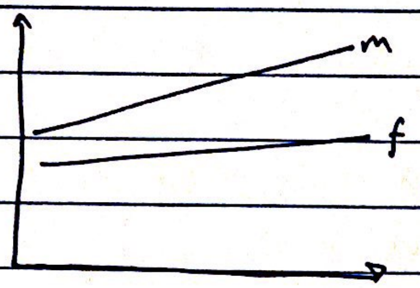
1.



Model 1

m: $\beta_0 + \beta_1 x_h$

f: $(\beta_0 + \beta_2) + (\beta_1 + \beta_3) x_h$

$$w_i \sim N(\beta_0 + \beta_1 x_h + \beta_2 I + \beta_3 I x_h, \tau^2)$$

Model 2

$$w_i \sim N(\beta_0' + \beta_1'(x_h - \bar{x}_h) + \beta_2' I + \beta_3' I(x_h - \bar{x}_h), \tau^2)$$

$$= (\beta_0' - \beta_1' \bar{x}_h) + \beta_1' x_h + (\beta_2' I - \beta_3' \bar{x}_h I) + \beta_3' I x_h$$

$$\therefore \boxed{\begin{array}{ll} \beta_1' = \beta_1 & \beta_3' = \beta_3 \\ \beta_0' - \beta_1' \bar{x}_h = \beta_0, & \beta_2' - \beta_3' \bar{x}_h = \beta_2 \end{array}}$$

$\Rightarrow$ only the constants change ~~th~~ between the two models, but we'll get the same slopes

2. True: $y \sim N(\beta_0 + \beta_1 x_1 + \beta_2 x_2, \sigma^2)$

x_1, x_2 uncorrelated, centered

Model 1: 2 predictors $\beta_0 + \beta_1 x_1 + \beta_2 x_2$

Model 2: only x_1 $\beta_0' + \beta_1' x_1$

(1) what is difference between β_1 and β_1'?

we expect $\beta_1 = \beta_1'$

because the slope value will be fit the same since x_1 and x_2 are uncorrelated

(2) what is difference between σ and σ'?

we expect $\sigma < \sigma'$

becase the noise must increase due to the unexplained effect of $\vec{x}_2$ on the heights $\vec{y}$

(3) what is difference between $var(\beta_1)$, $var(\beta_1')$?

we expect $var(\beta_1) < var(\beta_1')$ b/c we have less certainty about the value of β_1 in this model

3. true process: $\beta_{true} = 0$

$y_i \sim N(\beta_0 + \beta_1 x_1 \cancel{}, \sigma^2)$

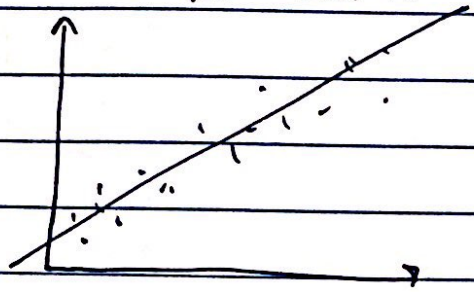
1 like before, we expect $\beta_1 = \beta_1'$
because x_1 and x_2 are ortogonal

2 now, $\sigma = \sigma'$ because the added information
is orthogonal

3 $var(\beta_1) = var(\beta_1')$

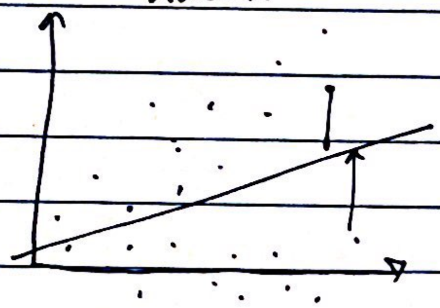
4. (1)

no hackers



$y \sim \beta_1 x$

hackers



$y \sim \beta_1' x$

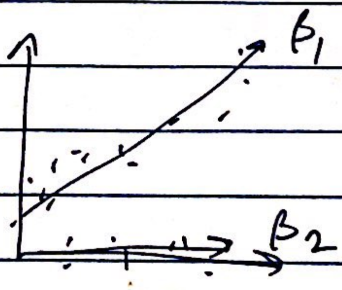
$$\boxed{\beta_1' < \beta_1}$$

* slope is shallower b/c fake data pulls line toward the horizontal

* since $\beta_0 = 0$ in the model and the hacker data is centered @ 0, the constant $\beta_0 = \beta_0'$

* Also, both β_1' & β_0' marginal posterior distributions will have higher variances

(2) we can add a 2nd parameter like an interaction term



$y' \sim \beta_1' x + \beta_2' I + \beta_2' I x$

now acts like the male / female problem