CS 736 : Assignment - Imaging and Image Reconstruction

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Maximum Marks: 55 + 5 (Submission) = 60 Points

1. (20 points) Filtered Backprojection.

   Consider the phantom image provided "SheppLogan256.png". Call this underlying image function $f(x, y)$. Associate (logically) a coordinate frame with the image with the origin located at the center pixel.

   (a) (8 points) Compute the Radon transform of $f(x, y)$ with the values of $\theta = 0, 3, 6, \cdots, 177$ degrees. Use the in-built function "radon()" in Matlab or from any standard library in Python (e.g., `https://scikit-image.org/docs/dev/auto_examples/transform/plot_radon_transform.html`).

   Next, using the in-built implementation of back-projection ("iradon()"), reconstruct the image from the Radon transform without any filter parameters.

   Now, implement the Ram-Lak filter, Shepp-Logan filter, and the Cosine filter (in the frequency domain), where L is a user-controlled parameter. Name the function as myFilter(). Use your function to filter (in all 3 ways) the Radon-transform data for 2 different values of L, i.e., $L = w_{\text{max}}$ and $L = w_{\text{max}}/2$, where w_{max} is the highest frequency in the (discrete) Fourier representation. Again, perform backprojection of the filtered data to reconstruct the image.

   Show all resulting images and justify the similarities and the differences observed between the different combinations of filters and parameter values.

   (b) (8 points) Generate two blurred versions of the image by convolving it with a Gaussian filter of standard deviations $\sigma = 1$ and $\sigma = 5$. Show the 3 different versions of the Shepp-Logan images, say S_0, S_1, S_5.

   For all these, compute the radon transform with $\theta = 0, 3, 6, \cdots, 177$ degrees, apply the Ram-Lak filter with $L = w_{\text{max}}$, and compute the backprojection. For all 3 filtered backprojections, say, R_0, R_1, R_5, compute the relative root-mean-squared errors (RRMSE) RRMSE(S_i, R_i), where the RRMSE for 2 images A and B is defined as:

   RRMSE$(A, B) = \sqrt{\sum_p (A(p) - B(p))^2} / \sqrt{\sum_p A(p)^2}$, where the summation is over all pixels p. In which of the 3 cases is the RRMSE the highest and the lowest ? Explain theoretically.

   (c) (4 points) For each of the 3 examples (i.e., Shepp-Logan phantoms S_0, S_1, S_5 and their filtered backprojections R_0, R_1, R_5 using the Ram-Lak filter), plot the RRMSE values as a function of L with $L = 1, 2, \cdots, w_{\text{max}}$. Explain your findings.

2. (25 points) Simulated Data Acquisition and Reconstruction (without and with prior).

We will try to simulate the data acquisition in CT imaging, followed by image reconstruction using simple filtered backprojection as well as prior-based iterative reconstruction. We wish to reconstruct the image on a grid size of 128 $\times$ 128. We will consider the Zubal phantom image ("ChestPhantom.png" (`http://noodle.med.yale.edu/zubal/`)) for this experiment.

(a) (8 points) Explicitly build a CT imaging / system matrix, say, A, by using the following angles: $\theta = 0, 1, 2, \cdots, 179$. You can use the in-built "radon()" function to populate the system matrix. Explain the strategy.

(b) (2 points) Use the generated system matrix to perform forward projection of the data to generate the radon transform. Add Gaussian noise to the sinogram data, where the standard deviation is 2% of the range of values in the radon transform.

(c) (2 points) Reconstruct the image from the noisy data using fitlered backprojection using suitable filter identified from Q1. Report the reconstructed images along with the RRMSE values.

(d) (4 points) Perform a Tikhonov regularized reconstruction using the matrix A. You may choose to solve it using the closed-form or perform iterative reconstruction. Report the optimal value of the regularization parameter such that the reconstructed image has minimum RRMSE. Similar to the previous assignment, report the RRMSE values of the reconstructed images obtained using regularization parameters around the optimal value (0.8*optimal and 1.2*optimal).

(e) (9 points) Perform regularized reconstruction using a Markov random Field (MRF) prior using a 4-neighborhood and the following 2-clique potential functions: (i) Squared difference, (ii) Huber, and (iii) Discontinuity-adaptive. Perform iterative reconstruction. Report the optimal value of the regularization parameter such that the reconstructed image has minimum RRMSE. Similar to the previous assignment, report the RRMSE values of the reconstructed images obtained using regularization parameters around the optimal value (0.8*optimal and 1.2*optimal).

3. (10 points) CT with Incomplete Data.

Load ground-truth images of attenuation coefficients ("ChestCT.png") and ("SheppLogan256.png").

Consider a CT imaging scenario where data can be acquired over 150 angles spanned over 150 degrees (not 180 degrees), in increments of 1 degree. That is, you can acquire data at the angles $[\theta, \theta+1, \cdots, \theta+150]$ degrees, for any given θ. For each of the given two datasets, which contigous set of 150 angles will you acquire the data to minimize the error in reconstruction ?

(a) (5 points) For each dataset, plot the RRMSE values between the ground-truth images and the reconstructed images for $\theta \in [0, 1, \cdots, 180]$.

(b) (5 points) For each dataset, show the reconstructed image for that value of θ that gives the least RRMSE.