# Constraint optimization

$f(x^2+y^2)$ min $\to 0$

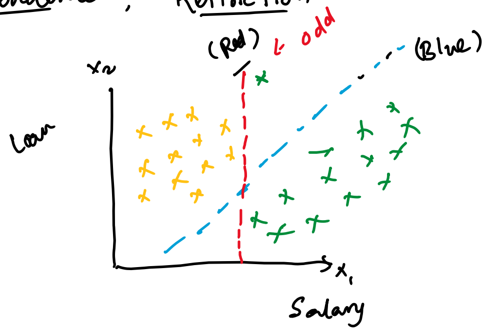
$L = f(x^2+y^2) + \lambda(x+2y+1) = 0$

$\boxed{x=1}$

→ optimization $f(x, y)$ $\nabla f \begin{bmatrix} x \\ y \end{bmatrix} \to (x, y)$ Minimization ✓

Motivation: limit, dependence, Restriction ← Optimization

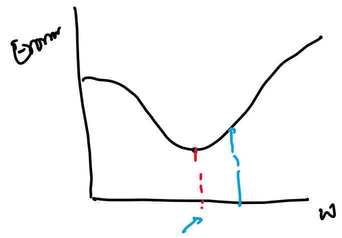
Intution :-




default.

x - Safe

x - Risk



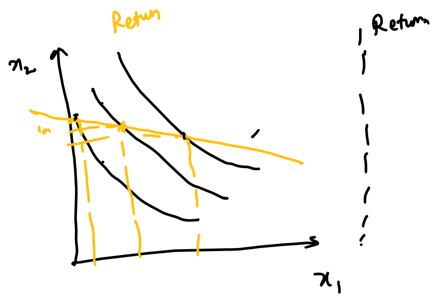

Cost function
Error
loss function

$$-\sum_{i=1}^{m} y_i(W^T x + w_0) < 0$$ (Minimize)

→ How do we make the optimization choose Blue over Red?

$f(x,y) = x^2 + y^2$

$g(x,y) = x + 2y - 1$




Investment 2-stocks ✓ Profit & loss

Budget? 2000 ✓ Stock-1 — 25 (Budget) ←

Stock-2 — 100

How many Stocks of 1 and 2?

(finding the right value of $x_1$ and $x_2$ Maximizing return?)

$$\boxed{25x_1 + 100x_2 = 2000}$$

$x_1$? $x_2$?

$\boxed{f(x^2+y^2)}$ Find minimum of $f(x,y)$ such that it also satisfies this condition

$g(x,y) \to \boxed{x+2y-1} = 0$ condition is also satisfied.

$f(x^2+y^2)$ $\nabla f \begin{bmatrix} 2x \\ 2y \end{bmatrix}$ $2x = 0$ $x = 0$ find min $f(x,y)$ ✓

$2y = 0$ $y = 0$

# Lagrange multipliers

error
Loss $= f(x,y)$

$$\mathcal{L} = x^2+y^2 + \lambda(x+2y-1)$$

($\lambda$ - langrange multiplier) $\lambda$ ←

(lagrange optimization) ✓ $f(x^2+y^2)$ $d=0$

$$\mathcal{L}(x,y,\lambda) = x^2+y^2+\lambda(x+2y-1)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 2x+\lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = 2y+2\lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = x+2y-1 = 0$$

$\lambda = -2x$
$\lambda = -y$ } $\lambda$ (c)

$2x = y$

$x+2y-1=0$

$x+2(2x)-1=0$

$5x-1=0$

$x = \frac{1}{5}$

$\frac{2}{5}$ ✓

→ optimization $-\sum_{i=1}^{n} y_i \frac{W^T.x+w_0}{\|W\|}$ distance based loss function

d - distance of a point to 'DB'

$$d = \frac{W^T.x+w_0}{\|W\|}$$

$\|W\| = \sqrt{w_1^2+w_2^2}$

multidimensional $= \sqrt{w_1^2+w_2^2+\dots w_m^2}$ m - parameters

$W \in R^m$

$$\mathcal{L} = -\sum_{i=1}^{n} y_i \frac{W^T.x+w_0}{\left(\sqrt{w_1^2+w_2^2+\dots w_m^2}\right)}$$

⇒

$y-2x+3=0$

$\boxed{y=2x-3}$

$x=0$ $y=-3$

$x=1$ $y=-1$

$2y = 4x-6$

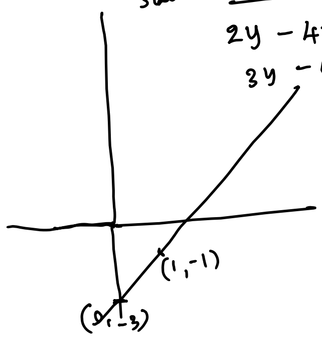
$y = \frac{4}{2}x - \frac{6}{2}$

$\boxed{y = 2x-3}$

Sol $(y-2x+3=0)$

$2y-4x+6=0$ ✓

$3y-6x+9=0$ ✓




$w_1=3,\ w_2=4,\ w_0=7$

Norm $= \sqrt{3^2+4^2} = 5$

(3,4)

and vector

$3x_1+4x_2+7=0$

$\boxed{\frac{3}{5}x_1+\frac{4}{5}x_2+\frac{7}{5}=0}$ → $\frac{1}{\|W\|}\times(3x_1+4x_2+7)$

$\left(\frac{3}{5},\frac{4}{5}\right)$

$$\sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25}+\frac{16}{25}} = \sqrt{\frac{25}{25}} = 1$$

Norm of the new line →

$\boxed{\|w\| = 1} \Rightarrow \boxed{\|w\| - 1 = 0}$ ✓ $\|w\| = 1$

$\|w\| = 1$

$\|w\|^2 = 1$

$\boxed{\|w\|^2 - 1 = 0}$

Constraint

$$\mathcal{L} = -\sum_{i=1}^{n} y_i (w^T \cdot x) + w_0 + \|w\|$$

$$\boxed{\mathcal{L} = -\sum_{i=1}^{n} \underline{y_i (w^T \cdot x) + w_0} + \lambda (w_m^2) = 0}$$

w

Lagrange multiplier

Lagrange optimization

$$\|w\| = \sqrt{w_1^2 + w_2^2 + \cdots w_m^2}$$

$$\|w\|^2 = \boxed{w_1^2 + w^2 + \cdots w_m^2}$$

$\mathcal{L}(w, w_0)$

( $f(w, w_0)$ , $g(w)$ s
minimize  Satisfy this function )

$g(x)$

norm(w) to loss equation.

$w_1^2 + w_2^2 + w_3^2 \cdots w_m^2 = 1$ only $\|w\| = 1$

$f(x,y) + \lambda (g(x,y))$

$f(x,y) = 0$

$x^2 + y^2 = 0$

$g(x)$

$x + 2y - 1 = 0$

$\|w\|^2 - 1 = 0$

$g(w)$ $\|w\|^2 - 1 = 0$ taking $\|w\|$ unit vector

$g(x,y) = 0$

$$\mathcal{L} = -\sum_{i=1}^{n} y_i (\underline{w^T \cdot x + w_0}) + \sum_{m=1}^{m} w_m^2 - 1 = 0$$