Phase transition of four-dimensional Ising model with higher-order tensor renormalization group

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(SA, Y. Kuramashi, T. Yamashita, and Y. Yoshimura, (2019), arXiv: 1906.06060 [hep-lat], Accepted by PRD)

We apply the higher-order tensor renormalization group (HOTRG) to the four-dimensional ferromagnetic Ising model, which has been attracting interests in the context of the triviality of the scalar ϕ^4 theory. We investigate the phase transition of this model with HOTRG enlarging the lattice size up to 1024^4 with parallel computation. The results for the internal energy and the magnetization are consistent with the weak first-order phase transition.

§ Introduction

ϕ^4 theory → Infinite-coupling limit → **Ising model**

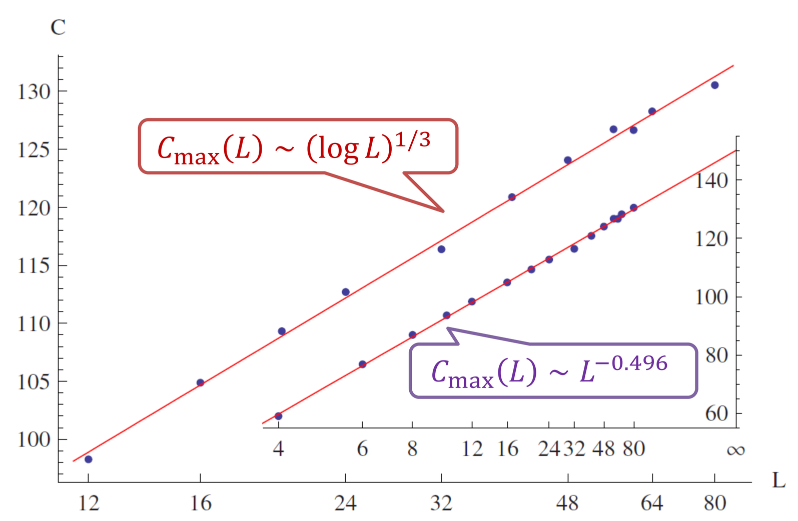
At the upper critical dimension, the perturbative RG predicts the leading scaling behaviors obey the mean-field exponents, but there emerge logarithmic corrections, e. g. $C \sim |t|^0 (\log|t|)^{1/3}$ Wegner-Riedel, PRB7(1973)248

If the leading scaling behavior is the mean-field type and it is modified just by the multiplicative logarithmic factor, then the theory is trivial in the continuum limit
Kenna-Lang NPB393(1993)461, Kenna NPB691(2004)292

The latest Monte Carlo study

- Finite-size scaling analysis with linear system sizes $L \leq 80$ Lundow-Markstrom PRE80(2009)031104

Maximum value of specific heat

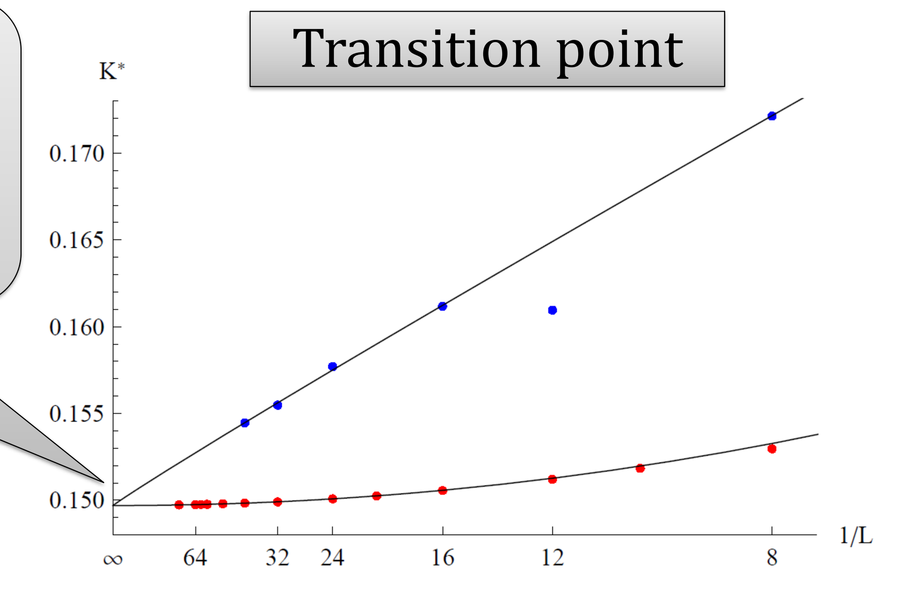


$L = 80$ is too small to catch the logarithmic divergence
OR
No logarithmic correction (specific heat is bounded also in the infinite volume)

- Non-vanishing boundary effect Lundow-Markstrom NPB845(2011)120

Fitting curves are forced to result in the same transition point in the infinite volume

Transition point



Worth trying different numerical approaches other than the Monte Carlo method!

Red : $L \in [4, 80]$ with the periodic boundary
Blue: $L \in [4, 40]$ with the open boundary

§ Tensor Network Scheme

- Application of higher-order tensor renormalization group (HOTRG) to the four-dimensional Ising model

1) Express the target function as tensor contraction
2) Evaluate it employing the low-rank approximation of tensor(s)
Advantage: we can directly deal with thermodynamic lattice!

※ HOTRG has been applied successfully to the three-dimensional Ising model Xie et al. PRB86(2012)045139, Wang et al. CPL31(2014)070503

The four-dimensional HOTRG

Straightforward application → Memory ~ $\mathcal{O}(D_{\mathrm{cut}}{}^8)$, Computational time ~ $\mathcal{O}(D_{\mathrm{cut}}{}^{15})$

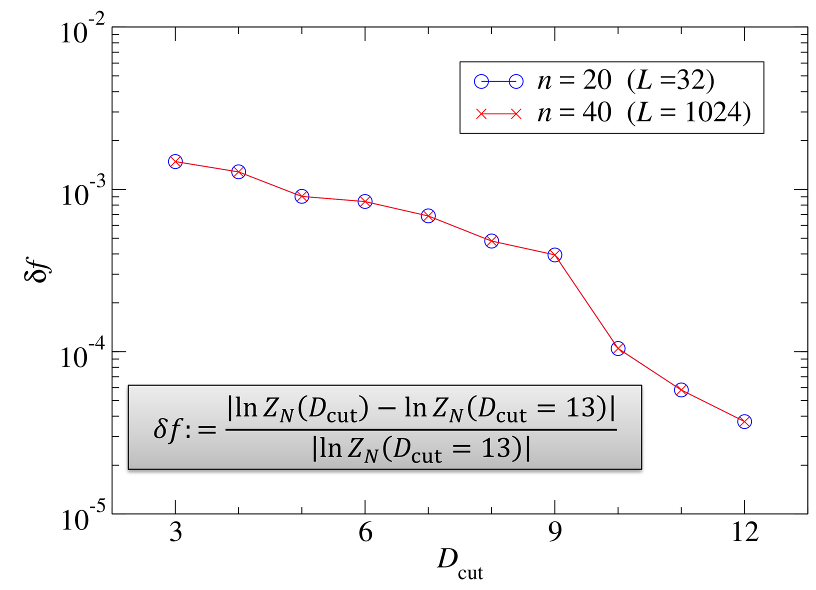
Our implementation → **Parallel computation for HOTRG with $D_{\mathrm{cut}}{}^2$ processes**

Memory/process ~ $\mathcal{O}(D_{\mathrm{cut}}{}^7)$, Computational time/process ~ $\mathcal{O}(D_{\mathrm{cut}}{}^{13})$

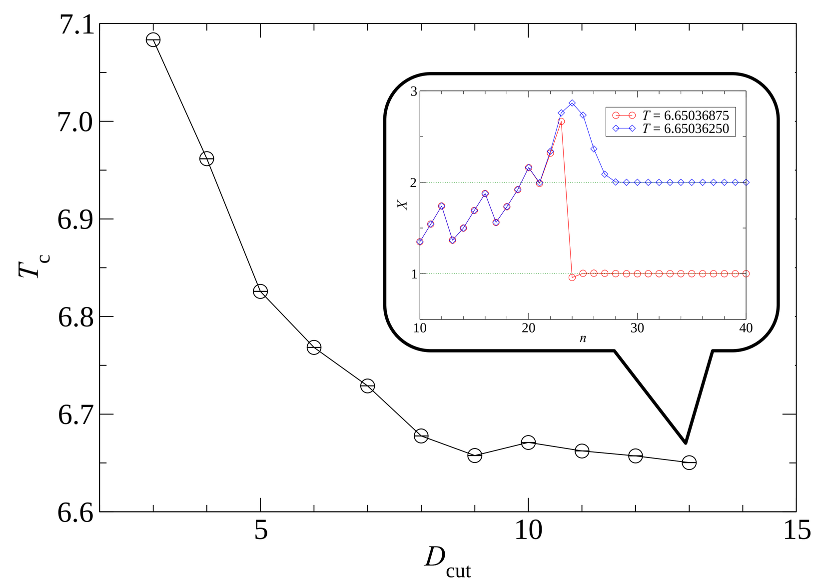
§ Numerical Results

Volume up to 1024^4 & reaching the bond dimension 13

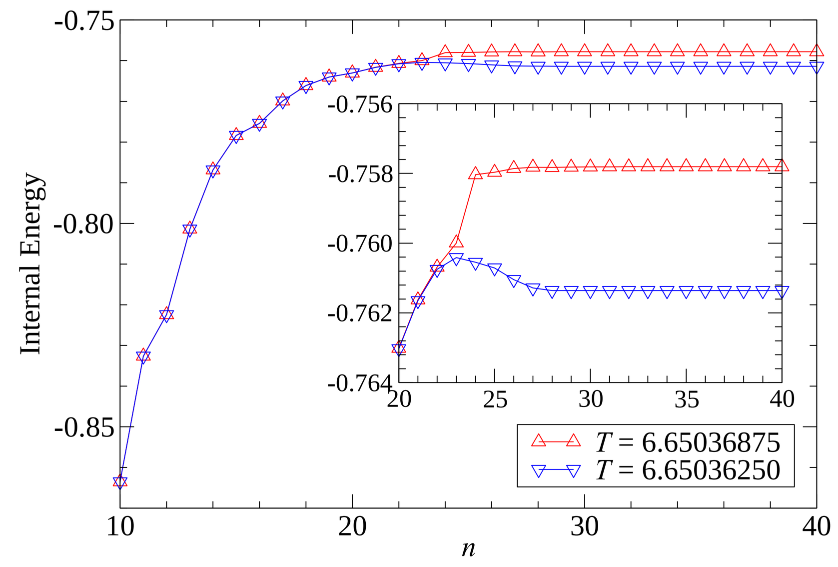
Free energy

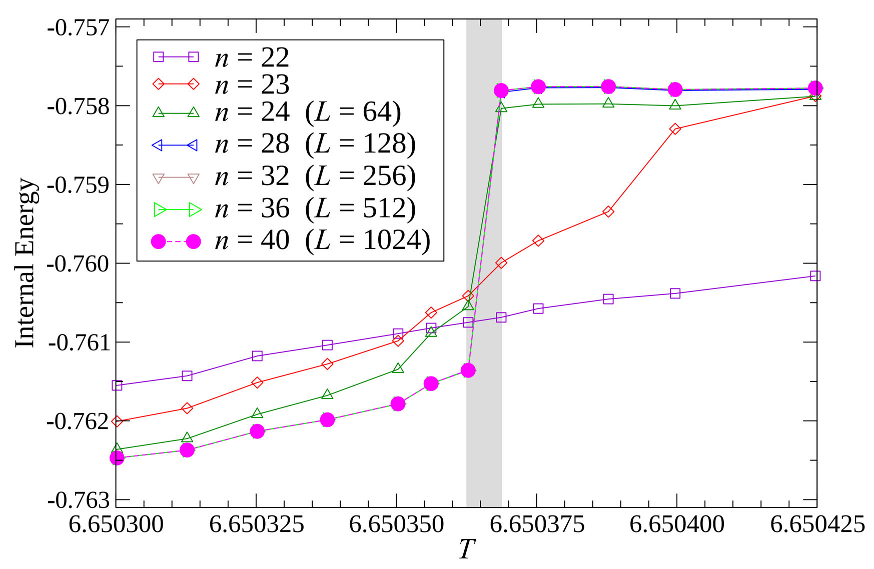


Transition point



Internal energy with $D_{\mathrm{cut}} = 13$

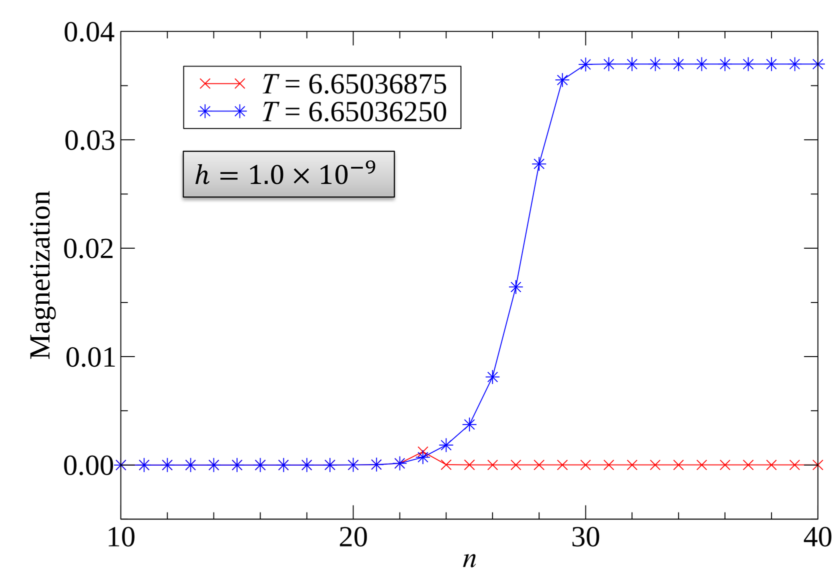


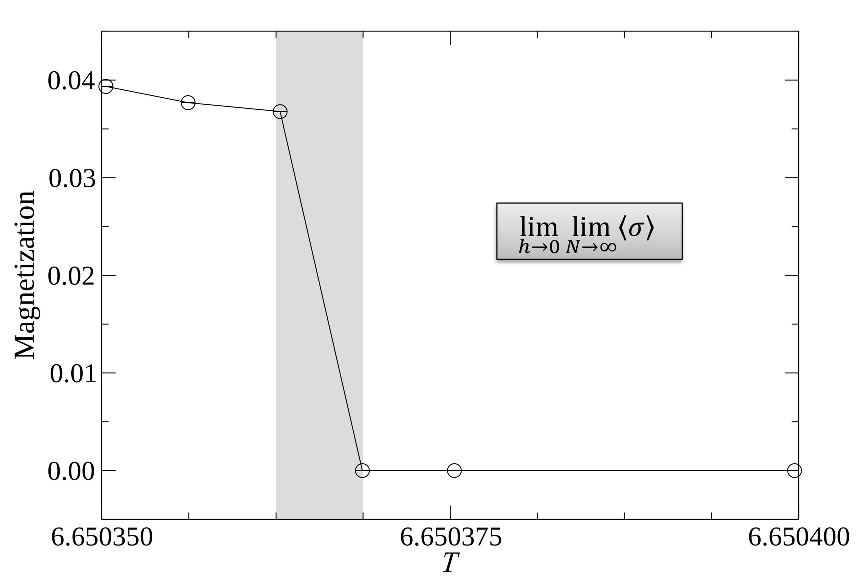


$\Delta\langle H\rangle(D_{\mathrm{cut}} = 13) = 0.0034(5)$ with $\Delta T = 6.25 \times 10^{-6}$

There emerges a finite jump with mutual crossings of curves for different volume, which is a characteristic feature of the first-order phase transition (cf. Fukugita et al. JSP59(1990)1397)

Magnetization with $D_{\mathrm{cut}} = 13$





$\Delta\langle\sigma\rangle(D_{\mathrm{cut}} = 13) = 0.037(2)$ with $\Delta T = 6.25 \times 10^{-6}$

A finite jump also emerges in the magnetization curve

§ Summary & Outlook

Monte Carlo ($L \leq 80$) Lundow-Markstrom PRE80(2009)031104	6.68026(2)
HOTRG with $D_{\mathrm{cut}} = 13$ ($L \leq 1024$)	6.650365(5)

- A finite jump for the internal energy has been found together with mutual crossings of curves among different lattice volumes around the transition point. A jump has also been observed in the magnetization. The numerical results obtained in this work are consistent with the weak first-order transition.
- Recently, "Anisotropic TRG" has been proposed. This is a potentially powerful algorithm to investigate the four-dimensional systems!

Adachi et al. arXiv: 1906.02007 [cond-mat.stat-mech]
Oba arXiv: 1908.07295 [cond-mat.stat-mech, hep-lat, physics.comp-ph]