Problems on Integrating factors

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2) $(xy^2+2x^2y^3)\,dx+(x^2y+x^3y^2)\,dy=0$ —— (1)

Sol^n: $M = xy^2+2x^2y^3$ | $N = x^2y+x^3y^2$

$\dfrac{\partial M}{\partial y} = 2xy+6x^2y^2$ | $\dfrac{\partial N}{\partial x} = 2xy+3x^2y^2$

$\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$, Hence Not Exact.

$\cancel{x\log y}$ $\log(xy^2)$ x^2

Check: $\boxed{(xy+2x^2y^2)\,y}\,dx + \underline{(xy+x^2y^2)}\,x\,dy = 0$

Hence it is $y\,dx+x\,dy$ form

Then $Mx-Ny = \cancel{x^2y^2}+2x^3y^3-[\cancel{x^2y^2}+x^3y^3]$

$= x^3y^3$

Now, multiply (1) by I.F.

$$\frac{1}{x^3y^3}[xy^2+2x^2y^3]\,dx+\frac{1}{x^3y^3}[x^2y+x^3y^2]\,dy=0$$

$$\therefore \left[\frac{1}{x^2y}+\frac{2}{x}\right]dx+\left[\frac{1}{xy^2}+\frac{1}{y}\right]dy=0$$

This is exact D.E.

Then sol^n is given by

$$\int\limits_{y\text{ const}} M\,dx+\int\limits_{\text{free of }x} N\,dy = C$$

$$\therefore \int\left(\frac{1}{x^2y}+\frac{1}{x}\right)dx+\int\frac{1}{y}\,dy = C$$

$$\therefore -\frac{1}{xy}+\log x+\log y = C$$

$$\boxed{-\frac{1}{xy}+\log(xy)=C}$$

3) $(x^3e^x-my^2)\,dx+(mxy)\,dy=0$ —— (1)

$1 \quad xy$

3) $(x^3 e^x - my^2)\,dx + (mxy)\,dy = 0$ — (1)

Sol^n: $M = x^3 e^x - my^2$ | $N = mxy$

$\frac{\partial M}{\partial y} = -2my$ | $\frac{\partial N}{\partial x} = my$, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Hence not exact

Then, Divide by N

Check $\left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}\right) = \frac{-3\cancel{m}y}{\cancel{m}xy} = \boxed{-\frac{3}{x}}$ — purely function of x

Then I.F. is $e^{\int -\frac{3}{x}dx} = e^{-3\int \frac{1}{x}dx} = e^{-3\log x} = e^{\log(x^{-3})}$

$= x^{-3} = \frac{1}{x^3}$

$\therefore$ Multiply eq^n (1) by I.F.

$(x^3 e^x - my^2)\frac{1}{x^3}dx + (mxy)\frac{1}{x^3}dy = 0$

$\left[e^x - m\frac{y^2}{x^3}\right]dx + \left[m\frac{y}{x^2}\right]dy = 0$

This is exact D.E.

$\therefore$ Sol^n is $\int_{\text{con}} M\,dx + \int N\,dy = C$

$-\int \frac{1}{x^3}dx$

$\int x^{-3}dx$

$\frac{x^{-2}}{-2}$

$\int\left(e^x - m\frac{y^2}{x^3}\right)dx + \int 0\,dy = C$

$\therefore \boxed{e^x + m\frac{y^2}{2x^2} = C}$

4) $(xy^3 + y)\,dx + 2(x^2y^2 + x + y^4)\,dy = 0$ — (1)

$M = xy^3 + y$, $N = 2(x^2y^2 + x + y^4)$

$\frac{\partial M}{\partial y} = 3xy^2 + 1$ | $\frac{\partial N}{\partial x} = 4xy^2 + 2$

not exact!

Check Divide by M, $\left(\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}\right) = \left(\frac{xy^2 + 1}{xy^3 + y}\right) = \frac{\cancel{(xy^2+1)}}{y\cancel{(xy^2+1)}}$

$= \frac{1}{y}$

Then, I.F. $= e^{\int \frac{1}{y}dy} = e^{\log y} = y$

Then, I.F. $= e^{\int \frac{1}{y} dy} = e^{\log y} = y$

Then multiply (1) by y

$\therefore (xy^4 + y^2)\,dx + (\underline{2x^2y^3} + \underline{2xy} + 2y^5)\,dy = 0$

This is exact D.E.

Then sol^n of D.E. is given by, $\int M\,dx + \int N\,dy = C$

$\therefore \int xy^4 + y^2\,dx + \int (2y^5)\,dy = C$

$$\frac{x^2y^4}{2} + xy^2 + 2\frac{y^6}{6} = C$$

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5) $\left[y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right] dx + \frac{1}{4}\left[x + xy^2\right] dy = 0$ (H.W.)

6) $(3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0$ (H.W)

7) $(\underset{M}{y + xy^2 + x^2y^3 + x^3y^4})\,dx + (\underset{N}{x - x^2y - x^3y^2 + x^4y^3})\,dy = 0$

Sol^n: $\frac{\partial M}{\partial y} = 1 + 2xy + 3x^2y^2 + 4x^3y^3$ | $\frac{\partial N}{\partial x} = 1 - 2xy - 3x^2y^2 + 4x^3y^3$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, Not exact D.E.

Then $\therefore \underline{[1 + xy + x^2y^2 + x^3y^3]}\,y\,dx + \underline{[1 - xy - x^2y^2 + x^3y^3]}\,x\,dy = 0$

Hence given eq^n is $y\,dx + x\,dy = 0$ form

$Mx - Ny = xy + x^2y^2 + x^3y^3 + x^4y^4 - [xy - x^2y^2 - x^3y^3 + x^4y^4]$

$= 2x^2y^2 + 2x^3y^3$

I.F $= \frac{1}{Mx - Ny} = \frac{1}{2x^2y^2 + 2x^3y^3} = \frac{1}{2x^2y^2(1 + xy)}$

multiply (1) by I.F.

$\left[\frac{y + xy^2 + x^2y^3 + x^3y^4}{2x^2y^2 + 2x^3y^3}\right] dx + \left[\frac{x - x^2y - x^3y^2 + x^4y^3}{2x^2y^2 + 2x^3y^3}\right] dy = 0$

This is exact.

$\left[\frac{y(1+\cancel{xy}) + x^2y^3\underline{(1+\cancel{xy})}}{}\right] dx + \frac{x[(1-xy) - x^2y^2(1-xy)]}{2x^2y^2(1+xy)}\,dy$

This is exact

$$\Rightarrow \left[\frac{y(1+\cancel{xy}) + x^2y^3(1+\cancel{xy})}{2x^2y^2(1+\cancel{xy})}\right]dx + \frac{x[(1-xy) + x^2y^2(1-xy)]}{2x^2y^2(1+xy)}\,dy$$

$$\left[\frac{1}{2x^2y} + \frac{y}{2}\right]dx + \frac{x\left[(1-xy)(1-x^2y^2)\right]}{2x^2y^2(1+xy)}\,dy$$

$$—\text{"}— \quad + \frac{\cancel{x}\left[(1-xy)(1-xy)\cancel{(1+xy)}\right]}{2x^{\cancel{2}}y^2\cancel{(1+xy)}}\,dy$$

$$—\text{"}— \quad + \left[\frac{1}{2xy^2}(1-2xy+x^2y^2)\right]dy$$

$$\left[\frac{1}{2x^2y} + \frac{y}{2}\right]dx + \left[\frac{1}{2xy^2} - \frac{1}{y} + \frac{x}{2}\right]dy$$

Now we find the sol^n

$$\int\left[\frac{1}{2x^2y} + \frac{y}{2}\right]dx + \int -\frac{1}{y}\,dy = C$$

$$\therefore\ -\frac{1}{2xy} + \frac{y}{2}x - \lg y = C$$

$$\boxed{-\frac{1}{xy} + xy - \lg y^2 = C'}$$