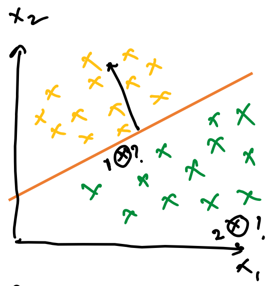
Recap

Data $\rightarrow$ classification

Data points (vectors)

Target



(slope, intercept)

Decision Boundary

weights, intercept

(w w_0)

scatterplot

$\rightarrow$ Find the best 'DB' such that Miss Classification is minimal

$\rightarrow$ Half Space $\quad w^T x + w_0 > 0 \quad | \quad w^T \cdot x + w_0 < 0$

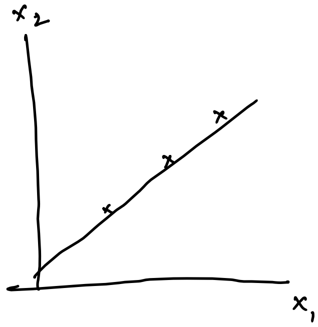
$\rightarrow$ Angle between Vectors, $\boxed{\cos\theta = \dfrac{w^T \cdot x}{\|w\| \, \|x\|}}$ — (1)

Obj $\rightarrow$ How far a point is from 'DB'?

Norm of a Vector

1. distance from a line
2. distance from another vector
3. distance from origin
4. distance to 'DB'

$$\boxed{w_1 x_1 + w_2 x_2 + w_0 = 0} \leftarrow \text{Hyperplane}$$



data point $\rightarrow$ plot  graph

(2,2)

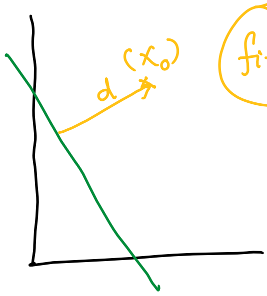
(2,2) (4,4) (6,6)

$\boxed{w_0, w_1, w_2}$

$$w_2 x_2 = -w_0 - w_1 x_1$$

$$\left[x_2 = \frac{(-w_0 - w_1 x_1)}{w_2} \right]$$

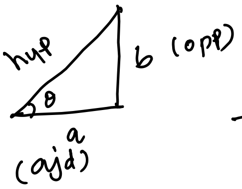
$$\begin{array}{cc} x_1 & x_2 \\ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} & \begin{bmatrix} \\ \\ \\ \end{bmatrix} \end{array}$$



find d ?

Distance from 'dB' to any vector?

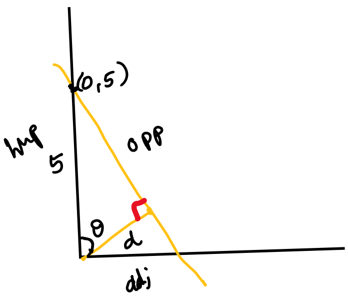
Trignometry :-



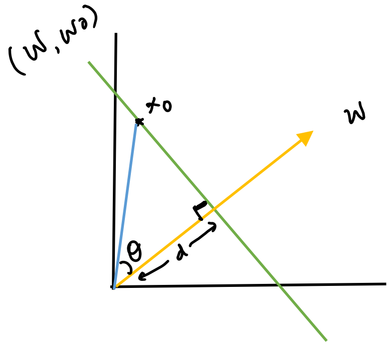
$$\tan\theta = \frac{b}{a}$$

✓ $\cos\theta = \frac{adj}{hyp}$ $\sin\theta = \frac{opp}{hyp}$ $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{opp}{\cancel{hyp}}}{\frac{adj}{\cancel{hyp}}} = \frac{opp}{adj} = \frac{b}{a}$

→ Distance from origin to 'DB'



$$\boxed{d = \cos\theta \cdot 5}$$



$$\boxed{w^T \cdot x_0 + w_0 = 0}$$

$w^T \cdot x_0 = -w_0$ — ①

Magnitude of x_0

Norm of a vector $\|x_0\|$

$\cos\theta = \frac{adj}{hyp} = \frac{d}{\|x_0\|}$

$d = \|x_0\| \cdot \cos\theta$

$\cos\theta$ w.r.t. 2 vectors x_1 and x_2

$\cos\theta$ w.r.t w^T, x

$\cos\theta = \frac{w^T \cdot x}{\|w\| \|x\|}$ — ②

$\cos\theta = \frac{w^T \cdot x_0}{\|w\| \|x_0\|}$

$d = \|x_0\| \cdot \cos\theta$

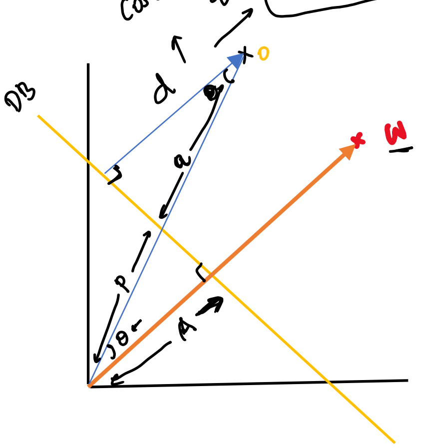
$= \cancel{\|x_0\|} \cdot \frac{w^T \cdot x_0}{\|w\| \cancel{\|x_0\|}}$

$= \frac{w^T \cdot x_0}{\|w\|}$ using ①

$\underline{d} = \frac{-w_0}{\|w\|}$

Note :- Distance from origin to 'DB' is given by $= \frac{-w_0}{\|w\|}$ ⊖ Sign of which side the boundary

$\cos\theta = \frac{d}{a}$ $\boxed{d = a \cdot \cos\theta}$



$d =$

Magnitude of vector x_0

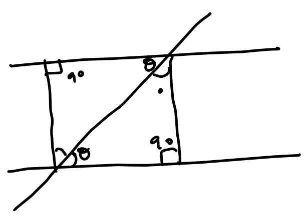
$\|x_0\|$

$A = \frac{-w_0}{\|w\|}$ — ③

$\boxed{\|x_0\| = p + q}$ ④

$\cos\theta = \frac{A}{P} \Rightarrow = \frac{A}{\cos\theta}$

$q = \|x_0\| - p \leftarrow$ from ④



$$q = \|x_0\| - P$$

$$\cos\theta = \frac{W^T \cdot x_0}{\|w\| \cdot \|x_0\|} \quad — ⑥$$

$$q = \|x_0\| - \frac{A}{\cos\theta} \quad — ⑤$$

$$= \frac{\|x_0\| \cdot \cos\theta - A}{\cos\theta}.$$

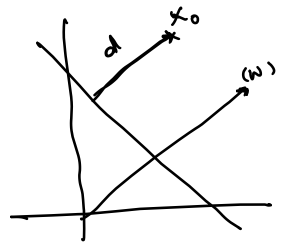
$$d = q \cdot \cos\theta$$

$$= \left(\|x_0\| - \frac{A}{\cos\theta}\right)\cos\theta$$

$$= \|x_0\| \cdot \cos\theta - A$$

$$= \cancel{\|x_0\|} \cdot \frac{W^T \cdot x_0}{\|w\| \cancel{\|x_0\|}} - \left(\frac{-w_0}{\|w\|}\right)$$

$$= \frac{W^T \cdot x_0 + w_0}{\|w\|}$$



$$d = \frac{[W^T \cdot x_0 + w_0]}{\|w\|} = \frac{\mathrm{dot}(w, x_0) + w_0}{\|w\|}$$

$$\cos\theta = \frac{d(w, x_0) + w_0}{\|w\|}$$

$$\boxed{\cos\theta = d}$$

$W^T \cdot x + w_0 > 0$ +ve half

$W^T \cdot x + w_0 < 0$ -ve half

$W^T \cdot x + w_0 = 0$ on boundary

$$\boxed{d = \frac{W^T \cdot x_0 + w_0}{\|w\|}}$$