Linear Algebra - 5

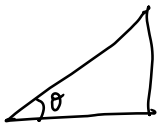
ReCap:- Classification (Binary classification)

→ Data : Vector , Target Vector: $[x_1, x_2 \dots x_m]$

→ Decision Boundary - Hyperplane (w, w_0)

→ $w^T x + w_0 = 0$ | $w^T \cdot x + w_0 > 0$ +ve

$w^T \cdot x + w_0 < 0$ -ve



$\cos\theta = \frac{adj}{hyp}$

→ Norm - length → $||x||$ → $\sqrt{x_1^2 + x_2^2}$

→ DoT Product → x_1, x_2 → $x_1^T \cdot x_2 = ||x_1||\,||x_2|| \cdot \cos\theta$

$\cos\theta = \frac{x_1^T \cdot x_2}{||x_1||\,||x_2||}$ → $\cos\theta = \frac{w^T \cdot x_0}{||w||\,||x_0||}$ ←

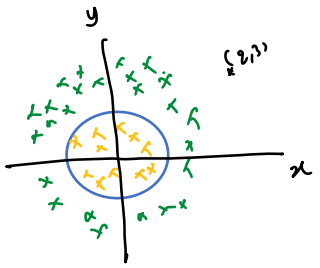
→ distance from origion to DB → $\frac{-w_0}{||w||}$

→ distance from datapoint (query point) to DB = $\frac{w^T x_q + w_0}{||w||}$

→ projection of a vector a on to b → $\frac{a \cdot b}{||b||}$

→ distance b/w two parallel hyperplanes → $\frac{w_1 - w_2}{||w||}$

→ Decision Boundary is non-linear

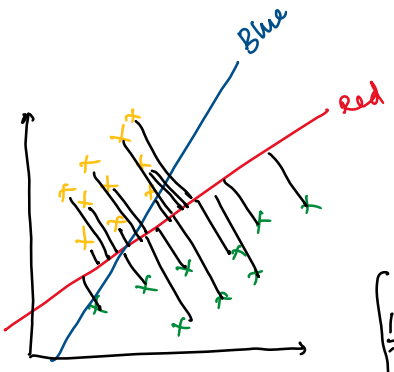


$x^2 + y^2 \leq r^2$ -ve

$x^2 + y^2 > r^2$ +ve space

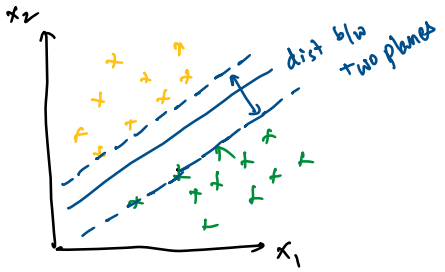
$(x-a)^2 + (y-b)^2 \leq r^2$

DB



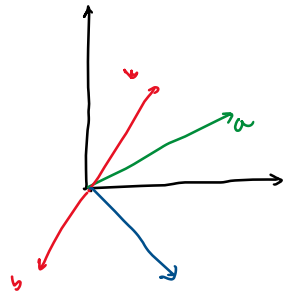
How do you decide 'DB' is a good one?

$\left[\frac{1}{m} \sum_{i=1}^{n} \frac{w^T \cdot x_i + w_0}{||w||} \times y_i \right]$ ← Max



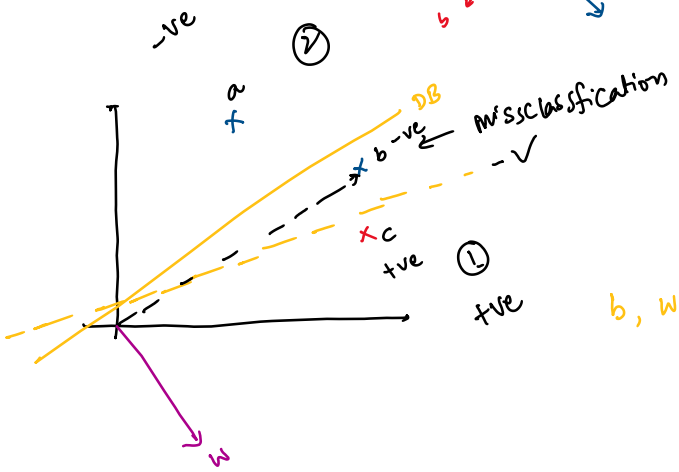
Max distance b/w two hyperplanes

→ Vector Additions



d : a + b → anti clock wise.

c : a − b → Clock



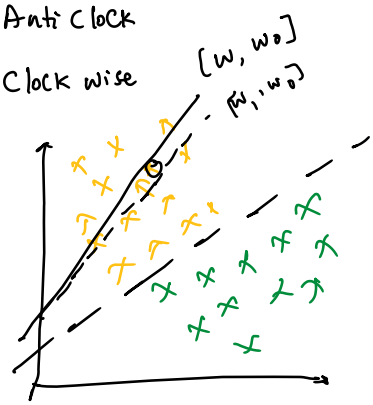
① Anti Clock

② Clockwise

b, w if w+b ?

① Anti Clock

② Clock wise



→ (w ± b) → DB will change?

(y : − or +)

→ [w + y; b]

w update → DB rotates / moves but which direction?

→ use the misclassified vector → To change the w vector

→ add / substract which direction DB has to rotate.

→ add / substract just use the sign of 'y' ? y +1 or −1

	x_1	x_2	y
① →	4	20	1
② →	5	35	−1

[w, w₀] ✓

① $w^T \cdot x + w_0 > 0$ correctly classified

② $w^T \cdot x + w_0 > 0$ wrongly classified

update w + 2 → DB

$w^T \cdot x + w_0 \leq 0$ false

→ $w_1 x_1 + w_2 x_2 + w_0 = 0$ → $w^T \cdot x + w_0 = 0$

w^T

$[w_1 \; w_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_0 \Rightarrow \; w_1 x_1 + w_2 x_2 + w_0 = 0$

w

$[w_1 \; w_2 \; w_0] \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} \rightarrow \boxed{w_1 x_1 + w_2 x_2 + w_0 = 0}$

DB → Rotate & Shift ✓

(perceptron)

→ $[w] \leftarrow [x_1 \; x_2 \; 1]$

$w \rightarrow [w_0 \; w_1 \; w_2] \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix} \Rightarrow [w_0 + w_1 x_1 + w_2 x_2] \geq$

① if a negative point is misclassified

update $w = w + (y_i^{-1}) x$

② if a positive point is miss classified.

update $w = w + y^1 \cdot x$