Unconventional Monetary Policy the ZLB

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Previous Literature

Learning and expectations in Macroeconomics- Evans & Honkapohja

A New Keynesian Model with Wealth in the Utility Function - Seaz & Michaillat

Tempered Particle Filtering - Herbst, Ed and Frank Schorfheide

Estimating Macroeconomic Models- Villaverde & Ramirez

Nonlinear Adventures at the Zero Lower Bound - Villaverde Et Al.

Fiscal Foundations of Inflation: Imperfect Knowledge- Eusepi & Preston

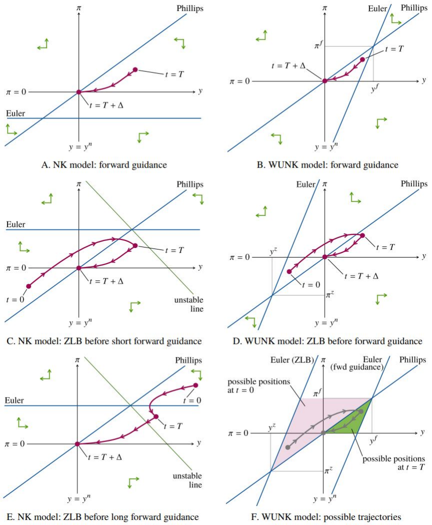
A New Keynesian Model with Wealth in the Utility Function - Piazzesi Et. Al.

Lit Overview: Wealth In Utility

$$\dot{b_j}(t) = i(t)b_j(t) + p_j(t)y_j(t) - \int_0^1 p_k(t)c_{jk}(t)dk$$

$$\int_0^{inf} e^{-\delta t}\{ln(c_j(t)) + u(\frac{b_j(t)}{p(t)}) - h_j(t) - \frac{}{2}\pi_j^2(t)\}dt$$

Lit Overview: Wealth



Lit Overview: Fiscal Policy and Learning

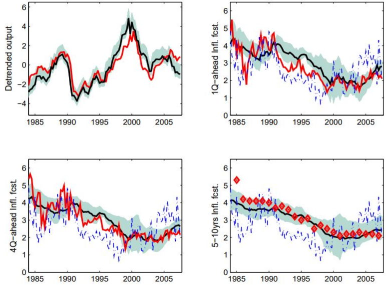
$$Z_t = [i_t, \pi_t, w_t, \tau_t^{LS}, \tau_t^{w}, b_t^{m}]'$$

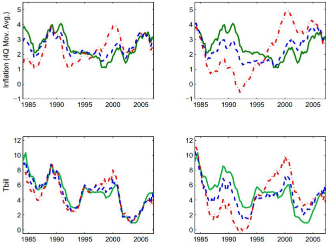
$$S_t = [A_t, \epsilon_t, G_t, m_t]'$$

$$Z_t = \Omega + \Phi_b b_{t-1} + \Phi_s S_{t-1} + e_t$$

$$S_t = FS_{t-1} + Q\epsilon_t$$

Fiscal Policy and Learning





Households:

$$U(c_t, b_t, b_k^k, L_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{(q_t^k b_t^k)^{1-n_2}}{1-n_2} + \frac{(q_t b_t)^{1-n_3}}{1-n_3} - \frac{L_t^{1+\psi}}{1+\psi}$$

$$c_t + b_t q_t + b_t^k q_t^k + m_t = (1-\tau_t^r)\{\pi_t^{-1}(1+\rho q_t)b_{t-1} + \pi_t^{-1}(1+\rho_k q_t^k)b_{t-1}^k + (1+R_{t-1}^m)m_{t-1}\} + (1-\tau_t^w)w_t L_t$$

Note: bonds have the following expression:

$q_t = E_t\{1+\rho q_{t+1}\}$

$q_t^k = E_t\{1+\rho_k q_{t+1}^k\}$

HH Focs:

$$\mathcal{L} = E_t[\Sigma_{t=0}^{inf}\beta^t\{U_t - \lambda_t(w_t)\}]$$

$$\frac{\partial\mathcal{L}}{\partial c_t} : c_t^{-\sigma} = \lambda_t$$

$$\frac{\partial\mathcal{L}}{\partial b_t} : (q_t b_t)^{-n_2} q_t = -c_t^{-\sigma} q_t + \beta E_t[c_{t+1}^{-\sigma}\pi_{t+1}^{-1}(1-\tau_{t+1}^r)R_{t+1}]$$

$$\frac{\partial\mathcal{L}}{\partial b_t^k} : (q_t^k b_t^k)^{-n_2} q_t^k = -c_t^{-\sigma} q_t^k + \beta E_t[c_{t+1}^{-\sigma}\pi_{t+1}^{-1}(1-\tau_{t+1}^r)R_{t+1}^k]$$

$$\frac{\partial\mathcal{L}}{\partial m_t} : c_t^{-\sigma} = \beta E_t[c_{t+1}^{-\sigma} i_t(1-\tau_{t+1}^r)\pi_{t+1}^{-1}]$$

$$\frac{\partial\mathcal{L}}{\partial L_t} : L_t^{\psi} = c_t^{-\sigma} w_t(1-\tau_t^w)$$

Firm pricing:

$$max_{p_t^*} \;\; E_t[\Sigma_{k=0}^{inf}(\theta\beta)^k \pi_{t+k}^{-1} B_{t+k}]$$

$$B_{t+k} \equiv p_t^{*\,1-\epsilon} p_{t+k}^{\epsilon-1} Y_{t+k} - p_{t+k}^{-\epsilon} p_t^{*\,-\epsilon} MC_{t+k}^r Y_{t+k}$$

After Log Linearizing around a Zero Inf S.S:

$$\pi_t = \tilde{\alpha}\theta_t[\pi_{t+1}] + \tilde{\alpha}\frac{(1-\beta\theta)}{1-\gamma} mc_t^r + u_t$$

$$u_t = \rho_u u_{t-1} + \nu_t^u$$

Government and MP:

$$i_t = max\{q_t + \phi_y y_t + \phi_\pi pi_t, 0\}$$

$$q_t = \rho_q q_{t-1} + \nu_t^q$$

$$q_t b_t = \pi_t^{-1}(1 + \rho q_t) b_{t-1} - s_t - \tau_t$$

$$s_t = \rho_s s_{t-1} + \nu_t^s$$

$$\tau_t = h\tau_t^r + (1-h)\tau_t^w = \frac{\phi_b}{\bar{b}}(q_t b_t)$$

Corporate Bond Demand

Firms Face the following problem

$min_{b_t^k, L_t} \; q_t^k b^k + w_t L_t$

$s.t. \; K_{t+1} = (1-\delta)K_t + (1 - S\{\frac{q_t b_t}{q_{t-1} b_{t-1}} \exp^{\zeta_t}\}) q_t b_t$

$s.t. \; \bar{Y}_t = \exp^{a_t} L_t^{1-\alpha} K_t^{\alpha}$

$\zeta_t = \rho_\zeta \zeta_{t-1} + \nu_t^\zeta$

ζ_t can be thought of as a "credit-freeze" shock.

Corporate Bond Demand

After Log-Linearizing, The following Conditions hold:

$b_t^k = w_t + q_{t-1}^k + b_{t-1}^k + (y_t - a_t - k_t)(1-\alpha)^{-1} - \zeta_t - q_t$

$mc_t^r = (1+\psi)^{-1}\alpha(\sigma c_t - w_t + E_t[\tau_{t+1}^w]) - \alpha k_t + w_t$

Putting Model into State Space Form:

$$\Gamma_0 E_t \begin{bmatrix} Y_{t+1} \\ S_{t+1} \end{bmatrix} = \Gamma_1 \begin{bmatrix} Y_t \\ S_t \end{bmatrix} + \Psi \bar{\epsilon_t}$$

After Solving, we have two Equilibria :

$$Y_{t+1} = A_{1,1} U_{t+1} + A_{1,2} M_{t+1} + \Psi_1 \bar{\epsilon_{t+1}}$$

$$M_{t+1} = A_{2,1} U_t + A_{2,2} M_t + \Psi_2 \bar{\epsilon_{t+1}}$$

$$Y_{t+1} = A^*_{1,1} U_{t+1} + A^*_{1,2} M_{t+1} + \Psi^*_1 \bar{\epsilon_{t+1}}$$

$$M_{t+1} = A^*_{2,1} U_t + A^*_{2,2} M_t + \Psi^*_2 \bar{\epsilon_{t+1}}$$

ZLB solution comes from demand shock expressed as:

$$r_t = (1 - \rho_r) r^*_t + p_r r_{t-1} + \nu^r_t$$

$$r^*_t = r^*_{t-1} + \nu^*_t$$

Why Add Learning?

"Under learning, the estimated present discounted value of taxes does not necessarily offset changes in debt holdings, as agents are uncertain about their long-run tax burden." - Preston ET. AL., 2013

...the effects of fiscal disturbances upon private sector budget constraints and hence upon aggregate demand. Such effects are neutralized by the existence of rational expectations...- Woodford, 1998

Putting Model in Learning form:

Decompose S_t into State and shocks: $S_t \equiv [M_t, U_t]'$

$$\begin{bmatrix} Y_{t+1} \\ M_{t+1} \end{bmatrix} = F * E_t \begin{bmatrix} Y_{t+1} \\ M_{t+1} \\ U_{t+1} \end{bmatrix} + \tilde{\Psi}\bar{\epsilon_t}$$

REE can be re-expressed as:

$$M_t = B_0 U_{t-1} + B_1 M_{t-1} + \Psi_2 \epsilon_t$$

Where:

$$Y_{t+1} = F_{1,1} E_t[Y_{t+1}] + F_{1,2} M_t + F_{1,3} U_t + B_{1,4} E_t[M_{t+1}]$$

$$M_t = F_{2,1} E_t[Y_{t+1}] + F_{2,2} E_t[M_{t+1}] + F_{2,3} U_t$$

$$U_t = R U_{t-1} + \tilde{\Psi}\bar{\epsilon_t}$$

Note: M_t is a ***not*** a jump variable... it is a function of the jump variables.

Putting Model in Learning form:

Similarly, The REEs for Y_t and S_t can be re-expressed as:

$$Y_t = \tilde{A}_0 M_t + \tilde{B}_0 U_t + \tilde{C}_0 \bar{\epsilon_t}$$

$$M_t = \tilde{A}_1 M_{t-1} + \tilde{B}_1 U_{t-1} + \tilde{C}_1 \bar{\epsilon_t}$$

The Perceived Law of Motion(PLM):

$$Y_t = G_t^0 M_t + G_t^1 U_t + \eta_{t+1}^1$$

$$M_t = Q_t^0 U_t + \eta_{t+1}^2$$

Iterating Forward, and Applying the Conditional Expectation:

$$Y_{t+1}^e = G_t^0 M_{t+1}^e + G_t^1 R U_t$$

$$M_{t+1}^e = Q_t^0 R U_t$$

Restricted Perceptions Mapping

$$M_t = \tilde{A}_1 M_{t-1} + \tilde{B}_1 U_t + \tilde{C}_1 \epsilon_t$$

$$\bar{M} = \tilde{A}_1 \bar{M} + \tilde{C} \bar{U}$$

$$\bar{M} = (I - \tilde{A}_1 L)^{-1} \tilde{C}_1 \bar{U}$$

$$E_t[M_{t+1}] = (I - \tilde{A}_1) \tilde{C}_1 R U_t$$

Ideally, we are interested in understanding what dynamics look like when beliefs are stable under learning... When:

$$Q_t^0 \to (I - \tilde{A}_1) \tilde{C}_1$$

$$G_t^0 \to \tilde{A}_0$$

$$G_t^1 \to \tilde{B}_0$$

Learning Initialization:

$$\phi_t^n \equiv \begin{bmatrix} G_t^0 \\ G_t^1 \\ Q_{0\,t} \end{bmatrix} \; \phi_t^z \equiv \begin{bmatrix} \tilde{G_t^0} \\ \tilde{G_t^1} \\ \tilde{Q_{0\,t}} \end{bmatrix}$$

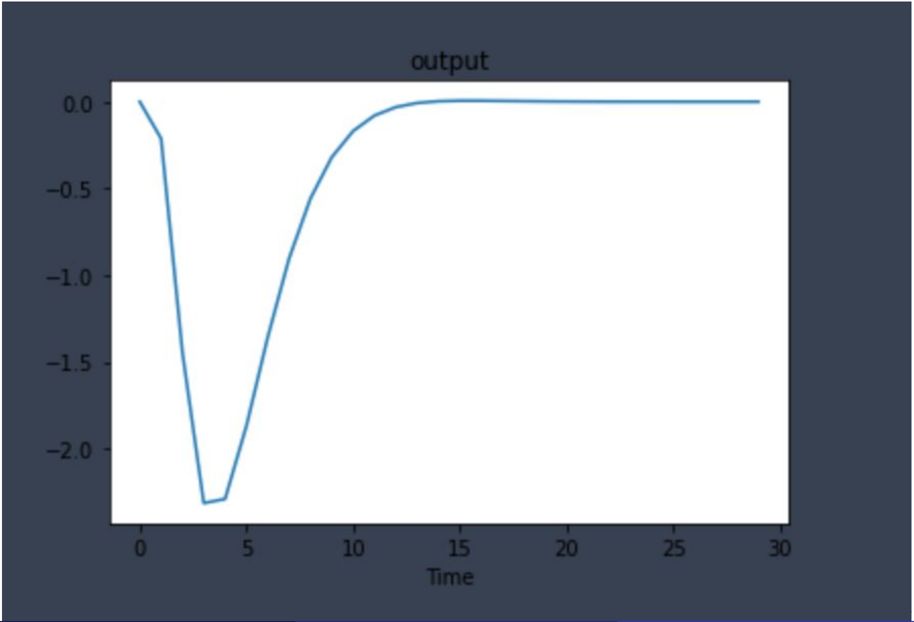
Where, $\phi_t^i \sim N(\phi_*, \Sigma)$

$$\phi_t \equiv \Theta(\{Y_{t-1}, M_{t-1}\})\phi_t^n + (1 - \Theta(\{Y_{t-1}, M_{t-1}\})\phi_t^z$$

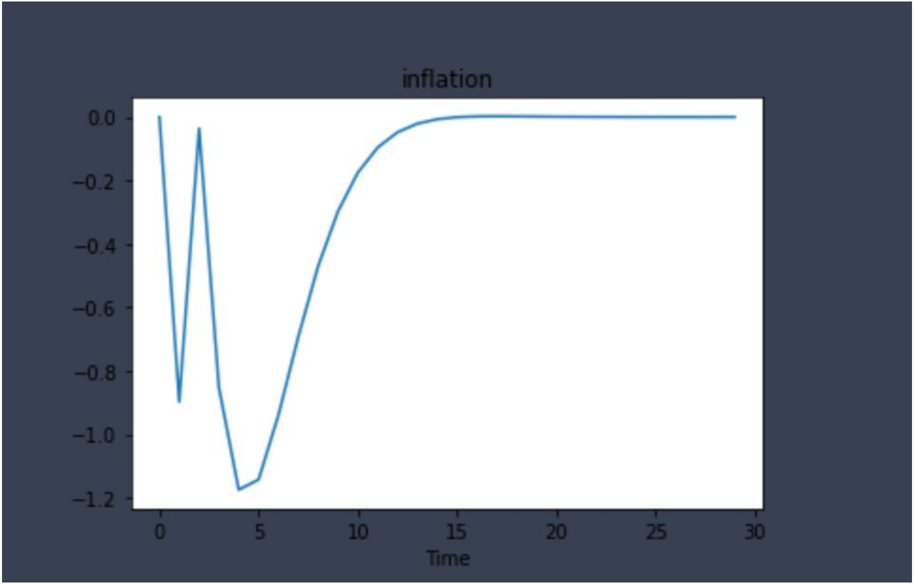
$$\Theta(\{Y_{t-1}, M_{t-1}\}) \equiv \hat{P}(i_t = 0 | Y_{t-1}, M_{t-1})$$

$$\phi_t^i = \phi_{t-1}^i + \gamma * Z_t(y_t - Z_t'\phi_{t-1}^i)$$

Demand Shock



Demand Shock cont.



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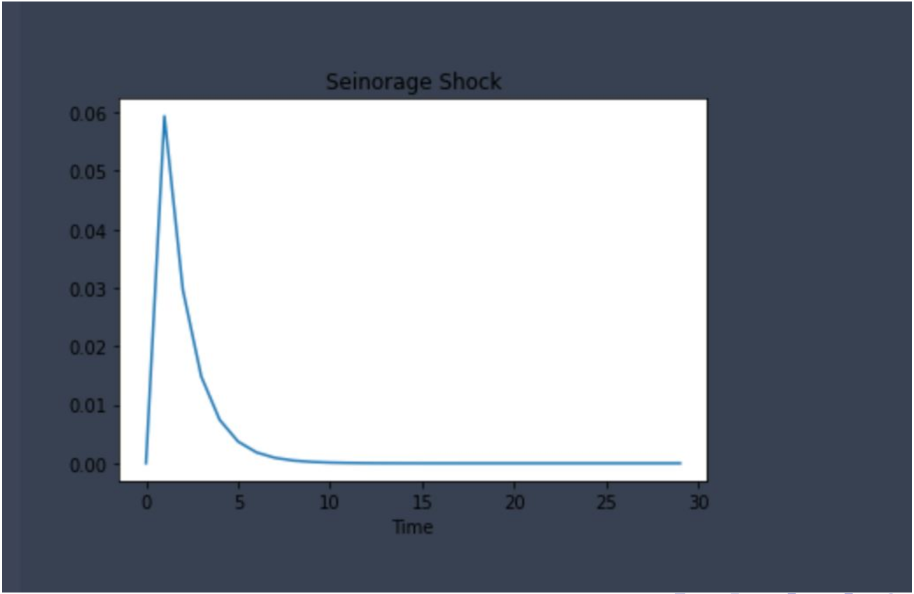

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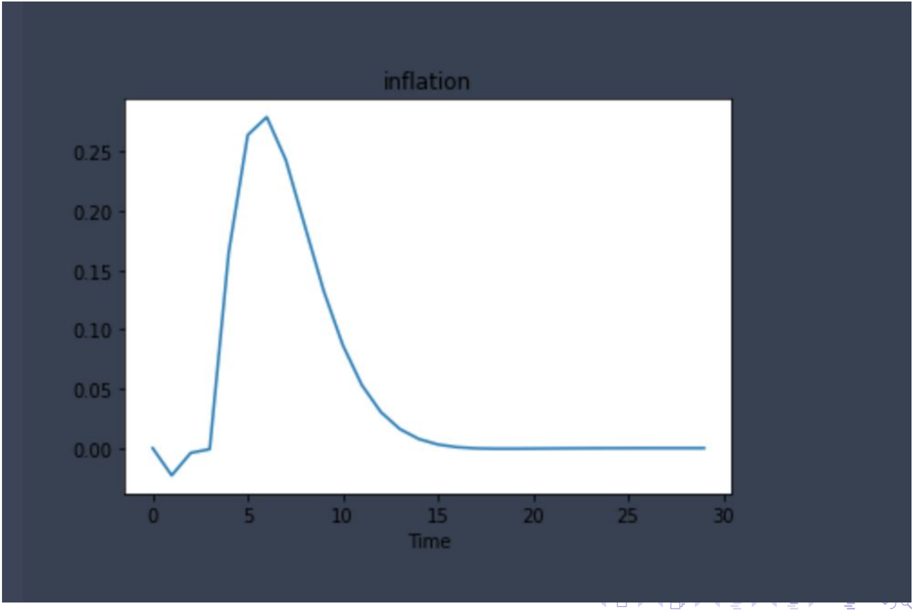

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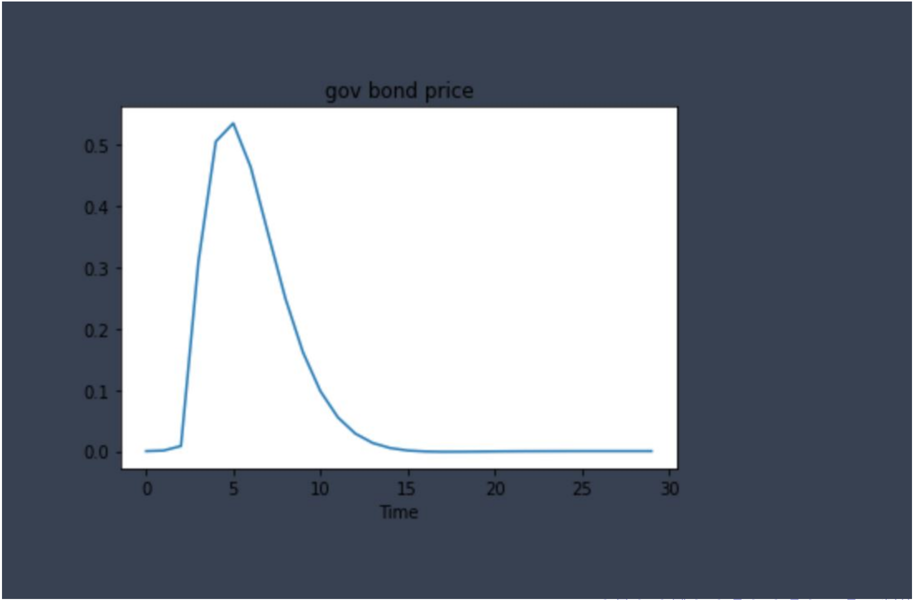

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