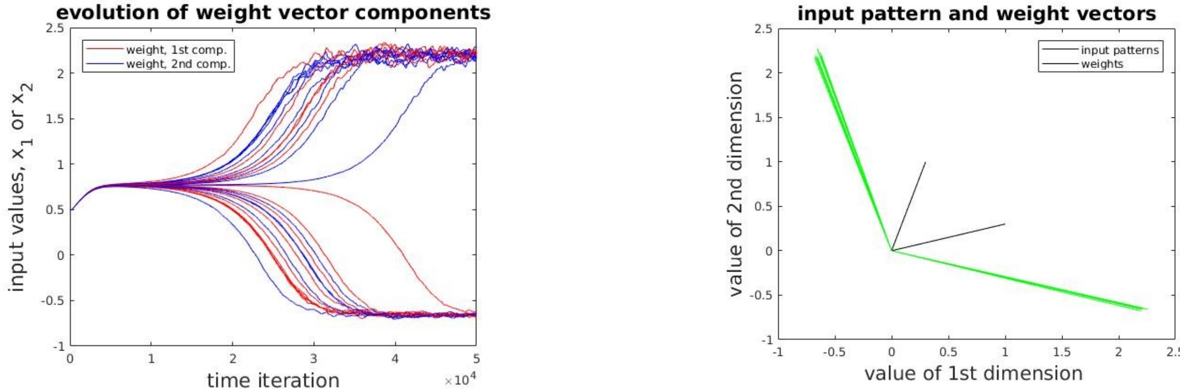
HW 3: BCM Rule. CAAM588. Alena Klindziuk

BCM learning rule maximizes input selectivity. It is characterized by a rule expressing synaptic change as a product of the presynaptic activity and a nonlinear function, $\phi(y;\theta_M)$, of postsynatic activity, y. For low values of the postsynaptic activity ($y<\theta_M$), ϕ is negative; for $y>\theta_M$, ϕ is positive. The rule stabilizes by allowing the modification threshold, θ_M, to vary as a super-linear function of the previous activity of the cell. Unlike traditional methods of stabilizing Hebbian learning, this "sliding threshold" provides a mechanism for incoming patterns, as opposed to converging afferents, to compete.

For K linearly independent input patterns, the BCM learning rule has in total 2^K fixed points. When those patterns are presented with probability p k = 1/K, there are K-j fixed points 2 of selectivity 1- j/K for j = 1, . . . , K 1 and there are 2 fixed points of selectivity
0.

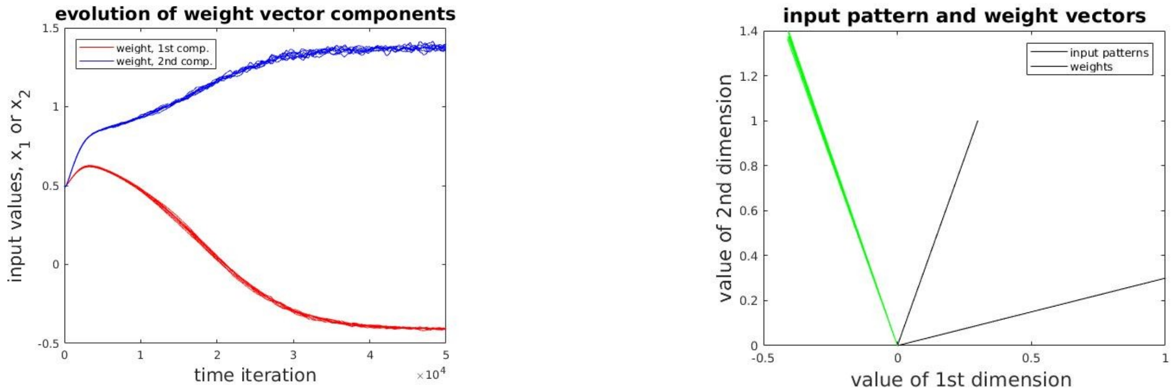
This can be proven by looking at simulations of various input scenarios. If there are two input patterns with the same probability of occurrence, than there are $2^2=4$ fixed points. ${}_2C_1 = 2$ of them are stable. This is illustrated in Figure 1. Running a simulation multiple times yields two different weight vectors (shown in green) that correspond to 2 different fixed points with about equal probability of occurrence.

Figure 1: Time evolution of components of weight vector (left) and weight vectors after 20000 time steps for 2 input patters with same probability.



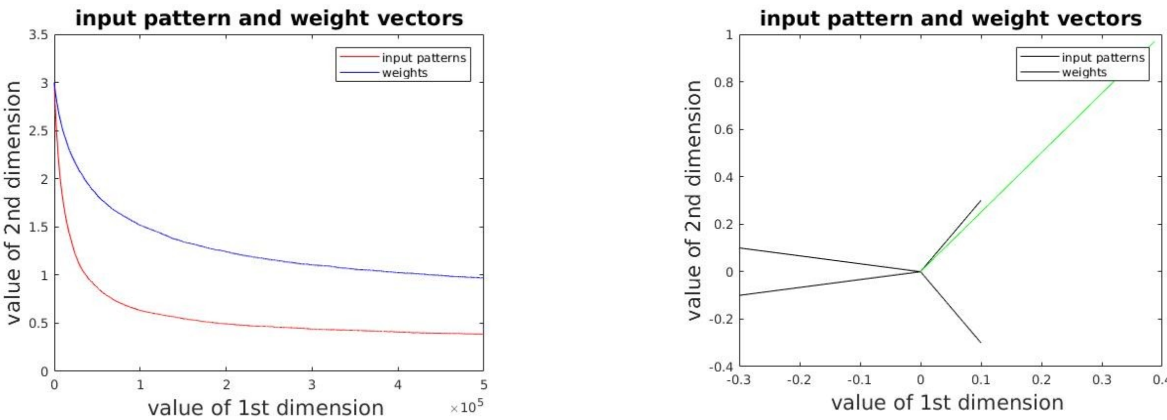
On the other hand, if the probabilities of getting the two patters are not equal, than the weight vector will converge to one point even after multiple runs, as shown in Figure 2.

Figure 2. Time evolution of components of weight vector (left) and weight vectors after 20000 time steps for 2 input patters with different probability.



If there are four patterns, with equal probabilities, than, there are four fixed points with highest selection, out of 16 fixed points in total. Since the probabilities of these points are equal, w can converge to 4 different points. Figure 3. shows this scenario for one run.

Figure 3: Time evolution of components of weight vector (left) and weight vectors after 20000 time steps for 4 input patters with same probability.



Extra credit simulation (part b):

Figure 4. Time evolution output y, synaptic weight, and sliding threshold for different values of tau.

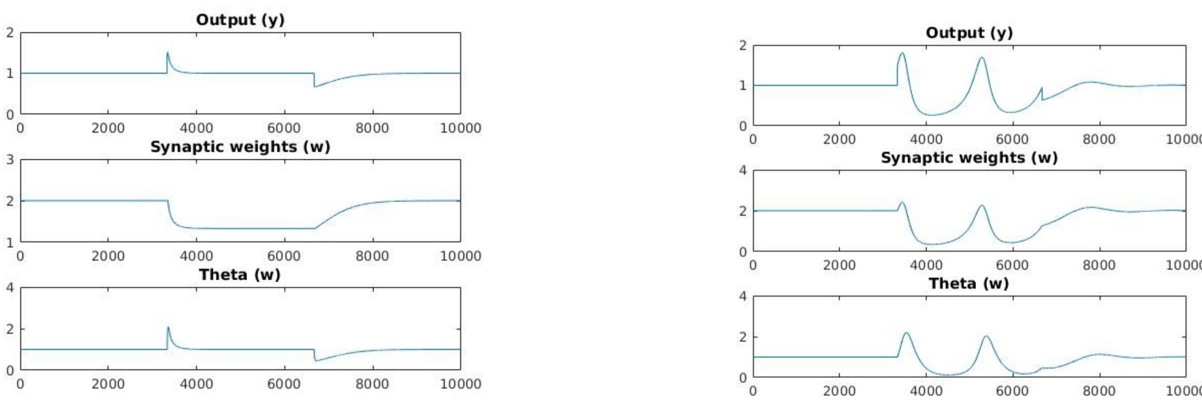


Figure 4. shows how output value y, synaptic weight, and threshold evolve with each iteration for a single 1d input with a perturbations at 3000th iteration and 5000th iterations. This shows that even if the system has been perturbed, system always converges to its fixed points. a) is for tau = 10 and b) is for tau = 200, which results in oscillations.