DIFFERENTIAL EQUATIONS

DEFINITIONS: An equation which contains one independent and one or more dependent variables and their derivatives $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots$ or the differential dx , dy is called a **Differential Equation**

The following are some differential equations:

(i) $\left(\frac{dy}{dx}\right)^2 - \cos x = 0$ **(ii)** $y^2\, dx + x^2 dy = 0$ **(iii)** $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - y = 0$

(iv) $\frac{d^3y}{dx^3} + 5\frac{dy}{dx} - y = 0$

ORDER: The order of differential equation is the order of the highest order derivative occurring in it.

DEGREE: The degree of differential equation is the degree of the highest order derivative occurring in it, when the D.E is so written that the derivative are free from negative or fractional indices.

Remark: The order and the degree of D.E are positive integers. Hence, in order to find the order and degree of a given D.E we have to remove the negative or fractional indices, if they occur in the D.E

EXAMPLES OF ORDER AND DEGREE

1. The given DE is $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = e^x$

This DE has highest order derivative $\frac{d^2y}{dx^2}$ with power 2

∴ the given DE is of order 2 and degree 2

2. The given DE is $\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 + 7 = 0$

This DE has highest order derivative $\frac{d^2y}{dt^2}$ with power 1

∴ the given DE is of order 2 and degree 1

3. The given DE is $\frac{d^4y}{dx^4} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3$

This DE has highest order derivative $\frac{d^4y}{dx^4}$ with power 1

∴ the given DE is of order 4 and degree 1

4. The given DE is $(y''')^2 + 2(y'')^2 + 3(y') + 4y = 0$

This can be written as: $\left(\frac{d^3y}{dx^3}\right)^2 + 2\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} + 4y = 0$

This DE has highest order derivative $\frac{d^3y}{dx^3}$ with power 2

$\therefore$ order $= 3$ and degree $= 2$

5. The given DE is $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$

This DE has highest order derivative $\frac{d^2y}{dx^2}$

$\therefore$ order $= 2$

Since this DE cannot be expressed as a polynomial in differential coefficients, the degree is not defined

6. The given DE is $\sqrt{1 + \frac{1}{\left(\frac{dy}{dx}\right)^2}} = \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$

On squaring both sides, we get, $1 + \frac{1}{\left(\frac{dy}{dx}\right)^2} = \left(\frac{d^2y}{dx^2}\right)^3$

$\therefore \left(\frac{dy}{dx}\right)^2 + 1 = \left(\frac{d^2y}{dx^2}\right)^3 \cdot \left(\frac{dy}{dx}\right)^2$

This DE has highest order derivative $\frac{d^2y}{dx^2}$ with power 3

$\therefore$ order $= 2$ and degree $= 3$

7. The given DE is $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}} = 7 \cdot \frac{d^2y}{dx^2}$

On cubing both sides, we get, $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^7 = 7^3 \cdot \left(\frac{d^2y}{dx^2}\right)^3$

This DE has highest order derivative $\frac{d^2y}{dx^2}$ with power 3

$\therefore$ order $= 2$ and degree $= 3$

8. The given DE is $\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{6}} \cdot \left(\frac{dy}{dx}\right)^{\frac{1}{3}} = 5$

$\therefore \left(\frac{d^3y}{dx^3}\right) \cdot \left(\frac{dy}{dx}\right)^2 = 5^6$

This DE has highest order derivative $\frac{d^3y}{dx^3}$ with power 1

$\therefore$ order $= 3$ and degree $= 1$

9. The given DE is $e^{\frac{dy}{dx}} + \frac{dy}{dx} = x$

This DE has highest order derivative $\frac{dy}{dx}$

$\therefore$ order $= 1$

Since this DE cannot be expressed as a polynomial in differential coefficients, the degree is not defined

10. The given DE is $\begin{vmatrix} x^3 & y^2 & 3 \\ 2x^2 & 3y\frac{dy}{dx} & 0 \\ 5x & 2\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] & 0 \end{vmatrix} = 0$

$\therefore x^3(0-0) - y^2(0-0) + 3\left[4x^2\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} - 15xy\frac{dy}{dx}\right] = 0$

$\therefore 4x^2y\frac{d^2y}{dx^2} + 4x^2\left(\frac{dy}{dx}\right)^2 - 15xy\frac{dy}{dx} = 0$

This DE has highest order derivative $\frac{d^2y}{dx^2}$ with power 1

$\therefore$ order $= 2$ and degree $= 1$

11. The given DE is $\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right) + y = 2\sin x$

In this equation the highest ordered derivative is $\frac{d^2y}{dx^2}$ and its power is one.

Therefore this equation has order 2 and degree 1

12. The given DE is $\left(\frac{d^2y}{dx^2}\right) = \sqrt[3]{1 + \left(\frac{dy}{dx}\right)^2}$

In this equation, the highest ordered derivative is $\frac{d^2y}{dx^2}$ and

therefore order of the differential equation is 2

But to determine the degree of this equation, first we have to remove cube root by raising the power on both side by 3

$\therefore \left(\frac{d^2y}{dx^2}\right)^3 = 1 + \left(\frac{dy}{dx}\right)^2$

Now we note that the power of the highest ordered derivative is 3 and therefore the degree of this differential equation is 3

13. The given DE is $\frac{dy}{dx} = \frac{2\sin x + 3}{\left(\frac{dy}{dx}\right)}$

The equation can be written in the form $\left(\frac{dy}{dx}\right)^2 = 2\sin x + 3$

Now, highest ordered derivative is $\frac{dy}{dx}$ and its power is two.

Hence, the order of the equation is 1 and degree is 2

14. The given DE is $\frac{d^2y}{dx^2} + \frac{dy}{dx} + x = \sqrt{1 + \frac{d^3y}{dx^3}}$

In the given equation, the highest ordered derivative is $\frac{d^3y}{dx^3}$ and therefore the order of this equation is 3

To determine the degree, square on both sides,

$\therefore \left(\frac{d^2y}{dx^2} + \frac{dy}{dx} + x\right)^2 = 1 + \frac{d^3y}{dx^3}$

Now, the power of highest ordered derivative is 1 and hence the degree of the equation is 1

15. The given DE is $\left[\frac{d^3y}{dx^3} + x\right]^{\frac{5}{2}} = \frac{d^2y}{dx^2}$

On squaring both sides, we get, $\left[\frac{d^3y}{dx^3} + x\right]^5 = \left(\frac{d^2y}{dx^2}\right)^2$

This DE has highest order derivative $\frac{d^3y}{dx^3}$ with power 5

$\therefore$ order $= 3$ and degree $= 5$

16. The given DE is $x + \frac{dy}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

On squaring both sides, we get, $\left(x + \frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$

$\therefore x^2 + 2x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$

$\therefore 2x\frac{dy}{dx} + x^2 = 1$

This DE has highest order derivative $\frac{dy}{dx}$ with power 1

$\therefore$ order $= 1$ and degree $= 1$