Calculo I

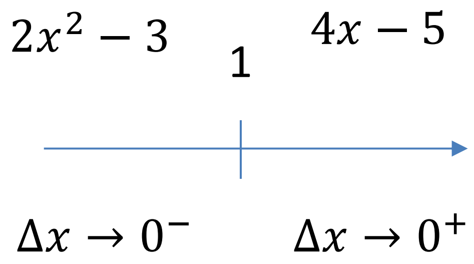
Revisão AV2

Prof. Pablo Vargas

$$f'_+(x_1) = \lim_{\Delta x \to 0^+} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

$$f'_-(x_1) = \lim_{\Delta x \to 0^-} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

Revisão AV2



1. Sendo f a função definida por partes e dada por $f(x) = \begin{cases} 2x^2 - 3 & se\ x < 1 \\ 4x - 5 & se\ x \geq 1 \end{cases}$, verifique se existe $f'(1)$.

$$f'_+(1) = \lim_{\Delta x \to 0^+} \frac{f(1 + \Delta x) - f(1)}{\Delta x}$$

$$f(1) = 4.1 - 5 = -1$$

$$f(1 + \Delta x) = 4.(1 + \Delta x) - 5 = 4 + 4\,\Delta x - 5 = 4\Delta x - 1$$

$$f'_+(1) = \lim_{\Delta x \to 0^+} \frac{4\Delta x - 1 - (-1)}{\Delta x} = \lim_{\Delta x \to 0^+} \frac{4\cancel{\Delta x}}{\cancel{\Delta x}} = \lim_{\Delta x \to 0^+} 4 = 4$$

$$f'_{+}(x_1) = \lim_{\Delta x \to 0^+} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

$$f'_{-}(x_1) = \lim_{\Delta x \to 0^-} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

Revisão AV2

$2x^2 - 3$ 1 $4x - 5$

$\Delta x \to 0^-$ $\Delta x \to 0^+$

1. Sendo f a função definida por partes e dada por $f(x) = \begin{cases} 2x^2 - 3 & se\ x < 1 \\ 4x - 5 & se\ x \geq 1 \end{cases}$, verifique se existe $f'(1)$.

$$f'_{-}(1) = \lim_{\Delta x \to 0^-} \frac{f(1 + \Delta x) - f(1)}{\Delta x}$$

$$f(1) = 4.1 - 5 = -1$$

$$f(1 + \Delta x) = 2(1 + \Delta x)^2 - 3 = 2.(1 + 2\Delta x + \Delta x^2) - 3$$

$$= 2 + 4\Delta x + 2\Delta x^2 - 3$$

$$= 2\Delta x^2 + 4\Delta x - 1$$

$$f'_{-}(1) = \lim_{\Delta x \to 0^-} \frac{2\Delta x^2 + 4\Delta x - 1 - (-1)}{\Delta x} = \lim_{\Delta x \to 0^-} \frac{2\Delta x^{\cancelto{1}{2}} + 4\cancel{\Delta x}}{\cancel{\Delta x}}$$

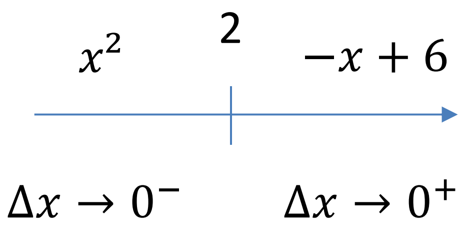
$$= \lim_{\Delta x \to 0^-} 2\cancelto{0}{\Delta x} + 4 = \lim_{\Delta x \to 0^-} 4 = 4$$

$$f'_{+}(1) = f'_{-}(1) = 4 \ \therefore portanto, f'(1) existe\ e\ \text{é}\ 4$$

$$f'_+(x_1) = \lim_{\Delta x \to 0^+} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

$$f'_-(x_1) = \lim_{\Delta x \to 0^-} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

Revisão AV2



1. Sendo f a função definida por partes e dada por $f(x) = \begin{cases} x^2 & se\ x < 2 \\ -x + 6 & se\ x \geq 2 \end{cases}$, verifique se existe $f'(2)$.

$$f'_-(2) = \lim_{\Delta x \to 0^-} \frac{f(2 + \Delta x) - f(2)}{\Delta x}$$

$$f(2) = -x + 6 = -2 + 6 = 4$$

$$f(2 + \Delta x) = x^2 = (2 + \Delta x)^2 = \Delta x^2 + 4\Delta x + 4$$

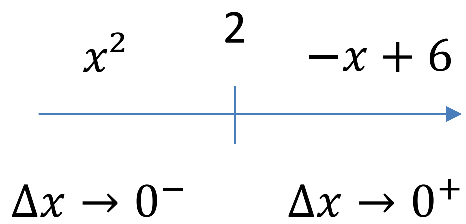
$$f'_-(2) = \lim_{\Delta x \to 0^-} \frac{\Delta x^2 + 4\Delta x + \cancel{4} \cancel{- 4}}{\Delta x}$$

$$= \lim_{\Delta x \to 0^-} \frac{\Delta x^{\cancel{2}} + 4\cancel{\Delta x}}{\cancel{\Delta x}} = \lim_{\Delta x \to 0^-} \overset{0}{\cancel{\Delta x}} + 4 = 4$$

$$f'_+(x_1) = \lim_{\Delta x \to 0^+} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

$$f'_-(x_1) = \lim_{\Delta x \to 0^-} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

Revisão AV2



1. Sendo f a função definida por partes e dada por $f(x) = \begin{cases} x^2 & se\ x < 2 \\ -x+6 & se\ x \geq 2 \end{cases}$, verifique se existe $f'(2)$.

$$f'_+(2) = \lim_{\Delta x \to 0^+} \frac{f(2+\Delta x) - f(2)}{\Delta x}$$

$$f(2) = -x + 6 = -2 + 6 = 4$$

$$f(2+\Delta x) = -(2+\Delta x) + 6 = -\Delta x + 4$$

$$f'_+(x_1)$$

$$= \lim_{\Delta x \to 0^+} \frac{-\Delta x + 4\ - 4}{\Delta x} = \lim_{\Delta x \to 0^+} \frac{-\Delta x}{\Delta x} = \lim_{\Delta x \to 0^+} -1 = -1$$

$$f'_+(2) \neq f'_-(2), f'(2) = não\ existe$$

Potência: $(x^n)' = n.x^{n-1}$
Soma/Subtração: $(f \pm g)' = f' \pm g'$
Produto por constante: $(Cf)' = C.f'$
Produto: $(fg)' = f'.g + f.g'$
Quociente: $\left(\frac{f}{g}\right)' = \frac{f'.g - f.g'}{g^2}$
Exponencial: $(e^x)' = e^x$

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2. Calcule y' sendo $y = \overset{f}{(x^3 - 2x)}\overset{g}{(x^2 - 1)}$

$(fg)' = f'.g + f.g'$

$f(x) = (x^3 - 2x) \therefore f'(x) = 3x^2 - 2$

$g(x) = (x^2 - 1) \therefore g'(x) = 2x$

$(fg)' = (3x^2 - 2).(x^2 - 1) + (x^3 - 2x).2x$

Potência: $(x^n)' = n.x^{n-1}$
Soma/Subtração: $(f \pm g)' = f' \pm g'$
Produto por constante: $(Cf)' = C.f'$
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Quociente: $\left(\frac{f}{g}\right)' = \frac{f'.g - f.g'}{g^2}$
Exponencial: $(e^x)' = e^x$

Revisão AV2

2. Calcule y' sendo $y = \overset{f}{(x^3 - 2x)}\overset{g}{(x^2 - 1)}$

$$(fg)'$$
$$= (x^3 - 2x)'.(x^2 - 1) + (x^3 - 2x).(x^2 - 1)'$$
$$= [(x^3)' - (2x^1)'].(x^2 - 1) + (x^3 - 2x).[(x^2)' - 1']$$
$$= (3x^2 - 2).(x^2 - 1) + (x^3 - 2x)(2x - 0)$$
$$= 3x^4 - 3x^2 - 2x^2 + 2 + (2x^4 - 4x^2)$$
$$y' = 5x^4 - 9x^2 + 2$$

Potência: $(x^n)' = n.x^{n-1}$
Soma/Subtração: $(f \pm g)' = f' \pm g'$
Produto por constante: $(Cf)' = C.f'$
Produto: $(fg)' = f'.g + f.g'$
Quociente: $\left(\frac{f}{g}\right)' = \frac{f'.g - f.g'}{g^2}$
Exponencial: $(e^x)' = e^x$

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$

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2. Calcule y' sendo $y = (x^{10} + 2)^{20}$

$u = x^{10} + 2 \therefore y = u^{20}$

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = \frac{d(u^{20})}{du}.\frac{d(x^{10}+2)}{dx} =$$

$20.u^{19}.(10x^9) = 20.(x^{10} + 2)^{19}.(10x^9) =$

$200x^9.(x^{10} + 2)^{19}$

Potência: $(x^n)' = n.x^{n-1}$
Soma/Subtração: $(f \pm g)' = f' \pm g'$
Produto por constante: $(Cf)' = C.f'$
Produto: $(fg)' = f'.g + f.g'$
Quociente: $\left(\frac{f}{g}\right)' = \frac{f'.g - f.g'}{g^2}$
Exponencial: $(e^x)' = e^x$

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$

Revisão AV2

3. *Calcule* $\frac{dy}{dx}$ *sendo* $x^3 + y^3 = 6xy$

$(x^3)' = 3x^2$

$(y^3)' = 3y^2.y'$

$f = 6x \text{ e } g = y$

$(6xy)' = (fg)' = (f'.g + f.g') = 6y + 6xy'$

$$3x^2 + 3y^2.y' = 6y + 6xy'$$

$$3y^2.y' + 6xy' = 6y - 3x^2$$

$$y'.(3y^2 + 6x) = 6y - 3x^2$$

$$y' = \frac{dy}{dx} = \frac{6y - 3x^2}{(3y^2 + 6x)}$$

$\int_b^a f(x).dx = -\int_a^b f(x).dx$
$\int_a^a f(x).dx = 0$
$\int_a^b kf(x).dx = \mathrm{k}\int_a^b f(x).dx$

$F'(x) = \frac{d}{dx}\int_a^x f(t).dt = f(x)$

Revisão AV2

4. *Determine* $\frac{dy}{dx}$ *se* $y = \int_{sen\,x}^{0} \frac{t}{\pi}.dt$

$\int_b^a f(x).dx = -\int_a^b f(x).dx$

$\int_a^a f(x).dx = 0$

$\int_a^b kf(x).dx = \mathrm{k}\int_a^b f(x).dx$

Revisão AV2

$F'(x) = \frac{d}{dx}\int_a^x f(t).dt = f(x)$

4. Determine $\frac{dy}{dx}$ se $y = \int_{sen\,x}^{0} \frac{t}{\pi}.dt$

$$y' = \frac{dy}{dx} = \frac{d}{dx}\int_{sen\,x}^{0} \frac{t}{\pi}.dt = \frac{1}{\pi}\frac{d}{dx}\int_{sen\,x}^{0} t.dt$$

$$= -\frac{1}{\pi}.\frac{d}{dx}\int_0^{sen\,x} t.dt = -\frac{1}{\pi}.\left(\frac{d}{du}\int_0^{u} t.dt\right).u'$$

$u = sen\,x$

$$-\frac{1}{\pi}.(u).\cos x = -\frac{1}{\pi}.sen\,x.cosx$$

I. $\int k.dx = kx + C$

II. $\int x^n.dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$

III. $\int \frac{1}{x}.dx = \ln|x| + C$

IV. $\int e^x.dx = e^x + \mathrm{C}$

V. $\int a^x.dx = \frac{a^x}{\ln} + \mathrm{C}$

$$\int f(g(x)).g'(x).dx = \int f(u).du$$

Revisão AV2

5. *Determine* $\int 2x(x^2+1)^3.dx$

I. $\int k.dx = kx + C$

II. $\int x^n.dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$

III. $\int \frac{1}{x}.dx = \ln|x| + C$

IV. $\int e^x.dx = e^x + \mathrm{C}$

V. $\int a^x.dx = \frac{a^x}{\ln a} + \mathrm{C}$

$$\int f(g(x)).g'(x).dx = \int f(u).du$$

Revisão AV2

5. *Determine* $\int 2x(\underbrace{x^2+1}_{u})^3.dx$

$$u = x^2 + 1 \ \therefore \ \frac{du}{dx} = 2x \rightarrow du = 2x.dx$$

$$\int du\,.u^3 = \int \underbrace{u^{3+1}}_{3+1}.du = \frac{u^4}{4} = \frac{(x^2+1)^4}{4} + C$$

Revisão AV2

I. $\int k.dx = kx + C$

II. $\int x^n.dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$

III. $\int \frac{1}{x}.dx = \ln|x| + C$

IV. $\int e^x.dx = e^x + C$

V. $\int a^x.dx = \frac{a^x}{\ln} + C$

VI. $\int sen\ x.dx = -\cos x + C$

VII. $\int cos\ x.dx = sen\ x + C$

VIII. $\int sec^2 x.dx = tg\ x + C$

IX. $\int cossec^2 x.dx = -cotg\ x + C$

X. $\int tg\ x.dx = \ln|\sec x| + C$

u

5. *Determine* $\int x.sen\ (3x^2).dx$

$$u = 3x^2 \therefore \frac{du}{dx} = 6x \rightarrow du = 6x.dx \rightarrow dx = \frac{du}{6x}$$

$$\int \cancel{x}.sen\ u.\frac{du}{6\cancel{x}} = \int sen\ u.\frac{du}{6} = \frac{1}{6}\int sen\ u.du$$

$$= \frac{1}{6}(-\cos u) = -\frac{1}{6}\cos 3x^2 + C$$

Revisão AV2

I. $\int k.dx = kx + C$

II. $\int x^n.dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$

III. $\int \frac{1}{x}.dx = \ln|x| + C$

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X. $\int tg\ x.dx = \ln|\sec x| + C$

5. *Determine* $\int x^2.\cos x.dx$

$$\int f(x).g(x).dx = \int u.dv = u.v - \int v.du$$

I. $\int k.dx = kx + C$

II. $\int x^n.dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$

III. $\int \frac{1}{x}.dx = \ln|x| + C$

IV. $\int e^x.dx = e^x + C$

V. $\int a^x.dx = \frac{a^x}{\ln} + C$

Revisão AV2

VI. $\int sen\,x.dx = -\cos x + C$

VII. $\int cos\,x.dx = sen\,x + C$

VIII. $\int sec^2x.dx = tg\,x + C$

IX. $\int cossec^2x.dx = -cotg\,x + C$

X. $\int tg\,x.dx = \ln|\sec x| + C$

u dv

5. *Determine* $\int x^2.\cos x.dx$

$$\int f(x).g(x).dx = \int u.dv = u.v - \int v.du$$

$$\int x^2.\cos x\,.dx = \int u.dv = u.v - \int v.du$$

$$u = x^2 \therefore \frac{du}{dx} = 2x \rightarrow du = 2x.dx$$

$$dv = \cos x.dx \therefore v = \int \cos x.dx = sen\,x$$

$$\int x^2.\cos x.dx = x^2.sen\,x - \int sen\,x\,.2x.dx$$

$$\int x^2.\cos x.dx = x^2.sen\,x - 2\int x.sen\,x\,.dx$$

u dv

Revisão AV2

I. $\int k.dx = kx + C$

II. $\int x^n.dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$

III. $\int \frac{1}{x}.dx = \ln|x| + C$

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V. $\int a^x.dx = \frac{a^x}{\ln} + C$

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VII. $\int \underline{cos\ x.dx = sen\ x} + C$

VIII. $\int sec^2 x.dx = tg\ x + C$

IX. $\int cossec^2 x.dx = -cotg\ x + C$

X. $\int tg\ x.dx = \ln|\sec x| + C$

f(x)

g(x)

5. Determine $\int x^2.\cos x.dx$

$$\int f(x).g(x).dx = \int u.dv = u.v - \int v.du$$

$$\int x.sen\ x\ .dx$$

$$u = x \ \therefore \frac{du}{dx} = 1 \rightarrow du = dx$$

$$dv = sen\ x.dx \ \therefore v = \int sen\ x.dx = -\cos x$$

$$\int x.sen\ x.dx = \overset{u}{x}.(\overset{v}{-\cos x}) - \int (\overset{v}{-\cos x}).\overset{du}{dx} = -x.\cos x + \int \cos x.dx$$

$$= \underline{-x.\cos x + sen\ x}$$

I. $\int k.dx = kx + C$

II. $\int x^n.dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$

III. $\int \frac{1}{x}.dx = \ln|x| + C$

IV. $\int e^x.dx = e^x + C$

V. $\int a^x.dx = \frac{a^x}{\ln} + C$

Revisão AV2

VI. $\int sen\, x.dx = -\cos x + C$

VII. $\int cos\, x.dx = sen\, x + C$

VIII. $\int sec^2 x.dx = tg\, x + C$

IX. $\int cossec^2 x.dx = -cotg\, x + C$

X. $\int tg\, x.dx = \ln|\sec x| + C$

f(x) g(x)

5. *Determine* $\int x^2.\cos x.dx$

$$\int x^2.\cos x.dx = x^2.sen\, x - 2(-x.\cos x + sen\, x)$$

$$\int x^2.\cos x.dx = x^2.sen\, x + 2x.\cos x - 2sen\, x + C$$

I. $\int k.dx = kx + C$

II. $\int x^n.dx = \frac{x^{n+1}}{n+1} + C \; (n \neq -1)$

III. $\int \frac{1}{x}.dx = \ln|x| + C$

IV. $\int e^x.dx = e^x + \mathrm{C}$

V. $\int a^x.dx = \frac{a^x}{\ln a} + \mathrm{C}$

Revisão AV2

$$\int_a^{+\infty} f(x).dx = \lim_{b \to +\infty} \int_a^b f(x).dx$$

$$\int_{-\infty}^b f(x).dx = \lim_{a \to -\infty} \int_a^b f(x).dx$$

$$\int_{-\infty}^{+\infty} f(x).dx = \lim_{a \to -\infty} \int_a^0 f(x).dx + \lim_{b \to +\infty} \int_0^b f(x).dx$$

6. *Calcule* $\int_{-\infty}^{0} x^3.dx$

I. $\int k.dx = kx + C$

II. $\int x^n.dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$

III. $\int \frac{1}{x}.dx = \ln|x| + C$

IV. $\int e^x.dx = e^x + \text{C}$

V. $\int a^x.dx = \frac{a^x}{\ln} + \text{C}$

Revisão AV2

$$\int_a^{+\infty} f(x).dx = \lim_{b\to+\infty}\int_a^b f(x).dx$$

$$\int_{-\infty}^{b} f(x).dx = \lim_{a\to-\infty}\int_a^b f(x).dx$$

$$\int_{-\infty}^{+\infty} f(x).dx = \lim_{a\to-\infty}\int_a^0 f(x).dx + \lim_{b\to+\infty}\int_0^b f(x).dx$$

6. *Calcule* $\int_{-\infty}^{0} x^3.dx$

$$\int_{-\infty}^{b} x^3.dx = \lim_{a\to-\infty}\int_a^0 x^3.dx$$

$$= \lim_{a\to-\infty}\int_a^0 \underset{3+1}{\underline{x^{3}}}\overset{+1}{}.dx = \lim_{a\to-\infty}\frac{x^4}{4}\Big|_a^0$$

$$= \lim_{a\to-\infty}\left(0 - \frac{a^4}{4}\right) = -\lim_{a\to-\infty}\left(\frac{a^4}{4}\right) = -\frac{(-\infty)^4}{4}$$

$$= -\infty$$

$$\int \frac{1}{x^2+1}.dx = arctg\, x + C = tg^{-1}\, x + C$$

Revisão AV2

$$\int_a^{+\infty} f(x).dx = \lim_{b\to+\infty} \int_a^b f(x).dx$$

$$\int_{-\infty}^{b} f(x).dx = \lim_{a\to-\infty} \int_a^b f(x).dx$$

$$\int_{-\infty}^{+\infty} f(x).dx = \lim_{a\to-\infty} \int_a^0 f(x).dx + \lim_{b\to+\infty} \int_0^b f(x).dx$$

$\frac{1}{1+x^2}.dx$

6. *Calcule* $\int_{-\infty}^{0} \frac{dx}{1+x^2}$

$$\int_{-\infty}^{0} \frac{1}{1+x^2}.dx = \lim_{a\to-\infty} \int_a^0 \frac{dx}{1+x^2} = \lim_{a\to-\infty} tg^{-1}\, x\Big|_a^0$$

$$= \lim_{a\to-\infty} (\cancel{tg^{-1}\, 0} - tg^{-1} a) = 0 - \lim_{a\to-\infty} tg^{-1} a$$

$$= 0 - \left(-\frac{\pi}{2}\right) = \frac{\pi}{2}$$