Lasers

The acronym Laser stands for Light amplification by stimulated emission of Radiation.

$$N = N_0 e^{-E/KT}$$

$$E_1 < E_2$$

$$N_1 = N_0 \, e^{-E_1/KT}$$

$$N_2 = N_0 \, e^{-E_2/KT}$$

$$\frac{N_1}{N_2} = \frac{e^{-E_1/KT}}{e^{-E_2/KT}} = e^{(E_2-E_1)/KT}$$

$$N_1 > N_2$$

$$N_{ab} \propto (N_1)(\theta)(\Delta t)$$

$$N_{ab} = \underbrace{B_{12}}_{E.C} (N_1)\,(\theta)(\Delta t)$$

$$\boxed{N_{sp} = A_{21} N_2 \Delta t}$$

⇒ when atom is presented in the excited state and when the quanta or photon is incident on it then it force the atom to come to the lower state. Due to this two identical photons are obtained (incident & another due to emission). This process is called stimulated emission.

⊙ Derivation of Eienstein coefficient :-

$$N_{ab} = B_{12} N_1 \theta \Delta t \quad \text{---} \quad (1)$$

$$N_{sp} = A_{21} N_2 \Delta t \quad \text{---} \quad (2)$$

$$N_{st} = B_{21} N_2 \theta \Delta t \quad \text{---} \quad (3)$$

upward transition = downward transition.

$$N_{ab} = N_{sp} + N_{st} \quad \text{---} \quad (4)$$

from eq^n (1), (2), (3), & (4)

$$B_{12} N N_1 \theta \Delta t = A_{21} N_2 \Delta t + B_{21} N_2 \theta \Delta t$$

$$B_{12} N_{12} \theta = A_{21} N_2 + B_{21} N_2 \theta$$

$$\theta (B_{12} N_1 - B_{21} N_2) = A_{21} N_2$$

$$\theta = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

dividing by $B_{21} N_2$

$$\theta = \frac{A_{21} N_2 / B_{21} N_2}{\frac{B_{12} N_1}{B_{21} N_2} - \frac{B_{21} N_2}{B_{21} N_2}}$$

$$\theta = \left(\frac{A_{21}}{B_{21}}\right) \frac{1}{\frac{B_{12} N_1}{B_{21} N_2}} \quad \text{———} \quad (5)$$

$$N_1 = N_0 e^{-E_1/KT}$$

$$N_2 = N_0 e^{-E_2/KT}$$

$$\therefore \frac{N_1}{N_2} = e^{(E_2 - E_1)/KT}$$

$$\because E_2 - E_1 = h\gamma$$

$$\frac{N_1}{N_2} = e^{h\gamma/KT} \quad \text{———} \quad (6)$$

from eq^n (5) and (6)

$$\theta = \left(\frac{A_{21}}{B_{21}}\right) \frac{1}{\left[\left(\frac{B_{12}}{B_{21}}\right)\left(e^{h\gamma/KT}\right)\right] - 1} \quad \text{———} \quad (7)$$

from black body Radiation

$$Q = \left(\frac{8\pi h\nu^3}{c^3}\right) \frac{1}{\left[e^{h\nu/KT}\right]} \quad \text{—— (8)}$$

from (7) and (8)

$$\boxed{\frac{A_{21}}{A_{21}} = \frac{8\pi h\nu^3}{c^3}} \quad \& \quad \boxed{\frac{B_{12}}{B_{21}} = 1 \Rightarrow B_{12} = B_{21}}$$

Note:-

(1)

$$\frac{N_{ab}}{N_{st}} = \frac{B_{12} N_1 Q \Delta t}{B_{21} N_2 Q \Delta t}$$

$$\because B_{12} = B_{21}$$

$$\frac{N_{ab}}{N_{st}} = \frac{N_1}{N_2}$$

$$\frac{N_{st}}{N_{ab}} = \frac{N_2}{N_1}$$

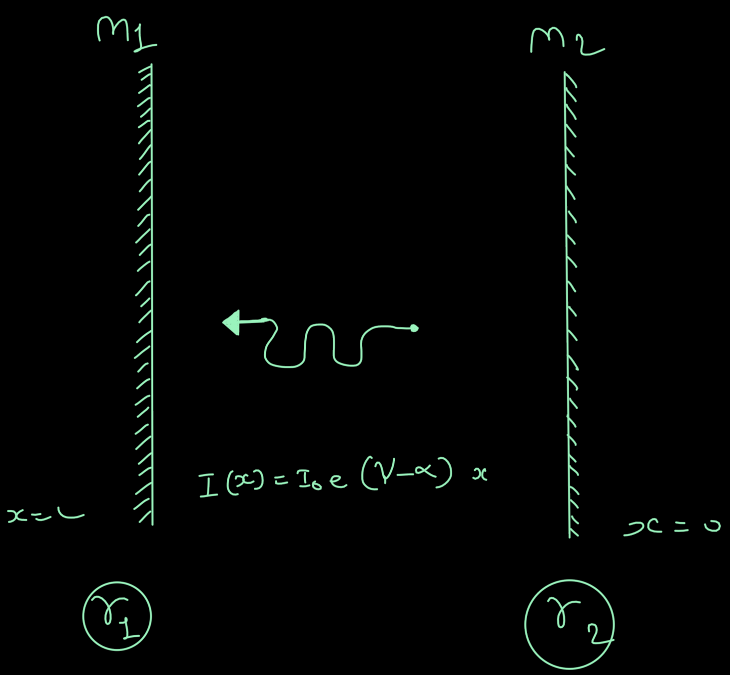
$$N_1 >> N_2$$

$$\frac{N_{st}}{N_{ab}} << 1 \qquad \boxed{N_{st} << N_{ab}}$$

② $$\frac{N_{st}}{N_{ab}} = \left(\frac{B_{21}}{A_{21}}\right) Q$$

$$N_{st} << N_{sp}$$

• Threshold condition :-



$$I_{(x)} = I_0 e^{(\gamma - \alpha)x}$$

after reflection from mirror m_1

$$I_1 = (r_1)(I) = (r_1) I_0 e^{(\gamma-\alpha)L}$$

after reflection from mirror m_2

$$I_2 = (r_2)(I_1)$$

$$I_2 = (r_2)(r_1)(I_0) e^{(\gamma-\alpha)(2L)} \quad \text{—— ①}$$

$$\text{Gain } G = \frac{I_2}{I_0}$$

$$G \geq 1$$

$$1 \leq \frac{(r_2)(r_1)(\cancel{I_0})\, e^{(\gamma-\alpha)(2L)}}{\cancel{I_0}}$$

$$1 \leq (r_1)(r_2)\, e^{(\gamma-\alpha)(2L)}$$

$$\frac{1}{(r_1 r_2)} \leq e^{(\gamma-\alpha)(2L)}$$

$$\therefore \ln\left(\frac{1}{r_1 r_2}\right) \leq (\gamma-\alpha)(2L)$$

$$(\gamma-\alpha) \leq \left(\frac{1}{2L}\right) \ln\left(\frac{1}{r_1 r_2}\right)$$

$$\gamma = \left(\frac{1}{2L}\right) \ln\left(\frac{1}{r_1 r_2}\right) + \alpha$$

or, $\gamma = \alpha - \left(\frac{1}{2L}\right) \ln(r_1 r_2)$

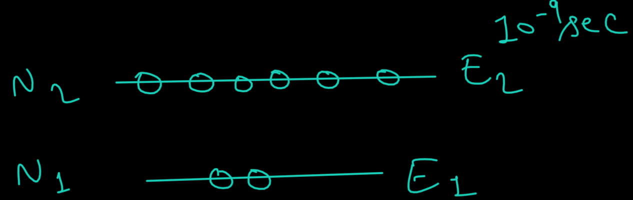
- Meta stable and energy level in which the life time of an atom is greater then 10^{-9} sec (but less than that of ground state) is called meta stable state.

This energy state is formed in some active medium b/w ground and excited state.

Pumping schemes :-

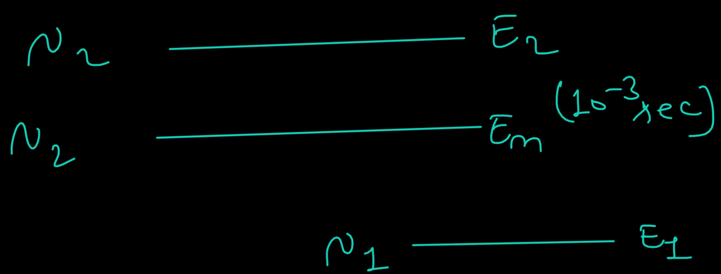
① Two Level pumping

$T_{max} \approx 10^{-9}$ sec , $\frac{N_2}{N_1} \neq \infty$



② Three level pumping

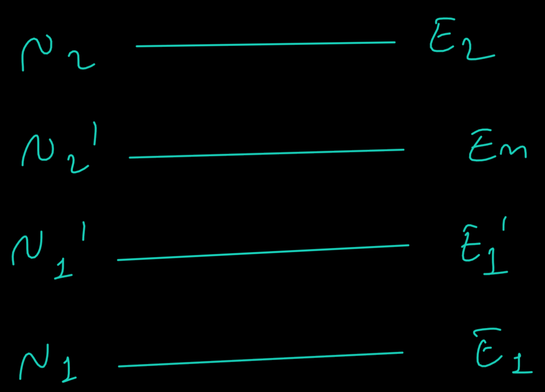
$T_{max} \approx 10^{-3}$, $\frac{N_2'}{N_1} \neq \infty$



③ four level pumping :-

$T_{max} \approx 10^{-3}$sec

$\frac{N_2'}{N_1'} = \infty$



Q.. find the ratio of the population of the two energy states of the active medium Producing laser transition b/w which has wavelength 694.3 nm at room temp.?

$\Rightarrow$ $\frac{N_1}{N_2} = ?$ $\quad \lambda = 694.3 \times 10^{-9}\, m$

$T = 27^\circ c = 300\, K$

$N_1 = N_0 e^{-E_1/KT}, \quad N_2 = N_0 e^{-E_2/KT}$

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/KT} \quad \text{——— (1)}$$

$$E_2 - E_1 = h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{-8}}{694.3 \times 10^{-9}}$$

$$= 2.864 \times 10^{-19}\, J = 1.79\, eV.$$

$$\frac{N_1}{N_2} = \frac{1.79}{e^{8.65} \times 10^{-5} \times 300} = e^{68.97}$$

$$\frac{N_1}{N_2} = 9.05 \times 10^{29}$$

$$N_1 >> N_2$$

Q. A laser source is emitting a laser beam with an average power of 4.5mW. find the number of photons emitted per second by the laser. The wavelength emitted is 6328 Å

$\Rightarrow$ $P = 4.5 \times 10^{-3}\, W$

$\frac{n}{t} = ?$

$\lambda = 6328 \times 10^{-10}\, m$

for 1 photon , $E = \frac{hc}{\lambda}$

for n photons , $E = \frac{nhc}{\lambda}$

power (P) $= \frac{E}{t} = \frac{nhc}{\lambda t}$

$$\frac{n}{t} = \frac{(P)\ (\lambda)}{hc} = \frac{4.5 \times 10^{-3} \times 6328 \times 10^{-10}}{6.63 \times 10^{-34} \times 3 \times 10^{8}}$$

$$\frac{n}{t} = 1.431 \times 10^{16}/sec$$