A Review of The Affine Particle-In-Cell Method

Presenter: Pin-Hua Ho

Introduction

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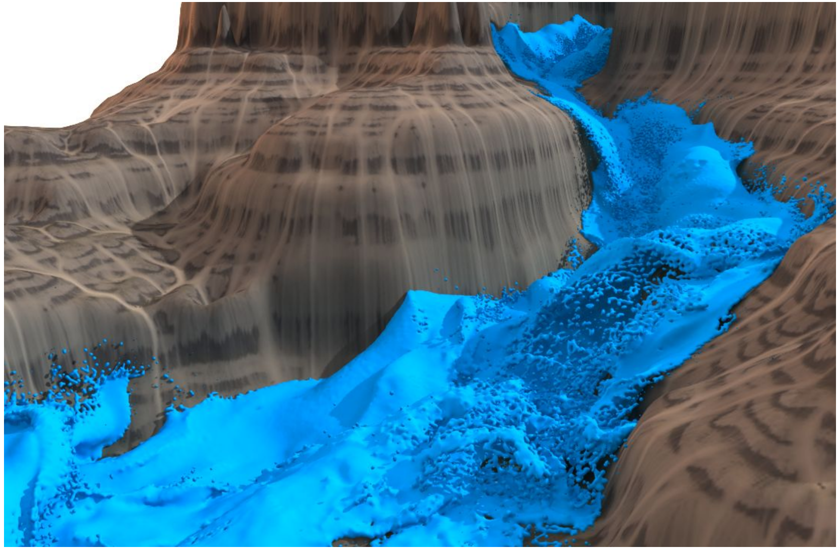
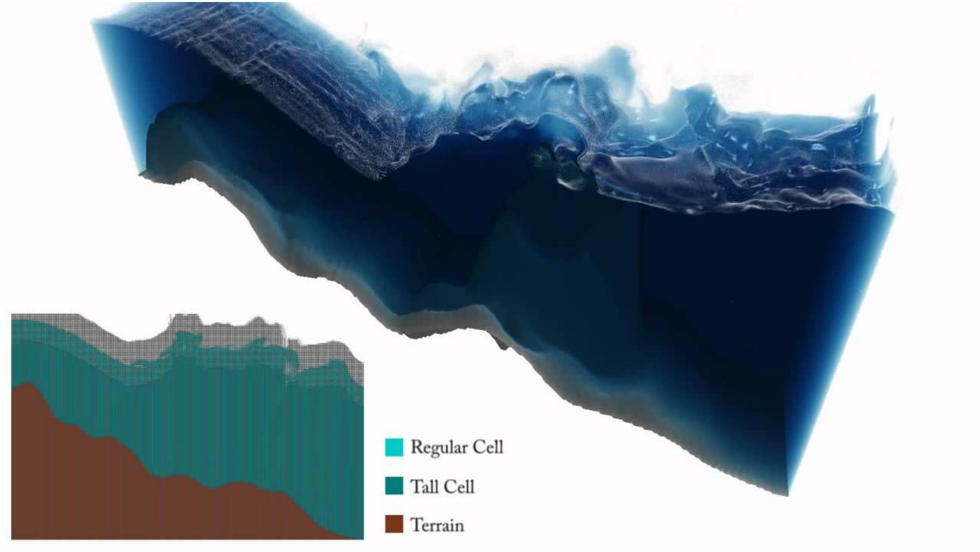
- transfer scheme? hybrid particle/grid method?

Fluid Simulation: Lagrangian vs. Eulerian



Lagrangian particles



Eulerian grid

image: RWTH Computer Animation Group

image: Chentanez et al., Real-Time Eulerian Water Simulation Using a Restricted Tall Cell Grid, Siggraph 2011

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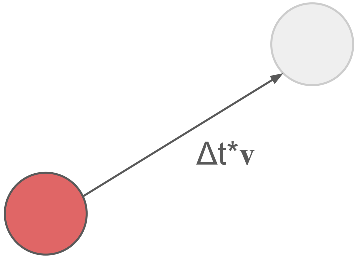

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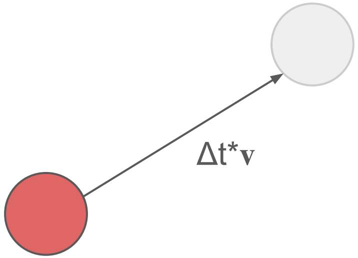

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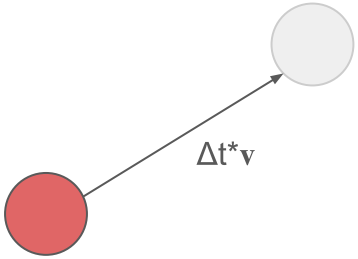
- Unstructured particle cloud
  - Hard to calculate spatial derivatives



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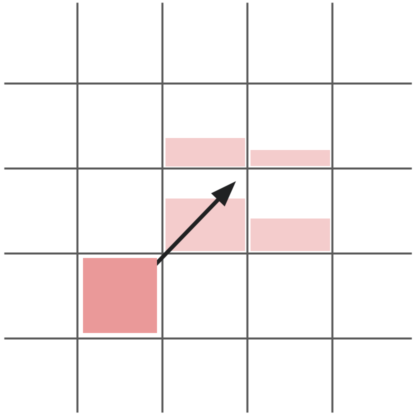

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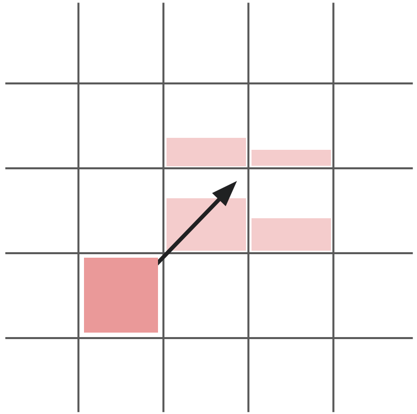

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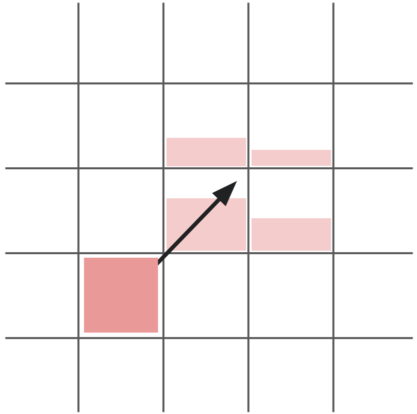

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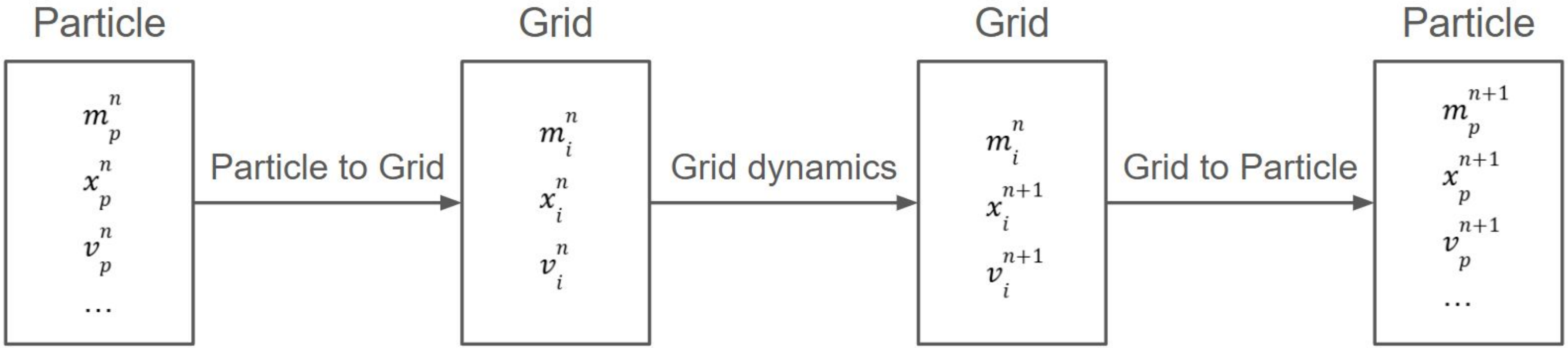
- How about we combine them and get both advantages?
- This is how the **hybrid particle/grid** method came from
- Use Lagrangian particles to perform advection
- And Eulerian grid to solve dynamics

Fluid Simulation: The Hybrid Method



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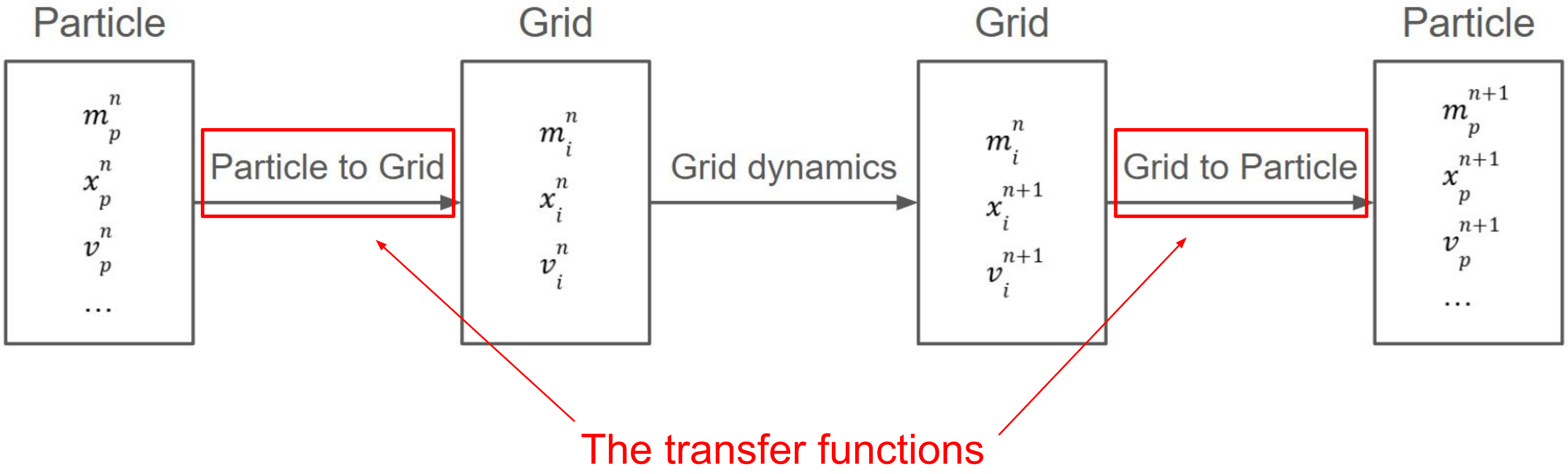

The Classics: PIC and FLIP

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- The Fluid Implicit Particle (FLIP) method

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- Quickly became prominent

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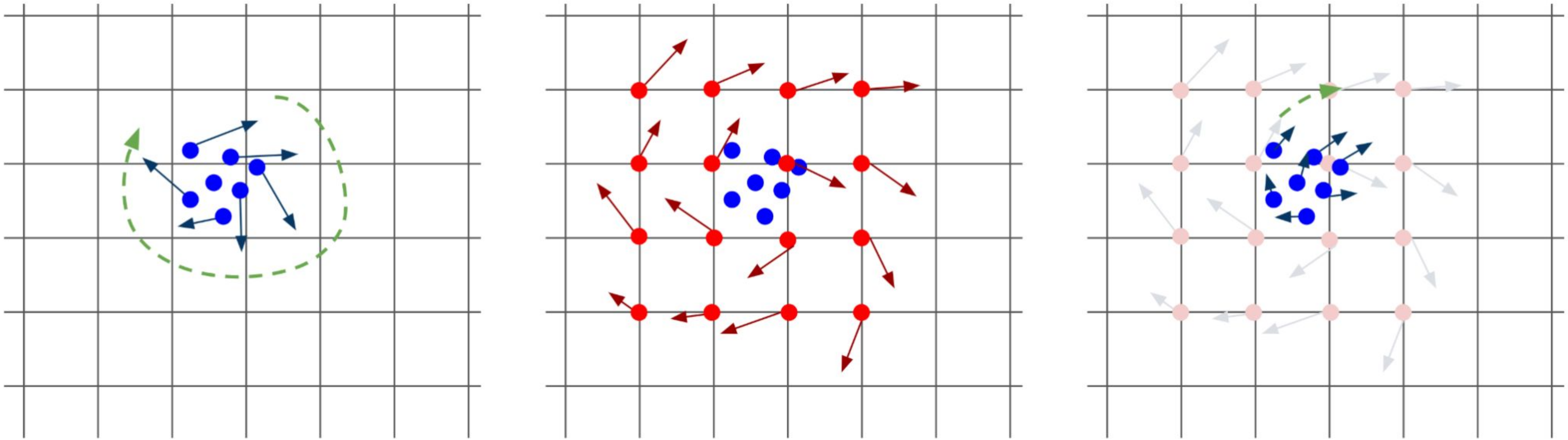
- The dissipation
- Total energy loss over time
- Mismatch of degree of freedom between particles and the grid
- Ususally, # of particles >> # of grid nodes
- Particles can carry more information then the grid could

The Classics: PIC (cont'd)

- When transfer between particles and the grid



(a) Before P2G

(b) Rasterized onto grid

(c) After G2P

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- Instead of interpolating and overriding the particle value at each time step
- Grid only calculate the increments then apply them to particles

The Classics: FLIP (cont'd)

- During the G2P transfers:

$$\mathbf{v}_p^{n+1} \leftarrow \sum_i w_{ip} \mathbf{v}_i^{n+1}.$$

PIC

$$\mathbf{v}_p^{n+1} \leftarrow \mathbf{v}_p^{n} + \sum_i w_{ip} \mathbf{\Delta v}_i^{n+1}.$$

FLIP

The Classics: FLIP (cont’d)

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- Massively improved conserving total energy

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- However, it comes with a cost

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- The mismatch of DoG between particles and the grid persists
- Some sub-grid fluid motions or **modes** couldn’t be seen by the grid
- Thus getting no proper physical response
- Results in so-called “ringing instability”
- The errors gets amplified overtime

$$\mathbf{v}_p^{n+1} \leftarrow \boxed{\mathbf{v}_p^n} + \sum_i w_{ip} \boldsymbol{\Delta} \mathbf{v}_i^{n+1}.$$

Using old values → errors persists!

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- During G2P transfers
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- How about making a single particle to carry more information?

The First Try: Rigid Particle-In-Cell (RPIC)

- Angular momentum loss was the primary observation from PIC
- Explicitly compensate for that angular momentum loss

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- Angular momentum loss was the primary observation from PIC
- Explicitly compensate for that angular momentum loss
- A particle could influence multiple nearby grid nodes
- inherently creating a “soft boundary”
- Treating the particle as a rigid body with volume

The First Try: Rigid Particle-In-Cell (RPIC) (cont’d)

- Let each particle remember its own angular momentum

The First Try: Rigid Particle-In-Cell (RPIC) (cont'd)

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- Then?

$$
\begin{array}{lcl}
m_p^n & \longrightarrow & m_i^n \\
x_p^n & \longrightarrow & x_i^n \\
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L_p^n & &
\end{array}
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P2G Transfer

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G2P Transfer

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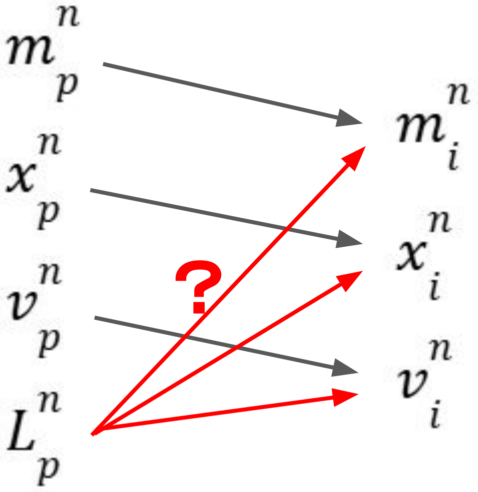

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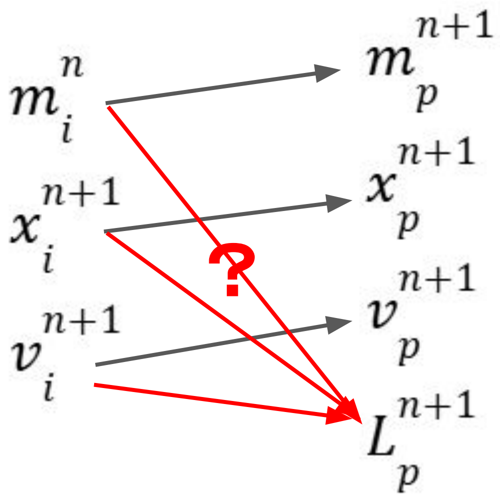

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$$\mathbf{L}_p^{n+1} = \sum_i w_{ip}^n (\mathbf{x}_i - \mathbf{x}_p^n) \times m_p \mathbf{v}_i^{n+1}.$$

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G2P Transfer

- Like a discrete integration over the piece-wise rigid body

The First Try: Rigid Particle-In-Cell (RPIC) (cont'd)

- For P2G transfer

$$m_i^n v_i^n = \sum_p w_{ip}^n m_p(\mathbf{v}_p^n + ((\mathbf{K}_p^n)^{-1}\mathbf{L}_p^n) \times (\mathbf{x}_i - \mathbf{x}_p^n).$$

- Matrix K is the inertia tensor
- The term $((\mathbf{K}_p^n)^{-1}\mathbf{L}_p^n)$ is the angular velocity of the particle
- The cross product gives the tangent velocity at grid location x_i

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- Thus dissipation persists
- Need a more powerful description for the local transform

The Affine Particle-In-Cell (APIC)

- If a single vector $\mathbf{L}_p^n$ is not enough
- How about a 3x3 matrix?

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- That can describe the full, locally affine velocity field

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- It's impossible to uniquely define 9 components from the 3x1 vector $\mathbf{L}_p^n$
- Divide the affine velocity matrix into 2 matrices
- Like p = mv?

The Affine Particle-In-Cell (APIC) (cont'd)

- The matrix division: $\mathbf{C}_p^n = \mathbf{B}_p^n (\mathbf{D}_p^n)^{-1}$.
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$$\mathbf{D}_p^n = \sum_i w_{ip}^n (\mathbf{x}_i - \mathbf{x}_p^n)(\mathbf{x}_i - \mathbf{x}_p^n)^T.$$

- D can be computed at the beginning of each time step

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- D can be computed at the beginning of each time step
- Then by definition, $\mathbf{B}_p^n$ is a momentum-like matrix

The Affine Particle-In-Cell (APIC) (cont'd)

- In G2P transfer, the term $\mathbf{B}_p^n$ is updated by:

$$\mathbf{B}_p^{n+1} = \sum_i w_{ip}^n \mathbf{v}_i^{n+1} (\mathbf{x}_i - \mathbf{x}_p^n)^T.$$

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- Then during the P2G transfer, anagolous to the RPIC:

$$m_i^n \mathbf{v}_i^n = \sum_p w_{ip}^n m_p (\mathbf{v}_p^n + \mathbf{B}_p^n (\mathbf{D}_p^n)^{-1} (\mathbf{x}_i - \mathbf{x}_p^n)).$$

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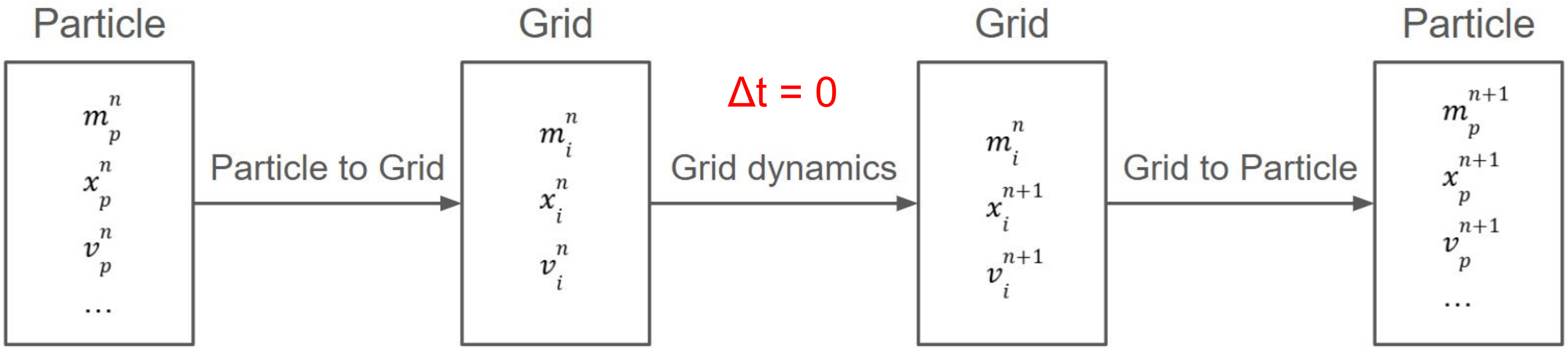

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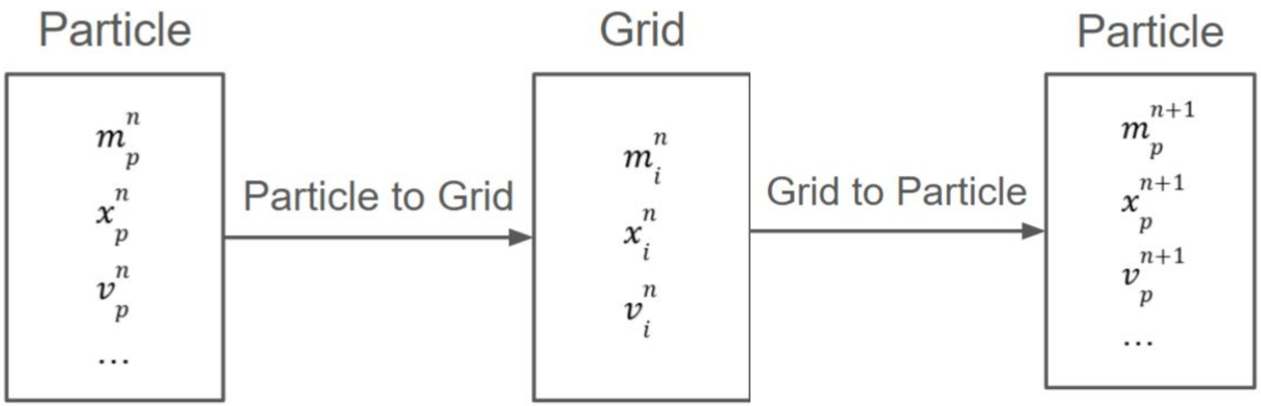

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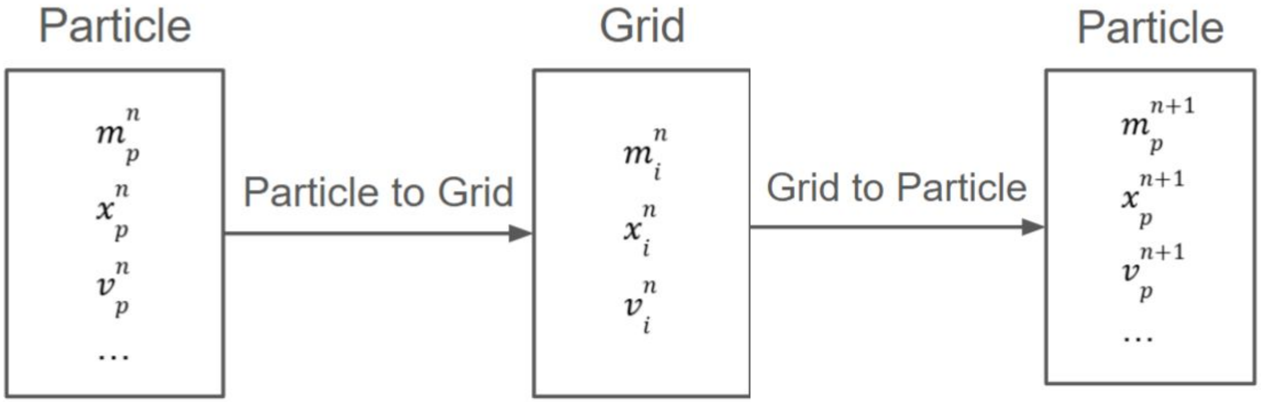
- Concept: Set $\Delta t = 0$
- Prove that during P2G and G2P transfers
  - Linear momentum conserves
  - Angular momentum conserves



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- APIC successfully improved dissipation based on PIC
- Didn’t rely on the unsafe direct data path in FLIP
  - Won’t have instability issue as FLIP does
- Compared with pure PIC, FLIP and the linear blend of PIC/FLIP
- The linear blend of PIC/FLIP is a practical use to mitigate both disadvantages

Results

image: Jiang et al. The Affine Particle-In-Cell Method

Discussion

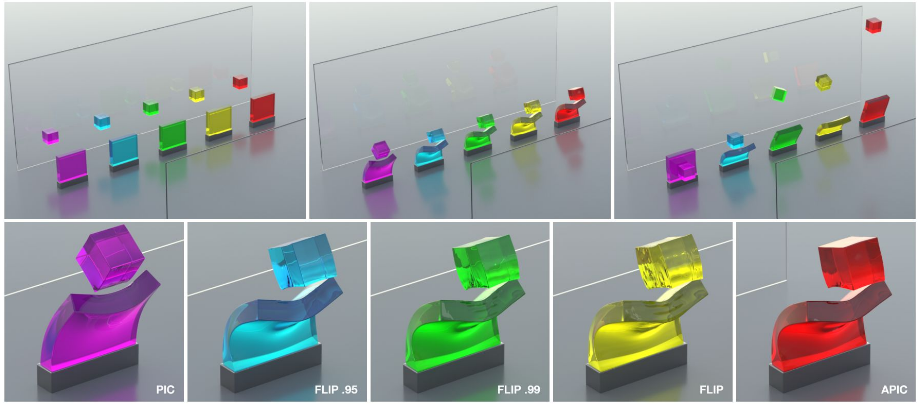
- APIC didn’t solve nor worsen ringing instability
- In terms of energy conservation, FLIP still does a better job
- The computational and memory overheads are dominated by other factor

Conclusion

- Traditional PIC suffers from dissipation
- FLIP fixed dissipation but introduced uncontrollable noise and instability
- The linear blend of PIC and FLIP mitigated but mixed both dissipation and noise
    - Moreover, finetuning the blending weight case by case adds another complexity for simulation

Conclusion (cont'd)

- APIC, based on PIC's framework, solved most of the dissipation
  - and being stable at the same time
  - provides a stable yet detailed, and controllable simulation
- APIC is versatile as a particle/grid transfer scheme
  - Can be use for other simulation frameworks as well, such as material point method (MPM)



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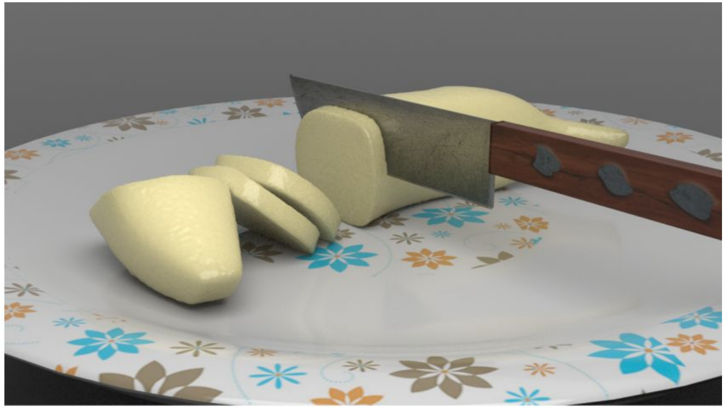

image: Hu et al. A Moving Least Squares Material Point Method (MLS-MPM) with Displacement Discontinuity and Two-Way Rigid Body Coupling

Thanks for listening!