4) finding PI when RHS is product (known function)

Monday, March 08, 2021 11:30 AM

$$\cancel{\left(\frac{1}{F(D)}x^m \sin 3x = \left(\frac{1}{F(D)}x^m\right)\left(\frac{1}{F(D)}\sin 3x\right)\right)}$$

Possibilities:

$\boxed{e^{ax}V} \rightarrow$ $e^{ax}\sin ax$, $e^{ax}\cos ax$ OR $e^{ax}(\text{Poly})$ $\boxed{e^{iax} = \cos ax + i\sin ax}$

$\downarrow$

$(\sin ax/\cos ax/\text{Poly}) \leftarrow (\text{poly})\left(\frac{\sin ax}{\cos ax}\right)$ (This can also reduced into $e^{ax}V$)

Formula:

$$\left[\frac{1}{F(D)}\right]e^{ax}V = e^{ax}\left[\frac{1}{F(D+a)}\right]V$$

$$\left[\frac{1}{F(D)}\right]e^{-ax}V = e^{-ax}\left[\frac{1}{F(D-a)}\right]V$$

$$y_p = (x^2)\left[\frac{1}{D^2+5D+2}\right]$$

zero

$f(x) \rightarrow$ $\cancel{x(f)}$

$$= \frac{x^2}{F''(a)}$$

$$x^2\left[\frac{1}{F''(a)}\right]e^{ax}$$

Problems:

1) $(D^2+3D+2)y = e^{2x}\sin x$

Sol^n: $y_c = c_1e^{-2x}+c_2e^{-x}$

$$y_p = \left[\frac{1}{D^2+3D+2}\right]e^{2x}\sin x = e^{2x}\left[\frac{1}{(D+2)^2+3(D+2)+2}\right]\sin x$$

(Replace D by $D+2$)

$$= e^{2x}\left[\frac{1}{D^2+4D+4+3D+6+2}\right]\sin x = e^{2x}\left[\frac{1}{D^2+7D+12}\right]\sin x$$

[Replace D^2 by -1]

$$= e^{2x}\left[\frac{1}{7D+11}\right]\sin x$$

$$= e^{2x}\left[\frac{1}{49D^2-121}\right](7D-11)\sin x$$

$$y_p = e^{2x}\left[\frac{1}{49D^2-121}\right](7\cos x - 11\sin x) = -\frac{e^{2x}}{170}(7\cos x - 11\sin x)$$

$$y_g = y_c + y_p =$$

$y_g = y_c + y_p =$

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2) $(D^3 - 7D - 6)\, y = (1+x^2)\, e^{2x} = \cancel{e^{2x} + x^2 e^{2x}}$

$y_c = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{3x}$

$y_p = \left[\dfrac{1}{D^3 - 7D - 6}\right] e^{2x}(1+x^2) = e^{2x}\left[\dfrac{1}{(D+2)^3 - 7(D+2) - 6}\right](1+x^2)$

[Replace D by $D+2$]

$= e^{2x}\left[\dfrac{1}{D^3 + 6D^2 + 12D + 8 - 7D - 14 - 6}\right](1+x^2)$

$= e^{2x}\left\{\dfrac{1}{-12 + 5D + 6D^2 + D^3}\right\}(1+x^2)$

$= -\dfrac{e^{2x}}{12}\left[\dfrac{1}{1 - \underbrace{\left(\dfrac{5D + 6D^2 + D^3}{12}\right)}_{\phi(D)}}\right](1+x^2)$

$\dfrac{1}{1-\phi(D)}$

$1 + \phi(D) + (\phi(D))^2$

$= -\dfrac{e^{2x}}{12}\left[1 + \left(\dfrac{5D}{12} + \dfrac{1}{2}D^2 + \cancel{\dfrac{D^3}{12}}\right) + \left(\dfrac{25}{144}D^2\right) + 0\right](1+x^2)$

$= -\dfrac{e^{2x}}{12}\left[(1+x^2) + \dfrac{5}{12}(2x) + \cancel{\dfrac{1}{2}}(\cancel{2}) + \dfrac{25}{144}(2)\right]$

$y_p = -\dfrac{e^{2x}}{12}\left[x^2 + \dfrac{5}{6}x + \dfrac{169}{72}\right]$

$y_g = y_c + y_p$

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3) $(D^2 + 4D + 4)\, y = \dfrac{e^{-2x}}{x^5}$

Sol^n: $y_c = (c_1 + c_2 x)\, e^{-2x}$

$y_p = \left[\dfrac{1}{D^2 + 4D + 4}\right] e^{-2x}\left(\dfrac{1}{x^5}\right) = e^{-2x}\left[\dfrac{1}{(D-2)^2 + 4(D-2) + 4}\right]\left(\dfrac{1}{x^5}\right)$

[Replace D by $D-2$]

$= e^{-2x}\left[\dfrac{1}{D^2 - \cancel{4D} + \cancel{4} + \cancel{4D} - \cancel{8} + \cancel{4}}\right]\left(\dfrac{1}{x^5}\right) = e^{-2x}\left(\dfrac{1}{D^2}\right)\dfrac{1}{x^5}$

$$= e^{-2x}\left[\frac{1}{D^2-\cancel{4D}+\cancel{4}+\cancel{4D}-\cancel{8}+\cancel{4}}\right]\left(\frac{1}{x^5}\right) = e^{-2x}\left(\frac{1}{D^2}\right)\frac{1}{x^5}$$

$$= e^{-2x}\left(\frac{1}{D}\right)\int x^{-5}\,dx = e^{-2x}\left(\frac{1}{D}\right)\left[\frac{x^{-4}}{-4}\right] = e^{-2x}\,\frac{x^{-3}}{12}$$

$$\boxed{y_p = \frac{e^{-2x}}{12x^3}} \qquad \& \quad y_g = y_c + y_p$$

H.W.

4) $(D^4-1)y = \cos x \cosh x = \cos x\left(\frac{e^x+e^{-x}}{2}\right) = \frac{1}{2}\left[\underline{e^x\cos x} + \underline{e^{-x}\cos x}\right]$

5) $(D^2-4D+3)y = 2xe^{3x} + 3e^x\cos 2x$

6) $(D^2-4D+3)y = e^x\cos 2x + \cos 3x$

7) $(D^2+1)y = \underline{x^2\sin 2x}$ $\quad$ ((poly) (sinax / cosax))

$y_c =$ H.W.

$$y_p = \left[\frac{1}{D^2+1}\right]x^2\,\underline{\sin 2x} = \underline{\text{Img}}\text{ Part of }\left[\frac{1}{D^2+1}\right]\left(x^2 e^{2ix}\right)$$

x^2 $\quad$ (Replace D by $D+2i$)

$$= \text{IP of } e^{2ix}\left[\frac{1}{D^2+4Di+4(i^2)+1}\right]x^2$$

$$= \text{IP of } e^{2ix}\left[\frac{1}{D^2+4Di-3}\right]x^2$$

$$= \text{IP of } \frac{e^{2ix}}{-3}\left[\frac{1}{1-\underbrace{\left(\frac{4Di}{3}+\frac{D^2}{3}\right)}_{Q(D)}}\right]x^2$$

$$= \text{IP of } \frac{e^{2ix}}{-3}\left[1+\left(\frac{4}{3}iD+\frac{D^2}{3}\right)+\left(\frac{16}{9}(i^2)D^2\right)\right]x^2$$

$$= \text{IP of } -\frac{e^{2ix}}{3}\left[(x^2)+\frac{4}{3}i(2x)+\frac{1}{3}(2)-\frac{16}{9}(2)\right]$$

$$= \text{IP of } -e^{2ix}\left[x^2+\frac{8}{3}xi-\frac{26}{9}\right]$$

$= \text{IP of } -\frac{1}{3}(\cos 2x + i\sin 2x)\left[x^2 + \frac{8}{3}xi - \frac{26}{9}\right]$

$$\boxed{y_p = -\frac{1}{3}\left[\frac{8}{3}x\cos 2x + x^2\sin 2x - \frac{26}{9}\sin 2x\right]}$$

$y_g = y_c + y_p$

all three together

8) $(D^2 - 2D + 1)y = xe^x \sin x = \boxed{e^x}\,\boxed{x\sin x}$

$y_c = $ H.W.

$y_p = \left[\frac{1}{D^2 - 2D + 1}\right] e^x x\sin x = e^x\left[\frac{1}{(D+1)^2 - 2(D+1) + 1}\right] x\sin x$

(Replace D by $D+1$)

$= e^x\left[\frac{1}{D^2 + \cancel{2D} + \cancel{1} - \cancel{2D} - \cancel{2} + \cancel{1}}\right] x\sin x$

$= e^x\left(\frac{1}{D}\right)\int \underset{u}{x}\,\underset{v}{\sin x}\,dx = e^x\left[\frac{1}{D}\right]\left[-x\cos x - \int -\cos x\,dx\right]$

$= e^x\left(\frac{1}{D}\right)\left[-x\cos x + \sin x\right]$

$= e^x\int(-x\cos x + \sin x)\,dx$

$= e^x\left[\left(-x\sin x - \int(-1)\sin x\,dx\right) + (-\cos x)\right]$

$= e^x\left[-x\sin x - \cos x - \cos x\right]$

$y_p = -e^x\left[x\sin x + 2\cos x\right]$