MODULE 3 : POLARIZATION

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\# Waves are basically of 2 types.

(i) Longitudinal waves :- A wave in which particles of the medium oscillate to & fro along the direction of propagation.

(ii) Transverse waves :- A wave in which every particle of the medium oscillates up & down at right angles to the direction of wave propagation.

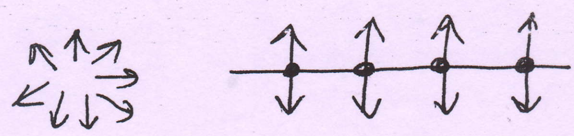
\# Unpolarised light : The ordinary light consists of very large number of vibrations in all planes with equal probability at right angles to the direction of propagation.

\# Plane Polarised light : In plane polarized light the vibrations are along a straight line in a plane ⊥ to the direction of propagation.

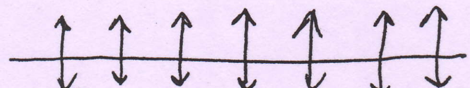
If the direction of vibration is parallel to the plane of paper, it is represented by a straight line arrow. Fig 1.(a).

If the direction of vibration is perpendicular to the plane of the paper, it is represented by a dot. Fig. 1 (b)

(i). Unpolarized light :

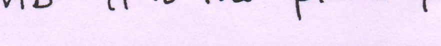


1.a. (ii) Plane polarized light :



(Vib^n ∥ to the plane of paper)

1.b. (iii) Plane polarized light :



(Vib^n ⊥ to the plane of paper)

Unpolarized light	Polarized light.
1. Consists of waves with planes of vibrations equally distributed in all directions about the ray direction.	1. Consists of waves having the electric field vector vibrating in a single plane normal to ray direction.
2. Symmetrical about the ray dir^n.	2. Asymmetrical about the ray direction.
3. Produced by conventional light sources	3. Is to be obtained from unpolarized light with the help of polarizers.
4. May be regarded as the resultant of 2 incoherent waves of equal intensity but polarized in mutually ⊥ planes.	4. May be regarded as the resultant of two mutually ⊥ coherent waves having zero phase difference.

PRODUCTION OF PLANE POLARIZED LIGHT

Plane polarized light may be produced from unpolarized light using the following optical phenomena.

(i) Polarization by reflection.

(ii). By double refraction.

Method 1: Polarization by reflection (Brewester's law)

The simplest way of producing a plane polarized light is by reflection.

When ordinary light is reflected from the surface of a transparent medium like glass or water it becomes partially polarized. The degree of polarization changes with the angle of incidence. At a particular angle of incidence the reflected light has the greatest percentage of polarised light, whereas the angle depends upon the nature of the reflecting surface.

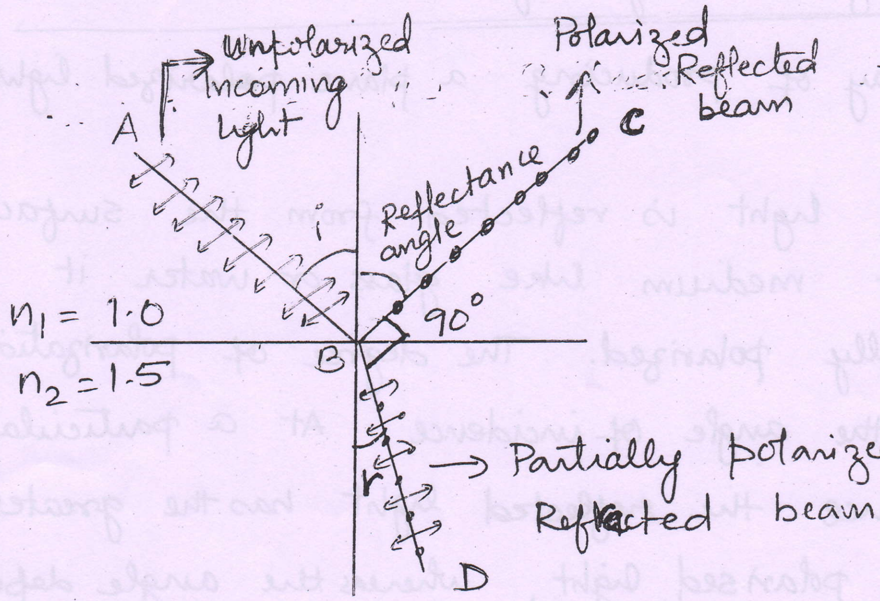
The angle of incidence is known as angle of polarization.

Brewester observed that for a particular angle of incidence known as angle of polarization, the reflected light is completely polarized in the in the plane of incidence. i.e having plane of vibration perpendicular to the plane of incidence.

Brewester proved that the tangent of the angle of polarization (P) is numerically equal to the refractive index (μ) of the medium.

i.e $\boxed{\mu = \tan P}$

This is known as **Brewester's law**. He also proved that the reflected & refracted rays are perpendicular to each other.



If a natural light is incident on a smooth surface at the polarizing angle, it is reflected along BC & refracted along BD. Brewester found that the maximum polarization of reflected ray occurs when it is at right angles to the refracted rays.

i.e $i + r = 90°$.

According to Snell's law,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

where μ_2 : Refractive index of the reflecting surface and μ_1: refractive index of the surrounding medium.

$$\frac{\sin i}{\sin(90-i)} = \frac{\mu_2}{\mu_1} = \frac{\sin i}{\cos i} = \frac{\mu_2}{\mu_1}$$

$$\therefore \tan i = \frac{\mu_2}{\mu_1} \Rightarrow \boxed{\tan p = \frac{\mu_2}{\mu_1}}$$

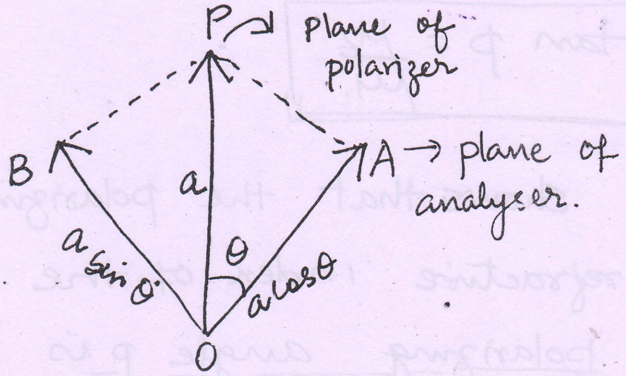
$\therefore$ The above equation shows that the polarizing angle depends on the refractive index of the reflecting surface. The polarizing angle p is known as Brewester's angle. Light reflected from any angle other than Brewester angle is partially polarized.

MALUS'S LAW

According to Malus, when a completely plane polarised light beam is incident on the analyser, the intensity of the polarized light transmitted through the analyser varies as the square of the cosine of the angle b/w the plane of transmission of the analyser & the plane of polariser.

Proof: Let $OP = a$ be the amplitude of the incident plane polarized light from a polarizer and θ, the angle b/w the planes of polarizer & analyser. The amplitude of incident plane polarized light can be resolved in two components, one parallel to the plane of transmission of analyser ($a\cos\theta$) and the other perpendicular to it ($a\sin\theta$). The component $a\cos\theta$ is transmitted through the analyser.

(3)



Intensity of the transmitted light through the analyser

$$I_\theta = (a\cos\theta)^2 = a^2\cos^2\theta.$$

If I be the intensity of incident polarized light, then

$$I = a^2$$

$$I_\theta = I\cos^2\theta \qquad \therefore \quad I_\theta \propto \cos^2\theta.$$

(i) When $\theta = 0$, i.e the two planes are parallel

$$I_\theta = 1 \text{ as } \cos\theta = 1.$$

(ii) when $\theta = \frac{\pi}{2}$, i.e the two planes are perpendicular

$$I_\theta = 0.$$

(iii) when $\theta = \pi$, the axes are parallel.

$$I_0 = \frac{I_0}{2}$$

(iv) when $\theta = 270^\circ$: the axes are perpendicular

$$\therefore I = 0.$$

Thus, we obtain two positions of maximum intensity and two positions of zero intensity when we rotate the axis of the analyzer with respect to that of the polarizer.

DOUBLE REFRACTION

Erasmus (1869) discovered that when a beam of ordinary unpolarised light is passed through a calcite crystal, the refracted light is split up into two refracted rays. The one which always obeys the ordinary laws of refraction + having vibrations ⊥ to the principal section is known as ordinary rays. The other ray in general doesn't obey the laws of refraction and having vibrations in the principal ~~axis~~ section is called as extraordinary rays. Both the rays are plane-polarized. This phenomenon is known as DOUBLE REFRACTION OR BIREFRINGENCE.

The crystal showing this phenomenon is known as Doubly refracting crystal. there are 2 types of doubly refracting crystals.

i) Uniaxial : In uniaxial crystals, there is only one direction (optic axes) along which the two refracted rays travel with the same velocity.

eg : Tourmaline, Calcite & Quartz crystal

(ii) Biaxial : In Biaxial crystals, there are two such directions along which the velocities are same.

Eg : Topaz & Aragonite etc.

POLARIZER AND ANALYZER.

*. Polarizer: It is an optical instrument, which utilizes the phenomenon of selective absorption or double refraction and transforms unpolarized light into polarized light. Plane polarized light is obtained by eliminating one of the two components in the unpolarized light.

When natural light is incident on a polarizer, the E-field component that is parallel to the chains of Iodine atoms induces current in the conducting chains and is therefore strongly absorbed. Hence, the light transmitted contains only the component that is perpendicular to the direction of molecular chains.

Effect of Polarizer on natural light

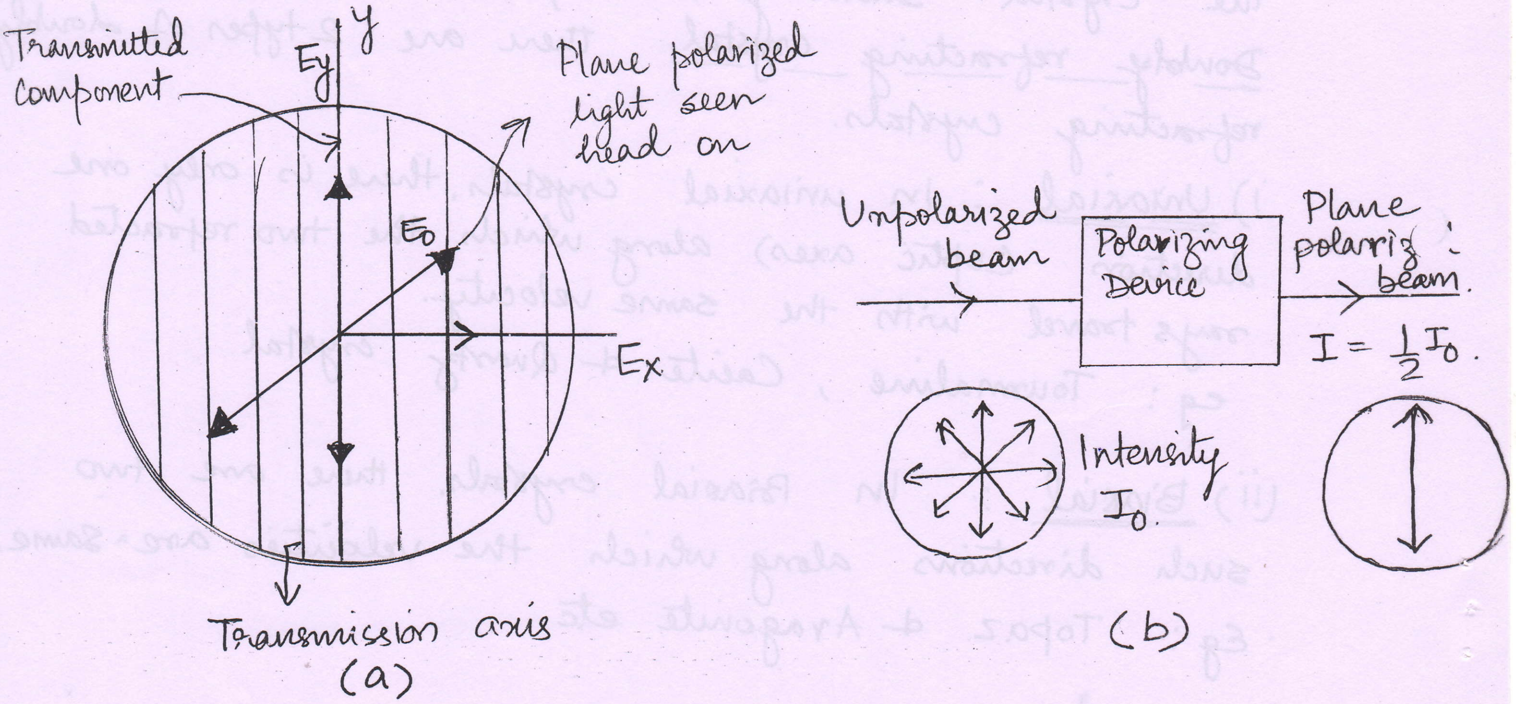


Figure: (a) Action of polarizer on linearly polarized wave.
(b) The intensity of an unpolarized beam reduces to half after passing through a polarizer.

* Action of polarizer on the incident unpolarized light.

If an unpolarized light is incident on a polarizer with electric field vector E_0 making an angle θ w.r.t the transmission axis of the polarizer. Then, E_0 may be resolved into its component vectors lying ∥ and ⊥ to the transmission axis of the polarizer. i.e E_y ∥ to the axis and E_x ⊥ to the transmission axis. The polarizer transmits the parallel component while blocking the perpendicular component.

As $E_y = E_0 \cos\theta$

Hence, intensity of the transmitted components is given by

$$I \propto E_y^2 = E_0^2 \cos^2\theta$$

In unpolarized light all the values of θ are equally probable. Therefore, the fraction of light transmitted through the polarizer equals the average value of $\cos^2\theta$, which is equal to $1/2$.

Thus, $I = E_0^2/2 = I_0/2$.

* Analyser :— It is an optical element, which is used to identify the plane of vibration of plane polarized light. It is not different in structure from the polarizer. Only its working differs.

NICOL'S PRISM.

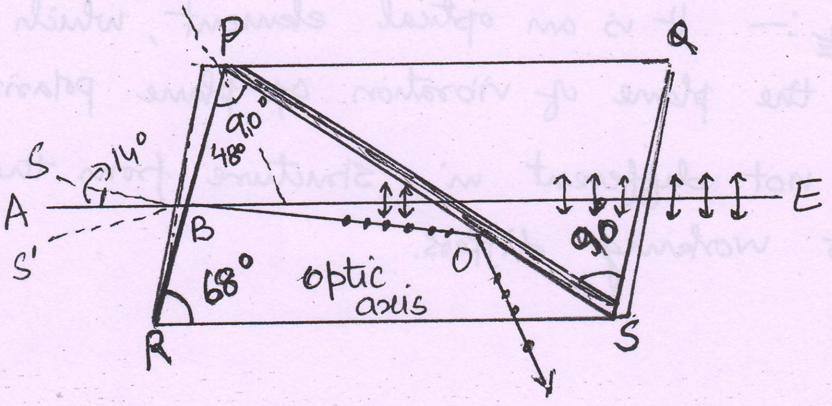
Principle:— It's a device to produce plane polarized light. It is known that when an ordinary line ~~pass~~ is transmitted through a calcite crystal, it splits in o-ray and e-ray which are completely plane polarized with vibrations in two mutually ⊥ planes.

If one beam is eliminated then the emergent beam from the calcite crystal will be plane-polarised light.

In 1828, Nicol eliminated the ordinary beam by utilizing the phenomenon of total reflection at thin film of canada balsam separating the two pieces of calcite.

The device is known as Nicol's prism.

Construction: A calcite crystal with length 3 times its width is taken. The end faces are grounded such that the angles in the principal section becomes 68° and 112° instead of 71° and 109°.



The crystal is cut into two pieces by a plane ⊥ to the principal section as well as the end faces PR and QS. The two cut surfaces are grounded and polished optically flat and then, cemented together by

The refractive index of canada Balsam lies b/w the refractive indices for the ordinary and extra-ordinary rays for calcite. For sodium D lines, the values are given below:

Refractive index for ordinary $\mu_0 = 1.6588$

Refractive index for canada balsam $\mu = 1.55$

Refractive index for extraordinary $\mu_e = 1.486$.

Action: When a beam of light AB enters the faces PR in direction parallel to the long side, it is doubly refracted into ordinary plane polarised beam BO and extra-ordinary plane polarised beam BE. It is clear that canada balsam layer acts a a rarer medium for an o-ray and denser medium for e-ray. The dimensions of the crystal are chosen such that the angle of incidence for o-ray at the CB surface becomes greater than the corresponding critical angle 69°.

Under these conditions, the o-ray is completely reflected at calcite-balsam surface and is absorbed by the tube containing the Nicol's prism. The e-ray is not totally reflected because it is travelling from a rarer to a denser medium and is thus transmitted with no-appreciable loss in intensity. It is slightly displaced laterally but emerges out of prism parallel to its original direction. Thus, only e-ray is transmitted. Since, e-ray is plane polarised having vibrations parallel to principal plane, the light emerging from the Nicol's prism is plane polarised.

Uses: Nicol's prism can be used as both polariser and an analyser.

→ When two Nicols are arranged co-axially, then, the first Nicol which produces plane polarised light is known as polariser while the second which analyses the polarised light is known as analyser.

→ When the Nicols are placed with their principal sections parallel to each other as shown in Fig (a) then the e-ray transmitted by one is freely transmitted by the other. If the second prism is gradually rotated, then the intensity of e-ray gradually decreases.

→ When the two Nicol's are at right angle to each other (Fig. b) i.e, they are in a crossed position, no light comes from second prism.

This is due to the fact that when the polarised e-rays enters the second Nicol's prism, it acts as o-ray & is totally internally reflected. Therefore, the first Nicol's prism N_1 produces plane polarised light & the second Nicol's prism N_2 detects it.

Fig (a):

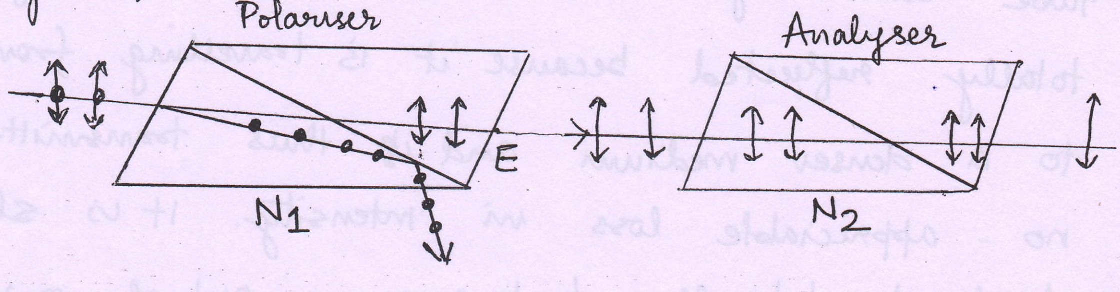
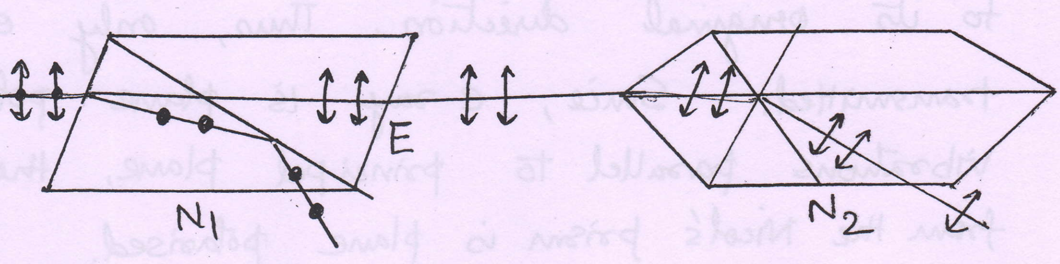


Fig (b):

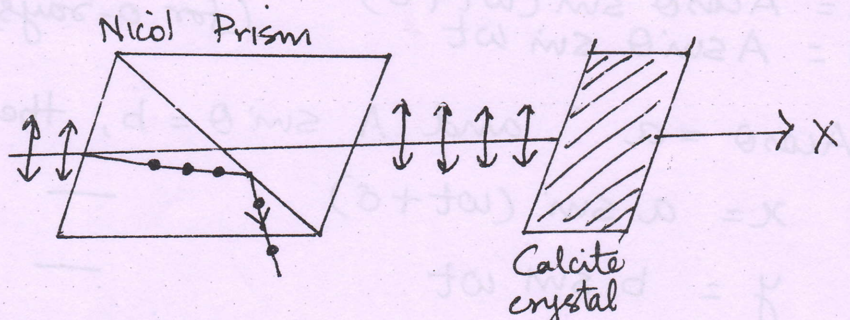


ELLIPTICALLY AND CIRCULARLY POLARISED LIGHT.

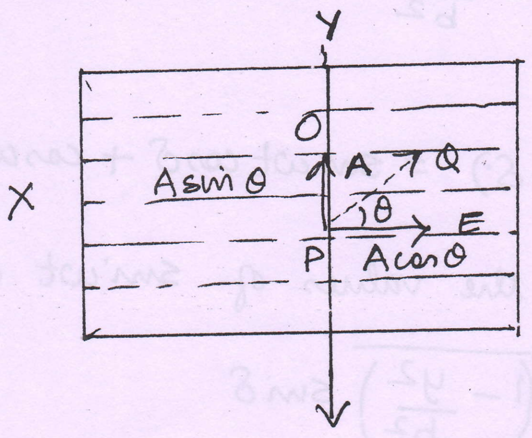
Consider a beam of plane polarized light falling normally on a calcite crystal cut with its optic axis parallel to its faces. Fig (a).

Let $A = PQ$ (b) be the maximum amplitude of incident light which makes angle θ with optic axis. The plane polarized light on entering the crystal is split up in two components o-ray and e-ray i.e. the amplitude A of incident plane polarized light is divided into two parts.

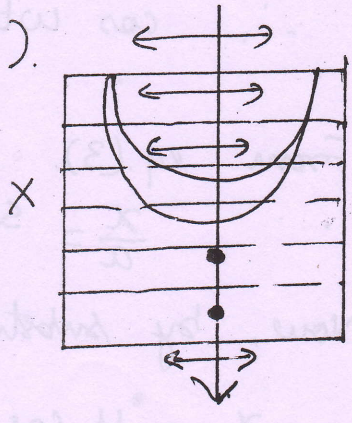
(a)



(b).



(c).



(i) The amplitude of o-rays (vib^n $\perp$ to optic axis) as $A\sin\theta$ along PO.

(ii) The amplitude of e-rays (vib^n along optic axis) as $A\cos\theta$ along PE

From the theory of double refraction, the o-ray and e-rays thus produced, traverses in the crystal in the same direction but with different velocities as in Fig.(c). On emerging from the crystal they have a phase difference (δ) depending upon the thickness of crystal.

Thus, we have 2 waves / 2 simple harmonic motions having amplitudes $A\cos\theta$ and $A\sin\theta$, vibrating along $\perp$ directions (along and $\perp$ to optic axis) and having a phase difference δ, depending upon the thickness of the crystal.

The equations for such waves can be written as

$$x = A\cos\theta \sin(\omega t + \delta) \quad \text{(for e-rays)} \quad — (1)$$
$$y = A\sin\theta \sin\omega t \quad \text{(for o-rays)} \quad — (2)$$

Let $A\cos\theta = a$ and $A\sin\theta = b$, then,

$$x = a\sin(\omega t + \delta) \quad — (3)$$
$$y = b\sin\omega t \quad — (4)$$

From eq. 4; $\sin\omega t = y/b$

$$\therefore \cos\omega t = \sqrt{1 - \frac{y^2}{b^2}}$$

From eq.(3).

$$\frac{x}{a} = \sin(\omega t + \delta) = \sin\omega t\cos\delta + \cos\omega t\sin\delta$$

Now, by substituting the values of $\sin\omega t$ and $\cos\omega t$

$$\frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{\left(1 - \frac{y^2}{b^2}\right)}\sin\delta$$

or $$\frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{\left(1 - \frac{y^2}{b^2}\right)}\sin\delta.$$

Now, squaring both the sides, we get,

$$\frac{x^2}{a^2} - 2\frac{xy}{ab}\cos\delta + \frac{y^2\cos^2\delta}{b^2} = \left(1-\frac{y^2}{b^2}\right)\sin^2\delta$$

or $$\frac{x^2}{a^2} - 2\frac{xy}{ab}\cos\delta + \frac{y^2}{b^2}(\cos^2\delta + \sin^2\delta) = \sin^2\delta$$

or $$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta. \quad \text{— (5)}$$

This is the general equation of ellipse.

Special Cases :

1.) When $\delta = 0°$; $\sin\delta = 0$ and $\cos\delta = 1$.

∴ From eq. 5

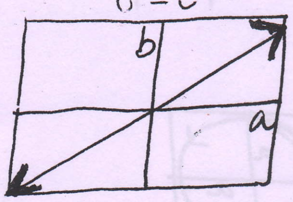
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

$$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0 \Rightarrow \frac{x}{a} = \frac{y}{b}$$

and $$\boxed{y = \frac{bx}{a}} \quad \text{— (6)}$$

This is the eq^n of straight line. Therefore, the light will be plane polarised light with vibrations in the same plane as in incident light as in Fig 1. (a)

$\delta = 0$



2) when $\delta = (2n+1)\frac{\pi}{2}$ where $n = (0, 1, 2, 3, \ldots)$

$\therefore \delta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

Here, eq^n (5) reduces to

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \quad \text{— (7.)}$$

(as $\sin\delta = 1$; $\cos\delta = 0$.)

This represents the equation of a symmetrical ellipse. Thus, the emergent light in this case will be elliptically polarised as in Fig 1(b) and (d)

Fig. 1(b)

$\delta = \pi/2$

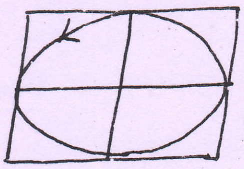
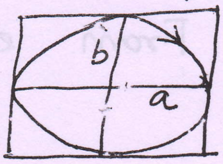


Fig 1(d).

$\delta = 3\pi/2$



3) When $\delta = \frac{\pi}{2}$ and $a = b$, then eq^n 5 becomes

$$x^2 + y^2 = a^2. \quad \text{— (8).}$$

This represents the equation of a circle. Thus, the emergent light will be circularly polarised. This happens when $\theta = 45°$, i.e, the incident plane polarised light on the crystal makes an angle of $45°$ with the direction of optic axis. Fig 1(f) and (g)

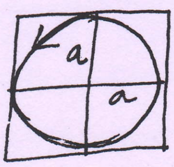


Fig 1.(f)

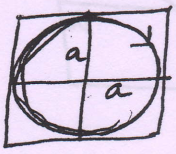


Fig. 1(g).

4) For all other values of δ, the nature of vibrations will be as shown in the above Figures.

Thus, in the polarised light:

(i) If the light vector vibrates along a straight line, it will be a plane polarised light.

(ii) If the light vector rotates along a circle i.e doesn't changes the magnitude, but traces a circular path while rotating (thus, circularly polarised).

(iii) If light vector rotates along ellipse i.e. changes in magnitude while rotating, it will be elliptically polarised. (i.e the path traversed by the light is an ellipse)