K J Somaiya College of Engineering, Vidyavihar, Mumbai

(A Constituent College of SVU)

Engineering Mechanics Notes

Module 4 – Equilibrium of Force System and Friction

Module Section 4.2 – Friction

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References: Engineering Mechanics, by M. D. Dayal & Engineering Mechanics – Statics and Dynamics, by N. H. Dubey.

Friction: When the surface of a body slides along/moves over or tends to slide along/move over the surface of another body, a force opposing the motion develops tangentially between the contact surfaces. This force which opposes the movement or tendency of movement is called a frictional force or simply friction.

Friction is due to the resistance offered to motion by minutely projecting portions at the contact surfaces. These microscopic projections get interlocked. To a small extent, the material of two bodies in contact also produces resistance to motion due to intramolecular force of attraction, i.e., adhesive properties.

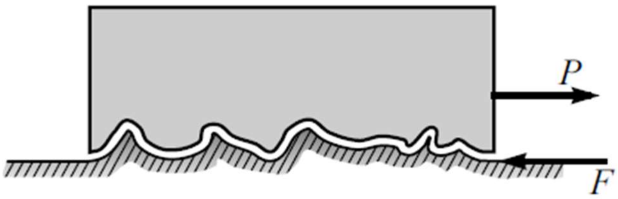


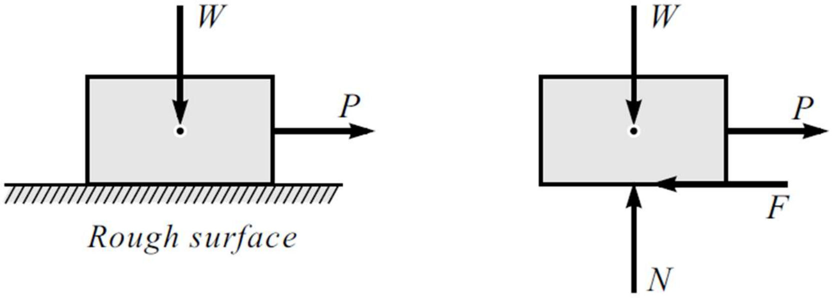
Fig. : Magnified Microscopic View of Rough Surface

Dry Friction: Dry friction develops when the unlubricated surface of two solids are in contact under a condition of sliding or a tendency to slide. A frictional force tangent to the surfaces of contact is developed both during the interval leading up to impending slippage and while slippage takes place. The direction of the frictional force always opposes the relative motion or impending motion. This type of friction is also known as Coulomb friction.

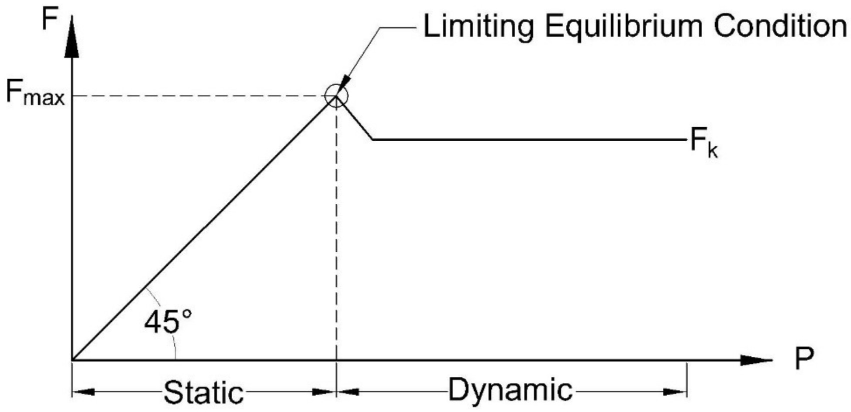
Fluid Friction: Fluid friction is developed when adjacent layers in a fluid (liquid or gas) are moving at different velocities. This motion gives rise to frictional forces between fluid elements and these forces depend on the relative velocity between layers. Fluid frictions depends not only on the velocity gradients within the fluid but also on viscosity of fluid, which is a measure of its resistance to shearing action between fluid layers. Fluid friction is treated in the study of fluid mechanics and is beyond the scope of this text. So, we are going to deal with dry friction only.

Limiting Equilibrium Condition:

Consider a block of weight W resting on a horizontal surface. The contacting surface possesses a certain amount of roughness. Let P be the horizontal force applied which will vary continuously from zero to a value sufficient to just move the block and then to maintain the motion. The free body diagram of the block shows active forces (i.e., Applied force P and weight of block W) and reactive forces (i.e., normal reaction N and tangential frictional force F).



As applied force P increases, the frictional force F is equal in magnitude and opposite in direction. However, there is a limit beyond which the magnitude of the frictional force cannot be increased. If the applied force is more than this maximum frictional force (F_{max}), there will be movement of one body over the other. Once the body begins to move, there is decrease in frictional force F from maximum value observed under static condition. The frictional force between the two surfaces when the body is in motion is called kinetic or dynamic friction (F_k).



<u>Limiting Frictional Force (F_{max})</u>: It is the maximum frictional force developed at the surface max when the block is at the verge of motion (impending motion).

<u>Coefficient of Friction</u>: By experimental evidence it is proved that limiting frictional force is directly proportional to normal reaction.

<u>Coefficient of Static Friction</u>: The ratio of limiting frictional force (F_{max}) and normal reaction (N) is called the coefficient of static friction (μ_s).

$$F_{max} \propto N \Rightarrow F_{max} = \mu_s N \Rightarrow \mu_s = \frac{F_{max}}{N}$$

Coefficient of Kinetic Friction: The ratio of kinetic frictional force (F_k) and normal reaction (N) is called the coefficient of static friction (μ_k).

$$F_k \propto N \Rightarrow F_k = \mu_k N \Rightarrow \mu_k = \frac{F_k}{N}$$

Kinetic friction is always less than limiting friction. And μ is always less than 1.

Laws of Friction:

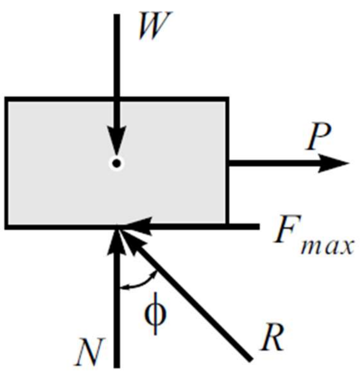
1. The frictional force is always tangential to the contact surface and acts in a direction opposite to that in which the body tends to move.
2. The magnitude of frictional force is self-adjusting to the applied force till the limiting frictional force is reached and at the limiting frictional force the body will have impending motion.
3. Limiting frictional force F_{max} is directly proportional to normal reaction ($F_{max} = \mu_s N$).
4. For a body in motion, kinetic frictional force F_k developed is less than that of limiting frictional force F_{max} and the relation $F_k = \mu_k N$ is applicable.
5. Frictional force depends upon the roughness of the surface and the material in contact.
6. Frictional force is independent of the area of contact between the two surfaces.
7. Frictional force is independent of the speed of the body.
8. Coefficient of static friction μ_s is always greater than the coefficient of kinetic friction μ_s.

Angle of Friction: It is the angle made by the "resultant of the limiting frictional force F_{max} and the normal reaction N" with the "normal reaction".

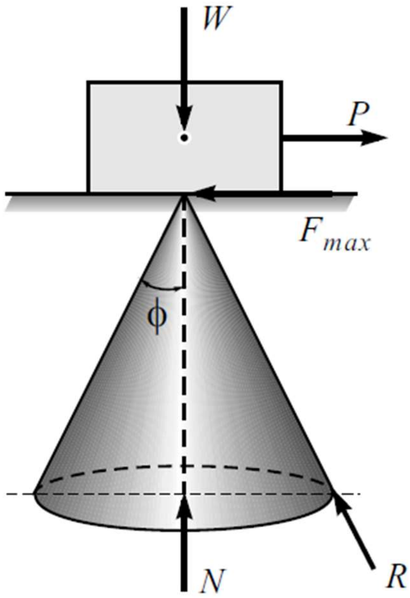
$$R = \sqrt{F_{max}^2 + N^2}$$

$$\tan \phi = \frac{F_{max}}{N} = \mu_s$$

$$\phi = \tan^{-1} \mu_s$$



Cone of Friction: When the applied force P is just sufficient to produce the impending motion of given body, angle of friction ϕ is obtained. If the direction of applied force P is gradually changed through 360°, the resultant R generates a right circular cone with semi vertex angle equal to ϕ. This is called the cone of friction.



Angle of Repose: It is the minimum angle of inclination of a plane with the horizontal at which the body so kept will just begin to slide down on it without the application of any external force (due to self-weight).

$$\sum F_x = 0 \Rightarrow$$

$$F_{max} - W \sin\alpha = 0$$

$$\mu_s N - W \sin\alpha = 0$$

$$W \sin\alpha = \mu_s N - (i)$$

$$\sum F_y = 0 \Rightarrow$$

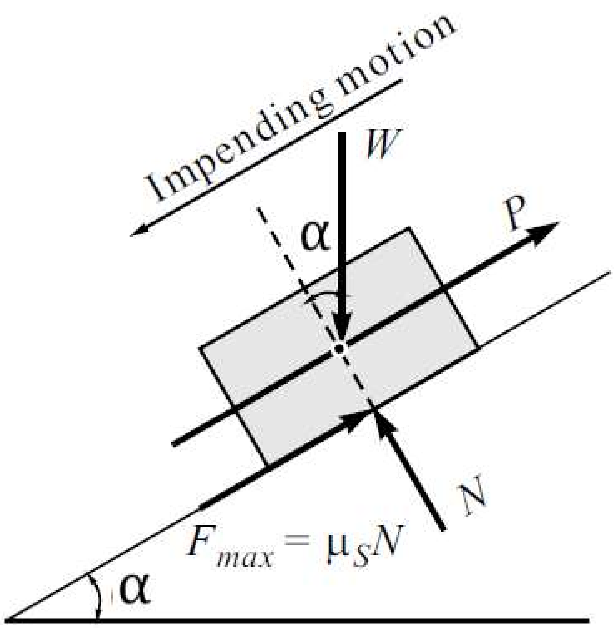
$$N - W\cos\alpha = 0$$

$$W\cos\alpha = N - (ii)$$

$$(i) \div (ii) \Rightarrow \frac{W\sin\alpha}{W\cos\alpha} = \frac{\mu_s N}{N}$$

$$\tan\alpha = \mu_s = \tan\phi$$

$$\therefore \alpha = \phi$$



Hence, the angle of repose is equal to the angle of friction.

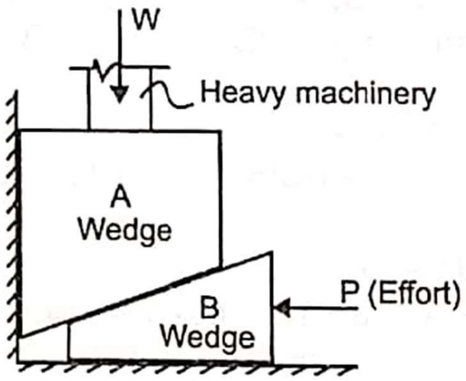
The above relation also shows that the angle of repose is independent of the weight of the body and it depends on μ.

Applications of Friction:

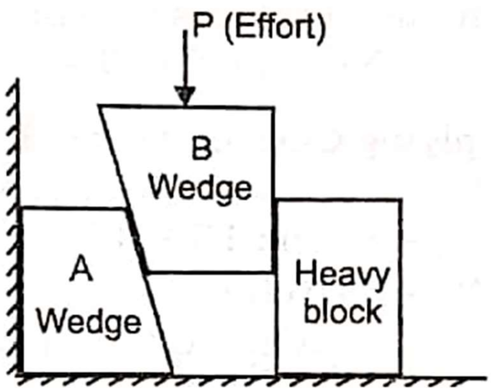
1. The running vehicle is controlled by applying brake to its tyre because of friction. The vehicle moves with better grip and does not slip due to appropriate friction between the tyre and the road.
2. One can walk comfortably on the floor because of proper gripping between floor and the sole of the shoes. It is difficult to walk on oily or soapy floor.
3. Belt and pulley arrangement permits loading and unloading of load effectively because of suitable friction.
4. Lift moves smoothly without slipping due to proper rope and pulley friction combination.
5. A simple lifting machine like screw jack functions effectively based on principle of wedge friction.

Wedges: A tapper shaped block (with very less angle of inclination) which are used for lifting or shifting or holding the heavy block by very less effort is called a wedge. The lifting or shifting of the distance is very small. While installing heavy machinery horizontal levelling is required with zero error. It is possible to adjust the small height by inserting wedge as a packing. Sometimes, combination of wedge is also used to push or shift heavy bodies by little distance. Simple lifting machine such as screw jack is based on principle of wedge which is used to raise or lower the heavy load by small effort.

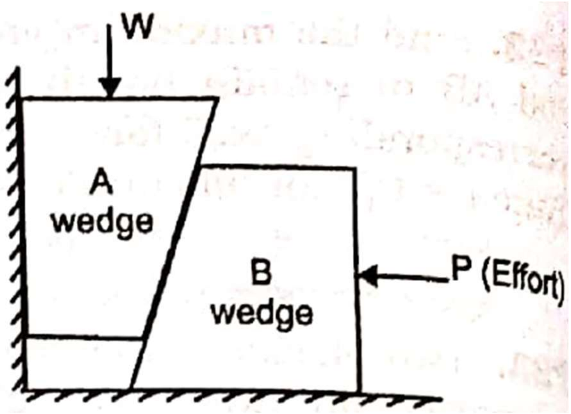
To lift heavy loads:



To slide heavy load:



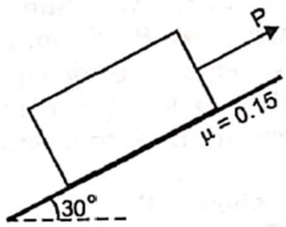
To hold the system in equilibrium:



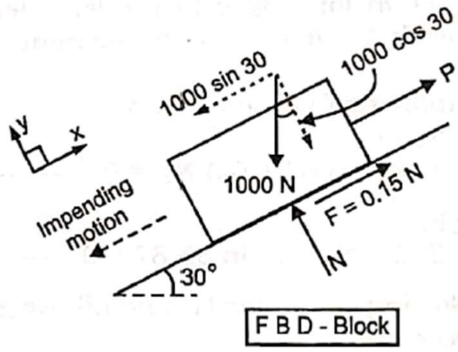
Ladders: Many a times, we come across the uses of ladder for reaching the higher height. Ladders are used by painters and carpenters who want to peg a nail in the wall for mounting a photo frame. We observe that care is taken to place the ladder at an appropriate angle with respect to ground and wall. We try to adjust the friction offered by the ground and wall in contact with ladder.

The forces acting on the ladder are normal reactions; frictional forces between the ground, the wall and the ladder, weight of the ladder and the weight of the man climbing the ladder. Considering the free body diagram of ladder, we get general force system. In ladder problems we need to use all 3 COEs.

Ex. 4.2 A block of weight 1000 N is kept on a rough inclined surface. A force P is applied parallel to plane to keep the block in equilibrium. Determine range of values of P for which the block will be in equilibrium. ***(MU May 13)***



Solution: Since we have to find range of values of P for equilibrium, let us first find P_{min}, which would be just sufficient to prevent the block from moving down the plane. The friction force therefore acts up the plane. Taking the axis as shown



Applying COE

$$\Sigma F_y = 0$$
$$N - 1000 \cos 30 = 0$$
$$\therefore \quad N = 866 \text{ N}$$

$$\Sigma F_x = 0$$
$$P - 1000 \sin 30 + 0.15 \text{ N} = 0$$
$$\therefore \quad P - 1000 \sin 30 + 0.15\,(866) = 0$$
$$\text{or} \quad P_{min} = 370.1 \text{ N}$$

When P is maximum for equilibrium of the block, it tends to just cause the block to move up the plane, thereby the friction force acts down the plane.

Applying COE

$\Sigma F_y = 0$

$N - 1000 \cos 30 = 0$

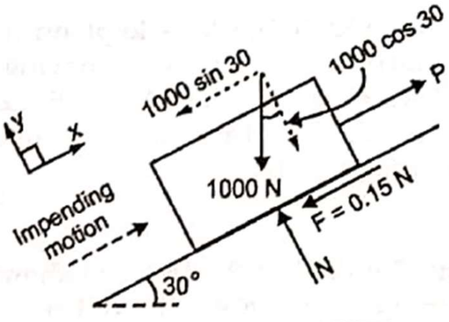
$\therefore \quad N = 866$ N

$\Sigma F_x = 0$

$P - 1000 \sin 30 - 0.15\ N = 0$

$\therefore \quad P - 1000 \sin 30 - 0.15\ (866) = 0$

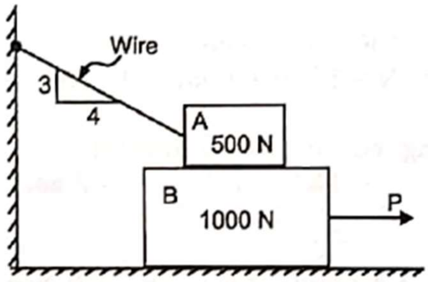
or $\quad P_{max} = 629.9$ N



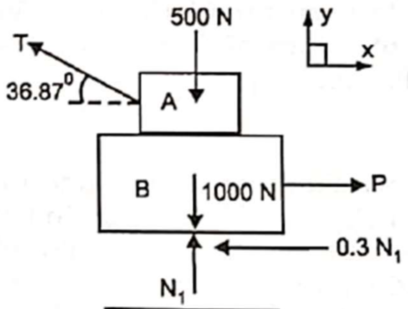
F B D - Block

The block is in equilibrium within the range $370.1\ N \le P \le 629.9\ N$ .... **Ans.**

Ex. 4.3 The upper block is tied to a vertical wall by a wire. Determine the horizontal force P required to just pull the lower block. Coefficient of friction for all surfaces is 0.3



Solution: Figure shows the FBD of the entire system. We find there are three unknowns viz. P, N_1 and T, and we have only two equations of equilibrium $\Sigma F_x = 0$ and $\Sigma F_y = 0$ for dimensionless blocks. We will therefore have to isolate the two blocks.



F B D - System

Figure shows the blocks isolated. Since block B tends to move to the right the friction force acts to the left. Hence for the block A, friction acts to the right.

Applying COE to block A

$\Sigma F_x = 0$

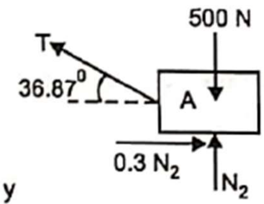
$-T \cos 36.87 + 0.3\ N_2 = 0$ -------- (1)

$\Sigma F_y = 0$

$-500 + N_2 + T \sin 36.87 = 0$ -------- (2)

Solving equations (1) and (2), we get

$N_2 = 408.2$ N



F B D - Block A

Applying COE to block B

$\Sigma F_y = 0$

$N_1 - N_2 - 1000 = 0$

$\therefore \quad N_1 - 408.2 - 1000 = 0$

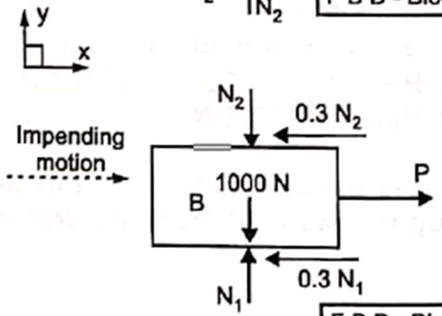
or $\quad N_1 = 1408.2$ N

$\Sigma F_x = 0$

$P - 0.3\ N_2 - 0.3\ N_1 = 0$

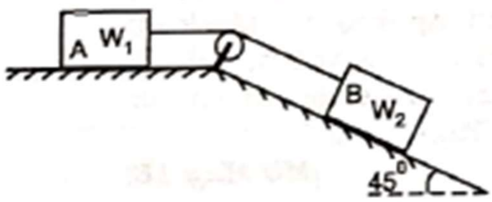
$\therefore \quad P - 0.3\ (408.2) - 0.3\ (1408.2) = 0$

or $\quad P = 544.9$ N -------- **Ans.**

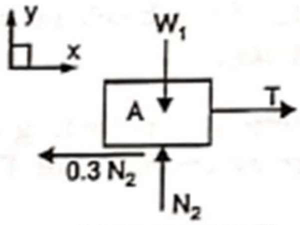


F B D - Block B

Ex. 4.4 Two blocks weighing W_1 and W_2 are connected by a string passing over a small smooth pulley as shown. If $\mu = 0.3$ for both the planes, find the minimum ratio W_1/W_2 required to maintain equilibrium.



Solution: The component of weight of block B is responsible for causing motion of the system to impend down the plane.



F B D - Block A

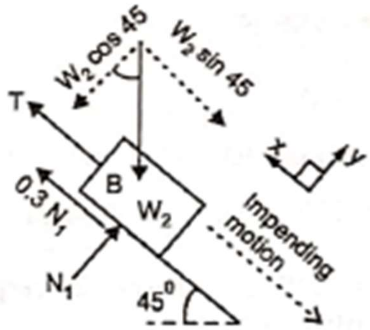
Isolating the two blocks

Taking different axes for A and B as shown

Applying COE to block B

$$\Sigma F_y = 0$$
$$N_1 - W_2 \cos 45 = 0$$
$$\therefore \quad N_1 = 0.707\ W_2 \qquad \text{--------(1)}$$

$$\Sigma F_x = 0$$
$$T + 0.3\ N_1 - W_2 \sin 45 = 0$$
$$\therefore \quad T + 0.3\ (0.707\ W_2) - W_2 \sin 45 = 0$$
$$\text{or} \quad T = 0.4949\ W_2 \qquad \text{--------(2)}$$



F B D - Block B

Applying COE to block A

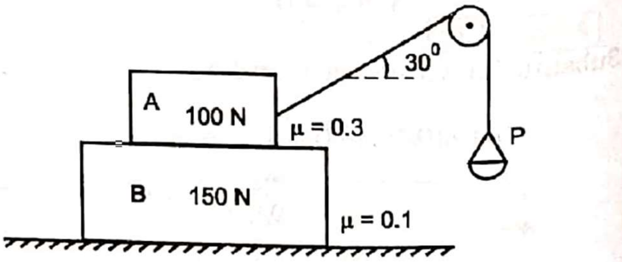
$$\Sigma F_y = 0$$
$$N_2 - W_1 = 0$$
$$\therefore \quad N_2 = W_1 \qquad \text{--------(3)}$$

$$\Sigma F_x = 0$$
$$T - 0.3\ N_2 = 0$$

Substituting values of T and N_2

$$0.4949\ W_2 - 0.3\ W_1 = 0$$
$$\therefore \quad \frac{W_1}{W_2} = 1.65 \qquad \text{----------} \textbf{ Ans.}$$

Ex. 4.6 Blocks A and B are resting on ground as shown. μ between ground and block is 0.1 and that between A and B is 0.3. Find the minimum value of P in the pan so that motion starts. ***(MU May 13)***



Solution: There are two possibilities. One is that block A moves over block B, while the other possibility is that both blocks A and B move together over the ground.

1st Possibility: Let block A move over block B

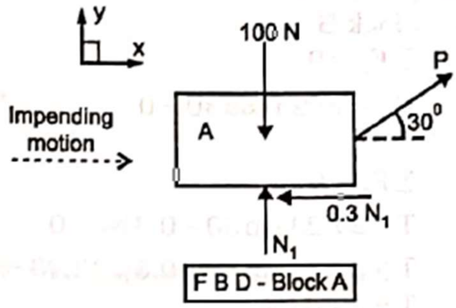
Applying COE

$\Sigma F_x = 0$

$P \cos 30 - 0.3\ N_1 = 0$ --------- (1)

$\Sigma F_y = 0$

$N_1 - 100 + P \sin 30 = 0$ --------- (2)



Solving equations (1) and (2)

$P = 29.53$ N

2nd Possibility: Both blocks A and B move together over the ground

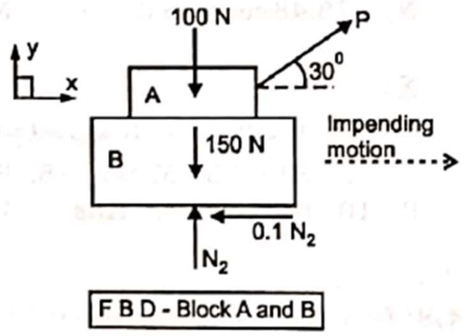
Applying COE

$\Sigma F_x = 0$

$P \cos 30 - 0.1\ N_2 = 0$ ----(3)

$\Sigma F_y = 0$

$N_2 - 100 - 150 + P \sin 30 = 0$ ----(4)

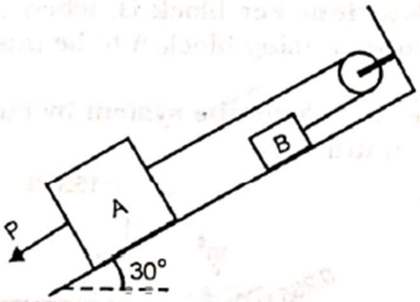


Solving equations (3) and (4)

$P = 27.29$ N

Since P required to move A and B together over the ground is less than P required for A to move over B, the system is set in motion at P = 27.29 N with both blocks moving together. .... **Ans.**

Ex. 4.7 Determine the force P to cause motion to impend. Take masses A and B as 8 kg and 4 kg respectively and coefficient of static friction as 0.3. The force P and rope are parallel to the inclined plane. Assume smooth pulley. ***(MU Dec 10)***



Solution: This is a system of two blocks connected to each other by a rope. As the force P applied to block A, impends to pull it down the plane, the block B impends to travel up the plane.

Isolating Block B

COE – Block B

$$\Sigma F_y = 0$$

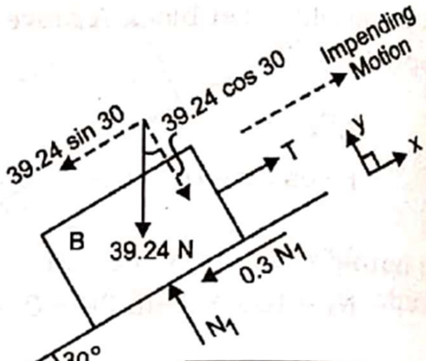
$$N_1 - 39.24\cos 30 = 0 \quad \therefore \quad N_1 = 33.98\,N$$

$$\Sigma F_x = 0$$

$$T - 39.24\sin 30 - 0.3\,N_1 = 0$$

$$T - 39.24\sin 30 - 0.3 \times 33.98 = 0$$

$$\therefore \quad T = 29.81\,N$$



F B D - Block B

Isolating Block A

COE – Block A

$$\Sigma F_y = 0$$

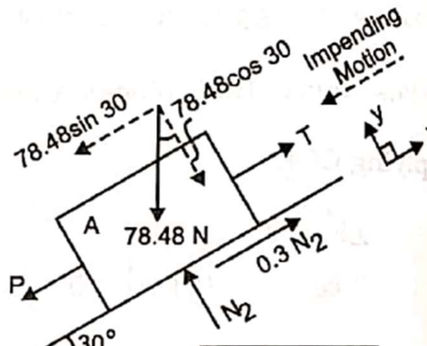
$$N_2 - 78.48\cos 30 = 0 \quad \therefore \quad N_2 = 67.96\,N$$

$$\Sigma F_x = 0$$

$$-P + T + 0.3\,N_2 - 78.48\sin 30 = 0$$

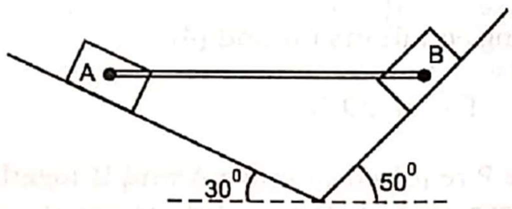
$$-P + 29.81 + 0.3 \times 67.96 - 78.48\sin 30 = 0$$

$$\therefore \quad P = 10.96\,N \quad \text{......... } \textbf{Ans.}$$



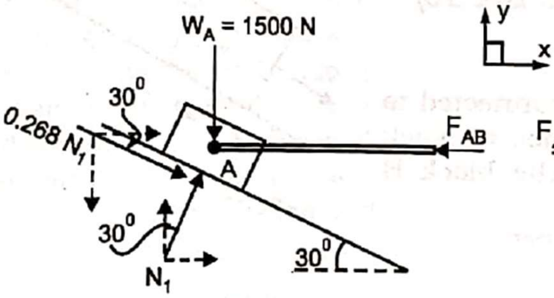
F B D - Block A

Ex. 4.8 Two blocks A and B are pin connected to a rod as shown. If block A weighs 1500 N, determine the maximum value of weight of block B for which equilibrium of the system is maintained. Angle of friction for all surfaces of contact is 15°.

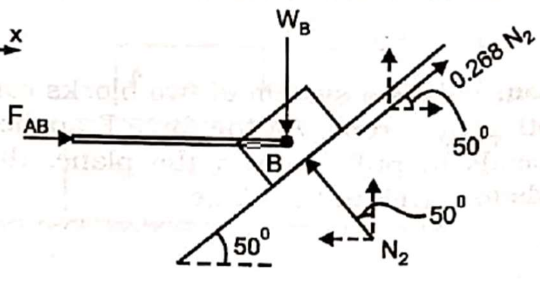


Solution: For block B, when its weight W_B is maximum, it will tend to slide down the slope causing block A to be pushed up its plane.

Let us isolate the system by cutting the rod. Assume force F_{AB} in the rod is compressive in nature.



F B D - Block A



F B D - Block B

Applying COE to Block A

$\Sigma F_Y = 0$

$N_1 \cos 30 - 0.268\ N_1 \sin 30 - 1500 = 0$

$\therefore \quad N_1 = 2049$ N

$\Sigma F_X = 0$

$-F_{AB} + 0.268\ N_1 \cos 30 + N_1 \sin 30 = 0$

Substituting $N_1 = 2049$ N, we get

$F_{AB} = 1500$ N

(+ ve value of F_{AB} indicates that force F_{AB} in the rod is compressive as assumed)

Applying COE to Block B

$\Sigma F_X = 0$

$F_{AB} + 0.268\ N_2 \cos 50 - N_2 \sin 50 = 0$

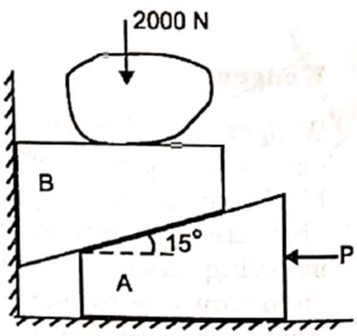
Substituting $F_{AB} = 1500$, we get, $\quad N_2 = 2526.2$ N

$\Sigma F_Y = 0$

$-W_B + N_2 \cos 50 + 0.268\ N_2 \sin 50 = 0$

Substituting $N_2 = 2526.2$ N, we get, $W_{B(maximum)} = 2142.4$ N .... **Ans.**

Ex. 4.9 To raise a heavy stone block weighing 2000 N, the arrangement shown is used. What horizontal force P is necessary to be applied to the wedge in order to raise the block. $\mu = 0.25$. Neglect the weight of the wedges.



Solution: In this wedge problem we need to find force P required to just lift the stone.

Isolating wedges A and B as shown.

Applying COE to wedge B

$\Sigma F_x = 0$

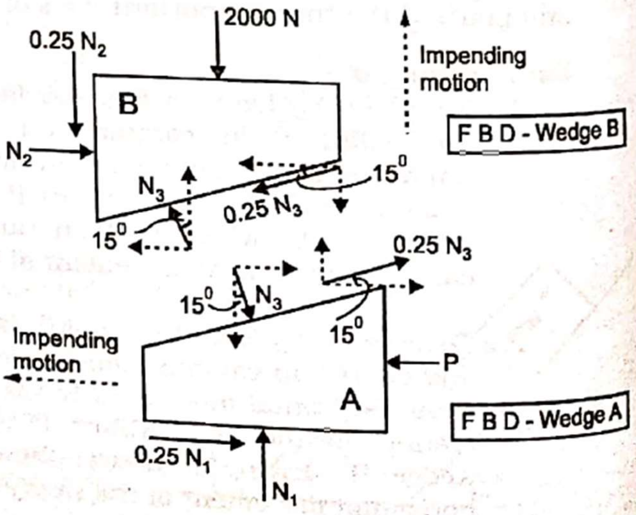
$N_2 - N_3 \sin 15 - 0.25\ N_3 \cos 15 = 0$ -------- (1)

$\Sigma F_y = 0$

$N_3 \cos 15 - 0.25\ N_3 \sin 15 - 0.25\ N_2 - 2000 = 0$ -------- (2)

Solving equations (1) and (2)

$N_3 = 2576.6$ N



Applying COE to wedge A

$\Sigma F_y = 0$

$N_1 - N_3 \cos 15 + 0.25\ N_3 \sin 15 = 0$

$\therefore N_1 - 2576.6 \cos 15 + 0.25\ (2576.6) \sin 15 = 0$

Or $\quad N_1 = 2322$ N

$\Sigma F_x = 0$

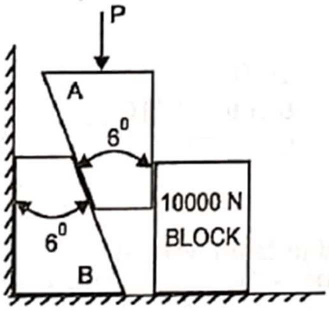
$-P + 0.25\ N_1 + N_3 \sin 15 + 0.25\ N_3 \cos 15 = 0$

$\therefore \quad -P + 0.25\ (2322) + 2576.6 \sin 15 + 0.25\ (2576.6) \cos 15 = 0$

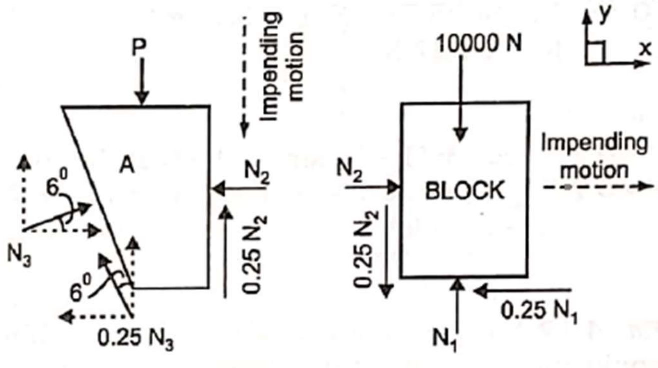
or $\quad P = 1869.6$ N .... **Ans.**

Ex. 4.10 Two 6° wedges are used to push the block horizontally as shown. Calculate the minimum force P required to push the block of weight 10000 N. $\mu = 0.25$ for all surfaces.

(MU Dec 08)



Solution: Figure below shows the FBD of the isolated wedge A and the block.



F B D - Wedge A   F B D - Block

Applying COE to block

$\Sigma F_x = 0$

$N_2 - 0.25\ N_1 = 0$ -------- (1)

$\Sigma F_y = 0$

$N_1 - 10000 - 0.25\ N_2 = 0$ -------- (2)

Solving equations (1) and (2)

$N_2 = 2666.7$ N

Applying COE to wedge A

$\Sigma F_x = 0$

$N_3 \cos 6 - 0.25\ N_3 \sin 6 - N_2 = 0$

$\therefore \quad N_3 \cos 6 - 0.25\ N_3 \sin 6 - 2666.7 = 0$

or $\quad N_3 = 2753.7$ N

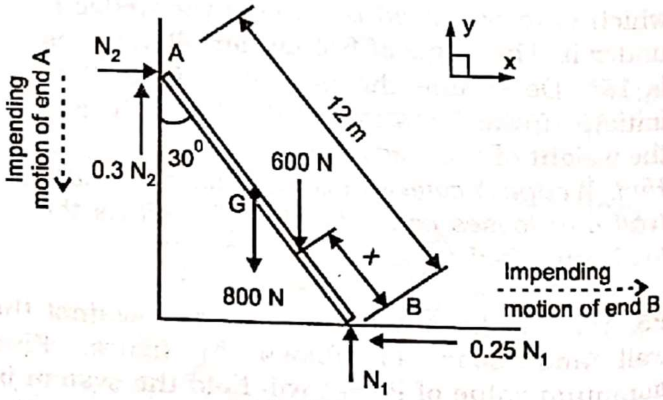
$\Sigma F_y = 0$

$-P + N_3 \sin 6 + 0.25\ N_3 \cos 6 + 0.25\ N_2 = 0$

$\therefore \quad -P + 2753.7 \sin 6 + 0.25\ (2753.7) \cos 6 + 0.25\ (2666.7) = 0$

or $\quad P = 1639.2$ N........... **Ans.**

Ex. 4.13 A 12 m ladder is resting against a vertical wall making 30° angle with the wall. Static friction between wall and ladder is 0.3 and that between ground and ladder is 0.25. A 600 N man ascends the ladder. How high will he be able to go before the ladder slips? Assume the weight of the ladder to be 800 N.



F B D - Ladder

Solution: Figure shows the FBD of ladder AB resting against the wall and floor. Let the person climb the distance x on the ladder when the ladder is on the verge of slipping. The weight of the ladder acts through its C.G.

Applying COE to the ladder

$\Sigma F_x = 0$

$N_2 - 0.25\ N_1 = 0$ ------ (1)

$$\Sigma F_y = 0$$

$$0.3\ N_2 + N_1 - 800 - 600 = 0 \qquad \text{------ (2)}$$

Solving equations (1) and (2)

$$N_1 = 1302.4\ N, \qquad N_2 = 325.6\ N$$

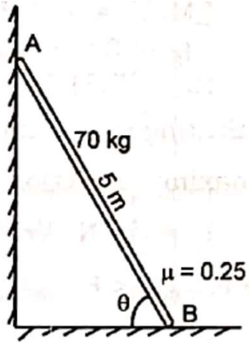
$$\Sigma M_B = 0 \quad + ve$$

$$-(N_2 \times 12 \cos 30) - (0.3\ N_2 \times 12 \sin 30) + (800 \times 6 \sin 30) + (600 \times x \sin 30) = 0$$

$$\therefore \quad x = 5.23\ m \qquad \text{............ } \textbf{Ans.}$$

Ex. 4.14 The rod AB of length 5 m and mass 70 kg is leaning against a wall. Find minimum θ for equilibrium. Take coefficient of friction = 0.25.

(KJS Nov 15)



Solution: For minimum value of θ, the ladder impends to slip down and away from the wall.

COE – ladder

$$\Sigma F_x = 0 \quad \rightarrow + ve$$

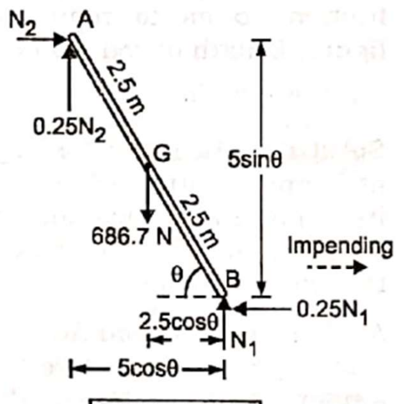
$$-0.25 N_1 + N_2 = 0 \qquad \text{......... (1)}$$

$$\Sigma F_y = 0 \quad \uparrow + ve$$

$$N_1 + 0.25 N_2 - 686.7 = 0 \qquad \text{......... (2)}$$

Solving equation (1) and (2), we get

$$N_1 = 646.3 N \quad \text{and} \quad N_2 = 161.6 N$$



F B D - Ladder

$$\Sigma M_B = 0 \quad + ve$$

$$-(N_2 \times 5 \sin\theta) - (0.25 N_2 \times 5 \cos\theta) + (686.7 \times 2.5 \cos\theta) = 0$$

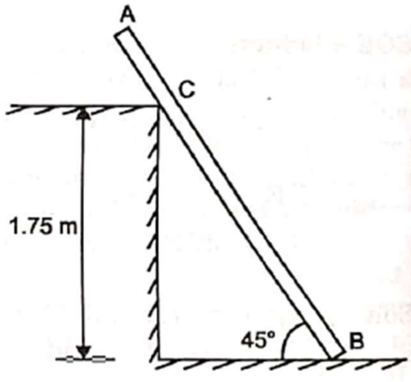
$$\therefore \quad -(161.6 \times 5 \sin\theta) - (0.25 \times 161.6 \times 5 \cos\theta) + 1716.65 \cos\theta = 0$$

$$\therefore \quad 1514.75 \cos\theta - 808 \sin\theta = 0$$

$$\therefore \quad \tan\theta = \frac{1514.75}{808} \quad \text{or} \quad \theta = 61.92° \qquad \text{......... } \textbf{Ans.}$$

Ex.4.16 Determine minimum value of coefficient of friction so as to maintain the position shown in figure. Length of rod AB is 3.5 m and it weighs 250 N.

(MU Dec 07)



Solution: The rod AB is supported by a rough surface at B and a rough edge at C. Since the ladder looses its equilibrium position by slipping to the right, frictional forces at B and C have to shown opposite to impending motion.

Applying COE to rod AC

$\Sigma M_B = 0$ ↺ +ve

$+ (250 \times 1.237) - (N_2 \times 2.475) = 0$

$\therefore \quad N_2 = 125$ N

$\Sigma F_x = 0 \rightarrow$ +ve

$N_2 \cos 45 - \mu\, N_2 \sin 45 - \mu N_1 = 0$

$\therefore \quad 125 \cos 45 - \mu \times 125 \sin 45 - \mu\, N_1 = 0$

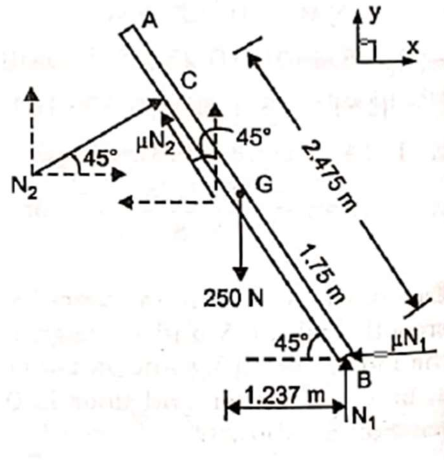
or $\quad \mu\,(N_1 + 88.39) = 88.39 \quad \ldots\ldots\ldots (1)$

$\Sigma F_y = 0 \uparrow$ + ve

$N_2 \sin 45 + \mu\, N_2 \cos 45 + N_1 - 250 = 0$

$\therefore \quad 125 \sin 45 + \mu \times 125 \cos 45 + N_1 - 250 = 0$

or $\quad N_1 = 161.61 - 88.39\,\mu \quad \ldots\ldots\ldots (2)$



F B D - Rod

Substituting value of N_1 from equation (2) in (1)

$\mu\,[(161.61 - 88.39\,\mu) + 88.39] = 88.39$

or $\quad -88.39\,\mu^2 + 250\,\mu - 88.39 = 0$

Solving the above quadratic equation , we get $\mu = 0.414$ or 2.414

since μ cannot be greater than 1 selecting the feasible value, we get $\mu = 0.414$...... **Ans.**