Reducible to Exact

If $Mdx + Ndy = 0$ is <u>not Exact</u>

Integrating factor (I.F.)

Let $F(x,y)$ be the factor ~~by which~~ Such that
$F(Mdx + Ndy = 0)$ becomes exact
Then $F(x,y)$ is said to be I.F.

Example: Consider $ydx - xdy = 0 \rightarrow$ Not exact
multiply by $\boxed{\frac{1}{y^2}}$ Then $\frac{1}{y}dx - \frac{x}{y^2}dy = 0$ is exact

$$\frac{\partial M}{\partial y} = -\frac{1}{y^2} \quad \Big| \quad \frac{\partial N}{\partial x} = -\frac{1}{y^2}$$

Hence $1/y^2$ is called I.F. of above D.E.
also $1/x^2$ is I.F. of above D.E.

This problem can also be solved by ~~ydx + xdy form~~ for Homogeneous I.F. is $\frac{1}{0}$ for $ydx + xdy$ I.F. is $\frac{1}{2xy}$ But sol^n is [illegible]

Rules for Finding I.F.

Rule 1 : (Homogeneous form)
If Given D.E. is Homogeneous Then

$$I.F. = \frac{1}{Mx + Ny} \quad \left(\text{Provided } Mx + Ny \neq 0\right)$$

Rule 2 : ($ydx + xdy$ form)
If Given D.E. is of form
$f_1(xy)\, ydx + f_2(xy)\, xdy = 0$
Then

$$I.F. = \frac{1}{Mx - Ny} \quad \left(\text{Provided } Mx - Ny \neq 0\right)$$

These functions are of product xy
e.g. (xy), x^2y^2, $\cos(xy)$, $\log(xy)$
not like x^2y, $x\sin y$

Rule 3 : (Divide by N)
~~Consider~~ Check if $\left(\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}\right)$ is purely function of x (Say $f(x)$)
Then $I.F. = e^{\int f(x)dx}$

Rule 4 (Divide by M)
Check if $\left(\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}\right)$ is purely function of y ($g(y)$)
Then $I.F. = e^{\int g(y)dy}$

Problems on Integrating factor

1) $\underbrace{(x^4 + y^4)}_{M}dx - \underbrace{xy^3}_{N}dy = 0$

Not exact, Homogeneous
$Mx + Ny = x^5 + xy^4 - xy^4 = x^5 \neq 0$
$\therefore I.F. = \frac{1}{x^5}$, multiply by $\frac{1}{x^5}$

$\therefore \boxed{\left(\frac{1}{x} + \frac{y^4}{x^5}\right)dx - \frac{y^3}{x^4}dy = 0}$ is exact

Can be solved By Divide by N Rule get Same I.F.

Then sol^n is

$$\boxed{\log x - \frac{y^4}{4x^4} + 0 = C}$$