E4301 Project: Adaptive Mesh Refinement Regridding Intervals

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Introduction

Adaptive mesh refinement (AMR) allows for efficient handling of problems that involve multiple scales by increasing mesh resolution only when and where it is needed. This provides the potential for substantial performance gains, as a coarser mesh can be used for the parts of the problem where it is appropriate. These performance gains offer the possibility not just of more convenient modelling, or higher accuracy solutions in the same time frame, but also opening new uses for numerical models (such as quickly running a model over a distribution of initial parameters that capture uncertainty in their values, and using the outcome to see find the uncertainty in the results).

AMR brings a new set of parameters to a numerical model, including how regions are chosen for refinement, how frequently refinement occurs, and how refined regions interact with each other as well as coarser meshes. Among these parameters, there exists a relationship between the time between refining the mesh and the regions chosen for refinement; the longer the interval between refinements, the larger the refined regions must be to ensure that any interesting feature that needs the higher resolution doesn't move somewhere else before the next refinement step is able to reposition the finer mesh appropriately. It is this relationship I am interesting in exploring, and its effect on running time of the model as a whole. If the refinement process is particularly costly, increasing the time between those steps is advantageous, but it is balanced by the need to increase the size of the refined regions.

To explore this trade-off, I used the Clawpack software, and its AMR implementation AMRClaw (see [2]), to look at two sample problems and

how their solutions react to changing the time between refinement and the buffer added around refined regions to counteract that time.

Adaptive Mesh Refinement in Clawpack

Clawpack is a suite of software tools for solving hyperbolic systems of PDEs using finite volume methods. The use of finite volume methods make it particularly suited to systems of conservation laws, and one of its uses is for modelling storm surges and tsunamis (where the differing scales between the expanse of the ocean and details of harbors make AMR a useful tool).

AMRClaw is Clawpack's AMR implementation (and its operation is described in more detail in [1]). The mesh takes the form of a rectilinear grid. Grid cells are flagged for refinement based on some criteria exceeding a threshold (in this case the maximum of the differences between neighbouring cells), and then clustered into rectangular patches that each act as a new, finer grid. A refinement also occurs in the time step, to ensure the CFL condition is still met, and thus several time steps occur on the finer grids for each step on the coarser grid. Boundary conditions are implemented using "ghost" cells which get their values either from the boundary conditions of the whole problem, from neighbouring refined grids, or by interpolating from coarser grids.

There are two runtime parameters provided by AMRClaw for specifying the interval between regridding (as the number of steps to take on each level before recomputing the refinement criteria and redrawing the patches) and a buffer width (as the number of cells in each direction around the flagged cells which also need to be included for refinement).

Running Parameter Sweeps and Gathering Data

To keep the scale of my project reasonable, I decided to focus on two examples provided with the AMRClaw software: the advection of a square region on a domain with periodic boundary conditions, and a gas dynamics problem using the Euler equations for non-viscous, adiabatic flow on a domain with four quadrants of differing density. The advection problem offers a simple and easily understandable problem for investigation; the expected behaviour of the solution is clear and the regions of interest for refinement are intuitively the boundary of the square. The Euler gas problem offers a more complex, physical problem that hopefully has features that make it closer to the kind of problem one would want to solve in practice.

Though the advection problem has an easy analytical solution, the gas problem does not. Thus for computing errors I used a run of the problem on which a single grid at the finest level was used as an approximate "true" solution. This choice is particularly suited to AMR, where the goal is effectively to get a solution like the one using a fine grid everywhere, without having to actually do so. Comparing solutions when differing grids are involved is not as straightforward as when a single uniform grid is involved, but AMRClaw allows one to specify "gauge" points where the solution will be logged independent of the actual grid, interpolating as necessary. A relative L_1 error was thus computed by sampling from a series of gauge points in both time and space (as a compromise between observing a few points closely over time or looking at the whole solution at only one time).

To run the problems over a series of parameters, I created a script `run_param_sweep.py`[1] to copy a base version of the problem directory repeatedly, modifying the `setrun.py` in each copy to have the necessary regridding parameters. A second script, `parse_runs.py`, provides functions to read the output files, aggregate the data, and plot the results.[2]

Regridding in a Simple Advection Problem

In the simple advection example provided in AMRClaw, a square region moves towards the upper right corner of a square domain, wraps around the space a bit, and returns to its original position at time $t = 2$. For a reference solution, I used a grid with 200 cells on each side and no refinement (see Figure 1). The AMR solutions all used a grid that started at 50 cells each side, and had two levels of refinement (by a factor of two each time, for a finest grid with cells one-fourth the width of the initial cells), making its finest cells the same size as those used everywhere in the reference solution.

First, I wanted to confirm the need to vary the regridding interval and buffer width in tandem with each other. To do this, I did ten runs of the AMR setup with buffer width of 1, but with regridding intervals from 1 to 10. We expect the regridding interval cannot exceed the buffer width by

[1] Professor Mandli directed me to some existing scripts for running parameter sweeps, but I was having trouble adapting it to my specific case. I gambled that with the time available to me it would be easier to create scripts for exactly what I needed than trying to reverse engineer a more complex, fully featured system to figure out how it worked.

[2] The images of the solutions themselves and the placement of various grids, however, come from the tools provided by Clawpack. I would like to thank Professor Mandli for providing code for visualizing the values of the refinement criteria and their location relative to the various grids.

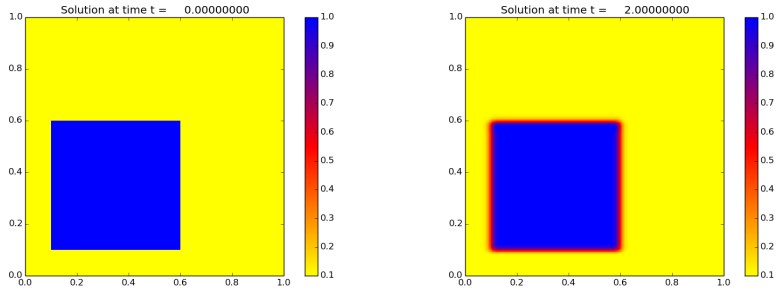


Figure 1: The start and end of the solution to the advection problem computed on the finest grid.

much without causing a non-trivial loss in accuracy. As the CFL condition constrains waves to moving through roughly one grid cell per time step, every step between regridding moves any finely resolved feature one cell closer to escaping a refined area, essentially losing information to the coarser grid.

The relative L_1 error (as compared to the fine grid reference solution) as a function of the regridding interval can be seen in Figure 2. As expected, the error increases with the regridding interval (roughly tripling once the interval reaches 9); the relative stability of the error until the interval reaches 4 can be expected from the "natural" buffering produced by aggregating cells flagged for refinement into rectilinear patches. The plateau that begins to form around an interval of 8 could indicate a limit to the "badness" of letting things escape from finer grids. Once the interval between regridding steps becomes large enough, most everything of interest may leave the refined region and become lost in the coarser grids. The point at which this occurs likely depends of the specifics of the problem, but in this advection problem, where everything moves at a constant rate and the regions of interest are rectilinear and at a fixed angle to the motion of the square, we might expect a relatively clear and well-defined plateau as things will move out of the refined regions at a predictable and unchanging rate.

The running time for these "bad" solutions can be seen in Figure 3a. The jump up at interval 2 appears to be an anomaly, and doesn't match with any change in the number of cells or the error. Otherwise, the running time is relatively constant, its peaks might be random variation in the running time, or the interactions between the decreased time from a larger interval, the slight increase in level 2 cells, and the slight decrease in level 3 cells as the interval increases. These trends in the number of cells can be seen in

Figure 2: Relative L_1 error for the advection problem, with the buffer width held constant at 1.

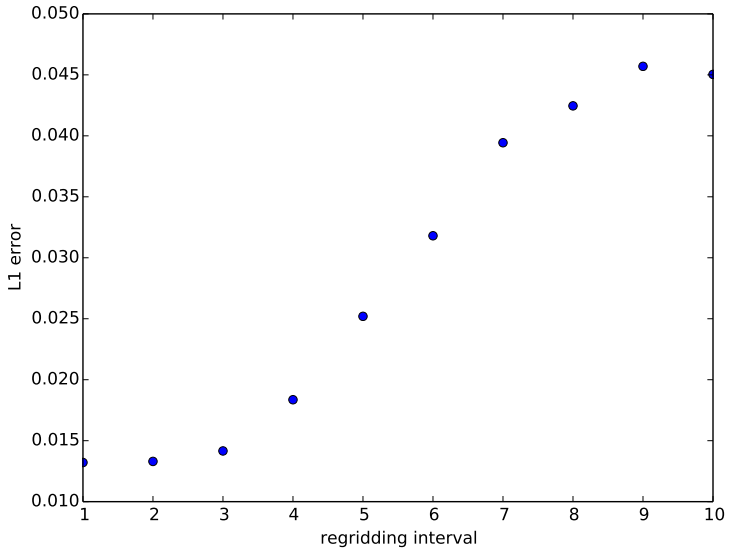
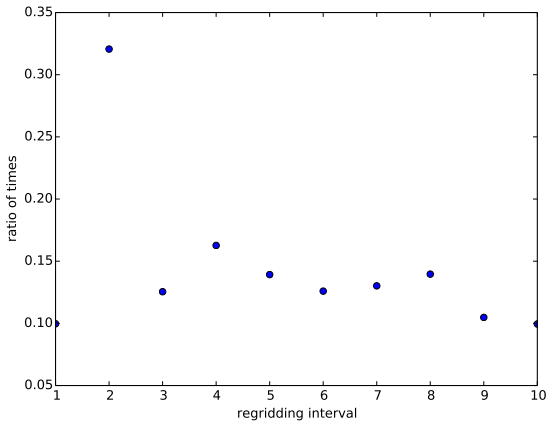


Figure 3b, and likely arise from the increasing "badness" of the solution, which manifest as a more dispersed square, which covers more area (thus more level 2 cells), but has a more gradual slope to its edges (thus smaller differences over short distances and fewer level 3 cells).

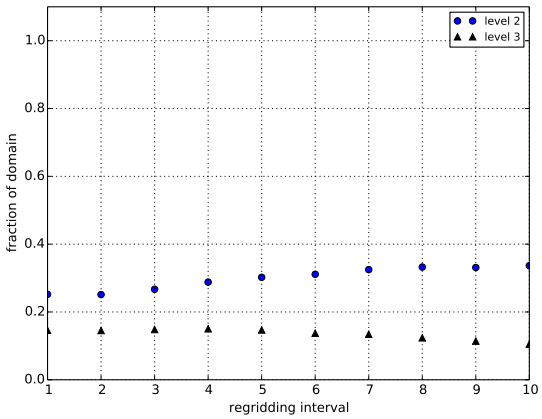
With the need to keep the regrid interval and buffer width in tandem confirmed, I did ten runs where the interval and width were equal to each other, ranging again from 1 to 10. The L_1 error as a function of the regrid interval and buffer width can be seen in Figure 4. Here we see the error decreasing as the interval/buffer increases, which is expected, as the higher buffer essentially corresponds to more of the problem being solved on finer grids at any given time. The drop in error is relatively minor compared to the blow-up of the error when the buffer was fixed, and it plateaus past an interval/buffer of 6. This corresponds to when the buffer is large enough for the second level to encompass essentially the entire domain (see Fugure 5).

It should be noted that this error is still relatively large, considering that with a regrid interval of 10 the problem is being solved on a level 2 grid everywhere, and a level 3 grid (the same resolution as the reference solution) over 60% of the domain (and that that 60% is focused around

Figure 3: Running time and cell coverage as a function of the regridding interval for the advection problem, with buffer width held at 1.



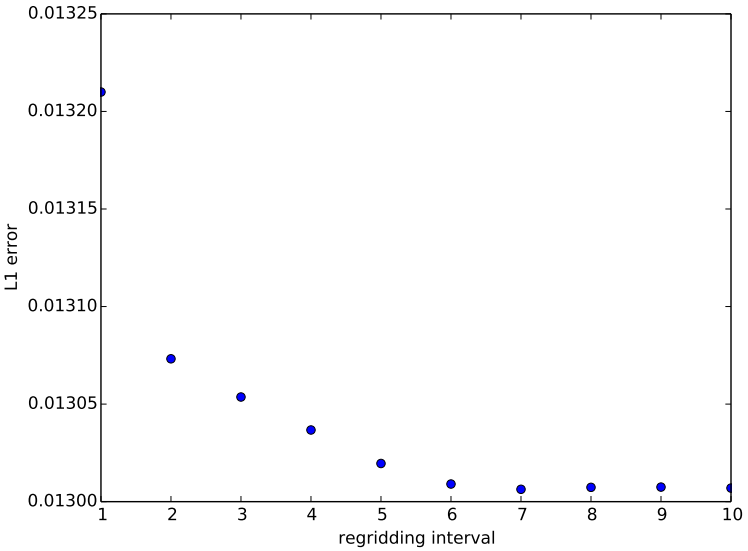
(a) Running time (as a fraction of the running time of the reference solution)



(b) Number of cells on each level (averaged over the whole computation) as a fraction of the total domain

the areas of interest). This may in part be due to the nature of the error estimate being used, combined with the specifics of the problem. Since the boundary of the square is ideally a step function, any slight difference in position or dispersion of the boundary could lead to very large errors, and since only 25 points are being sampled from, this may not be enough to

Figure 4: Relative L_1 error for the advection problem, with the number of buffer cells equal to the regrid interval.

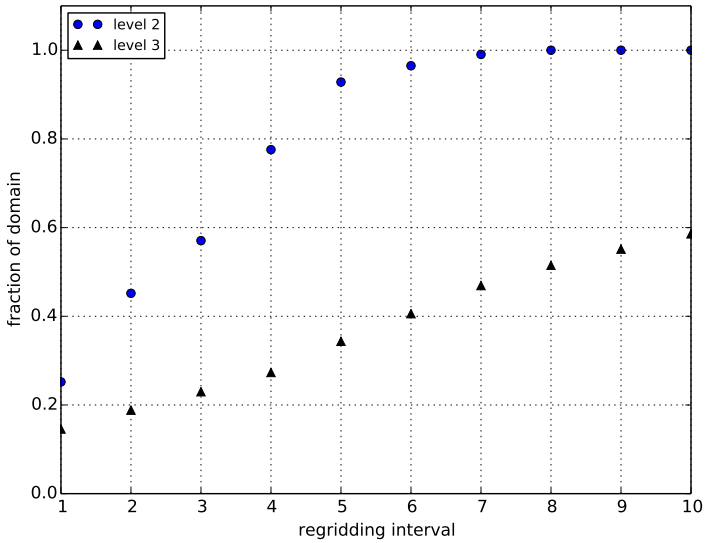


wash out those with the low errors of the vast areas that do agree.

Information on the time taken by each run is in Figure 6. The time is broken down into components: the time spent advancing the solution on the grids, the time spent regridding, and the time spent updating grids with information from more refined ones (the total running time is considered to be the sum of these components, ignoring any io or logging time). The time spent on the grids is clearly dominant, and as expected rises as the buffer width increases and more cells exist on each level. Similarly the general decrease of regridding time is expected as the interval between regrids increases[3], and as the number of cells on each level increases, we expect more time to be taken updating grids. The increase at an interval/buffer of 3 may be an anomaly (since it does appear in all three components) or it may be a particularly inefficient arrangement of flagged cells leading to unnecessarily large grid patches when combined with the buffer width.

[3]However, the plateau it seems to reach is less expected. Perhaps as more cells are flagged for inclusion by the increasing buffer, the algorithm that clusters them into patches becomes less efficient.

Figure 5: Average number of cells on each level for the advection problem, with the number of buffer cells equal to the regrid interval.



Regridding in an Euler Gas Problem

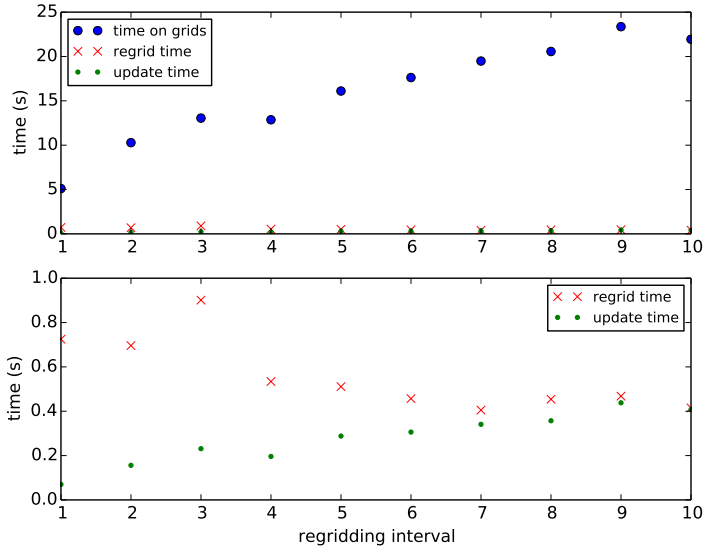
A similar analysis was then performed for the example Euler gas problem. The reference problem was run with a 320 by 320 grid; the AMR versions tarted with a 40 by 40 grid and a total of three levels, refining each dimension by a factor of 2 from the first to the second, and by a factor of 4 from the second to the third. Since the problem is more complex and runs on a finer grid, its runs took much longer and thus runs were only done for even regrid intervals, from 2 to 10.

Just like the advection problem, the Euler problem behaved poorly when the regrid interval was increased in excess of the buffer width (see Figure 7).[4] In this case, the increasing error manifested as more than just a blurring of the solution, but also in more qualitative discrepancies. Figure 8 shows a comparison between the reference solution and one where the regrid interval was allowed to exceed the buffer. Asymmetries arise in the "bad" solution, with the center appearing swirled or twisted relative to the reference.[5] The

[4]The data for these "bad" runs ends at an interval of 8, as I was having a problem parsing the output for the run at 10, and was unable to diagnose the problem in time.

[5]Presumably these asymmetries come from asymmetries in the clustered grid patches

Figure 6: Timing information for runs of the advection problem, with the with the number of buffer cells equal to the regrid interval. Note that at their slowest, the AMR runs take approximately 40% of the time of the reference solution.



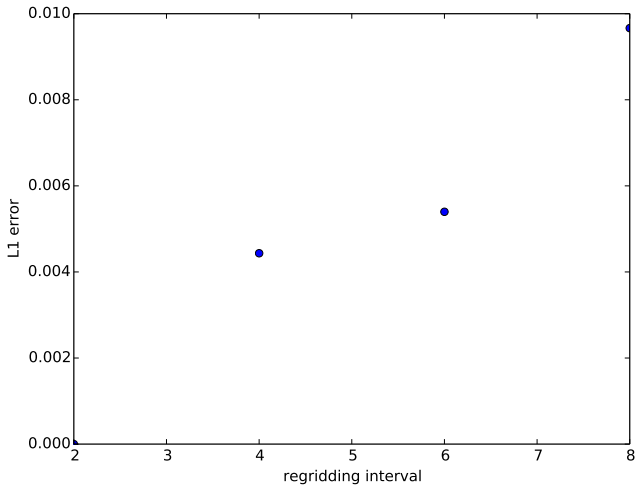
"bad" solution also has far more substantial curling structures on some of the boundaries between densities. I am not sure whether these curled features are supposed to be present or not, but the asymmetry is concerning, and there is clearly a danger in increasing the regridding interval beyond just coarsening or blurring the solution.

When keeping the buffer width tied to the regrid interval, the results from the Euler problem were again similar to the advection results above. The L_1 error drops then seems to roughly plateau, though again the drop is not too substantial (see Figure 9). Timing is even more dominated by the time spent on the grids (likely due to the more complex nature of computing the flux between cells, compared to the relatively trivial advection problem), but otherwise the overall direction and potential to plateau for each quantitys is the same as the interval/buffer increases (see Figure 10).

Behavior of the average number of grid cells on each level was overall

produced in the regridding step, as that clustering is not constrained to be necessarily balanced or symmetric. Some parts of the solution are thus refined over a wide enough area to track fine features for a longer time, while in other areas those features move out

Figure 7: Relative L_1 error for the Euler problem, with the buffer width held constant at 1.



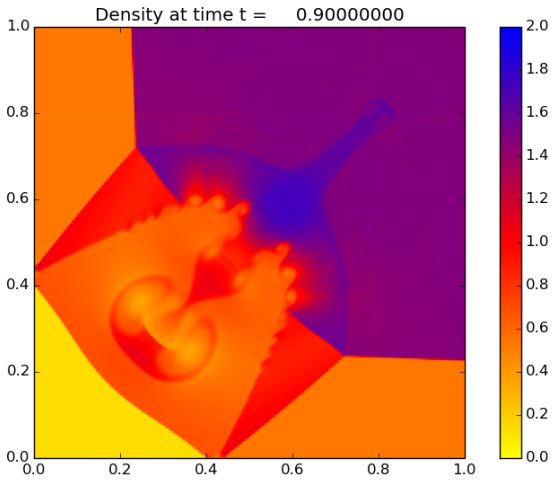
similar to the advection problem–level 2 cells eventually cover the whole grid, and the number of level 3 cells increases steadily (see Figure 11). However, it should be noted that the number of cells on each level over time in a single run was different between the advection and Euler problems. In the advection problem, the area of interest remains relatively constant in size and shape, but in the Euler problem that area grows substantially. Thus the number of cells on all levels grows to consume the whole domain of the Euler problem even in the case where the interval/buffer is only 2. Thus while the length of the time domain would likely not change much in the case of the advection problem in terms of effect of increasing the interval/buffer value, it could have a non-trivial impact for the Euler problem. (See Figure 12 for a comparison of the two problems, and the impact of interval/buffer size on the Euler problem.)

Conclusions

The relationship between the regridding interval and the buffer width was clearly seen in both problems by the dramatic increase in error when the regrid interval was allowed to increase without the buffer following it. As was

of the refined patch more readily.

(a) Reference solution



(b) Bad solution

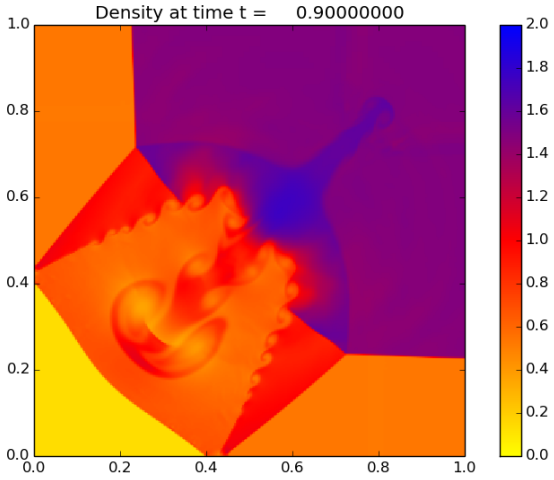
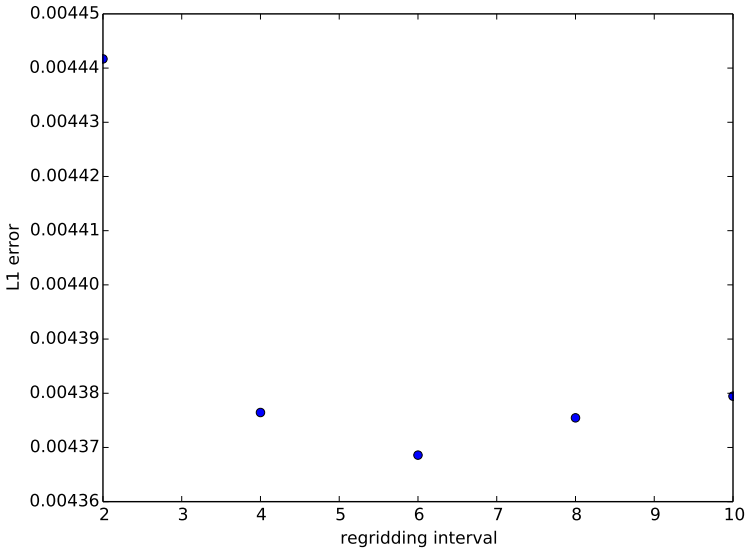


Figure 8: A comparison between the reference solution, and a "bad" solution (with a regrid interval of 8), showing qualitative distortions.

noted in the discussion of the Euler problem above, this error introduced by having interesting features escape their refined areas can manifest in qualitative impacts on the resulting solution.

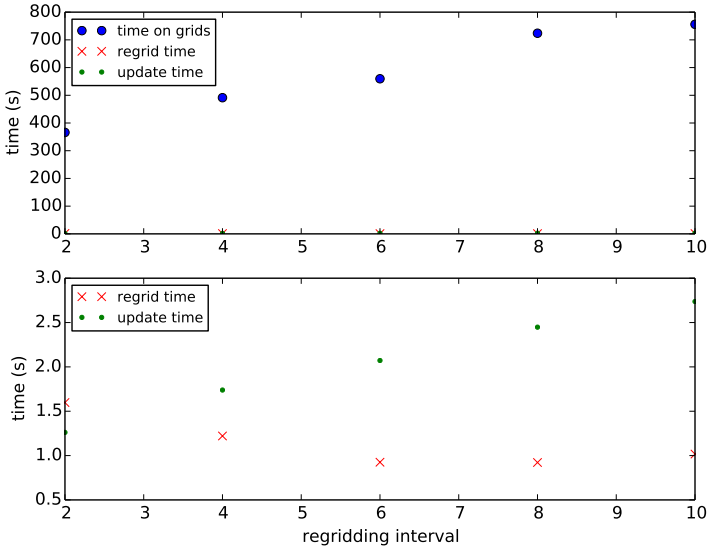
Figure 9: Relative L_1 error for the Euler problem, with the number of buffer cells equal to the regrid interval.



The trade-off between less time regridding but more time on finer grid cells that a higher interval/buffer would bring proved to be less interesting, with the grid time being dominant, and thus small regrid intervals and the consequently smaller buffers they allow seem unarguably preferable for these two problems. Indeed, the only reason to avoid the absolute minimum of interval/buffer seems to be the (relatively) substantial drop in error that accompanies a shift to a slightly higher interval/buffer (see Figures 4 and 9).

This relatively minor role for the regridding step does point in the direction of possibly more sophisticated regridding setups (either in the criteria for refining, or the clustering or updating of cells) being a worthwhile pursuit, if they could offer "smarter" refinement that cuts down on finer meshes without unduly increasing the error. Of course, it would need to be shown that similar relationships around the regrid interval and buffer width hold for problems of more interesting scale and complexity.

Figure 10: Timing information for runs of the Euler problem, with the with the number of buffer cells equal to the regrid interval. Note that at their slowest, the AMR runs take approximately 75% of the time of the reference solution.

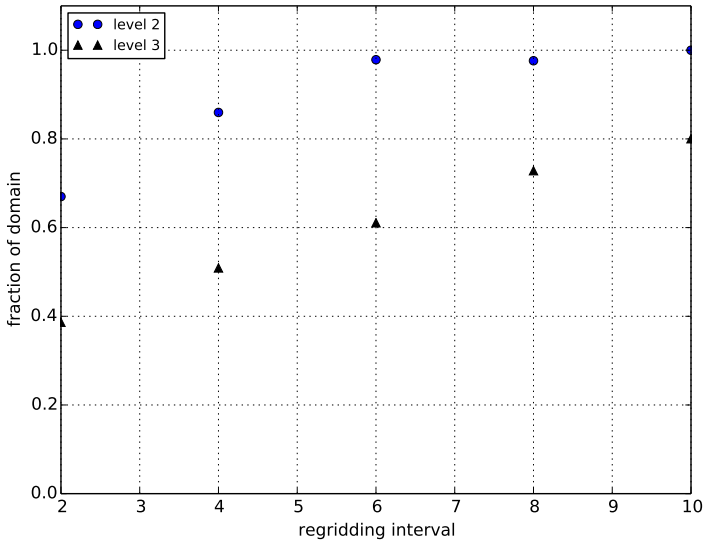


Ideas for Further Investigation

As a starting point, it would be worthwhile to slightly modify the parameter sweep and output parsing software to run multiple trials and aggregate the data; currently, the influence of random variation in the running times is unknown. The general trends seem to follow expectation, but to be sure more data should be gathered. If I had more time, I would have liked to use more gauges in computing the error in the solutions; as it is currently I feel not enough data is actually being pulled from the solutions to get an accurate picture of how close the solution is as a whole.

More time to investigate would also allow for runs over more parameters, perhaps of particular interest would be the number of levels and the refinement ratios between them. It would interesting to see what differences (if any) arise when comparing a set up that gradually refines over several levels to one that has only a couple of levels but a more dramatic refinement ratio. Simply running over wider ranges of existing parameters would also

Figure 11: Average number of cells on each level for the Euler problem, with the number of buffer cells equal to the regrid interval.



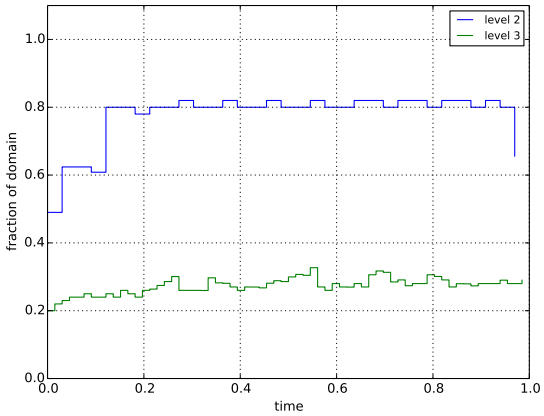
be beneficial[6] by giving a better sense of the rate at which running times and errors change in response to the regridding interval and buffer.

It would also be interesting to explore more computationally intensive regridding options, such as the Richardson extrapolation option provided in AMRClaw (which estimates the difference between advancing on a finer grid and the current grid, and flags cells where this difference exceeds a threshold; this is the primary method mentioned in [1]). A substantially more time-consuming regridding step could make the trade-off between the regrid interval and buffer width a more important consideration.

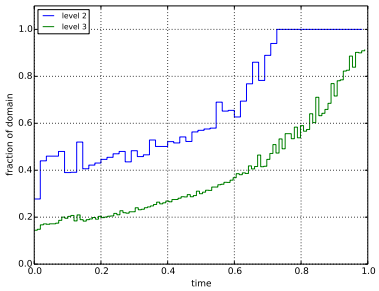
Problem-specific factors also no doubt play a large part in the interplay between regridding intervals and buffers. For example, the advection of the square produces regions of interest for refinement that align naturally with the grid. This likely reduces the number of grids produced by allowing more efficient grouping of flagged points, but it also may increase the need for buffering those regions. Suppose instead the region being advected were a diamond; the flagged regions would run diagonally, necessitating grids that

[6]However, larger domains would be needed to avoid filling in the whole space with the finest grids so quickly

(a) Advection with interval/buffer = 4



(b) Euler with interval/buffer = 2



(c) Euler with interval/buffer = 8

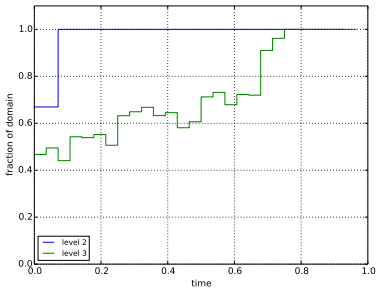


Figure 12: A comparison of the number of cells over time (as a fraction of the total domain) for three different runs: advection with a medium interval/buffer, Euler with a small one, and Euler with a larger one

would naturally leave more buffer around many of the flagged points. In contrast, for the square, the grid could cling much closer to flagged reasons, leaving little buffer without explicitly requiring it. Similarly, the Euler quadrants problem has structures that seem to align diagonally to the grid, and the opposite effects may arise if it were instead aligned 45^o from its present orientation.

Simply testing on a wider range of problems would be helpful to see which elements of the regridding interval and buffer relationship are more generalizable. In particular, in both the advection and Euler problems I examined, the areas of interest formed a sizeable proportion of the total

domain, leading to little distinction between comfortably encompassing the interesting areas and simply filling the entire domain with finer grids. Expanding the domain of the two problems already examined would allow for a preliminary investigation into those issues.

References

[1] M. J. Berger and R. J. LeVeque. "Adaptive Mesh Refinement using Wave-Propagation Algorithms for Hyperbolic Systems". In: *SIAM J. Numer. Anal.* 35 (1998), pp. 2298–2316.

[2] Clawpack Development Team. *Clawpack software*. Version 5.2. 2014. URL: `http://www.clawpack.org`.