EE2703: Assignment 10

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1 Introduction

In this assignment, we shall explore the convolution of two discrete-time signals. We shall look at *linear convolution* and *circular convolution* and their relationship. We shall also look at a certain sequence called *Zadoff-Chu sequence*, and it's special properties.

2 The FIR Filter

The given FIR Filter is this:

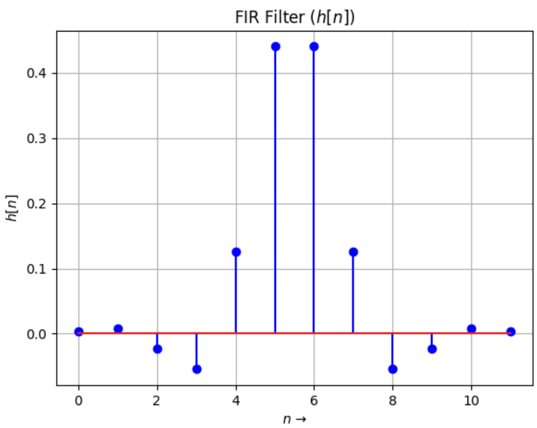


Figure 1: FIR Filter $h[.]$

If we observe carefully, we can see that the envelope of the filter looks like a $sinc(x)$ function, which means that it's DTFT will look similar to a $rect(\omega)$.

The magnitude and phase response of the given filter is given by:

$$H(e^{j\omega}) = \sum_{n=0}^{11} h[n]e^{-j\omega n} \tag{1}$$

Using the command `scipy.signal.freqz`, we shall be able to get the frequency response as shown below:

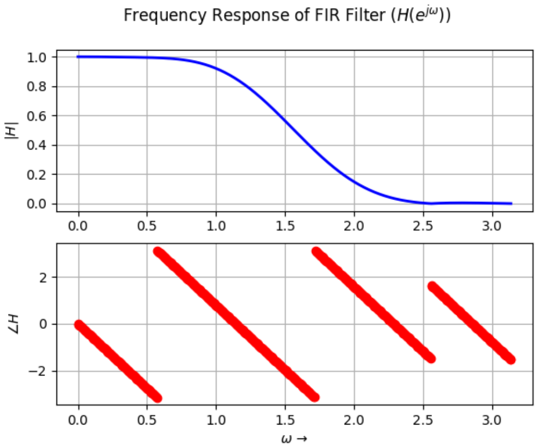


Figure 2: Frequency Response of FIR Filter $H(e^{j\omega})$

Since, the sequence $h[.]$ is real, it's DTFT $H(e^{j\omega})$ will be even[1]. So, we can see that the magnitude response indeed looks like similar to $rect(\omega)$.

3 Linear Convolution

Now, we shall take an input signal $x[n] = cos(0.2\pi n) + cos(0.85\pi n)$, and observe the output of the filter.

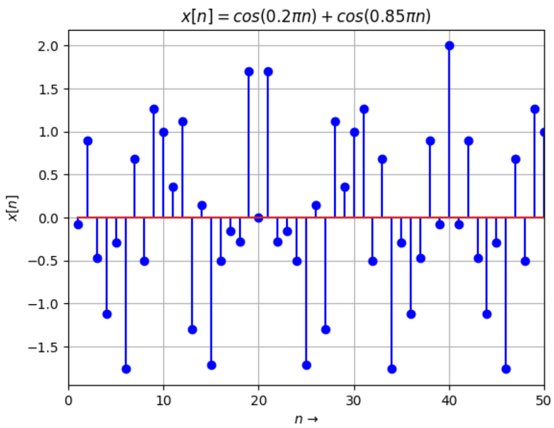


Figure 3: Input to filter, $x[.]$

The output of the filter $y[.]$ can be got by linear convolution of the input $x[.]$ and the filter's impulse response $h[.]$

$$y[n] = x * h = \sum_{\{k:\ h[k] \neq 0\}} x[n-k]h[k] \tag{2}$$

Using the command `numpy.convolve` to perform the linear convolution operation of $x[.]$ and $h[.]$, we get:

[1] $H(e^{-j\omega}) = \sum_{n=0}^{11} h[n]e^{j\omega n} = H^*(e^{j\omega}) \implies |H(e^{-j\omega})| = |H(e^{j\omega})|$

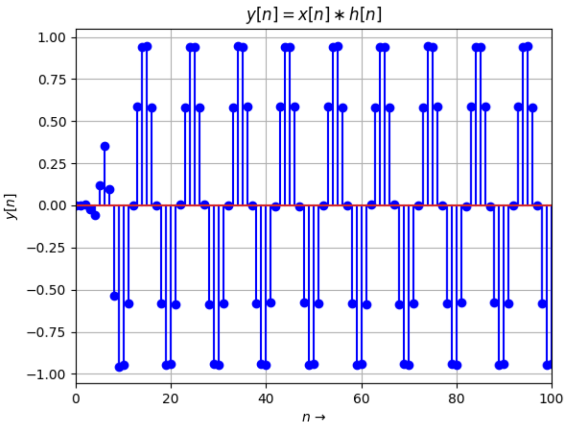


Figure 4: Output of Filter, $y[n]$

As expected, we see a sinusoid of frequency approximately 0.2π. This is because the filter has damped the high frequency component $cos(0.85\pi n)$ greatly. We can see from the magnitude response that $|H(e^{j\omega})|\Big|_{\omega=0.85\pi}$ is close to 0.

4 Circular Convolution

An N-point circular convolution is defined as:

$$y[.] = x[.] \; \text{(N)} \; h[.] \tag{3}$$

$$y[n] = \sum_{m=0}^{N-1} x[m]h[(n-m) \mod N], \; n \in [0, N-1] \tag{4}$$

Here, the length of all the three signals involved, $x[.]$, $h[.]$ and $y[.]$ are the same and equal to N.

We can easily see that the $y[.]$ obtained as a result of circular convolution and that obtained by linear convolution will not be same always. However, if we take an P-point circular convolution, by appropriately zero padding the two signals, such that $P \geq len(y_{lin}[.])$, then, the two will match. This is because $y_{cir}[.]$ is the principal period of $\tilde{y}_{lin}[.]$, which is a periodic extension of $y_{lin}[.]$, with period P. So, if the period $P < len(y_{lin}[.])$, then there will be time-aliasing, which will distort the output. Otherwise, if $P \geq len(y_{lin}[.])$, then, there will not be any time-aliasing and so, $y_{cir}[.]$ and $y_{lin}[.]$ will have the same information.

We get the following graphs, depending on the length of the circular convolution output.

`Case (a)` $N = 1024$

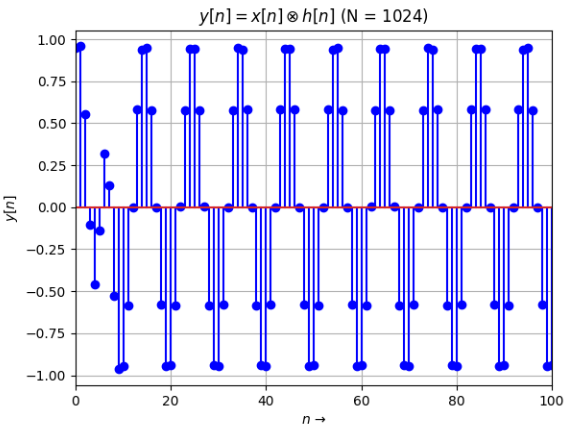


Figure 5: 1024-point circular convolution

`Case (b)` $N = 1034$

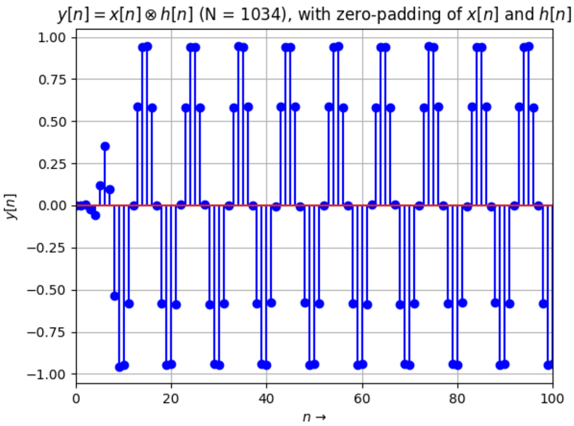


Figure 6: 1034-point circular convolution

Here, we can see that the 1034-point circular convolution results in the same output as linear convolution. This is because, the length of linear convolution signal is $1024 + 11 - 1 = 1034$. Thus, we have implemented linear convolution using circular convolution.

But, to efficiently implement linear convolution through circular convolution, we have to take 2^m-point circular convolution. This is because, the underlying algorithm, the *Fast Fourier Transform* (FFT) is most efficient when it is used for 2^m-point signals.

5 Circular Correlation of Zadoff-Chu Sequence

Consider the Zadoff-Chu sequence, a commonly used sequence in communication. The properties of the sequence are:

1. It is a complex sequence.
2. It is a constant amplitude sequence.
3. The auto correlation of a Zadoff-Chu sequence with a cyclically shifted version of itself is zero, except at the shift.
4. Correlation of Zadoff-Chu sequence with a delayed version of itself will give a peak at that delay.

The Zadoff-Chu sequence is shown below:

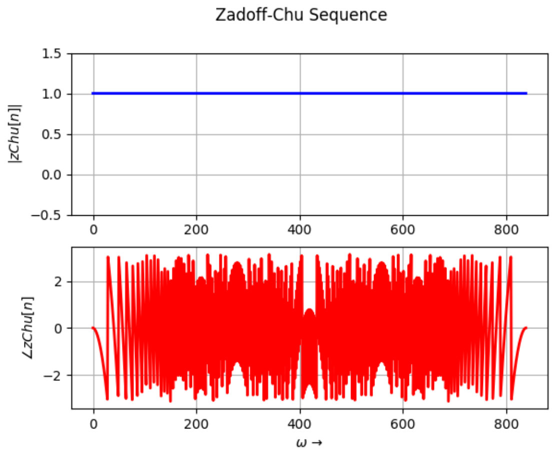
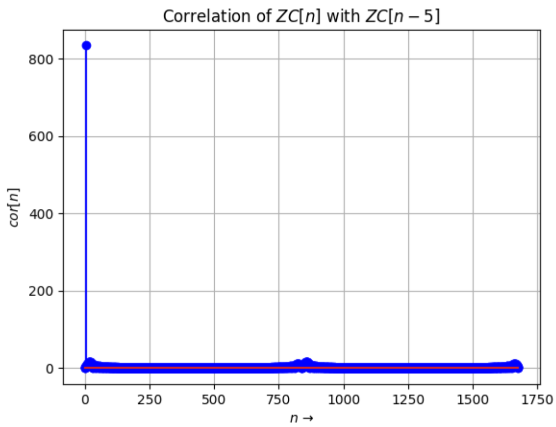


Figure 7: Zadoff-Chu Sequence

We can verify property (3) given above, by plotting the auto-correlation of $zChu[.]$ with a cyclically delayed version of itself. I have taken the auto-correlation with $zChu[.]$ cyclically rotated by 5. The result was:



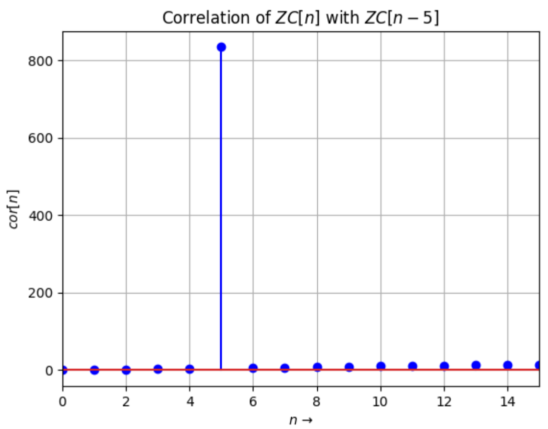


Figure 8: Auto-correlation of $zChu[.]$ with a cyclically delayed version of itself

In the above figure, we can see that the auto-correlation is non-zero only at $n = 5$, which corresponds to the delay value.

6 Conclusion

Linear convolution algorithm implemented using a direct summation is non-optimal and computationally expensive. A faster way to perform the convolution is to use the DFTs of the input and the filter. Circular convolution can be used for the implementation of linear convolution, with a much faster computation speed. The magnitude and phase response of a low pass filter were studied and the system's output, for a mixed frequency signal was obtained through three different methods. For the Zadoff-Chu sequence, it's auto-correlation with a cyclically shifted version of itself was found to be non-zero only at the point corresponding to the shift.