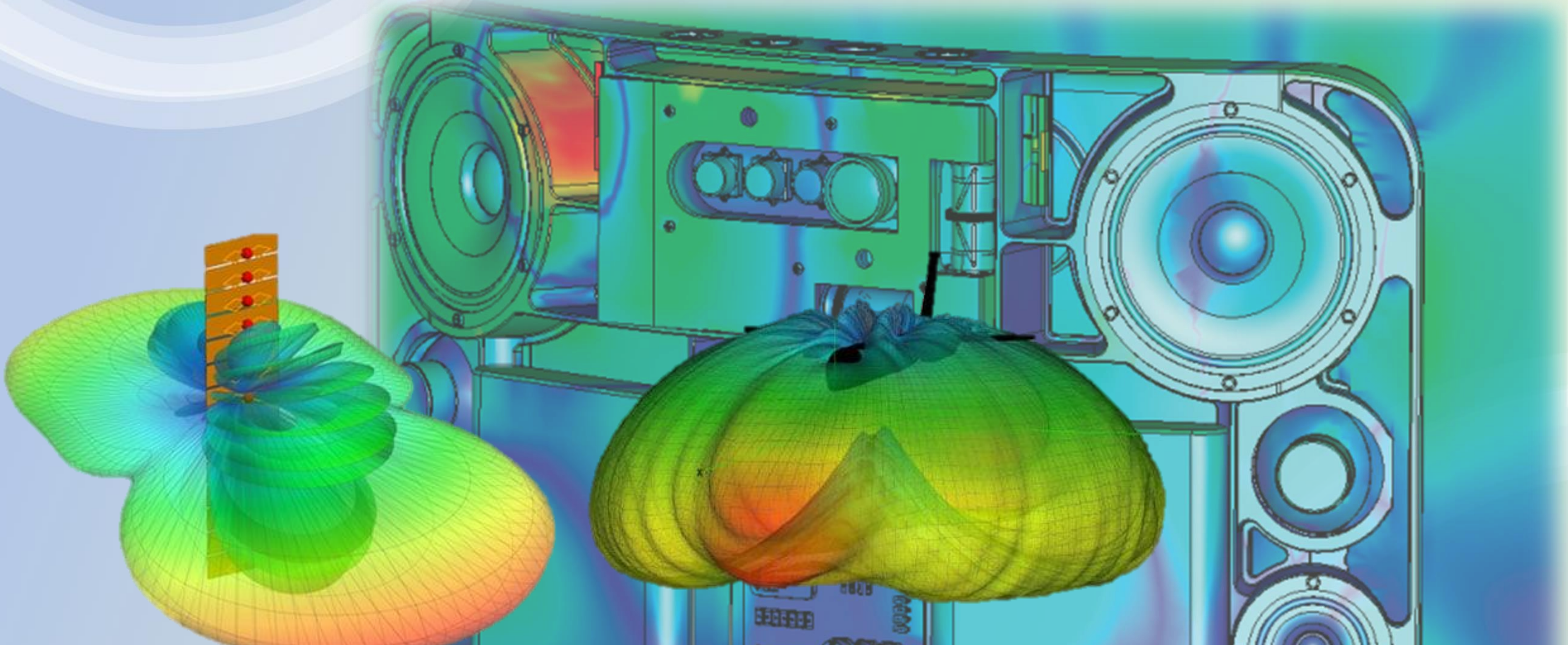
Pseudospectral Time-Domain Method in EM Simulations

Speaker: Jake W. Liu

Numerical Simulations in EM



- https://altair.com/feko-applications
- https://www.3ds.com/fileadmin/PRODUCTS-SERVICES/SIMULIA/PRODUCTS/CST/CST_Brochure_A4.pdf

🙃	IE	DE
TD	$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d}{dt}\int \boldsymbol{B} \cdot d\boldsymbol{s}$ $\oint \boldsymbol{H} \cdot d\boldsymbol{l} = \int (\boldsymbol{J} + \frac{d}{dt}\boldsymbol{D}) \cdot d\boldsymbol{s}$ $\oint \boldsymbol{D} \cdot d\boldsymbol{s} = \int q\ dv$ $\nabla \cdot \boldsymbol{B} = 0$	$\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$ $\nabla \times \boldsymbol{H} = \boldsymbol{J} + \partial \boldsymbol{D}/\partial t$ $\nabla \cdot \boldsymbol{D} = q$ $\nabla \cdot \boldsymbol{B} = 0$
FD	$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -j\omega \int \boldsymbol{B} \cdot d\boldsymbol{s}$ $\oint \boldsymbol{H} \cdot d\boldsymbol{l} = \int (\boldsymbol{J} + j\omega\boldsymbol{D}) \cdot d\boldsymbol{s}$ $\oint \boldsymbol{D} \cdot d\boldsymbol{s} = \int q\ dv$ $\nabla \cdot \boldsymbol{B} = 0$	$\nabla \times \boldsymbol{E} = -j\omega\boldsymbol{B}$ $\nabla \times \boldsymbol{H} = \boldsymbol{J} + j\omega\boldsymbol{D}$ $\nabla \cdot \boldsymbol{D} = q$ $\nabla \cdot \boldsymbol{B} = 0$

Numerical Simulations in EM

- Well known numerical methods are listed:

	IE	DE
TD	TDIE	FDTD
FD	MOM	FEM

Asymptotic Methods

- Asymptotic methods can be applied for the electrically-large cases:

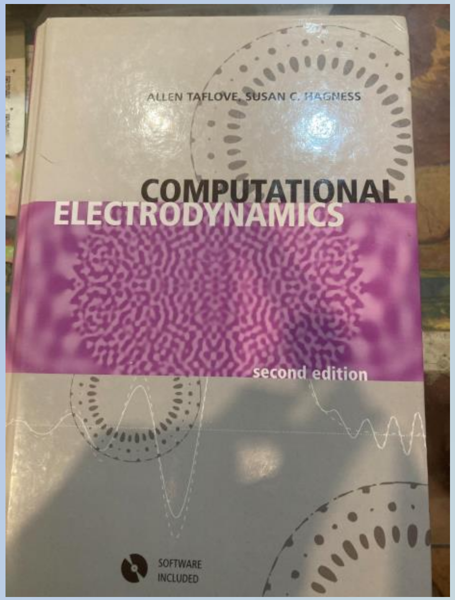
$\Rightarrow$ Geometrical optics (GO)
$\Rightarrow$ Physical Optics (PO)

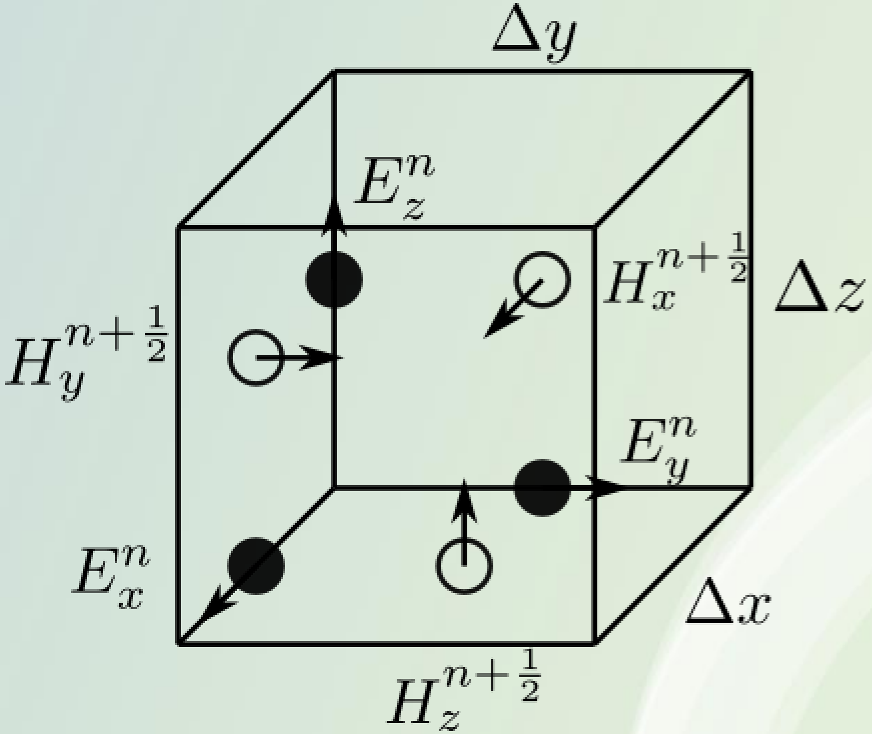
- And their advanced version:

$\Rightarrow$ Geometrical Theory of Diffraction (GTD)
$\Rightarrow$ Physical Theory of Diffraction (PTD)

Finite-Difference Time-Domain Method

- Kane Yee, 1966, "***Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media***“
- Allen Taflove, 1995, ***Computational Electrodynamics: The Finite-Difference Time-Domain Method***





FDTD Formulation

- Discretization in time => FD
- Discretization in space => FD

2nd order accuracy

$$\frac{E_x^{n+1/2}(k) - E_x^{n-1/2}(k)}{\Delta t} = -\frac{1}{\varepsilon_0}\frac{H_y^n\left(k+\frac{1}{2}\right) - H_y^n\left(k-\frac{1}{2}\right)}{\Delta x}$$

2nd order accuracy

Topics in FDTD

- Discretization
- Material
- Perfectly Matching Layer (PML)
- Hard Source / Soft Source
- Pure Scattered-Field / Total-Field Scattered-Field (TFSF)
- Near-to-Far Field Transformation (NTFF)

Pseudospectral Time-Domain Method

- Q. H. Liu, 1997, "***The PSTD algorithm: A time-domain method requiring only two cells per wavelength***"
- Discretization in time => FD
- Discretization in space => PS
  - using global basis function to approximate differential operations
    - Fourier => uniform collocated points
    - Multi-domain => non-uniform collocated points

PSTD Formulation

Denoting the Fourier transform of a function *f* as

$$F(k_\eta) \equiv \mathscr{F}_\eta[f] = \int_{-\infty}^{\infty} f(\mathbf{r}) e^{-ik_\eta \eta}\, d\boldsymbol{\eta},$$

then

$$\frac{\partial f(r)}{\partial \eta} = \mathfrak{F}_\eta^{-1}\{ik_\eta \mathfrak{F}_\eta[f(r)]\}$$

thus

$$a_\eta \mu \frac{\partial \mathbf{H}^{(\eta)}}{\partial t} + \mu \omega_\eta \mathbf{H}^{(\eta)} = \hat{\boldsymbol{\eta}} \times \mathscr{F}_\eta^{-1}\{ik_\eta \mathscr{F}_\eta[\mathbf{E}]\} - \mathbf{M}^{(\eta)}$$

$$a_\eta \epsilon \frac{\partial \mathbf{E}^{(\eta)}}{\partial t} + (a_\eta \sigma + \omega_\eta \epsilon)\mathbf{E}^{(\eta)} + \omega_\eta \sigma \mathbf{E}_I^{(\eta)}$$

$$= -\hat{\boldsymbol{\eta}} \times \mathscr{F}_\eta^{-1}\{ik_\eta \mathscr{F}_\eta[\mathbf{H}]\} - \mathbf{J}^{(\eta)},$$

Implementation of PSTD

- **FFT** can be applied to compute the derivatives.
- Be aware of the use of **fftshift/ifftshift**
- To avoid numerical errors, take the real part of the computed result!

```
dEy = real(ifft(ifftshift(1j*k.*fftshift(fft(Ey)))));
```

Advantages and Disadvantages of PSTD

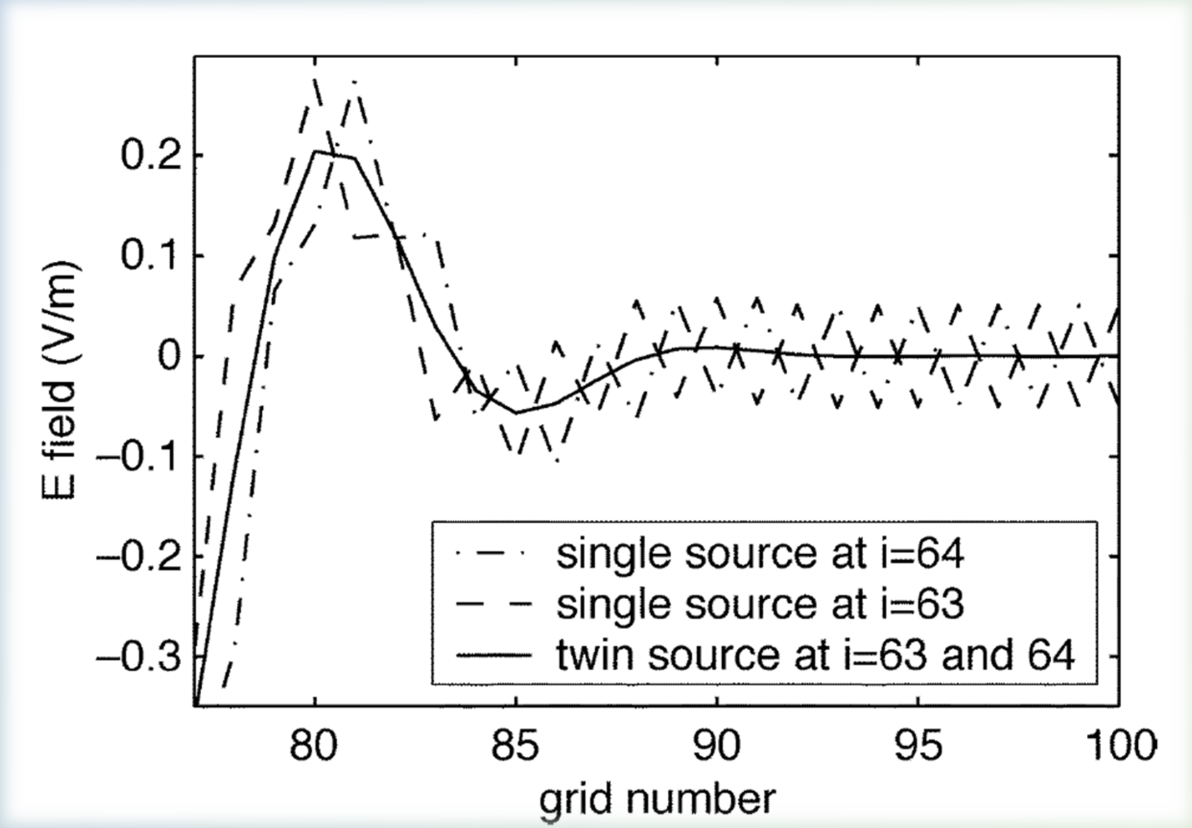
- Advantages
  - Requires only two cells per wavelength according to Nyquist theorem
  - Computations for E and H fields are collocated*
- Disadvantages
  - Difficult to deal with field discontinuities
    - Source
    - TFSF formulation
    - Metallic surfaces
    - High-contrast dielectrics

Topics in PSTD (vs. FDTD)

- Discretization
- Material
- Perfectly Matching Layer (PML)
- Hard Source / Soft Source
- Pure Scattered-Field / Total-Field Scattered-Field (TFSF)
- Near-to-Far Field Transformation (NTFF)

Compact Wave Source Condition

- Tae-Woo Lee and Susan C. Hagness, 2004, "***A Compact Wave Source Condition for the Pseudospectral Time-Domain Method***"



TFSF Formulation

- TFSF is a technique to introduce plane wave into the simulation space
- Decompose the total field into incident field and scattered field as

$$E_{tot} = E_{scat} + E_{inc}$$

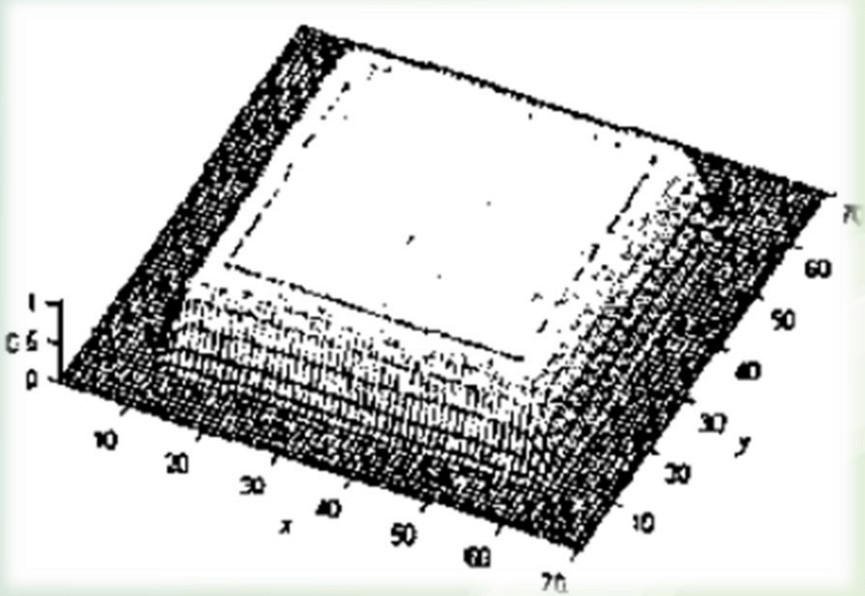
$\rightarrow$ assumed known

https://www.youtube.com/watch?v=tYyqLMU0WxU&ab_channel=meyavuz

TFSF Formulation in PSTD

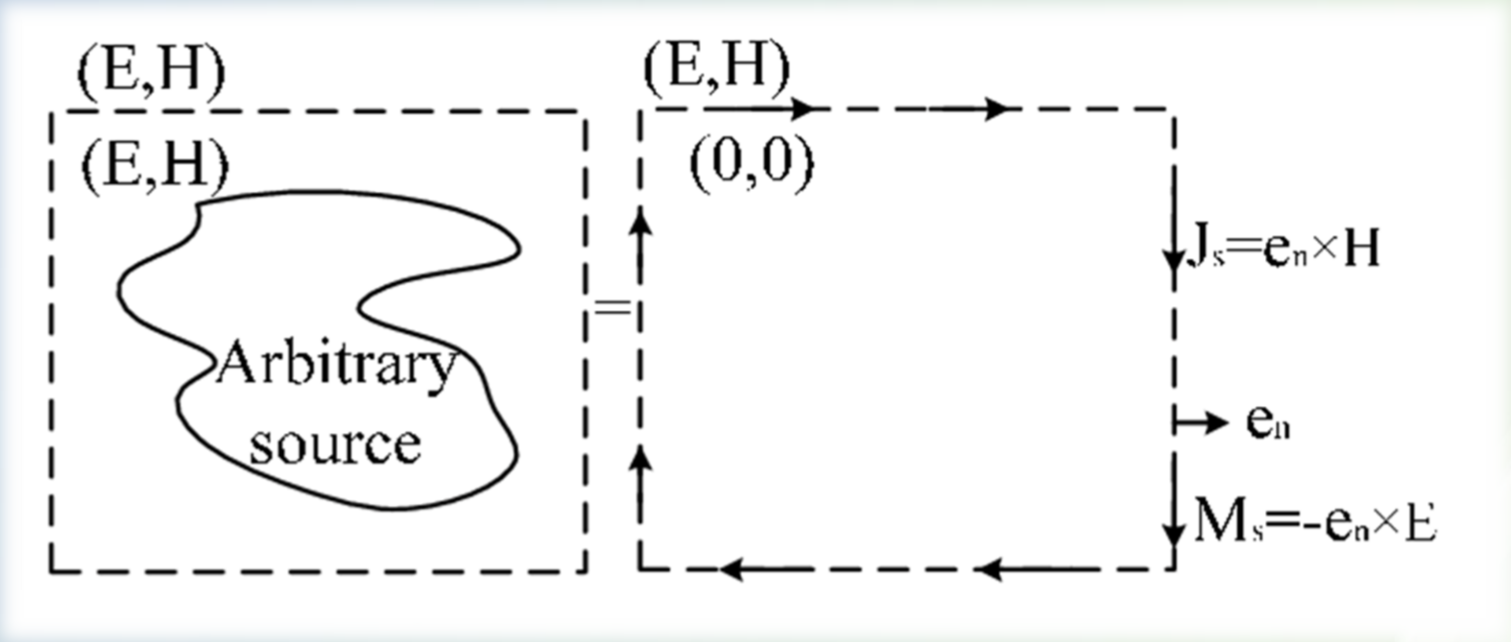
- Xiang Gao, Mark S. Mirolznik and Dennis W. Prather, 2004, "***Soft Source Generation in the Fourier PSTD Algorithm***"

$$
\begin{cases}
\hat{\hat{\vec{E}}}_{tot} = \hat{\hat{\vec{E}}}_{inc} + \vec{E}_{scat} = \zeta \vec{E}_{inc} + \vec{E}_{scat} \\
\hat{\vec{H}}_{tot} = \hat{\vec{H}}_{inc} + \vec{H}_{scat} = \zeta \vec{H}_{inc} + \vec{H}_{scat}
\end{cases}
$$



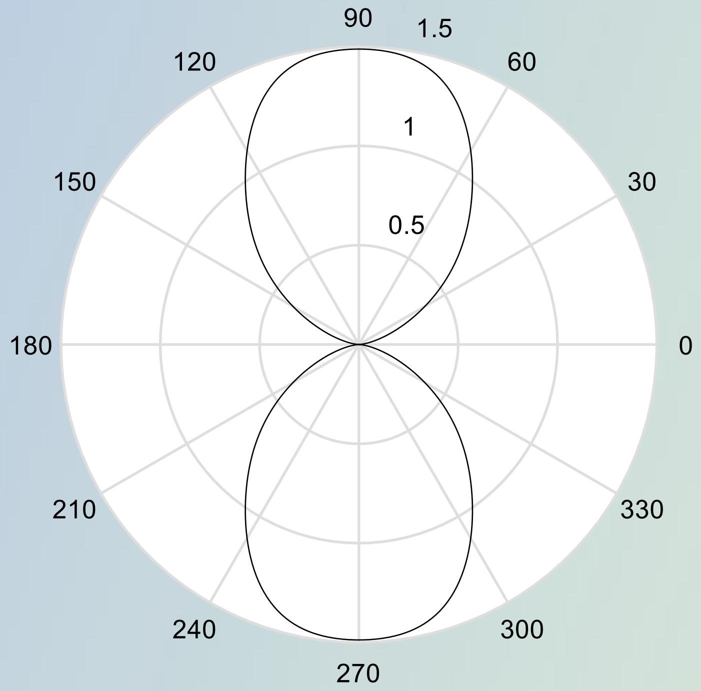
NTFF Transformation

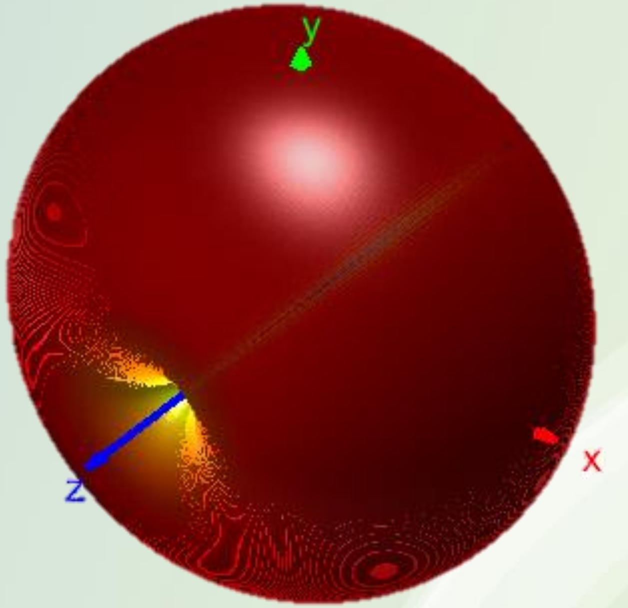
- From equivalence principle, when the surface currents on a selected imaginary surface are known, the fields inside the surface or outside the surface can be deduced from the imaginary currents.



NTFF Transformation

- From the imaginary currents, one can compute the far field of the source with NTFF transformation.



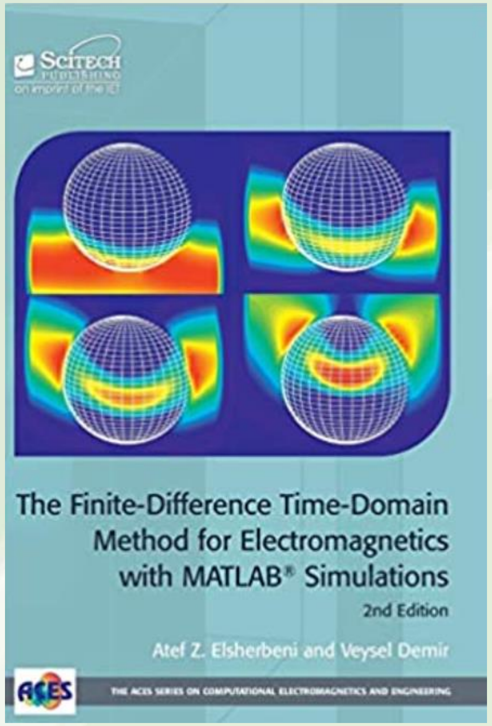


Exercises

- Try to modify your 1D FDTD code into PSTD formulation
- Find out the differences in the implementation
- Compare results of both methods with different simulation scenarios

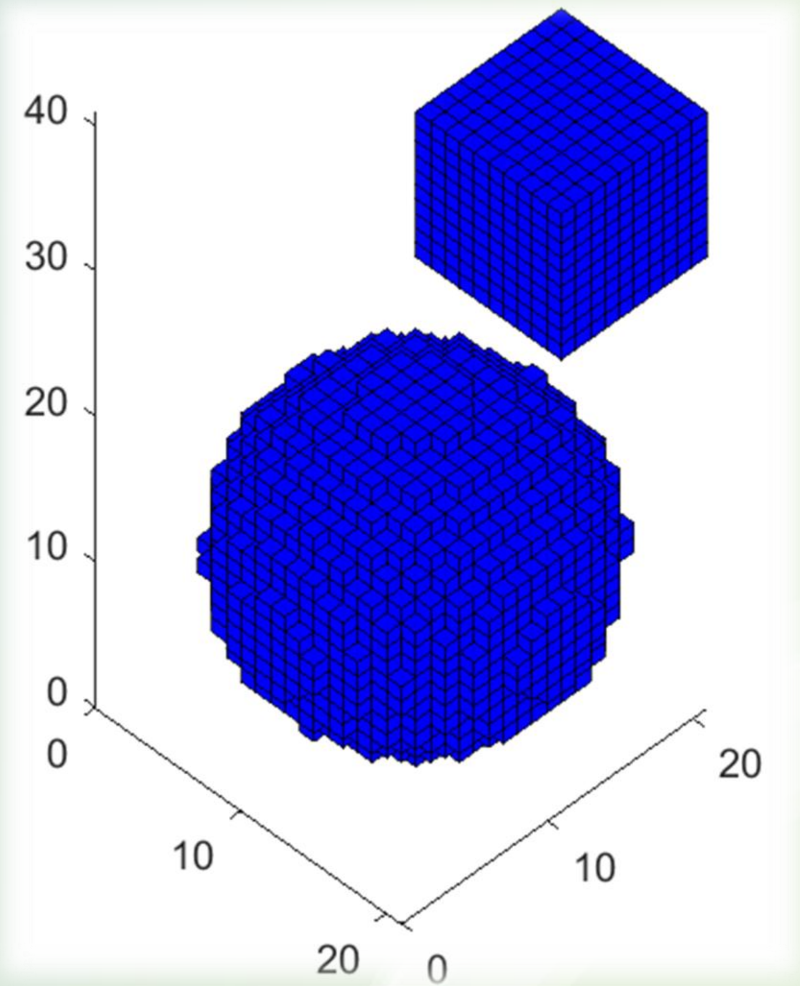
Some Useful References

- John B. Schneider, ***Understanding the FDTD Method***
- Atef Z. Elsherbeni and Veysel Demir, ***The Finite-Difference Time-Domain Method for Electromagnetics with MATLAB® Simulations***



Some Interesting Tools

- Voxelization tools:
  - Import arbitrary geometry (STL file)



Thank You!