Final Exam for Group B (20 Marks) **Apr 20, 2017**

Dept. of Electrical Engineering, IIT Madras
Applied Programming Lab Jan 2017 session

- ▷ **Time duration of exam is** 2.5 **hours**
- ▷ **Create a work directory called *your-roll-number* and make it accessible only by you:**

  ```
  chmod 700 your-roll-number
  ```

  Then change to that directory and do your work.
- ▷ **vector operations are a must or lose lots of marks!!**
- ▷ **Label all plots. Add legends. Make the plots professional looking.**
- ▷ **Comments are not optional. They are required.**
- ▷ **pseudocode should be readable and neatly formatted.**
- ▷ **code should be written as part of a LyX file. The LyX file should be professional in appearance, and I will give marks for it.**
- ▷ **LyX file should be named *your-roll-number.lyx***
- ▷ **PDF file should be named *your-roll-number.pdf***
- ▷ **Python code should be named *your-roll-number.py***
- ▷ **Python code should run!!**
- ▷ **Include the plots in the lyx file and generate a pdf.**
- ▷ **Internet will be turned off at the beginning of the exam.**
- ▷ **Leave the above three files in your working directory**
- ▷ **Late submission will result in reduced marks.**

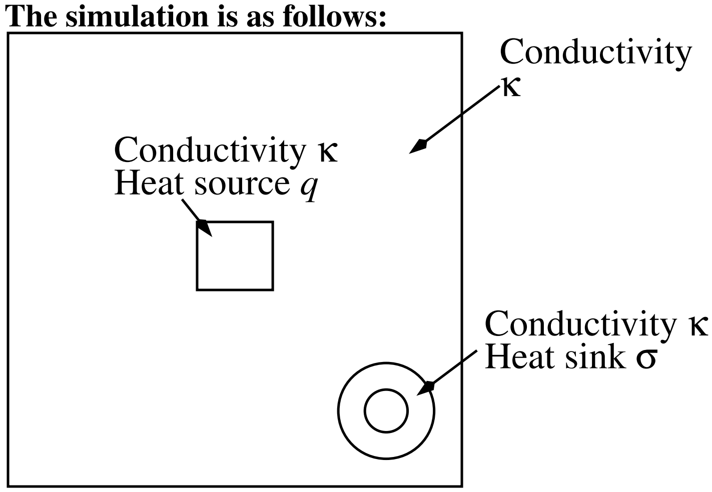
This is a problem in Poisson's equation applied to heat transfer

The IIT Madras satellite has a communications PBC which converts the digital signals to be sent to earth into the high power RF signals to be sent out of the antenna. In order to send down the required 1.1 Watts of power, the board actually consumes an extra 2.1 Watts of power which are converted into heat.

When this board is tested in our labs, it heats up about 10 to 15 degrees, which is quite alright. However, the problem is that most of the heat is removed by eddies of air that quickly cool the board. In space the board is in vacuum and the only way to cool the board is by conduction, or by radiation.

In this problem, we are going to solve for the temperature profile that will get rid of the 2.1 Watts via convection alone. This will over-estimate the temperature rise, but we will be able to get a worst case scenario.

The simulation is as follows:



The card is 10 cm by 10 cm, and the heat source is in the centre in a 2 cm by 2 cm square. The entire region has a copper plating with a uniform thermal conductivity κ. The heat source generates q Watts/sq metre over its surface ($q \times 4$cm$^2 = 2.1$Watts). At the bottom right is an annular region with centre at $(9,9)$ cm, and inner and outer radii of 0.4 and 0.8cm. This region is connected to a "heat sink" at T_0, which you can take to be zero. The region has a heat loss equal to $\sigma(T - T_0)$ Watts per sq m, where T is the temperature (yet to be found) at that point.

The equation to be solved is

$$\nabla \cdot (\kappa \nabla T) + q - \sigma T = 0$$

As done in the lab assignment, this can be rewritten as

$$\frac{\kappa}{\Delta^2}\left(\frac{T_{right}+T_{left}+T_{top}+T_{bottom}}{4}-T\right)+q-\sigma T=0$$

So the equation to be implemented is

$$\left(1+\Delta x\Delta y\sigma_{ij}\right)T_{ij}=\frac{1}{4}\left(T_{i-1,j}+T_{i+1,j}+T_{i,j-1}+T_{i,j+1}\right)+q\Delta x\Delta y$$

The boundary conditions are zero current, i.e., no heat flows out of the board. Take $\kappa=1$, $q=300$ and σ is an input variable of order 10^4.

- ▷ **Break the surface into a 101 by 101 mesh, with mesh points separated by 0.1 cm.**
- ▷ **Create an array σ_{ij} that is zero except at the annular region. How will you define those points?**
- ▷ **Create an array q_{ij} that is zero except in the heat source region where it is unity.**
- ▷ **Initialize the Temperature array to zero and iterate for 2000 steps. Try for 40000 steps next. What has changed in the two runs. Note, set all negative temperatures after an iteration to zero.**
- ▷ **Plot the per step error vs iteration number in a semilog plot for 40000 steps. Obtain the decay rate.**
- ▷ **Determine the number of steps to take to achieve 10^{-2} total error. Don't run the simulation - the number of steps will be too many. I only want your estimate.**
- ▷ **Plot a contour plot of $T(x,y)$ and label it.**
- ▷ **Convert the code into a function that takes one argument, namely σ. Freeze $N=20000$, since it is in the exponentially converging region.**
- ▷ **Now run the function for σ going from 10^{-3} to 10^5 in 25 steps and plot the variation of the maximum temperature vs σ as well as the minimum temperature vs σ in the same plot. Explain what you get.**
- ▷ **The entire analysis should be presented in a LyX report.**

Useful Python Commands (use “?” to get help on these from ipython)

```
from pylab import *
import system-function as name
Note: lstsq is found as scipy.linalg.lstsq
ones(List)
zeros(List)
range(N0,N1,Nstep)
arange(N0,N1,Nstep)
linspace(a,b,N)
logspace(log10(a),log10(b),N)
X,Y=meshgrid(x,y)
where(condition)
where(condition & condition)
where(condition | condition)
a=b.copy()
lstsq(A,b) to fit A*x=b
A.max() to find max value of numpy array (similalry min)
A.astype(type) to convert a numpy array to another type (eg int)
def func(args):
  ...
  return List
matrix=c_[vector,vector,...] to create a matrix from vectors
figure(n) to switch to, or start a new figure labelled n
plot(x,y,style,...,lw=...)
semilogx(x,y,style,...,lw=...)
semilogy(x,y,style,...,lw=...)
loglog(x,y,style,...,lw=...)
contour(x,y,matrix,levels...)
quiver(X,Y,U,V) # X,Y,U,V all matrices
xlabel(label,size=)
ylabel(label,size=)
title(label,size=)
```

```
xticks(size=) # to change size of xaxis numbers
yticks(size=)
legend(List) to create a list of strings in plot
annotate(str,pos,lblpos,...) to create annotation in plot
grid(Boolean)
show()
```