Japanese Business Cycles: Perceptions & Frictions *

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Abstract

This paper examines the Japanese Macro-economy through the lens of a behavioral DSGE equipped with financial frictions and a zero lower bound equilibrium. Armed with quarterly Macro & Financial data, the model finds both intermediary firm and banking credit frictions account for more than 50 % of the fluctuations in output and inflation from 2000-2020. Furthermore, I find via impulse response analysis, both equity and treasury LSAPs produce on average, increases in Output, Consumption, and Inflation by around .25%,.1%, and .05%, respectively. When the zero lower bound binds however, I find up to a 100-200 % increase in the effectiveness of both LSAPs & rate decreases under the binding zero lower bound regime, suggesting greater cause for action during such episodes during business cycles. Lastly, through counterfactual analysis via Particle filter methodology, I find evidence that had the BOJ coordinated interest rate, along with LSAP policy both in simultaneity and with higher magnitude during the Housing recession, this would prove to have further stabilized key macro-variables of interest, including 1 Quarter ahead Inflation expectations.

*William Branch has given structure and guidance

1 Introduction

Since the 1990s, Policymakers have taken more aggressive action towards combating low growth & inflation amidst an increasingly frequent binding zero interest rate. In particular, Japan has experienced historically long lasting zero lower bound periods where Fiscal & Monetary policy appeared to provide minimal impact in countering such negative business cycle effects.[1]. However, in recent developments, researchers have examined the role of fiscal and monetary policy multipliers under differing regimes. Miyamoto(2018) and Hiyoshi(2015) provide evidence of noticeably higher multipliers in the zlb equilibrium, along with the presence of time varying policy effects.[2] A number of papers attribute this occurrence to notions of pessimism or uncertainty[3] and address the ZLB constraint by advocating measures like forward guidance[4] & large scale asset purchases[5].[6]

In response to the slowing economic conditions since the 2000s, the BOJ has deployed aggressive unconventional monetary policies. From 2001-2006, the BOJ acquired 18 trillion yen worth of Japanese government bonds & held the policy rate near zero.[7] Furthermore, starting in 2010, the BOJ aggressively acquired Japanese government bonds & stocks(J-REITs & ETFs) via Large-Scale-Asset-Purchases(LSAP). During this period, the BOJ at one point held up to 80% and 50% of the Japanese ETF market & Treasury market, ratios unseen in any other developed nation. Recently, in efforts to combat the pandemic, the BOJ repeated this exercise along with its extended commitment hold a zero interest rate policy. Wang(2019) employs a non-linear DSGE using the shadow interest rate[8] and finds long lasting counterfactual effects from such unconventional monetary policy measures. Katagiri(2022) examines the BOJ's most recent ETF purchase program and finds a statistically

[1] Mishkin & Ito (2004)

[2] Ramey & Zubiary(2018) find similar results in the U.S.

[3] Benhabib et al.(2012) ;Planter et al.(2015)

[4] Woodford(2019);Swanson(2018)

[5] Chen(2011); Kim et al.(2020)

[6] Macroeconomics with Financial Frictions: A Survey(2011) & Brunnermeier. et. al. Forward Guidance, Monetary Policy Uncertainty, and the Term Premium Bundick et. al. (2021)

[7] See Iwata & Takenaka(2011) for a brief account of global monetary policy events

[8] Wu(2016)

significant decline in risk premia with an positive effect on equity prices. Kobuta & Shintani (2022) use high frequency identification via principal component analysis[9] on Japanese asset prices and finds statistically significant effects of LSAPs(via the path factor) on long term yields as well as stock prices, even in a low interest rate environment.

It is clear from the aforementioned literature, the effect of LSAPs & the ZLB are important to be taken into consideration together. Yet, it is not abundantly clear, how LSAPs drive the overall Japanese macro-economy or their effects on sentiment. Although the previous literature provides a foundational understanding that monetary policy has a sizeable impact, we also know from the literature, incorporating both financial frictions and behavioral models is imperative in obtaining a better assessment of the macro-economy. Fukunaga(2002) incorporates financial frictions in a DSGE model and finds a substantial portion of the output, investment, and inflation variance decomposition to have originated from a net worth shock. Hoshino et. al.(2021) obtain similar findings of financial shocks as a the main driver in fluctuations to output gap in Japan.

In addition to Financing Frictions, behavioral models that depart from the traditional Rational expectations assumption, offer a salient feature in describing how the macro-economy evolves over time. In most models, one typically assumes that agents form expectations over time based on perfect information along with the optimal forecast. In light of contrary evidence[10],and in line with Evans and Honkapohja (2001), agents learn about the economy over time and change their belief parameters in real time based on new observations each period. Indeed, Mishkin & Ito (2004) also document a historical account of Japan's persistent waves of optimism and pessimism due to the BOJ's lack of clear communication & corresponding policy uncertainty.

[9]See Gürkaynak et. al (2005) for the detailed methodology.
[10]See Milani(2006)

Motivated by previous analysis, I seek to answer the following questions: Firstly, to what degree do both equity and treasury LSAPs as well as standard policy impact key macro variables? Secondly, how strong of a role have credit frictions played in excerbating business cycles in Japan since the 2000s? How might the economy have been better positioned if policy implemented better coordinated LSAPs earlier on during the early 2000s and housing crisis? To answer each question, I employ a DSGE model via financial frictions, with agents who learn the economy over time. Additionally, I enable the economy to switch between zlb and non-zlb regimes where agents also form forecasted probabilities regarding the respective state in the next period. Hence, the paper is outlined as follows: Section 1 will discuss the model environment. Section 2 will cover the empirical methodology and corresponding statistical results. Section 3 with go over the model implied results. Lastly, Section 4 will discuss the Counterfactual analysis.

2 Model Environment

We have the following agents in the economy: households, intermediary firms, financial intermediaries, a fiscal authority, a labor-union, and a central bank. Households consume based on an anticipated utility framework[11] in which they take into account expectations of their future wealth net of taxes. They earn labor income, and investment income via return on the deposits made to the financial intermediary as well as equity income via dividend and equity price accumulation. Each period, Intermediary firms produce goods by employing labor and raising capital. Firms increase their capital stock by issuing equity to households or by issuing corporate debt to financial intermediaries subject to a collateral constraint. Firms determine their optimal labor demand as well as debt & equity issuance by maximizing flow profits subject to an exogenous investment friction and productivity process. I assume a representative final goods firm aggregates the j intermediate firm's good via CES preferences & a competitive market. All j firms exist in a monopolistically competitive

[11] Cogley & Seargant (2005)

environment. Wages are set by unions and prices are set by intermediary firms such that they hold a degree of market power and are subject to probabilities θ_w, θ_p, respectively, of being unable to optimize. Financial Intermediaries borrow from households subject to a zero-lower-bound constraint. They then make investments in both firm & government debt as well as reserves redeemed by the central bank. To overcome information asymmetry issues, Financial Intermediaries are subject to an incentive compatibility constraint imposed by the household as a fraction of net worth cannot fall bellow the total deposits issued. The fiscal authority levies a lump sum tax on households, and issues debt to the financial intermediary. Lastly, the Monetary authority follows a lagged Taylor rule and issues reserves to the Financial Intermediary.

2.1 Representative Household:

The household has additive preferences and derives utility from Consumption and Leisure.

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{H_t^{1+\nu}}{1+\nu} \tag{2.1}$$

$$c_t + s_t + q_t^{\psi}\psi_t = w_t H_t + \pi_t^{-1}(1 + R_{t-1}^s)s_{t-1} + \pi_t^{-1}(d_t + q_t^{\psi})\psi_{t-1} - \tau_t \tag{2.2}$$

Equation (2.1) is the household's utility function. Equation (2.2) is the household's budget constraint in which consumption net of investment in equities & deposits must equal the prior period's investment return plus labor income net of lump sump tax τ_t levied by the fiscal authority. Households maximize the following:

$$\max E_t \sum_{k=0}^{\inf} \beta^k U_{t+k} \text{ s.t. (2.1-2.3)} \tag{2.3}$$

First order conditions yield:

$$\frac{\partial \mathcal{L}}{\partial c_t} : c_t^{-\gamma} = \lambda_t \tag{2.4}$$

$$\frac{\partial \mathcal{L}}{\partial s_t} : \lambda_t = \beta \lambda_{t+1} \pi_{t+1}^{-1}(1 + R_t^s) \tag{2.5}$$

$$\frac{\partial \mathcal{L}}{\partial \psi_t} : q_t^{\psi} = E_t[\pi_{t+1}^{-1}(\frac{c_t}{c_{t+1}})^{\sigma}(q_{t+1}^{\psi} + d_{t+1})] = 0 \tag{2.6}$$

Where d_t are the real profits from the Intermediary firm:

$$d_t \equiv \Pi_t^{real,I}$$

2.2 Anticipated Utility:

As Eusepi & Preston(2013) and Woodford(2013) argue, households more realistically make consumption and savings decisions not based only on expected future consumption, but on expected discounted future wealth. Hence under the anticpated utlity framework, if households deviate from rational expectations, Ricardian equivalence no longer holds. Hence, future government expenditures then create disproportionate changes of expected future taxes. We start by linearizing the household budget constraint as follows:

$$s_{t-1} = \beta s_t + c_t + \tilde{\alpha_1} q_t^{\psi} + \tilde{\alpha_2}\psi_t + \tilde{\alpha_3}\pi_t + \tilde{\alpha_4} q_t^{\psi} + \tilde{\alpha_5} d_t + \tilde{\alpha_6}\psi_{t-1} + R_{t-1}^s + \tilde{\alpha_7}\tau_t \tag{2.7}$$

Next, we iterate this forward and obtain:

$$s_{t-1} = \sum_{k=0}^{\inf} \beta^k \Big(c_{t+k} + \tilde{\alpha_1} q_{t+k}^{\psi} + \tilde{\alpha_2}\psi_{t+k} + \tilde{\alpha_3}\pi_{t+k} + \tilde{\alpha_4} q_{t+k}^{\psi} + \tilde{\alpha_5} d_{t+k} + \tilde{\alpha_6}\psi_{t+k-1} + R_{t+k-1}^s + \tilde{\alpha_7}\tau_{t+k} \Big) \tag{2.8}$$

After plugging (2.4) into (2.5), iterating forward, and substituting into (2.7), I obtain the following expression for consumption:

$$c_t = (1-\beta)s_{t-1} + \nu_t$$

Where the expected infinite horizon of wealth is:

$$\nu_t = -(1-\beta)\Big(\tilde{\alpha_1} q_t^{\psi} + \tilde{\alpha_2}\psi_t + \tilde{\alpha_3}\pi_t + \tilde{\alpha_4} q_t^{\psi} + \tilde{\alpha_5} d_t + \tilde{\alpha_6}\psi_{t-1} + R_{t-1}^s + \tilde{\alpha_7}\tau_t\Big)$$

$$+\beta\gamma^{-1} E_t[\pi_{t+1} - R_t^s] + \beta E_t \nu_{t+1}$$

We see above that the household chooses its consumption based on the aggregate income flows conditional on the flow of not only future taxes/expenditures but also upon the perceived trajectory of asset prices, labor income, net of taxes. The flow of real wealth serves an important variable in amplifying business cycle dynamics for a given exogenous shock. Thus, I will henceforth label v_t the 'wealth channel'.

2.3 Monetary & Fiscal policy:

The Monetary authority holds a portion of government debt and private stocks. I restrict the central bank to these particular assets given that the BOJ has primarily used both of these as the primary tools since 1980. The Monetary authority creates excess reserves and purchases assets off the books of the FI. The mechanism as stated, is in line with the BOJ's stated open market operations guidelines.

The Central bank's flow budget constraint is:

$$q_t b_t^{cb} + q_t^{\psi} \psi_t^{cb} = re_t + \pi_t^{-1}(1 + \rho q_t) b_{t-1}^{cb} + (d_t + q_t^{\psi}) \psi_{t-1}^{cb} - \pi_t^{-1}(1 + R_{t-1}^{re}) re_{t-1} \tag{2.9}$$

Central bank profits are:

$$\Pi_t^{cb} \equiv \pi_t^{-1} \rho q_t b_{t-1}^{cb} + \pi_t^{-1} d_t \psi_{t-1}^{cb} - \pi_t^{-1}(1 + R_t^{re}) re_{t-1}$$

Assuming the Central bank returns its net profits to the Treasury, (2.9) becomes:

$$q_t b_t^{cb} + q_t^{\psi} \psi_t^{cb} = re_t + \pi_t^{-1} b_{t-1}^{cb} + \pi_t^{-1} q_t^{\psi} \psi_{t-1}^{cb} \tag{2.10}$$

The Treasury's flow budget constraint after receiving the Central bank's net revenues becomes:

$$q_t b_t = \pi_t^{-1}(1 + \rho q_t) b_{t-1} - \pi_t^{-1} \rho q_t b_{t-1}^{cb} - \pi_t^{-1} d_t \psi_{t-1}^{cb} + \pi_t^{-1}(1 + R_{t-1}^{re}) re_{t-1} - \tau_t \tag{2.11}$$

Above we see government debt issued each period equals the interest on debt issued in the previous period, plus treasury transfers to the household, minus lump sum taxes τ_t, minus dividends and coupon payments on stocks and treasuries held by the central bank, respectively, followed by a deduction of interest needed to pay off of reserves.

Market clearing for stocks and bonds are:

$$b_t = b_t^{cb} + b_t^{fi}$$

$$\psi_t = \psi_t^{cb} + \psi_t^{hh}$$

I assume that monetary policy uses an AR(1) rule for asset purchases:

$$q_t b_t^{cb} = (q_t b_{t-1}^{cb})^{\rho_b} \epsilon_t^{cb,b}$$

$$q_t^{\psi} \psi_t^{cb} = (q_t^{\psi} \psi_{t-1}^{cb})^{\rho_\psi} \epsilon_t^{cb,\psi}$$

Here, CB denotes the total quantity bonds and stocks Monetary policy owns at a given period, while ψ_t^{hh} denotes total equity holdings of households at time t.

Taxes evolve according to: $\tau_t = \phi_b b_{t-1}$.

2.4 A Note on Fiscal Policy:

In order to enable agents who's belief of the economy is consistent with the underlying equilibrium, I assume fiscal policy is passive. Doing so imposes that government deficits evolve based only on current and past variables, in similar functional form as the PLM, or perceived law of motion agents in the have over the dynamics of the economy.

I start with Fiscal policy's flow budget constraint:

$$q_t b_t = \pi_t^{-1}(1+\rho q_t)b_{t-1} - \pi_t^{-1}\rho q_t b_{t-1}^{cb} - \pi_t^{-1} d_t \psi_{t-1}^{cb} + \pi_t^{-1}(1+R_{t-1}^{re})re_{t-1} - \tau_t$$

If:

$$\frac{1+\rho q_t - \pi_t \phi_b}{q_{t-1}} \leq 1 \tag{2.12}$$

Then, I am able to express the Treasury's choice of debt($q_t b_t$) in terms of current and past-variables such that the expression below is stationary, similarly expressed in Leeper(1991):

$$q_t b_t = \sum_{i=0}^{t} (\frac{1+\rho q_{t-i} - \pi_{t-i}\phi_b}{\pi_{t-i}^{i} q_{t-i-1}})^i \Big(\pi_{t-i}^{-1}\rho q_{t-i} b_{t-i-1}^{cb} - \pi_{t-i}^{-1} D_{t-i}\psi_{t-i-1}^{cb} + \pi_{t-i}^{-1}(1+R_{t-i-1}^{re})re_{t-i-1} + g_{t-i}\Big)$$

In similar spirit to Grohé(2006), I take a steady-state approximation of (21) and impose:

$$\frac{1+\rho\bar{q} - \bar{\pi}\phi_b}{\bar{q}} \leq 1$$

$$\phi_b \geq 0$$

Hence, if the variables do not deviate too far from steady state, and the equilibrium is stable, the following restrictions ensure that Fiscal policy is passive and as a result, a PLM which is consistent with the Actual Law of Motion(ALM).

2.5 Firm Production Problem:

Each period, Intermediary firms select the optimal quantity of labor to employ, equity to issue, and corporate debt to issue(ψ_t, b_t^k, H_t). Motivated by Kiyotaki & Moore(1997), Because the Financial Intermediary is not privy to the profitability of each firm it lends b_t^k to, it then demands a level of collateral proportional to the market value of capital. This constraint is expressed where Q_t^k is Tobin's Q, which represents the shadow price of capital raised for the firm. Below, we see that the market value of the firm is expressed as the expected discounted sum of future firm profits.

The Market Value of the firm is:

$$V_t^I = E_t \sum_{k=0} \beta^k c_{t+k}^{-\delta} \Pi_{t+k} \tag{2.13}$$

Each time period, the firm earns real profit flows:

$$\Pi_t^I = A_t k_t^\alpha H_t^{1-\alpha} - \pi_t^{-1} d_t \psi_{t-1} - w_t H_t - \pi_t^{-1}(1+\rho_k q_t^k) b_{t-1}^k - \Omega_t^I \tag{2.14}$$

The Investment Adjustment cost is:

$$\Omega_t^I \equiv \zeta_t^I (\Delta_k (\frac{q_t^k B_t^k}{q_{t-1}^k B_{t-1}^k})^{c_I} + \Delta_\psi (\frac{q_t^\psi \Psi_t}{q_{t-1}^\psi \Psi_{t-1}})^{c_I}) \tag{2.15}$$

Re-expressed in terms of real adjustment cost:

$$\Omega_t^I = \pi_t^{c_I} \zeta_t^I (\Delta_k (\frac{q_t B_t^k}{q_{t-1}^k B_{t-1}^k})^{c_I} + \Delta_\psi (\frac{q_t^\psi \psi_t}{q_{t-1}^\psi \psi_{t-1}})^{c_I})$$

The Firm faces the following collateral constraint:

$$\theta_{2,t} Q_t^k k_t \geq \pi_t^{-1}(1+\rho_k q_t^k) b_{t-1} \tag{2.16}$$

Capital evolves according to:

$$k_t = (1-\delta) k_{t-1} + I_t \tag{2.17}$$

Firm Investment is:

$$I_t = q_t^\psi \psi_t + q_t^k b_t^k \tag{2.18}$$

Taking into account (2.14-2.18), the firm solves (2.13) with the following Lagrangian:

$$\mathcal{L}^I = E_t(\sum_{k=0} m^I_{t+k}\Pi_{t+k} - \sum_{k=0}^{\inf} Q^k_{t+k}[(1-\delta)K_{t+k-1} + I_{t+k} - K_{t+k}]$$

$$- \sum_{k=0} \mu^I_{t+k}(\theta_{2,t+k}Q^k_{t+k}K_{t+k} - (1-\rho_k q^k_{t+k})b_{t+k-1})) \tag{2.19}$$

<u>First order Conditions Yield:</u>

FOC for Capital Stock:

$$\frac{\partial \mathcal{L}^I}{\partial K_t} : \alpha m^I_t A_t k^{\alpha-1}_t h^{1-\alpha}_t + Q^k_t - (1-\delta)Q^k_{t+1} - \mu^I_{2,t}\theta_{2,t}Q^k_t = 0 \tag{2.20}$$

FOC for Corporate Bond Issuance:

$$\frac{\partial \mathcal{L}^I}{\partial b^k_t} : -m^I_{t+1}\pi^{-1}_{t+1}(1+\rho_k q^k_{t+1}) - m^I_t C^{I,b^k}_{1,t} + m^I_{t+1}C^{I,b^k}_{2,t+1} - Q^k_t q^k_t + \mu_{t+1}\pi^{-1}_{t+1}(1+\rho_k q^k_{t+1}) = 0 \tag{2.21}$$

FOC for Corporate Equity Issuance:

$$\frac{\partial \mathcal{L}^I}{\partial \psi_t} : -m^I_{t+1}\pi^{-1}_{t+1}d_{t+1} - m^I_t C^{I,\psi}_{1,t} + m^I_{t+1}C^{I,\psi}_{2,t+1} - Q^k_t q^\psi_t = 0 \tag{2.22}$$

FOC for Labor Demand:

$$\frac{\partial \mathcal{L}^I}{\partial H_t} : m^I_t(1-\alpha)k^\alpha_t h^{-\alpha}_t - m^I_t w_t = 0 \tag{2.23}$$

Where:

$$C^{I,x}_{1,t} \equiv \frac{\partial \Omega^I_t}{\partial x_t}$$

$$C^{I,x}_{2,t+1} \equiv \frac{\partial \Omega^I_{t+1}}{\partial x_t}$$

$$m^I_{t+k} = \beta^k c^{-\delta}_{t+k}$$

After re-arranging terms, I obtain an expression for the equilibrium corporate debt and stock issuance:

Expression for Firm Corporate Bond Issuance:

$$(m^I_{t+1} - \mu^I_{2,t+1})\pi^{-1}_{t+1}(1 + \rho_k q^k_{t+1}) + Q^k_t q^k_t + \pi^{c_I}_t \zeta^I_t c_I\{\Delta_k(\frac{q^k_t}{q^k_{t-1}b^k_{t-1}})^{c_I}(b^k_t)^{c_I-1}\}m^I_t =$$

$$\frac{\pi^{c_I}_{t+1} m^I_{t+1}\rho_I c_I \zeta^I_t\{\Delta_k(q^k_{t+1}b^k_{t+1})^{c_I}\}(q^k_t)^{-c_I}}{(b^k_t)^{c_I+1}} \tag{2.24}$$

Expression for Firm Equity Issuance:

$$\pi^{-1}_{t+1} m^I_{t+1} d_{t+1} + \pi^{c_f}_t \zeta^I_t c_I\{\Delta_\psi(\frac{q^\psi_t}{q^\psi_{t-1}\psi_{t-1}})^{c_I}(\psi_t)^{c_I-1}\}m^I_t + Q^k_t q^\psi_t$$

$$= \frac{\pi^{c_I}_{t+1} m^I_{t+1}\rho_I c_I \zeta^I_t\{\Delta_\psi(q^\psi_{t+1}\psi_{t+1})^{c_I}\}(q^\psi_t)^{-c_I}}{(\psi_t)^{c_I+1}} \tag{2.25}$$

When examining the firm's Foc's for bonds and stocks, I am able to show that holding all else constant, and increase in $E_t[\pi_{t+1}]$ yields a decline in both equity & debt issuance by the firm.

2.6 Financial Intermediary:

Each period the FI borrows deposits s_t from the household, makes various investments, and consumes her investment profits earned from the previous period. The FI Invests in: Corporate Bonds issued by the intermediary firm(b^k_t), Treasury Bonds issued by the fiscal authority(b_t), and Reserves issued by the central bank(re_t). When forming an investment portfolio, the FI is subject to the Gertler & Kiyotaki(2010) collateral constraint imposed by the household. That is, the FI cannot have a market value below a fraction $\theta_{1,t}s_t$ of real deposits, thereby enabling households to recoup losses given a heavy incurred loss to the FI.

Real Net worth evolves according to:

$$n_t = \pi_t^{-1}(re_{t-1}(1+R_{t-1}^{re}) + (1+\rho_k q_t^k)b_{t-1}^k + (1+\rho q_t)b_{t-1} + n_{t-1} - \tilde{\Omega}_t^f - (1+R_{t-1}^s)s_{t-1}) \quad (2.26)$$

The FI's adjustment cost is expressed as:

$$\Omega_t^f \equiv \zeta_t^f(\Delta_b(\frac{q_t B_t}{q_{t-1}B_{t-1}})^{c_f} + \Delta_k(\frac{q_t^k B_t^k}{q_{t-1}^k B_{t-1}^k})^{c_f}) \quad (2.27)$$

Re-expressed in terms of real adjustment cost:

$$\Omega_t^f = \pi_t^{-1}\tilde{\Omega}_t^f$$

$$\tilde{\Omega}_t^f = \pi_t^{1+c_f}\zeta_t^f(\Delta_b(\frac{q_t b_t}{q_{t-1}b_{t-1}})^{c_f} + \Delta_k(\frac{q_t^k b_t^k}{q_{t-1}^k b_{t-1}^k})^{c_f})$$

The Balance sheet identity is:

$$s_t = b_t^k q_t^k + b_t q_t + re_t \quad (2.28)$$

The FI's collateral constraint:

$$V_t \geq \theta_{1,t}s_t \quad (2.29)$$

Where the FI's Value is:

$$V_t \equiv \sum_{k=0}^{\inf} \sigma^k \beta^k c_{t+k}^{-\delta} n_{t+k}$$

Taking into account (2.26-2.29), The FI solves:

$$\mathcal{L} = E_t(\sum_{k=0}^{\inf}(\sigma)^k c_t^{-\gamma}\beta^k N_{t+k} - \sum_{k=0}^{\inf}\lambda_{t+k}^f[q_{t+k}b_{t+k} + q_{t+k}^k b_{t+k}^k + re_{t+k} - s_{t+k}]$$

$$-\sum_{k=0}^{\inf}\mu_{t+k}[V_{t+k} - \theta_{1,t+k}s_t]) \quad (2.30)$$

First Order Conditions Yield:

FOC for Corporate Bonds:

$$\frac{\partial \mathcal{L}}{\partial b_t^k}:\sum_{k=0} m^f_{t+k+1}(1+\rho_k q^k_{t+1})-\sum_{k=0} m^f_{t+k}C^{b_k}_{1,t}-\sum_{k=0} m^f_{t+k+1}C^{b_k}_{2,t+1}-\lambda^f_t q^k_t+(1+\rho_k q_{t+1})\sum_{k=0}\mu^f_{t+k+1}(\sum_{p=k+1} m^f_p)=0 \tag{2.31}$$

FOC for Government Bonds:

$$\frac{\partial \mathcal{L}}{\partial b_t^k}:\sum_{k=0} m^f_{t+k+1}(1+\rho q_{t+1})-\sum_{k=0} m^f_{t+k}C^{b}_{1,t}-\sum_{k=0} m^f_{t+k+1}C^{b}_{2,t+1}-\lambda^f_t q_t+(1+\rho q_{t+1})\sum_{k=0}\mu^f_{t+k+1}(\sum_{p=k+1} m^f_p)=0 \tag{2.32}$$

FOC for Deposits(borrowing):

$$\frac{\partial \mathcal{L}}{\partial s_t}:-(1+R^s_t)(\sum_{k=0} m^f_{t+k+1})+\lambda^f_t+\mu^f_{t+k}\theta_{1,t+k}+\sum_{k=0}\mu^f_{t+k+1}(\sum_{p=k} m^f_{t+p})(1+R^s_t)=0 \tag{2.33}$$

FOC for Reserves:

$$\frac{\partial \mathcal{L}}{\partial re_t}:\sum_{k=0} m^f_{t+k+1}(1+R^{re}_t)-\lambda^f_t-\sum_{k=0}\mu^f_{t+k+1}(\sum_{p=k} m_{t+p})(1+R^{re}_t)=0 \tag{2.34}$$

Expression for Corporate bond price:

$$\lambda^f_t q^k_t=(1+\rho_k q^k_{t+1})(\nu^m_{t+1}-\nu^{m,u}_{t+1})-(\nu^m_t-\nu^{m,u}_t)\pi_t^{1+c_f}\Delta_k c_f\zeta^f_t b_t^{k,c_f-1}(\frac{q^k_t}{q^k_{t-1}b^k_{t-1}})^{c_f}$$

$$+(\nu^m_{t+1}-\nu^{m,u}_{t+1})\frac{{\pi_{t+1}}^{1+c_f}c_f\rho_f\zeta^f_t\Delta_k(q^k_{t+1}b^k_{t+1})^{c_f}}{(b^k_{t+1})^{1+c_f}(q^k_t)^{c_f}} \tag{2.35}$$

Expression for Government bond price:

$$\lambda^f_t q_t=(1+\rho q_{t+1})(\nu^m_{t+1}-\nu^{m,u}_{t+1})-(\nu^m_t-\nu^{m,u}_t)\pi_t^{1+c_f}\Delta_b c_f\zeta^f_t b_t^{c_f-1}(\frac{q_t}{q_{t-1}b_{t-1}})^{c_f}$$

$$+(\nu^m_{t+1}-\nu^{m,u}_{t+1})\frac{{\pi_{t+1}}^{1+c_f}c_f\rho_f\zeta^f_t\Delta_b(q_{t+1}b_{t+1})^{c_f}}{(b_{t+1})^{1+c_f}(q_t)^{c_f}} \tag{2.36}$$

Expression for Reserves:

$$\lambda_t^f = (\nu_{t+1}^m - \nu_{t+1}^{m,u})(1 + R_t^{re}) \tag{2.37}$$

Expression for Deposits:

$$(1 + R_t^s) = (1 + R_t^{re}) + \theta_{1,t} \left(\frac{\mu_t^f}{\nu_{t+1}^m - \nu_{t+1}^{m,u}} \right) \tag{2.38}$$

Where:

$$C_{1,t}^x \equiv \frac{\partial \tilde{\Omega}_t^f}{\partial x_t}$$

$$C_{2,t+1}^x \equiv \frac{\partial \tilde{\Omega}_{t+1}^f}{\partial x_t}$$

$$m_{t+k}^f \equiv \sigma^k \beta^k \pi_{t+k}^{-1} c_{t+k}^{-\gamma}$$

$$\nu_t^m \equiv \sum_{k=0}^{\inf} m_{t+k}^f = \pi_t^{-1} c_t^{-\gamma} + \beta\sigma\nu_{t+1}^m$$

$$\nu_t^u \equiv \sum_{k=0}^{\inf} \mu_{t+k}^f = \mu_t^f + \nu_{t+1}^u$$

$$\nu_t^{m,u} \equiv \sum_{k=0}^{\inf} \mu_{t+k}^f \sum_{k=p}^{\inf} m_{t+k}^f = \mu_t^f \nu_t^m + \nu_{t+1}^{m,u}$$

We assume R_t^{re} is set by monetary policy via a Taylor rule.(Note: the Japanese Interest on reserve Data follows a similar trend to the historical Japanese nominal interest rate):

$$R_t^{re} = \rho R_{t-1}^{re} + (1-\rho)(\phi_x x_t + \phi_\pi \pi_t).$$

Using (15), we obtain the relationship that a higher R_t^{re} results in a decline in asset prices: q_t^k, q_t, q_t^ψ, holding all other variables constant. Intuitively, if monetary policy were to raise interest on reserves, the FI will divert a larger portion of funds in reserves and thus asset demand for all other products would decline, hence for the market to clear, asset prices would decline.

2.7 A Note on the Zero lower bound

In light of evidence presented by Hieder (2018), banks are unwilling to let $1 + R_t^s$ go below 0. In the model if $R_t^s < 0$, households would simply choose to not hold deposits and instead store income in terms of stored consumption goods or cash[12]. Thus, Bank behavior presents a natural "zero-lower-bound" equilibrium to explore. Using (2.33) & (2.37), I obtain an expression for R_t^s:

$$(1 + R_t^s) = \max\{0, (1 + R_t^{re}) + \theta_{1,t}\left(\frac{\mu_t^f}{\nu_{t+1}^m - \nu_{t+1}^{m,u}}\right)\}$$

We see from the above, absent the zero lower bound, the FI will set R_t^s equal to R_t^{re} but marked up by the extent of the financial friction, μ_t, seen in (6).

Aggregate consumption is expressed in terms of the sum of σ FI's who are forced to disburse their net worth to the household.

$$\bar{c}_t = c_1 c_t + \sigma n_t \tag{2.39}$$

Because the deposit rate departs from the traditional linear FI deposit demand Foc, the model, given most combinations of structural parameters, will not admit a unique, or determinate, solution. Furthermore, because I am interested in the exploring a richer set of equillibria, throughout the analysis, I only impose uniqueness for the Non-zlb equilibrium, while allowing indeterminacy in the zlb equilibrium. Though for a given set of parameters, the non-zlb is unique, because the zlb-case may not be unique, it follows then that the regime switching equilibrium need not be unique. In this paper however, I abstract from such issues, and only impose uniqueness for the non-zlb case. For all results, the equillibria obtained for the zlb is the MSV[13] implied solution.

[12]Note: I omit money balances in the model but I could just as easily add money in the budget constraint and obtain this result as well as the same exact equilibrium I solve for.

[13]McCallum(2004)

2.8 A note on indifference of bonds:

One point of concern is that when we study an economy where we assume a no-arbitrage condition, one would assume investors have no risk-adjusted preference between various investments. And as a consequence, large scale asset purchases would be rendered irrelevant if households and investors are able to costlessly shift from short to long term bonds, given a sudden swap on debt of differing maturities.[14] However, because the Financial Intermediary & Intermediary firms are subject to collateral constraints and face adjustment costs, agents are no longer able to costlessly shift their portfolio of holdings. Hence, for the purposes of evaluating the effectiveness of Large Scale Asset Purchases, b_t^{cb} & ψ_t^{cb}, we avoid the irrelevance pitfall.

2.9 Firm Pricing:

I Assume firms follow Calvo pricing with probability θ_p of being unable to change prices. Hence, firms price each period by solving:

$$\max_{p_t^*} \quad E_t[\sum_{k=0}^{\inf}(\theta\beta)^k \Lambda_{t+k} A_{t+k}] \tag{2.40}$$

$$A_{t+k} \equiv (\frac{p_t^*}{P_{t+k}})^{-\epsilon} Y_{t+k} - (\frac{p_t^*}{P_{t+k}})^{-\epsilon}(\frac{\Phi_{t+k}}{P_{t+k}})Y_{t+k} \tag{2.41}$$

Aggregate Price Identity:

$$P_t^{1-\epsilon} = [(1-\theta)(p_t^*)^{1-\epsilon} + \theta P_{t-1}^{1-\epsilon}] \tag{2.42}$$

$$\pi_t = (1-\theta)(1-\beta\theta)(\phi_t + \alpha_\pi^{-1}\delta c_t + \beta\theta E_t \pi_{t+1}) \tag{2.43}$$

With the follow New-Keynesian Phillips Curve, we see inflation is driven by aggregate firm marginal cost of production along with consumption and expected inflation in the next period.

[14] Woodford & Eggertson(2003), Wallace(1981)

2.10 Labor Union Wage Setting:

To introduce wage stickiness in the model, I assume that there exists a labor union which aggregates labor via CES preferences and leases out labor to firms in a competitive market. Hence, the labor union faces the following problem:

$$\psi_w \equiv \frac{\epsilon_w}{\epsilon_w - 1} \tag{2.44}$$

$$\max_{N_t(l)} W_t \left(\int_0^1 N_t^{\psi_w}(l) dl \right)^{\psi_w^{-1}} - \int_0^1 W_t(l) N_t(l) dl \tag{2.45}$$

Solving this yields:

$$N_t(l) = (\frac{W_t(l)}{W_t})^{-\epsilon_w} N_t \tag{2.46}$$

When households decide their labor supply, they take the above result from the union as given. Consequently, each household solves for their optimal labor supply via their optimal wage. In order to induce frictions in the wage setting process, I assume the household has probability θ_w of keeping the wage they set in the previous period[15]. Hence, the household optimizes wage considering this friction and seeks to maximize their discounted utility via the following problem:

$$\max_{w_t^*} \sum \Lambda_{t+k} \theta_w^k B_{t+k} \tag{2.47}$$

$$B_{t+k} = (-\frac{N_{t+k}^{1+\psi}}{1+\psi})(\frac{W_t^*}{W_{t+k}})^{-\epsilon_w(1+\psi)} + \lambda_{t+k} W_{t+k}(l) N_{t+k}(l) \tag{2.48}$$

$$N_{t+k}(l) = (\frac{W_t^*}{W_t})^{-\epsilon_w} N_t) \tag{2.49}$$

$$\frac{\partial}{\partial W_t^*} : (W_t^*)^{1-\alpha_0-\epsilon_w} = \sum (\beta\theta_w)^k \psi_w \{ \frac{N_{t+k}^{1+\psi} W_{t+k}^{\alpha_0}}{W_t^{\epsilon_w} N_{t+k} \lambda_{t+k}} \} \tag{2.50}$$

Using the Expression for Aggregate Wages:

$$W_t^{1-\epsilon_w} = (1-\theta_w)(W_t^*)^{1-\epsilon_w} + \theta_w W_{t-1}^{1-\epsilon_w} \tag{2.51}$$

[15]See Calvo(1983)

And after Log-Linearizing, I obtain:

$$w_{t+1} = (1-\theta_w)\alpha_1 \sum(\beta\theta_w)^k E_t\{(O^1_{t+k} - O^2_{t+k})\} + \theta_w w_{t-1} + \theta_w w_t \tag{2.52}$$

$$O^1_t = (1+\psi)n_t + \alpha_0 w_t \tag{2.53}$$

$$\begin{aligned} O^2_t &= \epsilon_w w_t + n_t + \lambda_t(1+\theta_w+\epsilon_w-\alpha_0)w_t \\ &= (1-\theta_w)\alpha_1\psi n_t - (1-\theta_w)\alpha_1\lambda_t + E_t w_{t+1} + \theta_w w_{t-1} \end{aligned} \tag{2.54}$$

$$(1+\theta_w)w_t = E_t w_{+1} + O^1_t - O^2_t + \theta_w w_{t-1}$$

2.11 Deriving Marginal Cost:

In order to derive the aggregate marginal cost of all firms, I can re-express the firm production problem as a cost-minimization problem. Expressed below, firms can solve the following:

$$\min_{N_{t+k},K_{t+k}} \sum \Lambda_{t+k}\{W_{t+k}N_{t+k} + \Phi(I_{t+k}, I_{t+k-1})I_{t+k} + (1+\rho^k)q^k_t - \Phi_{t+k}(Y_{t+k} - A_{t+k}K^{\alpha}_{t+k}N^{1-\alpha}_{t+k})\} \tag{2.55}$$

$$\frac{\partial}{\partial N_{t+k}} : W_{t+k} = MC_{t+k}(1-\alpha)K^{\alpha}_t N^{-\alpha}_t A_t \tag{2.56}$$

Log-Linearized, this becomes(note, I express the marginal cost as ϕ_t):

$$\phi_t = (\alpha)n_t - \alpha K_t - a_t + w_t + u_t \tag{2.57}$$

$$u_t = \rho_u u_{t-1} + \epsilon^u_t \tag{2.58}$$

Where u_t represents an exogenous cost push shock.

2.12 Market Clearing and Equilibrium:

Assuming all firms behave similarly, and that expectations are the same for each firm and household, the equilibrium can be defined as follows:

$w_t = \sum_j w_t(j)$, $n_t = \sum_j n_t(j)$, $c^{hh}_t = \sum_j c^{hh}_t(j)$, $q^k_t = \sum_j q^k_t(j)$, $q_t = \sum_j q_t(j)$, $b_t = \sum_j b_t(j)$,

$b_t^k = \sum_j b_t^k(j),\ b_t^s = \sum_j b_t^s(j),\ \phi_t = \sum_j \phi_t(j),\ Q_t = \sum_j Q_t(j),\ P_t = \sum_j P_t(j),\ K_t = \sum_j K_t(j),$ $y_t = \sum_j Y_t(j),\ h_t = \sum_j H_t(j),\ P_t = \sum_j P_t(j),\ k_t = \sum_j K_t(j),\ q_t^k = \sum_j q_t^k(j),\ q_t^{psi} = \sum_j q_t^{\psi}(j),$ $q_t = \sum_j q_t(j),\ R_t^s = \sum_j R_t^s(j),\ R_t^{re} = \sum_j R_t^{re}(j),\ \psi_t = \sum_j \psi_t(j),\ \psi_t^{hh} = \sum_j \psi_t^{hh}(j),\ \psi_t^{cb} = \sum_j \psi_t^{cb}(j),\ b_t = \sum_j b_t(j),\ b_t^{fi} = \sum_j b_t^{fi}(j),\ b_t^{cb} = \sum_j b_t^{cb}(j),\ s_t = \sum_j s_t(j),\ b_t^k = \sum_j b_t^k(j),$ $re_t = \sum_j re_t(j),\ Q_t = \sum_j Q_t(j),\ \mu_t = \sum_j \mu_t^j(j),\ \mu_t^2 = \sum_j \mu_t^2(j),\ w_t = \sum_j W_t(j),\ d_t = \sum_j d_t(j),$ $\nu_t^c = \sum_j \nu_t^c(j),\ \nu_t^m = \sum_j \nu_t^m(j),\ \nu_{m,ut} = \sum_j \nu_t^{m,u}(j),\ n_t = \sum_j n_t(j),\ \nu_t = \sum_j \nu_t(j),\ \phi_t = \sum_j \phi_t(j)$

The final equation the describes aggregate output:

$$Y_t = C_t + I_t. \tag{2.59}$$

Log Linearized, this becomes:

$$y_t = c_1 c_t + c_2 I_t. \tag{2.60}$$

Where $c_1 = \dfrac{C}{Y}$ and $c_2 = \dfrac{I}{Y}$.

In the model because we assume that a continuum of σ financial intermediaries dissolve their net worth and disburse their assets to the household. Aggregate consumption becomes:

$$c_t = \sigma n_t + (1-\sigma) c_t^{hh}. \tag{2.61}$$

The equilibrium is the sequence of prices q_t, q_t^k, R_t^s, P_t & allocation of variables described above and their corresponding evolution such that agents are optimizing subject to the corresponding, policy, household, fiscal, and firm level constraints.

3 Vector Representation of Equilibrium

After collecting all variables, I can now express the dynamics in the following state space form:

$$Z_t = AZ_{t-1} + BE_t Z_{t+1} + Q\bar{\epsilon}_t \tag{3.1}$$

$$Z_t = \begin{bmatrix} M_t \\ U_t \end{bmatrix} \tag{3.2}$$

$$M_t = \begin{bmatrix} c_t, y_t, h_t, \pi_t, k_t, q_t^k, q_t^{psi}, q_t, R_t^s, R_t^{re}, \psi_t, \psi_t^{hh}, \psi_t^{cb}, b_t, b_t^{fi}, b_t^{cb}, s_t, b_t^k, re_t, Q_t, \mu_t, \mu_t^2, w_t, \\ d_t, nu_t^c, \nu_t^m, \nu_{m,ut}, n_t, \nu_t \phi_t \end{bmatrix}' \tag{3.3}$$

$$U_t = \begin{bmatrix} r_t, u_t, a_t, \zeta_t^f, \zeta_t^I, \theta_{1,t}, \theta_{2,t} \end{bmatrix}' \tag{3.4}$$

Here, in equation (3.1), Z_t represents all variables previously described in the model. Where, M_t is a column vector that includes all endogenous variables. While U_t represents all exogenous AR(1) processes in the macro-economy. I solve for the Rational expectations equilibrium and obtain 2 equilibria: One when $i_t = 0$(Zero lower bound) and one when $i_t \geq 0$ (No Zero lower bound). Hence, after solving this becomes:

$$Z_t = CZ_{t-1} + D\bar{\epsilon}_t \tag{3.5}$$

Where:

$$M_t = C_{1,1}^j M_{t-1} + C_{1,2}^j U_{t-1} + D_1^j \bar{\epsilon}_t \tag{3.6}$$

$$U_t = RU_{t-1} + D_2^j \epsilon_t \tag{3.7}$$

for $j = \{z, n\}$

3.1 Adaptive Learning

We know from a body of literature [16] and in recent years, the difficulty of policy to meet expectations, and that the belief formation process is time varying. [17] Hence, it is imperative to examine how beliefs depart from rational expectations, drive business cycles and asset prices. Therefore, I allow for the parameters of the Rational Expectations Equilibrium(REE) to change over time via Stochastic Gradient learning[18] . Ultimately, this framework provides

[16] Branch and Evans(2005),Orphanides and Williams (2004)
[17] Evans and Honkapohja (2009)
[18] Evans(2010)

a more realistic dimension[19] and allows for waves of pessimism or optimism to influence the dynamics over time. Below, I display the set of equations that govern the economy in the model and explain the expectations formation process:

I assume that agents form the following beliefs about the economy.

Perceived Law of Motion:

$$M_t = \Lambda^m_{t-1} M_{t-1} + \Lambda^u_t U_t + \nu^m_t \tag{3.8}$$

Thus when forming expectations, I obtain:

$$\hat{E}_t M_{t+1} = \Lambda^m_{t-1} \hat{E}_t M_t + \Lambda^u_{t-1} R U_t \tag{3.9}$$

$$\hat{E}_t M_t = \Lambda^{z,m}_{t-1} M_{t-1} + \Lambda^{z,u}_{t-1} U_t \tag{3.10}$$

$$\Lambda_t \equiv \begin{bmatrix} vec(\Lambda^{z,m}_t) \\ vec(\Lambda^{z,u}_t) \end{bmatrix} \; ; \tag{3.11}$$

$$\Lambda^z_t = \Lambda_{t-1} + g P_z V'_t(\upsilon_t) \text{ if } i_t < 0 \tag{3.12}$$

$$\Lambda^n_t = \Lambda_{t-1} + g P_n V'_t(\upsilon_t) \text{ if } i_t \geq 0 \tag{3.13}$$

$$\upsilon_t \equiv \begin{bmatrix} \hat{E}_t M_t \end{bmatrix} - \begin{bmatrix} M_t \end{bmatrix} ; \; V_t \equiv \begin{bmatrix} I \otimes M'_{t-1} \\ I \otimes U'_t \end{bmatrix} \tag{3.14}$$

After substituting (3.9) into (3.1), I obtain the Actual Law of Motion:

$$M_t = A_t M_{t-1} + B_t U_t + Q \bar{\epsilon}_t \tag{3.15}$$

[19] Milani 2007

A note on constant gain learning:

In traditional models with learning, agents update beliefs in such a way that over time the changes in beliefs are stable and converge to the REE. This means agents dramatically update beliefs in earlier periods, and as time goes on, the changes to their belief parameters become incrementally smaller. In light of evidence by Branch and Evans(2005) as well as Orphanides and Williams (2004), I take seriously, the view that agents update beliefs each period with equal weight. This constant gain parameter g, determines the degree agents update beliefs based on forecast errors. Throughout the paper, this parameter is the subject of key interest in the proceeding sections.

3.2 Regime Switching Equillibria

To address both the zlb and non-ZLB equillibria, I equip agents with both the zlb and non-zlb parameters. Furthermore, I assume agents update their parameters by forming subjective probabilities. When doing so, I assume that the agent follows a binomial counting model a la' Cogley & Sargent (2005).[20] Hence, the model dynamics are expressed as follows:

Perceived Law of Motion

$$M_t = \Lambda^{*,m}_{t-1} M_{t-1} + \Lambda^{*,u}_{t-1} U_t + \nu^m_t \tag{3.16}$$

$$\Lambda^*_t \equiv \mu_t \Lambda^z_t + (1-\mu_t)\Lambda^n_t \tag{3.17}$$

$$\Lambda^n_t = \Lambda^n_{t-1} + gP_n V_t(\upsilon_t)' \tag{3.18}$$

$$\Lambda^z_t = \Lambda^z_{t-1} + gP_z V_t(\upsilon_t)' \tag{3.19}$$

Note, I initialize the Learning Parameters for $t = 0$ as follows:

$$\Lambda^z_0 = \begin{bmatrix} vec(C^{z,m}) \\ vec(C^{z,u} \end{bmatrix} \tag{3.20}$$

[20] Anticipated Utility and Rational Expectations as Approximations of Bayesian Decision Making

$$\Lambda_0^n = \begin{bmatrix} vec(C^{n,m}) \\ vec(C^{n,u}) \end{bmatrix} \tag{3.21}$$

$n_{0,0}^0 = 1;\ n_{0,1}^0 = 1;\ n_{1,0}^0 = 1;\ n_{1,1}^0 = 200;$

$$\mu_t = \mathbb{P}(i_t = 0|\mathcal{I}_{t-1}) = \begin{cases} \frac{n_{1,0}^t}{n_{1,1}^t + n_{1,0}^t} & i_t \neq 0 \\ \frac{n_{0,0}^t}{n_{0,1}^t + n_{0,0}^t} & i_t = 0 \end{cases}$$

(3.22)

Hence, when computing the expected future variables, I obtain the following expression:

$$\hat{E}_t M_{t+1} = \Lambda_{t-1}^{*,m} \hat{E}_t M_t + \Lambda_{t-1}^{*,u} R U_t \tag{3.23}$$

$$\hat{E}_t M_t = \Lambda_{t-1}^{*,m} M_{t-1} + \Lambda_{t-1}^{*,u} U_t \tag{3.24}$$

<u>Augmented law of motion:</u>

$$M_t = \begin{cases} A^z M_{t-1} + B^z \hat{E}_t M_{t+1} + C^z U_t + D^z \bar{\epsilon}_t & i_t = 0 \\ A M_{t-1} + B \hat{E}_t M_{t+1} + C U_t + D \bar{\epsilon}_t & i_t \geq 0 \end{cases} \tag{3.25}$$

In the new model with zlb dynamics, the subjective probability μ_t becomes an important feature driving values for $E_t M_{t+1}$. Consequently, each term $n_{i,j}^t$ represents a function which counts the number of times state i has moved to state j. Hence, when $i_t = 0$ we see this not only plays a role in driving the value of M_t through the non-expectation parameters seen in (3.1), but will also drive how $E_t M_{t+1}$ will evolve over time through Λ_t^*'s evolution.

4 Estimation

In order to properly understand the dynamics of the model in the context of the Japanese economy, I take Macro observable data from Japan and match this with the model generated nonlinear state space data. Using equations (3.1)-(3.24) The model is re-expressed as:

$$\begin{bmatrix} Y_t \\ M_t \end{bmatrix} = \tilde{K}_1(\Lambda^*_{t-1}(\mu_{t-1}), \Theta)M_{t-1} + \tilde{K}_2(\Lambda^*_{t-1}(\mu_t), \Theta)U_{t-1} + \tilde{K}_3(\Lambda^*_{t-1}(\mu_{t-1}), \Theta)\bar{\epsilon}_t \tag{4.1}$$

Above, we see that the variables of interest evolve in a nonlinear fashion. However, to properly empirically identify the likelihood, I must jointly estimate (3.5-3.6) with the learning equations (3.11-3.13) . Therefore the state transition equation and observation equations respectively, are:

$$V_t \equiv \begin{bmatrix} M_t \\ \Lambda^*_t \\ \mu_t \\ U_t \end{bmatrix} = \mathcal{F}(\Theta, Y_{t-1}, M_{t-1}, \Lambda^*_{t-1}, U_{t-1}, \bar{\epsilon}_t);\ \bar{\epsilon}_t \sim \mathcal{N}(0,\ \Psi(\Theta)\ ) \tag{4.2}$$

$$Y^{obs}_t = \bar{M}_1 V_t + \bar{M}_1 V_{t-1} + \epsilon^m_t; \tag{4.3}$$

$$\epsilon^m_t \sim \mathcal{N}(0, I) \tag{4.4}$$

$$\pi(Y^{obs}_t|\Theta, I_{t-1}) = N(Y^{obs}_t - \bar{M}_1 V_{t|t-1} - \bar{M}_1 V_{t-1|t-1}, I) \tag{4.5}$$

4.1 Particle Filter

In most models with learning, researchers hold the learning parameters fixed in order to utilize the Kalman filter[21]. This assumption has been shown to yield lower marginal &

[21] See: Milani 2002, Hommes,MavroMatris,Ozden(2018)

log likelihoods along with therefore inaccurate parameter estimates.[22] To overcome this challenge, I jointly estimate the learning parameters Λ_t^* and μ_t via a particle filter with re-sampling.[23]The importance of estimating the learning parameters as states can interpreted as follows: When agents form expectations of the future, the stochastic processes which drive macroeconomic variables are characterized by distributions that follow a mean and variance. Though the shocks are independent of the macro-economy, the distribution macroeconomic variables as well as the learning coefficients are endogenous to one another. By including the learning coefficients as part of the state variable, the researcher is in effect taking the stance that the distribution of macro variables are dependent on the distribution of beliefs and vice versa. Notice however, in the conditionally linear model, since we hold the learning parameters Λ_t^* and μ_t fixed, we assume that beliefs are driven by macro variables but not the reverse.

The particle filter simulates the structural error terms with a sufficient number of draws, places weights on the realized number states,and calculates the a weighted likelihood. Often this procedure places high weight on few particles and 0 weight on most particles which, in essence, induces a high variance of the Likelihood function.[24] In order to avoid this issue, I follow Rubio-Ramirez & Villaverde(2007) with the following algorithm:[25]

$$\textbf{Step 1}\text{(Initialize): Set } e_t^j \sim \bar{\epsilon}_t.$$

$$\textbf{Step 2}\text{(Propagate): } V_t^j = \mathcal{F}(V_{t-1}^j, \epsilon_t^j).$$

$$\textbf{Step 3}\text{(Evaluate): } w_t^j = \frac{P(Y_t|V_{t|t-1}^j, \Theta)}{\Sigma w_t^i}.$$

$$\textbf{Step 4}\text{(Re-sample): } q_i \sim \{w_t\}_{j=0}^J$$

[22]See: Kirpekar(2020)

[23]See:Rubio-Ramirez and Villaverde(2007) & Herbst and Schorfheide (2017)

[24]Kitigawa 1996

[25]For Convergence properties of the Likelihood, please refer to Rubio-Ramirez & Villaverde(2007) and Kitigawa(1996)

$$\text{and set: } V^j_{t|t-1} = V^i_t, \text{ for all } \{q_i\}^J_{i=0}$$

$$\text{Set } t = t + 1; \text{and repeat till } t = T.$$

Step 5(Calculate Likelihood): $P(Y^T|\theta) \approx \frac{1}{J}(\prod_{t=1}^{T}(\sum_{j=1}^{J} p(Y_t|w^j_t, V^j_{t|t-1}, Y^{t-1}, \theta)))$.

With the stated procedure, as the number of particles, J becomes greater, the Likelihood converges to the true distribution. However, because one must keep track of the states and their relative values across the sample, there exists a trade-off between and accuracy and computational time of the algorithm. In this paper, I find that the results of estimation do not change much between 1 - 10 thousand particles. Hence I assume J to be 5 thousand, a sufficient approximation to the true Likelihood function of interest in the proceeding results.

4.2 Ensemble Kalman Filter

Because the Particle Filter becomes infeasible for estimation as the dimension of the state space increases, I utilize an Ensemble Kalman Filter to estimate the likelihood with respect to the parameters. Drawing from the original Kalman filter, The Ensemble Kalman filter is an appropriate alternative in capturing the non-linearites because it allows for successive draws of the predictive distribution rather than relying on a single point estimate based on Gaussian error terms. Le Gland et. al(2009) examine the performance of the Ensemble Kalman filter and obtain statistical convergence of each state variable as the size of draws from the predictive covariance matrix increases. Below I outline a brief but instructive procedure of the Ensemble Kalman filter I use for the estimation of the empirical likelihood (ultimately utilized for the Metropolis Hastings algorithm). Though the Ensemble Filter can be generalized for a large class of nonlinear models, below I formulate a set of equations in the context of the previously outlined regime switching model via learning. The Observation equation of the model can also be expressed as follows:

$$Y^{obs}_t = H^1_{t-1} S_t + H^2_{t-1} S_{t-1} + \epsilon^m_t \tag{4.6}$$

Where:

$$H^1_{t-1} \equiv H^1(S_{t-1}, \theta)$$

$$H^2_{t-1} \equiv H^2(S_{t-1}, \theta)$$

Here, because we have expectation level data of Japan, $\hat{E}_t M_t$ is a function of $\Lambda_{t-1}, \mu_{t-1}, \& M_{t-1}$. Where, $\Lambda_{t-1}, \mu_{t-1}, \& M_{t-1} \in S_{t-1}$.

The State Transition equation is expressed as follows:

$$S_t = f(S_t, \epsilon^S_t) \tag{4.7}$$

First, we take a set of successive draws from ϵ^S_t, and call them $\epsilon^{j,S}_t$.

Next, we propagate the state space and obtain:

$$S^j_t = f(S^j_t, \epsilon^{j,S}_t)$$

$$\hat{Y}^j_t = H^1_{t-1} S^j_t + H^2_{t-1} S^j_{t-1|t-1}$$

After obtaining the model implied forecasts, we compute the recursive covariance matrix and empirical updates. The key difference in the Ensemble is that the Covariance matrix, is not known in closed form, and hence must be approximated via an empirical covariance computation. Below outlines the corresponding recursions:

$$\hat{P}^S_{t|t-1} = V\hat{A}R(S_t|\mathcal{I}_{t-1}) = \frac{1}{J}\sum_j (S^j_{t|t-1} - \hat{E}[S_{t|t-1}])'(S^j_{t|t-1} - \hat{E}[S_{t|t-1}])$$

$$D_t = V\hat{A}R(Y^{obs}_t|\mathcal{I}_{t-1}) = H^1_{t-1|t-1}\hat{P}^S_{t|t-1}(H^1_{t-1|t-1}) + I$$

$$L_t = C\hat{o}v(S_t, Y^{obs}_t|\mathcal{I}_{t-1}) = H^1_{t-1|t-1}\hat{P}^S_{t|t-1}$$

$$\hat{P}^S_{t|t} = V\hat{A}R(S_t|\mathcal{I}_t) = \hat{P}^S_{t|t-1} - L'_t(D_t)^{-1}L_t$$

$$S^j_{t|t} = S^j_{t|t-1} + L_t(D_t)^{-1}\widehat{e}^j_t$$

$$\widehat{e}^j_t \equiv Y^j_{t|t-1} - Y^m_t$$

26

4.3 Data Description

In order to properly identify the model and come up with meaningful insights, I utilize FRED ,TANKAI Survey & BOJ data. Equipped with expectations level data, the model is now better able to pin down equations (3) and (4). For all data except the Policy and Deposit rate, I apply the Christiano-Fitzgerald(1999) filter. I do this to ensure that the model is able to capture the zero lower bound periods more effectively using the respective interest rates.

$$Y^{obs}_t = \begin{bmatrix} H^{obs}_t, re^{obs}_t, s^{obs}_t, w^{obs}_t, b_t, b^{cb}_t, b^k_t, c^{obs}_t, d^{obs}_t, g^{obs}_t, I^{obs}_t, \psi^{cb}_t, q_t, q^k_t, q^{\psi}_t, y_t, \pi_t, \\ re_t, R^s_t, \Delta c^e_{t+1}, \Delta\pi^e_{t+1}\Delta y^e_{t+1} R^{re}_t \end{bmatrix} \tag{4.8}$$

4.4 Estimation Results

With the stated equations, I am now able to estimate the Likelihood function with non-linearity taken into account. Equipped with Priors given from the literature[27], I use the Chib & Ramamurthy(2009) Tailored randomized block random walk metropolis-hastings MCMC algorithm. This procedure allows for reduced simulations, while still sufficiently exploring the posterior distribution.[28]

[26]Details on the Model equations are outlined in the Appendix
[27]Christiano et. al.
[28]See Appendix for convergence results

Model Params.							
Para	Descr.	Prior Mean	Prior Std.	Post Mean	$5\%, 95\%$	Dist.	Bound
ψ	Frisch.	$.3\bar{3}$	.75	.66	.19,1.4	$\mathcal{N}$	$0 < x$
γ	CRRA	1.5	.375	2.4	1.9,2.78	Γ	$0 < x$
Δ_k	Adj Corp.	.	.	6.9	4.4,8.2	U	$0 < x$
Δ_ψ	Adj Eq.	.	.	5.0	6.1,9.1.	U	$0 < x$
Δ_b	Adj Treas.	.	.	7.3	5.0,7.2	U	$0 < x$
c_I	Adj Elas.	.01	.1	.32	.04,.64	γ	$0 < x$
θ_p	Calvo-prices	.66	.1	.65	.50,.80	Γ	$0 < x$
θ_w	Calvo-wages	.66	.1	.61	.41,.79	Γ	$0 < x$
ϕ_b	Tax Resp.	.	.	2.4	.99,3.7	U	$0 < x$
ρ_u	Dem.	.	.	.27	.03,.57	U	$0 < x$
ρ_r	Dem.	.	.	.45	.01,.91	U	$0 < x$
ρ_a	Prod.	.	.	.43	.07,.71	U	$0 < x$
ρ_ψ	lsap E.	.	.	.38	.08,.88	U	$0 < x$
ρ_b	lsap T.	.	.	.42	.05,.86	U	$0 < x$
ρ_{θ_1}	FI.	.	.	.45	.07,.92	U	$0 < x$
ρ_{θ_2}	Int. Firm.	.	.	.48	.08,.94	U	$0 < x$
ρ_f	FI Adj.	.	.	.42	.05,.87	U	$0 < x < 1$
ρ_I	Firm. Adj.	.	.	.32	.03,.88	U	$0 < x < 1$

Model Params Cont.							
Para	Descr.	Prior Mean	Prior Std.	Post Mean	$5\%, 95\%$	Dist.	Bound
σ_u	std dev.	.5	inf	.44	.11,1.0	Γ^{-1}	$0 < x$
σ_r	std dev.	.5	inf	.82	.12,1.9	Γ^{-1}	$0 < x$
σ_a	std dev.	.5	inf	.42	.1,1.1	Γ^{-1}	$0 < x$
σ_ψ	std dev.	.5	inf	.45	.11,1.3	Γ^{-1}	$0 < x$
σ_b	std dev.	.5	inf	.18	.07,1.06	Γ^{-1}	$0 < x$
σ_ψ	std dev.	.5	inf	.19	.1,.34	Γ^{-1}	$0 < x$
σ_{θ_1}	std dev.	.5	inf	.65	.11,1.5	Γ^{-1}	$0 < x$
σ_{θ_2}	std dev.	.5	inf	.45	.12,1.0	Γ^{-1}	$0 < x$
σ_{re}	std dev.	.5	inf	.49	.12,1.1	Γ^{-1}	$0 < x$
σ_τ	std dev.	.5	inf	.86	13,1.8	Γ^{-1}	$0 < x$
σ_w	std dev.	.5	inf	.47	.11,1.0	Γ^{-1}	$0 < x$
ρ	infl. pers.	.875	.1	.86	.71,.98	Γ	$0 < x$
ϕ_x	Output resp.	.15	.1	.14	.03,.33	Γ	$0 < x$
ϕ_π	Infl. resp.	1.7	.1	1.7	1.6,1.9	Γ	$0 < x$
g	rls gain.	.031	.022	.035	.012,.07	Γ	$0 < x$

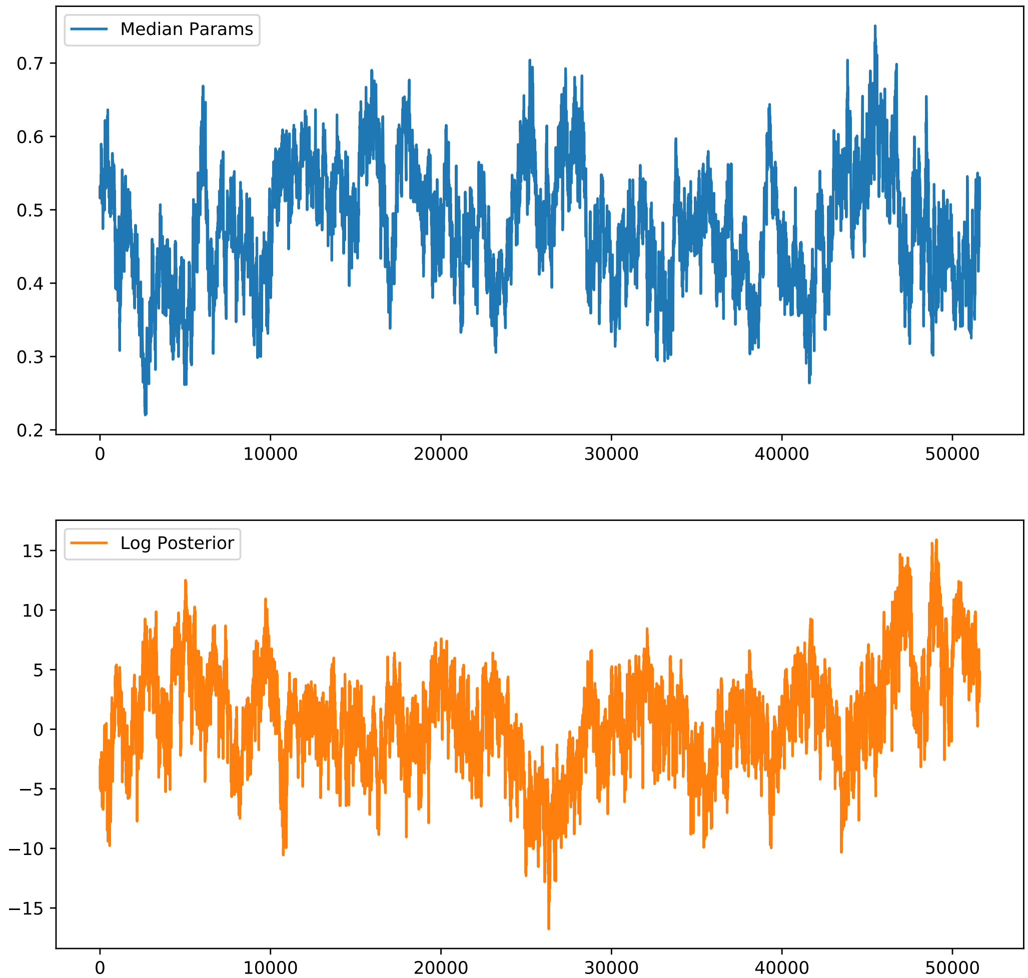

Figure 1: Traceplots

Figure 1 displays the rolling median of the last 50 thousand draws of the Markov Chain. Because we see the plots exhibit stationarity, this suggests strong evidence of model convergence. Likewise, the figure also shows the same but instead of the mean of each parameter draw, we see the log posterior exhibiting stationarity.

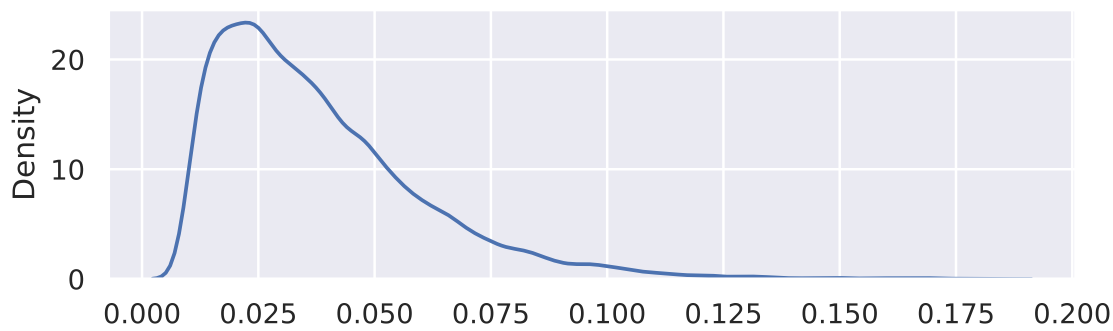

Figure 2: Posterior Dist. of RLS gain coeff.

Often in the learning literature, we see the RLS gain coefficient estimated to be close to .02. In line with this body of evidence, we see the posterior parameter distribution provides some evidence that although the mean is centered around to .025, there is a rightward skew. This suggests the coefficient could indeed be higher in Japan.[29]

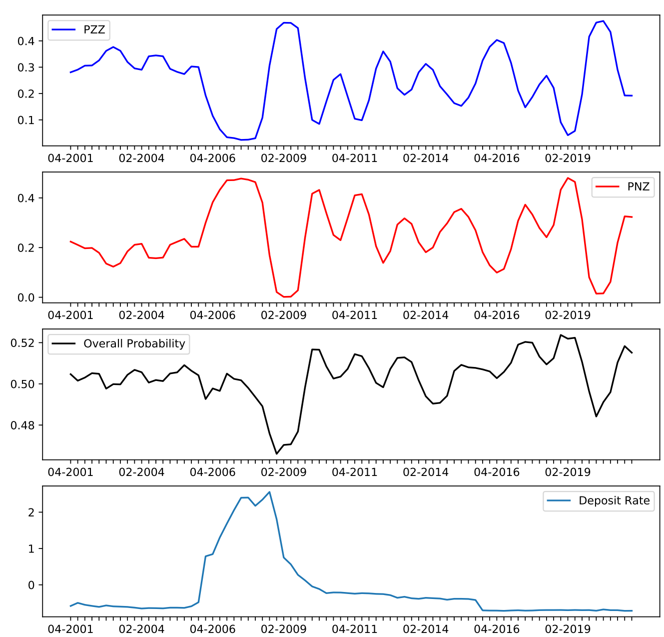

Figure 3: ZLB probability (μ_t)

[29]See: Cole and Milani(2020) & Milani(2004), , and Kirpekar(2020)

Above in Figure 3, we see the implied subjective probabilities evaluated at the posterior mean. Here PZZ represents the probability of entering in the zero lower bound the next period, given the current period was at the ZLB. PNZ represents the probability of entering the ZLB given the current period was not in the ZLB. Given that the model is not equipped with asset pricing sentiment level data, it does relatively well in capturing the strong comovement with the household Deposit rate. Intuitively, we know when the interest rate rises, all else equal, agents would reasonably expect that entering in the ZLB is likely to fall. Consequently, agents would expect the ZLB probability to rise as the deposit rate falls. These movements we see are reflected in the 2nd Quarter of 2009' along with the 2nd Quarter of 2019'. In both instances, it was clear to agents, the appropriate revision of the perceived ZLB probability.

5 Results

5.1 Variance Decomposition Evidence

Using the parameters of the model, I compute the generalized forecast error variance decomposition and thereby am able to obtain the overall contribution of variance from each exogenous shock the macro-variables of interest[30]. Following Lanne & Nyberg(2016) , I compute the following expression using the particle filter's forecast:

$Y_t = f(S_{t-1}, \epsilon_t),\ GI = E_t[Y_{t+h}|\epsilon^j_{t+1} = \sigma_j, I_{t-1}] - E[Y_{t+h}|I_{t-1}]$

$\lambda^j_t(h|e_{t+h}) = \frac{\sum_{l=1}^{h} GI(l|\epsilon^j_{t+h}=\sigma_j)}{\sum_{j=1}^{J}\sum_{l=1}^{h} GI(l|\epsilon_{t+1}=\sigma_j)^2}$

$FEVD^j_t \equiv \lambda^j_t / E[\lambda^j_t(h)] = \sum_{j=1}^{J} \lambda^j_t(h)$

We see from above, the Normalized measure of variance contribution relies on the information set I_{t-1} and hence is a function of the squared deviations of the mean forecast estimate. Below, I plot the relevant results for the contribution of Variance across the sample, and include only the top 5 shocks:

[30]see: Pesaran et. al 1997

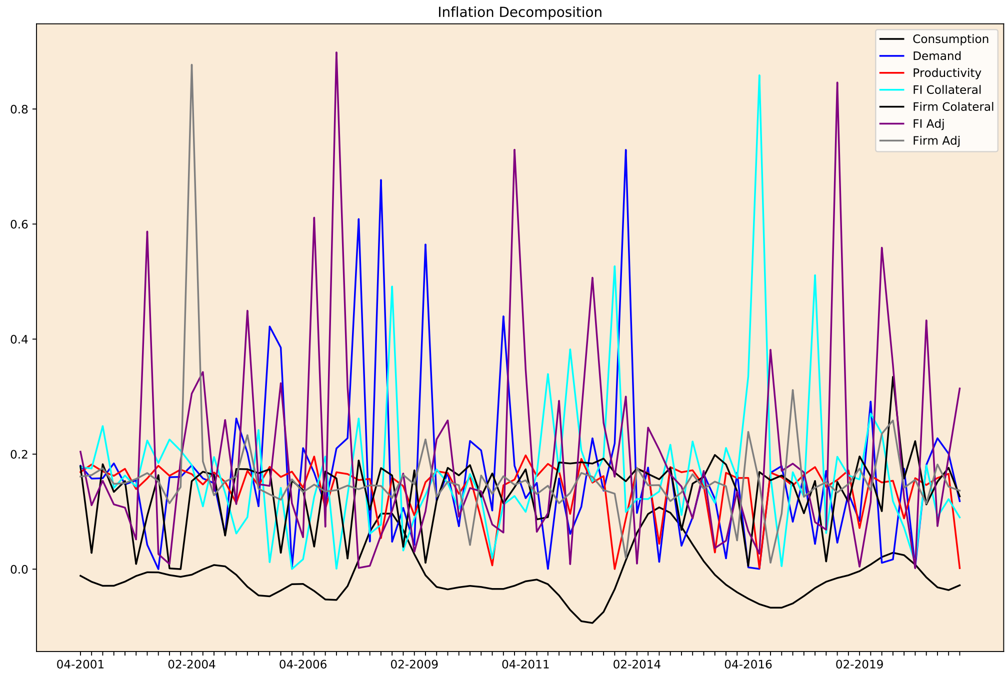

Figure 4: Shock decomposition for Inflation

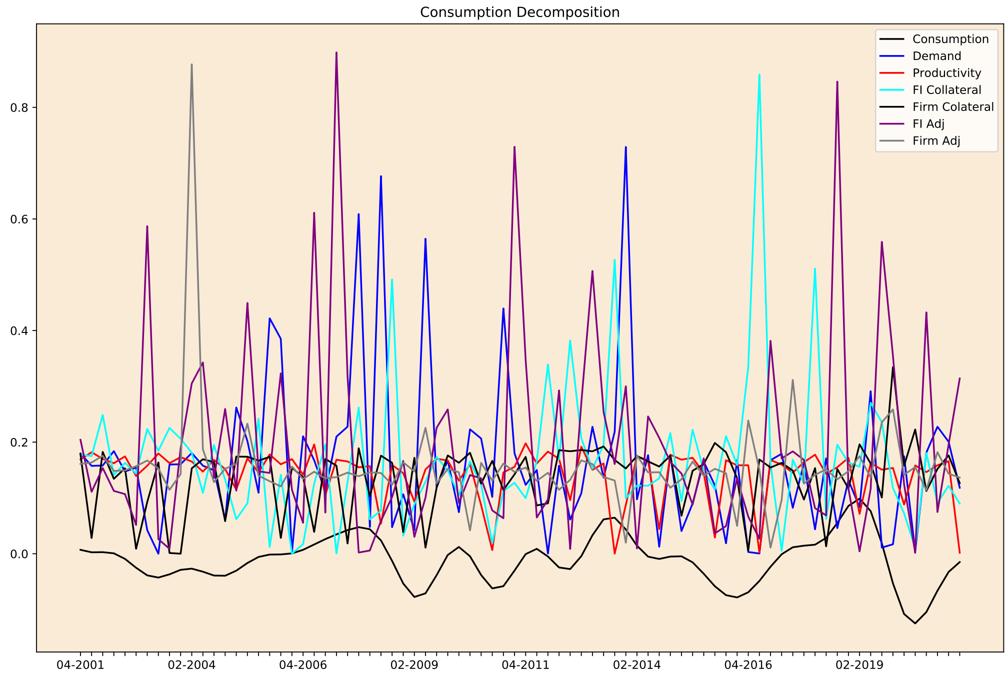

Figure 5: Shock decomposition for Output

With respect to Ouput Gap and Inflation, we see the model strongly attributes the dot-com and Great-recession to Demand, Bank collateral, and Financial Intermediary shocks. As confirmed by by Braun & Shoji(2007) along with Hirose(2010), much of Japan's output of the 1990's 'lost decade' era can be attributed to investment related adjustment costs. Within the context of the model, the exogenous shocks ζ_t^f & ζ_t^I represent both the implicit and explicit costs born by Financial Intermediaries and Firms buying or selling assets, respectively. When there is a financial panic, Intermediary firms find it harder for institutions such as banks or households to take on debt or equity positions given an increase in an overall flight to more liquid assets. Hence, a realization of ζ_t^f or ζ_t^I represent seller search frictions originated from jumps in investor asset liquidity preferences by households and banks alike. Likewise, we see the collateral constraint shock imposed by the household on the financial intermediary, $\theta_{1,t}$ played a key role in driving output shortfall in both '09 as well as in at the start of '17.

In both time periods, banks' net worth with respect their asset positions came into question by investors. Hence, given the higher uncertainty induced from such an environment, we see this contributed substantially in declined output gap. Lastly, we see for the dot-com, 2008', and covid-19 recessions, demand & productivity shocks also contributed substantially in driving these periods.

5.2 Impulse Responses

After observing the previous section of evidence, it is clear that demand & productivity shocks & credit frictions play a central role in driving business cycles in the Japanese economy. Futhermore, after observing plots of the ZLB probabilities and the policy rate, it is clear that the additional equilibrium is central in characterizing the evolution of the economy. Because other papers display differing economic outcomes at the zlb[31], I present impulse responses generated from the aforementioned regime switching model as well as plots directly evaluated when the zlb binds.

Large Scale Asset Purchase Shock b_t^{cb}:

[31] Ramey & Zubiary (2014), Bianchi et. al.(2017)

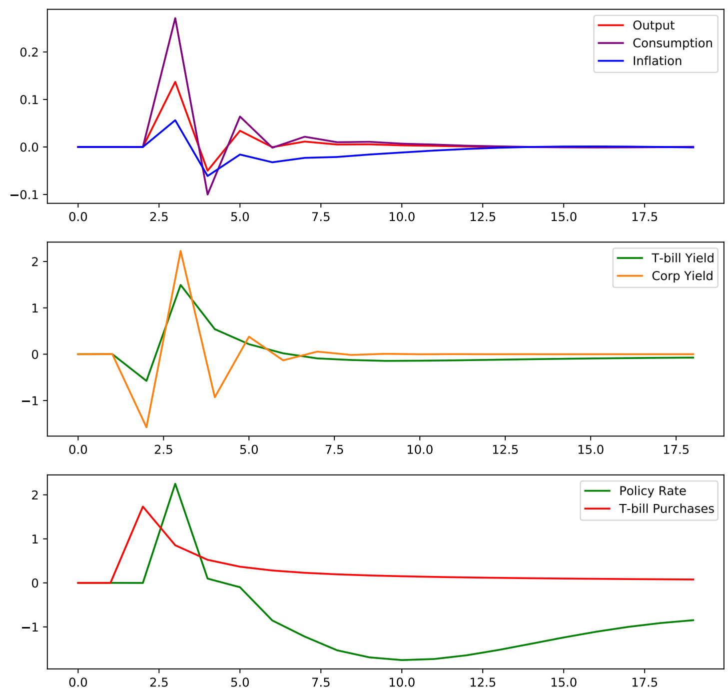

Figure 6: Treasury Bond LSAP

Figure 3 tells us that given a 150% increase in central bank purchases of Treasury bonds, Output gap & Inflation increase by .1% and .5% respectively, while Consumption increases by .2% on impact. While it may seem unusually low, we also see the policy rate responds quite high, thus tempering any additional stimulative effects agents may anticipate on key macro variables.

Large Scale Asset Purchase Shock ψ_t^{cb}:

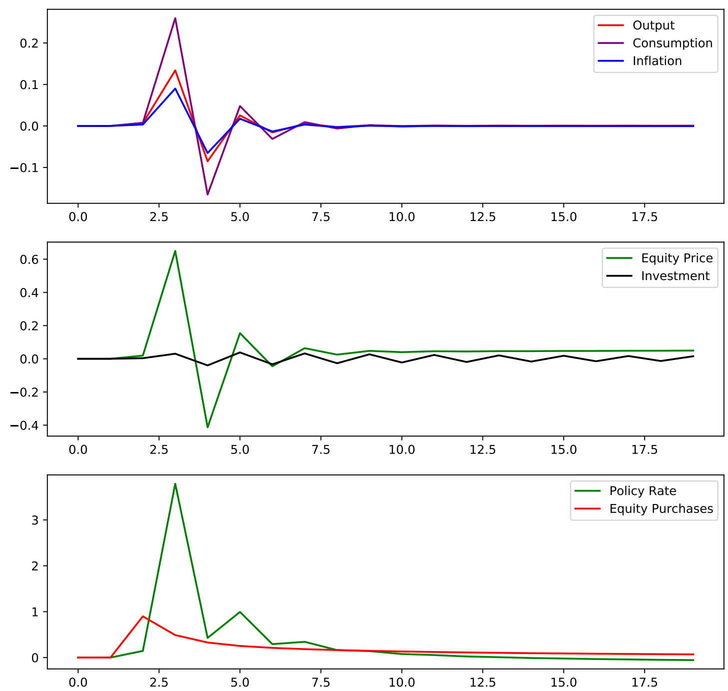

Figure 7: Equity LSAP

Figure 4 tells us that given a 300% increase in central bank purchases of Treasury bonds, Output gap & Inflation increase by .1 % while consumption increases by .2% on impact. In similar fashion to the Treasury LSAP, we see that the policy rate also increases quite aggressively thus attenuating any effects one might expect from the shock alone. Yet, as expected we see an increase in equity prices by .6% on impact followed by a decline by -.2%.

The volatility we see from both Figures 3 & 4, come from the regime switching process between the non-zlb and zlb equllibrium. Because the law of motions for some parameter draws can exhibit indeterminacy in the zlb-case, one would naturally expect such a result as a consequence. Below, I plot the zlb-equillibrium (with learning) implied IRFS:

ZLB Large Scale Asset Purchase Shock b_t^{cb}:

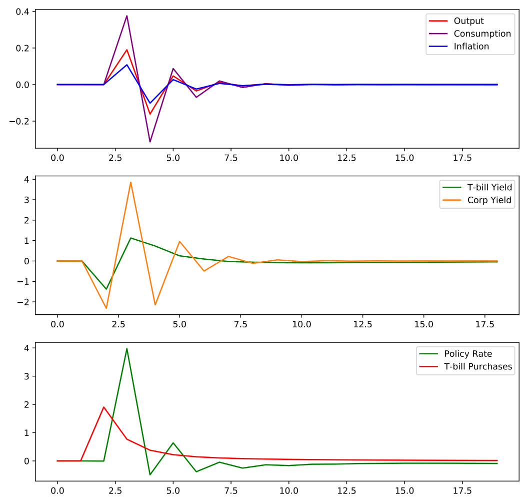

Figure 8: ZLB Treasury Bond LSAP

ZLB Large Scale Asset Purchase Shock ψ_t^{cb}:

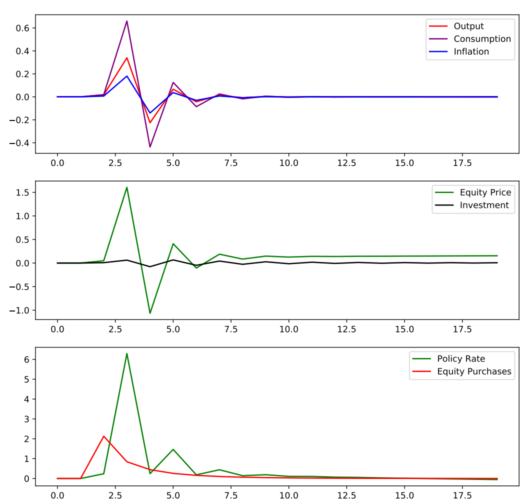

Figure 9: ZLB Equity LSAP

We see in figures 5 & 6 that zlb LSAPs provide roughly 100%-200% higher stimulus than the in the baseline model on the macro-variables of interest. And yet, we also see the same dynamic of the policy rate sharply increasing and nullifying potentially stronger effects of LSAPs. Another reason economically for the lack of strong response comes from both the low wage and price stickiness parameters θ_W & θ_p. Because the frequency of price adjustments are relatively quick, prices tend to adjust more frequently than the other developed countries. Thus, when firm & labor union response are less sluggish, consumption and investment decisions therefore take into account the "quick" price adjustments and hence are attenuated both on impact and in equilibrium.

Interest Rate Shock R_t^{re}:

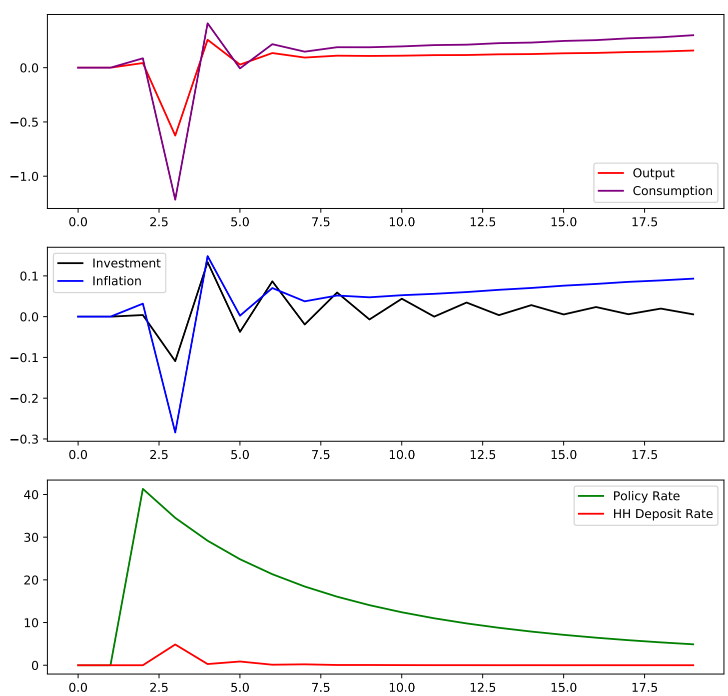

Figure 10: Interest Rate shock

Consistent with existing empirical evidence, we see the interest rate channel remains a fairly strong driver of macro variables. Figure 7, tell us that given a 40 basis point increase in the policy rate, Inflation declines by .2%, Consumption declines by .5% and output declines by 1%. We also see that following a positive MP shock, the household deposit rate increases by 10 basis points on impact but reverts back to equilibrium before the nominal interest rate does. This can be attributed in part to agents correctly anticipating the persistence of monetary policy and allocating assets away from deposits to stocks thus driving a downward pressure on demand for deposits.

Financial intermediary Collateral Shock $\theta_{1,t}$:

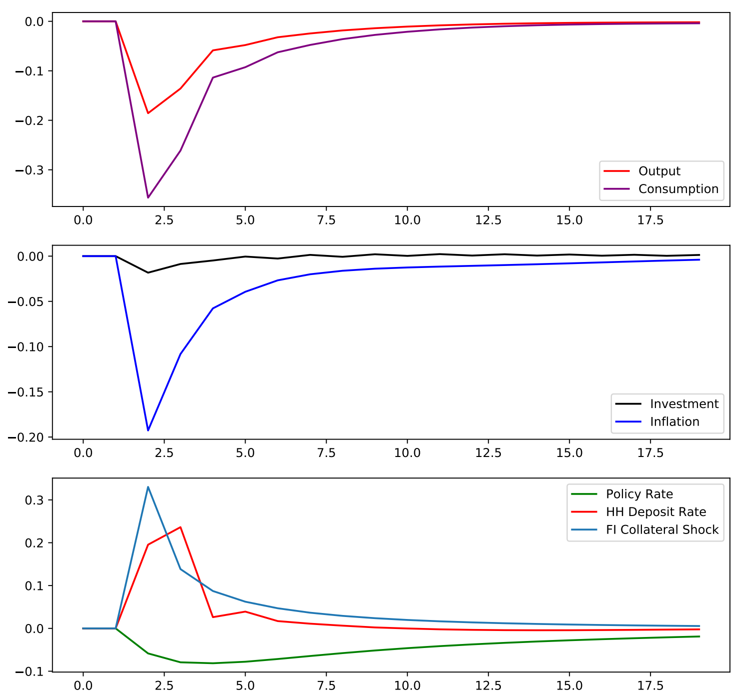

Figure 11: FI Collateral shock

In Figure 8 we see the equilibrium impacts from a 1 standard deviation shock to the financial intermediary collateral AR(1) process, $\theta_{1,t}$. On impact, we see it reduces all key macro-variables while raising the deposit rate. Inuitively, when the collateral shock is realized, households are less willing to supply deposits and thus all else equal, this drives up the price of borrowing for the Financial Intermediary. Hence, Intermediary firms are more capital constrained causing an erosion of investment and thus causing an overall contraction in the macro-economy.

Intermediary Firm Collateral Shock $\theta_{1,t}$:

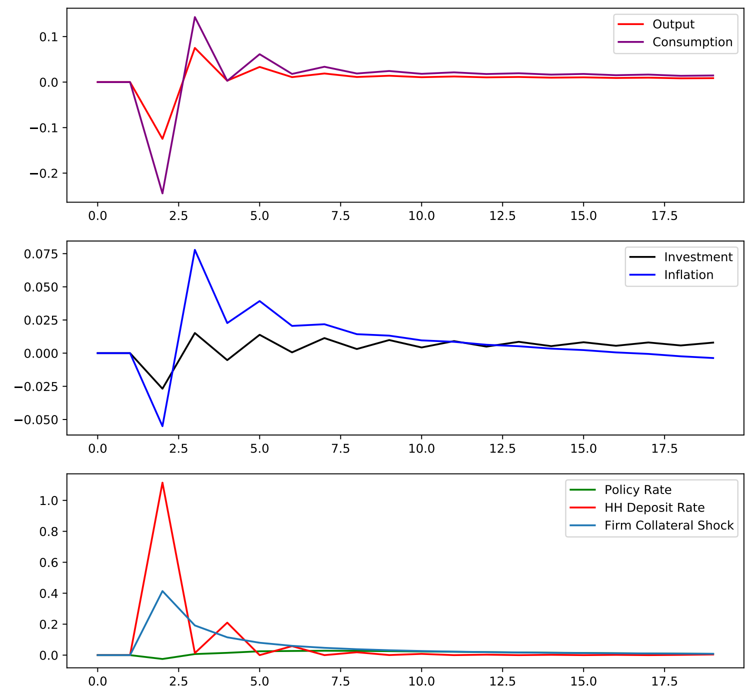

Figure 12: IF Collateral shock

In Figure 8 we see the equilibrium impacts from a 1 standard deviation shock to the Intermediary Firms' collateral AR(1) process, $\theta_{2,t}$. Similar to the previous shock, we see a strong contractionary episode on impact. Intuitively, when the Financial Intermediary is less willing to purchase corporate debt issued by the firm, firms naturally have less operating capital for production thus slowing down investment and economic activity. Yet, we see that shortly after, Consumption, Output, & Inflation all sharply rise and follow a relatively volatile trajectory. Furthermore, we also see that while the policy rate declines slightly to combat the contraction, the deposit rate increases. Economically, when credit conditions shrink-en, each unit of investible funds become more value-able and hence the opportunity cost in investing into bank deposits relative to equity markets from the household's perspective rises. Hence, we see in equilibrium, the deposit rate increases.

Financial Intermediary Adj Shock ϵ_t^f:

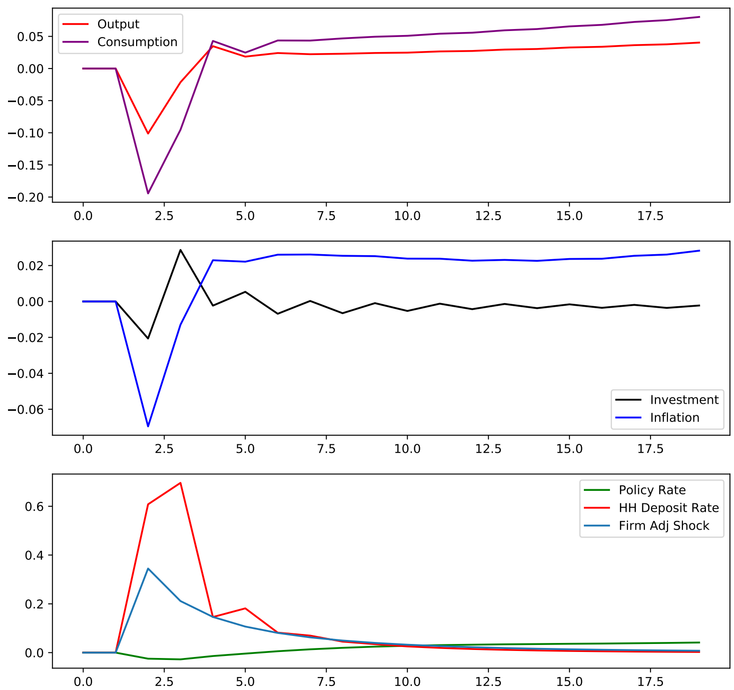

Figure 13: Financial Intedmeriary Adj. shock

We see in Figure 13, the effects of the Financial Intermediary's adjument cost. Like the collateral constraint, we also observe a contractionary impact, but with more of an output recovery and less of an inflation recovery. Intuitively, the friction represents the Financial intermediary's implicit and explicit transaction cost of T-bills and corporate debt. In a sense, the shock represents a period in which assets become relatively hard to liquidate when credit markets under go a "flight to safety".[32]

[32]Note: the Impulse response of the Intermediary firm's adjustment cost ϵ_{sh}^I looks the same in equilibrium.

5.3 Counterfactual Analysis

In light of the evidence presented, it is clear monetary policy's control over the Policy rate and LSAPs have the potential to stabilize or worsen economic welfare. Hence, in similar spirit to Del Negro et. al.(2015), I produce 2 counterfactual experiments using the smoothed estimates from the particle filter. Before describing them, I will first detail mathematically how I obtain estimates of each macro-variable given a different policy scenario.

State Transition & Observation Equation:

$$S_{t|t-1} = f(\theta, S_{t-1}, \epsilon_t | I_{t-1}) \;= f(\theta, S_{t-1|t-1}, \epsilon_{t|t-1}) \tag{5.1}$$

$$Y_{t|t-1} = M_1 S_{t|t-1} + M_2 S_{t-1|t-1} + \epsilon_t^m \tag{5.2}$$

Counterfactual State Transition & Observation Equation:

$$S^*_{t|t-1} = f(\theta, S^*_{t-1|t-1}, \epsilon^*_{t|t-1}) \tag{5.3}$$

$$Y^*_{t|t-1} = M_1 S^*_{t|t-1} + M_2 S^*_{t-1|t-1} + \epsilon_t^m \tag{5.4}$$

Counterfactual Difference:

$$D_t = Y^*_{t|t-1} - Y_{t|t-1} \tag{5.5}$$

Above, I have expressed the empirical model described in the beginning sections as the state transition equation (5.1) and measurement equation in (5.2). Equations (5.3) and (5.4) are the same but with the counterfactual shock ϵ_t^*, rather than the true shock ϵ_t implied from the observable data. For each counterfactual experiment, I generate the model implied macro variable forecast. Next I produce the counterfactual forecast conditioned on a policy counterfactual expressed in terms of a shock. For example, If I am interested in understanding the effects of contractionary monetary policy, I generate the model forecast via the particle filter. Then I generate the counterfactual forecast via the filter, but conditioned on draws of the ϵ_t^i which have a positive mean σ_i rather than 0 in the base case. Then, by taking the difference of the two forecasts, I obtain the counterfactual effect. For each experiment, on the left, I plot both true forecast and the counterfactual variables. While on the right, I compute the difference between the two, D_t.

Counterfactual 1: How would the Late 2000s crash look like had the BOJ not undertaken unexpected rate hikes?

As discussed in Mishkin & Ito(2004), during the start of the dot-com crash in Japan, banks and households seemed confident of a low rate interest rate environment in the near future. Hence to the surprise of agents in the economy, the BOJ pursued rate hikes to adress the pre-emptive onset of inflation. As a result, Japan's economic woes worsened in the form of low inflation that had taken years to re-stabilize. Hence when conducting the following experiment, I set $\epsilon^*_{i,t} \sim \mathcal{N}(0, \sigma_i) + \sigma_i$, for:$t = 03/2000$

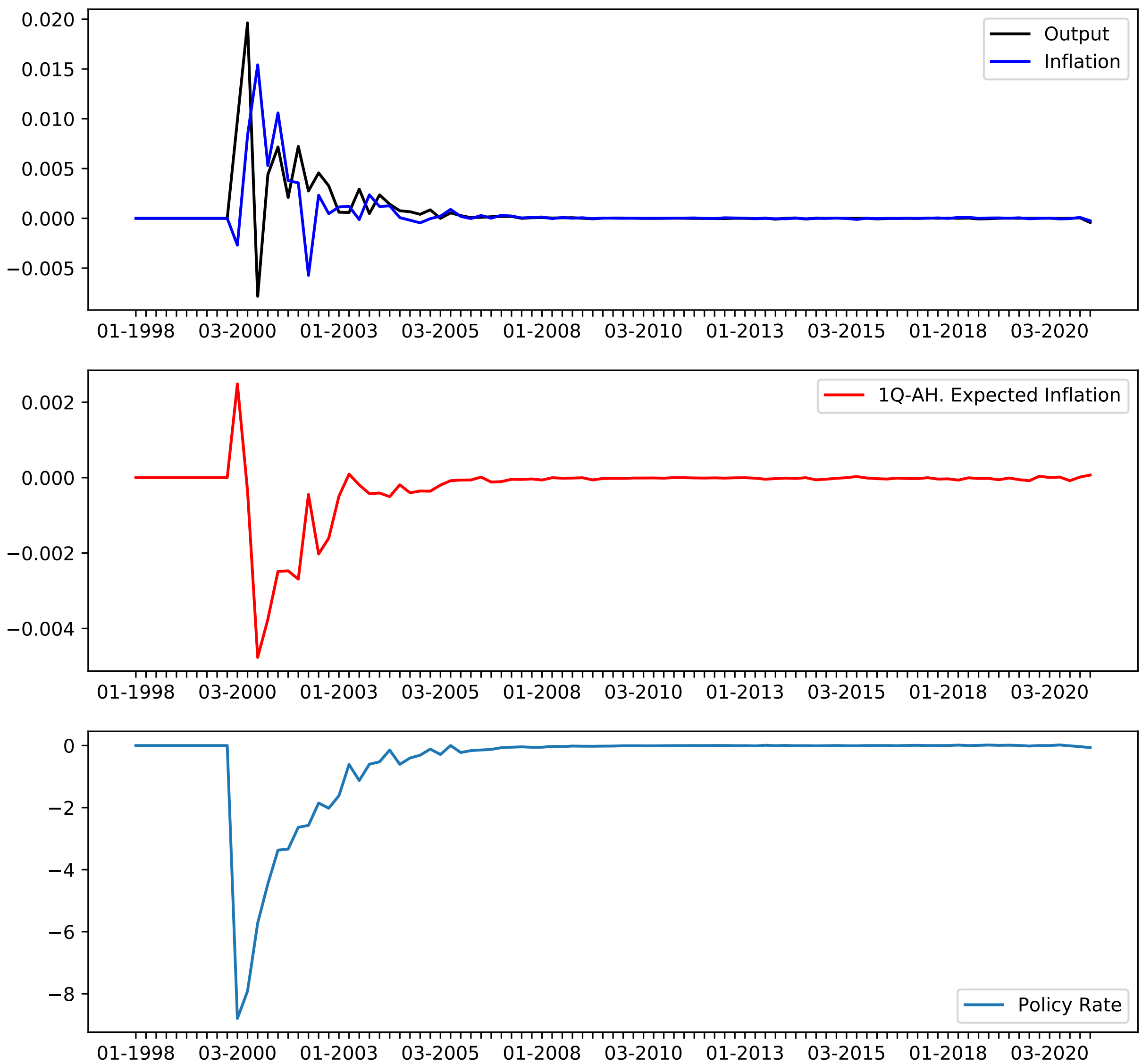

Figure 14: Effect of Avoiding Rate Hikes

Figure 14 reports the counterfactual macroeconomic outcome of avoiding rate hikes. We see for a that given a reduction of the counterfactual policy rate by 8 percent, Output and Inflation increase cumulatively by 6.5% & 5.0% ,respectively. Interestingly we see that Inflation expectations are largely unaffected by such a policy manuever, suggesting that rate hikes during this period may not have triggered a deflationary epsiode, but rather the inherent exogenous shocks of the economy were the main force in influencing future inflation expectations. Nonetheless, it is clear that had the BOJ responded in a less contractionary manner, Prices and Output growth would be better positioned as a result of eased credit conditions amidst the negative shocks to the economy.

<u>Counterfactual 2:</u> **How would the Housing crisis look like had the BOJ undertaken Treasury & Equity LSAPs together with rate drops in unison?**

If one observes the BOJ's balance sheet of Treasurys,Equitys along, and Policy rate. It can be argued that more swift measures could have been taken to combat the negative financial shocks exhibited in the banking and firm level credit markets. It was not until a few quarters after consumption and output growth fell that the BOJ underwent LSAPs. Hence, the counterfactual is defined as: I set:

$\epsilon^*_{i,t} \sim \mathcal{N}(0, \sigma_i) + \sigma_i$, for:$t = 03/2008$

$\epsilon^*_{b,t} \sim \mathcal{N}(0, \sigma_b) + 10\sigma_b$, for:$t = 03/2008$

$\epsilon^*_{psi,t} \sim \mathcal{N}(0, \sigma_{psi}) + 10\sigma_{psi}$, for:$t = 03/2008$

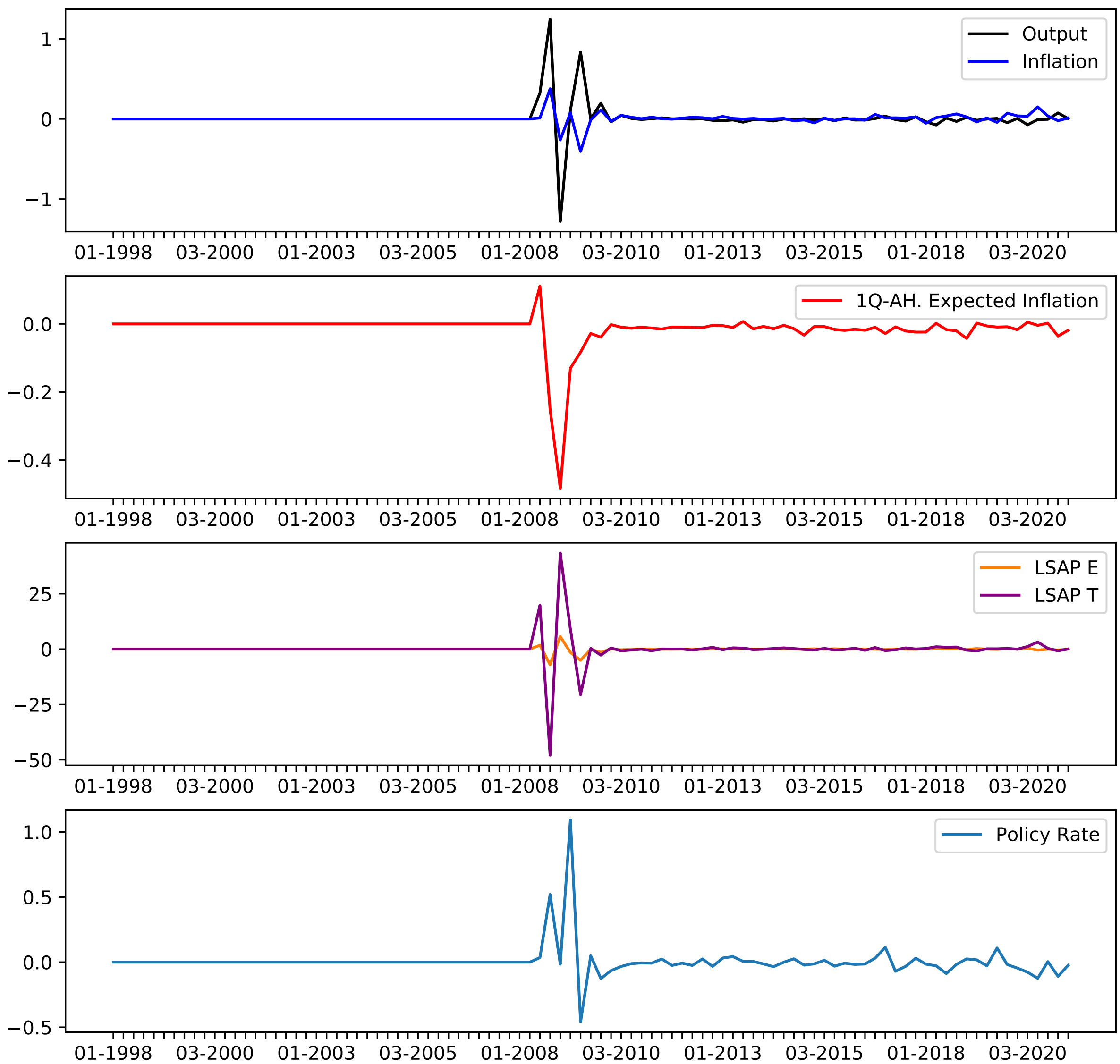

Figure 15: Simulateous MP Response in Housing Crash

In Figure 15, we observe the counterfactual effect of the BOJ's hypothetical 'agressive' policy measures. Output and Inflation cumulatively increase by 114% and 39%, respectively. While expected inflation drops by 149%. Although these offer puzzling and somewhat contradictory results. Similar to the Impulse responses, the Policy rate sharply increases shortly following the LSAP policy. Hence, reducing future inflation expectations while still enabling higher impact on Output and Inflation.

In summary, LSAPs add flexibility to monetary policy. However, it need not be considered a panacea for any and all economic sluggishness. This is clear from observing results in great recession counterfactual. One potential reasoning for the lack of strong results for this may come from monetary policy's tightening of R_t^{re}. It perhaps the case that the Output gap along with future Expected Inflation would appear stronger, had the fed announced its commitment to keep R_t^{re} fixed while conducting LSAPs. However, I leave such hypotheses for future work.

6 Conclusion

Using a the Ensemble Kalman Filter , I estimate a a medium Scale DSGE with regime switching behavior. Utilizing the Particle Filter procedure for post-estimation, I display the importance of credit frictions and traditional Demand & Productivity effects playing a strong role in reducing output, and inflation. Through impulse response and counterfactual analysis, I provide a cautionary tale for LSAP's. Though they carry weight in raising expected output, consumption, and inflation in a positive direction, they ought to coordinated with interest-rate policy. Otherwise contractionary monetary policy may nullify desired stabilization effects. Through impulse response analysis, I find strikingly differing equillibria of LSAPs. Thus implying the importance of policy to act switfly during the zlb equilibrium. The paper ultimately seeks to address the importance of merging behavioral models with filtering methods to isolate sources of business cycles and generate accurate policy conclusions.

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8 Plots

Data_Fit.pdf

9 Appendix A: Model Linearization

9.1 Households:

The Representative Household maximizes the following Objective Function:[33]

$$\mathcal{L} = \max E_t \sum_{k=0}^{\inf} \beta^k \mathcal{Q}_{t+k}$$

$$\begin{aligned}\mathcal{Q}_t = U_t - \lambda_t(c_t + b_t q_t + \sum_j b_t^k q_t^k + b_t^s \\ - \pi_t^{-1}\{(1+\rho q_t)b_{t-1} + \sum_j (1+\rho_j q_t^k(j))b_{t-1}^k(j) + b_{t-1}^s(1+R_{t-1}) + W_t(l)N_t(l) - T_t\})\end{aligned}$$

After evaluating (2.7) at steady state, I obtain:

$$1 + \bar{R} = \bar{\pi}\beta^{-1}$$

Linearizing (2.6) & (2.7) becomes:

$$-\delta c_t = E_t(-\delta c_{t+1} - \pi_{t+1} + i_t)$$

Linearizing (2.8) & (2.9) becomes:

$$\partial log(q_t b_t)^{-\nu} = \partial log(\lambda_t - \beta E_t \lambda_{t+1}\pi_{t+1}^{-1}(1 + q_{t+1}))$$

$$(q_t + b_t)(-\nu) = \frac{\partial\lambda_t - \beta\partial E_t \lambda_{t+1}\pi_{t+1}^{-1}(1+\rho q_{t+1}))}{\lambda q - \beta\lambda\bar{\pi}^{-1}(1+)}$$

$$(-\nu)(q_t + b_t) = d_1\{\delta c_t + q_t\} - d_2 E_t\{\delta c_{t+1} - \pi_{t+1} + \rho q_{t+1}\}$$

$$(\nu + c_1)q_t = -\nu b_t + d_1\delta c_t - d_2 E_t(-\delta c_{t+1} - \pi_{t+1} + \rho q_{t+1})$$

$$d_1 \equiv \frac{C^\delta q}{C^\delta q -^{-\delta} \bar{\pi}_{-1} q}$$

$$d_2 \equiv \frac{\beta C^\delta q}{C^\delta q -^{-\delta} \bar{\pi}_{-1} q}$$

[33]each l household has a specific wage. I drop the l for all other FOCs since they are the same for each household.

9.2 Firm Level Equations:

$$\frac{\partial \Phi_{t+k}}{\partial I_t} = \eta(\frac{I_{t+1}}{I_t} - \sigma)\frac{I_{t+1}}{I_t}I_t^{-1}$$

$$\partial log(\frac{\partial \Phi_t}{\partial I_t}) = \partial log(\frac{I_{t+1}}{I_t}I_t^{-1}) + \partial log(\eta\frac{I_{t+1}}{I_t} - \sigma)$$

$$\partial log(\frac{\partial \Phi_t}{\partial I_t}) = \partial log(\Delta I_{t+1}) = (\frac{2-\sigma}{1-\sigma})\Delta I_{t+1} - I_t$$

After using the partial derivative of Φ_t, I Linearize Equation (2.34) and get the following expression:[34]

$$\partial log E_t\{-R^k_{t+1} - \eta I_t(\frac{I_t}{I_{t-1}} - \sigma) - (\frac{\eta}{2})(\frac{I_t}{I_{t-1}}^2) + Q_t\zeta_t\} = \partial log E_t\{(\beta\frac{C_t}{C_{t+1}})^\delta(\eta(\frac{I_{t+1}}{I_t}) - \sigma)\frac{I_{t+1}}{I_t}^2\}$$

$$\bar{Z} \equiv \bar{R}^k + \beta(\eta I(1-\sigma) - (\frac{\eta}{2})(1-\sigma)^2) + \bar{Q}$$

$$\nu_1 E_t R^k_{t+1} + \nu_2 I_{t-1} + \nu_3 I_t - \nu_3 I_{t-1} + \nu_4(Q_t + \zeta_t) = \delta(E_t c_{t+1} - c_t) + (\frac{3\eta - 2\sigma}{\eta - \sigma})E_t I_{t+1}$$

$$\nu_1 \equiv \frac{\bar{R}^k}{\bar{Z}}$$

$$\nu_2 \equiv \frac{\eta I(1-\sigma)}{\bar{Z}}$$

$$\nu_3 \equiv \frac{\eta(1-\sigma)}{2\bar{Z}}$$

$$\nu_4 \equiv \frac{\bar{Q}}{\bar{Z}}$$

Linearizing (2.33) becomes:

$$\partial log(Q_t) = \frac{\partial(f'_t + (\beta(\frac{C_t}{C_{t+1}})^\delta(1-\sigma)E_t Q_{t+1}))}{\bar{Q}}$$

Where:

$$\partial log(f'_t) = a_t + (\alpha - 1)k_t + (1-\alpha)n_t$$

$$\bar{Q} = f' + (1-\sigma)\beta\bar{Q}$$

[34]Note: $R^k_{t+1} = \frac{1 + \rho^k q_{t+1}}{q_t}$

9.3 Wages:

Linearizing (2.51) becomes:

$$(1-\epsilon_w)w_t = \partial log\{(1-\theta_w)(w_t^*)^{1-\epsilon_w} + \theta_w(w_{t-1})^{1-\epsilon_w}\}$$

$$w_t = (1-\theta_w)w_t^* + \theta_w w_{t-1}$$

Linearizing (2.50) becomes:

$$(w_t^*)(1+\alpha_0-\epsilon_w) = \partial log\{\sum_k(\beta\theta_w)^k(\frac{\epsilon_w}{\epsilon_w-1})\frac{N_{t+k}^{1+\psi}W_{t+k}^{\alpha_0}}{W_{t+k}^{\epsilon_w}N_{t+k}\lambda_{t+k}}\}$$

$$(w_t^*)(1+\alpha_0-\epsilon_w) = \partial log\{\sum_k(\beta\theta_w)^k(\frac{\epsilon_w}{\epsilon_w-1})\frac{N_{t+k}^{1+\psi}W_{t+k}^{\alpha_0}}{W_{t+k}^{\epsilon_w}N_{t+k}\lambda_{t+k}}\}$$

$$\partial log(\sum(\theta_w\beta)^k O_{t+k}^1) = \frac{\partial\sum(\theta_w\beta)^k O_{t+k}^1}{O^1(1-\theta\beta)^{-1}}$$

$$\partial log(\sum(\theta_w\beta)^k O_{t+k}^2) = \frac{\partial\sum(\theta_w\beta)^k O_{t+k}^2}{O^2(1-\theta\beta)^{-1}}$$

$$w_t^* = (1-\epsilon_w\psi)^{-1}(1-\theta_w\beta)^{-1}\sum_k(\theta_w\beta)^k\{O_{t+k}^1 - O_{t+k}^2\}$$

After some re-arranging, I obtain (2.54) where:

$$\alpha_1 \equiv ((1-\theta_w\beta)(1-\epsilon_w\psi))^{-1}$$

9.4 Phillips Curve

$$max_{p_t^*}\ E_t[\sum_{k=0}^{\inf}(\theta\beta)^k Q_{t+k}A_{t+k}]$$

$$A_{t+k} \equiv (\frac{p_t^*}{P_{t+k}})^{-\epsilon}Y_{t+k} - (\frac{p_t^*}{P_{t+k}})^{-\epsilon}(\frac{\Phi_{t+k}}{P_{t+k}})Y_{t+k}$$

$$\frac{\partial}{\partial p_t^*} := E_t[\sum_{k=0}^{\inf}(\beta\theta)^k\{C_{t+k}^{-\sigma}B_{t+k}\}] = 0$$

$$B_{t+k} \equiv (1-\epsilon)(p_t^*)^{-\epsilon}p_{t+k}^{\epsilon-1}Y_{t+k} + \epsilon(p_t^*)^{-\epsilon-1}p_{t+k}^{\epsilon}\Phi_{t+k}Y_{t+k}$$

$$\sum_{k=0}^{\inf}(\beta\theta)\bar{C}^{-\sigma}\bar{B} = 0$$

$$(1-\beta\theta)^{-1}\bar{C}^{-\sigma}\bar{B} = 0$$

Hence in Steady State:

$$\bar{B} = 0$$

$$\bar{B}_1 = -\bar{B}_2$$

Where:

$$\bar{B}_1 \equiv (1-\epsilon)pY$$

$$\bar{B}_2 \equiv -\epsilon p\Phi Y$$

$$\partial log(\sum(\beta\theta)^k C_{t+k}^{-\sigma} B_{t+k}) = 0$$

$$\frac{\partial(\sum(\beta\theta)^k C_{t+k}^{-\sigma} B_{t+k})}{(1-\beta\theta)^{-1}C^{-\sigma}\bar{B}} = 0$$

$$\sum(\beta\theta)^k \partial log(C_{t+k}^{-\sigma})\partial log(B_{t+k}) = 0$$

$$\partial log(B_{t+k}) = \partial log(B_{1,t+k} + B_{2,t+k})$$

$$\partial log(B_{t+k}) = \frac{\partial(\bar{B}_1 B_{1,t+k} - \bar{B}_2 B_{2,t+k})}{\bar{B}}$$

$$\partial log(B_{t+k}) = \frac{B_1 \partial(B_{1,t+k} - B_{2,t+k})}{B_1 + B_2}$$

$$\partial log(B_{t+k}) = \alpha_\pi(\partial log(B_{1,t+k}) - \partial log(B_{2,t+k}))$$

$$\alpha_\pi \equiv \frac{B_1}{B_1+B_2} = \frac{1}{1+\dfrac{B_2}{B_1}} = \frac{1}{1+(\frac{\epsilon}{\epsilon-1})\bar{\phi}}$$

Using the following linearization, I now obtain the following result:

$$\sum_{k=0}^{\inf}(\beta\theta)^k p_t^* =_t \sum_{k=0}^{\inf}(\beta\theta)^k(p_{t+k} + \alpha_\pi^{-1}c_{t+k+1} + \phi_{t+k})$$

$$p_t^* = (1-\beta\theta)(p_t + \alpha_\pi^{-1}E_t c_{t+1} + \phi_t) + (1-\beta\theta)\sum_{k=1}^{\inf}(\beta\theta)^k(p_{t+k+1} + \alpha_\pi^{-1}\sigma c_{t+k+2} + \phi_{t+1})$$

$$p_t^* = (1-\beta\theta)(p_t + \delta\alpha_\pi^{-1}E_t c_{t+1} + \phi_t) + \beta\theta E_t p_{t+1}^*$$

Aggregate Price Identity:

$$P_t^{1-\epsilon} = [(1-\theta)(p_t^*)^{1-\epsilon} + \theta P_{t-1}^{1-\epsilon}]$$

Log-Linearized, this becomes:

$$\frac{\pi_t}{1-\theta} = p_t^* - p_t$$

$$\pi_t = (1-\theta)(1-\beta\theta)(\phi_t + \alpha_\pi^{-1}\delta E_t c_{t+1} + \beta\theta E_t \pi_{t+1})$$

9.5 Anticipated Utility of Consumption:

Linearizing (2.2) yields:

$$c_t = -\eta_1(q_t+b_t)-\eta_2(b_t^k+q_t^k)-\eta_3 b_t^s-\eta_4(\rho q_t+b_{t-1})+\eta_5(\rho_k q_t^k+b_{t-1}^k)+\eta_6(b_{t-1}+i_{t-1})+\eta_7\{w_t+n_t\}-\eta_8\tau_t-\eta_9\pi_t$$

$$\eta_1 = \frac{qb}{c}$$

$$\eta_2 = \frac{q^k b^k}{c}$$

$$\eta_3 = \frac{(1+R^s)b_s}{\bar{\pi}C}$$

$$\eta_4 = \frac{(1+\rho_k q^k)b^k}{C\bar{\pi}}$$

$$\eta_5 = \frac{(1+\rho q)b}{C\bar{\pi}}$$

$$\eta_6 = \frac{(1+R^s)b_s}{C\bar{\pi}}$$

$$\eta_7 = \frac{\bar{W}\bar{N}}{C\bar{\pi}}$$

$$\eta_8 = \frac{\tau}{C\bar{\pi}}$$

$$\eta_9 = \eta_4 + \eta_5 + \eta_6$$

Using the the expression $1 + R^s = \bar{\pi}\beta^{-1}$, I obtain:

$$b_{t-1}^s = (\eta_6)^{-1}c_t + \tilde{\eta}_1(q_t + b_t) + \tilde{\eta}_2(q_t^k + b_t^k) - \tilde{\eta}_4(\rho q_t + b_{t-1}) + \beta b_t$$
$$-\tilde{\eta}_5(\rho_k q_t^k + b_{t-1}^k) + \tilde{\eta}_6(b_{t-1}^s + i_{t-1}) + \tilde{\eta}_7(w_t + N_t) - \tilde{\eta}_8\tau_t - \tilde{\eta}_9\pi_t$$

Iterating this equation forward, I obtain:

$$b_{t-1}^s = E_t\sum_k \beta^k\{\eta_6^{-1}c_{t+k} + \tilde{\eta}_1(q_{t+k} + b_{t+k}) + \tilde{\eta}_2(q_{t+k}^k + b_{t+k}^k) - \tilde{\eta}_4(\rho q_{t+k} + b_{t+k-1})$$
$$- \tilde{\eta}_5(\rho_k q_{t+k}^k + b_{t+k-1}^k) + \tilde{\eta}_6(b_{t+k-1}^s + i_{t+k-1}) + \tilde{\eta}_7(w_{t+k} + N_{t+k}) - \tilde{\eta}_8\tau_{t+k} - \tilde{\eta}_9\pi_{t+k}\}$$

After plugging (2.6) into this result. I obtain (2.61-2.62)

9.6 Macaulay duration:

In order to properly evaluate the consol bond model to the observed expectations & real bond yield data, I follow a similar metholodology to Matveev(2016) to compute the Mcauley duration(evaluated at the steady state) as a function of the ρ_k & ρ I seek to use for estimation.

$$D_t^k = \sum_j (\frac{\beta\rho_k}{R})^j \frac{q_{t+j}^k}{q_t^k}$$

$$D_t^k = (\frac{\beta\rho_k}{R})^{-1} \sum_j (\frac{\beta\rho_k}{R})^{j+1} \frac{q_{t+j}^k}{q_t^k}$$

$$D_t^k = (\frac{\beta\rho_k}{R})^{-1} \frac{\partial}{\partial x} \sum_{j=0} (\frac{\beta\rho_k}{R}^j))$$

10 Appendix B: Stochastic Gradient Learning:

As described in Evans(2010). I use the Generalized Stochastic Gradient algorithm 1. This means I must derive a value for the inverse of the co-variance of regressors. I start with the perceived law of motion:

$$Z_t = M_m^t Z_{t-1} + M_u^t U_t$$

$$U_t = RU_{t-1} + \epsilon_t^u$$

Hence, the perceived law of motion can be characterized as:

$$H_t \equiv \begin{bmatrix} Z_t \\ U_t \end{bmatrix}$$

$$H_t = K_1 H_{t-1} + K_2 \epsilon_t^u + \eta_t^h$$

$$var(H^*) = K_1 var(H^*) K_1' + K_2 \epsilon_t^u K_2' + c_s^2 I$$

$$var(H^*) = (N_1 K_2)\Omega(K_2 N_1)' + c_s^2 (N_1 K_2)(K_2 N_1)'$$

Note in the above, I define $var(H^*)$ as the variance-covariance matrix of the regressor, H_t. The above perceived law of motion contains the term η_t^h that accounts for the uncertainty the agent perceives in her regression specification. I set the standard deviation of the forecast error, $c_s = 1e3$. I choose this number because this is the lowest value such that I can use a constant gain of .02 without yielding an explosive equillibria. Though this may be considered an ad-hoc choice, for empirical purposes, we are interested in an R matrix centered around the model equations such that they behave stable with a well supported constant gain learning parameter within the neighborhood of 0.02.

Because $Eh_t'h_t$ is equivalent to $var(h)$, I am able to define both R and R_z that go into the stochastic gradient algorithm as $R = var(H^*)^{-1}$ and $R_z = var(H_z^*)^{-1}$. Where:

$$var(H_z^*) = N_1^z K_2^z \Omega (K_2^z N_1^z)' + c_s^2 (N_1^z K_2^z)(K_2^z N_1^z)'$$

Note the superscript z denotes the entries of the matrix that come from the solution to the rational expectations equilibrium when $i_t = 0$. Hence, the parameter update follows the following process as described in Evans(2010):

$$\phi_t = \phi_t + gRh_{t-1}(y_{t-1} - \phi_{t-1}' h_{t-1})'$$

Because the agent has two equillbria to consider when forming expectations, I modify the learning rule in the following manner:

$$\tilde{\phi}_{t-1} = \mu_{t-1}\phi_{t-1}^z + (1 - \mu_{t-1})\phi_{t-1}$$

$$\phi_t = \phi_t + gRh_{t-1}(y_{t-1} - \tilde{\phi}_{t-1}' h_{t-1})'$$

$$\phi_t^z = \phi_{t-1}^z + gR_z h_{t-1}(y_{t-1} - \tilde{\phi}_{t-1}' h_{t-1})' \text{ if } t = 0$$

Note, in the model, I assume the agent updates ϕ_t^z only when $i_t = 0$. And I also assume ϕ_t updates at all times.