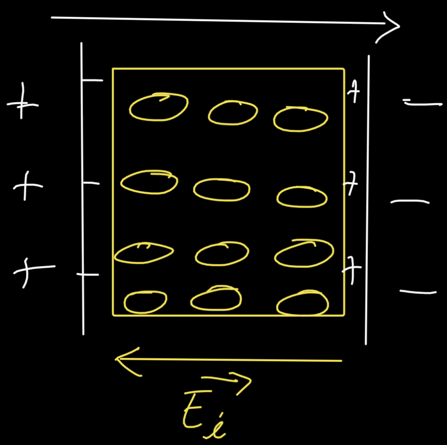
Relation b/w $\vec{D}$, $\vec{E}$ and $\vec{P}$



$$E_0 = \frac{\sigma}{\varepsilon_0} \longrightarrow (1)$$

$$E_i = \frac{\sigma_p}{\varepsilon_0} \longrightarrow (2)$$

$$E = E_0 - E_i$$

$$E = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_p}{\varepsilon_0}$$

$$E = \frac{\sigma - \sigma_p}{\varepsilon_0}$$

$$\varepsilon_0 E = \sigma - \sigma_p$$

$$\varepsilon_0 E = D - P$$

$$D = \varepsilon_0 E + P$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

Relation between ε_r & χ

$\because D = \varepsilon_0 E + P$

$\therefore P = D - \varepsilon_0 E$

$$P = \underbrace{D}_{\sigma} - \varepsilon_0 E$$

$P = \sigma - \varepsilon_0 E \longrightarrow (1)$

$\because E = \dfrac{\sigma}{\varepsilon_0 \varepsilon_r}$

$\therefore \sigma = \varepsilon_0 \varepsilon_r E \longrightarrow (2)$

from (1) and (2)

$P = \varepsilon_0 \varepsilon_r E - \varepsilon_0 E$

$P = \varepsilon_0 E (\varepsilon_r - 1) \rightarrow (3)$

$P = \chi \varepsilon_0 E \rightarrow (4)$

Compare (3) & (4)

$\varepsilon_0 E (\varepsilon_r - 1) = \chi \varepsilon_0 E$

$\therefore \chi = \varepsilon_r - 1$

$$\varepsilon_r = \chi + 1$$

⊙ Polarizability (α) :-

dipole moment

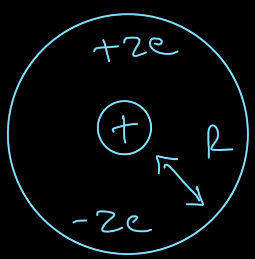
$$\mu \propto E$$

$$\mu = \underbrace{\alpha}_{\text{Polarizability}} E$$

$$\boxed{\alpha = \frac{\mu}{E}}$$

⊙ Expression for electronic polarizability (α) :-

1) Consider an atom as shown below.



z = atomic number

$\therefore$ charge on nucleus $= +ze$

charge due to electron $= -ze$

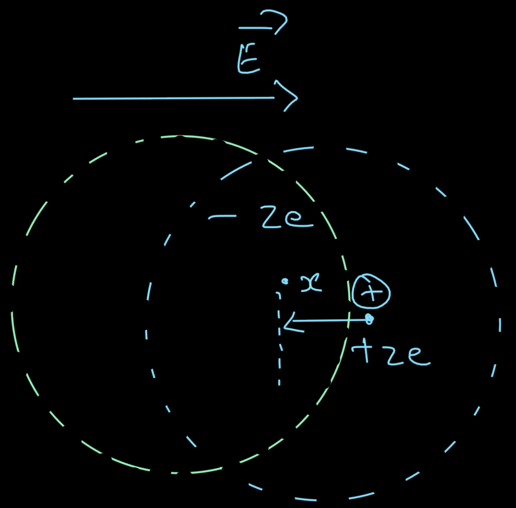
Let R = atomic radius

$\therefore$ Volume charge density (δ) $= \dfrac{(-ze)}{\left(\frac{4}{3}\right)\pi R^3}$

$$\rho = -\left(\frac{3}{4}\right)\frac{Ze}{\pi R^3} \longrightarrow (1)$$

2) When it is placed in an external electric feild $\vec{E}$, then Lorentz force.

$$\vec{f_2} = -(Ze)(E) \longrightarrow (2)$$



Due to this the electron cloud will shift.

Let it shift by distance 'x' and charge on electron cloud is Q.

$$\therefore Q = \begin{bmatrix}\text{Volume charge density } (\rho)\end{bmatrix}\begin{bmatrix}\text{Volume of sphere having radius } x\end{bmatrix}$$

$$Q = \left[-\cancel{\left(\frac{3}{4}\right)}\frac{Ze}{\cancel{\pi}R^3}\right]\left[\cancel{\frac{4}{3}}\cancel{\pi}x^3\right]$$

$$Q = (-Ze)\frac{x^3}{R^3} \longrightarrow (3)$$

R

③ ∴ Coulomb force

$$f_c = \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{\boxed{(Q)}\,(Ze)}{x^2} \longrightarrow \text{from ③}$$

$$f_c = \left(\frac{1}{4\pi\varepsilon_0}\right) \left[(-Ze)\frac{x^{\cancel{3}}}{R^3}\right] \frac{Ze}{\cancel{x^2}}$$

$$f_c = \left(\frac{1}{4\pi\varepsilon_0}\right) \left[(-Ze)\frac{x}{R^3}\right] (Ze) \longrightarrow ④$$

④ At eq^{llm}

$$f_L = f_c$$

$$(-Ze)(E) = \left(\frac{1}{4\pi\varepsilon_0}\right) \left[(-Ze)\frac{x}{R^3}\right] (Ze)$$

$$x = \frac{(4\pi\varepsilon_0)(R^3)\,E}{Ze}$$

$$\therefore \mu = (Ze)\,(x)$$

$$\mu = \frac{(\cancel{Ze})(4\pi\varepsilon_0)(R^3)(E)}{(\cancel{Ze})}$$

$$\mu = (4\pi\varepsilon_0)(R^3)E$$

$\because \mu = \alpha E$

$$\boxed{\therefore \alpha = (4\pi\varepsilon_0)(R^3)}$$

Expression for ε_r:-

$$\varepsilon_r = 1 + \chi \longrightarrow (1)$$

$$\because P = \chi \varepsilon_0 E$$

$$\therefore \chi = \frac{P}{\varepsilon_0 E} \longrightarrow (2)$$

(2) $\rightarrow$ (1), $$\varepsilon_r = 1 + \frac{P}{\varepsilon_0 E} \longrightarrow (3)$$

$$\because P = N\alpha E \longrightarrow (4)$$

(4) $\rightarrow$ (3), $$\varepsilon_r = 1 + \frac{N\alpha \cancel{E}}{\varepsilon_0 \cancel{E}}$$

$$\boxed{\varepsilon_r = 1 + \frac{N\alpha}{\varepsilon_0}}$$

Clausius Mosotti Equation:-

$$P = N\alpha E$$

$$\alpha_e = \frac{P}{NE_i} = \frac{P}{N\left[E + \frac{P}{3\varepsilon_0}\right]} \longrightarrow (1)$$

$$P = \varepsilon_0 \chi E = \varepsilon_0 (\varepsilon_r - 1) E$$

$$\therefore E = \frac{P}{\varepsilon_0(\varepsilon_r - 1)} \longrightarrow (2)$$

$$\alpha_e = \frac{P}{N\left[\frac{P}{\varepsilon_0(\varepsilon_r-1)} + \frac{P}{3\varepsilon_0}\right]}$$

$$\frac{N\alpha_e}{\varepsilon_0} = \frac{1}{\left[\frac{1}{\varepsilon_r - 1} + \frac{1}{3}\right]} = \frac{1}{\left[\frac{\varepsilon_r+2}{3\varepsilon(r-1)}\right]}$$

$$\boxed{\frac{3(\varepsilon_r - 1)}{\varepsilon_r + 2} = \frac{N\alpha_0}{\varepsilon_0} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2}}$$