cR2

version 2.0 (November 2018)

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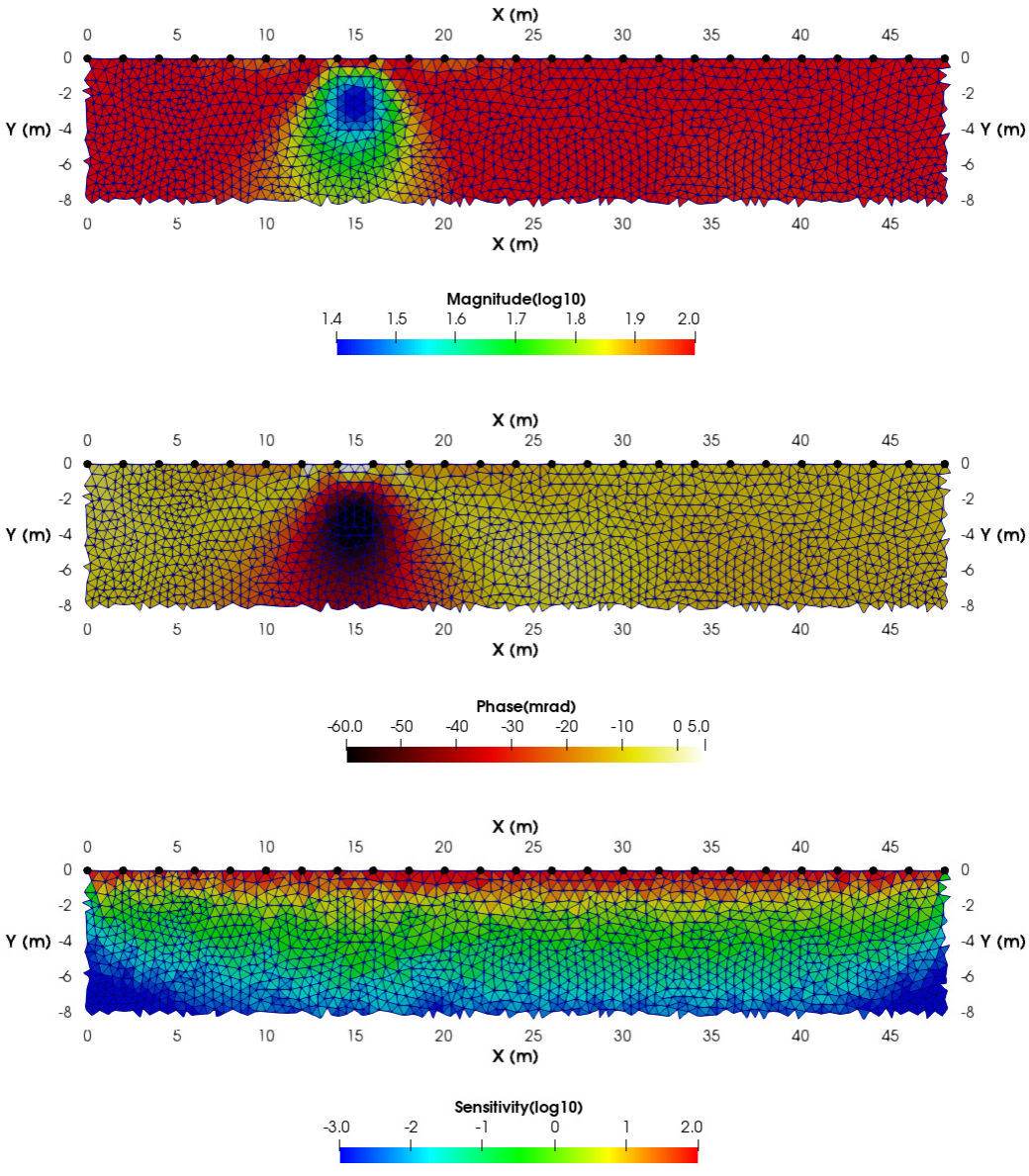


Contents

1. Version history

Changes to c*R2* from v1.9

Allocatable (dynamic) arrays are now used, meaning that there is (in theory) no problem size limit. A sensitivity map is now produced (as a separate file and embedded with the vtk). The input of starting resistivity and phase in ***cR2.in*** is now consistent with **R2 v3.2**. A bug fix has been corrected that affected calculations with a triangular mesh when node 1 was close to (or at) an electrode (that resulted in high errors in forward model calculations for measurements involving such an electrode). Error checks on input of the data file (**protocol.dat**) are now made (to avoid incorrect electrode numbering). An option for a more gradual progression of the inversion is now added (consistent with **R2 v3.2**). Documentation has been improved by adding more illustration of meshing and parameterisation.

Changes to cR2 from v1.8a

Vtk output is now created. The output region can also be selected.

Changes to cR2 from v1.8

An option to output the Jacobian in a forward model is added.

Changes to cR2 from v1.7

The mesh input is completely changed and now follows a similar format to R2 v2.5
Linear solver for forward problem now uses PARDISO solver. This may result in slower execution but much larger problems can be solved.

Changes to cR2 from v1.6

For a quadrilateral mesh it is now necessary to define the elevations of all electrodes – see line 22 in cR2.in details below. This now allows topographic variation in the mesh.

2. **Computer requirements for *cR2* v2.0**

In this release only one version has been compiled for the Windows environment: a 64bit version, cR2.exe. A 32bit version is not provided because of the limited array allocation. Linux users should be able to run c*R2* with the command "wine cR2.exe" (thanks to Rodolphe Cattin for this tip). I have not tested this, however. Users requiring a version compiled for other processors should contact the author.

NOTE 1: ***cR2*** *is provided as a standalone executable. It does not need to be installed – the executable is put in the folder containing the input files and run from there. Output files will be created in the same folder.*

NOTE 2: *You will be able to run* ***cR2*** *by double clicking the executable. However, if the program stops abruptly (for example, due to an error in the input file or if you are trying to run an executable compiled for a different processor architecture) then you will not see any error message on the screen since the window will disappear. Therefore, it is advisable to run* ***cR2*** *from the Command Prompt (just run CMD from the Start Menu – you may need to move your working directory and run* ***cR2*** *from there).*

NOTE 3: *all input files should be prepared with a text editor. [I prefer to use TextPad (www.textpad.com) because it allows much greater editing facilities although any text editor will work]. It is important that you do not include tabs in the files. These are often inserted if you copy and paste from Excel, for example. You should convert these tabs to spaces (TextPad will allow you to set this up to happen automatically).*

3. **Introduction to *cR2* v2.0**

cR2 has been developed for imaging complex resistivity (i.e. induced polarisation) using arbitrary electrode arrays. ***cR2*** will compute a forward model for a given distribution of complex resistivity (defined in terms of magnitude and phase angle). ***cR2*** can also provide an inverse solution for a 2-D complex resistivity distribution based on computation of 3-D current flow using a finite element mesh based on either triangle or rectangle elements. The inverse solution is based on a regularised objective function combined with weighted least squares (an 'Occams' type solution) as defined in Binley and Kemna (2005) and Kemna *et al.*(2004).

Parameters (for the inverse solution) are made up of one or more elements. Electrodes are specified at node points. These are the corners of the elements. The boundary conditions along all four boundaries of the mesh are Neumann conditions (zero flux) and therefore if you are investigating a half space you must extend left, right and lower boundaries of the mesh to some distance away from the area of investigation (typically 5 to 10 times the distance – see later). The mesh can be made up of either quadrilateral elements or triangular elements.

The current version does not have upper limits set for the size of the problem that can be solved. However, it is important that the user has some appreciation of whether the problem they are trying to solve is realistic for their given hardware. Large problems in inverse mode can be memory hungry. As soon as the user's RAM is used then the computer will start using virtual memory (paging to disk) which can be very slow. To help the user asses memory needs ***cR2*** will output an estimate of the memory needs early on in its execution. For large problems it is important that the user compares this with physical memory (RAM) that is available.

For information on solving resistivity forward and inverse problems see Binley (2015) and Binley and Kemna (2005), the latter being useful for the full complex resistivity description. Contact the author for a digital copy of the former.

It is strongly recommended that the user becomes familiar with inverting DC resistivity data with sister code ***R2*** before working with ***cR2***. Many of the concepts about meshing, parameterisation are similar.

In ***cR2*** complex resistivity is defined by a magnitude and phase angle. The magnitude is equivalent to a DC resistivity since the phase angles are likely to be small. In an inversion, ***cR2*** also computes the real and imaginary conductivity since these are more useful for analysis of the conduction and polarization of the subsurface. Data for ***cR2*** are supplied in terms of magnitude and phase angle of the measured impedance. A negative phase angle is a positive IP effect. It is important to understand that the magnitude is a positive number, unlike DC resistance, which can be positive or negative. For example, a surface DC resistivity dipole-dipole survey with four adjacent electrodes, 1,2,3 and 4 in AB-MN configuration 1,2 – 3,4 will give a negative resistance since the geometric factor is negative. Let's say that the value is -1.0Ω. If a complex resistivity survey is carried out with the same configuration then the magnitude would be 1.0Ω and the phase angle (for a non-polarizing subsurface) would not be zero but would be $-\pi$ radians, i.e. -3,141.6 mrad. Preparing data in this way can be messy and lead to confusion and so measurements with a negative geometric factor it may be easier to express the measurement as the equivalent with a positive geometric factor. The values below are equivalent.

A	B	M	N	Magnitude (Ω)	Phase angle (mrad)
1	2	3	4	1.0	-3,141.6
1	2	4	3	1.0	0.0
2	1	3	4	1.0	0.0

IP data measured in the time domain will give a resistance and chargeability (M). The resistance is easily transformed to a magnitude taking care of the polarity issue above. The chargeability can be transformed to an equivalent phase angle using the method described in Kemna et al. (1997). The

conversion is a function of the chargeability sampling and current injection frequency. Typically the phase angle (in mrad) is equivalent to ~-1.3M (in mV/V). See Mwakanyamale et al.(2012) as an example of such a conversion for application of ***cR2*** to time domain IP data.

4. **Designing meshes**

cR2 uses a mesh of finite elements to compute the forward model (in both forward and inverse mode). As stated, earlier, the mesh can be based on quadrilateral (often referred to as a *structured* mesh) or triangular (*unstructured* mesh) finite elements. Each element is defined by node points on its perimeter (see Figure 4.1). ***cR2*** computes the voltage at each node point given a dipole current source applied at a pair of nodes. Therefore, the minimum requirement of the mesh is that the electrodes must be sited at node points. The accuracy of the computed voltage field is strongly dependent on the discretisation of the nodes: nodes should be closely spaced near electrode sites as voltage gradients will be high.

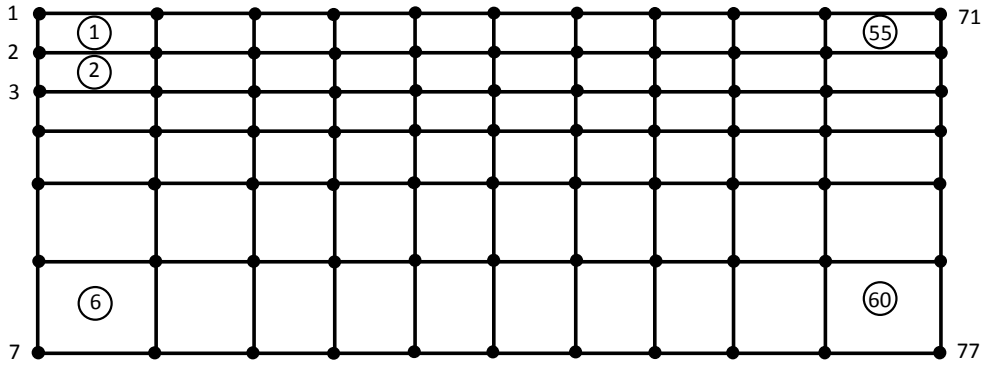


Figure 4.1: Example finite element showing ordering of nodes and elements (in circles).

The mesh should clearly cover the survey area laterally and the expected depth of investigation, however, given that injected current will transfer further laterally and vertically, the mesh should extend sufficiently in order to account for this (mimicking infinite boundaries). There is no need to retain a fine discretisation in these 'infinite' boundary regions: it is good practice to let the elements gradually increase in size laterally and vertically outside the region of investigation. Figure 4.2 illustrates the zoning of discretisation. Note that this example is for a surface electrode array. For others, e.g. cross-borehole, similar concepts are applied.

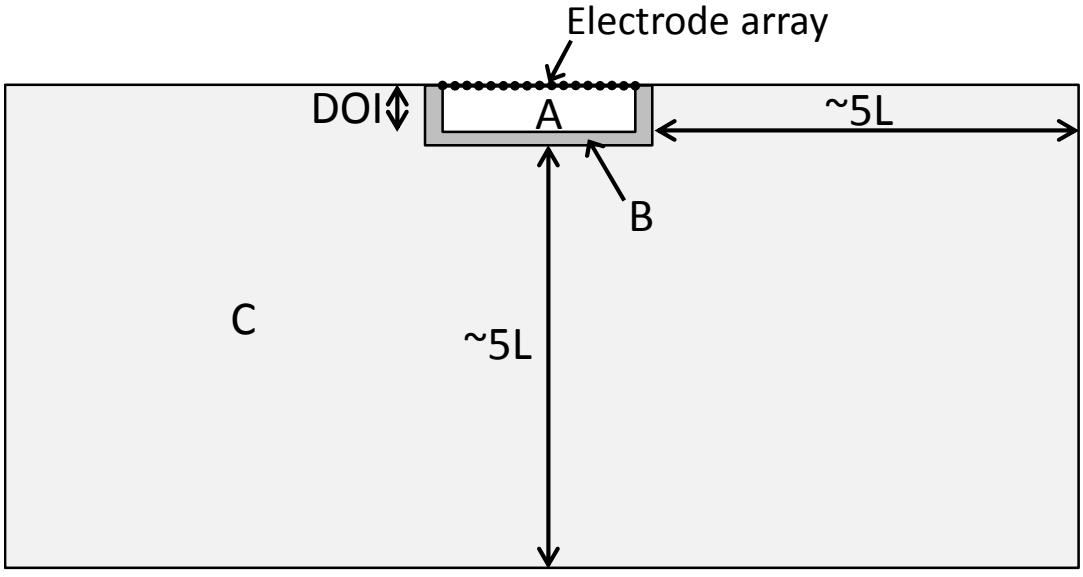


Figure 4.2: Extending the finite element mesh to account for 'infinite' boundaries. DOI is the depth of investigation. Zone A is the region of interest; elements should be finely discretised especially near the electrodes. Zone B has the same discretisation as A and is included to ensure good accuracy of the forward calculations; this zone typically extends two or three time the electrode spacing. Zone C

typically extends ~5L where L is the length of the longest current dipole. Discretisation should gradually get coarser in Zone C, moving away from the region of investigation.

A quadrilateral mesh in ***cR2*** is defined by a grid of numnp_z rows and numnp_x columns of nodes. The mesh can be set in three different ways (Figure 4.3). The simplest type is a rectangular grid (Figure 4.3a). In **cR2.in** we define this as mesh_type = 4 (see later). In this case, the elevation of all nodes in a given row is constant. The mesh is defined by the x co-ordinates of each column, the z co-ordinates of the nodes in the top row (i.e. the topography) and the depth of each row relative to the top row. If the topography varies across the survey area then a mesh shown in Figure 4.3b results. Again this is mesh_type = 4. A more complex mesh structure (see Figure 4.3c) can also be created. In **cR2.in** we define this as mesh_type = 5. In this case the x co-ordinates are still fixed for each column but the elevation of all nodes in each column must be input. In comparison to mesh_type = 5 more input defining the geometry is need but the co-ordinates of every node is not needed.

For more complex geometry ***cR2*** allows the definition of a triangular mesh – in this case the co-ordinates for all nodes must be defined, along with the element definitions (i.e. the three node numbers for each element). In **cR2.in** we define this as mesh_type = 3.

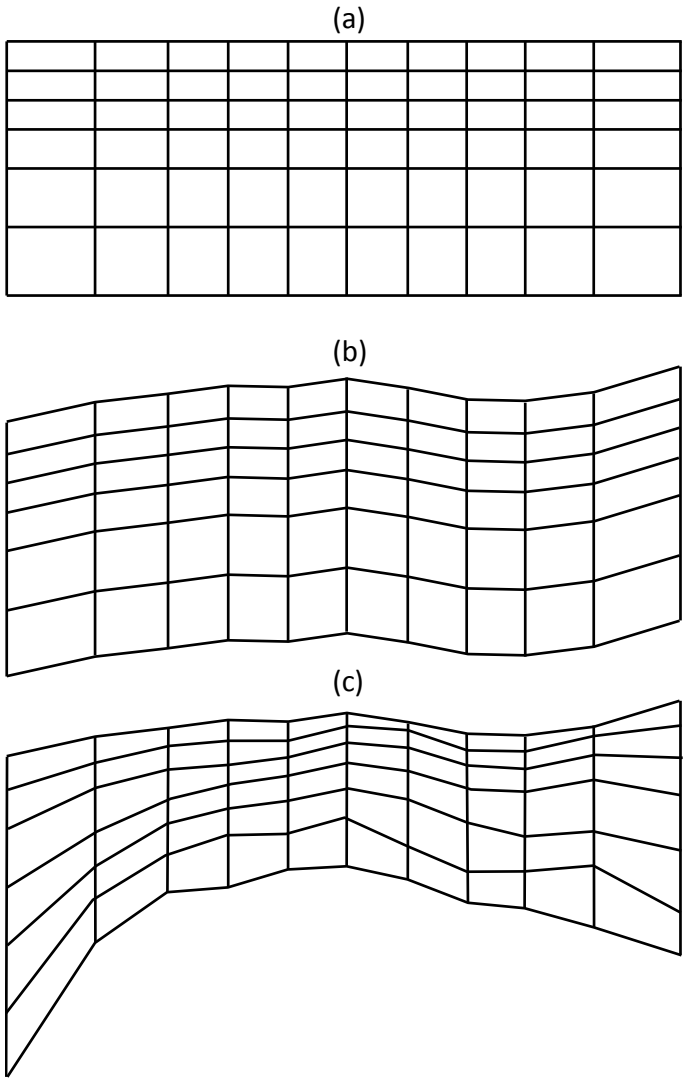


Figure 4.3: Quadrilateral mesh types.

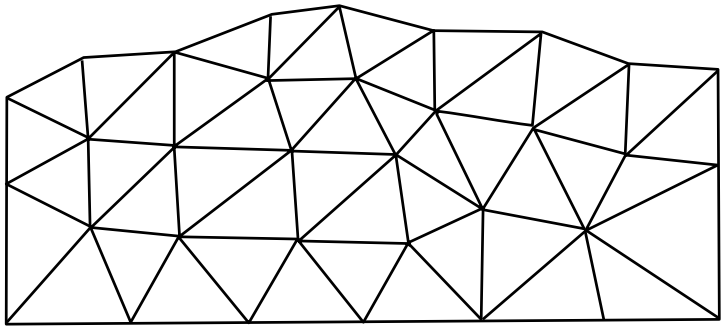


Figure 4.4: Triangular mesh example.

Triangular meshes not only allow full flexibility in defining geometry but also result in more computationally efficient meshes since the extremities of the mesh can be made to have coarser discretisation (see Figure 4.5). If you are working with a triangular mesh then you must create the mesh and store details of the geometry of the mesh in a file **mesh.dat**. There are a number of good meshing tools available. Gmsh (see http://www.geuz.org/gmsh/) is a powerful finite element mesh generator with a large user base with video tutorials available online. Alternatively, software for general finite element analysis (e.g. COMSOL) contain mesh generators, as do software for specific applications (e.g. groundwater code environments like GMS).

Gmsh mesh utility codes for *cR2*

The ***cR2*** download package contains some simple codes for working with Gmsh. These are located in the **Mesh_utilities/Binley** folder. ***GenGmshGeo2D*** creates a geometry file for Gmsh. This can then be loaded in Gmsh and a 2D mesh easily created. The mesh in Gmsh is saved as a file **GenGmshGeo2D.msh** which can be transformed to the **mesh.dat** format for ***cR2*** using the partner code ***GmshMsh2R2***. The code ***Check_R2_Tmesh*** can be run to check the **mesh.dat** file that is created. One thing that this code checks is the node numbering of the elements in the mesh. All finite elements should have node numbers in a counter-clockwise direction. ***Check_R2_Tmesh*** will check this and correct any such errors.

A Gmsh tutorial (**gmsh R2 tutorial.pdf**) that was written by Judy Robinson (Rutgers University) is also supplied (see **Mesh_utilities/Robinson**). Although this was written for sister code **R2,** the mesh setup is the same for ***cR2*****.** Judy has also kindly provided two Matlab scripts for working with Gmsh and ***R2***, along with an example (based on the Surface1 example below).

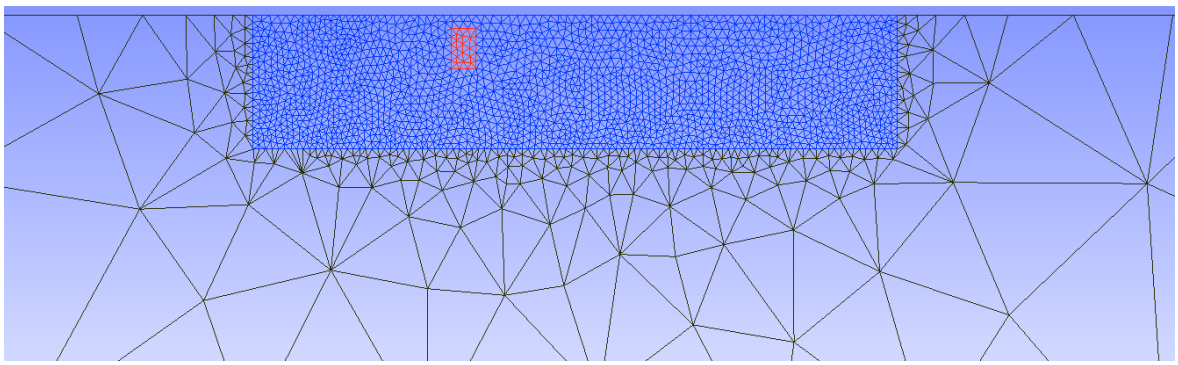


Figure 4.5: Example triangular mesh with variable discretisation.

Niels Claes (University of Wyoming) has provided a Matlab script (see folder **Mesh_utilities/Claes**) that will create a mesh file using Gmsh.

Jimmy Boyd (British Geological Survey/ Lancaster University) has written a python script to covert Gmsh msh files to **mesh.dat** format for ***R2*** (or ***cR2***). See **Mesh_utilities/Boyd**

5. **Parameterisation**

For a forward model calculation the resistivity for each element must be defined. For an inverse solution the starting resistivity model must be defined. This is normally a uniform model. In inverse mode the discretisation of parameters must be set. The simplest way to do this is to set each finite element as a parameter. In some cases a more complex parameterisation is required. In a quadrilateral mesh we can define 'patch' sizes for parameters. A patch is a group of elements, which is illustrated in Figure 4.6. The advantage of such patching of elements is the reduced computational time, since the number of parameters is reduced.

In some instances we may wish to fix the resistivity in part of the mesh, i.e. the parameters may only cover a subset of the finite element mesh (see, for example, Figure 4.7). ***cR2*** allows this type of parameterisation by stating where the patching starts and ends in both x and y direction.

Greater control of parameterisation is allowed for a triangular mesh (e.g. Figure 4.8) by defining a parameter number for each element in **mesh.dat**. Note that if a parameter number 0 is assigned for an element then the inverse solution fixes the resistivity of this element to the starting value.

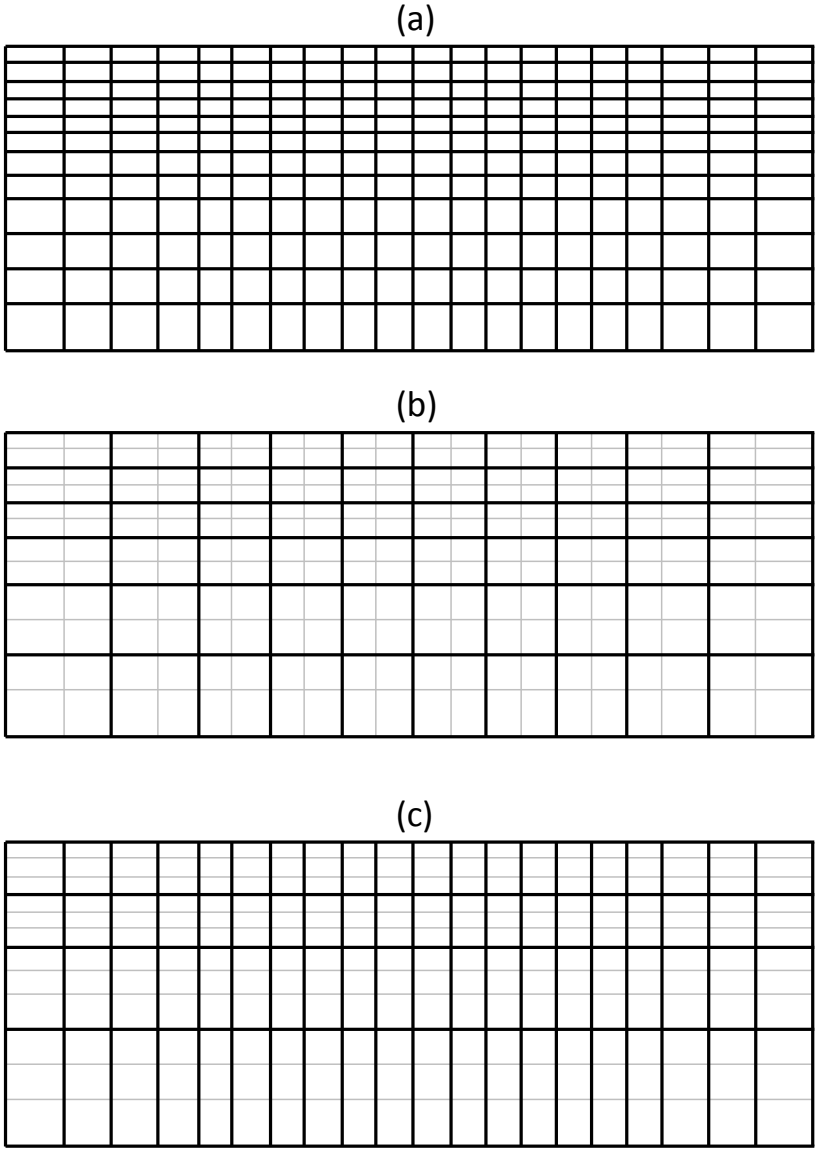


Figure 4.6: Defining parameter boundaries. The grey lines show the finite element mesh, the black lines show the parameter zones. (a) Each element is a parameter. (b) Each parameter is a 2 x 2 patch of elements. (c) Each parameter is a 3 x 1 patch of elements.

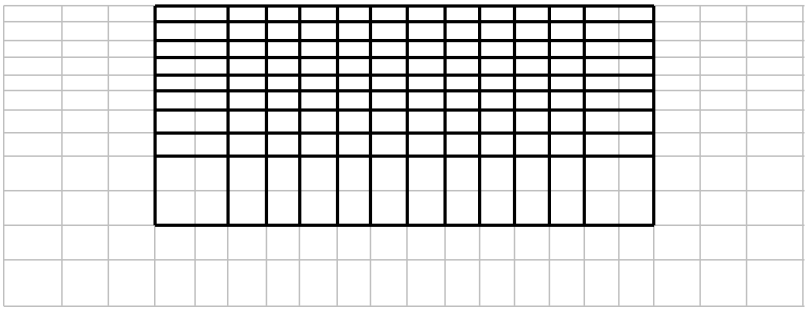


Figure 4.7: Parameterisation of a subset of the finite element mesh. The grey lines show the finite element mesh, the black lines show the parameter zones.

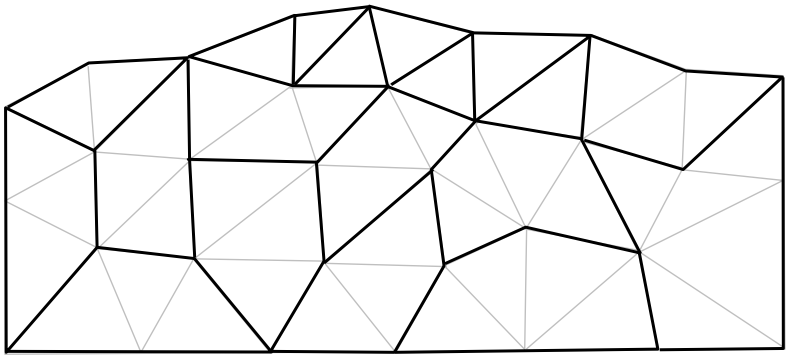


Figure 4.8: Parameterisation of a triangular mesh. The grey lines show the finite element mesh, the black lines show the parameter zones.

Further control of parameterisation is achieved through 'zoning' of parameters. In ***cR2*** a zone is defined as a congruent collection of parameters. Smoothing (in the inverse solution) does not occur across zones. This can be useful if *a priori* information allows the user to define sharp contrasts (e.g. at a water table boundary).

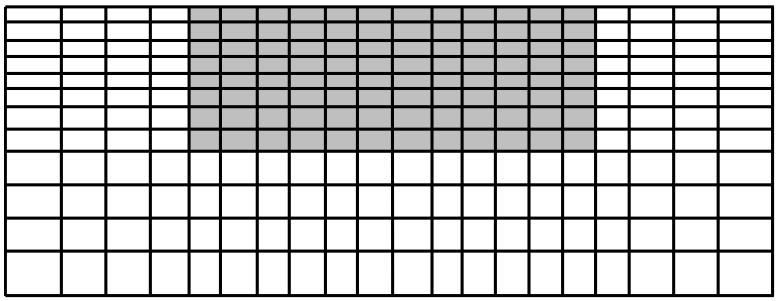
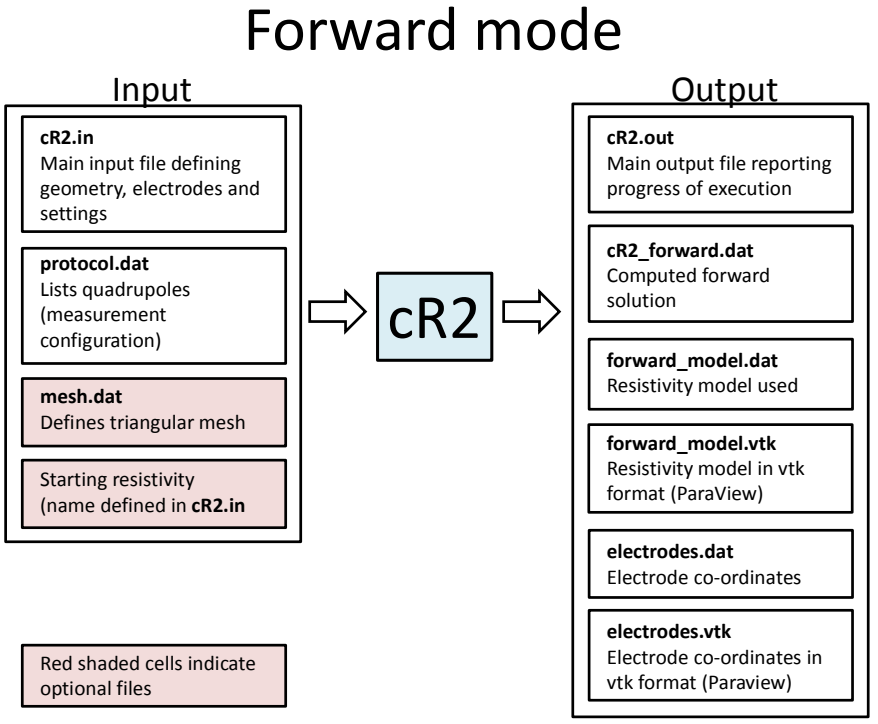


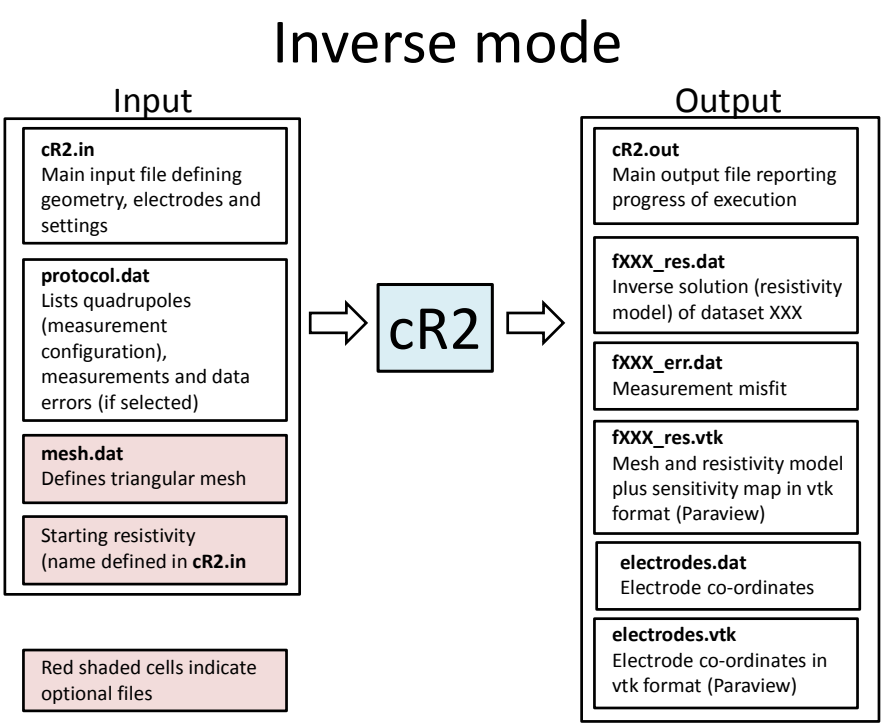
Figure 4.9: Zoning of parameterisation. The grey shaded region is a different zone to the rest of the mesh.

Again, full flexibility is possible with a triangular mesh. For each element the parameter and zone is defined in the **mesh.dat** file.

6. **Input and output files**

The input and output files used by ***cR2*** are shown schematically below, and described in detail later.





cR2 requires at least two input files: **cR2.in** and **protocol.dat.** If a triangular mesh is used then an additional input file – **mesh.dat** – is required. **cR2.in** contains information on the geometry of the problem to be solved. **protocol.dat** contains the measurement

cR2 will output a number of files:

- **cR2.out** which will contain main log of execution.

- **electrodes.dat** contains the coordinates of the electrodes.

If the problem to be run is a forward model then ***cR2*** will output:

- **cR2_forward.dat** which will contain the forward model for the electrode configuration in protocol.dat The format of **cR2_forward.dat** is the same as **protocol.dat:** the first column is a measurement number, the next 4 columns contain the quadrupole electrode numbers, column 6 contains the calculated magnitudes, column 7 contains the phase angles in mrad, and column 8 contains the calculated apparent resistivities.

- **forward_model.da**t which will contain the resistivity distribution for your forward model (i.e. what you specified in the input for ***cR2***). Note that the format of these will be the same as described below for inverse mode.

- **forward_model.vtk** as above but vtk format (allowing plotting in ParaView, for example).

If the problem to be run is an inverse model then ***cR2*** will output:

- **f001_res.dat** which will contain the resistivity result of the inverse solution. **f001_res.dat** will contain seven columns: x, z, resistivity magnitude, phase angle (in mrad), $\log_{10}$(resistivity magnitude), $\log_{10}$(real conductivity, in S/m), $\log_{10}$(imaginary conductivity, in S/m), where x,y are coordinates at centroid of each element and the other properties are for the that element. The format is setup to work directly with Surfer.

- **f001_err.dat** will contain nine columns. The first four columns contain the electrode numbers. In the fifth column is the normalised data misfit, the next column contains the observed data recorded as an apparent resistivity, the next column contains the equivalent apparent resistivities for the computed model, the next column is the observed phase angles, the next column is the computed phase angles, the next column shows the original data weight (i.e. data standard deviation in same units as data), the next column is the final data weight, the last columns shows a "1" if any weights have been changed during the inversion, otherwise a "0" will appear.

- **f001_res.vtk** will contain resistivity magnitude, $\log_{10}$ resistivity magnitude, phase angle (in mrad), $\log_{10}$ (real conductivity, in S/m), $\log_{10}$ (imaginary conductivity, in S/m), $\log_{10}$ (sensitivity) all in vtk format (allowing plotting in ParaView, for example). Here, sensitivity is defined as the diagonal of the matrix $[J^T W^T W J]$ which gives an idea of the mesh sensitivity coverage (see Binley and Kemna, 2005).

- If you have more than one dataset in protocol.dat (see later) then the files **f001_res.dat, f002_res.dat, f003_res.dat,** etc will be created. Similarly a set of **_err.dat** files will be output.

In addition ***cR2*** will output:

- **electrodes.dat** , which contains the co-ordinates of the electrodes. The values are in three columns: x,z,y (where y is a dummy value – set to 0.0).

- **electrodes.vtk** contains the co-ordinates of the electrodes in vtk format. The values are in three columns: x,z,y (the latter being set to zero). Use this file if you are working with Paraview to look at the resistivity images. Once you have opened the electrodes.vtk file in Paraview you select "apply" then you select the "Glyph" icon; this allows you to plot the electrodes as small spheres (or other objects).

NOTE: If ***cR2*** fails to converge in inverse mode, all output files except the sensitivity map/resolution matrix will still be output in order to allow the user to assess the source of the problem (the **fXXX.err** file is useful for this) but the user should check the **cR2.out** file to ensure convergence is achieved before using any computed resistivity models.

Some comments on co-ordinate sign convention

When a quadrilateral mesh is created the user specifies the horizontal and vertical co-ordinates of the rows and columns forming the mesh. The convention here (see line 9 in **cR2.in**) is for the vertical co-ordinates to be specified as depth (not elevation). When these are output in a **.dat** file (e.g. **forward_model.dat**, for a forward model, or **f001.dat**, for an inversion) then the vertical co-ordinate sign is changed. This is so that programs like Surfer will show the section properly. The same switching of sign is changed in the vtk output (e.g. **forward_model.vtk** and **f001.vtk**). The electrode co-ordinates (output in **electrodes.dat** and **electrodes.vtk**) will also have a switched sign for the vertical co-ordinates.

Triangular meshes should be created with the vertical co-ordinate representing elevation, not depth. And so when a triangular mesh is used, the sign change is not made in any of the output files listed above.

7. Details of cR2.in

Line1: (Char*80) header

where header is a title of up to 80 characters

Line 2: (2 Int, Real, 2 Int) job_type, mesh_type, flux_type

where job_type is 0 for forward solution only or 1 for inverse solution; mesh_type is 3 for triangular mesh or 4 for a regular quadrilateral mesh or 5 for a more generalised quadrilateral mesh (see Figure 4.3c); flux_type is 2.0 for 2D current flow (i.e. line electrodes, which are infinitely long orthogonal to the section) or 3.0 (usual mode) for fully 3D current flow (i.e. point electrodes).

If mesh_type is 3 then a triangular mesh is to be used. This allows much greater flexibility of defining geometry but requires creation of a finite element mesh. The file **mesh.dat** must be supplied which contains the mesh details including node coordinates and element indices (see details later).

If (mesh_type = 4) then a regular quadrilateral mesh is to be used and the following are read:

Line 3: (2 Int) numnp_x, numnp_z

where numnp_x is number of nodes in the x direction (horizontal) and numnp_z is the number of nodes in the z (vertical) direction

Line 4: (numnp_x Real) xx

where xx is an array containing x coordinates of each of numnp_x node columns

Line 5: (numnp_x Real) topog

where topog is an array containing elevations of each of numnp_x node columns. If the topography is flat then set topog to zero for all values.

Line 6: (numnp_y Real) zz

where zz is an array containing the depths of each of numnp_z node rows relative to the topog array. Set zz(1) to zero and the other values to a positive number (i.e. zz represents depth, not topography).

Else if (mesh_type = 5) then a more generalised quadrilateral mesh is to be used and the following are read:

Line 7: (2 Int) numnp_x, numnp_z
where numnp_x is number of nodes in the x direction (horizontal) and numnp_z is the number of nodes in the z (vertical) direction

Line 8: (numnp_x Real) xx

where xx is an array containing x co-ordinates of each of numnp_x node columns

Line 9: (numnp_y Real) zz

where zz is an array containing elevations (not depths) of each of numnp_z node rows for column 1 in the x direction. For each column of nodes zz(1) is the topography.

Repeat Line 9 for all numnnp_x columns.

End if

Note: It is wise to add a carriage returns to break up a long list of input values (in Line 4, 5, 6, 8 and 9, for example). Don't write more than 20 numbers on each line as the compiler may not like it.

If (mesh_type = 3) then read the following

Line 10: (Real) scale

where scale is a scaling factor for the mesh co-ordinates. This is usually 1.0 but if a standardised mesh is used, say for a unit circle, then this scaling factor is useful to adjust the mesh for a specific problem. Set scale=1 if you do not wish to change the coordinates of the mesh defined in **mesh.dat.**

End if

Line 11: (Int) num_regions

where num_regions is number of resistivity regions that will be specified either as starting condition for inverse solution or actual model for forward solution. The term "region" has no significance in the inversion – it is just a means of inputting a non-uniform resistivity as a starting model for inversion or for forward calculation.

If (num_regions = 0) then read the following

Line 12: (15*Char) file_name

where file_name is the name of the file containing the resistivitities from a previous inversion (the **_res.dat** file that had been produced). Note that the file_name must be no more than 15 characters and there should be no spaces before the file name and no characters in the line after the file name.

Else

Line 13: (2 Int, Real) elem_1, elem_2, mag, phase

where the resistivity expressed as a magnitude (mag) and phase angle (phase) (in mrad) will be assigned for all elements from elem_1 to elem_2 (inclusive). Note that for a quadrilateral mesh the elements are numbered down columns first (top to bottom) then along rows (left to right).

Repeat Line 13 for all num_regions

End if

NOTE: *you must assign all elements a starting value. The number of elements in the mesh is (numnp_x-1) x (numnp_y-1) for a quadrilateral mesh. All these elements must be assigned a resistivity. Note also that if you assign an element a value, it will overwrite any previous assignment.*

If (job_type = 1. i.e. an inverse solution) then read the following

If (mesh_type = 4 or 5) then read the following

Line 14: (2 Int) patch_size_x, patch_size_z

where patch_size_x and patch_size_z are the parameter block sizes in the x and z direction, respectively. We differentiate between parameter size and element size to allow faster computation. The larger the patch size the few parameters and the faster the inversion, however, if we increase it too much we will reduce the flexibility to create variation of resistivity. If computational time is not a problem then use a patch size of 1 for x and z. Note that the number of elements in the x direction must be perfectly divisible by patch_size_x and the number of elements in the z direction must be perfectly divisible by patch_size_z otherwise set them both to zero. See examples in Figure 4.6. For Figure 4.6a patch_size_x =1 and patch_size_z = 1. For Figure 4.6b patch_size_x =2 and patch_size_z = 2. For Figure 4.6c patch_size_x =1 and patch_size_z = 3.

If (patch_size_x = 0) and (patch_size_z = 0) then read the following

Line 15: (2 Int) num_param_x, num_param_z

where num_param_x and num_param_z are the number of parameter blocks in the x and z directions

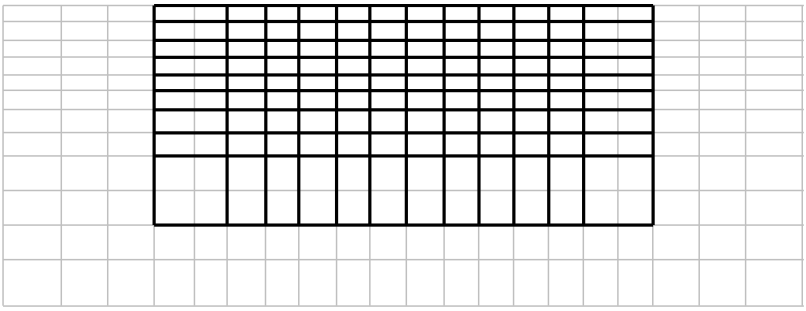
Line 16: (1+num_param_x Int) npxstart, npx(i), i=1,num_param_x

where npxstart is the column number in the mesh where the parameters start; npx specifies the number of elements in each parameter block in the x direction

Line 17: (1+num_param_y Int) npzstart, npz(i), i=1,num_param_z

where npzstart is the row number in the mesh where the parameters start; npz specifies the number of elements in each parameter block in the z direction

See example in Figure 4.7 (copied below). For this example we would set num_param_x = 12 and num_param_z = 9. Then we would set Line 17 as 3, 2,1,1,1,1,1,1,1,1,1,2 and Line 17 as 1, 1,1,1,1,1,1,1,1,2.



End if

End if

NOTE: *the following line input is different to v1.9 and older versions of* ***cR2***

Line 18: (Int, Real) inverse_type, target_decrease

where inverse_type is 0 for pseudo-Marquardt solution or 1 for regularised solution with linear filter (usual mode) or 4 for blocked linear regularised type (see also line 21). Note that the blocking defined here is only for a quadrilateral mesh – for blocking within a triangular mesh see the details for preparing **mesh.dat** later. target_decrease is a real number which allows the user to specify the relative reduction of misfit in each iteration. A value of 0.25 will mean that ***cR2*** will

aim to drop the misfit by 25% (and no more) of the value at the start of the iteration. This allows a slower progression of the inversion, which can often result in a better convergence. If you set target_decrease to 0.0 then ***cR2*** will try to achieve the maximum reduction in misfit in the iteration.

Line 19: (Real, 2 Int, Real) tolerance, max_iterations, error_mod, alpha_aniso

where tolerance is desired misfit (usually 1.0); max_iterations is the maximum number of iterations; error_mod is 0 if you wish to preserve the data weights, or 2 if you wish the inversion to update the weights as the inversion progresses based on how good a fit each data point makes. The routine used is based on Morelli and LaBrecque (1996). Note that no weights will be increased. The smoothing factor used in the code (alpha) is searched for at each iteration. The search is done over a range of steps in alpha, the number of steps is 10. alpha_aniso is the anisotropy of the smoothing factor, set alpha_aniso > 1 for smoother horizontal models, alpha_aniso < 1 for smoother vertical models, or alpha_aniso=1 for normal (isotropic) regularisation.

Line 20: (5 Real) min_error, a_wgt, b_wgt, rho_min, rho_max

where min_error is the minimum magnitude error (this is to ensure that very low errors are not assigned and is only used if a_wgt and b_wgt are both zero), a_wgt and b_wgt are error variance model parameters: a_wgt is the relative error of magnitudes; b_wgt is absolute error of phase values (in mrad); rho_min and rho_max are the minimum and maximum observed apparent resistivity magnitude to be used for inversion (use large extremes if you want all data to be used). **NOTE** that if your mesh contains topography, or the surface elevation is not zero, or the left, right and lower extent of the mesh does not represent infinite boundaries then the geometric factor computed in the code will be incorrect and thus any comparison of apparent resistivities against upper and lower limits will be invalid. For such a case you should set rho_min and rho_max to be very low and very high values, e.g. -10e10 and 10e10, respectively. Note also that you can select to include individual errors for each measurement in the data input file **protocol.dat** – to do this a_wgt and b_wgt should be set to 0.0. If a_wgt and b_wgt are set to zero then **protocol.dat** must contain errors for the data weights (see next section). It is advisable to estimate a_wgt and b_wgt from error checks in the field data (ideally from reciprocal measurements - not measures of repeatability). Typically for surface data a_wgt will be about 0.02 (equivalent to 2% error), b_wgt will be typically 2mrad for good data, but could be much higher.

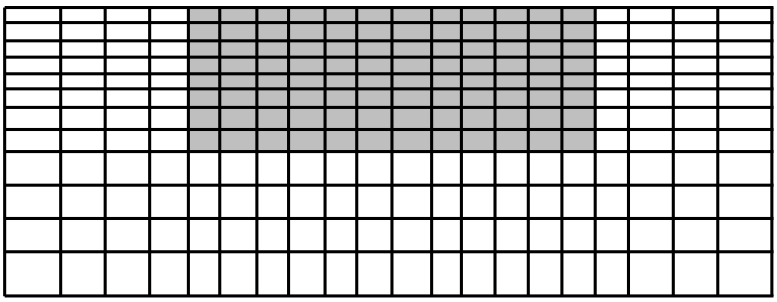
Line 21: (num_param_x Int) param_symbol

If you have specified zoning of parameters (inverse_type = 4 in line 18) so that each zone is disconnected from other zones then for a quadrilateral mesh (see Figure 4.9) the zones are specified by producing a simple plan of the parameter mesh. You must input for each row of parameters an integer representing the parameters. This is repeated for each row. Make sure that you put a space between each integer. As an example consider the zoned mesh in Figure 4.9 (copied below) with 20 elements in the x direction and 12 elements in the z direction. In this example we wish to zone the region shaded. As each element is a parameter then the patch size in x and y is 1, so in total there are 20 parameters (x) by 12 parameters (z). If we want to set the boundary of the shaded region so that there is no smoothing to the unshaded region then we would input:

1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1
1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1
1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1
1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1
1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1
1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1

1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1
1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Note that we have used 1s and 2s to define the regions. We could have used any other integer.



If for the problem above we had a patch size of 2 in x and z then Line 21 would be:

1 1 2 2 2 2 2 2 1 1
1 1 2 2 2 2 2 2 1 1
1 1 2 2 2 2 2 2 1 1
1 1 2 2 2 2 2 2 1 1
1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1

If we had defined the problem to have a patch size in x of 4 and a patch size in z of 2 then Line 21 would be:

1 2 2 2 1
1 2 2 2 1
1 2 2 2 1
1 2 2 2 1
1 1 1 1 1
1 1 1 1 1

Repeat line 21 for all num_param_z

End if

End if

Line 22: (Integer) num_xz_poly
where num_xz_poly is the number of x,y co-ordinates that define a polyline bounding the output volume. If num_xz_poly is set to zero then no bounding is done in the x-z plane. The co-ordinates of the bounding polyline follow in the next line. **NOTE:** the first and last pair of co-ordinates must be identical (to complete the polyline). So, for example, if you define a bounding square in x,y then you must have 5 co-ordinates on the polyline. The polyline must be defined as a series of co-ordinates in sequence, although the order can be clockwise or anti-clockwise (see examples later). **NOTE: *cR2*** stores the vertical co-ordinates for nodes in a quadrilateral mesh with a convention positive upwards. For example, if the ground surface has an elevation of 0m and you wish to output to a depth of 8m then z=-8m must be used for the lower boundary of the polygon. Similarly, if the ground surface elevation is 100m and you wish to output to a depth of 8m then z=-92m must be used for the lower boundary of the polygon. If a

triangular mesh is used then the co-ordinates specified in the mesh file are used and the above comments about sign convention do not apply.

Line 23: (2 Real) x_poly(1), z_poly(2)
where x_poly(1), z_poly(1) are the co-ordinates of the first point on the polyline.
Repeat line 23 for all num_xz_poly co-ordinates.

Line 24: (Int) num_electrodes

where num_electrodes is number of electrodes

If (mesh_type = 3) then

Line 25: (2 Int) j, node

where j is the electrode number and node is the node number in the finite element mesh

Else

Line 26: (3 Int) j, column, row

where j is the electrode number, column is the column index for the node the finite element mesh and row is the row index for the node in the finite element mesh. The column value must be in the range 1 to numnp_x and the row value must be in the range 1 to numnp_z. Both values must be integer values.

End If

Repeat Line 25/26 for all num_electrodes

END OF INPUT FOR **cR2.in**

8. Details of protocol.dat

protocol.dat contains measurement schedule (and data for inverse if selected)

Line 1: (Int) num_ind_meas

where num_ind_meas is number of measurements to follow in file

If (job_type = 1) then

If (a_wgt = 0 AND b_wgt = 0) then

Line 2: (5 Int, 4 Real) j, elec(1,k), elec(2,k), elec(3,k), elec(4,k), mag, phase, mag_error, phase_error
where j is not used (but usually is used as a measurement number in the file); elec(1,k) is the electrode number for the P+ electrode; elec(2,k) is the electrode number for the P- electrode; elec(3,k) is the electrode number for the C+ electrode; elec(4,k) is the electrode number for the C- electrode; measured resistivity has magnitude mag (in Ω) and phase angle phase (in mrad), mag_error is absolute error in magnitude (in Ω), phase_error is the phase error (in mrad).

Else

Line 2: (5 Int, 2 Real) j, elec(1,k), elec(2,k),elec(3,k),elec(4,k), mag, phase
where j is not used (but usually is used as a measurement number in the file); elec(1,k) is the electrode number for the P+ electrode; elec(2,k) is the electrode number for the P- electrode; elec(3,k) is the electrode number for the C+ electrode; elec(4,k) is the electrode number for the C- electrode; measured resistivity has magnitude mag (in Ω) and phase angle phase (in mrad).

End if

Repeat Line 2 for all num_ind_meas

Else (for forward solution only)

Line 3: (5 Int) j, elec(1,k), elec(2,k), elec(3,k), elec(4,k)
where j is not used (but usually is used as a measurement number in the file); elec(1,k) is the electrode number for the P+ electrode; elec(2,k) is the electrode number for the P- electrode; elec(3,k) is the electrode number for the C+ electrode; elec(4,k) is the electrode number for the C- electrode.

Repeat Line 3 for all num_ind_meas

End if

You can add as many datasets to the file **protocol.dat**. Just concatenate the datasets into one file. ***cR2*** will continue to read and process data using the settings defined in **cR2.in**

END OF INPUT FOR **protocol.dat**

9. Details of mesh.dat

It is useful if your mesh generator permits 'materials' to be defined, allowing some zoning of the mesh (to permit blocking at interfaces). Also, you may find it beneficial (for computational efficiency) to create a coarse mesh to define the parameters and then refine this mesh (splitting a triangle element into more elements) to have more elements for the forward solution. The simplest mesh consists of an equal number of parameters and elements and one zone. More complex arrangements allow for grouping of elements into parameters and multiple zones. Regularisation is not applied at the interface of zones.

Line 1: (2 Int) numel, numnp

Where numel is the number of triangle elements and numnp is the number of nodes.

Line 2: (6 Int) n, index(1,n), index(2,n), index(3,n), param(n), zone(n)

Where n is the element number; index(1,n), index(2,n) and index(3,n) are the node numbers of the element, numbered in a counter-clockwise direction; param(n) is the parameter number of the element (to make every element a parameter then make this value equal to the element number); zone(n) is the zone number for element n. To have one zone make zone(n) equal to 1 for all elements. Zones must be connected elements. Parameters cannot occupy more than one zone. NOTE also, to make an parameter fixed to the starting resistivity, set param(n) to zero but note that if this is done all elements with param(n) = 0 must be at the end of the block of elements (see Surface 8 example below). This will involve reordering elements and care must be taken to ensure that any associated files with the element mapping (e.g. a start resistivity file, if used) but follow the same new element numbering.

Repeat line 2 for all numel elements.

Line 3: (Int, 2 Real) n, x(n), z(n)

Where n is the node number; x(n), z(n) are the coordinates of node n.

Repeat line 3 for all numnp nodes.

END OF INPUT FOR **mesh.dat**

10. **Examples**

The folder "Examples" contains a number of worked examples of ***cR2*** to illustrate how to setup input files and work with model output.

Surface electrode array 1 – getting started

The subfolder "Examples/Surface_1" contains an example synthetic model of a surface electrode array using a dipole-dipole measurement scheme. The example is a modification of the one in Binley and Kemna (2005). For this problem 25 electrodes are positioned at 2m spacing on a flat surface of a half space. The electrodes are numbered 1 to 25 from left to right. A forward model is setup to determine the measured transfer resistances for a dipole-dipole scheme with 117 measurements. The resistivity model is shown in Figure 10.1. A small target with resistivity 10 Ωm, -10mrad lies within a 100 Ωm. -10mrad half space: positioned vertically between depths 1m and 4m and horizontally between 14m and 16m.

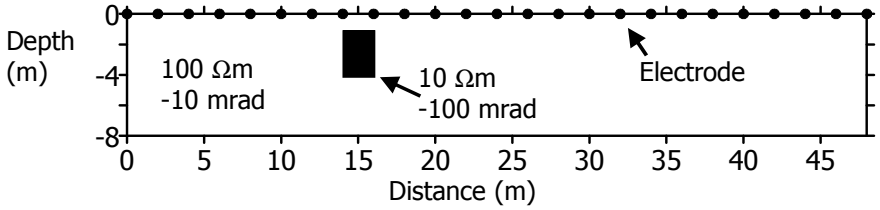


Figure 10.1: Definition of synthetic model for surface array 1 problem

The subfolder "Examples/Surface_1/Forward" contains the **protocol.dat** file for the forward problem. Also contained in the folder is the file **cR2.in** which defines the geometry and resistivity model. Since the model is a half space the finite element mesh must extend significantly away from the region of investigation (horizontally and vertically downwards). The mesh developed consists of 225 node columns and 49 node rows (i.e. 11,025 nodes, 10,752 elements). The file **cR2.in** shows how the mesh is designed to get progressively coarser away from the region of study. Note that the co-ordinates of the mesh have been set so that electrode 1 is at (0,0) for this problem. In the mesh electrode 1 is located at node column 17 (i.e. there are 16 elements to the left of the electrode array to represent an infinite boundary condition to the left. For this example 8 elements are placed between electrodes and so node 2 is at node column 25, node 3 is at column 33, etc. Since the electrodes are located on the ground surface the row node for all electrodes is 1. All the electrode positions are assigned in **cR2.in**. The file also assigns the resistivity for all elements. For this problem it is done by defining the resistivity of 9 congruent blocks of elements. First all elements in the mesh are set to 100 Ωm, -10mrad and then 8 columns of vertically adjacent elements are defined to set the 10 Ωm, -100mrad anomaly (remember that the elements are numbered vertically then horizontally).

When ***cR2*** is run the output files are:

cR2.out, which contains the main log of execution
electrodes.dat, which contains the electrode co-ordinates
electrodes.vtk, which contains the electrode co-ordinates in vtk format
cR2_forward.dat, which contains the forward model, i.e. the 117 impedances. Note that the apparent resistivity for each of the 117 measurements is also stored.
forward_model.dat, which contains the co-ordinates of the centroid of each finite element in the mesh, the resistivity magnitude and phase angle of each finite element along with the logarithm (to base 10) of the resistivity and the real and imaginary conductivity. This file is useful for checking if the resistivities were defined correctly in **cR2.in**
forward_model.vtk, which contains the forward model in vtk format (see Figure 10.2).

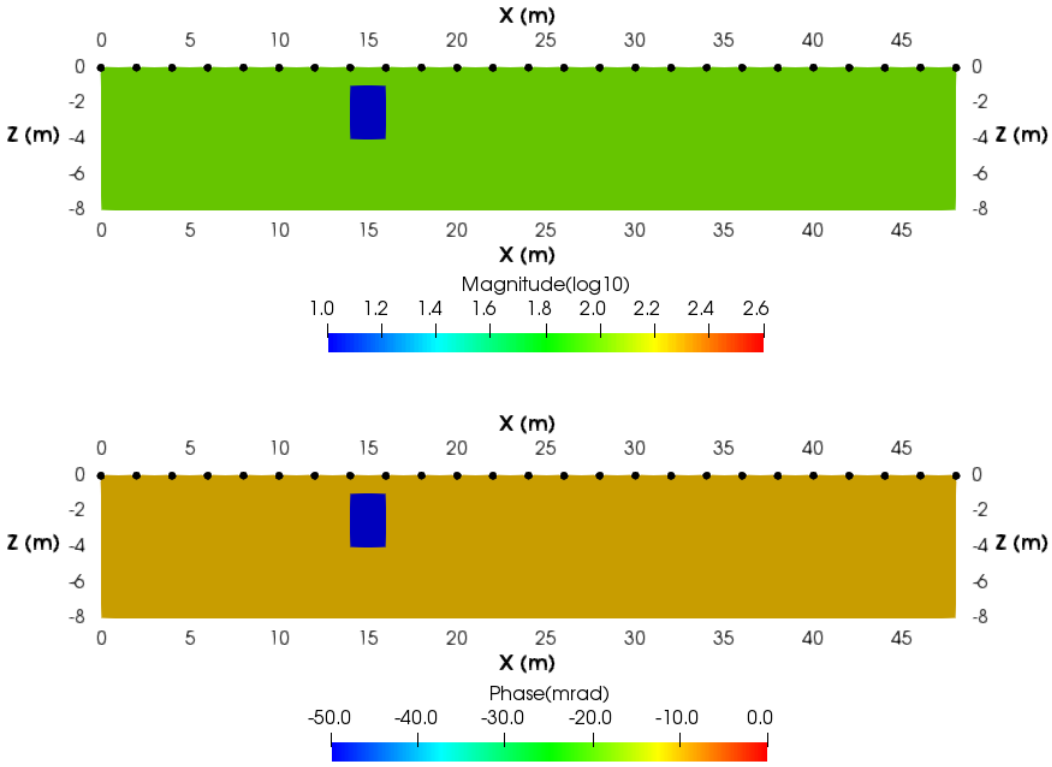


Figure 10.2: Paraview plot of **forward_model.vtk**

The subfolder "Surface_1/Inverse" contains files for running the inversion of the transfer resistances determined above. For this a uniform starting resistivity of 100 Ωm, 0mrad is defined in the file **cR2.in**. The 'data' to be inverted are stored in file **protocol.dat**: here the values are simply the impedances (magnitude and phase angle) that appeared in the **cR2_forward.dat** file described earlier.

For the inverse problem we have used a patch_size of 4 in both x and z directions, i.e. each inverse parameter is a 2 by 2 block of finite elements.

When ***cR2*** is run in this case the output files are:

cR2.out, which contains the main log of execution;
electrodes.dat, which contains the electrode co-ordinates;
electrodes.vtk, which contains the electrode co-ordinates in vtk format
f001_res.dat, which contains the computed resistivity magnitude and phase angle, $\log_{10}$ resistivity magnitude, real and imaginary conductivity for each finite element in the output region defined in **cR2.in**;
f001_res.vtk, which, in vtk format, contains the values in **f001_res.dat** and also the sensitivity map;
f001_err.dat, which contains the misfit for each of the 117 measurements.

Figure 10.3 shows the results of the inversion (compare with Fig 5.8 of Binley and Kemna(2005)). This is an image map of the results in **f001_res.vtk**. The file **f001_res.dat** could also have been used, e.g. with Surfer. Note that only the region within the electrode array and to a depth of 8m has been plotted.

In Figure 10.4 the sensitivity map is also shown. The values are computed with the equation 5.20 of Binley and Kemna (2005). High values indicate areas of high measurement sensitivity.

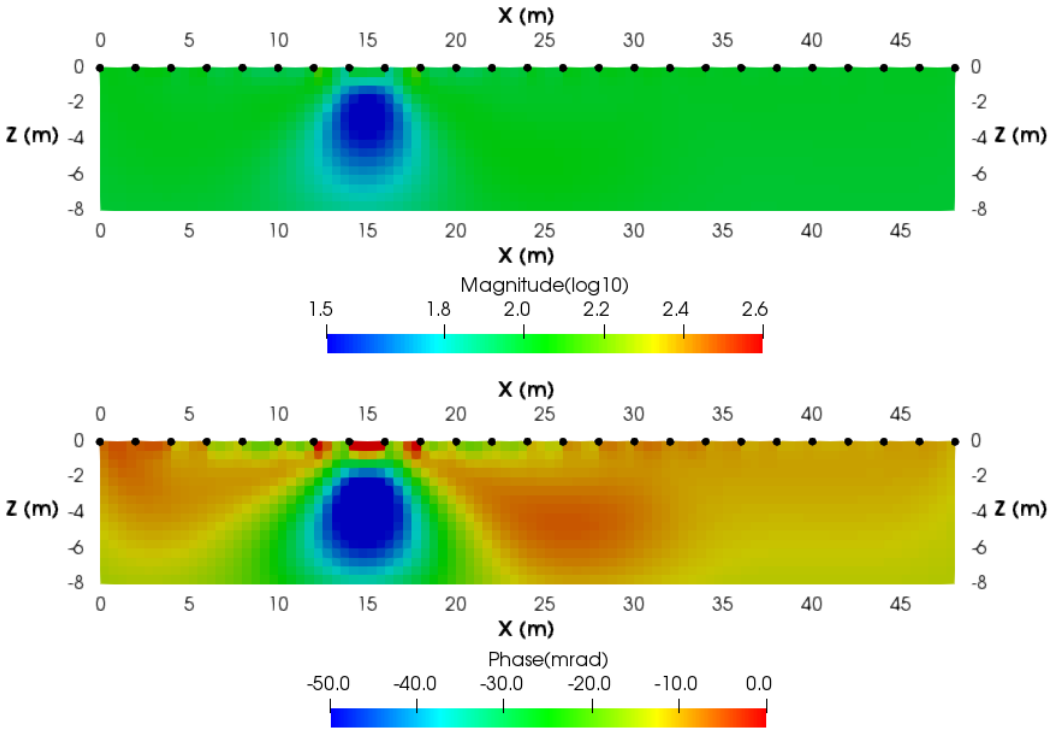


Figure 10.3: Inverse model for surface array 1 problem with dp-dp array

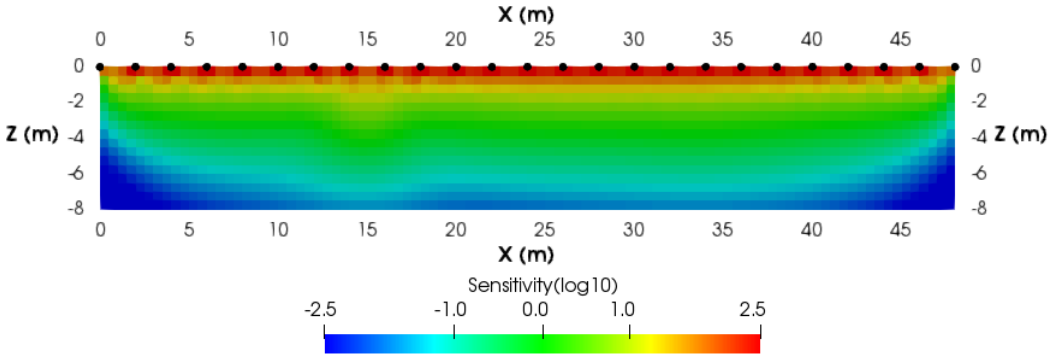


Figure 10.4: Sensitivity map for inverse model for surface array 1 problem with dp-dp array

Surface electrode array 2 – adding topography

The subfolder "Examples/Surface_2" contains an example similar to the previous case but with varying surface topography. Here the ground surface slopes from 0m at electrode 1 to 1m in the centre of the electrode array and then drops back to 0m at electrode 25 (see Figure 10.5). The file **cR2.in** is now different for the forward and inverse model runs through the addition of topography data. Figure 10.6 shows the inverse solution for this case.

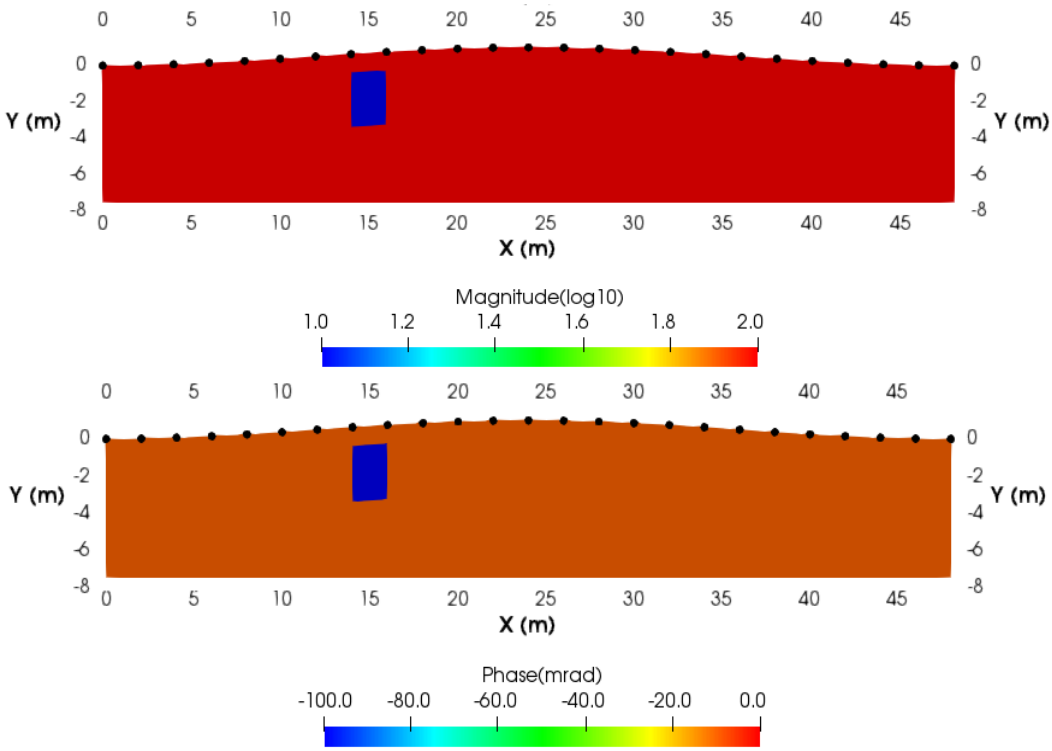


Figure 10.5: Definition of synthetic model for surface array 2 problem

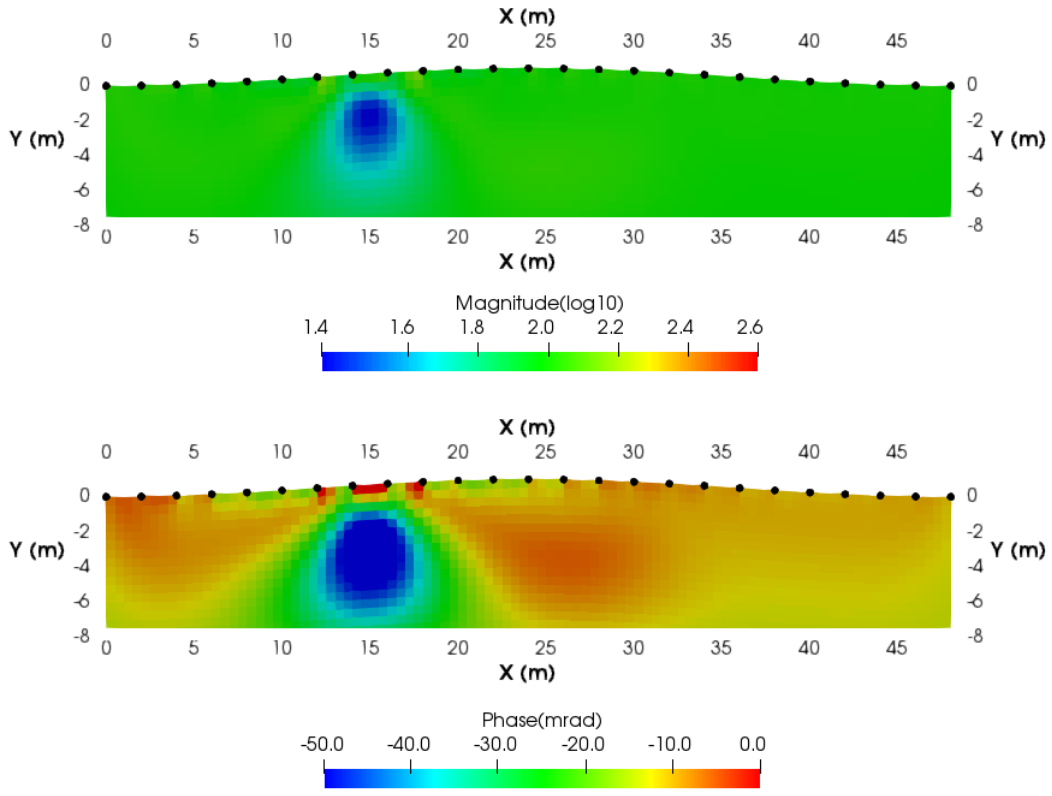


Figure 10.6: Inverse model for surface array 2 problem

Surface electrode array 3 – triangular meshing

The folder Examples/Surface_3/ contains input files for running a forward and inverse problems for the dipole-dipole survey (from Surface electrode array 1) using a triangular mesh. The mesh is defined in **mesh.dat**. It contains 4,204 elements and 2,160 nodes. The region modelled extends approximately

200m to the left and right of the electrode array, and approximately 200m beyond the zone of investigation.

The folder Examples/Surface_3/Forward contains the input files for a forward model. In this mesh the first 40 elements represent the 10Ωm, -100mrad anomaly: in **cR2.in** the two regions are defined. Figure 10.7 shows a plot of the forward model definition using ParaView. Note that the region extracted for plotting is based on the position of the centroid of elements and consequently a 'jagged' boundary often exists for triangular mesh output.

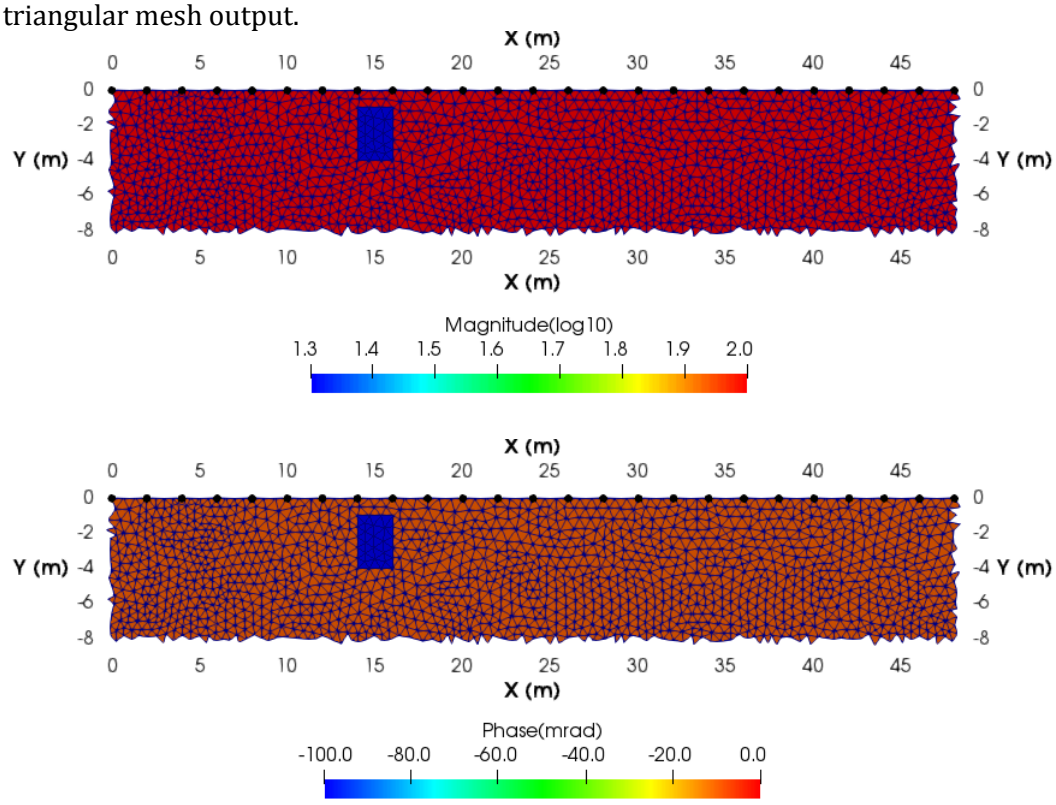
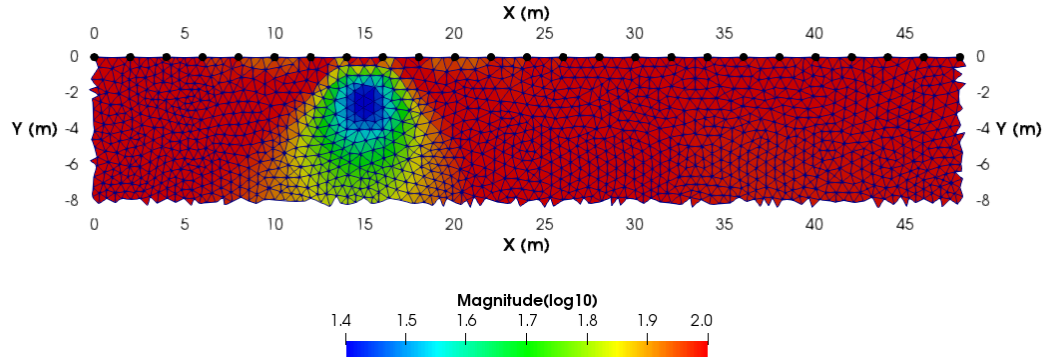


Figure 10.7: Definition of forward model using a triangular mesh

The folder Examples/Surface_3/Inverse_1 contains the input files for an inversion of the data using a triangular mesh. Figure 10.8 shows the result, plotted in ParaView. Figure 10.9 shows the sensitivity map for this problem.



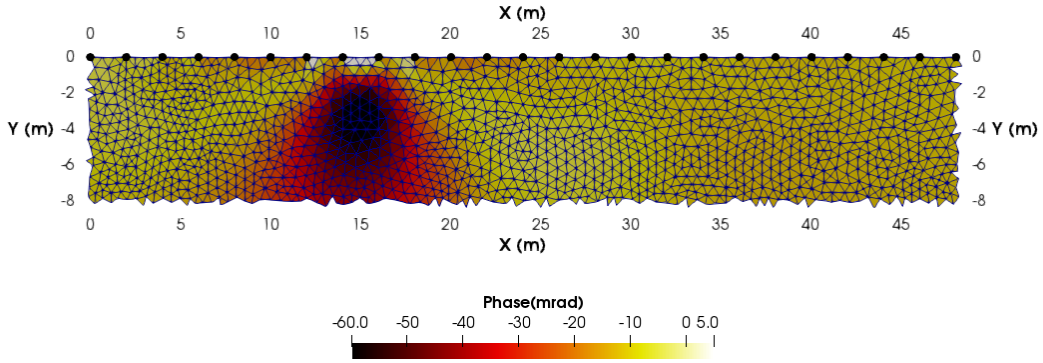


Figure 10.8: Inversion of dipole-dipole data.

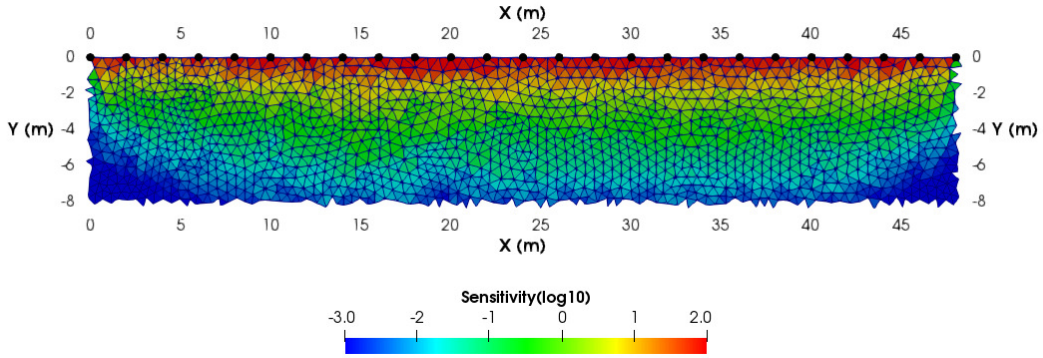


Figure 10.9: Sensitivity map for triangular mesh problem

The folder Examples/Surface_3/Inverse_2 contains the input files for an inversion of the same data using anisotropic regularisation to minimise lateral smoothing (see Figure 10.10).

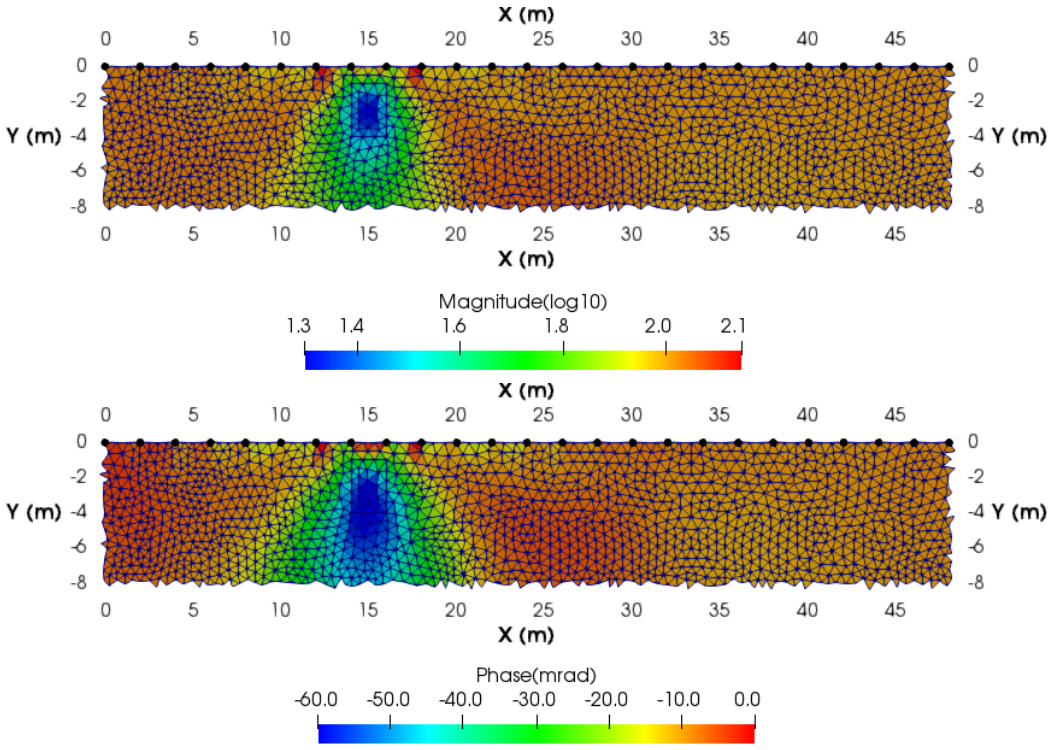


Figure 10.10: Inversion of dipole-dipole data with enhanced vertical smoothing.

The folder Examples/Surface_3/Inverse_3 contains the input files for an inversion of the same data as above but in this example the inverse region is blocked into two zones: one representing the low resistivity magnitude/high phase angle zone in Figure 10.7 and the other representing the remainder of the mesh. In this case the 40 elements that occupy the area where the low resistivity feature exists are given a different zone to the other elements in the mesh (see input for **mesh.dat**). These 40 elements are 4165 to 4204 in **mesh.dat**: a zone number of 2 is assigned to them (zone = 1 is assigned to the other elements). Figure 10.11 shows the resulting inversion. The inversion has been forced to honour the known boundaries and as a result the solution is near perfect. There is subtle variation in the zones, however – see lower image in Figure 10.11.

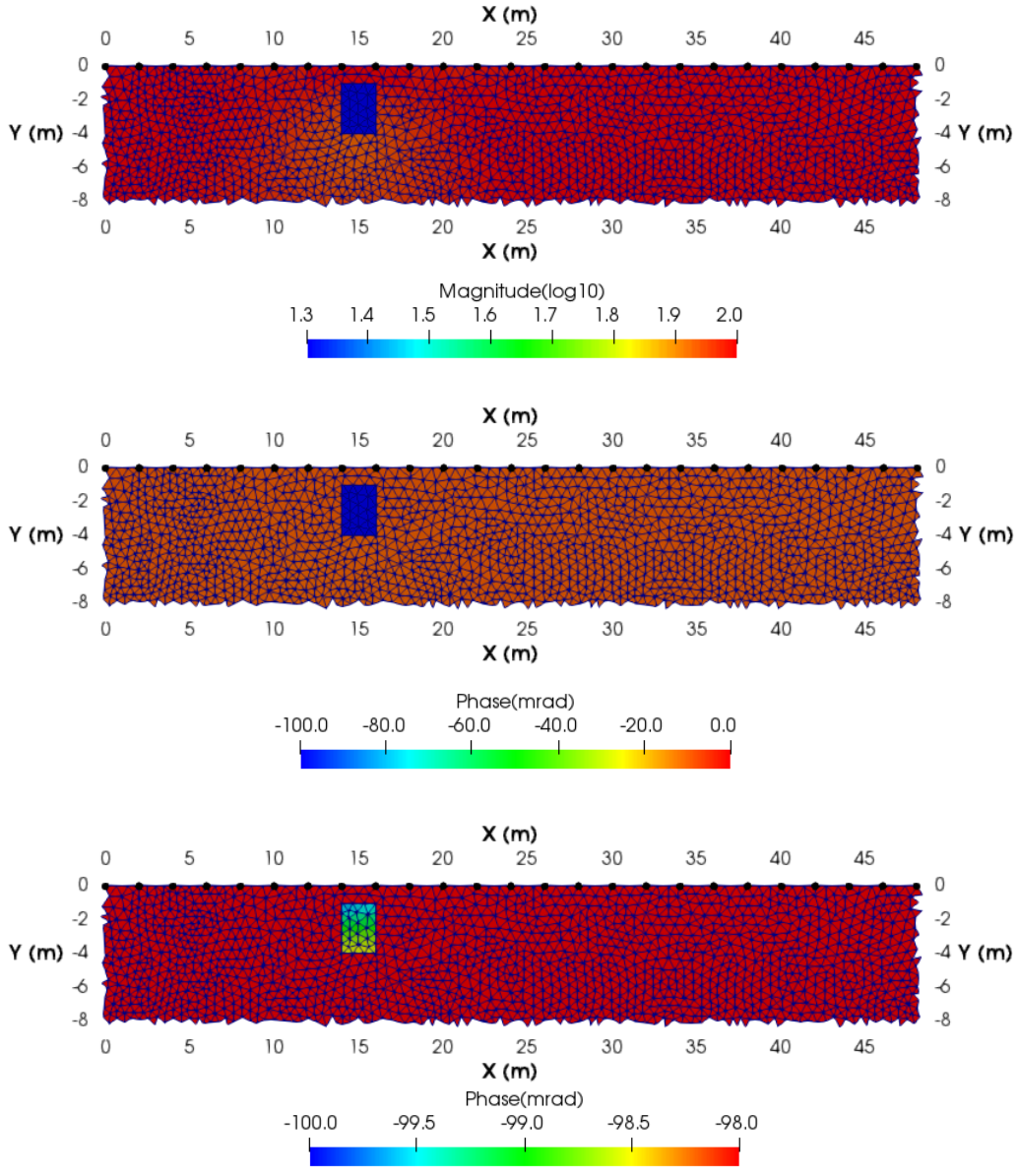


Figure 10.11: Inversion of dipole-dipole data with region blocking. The lower figure is included to show that variation does exist within the zones.

The folder Examples/Surface_3/Inverse_4 contains input files to illustrate the effect of fixing resistivity using a triangular mesh. In this example we set the resistivity of some elements to a fixed value. To do this we set their parameter number to zero in **mesh.dat**. Note that to do this we have to move this block of elements in **mesh.dat** to the end of the list of elements. In this example we set the 40 elements in the 'anomaly' area to be fixed to whatever value is assigned as the starting model (in **cR2.**in) (in this case 20

Ωm, -20mrad). All other elements can change in the inversion. The inversion is show in Figure 10.12. Note that the true target is 10 Ωm and -100 mrad and so by forcing the region to be 20 Ωm and -20mrad the adjacent elements are affected as the inversion compensates for the difference.

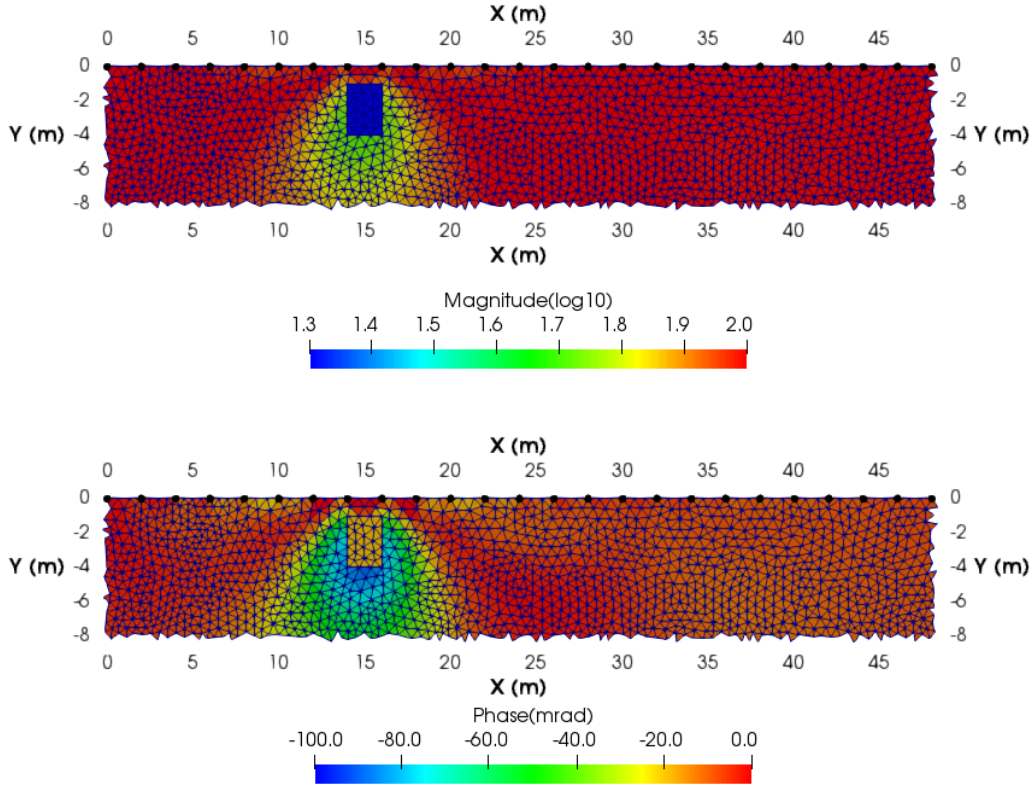
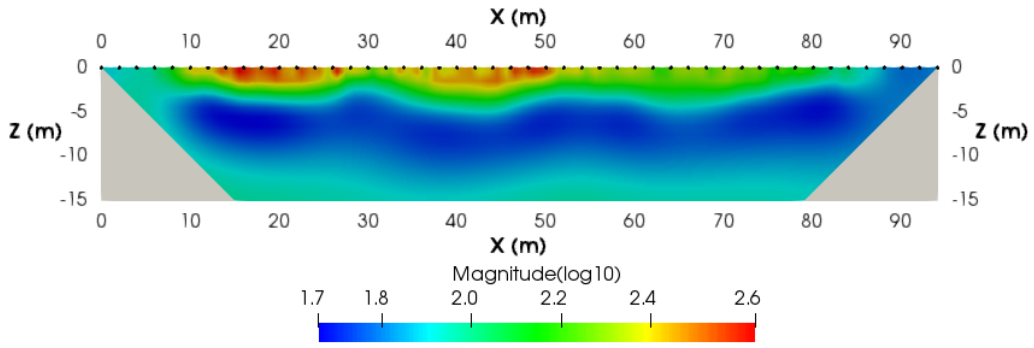


Figure 10.12: Inversion of dipole-dipole data with 40 elements (where the 'target' is located) set to 20 Ωm, -20mrad (the true value is 10 Ωm, -100mrad and so some smearing around the 'target zone' exists).

Surface electrode array 4 – field example

The folder Examples/Surface_4/ contains input files for a field survey from a UK site. The purpose of the survey was to identify the depth of a sandstone aquifer and the nature of the superficial (glacially deposited) sediments above it, with a view to assessing the vulnerability of the sandstone to surface sourced contamination. The survey was carried out in time domain IP mode with an Iris Syscal Pro using 48 electrodes at 2m spacing. Measured transfer resistances and chargeabilities were converted to equivalent magnitudes and phase angles. Figure 10.13 shows the resulting inversion. Figure 10.13 also shows the sensitivity map for the inversion. The sandstone boundary is approximately 10m deep (confirmed by drilling). The clay rich sediments above are revealed by low resistivity magnitude and a low phase angle. For more information see Mejus (2014). In this example the data errors are set for each measurement in **protocol.dat.**



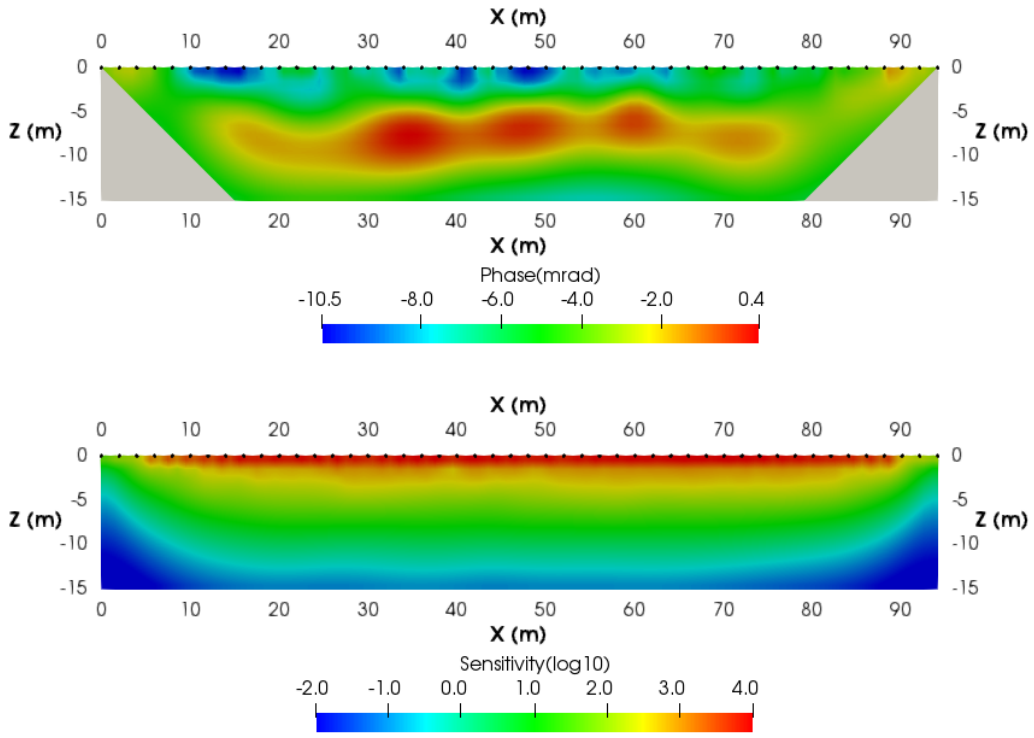


Figure 10.13: Inversion of field data.

Final note on examples

The examples included should give the user a good start in setting up files for their own datasets. Again, it is strongly recommended that the user becomes familiar with sister code ***R2*** before working with ***cR2***. The examples provided with ***R2*** show how cross-borehole surveys can be inverted – exactly the same approach can be followed with ***cR2***. When working with inversion of field data if problems occur then it is recommended that the user runs a forward model with the measurement set to check that computed values from this make sense. Many problems I hear about could have been avoided by running such tests.

Finally, as ***R2*** has a 3D equivalent ***R3t***, ***cR2*** also has a 3D version ***cR3t***. Copies are available from the author

11. **Additional codes and scripts**

The ***cR2*** package contains a number of codes and scripts that may help the user in creating input files and visualising output from ***cR2***. If you are interested in other scripts then download the sister code ***R2*** – a number of additional third party scripts and codes are included.

Meshing

Folder **Mesh_Utilities/Binley**

GenGmshGeo2D.exe
Creates a Gmsh geo (geometry file) which may be used for creating a triangular mesh. Note that the geo file can be edited before meshing, e.g. to add topography. The geo file created is **GenGmshGeo2D.geo**

GmshMsh2R2.exe
Reads the **GenGmshGeo2D.geo** created (and, perhaps, edited) in Gmsh, which can be meshed, creating a msh (mesh file) **mesh.dat.** Note that the first line will contain the number of elements, which is needed in **cR2.in** for defining the resistivity (starting model or forward model definition).

Check_R2_Tmesh
Run to check the **mesh.dat** file that is created and fix errors, in particular node numbering of the elements in the mesh.

Folder **Mesh_Utilities/Boyd**

gmsh2R2msh
This python code and executable (written by Jimmy Boyd (British Geological Survey/Lancaster University) with convert a Gmsh msh file to an ***R2*** (or ***cR2***) **mesh.dat** file.

Folder **Mesh_Utilities/Claes**

create_mesh
A Matlab script written by Niels Claes (Wyoming University) will create an ***R2*** (or **cR2**) **mesh.dat** file for a triangular mesh.

12. **Getting started with ParaView**

The vtk files created by ***cR2*** have been structured to work in ParaView - an opensource visualisation application that can be downloaded from https://www.paraview.org/ Users are advised to study online tutorials on ParaView. Here are a few simple tips to help the user get going.

The **fXXX_res.vtk** file contains the inverted resistivity magnitude, phase angle, log transformed resistivity magnitude, log real conductivity, low imaginary conductivity and a sensitivity map. The file also contains the finite element mesh structure.

Open a **fXXX_res.vtk** file in ParaView, click Apply under Properties and you will get a map of the resistivity. Under Coloring in Properties you can select one of other variables stored, e.g. log10resistivity.

The default Representation of the image is Surface. Change to Surface with edges to see the finite element mesh and the resistivity image.

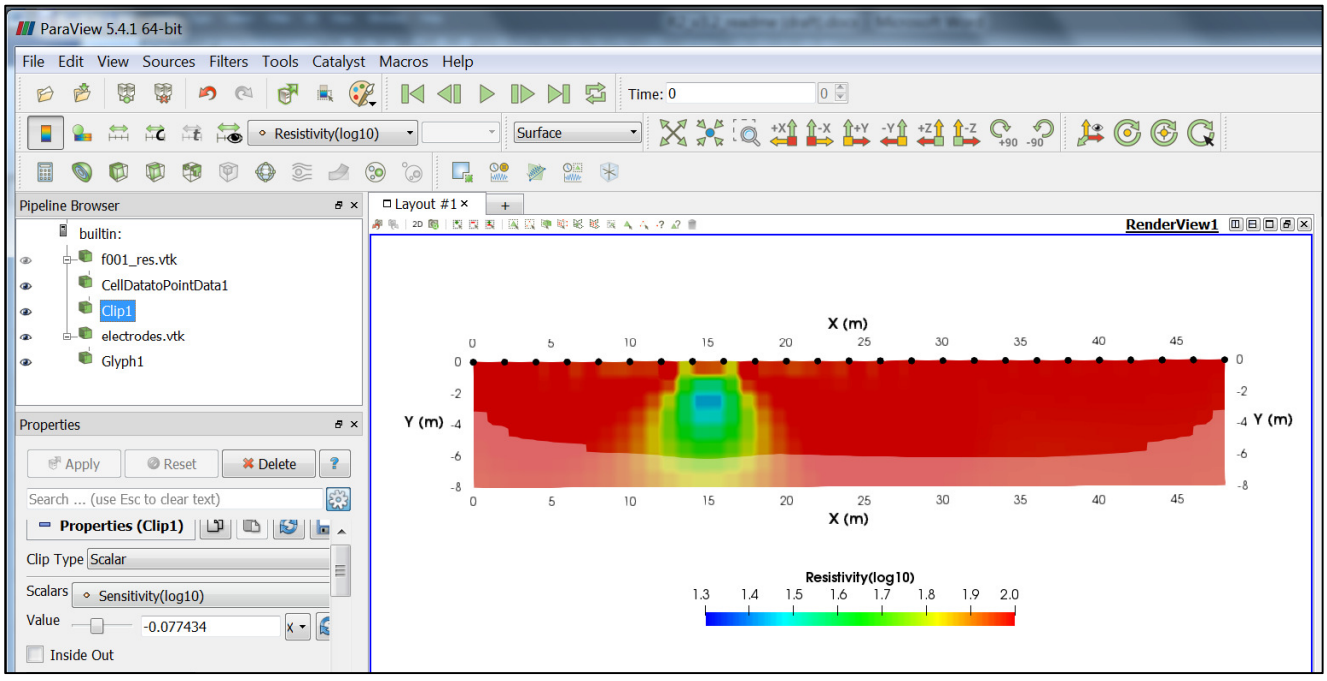
Axis labels and the colour legend are easily changed to suit the user.

To get an interpolated image (rather than one that shows element by element) then highlight the vtk file in the Pipeline Browser and select the Cell Data to Point Data Filter. Then select Apply in Properties.

If you want to show an image with thresholded values based on the sensitivity map then select an Opacity of 0.6 under Styling in Properties. Now select the Clip Filter. Select Clip Type as Scalar under Properties and select Sensitivity(log10) as Scalars. Select a mid-range value in the slider bar for Value. Under Coloring select Resistivity(log10). You should now see the thresholded region as a solid colour and the outer area as opaque.

To show the electrodes, open the **electrodes.vtk** file and click Apply under Properties. Now select the Glyph filter and under Glyph type select Sphere. Make the radius of the sphere smaller than the default, say 0.05, and select Apply. You can change the colour of the glyph in the Properties box. If not all electrodes are displayed then select Masking under Properties and select Glyph mode to All Points.

The screenshot below shows an sample from the Surface 1 example (this uses ***R2*** results but ***cR2*** is similar).



13. **Common User Errors**

Below is a list of some common user errors that I have encountered. This may be useful for new users.

A common mistake is for a new user to go straight into trying to run an inverse solution without getting a good feeling for the model that is being used. New (and old) users working on new problems should first try run a forward model for a uniform resistivity. This will help sort out any problems with the definition of the mesh, etc. It will also be useful in understanding the quality of the forward model and help judge this against the quality of the data.

If you can, run the code from the command line. You will need to run CMD in Windows, then move to the correct folder and then type ***cR2.*** Doing it like this help see any errors if the program crashes unexpectedly because of incorrect input.

In the example input files provided there are comments at the end of most lines in the form "<< comment" . Note that these are always at the end of a line. You cannot have these appearing on their own in a line. If you do then ***cR2*** will try read this comment when it is expecting numerical input and simply crash.

The mesh is based on elements and nodes. In **cR2.in** Lines 3 to 9 are based on nodes, whereas Line 13 is based on elements. It is important to understand the difference and not mix the two.

On Line 18 in **cR2.in**, specifying a tolerance of 1.0 means that you are happy that you have estimated your errors correctly (Line 19). Don't just use the a_wgt and b_wgt values in the example files – spend time to understand the likely errors in your measurements and model.

Setting the minimum and maximum apparent resistivity (Line 19 of **cR2.in**) is only valid if you have a flat surface and an infinite half space problem, otherwise the geometric factors that ***cR2*** will compute will be incorrect.

For a quadrilateral mesh the electrode positions are defined by their column and row positions in the mesh (Line 25 of **cR2.in**). These are not the co-ordinates of the electrodes but their position in the mesh.

In the definition of the input files, each line has been defined in terms of the type of numbers that are required. For example, (Real, 2 Int, 2 Real) means one real number, followed by two integers, followed by two reals. You can substitute integers for reals but not the other way round. So if the code is expecting an integer and your line entry has 1.3, for example, then the code will crash.

Note that the data in **protocol.dat** should be provided in transfer resistances, NOT apparent resistivities. Also note that the polarity should be reflected in the phase angle (see section 3). It is very wise to check the polarity of your measurements – you can do this by computing the geometric factor for your measurement configuration (provided topographic and non-infinite boundaries are not significant). If you don't know how to compute the geometric factors then you should run a forward model with ***cR2*** for a uniform half space and compare the computed polarities with those in your data. For a surface electrode array your data should be the same polarity as the model, otherwise the measurements will not be included in the inversion. For electrodes not on the surface the polarity can change as the resistivity structure changes in the inversion.

Make sure you check that the solution has converged in inverse mode (see **cR2.out**). Just because a resistivity model has been computed it does not mean that convergence has been reached. If the solution has not converged then go through the **fXXX.err** file and look at see if any particular measurements are problematic. Also check that you are confident with the a_wgt and b_wgt error settings you have applied (Line 19, **cR2.in**). A common mistake is to set these too low. A good estimate

of a_wgt is important for the magnitude inversion and b_wgt is critical for the phase angle . Normally, you should be able to get convergence in less than 5 iterations for the magnitude and typically only one iteration for the phase angle improvement. It is not wise to increase the maximum number of iterations to a large number. If you don't get convergence in 10 iterations then there is definitely some problem with the data, the assumptions or the input files.

14. **References**

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*If you make use of **cR2** then please contact the author (a.binley@lancaster.ac.uk) so that you can be added to a mailing list for future updates, fixes, etc.*

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