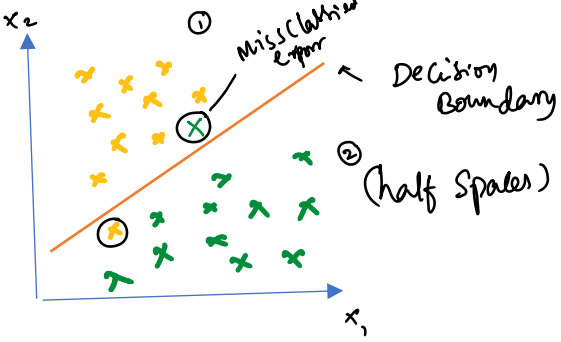
Linear Algebra

Recap

Classification — features — output/Target

	x_1	x_2	x_3	x_4	Target (y)
Vector ①	10	20	1	5.1	0
	2	15	4	6.9	1

← Data point · one observation.



$y = mx + c$ ← General

$w_1 x_1 + w_2 x_2 + w_0 = 0$

$y = wx + b$

Objective :- Find Decision boundary (w, b)

Hyperplane

$$w_1 x_1 + w_2 x_2 + \dots + w_m x_m + w_0 = 0 \quad — ①$$

$$\Rightarrow \checkmark \boxed{w^T \cdot x + b = 0} \quad — ②$$

$\checkmark$ $w^T \cdot x + w_0 = 0$

vectors :- Columns Vectors

$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$x \begin{bmatrix} 10 \\ 20 \\ 1 \\ 5.1 \end{bmatrix}$ $\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$ (w)

4×1 4×1

with four input features (dimensions)

$w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + w_0 = 0$

1×4

$[w_1 \; w_2 \; w_3 \; w_4] \times \begin{bmatrix} 10 \\ 20 \\ 1 \\ 5.1 \end{bmatrix}$

$(W^T \times x)$ 4×1

→ matrices

Matrix multiplication

$1 \times 5 + 2 \times 7$

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$

2×2 2×2 2×2

$n \times m$ $m \times p$ → $n \times p$

Dot product $[x_1 \; x_2] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \rightarrow [x_1 y_1 + x_2 y_2]$ (x: X, y: Y) $\rightarrow x^T \cdot y$

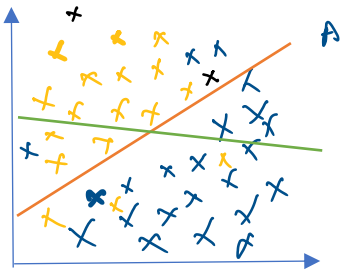
$[a^T \cdot b]$ | $a \cdot b$ $\langle a, b \rangle$

Dot product Notation $W^T \cdot x + w_0 = 0$

$[w_1 \; w_2 \; w_3 \; w_4] \times \begin{bmatrix} 10 \\ 20 \\ 1 \\ 5.1 \end{bmatrix}$

$w_1 \times 10 + w_2 \times 20 + w_3 \times 1 + w_4 \times 5.1 + w_0 = 0$

$[w_1, w_2, w_3, w_4, w_0]$ that given Minimum error points



Find a decision boundary that has Minimum error

$w_1 10 + w_2 20 + w_3 1 + w_4 5.1 + w_0 \; > 0 \rightarrow 1$

$< 0 \rightarrow 0$

→ Relation between Decision boundary and vector $W[w_0 \; w_1 \cdots w_m]$

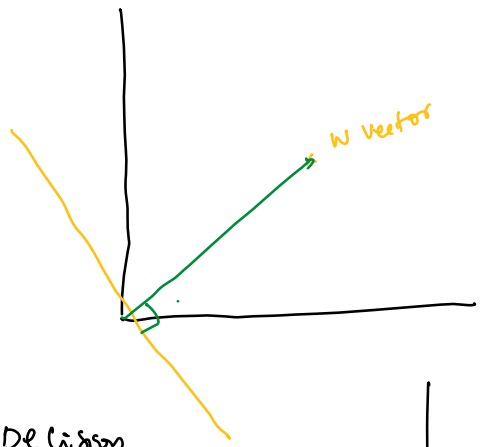
hyperplane $W^T \cdot x + w_0 = 0 \longrightarrow W[w_0, w_1 \cdots w_m]$

Can we consider the weight vector as a point on the space?

$w_1 x_1 + w_2 x_2 + w_0 = 0$

$y = mx + c$

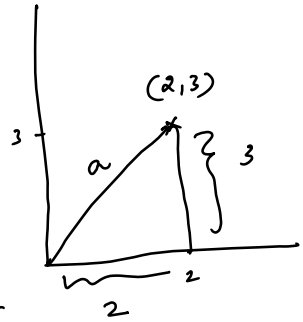
$\frac{w_1}{w_2} \; c : \frac{w_0}{w_2}$ ↑ changing intercept



W vector ⊥ Decision boundary

Vector (weight vector) → Decision Boundary

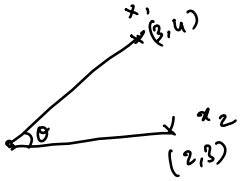
→ length of a Vector? (2,3)



$a = \sqrt{2^2 + 3^2} =$

Norm of a Vector $\|x\| = \sqrt{2^2 + 3^2} = \sqrt{4+9}$

$=$



$\|x_1\| \rightarrow \sqrt{3^2+4^2}$

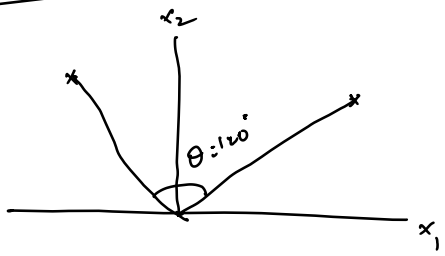
$\|x_2\| \rightarrow \sqrt{2^2+3^2}$

→ Dot product of two (vectors)

$$x_1^T \cdot x_2 = \|x_1\| \, \|x_2\| \cos\theta$$

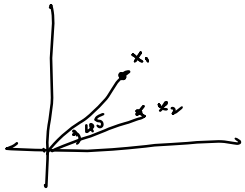
$$\cos\theta = \frac{x_1^T \cdot x_2}{\|x_1\| \, \|x_2\|}$$

Angle	Cos θ	value
0		1
45		$\frac{1}{\sqrt{2}}$
60		$\frac{1}{2}$
90		0
⋮		⋮
180		-1



Cos 120° → − ve value

cos180 = −1



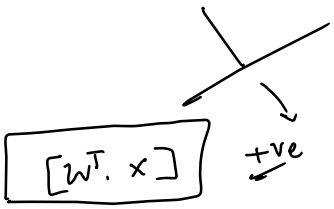
$\cos 45 = \frac{1}{\sqrt{2}}$ (+ve)

θ - acute angle

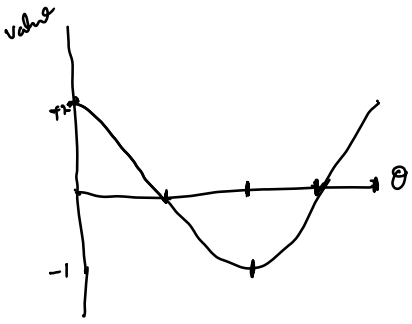
$x_1^T \cdot x_2$ +ve

θ - obtuse

$x_1^T \cdot x_2$ → -ve



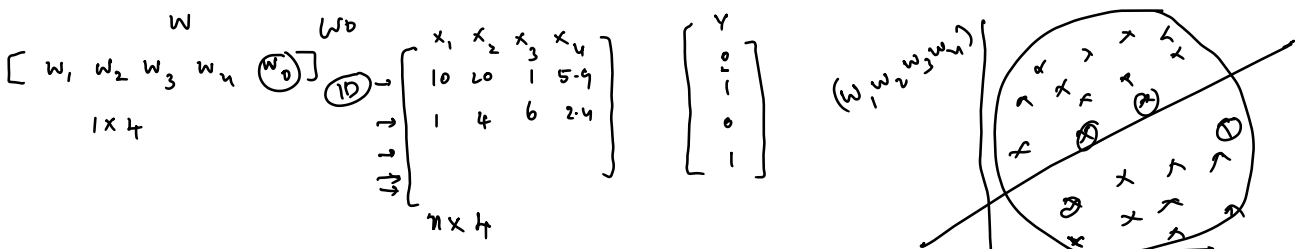
$\|w\| \, \|x\| \cos\theta$



$$w \cdot x^T + w_0 = 0$$

$$w^T \cdot x = \|w\| \cdot \|x\| \cdot \cos\theta \quad — (1)$$

$$\cos\theta = \frac{w^T \cdot x}{\|w\| \cdot \|x\|} \quad — (2)$$



$w_1 \times 10 + w_2 20 + w_3 1 + w_4 5.9 + w_0 < 0$

$w_1 \times 1 + w_2 4 + w_3 6 + w_4 2.4 + w_0 > 0$

Vector (Colum vector)

Vector (datapoint) → one observation

$[w_1\ w_2\ w_3\ w_4] \begin{bmatrix} 10 \\ 20 \\ 1 \\ 5.9 \end{bmatrix}$ ← one data point

$[w_1 \times 10 + w_2 20 + w_3 1 + w_4 5.9]$

$w^T x + w_0 = 0$

$[X \cdot W]$

→ $\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ 10 & 20 & 1 & 5.9 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} \Rightarrow$

$10 \times w_1 + 20 \times w_2 + 1 \times w_3 + 5.9 \times w_4 > 0$

< 0

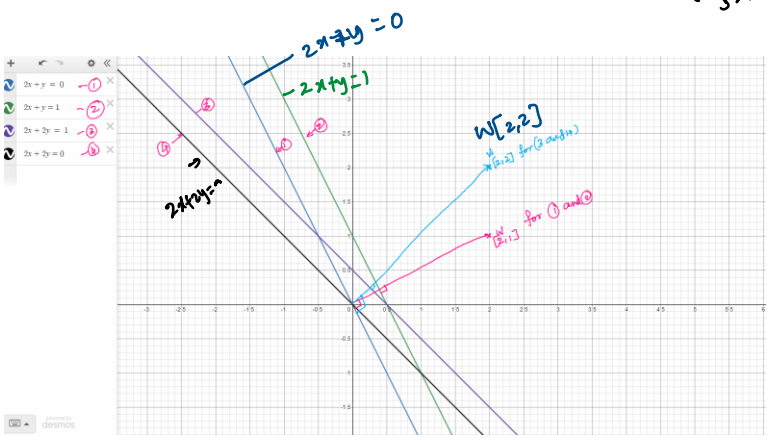
→ [Column] $(W^T \cdot x + w_0] = 0$ ✓

obs1 obs2 obs3 ... obsn

$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} \times \begin{bmatrix} 10 \\ 20 \\ 1 \\ 5.9 \end{bmatrix}$ Vector

$W^T\ [w_1\ w_2\ w_3\ w_4] \times \begin{bmatrix} 10 \\ 20 \\ 1 \\ 5.9 \end{bmatrix}$

$W^T \cdot x + w_0 = 0$



W vector ⊥ To Decision Boundary (line)