# Mediation with Repeated-Measures and Multilevel Data

Amanda K. Montoya
University of California, Los Angeles

Workshop: 11:30pm – 2:20pm

Please go to https://github.com/akmontoya/APS2019.git, download the folder and open SPSS.

# Workshop Procedures

**Download files at**
https://github.com/akmontoya/APS2019.git

Assuming some familiarity with:

- Regression & Multilevel Models
- Mediation
- SPSS

What we will learn:

- Mediation in Between Subjects Designs
- Mediation in Two-Instance Within-Participant Designs
- Introduction to Multilevel Modeling
- Mediation with Multilevel Data
- Q&A

Short breaks throughout

How we will learn:

- Combination of theory and practice
- Follow along with the analysis as we go
  - Use syntax!
  - **Ask questions** about concepts or anything that is confusing
- Make friends, if you have troubles as you go through you can work together.

# Mediation

- Between Subjects Mediation
  - Path analytic approach
  - Interpretation
  - Estimation
  - Inference

- Repeated Measures Data

- Two-Instance Repeated-Measures Mediation
  - Judd Kenny and McClelland (2001)
  - Path analytic approach
  - Estimation of Indirect Effects
  - MEMORE
  - Reporting (Writing and Figures)
  - Common Questions

# Running Example: Group Work in Computer Science (BS)

Montoya, A. K. (2013) Increasing Interest in Computer Science thought Group Work: A Goal Congruity Approach.

**Between-Subjects Version (CompSci_BS.sav) :**
Female participants (N = 107) read *one of two* syllabi for a computer science class. One of the syllabi reported the class would have group projects throughout (`cond = 1`), and the other syllabi stated that there would be individual projects (`cond = 0`) throughout the class.

**Measured Variables:**

- `Interest` in the class ($\alpha = .89$)
  - How interested are you in taking the class you read about?
  - How much would you want to take the class you read about?
  - How likely would you be to choose the class you read about?
  - How interested are you in majoring in computer science?
  - 1 Not at All – 7 Very much
- `CSComm`: Perceptions that computer science is communal ($\alpha = .90$)
  - Computer science would assist me in _______________.
  - Helping others, serving the community, working with others, connecting with others, caring for others.
  - 1 Strongly Disagree – 7 Strongly Agree

# University of Washington
# Computer Science & Engineering 142: Introduction to Programming I
# Course Syllabus

## Instructor

**name:** John Johnson
**email:** j.johnson@uw.edu
**office:** CSE 800
**office phone:** (206)555-1234
**office hours:** see course website

## Course Overview

This course provides an introduction to computer science using the Java programming language. CSE 142 is primarily a programming course that focuses on common computational problem solving techniques. No prior programming experience is assumed, although students should know the basics of using a computer (e.g., using a web browser and word processing program) and should be competent with math through Algebra 1. The information, concepts, and analytical thinking introduced in lecture provide a unifying framework for the topics covered in CSE 143.

## Lecture Time

MWF 12:00 PM - 1:00 PM, Classroom TBA

## Discussion Sections

You will be expected to participate in a weekly discussion section, held on Thursdays (see course website for details). The TA who runs your section will grade your homework assignments. In section, we will answer questions, go over common errors in homework, and discuss sample problems in more detail than lecture.

## Course Web Site

- http://www.cs.washington.edu/142/

## Textbook

- Reges/Stepp, *Building Java Programs: A Back to Basics Approach (2nd Edition)*.

## Grading

The primary assessment for your success in this class is exams. There will be 2 midterms and 1 final, and together they make up 85% of your grade. The homework assignments are designed to prepare you for your exams. The exams are designed to assess your ability to utilize the concepts you've learned from your homework and in lecture in new contexts.

5% participation
10% weekly homework assignments
25% midterm 1
25% midterm 2
35% final exam

## Exams

Our exams are closed-book and closed-notes, although each student will be allowed to bring a single index card with hand-written notes (no larger than 5" by 8"). No electronic devices may be used, including calculators. Make-up exams will not be given except in case of a serious emergency.

## Homework

Homework consists of weekly assignments done in optional groups and submitted electronically on the course web site. Disputes about homework grading must be made within 2 weeks of receiving the grade. If you don't make an honest effort on the homework, your exam score will reflect it.

## Academic Integrity and Collaboration

Computer Science is best learned through interacting with your fellow students to ensure that you thoroughly understand each concept. Homework assignments may be completed with other students. You are strongly encouraged to discuss general ideas of how to approach an assignment with other students, and may discuss specific details about what to write with other students. Any help you receive from or provide to classmates should be cited in your assignment. You may seek help from University of Washington CSE 142 TAs, professors, and classmates.

You must abide by the following rules:

- You are highly encouraged to work with another student on homework assignments.
- You may not show another student outside of your class your solution to an assignment, nor look at his/her solution.
- You may not have anyone outside of your class describe in detail how to solve an assignment or sit with you as you write it.
- You may not post online about your homework, other than on the class discussion board, to ask others for help.

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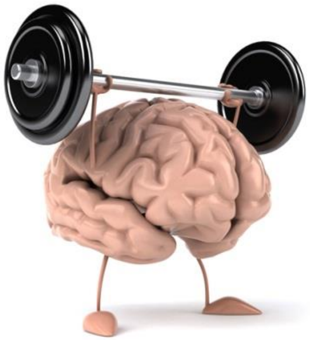
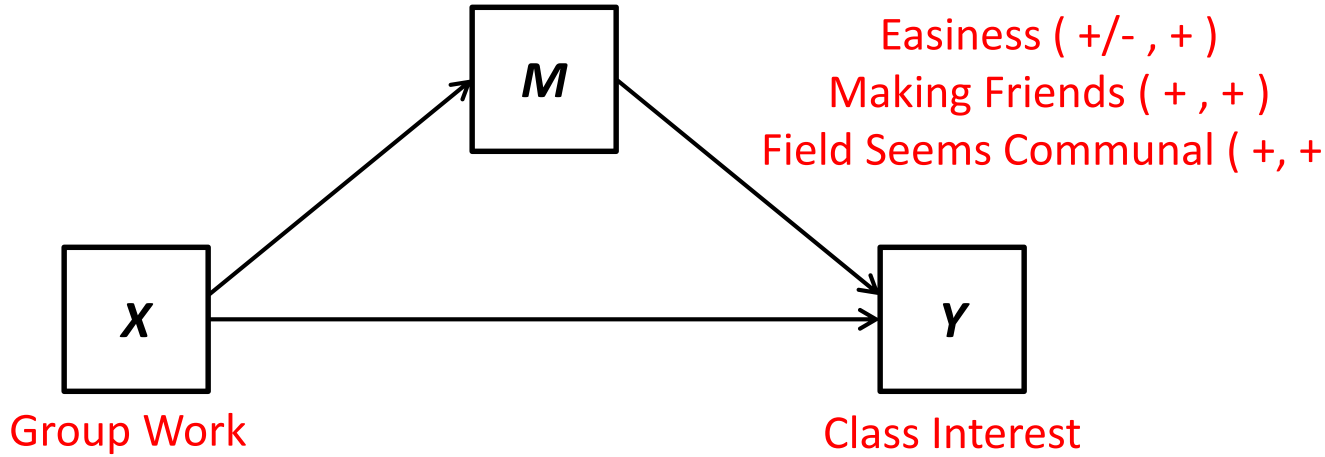
- You may not post online about your homework to ask others for help.



# Mediation




A simple mediation model connects an **assumed** causal variable (*X*) to an **assumed** outcome variable (*Y*), through some mechanism (*M*).

*M* is frequently referred to as a *mediator, intermediary variable,* or *surrogate variable*.

Many different kind of variables may act as mediators. Emotional variables, situational, individual level variables, cognitive variables, environmental variables, etc.

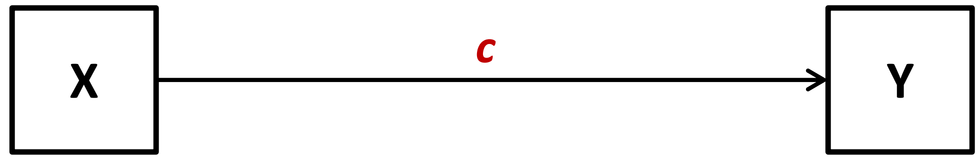
Mediation can be found throughout the psychology literature and is particularly common in social psychology

A quick example: Name some possible mediators!

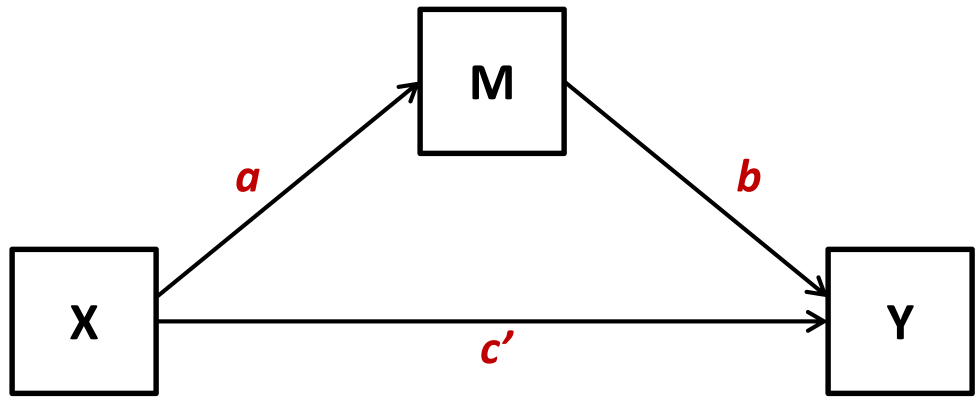
# Mediation: Path Analysis

Consider *a*, *b*, *c*, and *c′* to be measures of the effect of the variables in the mediation model.

These could be measured using regression coefficients from OLS or path estimates in a structural equation model using maximum likelihood estimation.




$$Y_i = i_{Y^*} + cX_i + e_{Y_i^*}$$




$$M_i = i_M + aX_i + e_{M_i}$$

$$Y_i = i_Y + c'X_i + bM_i + e_{Y_i}$$

Indirect effect of *X* on *Y* (through *M*) = *a* × *b*

Direct effect of *X* on *Y* (not through *M*) = *c′*

Indirect effect = total effect - direct effect

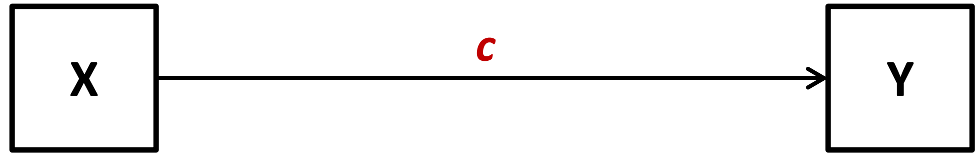
*a* × *b* = *c* - *c′*

Total effect = direct effect + indirect effect

*c* = *c′* + *a* × *b*

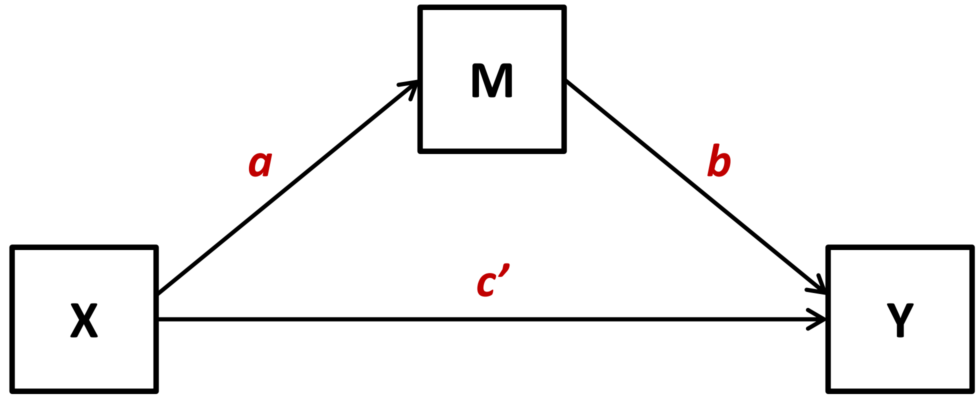
# Interpreting the Coefficients

**Total Effect (*c*):** The effect of our presumed cause (*X*) on our outcome (*Y*), without controlling for any other variables.




***a*-path:** The effect of our presumed cause (*X*) on our mediator *(M).*

***b*-path:** The effect of our mediator (*M*) on the outcome (*Y*) while controlling for *X*. (i.e. predicted difference in *Y* for two people with the <u>same score on *X*</u> but who differ on *M* by one unit).

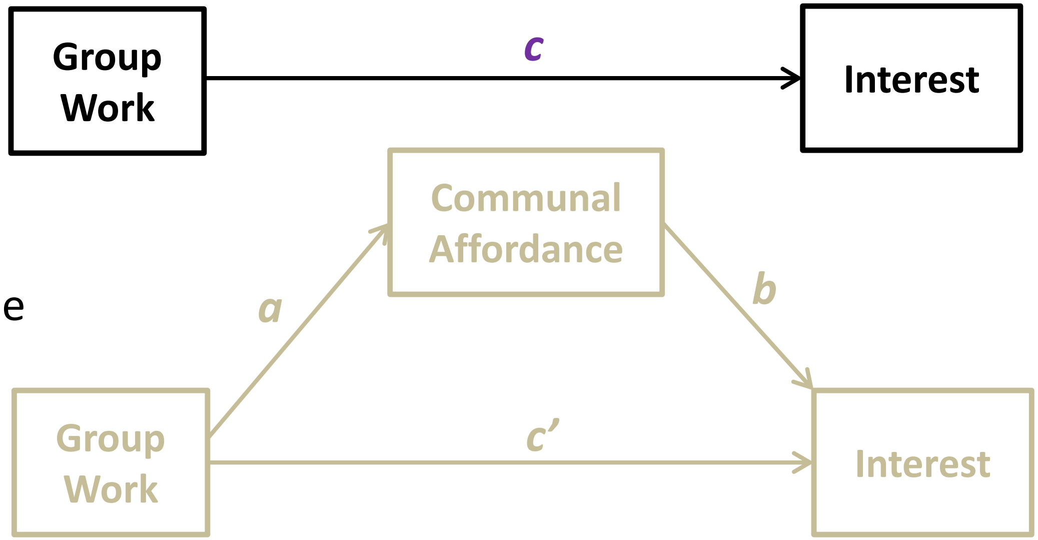



**Direct effect (*c'*):** The effect of our presumed cause (*X*) on *Y* while controlling for *M*. (i.e. predicted difference in *Y* for two people who differ by one unit on *X* but with the <u>same score on M</u>)

**Indirect Effect (*ab*):** Product of effect of *X* on *M*, and effect of *M* on *Y* controlling for *X*. The effect of *X* on *Y* through *M*.

# Estimation with CompSci_BS Data

**Research Question:** **Can <u>group work</u> in computer science classes increase women's <u>interest</u> by increasing their perception that computer science is <u>communal</u>?**




The *c*-path can be estimated in a sample using the regression equation below.

$$Y_i = i_{Y^*} + cX_i + e_{Y_i^*}$$

```
regression /dep = interest /method = enter cond.
```

**Coefficients[a]**

| Model | | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. |
|---|---|---|---|---|---|---|
| | | B | Std. Error | Beta | | |
| 1 | (Constant) | 2.701 | .193 | | 14.002 | .000 |
| | Cond | .462 | .285 | .156 | 1.621 | .108 |

a. Dependent Variable: Interest

Overall women were .462 units more interested in the class with group work.

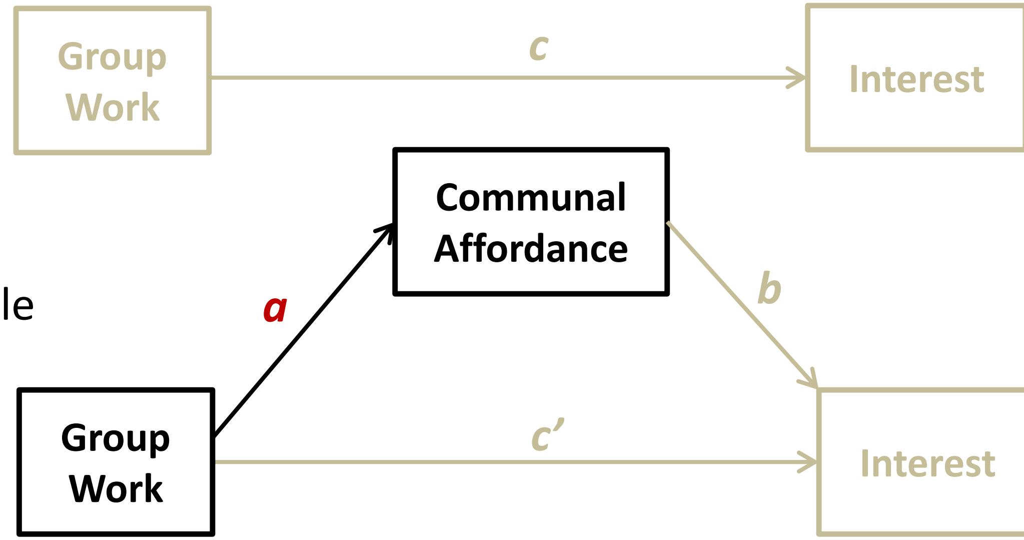
*c* = .462

# Estimation with CompSci_BS Data

**Research Question:** **Can group work in computer science classes increase women's interest by increasing their perception that computer science is communal?**

The *a*-path can be estimated in a sample using the regression equation below.

$$M_i = i_M + aX_i + e_{M_i}$$




```
regression /dep = CScomm /method = enter cond.
```

**Coefficients**[a]

| Model | | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. |
|---|---|---|---|---|---|---|
| | | B | Std. Error | Beta | | |
| 1 | (Constant) | 3.421 | .159 | | 21.472 | .000 |
| | Cond | .488 | .237 | .198 | 2.060 | .042 |

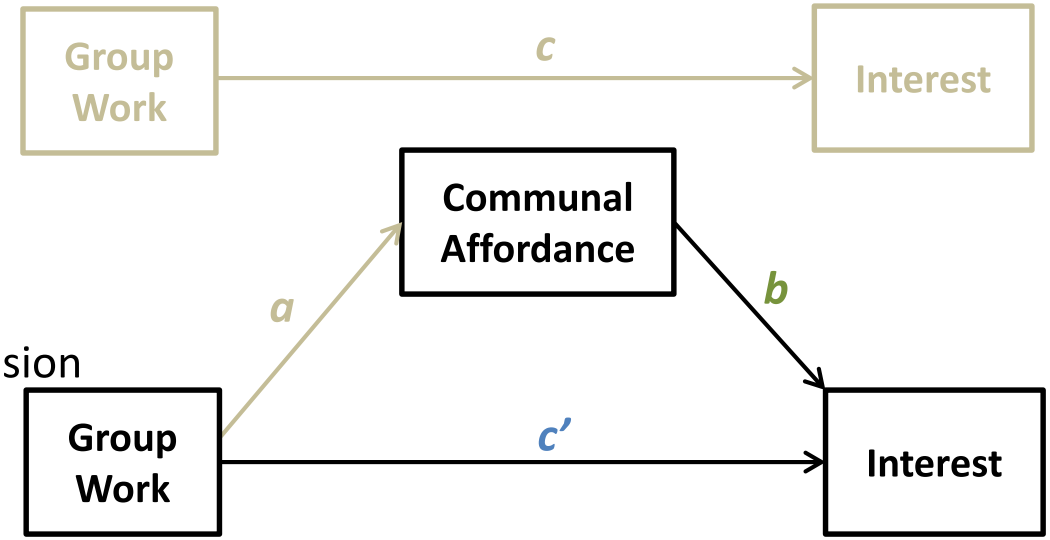
a. Dependent Variable: CSComm

Women saw computer science as .488 units more communal after reading a syllabus with group work.

*a* = .488

# Estimation with CompSci_BS Data

**Research Question: Can group work in computer science classes increase women's interest by increasing their perception that computer science is communal?**




The *b*-path and direct effect can be estimated in a sample using the regression equation below.

$$Y_i = i_Y + c'X_i + bM_i + e_{Y_i}$$

```
regression /dep = interest /method = enter cond CScomm.
```

$c' = 0.218$

**Coefficients**[a]

| Model | | Unstandardized Coefficients B | Unstandardized Coefficients Std. Error | Standardized Coefficients Beta | t | Sig. |
|---|---|---|---|---|---|---|
| 1 | (Constant) | .964 | .413 | | 2.336 | .021 |
| | Cond | .218 | .268 | .073 | .812 | .419 |
| | CSComm | .508 | .109 | .421 | 4.663 | .000 |

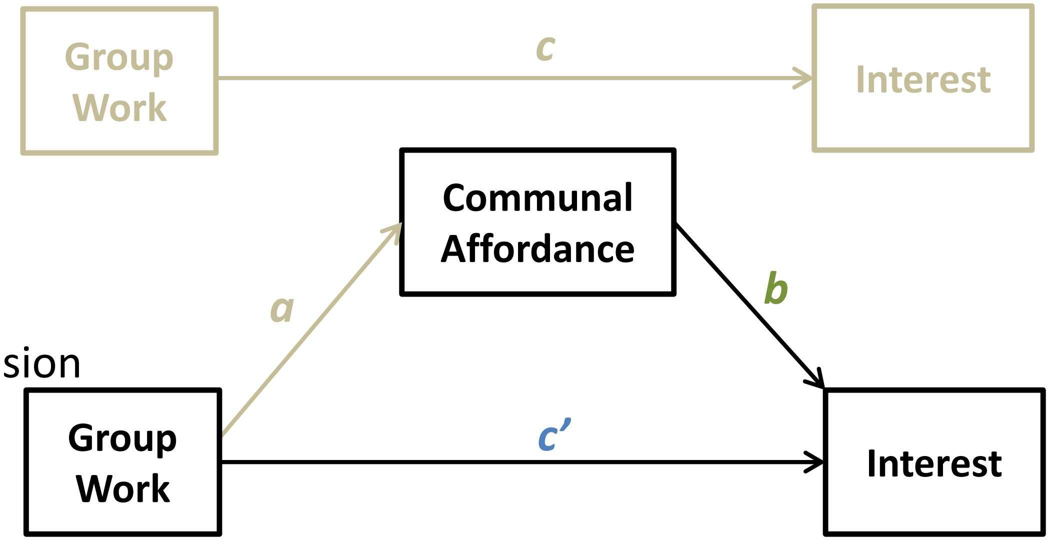
a. Dependent Variable: Interest

Controlling for communal affordance, women in the group work condition were .218 units more interested in the class with group work.

$b = .508$

# Estimation with CompSci_BS Data

**Research Question: Can group work in computer science classes increase women's interest by increasing their perception that computer science is communal?**

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**Coefficients**[a]

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| | Cond | .218 | .268 | .073 | .812 | .419 |
| | CSComm | .508 | .109 | .421 | 4.663 | .000 |

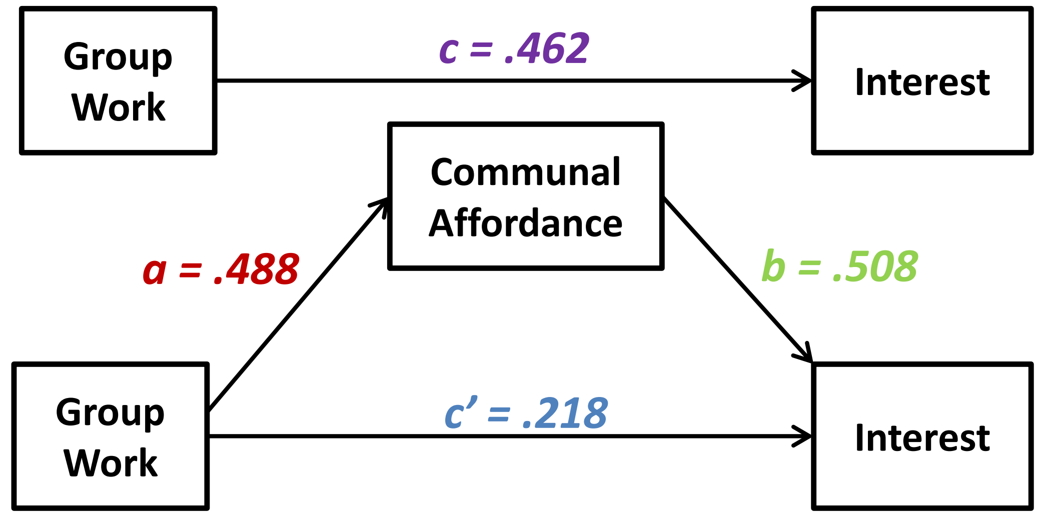
a. Dependent Variable: Interest

For two people in the same condition, a one unit difference in communal goals results in a 0.51 unit difference in interest, on average.

# Interpreting the Coefficients

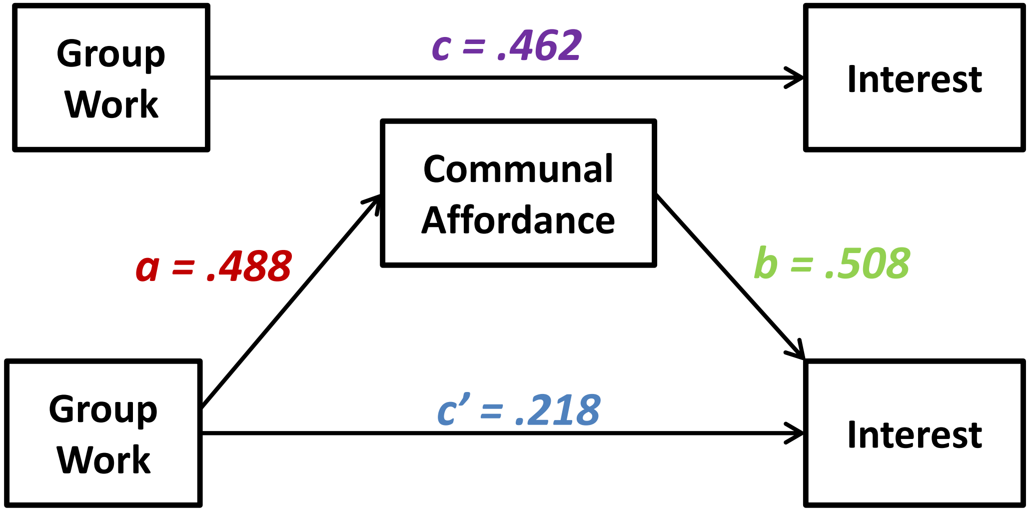
**<u>Research Question:</u> Can group work in computer science classes increase women's interest by increasing their perception that computer science is communal?**

On average, women were .46 units more interested in the class with group work ($p$ = .108). Similarly, computer science was perceived as .49 units more communal after reading a syllabus with group work ($p$ = .042). Controlling for condition, a one unit increase in communal affordance resulted in a .508 unit increase in interest ($p$ < .001). Controlling for communal affordance, group work did not predict additional interest ($c'$ = .22, $p$ = .42).




**But what about the indirect effect?**

# Interpreting Indirect, Direct, and Total Effects




**Indirect Effect**

$a \times b = .488 \times .508 = .249$

Group work increased interest by .249 units indirectly through communal affordance. Where group work increased perceptions of communal affordance by .488 units, and a one unit increase in communal affordance resulted in a .508 unit increase in interest.

**Direct Effect**

$c' = .218$

Group work increased interest by .218 units directly (not through communal affordance).

$p = .419$

**Total Effect**

$c = .462$

Group work increased interest by .462 units in total.

$p = .108$

*Inference for the direct and total effects can be drawn from the regression results because these are based on a single regression parameter.*

# Inference about the Indirect Effect

- How to make proper inference about the indirect effect may be the most active area of research in mediation analysis
- Some methods you may have heard of
  - Causal Steps / Baron and Kenny Method / Baron and Kenny Steps
  - Test of Joint Significance
  - Sobel Test / Multivariate Delta Method
  - Monte Carlo Confidence Intervals
  - Distribution of the Product Method
  - Bootstrap Confidence Intervals
    - Percentile Bootstrap
    - Bias-Corrected Bootstrap
    - Bias Corrected and Accelerated Bootstrap

- Why is this so hard?
  - The product of two normal distributions is not necessarily normal. The shape of the distribution of the indirect effect depends on the true indirect effect.
  - There are many instances where the indirect effect could be zero (either *a* or *b* could be zero, or both could be zero).

# Causal Steps Method

**Method**

1. Test if there is a significant total effect ($c \neq 0$).
2. Test if there is a significant effect of $X$ on $M$ ($a \neq 0$).
3. Test if there is a significant effect of $M$ on $Y$ controlling for $X$ ($b \neq 0$).
4. If all three steps are confirmed, test for partial vs. complete mediation.
    1. If $X$ still has an effect on $Y$ controlling for $M$ ($c' \neq 0$), this is partial mediation
    2. If $X$ does not have a significant effect on $Y$ controlling for $M$, complete mediation

**Appeal**

- Easy to do, just need regression
- Intuitive

**What's wrong with it?**

- No estimate of the indirect effect
- No quantification of uncertainty about conclusion
    - $p$-value
    - Confidence Interval
- Requirement that the total effect is significant before looking for indirect effect
- Issues with *complete* and *partial* mediation

# Joint Significance

**Method**

1. Test if there is a significant effect of $X$ on $M$ ($a \neq 0$).
2. Test if there is a significant effect of $M$ on $Y$ controlling for $X$ ($b \neq 0$).

**Appeal**

- Easy to do, just need regression
- Intuitive
- Solves issues of requirement of significant total effect to claim an indirect effect.
- Good method balance Type I Error and Power

**What's wrong with it?**

- No estimate of the indirect effect
- No quantification of uncertainty about conclusion
  - *p*-value
  - Confidence Interval

# Bootstrap Confidence Intervals (Percentile)

Empirically estimate sampling distribution of the indirect effect. From this distribution compute confidence intervals which can be used for estimation and hypothesis testing.

**Method**

1. Randomly sample $n$ cases from your dataset with replacement.
2. Estimate the indirect effect using resampled dataset, call this $ab^{(1)}$
3. Repeat steps 1 and 2 a total of $K$ times where $K$ is many (10,000 recommended), each time calculated $ab^{(k)}$.
4. The sampling distribution of the $ab^{(i)}$'s can be used as an estimate of the sampling distribution of the indirect effect.
5. For a 95% confidence interval the lower and upper bounds will be the $2.5^{th}$ and $97.5^{th}$ percentiles of the $K$ estimates of the indirect effect.

**Appeal**

- No assumptions about the sampling distribution of the indirect effect
- Provides point estimate of indirect effect
- Can calculate confidence intervals
- Good method balance Type I Error and Power

**What's wrong with it?**

- Most software does not have this functionality built in
- Requires original data

# Bootstrap Confidence Intervals

Original Data

| X | M | Y |
|---|---|---|
| -0.35 | -0.58 | 0.25 |
| 0.31 | -0.50 | 1.89 |
| -0.19 | 2.61 | 2.08 |
| -1.30 | -1.49 | -0.54 |
| 0.59 | 1.14 | 1.74 |
| -0.29 | -0.29 | 1.04 |
| 1.80 | 0.08 | 1.23 |
| -0.01 | 1.20 | 1.30 |
| 0.30 | 1.35 | 1.31 |
| -0.98 | 0.90 | -0.76 |

$a$ = .2931 $b$ = .3099

$ab$ = .0908

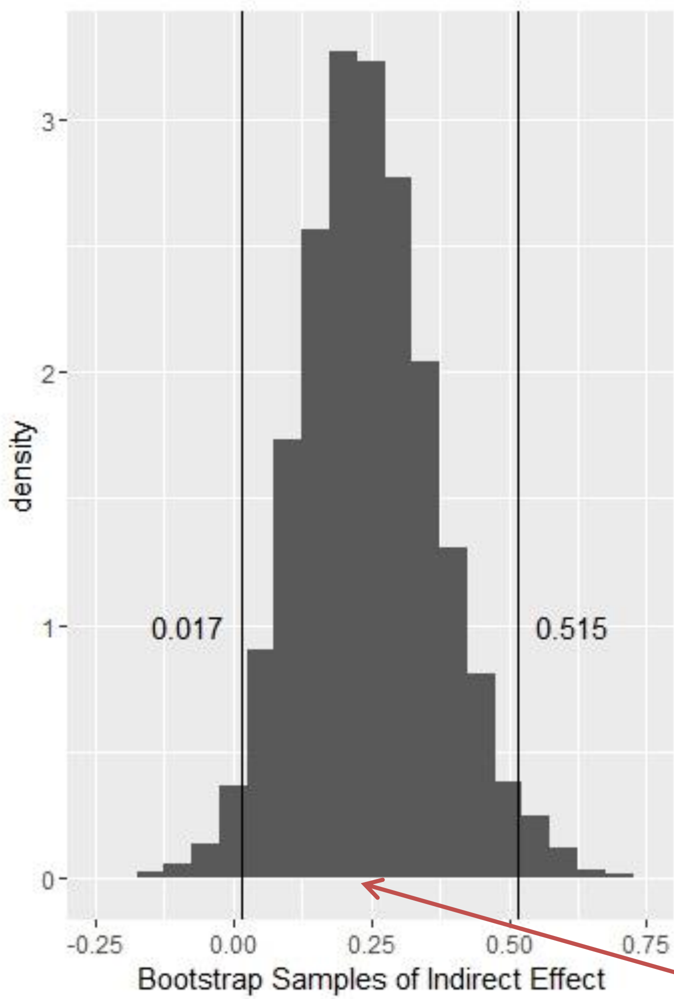
Bootstrap Sample

| X | M | Y |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

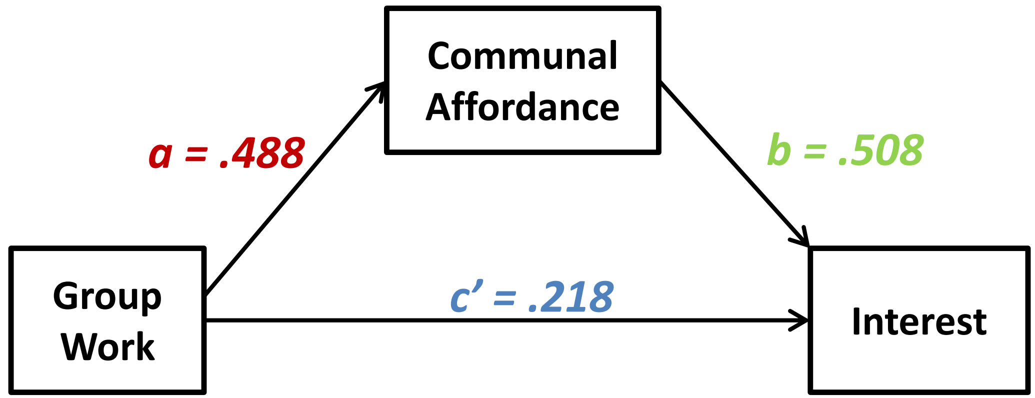
$a$ = -.1035 $b$ = .1495

$ab$ = -.0155

# Bootstrap Confidence Intervals (CompSci Data)




$ab$ = .249




$ab$ = .249

Zero is not contained in the confidence interval [0.017, 0.515] so we conclude the indirect effect is different from zero with 95% confidence. This is similar to rejecting the null hypothesis at $\alpha$ = .05.

# The Monte Carlo Interval

Monte Carlo empirically estimate the sampling distribution of the indirect effect and generate a confidence interval (CI) for estimation and hypothesis testing. This simulation based method assumes each individual path (*a* and *b*) are normally distributed.

(1) Generate *k* samples from a normal distribution with mean *a* and standard deviation $s_a$

(2) Generate *k* samples from a normal distribution with mean *b* and standard deviation $s_b$

(3) Multiply samples together to get a distribution of *k* estimates of *ab.*

(4) Rank order estimates and select estimates which define the lower percentile of sorted *k* estimates and upper percentile of sorted estimates which define CI of interest.

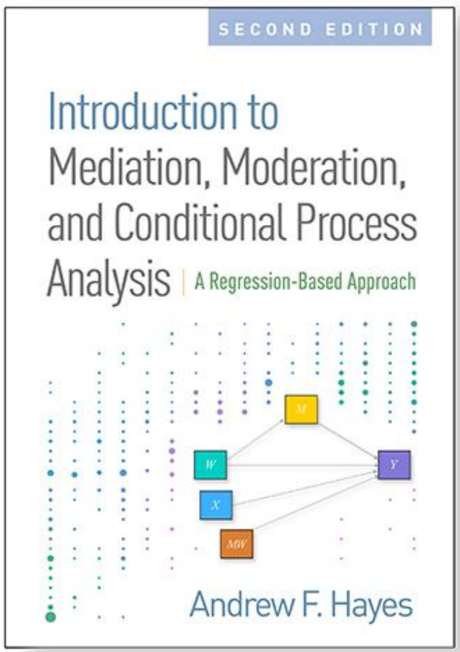
(5) For 95% CI lower and upper bounds are $2.5^{th}$ and $97.5^{th}$ percentile in *k* bootstrap estimates of the indirect effect.

This method performs well (similarly to bootstrapping) in a variety of simulation studies, but is still less popular.

This method makes stronger assumptions than bootstrapping, but does not seem to result in greater power.

# PROCESS

PROCESS is a macro available for SPSS and SAS written by Andrew F. Hayes, documented in *Mediation, Moderation, and Conditional Process Analysis*, and available for free online at *processmacro.org*

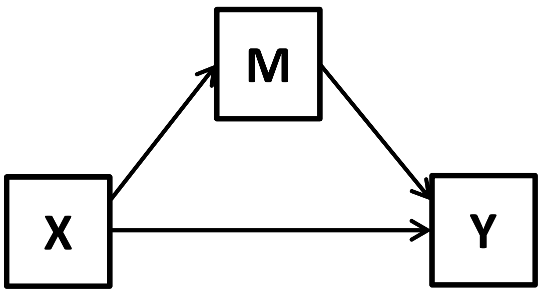



**Published in January 2018 and available through The Guilford Press, Amazon.com, and elsewhere.**

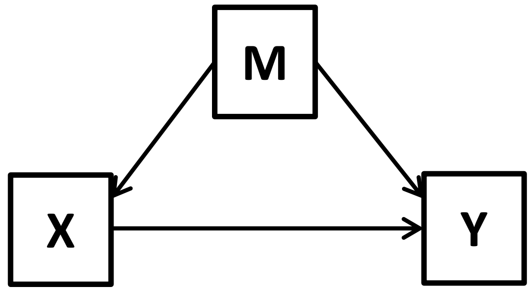
- PROCESS integrates a variety of macros previously developed by Hayes: SOBEL, INDIRECT, MODMED, MODPROBE, MED3C. If you are using any of these now, switch to PROCESS.
- Current version is 3.0
- PROCESS can assess a variety of models. Find the model you are interested in in the templates file, then use that model number.
- Appendix A of IMMCPA provides complete documentation of options in PROCESS and how to use them.
- Version 3 allows for specifying your own models (not from templates)

# A Brief Caution on Causality

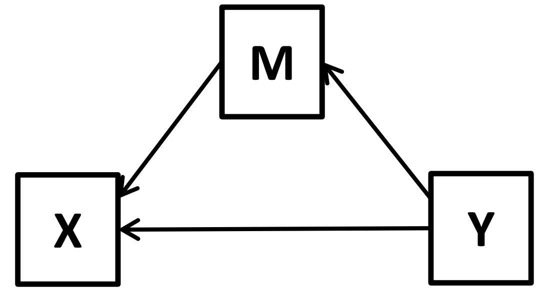
There are a number of alternative causal processes that may be occurring when a *statistical indirect effect* is present:



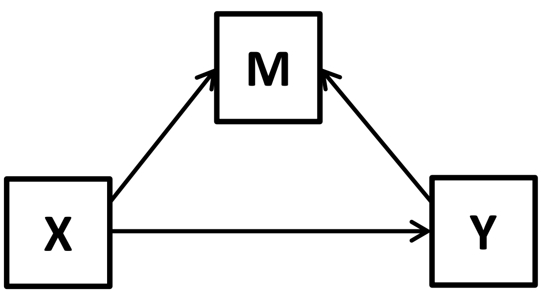

*X* affects *Y* through *M*



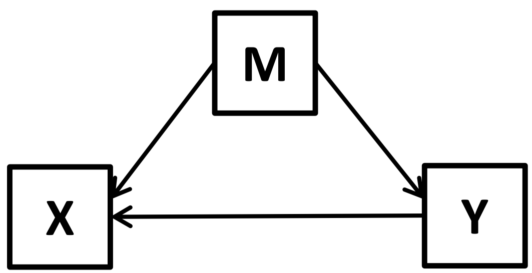

*M* affects *Y* through *X*



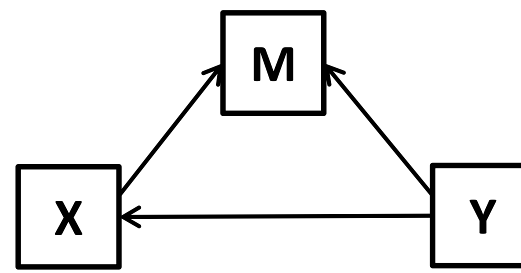

*Y* affects *X* through *M*




*X* affects *M* through *Y*



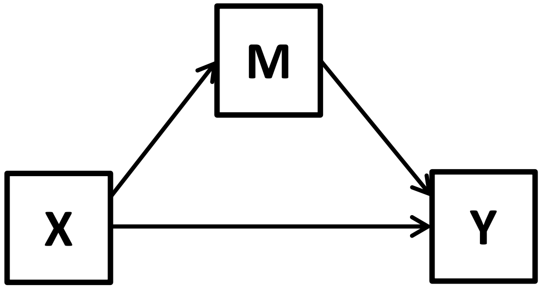

*M* affects *X* through *Y*



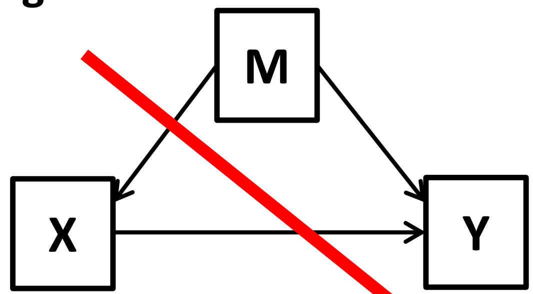

*Y* affects *M* through *X*

# A Brief Caution on Causality

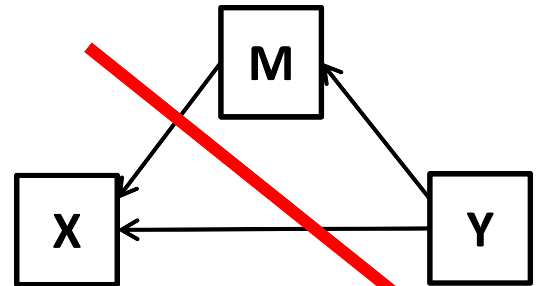
**What you get by manipulating *X*.**



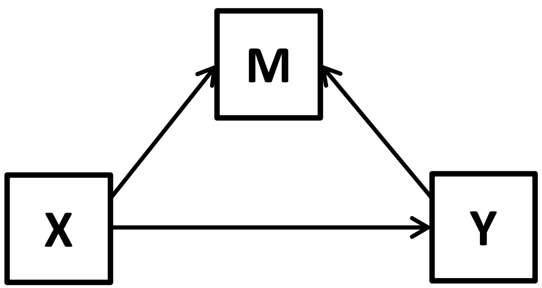

*X* affects *Y* through *M*



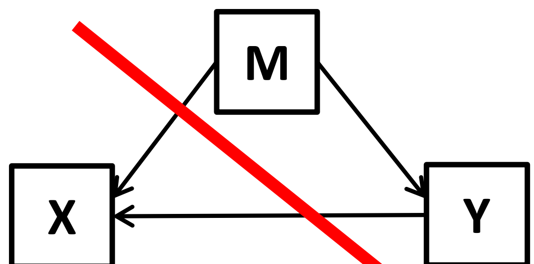

*M* affects *Y* through *X*



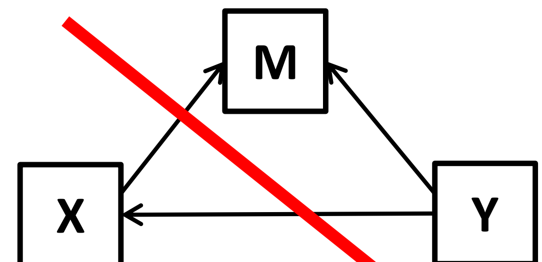

*Y* affects *X* through *M*




*X* affects *M* through *Y*




*M* affects *X* through *Y*




*Y* affects *M* through *X*

Even when *X* is manipulated, we can not provide evidence for the causal order between *M* and *Y*. This can only be supported using other experiments or previous research. *A statistically significant indirect effect does not lend credence to one model over another (*Thoemmes, 2015, *Basic and Applied Social Psychology).*

# Repeated Measures Data

There are many different kinds of "repeated measures data." What type of data you have will determine what kind of mediation analysis is appropriate.

Types of *Repeated Measurements:*

- Each person *over time*
- *Nested/Multilevel* data (individuals within schools, cohorts, etc)
- *Dyadic* data (twins, couples, labmates, roommates)
- Each person in a *variety of circumstances*
- and many more…

What is measured repeatedly?

- Specifically in mediation, it's important to think about how/when/how many times the variables in your mediation model are measured
- *Multilevel* has a nice system referring to levels (1-1-1 mediation, 2-2-1, mediation etc.
- Is your causal variable measured repeatedly?
- Is your causal variable what differentiates your repeated measurements?

# Repeated Measures Data

MEMORE is for **two-instance repeated measures** mediation analysis, where the causal variable of interest is the factor which differs by repeated measures.

*X*: varies between repeated measurements
*M*: measured in each of the two instances
*Y*: measured in each of the two instances

Examples:

- Participants read two scenarios. Interested in how scenario influences *Y* through *M*. Measure *M* and *Y* in each scenario.
- Pre-post test: Therapist measures certain symptoms and various outcomes before administering some intervention, and after administering the intervention.
- Researcher interested in if male partners in heterosexual relationships believe fights are less severe because they are less perceptive of small “squabbles”. Measure both male and female partners in relationships, self report number of small “squabbles” and severity of last fight.

Non-Examples:

- Does calorie consumption impact body image through weight gain over time?
- Any instance where repeated-measure factor is a “nuisance” (e.g. studying schools, but not interested in comparing schools directly).

# Running Example: Group Work in Computer Science (WS)

Montoya, A. K. (2013) Increasing Interest in Computer Science thought Group Work: A Goal Congruity Approach (Undergraduate Honors Thesis).

**Within-Subjects Version (CompSci_WS.sav) :**
Female participants (N = 51) read two syllabi for a different computer science classes. One of the syllabi reported the class would have group projects throughout, and the other syllabi stated that individual project would be scheduled throughout.

- Syllabi also differed in professor's name (but not gender), and the primary programming language used in the class.

**Measured Variables:**

- Interest in each the class (same as BS version)
  - Two measures: `int_i int_g`
- Perceptions that the class has a communal environment.
  - Two measures: `comm_i comm_g`
  - Taking this class would assist me in _______________.
  - Helping others, serving the community, working with others, connecting with others, caring for others.
- How difficult would you rate the class you read about?
  - Two measures: `diff_i diff_g`

# Judd, Kenny, and McClelland (2001)

Judd, C. M., Kenny, D. A., & McClelland, G. H. (2001). Estimating and testing mediation and moderation in within-subject designs. *Psychological Methods*, *6*, 115-134.

Psychological Methods
2001, Vol. 6, No. 2, 115-134


1082-989X/01/$5.00 DOI: 10.1037//1082-989X.6.2.115

## Estimating and Testing Mediation and Moderation in Within-Subject Designs

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Analyses designed to detect mediation and moderation of treatment effects are increasingly prevalent in research in psychology. The mediation question concerns the processes that produce a treatment effect. The moderation question concerns factors that affect the magnitude of that effect. Although analytic procedures have been reasonably well worked out in the case in which the treatment varies between participants, no systematic procedures for examining mediation and moderation have been developed in the case in which the treatment varies within participants. The authors present an analytic approach to these issues using ordinary least squares estimation.

The issues of mediation and moderation have received considerable attention in recent years in both basic and applied research (Baron & Kenny, 1986; James & Brett, 1984; Judd & Kenny, 1981b; MacKinnon & Dwyer, 1993). In addition to knowing whether a particular intervention has an effect, the researcher typically wants to know about factors that affect the magnitude of that effect (i.e., moderation) and mechanisms that produce the effect (i.e., mediation). Such knowledge helps in both theory development and intervention application.

To illustrate the difference between mediation and moderation, consider a design in which a researcher is interested in whether students who are taught with a new curriculum (the treatment condition) show higher performance on a subsequent standardized test than students taught under the old curriculum (the control condition). Assuming that a performance difference is found, one might plausibly hypothesize different mediating mechanisms for this effect. The new curriculum might increase students' interest in the subject matter; it might cause students to study harder outside of class; or it might convey the material more clearly. These are alternative reasons why the performance difference is found, that is, alternative mediators of the treatment effect. The researcher might also be interested in factors that affect the magnitude of the difference between performance following the old curriculum and performance following the new one. That difference might be larger or smaller for different types of students or in different types of classrooms or when taught by different kinds of teachers. All of these then are potential moderators of the treatment effect.

It is possible that the same variable may serve as both a mediator and a moderator. For instance, study time might serve both roles. First, as a mediator, the new curriculum might lead to higher performance because it causes students to study more. Second, as a moderator, the treatment might be especially effective for students who spend more time studying.

Procedures for assessing mediation and moderation have been relatively well worked out through ordinary least squares regression and analysis of variance procedures. Mediation is assessed through a four-step procedure (Baron & Kenny, 1986; Judd & Kenny,

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One of the few treatments of mediation analysis in this common research design.

A "causal steps", Baron and Kenny type logic to determining whether *M* is functioning as a mediator of *X*'s effect on *Y* when both *M* and *Y* are measured twice in difference circumstances but on the same people.

1. On average, does *Y* differ by condition?
2. On average, does *M* differ by condition?
3. Does difference in *M* predict a difference in *Y*?
4. Does the difference in *M* account for all the difference in *Y*?

# Computer Science Within-Subjects Data Example

Montoya, A. K. (2013) Increasing Interest in Computer Science thought Group Work: A Goal Congruity Approach (Undergraduate Thesis).

**Research Question: Can <u>group work</u> in computer science classes increase women's <u>interest</u> by increasing their perception that computer science is <u>communal</u>?**

Data is in *wide form*: repeated measurements of the same variables are saved as separate variables (one row per participant). *Long form* is when there is a variable coding instance of repeated measurements (multiple rows per participant, one for each instance).

CompSci_WS.sav

| int_I | int_G | comm_I | comm_G |
|---|---|---|---|
| 1.50 | 4.00 | 1.00 | 6.80 |
| 2.75 | 3.25 | 2.00 | 5.40 |
| 5.75 | 2.50 | 3.20 | 3.60 |
| 3.50 | 5.75 | 1.60 | 5.20 |
| 2.25 | 2.00 | 4.40 | 4.60 |
| 1.50 | 1.75 | 3.00 | 5.00 |
| 2.50 | 4.25 | 4.20 | 4.40 |
| 6.00 | 1.75 | 4.80 | 2.40 |
| 3.00 | 2.00 | 2.60 | 5.80 |
| 4.00 | 5.25 | 1.60 | 5.00 |
| 5.00 | 5.00 | 4.60 | 6.20 |
| 2.00 | 1.75 | 3.80 | 4.20 |
| 1.00 | 1.75 | 2.60 | 3.20 |
| 1.25 | 4.50 | 1.00 | 6.00 |
| 5.75 | 4.50 | 2.60 | 6.00 |
| 3.25 | 4.75 | 3.00 | 6.20 |
| 2.75 | 2.25 | 4.80 | 4.60 |
| 5.50 | 2.00 | 4.00 | 7.00 |
| 1.75 | 5.25 | 1.60 | 5.60 |
| 4.00 | 5.50 | 1.80 | 5.40 |
| 2.25 | 4.00 | 2.20 | 4.80 |
| 4.00 | 6.50 | 2.00 | 6.80 |
| 5.00 | 4.50 | 3.20 | 6.00 |

# Analysis using Judd et al. (2001)

1. On average, does $Y$ differ by condition?

Setup a model of the outcome in each condition:

$$Y_{1i} = c_1 + \epsilon_{Y_{1i}^*}$$
$$Y_{2i} = c_2 + \epsilon_{Y_{2i}^*}$$

Is $c_1$ different from $c_2$?

Based on these models, setup a new model where you can directly estimate and conduct inference on what you are interested in (in this case $c_2 - c_1$):

$$Y_{2i} - Y_{1i} = (c_2 - c_1) + \left(\epsilon_{Y_{2i}^*} - \epsilon_{Y_{1i}^*}\right) = c + \epsilon_{Y_i^*}$$

Use intercept only regression analysis, or a paired sample t-test, or a one sample t-test on the differences to conduct inference on $c_2 - c_1$

With the data: On average, is class interest higher in the group work condition?

```
T-TEST PAIRS=int_G WITH int_I (PAIRED).
```

**Paired Samples Test**

| | Paired Differences | | | | | t | df | Sig. (2-tailed) |
|---|---|---|---|---|---|---|---|---|
| | | | | 95% Confidence Interval of the Difference | | | | |
| | Mean | Std. Deviation | Std. Error Mean | Lower | Upper | | | |
| Pair 1 int_G - int_I | .37255 | 1.99585 | .27948 | -.18879 | .93389 | 1.333 | 50 | .189 |

# Analysis using Judd et al. (2001)

2. On average, does *M* differ by condition?

Setup a model of the mediator in each condition:

$$M_{1\mathrm{i}} = a_1 + \epsilon_{M_{1i}}$$
$$M_{2\mathrm{i}} = a_2 + \epsilon_{M_{2i}}$$

Is $a_1$ different from $a_2$?

Based on these models, setup a new model where you can directly estimate and conduct inference on what you are interested in (in this case $a_2 - a_1$):

$$M_{2i} - M_{1i} = (a_2 - a_1) + \left(\epsilon_{M_{2i}} - \epsilon_{M_{1i}}\right) = a + \epsilon_{M_i}$$

Use intercept only regression analysis, or a paired sample t-test, or a one sample t-test on the differences to conduct inference on $a_2 - a_1$

With the data: On average, is communal goal affordance higher in the group work condition?

```
T-TEST PAIRS=comm_G WITH comm_I (PAIRED).
```

**Paired Samples Test**

| | Paired Differences | | | | | t | df | Sig. (2-tailed) |
|---|---|---|---|---|---|---|---|---|
| | | | | 95% Confidence Interval of the Difference | | | | |
| | Mean | Std. Deviation | Std. Error Mean | Lower | Upper | | | |
| Pair 1 comm_G - comm_I | 2.29412 | 1.77870 | .24907 | 1.79385 | 2.79438 | 9.211 | 50 | .000 |

# Analysis using Judd et al. (2001)

3. Does difference in *M* predict a difference in *Y*? / Does *M* predict *Y* controlling for condition?

Setup a model of the outcome in each condition:

$$Y_{1\mathrm{i}} = g_{10} + g_{11}M_{1i} + \epsilon_{Y_{1i}}$$
$$Y_{2\mathrm{i}} = g_{20} + g_{21}M_{2i} + \epsilon_{Y_{2i}}$$

Note that there are **two estimates** of the effect of *M* on *Y*. Let's average them to estimate an average effect of *M* on *Y*. Setup a new model where you can directly estimate and conduct inference on what you are interested in (in this case $\frac{1}{2}(g_{21} + g_{11})$):

Optional board work

$$Y_{2i} - Y_{1i} = (g_{20} - g_{10}) + g_{21}M_{2i} - g_{11}M_{1i} + (\epsilon_{Y_{2i}} - \epsilon_{Y_{1i}})$$
$$Y_{2i} - Y_{1i} = (g_{20} - g_{10}) + \underbrace{\frac{g_{21}+g_{11}}{2}}_{b}(M_{2i} - M_{1i}) + \underbrace{\frac{(g_{21}-g_{11})}{2}}_{d}(M_{2i} + M_{1i}) + (\epsilon_{Y_{2i}} - \epsilon_{Y_{1i}})$$

# Analysis using Judd et al. (2001)

3. Does *M* predict *Y* controlling for condition?

With the data: Does communal goal affordance predict interest in the class?

```
compute int_diff = int_G - int_I.
compute comm_diff = comm_G - comm_I.
compute comm_sum = comm_G+comm_I.
EXECUTE.
regression dep = int_diff /method = enter comm_diff comm_sum.
```

**Coefficients**[a]

| Model | | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. |
|---|---|---|---|---|---|---|
| | | B | Std. Error | Beta | | |
| 1 | (Constant) | 1.310 | 1.877 | | .698 | .489 |
| | comm_diff | .590 | .135 | .526 | 4.385 | .000 |
| | comm_sum | -.275 | .216 | -.153 | -1.272 | .210 |

a. Dependent Variable: int_diff

✓

# Analysis using Judd et al. (2001)

4. Does the difference in communal goal affordance account for all the difference in interest?

$$Y_{2i} - Y_{1i} = (g_{20} - g_{10}) + \underbrace{\frac{g_{21}+g_{11}}{2}}_{b}(M_{2i} - M_{1i}) + \underbrace{\frac{(g_{21}-g_{11})}{2}}_{d}(M_{2i} + M_{1i}) + (\epsilon_{Y_{2i}} - \epsilon_{Y_{1i}})$$

Next we center the sum term, so the intercept has the interpretation of the predicted difference in *Y* for someone with no difference in *M*'s but is average on *M*'s.

$$Y_{2i} - Y_{1i} = c' + b(M_{2i} - M_{1i}) + d(M_{2i} + M_{1i} \ - (\overline{M_2 + M_1})) + (\epsilon_{Y_{2i}} - \epsilon_{Y_{1i}})$$

where $\quad c' = \left(g_{20} - g_{10} + d(\overline{M_2 + M_1})\right)$

Intercept is predicted *outcome* when all regressors are zero. This means predicted difference in *Y* when there is no difference in *M* and a person is average on the sum of *M*.

# Analysis using Judd et al. (2001)

4. Does the difference in communal goal affordance account for all the difference in interest?

With the data: Is there a significance difference in interest predicted when there is no difference in communal goals?

```
compute comm_sumc = comm_G+comm_I- 8.325490.
EXECUTE.
regression dep = int_diff /method = enter comm_diff comm_sumc.
```

**Coefficients**[a]

| Model | | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. |
|---|---|---|---|---|---|---|
| | | B | Std. Error | Beta | | |
| 1 | (Constant) | -.981 | .388 | | -2.527 | .015 |
| | comm_diff | .590 | .135 | .526 | 4.385 | .000 |
| | comm_sum | -.275 | .216 | -.153 | -1.272 | .210 |

a. Dependent Variable: int_diff

# Analysis using Judd et al. (2001)

1. On average, is interest higher in the group work condition?
2. On average, is communal goal affordance higher in the group work condition?
3. Does difference in communal affordance predict a difference in interest?
4. Does the difference in communal goal affordance account for all the difference in interest?

**According to Judd, Kenny, and McClelland we do not have a mediated effect!**

Because there is no evidence that interest is higher in the group work condition, the Judd et al. (2001) method would conclude there is not mediation.

# Judd et al. Criticisms and Misuses

All criticisms of the causal steps approach apply to this approach:

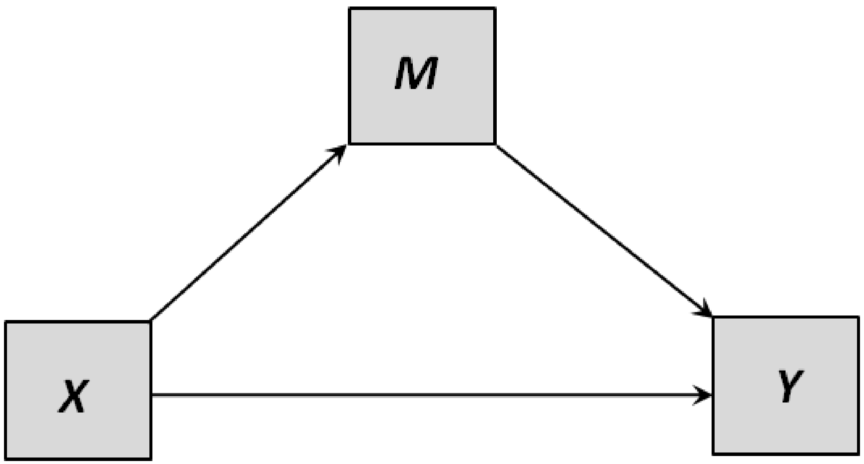
- There is no explicit quantification of the indirect effect
  - Inference about an indirect effect should be the result of a test on a *quantification* of the indirect effect
- Requiring that there must be a total effect is too restrictive
  - The direct and indirect effect could be of opposite sign
  - There is greater power to detect the indirect effect than total effect *(Judd, Kenny, 2014, Psych Science)*

This method has been used by a variety of researchers:

- Approximately 800 citing papers, with around 300 using this method
- Many researchers do not report or estimate the partial regression coefficient for the sum of the mediators
- Because the estimate of the indirect effect is not made explicit, researchers often misinterpret the coefficients
  - *b* path is often interpreted as indirect effect
- Extensions to more complicated models have been poorly implemented until recently

# Can we think about it like a path analysis?

**Analytic Goal:** **Can <u>group work</u> in computer science classes increase women's <u>interest</u> by increasing their perception that computer science is <u>communal</u>?**




Where is *X* in the data?

| $Y_1$ int_I | $Y_2$ int_G | $M_1$ comm_I | $M_2$ comm_G |
|---|---|---|---|
| 1.50 | 4.00 | 1.00 | 6.80 |
| 2.75 | 3.25 | 2.00 | 5.40 |
| 5.75 | 2.50 | 3.20 | 3.60 |
| 3.50 | 5.75 | 1.60 | 5.20 |
| 2.25 | 2.00 | 4.40 | 4.60 |
| 1.50 | 1.75 | 3.00 | 5.00 |
| 2.50 | 4.25 | 4.20 | 4.40 |
| 6.00 | 1.75 | 4.80 | 2.40 |
| 3.00 | 2.00 | 2.60 | 5.80 |
| 4.00 | 5.25 | 1.60 | 5.00 |
| 5.00 | 5.00 | 4.60 | 6.20 |
| 2.00 | 1.75 | 3.80 | 4.20 |
| 1.00 | 1.75 | 2.60 | 3.20 |
| 1.25 | 4.50 | 1.00 | 6.00 |
| 5.75 | 4.50 | 2.60 | 6.00 |
| 3.25 | 4.75 | 3.00 | 6.20 |
| 2.75 | 2.25 | 4.80 | 4.60 |
| 5.50 | 2.00 | 4.00 | 7.00 |
| 1.75 | 5.25 | 1.60 | 5.60 |
| 4.00 | 5.50 | 1.80 | 5.40 |
| 2.25 | 4.00 | 2.20 | 4.80 |
| 4.00 | 6.50 | 2.00 | 6.80 |
| 5.00 | 4.50 | 3.20 | 6.00 |
| 5.00 | 3.75 | 4.00 | 4.80 |
| 4.75 | 5.25 | 1.20 | 6.60 |

# Advantages of a path analytic approach

Provides an estimate of the indirect, total, and direct effects

- Allows us to conduct inferential tests directly on an estimate of the indirect effect

Connects researchers understanding of between-subjects mediation to within-subjects mediation

- Reduce misinterpretation of regression coefficients

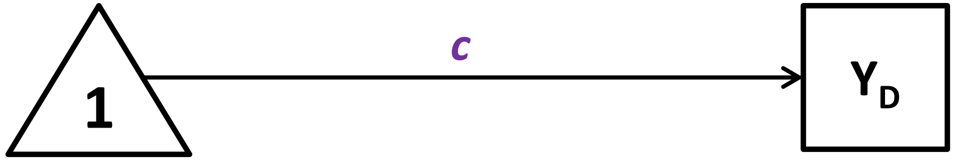
Using a path analytic framework will help extend the simple mediation model to more complicated questions

- Multiple mediators
- Moderated mediation
- Integration of between and within-subjects designs

# Path-Analytic Approach

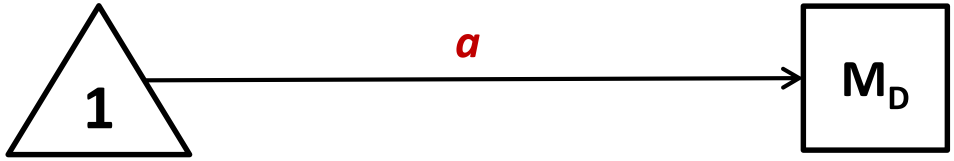
**Total Effect (*c*):** The effect of our presumed cause (*X*) on our outcome (*Y*), without controlling for any other variables. (i.e. mean difference in outcome between the two conditions).

$$Y_{2i} - Y_{1i} = c + \epsilon_{Y_i^*}$$




***a*-path:** The effect of our presumed cause (*X*) on our mediator *(M).* (i.e. mean difference in mediator between the two conditions).

$$M_{2i} - M_{1i} = a + \epsilon_{M_i}$$

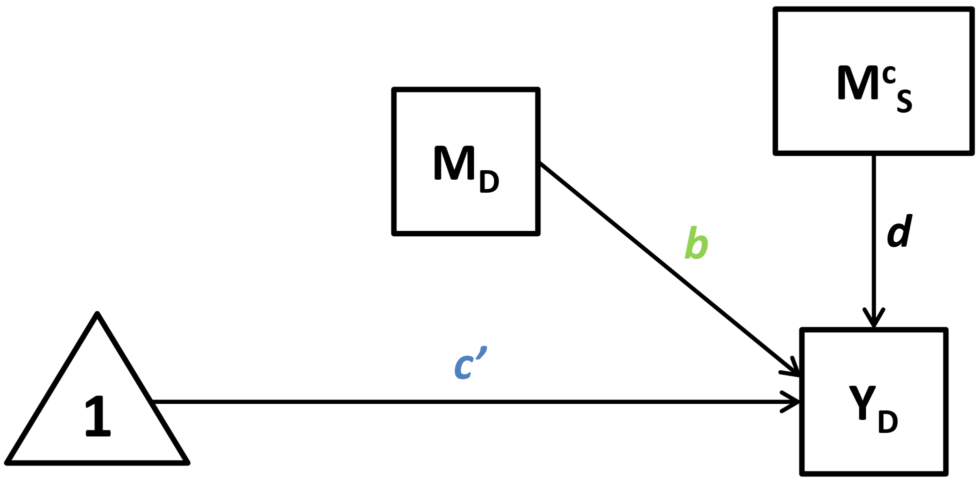


# Path-Analytic Approach

***b*-path:** The effect of our mediator (*M*) on the outcome (*Y*) while controlling for *X*. (i.e. predicted difference in *Y* for two people with the same score on *X* but who differ on *M* by one unit).

**Direct effect (*c'*):** The effect of our presumed cause (*X*) on *Y* while controlling for *M*. (i.e. predicted difference in *Y* for two people who differ by one unit on *X* but with the same score on *M*)

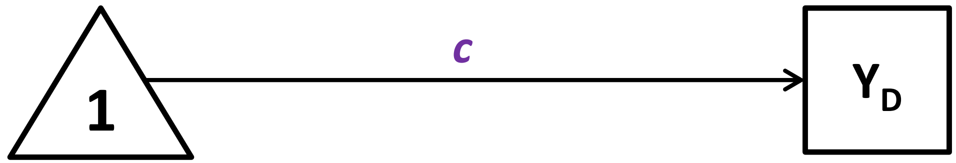
$$Y_{2i} - Y_{1i} = c' + b(M_{2i} - M_{1i}) + d(M_{2i} + M_{1i} - (\overline{M_2 + M_1})) + \epsilon_{Y_i}$$



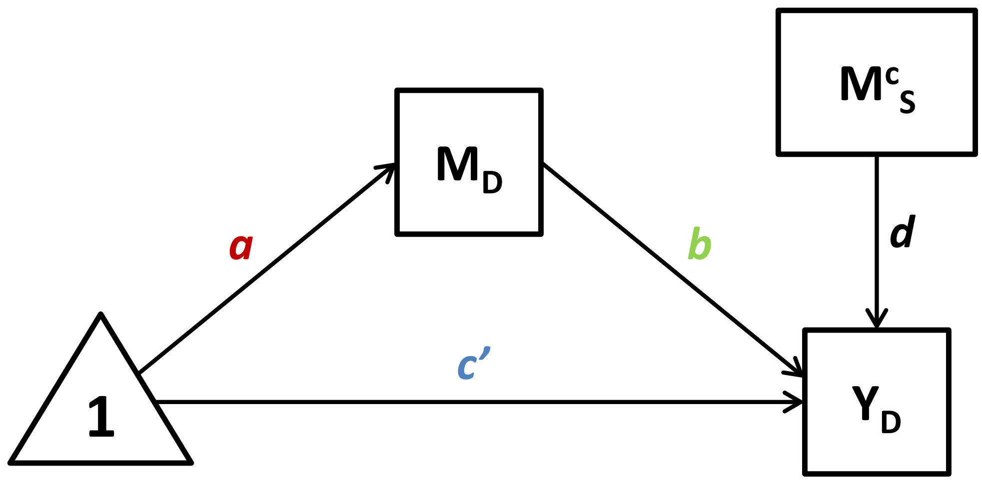

[c] Indicates mean centered

# Path-Analytic Approach

**Indirect Effect (*ab*):** Product of effect of *X* on *M*, and effect of *M* on *Y* controlling for *X*. The effect of *X* on *Y* through *M*.



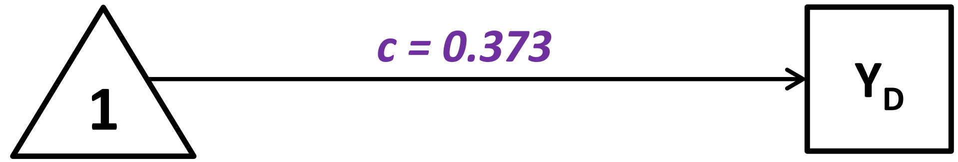

$^{c}$ Indicates mean centered



# Within Subjects: Path Estimates

Total Effect *c*: (Regress $Y_D$ on a constant)

$$\widehat{Y_D} = c$$
$$\widehat{Y_D} = .373$$



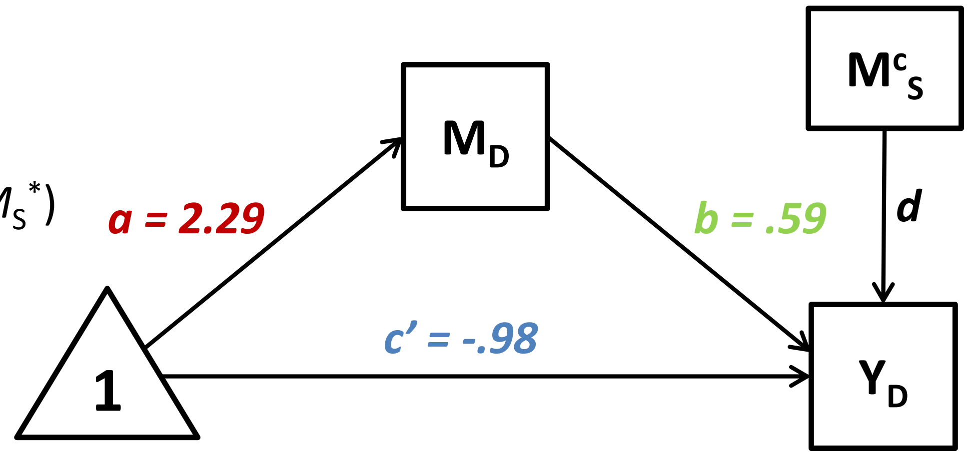

*a* path: (Regress $M_D$ on a constant)

$$\widehat{M_D} = a$$
$$\widehat{M_D} = 2.29$$

$^c$ Indicates mean centered

*b* path and *c′* path: (Regress $Y_D$ on $M_D$ and $M_S{}^*$)

$$\widehat{Y_D} = c' + b_1 M_D + d M_S^c$$
$$\widehat{Y_D} = -.98 + .59 M_D - .28 M_S^c$$




A one unit increase in the difference in communal goal affordance is expected to result in a .59 unit increase in the difference in interest.
People with no difference in communal goal affordance perceptions are expected to be .98 units more interested in the individual class than the group work class .

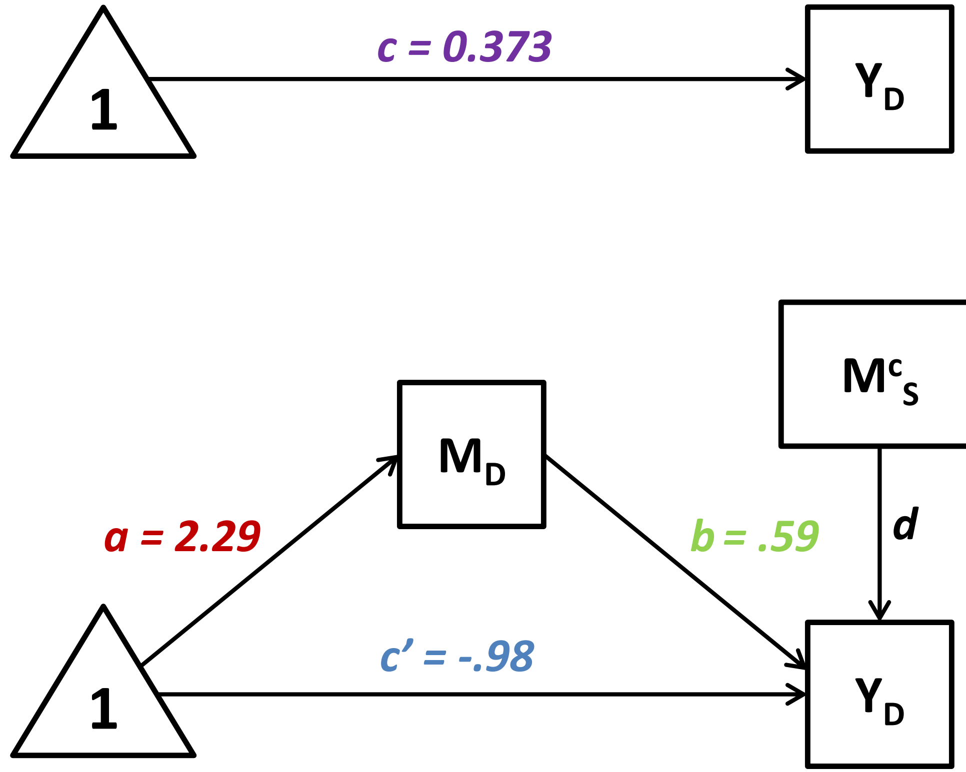
Note: $M_s$ must be mean centered for c′ to have intended interpretation

# Data Example: Partitioning effect of X on Y

**The effect of *X on Y* partitions into two components: direct and indirect, in the usual way.**

| | | | | | | |
|---|---|---|---|---|---|---|
| c | = | c' | + | a | x | b |
| .373 | = | -.98 | + | 2.29 | x | .59 |
| .373 | = | -.98 | + | 1.35 | | |

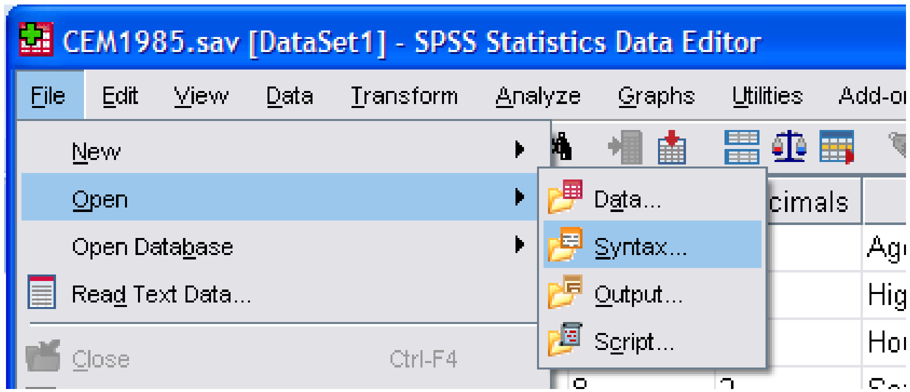
**We can conduct inferential tests on the estimate of the indirect effect as in any other mediation analysis.**

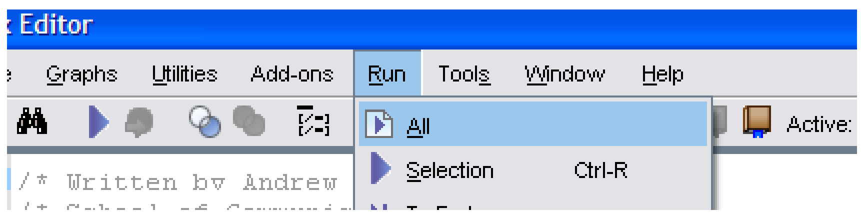



MEMORE has three methods of inference for the indirect effect available: bootstrapping, Monte Carlo confidence intervals, Sobel Tests

# Teaching your package MEMORE

MEMORE is a command which must be taught and re-taught to your statistical package (SPSS) every time you open the package. To teach your program the MEMORE command, open the memore.sps file and run the script exactly as is.




SPSS now knows a new command called MEMORE

# Writing MEMORE Syntax

MEMORE has 2 required arguments: `Y` and `M`

```
MEMORE m= comm_G comm_I /y = int_G int_I /normal=1/samples=10000
    /conf = 90 /model = 1.
```

**M** is your list of mediators (order matters)
**Y** is you list of outcomes (order should be matched to the order in the M list)

Arguments:

- **model** specifies the model you are interested. The default is 1, mediation. Moderation models are 2 and 3.
- **normal = 1** asks for Sobel test
- **samples** corresponds to the number of bootstrap/MC samples you would like
- **conf** specifies level of confidence you want (default is 95)
- **mc = 1** asks for Monte Carlo confidence intervals
- **bc = 1** asks for bias corrected bootstrap confidence intervals

# Using MEMORE for CompSci WS data

```
MEMORE m= comm_G comm_I /y = int_G int_I/model = 1.
```

```
********************** MEMORE Procedure for SPSS Version 2.Beta ************************

                           Written by Amanda Montoya

                    Documentation available at akmontoya.com

*************************************************************************************

Model:
  1

Variables:
Y = int_G    int_I
M = comm_G   comm_I

Computed Variables:
Ydiff =          int_G     -       int_I
Mdiff =          comm_G    -       comm_I
Mavg =  (        comm_G    +       comm_I    )        /2       Centered

Sample Size:
  51

*************************************************************************************
```

First part of output repeats what you told MEMORE to do. Always double check that this is correct!

# Using MEMORE for CompSci WS data

```
MEMORE m= comm_G comm_I /y = int_G int_I/model = 1.
```

```
**************************************************************************
Outcome: Ydiff =  int_G       -       int_I

Model
          Effect         SE          t          p       LLCI       ULCI
'X'        .3725      .2795     1.3330      .1886     -.1888      .9339

Degrees of freedom for all regression coefficient estimates:
  50

**************************************************************************
Outcome: Mdiff =  comm_G      -       comm_I

Model
          Effect         SE          t          p       LLCI       ULCI
'X'       2.2941      .2491     9.2108      .0000     1.7938     2.7944

Degrees of freedom for all regression coefficient estimates:
  50

**************************************************************************
```

Outcome variable

$c = .37$

$a = 2.29$

First few sections are regression models involved in the mediation analysis. This is the model of *Y* from *X*, therefore this is the model which produces the estimate of *c*

# Using MEMORE for CompSci WS data

```
MEMORE m= comm_G comm_I /y = int_G int_I/model = 1.
```

```
*****************************************************************************
Outcome: Ydiff =  int_G      -        int_I

Model Summary
          R        R-sq        MSE          F        df1        df2          p
      .5639       .3180     2.8299    11.1909     2.0000    48.0000      .0001

Model
              coeff          SE           t           p        LLCI        ULCI
'X'          -.9814       .3884     -2.5269       .0149     -1.7623      -.2005
Mdiff         .5902       .1346      4.3845       .0001       .3195       .8608
Mavg         -.5505       .4328     -1.2718       .2096     -1.4208       .3198

Degrees of freedom for all regression coefficient estimates:
  48
```

This is the model predicting $Y_D$ from a constant, $M_D$, and $M^c_{avg}$ therefore this model gives us an estimate of *b* and *c'*

$c' = -.98$

$b = .590$

# Using MEMORE for CompSci WS data

```
MEMORE m= comm_G comm_I /y = int_G int_I/model = 1.
```

```
********************** TOTAL, DIRECT, AND INDIRECT EFFECTS **********************

Total effect of X on Y
     Effect         SE          t         df          p       LLCI       ULCI
      .3725      .2795     1.3330    50.0000      .1886     -.1888      .9339

Direct effect of X on Y
     Effect         SE          t         df          p       LLCI       ULCI
     -.9814      .3884    -2.5269    48.0000      .0149    -1.7623     -.2005

Indirect Effect of X on Y through M
          Effect     BootSE   BootLLCI   BootULCI
Ind1      1.3540      .3260      .6827     1.9653

Indirect Key
Ind1   X        ->       M1diff   ->       Ydiff
```

Important effects for mediation and inference about these effects

Based on a 95% bootstrap confidence interval we have evidence of mediation!

# Turning off the *XM* interaction

$$Y_{2i} - Y_{1i} = c' + b(M_{2i} - M_{1i}) + d(M_{2i} + M_{1i} - (\overline{M_2 + M_1})) + \epsilon_{Y_i}$$

When we estimate this regression model, we allow the relationship between *M* and *Y* to differ by instance (*X*). This is like allowing for an interaction between *X* and *M* when estimating *Y*.

We do this by including the sum term in the regression model.

*d* estimates the difference in the relationship between $M_1 \rightarrow Y_1$ and $M_2 \rightarrow Y_2$. If we fix this coefficient to zero (do not include the sum term in the model) we fix the interaction to zero.

```
MEMORE m= comm_G comm_I /y = int_G int_I /xmint = 0 /model = 1.
```

# Turning off the XM interaction

```
MEMORE m= comm_G comm_I /y = int_G int_I /xmint = 0 /model = 1.
```

**No interaction**

**Interaction**

```
Outcome: Ydiff =  int_G     -       int_I

Model
        Effect         SE          t          p       LLCI       ULCI
'X'      .3725      .2795     1.3330      .1886     -.1888      .9339

Degrees of freedom for all regression coefficient estimates:
  50
```

*c* = .3725 (.2795)

```
**************************************************************************
Outcome: Mdiff =  comm_G    -       comm_I

Model
        Effect         SE          t          p       LLCI       ULCI
'X'     2.2941      .2491     9.2108      .0000     1.7938     2.7944

Degrees of freedom for all regression coefficient estimates:
  50

**************************************************************************
```

*a* = 2.2941 (.2491)

# Turning off the XM interaction

```
MEMORE m= comm_G comm_I /y = int_G int_I /xmint = 0 /model = 1.
```

**No interaction** | **Interaction**

```
**************************************************************************
Outcome: Ydiff =  int_G      -        int_I

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5432      .2950     2.8655    20.5060     1.0000    49.0000      .0000

Model
           coeff         SE          t          p       LLCI       ULCI
'X'      -1.0257      .3893    -2.6349      .0112    -1.8079     -.2434
Mdiff      .6095      .1346     4.5284      .0000      .3390      .8799
```

*c* = -.9814(.2795)

*b* = .5902 (.1346)

*d* = -.5505 (.4328)

```
 Indirect Effect of X on Y through M
           Effect     BootSE   BootLLCI   BootULCI
 Ind1      1.3982      .3082      .8034     2.0156
```

*ab* = 1.3540 [.6827,1.9653]

Ultimately results are mostly unchanged, but that is not always the case.

# Writing up a Repeated Measures Mediation Analysis

Tips:

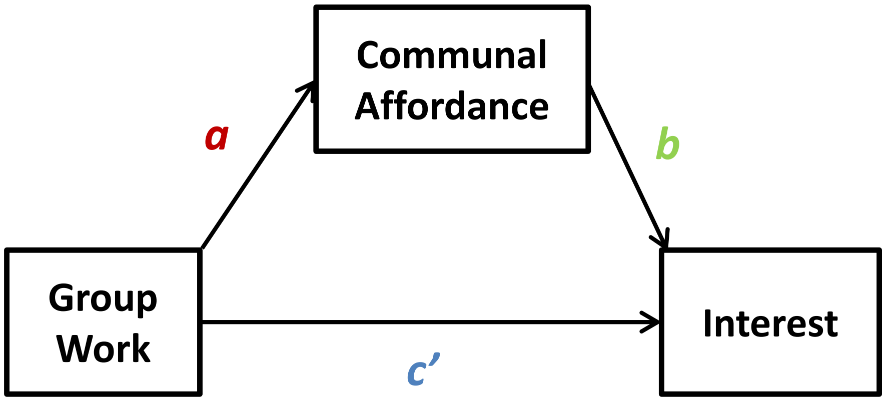
- Walk the reader through the steps of the mediation in a way that is intuitive.
  - Include interpretations of the results: b.e.g. “The total effect was significant, $p < .05$”
- Use equations and numbers *where helpful*.
- Avoid using computational variable names (e.g. RESPAPPR)
- Avoid causal language if it is not supported by your research design.
- Pick one inferential method and report it
- Read the write ups of other’s mediation analyses

**Is the effect of group work on class interest mediated by communal goal affordance of the class?**

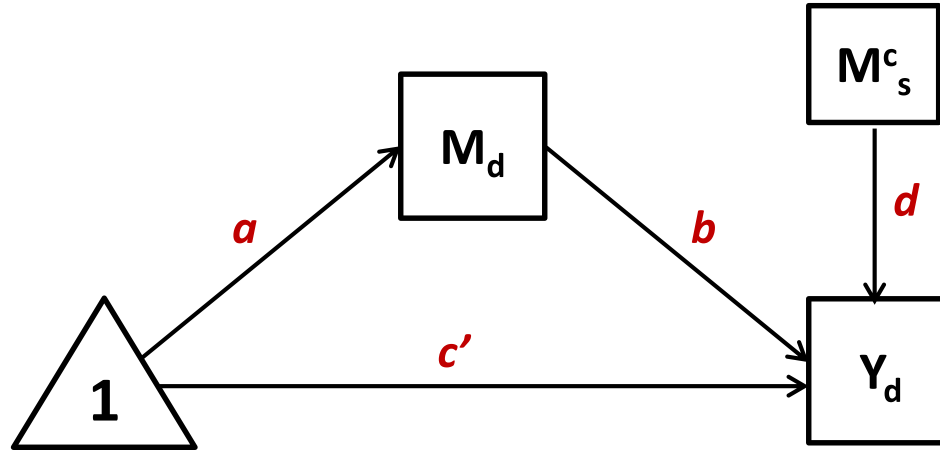
Overall there was no evidence of a total effect of group work on interest in computer science classes, we estimate that individuals were .37 units higher on interest in group work than individual work classes ($p = .19$). The class with group work was rated 2.29 units higher on communal goal affordance than the class with individual work ($p < .001$). A one unit increase in perception of communal goal affordance increased interest in the class by .59 units ($p = .0001$), and the relationship between communal goal affordance and interest in a class did not depend on condition ($p = .21$). The effect of group work on interest through communal goal fulfillment was different from zero ($ab = 1.35$, 95% Bootstrap CI [.68, 1.96]). This means that we expect women to be 1.35 units more interested in a computer science class with group work compared to one without group work, through the effect of group work on communal goal affordance, and the subsequent effect of communal goal affordance on interest. There was a significant direct effect between group work and interest ($c' = -.98$, $p = .01$). This indicates that there may be some other process, separate from communal goal affordance, which is actually deterring women from computer science classes with group work.

# Visualizations

I suggest using both a conceptual and statistical visualization in order to help the reader understand the process you are testing.




**Conceptual**




**Statistical**

Tips:

- Providing a conceptual diagram helps the readers understand the process you are interested in.
- Providing a statistical diagram helps readers understand how you estimated the model, and that you did it correctly.
- Provide path estimates on statistical diagram or in a table.
- Don't forget to report the path estimates and statistics for the *d* path. It's important!

# Common Questions

**Can this method be used for more than two conditions?**

YES! Judd, Kenny, and McClelland (2001) describe a system for setting up contrasts among conditions, and testing the indirect effects of those contrasts.

I recommend reading Hayes & Preacher (2014) on mediation analysis with a multicategorical IV if you want to try this out. I am happy to give instructions on how to trick MEMORE into doing this. There will be functionality (soonish) for MEMORE to do this.

**ALTERNATIVES:** Some of the other repeated-measures mediation options are more appropriate if you have more than two conditions (especially longitudinal), so take a look at those when thinking about these options.

**Can I use multiple mediators?**

YES! MEMORE is already set up to do parallel mediation with up to 10 sets of mediators and serial mediation with up to **five** sets of mediators (See Montoya & Hayes 2017 for instructions).

**Can we do conditional process models?**

VERY SOON!

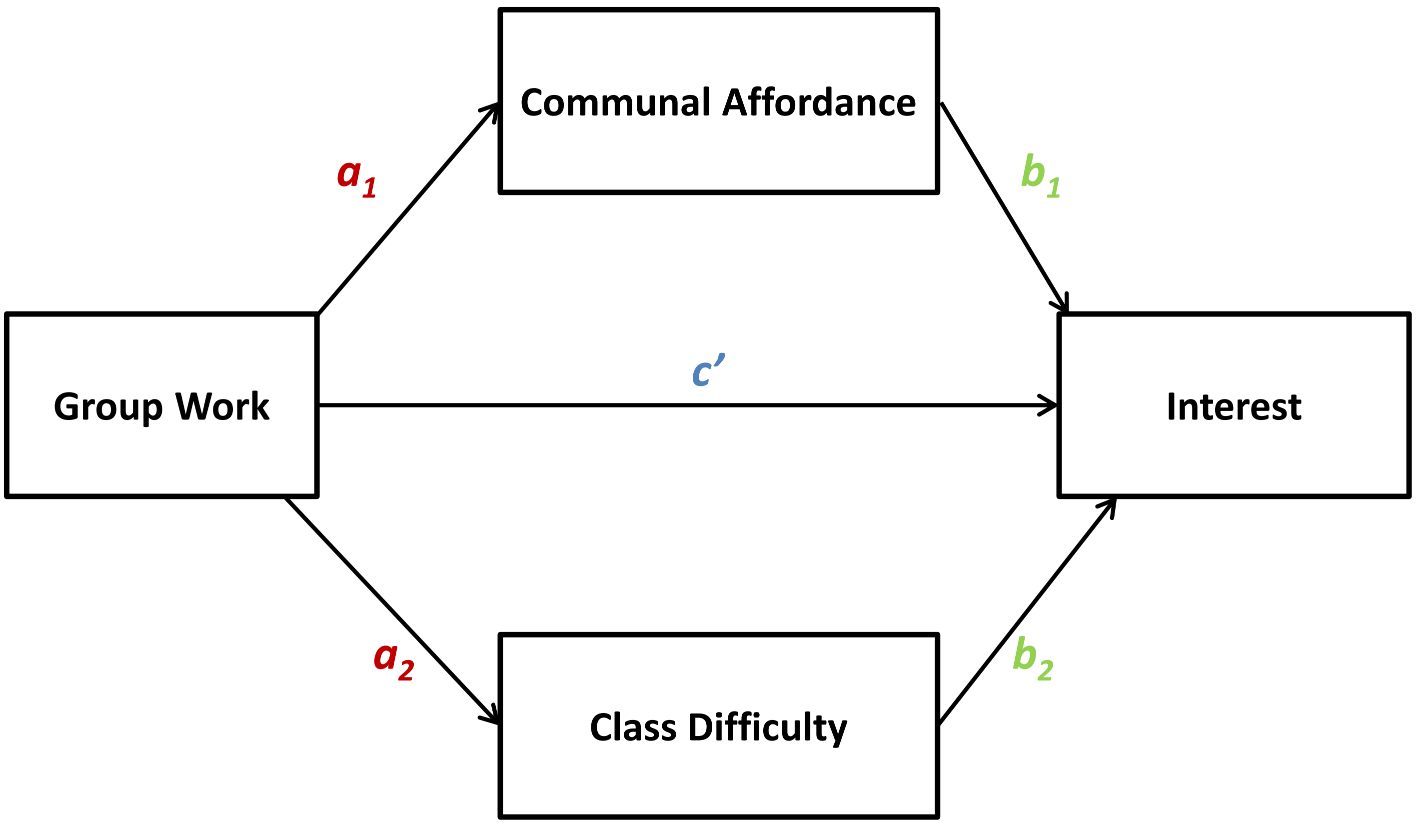
**How do I control for covariates?**

All of MEMORE's mediation analyses are within-person models, so you do not need to control for any between subjects variables such as age, gender, big-5.

Sometimes there are covariates which change within a person across conditions that you want to account for, this can be done by treating this additional variable as another set of mediators.

# Using MEMORE for CompSci WS data

**Do people just like group work classes because they are easier?**



# Using MEMORE for CompSci WS data

**Do people just like group work classes because they are easier?**

```
MEMORE m = comm_I comm_G diff_I diff_G /y = int_I int_G.
```

Notice that we are now **controlling** for difficulty of the class when estimating the effect of communal goal affordance on interest!

```
**************************************************************************
Outcome: Ydiff =  int_I     -       int_G

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .6307      .3978     2.6073     7.5978     4.0000    46.0000      .0001

Model
             coeff         SE          t         df          p       LLCI       ULCI
'X'          .9172      .3815     2.4042    46.0000      .0203      .1493     1.6851
M1diff       .4847      .1448     3.3460    46.0000      .0016      .1931      .7762
M2diff      -.4123      .1878    -2.1952    46.0000      .0332     -.7904     -.0342
M1avg        .5160      .4157     1.2411    46.0000      .2209     -.3209     1.3528
M2avg       -.3781      .2879    -1.3133    46.0000      .1956     -.9577      .2014
```

# Using MEMORE for CompSci WS data

**Do people just like group work classes because they are easier?**

```
MEMORE m = comm_I comm_G diff_I diff_G /y = int_I int_G.
```

```
********************** TOTAL, DIRECT, AND INDIRECT EFFECTS **********************

Total effect of X on Y
     Effect         SE          t         df          p       LLCI       ULCI
     -.3725      .2795    -1.3330    50.0000      .1886     -.9339      .1888

Direct effect of X on Y
     Effect         SE          t         df          p       LLCI       ULCI
      .9172      .3815     2.4042    46.0000      .0203      .1493     1.6851

Indirect Effect of X on Y through M
          Effect     BootSE   BootLLCI   BootULCI
Ind1     -1.1119      .3812    -1.8531     -.3522
Ind2      -.1779      .1160     -.4465      .0000
Total    -1.2897      .3507    -1.9566     -.5612

Indirect Key
Ind1   X        ->       M1diff   ->       Ydiff
Ind2   X        ->       M2diff   ->       Ydiff
```

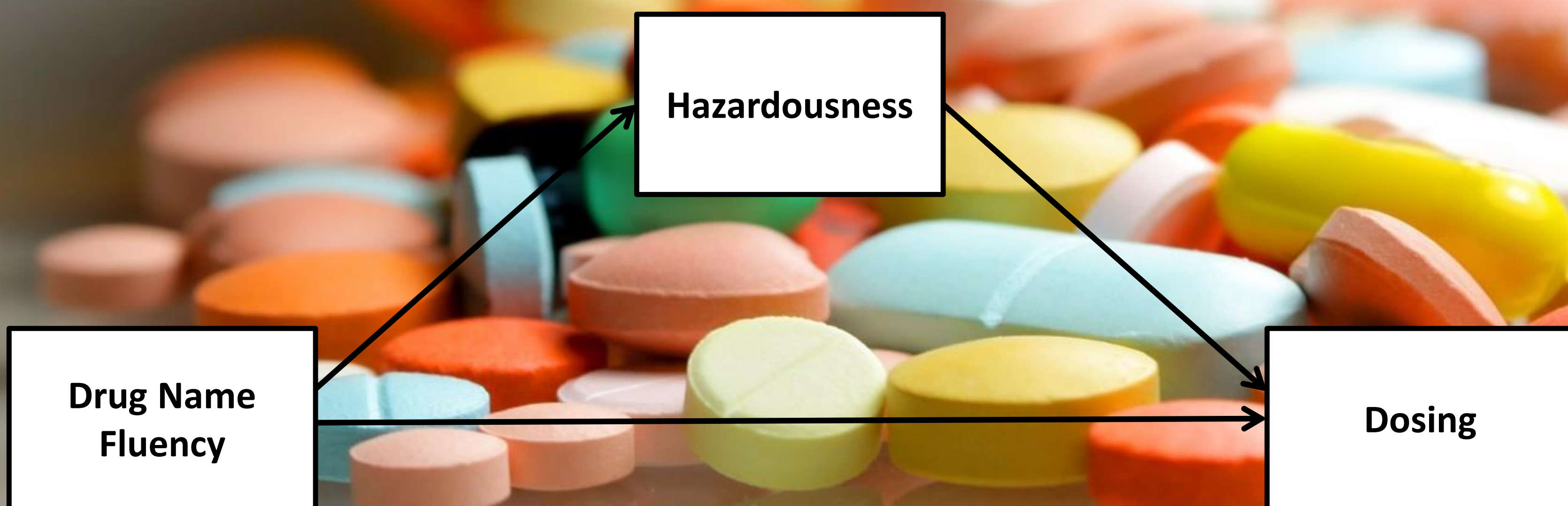
Controlling for difficulty, there is still a significant indirect effect through communal affordance!

# Example: Drug Name Fluency

Dohle, S., & Montoya, A. K. (2017). The dark side of fluency: Fluent names increase drug dosing. *Journal of Experimental Psychology: Applied*, *23*(3), 231 – 239.

**Research Question:** Can the name of drugs impact how hazardous they seem and how much people are willing to dose the drugs?

Imagine you have a cold, and there are a variety of medications available including (a) Fastinorbine and (b) Cytrigmcmium. Which seems more hazardous? Which are you willing to dose more of?



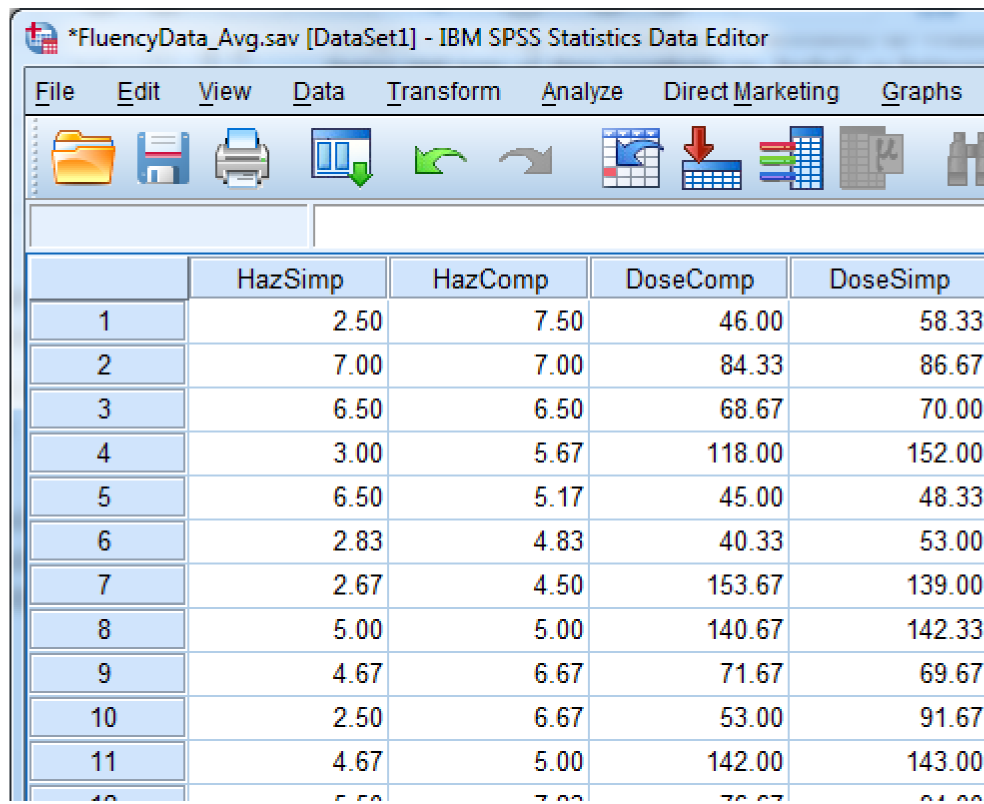
# Example: Drug Name Fluency

Dohle, S., & Montoya, A. K. (2017). The dark side of fluency: Fluent names increase drug dosing. *Journal of Experimental Psychology: Applied*, *23*(3), 231 – 239.

Participants (N = 70) were asked to imagine they had the flu, and 6 different drugs were provided to treat the drug. Participants poured the dose they would feel comfortable taking at maximum into a plastic cup. Each person judged drugs with simple or complex names (3 of each). Responses on the measured variables were averaged across the 3 drugs (but later we'll look at what happens when we treat these separately).

**Measured Variables:**

- Dosage in mL
  - Variable name: `Dose`
  - 0 mL – 200mL
- Hazardousness of drug
  - Variable name: `Haz`
  - Average of two questions:
    - Hazardousness (1-7)
    - Dangerousness (1-7)




| | HazSimp | HazComp | DoseComp | DoseSimp |
|---|---|---|---|---|
| 1 | 2.50 | 7.50 | 46.00 | 58.33 |
| 2 | 7.00 | 7.00 | 84.33 | 86.67 |
| 3 | 6.50 | 6.50 | 68.67 | 70.00 |
| 4 | 3.00 | 5.67 | 118.00 | 152.00 |
| 5 | 6.50 | 5.17 | 45.00 | 48.33 |
| 6 | 2.83 | 4.83 | 40.33 | 53.00 |
| 7 | 2.67 | 4.50 | 153.67 | 139.00 |
| 8 | 5.00 | 5.00 | 140.67 | 142.33 |
| 9 | 4.67 | 6.67 | 71.67 | 69.67 |
| 10 | 2.50 | 6.67 | 53.00 | 91.67 |
| 11 | 4.67 | 5.00 | 142.00 | 143.00 |

# Example: Drug Name Fluency

Dohle, S., & Montoya, A. K. (2017). The dark side of fluency: Fluent names increase drug dosing. *Journal of Experimental Psychology: Applied*, *23*(3), 231 – 239.

1. Estimate the proposed model (Fluency -> Hazardousness -> Dosage) using MEMORE
2. Turn off the *XM* interaction
3. Find estimates of the following paths: *a*, *b*, *c*, *c'*
4. Of the following inferential methods, which support the hypothesized mediation model (use $\alpha = 0.05$ or 95% confidence intervals):
   Percentile bootstrap CIs, Monte Carlo CIs, Sobel Test / Normal Theory
5. Practice writing up some of the results explored above.

**Take a break and work on this, I'll wander around to help.**

# Using MEMORE for Drug Fluency data

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp /xmint = 0.
```

```
**************************************************************************
Outcome: Ydiff =  DoseSimp  -       DoseComp

Model
         Effect          SE           t           p        LLCI        ULCI
'X'     11.0476      1.5770      7.0055       .0000      7.9016     14.1937

Degrees of freedom for all regression coefficient estimates:
  69

**************************************************************************
Outcome: Mdiff =  HazSimp   -       HazComp

Model
         Effect          SE           t           p        LLCI        ULCI
'X'     -2.1048       .1848    -11.3893       .0000     -2.4734     -1.7361

Degrees of freedom for all regression coefficient estimates:
  69

**************************************************************************
```

*c = 11.05*

*Interpretation?*

*a = -2.10*

*Interpretation?*

On average, participants dosed 11.05 mL more of the simply named drug than the complex named drug ($t(69) = 7.06$, $p < .001$).

Participants thought the complex drug was 2.10 points more hazardous than the simply named drug, on average ($t(69) = 11.39$, $p < .001$).

# Using MEMORE for Drug Fluency data

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp.
```

```
**************************************************************************************
Outcome: Ydiff =  DoseSimp  -       DoseComp

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .4020      .1616   148.1029    13.1047     1.0000    68.0000      .0006

Model
            coeff         SE          t          p       LLCI       ULCI
'X'        3.8280     2.4684     1.5508      .1256    -1.0978     8.7537
Mdiff     -3.4302      .9475    -3.6200      .0006    -5.3210    -1.5393

Degrees of freedom for all regression coefficient estimates:
  68
```

*c' = 3.83* *Interpretation?*

*b = -3.43* *Interpretation?*

After controlling for hazardousness, participants were expected to dose 3.8 mL more of the simple drug. This effect was not significantly different than zero ($t(68) = 1.55$, $p = .13$).

A one unit increase in the difference in perceived hazardousness between conditions results in a 3.43 unit decrease in the difference in dosage ($t(68) = 3.62$, $p < .001$).

A one unit increase in perceived hazardousness results in a 3.43 unit decrease in dosage, averaged across fluency conditions ($t(68) = 3.62$, $p < .001$).

# Using MEMORE for Drug Fluency data

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp.
```

```
************************* TOTAL, DIRECT, AND INDIRECT EFFECTS *************************

Total effect of X on Y
     Effect         SE          t         df          p       LLCI       ULCI
    11.0476     1.5770     7.0055    69.0000      .0000     7.9016    14.1937

Direct effect of X on Y
     Effect         SE          t         df          p       LLCI       ULCI
     3.8280     2.4684     1.5508    68.0000      .1256    -1.0978     8.7537

Indirect Effect of X on Y through M
          Effect     BootSE   BootLLCI   BootULCI
Ind1      7.2197     1.8940     3.8590    11.1609

Indirect Key
Ind1   'X'     ->      M1diff   ->      Ydiff
```

*ab = 7.22*   *Interpretation?*

Participants were dosed simple drugs 7.22 mL more, through the effect of simple drugs on hazardousness and the subsequent effect of hazardousness on dosage (Percentile CI = [3.86, 11.16]).

Drug name fluency increased dosage indirectly effect through hazardousness by 7.22 mL (Percentile CI = [3.86, 11.16]).

Simple names were perceived as less hazardous, which then increased dosage, resulting in an indirect effect of 7.22 mL on dosage (Percentile CI = [3.86, 11.16]).

# Using MEMORE for Drug Fluency data

Methods of Inference

Percentile Bootstrap CI

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp /xmint = 0.
```

Indirect Effect of X on Y through M

| | Effect | BootSE | BootLLCI | BootULCI |
|---|---|---|---|---|
| Ind1 | 7.2197 | 1.8940 | 3.8590 | 11.1609 |

Monte Carlo CI

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp /xmint = 0 /mc = 1.
```

Indirect Effect of X on Y through M

| | Effect | MCSE | MCLLCI | MCULCI |
|---|---|---|---|---|
| Ind1 | 7.2197 | 2.0916 | 3.2369 | 11.3834 |

Sobel Test / Normal Theory

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp /xmint = 0 /normal = 1.
```

Normal Theory Tests for Indirect Effect

| | Effect | SE | Z | p |
|---|---|---|---|---|
| Ind1 | 7.2197 | 2.0927 | 3.4500 | .0006 |

# Mediation

- Between Subjects Mediation
  - Path analytic approach
  - Interpretation
  - Estimation
  - Inference

- Repeated Measures Data

- Two-Instance Repeated-Measures Mediation
  - Judd Kenny and McClelland (2001)
  - Path analytic approach
  - Estimation of Indirect Effects
  - MEMORE
  - Reporting (Writing and Figures)
  - Common Questions

New York Elegance

# Part II: Mediation and Multilevel Modeling

One of the primary assumptions of Ordinary least squares (OLS) regression is that each observation is independent of all other observations.

Ordinary least squares (OLS) regression is not directly applicable when data are nested.

- Students nested within classrooms
- Employees nested within companies
- Repeated measurements nested within individuals

Responses from employees within the same company tend to be more related to each other than responses from employees in different companies.

This violates the assumption of independence.

Several methods are available for accounting for this dependence, but today we will focus on multilevel/mixed modeling.

# Multilevel Modeling

What's a level?

Students (Level 1) within classrooms (Level 2)
Employees (Level 1) within companies (Level 2)
Repeated measurements (Level 1) within individuals (Level 2)

Multilevel models are often expressed either as separate equations for the different levels of the model, or as one combined model.

Let *i* denote Level 1 units and *j* denote Level 2 units

$X_{ij}$: Person *i* in group *j*'s observation on *X*
$Y_{ij}$: Person *i* in group *j*'s observation on *Y*
$W_j$: Group *j*'s observation on *W* (Level 2 characteristics (e.g., Company size))

# Two-Level Unconditional Model

Let's predict an outcome at Level 1 using a predictor from Level 1

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$b_{0j}$: The expected value of *Y* for someone in group *j* with $X_{ij} = 0$. Notice this is allowed to vary by group! This is the **intercept** for group *j*.

$b_{1j}$: The expected difference in *Y* for two people in the same group *j* that differ by 1 unit on $X_{ij}$. This is the **slope** for group *j.*

$e_{ij}$: The error in estimating $Y_{ij}$. $e_{ij} \sim N(0, \sigma^2)$

Even if you're not familiar with multilevel models, this should look familiar to what we do in regression. Except the intercept and slope are allowed to randomly vary across groups. We call these *random effects*.

# Two-Level Unconditional Model

We also create a Level 2 Model, for the intercept and slope:

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$$b_{0j} = b_0 + u_{0j}$$

$$b_{1j} = b_1 + u_{1j}$$

Notice that it's the *u*'s that make the "random effects" random. By allowing the intercept and slope to vary across groups, we soak up all the "dependence" in the data.

$b_0$ is the grand-mean intercept (i.e., the average intercept across groups)

$b_1$ is the grand-mean slope (i.e., the average slope across groups)

We assume that $(u_{0j}, u_{1j}) \sim MVN(0, T)$ where $T = \begin{bmatrix} \tau_{00} & \tau_{01} \\ \tau_{01} & \tau_{11} \end{bmatrix}$

The random effects are not individuals estimated, but rather we estimate their covariance matrix as well as the grand-mean intercept and slope.

# Simplifying the Model

Not all coefficients need to be random. For example the intercept could be random but the slope could vary across groups:

$$Y_{ij} = b_{0j} + b_1 X_{ij} + e_{ij}$$

$b_{0j} = b_0 + u_{0j}$

$b_1 = b_1$

$b_0$ is the grand-mean intercept (i.e., the average intercept across groups)

$b_1$ is the slope (assumed to be the same for all groups)

This is like a special case where we assume $\tau_{11} = 0$

We will mostly use the case where we have random-slopes as this is what adds complexity to mediation in multilevel models.

# The Combined Model

Sometimes it's clearer to represent both the Level 1 and Level 2 equations together in a *combined model*. We plug in the Level 2 equations in their spots in the Level 1 model

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$$Y_{ij} = (b_0 + u_{0j}) + (b_1 + u_{1j})X_{ij} + e_{ij}$$

We can combine and rearrange terms to separate the parts of the model which are random and those which are not random (i.e., fixed).

$$Y_{ij} = \underbrace{\left(b_0 + b_1X_{ij}\right)}_{\text{Fixed: does not vary by group}} + \underbrace{u_{0j} + u_{1j}X_{ij} + e_{ij}}_{\text{Random: varies by group}}$$

You can see how each individual's response is a function of the **overall intercept** the **overall slope** as well as their **group's deviations** from the overall intercept and slope and a **individual-specific error.**

# Adding Level 2 Predictors

We can explain variability in the group intercept or slope using characteristics of the Level 2 units.

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$b_{0j} = b_0 + g_{01}W_j + u_{0j}$

$b_{1j} = b_1 + g_{11}W_j + u_{1j}$

$b_0$ is the expected group intercept when $W_j$ is zero.

$b_1$ is the expected group slope when $W_j$ is zero.

$g_{01}$ is how much we expect the intercept to change with a one unit change in $W_j$

$g_{11}$ is how much we expect the slope (relationship between X and Y) to change with a one unit change in $W_j$

# Adding Level 2 Predictors

We can rewrite the model as a *combined* model, by combining Level 1 and Level 2 equations:

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$$Y_{ij} = (b_0 + g_{01}W_j + u_{0j}) + (b_1 + g_{11}W_j + u_{1j})X_{ij} + e_{ij}$$

$$Y_{ij} = \underbrace{b_0 + g_{01}W_j + b_1X_{ij} + g_{11}W_jX_{ij}}_{\text{Fixed: does not vary by group}} + \underbrace{u_{1j}X_{ij} + u_{0j} + e_{ij}}_{\text{Random: varies by group}}$$

You can see in the combined equation that by including $W_j$ as a predictor of the *slope* we include an interaction between $W_j$ and $X_{ij}$.

This means the effect of *X* on *Y* depends on the value of *W*.

# Fluency Data

The Fluency data we used for within-subjects mediation (FluencyData_Avg.sav) is in wide form and we must convert it to long-form for multilevel modeling (FluencyData_Avg_long.sav).

```
VARSTOCASES
  /ID=id
  /MAKE Hazard FROM HazSimp HazComp
  /MAKE Dose FROM DoseSimp DoseComp
  /INDEX=Simple(2)
  /KEEP=
  /NULL=KEEP.

RECODE Simple (2=0) (1=1).
EXECUTE.
```

# Fluency Data

We can use the SPSS MIXED procedure to fit a multilevel model.

Let's look at the relationship between Dosage and Hazardousness using a model with a random intercept and a random slope.

```
MIXED Dose WITH Hazard
    /Fixed = Hazard | SSTYPE(3)
    /Method = REML
    /Print = G Solution Testcov
    /Random = INTERCEPT Hazard | Subject(id) COVTYPE(UN).
```

$$Y_{ij} = \left(b_0 + b_1 X_{ij}\right) + u_{0j} + u_{1j} X_{ij} + e_{ij}$$

$Y_{ij}$: Dosage for observation *i* for person *j*

$X_{ij}$: Hazardousness for observation *i* for person *j*

**Give it a try!**

# Fluency Data: Fixed Effects

```
MIXED Dose WITH Hazard
    /Fixed = Hazard | SSTYPE(3)
    /Method = REML
    /Print = G Solution Testcov
    /Random = INTERCEPT Hazard | Subject(id) COVTYPE(UN).
```

**Estimates of Fixed Effects**[a]

| Parameter | Estimate | Std. Error | df | t | Sig. | 95% Confidence Interval Lower Bound | 95% Confidence Interval Upper Bound |
|---|---|---|---|---|---|---|---|
| Intercept | 109.484321 | 4.932038 | 52.923 | 22.199 | .000 | 99.591570 | 119.377072 |
| Hazard | -4.838863 | .615111 | 33.198 | -7.867 | .000 | -6.090033 | -3.587694 |

a. Dependent Variable: Dosing Simple.

The expected dose administration of drugs is 109.48 mL given a hazardousness rating of zero ($X_{ij} = 0$). But remember this is an average across all people.

For each one unit increase in harazardousness, dose administration of drugs is expected to decrease by 4.84 mL. Remember this is an average across all people.

# Fluency Data: Random Effects

$$(u_{0j}, u_{1j}) \sim MVN(0, T) \text{ where } T = \begin{bmatrix} \tau_{00} & \tau_{01} \\ \tau_{01} & \tau_{11} \end{bmatrix}$$

**Estimates of Covariance Parameters**[a]

| Parameter | | Estimate | Std. Error | Wald Z | Sig. | 95% Confidence Interval | |
|---|---|---|---|---|---|---|---|
| | | | | | | Lower Bound | Upper Bound |
| Residual | | 58.509962 | 18.167383 | 3.221 | .001 | 31.836928 | 107.529709 |
| Intercept + Hazard [subject = id] | UN (1,1) | 1062.426086 | 309.657876 | 3.431 | .001 | 600.074242 | 1881.015899 |
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There is substantial between-person variability ($\tau_{00} = 1062.43$) in dosage of drugs with a hazardousness rating of zero.

The relationship between hazardousness and dosage varies across individuals ($\tau_{11} = 5.02$)

Those with higher-than-average dose values at $X_{ij} = 0$ (hazardousness is zero) have lower-than-average slopes for the relationship between hazardousness and dosage ($\tau_{01} = -30.09$)

# Centering Variables

There is substantial between-person variability ($\tau_{00} = 1062.43$) in dosage of drugs with a hazardousness rating of zero.

When we interpret $\tau_{00}$ we condition on the predictor being zero (i.e., Hazardousness is zero).

In this data a score of zero is impossible for hazardousness because it's the average of two items scored 1 – 7. So the intercept and it's variance are not interpretable.

For multilevel models, there are two common centering options (grand mean centering and **group mean centering**).

The choice of centering has a big impact on the parameter estimates and their substantive meaning.

Enders, C. K. & Tofighi, D. (2007). Centering predictor variables in cross-sectional multilevel models: a new look at an old issue. *Psychological Methods, 12*(2), 121-138.

# Within-Group Centering

Typically variables at Level 1 contain information about Level 1 and Level 2.

Consider the hazardousness ratings: $X_{ij}$

Part of $X_{ij}$ has to do with how hazardous **that specific drug** is compared to other drugs. (Level 1)

But another part has to do with how hazardous the person sees **drugs in general.** (Level 2)

$$X_{ij} = \underbrace{X_{ij} - \bar{X}_{.j}}_{\text{Within-group/ Level 1}} + \underbrace{\bar{X}_{.j}}_{\text{Between-group/ Level 2}}$$

$\bar{X}_{.j}$ is the group *j*'s mean of $X_{ij}$

Within-group centering divides these two pieces of information, so we can see what is Level 1 variance and what is Level 2 variance, separately.

The within and between group pieces are uncorrelated.

# Within-Group Centering

To group mean center we subtract the group's mean of *X* from each observation on that predictor.

Person 1

| Simple | Hazard | Hazard_Centered |
|---|---|---|
| 0 | 7.50 | 2.50 |
| 1 | 2.50 | -2.50 |
| **Group mean->** | 5 | |

Person 34

| Simple | Hazard | Hazard_Centered |
|---|---|---|
| 0 | 3.83 | .58 |
| 1 | 2.67 | -.58 |
| **Group mean->** | 3.25 | |

# Within-Group Centering

```
AGGREGATE
   /OUTFILE = * MODE = ADDVARIABLES
   /BREAK = id
   /Hazard_m = MEAN(Hazard).

COMPUTE Hazard_groupc = Hazard - Hazard_m.
Execute.
```

Compute a new variable called Hazard_m, which will be the group mean of hazard.

Next we compute the group-mean centered hazard ratings, and call these Hazard_groupc.

*FluencyData_Avg_long.sav [DataSet12] - IBM SPSS Statistics Data Editor

File Edit View Data Transform Analyze Graphs Utilities Extensions Window Help

18 :

| | id | Simple | Hazard | Dose | Hazard_m | Hazard_groupc |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 2.50 | 58.33 | 5.00 | -2.50 |
| 2 | 1 | 0 | 7.50 | 46.00 | 5.00 | 2.50 |
| 3 | 2 | 1 | 7.00 | 86.67 | 7.00 | .00 |
| 4 | 2 | 0 | 7.00 | 84.33 | 7.00 | .00 |
| 5 | 3 | 1 | 6.50 | 70.00 | 6.50 | .00 |

# Within-Group Centering

Thinking about within and between group variance, we can see how there may be **two relationships** of interest:

(1) How does within-group variance in X predict variance in Y?
(2) How does between-group variance in X predict variance in Y?

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$$Y_{ij} = b_{0j} + b_{1j}(X_{ij} - \bar{X}_{.j} + \bar{X}_{.j}) + e_{ij}$$

$$Y_{ij} = b_{0j} + b_{1j}(X_{ij} - \bar{X}_{.j}) + b_{1j}\bar{X}_{.j} + e_{ij}$$

When we don't use any centering (or use grand mean centering) we're fixing the relationship between the within-group part of *X* and *Y* to be equal to the relationship between the between-group part of *X* and *Y*.

Ultimately this makes these coefficients difficult to interpret because they're a blend of these two relationship (Raudenbush & Bryk, 2002).

# Contextual Effects

Sometimes we are interested in the within-group relationship between a Level 1 predictor and an outcome as well as the between-group relationship.

When the between-group effect is different from the within-group effect, we call this a **contextual effect** (Raudenbush & Bryk, 2002).

The within-group relationship is tested by including the group-mean centered Level 1 predictor.

The between-group relationship can be tested by adding the group mean of the Level 1 predictor as a Level 2 predictor for the random intercept.

$$Y_{ij} = b_{0j} + b_{1j}(X_{ij} - \bar{X}_{.j}) + e_{ij}$$

$$b_{0j} = b_0 + g_{01}\bar{X}_{.j} + u_{0j}$$

$$b_{1j} = b_1 + u_{1j}$$

# Contextual Effects

The combined contextual effects model:

$$Y_{ij} = (b_0 + g_{01}\bar{X}_{.j} + u_{0j}) + (b_1 + u_{1j})(X_{ij} - \bar{X}_{.j}) + e_{ij}$$

$$Y_{ij} = \underbrace{b_0 + g_{01}\bar{X}_{.j} + b_1\left(X_{ij} - \bar{X}_{.j}\right)}_{\text{Fixed}} + \underbrace{u_{1j}\left(X_{ij} - \bar{X}_{.j}\right) + u_{0j} + e_{ij}}_{\text{Random}}$$

$b_1$ represents the **average within-group effect** of $X_{ij}$ on $Y_{ij}$
The variance in the **within-group effect** is $Var\left(b_{1j}\right) = Var\left(u_{1j}\right) = \tau_{11}$

$g_{01}$ represents the **between group effect** of $X_{ij}$ on $Y_{ij}$.

When $b_1$ and $g_{01}$ differ from each other, this means there is a contextual effect.

# Contextual Effects

```
MIXED Dose WITH Hazard_groupc Hazard_m
    /Fixed = Hazard_groupc Hazard_m | SSTYPE(3)
    /Method = REML
    /Print = G Solution Testcov
    /Random = INTERCEPT Hazard_groupc |
Subject(id) COVTYPE(UN).
```

Var: DOSE
Dose for drug *i* for person *j*

Var: Hazard_groupc
Drug *i*'s deviation from Person *j*'s average hazardousness rating

$$Y_{ij} = b_0 + g_{01}\bar{X}_{.j} + b_1\left(X_{ij} - \bar{X}_{.j}\right) + u_{1j}\left(X_{ij} - \bar{X}_{.j}\right) + u_{0j} + e_{ij}$$

Var: Hazard_m
Person *j*'s average hazardousness rating

# Contextual Effects

$$Y_{ij} = \boxed{b_0} + \boxed{g_{01}}\bar{X}_{.j} + \boxed{b_1}(X_{ij} - \bar{X}_{.j}) + u_{1j}(X_{ij} - \bar{X}_{.j}) + u_{0j} + e_{ij}$$

```
MIXED Dose WITH Hazard_groupc Hazard_m
     /Fixed = Hazard_groupc Hazard_m | SSTYPE(3)
     /Method = REML
     /Print = G Solution Testcov
     /Random = INTERCEPT Hazard_groupc | Subject(id)
COVTYPE(UN).
```

**Estimates of Fixed Effects**[a]

| Parameter | Estimate | Std. Error | df | t | Sig. | 95% Confidence Interval Lower Bound | 95% Confidence Interval Upper Bound |
|---|---|---|---|---|---|---|---|
| Intercept | 115.262328 | 19.454675 | 68.013 | 5.925 | .000 | 76.441268 | 154.083388 |
| Hazard_groupc | -4.827233 | .669329 | 22.424 | -7.212 | .000 | -6.213817 | -3.440649 |
| Hazard_m | -5.939038 | 3.603082 | 68.013 | -1.648 | .104 | -13.128853 | 1.250776 |

a. Dependent Variable: Dosing Simple.

For drug's at each individual's group mean the expected dosage is 115.26 mL.

For two drugs that differ by 1 unit on hazardousness, the more hazardous drug is expected to be dosed 4.83 mL less, controlling for average hazardousness rating.

Individuals 1 unit higher on average rating of hazardousness, are expected to dose drugs 5.94 units less, controlling for deviation of the drug from the individual's average.

# Other Types of Repeated Measures Mediation

- Multilevel Models
  - Bauer, Preacher, Gil (2006) *Psychological Methods*
    Covers Mediation and Moderated Mediation for 1-1-1 multilevel mediation
  - Kenny, Korchmaros, Bolger (2003) *Psychological Methods*
    Covers mediation for 1-1-1 multilevel models
  - **COMING SOON**: Nick Rockwood's MLMediation Macro (see afhayes.com for updates)
- Latent Growth Curve Models (Longitudinal Processes M-Y measured over time)
  - Choeng, MacKinnon, Khoo (2003) *Structural Equation Modeling*
- Structural Equation Modeling (Can be used for a variety of data types)
  - Cole & Maxwell (2003) *Journal of Abnormal Psychology*
    X, M, and Y all measured over time
  - Newsom (2009) *Structural Equation Modeling*
    Dyadic data using LGMs
  - Selig & Little (2012) *Handbook of Developmental Research Methods*
    Autoregressive models and cross-lagged panel models for longitudinal data X, M, and Y all measured over time.
- **Selig & Preacher (2009) *Research in Human Development***
  - **Longitudinal Models X, M, and Y measured across time. Cross-lagged panel models, latent growth models, latent difference score models**
- Multilevel SEM
  - Preacher, Zyphyr, Zhang, 2010
  - Preacher, Zhang, Zyphur, 2011

# Other Kinds of Bootstrap Confidence Intervals

All bootstrap confidence intervals use the same basic sampling technique, just use different methods for choosing the end points of the confidence intervals

**Bias-Corrected Confidence Interval**

- Percentile bootstrapping assumes that your sample estimate (*ab*) is unbiased in estimating the population indirect effect
- Bias-corrected reduces this assumption to assuming that the bias of *ab* is a constant (i.e. as N goes to infinity *ab* will go to the population indirect effect plus some constant)
- Bias-corrected confidence intervals estimate the bias of *ab* then adjust edges of confidence interval to be "bias-corrected" (i.e. centered not around your original estimate of *ab*), but around the point based on the bias estimation.

**Bias-Corrected and Accelerated**

- Same principles as BC regarding bias correction
- Acceleration allows for the assumption that the standard error of the indirect effect depends on the population value of the indirect effect
- Acceleration parameter, which is used to adjust the ends of the confidence interval is estimated using leave-one-out estimates of skew of the estimates of the indirect effect.

# Part II: Mediation and Multilevel Modeling

One of the primary assumptions of Ordinary least squares (OLS) regression is that each observation is independent of all other observations.

Ordinary least squares (OLS) regression is not directly applicable when data are nested.

- Students nested within classrooms
- Employees nested within companies
- Repeated measurements nested within individuals

Responses from employees within the same company tend to be more related to each other than responses from employees in different companies.

This violates the assumption of independence.

Several methods are available for accounting for this dependence, but today we will focus on multilevel/mixed modeling.

# Multilevel Modeling

What's a level?

Students (Level 1) within classrooms (Level 2)
Employees (Level 1) within companies (Level 2)
Repeated measurements (Level 1) within individuals (Level 2)

Multilevel models are often expressed either as separate equations for the different levels of the model, or as one combined model.

Let *i* denote Level 1 units and *j* denote Level 2 units

$X_{ij}$: Person *i* in group *j*'s observation on *X*
$Y_{ij}$: Person *i* in group *j*'s observation on *Y*
$W_j$: Group *j*'s observation on *W* (Level 2 characteristics (e.g., Company size))

# Two-Level Unconditional Model

Let's predict an outcome at Level 1 using a predictor from Level 1

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$b_{0j}$: The expected value of *Y* for someone in group *j* with $X_{ij} = 0$. Notice this is allowed to vary by group! This is the **intercept** for group *j*.

$b_{1j}$: The expected difference in *Y* for two people in the same group *j* that differ by 1 unit on $X_{ij}$. This is the **slope** for group *j.*

$e_{ij}$: The error in estimating $Y_{ij}$. $e_{ij} \sim N(0, \sigma^2)$

Even if you're not familiar with multilevel models, this should look familiar to what we do in regression. Except the intercept and slope are allowed to randomly vary across groups. We call these *random effects*.

# Two-Level Unconditional Model

We also create a Level 2 Model, for the intercept and slope:

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$$b_{0j} = b_0 + u_{0j}$$

$$b_{1j} = b_1 + u_{1j}$$

Notice that it's the *u*'s that make the "random effects" random. By allowing the intercept and slope to vary across groups, we soak up all the "dependence" in the data.

$b_0$ is the grand-mean intercept (i.e., the average intercept across groups)

$b_1$ is the grand-mean slope (i.e., the average slope across groups)

We assume that $(u_{0j}, u_{1j}) \sim MVN(0, T)$ where $T = \begin{bmatrix} \tau_{00} & \tau_{01} \\ \tau_{01} & \tau_{11} \end{bmatrix}$

The random effects are not individually estimated, but rather we estimate their covariance matrix as well as the grand-mean intercept and slope.

# The Combined Model

Sometimes it's clearer to represent both the Level 1 and Level 2 equations together in a *combined model*. We plug in the Level 2 equations in their spots in the Level 1 model

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$$Y_{ij} = (b_0 + u_{0j}) + (b_1 + u_{1j})X_{ij} + e_{ij}$$

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You can see how each individual's response is a function of the **overall intercept** the **overall slope** as well as their **group's deviations** from the overall intercept and slope and a **individual-specific error.**

# Simplifying the Model

Not all coefficients need to be random. For example the intercept could be random but the slope could vary across groups:

$$Y_{ij} = b_{0j} + b_1 X_{ij} + e_{ij}$$

$b_{0j} = b_0 + u_{0j}$

$b_1 = b_1$

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$X_{ij}$: Hazardousness for observation *i* for person *j*

**Give it a try!**

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Consider the hazardousness ratings: $X_{ij}$

Part of $X_{ij}$ has to do with how hazardous **that specific drug** is compared to other drugs. (Level 1)

But another part has to do with how hazardous the person sees **drugs in general.** (Level 2)

$$X_{ij} = \underbrace{X_{ij} - \bar{X}_{.j}}_{\text{Within-group/ Level 1}} + \underbrace{\bar{X}_{.j}}_{\text{Between-group/ Level 2}}$$

$\bar{X}_{.j}$ is the group *j*'s mean of $X_{ij}$

Within-group centering divides these two pieces of information, so we can see what is predicted by Level 1 variance and what is predicted by Level 2 variance, separately.

The within and between group pieces are uncorrelated.

# Within-Group Centering

To group mean center we subtract the group's mean of *X* from each observation on that predictor.

Person 1

| Simple | Hazard | Hazard_Centered |
|---|---|---|
| 0 | 7.50 | 2.50 |
| 1 | 2.50 | -2.50 |
| **Group mean->** | 5 | |

Person 34

| Simple | Hazard | Hazard_Centered |
|---|---|---|
| 0 | 3.83 | .58 |
| 1 | 2.67 | -.58 |
| **Group mean->** | 3.25 | |

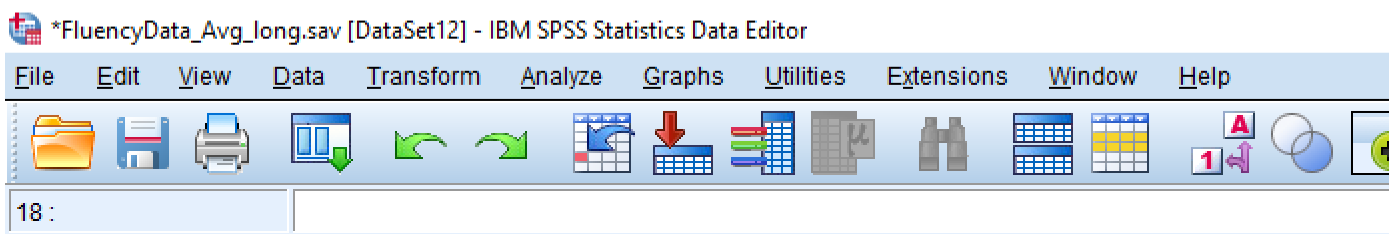
# Within-Group Centering

```
AGGREGATE
   /OUTFILE = * MODE = ADDVARIABLES
   /BREAK = id
   /Hazard_m = MEAN(Hazard).

COMPUTE Hazard_groupc = Hazard - Hazard_m.
Execute.
```

Compute a new variable called Hazard_m, which will be the group mean of hazard.

Next we compute the group-mean centered hazard ratings, and call these Hazard_groupc.




| | id | Simple | Hazard | Dose | Hazard_m | Hazard_groupc |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 2.50 | 58.33 | 5.00 | -2.50 |
| 2 | 1 | 0 | 7.50 | 46.00 | 5.00 | 2.50 |
| 3 | 2 | 1 | 7.00 | 86.67 | 7.00 | .00 |
| 4 | 2 | 0 | 7.00 | 84.33 | 7.00 | .00 |
| 5 | 3 | 1 | 6.50 | 70.00 | 6.50 | .00 |

# Within-Group Centering

Thinking about within and between group variance, we can see how there may be **two relationships** of interest:

(1) How does within-group variance in X predict variance in Y?
(2) How does between-group variance in X predict variance in Y?

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$$Y_{ij} = b_{0j} + b_{1j}(X_{ij} - \bar{X}_{.j} + \bar{X}_{.j}) + e_{ij}$$

$$Y_{ij} = b_{0j} + b_{1j}(X_{ij} - \bar{X}_{.j}) + b_{1j}\bar{X}_{.j} + e_{ij}$$

When we don't use any centering (or use grand mean centering) we're fixing the relationship between the within-group part of *X* and *Y* to be equal to the relationship between the between-group part of *X* and *Y*.

Ultimately this makes these coefficients difficult to interpret because they're a blend of these two relationship s(Raudenbush & Bryk, 2002).

# Contextual Effects

Sometimes we are interested in the within-group relationship between a Level 1 predictor and an outcome as well as the between-group relationship.

When the between-group effect is different from the within-group effect, we call this a contextual effect (Raudenbush & Bryk, 2002).

The within-group relationship is tested by including the group-mean centered Level 1 predictor.

The between-group relationship can be tested by adding the group mean of the Level1 predictor as a Level 2 predictor for the random intercept.

$$Y_{ij} = b_{0j} + b_{1j}(X_{ij} - \bar{X}_{.j}) + e_{ij}$$

$$b_{0j} = b_0 + g_{01}\bar{X}_{.j} + u_{0j}$$

$$b_{1j} = b_1 + u_{1j}$$

# Contextual Effects

The combined contextual effects model:

$$Y_{ij} = (b_0 + g_{01}\bar{X}_{.j} + u_{0j}) + (b_1 + u_{1j})(X_{ij} - \bar{X}_{.j}) + e_{ij}$$

$$Y_{ij} = \underbrace{b_0 + g_{01}\bar{X}_{.j} + b_1\left(X_{ij} - \bar{X}_{.j}\right)}_{\text{Fixed}} + \underbrace{u_{1j}\left(X_{ij} - \bar{X}_{.j}\right) + u_{0j} + e_{ij}}_{\text{Random}}$$

$b_1$ represents the **average within-group effect** of $X_{ij}$ on $Y_{ij}$
The variance in the **within-group effect** is $Var\left(b_{1j}\right) = Var\left(u_{1j}\right) = \tau_{11}$

$g_{01}$ represents the **between group effect** of $X_{ij}$ on $Y_{ij}$.

When $b_1$ and $g_{01}$ differ from each other, this means there is a contextual effect.

# Contextual Effects

```
MIXED Dose WITH Hazard_groupc Hazard_m
    /Fixed = Hazard_groupc Hazard_m | SSTYPE(3)
    /Method = REML
    /Print = G Solution Testcov
    /Random = INTERCEPT Hazard_groupc |
Subject(id) COVTYPE(UN).
```

Var: DOSE
Dose for drug *i* for person *j*

Var: Hazard_groupc
Drug *i*'s deviation from Person *j*'s average hazardousness rating

$$Y_{ij} = b_0 + g_{01}\bar{X}_{.j} + b_1(X_{ij} - \bar{X}_{.j}) + u_{1j}(X_{ij} - \bar{X}_{.j}) + u_{0j} + e_{ij}$$

Var: Hazard_m
Person *j*'s average hazardousness rating

# Contextual Effects

$$Y_{ij} = \boxed{b_0} + \boxed{g_{01}}\bar{X}_{.j} + \boxed{b_1}(X_{ij} - \bar{X}_{.j}) + u_{1j}(X_{ij} - \bar{X}_{.j}) + u_{0j} + e_{ij}$$

```
MIXED Dose WITH Hazard_groupc Hazard_m
    /Fixed = Hazard_groupc Hazard_m | SSTYPE(3)
    /Method = REML
    /Print = G Solution Testcov
    /Random = INTERCEPT Hazard_groupc | Subject(id)
COVTYPE(UN).
```

**Estimates of Fixed Effects**[a]

| Parameter | Estimate | Std. Error | df | t | Sig. | 95% Confidence Interval Lower Bound | 95% Confidence Interval Upper Bound |
|---|---|---|---|---|---|---|---|
| Intercept | 115.262328 | 19.454675 | 68.013 | 5.925 | .000 | 76.441268 | 154.083388 |
| Hazard_groupc | -4.827233 | .669329 | 22.424 | -7.212 | .000 | -6.213817 | -3.440649 |
| Hazard_m | -5.939038 | 3.603082 | 68.013 | -1.648 | .104 | -13.128853 | 1.250776 |

a. Dependent Variable: Dosing Simple.

For drugs at each individual's group mean the expected dosage is 115.26 mL.

For two drugs that differ by 1 unit on hazardousness, the more hazardous drug is expected to be dosed 4.83 mL less, controlling for average hazardousness rating.

Individuals 1 unit higher on average rating of hazardousness, are expected to dose drugs 5.94 units less, controlling for deviation of the drug from the individual's average.

# Mediation Modeling with Multilevel Data

Multilevel mediation processes are often labeled by the level at which each variable varies.

- 1-1-1 implies that *X*, *M*, and *Y* are all measured at Level 1.
  **Example:** Measuring individuals on a variety of days, we may wonder if number of calories eaten before noon each day (*X*) predicts daily stress (Y) through improved productivity in the afternoon (*M*).

- 2-1-1 implies that *X* is measured at Level 2, but *M* and *Y* are at Level 1
  **Example:** Individuals are randomly assigned to either a healthy breakfast supplement (*X*) and tracked over a variety of days to see if their daily stress (*Y*) is improved through improved through afternoon productivity (*M*).

- 2-2-1 implies *X* and *M* are Level 2, but *Y* is measured at Level 1.
  **Example:** Perhaps individuals were randomly assigned to either a healthy breakfast supplement (*X*) and asked at the end of the week whether they felt able accomplish what they needed to in the afternoon this week (*M*) and we track their daily stress (*Y*).

# Mediation Modeling with Multilevel Data

You may notice that when some variables are at Level 2, they should not be able to predict Level 1 variability.

For example, knowing whether someone is in the breakfast supplement condition, should help us predict their average daily stress, but none of the deviation of day-to-day stress from the person's average.

As such we'll focus on the 1-1-1 model, as it is the most general multilevel mediation model that exists when all variables are measured at Level 1.

We can explore between-group and within-group variability. **Indirect effects** can occur at both levels!

# General 1-1-1 Mediation Model

Recall the single-level (between-subjects) mediation model:

$$M_i = a_0 + a_1 X_i + e_{M_i}$$

$$Y_i = b_0 + c' X_i + b_1 M_i + e_{Y_i}$$

Let's make it multilevel, where *X*, *M*, and *Y* are Level 1 variables:

$$M_{ij} = a_{0j} + a_{1j} X_{ij} + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c'_j X_{ij} + b_{1j} M_{ij} + e_{Y_{ij}}$$

Let's use group-mean centering, because we're interested in differentiating within and between effects:

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c'_j(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

# 1-1-1 Mediation Model: Within-Group Effects

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c_j'(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

$a_{1j} = a_W + u_{a_1 j}$ This is the within-group effect of *X* on *M* for group *j*

$b_{1j} = b_W + u_{b_1 j}$ This is the within-group effect of *M* on *Y*, controlling for *X* in group *j*

$c_j' = c'_W + u_{c' j}$ This is the within-group effect of *X* on *Y*, controlling for *M* in group *j*

$a_W$, $b_W$, and $c_W'$ are the **average within-group effects** of *X* and *M, M* on *Y* controlling for *X*, and *X* and *Y* controlling for *M* respectively.

# 1-1-1 Mediation Model: Between-Group Effects

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c_j'(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) \; + e_{Y_{ij}}$$

BUT WAIT! We haven't included between-group effects. How did we do that before?

**Include $\bar{X}_{.j}$ as a Level 2 predictor of the intercept.**

$$a_{0j} = a_M + a_B \, \bar{X}_{.j} + u_{a_0 j}$$

$$b_{0j} = b_Y + c_B' \, \bar{X}_{.j} + b_B \, \bar{M}_{.j} + \; u_{b_0 j}$$

$a_B$, $b_B$, and $c_B'$ are the **between-group effects** of *X* and *M, M* on *Y* controlling for *X*, and *X* and *Y* controlling for *M* respectively.

# 1-1-1 Mediation Full Model

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c_j'(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

$$a_{0j} = a_M + a_B\, \bar{X}_{.j} + u_{a_0 j}$$

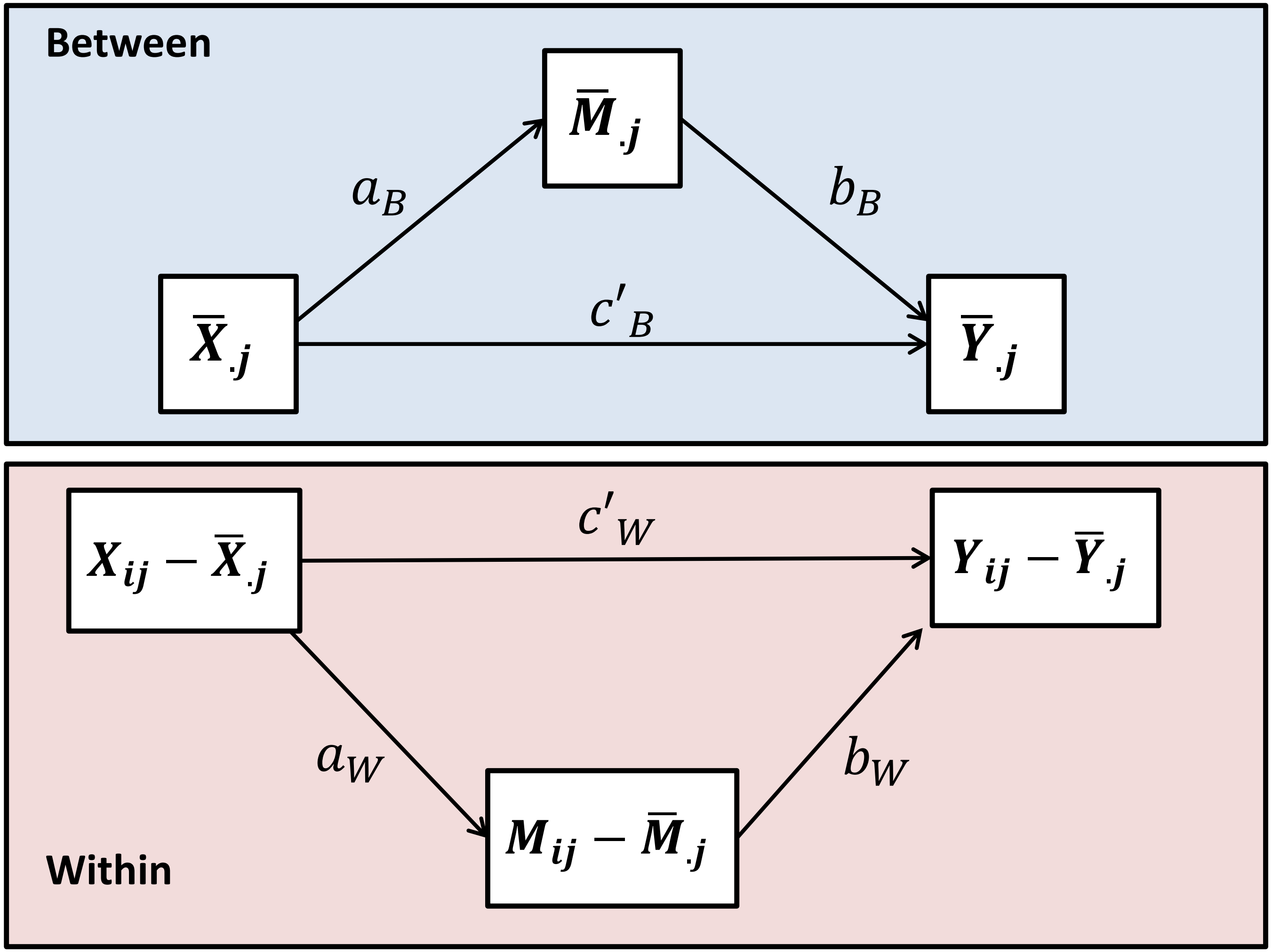
$$b_{0j} = b_Y + c_B'\, \bar{X}_{.j} + b_B\, \bar{M}_{.j} + u_{b_0 j}$$

$$a_{1j} = a_W + u_{a_1 j}$$

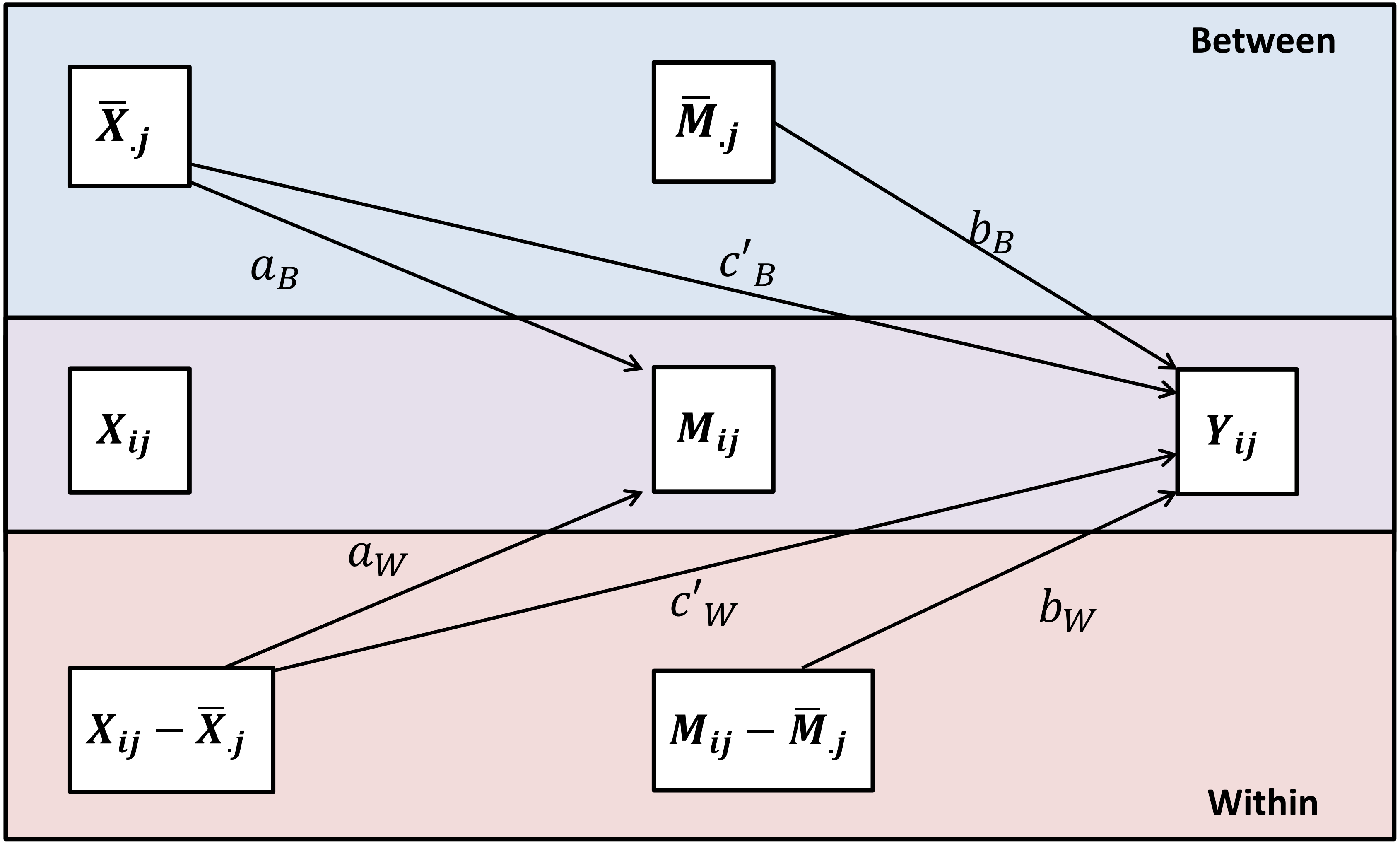
$$b_{1j} = b_W + u_{b_1 j}$$

$$c_j' = c'_W + u_{c' j}$$

# 1-1-1 Mediation Model: Conceptual Diagram



# 1-1-1 Mediation Model: Statistical Diagram

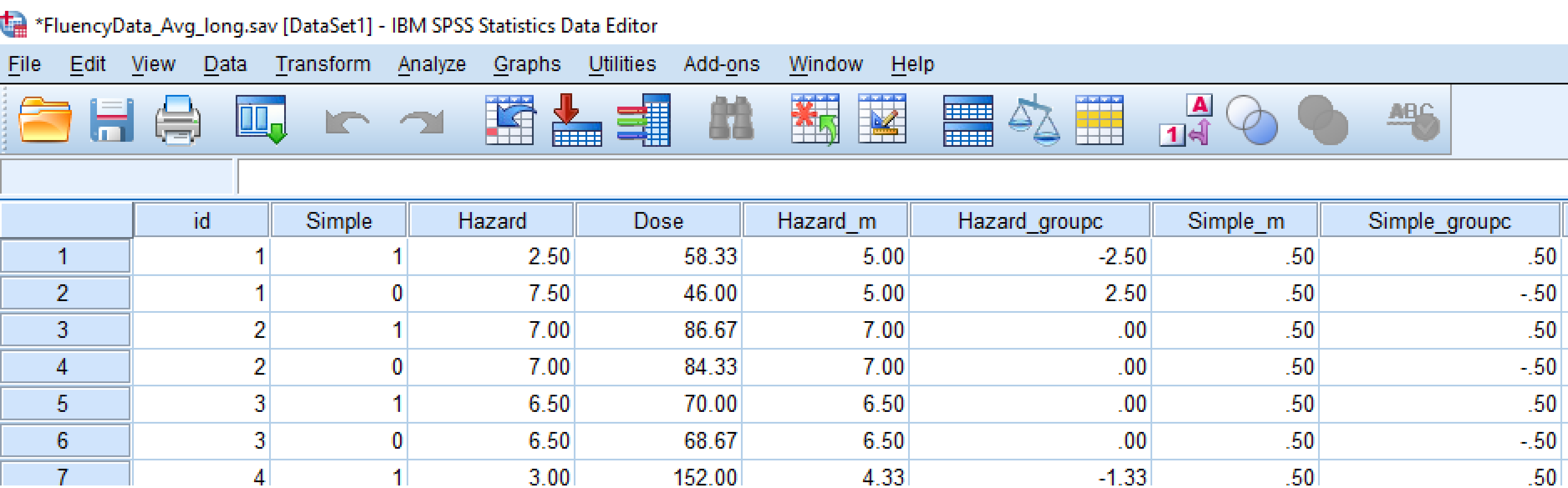


# Estimating the *M* Equation

First we have to group-mean center `Simple` and create a variable which is the group mean of `Simple`.

```
AGGREGATE
/OUTFILE = * MODE = ADDVARIABLES
/BREAK = id
/Simple_m = MEAN(Simple).

COMPUTE Simple_groupc = Simple - Simple_m.
EXECUTE.
```



*FluencyData_Avg_long.sav [DataSet1] - IBM SPSS Statistics Data Editor

File Edit View Data Transform Analyze Graphs Utilities Add-ons Window Help

| | id | Simple | Hazard | Dose | Hazard_m | Hazard_groupc | Simple_m | Simple_groupc |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 2.50 | 58.33 | 5.00 | -2.50 | .50 | .50 |
| 2 | 1 | 0 | 7.50 | 46.00 | 5.00 | 2.50 | .50 | -.50 |
| 3 | 2 | 1 | 7.00 | 86.67 | 7.00 | .00 | .50 | .50 |
| 4 | 2 | 0 | 7.00 | 84.33 | 7.00 | .00 | .50 | -.50 |
| 5 | 3 | 1 | 6.50 | 70.00 | 6.50 | .00 | .50 | .50 |
| 6 | 3 | 0 | 6.50 | 68.67 | 6.50 | .00 | .50 | -.50 |
| 7 | 4 | 1 | 3.00 | 152.00 | 4.33 | -1.33 | .50 | .50 |

# Estimating the *M* Equation

Next we predict `Hazard` from the group-mean centered `Simple` (`Simple_groupc`) and the group means of `Simple` (`Simple_m`)

```
MIXED Hazard WITH Simple_groupc Simple_m
/FIXED = Simple_groupc Simple_m | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept Simple_groupc| Subject(id) COVTYPE(UN).
```

**Estimates of Fixed Effects**[b]

| Parameter | Estimate | Std. Error | df | t | Sig. | 95% Confidence Interval | |
|---|---|---|---|---|---|---|---|
| | | | | | | Lower Bound | Upper Bound |
| Intercept | 5.304762 | .121182 | 69.000 | 43.775 | .000 | 5.063010 | 5.546513 |
| Simple_groupc | -2.104762 | .184802 | 69.000 | -11.389 | .000 | -2.473433 | -1.736091 |
| Simple_m | 0[a] | 0 | . | . | . | . | . |

a. This parameter is set to zero because it is redundant.
b. Dependent Variable: Hazardousness Simple.

**UH OH! Something went wrong? What happened?**

# Estimating the *M* Equation

"Redundant" group-mean for simple

Person 1

| Simple | Simple_Centered |
|---|---|
| 0 | -.50 |
| 1 | .50 |
| **Group mean-> 0.5** | |

Person 34

| Simple | Hazard_Centered |
|---|---|
| 0 | -.50 |
| 1 | .50 |
| **Group mean-> 0.5** | |

# Estimating the *M* Equation

"Redundant" group-mean for simple!

Because each participant complete both levels of `Simple` (0 and 1) one time each, all participants have a person-level mean of 0.5 (see `Simple_m`)

This means there is no between-person variability on `Simple`, so the group means and the intercept are linear combinations of one another (`Intercept = 2*Simple_m`; ., the model is not identified).

We can remove the between-person effect of *X* from the model, meaning there will not be a between-person indirect effect.

$$a_{0j} = a_M + \cancel{a_B \bar{X}_{.j}} + u_{a_0 j} = a_M + u_{a_0 j}$$

$$b_{0j} = b_Y + \cancel{c'_B \bar{X}_{.j}} + b_B \bar{M}_{.j} + u_{b_0 j} = b_Y + b_B \bar{M}_{.j} + u_{b_0 j}$$

| Simple_m |
|---|
| .50 |
| .50 |
| .50 |
| .50 |
| .50 |
| .50 |
| .50 |
| .50 |
| .50 |
| .50 |
| .50 |
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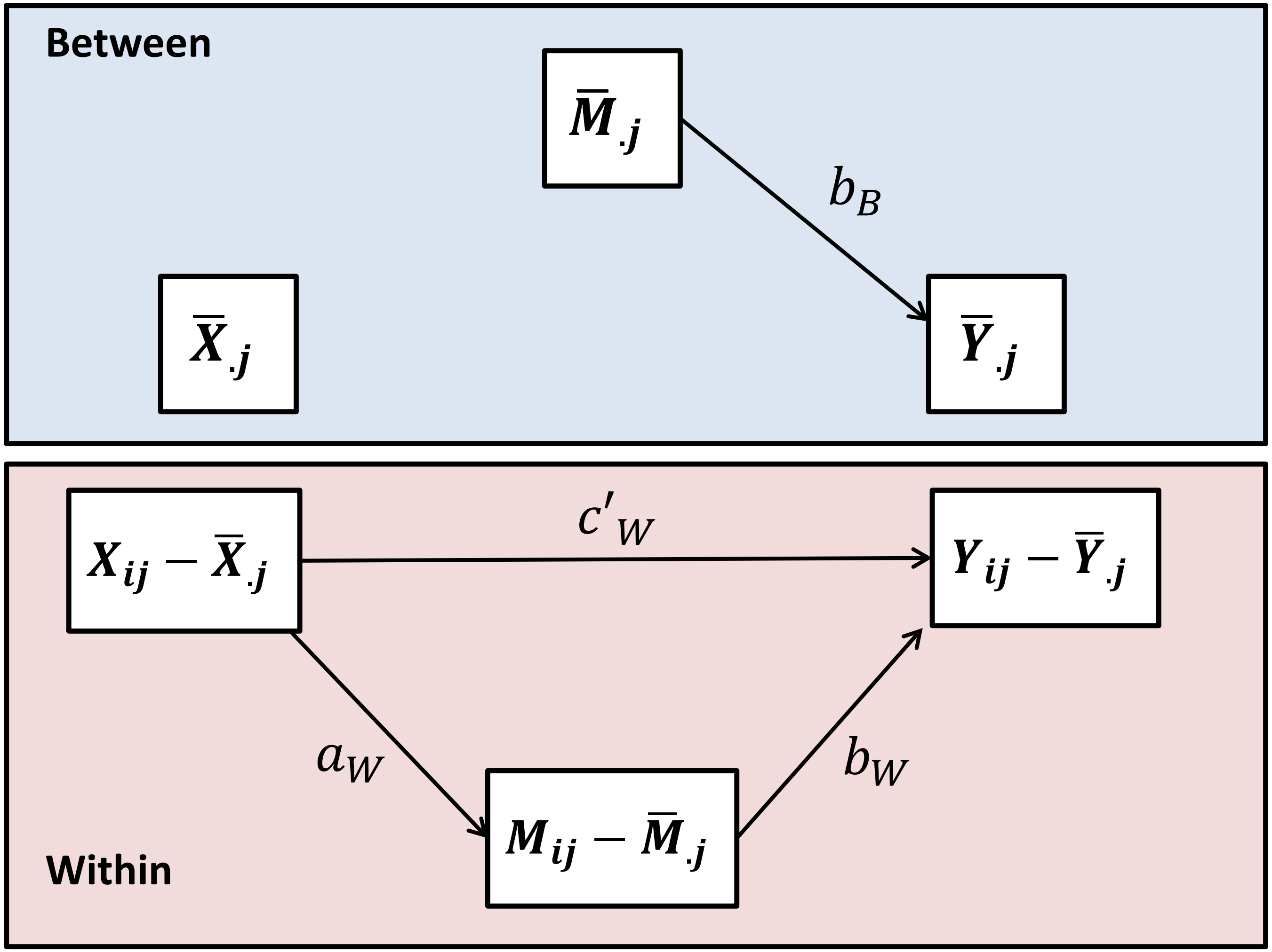
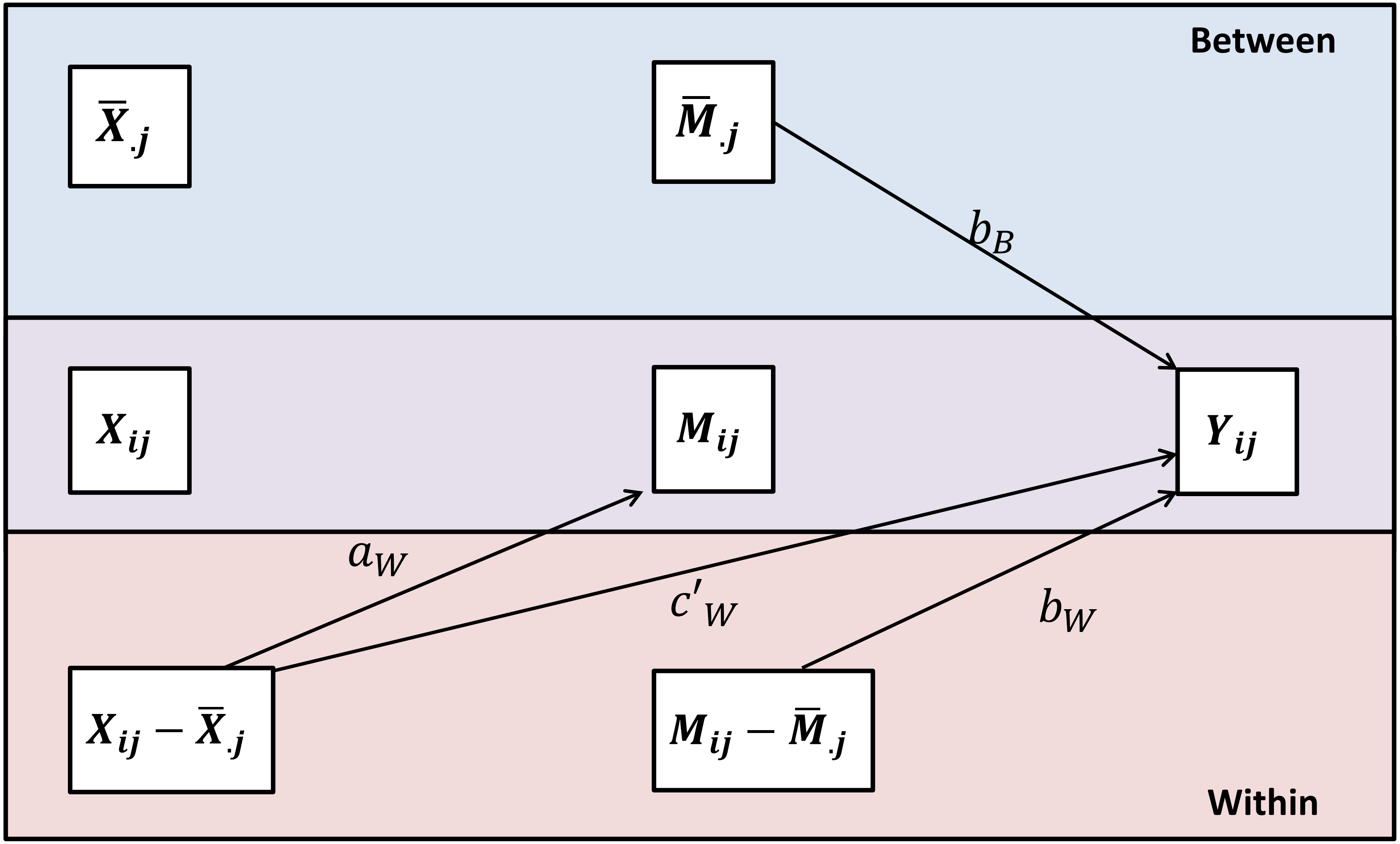
| .50 |
| .50 |

# Revised: Conceptual Diagram



# Revised: Statistical Diagram



# Estimating the *M* Equation (Again)

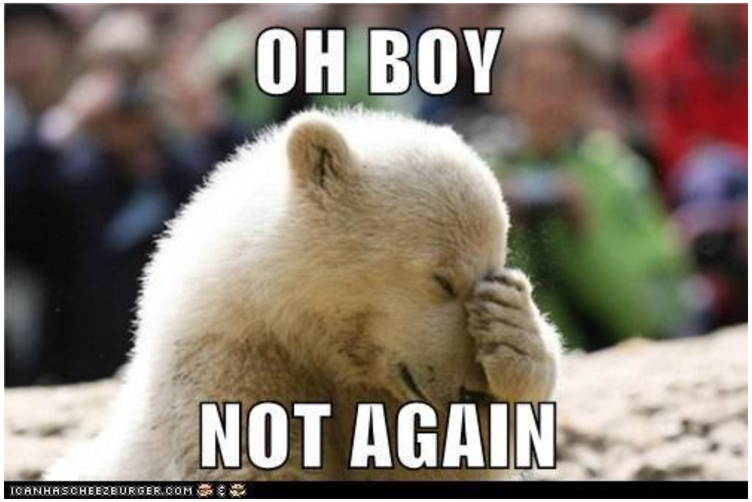
```
MIXED Hazard WITH Simple_groupc
/FIXED = Simple_groupc | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept Simple_groupc| Subject(id)
COVTYPE(UN).
```

**Estimates of Covariance Parameters**[b]

| Parameter | | Estimate | Std. Error | Wald Z | Sig. | 95% Confidence Interval | |
|---|---|---|---|---|---|---|---|
| | | | | | | Lower Bound | Upper Bound |
| Residual | | .653447 | .203505 | 3.211 | .001 | .354910 | 1.203101 |
| Intercept + Simple_groupc [subject = id] | UN (1,1) | .701233 | .202258 | 3.467 | .001 | .398426 | 1.234173 |
| | UN (2,1) | .071440 | .188917 | .378 | .705 | -.298830 | .441710 |
| | UN (2,2) | 1.083744[a] | .000000 | . | . | . | . |

a. This covariance parameter is redundant. The test statistic and confidence interval cannot be computed.
b. Dependent Variable: Hazardousness Simple.

Something has gone wrong with the variance for the slope!



# 1-1-1 Mediation Full Model

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$a_{0j} = a_M + u_{a_0 j}$$

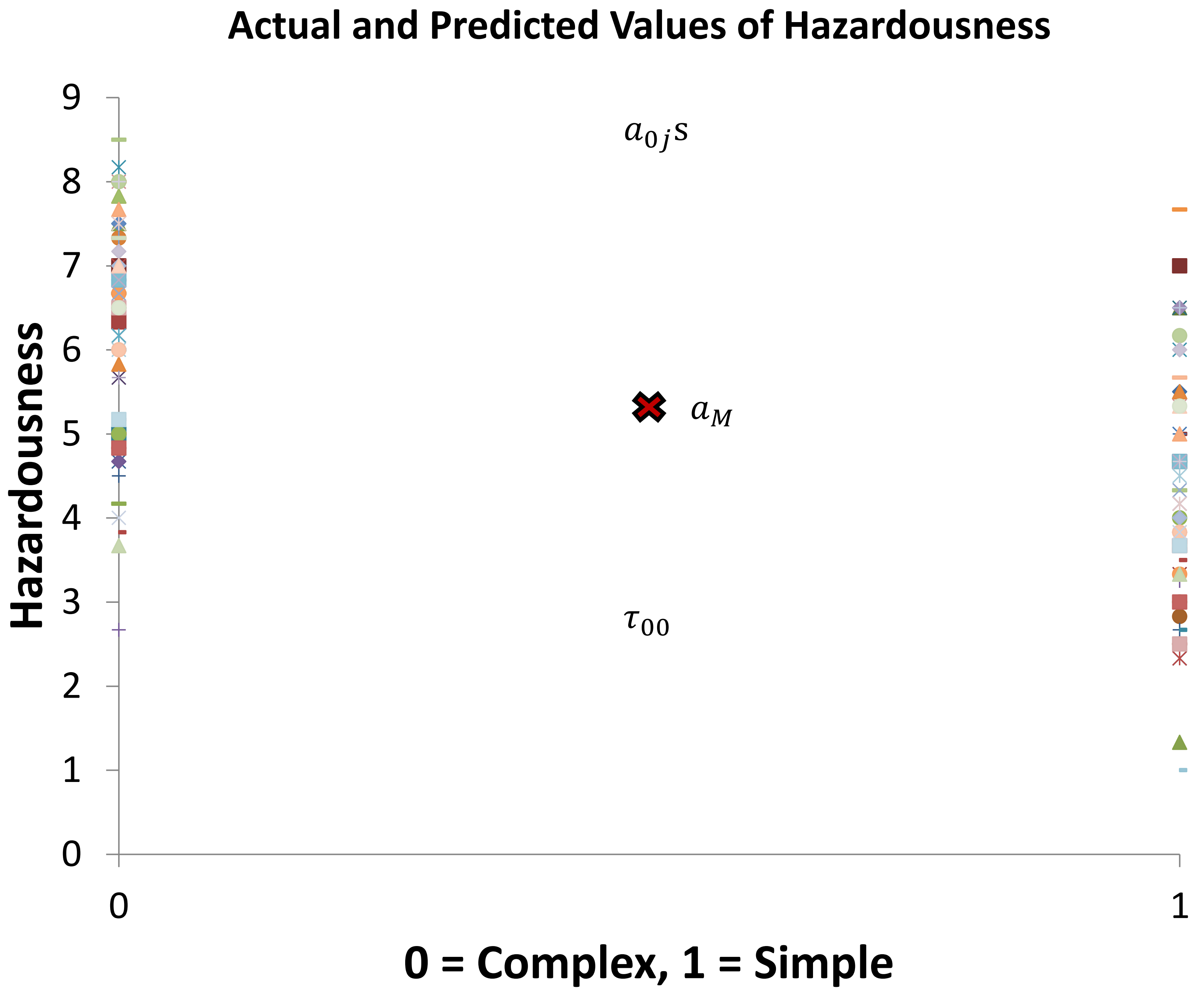
$$a_{1j} = a_W + u_{a_1 j}$$

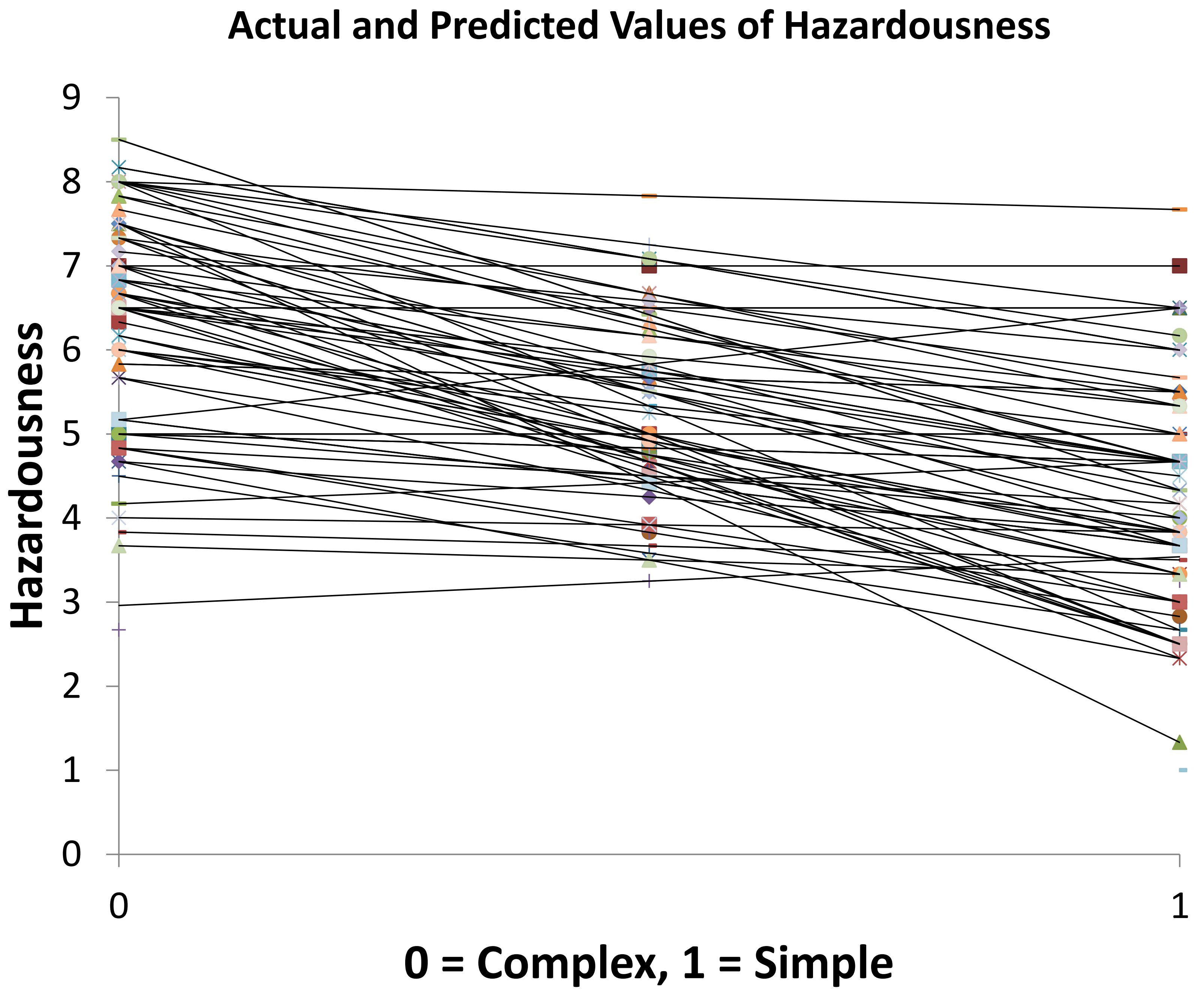
$$M_{ij} = (a_M + u_{a_0 j}) + (a_W + u_{a_1 j})(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

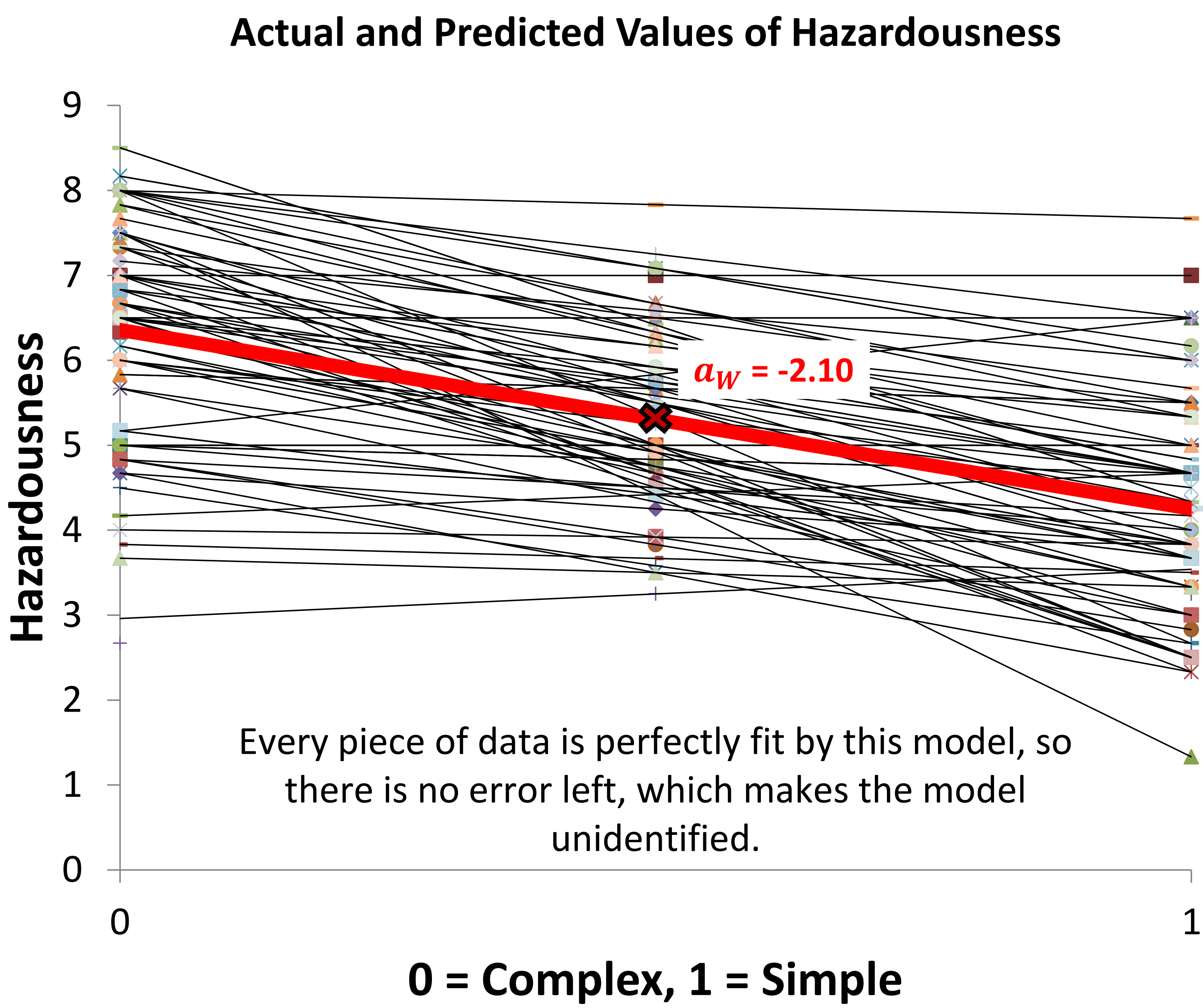
Each person get's their own intercept

Each person get's their own slope

We only have two observations per person, so giving each person their own intercept and their own slope would perfectly fit the data, and there will be no error left over!


Actual and Predicted Values of Hazardousness
Hazardousness
9
8
7
6
5
4
3
2
1
0
$a_{0j}s$
$a_M$
$\tau_{00}$
0
1
0 = Complex, 1 = Simple


Actual and Predicted Values of Hazardousness
Hazardousness
0
1
2
3
4
5
6
7
8
9
0
1
0 = Complex, 1 = Simple



Actual and Predicted Values of Hazardousness
Hazardousness
0 = Complex, 1 = Simple
$a_W$ = -2.10
Every piece of data is perfectly fit by this model, so there is no error left, which makes the model unidentified.
0
1
2
3
4
5
6
7
8
9

# Estimating the *M* Equation (Again, Again)

Get rid of the random slope, assuming there is no variance in $a_W$

```
MIXED Hazard WITH Simple_groupc
/FIXED = Simple_groupc | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept | Subject(id) COVTYPE(VC).
```

**Estimates of Fixed Effects[a]**

| Parameter | Estimate | Std. Error | df | t | Sig. | 95% Confidence Interval | |
|---|---|---|---|---|---|---|---|
| | | | | | | Lower Bound | Upper Bound |
| Intercept | 5.304762 | .121182 | 69.000 | 43.775 | .000 | 5.063010 | 5.546513 |
| Simple_groupc | -2.104762 | .184802 | 69.000 | -11.389 | .000 | -2.473433 | -1.736091 |

a. Dependent Variable: Hazardousness Simple.

An one unit increase in `Simple_groupc` (i.e., moving from the complex to simple condition) predicts a 2.10 unit decrease in perceptions of hazardousness averaged across individuals. $a_W = -2.10$

# Estimating the *Y* Equation

$$Y_{ij} = b_{0j} + c_j'(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) \; + e_{Y_{ij}}$$

$$b_{0j} = b_Y + \cancel{c_B' \, \bar{X}_{.j}} + b_B \, \bar{M}_{.j} + \; u_{b_0 j}$$

$$c_j' = c'_W \cancel{+ u_{c'j}}$$

$$b_{1j} = b_W \cancel{+ u_{b_1 j}}$$

Cut out terms involving group mean of *X*, remove random slopes

**Why do we keep the term involving group mean of *M*?**

```
MIXED Dose WITH Hazard_groupc Hazard_m Simple_groupc
/FIXED = Hazard_groupc Hazard_m Simple_groupc | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept | Subject(id) COVTYPE(VC).
```

# Estimating the $Y$ Equation

$$Y_{ij} = b_Y + b_B\,\bar{M}_{.j} + u_{b_0 j} + c'_W(X_{ij} - \bar{X}_{.j}) + \boxed{b_W}(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

```
MIXED Dose WITH Hazard_groupc Hazard_m Simple_groupc
/FIXED = Hazard_groupc Hazard_m Simple_groupc | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept | Subject(id) COVTYPE(VC).
```

**Estimates of Fixed Effects**[a]

| Parameter | Estimate | Std. Error | df | t | Sig. | 95% Confidence Interval | |
|---|---|---|---|---|---|---|---|
| | | | | | | Lower Bound | Upper Bound |
| Intercept | 115.420197 | 19.457732 | 68 | 5.932 | .000 | 76.592897 | 154.247498 |
| Hazard_groupc | -3.430154 | .947546 | 68.000 | -3.620 | .001 | -5.320953 | -1.539354 |
| Hazard_m | -5.968798 | 3.603669 | 68 | -1.656 | .102 | -13.159807 | 1.222211 |
| Simple_groupc | 3.827962 | 2.468446 | 68.000 | 1.551 | .126 | -1.097745 | 8.753670 |

a. Dependent Variable: Dosing Simple.

A one unit increase in deviation from the group mean on hazardousness, predicts a 3.43 mL decrease in dosage, controlling for group mean hazardousness and name complexity.

$b_W = -3.43$

# Estimating the *Y* Equation

$$Y_{ij} = b_Y + \boxed{b_B}\,\bar{M}_{.j} + u_{b_0 j} + c'_W(X_{ij} - \bar{X}_{.j}) + b_W(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

```
MIXED Dose WITH Hazard_groupc Hazard_m Simple_groupc
/FIXED = Hazard_groupc Hazard_m Simple_groupc | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept | Subject(id) COVTYPE(VC).
```

**Estimates of Fixed Effects**[a]

| Parameter | Estimate | Std. Error | df | t | Sig. | 95% Confidence Interval | |
|---|---|---|---|---|---|---|---|
| | | | | | | Lower Bound | Upper Bound |
| Intercept | 115.420197 | 19.457732 | 68 | 5.932 | .000 | 76.592897 | 154.247498 |
| Hazard_groupc | -3.430154 | .947546 | 68.000 | -3.620 | .001 | -5.320953 | -1.539354 |
| Hazard_m | -5.968798 | 3.603669 | 68 | -1.656 | .102 | -13.159807 | 1.222211 |
| Simple_groupc | 3.827962 | 2.468446 | 68.000 | 1.551 | .126 | -1.097745 | 8.753670 |

a. Dependent Variable: Dosing Simple.

A one unit increase in the group-mean hazard rating predicts a 5.97 mL decrease in dosage, controlling for deviation from the group-mean in hazard rating and name complexity. $b_B = -5.97$

# Estimating the *Y* Equation

$$Y_{ij} = b_Y + b_B\,\bar{M}_{.j} + u_{b_0 j} + \boxed{c'_W}(X_{ij} - \bar{X}_{.j}) + b_W(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

```
MIXED Dose WITH Hazard_groupc Hazard_m Simple_groupc
/FIXED = Hazard_groupc Hazard_m Simple_groupc | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept | Subject(id) COVTYPE(VC).
```

**Estimates of Fixed Effects**[a]

| Parameter | Estimate | Std. Error | df | t | Sig. | 95% Confidence Interval | |
|---|---|---|---|---|---|---|---|
| | | | | | | Lower Bound | Upper Bound |
| Intercept | 115.420197 | 19.457732 | 68 | 5.932 | .000 | 76.592897 | 154.247498 |
| Hazard_groupc | -3.430154 | .947546 | 68.000 | -3.620 | .001 | -5.320953 | -1.539354 |
| Hazard_m | -5.968798 | 3.603669 | 68 | -1.656 | .102 | -13.159807 | 1.222211 |
| Simple_groupc | 3.827962 | 2.468446 | 68.000 | 1.551 | .126 | -1.097745 | 8.753670 |

a. Dependent Variable: Dosing Simple.

A one unit increase simplicity rating (i.e., going from a complex to simple name) increases dosage by 3.83 mL, controlling for hazardousness ratings $c'_W = 3.83$

# Indirect Effects in Multilevel Modeling

$$M_{ij} = a_M + u_{a_0 j} + a_W(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_Y + b_B\,\bar{M}_{.j} + \; u_{b_0 j} + c'_W(X_{ij} - \bar{X}_{.j}) + b_W(M_{ij} - \bar{M}_{.j}) \; + e_{Y_{ij}}$$

Now we have estimates of everything needed for a mediation model.

There's a lot more coefficients here than when we did between-subjects or two instance repeated-measures.

Generally there are going to be two types of indirect effects in MLMs:

**Within-Indirect Effects**

**Between-Indirect Effects**

Because there is no group-mean variation in *X* in this data, we'll only look at the within-indirect effect.

# Within-Indirect Effects

$$M_{ij} = a_M + u_{a_0 j} + a_W(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_Y + b_B\,\bar{M}_{.j} + \ u_{b_0 j} + c'_W(X_{ij} - \bar{X}_{.j}) + b_W(M_{ij} - \bar{M}_{.j}) \ + e_{Y_{ij}}$$

A within-group indirect effect quantifies the expected difference in *Y* through *M* for two Level 1 units in the **same Level 2 unit** who differ by one unit on *X*.

When $a_{1j}$ and $b_{1j}$ don't randomly vary the indirect effect is: $a_W b_W$

From our data $a_W = -2.1048$, $b_W = -3.4301$,

$$a_W b_W = (2.1048)(-3.4301) = 7.2197$$

Within a given Level 2 unit (within a specific person), we expect dosage to be 7.22 mL higher in the simple name condition as compared to the complex name condition, through the specific mechanism where name complexity influences perceived hazardousness which then in turn affects dosage.

# Between-Indirect Effects

$$M_{ij} = a_M + a_B\,\bar{X}_{.j} + u_{a_0 j} + a_W(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_Y + c'_B\,\bar{X}_{.j} + b_B\,\bar{M}_{.j} + \; u_{b_0 j} + c'_W(X_{ij} - \bar{X}_{.j}) + b_W(M_{ij} - \bar{M}_{.j}) \; + e_{Y_{ij}}$$

We don’t have a between indirect effect in the model that we estimated.

But we could think of a similar study where some people saw lots of complex drugs and a few simple drugs and others saw lots of simple drugs and a few complex ones.

The between-indirect effect quantifies the expected difference in the group-mean of *Y* through the group-mean of *M* for two Level 2 units that differ by 1 unit on the average of *X*.

In the above example this would be the expected difference in average dosage through average hazardousness for two individuals two differ by 1 on the average simple exposure.

The estimate of the between indirect effect is always $a_B b_B$

# Between-Indirect Effects

$$M_{ij} = a_M + a_B\,\bar{X}_{.j} + u_{a_0 j} + a_W(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_Y + c'_B\,\bar{X}_{.j} + b_B\,\bar{M}_{.j} + u_{b_0 j} + c'_W(X_{ij} - \bar{X}_{.j}) + b_W(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

Often the between-indirect effect can be difficult to interpret, as the meaning of the group-aggregate of a variable may differ from the meaning of the variable at the individual level.

Example by Preacher et al. (2010) on differentiating individual efficacy and collective efficacy of a group:

- The aggregate individual efficacy for a given group is a group-level variable (in that it only varies between groups).
- But the focus is still at the individual level and the meaning of such a variable is likely to differ from the meaning of a variable characterizing the dynamics of the self efficacy of the group as a collective.

It's okay not to estimate or be interested in the between-indirect effect, often times in psychology we're interested is within-individual change.

# Inference about Indirect Effects

As with single-level mediation models, the Sobel/normal theory methods are not appropriate due to the non-normal sampling distribution of the indirect effect.

Bootstrapping in multilevel models can be very difficult, as we want to bootstrapping to mimic the way data is collected from the population. It's unclear if we should be resampling at the group level, or resampling groups and then sample Level 1 units from the group.

For inference in multilevel models, we'll rely on **Monte Carlo Confidence Intervals**

**Monte Carlo Confidence Intervals (MCCIs)** are constructed by simulating data from the estimated sampling distribution of the model parameters and constructing an estimate of the sampling distribution of the indirect effect(s) using the simulated distribution of each part of the indirect effect.

# Inference about Indirect Effects

**Monte Carlo Confidence Intervals (MCCIs)** are constructed by simulating data from the estimated sampling distribution of the model parameters and constructing an estimate of the sampling distribution of the indirect effect(s) using the simulated distribution of each part of the indirect effect.

$$\begin{bmatrix} \hat{\boldsymbol{f}} \\ \hat{\boldsymbol{r}} \end{bmatrix}$$

- Consider two vectors: $\hat{\boldsymbol{f}}$ is a vector containing all of the **FIXED** effect estimates. $\hat{\boldsymbol{r}}$ is a vector containing all of the **RANDOM** effect estimates.
- If we did the study again we would get different estimates for $\hat{\boldsymbol{f}}$ and $\hat{\boldsymbol{r}}$ so let's represent their sampling covariance matrices as $\widehat{\boldsymbol{\Sigma}_{\hat{f}}}$ (estimated sampling variances and covariances among fixed effects) and $\widehat{\boldsymbol{\Sigma}_{\hat{r}}}$ (estimated sampling variances and covariances among random effects)
- We know that both random and fixed effects are *normally distributed* and we know they are independent of each other.
- We generate $\boldsymbol{f}^*$ **and** $\boldsymbol{r}^*$ to have a multivariate normal distribution with means, variances, and covariances set by the estimates from the model.

# Inference about Indirect Effects

$$\begin{bmatrix} \boldsymbol{f}^* \\ \boldsymbol{r}^* \end{bmatrix} \sim MVN\left( \begin{bmatrix} \hat{\boldsymbol{f}} \\ \hat{\boldsymbol{r}} \end{bmatrix}, \begin{bmatrix} \widehat{\boldsymbol{\Sigma}_{\hat{\boldsymbol{f}}}} & \boldsymbol{0} \\ \boldsymbol{0} & \widehat{\boldsymbol{\Sigma}_{\hat{\boldsymbol{r}}}} \end{bmatrix} \right)$$

We generate a large number of samples of $\boldsymbol{f}^*$ and $\boldsymbol{r}^*$ (e.g., 10,000)

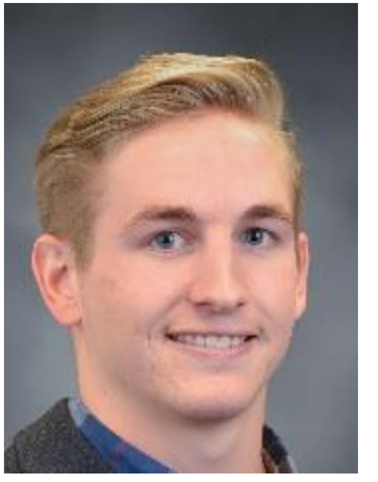
For each sample we calculate the within-indirect effect (and/or between-indirect effect), giving us 10,000 estimates of the indirect effect, which approximates the sampling distribution of the indirect effect.

A $100(1-\alpha)\%$ confidence interval is obtained by using the $100\left(\frac{\alpha}{2}\right)$and $100\left(1-\frac{\alpha}{2}\right)$ percentiles of the simulated sampling distribution.

This method has some similarities to bootstrapping, and is sometimes called the **parametric bootstrap.**

# Application in SPSS: MLmed

`Mlmed` is a package for SPSS which can do all of the analysis for you. It does all the recentering, estimates the indirect effects, and does the MCCI on your behalf.



MLmed is written and maintained by Nick Rockwood (PhD Candidate at Ohio State working with Dr. Andrew Hayes). It can be found at njrockwood.com. You also have a copy in your folder.

Just like MEMORE, you need to open the MLmed.sps file, select run all, and now SPSS knows what to do when you use an MLmed command.

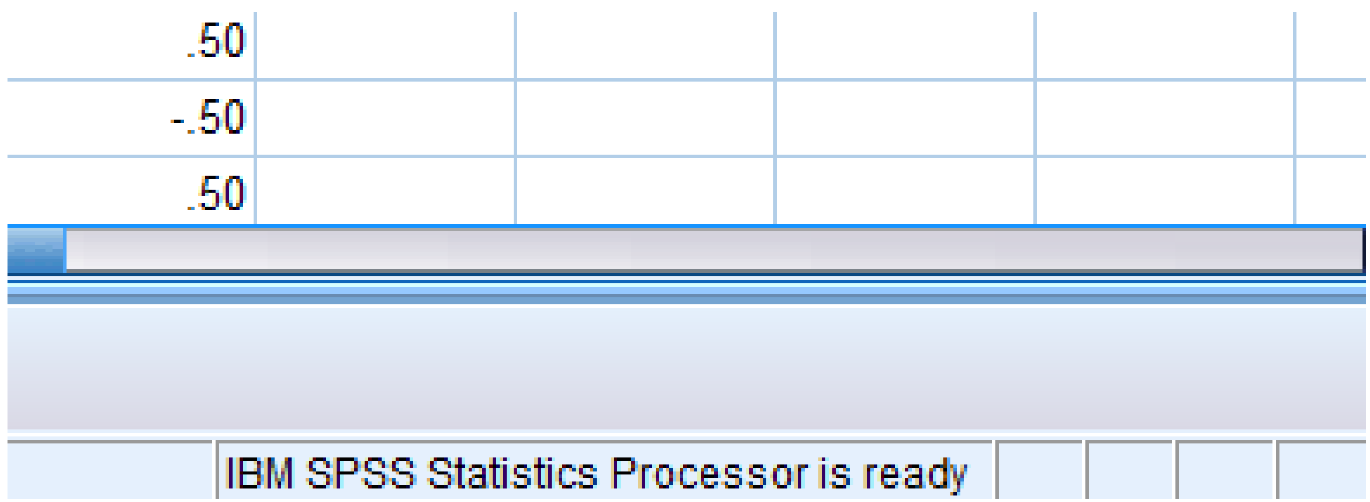
The macro does much more than what I describe here today. Check out the User Guide as well as Rockwood & Hayes (2018).

I'll explain the syntax as we go along.

# Doing it in SPSS: MLmed

```
Mlmed data = dataset1
/x = Simple
/xB = 0
/m1 = Hazard
/y = Dose
/cluster = id
/covmat = UN
/folder = /Users/Akmontoya/Desktop/
```

**WARNING:** When you run the code, some windows may pop up on your screen. Let everything resolve, and don't try to interact with those windows.



# Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /m1 = Hazard /y = Dose
/cluster = id /covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
*********************** MLMED - BETA VERSION ************************

                     Written by Nicholas J. Rockwood

              Documentation available at www.njrockwood.com

              Please report any bugs to rockwood.19@osu.edu

**********************************************************************

Model Specification
N             140
Fixed           6
Rand(L1)        2
Rand(L2)        2
Total          10

Model Fit Statistics
        Value
-2LL 1669.977
AIC  1677.977
AICC 1678.126
CAIC 1696.430
BIC  1692.430
```

# Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /m1 = Hazard /y = Dose
/cluster = id /covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
***************************  FIXED EFFECTS  ***************************

**********************************************************************
 Outcome: Hazard

Within- Effects
         Estimate      S.E.        df         t         p        LL        UL
constant   5.3048     .1212   69.0000   43.7751     .0000    5.0630    5.5465
Simple    -2.1048     .1848   69.0000  -11.3893     .0000   -2.4734   -1.7361

Note: No Between- Effect(s) Specified.

**********************************************************************
 Outcome: Dose

Within- Effects
         Estimate      S.E.        df         t         p        LL        UL
constant 115.4202   19.4577   68.0000    5.9318     .0000   76.5929  154.2475
Simple     3.8280    2.4684   68.0000    1.5508     .1256   -1.0977    8.7537
Hazard    -3.4302     .9475   68.0000   -3.6200     .0006   -5.3210   -1.5394

Between- Effects
       Estimate      S.E.        df         t         p        LL        UL
Hazard  -5.9688    3.6037   68.0000   -1.6563     .1023  -13.1598    1.2222
```

$a_W$

$c'_W$

$b_W$

$b_B$

# Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /m1 = Hazard /y = Dose
/cluster = id /covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
*************************  RANDOM EFFECTS  **************************

Level-1 Residual Estimates
        Estimate      S.E.    Wald Z         p        LL        UL
Dose     74.0515   12.6997    5.8310     .0000   52.9120  103.6367
Hazard    1.1953     .2035    5.8737     .0000     .8562    1.6688

Random Effect Estimates
    Estimate      S.E.    Wald Z         p        LL        UL
1      .4303     .2024    2.1255     .0335     .1711    1.0820
2   884.0882  158.0973    5.5921     .0000  622.7006  1255.197

Random Effect Key
1     Int        Hazard
2     Int        Dose
```

# Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /m1 = Hazard /y = Dose
/cluster = id /covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
************************ INDIRECT EFFECT(S) *************************

Within- Indirect Effect(s)
          E(ab)   Var(ab)    SD(ab)
Hazard   7.2197    .0000     .0000

Within- Indirect Effect(s)
          Effect        SE         Z          p      MCLL      MCUL
Hazard   7.2197    2.1000    3.4379     .0006    3.2952   11.4551
```

$a_W b_W$

```
Note: No Between- Indirect Effect(s) Specified.
```

$a_B b_B$

```
------ END MATRIX -----
```

# A Comparison: MEMORE vs. MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /m1 = Hazard /y = Dose
/cluster = id /covmat = UN /folder = /Users/Akmontoya/Desktop/
```

**MLmed**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $a_W$ | Simple | -2.1048 | .1848 | 69.0000 | -11.3893 | .0000 | -2.4734 | -1.7361 |
| $c'_W$ | Simple | 3.8280 | 2.4684 | 68.0000 | 1.5508 | .1256 | -1.0977 | 8.7537 |
| $b_W$ | Hazard | -3.4302 | .9475 | 68.0000 | -3.6200 | .0006 | -5.3210 | -1.5394 |

Within- Indirect Effect(s)

| $a_W b_W$ | | Effect | SE | Z | p | MCLL | MCUL |
|---|---|---|---|---|---|---|---|
| | Hazard | 7.2197 | 2.1000 | 3.4379 | .0006 | 3.2952 | 11.4551 |

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp /xmint = 0.
```

**MEMORE**

| | | Effect | SE | t | p | LLCI | ULCI |
|---|---|---|---|---|---|---|---|
| $a$ | 'X' | -2.1048 | .1848 | -11.3893 | .0000 | -2.4734 | -1.7361 |
| $c'$ | 'X' | 3.8280 | 2.4684 | 1.5508 | .1256 | -1.0978 | 8.7537 |
| $b$ | Mdiff | -3.4302 | .9475 | -3.6200 | .0006 | -5.3210 | -1.5393 |

Indirect Effect of X on Y through M

| | | Effect | BootSE | BootLLCI | BootULCI |
|---|---|---|---|---|---|
| $ab$ | Ind1 | 7.2197 | 1.8940 | 3.8590 | 11.1609 |

# A Comparison: MEMORE vs. MLmed

The model MEMORE fits is equivalent to a random intercept only 1-1-1 mediation model:

- when we have 2 observations per person
- *X* is dichotomous
- each person is observed once for each level of *X*

MLmed is a more general multilevel mediation tool

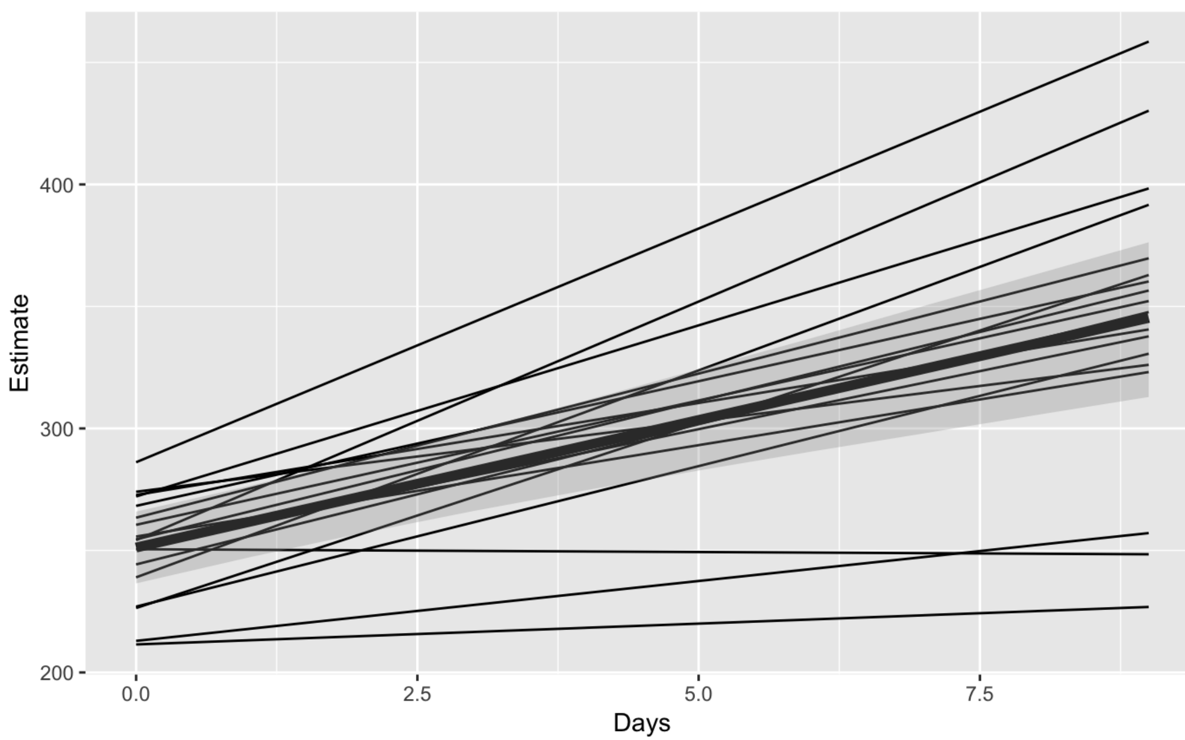
- Syntax is more verbose
- Much more flexible
- Can fit 1-1-1 or 2-1-1 mediations
- Can include covariates, multiple mediators, Level 2 moderators
- Can include random slopes

# Adding random slopes

One of the major benefits of multilevel modeling is the ability to incorporate **random slopes**

We can allow the relationship between two variables to vary across groups.

This often more closely resembles the reality of the world as we understand it, where a relationship is not constant but rather has some variance around a mean slope.



# Random slopes: Indirect Effect

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c_j'(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) \; + e_{Y_{ij}}$$

When the slopes have random variance what does this do to the indirect effect?

**Between Indirect Effect:** unchanged
**Within Indirect Effect**

When $a_{1j}$ and $b_{1j}$ vary across groups, we may want to estimate the **average within-group indirect effect** and it's variance.

Expected Value (i.e. average) of group j's indirect effect

$$E(a_j b_j) = a_W b_W + \sigma_{a_j, b_j}$$

Even when both $a_W$ and $b_W$ are zero, the average within-group indirect effect can be non-zero if the two random effects covary.

This is also true when one or more slope is fixed, but in that case the covariance is zero, so the equation simplifies to $a_W b_W$

# Random slopes: Indirect Effect

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c_j'(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

If $a_j$ and $b_j$ is random, the within-group indirect effect is also random.

We can calculate the variance of the within-group indirect effect across groups as:

$$Var\left(a_j b_j\right) = b^2\sigma^2_{a_j} + a^2\sigma^2_{b_j} + \sigma^2_{a_j}\sigma^2_{b_j} + 2ab\sigma_{a_j,b_j} + \sigma^2_{a_j,b_j}$$

This tells us how much we can expect the within-group indirect effect to vary across groups.

When $a_j$ or $b_j$ is fixed, this variance is zero.

The way we do inference for the indirect effect is unchanged, we continue to use the MCCI and MLmed will include the relevant factors.

# Random slopes: Indirect Effect

$$E(a_j b_j) = a_W b_W + \boxed{\sigma_{a_j,b_j}}$$

$$Var(a_j b_j) = b^2\sigma^2_{a_j} + a^2\sigma^2_{b_j} + \sigma^2_{a_j}\sigma^2_{b_j} + 2ab\sigma_{a_j,b_j} + \boxed{\sigma^2_{a_j,b_j}}$$

When we estimate the *M* equation and *Y* equation separately we do not estimate the covariance $\sigma_{a_j,b_j}$

In these circumstances, it is not possible to estimate the average within group indirect effect.

Instead the equations need to be estimated simultaneously.

Some SEM packages (e.g., Mplus) can estimate multilevel models simultaneously.

Bauer, Preacher, and Gil (2006) demonstrate how the equations can be estimated simultaneously using traditional (univariate) multilevel modeling software.

MLmed utilizes this method when estimating mediation models with random slopes

# Example: Drug Name Fluency

Dohle, S., & Montoya, A. K. (2017). The dark side of fluency: Fluent names increase drug dosing. *Journal of Experimental Psychology: Applied*, *23*(3), 231 – 239.

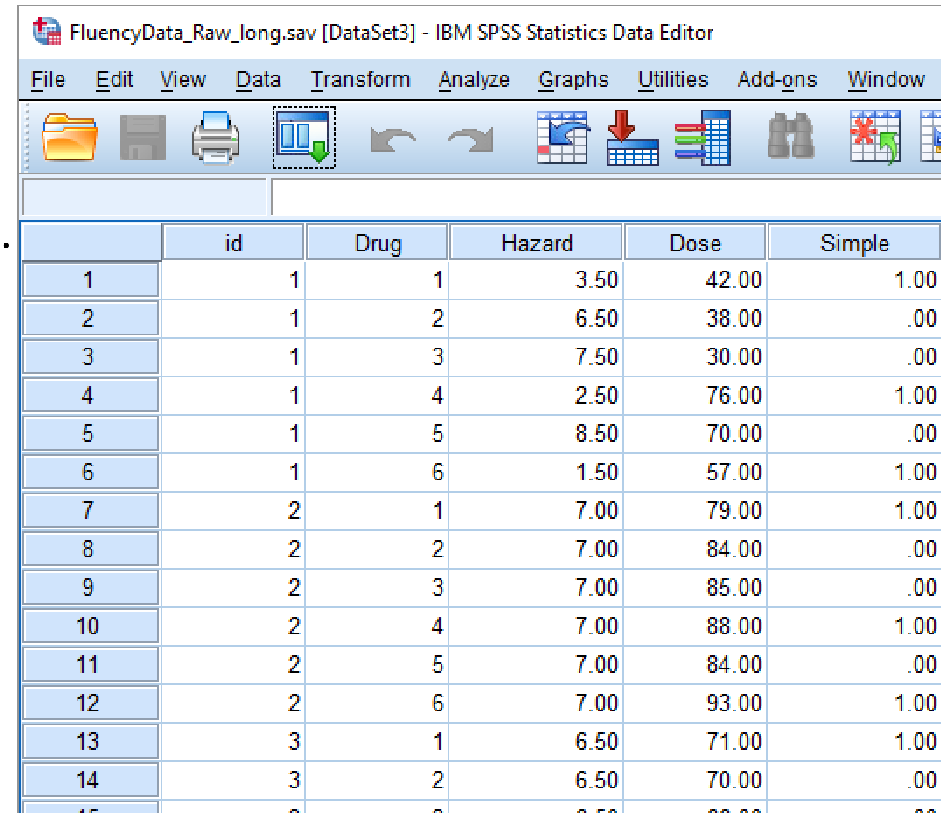
Participants (N = 70) were asked to imagine they had the flu, and 6 different drugs were provided to treat the drug. Participants poured the dose they would feel comfortable taking at maximum into a plastic cup. Each person judged drugs with simple or complex names (**3 of each**).

Open the dataset FluencyData_Raw_long.sav

There are six drugs (Drug = 1 – 6)

Each person saw 3 simple and 3 complex drugs.

We will treat each of these as repeated observations of the same person (6 instead of 2).




| | id | Drug | Hazard | Dose | Simple |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 3.50 | 42.00 | 1.00 |
| 2 | 1 | 2 | 6.50 | 38.00 | .00 |
| 3 | 1 | 3 | 7.50 | 30.00 | .00 |
| 4 | 1 | 4 | 2.50 | 76.00 | 1.00 |
| 5 | 1 | 5 | 8.50 | 70.00 | .00 |
| 6 | 1 | 6 | 1.50 | 57.00 | 1.00 |
| 7 | 2 | 1 | 7.00 | 79.00 | 1.00 |
| 8 | 2 | 2 | 7.00 | 84.00 | .00 |
| 9 | 2 | 3 | 7.00 | 85.00 | .00 |
| 10 | 2 | 4 | 7.00 | 88.00 | 1.00 |
| 11 | 2 | 5 | 7.00 | 84.00 | .00 |
| 12 | 2 | 6 | 7.00 | 93.00 | 1.00 |
| 13 | 3 | 1 | 6.50 | 71.00 | 1.00 |
| 14 | 3 | 2 | 6.50 | 70.00 | .00 |

# Doing it in SPSS: MLmed

```
Mlmed data = dataset1
/x = Simple
/xB = 0
/randx = 01
/m1 = Hazard
/randm = 1
/y = Dose
/cluster = id
/covmat = UN
/folder = /Users/Akmontoya/Desktop/
```

First number is random effect of *X* on *Y* ($c_j'$), second number is random effect of *X* on *M* ($a_j$)

Random effect of *M* on *Y* ($b_j$). When you have *k* mediators this list should be *k* long (e.g., 3 mediators 010)

# Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /randx =
01 /m1 = Hazard /randm = 1 /y = Dose /cluster = id
/covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
***************************  FIXED EFFECTS  ***************************

************************************************************************
 Outcome: Hazard

Within- Effects
         Estimate      S.E.        df         t         p        LL        UL
constant   5.3048     .1212   69.0000   43.7751     .0000    5.0630    5.5465
Simple    -2.1048     .1848   69.0000  -11.3893     .0000   -2.4734   -1.7361

Note: No Between- Effect(s) Specified.

************************************************************************
 Outcome: Dose

Within- Effects
         Estimate      S.E.        df         t         p        LL        UL
constant 115.4202   19.4577   68.0000    5.9318     .0000   76.5929  154.2475
Simple     3.8946    2.0466  342.6421    1.9029     .0579    -.1309    7.9201
Hazard    -3.0849     .6996   76.2642   -4.4094     .0000   -4.4782   -1.6916

Between- Effects
       Estimate      S.E.        df         t         p        LL        UL
Hazard  -5.9688    3.6037   68.0000   -1.6563     .1023  -13.1598    1.2222
```

Fixed effects are not different from when we used the averaged data.

# Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /randx = 01 /m1 =
Hazard /randm = 1 /y = Dose /cluster = id
/covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
**************************  RANDOM EFFECTS  ***************************

Level-1 Residual Estimates
       Estimate      S.E.    Wald Z         p        LL        UL
Dose   245.4574   20.2443   12.1247     .0000  208.8202  288.5225
Hazard   1.3619     .1151   11.8322     .0000    1.1540    1.6073

Random Effect Estimates
       Estimate      S.E.    Wald Z         p        LL        UL
(1,1)     .8010     .1761    4.5494     .0000     .5206    1.2323
(2,2)  880.2044  158.0058    5.5707     .0000  619.1332  1251.362
(3,3)    1.4827     .4142    3.5799     .0003     .8576    2.5635
(4,3)     .6696     .8673     .7720     .4401   -1.0303    2.3695
(4,4)    3.9093    3.5234    1.1095     .2672     .6682   22.8701

Random Effect Covariance Matrix
            1          2          3          4
1       .8010      .0000      .0000      .0000
2       .0000   880.2044      .0000      .0000
3       .0000      .0000     1.4827      .6696
4       .0000      .0000      .6696     3.9093

Random Effect Correlation Matrix
            1          2          3          4
1      1.0000      .0000      .0000      .0000
2       .0000     1.0000      .0000      .0000
3       .0000      .0000     1.0000      .2781
4       .0000      .0000      .2781     1.0000
```

```
Random Effect Key
1     Int       Hazard
2     Int       Dose
3     Slope     Simple     ->     Hazard
4     Slope     Hazard     ->     Dose
```

# Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /randx = 01 /m1 =
Hazard /randm = 1 /y = Dose /cluster = id
/covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
************************  INDIRECT EFFECT(S)  *************************

Within- Indirect Effect(s)
           E(ab)   Var(ab)    SD(ab)
Hazard    7.1626   46.3690    6.8095

Within- Indirect Effect(s)
          Effect        SE         Z         p      MCLL      MCUL
Hazard    7.1626    1.8403    3.8921     .0001    3.6753   10.8879

Note: No Between- Indirect Effect(s) Specified.
```

On average, within an individual, the difference in dosage between sample drugs and complex drugs that operates indirect through perceived hazardousness is estimated to be 7.16 (MCCI = [3.68, 10.89]), where simple drugs are administered at higher dosages than complex drugs. However there is substantial between-person variability in this indirect effect (SD = 6.81).

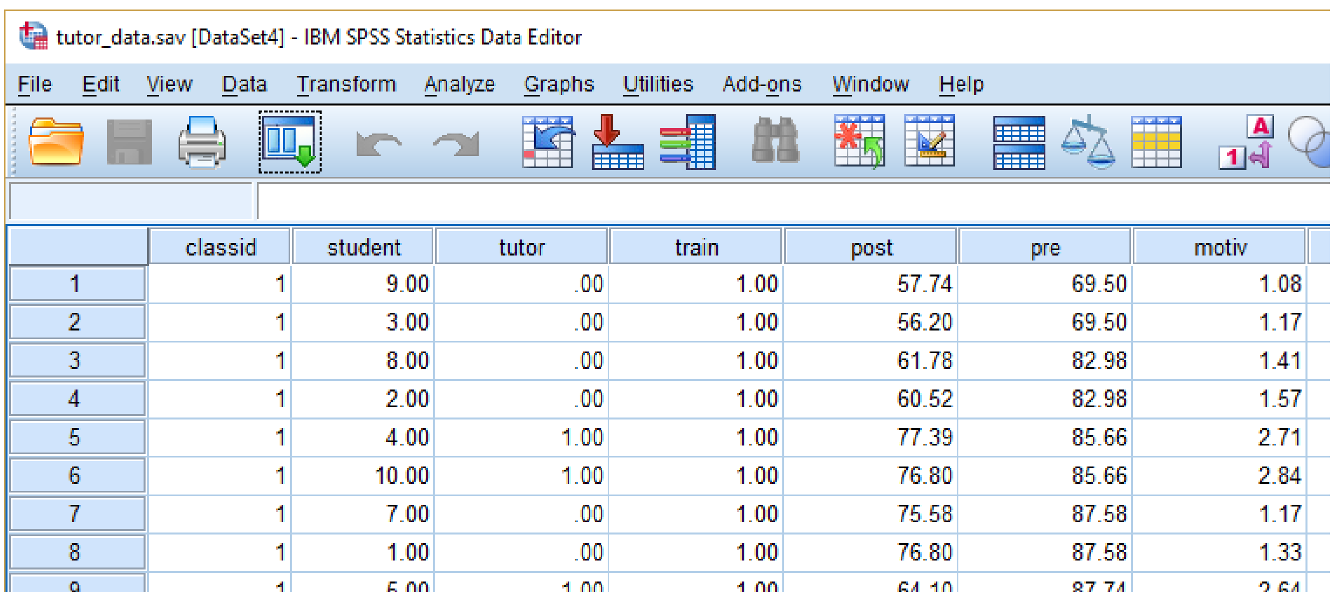
# Tutor Data

This example uses a simulated dataset (tutor_data.sav) based on an educational experiment.

Suppose 48 classrooms were randomly sampled where no classrooms were in the same school.

Next, students within each classroom were randomly sampled to participate in an after-school tutoring program throughout the school year.

The total number of students is 450, where 223 students are assigned to tutoring and 227 are assigned to control (no tutoring program).



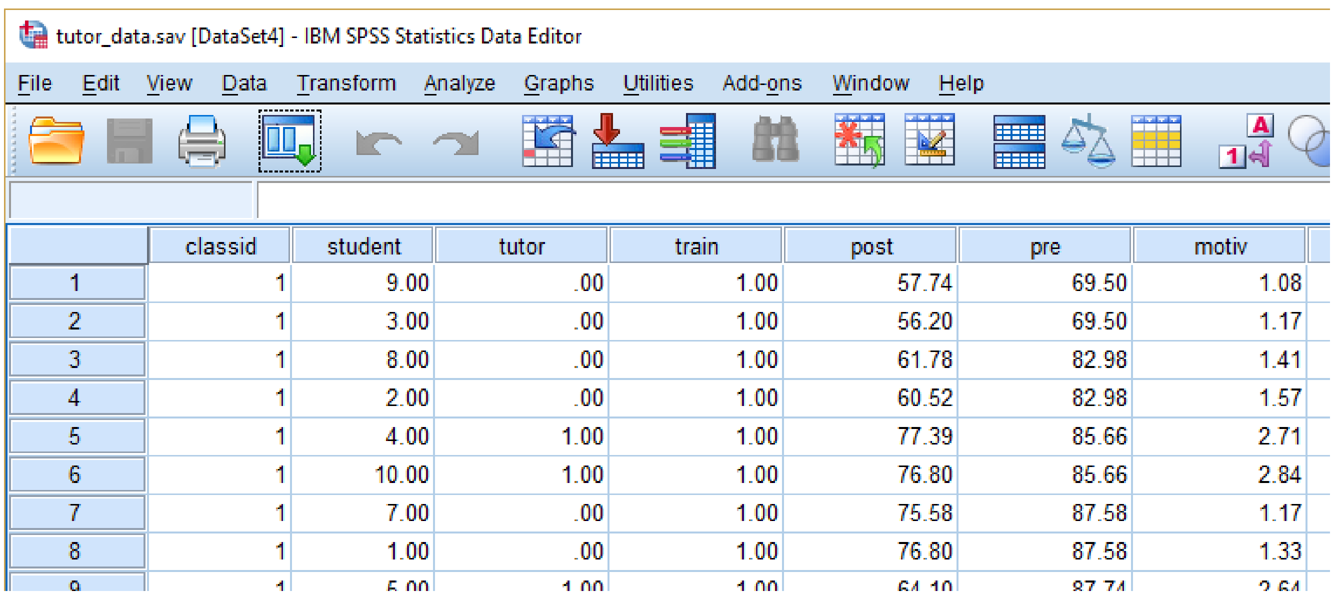

| | classid | student | tutor | train | post | pre | motiv |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 9.00 | .00 | 1.00 | 57.74 | 69.50 | 1.08 |
| 2 | 1 | 3.00 | .00 | 1.00 | 56.20 | 69.50 | 1.17 |
| 3 | 1 | 8.00 | .00 | 1.00 | 61.78 | 82.98 | 1.41 |
| 4 | 1 | 2.00 | .00 | 1.00 | 60.52 | 82.98 | 1.57 |
| 5 | 1 | 4.00 | 1.00 | 1.00 | 77.39 | 85.66 | 2.71 |
| 6 | 1 | 10.00 | 1.00 | 1.00 | 76.80 | 85.66 | 2.84 |
| 7 | 1 | 7.00 | .00 | 1.00 | 75.58 | 87.58 | 1.17 |
| 8 | 1 | 1.00 | .00 | 1.00 | 76.80 | 87.58 | 1.33 |
| 9 | 1 | 5.00 | 1.00 | 1.00 | 64.10 | 87.74 | 2.64 |

# Tutor Data

The **tutor** variable in the dataset codes the assignment of each student (0 = control, 1 = tutoring)

Before completing an end of year mathematics exam (**post**), the students' academic motivation was measured (**motiv**)

There is also data on the students' test scores from the previous year (**pre**).




| | classid | student | tutor | train | post | pre | motiv |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 9.00 | .00 | 1.00 | 57.74 | 69.50 | 1.08 |
| 2 | 1 | 3.00 | .00 | 1.00 | 56.20 | 69.50 | 1.17 |
| 3 | 1 | 8.00 | .00 | 1.00 | 61.78 | 82.98 | 1.41 |
| 4 | 1 | 2.00 | .00 | 1.00 | 60.52 | 82.98 | 1.57 |
| 5 | 1 | 4.00 | 1.00 | 1.00 | 77.39 | 85.66 | 2.71 |
| 6 | 1 | 10.00 | 1.00 | 1.00 | 76.80 | 85.66 | 2.84 |
| 7 | 1 | 7.00 | .00 | 1.00 | 75.58 | 87.58 | 1.17 |
| 8 | 1 | 1.00 | .00 | 1.00 | 76.80 | 87.58 | 1.33 |

# Tutor Data

We are interested in testing whether there is evidence that the participation in after-school tutoring program ($X$ = tutor) results in higher mathematics post-test scores ($Y$ = post), on average, due to an increase in student motivation ($M$ = motiv).

Further we are interested in whether this effect is consistent across classrooms, or whether there is between-classroom variability in the effect.

Throughout, we will use the previous year's math test score ($Q$ = pre) as a covariate.

All variables are level-1 (student level).

The proportion of students assigned to tutoring in each class is not constant, so there is between-class variability in $X$. Additionally there will be between class variability in $M$, $Q$, and $Y$.

We will use group-mean centering to remove between-class variability and add this back into the model using the classroom means as predictors of the random intercepts to that within-class and between-class effect can be estimated separately.

# Doing it in SPSS: MLmed

```
Mlmed data = dataset1
/x = tutor
/m1 = motiv
/y = post
/cov1 = pre
/cluster = classid
/covmat = UN
/folder = /Users/Akmontoya/Desktop/
```

Add in a Level 1 covariate

# Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = tutor /m1 = motiv /y = post /cov1 = pre
/cluster = classid /covmat = UN /folder = Users/Akmontoya/Desktop/
```

Outcome: motiv

Within- Effects

| | Estimate | S.E. | df | t | p | LL | UL |
|---|---|---|---|---|---|---|---|
| constant | 1.2844 | .5128 | 46.8208 | 2.5049 | .0158 | .2528 | 2.3160 |
| tutor | 1.3517 | .0462 | 398.0015 | 29.2783 | .0000 | 1.2610 | 1.4425 |
| pre | .0194 | .0022 | 398.0015 | 8.7609 | .0000 | .0151 | .0238 |

Between- Effects

| | Estimate | S.E. | df | t | p | LL | UL |
|---|---|---|---|---|---|---|---|
| tutor | .8443 | .5654 | 47.1328 | 1.4933 | .1420 | -.2930 | 1.9817 |
| pre | .0039 | .0058 | 45.9489 | .6771 | .5017 | -.0077 | .0156 |

************************************************************************

Outcome: post

Within- Effects

| | Estimate | S.E. | df | t | p | LL | UL |
|---|---|---|---|---|---|---|---|
| constant | 50.3689 | 7.2512 | 43.9803 | 6.9463 | .0000 | 35.7548 | 64.9830 |
| tutor | -3.2913 | 1.4885 | 394.8792 | -2.2112 | .0276 | -6.2176 | -.3649 |
| motiv | 4.4675 | .9097 | 394.8792 | 4.9112 | .0000 | 2.6791 | 6.2559 |
| pre | .3746 | .0440 | 394.8792 | 8.5215 | .0000 | .2882 | .4611 |

Between- Effects

| | Estimate | S.E. | df | t | p | LL | UL |
|---|---|---|---|---|---|---|---|
| tutor | -24.6103 | 7.7505 | 48.0227 | -3.1753 | .0026 | -40.1935 | -9.0271 |
| motiv | 12.3961 | 1.9524 | 43.5166 | 6.3490 | .0000 | 8.4600 | 16.3323 |
| pre | .0119 | .0775 | 44.9123 | .1532 | .8789 | -.1443 | .1680 |

# Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = tutor /m1 = motiv /y = post /cov1 = pre
/cluster = classid /covmat = UN /folder = Users/Akmontoya/Desktop/
```

```
************************  INDIRECT EFFECT(S)  ****************

Within- Indirect Effect(s)
          E(ab)   Var(ab)    SD(ab)
motiv    6.0388     .0000     .0000

Within- Indirect Effect(s)
         Effect        SE         Z         p      MCLL      MCUL
motiv    6.0388    1.2475    4.8408     .0000    3.6273    8.5073

Between- Indirect Effect(s)
         Effect        SE         Z         p      MCLL      MCUL
motiv   10.4665    7.2843    1.4368     .1508   -3.3701   25.5751

Test of Indirect Contextual Effect(s): Between - Within
            Dif      MCLL      MCUL
motiv    4.4276   -9.5305   19.8116
```

# Doing it in SPSS: MLmed

```
************************  INDIRECT EFFECT(S)  ****************

Within- Indirect Effect(s)
         E(ab)   Var(ab)   SD(ab)
motiv   6.0388     .0000     .0000

Within- Indirect Effect(s)
        Effect        SE         Z         p      MCLL      MCUL
motiv   6.0388    1.2475    4.8408     .0000    3.6273    8.5073

Between- Indirect Effect(s)
        Effect        SE         Z         p      MCLL      MCUL
motiv  10.4665    7.2843    1.4368     .1508   -3.3701   25.5751

Test of Indirect Contextual Effect(s): Between - Within
           Dif      MCLL      MCUL
motiv   4.4276   -9.5305   19.8116
```

Within a given classroom, there is a significant indirect effect of tutoring on posttest through motivation controlling for pretest ($E\left(a_j b_j\right) = 6.04,\ MCCI = [3.53{,}8.50]$), where students who participated in tutoring performed better on the post test.

There was not significant evidence that between classroom variability in proportion of students assigned to tutoring influenced average classroom performance through average motivation ($a_B b_B = 10.47\ \mathrm{MCCI} = [-3.42\ 25.32]$). There's not significant evidence that the within and between indirect effects significantly different.

# Exercise: Adding random slopes

In addition to being interested in the average within-class indirect effect, we are also interested in determining if that within-class indirect effect varies across classrooms.

- Using MLmed, expand the model to include a random $a_j$ and $b_j$, as well as the covariance between these paths
- Interpret the individual coefficients and their variances making up the mediation model.
- Interpret the average and variance of the within-group indirect effect in the context of the specific example.

# 2-1-1 Models

The 1-1-1 model is for the general data design where *X*-*M*-*Y* all contain within and between group variability.

In the dosage data, the model we fit only had within group variability in *X*.

MLmed can also be used to fit models where *X* only contains information about between-group variability (2-1-1 models). This  type of model is useful for cluster-randomized designs (each group assigned to a condition).

MLmed does not fit 2-2-1 models but these can be fit piecewise, where *X* → *M* is an OLS regression and the *Y* equation is fit using MLM.

Models with "upward effects" (e.g., 1-2-1) cannot be fit in MLM software and require multilevel structural equation modeling.

# 2-1-1 Example

Suppose that rather than students assigned to tutoring, teachers completed a training program deisgned to teach a number of skills focused on engaging their students through the use of interactive real-world applications.

It is thought that students who are exposed to the interactive real-world applications will see the utility of the content being taught and they will be more motivatted, leading to an increase in their post-test scores.

The variable **train** is a teacher-level (Level 2) training identifier (1 = completed training, 0 = control)

# 2-1-1 Example

In the tutor dataset we may be interested in testing if the average amount of student motivation mediates the relationship between the teacher's completion of a training program and the average post-test score of their students.

This is a 2-1-1 model since training (*X*) is a Level 2 variable (classroom level), motivation and posttest scores are both at Level 1 (student level).

There can only be a between-group indirect effect.

```
MLmed data = dataset4
/x = train
/xW = 0
/m1 = motiv
/y = post
/cov1 = pre
/cluster = classid
/folder = /Users/Akmontoya/Desktop/
```

Set the within effect of X to zero

# 2-1-1 Example

```
MLmed data = dataset4 /x = train /xW = 0 /m1 = motiv /y = post
/cov1 = pre /cluster = classid /folder = /Users/Akmontoya/Desktop/
```

```
**************************************************************************
 Outcome: motiv

Within- Effects
         Estimate     S.E.        df         t         p        LL        UL
constant   2.2615    .4840   56.7024    4.6724     .0000    1.2921    3.2308
pre         .0295    .0039  398.6255    7.6001     .0000     .0219     .0372

Between- Effects
      Estimate     S.E.        df         t         p        LL        UL
train     .3489    .1743   46.2330    2.0014     .0512    -.0019     .6997
pre      -.0065    .0078   55.2377    -.8368     .4063    -.0220     .0090

**************************************************************************
 Outcome: post

Within- Effects
         Estimate     S.E.        df         t         p        LL        UL
constant  48.8441   8.4186   45.1925    5.8019     .0000   31.8902   65.7980
motiv      2.8059    .5150  396.2348    5.4481     .0000    1.7934    3.8185
pre         .3991    .0427  396.2348    9.3352     .0000     .3150     .4831

Between- Effects
      Estimate     S.E.        df         t         p        LL        UL
train    5.2720   2.6375   43.0894    1.9989     .0520    -.0467   10.5907
motiv    9.8785   2.1110   43.3758    4.6796     .0000    5.6224   14.1346
pre      -.0992    .1109   47.9342    -.8953     .3751    -.3221     .1236

**************************************************************************
```

# 2-1-1 Example

```
MLmed data = dataset4 /x = train /xW = 0 /m1 = motiv /y = post
/cov1 = pre /cluster = classid /folder = /Users/Akmontoya/Desktop/
```

```
*************************  RANDOM EFFECTS  **************************

Level-1 Residual Estimates
      Estimate      S.E.    Wald Z         p        LL        UL
post   75.1677    5.3404   14.0754     .0000   65.3968   86.3983
motiv    .7104     .0503   14.1178     .0000     .6184     .8162

Random Effect Estimates
   Estimate      S.E.    Wald Z         p        LL        UL
1      .0818     .0358    2.2833     .0224     .0347     .1931
2    26.8186    7.9874    3.3576     .0008   14.9596   48.0784

Random Effect Key
1     Int          motiv
2     Int          post
```

# 2-1-1 Example

```
MLmed data = dataset4 /x = train /xW = 0 /m1 = motiv /y = post
/cov1 = pre /cluster = classid /folder = /Users/Akmontoya/Desktop/
```

```
************************  INDIRECT EFFECT(S)  *************************

Note: No Within- Indirect Effect(s) Specified.

Between- Indirect Effect(s)
        Effect        SE         Z         p      MCLL      MCUL
motiv   3.4462    1.9085    1.8057     .0710     .1184    7.4554
```

There is a significant between-group indirect effect of teacher training on student posttest, by way of student motivation ($a_B b_B = 3.45, MCCI = [0.05, 7.47]$). Specifically, the students of teachers who participated in the training had higher motivation on average, than students of teachers who did not participate in the training, and higher average motivation led to higher average posttest scores.

# Other Types of Repeated Measures Mediation

- Latent Growth Curve Models (Longitudinal Processes M-Y measured over time)
  - Choeng, MacKinnon, Khoo (2003) *Structural Equation Modeling*

- Structural Equation Modeling (Can be used for a variety of data types)
  - Cole & Maxwell (2003) *Journal of Abnormal Psychology*
    X, M, and Y all measured over time
  - Newsom (2009) *Structural Equation Modeling*
    Dyadic data using LGMs
  - Selig & Little (2012) *Handbook of Developmental Research Methods*
    Autoregressive models and cross-lagged panel models for longitudinal data X, M, and Y all measured over time.

- Multilevel SEM
  - Preacher, Zyphyr, Zhang, 2010
  - Preacher, Zhang, Zyphur, 2011

Selig & Preacher (2009) *Research in Human Development*

- Longitudinal Models X, M, and Y measured across time. Cross-lagged panel models, latent growth models, latent difference score models

# Wrapping Up

Where to learn more:

| | | |
|---|---|---|
| Amanda Kay Montoya | akmontoya.com | akmontoya@ucla.edu |
| Nick Rockwood | njrockwood.com | rockwood.19@osu.edu |
| | | |
| @AmandaKayMontoya | @njrockwood | |

MLMED and MEMORE both have many features not described here

MLMED does moderated mediation

MEMORE does moderation and (coming soon) moderated mediation

**Mediation for Dyadic Data!** http://afhayes.com/public/chj2019.pdf

**Jacob Coutts: Poster Friday 2:30 – 3:30 V83**

**Andrew Hayes: Talk Saturday 1:30 – 1:55 Wilson AB**

Github.com/akmontoya/APS2019