SOMAIYA VIDYAVIHAR UNIVERSITY

Somaiya TRUST

K J Somaiya College of Engineering, Vidyavihar, Mumbai

(A Constituent College of SVU)

Engineering Mechanics Typical Theory Questions

Module 1 – System of Forces

Module Section 1.1 – System of Coplanar Forces

Class: F.Y. B. Tech Division: C3
Professor: Parag S. Sarode Date: 20/06/2023

References: Engineering Mechanics, by M. D. Dayal & Engineering Mechanics – Statics and Dynamics, by N. H. Dubey.

Module 1 – System of Forces

Module 1.1 Coplanar forces

1. State and prove Varignon's theorem Marks 2 to 5
2. State and explain varignon's theorem with suitable example. Marks 2

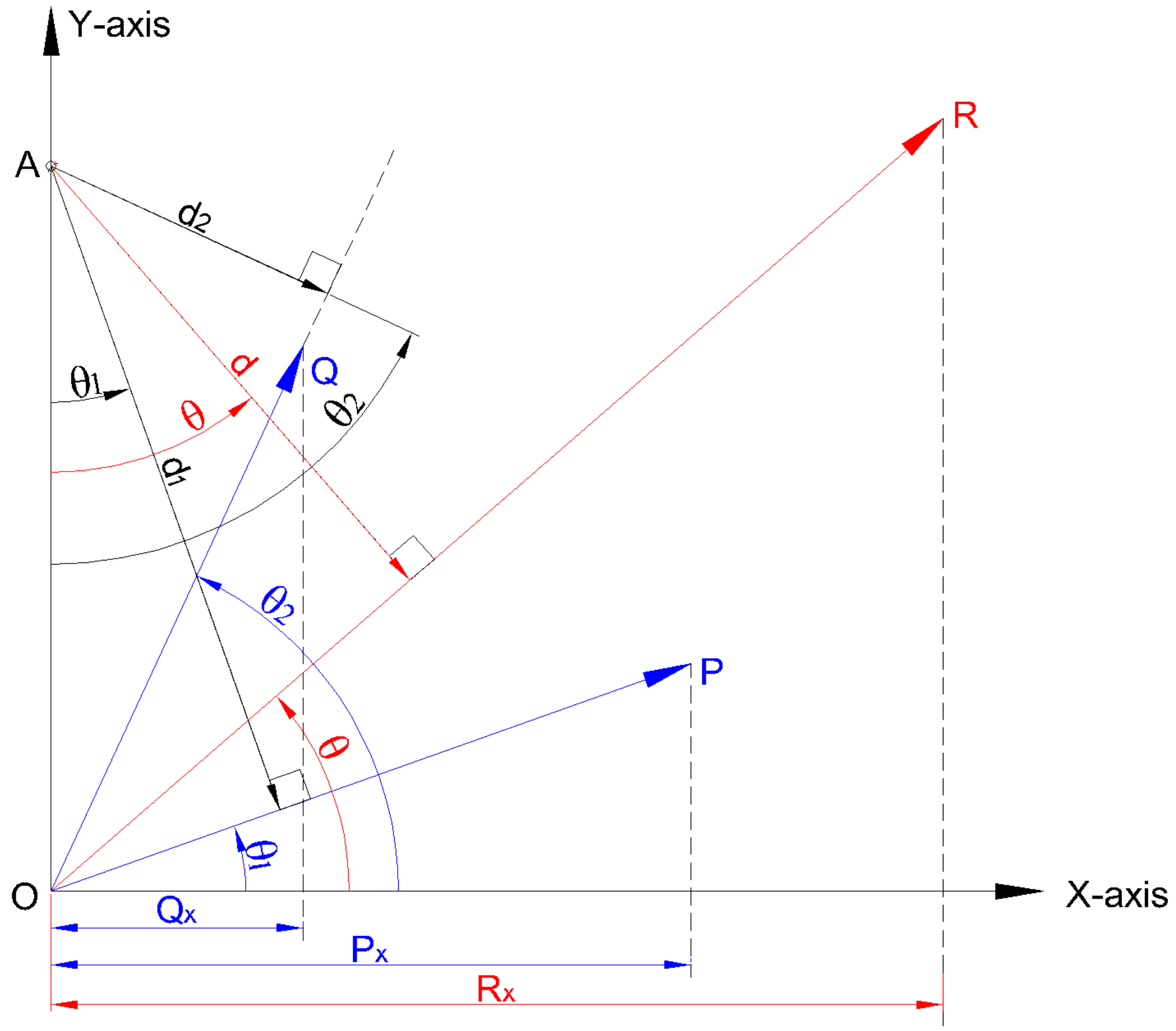
Varignon's Theorem: The algebraic sum of the moments of a system of coplanar forces about any point in the plane is equal to the moment of the resultant force of the system about the same point.

Proof:

Let P & Q be 2 concurrent forces acting at O (origin), making angles θ_1 & θ_2 with x-axis. Let their resultant be R making angle θ with x-axis.

Let A be a point on the y-axis about which moments are to be taken. Let d_1, d_2, & d be the moment arms of P, Q & R respectively from moment centre A.

Let the components of the forces in x-direction be denoted by adding a 'subscript x'.



Moment due to force P about A,

$$M_A^P = +P \times d_1 M_A^P = P \times OA \cos \cos \theta_1 \; M_A^P = P \cos \cos \theta_1 \; \times OA M_A^P = P_x OA$$

Similarly, moment due to Q about A, $M_A^Q = +Q \times d_2 = Q_x OA$

and, moment due to R about A, $M_A^R = +R \times d = R_x OA$

Now, the sum of moments of forces P & Q about A is given by,

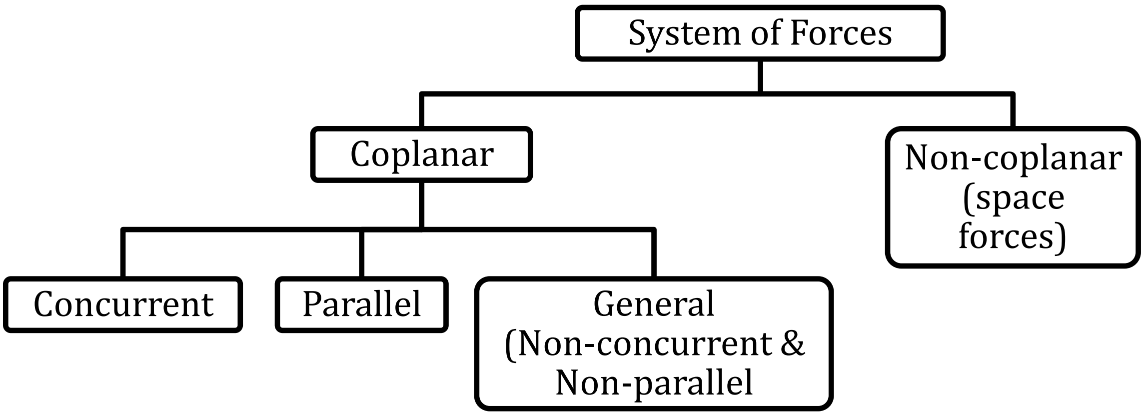
$$\sum M_A^F = M_A^P + M_A^Q \sum M_A^F = P_x OA + Q_x OA \sum M_A^F = (P_x + Q_x) OA \sum M_A^F = R_x OA \sum M_A^F = M_A^R$$

This means that the sum of moments of the two forces about a point is equal to the moment of resultant force about that point.

Hence, the theorem is proved.

3. List the different types of system of forces and explain any one of them. Marks 2

System of Forces: When a number of forces act simultaneously on a body then they are said to form a system of forces or force system.



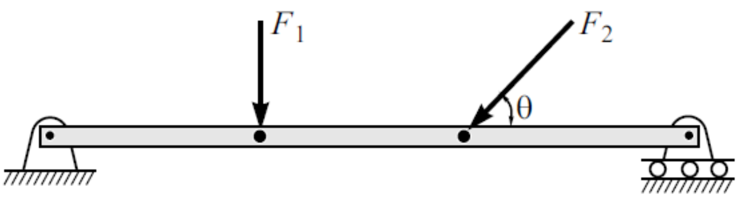
In Coplanar System of Forces, all forces lie in one plane; while in Non-coplanar System of Forces, all the forces in the system do not lie in a single plane.

Both systems can be subdivided into:

a) Concurrent Force System: In this, the lines of action of all the forces in the system pass through the same point.
   e.g., lamp hanging from string, electric pole supporting heavy cables, forces on a tripod, etc.
b) Parallel Force System: In this, the lines of action of all the forces in the system are parallel to each other.
   e.g., weighing scale, things places on a table, people sitting on a bench, etc.
c) General Force System: In this, the lines of action of all the forces in the system are neither concurrent nor parallel to each other.
   e.g., a moving vehicle has engine power, friction due to road, wind resistance, weight of vehicle and passengers, etc. acting in various directions.

4. Discuss on different types of load with neat sketches. Marks 5

1. Point Load: If the whole intensity of load is assumed to be concentrated at a point, then it is known as point load.

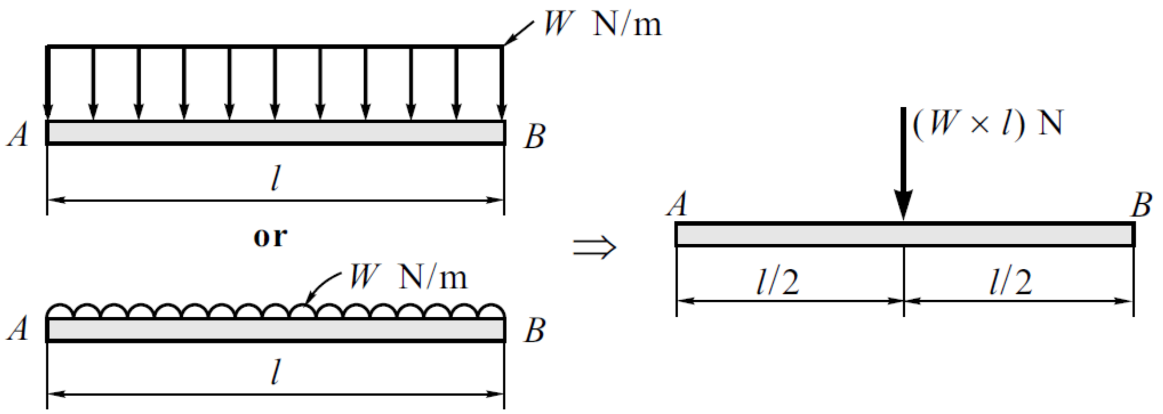


2. Distributed Load: When a load acts throughout the length of a beam or body in varying degree, it can be called as distributed load. This load may consist

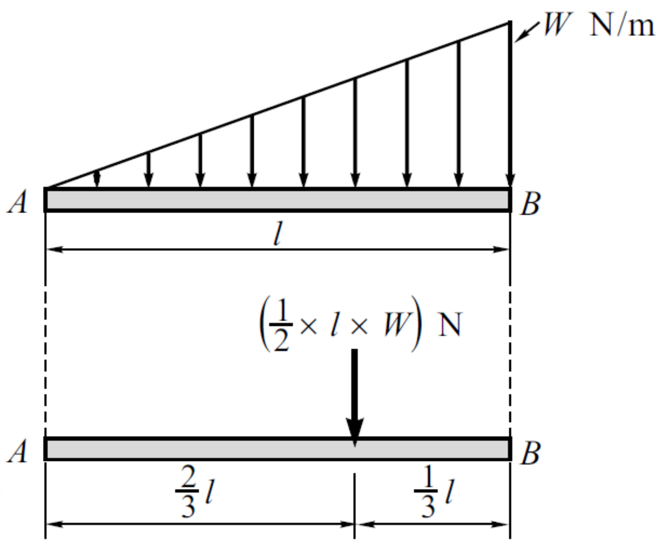
of the weight of materials supported directly or indirectly by the beam or it may be caused by wind or hydraulic pressure.

A distributed load on a beam can be replaced by a concentrated point load. The magnitude of this equivalent point load is equal to the area under loading diagram and it acts through the centroid.

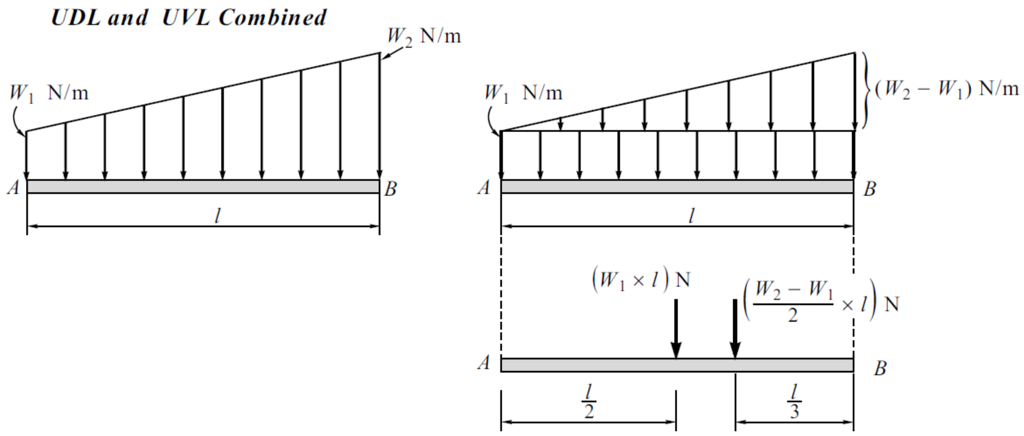
a) Uniformly Distributed Load (UDL): If the whole intensity of load is distributed uniformly along the length of loading, then it is called uniformly distributed load. E.g., weight of a slab of a building flooring.



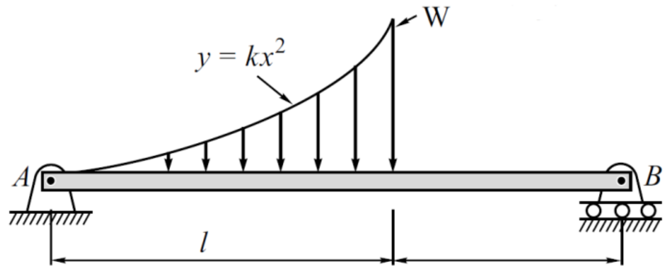
b) Uniformly Varying Load (UVL): If the whole intensity of load is distributed uniformly at varying rate along the length of loading, then, it is known as uniformly varying load. E.g., in a dam the hydraulic pressure varies linearly with the depth.



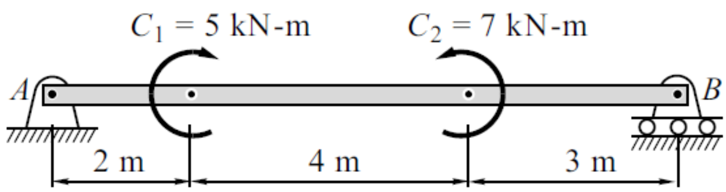
c) Trapezoidal Load (UDL + UVL): If the whole intensity of load is distributed uniformly at varying rate along the length of loading from some lower intensity at on end to a higher intensity at the other end, then, it is known as trapezoidal load.



d) <u>Varying Load</u>: The varying load is given by some relation.

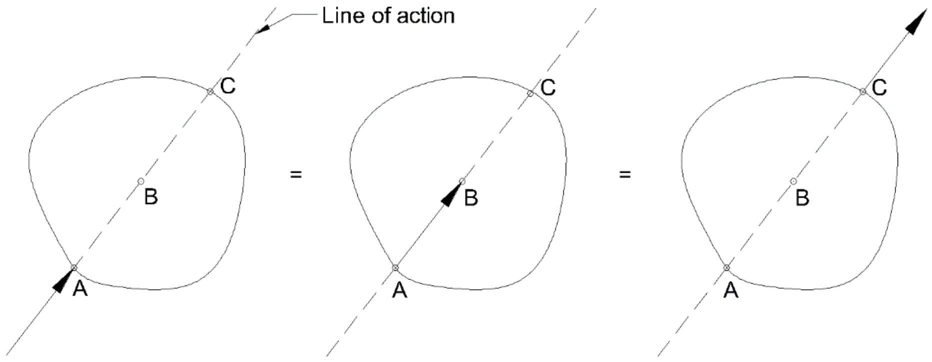


3. <u>Couple</u>: A couple load acting on a body tends to cause rotation of the body. Its location on the body is of no significance because couples are free vectors.



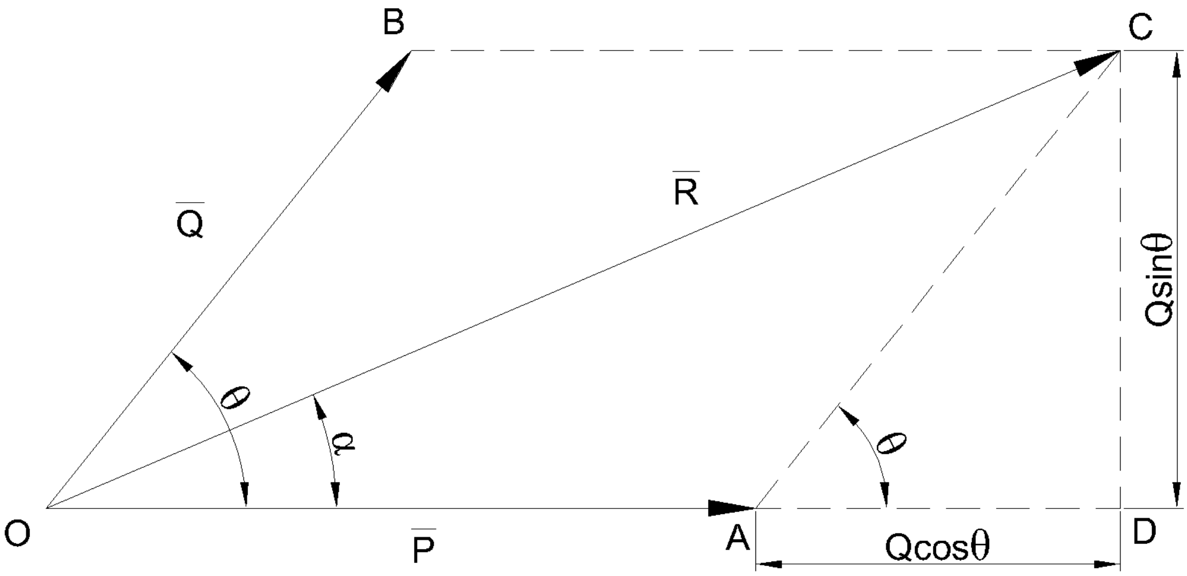
5. Explain principle of transmissibility of forces with a neat sketch

Principle of Transmissibility of Force: A force being a sliding vector will not affect the state of a rigid body (whether at rest or in motion) if the force acts from a different point along its line of action. E.g., in a train, the engine can be located at the front pulling the other cars with it or at the back pushing them forward.



6. Explain law of parallelogram of forces.

Law of Parallelogram for Vectors: If two vectors acting simultaneously at a point are represented in magnitude and direction by two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the vectors.



In ΔOCD,

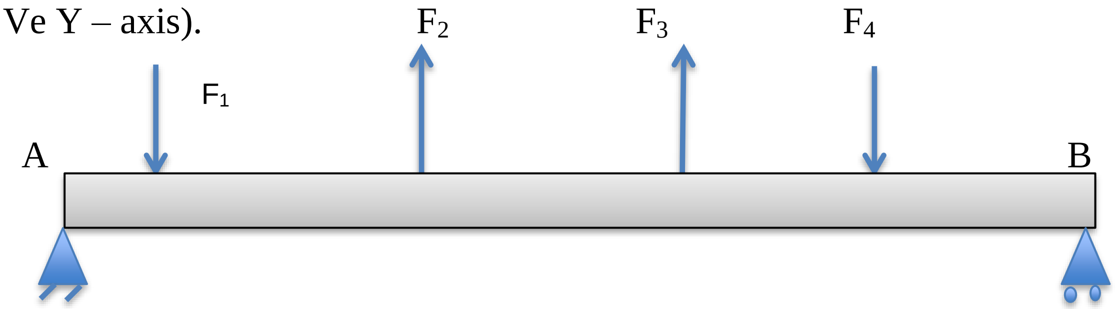
$$OC^2 = OD^2 + CD^2 = (OA + AD)^2 + CD^2$$
$$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$
$$R^2 = P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$$
$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$tan\, \tan\alpha = \frac{CD}{OD} = \frac{CD}{OA+AD}$$

$$\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

7. The resultant of a system of parallel of forces is zero. What does it signify?

For a system of two parallel forces, the resultant of parallel forces zero, signify that the system is acted with summation of forces in upward direction (towards +ve Y – axis) is equal to summation of forces in downward direction (towards –Ve Y – axis).



In the above fig, shows unlike parallel force F_1, F_2, F_3 and F_4. Forces F_1 and F_4 are acting in downward direction and forces F_2 and F_3 are acting in upward direction. The resultant of parallel system in this case will be zero, if $\sum (F_1 + F_4) = \sum (F_2 + F_3)$. So the system is acted by a couple.

Module 1.2 Forces in Space

1. Discuss resultant of concurrent forces in space Marks 4

1) Force Vector:

$$\underline{F} = (F)(\hat{e}_{AB})\underline{F} = (F)\left[\frac{(x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\right]\underline{F}$$

$$= F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$$

($where$ $\hat{e}_{AB}$ is the unit vector
in the direction of AB)

2) Resultant of vector forces F_1, F_2, F_3….

3) Magnitude of Resultant forces $R = \sqrt{{R_x}^2 + {R_y}^2 + {R_z}^2}$

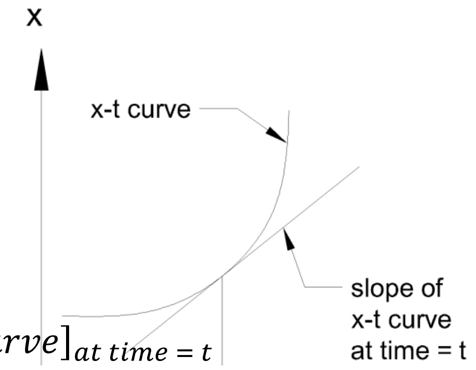
4) Direction of resultant forces: Direction of the resultant force is given by the angles,θ_x, θ_y, and θ_z.$R_x = R\, cos\, cos\, \theta_x$, $R_y = R\, cos\, cos\, \theta_y$, $R_z = R\, cos\, cos\, \theta_z$

Module 2 – Kinematics of particles and Rigid bodies

Module 2.1 Kinematics of particles

1. Explain x-t, v-t and a-t curves in kinematics. Marks 5/6

The motion of a particle along a straight line can be represented by motion curves. They are the graphical representation of position, displacement, velocity and acceleration with time.

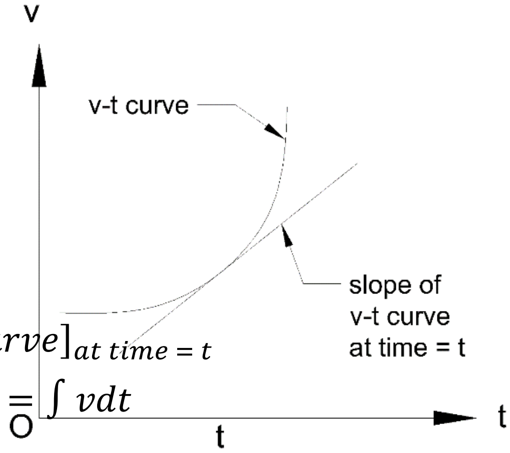


Position-Time (x-t) curve

Since, $v = \frac{dx}{dt}$

at any instant of time,
the slope of x-t curve gives
the velocity of the particle
at that instant.

$$\therefore v = [slope\ x - t\ curve]_{at\ time\ =\ t}$$



Velocity-Time (v-t) curve

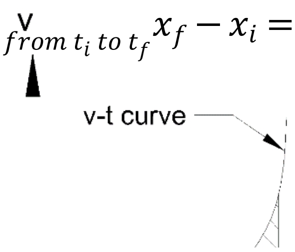
Since, $a = \frac{dv}{dt}$

at any instant of time,
the slope of v-t curve gives
the velocity of the particle
at that instant.

$$\therefore a = [slope\ v - t\ curve]_{at\ time\ =\ t}$$

Now, $v = \frac{dx}{dt} \to dx = vdt Integrating, \int dx = \int vdt$

$\int vdt$ represents the area under
the v-t curve from t_i to $t_f \int_{x_i}^{x_f} dx = [AUC\ v - t]_{from\ t_i\ to\ t_f} x_f - x_i = [AUC\ v - t]_{t_i - t_f} \therefore x_f = x_i + [AUC\ v - t]_{t_i - t_f}$



Acceleration-Time (a-t) curve

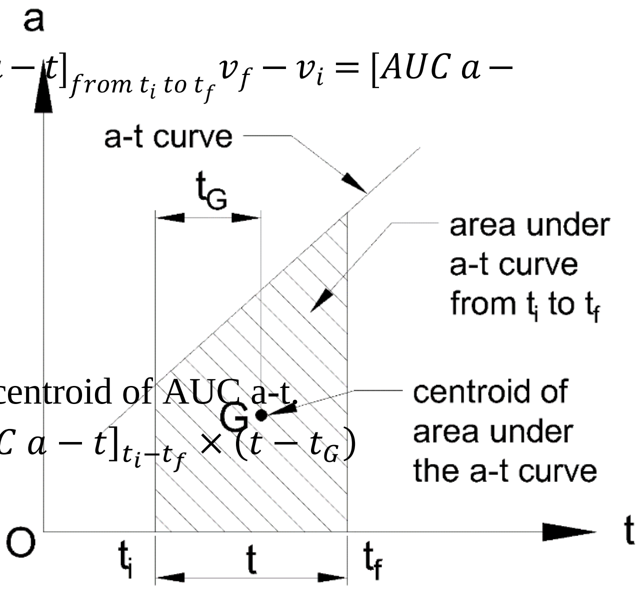
Since, $a = \frac{dv}{dt} \to dv = adt \to \int dv = \int adt$

$\int vdt$ represents the area under
the a-t curve from t_i to $t_f \int_{v_i}^{v_f} dx = [AUC\ a - t]_{from\ t_i\ to\ t_f} v_f - v_i = [AUC\ a - t]_{t_i - t_f} \therefore v_f = v_i + [AUC\ a - t]_{t_i - t_f}$

From a-t curve particle's position
can also be known at an instant,
using area moment method.

Here, t $= t_f - t_i$ and t_G is time from t_i to the centroid of AUC a-t.

$$x_f = x_i + v_i \times t + [AUC\ a - t]_{t_i - t_f} \times (t - t_G)$$



2. Explain i) curvilinear motion by rectangular component method. Ii) curvilinear motion by tangential and normal component method.

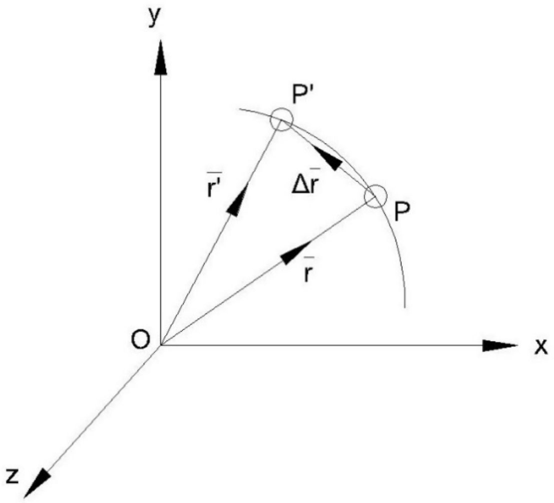
Curvilinear Motion: A particle which travels on a curved path is said to be performing curvilinear motion.

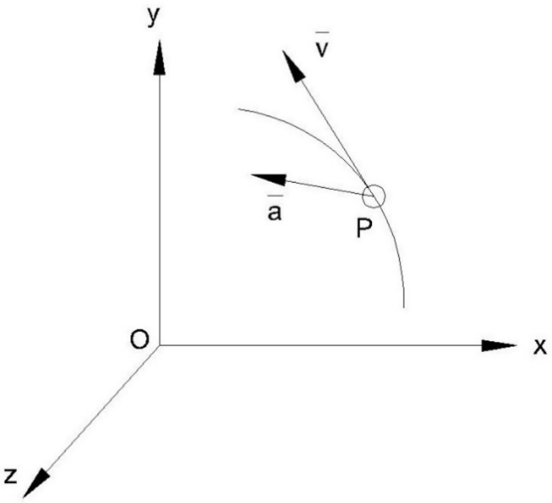
Position: It is represented by a position vector $\underline{r}$ with a starting point from the origin of the reference axis till the particle P. As the particle travels along the curved path, the value of $\underline{r}$ keeps changing.

Velocity: Suppose a particle changes position from P to P', i.e., the position vectors from $\underline{r}$ to $\underline{r}'$, in a time interval of Δt.

$$Average\ velocity, v_{av} = \frac{\Delta \underline{r}}{\Delta t} Instantaneous\ velocity, \underline{v} = \frac{\Delta \underline{r}}{\Delta t} = \frac{d\underline{r}}{dt}$$

In curvilinear motion, instantaneous velocity of a particle is always tangent to the curved path at that instant.





Acceleration: As the direction of velocity continuously changes in a curvilinear motion, there exists acceleration at every instant of the motion.

$$Average\ acceleration, a_{av} = \frac{\Delta \underline{v}}{\Delta t} Instantaneous\ acceleration, \underline{a} = \frac{\Delta \underline{v}}{\Delta t} = \frac{d\underline{v}}{dt}$$

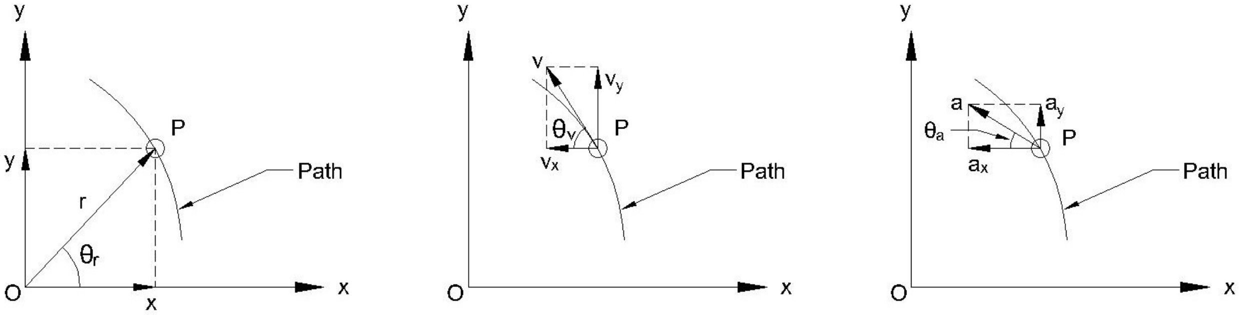
Curvilinear Motion by Rectangular Component System: Curvilinear motion can be split into motion along x, y, z directions, which can be independently considered as three rectilinear motions along those directions respectively.

$$\underline{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad \& \quad r = \sqrt{x^2+y^2+z^2} \quad \underline{v} = \frac{d\underline{r}}{dt}$$

$$= v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k} \quad \& \quad v = \sqrt{v_x^2+v_y^2+v_z^2} \quad \underline{a} = \frac{d\underline{v}}{dt}$$

$$= a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k} \quad \& \quad a = \sqrt{a_x^2+a_y^2+a_z^2}$$

For a particle in xy plane, the rectangular components will be as shown below:



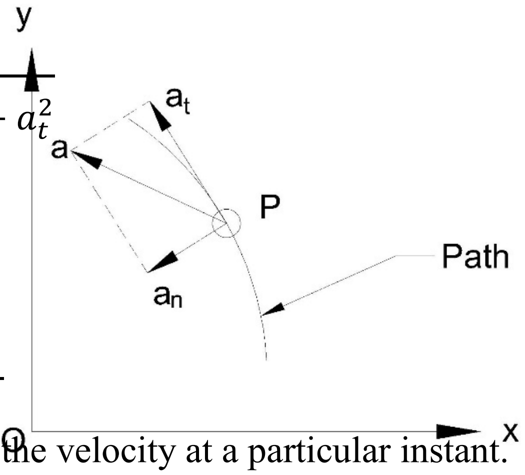
$tan\,tan\,\square_r = \frac{y}{x}$; $\qquad tan\,tan\,\square_v = \frac{v_y}{v_x}$; $\qquad tan\,tan\,\square_a = \frac{a_y}{a_x}$

Curvilinear Motion by Tangential & Normal Component System (N-T System): Curvilinear motion can also be studied by splitting the acceleration along the tangent to the path and normal to the path.

The velocity vector is always directed towards the tangential direction. But the net acceleration may be in any direction. So, it is convenient to express acceleration as *tangential acceleration* (a_t) and *normal acceleration* (a_n).

$$\underline{a} = a_n\hat{e}_n + a_t\hat{e}_t$$

$$a = \sqrt{a_n^2 + a_t^2}$$



a_n represents the change in direction and is always directed towards the centre of curvature.

$$a_n = \frac{v^2}{\rho}$$

Where ρ is the radius of curvature and v is the velocity at a particular instant.

a_t represents the change in velocity and is given by,

$$a_t = \frac{dv}{dt}$$

For curves which are defined as $y = f(x)$,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

And, in terms of rectangular components,

$$\rho = \left|\frac{v^3}{a_x v_y - a_y v_x}\right|$$

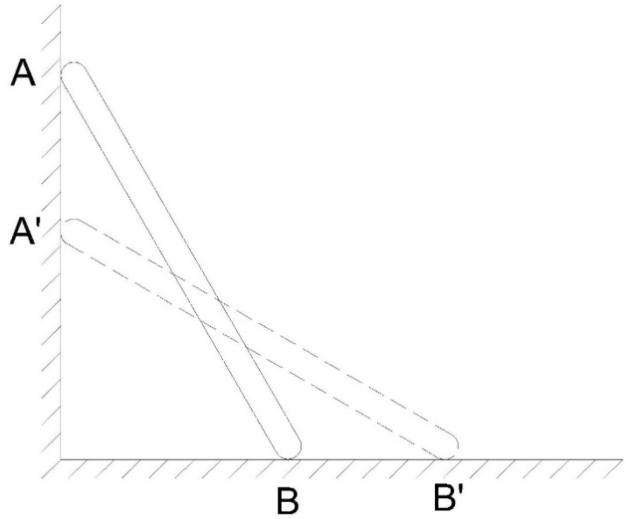
Module 2.2: Kinematics of Rigid bodies

1. Define general plane motion & ICR. What are the properties of an ICR. Marks 5

General Plane Motion: It is a combination of translation motion and rotational motion happening at the same time.

Example 1: Consider a ladder AB having the top end A on a vertical wall and bottom end B on the floor and its sliding. Hence, the velocity of A will be vertically down and that of B will be horizontally towards right.

Here, since A and B are not moving in the same direction, it is not a translation motion, but it is not strictly rotation either even though there is some rotation involved. Hence, it is a combination of both, i.e., general plane (GP) motion



Instantaneous Centre of Rotation (I.C.R.): For general plane motion, at a particular instant, the body can be said to be rotating about a specific point. This point keeps changing as the body moves through the plane. This is called the instantaneous centre of rotation.

It is defined as the point about which a general plane moving body rotates at any given instant. The locus of the ICR's throughout the motion is known as centrode. ICR's are usually denoted by the letter I.

2. Classify types of motion for rigid body using suitable example. Marks 5

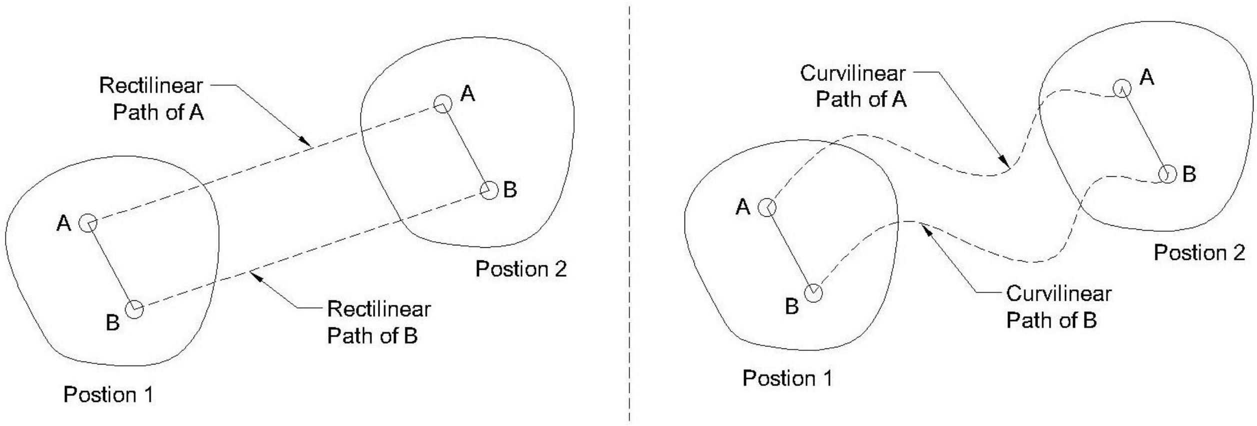
List the rigid body motion, and then brief each one with suitable example.

Types of Rigid Body Motion:

1. Translation Motion
2. Rotation about a fixed axis
3. General Plane Motion
4. Motion about a fixed point
5. General Motion

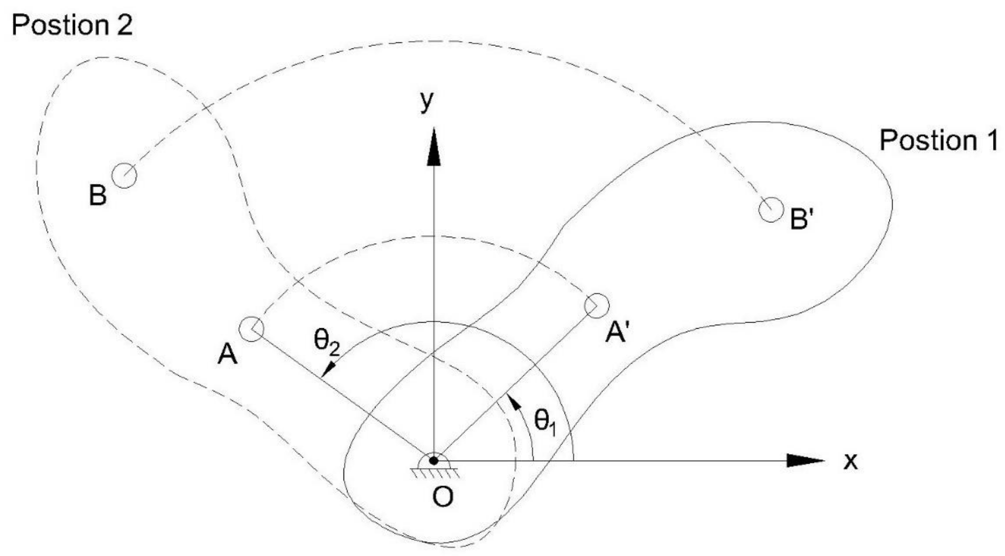
Translation Motion: In this, all the particles forming the body travel along parallel paths, and the orientation on the body does not change during the motion. The motion may be rectilinear or curvilinear.

Let a body move from position 1 to position 2, with two points A & B labelled for reference. The line joining A & B maintains the same direction orientation in both positions. The path travelled by A is parallel to the path travelled by B, be it straight or curved.



At any given instant, in a translational motion, all particles of the body have the same displacement, same velocity and same acceleration. Hence, at its centre of gravity G, a rigid body is similar to a particle in translation motion.

Rotation about Fixed Axis: In this, all the particles of the body travel along concentric circular paths about a common centre of rotation. The axis of rotation is perpendicular to the plane of motion.



Angular Position θ is measured in anticlockwise direction from x-axis in radians.

Angular Displacement is the change in angular position. It is also labelled with θ and measured in radians (rad) given by, $\theta \text{ or } \Delta\theta = \theta_2 - \theta_1$.

$$1\, revolution = 2\pi\, radians = 360°$$

Angular Velocity is the rate of change of angular position with respect to time measured in radians per second (rad/s).

$$\omega = \frac{d\theta}{dt} \circlearrowleft +ve \qquad 1\, rpm = \frac{2\pi}{60}\, rad/s$$

Angular Acceleration is the rate of change of angular velocity with respect to time measured in radians per second squared (rad/s^2).

$$\alpha = \frac{d\omega}{dt}$$

Types of Rotational about Fixed Axis:

Uniform Angular Velocity Motion

$$\omega = \frac{\theta}{t}$$

Uniform Angular Acceleration Motion

$$\omega = \omega_0 + \alpha t \theta = \omega_0 t + \frac{1}{2}\alpha t^2 \omega^2 = \omega_0^2 + 2\alpha\theta$$

Variable Angular Acceleration Motion

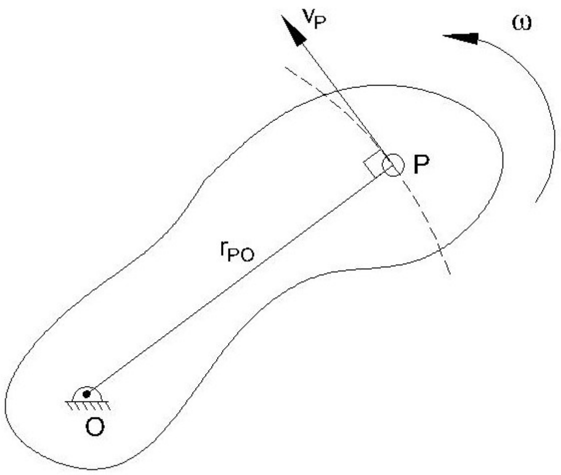
$$\omega = \frac{d\theta}{dt} \quad \& \quad \alpha = \frac{d\omega}{dt} \quad \rightarrow \quad \alpha = \omega\frac{d\omega}{d\theta}$$

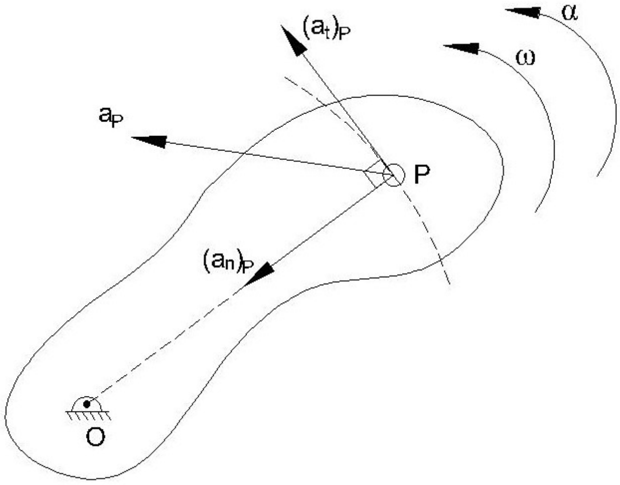
Relations between Linear and Angular Parameters:

All particles in a rotating body will have the same angular velocity but different linear velocities. For a point P, if v_P is the linear velocity and r_{PO} is the radial distance from P to O, then $v_P = r_{PO} \times \omega$.

In general, for any particle with linear velocity v located at a radial distance of r from the axis of rotation with the body having an angular velocity of ω,

$$v = r\omega$$





If the particle has a linear acceleration of a_P which can be resolved into normal component $(a_n)_P$ and tangential component $(a_t)_P$, and the body has an angular acceleration of α, then,

$$\because a_n = \frac{v^2}{\rho} \quad \Rightarrow \quad (a_n)_P = \frac{(v_P)^2}{r_{PO}} = \frac{(r_{PO} \times \omega)^2}{r_{PO}} = \frac{(r_{PO})^2(\omega)^2}{r_{PO}} = r_{PO} \times \omega^2 \because a_t$$

$$= \frac{dv}{dt} \quad \Rightarrow \quad (a_t)_P = \frac{dv_P}{dt} = \frac{d(r_{PO} \times \omega)}{dt} = r_{PO}\frac{d\omega}{dt} = r_{PO} \times \alpha$$

In general, for any particle with linear acceleration a located at a radial distance of r from the axis of rotation with the body having an angular velocity of ω,

$$a_n = r\omega^2 \quad \& \quad a_t = r\alpha$$

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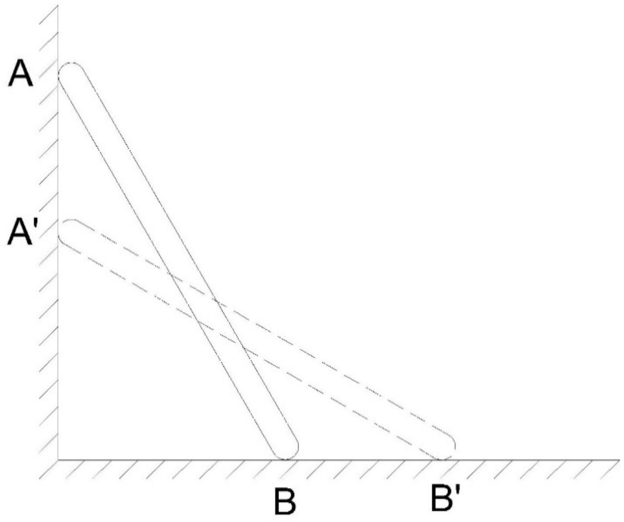
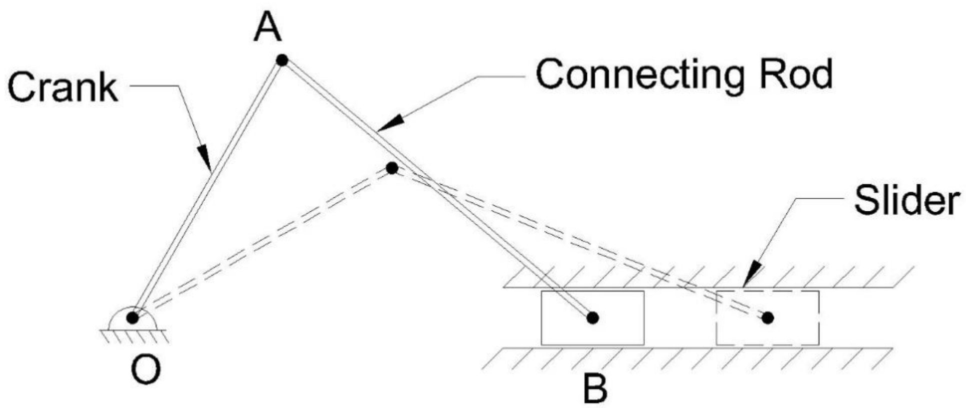
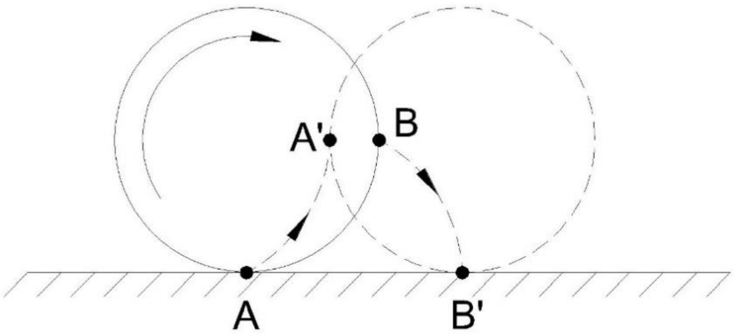


Example 2: In a slider-crank mechanism, shown below, the crank undergoes rotational motion about a fixed hinge support and the slider undergoes back and forth translation motion. The connecting rod linking the crank and slider undergoes GP motion.



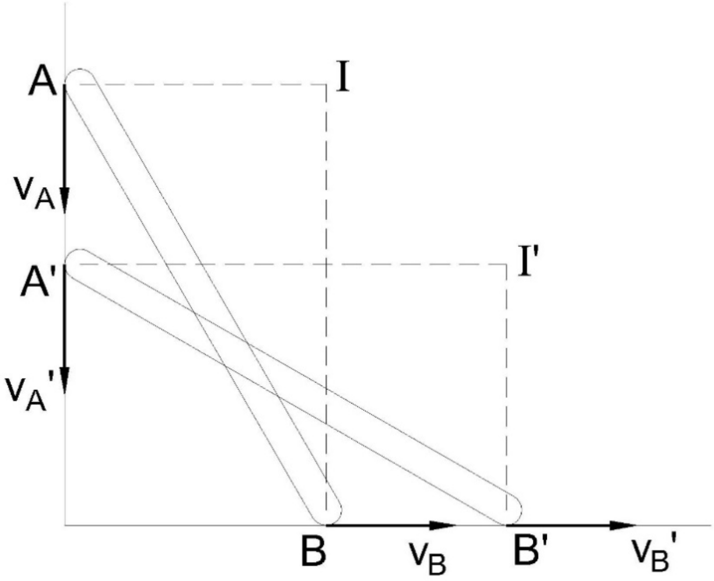
Example 3: When a wheel rolls without slipping on the ground, the wheel rotates as well as translates. Hence, it undergoes GP motion.

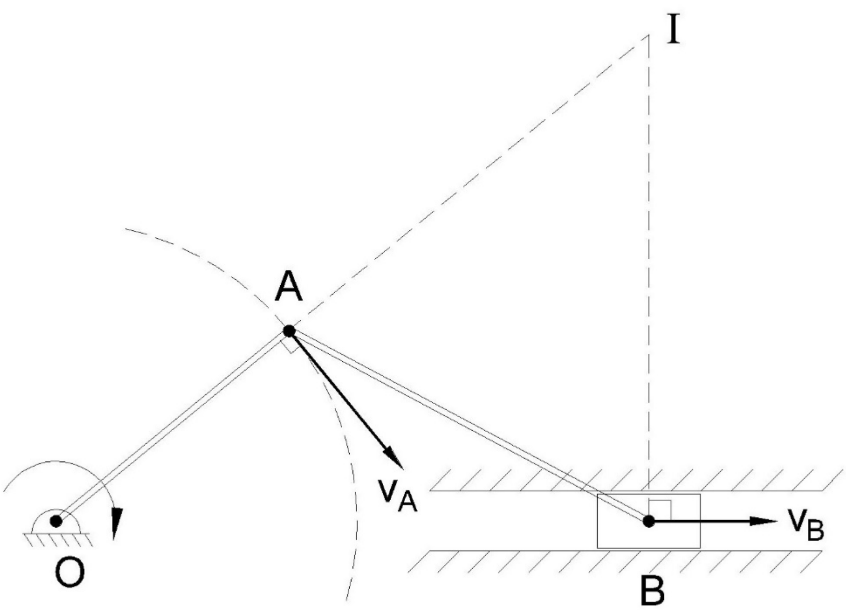


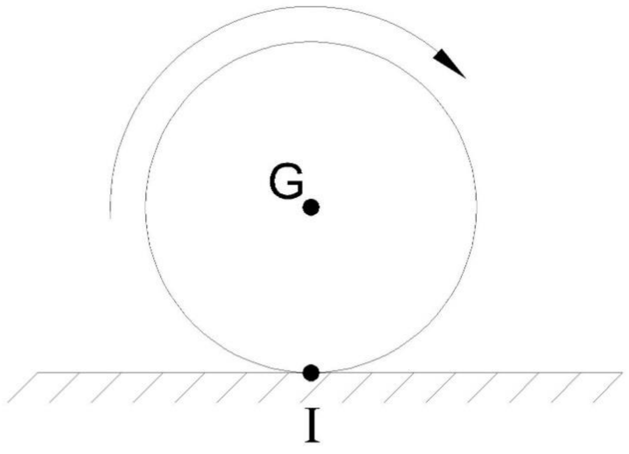
Instantaneous Centre of Rotation (I.C.R.): For general plane motion, at a particular instant, the body can be said to be rotating about a specific point. This point keeps changing as the body moves through the plane. This is called the instantaneous centre of rotation.

It is defined as the point about which a general plane moving body rotates at any given instant. The locus of the ICR's throughout the motion is known as centrode. ICR's are usually denoted by the letter I.

Instantaneous Centre Method: To find the angular velocity of a GP body, we use this method. Find the points on a GP body whose velocity is known. If we draw perpendiculars to the direction of velocity of those points, they will intersect at a certain point. This point is the I.C.R. and we can find the radial lengths to those points. Depending on the known quantities, we can use the relation $v = r\omega$ to find the unknown quantities in a given problem.







For a wheel which rolls without slipping (performing a general plane motion), the Instantaneous Centre of Rotation is the point of contact with the ground. This is because the centre of rotation should have zero velocity and the point in contact with the ground has velocity of the ground, which is zero.

3. Explain Instantaneous centre of rotation. Marks 5/6

Module 3: Centroid

1. Describe the method of finding centroids of composite areas.

Take a suitable example and show with the help of example.

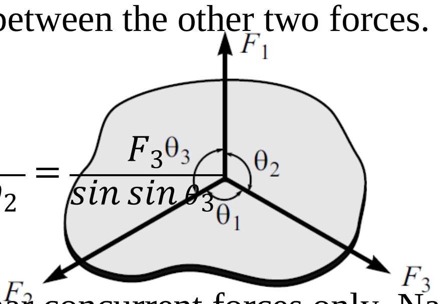
Module 4 – Equilibrium of Force System & Friction

Module 4.1 Equilibrium of Force System

1. State Lami's theorem. State the necessary condition for application of Lami's theorem. Marks 5
2. State and prove Lami's theorem. Marks 5

Lami's Theorem: If three concurrent coplanar forces acting on a body having same nature (i.e., pulling or pushing) are in equilibrium, then each force is proportional to the sine of angle included between the other two forces.

$$\frac{F_1}{sin\,sin\,\theta_1} = \frac{F_2}{sin\,sin\,\theta_2} = \frac{F_3}{sin\,sin\,\theta_3}$$



It is applicable to three non-parallel coplanar concurrent forces only. Nature of three forces must be same (i.e., pulling or pushing). If any force is in the opposite sense, then simply placing a negative sign with it, Lami's theorem can be applied.

Proof

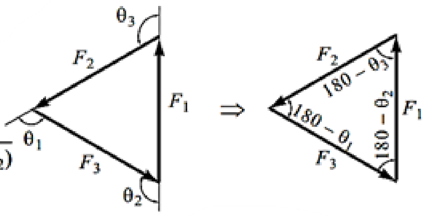
By three-force triangle theorem, we get a closed triangle. Refer to Fig.

By sine rule, we have

$$\frac{F_1}{\sin(180^\circ - \theta_1)} = \frac{F_2}{\sin(180^\circ - \theta_2)} = \frac{F_2}{\sin(180^\circ - \theta_2)}$$

$$\therefore \quad \frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

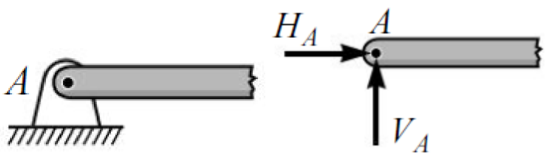
Hence proved.

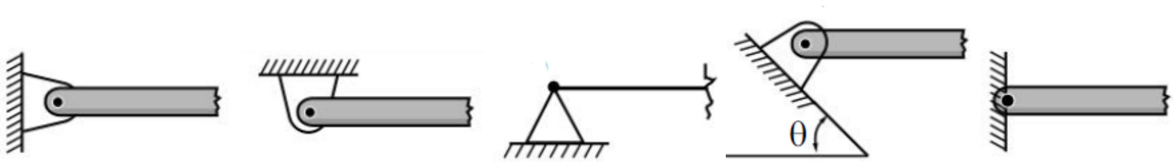


3. Describe types of supports used for beams. Marks 2
4. Discuss on different types of supports with neat sketches. Marks 5

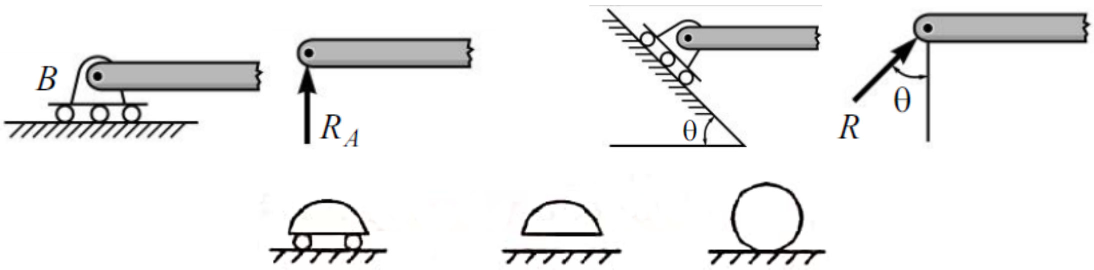
Types of Support:

1. Hinge (Pin) Support: The hinge support allows free rotation about the pin end but it does not allow linear displacement of that end. Since linear displacements are restricted in horizontal and vertical directions, the reactions offered at hinge support are H_A and V_A. E.g., doors on hinges, laptops, suitcases, etc.

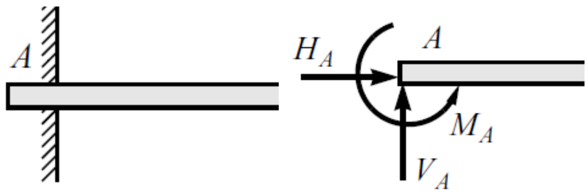




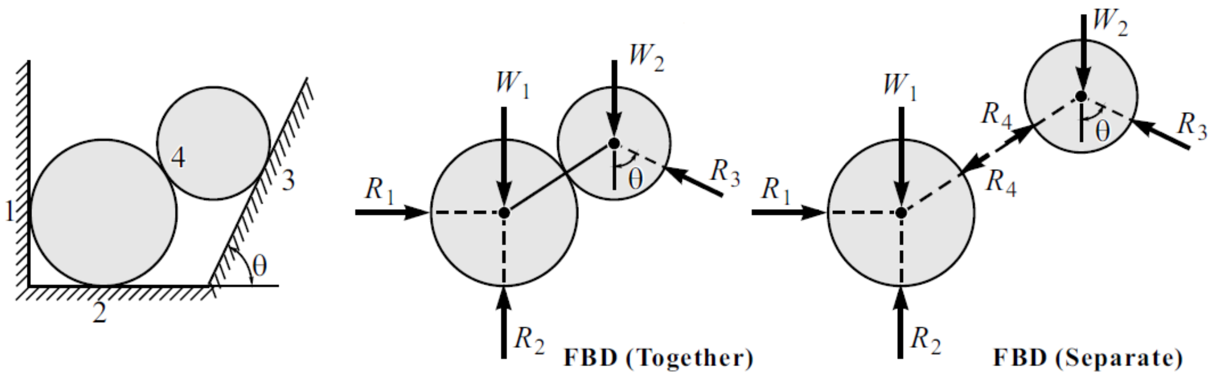
2. Roller Support: A roller support is equivalent to a frictionless surface. It can only exert a force that is perpendicular to the supporting surface. The roller support is free to roll along the surface on which it rests. Since the linear displacement in normal direction to surface of roller is restricted, it offers a reaction in normal direction to surface of roller (R_A). E.g., sliding doors, drawers, etc. Collar or slider free to move along smooth guides are also similar to roller support since they can support force normal to guide only.



3. Fixed Support: When the end of a beam is fixed then that support is said to be fixed support. Fixed support neither allows linear displacement nor rotation of a beam. Due to these restrictions, the reactions offered at fixed supports are horizontal component H_A, vertical component V_A and couple component M_A.



4. Smooth Surface Contact: When a body is in contact with a smooth (frictionless) surface at only one point, the reaction is a force normal to the surface, acting at the point of contact.



5. Inextensible String, Cable, Belt Rope, Cord, Chain or Wire: The force developed in rope is always a tension away from the body in the direction of rope. When one end of a rope is connected to a body, then the rope is not to be considered as a part of the system and it is replaced by tension in FBD.

5. What are the conditions of equilibrium for concurrent, parallel & general force system?

COE for Various Force System:

Parallel Force System: One of the following sets of equations can be used

$$\sum F = 0 \;\&\; \sum M = 0$$

$\sum M_A = 0$ & $\sum M_B = 0$ (line AB should not be parallel to the forces)

General Force System: One of the following sets of equations can be used

$\sum F_x = 0, \sum F_y = 0$ & $\sum M = 0$

$\sum F_x = 0, \sum M_A = 0$ & $\sum M_B = 0$ (line AB should not be perpendicular to the x-axis)

$\sum M_A = 0, \sum M_B = 0$ & $\sum M_C = 0$ (A, B, & C should not be collinear)

6. Describe F.B.D. and its importance in the analysis of problems.

Free Body Diagram (FBD):

The Free Body Diagram (FBD) is a sketch of the body showing all active and reactive forces that acts on it after removing all supports with consideration of geometrical angles and distance given.

To investigate the equilibrium of a body, we remove the supports and replace them by the reactions which they exert on the body. The first step in equilibrium analysis is to identify all the forces that act on the body, which is represented by a free body diagram. Therefore, the free body diagram is the most important step in the solution of problems in mechanics.

Importance of FBD:

1. The sketch of FBD is the key step that translates a physical problem into a form that can be analysed mathematically.
2. The FBD is the sketch of a body, a portion of a body or two or more connected bodies completely isolated or free from all other bodies, showing the force exerted by all other bodies on the one being considered.
3. FBD represents all active (applied) forces and reactive (reactions) forces. Forces acting on the body that are not provided by the supports are called active force (weight of the body and applied forces). Reactive forces are those that are exerted on a body by the supports to which it is attached.
4. FBD helps in identifying known and unknown forces acting on a body.
5. FBD helps in identifying which type of force system is acting on the body so by applying appropriate condition of equilibrium, the required unknowns are calculated.

Module 4.2 Friction:

1. Define laws of friction or State laws of dry friction. Marks 5
2. Discuss laws of friction. Marks 5

Laws of Friction:

1. The frictional force is always tangential to the contact surface and acts in a direction opposite to that in which the body tends to move.
2. The magnitude of frictional force is self-adjusting to the applied force till the limiting frictional force is reached and at the limiting frictional force the body will have impending motion.
3. Limiting frictional force F_{max} is directly proportional to normal reaction ($F_{max} = \mu_s N$).
4. For a body in motion, kinetic frictional force F_k developed is less than that of limiting frictional force F_{max} and the relation $F_k = \mu_k N$ is applicable.
5. Frictional force depends upon the roughness of the surface and the material in contact.
6. Frictional force is independent of the area of contact between the two surfaces.
7. Frictional force is independent of the speed of the body.
8. Coefficient of static friction μ_s is always greater than the coefficient of kinetic friction μ_s.

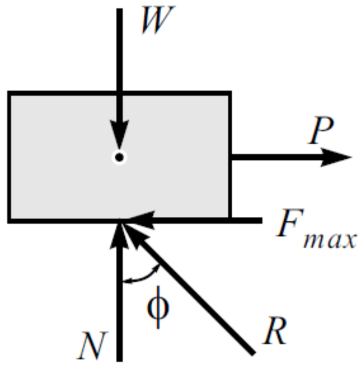
3. Describe angle of friction and cone of friction of with neat sketches. Marks 5
4. Define angle of friction and angle of repose. Show that angle of angle of fraction is equal to angle of repose. Marks 4
5. Explain angle of friction, angle of repose and the relation between the two. Marks 4

Angle of Friction: It is the angle made by the “resultant of the limiting frictional force F_{max} and the normal reaction N” with the “normal reaction”.

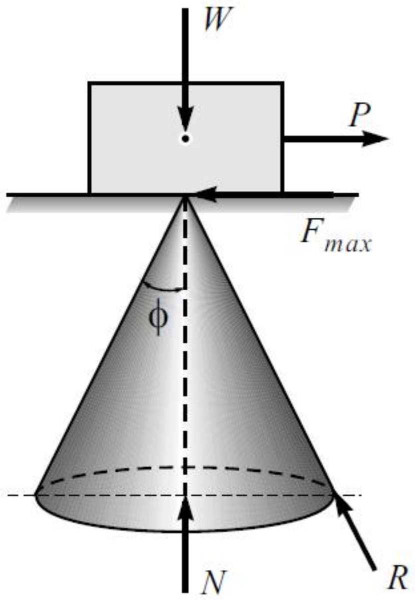
$$R = \sqrt{F_{max}^2 + N^2}$$

$$tan\, tan\, \phi = \frac{F_{max}}{N} = \mu_s$$

$$\phi = \mu_s$$



Cone of Friction: When the applied force P is just sufficient to produce the impending motion of given body, angle of friction ϕ is obtained. If the direction of applied force P is gradually changed through 360°, the resultant R generates a right circular cone with semi vertex angle equal to ϕ. This is called the cone of friction.



Angle of Repose: It is the minimum angle of inclination of a plane with the horizontal at which the body so kept will just begin to slide down on it without the application of any external force (due to self-weight).

$\sum F_x = 0 \Rightarrow$

$$F_{max} - W \sin \sin \alpha = 0$$

$$\mu_s N - W \sin \sin \alpha = 0$$

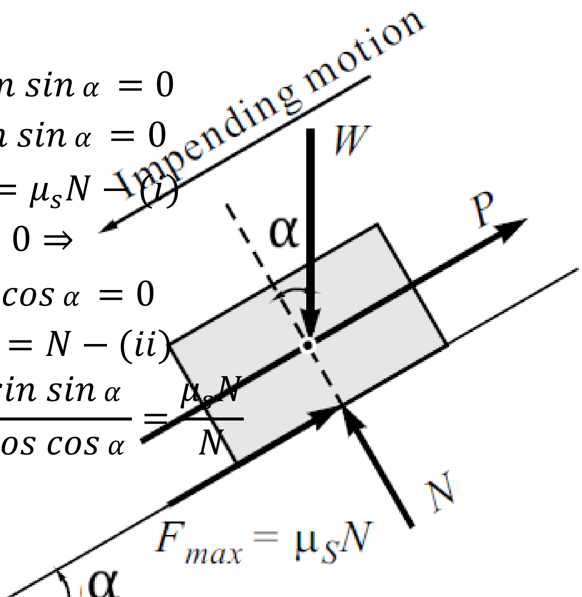
$$W \sin \sin \alpha = \mu_s N - (i)$$

$$\sum F_y = 0 \Rightarrow$$

$$N - W \cos \cos \alpha = 0$$

$$W \cos \cos \alpha = N - (ii)$$

$$(i) \div (ii) \Rightarrow \frac{W \sin \sin \alpha}{W \cos \cos \alpha} = \frac{\mu_s N}{N}$$



$$\tan \tan \alpha = \mu_s = \tan \tan \phi$$
$$\therefore \alpha = \phi$$

Hence, the angle of repose is equal to the angle of friction.

The above relation also shows that the angle of repose is independent of the weight of the body and it depends on μ.

6. Why coefficient of static friction is greater than coefficient of kinetic friction?

Static friction has greater value than the kinetic friction because static friction acts when the body is at rest and there is much more inter molecular attraction between the object and the surface for a long time which is required to be overcome first.

Whereas in kinetic friction an object is in moving condition. The contact of the object with the surface is for short time duration and have a less inter molecular attraction between them. As a result, only a less force is needed to move a moving object than the static object.

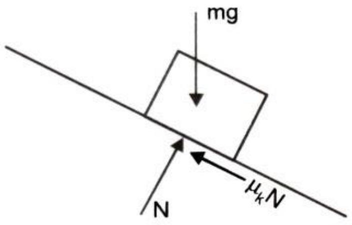
Module 5 – Kinetics of particle

Module 5.1: Force and acceleration

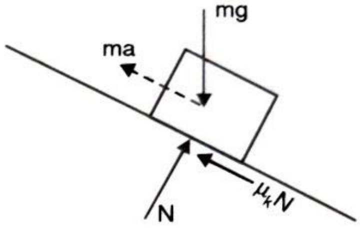
1. Explain D'Alembert's Principle Marks 2

D'Alembert's Principle: If the equation $\sum F = ma$ is rearranged as $\sum F - ma = 0$, treating the "– ma" as an inertia force, the system can be considered to be in dynamic equilibrium.

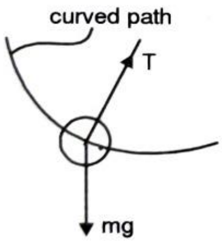
This is useful only in that COE can be used just like in static equilibrium situations, but it is not a realistic analysis.



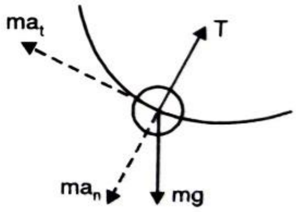
Actual forces acting on the block



Actual forces + Inertia force creates a state of dynamic equilibrium.



Actual forces acting on the pendulum



Actual forces + Inertia forces create a state of dynamic equilibrium

Applying the D'Alembert's Principle, we get a very similar equation as in NSL,

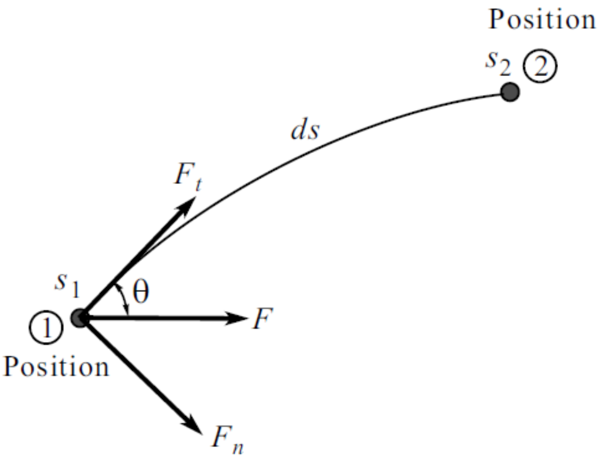
$$\sum F_x - ma_x = 0$$
$$\Rightarrow P - mg\ sin\ sin\ \theta - \mu_k N - ma = 0$$

Module 5.2: Work energy principle

1. State and derive work energy principle Marks 5
2. Explain work energy principle for a particle Marks 2
3. Explain work energy principle and write its mathematical expression. Marks 5

Work Energy Principle:

Work done by the forces acting on a particle during some displacement is equal to the change in kinetic energy during that displacement.



Consider the particle having mass m is acted upon by a force F and moving along a path as shown. Let v_1 and v_2 be the velocities of the particle at position 1 and position 2 and the corresponding displacement s_1 and s_2 respectively.

By Newton's second law in the tangential direction, we have,

$$\sum F_t = ma_t$$

$$F\ cos\ cos\ \theta\ = ma_t = m\frac{dv}{dt} = m\frac{dv}{ds} \times \frac{ds}{dt} = mv\frac{dv}{ds}$$

$$F\ cos\ cos\ \theta\ ds = mv\ dv$$

Integrating both sides, we have,

$$\int_{s_1}^{s_2} F\ cos\ cos\ \theta\ ds = \int_{v_1}^{v_2} mv\ dv$$

$$U_{1-2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

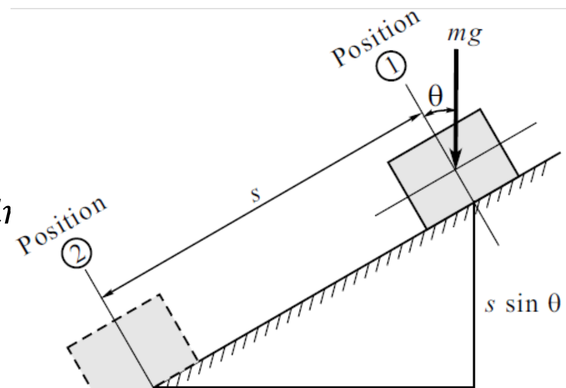
$$\therefore Work\ done = \Delta Kinetic\ Energy$$

4. Explain the concept of work of spring. Also explain when it will be positive and when it will be negative. Marks 2
5. Explain: a) work done by a force
   b) Work done by a weight force
   c) Work done by a frictional force
   d) Work done by a spring force

Work Done by Weight Force:

Work done = Component of weight in the direction of displacement × Displacement

$$U = mg\ sin\ si$$

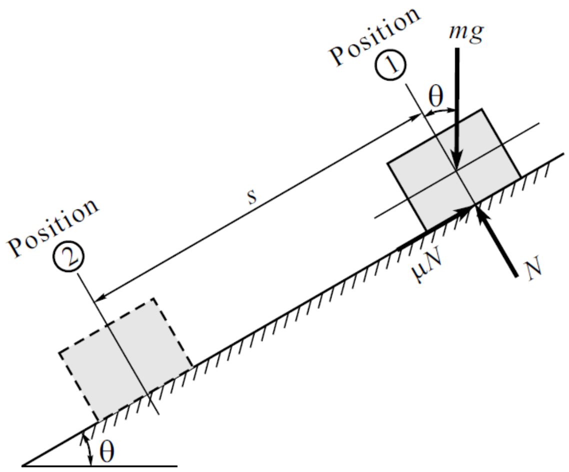


OR

Work done = Weight force × Displacement
in the direction of weight force

$$U = mg \times s \sin \sin \theta$$

Work Done by Frictional Force:



Work done = – Friction × Displacement

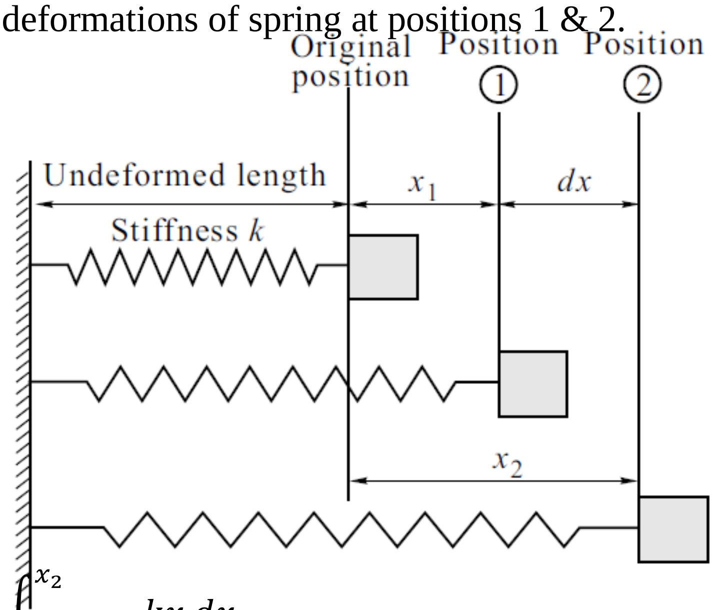
$$U = \mu N ›$$

Work done by friction force is negative because direction of frictional force and displacement is opposite.

Work done by normal reaction (N) and component of weight force perpendicular to inclined plane (mg cos θ) is zero.

Work Done by Spring Force:

Consider a spring of stiffness k as shown in the figure, with some undeformed (free/original) length. Let x_1 & x_2 be deformations of spring at positions 1 & 2.



∴ Spring force F = −k × x

where k is the spring stiffness (N/m)
x is the deformation of spring (m)
– ve sign indicates direction of spring force acting towards original position.

Work done = Spring force × Deformation

$$U = \int_{x_1}^{x_2} - kx\, dx$$

$$U = -\frac{1}{2}k(x_2^2 - x_1^2) \quad OR \quad U = \frac{1}{2}k(x_1^2 - x_2^2)$$

Module 5.3: Impulse Momentum Principle

1. Derive an equation for law of conservation of momentum.

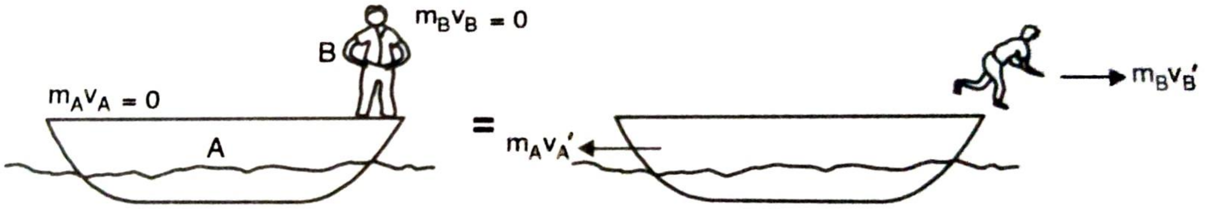
Marks 2

Conservation of Momentum Equation:

In a system, if the resultant force is zero, the impulse momentum equation reduces to final momentum equal to initial momentum. Such situation arises in many cases because the force system consists of only action and reaction on the elements of the system. The resultant force is zero, only when entire system is considered, but not when the free body of each element of the system is considered.

If a man jumps off a boat, the action of the man is equal and opposite to the reaction of the boat. Hence, the resultant is zero in the system. Similar equation holds good when we consider the system of a gun and shell.

Initial momentum = Final momentum



$$mv_1 + Impulse_{1-2} = mv_2$$
$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \ (\because Impulse_{1-2} = 0)$$
$$m_A v_A' = -m_B v_B' \ (\because m_A v_A = 0, m_B v_B = 0)$$

"*For dynamic situations involving a system of particles, if the net impulse is zero, the momentum of the system is conserved*."

2. Describe elastic and inelastic collision

Marks 5

A collision occurs when two objects come in direct contact with each other. It is the situation in which two or more bodies exert forces on each other in about a relatively short time.
There are two types of collision, such as:

1. Elastic collision
2. Inelastic collision

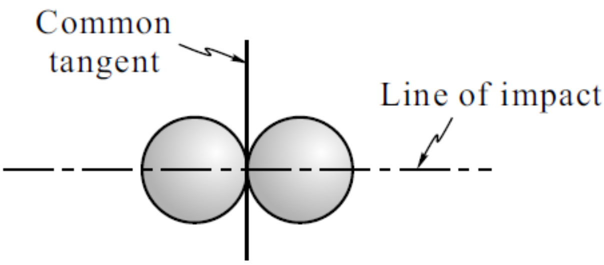
Elastic collisions:
A state where there is no net loss in kinetic energy in the system as the result of the collision is called an elastic collision.

Inelastic collisions:
A type of collision where this is a loss of kinetic energy is called an inelastic collision. The lost kinetic energy is transformed into thermal energy, sound energy, and material deformation.

3. Discuss on direct central and oblique central impact with neat sketches. Marks 5
4. Define the terms neat sketches: direct impact, oblique impact and line of impact. Marks 6

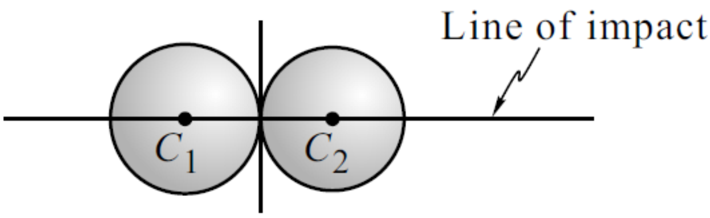
Impact: A collision of two bodies, which occurs for a very small interval of time and during which the two bodies exert relatively very large forces on each other, is called an impact.

Line of Impact: The common normal to the surfaces of two bodies in contact during the impact is called line of impact, and is perpendicular to the common tangent.

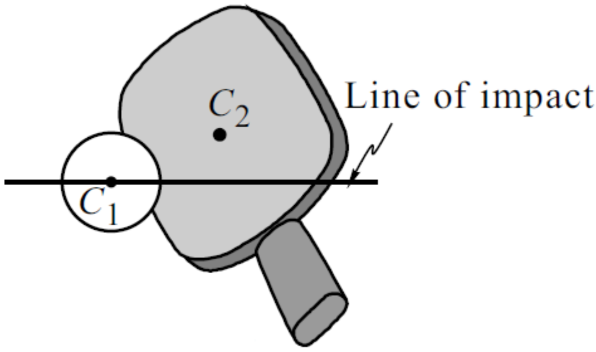


Types of Impact:

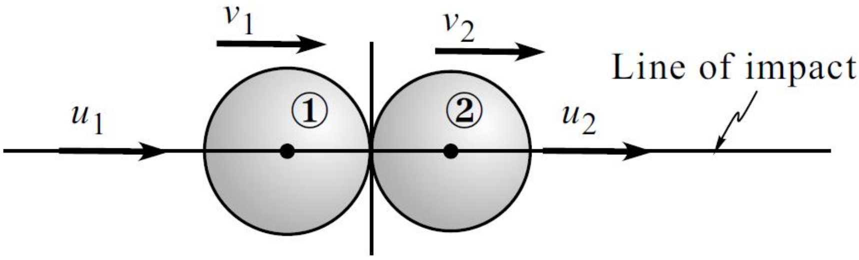
Central Impact: When the mass centres C_1 and C_2 of the colliding bodies lie on the line of impact, it is called central impact.



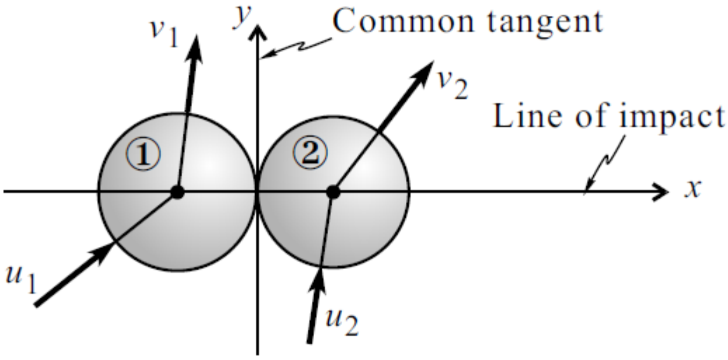
Non-Central Impact: When the mass centres C_1 and C_2 of the colliding bodies do not lie on the line of impact, it is called non-central or eccentric impact.



Direct Central Impact: When the direction of motion of the mass centres of two colliding bodies is along the line of impact then we say it is direct central impact. Here, the velocities of two bodies collision are collinear with the line of impact.

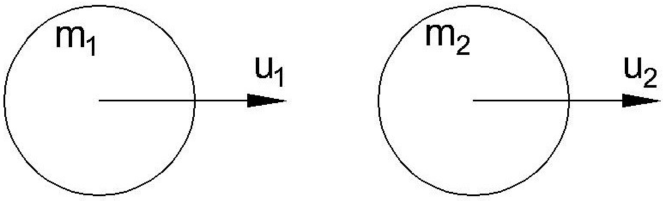


Oblique Central Impact: When the direction of motion of the mass centres of one or two colliding bodies is not along the line of impact (i.e., at the same angle with the line of impact) then we say it is oblique central impact. Here the velocities of two bodies collision are not collinear with the line of impact.

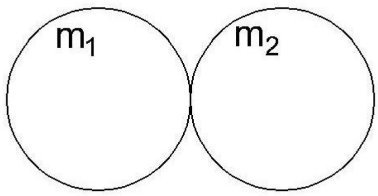


Direct Central Impact:

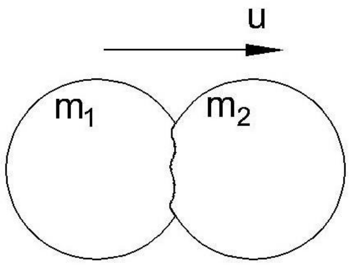
1. Two particles with masses m_1 & m_2 are traveling at velocities u_1 & u_2. If u_1 is greater than u_2, impact will occur.



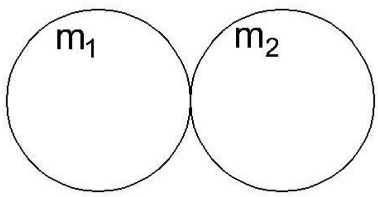
2. When impact takes place, the period of impact is made of period of deformation and the period of restitution (regaining the original shape).



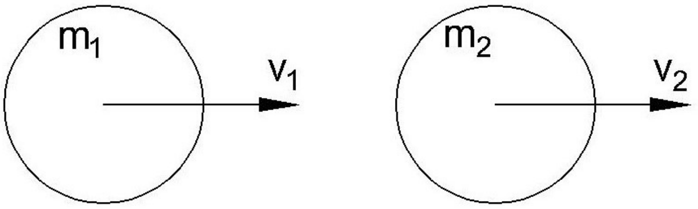
3. During the period of deformation, the particles exert large impulsive forces on each other. The deformation of both particles continues till maximum deformation, when both particles are momentarily united and are moving together with common velocity u.



4. Now the period of restitution begins, during which the particles restore their shape. The particles may restore their shape completely, partially, or not at all, depending upon their properties. During this period also some impulsive force is exerted by the particles on each other. At the end of this period the particles separate from each other.



5. The particles now move independently with new velocities v_1 and v_2.



5. State and derive the impulse momentum principle.

Impulse Momentum Equation:

From Newton's Second Law, we have,

$$F = \frac{d(mv)}{dt} \Rightarrow F\,dt = d(mv)$$

$$\int_{t_1}^{t_2} F\,dt = \int_{v_1}^{v_2} d(mv) = mv_2 - mv_1$$

$$\therefore Impulse_{1-2} = \Delta Momentum$$

$$mv_1 + Impulse_{1-2} = mv_2$$

Or $\quad$ Initial Momentum + Impulse Imparted = Final Momentum

This gives rise to the Principle of Impulse Momentum which states that, "*for a particle or a system of particles acted upon by forces during a time interval, the total impulse acting on the system is equal to the difference between the final momentum and initial momentum during that period*

6. State the principle of conservation of momentum.

Discussed above in Q.1