Pattern Recognition (PR)

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This is a printable version of the slides of the lecture

Pattern Recognition (PR)
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Prof. Dr.-Ing. Andreas Maier

AdaBoost

Boosting Methods

- Boosting is one of the most powerful learning techniques introduced in the last twenty years.
- Combines output of many *weak* classifiers to produce a powerful committee.
- The most popular boosting algorithm is called *AdaBoost* (Freund and Schapire, 1997).

Boosting Methods (cont.)

Definition

A weak classifier is one whose error rate is only slightly better than random guessing.

Idea of boosting:

- Sequentially apply the *weak* classifier to repeatedly modified versions of the data.
- This produces a sequence of classifiers.
- Weighted majority vote yields the final prediction.

Boosting Methods (cont.)

- Consider a two-class problem with $y \in \{-1, +1\}$.
- Given a set of observations $\mathscr{D} = \{(\boldsymbol{x}_i, y_i); \quad i = 1, \dots, N\}$ and a classifier $G(\boldsymbol{x})$, the error rate on the training sample is:

$$\overline{\text{err}} = \frac{1}{N} \sum_{i=1}^{N} I(y_i \neq G(\boldsymbol{x}_i))$$

where I is the indicator function.

Boosting Methods (cont.)

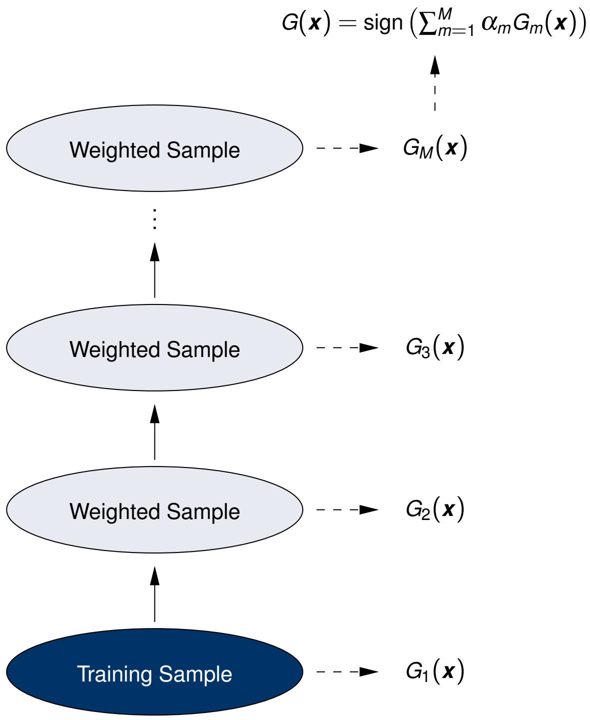
- Sequentially applied weak classifiers produce a sequence $G_m(\boldsymbol{x})$ with $m = 1, 2, \ldots, M$
- The combined prediction is then:

$$G(\boldsymbol{x}) = \operatorname{sign}\left(\sum_{m=1}^{M} \alpha_m G_m(\boldsymbol{x})\right)$$

- The weighting factors $\alpha_1, \ldots, \alpha_M$ are computed by the boosting algorithm.
- Each α_m weights the output of the corresponding classifier $G_m(\boldsymbol{x})$.

Boosting Methods

Schematic of boosting procedure:



Boosting Methods (cont.)

Modifications on the data:

- Each boosting step consists of applying weights $w_1, w_2, \ldots, w_N$ to the training samples.
- Initially, the weights are set to $w_i = 1/N$.
- Thus the first classifier in the sequence is trained the usual way.
- For $m \geq 2$ the weights are individually modified.
- The classifiers G_m, with $m \geq 2$ are trained on differently-weighted samples.

Boosting Methods

Weighting scheme:

- At step m the misclassified observations from G_{m-1} have increased weights.
- The weights for correctly classified samples from G_{m-1} have decreased weights.

Effects of the weighting scheme:

- Observations that are difficult to classify correctly get ever-increasing influence.
- Each successive classifier is forced to concentrate more on those observations that were misclassified by the previous one.

AdaBoost

AdaBoost algorithm:

Initialize weights: $w_i = \leftarrow 1/N$, $i = 1, \ldots, N$
$m \leftarrow 1$
Fit classifier $G_m(\boldsymbol{x})$ to training data using $\boldsymbol{w}$
Compute classification error: $\text{err}_m = \frac{\sum_{i=1}^N w_i I(y_i \neq G_m(x_i))}{\sum_{i=1}^N w_i}$
Compute classifier weights: $\alpha_m = \log\left(\frac{1-\text{err}_m}{\text{err}_m}\right)$
Compute new sample weights: $w_i \leftarrow w_i \exp\left[\alpha_m I(y_i \neq G_m(x_i))\right]$, $i = 1, \ldots, N$
$m \leftarrow m + 1$
$m = M$
Output: $G(\boldsymbol{x}) = \text{sign}\left(\sum_{m=1}^M \alpha_m G_m(\boldsymbol{x})\right)$

AdaBoost (cont.)

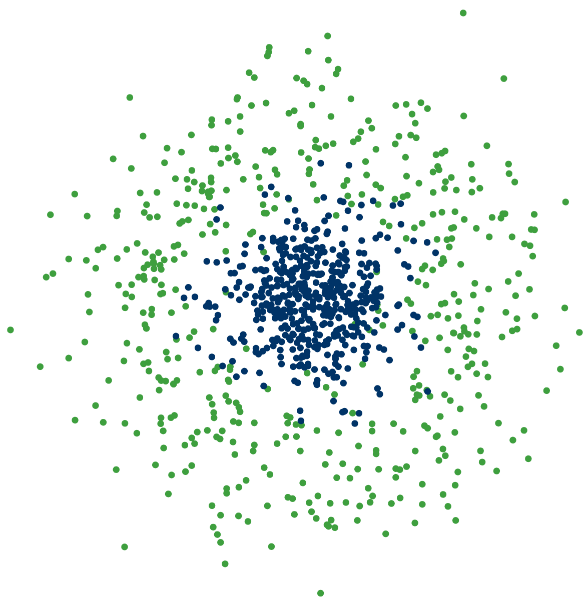
Notes:

- This version of AdaBoost is called discrete, because each $G_m(\boldsymbol{x})$ returns a discrete class label.
- AdaBoost can be modified to return also a real-valued prediction in the interval $[-1, +1]$.
- Instead of taking just any classifier for $G_m(\boldsymbol{x})$, that classifier may be used that results in the smallest error at step m.

AdaBoost dramatically increases the performance of even a very weak classifier.

AdaBoost (cont.)

Example from J. Matas and J. Šochman
(Centre for Machine Perception, Technical University, Prague):



AdaBoost (cont.)

Example from J. Matas and J. Šochman
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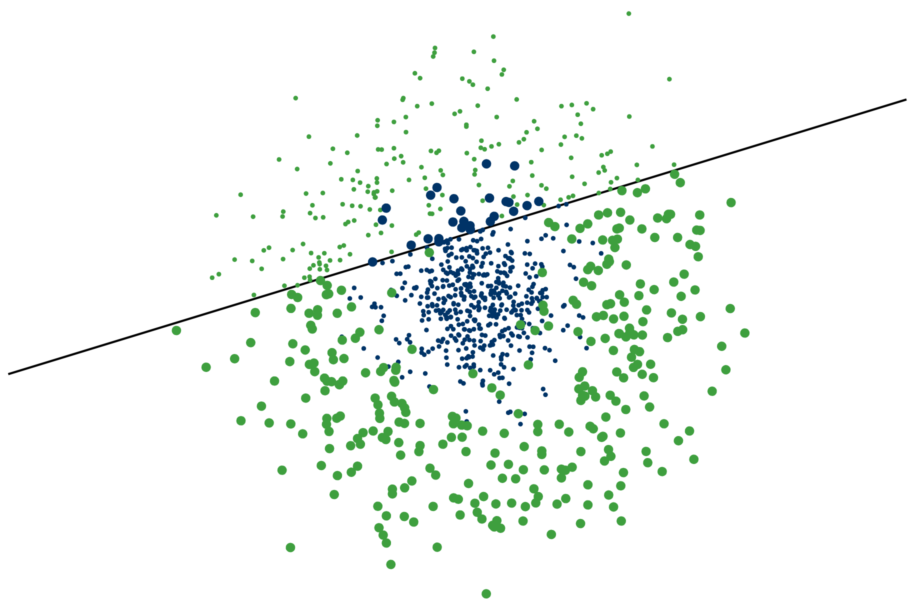

AdaBoost (cont.)

Example from J. Matas and J. Šochman
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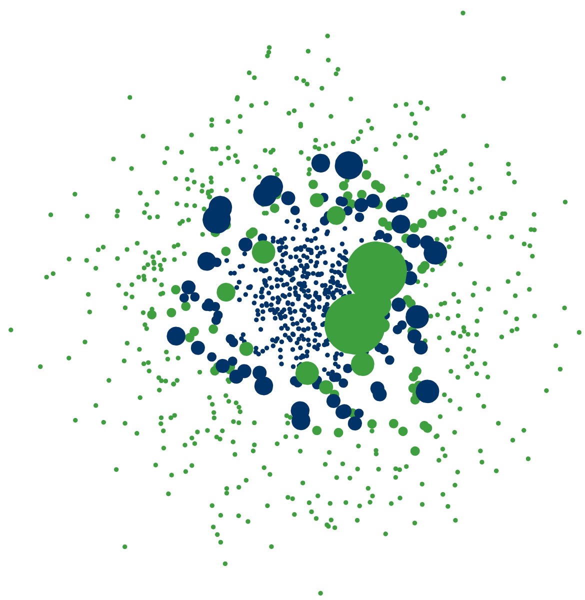

AdaBoost (cont.)

Example from J. Matas and J. Šochman
(Centre for Machine Perception, Technical University, Prague):

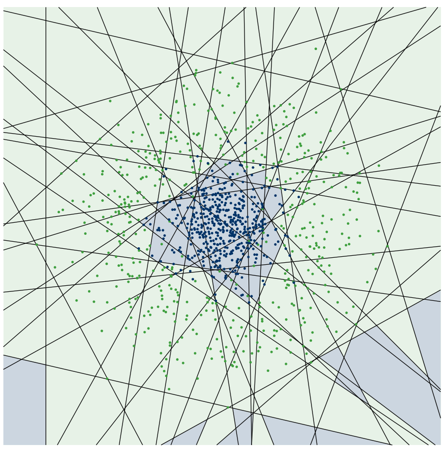

Fig.: Final classifier (based on a perceptron).

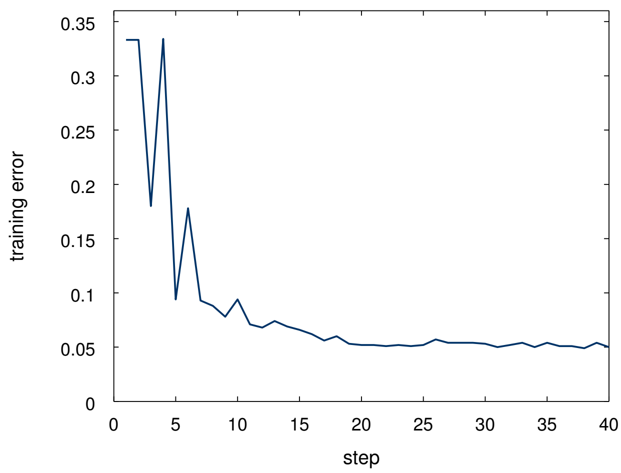

Fig.: Reduction of the classification error w. r. t. the sequence length.

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Next Time in Pattern Recognition

Exponential Loss

- Boosting fits an additive model in a set of elementary basis functions:

$$f_M(\boldsymbol{x}) = \sum_{m=1}^{M} \beta_m \, b(\boldsymbol{x}; \gamma_m)$$

  where β_m are expansion coefficients and $b(\boldsymbol{x}; \gamma_m)$ is a basis function given a set of parameters γ_m.

- Additive expansions are very popular in learning techniques:
  - single-hidden-layer neural networks (perceptron)
  - wavelets
  - classification trees
  - etc.

Exponential Loss (cont.)

- Expansion models are typically fit by minimizing a loss function L averaged over the training data:

$$\min_{\{\beta_m,\gamma_m\}_1^M} \left\{ \sum_{i=1}^{N} L\big(y_i, f_M(\boldsymbol{x}_i)\big) = \sum_{i=1}^{N} L\left(y_i, \sum_{m=1}^{M} \beta_m b(\boldsymbol{x}_i; \gamma_m)\right) \right\} \qquad (1)$$

- Forward stagewise modeling approximates the solution to (1):
  - New basis functions are sequentially added.
  - Parameters and coefficients of already added functions are not changed.
  - At each iteration, the subproblem of fitting just a single basis function is solved.

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Exponential Loss (cont.)

- The *m*-th subproblem may be rewritten as:

$$(\beta_m, \gamma_m) = \operatorname*{argmin}_{\beta,\gamma} \sum_{i=1}^{N} L\big(y_i, f_{m-1}(\boldsymbol{x}_i) + \beta\, b(\boldsymbol{x}_i; \gamma)\big)$$

- AdaBoost is equivalent to a forward stagewise additive modeling using an exponential loss function:

$$L(y, f(\boldsymbol{x})) = \exp(-y\, f(\boldsymbol{x}))$$

Exponential Loss (cont.)

Proof:

- For AdaBoost, the basis functions are the classifiers $G_m(\boldsymbol{x}) \in \{-1, +1\}$.
- Using the exponential loss function, one must solve at each step:

$$(\beta_m, G_m) = \operatorname*{argmin}_{\beta, G} \sum_{i=1}^{N} \exp\left[-y_i\left(f_{m-1}(\boldsymbol{x}_i) + \beta G(\boldsymbol{x}_i)\right)\right]$$

- Using the weight $w_i^{(m)} = \exp(-y_i f_{m-1}(\boldsymbol{x}_i))$ this can be rewritten as:

$$(\beta_m, G_m) = \operatorname*{argmin}_{\beta, G} \sum_{i=1}^{N} w_i^{(m)} \exp(-\beta y_i G(\boldsymbol{x}_i))$$

Exponential Loss (cont.)

Observations:

- Since $w_i^{(m)}$ is independent of β and $G(\boldsymbol{x})$, it can be seen as a weight applied to each observation.
- However, this weight depends on f_{m-1}, so the weights change with each iteration m.

Exponential Loss (cont.)

$$
\begin{aligned}
(\beta_m, G_m) &= \underset{\beta, G}{\operatorname{argmin}} \sum_{i=1}^{N} w_i^{(m)} \exp(-\beta y_i G(\boldsymbol{x}_i)) \\
&= \underset{\beta, G}{\operatorname{argmin}}\ e^{-\beta} \sum_{y_i = G(\boldsymbol{x}_i)} w_i^{(m)} + e^{\beta} \sum_{y_i \neq G(\boldsymbol{x}_i)} w_i^{(m)} \\
&= \underset{\beta, G}{\operatorname{argmin}} \left(e^{\beta} - e^{-\beta}\right) \sum_{i=1}^{N} w_i^{(m)} I(y_i \neq G(\boldsymbol{x}_i)) + e^{-\beta} \sum_{i=1}^{N} w_i^{(m)}
\end{aligned}
$$

- Thus, for any value $\beta > 0$ the solution for $G_m(\boldsymbol{x})$ is:

$$
G_m = \underset{G}{\operatorname{argmin}} \sum_{i=1}^{N} w_i^{(m)} I(y_i \neq G(\boldsymbol{x}_i))
$$

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Exponential Loss (cont.)

- Plugging the reformulated $G_m(\boldsymbol{x})$ into the objective function and solving for β_m yields:

$$
\beta_m = \frac{1}{2} \log \frac{1 - \mathrm{err}_m}{\mathrm{err}_m}
$$

with err_m being the minimized weighted error rate:

$$
\mathrm{err}_m = \frac{\sum_{i=1}^{N} w_i^{(m)} I(y_i \neq G_m(\boldsymbol{x}_i))}{\sum_{i=1}^{N} w_i^{(m)}}
$$

Exponential Loss (cont.)

- From the update formula of the approximation:

$$f_m(\boldsymbol{x}) = f_{m-1}(\boldsymbol{x}) + \beta_m G_m(\boldsymbol{x})$$

we can calculate the weights for the next iteration:

$$\begin{aligned} w_i^{(m+1)} &= w_i^{(m)} e^{-\beta_m y_i G_m(\boldsymbol{x}_i)} \\ & \quad (\text{using: } -y_i G_m(\boldsymbol{x}_i) = 2I(y_i \neq G_m(\boldsymbol{x}_i)) - 1) \\ &= w_i^{(m)} e^{\alpha_m I(y_i \neq G_m(\boldsymbol{x}_i))} e^{-\beta_m} \end{aligned}$$

with $\alpha_m = 2\beta_m$.

□

Exponential Loss (cont.)

Compare result to initial AdaBoost algorithm:

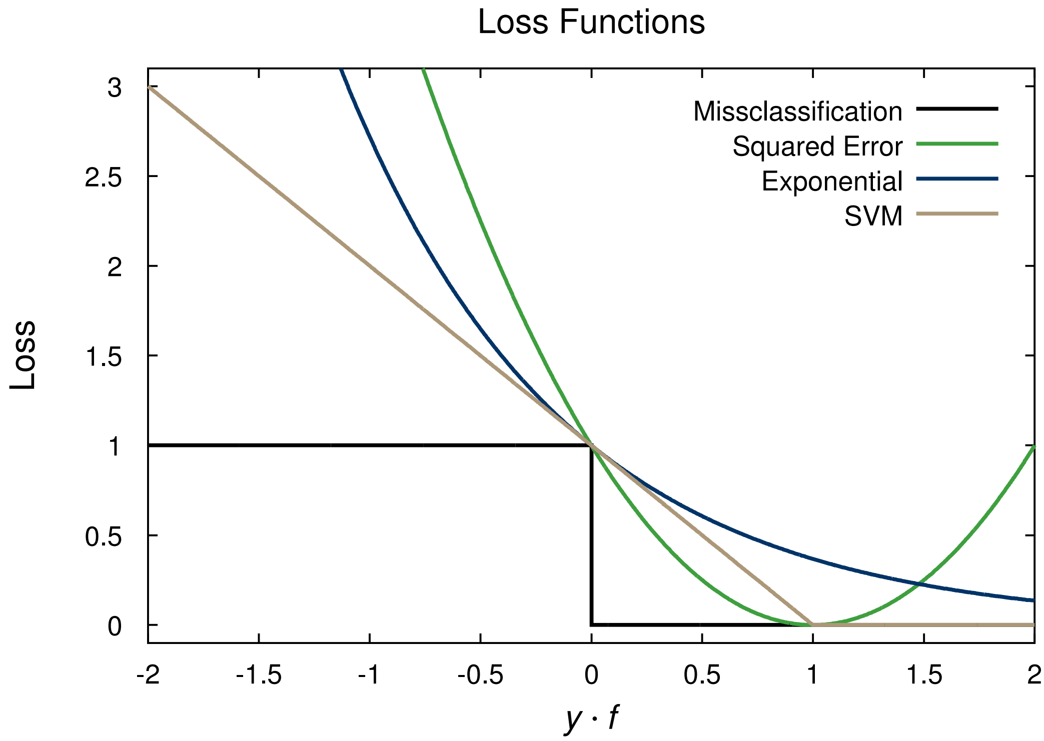
- Exponential loss:

$$\begin{aligned} \beta_m &= \frac{1}{2} \log \frac{1 - \text{err}_m}{\text{err}_m} \\ \alpha_m &= 2\beta_m \\ w_i^{(m+1)} &= w_i^{(m)} e^{\alpha_m I(y_i \neq G_m(\boldsymbol{x}_i))} e^{-\beta_m} \end{aligned}$$

- AdaBoost:

$$\begin{aligned} \alpha_m &= \log \frac{1 - \text{err}_m}{\text{err}_m} \\ w_i &\leftarrow w_i e^{\alpha_m I(y_i \neq G_m(\boldsymbol{x}_i))} \end{aligned}$$

Exponential Loss (cont.)



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Exponential Loss (cont.)

Discussion:

- The equivalence of AdaBoost to forward stagewise additive modeling was discovered five years after its invention.
- The AdaBoost criterion yields a monotone decreasing function of the margin: $y\, f(\boldsymbol{x})$.
- In $\{-1, 1\}$ classification, the margin plays a role similar to the residuals $y - f(\boldsymbol{x})$ in regression:
  - Observations with $y_i\, f(\boldsymbol{x}_i) > 0$ are classified correctly
  - Observations with $y_i\, f(\boldsymbol{x}_i) < 0$ are misclassified
  - The decision boundary is at $f(\boldsymbol{x}) = 0$

Exponential Loss (cont.)

Discussion (cont.):

- The goal of the classification algorithm is to produce positive margins as frequently as possible.
- Thus, any loss criterion should penalize negative margins more heavily than positive ones!
- The exponential criterion concentrates much more influence on observations with large negative margins.

Due to the exponential loss, AdaBoost performance is known to degrade rapidly:

- in situations of noisy data
- when there are wrong class labels in the training data

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Next Time in Pattern Recognition

Face Detection

Famous algorithm developed by Viola and Jones in 2001.

Contributions:

- Integral images for feature computation.
- Usage of AdaBoost for boosting.
- Classifier cascade for fast rejection of non-face regions.

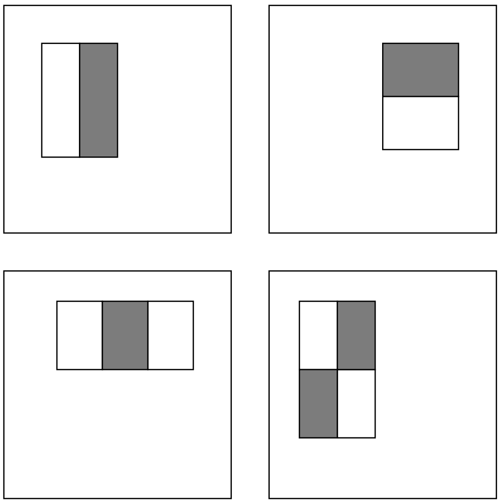
Various implementations available: for example look into *OpenCV FaceDetector* sample code.

Face Detection (cont.)

Features:

- Features are adapted from Haar basis functions.
- They are calculated by subtracting the sum of the pixels in the white from the sum of the pixels in the gray rectangles:

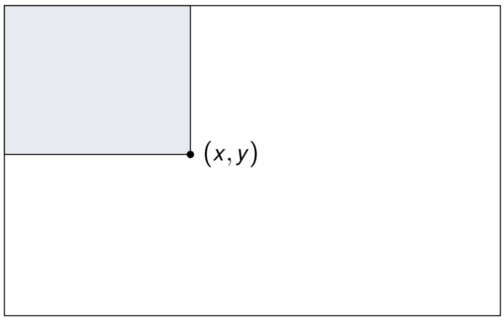
4 types of features used:



Face Detection (cont.)

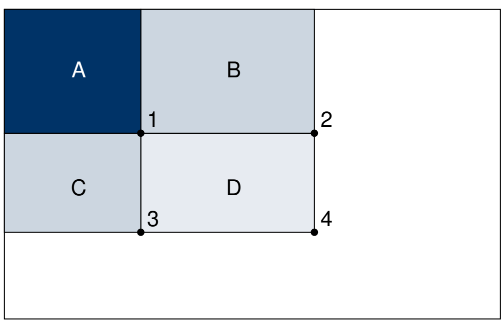
- The exhaustive set of features is very large: $> 45\,000$ for a 380x280 image.
- Rectangle features can be efficiently computed using an Integral Image II:

$$II(x,y) = \sum_{x' \le x, y' \le y} I(x', y')$$



Face Detection (cont.)

Computation of the sum of intensities for any rectangle D with just 3 basis operations based on the integral image:



- Value at 1: A
- Value at 2: A + B
- Value at 3: A + C
- Value at 4: A + B + C + D
- Sum within D: $4 + 1 - (2 + 3)$

Face Detection (cont.)

Boosting:

- The classification functions are restricted to each depend on a single feature only.
- This way, AdaBoost can be interpreted as an effective feature selecting algorithm:
  - The single rectangle feature is selected which best separates the observations.
  - For each feature, the weak learner determines the optimal threshold classification function.

Face Detection (cont.)

- A weak classifier $G_j(\boldsymbol{x})$ consists of a feature x_j, an optimal threshold θ_j, and a parity s_j to indicate the direction of the inequality:

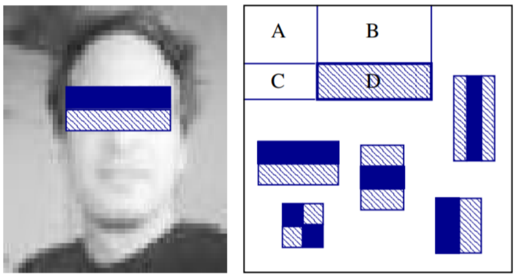
$$G_j(\boldsymbol{x}) = \begin{cases} 1 & \text{if } s_j f_j(\boldsymbol{x}) < s_j \theta_j \\ 0 & \text{otherwise} \end{cases}$$

  with $\boldsymbol{x}$ being a sub-window of an image.

- Very aggressive process to discard the vast majority of features.

Face Detection (cont.)

- Best feature selected by AdaBoost:



Christian Hacker, Anton Batliner, and Elmar Noeth. Are You Looking at Me, Are You Talking with Me: Multimodal Classification of the Focus of Attention.

Face Detection (cont.)

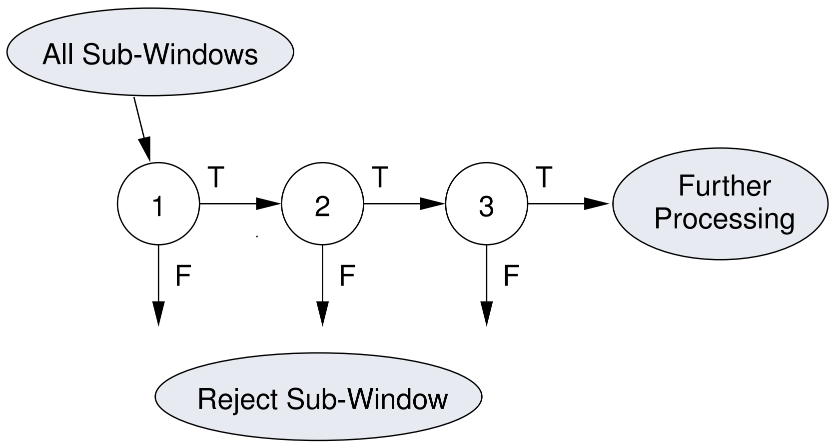
Classifier Cascade:

- Despite the efficiency of the feature computation, the complexity of the problem is still huge.
- Evaluating the full AdaBoost sequence on all sub-windows of the image takes too much time.

- Idea of cascaded classifiers:
  - Simpler classifiers used to reject the majority of sub-windows.
  - Each stage is again created by AdaBoost.
  - Stage 1 uses the two features shown before; detects 100 % faces with false positive rate (FPR) of around 40 %.

Face Detection (cont.)

- The cascade has the overall shape of a degenerated decision tree:



- A negative classification at any stage yields an immediate rejection of the sub-window.

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Face Detection (cont.)

- Structure of cascade reflects the (usually) overwhelming majority of negative sub-windows in an image.
- Goal is to reject as many negatives as possible at the earliest stage of the processing.

Training of the cascade:

- Subsequent classifiers are trained using only those examples which pass through all the previous stages.
- The next classifier faces a more difficult task than the previous one.
- Trade-off between more features achieving higher detection rates and lower false positive rates while requiring more computation time.

Face Detection (cont.)

Final classifier:

- 32 layer cascade of increasingly strong boosted classifiers

Layer	# Features	TPR	FPR
1	2	100 %	40 %
2	5	100 %	20 %
3-5	20		
6-7	50		
8-12	100		
13-32	200		

- Number of stages and features adapted until false positive rate on validation was nearly zero while maintaining high correct rate.

Face Detection (cont.)

Example images from the training set:

Paul Viola and Michael Jones. Rapid Object Detection using a Boosted Cascade of Simple Features. IEEE Conf Comput Vis Pattern Recognit.

Face Detection (cont.)

Results on test set:

Christian Hacker, Anton Batliner, and Elmar Noeth. Are You Looking at Me, Are You Talking with Me: Multimodal Classification of the Focus of Attention.

Lessons Learned

- Boosting of weak classifiers can yield a powerful combined classifier
- AdaBoost algorithm
- Weights of the classifiers and on the data are adjusted
- AdaBoost minimizes an exponential loss function
- Viola & Jones face detection algorithm

Further Reading

Most of the content of this lecture has been taken from:

- T. Hastie, R. Tibshirani, J. Friedman:
  The Elements of Statistical Learning,
  2nd Edition, Springer, 2009.
- Y. Freund, R. E. Schapire:
  A decision-theoretic generalization of on-line learning and
  an application to boosting,
  Journal of Computer and System Sciences, 55(1):119-139, 1997.
- P. A. Viola, M. J. Jones: Robust Real-Time Face Detection,
  International Journal of Computer Vision 57(2): 137-154, 2004.
- J. Matas and J. Šochman: AdaBoost,
  Centre for Machine Perception, Technical University, Prague.
  ☞ http://cmp.felk.cvut.cz/~sochmj1/adaboost_talk.pdf