Start filling in and adding! Github location is: here at my account.

1 Definitions

Intervals

An interval is a set I where if $a, b \in I$ then $\forall c$ such that $a < b < c,\ c \in I$.

Compact

A set is compact if it is closed and bounded.

Closed

There are a number of definitions for $A \subseteq \mathbb{R}$ being closed:

- A set is closed if it contains all of its limit points.
- $\forall \{a_n\}$ such that $a_n \in A$, if $\lim_{n\to\infty} a_n = a$, then $a \in A$.
- A contains all of its accumulation points.
- The complement of A is open (int $\mathcal{U}$.)

Bounded

- The set A is bounded if $\exists M \in \mathbb{R}$ such that $\forall a \in A, |a| < M$.
- The function $f : A \to \mathbb{R}$ is bounded if $\exists M \in \mathbb{R}$ such that $\forall a \in A, |f(a)| < M$.

Bounded Function Set

If $A \subseteq \mathbb{R}$, $b(A)$ is the set of all bounded functions from $A \to \mathbb{R}$.

Continuous Function Set

If $A \subseteq \mathbb{R}$, $c(A)$ is the set of all continuous functions from $A \to \mathbb{R}$.

Partition

$\mathcal{P} = (x_0, x_1, \ldots, x_n)$ is said to be a partition of $[a, b]$ if $a = x_0, x_1, \ldots, x_n = b$.

Tagged Partition

$\dot{\mathcal{P}} = (x_0, x_1, \ldots, x_n)$ is said to be a tagged partition of $[a, b]$ if it is a partition and $t_i \in [x_{i-1}, x_i]$.

Riemann Sum

If $f \in b[a, b]$ and $\dot{\mathcal{P}}$ is a tagged partition, then

$$s(f, \dot{\mathcal{P}}) = \sum_{i=1}^{n} f(t_i)(x_i - x_{i-1})$$

is called a Riemann sum of f for tagged partition $\dot{\mathcal{P}}$.

Riemann Integrable

Let $f \in b[a, b]$, we say f is Riemann Integrable, written $f \in \mathcal{R}[a, b]$ if $\exists L \in \mathbb{R}$ such that $\forall \epsilon > 0,\ \exists \delta_\epsilon > 0$ such that

$$|s(f, \dot{\mathcal{P}}) - L| < \epsilon \text{ whenever } ||\dot{\mathcal{P}}|| < \delta_\epsilon.$$

Thus we can say that

$$\int_a^b f = L.$$

Norm of a Partition

If $\mathcal{P}$ is a partition of $[a, b]$ then

$$||\mathcal{P}|| = \max_{1 \leq i \leq n} \{x_1 - x_0, x_2 - x_1, \ldots, x_i - x_{i-1}, \ldots, x_n - x_{n-1}\}$$

is called the "norm" of $\mathcal{P}$.

Indicator Function

Lipschitz Continuous

Pointwise Convergence of Functions

Uniform Convergence of Functions

Uniform Norm

Cauchy wrt Uniform Norm

Let $\{f_n\}$ be a sequence with $f_n \in b(A) \forall n$.
We say $\{f_n\}$ is Cauchy wrt the uniform norm if $\forall\, \epsilon > 0 \,\exists\, N \in \mathbb{N}$ such that if $n, m \geq N$, then

$$||f_n - f_m||_u < \epsilon.$$

2 Theorems

1. Intermediate Value Theorem

2. Bolzano-Weirstrauss Theorem

3. Riemann Integrable Function Uniqueness

4. Squeeze Theorem

5. Cauchy Criterion for Integration

6. All continuous functions are Riemann Integrable

7. Monotonic functions with the reals as an image are Riemann Integrable

8. Integral of an interval equals the sum of the integrals between two "connected" intervals
   Suppose $f : [a, b] \to \mathbb{R}$ and $c \in (a, b)$, then $f \in \mathcal{R}[a, b]$ iff $f \in \mathcal{R}[a, c]$ and $f \in \mathcal{R}[c, b]$.

9. Fundamental Theorem of Calculus, part I

10. Fundamental Theorem of Calculus, part II

11. Lipschitz Continuity Criteria

12. Change of Variables (substitution)

13. Integration by Parts

14. Sequences that imply continuity
    If $\{f_n\}$ is a sequence of functions $f_n \in c(A)$ and $f_n \stackrel{u}{\to} f$ on A, then $f \in c(A)$.

15. More with sequences, I guess these don't all need names... or labels.
    Suppose for all n $f_n \in \mathcal{R}[a, b]$ and $f_n \stackrel{u}{\to} f$ on $[a, b]$. Then $f \in \mathcal{R}[a, b]$ and $\lim\limits_{x\to\infty} \displaystyle\int_a^b f_n = \int_a^b f$..

16. Every Cauchy sequence in $b(A)$ converges in $b(A)$ with respect to $||\cdot||_u$. That is, $b(A)$ is complete wrt $||\cdot||_u$.

3 Examples

Add examples in class or other examples you may think of here

4 Homework

Add homework problems here. Not necessarily every one, but the ones in particular that would make great examples. The homework problems that give a theorem, or lemma, or other super-cool results, feel free to put in the Theorems section.