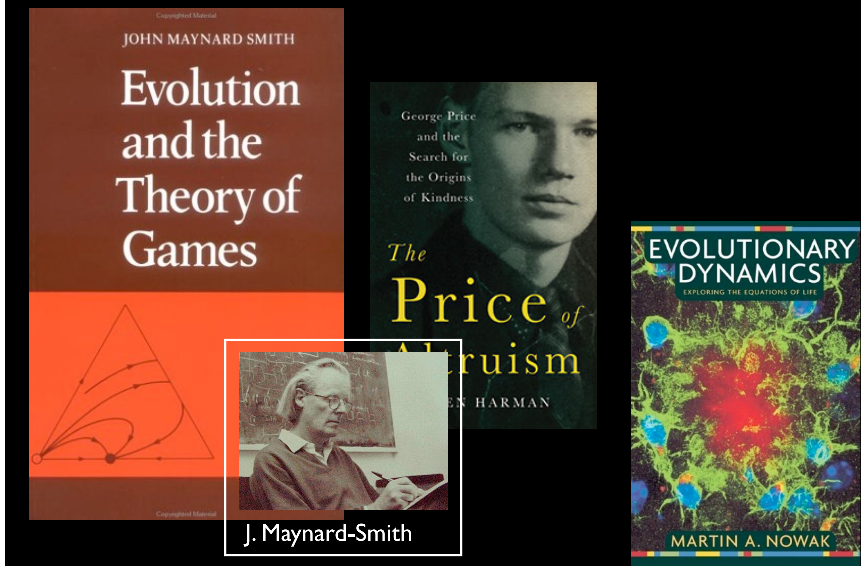

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Social dilemmas

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… attempt to define

Social dilemmas are situations in which each member of a group has a clear and unambiguous incentive to make a choice that—when made by all members—provides poorer outcomes for all than they would have received if none had made the choice. Thus, by doing what seems individually reasonable and rational, people end up doing less well than they would have done if they had acted unreasonably or irrationally. This paradoxical pos-

R.M. Dawes and D.M. Messick (2000) Social Dilemmas. International Journal of Psychology 35(2):111-116

THE QUESTION OF COOPERATION

Social dilemmas are situations in which individual rationality leads to collective irrationality. That is, individually reasonable behavior leads to a situation in which everyone is worse off than they might have been otherwise. Many of

P. Kollock (1998) Social Dilemmas: the anatomy of cooperation Ann. Rev. Sociol. 24:183-214

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The Tragedy of the Commons

The population problem has no technical solution; it requires a fundamental extension in morality.

Garrett Hardin

sional judgment. . . ." Whether they were right or not is not the concern of the present article. Rather, the concern here is with the important concept of a class of human problems which can be called "no technical solution problems," and, more specifically, with the identification and discussion of one of these.

It is easy to show that the class is not a null class. Recall the game of tick-tack-toe. Consider the problem, "How can I win the game of tick-tack-toe?" It is well known that I cannot, if I assume (in keeping with the conventions of game theory) that my opponent understands the game perfectly. Put another way, there is no "technical solution" to the problem. I can win only by giving a radical meaning to the word "win." I can hit my opponent over the head; or I can drug him; or I can falsify the records. Every way in which I "win" involves, in some sense, an abandonment of the game, as we intuitively understand it. (I can also, of course, openly abandon the game—refuse to

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Using common and public goods …

	Excludable	Non-excludable
Rivalrous	Private goods food, clothing, car, TV, house, ...	Common goods fish stocks, timber, oil, coal, ...
Non-rivalrous	Club goods cinemas, private parks, satelite tv, ...	Public goods air, knowledge, national defense, street lightning, social welfare,..

Rivalry; whether the consumption of a good by one person precludes its consumption by another person

Excludability; whether it is possible to exclude a person from consumption of the goods

Using common and public goods …

Neither state control nor privatisation are necessary to protect common resources

Bottom-up small-scale institutions, created by communes relying on or living close to the resource, that govern access to the resource will ensure the well-being of themselves and the survival of the resource.

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28

Using common and public goods …

BUT

Do the people in the commune really have the **capacity to organize** themselves for any common resource?

What about **individual differences in benefits**?

What about **free-riders**?

What if **others** not belonging to that commune also want access?

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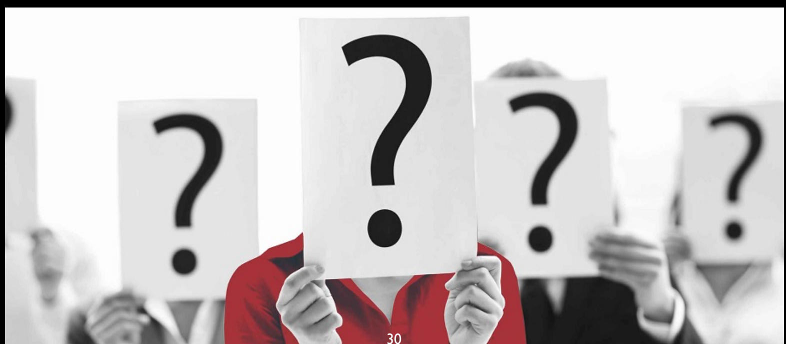
Central question:

How to reach cooperation in social dilemmas?



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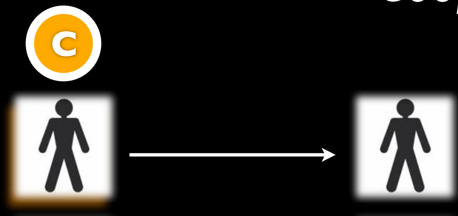
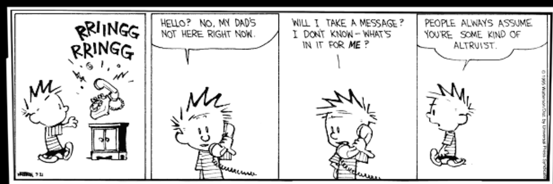
Cooperation?



pays a cost c receives a benefit $b > c$



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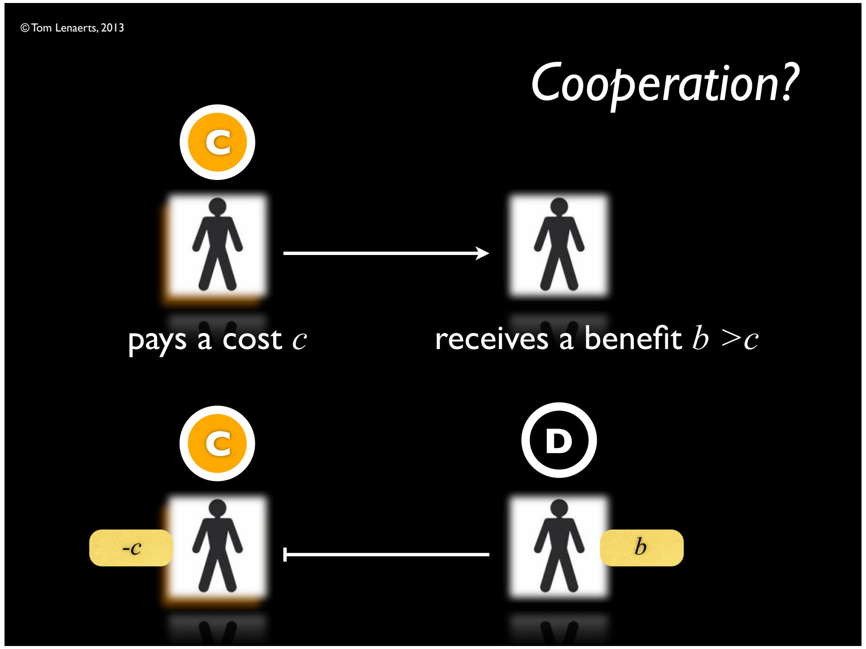

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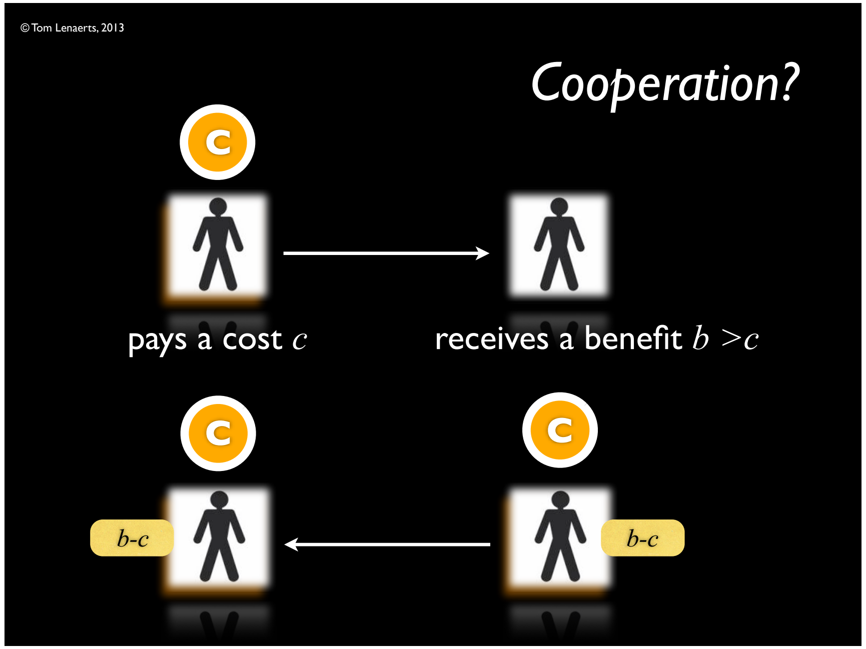

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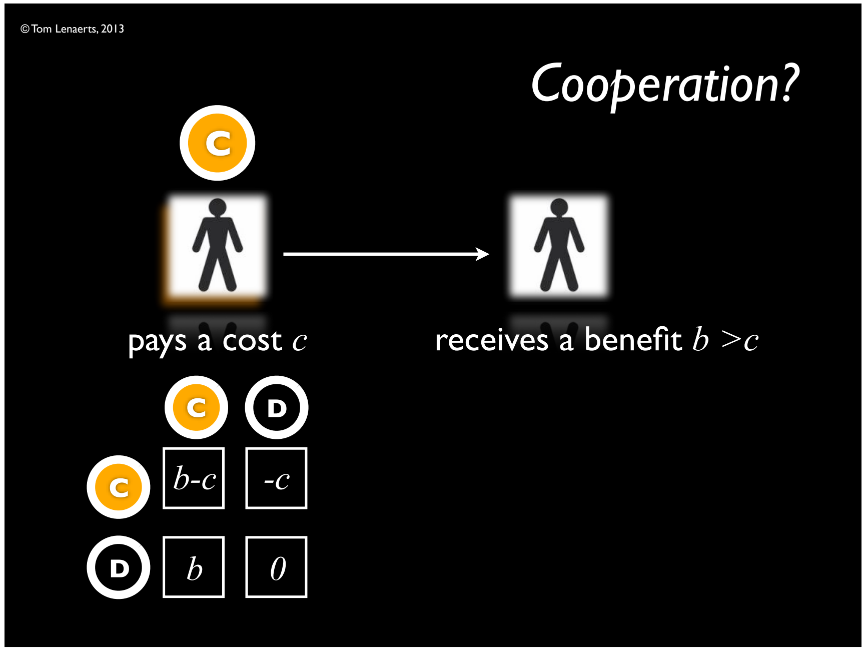

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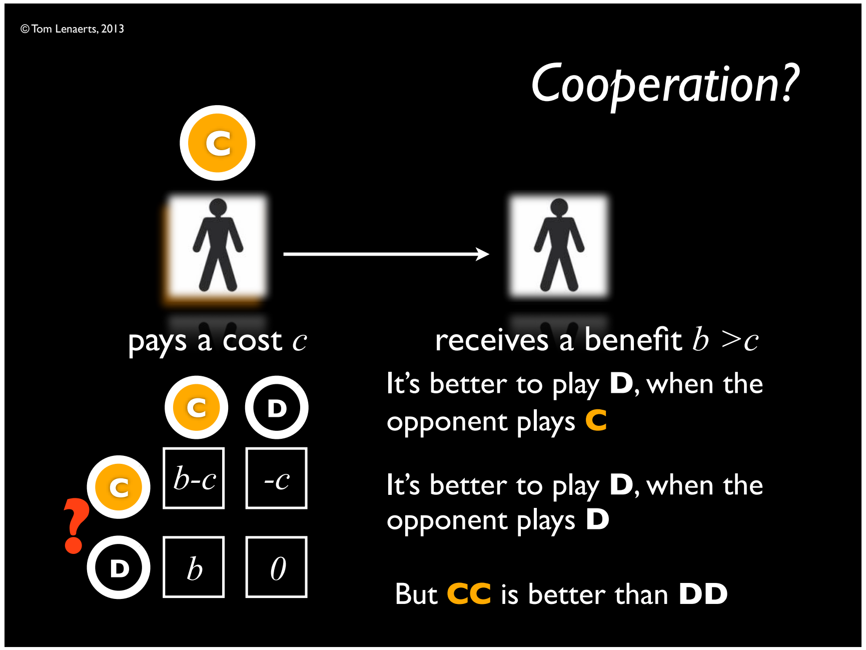

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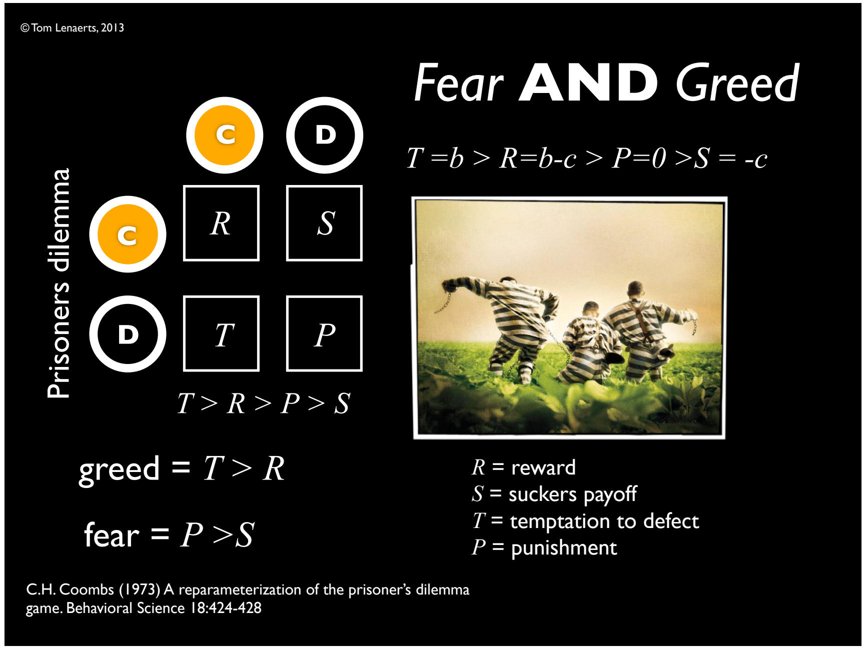

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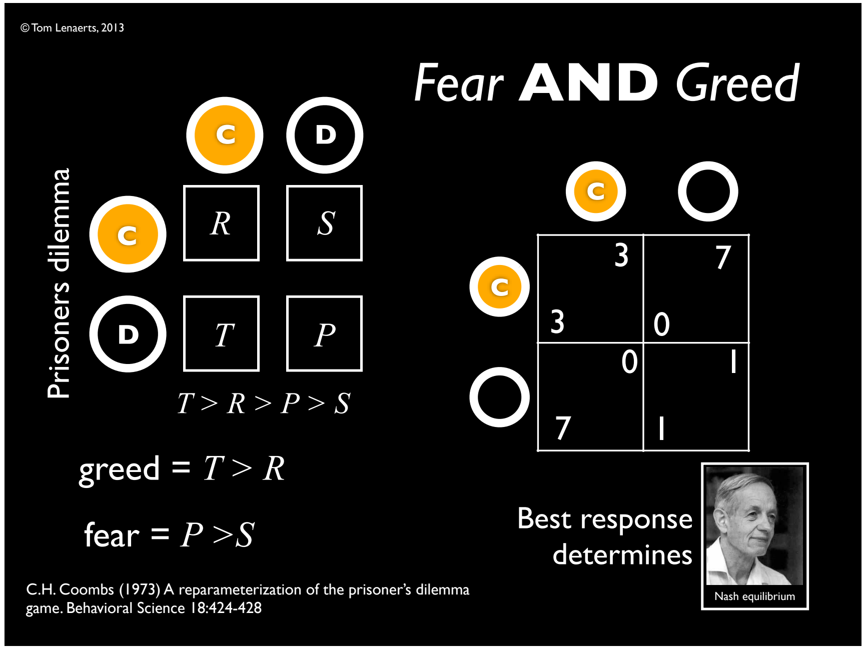

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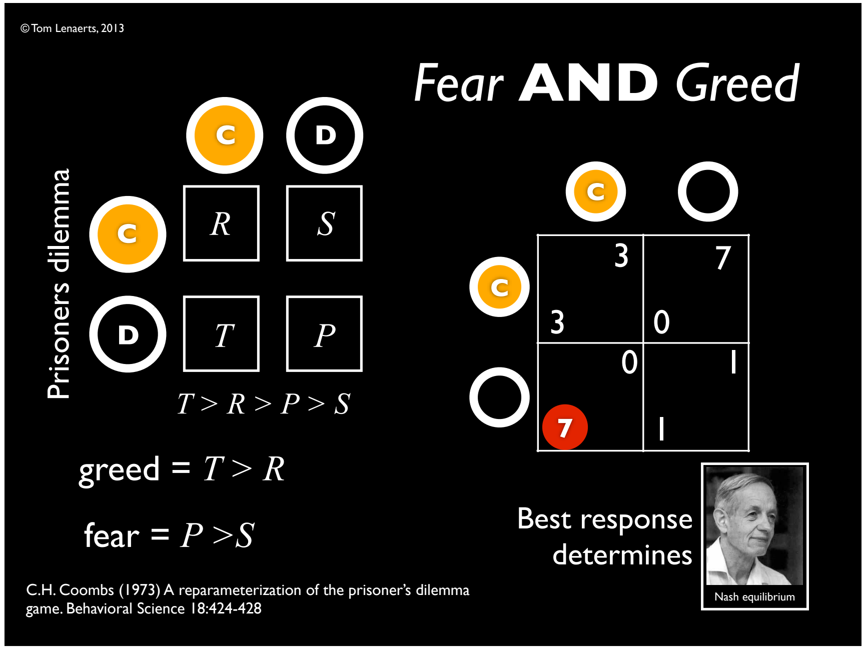

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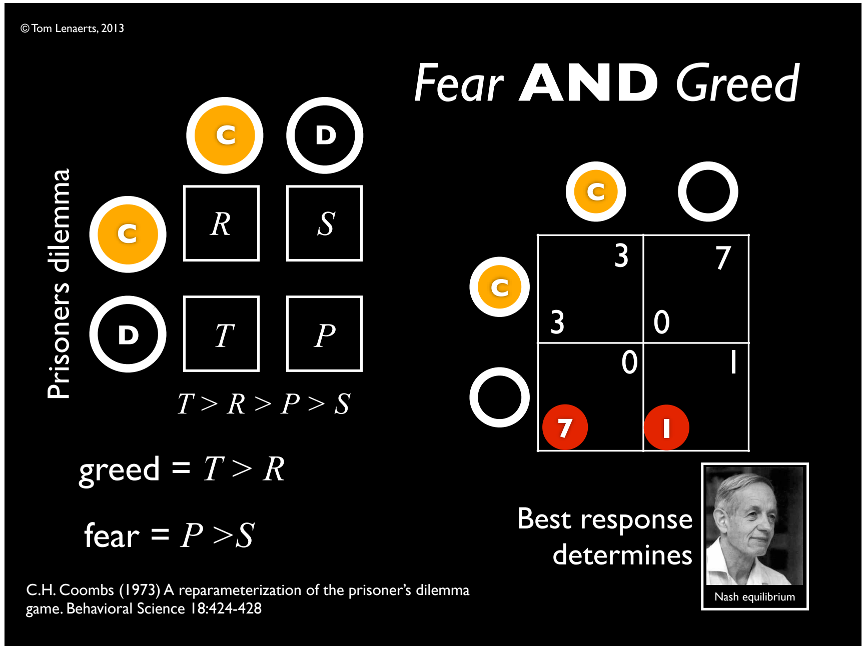

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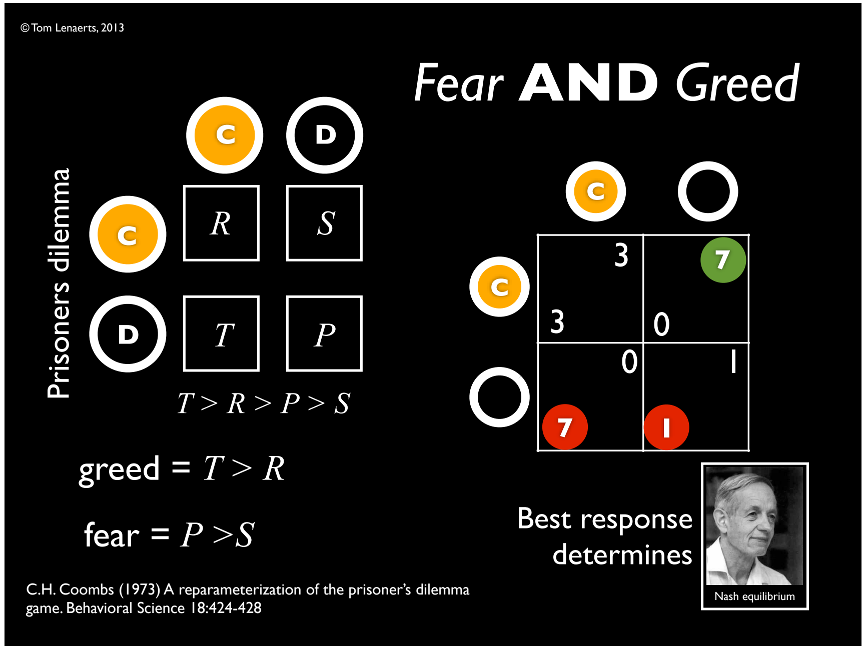

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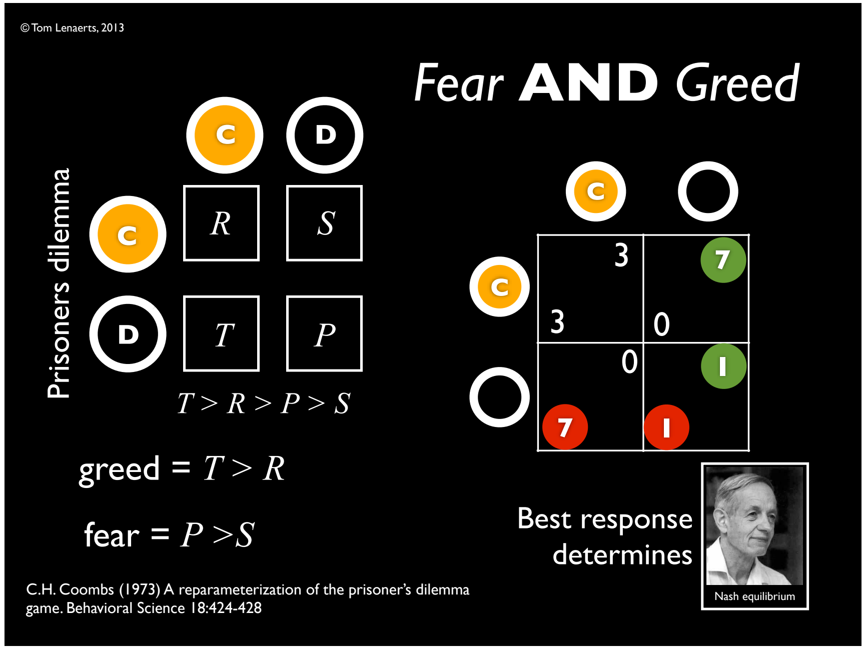

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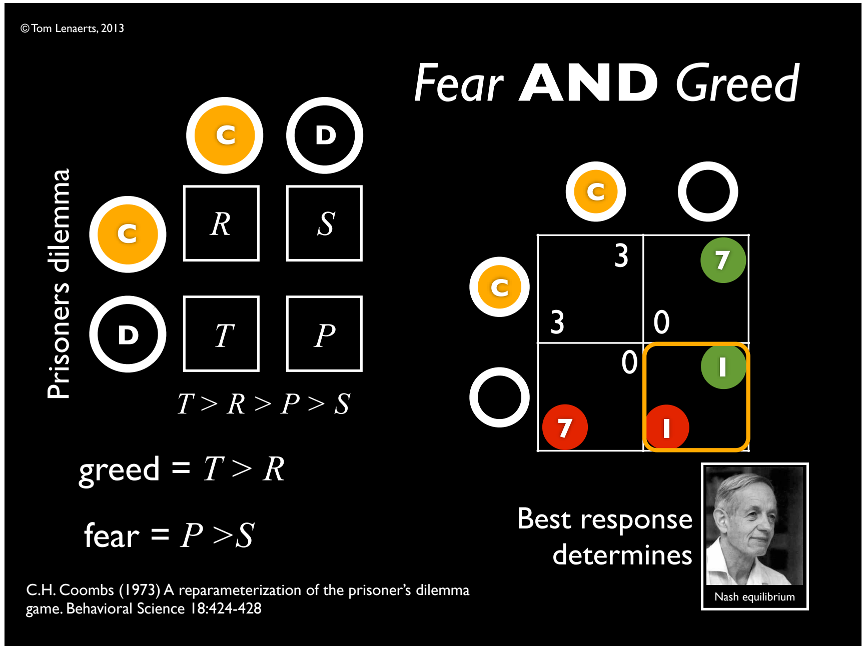

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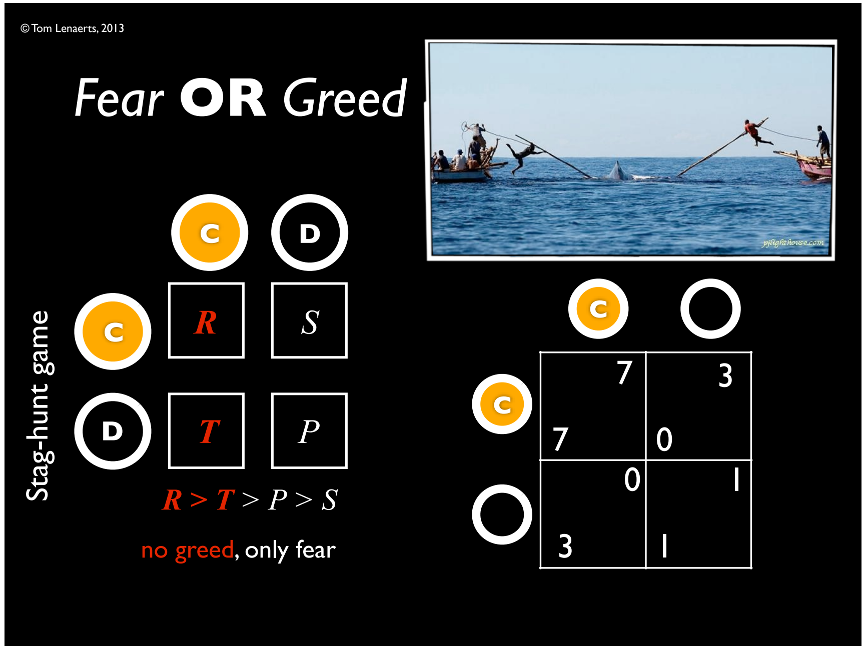

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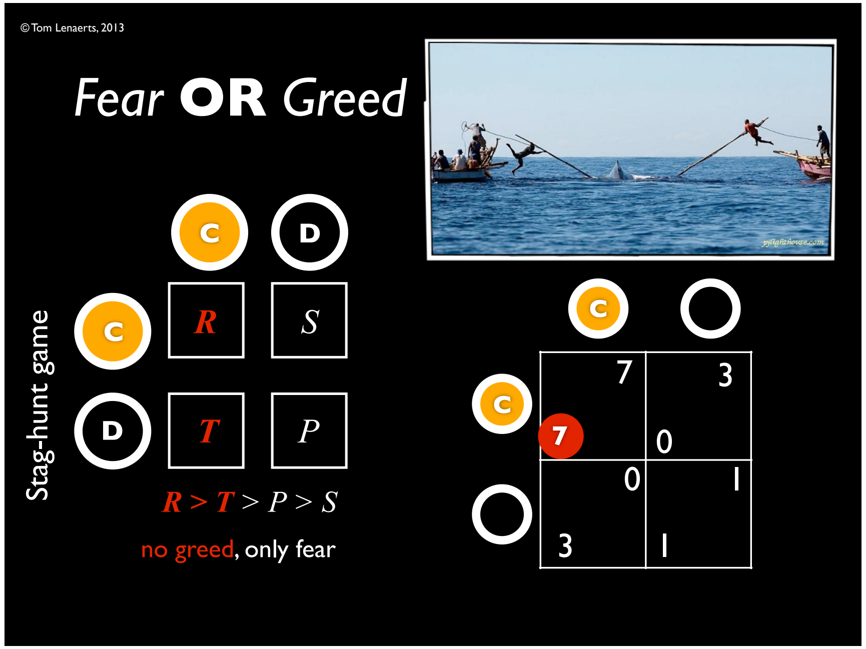

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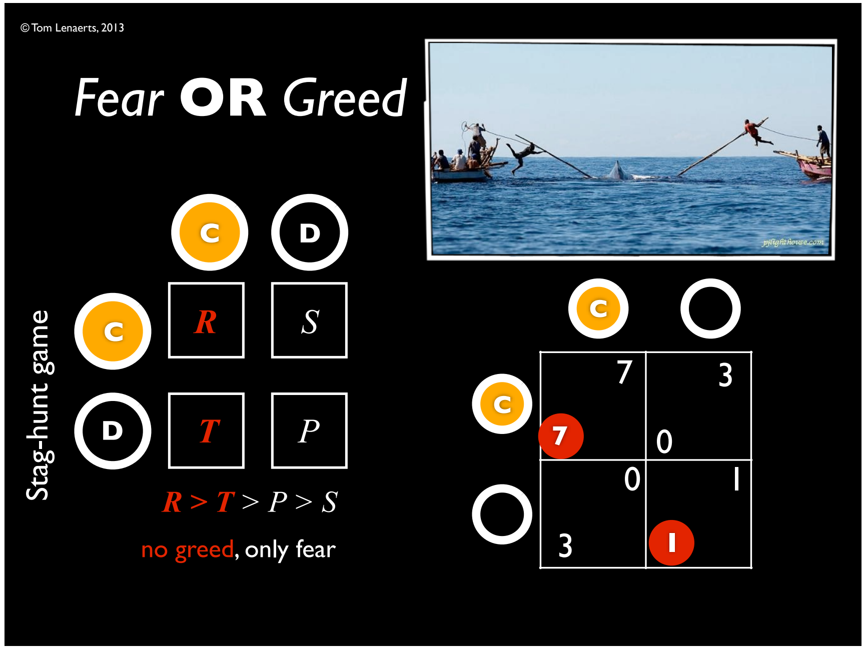

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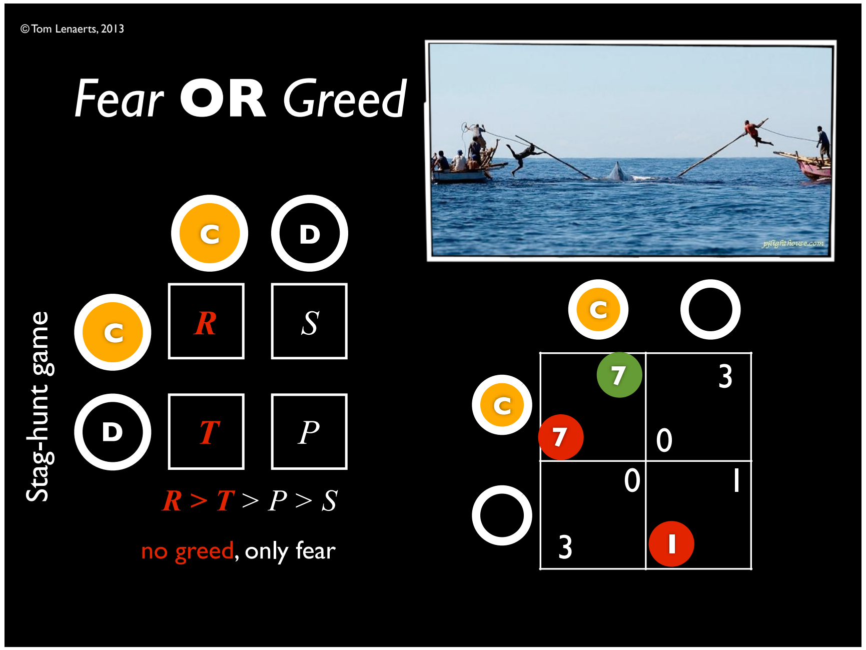

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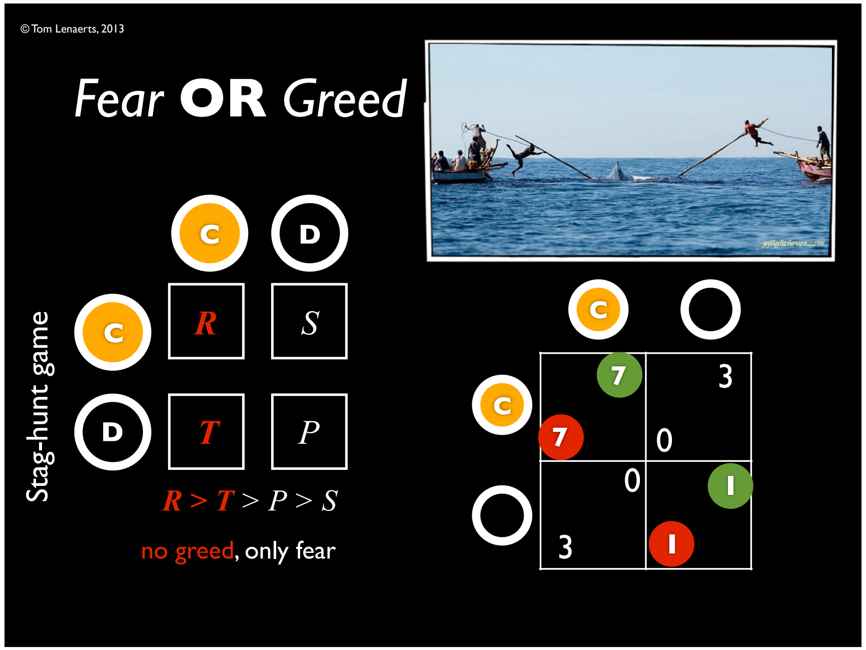

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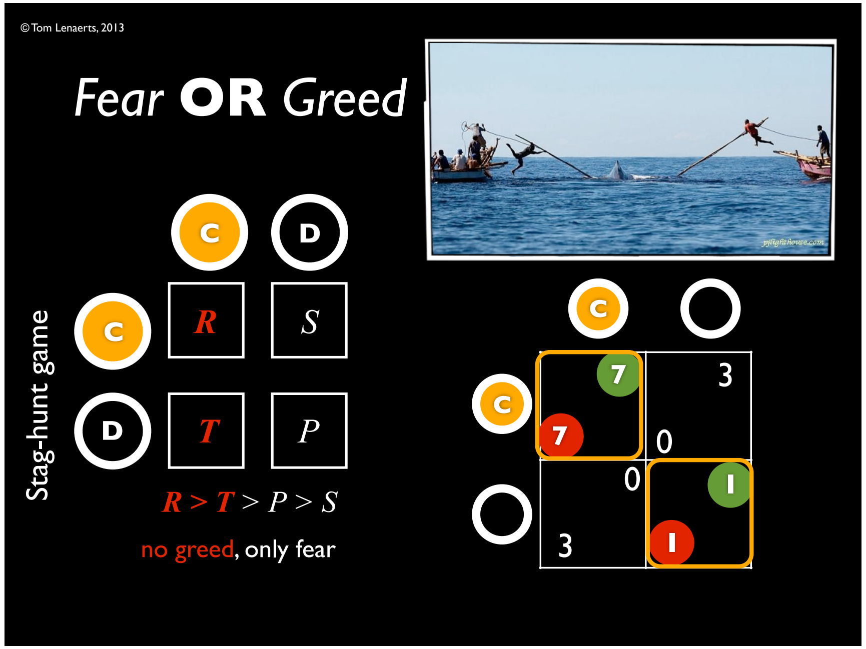

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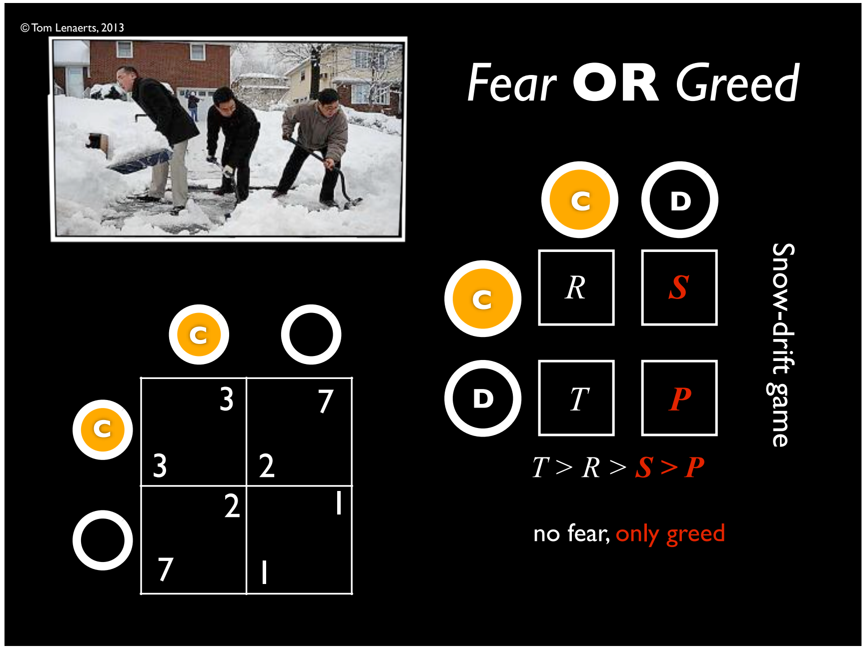

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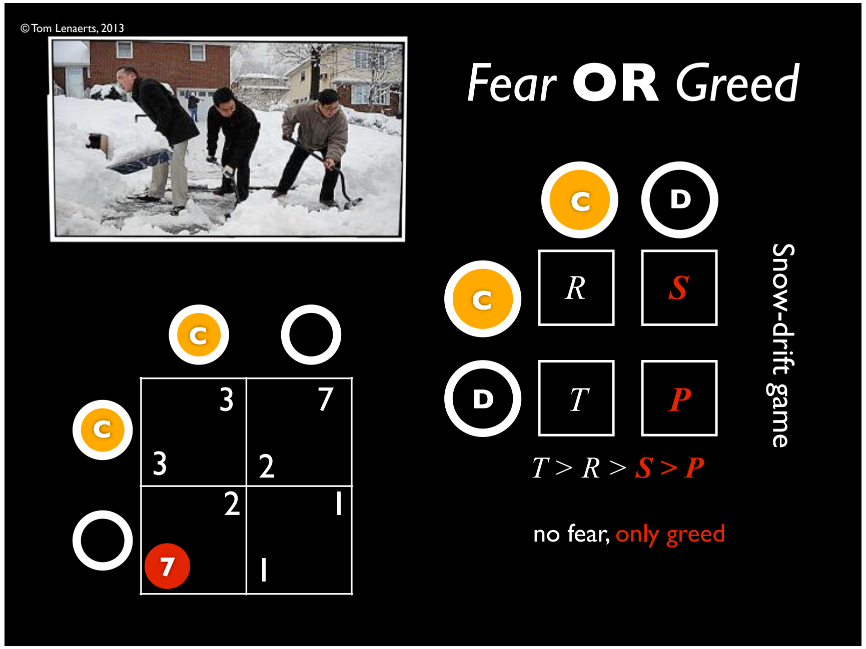

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37-3

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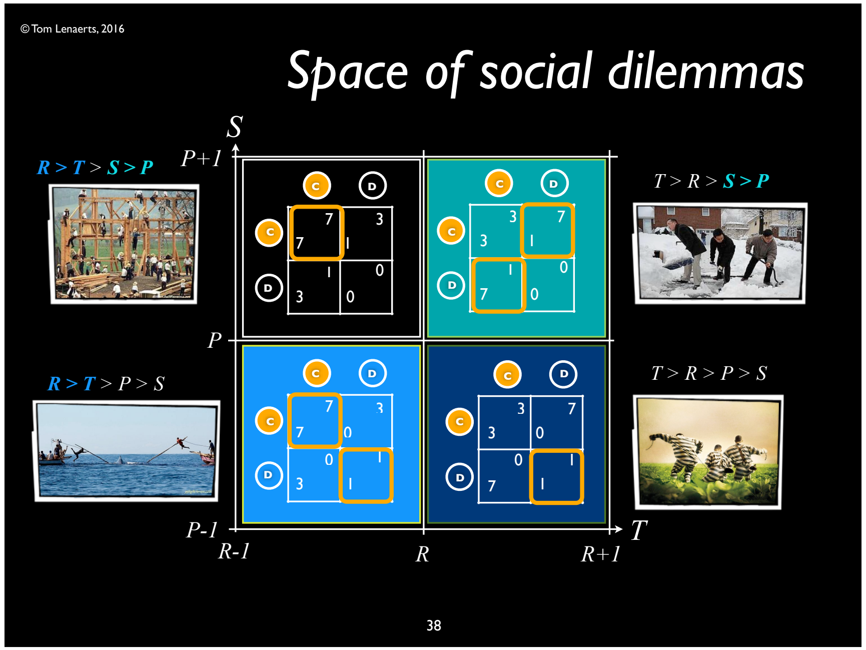
37-5

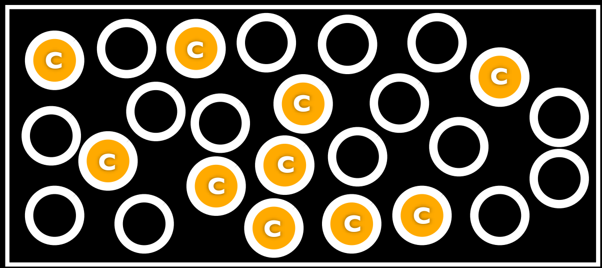
37-6



38

Evolutionary stable strategies …

Can a **C** player invade a population of **D** players?



The fraction of **C** (**D**) players is ε $(1\text{-}\varepsilon)$

J. Maynard-Smith and G.R. Price (1973) The logic of animal conflict. Nature 246:15-18

39

Evolutionary stable strategies …

Can a **C** player invade a population of **D** players?

success of **C** in a **D** population

success of **D** population against a **C** invader

$$\boxed{S(1\text{-}\varepsilon)+R\varepsilon} > \boxed{P(1\text{-}\varepsilon)+T\varepsilon}$$

	C	D
C	R	S
D	T	P

C can invade when:

i) $S>P$ or ii) $S=P$ and $R>T$

D can invade when:

i) $T>R$ or ii) $T=R$ and $P>S$

J. Maynard-Smith and G.R. Price (1973) The logic of animal conflict. Nature 246:15-18

40

Can C invade D?

	C	D
C	3, 3	0, 7
D	7, 0	1, 1

	C	D
C	7, 7	0, 3
D	3, 0	1, 1

	C	D
C	3, 3	2, 7
D	7, 2	1, 1

41-1

Can C invade D?

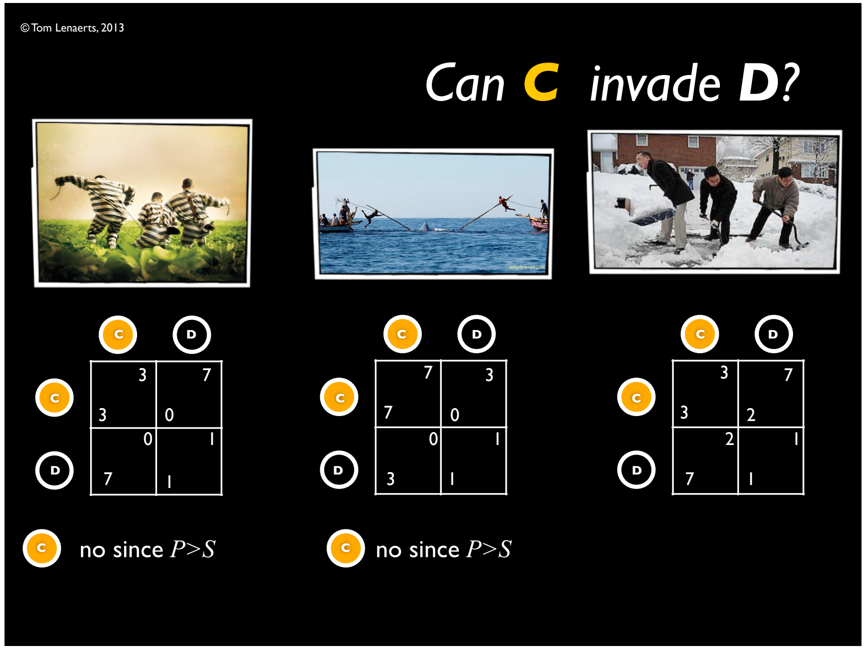
	C	D
C	3, 3	0, 7
D	7, 0	1, 1

	C	D
C	7, 7	0, 3
D	3, 0	1, 1

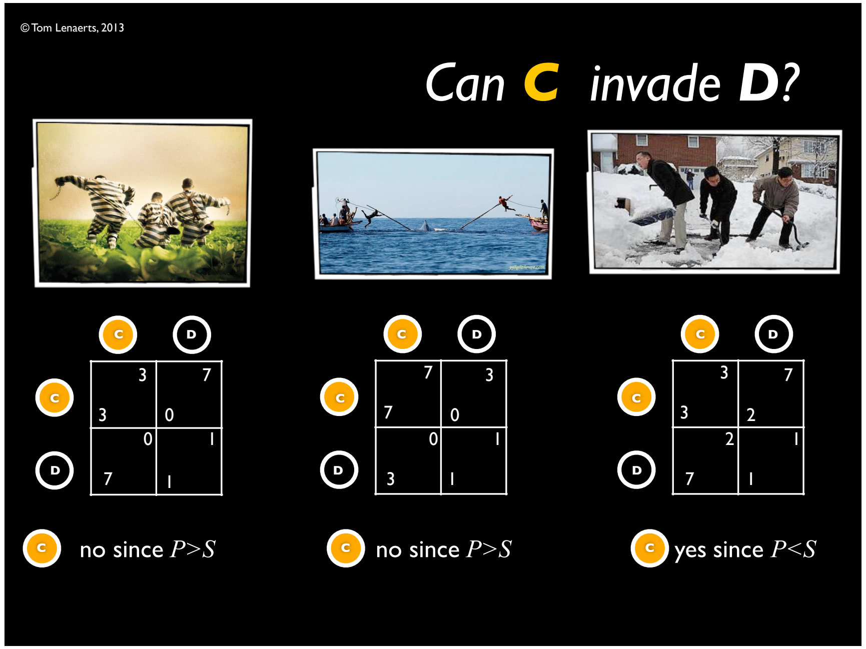
	C	D
C	3, 3	2, 7
D	7, 2	1, 1

C no since $P>S$

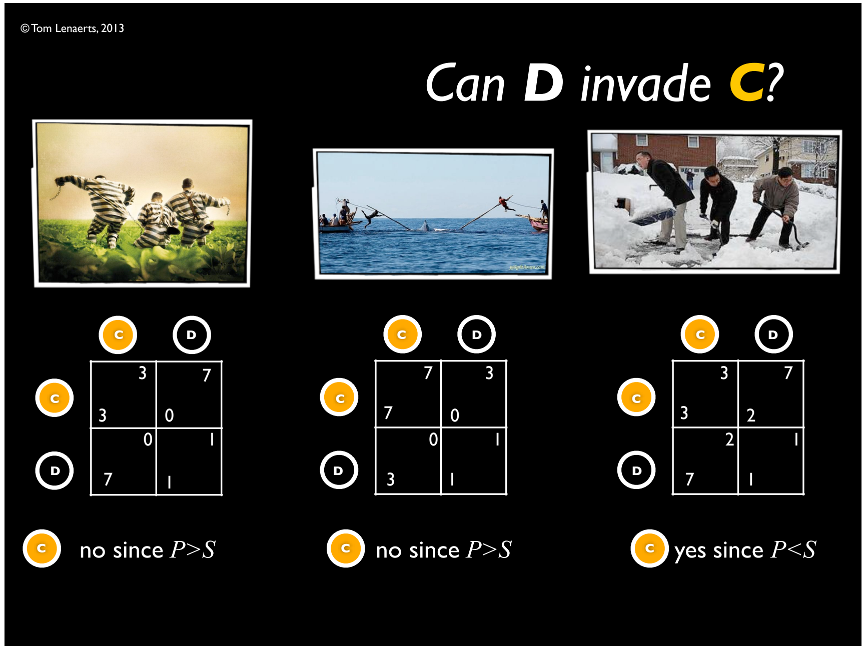
41-2



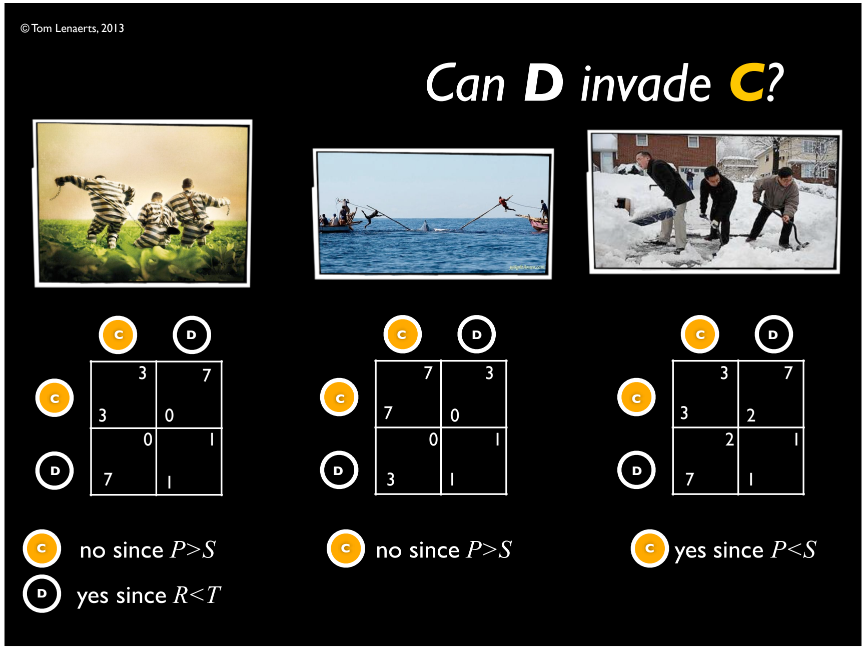
41-3



41-4



41-5



41-6

©Tom Lenaerts, 2013

*Can **D** invade **C**?*

	C	D
C	3, 3	0, 7
D	7, 0	1, 1

	C	D
C	7, 7	0, 3
D	3, 0	1, 1

	C	D
C	3, 3	2, 7
D	7, 2	1, 1

C no since $P>S$
D yes since $R<T$

C no since $P>S$
D no since $R>T$

C yes since $P<S$

41-7

©Tom Lenaerts, 2013

*Can **D** invade **C**?*

	C	D
C	3, 3	0, 7
D	7, 0	1, 1

	C	D
C	7, 7	0, 3
D	3, 0	1, 1

	C	D
C	3, 3	2, 7
D	7, 2	1, 1

C no since $P>S$
D yes since $R<T$

C no since $P>S$
D no since $R>T$

C yes since $P<S$
D yes since $R<T$

41-8

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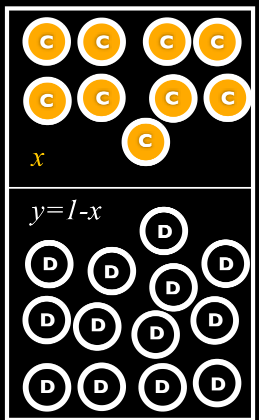
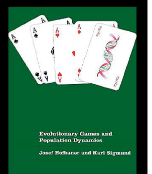
Evolutionary dynamics

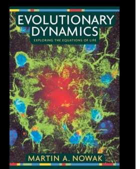


Replicator equation ...

$$\frac{dx}{dt} = x(1-x)[f_C(x)-f_D(x)]$$
$$= x(1-x)[(b-c+c-b+0)x-c-0]$$
$$= -cx(1-x)$$

C (x=1) $\longrightarrow$ D (x=0)





P.D. Taylor and L.B. Jonker (1978) Evolutionary stable strategies and game dynamics. Mathematical biosciences 40(1-2):145-156

42

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Dynamics of social dilemmas

$$\frac{dx}{dt} = x(1-x)[(R-S-T+P)x+S-P]$$

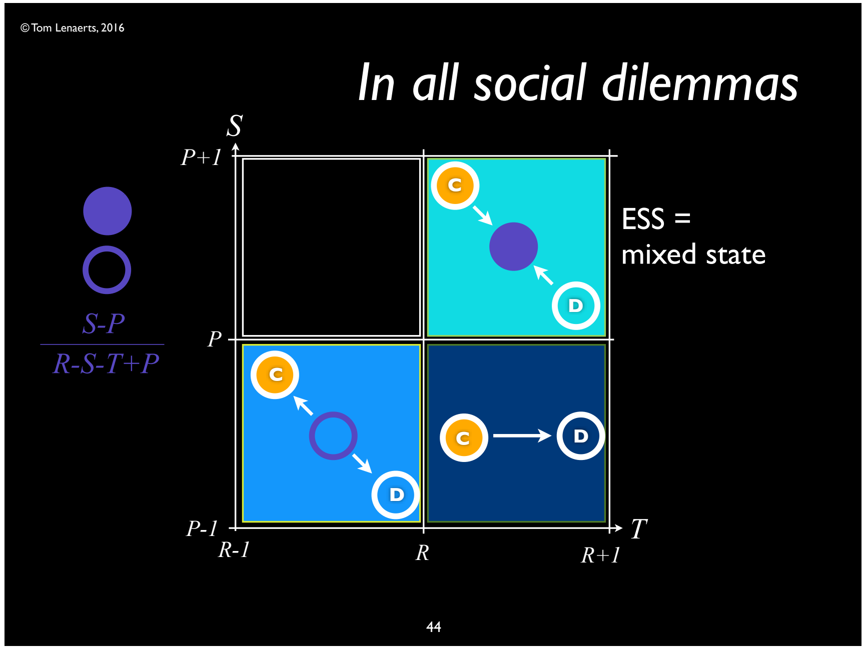
D dominates **C**: C $\longrightarrow$ D — fear and greed

D and **C** are bistable: C $\longleftarrow$ ○ $\longrightarrow$ D — only fear

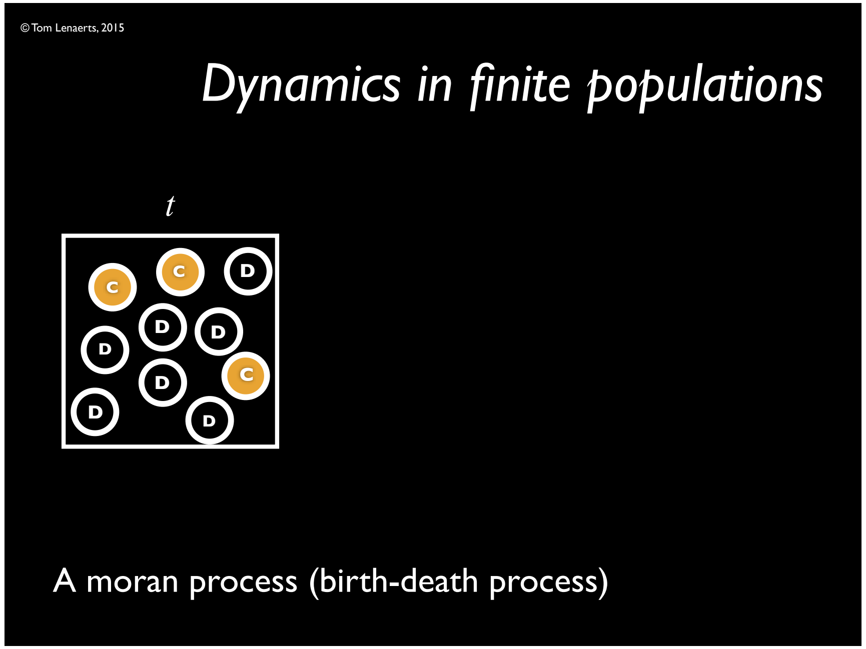
D and **C** coexist: C $\longrightarrow$ ● $\longleftarrow$ D — only greed

$$x^*=\frac{S-P}{R-S-T+P}$$

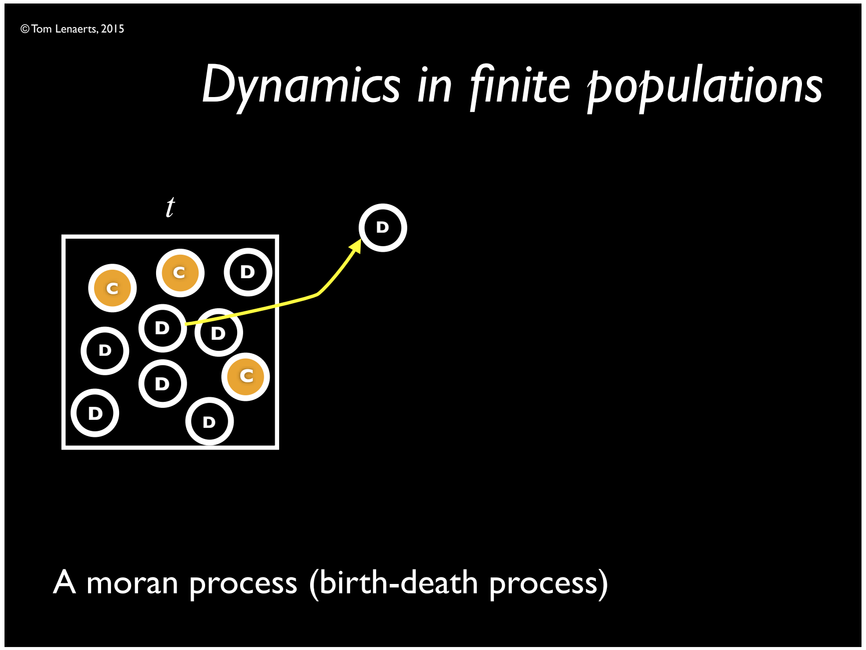
43



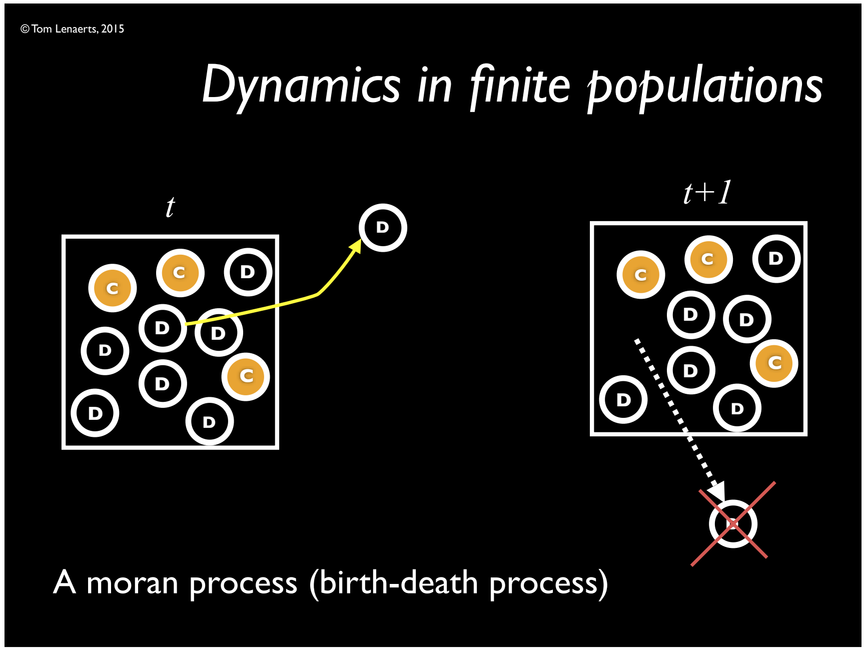
44



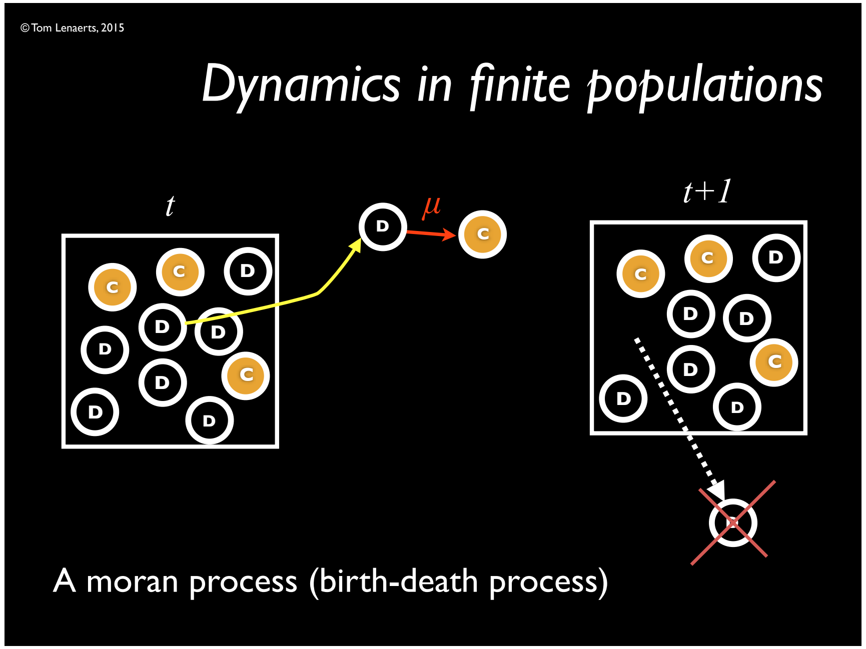
45-1



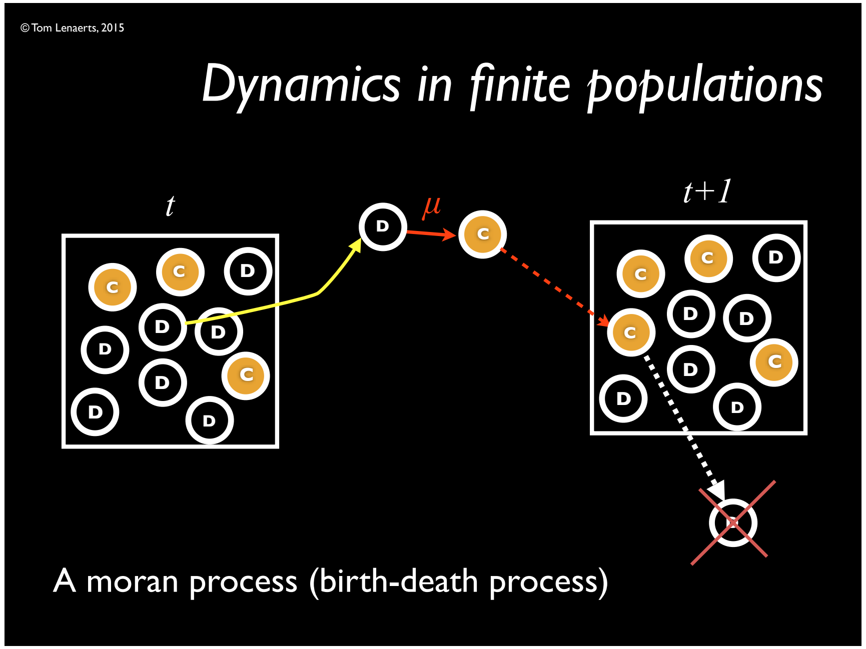
45-2



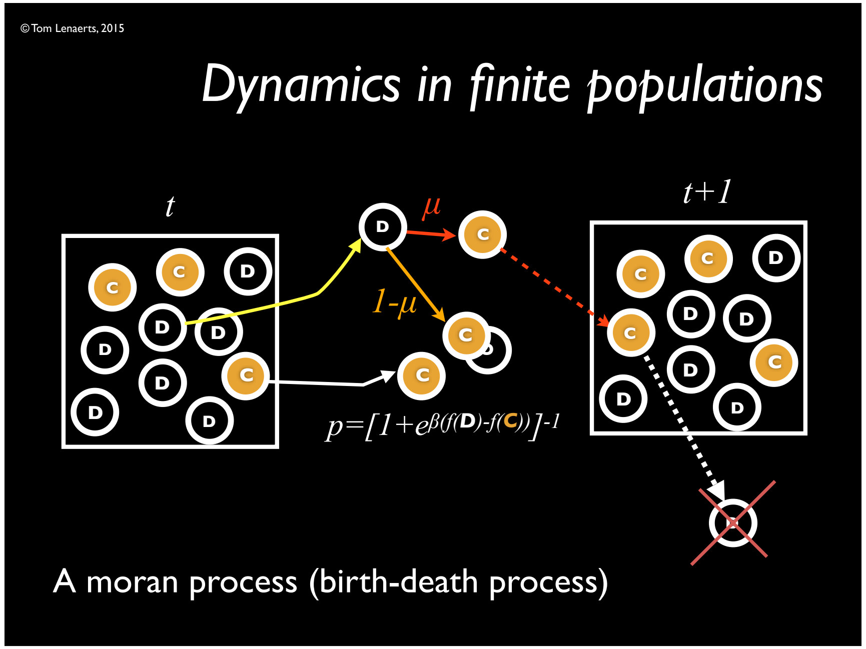
45-3



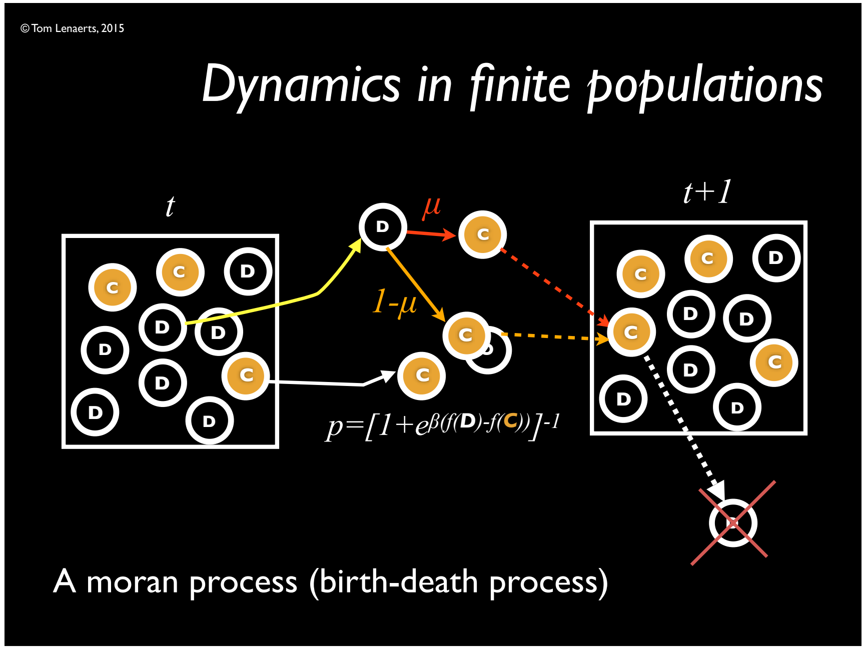
45-4



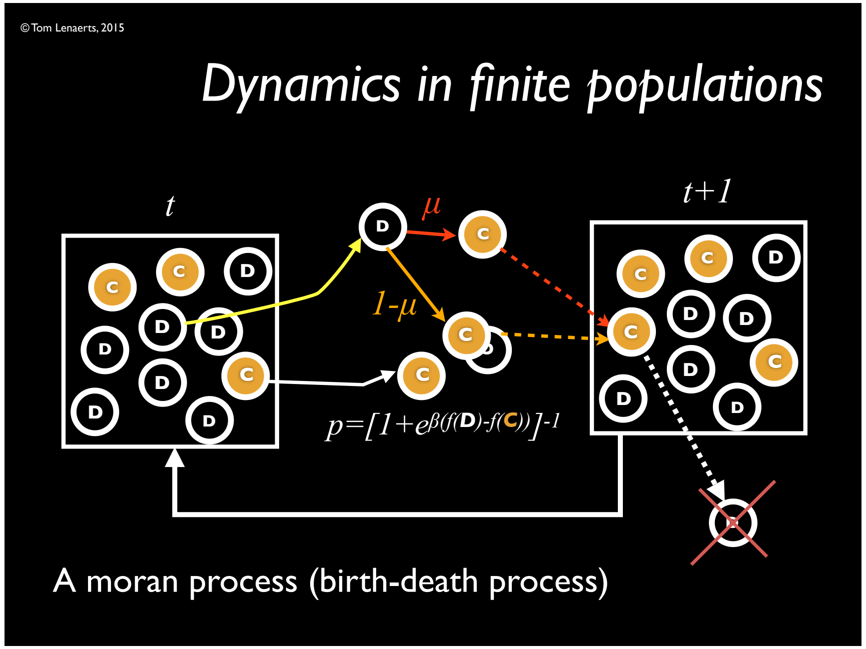
45-5



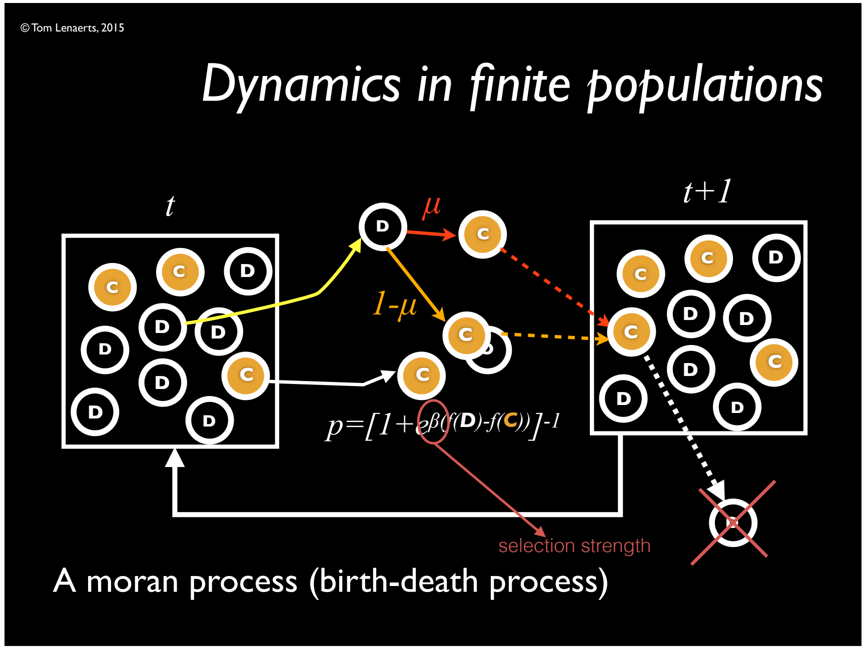
45-6



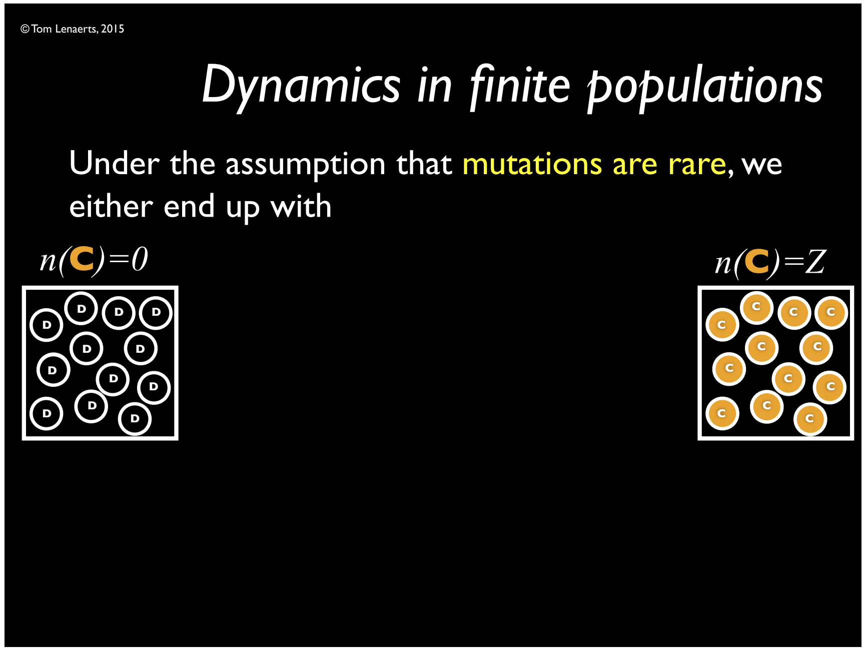
45-7



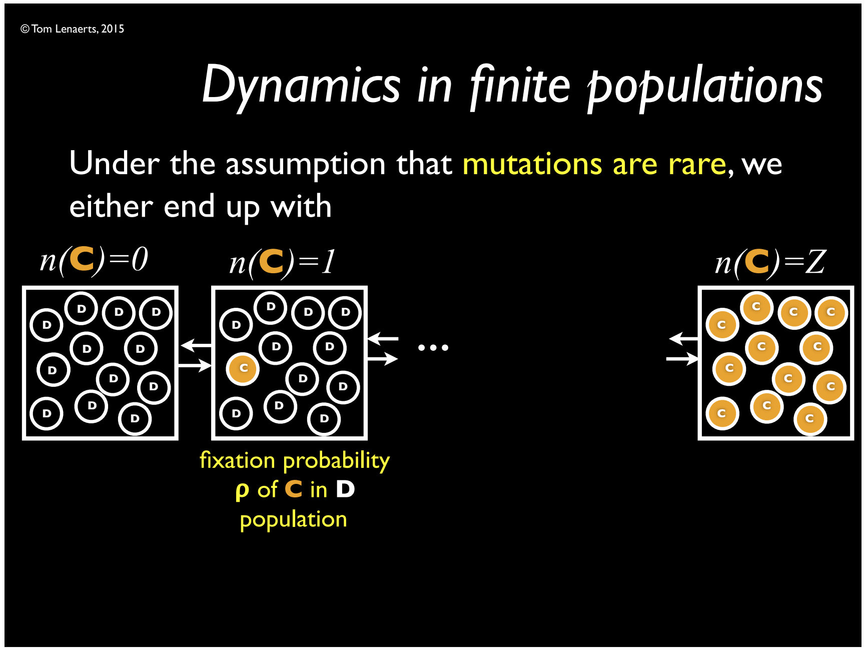
45-8



45-9



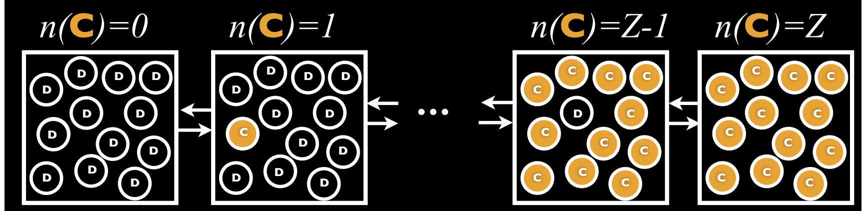
46-1



46-2

Dynamics in finite populations

Under the assumption that mutations are rare, we either end up with



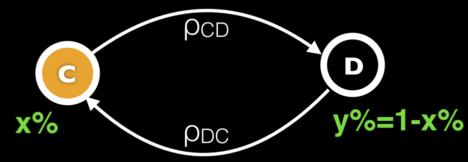
fixation probability ρ of **C** in **D** population

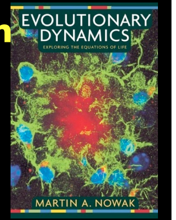
fixation probability ρ of **D** in **C** population

46-3

Dynamics in finite populations

Which produces a **reduced Markov chain**





For which the **stationary distributions** can be calculated = *how likely it is to end up in either monomorphic state*

Fudenberg, D., & Imhof, L.A. (2006). Imitation processes with small mutations. J. Econ.Theo, 131,251–262.

Imhof, L.A., Fudenberg, D., & Nowak, M.A. (2005). Evolutionary cycles of cooperation and defection. Proc Nat Acad Sci USA, 102(31), 10797–10800.

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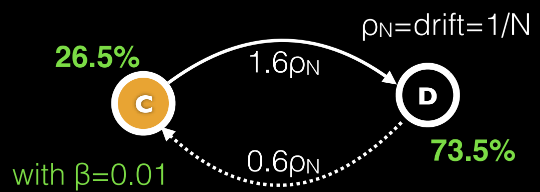
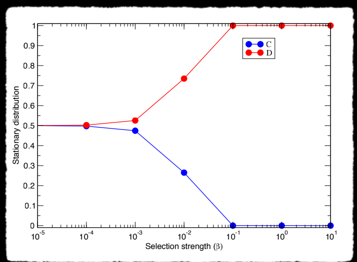
Dynamics in finite populations

	C	D
C	3, 3	0, 4
D	4, 0	1, 1



with β=0.01

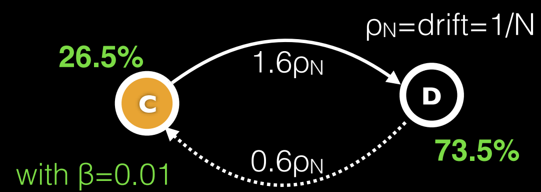
for varying β



48-1

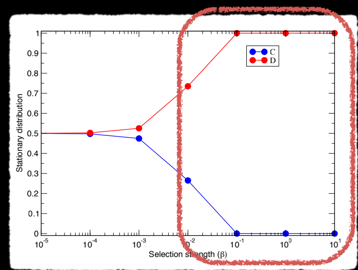
Dynamics in finite populations

	C	D
C	3, 3	0, 4
D	4, 0	1, 1



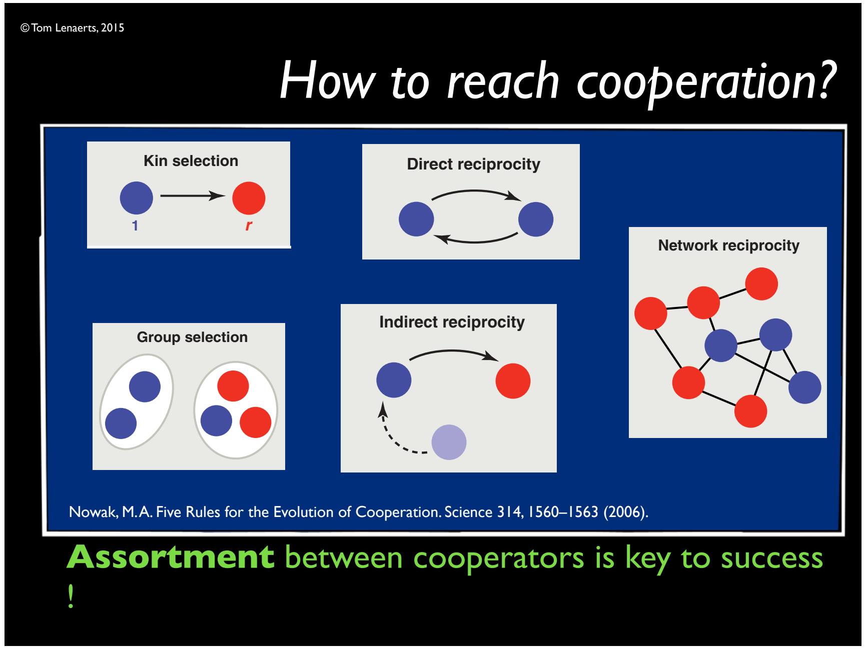
with β=0.01

for varying β

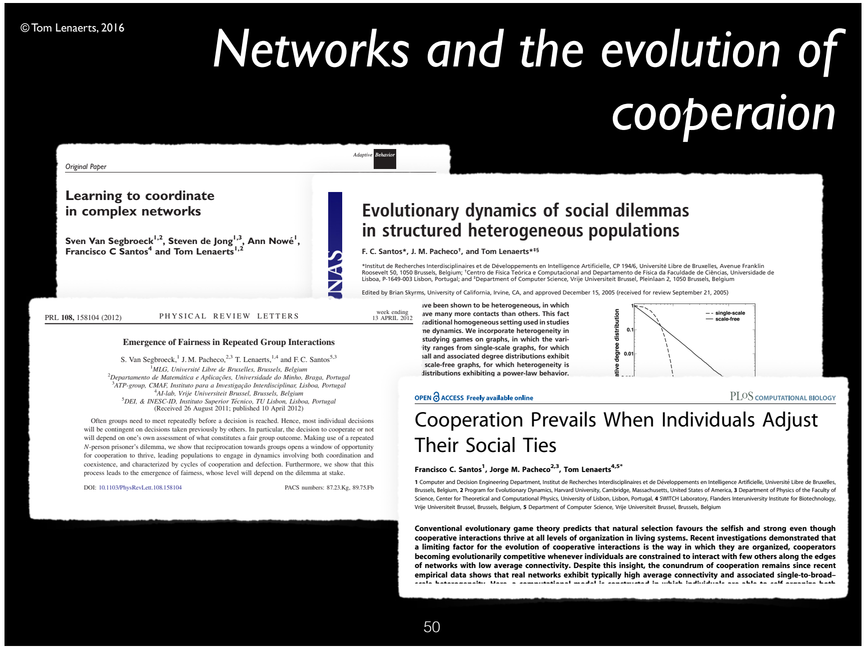


How can we achieve cooperation?

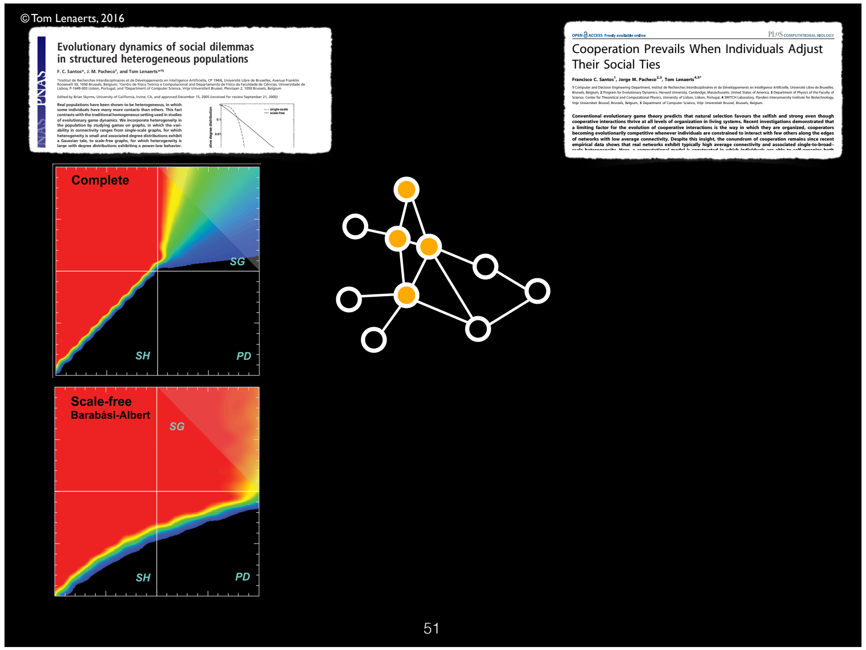
48-2


©Tom Lenaerts, 2015
How to reach cooperation?
Kin selection
1
r
Direct reciprocity
Network reciprocity
Group selection
Indirect reciprocity
Nowak, M.A. Five Rules for the Evolution of Cooperation. Science 314, 1560–1563 (2006).
Assortment between cooperators is key to success !

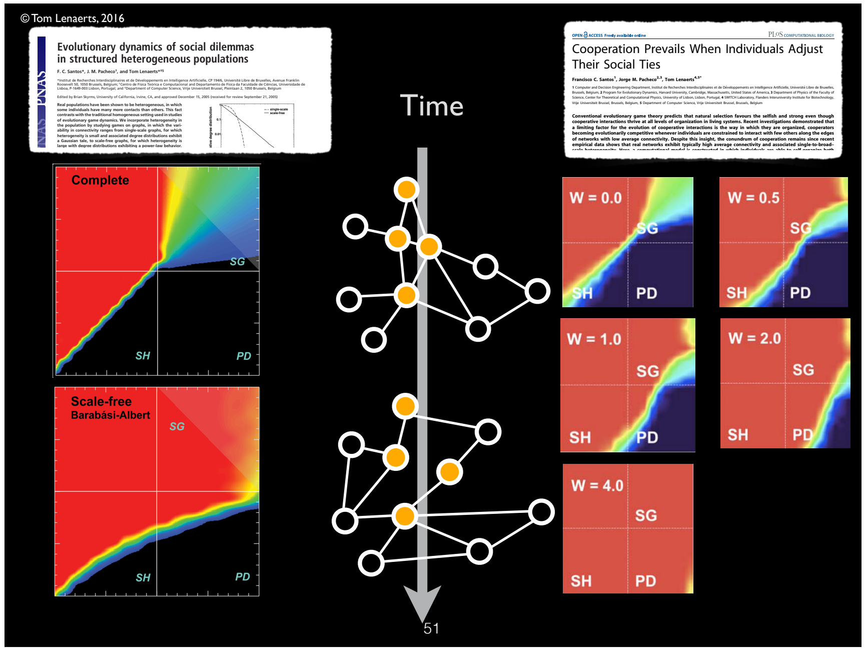
49


©Tom Lenaerts, 2016
Networks and the evolution of cooperaion
Learning to coordinate in complex networks
Sven Van Segbroeck, Steven de Jong, Ann Nowé, Francisco C Santos and Tom Lenaerts
Evolutionary dynamics of social dilemmas in structured heterogeneous populations
F. C. Santos, J. M. Pacheco, and Tom Lenaerts
PHYSICAL REVIEW LETTERS
Emergence of Fairness in Repeated Group Interactions
S. Van Segbroeck, J. M. Pacheco, T. Lenaerts, and F. C. Santos
Cooperation Prevails When Individuals Adjust Their Social Ties
Francisco C. Santos, Jorge M. Pacheco, Tom Lenaerts
50

50


©Tom Lenaerts, 2016
Evolutionary dynamics of social dilemmas in structured heterogeneous populations
Cooperation Prevails When Individuals Adjust Their Social Ties
Complete
SG
SH
PD
Scale-free
Barabási-Albert
SG
SH
PD
51

51-1


©Tom Lenaerts, 2016
Evolutionary dynamics of social dilemmas in structured heterogeneous populations
Cooperation Prevails When Individuals Adjust Their Social Ties
Time
Complete
SG
SH
PD
Scale-free
Barabási-Albert
SG
SH
PD
W = 0.0
W = 0.5
W = 1.0
W = 2.0
W = 4.0
51

51-2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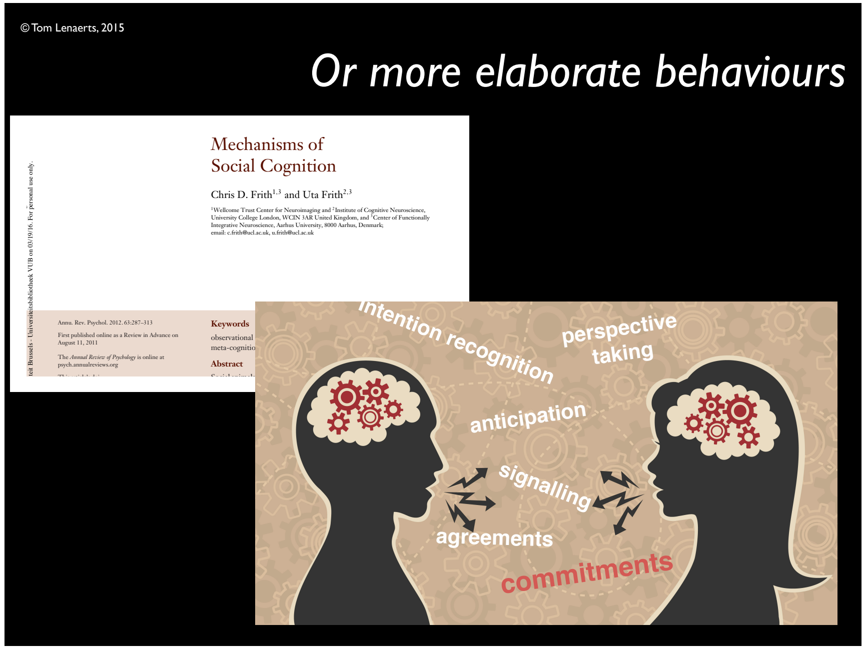
Evolutionary dynamics of social d
in structured heterogeneous popu
ils When Individuals Adjust
Complete
SH
Scale-free
Barabási-Albert
SG
SH
W = 0.5
SG
SH
PD
W = 2.0
SG
SH
PD
KEEP
CALM
AND
maybe more on this
topic later (if there
is time)

51-3


©Tom Lenaerts, 2015
Or more elaborate behaviours
Mechanisms of
Social Cognition
Chris D. Frith and Uta Frith
Keywords
Abstract
intention recognition
perspective
taking
anticipation
signalling
agreements
commitments

52


©Tom Lenaerts, 2016
Trust drives social interactions ...
integrity
sincerity
Trust
reliability
commitment
consistency
competence
How to make sure we can trust a partner?
53

53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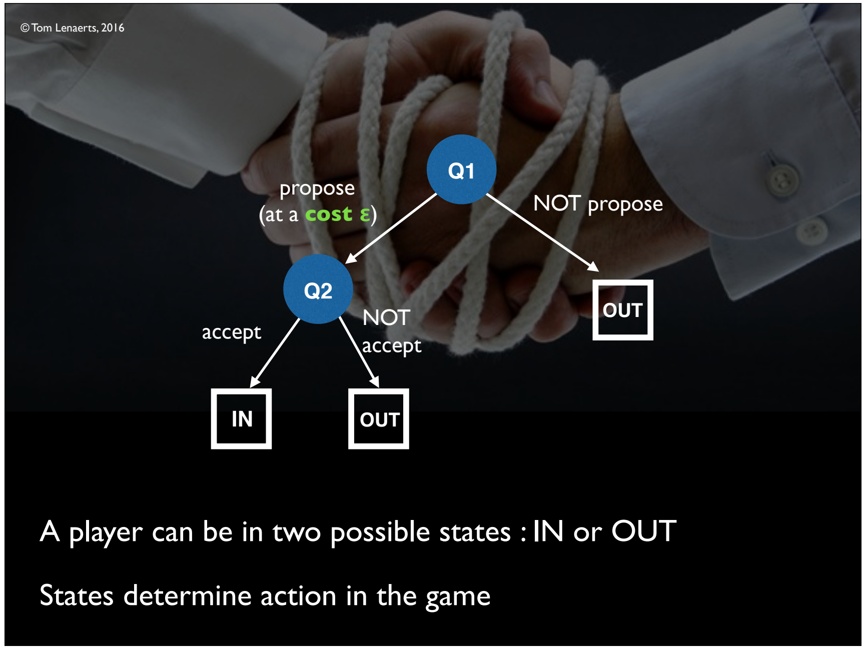
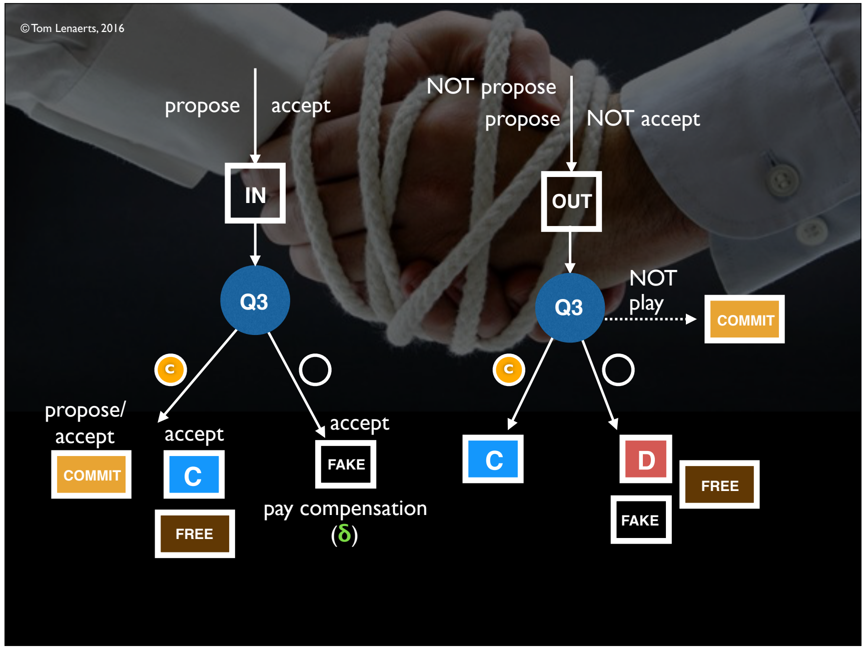
“Commitment is giving up options to
change incentives in a situation”
“specialised capacities to make
commitments may have evolved by
natural selection”
EVOLUTION
and the CAPACITY
for COMMITMENT
RANDOLPH M. NESSE

54

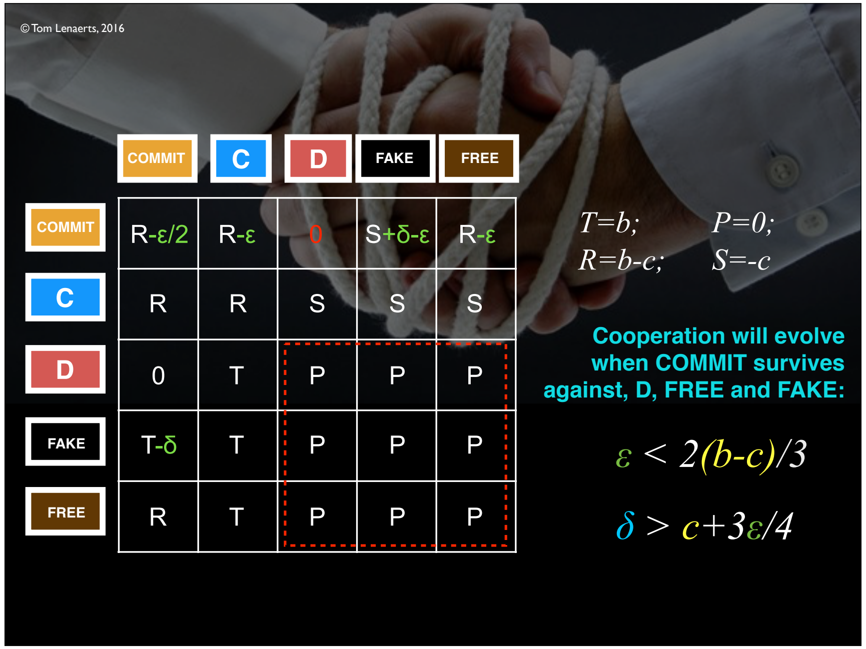
55



56

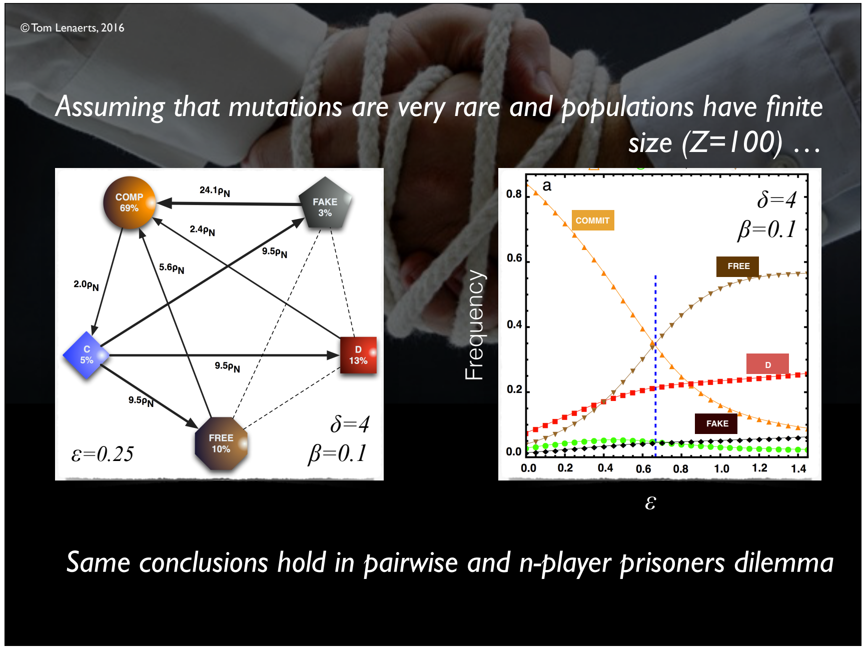


57



	COMMIT	C	D	FAKE	FREE
COMMIT	R-ε/2	R-ε	0	S+δ-ε	R-ε
C	R	R	S	S	S
D	0	T	P	P	P
FAKE	T-δ	T	P	P	P
FREE	R	T	P	P	P

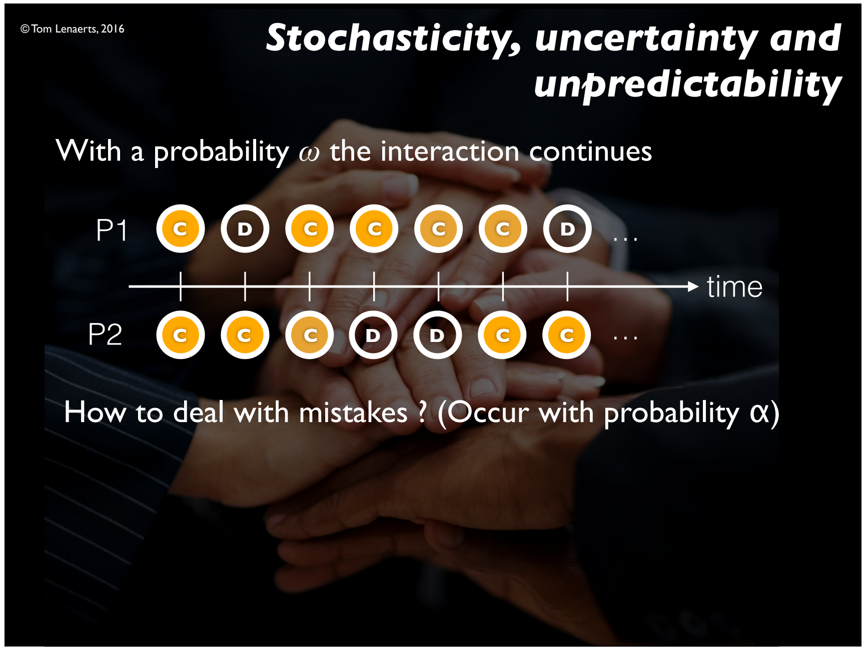
58


©Tom Lenaerts, 2016
Assuming that mutations are very rare and populations have finite size (Z=100) …
COMP 69%
FAKE 3%
C 5%
D 13%
FREE 10%
24.1ρN
2.4ρN
9.5ρN
5.6ρN
2.0ρN
9.5ρN
9.5ρN
ε=0.25
δ=4
β=0.1
Frequency
a
COMMIT
FREE
D
FAKE
δ=4
β=0.1
ε
Same conclusions hold in pairwise and n-player prisoners dilemma

59


©Tom Lenaerts, 2016
Commitments are often long-term
With a probability ω the interaction continues
P1
P2
time

60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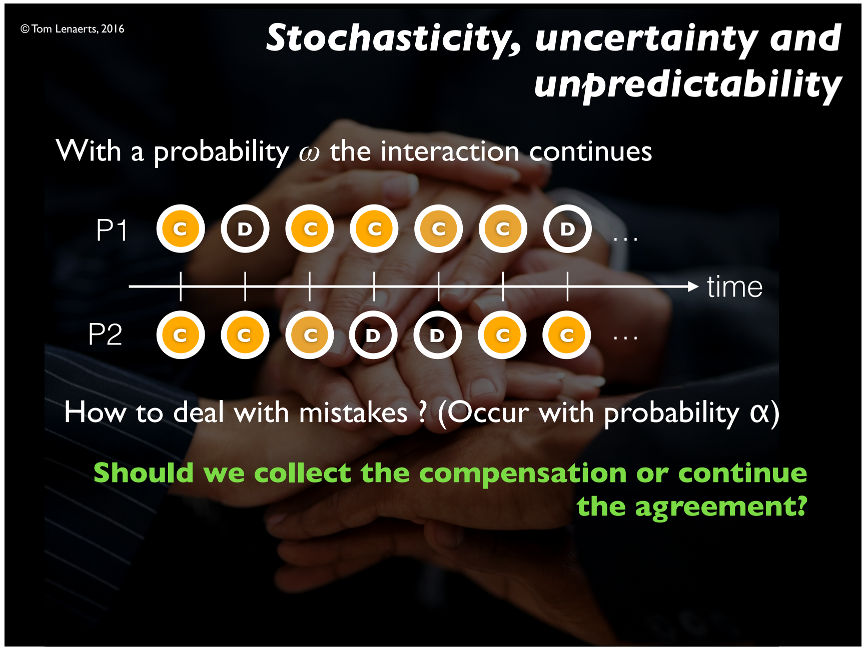
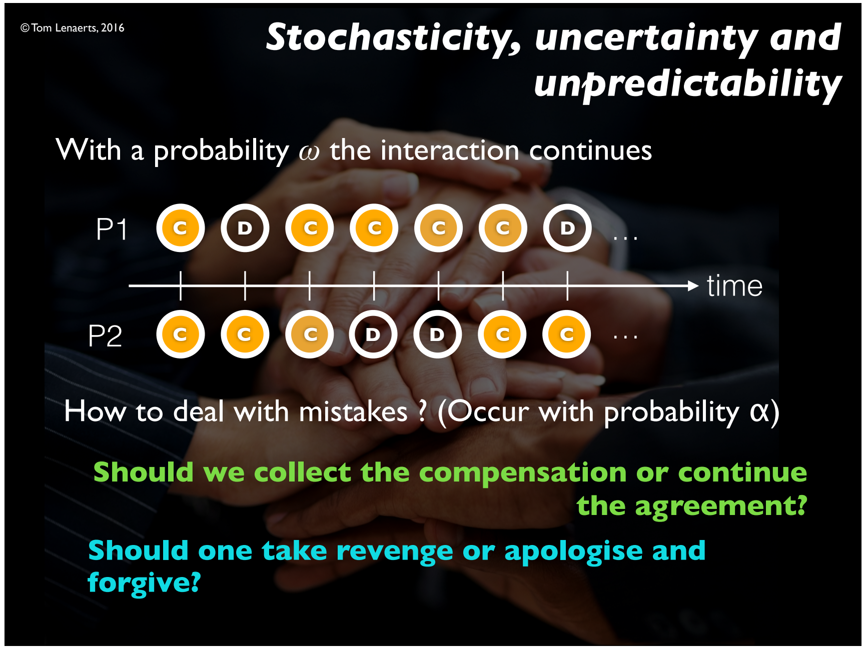
Stochasticity, uncertainty and unpredictability
With a probability ω the interaction continues
P1
P2
time
How to deal with mistakes ? (Occur with probability α)

61-1


©Tom Lenaerts, 2016
Stochasticity, uncertainty and unpredictability
With a probability ω the interaction continues
P1
P2
time
How to deal with mistakes ? (Occur with probability α)
Should we collect the compensation or continue the agreement?

61-2


©Tom Lenaerts, 2016
Stochasticity, uncertainty and unpredictability
With a probability ω the interaction continues
P1 C D C C C C D …
time
P2 C C C D D C C …
How to deal with mistakes ? (Occur with probability α)
Should we collect the compensation or continue the agreement?
Should one take revenge or apologise and forgive?

61-3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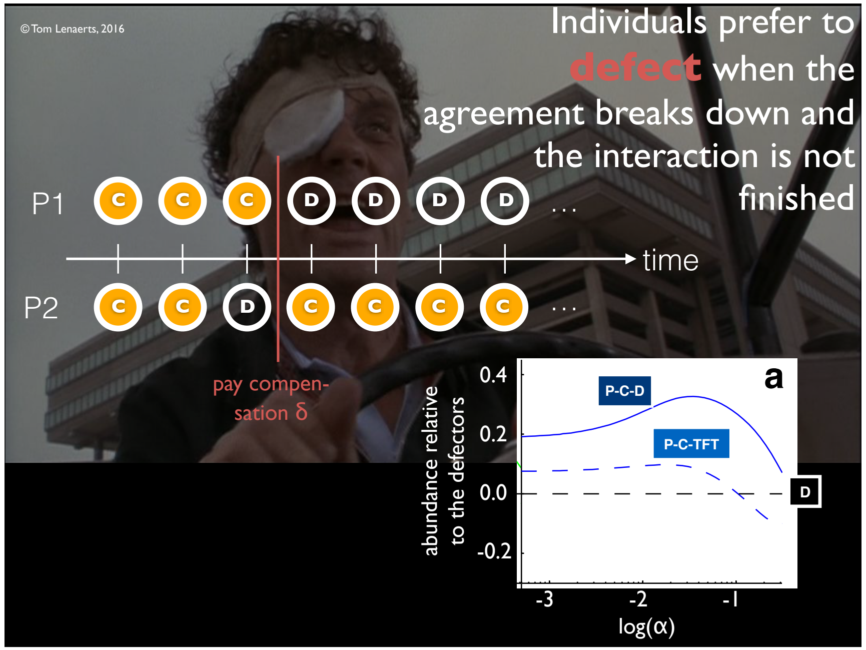
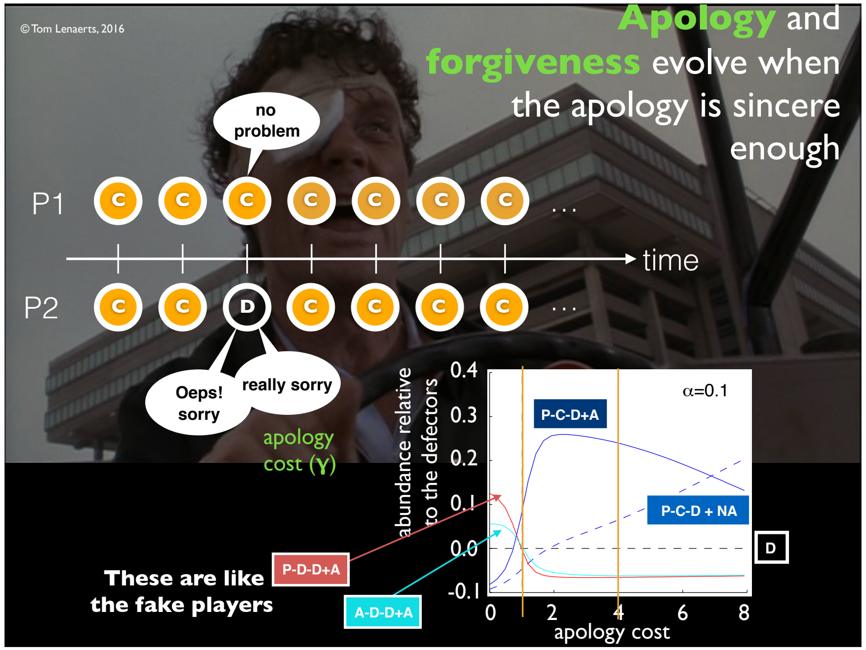
www.nature.com/scientificreports
SCIENTIFIC REPORTS
OPEN
Apology and forgiveness evolve to resolve failures in cooperative agreements
Received: 23 February 2015
Accepted: 22 April 2015
Published: xx xx xxxx
Michael E. McCullough
BEYOND REVENGE
The Evolution of the FORGIVENESS INSTINCT

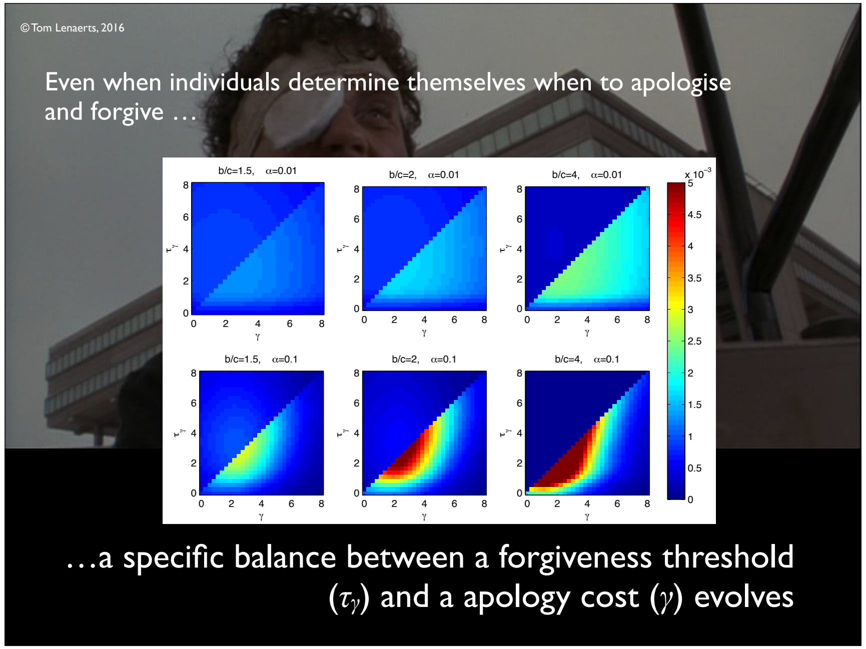
62


©Tom Lenaerts, 2016
Individuals prefer to defect when the agreement breaks down and the interaction is not finished
P1 C C C D D D D …
time
P2 C C D C C C C …
pay compen-sation δ
a
P-C-D
P-C-TFT
D
abundance relative to the defectors
0.4
0.2
0.0
-0.2
-3
-2
-1
log(α)

63


©Tom Lenaerts, 2016
Apology and forgiveness evolve when the apology is sincere enough
no problem
P1 C C C C C C C …
time
P2 C C D C C C C …
Oeps! sorry
really sorry
apology cost (γ)
These are like the fake players
P-D-D+A
A-D-D+A
α=0.1
P-C-D+A
P-C-D + NA
D
abundance relative to the defectors
0.4
0.3
0.2
0.1
0.0
-0.1
0
2
4
6
8
apology cost

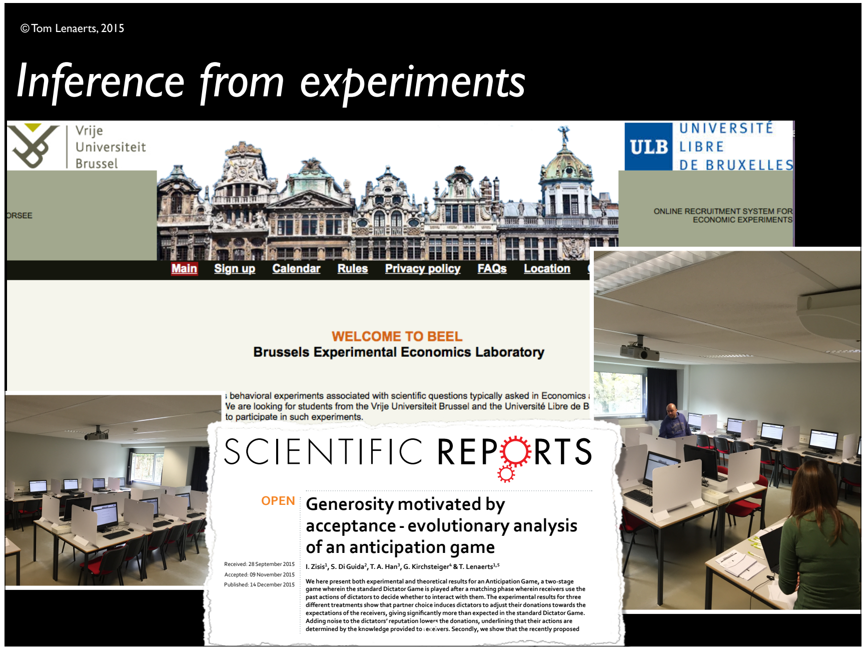
64


©Tom Lenaerts, 2016
Even when individuals determine themselves when to apologise and forgive …
b/c=1.5, α=0.01
b/c=2, α=0.01
b/c=4, α=0.01
b/c=1.5, α=0.1
b/c=2, α=0.1
b/c=4, α=0.1
x 10^-3
…a specific balance between a forgiveness threshold (τ_γ) and a apology cost (γ) evolves

65


©Tom Lenaerts, 2016
How do people really behave in games?
66

66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Inference from experiments
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ULB UNIVERSITÉ LIBRE DE BRUXELLES
ONLINE RECRUITMENT SYSTEM FOR ECONOMIC EXPERIMENTS
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Brussels Experimental Economics Laboratory
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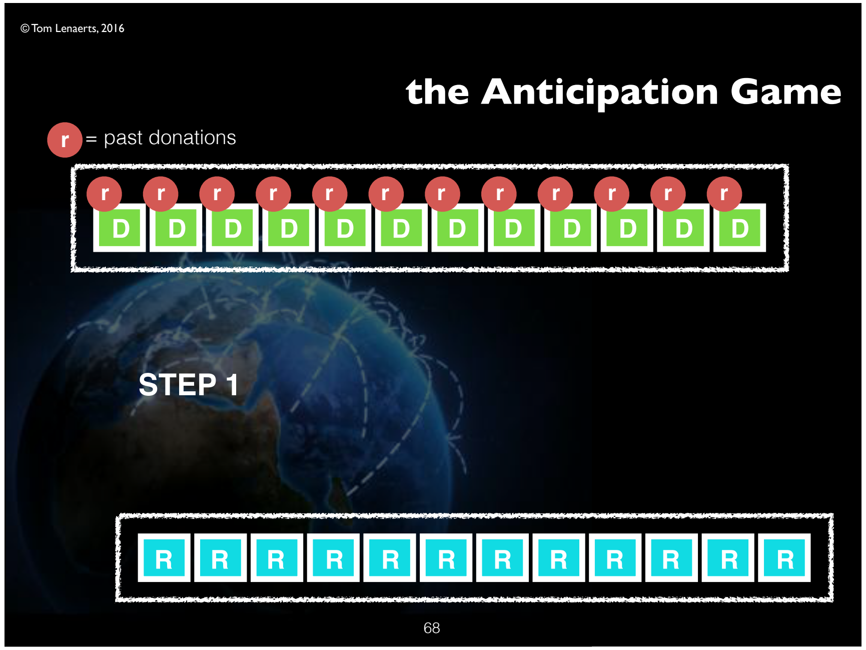
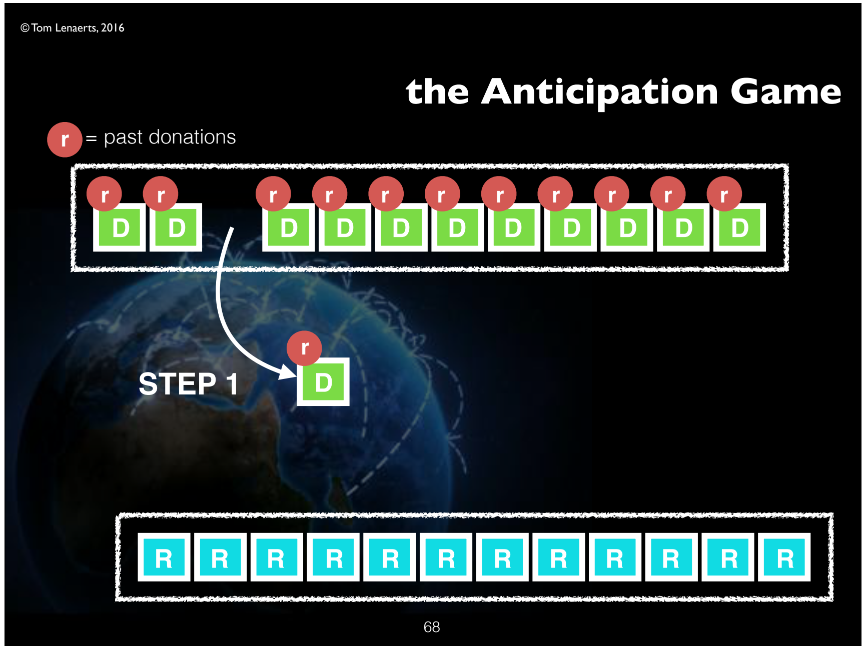
Generosity motivated by acceptance - evolutionary analysis of an anticipation game

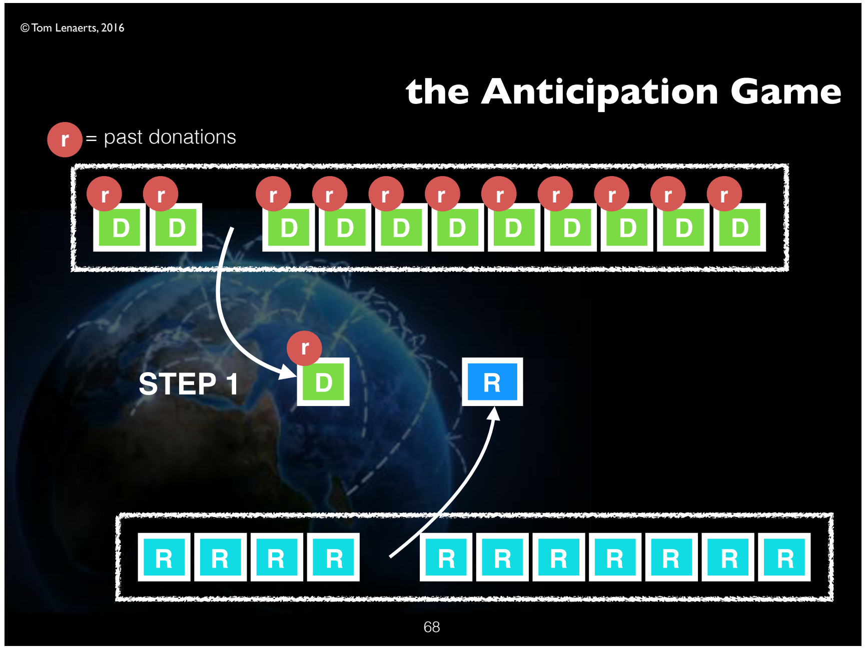
67


©Tom Lenaerts, 2016
the Anticipation Game
r = past donations
r r r r r r r r r r r r
D D D D D D D D D D D D
STEP 1
R R R R R R R R R R R R
68

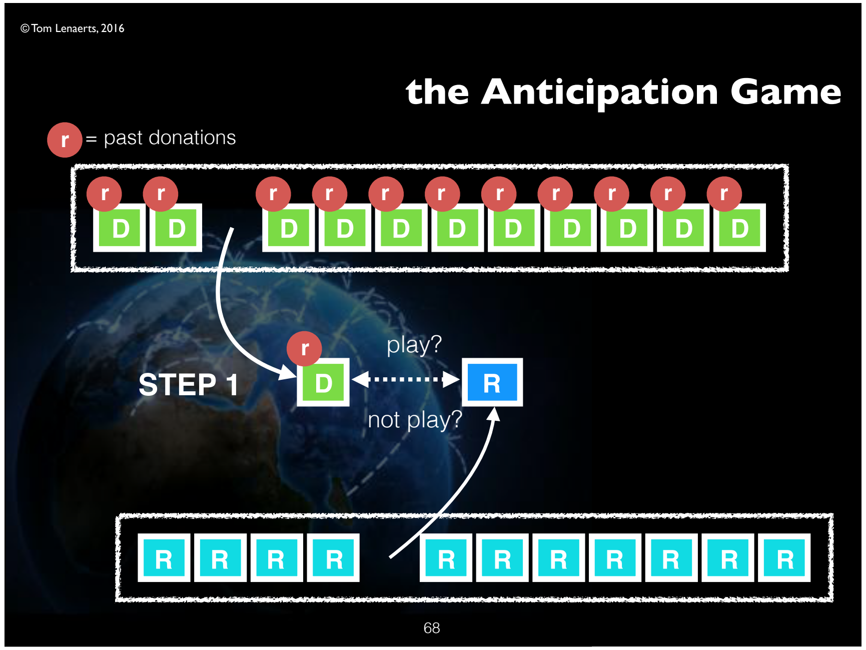
68-1



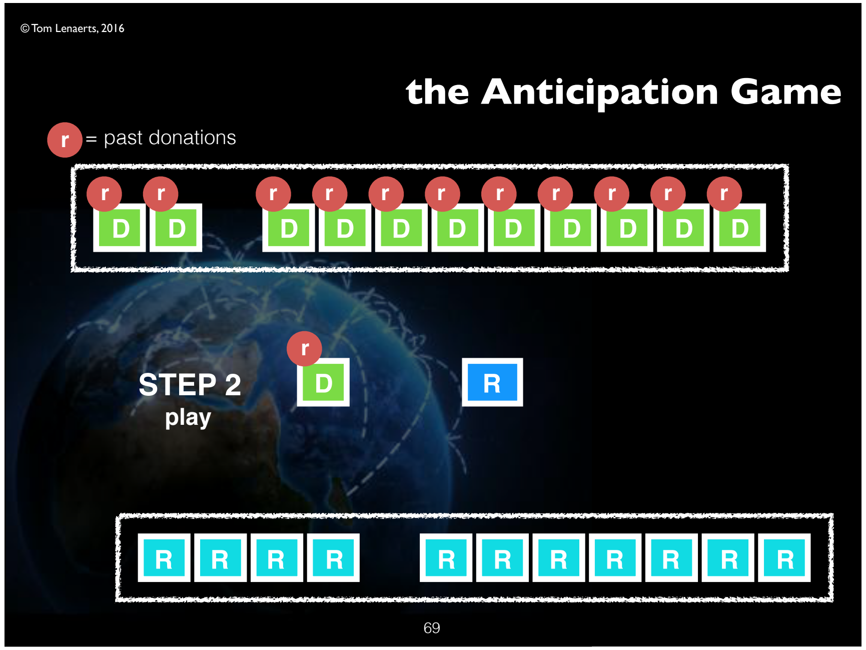
68-2



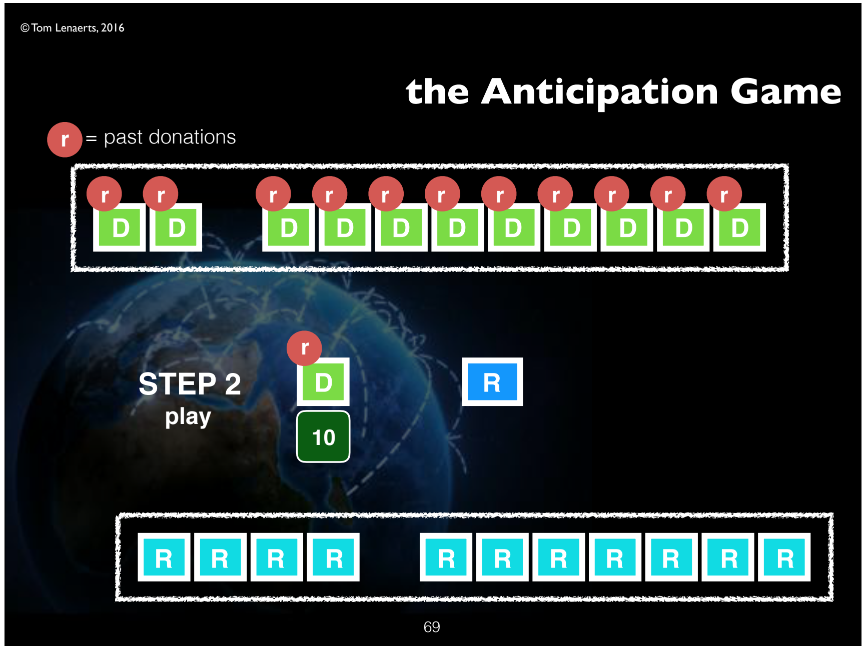
68-3



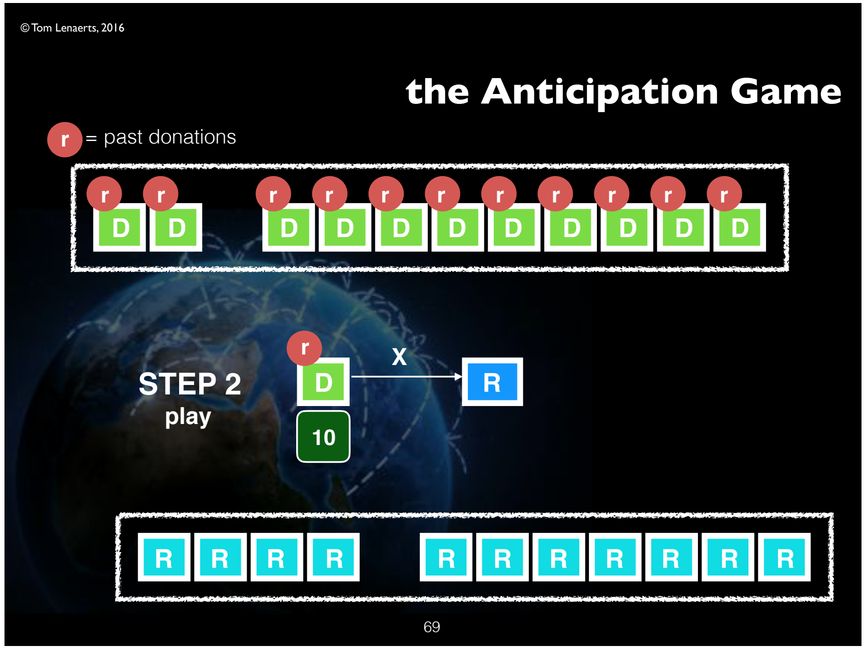
68-4



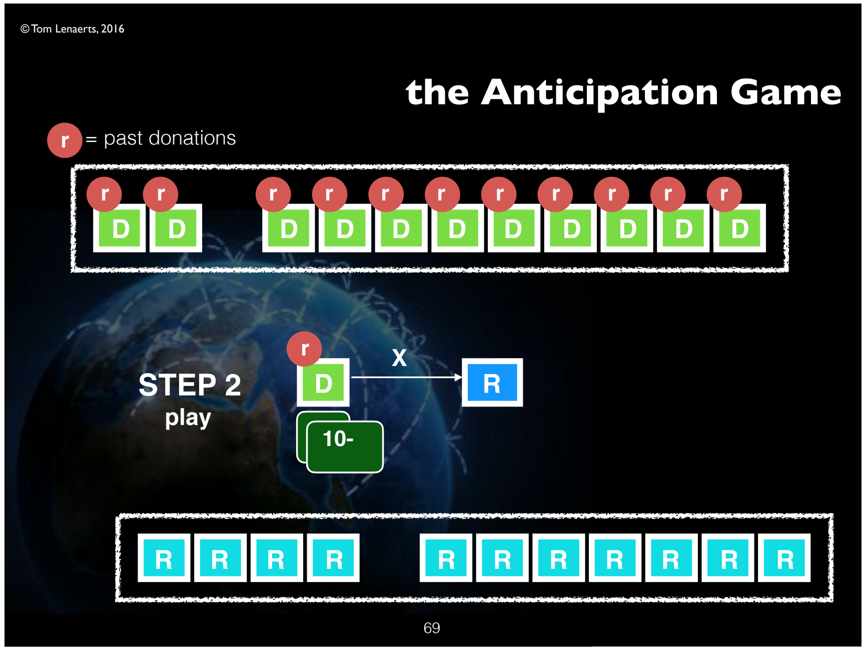
69-1



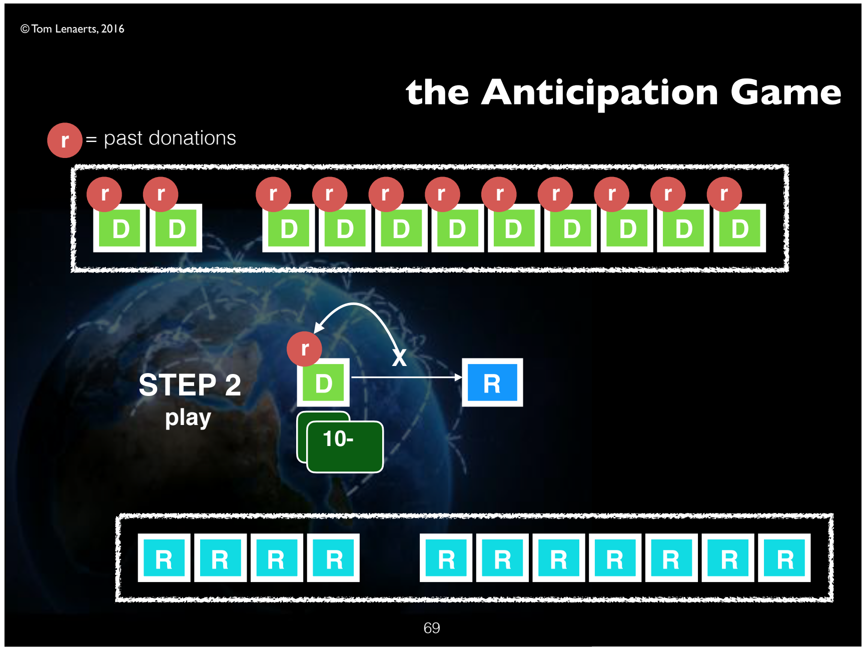
69-2



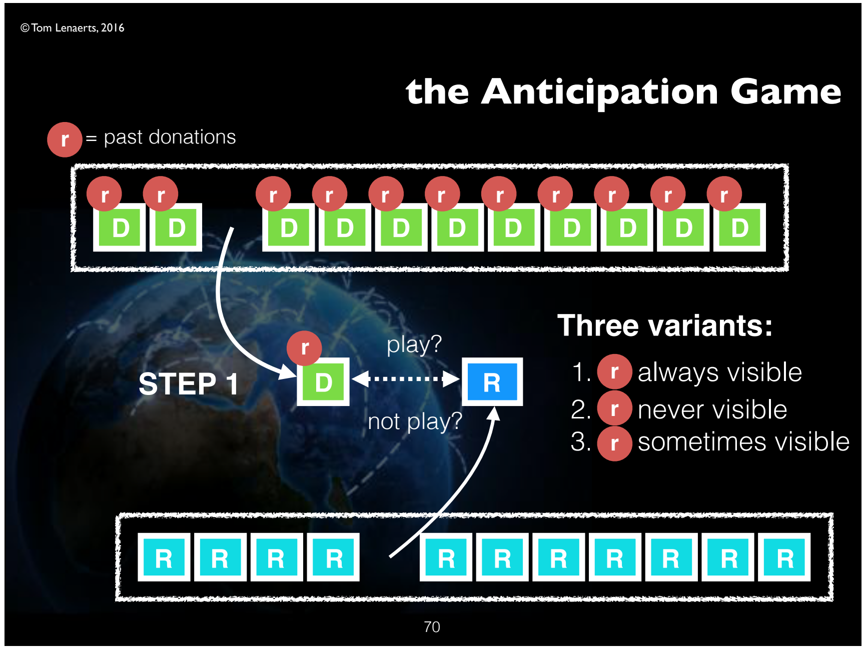
69-3



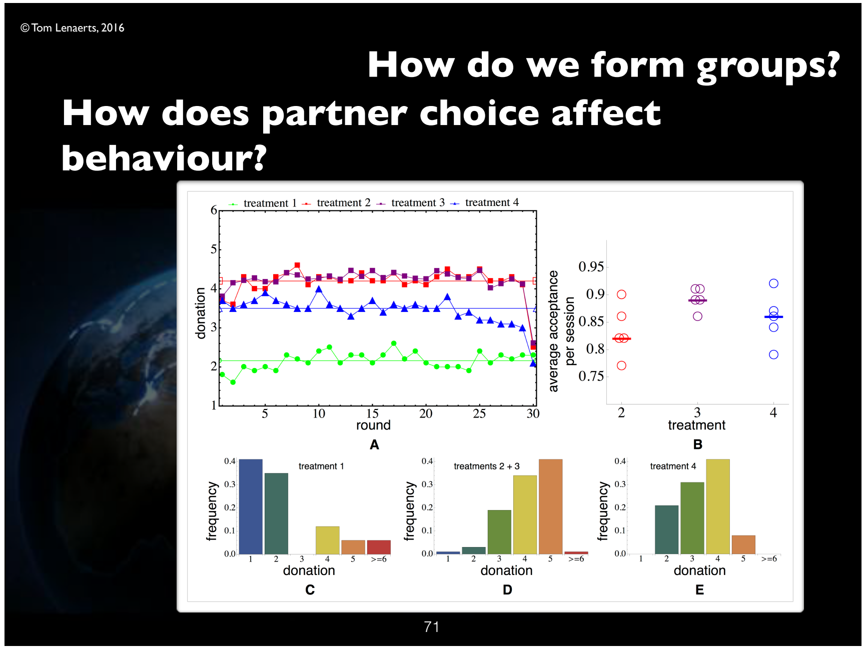
69-4



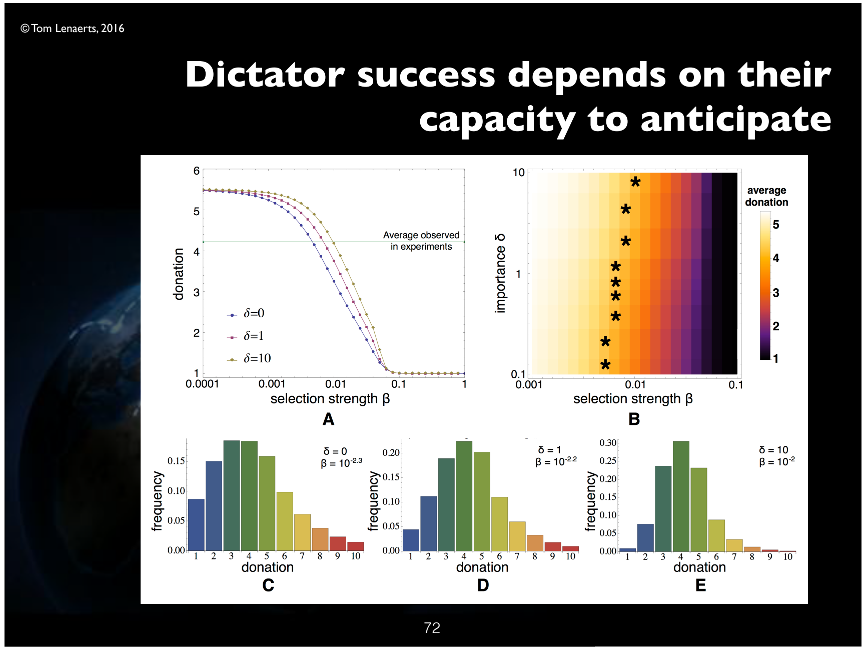
69-5


©Tom Lenaerts, 2016
the Anticipation Game
r = past donations
STEP 1
play?
not play?
Three variants:
1. r always visible
2. r never visible
3. r sometimes visible
70

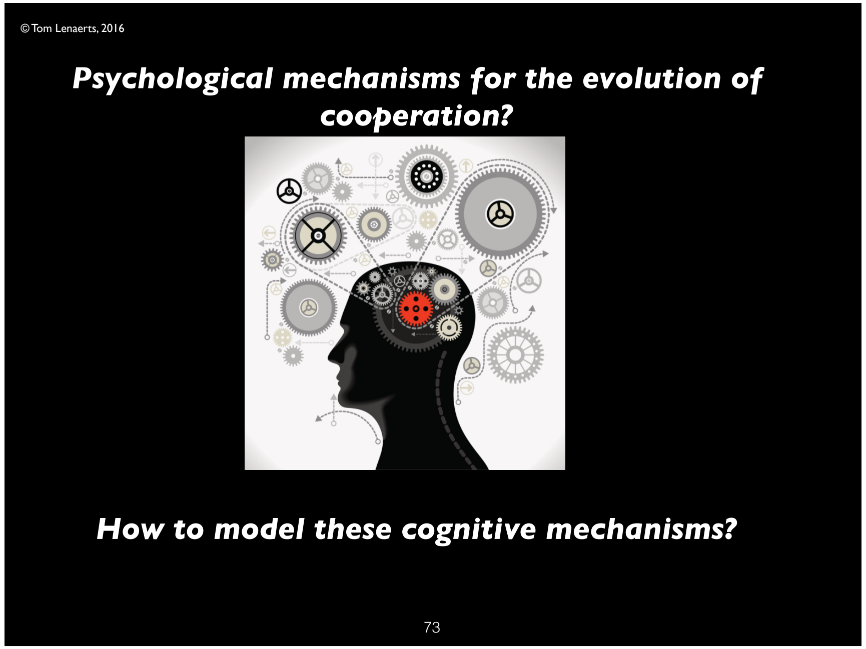
70


©Tom Lenaerts, 2016
How do we form groups?
How does partner choice affect behaviour?
treatment 1
treatment 2
treatment 3
treatment 4
donation
round
A
average acceptance per session
treatment
B
frequency
donation
C
treatments 2 + 3
D
E
71

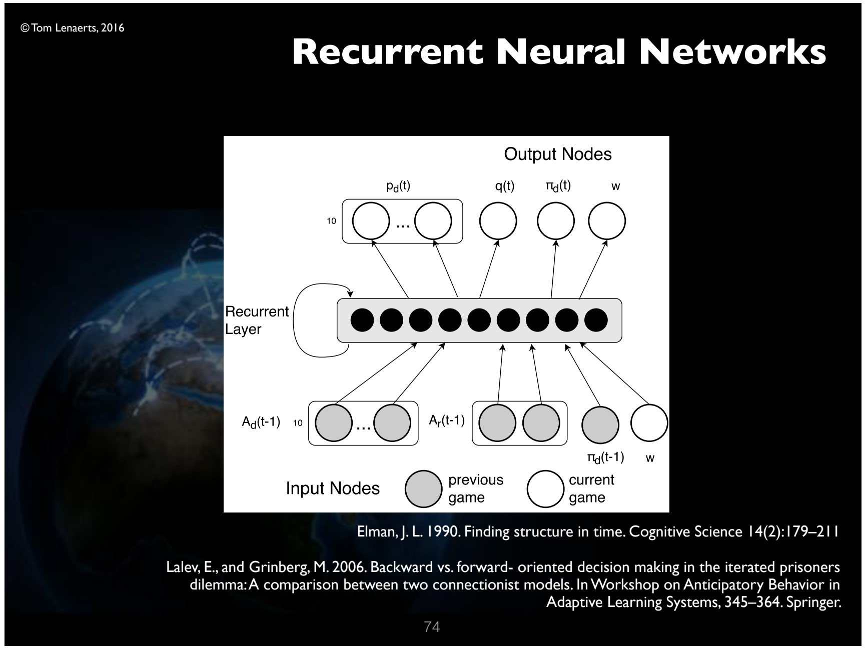
71


©Tom Lenaerts, 2016
Dictator success depends on their capacity to anticipate
donation
Average observed in experiments
δ=0
δ=1
δ=10
selection strength β
A
importance δ
average donation
B
frequency
donation
C
D
E
72

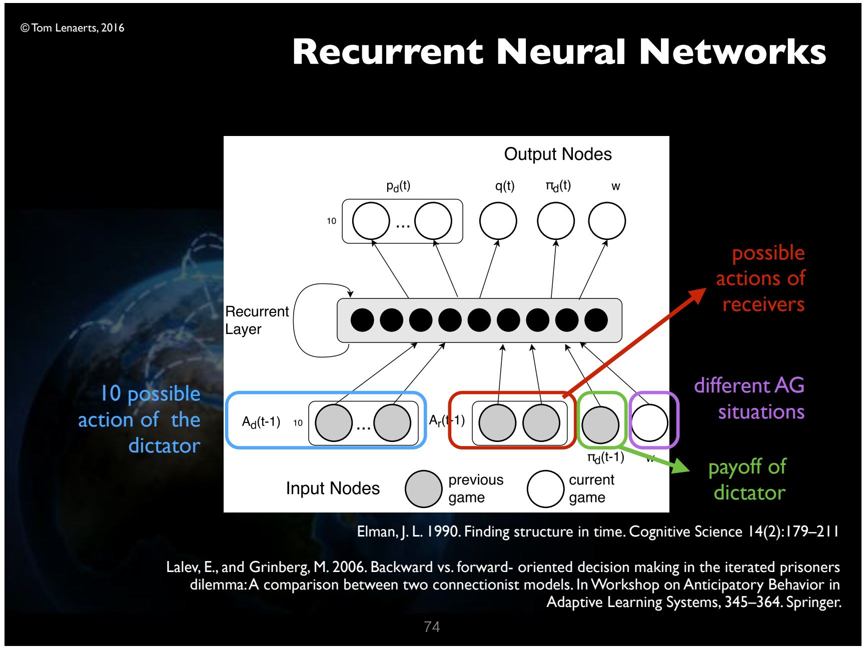
72


©Tom Lenaerts, 2016
Psychological mechanisms for the evolution of cooperation?
How to model these cognitive mechanisms?
73

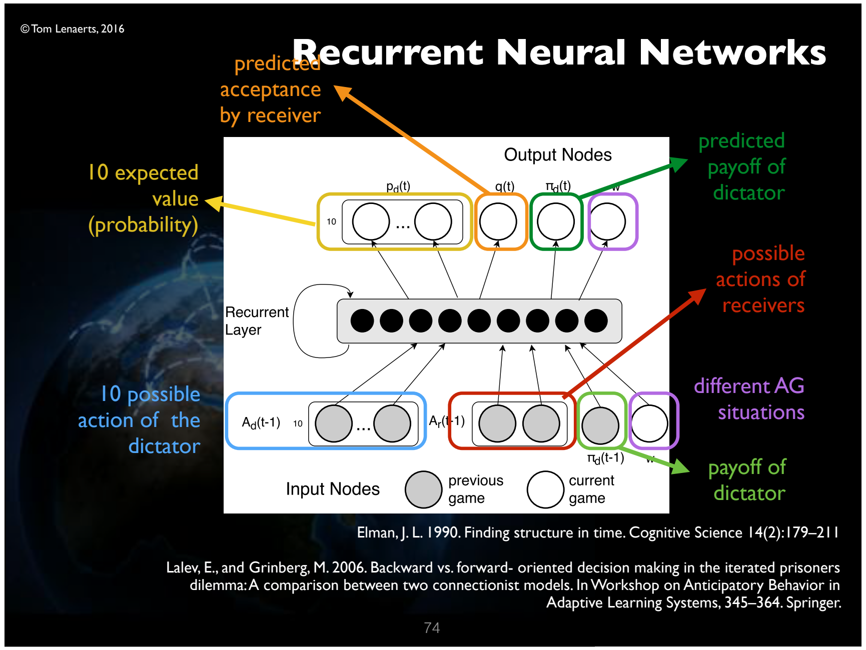
73



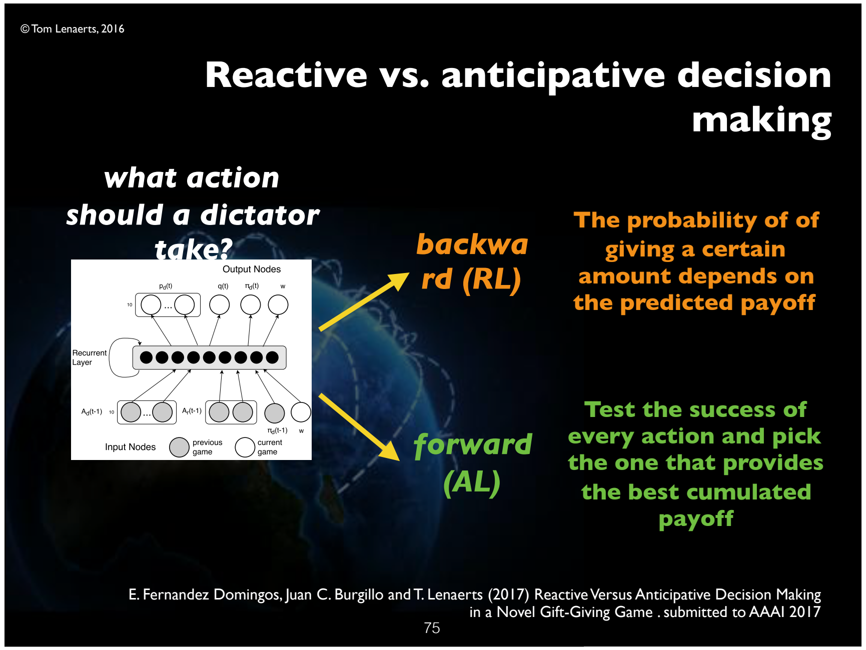
74-1



74-2



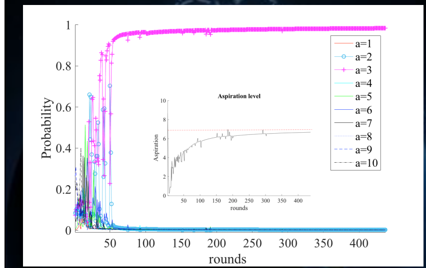
74-3



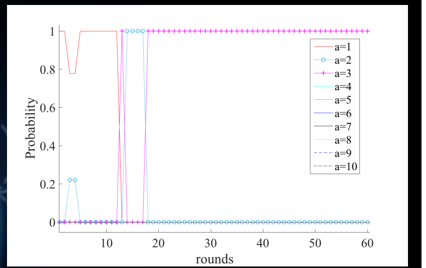
75

Reactive vs. anticipative decision making

backward (RL)



forward (AL)

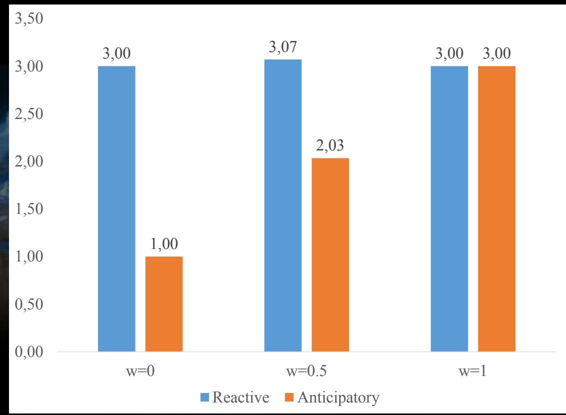


E. Fernandez Domingos, Juan C. Burgillo and T. Lenaerts (2017) Reactive Versus Anticipative Decision Making in a Novel Gift-Giving Game . submitted to AAAI 2017

76

76

Reactive vs. anticipative decision making



E. Fernandez Domingos, Juan C. Burgillo and T. Lenaerts (2017) Reactive Versus Anticipative Decision Making in a Novel Gift-Giving Game . submitted to AAAI 2017

77

77

What to conclude?

Changes in **r** changes the environment for decision making

To reproduce individual behaviour → introduce payoff received for future gains

Anticipatory decision making produces correct response to a changing environment

"Every living systems is an anticipating system"
(R. Rosen 1985)

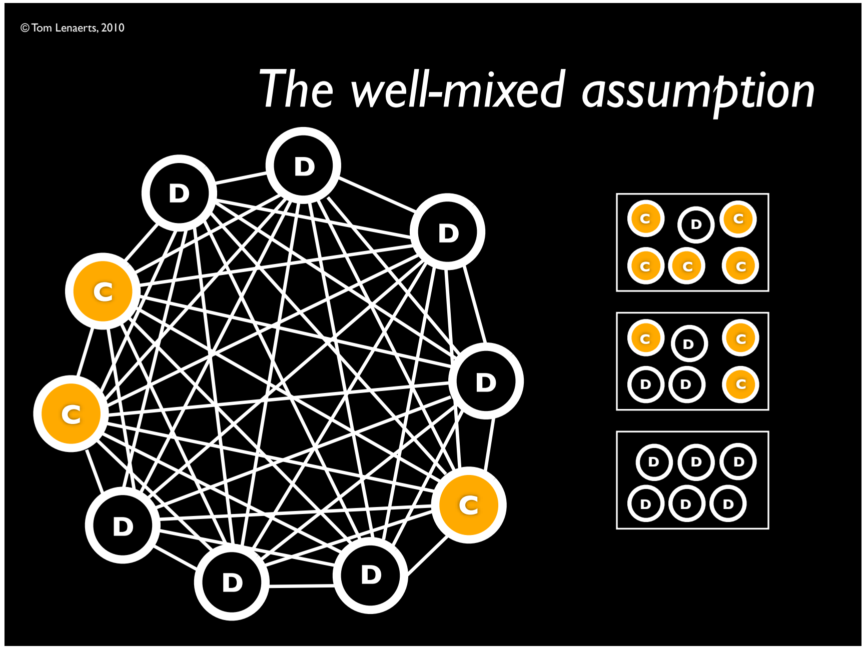
78

78

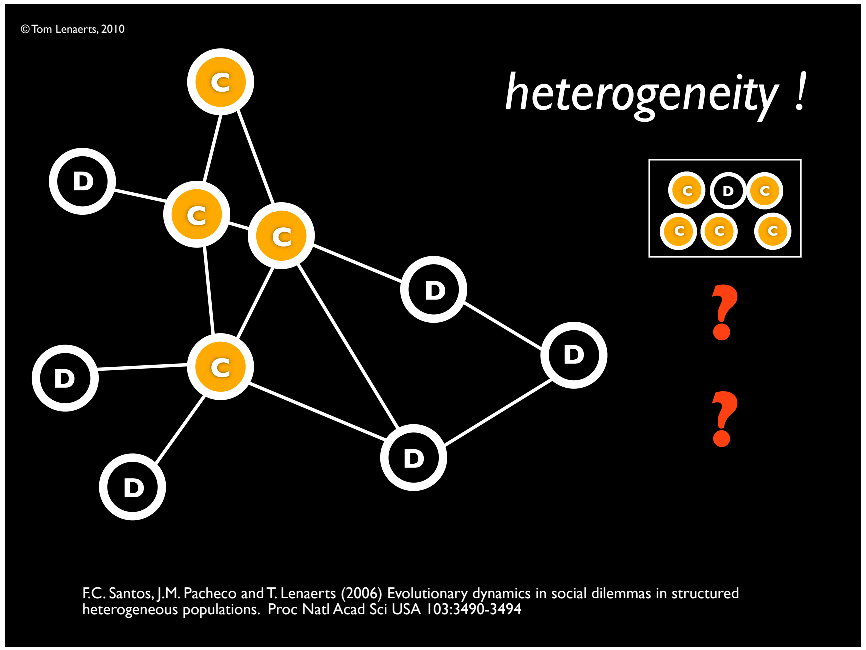
Questions

79

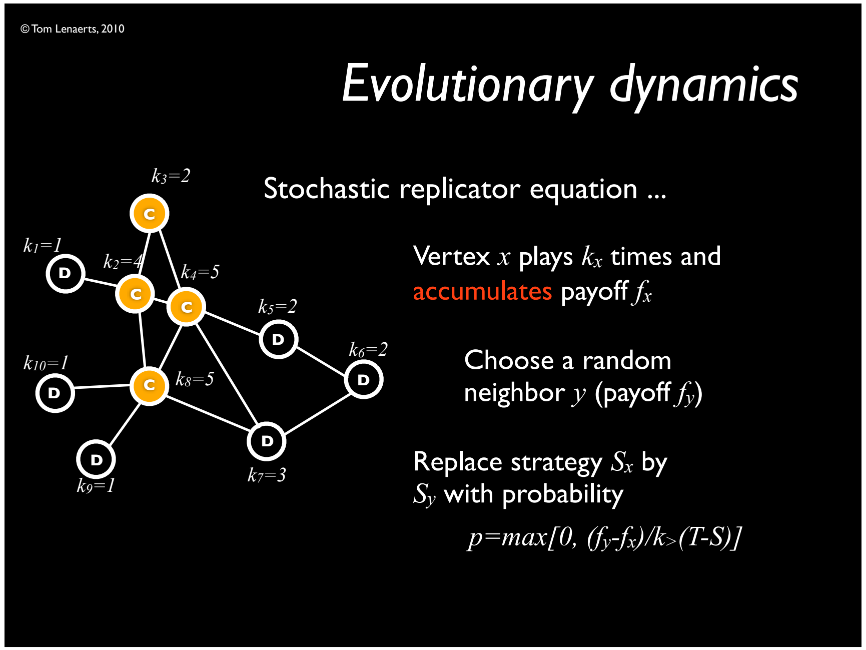
80


©Tom Lenaerts, 2010
The well-mixed assumption
C
D

81


©Tom Lenaerts, 2010
heterogeneity !
C
D
?
?
F.C. Santos, J.M. Pacheco and T. Lenaerts (2006) Evolutionary dynamics in social dilemmas in structured heterogeneous populations. Proc Natl Acad Sci USA 103:3490-3494

82


©Tom Lenaerts, 2010
Evolutionary dynamics
Stochastic replicator equation ...
Vertex x plays k_x times and accumulates payoff f_x
Choose a random neighbor y (payoff f_y)
Replace strategy S_x by S_y with probability
$p=max[0, (f_y-f_x)/k_>(T-S)]$
$k_1=1$
$k_2=4$
$k_3=2$
$k_4=5$
$k_5=2$
$k_6=2$
$k_7=3$
$k_8=5$
$k_9=1$
$k_{10}=1$
C
D

83

Simulation I

The EGT assumption:

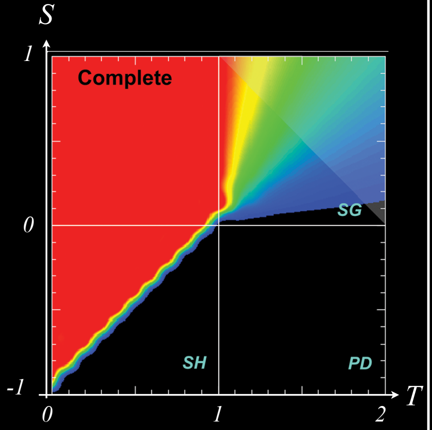
Everyone interacts with everyone

$N=10^4$

100 runs

50% **C**, 50% **D**

$R=1,\ P=0$



84

Which networks?

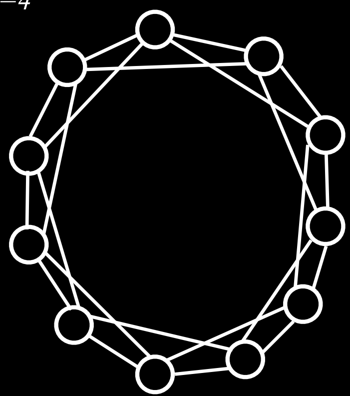
Which models have people been using?

What does data tell us about real networks?

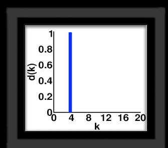
85

Regular graphs

Every node has exactly the same degree $<k>=4$



regular and democratic network



86

Simulation II

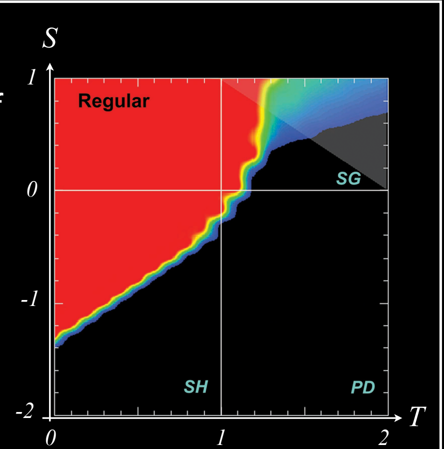
There is a limit to the number of interactions but its democratic

$N=10^4\ \ <k>=4$

100 runs

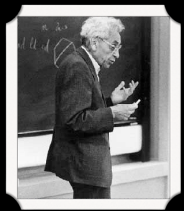
50% **C**, 50% **D**

$R=1,\ P=0$



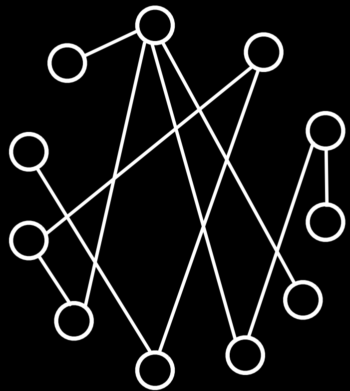
87

Random graphs

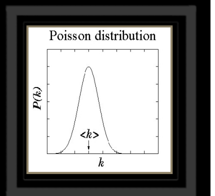


P. Erdős
(1913-1996)

Connect with probability p



$p=1/6, N=10$
$\Rightarrow \langle k \rangle \approx 1.5$



random and
democratic
network

88

Simulation III

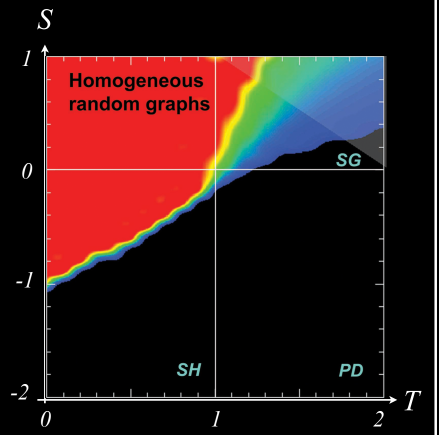
There is a limit to the number of interactions but its democratic and random

$N=10^4$ $\langle k \rangle=4$

100 runs

50% **C**, 50% **D**

$R=1, P=0$



89

Which networks?

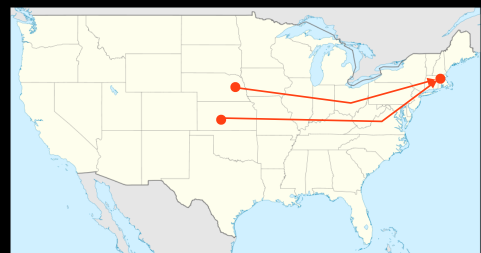
Which models have people been using?

What does data tell us about real networks?

90

Small world experiment

What is the average number of connections between any two people?

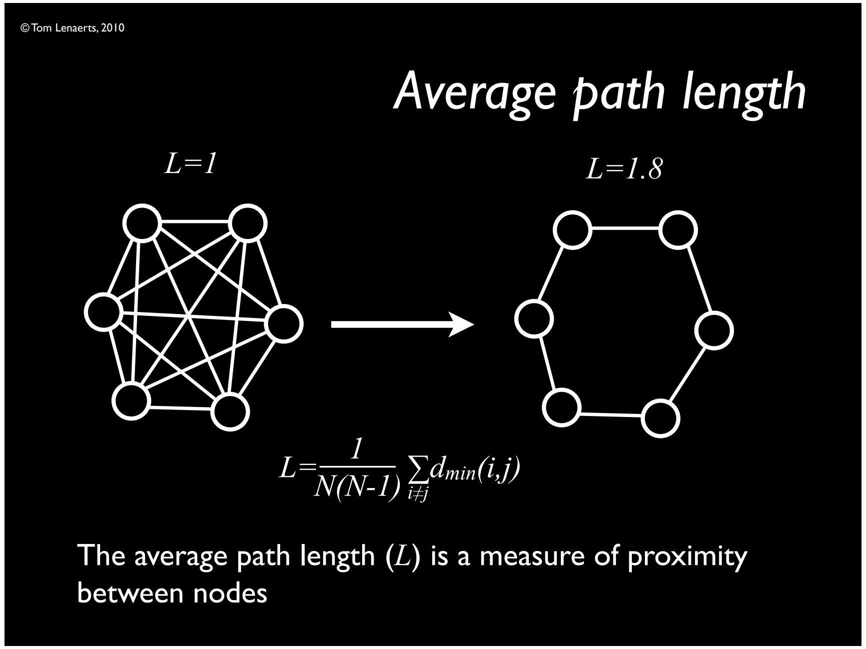


S. Milgram
(1933-1984)

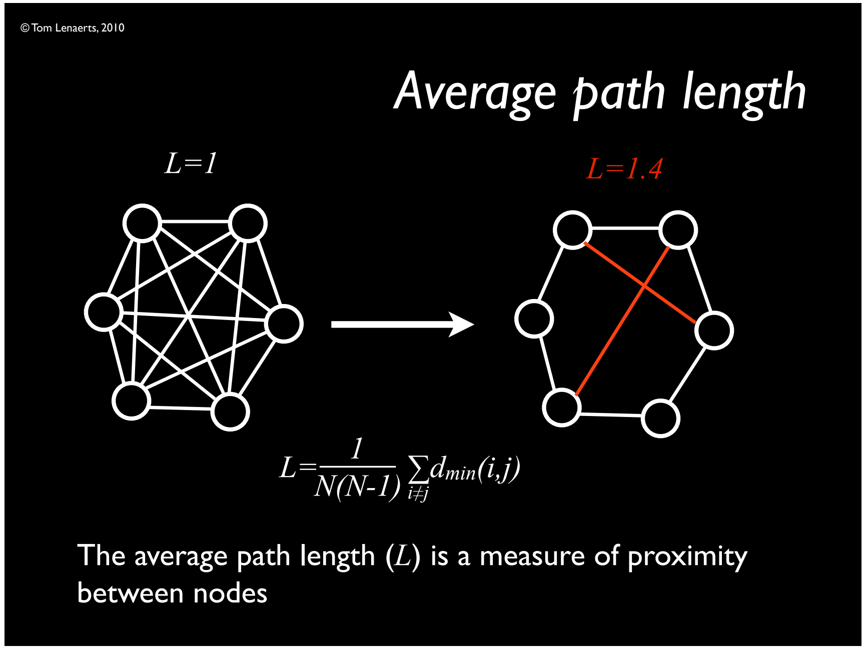
"Six degrees of separation" (J. Guare, 1990)

J. Travers and S. Milgram (1969) An experimental study of the small-world problem. Sociometry 32(4):425-443

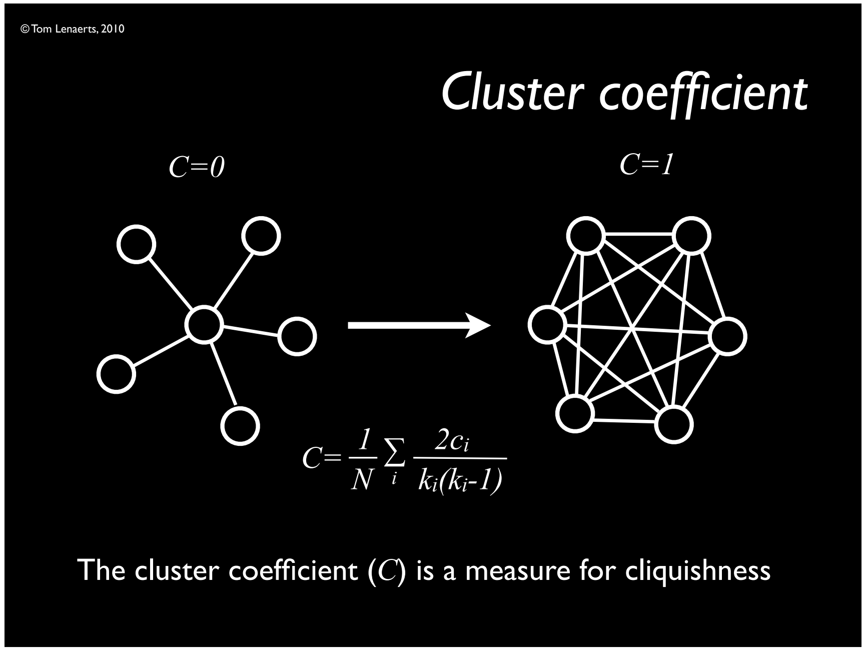
91



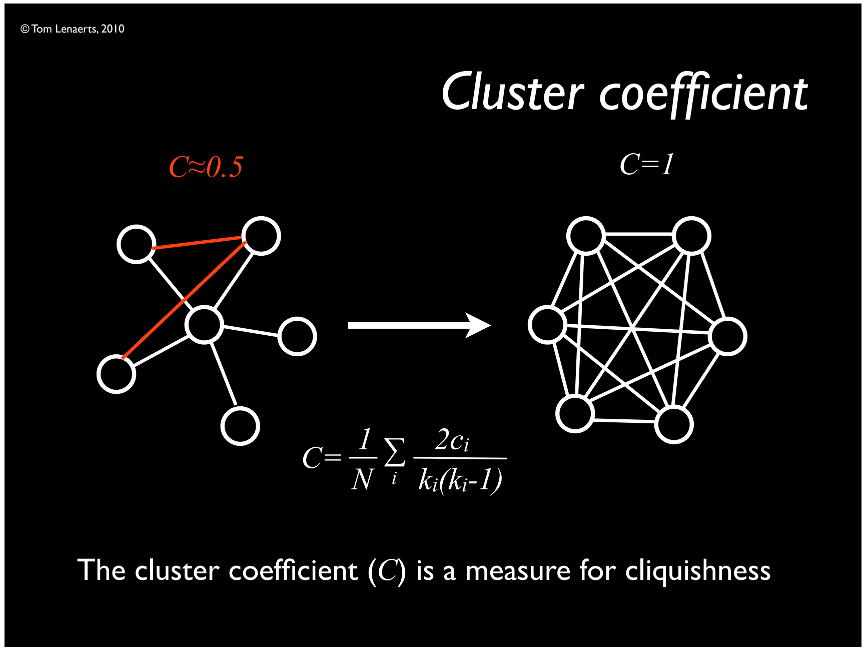
92-1



92-2

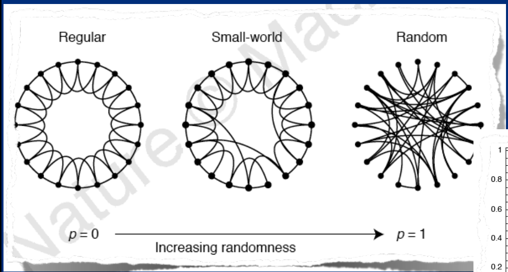


93-1

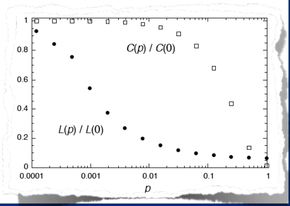


93-2

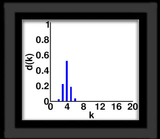
Small world networks



electrical power grid of South California, network of world airports, movie-actor network, the neuronal network of the worm C.elegans



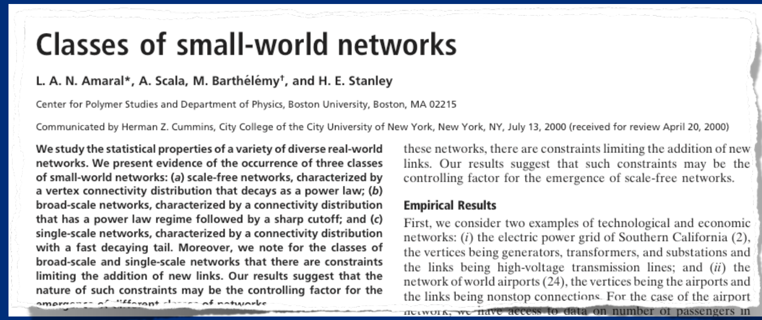
D.Watts and S. Strogatz (1998) Collective dynamics of 'small-world' networks. Nature 393:440-442



Mechanism:

1. take a regular graph
2. randomly rewire every edge with probability p
3. avoid loops and double edges

Network classes



Classes of small-world networks

L. A. N. Amaral*, A. Scala, M. Barthélémy†, and H. E. Stanley

Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215

Communicated by Herman Z. Cummins, City College of the City University of New York, New York, NY, July 13, 2000 (received for review April 20, 2000)

We study the statistical properties of a variety of diverse real-world networks. We present evidence of the occurrence of three classes of small-world networks: (*a*) scale-free networks, characterized by a vertex connectivity distribution that decays as a power law; (*b*) broad-scale networks, characterized by a connectivity distribution that has a power law regime followed by a sharp cutoff; and (*c*) single-scale networks, characterized by a connectivity distribution with a fast decaying tail. Moreover, we note for the classes of broad-scale and single-scale networks that there are constraints limiting the addition of new links. Our results suggest that the nature of such constraints may be the controlling factor for the emergence of different classes of networks.

these networks, there are constraints limiting the addition of new links. Our results suggest that such constraints may be the controlling factor for the emergence of scale-free networks.

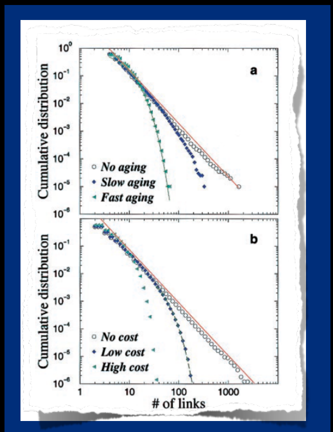
Empirical Results

First, we consider two examples of technological and economic networks: (*i*) the electric power grid of Southern California (2), the vertices being generators, transformers, and substations and the links being high-voltage transmission lines; and (*ii*) the network of world airports (24), the vertices being the airports and the links being nonstop connections. For the case of the airport network, we have access to data on number of passengers in

L.A.N. Amaral, A. Scala, M. Barthelemy and H.E. Stanley (2000) Classes of small-world networks. Proc Natl Acad Sci USA 97(21): 11149-11152

electrical power grid of South California, network of world airports, movie-actor network, acquintance network of mormons, friendship network of 417 Madison Junior High school students, the neuronal network of the worm C.elegans, the conformational space of a lattice polymer chain

Network classes

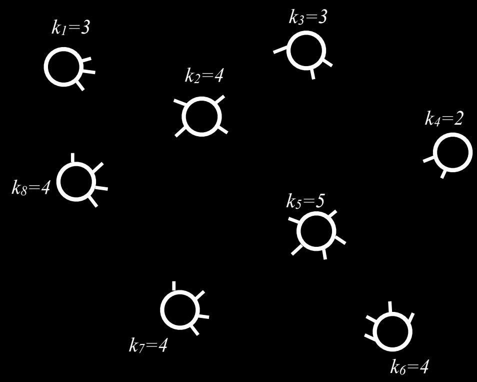


Aging of vertices as in the movie-actor network

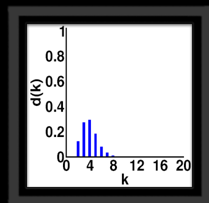
cost of adding links or the limited capacity of vertices as in the airport network

L.A.N. Amaral, A. Scala, M. Barthelemy and H.E. Stanley (2000) Classes of small-world networks. Proc Natl Acad Sci USA 97(21): 11149-11152

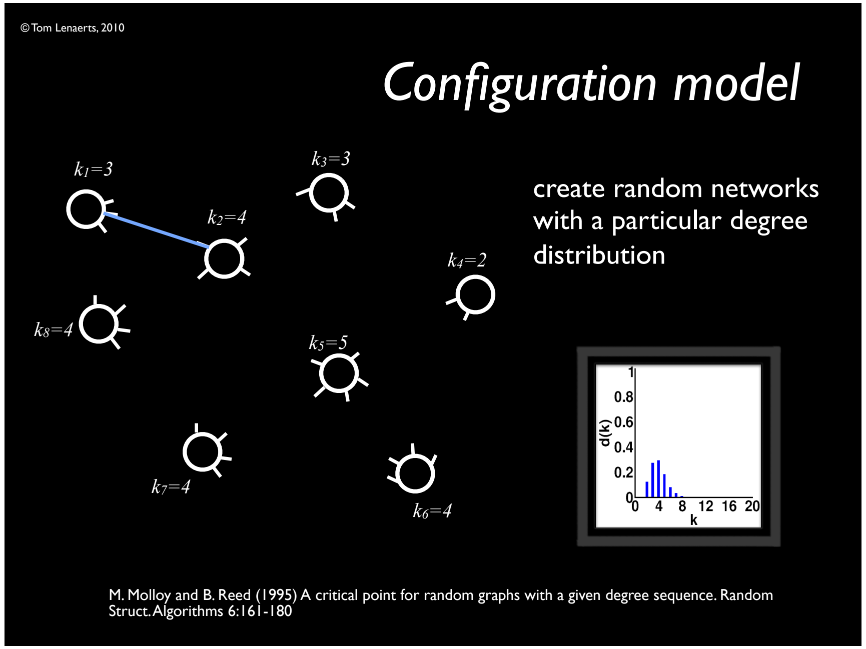
Configuration model



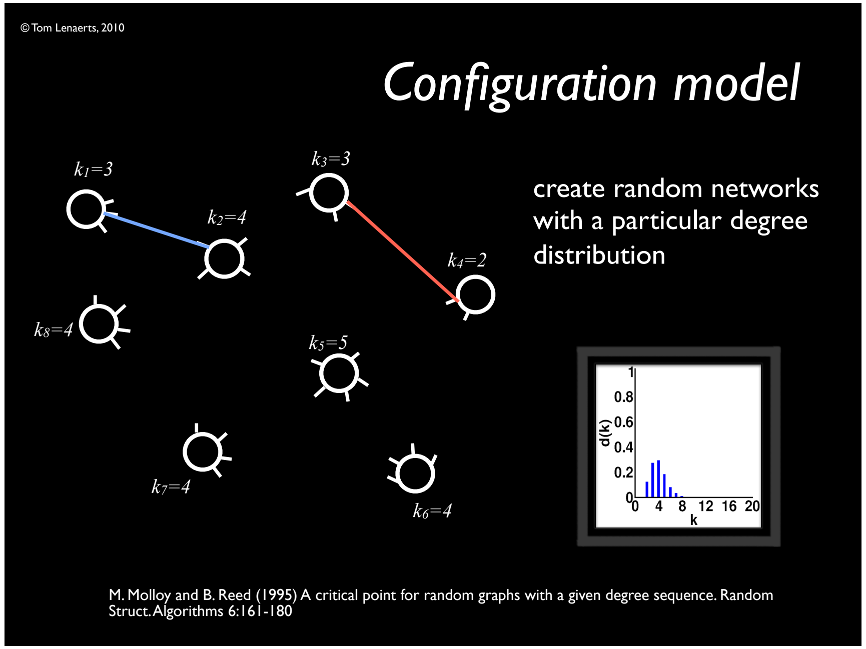
create random networks with a particular degree distribution



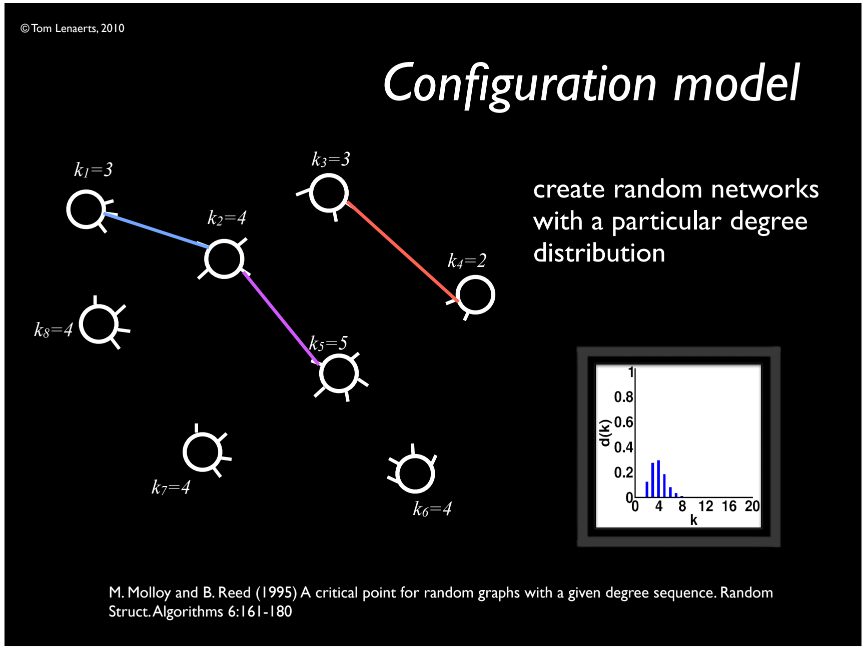
M. Molloy and B. Reed (1995) A critical point for random graphs with a given degree sequence. Random Struct. Algorithms 6:161-180



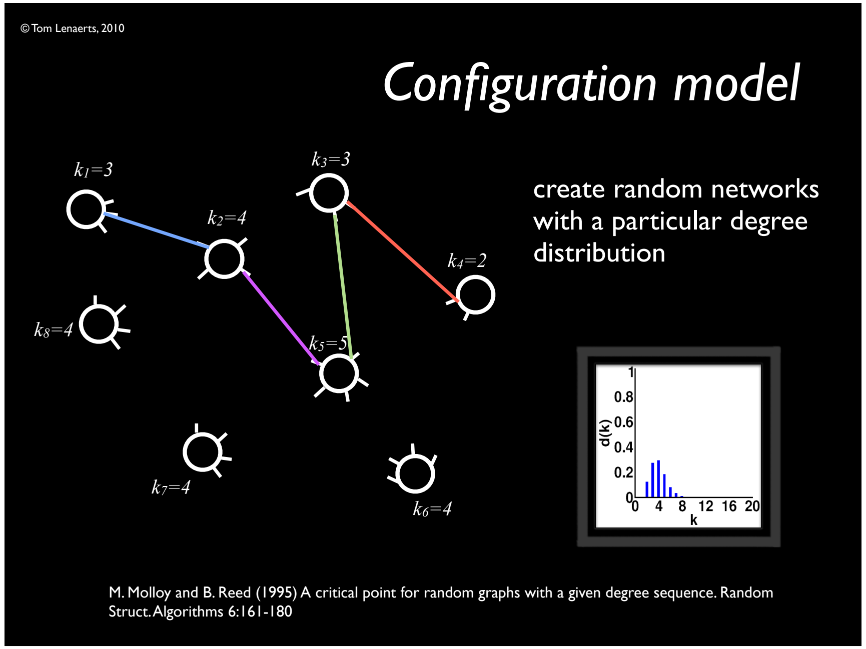
97-2



97-3



97-4



97-5

Simulation IV

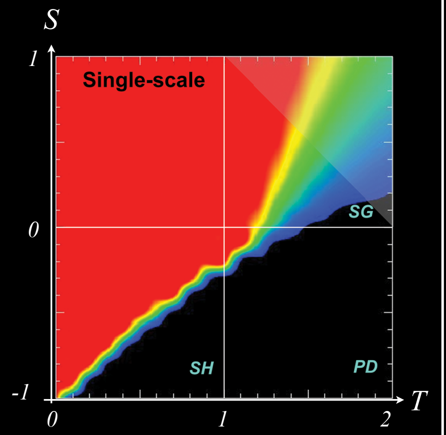
low heterogeneity assumption

$N=10^4$ $<k>=4$

100 runs

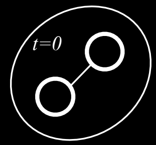
50% **C**, 50% **D**

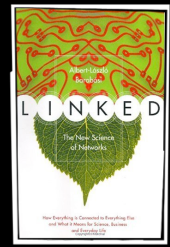
$R=1,\ P=0$



98

Scale-free Networks



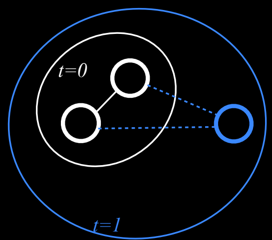


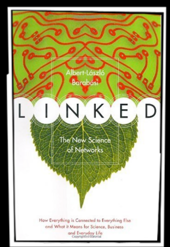
"the rich get richer"

A.-L. Barabási and R. Albert (1999) Emergence of Scaling in Random Networks. Science 286:509-512

99-1

Scale-free Networks



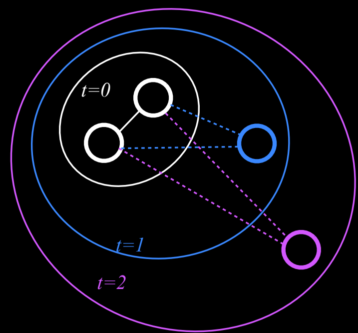


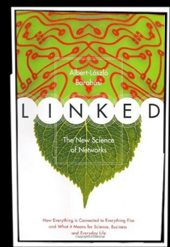
"the rich get richer"

A.-L. Barabási and R. Albert (1999) Emergence of Scaling in Random Networks. Science 286:509-512

99-2

Scale-free Networks

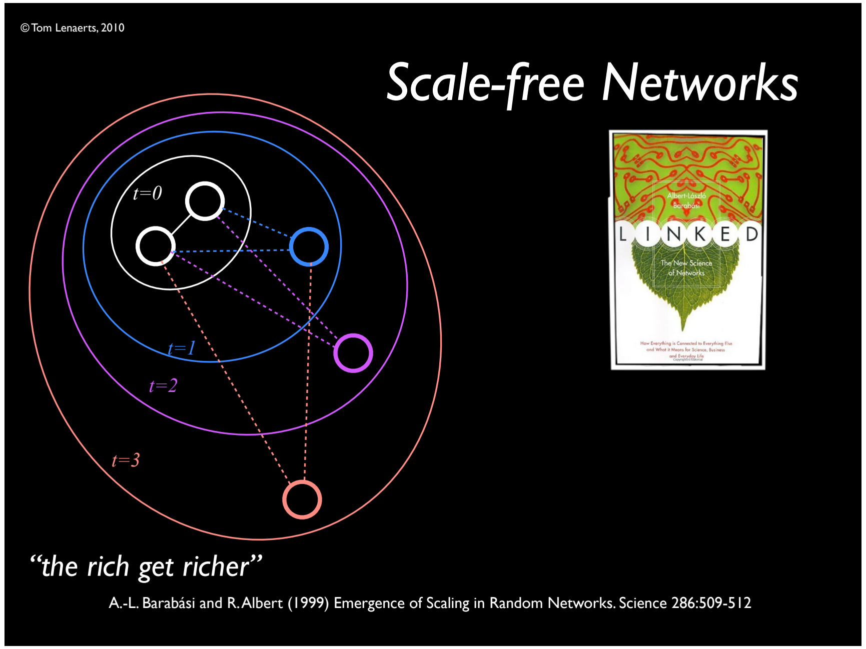




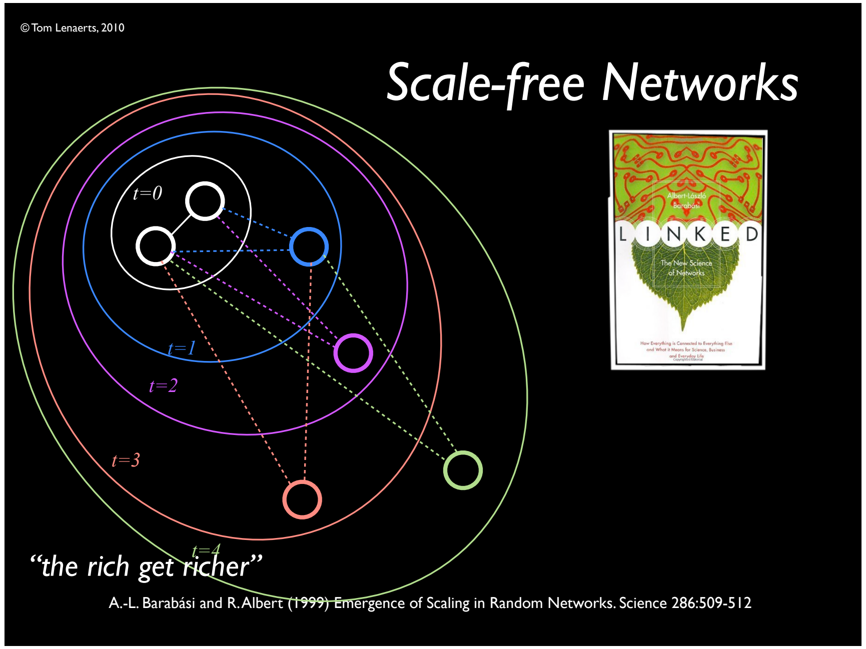
"the rich get richer"

A.-L. Barabási and R. Albert (1999) Emergence of Scaling in Random Networks. Science 286:509-512

99-3



99-4



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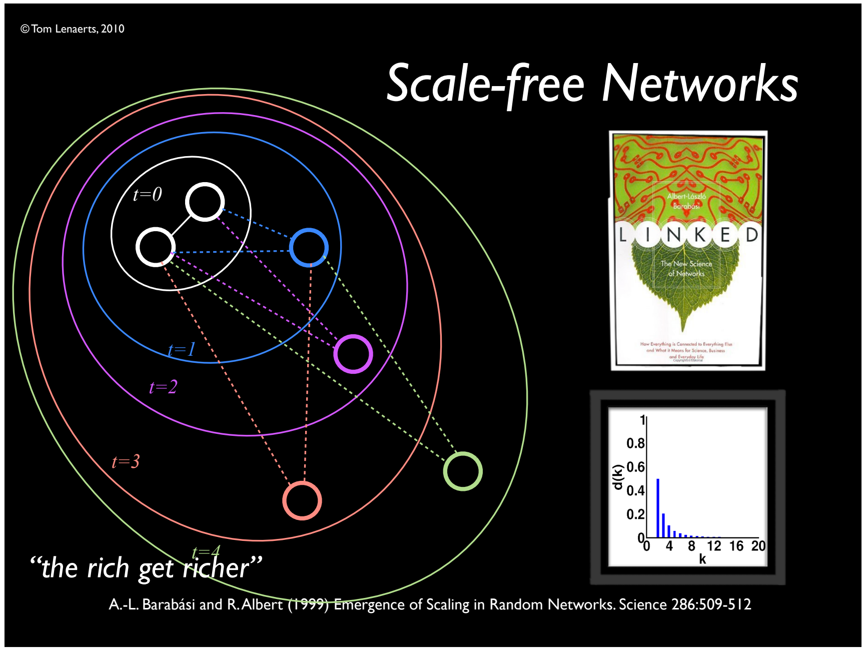

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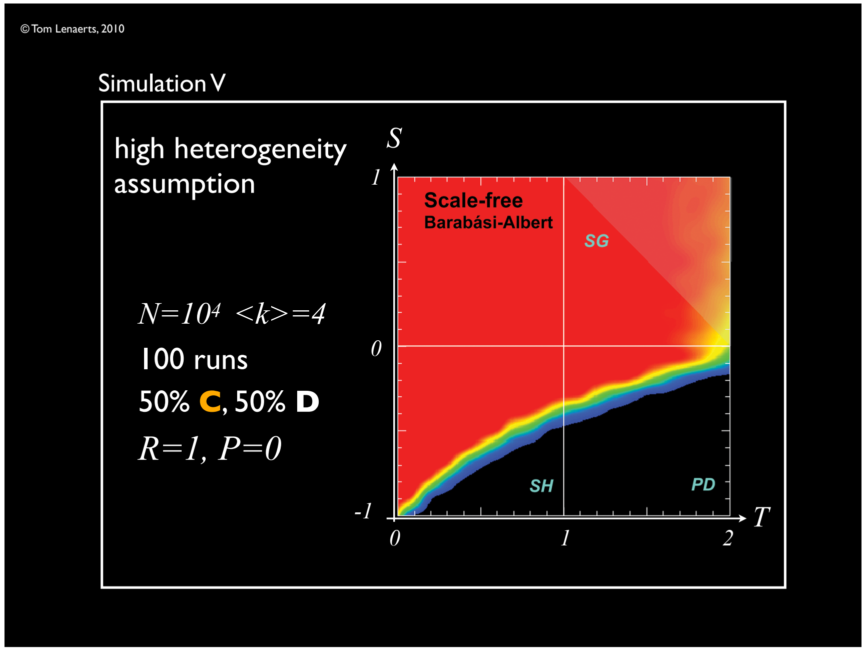
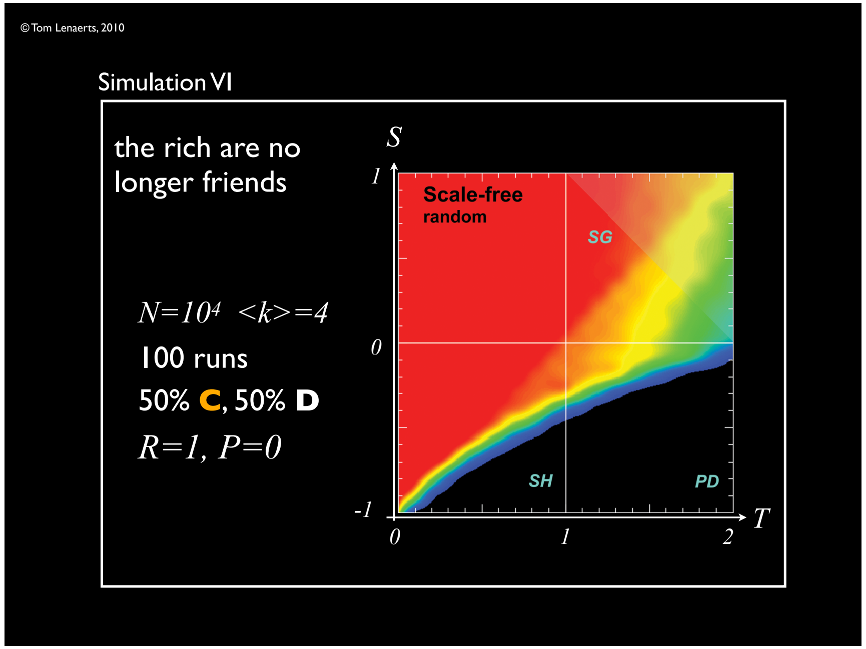
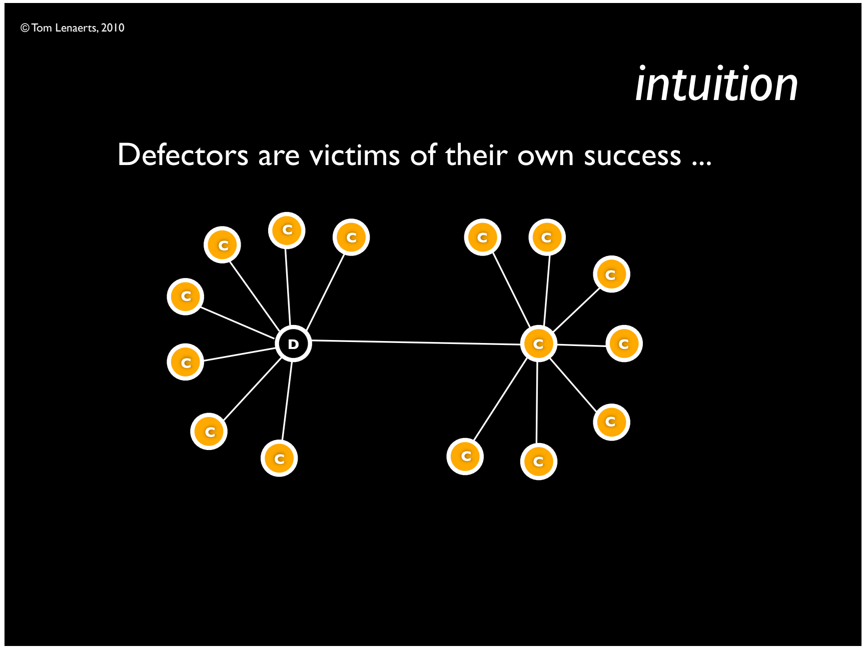


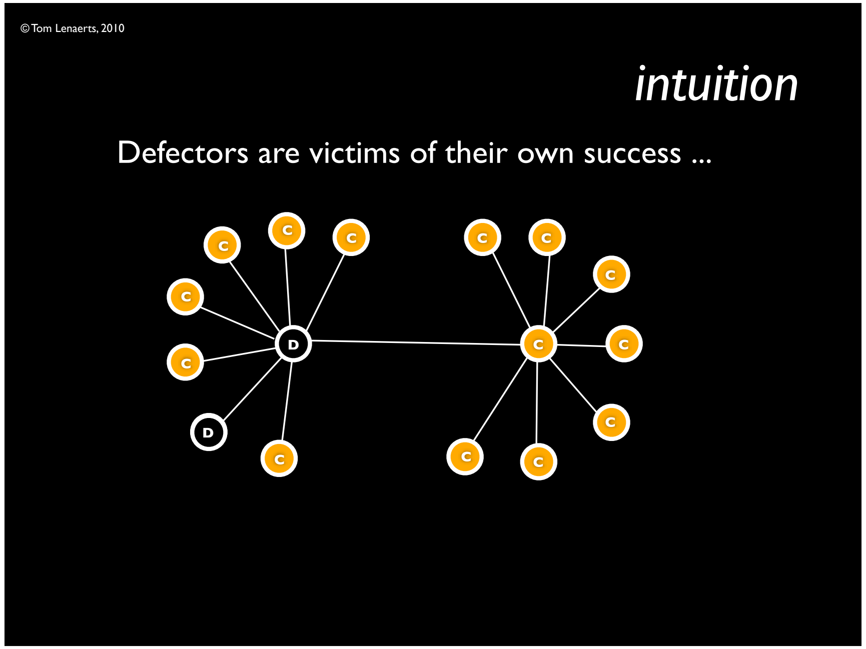
100



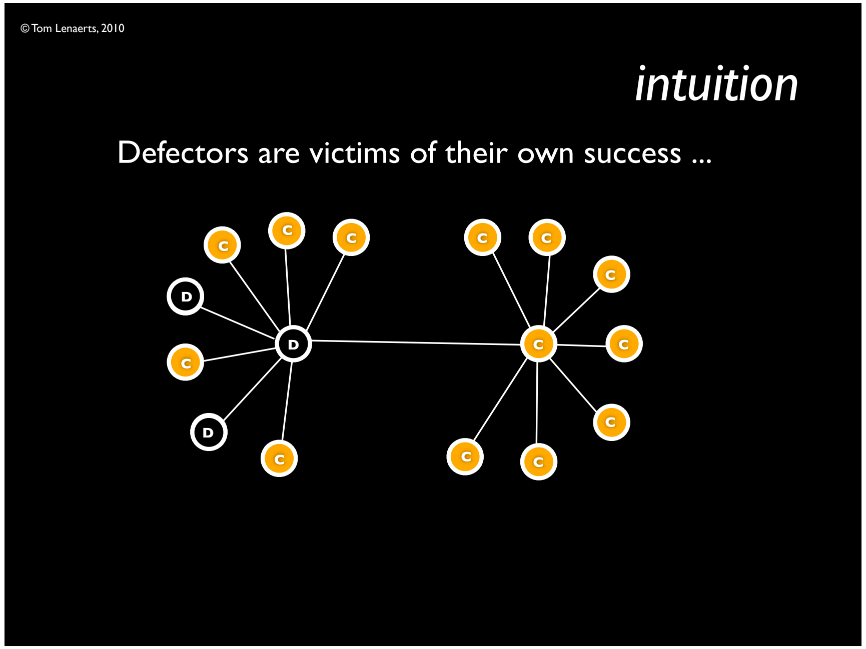
101



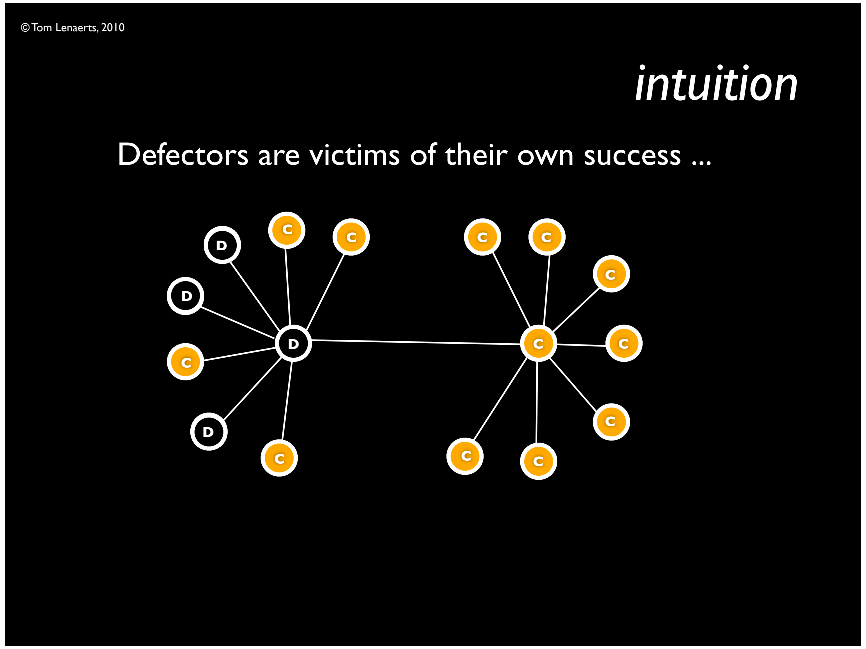
102-1



102-2



102-3



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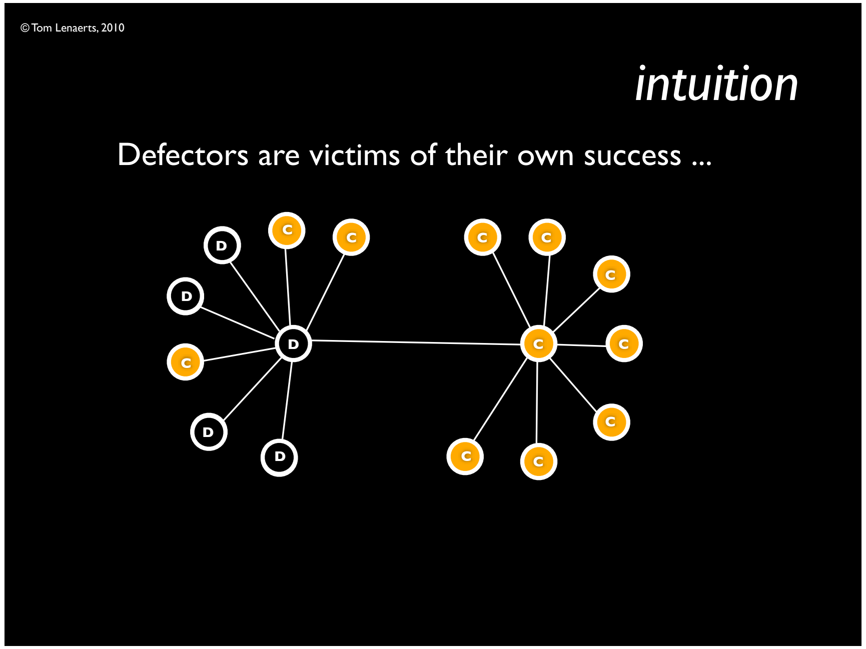

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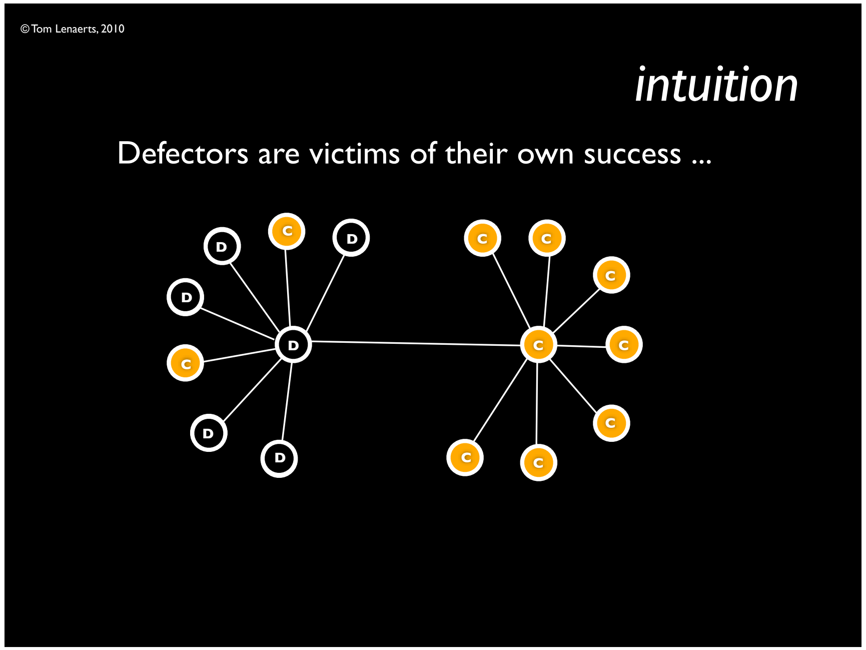

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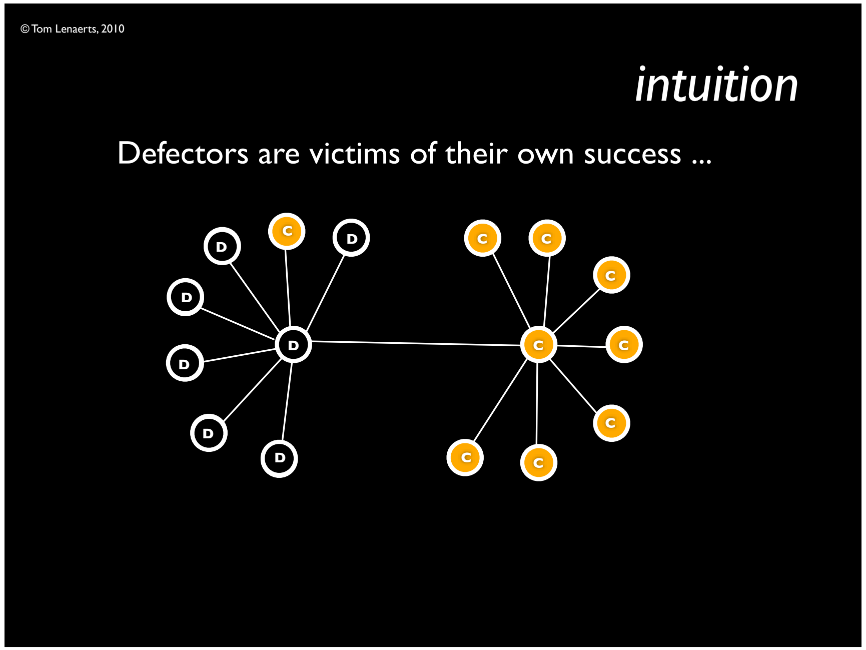
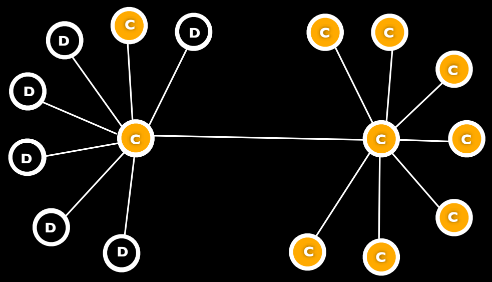


102-7

intuition

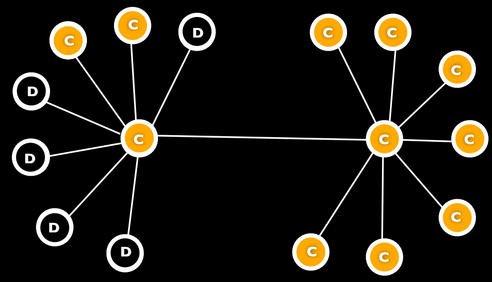
Defectors are victims of their own success ...



102-8

intuition

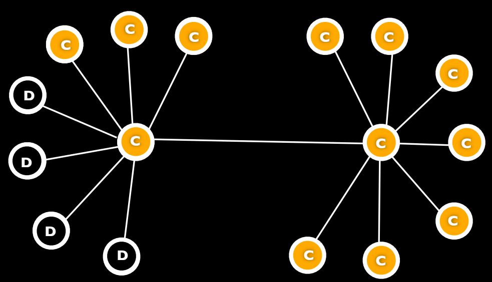
Defectors are victims of their own success ...



102-9

intuition

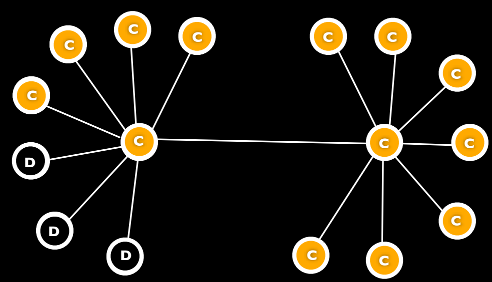
Defectors are victims of their own success ...



102-10

intuition

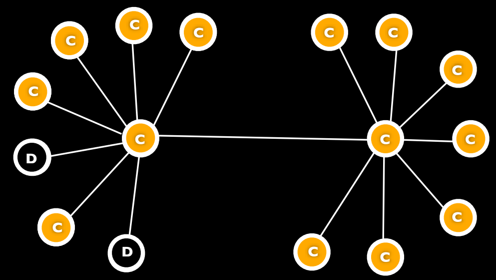
Defectors are victims of their own success ...



102-11

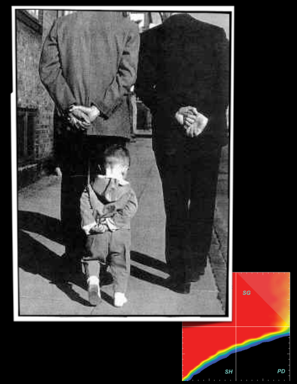
intuition

Defectors are victims of their own success …



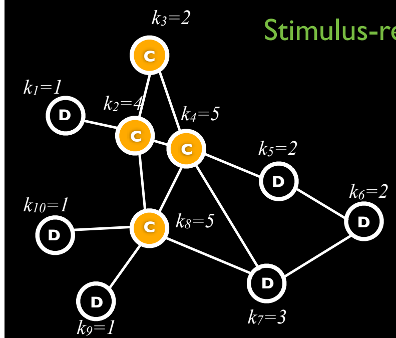
102-12

social versus individual learning




103

social versus individual learning



Stimulus-response learning …

Vertex x plays **once** with a random neighbor y and both receive a payoff f_x (and f_y)

update strategy using the following model $i \in \{x,y\}$:

$$p_i(t+1) = p_i(t) + \lambda\beta_i(t)*(1-p_i(t))$$
when i played **C** at time t

$$p_i(t+1) = p_i(t) - \lambda\beta_i(t)*p_i(t)$$
when i played **D** at time t

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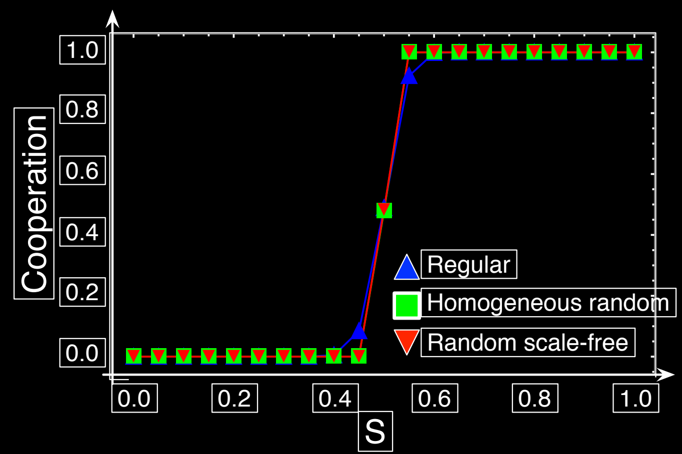
social versus individual learning



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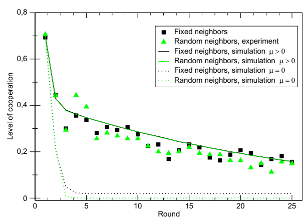
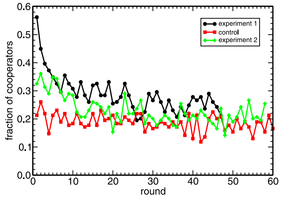
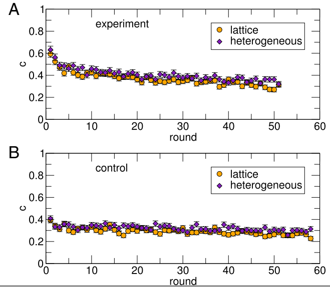
Presentation next week!







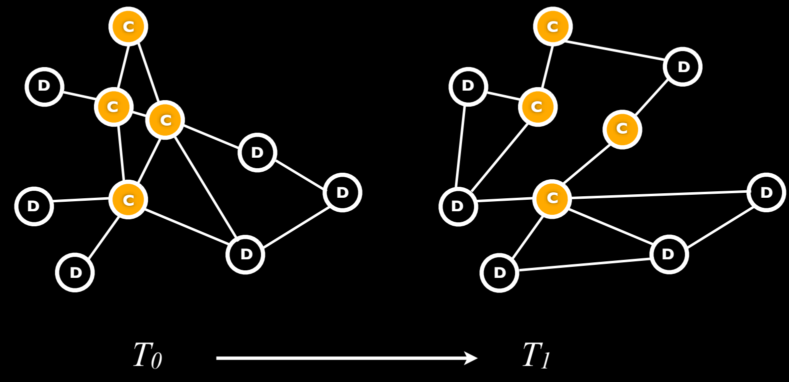
Traulsen, A., Semmann, D., Sommerfeld, R. D., Krambeck, H.-J., & Milinski, M. (2010). **Human strategy updating in evolutionary games.** Proceedings of the National Academy of Sciences, 107(7), 2962–2966.

Grujić, J., Fosco, C., Araujo, L., Cuesta, J. A., & Sánchez, A. (2010). **Social Experiments in the Mesoscale: Humans Playing a Spatial Prisoner's Dilemma.** PLoS ONE, 5(11), e13749.

Gracia-Lazaroa, C., Ferrera, A., Ruiza, G., Alfonso Tarancona, B., Jose A Cuestaa, C., Angel Sancheza, C., & Moreno, Y. (2012). **Heterogeneous networks do not promote cooperation when humans play a Prisoner's Dilemma.** Proceedings of the National Academy of Sciences, 109(32), 12922–12926.

106

Networks are dynamic



107

Networks are dynamic

Agent-based simulations

F.C. Santos, J.M. Pacheco and T. Lenaerts (2006) Cooperation previals when individuals adjust their social ties. PLoS Comp Biol 2(12):e178

S. Van Segbroeck, F.C. Santos, A. Nowé, J.M. Pacheco and T. Lenaerts (2008) The evolution of prompt reactions to adverse ties. BMC Evol Biol 8:287

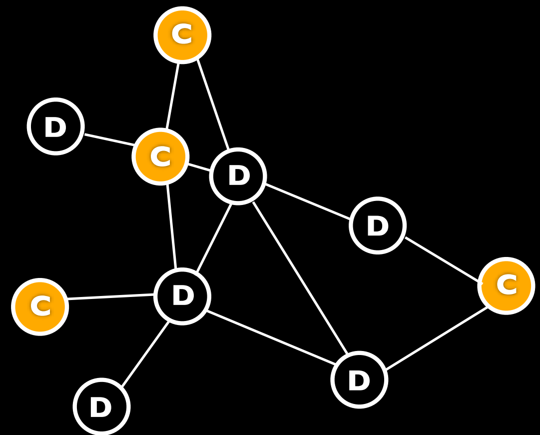
Analytics and numerical approximations

J.M Pacheco, A. Traulsen and M. Nowak (2006) Coevolution of strategy and structure in complex networks with dynamical linking. Phys Rev Lett 97:258103

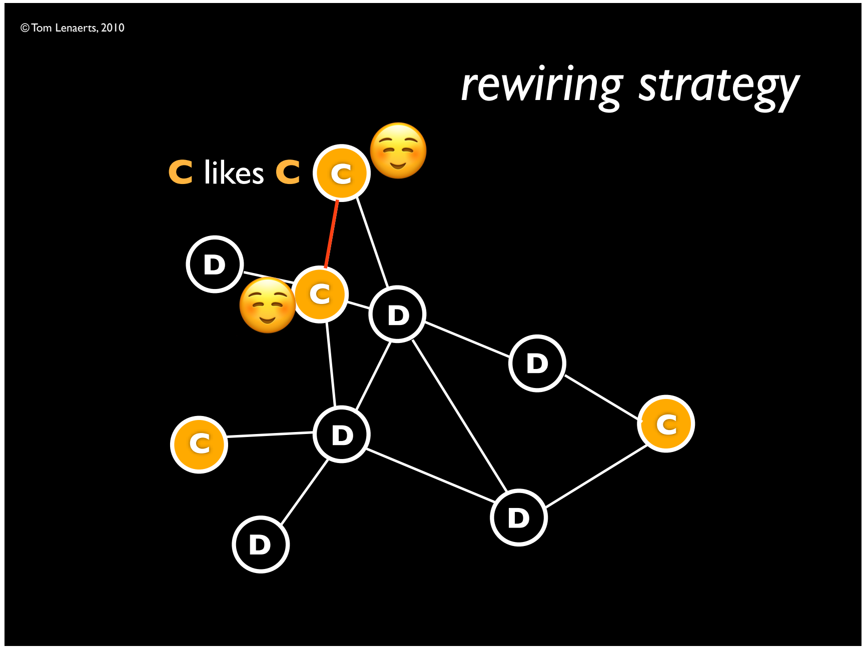
S. Van Segbroeck, F.C. Santos, T. Lenaerts and J.M. Pacheco (2009) Reacting differently to adverse ties promotes the evolution of cooperation. Phys Rev Lett 102:058105

108

rewiring strategy



109-1



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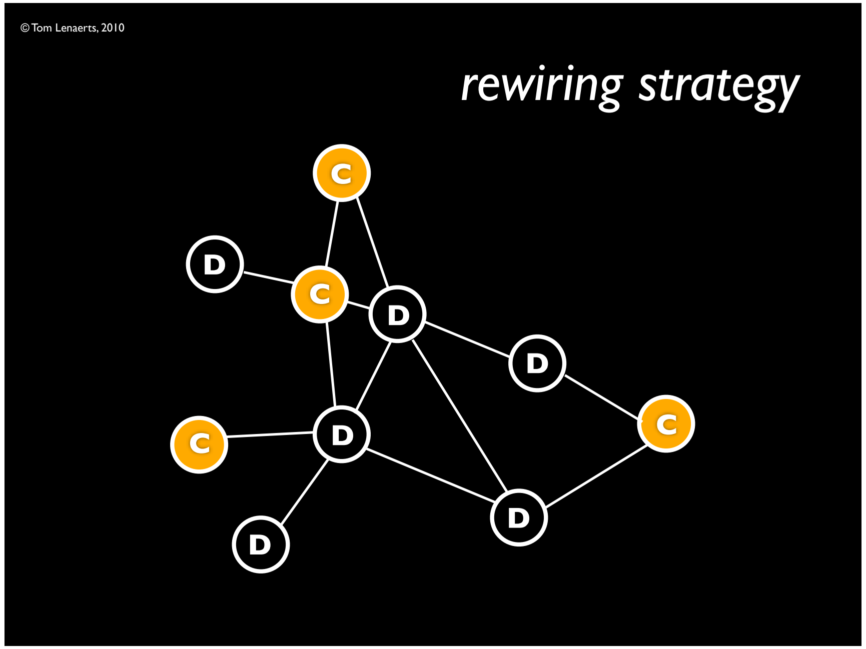

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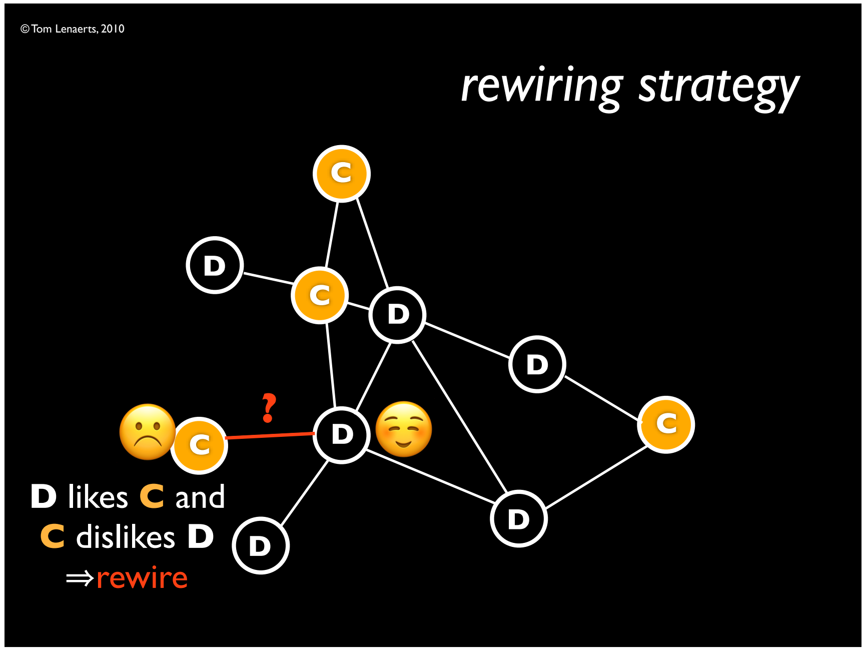

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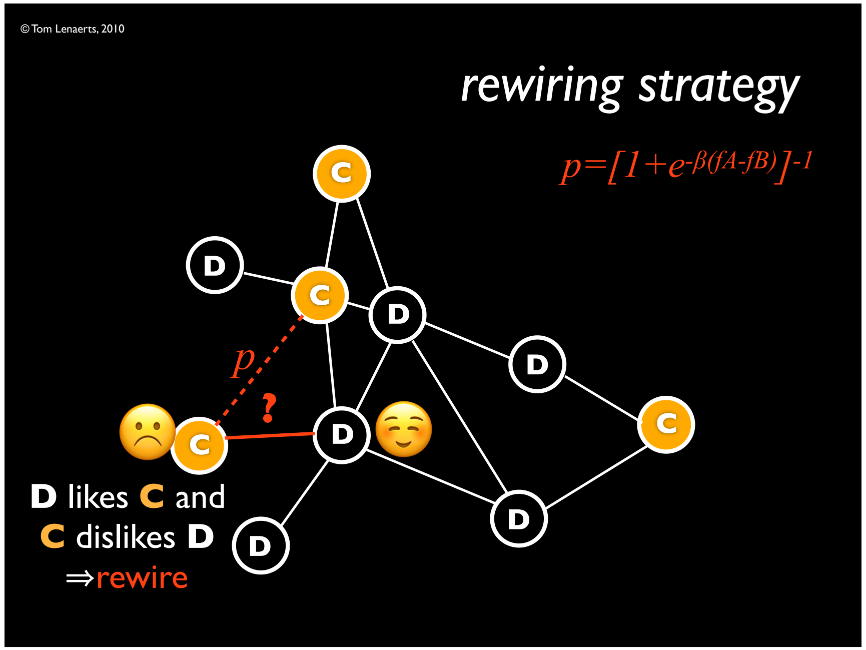

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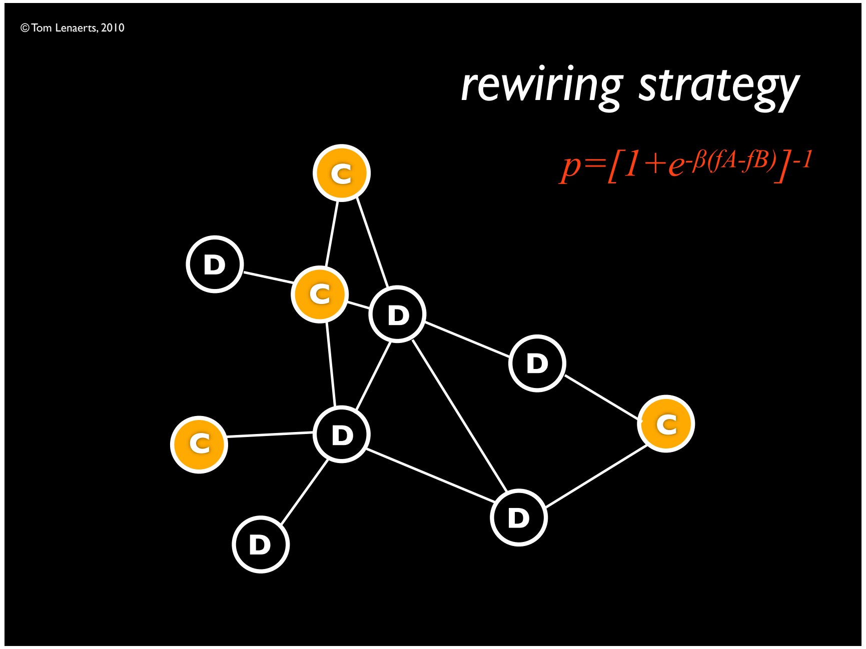

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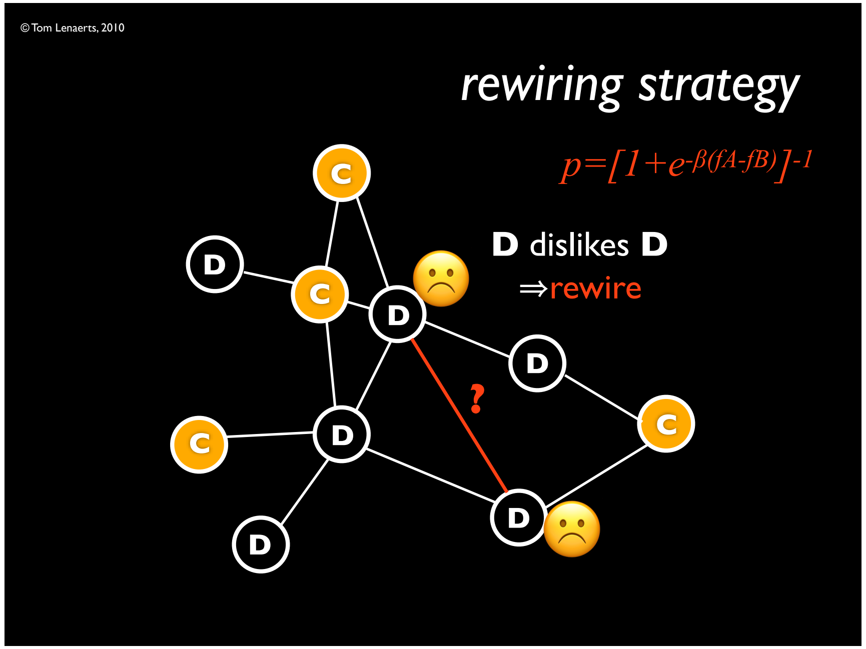

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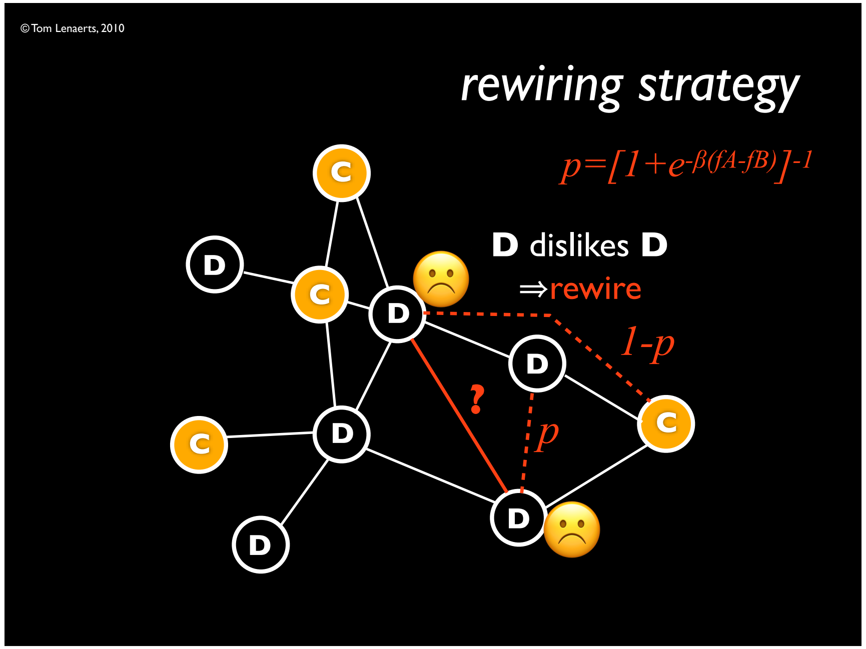

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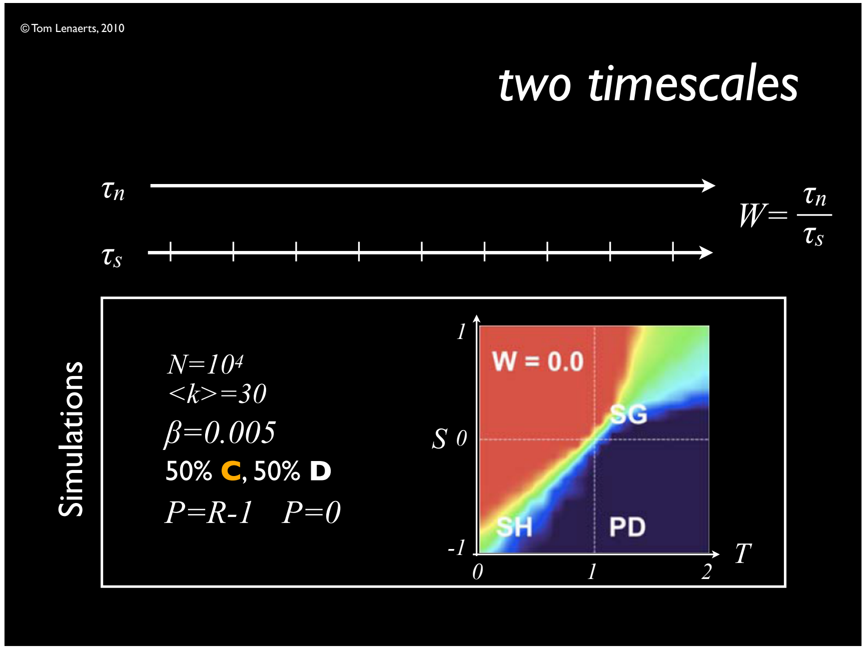
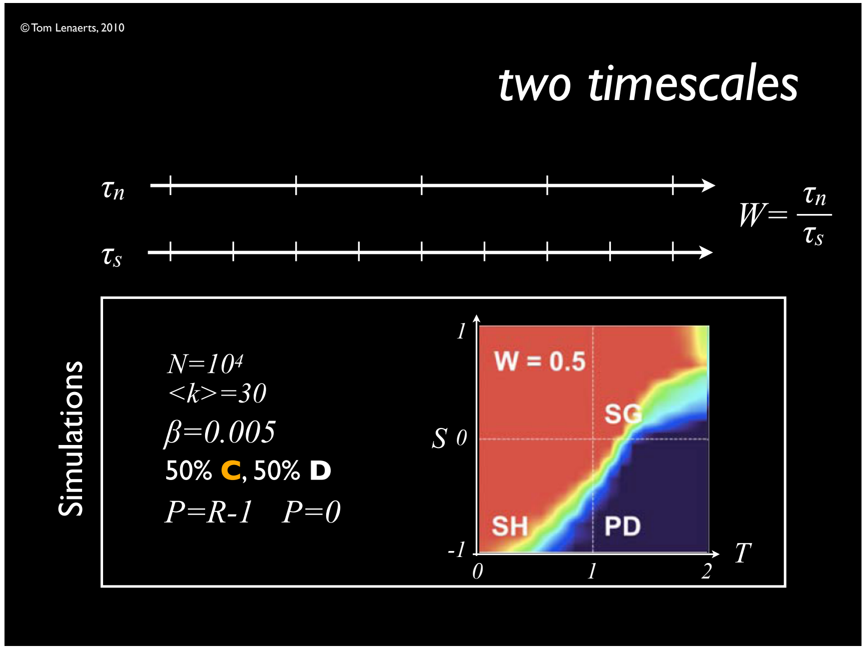


110-1



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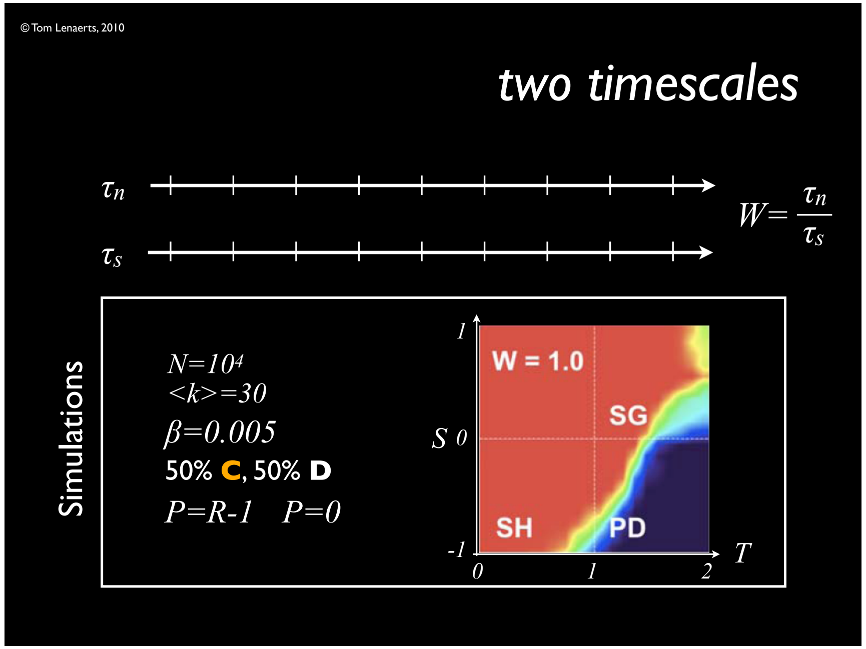

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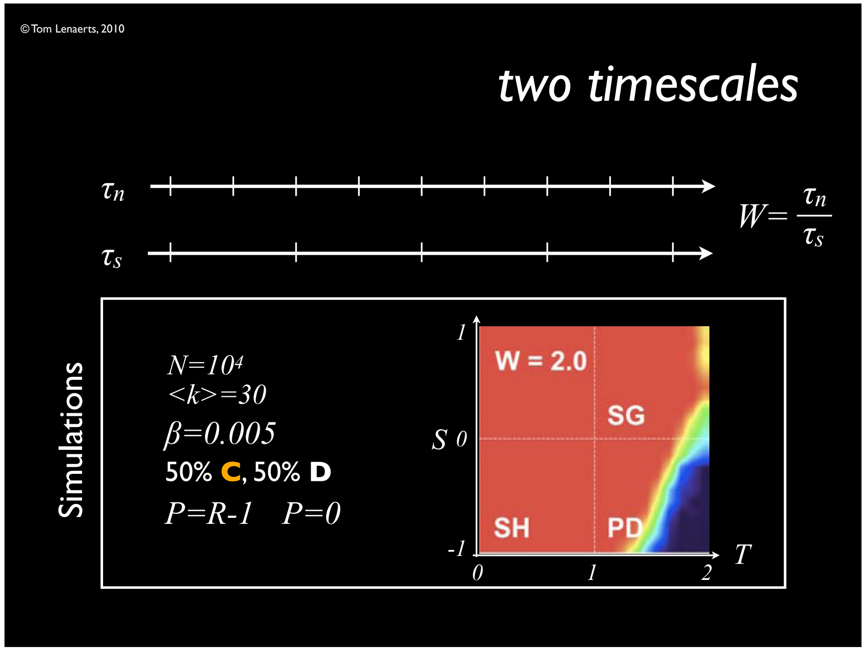

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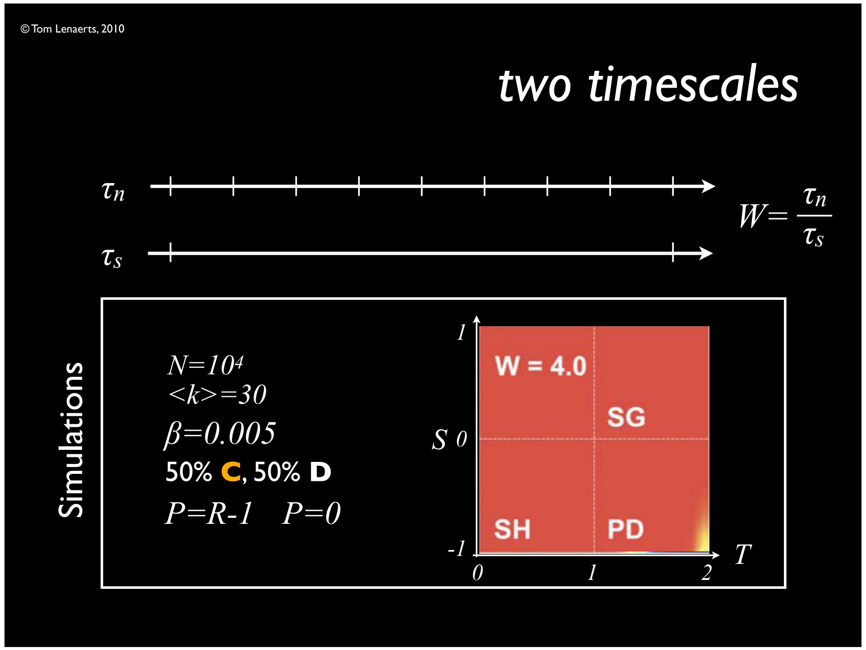

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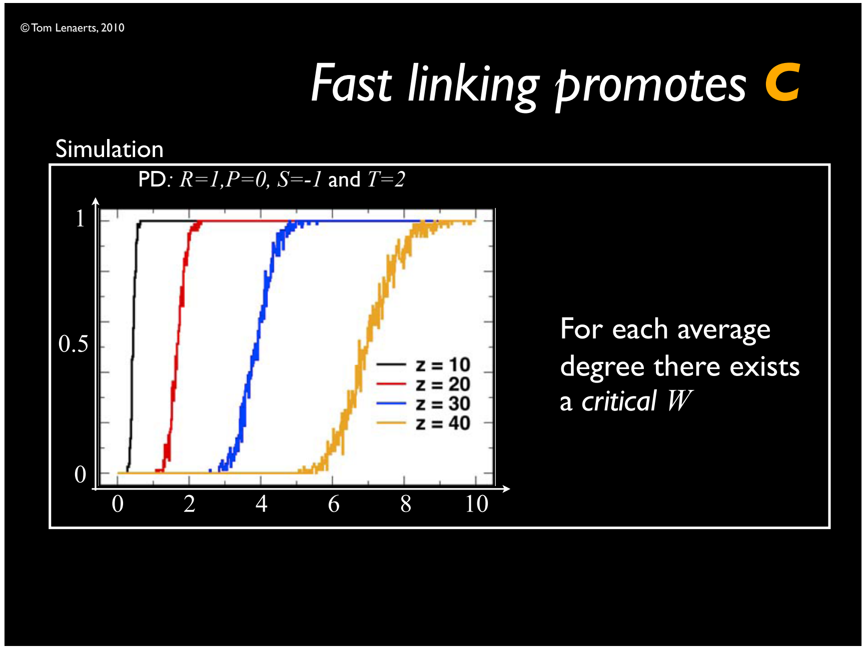

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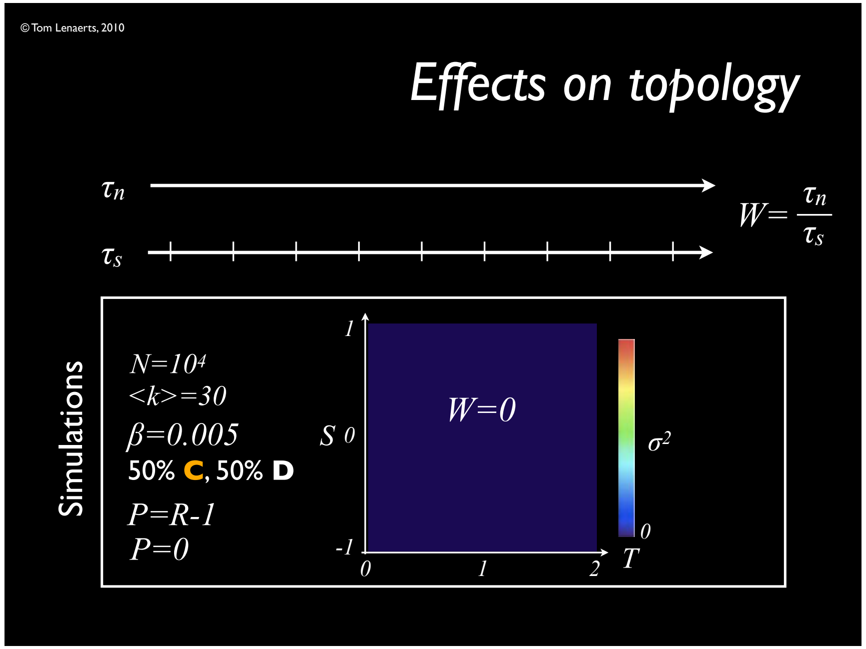
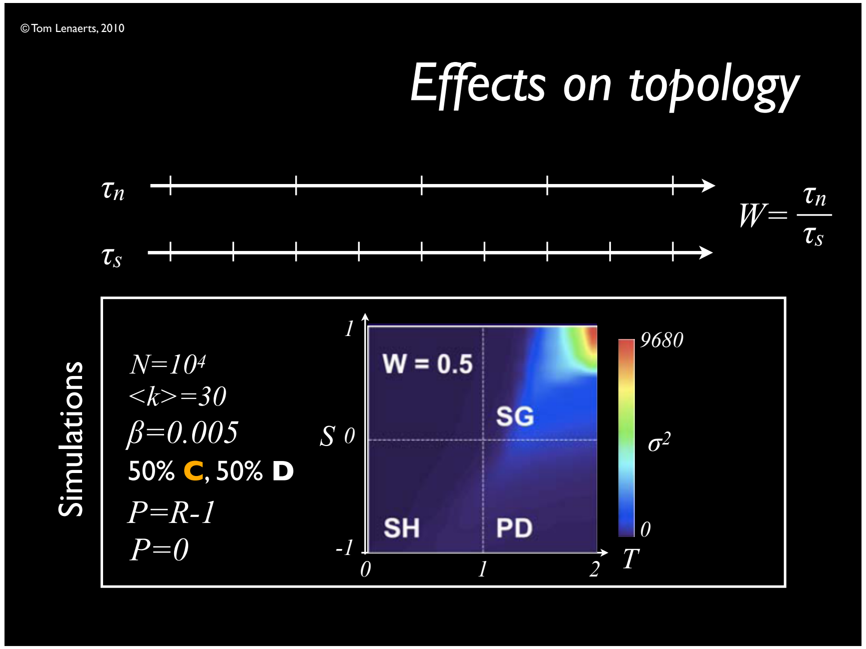
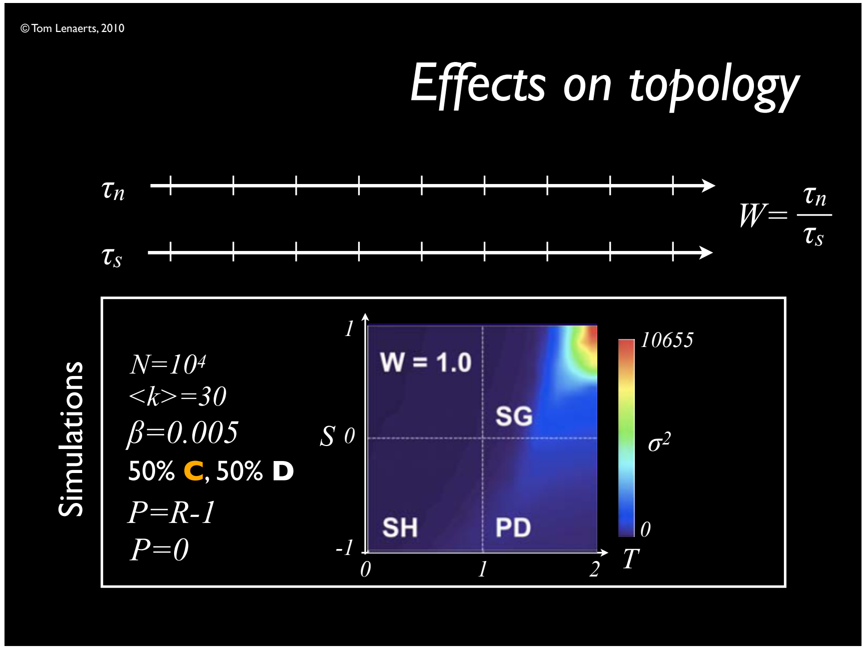


112-1



112-2



112-3

Effects on topology

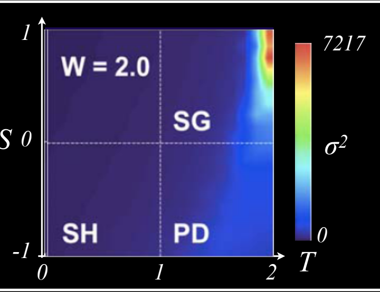
τ_n

τ_s

$W = \frac{\tau_n}{\tau_s}$

Simulations

$N=10^4$
$<k>=30$
$\beta=0.005$
50% **C**, 50% **D**
$P=R-1$
$P=0$



112-4

Effects on topology

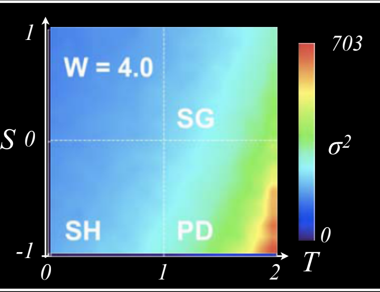
τ_n

τ_s

$W = \frac{\tau_n}{\tau_s}$

Simulations

$N=10^4$
$<k>=30$
$\beta=0.005$
50% **C**, 50% **D**
$P=R-1$
$P=0$



112-5

Effects on topology

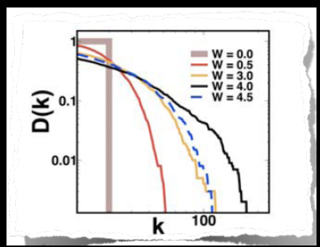
τ_n

τ_s

$W = \frac{\tau_n}{\tau_s}$

Simulations

$N=10^4$
$<k>=30$
$\beta=0.005$
50% **C**, 50% **D**
$T=2,\ R=1,$
$P=0,\ S=1$



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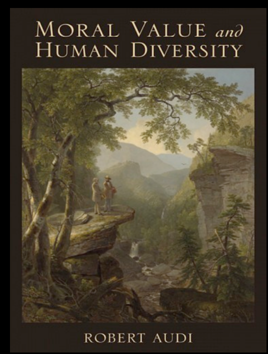
Everyone reacts differently



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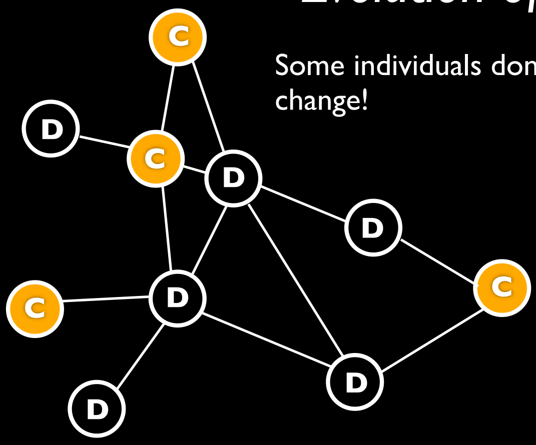
Evolution of rewiring

Some individuals don't like change!



Change introduces uncertainty!

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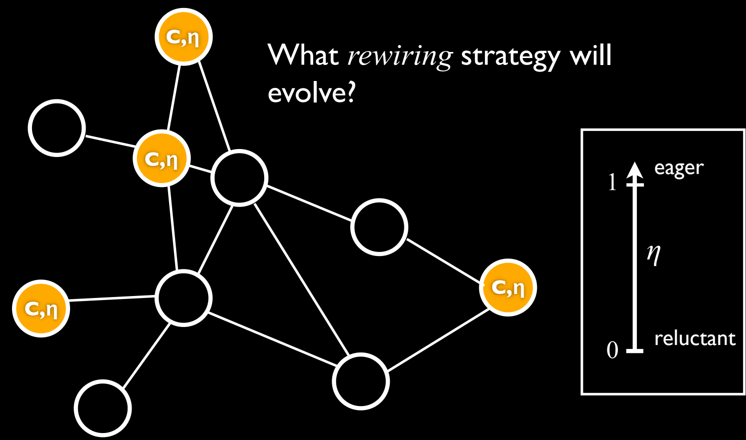
Evolution of rewiring

What *rewiring* strategy will evolve?



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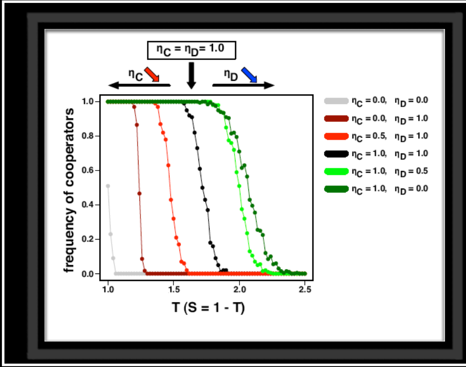
Fast versus slow

Simulation I



Assume fixed η for **C** and **D**

PD game

$N=10^3$ $z=30$

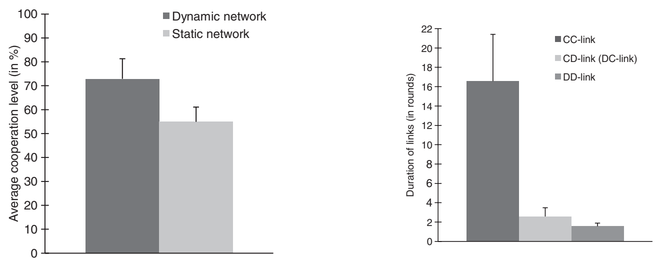
100 runs

50% **C**, 50% **D**

$W=2.5$ $\beta=0.005$

117

Recent experiments



Fehl, K., Van Der Post, D. J., & Semmann, D. (2011). **Co-evolution of behaviour and social network structure promotes human cooperation.** Ecology Letters, 14(6), 546–551.

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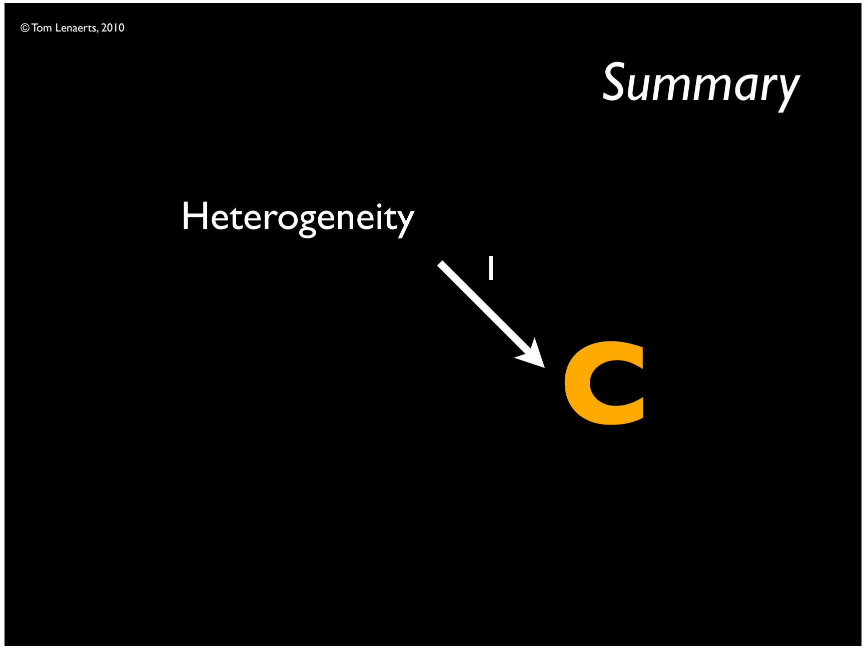
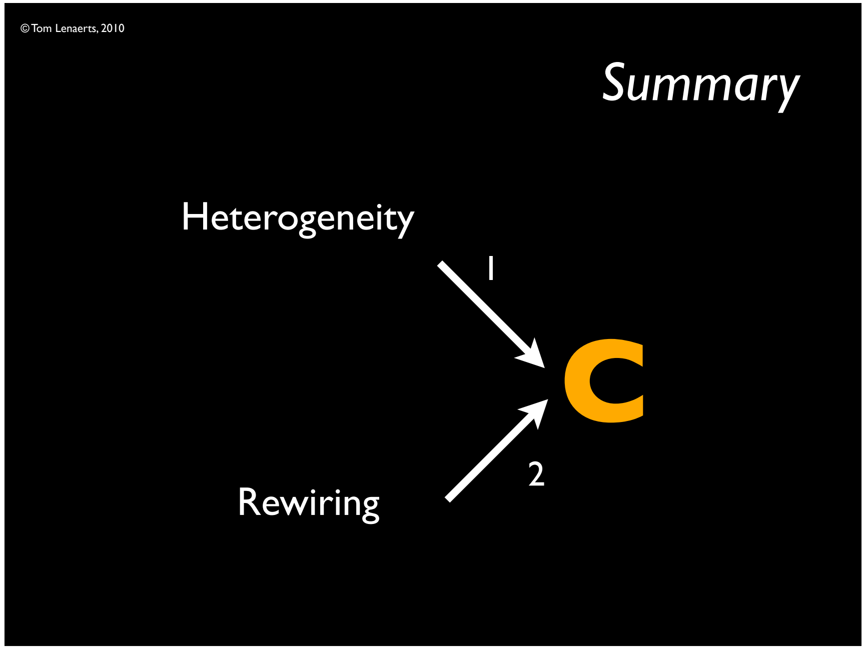
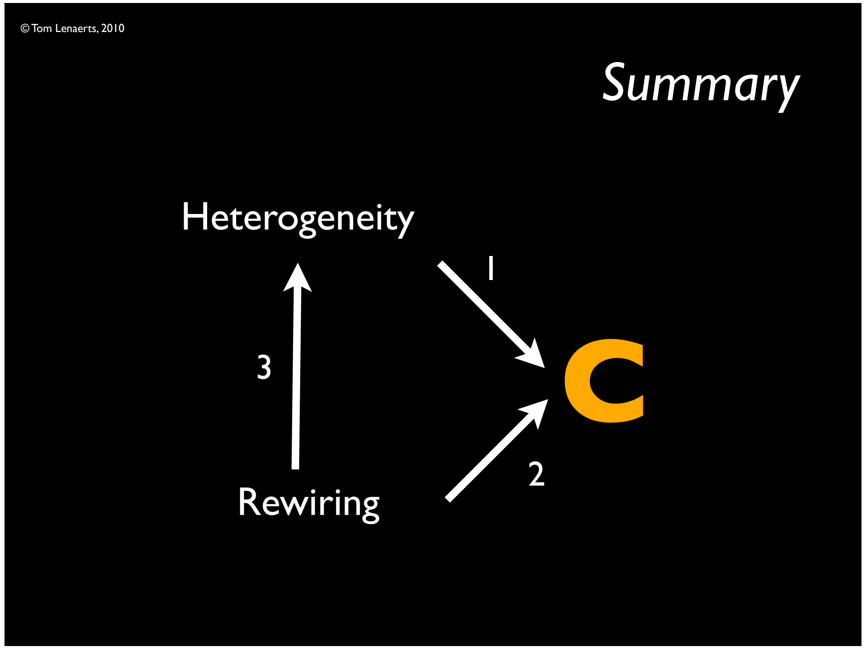
119-1



119-2



119-3



119-4