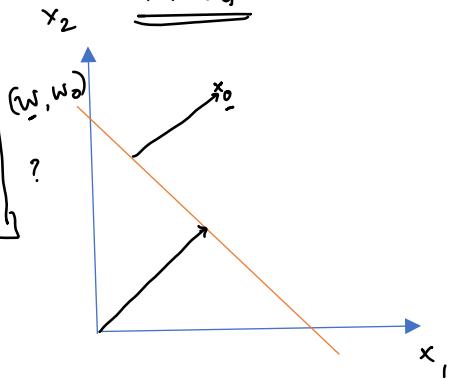


## Decision Boundary

- Error Minimization ✓
- Unit Vector ✓
- Vector projection
- Vector Addition

$$d = \frac{\mathbf{w}^T \mathbf{x}_0 + w_0}{\|\mathbf{w}\|}$$

Recap



$$\text{Origin DB} = \frac{-w_0}{\|\mathbf{w}\|} \quad \checkmark$$

### Half Space

$$\begin{array}{ll} \mathbf{w}^T \mathbf{x} + w_0 > 0 & \text{+ve Space} \\ \mathbf{w}^T \mathbf{x} + w_0 < 0 & \text{-ve Space} \end{array}$$

→ Which one is Positive half Space? Which is Negative half Space

① Top (above DB) +ve Space, below (DB) Negative

② Top (above DB) -ve Space, below DB positive



## Classification (Binary classification)

→ Which one is a better DB?

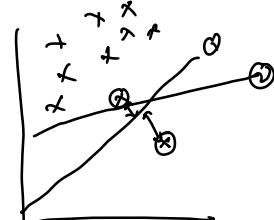
→ Sum of distances of all points to the Boundary

(Maximum) ✓

Blue → 100 ✓

Orange → less 80 ✓

Which one is better



→ Avg distance (mean distance)  $\text{Max} \uparrow$

→ Max (Min distance b/w one positive and one negative point)

$$\sum_{i=1}^n \frac{\mathbf{w}^T \mathbf{x}_i + w_0}{\|\mathbf{w}\|} \quad (d) \quad d \text{ +ve or -ve}$$

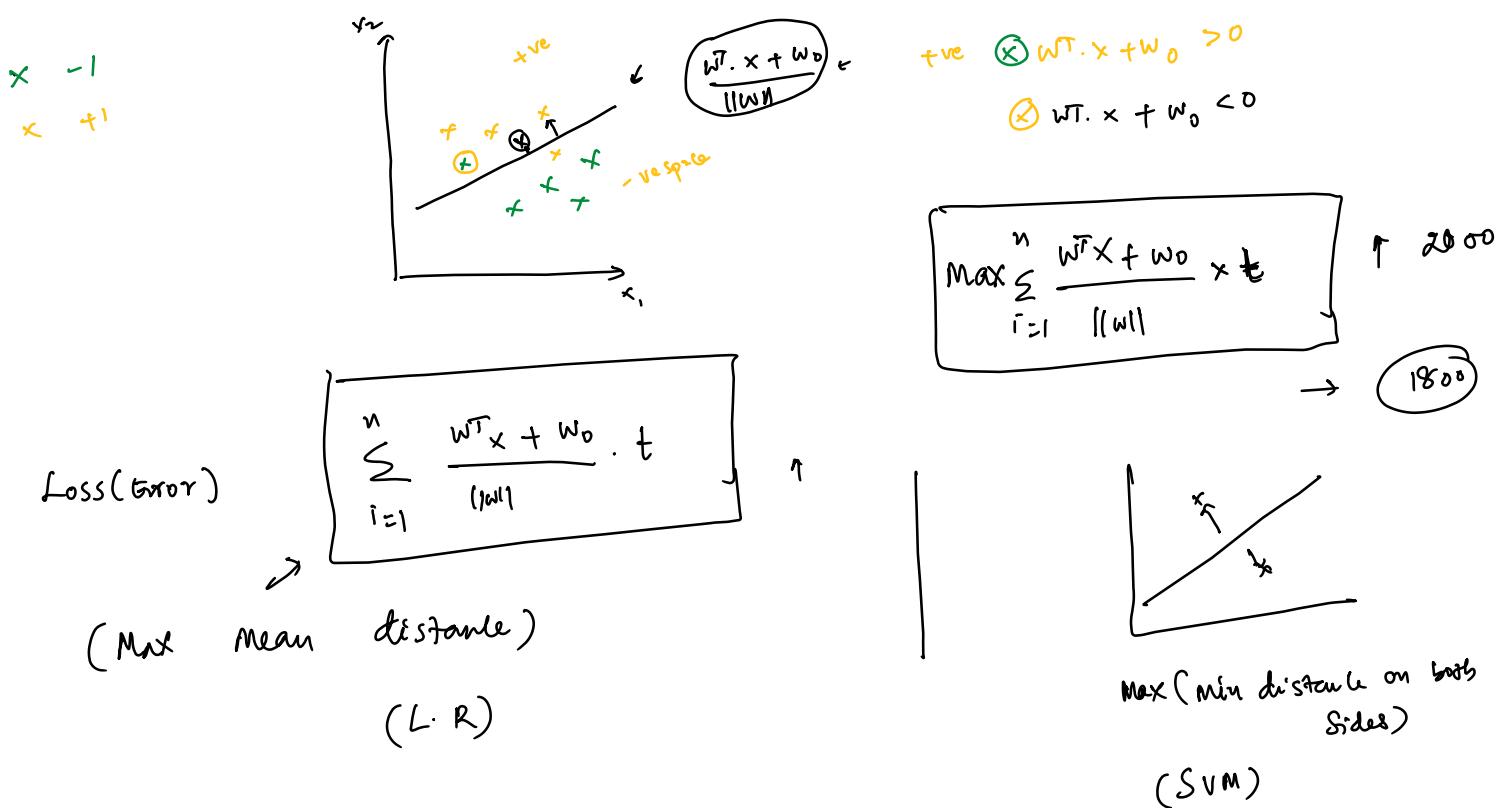
EMI Monthly Salary

n - # of obs

f <sub>1</sub>	f <sub>2</sub>	Target	-1
10	20	0	-1
15	16	1	+1

Mean distance  
from target

$$\text{Mean distance} = \sum \frac{\mathbf{w}^T \mathbf{x}_i + w_0}{\|\mathbf{w}\|} \times T + 1$$

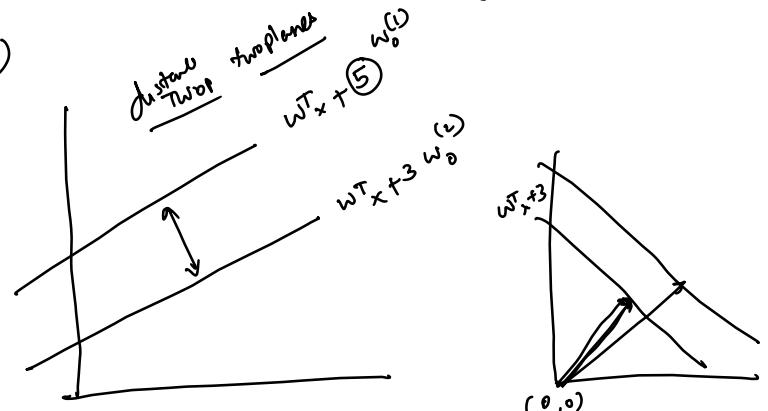


→ Distance b/w two planes (Parallel)

→

→ If two planes are Parallel.

- ①  $w$  and  $w_0$  will be same
- ②  $w$  is same  $w_0$  will change
- ③ both  $w$  and  $w_0$  will change

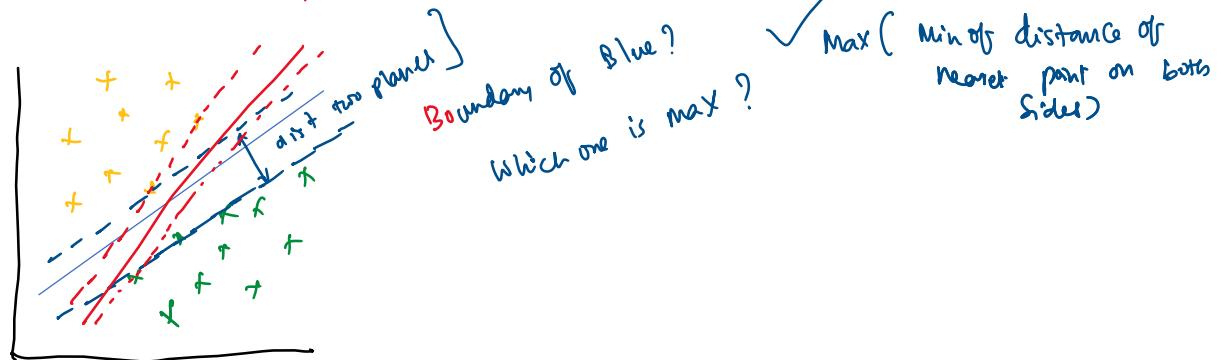


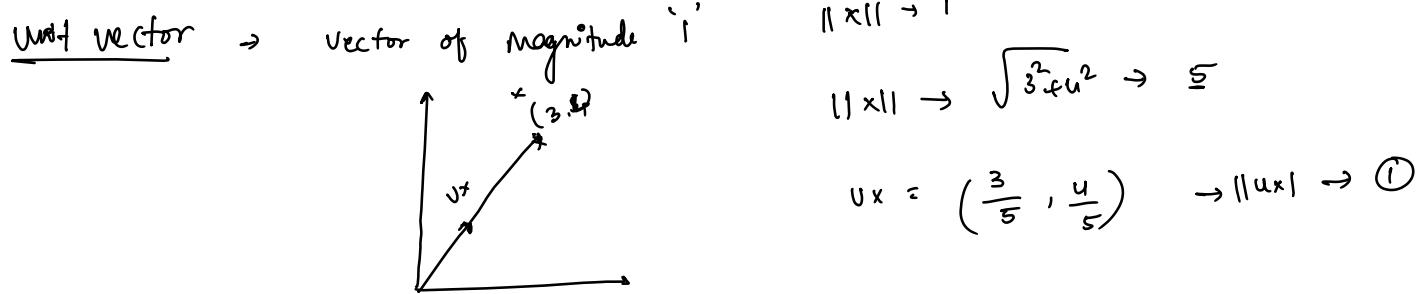
$$\rightarrow \text{origin to plane} = \frac{-w_0}{\|w\|} = \frac{-3}{\|w\|} = \frac{-5}{\|w\|} = \frac{5}{\|w\|} - \frac{3}{\|w\|}$$

$$\rightarrow \frac{w_0' - w_0''}{\|w\|}$$

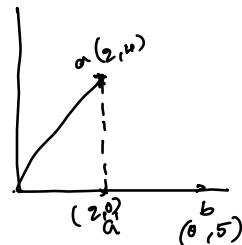
Boundary of Red

Svm

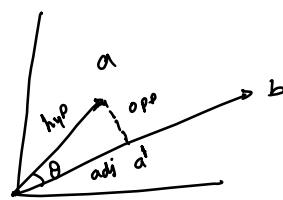




### Projection of a vector



$a'$  is projection of  $a$  onto  $b$  vector



$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{a'}{\|a\|}$$

$$a' = \|a\| \cdot \cos \theta$$

$$\rightarrow \text{hyp} \rightarrow \|a\|$$

$\cos \theta$  (w.r.t. two vectors  $a, b$ )?

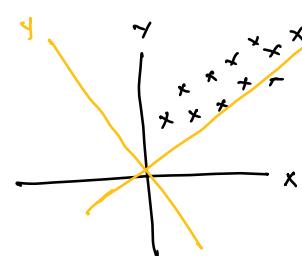
$$\cos \theta = \frac{a \cdot b}{\|a\| \|b\|} \quad -②$$

$$a' = \|a\| \cdot \frac{a \cdot b}{\|a\| \|b\|}$$

$$\cos \theta = \frac{a^T \cdot b}{\|a\| \|b\|}$$

$$a' = \frac{a \cdot b}{\|b\|}$$

← Projection of vector  $a$  on  $b$  is calculated.

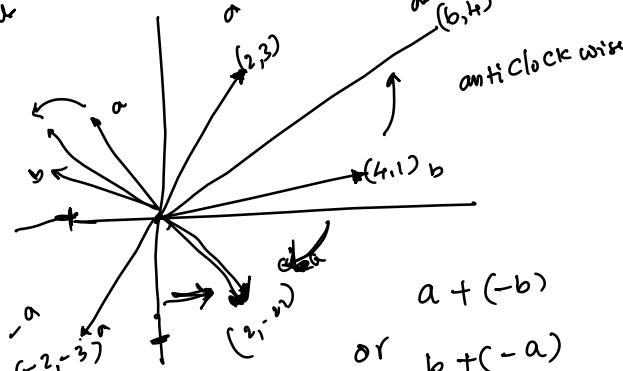


addition → anti-clockwise  
Sub → clockwise

### Vector Addition

Can we subtract vectors?

Given two vectors  
Add them  
it moves in anti clockwise  
direction.



$$b + (-a) =$$

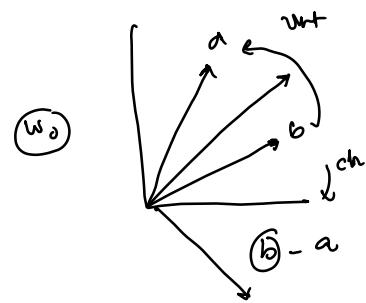
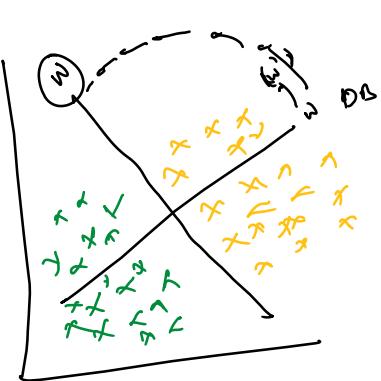
$$a_4 = (a - b)$$

$$\begin{aligned} a &= (2, 3) \\ b &= (4, 1) \\ a - b &= (6, 4) \end{aligned}$$

$\alpha$  addition

Perception

shift, rotate  
rotation ( $w$ )



DB, Hyperplanes  
half spaces

distance from origin to  
plane  
distance of vector from

Maximize  $\uparrow f(x)$  or Minimize  $f(x)$



Maximize or Minimize

$$f(w, x) \rightarrow \sum_{i=1}^n \frac{w^T x + w_0}{\|w\|} \times t_i$$

$f(w, w_0, x)$  the values of  $w$  and  $w_0$  such that

$f(w, w_0, x)$  taken max possible value  
 $f(w, w_0, x)$  taken min possible value

Sum / mean distance of  
all data points

avoid +ve and  
-ve distance

Maximizing Sum / mean  
Max (dist b/w the  
nearest

distance b/w  
two planes  
Projection  
Addition

Forward  
optimization

$$\begin{aligned} 9x + 2y - 10 &= 0 \\ 2x + 20y - 100 &= 0 \end{aligned}$$

elimination

(Gauss Seidel  
Newton Raphson method)

(Gradient descent)  
optimization

Calculation  
 $w_0, w$

$$0 = 10x_1 + 20x_2 + 30x_3 + \dots + 10x_{10} + 90 = 0$$

Minimize error  
Maximize distance of all points  
DB

Unit vector  
projection  
distance b/w parallel planes  
vector addition