HW 8: Hebb and Oja

In this homework assignment, we learned to construct covariance matrix from a bivariate distribution and solve for its eigenvalues and eigenvectors. We also implemented a Hebb and an Oja neurons in MATLAB and showed the evolution of their synaptic weights.

For problem 1) see attached pdf.
Problems 2) and 3) are explained below:

Hebb neuron follows a simple learning rules. However, its synaptic weights keep increasing without bound with each time iteration. In Oja neuron, the synaptic weights are normalized such that they converge to the principle component of its covariance matrix. Thus Oja's rule becomes the principle component analyzer.

Figure 1 shows the synaptic weight evolution for a Hebb rule in black and for the Oja rule in green given a simple input with variances 1 and 3 and tiled at 30 degrees. We can see that the synaptic weights of both Hebb and Oja rule evolve in the direction of the principle component. However, Hebb synaptic weights indeed increase without bound, while Oja weights shortly stop increasing. Moreover, both point in the direction of highest variance and are tilted at an angle of 30 degrees about x-axis, as is expected given the distribution.

Figure 2 shows the eigenvectors of the correlation matrix the bivariate distribution that was used as input in our Hebb and Oja neurons. The correlation matrix was calculated to be:

$$Q = \begin{matrix} 2.5678 & 0.8349 \\ 0.8349 & 1.6099 \end{matrix}$$

with eigenvalues 1.1263 and 3.0514 and eigenvectors:

$$V1 = \begin{matrix} 0.5012 \\ -0.8653 \end{matrix} \qquad V2 = \begin{matrix} -0.8653 \\ -0.5012 \end{matrix}$$

The eigenvecors point in the directions of largest and smallest variaces with the eigenvector with highest eigenvalue indicating the direction of the largest. This is also the direction (30 degree rotation about x-axis) in which the Hebb and Oja weights are evolving.

Figure 1:

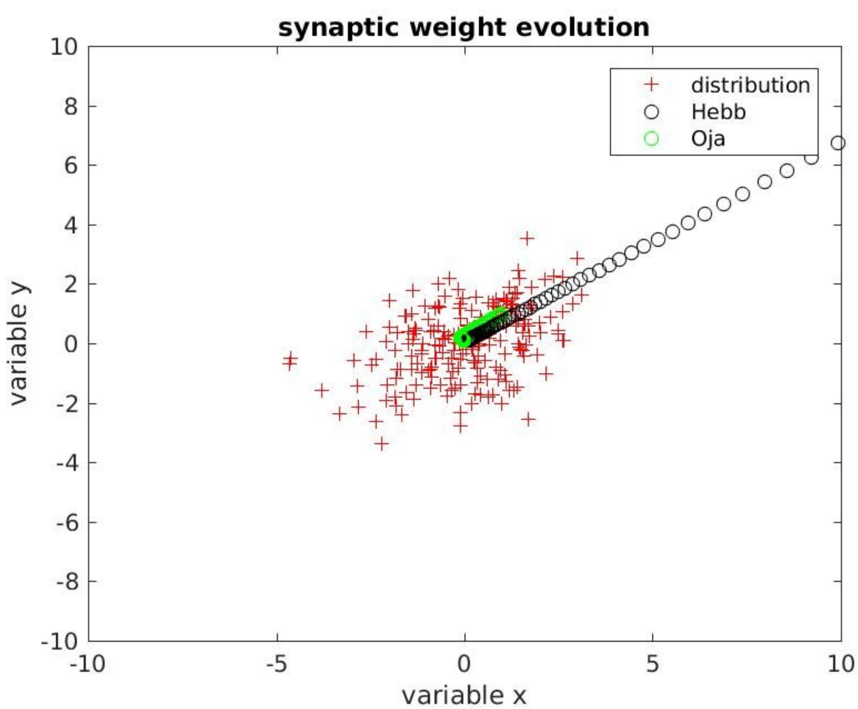
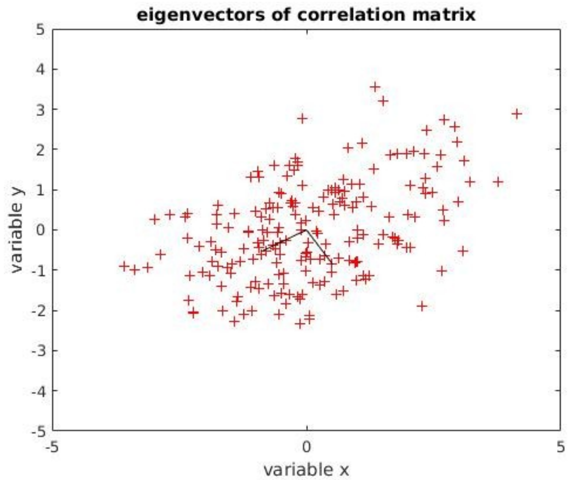


Figure 2:



eigenvectors of correlation matrix
variable y
variable x
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5
0
5