EE1103:Assignment 9 (LU Decomposition)

1. (a) Derive an LU decomposition based on the Gauss elimination

$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$

$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$

$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

(b)Then, multiply the resulting [L] and [U] matrices to determine that [A] is produced.

(c) Complete the problem by generating the final solution with forward and back substitution.

Hint:

Pseudo code:

(a) subroutine to implement the decomposition phase:

```
SUB Decompose (a, n)
    DOFOR k =1, n -1
        DOFOR i =k+ 1, n
            factor =a_{i,k}/a_{k,k}
            a_{i,k} =factor
            DOFOR j =k +1, n
                a_{i,j} =a_{i,j}-factor * a_{k,j}
            END DO
        END DO
    END DO
END Decompose
```

(b) subroutine to implement both substitution phases:

```
SUB Substitute (a, n, b, x)
    'forward substitution
    DOFOR i =2, n
        sum=b_i
        DOFOR j =1, i -1
```

```
                sum=sum-a_{i,j} * b_j
            END DO
            b_i=sum
        END DO
        'back substitution
        x_n=b_n/a_{n,n}
        DOFOR i =n -1, 1, -1
            sum=0
            DOFOR j =i +1, n
                sum=sum+a_{i,j} * x_j
            END DO
            x_i=(b_i-sum)/a_{i,i}
        END DO
END Substitute
```

(c) Pseudocode for an LU decomposition algorithm

```
SUB Ludecomp (a, b, n, tol, x, er)
        DIM o_n, s_n
        er =0
        CALL Decompose(a, n, tol, o, s, er)
        IF er <>-1 THEN
            CALL Substitute(a, o, n, b, x)
        END IF
END Ludecomp

SUB Decompose (a, n, tol, o, s, er)
        DOFOR i =1, n
            o_i=i
            s_i=ABS(a_{i,1})
            DOFOR j =2, n
                IF ABS(a_{i,j})>s_i THEN s_i=ABS(a_{i,j})
            END DO
        END DO
        DOFOR k =1, n -1
            CALL Pivot(a, o, s, n, k)
            IF ABS(a_{o(k),k}/s_{o(k)}) <tol THEN
                er =-1
                PRINT a_{o(k),k}/s_{o(k)}
                EXIT DO
```

```
            END IF
            DOFOR i =k +1, n
                factor =a_o(i),k/a_o(k),k
                a_o(i),k =factor
                DOFOR j =k +1, n
                    a_o(i),j =a_o(i),j -factor*a_o(k),j
                END DO
            END DO
        END DO
        IF ABS(a_o(k),k/s_o(k)) <tol THEN
        er =-1
        PRINT a_o(k),k/s_o(k)
        END IF
END Decompose

SUB Pivot (a, o, s, n, k)
        p =k
        big =ABS(a_o(k),k/s_o(k))
        DOFOR ii=k +1, n
            dummy =ABS(a_o(ii),k/s_o(ii))
            IF dummy >big THEN
                big =dummy
                p=ii
            END IF
        END DO
        dummy =o_p
        o_p=o_k
        o_k =dummy
END Pivot

SUB Substitute (a, o, n, b, x)
        DOFOR i =2, n
            sum =b_o(i)
            DOFOR j =1, i -1
                sum=sum-a_o(i),j*b_o(j)
            END DO
            b_o(i) =sum
        END DO
        x_n =b_o(n)/a_o(n),n
        DOFOR i =n-1, 1, -1
```

```
            sum =0
            DOFOR j =i +1, n
                  sum=sum +a_o(i),j *x_j
            END DO
            x_i=(b_o(i) -sum)/a_o(i),i
      END DO
END Substitute
```