Sampling non-isotropic Gaussian random fields

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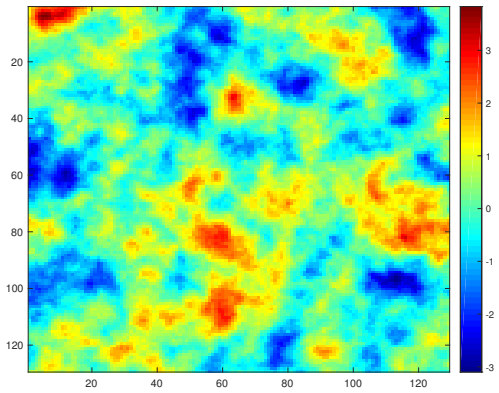
`RandField_Matern.m` script uses the following stationary non-isotropic Matérn model

$$C_{\Phi}(\mathbf{x_1}, \mathbf{x_2}) = \sigma_c^2 \frac{2^{1-\nu_c}}{\Gamma(\nu_c)} \left(2\sqrt{\nu_c}\tilde{r}\right)^{\nu_c} K_{\nu_c}\left(2\sqrt{\nu_c}\tilde{r}\right) \tag{1}$$

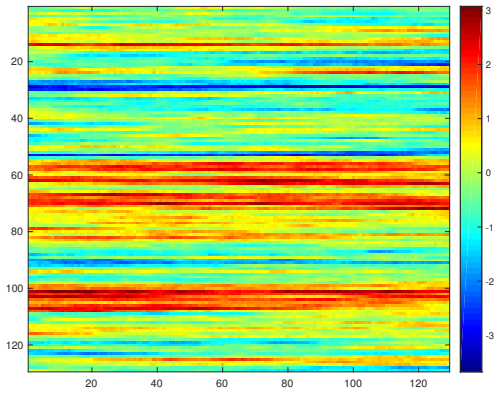
$$\tilde{r} = \sqrt{\frac{(x_1 - x_2)^2}{\lambda_{cx}^2} + \frac{(y_1 - y_2)^2}{\lambda_{cy}^2}} \quad \text{with} \quad \mathbf{x_1} = (x_1, y_1), \mathbf{x_2} = (x_2, y_2). \tag{2}$$

where λ_{cx} and λ_{cy} are correlation lengths along x- and y-coordinates, respectively, ν_c is the smoothness of the random field and σ_c^2 is the marginal variance. Some examples of usage:

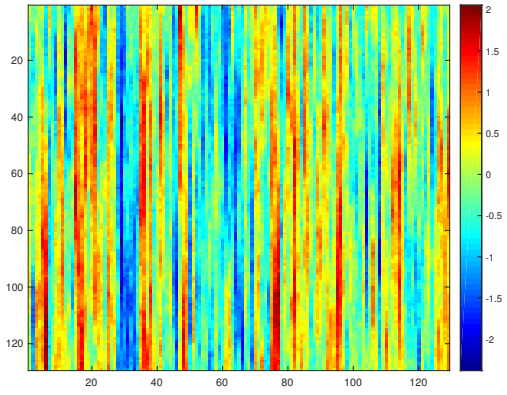
```
»[F]= RandField_Matern(0.1,0.1,1,1,7,1)% isotropic
```




```
»[F]= RandField_Matern(2,0.02,0.5,1,7,1)% layering along x
```




```
»[F]= RandField_Matern(0.01,1,0.5,0.5,7,1) % layering along y
```



References

[1] P. Kumar, P. Luo, F. J. Gaspar, C. W. Oosterlee, *A multigrid multilevel Monte Carlo method for transport in the Darcy-Stokes system*, Journal of Computational Physics 371 (2018) 382 – 408.

[2] C. Dietrich, G. Newsam, *Fast and exact simulation of stationary Gaussian processes through circulant embedding of the covariance matrix*, SIAM J. Sci. Comput. 18 (1997) 1088–1107.

[3] A. Wood, G. Chan, *Simulation of stationary Gaussian processes in* $[0, 1]^d$, Journal of Computational and Graphical Statistics 3 (1994) 409–432.