Nonparametric Tail Risk Analysis of Global Equities

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Motivation

Almeida et al. (2017): higher-order moments of asset returns linked to tail risks

Global tail risks not observed

This paper:

- forecast small set of global stock returns
- extract investment implications

Main ingredients:

- asymmetric shocks, stochastic volatility, restricted information of beliefs
- covariance with business cycles
- dual problem: minimum-distance discount factor and optimal portfolio weights

Motivation

Main findings:

- EM filter offers flexibility in characterizing distributions
- higher-order moments in macro variables strongly correlated with asset prices

Related Literature

Nonlinear stochastic discount factor

- Almeida et al. (2017)
- Nicolini et al. (2015)

Higher-order moments in asset pricing

- Kraus & Litzenberger (1976, 1983), Dittmar (2002)
- Chabi-Yo (2019)
- Barro & Liao (2020)

Estimation:

- Chib & Ramamurthy (2010)
- Chen & Liu (2000): Mixture Kalman filters
- Ehrmann, Fratzscher, Rigobon (2006): Macro shocks

Minimum-distance stochastic discount factor

Almeida et. al (2016): given Cressie-Read discrepancy function $\phi(m) = \frac{m^{\gamma+1} - a^{\gamma+1}}{\gamma(\gamma+1)}$, solving

$$\min_{m_1, m_2, .., m_T} \frac{1}{T} \sum_{t=1}^{T} \phi(m_t) \text{ s.t. } \frac{1}{T} \sum_{t=1}^{T} m_t R_t = 1_K; \frac{1}{T} \sum_{t=1}^{T} m_t = a$$

is equivalent to solving for

$$\hat{\lambda} = \underset{\lambda \in \mathbb{R}^K}{\operatorname{argsup}} \frac{1}{T} \sum_{t=1}^{T} \left(\frac{a^\gamma}{1+\gamma} - \frac{1}{1+\gamma} \left(a^\gamma + \gamma \lambda' \left(R_t - \frac{1}{a} 1_K \right) \right)^{\frac{\gamma+1}{\gamma}} \right).$$

Then

$$\widehat{m}_t = a \frac{\left(a^\gamma + \gamma \hat{\lambda}' \left(R_t - \frac{1}{a} 1_K \right) \right)^{\frac{1}{\gamma}}}{\frac{1}{T} \sum_{t=1}^{T} \left(a^\gamma + \gamma \hat{\lambda}' \left(R_t - \frac{1}{a} 1_K \right) \right)^{\frac{1}{\gamma}}}$$

Asset Pricing

We first assume the SDF has a discount has the following form:

$$m_{t+1} = \beta + \sum_j \tau^j_{t+1}$$

$$P^i_t = \widehat{E}_t \left(m_{t+1} P^i_{t+1} \right)$$

$$P^i_t = \widehat{E}_t \left(\beta P^i_{t+1} + P^i_{t+1} \sum_j \theta_j \tau^j_{t+1} \right)$$

Investors perceive asset prices to follow:

$$\widetilde{P}^i_t = \mu_i + \psi_i P^i_{t-1} + \sigma^i_t \varepsilon^{s,i}_t$$

Iterating forward:

$$\widehat{E}_t \widetilde{P}^i_{t+1} = \mu_i + \psi_i \widehat{E}_t P^i_t + \sigma^i_t \varepsilon^{s,i}_t$$

Stochastic Process

Joint Normality of Equity and Macro shocks:

$$\begin{bmatrix} \varepsilon^{s}_{t+1} \\ \varepsilon^{j,N}_{t+1} \end{bmatrix} \sim N\left(\overline{0},\ \begin{bmatrix} \left(\sigma^{s,i}_{t+1}\right)^2 & h^{i,j}_t \\ h^{i,j}_t & \left(\sigma^{N,j}_{t+1}\right)^2 \end{bmatrix}\right)$$

Inflation and Output Growth Process:

$$z^j_t = \begin{bmatrix} x^j_t \\ \pi^j_t \end{bmatrix} = \beta^j_{0,t} + \beta^j_{1,t} z^j_{t-1}$$

$$\beta^j_{0,t} \sim \tau^j_{0,t}$$

$$\beta^j_{0,t} \sim \tau^j_{1,t}$$

Tail Risk Errors:

$$\tau^j_{t+1} = \begin{cases} \varepsilon^{j,N}_{t+1} \ pr.\ 1-\lambda_j \\ q^j_{t+1} \ pr.\ \lambda_j \end{cases}$$

Mixture Model Estimation

Step 1: Generate Random Variables

$$\widetilde{C} \equiv \{c_n\}_{n=1}^{N} \sim \overline{\tau}_{t+1}$$

Step 2: Approximate distribution via EM algorithm

$$(w^*, \mu^*, \Sigma^*) = \max_{w,\mu,\Sigma} J\left(\widetilde{C}, w^*, \mu^*, \Sigma^*\right)$$

$$J\left(\widetilde{C}, w^*, \mu^*, \Sigma^*\right) \equiv \prod_{n=0}^{N} \sum_{k} w_k \left\{f_N\left(c_n - \mu_k, \Sigma_k\right)\right\}$$

Step 3: Generate macro variables via Ensemble kalman filter:

$$E\left[z_t^j | \mathcal{I}_{t-1}\right] = \sum_{k} w_k \left(\beta_{0,t|t-1}^{j,k} + \beta_{1,t|t-1}^{j,k} z_{t-1|t-1}^{j,k}\right)$$

Empirical Likelihood

Observation Equation:

$$\widetilde{Y}_t = \begin{bmatrix} Y_t^{obs,stock} \\ Y_t^{macro,stock} \end{bmatrix} = M_1 T_t + M_2 T_{t-1} + \varepsilon_t^m \ ; \ \varepsilon_t^m \sim \mathcal{N}(0, D)$$

Likelihood:

$$P\left(\widetilde{Y}|\Theta\right) = \prod_{t=1}^{T} \mathcal{N}\left(\widetilde{Y}_t - M_1 T_t + M_2 T_{t-1}, D\right)$$

Tail Model:

$$T_t \equiv \sum_k w_k \{A_t^k + B_t^k T_{t-1}^k + \varepsilon_t^{k,T}\}$$

Normal Model:

$$T_t = A_t + B_t T_{t-1} + \varepsilon_t^T$$

Prior Specification

$$\text{Rare Disaster Prob: } \lambda_j \sim \Gamma(\mu = .1, \sigma^2 = .05) \;, \forall j$$

$$vec(\Omega) \sim \mathcal{N}(\mu = vec(\Sigma_M^{OLS}), \sigma^2 = 1000 \times I)$$

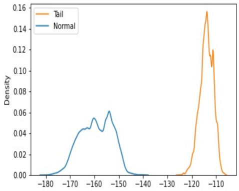
$$\text{Pareto Curvature: } \alpha \sim \Gamma(\mu = 7.5, \sigma^2 = .05)$$

$$\lambda_j \in [0, 1] \;\; \forall j$$

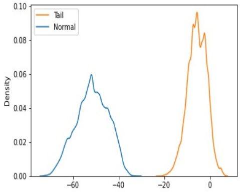
$$\alpha \in [5, 8]$$

$$\text{Macro Sensitivity: } \theta_{i,j} \in [0, \infty] \;\; \& \theta_{i,i} = 1 \; \forall i, j$$

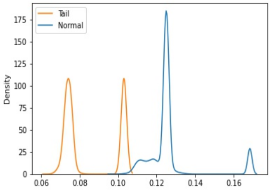
Estimation Results



(a) Likelihood



(b) Log Posterior



(c) SSE

Predictive Distribution

Utilizing the Equity forecast distribution, we obtain:

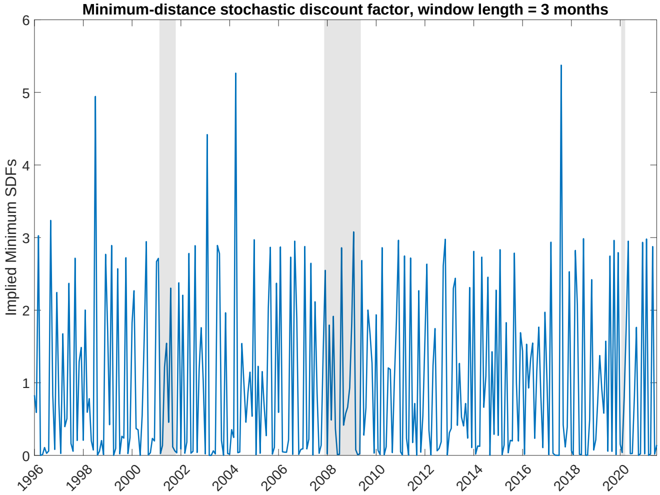
$$\widehat{m}_t = E\left[\mathcal{G}\left(\{R_\tau\}_\tau^T, \gamma, a\right)\right] = E\left[\mathcal{G}\left(\{\mathcal{F}_{t-1}(R_\tau)\}_\tau^T, \gamma, a\right)\right]$$

$$\begin{aligned}\mathcal{F}_{t-1}(\overline{P}_t - \overline{P}_{t-1|t-1}) &= \mathcal{F}(m_{t+1} \circ \overline{P}_{t+1} - \overline{P}_{t-1|t-1}) \\ &= \mathcal{F}_{t-1}(m_{t+1}(\mu + \Psi\overline{P}_{t-1|t-1} + (\Lambda_S S_{t-1|t-1} + \varepsilon_i^s))\widetilde{\varepsilon}_t - \overline{P}_t)\end{aligned}$$

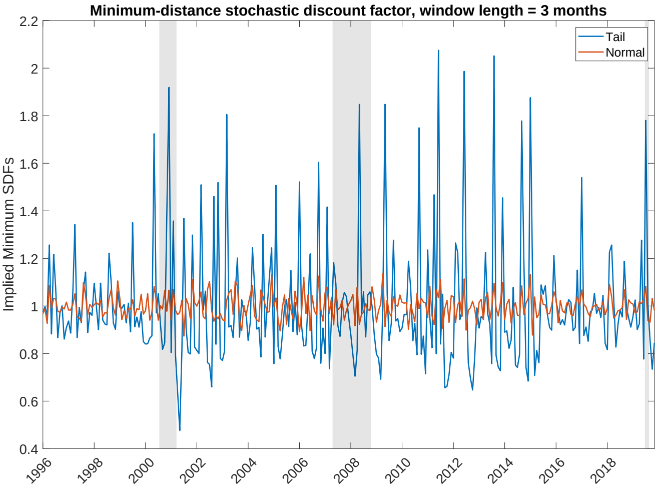
$$R_t \approx \overline{P}_t - \overline{P}_{t-1}$$

$$\mathcal{F}(m_t) \sim w_k \mathcal{N}_k(\mu_k, \Sigma_k)$$

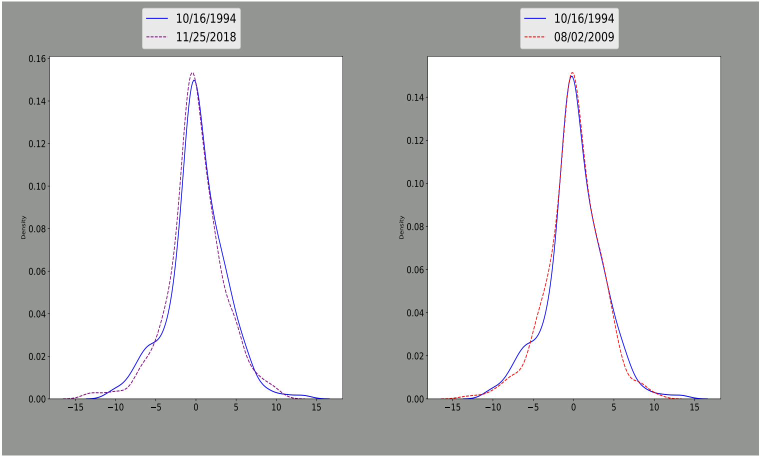
SDF Results (Data)



SDF Results (Model)



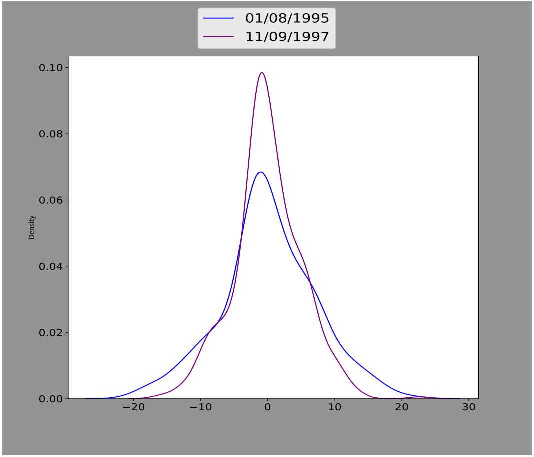
Predictive Distribution Results



(d) Nikkei 225 Covid

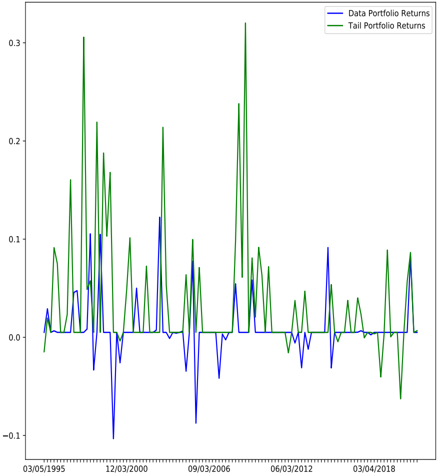
(e) Nikkei 225 Global Financial Crisis

Predictive Distribution Results

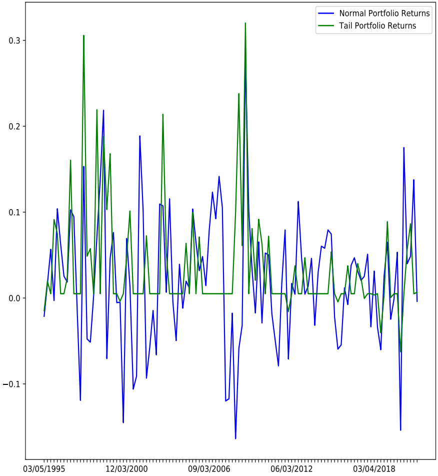


(f) KOSPI Asian Financial Crisis

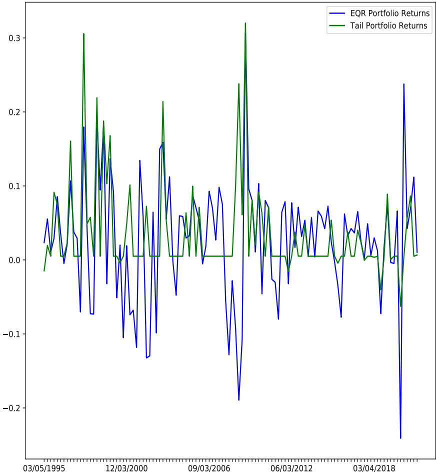
Portfolio Performance (TvD)



Portfolio Performance (TvN)



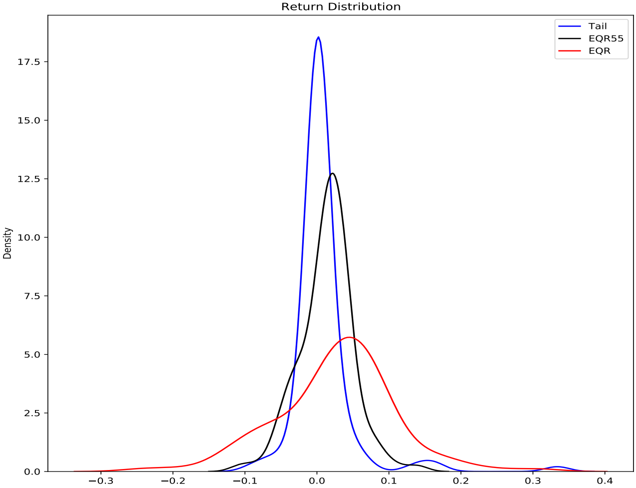
Portfolio Performance (TvEQR)



Return Performance Stats

Portfolio	Sortino Ratio	Sharpe Ratio	Sortino Std Dev.	Avg Return
Tail	.245	.134	2.39%	.83%
Normal	.09	.06	2.57%	.47%
MD SDF Data	.11	.11	2.60 %	0.54 %
55 Risk Free Equal Cap	.452	.293	2.92%	1.57%
Equal Cap	0.447	0.293	5.37 %	2.66 %
Tail MV	0.004	0.003	6.46 %	.526 %
Norm MV	-0.013	-0.009	6.28 %	0.418 %

Return Distribution



Tail & Normal Return Distribution