INTEGRATION

1	**Def :** If $\frac{d}{dx}$ (f(x)) = g (x) then $\int g(x)dx$ = f(x) + c Where, c is called constant of integration
2	$\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$
3	$\int \left(f(x) - g(x)\right)dx = \int f(x)dx - \int g(x)dx$
4	$\int k\, f(x)dx = k\int f(x)dx$ Where , k = constant.
5	If $\int f(x)dx = g(x) + c$ then, $\int f(lx + m)dx = \frac{1}{l}g(lx + m) + c$
6	$\int \frac{f'(x)}{f(x)}dx = \log\lvert f(x)\rvert + c$
7	$\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c$
8	$\int \left(f(x)\right)^n f'(x)dx = \frac{\left(f(x)\right)^{n+1}}{n+1} + c$
9	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ Where, $n \neq -1$
10	$\int 1\, dx = x + c$
11	$\int \frac{1}{x^2}dx = \frac{-1}{x} + c$
12	$\int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c$
13	$\int \frac{1}{x}\, dx = \log\lvert x\rvert + c$
14	$\int e^x dx = e^x + c$
15	$\int a^x dx = \frac{a^x}{\log a} + c$ Where, $a > 0$, $a \neq 1$
16	$\int \cos x\, dx = \sin x + c$
17	$\int \sin x\, dx = -\cos x + c$
18	$\int \sec^2 x\, dx = \tan x + c$
19	$\int \mathrm{cosec}^2 x\, dx = -\cot x + c$
20	$\int \sec x \tan x\, dx = \sec x + c$

21	$\int \text{cosec}\, x \cot x\, dx = -\text{cosec}\, x + c$
22	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x + c$ **OR** $-\cos^{-1}x + c$
23	$\int \frac{1}{1+x^2} dx = \tan^{-1}x + c$ **OR** $-\cot^{-1}x + c$
24	$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}x + c$ **OR** $-\text{cosec}^{-1}x + c$
25	$\int \tan x\, dx = \log\lvert\sec x\rvert + c = -\log\lvert\cos x\rvert + c$
26	$\int \cot x\, dx = \log\lvert\sin x\rvert + c = -\log\lvert\text{cosec} x\rvert + c$
27	$\int \sec x\, dx = \log\lvert\sec x + \tan x\rvert + c = \log\left\lvert\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right\rvert + c$
28	$\int \text{cosec}\, x\, dx = \log\lvert\text{cosec}\, x - \cot x\rvert + c = \log\left\lvert\tan\frac{x}{2}\right\rvert + c$
29	$\int \frac{1}{x^2+a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
30	$\int \frac{1}{a^2-x^2} dx = \frac{1}{2a}\log\left\lvert\frac{a+x}{a-x}\right\rvert + c$
31	$\int \frac{1}{x^2-a^2} dx = \frac{1}{2a}\log\left\lvert\frac{x-a}{x+a}\right\rvert + c$
32	$\int \frac{1}{\sqrt{x^2+a^2}} dx = \log\left\lvert x + \sqrt{x^2+a^2}\right\rvert + c$
33	$\int \frac{1}{\sqrt{x^2-a^2}} dx = \log\left\lvert x + \sqrt{x^2-a^2}\right\rvert + c$
34	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
35	$\int \sqrt{a^2-x^2} dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$
36	$\int \sqrt{x^2+a^2} dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left\lvert x + \sqrt{x^2+a^2}\right\rvert + c$
37	$\int \sqrt{x^2-a^2} dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left\lvert x + \sqrt{x^2-a^2}\right\rvert + c$
38	$\int e^x[f(x) + f'(x)]dx = e^x f(x) + c$

	Integral / Type	Method
1.	$\int \tan^2 x \, dx$	Put $\tan^2 x = \sec^2 x - 1$
2.	$\int \cot^2 x \, dx$	Put $\cot^2 x = \operatorname{cosec}^2 x - 1$
3.	$\int \sin^2 x \, dx$	Put $\sin^2 x = \left(\frac{1-\cos 2x}{2}\right)$
4.	$\int \cos^2 x \, dx$	Put $\cos^2 x = \left(\frac{1+\cos 2x}{2}\right)$
5.	$\int \sin^3 x \, dx$	Put $\sin^3 x = \frac{3}{4}\sin x - \frac{1}{4}\sin(3x)$
6.	$\int \cos^3 x \, dx$	Put $\cos^3 x = \frac{3}{4}\cos x + \frac{1}{4}\cos(3x)$
7.	$\int \frac{1}{1 \pm \sin x} dx$, $\int \frac{1}{\sec x \pm \tan x} dx$ $\int \frac{1}{\operatorname{cosec} x \pm \cot x} dx$	Rationalize it
8.	$\int \frac{1}{1 \pm \cos x} dx$	Either rationalize it or use half angle formula $1 + \cos x = 2\cos^2\left(\frac{x}{2}\right)$ & $1 - \cos x = 2\sin^2\left(\frac{x}{2}\right)$
9.	To integrate, the product of Sine and sine, sine and cosine, cosine and cosine.	Use defactorisation formula
10.	$\int \sqrt{1 \pm \sin x} \, dx$	Use half angle formula, $1 \pm \sin 2\theta = (\cos\theta \pm \sin\theta)^2$
11.	$\int \sqrt{1 \pm \cos x} \, dx$	Use half angle formula, $1 + \cos x = 2\cos^2\left(\frac{x}{2}\right)$, $1 - \cos x = 2\sin^2\left(\frac{x}{2}\right)$
12.	$\int \frac{p(x)}{Q(x)} dx$ Where P(x) and Q(x) are polynomial in x (degree of P(x) $\geq$ degree of Q(x))	**1.** Divide P(x) by Q(x) **2.** Write P(x) = quotient. Q(x) + remainder **3.** Put value of P(x) in integral and take separate division.
13.	$\int \frac{P(x)}{ax^2+b} dx$ where $P(x)$ is a polynomial in x.	If degree of $P(x) \leq 1$ **1.** Separate division & Separate integrals **2.** Use $\int \frac{f'(x)}{f(x)} dx = \log f(x) + c$ and standard formulae. If degree of $P(x) \geq 2$ **1.** First divide $P(x)$ by $ax^2 + b$ **2.** Write P(x) = quotient $(ax^2 + b)$ + remainder **3.** Put value of P(x) in integral and take separate division and then integrate.

14.	$\int P(x)\sqrt{ax+b}\,dx$, $\int \frac{P(x)}{\sqrt{ax+b}}\,dx$ $\int P(x)(ax+b)^n dx$, $\int \frac{P(x)}{(ax+b)^n}\,dx$ Where, P(x) is polynomial in x	Put $(ax+b) = t$
15.	$\int \frac{a\sin x + b\cos x}{c\sin x + d\cos x}\,dx$, $\int \frac{ae^x+b}{ce^x+d}\,dx$ **Note :** Answer is always $Ax + B\,log\,\lvert DR\rvert + c$	**1.** write NR = A(DR)+B $\frac{d}{dx}$(DR) **2.** Find A and B **3.** Put value of NR in integral. **4.** Take separate division and separate integral. **5.** use $\int \frac{f'(x)}{f(x)}dx = \log\lvert f(x)\rvert + c$
16.	$\int \frac{\sin(x+a)}{\sin(x+b)}dx$, $\int \frac{\sin(x+a)}{\cos(x+b)}dx$ $\int \frac{\cos(x+a)}{\sin(x+b)}dx$, $\int \frac{\cos(x+a)}{\cos(x+b)}dx$	**1.** Adjust angle of DR in NR **2.** Apply formula sin (A $\pm$ B), cos (A $\pm$ B) in NR only. **3.** Take separate division and Separate integral.
17.	$\int \frac{1}{\sin(x-a)\cos(x-b)}dx$	**1.** Multiply and divide by cos (b – a). **2.** Write (b – a) = $[(x-a)-(x-b)]$ in NR. **3.** Apply formula cos (A – B) in NR only. **4.** Take separate division and Separate integral.
18.	$\int \frac{1}{\sin(x-a)\sin(x-b)}\,dx$ $\int \frac{1}{\cos(x-a)\cos(x-b)}\,dx$	**1.** Multiply and divide by sin (b – a) **2.** Write (b – a) = $[(x-a)-(x-b)]$ in NR. **3.** Apply formula sin (A – B) in NR only. **4.** Take separate division and Separate integral. **Note :** In above method, instead of (b – a) we can take (a – b)
19.	$\int \frac{1}{a\sin x + b\cos x}dx$	**1.** Find $\sqrt{a^2+b^2} = r$ **2.** Divide and multiply by r in DR only. **3.** Put $\frac{a}{r} = \cos\propto$, $\frac{b}{r} = \sin\propto$ **OR** $\frac{a}{r} = \sin\propto$, $\frac{b}{r} = \cos\propto$ **4.** Put DR in form sin (A + B) **OR** In form cos (A + B). **5.** use formula $\int \mathrm{cosec}\,dx$ **OR** $\int \sec x\,dx$ **6.** Replace $\propto = \tan^{-1}\left(\frac{b}{a}\right)$ **OR** $\propto = \tan^{-1}\left(\frac{a}{b}\right)$
20.	$\int \frac{1}{ax^2+bx+c}dx$, $\int \frac{1}{\text{quadratic}}$	**1.** Find LT = $\frac{(MT)^2}{4(FT)}$ **OR** $= \left(\frac{1}{2} \times coefficient\ of\ x\right)^2$ When coefficient of x^2 is 1 **2.** Make complete square **3.** Apply formula, $\int \frac{1}{x^2+a^2}dx$, $\int \frac{1}{a^2-x^2}dx$, $\int \frac{1}{x^2-a^2}dx$

21.	$\int \frac{\text{linear}}{\text{quadratic}}$, $\int \frac{lx+m}{ax^2+bx+c}dx$	**1.** Express $lx+m = A\frac{d}{dx}(ax^2+bx+c)+B$ **2.** Obtain the value of A and B by equating the coefficient of like powers of x on both sides **3.** Replace $lx+m = A\frac{d}{dx}(ax^2+bx+c)+B$ in given integral **4.** Take separate division and separate integral. **5.** In one integral , use formula, $\int \frac{f'(x)}{f(x)}dx = \log\|f(x)\|+c$ **6.** In another integral, use type, $\int \frac{1}{\text{quadratic}}$
22.	$\int \frac{1}{a\sin^2x+b\cos^2x+c}dx$ $\int \frac{1}{a\sin^2x+b\cos^2x}dx$ $\int \frac{1}{a\sin^2x+c}dx$, $\int \frac{1}{b\cos^2x+c}dx$	**1.** Divide NR and DR by $\cos^2x$. **2.** Convert each term of DR in tanx using $1+\tan^2\theta = \sec^2\theta$. **3.** Put tanx = t. **4.** Convert integral into type $\int \frac{1}{\text{quadratic}}$
23.	$\int \frac{1}{a\sin x+b\cos x+c}dx$ $\int \frac{1}{a\sin x+b\cos x}dx$ $\int \frac{1}{a\sin x+c}dx$, $\int \frac{1}{b\cos x+c}dx$	**1.** Put $\tan\left(\frac{x}{2}\right) = t$ (angle of tan is half of angle of sin & cos) **2.** dx = $2\left(\frac{1}{1+t^2}\right)dt$ **3.** sin x = $\frac{2\tan(x/2)}{1+\tan^2(x/2)} = \frac{2t}{1+t^2}$, Cos x = $\frac{1-\tan^2(x/2)}{1+\tan^2(x/2)} = \frac{1-t^2}{1+t^2}$ **4.** Covert integral in type $\int \frac{1}{\text{quadratic}}$
24.	$\int \frac{1}{x^{\frac{1}{n}}+x^{\frac{1}{m}}}dx$	Put x = t^l Where l is LCM of n and m.
25.	**Odd power of sin x or cos x** $\int \sin^n x\,dx$, $\int \cos^n x\,dx$ Where, n is odd natural number	**1.** Separate one sin x **or** cos x. **2.** Put remaining in terms of power of $\sin^2x$ **or** $\cos^2x$. **3.** Replace $\sin^2x = 1-\cos^2x$ **or** $\cos^2x = 1-\sin^2x$ **4.** Put cos x = t **or** sin x = t
26.	**Even power of sin x or cos x.** $\int \sin^n x\,dx, \int \cos^n x\,dx$ Where, n is even natural number	**1.** Write integrand in power of $\sin^2x$ **or** $\cos^2x$. **2.** Replace $\sin^2x = \left(\frac{1-\cos 2x}{2}\right)$ **or** $\cos^2x = \left(\frac{1+\cos 2x}{2}\right)$ **3.** Go to reducing power by step (2).
27.	**Any power of tan x or cot x** $\int \tan^n x\,dx, \int \cot^n x\,dx$ Where, n is even or odd.	**1.** Separate one $\tan^2x$ **or** $\cot^2x$. **2.** Replace that $\tan^2x = \sec^2x - 1$ **or** $\cot^2x = \text{cosec}^2x - 1$. **3.** Separate integral. **4.** In one integral put tan x = t **or** cot x = t **5.** In another integral, repeat above steps, if require.

<table>
<tr><td>28.</td><td>Even power of sec x or cosec x
$\int \sec^n x\,dx, \int \operatorname{cosec}^n x\,dx$
Where, n is even natural number</td><td colspan="2">1. Separate one $\sec^2 x$ or $\operatorname{cosec}^2 x$
2. Write remaining in power of $\sec^2 x$ or $\operatorname{cosec}^2 x$.
3. Replace $\sec^2 x = 1 + \tan^2 x$ or $\operatorname{cosec}^2 x = 1 + \cot^2 x$.
4. Put tan x = t or cot x = t</td></tr>
<tr><td>29.</td><td>Odd power of sec x or cosec x.
$\int \sec^n x\,dx, \int \operatorname{cosec}^n x\,dx$
Where, n = 3</td><td colspan="2">Use integration by parts.</td></tr>
<tr><td rowspan="7">30.</td><td rowspan="7">Example involving square root</td><td>If integrand Contains</td><td>Substitute</td></tr>
<tr><td>1. $\sqrt{a^2 - x^2}$</td><td>x = a sin θ, x = a cos θ</td></tr>
<tr><td>2. $\sqrt{x^2 - a^2}$</td><td>x = a sec θ, x = a cosec θ</td></tr>
<tr><td>3. $\sqrt{a^2 + x^2}$</td><td>x = a tan θ, x = a cot θ</td></tr>
<tr><td>4. $\sqrt{(a-x)/(a+x)}$</td><td>x = a cos θ</td></tr>
<tr><td>5. $\sqrt{(a-x)/x}$</td><td>x = a $\sin^2\theta$</td></tr>
<tr><td>6. $\sqrt{2ax - x^2}$</td><td>x = 2a $\sin^2\theta$</td></tr>
<tr><td>31.</td><td>$\int \frac{1}{\sqrt{\text{quadratic}}}$, $\int \frac{1}{\sqrt{ax^2+bx+c}}$ dx</td><td colspan="2">1. Find LT = $\frac{(MT)^2}{4(FT)}$ OR $= \left(\frac{1}{2} \times coefficient\ of\ x\right)^2$
When coefficient of x^2 is 1
2. Adjust LT and make complete square.
3. Use formula $\int \frac{1}{\sqrt{x^2+a^2}}dx$, $\int \frac{1}{\sqrt{x^2-a^2}}dx$, $\int \frac{1}{\sqrt{a^2-x^2}}dx$</td></tr>
<tr><td>32.</td><td>$\int \frac{\text{linear}}{\sqrt{\text{quadratic}}}$, $\int \frac{lx+m}{\sqrt{ax^2+bx+c}}$ dx</td><td colspan="2">1. Express $lx + m = A\frac{d}{dx}(ax^2 + bx + c) + B$
2. Obtain the value of A and B by equating the coefficient of like powers of x on both sides
3. Replace $lx + m = A\frac{d}{dx}(ax^2 + bx + c) + B$ in the given integral
4. Take separate division & separate integral.
5. In one integral, substitute quadratic = t or use formula $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c$
6. In another integral, use type $\int \frac{1}{\sqrt{\text{quadratic}}}$</td></tr>
<tr><td>33.</td><td>$\int \frac{1}{\text{linear}\sqrt{\text{linear}}}$ $\int \frac{1}{(px+q)\sqrt{ax+b}}$ dx</td><td colspan="2">Put ax + b = t^2 i.e. $\sqrt{\text{linear}} = t$</td></tr>
</table>

34.	$\int \frac{1}{\text{linear}\sqrt{\text{quadratic}}}$ $\int \frac{1}{(px+q)\sqrt{ax^2+bx+c}}\,dx$	Put $px+q=\frac{1}{t}$
35.	$\int \sqrt{\text{quadratic}}$ $\int \sqrt{ax^2+bx+c}\,dx$	**1.** Find LT = $\frac{(MT)^2}{4(FT)}$ **OR** $=\left(\frac{1}{2}\times \textit{coefficient of } x\right)^2$ When coefficient of x^2 is 1 **2.** Adjust LT and make complete square. **3.** Use formula $\int\sqrt{x^2+a^2}\,dx$, $\int\sqrt{x^2-a^2}\,dx$, $\int\sqrt{a^2-x^2}\,dx$
36.	$\int linear\sqrt{\text{quadratic}}$ $\int (lx+m)\sqrt{ax^2+bx+c}\,dx$	**1.** Express $lx+m=A\frac{d}{dx}(ax^2+bx+c)+B$ **2.** Obtain the value of A and B by equating the coefficient of like powers of x on both sides **3.** Replace $lx+m=A\frac{d}{dx}(ax^2+bx+c)+B$ in the given integral **4.** Take separate integral. **5.** In one integral, substitute quadratic = t **6.** In another integral, use type $\int\sqrt{\text{quadratic}}$
37.	**INTEGRATION BY PARTIAL FRACTION :**	
(i)	$\int \frac{px+q}{(x-a)(x-b)}\,dx$	**distinct linear factor** Express : $\frac{px+q}{(x-a)(x-b)}=\frac{A}{(x-a)}+\frac{B}{(x-b)}$
(ii)	$\int \frac{px^2+qx+r}{(x-a)(x-b)(x-c)}\,dx$	**distinct linear factor** Express : $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}=\frac{A}{(x-a)}+\frac{B}{(x-b)}+\frac{C}{(x-c)}$
(iii)	$\int \frac{px+q}{(x-a)^2}\,dx$	**repetitive linear factor** Express : $\frac{px+q}{(x-a)^2}=\frac{A}{(x-a)}+\frac{B}{(x-a)^2}$
(iv)	$\int \frac{px^2+qx+r}{(x-a)^3}\,dx$	**repetitive linear factor** Express : $\frac{px^2+qx+r}{(x-a)^3}=\frac{A}{(x-a)}+\frac{B}{(x-a)^2}+\frac{C}{(x-a)^3}$
(v)	$\int \frac{px^2+qx+r}{(x-a)^2(x-b)}\,dx$	**repetitive linear factor** Express : $\frac{px^2+qx+r}{(x-a)^2(x-b)}=\frac{A}{(x-a)}+\frac{B}{(x-a)^2}+\frac{C}{(x-b)}$
(vi)	$\int \frac{px^2+qx+r}{(x-a)(x^2+bx+c)}\,dx$	**Linear & quadratic factor**

		Express: $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{(x-a)} + \frac{Bx+C}{(x^2+bx+c)}$
38.	**Inside power** $\int \frac{1}{x(ax^n+b)} dx$	**1.** Multiply x inside the bracket. **2.** Divide NR and DR by x^{n+1}. **3.** Adjust derivative of DR in NR. **4.** Use formula $\int \frac{f'(x)}{f(x)} dx = \log\lvert f(x)\rvert + c$.
39.	**Outside power** $\int \frac{1}{x^n(ax+b)} dx$	**1.** Adjust x and (ax + b) in NR **2.** Take separate division and Separate integral **3.** Repeat above steps 'n' times.
40.	**INTEGRATION BY PARTS :** To integrate, the product of two different functions.	$\int uv\, dx = u \int v\, dx - \int \left(\frac{du}{dx} \int v\, dx\right) dx$
	1. While using integration by parts we select the function 'v' such that $\int v\, dx$ is easily possible **2.** To select 'u' and 'v' we use LIATE rule **L –** Log, **I –** Inverse, **A –** Algebraic, **T –**Trigo, **E –** Exponential Function. We select 'u' as that function which come first in **LIATE order** **3.** To integrate, $\sin^{-1}x, \cos^{-1}x$……..& log x we take v = 1 and apply integration by parts	
41.	$\int \frac{\sin x \pm \cos x}{a+b\sin 2x} dx$	**1.** Find the function $f(x)$ such that $\frac{d}{dx}[f(x)] =$ NR **2.** Put $f(x) = t$ **3.** Squaring $f(x) = t$ obtain value of $sin 2x$ **4.** Use type $\int \frac{1}{\text{quadratic}}$
42.	$\int \frac{x^2+1}{x^4+1} dx$	Divide by x^2 and put $\left(x \pm \frac{1}{x}\right) = t$
43.	$\int \sqrt{\frac{linear}{linear}}$, $\int \sqrt{\frac{ax+b}{cx+d}} dx$	**1.** Multiply & divide by $ax + b$ into root **2.** Convert into type $\int \frac{\text{linear}}{\sqrt{\text{quadratic}}}$

DEFINITE INTEGRALS

1	If $\int f(x)dx = g(x) + c$, then $\int_a^b f(x)dx = [g(x)]_a^b = g(b) - g(a)$
2	$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
3	$\int_a^b k.f(x)dx = k\int_a^b f(x)dx$, where k is a constant.
4	$\int_a^b uv\,dx = (u\int v\,dx)_a^b - \int_a^b \left(\frac{du}{dx}\int v\,dx\right)dx$
5	$\int_a^b f(x)dx = \int_a^b f(t)dt$
6	$\int_a^b f(x)dx = -\int_b^a f(x)dx$
7	If $a < c < b$, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
8	$\int_0^a f(x)dx = \int_0^a f(a-x)dx$
9	$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
10	$\int_0^{2a} f(x)dx = \int_0^a [f(x) + f(2a-x)]dx$
11	$\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ if $f(x) = f(2a-x)$
12	$\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)dx & \text{If f is an even functions} \\ 0 & \text{If f is an odd functions} \end{cases}$