EXPANSION OF FUNCTIONS

EXERCISE – I

1. Express f(x) $= x^5 - 5x^4 + 6x^3 - 7x^2 + 8x - 9$ in powers of$(x-1)$.

2. Expand f(x) $= x^4 - 3x^3 + 2x^2 - x + 1$ in powers of $(x-3)$.

3. Expand f(x) $= x^3 - 2x^2 + 3x + 5$ in powers of $(x-2)$

4. Expand f(x) $= 2x^2 + 3x^2 - 8x + 7$ in terms of $(x-2)$

5. Expand in f(x) $= \sqrt{1 + x + 2x^2}$ powers of $(x-1)$ using Taylor's series.

6. Expand log x in powers of $(x-2)$

7. Expand log $tan\left(\frac{\pi}{4} + x\right)$ in powers of x .

8. By using Taylor's Theorem arrange in powers of x, $7 + (x+2) + 3(x+2)^3 + (x+2)^4$.

9. Expand $tan\left(\frac{\pi}{4} + x\right)$ and hence find the value of $tan(46°, 36')$ upto four places of decimals .

10. Using Taylor's series Theorem find $\sqrt{9.12}$ correct to five places of decimals.

ANSWERS

1. $\mathrm{f(x)} = -6 - 3(\mathrm{x}-1) - 9(\mathrm{x}-1)^2 - 4(\mathrm{x}-1)^3 + (\mathrm{x}-1)^5$

2. $\mathrm{f(x)} = 16 + 38(\mathrm{x}-3) + 29(\mathrm{x}-3)^2 + 9(\mathrm{x}-3)^3 + (\mathrm{x}-3)^4$

3. $11 + 7(x-2) + 4(x-2)^2 + (x-2)^3$

4. $19 + 28(x-2) + 15(x-2)^2 + 2(x-2)^3$

5. $\mathrm{f(x)} = 2 + \frac{5}{4}(x-1) + \frac{7}{64}(x-1)^2 + \ldots\ldots\ldots\ldots$

6. $\log x = \log 2 + \frac{1}{2}(x-2) - \frac{1}{2!}\cdot\frac{1}{4}(x-2)^2 + \frac{1}{3!}\cdot\frac{1}{4}(x-2)^3 + \ldots\ldots\ldots.$

7. $2x + \frac{4}{3}x^3 + \frac{4}{3}x^5 + \ldots\ldots\ldots\ldots$

8. $49 + 69x + 42x^2 + 11x^3 + x^4$.

9. 1.0574

10. 3.01993

EXERCISE - II

1. Expand in powers of $x, e^x \sec x$

2. Expand in powers of $x, e^{x\cos x}$

3. Show that $e^x \log(1+x) = x + \frac{x^2}{2!} + \frac{2x^3}{3!} + \cdots$

4. If $x^3 + 2xy^2 - y^3 + x = 1$, prove that $y = -1 + x - \frac{x^2}{3} + \cdots$

5. Using Maclaurin's Series prove that $5^x = 1 + x\log 5 + \frac{x^2}{2!}(\log 5)^2 + \cdots$

6. Using Maclaurin's Series prove that $\log\cos x = -\frac{x^2}{2} - \frac{x^4}{12} - \frac{x^6}{45} - \cdots$

7. Using Maclaurin's Series prove that $e^{\sin x} = 1 + x + \frac{x^2}{2} - \frac{x^4}{8} - \frac{x^5}{15} + \cdots$

8. Using Maclaurin's Series prove that $\log(1 + \tan x) = x - \frac{x^2}{2} + \frac{2x^3}{3} - \cdots$

9. Using Maclaurin's Series prove that $e^x \sin x = x + x^2 + \frac{2x^3}{3} + \cdots$

10. If $x^3 + y^3 + xy - 1 = 0$, prove that $y = 1 - \frac{x}{3} - \frac{26x^3}{81} - \cdots$

11. Prove that $\log(1 - x + x^2 - x^3) = -x + \frac{x^2}{2} - \frac{x^3}{3} + \cdots$

12. Expand $(1 + x)^{1/x}$ upto the term x^2

13. Prove that $(1 + x)^{(1+x)} = 1 + x + x^2 + \frac{x^3}{3} + \cdots$

14. $\log\left(\frac{1+x}{1-x}\right) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots\right]$ and hence find $\log_e\left(\frac{11}{9}\right)$

15. Prove that $\cos^2 x = 1 - x^2 + \frac{1}{3}x^4 - \frac{2}{45}x^6 + \cdots$

16. Prove that $\log\left(\frac{\tan x}{x}\right) = \frac{x^2}{3} + \frac{7}{90}x^4 + \cdots$

17. Prove that $\log(1 + x + x^2 + x^3 + x^4) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$

ANSWERS

1. $e^x \sec x = 1 + x + \frac{2x^2}{2!} + \cdots$

2. $e^{x \cos x} = 1 + x + \frac{x^2}{2} + \cdots$

12. $e \cdot \left[1 - \frac{x}{2} + \frac{11x^2}{24} - \cdots\right]$

14. 0.20067