

Molecular Dynamics of Lennard Jones Fluid

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PHYS 610 Methods of Molecular Simulation

We investigate the properties of particles in a Lennard-Jones potential in a molecular dynamics simulation. Lennard Jones potential represents an interaction between two particles and its associated energy function is

$$U = \frac{1}{2} \sum_{i \neq j} u_{ij}(|\mathbf{r}_i - \mathbf{r}_j|)$$

where the intermolecular potential is

$$u(r)_{LJ} = 4[r^{-12} - r^{-6}]$$

where r is the interparticle separation. To avoid the computational costs associated with an infinite potential, it is reasonable to implement an approximation that cuts off the potential at some length, r_c and to add the approximate amount of energy that has been cut off for all particles at once. In this case, potential becomes

$$u(r) = \begin{cases} \infty & \text{for } r \leq r_c \\ 0 & \text{for } r \geq r_c \end{cases}$$

The so called tail correction for potential energy in this case is

$$u_{\text{tail}} = \frac{8}{3} \pi \rho \left[\frac{1}{3} \left(\frac{1}{r_c} \right)^9 - \left(\frac{1}{r_c} \right)^3 \right]$$

and the tail correction for pressure is

$$P_{\text{tail}} = \frac{16}{3} \pi \rho^2 \left[\frac{2}{3} \left(\frac{1}{r_c} \right)^9 - \left(\frac{1}{r_c} \right)^3 \right]$$

where ρ is the number density. But we can also find the pressure without using the tail end correction by using the virial

$$P = \frac{\rho}{\beta} + \frac{\text{vir}}{V}$$

where V is the volume, and vir is the virial, which is

$$\text{vir} = \frac{1}{3} \sum_i \sum_{i \geq j} f(r_{ij}) \cdot r_{ij}$$

the pressure-density plot calculated in the simulation using the virial is shown in Figure 6 (right).

where ρ is the number density and $g(d^+)$ is the value of the RDF at contact. The RDF, $g(r)$, is a pair correlation function that describes how atoms in a disordered system are packed in relation to each other.

Radial distribution function is an important measure of the static structural properties of the liquid. It can be computed as

$$g(r) = \frac{1}{N} \frac{\sum_{i=1}^N \text{number of particles between } r \text{ and } r + \Delta r}{4\pi r^2 \rho \Delta r}$$

where N is the number of particles. The radial distribution functions obtained in the simulation for various values of density are shown in Figures 1 and 2.

An important dynamical property to evaluate is the diffusion coefficient, which is related to the concentration c by

$$\frac{\partial c}{\partial t} = D \nabla^2 c$$

then, the diffusion coefficient is

$$D = \frac{1}{6} \frac{\partial}{\partial t} \langle r^2 \rangle$$

where $\langle r^2 \rangle$ is mean squared displacement. Figure 4 shows the $\langle r^2 \rangle$ trajectory for various number densities and the approximated slope of the linear portion of the curve. Figure 5 shows the log linear plot of $\frac{1}{6} \frac{\partial}{\partial t} \langle r^2 \rangle$ vs time with a horizontal line to approximate the value of the diffusion coefficient. Figure 6 (right) plots the D for each density.

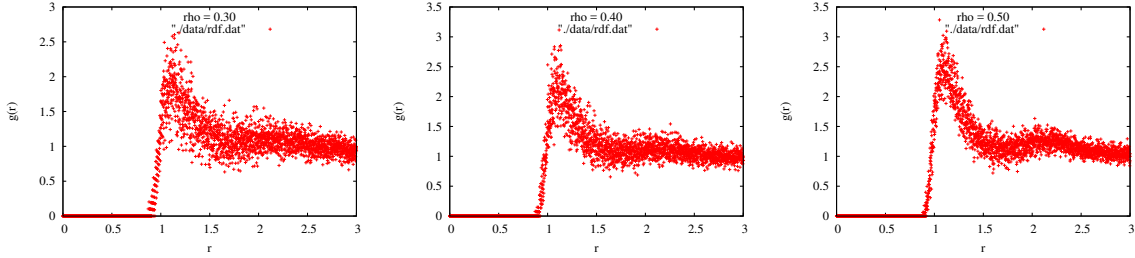


Figure 1: Radial distribution function for $\rho = 0.3$, $\rho = 0.4$, $\rho = 0.5$.

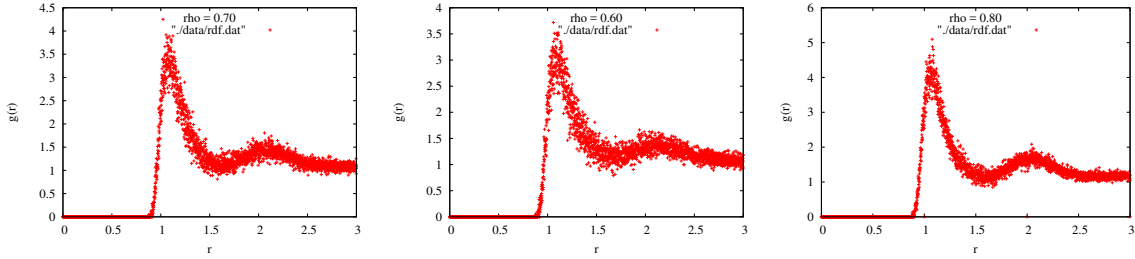


Figure 2: Radial distribution function for $\rho = 0.6$, $\rho = 0.7$, $\rho = 0.8$.

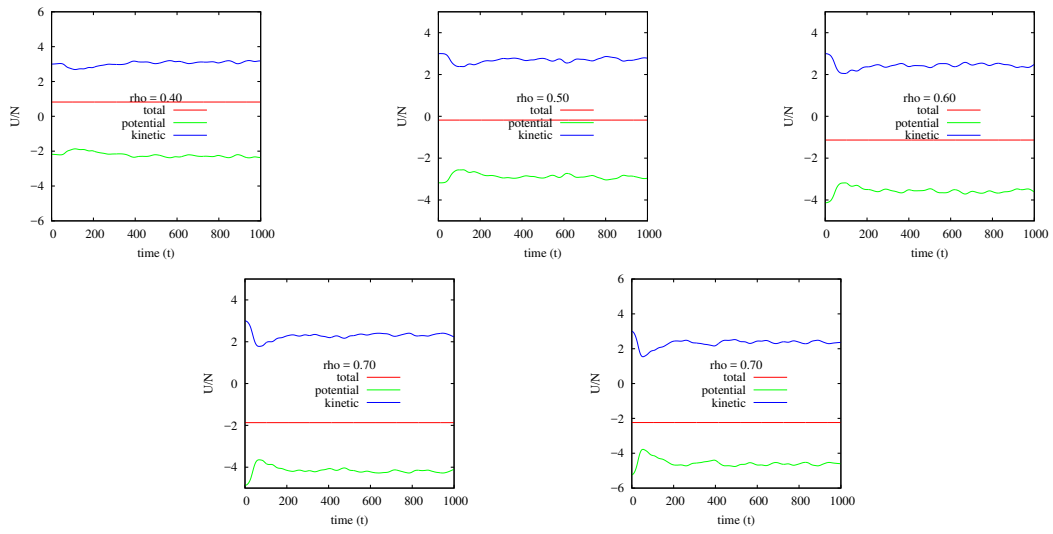


Figure 3: Time evolution of kinetic, potential and total energy for $\rho = 0.4$, $\rho = 0.5$, $\rho = 0.6$, $\rho = 0.7$, $\rho = 0.8$.

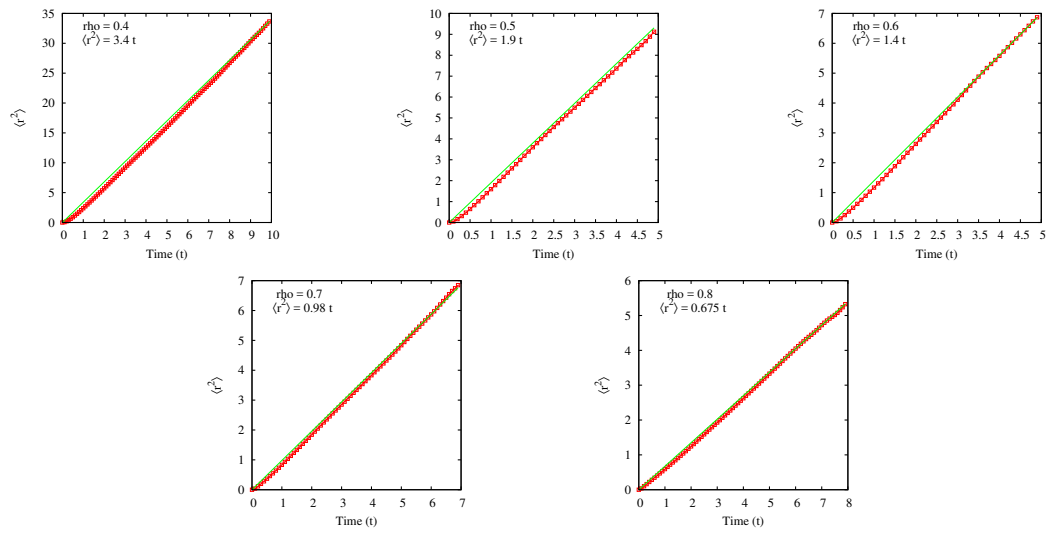


Figure 4: Mean squared displacement vs time for $\rho = 0.4$, $\rho = 0.5$, $\rho = 0.6$, $\rho = 0.7$, $\rho = 0.8$.

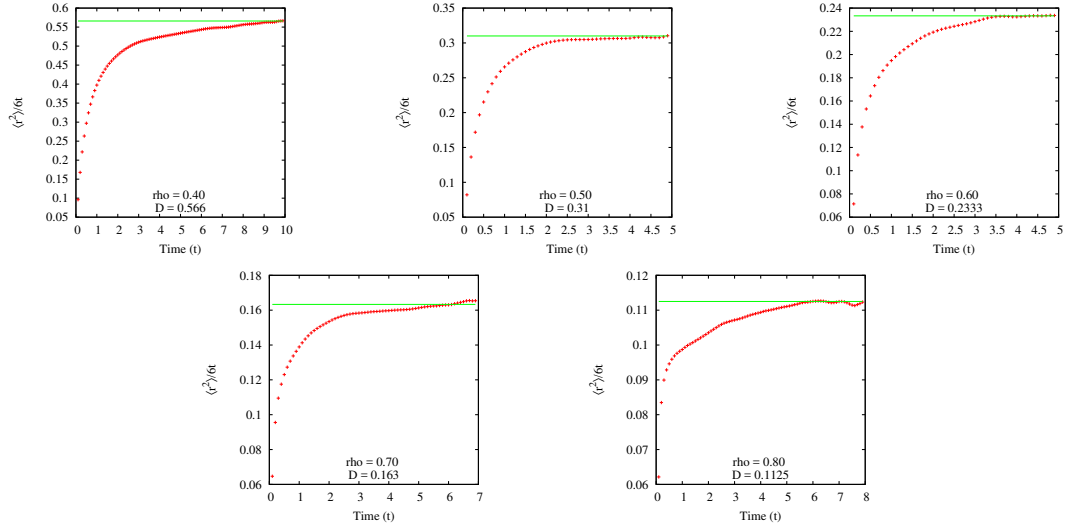


Figure 5: Diffusion coefficient vs time for $\rho = 0.4$, $\rho = 0.5$, $\rho = 0.6$, $\rho = 0.7$, $\rho = 0.8$.

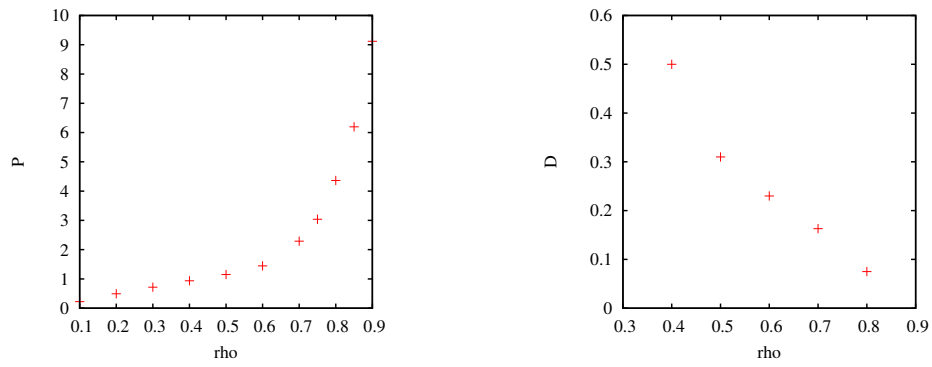


Figure 6: (left) Pressure vs density plot (right) diffusion coefficient vs density plot