Lista 3 Álgebra Linear

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Curso: Ciência da Computação

/ / $\vec{A}$

1- Explique por que cada uma das seguintes regras algébricas não funciona em geral quando os números reais a e b são substituídos por matrizes n×n, A e B. Use o exemplo para deduzir as equações

$\vec{A}$

a) $(a+b)^2 = a^2 + 2.a.b + b^2$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$ $\quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$A+B = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix} \Rightarrow (A+B)^2 = \begin{bmatrix} 115 & 144 \\ 180 & 224 \end{bmatrix}$

$a^2 + 2.a.b + B \cdot B = \begin{bmatrix} 112 & 132 \\ 192 & 228 \end{bmatrix}$ diferente

A e B sendo matrizes

$(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = a^2 + \vec{a}\vec{b} + \vec{b}\vec{a} + b^2$
$= a^2 + \vec{a}\vec{b} + \vec{b}.\vec{a} + b^2$

$\vec{a}.\vec{b} \neq \vec{b}.\vec{a}$ que não são iguais, logo $a^2 + 2a.b + b^2$ é errado

b) $(a+b)(a-b) = a^2 - b^2$

provando matematicamente

$a^2 - \vec{a}\vec{b} + \vec{b}\vec{a} - b^2 \neq a^2 - b^2$

mesmo princípio, a multiplicação não é comutativa

Credeal

2- Encontre a matriz inversa para cada uma das ~~resultados~~ matrizes

a) $\begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 7.a + 2c & 7b + 2d \\ 3.a + c & 3b + d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\qquad \begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix}$ //

$\begin{cases} 7.a + 2c = 1 \\ 3a + c = 0 \end{cases}$

$c = -3a$

$7a + 2.(-3a) = 1$

$7a - 6a = 1$

$a = 1$

$-3.1 - c = 0 \quad (-1)$

$-3 = c$

$\begin{cases} 7b + 2d = 0 \\ 3b + d = 1 \end{cases} \rightarrow 7b + 2(1 - 3.b) = 0$

$7b + 2 - 5b = 0$

$1b = -2$

$d = 1 - 3b \qquad b = -2$

$d = 1 - 3.(-2)$

$d = 1 + 6 = 7$

b) $\begin{pmatrix} 3 & 5 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 3a + 5.c & 3b + 5d \\ 2a + 3c & 2b + 3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 2 & -3 \end{bmatrix}$

$3a + 5c = 1$

$2a + 3c = 0$

$(-2) \; -6a - 10c = -2$

$(3) \; 6a + 9c = 0$

$0 - 1c = -2$

$c = 2$

$2.a + 3.2 = 0$

$2a + 6 = 0$

$2a = -5$

$a = -3$

$\begin{cases} 3b + 5d = 0 \\ 2b + 3d = 1 \end{cases}$

$(-2) \; -6b - 10d = 0$

$(3) \; 6b + 9d = 3$

$0 - 1d = 3$

$d = -3$

$2b + 3.(-3) = 1$

$2b - 9 = 1$

$2b = 10$

$b = 5$

Credeal

$$\begin{bmatrix} 1 & -1.5 \\ -1 & 2 \end{bmatrix}$$

c) $\begin{pmatrix} 4 & 3 \\ 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 4a+3c & 4b+3d \\ 2a+2c & 2b+2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$4a+3c=1 \quad (-2)$
$2a+2c=0 \quad (3)$

$-8a-6c=-2$
$6a+6c=0$
$-2a+0=-2$
$a=1$

$2.1+2c=0$
$2c=-2$
$c=-1$

$4b+3d=0 \quad (-2)$
$2b+2d=1 \quad (3)$

$-8b-6d=0$
$6b+6d=3$
$-2b=3$
$b=\frac{-3}{2}$

$2.\left(\frac{-3}{2}\right)+2d=1$
$d=2$

4 - Quais das matrizes são elementares? Classifique cada matriz elementar, em tipo

a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Tipo I

b) $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$
Tipo II

c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{pmatrix}$
Tipo III

d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Tipo II

Credeal

6 - Calcule a fatoração LU de cada uma das seguintes matrizes

a) $\begin{pmatrix} 3 & 1 \\ 9 & 5 \end{pmatrix}$ $\quad I_{21} = \dfrac{-9}{3_{a_{11}}} = -3 \Rightarrow L_2 = (9 \;\; 5)$

$$\begin{array}{rrr} & 9 & 5 \\ + & -9 & -3 \\ \hline & 0 & 2 \end{array}$$

$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} = U \quad L = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

↳ matriz triangular

$\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3+0 & 1+0 \\ -9+0 & -3+2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -9 & -1 \end{bmatrix}$

b) $\begin{pmatrix} 2 & 4 \\ -2 & 1 \end{pmatrix}$ $\quad I_{21} = \dfrac{a_{21}}{a_{11}} = -\dfrac{(-2)}{2} = 1$

$L = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 4 \\ 0 & 5 \end{bmatrix}$ $\quad L_2 = -2 \;\; 1$

$$\begin{array}{rrr} & -2 & 1 \\ + & 2 & 4 \\ \hline & 0 & 5 \end{array}$$

$\overset{L}{\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}} \cdot \overset{U}{\begin{bmatrix} 2 & 4 \\ 0 & 5 \end{bmatrix}} = \begin{bmatrix} 2+0 & 4+0 \\ 2+0 & 4+5 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 2 & 9 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 5 \\ -2 & 2 & 7 \end{bmatrix}$ $\quad I_{21} = -\dfrac{a_{21}}{a_{11}} = \dfrac{-3}{1} = -3 \qquad L_2 = 3 \;\; 5 \;\; 5$

$$\begin{array}{rrrr} & 3 & 5 & 5 \\ + & -3 & -3 & -3 \\ \hline & 0 & 2 & 3 \end{array}$$

$I_{31} = -\dfrac{a_{31}}{a_{11}} = \dfrac{-2}{1} = +2 \qquad L_3 = -2 \;\; 2 \;\; 7$

$$\begin{array}{rrrr} & -2 & 2 & 7 \\ + & 2 & 2 & 2 \\ \hline & 0 & 4 & 9 \end{array}$$

$I_{32} = -\dfrac{2}{2} = \dfrac{-2}{2} = +2$

$L_3 = 0 \;\; 4 \;\; 9$

$$\begin{array}{rrrr} & 0 & 4 & 9 \\ + & 0 & -4 & -18 \\ \hline & 0 & 0 & -9 \end{array}$$

$L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -9 \end{bmatrix}$

Credeal

$$LU = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 0 \\ 2 & -2 & 13 \end{bmatrix}$$

3- Uma matriz é dita idempotente se $A^2 = A$. Qual me satisf

a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ = é idempotente

b) $\begin{bmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$ = é idempotente

c) $\begin{bmatrix} 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$ = é idempotente

5- Para cada um dos pares de matrizes, ache a matriz elementar, tal que $E \times A = B$

a)

$A = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 2 \\ 5 & 3 \end{bmatrix}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 5 & 3 \end{bmatrix}$

$\begin{bmatrix} 2.a + 5b & -a + 3b \\ 2c + 5d & -c + 3d \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 5 & 3 \end{bmatrix}$ $\begin{bmatrix} -2 & 0 \\ 30 & -11 \end{bmatrix}$

$2.a + 5b = -4$

$-a + 3b = 2$

$a = -2 + 3b$

$2.(-2 + 3b) + 5b = -4$

$-4 + 5b + 5b = -4$

$-4 + 11b = -4$

$b = 0$

$a = -2$

$2c + 5d = 5$

$c = -3 - 3d$

$2.(-3 - 3d) + 5d = 5$

$-6 - 6d + 5d = 5$

$-6 - 1d = 5$

$d = \frac{11}{-1} = -11$

$c = -3 - 3(-11)$

$= -3 + 33 = 30$

Credeal

b) $A = \begin{bmatrix} 2 & 1 & 3 \\ -2 & 4 & 5 \\ 3 & 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ -2 & 4 & 5 \end{bmatrix}$ outra

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 3 \\ -2 & 4 & 5 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ -2 & 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

c) $A = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 0 & 2 \\ -2 & 3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 0 & 2 \\ 0 & 3 & 5 \end{bmatrix}$ outra

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 & 3 \\ 1 & 0 & 2 \\ -2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 0 & 2 \\ 0 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

7- Efetue a multiplicação de blocos de matrizes abaixo

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 2 & 1 & 2 & -1 \end{array} \right) \cdot \begin{bmatrix} 4 & -2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 2 \\ \hline 1 & 2 & 3 \end{bmatrix} \quad R = \begin{bmatrix} 6 & 0 & 1 \\ 11 & -1 & 4 \end{bmatrix}$$

$[A \;\; B]_{1\times 2} \cdot \begin{bmatrix} C \\ D \end{bmatrix}_{2\times 1}$ $\quad A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$[A \cdot C + B D]_{1\times 1}$

$C = \begin{bmatrix} 4 & -2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad D = [1 \;\; 2 \;\; 3]$

$$\begin{bmatrix} 7 & 2 & 4 \\ 12 & 1 & 7 \end{bmatrix} + \begin{bmatrix} -1 & -2 & -3 \\ -1 & -2 & -3 \end{bmatrix}$$

$B \cdot D = \begin{bmatrix} -1 & -2 & -3 \\ -1 & -2 & -3 \end{bmatrix} \quad A \cdot C = \begin{bmatrix} 7 & 2 & 4 \\ 12 & 1 & 7 \end{bmatrix}$

Credeal

8- Para cada um dos seguintes itens, calcule (I) $\det(A)$, (II) $\operatorname{adj}(A)$ e (III) A^{-1}

a) $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ $\det(A) = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} = -1 - 6 = -7$

$\operatorname{adj}(A) = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -3 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{-7} \cdot \begin{pmatrix} -1 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 1/7 & 2/7 \\ 3/7 & -\frac{1}{7} \end{pmatrix}$

c) $\begin{pmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ -2 & 2 & -1 \end{pmatrix} = C$

$\det(C) = \left|\begin{matrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ -2 & 2 & -1 \end{matrix}\right| \begin{matrix} 1 & 3 \\ 2 & 1 \\ -2 & 2 \end{matrix}$

$\det(C) = -1 - 6 + 4 - (-2 + 2 - 6) = -3 + 2 - 2 + 5 = 3$

$\operatorname{adj}(C) = \begin{vmatrix} \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ -2 & -1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ -2 & 2 \end{vmatrix} \\ \begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ -2 & 2 \end{vmatrix} \\ \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} \end{vmatrix}^T = \begin{vmatrix} -3 & 0 & 6 \\ -5 & 1 & 8 \\ 2 & -1 & -5 \end{vmatrix}^T$

$\begin{vmatrix} -3 & -5 & 2 \\ 0 & 1 & -1 \\ 6 & 8 & -5 \end{vmatrix}$

$A^{-1} = \frac{1}{3} \cdot \begin{vmatrix} -3 & -5 & 2 \\ 0 & 1 & -1 \\ 6 & 8 & -5 \end{vmatrix} = \begin{vmatrix} -1 & -5/3 & 2/3 \\ 0 & 1/3 & -1/3 \\ 2 & 8/3 & -5/3 \end{vmatrix}$

Credeal

9- Use a regra de Cramer

a) $x_1 + 2x_2 = 3$
$3x_1 - x_2 = 1$

$$\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = A$$

$$\det(A) = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}_{2\times 2} = -1 - 6 = -7$$

$$\det(A_1) = \begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} = -3 - 2 = -5$$

$$\det(A_2) = \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = 1 - 9 = -8$$

$$x_1 = \frac{A_1}{A} \qquad x_2 = \frac{A_2}{A}$$

$$x_1 = \frac{-5}{-7} = \frac{5}{7} \qquad x_2 = \frac{-8}{-7} = \frac{8}{7}$$

b) $2x_1 + 3x_2 = 2$
$3x_1 + 2x_2 = 5$

$$A = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}$$

$$\det(A) = 4 - 9 = -5$$

$$\det(A_1) = \begin{vmatrix} 2 & 3 \\ 5 & 2 \end{vmatrix} = 4 - 15 = -11$$

$$\det(A_2) = \begin{vmatrix} 2 & 2 \\ 3 & 5 \end{vmatrix} = 10 - 5 = 4$$

$$x_1 = \frac{-11}{-5} = \frac{11}{5}$$

$$x_2 = \frac{4}{-5} = \frac{-4}{5}$$

Credeal