Application of 1st order and 1st deg DE (Mechanical and Electrical)

Monday, February 22, 2021 11:29 AM

I} Mechanical Engg Application :

1) $\frac{dv}{dt} = g\cos\alpha - kv$, $v \Rightarrow$ velocity , $t \Rightarrow$ time
g, α, k are constants

at $t = 0$, $v = 0$, Then solve & find v.

Sol^n: Consider $\frac{dv}{dt} + kv = g\cos\alpha$

$P = k$, $Q = g\cos\alpha$

$I.F. = e^{\int P dt} = e^{\int k dt} = e^{kt}$

then, Sol^n is given by,

$$v e^{\int P dt} = \int \left(Q e^{\int P dt}\right) dt + C$$

$$v e^{kt} = \int (g\cos\alpha) e^{kt} dt + C$$

$$\boxed{v e^{kt} = g\cos\alpha \frac{e^{kt}}{k} + C}$$ is general sol^n

Use, at $t = 0$, $v = 0$, substitute

$$0 = \frac{g\cos\alpha}{k}(1) + C$$

$$\boxed{C = -\frac{g\cos\alpha}{k}}$$

$\therefore$ Perticular Sol^n is given by,

$$v e^{kt} = \frac{g\cos\alpha}{k} e^{kt} - \frac{g\cos\alpha}{k}$$

$$v e^{kt} = \frac{g\cos\alpha}{k}(e^{kt} - 1)$$

$$\boxed{v = \frac{g\cos\alpha}{k}(1 - e^{-kt})}$$

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2) mass m, eq^n is $mv\frac{dv}{dx} = ka^2 - kv^2$, distance x Then prove that

$$\boxed{\frac{2kx}{m} = \log\left(\frac{a^2}{a^2 - v^2}\right)}$$ where $mg = ka^2$

Sol^n: $mv\frac{dv}{dx} = k(a^2 - v^2)$

rearrange to get variable separable

$$\left(\frac{v}{a^2 - v^2}\right) dv = \frac{k}{m} dx$$

Integrate both sides.

$$\int \frac{v}{a^2 - v^2} dv = \int \frac{k}{m} dx$$

$p = a^2 - v^2$, $-\frac{1}{2} dp = v\,dv$

$$\therefore \quad = \frac{k}{m}x + C$$

$$-\frac{1}{2}\int \frac{dp}{p}$$

Initial conditions $t = 0$, $v = 0$, $x = 0$

$$\therefore \boxed{-\frac{1}{2}\log(a^2) = C}$$

$$-\frac{1}{2}\log(a^2 - v^2) = \frac{kx}{m} - \frac{1}{2}\log(a^2)$$

$$\log(a^2) - \log(a^2 - v^2) = \frac{2kx}{m}$$

$$\boxed{\log\left(\frac{a^2}{a^2 - v^2}\right) = \frac{2kx}{m}}$$

$$\therefore \ p = a^2 - v^2 \quad = \frac{k}{m}x + C$$

$$-\frac{1}{2}\int \frac{1}{p}\,dp = \frac{k}{m}x + C$$

$$\boxed{-\frac{1}{2}\log(a^2 - v^2) = \frac{kx}{m} + C}$$

$$\boxed{d(a^2 - v^2) \qquad m}$$

II] Electrical Engg Problems : (R-L / R-L-C Circuits)

Here Refer $R \Rightarrow$ Resistance, $L \Rightarrow$ Inductance,

$C \Rightarrow$ Capacitance, $E/V \Rightarrow$ Electromotive force/voltage.

$q \Rightarrow$ charge, $i \Rightarrow$ current

Rules / Law's Governed by Circuits

1) $i = \frac{dq}{dt} \quad \therefore \ q = \int i\,dt$

2) Voltage drop accross Resistor $(R) \Rightarrow \boxed{V_R = iR}$

3) Voltage drop accross Inductor $(L) \Rightarrow \boxed{V_L = L\frac{di}{dt}}$

4) Voltage drop accross Capacitor $(C) \Rightarrow \boxed{V_C = \frac{q}{C}}$

Kirchhoff's Law

R-L-E circuits : Rule $\Rightarrow V_R + V_L = E$ ✓

$$\therefore \ \boxed{iR + L\frac{di}{dt} = E} \Rightarrow \boxed{\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}} \ ✓$$

R-L-C-E Circuits : Rule $\Rightarrow V_R + V_L + V_C = E$

$$iR + L\frac{di}{dt} + \frac{q}{C} = E \Rightarrow \frac{di}{dt} + \frac{R}{L}i + \frac{q}{LC} = \frac{E}{L} \ ✓$$

$$\times \quad \frac{dq}{dt} \qquad \frac{d^2q}{dt^2} \Rightarrow \boxed{\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = \frac{E}{L}} \ \times$$

Problems

1) current i is given by $L\frac{di}{dt} + Ri = E$, Then Find Express of i if at $t=0$, $i=0$, L, R, E constants

$\underline{\text{Sol}^n}$: $\underline{\text{Consider}}$ eq^n $\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}$ is linear eq^n

$P = \frac{R}{L}$, $Q = E/L$

$\therefore$ I.F. $= e^{\int P\,dt} = e^{\int \frac{R}{L} dt} = e^{\frac{Rt}{L}}$

Then sol^n is given by,

$$i e^{\int P dt} = \int \left(Q e^{\int P dt}\right) dt + C$$

$$i e^{\frac{Rt}{L}} = \int \frac{E}{L} e^{\frac{Rt}{L}} dt + C$$

$$\boxed{i e^{\frac{Rt}{L}} = \frac{E}{\cancel{L}} \frac{e^{Rt/L}}{R/\cancel{L}} + C}$$

$$\boxed{i e^{\frac{Rt}{L}} = \frac{E}{R} e^{Rt/L} + C}$$

Put $t = 0$, $i = 0$

$$0 = \frac{E}{R} + C \quad \therefore \boxed{C = -\frac{E}{R}}$$

$$\therefore i e^{Rt/L} = \frac{E}{R}\left(e^{Rt/L}\right) - \frac{E}{R}$$

$$i = \frac{E}{R} \frac{\left(e^{Rt/L} - 1\right)}{e^{Rt/L}}$$

$$\boxed{i = \frac{E}{R}\left(1 - e^{-Rt/L}\right)}$$

2) $R = 100\,\Omega$ & $L = 0.5\,H$, $E = 20V$, If $\boxed{L\frac{di}{dt} + Ri = E}$

$\underline{\text{Sol}^n}$ Solve & Find answer (Same as above)

$$i = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$$

$$\boxed{i = (0.2)\left(1 - e^{-200t}\right)}$$