Practice problems.

Friday, February 19, 2021 10:35 AM

1) $y\frac{dx}{dy} = x - yx^2\cos y$ ✓ $\longrightarrow$ $y = \cancel{(x - yx^2\cos y)}\frac{dy}{dx}$

2) $\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^3$ ✓

1) $\boxed{y}\frac{dx}{dy} - x = -yx^2\cos y$

Divide by yx^2

$$\boxed{\frac{1}{x^2}\frac{dx}{dy} - \frac{1}{y}\frac{1}{x} = -\cos y}$$

Then put $v = \frac{1}{x}$

$$\frac{dv}{dy} = -\frac{1}{x^2}\frac{dx}{dy}$$

$$\boxed{-\frac{dv}{dy} = \frac{1}{x^2}\frac{dx}{dy}}$$

$\therefore -\frac{dv}{dy} - \frac{1}{y}v = -\cos y$

Cancel -1

$$\boxed{\frac{dv}{dy} + \frac{1}{y}v = \cos y}$$

linear in v

$P = \frac{1}{y}$, $Q = \cos y$

$e^{\int P dy} = e^{\int \frac{1}{y}dy} = y$

Then solⁿ is given by,

$$v e^{\int P dy} = \int Q e^{\int P dy} dy + C$$

$$v y = \int \underset{v}{\cos y} \cdot \underset{u}{y}\, dy + C$$

$$= y\sin y - \int \sin y\, dy + C$$

$$\frac{1}{x}y = y\sin y + \cos y + C$$

$$\boxed{\frac{y}{x} = y\sin y + \cos y + C}$$ ✓

2) Divide by $z(\log z)^3$

$$\frac{1}{z(\log z)^3}\frac{dz}{dx} + \frac{1}{x}\boxed{\frac{1}{(\log z)^2}} = \frac{1}{x^2} \quad (1)$$

put $\frac{1}{(\log z)^2} = t$

$$-\frac{2}{(\log z)^3}\left(\frac{1}{z}\right)\frac{dz}{dx} = \frac{dt}{dx}$$

Here

$P = -\frac{2}{x}$, $Q = -\frac{2}{x^2}$

$$e^{\int P dx} = e^{\int -\frac{2}{x}dx}$$

$$= e^{\log x^{-2}}$$

$$= \frac{1}{x^2}$$

solⁿ is given by

$$-\frac{2}{(\log z)^3}\left(\frac{1}{z}\right)\frac{dz}{dx} \quad dx$$

$$\therefore \frac{1}{2(\log z)^3}\frac{dz}{dx} = -\frac{1}{2}\frac{dt}{dx} \quad \text{put in (1)}$$

$$\therefore -\frac{1}{2}\frac{dt}{dx} + \frac{1}{x}t = \frac{1}{x^2}$$

Coefficient should be 1

$$\boxed{\frac{dt}{dx} - \frac{2}{x}t = -\frac{2}{x^2}}$$

linear in t

Sol^n is given by

$$t\,e^{\int P\,dx} = \int Q\,e^{\int P\,dx}\,dx + c$$

$$\boxed{\frac{1}{x^2(\log z)^2} = +\frac{2}{3}x^{-3} + C}$$

How to identify

3 terms (linear / reducible)

✓ linear $\quad y/x$

✓ reducible

Not 3 terms (exact / I.F. 4 rules)

✓ exact

↳ Rules:

1) Homogeneous
2) $y\,dx + x\,dy = 0$
3) Divide by $\frac{M}{N}$

Mixed Problems

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1) $x\frac{dy}{dx} + y\log y = xye^x$

2) $y\,dx - x\,dy + \log x\,dx = 0$ (HW)

3) $(2x^3y+3)\,dy + (3x^2y^2+2x)\,dx = 0$ (HW)

4) $\cos x\frac{dy}{dx} + y\sin x = \sqrt{y\sec x}$ $\cos x$

5) $(y+e^y-e^{-x})\,dx + x(1+e^y)\,dy = 0$

6) $\left[2x\sinh\left(\frac{y}{x}\right) + 3y\cosh\left(\frac{y}{x}\right)\right]dx - 3x\cosh\left(\frac{y}{x}\right)dy = 0$

7) $\sin y\frac{dy}{dx} = (1-x\cos y)\cos y \longrightarrow \sin y\frac{dy}{dx} = \cos y - x\cos^2 y$

8) $\left(\frac{\log(\log y)}{x} + \frac{2}{3}xy^3\right)dx + \left[\frac{\log x}{y\log y} + x^2y^2\right]dy = 0$

Sol^n:

1) $\boxed{x}\frac{dy}{dx} + y\log y = x\boxed{y}e^x$

Divide by xy

$$\boxed{\frac{1}{y}\frac{dy}{dx}} + \frac{1}{x}\boxed{\log y} = e^x$$

put $v = \log y$

$$\frac{dv}{dx} = \frac{1}{y}\frac{dy}{dx}$$

$$\boxed{1\frac{dv}{dx} + \frac{1}{x}v = e^x}$$

linear in v

$P = \frac{1}{x}$, $Q = e^x$

$\therefore e^{\int P\,dx} = e^{\int \frac{1}{x}dx} = x$

Sol^n is given by,

$$ve^{\int P\,dx} = \int Q e^{\int P\,dx}dx + C$$

$$x\,dy + (y\log y - xye^x)\,dx = 0$$

N $\quad M$

$$\frac{\partial M}{\partial y} = y\left(\frac{1}{y}\right) + \log y(1) - xe^x(1)$$

$$= 1 + \log y - xe^x$$

$$\frac{\partial N}{\partial x} = 1$$

Divide by M

$$\left(\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}\right) = \frac{-\log y + xe^x}{y(\log y - xe^x)} = -\frac{1}{y}$$

$$e^{\int g(y)\,dy} = e^{\int -\frac{1}{y}dy} = \frac{1}{y}$$

$\therefore$ multiply by $\frac{1}{y}$

$$\frac{x}{y}\,dy + (\log y - xe^x)\,dx = 0$$

Sol^n is given by

$$ve^{\int P\,dx} = \int Q e^{\int P\,dx}\,dx + C$$

$$vx = \int e^x x\,dx + C$$

$$\boxed{x \log y = e^x(x-1) + C}$$

$$\boxed{x \log y - e^x(x-1) = C}$$

$$\underbrace{\frac{x}{y}}_{N}\,dy + \underbrace{(\log y - xe^x)}_{M}\,dx = 0$$

is exact eq^n

Sol^n is given by,

$$\int_{y\ \text{const}} M\,dx + \int_{\text{free of } x} N\,dy = C$$

$$\int (\log y - xe^x)\,dx + 0 = C$$

$$\therefore \boxed{x \log y - e^x(x-1) = C}$$

2) $\cos x \dfrac{dy}{dx} + y \sin x = \boxed{\sqrt{y}}\sec x$

Divide by $\cos x \sqrt{y}$

$$\frac{1}{\sqrt{y}}\frac{dy}{dx} + \frac{1}{\sqrt{y}}\tan x = (\sqrt{\sec x})(\sec x)$$

$$\boxed{\frac{1}{\sqrt{y}}\frac{dy}{dx}} + \tan x \boxed{\frac{1}{\sqrt{y}}} = (\sec x)^{3/2}$$

$$v = \frac{1}{\sqrt{y}}, \quad \frac{dv}{dx} = \frac{1}{2\sqrt{y}}\frac{dy}{dx} \Rightarrow 2\frac{dv}{dx} = \frac{1}{y}\frac{dy}{dx}$$

$$\cancel{2}\frac{dv}{dx} + \frac{\tan x}{2} v = \frac{(\sec x)^{3/2}}{2}$$

exact

Homogeneous!

6) $Mx + Ny = 2x^2 \sinh\left(\frac{y}{x}\right) + 3xy \cancel{\cosh}\left(\frac{y}{x}\right) - 3xy \cancel{\cosh}\left(\frac{y}{x}\right)$

$$= 2x^2 \sinh\left(\frac{y}{x}\right)$$

$$\therefore \text{IF} = \frac{1}{Mx+Ny} = \frac{1}{2x^2 \sinh\left(\frac{y}{x}\right)}$$

multiply to eq^n (1)

$\therefore \left[\frac{1}{x} + \frac{3y}{2x^2}\frac{\cosh(y/x)}{\sinh(y/x)}\right]dx + -\frac{3}{2x}\frac{\cosh(y/x)}{\sinh(y/x)}$ exact D.E.

Sol^n:

$$\underset{x\text{ const}}{\int N\,dy} + \underset{\text{free of } y}{\int M\,dx} = C$$

$$-\frac{3}{2}\int \frac{\frac{1}{x}\cosh(y/x)}{\sinh(y/x)}\,dy + \int \frac{1}{x}\,dx = C$$

put $\sinh\left(\frac{y}{x}\right) = t$, $\frac{1}{x}\cosh\left(\frac{y}{x}\right)dx = dt$

$$-\frac{3}{2}\int \frac{dt}{t} + \log x = C$$

$$-\frac{3}{2}\log\left(\sinh\left(\frac{y}{x}\right)\right) + \log x = \log c'$$

$$\boxed{x\,\sinh\left(\frac{y}{x}\right)^{-3/2} = c'}$$ Final ans.

If tackeled by Divide by N

$$\log\left(\sinh\frac{y}{x}\right) - \frac{2}{3}\log x = -\frac{2}{3}\log c'$$

$$\boxed{x^{-2/3}\sinh\left(\frac{y}{x}\right) = C''}$$