PRACTICE PROBLEMS ON GAMMA FUNCTION

1. Given $\overline{|1\cdot 8} = 0.9314$, find the value of $\overline{|-2\cdot 2}$.

2. Compute $\overline{|-2\cdot 5}$.

3. Prove that $\overline{\left|n+\frac{1}{2}\right.} = \frac{1\cdot 3\cdot 5\ldots\ldots(2n-1)}{2^n}\sqrt{\pi}$. Hence or otherwise prove that $\overline{\left|n+\frac{1}{2}\right.} = \frac{(2n)!\,\sqrt{\pi}}{n!\,4^n}$.

4. If $I_n = \dfrac{\frac{\sqrt{\pi}}{2}\overline{\left|\frac{n+1}{2}\right.}}{\overline{\left|\frac{n}{2}+1\right.}}$, show that $I_{n+2} = \frac{n+1}{n+2} I_n$ and hence, find I_5.

Evaluate the following integrals **(5 to 11)**

5. $\int_0^\infty e^{-h^2x^2}\,dx$

6. $\int_0^\infty e^{-x^2/4}\,dx$

7. $\int_0^\infty e^{-x^5}\,dx$

8. $\int_0^\infty \sqrt{x}\,e^{-x^2}\,dx$

9. $\int_0^\infty (2x^2+4)\,e^{-2x^2}\,dx$

10. $\int_0^\infty x^{1/4}\ e^{-\sqrt{x}}dx$

11. $\int_0^\infty x^n\,e^{-\sqrt{ax}}\,dx$

12. Prove that $\int_0^\infty x^2\,e^{-x^4}\,dx \,.\int_0^\infty\ e^{-x^4}\,dx = \frac{\pi}{\mathbf{8}\sqrt{2}}$

13. Prove that $\int_0^\infty x\,e^{-x^4}\,dx\,.\int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}\,dx = \frac{\sqrt{\pi}}{8}\overline{\left|\frac{1}{4}\right.}$

14. Prove that $\int_0^\infty x\,e^{-x^8}\,dx\,.\int_0^\infty x^2\,e^{-x^8}\,dx = \frac{1}{64}\overline{\left|\frac{1}{4}\right.}\,\overline{\left|\frac{3}{8}\right.}$

15. Prove that $\int_0^\infty xe^{-x^8}\,.dx\,.\int_0^\infty x^2e^{-x^4}\ dx = \frac{\pi}{16\sqrt{2}}$

16. Prove that $\int_0^\infty \sqrt{y}\ .\,e^{-y^2}dy\,.\int_0^\infty \frac{e^{-y^2}}{\sqrt{y}}\ dy = \frac{\pi}{2\sqrt{2}}$

17. Prove that $\int_0^\infty \frac{e^{-x^3}}{\sqrt{x}}\ dx\,.\int_0^\infty y^4e^{-y^6}\,dy = \frac{\pi}{9}$

Evaluate the following integrals **(18 to 29)**

18. $\int_0^1 x^m.\left(\log\,\frac{1}{x}\right)^n\ dx$

19. $\int_0^1 (x\,\log x)^3\,dx$

20. $\int_0^1 x^3\left(\log\,\frac{1}{x}\right)^4 dx$

21. $\int_0^1 \frac{dx}{\sqrt{x.\log(1/x)}}$

22. $\int_0^1 \sqrt{x\log{(1/x)}}\ .\,dx$

23. $\int_0^1 \frac{dx}{\sqrt{-\log x}}$

24. $\int_0^1 \sqrt[3]{\log(1/x)}\ .\,dx$

25. $\int_0^1 \sqrt{\ \log\,(1/x)}\,.\,dx$

26. $\int_0^1 (\log x)^5\,dx$

27. $\int_0^\infty \frac{x^7}{7^x}\ .\,dx$

28. $\int_0^\infty\ 7^{-4\,x^2}\,dx$

29. $\int_0^\infty 3^{-4\,x^2}\,dx$

30. Show that **(i)** $\int_0^\infty x^{m-1}\cos ax\,dx = \frac{\overline{|m}}{\mathrm{a^m}}\cos\left(\frac{\mathrm{m}\pi}{2}\right)$. **(ii)** $\int_0^\infty \mathrm{x^{m-1}\,sinax\,dx} = \frac{\overline{|m}}{\mathrm{a^m}}\ \sin\left(\frac{\mathrm{m}\pi}{2}\right)$.

31. Show that $\int_0^\infty e^{-ax}\,x^{n-1}\,dx = \frac{\overline{|n}}{a^n}$ where a, n are positive. Deduce that

(i) $\int_0^\infty e^{-ax}\,x^{n-1}\cos bx\,dx = \frac{\overline{|n}}{r^n}\cdot\ \cos n\,\theta$ **(ii)** $\int_0^\infty e^{-ax}\,x^{n-1}\sin bx\,dx = \frac{\overline{|n}}{r^n}\cdot\ \sin n\,\theta$

Where $r^2 = a^2+b^2, \theta = tan^{-1}(b/a)$

32. Prove that **(i)** $\int_0^\infty \cos\left(ax^{1/n}\right)\ dx = \frac{\overline{|n+1}}{a^n}\ \cos\left(\frac{n\pi}{2}\right)$ **(ii)** $\int_0^\infty \sin\left(ax^{1/n}\right)\ dx = \frac{\overline{|n+1}}{a^n}\ \sin\left(\frac{n\pi}{2}\right)$

33. Prove that **(i)** $\int_0^\infty \mathrm{xe^{-ax}\cos bx\,dx} = \frac{\mathrm{a^2-b^2}}{\mathrm{(a^2+b^2)^2}}$ **(ii)** $\int_0^\infty xe^{-ax}\sin bx\,dx = \frac{2ab}{(a^2+b^2)^2}$

ANSWERS

1.	-2.21	**2.**	$-\frac{8}{15}\sqrt{\pi}$	**4.**	$\frac{8}{15}$	**5.**	$\frac{\sqrt{\pi}}{2h}$	**6.**	$\overline{\left\|\frac{1}{2}\right.} = \sqrt{\pi}$
7.	$\frac{1}{5}\cdot\overline{\left\|\frac{1}{5}\right.}$	**8.**	$\frac{1}{2}\overline{\left\|\frac{3}{4}\right.}$	**9.**	$\frac{9\sqrt{\pi}}{4\sqrt{2}}$	**10.**	$\frac{3}{2}\sqrt{\pi}$	**11.**	$\frac{2\overline{\|2n+2}}{a^{n+1}}$
18.	$\frac{\overline{\|n+1}}{(m+1)^{n+1}}$	**19.**	$-\frac{3}{128}$	**20.**	$\frac{3}{128}$	**21.**	$\sqrt{2\pi}$	**22.**	$\frac{\sqrt{\pi}}{3\sqrt{3/2}}$
23.	$\sqrt{\pi}$	**24.**	$\frac{1}{3}\overline{\left\|\frac{1}{3}\right.}$	**25.**	$\frac{\sqrt{\pi}}{2}$	**26.**	-120	**27.**	$\frac{7!}{(\log 7)^8}$
28.	$\frac{\sqrt{\pi}}{4\sqrt{\log 7}}$	**29.**	$\frac{\sqrt{\pi}}{4\sqrt{\log 3}}$						