Final Exam for Group A (20 Marks) **Apr 19, 2017**

Dept. of Electrical Engineering, IIT Madras
Applied Programming Lab Jan 2017 session

- **Time duration of exam is** 2.5 **hours**
- **Create a work directory called** ***your-roll-number (lower case)*** **and make it accessible only by you:**

  ```
  chmod 700 your-roll-number
  ```

  Then change to that directory and do your work.
- **vector operations are a must.**
- **Label all plots. Add legends. Make the plots professional looking.**
- **Comments are not optional. They are required.**
- **pseudocode should be readable and neatly formatted.**
- **code should be written as part of a LyX file. The LyX file should be professional in appearance, and I will give marks for it.**
- **LyX file should be named** ***your-roll-number.lyx*** **(if you submit a .tex file it should be named** ***your-roll-number.tex*****)**
- **PDF file should be named** ***your-roll-number.pdf***
- **Python code should be named** ***your-roll-number.py***
- **Python code should run!!**
- **Include the plots in the lyx file and generate a pdf.**
- **Internet will be turned off at the beginning of the exam.**
- **Leave the above three files in your working directory**
- **Late submission will result in reduced marks.**

To compute the Radiation Profile of a dipole antenna of length $2l = 3\lambda/2$.

A long wire carries a current defined (in phasor form) by

$$I = \frac{4\pi}{\mu_0} I_0 \sin\left(k\left(l - |z|\right)\right) \tag{1}$$

where I_0 may be taken to be unity, and $k = 2\pi/\lambda$ is the wave number in vacuum. The half-length of the wire is $l = 3\lambda/4$, so that $kl = 3\pi/2$. The current is "centre fed" via a transmission line. The wire is along the z-axis and centered at the origin. The length of the wire is $2l = 3\lambda/2$. The problem is to compute and plot the magnetic field $\left|\vec{B}\right|^2$ at $r = 10\lambda$ vs θ, and to compare with theory by plotting the expression

$$\frac{1}{2}\mathbf{Re}\left\{\vec{E} \times \vec{H}\right\} \propto |B|^2 \propto \left[\frac{\cos\left(kl\cos\theta\right) - \cos\left(kl\right)}{\sin\theta}\right]^2 \tag{2}$$

in the same plot (suitably scaled).

The computation involves the calculation of the vector potential

$$\vec{A}(x,y) = \frac{\mu_0}{4\pi}\int \frac{I(z)e^{jkR}dz}{R}$$

where $\vec{R} = \vec{r} - \vec{r}'$ and $k = \omega/c = 0.1$. $\vec{r}$ is the point where we want the field, and $\vec{r}' = z'\hat{z}'$ is the point on the wire. This can be reduced to a sum:

$$A_{mnp} = \sum_{q=0}^{N-1} \frac{\sin\left(k\left(l - |z|\right)\right)\exp\left(jkR_{mnpq}\right)\vec{dl}'}{R_{mnpq}} \tag{3}$$

where $\vec{r}$ is at (x_m, y_n, z_p) and $\vec{r'}$ is at $z_q\hat{z}$. Note that A_{mnp} is the z component of the vector potential $\vec{A}$, i.e., $A_z\left(x_m, y_n, z_p\right)$, and $R_{mnpq} = \left|\vec{r}_{mnp} - \vec{r}'_q\right|$.

From $\vec{A}$, you can obtain $\vec{B}$ as

$$\vec{B} = \nabla \times \vec{A}$$

In the $x - z$ plane this becomes

$$\vec{B} = \hat{x}\frac{\partial A_z}{\partial y} - \hat{y}\frac{\partial A_z}{\partial x}$$

i.e.,

$$B_x = \frac{A_z(x, y+\Delta y, z) - A_z(x, y-\Delta y, z)}{2\Delta y} \tag{4}$$

and

$$B_y = -\frac{A_z(x+\Delta x, y, z) - A_z(x-\Delta x, y, z)}{2\Delta x} \tag{5}$$

The value of $\left|\vec{B}\right|^2 = B_x^2 + B_y^2$ **at** $r = 10\lambda$ **should be correct to** 2 **digits.**

[part 1:5 marks] Write down the pseudocode to obtain the desired $|B|^2$**. Indicate the number of sections needed along the antenna, the** Δx **and** Δy **needed in order to achieve** 2 **digit accuracy. Also indicate how you will compute the expressions in Eqs. 3, 4 and 5.**

[part 2:6 marks] Create the arrays needed to compute A_z**. Define a vector function that returns** $A_z(x, y, z)$**, and using it, define vector functions that return** $B_x(x, y, z)$ **and** $B_y(x, y, z)$**.**

[part 3:3 marks] For 100 **values of** θ **varying from** 0 **to** π**, obtain** $|B|^2$ **and plot the same in Figure 1.**

[part 4:3 marks] Fit the array of $|B|^2$ **values against Eq. 2 and obtain the scaling factor. Using this plot the theoretical fit in Figure 1. What is the error between the two? Is the answer correct to** 2 **digits?**

[part 4:3 marks] Compute $|B|^2$ **along the** x **axis at** 100 **points logarithmically varied from** $x = 0.1\lambda$ **to** $x = 100\lambda$**. Plot the values vs** x **in a log-log plot in Figure 2. Fit the values to a model of the form** Ae^{bx} **and determine the best** A **and** b**. Note: You may need to fit over a part of the range only. Does the answer agree with Ampere's law?**

$$B_\theta = \frac{\mu_0 I}{2\pi x}$$

Why or why not?

Useful Python Commands (use "?" to get help on these from ipython)

```
from pylab import *
import system-function as name
Note: lstsq is found as scipy.linalg.lstsq
ones(List)
zeros(List)
```

```
where(condition)
where(condition & condition)
where(condition | condition)
a=b.copy()
lstsq(A,b) to fit A*x=b
A.max() to find max value of numpy array (similalry min)
A.astype(type) to convert a numpy array to another type (eg int)
def func(args):
  ...
  return List
matrix=c_[vector,vector,...] to create a matrix from vectors
figure(n) to switch to, or start a new figure labelled n
plot(x,y,style,...,lw=...)
semilogx(x,y,style,...,lw=...)
semilogy(x,y,style,...,lw=...)
loglog(x,y,style,...,lw=...)
contour(x,y,matrix,levels...)
quiver(X,Y,U,V) # X,Y,U,V all matrices
xlabel(label,size=)
ylabel(label,size=)
title(label,size=)
xticks(size=) # to change size of xaxis numbers
yticks(size=)
legend(List) to create a list of strings in plot
annotate(str,pos,lblpos,...) to create annotation in plot
grid(Boolean)
show()
```