**Online Appendix A: Mathematical results for “Indirect effects of environmental change in resource competition models”**

1. **Niche overlap in the essential resource model:**

Chesson (2013) defines niche differences and niche overlap,**, in terms of phenomenological competition coefficients (found in equation 1 in main text) from a two species Lotka-Volterra competition model with intraspecific competition coefficients:

|  |  |  |
| --- | --- | --- |
|  |  | (A1) |
|  |  | (A2) |

Here we use Chesson’s definition to quantify niche overlap in Tilman’s essential resource competition model. To do this, we translate the essential resource model into a Lotka-Volterra competition model following Tilman (1982; pp 190-204). Tilman used a Lotka-Volterra model with a different parameterization than Chesson:

|  |  |  |
| --- | --- | --- |
|  |  | (A3) |
|  |  | (A4) |

Where *K*F and *K*C gives the carrying capacity of each species, **’ gives the per capita effect of the competitor on the focal species and **’ gives the per capita competitive effect of focal species on the competitor (we use alpha prime and beta prime to distinguish these from the competition coefficients in Chesson’s parameterization). If we assume that species coexist at equilibrium and the focal species is limited by resource one and the competitor by resource two, then the following translation between the mechanistic and phenomenological model is possible (see Tilman 1982; pp 190 -204 for more details):

|  |  |  |
| --- | --- | --- |
|  |  | (A5) |
|  |  | (A6) |
|  |  | (A7) |
|  |  | (A8) |

We can substitute the mechanistic definitions of the competition coefficients from eqq. (A5) - (A8) above into Chesson’s definition of niche overlap (Chesson 2013, p. 233),

|  |  |  |
| --- | --- | --- |
|  |  | (A9) |

1. **Comparison between niche overlap and angle between consumption vectors in the essential resource model:**

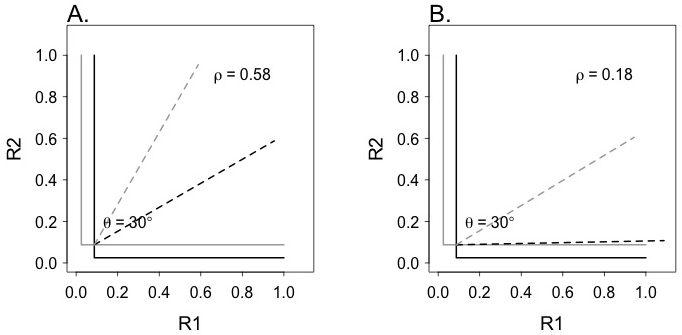


Figure A1: ZNGI plots showing how niche overlap, **, and the angle between resource consumption vectors change in an essential resource model. The x-axis is the concentration of resource one and the y-axis is the concentration of resource two. The solid lines show the ZNGI for two different species. The dashed lines show the consumption vectors for each species. Theta is the angle in degrees between the consumption vectors. In figure 2 in the main text we show how the angle and niche overlap relate. We include the figure above to show that the two are not perfectly equivalent. The angle between resource consumption vectors is the same (** = 30°) in both panels above but niche overlap as given by our measure of ** is larger in the second figure. In A the consumption vectors for the black line is equal to [0.5, 0.866] and for the gray line [0.866, 0.5]. In B. the consumption vector for the black line is [0.980, 0.2] and for the gray line [0.749, 0.663].

1. **Net and indirect effects in the essential resource model:**

We can solve for the equilibrium abundances of the competing species, *N*F\* and *N*C\*, by setting the differential equations in the main text to zero and setting the resource concentrations to their equilibrium values, *R*1\* and *R*2\*

|  |  |  |
| --- | --- | --- |
|  |  | (A10) |
|  |  | (A11) |

Equations (A10) and (A11) are equivalent to the functions *f* and *g* that define the effects of resources and competitor on each species’ equilibrium (equations 2 and 3 in the main text). In order to find the partial derivatives needed to apply equation 5 in the main text, we can differentiate the equations (A10) and (A11) above:

|  |  |  |
| --- | --- | --- |
|  |  | (A12) |
|  |  | (A13) |
|  |  | (A14) |
|  |  | (A15) |
|  |  | (A16) |
|  |  | (A17) |

We can substitute these partial derivatives into the equation for the indirect effect of a change in resource supply on the focal species (equation 5 in the main text). Note that all these partial derivatives are evaluated at the coexistence equilibrium (*R*\*1, *R*\*2).

1. **Niche overlap in the substitutable resource model:**

Tilman (1982, pp. 270-272) showed how Lotka-Volterra competition parameters are defined in terms of the substitutable resource model. As in the essential resource model, we can make this conversion and then use the phenomenological competition coefficients to calculate the niche overlap using Chesson’s definition. Given the parameters for the substitutable resource competition model established in equations 27 through 30 in the main text the phenomenological competition coefficients are given by the following equations,

|  |  |  |
| --- | --- | --- |
|  |  | (A18) |
|  |  | (A19) |
|  |  | (A20) |
|  |  | (A21) |

Given the identities above, and assuming that the focal species’ relative consumption rate of resource two is less than the relative consumption rate of resource two by the competitor, then niche overlap for this model is,

|  |  |  |
| --- | --- | --- |
|  |  | (A22) |

1. **Net and indirect effects in the substitutable resource model:**

Equilibrium populations in the substitutable resource model can be found by setting the differential equations (29) and (30) in the main text to zero and solving for the abundances of each species (Tilman 1982. pp. 270 - 272):

|  |  |  |
| --- | --- | --- |
|  |  | (A23) |
|  |  | (A24) |

where *B*F and *B*C are defined by (Tilman 1982, p. 270):

|  |  |  |
| --- | --- | --- |
|  |  | (A25) |
|  |  | (A26) |

Equations (A23) and (A24) are equivalent to the functions *f* and *g* that define the effects of resources and competitor on each species’ equilibrium (equations 2 and 3 in main text). In order find the partial derivatives needed to apply equation 5 in the main text, we differentiate the equations (A23) and (A24) above:

|  |  |  |
| --- | --- | --- |
|  |  | (A27) |
|  |  | (A28) |
|  |  | (A29) |
|  |  | (A30) |
|  |  | (A31) |
|  |  | (A32) |

Note, as in the essential resource model, these partial derivatives are evaluated at the coexistence equilibrium (). We can use these partial derivatives to find the net effects of a change in *S*1 on the focal species following the general formula (eq. 4 in the main text):

|  |  |  |
| --- | --- | --- |
|  |  | (A33) |

In order to clarify the connection to niche overlap, we re-write equation (A33), using the definition of niche overlap (eq. (A21)) for this model,

|  |  |  |
| --- | --- | --- |
|  |  | (A34) |

The indirect effects are the net effects minus the direct effects on the focal species,

|  |  |  |
| --- | --- | --- |
|  |  | (A35) |

which can be rewritten so that it more closely resembles the definition of indirect effects in the essential resource model (see equations 24 and 26 in the main text),

|  |  |  |
| --- | --- | --- |
|  |  | (A36) |

**References:**

Chesson, P. 2013. Species competition and predation. Ecological Systems (pp. 223–256). Springer.

Tilman, D. 1982. Resource competition and community structure. Princeton University Press.