EE2703: Assignment 9

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1 Introduction

In this assignment, we continue our explorations on the DFT of a finite-length sequence with the FFT algorithm, using the `numpy.fft` module. We shall look at finding the DFTs of non-periodic functions, and problems associated with them, namely the *Gibbs Phenomenon* and how to overcome them, using *windowing*.

2 Spectrum of $sin(\sqrt{2}t)$

Using the method we followed for obtaining the DFT of a periodic signal, we get the following spectrum for $sin(\sqrt{2}t)$:

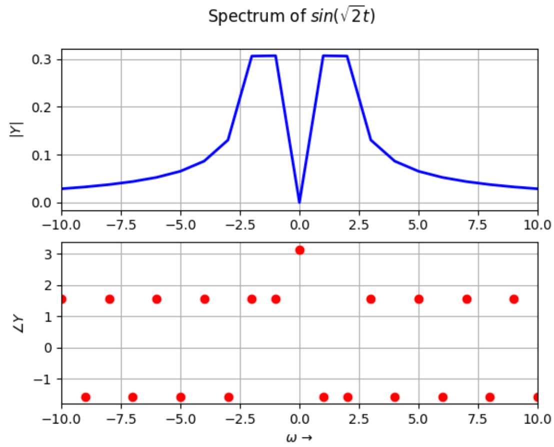

Figure 1: Spectrum of $sin(\sqrt{2}t)$

But, this is not what we expected! We expected two peaks, but that is not what we got. This is because, we aren't finding out the DFT of the required function, $sin(\sqrt{2}t)$:

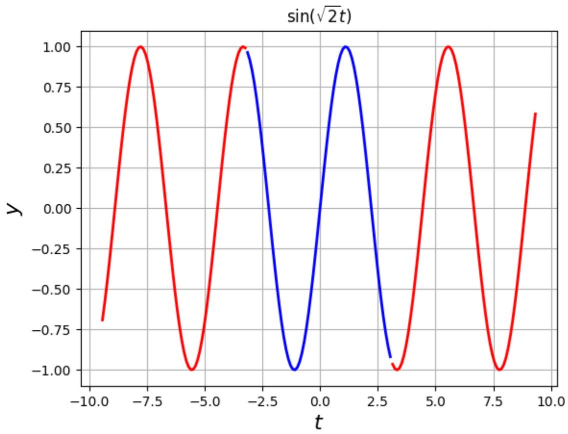

Figure 2: $sin(\sqrt{2}t)$

We can see that this is not what we want to calculate the DFT for. The discontinuities in the function has led to the very problematic *Gibb's Phenomenon*. So, we do not observe a sharp peak, but rather a flat one.

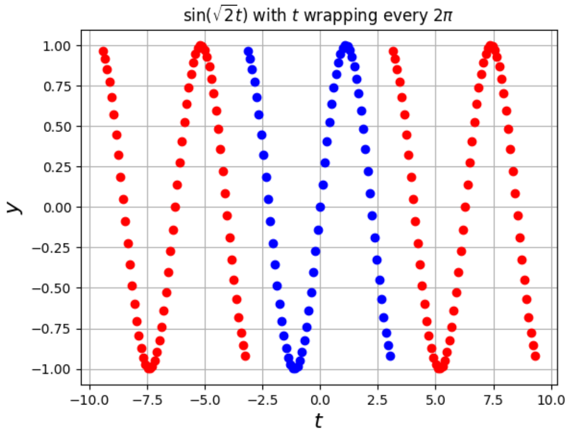

Figure 3: The function for which we are calculating DFT

This problem can be fixed by **windowing**. Windowing is an operation in which we multiply the time-domain function with a suitable window function. In this assignment, we choose the *Hamming Window*, defined as:

$$W_N[n] = \begin{cases} 0.54 + 0.46\ cos(\frac{2\pi n}{N-1}), & |n| < N \\ 0, & otherwise \end{cases}$$

The result in time domain is that the magnitude of the jump is greatly reduced, thus minimizing the effect of Gibb's phenomenon in the frequency domain:

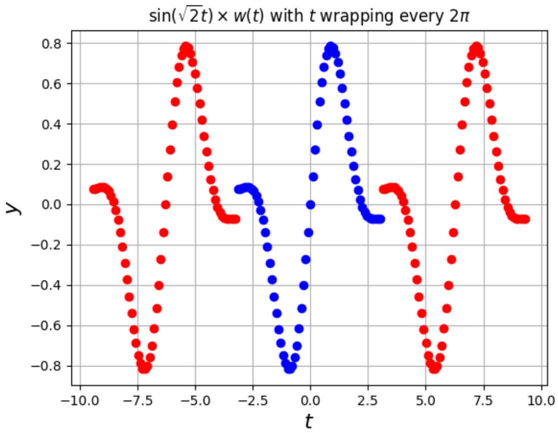

Figure 4: Windowed $sin(\sqrt{2}t)$

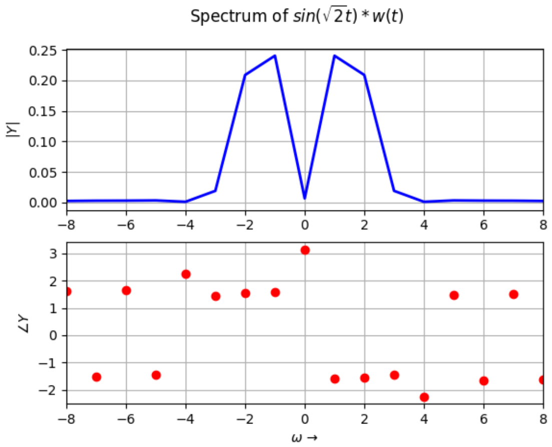

Figure 5: Spectrum of windowed $sin(\sqrt{2}t)$

3 Spectrum of $cos^3(0.86t)$

We can see the effect of windowing even better for $cos^3(0.86t)$. We get the following spectra before and after windowing:

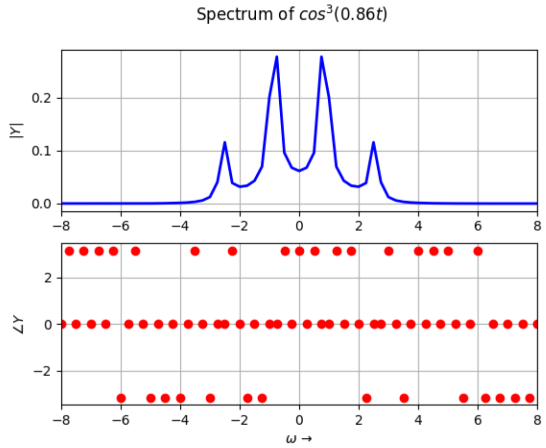

Figure 6: Spectrum of $cos^3(0.86t)$ without windowing

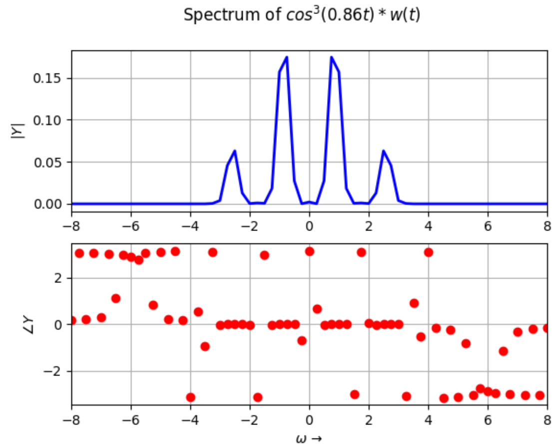

Figure 7: Spectrum of $cos^3(0.86t)$ after windowing

We can see narrower and sharper peaks at the frequencies that are present in the signal.

4 Parameter Estimation using DFT

We are given a 128-element vector with sampled values of the signal $cos(\omega_o t + \delta)$, where ω_o and δ are to be estimated.

We cannot extract ω_o from the DFT spectrum because the sampling rate is very low (only 128 points). The peaks will overlap and so we will not be able to extract their values.

I used an approach as illustrated below:

$$\omega_{o_{est}} = \frac{\sum_\omega |Y(j\omega)|^k \cdot \omega}{\sum_\omega |Y(j\omega)|^k}$$

$$y(t) = Acos(\omega_o t) - Bsin(\omega_o t)$$

$$\delta_{est} = \arctan(-\frac{B}{A})$$

In the above equations, I used a least squares approach to find A, B.

I varied the parameter k manually to find out which worked best for the given range of ω_o. I used $k = 1.7$, for estimation in absence of noise, and $k = 2.4$, for estimation in presence of noise, to give the following results (I had set $\omega_o = 1.35$ and $\delta = \frac{\pi}{2}$ in both cases) :

Noise	ω_o	$\omega_{o_{est}}$	δ	δ_{est}
0	1.35	1.453	1.571	1.571
0.1*`rand(128)`	1.35	1.437	1.571	1.571

5 DFT of chirp $cos(16t(1.5+\frac{t}{2\pi}))$

A chirp is a signal in which the frequency increases or decreases with time[1]. We are going to analyse the chirp signal $cos(16t(1.5+\frac{t}{2\pi}))$. First, we shall plot the signal versus time, to get an idea of how the signal looks like:

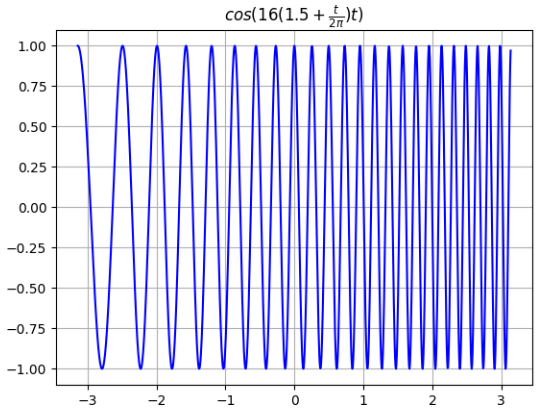

Figure 8: $cos(16t(1.5+\frac{t}{2\pi}))$

We see that the frequency varies from 16 rad/sec to 32 rad/sec as t goes from $-\pi$ sec to π sec. On finding the DFT of the above signal, we get:

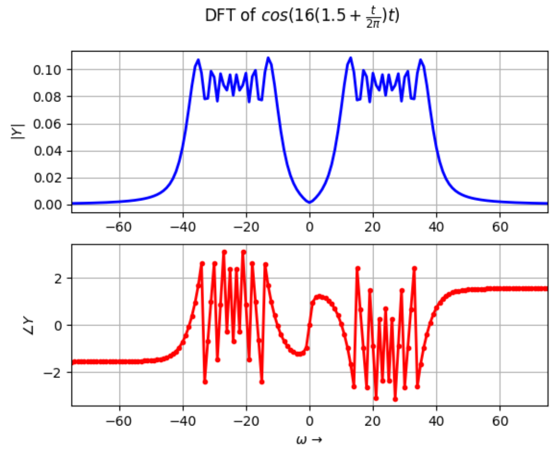

Figure 9: Spectrum of $cos(16t(1.5+\frac{t}{2\pi}))$

Applying the Hamming Window to the chirp results in the following:

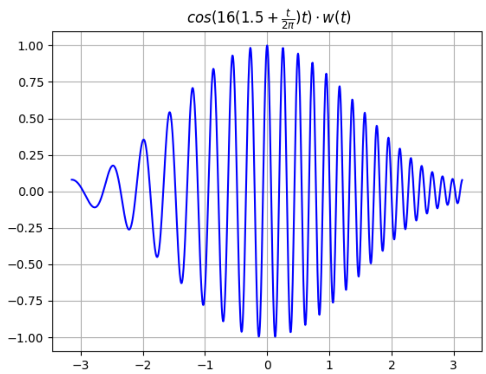

Figure 10: Windowed $cos(16t(1.5+\frac{t}{2\pi}))$

[1] *Source*: https://en.wikipedia.org/wiki/Chirp

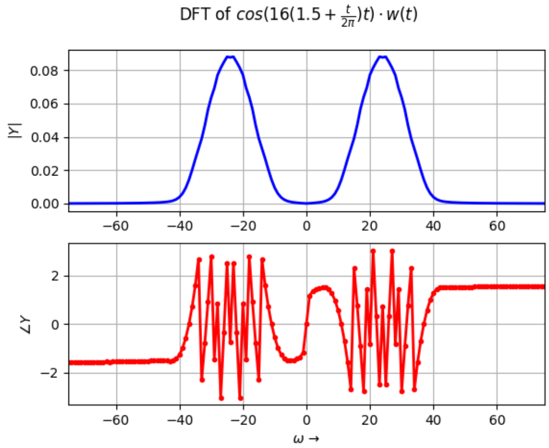

Figure 11: Spectrum of windowed $cos(16t(1.5+\frac{t}{2\pi}))$

The variations in the frequency have been smoothed out by the window, and also, we can see that the frequencies are more accurately confined to the range 16-32 rad/sec.

6 Time-frequency plot of $cos(16t(1.5+\frac{t}{2\pi}))$

We shall split the chirp in the time interval $[-\pi,\ \pi]$ into smaller intervals of time, and observe how the frequency of the signal varies with time.

Initially we had a 1024-length vector with the values of the chirp signal. We shall split it into 64-length vectors, take the DFTs of these localized vectors, and plot a time-frequency surface plot to observe the variation of the frequency with time.

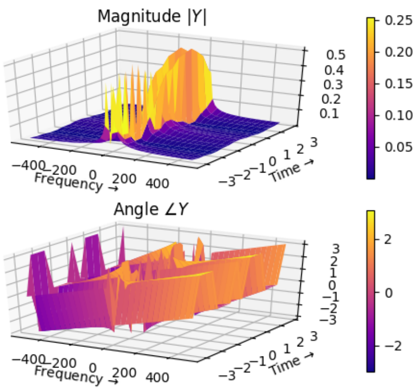

Figure 12: Spectrum of windowed $cos(16t(1.5+\frac{t}{2\pi}))$

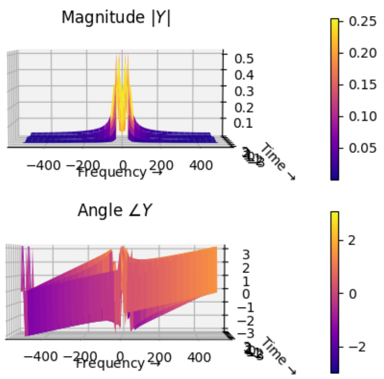

Figure 13: Spectrum of windowed $cos(16t(1.5+\frac{t}{2\pi}))$

Now, we shall do the same, but with a windowed version of the chirp.

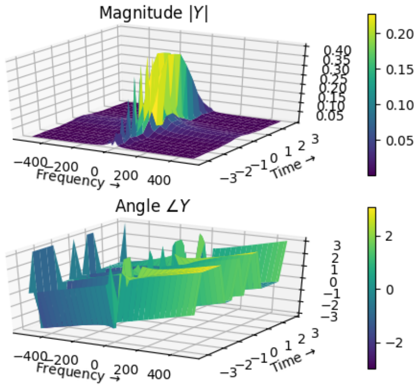

Figure 14: Spectrum of windowed $cos(16t(1.5 + \frac{t}{2\pi}))$

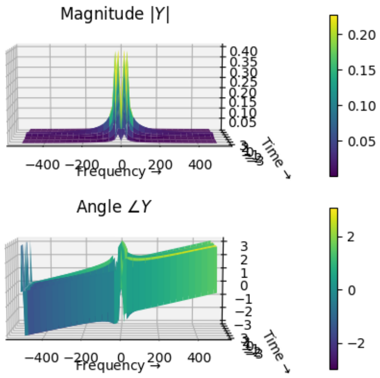

Figure 15: Spectrum of windowed $cos(16t(1.5 + \frac{t}{2\pi}))$

We can see that the change in the magnitude is more gradual in the windowed case.

7 Conclusion

The DFT was obtained using a 2π periodic extension of the signal, and thus the spectrum was found to be erroneous for a non periodic function. The spectrum was rectified by the using a windowing technique, by employing the Hamming window. Given a vector of cosine values in the a time interval, the frequency and phase were estimated from the DFT spectrum, by using the expectation value of the frequency and a parameter tuning for optimum values. The DFT of a chirped signal was analysed and its time-frequency plot showed the gradual variation of peak frequency of the spectrum with time.