

# **Introduction to Data Structures**



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# What is Algorithm

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- An algorithm is “a finite set of precise instructions for performing a computation or for solving a problem”. It will take some value or set of values, as input, and produces some value or set of values, as output.
- Example:
  - Directions to somebody's house is an algorithm
  - A sorted, non-decreasing sequence of natural numbers of non-zero, finite length

# Some algorithms are harder than others



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- Some algorithms are easy
  - Finding the largest (or smallest) value in a list
  - Finding a specific value in a list
- Some algorithms are a bit harder
  - Sorting a list
- Some algorithms are very hard
  - Finding the shortest path between Miami and Seattle
- Some algorithms are essentially impossible
  - Factoring large composite numbers



# Characteristics of an Algorithm

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- Well-Defined Inputs: Inputs, it should be well-defined.
- Well-Defined Outputs: Must clearly define the output.
- Definiteness: the steps are defined precisely.
- Correctness: should produce the correct output.
- Finiteness: should end up with finite number of steps.
- Effectiveness: each step must be able to be performed in a finite amount of time.
- Generality: the algorithm should be applicable to all problems of a similar form.
- Language Independent: Just plain instructions that can be implemented in any language.



# Algorithm: Example

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## Algorithm to add two numbers

Step 1: Start  
Step 2: Declare variables num1, num2 and sum.  
Step 3: Read values num1 and num2.  
Step 4: Add num1 and num2 and assign the result to sum.  
           $\text{sum} \leftarrow \text{num1} + \text{num2}$   
Step 5: Display sum  
Step 6: Stop

## Algorithm to find all roots of a quadratic equation $ax^2+bx+c=0$ .

Step 1: Start  
Step 2: Declare variables a, b, c, D, x1, x2, rp and ip;  
Step 3: Calculate discriminant  
           $D \leftarrow b^2 - 4ac$   
Step 4: If  $D \geq 0$   
           $r1 \leftarrow (-b + \sqrt{D}) / 2a$   
           $r2 \leftarrow (-b - \sqrt{D}) / 2a$   
          Display r1 and r2 as roots.  
Else  
          Calculate real part and imaginary part  
           $rp \leftarrow -b / 2a$   
           $ip \leftarrow \sqrt{(-D)} / 2a$   
          Display  $rp + j(ip)$  and  $rp - j(ip)$  as roots  
Step 5: Stop



# What is a Good Algorithm?

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- Efficient:
  - Running time (**small**)
  - Space used (**less**)
- Efficiency as a function of input size:
  - The input size
  - Number of data elements



# Space Complexity of Algorithms

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For any algorithm memory may be used for the following:

- Variables (includes constant values, temporary values)
  - Program Instruction
  - Execution
- 
- **Space complexity:** Amount of memory used by the algorithm (including the input values to the algorithm) to execute and produce the output.
  - While executing, algorithm uses memory space for three reasons:
    - **Instruction Space:** memory used to save the compiled version of instructions.
    - **Environmental Stack:** In function call, a system stack is maintained.
    - **Data Space:** Amount of space used by the variables and constants.
  - while calculating the Space Complexity of any algorithm,
    - **only Data Space is considered** (neglecting Instruction Space and Environmental Stack.)



# Calculating the Space Complexity

## Example:

```
{  
    int z = a + b + c;  
    return(z);  
}
```

In this expression,

- variables a, b, c and z are all integer types, take 4 bytes each.
- Total memory =  $(4(4) + 4) = 20$  bytes,
- Additional 4 bytes is for return value.
- Because this space requirement is **fixed**, called **Constant Space Complexity**.

## Example:

```
// n is the length of array a[]  
int sum(int a[], int n) {  
    int x = 0;                // 4 bytes for x  
    for(int i = 0; i < n; i++) { // 4 bytes for i  
        x = x + a[i];  
    }  
    return(x);  
}
```

- In this code,  $4*n$  bytes of space is required for the array a[] elements.
- 4 bytes each for x, n, i and the return value.
- Total memory requirement is  $(4n + 16)$ , **increasing linearly** with increase in the input value n. Called as **Linear Space Complexity**.





# Data Structures and Algorithms

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- **Algorithm:** Outline, the essence of a computational procedure, step-by-step instructions
- **Program:** an implementation of an algorithm in some programming language
- **Data structure:**
  - Organization of data needed to solve the problem
  - A way of organizing all data items that considers not only the elements stored but also their relationship to each other.



# Time Complexity

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- Time complexity of an algorithm signifies the **total time required** by the program to run till its completion.
- Time Complexity is most commonly estimated by **counting the number of elementary steps performed** by any algorithm to finish execution.
- Any problem can have number of solutions.
- **Two different algorithms** to find square of a number:

```
m=0
```

```
for i=1 to n
```

```
    do m = m + n
```

```
return m
```

- loop will run n number of times
- time complexity will be n atleast
- As the value of n will increase the time taken will also increase

```
return n*n
```

- For this code, time complexity is constant
- Never be dependent on the value of n, so always give the result in 1 step.



# Time Complexity

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- Performance of algorithm may vary with different input data
- **worst-case Time complexity** is considered because that is the maximum time taken for any input size.
- The most common metric for calculating time complexity is **Big O notation**.
- This **removes all constant factors** so that the running time can be estimated in relation to  $n$ , as  $n$  approaches infinity.



# Time Complexity

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statement;

- A single statement.
- Its Time Complexity will be **Constant**.
- The running time of the statement will **not change in relation to n**.

```
for(i=0; i < n; i++) {  
    statement;  
}
```

- The time complexity for this algorithm will be **Linear**.
- The running time of the loop is **directly proportional to n**.
- When n doubles, so does the running time.



# Time Complexity

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```
for(i=0; i < n; i++) {  
    for(j=0; j < n; j++) {  
        statement;  
    }  
}
```

- Time complexity will be **Quadratic**.
- Running time of the two loops is **proportional to the square of n**.
- When n doubles, the running time increases by  $n * n$ .

```
while(low <= high) {  
    mid = (low + high) / 2;  
    if (target < list[mid])  
        high = mid - 1;  
    else if (target > list[mid])  
        low = mid + 1;  
    else break;  
}
```

- This is an algorithm to **break a set of numbers into halves**, to search a particular value.
- **Logarithmic Time Complexity**.
- The running time of the algorithm is proportional to the number of times n can be divided by 2.
- Because the algorithm **divides the working area in half with each iteration**.



# Asymptotic Analysis

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- Asymptotic analysis of an algorithm refers to defining the **mathematical boundation** of its **run-time performance**.
- Asymptotic analysis is input bound
  - no input to the algorithm, concluded to work in a **constant time**.
  - Other than the "**input**" **all other factors are considered constant**.
- Asymptotic analysis refers to computing the **running time of any operation in mathematical units of computation**.
  - Example: running time of one operation is computed as  $f(n)$  and may be for another operation it is computed as  $g(n^2)$
- Describes the algorithm efficiency and performance in a meaningful way.
- Describes the behaviour of time or space complexity for an algorithm



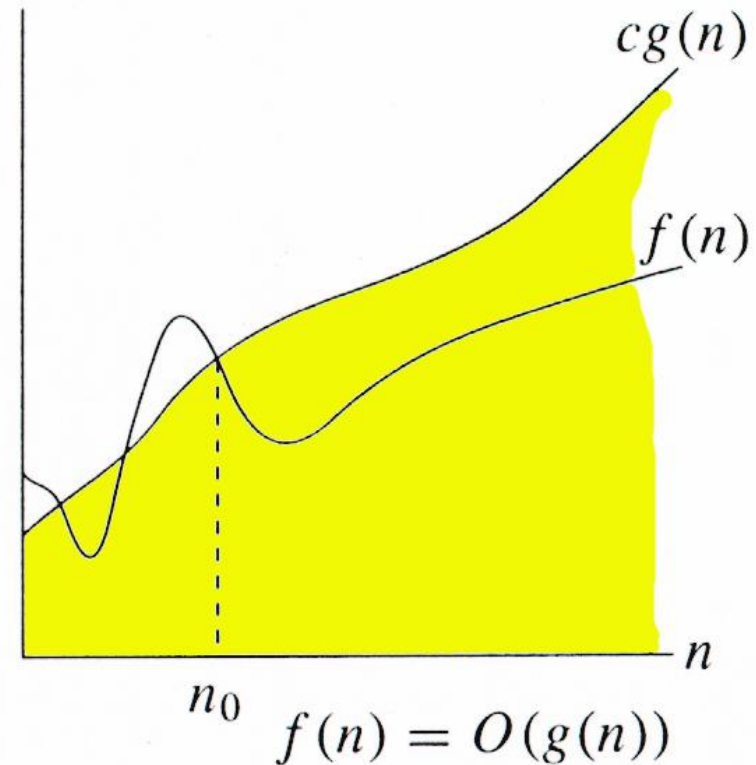
# Asymptotic Notation

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- Commonly used asymptotic notations to calculate the running time complexity of an algorithm.
  - $O$  Notation
  - $\Omega$  Notation
  - $\theta$  Notation

# Big Oh Notation, O

- The Big Oh notation is the formal way to express the **upper bound of an algorithm's running time**.
- It measures the worst case time complexity or the longest amount of time an algorithm can possibly take to complete.
- Definition:  $O(g(n)) = \{ f(n) : \exists \text{ positive constants } c \text{ and } n_0, \text{ such that } \forall n \geq n_0, \text{ we have } 0 \leq f(n) \leq cg(n) \}$
- Intuitively: Set of all functions whose rate of growth is the same as or lower than that of  $g(n)$ .







# Big Oh Notation: Example

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## Example:

Compute Big-Oh notation for

$$f(n) = 10n^2 + 4n + 2$$

$$\text{Given } f(n) = 10n^2 + 4n + 2$$

$$f(n) \leq c * g(n)$$

$$10n^2 + 4n + 2 \leq 10n^2 + 4n + n, \text{ for } n \geq 2$$

$$10n^2 + 4n + 2 \leq 10n^2 + 5n$$

$$10n^2 + 4n + 2 \leq 10n^2 + n^2, \text{ for } n \geq 5$$

$$10n^2 + 4n + 2 \leq 11n^2 \text{ where } c = 11,$$
$$g(n) = n^2 \text{ and } n_0 = 5$$

$$\text{Hence } f(n) = O(n^2)$$

## Example:

Compute Big-Oh notation for

$$f(n) = 1000n^2 + 100n - 6$$

$$\text{Given } f(n) = 1000n^2 + 100n - 6$$

$$f(n) \leq c * g(n)$$

$$1000n^2 + 100n - 6 \leq 1000n^2 + 100n \text{ for all values of } n$$

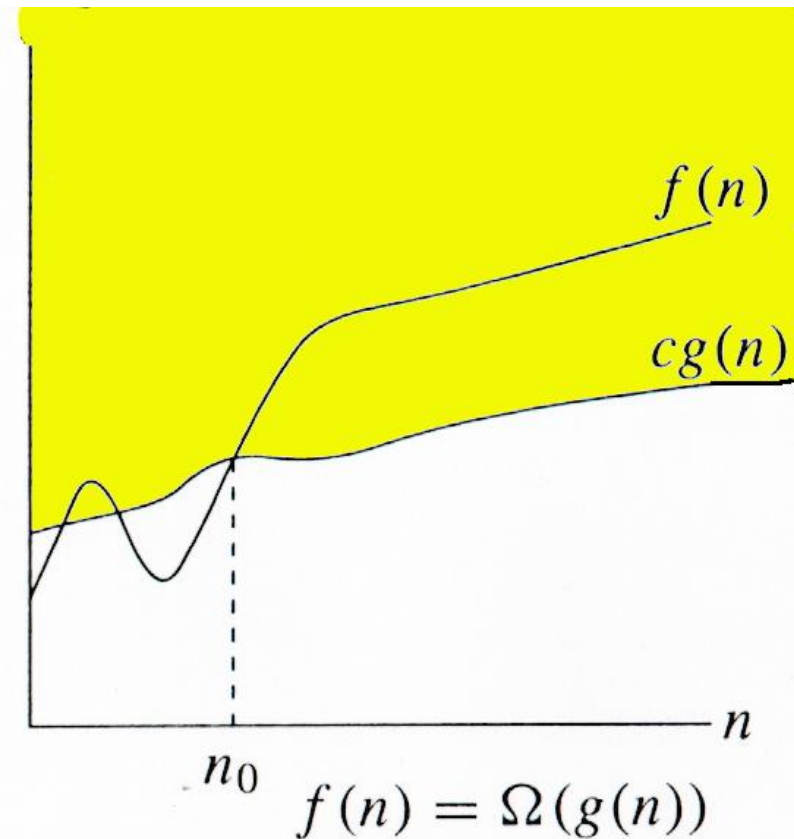
$$1000n^2 + 100n - 6 \leq 1000n^2 + n^2, \text{ for } n \geq 100$$

$$1000n^2 + 100n - 6 \leq 1001n^2, \text{ where } c = 1001, g(n) = n^2 \text{ and } n_0 = 100$$

$$\text{Hence } f(n) = O(n^2)$$

# Omega Notation, $\Omega$

- The Omega notation is the formal way to express the lower bound of an algorithm's running time.
- It measures the best case time complexity or the best amount of time an algorithm can possibly take to complete.
- **Definition:**  $\Omega(g(n)) = \{f(n) :$   
 $\exists$  positive constants  $c$  and  $n_0$ , such that  
 $\forall n \geq n_0,$
- we have  $0 \leq cg(n) \leq f(n)\}$
- **Intuitively:** Set of all functions whose rate of growth is the same as or higher than that of  $g(n)$ .





# Omega Notation, $\Omega$

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## Example:

Compute omega notation for  $f(n) = 10n^2 + 4n + 2$

Given  $f(n) = 10n^2 + 4n + 2$

$$f(n) \geq c * g(n)$$

$10n^2 + 4n + 2 \geq 10n^2$  for all values of  $n$  ( $n \geq 0$ )

where  $c=10$ ,  $g(n)=n^2$  and  $n_0=0$

Hence  $f(n) = \Omega(n^2)$

## Example:

Compute omega notation for  $f(n) = 4n^3 + 2n + 3$

Given  $f(n) = 4n^3 + 2n + 3$

$$f(n) \geq c * g(n)$$

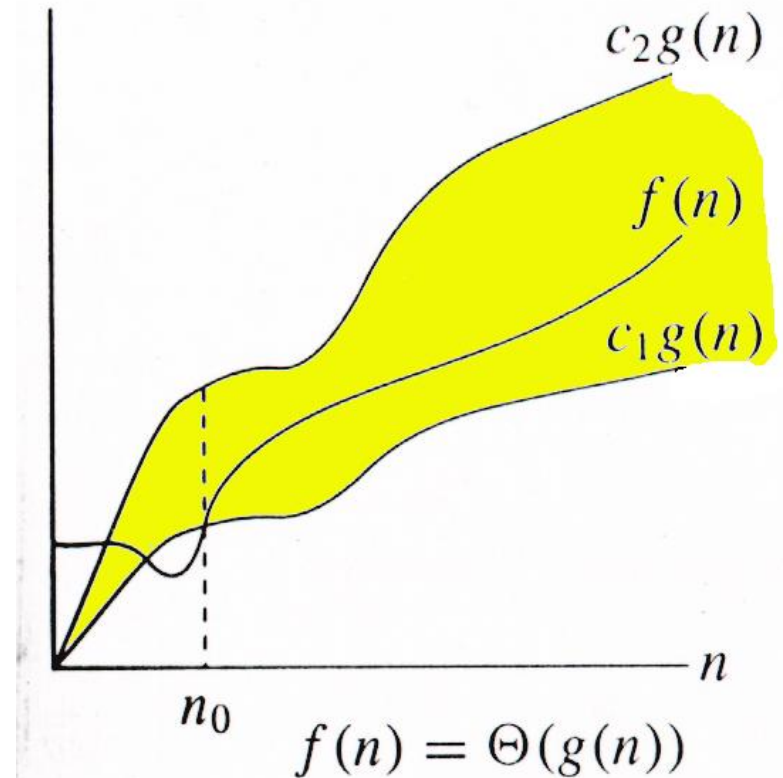
$4n^3 + 2n + 3 \geq 4n^3$  for all values of  $n$  ( $n \geq 0$ )

where  $c=4$ ,  $g(n)=n^3$

and  $n_0=0$

# Theta Notation, $\Theta$

- The Theta notation is the formal way to express **both the lower bound and the upper bound** of an algorithm's running time.
- **Definition:**  $\Theta(g(n)) = \{f(n) :$   
 $\exists$  positive constants  $c_1, c_2$ , and  $n_0$ ,  
such that  $\forall n \geq n_0, 0 \leq c_1g(n) \leq f(n) \leq c_2g(n)\}$
- **Intuitively:** Set of all functions that have the same *rate of growth* as  $g(n)$ .
- $g(n)$  is an asymptotically tight bound for  $f(n)$ .





# Theta Notation, $\theta$

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## Example:

Compute theta notation for  $f(n)=3n+2$

Given  $f(n)=3n+2$

$$c_1 * g(n) \leq f(n) \leq c_2 * g(n)$$

Compute  $f(n) \leq c_2 * g(n)$

$$3n+2 \leq 3n+n, \text{ for } n \geq 2$$

$$3n+2 \leq 4n \text{ where } c_2=4 \text{ and } g(n)=n$$

Compute  $c_1 * g(n) \leq f(n)$

$$3n \leq 3n+2 \text{ for all values of } n$$

$$\text{where } c_1=3, g(n)=n$$

$$\text{Hence, } f(n) = \theta(n)$$

## Example:

Compute theta notation for  $f(n)=10n^2+4n+2$

Given  $f(n)=10n^2+4n+2$

$$c_1 * g(n) \leq f(n) \leq c_2 * g(n)$$

Compute  $f(n) \leq c_2 * g(n)$

$$10n^2+4n+2 \leq 10n^2+4n+n, \text{ for } n \geq 2$$

$$10n^2+4n+2 \leq 10n^2+5n$$

$$10n^2+4n+2 \leq 10n^2+n^2, \text{ for } n \geq 5$$

$$10n^2+4n+2 \leq 11n^2, \text{ where } c_2=11 \text{ and } g(n)=n^2$$

Compute  $c_1 * g(n) \leq f(n)$

$$10n^2 \leq 10n^2+4n+2 \text{ for all values of } n$$

$$\text{where } c_1=10, g(n)=n^2$$

$$\text{Hence, } f(n) = \theta(n^2)$$



# Common Asymptotic Notations

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Following is a list of some common asymptotic notations:

- constant —  $O(1)$
- logarithmic —  $O(\log n)$
- linear —  $O(n)$
- $n \log n$  —  $O(n \log n)$
- quadratic —  $O(n^2)$
- cubic —  $O(n^3)$
- polynomial —  $n^{O(1)}$
- exponential —  $2^{O(n)}$



# Analysis of Algorithms

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1.  **$O(1)$** : Time complexity of a function (or set of statements) is considered as  $O(1)$  if it doesn't contain **loop**, **recursion** and **call to any other non-constant time function**.

// set of non-recursive and non-loop statements

- Example: swap() function has  $O(1)$  time complexity.
- A **loop** or **recursion** that runs a constant number of times is also considered as  $O(1)$ . For example the following loop is  $O(1)$ .

// Here c is a constant

```
for (int i = 1; i <= c; i++) {  
    // some  $O(1)$  expressions }  
}
```



# Analysis of Algorithms

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- 2)  **$O(n)$** : Time Complexity of a loop is considered as  $O(n)$  if the loop variables is **incremented** / **decremented** by a constant amount.

## Example:

// Here c is a positive integer constant

```
for (int i = 1; i <= n; i += c) {  
    // some  $O(1)$  expressions
```

```
}
```

```
for (int i = n; i > 0; i -= c) {  
    // some  $O(1)$  expressions }
```





# Analysis of Algorithms

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3.  $O(n^c)$ : Time complexity of nested loops is equal to the number of times the innermost statement is executed.

## Example:

```
for (int i = 1; i <= n; i += c) {  
    for (int j = 1; j <= n; j += c) {  
        // some O(1) expressions    }  
}
```

```
for (int i = n; i > 0; i -= c) {  
    for (int j = i+1; j <= n; j += c) {  
        // some O(1) expressions  
    }  
}
```

For example Selection sort and Insertion Sort have  $O(n^2)$  time complexity.



# Analysis of Algorithms

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- 4)  **$O(\text{Log}n)$** : Time Complexity of a loop is considered as  $O(\log n)$  if the loop variables is divided / multiplied by a constant amount.

```
for (int i = 1; i <=n; i *= c) {  
    // some  $O(1)$  expressions }  
for (int i = n; i > 0; i /= c) {  
    // some  $O(1)$  expressions }
```

Example: Binary Search (refer iterative implementation) has  $O(\log n)$  time complexity.



# Analysis of Algorithms

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5.  **$O(\text{LogLog}n)$**  Time Complexity of a loop is considered as  $O(\log \log n)$  if the loop variables is reduced / increased exponentially by a constant amount.

```
// Here c is a constant greater than 1
for (int i = 2; i <= n; i = pow(i, c)) {
    // some  $O(1)$  expressions
}

// Here func() is sqrt or cuberoot or any other constant root
for (int i = n; i > 0; i = func(i)) {
    // some  $O(1)$  expressions
}
```



# Analysis of Algorithms

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## How to combine time complexities of consecutive loops?

When there are consecutive loops, we calculate time complexity as sum of time complexities of individual loops.

```
for (int i = 1; i <= m; i += c) {  
    // some O(1) expressions  
}  
for (int i = 1; i <= n; i += c) {  
    // some O(1) expressions  
}
```

- Time complexity of above code is  $O(m) + O(n)$  which is  $O(m+n)$
- If  $m == n$ , the time complexity becomes  $O(2n)$  which is  $O(n)$ .



# Question Discussion

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What is time complexity of func()?

```
int func(int n) {  
    int count = 0;  
    for (int i = n; i > 0; i /= 2)  
        for (int j = 0; j < i; j++)  
            count += 1;  
    return count; }
```

- (A)  $O(n^2)$
- (B)  $O(n \log n)$
- (C)  $O(n)$
- (D)  $O(n \log n \log n)$



# Question Discussion

---

What is time complexity of func()?

```
int func(int n) {  
    int count = 0;  
    for (int i = n; i > 0; i /= 2)  
        for (int j = 0; j < i; j++)  
            count += 1;  
    return count; }
```

- (A)  $O(n^2)$
- (B)  $O(n \log n)$
- (C)  $O(n)$
- (D)  $O(n \log n \log n)$

Answer: (C)

Explanation:

- For a input integer  $n$ , the innermost statement of func() is executed following times.  
$$n + n/2 + n/4 + \dots 1$$
- So time complexity  $T(n)$  can be written as:  $T(n) = O(n + n/2 + n/4 + \dots 1) = O(n)$



# Question Discussion

---

What is the time complexity of func()?

```
int func(int n) {  
    int count = 0;  
    for (int i = 0; i < n; i++)  
        for (int j = i; j > 0; j--)  
            count = count + 1;  
    return count;  
}
```

- (A)  $O(n)$
- (B)  $O(n^2)$
- (C)  $O(n \cdot \log n)$
- (D)  $O(n \log n \log n)$



# Question Discussion

---

What is the time complexity of func()?

```
int func(int n) {  
    int count = 0;  
    for (int i = 0; i < n; i++)  
        for (int j = i; j > 0; j--)  
            count = count + 1;  
    return count;  
}
```

- (A)  $O(n)$
- (B)  $O(n^2)$
- (C)  $O(n \cdot \log n)$
- (D)  $O(n \log n \log n)$

Answer: (B)

Explanation:

- Time complexity can be calculated by counting number of times the expression “count = count + 1;” is executed.
- The expression is executed  $0 + 1 + 2 + 3 + 4 + \dots + (n-1)$  times.
- Time complexity =  $O(0 + 1 + 2 + 3 + \dots + n-1) = O(n \cdot (n-1)/2) = O(n^2)$





# Question Discussion

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Which of the given options provides the increasing order of asymptotic complexity of functions f1, f2, f3 and f4?

$$f1(n) = 2^n$$

$$f2(n) = n^{(3/2)}$$

$$f3(n) = n \text{Log} n$$

$$f4(n) = n^{(\text{Log} n)}$$

- (A) f3, f2, f4, f1
- (B) f3, f2, f1, f4
- (C) f2, f3, f1, f4
- (D) f2, f3, f4, f1



# Question Discussion

---

Which of the given options provides the increasing order of asymptotic complexity of functions  $f_1$ ,  $f_2$ ,  $f_3$  and  $f_4$ ?

$$\begin{array}{ll} f_1(n) = 2^n & f_2(n) = n^{(3/2)} \\ f_3(n) = n \text{Log} n & f_4(n) = n^{(\text{Log} n)} \end{array}$$

- (A)  $f_3$ ,  $f_2$ ,  $f_4$ ,  $f_1$
- (B)  $f_3$ ,  $f_2$ ,  $f_1$ ,  $f_4$
- (C)  $f_2$ ,  $f_3$ ,  $f_1$ ,  $f_4$
- (D)  $f_2$ ,  $f_3$ ,  $f_4$ ,  $f_1$

Ans: (A)

Explanation:

$$\begin{array}{ll} f_1(n) = 2^n & f_2(n) = n^{(3/2)} \\ f_3(n) = n \text{Log} n & f_4(n) = n^{(\text{Log} n)} \end{array}$$

- Except  $f_3$ , all other are polynomial/ exponential.
- $f_3$  is definitely first in output. Among remaining,  $n^{(3/2)}$  is next.
- One way to compare  $f_1$  and  $f_4$  is to take Log of both functions.
  - Order of growth of  $\text{Log}(f_1(n))$  is  $O(n)$  and order of growth of  $\text{Log}(f_4(n))$  is  $O(\text{Log} n * \text{Log} n)$ .
  - Since  $O(n)$  has higher growth than  $\Theta(\text{Log} n * \text{Log} n)$ ,  $f_1(n)$  grows faster than  $f_4(n)$ .



# Question Discussion

---

Which of the following is not  $O(n^2)$ ?

(A)  $(15^{10}) * n + 12099$

(B)  $n^{1.98}$

(C)  $n^3 / (\text{sqrt}(n))$

(D)  $(2^{20}) * n$



# Question Discussion

---

Which of the following is not  $O(n^2)$ ?

(A)  $(15^{10}) * n + 12099$

(B)  $n^{1.98}$

(C)  $n^3 / (\text{sqrt}(n))$

(D)  $(2^{20}) * n$

Answer: (C)

Explanation: The order of growth of option c is  $n^{2.5}$  which is higher than  $n^2$ .



# Question Discussion

---

What is the time, space complexity of following code:

```
int a = 0, b = 0;
for (i = 0; i < n; i++) {
    a = a + rand();
}
for (j = 0; j < m; j++) {
    b = b + rand();
}
```

- A.  $O(n * m)$  time,  $O(1)$  space
- B.  $O(n + m)$  time,  $O(n + m)$  space
- C.  $O(n + m)$  time,  $O(1)$  space
- D.  $O(n * m)$  time,  $O(n + m)$  space



# Question Discussion

---

What is the time, space complexity of following code:

```
int a = 0, b = 0;
for (i = 0; i < n; i++) {
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    b = b + rand();
}
```

- A.  $O(n * m)$  time,  $O(1)$  space
- B.  $O(n + m)$  time,  $O(n + m)$  space
- C.  $O(n + m)$  time,  $O(1)$  space
- D.  $O(n * m)$  time,  $O(n + m)$  space

C.  $O(n + m)$  time,  $O(1)$  space

**Explanation:** The first loop is  $O(n)$  and the second loop is  $O(m)$ . Since we don't know which is bigger, we say this is  $O(n + m)$ . This can also be written as  $O(\max(n, m))$ .

Since there is no additional space being utilized, the space complexity is constant /  $O(1)$



# Question Discussion

---

What is the time complexity of following code:

```
int a = 0;
for (i = 0; i < n; i++) {
    for (j = n; j > i; j--) {
        a = a + i + j;
    }
}
```

- A.  $O(n)$
- B.  $O(n \cdot \log(n))$
- C.  $O(n \cdot \sqrt{n})$
- D.  $O(n \cdot n)$



# Question Discussion

---

What is the time complexity of following code:

```
int a = 0;
for (i = 0; i < n; i++) {
    for (j = n; j > i; j--) {
        a = a + i + j;
    }
}
```

- A.  $O(n)$
- B.  $O(n * \log(n))$
- C.  $O(n * \sqrt{n})$
- D.  $O(n * n)$

D.  $O(n * n)$

**Explanation:** The above code runs total no of times

$$= n + (n - 1) + (n - 2) + \dots 1 + 0$$

$$= n * (n + 1) / 2$$

$$= 1/2 * n^2 + 1/2 * n$$

$O(n^2)$  times.





# Question Discussion

---

What is the time complexity of following code:

```
int i, j, k = 0;
for (i = n / 2; i <= n; i++) {
    for (j = 2; j <= n; j = j * 2) {
        k = k + n / 2;
    }
}
```

- A.  $O(n)$
- B.  $O(n \log n)$
- C.  $O(n^2)$
- D.  $O(n^2 \log n)$



# Question Discussion

---

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- A.  $O(n)$
- B.  $O(n \log n)$
- C.  $O(n^2)$
- D.  $O(n^2 \log n)$

B.  $O(n \log n)$

**Explanation:** If you notice,  $j$  keeps doubling till it is less than or equal to  $n$ . Number of times, we can double a number till it is less than  $n$  would be  $\log(n)$ .

Let's take the examples here.

for  $n = 16$ ,  $j = 2, 4, 8, 16$

for  $n = 32$ ,  $j = 2, 4, 8, 16, 32$

So,  $j$  would run for  $O(\log n)$  steps.

$i$  runs for  $n/2$  steps.

So, total steps =  $O(n/2 * \log(n)) = O(n * \log n)$



# Question Discussion

---

What is the time complexity of following code:

```
int a = 0, i = n;  
while (i > 0) {  
    a += i;  
    i /= 2;  
}
```

- A.  $O(n)$
- B.  $O(\sqrt{n})$
- C.  $O(n / 2)$
- D.  $O(\log n)$



# Question Discussion

---

What is the time complexity of following code:

```
int a = 0, i = n;  
while (i > 0) {  
    a += i;  
    i /= 2;  
}
```

- A.  $O(n)$
- B.  $O(\sqrt{n})$
- C.  $O(n / 2)$
- D.  $O(\log n)$

D.  $O(\log n)$

**Explanation:** We have to find the smallest  $x$  such that  $n / 2^x \leq 1$   
 $x = \log(n)$



# Question Discussion

---

```
for(i=1; i<=n; i=i*2)
    for(j=n; j>=1; j=j/2)
        statement;
```

Time Complexity  $O((\log n)^2)$



# Question Discussion

---

```
void function(int n){  
    int i,j,k;  
    for(i=n/2; i<=n; i++)  
        for(j=1; j + n/2<=n; j++)  
            for(k=1; k<=n; k= k * 2)  
                count++;  
}
```

Ans:  $O(n^2 * \log n)$  . In both these type of answers, we should give the marks because students do not know mathematical computation of time complexity.



# Question Discussion

---

What is time complexity of the function func()?

```
void func(int arr[], int n) {  
    int i, j = 0;  
    for(i=0; i<n; i++)  
        while(j < n && arr[i] < arr[j])  
            j++;  
}
```

Ans:  $O(n)$  (Note: Since the  $j$  is not initialized inside the outer while loop, the inner while loop will be executed nearly  $n$  number of times)



# Question Discussion

---

What does it mean when we say that an algorithm X is asymptotically more efficient than Y?

- A. X will always be a better choice for small inputs
- B. X will always be a better choice for large inputs
- C. Y will always be a better choice for small inputs
- D. X will always be a better choice for all inputs

B. X will always be a better choice for large inputs

**Explanation:** In asymptotic analysis we consider growth of algorithm in terms of input size. An algorithm X is said to be asymptotically better than Y if X takes smaller time than Y for all input sizes  $n$  larger than a value  $n_0$  where  $n_0 > 0$ .





# Data Structures and Algorithms

---

- **Data structure** affects the design of both **structural** & **functional** aspects of a program.

Program=Algorithm + Data Structure

- Algorithm is a step by step procedure to solve a particular function.



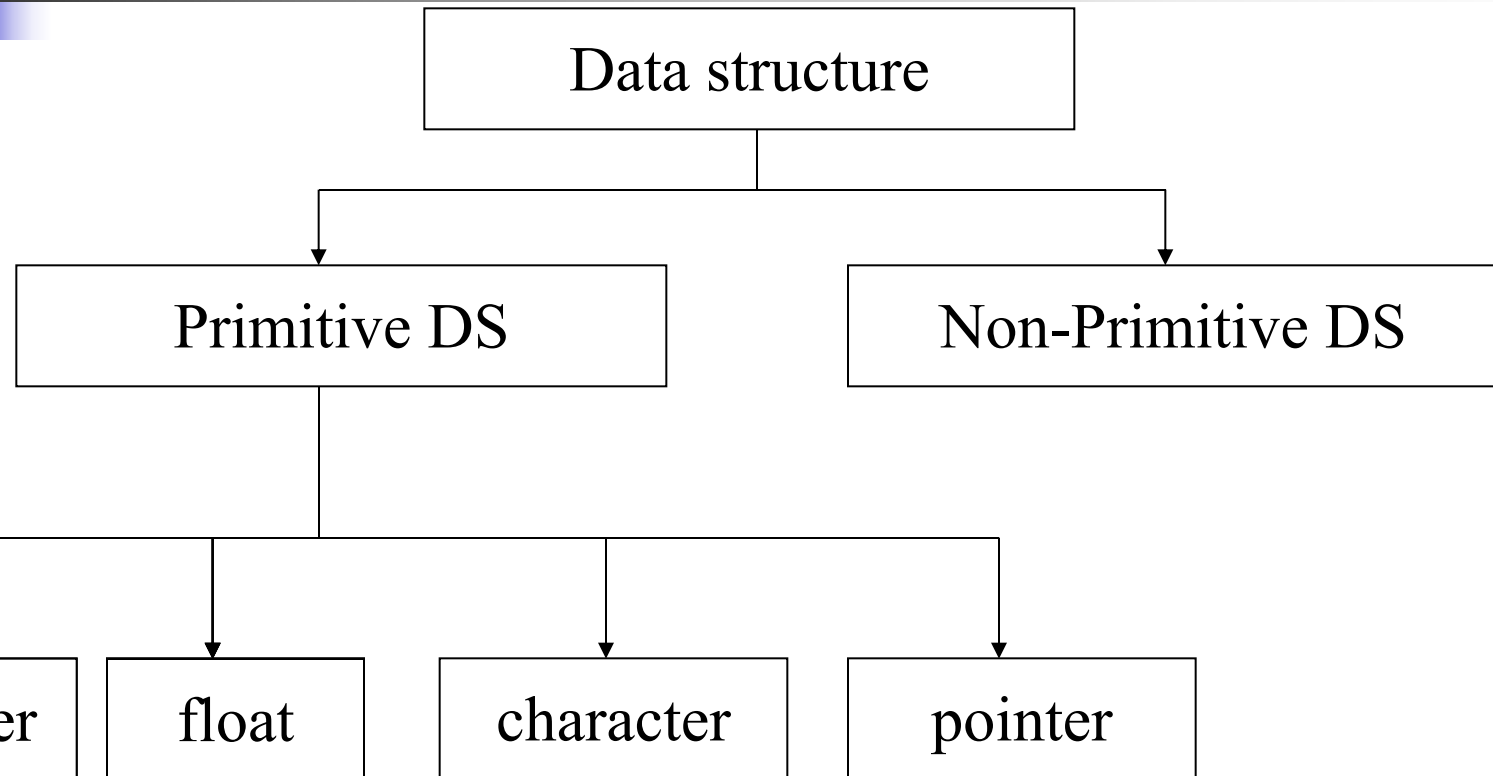
# Classification of Data Structure

---

- Data structure are normally divided into two broad categories:
  - Primitive Data Structure
  - Non-Primitive Data Structure

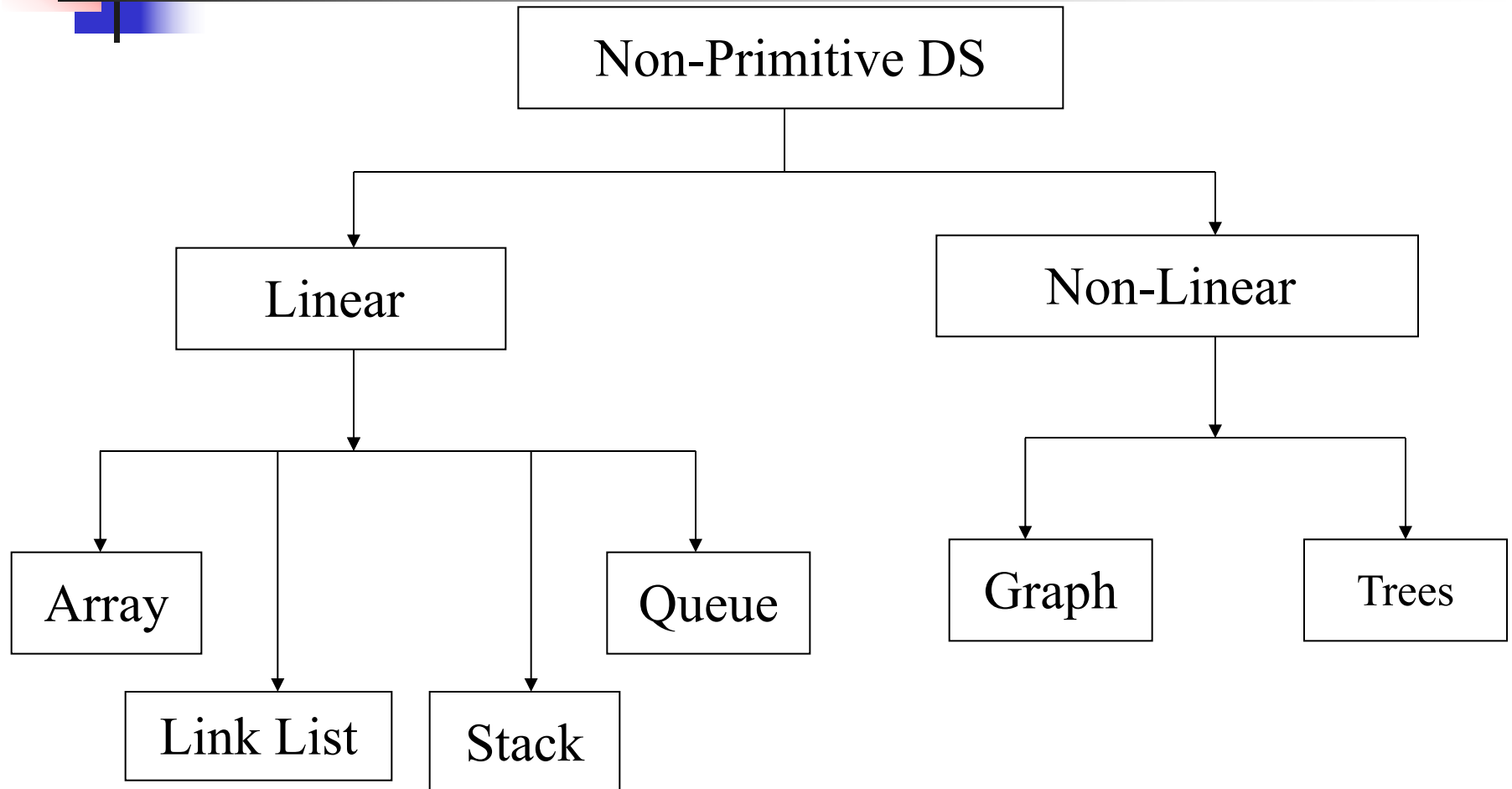


# Classification of Data Structure





# Classification of Data Structure





# Primitive Data Structure

---

- There are basic structures and directly operated upon by the machine instructions.
- In general, there are different representation on different computers.
- Integer, Floating-point number, Character constants, string constants, pointers etc, fall in this category.



# Non-Primitive Data Structure

---

- There are more sophisticated data structures.
- These are **derived** from the primitive data structures.
- The non-primitive data structures emphasize on structuring of a group of **homogeneous** (**same type**) or **heterogeneous** (**different type**) data items.



# Non-Primitive Data Structure

---

- Lists, Stack, Queue, Tree, Graph are example of non-primitive data structures.



# Non-Primitive Data Structure

---

- The most commonly used operation on data structure are broadly categorized into following types:
  - Create
  - Selection
  - Updating
  - Searching
  - Sorting
  - Merging
  - Destroy or Delete





# Difference between them

---

- A primitive data structure is generally a basic structure that is usually built into the language, such as an integer, a float.
- A non-primitive data structure is built out of primitive data structures linked together in meaningful ways, such as
  - linked-list, binary search tree, AVL Tree, graph etc.



# Description of various Data Structures: Arrays

---

- An array is defined as a set of finite number of homogeneous elements or same data items.
- It means an array can contain one type of data only, either all integer, all float-point number or all character.



# Arrays

---

- Simply, declaration of array is as follows:

```
int arr[10]
```

- Where int specifies the data type or type of elements arrays stores.
- “arr” is the name of array & the number specified inside the square brackets is the number of elements an array can store, this is also called sized or length of array.



# Arrays

---

- Following are some of the concepts to be remembered about arrays:
  - The individual element of an array can be accessed by specifying name of the array, following by index or subscript inside square brackets.
  - The first element of the array has index zero[0]. It means the first element and last element will be specified as:arr[0] & arr[9] respectively.



# Arrays

---

- The elements of array will always be stored in the consecutive (continues) memory location.
- The number of elements that can be stored in an array, that is the size of array or its length is given by the following equation:  
$$(\text{Upperbound} - \text{lowerbound}) + 1$$



# Arrays

---

- For this array it would be  $(9-0)+1=10$ , where 0 is the lower bound of array and 9 is the upper bound of array.
- Array can always be read or written through loop.
  - If we read a one-dimensional array it require one loop for reading and other for writing the array.



# Arrays

---

- For example: Reading an array

```
for(i=0;i<=9;i++)  
    scanf("%d", &arr[i]);
```

- For example: Writing an array

```
for(i=0;i<=9;i++)  
    printf("%d", arr[i]);
```



# Arrays

---

- If we are reading or writing two-dimensional array it would require two loops.
  - And similarly the array of a N dimension would required N loops.
- Some common operation performed on array are:
  - Creation of an array
  - Traversing an array





# Arrays

---

- Insertion of new element
- Deletion of required element
- Modification of an element
- Merging of arrays



# Lists

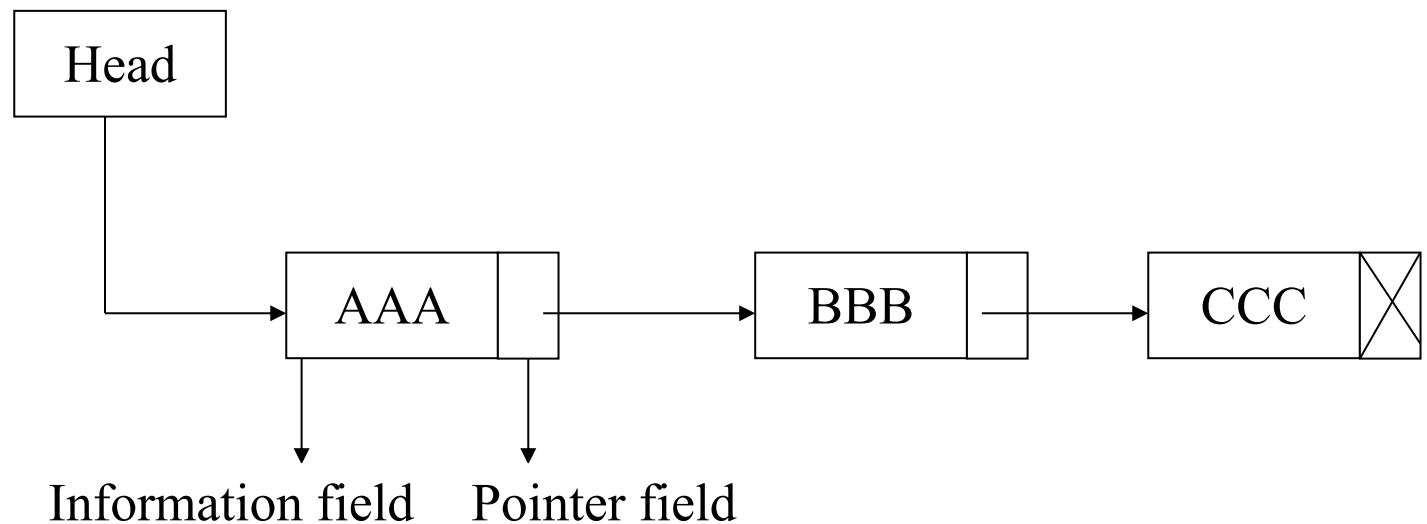
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- A **lists** (**Linear linked list**) can be defined as a collection of variable number of data items.
- Lists are the most commonly used non-primitive data structures.
- An element of list must contain at least two fields:
  - one for storing data or information and other for storing address of next element.
- For storing address, a special data structure of list the address must be pointer type.

# Lists

- Technically each such element is referred to as a node, therefore a list can be defined as a collection of nodes as show bellow:

[Linear Liked List]





# Lists

---

- Types of linked lists:
  - Single linked list
  - Double linked list
  - Single circular linked list
  - Double circular linked list



# Stack

---

- A stack is also an ordered collection of elements like arrays, but it has a special feature that deletion and insertion of elements can be done only from one end called the top of the stack (TOP)
- Due to this property it is also called as last in first out type of data structure (LIFO).



# Stack

---

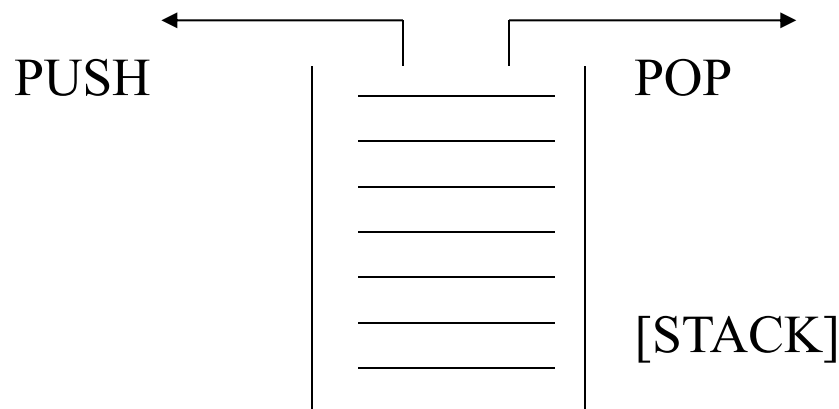
- It could be thought of just like a stack of plates placed on table in a party, a guest always takes off a fresh plate from the top and the new plates are placed on to the stack at the top.
- It is a non-primitive data structure.
- When an element is inserted into a stack or removed from the stack, its base remains fixed where the top of stack changes.



# Stack

---

- Insertion of element into stack is called PUSH and deletion of element from stack is called POP.
- The bellow show figure how the operations take place on a stack:





# Stack

---

- The stack can be implemented into two ways:
  - Using arrays (Static implementation)
  - Using pointer (Dynamic implementation)





# Queue

---

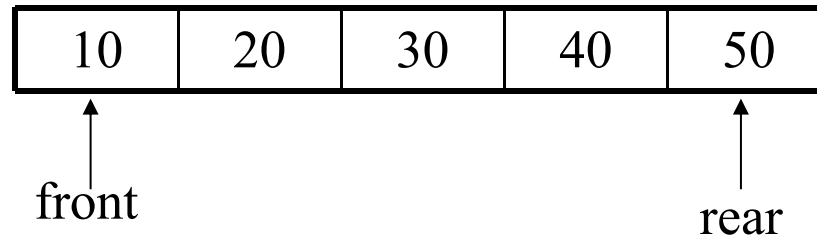
- Queue are first in first out type of data structure (i.e. FIFO)
- In a queue new elements are added to the queue from one end called REAR end and the element are always removed from other end called the FRONT end.
- The people standing in a railway reservation row are an example of queue.



# Queue

---

- Each new person comes and stands at the end of the row and person getting their reservation confirmed get out of the row from the front end.
- The bellow show figure how the operations take place on a stack:





# Queue

---

- The queue can be implemented into two ways:
  - Using arrays (Static implementation)
  - Using pointer (Dynamic implementation)



# Trees

---

- A tree can be defined as finite set of data items (nodes).
- Tree is non-linear type of data structure in which data items are arranged or stored in a sorted sequence.
- Tree represent the hierarchical relationship between various elements.



# Trees

---

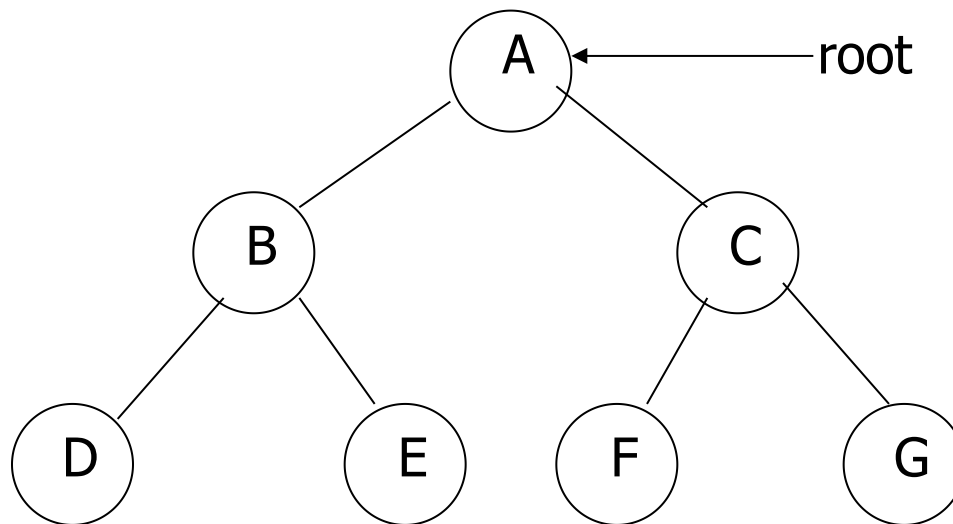
- In trees:
  - There is a special data item at the **top of hierarchy** called the *root* of the tree.
  - The remaining data items are partitioned into number of **mutually exclusive subset**, each of which is itself, a tree which is called the sub tree.
  - The tree always grows in length towards bottom in data structures, unlike natural trees which grows upwards.



# Trees

---

- The tree structure organizes the data into branches, which related the information.





# Graph

---

- Graph is a mathematical non-linear data structure capable of representing many kind of physical structures.
- It has found application in Geography, Chemistry and Engineering sciences.
- Definition: A graph  $G(V,E)$  is a set of vertices  $V$  and a set of edges  $E$ .



# Graph

---

- An edge connects a pair of vertices and many have weight such as length, cost and another measuring instrument for according the graph.
- Vertices on the graph are shown as point or circles and edges are drawn as arcs or line segment.

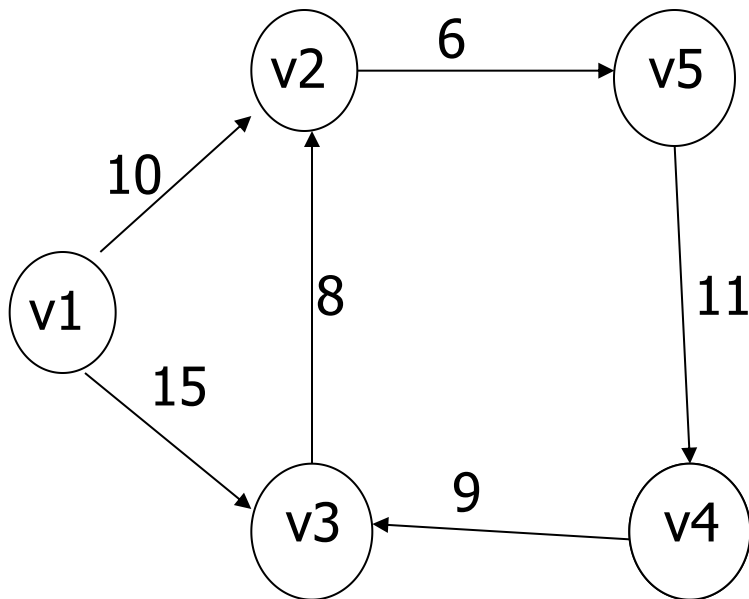




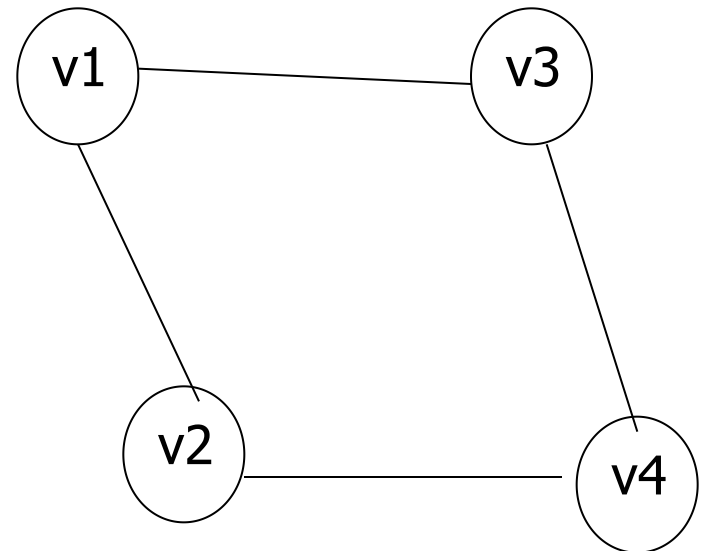
# Graph

---

- Example of graph:



[a] Directed & Weighted Graph



[b] Undirected Graph



# Graph

---

- Types of Graphs:
  - Directed graph
  - Undirected graph
  - Simple graph
  - Weighted graph
  - Connected graph
  - Non-connected graph