**A more technical definition of emergence in multivariate functions:**

In general, the question of whether a model with multiple species of competitor contains HOIs is equivalent to asking whether the model can be decomposed into a set functions one for each competitor species. If so the dynamics of the multicompetitor community are effectively the “sum” of its parts.

To make this definition more precise, we can borrow some of the concepts developed by electrical engineers to reason about emergent behaviors in the design of complex electronic systems (vague I know but I don’t really know what to call this). For engineers, t is critical to ensure that the behavior of a large composite function can be predicted based on the behavior of the simpler functions that make it up. Otherwise there is some kind of emergence in the system that can lead to unexpected (and for engineers this means dangerous) behavior (Hinton 1997).

The analogy with ecological systems is straightforward. If a researcher understands how performance depends on the density of each separate species, can the multispecies community be but together from these separate behaviors or not?

**Here we propose the following rules for decomposing any multispecies model of community interactions without HOIs:**

1. **The multispecies function can be decomposed into separate univariate functions. (no multivariate functions!)**
2. **Terms for the density of each competitor are only found in one function. (no repeats!)**
3. **The functions are linked by addition (additivity!).**

Consider the following generic multispecies competition model:

the per capita growth rate of the focal species, is on the RHS, and *F* is a function of the density of three competitor species, , , and .

As an example, let us decompose the following model for competition

where , , and

What about a more complex function such as the following:

,

where

Or another even more complex form:

,

where , and , and g(x) is defined as above.

Note that in these decompositions **1) each function is univariate; 2) each competitor’s density appears in only one function, 3) functions are combined by addition only**. Therefore, none of these above functions have HOIs. Despite their complexity and non-linearity, we can reason about each as the sum of their constituent functions.

***Now consider another function:***

Because of the multiplicative term**:**  the function cannot be decomposed into separate univariate functions without the competitor densities appearing in more than one function.

**Functional Trees to trace routes of influence**

The following figure demonstrates all the ways in which a three species competition function can be decomposed. There are only a finite number of ways to do this in a pairwise model. Perhaps this a useful realization.

I believe we can map out all the ways multivariate functions can be decomposed while retaining a “pairwise” nature. Diagram in next page.







