E0-226: Linear Algebra and Probability **(Due: 15/09/2019 11:59 P.M.)**

Assignment-1

Instructor: Prof. M. N. Murty *TAs:* Kishalay Das, Paarth Gupta

Assignment Policy:

Welcome to the First Assignment of Linear Algebra, where you will implement your first assignment on Gauss Elimination and System of Linear Equations. **Read all the instructions below carefully before you start working on the assignment, and before you make a submission.**

- This is a coding assignment and the code has to be written in Python. Use Python version 3 to solve all the problems.The python file name should be **main.py**.
- **For computational part you are not supposed to use any library function.Do code from scratch.**
- The assignment deliverable is your python code (main.py) and your report(pdf format).
- Send above files in a zip file to lap_e0226@outlook.com with the following format for both zip file name and the subject name of email: **LAP_1_XXXXX**(Last 5 digits of your Student ID)
- **Read the complete assignment carefully, before attempting to solve it.**
- You are encouraged to discuss among yourselves, but **DO NOT COPY** solutions or code.**The consequences will be severe**.
- The submission deadline is **15th September,2019 11:59 P.M.**.

Problem 1: Gauss Elimination and Elementary Matrices (0.5+1+1+0.5=3 points)

Gauss Elimination is an algorithm in linear algebra used for various purposes like solving a system of linear equations, finding the rank of a matrix, calculating the determinant of a matrix etc. In Gauss elimination method, given any matrix, we use a sequence of elementary row operations and transform the matrix into Row echelon form.
And for each of such elementary row operation, there will be elementary matrix you need to multiply with the matrix. An **elementary matrix** is a matrix which differs from the identity matrix by one single elementary row operation and Left multiplication by an elementary matrix represents elementary row operations.Here is an example:-

Step1 :

$$\begin{bmatrix} 1 & 0 & 0 \\ 5 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{1} \\ \mathbf{5} & \mathbf{10} & \mathbf{2} \\ \mathbf{1} & \mathbf{8} & \mathbf{9} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{1} \\ \mathbf{0} & \mathbf{5} & \mathbf{3} \\ \mathbf{1} & \mathbf{8} & \mathbf{9} \end{bmatrix}$$

Step2 :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{1} \\ \mathbf{0} & \mathbf{5} & \mathbf{3} \\ \mathbf{1} & \mathbf{8} & \mathbf{9} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{1} \\ \mathbf{0} & \mathbf{5} & \mathbf{3} \\ \mathbf{0} & \mathbf{5} & \mathbf{8} \end{bmatrix}$$

Step3 :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{1} \\ \mathbf{0} & \mathbf{5} & \mathbf{3} \\ \mathbf{0} & \mathbf{5} & \mathbf{8} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{1} \\ \mathbf{0} & \mathbf{5} & \mathbf{3} \\ \mathbf{0} & \mathbf{0} & \mathbf{5} \end{bmatrix}$$

Task I:

You have to write a code for Gauss-Jordan Elimination which will take a matrix as input and as output the code will print the following:

- Rank of the given Matrix.
- Row echelon form of the matrix.
- The sequence of elementary matrices used in each step **(print in order)**.

You are not allowed to use any library function. You need to implement it from scratch.Your code will take input matrix from a input.txt file (Sample file here under problem1 folder).

Your runnable code should use following format:

`python main.py problem1 input_file_path`

`OUTPUT FORMAT` `:(For above matrix)`
`RANK OF THE MATRIX: 3`
`ROW ECHELON FORM:`

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 5 & 3 \\ 0 & 0 & 5 \end{bmatrix}$$

`SEQUENCE OF ELEMENTARY MATRICES USED:`

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 5 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Task II:

Compare Row Space,Column Space and Null Space of the original matrix and it's echelon form.What do you think about it? Whether they will be same or different? Write down your answer in the report. Give appropriate reason for your answer. **No need to code in this task.**

Problem 2: System Of Linear Equations

(0.5+0.5+0.5+0.5=2 points)

A **system of linear equations** (or linear system) is a finite collection of linear equations in a collection of variables. For instance, a linear system of m equations in n variables $x_1, x_2, ..., x_n$ can be written as

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n & = & \mathrm{b}_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n & = & \mathrm{b}_2 \\ . & . & . \\ . & . & . \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n & = & \mathrm{b}_m \end{cases}$$

A solution of the above linear system is a tuple $(s_1, s_2, ..., s_n)$ of numbers that makes each equation a true statement when the values $s_1, s_2, ..., s_n$ are substituted for $x_1, x_2, ..., x_n$ respectively. The set of all solutions of a linear system is called the solution set of the system.
Any system of linear equations has one of the following exclusive conclusions.

(a) No solution.

(b) Unique solution.

(c) Infinitely many solutions.

A linear system is said to be **consistent** if it has at least one solution; and is said to be **inconsistent** if it has no solution.

Task I:
You will be given the augmented coefficient matrix [A | B] of the linear system in input.txt file (Sample file here under problem2 folder). Last column is the "B" part of augmented coefficient matrix . You need to write a python code to find out solution of this linear system.**(You are supposed to write the code without using any library function.)** Your code will take the augmented coefficient matrix as input and output as follows:

(a) If system is inconsistent it will print **"NO SOLUTION EXISTS !"**

(b) If system is consistent and has unique solution, print **"UNIQUE SOLUTION EXISTS !"** and print that solution in the next line.

(c) If system is consistent and infinitely many solutions, print **"MANY SOLUTIONS EXIST !"** and print any one of the solutions in the next line.

Your runnable code should employ the following format:

```
python main.py problem2 input_file_path
```

Task II:
What is the time complexity of the algorithm that you have used to solve the linear system? Report it in your report and explain your reason properly.**No need to code in this task.**