**Министерство образования и науки РФ Санкт-Петербургский политехнический университет Петра Великого Институт компьютерных наук и технологий Высшая школа программной инженерии**

**КУРСОВАЯ РАБОТА**

по дисциплине «Вычислительная математика»

по теме «Исследование движения космического корабля»

Выполнил

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Санкт-Петербург

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Формулировка задания

Рассмотрим движение космического корабля в гравитационном поле двух фиксированных притягивающих центров. Масса Земли Mз много больше массы Луны Mл. В декартовой прямоугольной системе, показанной на рисунке, безразмерные уравнения имеют вид:

0

1

Земля

Луна

(x, y) – космический корабль

x

y

Найти траекторию космического корабля и построить графики в плоскостях (x, t), (y, t), (x, y).

Оценить погрешность результата и влияние на точность погрешности исходных данных.

Выполнение работы

Рассчитаем начальные условия – для этого найдем корень уравнения с интегралом, использую самый точный метод решения нелинейных уравнений из библиотеки scipy - toms748 (кубическая интерполяция и метод Ньютона). Для нахождения самого интеграла используем метод quad. Относительную погрешность решения выбираем исходя из точности чисел, данных в условии задачи. После того, как начальные условия рассчитаны, нам необходимо решить систему дифференциальных уравнений второго порядка. Используем метод приведения системы дифференциальных уравнений высокого порядка к системе дифференциальных уравнений первого порядка:

Полученную систему решаем методами Рунге-Кутты 4 и 5 степени, заносим результаты в таблицу и по ней строим графики – координат в пространстве и скоростей от времени. Решаем систему для исходных данных, отличающихся от начальных в 1.05 и 1.5 раза и строим таблицу и графики для этих решений.

Результат работы

## Вывод программы:

Relative tolerance = 1e-06

0.10 : [ 1.19512718 -0.09606556 0.09967227 0.990539 ]

±[4.87274336e-09 9.60655641e-08 9.96722719e-08 9.46099755e-09]

0.20 : [ 1.18117953 -0.18092839 0.19772077 0.96935806]

±[1.88203890e-08 1.80928386e-07 1.97720768e-07 3.06419421e-08]

0.30 : [ 1.15925656 -0.25634283 0.29341607 0.94410672]

±[4.07433578e-08 2.56342831e-07 2.93416068e-07 5.58932824e-08]

0.40 : [ 1.1300931 -0.32621263 0.38645605 0.91622656]

±[6.99068190e-08 3.26212628e-07 3.86456046e-07 8.37734370e-08]

0.50 : [ 1.0941356 -0.39239453 0.47655734 0.88522934]

±[1.05864320e-07 3.92394533e-07 4.76557337e-07 1.14770656e-07]

0.60 : [ 1.05171785 -0.45544982 0.56338057 0.85060132]

±[1.48282069e-07 4.55449821e-07 5.63380573e-07 1.49398680e-07]

0.70 : [ 1.00314948 -0.5153832 0.6465479 0.81208946]

±[1.96850441e-07 5.15383199e-07 6.46547895e-07 1.87910537e-07]

0.80 : [ 0.94875235 -0.5719802 0.72566776 0.76966153]

±[2.51247571e-07 5.71980198e-07 7.25667759e-07 2.30338469e-07]

0.90 : [ 0.8888743 -0.62495112 0.80035368 0.72344144]

±[3.11125620e-07 6.24951116e-07 8.00353677e-07 2.76558556e-07]

1.00 : [ 0.82389303 -0.6739956 0.87023731 0.67366166]

±[3.76106888e-07 6.73995595e-07 8.70237308e-07 3.26338336e-07]

1.10 : [ 0.75421543 -0.7188338 0.93497758 0.6206307 ]

±[4.45784488e-07 7.18833796e-07 9.34977579e-07 3.79369302e-07]

1.20 : [ 0.68027465 -0.75922232 0.99426709 0.56471018]

±[5.19725275e-07 7.59222317e-07 9.94267090e-07 4.35289816e-07]

1.30 : [ 0.60252597 -0.79496241 1.04783653 0.50629775]

±[5.97473950e-07 7.94962415e-07 1.04783653e-06 4.93702246e-07]

1.40 : [ 0.52144217 -0.82590381 1.09545757 0.44581349]

±[6.78557746e-07 8.25903812e-07 1.09545757e-06 5.54186509e-07]

1.50 : [ 0.43750858 -0.85194573 1.13694454 0.38368883]

±[7.62491342e-07 8.51945734e-07 1.13694454e-06 6.16311168e-07]

1.60 : [ 0.35121813 -0.87303603 1.17215511 0.32035731]

±[8.48781787e-07 8.73036030e-07 1.17215511e-06 6.79642694e-07]

1.70 : [ 0.26306667 -0.88916892 1.20099005 0.25624678]

±[9.36933253e-07 8.89168915e-07 1.20099005e-06 7.43753223e-07]

1.80 : [ 0.1735484 -0.90038174 1.22339238 0.19177302]

±[1.02645152e-06 9.00381736e-07 1.22339238e-06 8.08226978e-07]

1.90 : [ 0.08315181 -0.90675104 1.23934585 0.12733453]

±[1.11684811e-06 9.06751040e-07 1.23934585e-06 8.72665472e-07]

2.00 : [-0.00764404 -0.90838822 1.24887306 0.06330842]

±[1.20764396e-06 9.08388225e-07 1.24887306e-06 9.36691580e-07]

2.10 : [-9.83727915e-02 -9.05434946e-01 1.25203307e+00 4.74283972e-05]

±[1.29837271e-06 9.11341503e-07 1.25203307e-06 9.99952572e-07]

2.20 : [-0.18858357 -0.8980585 1.24891895 -0.06212221]

±[1.38858349e-06 9.18717951e-07 1.25514719e-06 1.06212221e-06]

2.30 : [-0.27784331 -0.88644728 1.23965499 -0.12290201]

±[1.47784323e-06 9.30329168e-07 1.26441116e-06 1.12290201e-06]

2.40 : [-0.36573869 -0.87080648 1.22439392 -0.18202174]

±[1.56573861e-06 9.45969970e-07 1.27967223e-06 1.18202174e-06]

2.50 : [-0.45187754 -0.85135404 1.20331408 -0.23923935]

±[1.65187747e-06 9.65422405e-07 1.30075207e-06 1.23923935e-06]

2.60 : [-0.53589001 -0.82831703 1.17661663 -0.29434039]

±[1.73588993e-06 9.88459423e-07 1.32744952e-06 1.29434039e-06]

2.70 : [-0.61742922 -0.80192829 1.14452284 -0.34713692]

±[1.81742914e-06 1.01484816e-06 1.35954330e-06 1.34713692e-06]

2.80 : [-0.6961718 -0.77242365 1.10727151 -0.39746631]

±[1.89617172e-06 1.04435280e-06 1.39679464e-06 1.39746631e-06]

2.90 : [-0.77181797 -0.74003939 1.06511648 -0.44518959]

±[1.97181789e-06 1.07673706e-06 1.43894967e-06 1.44518959e-06]

3.00 : [-0.84409152 -0.70501019 1.01832438 -0.49018985]

±[2.04409144e-06 1.11176626e-06 1.48574176e-06 1.49018985e-06]

3.10 : [-0.91273956 -0.66756741 0.96717252 -0.53237041]

±[2.11273948e-06 1.14920904e-06 1.53689363e-06 1.53237041e-06]

3.20 : [-0.97753211 -0.62793775 0.91194691 -0.57165308]

±[2.17753203e-06 1.18883870e-06 1.59211923e-06 1.57165308e-06]

3.30 : [-1.03826158 -0.58634216 0.85294056 -0.60797636]

±[2.23826150e-06 1.23043429e-06 1.65112558e-06 1.60797636e-06]

3.40 : [-1.09474217 -0.54299509 0.79045185 -0.64129371]

±[2.29474209e-06 1.27378136e-06 1.71361429e-06 1.64129371e-06]

3.50 : [-1.14680914 -0.49810395 0.72478314 -0.67157191]

±[2.34680906e-06 1.31867250e-06 1.77928301e-06 1.67157191e-06]

3.60 : [-1.19431816 -0.45186876 0.6562395 -0.69878953]

±[2.39431808e-06 1.36490769e-06 1.84782664e-06 1.69878953e-06]

3.70 : [-1.23714452 -0.40448208 0.58512763 -0.72293544]

±[2.43714444e-06 1.41229437e-06 1.91893851e-06 1.72293544e-06]

3.80 : [-1.27518237 -0.35612893 0.51175488 -0.74400754]

±[2.47518229e-06 1.46064752e-06 1.99231126e-06 1.74400754e-06]

3.90 : [-1.30834402 -0.30698701 0.4364284 -0.76201153]

±[2.50834394e-06 1.50978944e-06 2.06763774e-06 1.76201153e-06]

4.00 : [-1.33655917 -0.25722689 0.35945444 -0.77695982]

±[2.53655910e-06 1.55954956e-06 2.14461171e-06 1.77695982e-06]

4.10 : [-1.35977426 -0.20701234 0.28113769 -0.78887059]

±[2.55977418e-06 1.60976411e-06 2.22292845e-06 1.78887059e-06]

4.20 : [-1.37795175 -0.15650074 0.20178081 -0.79776692]

±[2.57795167e-06 1.66027571e-06 2.30228534e-06 1.79776692e-06]

4.30 : [-1.39106956 -0.10584349 0.12168389 -0.80367607]

±[2.59106948e-06 1.71093296e-06 2.38238226e-06 1.80367607e-06]

4.40 : [-1.39912046 -0.05518651 0.04114414 -0.80662888]

±[2.59912038e-06 1.76158994e-06 2.46292200e-06 1.80662888e-06]

4.50 : [-1.40211157 -0.00467071 -0.03954447 -0.80665921]

±[2.60211149e-06 1.81210574e-06 2.54361062e-06 1.80665921e-06]

4.60 : [-1.40006386 0.04556748 -0.1200915 -0.8038035 ]

±[2.60415920e-06 1.86234393e-06 2.62415764e-06 1.80951491e-06]

4.70 : [-1.39301175 0.09539558 -0.20021026 -0.79810049]

±[2.61121131e-06 1.91217203e-06 2.70427640e-06 1.81521792e-06]

4.80 : [-1.38100272 0.14468458 -0.27961804 -0.78959093]

±[2.62322034e-06 1.96146103e-06 2.78368419e-06 1.82372748e-06]

4.90 : [-1.36409701 0.19330838 -0.35803631 -0.77831742]

±[2.64012605e-06 2.01008483e-06 2.86210246e-06 1.83500100e-06]

5.00 : [-1.34236734 0.24114331 -0.43519087 -0.76432429]

±[2.66185572e-06 2.05791976e-06 2.93925701e-06 1.84899412e-06]

5.10 : [-1.3158987 0.28806762 -0.51081205 -0.74765764]

±[2.68832436e-06 2.10484407e-06 3.01487819e-06 1.86566077e-06]

5.20 : [-1.28478818 0.33396103 -0.58463488 -0.72836535]

±[2.71943488e-06 2.15073748e-06 3.08870102e-06 1.88495306e-06]

5.30 : [-1.24914483 0.37870428 -0.65639925 -0.70649719]

±[2.75507822e-06 2.19548073e-06 3.16046540e-06 1.90682122e-06]

5.40 : [-1.20908962 0.42217868 -0.72585018 -0.68210503]

±[2.79513344e-06 2.23895513e-06 3.22991632e-06 1.93121339e-06]

5.50 : [-1.16475534 0.46426574 -0.79273793 -0.65524307]

±[2.83946772e-06 2.28104219e-06 3.29680408e-06 1.95807534e-06]

5.60 : [-1.11628667 0.5048468 -0.85681836 -0.62596818]

±[2.88793639e-06 2.32162325e-06 3.36088450e-06 1.98735023e-06]

5.70 : [-1.06384015 0.54380272 -0.91785313 -0.59434023]

±[2.94038290e-06 2.36057917e-06 3.42191927e-06 2.01897818e-06]

5.80 : [-1.00758429 0.58101359 -0.97561007 -0.56042255]

±[2.99663876e-06 2.39779004e-06 3.47967622e-06 2.05289586e-06]

5.90 : [-0.94769963 0.61635851 -1.02986355 -0.52428241]

±[3.05652343e-06 2.43313496e-06 3.53392969e-06 2.08903600e-06]

6.00 : [-0.88437886 0.64971543 -1.08039486 -0.4859916 ]

±[3.11984420e-06 2.46649188e-06 3.58446101e-06 2.12732681e-06]

6.10 : [-0.81782692 0.680961 -1.12699274 -0.445627 ]

±[3.18639613e-06 2.49773745e-06 3.63105888e-06 2.16769141e-06]

6.20 : [-0.74826119 0.70997059 -1.16945388 -0.40327128]

±[3.25596187e-06 2.52674704e-06 3.67352003e-06 2.21004713e-06]

6.30 : [-0.67591153 0.73661831 -1.20758359 -0.35901359]

±[3.32831153e-06 2.55339476e-06 3.71164973e-06 2.25430482e-06]

6.40 : [-0.60102049 0.76077715 -1.24119641 -0.31295036]

±[3.40320257e-06 2.57755360e-06 3.74526255e-06 2.30036805e-06]

6.50 : [-0.52384333 0.78231922 -1.27011695 -0.2651861 ]

±[3.48037973e-06 2.59909567e-06 3.77418309e-06 2.34813231e-06]

6.60 : [-0.44464815 0.80111613 -1.2941807 -0.21583426]

±[3.55957491e-06 2.61789258e-06 3.79824684e-06 2.39748415e-06]

6.70 : [-0.36371589 0.81703947 -1.31323498 -0.16501807]

±[3.64050717e-06 2.63381592e-06 3.81730112e-06 2.44830034e-06]

6.80 : [-0.2813403 0.82996146 -1.32713995 -0.11287151]

±[3.72288276e-06 2.64673791e-06 3.83120610e-06 2.50044690e-06]

6.90 : [-0.19782785 0.83975575 -1.33576977 -0.05954012]

±[3.80639520e-06 2.65653220e-06 3.83983591e-06 2.55377829e-06]

7.00 : [-0.11349754 0.8462984 -1.33901373 -0.00518192]

±[3.89072552e-06 2.66307485e-06 3.84307988e-06 2.60813649e-06]

7.10 : [-0.02868058 0.84946904 -1.33677762 0.05003177]

±[3.97554248e-06 2.66624548e-06 3.84531599e-06 2.66335018e-06]

7.20 : [ 0.05628001 0.84915225 -1.32898503 0.10591561]

±[4.06050307e-06 2.66656227e-06 3.85310859e-06 2.71923402e-06]

7.30 : [ 0.14102999 0.84523918 -1.31557881 0.16226962]

±[4.14525304e-06 2.67047534e-06 3.86651480e-06 2.77558803e-06]

7.40 : [ 0.22520461 0.83762929 -1.29652258 0.21887871]

±[4.22942767e-06 2.67808523e-06 3.88557103e-06 2.83219712e-06]

7.50 : [ 0.3084296 0.8262324 -1.27180223 0.27551244]

±[4.31265265e-06 2.68948212e-06 3.91029138e-06 2.88883085e-06]

7.60 : [ 0.39032225 0.81097095 -1.24142745 0.33192513]

±[4.39454531e-06 2.70474357e-06 3.94066617e-06 2.94524354e-06]

7.70 : [ 0.47049286 0.79178232 -1.20543324 0.38785624]

±[4.47471592e-06 2.72393220e-06 3.97666037e-06 3.00117465e-06]

7.80 : [ 0.54854629 0.76862145 -1.16388138 0.44303133]

±[4.55276934e-06 2.74709307e-06 4.01821223e-06 3.05634975e-06]

7.90 : [ 0.62408389 0.7414634 -1.11686172 0.49716356]

±[4.62830694e-06 2.77425112e-06 4.06523189e-06 3.11048197e-06]

8.00 : [ 0.69670562 0.7103059 -1.06449333 0.54995595]

±[4.70092868e-06 2.80540862e-06 4.11760029e-06 3.16327436e-06]

8.10 : [ 0.76601248 0.67517173 -1.00692532 0.60110465]

±[4.77023554e-06 2.84054279e-06 4.17516829e-06 3.21442306e-06]

8.20 : [ 0.83160904 0.63611051 -0.94433739 0.6503035 ]

±[4.83583210e-06 2.87960401e-06 4.23775622e-06 3.26362191e-06]

8.30 : [ 0.89310623 0.59319958 -0.87693971 0.69725024]

±[4.89732929e-06 2.92251494e-06 4.30515391e-06 3.31056865e-06]

8.40 : [ 0.95012408 0.54654309 -0.8049721 0.7416553 ]

±[4.95434713e-06 2.96917143e-06 4.37712152e-06 3.35497371e-06]

8.50 : [ 1.0022942 0.49626763 -0.72870223 0.78325411]

±[5.00651726e-06 3.01944689e-06 4.45339138e-06 3.39657252e-06]

8.60 : [ 1.04926158 0.44251133 -0.64842228 0.82182501]

±[5.05348464e-06 3.07320319e-06 4.53367133e-06 3.43514342e-06]

8.70 : [ 1.09068467 0.38539928 -0.56444335 0.85721592]

±[5.09490773e-06 3.13031524e-06 4.61765026e-06 3.47053433e-06]

8.80 : [ 1.12623169 0.32498895 -0.47708671 0.88938376]

±[5.13045475e-06 3.19072557e-06 4.70500690e-06 3.50270217e-06]

8.90 : [ 1.15556853 0.26114584 -0.38667032 0.91844685]

±[5.15979158e-06 3.25456868e-06 4.79542329e-06 3.53176526e-06]

9.00 : [ 1.17832705 0.19325313 -0.29349064 0.94471345]

±[5.18255010e-06 3.32246138e-06 4.88860298e-06 3.55803186e-06]

9.10 : [ 1.19402875 0.11957795 -0.19781091 0.96844306]

±[5.19825181e-06 3.39613657e-06 4.98428271e-06 3.58176147e-06]

9.20 : [ 1.2019309 0.0365328 -0.09992472 0.98829313]

±[5.20615396e-06 3.47918172e-06 5.08216889e-06 3.60161154e-06]

9.30 : [ 1.20092912e+00 -5.82044261e-02 -5.05203050e-04 9.97384531e-01]

±[5.20715574e-06 3.57391895e-06 5.18158841e-06 3.61070294e-06]

9.40 : [ 1.19021596 -0.15499037 0.09890434 0.9877517 ]

±[5.21786890e-06 3.67070488e-06 5.28099795e-06 3.62033577e-06]

9.50 : [ 1.17029648 -0.24145025 0.19662298 0.96543923]

±[5.23778839e-06 3.75716477e-06 5.37871659e-06 3.64264825e-06]

9.60 : [ 1.14224096 -0.31849702 0.29185654 0.93878385]

±[5.26584390e-06 3.83421154e-06 5.47395015e-06 3.66930362e-06]

9.70 : [ 1.10676845 -0.39031239 0.38429073 0.90939927]

±[5.30131641e-06 3.90602691e-06 5.56638434e-06 3.69868820e-06]

9.80 : [ 1.06428437 -0.45890373 0.47362066 0.87654817]

±[5.34380050e-06 3.97461825e-06 5.65571427e-06 3.73153930e-06]

9.90 : [ 1.01507746 -0.52478248 0.55945886 0.83946406]

±[5.39300740e-06 4.04049700e-06 5.74155248e-06 3.76862341e-06]

10.00 : [ 0.95942268 -0.58780885 0.64135753 0.79771111]

±[5.44866218e-06 4.10352337e-06 5.82345114e-06 3.81037636e-06]

10.10 : [ 0.8976241 -0.64758021 0.71884023 0.75113962]

±[5.51046077e-06 4.16329473e-06 5.90093385e-06 3.85694785e-06]

10.20 : [ 0.83003177 -0.70359721 0.79142678 0.69981318]

±[5.57805310e-06 4.21931173e-06 5.97352039e-06 3.90827429e-06]

10.30 : [ 0.75704711 -0.75534143 0.85865172 0.64395652]

±[5.65103776e-06 4.27105595e-06 6.04074534e-06 3.96413096e-06]

10.40 : [ 0.67912248 -0.80231732 0.92007864 0.58392001]

±[5.72896238e-06 4.31803184e-06 6.10217225e-06 4.02416747e-06]

10.50 : [ 0.59675756 -0.84407794 0.9753113 0.52015336]

±[5.81132731e-06 4.35979246e-06 6.15740491e-06 4.08793412e-06]

10.60 : [ 0.51049352 -0.88024233 1.02400248 0.45318375]

±[5.89759134e-06 4.39595685e-06 6.20609610e-06 4.15490373e-06]

10.70 : [ 0.42090584 -0.9105072 1.06586074 0.38359602]

±[5.98717903e-06 4.42622171e-06 6.24795435e-06 4.22449145e-06]

10.80 : [ 0.32859611 -0.93465422 1.10065518 0.31201392]

±[6.07948876e-06 4.45036874e-06 6.28274880e-06 4.29607355e-06]

10.90 : [ 0.23418335 -0.95255347 1.12821853 0.23908215]

±[6.17390151e-06 4.46826799e-06 6.31031214e-06 4.36900532e-06]

11.00 : [ 0.13829516 -0.96416326 1.14844826 0.16544949]

±[6.26978971e-06 4.47987778e-06 6.33054188e-06 4.44263799e-06]

11.10 : [ 0.04155899 -0.9695269 1.16130633 0.09175333]

±[6.36652587e-06 4.48524142e-06 6.34339995e-06 4.51633414e-06]

11.20 : [-0.05540602 -0.96876675 1.16681727 0.01860608]

±[6.46349088e-06 4.48600158e-06 6.34891088e-06 4.58948139e-06]

11.30 : [-0.15199661 -0.96207617 1.1650651 -0.0534163 ]

±[6.56008147e-06 4.49269216e-06 6.35066305e-06 4.66150377e-06]

11.40 : [-0.24763201 -0.94971004 1.15618924 -0.12378335]

±[6.65571687e-06 4.50505829e-06 6.35953890e-06 4.73187082e-06]

11.50 : [-0.34175958 -0.93197426 1.14037961 -0.19201671]

±[6.74984445e-06 4.52279407e-06 6.37534853e-06 4.80010418e-06]

11.60 : [-0.43385938 -0.90921506 1.11787116 -0.25769435]

±[6.84194425e-06 4.54555327e-06 6.39785699e-06 4.86578182e-06]

11.70 : [-0.52344766 -0.8818084 1.08893812 -0.32045252]

±[6.93153252e-06 4.57295993e-06 6.42679002e-06 4.92854000e-06]

11.80 : [-0.61007932 -0.85014994 1.05388821 -0.37998586]

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±[7.10143432e-06 4.64012235e-06 6.50267136e-06 5.04413323e-06]

12.00 : [-0.77289397 -0.7757054 0.96680139 -0.48843749]

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1.10 : [ 0.87449728 -0.62019049 1.02189555 0.74088463]

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2.10 : [ 0.13720325 -0.80080593 1.54119443 0.30060599]

±[1.12279667e-06 8.00805931e-07 1.54119443e-06 7.49394009e-07]

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2.70 : [-0.3422278 -0.78664361 1.65109272 0.07298535]

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2.80 : [-0.42050953 -0.77878026 1.65670085 0.03938751]

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2.90 : [-0.49794377 -0.76971408 1.65901214 0.00704676]

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3.00 : [-0.57441612 -0.75956082 1.65815168 -0.02404986]

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3.10 : [-0.64982325 -0.74842673 1.65424303 -0.0539208 ]

±[1.90982316e-06 8.58374574e-07 1.66378125e-06 1.10392080e-06]

3.20 : [-0.72407202 -0.73640906 1.64740762 -0.0825888 ]

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±[2.05707849e-06 8.83204680e-07 1.68025977e-06 1.16007988e-06]

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±[2.19907123e-06 9.10897415e-07 1.70750698e-06 1.21164665e-06]

3.60 : [-1.00792923 -0.6811641 1.59313684 -0.18578369]

±[2.26792915e-06 9.25637199e-07 1.72488743e-06 1.23578369e-06]

3.70 : [-1.07528705 -0.66591168 1.57339573 -0.20886544]

±[2.33528697e-06 9.40889621e-07 1.74462854e-06 1.25886544e-06]

3.80 : [-1.14109632 -0.6502016 1.55139787 -0.23092393]

±[2.40109623e-06 9.56599699e-07 1.76662641e-06 1.28092393e-06]

3.90 : [-1.20531378 -0.63408365 1.52724398 -0.25199111]

±[2.46531370e-06 9.72717653e-07 1.79078029e-06 1.30199111e-06]

4.00 : [-1.26790096 -0.61760284 1.50103163 -0.27209856]

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4.10 : [-1.32882362 -0.60079988 1.47285523 -0.29127726]

±[2.58882353e-06 1.00600142e-06 1.84516904e-06 1.34127726e-06]

4.20 : [-1.38805142 -0.58371152 1.44280613 -0.30955747]

±[2.64805134e-06 1.02308978e-06 1.87521814e-06 1.35955747e-06]

4.30 : [-1.44555752 -0.56637093 1.4109727 -0.32696858]

±[2.70555743e-06 1.04043037e-06 1.90705157e-06 1.37696858e-06]

4.40 : [-1.5013182 -0.54880806 1.37744043 -0.34353903]

±[2.76131812e-06 1.05799324e-06 1.94058384e-06 1.39353903e-06]

4.50 : [-1.55531262 -0.53104988 1.342292 -0.35929624]

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4.60 : [-1.60752248 -0.5131207 1.30560741 -0.37426656]

±[2.86752240e-06 1.09368060e-06 2.01241686e-06 1.42426656e-06]

4.70 : [-1.6579318 -0.49504244 1.26746408 -0.38847525]

±[2.91793171e-06 1.11175886e-06 2.05056019e-06 1.43847525e-06]

4.80 : [-1.70652666 -0.47683479 1.22793695 -0.40194646]

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5.00 : [-1.79822657 -0.44010051 1.14501938 -0.42676754]

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5.10 : [-1.84131243 -0.42160415 1.10176748 -0.43816018]

±[3.10131234e-06 1.18519716e-06 2.21625679e-06 1.48816018e-06]

5.20 : [-1.88254512 -0.40303927 1.05740907 -0.44890096]

±[3.14254503e-06 1.20376203e-06 2.26061520e-06 1.49890096e-06]

5.30 : [-1.92191838 -0.38441742 1.01200839 -0.45900859]

±[3.18191830e-06 1.22238388e-06 2.30601588e-06 1.50900859e-06]

5.40 : [-1.95942705 -0.36574896 0.96562786 -0.46850077]

±[3.21942697e-06 1.24105234e-06 2.35239641e-06 1.51850077e-06]

5.50 : [-1.99506693 -0.34704317 0.91832819 -0.47739419]

±[3.25506685e-06 1.25975813e-06 2.39969608e-06 1.52739419e-06]

5.60 : [-2.02883472 -0.32830834 0.87016846 -0.48570457]

±[3.28883463e-06 1.27849296e-06 2.44785581e-06 1.53570457e-06]

5.70 : [-2.06072788 -0.30955192 0.82120622 -0.49344668]

±[3.32072780e-06 1.29724938e-06 2.49681805e-06 1.54344668e-06]

5.80 : [-2.0907446 -0.29078056 0.7714976 -0.50063438]

±[3.35074452e-06 1.31602074e-06 2.54652667e-06 1.55063438e-06]

5.90 : [-2.11888369 -0.27200019 0.72109739 -0.50728063]

±[3.37888361e-06 1.33480111e-06 2.59692688e-06 1.55728063e-06]

6.00 : [-2.14514452 -0.25321612 0.67005912 -0.51339754]

±[3.40514444e-06 1.35358518e-06 2.64796515e-06 1.56339754e-06]

6.10 : [-2.16952695 -0.2344331 0.61843515 -0.51899638]

±[3.42952687e-06 1.37236821e-06 2.69958912e-06 1.56899638e-06]

6.20 : [-2.19203131 -0.21565534 0.56627677 -0.5240876 ]

±[3.45203123e-06 1.39114596e-06 2.75174751e-06 1.57408760e-06]

6.30 : [-2.21265832 -0.19688663 0.51363423 -0.52868089]

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6.40 : [-2.23140906 -0.17813035 0.46055689 -0.53278516]

±[3.49140897e-06 1.42867096e-06 2.85746739e-06 1.58278516e-06]

6.50 : [-2.24828491 -0.1593895 0.40709322 -0.53640858]

±[3.50828482e-06 1.44741180e-06 2.91093105e-06 1.58640858e-06]

6.60 : [-2.26328756 -0.14066681 0.35329095 -0.5395586 ]

±[3.52328748e-06 1.46613449e-06 2.96473332e-06 1.58955860e-06]

6.70 : [-2.27641896 -0.1219647 0.29919706 -0.54224198]

±[3.53641887e-06 1.48483661e-06 3.01882721e-06 1.59224198e-06]

6.80 : [-2.28768126 -0.10328536 0.24485791 -0.5444648 ]

±[3.54768118e-06 1.50351594e-06 3.07316637e-06 1.59446480e-06]

6.90 : [-2.29707685 -0.08463081 0.19031927 -0.54623245]

±[3.55707677e-06 1.52217049e-06 3.12770500e-06 1.59623245e-06]

7.00 : [-2.30460831 -0.06600286 0.13562643 -0.54754969]

±[3.56460823e-06 1.54079844e-06 3.18239784e-06 1.59754969e-06]

7.10 : [-2.31027837 -0.04740321 0.08082421 -0.54842064]

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7.20 : [-2.31408995 -0.02883343 0.02595706 -0.54884879]

±[3.57408986e-06 1.57796787e-06 3.29206721e-06 1.59884879e-06]

7.30 : [-2.3160461 -0.01029504 -0.02893089 -0.548837 ]

±[3.57604602e-06 1.59650626e-06 3.34695516e-06 1.59886058e-06]

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±[3.57614996e-06 1.61501183e-06 3.40182002e-06 1.59931004e-06]

7.50 : [-2.31440514 0.02668185 -0.13859386 -0.54750206]

±[3.57789488e-06 1.63348315e-06 3.45661813e-06 1.60019552e-06]

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±[3.58148515e-06 1.65191881e-06 3.51130594e-06 1.60151593e-06]

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7.80 : [-2.29811294 0.08187603 -0.30215253 -0.54223729]

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7.90 : [-2.289009 0.10019585 -0.35624866 -0.53961255]

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8.00 : [-2.27807516 0.11847385 -0.41006049 -0.53655124]

±[3.61422485e-06 1.72527515e-06 3.72808476e-06 1.61114633e-06]

8.10 : [-2.26531569 0.13670826 -0.46354429 -0.5330515 ]

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8.20 : [-2.25073503 0.15489713 -0.51665609 -0.52911087]

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8.30 : [-2.23433785 0.17303836 -0.56935167 -0.52472628]

±[3.65796217e-06 1.77983966e-06 3.88737594e-06 1.62297129e-06]

8.40 : [-2.21612902 0.19112961 -0.62158643 -0.5198941 ]

±[3.67617099e-06 1.79793091e-06 3.93961070e-06 1.62780347e-06]

8.50 : [-2.19611368 0.20916833 -0.67331542 -0.51461006]

±[3.69618634e-06 1.81596963e-06 3.99133970e-06 1.63308751e-06]

8.60 : [-2.1742972 0.22715169 -0.72449322 -0.5088693 ]

±[3.71800281e-06 1.83395299e-06 4.04251749e-06 1.63882828e-06]

8.70 : [-2.15068529 0.24507654 -0.77507388 -0.50266632]

±[3.74161472e-06 1.85187784e-06 4.09309815e-06 1.64503125e-06]

8.80 : [-2.12528396 0.2629394 -0.82501087 -0.49599501]

±[3.76701605e-06 1.86974070e-06 4.14303515e-06 1.65170256e-06]

8.90 : [-2.0980996 0.28073643 -0.87425704 -0.48884861]

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9.00 : [-2.06913901 0.29846333 -0.92276451 -0.4812197 ]

±[3.82316100e-06 1.90526463e-06 4.24078878e-06 1.66647788e-06]

9.10 : [-2.03840943 0.31611536 -0.97048463 -0.4731002 ]

±[3.85389058e-06 1.92291666e-06 4.28850890e-06 1.67459737e-06]

9.20 : [-2.00591861 0.33368726 -1.01736791 -0.46448136]

±[3.88638140e-06 1.94048856e-06 4.33539218e-06 1.68321621e-06]

9.30 : [-1.97167485 0.35117318 -1.06336394 -0.45535373]

±[3.92062517e-06 1.95797448e-06 4.38138822e-06 1.69234384e-06]

9.40 : [-1.93568705 0.36856668 -1.10842136 -0.44570715]

±[3.95661296e-06 1.97536798e-06 4.42644563e-06 1.70199043e-06]

9.50 : [-1.89796483 0.38586059 -1.15248771 -0.43553073]

±[3.99433518e-06 1.99266189e-06 4.47051199e-06 1.71216685e-06]

9.60 : [-1.85851852 0.40304702 -1.19550946 -0.42481286]

±[4.03378149e-06 2.00984832e-06 4.51353373e-06 1.72288472e-06]

9.70 : [-1.8173593 0.42011722 -1.23743183 -0.41354116]

±[4.07494071e-06 2.02691852e-06 4.55545610e-06 1.73415641e-06]

9.80 : [-1.77449927 0.43706154 -1.27819879 -0.4017025 ]

±[4.11780074e-06 2.04386284e-06 4.59622306e-06 1.74599507e-06]

9.90 : [-1.72995154 0.45386932 -1.31775296 -0.38928296]

±[4.16234847e-06 2.06067063e-06 4.63577724e-06 1.75841461e-06]

10.00 : [-1.68373035 0.47052881 -1.35603553 -0.37626782]

±[4.20856967e-06 2.07733011e-06 4.67405980e-06 1.77142975e-06]

10.10 : [-1.63585115 0.48702704 -1.39298616 -0.36264158]

±[4.25644886e-06 2.09382834e-06 4.71101043e-06 1.78505600e-06]

10.20 : [-1.58633079 0.50334971 -1.42854293 -0.34838791]

±[4.30596922e-06 2.11015101e-06 4.74656720e-06 1.79930967e-06]

10.30 : [-1.53518758 0.5194811 -1.46264226 -0.33348968]

±[4.35711243e-06 2.12628240e-06 4.78066653e-06 1.81420790e-06]

10.40 : [-1.48244152 0.53540385 -1.49521878 -0.31792894]

±[4.40985849e-06 2.14220515e-06 4.81324306e-06 1.82976863e-06]

10.50 : [-1.4281144 0.5510989 -1.52620534 -0.30168696]

±[4.46418561e-06 2.15790020e-06 4.84422961e-06 1.84601061e-06]

10.60 : [-1.37223003 0.56654524 -1.55553282 -0.28474421]

±[4.52006998e-06 2.17334654e-06 4.87355709e-06 1.86295336e-06]

10.70 : [-1.31481441 0.58171977 -1.58313014 -0.26708039]

±[4.57748560e-06 2.18852107e-06 4.90115442e-06 1.88061719e-06]

10.80 : [-1.25589598 0.59659711 -1.60892416 -0.24867448]

±[4.63640403e-06 2.20339841e-06 4.92694843e-06 1.89902309e-06]

10.90 : [-1.19550583 0.61114933 -1.63283958 -0.2295048 ]

±[4.69679419e-06 2.21795063e-06 4.95086385e-06 1.91819278e-06]

11.00 : [-1.13367797 0.62534576 -1.65479892 -0.20954903]

±[4.75862204e-06 2.23214706e-06 4.97282319e-06 1.93814855e-06]

11.10 : [-1.07044965 0.63915271 -1.67472242 -0.18878434]

±[4.82185036e-06 2.24595401e-06 4.99274670e-06 1.95891323e-06]

11.20 : [-1.00586164 0.65253319 -1.69252805 -0.16718752]

±[4.88643837e-06 2.25933449e-06 5.01055232e-06 1.98051006e-06]

11.30 : [-0.93995858 0.6654466 -1.70813141 -0.14473503]

±[4.95234144e-06 2.27224790e-06 5.02615568e-06 2.00296254e-06]

11.40 : [-0.87278937 0.67784841 -1.72144575 -0.12140328]

±[5.01951065e-06 2.28464972e-06 5.03947002e-06 2.02629429e-06]

11.50 : [-0.80440757 0.68968981 -1.73238197 -0.09716878]

±[5.08789244e-06 2.29649111e-06 5.05040624e-06 2.05052879e-06]

11.60 : [-0.73487186 0.70091732 -1.74084864 -0.07200843]

±[5.15742815e-06 2.30771863e-06 5.05887291e-06 2.07568915e-06]

11.70 : [-0.66424651 0.71147242 -1.74675205 -0.04589981]

±[5.22805350e-06 2.31827372e-06 5.06477632e-06 2.10179777e-06]

11.80 : [-0.59260192 0.72129108 -1.74999628 -0.01882162]

±[5.29969809e-06 2.32809239e-06 5.06802055e-06 2.12887596e-06]

11.90 : [-0.52001517 0.73030341 -1.75048339 0.00924592]

±[5.37228485e-06 2.33710471e-06 5.06850766e-06 2.15694349e-06]

12.00 : [-0.44657065 0.73843313 -1.74811353 0.03832045]

±[5.44572936e-06 2.34523443e-06 5.07087752e-06 2.18601802e-06]

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±[2.21848051e-09 4.42736940e-08 1.29929048e-07 2.12038833e-09]

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±[8.81826199e-09 8.74639429e-08 2.59441647e-07 8.25844358e-09]

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±[1.96482107e-08 1.28773920e-07 3.88161043e-07 1.78843730e-08]

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±[5.31078749e-08 2.04087058e-07 6.42003256e-07 4.52539314e-08]

0.60 : [ 1.48477939 -0.23770272 0.76665612 1.23804104]

±[7.52205068e-08 2.37702719e-07 7.66656122e-07 6.19589617e-08]

0.70 : [ 1.45944455 -0.26852845 0.88956462 1.21992946]

±[1.00555347e-07 2.68528450e-07 8.89564621e-07 8.00705443e-08]

0.80 : [ 1.43116626 -0.29657933 1.01060867 1.20081624]

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1.00 : [ 1.36686671 -0.3446952 1.24681099 1.16099235]

±[1.93133182e-07 3.44695201e-07 1.24681099e-06 1.39007646e-07]

1.10 : [ 1.33136069 -0.36503489 1.3619034 1.14086676]

±[2.28639202e-07 3.65034891e-07 1.36190340e-06 1.59133241e-07]

1.20 : [ 1.29393476 -0.3831242 1.47498991 1.12090685]

±[2.66065136e-07 3.83124201e-07 1.47498991e-06 1.79093152e-07]

1.30 : [ 1.2548046 -0.39915111 1.58609609 1.10128519]

±[3.05195293e-07 3.99151105e-07 1.58609609e-06 1.98714808e-07]

1.40 : [ 1.21416695 -0.41330567 1.69526264 1.08213285]

±[3.45832951e-07 4.13305672e-07 1.69526264e-06 2.17867146e-07]

1.50 : [ 1.17219969 -0.42577336 1.80254151 1.06354504]

±[3.87800209e-07 4.25773359e-07 1.80254151e-06 2.36454957e-07]

1.60 : [ 1.12906261 -0.43673042 1.90799266 1.04558699]

±[4.30937282e-07 4.36730421e-07 1.90799266e-06 2.54413011e-07]

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1.80 : [ 1.03983424 -0.45475596 2.11367566 1.01170417]

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2.00 : [ 0.94744308 -0.46853142 2.31286027 0.98060432]

±[6.12556818e-07 4.68531424e-07 2.31286027e-06 3.19395677e-07]

2.10 : [ 0.90030382 -0.47412457 2.41018897 0.96608174]

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2.20 : [ 0.85264245 -0.4789887 2.50609866 0.95222043]

±[7.07357444e-07 4.78988700e-07 2.50609866e-06 3.47779568e-07]

2.30 : [ 0.80452749 -0.48321007 2.60065433 0.93899711]

±[7.55472407e-07 4.83210066e-07 2.60065433e-06 3.61002885e-07]

2.40 : [ 0.75601932 -0.4868648 2.69391849 0.92638588]

±[8.03980580e-07 4.86864797e-07 2.69391849e-06 3.73614120e-07]

2.50 : [ 0.70717117 -0.49001995 2.785951 0.91435932]

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2.60 : [ 0.65803001 -0.49273451 2.87680891 0.90288939]

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2.70 : [ 0.60863722 -0.49506033 2.96654649 0.89194801]

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2.80 : [ 0.55902937 -0.49704301 3.05521521 0.88150751]

±[1.00097053e-06 4.97043009e-07 3.05521521e-06 4.18492491e-07]

2.90 : [ 0.50923872 -0.49872259 3.14286379 0.871541 ]

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3.00 : [ 0.45929378 -0.50013428 3.22953834 0.86202253]

±[1.10070612e-06 5.00134284e-07 3.22953834e-06 4.37977474e-07]

3.10 : [ 0.40921976 -0.50130908 3.3152824 0.85292726]

±[1.15078014e-06 5.01309078e-07 3.31528240e-06 4.47072738e-07]

3.20 : [ 0.35903895 -0.50227424 3.40013711 0.84423155]

±[1.20096095e-06 5.02274237e-07 3.40013711e-06 4.55768446e-07]

3.30 : [ 0.30877109 -0.50305378 3.48414128 0.83591297]

±[1.25122880e-06 5.03053784e-07 3.48414128e-06 4.64087025e-07]

3.40 : [ 0.25843367 -0.5036689 3.56733157 0.82795033]

±[1.30156623e-06 5.03668902e-07 3.56733157e-06 4.72049669e-07]

3.50 : [ 0.20804217 -0.50413829 3.64974255 0.82032364]

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3.60 : [ 0.15761032 -0.50447847 3.73140687 0.8130141 ]

±[1.40238958e-06 5.04478469e-07 3.73140687e-06 4.86985897e-07]

3.70 : [ 0.10715029 -0.50470406 3.81235535 0.80600406]

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3.80 : [ 0.05667289 -0.50482799 3.89261711 0.79927693]

±[1.50332701e-06 5.04827991e-07 3.89261711e-06 5.00723071e-07]

3.90 : [ 0.0061877 -0.50486175 3.97221965 0.79281718]

±[1.55381220e-06 5.04861749e-07 3.97221965e-06 5.07182825e-07]

4.00 : [-0.04429679 -0.50481551 4.05118897 0.78661023]

±[1.60429669e-06 5.04907986e-07 4.05118897e-06 5.13389770e-07]

4.10 : [-0.09477304 -0.50469833 4.12954966 0.78064245]

±[1.65477294e-06 5.05025170e-07 4.12954966e-06 5.19357554e-07]

4.20 : [-0.14523436 -0.50451824 4.207325 0.77490104]

±[1.70523426e-06 5.05205254e-07 4.20732500e-06 5.25098960e-07]

4.30 : [-0.19567483 -0.50428242 4.28453702 0.76937404]

±[1.75567473e-06 5.05441076e-07 4.28453702e-06 5.30625963e-07]

4.40 : [-0.2460892 -0.50399725 4.36120658 0.76405022]

±[1.80608910e-06 5.05726248e-07 4.36120658e-06 5.35949783e-07]

4.50 : [-0.29647283 -0.50366842 4.43735348 0.75891907]

±[1.85647273e-06 5.06055076e-07 4.43735348e-06 5.41080932e-07]

4.60 : [-0.3468216 -0.50330103 4.51299649 0.75397074]

±[1.90682150e-06 5.06422473e-07 4.51299649e-06 5.46029264e-07]

4.70 : [-0.3971319 -0.50289961 4.58815341 0.74919598]

±[1.95713180e-06 5.06823892e-07 4.58815341e-06 5.50804018e-07]

4.80 : [-0.44740053 -0.50246823 4.66284118 0.74458614]

±[2.00740042e-06 5.07255269e-07 4.66284118e-06 5.55413861e-07]

4.90 : [-0.49762467 -0.50201054 4.73707587 0.74013307]

±[2.05762457e-06 5.07712962e-07 4.73707587e-06 5.59866927e-07]

5.00 : [-0.54780187 -0.50152979 4.81087277 0.73582915]

±[2.10780176e-06 5.08193711e-07 4.81087277e-06 5.64170854e-07]

5.10 : [-0.59792996 -0.50102891 4.88424643 0.73166718]

±[2.15792985e-06 5.08694593e-07 4.88424643e-06 5.68332816e-07]

5.20 : [-0.64800706 -0.50051051 4.95721071 0.72764044]

±[2.20800696e-06 5.09212985e-07 4.95721071e-06 5.72359558e-07]

5.30 : [-0.69803155 -0.49997696 5.02977881 0.72374258]

±[2.25803145e-06 5.09746534e-07 5.02977881e-06 5.76257423e-07]

5.40 : [-0.74800202 -0.49943037 5.10196332 0.71996762]

±[2.30800192e-06 5.10293126e-07 5.10196332e-06 5.80032381e-07]

5.50 : [-0.79791726 -0.49887264 5.17377625 0.71630995]

±[2.35791715e-06 5.10850862e-07 5.17377625e-06 5.83690051e-07]

5.60 : [-0.84777623 -0.49830546 5.24522904 0.71276427]

±[2.40777613e-06 5.11418037e-07 5.24522904e-06 5.87235732e-07]

5.70 : [-0.89757809 -0.49773038 5.31633267 0.70932558]

±[2.45757798e-06 5.11993116e-07 5.31633267e-06 5.90674416e-07]

5.80 : [-0.94732209 -0.49714878 5.38709757 0.70598919]

±[2.50732199e-06 5.12574719e-07 5.38709757e-06 5.94010813e-07]

5.90 : [-0.99700767 -0.49656189 5.45753376 0.70275063]

±[2.55700756e-06 5.13161606e-07 5.45753376e-06 5.97249370e-07]

6.00 : [-1.04663433 -0.49597084 5.52765082 0.69960571]

±[2.60663423e-06 5.13752663e-07 5.52765082e-06 6.00394287e-07]

6.10 : [-1.09620173 -0.49537661 5.5974579 0.69655047]

±[2.65620162e-06 5.14346884e-07 5.59745790e-06 6.03449535e-07]

6.20 : [-1.14570958 -0.49478013 5.66696377 0.69358113]

±[2.70570948e-06 5.14943369e-07 5.66696377e-06 6.06418866e-07]

6.30 : [-1.19515771 -0.49418219 5.73617687 0.69069417]

±[2.75515760e-06 5.15541305e-07 5.73617687e-06 6.09305835e-07]

6.40 : [-1.244546 -0.49358353 5.80510524 0.6878862 ]

±[2.80454589e-06 5.16139965e-07 5.80510524e-06 6.12113804e-07]

6.50 : [-1.29387441 -0.4929848 5.87375663 0.68515404]

±[2.85387431e-06 5.16738694e-07 5.87375663e-06 6.14845960e-07]

6.60 : [-1.34314297 -0.49238659 5.94213848 0.68249467]

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6.70 : [-1.39235176 -0.49178943 6.0102579 0.67990523]

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6.90 : [-1.49059059 -0.49060008 6.14573665 0.67492538]

±[3.05059048e-06 5.19123414e-07 6.14573665e-06 6.25074618e-07]

7.00 : [-1.539621 -0.4900087 6.21310891 0.67252993]

±[3.09962090e-06 5.19714798e-07 6.21310891e-06 6.27470066e-07]

7.10 : [-1.58859241 -0.48941997 6.28024463 0.67019432]

±[3.14859231e-06 5.20303524e-07 6.28024463e-06 6.29805681e-07]

7.20 : [-1.6375051 -0.48883421 6.34714969 0.66791632]

±[3.19750499e-06 5.20889291e-07 6.34714969e-06 6.32083682e-07]

7.30 : [-1.68635936 -0.48825166 6.41382975 0.66569382]

±[3.24635926e-06 5.21471834e-07 6.41382975e-06 6.34306183e-07]

7.40 : [-1.73515555 -0.48767259 6.48029024 0.6635248 ]

±[3.29515544e-06 5.22050913e-07 6.48029024e-06 6.36475196e-07]

7.50 : [-1.783894 -0.48709718 6.54653643 0.66140736]

±[3.34389390e-06 5.22626321e-07 6.54653643e-06 6.38592638e-07]

7.60 : [-1.83257511 -0.48652563 6.61257337 0.65933966]

±[3.39257501e-06 5.23197870e-07 6.61257337e-06 6.40660336e-07]

7.70 : [-1.88119926 -0.4859581 6.67840596 0.65731997]

±[3.44119916e-06 5.23765401e-07 6.67840596e-06 6.42680033e-07]

7.80 : [-1.92976687 -0.48539473 6.74403891 0.65534661]

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7.90 : [-1.97827835 -0.48483565 6.80947677 0.653418 ]

±[3.53827825e-06 5.24887853e-07 6.80947677e-06 6.46582003e-07]

8.00 : [-2.02673414 -0.48428095 6.87472395 0.65153262]

±[3.58673404e-06 5.25442546e-07 6.87472395e-06 6.48467381e-07]

8.10 : [-2.07513469 -0.48373074 6.93978469 0.64968902]

±[3.63513459e-06 5.25992756e-07 6.93978469e-06 6.50310975e-07]

8.20 : [-2.12348044 -0.48318509 7.0046631 0.64788583]

±[3.68348034e-06 5.26538408e-07 7.00466310e-06 6.52114172e-07]

8.30 : [-2.17177186 -0.48264406 7.06936316 0.6461217 ]

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8.40 : [-2.22000941 -0.48210771 7.1338887 0.64439538]

±[3.78000931e-06 5.27615785e-07 7.13388870e-06 6.55604620e-07]

8.50 : [-2.26819356 -0.48157608 7.19824345 0.64270565]

±[3.82819346e-06 5.28147417e-07 7.19824345e-06 6.57294353e-07]

8.60 : [-2.31632479 -0.4810492 7.26243101 0.64105134]

±[3.87632468e-06 5.28674295e-07 7.26243101e-06 6.58948661e-07]

8.70 : [-2.36440356 -0.4805271 7.32645486 0.63943134]

±[3.92440346e-06 5.29196396e-07 7.32645486e-06 6.60568657e-07]

8.80 : [-2.41243037 -0.4800098 7.39031839 0.63784459]

±[3.97243027e-06 5.29713703e-07 7.39031839e-06 6.62155412e-07]

8.90 : [-2.46040568 -0.47949729 7.45402486 0.63629005]

±[4.02040558e-06 5.30226206e-07 7.45402486e-06 6.63709949e-07]

9.00 : [-2.50832999 -0.4789896 7.51757744 0.63476675]

±[4.06832988e-06 5.30733901e-07 7.51757744e-06 6.65233252e-07]

9.10 : [-2.55620376 -0.47848671 7.58097922 0.63327374]

±[4.11620366e-06 5.31236790e-07 7.58097922e-06 6.66726263e-07]

9.20 : [-2.60402749 -0.47798862 7.64423317 0.63181011]

±[4.16402739e-06 5.31734882e-07 7.64423317e-06 6.68189891e-07]

9.30 : [-2.65180165 -0.47749531 7.70734219 0.630375 ]

±[4.21180154e-06 5.32228187e-07 7.70734219e-06 6.69625004e-07]

9.40 : [-2.69952671 -0.47700678 7.77030909 0.62896756]

±[4.25952661e-06 5.32716723e-07 7.77030909e-06 6.71032439e-07]

9.50 : [-2.74720316 -0.47652299 7.8331366 0.627587 ]

±[4.30720306e-06 5.33200510e-07 7.83313660e-06 6.72413001e-07]

9.60 : [-2.79483147 -0.47604393 7.89582736 0.62623254]

±[4.35483136e-06 5.33679570e-07 7.89582736e-06 6.73767461e-07]

9.70 : [-2.8424121 -0.47556957 7.95838395 0.62490343]

±[4.40241200e-06 5.34153930e-07 7.95838395e-06 6.75096565e-07]

9.80 : [-2.88994553 -0.47509988 8.02080887 0.62359897]

±[4.44994543e-06 5.34623621e-07 8.02080887e-06 6.76401028e-07]

9.90 : [-2.93743223 -0.47463483 8.08310454 0.62231846]

±[4.49743213e-06 5.35088672e-07 8.08310454e-06 6.77681539e-07]

10.00 : [-2.98487265 -0.47417438 8.14527333 0.62106124]

±[4.54487255e-06 5.35549119e-07 8.14527333e-06 6.78938762e-07]

10.10 : [-3.03226726 -0.4737185 8.20731754 0.61982666]

±[4.59226716e-06 5.36004997e-07 8.20731754e-06 6.80173336e-07]

10.20 : [-3.0796165 -0.47326715 8.2692394 0.61861412]

±[4.63961640e-06 5.36456344e-07 8.26923940e-06 6.81385877e-07]

10.30 : [-3.12692084 -0.4728203 8.33104108 0.61742302]

±[4.68692074e-06 5.36903198e-07 8.33104108e-06 6.82576978e-07]

10.40 : [-3.17418071 -0.4723779 8.3927247 0.61625279]

±[4.73418061e-06 5.37345601e-07 8.39272470e-06 6.83747214e-07]

10.50 : [-3.22139657 -0.47193991 8.45429232 0.61510287]

±[4.78139646e-06 5.37783592e-07 8.45429232e-06 6.84897135e-07]

10.60 : [-3.26856884 -0.47150628 8.51574593 0.61397272]

±[4.82856874e-06 5.38217216e-07 8.51574593e-06 6.86027276e-07]

10.70 : [-3.31569797 -0.47107698 8.5770875 0.61286185]

±[4.87569786e-06 5.38646515e-07 8.57708750e-06 6.87138150e-07]

10.80 : [-3.36278438 -0.47065197 8.63831893 0.61176975]

±[4.92278428e-06 5.39071532e-07 8.63831893e-06 6.88230254e-07]

10.90 : [-3.4098285 -0.47023118 8.69944206 0.61069593]

±[4.96982840e-06 5.39492314e-07 8.69944206e-06 6.89304069e-07]

11.00 : [-3.45683076 -0.46981459 8.76045871 0.60963994]

±[5.01683065e-06 5.39908904e-07 8.76045871e-06 6.90360058e-07]

11.10 : [-3.50379156 -0.46940215 8.82137063 0.60860133]

±[5.06379146e-06 5.40321348e-07 8.82137063e-06 6.91398668e-07]

11.20 : [-3.55071132 -0.46899381 8.88217954 0.60757967]

±[5.11071122e-06 5.40729692e-07 8.88217954e-06 6.92420333e-07]

11.30 : [-3.59759046 -0.46858952 8.94288711 0.60657453]

±[5.15759035e-06 5.41133981e-07 8.94288711e-06 6.93425471e-07]

11.40 : [-3.64442936 -0.46818924 9.00349498 0.60558551]

±[5.20442926e-06 5.41534263e-07 9.00349498e-06 6.94414488e-07]

11.50 : [-3.69122843 -0.46779292 9.06400474 0.60461223]

±[5.25122833e-06 5.41930582e-07 9.06400474e-06 6.95387774e-07]

11.60 : [-3.73798807 -0.46740051 9.12441794 0.60365429]

±[5.29798797e-06 5.42322985e-07 9.12441794e-06 6.96345710e-07]

11.70 : [-3.78470867 -0.46701198 9.1847361 0.60271134]

±[5.34470856e-06 5.42711518e-07 9.18473610e-06 6.97288661e-07]

11.80 : [-3.8313906 -0.46662727 9.2449607 0.60178302]

±[5.39139049e-06 5.43096227e-07 9.24496070e-06 6.98216984e-07]

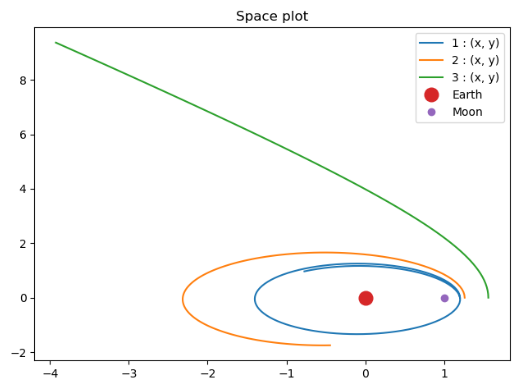
11.90 : [-3.87803425 -0.46624634 9.30509318 0.60086898]

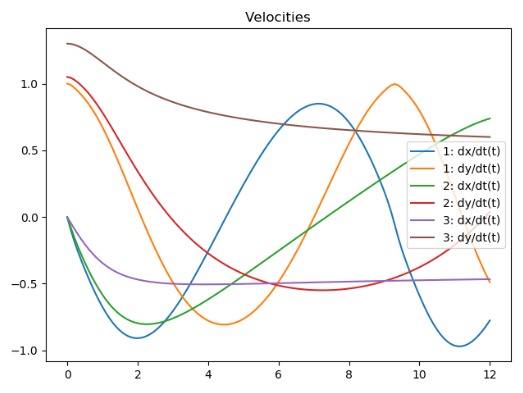
±[5.43803414e-06 5.43477157e-07 9.30509318e-06 6.99131023e-07]

12.00 : [-3.92463999 -0.46586914 9.36513496 0.59996889]

±[5.48463989e-06 5.43854355e-07 9.36513496e-06 7.00031110e-07]

Графики:





Вывод

Заметим, что полученная траектория вполне удовлетворяет законам физики, а скорости меняются периодически с выполнением закона сохранения энергии. Тело движется по эллипсу, что подтверждается 1 законом Кеплера. Оцененная погрешность вычислений позволяет увеличить время наблюдения на несколько порядков, не теряя при этом в точности. Зависимость решения от начальных условий очень сильная – в некоторых случаях орбита выходит на эллипс, в некоторых тело по параболе улетает в бесконечность.

Исходный код программы:

import numpy

from matplotlib import pyplot

from scipy.integrate import quad, RK45

from scipy.optimize import toms748

TOLERANCE = 1e-6

MU = 1 / 82.3

T = 12

def r3(x, y):

return numpy.sqrt(x \*\* 2 + y \*\* 2)

def rl(x, y):

return numpy.sqrt((x - 1) \*\* 2 + y \*\* 2)

def motion\_equation(t, x):

r3\_ = r3(x[0], x[2]) \*\* 3

rl\_ = rl(x[0], x[2]) \*\* 3

return numpy.array([

x[1],

-(1 - MU) \* x[0] / r3\_ - MU \* (x[0] - 1) / rl\_,

x[3],

-(1 - MU) \* x[2] / r3\_ - MU \* x[2] / rl\_

])

def x\_star\_equation(x):

def x\_star\_equation\_integral(z):

return 1 / (numpy.e \*\* (1.7 \* (z \*\* 2)) + x)

(integral, \_) = quad(x\_star\_equation\_integral, 0, 1)

return integral - 0.3707355 \* x

x\_star = toms748(x\_star\_equation, 0, 3)

a = 1.2 \* x\_star

b = 0

c = 0

d = 1

initial = numpy.array([

a, b, c, d

])

lower\_bound = 0

upper\_bound = T

step = 0.1

\_, space\_plot = pyplot.subplots()

\_, velocities\_plot = pyplot.subplots()

def simulate(initial\_state, name=""):

current = lower\_bound

res = initial\_state.copy()

total\_tol = [0, 0, 0, 0]

t = numpy.arange(lower\_bound, upper\_bound + TOLERANCE, step)

x, dx, y, dy = [initial\_state[0]], [initial\_state[1]], [initial\_state[2]], [initial\_state[3]]

while numpy.abs(current - upper\_bound) > TOLERANCE:

integrator = RK45(

fun=motion\_equation,

t0=current,

y0=res,

t\_bound=current + step,

rtol=TOLERANCE

)

while integrator.status == "running":

integrator.step()

previous\_res = res

res = integrator.y

x\_, dx\_, y\_, dy\_ = res

x.append(x\_)

dx.append(dx\_)

y.append(y\_)

dy.append(dy\_)

current += step

tolerance = abs(res - previous\_res) \* TOLERANCE

total\_tol += tolerance

print("{0:1.2f} : {1}\n ±{2}".format(current, res, total\_tol))

space\_plot.plot(x, y, label="{0} : (x, y)".format(name))

velocities\_plot.plot(t, dx, label="{0}: dx/dt(t)".format(name))

velocities\_plot.plot(t, dy, label="{0}: dy/dt(t)".format(name))

print("Relative tolerance = {0}".format(TOLERANCE))

simulate(initial\_state=initial, name="1")

simulate(initial\_state=initial \* 1.05, name="2")

simulate(initial\_state=initial \* 1.3, name="3")

space\_plot.set\_title("Space plot")

space\_plot.plot(0, 0, 'o', label="Earth", markersize=12)

space\_plot.plot(1, 0, 'o', label="Moon", markersize=6)

space\_plot.legend()

velocities\_plot.set\_title("Velocities")

velocities\_plot.legend()

pyplot.show()