Lista 3 Álgebra Linear

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Curso: Ciência da Computação

1- Explique por que cada uma das seguintes regras algébricas não funciona em geral quando os números reais a e b são substituídos por matrizes nxn, A e B. Use o caso para testar as equações

a) $(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \qquad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$A+B = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix} \Rightarrow (A+B)^2 = \begin{bmatrix} 115 & 144 \\ 180 & 224 \end{bmatrix}$

$a^2 + 2 \cdot a \cdot b + B \cdot B = \begin{bmatrix} 112 & 132 \\ 198 & 228 \end{bmatrix}$ ← diferente

A e B sendo matrizes

$(\vec{a}+\vec{b}) \cdot (\vec{a}+\vec{b}) = \vec{a}^2 + \vec{a}\vec{b} + \vec{b}\vec{a} + \vec{b}^2$

$= a^2 + \vec{a}\vec{b} + \vec{b} \cdot \vec{a} + b^2$

$\vec{a} \cdot \vec{b} \neq \vec{b} \cdot \vec{a}$ { qual não são iguais, logo $a^2 + 2ab + b^2$ é outro

b) $(a+b)(a-b) = a^2 - b^2$

provando matematicamente

$a^2 - \vec{a}\vec{b} + \vec{b}\vec{a} - b^2 \neq a^2 - b^2$

(mesmo princípio, a multiplicação não é comutativa

2- Encontre a matriz inversa para cada uma das matrizes

a) $\begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix}$

$\begin{bmatrix} 7.a+2c & 7b+2d \\ 3.a+c & 3b+d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$7.a+2c=1$ | $7b+2d=0$ → $7b+2(1-3b)=0$

$3a+c=0$ | $3b+d=1$ → $d=1-3b$

$7b+2-5b=0$

$2b=-2$

$b=-2$

$d=1-3.(-2)$

$d=1+5=7$

$c=-3a$

$7a+2.(-3a)=1$

$7a-6a=1$

$a=1$

$-3.1=-c=0 \quad (-1)$

$-3=C$

b) $\begin{pmatrix} 3 & 5 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 3a+5c & 3b+5d \\ 2a+3c & 2b+3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 2 & -3 \end{bmatrix}$

$3a+5c=1$ | $3b+5d=0$

$2a+3c=0$ | $2b+3d=1$

$(-2)\ -6a-10c=-2$ $\quad (-2)\cdot 6b-10d=0$

$(3)\ 6a+9c=0$ $\quad (3)\ 6b+9d=3$

$0-1c=-2$ $\quad 0-1d=3$

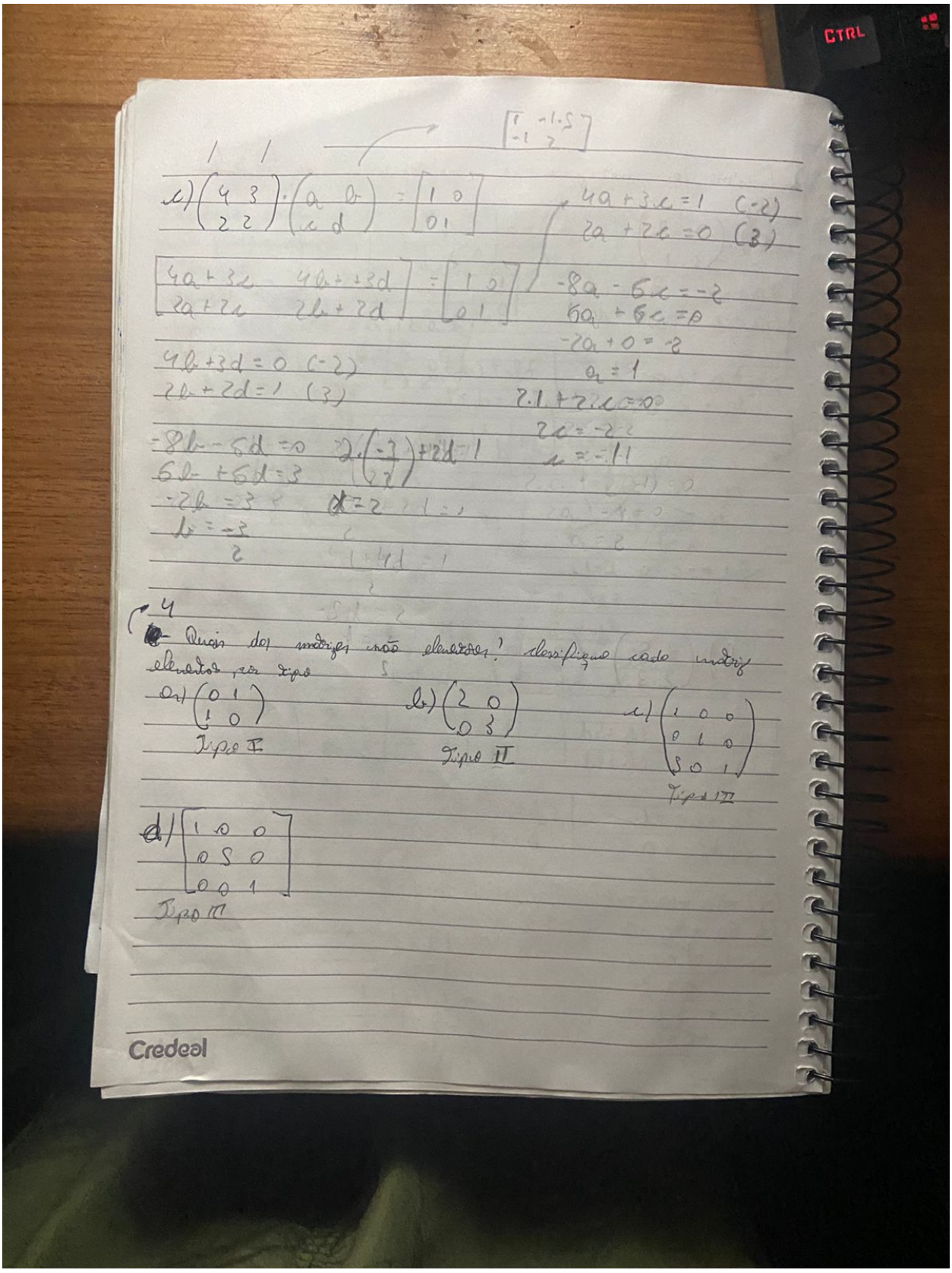
$c=2$ $\quad d=-3$

$2.a+3.2=0$ $\quad 2b+3.(-3)=1$

$2a+6=0$ $\quad 2b-9=1$

$2a=-5$ $\quad 2b=10$

$a=-3$ $\quad b=5$



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6- Calcule a fatoração LU de cada uma das seguintes matrizes

a) $\begin{pmatrix} 3 & 1 \\ 9 & 5 \end{pmatrix}$ $I_{11} = \frac{a_{21}}{a_{11}} = \frac{9}{3} = 3$ $L_2 = (9\ 5) + (-9\ -3) = (0\ 2)$

$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} = U \quad L = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

U matriz triangular

$\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3+0 & 1+0 \\ -9+0 & -3+2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -9 & -1 \end{bmatrix}$

b) $\begin{pmatrix} 2 & 4 \\ -2 & 1 \end{pmatrix}$ $I_{21} = \frac{a_{21}}{a_{11}} = \frac{-(-2)}{2} = 1$

$L = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 4 \\ 0 & 5 \end{bmatrix}$ $L_2 = (-2\ 1) + (2\ 4) = (0\ 5)$

L U

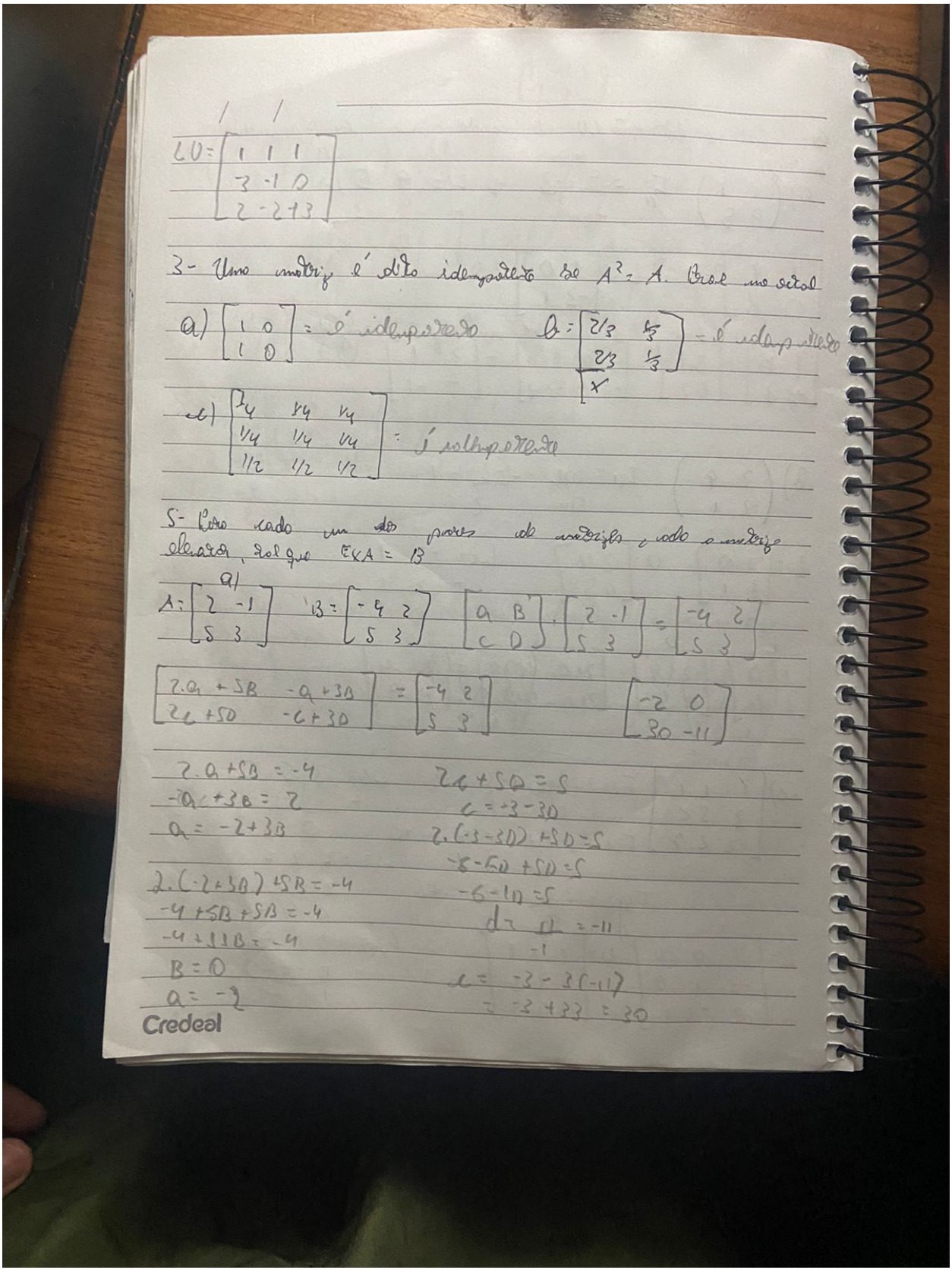
$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 2+0 & 4+0 \\ 2+0 & 4+5 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 2 & 9 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 6 \\ -2 & 2 & 7 \end{bmatrix}$ $I_{21} = \frac{-a_{21}}{a_{11}} = \frac{-3}{1} = -3$ $L_2 = (3\ 5\ 6) + (-3\ -3\ -3) = (0\ 2\ 3)$

$I_{31} = \frac{-a_{31}}{a_{11}} = \frac{-(-2)}{1} = +2$ $L_3 = (-2\ 2\ 7) + (2\ 2\ 2) = (0\ 4\ 9)$

$I_{32} = \frac{-2}{2} = -2 = +2$ $L_3 = (0\ 4\ 9) + (0\ -4\ -12) = (0\ 0\ -9)$

$L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -9 \end{bmatrix}$



$$LD = \begin{bmatrix} 1 & 1 & 1 \\ -3 & -1 & 0 \\ 2 & -2 & 3 \end{bmatrix}$$

3- Uma matriz é dita idempotente se $A^2 = A$. Crie uma tal

a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ = é idempotente b) $\begin{bmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix}$ = é idempotente

c) $\begin{bmatrix} 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$ = é idempotente

5- Para cada um dos pares de matrizes, ache a matriz elementar, tal que $EA = B$

a)

$A = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 2 \\ 5 & 3 \end{bmatrix}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 5 & 3 \end{bmatrix}$

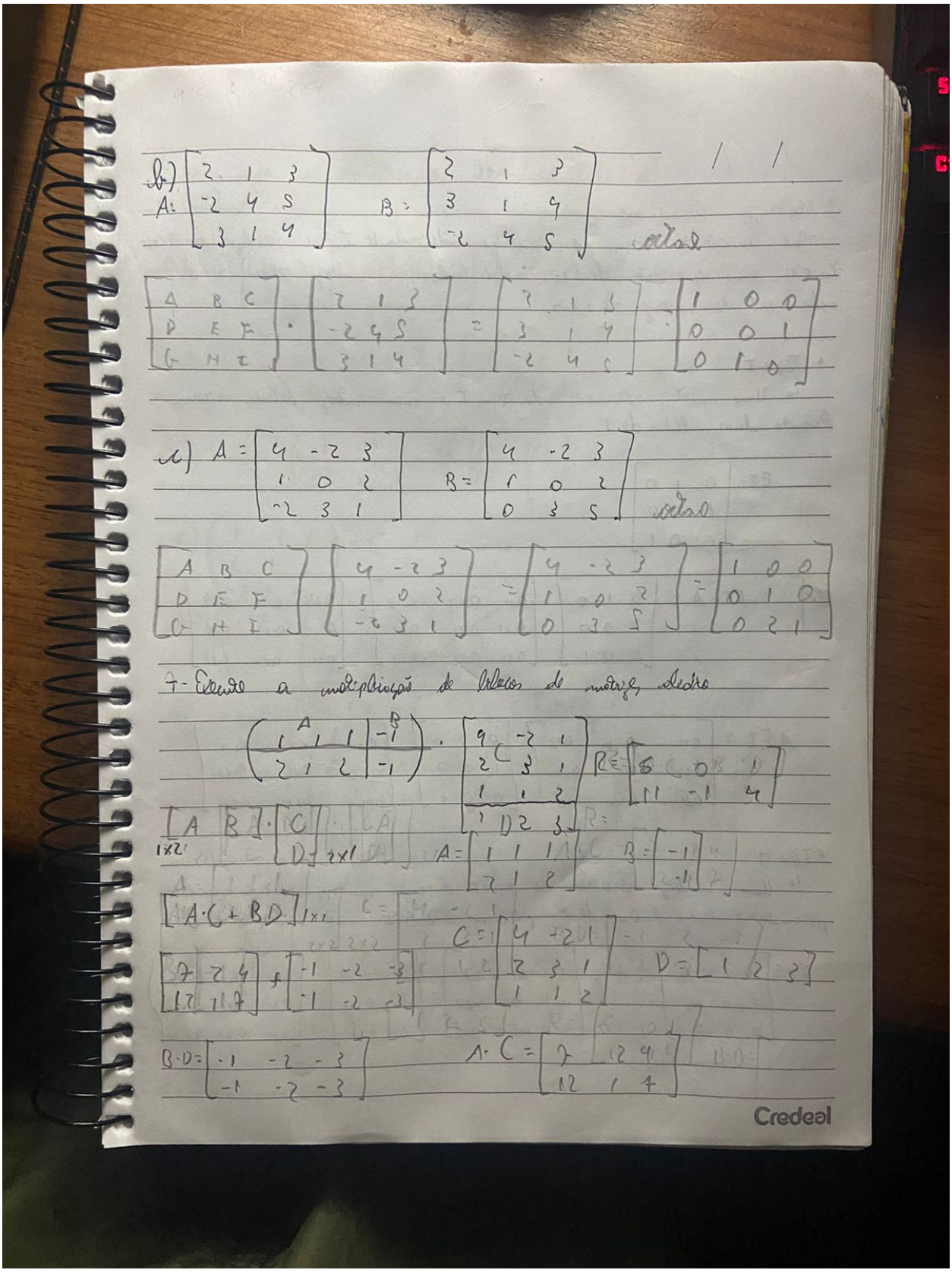
$\begin{bmatrix} 2a+5b & -a+3b \\ 2c+5d & -c+3d \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 5 & 3 \end{bmatrix}$ $\begin{bmatrix} -2 & 0 \\ 30 & -11 \end{bmatrix}$

$2a + 5b = -4$
$-a + 3b = 2$
$a = -2 + 3b$

$2(-2+3b) + 5b = -4$
$-4 + 5b + 5b = -4$
$-4 + 11b = -4$
$b = 0$
$a = -2$

$2c + 5d = 5$
$c = -3 - 3d$
$2(-3-3d) + 5d = 5$
$-6 - 6d + 5d = 5$
$-6 - d = 5$
$d = \frac{11}{-1} = -11$
$c = -3 - 3(-11)$
$= -3 + 33 = 30$

Credeal



b) $A = \begin{bmatrix} 2 & 1 & 3 \\ -2 & 4 & 5 \\ 3 & 1 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ -2 & 4 & 5 \end{bmatrix}$ não

$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 3 \\ -2 & 4 & 5 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ -2 & 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

c) $A = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 0 & 2 \\ -2 & 3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 0 & 2 \\ 0 & 3 & 5 \end{bmatrix}$ não

$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} 4 & -2 & 3 \\ 1 & 0 & 2 \\ -2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 0 & 2 \\ 0 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

4- Execute a multiplicação de blocos de matriz, dada

$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & 1 & 2 \\ \hline 1 & 0 & 2 \end{array}\right) \cdot \left(\begin{array}{cc|c} 9 & -2 & 1 \\ 2 & 3 & 1 \\ \hline 1 & 1 & 2 \end{array}\right) = \begin{bmatrix} 8 & 0 & 1 \\ 11 & -1 & 4 \end{bmatrix}$

$\begin{bmatrix} A & B \end{bmatrix} \cdot \begin{bmatrix} C \\ D \end{bmatrix}$ 1×2 2×1

$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$[A \cdot C + B \cdot D]_{1\times 1}$

$C = \begin{bmatrix} 4 & -2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

$\begin{bmatrix} 7 & 2 & 4 \\ 12 & 1 & 7 \end{bmatrix} + \begin{bmatrix} -1 & -2 & -3 \\ -1 & -2 & -3 \end{bmatrix}$

$B \cdot D = \begin{bmatrix} -1 & -2 & -3 \\ -1 & -2 & -3 \end{bmatrix}$ $A \cdot C = \begin{bmatrix} 7 & 2 & 4 \\ 12 & 1 & 7 \end{bmatrix}$

Credeal

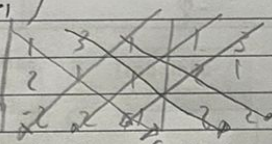
8- Para cada um dos seguintes itens, calcule (I) $\det(A)$, (II) $\text{adj}(A)$ e (III) A^{-1}

a) $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ $\det(A) = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = -1 - 6 = -7$

$\text{adj}(A) = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -3 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{-7} \cdot \begin{pmatrix} -1 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 1/7 & 2/7 \\ 3/7 & -\frac{1}{7} \end{pmatrix}$

c) $\begin{pmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ -2 & 2 & -1 \end{pmatrix} = C$

$\det(C) =$ 

$\det(C) = -1 - 6 + 4 - (-2 + 2 - 5) = -3 + 2 - 2 + 5 = 3$

$\text{adj}(C) = \begin{vmatrix} \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ -2 & -1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ -2 & 2 \end{vmatrix} \\ \begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ -2 & 2 \end{vmatrix} \\ \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} \end{vmatrix}^T = \begin{vmatrix} -3 & 0 & 6 \\ -5 & 1 & 8 \\ 2 & -1 & -5 \end{vmatrix}^T = \begin{vmatrix} -3 & -5 & 2 \\ 0 & 1 & -1 \\ 6 & 8 & -5 \end{vmatrix}$

$A^{-1} = \frac{1}{3} \cdot \begin{vmatrix} -3 & -5 & 2 \\ 0 & 1 & -1 \\ 6 & 8 & -5 \end{vmatrix} = \begin{vmatrix} -1 & -5/3 & 2/3 \\ 0 & 1/3 & -1/3 \\ 2 & 8/3 & -5/3 \end{vmatrix}$

9- Use a regra de cramer

a) $x_1 + 2x_2 = 3$

$3x_1 - x_2 = 1$

$$\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = A$$

$$\det(A) = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}_{2\times 2} = -1 - 6 = -7$$

$$\det(A_1) = \begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} = -3 - 2 = -5$$

$$\det(A_2) = \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = 1 - 9 = -8$$

$$x_1 = \frac{A_1}{A} \qquad x_2 = \frac{A_2}{A}$$

$$x_1 = \frac{-5}{-7} = \frac{5}{7} \qquad x_2 = \frac{-8}{-7} = \frac{8}{7}$$

b) $2x_1 + 3x_2 = 2$

$3x_1 + 2x_2 = 5$

$$A = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}$$

$\det(A) = 4 - 9 = -5$

$$\det(A_1) = \begin{vmatrix} 2 & 3 \\ 5 & 2 \end{vmatrix} = 4 - 15 = -11 \qquad x_1 = \frac{-11}{-5} = \frac{11}{5}$$

$$\det(A_2) = \begin{vmatrix} 2 & 2 \\ 3 & 5 \end{vmatrix} = 10 - 6 = 4 \qquad x_2 = \frac{4}{-5} = \frac{-4}{5}$$

Credeal