In answering whether a model with two or more competitors includes HOIs we must first acknowledge an important point:  the form of the model itself cannot always be arrived at by only observing the single competitor cases. In these cases the form of the multi-competitor model in fact entails an implicit HOI.

Consider the following single competitor model, , where denotes the function giving the effect of competitor one on the focal species *i*.

Now consider the problem of including the effects of a second competitor whose effect on the focal species is defined by the following single competitor model, The question now is how to model the joint effects of competitors one and two together in a two competitor function . We can write out a general functional form for this as follows,

where *Pn* = (0, 1) and *P*1 + *P*2 = 1. (In other words, *Pn* has two elements that are either 0 or 1 and these elements always sum to one). Note that the form of the function *Gi* depends on the value of the parameter *Pn*. Thus if *P*1 = 1 and *P*2 = 0 we get a fundamentally different model than if *P*1 = 0 and *P*2 = 1. Moreover, the value of the *P* parameter is NOT available to us in the single competitor models: the functional form for the effect of competitor 2 in isolation is the same.

In general, when two single competitor models can be combined in more than one way into a multi-competitor model, there is an implicit HOI. We can make this HOI explicit, by defining a parameter P as above which captures the choice.

We have been defining a model without HOIs as one that can be reconstructed from a set of single competitor models without any additional parameters not found in the set of single competitor models. I believe this is correct, but perhaps the above example shows that we have not been counting up all the possible parameters involved.

This issue does not show up when we are working with only two competitors because any implicit HOI remains hidden when we apply say the A&M criteria. However, in a model with three competitors where the relationship between any pair of competitors is ambiguous, as in the cases you sent to me this week, I think the A&M criteria actually may be correct in telling us that there is an HOI. The A&M criteria are telling us that we cannot arrive at the multi-competitor model from the set of all parameters (implicit and explicit) in the separate single competitor models. Thus, there is in fact an HOI even by our original conceptual definition.