Do we need detailed demographic data to forecast pop-

² ulation responses to climate change? Probably.

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8 Abstract

The ability of population models to skillfully forecast future states under climate change is constrained by the limited spatial and temporal extent of demographic data. This data is often limited because it is difficult and costly to collect. An alternative is to rely on 11 aggregate, population-level data that is easier and less costly to collect. Doing so requires 12 assuming that population-level data accurately represents the aggregate response of the 13 individuals that actually respond to weather. We tested this assumption using population models of four Montana grassland species fit using individual and aggregated forms of the same data. We fit population models with interannual variation in vital rates explained, in part, by climate covariates and then perturbed observed climate to compare model forecasts. 17 Population models based on individual-level demographic data outperformed the models based on aggregate-level data in terms of accuracy and precision. The two model types produced inconsistent forecasts when we perturbed climate. Thus, it seems that, at least for the species in this location, demographic data is necessary to pick up key climate drivers of vital rates like survival that are not well resolved in aggregated data. On a pessimistic note, our work shows that even with detailed demographic data, model forecasts are extremely

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- uncertain. It is becoming clear the long time series (over two decades) are necessary to produce meaningful forecasts.
- 26 Keywords: forecasting, climate change, grassland, integral projection model, population model

27 Introduction

Population models are important tools for predicting the impacts of environmental change on species. But reconciling the scales at which population models are parameterized and the scales at which environmental changes play out remains a challenge (Clark et al. 2010, 2012, Freckleton et al. 2011, Queenborough et al. 2011). The major hurdle is that most population models, at least for plant species, are built using data from small, localized plots because parameterizing traditional population models requires tracking the fates of individuals. These 33 models are difficult to scale up from the micro to meso-scales because the fitted parameters do not fully represent the spatial variation present at scales beyond that at which the data are collected (Sæther et al. 2007). At the same time, most demographic data is collected over short time spans. For example, the most common study duration in the COMPADRE matrix population model database is 4 years and only a few exceed 10 years (Salguero-Gómez et al. 2015). The constrained spatio-temporal extent of most demographic datasets reflects the difficulty of collecting such data, but those constraints limit our ability to extrapolate population models. Thus, our ability to use population models to predict the consequences of climate change is limited when we rely on individual-level data.

Aggregate measures of individual plant performance, such as those typically collected as part of large-scale census efforts, offer an alternative to detailed demographic data for modeling populations (Clark and Bjørnstad 2004, Freckleton et al. 2011). Such population-level data will never match the precision of individual-level data, but it is more feasible to attain a broad coverage sample when collecting coarse-scale data. This presents a difficult trade-off: on the one hand, individual-level data leads to more reliable models; on the other hand,

population-level data leads to models that will produce less precise predictions but can be applied over greater spatial and temporal extents. An open question is how well models based on population-level data compare to models based on individual-level data.

To date, relatively few studies have tried to model populations based on data other than
detailed individual-level data. An important exception is an effort by Taylor and Hastings
(2004) to model the population growth rate of an invasive species to investigate the best
strategies for invasion control. They used a "density-structured" model where the state variable
is a discrete density state rather than a continuous density measure. Building on this work,
Freckleton et al. (2011) showed that density-structured models compare well to continuous
models in theory, and Queenborough et al. (2011) showed the application of such methods
in a study on arable weeds. In particular, Queenborough et al. (2011) provide empirical
evidence that density-structured models are capable of reproducing population dynamics,
even if some precision is lost when compared to fully continuous models. Thus, population
models based on coarse, population-level data show promise for producing ecological forecasts
at landscape and regional scales (Queenborough et al. 2011). However, none of these models
included environmental covariates.

Basing population models on aggregated individual-level data in a climate change context is hampered by the fact that it is individuals that respond to climate, not populations (Clark et al. 2012). This fact puts us in uneasy proximity to an "ecological fallacy" where one deduces inference on the individual from statistical inference on the group (Piantadosi et al. 1988). For example, individual plants may respond positively to precipitation but a negative trend is observed at the population level due to increased competition among plants as they grow larger and consume more resources. Thus, it is important to ask the question:

Can aggregated data be used to detect climate signals of the same sign and magnitude as individual-level data? If not, then building population models with climate covariates on aggregated data will lead to incorrect forecasts.

Here, we test the assumption that statistical and population models based on aggregated data can detect climate signals as wells as models based on individual-level data. We use a unique demographic dataset that tracks the fates of individual plants from four species over 14 years to build single-species population models, since those are often used tools for ecological forecasts and climate vulnerability assessments. We first fit population models with interannual variation in vital rates explained, in part, by climate covariates. We then perturb the climate covariates to test the sensitivities of species to climate change. By doing these analyses using both individual and aggregated forms of the same data we can directly compare the two types of models.

In general, we find that population models based on detailed demographic data are more accurate and precise than models based on aggregated data. Both types of models are able to detect climate signals, as evidenced by the sensitivity of simulated equilbrium plant cover under a perturbed climate scenario. But the two types of models produce inconsistent forecasts, in some cases producing completely opposing predictions. This leads us to conclude that, at least for these species at this location, detailed demographic data is necessary to detect the "right" climate signal. A worrying caveat to our work is that forecasts from both models were very uncertain. It seems that even 14 years worth of demographic data is not enough to produce meaningful forecasts when model uncertainty is explicitly considered.

93 Materials and Methods

94 Study site and data

Our demographic data comes from the Fort Keogh Livestock and Range Research Laboratory in eastern Montana's northern mixed prairie near Miles City, Montana, USA (46° 19' N, 105° 48' W). The dataset is freely available on Ecological Archives² (Anderson et al. 2011), and interested readers should refer to the metadata therein for a complete description. The site is

²http://esapubs.org/archive/ecol/E092/143/

about 800 m above sea level and mean annual precipitation (1878-2009) is 334 mm, with most annual precipitation falling from April through September. The site is grass dominated and, for the purposes of our study, we focus on the four most abundant graminoid species:

Bouteloua gracilis (BOGR), Hesperostipa comata (HECO), Pascopyrum smithii (PASM), and Poa secunda (POSE).

From 1932 to 1945 individual plants were identified and mapped annually in 44 1-m² quadrats 104 using a pantograph. The quadrats were distributed in six pastures, each assigned a grazing 105 treatment of light (1.24 ha/animal unit month), moderate (0.92 ha/aum), and heavy (0.76 106 ha/aum) stocking rates (two pastures per treatment). In this analysis we account for potential 107 differences among the grazing treatments, but do not focus on grazing×climate interactions. 108 The annual maps of the quadrats were digitized and the fates of individual plants tracked and 109 extracted using a computer program. Daily climate data, which we aggregated into climate 110 variables of interest, are available for the duration of the data collection period (1932 - 1945) 111 from the Miles City airport, Wiley Field, 9 km from the study site. 112

In this paper, we model populations based on two levels of data: individual and quadrat (Figure 1). The individual data is the "raw" data. For the quadrat level we data we simply sum individual areal cover for each quadrat by species. This is equivalent to a perfect census of quadrat percent cover, so we do not need to consider measurement error. Based on these two datasets we can compare population models built using individual level data and aggregated quadrat level data.

All R code and data necessary to replicate our analysis is archived on GitHub as release
120 1.0 (http://github.com/atredennick/MicroMesoForecast/releases). That stable release will
121 remain static as a record of this analysis, but subsequent versions may appear if we update
122 this work.

123 Stastical models of vital rates

At both levels of inference (individual and quadrat), the building blocks of our population 124 models are vital rate regressions. For individual level data we fit models for survival, growth, 125 and recruitment of new individuals for each species. At the quadrat level we fit a single 126 regression model for population growth. We describe the statistical models separately since 127 fitting the models required different approaches. All models contain five climate covariate that 128 we chose a priori: "water year" precipitation at t-1 (lagppt); fall through spring precipitation 129 at t-1 and t-2 (ppt1 and ppt2, respectively) and mean spring temperature at t-1 and t-2130 (TmeanSpr1 and TmeanSpr2, respectively), where t is the observation year. 131 We fit all models using a hierarchical Bayesian approach, which we describe in more detail

We fit all models using a hierarchical Bayesian approach, which we describe in more detail below. However, for each vital rate statistical model we also define the likelihood model we use. For the likelihood models, \mathbf{Y} is always the relevant vector of observations (e.g., whether a genet survived [1] or not [0] from year t to t+1).

Vital rate models at the individual level We used logistic regression to model survival probability (S) of genet i from species j in quadrat group Q from time t to t+1:

$$\operatorname{logit}(S_{ijQ,t}) = \gamma_{j,t}^S + \phi_{jQ}^S + \beta_{j,t}^S x_{ij,t} + \omega_j^S w_{ij,t} + \theta_{jk}^S C_{k,t} + \varepsilon_t^S$$
(1)

$$Y_{ijQ,t}^S \sim \text{Bernoulli}(S_{ijQ,t})$$
 (2)

where $x_{ij,t}$ is the log of genet size, $\gamma_{j,t}^S$ is a year-specific intercept, $\beta_{j,t}^S$ is the year-specific slope parameter for size, ϕ_{jQ}^S is the random effect of quadrat group location, and θ_k^S is the fixed parameter for the effect of the kth climate covariate at time t ($C_{k,t}$). We include densitydependence by estimating the effect of crowding on the focal individual by other individuals of the same species. ω is the effect of crowding and $w_{t,Q}$ is the crowding experienced by the focal individual at time t in quadrat group Q.

We modeled growth as Gaussian process describing genet size at time t+1 as a function of size at t and climate covariates:

$$x_{ijQ,t+1} = \gamma_{j,t}^G + \phi_{jQ}^G + \beta_{j,t}^G x_{ij,t} + \omega_j^G w_{ij,t} + \theta_{jk}^G C_{k,t}$$
 (3)

$$Y_{ijQ,t}^G \sim \text{Normal}(x_{ijQ,t+1}, \sigma_j)$$
 (4)

where x is log genet size and all other parameters are as described for the survival regression.

Our data allows us to track new recruits, but we cannot assign a specific parent to new genets.

So, for recruitment, we work at the quadrat level and model the number of new individuals of species j in quadrat q recruiting at time t+1 as a function of quadrat "effective cover" (A') in the previous year (t). Effective cover is a mixture of observed cover (A) in the focal quadrat (q) and the mean cover across the entire group (\bar{A}) of Q quadrats in which q is located:

$$A'_{jq,t} = p_j A_{jq,t} + (1 - p_j) \bar{A}_{jQ,t}$$
(5)

where p is a mixing fraction between 0 and 1 that is estimated within the model.

We assume the number of individuals, Y^R , recruiting at time t+1 follows a negative binomial distribution:

$$Y_{jq,t+1}^R \sim \text{NegBin}(\lambda_{jq,t+1}, \zeta)$$
 (6)

where λ is the mean intensity and ζ is the size parameter. We define λ as:

$$\lambda_{jq,t+1} = A'_{jq,t} e^{(\gamma_{j,t}^R + \phi_{jQ}^R + \theta_{jk}^R C_{k,t} + \omega^R \sqrt{A'_{q,t}})}$$
 (7)

where A' is effective cover (cm²) of species j in quadrat q and all other terms are as in the survival and growth regressions. 157

Population model at the quadrat level The statistical approach used to model vital 158 rates using aggregated data depends on the type of data collected. In our case, and as is 159 often the case with census data, we have percent cover data (which can easily be transformed 160 to proportion data, of course). We first considered fitting three vital rate models analogous 161 to those we fit at the individual level: one for probability of extirpation within a quadrat 162 (analagous to survival), one for cover change within a quadrat (analagous to growth), and 163 one for probability of colonization within a quadrat (analogous to recruitment). However, 164 within-quadrat extirpation and colonization events were rare in our time series (N=9 and 165 N=10, respectively across all species). Given the broad spatial distribution of the quadrats 166 we are studying, it is safe to assume that these events are in fact rare enough to be ignored 167 for our purposes. So we constrained our statistical modeling of vital rates at the population 168 level to change in percent cover within quadrats. For the remaining discussion of statistical 169 modeling we refer to proportion data, which is simply percent data divided by 100. 170 An obvious choice for fitting a linear model to proportion data is beta regression because the 171 support of the beta distribution is [0,1], not including true zeros or ones. However, when we 172 used fitted model parameters from a beta regression in a quadrat-based population model 173 the simulated population tended toward 100% cover for all species. We therefore chose a 174 more constrained modeling approach based on a truncated log-normal likelihood. The model

175

$$x_{jq,t+1} = \gamma_{j,t}^G + \phi_{jQ}^G + \beta_{j,t}^G x_{jq,t} + \theta_{jk}^S C_{k,t}$$
 (8)

$$Y_{jq,t+1}^G \sim \text{LogNormal}(x_{jq,t+1}, \tau j) T[0, 1]$$
(9)

for quadrat cover change (G) from time t to t+1 is

where $x_{jq,t}$ is the log of species' j proportional cover in quadrat q at time t and all other parameters are as in the individual-level growth model (Eq. #). The log normal likelihood includes a truncation (T[0,1]) to ensure that predicted values do not exceed 100% cover.

180 Model fitting

Our Bayesian approach to fitting the vital rate models required choosing appropriate priors for unknown parameters and deciding which, if any, of those priors should be hierarchical. 182 We decided to fit models where all terms were fit by species. Within a species, we fit yearly 183 size effects and yearly intercepts hierarchically where year-specific coefficients were drawn 184 from global distributions representing the mean size effect and intercept. We used flat, 185 uninformative priors for all unknown parameters (Appendix X). 186 All of our analyses (model fitting and simulating) were conducted in R (R Core Development 187 Team 2013). We used the 'No-U-Turn' MCMC sampler in Stan (Stan Development Team 188 2014a) to estimate the posterior distributions of model parameters using the package 'rstan' 189 (Stan Development Team 2014b). We obtained posterior distributions for all model parameters 190 from three parallel MCMC chains run for 1,000 iterations after discarding an initial 1,000 191 iterations. We recignize such short MCMC chains may surprise those more familiar with 192 other MCMC samplers (i.e. JAGS or WinBUGS), but the Stan sampler is exceptionally 193 efficient, which reduces the number of iterations needed to achieve convergence. We assessed 194 convergence visually and made sure scale reduction factors for all parameters were less than 195 1.01. For the purposes of including parameter uncertainty in our population models, we saved 196 the final 1,000 iterations from each of the three MCMC chains for all parameters to be used 197 as randomly drawn values during population simulation. This step alleviates the need to 198 reduce model parameters by model selection since sampling from the full parameter space in 199 the MCMC ensures that if a parameter broadly overlaps zero, on average the effect in the 200

population models will also be near zero.

202 Population models

With the posterior distribution of the vital rate statistical models in hand, it is straightforward to simulate the population models. We used an Integral Projection Model (IPM) to model populations based on individual level data and an quadrat based version of an individually-based model (Quadrat-Based Model, QBM) to model populations based on quadrat level data. We describe each in turn.

Integral projection model We use an environmentally stochastic IPM (Rees and Ellner 2009) that includes the random year effects and the climate covariates from the vital rate statistical models. But note that we can, and do for some simulations, ignore the random year effects so that only the climate effects can drive interannual variation. Our IPM follows the specification of Chu and Adler (2015) where the population of species j is a density function $n(u_j, t)$ giving the density of sized-u genets at time t. Genet size is on the natural log scale, so that $n(u_j, t)du$ is the number of genets whose area (on the arithmetic scale) is between e^{u_j} and e^{u_j+du} . So, the density function for any size v at time t+1 is

$$n(v_j, t+1) = \int_{L_i}^{U_j} k_j(v_j, u_j, \bar{\mathbf{w}}_j(u_j)) n(u_j, t)$$
(10)

where $k_j(v_j, u_j, \bar{\mathbf{w_j}})$ is the population kernal that describes all possible transitions from size u to v and $\bar{\mathbf{w_j}}$ is a vector of estimates of average crowding experienced from all other species by a genet of size u_j and species j. The integral is evaluated over all possible sizes between predefined lower (L) and upper (U) size limits that extend beyond the range of observed genet sizes.

The population kernal is defined as the joint contributions of survival (S), growth (G), and recruitment (R):

$$k_j(v_j, u_j, \bar{\mathbf{w}}_j) = S_j(u_j, \bar{\mathbf{w}}_j(u_j))G_j(v_j, u_j, \bar{\mathbf{w}}_j(u_j)) + R_j(v_j, u_j, \bar{\mathbf{w}}_j), \tag{11}$$

which, said plainly, means we are calculating growth (G) for individuals that survive (S)from time t to t+1 and adding in newly recruited (R) individuals of an average sized one-year-old genet for the focal species. Our stastical model for recruitment (R), described above) returns the number of new recruit produced per quadrat. Following previous work (Adler et al. 2012, Chu and Adler 2015), we assume that fecundity increases linearly with size $(R_j(v_j, u_j, \bar{\mathbf{w}_j}) = e^{u_j} R_j(v_j, \bar{\mathbf{w}_j})$ to incorporate the recruitment function in the spatiallyimplicit IPM.

We used random draws from the final 1,000 iterations from each of three MCMC chains to 230 introduce stochasticity into our population models. We thinned these chains by 10 to reduce 231 potential autocorrelation, leaving us with 300 samples for each parameter. At each time step, 232 we randomly selected climate covariates from one of the 14 observed years. Then, we drew 233 the full parameter set (climate effects and density-dependence fixed effects) from a randomly 234 selected MCMC iteration. Using this approach, rather than simply using coefficient point 235 estimates, ensures that relatively unimportant climate covariates (those that broadly overlap 236 0) have little effect on the simulation results. Since our focus was on the contribution of 237 climate covariates to population states, we set the random year effects and the random group 238 effects to zero. 239

Quad-based model Our quad-based model (QBM) perfectly mirrors its statistical description (Eq. #). We use the same approach for drawing parameter values as described for the IPM.

243 Model validation

To test each model's ability to forecast the population state we used leave-one-year-out cross validation. For both levels of modeling, we fit the vital rate models using observations from all years except one, and then used those fitted parameters in the population models to perform a one-step-ahead forecast for the year whose observations were withheld from model fitting. We repeated this procedure for all 13 observation years. This model validation allowed us to compare accuracy and precision of the two modeling approaches (individual-level versus population-level).

Results $\mathbf{Results}$

252 Comparison of Forecast Models

We performed one-step-ahead forecasts to compare the two models (IPM vs. QBM). We did not use the random year effects for these forecasts. Thus, beyond the effect of genet size or quadrat cover, only the climate covariates could affect expect plant cover. The IPM had significantly lower overall error (MAE, mean absolute error) for two species (H. comata and P. smithii; Table 1). In no case did the QBM significantly outperform the IPM (Table 257 1). As a metric of model uncertainty, we calculated the euclidean distance between the 258 upper and lower 90% quantiles for each model's forecasts. The IPM always outperformed the 259 QBM in this respect (Table 1), with consistently lower distances between the 90% quantiles. 260 In general the IPM outperformed the QBM because it had (i) lower MAE for two species, 261 (ii) statistically similar MAE for the other two species, and (iii) considerably more precise 262 forecasts for all species.

Vital rate sensitivities to climate (IPM)

²⁶⁵ Forecasting Climate Change Impacts

Equilibrium cover simulated from the models was sensitive to climate perturbations, but the 266 IPM and QBM produced inconsistent results (Figure 3). For B. qracilis, the IPM predicted a 267 modest increase in cover with a 1% increase in the mean of precipitation or a 1% increase 268 in the mean of temperature, the compounding effect of both being a 20% increase in cover. 269 This reflects the relatively strong effects of precipitation and climate on B. gracilis genet 270 growth and recruitment (Figure 1). The QBM also predicted increased B. gracilis cover with 271 a precipitation increase, but increasing temperature decreased equilibrium cover (Figure 3). The IPM and QBM produced consistent predictions for H. comata under increased precipi-273 tation and when both precipitation and temperature were increased (Figure 3). However, 274 the IPM predicted more modest changes than the QBM, and for a temperature increase the 275 two models differed: the IPM predicted an increase in cover while the QBM predicted the 276 opposite. 277

278 Discussion

We sought to test the assumption that the sensitivities of plant populations to climate 279 variables can be detected equally well using either individual level data or population level 280 data. This is an important question to answer because population models are key tools for 281 predicting the consequences of global climate change. However, they can be of limited use 282 when built on data from a small subset of a population in space or time. If population level 283 data (i.e., some aggregated form of individual level data) can be used to detect climate effects 284 on population dynamics, then we would have a cheaper and easier option for data collection 285 over relatively large temporal and spatial extents (e.g. Freckleton et al. 2011). 286

287 It is perhaps startling that two different types of models fit using the same data produce

 $_{288}$ drastically different forecasts (Figure 3).

Tables Tables

Table 1: Mean absolute error (MAE) and accuracy (Pearson's ρ) for one-step-ahead forecast from both model types. Forecasts were made without random year effects; only climate covariates could explain year-to-year variation. 90 Distance refers to the average distance between the upper and lower 90 percentiles of the 100 predicted values for each quadrat-year combination.

Species	Model	MAE	90% Distance	Mean Obs. Cover
BOGR	IPM	4.77	29.33	9.16
BOGR	QBM	5.30	44.63	9.22
HECO	IPM	0.60	2.82	1.22
HECO	QBM	2.03	12.05	1.25
PASM	IPM	0.18	0.49	0.40
PASM	QBM	0.17	1.34	0.40
POSE	IPM	0.78	2.16	1.23
POSE	QBM	1.30	7.04	1.25

The IPM MAE is significantly lower for HECO $(p = 3.3 \times 10^{-10})$ and POSE (p = 0.00013).

MAEs are statisticially similar between models for BOGR and PASM.

₂₉₂ Figures

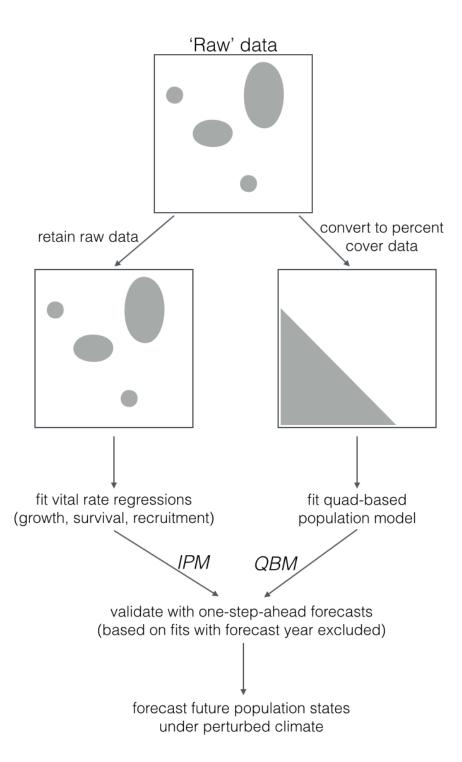


Figure 1: Work flow of the data aggregation, model fitting, and population simulating.

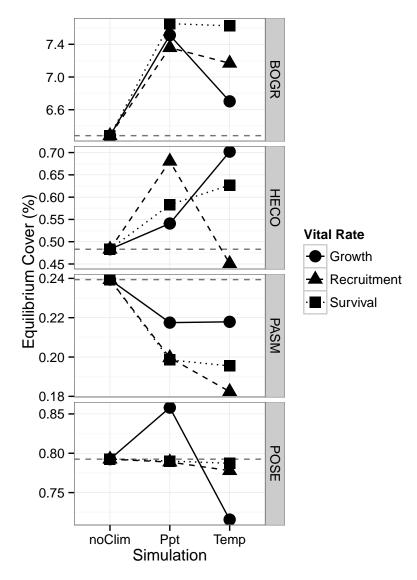


Figure 2: Sensitivity of equilibrium cover to removal of climate effects from each vital rate regression.

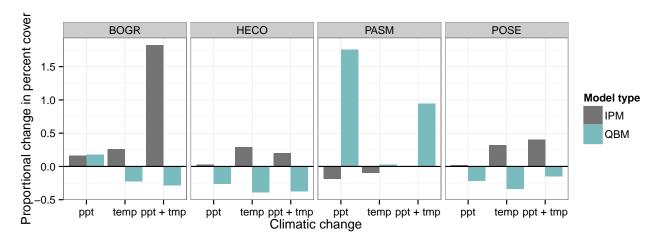


Figure 3: Proportional change in species' mean cover caused by a 1% increase in observed precipitation, temperature, or both as predicted by the individual-based IPM and the aggregate-based QBM. Note that the change for POSE due to a precipitation increase predicted by the QBM is almost zero and so does not show up in the figure. Labels on x-axis refer to: 'ppt' = 1% increase in mean precipitation; 'temp' = 1% increase in mean temperature: 'ppt + temp' = 1% increase in mean precipitation and mean temperature.

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