$$P(A \mid \text{data}) = \frac{P(\text{data} \mid A)\, P(\cancel{A})}{P(\cancel{A}) \text{ data}}$$

\# of water

1. $P_1(w) = 0.25$ $\quad N_1 = 100$ $\quad$ data $= n_1, n_2$

$P_2(w) = 0.75$ $\quad N_2 = 50$

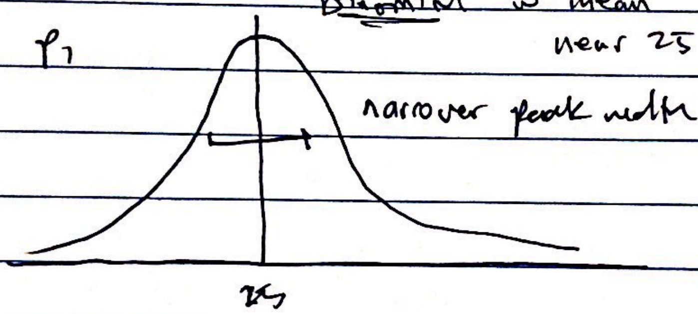
binomial distribution (either l or w)

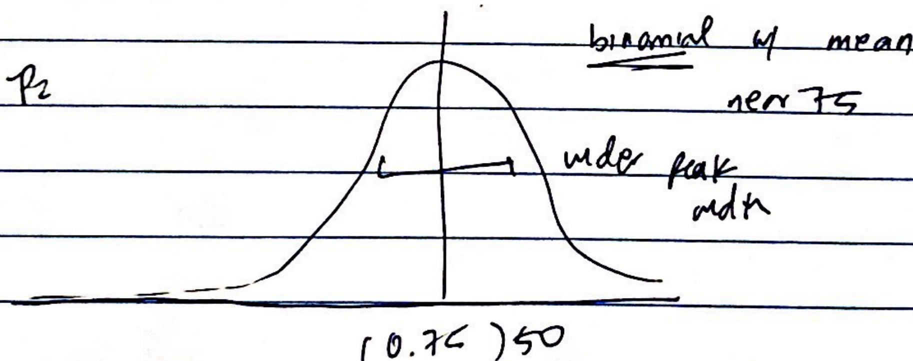
uniform prior

likelihood fn. is binomial

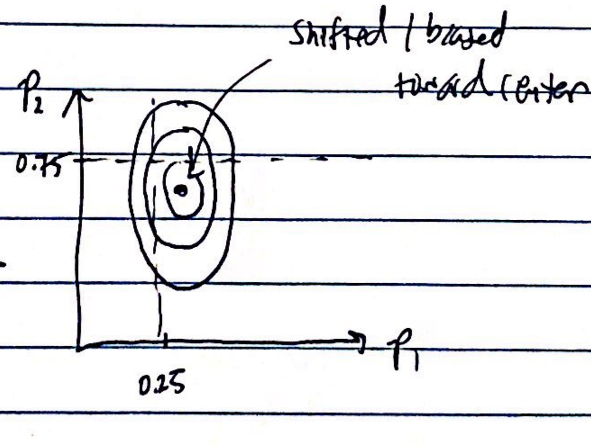
$P_i(n_i) = \binom{N_i}{n_i} p_i^{n_i} q_i^{N_i - n_i}$

posterior is 2D





combine into 1 2D plot :



more spread on P_2 estimate b/c less data

2. (1) Prior : we expect 10% of claims to be true
∴ we can choose a gaussian prior centered @ 0.1

b/c the prior assigns a probability to the validity of the claim

$P(\theta) = \text{gaussian}(\mu = 0.10, \sigma = \ )$

(2) The likelihood function is a binomial distribution

~ θ small ⇒ false $\theta \sim 1$ ⇒ true

θ = prob that the claim holds for one person

natural priors : Beta$(\alpha, 9\alpha)$ for $\alpha > 0$

b/c mean $= \frac{\alpha}{\alpha + 9\alpha} = 0.1$

can we pool data from both studies — yes; in this case

⇒ $N = 100$ $n = 80$ ∴ binomial likelihood

Posterior for beta prior = beta

$$\beta(\alpha + 52 + 28, 9\alpha + 48 + 22)$$

Problem 3 (1 point)

Consider the same setting as in problem 2, with two differences.

mean @ 50%?

First, you are required to use uninformative priors to evaluate the claims.

Second, the first study has 1000 subjects, out of which 580 exhibit an increase of at least 10% in the exams scores with the medication, and the second study has 50 subjects, but the data is corrupted and you can't read the results.

Suppose also that you are **only** asked to judge if Omniscient's hypothesis is more likely than not to be valid.

QUESTION: Would you feel confident making a conclusion under these circumstances?

uninformative prior ⇒ can't use info abt

Pusso claims being low prob

can the smaller test change results significantly?

... not too much even if 0% ; we'd still have

a higher proportion than original 100, 52

∴ w a uniform prior we'd have binomial result

$P(\theta|data) \approx P(data|\theta) = \binom{N}{n} \theta^n (1-\theta)^{N-n}$

~~I wouldn't feel comfortable if 2 tests~~

would prior even affect results? I'd have to check

if prior doesn't affect results b/c N large, then I'd be confident b/c even if 2nd trial had 0/50, we'd get a result similar to the previous one

3. repeating w/ uniform prior:

we again assume a binomial likelihood function:

$p(\text{data}) | \theta) = \binom{N}{n} \theta^n (1-\theta)^{N-n}$

$N = 1050$

$n \in [530, 630]$

to choose lower bound to be safe

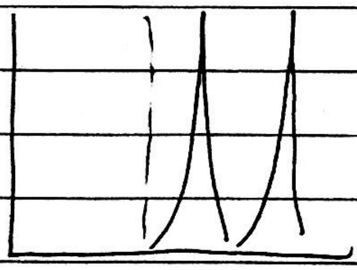
$\Rightarrow p(\theta | \text{data}) = \text{uniform} \times p(\text{data} | \theta)$

$=$ ~~[illegible]~~

where does this peak? → can't be worse than previous problem

~~[illegible]~~

if $\theta > \frac{1}{2}$ for both cases, then $p(\theta > \frac{1}{2} | \text{data}) \approx 1$ and we accept ✓



∴ yes, I'd be comfortable

$$\Rightarrow \boxed{\left(\frac{a}{a+b}\right)\left(\frac{a+1}{a+1+b}\right) + \left(1-\frac{a}{a+b}\right)\left(\frac{a}{a+b+1}\right)}$$

Problem 4 (1 point)

simplifies to $\boxed{\frac{a}{a+b}}$

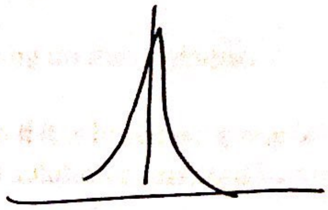
Consider a binomial inference problem with Beta priors.

Suppose that after observing the data, your current posteriors are $Beta\,(a, b)$.

QUESTION: What is the expected change in the posterior mean if you were to receive one additional independent observation from the same data generating process?

posterior → prior

$\beta(a,b) \Rightarrow$ mean $= \frac{a}{a+b}$



$\frac{a}{a+b}$

new result : is it a yes
or a no?

data : n given N

new data : either n+1 given N+1

or n given N+1

what determines
where new point will
lie — true prob(θ)
which is expected value
or prior

basically, what is prob of these cases?

$\beta'(a', b') \longrightarrow \beta(a, b)$

$a = a' + n \qquad b = b' + (N-n)$

∴ adding 1 $\quad a' + n + 1 = a+1, \quad b' + (N+1 - (n+1)) = b \qquad \beta(a+1, b)$

or $a' + n = a \qquad b' + (N+1 - n) = b+1 \qquad$ or $\beta(a, b+1)$

∴ expect $\theta = \frac{a}{a+b}$

∴ prob new data is true is $\theta = \beta(a,b)$

$P(\text{true}) \times \overset{\text{mean}}{\beta(a+1, b)} + P(\text{false}) \cdot \overset{\text{mean}}{\beta(a, b+1)}$

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4. your posterior $\beta(a,b)$ respects all info in data

⇒ expect no change in mean b/c you already have your data

$$p(y=1) = \int_0^1 \theta \, d\beta(a,b) = \frac{a}{a+b} \quad ✓$$

$$p(y=0) = 1 - \frac{a}{a+b} \quad ✓$$

~~put on handout~~

$$\text{expected posterior} = \left(\frac{a}{a+b}\right)\left(\frac{a+1}{a+1+b}\right) + \left(1 - \frac{a}{a+b}\right)\left(\frac{a}{a+b+1}\right)$$

$$\text{simplifies to: } \left(\frac{a}{a+b}\right)\left(1\right)$$

⇒ no change to posterior