Reducible to linear DE

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Since $\boxed{\frac{dy}{dx}} + P\boxed{y} = Q$

$\boxed{f'(y)\frac{dy}{dx}} + P\,\boxed{f(y)} = Q$

Two types

Reducible to linear in y

If D.E. is of the form

$\boxed{f'(y)\frac{dy}{dx} + P\,f(y) = Q}$ — ①

No function of y

where, P & Q are purely function of x / constant

Then it can be reduced to linear

By substitution, $f(y) = v$

Then, $f'(y)\frac{dy}{dx} = \frac{dv}{dx}$

Then put in ①

$\frac{dv}{dx} + Pv = Q$

This is linear in v can be solved by prev. method.

Reducible to linear in x

If D.E. is of form

$\boxed{g'(x)\frac{dx}{dy} + P'g(x) = Q'}$ — ②

where, P' & Q' are purely function of y / constant

It reduced to linear

$g(x) = v$

$g'(x)\frac{dx}{dy} = \frac{dv}{dy}$

put in ②

$\frac{dv}{dy} + P'v = Q'$

linear in v

Problems:

1) $\sec y\frac{dy}{dx} + 2x\sin y = 2x\overset{\times}{\boxed{\cos y}}$

Divide by $\boxed{\cos y}$?

$\frac{\sec y}{\cos y}\frac{dy}{dx} + 2x\frac{\sin y}{\cos y} = 2x$

Then $\boxed{\sec^2 y\frac{dy}{dx}} + 2x\,\boxed{\tan y} = 2x$ ①

Compare with $f'(y)\frac{dy}{dx} + P\,f(y) = Q$ | ∴ $\int P\,dx = \int 2x\,dx = x^2$

Tricks (Hints)

1) 3 terms

2) ✓ $f'(y)\frac{dy}{dx} + P\,f(y) = Q$ ✓

No function of x

No function of y

Compare with $f'(y)\frac{dy}{dx} + Pf(y) = Q$

Consider $v = \tan y$

$$\frac{dv}{dx} = \sec^2 y \frac{dy}{dx}$$

Then put in ①

$$\frac{dv}{dx} + 2x\underline{\underline{v}} = 2x$$

linear in v

$p = 2x \ , \ Q = 2x$

$\therefore \int P\,dx = \int 2x\,dx = x^2$

$$e^{\int P dx} = e^{x^2}$$

Then solⁿ is given by,

$$v e^{\int P dx} = \int \left[Q e^{\int P dx} \right] dx + C$$

$$v e^{x^2} = \int (2x) e^{x^2} dx + C$$

$t = x^2 \ , \ dt = 2x\,dx$

$$= \int e^t dt + C$$

$$v e^{x^2} = e^{x^2} + C$$

$$\boxed{e^{x^2}\left[\tan y - 1\right] = C}$$

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2) $\underline{\underline{\boxed{x}\frac{dy}{dx}}} - 1 = x\boxed{e^{-y}}$ ×

Divide by xe^{-y} to entire eqⁿ

Then $\frac{1}{e^{-y}}\frac{dy}{dx} - \frac{1}{xe^{-y}} = 1$

Then $\boxed{e^y \frac{dy}{dx}} - \underline{\underline{\frac{1}{x}}}\boxed{e^y} = \underline{\underline{1}}$ —①

Compare with $f'(y)\frac{dy}{dx} + Pf(y) = Q$

Put $e^y = v$

$$e^y \frac{dy}{dx} = \frac{dv}{dx}$$

Put in ①, $\frac{dv}{dx} - \frac{1}{x}v = 1$

This is linear in v

$P = -\frac{1}{x} \ , \ Q = 1$

$$e^{\int P dx} = e^{\int -\frac{1}{x} dx}$$

$$= e^{-\log x} = \frac{1}{x}$$

The solⁿ is given by,

$$v e^{\int P dx} = \int \left[Q e^{\int P dx} \right] dx + C$$

$$v\left(\frac{1}{x}\right) = \int (1)\frac{1}{x} dx + C$$

$$\boxed{\frac{e^y}{x} = \log x + C}$$

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3) $\frac{dx}{dy} = e^{y-x}\left(e^y - e^x\right)$

3) $\frac{\quad}{dy} - e \quad (e - e)$

4) $\underline{e^{x}(x+1)}\,dx + (y^2 e^{2y} - xe^{x})\,dy = 0$

HW 5) $y\frac{dx}{dy} = x - yx^2\cos y$ HW 6) $\frac{dz}{dx} + \frac{z}{x}\lg z = \frac{z}{x^2}(\lg z)^2$

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3) $\frac{dx}{dy} = \frac{e^{y}}{e^{x}}\left[e^{y} - e^{x}\right]$

$e^{x}\frac{dx}{dy} = e^{2y} - e^{y}e^{x}$

$\boxed{e^{x}\frac{dx}{dy}} + \underset{=}{e^{y}}\,\boxed{e^{x}} = \underset{=}{e^{2y}}$

$\downarrow$

v

$\frac{dv}{dy} + \underset{=}{e^{y}}\,V = \underset{=}{e^{2y}}$

$e^{\int P dy} = \underline{e^{e^{y}}}$

Then

$V e^{e^{y}} = \int \underline{\boxed{e^{e^{y}}}}\,\underline{e^{2y}}\,dy + C$

put $t = e^{y}$, $\underline{dt = e^{y}dy}$

$Ve^{e^{y}} = \int e^{t}\,t\,dt + C$

$Ve^{e^{y}} = e^{t}(t-1) + C$

put back

$t = e^{y}, v = e^{x}$

$e^{x}e^{e^{y}} = e^{e^{y}}(e^{y} - 1) + C$

$\boxed{e^{e^{y}}\left[e^{x} - e^{y} + 1\right] = C}$

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4) $e^{x}(x+1)\frac{dx}{dy} + (\boxed{\underline{y^2 e^{2y}}} - \underline{xe^{x}}) = 0$

$e^{x}(x+1)\frac{dx}{dy} - \underline{xe^{x}} = -y^2 e^{2y}$

V

$\frac{dv}{dy} - v = -y^2 \underset{=}{e^{2y}}$

$P = -1$, $Q = -y^2 e^{2y}$

$e^{\int P dy} = e^{\int -1\,dy} = e^{-y}$

Sol^n is given by

$Ve^{-y} = \int\left[-y^2 e^{2y}\right](e^{-y})\,dy + C$

$Ve^{-y} = -\int \underset{u}{y^2}\,\underset{v}{e^{y}}\,dy + C$

$= -\left[y^2 e^{y} - (2y)(e^{y}) + (2)(e^{y}) + 0\right]$

Ve^{-y}

e

ve^{-y} $+ (2)(e^{y}) + 0]$

$= (-y^{2} + 2y - 2)e^{y} + c$

Generalised $\underline{uv}$ Rule ?

$$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \text{------}$$

$\boxed{\text{ILATE}}$ v_1, v_2 successive integration

$xe^{x}e^{-y}$

$= e^{y}(-y^{2}+2y-2) + C$

$$xe^{x} = e^{2y}(-y^{2}+2y-2) + \underline{\underline{Ce^{y}}}$$