

# Across the Great Divide: from ML Theory to Practice

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Google NYC

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# Introduction

Google Research

Research Scientist at Google NYC

Working on machine learning algorithm design and analysis



Past lives:

- USyd
- UCSD
- NICTA/CSIRO Data61/ANU



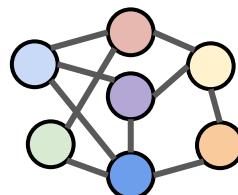
# Supervised learning in theory

Google Research

Training data



Model training



Model predictions



0.2	0.1	0.4	0.2	0.1
-----	-----	-----	-----	-----

# Supervised learning in theory

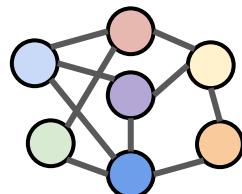
Google Research

Training data



$$\{(x_n, y_n)\}_{n=1}^N$$

Model training



$$\min_{f \in \mathcal{F}} \frac{1}{N} \sum_{n \in N} \ell(y_n, f(x_n))$$

Model predictions



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$$f(x^*)$$

# Supervised learning in practice

Google Research

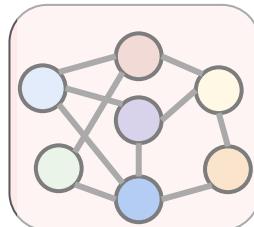
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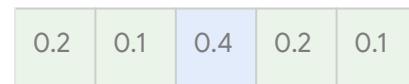
Model training

What if the model size is **too large**?



$$\min_{f \in \mathcal{F}} \frac{1}{N} \sum_{n \in N} \ell(y_n, f(x_n))$$

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$$f(x^*)$$

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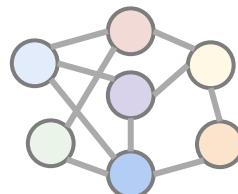
Google Research

Training data



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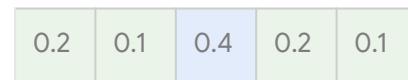
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$$\min_{f \in \mathcal{F}} \frac{1}{N} \sum_{n \in N} \ell(y_n, f(x_n))$$

What if this loss  
is **expensive** to  
compute?

Model predictions



$$f(x^*)$$

# Supervised learning in practice

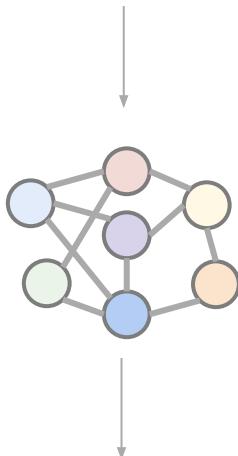
Google Research

Training data



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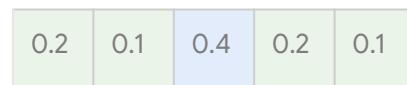
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$$\min_{f \in \mathcal{F}} \frac{1}{N} \sum_{n \in N} \ell(y_n, f(x_n))$$

What if this  
operation is  
stochastic?

Model predictions



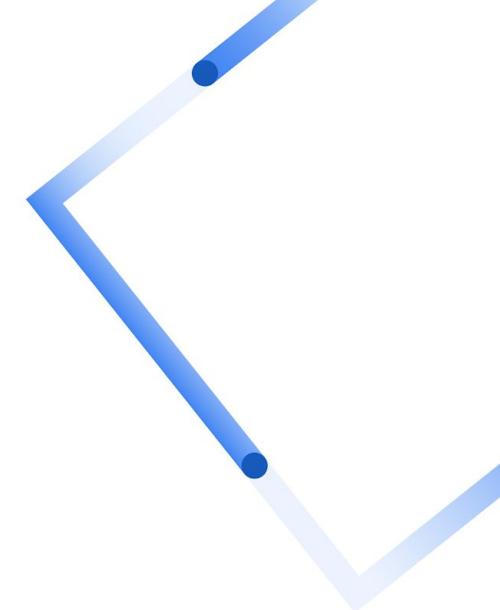
$$f(x^*)$$

# Agenda

- 01 Background
- 02 Distillation
- 03 Extreme classification
- 04 Churn
- 05 Summary

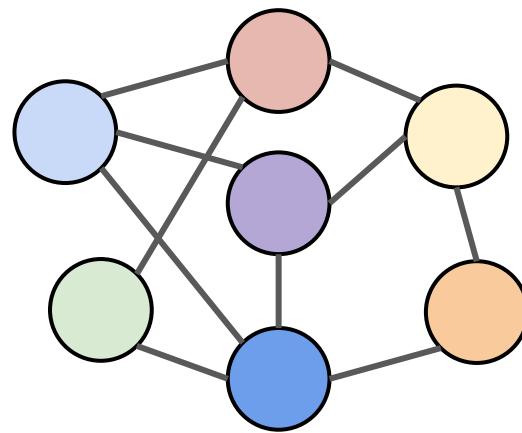
01

# Background



# Neural networks for classification

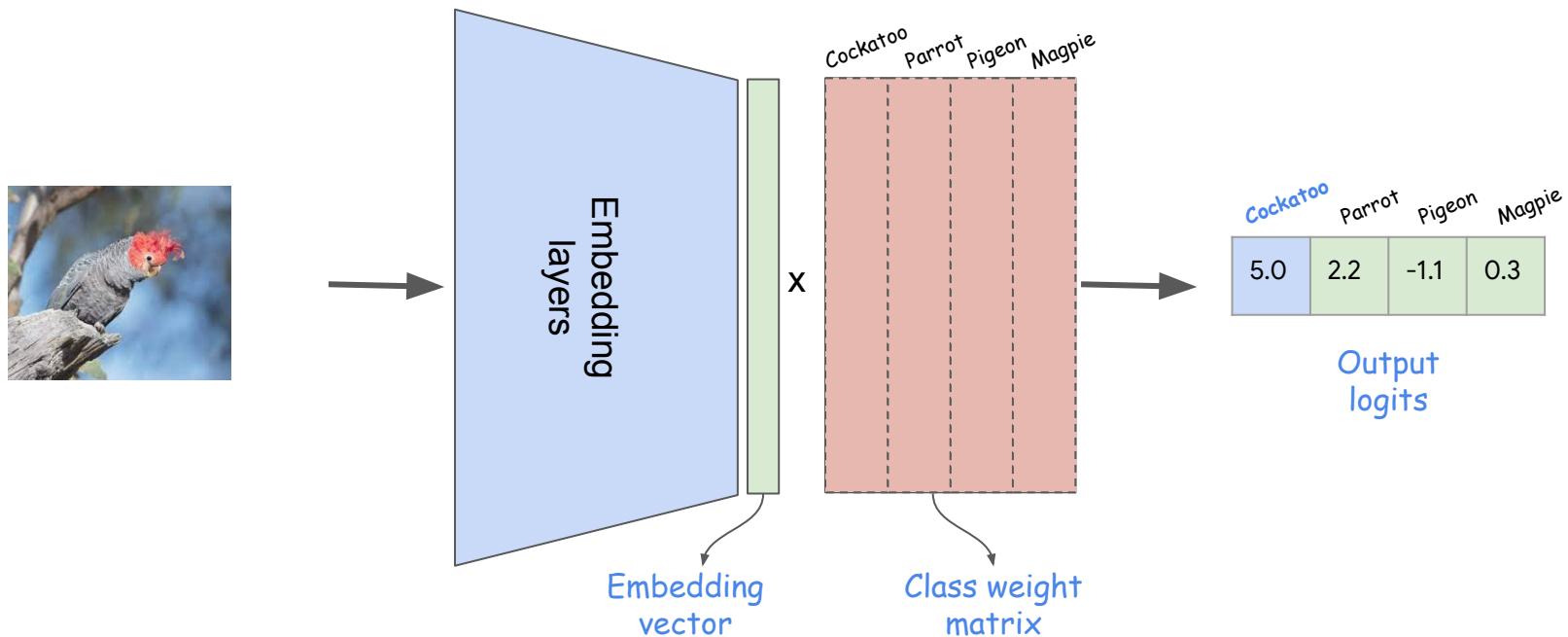
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*"Cockatoo"*

# Neural networks for classification

Google Research



# Neural networks for classification

Google Research

Training objective: minimise **softmax cross-entropy**

$$\log \sum \exp \begin{matrix} & \text{Cockatoo} \\ & \text{Parrot} \\ & \text{Pigeon} \\ & \text{Magpie} \\ \hline 5.0 & 2.2 & -1.1 & 0.3 \end{matrix} - \boxed{5.0}$$

This approximately minimises the (negative) **prediction margin**:

$$\max \begin{matrix} & \text{Parrot} \\ & \text{Pigeon} \\ & \text{Magpie} \\ \hline 2.2 & -1.1 & 0.3 \end{matrix} - \boxed{5.0}$$

 Highest score of  
"wrong" label

# Neural networks for classification

Google Research

Training objective: minimise **softmax cross-entropy**

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This equivalently minimises the **KL divergence**:

$$\text{KL}(\mathbf{e}_y \| \mathbf{p}(x))$$

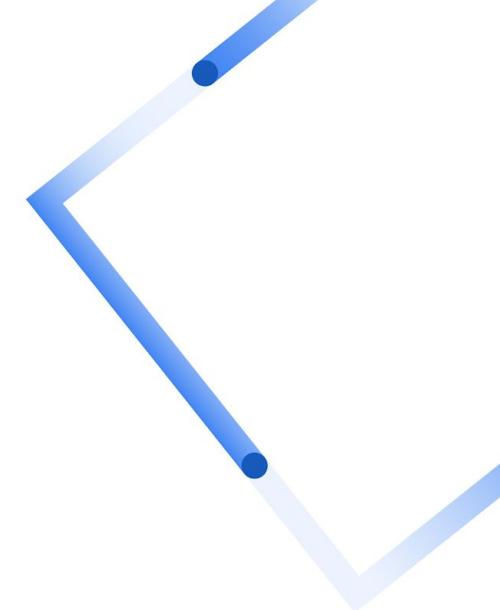
“One-hot” label vector       $\begin{matrix} 1.0 & 0.0 & 0.0 & \dots \end{matrix}$

“Softmax” probability vector       $\begin{matrix} 0.3 & 0.5 & 0.1 & \dots \end{matrix}$

$$p_i(x) = \frac{\exp(s_i(x))}{\sum_{j \in [L]} \exp(s_j(x))}$$

02

# Distillation



# Supervised learning in theory

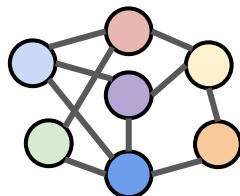
Google Research

Training data



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Model training



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Model predictions



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$$f(x^*)$$

# Supervised learning in practice

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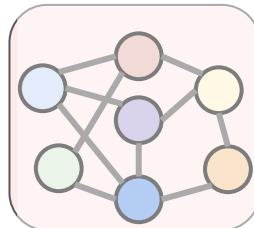
Training data



$$\{(x_n, y_n)\}_{n=1}^N$$

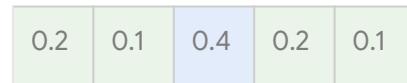
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Model predictions



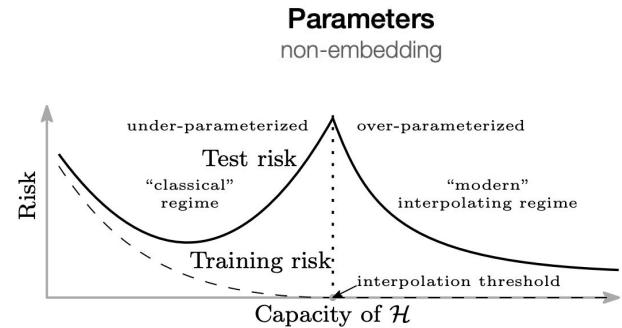
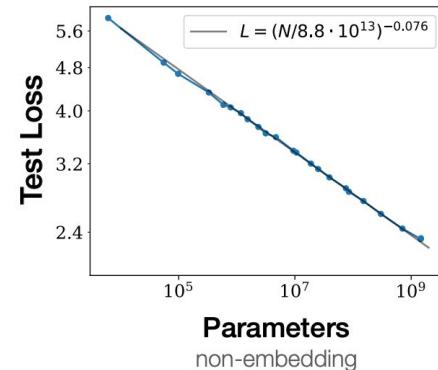
$$f(x^*)$$

# Why increase model size?

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😊 Can **work better!**

Particularly for complex tasks, e.g.,  
language modelling



# Why (not) increase model size?

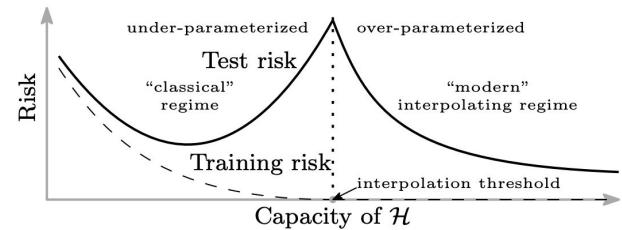
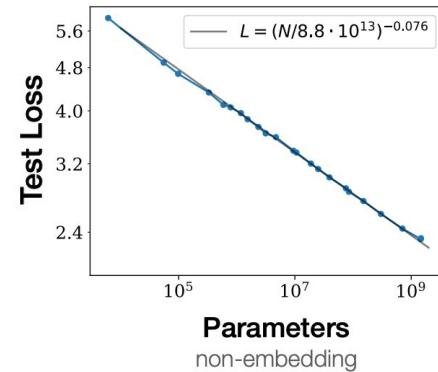
Google Research

😊 Can **work better!**

Particularly for complex tasks, e.g.,  
language modelling

😢 More expensive to **train**

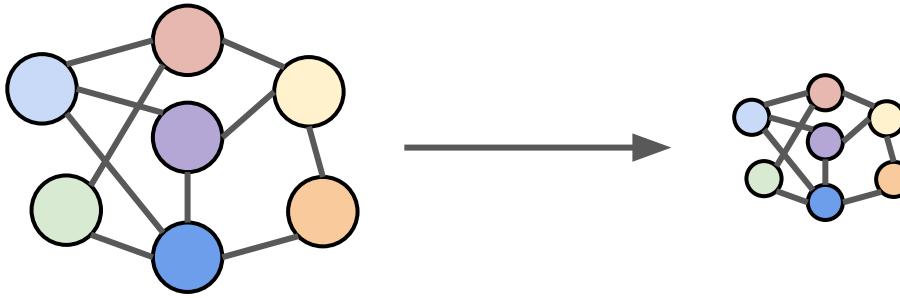
😴 More expensive to **predict**



# Idea: model compression

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Ideally, **compress** our model while **preserving** performance

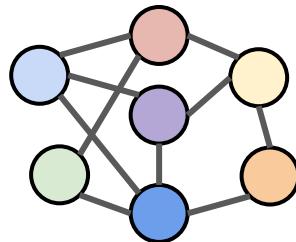


Many options: quantisation, architecture optimisation, **distillation**,....

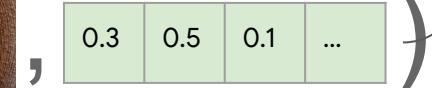
# Distillation in a nutshell

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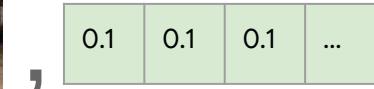
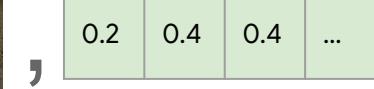
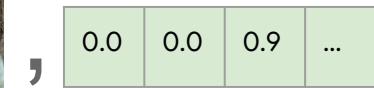
Train a “student” model using **soft predictions** from “teacher” model



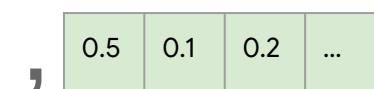
Trained on  
one-hot labels



Predictions from  
“teacher”



Trained on teacher  
predictions



# Distillation loss function

Google Research

Minimise

softmax cross-entropy

$$\log \sum \exp \begin{matrix} & \text{Cockatoo} & \text{Parrot} & \text{Pigeon} & \text{Magpie} \\ & 5.0 & 2.2 & -1.1 & 0.3 \end{matrix} - \boxed{5.0}$$

# Distillation loss function

Google Research

Minimise **teacher-weighted** softmax cross-entropy

$$p^t(\text{Cockatoo}) \times \log \sum \exp \begin{matrix} & \text{Cockatoo} & \text{Parrot} & \text{Pigeon} & \text{Magpie} \\ \hline 5.0 & 2.2 & -1.1 & 0.3 \end{matrix} - \boxed{5.0} +$$

$$p^t(\text{Parrot}) \times \log \sum \exp \begin{matrix} & \text{Cockatoo} & \text{Parrot} & \text{Pigeon} & \text{Magpie} \\ \hline 5.0 & 2.2 & -1.1 & 0.3 \end{matrix} - \boxed{2.2} +$$

$$p^t(\text{Pigeon}) \times \log \sum \exp \begin{matrix} & \text{Cockatoo} & \text{Parrot} & \text{Pigeon} & \text{Magpie} \\ \hline 5.0 & 2.2 & -1.1 & 0.3 \end{matrix} - \boxed{-1.1} +$$

$$p^t(\text{Magpie}) \times \log \sum \exp \begin{matrix} & \text{Cockatoo} & \text{Parrot} & \text{Pigeon} & \text{Magpie} \\ \hline 5.0 & 2.2 & -1.1 & 0.3 \end{matrix} - \boxed{0.3} +$$

# Distillation loss function: formally

Google Research

Suppose the teacher's predictions are  $p^t$

Then, we may minimise:

$$\frac{1}{N} \sum_{n=1}^N [(1 - \alpha) \cdot \text{KL}(\mathbf{e}_{y_n} \| p(x_n)) + \alpha \cdot \text{KL}(p^t(x_n) \| p(x_n))]$$

Mixing weight



Input  
data

1.0	0.0	0.0	...
-----	-----	-----	-----

"One-hot"  
label

0.3	0.5	0.1	...
-----	-----	-----	-----

"Soft"  
label

# Why does distillation help?

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Transfers **class relationship** information

“Dark knowledge”

Learns which errors to penalise more

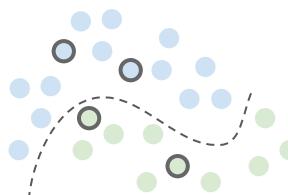
Per-sample **label smoothing**

Prevents over-confident predictions

	0.8	0.2	0.0	0.0
	0.1	0.9	0.0	0.0
	0.0	0.1	0.8	0.1
	0.0	0.0	0.3	0.7

Can be used on **unlabelled samples**

Form of semi-supervised learning!

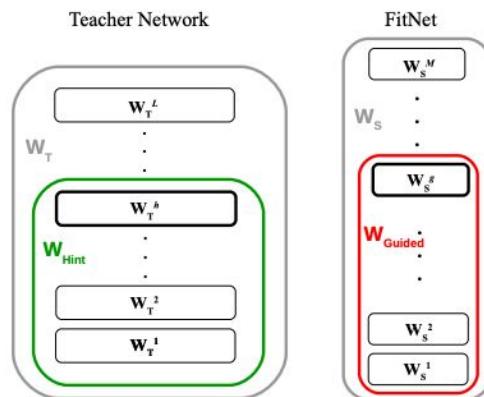


# Beyond probability matching

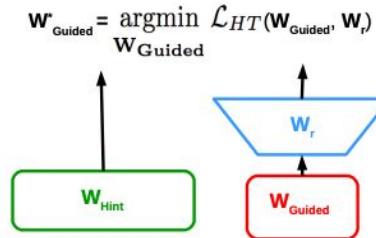
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Can match **more structure** in teacher model

e.g., match embeddings, pairwise similarities, ...



(a) Teacher and Student Networks



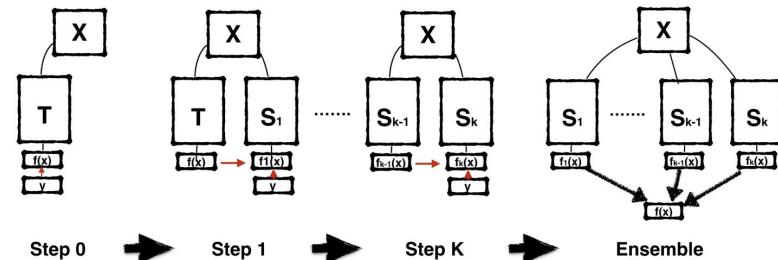
(b) Hints Training

# Do we need complex teachers?

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No. You can “self-distill” (!)

Can give non-trivial gains



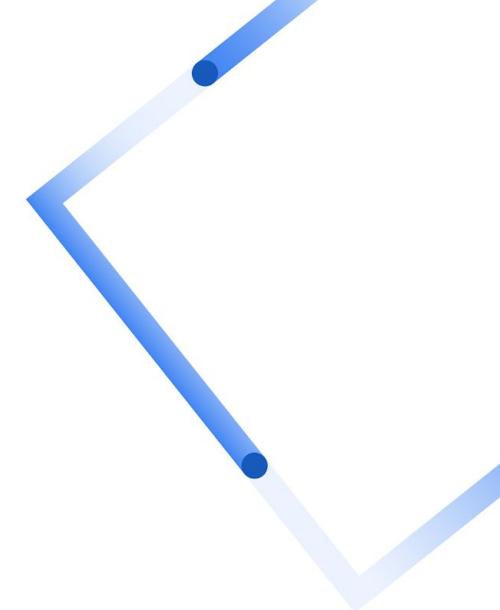
Why does this help?

Mostly an active area of research

One view: sample-dependent regularisation

03

# Extreme classification



# Supervised learning in theory

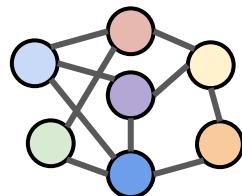
Google Research

Training data



$$\{(x_n, y_n)\}_{n=1}^N$$

Model training



$$\min_{f \in \mathcal{F}} \frac{1}{N} \sum_{n \in N} \ell(y_n, f(x_n))$$

Model predictions



0.2	0.1	0.4	0.2	0.1
-----	-----	-----	-----	-----

$$f(x^*)$$

# Supervised learning in practice

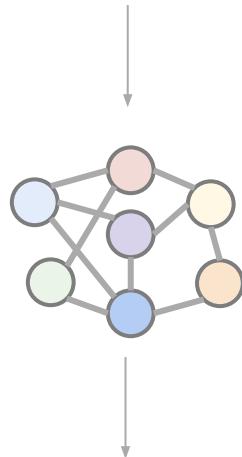
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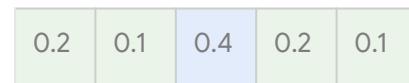
Model training



$$\min_{f \in \mathcal{F}} \frac{1}{N} \sum_{n \in N} \ell(y_n, f(x_n))$$

What if this loss  
is **expensive** to  
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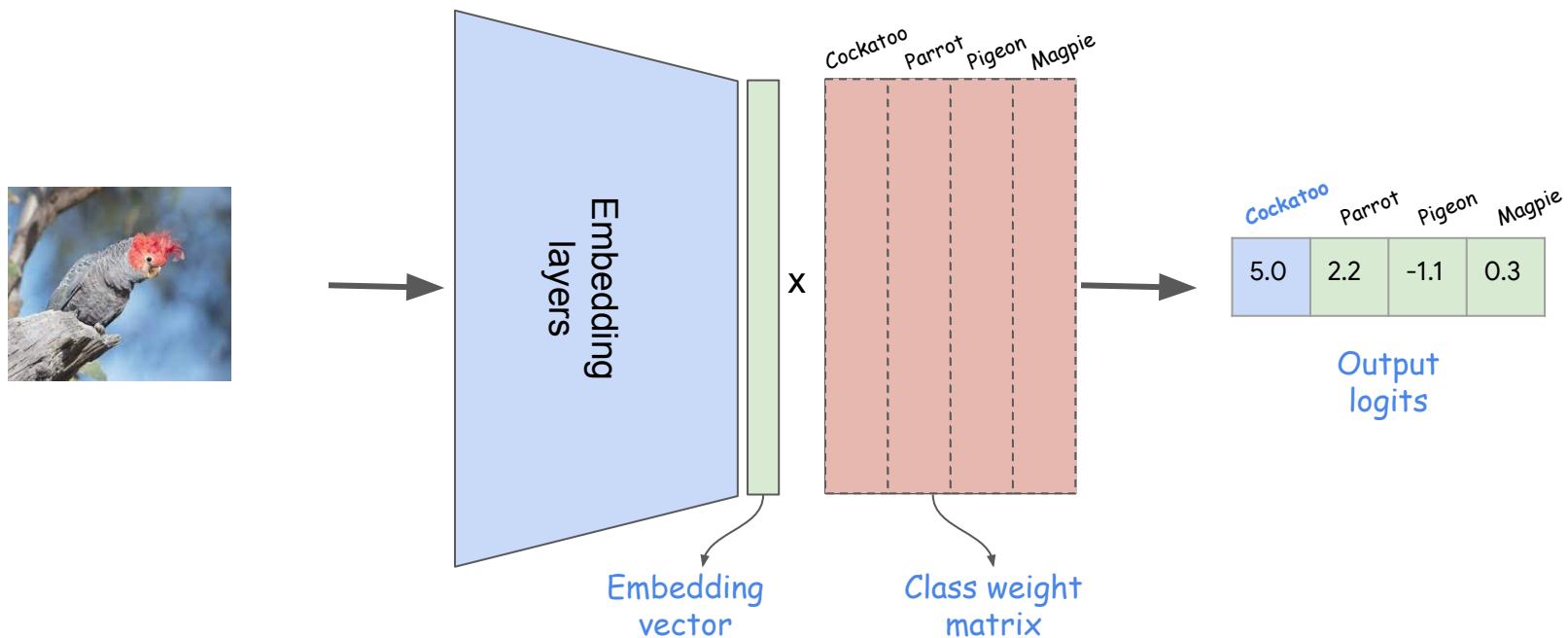
Model predictions



$$f(x^*)$$

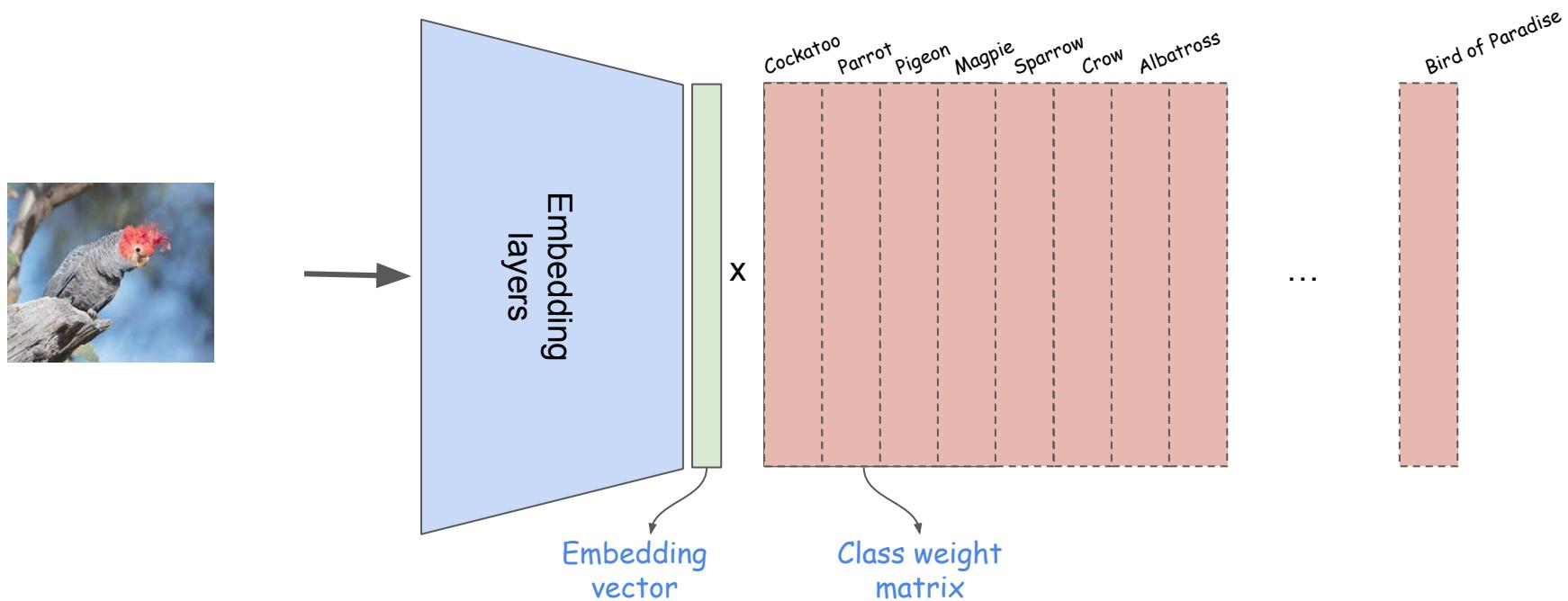
# Neural networks for classification

Google Research



# Neural networks for **extreme** classification

Google Research



# Neural networks for **extreme** classification

Google Research

Training objective: minimise **softmax cross-entropy**

$$\log \sum \exp \left[ \begin{array}{c} \text{Cockatoo} \\ \text{Parrot} \\ \text{Pigeon} \\ \text{Magpie} \\ \text{Sparrow} \\ \text{Crow} \\ \text{Albatross} \\ \text{Bird of Paradise} \end{array} \right] - \boxed{5.0}$$

The diagram shows a softmax cross-entropy calculation. On the left, there is a mathematical expression:  $\log \sum \exp$ . To its right is a horizontal vector of scores for eight bird species: Cockatoo (5.0), Parrot (2.2), Pigeon (-1.1), Magpie (0.3), Sparrow (0.1), Crow (-4.8), Albatross (-1.9), and Bird of Paradise (...). The last score is followed by three dots, indicating more items. To the right of the vector is a minus sign (-) and a blue box containing the value 5.0.

**Hard to compute** even for a single sample!

# Negative sampling

Google Research

Select a subset of “**negative**” labels to contrast against “**positive**”

“Positive” label



“Negative” labels

# Negative sampling

Google Research

Select a subset of “**negative**” labels to contrast against “**positive**”

“Positive” label



“Negative” labels

Ideally, we would like the sampling to:

- Be **easy** to compute
- Result in **informative** negatives

# Choosing the sampling distribution

Google Research

## Solution #1: within-batch negatives

"Positive" label  
 $(x_1, )$   
  
 $(x_2, )$   
  
 $(x_3, )$   


"Negative" labels



-  Easy to compute
-  Biased towards frequent labels

# Choosing the sampling distribution

Google Research

## Solution #2: uniform random negatives

"Positive" label  
 $(x_1, )$   
  
 $(x_2, )$   
  
 $(x_3, )$   


### "Negative" labels



Easy to compute



Not biased towards any label



May not be informative

# Choosing the sampling distribution

Google Research

## Solution #3: hard negative mining

"Positive" label  
 $(x_1,$   )

"Negative" labels



Maximally informative

$(x_2,$   )



Hard to compute

$(x_3,$   )



# Finding hard-negatives

Google Research

Ideally, find labels that are **maximally confusing** for model



this set changes as training progresses

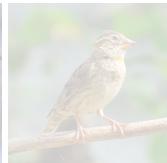


finding these exactly still requires sweeping over all labels!



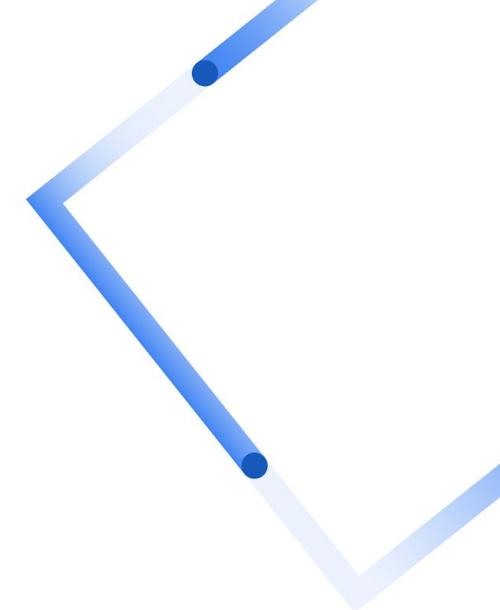
can **approximate**: find hardest labels **within a large batch** of uniformly sampled labels

$(x_1,$



04

# Model churn



# Supervised learning in theory

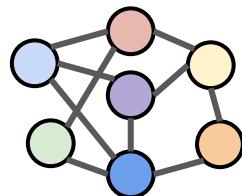
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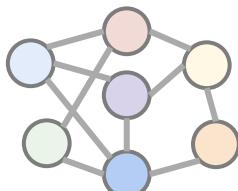
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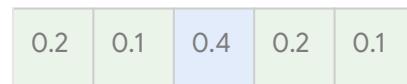
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Model predictions

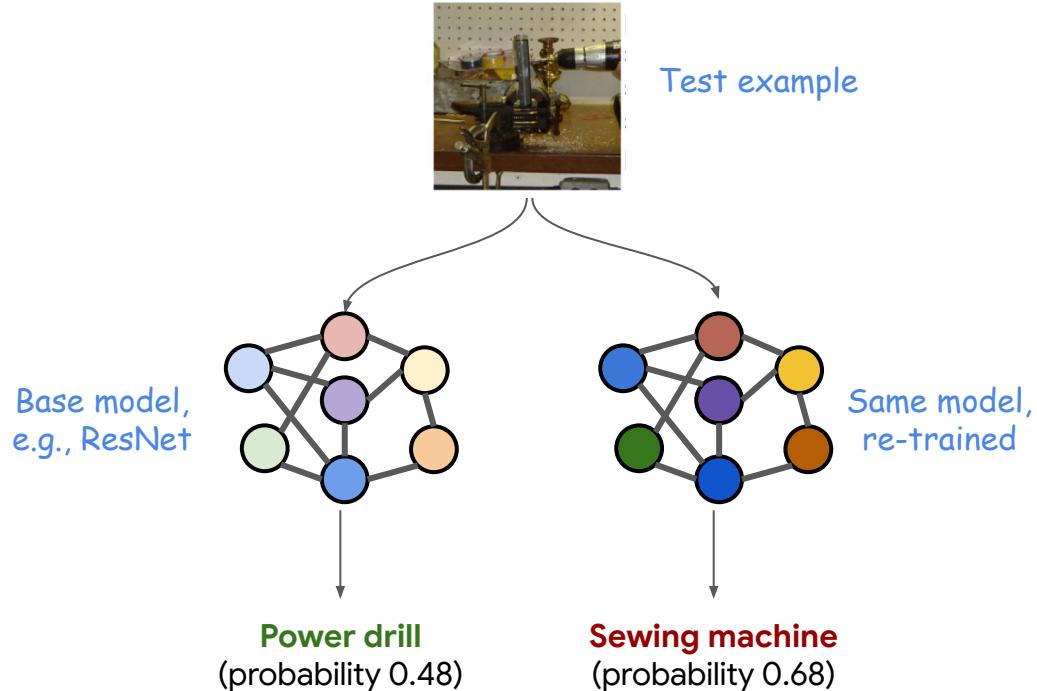


$$f(x^*)$$

# Churn in a nutshell

Google Research

Model prediction disagreement under different training and/or inference conditions



# Churn for classification

Google Research

Suppose we have two classification models,  $M_1$  and  $M_2$   
e.g., two independently trained models on the same data

The corresponding churn is the probability of disagreement:

$$\text{Churn}(M_1, M_2) = \Pr(M_1(x) \neq M_2(x))$$

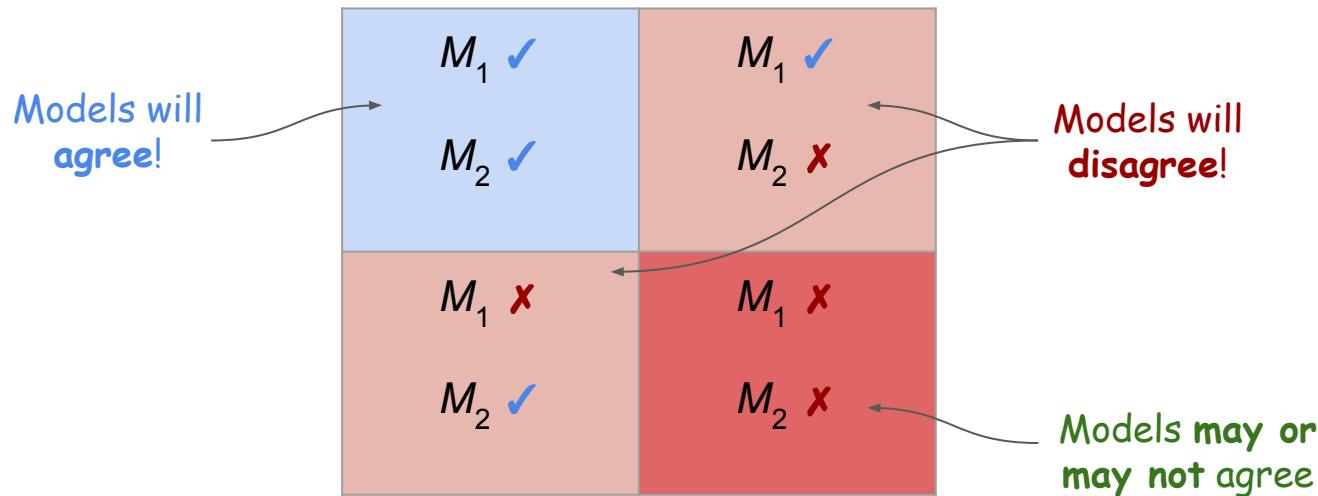
Fraction of times  
they predict a  
different label

# Churn versus accuracy

Google Research

Churn can only occur when one or both models is wrong

The better the individual models, the lower the churn

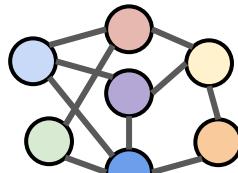


# Churn versus accuracy variation

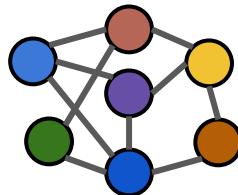
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Churn ≠ variation in accuracy

Models may  
disagree here!



60% accuracy



60% accuracy



✓



✓



✓



✗



✗



✗



✗



✓



✓

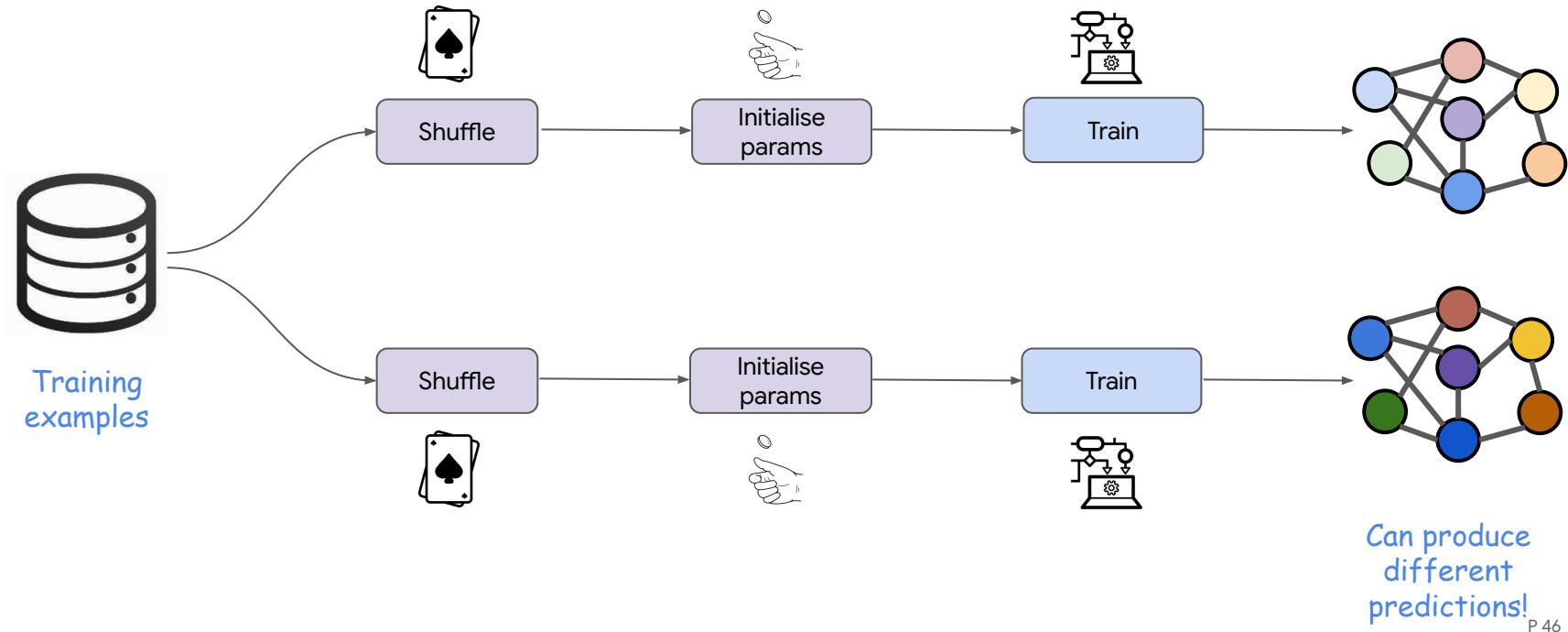


✓

# Churn from model training

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Churn exists even when training on the **same** data, due to several sources of randomness:



# Churn from computing platform

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Inherent non-determinism in GPU and TPU

Floating-point addition is not associative!



```
[1] (0.1+0.2)+0.3  
0.6000000000000001  
  
[2] 0.1+(0.2+0.3)  
0.6
```

# Do neural models exhibit churn?

Google Research

Unfortunately, **yes**

Predictions from 5x independently trained ResNet models on ImageNet

76.0% accuracy with 0.1% standard deviation

Disagreement on **15%** of examples!



**power drill** - 0.48  
sewing machine - 0.68  
sewing machine - 0.28  
sewing machine - 0.53  
**power drill** - 0.87



**wooden spoon** - 0.24  
spaghetti squash - 0.71  
**French loaf** - 0.67  
French loaf - 0.57  
French loaf - 0.63



**swing** - 0.82  
**lawn mower** - 0.56  
tricycle - 0.49  
balance beam - 0.75  
lawn mower - 0.45



**fountain pen** - 0.46  
can opener - 0.28  
crossword - 0.62  
hammer - 0.22  
crossword - 0.5

How do we mitigate such prediction differences?

# Co-distillation

Google Research

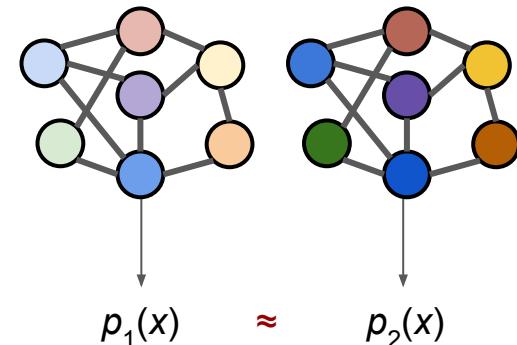
**Motivation:** churn is partly a result of randomness in training

**Idea:** explicitly try to smooth out this randomness!

**Approach:** train two independent models, and encourage their predictions to be similar to each other

Can be seen as “co-distillation”

**Bonus:** also improves performance!



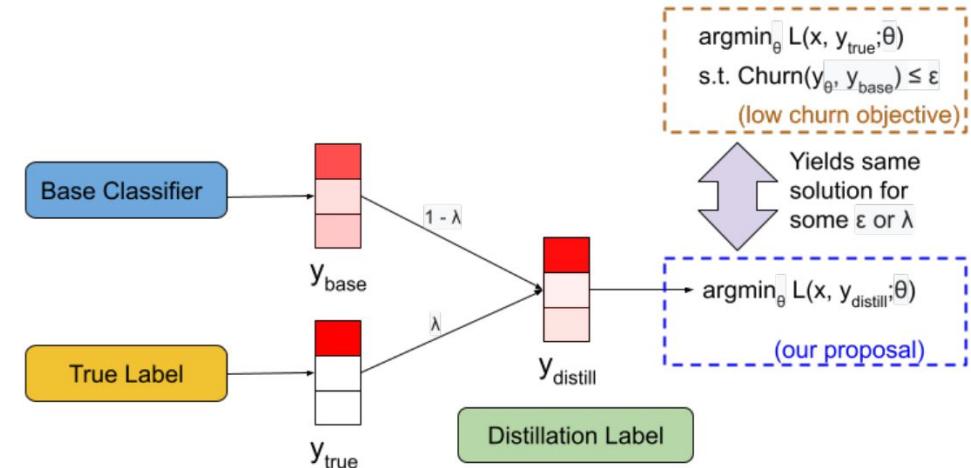
# Distillation for churn

Google Research

Churn can also occur more generally between model versions  
e.g., models trained on different weeks, with different architectures, ...

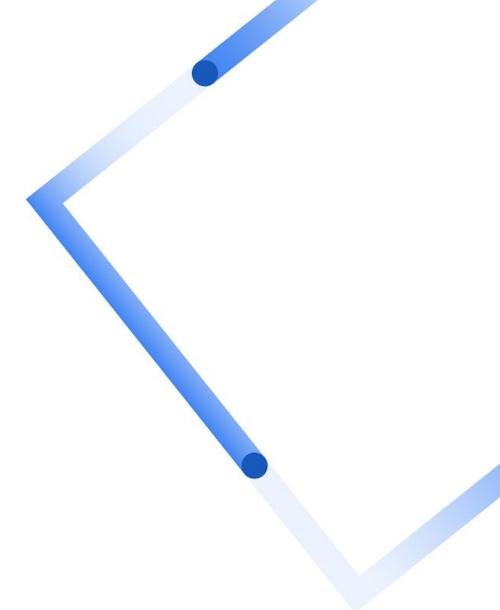
Idea: constrain predictions to be similar to original model

Implementation: distillation!



05

# Summary



# Supervised learning in practice!

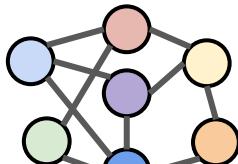
Google Research

Training data



$$\{(x_n, y_n)\}_{n=1}^N$$

Model training



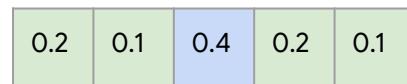
$$\min_{f \in \mathcal{F}} \frac{1}{N} \sum_{n \in N} \ell(y_n, f(x_n))$$

Churn  
reduction

Negative  
mining

Distilled  
label

Model predictions



$$f(x^*)$$

# Thank you!

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# Acknowledgements

Google Research

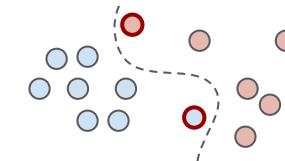
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# Entropy regularisers

Google Research

**Motivation:** churn occurs when samples' labels flip

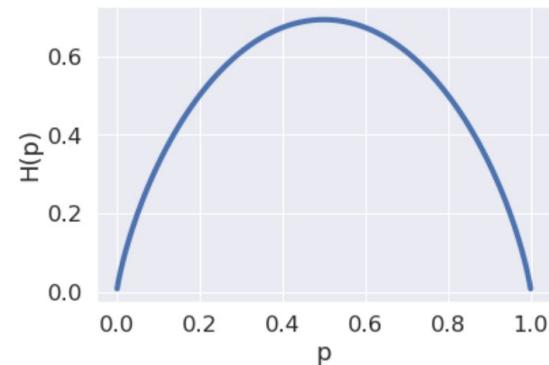


**Idea:** move examples away from the classifier boundary!

**Approach:** reduce prediction entropy: for logits  $p$ , penalise

$$H(p) = -\sum_i p_i \log p_i$$

discourage highly uncertain predictions



# Churn from data changes

Google Research

Refreshes of the data can change the learned model

