Japanese Business Cycles: Perceptions & Frictions *

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Abstract

From the late 1990s to the present, Japan has suffered prolonged periods of economic stagnation and recurrent bouts at the Zero Lower Bound. Both inflation and output growth have remained on average 2% percent lower than most developed nations. To address this, the Bank of Japan has pursued interest rate reductions along with an expansion of Treasury Bonds and domestic ETF purchases on its balance sheet. Such policies manipulate perceptions of future wealth and thereby drastically alter consumption and investment decisions. Hence, understanding how household expectations and the banking sector affect the overall macro-economy is crucial. Japan provides an ideal laboratory to study the quantitative impact of financial frictions vis a vis private-sector misperceptions and learning. To that end, I estimate a nonlinear medium-scale DSGE using Japanese macro-finance data via the Ensemble Kalman Filter. I find central bank purchases of equities lead to a peak in output growth and inflation of 6.6% and 2.8 %. While central bank purchases of Treasuries lead to peak output and inflation by 6% and 2.9%, respectively. When the household deposit rate is at the ZLB, I find a 30% decrease in the peak output growth and inflation for both Treasury and equity LSAPs compared to the baseline non-ZLB case. Using the Particle Filter, I find counterfactual evidence that had the BOJ coordinated interest rate and LSAP policies together during the Global Financial Crisis, this would have induced a 1.25% higher cumulative output growth and 6.58% higher inflation. I also find that had the BOJ initiated equity LSAPs during the Dot-Com bubble in the early 2000s, this would have induced a 1.5% higher cumulative output growth and 4.5% higher inflation.

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1 Introduction

Since the start of the Global Financial Crisis, developed economies have grappled with low economic growth and inflation. In response, policymakers have adopted low-interest rate policies and Large-Scale-Asset-Purchases(LSAPs) to combat economic sluggishness. In particular, Japan faced these same challenges starting in the early 1990s. In response, the Bank of Japan(BOJ) deployed aggressive fiscal and unconventional monetary policies. From 2001-2006, the BOJ acquired 18 trillion yen worth of Japanese government bonds & held the policy rate near zero.[1] Then in 2010, the BOJ acquired both Japanese government bonds and stocks(J-REITs & ETFs) via LSAPs. During this period, the BOJ at one point held up to 50% of the Japanese ETF market and Treasury market, ratios unseen in other developed nations. Japan's aggressive LSAP policies and long history of a 0% floor on the deposit rate(ZLB), offer an instructive case study for policymakers in other developed nations to examine their own LSAP and interest rate toolkit.

In a standard model where agents have perfect information(rational expectations), LSAPs have little effect on the macro-economy. As a result, economists have departed from traditional models by employing financial frictions with limited participation. As outlined in Kiyotaki & Gertler (2017), Gertler & Karadi (2011), and Bernanke et. al. (1998), financial frictions capture the investment formation process through banks that propagate disruptions to the broader economy. Enabling this feature magnifies LSAP effects through investment demand and firm-level issuance decisions. In Japan, Fukunaga (2010) and Hoshino et. al. (2021) confirm this and find evidence of financing frictions driving output growth and inflation.

However, there is still no consensus on the causal outcome of such policies. Wang (2021) and Basu & Wada (2018) employ pricing and financial frictions to find a positive LSAP shock that generates a 2% and 0.4% peak increase in output growth(respectively), both estimates being starkly inconsistent. One source for the difference can from the nature of financial frictions. Wang (2021) models frictions in terms of financial adjustment costs, whereas Basu & Wada (2018) include a banking sector to account for lending frictions. In this paper, I include both types of financial frictions along with pricing frictions to answer the following novel questions: First, to what degree do equity LSAPs, Treasury LSAPs, and interest rate policy impact key macro variables? Second, how might the economy have been better positioned if the BOJ pursued alternative policies during the Dot-Com and Global Financial Crisis? To resolve the inconsistencies in the literature, I relax the rational expectations assumption and incorporate the ZLB on the household deposit rate.

In their analysis of the U.S., Eusepi and Preston (2018) assume households, that finance government liabilities, hold an imperfect view of the effects of fiscal policy, and learn this over time via recursive updat-

[1] See Iwata & Takenaka (2011) for a brief account of global monetary policy events

ing. As a consequence, households no longer know the true nature of the government's long-run solvency condition which equates the current level of debt to the present discounted value of future primary surpluses. This misperception of government policy and solvency causes households to erroneously treat increases in government debt issuance as increases in future wealth, thereby amplifying policy.

Like Eusepi & Preston (2018), I allow households to misperceive the government's budget constraint and the true policy effects. As a result, LSAPs will alter households' anticipated flows of wealth and therefore disproportionately change consumption compared to a rational expectations model. Next, I apply the adaptive learning framework of Evans and Honkapohja (2001) and initialize agents with beliefs near the rational expectations equilibrium (REE). This feature enables agents' beliefs to start near the true parameters of the economy as they recursively update over time to new information. Consequently, the adaptive learning process generates waves of pessimism or optimism in the beliefs of the economy.

Next, I incorporate a Zero Lower Bound equilibrium on the household's deposit rate. Benhabib, Schmitt-Grohé, and Uribe(1998) describe how the ZLB generates negative perceptions of policy, which then gives rise to a deflationary equilibrium. Evans, Honkapohja, and Guse (2007) describe an economy where households can drive an economy to the ZLB through pessimistic beliefs formed from adaptive learning. In Japan, Miyamoto (2018) and Iiboshi (2020) provide evidence of noticeably higher impacts on output and inflation in the ZLB equilibrium, along with the presence of time-varying policy effects.[2]. Thus, if one wishes to properly quantify monetary policy, it is crucial to explore the equilibrium outcomes of both the ZLB and non-ZLB of the household deposit rate.

Appending behavioral expectations to frictions and the ZLB is imperative in identifying monetary policy. Intuitively, pricing and financial frictions enable investment and labor demand to depend on the future beliefs of the economy. Attaching a behavioral framework allows uncertainty of the future macro-economy to drive lending and pricing decisions. Because LSAPs manipulate the future path of asset prices, in a world with perfect information, this path of evolution is perfectly known. However, when households misperceive the government's budget constraint and policy effects, household expected income flows disproportionately change, resulting in self-fulling outcomes which influence the macro-economy.

Using Japanese financial and macroeconomic data, I estimate the parameters of the model via Bayesian Monte Carlo Simulation. Because the model contains nonlinear features, one cannot rely on a traditional linear Kalman Filter. Hence, I use the Ensemble Kalman Filter which asymptotically approaches the true likelihood for the parameter estimation. For the post-estimation results, I use the Particle Filter via re-sampling outlined in Villaverde & Rubio-Ramirez (2007). The model produces three conclusions. The first result finds an increase in central bank balances of 222% in equities leads to peak output and inflation by

[2]Ramey & Zubiary (2018) find similar results in the U.S.

6.6% and 2.8%. While central bank purchases of 227% in Treasuries lead to peak output and inflation by 6% and 2.9%, respectively. When the household deposit rate is at the ZLB, I find a 30% decrease in output growth and inflation for both Treasury and equity LSAPs compared to the baseline non-ZLB case. The second result finds that 23.6% of the variance of output is attributed to LSAPs. While 22.4% of the variance of inflation is attributed to LSAPs. The third result finds if the BOJ coordinated interest rate and LSAP policies together during the Global Financial Crisis via a positive standard deviation shock to LSAPs and negative standard deviation shock to interest on bank reserves, this would have induced a 1.25% higher cumulative output growth and 6.58% higher inflation than the observed historical outcome. Furthermore, if the BOJ conducted equity LSAPs During the Dot-Com crash in the early 2000s in the form of a standard deviation shock, this would have induced a 1.5% higher cumulative output growth and 4.5% higher inflation than the observed historical outcome.

The paper is outlined as follows: Section 2 will cover the surrounding literature. Section 3 will discuss the model environment. Section 4 will cover the empirical methodology and corresponding statistical results. Section 5 with go over the model's implied results. Lastly, Section 6 will discuss the Counterfactual analysis.

2 Related Literature:

This paper fits into a larger class of literature surrounding equilibrium models and more general structural approaches in explaining macroeconomic dynamics. Within the class of General equilibrium models, Gertler et. al (2015), Kiyotaki & Moore (1997), as well as Bernanke et. al. (1999) highlight the importance of financial sector credit conditions to the macro-economy. These papers underscore the importance of lending contracts on the part of financial intermediaries which result from lenders having imperfect information about the creditworthiness of their borrowers and the propagation of shocks on investment and consumption resulting from declining credit conditions. This paper most closely follows Sims and Wu (2022), who provide a framework to evaluate LSAPs after taking into account the banking sector. We extend their exercise by relaxing the rational expectations assumption, adding a ZLB regime, and accounting for the differential effects of equity versus Treasury LSAPs. Chen et. al. (2011) models U.S. LSAPs similarly, but with segmented financial markets and finds on average a .37% and .5% increase to output growth from an LSAP shock and interest rate shock, respectively. Benhabib, Schmitt-Grohe, and Uribe (2001) make the case for taking the zero lower bound into account finding an endogenous mechanism for entering a self-fulfilling deflationary equilibrium. Indeed, Hirose et. al (2014) empirically confirms the importance of modeling the ZLB in a nonlinear DSGE for the Japanese Economy. On the behavioral New-Keynesian side, the adaptive learning and infinite horizon literature grow with increasing attention. Branch & Evans (2016) use a New-Keynesian model where agents recursively update expectations and highlight the likely outcome of deflation arising from self-confirming beliefs. Mcclung (2020) similarly models expectation formation in a ZLB regime-switching environment to better capture economic dynamics resulting from monetary policy. Benhabib, Evans, and Honkapohja (2014) and Eusepi and Preston (2010) highlight the importance of agents who form expectations based on an infinite discounted horizon of variables. As a consequence, they also confirm the presence of deflationary episodes in the economy from self-fulfilling expectations formations. Du, Eusepi, and Preston (2021) incorporate an infinite horizon model to evaluate exchange rate dynamics and find learning dynamics help explain financial co-movements and volatility. Gaus and Gibbs (2018) along with Eusepi & Preston (2018) confirm that employing an infinite horizon model with adaptive learning better explains U.S. macroeconomic data than alternative models with learning or without.

With respect to more general Structural Vector Autoregressive(SVAR) models, there is a large body of work that explores the effects of monetary policy on asset prices as well as the overall economy. Swanson (2018) uses high-frequency changes in asset prices to identify the effect of LSAPs and interest rate changes on long-term yields and equity prices in the United States. Kim, Laubach, & Wei (2020) follow a similar approach and conduct a counterfactual analysis to find without LSAPs between late 2012-2014, U.S. CPI inflation would have been 1% lower and the unemployment rate 4% higher. Bauer & Swanson (2022) use high-frequency changes in asset prices as instruments and find for a 25bp increase in the Policy rate, U.S. industrial production decreases by 0.4%. Plagborg-Møller and Wolf (2021) use local projection Instrument variable methods and find similar results of monetary policy impacts. Miranda-Agrippino and Ricco (2022) append a traditional local projection SVAR model with imperfect information and find different results for the U.S. economy.

This paper builds on the existing methodologies by incorporating key theoretical ingredients together into a nonlinear process and quantitatively evaluates LSAP policy outcomes. Motivated by the SVAR literature, this paper equips historical Japanese macro-finance data with existing economic theory. Using Bayesian Monte Carlo estimation to determine the parameters of the model, the paper conducts impulse response along with historical variance decomposition analysis. Furthermore, because the data set includes household and firm-level expectations, the learning parameters are thus well anchored to estimate the counterfactual effects of more aggressive monetary policy actions.

3 Model Environment

The model has the following agents in the economy: households, intermediary and final-goods firms, financial intermediaries, a fiscal authority, a labor union, and a central bank. A continuum $[0, L]$ of households consume based on an infinite horizon learning framework[3], where they take into account expectations of their discounted future wealth infinite periods ahead. They earn labor income, and investment income via return on the deposits made to the financial intermediary and from dividend income as well as stock price appreciation. In each period, a continuum of monopolistically competitive $[0, J]$ intermediary firms produce goods by employing labor and raising capital. I assume they share a degree ϵ of market power. Furthermore, I assume a representative final-goods firm aggregates the intermediary firm goods via CES preferences & a competitive market. I assume that a labor union aggregates household wages via CES preferences and each l household has a degree ϵ_w of market power on its wages. In order to introduce wage and price rigidity, I assume both households and intermediary firms, when determining wages and prices, are subject to probabilities θ_w, θ_p, respectively, of being unable to re-set wages or prices in the next period.

Intermediary firms increase their capital stock from external financing via equity obtained from households or corporate debt from financial intermediaries. Because the financial intermediary is not privy to the profitability of the intermediary firms, it requires a level of collateral to hold in the event of the intermediary firm's bankruptcy. Hence, intermediary firms determine labor demand as well as debt & equity issuance by maximizing expected future discounted profits subject to an exogenous investment friction, productivity process, and collateral constraint. Financial intermediaries borrow from households subject to a zero lower bound constraint. Because corporate and Treasury-issued debt are illiquid assets in comparison to equity holdings bought and sold on the stock exchange, the household relies on the financial intermediary to invest in such assets on its behalf. Hence, the financial intermediary makes investments in both firm & government debt as well as reserves redeemed by the central bank. Because the financial intermediary at any period can liquidate its assets and renege on its obligations to the household, there exists a moral hazard problem. To ensure the financial intermediary is unwilling to liquidate its assets, the household imposes a fraction of the financial intermediary's net worth that cannot fall below a fraction of the total deposits issued. The fiscal authority levies a lump sum tax on households and issues debt that the financial intermediaries acquire in their investment portfolios. Lastly, the Central bank follows a lagged taylor rule and issues reserves to the financial intermediary.

3.1 Representative household:

The representative household has additive preferences and derives utility from consumption and leisure.

$$U_t(l) = \frac{C_t(l)^{1-\gamma}}{1-\gamma} - \frac{H_t(l)^{1+\nu}}{1+\nu} \tag{3.1}$$

$$c_t(l) + s_t(l) + q_t^{\psi}\psi_t(l) = w_t(l)H_t(l) + \pi_t^{-1}(1 + R_{t-1}^s)s_{t-1}(l) + \pi_t^{-1}(d_t + q_t^{\psi})\psi_{t-1}(l) - \tau_t \tag{3.2}$$

[3] Cogley & Sargent (2005)

Equation (3.1) is the household's utility function. Equation (3.2) is the household's budget constraint in which consumption net of investment in equities & deposits must equal the prior period's investment return plus labor income net of lump sump tax τ_t levied by the fiscal authority. Note all lowercase variables denote the nominal value divided by the overall price level (for example $c_t \equiv \frac{C_t}{P_t}$). Here, c_t, s_t, H_t, and ψ_t are the consumption, deposits, labor hours, and equity holdings the household decides each period. w_t, d_t, q_t^{ψ}, R_t^s are wages, dividend yield, equity price, and the household deposit rate.

Households maximize the following:

$$\max \hat{E}_t \sum_{k=0}^{\infty} \beta^k U(l)_{t+k} \text{ s.t. } (3.1\text{-}3.2) \tag{3.3}$$

Note: $\hat{E}_t$ denotes the household's subjective expectations formed from imperfect information.

First-order conditions yield:

$$\frac{\partial \mathcal{L}}{\partial c_t(l)} : c_t(l)^{-\gamma} = \lambda_t(l) \tag{3.4}$$

$$\frac{\partial \mathcal{L}}{\partial s_t(l)} : \lambda_t(l) = \beta \lambda_{t+1}(l) \pi_{t+1}^{-1} (1 + R_t^s) \tag{3.5}$$

$$\frac{\partial \mathcal{L}}{\partial \psi_t(l)} : q_t^{\psi} = \hat{E}_t[\pi_{t+1}^{-1} (\frac{c_t(l)}{c_{t+1}(l)})^{\sigma} (q_{t+1}^{\psi} + d_{t+1})] = 0 \tag{3.6}$$

Where d_t are the real profits from the Intermediary firm:

$$d_t \equiv \Pi_t^{real,I}$$

If we assume an economy where households are perfectly rational and are equipped with perfect information, then replacing $\hat{E}_t$ with E_t and using equations (3.4) and (3.5) are sufficient in characterizing aggregate consumption choices. However, when I relax this assumption and allow households to take into account their intertemporal budget constraint (3.2), consumption will have a different functional form. The next section outlines the household's consumption decision rule.

Note: Erceg, Henderson, and Levin (2000) show when households have separable preferences in leisure and consumption, then they differ only in labor supply. Hence, for equations (3.3-3.6) I drop the term l.

Under rational expectations, Note that $E_t[x_t] \equiv E[x_{t+1}|I_t]$. While with subjective expectations, $\hat{E}_t[x_t] \equiv E[x_{t+1}|\tilde{I}_t, \tilde{\theta}]$. Here $\tilde{\theta}$ represents the true parameters of the model. Economically, this means that under rational expectations, agents know the correlation structure of all variables. Furthermore, I_t expresses that all variables are known at time t. While $\tilde{I}_t$, represents the idea that agents have limited knowledge of the variables known at time t. Section 4 details the formation of subjective beliefs.

3.2 Anticipated utility framework:

Building on Eusepi & Preston(2018) and Woodford(2013), I enable households to form consumption and savings decisions based not only on expected future consumption but on their infinite discounted future utility. Under the anticipated utility framework, if households deviate from rational expectations, Ricardian equivalence may not necessarily hold. That is, future government expenditures can then create disproportionate changes in expected future taxes. Similarly, for a given Treasury LSAP, future government expenditures may be disproportionately affected in the infinite horizon learning model. I start by linearizing the household budget constraint as follows:

$$s_{t-1}(l) = \beta s_t(l) + c_t + \tilde{\alpha_1} q_t^{\psi} + \tilde{\alpha_2}\psi_t + \tilde{\alpha_3}\pi_t + \tilde{\alpha_4} q_t^{\psi} + \tilde{\alpha_5} d_t + \tilde{\alpha_6}\psi_{t-1} + R_{t-1}^s + \tilde{\alpha_7}\tau_t + \tilde{\alpha_8}(w_t + H_t(l)) \quad (3.7)$$

Next, I iterate this forward and obtain:

$$s_{t-1}(l) = \sum_{k=0}^{\infty} \beta^k \Big(c_{t+k} + \tilde{\alpha_1} q_{t+k}^{\psi} + \tilde{\alpha_2}\psi_{t+k} + \tilde{\alpha_3}\pi_{t+k} + \tilde{\alpha_4} q_{t+k}^{\psi} + \tilde{\alpha_5} d_{t+k} + \tilde{\alpha_6}\psi_{t+k-1} + R_{t+k-1}^s + \tilde{\alpha_7}\tau_{t+k} + \tilde{\alpha_8}(w_t + H_t(l)) \Big)$$

After plugging (3.4) into (3.5), iterating forward, and substituting into (3.7), I obtain the following expression for consumption:

$$c_t(l) = (1-\beta)s_{t-1}(l) + \nu_t^c(l)$$

Where the expected infinite horizon of income flows is:

$$\nu_t^c(l) = (1-\beta)\Big(\alpha_4 q_t^{\psi} + \alpha_5 d_t + \alpha_6\psi_{t-1} + R_{t-1}^s + \alpha_8 w_t + \alpha_8 H_t(l) - \alpha_1 q_t^{\psi} - \alpha_2\psi_t - \alpha_3\pi_t - \alpha_7\tau_t\Big)$$

$$+\beta\gamma^{-1}\hat{E}_t[\pi_{t+1} - R_t^s] + \beta\hat{E}_t\nu_{t+1}^c(l)$$

we see above that the household chooses its consumption based on the aggregate income flows conditional on the flow of not only future taxes/expenditures but also on the perceived trajectory of asset prices, labor income, and net of taxes. The flow of real wealth serves as a key variable in amplifying business cycle dynamics for a given exogenous shock. Thus, I will henceforth label v_t^c the 'wealth channel'.

3.3 Monetary & fiscal policy:

The Central bank holds a portion of government debt and private stocks. I restrict the central bank to these particular assets given that the BOJ has primarily used both of these as the primary tools since 1980. The Central bank creates excess reserves and purchases assets on the open market composed of only financial intermediaries. The mechanism as stated is closely in line with the BOJ's stated open market operations guidelines.

The Central bank's flow budget constraint is:

$$q_t b_t^{cb} + q_t^{\psi} \psi_t^{cb} = re_t + \pi_t^{-1}(1+\rho q_t) b_{t-1}^{cb} + (d_t + q_t^{\psi}) \psi_{t-1}^{cb} - \pi_t^{-1}(1+R_{t-1}^{re}) re_{t-1} \tag{3.8}$$

Central bank's profits are:

$$\Pi_t^{cb} \equiv \pi_t^{-1} \rho q_t b_{t-1}^{cb} + \pi_t^{-1} d_t \psi_{t-1}^{cb} - \pi_t^{-1}(1+R_t^{re}) re_{t-1}$$

Assuming the Central bank returns its net profits to the Treasury, (3.8) becomes:

$$q_t b_t^{cb} + q_t^{\psi} \psi_t^{cb} = re_t + \pi_t^{-1} b_{t-1}^{cb} + \pi_t^{-1} q_t^{\psi} \psi_{t-1}^{cb} \tag{3.9}$$

The Treasury's flow budget constraint after receiving the Central bank's net revenue becomes:

$$q_t b_t = \pi_t^{-1}(1+\rho q_t) b_{t-1} - \pi_t^{-1} \rho q_t b_{t-1}^{cb} - \pi_t^{-1} d_t \psi_{t-1}^{cb} + \pi_t^{-1}(1+R_{t-1}^{re}) re_{t-1} - \tau_t$$

Above we see government debt issued each period equals the interest on debt issued in the previous period, plus treasury transfers to the household, minus lump sum taxes τ_t, minus dividends and coupon payments on stocks and treasuries held by the Central bank, respectively, followed by a deduction of interest needed to pay off of reserves.

Market clearing for stocks and bonds are:

$$b_t = b_t^{cb} + b_t^{fi}$$

$$\psi_t = \psi_t^{cb} + \psi_t^{hh}$$

The Central bank sets interest on bank reserves they offer to the Financial intermediary with the following lagged Taylor Rule:

$$R_t^{re} = \rho_{re} R_{t-1}^{re} + (1-\rho_{re})(\phi_x x_t + \phi_\pi \pi_t) + \epsilon_t^{re}$$

I assume the Central bank uses an AR(1) rule during the Non-ZLB period for asset purchases and a Taylor Rule during ZLB periods:

$$b_t^{cb} = \begin{cases} \rho_b b_{t-1}^{cb} + \epsilon_t^{cb,b} & R_t^s \geq 0 \\ (1-\rho_b)(\phi_x x_t + \phi_\pi \pi_t) + \epsilon_t^{cb,b} & R_t^s = 0 \end{cases}$$

$$\psi_t^{cb} = \begin{cases} \rho_b \psi_{t-1}^{cb} + \epsilon_t^{cb,\psi} & R_t^s \geq 0 \\ (1-\rho_b)(\phi_x x_t + \phi_\pi \pi_t) + \epsilon_t^{cb,\psi} & R_t^s = 0 \end{cases}$$

Here, CB denotes the total quantity of bonds and stocks the Central bank owns at a given period, while ψ_t^{hh} denotes the total equity holdings of households at time t.

Taxes evolve according to: $\tau_t = \phi_b b_{t-1}$.

3.4 Intermediary firm production problem:

Each period, Intermediary firms select the optimal quantity of labor to employ, equity to issue, and corporate debt to issue(ψ_t, b_t^k, H_t). Motivated by Kiyotaki & Moore (1997), Because the Financial intermediary is not privy to the profitability of each firm it lends b_t^k to, it then demands a level of collateral proportional to the market value of capital. This constraint is expressed where Q_t^k is Tobin's Q, which represents the shadow price of capital raised for the firm. Below, we see that the market value of the firm is expressed as the expected discounted sum of future firm profits.

The Market Value of the firm is:

$$V_t^I = \hat{E}_t \sum_{k=0} \beta^k c_{t+k}^{-\delta} \Pi_{t+k} \tag{3.10}$$

Each period, the firm earns real profit flows:

$$\Pi_t^I = A_t k_t^\alpha H_t^{1-\alpha} - \pi_t^{-1} d_t \psi_{t-1} - w_t H_t - \pi_t^{-1}(1 + \rho_k q_t^k) b_{t-1}^k - \Omega_t^I \tag{3.11}$$

The investment adjustment cost is:

$$\Omega_t^I = \pi_t^{c_I} \zeta_t^I \left(\left(\frac{q_t B_t^k}{q_{t-1}^k B_{t-1}^k} \right)^{c_I} + \left(\frac{q_t^\psi \psi_t}{q_{t-1}^\psi \psi_{t-1}} \right)^{c_I} \right)$$

The intermediary firm faces the following collateral constraint:

$$\theta_{2,t} Q_t^k k_t \geq \pi_t^{-1}(1 + \rho_k q_t^k) b_{t-1} \tag{3.12}$$

Capital evolves according to:

$$k_t = (1 - \delta) k_{t-1} + I_t \tag{3.13}$$

Intermediary firm investment is:

$$I_t = q_t^\psi \psi_t + q_t^k b_t^k \tag{3.14}$$

Taking into account (3.10-3.14), the firm solves (3.10) with the following Lagrangian:

$$\mathcal{L}^I = \hat{E}_t(\sum_{k=0} m^I_{t+k}\Pi_{t+k} - \sum_{k=0}^{\infty} Q^k_{t+k}[(1-\delta)K_{t+k-1} + I_{t+k} - K_{t+k}]$$

$$-\sum_{k=0} \mu^I_{t+k}(\theta_{2,t+k}Q^k_{t+k}K_{t+k} - (1-\rho_k q^k_{t+k})b_{t+k-1}))$$

Solving the Lagrangian above yields the equilibrium corporate bond (b^k_t), equity supply (ψ_t), and labor demand (H_t) each period. Details of equilibrium conditions can be found in Appendix B. In the next section, the paper outlines the equilibrium corporate bond (b^k_t), Treasury bond (b_t), and bank reserve (re_t) demand as well as bank deposit (s_t) issuance derived from the banking sector optimization.

3.5 Financial intermediary:

Each period the FI borrows deposits (s_t) from the household, invests in a portfolio of assets, and consumes investment profits earned from the previous period. The FI invests in: corporate bonds issued by the intermediary firm (b^k_t), Treasury Bonds issued by the fiscal authority (b_t), and reserves issued by the Central bank (re_t). When forming an investment portfolio, the FI is subject to a collateral constraint imposed by the household. That is, the FI cannot have a market value below a fraction $\theta_{1,t}s_t$ of real deposits, thereby enabling households to recoup losses given a heavy incurred loss to the FI. As in Gertler & Kiyotaki (2010), to prevent the FIs from overcoming their financial constraints, each period a fraction of σ of financial intermediaries will be forced to consume their accumulated wealth.

real net worth evolves according to:

$$n_t = \pi_t^{-1}(re_{t-1}(1+R^{re}_{t-1}) + (1+\rho_k q^k_t)b^k_{t-1} + (1+\rho q_t)b_{t-1} + n_{t-1} - \tilde{\Omega}^f_t - (1+R^s_{t-1})s_{t-1}) \tag{3.15}$$

The FI's adjustment cost is expressed as:

$$\tilde{\Omega}^f_t = \pi_t^{1+c_f}\zeta^I_t((\frac{q_t b_t}{q_{t-1}b_{t-1}})^{c_f} + (\frac{q^k_t b^k_t}{q^k_{t-1}b^k_{t-1}})^{c_f})$$

The balance sheet identity is:

$$s_t = b^k_t q^k_t + b_t q_t + re_t \tag{3.16}$$

The FI's collateral constraint:

$$V_t \geq \theta_{1,t}s_t \tag{3.17}$$

Where the FI's Value is:

$$V_t \equiv \sum_{k=0}^{\infty}(1-\sigma)^k \beta^k c_{t+k}^{-\delta} n_{t+k} \tag{3.18}$$

Taking into account (3.15-3.17), The FI solves (3.18):

$$\mathcal{L} = \hat{E}_t(\sum_{k=0}^{\infty}(1-\sigma)^k c_t^{-\gamma}\beta^k N_{t+k} - \sum_{k=0}^{\infty}\lambda_{t+k}^f[q_{t+k}b_{t+k} + q_{t+k}^k b_{t+k}^k + re_{t+k} - s_{t+k}]$$

$$-\sum_{k=0}^{\infty}\mu_{t+k}[V_{t+k} - \theta_{1,t+k}s_t]) \tag{3.19}$$

FOC for deposits(borrowing):

$$\frac{\partial \mathcal{L}}{\partial s_t} : -(1+R_t^s)(\sum_{k=0}^{\infty} m_{t+k+1}^f) + \lambda_t^f + \mu_{t+k}^f\theta_{1,t+k} + \sum_{k=0}^{\infty}\mu_{t+k+1}^f(\sum_{p=0}^{k} m_{t+p}^f)(1+R_t^s) = 0 \tag{3.20}$$

Re-arranging the FOC for Central bank's reserves:

$$\lambda_t^f = (\nu_{t+1}^m - \nu_{t+1}^{m,u})(1+R_t^{re}) \tag{3.21}$$

Re-arranging the FOC for deposits yields:

$$(1+R_t^s) = (1+R_t^{re}) + \theta_{1,t}\left(\frac{\mu_t^f}{\hat{E}_t\nu_{t+1}^m - \hat{E}_t\nu_{t+1}^{m,u}}\right) \tag{3.22}$$

m_{t+k}^f represents the stochastic discount factor the Financial intermediary weighs on returns received at time $t+k$:

$$m_{t+k}^f \equiv (1-\sigma)^k\beta^k\pi_{t+k}^{-1}c_{t+k}^{-\gamma}$$

ν_t^m represents the stochastic discount factor the Financial intermediary weighs on the total returns received from both corporate and treasury consol bonds:

$$\nu_t^m \equiv \sum_{k=0}^{\infty} m_{t+k}^f = \pi_t^{-1}c_t^{-\gamma} + \beta\sigma\nu_{t+1}^m$$

μ_{t+k}^f represents the shadow value the financial intermediary obeys abiding by the collateral constraint. While ν_t^u shadow value of abiding by the collateral constraint over an infinite horizon.

$$\nu_t^u \equiv \sum_{k=0}^{\infty}\mu_{t+k}^f = \mu_t^f + \nu_{t+1}^u$$

$\nu_t^{m,u}$ represents the discounted value the financial intermediary takes into account of abiding by the collateral constraint.

$$\nu_t^{m,u} \equiv \hat{E}_t \sum_{k=0}^{\infty} \mu_{t+k}^f \sum_{k=p}^{\infty} m_{t+k}^f = \mu_t^f \nu_t^m + \hat{E}_t \nu_{t+1}^{m,u}$$

Using (3.21), we obtain the relationship that a higher R_t^{re} results in a decline in asset prices: q_t^k, q_t, q_t^ψ, holding all other variables constant. Intuitively, if monetary policy were to raise interest on bank reserves, the FI will divert a larger portion of funds into reserves and thus asset demand for all other products would decline, hence for the market to clear, asset prices would decline. Details of equilibrium conditions can be found in Appendix C.

3.6 A note on the Zero Lower Bound

In light of the evidence presented by Heider et. al. (2018), banks are unwilling to let $1 + R_t^s$ go below 0. In the model if $R_t^s < 0$, households would simply choose to not hold deposits[4]. Thus, bank behavior presents a natural "zero-lower-bound" equilibrium to explore. Using (3.20) & (3.21), I obtain an expression for R_t^s:

$$(1 + R_t^s) = \max\{0, (1 + R_t^{re}) + \theta_{1,t} \left(\frac{\mu_t^f}{\hat{E}_t \nu_{t+1}^m - \hat{E}_t \nu_{t+1}^{m,u}} \right) \}$$

We see from above, absent the ZLB, the household will demand R_t^s equal to R_t^{re} but marked up by the extent of the financial friction, μ_t, observed in (3.19).

Aggregate consumption is expressed in terms of the sum of σ FI's who are forced to disburse their net worth to the household.

$$\bar{c}_t = c_1 c_t^{hh} + \sigma n_t \tag{3.23}$$

Because the deposit rate departs from the traditional linear FI deposit demand FOC, the model, given most combinations of structural parameters, will not admit a unique, or determinate, solution. Furthermore, because I am interested in exploring a richer set of equilibria, throughout the analysis, I only impose uniqueness for the Non-ZLB equilibrium, while allowing indeterminacy in the ZLB equilibrium. Though for a given set of parameters, the non-ZLB is unique, because the ZLB-case may not be unique, it follows then that the regime-switching equilibrium need not be unique. In this paper, however, I abstract from such issues and only impose uniqueness for the non-ZLB case. For all results, the equilibrium obtained for the ZLB is the MSV[5] implied solution.

[4]Note: I omit money balances in the model but I could just as easily add money in the budget constraint and obtain this result as well as the same exact equilibrium I solve for.

[5]McCallum(2004)

3.7 A note on the Relevance of LSAPs:

One point of concern is whether investors have no risk-adjusted preference between various investments. If this result holds in a model, large-scale asset purchases would be rendered irrelevant assuming households and investors can costlessly swap from short to long-term bonds, given a sudden swap on debt of differing maturities.[6] However, because the financial intermediary & intermediary firms are subject to collateral constraints and face adjustment costs, agents cannot shift their portfolio of holdings without incurring costs. Hence, to evaluate the effectiveness of Large Scale Asset Purchases, b_t^{cb} & ψ_t^{cb}, we avoid the irrelevance pitfall, which means that LSAP policies cannot be replicated from alternative fiscal and interest rate on bank reserve policies.

Taking into account the first-order conditions of each agent as well as fiscal & monetary policy, we obtain the following results for LSAPs:

If the Central bank engages in Treasury bond purchases or Stock purchases, and if the persistence(ρ_I) of the adjustment cost(ζ_t) banks & intermediary firms incur is sufficiently low, then holding all else constant, this will yield an increase in treasury bond and stock prices, respectively. Note: for the positive effect on stock prices, I assume ρ_I is sufficiently low.

$$\frac{\partial q_t^{\psi}}{\partial \psi_t^{cb}} \geq 0$$

$$\frac{\partial q_t}{\partial b_t^{cb}} \geq 0$$

Details of this result can be seen in Appendix J.

3.8 Wage and Price rigidity:

I Assume Intermediary firms follow Calvo pricing with probability θ_p of being unable to change prices. Hence, firms price each period by solving:

$$\max_{p_t^*} \quad \hat{E}_t[\sum_{k=0}^{\infty}(\theta\beta)^k \Lambda_{t+k} A_{t+k}] \tag{3.24}$$

$$A_{t+k} \equiv (\frac{p_t^*}{P_{t+k}})^{-\epsilon} Y_{t+k} - (\frac{p_t^*}{P_{t+k}})^{-\epsilon}(\frac{\Phi_{t+k}}{P_{t+k}})Y_{t+k} \tag{3.25}$$

Aggregate Price Identity:

$$P_t^{1-\epsilon} = [(1-\theta)(p_t^*)^{1-\epsilon} + \theta P_{t-1}^{1-\epsilon}] \tag{3.26}$$

[6] Woodford & Eggertson(2003), Wallace(1981)

$$\pi_t = (1-\theta)(1-\beta\theta)(\phi_t + \alpha_\pi^{-1}\delta c_t + \beta\theta\hat{E}_t\pi_{t+1}) \tag{3.27}$$

With the following New-Keynesian Phillips Curve, we see inflation is driven by aggregate firm marginal cost of production along with consumption and expected inflation in the next period.

Labor union wage setting:

Muto & Shintani (2014) find that modeling wage frictions in terms of a Calvo (1983) style framework accurately characterizes wage inflation and unemployment interactions. Hence to introduce wage stickiness in the model, I assume that there exists a labor union that aggregates labor via CES preferences and leases out labor to firms in a competitive market. Details of Calvo-Pricing can be found in Appendix H. The household optimizes wage considering this friction and seeks to maximize their discounted utility via the following problem:

$$\max_{w_t^*} \hat{E}_t \sum_{k=0}^{\infty} \Lambda_{t+k}\theta_w^k B_{t+k}$$

$$B_{t+k} = (-\frac{H_{t+k}^{1+\psi}}{1+\psi})(\frac{W_t^*}{W_{t+k}})^{-\epsilon_w(1+\psi)} + \lambda_{t+k}W_{t+k}(l)H_{t+k}(l)$$

After solving the household's dynamic optimization problem with respect to labor union decisions outlined with the above equations, we obtain an expression for the aggregate wage, similar to (3.27). Details of Calvo-Wages can be found in Appendix G.

3.9 Deriving Marginal Cost:

To derive the aggregate marginal cost of all firms, I can re-express the intermediary firm production problem as a cost-minimization problem. As expressed below, firms can solve the following:

$$\min_{H_{t+k},K_{t+k}} \hat{E}_t \sum_{k=0}^{\infty} \Lambda_{t+k}\{W_{t+k}H_{t+k} + \Phi(I_{t+k}, I_{t+k-1})I_{t+k} + (1+\rho^k)q_t^k - \Phi_{t+k}(Y_{t+k} - A_{t+k}K_{t+k}^{\alpha}H_{t+k}^{1-\alpha})\}$$

$$\frac{\partial}{\partial H_{t+k}} : W_{t+k} = MC_{t+k}(1-\alpha)K_t^{\alpha}H_t^{-\alpha}A_t$$

Log-Linearized, this becomes(note, I express the marginal cost as ϕ_t):

$$\phi_t = (\alpha)H_t - \alpha K_t - a_t + w_t + u_t \tag{3.28}$$

$$u_t = \rho_u u_{t-1} + \epsilon_t^u$$

Where u_t represents an exogenous cost-push shock.

3.10 Market Clearing and Equilibrium:

Assuming all agents are optimizing after taking into account their respective budget constraints or collateral constraints, expectations are homogeneous($\hat{E}_t^j$ is the same $\forall j$), monetary & fiscal policy is set according to the aforementioned rules, and the market clears at each time t, then the equilibrium can be defined as the sequence of prices q_t, q_t^k, R_t^s, P_t and allocation of variables described above. See Appendix K for aggregation of the continuum. The final equation that describes aggregate output is:

$$Y_t = C_t + I_t.$$

Log Linearized, this becomes:

$$y_t = c_1 c_t + c_2 I_t.$$

Where $c_1 = \dfrac{C}{Y}$ and $c_2 = \dfrac{I}{Y}$.

4 Vector Representation of Equilibrium

After collecting all variables, I can now express the dynamics in the following state space form:

$$Z_t = AZ_{t-1} + B\hat{E}_t Z_{t+1} + Q\bar{\epsilon}_t \tag{4.1}$$

$$Z_t = \begin{bmatrix} M_t \\ U_t \end{bmatrix} \tag{4.2}$$

$$M_t = \begin{bmatrix} c_t, y_t, h_t, \pi_t, k_t, q_t^k, q_t^{psi}, q_t, R_t^s, R_t^{re}, \psi_t, \psi_t^{hh}, \psi_t^{cb}, b_t, b_t^{fi}, b_t^{cb}, s_t, b_t^k, re_t, Q_t, \mu_t, \mu_t^2, w_t, \\ d_t, nu_t^c, \nu_t^m, \nu_{m,ut}, n_t, \nu_t \phi_t \end{bmatrix}' \tag{4.3}$$

$$U_t = \begin{bmatrix} r_t, u_t, a_t, \zeta_t^I, \theta_{1,t}, \theta_{2,t} \end{bmatrix}' \tag{4.4}$$

Here, in equation (4.1), Z_t represents all variables previously described in the model. Where M_t is a column vector that includes all endogenous variables. While U_t represents all exogenous AR(1) processes in the macro-economy. I solve for the REE and obtain 2 equilibria: One when $R_t^s = 0$(ZLB) and one when $R_t^s \geq 0$ (No Zero lower bound). Hence, after solving this becomes:

$$Z_t = CZ_{t-1} + D\bar{\epsilon}_t \tag{4.5}$$

Where:

$$M_t = C_{1,1}^j M_{t-1} + C_{1,2}^j U_{t-1} + D_1^j \bar{\epsilon}_t \tag{4.6}$$

$$U_t = RU_{t-1} + D_2^j \epsilon_t \tag{4.7}$$

for $j = \{z, n\}$

4.1 Adaptive learning

We know from previous literature [7] the difficulty of policy to meet expectations and that participants in the economy form beliefs which change over time[8]. Under adaptive learning, agents behave as statisticians and use parameters to construct forecasts. As new information is taken into account, agents update their parameters. Economically this means when new information is processed, agents change their beliefs regarding how the economy evolves. For the proceeding analysis, I initialize agents with parameters near REE and let them update their parameters via Recursive least squares (RLS), a procedure closely derived from Ordinary least squares.

Because the RLS procedure requires a high degree of computation via the sample covariance matrix, I follow Evans el. al. (2010) 's Stochastic Gradient learning model.[9] I modify the RLS procedure and set the sample covariance matrix as a constant matrix based on the steady-state implied by the model parameters. See Appendix I for details. Below, I display the set of equations that govern the economy in the model and explain the expectations formation process:

I assume that agents form the following beliefs about the economy.

Perceived Law of Motion:

$$M_t = \Lambda^m_{t-1} M_{t-1} + \Lambda^u_t U_t + \nu^m_t \tag{4.8}$$

Under the subjective expectations process, agents have a limited information set where they only observe shocks (U_t) in the same time period:

$$\hat{E}_t[M_{t+1}] = \hat{E}_t[M_{t+1}|U_t, M_{t-1}]$$

Thus when forming expectations, I obtain:

$$\hat{E}_t M_{t+1} = \Lambda^m_{t-1} \hat{E}_t M_t + \Lambda^u_{t-1} R U_t \tag{4.9}$$

$$\hat{E}_t M_t = \Lambda^m_{t-1} M_{t-1} + \Lambda^u_{t-1} U_t \tag{4.10}$$

$$\Lambda_t \equiv \begin{bmatrix} vec(\Lambda^m_t) \\ vec(\Lambda^u_t) \end{bmatrix} ; \tag{4.11}$$

$$\Lambda^z_t = \Lambda_{t-1} + g P_z V'_t(\upsilon_t) \text{ if } i_t < 0 \tag{4.12}$$

[7] Branch and Evans(2017), Orphanides and Williams (2004)

[8] Evans and Honkapohja (2009)

[9] Detials of Stochastic Gradient Learning can be found in Appendix I

$$\Lambda_t^n = \Lambda_{t-1} + gP_nV_t'(\upsilon_t) \text{ if } i_t \geq 0 \tag{4.13}$$

$$\upsilon_t \equiv \begin{bmatrix} \hat{E}_t M_t \end{bmatrix} - \begin{bmatrix} M_t \end{bmatrix}; \; V_t \equiv \begin{bmatrix} I \otimes M_{t-1}' \\ I \otimes U_t' \end{bmatrix} \tag{4.14}$$

After substituting (3.9) into (3.1), I obtain the Actual Law of Motion:

$$M_t = A_t M_{t-1} + B_t U_t + Q\bar{\epsilon}_t \tag{4.15}$$

A note on constant gain learning:

In traditional models with learning, agents act as statisticians who form beliefs in such a way that over time, changes in beliefs are stable and converge to the REE. This means agents dramatically respond to their forecast errors in earlier periods, and as time goes on, changes in parameters become geometrically smaller. In light of evidence by Branch and Evans (2006) as well as Orphanides and Williams (2004), I take seriously, the view that agents revise their forecast each period with a constant weight. The constant gain parameter g, determines the degree agents update beliefs based on forecast errors each period. Throughout the paper, this parameter is the subject of key interest and is commonly set at 0.02[10].

4.2 Regime Switching Equillibria

To address both the ZLB and non-ZLB equilibria, I equip agents with both the ZLB and non-ZLB parameters. Furthermore, I assume agents update their parameters by forming subjective probabilities. When doing so, I assume that the agent follows a binomial counting model a la' Cogley & Sargent (2008). Hence, the model dynamics are expressed as follows:

Perceived Law of Motion

$$M_t = \Lambda_{t-1}^{*,m} M_{t-1} + \Lambda_{t-1}^{*,u} U_t + \nu_t^m \tag{4.16}$$

$$\Lambda_t^* \equiv \mu_t \Lambda_t^z + (1 - \mu_t)\Lambda_t^n \tag{4.17}$$

$$\Lambda_t^n = \Lambda_{t-1}^n + gP_nV_t(\upsilon_t)' \tag{4.18}$$

$$\Lambda_t^z = \Lambda_{t-1}^z + gP_zV_t(\upsilon_t)' \tag{4.19}$$

[10]See Milani 2006

Note, I initialize the Learning Parameters for $t = 0$ as follows:

$$\Lambda_0^z = \begin{bmatrix} vec(C^{z,m}) \\ vec(C^{z,u} \end{bmatrix} \tag{4.20}$$

$$\Lambda_0^n = \begin{bmatrix} vec(C^{n,m}) \\ vec(C^{n,u}) \end{bmatrix} \tag{4.21}$$

$n_{0,0}^0 = 1;\ n_{0,1}^0 = 1;\ n_{1,0}^0 = 1;\ n_{1,1}^0 = 200;$

$$\mu_t = \mathbb{P}(i_t = 0|\mathcal{I}_{t-1}) = \begin{cases} \frac{n_{1,0}^t}{n_{1,1}^t + n_{1,0}^t} & i_t \neq 0 \\ \frac{n_{0,0}^t}{n_{0,1}^t + n_{0,0}^t} & i_t = 0 \end{cases}$$

(4.22)

Hence agents use the ZLB forecast probability μ_{t-1}, to obtain the weighted belief parameters Λ_{t-1}^*, to form expectation about the future:

$$\hat{E}_t M_{t+1} = \Lambda_{t-1}^{*,m} \hat{E}_t M_t + \Lambda_{t-1}^{*,u} R U_t \tag{4.23}$$

$$\hat{E}_t M_t = \Lambda_{t-1}^{*,m} M_{t-1} + \Lambda_{t-1}^{*,u} U_t \tag{4.24}$$

Augmented law of motion:

$$M_t = \begin{cases} A^z M_{t-1} + B^z \hat{E}_t M_{t+1} + C^z U_t + D^z \bar{\epsilon}_t & i_t = 0 \\ A M_{t-1} + B \hat{E}_t M_{t+1} + C U_t + D \bar{\epsilon}_t & i_t \geq 0 \end{cases} \tag{4.25}$$

In the new model with ZLB dynamics, the subjective probability μ_t becomes an important feature driving values for $\hat{E}_t M_{t+1}$. Consequently, each term $n_{i,j}^t$ represents a function that counts the number of times state i has moved to state j. Hence, when $R_t^s = 0$ we see this not only plays a role in driving the value of M_t through the Non-expectation parameters seen in matrices A, B in equation (4.1) but will also drive how $\hat{E}_t M_{t+1}$ will evolve over time through Λ_t^*'s evolution.

5 Estimation

In order to properly understand the dynamics of the model in the context of the Japanese economy, I take Macro observable data from Japan and match this with the model-generated nonlinear state space data. Using equations (4.1-4.24) The model is re-expressed as:

$$\begin{bmatrix} Y_t \\ M_t \\ U_t \end{bmatrix} = \tilde{K}_1(\Lambda^*_{t-1}(\mu_{t-1}), \Theta)M_{t-1} + \tilde{K}_2(\Lambda^*_{t-1}(\mu_t), \Theta)U_{t-1} + \tilde{K}_3(\Lambda^*_{t-1}(\mu_{t-1}), \Theta)\bar{\epsilon}_t \tag{5.1}$$

Above, we see that the variables of interest evolve in a nonlinear fashion. However, to properly empirically identify the likelihood, I must jointly estimate (4.5-4.6) with the learning equations (4.11-4.13). Therefore the state transition equation and observation equations respectively, are:

$$V_t \equiv \begin{bmatrix} M_t \\ \Lambda^*_t \\ \mu_t \\ U_t \end{bmatrix} = \mathcal{F}(\Theta, Y_{t-1}, M_{t-1}, \Lambda^*_{t-1}, U_{t-1}, \bar{\epsilon}_t);\ \bar{\epsilon}_t \sim \mathcal{N}(0,\ \Psi(\Theta)\ ) \tag{5.2}$$

$$Y^{obs}_t = \bar{M}_1 V_t + \bar{M}_1 V_{t-1} + \epsilon^m_t; \tag{5.3}$$

$$\epsilon^m_t \sim \mathcal{N}(0, I) \tag{5.4}$$

$$\pi(Y^{obs}_t|\Theta, I_{t-1}) = N(Y^{obs}_t - \bar{M}_1 V_{t|t-1} - \bar{M}_1 V_{t-1|t-1}, I) \tag{5.5}$$

5.1 Particle Filter

In most models with learning, researchers hold the learning parameters fixed in order to utilize the Kalman filter[11]. This assumption has been shown to yield lower marginal & log-likelihoods along with inaccurate parameter estimates.[12] To overcome this challenge, I jointly estimate the learning parameters Λ^*_t and μ_t via a particle filter with re-sampling.[13]The importance of estimating the learning parameters as states can be interpreted as follows: When agents form expectations of the future, the stochastic processes which drive macroeconomic variables are characterized by distributions that follow a mean and variance. Though the shocks are independent of the macro-economy, the distribution macroeconomic variables as well as the learning coefficients are endogenous to one another. By including the learning coefficients as part of the state

[11]For example, Milani & Favero (2001) and Hommes el. al.(2018) use a Kalman filter.
[12]See: Kirpekar(2020)
[13]See: Rubio-Ramirez and Villaverde(2007) & Herbst and Schorfheide (2017)

variable, the researcher is in effect taking the stance that the distribution of macro variables is dependent on the distribution of beliefs and vice versa. Notice, however, in the conditionally linear model, since we hold the learning parameters Λ_t^* and μ_t fixed, we assume that beliefs are driven by macro variables but not the reverse.

The particle filter simulates the structural error terms with a sufficient number of draws, places weights on the realized number states, and calculates the weighted likelihood. Often this procedure places a high weight on few particles and 0 weight on most particles which, in essence, induces a high variance of the Likelihood function.[14] [15] In order to avoid this issue, I follow Rubio-Ramirez & Villaverde(2007) with the following algorithm. Details of the Particle Filter can be found in Appendix F.

5.2 Ensemble Kalman Filter

Because the Particle Filter becomes infeasible for estimation as the dimension of the state space increases, I utilize an Ensemble Kalman Filter to estimate the likelihood with respect to the parameters. Drawing from the original Kalman filter, The Ensemble Kalman filter is an appropriate alternative in capturing the Non-linearity because it allows for successive draws of the predictive distribution rather than relying on a single point estimate based on Gaussian error terms. Le Gland et. al(2009) examine the performance of the Ensemble Kalman filter and obtain statistical convergence of each state variable as the size of draws from the predictive covariance matrix increases. Hence, in the Bayesian parameter estimation procedure, I use the Ensemble Kalman Filter likelihood. Details of the Ensemble Kalman Filter can be found in Appendix F.

5.3 Data Description

To properly identify the model and come up with meaningful insights, I utilize FRED, TANKAI Survey & BOJ data. Equipped with expectations-level data, the model is now better able to pin down equations (5.1) and (5.2). For all data except the policy and deposit rate, I apply the Christiano-Fitzgerald(1999) filter. I do this to ensure that the model can capture the zero lower bound periods more effectively using the respective interest rates.

$$Y_t^{obs} = \begin{bmatrix} H_t^{obs}, re_t^{obs}, s_t^{obs}, w_t^{obs}, b_t, b_t^{cb}, b_t^k, c_t^{obs}, d_t^{obs}, g_t^{obs}, I_t^{obs}, \psi_t^{cb}, q_t, q_t^k, q_t^{\psi}, y_t, \pi_t, n_t, \\ re_t, R_t^s, \Delta c_{t+1}^e, \Delta y_{t+1}^e R_t^{re} \end{bmatrix} \tag{5.6}$$

[14]For Convergence properties of the Likelihood, please refer to Rubio-Ramirez & Villaverde(2007) and Kitigawa(1996)

[15]See Kitigawa (1996)

6 Results

6.1 Post Estimation Results:

Constant Gain Learning

Often in the learning literature, we see the RLS gain coefficient is estimated close to 0.02. Shown below in Figure 1, we see the posterior parameter distribution provides some evidence that although the mean is centered at 0.025, there is a rightward skew. This suggests the coefficient could indeed be higher in Japan.[16]

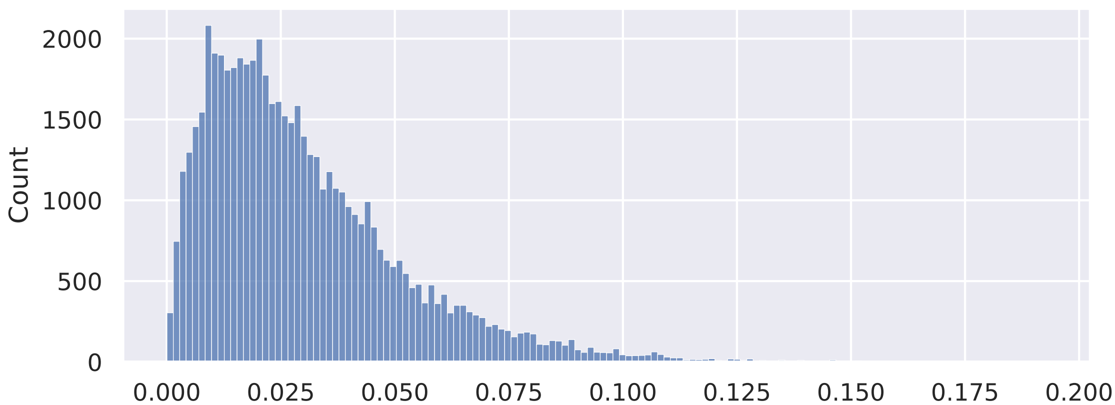

Figure 1: Posterior Dist. of RLS gain coeff.

Regime Switching ZLB probability

In Figure 3, we see the model-implied probabilities evaluated at the posterior mean. Using regime-switching dynamics, the overall probability is expressed as the weighted average of the regimes. That is:

$$P(R^s_t = 0|_t) = \sum_k \tilde{w}^{k,z}_t P(R^s_t|R^s_{t-1}, I_t) + \sum_p \tilde{w}^{p,n}_t P(R^s_t|R^s_{t-1}, I_t)$$

Where $\tilde{w}^{k,z}_t$ is the normalized empirical likelihood of draw k staying at the ZLB. Conversely, $\tilde{w}^{p,n}_t$ is the normalized empirical likelihood of draw p entering the ZLB.

[16]See: Cole and Milani(2021) & and Kirpekar(2020)

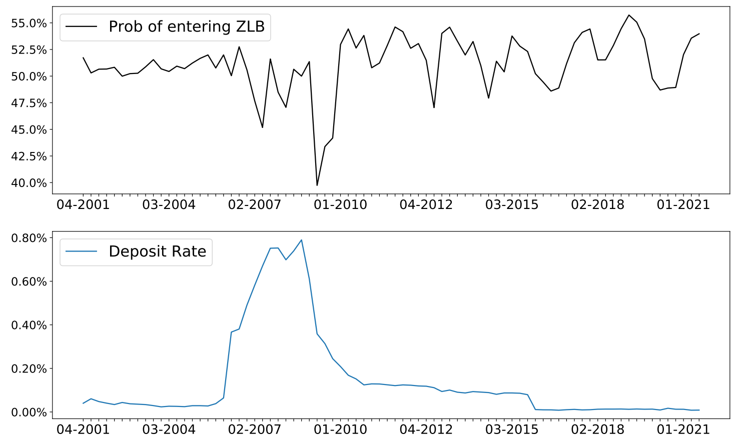

Figure 2: ZLB probability (μ_t)

Even without data on expected future bond yield or stock prices, the model does well in capturing the co-movement of the household deposit rate with the interest rate on bank reserves. Intuitively when the central bank raises the interest rate paid on bank reserves, agents would reasonably expect that entering the ZLB is likely to fall. Consequently, agents expect the ZLB probability to rise as the deposit rate falls. These movements we see are reflected in the 1rst Quarter of 2009' along with the 2nd Quarter of 2019'. In both instances, it was clear to agents, the appropriate revision of the perceived ZLB probability.

6.2 Impulse Responses

After estimating the model using Japanese macroeconomic data, we are able to examine credit and LSAP shocks. After observing plots of the ZLB probabilities and the policy rate, it also is clear that the ZLB is central in characterizing the evolution of the economy. Because other papers display differing policy impacts at the ZLB, I present impulse responses generated from the aforementioned regime switching model as well as plots directly evaluated when the ZLB binds.[17]

Large Scale Asset Purchase shocks:

Figure 4 tells us for a given 222% increase in central bank purchases of equities, output growth & inflation peak at 6.6% and 2.8% respectively, while consumption increases by 13.55% on impact. Figure 5 tells us that for a given 227% increase in central bank purchases of Treasuries, output growth & inflation peak at by 6% and 2.9% respectively, while consumption increases by 11.8% on impact. While these magnitudes may seem similar, when we look at the cumulative effect across the entire time horizon, the effects are slightly

[17] See Ramey & Zubiary (2014)

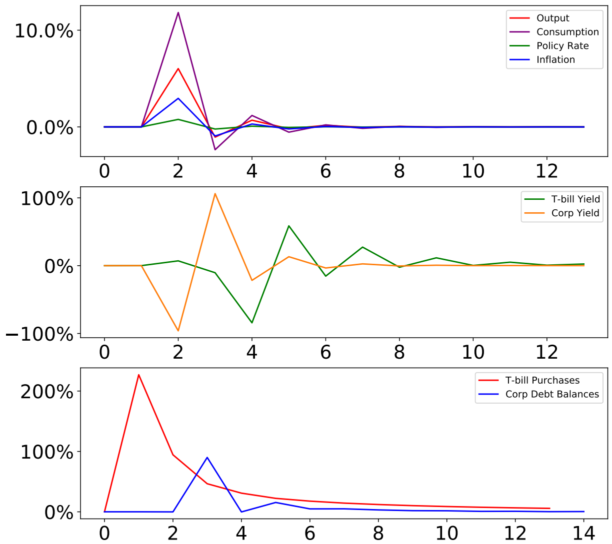

Figure 3: Treasury Purchases

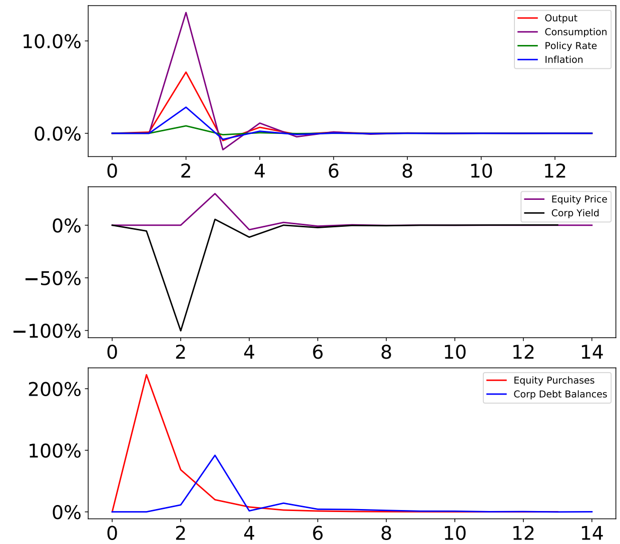

Figure 4: Equity Purchases

different. For the same equity LSAP, cumulative output growth & inflation increase by 6.7% and 2.2%. While cumulative consumption increases by 12.1% on impact. For the same Treasury LSAP, cumulative output growth & inflation increase by 5.7% and 2.05%. While cumulative consumption increases by 10.17% on impact. Intuitively, because the equity LSAP directly affects the equity holdings in the household's budget constraint, consumption choices respond with a slightly higher magnitude. This in turn induces higher effects to output growth.

While the overall LSAP magnitudes may seem high, it is important to note in reality, expectation level parameters agents form in the model, can shift depending on the presence of exogenous shocks in the economy and thereby attenuate such effects. Consequently, as discussed in the next section, historically, the presence of contractionary shocks has altered much of the historical macro dynamics, thus inducing deviations in perceptions of future economic conditions.

ZLB Large Scale Asset Purchase shocks:

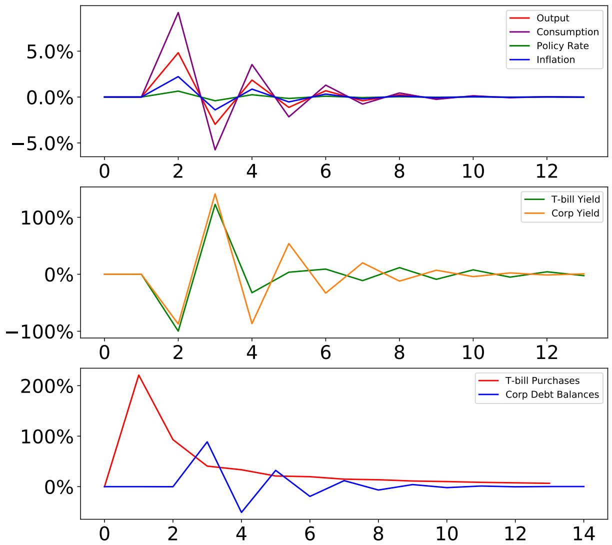

Figure 5: ZLB Treasury Purchases

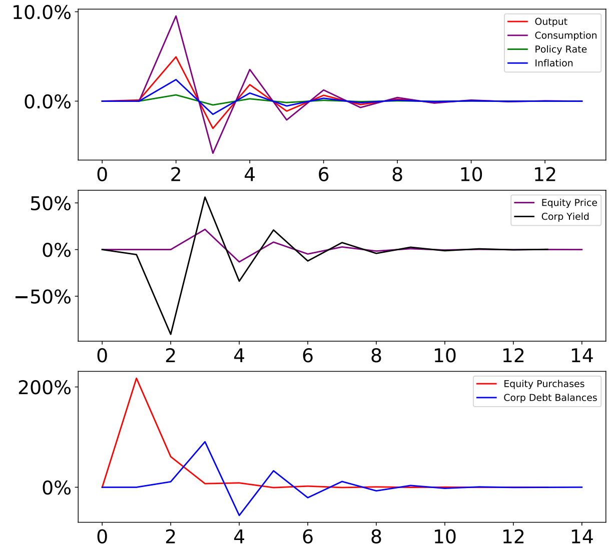

Figure 6: ZLB Equity Purchases

Figure 6 tells us that for a given 217% increase in central bank purchases of equities, output growth & inflation peak at by 4.95% and 2.4% respectively, while consumption increases by 9.5% on impact. Figure 5 tells us for a given 220% increase in central bank purchases of Treasuries, output growth & inflation peak at by 4.8% and 2.2% respectively. While consumption increases by 9.2% on impact. In comparison to the non-ZLB equilibrium, we see that while inflation responses are roughly the same, output responses are around 25% to 30% lower for Treasury and equity LSAPs, respectively. One reason for this comes from the fact that in Japan, the Calvo parameters(θ_p and θ_w) are higher relative to other developed nations. Hence, within the model, for a given rise in consumption and investment, wage and price adjustments will occur with relative speed thereby muting the inflation responses regardless of the ZLB. On the other hand, because the ZLB presents a floor to the deposit rate, this feature reduces the financial intermediary's ability to borrow thereby leading to a trajectory fraught with uncertainty. As we observe in the plots, LSAPs will on the net produce a positive impact on the economy. But the impulse response path, given the presence of adaptive learning and financing adjustment costs, endogenously produces higher volatility than the non-ZLB case.

Interest Rate Shock R_t^{re}:

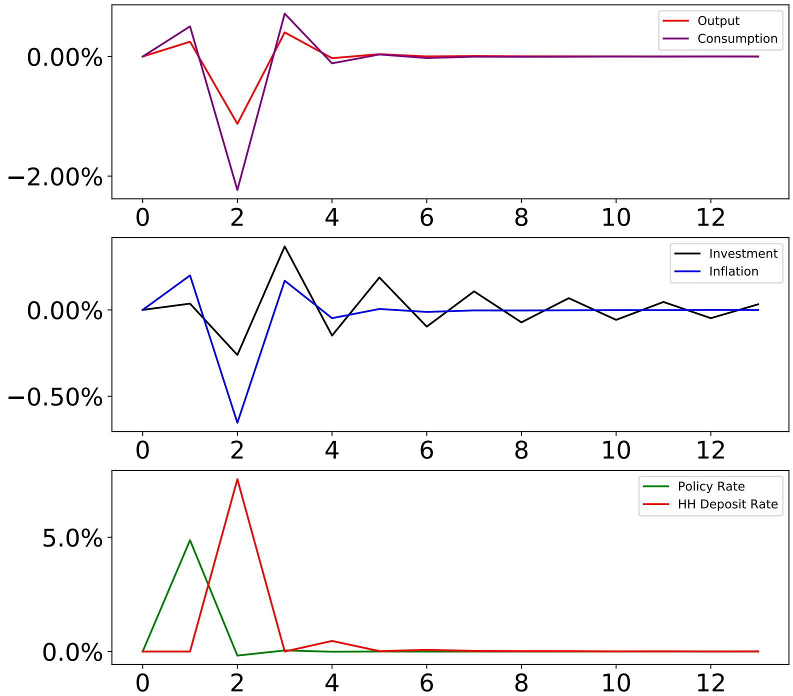

Figure 7: Interest Rate shock

Figure 8, tells us for a 4.7 % increase to the interest rate on bank reserves, inflation peaks at -1.56%, consumption peaks at -0.65%, and output peaks at -1.12%. Furthermore, for a positive MP shock, the household deposit rate increases by 7.89 % on impact and reverts back to equilibrium 2-3 quarters after the nominal interest rate does. This can be attributed in part to agents correctly anticipating the persistence of monetary policy and allocating assets away from deposits to stocks thus driving downward pressure on demand for deposits. One reason economically for the lack of strong response to inflation comes from both the low wage and price stickiness parameters θ_W & θ_p. Because the frequency of price adjustments is relatively quick, prices tend to adjust more frequently. Thus, when firm & labor union responses are less sluggish, firms & unions respond sharply to household consumption decisions and therefore make "quick" downward price adjustments and hence attenuate the impact in equilibrium. Additionally, because lower investment places upward pressure on prices when investment falls, we have the opposing forces of declining consumption along with the high marginal cost that induces a relatively weak response to inflation.

Financial friction shocks:

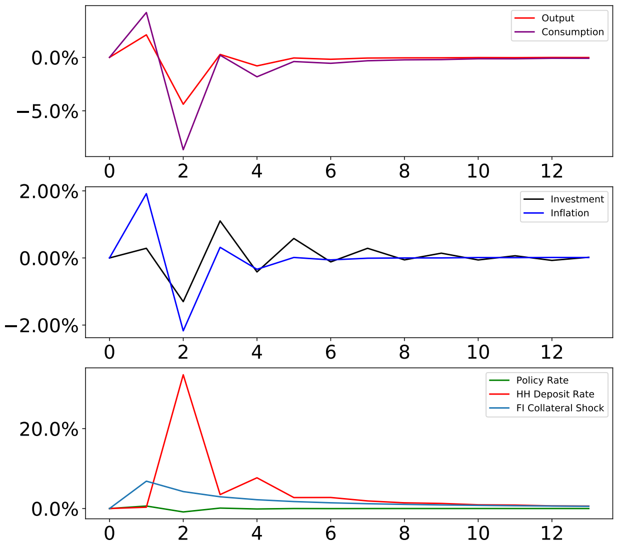

Figure 8: FI collateral shock

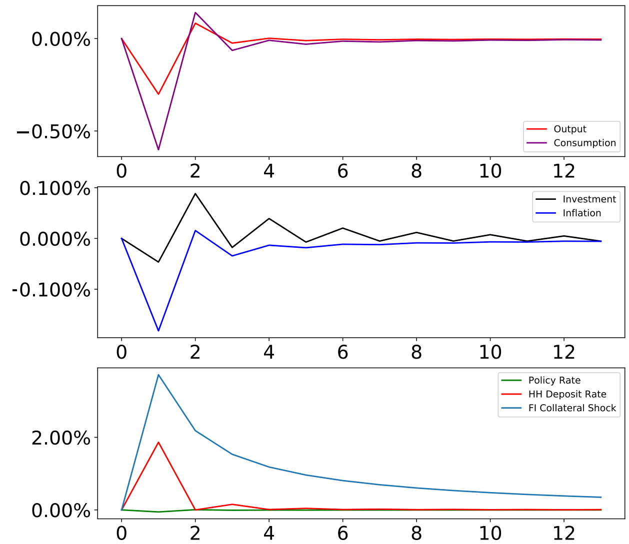

Figure 9: Intermediary firm collateral shock

Financial friction shocks:

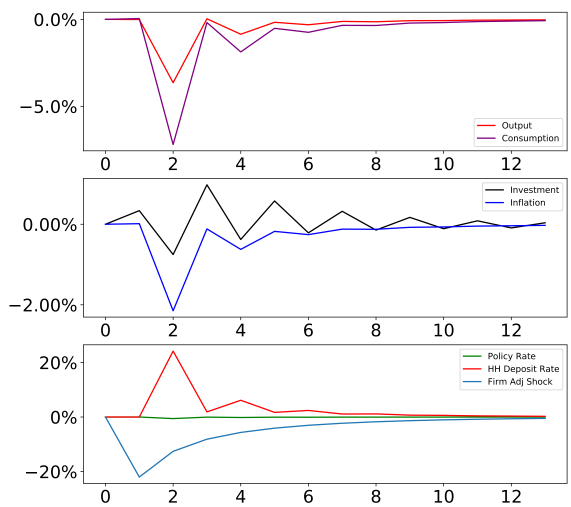

Figure 10: Fin. Adj shock

Figures 9, 10, and 11 provide quantitative effects of the collateral shocks imposed on the intermediary firm and financial intermediaries along with the financial adjustment cost shock. We see for all shocks, there is a sharp decline in both inflation, consumption, and output growth on impact, followed by volatile movements produced from the adaptive learning framework. In the next section, we will explore to what degree these shocks have played a role in the Japanese macro-economy since the 1990s.

6.3 Variance Decomposition Evidence

Using the parameters of the model, I compute the generalized forecast error variance decomposition and thereby am able to obtain the overall contribution of variance from each exogenous shock to the macro-variables of interest[18]. Following Lanne & Nyberg (2016), I compute the following expression using the particle filter's forecast:

$Y_t = f(S_{t-1}, \epsilon_t),\ GI = E_t[Y_{t+h}|\epsilon^j_{t+1} = \sigma_j, I_{t-1}] - E[Y_{t+h}|I_{t-1}]$

$\lambda^j_t(h|e_{t+h}) = \frac{\sum_{l=1}^{h} GI(l|\epsilon^j_{t+h}=\sigma_j)}{\sum_{j=1}^{J}\sum_{l=1}^{h} GI(l|\epsilon_{t+1}=\sigma_j)^2}$

$FEVD^j_t \equiv \lambda^j_t / E[\lambda^j_t(h)] = \sum_{j=1}^{J} \lambda^j_t(h)$

We see from above, the normalized measure of variance contribution relies on the information set I_{t-1} and hence is a function of the squared deviations of the mean forecast estimate. Below, I plot the relevant results for the contribution of variance across the sample:

[18] see: Pesaran et. al (1998)

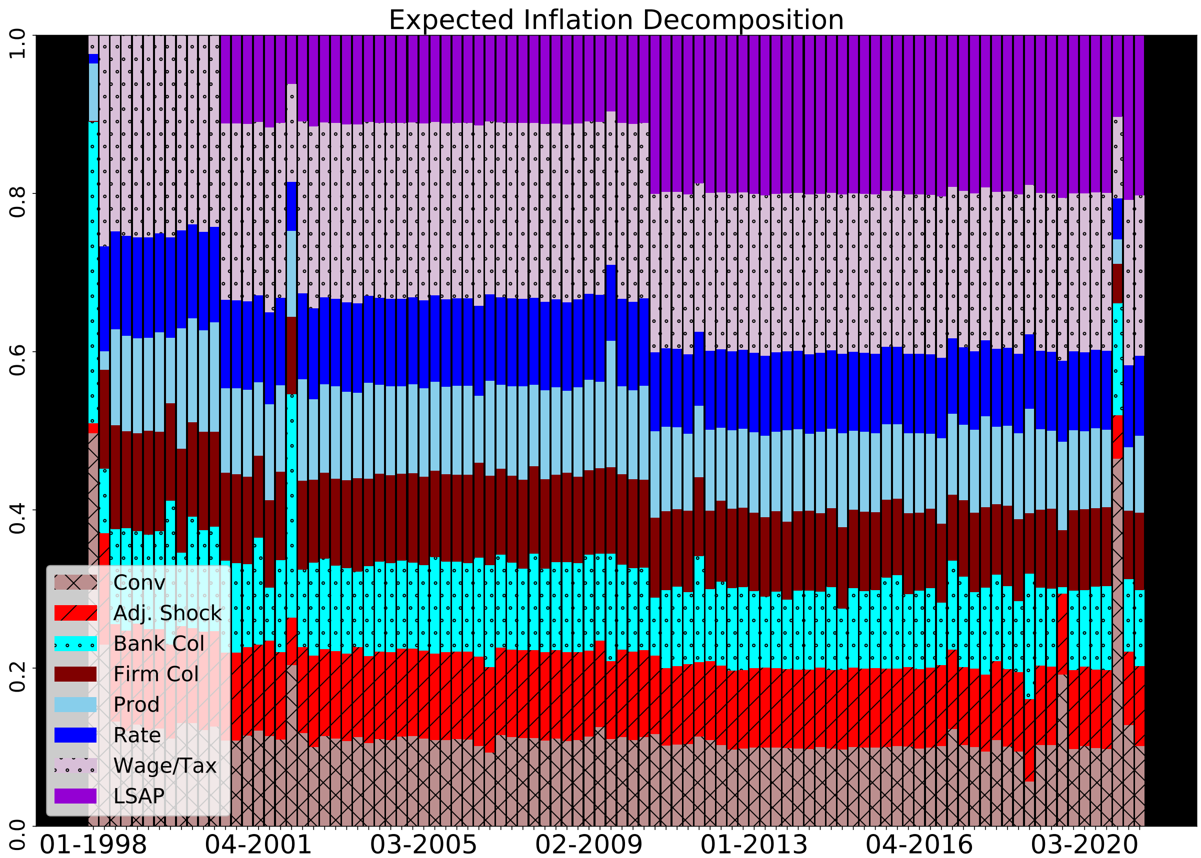

Figure 11: Shock decomposition for Expected Inflation

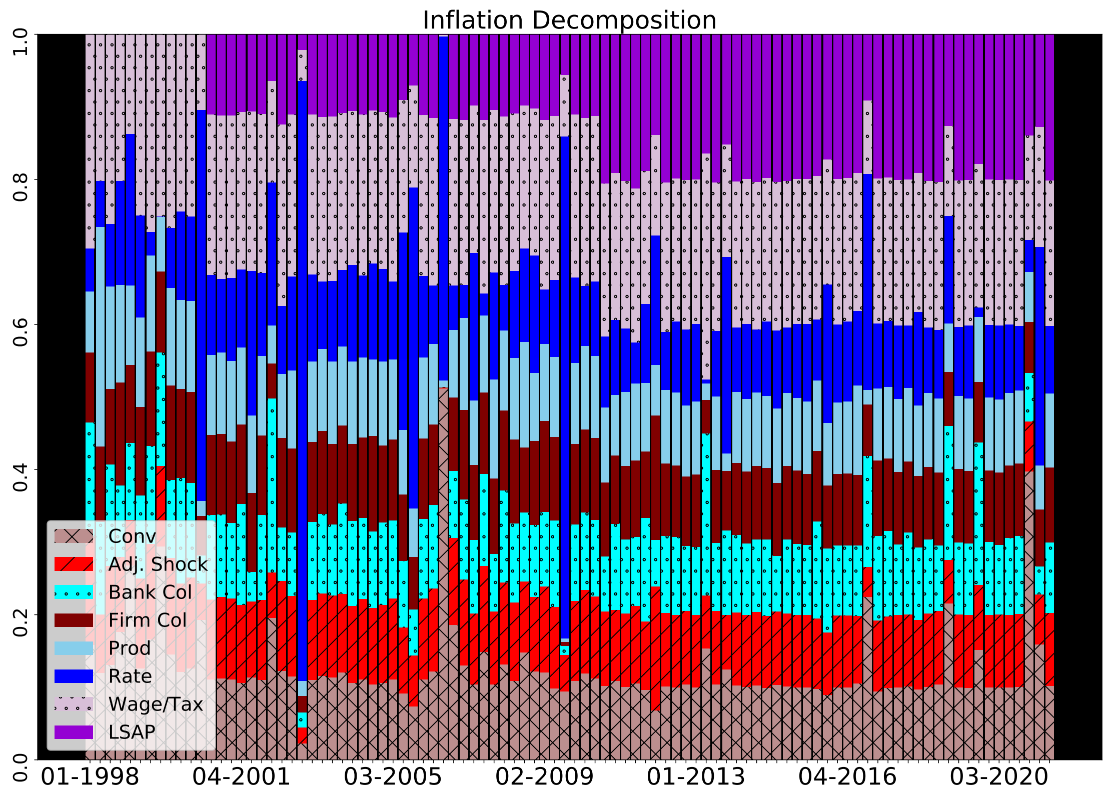

Figure 12: Shock decomposition for Inflation

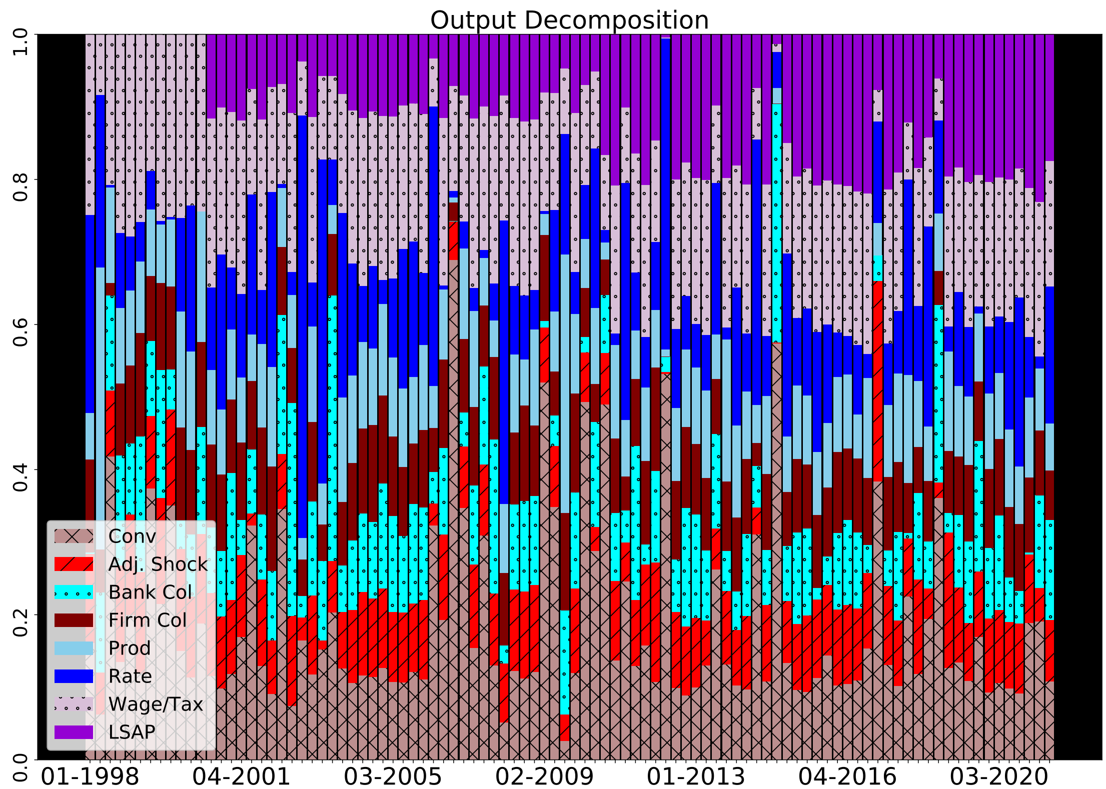

Figure 13: Shock decomposition for Output

Figures 11 and 12 tell us conventional demand shocks, bank collateral $\theta_{1,t}$, and intermediary firm shocks $\theta_{2,t}$ are all crucial drivers in the formation of inflation and expected inflation. Within the previously derived expression for inflation(3.27) and marginal cost(3.28), we see there are two opposing forces at work. On one hand, we have reductions to capital investment decrease K_t and thereby increase the marginal cost. However, at the same time, we have that consumption endogenously enters the Phillips curve in (3.28). Hence, because credit frictions reduce net worth in the banking sector, this delivers a strong negative effect on asset prices and thereby drives down consumption choices today and tomorrow. Consequently, delivering negative inflation and expected inflation. Economically, when there is a financial crisis, asset prices decline and lead to negative perceptions of wealth for households and investors alike. Taking this knowledge into account, the retail sector will reduce prices in response to sharp reductions in aggregate consumption.

Figure 13 attributes the Dot-com and Global Financial Crisis to demand, bank collateral, and financial intermediary shocks. As confirmed by Braun & Wauki (2006) along with Kaihatsu & Kurozumi (2010), much of Japan's output of the 1990s 'lost decade' era is attributed to adjustment costs. The exogenous shock ζ_t represents implicit and explicit costs born firms who either issue or sell off assets. When there is a financial panic, intermediary firms find it harder for institutions such as banks or households to take on debt or equity positions giving an increase in an overall flight to more liquid assets. Hence, a realization of ζ_t represents buyer & seller frictions originating from jumps in investor asset liquidity preferences by households and banks alike. Likewise, we see the collateral constraint shock imposed by the household on the financial intermediary, $\theta_{1,t}$(Teal) played a key role in driving output shortfall from 09' to the start of '20. During this period, financial intermediary net worth with respect to their asset positions came into question by investors. A similar story arises for intermediary firm collateral, $\theta_{2,t}$ (dark red) but with a lower magnitude. Lastly, we see that conventional demand shocks also contributed substantially to driving the pandemic, but interestingly we see the bank collateral constraint remained a key driver at the onset of the pandemic, suggesting that the constrained lending environment during this time exacerbated the decline in output growth.

6.4 Counterfactual monetary policy analysis:

In light of the evidence presented, we see that monetary policy's control over the interest rate on bank reserves and LSAPs has the potential to stabilize and improve economic welfare. Hence, in a similar spirit to Del Negro et. al.(2012), I produce 2 counterfactual experiments using the smoothed estimates from the particle filter. Before describing them, I will first detail how I obtain estimates of macro-variables under different policy scenarios.

<u>State Transition & Observation Equation:</u>

$$S_{t|t-1} = f(\theta, S_{t-1}, \epsilon_t | I_{t-1}) \ = f(\theta, S_{t-1|t-1}, \epsilon_{t|t-1}) \tag{6.1}$$

$$Y_{t|t-1} = M_1 S_{t|t-1} + M_2 S_{t-1|t-1} + \epsilon_t^m \tag{6.2}$$

<u>Counterfactual State Transition & Observation Equation:</u>

$$S^*_{t|t-1} = f(\theta, S^*_{t-1|t-1}, \epsilon^*_{t|t-1}) \tag{6.3}$$

$$Y^*_{t|t-1} = M_1 S^*_{t|t-1} + M_2 S^*_{t-1|t-1} + \epsilon_t^m \tag{6.4}$$

<u>Counterfactual Difference:</u>

$$D_t = Y^*_{t|t-1} - Y_{t|t-1} \tag{6.5}$$

Above, I have expressed the empirical model described in the beginning sections as the state transition equation (5.1) and measurement equation in (5.2). Equations (5.3) and (5.4) are the same but with the counterfactual shock ϵ_t^*, rather than the true shock ϵ_t implied from the observable data. For each counterfactual experiment, I generate the model-implied macro variable forecast. Next, I produce the counterfactual forecast conditioned on a policy counterfactual expressed in terms of a shock. For example, if I am interested in understanding the effects of contractionary monetary policy, I generate the model forecast via the Particle Filter. Then I generate the counterfactual forecast via the Particle Filter but conditioned on draws of the ϵ_t^i which have a positive mean σ_i rather than 0 in the base case. Then, by taking the difference between the two forecasts, I obtain the counterfactual effect. For each experiment, on the left, I plot both the true forecast and the counterfactual variables. While on the right, I compute the difference between the two, D_t.

Counterfactual 1: How would the Dot-Com crash of the early 2000s look like had the BOJ undertaken equity LSAPs?

As discussed in Mishkin & Ito (2004), during the start of the Dot-Com crash in Japan, internal policy proposals at the BOJ for swift asset purchases were quickly rejected and only adopted following worsening economic conditions. Hence, one may ask, to what degree would we see improvement in this period if the BOJ hypothetically accepted such equity LSAP measures?

When conducting the following experiment, I set $\underline{\psi_t^{cb} = \psi_t^{cb,obs} + \sigma_\psi, \text{ for } t = 01/2001:}$

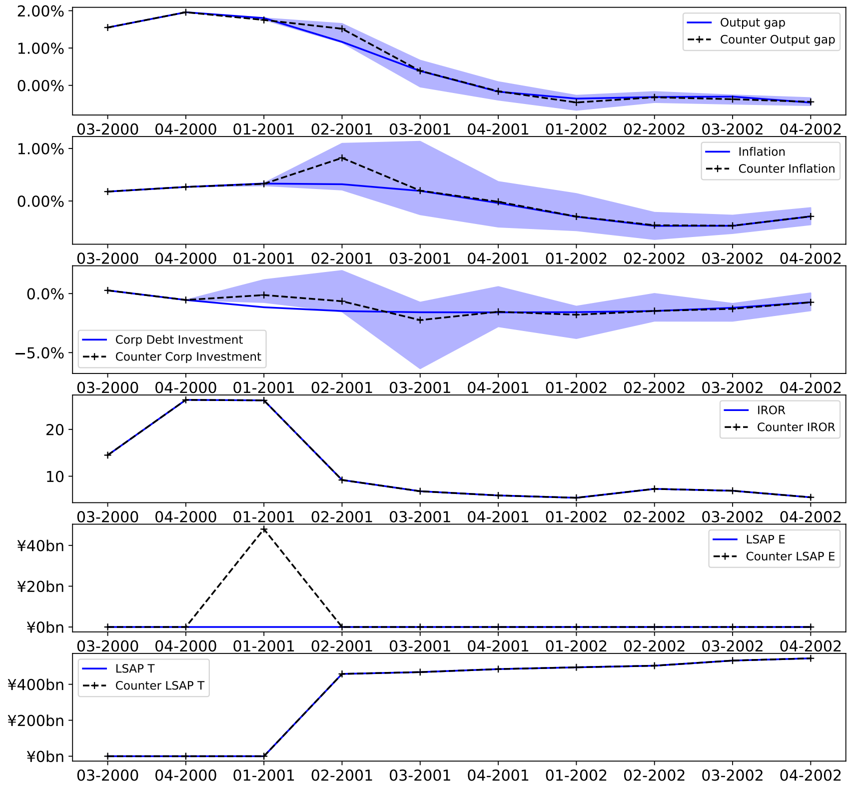

Figure 14: Effect of equity LSAPs

Figure 14 reports the counterfactual outcome of including equity LSAPs. We see for the given cumulative increase of the BOJ's ETF balances shown above, output growth and inflation increase by 1.5% and 2.0%, respectively. We also observe that the corporate debt investment increases by 16.8% more than the observed historical outcome. Meanwhile, the bank reserve rate set by the BOJ along with the BOJ's Treasury bond holdings is set to be the same as the observed historical outcome. We see that despite increased efforts from the policy counterfactual, the Dot-Com bubble could not have been avoided, but it is apparent the sharp decline in inflation and output growth would have been attenuated by a standard equity LSAP policy shock response. **Counterfactual 2: What economic conditions would arise during the Global**

Financial Crisis had the BOJ undertaken Treasury & equity LSAPs together with a decline in the interest rate on bank reserves?

If one observes the BOJ's balance sheet of Treasurys, equities, and interest rate on reserves policy, there seemed to have been an overall unwillingness to implement increases in its balance sheet to address the worsening macroeconomic climate. Similar to the Dot-Com crash, the BOJ also chose to avoid implementing any equity LSAP measures till 2016. Hence in the counterfactual, the BOJ treats the start of GFC with immediate attention by setting:

$\underline{b_t^{cb} = b_t^{cb,obs} + \sigma_b, \text{ for } t = 03/2008 :}$, $\underline{\psi_t^{cb} = \psi_t^{cb,obs} + \sigma_\psi, \text{ for } t = 03/2008 :}$, $\underline{R_t^{re} = R_t^{re,obs} - .1 * \sigma_{re}, \text{ for } t = 03/2008 :}$

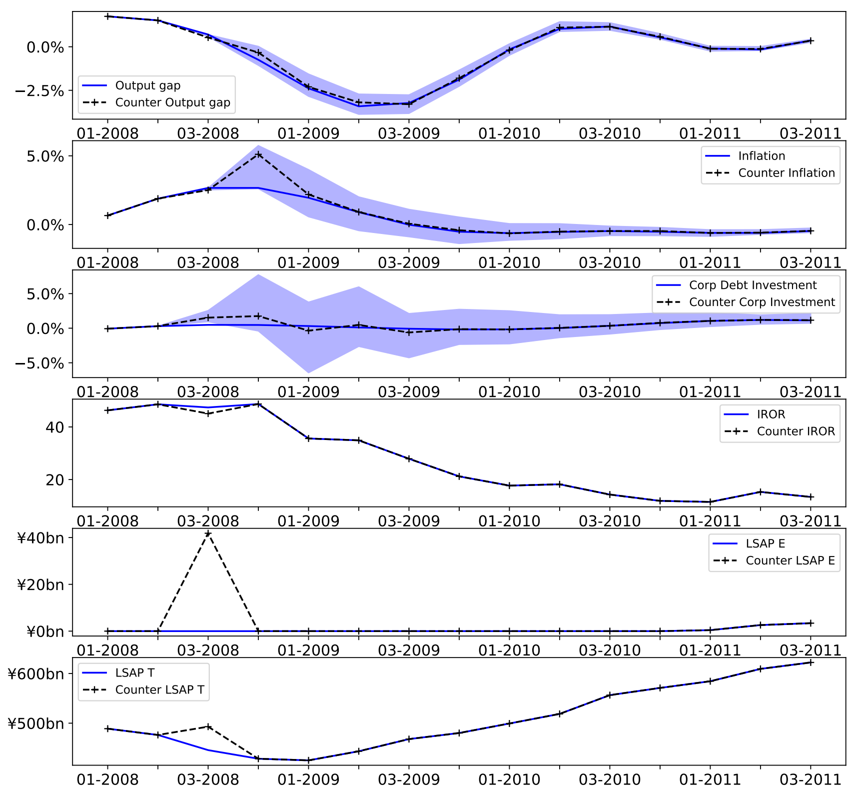

Figure 15: Swift MP Response to the Global Financial Crisis

In Figure 16, we observe the counterfactual effect of the BOJ's alternative LSAP and interest rate policy measures. For the policy experiment, we observe a 72% increase in Treasury LSAPs, a 53% increase in equity LSAPs(compared to the observed outcome), and a 2.33% decline in the interest rate on bank reserves. Such a policy measure implemented in the third quarter of 2008 implies output growth and inflation cumulatively increase by 1.2% and 6.6%, respectively. Meanwhile, the corporate debt investment increases by 12.8%. compared to the counterfactual. Similar to the Dot-Com crash, we see that if the BOJ pursued more aggressive measures, this would have reduced the downward declines in output growth and inflation. But we still see in some sense, the decline would have been inevitable nonetheless.

In summary, LSAPs add flexibility to monetary policy. However, it need not be considered a panacea for any and all economic sluggishness. This is clear from observing results for both counterfactual experiments. While LSAP and interest rates on reserve policies can attenuate sharp drops in output and inflation, they are unable to prevent declines resulting from exogenous credit frictions and conventional demand shocks.

7 Conclusion

Utilizing a medium-scale New-Keynesian model with financial frictions, learning, and a regime-switching equilibrium, this paper empirically investigates Japan's macroeconomic climate over the last three decades. Using an Ensemble Kalman Filter to estimate the nonlinear likelihood, the paper finds that agents in the Japanese macro-economy have a constant gain parameter noticeably higher than the conventional estimates of 0.02. Furthermore, much of the variation in output, inflation, and inflation expectations are largely explained by conventional and financial friction shocks, suggesting that firm and bank-level liquidity is crucial to understanding how output and inflation evolve over time. Using Impulse response analysis, we observe differential effects at the ZLB for LSAPs along with a somewhat muted effect on contractionary monetary policy, implied by the estimated parameters. Lastly, through counterfactual analysis, the paper finds evidence that interest rate policy and LSAP policy, when coordinated together, provide a noticeably higher stimulative impact on the macro-economy than the baseline outcome during the Global Financial Crisis. Furthermore, had the BOJ acquired equities during the Dot-Com crash, this would have stabilized output growth and inflation more than the actual historical outcome. In summary, the paper provides an instructive methodology for other central banks to evaluate LSAPs and resolves the existing literature's mixed results in Japanese LSAPs by taking into account novel financial and pricing frictions, the ZLB, and adaptive learning via Anticipated utility.

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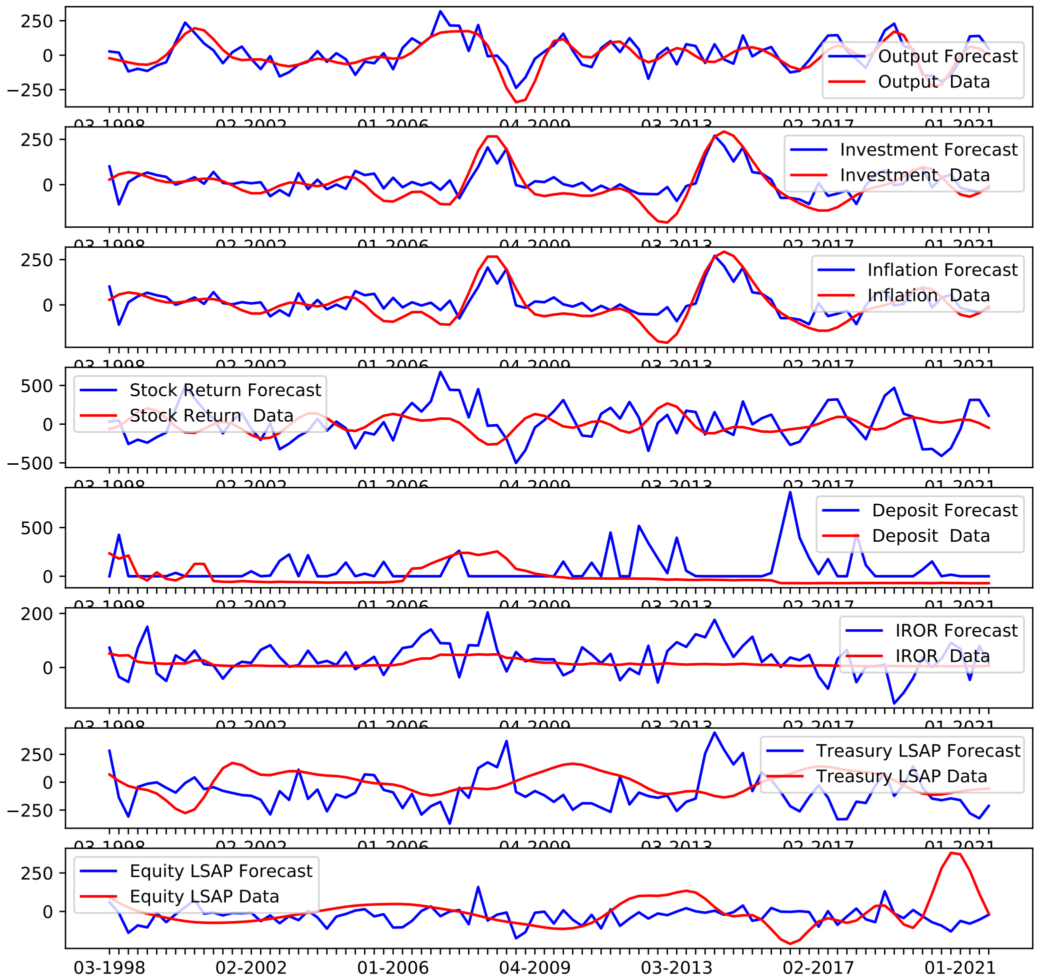
*Appendix

Appendix A: Plots



Appendix B: Intermediary Firm First Order Conditions

First order Conditions Yield:

FOC for capital stock:

$$\frac{\partial \mathcal{L}^I}{\partial K_t} : \alpha m_t^I A_t k_t^{\alpha-1} h_t^{1-\alpha} + Q_t^k - (1-\delta) Q_{t+1}^k - \mu_{2,t}^I \theta_{2,t} Q_t^k = 0$$

FOC for corporate bond issuance:

$$\frac{\partial \mathcal{L}^I}{\partial b_t^k} : -m_{t+1}^I \pi_{t+1}^{-1}(1+\rho_k q_{t+1}^k) - m_t^I C_{1,t}^{I,b^k} + m_{t+1}^I C_{2,t+1}^{I,b^k} - Q_t^k q_t^k + \mu_{t+1}\pi_{t+1}^{-1}(1+\rho_k q_{t+1}^k) = 0$$

FOC for corporate equity issuance:

$$\frac{\partial \mathcal{L}^I}{\partial \psi_t} : -m_{t+1}^I \pi_{t+1}^{-1} d_{t+1} - m_t^I C_{1,t}^{I,\psi} + m_{t+1}^I C_{2,t+1}^{I,\psi} - Q_t^k q_t^\psi = 0 \tag{7.1}$$

FOC for labor demand:

$$\frac{\partial \mathcal{L}^I}{\partial H_t} : m_t^I (1-\alpha) k_t^\alpha h_t^{-\alpha} - m_t^I w_t = 0$$

After re-arranging terms, I obtain an expression for the equilibrium corporate debt and stock issuance:

Expression for intermediary firm corporate bond issuance:

$$(m_{t+1}^I - \mu_{2,t+1}^I)\pi_{t+1}^{-1}(1+\rho_k q_{t+1}^k) + Q_t^k q_t^k + \pi_t^{c_I} \zeta_t^I c_I \{ (\frac{q_t^k}{q_{t-1}^k b_{t-1}^k})^{c_I} (b_t^k)^{c_I - 1} \} m_t^I =$$

$$\frac{\pi_{t+1}^{c_I} m_{t+1}^I \rho_I c_I \zeta_t^I \{ (q_{t+1}^k b_{t+1}^k)^{c_I} \} (q_t^k)^{-c_I}}{(b_t^k)^{c_I + 1}}$$

expression for firm equity issuance:

$$\pi_{t+1}^{-1} m_{t+1}^I d_{t+1} + \pi_t^{c_f} \zeta_t^I c_I \{ (\frac{q_t^\psi}{q_{t-1}^\psi \psi_{t-1}})^{c_I} (\psi_t)^{c_I - 1} \} m_t^I + Q_t^k q_t^\psi$$

$$= \frac{\pi_{t+1}^{c_I} m_{t+1}^I \rho_I c_I \zeta_t^I \{ (q_{t+1}^\psi \psi_{t+1})^{c_I} \} (q_t^\psi)^{-c_I}}{(\psi_t)^{c_I + 1}}$$

Here, $C_{1,t}^x$ represents the optimal choice to incur cost of accumulating asset x_t:

$$C_{1,t}^x \equiv \frac{\partial \tilde{\Omega}_t^f}{\partial x_t}$$

$C^x_{2,t}$ represents the optimal choice to incur the future cost of accumulating asset x_t:

$$C^x_{2,t+1} \equiv \frac{\partial \tilde{\Omega}^f_{t+1}}{\partial x_t}$$

When examining the firm's Foc's for bonds and stocks, I am able to show that holding all else constant, and increase in $\hat{E}_t[\pi_{t+1}]$ yields a decline in both equity & debt issuance by the firm.

Appendix C: Financial Intermediary First Order Conditions

First Order Conditions Yield:

FOC for corporate Bonds:

$$\frac{\partial \mathcal{L}}{\partial b^k_t} : \sum_{k=0}^{\infty} m^f_{t+k+1}(1+\rho_k q^k_{t+1}) - \sum_{k=0}^{\infty} m^f_{t+k} C^{b_k}_{1,t} - \sum_{k=0}^{\infty} m^f_{t+k+1} C^{b_k}_{2,t+1} - \lambda^f_t q^k_t + (1+\rho_k q_{t+1}) \sum_{k=0}^{\infty} \mu^f_{t+k+1} (\sum_{p=0}^{k+1} m^f_p) = 0$$

FOC for government Bonds:

$$\frac{\partial \mathcal{L}}{\partial b^k_t} : \sum_{k=0}^{\infty} m^f_{t+k+1}(1+\rho q_{t+1}) - \sum_{k=0}^{\infty} m^f_{t+k} C^{b}_{1,t} - \sum_{k=0}^{\infty} m^f_{t+k+1} C^{b}_{2,t+1} - \lambda^f_t q_t + (1+\rho q_{t+1}) \sum_{k=0}^{\infty} \mu^f_{t+k+1} (\sum_{p=0}^{k+1} m^f_p) = 0 \tag{7.2}$$

FOC for reserves:

$$\frac{\partial \mathcal{L}}{\partial re_t} : \sum_{k=0}^{\infty} m^f_{t+k+1}(1+R^{re}_t) - \lambda^f_t - \sum_{k=0}^{\infty} \mu^f_{t+k+1} (\sum_{p=0}^{k} m_{t+p})(1+R^{re}_t) = 0 \tag{7.3}$$

Re-arranging the FOC for corporate bond holdings yields:

$$\lambda^f_t q^k_t = (1+\rho_k q^k_{t+1})(\nu^m_{t+1} - \nu^{m,u}_{t+1}) - (\nu^m_t - \nu^{m,u}_t)\pi^{1+c_f}_t c_f \zeta^f_t b^{k,c_f-1}_t (\frac{q^k_t}{q^k_{t-1} b^k_{t-1}})^{c_f}$$

$$+(\nu^m_{t+1} - \nu^{m,u}_{t+1}) \frac{\pi^{1+c_f}_{t+1} c_f \rho_f \zeta^f_t (q^k_{t+1} b^k_{t+1})^{c_f}}{(b^k_{t+1})^{1+c_f} (q^k_t)^{c_f}}$$

Re-arranging the FOC for government bond holdings yields:

$$\lambda^f_t q_t = (1+\rho q_{t+1})(\nu^m_{t+1} - \nu^{m,u}_{t+1}) - (\nu^m_t - \nu^{m,u}_t)\pi^{1+c_f}_t c_f \zeta^f_t b^{c_f-1}_t (\frac{q_t}{q_{t-1} b_{t-1}})^{c_f}$$

$$+(\nu^m_{t+1} - \nu^{m,u}_{t+1}) \frac{\pi^{1+c_f}_{t+1} c_f \rho_f \zeta^f_t (q_{t+1} b_{t+1})^{c_f}}{(b_{t+1})^{1+c_f} (q_t)^{c_f}}$$

Appendix D: MCMC Traceplots

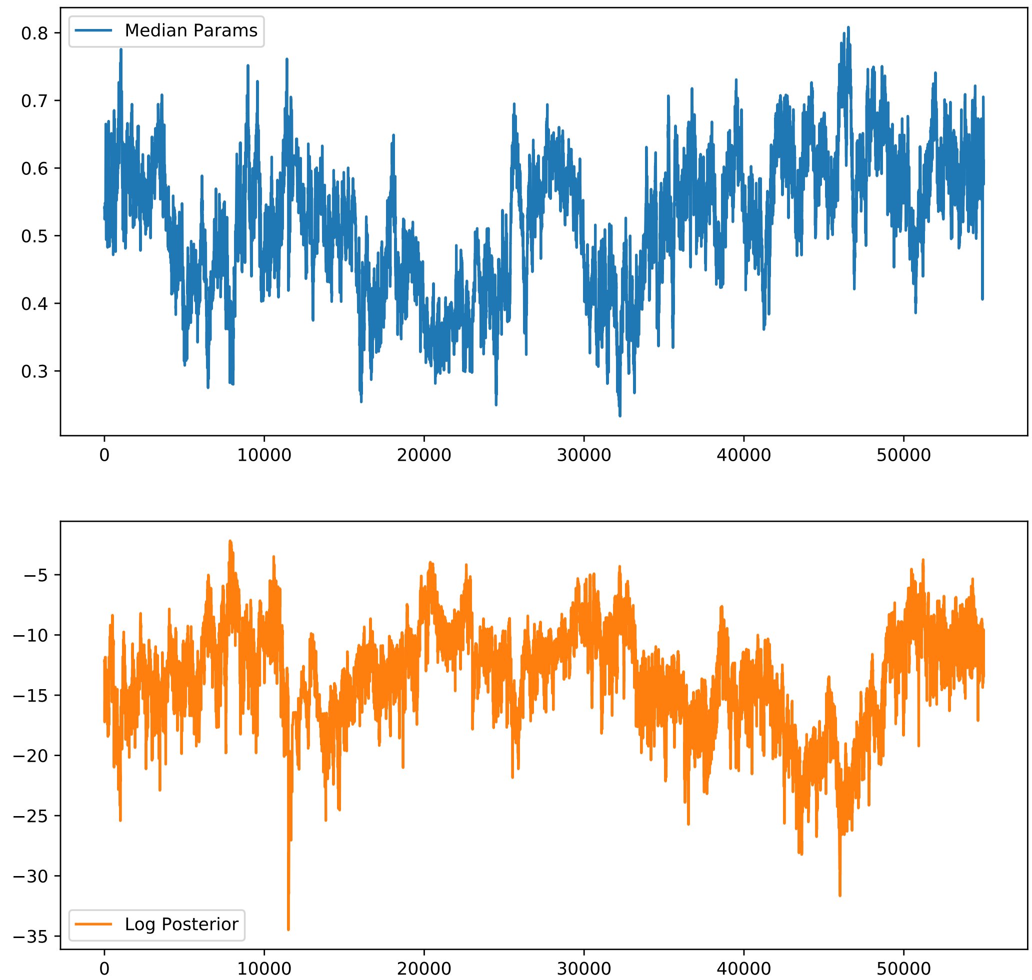


Figure 16: Traceplots

Figure 1 displays the rolling median of the last 50 thousand draws of the Markov Chain. Because we see the plots exhibit stationarity, this suggests strong evidence of model convergence. Likewise, the figure also shows the same but instead of the mean of each parameter draw, we see the log posterior exhibiting stationarity.

Appendix E: Estimation Results

With the stated equations, I am now able to estimate the Likelihood function with Non-linearity taken into account. Equipped with Priors given from the literature[19], I use the Chib & Ramamurthy's (2009) Tailored randomized block random walk Metropolis-Hastings MCMC algorithm. This procedure allows for reduced simulations, while still sufficiently exploring the posterior distribution.[20]

Model Params.							
Para	Descr.	Prior Mean	Prior Std.	Post Mean	$5\%, 95\%$	Dist.	Bound
ψ	Frisch.	$3.3\bar{3}$	.75	3.23	1.7,4.4	$\mathcal{N}$	$0 < x$
γ	CRRA	1.5	.375	2.2	2.03,2.37	Γ	$0 < x$
θ_p	Calvo-prices	.66	.1	.66	.50,.82	Γ	$0 < x$
θ_w	Calvo-wages	.66	.1	.65	.49,.82	Γ	$0 < x$
c_I	Fin Adj.	.	.	.50	.05,.88	U	$0 < x$
ϕ_b	Tax Resp.	.	.	1.19	.05,4.54	U	$0 < x$
ρ_u	Dem.	.	.	.51	.05,.95	U	$0 < x$
ρ_r	Dem.	.	.	.43	.05,.91	U	$0 < x$
ρ_a	Prod.	.	.	.39	.04,.81	U	$0 < x$
ρ_ψ	lsap E.	.	.	.43	.08,.88	U	$0 < x$
ρ_b	lsap T.	.	.	.46	.03,.94	U	$0 < x$
ρ_{θ_1}	FI.	.	.	.49	.06,.92	U	$0 < x$
ρ_{θ_2}	Int. Firm.	.	.	.49	.05,.94	U	$0 < x$
ρ_I	AR(1) Adj.	.	.	.27	.07,.86	U	$0 < x < 1$

[19] Del Negro & Schorfheide (2008)
[20] See Appendix for convergence results

Model Params Cont.							
Para	Descr.	Prior Mean	Prior Std.	Post Mean	$5\%, 95\%$	Dist.	Bound
σ_u	std dev.	.5	inf	.44	.11,1.0	Γ^{-1}	$0 < x$
σ_r	std dev.	.5	inf	.82	.12,1.9	Γ^{-1}	$0 < x$
σ_a	std dev.	.5	inf	.42	.1,1.1	Γ^{-1}	$0 < x$
σ_ψ	std dev.	.5	inf	.45	.11,1.3	Γ^{-1}	$0 < x$
σ_b	std dev.	.5	inf	.18	.07,1.06	Γ^{-1}	$0 < x$
σ_ψ	std dev.	.5	inf	.19	.1,.34	Γ^{-1}	$0 < x$
σ_{θ_1}	std dev.	.5	inf	.65	.11,1.5	Γ^{-1}	$0 < x$
σ_{θ_2}	std dev.	.5	inf	.45	.12,1.0	Γ^{-1}	$0 < x$
σ_{re}	std dev.	.5	inf	.49	.12,1.1	Γ^{-1}	$0 < x$
σ_τ	std dev.	.5	inf	.86	13,1.8	Γ^{-1}	$0 < x$
σ_w	std dev.	.5	inf	.47	.11,1.0	Γ^{-1}	$0 < x$
ρ	infl. pers.	.875	.1	.86	.71,.98	Γ	$0 < x$
ϕ_x	Output resp.	.15	.1	.14	.03,.33	Γ	$0 < x$
ϕ_π	Infl. resp.	1.7	.1	1.7	1.6,1.9	Γ	$0 < x$
g	rls gain.	.031	.022	.035	.012,.07	Γ	$0 < x$

Appendix F: Ensemble Kalman Filter and Particle Filter Algorithm:

Ensemble Kalman Filter Algorithm:

The Observation equation of the model can also be expressed as follows:

$$Y_t^{obs} = H_{t-1}^1 S_t + H_{t-1}^2 S_{t-1} + \epsilon_t^m$$

Where:

$$H_{t-1}^1 \equiv H^1(S_{t-1}, \theta)$$

$$H^2_{t-1} \equiv H^2(S_{t-1}, \theta)$$

Here, because we have expectation level data of Japan, $\hat{E}_t M_t$ is a function of $\Lambda_{t-1}, \mu_{t-1}, \& M_{t-1}$. Where, $\Lambda_{t-1}, \mu_{t-1}, \& M_{t-1} \in S_{t-1}$.

The State Transition equation is expressed as follows:

$$S_t = f(S_t, \epsilon^S_t)$$

First, I take a set of successive draws from ϵ^S_t, and call them $\epsilon^{j,S}_t$.

Next, I propagate the state space and obtain:

$$S^j_t = f(S^j_t, \epsilon^{j,S}_t)$$

$$\hat{Y}^j_t = H^1_{t-1} S^j_t + H^2_{t-1} S^j_{t-1|t-1}$$

After obtaining the model-implied forecasts, we compute the recursive covariance matrix and empirical updates. The key difference in the Ensemble is that the Covariance matrix, is not known in closed form, and hence must be approximated via an empirical covariance computation. Below outlines the corresponding recursions:

$$\hat{P}^S_{t|t-1} = V\hat{A}R(S_t|\mathcal{I}_{t-1}) = \frac{1}{J}\sum_j (S^j_{t|t-1} - \hat{E}[S_{t|t-1}])'(S^j_{t|t-1} - \hat{E}[S_{t|t-1}])$$

$$D_t = V\hat{A}R(Y^{obs}_t|\mathcal{I}_{t-1}) = H^1_{t-1|t-1}\hat{P}^S_{t|t-1}(H^1_{t-1|t-1}) + I$$

$$L_t = C\hat{o}v(S_t, Y^{obs}_t|\mathcal{I}_{t-1}) = H^1_{t-1|t-1}\hat{P}^S_{t|t-1}$$

$$\hat{P}^S_{t|t} = V\hat{A}R(S_t|\mathcal{I}_t) = \hat{P}^S_{t|t-1} - L'_t(D_t)^{-1}L_t$$

$$S^j_{t|t} = S^j_{t|t-1} + L_t(D_t)^{-1}\hat{e}^j_t$$

$$\hat{e}^j_t \equiv Y^j_{t|t-1} - Y^m_t$$

Particle Filter Algorithm:

Step 1(Initialize): Set $e^j_t \sim \bar{\epsilon}_t$.

Step 2(Propagate): $V^j_t = \mathcal{F}(V^j_{t-1}, \epsilon^j_t)$.

Step 3(Evaluate): $w^j_t = \dfrac{P(Y_t|V^j_{t|t-1}, \Theta)}{\Sigma w^i_t}$.

$$\textbf{Step 4}(\text{Re-sample}):\ q_i \sim \{w_t\}_{j=0}^{J}$$

$$\text{and set: } V_{t|t-1}^{j} = V_t^i, \text{ for all } \{q_i\}_{i=0}^{J}$$

$$\text{Set } t = t+1; \text{and repeat till } t = T.$$

$$\textbf{Step 5}(\text{Calculate Likelihood}):\ P(Y^T|\theta) \approx \frac{1}{J}(\prod_{t=1}^{T}(\sum_{j=1}^{J} p(Y_t|w_t^j, V_{t|t-1}^j, Y^{t-1}, \theta))).$$

With the stated procedure, as the number of particles, J becomes greater, the Likelihood converges to the true distribution. However, because one must keep track of the states and their relative values across the sample, there exists a trade-off between and accuracy and computational time of the algorithm. In this paper, I find that the results of estimation do not change much between 1 - 10 thousand particles. Hence I assume J to be 5 thousand, a sufficient approximation to the true Likelihood function of interest in the proceeding results.

Appendix G: Calvo-Wages:

Hence, the labor union faces the following problem:

$$\psi_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$$

$$\max_{H_t(l)} W_t \left(\int_0^1 H_t(l)^{\psi_w} dl \right)^{\psi_w^{-1}} - \int_0^1 W_t(l) H_t(l) dl$$

Solving this yields:

$$H_t(l) = (\frac{W_t(l)}{W_t})^{-\epsilon_w} H_t$$

When households decide their labor supply, they take the above result from the union as given. Consequently, each household solves for its optimal labor supply via its optimal wage. To induce friction in the wage setting process, I assume the household has probability θ_w of keeping the wage they set in the previous period[21]. Hence, the household optimizes wage considering this friction and seeks to maximize their discounted utility via the following problem:

$$\max_{w_t^*} \hat{E}_t \sum_{k=0}^{\infty} \Lambda_{t+k} \theta_w^k B_{t+k}$$

[21] See Calvo(1983)

$$B_{t+k} = (-\frac{H_{t+k}^{1+\psi}}{1+\psi})(\frac{W_t^*}{W_{t+k}})^{-\epsilon_w(1+\psi)} + \lambda_{t+k}W_{t+k}(l)H_{t+k}(l)$$

$$H_{t+k}(l) = (\frac{W_t^*}{W_t})^{-\epsilon_w}H_t)$$

$$\frac{\partial}{\partial W_t^*} : (W_t^*)^{1-\alpha_0-\epsilon_w} = \sum_{k=0}^{\infty}(\beta\theta_w)^k\psi_w\{\frac{N_{t+k}^{1+\psi}W_{t+k}^{\alpha_0}}{W_t^{\epsilon_w}N_{t+k}\lambda_{t+k}}\}$$

Using the Expression for Aggregate Wages:

$$W_t^{1-\epsilon_w} = (1-\theta_w)(W_t^*)^{1-\epsilon_w} + \theta_w W_{t-1}^{1-\epsilon_w}$$

And after Log-Linearizing, I obtain:

$$w_{t+1} = (1-\theta_w)\alpha_1\sum_{k=0}^{\infty}(\beta\theta_w)^k\hat{E}_t\{(O_{t+k}^1 - O_{t+k}^2)\} + \theta_w w_{t-1} + \theta_w w_t$$

$$O_t^1 = (1+\psi)n_t + \alpha_0 w_t$$

$$\begin{aligned} O_t^2 &= \epsilon_w w_t + n_t + \lambda_t(1+\theta_w+\epsilon_w-\alpha_0)w_t \\ &= (1-\theta_w)\alpha_1\psi n_t - (1-\theta_w)\alpha_1\lambda_t + \hat{E}_t w_{t+1} + \theta_w w_{t-1} \end{aligned}$$

$$(1+\theta_w)w_t = \hat{E}_t w_{+1} + O_t^1 - O_t^2 + \theta_w w_{t-1}$$

Appendix H: Calvo-Prices:

$$max_{p_t^*}\ E_t[\sum_{k=0}^{\inf}(\theta\beta)^k Q_{t+k}A_{t+k}]$$

$$A_{t+k} \equiv (\frac{p_t^*}{P_{t+k}})^{-\epsilon}Y_{t+k} - (\frac{p_t^*}{P_{t+k}})^{-\epsilon}(\frac{\Phi_{t+k}}{P_{t+k}})Y_{t+k}$$

$$\frac{\partial}{\partial p_t^*} := E_t[\sum_{k=0}^{\inf}(\beta\theta)^k\{C_{t+k}^{-\sigma}B_{t+k}\}] = 0$$

$$B_{t+k} \equiv (1-\epsilon)(p_t^*)^{-\epsilon}p_{t+k}^{\epsilon-1}Y_{t+k} + \epsilon(p_t^*)^{-\epsilon-1}p_{t+k}^{\epsilon}\Phi_{t+k}Y_{t+k}$$

$$\sum_{k=0}^{\inf}(\beta\theta)\bar{C}^{-\sigma}\bar{B} = 0$$

$$(1-\beta\theta)^{-1}\bar{C}^{-\sigma}\bar{B} = 0$$

Hence in Steady State:

$$\bar{B} = 0$$

$$\bar{B}_1 = -\bar{B}_2$$

Where:

$$\bar{B}_1 \equiv (1-\epsilon)pY$$

$$\bar{B}_2 \equiv -\epsilon p \Phi Y$$

$$\partial log(\sum (\beta\theta)^k C_{t+k}^{-\sigma} B_{t+k}) = 0$$

$$\frac{\partial(\sum (\beta\theta)^k C_{t+k}^{-\sigma} B_{t+k})}{(1-\beta\theta)^{-1} C^{-\sigma} \bar{B}} = 0$$

$$\sum (\beta\theta)^k \partial log(C_{t+k}^{-\sigma}) \partial log(B_{t+k}) = 0$$

$$\partial log(B_{t+k}) = \partial log(B_{1,t+k} + B_{2,t+k})$$

$$\partial log(B_{t+k}) = \frac{\partial(\bar{B}_1 B_{1,t+k} - \bar{B}_2 B_{2,t+k})}{\bar{B}}$$

$$\partial log(B_{t+k}) = \frac{B_1 \partial(B_{1,t+k} - B_{2,t+k})}{B_1 + B_2}$$

$$\partial log(B_{t+k}) = \alpha_\pi(\partial log(B_{1,t+k}) - \partial log(B_{2,t+k}))$$

$$\alpha_\pi \equiv \frac{B_1}{B_1 + B_2} = \frac{1}{1 + \dfrac{B_2}{B_1}} = \frac{1}{1 + (\frac{\epsilon}{\epsilon - 1})\bar{\phi}}$$

Using the following linearization, I now obtain the following result:

$$\sum_{k=0}^{\inf} (\beta\theta)^k p_t^* =_t \sum_{k=0}^{\inf} (\beta\theta)^k (p_{t+k} + \alpha_\pi^{-1} c_{t+k+1} + \phi_{t+k})$$

$$p_t^* = (1-\beta\theta)(p_t + \alpha_\pi^{-1} E_t c_{t+1} + \phi_t) + (1-\beta\theta) \sum_{k=1}^{\inf} (\beta\theta)^k (p_{t+k+1} + \alpha_\pi^{-1} \sigma c_{t+k+2} + \phi_{t+1})$$

$$p_t^* = (1-\beta\theta)(p_t + \delta\alpha_\pi^{-1} E_t c_{t+1} + \phi_t) + \beta\theta E_t p_{t+1}^*$$

Aggregate Price Identity:

$$P_t^{1-\epsilon} = [(1-\theta)(p_t^*)^{1-\epsilon} + \theta P_{t-1}^{1-\epsilon}]$$

Log-Linearized, this becomes:

$$\frac{\pi_t}{1-\theta} = p_t^* - p_t$$

$$\pi_t = (1-\theta)(1-\beta\theta)(\phi_t + \alpha_\pi^{-1} \delta E_t c_{t+1} + \beta\theta E_t \pi_{t+1})$$

Appendix I: Stochastic Gradient Learning:

As described in Evans(2010). I use the Generalized Stochastic Gradient algorithm 1. This means I must derive a value for the inverse of the co-variance of regressors. I start with the perceived law of motion:

$$Z_t = M_m^t Z_{t-1} + M_u^t U_t$$

$$U_t = RU_{t-1} + \epsilon_t^u$$

Hence, the perceived law of motion can be characterized as:

$$H_t \equiv \begin{bmatrix} Z_t \\ U_t \end{bmatrix}$$

$$H_t = K_1 H_{t-1} + K_2 \epsilon_t^u + \eta_t^h$$

$$var(H^*) = K_1 var(H^*) K_1' + K_2 \epsilon_t^u K_2' + c_s^2 I$$

$$var(H^*) = (N_1 K_2)\Omega(K_2 N_1)' + c_s^2 (N_1 K_2)(K_2 N_1)'$$

Note in the above, I define $var(H^*)$ as the variance-covariance matrix of the regressor, H_t.
The above perceived law of motion contains the term η_t^h that accounts for the uncertainty the agent perceives in her regression specification. I set the standard deviation of the forecast error, $c_s = 1e3$. I choose this number because this is the lowest value such that I can use a constant gain of .02 without yielding an explosive equilibrium. Though this may be considered an ad-hoc choice, for empirical purposes, we are interested in an R matrix centered around the model equations such that they behave stable with a well-supported constant gain learning parameter within the neighborhood of 0.02.
Because $Eh_t' h_t$ is equivalent to $var(h)$, I am able to define both R and R_z that go into the stochastic gradient algorithm as $R = var(H^*)^{-1}$ and $R_z = var(H_z^*)^{-1}$. Where:

$$var(H_z^*) = N_1^z K_2^z \Omega(K_2^z N_1^z)' + c_s^2 (N_1^z K_2^z)(K_2^z N_1^z)'$$

Note the superscript z denotes the entries of the matrix that come from the solution to the rational expectations equilibrium when $i_t = 0$. Hence, the parameter update follows the following process as described in Evans(2010):

$$\phi_t = \phi_t + gRh_{t-1}(y_{t-1} - \phi'_{t-1}h_{t-1})'$$

Because the agent has two equilibria to consider when forming expectations, I modify the learning rule in the following manner:

$$\tilde{\phi}_{t-1} = \mu_{t-1}\phi^z_{t-1} + (1 - \mu_{t-1})\phi_{t-1}$$

$$\phi_t = \phi_t + gRh_{t-1}(y_{t-1} - \tilde{\phi}'_{t-1}h_{t-1})'$$

$$\phi^z_t = \phi^z_{t-1} + gR_z h_{t-1}(y_{t-1} - \tilde{\phi}'_{t-1}h_{t-1})' \text{ if } t = 0$$

Note, in the model, I assume the agent updates ϕ^z_t only when $i_t = 0$. And I also assume ϕ_t updates at all times.

Appendix J: LSAP Comparative Statics

Comparative Statics: Expected Future Inflation

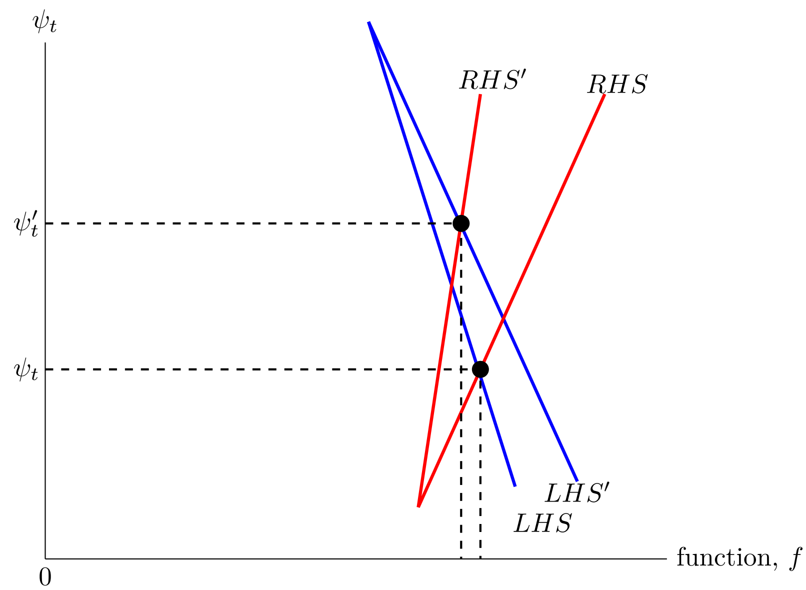
When examining the firm's First order conditions for bonds and stocks, I am able to show that holding all else constant, and increase in $E_t[\pi_{t+1}]$ yields a decline in total equities issued by the firm. I obtain the following relationship for stock issuance after arranging terms in (A0):

$$\text{(RHS) } m^I_{t+1}\pi^{-1}_{t+1}d_{t+1}\psi^{1-c_I}_t + \pi^{c_I}_t\zeta^I_t c_I\{\Delta_\psi(\frac{q^\psi_t}{q^\psi_{t-1}\psi_{t-1}})^{c_I}\}m^I_t + Q^k_t q^\psi_t \psi^{1-c_I}_t$$

$$= \frac{\pi^{c_f}_{t+1}m^I_{t+1}\rho_I c_I \zeta^I_t\{\Delta_\psi(q^\psi_{t+1}\psi_{t+1})^{c_I}\}(q^\psi_t)^{-c_I}}{(\psi_t)^{2c_I}} \tag{7.4}$$

Above, we can see that the RHS is increasing in ψ_t, while the LHS is decreasing in ψ_t. Hence, holding all else constant, an increase in π^{-1}_{t+1}, will result in a decline in ψ_t for the equality to hold. We can see this graphically on the next page.

Equity Issuance after an increase in $E_t\pi_{t+1}$:



Below, I have plotted the LHS and RHS of (7.4). Given an increase in $E_t\pi_{t+1}$, the RHS decreases, while the LHS shifts downward.

Comparative Statics: LSAP Given the stated structure for asset prices and firm capital issuance:

$$\frac{\partial q_t^{\psi}}{\partial \psi_t^{cb}} \geq 0$$

$$\frac{\partial q_t}{\partial b_t^{cb}} \geq 0$$

That is, if the Central bank engages in Treasury bond purchases or Stock purchases, holding all else constant, this will yield an increase in treasury bond and stock prices, respectively. Note: for the positive effect on stock prices, I assume ρ_I is sufficiently low.

Recall the Treasury Flow budget constraint is:

$$q_t b_t = \pi_t^{-1}(1+\rho q_t)b_{t-1} - \pi_t^{-1}\rho q_t b_{t-1}^{cb} - \pi_t^{-1} d_t \psi_{t-1}^{cb} + \pi_t^{-1}(1+R_{t-1}^{re})re_{t-1} - \tau_t$$

Re-arranging the Treasury's flow budget constraint (A2), or treasury bond supply, I obtain:

$$q_t = b_t^{-1}\Big(\pi_t^{-1}\{(1+\rho q_t)b_{t-1} - \rho q_t b_{t-1}^{cb} - d_t\psi_{t-1}^{cb} + (1+R_t^{re})re_{t-1}\} - \tau_t\Big) \tag{7.5}$$

Substituting the expression for Government bond price (7.2) into the expression for bank reserves (7.3) obtained from the financial intermediary's First order conditions, I obtain:

$$(1+R_t^{re})(\nu_t^m-\nu_t^{m,u})q_t=(1+\rho_k q_{t+1})(\nu_{t+1}^m-\nu_{t+1}^{m,u})-\zeta_t\Delta_b(\frac{q_t}{q_{t-1}b_{t-1}^{fi}})^{c_f}c_f(b_t^{fi})^{c_f-1}(\nu_t^m-\nu_t^{m,u})$$

$$+\rho_f\zeta_t^f\Delta_b(q_{t+1}b_{t+1}^{fi})^{c_f}q_t^{-c_f}(b_t^{fi})^{-c_f-1}(\nu_{t+1}^m-\nu_{t+1}^{m,u}) \tag{7.6}$$

After dividing both sides of (7.6) by $(1+R_t^{re})(\nu_{t+1}^m-\nu_{t+1}^{m,u})$, I obtain:

$$q_t=(1+R_t^{re})^{-1}\Big((1+\rho q_{t+1})(\frac{\nu_{t+1}^m-\nu_{t+1}^{m,u}}{\nu_t^m-\nu_t^{m,u}})+\frac{\rho_f\zeta_t^f\Delta_b(q_{t+1}b_{t+1}^{fi})^{c_f})}{q_t^{c_f}(b_t^{fi})^{c_f+1}}\Big)$$

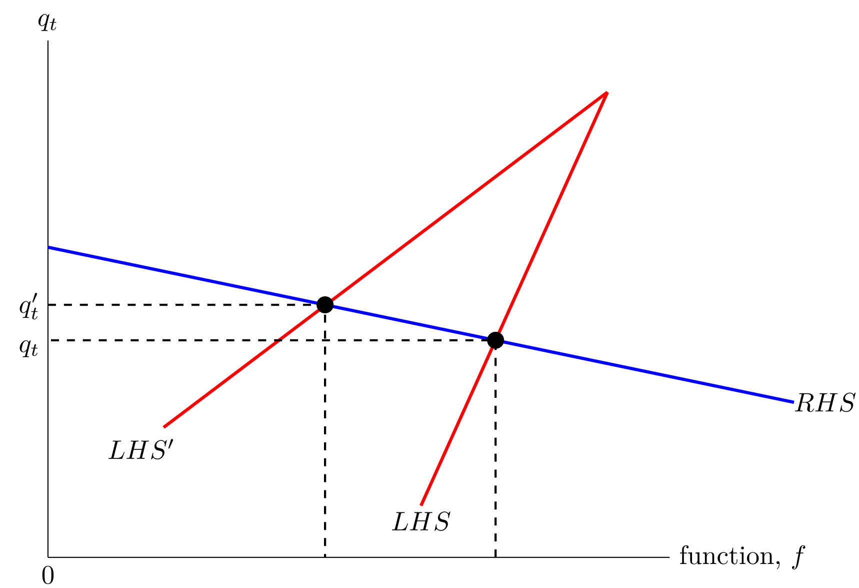
$$-(1+R_t^{re})^{-1}\Big(q_t^{c_f}(b_t^{fi})^{c_f-1}\zeta_t^f\Delta_b c_f(q_{t-1}b_{t-1}^{fi})^{-c_f}(\frac{\nu_{t+1}^m-\nu_{t+1}^{m,u}}{\nu_t^m-\nu_t^{m,u}})\Big) \tag{7.7}$$

Next, I substitute (7.5) into the LHS of (7.7) and add the bottom term of (7.7) to both sides, and obtain:

$$\begin{aligned}\text{(RHS) } (1+R_t^{re})^{-1}\{&\frac{\rho_f\zeta_t^f\Delta_b(q_{t+1}b_{t+1}^{fi})^{c_f})}{q_t^{c_f}(b_t^{fi})^{c_f+1}}+(1+\rho q_{t+1})(\frac{\nu_{t+1}^m-\nu_{t+1}^{m,u}}{\nu_t^m-\nu_t^{m,u}})\}\\ &=\Big((1+R_t^{re})^{-1}q_t^{c_f}(b_t^{fi})^{c_f-1}\zeta_t\Delta_b c_f(q_{t-1}b_{t-1}^{fi})^{-c_f}(\frac{\nu_{t+1}^m-\nu_{t+1}^{m,u}}{\nu_t^m-\nu_t^{m,u}})+\\ &\frac{(\pi_t^{-1}\{(1+\rho q_t)b_{t-1}-\rho q_t b_{t-1}^{cb}-d_t\psi_{t-1}^{cb}+(1+R_t^{re})re_{t-1}\}-\tau_t)}{b_t^{cb}+b_t^{fi}}\Big)\text{ (LHS)}\end{aligned} \tag{7.8}$$

(7.8) tells us the LHS is increasing in q_t while the RHS is decreasing in q_t. Given an increase in b_t^{cb}, we know the LHS decreases while the RHS remains unchanged. Note: when arriving at this conclusion, I use the fact that $b_t=b_t^{fi}+b_t^{cb}$. Hence, all else equal, when b_t^{cb} increases, the denominator of the LHS decreases while RHS remains unchanged, leading to a higher equilibrium bond price. Below is an illustration of the comparative statics:

Bond Price Dynamics:



<u>Stock Price Dynamics:</u>

Recall, The household's stock demand via the FOC yields:

$$q_t^{\psi} = E_t[\pi_{t+1}^{-1}(\frac{c_t}{c_{t+1}})^{\sigma}(q_{t+1}^{\psi} + D_{t+1})]$$

Because the above expression is not a function of ψ_t, we need only examine the Intermediary firm's FOC for equity issuance (32).

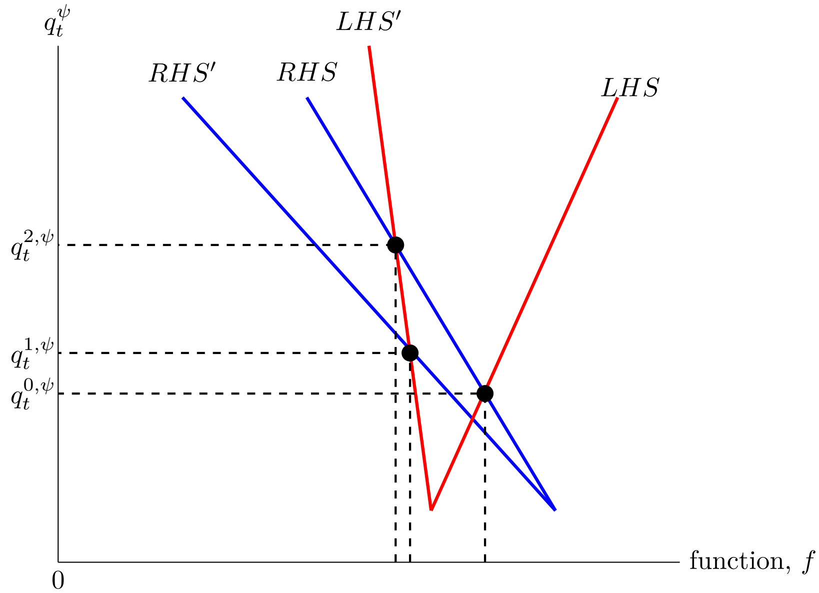
Recall (7.1), The equity supply, is:

$$\text{(LHS) } \pi_{t+1}^{-1} m_{t+1}^I d_{t+1} + \pi_t^{c_f} \zeta_t^I c_I \{\Delta_{\psi}(\frac{q_t^{\psi}}{q_{t-1}^{\psi}\psi_{t-1}})^{c_I}(\psi_t)^{c_I - 1}\} m_t^I + Q_t^k q_t^{\psi}$$

$$= \frac{\pi_{t+1}^{c_I} m_{t+1}^I \rho_I c_I \zeta_t^I \{\Delta_{\psi}(q_{t+1}^{\psi}\psi_{t+1})^{c_I}\}(q_t^{\psi})^{-c_I}}{(\psi_t)^{c_I+1}} \text{ (RHS)} \tag{7.9}$$

(7.9) tells us the LHS is increasing in q_t^{ψ}, while the RHS is decreasing in q_t^{ψ}. Given an increase in ψ_t^{cb}, we know $\psi_t(= \psi_t^{hh} + \psi_t^{cb})$ increases. Thus the RHS decreases and the LHS decreases. Meaning that one cannot determine the outcome of the resulting price. However, if we assume ρ_I, the persistence of the investment adjustment cost shock, is sufficiently 'low'. This would allow us to say q_t^{ψ} will increase. In the next page, I plot the corresponding results:

Stock Price Dynamics:



Below I have plotted the resulting outcome given a small enough ρ_I. Here, $q_t^{0,\psi}$ represents the initial price without a change in central bank equity purchases. Now, let's suppose the central bank increases ψ_t^{cb}. This will increase ψ_t, and lead to the RHS shifting to RHS' and the LHS shifting to LHS' seen in equation (7.9). In the case where $\rho_I = 0$, the equilibrium price is $q_t^{2,\psi}$. In the case where ρ_I is small but not zero, the resulting price is $q_t^{1,\psi}$. Hence depending on the magnitude of ρ_I, RHS' can shift outward by any possible range, but with a conservative estimate for this parameter, we will have the possible equilibrium prices where: $(q_t^{1,\psi}, q_t^{2,\psi}) > q_t^{0,\psi}$.

Appendix K: Aggregation

$$\int_0^1 c_t(j)dj = c_t$$

$$\int_0^1 y_t(j) = y_t$$

$$\int_0^1 h_t(j)dj = h_t$$

$$\int_0^1 P_t(j)dj = P_t$$

$$\int_0^1 k_t(j)dj = k_t$$

$$\int_0^1 \psi_t^{hh}(j)dj = \psi_t^{hh}$$

$$\int_0^1 b_t^{fi}(j)dj = b_t^{fi}$$

$$\int_0^1 s_t(j)dj = s_t$$

$$\int_0^1 b_t^k(j)dj = b_t^k$$

$$\int_0^1 re_t(j)dj = re_t$$

$$\int_0^1 Q_t(j)dj = Q_t$$

$$\int_0^1 \mu_t(j)dj = mu_t$$

$$\int_0^1 \mu_t^2(j)dj = mu_t^2$$

$$\int_0^1 W_t(j)dj = w_t$$

$$\int_0^1 d_t(j)dj = d_t$$

$$\int_0^1 nu_t^c(j)dj = nu_t^c$$

$$\int_0^1 \nu_t^m(j)dj = \nu_t^m$$

$$\int_0^1 \nu_{m,ut}(j)dj = \nu_{m,ut}$$

$$\int_0^1 n_t(j)dj = n_t$$

$$\int_0^1 \nu_t(j)dj = \nu_t$$

$$\int_0^1 \phi_t(j)dj = phi_{tt}$$

Appendix L: Macaulay duration

In order to properly evaluate the parameters of the corporate and Treasury consol bonds ($\rho \& \rho_k$) to the observed expectations & real bond yield data, I follow a similar methodology to Matveev(2016) to compute the Macaulay duration(evaluated at the steady state) as a function of the ρ_k & ρ I seek to use for estimation.

$$D_t^k = \sum_j (\frac{\beta\rho_k}{R})^j \frac{q_{t+j}^k}{q_t^k}$$

$$D_t^k = (\frac{\beta\rho_k}{R})^{-1} \sum_j (\frac{\beta\rho_k}{R})^{j+1} \frac{q_{t+j}^k}{q_t^k}$$

$$D_t^k = (\frac{\beta\rho_k}{R})^{-1} \frac{\partial}{\partial x} \sum_{j=0} (\frac{\beta\rho_k}{R}^j))$$

Likewise, for Treasury bonds, I obtain:

$$D_t = (\frac{\beta\rho}{R})^{-1} \frac{\partial}{\partial x} \sum_{j=0} (\frac{\beta\rho}{R}^j))$$

Here, D_t represents the observed average duration of Japanese corporate or Treasury debt obtained from the World Bank. I use a numerical solution to find ρ and ρ_k.