

The background of the slide features a wide-angle photograph of the Swiss Alps under a clear blue sky. In the foreground, a small town with white buildings and red roofs is nestled at the base of the mountains. A church with a tall, light-colored steeple stands prominently on the right side of the town.

Mediation, Moderation, and Conditional Process Analysis I

Instructor: Amanda K. Montoya



14-18 June 2021

Global School in Empirical Research Methods, St. Gallen, Switzerland

Understanding causal effects

Hagtvedt, H., & Patrick, V. M. (2008). Art infusion: The influence of visual art on the perception and evaluation of consumer products. *Journal of Marketing Research*, 45, 379-389.



HENRIK HAGTVEDT and VANESSA M. PATRICK*

In this research, the authors investigate the phenomenon of "art infusion," in which the presence of visual art has a favorable influence on the evaluation of consumer products through a content-independent spillover of luxury perceptions. In three studies, the authors demonstrate the art infusion phenomenon in both real-world and controlled environments using a variety of stimuli in the contexts of packaging, advertising, and product design.

Keywords: visual art, luxury, aesthetics, spillover effects, packaging, advertising, product design

Art Infusion: The Influence of Visual Art on the Perception and Evaluation of Consumer Products

How does the presence of visual art alter the way people view a consumer product? Throughout history, art has had the ability to arouse the imagination and capture the attention. Therefore, it is not surprising that art images are often used to promote unrelated products—for example, by being displayed in advertisements (Hetsroni and Tukachinsky 2005). It is proposed that such "high-culture" images reach more people more often through advertising than through any other medium" (Hoffman 2002, p. 6). Other times, art becomes an integrated part of a product, such as when furniture is artistically designed or a painting is printed on a shirt. Some companies, such as De Beers, use art in image promotion, conveying the idea that diamonds, like paintings, are unique works of art (Epstein 1982). Sometimes, art is even created for the sole purpose of marketing a product, such as in the enduring Absolut Vodka advertising campaign (Lewis 1996).

It is clear that influential marketing practitioners believe that art somehow has the power to influence consumer per-

ceptions. Vast amounts of money are spent on representing visual art in conjunction with products, in the hope that the products will become more marketable as a result. However, the issue of whether these beliefs are well founded remains unresolved. Furthermore, there is little evidence to suggest that marketing professionals have been provided with the scientific basis necessary to use visual art in a strategic manner rather than purely on the basis of experience and intuition. Supplying this basis is a complex endeavor. However, the current research represents an initial step to analyze systematically the influence of visual art on consumer evaluations of the products with which it is associated. This influence represents a fundamental gap in current understanding, not only in terms of the \$23.5 billion global art market (Kusin & Company 2002) but also in terms of the potential impact of art on other markets and marketing activities.

In this research, we examine the phenomenon of "art infusion," which we broadly define as the general influence of the presence of art on consumer perceptions and evaluations of products with which it is associated. More specifically, we theorize that perceptions of luxury associated with visual art spill over from the artwork onto products with which it is associated, leading to more favorable evaluations of these products. Furthermore, we propose that this influence does not depend on the content of the specific artwork—that is, what is depicted in the artwork—but rather on general connotations of luxury associated with visual art.

In Study 1, we demonstrate the art infusion phenomenon in a real-world setting. In this study, consumers are briefly exposed to art or nonart images, which are matched for

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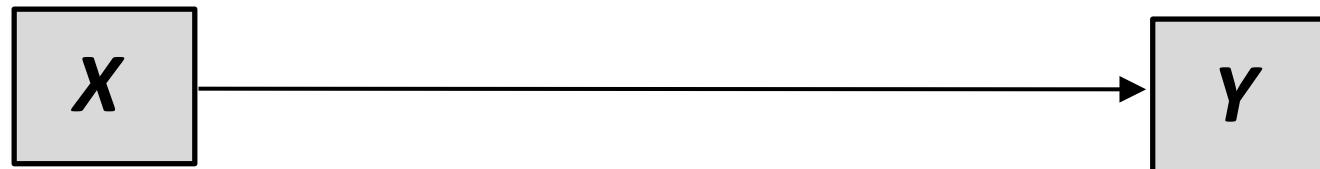


PHOTO or PAINTING

PRODUCT
EVALUATION

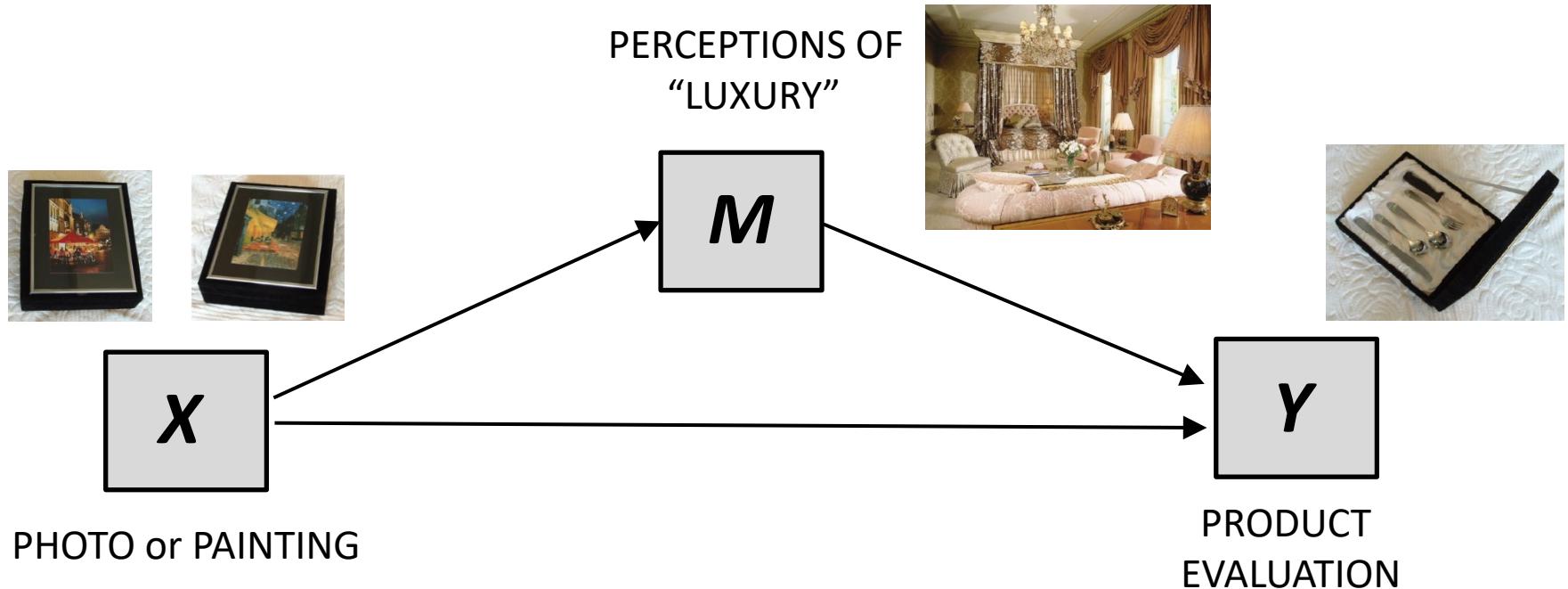
The product with the impressionist painting on the box was evaluated more favorably than the product with a photograph of a similar scene.

Remaining Questions:

- How does this effect occur? What is the *mechanism* that produces it?
- Is the effect consistent across type of product, type of consumer, and so forth.

These are the important questions!

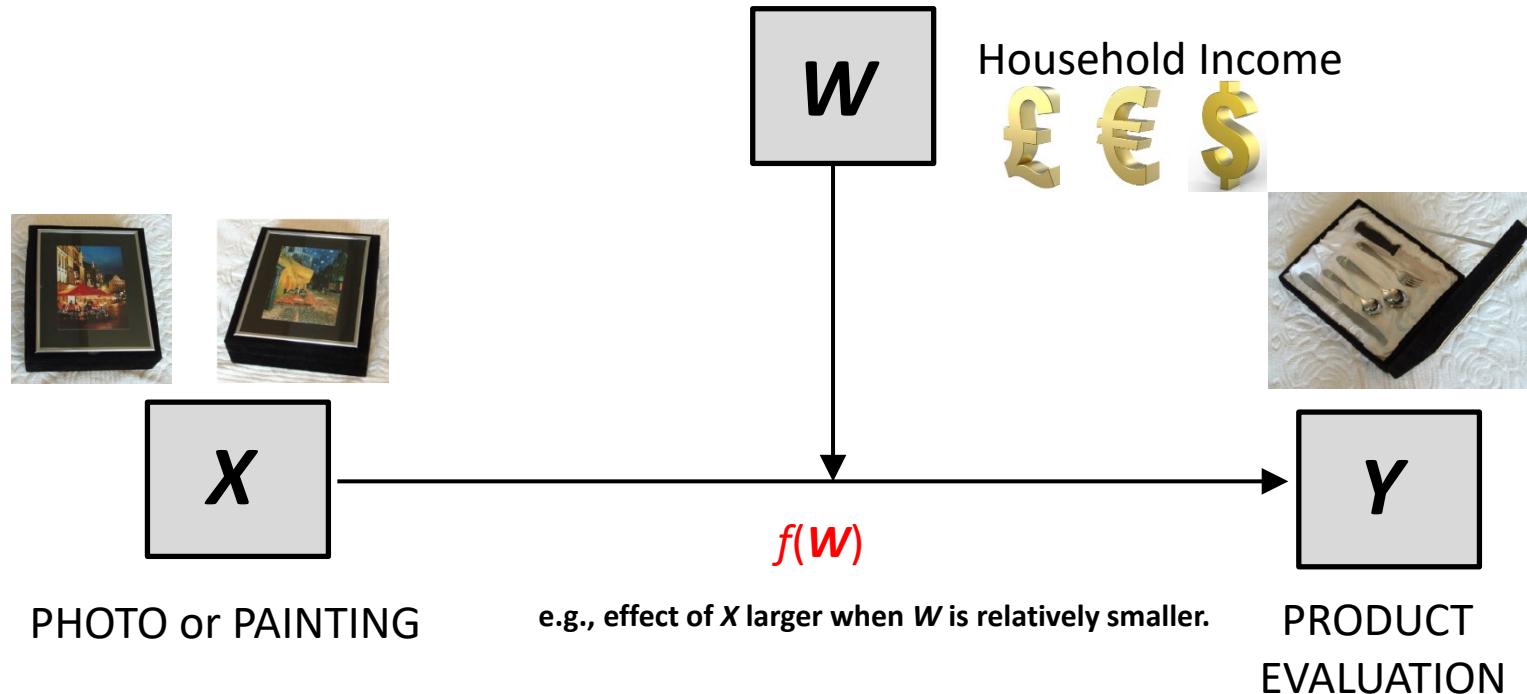
A simple mediation model



Art-infused product was perceived as more “luxurious,” and this greater perceived luxury translated into a more favorable product evaluation. Thus, the infusion of the product with art influenced product evaluation at least partly through the “mechanism” of perceived luxuriousness.

Mediation analysis is about estimating and making inferences about such *indirect effects*.

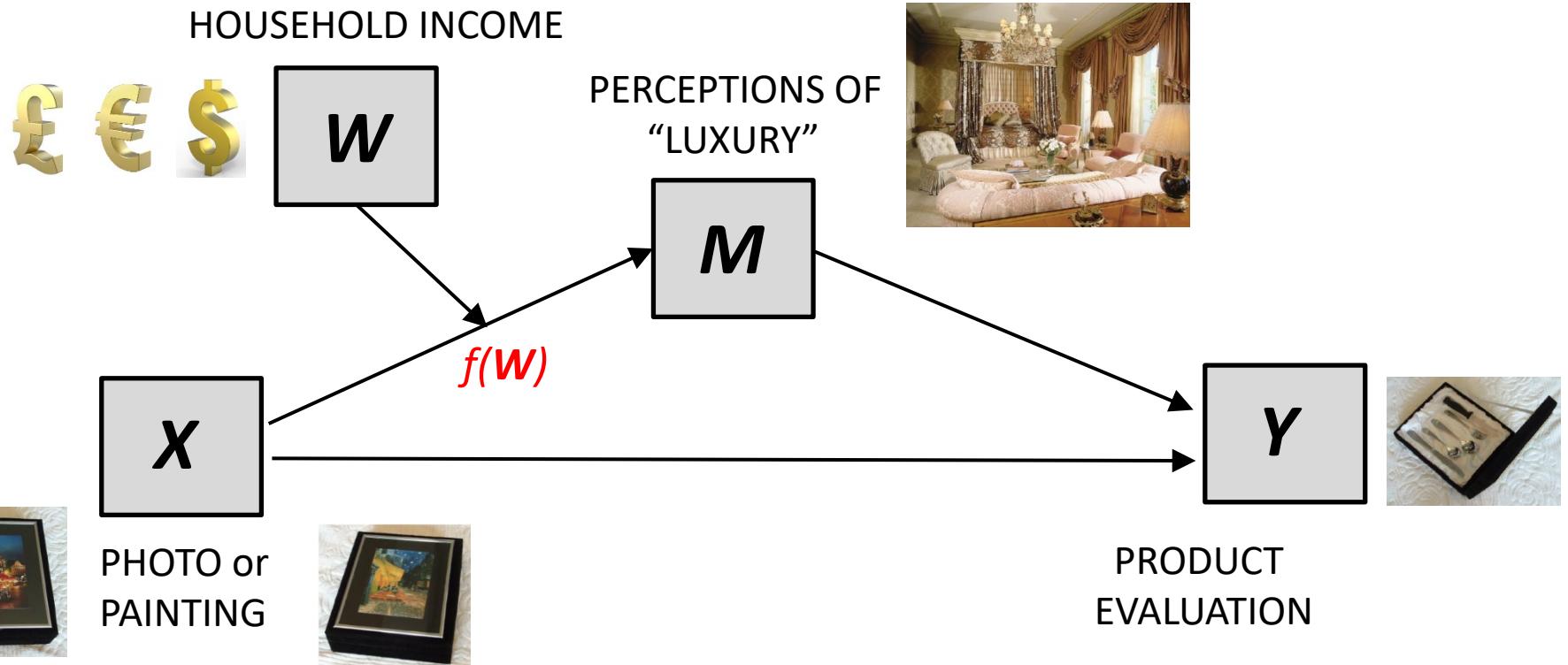
Moderation



Is this effect of art infusion larger among those with less income? Is the size of the effect dependent on (or a function of) income? In this case, income is a *moderator* of the effect of art infusion on product evaluation.

Moderation analysis is about the estimation of *contingent effects*, i.e., examining the boundary conditions of effects or the factors that make effects large versus small, positive versus negative.

Combining moderation and mediation



Does art infusion result in greater perceptions of luxury more so among those with less income? If so, then the indirect effect of art infusion on product evaluation through perceptions of luxury depends on income. Thus, the strength of this “mechanism” may depend on income. Mediation can be moderated.

In this class

- After a review of OLS regression, we start with questions of “**HOW**”—
statistical mediation analysis

“Direct,” “indirect,” and “total effects” in path models and how to test hypotheses in such models using OLS regression and various computational tools developed for this purpose.

- We then move to questions of “**WHEN**”—**moderation analysis**

Estimation and interpretation of models in which a predictor can have different effects on an outcome depending on the value of another variable in the model.

- We then explore models that combine moderation and mediation—
“conditional process analysis”
- With fundamentals covered, we address more complex models and issues in mediation and moderation analysis, such as multiple mediators and multi-categorical independent variables and repeated-measures mediation.

What you'll need

- This course is hands-on. Hopefully you brought a laptop with SPSS 19+ or SAS 9.1+ with PROC IML. If not, that is ok. You'll still benefit.

[SPSS Code](#)

[SAS Code](#)

- Various files available on a USB drive or github.com/akmontoya/GSERM21SG.

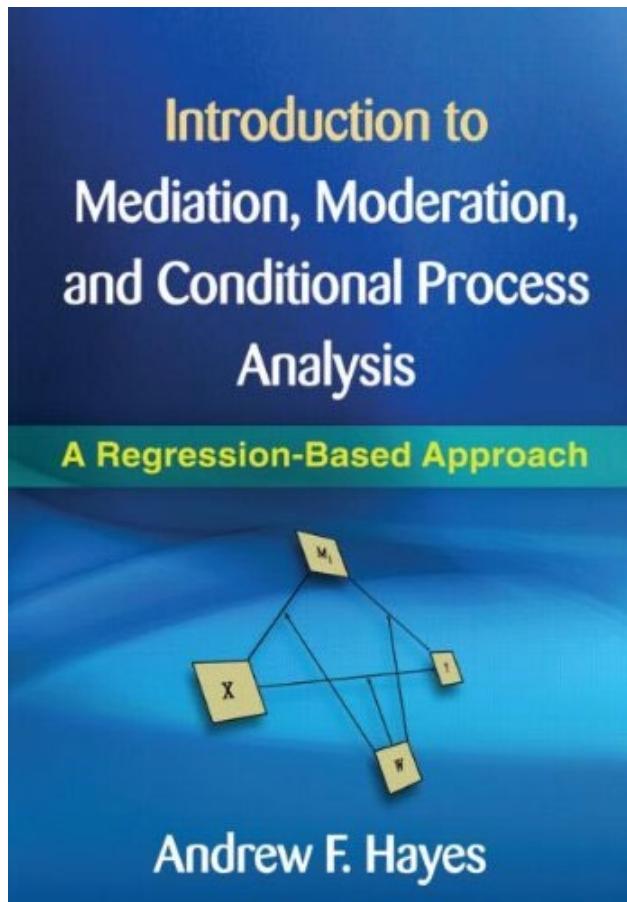
- SPSS and SAS data folders. SPSS data files are ready to go. SAS files are programs thus must be executed to make them “work” files.
 - SPSS and SAS PROCESS folders. This contains the PROCESS macro we'll heavily rely on, and some documents related to it.
 - Miscellaneous folder. Various files, including some PDFs and other miscellaneous things of relevance to this course.
-
- A lot of stamina.

What we will and won't do

- We will stick with fairly simple models to cover basic principles, with continuous outcomes, and cross-sectional or experimental data.
- Statistical mediation analysis. No discussion of counterfactuals, “potential outcomes,” directed graphs, or other approaches to thinking about cause.
- Everything OLS-regression-based.
- No dichotomous outcomes, nothing multilevel.

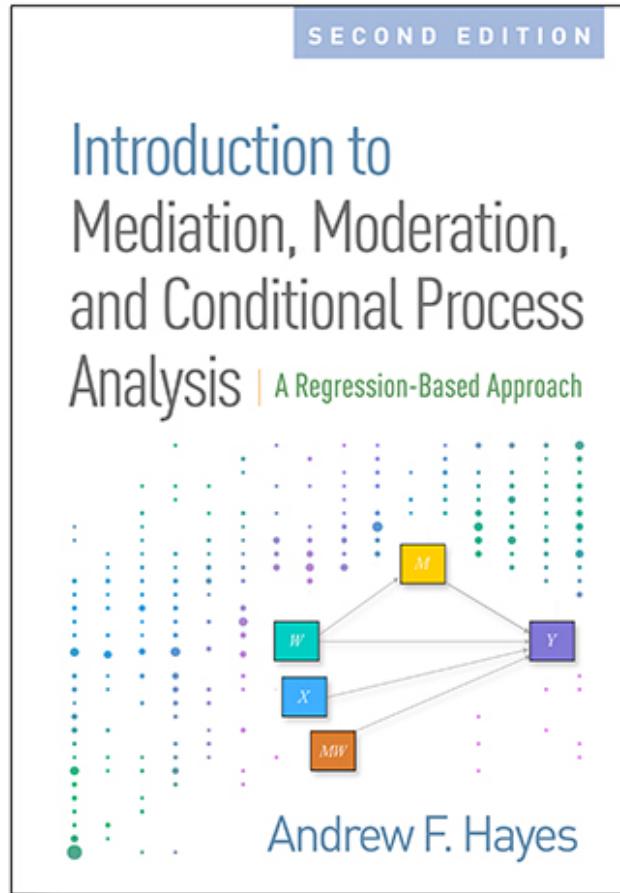
Although the principles are not software specific, their implementation is facilitated with the use of a “macro” which makes otherwise tedious things very simple and effortless. You will learn about PROCESS.

This course is a companion to...



2013

SECOND EDITION



December 2017

Example for the class “inspired by”...

Bayram-Ozdemir, S. & Stattin, H. (2014). Why and when is ethnic harassment a risk for immigrant adolescents' school adjustment? Understanding the processes and conditions. *Journal of Youth and Adolescence*, 43, 1252-1265.



J Youth Adolescence (2014) 43:1252–1265
DOI 10.1007/s10964-013-0038-y

EMPIRICAL RESEARCH

Why and When is Ethnic Harassment a Risk for Immigrant Adolescents' School Adjustment? Understanding the Processes and Conditions

Sevgi Bayram Özdemir · Håkan Stattin

Received: 5 July 2013 / Accepted: 7 October 2013 / Published online: 17 October 2013
© Springer Science+Business Media New York 2013

Abstract Ethnically harassed immigrant youth are at risk for experiencing a wide range of school adjustment problems. However, it is still unclear why and under what conditions experiencing ethnic harassment leads to school adjustment difficulties. To address this limitation in the literature, we examined two important questions. First, we investigated whether self-esteem and/or depressive symptoms would mediate the associations between ethnic harassment and poor school adjustment among immigrant youth. Second, we examined whether immigrant youths' perception of school context would play a buffering role in the pathways between ethnic harassment and school adjustment difficulties. The sample ($n = 330$; $M_{age} = 14.07$, $SD = .90$; 49 % girls at T1) was drawn from a longitudinal study in Sweden. The results revealed that experiencing ethnic harassment led to a decrease in immigrant youths' self-esteem over time, and that youths' expectations of academic failure increased. Further, youths' relationships with their teachers and their perceptions of school democracy moderated the mediation processes. Specifically, when youth had poor relationships with their teachers or perceived their school context as less democratic, being exposed to ethnic harassment led to a decrease in their self-esteem. In turn, they reported low school satisfaction and perceived themselves as being unsuccessful in school. Such indirect effects were not observed when youth had high positive relationships with their teachers or perceived their school as offering a democratic environment. These findings highlight the importance of understanding underlying processes and conditions in the examination of the effects of ethnic devaluation experiences in order to reach a more comprehensive understanding of immigrant youths' school adjustment.

Keywords Immigrant youth · School adjustment · Ethnic harassment · Ethnic victimization · Depression · Self-esteem

Introduction

Adjustment and success in academic life is a key factor for immigrant youths' integration into the host culture and their future prospects (Health et al. 2008). Thus, this issue has become one of the policy priorities for immigrant-receiving countries, and extensive efforts have been made to identify the factors that may play a role in the school adjustment and performance of immigrant youth. Experience of ethnic harassment (i.e., negative treatments or derogatory comments in relation to ethnic background) is one of the major contextual stressors for immigrant youth (Garcia Coll et al. 1996) and poses a threat to their school adjustment.

Research on ethnic minority adolescents in the U.S. and Europe has shown that a substantial number of youth are treated badly and victimized by their peers, teachers, and neighbors, at school and in other contexts (e.g., Hunyadi and Fulgini 2010; Liebkind et al. 2004; Verkuyten and Thijss 2002). Such negative experiences have been linked to a wide range of school outcomes. Youth who are harassed on the basis of their ethnic origin tend to develop negative beliefs about their academic competence and rewards of schooling (Eccles et al. 2006; Wong et al. 2003), display

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Springer



Example for the class “inspired by...”

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330 7th to 9th grade students in Sweden measured in the spring term (T1) and again one year later (T2).
All were first or second generation immigrants who reported at least some ethnic harassment.

HARASS: 6-item measure of ethnicity-related harassment frequency (scaled 1 to 5). T1 only.

POSREL: 6-item measure of positivity of relationships with teachers (scaled 1 to 4). T1 only.

SE: 10-item Rosenberg self-esteem scale (scaled 1 to 4). T1 and T2.

DEP: 20-item Center for Epidemiological Studies Depression Scale for Children (scaled 1 to 4). T1 and T2.

FAIL: 4-item measure of perceived academic failure at school (scaled 1 to 4). T1 and T2.

SATIS: 5-item measure of satisfaction in school (scaled 1 to 5). T1 and T2.

All are continuous variables scaled such that higher = more.

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The Data: HARASS

SPSS

	harass	se2	dep2	satis2	fail2	posrel	se1	dep1	satis1	fail1
1	2.16	3.80	1.25	4.80	1.00	1.66	3.60	1.10	4.40	2.00
2	2.33	2.90	1.95	4.20	1.75	2.83	2.80	2.25	3.60	2.25
3	1.16	3.80	1.95	5.00	1.50	3.16	3.80	1.95	4.80	1.00
4	2.50	2.50	2.15	3.00	2.00	2.00	3.10	1.25	3.20	2.00
5	1.33	3.00	1.90	2.60	1.00	2.83				
6	1.16	2.50	1.25	3.40	1.25	2.83				
7	1.50	2.10	2.05	2.40	1.50	3.00				
8	1.50	3.30	1.75	2.80	1.25	3.00				
9	1.83	2.70	2.15	3.20	1.50	3.50				
10	1.33	2.80	2.45	4.00	2.00	3.16				

The SPSS file is ready for analysis. The SAS version is a SAS program that must be executed to produce a temporary work data file.

“1” and “2” in the variable name refers to time 1 and time 2, respectively. The absence of a number means the data were available only at time 1.

SAS

```
data harass;
  input harass se2 dep2 satis2 fail2 posrel se1 dep1 satis1 fail1;
datalines;
2.16 3.80 1.25 4.80 1.00 1.66 3.60 1.10 4.40 2.00
2.33 2.90 1.95 4.20 1.75 2.83 2.80 2.25 3.60 2.25
1.16 3.80 1.95 5.00 1.50 3.16 3.80 1.95 4.80 1.00
2.50 2.50 2.15 3.00 2.00 2.00 3.10 1.25 3.20 2.00
1.33 3.00 1.90 2.60 1.00 2.83 2.60 1.55 2.40 1.75
1.16 2.50 1.25 3.40 1.25 2.83 2.80 1.45 3.00 2.00
1.50 2.10 2.05 2.40 1.50 3.00 1.80 2.10 2.40 2.00
1.50 3.30 1.75 2.80 1.25 3.00 3.40 1.55 3.80 1.25
1.83 2.70 2.15 3.20 1.50 3.50 2.90 2.20 3.40 1.50
1.33 2.80 2.45 4.00 2.00 3.16 2.80 1.25 4.00 1.25
1.33 2.60 1.70 3.60 1.75 3.16 3.00 2.05 4.40 2.25
```

These are not the actual data from this study. They were generated to produce similar results to the published study.

A quick review of regression analysis

- Linear regression is the foundation of this class.
- Used throughout science as a means of “modeling” the relationship between variables.
- Many of the kinds of analyses and statistics you already know about can be expressed in the form of a linear regression model
 - independent groups t test
 - analysis of variance

Pearson's coefficient of correlation (r) is the building block of linear regression analysis. Consider the correlation between positivity of relationships with teachers and satisfaction at school (measured contemporaneously).

Correlations

		posrel	satis1
posrel	Pearson Correlation	1	.458
	Sig. (2-tailed)		.000
	N	330	330
satis1	Pearson Correlation	.458	1
	Sig. (2-tailed)	.000	
	N	330	330

SPSS code in black box

```
correlations variables = posrel satis1.
```

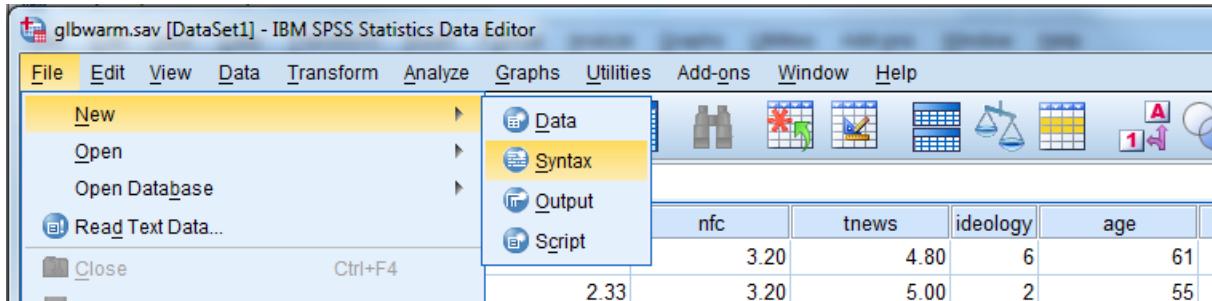
SAS code in white box

```
proc corr data=harass;var posrel satis1;run;
```

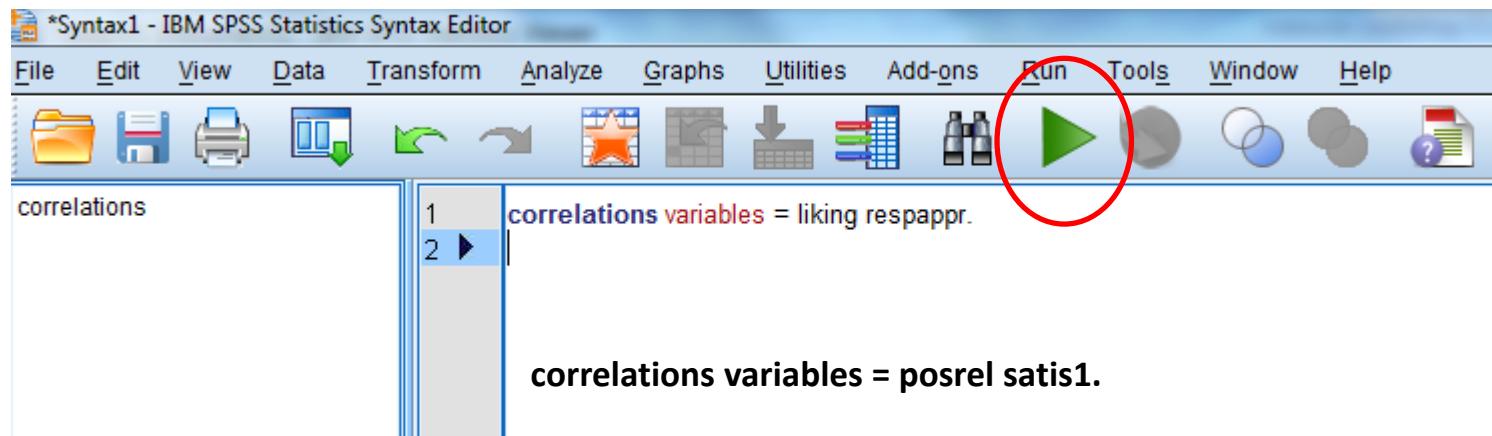
Using SPSS syntax

We will use syntax to instruct SPSS what to do in this class. There are many benefits of learning how to write SPSS syntax.

(1) Open a new syntax window (File > New > Syntax)



(2) Type your command(s) into the blank window that opens



(3) Click and drag to highlight code you want to execute and press the “play” button or select various options under “Run” in the syntax window menu.



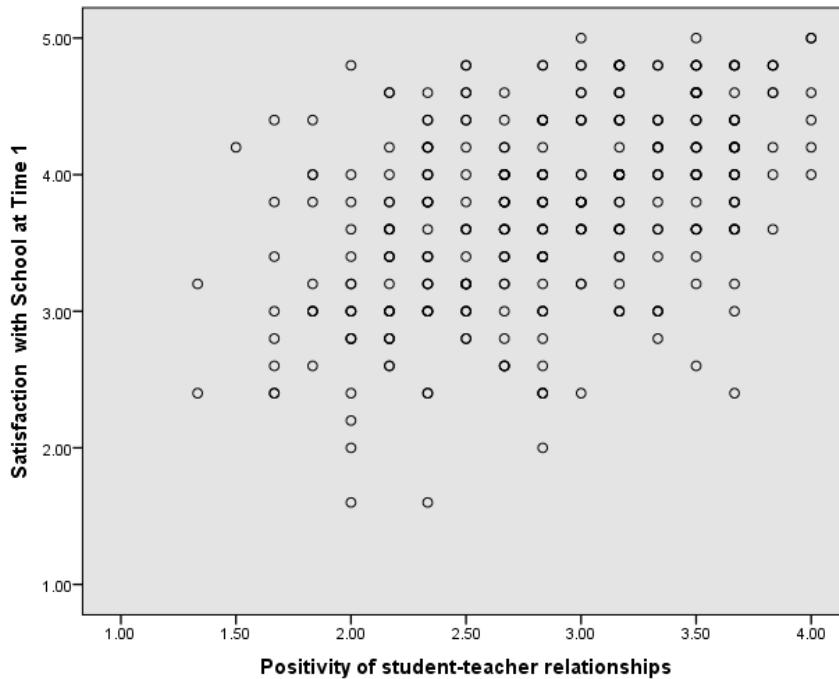
A scatterplot

Consider a scatterplot visually depicting this relationship:

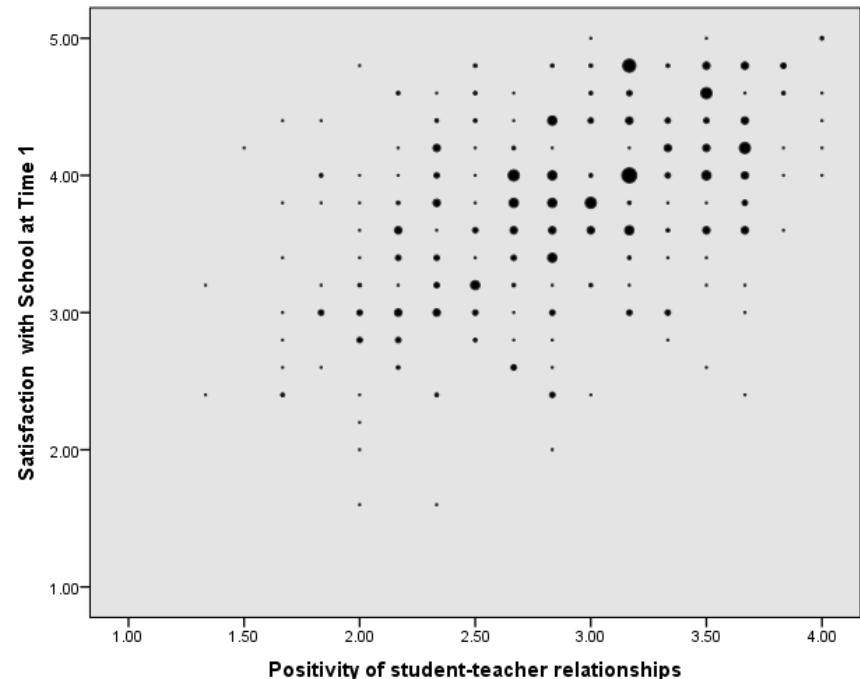
```
graph/scatterplot=posrel with satis1.
```

```
proc sgscatter data=harass;plot satis1*posrel;run;
```

Overlapping points make this unclear



After “binning” to see the overlap:



If you had to draw a single straight line through this plot that “best fits” the relationship, where would you draw it? At its heart, this is the problem regression analysis solves.

OLS (Ordinary Least Squares) linear regression

Goal: Derive the equation (“model”) for the line representing the association between independent variable X and dependent variable Y that “best fits” the data.

The “simple regression model” (i.e., only one variable on the right hand side) takes the form

$$Y_i = b_0 + b_1 X_i + e_i$$

Using the ordinary least squares criterion, there is only one line described by the function

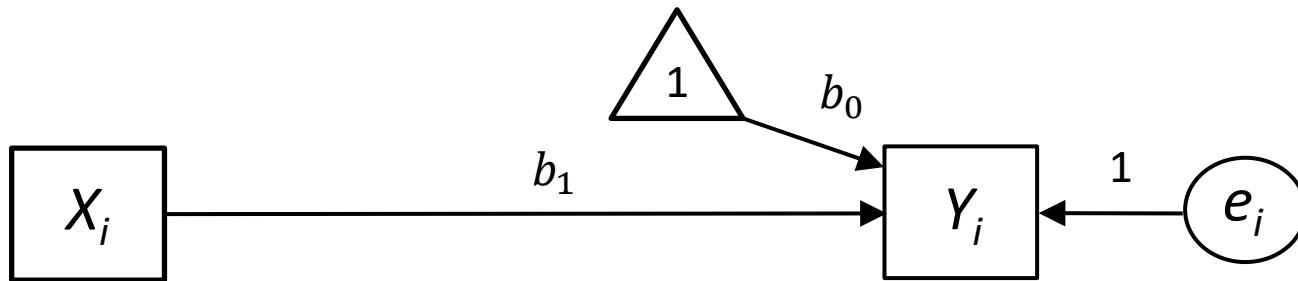
$$\hat{Y}_i = b_0 + b_1 X_i \quad e_i = Y_i - \hat{Y}_i$$

that “best fits” the data, where \hat{Y}_i is the **estimated** or **fitted** value of Y_i , and “best fit” is defined as the line that minimizes the **sum of the squared residuals** ($SS_{residual}$), summed over all n cases in the data. This is called the **LEAST SQUARES** criterion.

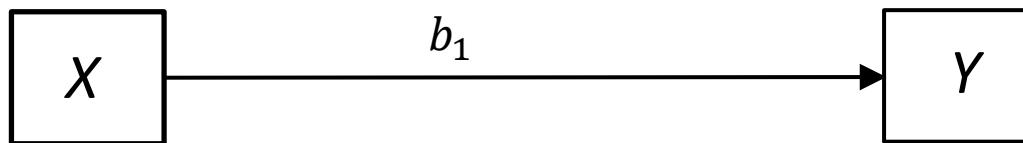
$$SS_{residual} = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^n e_i^2$$

A visual representation

$$Y_i = b_0 + b_1 X_i + e_i$$



or, in shorthand,



In a diagram such as this, \longrightarrow represents “predictor of” or “component of” but not necessarily “cause of,” although the association could be causal. So X is a predictor of Y (and *perhaps* a cause of Y) in this diagram.

Easier to do in SPSS and then explain

This is an easy problem for a computer with an OLS regression routine. We estimate Y from X , or **regress Y on X**. X = POSREL (positivity of teacher-student relationships), Y = SATIS1 (satisfaction with school).

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.458 ^a	.210	.207	.61840

a. Predictors: (Constant), posrel

ANOVA^a

Model	Sum of Squares	df	Mean Square	F	Sig.
1 Regression	33.263	1	33.263	86.980	.000 ^b
Residual	125.433	328	.382		
Total	158.696	329			

a. Dependent Variable: satis1

b. Predictors: (Constant), posrel

$$SS_{\text{residual}} = 125.433$$

No other $\{b_0, b_1\}$ pair would produce a smaller value of SS_{residual} .

$$\hat{Y}_i = b_0 + b_1 X_i$$

$$\hat{Y}_i = 2.245 + 0.531 X_i$$

This is the best fitting OLS regression model, assuming a linear association between X and Y .

Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.	95.0% Confidence Interval for B	
	B	Std. Error				Lower Bound	Upper Bound
1 (Constant)	2.245	.166		13.527	.000	1.918	2.571
X posrel	.531	.057	.458	9.326	.000	.419	.643

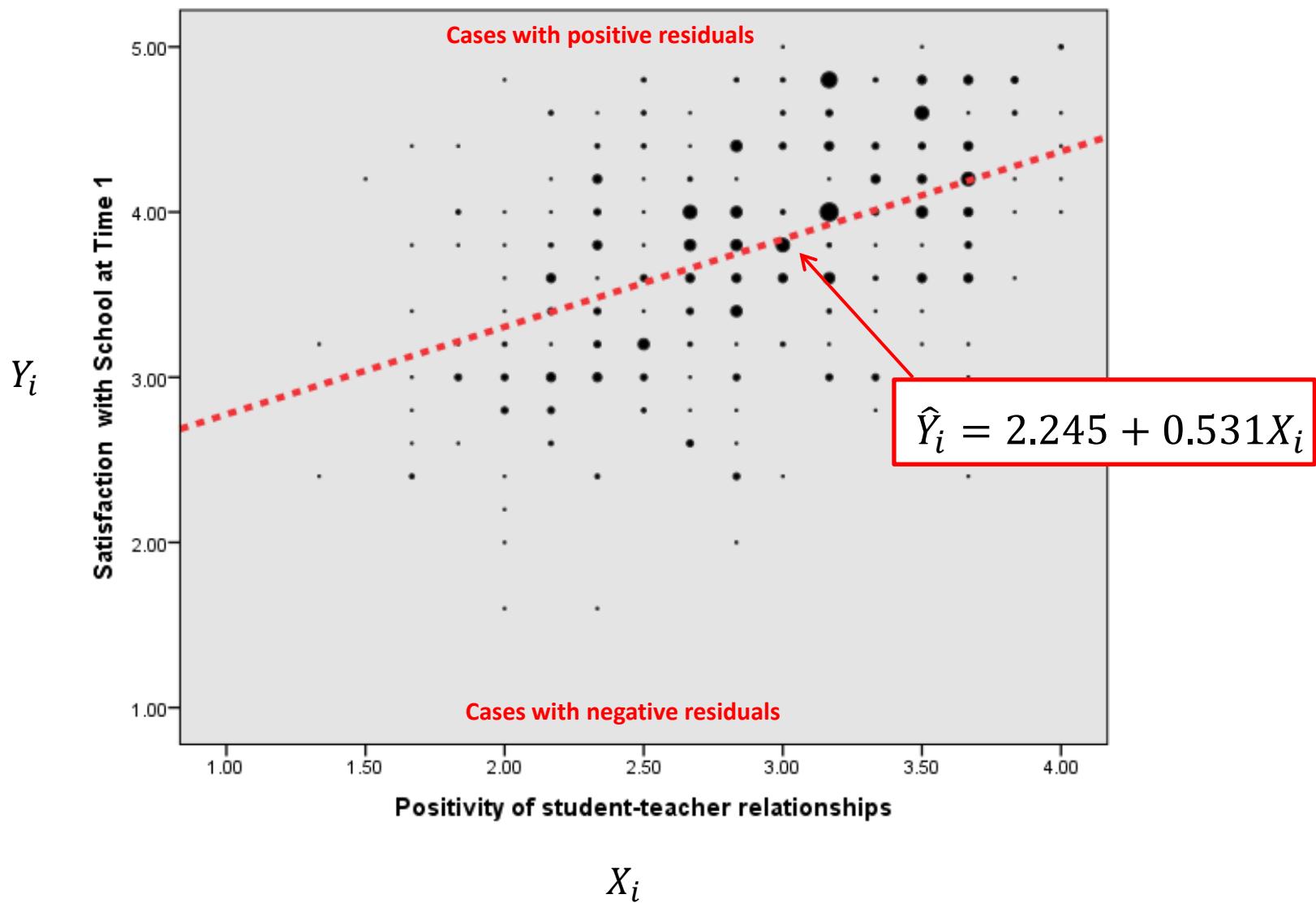
a. Dependent Variable: satis1

```
regression/statistics defaults ci/dep=satis1/method=enter posrel.
```

```
proc reg data=harass;model satis1 = posrel/stb clb;run;
```

Output A

The model in visual form



Interpretation of b_0 and b_1

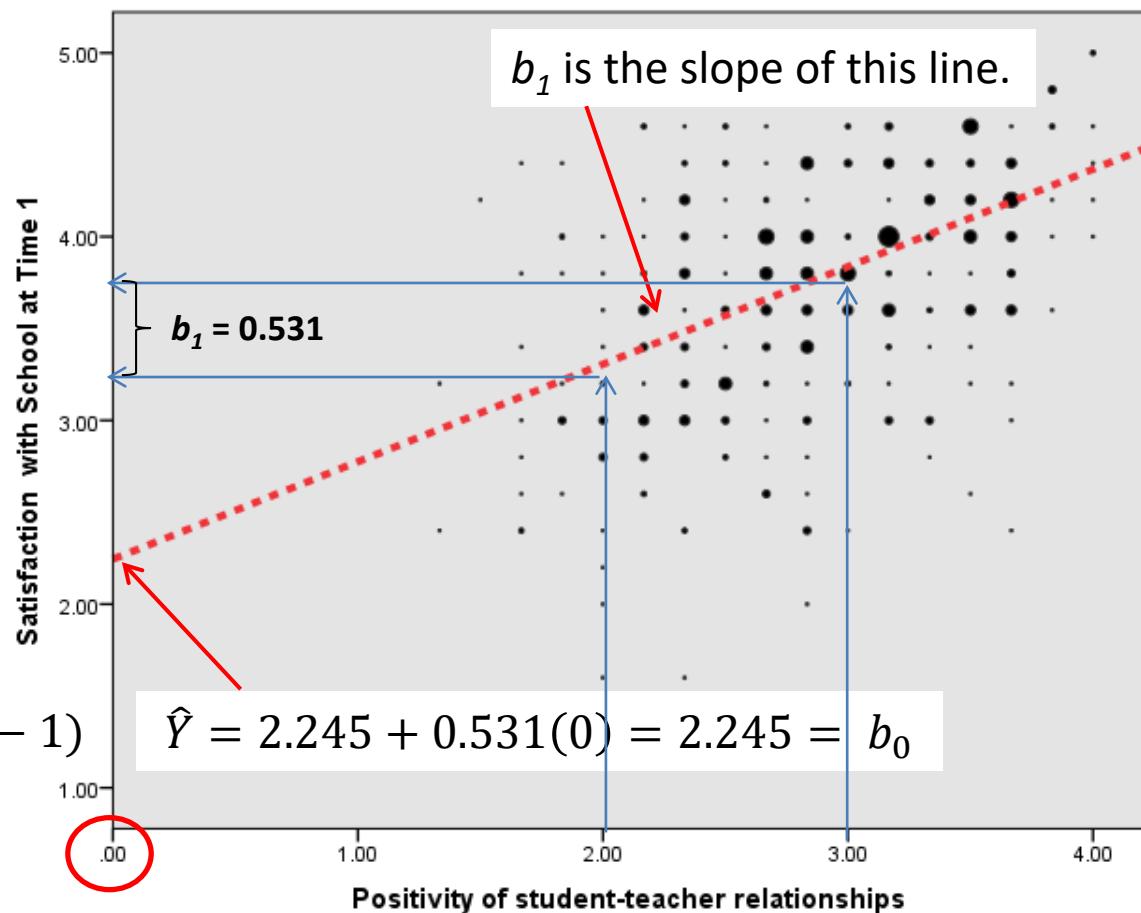
$$\hat{Y}_i = b_0 + b_1 X_i$$

$$\hat{Y}_i = 2.245 + 0.531 X_i$$

b_1 = estimated difference in Y between two cases that differ by one unit on X . The sign of b_1 speaks to the sign of the association between X and Y .

$$b_1 = \hat{Y}|(X = \theta) - \hat{Y}|(X = \theta - 1)$$

b_0 = estimated value of Y when $X = 0$. This is not meaningful here.



Two kids that differ by one unit in the positivity of their student-teacher relationships are estimated to differ by $b_1 = 0.531$ units in satisfaction with school. The kid that is **higher** in positivity of those relationships is estimated to be *more* satisfied (because b_1 is positive)

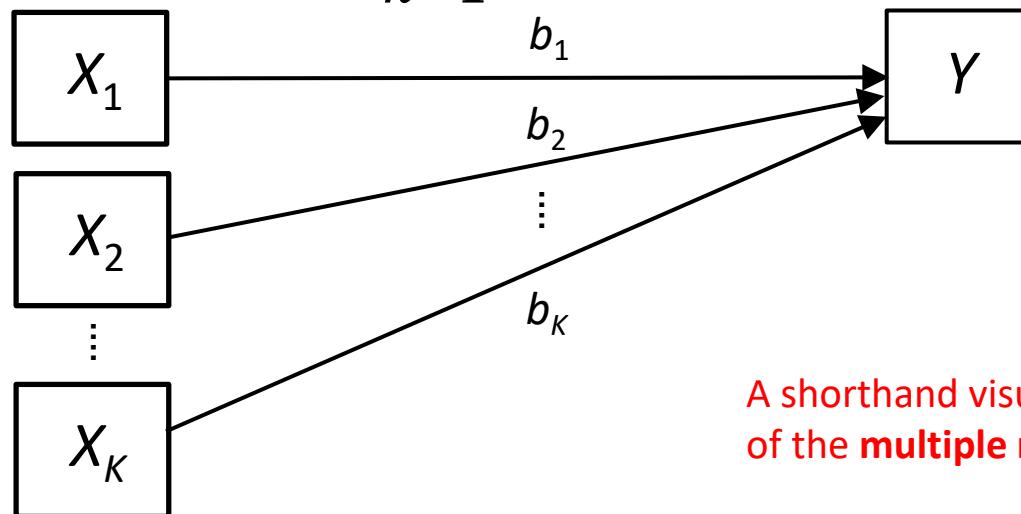
Multiple predictors

Multiple predictors variables are handled with ease, without modification to the estimation process. But this results in some interpretational changes.

$$Y_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + b_3 X_{3i} + \dots + b_K X_{Ki} + e_i$$

or, more concisely,

$$Y_i = b_0 + \sum_{k=1}^K b_k X_{ki} + e_i$$



A shorthand visual representation
of the **multiple regression** model.

A multiple regression model (SPSS)

```
regression/statistics defaults ci/dep=satis1/method=enter posrel harass sel.
```

```
proc reg data=harass;model satis1 = posrel harass sel/stb clb;run;
```

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.545 ^a	.297	.290	.58503

a. Predictors: (Constant), sel, posrel, harass

ANOVA^a

Model	Sum of Squares	df	Mean Square	F	Sig.
1 Regression	47.120	3	15.707	45.891	.000 ^b
Residual	111.576	326	.342		
Total	158.696	329			

a. Dependent Variable: satis1

b. Predictors: (Constant), sel, posrel, harass

$$SS_{\text{residual}} = 111.576$$

No other set of b values would produce a smaller value of SS_{residual} .

X_1 = positivity of teacher-student relationships

X_2 = harassment frequency

X_3 = self-esteem.

Coefficients^a

Model	Unstandardized Coefficients		Beta	t	Sig.	95.0% Confidence Interval for B	
	B	Std. Error				Lower Bound	Upper Bound
1 (Constant)	1.479	.309		4.788	.000	.871	2.087
posrel	.472	.055	.408	8.519	.000	.363	.581
harass	-.145	.090	-.078	-1.613	.108	-.322	.032
sel	.373	.064	.276	5.828	.000	.247	.499

a. Dependent Variable: satis1

$$\widehat{Y}_i = 1.479 + 0.472X_{1i} - 0.145X_{2i} + 0.373X_{3i}$$

Output B

The meaning of the values of b

$$\hat{Y}_i = 1.479 + 0.472X_{1i} - 0.145X_{2i} + 0.373X_{3i}$$


Two kids *the same on all predictors except the positivity of their teacher relationships (X_1)* but who differ by one unit in such positivity will differ by $b_1 = 0.472$ units in estimated satisfaction with school (\hat{Y}), where more positivity corresponds to higher satisfaction.

b_1 , the **partial regression coefficient** for X_1 , quantifies how differences in positivity of the student-teacher relationship relates to differences in satisfaction with school when *all other predictor variables in the model are held constant*, or “statistically controlling for” those other variables.

Two people the same on all predictors except ethnic harassment (X_2) who *differ by one unit in harassment* will differ by $b_2 = 0.145$ units in estimated satisfaction with school (\hat{Y}), where higher harassment corresponds to lower satisfaction.

b_2 , the **partial regression coefficient** for X_2 , quantifies how *differences in frequency in the experience of ethnic harassment relate to differences in school satisfaction* when *all other predictor variables in the model are held constant*, or “statistically controlling for” those other variables. The negative sign for b_2 means those who experience *more* harassment are estimated to be *less* satisfied with school.

Statistical inference

$$Y_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + b_3 X_{3i} + \dots + b_K X_{Ki} + e_i$$

The constant, values of b are sample-specific. They are sample-specific estimates of a corresponding population or process model (the “true” model):

$$Y_i = \widetilde{b}_0 + \widetilde{b}_1 X_{1i} + \widetilde{b}_2 X_{2i} + \widetilde{b}_3 X_{3i} + \dots + \widetilde{b}_K X_{Ki} + \widetilde{e}_i$$

Departures between the “true model” and the obtained model resulting from our data are used to test hypotheses about the “true values” of b .

Departures between the true and the obtained model are assumed to be driven by **“random” processes**, such as random sampling, random assignment variation, measurement error, etc., unless the data suggest otherwise. We attempt to estimate the true model using our data, hoping that our estimates of the true values of that model are accurate.

Null hypothesis testing for \tilde{b}

$$Y_i = \tilde{b}_0 + \tilde{b}_1 X_{1i} + \tilde{b}_2 X_{2i} + \tilde{b}_3 X_{3i} + \dots + \tilde{b}_K X_{Ki} + \tilde{e}_i$$

In any study, we observe only b_j , the sample estimate of \tilde{b}_j . We often are interested in making an inference about the size of \tilde{b}_j , or testing a hypothesis about its value.

e.g., Null hypothesis test about \tilde{b}_j :

Assume \tilde{b}_1 equals some specific value. Typically, we assume $\tilde{b}_1 = 0$ under the **null hypothesis** (i.e., X_1 is unrelated to Y when all other variables in the model are held constant).

$$\begin{aligned} H_0: \tilde{b}_1 &= 0 \\ H_a: \tilde{b}_1 &\neq 0 \end{aligned}$$

If H_0 is true, then b_1 / s_{b_1} follows the $t(df_{residual})$ distribution, where s_{b_1} is the estimated standard error of b_1 . Using the t distribution, we generate a p -value and reject H_0 in favor of H_a if $p \leq \alpha$ -level chosen for the test (usually .05). In that case, the result is “statistically significant.”

Statistical inference for partial regression coefficients

ANOVA^a

Model	Sum of Squares	df	Mean Square	F	Sig.
1 Regression	47.120	3	15.707	45.891	.000 ^b
Residual	111.576	326	.342		
Total	158.696	329			

a. Dependent Variable: satis1

b. Predictors: (Constant), se1, posrel, harass

Coefficients^a

Model	Unstandardized Coefficients		Beta	t	Sig.	95.0% Confidence Interval for B	
	B	Std. Error				Lower Bound	Upper Bound
1 (Constant)	1.479	.309		4.788	.000	.871	2.087
posrel	.472	.055	.408	8.519	.000	.363	.581
harass	-.145	.090	-.078	-1.613	.108	-.322	.032
se1	.373	.064	.276	5.828	.000	.247	.499

a. Dependent Variable: satis1

$$H_0: \tilde{b}_1 = 0$$

$$H_a: \tilde{b}_1 \neq 0$$

$b_1 = 0.472$, $se(b_1) = 0.055$,
 $t(326) = 8.519$, $p < 0.001$

Reject H_0 in favor of H_a

$$H_0: \tilde{b}_2 = 0$$

$$H_a: \tilde{b}_2 \neq 0$$

$b_2 = -0.145$, $se(b_2) = 0.090$,
 $t(326) = -1.613$, $p = 0.108$

Do not reject H_0

Output B

Two kids equal in ethnic harassment frequency and self esteem but who differ in the positivity of their student-teacher relationships differ from each other in satisfaction more than can be explained by chance. The observed and statistically significant positive partial relationship tells us that kids with more positive student-teacher relationships are more satisfied with school.

Two kids equal in the positivity of their student-teacher relationships and their self-esteem but who differ in ethnic harassment frequency do not differ in their satisfaction *any more than would be expected by "chance."*

Interval estimation

Inferences can also be framed as an interval such that this interval will capture the true value a certain percentage of the time. As a rough rule-of-thumb, we can be 95% confident that the true value resides within about 2 standard errors of the obtained estimate.

$$b_j - 2s_{b_j} \leq \tilde{b}_j \leq b_j + 2s_{b_i}$$

Output B

Coefficients^a

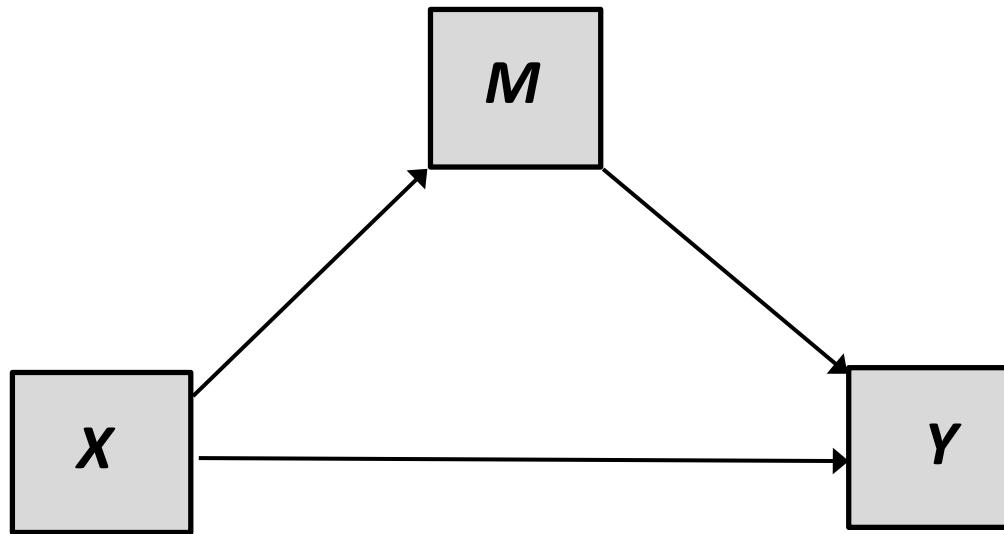
Model	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.	95.0% Confidence Interval for B	
	B	Std. Error				Lower Bound	Upper Bound
1	(Constant)	1.479	.309	4.788	.000	.871	2.087
	posrel	.472	.055	.408	8.519	.000	.363
	harass	-.145	.090	-.078	-1.613	.108	-.322
	se1	.373	.064	.276	5.828	.000	.247

a. Dependent Variable: satis1

We can be 95% confident \tilde{b}_1 is somewhere between 0.363 and 0.581
We can be 95% confident \tilde{b}_2 is somewhere between -0.322 and 0.032.

Question: Is it fair to say we have 'no evidence' that kids who are harassed more frequently are less satisfied with school?

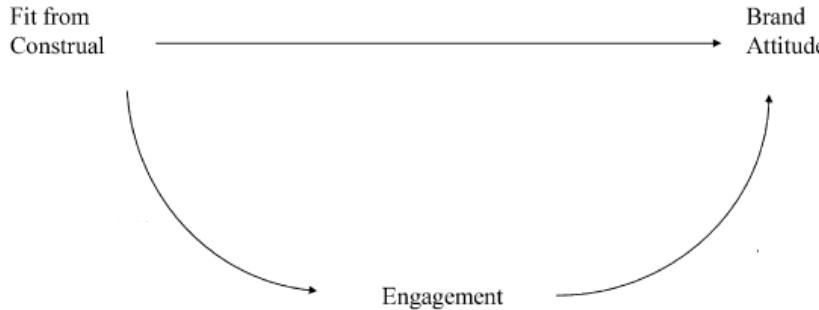
Statistical mediation analysis



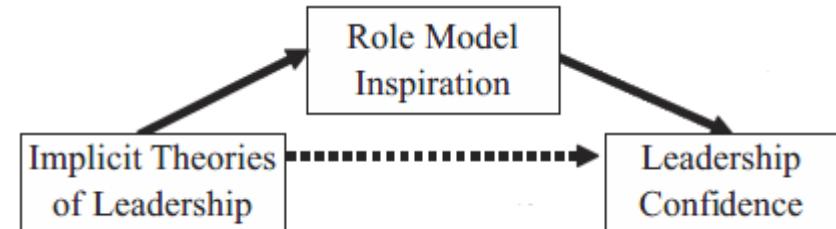
The “simple mediation” model

A mediation model links an assumed cause (X) to an assumed effect (Y) at least in part via an intermediary variable (M). An intermediary variable can be a psychological state, a cognitive process, an affective response, or any other conceivable “mechanism” through which X exerts an effect on Y . X affects M which in turn affects Y .

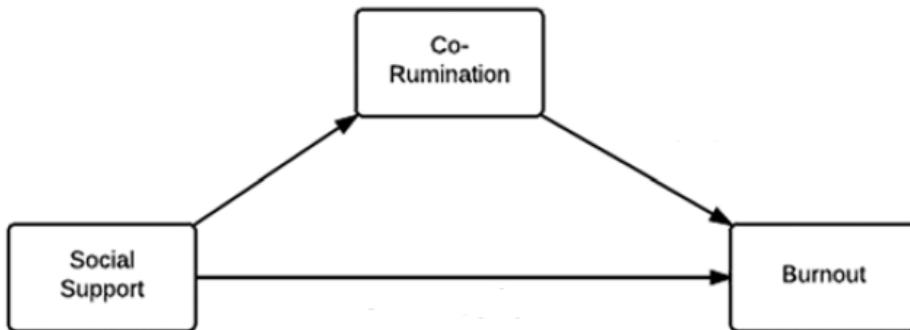
Some examples in the literature



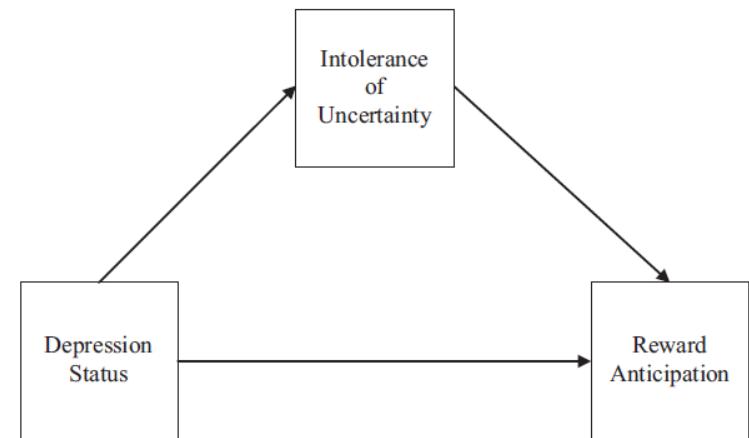
Lee, A. Y., Keller, P. A., & Sternthal, B. (2010). Value from regulatory construal fit: Persuasive impact of fit between consumer goals and message concreteness. *Journal of Consumer Research*, 36, 735-747.



Hoyt, C. L., Burnette, J. L., & Innella, A. N. (2012). I can do that: The impact of implicit theories on leadership model effectiveness. *Personality and Social Psychology Bulletin*, 38, 257-268.



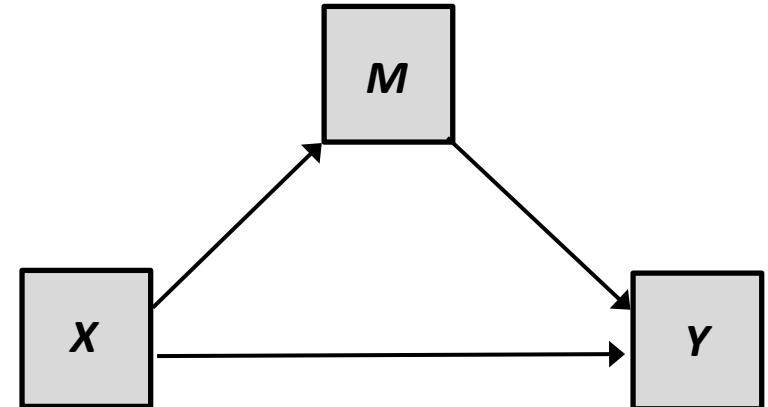
Boren, J. P. (2014). The relationship between co-rumination, social support, stress, and burnout among working adults. *Management Communication Quarterly*, 28, 3-25.



Nelson, B. D., Shankman, S. A., & Proudfoot, G. H. (2014). Intolerance of uncertainty mediates reduced reward anticipation in major depressive disorder. *Journal of Affective Disorders*, 158, 108-113.

The most basic intervening variable model

- ❑ For M to be an intermediary, it must be located *causally between* X and Y .
- ❑ M is sometimes called a “mediator”, but it goes by other names as well.



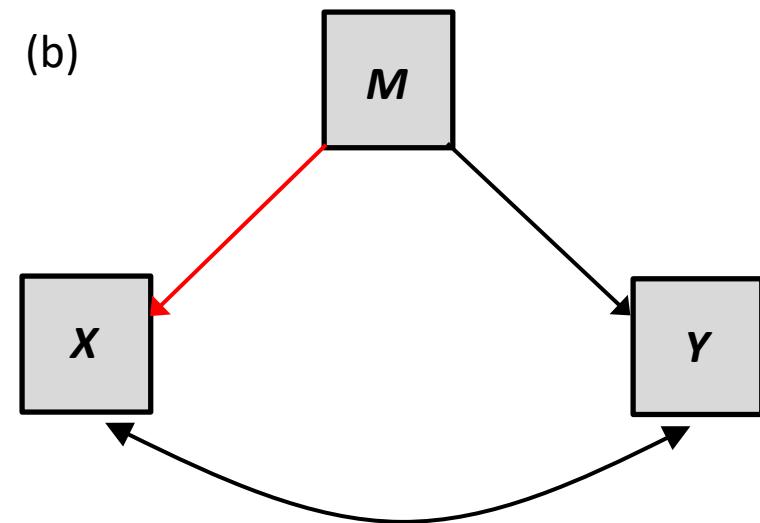
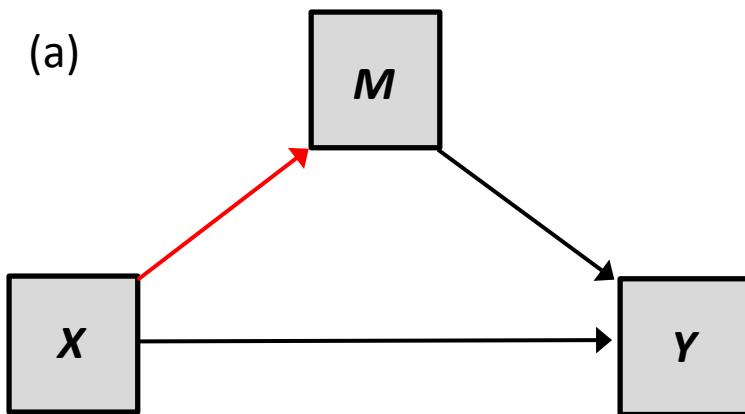
- ❑ Mediator models are causal models and carry with them the usual criteria for making causal claims.

Difficult to establish cause statistically or otherwise.

Theory is sometimes the sole foundation upon which our causal claims rest. **That's ok so long as we recognize this.**

Mediation and spuriousness

Mediation analysis cannot distinguish between (a)mediation and (b)spuriousness. If (b) can be deemed plausible, that weakens the case for (a) regardless of what the data analysis tells you.

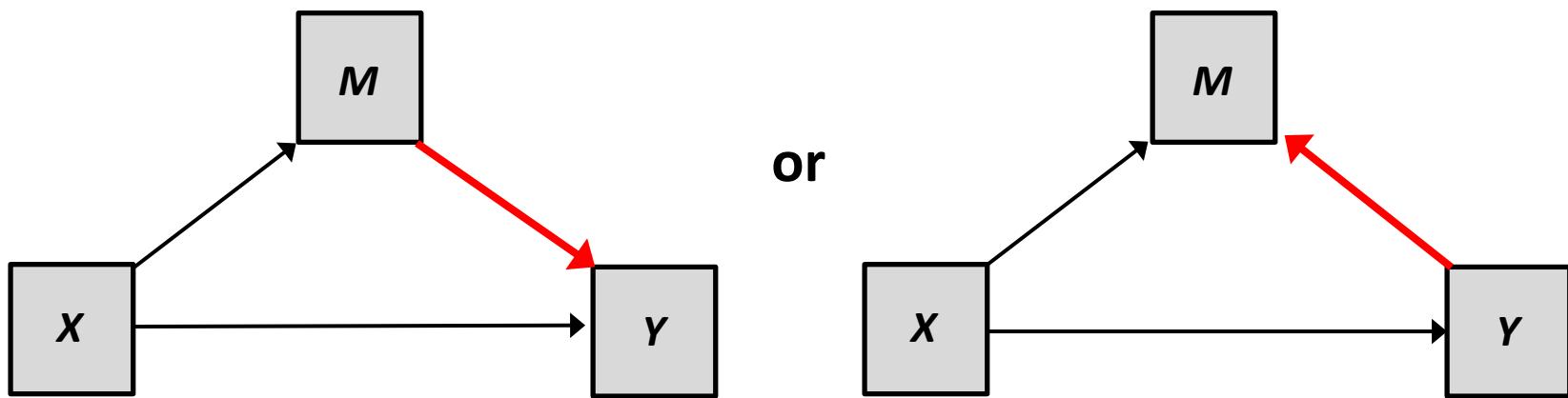


Inferences are always design-bound.

Mediation is a causal process, but causal claims are only justified if the design allows such claims, regardless of what the statistics say.

Causal order

Manipulation of and random assignment to X affords causal inference for the effect of X on M and Y , but not the effect of M on Y . We cannot establish causal order for the $M-Y$ path using the methods that are the focus here. Theory is important. Multiple studies can help, one of which involves manipulation of M .



When X is not experimentally manipulated, all paths are subject to potential alternative causal orders.

Path analysis: Total, direct, and indirect effects

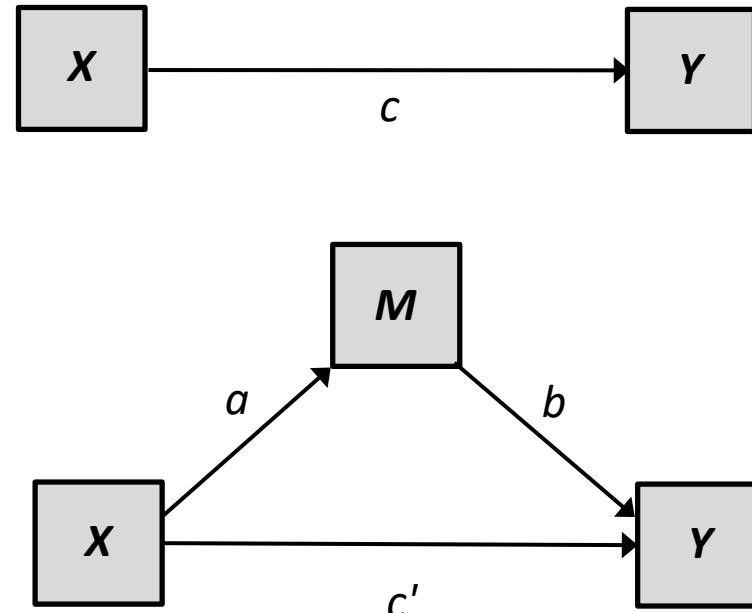
Let a , b , c , and c' be quantifications of causal effects, such as regression coefficients in an OLS model (or maximum likelihood path estimates in a structural equation model)

$$\widehat{Y}_i = c_0 + cX_i$$

$$\widehat{M}_i = a_0 + aX_i$$

$$\widehat{Y}_i = c'_0 + c'X_i + bM_i$$

A “simple mediation” model



total effect = direct effect + indirect effect

$$c = c' + (a \times b)$$

indirect effect = total effect – direct effect

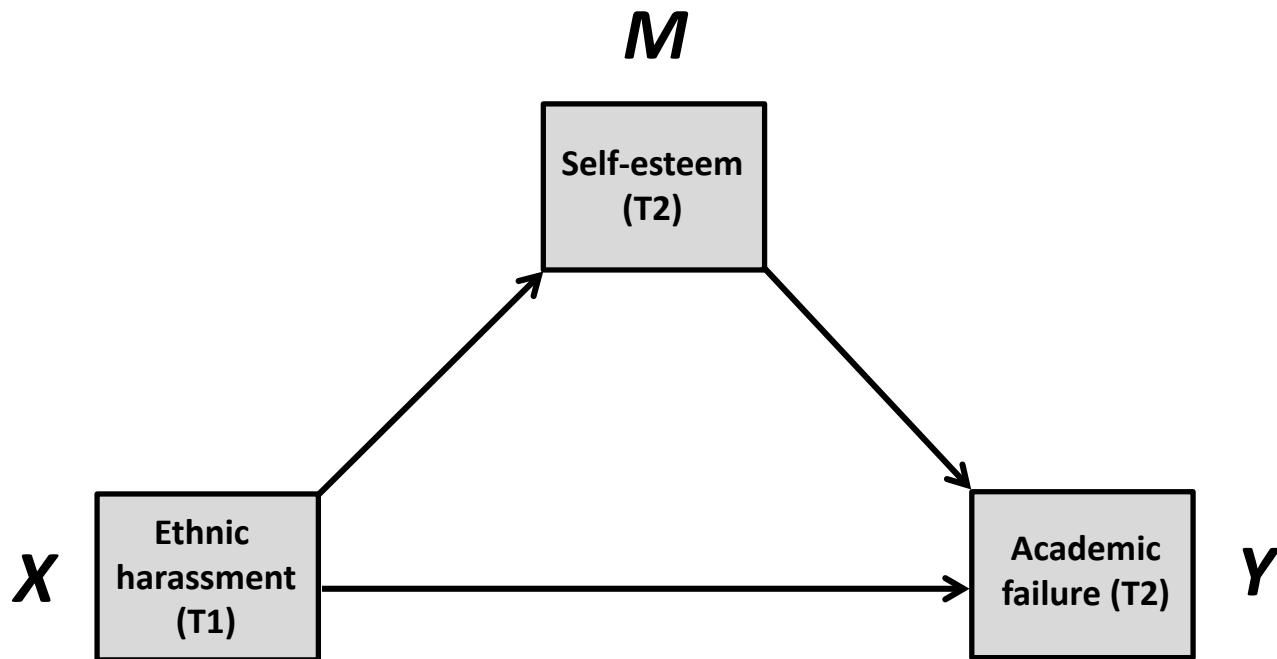
$$(a \times b) = c - c'$$

c = “total effect” of X on Y

$a \times b$ = “indirect effect” of X on Y

c' = “direct effect” of X on Y

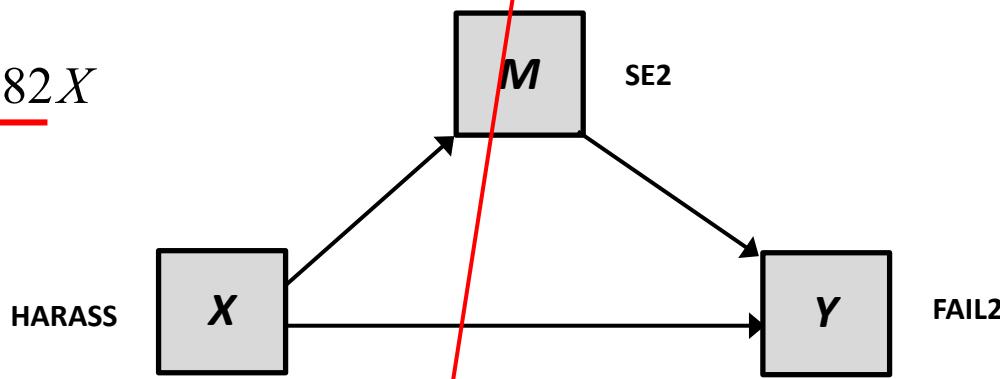
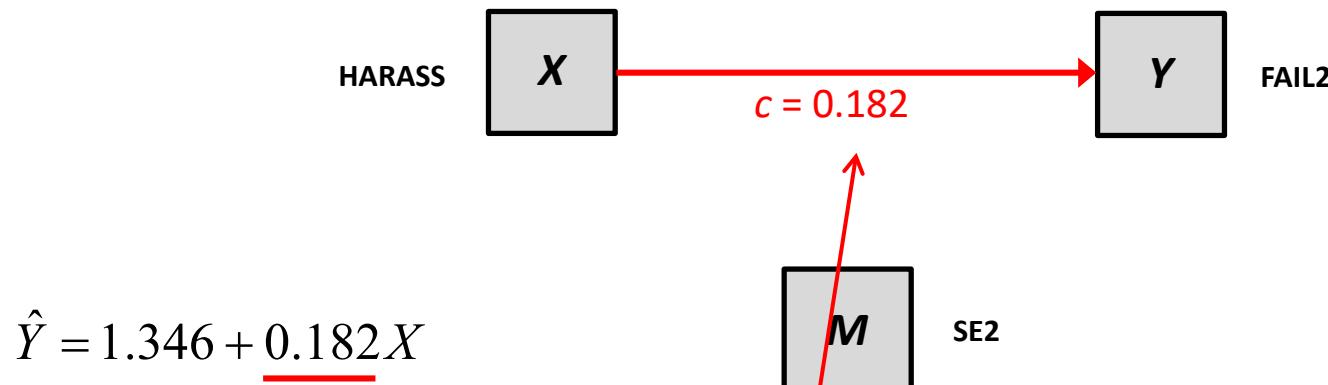
Our question



Does ethnic harassment influence school performance by affecting self-esteem which in turn affects performance.

Asking this question does not require evidence that there is a bivariate relationship between X (ethnic harassment) and Y (performance)

Using a set of OLS regression analyses



Coefficients^a

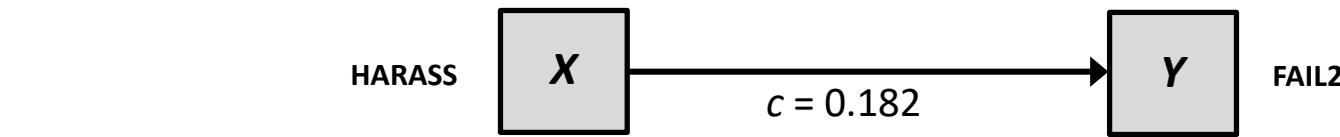
Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
1	(Constant)	1.346	.113		.000
	harass	.182	.073	.136	.014

a. Dependent Variable: fail2

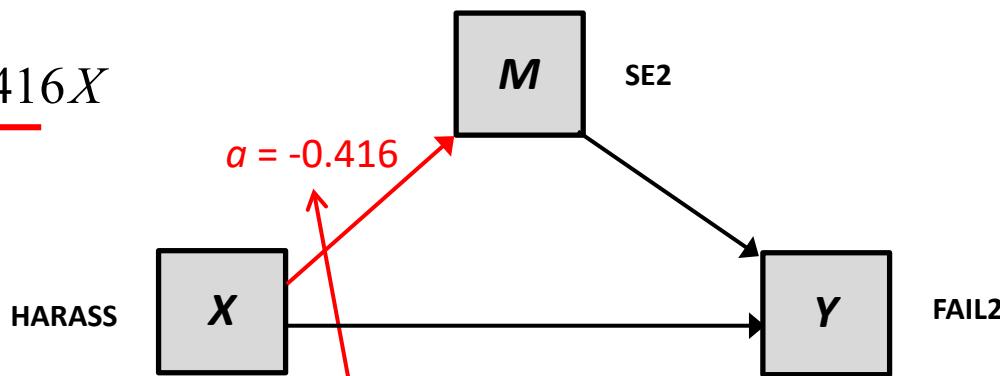
SPSS: **regression/dep=fail2/method=enter harass.**

SAS: **proc reg data=harass;model fail2=harass;run;**

Using a set of OLS regression analyses



$$\hat{M} = 3.597 - 0.416X$$



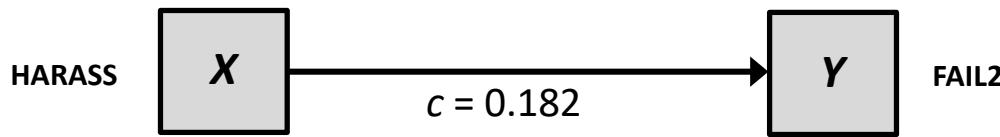
Model	Coefficients ^a			t	Sig.
	B	Std. Error	Standardized Coefficients Beta		
1	3.597	.123		29.123	.000
harass	-416	.080	-.276	-5.209	.000

a. Dependent Variable: se2

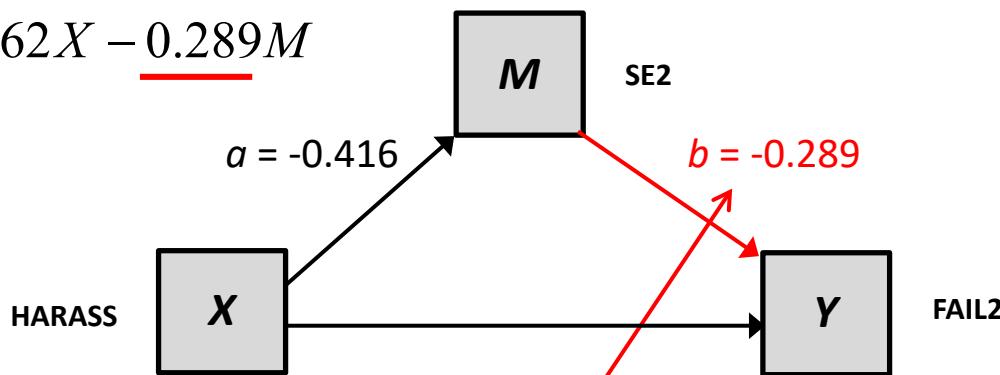
```
regression/dep=se2/method=enter harass.
```

```
proc reg data=harass;model se2=harass;run;
```

Using a set of OLS regression analyses



$$\hat{Y} = 2.385 + 0.062X - 0.289M$$



Coefficients^a

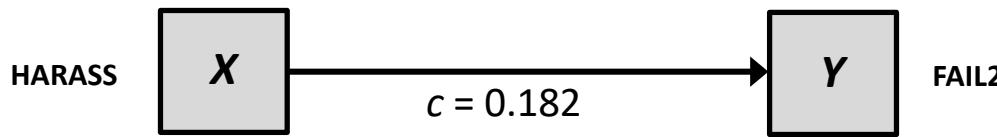
Model	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.
	B	Std. Error			
1	(Constant)	2.385	.204		
	harass	.062	.072	.046	.850
	se2	-.289	.048	-.324	-5.988

a. Dependent Variable: fail2

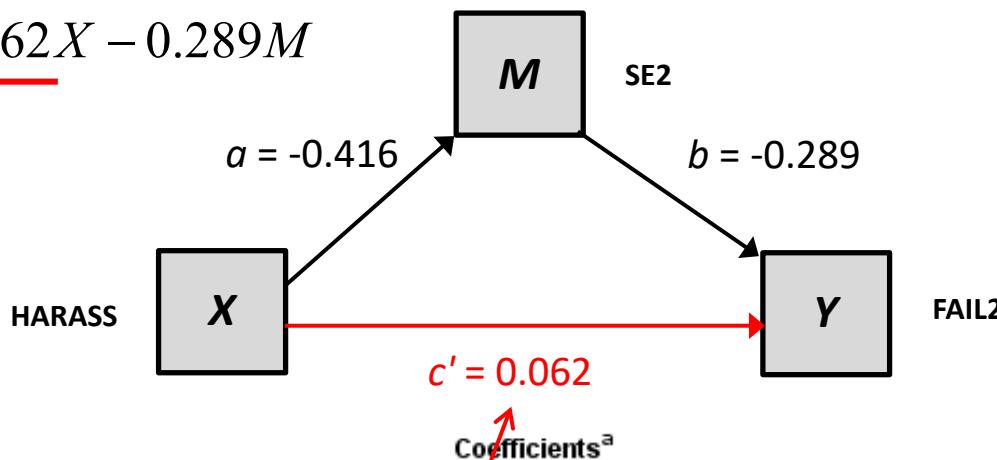
regression/dep=fail2/method=enter harass se2.

proc reg data=harass;model fail2=harass se2;run;

Using a set of OLS regression analyses



$$\hat{Y} = 2.385 + 0.062X - 0.289M$$



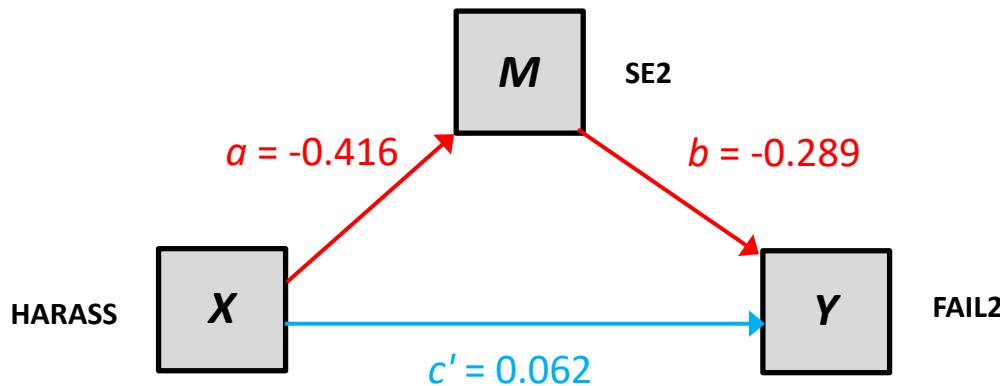
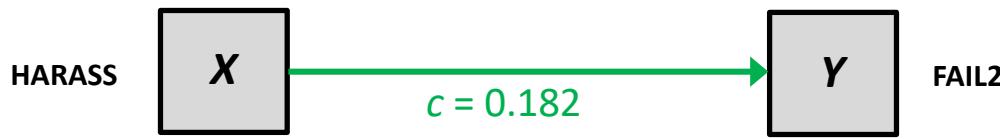
Model	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.
	B	Std. Error			
1 (Constant)	2.385	.204		11.676	.000
harass	.062	.072	.046	.850	.396
se2	-.289	.048	-.324	-5.988	.000

a. Dependent Variable: fail2

regression/dep=fail2/method=enter harass se2.

proc reg data=harass;model fail2=harass se2;run;

Using a set of OLS regression analyses



Direct effect of X on $Y = c' = 0.062$

Indirect effect of X on Y via $M = ab = -0.416(-0.289) = 0.120$

Total effect of X on $Y = c' + ab = 0.062 + 0.120 = 0.182 = c$

Interpretation of the total, direct, and indirect effects

Generic

Total: Two people who differ by one unit on X are estimated to differ by c units on Y on average.

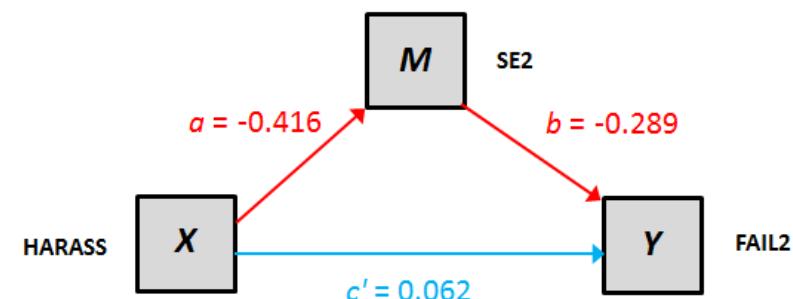
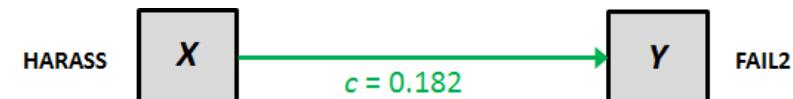
Indirect: They differ by ab units on average as a result of the effect of X on M which in turn affects Y .

Direct: The rest of the difference, the difference of c' units, is due to the effect of X on Y independent of M .

$$\text{Direct effect} = c' = 0.062$$

$$\text{Indirect effect} = ab = -0.416(-0.289) = 0.120$$

$$\text{Total effect} = c = 0.062 + 0.120 = 0.182$$



Specific

Total: Two kids who differ by one scale point in ethnic harassment are estimated to differ by **0.182** units in perceived academic failure one year later, with the more frequently-harassed kid perceiving greater failure.

Indirect: They differ by **0.120** units in perceived failure as a result of the negative effect of harassment on self esteem a year later, which in turn increases perceived failure.

Direct: Independent of this mechanism, the more harassed kid is estimated to be **0.062** units higher in perceived failure.

It works for dichotomous X too.

Garcia, D. M., Schmitt, M. T., Branscombe, N. R., & Ellemers, N. (2010). Women's reactions to ingroup members who protest discriminatory treatment: The importance of beliefs about inequality and response appropriateness. *European Journal of Social Psychology*, 49, 733-745.

European Journal of Social Psychology
Eur. J. Soc. Psychol. 40, 733–745 (2010)
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(www.interscience.wiley.com) DOI: 10.1002/ejsp.644

Research article

Women's reactions to ingroup members who protest discriminatory treatment: The importance of beliefs about inequality and response appropriateness

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²Simon Fraser University, Canada

³University of Kansas, USA

⁴Leiden University, The Netherlands

Abstract

Our goal was to identify factors that shape women's responses to ingroup members who protest gender discrimination. We predicted and found that women who perceived gender discrimination as pervasive regarded a protest response as being more appropriate than a no protest response and expressed greater liking and less anger towards a female lawyer who protested rather than did not protest an unfair promotion decision. Further, beliefs about the appropriateness of the response to discrimination contributed to evaluations of the protesting lawyer. Perceptions that the complaint was an appropriate response to the promotion decision led to more positive evaluations of an ingroup discrimination protester. Copyright © 2009 John Wiley & Sons, Ltd.

Protest can be an effective means of improving the plight of a devalued group. Historically, there are many examples of protest, even from a single individual, that have advanced a group's social position (e.g. Mentor Savings Bank vs. Vinson, 477 US 57, 1986; Dekker vs. VIV-Centrum ECJ, 1992). Despite the potential gains to be obtained by protesting illegitimate treatment, protestors might not always be appreciated by members of their own group. Whether disadvantaged group members respond positively or negatively to ingroup protestors will likely depend upon the *perceived* implications that the protestor's action has for the ingroup. Unless protest is seen as justified by the social circumstances and an effective means of bringing about positive change, a protestor might be seen as making the ingroup look like complainers. Such threat to the ingroup's reputation could evoke the ire and disdain of the disadvantaged group towards the protestor. Hence, perceptions of the justification for and likely consequences of protest will be critical to others' reactions to an ingroup discrimination claimer. We propose that protest by an ingroup member will be seen as appropriate and thus appreciated to the extent that observers perceive that their ingroup is targeted by pervasive discrimination.

SOCIAL CONSEQUENCES OF CLAIMING DISCRIMINATION

Gender discrimination continues to be widespread throughout Western employment settings (see Charles & Grusky, 2004). The continuation of gender discrimination has substantive negative implications for women's economic and

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E-mail: donnagarcia3@gmail.com

Participants (all female) read a narrative about a female attorney who lost a promotion at her firm to a much less qualified male through unequivocally discriminatory actions of the senior partners.

Participants assigned to the 'protest' condition were then told she protested the decision by presenting an argument to the partners about how unfair the decision was.

Participants assigned to the 'no protest' condition were told that although she was disappointed, she accepted the decision and continued working at the firm.

After reading the narrative, the participants evaluated **how appropriate they perceived her response to be**, and also evaluated the characteristics of the attorney, the responses of which were aggregated to produce a measure of "liking." Prior to the study, the participants filled out the Modern Sexism Scale.

The data: PROTEST

The screenshot shows the IBM SPSS Statistics Data Editor window. The title bar reads "*protest.sav [DataSet1] - IBM SPSS Statistics Data Editor". The menu bar includes File, Edit, View, Data, Transform, Analyze, Graphs, Utilities, Add-ons, Window, and Help. Below the menu is a toolbar with various icons. The data view shows 14 rows of data across 8 columns. The columns are labeled: subnum, cond, sexism, angry, liking, respappr, and protest. The data values range from 0 to 209.

	subnum	cond	sexism	angry	liking	respappr	protest
1	209	2	4.87	2	4.83	4.25	1.00
2	44	0	4.25	1	4.50	5.75	.00
3	124	2	5.00	3	5.50	4.75	1.00
4	232	2	5.50	1	5.66	7.00	1.00
5	30	2	5.62	1	6.16	6.75	1.00
6	140	1	5.75	1	6.00	5.50	1.00
7	27	2	5.12	2	4.66	5.00	1.00
8	64	0	6.62	1	6.50	6.25	.00
9	67	0	5.75	6	1.00	3.00	.00
10	182	0	4.62	1	6.83	5.75	.00
11	85	2	4.75	2	5.00	5.25	1.00
12	109	2	6.12	5	5.66	7.00	1.00
13	122	0	4.87	2	5.83	4.50	.00
14	69	1	5.87	1	6.50	6.25	1.00

SAS users, run this program to make a temporary or “work” data file named PROTEST.

The screenshot shows a SAS code editor window titled "protest". The code is a DATA step named "protest" with an INPUT statement and a CARDS statement. The INPUT statement specifies variables: subnum, cond, sexism, angry, liking, respappr, and protest. The CARDS statement contains 14 data records, which are identical to the data in the SPSS table above.

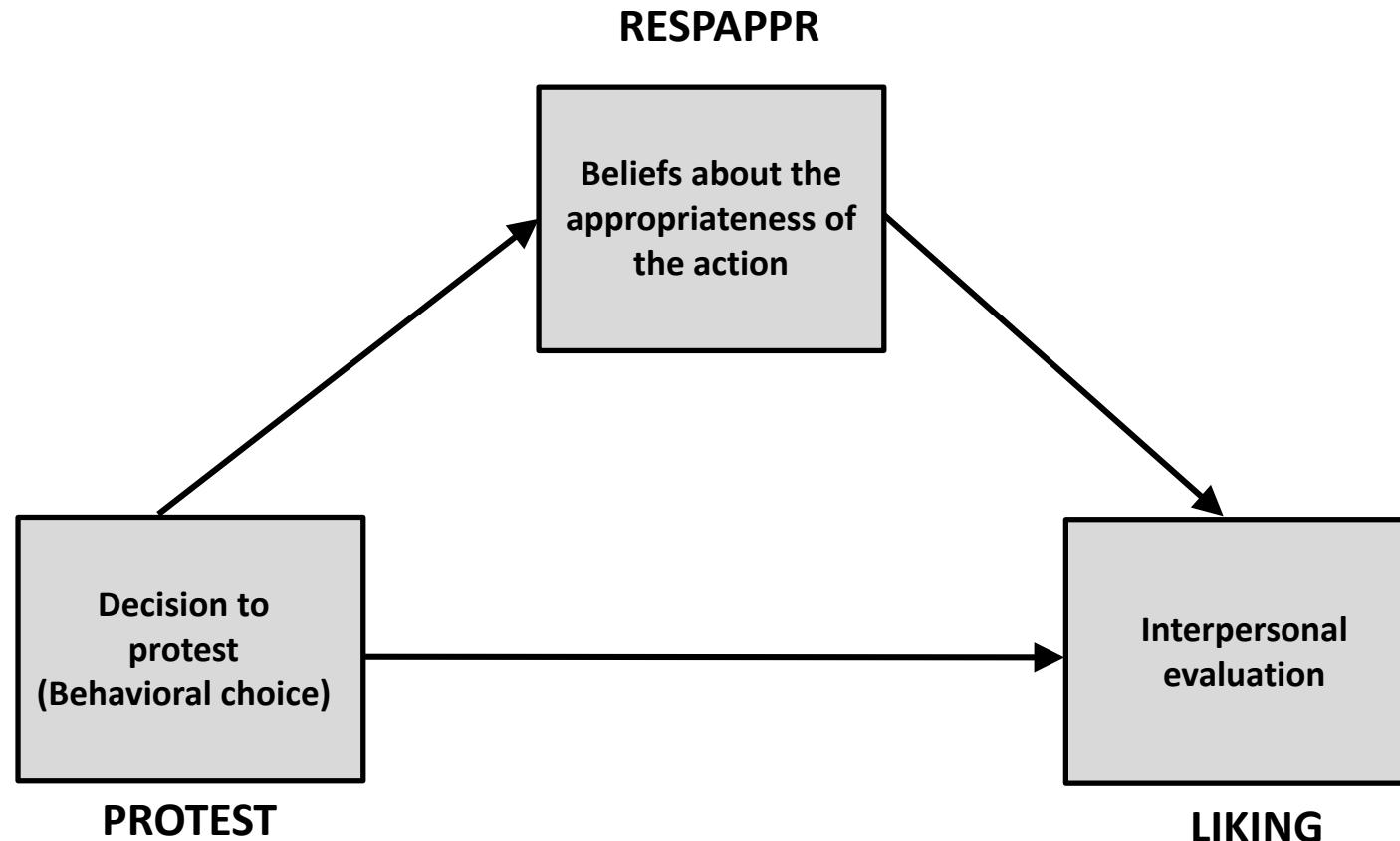
```
data protest;
  input subnum cond sexism angry liking respappr protest;
  cards;
  209 2 4.87 2 4.83 4.25 1.00
  44 0 4.25 1 4.50 5.75 .00
  124 2 5.00 3 5.50 4.75 1.00
  232 2 5.50 1 5.66 7.00 1.00
  30 2 5.62 1 6.16 6.75 1.00
  140 1 5.75 1 6.00 5.50 1.00
  27 2 5.12 2 4.66 5.00 1.00
  64 0 6.62 1 6.50 6.25 .00
  67 0 5.75 6 1.00 3.00 .00
  182 0 4.62 1 6.83 5.75 .00
  85 2 4.75 2 5.00 5.25 1.00
  109 2 6.12 5 5.66 7.00 1.00
  122 0 4.87 2 5.83 4.50 .00
  69 1 5.87 1 6.50 6.25 1.00
  
```

PROTEST: Experimental condition (1 = protest, 0 = no protest)

LIKING : Evaluation (liking) of the lawyer (higher = more positive evaluation, i.e. like more)

RESPAPPR: A measure of how appropriate the lawyer's behavior in response to the action of the partners was perceived to be for the situation (higher = more appropriate)

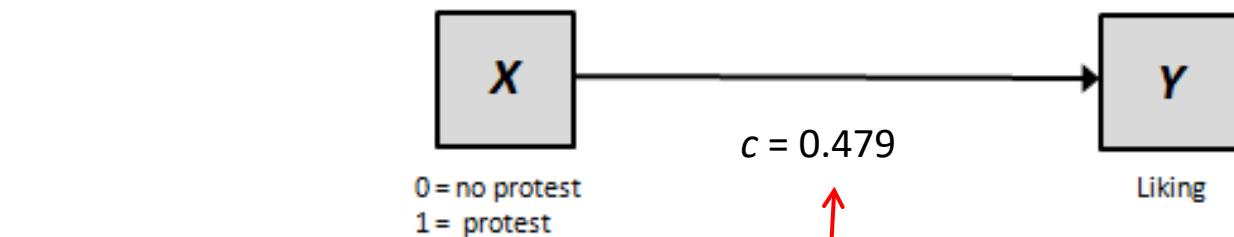
Our question



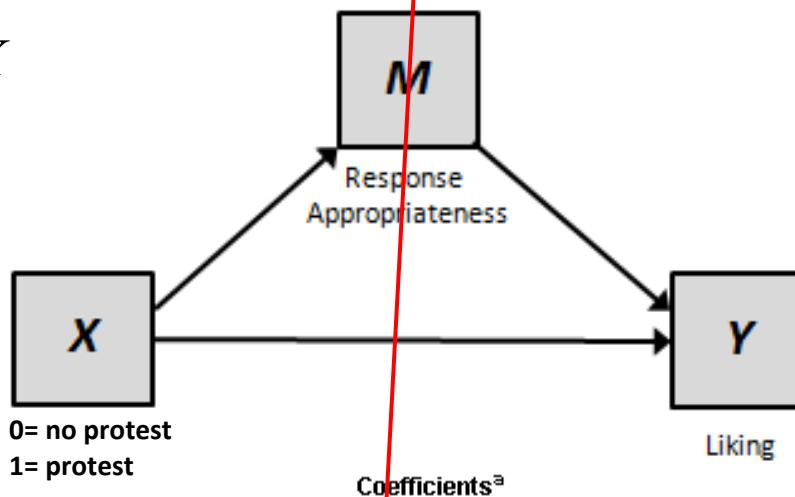
Do perceptions of the appropriateness of the response act as the mechanism through which that choice influences interpersonal evaluation?

Notice that this question is not asked contingent on evidence of simple association between the choice and the evaluation.

Using a set of OLS regression analyses



$$\hat{Y} = 5.313 + \underline{0.479}X$$



Model	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.
	B	Std. Error			
1 (Constant)	5.313	.161		33.055	.000
X: experimental condition (0 = no protest, 1 = protest)	.479	.195	.213	2.460	.015

a. Dependent Variable: Y: liking of the target

SPSS: **regression/dep=liking/method=enter protest.**

SAS: **proc reg data=protest;model liking=protest;run;**

Because the two groups differ by one unit on **X**, this is the difference between the group means. The attorney was liked more when she protested than when she did not.

Interpretation when X is dichotomous

$$\hat{Y}_i = 5.310 + 0.479X_i$$

means tables = liking by protest.

When $X = 1$ (protest), $\hat{Y} = 5.310 + 0.479(1) = 5.789$

When $X = 0$ (no protest), $\hat{Y} = 5.310 + 0.479(0) = 5.310$

LIKING: liking of the target

PROTEST: experimental condition (0 = no protest, 1 = protest)	Mean	N	Std. Deviation
no protest	5.3102	41	.130158
protest	5.7889	88	.87669
Total	5.6367	129	1.04970

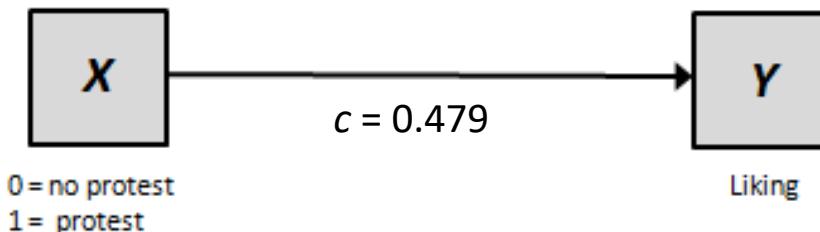
Notice that with X coded 0 and 1, the model yields the group means, b is the difference between the group means, and *the regression constant* is the mean for the group coded $X = 0$ (no protest condition).

More generally, if the two groups are coded by a difference of λ units, such that $X = \theta + \lambda$ for group 1 and $X = \theta$ for group 2,

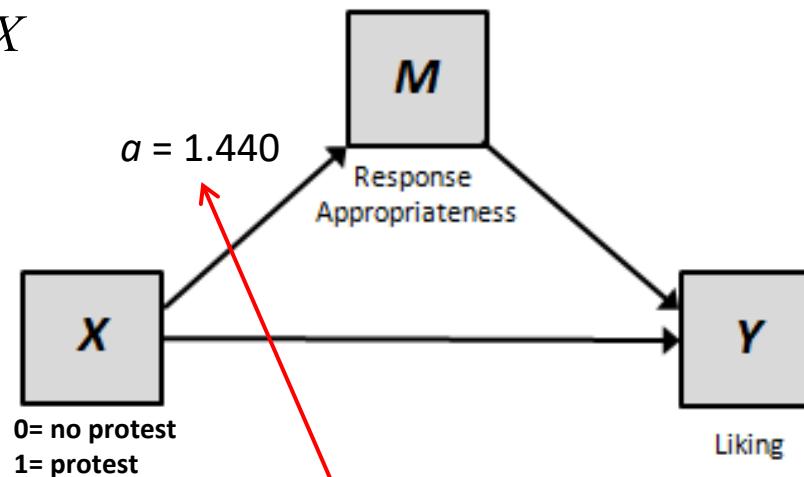
$$b = (\bar{Y}_1 - \bar{Y}_2) / \lambda$$

If you get in the habit of coding a dichotomous variable such that the groups differ by one unit on X , b will always be a difference between group means.

Using a set of OLS regression analyses



$$\hat{M} = 3.884 + \underline{1.440X}$$



Because the two groups differ by one unit on X, this is the difference between the group means. Protesting was seen as more appropriate for the situation than doing nothing.

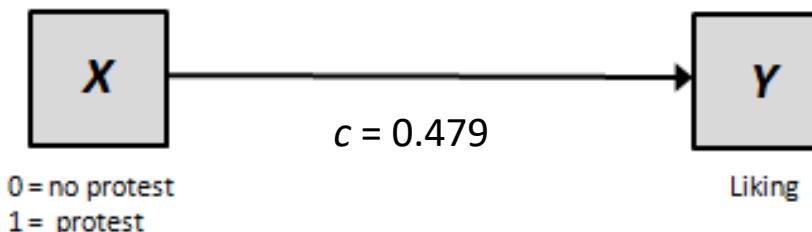
Model	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.
	B	Std. Error			
1 (Constant)	3.884	.183		21.208	.000
X: experimental condition (0 = no protest, 1 = protest)	1.440	.222	.499	6.493	.000

a. Dependent Variable: M: appropriateness of response

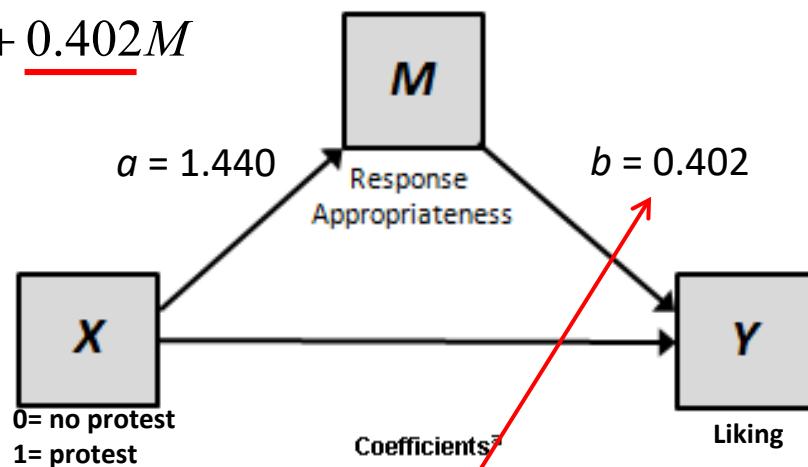
regression/dep=respappr/method=enter protest.

proc reg data=protest;model respappr=protest;run;

Using a set of OLS regression analyses



$$\hat{Y} = 3.751 - 0.100X + \underline{0.402M}$$



Holding constant what the attorney did, she was liked more by those who saw her behavior as more appropriate for the situation.

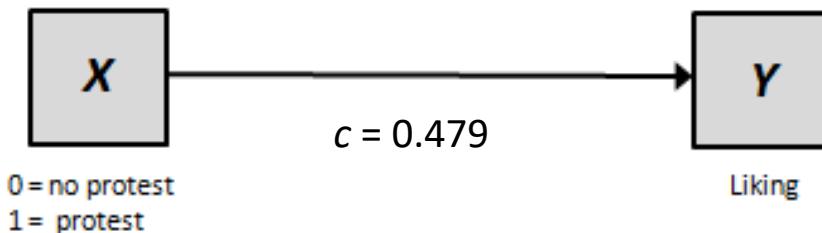
Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error			
1 (Constant)	3.751	.306		12.271	.000
M: appropriateness of response	.402	.069	.517	5.789	.000
X: experimental condition (0 = no protest, 1 = protest)	-.100	.200	-.045	-.501	.617

a. Dependent Variable: Y: liking of the target

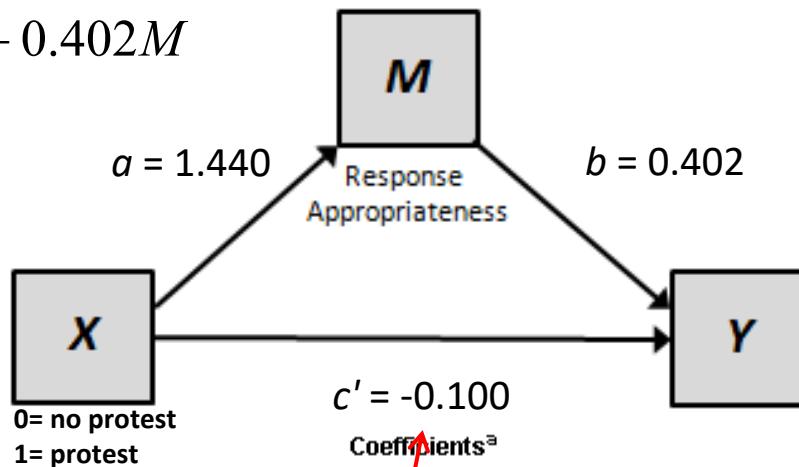
```
regression/dep=liking/method=enter respappr protest.
```

```
proc reg data=protest;model liking=respappr protest;run;
```

Using a set of OLS regression analyses



$$\hat{Y} = 3.751 - 0.100X + 0.402M$$



Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error			
1	(Constant)	3.751	.306		12.271	.000
	M: appropriateness of response	.402	.069	.517	5.789	.000
	X: experimental condition (0 = no protest, 1 = protest)	-.100	.200	-.045	-.501	.617

a. Dependent Variable: Y: liking of the target

```
regression/dep=liking/method=enter respappr protest.
```

```
proc reg data=protest;model liking=respappr protest;run;
```

Because the two groups differ by one unit on **X**, this is the difference between the group means adjusted for differences between the groups in how appropriate her behavior was perceived as being for the situation (i.e., holding it constant)

Interpretation of total, direct, and indirect effects

Generic

Total: Two people who differ by one unit on X are estimated to differ by c units on Y on average.

Indirect: They differ by ab units on average as a result of the effect of X on M which in turn affects Y .

Direct: The rest of the difference, the difference of c' units, is due to the effect of X on Y independent of M .

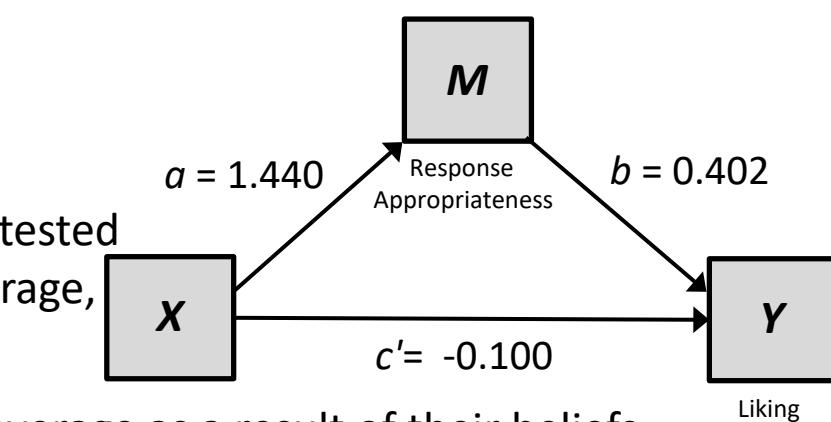
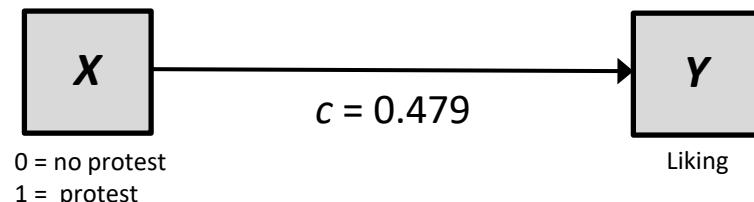
Specific

Total: Participants who were told the lawyer protested ($X = 1$) liked the lawyer 0.479 units **more**, on average, than those who were told she did not protest.

Direct effect = -0.100

Indirect Effect = $1.440(0.402) = 0.579$

Total effect = $-0.100 + 0.579 = 0.479$



Indirect: They liked her by 0.579 units **more** on average as a result of their beliefs about the appropriateness of her response, which in turn affected their liking.

Direct: Among those equal in their beliefs about the appropriateness of her response, those who were told the lawyer protested liked her 0.100 units **less** (because the sign is negative) than those who were told she did not protest the decision.

Statistical Inference

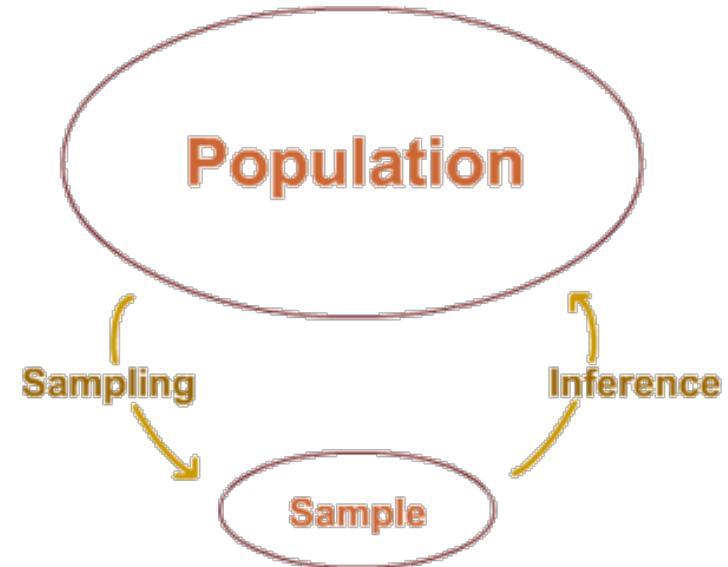
Statistical inference: The indirect effect

Statistical inference is how we take information from our sample and generalize to the population.

Up to this point we have made estimates based on the sample, but we were unable to make any claims about what we think the population looks like.

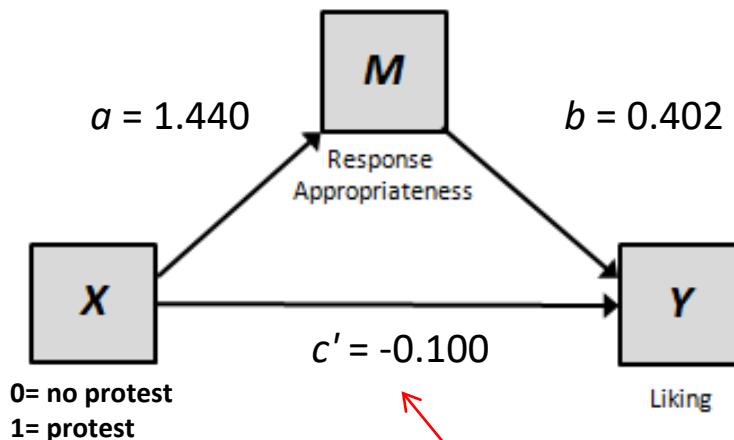
Methods of inference:

- Sobel Test (Normality Test / Delta Method)
- Bootstrap confidence intervals
- Monte Carlo confidence intervals
- Causal Steps Method



Statistical inference: The direct effect

Inference for the direct effect is simple and noncontroversial. The inference can be framed in terms of a hypothesis test or a confidence interval. Any OLS regression program will provide both.



No statistically significant evidence of a direct effect of protest on liking,
 $c' = -0.100, p = 0.617,$
95% CI = (-0.497 to 0.296)

regression/statistics defaults ci/ dep=liking/method=enter respappr protest.

proc reg data=protest;model liking=respappr protest/stb clb; run;

Model	Coefficients ^a							
	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B		
	B	Std. Error	Beta			Lower Bound	Upper Bound	
1	(Constant)	3.747	.306	12.255	.000	3.142	4.352	
	RESPAPPR: appropriateness of response	.402	.070	.517	5.788	.000	.265	.540
	PROTEST: experimental condition (0 = no protest, 1 = protest)	-.101	.200	-.045	-.502	.616	-.497	.296

a. Dependent Variable: LIKING: liking of the target

Statistical inference: The indirect effect

The indirect effect estimates the influence of X on Y through the mechanism represented by M (i.e., the $X \rightarrow M \rightarrow Y$ sequence). 21st-century mediation analysis bases claims of mediation on evidence that the indirect effect is different from zero.

A popular “20th-century” approach to inference: The Sobel test

$$Z = \frac{ab}{\sqrt{b^2 s_a^2 + a^2 s_b^2 + S_a^2 S_b^2}}$$

Indirect effect → ab

“Second order” estimator of the standard error of ab → $\sqrt{b^2 s_a^2 + a^2 s_b^2 + S_a^2 S_b^2}$

One version eliminates this term (“first order” estimator), $S_a^2 S_b^2$

A p -value is derived by assuming normality of the sampling distribution of the indirect effect and using the standard normal distribution for derivation of p . A p -value no greater than α leads to the claim that the indirect effect is statistically different from zero at the α level of significance.

Computation with Protest Data

$$a = 1.440 \quad s_a = 0.222$$

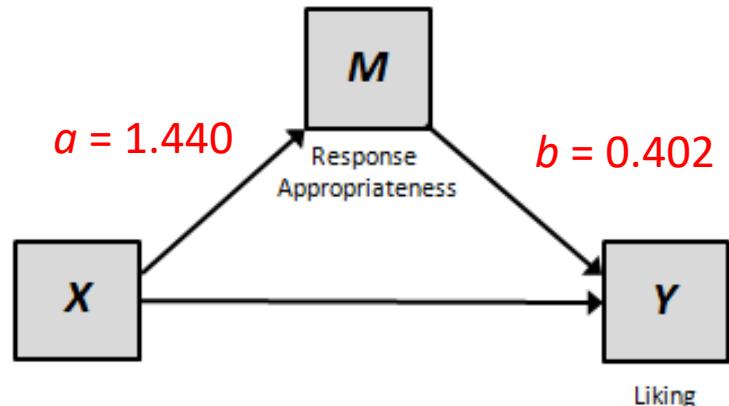
Model	Unstandardized Coefficients		Standardized Coefficients		t	Sig.
	B	Std. Error	Beta			
1	(Constant)	3.984	.183		21.208	.000
	X: Protest	1.440	.222	.499	6.493	.000

a. Dependent Variable: RESPAPPR: appropriateness of response

Model	Unstandardized Coefficients		Standardized Coefficients		t	Sig.
	B	Std. Error	Beta			
1	(Constant)	3.747	.306		12.255	.000
	RESPAPPR: appropriateness of response	.402	.070	.517	5.788	.000
	PROTEST: experimental condition (0 = no protest, 1 = protest)	-.101	.200	-.045	-.502	.616

a. Dependent Variable: LIKING: liking of the target

$$b = 0.402 \quad s_b = 0.070$$



$$Z = \frac{ab}{\sqrt{b^2 s_a^2 + a^2 s_b^2 + s_a^2 s_b^2}}$$

$$(1.440)(0.402)$$

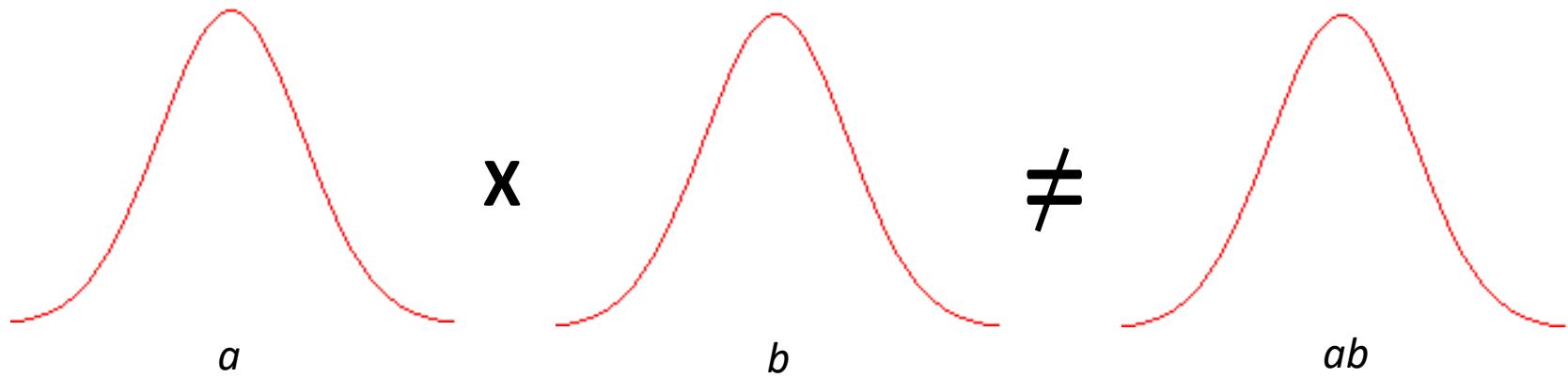
$$\frac{0.5789}{0.1355} = 4.27, p < .0001$$

$$Z = \frac{(1.440)(0.402)}{\sqrt{(0.402)^2(0.222)^2+(1.440)^2(0.070)^2+(0.222)^2(0.070)^2}} = \frac{0.5789}{0.1355} = 4.27, p < .0001$$

The indirect effect is statistically significant. But this test has serious problems.

What's wrong with the Sobel test?

For the Sobel test, the p -value is derived by assuming normality of the sampling distribution of the indirect effect and using the standard normal distribution. Although this assumption is fairly sensible in large samples, it is not in smaller ones. What is a sufficiently large sample is situationally-specific, and typically you won't know going into the analysis whether or not to trust large sample theory.



This assumption, which typically will not hold, yields a test that is lower in power than alternatives. **Experts in mediation analysis don't recommend the use of this test, though it remains popular.** Eventually, researchers will get the message.

The bootstrap confidence interval

Bootstrapping allows us to empirically estimate the sampling distribution of the indirect effect and generate a confidence interval (CI) for estimation and hypothesis testing. **It has become the preferred inferential method for estimating and testing indirect effects.**

- (1) Take a random sample of size n from the sample *with replacement*.
- (2) Estimate the indirect effect in this “resample”.
- (3) Repeat (1) and (2) a total of k times, where k is at least 1,000. The larger k , the better. I recommend at least 5,000.
- (4) Use distribution of the indirect effect over multiple resamples as an approximation of the sampling distribution of the indirect effect.
- (5) For 95% CI using “percentile” method, lower and upper bounds are 2.5th and 97.5th percentile in k bootstrap estimates of the indirect effect. Variations exist (e.g., ‘bias corrected’ or ‘bias-corrected and accelerated’ confidence intervals but they do not perform as well as percentile.)

Bootstrapping

Your data

X	M	Y
4.3	1.4	9.1
1.4	5.4	6.4
4.9	4.3	1.3
5.9	2.3	5.4
6.1	3.3	3.9
3.8	3.1	6.3
2.8	3.2	1.5
9.4	4.1	2.3
4.3	1.3	4.4
4.9	3.7	2.1

A resampling of your data

X	M	Y
5.9	2.3	5.4
4.9	4.3	1.3
9.4	4.1	2.3
4.9	4.3	1.3
4.3	1.4	9.1
1.4	5.4	6.4
3.8	3.1	6.3
9.4	4.1	2.3
6.1	3.3	3.9
4.3	1.3	4.4

$$a = -0.051 \quad b = -0.844$$

$$ab = 0.043$$

$$a = 0.020 \quad b = -0.921$$

$$ab = -0.018$$

Bootstrapping

Your data

X	M	Y
4.3	1.4	9.1
1.4	5.4	6.4
4.9	4.3	1.3
5.9	2.3	5.4
6.1	3.3	3.9
3.8	3.1	6.3
2.8	3.2	1.5
9.4	4.1	2.3
4.3	1.3	4.4
4.9	3.7	2.1

Another resampling of your data

X	M	Y
6.1	3.3	3.9
4.9	4.3	1.3
2.8	3.2	1.5
4.9	3.7	2.1
3.8	3.1	6.3
9.4	4.1	2.3
2.8	3.2	1.5
4.9	4.3	1.3
4.9	3.7	2.1
1.4	5.4	6.4

$$a = -0.051 \quad b = -0.844$$

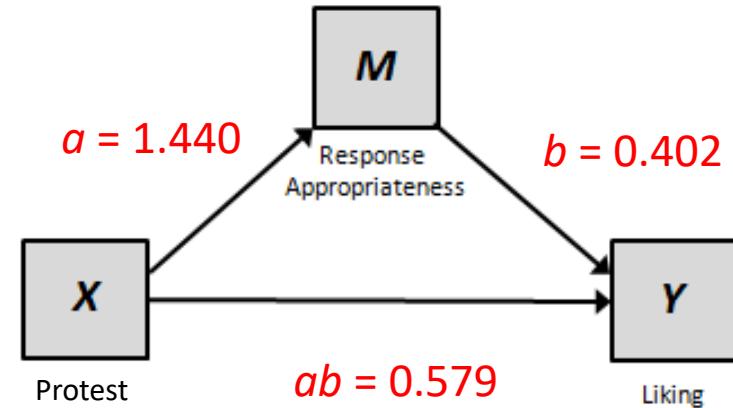
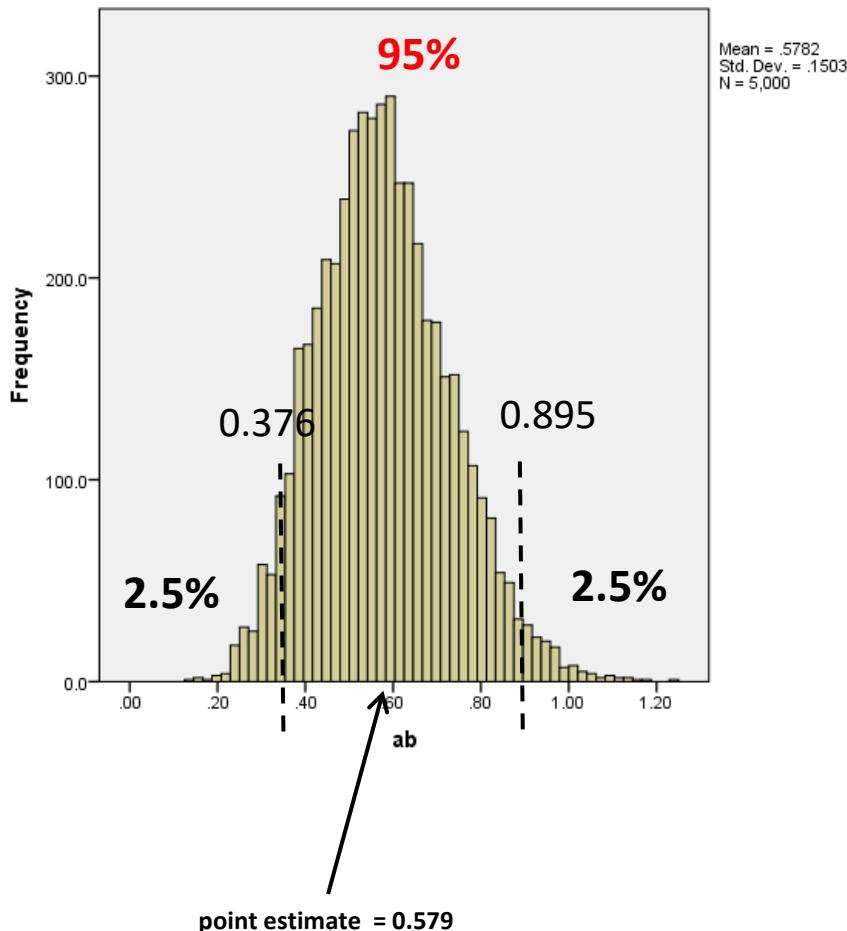
$$ab = 0.043$$

$$a = -0.034 \quad b = 0.523$$

$$ab = -0.017$$

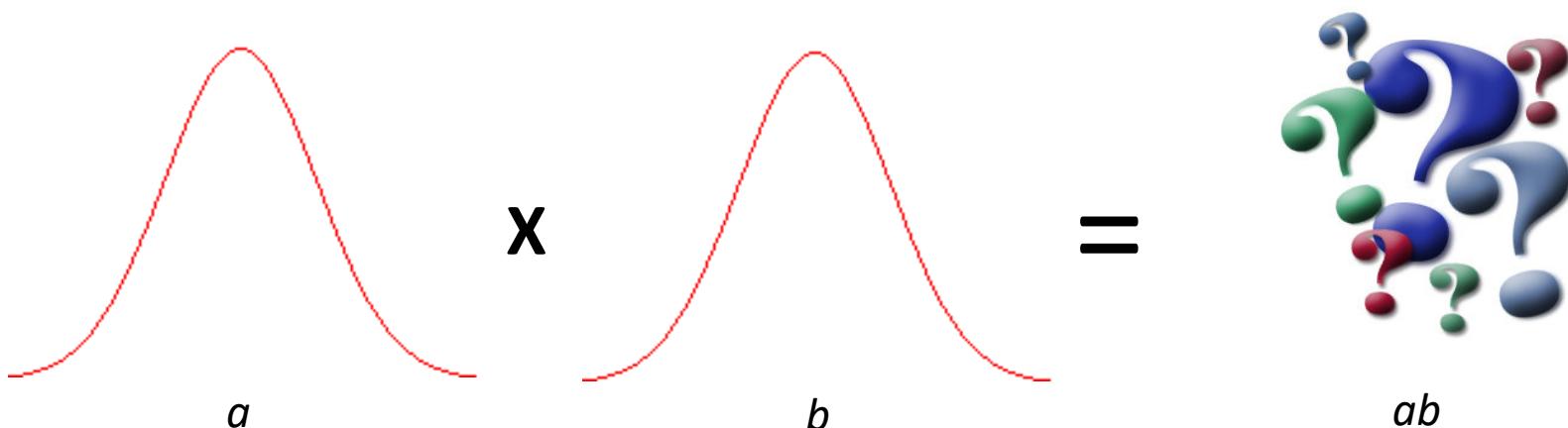
5,000 bootstrap estimates of the indirect effect

95% of the 5,000 bootstrap estimates of the indirect effect were between 0.376 and 0.895. This is our 95% confidence interval.



Zero is not in the confidence interval, so we **can** claim an indirect effect different from zero with 95% confidence. This is akin to (though not exactly the same as) rejecting the null hypothesis of no indirect effect at the $\alpha = 0.05$ level of significance.

The Monte Carlo interval



If all of the assumptions of linear regressions are met (or sample sizes are sufficiently large), then we know that a and b will have normal distributions.

The Monte Carlo confidence intervals takes advantage of this knowledge by simulating normal distributions for a and b then calculating their product to get an estimated distribution of the indirect effect (ab).

The Monte Carlo interval

Monte Carlo empirically estimates the sampling distribution of the indirect effect and generates a confidence interval (CI) for estimation and hypothesis testing. This simulation based method assumes each individual path (a and b) is normally distributed.

- (1) Generate k samples from a normal distribution with mean a and standard deviation s_a**
- (2) Generate k samples from a normal distribution with mean b and standard deviation s_b**
- (3) Multiply samples together to get a distribution of k estimates of ab .**
- (4) Rank order estimates and select estimates which define the lower percentile of ranked k estimates and upper percentile of sorted estimates which define CI of interest.**
- (5) For 95% CI lower and upper bounds are 2.5th and 97.5th percentile in k bootstrap estimates of the indirect effect.**

The Monte Carlo interval

This method performs well (similarly to bootstrapping) in a variety of simulation studies, but is still less popular.

This method makes stronger assumptions than bootstrapping, but does not result in great power.

Add `mc = 1` to PROCESS command line to request Monte Carlo CIs

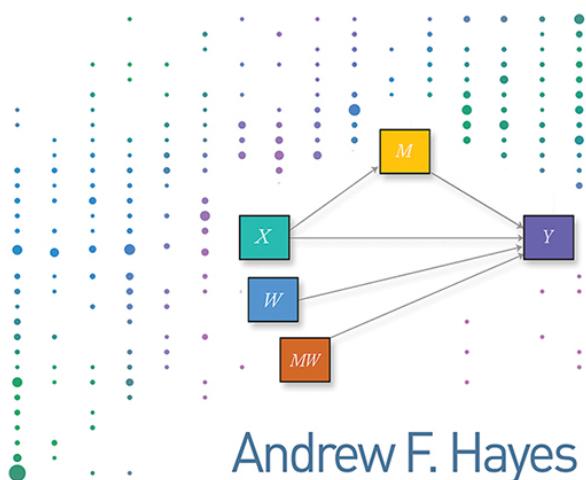
```
Indirect effect(s) of X on Y:  
Effect      MC SE      MC LLCI      MC ULCI  
respappr   .5793     .1332     .3369     .8469
```

The Monte Carlo confidence interval also suggests that we are confident the indirect effect is not zero. So all three inferential methods come to the same conclusion, which is comforting.

PROCESS

SECOND EDITION

Introduction to Mediation, Moderation, and Conditional Process Analysis | A Regression-Based Approach

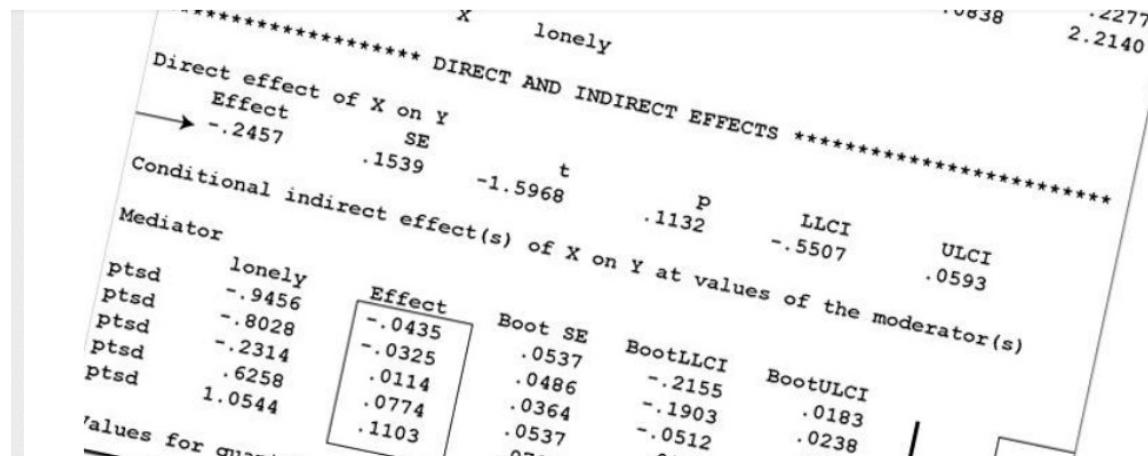


Andrew F. Hayes

Published in December 2017 and available through
The Guilford Press, Amazon.com, and elsewhere.

- First released in beta form in March of 2012 and later documented in Hayes (2013, IMMCPA, published by The Guilford Press).
- Available for both SPSS (in macro and “custom dialog” form) and SAS.
- An integration of functions available in my other published macros for mediation and moderation analysis (SOBEL, INDIRECT, MODMED, MODPROBE, MED3C) and a whole lot more, all in one command.
- A handy tool for both “confirmatory” and “exploratory” approaches to data analysis.
- Freely available at www.processmacro.org.
The current release is v3.5

The PROCESS macro for SPSS and SAS



PROCESS is an easy to use add-on for SPSS and SAS for statistical mediation, moderation, and conditional process analysis. The use of **PROCESS** is described and documented in *Introduction to Mediation, Moderation, and Conditional Process Analysis*, published by The Guilford Press.

PROCESS uses an ordinary least squares or logistic regression-based path analytic framework for estimating direct and indirect effects in simple and multiple mediator models, two and three way interactions in moderation models along with simple slopes and regions of significance for probing interactions, conditional indirect effects in moderated mediation models with a single or multiple mediators and moderators, and indirect effects of interactions in mediated moderation models also with a single or multiple mediators. Bootstrap and Monte Carlo confidence intervals are implemented for inference about indirect effects.

PROCESS can estimate moderated mediation models with multiple mediators, multiple moderators of individual paths, interactive effects of moderators on individual paths, and models with dichotomous outcomes.

PROCESS was written by Andrew F. Hayes, Professor of Quantitative Psychology at The Ohio State University.

Facebook users can stay apprised of latest developments in **PROCESS** by liking [here](#). Tweeters about **PROCESS** can use the hashtag [#processmacro](#)

Read the documentation (eventually)

The PROCESS documentation is an eventual must-read. It describes how to use PROCESS, as well as its various options, capabilities, and limitations. It is available as Appendix A in Hayes (2017). *Introduction to Mediation, Moderation, and Conditional Process Analysis (2nd Ed)*. At a minimum, you must have the model templates handy, as PROCESS expects you to tell it which model number you are estimating and which variables play what role. **You have a mini version of the templates PDF.**

Appendix A Using PROCESS

This appendix describes how to install and execute PROCESS, how to set up a PROCESS command, and it documents its many features, some of which are not described elsewhere in this book. As PROCESS is modified and features are added, supplementary documentation will be released at www.afhayes.com. Check this web page regularly for updates. Also available at this page is a complete set of model templates identifying each model that PROCESS can estimate.

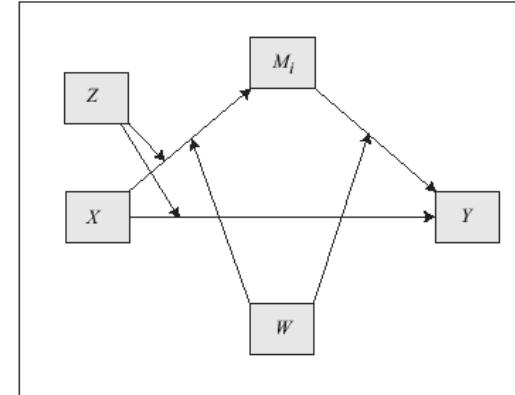
This documentation focuses on the SPSS version of PROCESS. All features and functions described below are available in the SAS version as well and work as described here, with minor modifications to the syntax. At the end of this documentation (see page 438), a special section devoted to SAS describes some of the differences in syntax structure for the SAS version compared to what is described below.

Overview

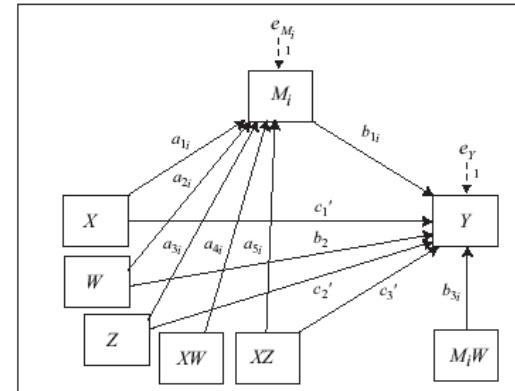
PROCESS is a computational tool for path analysis-based moderation and mediation analysis as well as their integration in the form of a conditional process model. In addition to estimating unstandardized model coefficients, standard errors, *t* and *p*-values, and confidence intervals using either OLS regression (for continuous outcomes) or maximum likelihood logistic regression (for dichotomous outcomes), PROCESS generates direct and indirect effects in mediation models, conditional effects (i.e., "simple slopes") in moderation models, and conditional indirect effects in conditional process models with a single or multiple mediators. PROCESS offers various methods for probing two- and three-way interactions and can construct percentile bootstrap, bias-corrected bootstrap, and Monte Carlo confidence intervals for indirect effects. In mediation models, multiple mediator variables can be specified to operate in parallel or in serial. Heteroscedasticity-consistent standard errors are available for inference about model coeffi-

Model 62

Conceptual Diagram

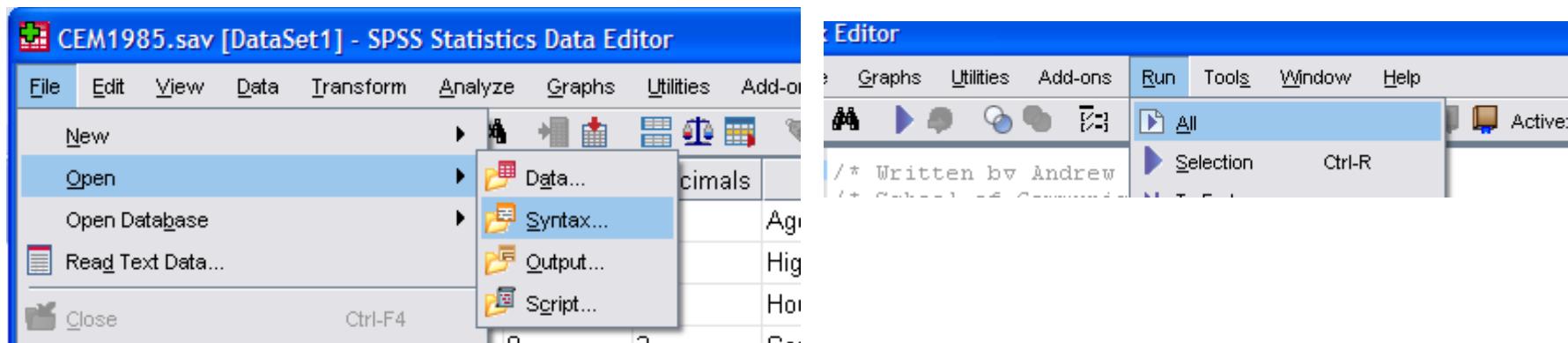


Statistical Diagram



PROCESS as a syntax-driven macro

Open process.sps as a syntax file and run the entire program **exactly as is**. This produces a new SPSS command called PROCESS. See the documentation for details on the syntax structure. PROCESS goes away when you close SPSS.

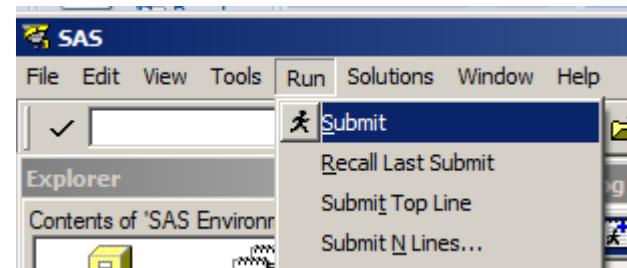
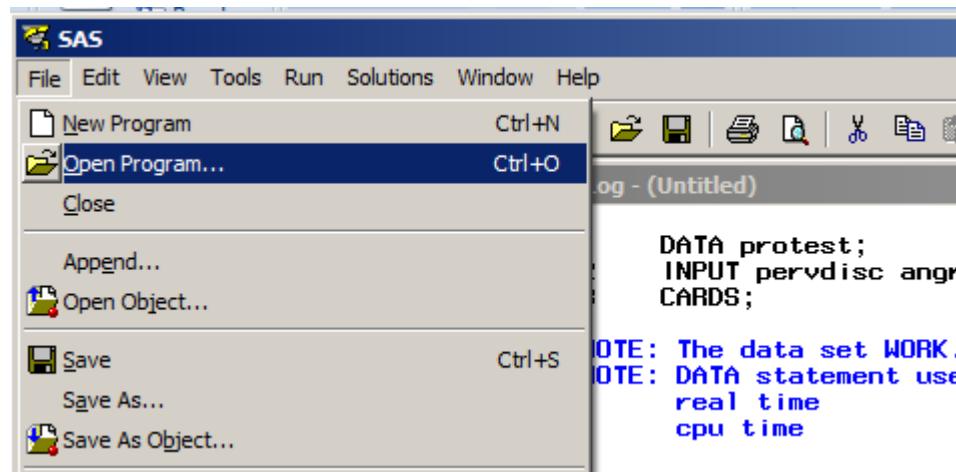


Once PROCESS is defined, you can run a properly-formatted PROCESS command in a new syntax window. Such a command might look something like below. Not all of the arguments below are required.

```
process y=liking/x=protest/m=respappr/total=1/normal=1  
/model=4/boot=10000.
```

PROCESS for SAS

In SAS, open process.sas and submit the entire program **exactly as is**. This produces a new SAS command called %PROCESS. The syntax structure is described in the documentation.



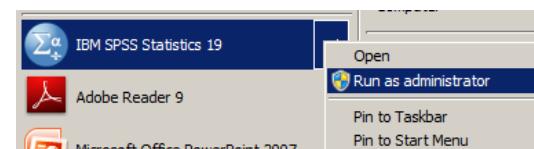
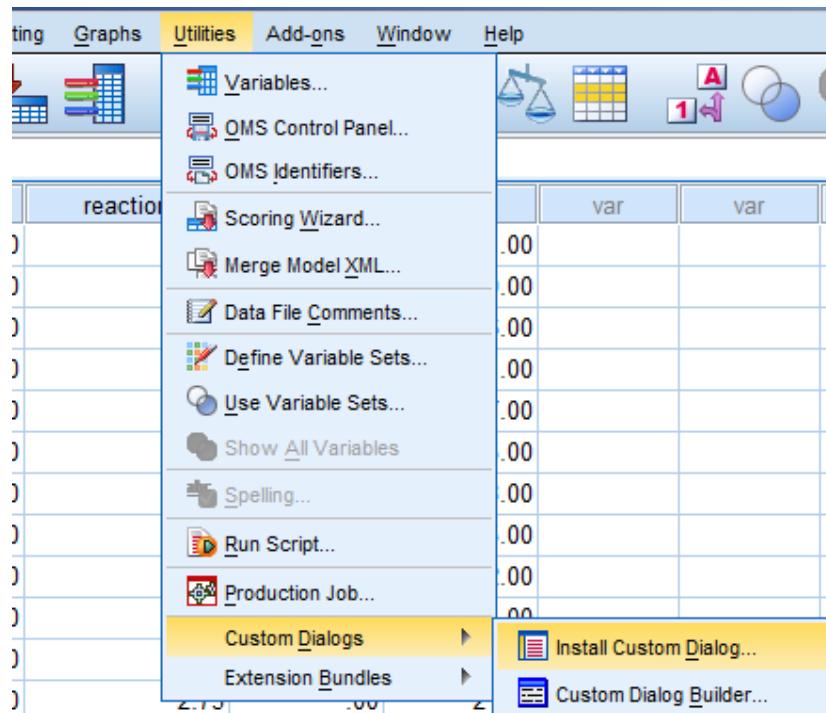
Once PROCESS is defined, you can run a properly-formatted PROCESS command in a new syntax window. Such a command might look something like below. Not all of the arguments below are required.

```
%process (data=protest,y=liking,x=protest,m=respappr,total=1,  
normal=1,boot=10000,model=4);
```

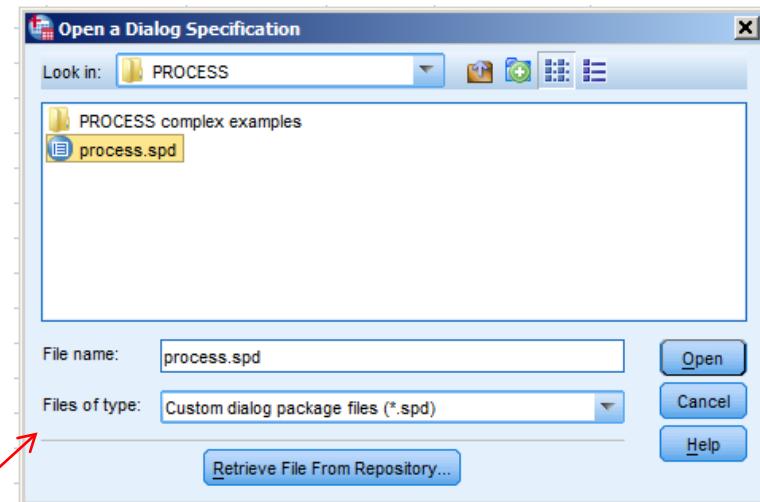
PROCESS “Custom Dialog”

The PROCESS macro must be run at least once in your SPSS session to activate the PROCESS command. Custom Dialog files are permanently installed in SPSS, integrating the procedure into SPSS menus. Use the procedure below. In Windows, installation requires administrative access to your machine. You **probably have to open SPSS as an administrator as well. You may not have access to do so.**

In SPSS 23 and earlier:



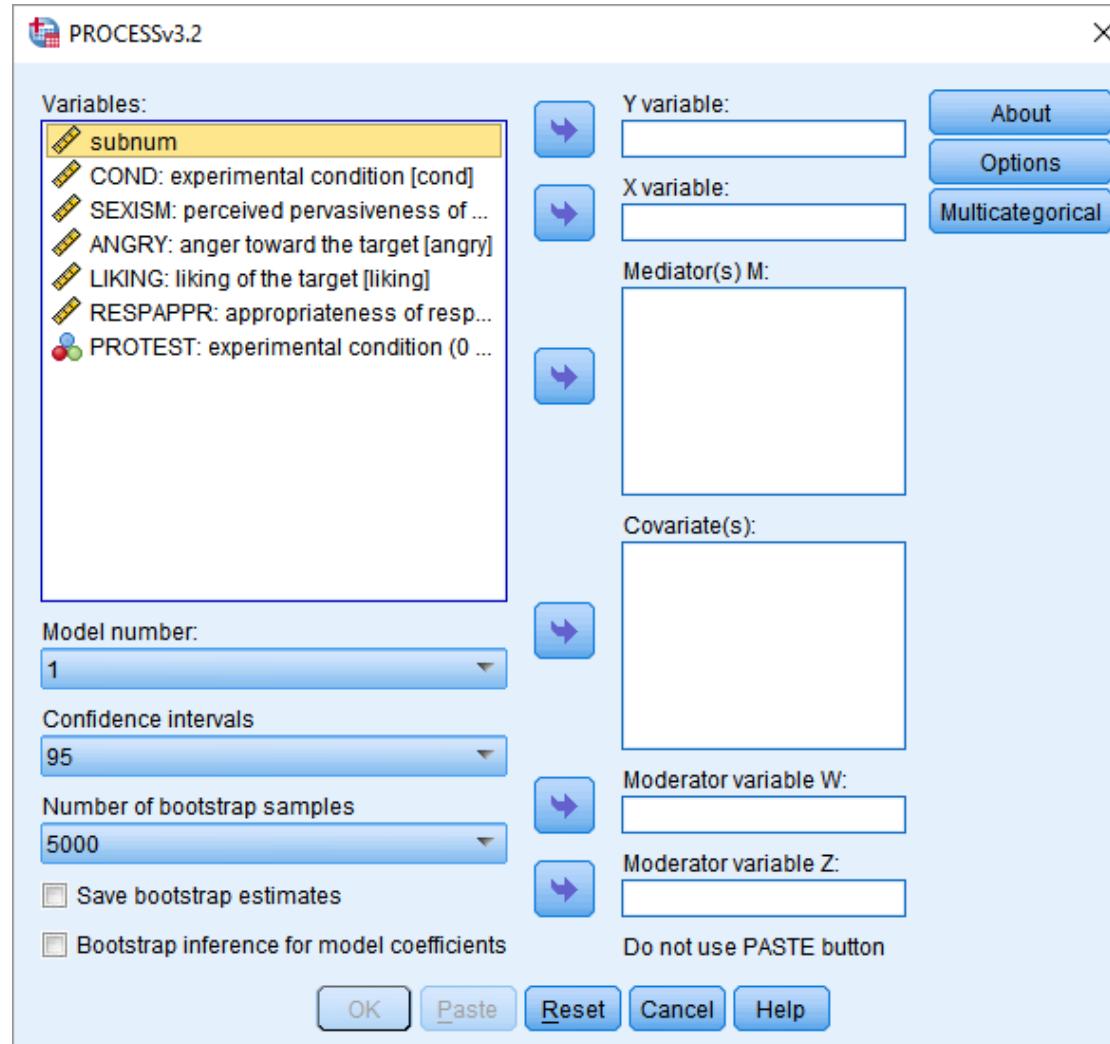
Right click SPSS
to select



When installed, PROCESS can be found under “Analyze”→“Regression”

In SPSS24, look under “Extensions” for the Utilities option

PROCESS “Custom Dialog”

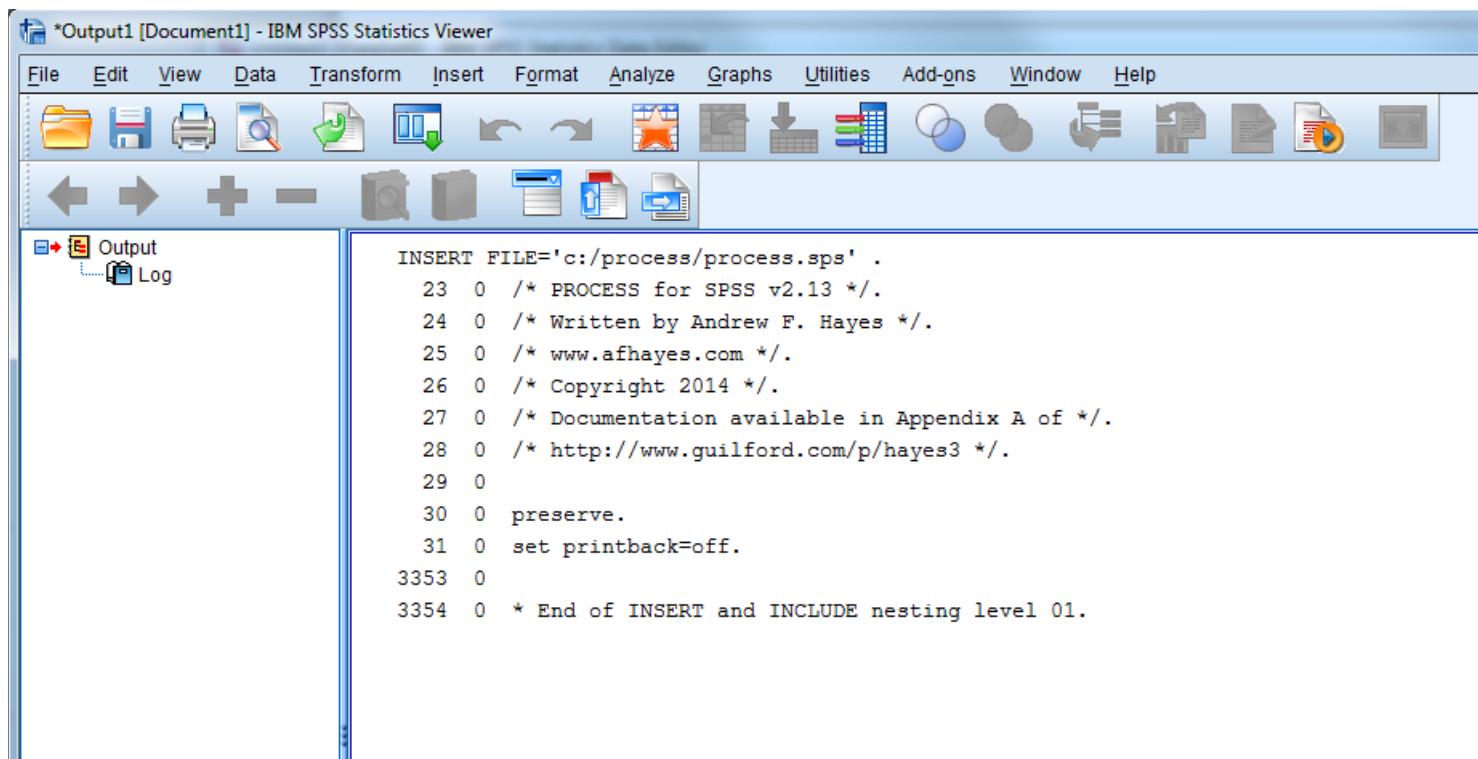


Installing the dialog box does not eliminate the need to run the PROCESS code if you plan on executing with syntax. And don't use the PASTE button.

Autoexecution

It is possible to get SPSS to execute the PROCESS code on its own when SAS/SPSS executes. A document is provided to you with the course files that provides instructions for SPSS for Windows and Mac.

When successful, SPSS for Windows users typically see something like the following in the output window when SPSS is opened:



```
INSERT FILE='c:/process/process.sps'.
23 0 /* PROCESS for SPSS v2.13 */.
24 0 /* Written by Andrew F. Hayes */.
25 0 /* www.afhayes.com */.
26 0 /* Copyright 2014 */.
27 0 /* Documentation available in Appendix A of */.
28 0 /* http://www.guilford.com/p/hayes3 */.
29 0
30 0 preserve.
31 0 set printback=off.
3353 0
3354 0 * End of INSERT and INCLUDE nesting level 01.
```

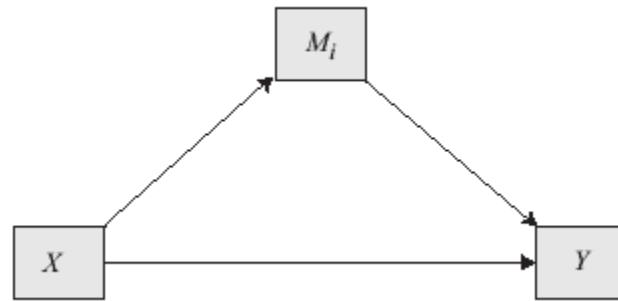
Model template system

PROCESS has 56 model templates, referred to by number in PROCESS, which specify the form of the model (moderation, mediation, moderated mediation, and so forth) and which variables play what roles.

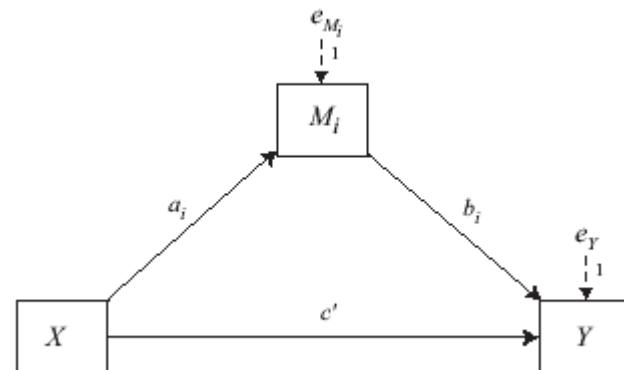
Conceptual diagram

Example #1:

Model 4 is a simple or parallel multiple mediator model, which estimates the direct and indirect effect(s) of X on Y through one or more mediators (M) (up to 10 mediators at once)



Statistical diagram



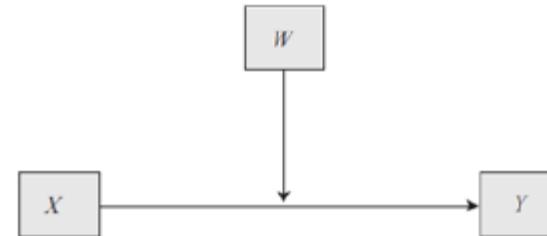
Model template system

PROCESS has 56 model templates, referred to by number in PROCESS, which specify the form of the model (moderation, mediation, moderated mediation, and so forth) and which variables play what roles.

Conceptual diagram

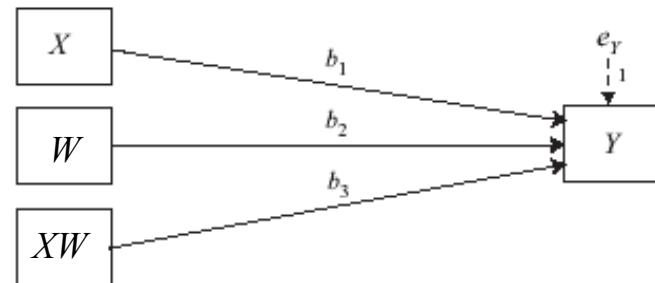
Example #2:

Model 1 is a simple moderation model, with W moderating the effect of X on Y .



The statistical diagram shows the model in the form of a path diagram. This is the form in which the model is estimated.

Statistical diagram



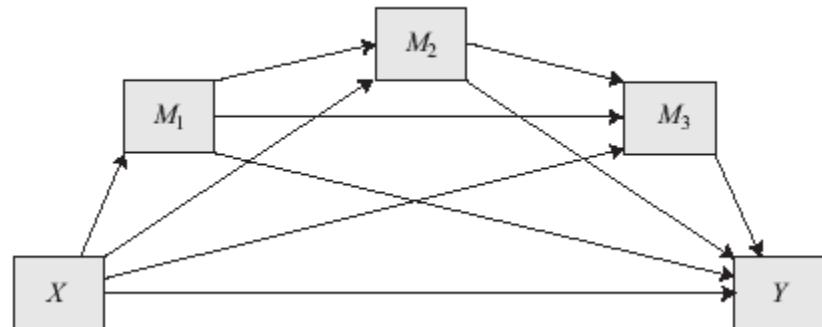
Model template system

PROCESS has 56 model templates, referred to by number in PROCESS, which specify the form of the model (moderation, mediation, moderated mediation, and so forth) and which variables play what roles.

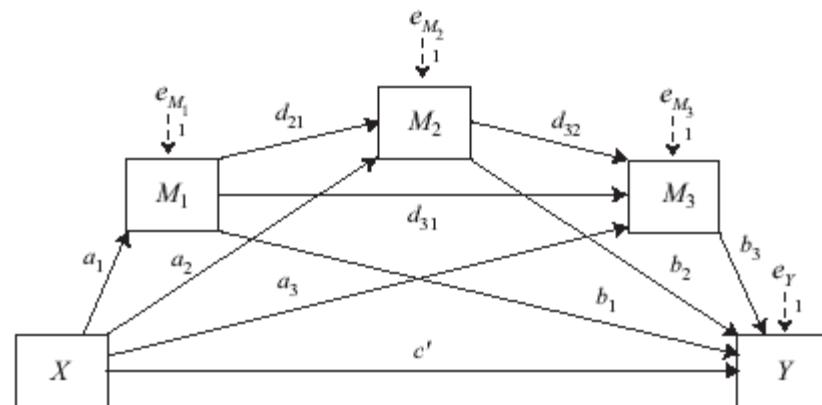
Example #3:

Model 6 is a serial multiple mediator model, which estimates the direct and indirect effect(s) of X on Y through up to 4 mediators (M) chained together in serial. An example with **three** mediators is depicted to the right.

Conceptual diagram



Statistical diagram



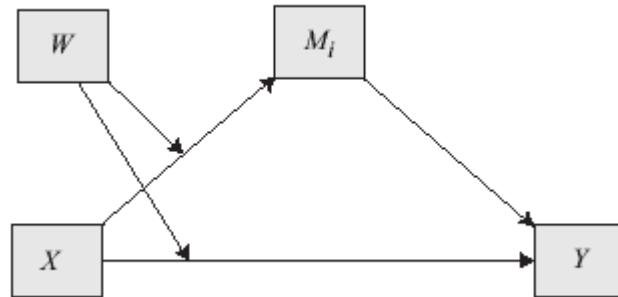
Model template system

PROCESS has 56 model templates, referred to by number in PROCESS, which specify the form of the model (moderation, mediation, moderated mediation, and so forth) and which variables play what roles.

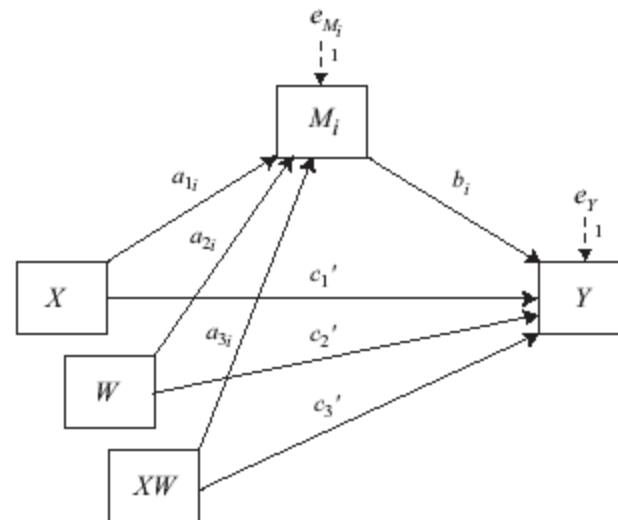
Example #4:

Model 8 is a conditional process model which estimates the conditional direct and indirect effects of X on Y through M , with direct effect and “first stage” moderation by W .

Conceptual diagram



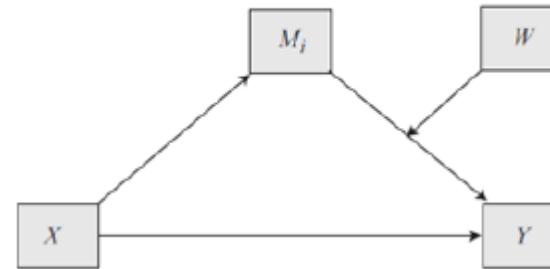
Statistical diagram



Model template system

PROCESS has 56 model templates, referred to by number in PROCESS, which specify the form of the model (moderation, mediation, moderated mediation, and so forth) and which variables play what roles.

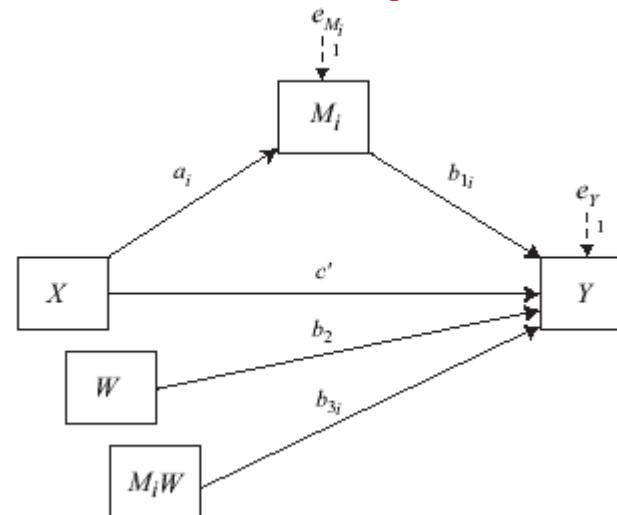
Conceptual diagram



Example #5:

Model 14 is a conditional process model which estimates the direct effect of X on Y and conditional indirect effects of X on Y through M , with “second stage” moderation by W .

Statistical diagram



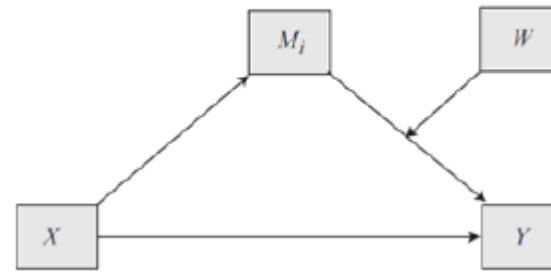
Model template system

PROCESS has 56 model templates, referred to by number in PROCESS, which specify the form of the model (moderation, mediation, moderated mediation, and so forth) and which variables play what roles.

Minimum required specifications

- Which variables play which role in the model (**y=** **x=** **m=** **w =** and so forth)
- Model number (**model=**)
- SAS only: Data file (**data=**)

Conceptual diagram



SPSS

```
PROCESS y=yvar/x=xvar/m=mvlist/w=wvar/model=14 .
```

SAS

```
%process (data=filename,y=yvar,x=xvar,m=mvlist,w=wvar,model=14) ;
```

Limitations and constraints

- Only one X and one Y allowed in a model.
- PROCESS is an OLS or logistic regression modeling tool. Categorical mediators are not allowed.
- Up to 10 mediators in numbered models, 6 in custom models.
- No more than two moderators can be used in any model.
- Most variables can play only one role in the model. Except, a variable can be both a moderator and a covariate in separate equations.
- PROCESS is a single-level observed variable modeling system. No multilevel problems can be analyzed with PROCESS.
- PROCESS requires complete data. Listwise deletion is used for cases missing on any variable in the model.
- Although PROCESS will accept them, it is safer to restrict variable names to eight characters or fewer.

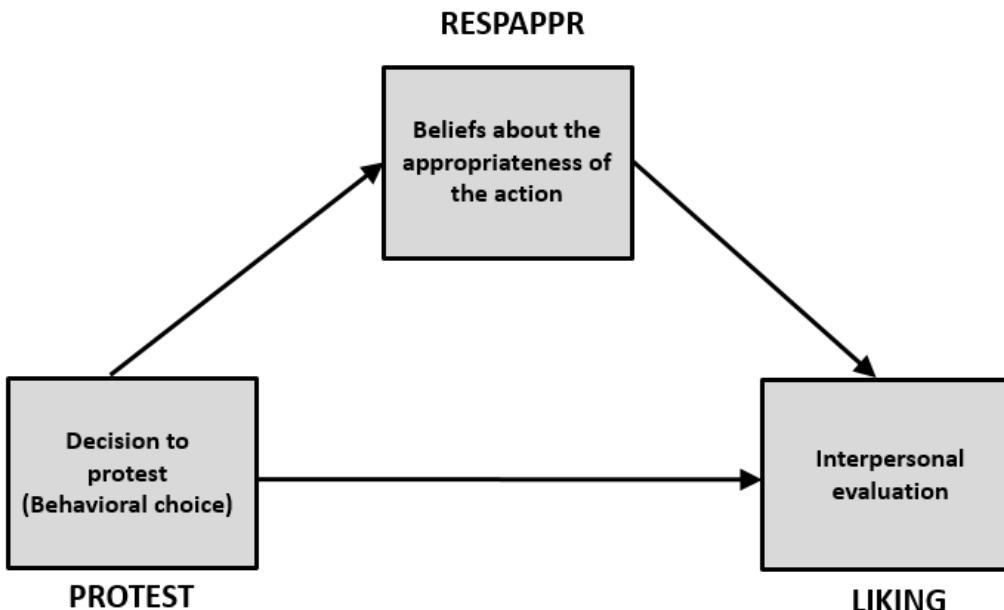
If you are familiar with PROCESS v2, see the “What’s new in PROCESS 3” pages in your course book.

Differences between V2 & V3

- No longer need the `vars` list
- Covariates now listed in `cov` list
- Moderators are always `W` and `Z`, no more `V`, `M` or `Q` moderators
- Dichotomous `Y` now available for some models
- A variety of models have been cut, but new ability to create and edit models
- New models for serial moderated mediation and serial and parallel mediation
- Probing option now defaults to what used to be `quantiles`, can use `moments` argument for legacy output
- Probing and plotting for models with any moderation
- Default is now percentile bootstrap, no more BC or ABC
- Multicategorical `X` or Moderators
- `Wmodval` and `zmodval` allow for multiple values
- Covariate assignment
- Bootstrap CIs for regression coefficients
- Model construction
- `Cluster`, `ws`, `varorder`, and `percent` are no longer options
- `Stand` option for standardizing coefficients

Estimation of the PROTEST model in PROCESS

PROCESS Model 4



What should the PROCESS command look like?

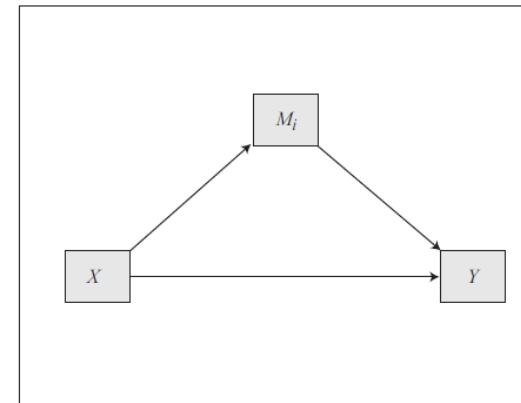
```
process y=liking/x=protest/m=respappr/model=4/boot=10000/normal=1/total=1.
```

not required

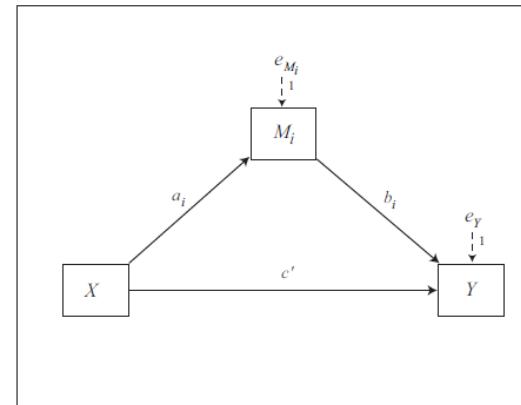
```
%process (data=protest, y=liking, x=protest, m=respappr, model=4, boot=10000, normal=1, total=1);
```

Model 4

Conceptual Diagram



Statistical Diagram



PROCESS output

***** PROCESS Procedure for SPSS Version 3.00 *****

Written by Andrew F. Hayes, Ph.D. www.afhayes.com
Documentation available in Hayes (2018). www.guilford.com/p/hayes3

Model : 4
Y : liking
X : protest
M : respappr

Sample

Size: 129

OUTCOME VARIABLE:

respappr

$$\hat{M} = 3.884 + 1.440 X$$

Model Summary

R	R-sq	MSE	F	df1	df2	P
.4992	.2492	1.3753	42.1550	1.0000	127.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	3.8841	.1831	21.2078	.0000	3.5217	4.2466
protest	1.4397	.2217	6.4927	.0000	1.0009	1.8785

path a

Output C

PROCESS output

Outcome: liking

$$\hat{Y} = 3.747 - 0.101X + 0.402M$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.4959	.2459	.8441	20.5483	2.0000	126.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	3.7473	.3058	12.2553	.0000	3.1422	4.3524
respappr	.4024	.0695	5.7884	.0000	.2648	.5400
protest	-.1007	.2005	-.5023	.6163	-.4975	.2960

path b
path c'

***** TOTAL EFFECT MODEL *****

Outcome: liking

$$\hat{Y} = 5.310 + 0.479X$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.2131	.0454	1.0601	6.0439	1.0000	127.0000	.0153

Model

	coeff	se	t	p	LLCI	ULCI
constant	5.3102	.1608	33.0244	.0000	4.9921	5.6284
protest	.4786	.1947	2.4584	.0153	.0934	.8639

path c

Output C

PROCESS output

Output C

***** TOTAL, DIRECT, AND INDIRECT EFFECTS *****

Total effect of X on Y

Effect	SE	t	p	LLCI	ULCI
.4786	.1947	2.4584	.0153	.0934	.8639

path c

Direct effect of X on Y

Effect	SE	t	p	LLCI	ULCI
-.1007	.2005	-.5023	.6163	-.4975	.2960

path c'

Indirect effect of X on Y

Effect	Boot SE	BootLLCI	BootULCI
respappr	.5793	.1519	.3113 .9067

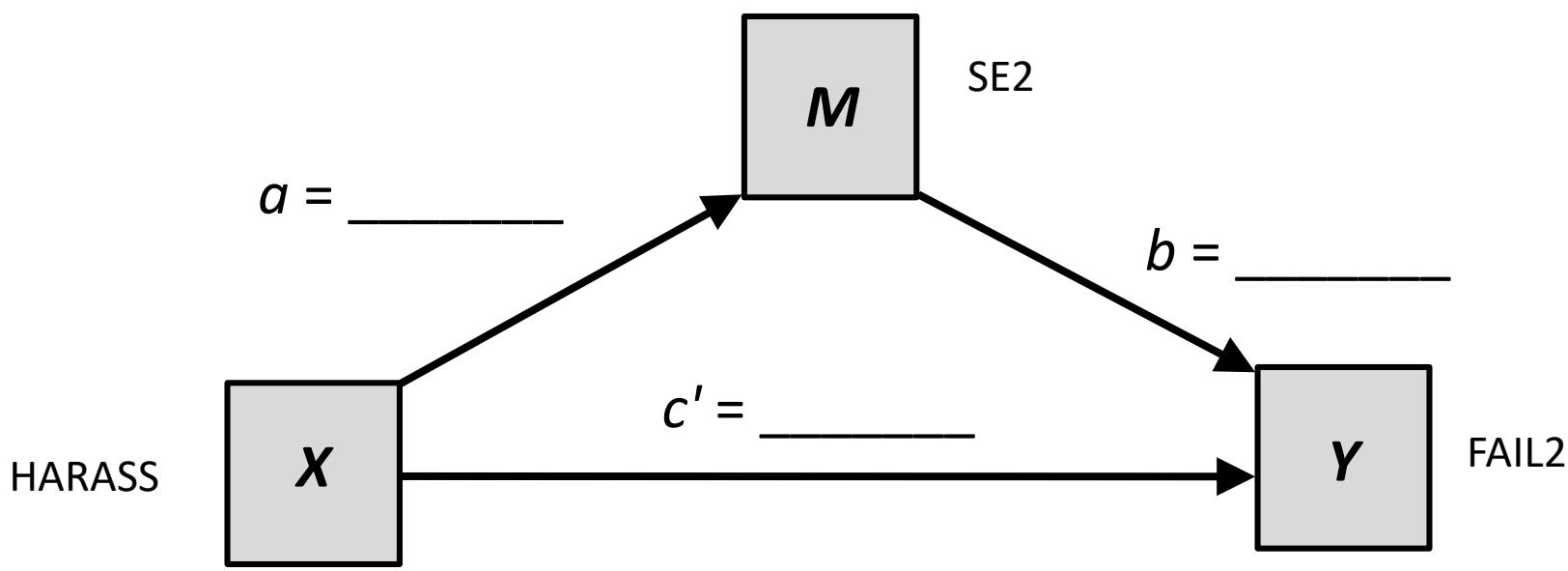
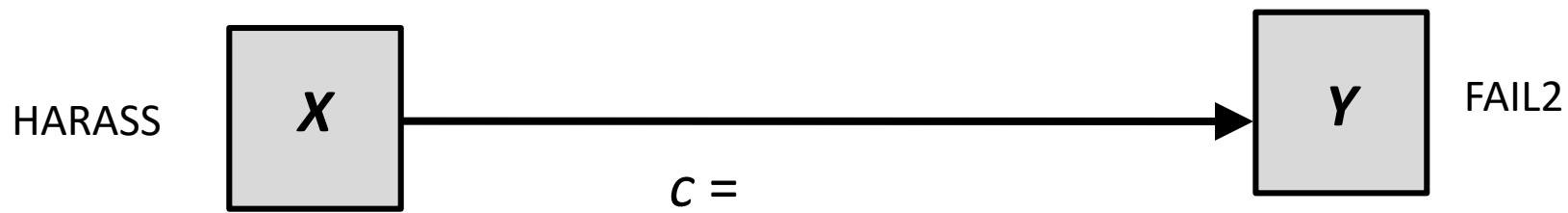
ab with 95% bootstrap
confidence interval

Normal theory tests for indirect effect

Effect	se	z	p
.5793	.1350	4.2924	.0000

Sobel test

Her behavior was perceived as more appropriate if she protested relative to when she did not ($a = 1.440$), and the more appropriate her behavior, the more positively she was perceived ($b = 0.402$). Her choice to protest had a positive effect on how favorably she was perceived indirectly through perceived appropriateness of the response (point estimate: 0.579, 95% CI = 0.311 to 0.907). After accounting for this mechanism, there was no significant effect of her choice to protest on how she was evaluated (direct effect = -0.101, $p = 0.62$)

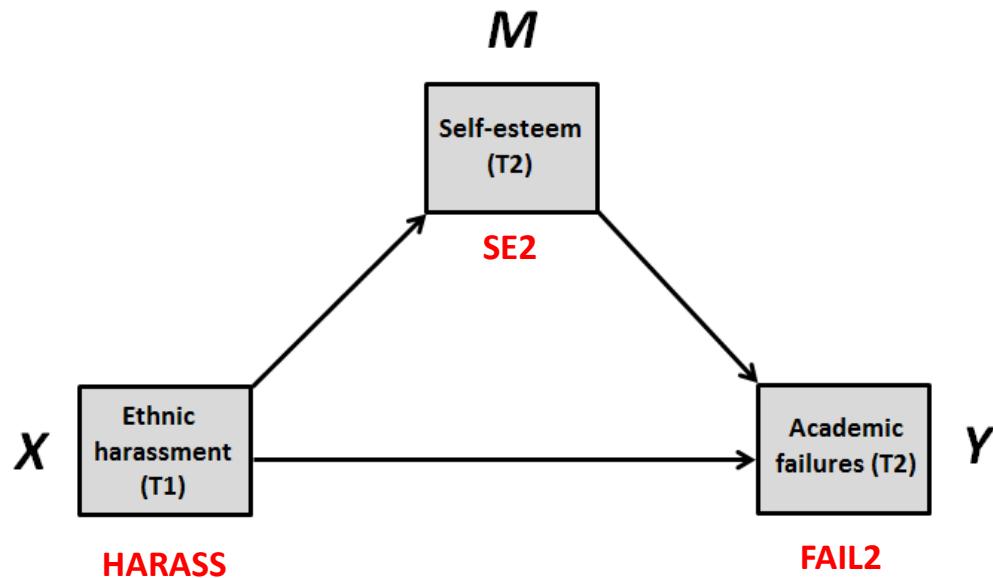


Indirect effect = _____, 95% bootstrap CI = _____

Your CI will not
exactly match. Why?

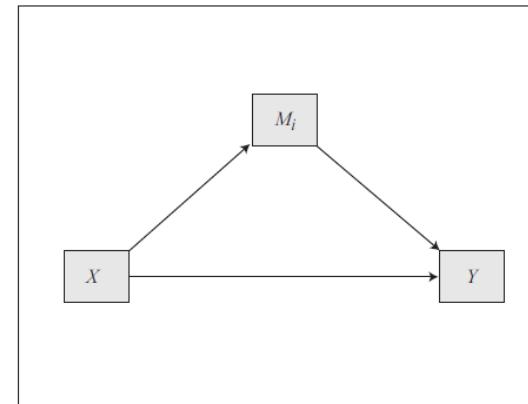
Estimation of the harassment model in PROCESS

PROCESS Model 4

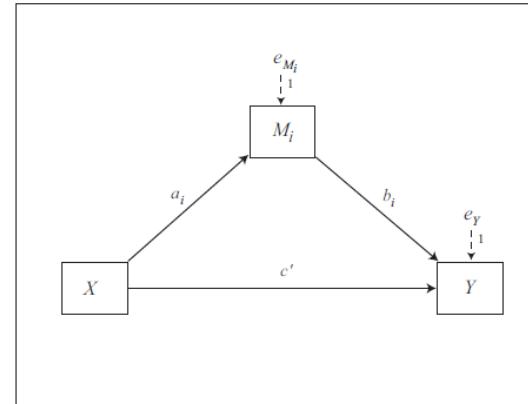


Model 4

Conceptual Diagram



Statistical Diagram



```
Process y=fail2/x=harass/m=se2/model=4/boot=10000  
/normal=1/total=1.
```

```
%process (data=harass,y=fail2,x=harass,m=se2,model=4,  
boot=10000,normal=1,percent=1,total=1);
```

PROCESS output

***** PROCESS Procedure for SPSS Version 3.00 *****

Written by Andrew F. Hayes, Ph.D. www.afhayes.com
Documentation available in Hayes (2018). www.guilford.com/p/hayes3

Model : 4
Y : fail2
X : harass
M : se2

Sample
Size: 330

OUTCOME VARIABLE: $\hat{M} = 3.597 - 0.416X$
se2

Model Summary

R	R-sq	MSE	F	df1	df2	p
.2764	.0764	.2905	27.1349	1.0000	328.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	3.5966	.1235	29.1227	.0000	3.3536	3.8395
harass	-.4156	.0798	-5.2091	.0000	-.5725	-.2586

path a

PROCESS output

Outcome: fail2

$$\hat{Y} = 2.385 + 0.062X - 0.289M$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.3397	.1154	.2215	21.3247	2.0000	327.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	2.3845	.2042	11.6757	.0000	1.9827	2.7863
se2	-.2887	.0482	-5.9879	.0000	-.3836	-.1939
harass	.0616	.0725	.8499	.3960	-.0810	.2042

path b
path c'

***** TOTAL EFFECT MODEL *****

Outcome: fail2

$$\hat{Y} = 1.346 + 0.182X$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.1356	.0184	.2451	6.1419	1.0000	328.0000	.0137

Model

	coeff	se	t	p	LLCI	ULCI
constant	1.3460	.1134	11.8660	.0000	1.1229	1.5692
harass	.1816	.0733	2.4783	.0137	.0374	.3257

path c

PROCESS output

***** TOTAL, DIRECT, AND INDIRECT EFFECTS *****

Total effect of X on Y

Effect	SE	t	p	LLCI	ULCI
.1816	.0733	2.4783	.0137	.0374	.3257

path c

Direct effect of X on Y

Effect	SE	t	p	LLCI	ULCI
.0616	.0725	.8499	.3960	-.0810	.2042

path c'

Indirect effect of X on Y

Effect	Boot SE	BootLLCI	BootULCI
se2	.1200	.0321	.0629 .1899

ab with 95% bootstrap confidence interval

Normal theory tests for indirect effect

Effect	se	z	p
.1200	.0308	3.8993	.0001

Sobel test

Kids one unit higher in harassment frequency were 0.416 units lower in self esteem one year later ($a = -0.416$), and lower self-esteem was related to higher perceived academic failure ($b = -0.289$). So harassment indirectly affected perceived academic failure (point estimate: 0.120, 95% bootstrap CI = 0.063 to 0.190). After accounting for this mechanism, there was no evidence of an effect of harassment on perceived academic failure (direct effect = 0.062, $p = 0.396$, 95% CI = -0.081 to 0.204)

Some additional options

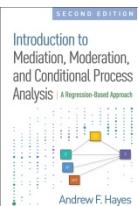
SPSS

```
process y=liking/x=protest/m=respappr/model=4/boot=10000/normal=1/total=1/  
effsize=1/conf=99/save=1/seed=25545.
```

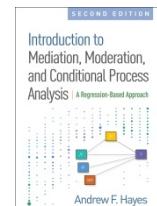
SAS

```
%process (data=protest, y=liking,x=protest,m=respappr,model=4,boot=10000,  
normal=1,total=1,effsize=1,conf=99,save=boots,seed=25545);
```

- EFFSIZE=1:** Generates various effect size measures for the indirect effect.
- CONF=z:** Changes level of confidence to z% for confidence intervals.
- SAVE=1 or
SAVE=fn** in SPSS, produces a file of all bootstrap estimates of all regression coefficients in the model. In SAS, saves bootstrap estimates to a file named “fn”
- SEED=xxxx:** Seeds the random number generator for replication of resamples over repeated runs of PROCESS.



See the documentation in Appendix A of IMCPA for details.



Confounding

Kids who reported greater harassment earlier reported lower in self-esteem later, but they also reported lower self-esteem earlier ($r = -0.18$), and self-esteem was temporally consistent over time ($r = 0.51$). Furthermore, students who reported lower self esteem later reported greater academic failure later, but these low self-esteem students at time 2 also reported greater academic failure earlier ($r = -0.26$), and academic failure was temporally consistent ($r = 0.30$)

		Correlations				
		harass	se1	fail1	se2	fail2
harass	Pearson Correlation	1	-.176	.196	-.276	.136
	Sig. (2-tailed)		.001	.000	.000	.014
	N	330	330	330	330	330
se1	Pearson Correlation	-.176	1	-.306	.505	-.259
	Sig. (2-tailed)	.001		.000	.000	.000
	N	330	330	330	330	330
fail1	Pearson Correlation	.196	-.306	1	-.255	.297
	Sig. (2-tailed)	.000	.000		.000	.000
	N	330	330	330	330	330
se2	Pearson Correlation	-.276	.505	-.255	1	-.337
	Sig. (2-tailed)	.000	.000	.000		.000
	N	330	330	330	330	330
fail2	Pearson Correlation	.136	-.259	.297	-.337	1
	Sig. (2-tailed)	.014	.000	.000	.000	
	N	330	330	330	330	330

Pre-existing self-esteem and failure confound the relationships we believe to be causal. We want to know whether later self-esteem and academic failure are related to ethnic harassment frequency after accounting for initial self-esteem and failure.

Confounding

Some effects in a mediation model are subject to 'confounding' even when X is based on random assignment, making causality harder to establish. Partialing out various confounders can help though won't solve the problem entirely.

$$\hat{Y} = c_0 + cX + c_2U$$

$$\hat{M} = a_0 + aX + a_2U$$

$$\hat{Y} = c'_0 + c'X + bM + c'_2U$$

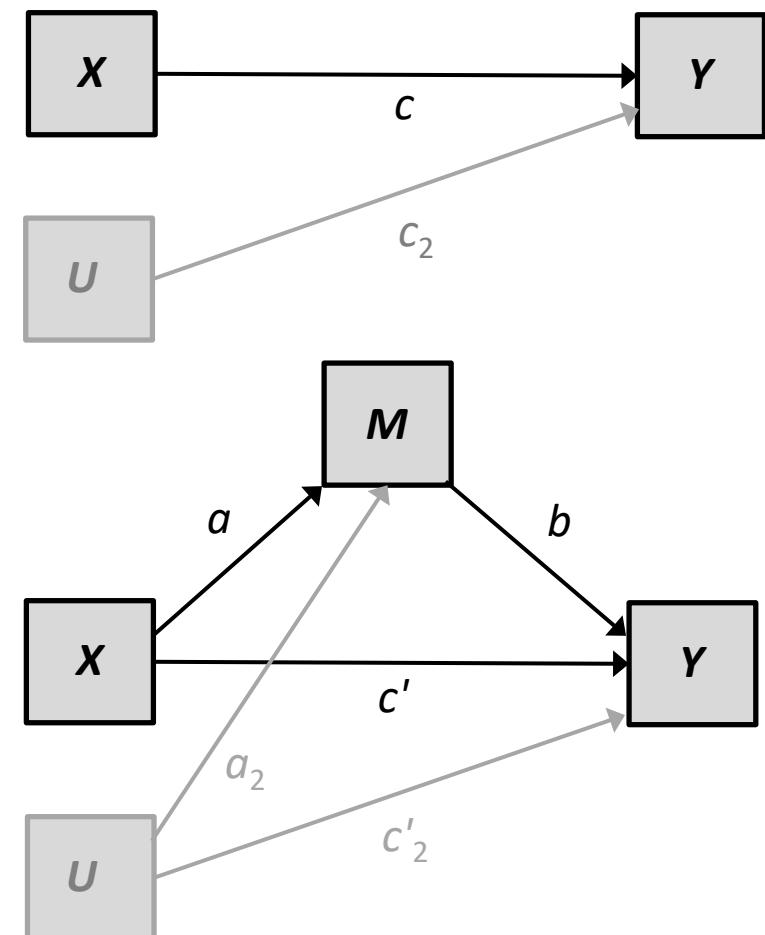
total effect = direct effect + indirect effect

$$c = c' + (a \times b)$$

indirect effect = total effect – direct effect

$$a \times b = c - c'$$

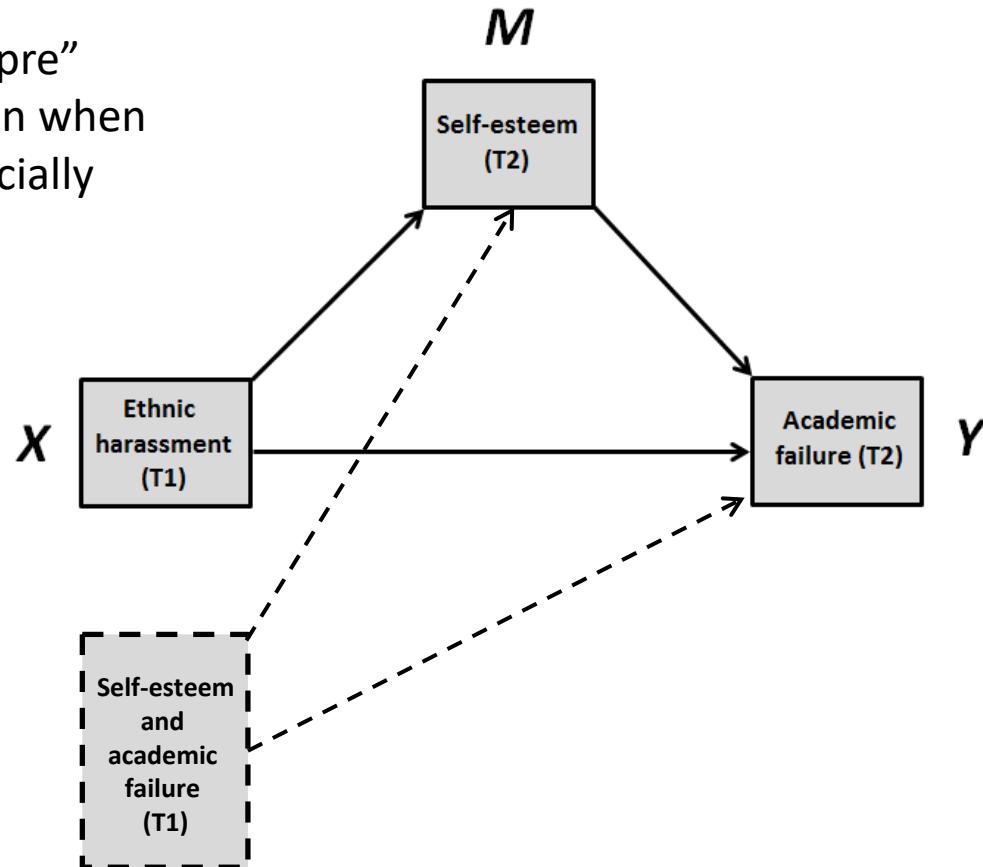
A simple mediation model, adjusting for a potential confounding variable (U)



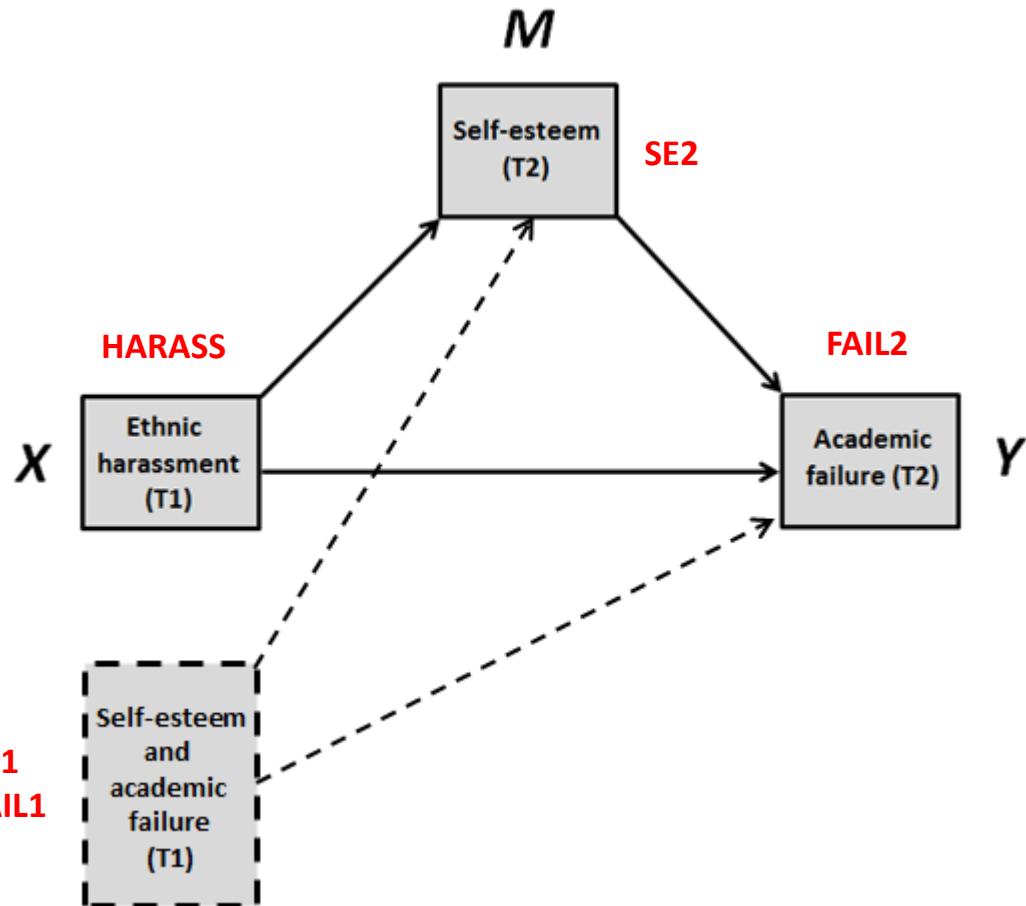
Some rationales for adjusting for prior state

When available, it is desirable to include “pre” measures of M and/or Y as covariates, even when X is experimentally manipulated, but especially when it is not.

- (a) Doing so can **increase precision** in the estimation of X ’s effect on M and/or Y if pre-measures are correlated with later measures (as they typically are).
- (b) Prior states often are correlated with X , M , or Y , introducing a **“self-selection threat”** to causal claims. Including prior state helps to reduce that threat.
- (c) It gives an interpretation to paths that are **closer to a “change” interpretation** without regression artifacts that can be introduced with the use of difference scores. In this example, the b path estimates the relationship between later self-esteem and how much higher or lower a student’s failure is given expected later failure from prior self-esteem and failure. Path a estimates the relationship between ethnic harassment frequency and how much lower or higher a student’s self-esteem is later relative to what would be expected given his or her earlier self-esteem and academic failure.

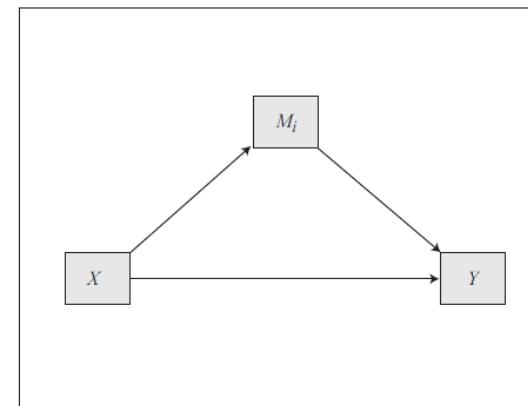


Adding covariates to a model using PROCESS

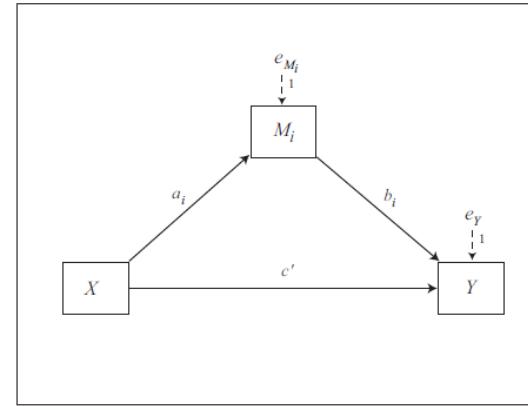


Model 4

Conceptual Diagram



Statistical Diagram



```
process cov=se1 fail1/y=fail2/x=harass/m=se2/model=4/
boot=10000/total=1.
```

```
%process (data=harass, cov=se1 fail1,y=fail2,x=harass,m=se2,
model=4,boot=10000,total=1);
```

PROCESS output

***** PROCESS Procedure for SPSS Version 3.00 *****

Written by Andrew F. Hayes, Ph.D. www.afhayes.com
Documentation available in Hayes (2018). www.guilford.com/p/hayes3

Model : 4
Y : fail2
X : harass
M : se2

Covariates:
sel fail1

Output D

Sample
Size: 330

OUTCOME VARIABLE:
se2

Model Summary

R	R-sq	MSE	F	df1	df2	p
.5450	.2971	.2224	45.9259	3.0000	326.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	2.0280	.2412	8.4095	.0000	1.5536	2.5024
harass	-.2728	.0717	-3.8025	.0002	-.4139	-.1317
sel	.4879	.0536	9.1081	.0000	.3825	.5933
fail1	-.1010	.0606	-1.6661	.0967	-.2202	.0182

path a

PROCESS output

Outcome: fail2

Output D

Model Summary

R	R-sq	MSE	F	df1	df2	p
.4059	.1648	.2105	16.0276	4.0000	325.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	2.0934	.2588	8.0900	.0000	1.5843	2.6025
se2	-.2175	.0539	-4.0375	.0001	-.3235	-.1115
harass	.0196	.0713	.2754	.7832	-.1207	.1599
sel	-.0672	.0584	-1.1517	.2503	-.1820	.0476
fail1	.2307	.0592	3.8966	.0001	.1142	.3471

path b
path c'

***** TOTAL EFFECT MODEL *****

Outcome: fail2

Model Summary

R	R-sq	MSE	F	df1	df2	p
.3505	.1229	.2203	15.2219	3.0000	326.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	1.6523	.2400	6.8842	.0000	1.1801	2.1245
harass	.0790	.0714	1.1060	.2695	-.0615	.2194
sel	-.1733	.0533	-3.2513	.0013	-.2782	-.0685
fail1	.2526	.0603	4.1886	.0000	.1340	.3713

path c

PROCESS output

***** TOTAL, DIRECT, AND INDIRECT EFFECTS *****

Output D

Total effect of X on Y

Effect	SE	t	p	LLCI	ULCI
.0790	.0714	1.1060	.2695	-.0615	.2194

path c

Direct effect of X on Y

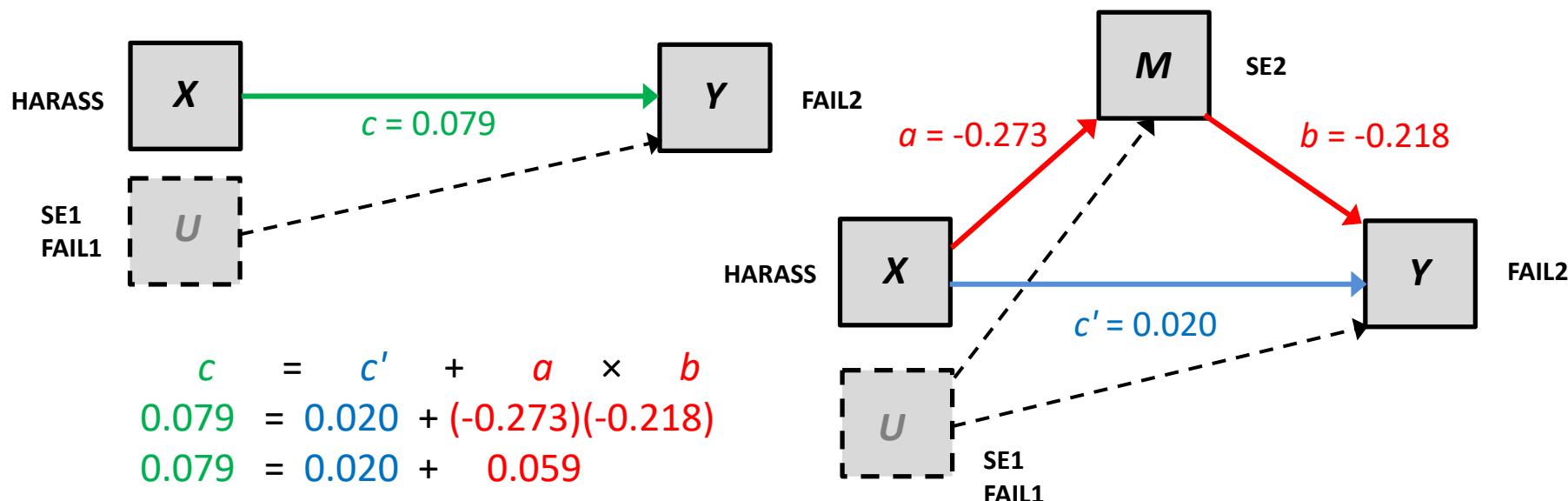
Effect	SE	t	p	LLCI	ULCI
.0196	.0713	.2754	.7832	-.1207	.1599

path c'

Indirect effect of X on Y

Effect	Boot SE	BootLLCI	BootULCI
se2	.0593	.0229	.0203

ab with 95% bootstrap
confidence interval



Discussion: Comparing Results

We ran the mediation analysis with the harass data twice. Once without covariates and once with covariates.

- What are some key differences between the results?
- Which analysis do you think best approximates the “truth”?
- If the results had not changed while including the covariates, what would that mean?

What about Baron & Kenny?

Also called the “causal steps” approach, it was popularized by Baron and Kenny (1986) as a test of mediation.

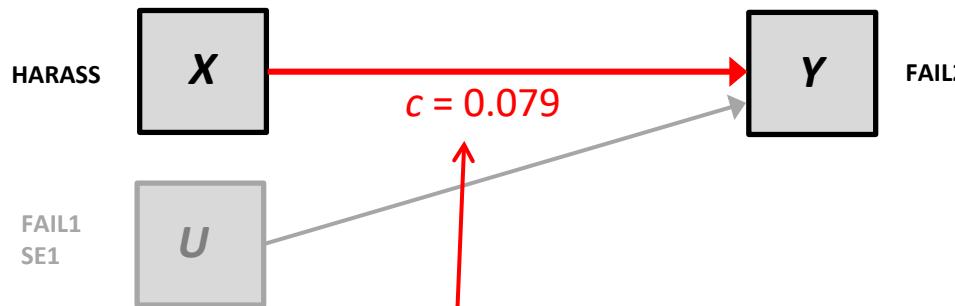
Conditions required to claim M functions as a mediator of the relationship between X and Y :

- (1) Does X affect Y ?
- (2) Does X affect M ?
- (3) Does M affect Y holding X constant ?
- (4) Is the direct effect of X closer to zero than the total effect?
 - (i) If direct effect is closer to zero than total effect but statistically different from zero, claim “partial mediation”
 - (ii) If direct effect is closer to zero than total effect and not statistically different from zero, claim “complete mediation”
 - (iii) Otherwise:

} ..as gauged by a hypothesis test.

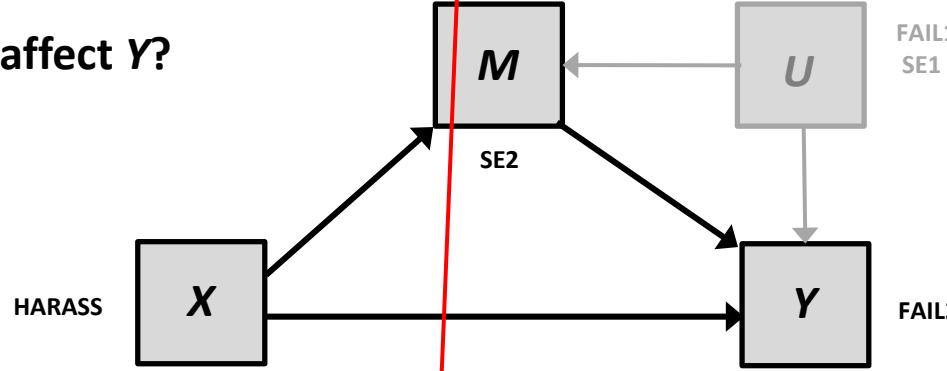


Using a set of OLS regression analyses



Condition 1: Does X affect Y ?

**NO. THUS, NO
MEDIATION BY
THE CAUSAL
STEPS STRATEGY**



Coefficients^a

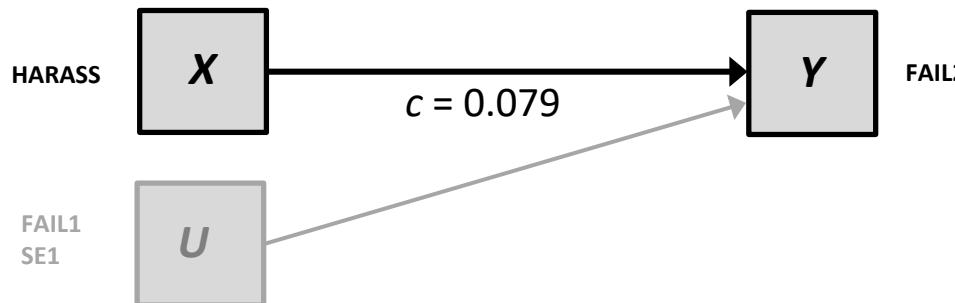
Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
1	(Constant)	1.652	.240		.000
	harass	.079	.071	.059	.270
	fail1	.253	.060	.231	.000
	se1	-.173	.053	-.179	.001

a. Dependent Variable: fail2

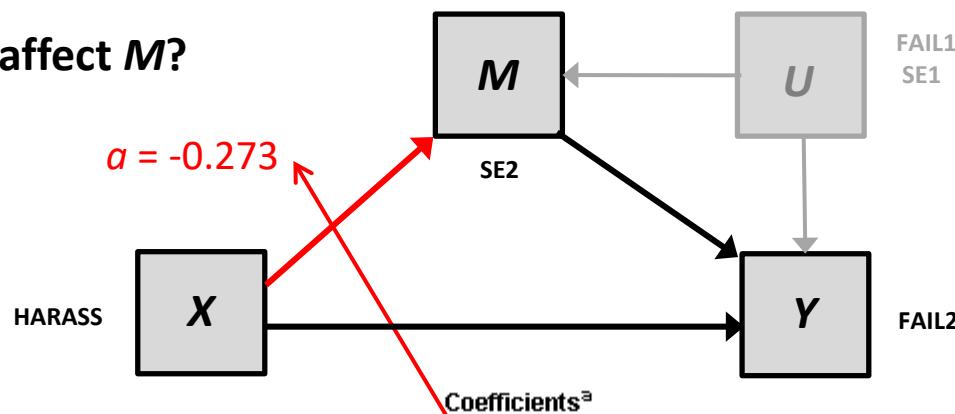
```
regression/dep=fail2/method=enter harass fail1 sel.
```

```
proc reg data=harass;model fail2=harass fail1 sel;run;
```

Using a set of OLS regression analyses



Condition 2: Does X affect M ?



Coefficients^a

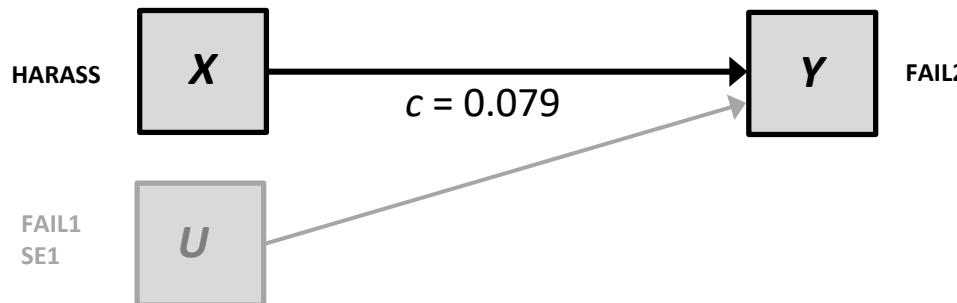
Model	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.
	B	Std. Error			
1	(Constant)	2.028	.241	8.410	.000
	harass	-.273	.072	-.181	-3.802 .000
	se1	.488	.054	.448	9.108 .000
	fail1	-.101	.061	-.082	-1.666 .097

a. Dependent Variable: se2

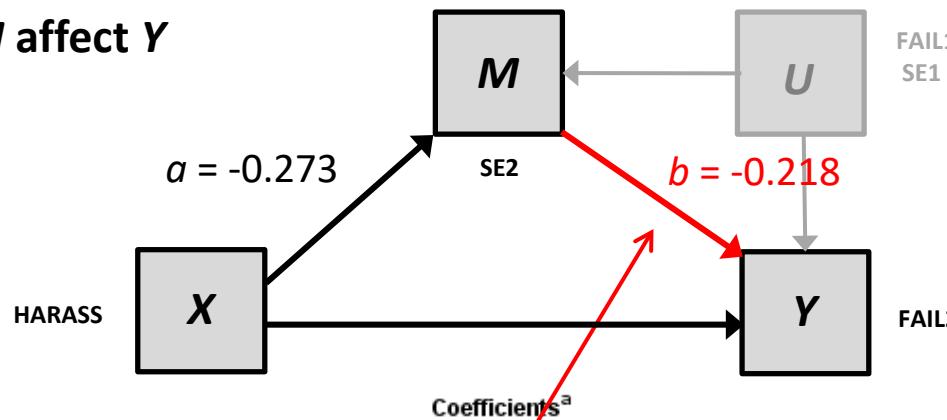
```
regression/dep=se2/method=enter harass se1 fail1.
```

```
proc reg data=harass;model se2=harass se1 fail1;run;
```

Using a set of OLS regression analyses



Condition 3: Does M affect Y holding X constant?



Model	Unstandardized Coefficients			Standardized Coefficients	t	Sig.
	B	Std. Error	Beta			
1	(Constant)	2.093	.259		8.090	.000
	harass	.020	.071	.015	.275	.783
	se2	-.218	.054	-.244	-4.038	.000
	fail1	.231	.059	.211	3.697	.000
	se1	-.067	.058	-.069	-1.152	.250

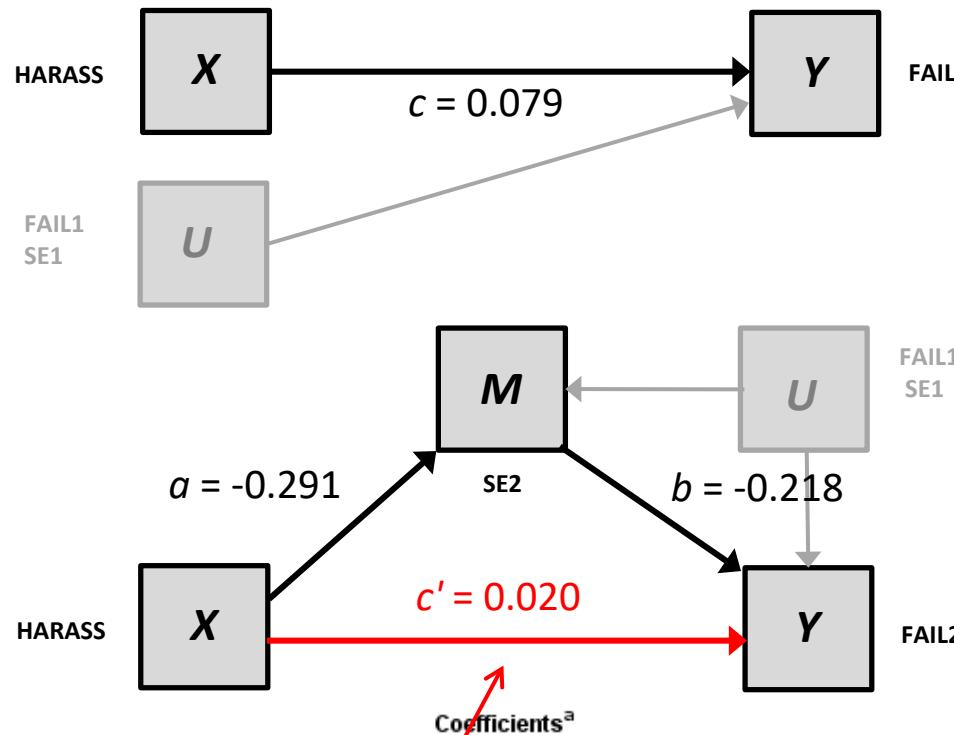
a. Dependent Variable: fail2

```
regression/dep=fail2/method=enter harass se2 fail1 se1.
```

```
proc reg data=harass;model fail2=harass se2 fail1 se1;run;
```

Using a set of OLS regression analyses

4. Qualitatively compare c to c'



Direct effect (c') is closer to zero than the total effect (c) and is not statistically different from zero. But can we really call this 'complete mediation' given that there was no total effect of X in the first place by commonly-used inferential rules?

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
1	(Constant)	2.093	.259	8.090	.000
	harass	.020	.071	.275	.783
	se2	-.218	.054	-.244	.000
	fail1	.231	.059	.211	.000
	se1	-.067	.058	-.069	.250

a. Dependent Variable: fail2

```
regression/dep=fail2/method=enter harass se2 fail1 sel.
```

```
proc reg data=harass;model fail2=harass se2 fail1 sel;run;
```

Problems with the causal steps approach

❑ Indirect effect is logically inferred rather than directly estimated

But typically, we make inferences from data using estimates of quantities pertinent to the question. Why should inferences about indirect effects be any different?

A fallacious rebuttal: if a and b are both different from zero (as established by rejection of the null hypothesis) so too must their product, so no estimate or test of indirect effect is needed.

- a) Although this is true at the population level, it isn't necessarily true at the sample level.
- b) An indirect effect may be different from zero even in the absence of evidence that both paths a and b are.

❑ If data fail to meet a single criterion, **game over--no indirect effect through M .**

The use of multiple, fallible hypothesis tests gives this approach the **lowest power** among competing methods for testing intervening variable effects. **Tests or claims of mediation should not be based on the significance of individual paths in the model.**

What about Baron & Kenny?

Consider:

$$X + Y = Z$$

If $Z > 0$ what does this tell us about X?
What does this tell us about Y?

Problems with the causal steps approach

- If total effect (path c) is not detectably different from zero, the **game doesn't even begin.**

This is logically sensible if you accept one definition of a mediator variable – a variable that is causally between X and Y and that **accounts for their association**.

- By this definition, an effect that does not exist can't be mediated. But the significance of c neither constrains nor determines the size of the product of paths a and b , nor does it tell us whether that product is different from zero.
- Kenny and Judd (2014, *Psychological Science*) illustrate that a hypothesis test about the total effect is generally less powerful than a hypothesis test about the indirect effect.

- Because the indirect effect is not quantified, this method does not lend itself well to comparisons between indirect effects in multiple mediators models, or to modeling of the size of indirect effects ('conditional process analysis')

“Complete”/“full” and “partial” mediation

The causal steps strategy is often used as a means of labeling a process as “complete” or “partial mediation”. There is little value to this semantic labeling exercise.

- ❑ What if there is no evidence of a total effect (i.e., c non-significant)? This can happen, and actually does more often than people probably realize. Thus, these concepts don't have a place much of the time.
- ❑ The reliance on statistical significance criteria means that when power is high for the test on c' partial mediation is the best you can hope for, and when power is relatively low, complete mediation is more likely. So if establishing complete mediation is your goal, you should intentionally limit the size of your sample to as small as necessary
- ❑ Establishing complete mediation by your favored mediator does not preclude others from being able to make the same claim with their own favored mediator.
- ❑ “Direct effects” don't exist in reality. All effects are mediated by something. Thus, a claim of ‘partial mediation’ is a claim that one has not specified the model correctly.

Experts in mediation analysis have abandoned these concepts, and so should you. They are of historical interest only these days.

Mediation analysis summary thus far

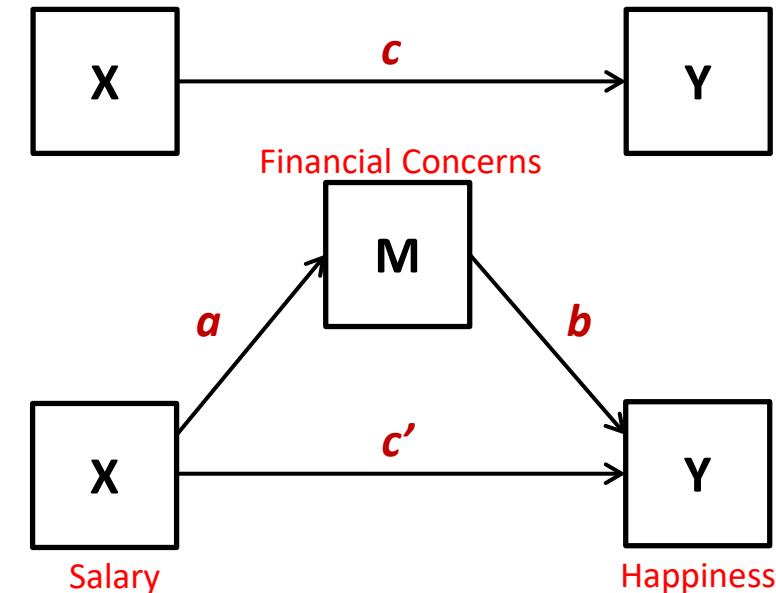
- Mediators are variables which are causally located between two variables X and Y and that explain, in part, the effect of X on Y . X affects M which in turn affects Y .
- The causal steps strategy popularized by Baron and Kenny (1986) remains a popular method for mediation analysis.
 - Yet it is among the lowest in power, in some circumstances, massively so.
 - It is not consistent with modern thinking about mediation analysis.
 - Its use is not recommended. Soon you won't be able to get away with it.
- Tests of mediation should be based on an estimate of the indirect effect.
 - Sobel test for inference in large samples only, but we don't know how large is large enough.
 - Bootstrap or Monte Carlo confidence intervals in a sample of any size.
- There is no need to condition the hunt for an indirect effect on a statistically significant total effect (path c).
- Focus interpretation on the size and sign of the indirect effect. Tests of significance for the individual paths ($X \rightarrow M$ and $M \rightarrow Y$) are useful as supplemental information but need not be part of the story.



Path Analysis: Exercise Example

Suppose the true state of the world is such, and salary is measured in thousands of dollars per year (i.e. a one unit increase in salary corresponds to a \$1000 increase in salary/year): An increase in salary of **\$2,000/year** is associated with an overall increase in happiness of **3**.

Suppose also that an increase in salary of **\$1,000/year** is associated with a decrease in financial concerns by **2**. It is known that increasing financial concerns by **1** decreases happiness by **.5** when controlling for salary.



$$Y_i = i_{Y*} + cX_i + e_{Y_i}$$

$$M_i = i_M + aX_i + e_{M_i}$$

$$Y_i = i_Y + c'X_i + bM_i + e_{Y_i}$$

Direct effect of X on Y (not through M) = ***c'***

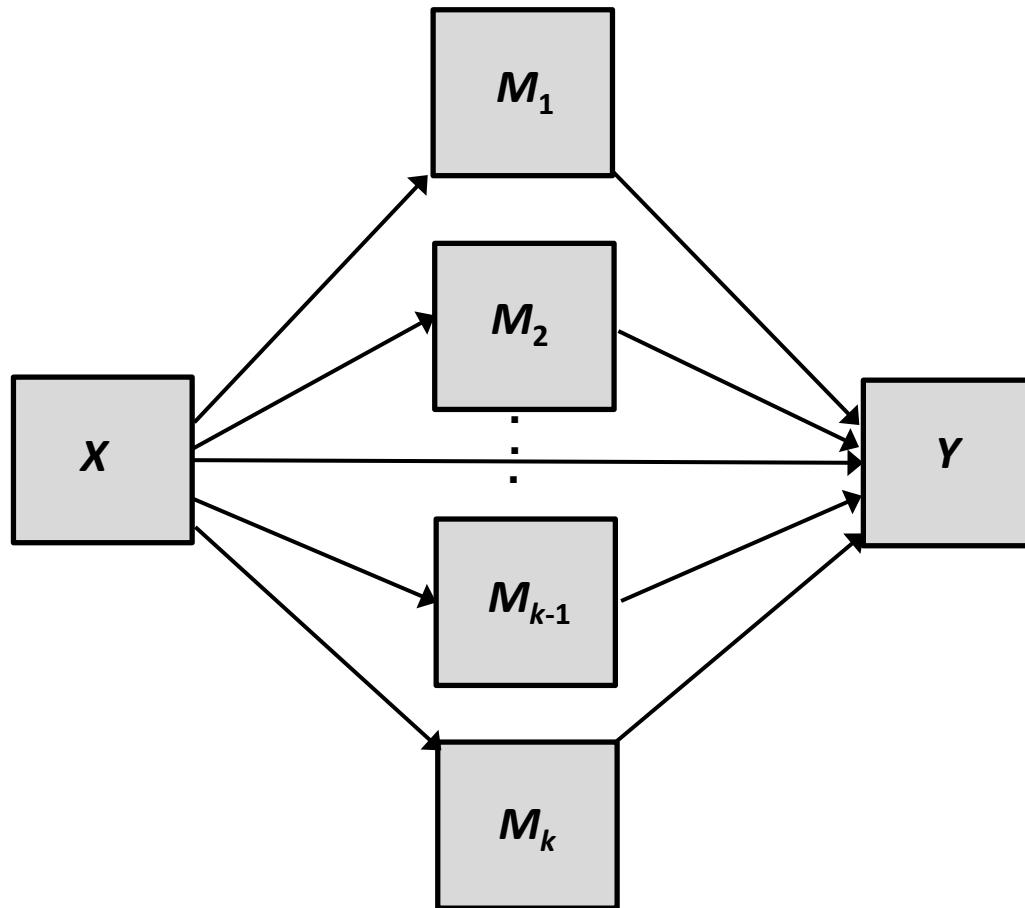
Indirect effect of X on Y (through M) = ***a × b***

Total effect = direct effect + indirect effect

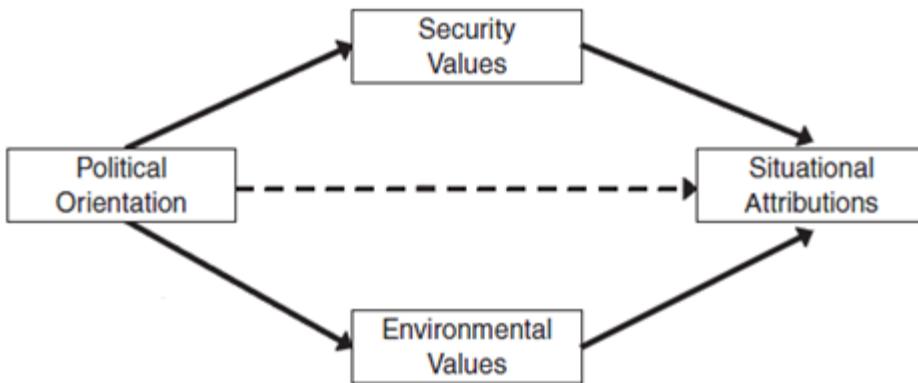
$$\begin{aligned} \text{Indirect effect} &= \text{Total effect} - \text{direct effect} \\ \text{Indirect effect} &= c' + a \times b \\ a \times b &= c - c' \end{aligned}$$

Models with More Than One Mediator

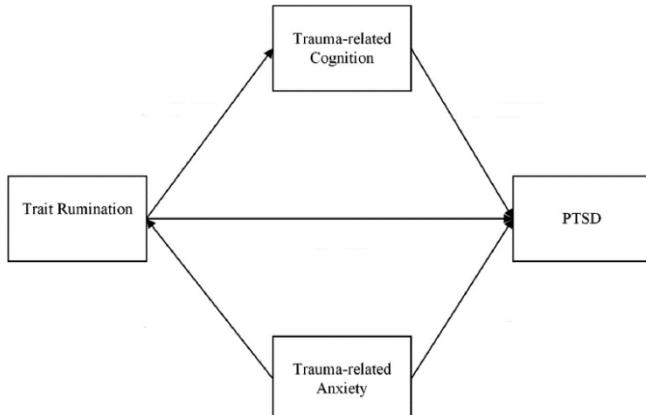
A parallel multiple mediator model



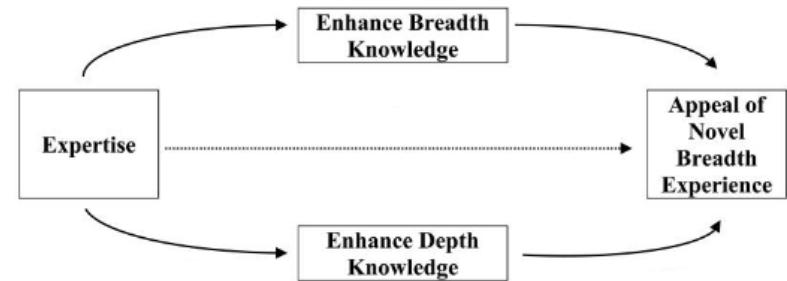
Some examples From the literature with 2 mediators



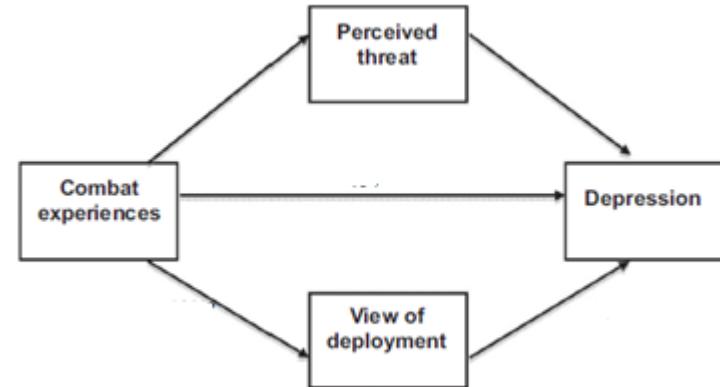
Morgan, G. S., Mullen, E., & Skitka, L. J. (2010). When values and attributions collide: Liberals' and conservatives' values motivate attributions for alleged misdeeds. *Personality and Social Psychology Bulletin*, 36, 1241-1254.



Spinhoven, P., Penninx, B. W., Krempeniou, A., et al. (2015). Trait rumination predicts onset of post-traumatic stress disorder through trauma-related cognitive appraisals: A 4-year longitudinal study. *Behaviour Research and Therapy*, 71, 101-109.

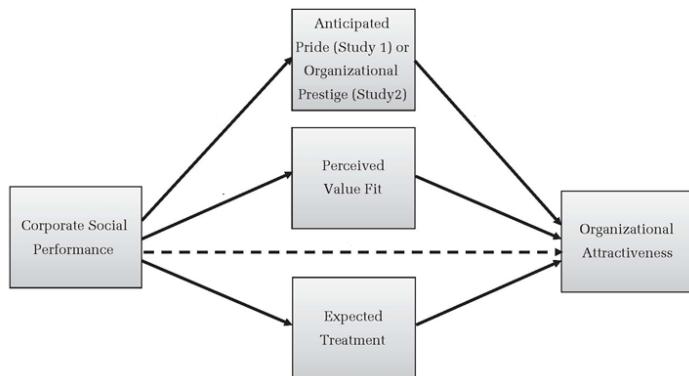


Clarkson, J. J., Janiszewski, C., & Cinelli, M. D. (2013). The desire for consumption knowledge. *Journal of Consumer Research*, 39, 1313-1329.

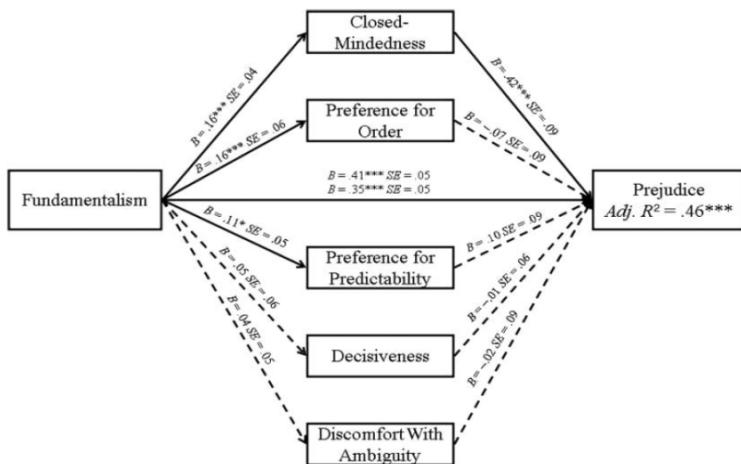


Pitts, B. L., & Safer, M. A. (2016). Retrospective appraisals mediate the effect of combat experiences on PTS and depression symptoms in U.S. Army medics. *Journal of Traumatic Stress*, 29, 65-71.

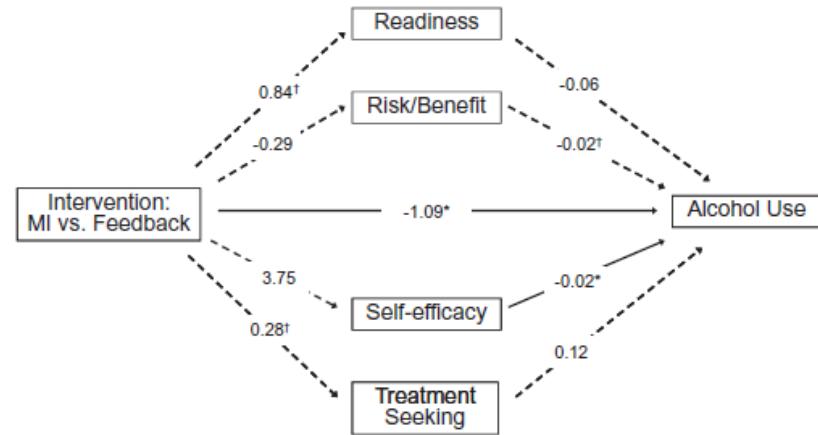
Some examples from the literature with several mediators



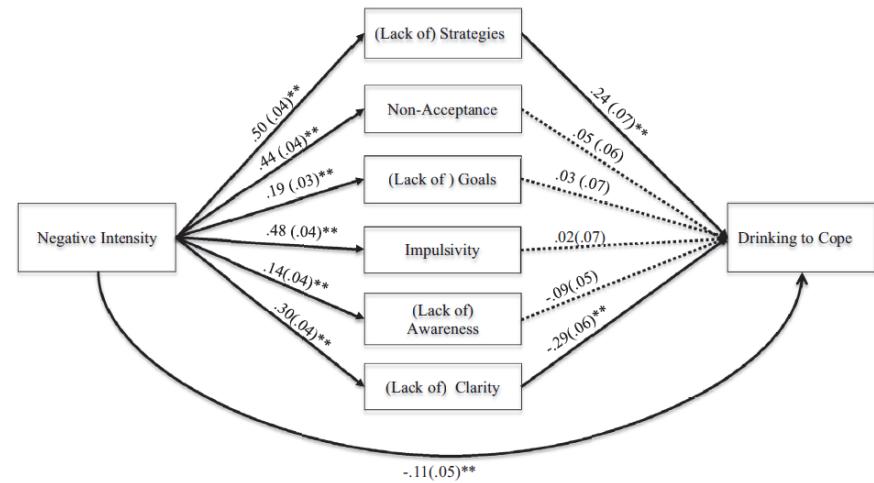
Jones, D. A., Willness, C. R., & Madey, S. (2014). Why are job seekers attracted by corporate social performance? Experimental and field tests of three signal-based mechanisms. *Academy of Management Journal*, 57, 383-404.



Brandt, M. J., & Reyna, C. (2010). The role of prejudice and the need for closure in religious fundamentalism. *Personality and Social Psychology Bulletin*, 36, 715-725.



Barnett, N. P., Apodaca, T. R., et al. (2010). Moderators and mediators of two brief interventions for alcohol in the emergency department. *Addiction*, 105, 452-465.



Veilleux, J. C., Skinner, K. D., Reese, E. D., & Shaver, J. A. (2014). Negative affect intensity influences drinking to cope through facets of emotion dysregulation. *Personality and Individual Differences*, 49, 96-101.

Why estimate such a model?

- Many causal effects probably operate through multiple mechanisms simultaneously. Better to estimate a model **consistent with such real-world complexities**.
- If your proposed mediator is correlated with the real mediator but not caused by the independent variable, a model with only your proposed mediator in it will be a **misspecification** and will potentially misattribute the process to your proposed mediator rather than the real mediator—“epiphenomenality.”
- Different theories may postulate different mediators as mechanisms. Including them all in a model simultaneously allows for a formal statistical comparison of indirect effects **representing different theoretical mechanisms**.

Path Analysis: Total, Direct, and Indirect Effects

$$\hat{Y} = c_0 + cX$$

$$\widehat{M}_j = a_{0j} + a_j X$$

$$\hat{Y} = c'_0 + c'X + \sum_{j=1}^K b_j M_j$$

c = “total effect” of X on Y

$a_j \times b_j$ = “specific indirect effect” of X on Y through M_i

$\sum (a_j \times b_j)$ = “total indirect effect” of X on Y

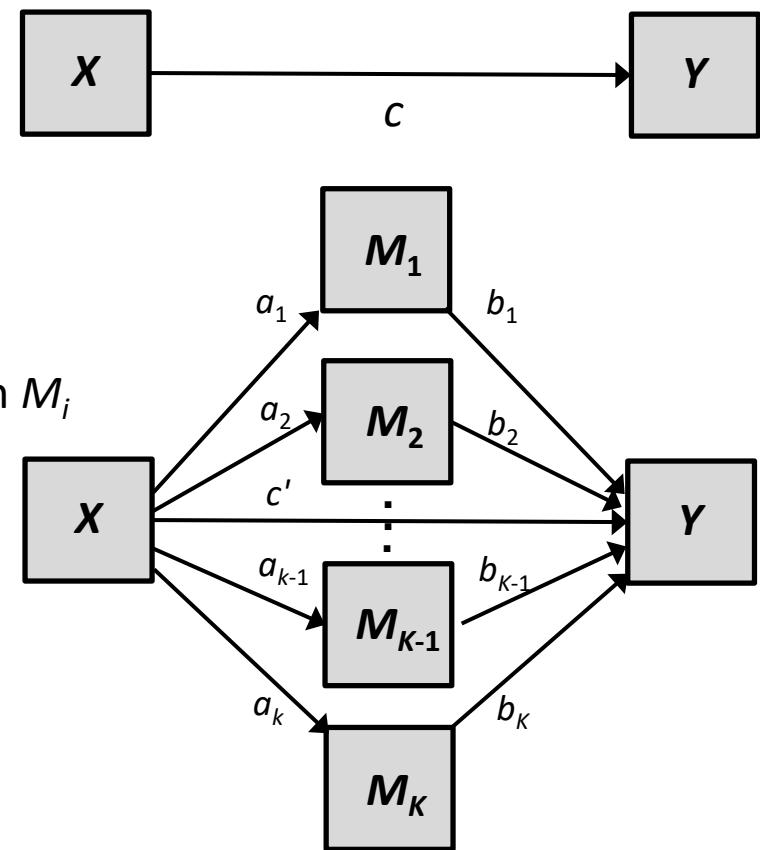
c' = “direct effect” of X on Y

total effect = direct effect + total indirect effect

$$c = c' + \sum (a_j \times b_j)$$

total indirect effect = total effect – direct effect

$$\sum (a_j \times b_j) = c - c'$$



Example: Science

Participants read a syllabus for a computer science class. The syllabus one of two policies: **procollaboration or no collaboration.**

Participants were randomly assigned to condition.

Participants completed questionnaire (Higher = greater):

- (1) **interest in the class** (this is the primary DV).
- (2) how much they felt the class would help them in achieving **communal goals** (helping others, working with others)
- (3) how **difficult** they expected the class to be.



Question: Does group work in computer science classes increase interest in the class indirectly through perceived communal goal fulfillment, through class difficulty, or both?

Would people who read about the procollaboration policy think the class is more communal and would that communality then predict greater interest? Would the procollaboration policy make students think the course is easier, and this would increase interest?

The data: Science

ProNo	comm	diff	interest
1.00	5.20	5	6.0
1.00	1.00	4	2.2
1.00	4.00	4	2.5
1.00	4.00	2	3.5
1.00	7.00	7	7.0
1.00	6.00	1	6.0
1.00	4.00	7	2.7
1.00	3.40	6	4.2
1.00	4.20	3	3.5
1.00	4.60	6	1.5
1.00	4.20	5	2.0
1.00	4.40	5	1.0
1.00	5.40	3	7.0
1.00	5.00	4	2.2
1.00	4.60	6	2.7
1.00	3.40	4	1.2
1.00	3.00	5	2.7

data science;

```
input Subject Cond sex ProNo comm diff interest;
```

datalines;

106	1	1	1	5.2	5	6
109	1	1	1	1	4	2.25
112	1	1	1	4	4	2.5
114	1	1	1	4	2	3.5
115	1	1	1	7	7	7
121	1	1	1	6	1	6
131	1	1	1	4	7	2.75
132	1	1	1	3.4	6	4.25
148	1	1	1	4.2	3	3.5
161	1	1	1	4.6	6	1.5
162	1	1	1	4.2	5	2
164	1	1	1	4.4	5	1
174	1	1	1	5.4	3	7
176	1	1	1	5	4	2.25
177	1	1	1	4.6	6	2.75
178	1	1	1	3.4	4	1.25
190	1	1	1	3.8	5	2.75
206	1	1	1	4.2	6	1.75
216	1	1	1	7	7	6
217	1	1	1	5	4	4.75

ProNo: Experimental condition (1 = procollaboration, 0 = no collaboration)

interest : interest in class (higher = greater interest)

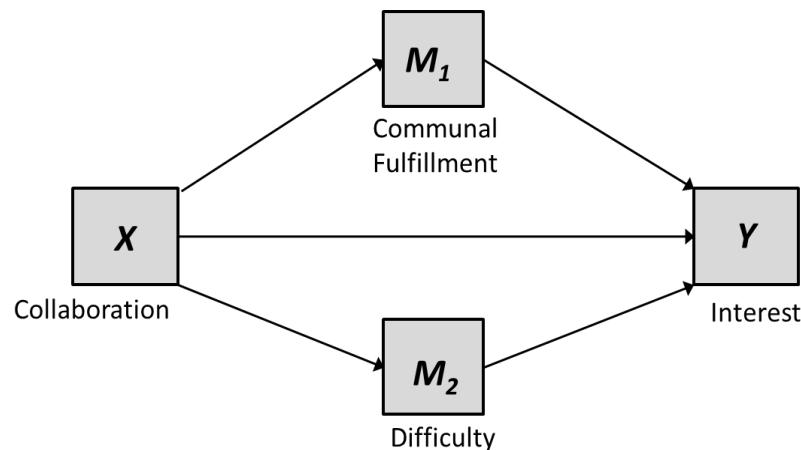
comm: Perceived fulfillment of communal goals (higher = more fulfillment)

diff: Perceived difficulty of the class (higher = more difficult)

gender: 1 = Male, 2 = Female

Estimation and inference using PROCESS

PROCESS model 4 is used for the parallel multiple mediator model.



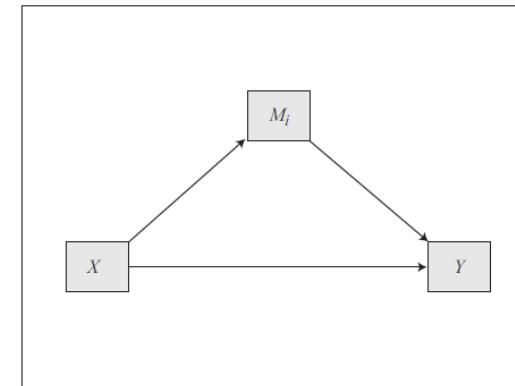
Up to 10 mediators can be listed in the “m =” list. Order does not matter.

```
process y=interest/x=ProNo/m=comm diff/total=1/boot=10000/model=4/normal=1/contrast=1.
```

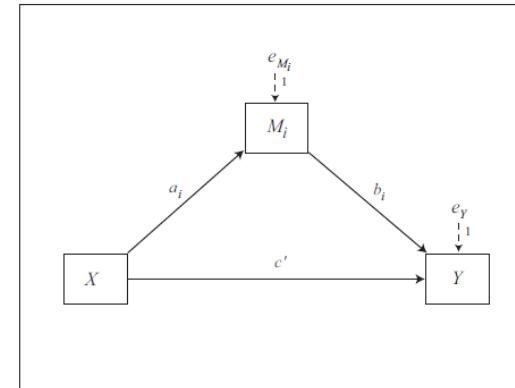
```
%process (data=science,y=interest,x=ProNo,m=comm diff,total=1,boot=10000,model=4,  
normal=1,contrast=1);
```

Model 4

Conceptual Diagram



Statistical Diagram



PROCESS output

Model : 4
 Y : interest
 X : ProNo
 M1 : comm
 M2 : diff
 Sample Size: 232

OUTCOME VARIABLE:

comm

$$\widehat{M}_1 = 3.12 + 0.78X$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.3031	.0919	1.5279	23.2670	1.0000	230.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	3.1160	.1133	27.4994	.0000	2.8927	3.3392
ProNo	.7831	.1624	4.8236	.0000	.4632	1.1030

← a_1 path

OUTCOME VARIABLE:

diff

$$\widehat{M}_2 = 4.94 - 0.15X$$

Model Summary

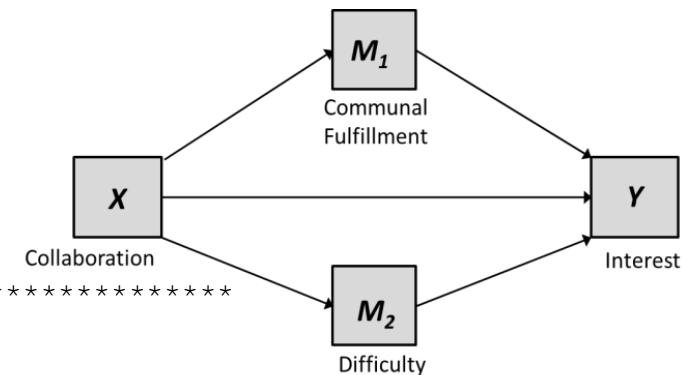
R	R-sq	MSE	F	df1	df2	p
.0594	.0035	1.6760	.8155	1.0000	230.0000	.3674

Model

	coeff	se	t	p	LLCI	ULCI
constant	4.9412	.1187	41.6352	.0000	4.7073	5.1750
ProNo	-.1536	.1700	-.9031	.3674	-.4886	.1815

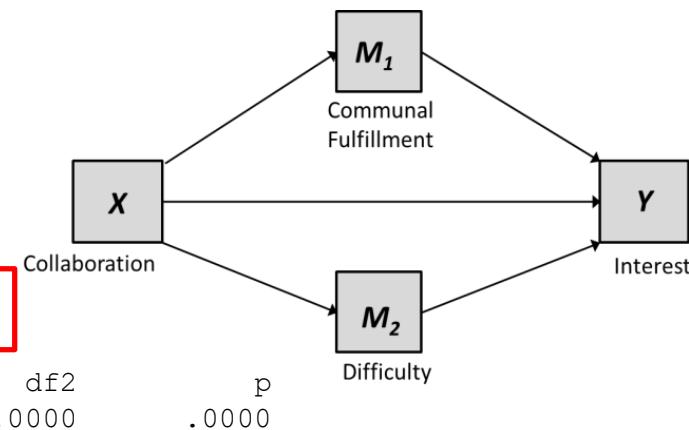
← a_2 path

Output E



PROCESS output

Output E



OUTCOME VARIABLE:

interest

$$\hat{Y} = 0.49 - 0.09X + 0.54M_1 + 0.14M_2$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.4418	.1952	1.9659	18.4348	3.0000	228.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	.4898	.4618	1.0607	.2899	-.4201	1.3996
ProNo	-.0895	.1933	-.4630	.6438	-.4705	.2914
comm	.5367	.0752	7.1418	.0000	.3886	.6848
diff	.1364	.0718	1.9008	.0586	-.0050	.2778

***** TOTAL EFFECT MODEL *****

OUTCOME VARIABLE:

interest

$$\hat{Y} = 2.84 + 0.31X$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.1000	.0100	2.3974	2.3217	1.0000	230.0000	.1290

Model

	coeff	se	t	p	LLCI	ULCI
constant	2.8361	.1419	19.9817	.0000	2.5565	3.1158
ProNo	.3099	.2034	1.5237	.1290	-.0908	.7106

c' path
b₁ path
b₂ path

c path

PROCESS output

Output E

***** TOTAL, DIRECT, AND INDIRECT EFFECTS OF X ON Y *****

Total effect of X on Y

Effect	se	t	p	LLCI	ULCI
.3099	.2034	1.5237	.1290	-.0908	.7106

c path

Direct effect of X on Y

Effect	se	t	p	LLCI	ULCI
-.0895	.1933	-.4630	.6438	-.4705	.2914

c' path

Indirect effect(s) of X on Y:

	Effect	BootSE	BootLLCI	BootULCI
TOTAL	.3994	.1135	.1950	.6399
comm	.4203	.1128	.2171	.6594
diff	-.0209	.0282	-.0858	.0280
(C1)	.4413	.1189	.2251	.6927

$a_1 b_1 + a_2 b_2$ with bootstrap CI

$a_1 b_1$ with bootstrap CI

$a_2 b_2$ with bootstrap CI

Normal theory test for indirect effect(s):

	Effect	se	Z	p
comm	.4203	.1059	3.9706	.0001
diff	-.0209	.0284	-.7367	.4613

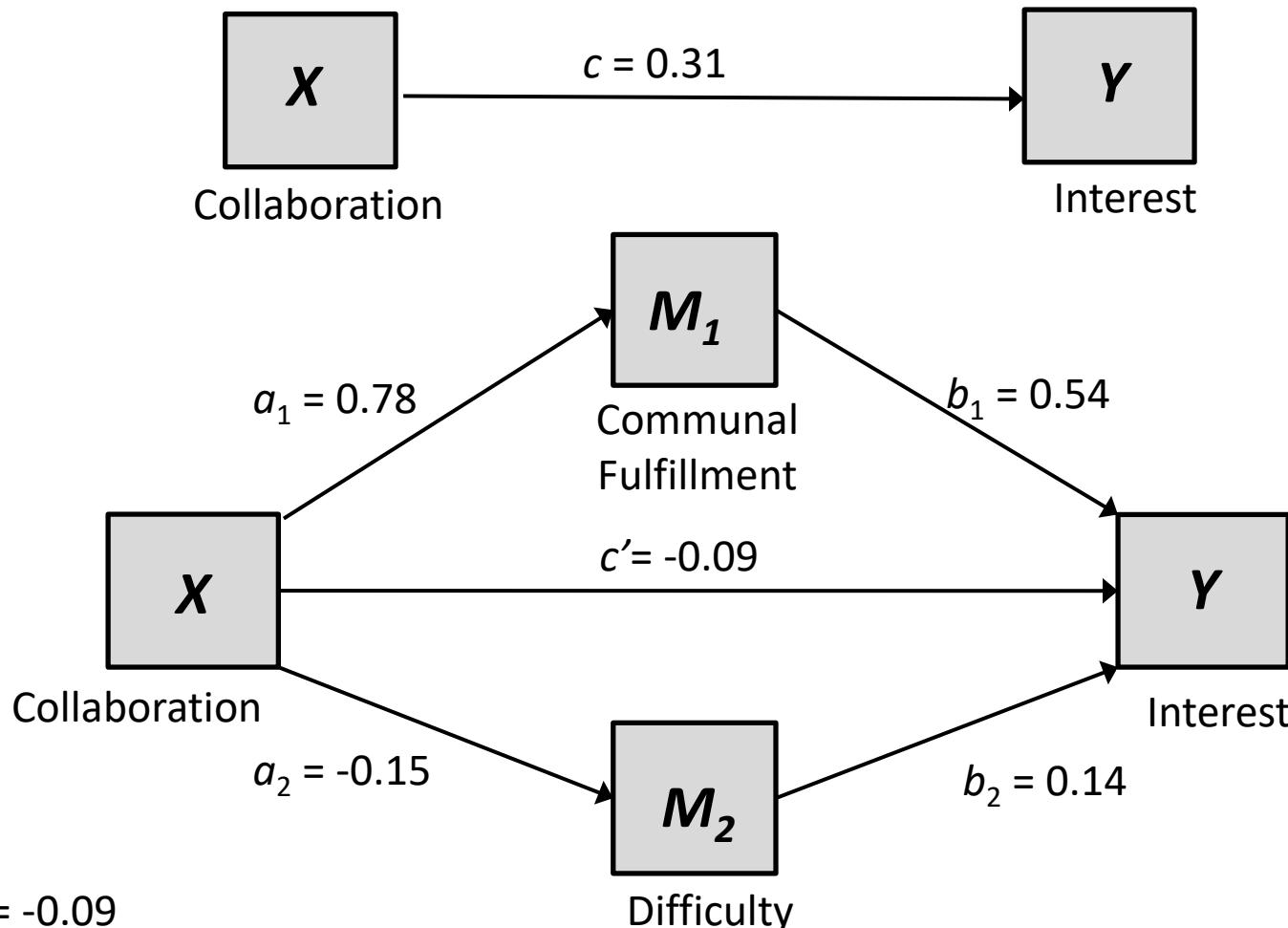
Sobel tests (less trustworthy than bootstrap CIs)

Specific indirect effect contrast definition(s):

(C1) comm minus diff

The data are consistent with the claim that group work influences interest indirectly through communal goal fulfillment controlling for difficulty (0.420; 95% CI = 0.217 to 0.659) but not through difficulty controlling for goal fulfillment (-0.0209; 95% CI = -0.086 to 0.028).

Example: Science



Direct effect = -0.09

Specific indirect effect via Comm: $0.78(0.54) = 0.42$

Specific indirect effect via Diff: $-0.15(0.14) = -0.02$

Total indirect effect = $0.42 - 0.02 = 0.40$

Total effect = $-0.09 + .42 - 0.02 = 0.31$

Things to consider

- (1) In a multiple mediator model, the specific indirect effect through M_k quantifies the component of the total indirect effect that is unique to M_k . Each specific indirect effect is estimated **controlling for all other mediators**.

M_k may function as a mediator variable when considered in isolation but not when considered with other mediator variables in the same model. If the intervening variables are highly intercorrelated, they can “cancel out” each others’ effects.

- (2) It is possible for a total indirect effect to be not detectably different from zero even when one or more specific indirect effects is.

total indirect effect = sum of specific indirect effects

$$\Sigma (a_i b_i) = a_1 b_1 + a_2 b_2 + \dots a_k b_k.$$

Scenario (a): A single large specific indirect combined with several tiny ones.

Scenario (b): Specific indirect effects that have different signs and add to near zero.

In multiple mediator models, the total indirect effect is not always of great interest.

Comparing specific indirect effects

Indirect effects quantify how Y changes as X changes by one unit through a mediator. **They are free of the scale of measurement of the mediators.** So in multiple mediator models, indirect effects linking the same X to the same Y are directly comparable even if the mediators are measured on different scales. We can statistically compare them if so desired. No standardization or other arithmetic gymnastics is required.

Approach #1: Calculate the ratio of the difference between the indirect effect through M_i and the indirect effect through M_j to its standard error. Assuming a normally distributed sampling distribution of the difference, a p -value for the null hypothesis that the difference equals zero can be derived from the standard normal distribution.

$$Z = \frac{a_i b_i - a_j b_j}{se_{a_i b_i - a_j b_j}}$$

Approach #2: Bootstrap a confidence interval for the $a_i b_i - a_j b_j$ and ascertain whether 0 is in the confidence interval as a pseudo null hypothesis test that the difference is zero.

PROCESS can generate a bootstrap confidence interval for all possible pairwise comparisons between specific indirect effects

PROCESS output

```
process y=interest/x=prono/m=comm diff/total=1/boot=10000/model=4/normal=1/contrast=1.
```

```
%process (data=science,y=interest,x=prono,m=comm diff, total=1,boot=10000,model=4,  
normal=1,contrast=1);
```

Indirect effect(s) of X on Y:

	Effect	BootSE	BootLLCI	BootULCI
TOTAL	.3994	.1135	.1950	.6399
comm	.4203	.1128	.2171	.6594
diff	-.0209	.0282	-.0858	.0280
(C1)	.4413	.1189	.2251	.6927

$$a_1b_1 - a_2b_2$$

Normal theory test for indirect effect(s) :

	Effect	se	Z	p
comm	.4203	.1059	3.9706	.0001
diff	-.0209	.0284	-.7367	.4613

Specific indirect effect contrast definition(s) :

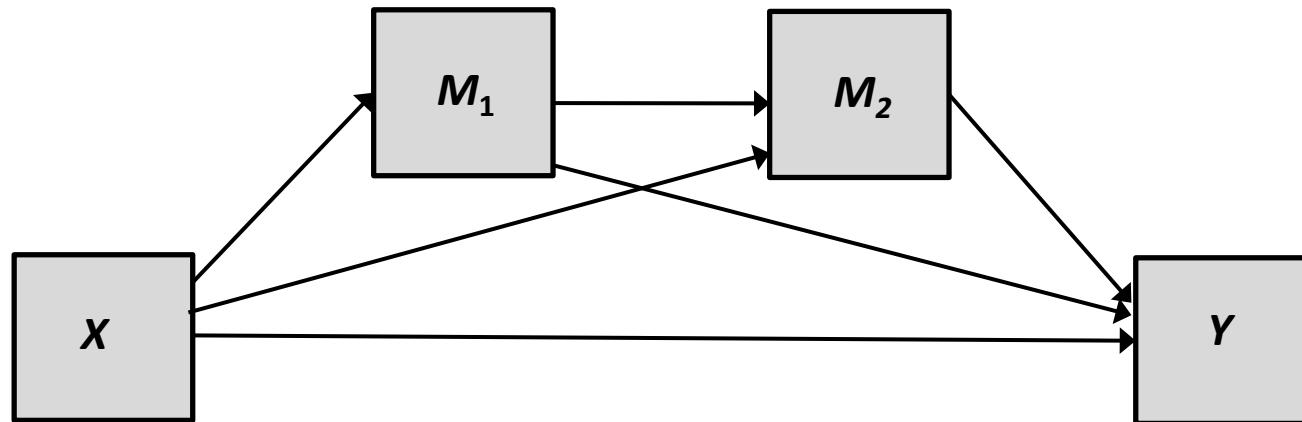
(C1) comm minus diff

Output E

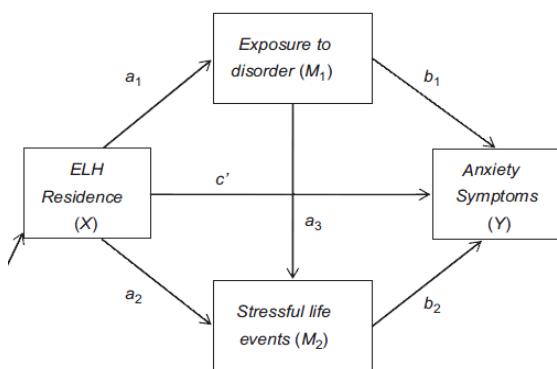
The specific indirect effect of collaboration on interest through perceived Communal goal fulfillment is different from the specific indirect effect through perceived class difficulty (difference = 0.441; 95% CI = 0.225 to .6927).

The serial multiple mediator model

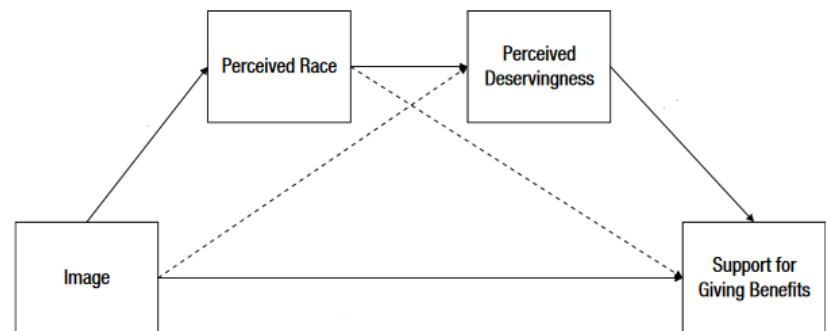
A serial multiple mediator model with two mediators and all possible direct and indirect effects freely estimated.



Some examples in the literature:

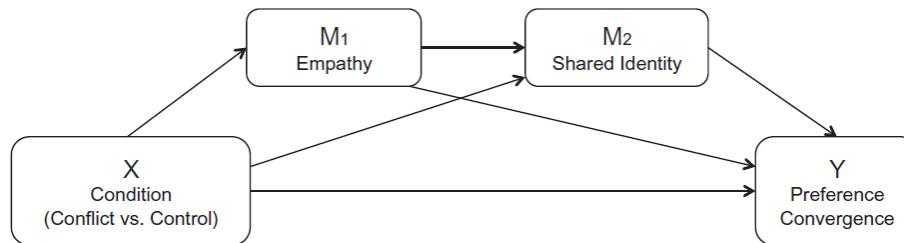


Casciano, R., & Massey, D. S. (2012). Neighborhood disorder and anxiety symptoms: New evidence from a quasiexperimental study. *Health and Place*, 18, 180-190.

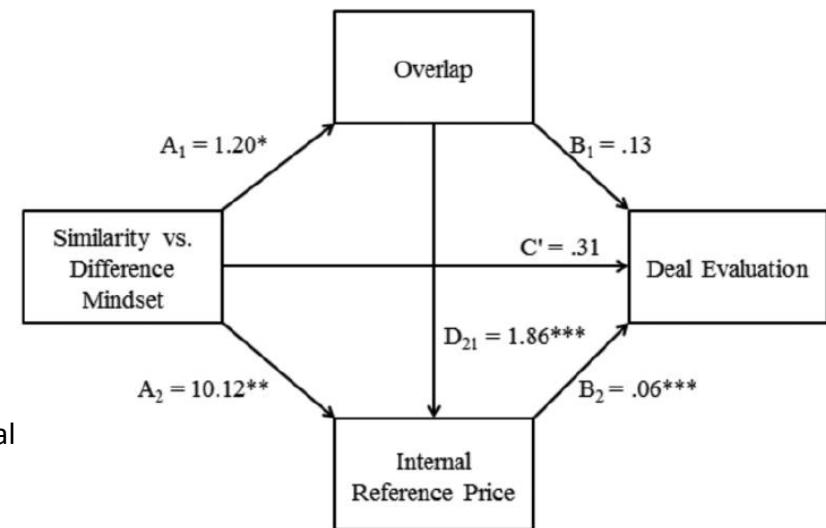


Brown-Iannuzzi, J. L., Dotsch, R., Cooley, E., & Payne, B. K. (2017). The relationship between mental representations of welfare recipients and attitudes toward welfare. *Psychological Science*, 28, 92-103.

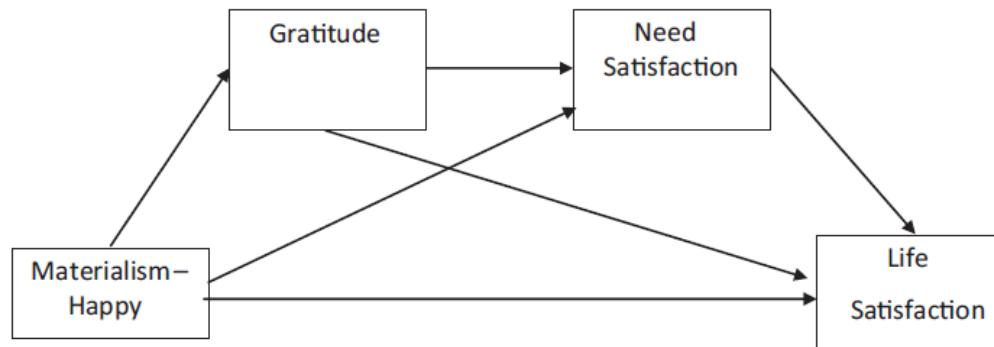
More examples from the literature



Schrift, R. Y., & Moty, A. (2015). Pain and preferences: Observed decisional conflict and the convergence of preferences. *Journal of Consumer Research*, 42, 515-534..

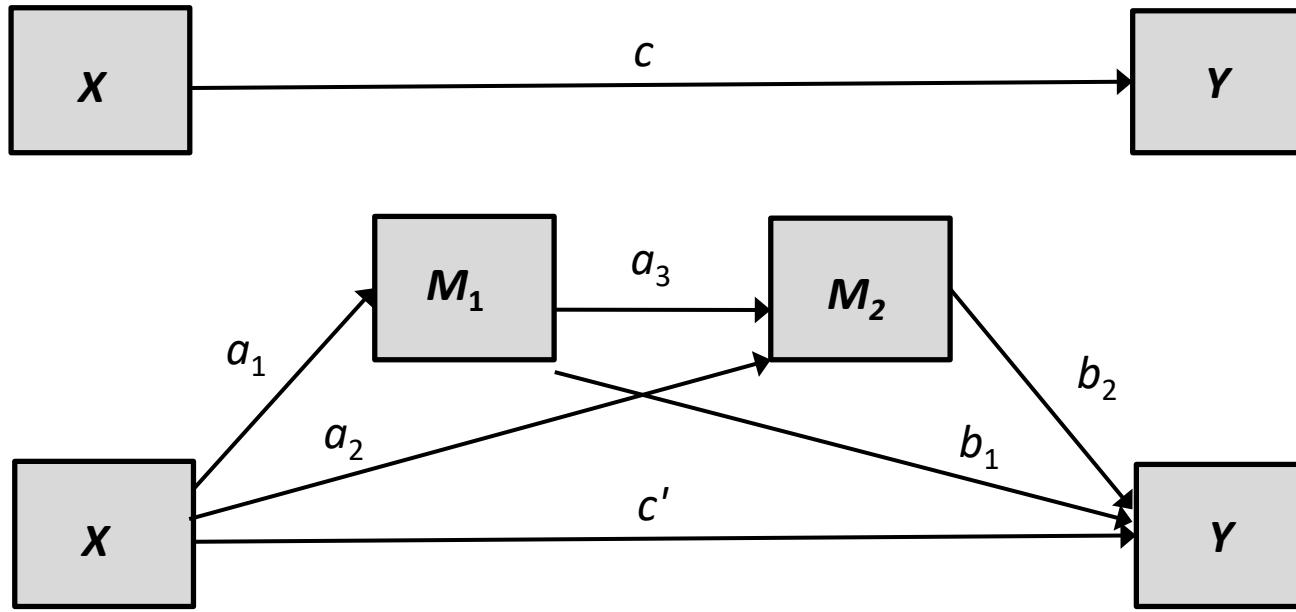


Kan, C., Lichtenstein, D. R., Grant, S. J., & Janiszewski, C. (2014). Strengthening the influence of advertised reference prices through information priming. *Journal of Consumer Research*, 40, 1078-1096.



Tsang, J-A., Carpenter, T. P., Roberts, J. A., Frisch, M. B., & Carlisle, R. D. (2014). Why are materialists less happy? The role of gratitude and need satisfaction in the relationship between materialism and life satisfaction. *Personality and Individual Differences*, 64, 62-66.

Serial mediation: Path analysis rules



The total effect of X on Y is equal to the direct effect of X plus the sum of all specific indirect effects (there are three of them here).

$$\hat{Y} = c_0 + cX$$

$$\hat{M}_1 = a_{01} + a_1 X$$

$$\hat{M}_2 = a_{02} + a_2 X + a_3 M_1$$

$$\hat{Y} = c'_0 + c'X + b_1 M_1 + b_2 M_2$$

Direct effect of X : c'

Specific indirect effect of X through M_1 : $a_1 b_1$

Specific indirect effect of X through M_2 : $a_2 b_2$

Specific indirect effect of X through M_1 and M_2 : $a_1 a_3 b_2$

Total indirect effect of X : $a_1 b_1 + a_2 b_2 + a_1 a_3 b_2$

Total effect of X : $c = c' + a_1 b_1 + a_2 b_2 + a_1 a_3 b_2$

Example

May 1st, 2011, 11:30PM



Professor Erik Nisbet (OSU School of Communication) had an national telephone survey in the field examining perceptions of Muslims in the U.S. when Obama announces the death of bin Laden; 390 respondents prior to announcement (**BINLADEN = 0**) and 271 after announcement (**BINLADEN = 1**). See report in materials provided for details.

Measures

STEREO: Stereotype endorsement, 4 items (5-pt semantic differential)

“Please tell us how much you associate each of the following sets of characteristics with Muslims”

e.g., Peaceful – Violent
Tolerant – Fanatical

RTHREAT: Realistic threat, 5 items (5-pt Likert)

“Below are a few statements expressing different views about Muslims living in the U.S. Please read and tell us how much you agree with each statement”

e.g., “Muslims in the U.S. sympathize with terrorists”
“Muslims make America a more dangerous place to live”

MCIVIL: Restriction of Muslim civil liberties, 5 items (5-pt Likert)

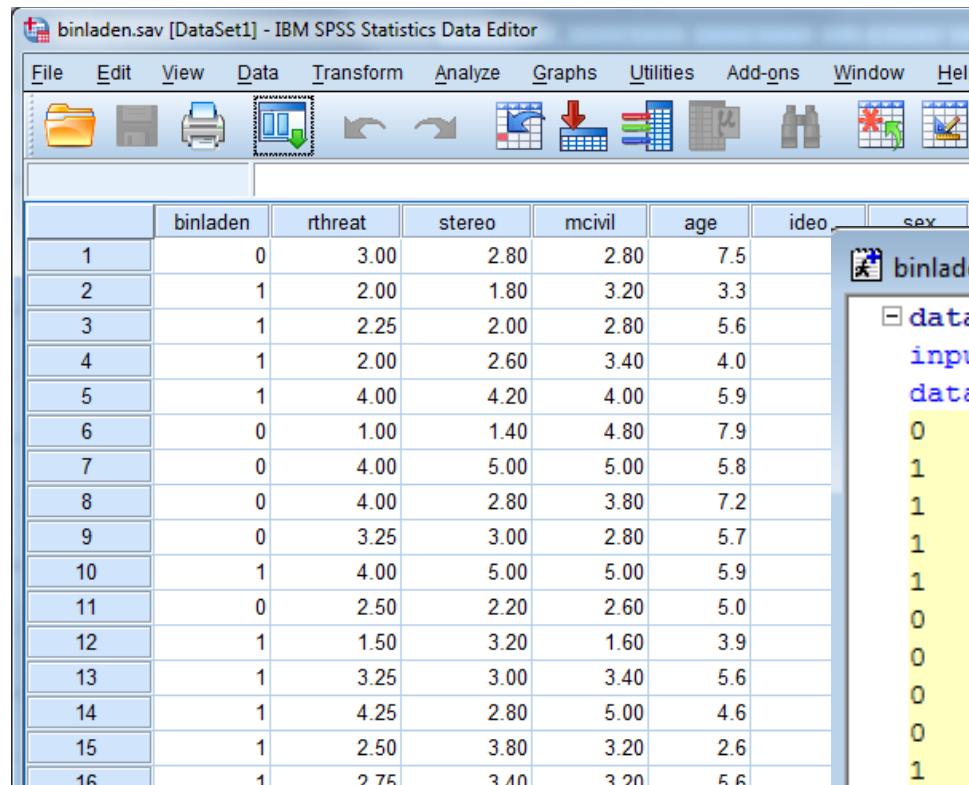
“Below are some statements people have expressed about Muslim civil liberties and terrorism in the U.S. Please read each and tell us how much you agree or disagree...”

e.g., “All Muslims in the U.S. should be required to carry a special ID card”
“Muslims in the U.S. should register their whereabouts with the U.S. government”

Higher reflect greater negative stereotype endorsement/threat/willingness to restrict...”

The Data: BINLADEN

BINLADEN.SAV

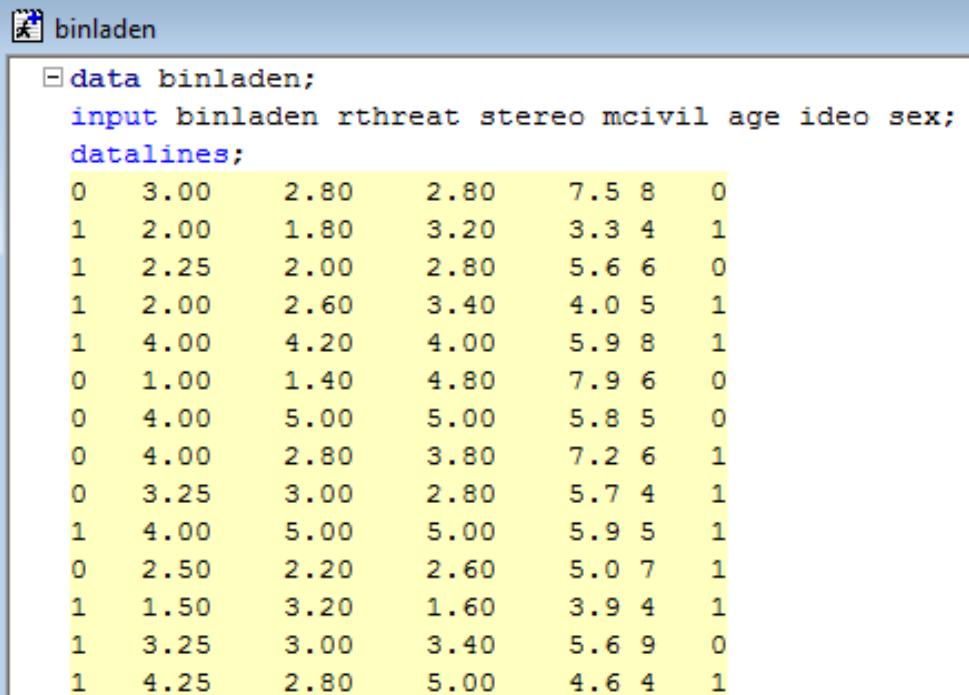


The screenshot shows the IBM SPSS Statistics Data Editor window titled "binladen.sav [DataSet1]". The menu bar includes File, Edit, View, Data, Transform, Analyze, Graphs, Utilities, Add-ons, Window, and Help. The toolbar below has icons for file operations like Open, Save, Print, and Data View. The data view shows a table with 16 rows and 8 columns. The columns are labeled: binladen, rthreat, stereo, mcivil, age, ideo, and sex. The data consists of various numerical values.

	binladen	rthreat	stereo	mcivil	age	ideo	sex
1	0	3.00	2.80	2.80	7.5	8	0
2	1	2.00	1.80	3.20	3.3	4	1
3	1	2.25	2.00	2.80	5.6	6	0
4	1	2.00	2.60	3.40	4.0	5	1
5	1	4.00	4.20	4.00	5.9	8	1
6	0	1.00	1.40	4.80	7.9	6	0
7	0	4.00	5.00	5.00	5.8	5	0
8	0	4.00	2.80	3.80	7.2	6	1
9	0	3.25	3.00	2.80	5.7	4	0
10	1	4.00	5.00	5.00	5.9	5	1
11	0	2.50	2.20	2.60	5.0	7	0
12	1	1.50	3.20	1.60	3.9	4	1
13	1	3.25	3.00	3.40	5.6	9	0
14	1	4.25	2.80	5.00	4.6	8	1
15	1	2.50	3.80	3.20	2.6	7	0
16	1	2.75	3.40	3.20	5.6	4	1

Also included in the data file are respondent **age** in decades, political **ideology** (7 point scale, higher = more conservative) and **gender** (0 = female, 1 = male).

BINLADEN.SAS

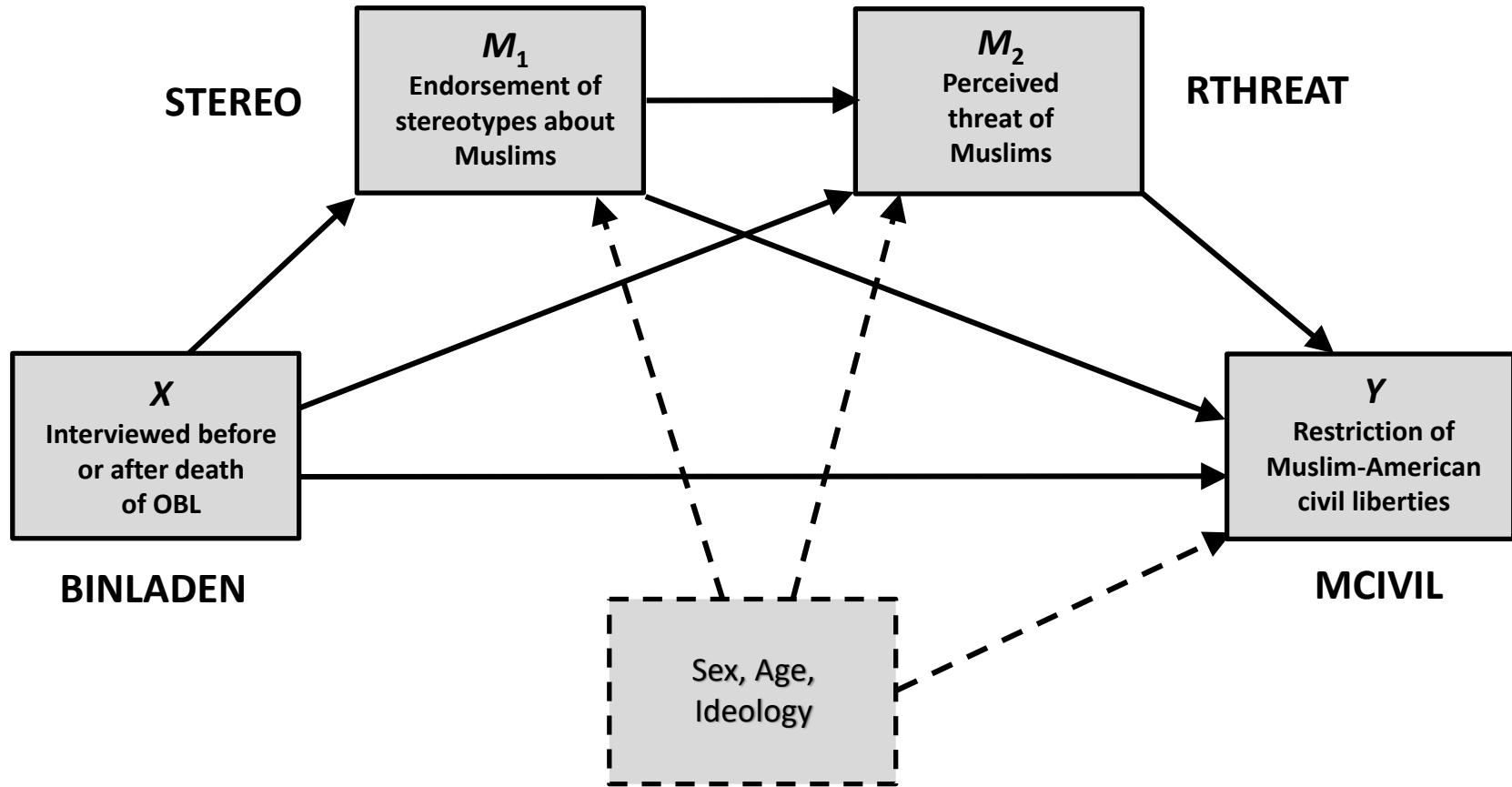


The screenshot shows a SAS code editor window titled "binladen". The code defines a data step "data binladen;" followed by an "input" statement listing variables: binladen, rthreat, stereo, mcivil, age, ideo, and sex. Below the code is a data preview window showing 16 rows of data corresponding to the SPSS table above.

```
data binladen;
  input binladen rthreat stereo mcivil age ideo sex;
  datalines;
```

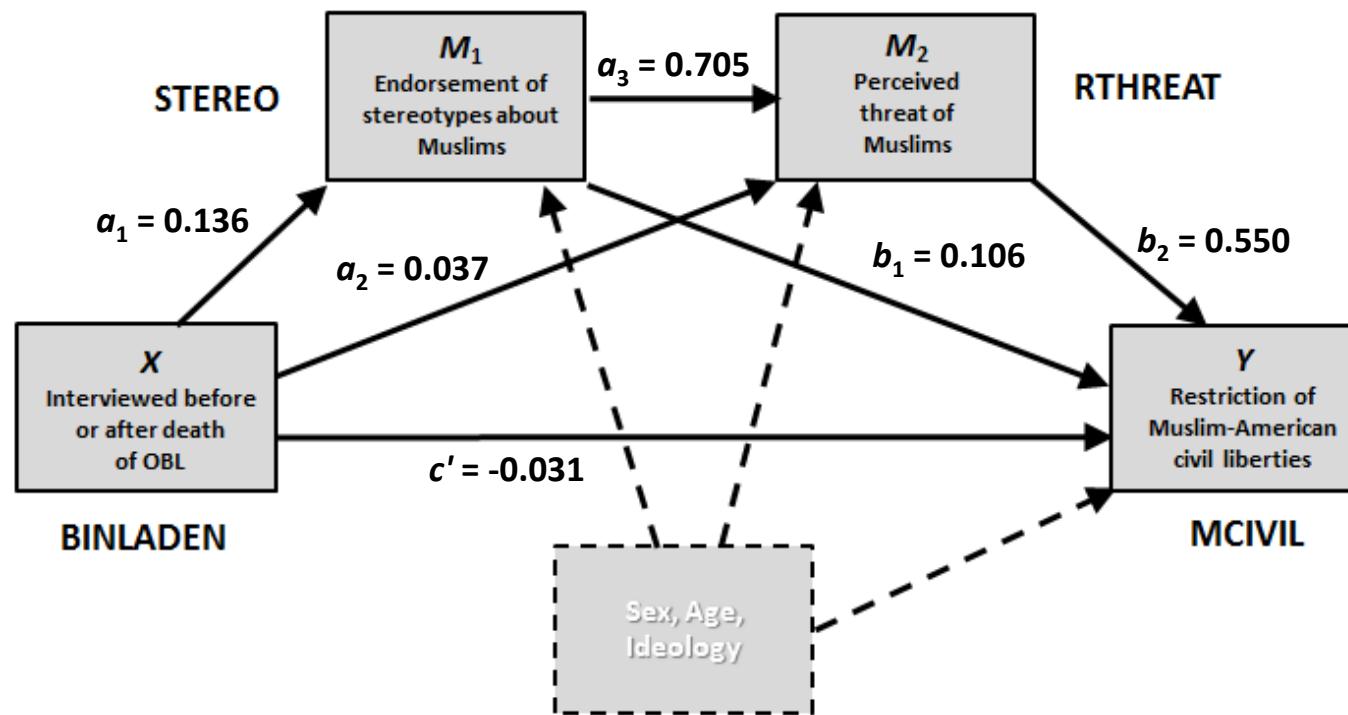
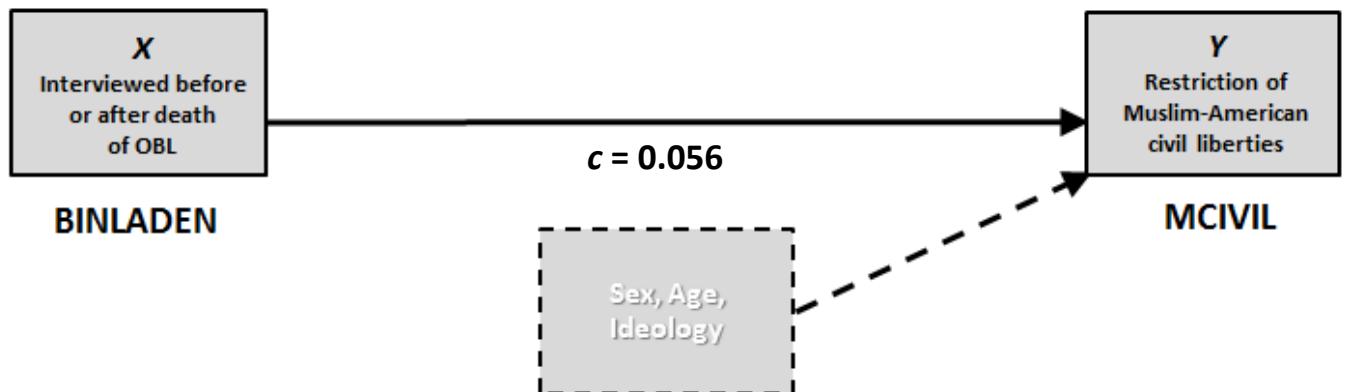
	binladen	rthreat	stereo	mcivil	age	ideo	sex
1	0	3.00	2.80	2.80	7.5	8	0
2	1	2.00	1.80	3.20	3.3	4	1
3	1	2.25	2.00	2.80	5.6	6	0
4	1	2.00	2.60	3.40	4.0	5	1
5	1	4.00	4.20	4.00	5.9	8	1
6	0	1.00	1.40	4.80	7.9	6	0
7	0	4.00	5.00	5.00	5.8	5	0
8	0	4.00	2.80	3.80	7.2	6	1
9	0	3.25	3.00	2.80	5.7	4	0
10	1	4.00	5.00	5.00	5.9	5	1
11	0	2.50	2.20	2.60	5.0	7	0
12	1	1.50	3.20	1.60	3.9	4	1
13	1	3.25	3.00	3.40	5.6	9	0
14	1	4.25	2.80	5.00	4.6	8	1
15	1	2.50	3.80	3.20	2.6	7	0
16	1	2.75	3.40	3.20	5.6	4	1

Example

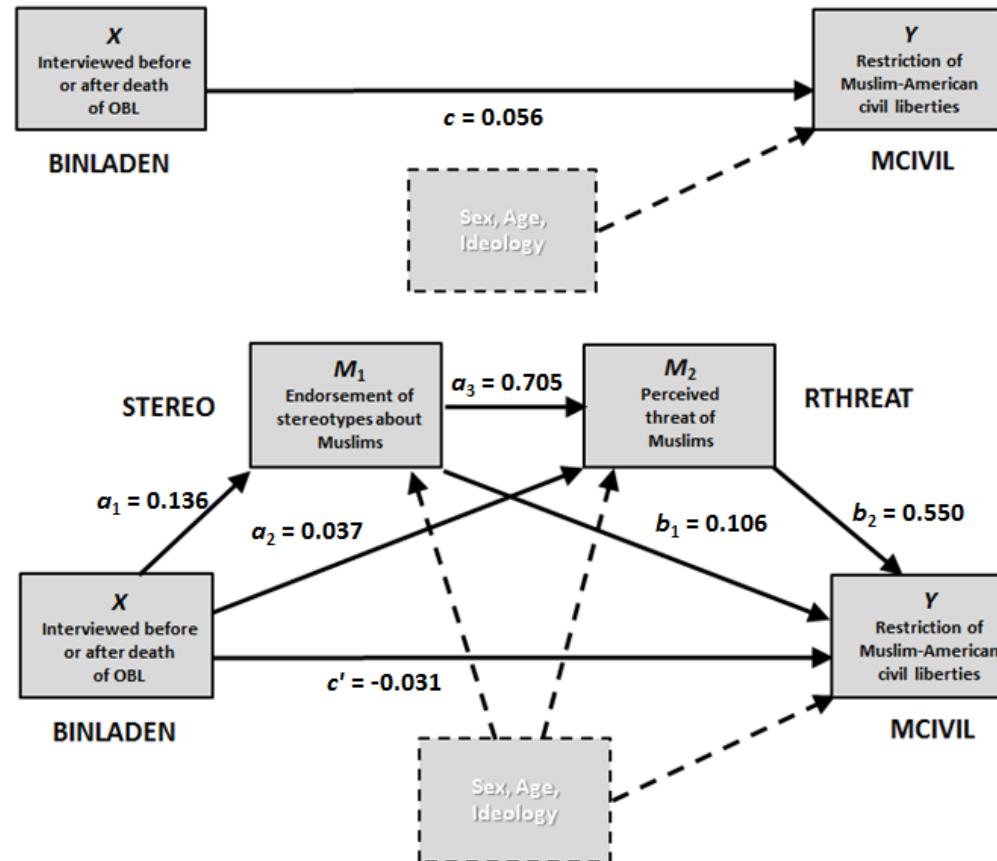


This model includes four pathways of influence of news coverage of OBL death, two through a single mediator, one through both mediators in serial, and one direct.

Example



Example



Direct effect = -0.031

Specific indirect effect via stereotype endorsement: $0.136(0.106) = 0.014$

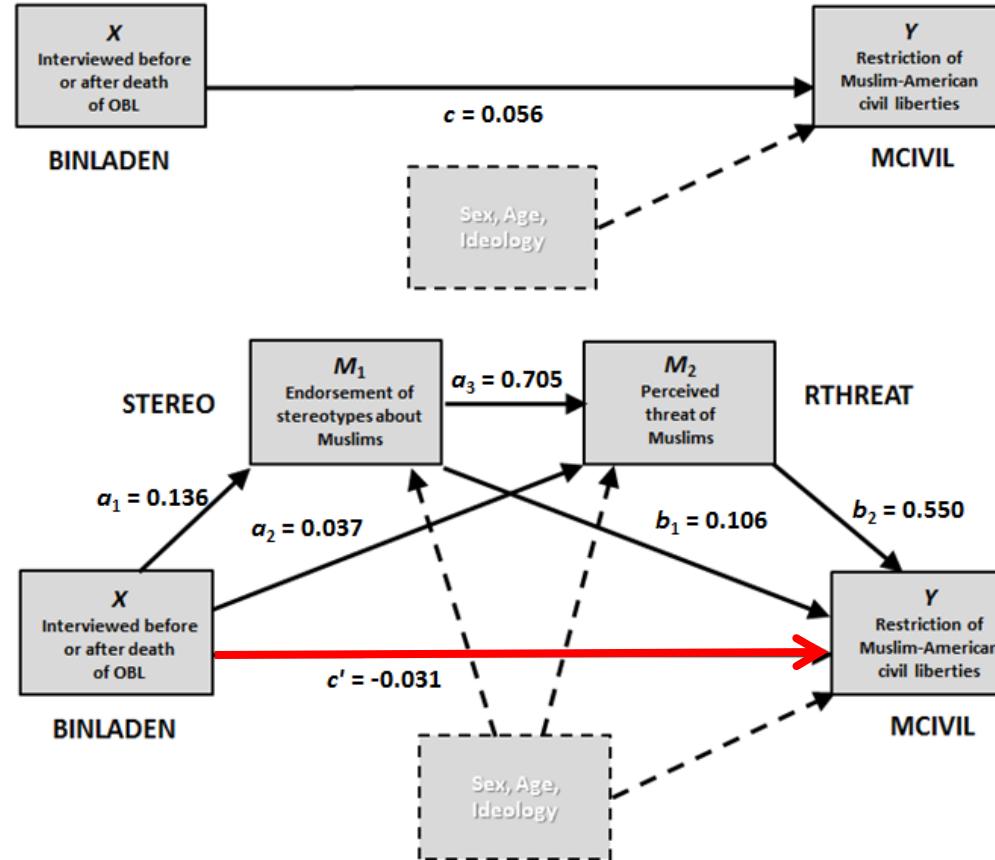
Specific indirect effect via perceived threat: $0.037(0.550) = 0.020$

Specific indirect effect via stereotype endorsement and threat: $0.136(0.705)(0.550) = 0.053$

Total indirect effect = $0.014 + 0.020 + 0.053 = 0.087$

Total effect = $-0.031 + 0.087 = 0.056$

Example



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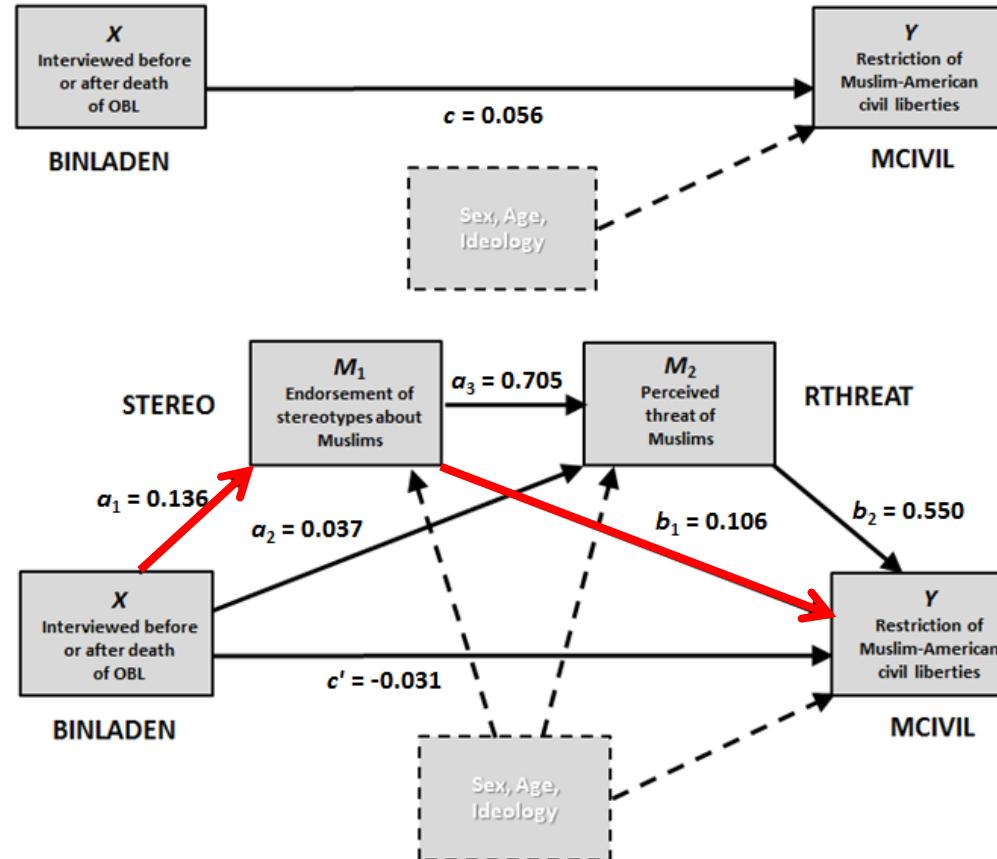
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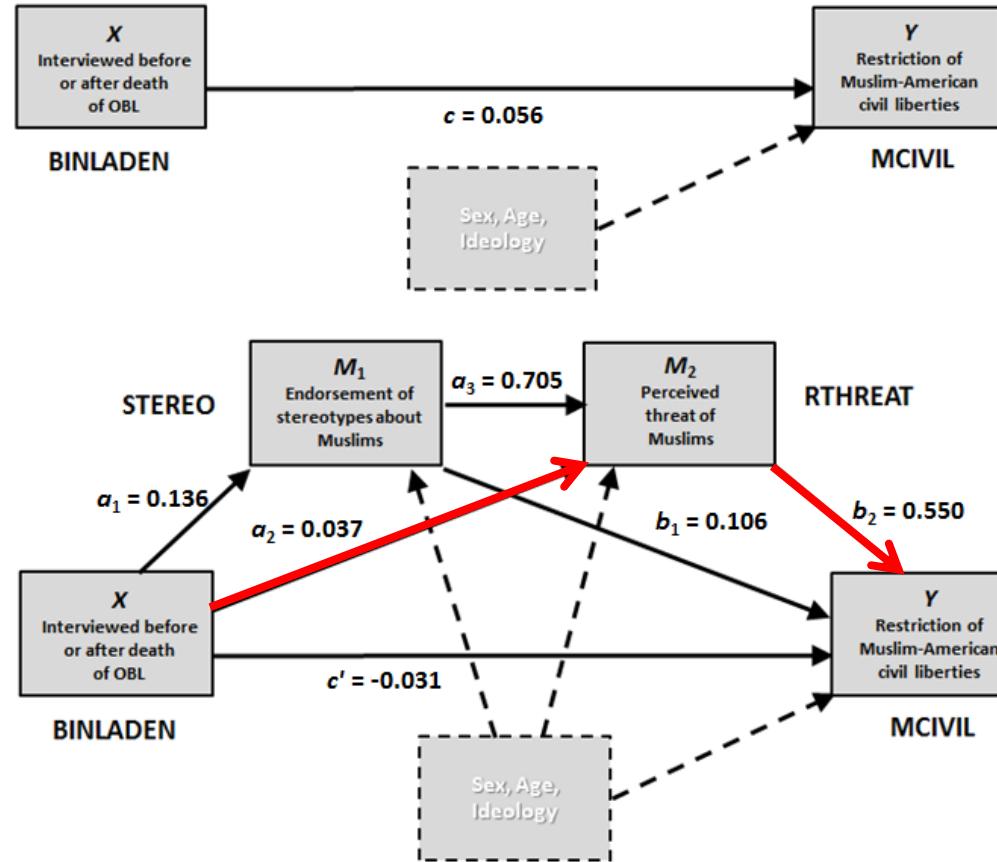
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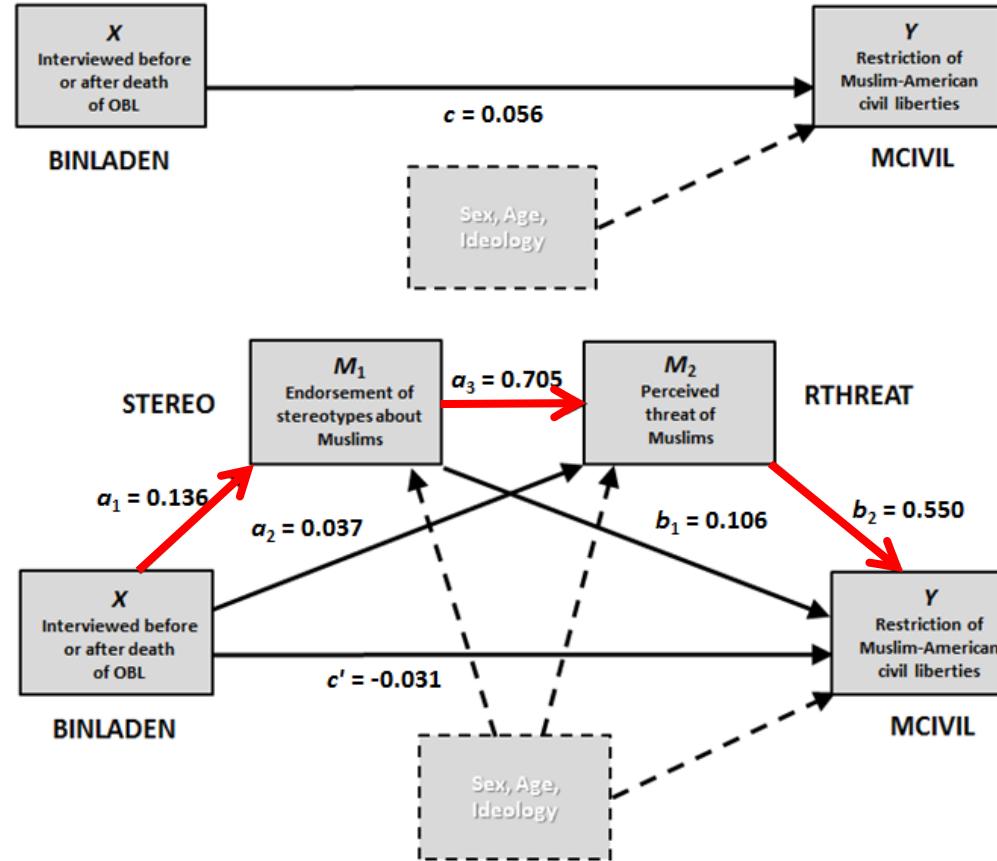
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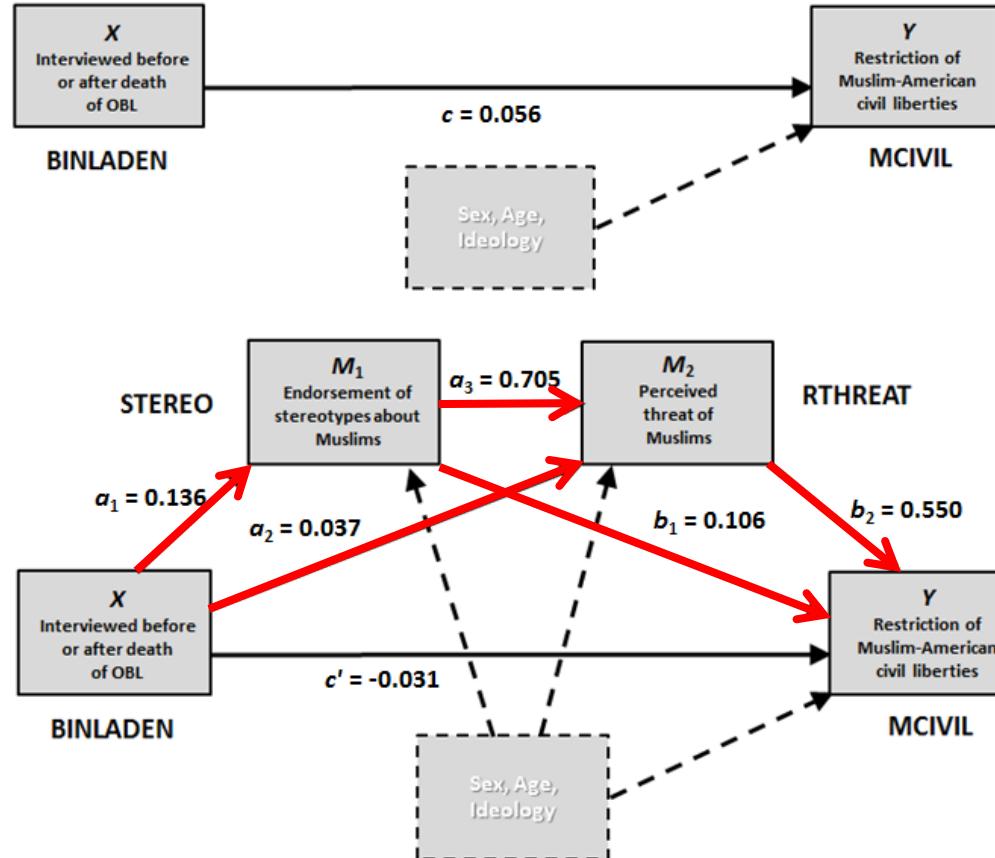
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Example



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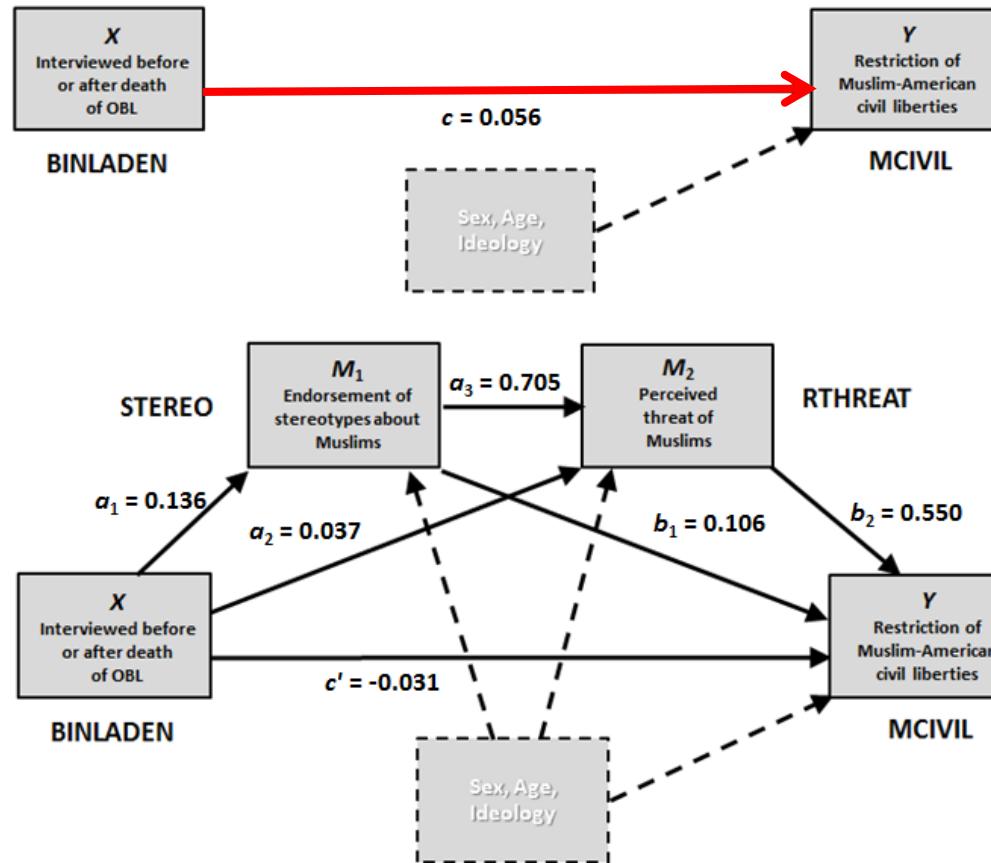
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Example



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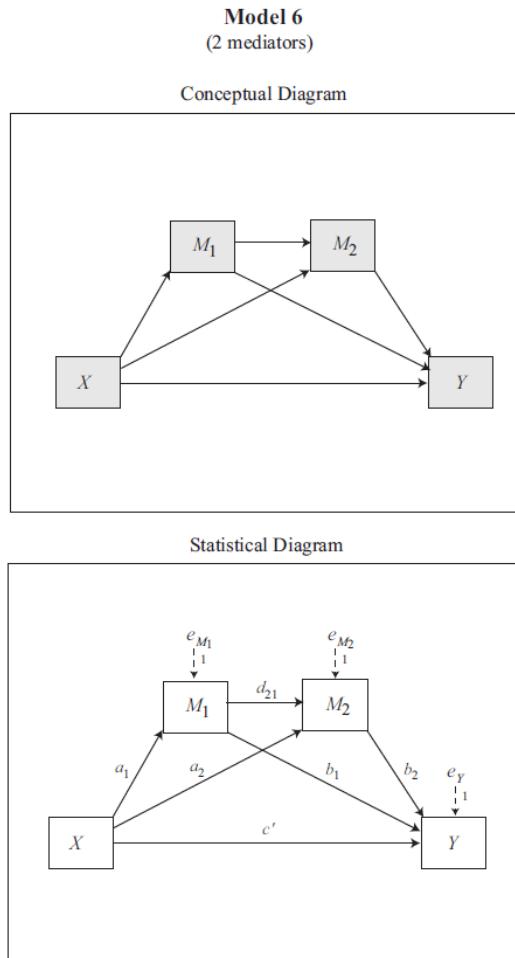
Specific indirect effect via perceived threat: $0.037(0.550) = 0.020$

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Total indirect effect = $0.014 + 0.020 + 0.053 = 0.087$

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Estimation and inference using PROCESS



PROCESS model 6 is the serial multiple mediator model.

In model 6, order of the variables in the "m=" list matters. Variables listed earlier are causally prior to those listed later. PROCESS allows up to four mediators to be linked in a causal chain. All possible indirect and direct effects are estimated.

```
*process cov=sex age ideo/y=mcivil/x=binladen/m=stereo rthreat /boot=10000/model=6/total=1.
```

```
%process (data=binladen,cov=sex age ideo,y=mcivil,x=binladen,m=stereo rthreat,boot=10000,  
model=6,total=1);
```

PROCESS output

```
*****
```

Model = 6

Y = mcivil

X = binladen

M1 = stereo

M2 = rthreat

Output F

Statistical Controls:

CONTROL= sex age ideo

Sample size

661

Outcome: stereo

$$\hat{M}_1 = 1.905 + 0.136X + \dots$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.3557	.1265	.6495	23.7609	4.0000	656.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	1.9045	.1322	14.4084	.0000	1.6449	2.1640
binladen	.1358	.0639	2.1258	.0339	.0104	.2613
sex	.0398	.0635	.6262	.5314	-.0849	.1644
age	.0504	.0192	2.6220	.0089	.0127	.0882
ideo	.1293	.0143	9.0483	.0000	.1012	.1574

a_1 path

```
*****
```

PROCESS output

Output F

Outcome: rthreat

$$\hat{M}_2 = -0.255 + 0.037X + 0.705M_1 + \dots$$

R	R-sq	MSE	F	df1	df2	p
.6764	.4575	.6076	110.4916	5.0000	655.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	-.2548	.1467	-1.7369	.0829	-.5428	.0332
stereo	.7047	.0378	18.6630	.0000	.6306	.7789
binladen	.0374	.0620	.6038	.5462	-.0843	.1592
sex	.1286	.0614	2.0938	.0367	.0080	.2492
age	.0451	.0187	2.4135	.0161	.0084	.0818
ideo	.0898	.0147	6.1257	.0000	.0610	.1186

a_3 path
 a_2 path

PROCESS output

Output F

Outcome: mcivil

$$\hat{Y} = 0.717 - 0.031X + 0.106M_1 + 0.549M_2 + \dots$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.6727	.4526	.5890	90.1100	6.0000	654.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	.7165	.1448	4.9499	.0000	.4323	1.0008
stereo	.1057	.0460	2.2965	.0220	.0153	.1960
rthreat	.5491	.0385	14.2732	.0000	.4736	.6247
binladen	-.0311	.0611	-.5095	.6106	-.1510	.0888
sex	-.1001	.0607	-1.6504	.0993	-.2193	.0190
age	-.0103	.0185	-.5599	.5758	-.0466	.0259
ideo	.0545	.0148	3.6696	.0003	.0253	.0836

*b*₁ path

*b*₂ path

c' path

PROCESS output

Output F

***** TOTAL EFFECT MODEL *****

Outcome: mcivil

$$\hat{Y} = 1.515 + 0.056X + \dots$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.3675	.1351	.9278	25.6100	4.0000	656.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	1.5149	.1580	9.5894	.0000	1.2047	1.8251
binladen	.0564	.0764	.7380	.4608	-.0936	.2063
sex	-.0099	.0759	-.1310	.8958	-.1589	.1391
age	.0393	.0230	1.7085	.0880	-.0059	.0844
ideo	.1675	.0171	9.8053	.0000	.1339	.2010

c path

PROCESS output

***** TOTAL, DIRECT, AND INDIRECT EFFECTS *****

Output F

Total effect of X on Y

Effect	SE	t	p	LLCI	ULCI
.0564	.0764	.7380	.4608	-.0936	.2063

c path

Direct effect of X on Y

Effect	SE	t	p	LLCI	ULCI
-.0311	.0611	-.5095	.6106	-.1510	.0888

c' path

Indirect effect(s) of X on Y:

Effect	BootSE	BootLLCI	BootULCI
TOTAL	.0460	.0005	.1798
Ind1	.0098	-.0002	.0373
Ind2	.0338	-.0446	.0895
Ind3	.0248	.0050	.1021

$a_1b_1 + a_2b_2 + a_1a_3b_2$

a_1b_1

a_2b_2

and bootstrap CIs

$a_1a_3b_2$

Indirect effect key

Ind1 :	binladen ->	stereo	->	mcivil
Ind3 :	binladen ->	stereo	->	rthreat -> mcivil
Ind2 :	binladen ->	rthreat	->	mcivil

The data are consistent with the claim that coverage of OBL's death increased endorsement of restriction of Muslim civil liberties serially through stereotype endorsement and perceived threat of Muslims (0.053, 95% CI=0.005 to 0.102) but not through stereotype endorsement independent of perceived threat (.014, 95% CI = -0.0002 to 0.037) or perceived threat independent of stereotype endorsement (0.021, 95% CI = -0.045 to 0.089). There is no evidence of a direct effect of his death independent of these pathways of influence.

Moderation

Moderation. The effect of X on Y can be said to be *moderated* if its size or direction is dependent on some third variable W . It tells us about the conditions that facilitate, enhance, or inhibit the effect, or for whom or what the effect is large vs. small, present vs. absent, positive vs. negative vs. zero.

The simple regression coefficient

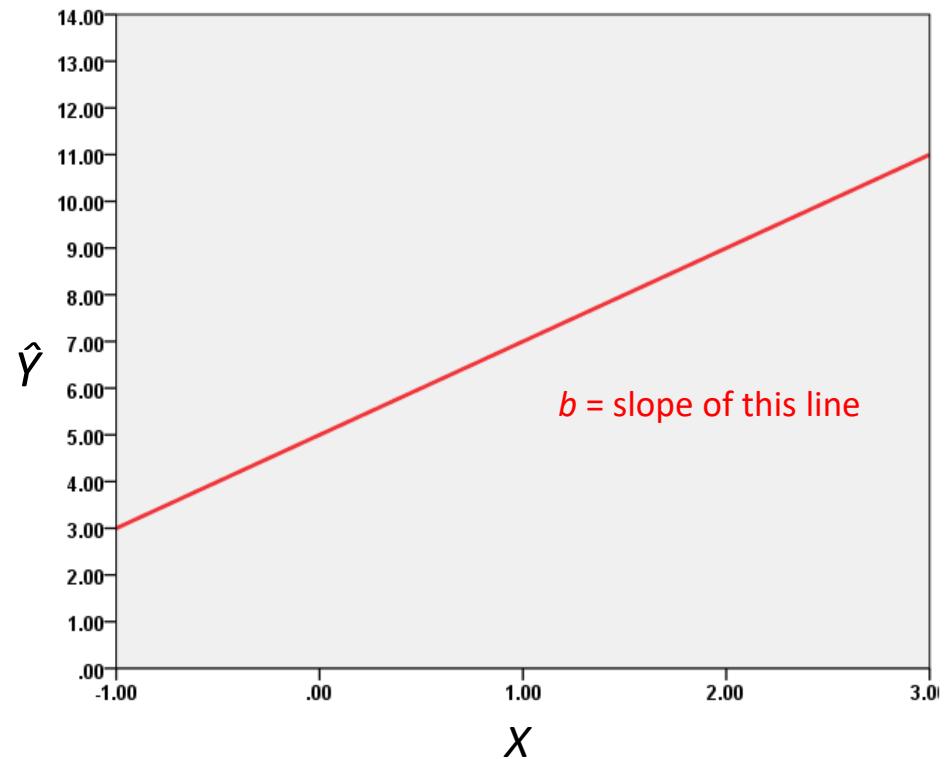
Consider a simple regression model with predictor variable X .

$$\hat{Y} = b_0 + bX \quad \text{such as} \quad \hat{Y} = 5.00 + 2.00X$$

Two cases that differ by one unit on X are estimated to differ by $b = 2.00$ units on Y . b is a “**global property**” of the model, in that makes no difference which value of X you start at--- b is the estimated difference in Y between two cases who differ by a unit on X .

Most generally, $b = \hat{Y}|(X = \omega + 1) - \hat{Y}|(X = \omega)$ for all ω .

X	\hat{Y}
-1	3.00
0	5.00
1	7.00
2	9.00
3	11.00



Partial regression coefficients as **unconditional** effects

Consider a multiple regression model with two predictors, X and W .

$$\hat{Y} = b_0 + b_1 X + b_2 W \quad \text{such as} \quad \hat{Y} = 4.50 + 2.00X + 0.50W$$

Regardless of W , a one unit difference in X is associated with the same expected difference on \hat{Y} . And regardless of the value of X , a one unit difference in W is associated with the same expected difference on \hat{Y} . This is true regardless of which value of X or W you choose. b_1 and b_2 are **global properties** of the model.

Most generally,

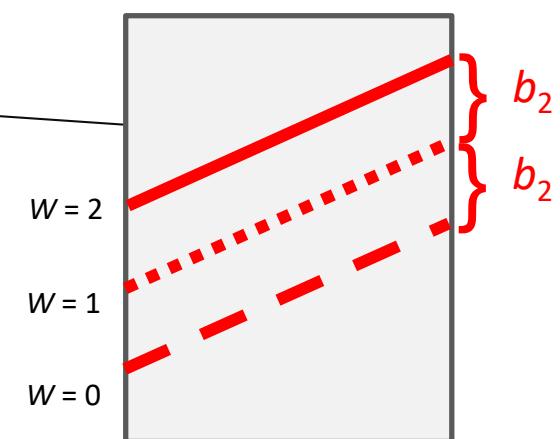
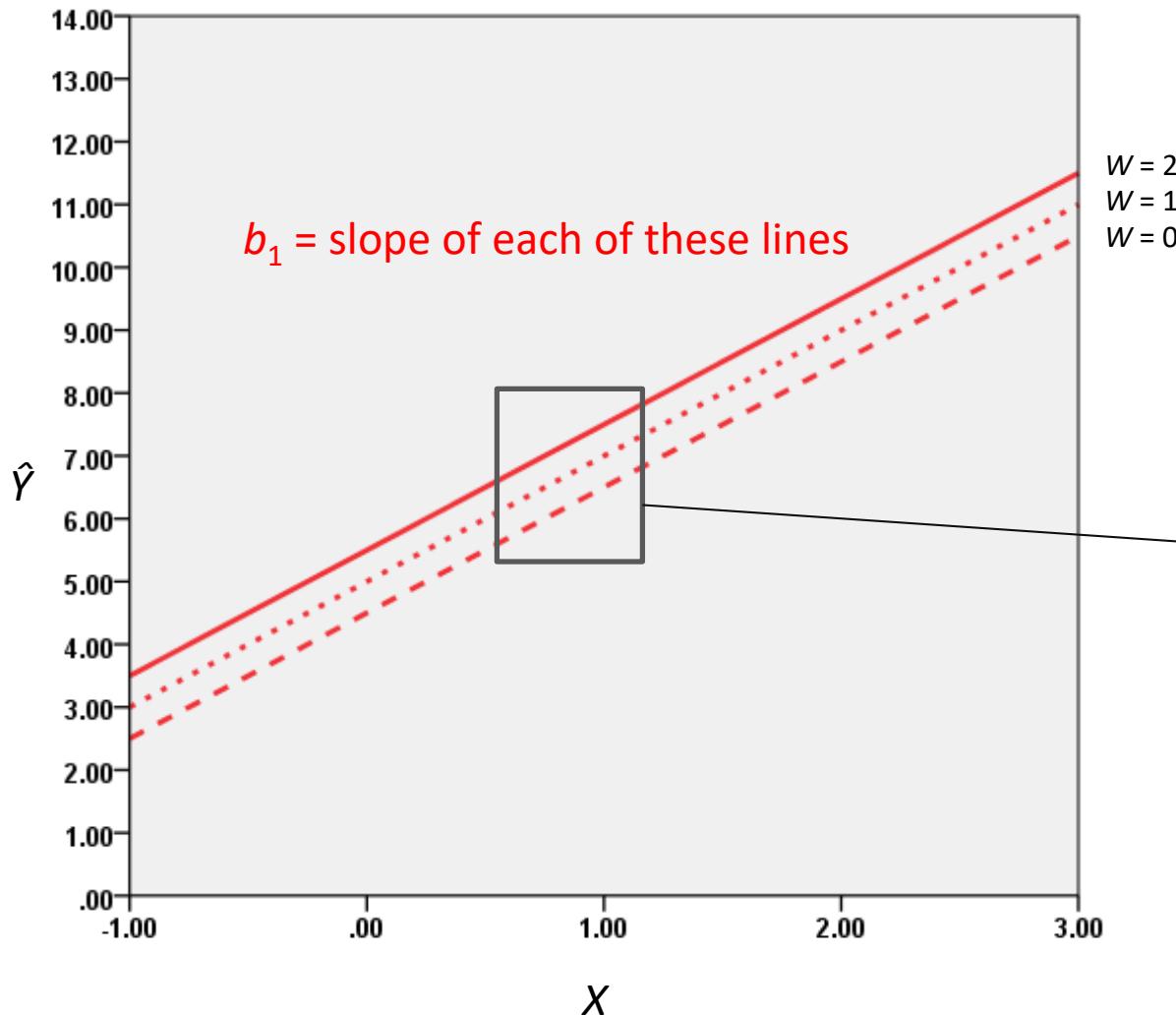
$$b_1 = \hat{Y}|(X = \omega + 1, W = \lambda) - \hat{Y}|(X = \omega, W = \lambda) \text{ for all } \omega, \lambda$$

$$b_2 = \hat{Y}|(W = \lambda + 1, X = \omega) - \hat{Y}|(W = \lambda, X = \omega) \text{ for all } \lambda, \omega$$

X	W	\hat{Y}
-1	0	2.50
-1	1	3.00
-1	2	3.50
0	0	4.50
0	1	5.00
0	2	5.50
1	0	6.50
1	1	7.00
1	2	7.50
2	0	8.50
2	1	9.00
2	2	9.50

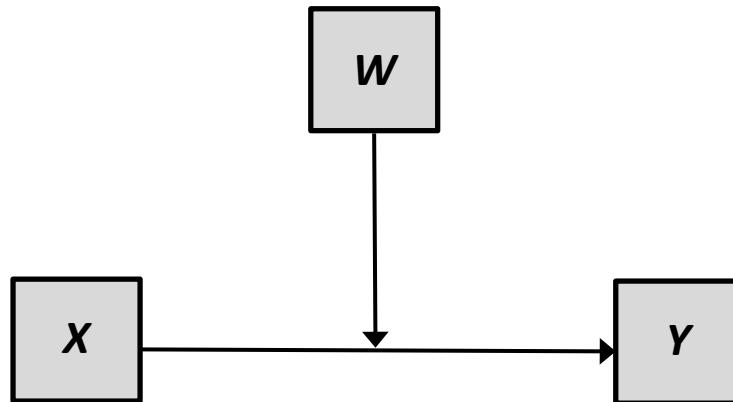
Partial regression coefficients as **unconditional** effects

$$\hat{Y} = 4.50 + 2.00X + 0.50W$$



Moderation

Moderation. The effect of X on Y can be said to be *moderated* if its size or direction is dependent on some third variable W . It tells us about the conditions that facilitate, enhance, or inhibit the effect, or for whom or what the effect is large vs. small, present vs. absent, positive vs. negative vs. zero.



In this diagram, W is depicted to *moderate* the size of the effect of X on Y , meaning that the size of the effect of X on Y depends on W . In such a case, we say W is the *moderator* of the $X \rightarrow Y$ relationship, or that X and W *interact* in their influence on Y . X is sometimes called the **focal predictor**, and W the **moderator**.

Releasing this constraint on the model

Suppose we let X 's effect be a function of W , $f(W)$, as in

$$\hat{Y} = b_0 + f(W)X + b_2W$$

For instance, let $f(W)$ be a linear function of W , $b_1 + b_3W$. Thus,

$$\hat{Y} = b_0 + (b_1 + b_3W)X + b_2W$$

This can be rewritten in an equivalent form as

$$\hat{Y} = b_0 + b_1X + b_2W + b_3XW$$

This model, the “simple moderation model,” allows X 's effect on Y to depend linearly on W . Other forms of moderation are possible, but this form is the one most frequently estimated.

Moderation

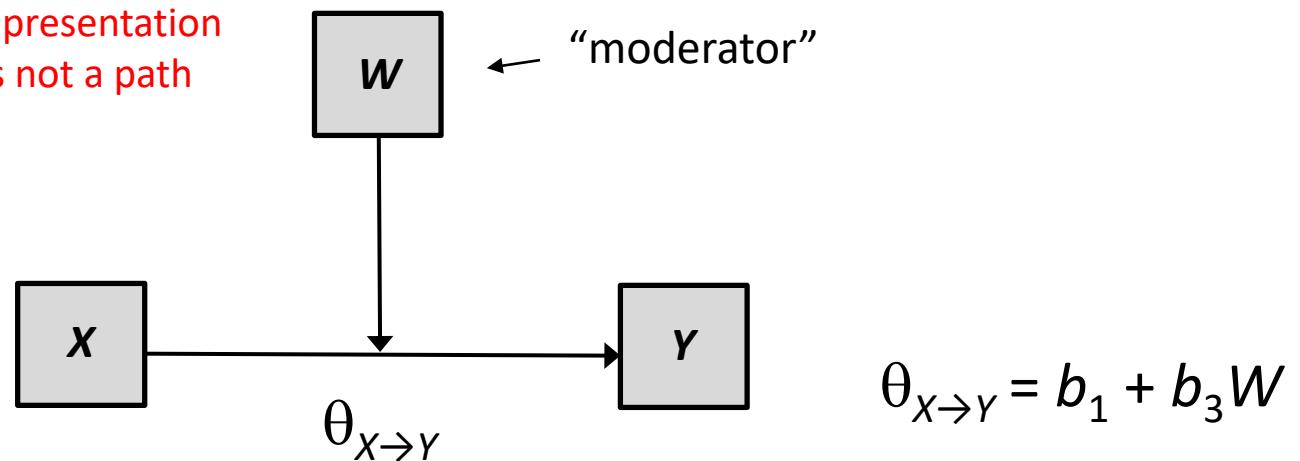
$$\hat{Y} = b_0 + b_1X + b_2W + b_3XW$$

can be written as

$$\hat{Y} = b_0 + (b_1 + b_3W)X + b_2W$$

This is a conceptual representation of moderation. This is not a path diagram.

“focal predictor”



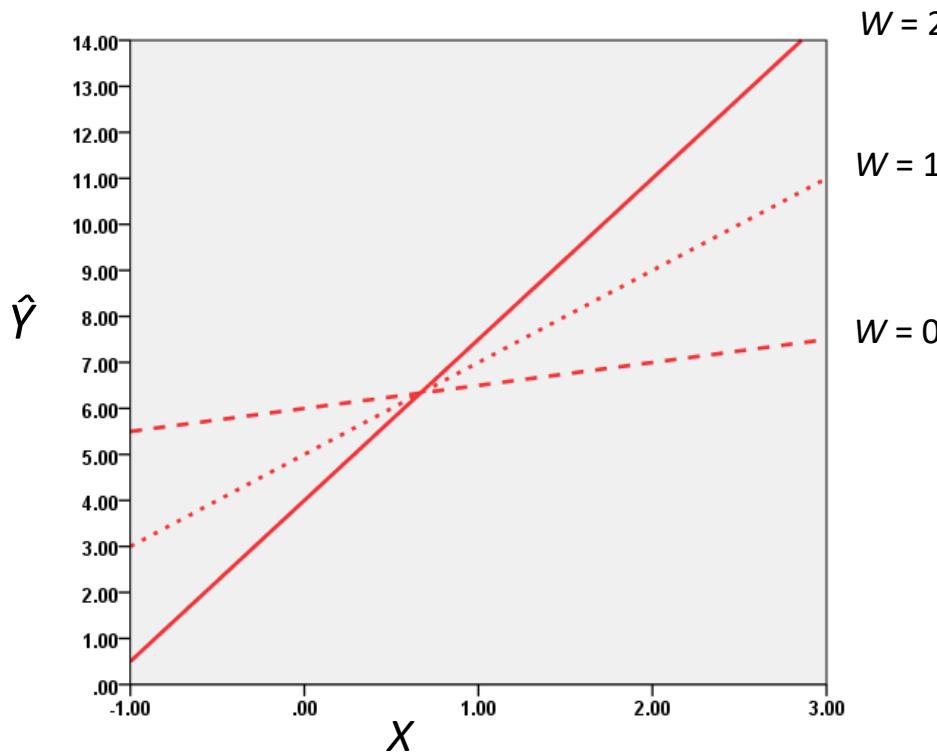
$\theta_{X \rightarrow Y}$ is the “conditional effect of X” defined by the function $b_1 + b_3W$

X 's effect as a function of W

$$\begin{aligned}b_0 &= 6.00 \\b_1 &= 0.50 \\b_2 &= -1.00 \\b_3 &= 1.50\end{aligned}$$

$$\hat{Y} = 6.00 + 0.50X - 1.00W + 1.50XW$$

Observe that the amount by which two cases that differ by one unit on X are estimated to differ on Y **depends on W** .



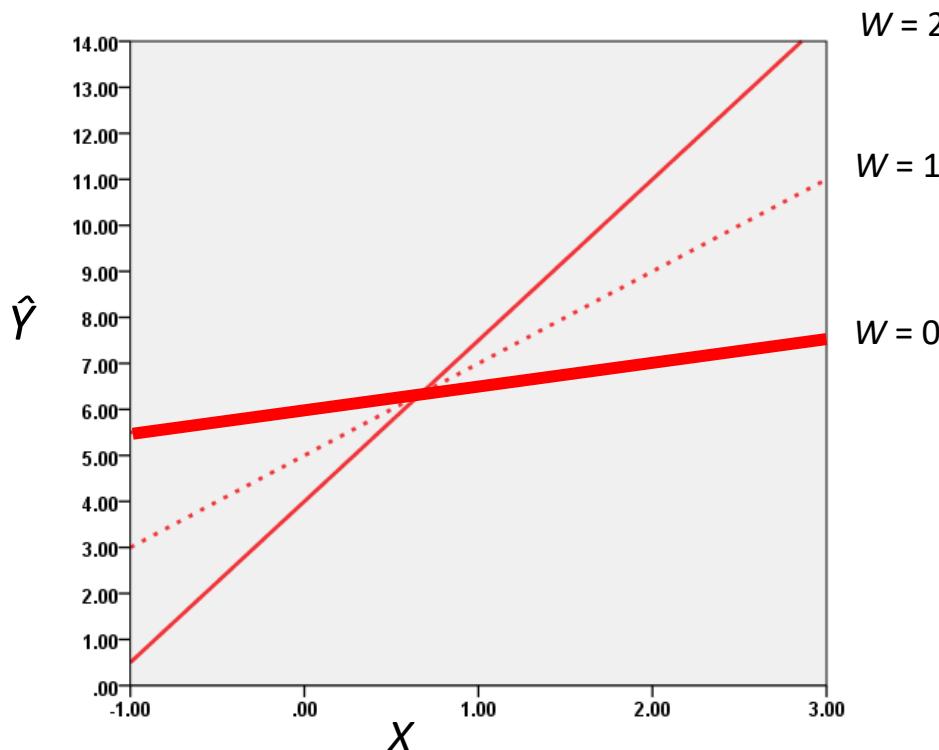
X	W	\hat{Y}
-1	0	5.50
-1	1	3.00
-1	2	0.50
0	0	6.00
0	1	5.00
0	2	4.00
1	0	6.50
1	1	7.00
1	2	7.50
2	0	7.00
2	1	9.00
2	2	11.00

X's effect as a function of W

$$\begin{aligned}b_0 &= 6.00 \\b_1 &= 0.50 \\b_2 &= -1.00 \\b_3 &= 1.50\end{aligned}$$

$$\hat{Y} = 6.00 + 0.50X - 1.00W + 1.50XW$$

Observe that the amount by which two cases that differ by one unit on X are estimated to differ on Y depends on W .



X	W	\hat{Y}
-1	0	5.50
-1	1	3.00
-1	2	0.50
0	0	6.00
0	1	5.00
0	2	4.00
1	0	6.50
1	1	7.00
1	2	7.50
2	0	7.00
2	1	9.00
2	2	11.00

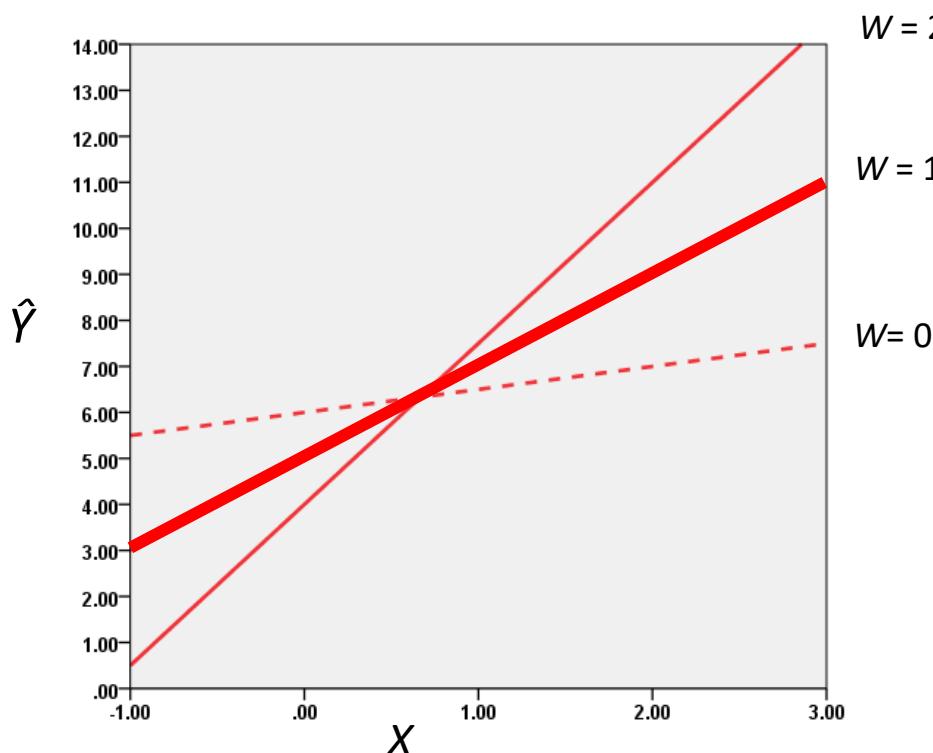
$$\begin{aligned}\theta_{X \rightarrow Y} &= b_1 + b_3 W \\&= 0.50 + 1.50W \\&= 0.50 + 1.50(0) = 0.50\end{aligned}$$

X's effect as a function of W

$$\begin{aligned}b_0 &= 6.00 \\b_1 &= 0.50 \\b_2 &= -1.00 \\b_3 &= 1.50\end{aligned}$$

$$\hat{Y} = 6.00 + 0.50X - 1.00W + 1.50XW$$

Observe that the amount by which two cases that differ by one unit on X are estimated to differ on Y depends on W .



X	W	\hat{Y}
-1	0	5.50
-1	1	3.00
-1	2	0.50
0	0	6.00
0	1	5.00
0	2	4.00
1	0	6.50
1	1	7.00
1	2	7.50
2	0	7.00
2	1	9.00
2	2	11.00

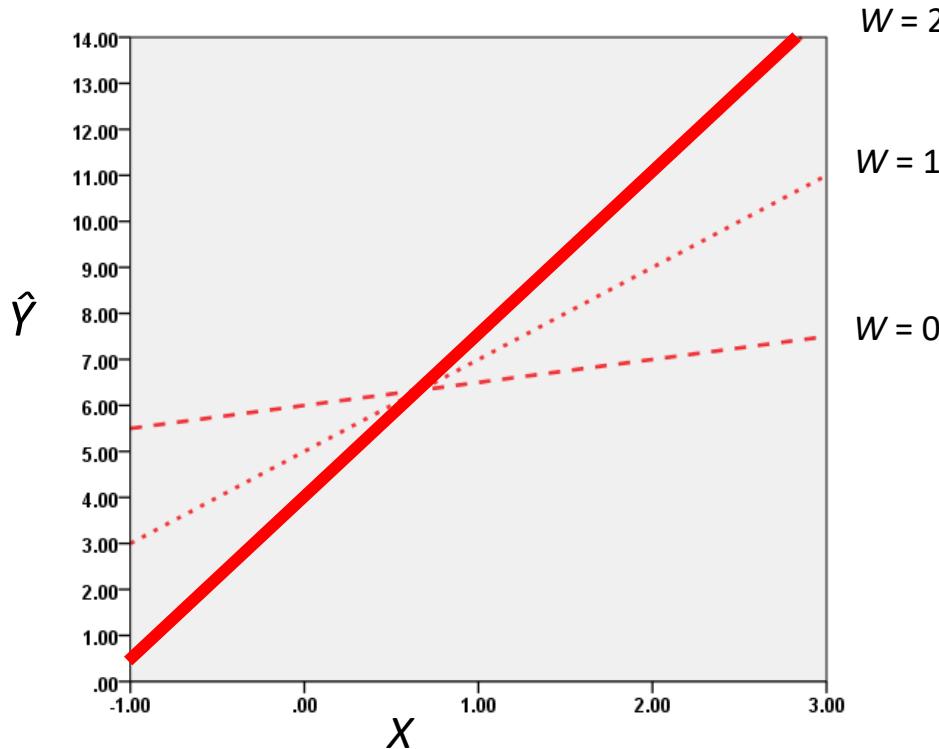
$$\begin{aligned}\theta_{X \rightarrow Y} &= b_1 + b_3 W \\&= 0.50 + 1.50W \\&= 0.50 + 1.50(1) = 2.00\end{aligned}$$

X's effect as a function of W

$$\begin{aligned}b_0 &= 6.00 \\b_1 &= 0.50 \\b_2 &= -1.00 \\b_3 &= 1.50\end{aligned}$$

$$\hat{Y} = 6.00 + 0.50X - 1.00W + 1.50XW$$

Observe that the amount by which two cases that differ by one unit on X are estimated to differ on Y depends on W .



X	W	\hat{Y}
-1	0	5.50
-1	1	3.00
-1	2	0.50
0	0	6.00
0	1	5.00
0	2	4.00
1	0	6.50
1	1	7.00
1	2	7.50
2	0	7.00
2	1	9.00
2	2	11.00

$$\begin{aligned}\theta_{X \rightarrow Y} &= b_1 + b_3 W \\&= 0.50 + 1.50W \\&= 0.50 + 1.50(2) = 3.50\end{aligned}$$

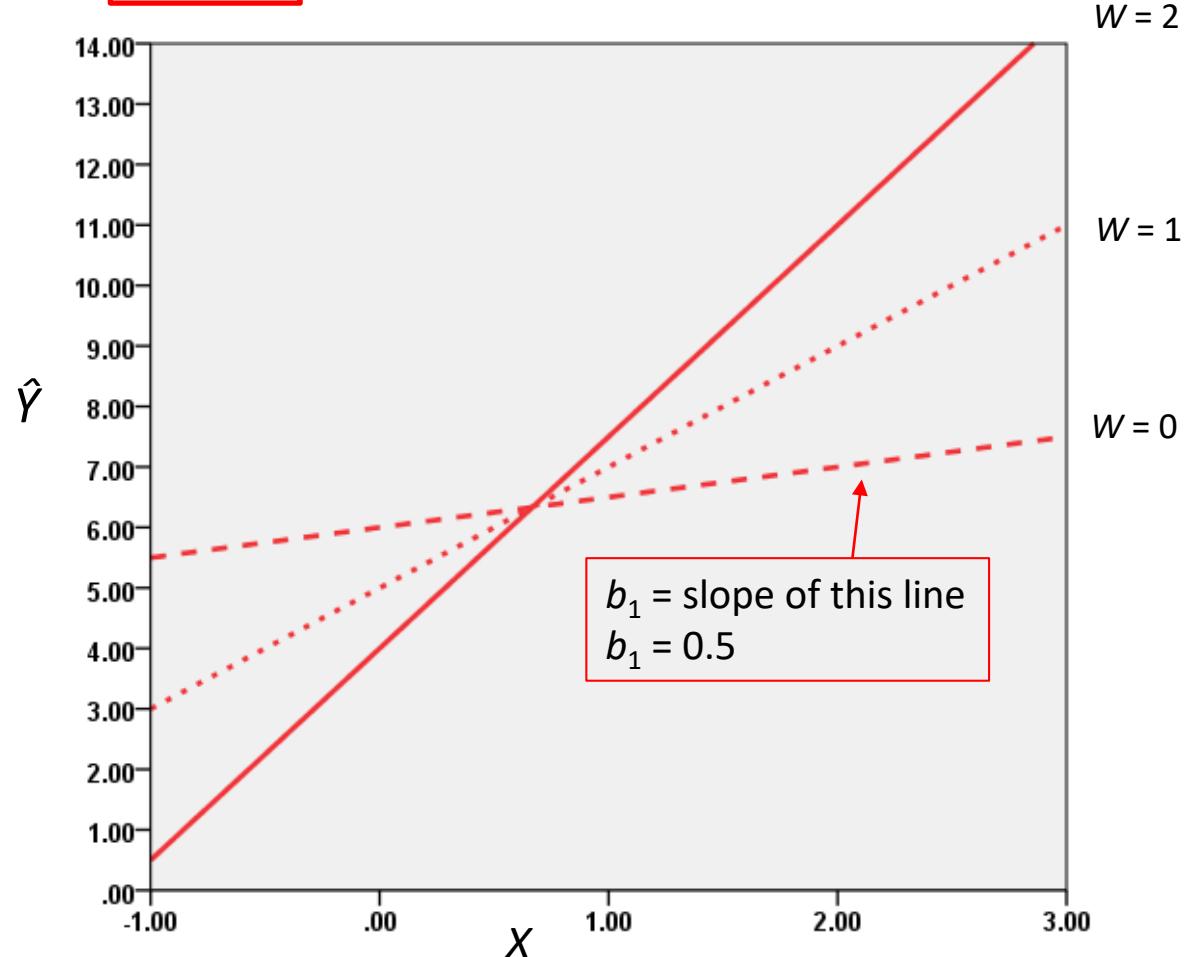
Interpretation of b_1 as a conditional effect

$$\begin{aligned}b_0 &= 6.00 \\b_1 &= 0.50 \\b_2 &= -1.00 \\b_3 &= 1.50\end{aligned}$$

$$\hat{Y} = 6.00 + \boxed{0.50X} - 1.00W + 1.50XW$$

b_1 is the effect of X on Y when $W = 0$. It quantifies how much two cases that differ by one unit on X but with $W = 0$ are estimated to differ on Y .

b_1 is a **local property** of the model. It characterizes the association between X and Y only when $W = 0$.



$$b_1 = \hat{Y}|(X = \omega + 1, W = 0) - \hat{Y}|(X = \omega, W = 0) \text{ for all } \omega.$$

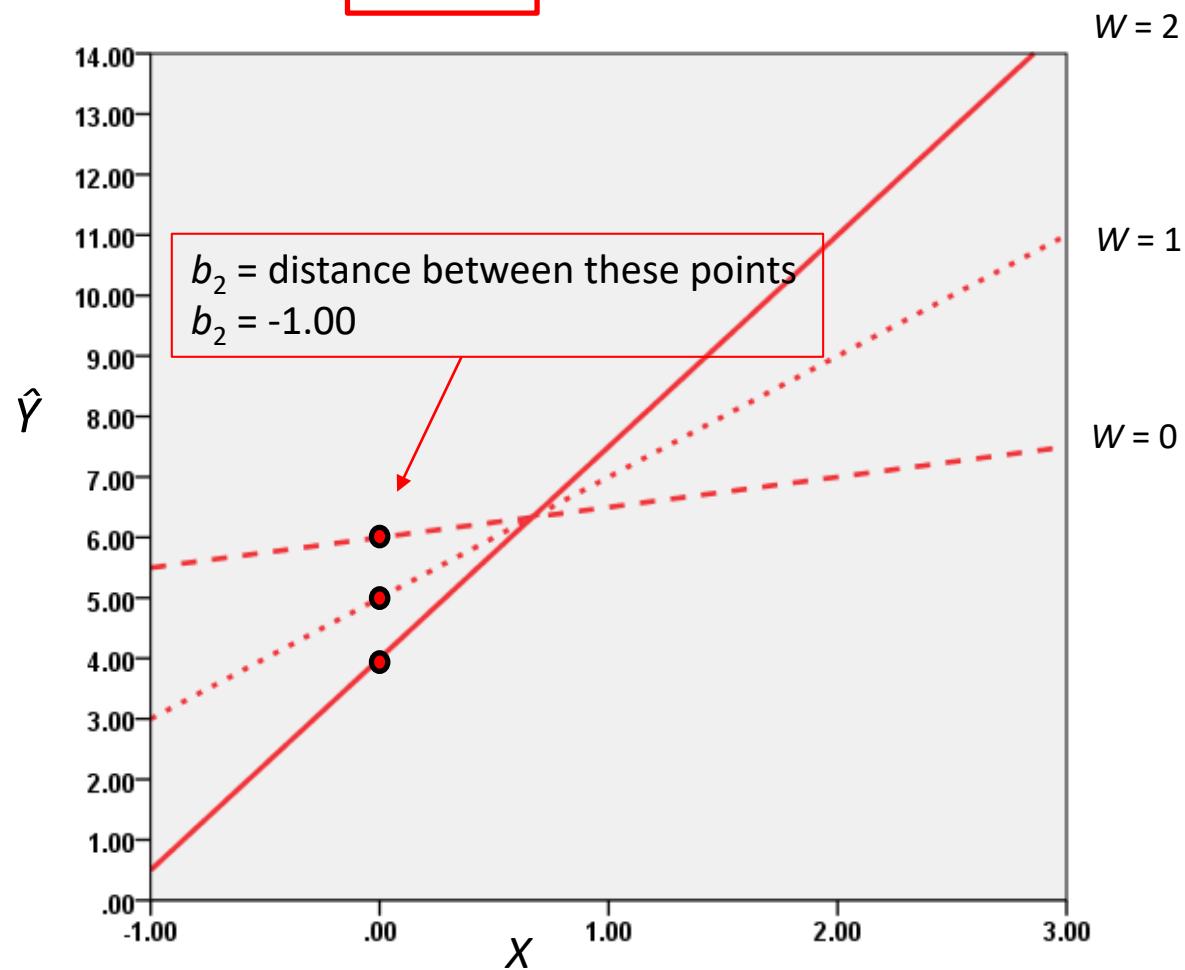
Interpretation of b_2 as a conditional effect

$$\begin{aligned}b_0 &= 6.00 \\b_1 &= 0.50 \\b_2 &= -1.00 \\b_3 &= 1.50\end{aligned}$$

$$\hat{Y} = 6.00 + 0.50X - 1.00W + 1.50XW$$

b_2 is the effect of W when $X = 0$. It quantifies how much two cases that differ by one unit on W but with $X = 0$ are estimated to differ on Y .

b_2 is a **local property** of the model. It characterizes the association between W and Y only when $X = 0$.



$$b_2 = \hat{Y}|(W = \lambda + 1, X = 0) - \hat{Y}|(W = \lambda, X = 0) \text{ for all } \lambda$$

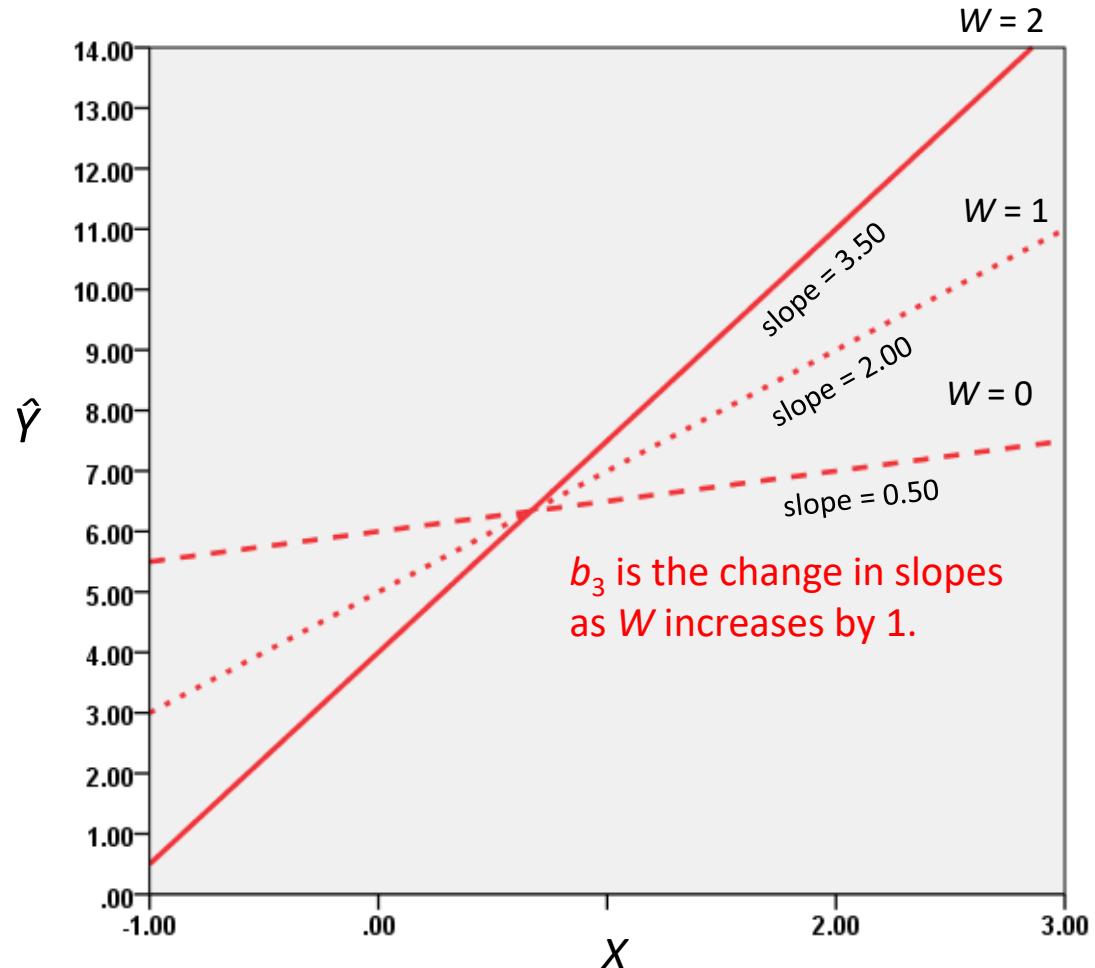
Interpretation of b_3

$$\hat{Y} = 6.00 + 0.50X - 1.00W + 1.50XW$$

b_3 is the amount by which the conditional effect of X changes as W changes by one unit.

$$\theta_{X \rightarrow Y} = b_1 + b_3 W \\ = 0.50 + 1.50W$$

$\theta_{X \rightarrow Y}$	W
0.50	0
2.00	1
3.50	2



$$b_3 = (\theta_{X \rightarrow Y} | W = \lambda + 1) - (\theta_{X \rightarrow Y} | W = \lambda) \text{ for all } \lambda$$

Differences in interpretation

$$\hat{Y} = b_0 + b_1X + b_2W$$

b_0

The estimated value of Y when X and $W = 0$.

b_1

The effect of X on Y holding W constant. This is a *partial* effect.

b_2

The effect of W on Y holding X constant. This is a *partial* effect.

b_3

$$\hat{Y} = b_0 + b_1X + b_2W + b_3XW$$

The estimated value of Y when X and $W = 0$.

The effect of X on Y when $W = 0$.
This is a *conditional* effect. It is
Not a “main effect” or “average
effect” of X .

The effect of W on Y when $X = 0$.
This is a *conditional* effect. It is
not a “main effect” or “average
effect” of W .

How much the effect of X on Y changes as W changes by 1 unit.

Symmetry in moderation

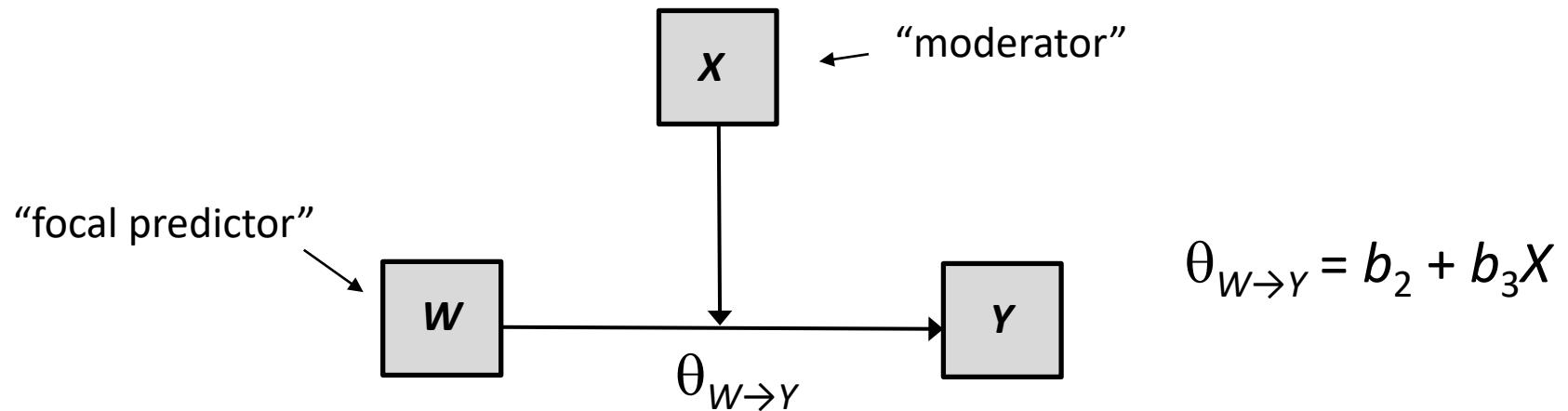
$$\hat{Y} = b_0 + b_1X + b_2W + b_3XW$$

We saw that this is alternative representation of

$$\hat{Y} = b_0 + (b_1 + b_3W)X + b_2W$$

But it is also an alternative representation of

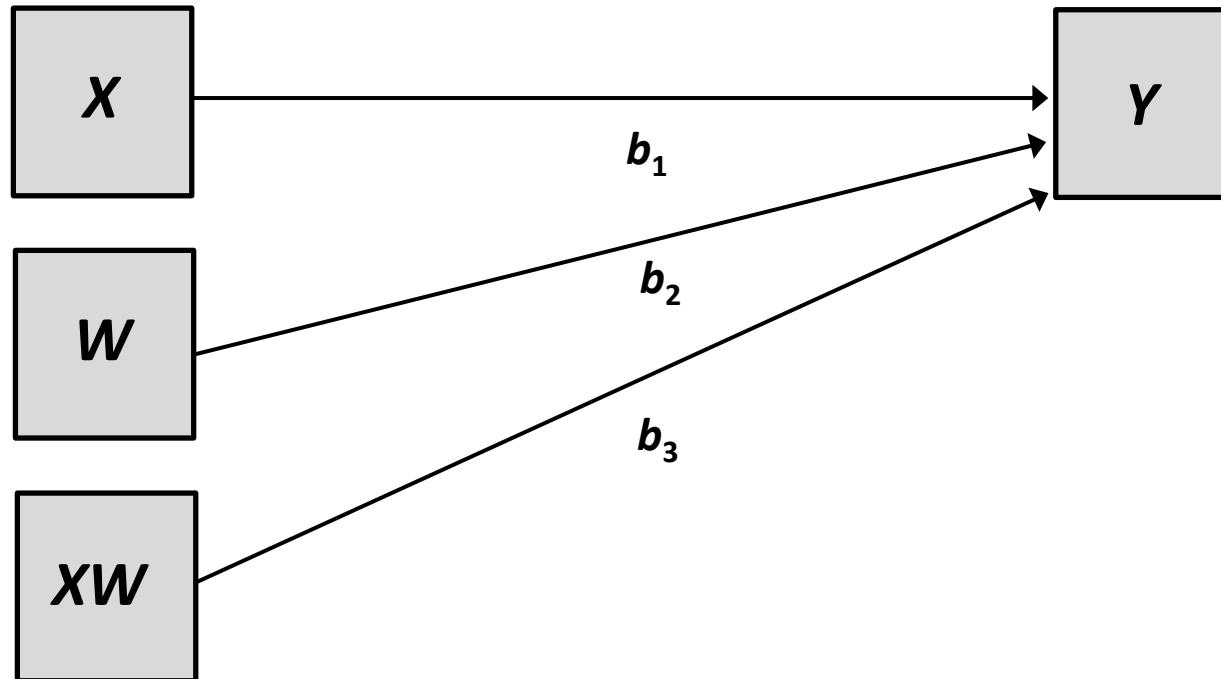
$$\hat{Y} = b_0 + (b_2 + b_3X)W + b_1X$$



Here, X moderates the size of the effect of W on Y . Now X is the moderator. Ultimately, which variable X or W we think of as the moderator depends on substantive concerns. Statistically, it makes no difference as they are mathematically equivalent models.

In path diagram form

$$\hat{Y} = b_0 + b_1X + b_2W + b_3XW$$

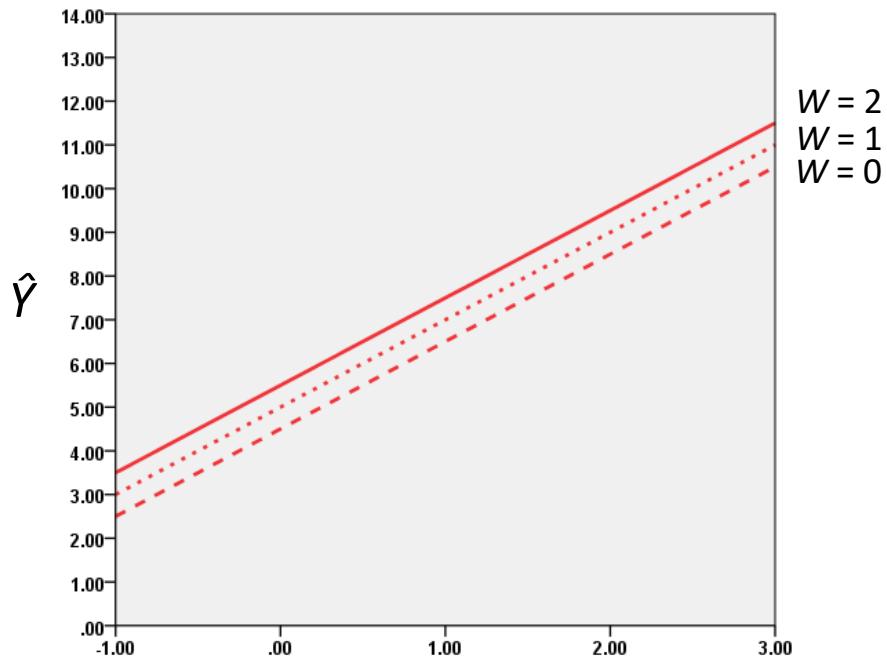


Remember

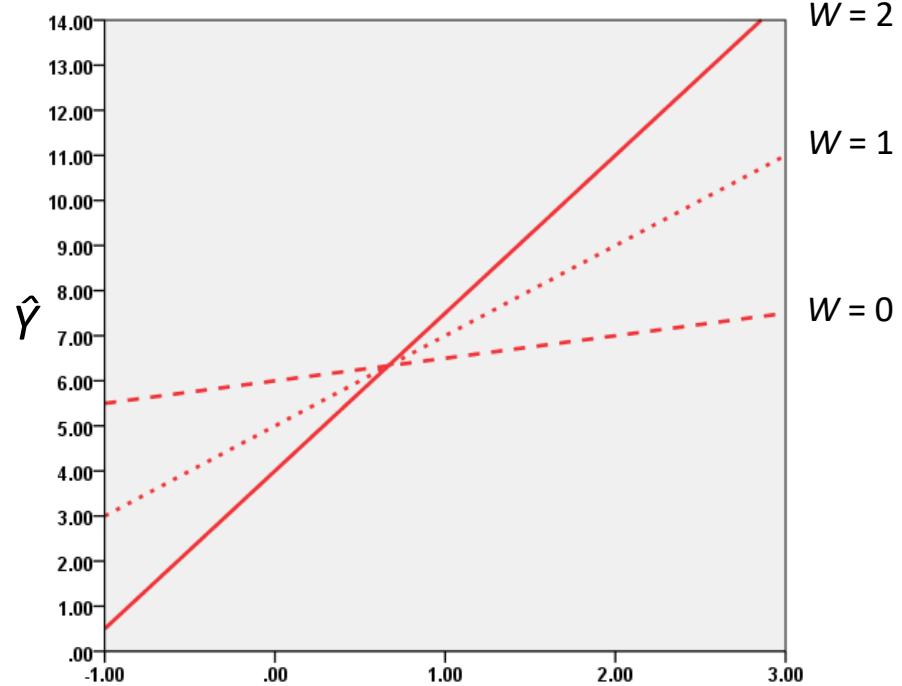
b_1 is NOT the effect of X on Y . The effect of X is $b_1 + b_3W$
 b_2 is NOT the effect of W on Y . The effect of W is $b_2 + b_3X$

The importance of b_3 when testing a moderation hypothesis

$$\hat{Y} = 4.50 + 2.00X + 0.50W + 0XW$$



$$\hat{Y} = 6.00 + 0.50X - 1.00W + 1.50XW$$



$$\begin{aligned}\theta_{X \rightarrow Y} &= b_1 + b_3 W & X \\ &= 2.00 + 0W & \theta_{W \rightarrow Y} = b_2 + b_3 X \\ &&= 0.50 + 0X\end{aligned}$$

$$\begin{aligned}\theta_{X \rightarrow Y} &= b_1 + b_3 W & X \\ &= 0.50 + 1.50W & \theta_{W \rightarrow Y} = b_2 + b_3 X \\ &&= -1.00 + 1.50X\end{aligned}$$

When $b_3 = 0$, a one unit change in X has the same effect on Y regardless of W , and a one unit change in W has the same effect on Y regardless of X . When $b_3 \neq 0$, the effect of a change in X on Y depends on W , and the effect of a change in W on Y depends on X . So we test a moderation hypothesis by testing whether b_3 is different from zero.

Example inspired by ...

Witkiewitz, K., & Bowen, S. (2010). Depression, craving, and substance use following a randomized trial of mindfulness-based relapse prevention. *Journal of Consulting and Clinical Psychology, 78*, 362-374.

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2010, Vol. 78, No. 3, 362-374

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Depression, Craving, and Substance Use Following a Randomized Trial of Mindfulness-Based Relapse Prevention

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Sarah Bowen
University of Washington

Objective: A strong relation between negative affect and craving has been demonstrated in laboratory and clinical studies, with depressive symptomatology showing particularly strong links to craving and substance abuse relapse. Mindfulness-based relapse prevention (MBRP), shown to be efficacious for reduction of substance use, uses mindfulness-based practices to teach alternative responses to emotional discomfort and lessen the conditioned response of craving in the presence of depressive symptoms. The goal in the current study was to examine the relation between measures of depressive symptoms, craving, and substance use following MBRP. **Method:** Individuals with substance use disorders ($N = 168$; mean age 40.45 years, $SD = 10.28$; 36.3% female; 46.4% non-White) were recruited after intensive stabilization, then randomly assigned to either 8 weekly sessions of MBRP or a treatment-as-usual control group. Approximately 73% of the sample was retained at the final 4-month follow-up assessment. **Results:** Results confirmed a moderated-mediation effect, whereby craving mediated the relation between depressive symptoms (Beck Depression Inventory) and substance use (Timeline Follow-Back) among the treatment-as-usual group but not among MBRP participants. MBRP attenuated the relation between posttreatment depressive symptoms and craving (Penn Alcohol Craving Scale) 2 months following the intervention ($\beta^2 = .21$). This moderation effect predicted substance use 4 months following the intervention ($\beta^2 = .18$). **Conclusion:** MBRP appears to influence cognitive and behavioral responses to depressive symptoms, partially explaining reductions in posttreatment substance use among the MBRP group. Although results are preliminary, the current study provides evidence for the value of incorporating mindfulness practice into substance abuse treatment and identifies a potential mechanism of change following MBRP.

Keywords: mindfulness based relapse prevention, substance use, craving, negative affect, depression

Addiction has generally been characterized as a chronic and relapsing condition (Connors, Maiso, & Zwykai, 1996; Lester, 1999). Research on the relapse process has implicated numerous risk factors that appear to be the most robust and immediate predictors of posttreatment substance use, including negative affect, craving or urges, interpersonal stress, motivation, self-efficacy, and ineffective coping skills in high-risk situations (Connors et al., 1996; Witkiewitz & Marlatt, 2004). Targeting these risk factors during treatment, either pharmacologically (e.g., naltrexone to reduce alcohol craving; Richardson et al., 2008) or behaviorally (e.g., coping skills training; Monti et al., 2001), has become a priority for substance abuse researchers and clinicians.

Katie Witkiewitz, Department of Psychology, Washington State University Vancouver; Sarah Bowen, Department of Psychology, University of Washington.

This research was supported by National Institute on Drug Abuse Grant R21 DA010562 (G. Alan Marlatt, principal investigator). We gratefully acknowledge G. Alan Marlatt for his leadership and support and the mindfulness-based relapse prevention research team for its dedication to this project. Without these efforts and talents, this project would not have been possible.

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Depression, Craving, and Relapse

The significant roles of negative affective states and craving in the substance use relapse process have been described for over 30 years (e.g., Ludwig & Wikler, 1974; Solomon & Corbit, 1974). Craving, the subjective experience of an urge or desire to use substances (Kozlowski & Wilkinson, 1987), has been shown to strongly predict reinstatement of substance use for all major drugs of abuse (e.g., Hartz, Frederick-Osborne, & Galloway, 2001; Hopper et al., 2006; Shiffman et al., 2002). Negative affect has been shown to be a prominent cue for craving in both laboratory and clinical studies (e.g., Cooney, Litt, Morse, Bauer, & Gaupp, 1997; Perkins & Grobe, 1992; Shiffman & Waters, 2004; Sinha & O'Malley, 1999; Stewart, 2000; Wheeler et al., 2008), and both the experience of negative affective states and the desire to avoid these aversive states have been described as primary motives for substance use (e.g., Wikler, 1948). Depressive symptomatology has been linked to reinstitution of drug use following periods of abstinence (e.g., Curran, Booth, Kirchner, & Denke, 2007; Witkiewitz & Villarroel, 2009), and self-reported depression has been shown to predict substance use treatment outcomes (e.g., Cornelius et al., 2004; Greenfield et al., 1998; Hodgins, el-Guebaly, & Armstrong, 1995).

The relation between depression and substance use is also evident in the disproportionately higher rates of substance use relapse in individuals with affective disorders (Conner, Sorensen, & Leonard, 2005; Hasin & Grant, 2002; Kool et al., 2008).

168 clients of a public service agency providing treatment for alcohol and substance use disorders.

MBRP : Randomly assigned to treatment as usual (0) or mindfulness-based relapse prevention therapy (1)

BDI0: Beck Depression Inventory at start of therapy (0 to 3; multiply by 21 to see BDI in its original 0 to 63 metric). This is also available at the termination of therapy (**BDIP**)

CRAVE2: Score on the Penn Alcohol Craving Scale at 2 month follow-up (0 to 6). Also available at baseline, prior to start of therapy (**CRAVE0**)

USE4: Alcohol and other substance use at 4-month follow-up. (0 to 5)

TREATHRS: Hours of therapy administered.

The data file is MBRP

The Data: MBRP

SPSS

mbrp.sav [DataSet1] - IBM SPSS Statistics Data Editor

	mbrp	bdi0	bdip	crave0	crave2	use4
1	0	1.28	1.09	4.0	.8	.95
2	0	1.47	1.52	2.4	3.8	1.30
3	0	.66	1.14	2.2	1.4	1.09
4	1	1.66	1.23	2.2	2.4	1.17
5	0	1.28	.85	4.2	2.4	.91
6	1	.95	1.04	1.0	1.0	1.31
7	0	1.38	.85	2.0	.8	.36
8	0	1.76	.95	3.2	2.0	.83
9	0	.80	.71	3.0	1.2	.57
10	1	1.47	1.14	1.2	1.6	1.24
11	1	1.38	2.00	2.4	2.6	1.54
12	0	1.00	.61	1.0	.6	.63
13	1	1.38	1.66	3.8	1.2	.54
14	1	1.09	.95	1.8	1.2	1.81

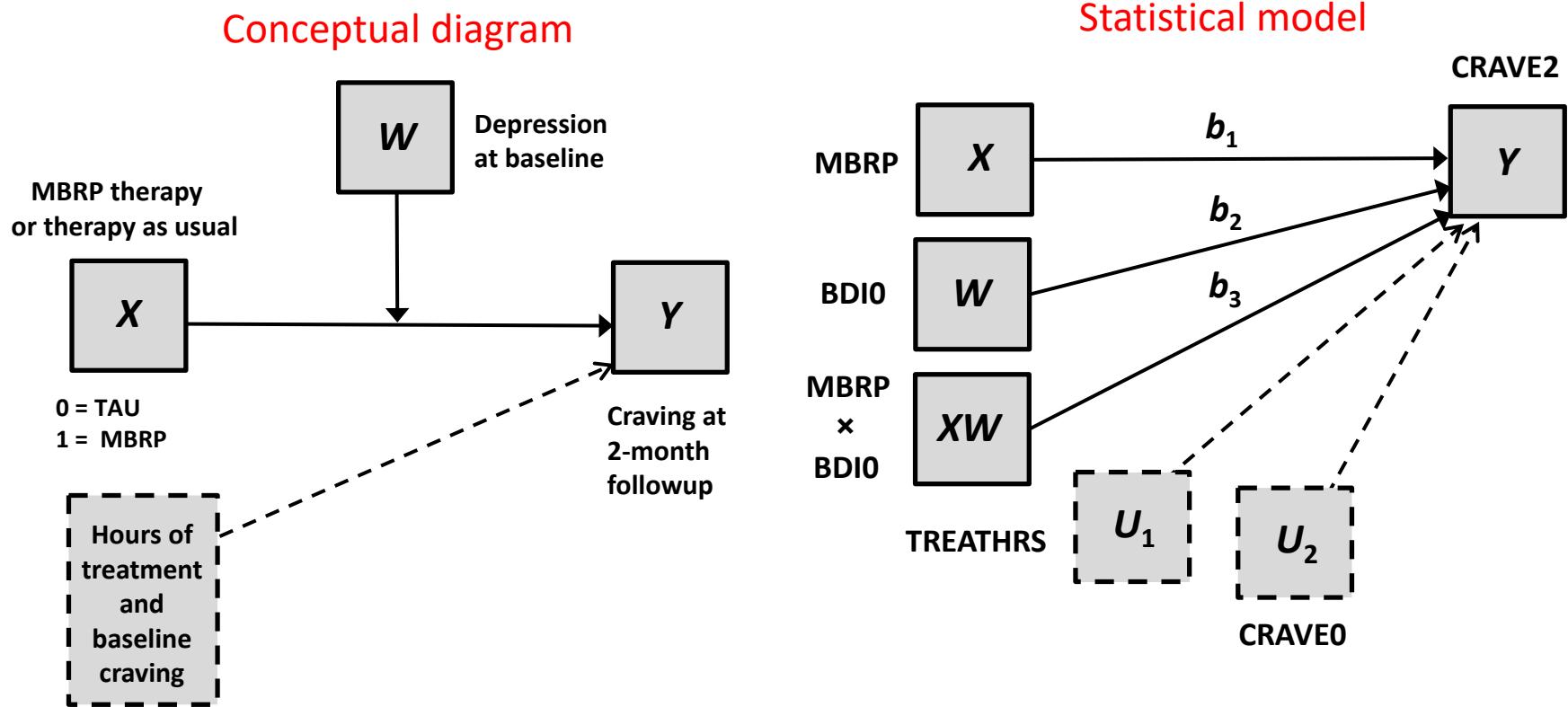
SAS

```
mbrp *
```

```
data mbrp;
input mbrp bdi0 bdip crave0 crave2 use4 treathrs;
datalines;
0 1.28 1.09 4.0 .8 .95 32
0 1.47 1.52 2.4 3.8 1.30 36
0 .66 1.14 2.2 1.4 1.09 34
1 1.66 1.23 2.2 2.4 1.17 37
0 1.28 .85 4.2 2.4 .91 27
1 .95 1.04 1.0 1.0 1.31 32
0 1.38 .85 2.0 .8 .36 38
0 1.76 .95 3.2 2.0 .83 26
0 .80 .71 3.0 1.2 .57 42
1 1.47 1.14 1.2 1.6 1.24 32
1 1.38 2.00 2.4 2.6 1.54 25
0 1.00 .61 1.0 .6 .63 24
1 1.38 1.66 3.8 1.2 .54 35
1 1.09 .95 1.8 1.2 1.81 34
```

These aren't their actual data. But the analyses we do yield similar results to what they report.

Example



Does the effect of MBRP therapy relative to therapy as usual on craving depend on initial depression? That is, is the therapy more or less effective as a function of depression prior to start of therapy?

Estimation using OLS regression

```
compute mbrpdep = mbrp*bdi0.
regression/dep = crave2/method = enter mbrp bdi0 mbrpdep treathrs crave0.
```

```
data mbrp; set mbrp; mbrpdep=mbrp*bdi0; run;
proc reg data=mbrp; model crave2=mbrp bdi0 mbrpdep treathrs crave0; run;
```

X = MBRP
W = BDI0
Y = CRAVE2

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \dots$$

Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.
	B	Std. Error			
1 (Constant)	1.038	.470		2.209	.029
MBRP: Therapy as usual (0) or MBRP therapy (1)	.587	.524	.299	1.120	.264
BDI0: Beck Depression Inventory baseline	1.122	.276	.366	4.063	.000
mbrpdep	-.948	.423	-.598	-2.240	.026
TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

a. Dependent Variable: CRAVE2: Craving at two month follow-up

Output G

“conditional effects”, not
“main effects”

$$\begin{aligned}b_1 &= 0.587 \\b_2 &= 1.122 \\b_3 &= -0.948\end{aligned}$$

The coefficient for the product is statistically different from zero. This means that the effect of MBRP therapy on craving depends on the person's level of depression at the start of therapy. But to really understand what is happening, we need a picture.

Visualizing the model

Rejecting the null hypothesis that “true b_3 ” is equal to zero tells you that the focal predictor’s effect is indeed moderated by the proposed moderator. But moderation can take many different forms. We need to visualize the effect in order to interpret the result.

Step 1: Select various combinations of values of the focal predictor and moderator. The selection is sometimes arbitrary, but it may not be. Just make sure the values chosen are within the range of the data.

Step 2: Using the model, generate the estimates of Y using your selected values of the focal predictor and moderator. If your model includes covariates, use the sample mean for each of those.

Step 3: Graph, using whatever graphics program you prefer.

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW - 0.018U_1 + 0.192U_2$$

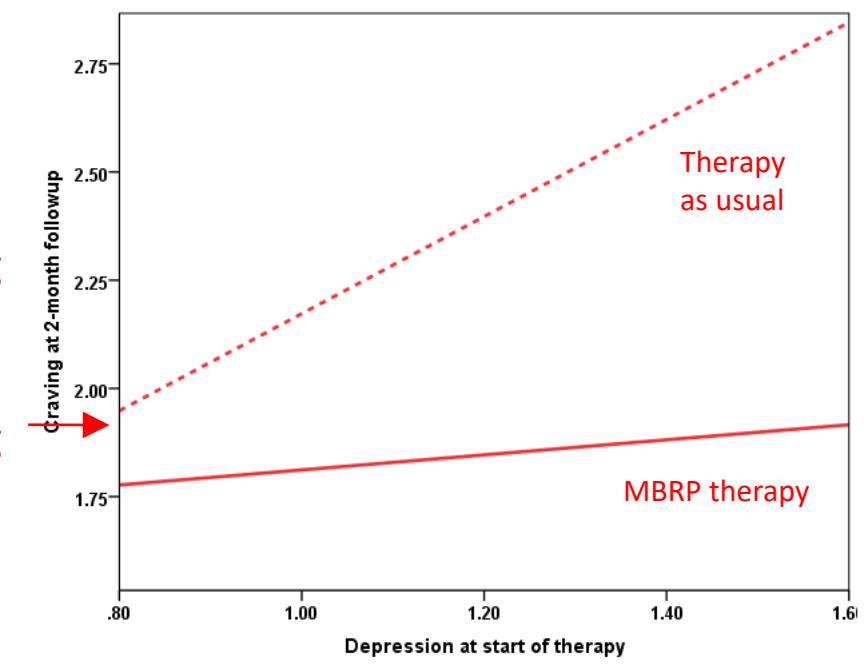
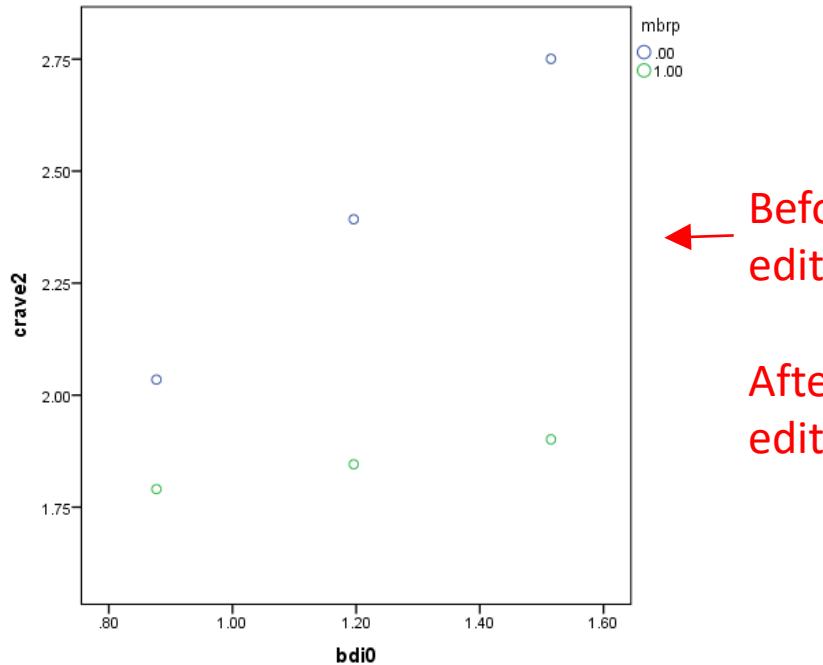
MBRP (X)	BDI0 (W)	TREATHRS (U_1)	CRAVE0(U_2)	\hat{Y}
0	0.877	30.685	2.943	2.035
0	1.196	30.685	2.943	2.393
0	1.515	30.685	2.943	2.751
1	0.877	30.685	2.943	1.790
1	1.196	30.685	2.943	1.846
1	1.515	30.685	2.943	1.901

I used one standard deviation below the mean, the mean, and one standard deviation above the mean. It really makes no difference what you choose, except you want to make sure that your resulting graph is not extrapolating beyond the available data.



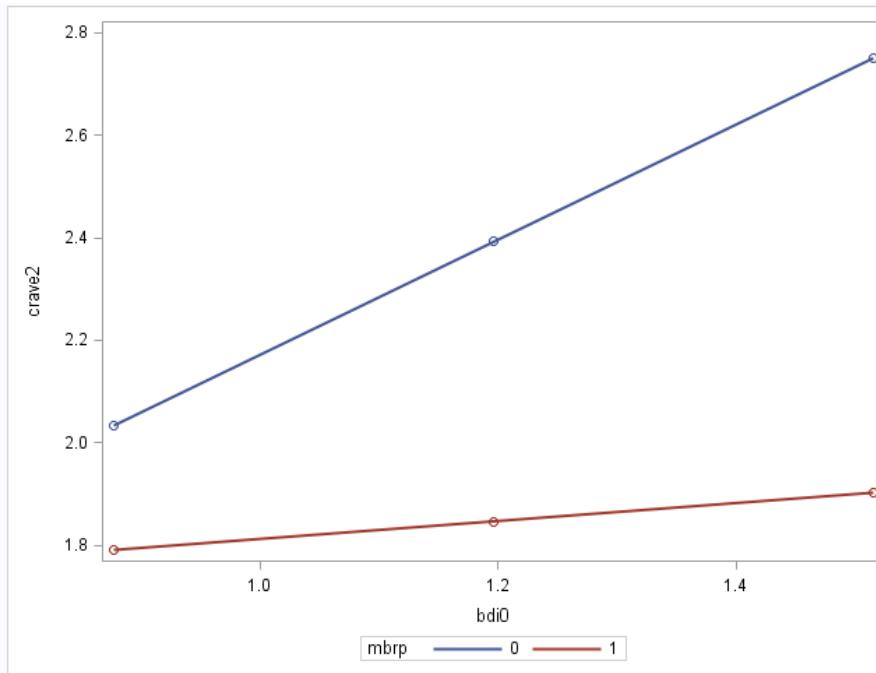
Example code in SPSS

```
data list free/mbrp bdi0.  
begin data.  
0 0.877  
0 1.196  
0 1.515  
1 0.877  
1 1.196  
1 1.515  
end data.  
compute crave2=1.038+0.587*mbrp+1.122*bdi0-0.948*mbrp*bdi0-0.018*30.685+0.192*2.943.  
graph/scatterplot = bdi0 with crave2 by mbrp.
```



Example code in SAS

```
data;
input mbrp bdi0;
crave2=1.038+0.587*mbrp+1.122*bdi0-0.948*mbrp*bdi0-0.018*30.685+0.192*2.943;
datalines;
0  0.877
0  1.196
0  1.515
1  0.877
1  1.196
1  1.515
run;
proc sgplot;reg x=bdi0 y=crave2/group=mbrp;run;
```



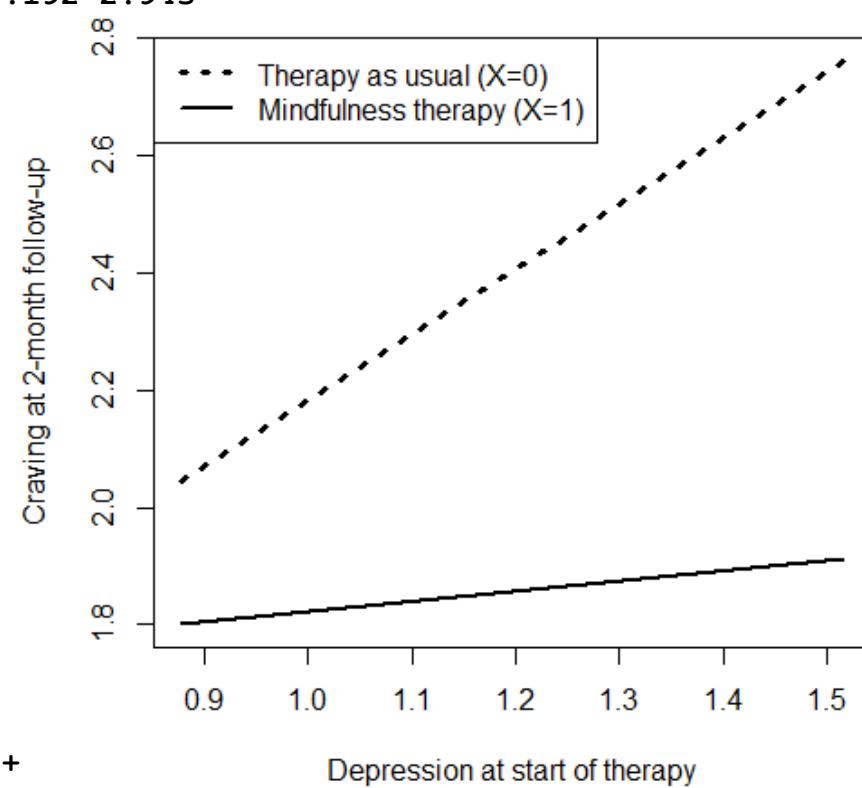
Example code in R

Although hard to learn at first, once you learn how to use R, you will find it very helpful in the construction of visual depictions of models.

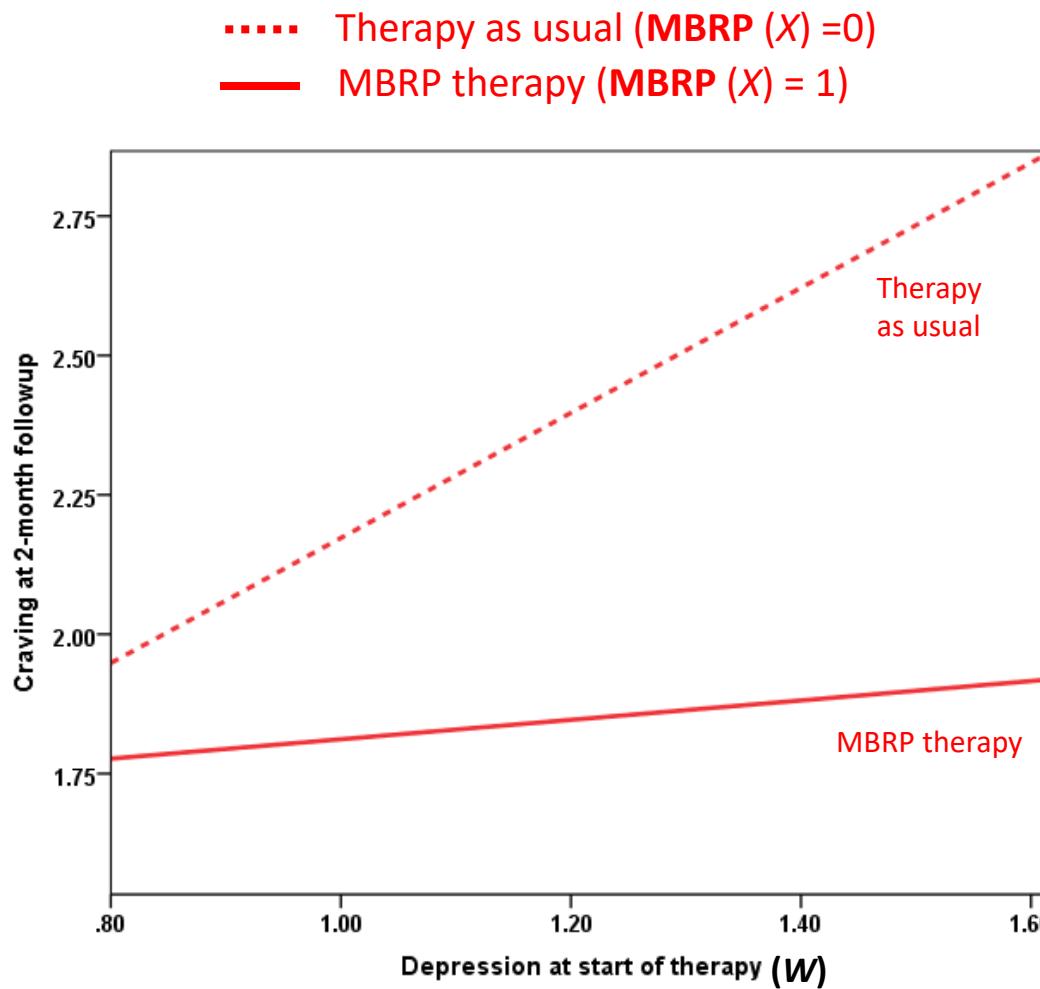
```
x<-c(0,1,0,1,0,1)
w<-c(0.877,0.877,1.196,1.196,1.515,1.515)
y<-1.038+0.587*x+1.122*w-0.948*x*w-0.018*30.685+0.192*2.943
plot(y=y,x=w,pch=15,col="white",
xlab="Depression at start of therapy",
ylab="Craving at 2-month follow-up")
legend.txt<-c("Therapy as usual (X=0)",
"Mindfulness therapy (X=1)")
legend("topleft",legend=legend.txt,
lty=c(3,1),lwd=c(3,2))
lines(m[x==0],y[x==0],lwd=3,lty=3)
lines(m[x==1],y[x==1],lwd=2,lty=1)
```

OR

```
library(ggplot2)
qplot(x = w, y = y, linetype = as.factor(x),
      geom = "line")+
  xlab("Depression at start of therapy")+
  ylab("Craving at 2-month follow-up")+
  scale_linetype_discrete(name=element_blank(),
    breaks=c("0", "1"),
    labels=c("Therapy as Usual (X = 0)",
            "Mindfulness Therapy (X = 1)"))+
  theme(legend.justification=c(-0.1,1.1),
        legend.position=c(0,1),
        panel.background = element_rect("white", "black"),
        panel.grid.major = element_blank())
```



Substantive interpretation of the pattern



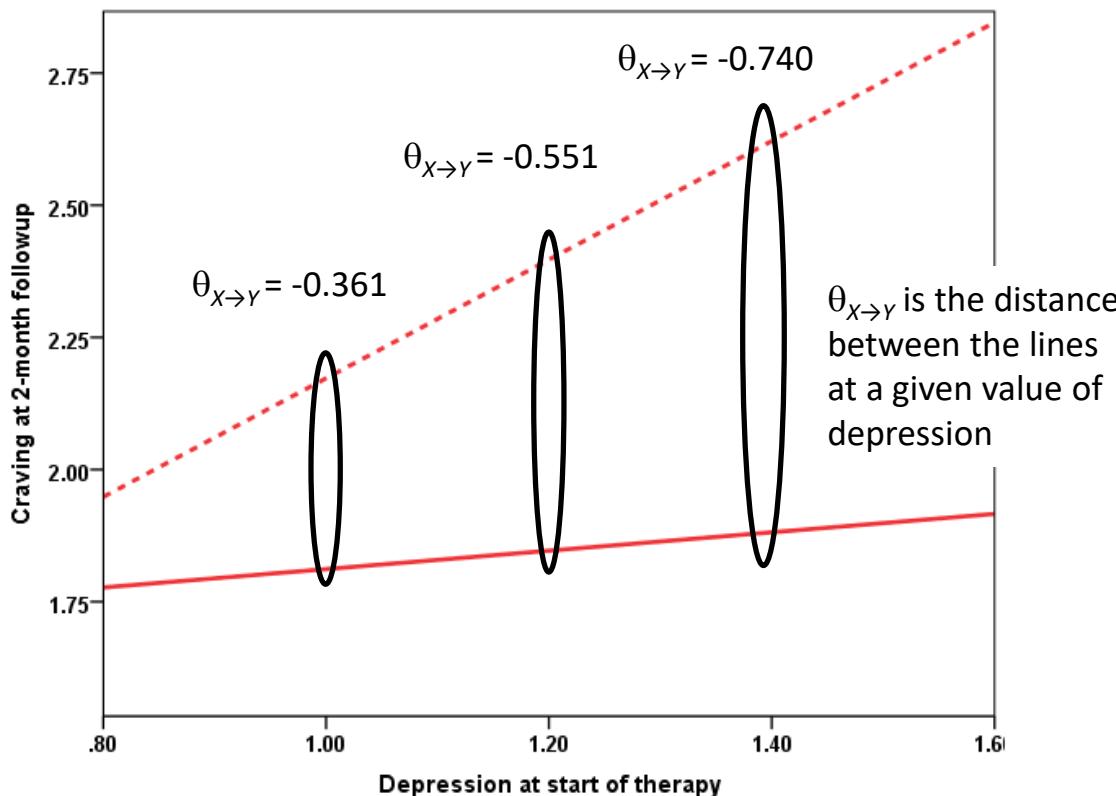
Those who receive MBRP therapy crave substances less than those who receive therapy as usual, but this difference is larger among those more depressed at the start of therapy.

A graphical depiction of the model

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \dots \quad \text{or, equivalently,}$$

Therapy as usual (MBRP ($X = 0$)
MBRP therapy (MBRP ($X = 1$)

$$\hat{Y} = 1.038 + (0.587 - 0.948W)X + 1.122W + \dots$$



The conditional effect of MBRP therapy ($\theta_{x \rightarrow y}$) is defined by the function

$$\theta_{x \rightarrow y} = 0.587 - 0.948W$$

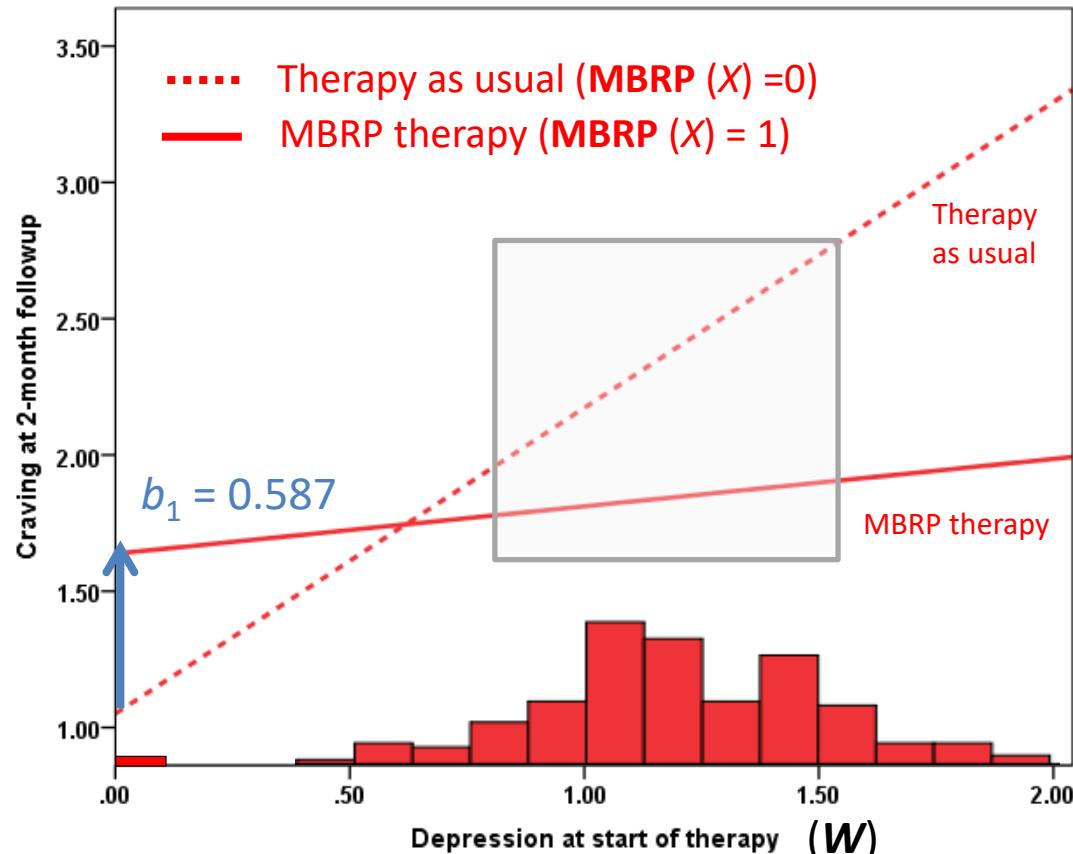
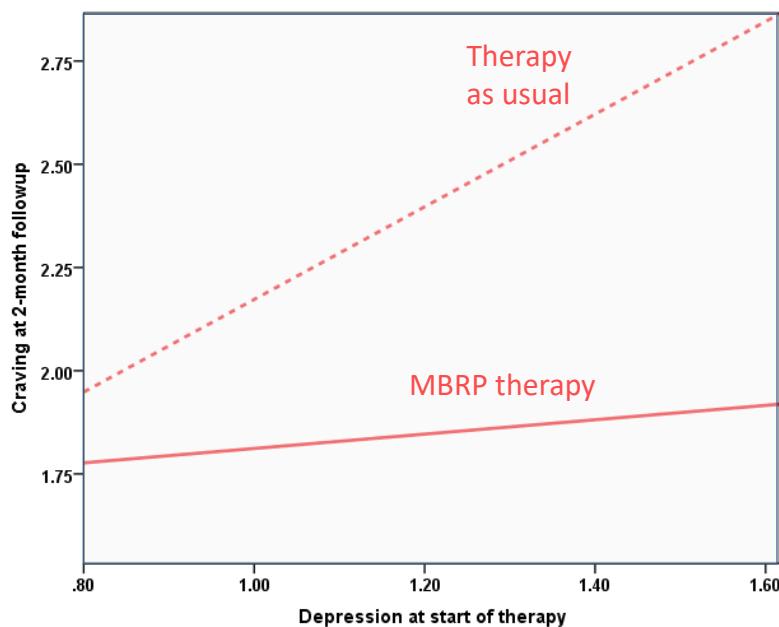
BDI0 (W)	$\theta_{x \rightarrow y}$
1.00	-0.361
1.20	-0.551
1.40	-0.740

You can plug any value of BDI0 you want into the function to get the conditional effect of MBRP therapy

Interpretation of b_1

$$b_1 = 0.587$$

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \dots$$



b_1 is the effect of X on Y when $W = 0$. It is a conditional effect, and a local term of the model.

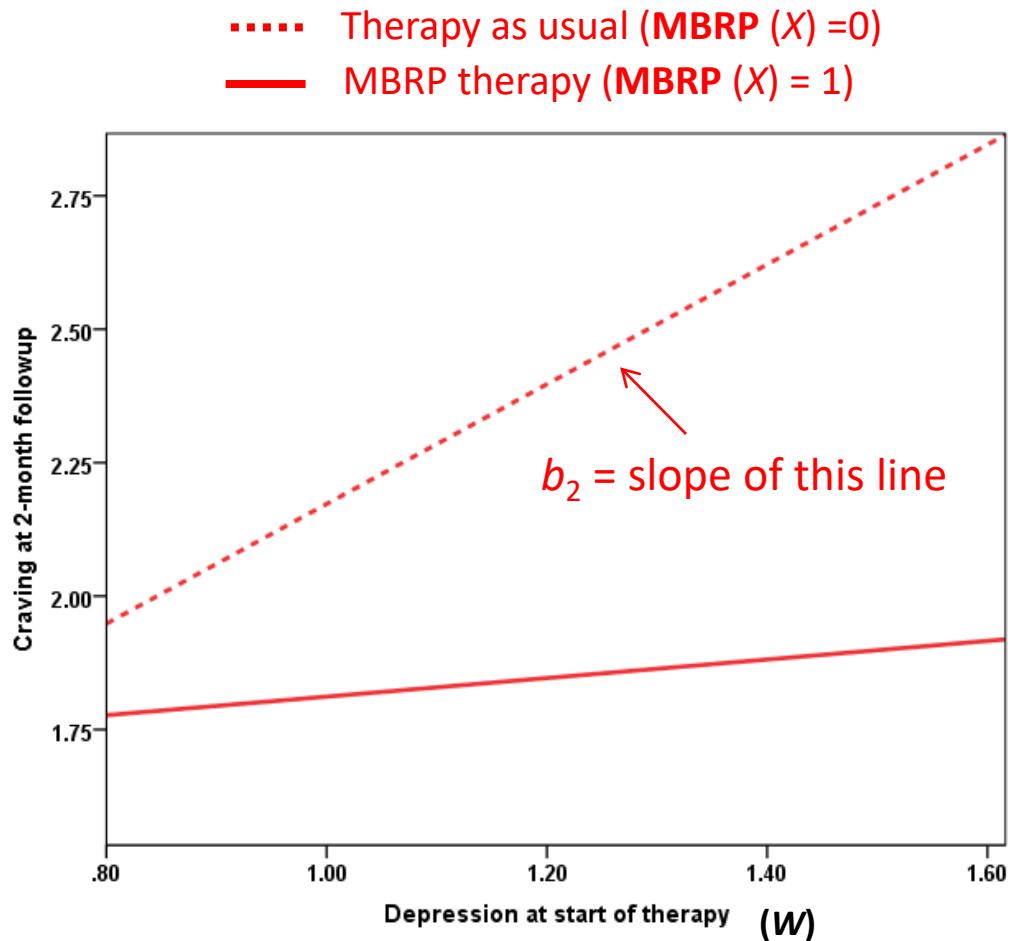
Interpretation of b_2

$$b_2 = 1.122$$

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \dots$$

b_2 is the conditional effect of W when $X = 0$. It is a conditional effect and a local term of the model.

Among those given therapy as usual, those who were relatively more depressed at the start of therapy had relatively higher craving at two months follow-up



Interpretation of b_3

$$b_3 = -0.948$$

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \dots$$

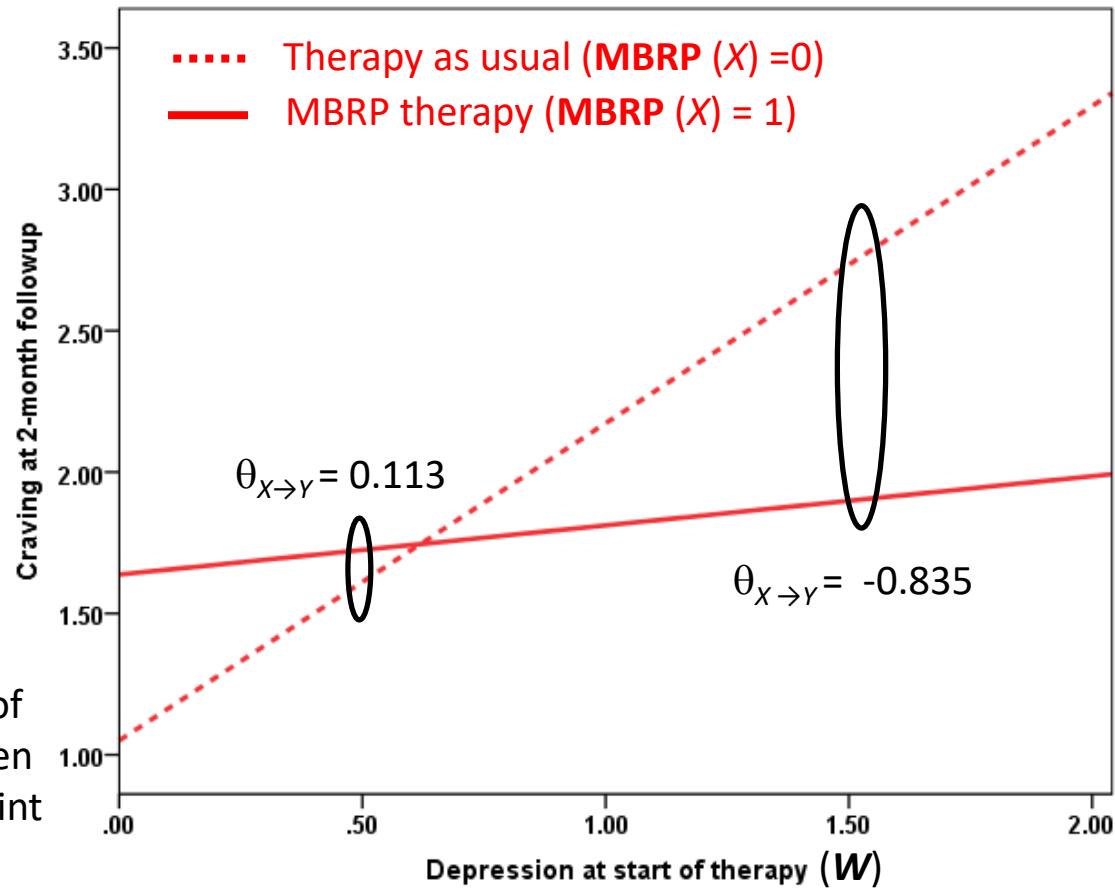
$$\theta_{X \rightarrow Y} = 0.587 - 0.948W$$

W	$\theta_{X \rightarrow Y}$
0.50	0.113
1.00	-0.361
1.50	-0.835
2.00	-1.309

$$-0.835 - (0.113) = -0.948 = b_3$$

$$-1.309 - (-0.361) = -0.948 = b_3$$

b_3 is the difference in the effect of MBRP therapy on craving between those who differ by one scale point in their pre-therapy depression.



$$\theta_{X \rightarrow Y}|(W = \lambda+1) - \theta_{X \rightarrow Y}|(W = \lambda) = -0.948, \text{ for all } \lambda$$

Interpretation of b_3

$$b_3 = -0.948$$

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \dots$$

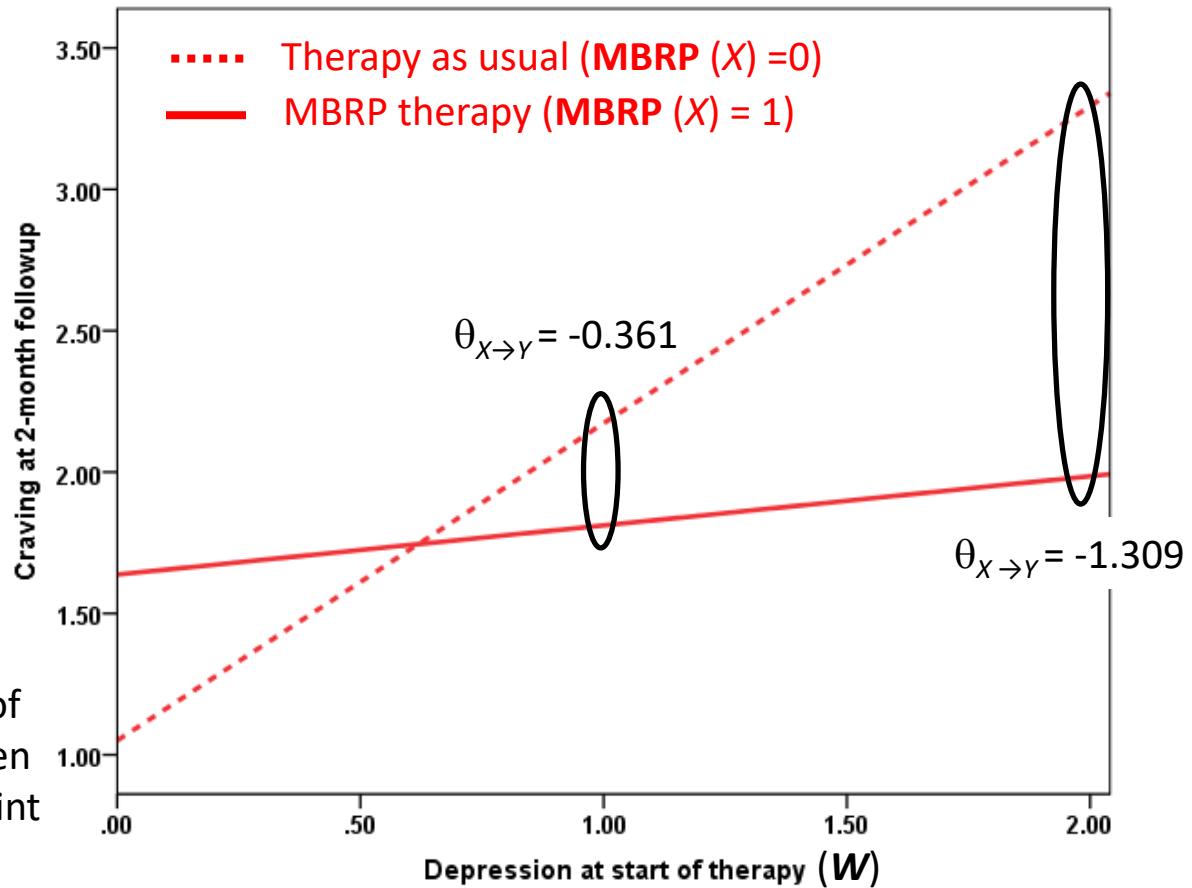
$$\theta_{X \rightarrow Y} = 0.587 - 0.948W$$

W	$\theta_{X \rightarrow Y}$
0.50	0.113
1.00	-0.361
1.50	-0.835
2.00	-1.309

$$-0.835 - (0.113) = -0.948 = b_3$$

$$-1.309 - (-0.361) = -0.948 = b_3$$

b_3 is the difference in the effect of MBRP therapy on craving between those who differ by one scale point in their pre-therapy depression.



$$\theta_{X \rightarrow Y}|(W=\lambda+1) - \theta_{X \rightarrow Y}|(W = \lambda) = -0.948, \text{ for all } \lambda$$

Interpretation of b_3

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \dots$$

$$b_3 = -0.948$$

$$\theta_{x \rightarrow y} = 0.587 - 0.948W$$

b_3 is the difference in the effect of MBRP therapy on craving between those who differ by one scale point in their pre-therapy depression.

$$\theta_{x \rightarrow y}|(W=\lambda) - \theta_{x \rightarrow y}|(W = \lambda - 1) = -0.948, \text{ for all } \lambda$$

Select a value of λ then calculate the conditional effect of X on Y at $\lambda-1$, λ , and the difference between the two. Then answer the questions below.

λ	$\theta_{x \rightarrow y} (W=\lambda)$	$\theta_{x \rightarrow y} (W = \lambda - 1)$	$\theta_{x \rightarrow y} (W=\lambda) - \theta_{x \rightarrow y} (W = \lambda - 1)$

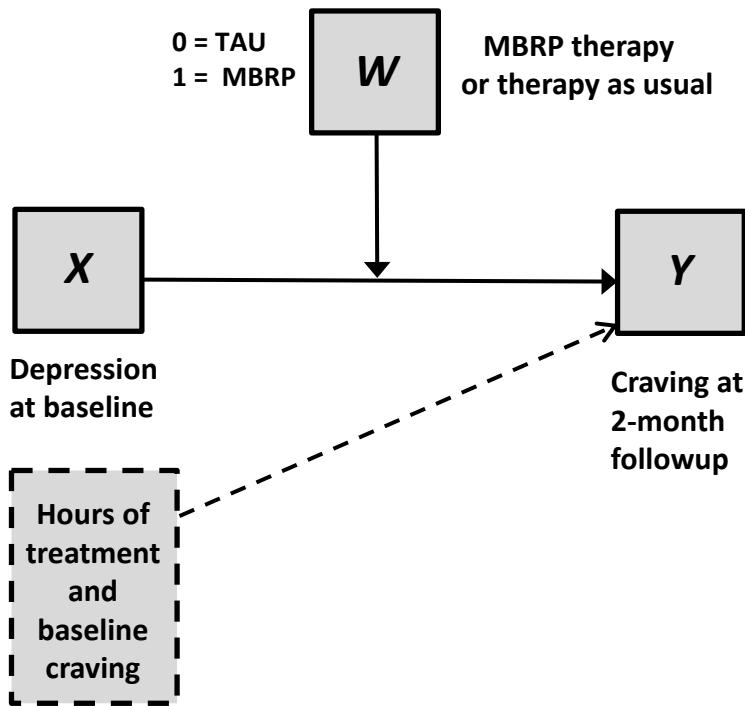
1. Write an interpretation for $\theta_{x \rightarrow y}|(W=\lambda)$ given your specific value of λ .
2. Write an interpretation for $\theta_{x \rightarrow y}|(W = \lambda - 1)$ given your specific value of λ .
3. Write an interpretation for $\theta_{x \rightarrow y}|(W=\lambda) - \theta_{x \rightarrow y}|(W = \lambda - 1)$ given your specific value of λ .
4. Use your own words to describe what it means that $\theta_{x \rightarrow y}|(W=\lambda) - \theta_{x \rightarrow y}|(W = \lambda - 1) = -0.948$ for all λ .

λ	$\theta_{x \rightarrow y} (W=\lambda)$	$\theta_{x \rightarrow y} (W = \lambda-1)$	$\theta_{x \rightarrow y} (W=\lambda) - \theta_{x \rightarrow y} (W = \lambda-1)$
3	0.587 – 0.948(3) =-2.257	0.587-.948(2) = -1.309	-2.257-(-1.309) = -.948

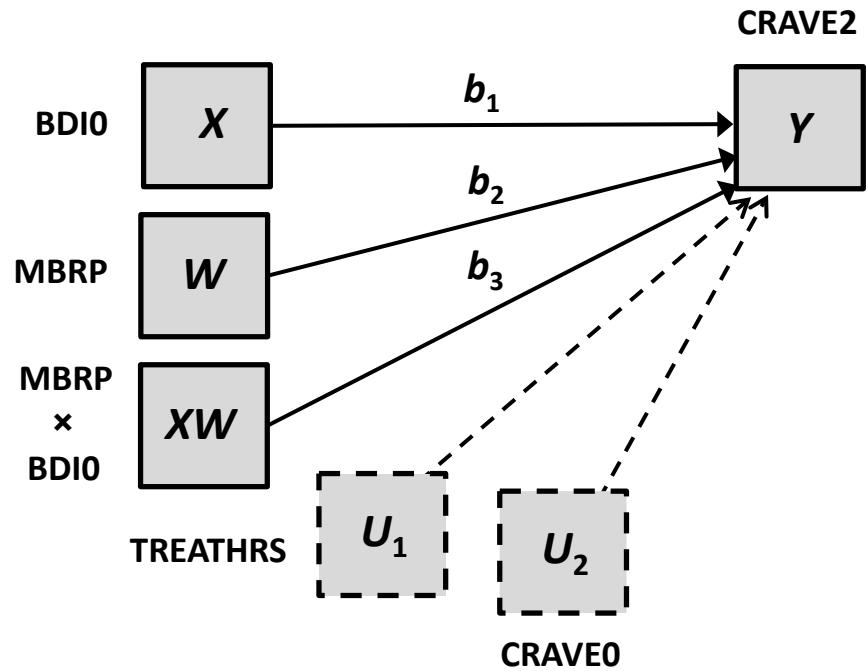
1. Write an interpretation for $\theta_{x \rightarrow y}|(W=\lambda)$ given your specific value of λ . Cravings are 2.257 units lower in the MPBR therapy condition compared to the TAU condition, when pre-therapy depression level is 3.
1. Write an interpretation for $\theta_{x \rightarrow y}|(W = \lambda-1)$ given your specific value of λ . Cravings are 1.309 units lower in the MPBR therapy condition compared to the TAU condition, when pre-therapy depression level is 2.
1. Write an interpretation for $\theta_{x \rightarrow y}|(W=\lambda) - \theta_{x \rightarrow y}|(W = \lambda-1)$ given your specific value of λ . The effect of MBRP therapy (relative to TAU) is 0.948 units more effective at decreasing cravings for the pretherapy depression group with score 3 compared to those with score 2.
1. Use your own words to describe what it means that $\theta_{x \rightarrow y}|(W=\lambda) - \theta_{x \rightarrow y}|(W = \lambda-1) = -.948$ for all λ . The effect of MBRP therapy (relative to TAU) is 0.948 units more effective at decreasing cravings for the pretherapy depression group with a one unit higher score than another group.

A Dichotomous Moderator

Conceptual diagram



Statistical model



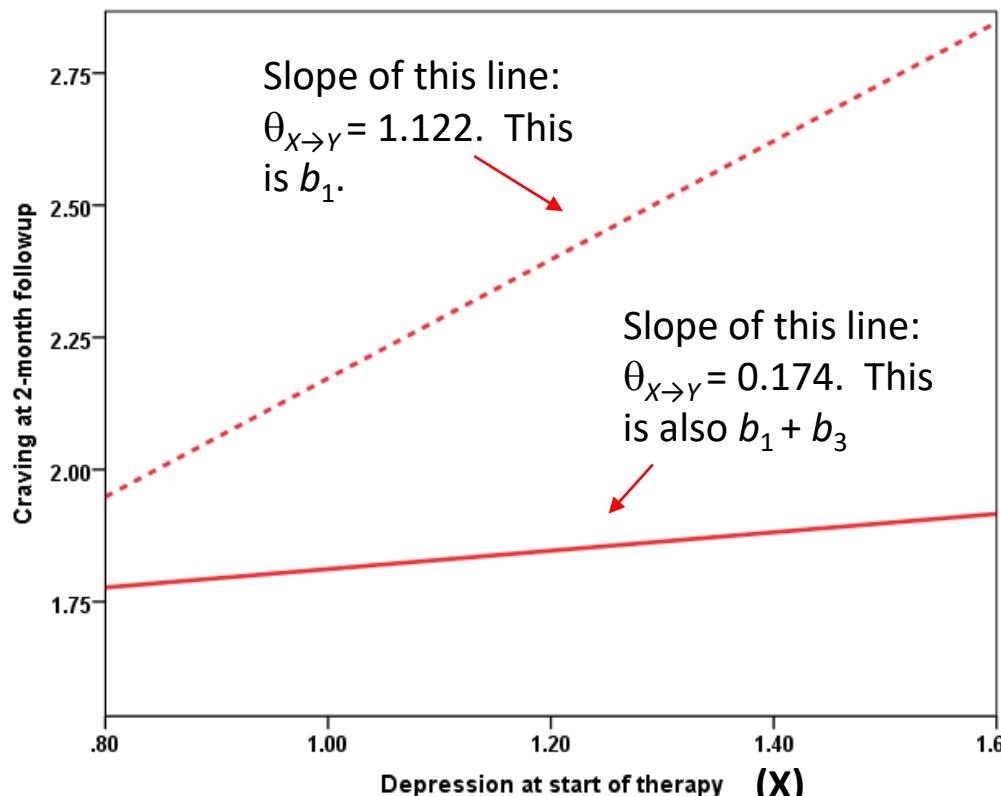
Does the association between pre-treatment depression and later craving differ between those who receive MBRP therapy versus therapy as usual?

A graphical depiction of the model

$$\hat{Y} = 1.038 + 1.122X + 0.587W - 0.948XW + \dots \quad \text{or, equivalently,}$$

- Therapy as usual (**MBRP (W) = 0**)
- MBRP therapy (**MBRP (W) = 1**)

$$\hat{Y} = 1.038 + (1.122 - 0.948W)X + 0.587W + \dots$$



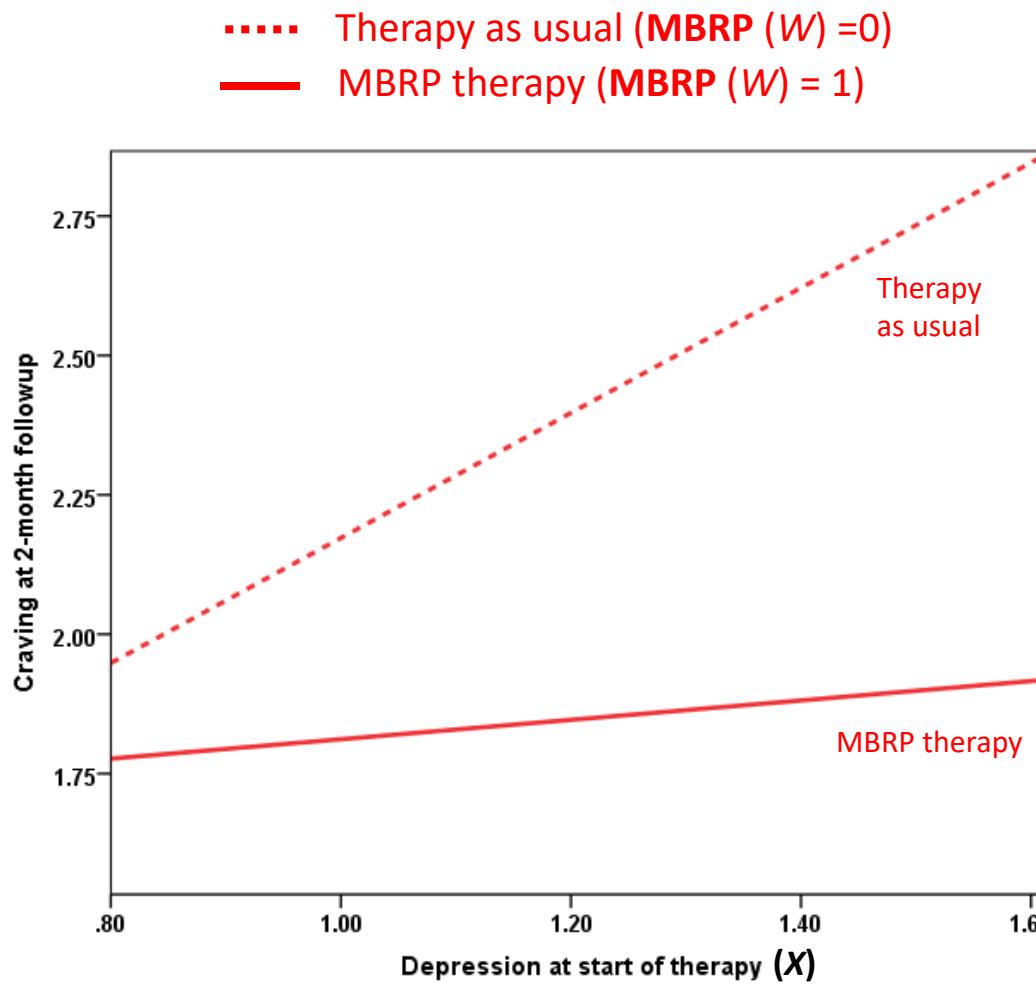
The conditional effect of pre-therapy depression ($\theta_{X \rightarrow Y}$) is defined by the function

$$\begin{aligned}\theta_{X \rightarrow Y} &= b_1 + b_3 W \\ &= 1.122 - 0.948W\end{aligned}$$

MBRP (W)	$\theta_{X \rightarrow Y}$
0	1.122
1	0.174

Two cases that differ by one unit on W are estimated to differ by $\theta_{X \rightarrow Y}$ units on Y. $\theta_{X \rightarrow Y}$ depends on W.

Substantive interpretation of the pattern

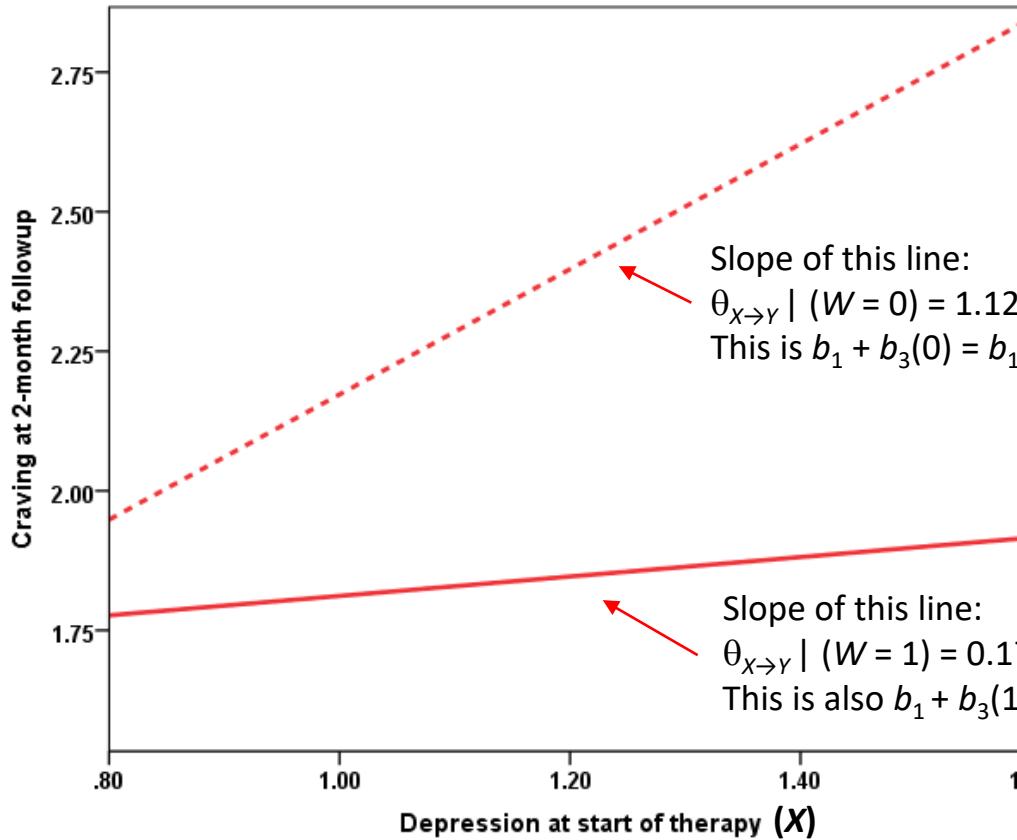


A larger effect of pre-therapy depression on later craving among those who experienced therapy as usual compared to those who received mindfulness behavioral relapse prevention therapy. MBRP therapy seems to have disrupted the link between depression and craving.

Interpreting b_3

- Therapy as usual (MBRP ($W = 0$)
- MBRP therapy (MBRP ($W = 1$)

$$\begin{aligned}\hat{Y} &= 1.038 + 1.122X + 0.587W - 0.948XW \\ &= 1.038 + (1.122 - 0.948W)X + 0.587W + \dots\end{aligned}$$



$$\begin{aligned}\theta_{X \rightarrow Y} | W &= b_1 + b_3 W \\ \theta_{X \rightarrow Y} | W &= 1.122 - 0.948 W\end{aligned}$$

$$\begin{aligned}b_3 &= \theta_{X \rightarrow Y} | (W = 1) - \theta_{X \rightarrow Y} | (W = 0) \\ &= (b_1 + b_3) - (b_1) \\ &= (0.174) - (1.122) \\ &= -0.948\end{aligned}$$

So b_3 is the difference in the slopes of these two lines. As W increases by one unit, $\theta_{X \rightarrow Y}$ decreases by 0.948 units. This difference is statistically different from zero.

Probing an interaction

The coefficient for the product term carries information about how changes in one variable are related to changes in the effect of the other. A picture helps to understand how the focal variable's effect changes as a function of the moderator variable.

It is typically desirable to conduct statistical tests of the focal predictor variable's effect at values of the moderator. This allows you to make more definitive claims about where the focal predictor variables effect is zero versus where it is not.

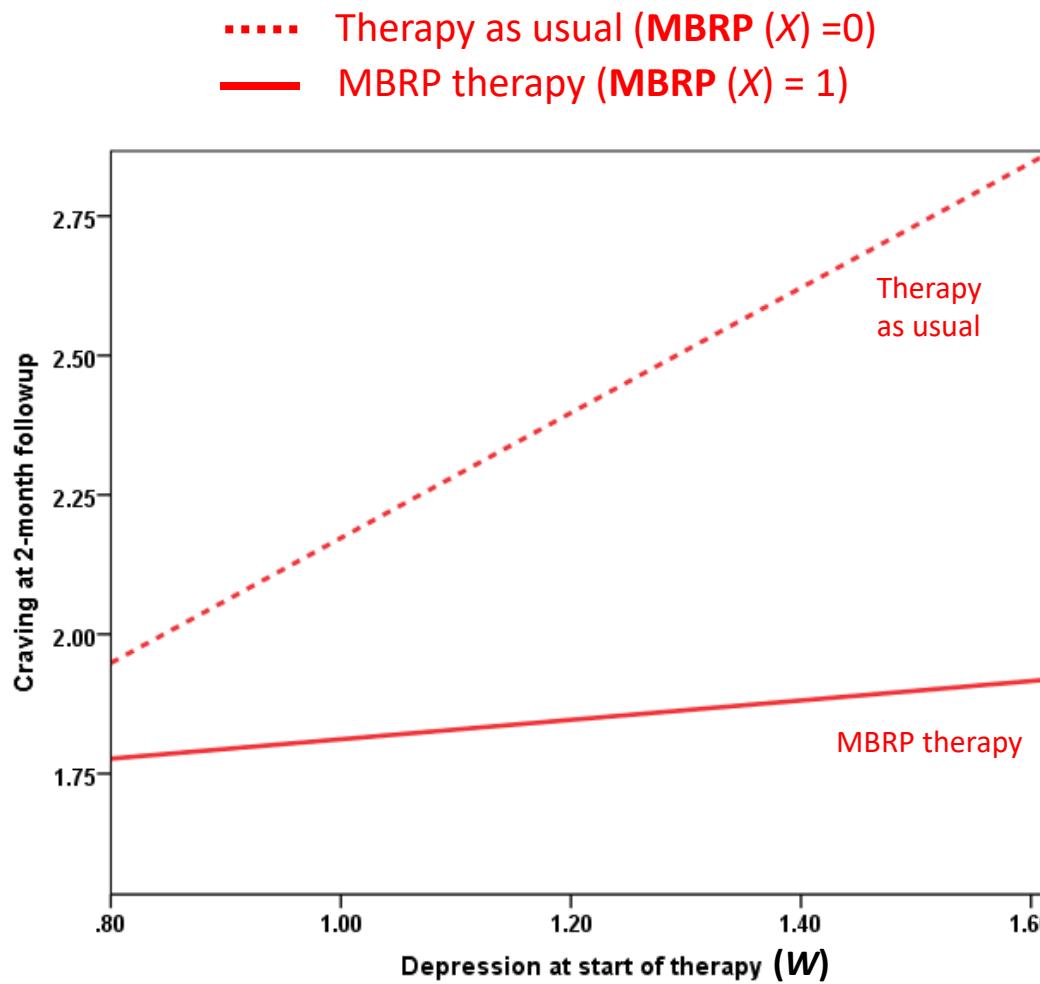
“Pick-a-Point” Approach

Select values of the moderator and estimate the conditional effect of the focal predictor at those values of the moderator, along with a hypothesis test or confidence interval.

Johnson-Neyman Technique

Derive mathematically where on the moderator variable continuum the focal variable's effect transitions between statistically significant and nonsignificant.

Substantive interpretation of the pattern



Those who receive MBRP therapy crave substances less than those who receive therapy as usual, and this difference is larger among those more depressed at the start of therapy.

Pick-a-point approach

$$\hat{Y} = b_0 + b_1X + b_2W + b_3XW$$

Select a value of the moderator (W) at which you'd like to have an estimate of $\theta_{X \rightarrow Y}$, the focal predictor variable's (X) effect. Then derive its standard error. The ratio of the effect to its standard error is distributed as $t(df_{\text{residual}})$ under the null hypothesis that the effect of the focal predictor is zero at that moderator value, where df_{residual} is the residual degrees of freedom from the regression model.

We already know that

$$\theta_{X \rightarrow Y} = b_1 + b_3W$$

The estimated standard error of $\theta_{X \rightarrow Y}$ is

$$s_{\theta_{X \rightarrow Y}} = \sqrt{s_{b_1}^2 + 2W s_{b_1 b_3} + W^2 s_{b_3}^2}$$

Squared standard error of b_1 Covariance of b_1 and b_3 Squared standard error of b_3

You could do this by hand, and instructions are available in various books on regression analysis (e.g., Aiken and West, 1991; Cohen et al., 2003). But there is no reason to, and the potential for mistakes is high. It is made easier using “**regression centering**.”

Pick-a-point: Regression centering approach

$$\hat{Y} = b_0 + b_1X + b_2W + b_3XW$$

In the above model, b_1 estimates the conditional effect of X when $W = 0$. If we desire the conditional effect of X when W equals some value λ , we can produce a new variable W' that is W centered around λ , such that $W' = 0$ when $W = \lambda$. Then substitute W' for W in the model above. That is, we will estimate

$$\hat{Y} = b_0 + b_1X + b_2(W - \lambda) + b_3X(W - \lambda)$$

as

$$\hat{Y} = b_0 + b_1X + b_2W' + b_3XW' \text{ where } W' = W - \lambda$$

In this model, b_1 is the conditional effect of X when $W' = 0$. But $W' = 0$ when $W = \lambda$. So b_1 estimates the conditional effect of X when $W = \lambda$. A common (but arbitrary) convention is to use $\lambda = \bar{W}$, $\lambda = \bar{W} - SD_W$, and $\lambda = \bar{W} + SD_W$

Pick-a-point: Regression centering approach

```
compute bdi0_p = bdi0-1.196. <
compute interact = bdi0_p*mbrp.
regression/dep = crave2/method = enter mbrp bdi0_p interact treathrs crave0.
```

$\lambda = 1.196$
(the sample mean)

```
data mbrp;set mbrp;
bdi0_p=bdi0-1.196;
interact=bdi0_p*mbrp;
proc reg data=mbrp;model crave2=mbrp bdi0_p interact treathrs crave0;run;
```

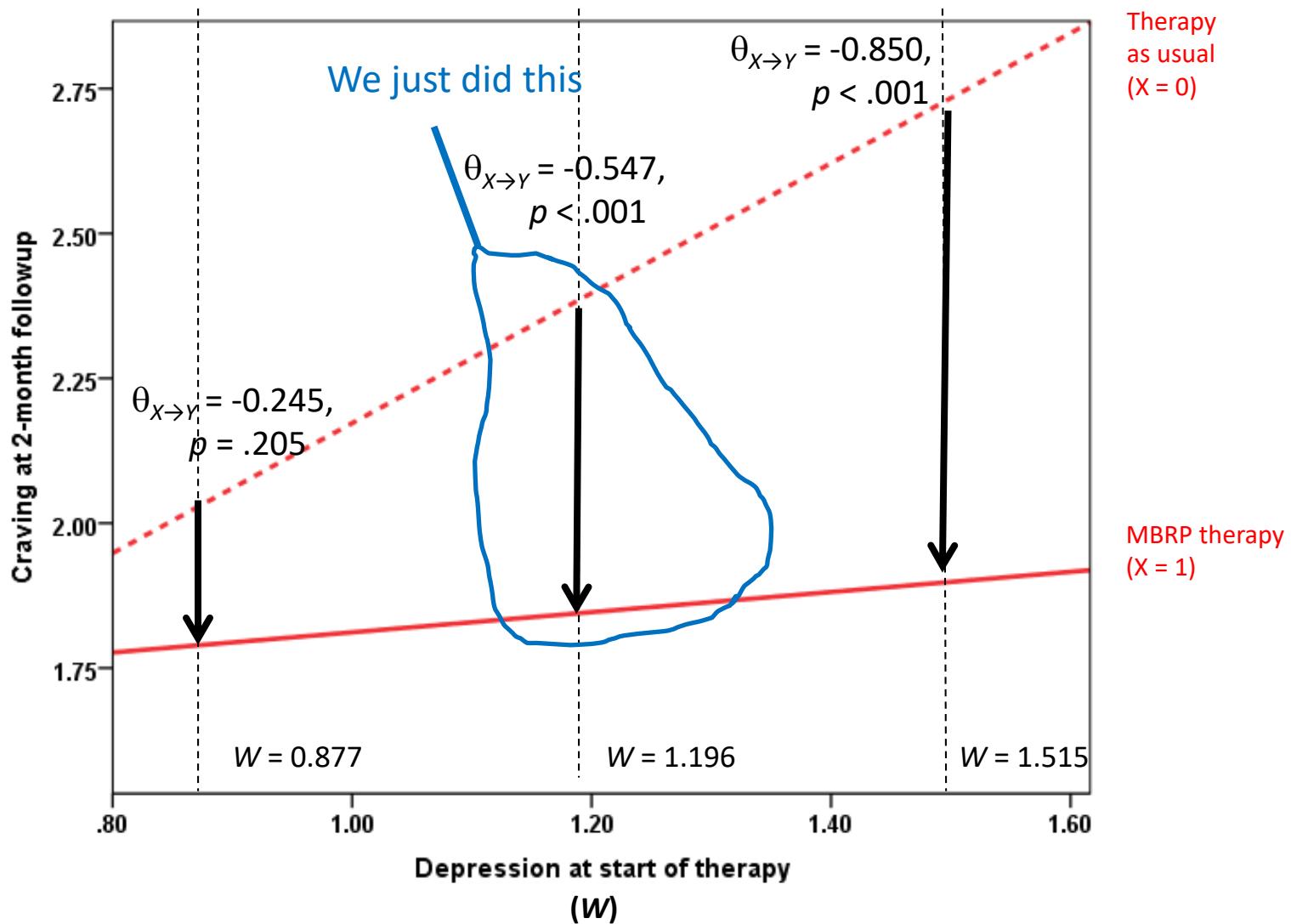
Model	Coefficients ^a					
	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	
	B	Std. Error	Beta			
1 (Constant)	2.380	.364		6.534	.000	
MBRP: Therapy as usual (0) or MBRP therapy (1)	-.547	.137	-.279	-3.980	.000	
bdi0_p	1.122	.276	.366	4.063	.000	
interact	-.948	.423	-.197	-2.240	.026	
TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088	
CRAVE0: Baseline craving	.192	.073	.183	2.614	.010	

a. Dependent Variable: CRAVE2: Craving at two month follow-up

$$\theta_{X \rightarrow Y} |(W = 1.196) \quad S_{\theta_{X \rightarrow Y}}$$

MBRP therapy reduces craving relative to therapy as usual among people “average” in pre-therapy depression, $\theta_{X \rightarrow Y} = -0.547, p < .001$.

Repeat for other values of the moderator



Pick-a-point: Regression centering approach

```
compute bdi0_p = bdi0-0.877.  
compute interact = bdi0_p*mbrp.  
regression/dep = crave2/method = enter mbrp bdi0_p interact treathrs crave0.
```

$\lambda = 0.877$
(One SD below the sample mean)

```
data mbrp;set mbrp;  
bdi0_p=bdi0-0.877;  
interact=bdi0_p*mbrp;  
proc reg data=mbrp;model crave2=mbrp bdi0_p interact treathrs crave0;run;
```

Model	Coefficients ^a				
	B	Std. Error	Standardized Coefficients	t	Sig.
1 (Constant)	2.023	.367		5.506	.000
MBRP: Therapy as usual (0) or MBRP therapy (1)	-.245	.192	-.124	-1.272	.205
bdi0_p	1.122	.276	.366	4.063	.000
interact	-.948	.423	-.242	-2.240	.026
TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

a. Dependent Variable: CRAVE2: Craving at two month follow-up

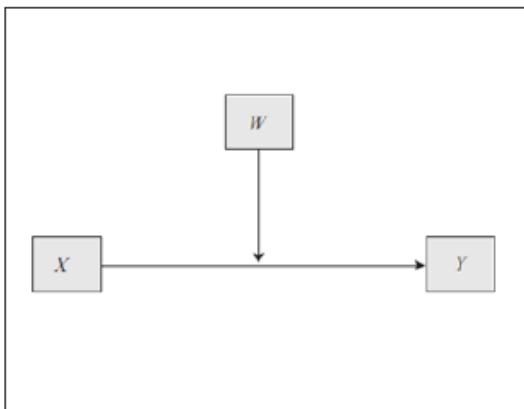
$$\theta_{X \rightarrow Y} |(W = 0.877) \quad S_{\theta_{X \rightarrow Y}}$$

MBRP therapy does not reduce craving relative to therapy as usual among people “relatively low” in pre-therapy depression, $\theta_{X \rightarrow Y} = -0.245$, $p = .21$.

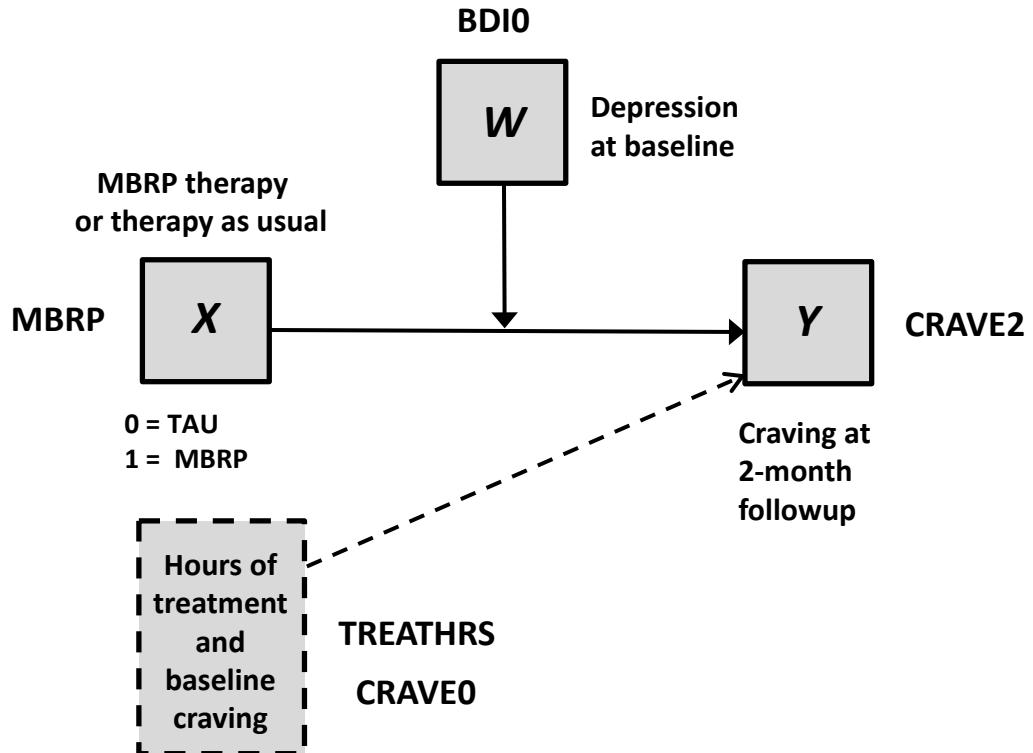
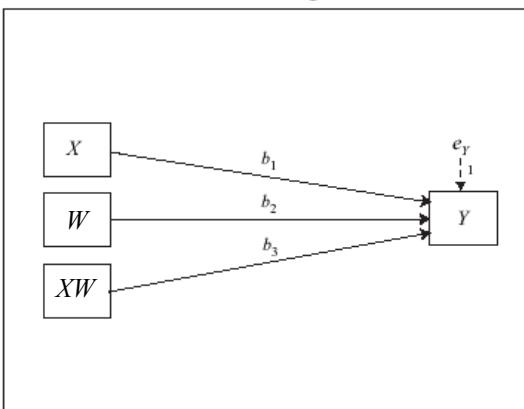
Using PROCESS

Model 1

Conceptual Diagram



Statistical Diagram



```

process cov = treachrs crave0/y=crave2/x=mbrp/w=bdi0
  /model=1/jn=1/plot=1/moments=1.
  
```

```

%process (data=mbrp,cov=treachrs crave0,y=crave2,x=mbrp,
  w=bdi0,model=1,jn=1,plot=1,moments=1);
  
```

PROCESS output

Model : 1
Y : crave2
X : mbrp
W : bdi0

Output H

Covariates:
treathrs crave0

Sample
Size: 168

OUTCOME VARIABLE:

$$\hat{Y} = 1.038 + 0.587X + 1.122M - 0.948XM + \dots$$

PROCESS
generates
the product
term for you.

Model Summary

R	R-sq	MSE	F	df1	df2	P
.5140	.2642	.7277	11.6319	5.0000	162.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	1.0385	.4701	2.2090	.0286	.1102	1.9668
mbrp	.5872	.5241	1.1204	.2642	-.4478	1.6222
bdi0	1.1221	.2762	4.0625	.0001	.5767	1.6675
Int_1	-.9485	.4235	-2.2398	.0265	-1.7847	-.1122
treathrs	-.0177	.0103	-1.7190	.0875	-.0380	.0026
crave0	.1920	.0735	2.6138	.0098	.0470	.3371

Product terms key:

Int_1 : mbrp x bdi0

Test(s) of highest order unconditional interaction(s):

	R2-chng	F	df1	df2	P
X*W	.0228	5.0166	1.0000	162.0000	.0265

PROCESS output

PROCESS sees that the moderator is quantitative (because it has more than 2 values) so it automatically implements the pick-a-point procedure. When moments = 1 moderator values equal to the mean of the moderator as well as \pm one SD from the mean.

```
*****
```

Conditional effect of X on Y at values of the moderator(s):

bdi0	Effect	se	t	p	LLCI	ULCI
.8772	-.2447	.1922	-1.2733	.2047	-.6243	.1348
1.1963	-.5473	.1375	-3.9818	.0001	-.8188	-.2759
1.5153	-.8500	.1933	-4.3973	.0000	-1.2317	-.4683

Values for quantitative moderators are the mean and plus/minus one SD from mean.
Values for dichotomous moderators are the two values of the moderator.

```
*****
```

$$\theta_{X \rightarrow Y} = 0.587 - 0.948W$$

Output H

MBRP therapy resulted in lower craving than did therapy as usual among those relatively “moderate” ($\theta_{X \rightarrow Y|W=1.196} = -0.547, p < .001$) or “relatively high” ($\theta_{X \rightarrow Y|W=1.515} = -0.850, p < .001$) in pre-therapy depression. Among those “relatively low” in pre-therapy depression, MBRP therapy had no statistically significant effect on craving relative to therapy as usual. ($\theta_{X \rightarrow Y|W=0.877} = -0.245, p = .205$)

PROCESS output: PLOT option

```
process cov = treathrs crave0/y=crave2/x=mbrp/w=bdi0  
/model=1/jn=1/plot=1/moments=1.
```

```
%process (data=mbrp,cov=treathrs crave0,y=crave2,x=mbrp,  
w=bdi0,model=1,jn=1,plot=1,moments=1);
```

Both the SPSS and SAS versions produce a table of estimated values of Y for different combinations of X and W . Plug these into your preferred graphing program to generate a plot, or use SPSS or SAS's graphics features. SPSS writes the code for you. Just cut and paste this into an SPSS syntax file and execute:

```
DATA LIST FREE/mbrp bdi0 crave2.  
BEGIN DATA.  
    .0000      .8772      2.0456  
    1.0000     .8772      1.8009  
    .0000      1.1963      2.4037  
    1.0000     1.1963      1.8563  
    .0000      1.5153      2.7617  
    1.0000     1.5153      1.9117  
END DATA.  
GRAPH/SCATTERPLOT=bdi0 WITH crave2 BY mbrp.
```

Output H

Generating a graph from PROCESS “PLOT” option: SAS

```
data;  
input mbrp bdi0 crave2;  
datalines;  
  .0000      .8772    2.0456  
  1.0000     .8772    1.8009  
  .0000      1.1963   2.4037  
  1.0000     1.1963   1.8563  
  .0000      1.5153   2.7617  
  1.0000     1.5153   1.9117  
run;  
proc sgplot;reg x=bdi0 y=crave2/group=mbrp;run;
```

Output generated
by the PLOT option.

Generating a graph from PROCESS “PLOT” option: R

```
x<-c(0,1,0,1,0,1)
w<-c(0.877,0.877,1.196,1.196,1.515,1.515)
y<-c(2.046,1.801,2.404,1.856,2.762,1.912)
plot(y=y,x=w,pch=15,col="white",
xlab="Depression at start of therapy",
ylab="Craving at 2-month follow-up")
legend.txt<-c("Therapy as usual (X=0)","Mindfulness therapy (X=1)")
legend("topleft",legend=legend.txt,
lty=c(3,1),lwd=c(3,2))
lines(w[x==0],y[x==0],lwd=3,lty=3)
lines(w[x==1],y[x==1],lwd=2,lty=1)
```

} From the PLOT option in PROCESS.

Additional probing options

Setting moments = 0 or leaving it out, produces estimates of the conditional effect of X at the 16th, 50th, and 84th percentiles of the moderator rather than the mean and plus/minus one standard deviation. Or use the wmodval option to request a specific value of the moderator at which you'd like the conditional effect of X.

```
process y = ... /moments = 0.
```

```
%process (data = ... , moments = 0);
```

Conditional effects of the focal predictor at values of the moderator(s) :

bdi0	Effect	se	t	p	LLCI	ULCI
.9020	-.2683	.1850	-1.4500	.1490	-.6336	.0971
1.1900	-.5414	.1375	-3.9384	.0001	-.8129	-.2699
1.5180	-.8525	.1941	-4.3923	.0000	-1.2358	-.4692

W values in conditional tables are the 16th, 50th, and 84th percentiles.

```
process y = ... /wmodval = 1.5.
```

```
%process (data = ... , wmodval = 1.5);
```

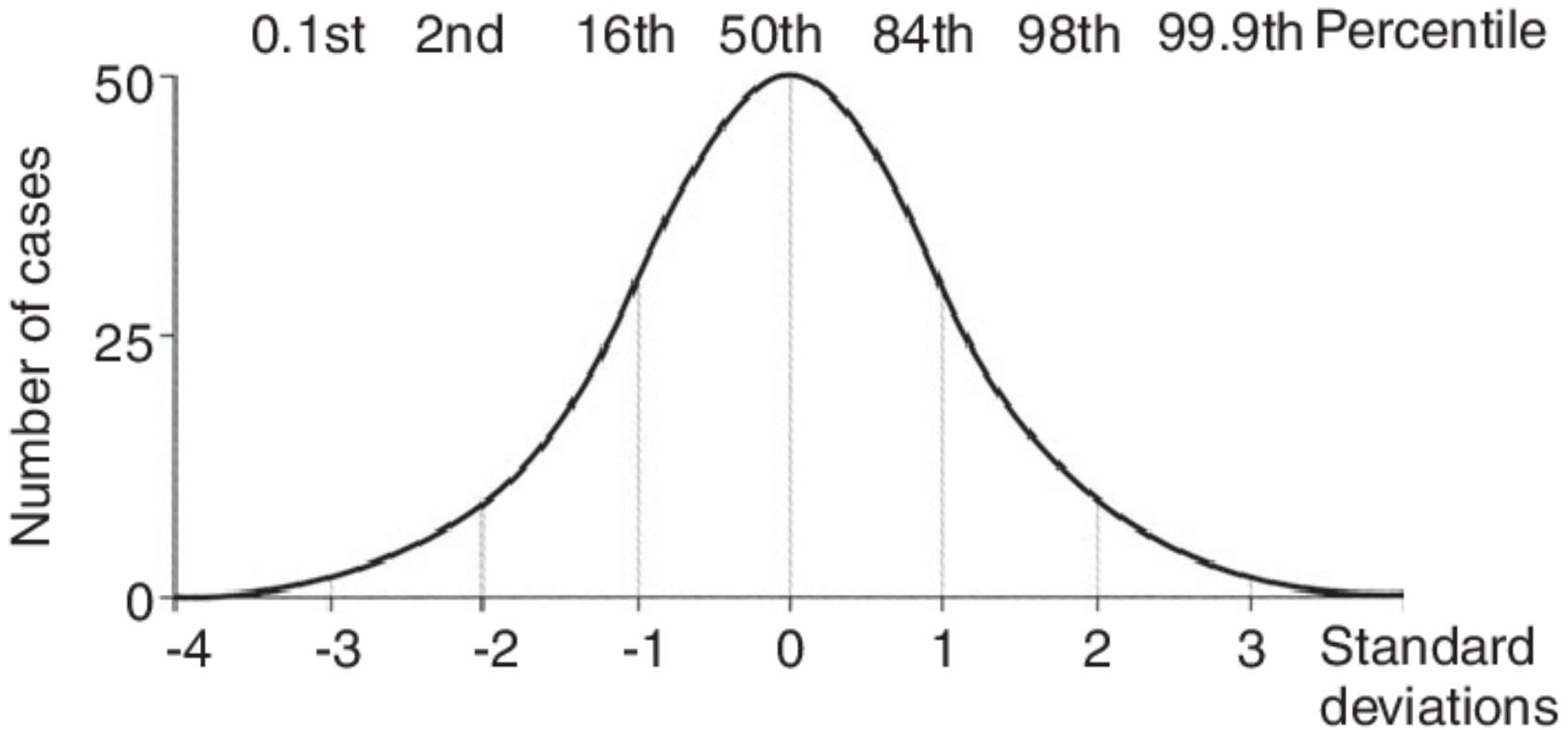
Conditional effect of X on Y at values of the moderators(s)

bdi0	Effect	se	t	p	LLCI	ULCI
1.5000	-.8354	.1888	-4.4253	.0000	-1.2082	-.4626

Why the 16th, 50th, and 84th Percentile?

The 16th, 50th, and 84th percentiles of a normal distribution correspond to the Mean \pm 1SD.

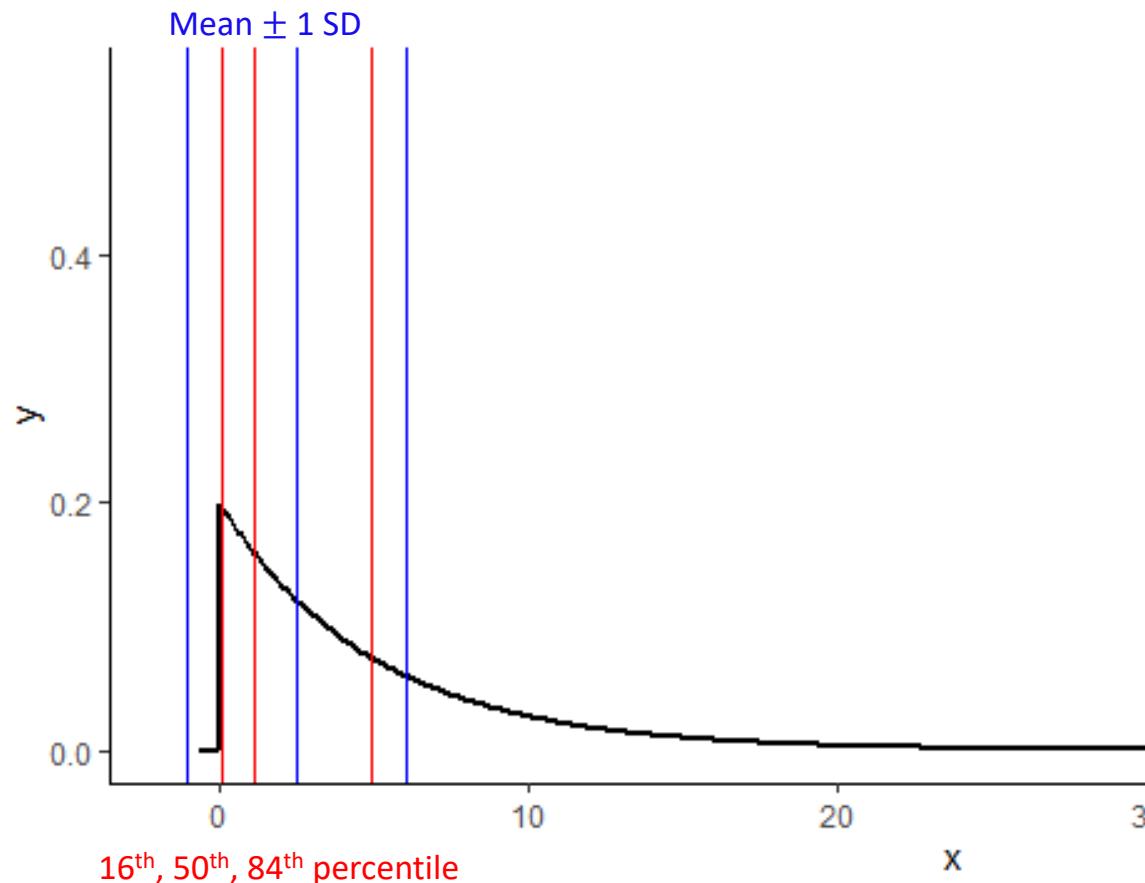
So **assuming your moderator is normally distributed, you would get the same answer either way**, and if the moderator is not normally distributed probing at percentiles guarantees they are within the range of the observed data.



Why the 16th, 50th, and 84th Percentile?

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So assuming your moderator is normally distributed, you would get the same answer either way, and **if the moderator is not normally distributed probing at percentiles guarantees they are within the range of the observed data.**



The Johnson-Neyman technique

The Johnson-Neyman technique seeks to find the value or values of the moderator (W) within the data, if they exist, such that the p -value for the ratio of the conditional effect of the focal predictor at that value or values of W is exactly equal to some chosen level of significance α

To do so, we ask what value of W produces a ratio exactly equal to the critical t value (t_{crit}) required to reject the null hypothesis that the conditional effect of X is equal to zero?

$$t_{crit} = \frac{b_1 + b_3 W}{\sqrt{s_{b_1}^2 + 2W s_{b_1 b_3}^2 + W^2 s_{b_3}^2}}$$

Isolate W and solve the polynomial that results. The quadratic formula finds the solutions:

$$W = \frac{-2(t_{crit}^2 s_{b_1 b_3}^2 - b_1 b_3) \pm \sqrt{(2t_{crit}^2 s_{b_1 b_3}^2 - 2b_1 b_3)^2 - 4(t_{crit}^2 s_{b_3}^2 - b_3^2)(t_{crit}^2 s_{b_1}^2 - b_1^2)}}{2(t_{crit}^2 s_{b_3}^2 - b_3^2)}$$

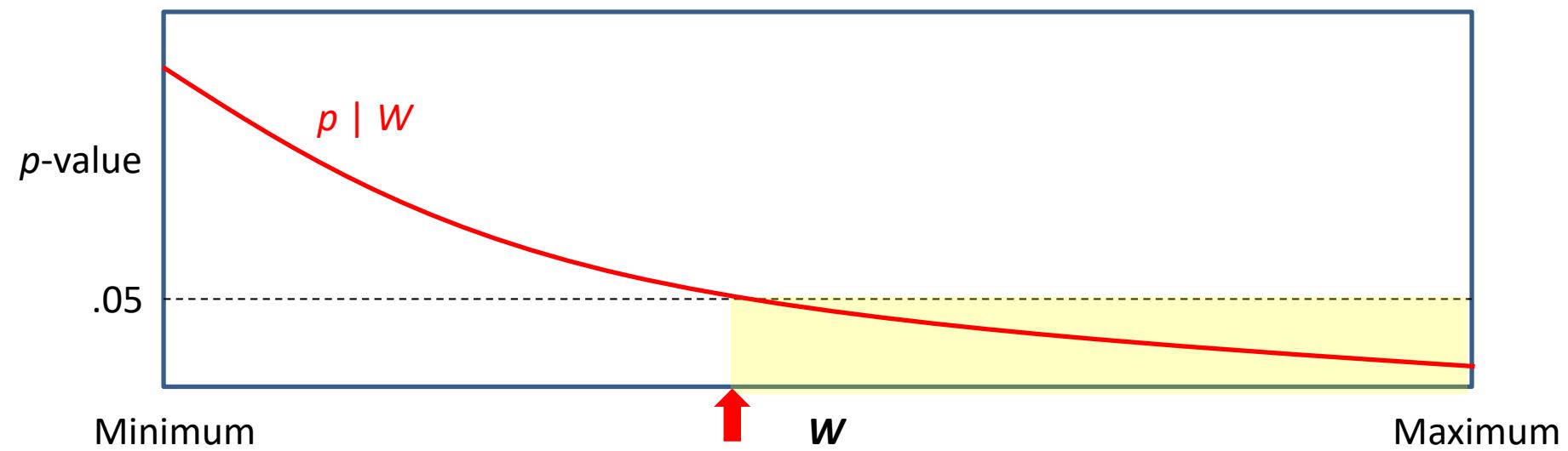
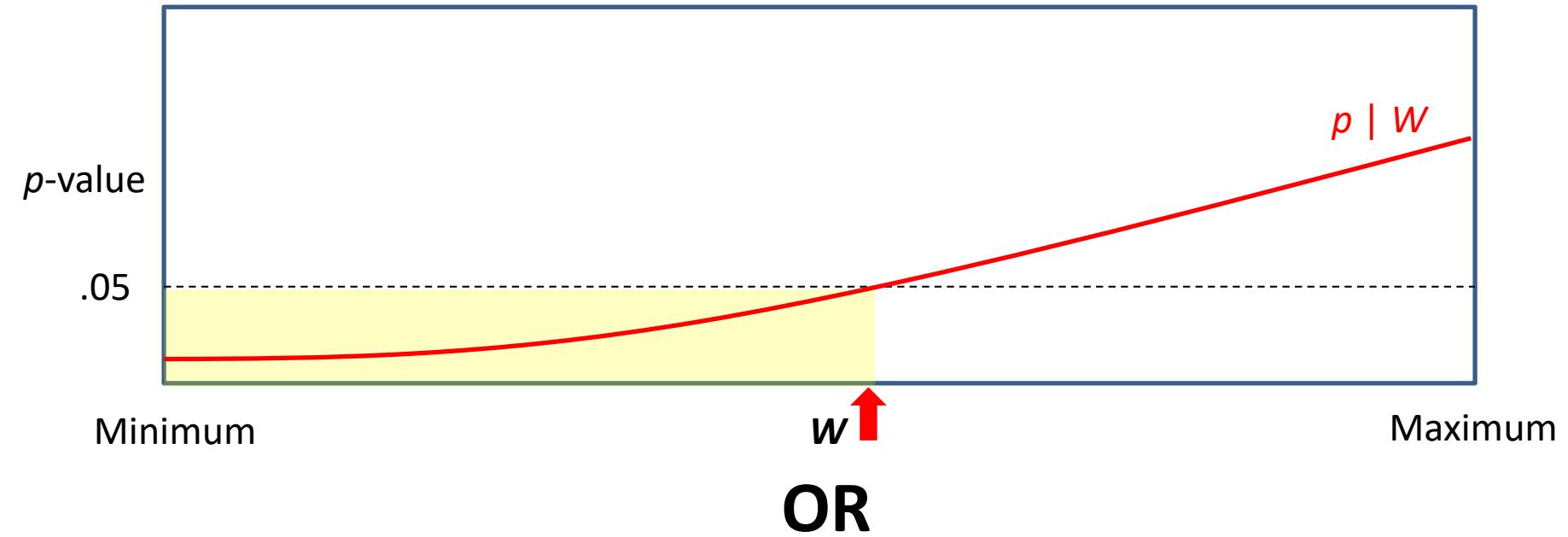
The Johnson-Neyman technique

This will produce no values, one value, or two values of W that are within the range of the moderator variable data.

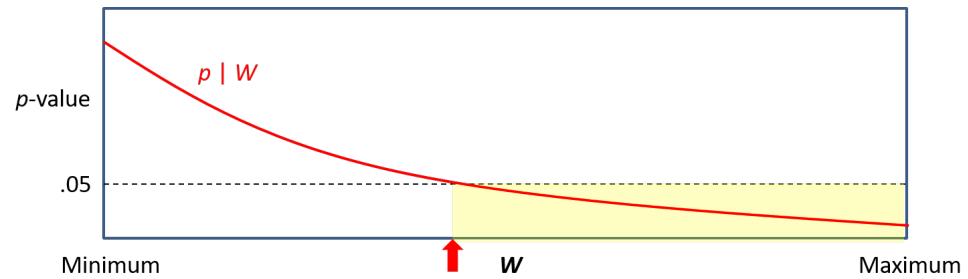
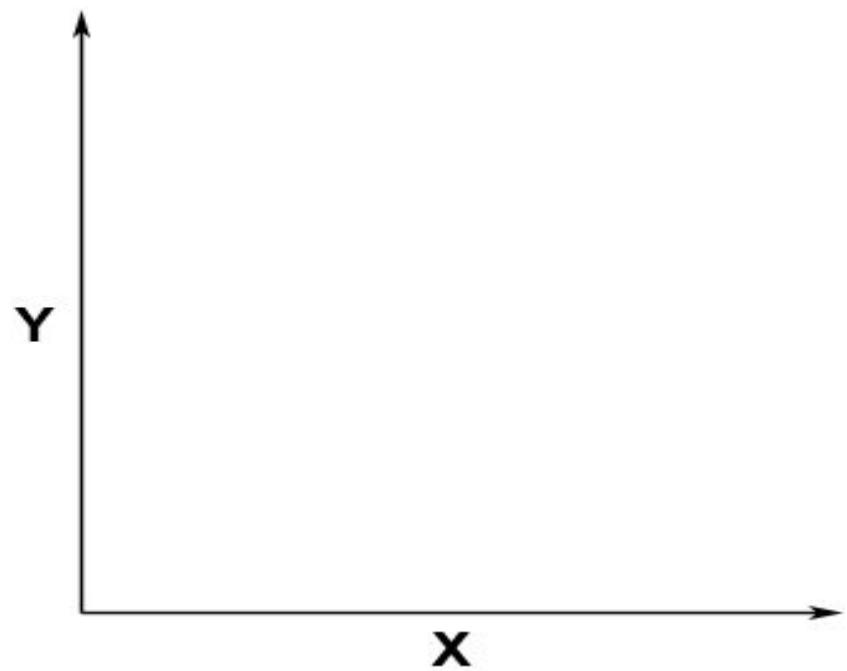
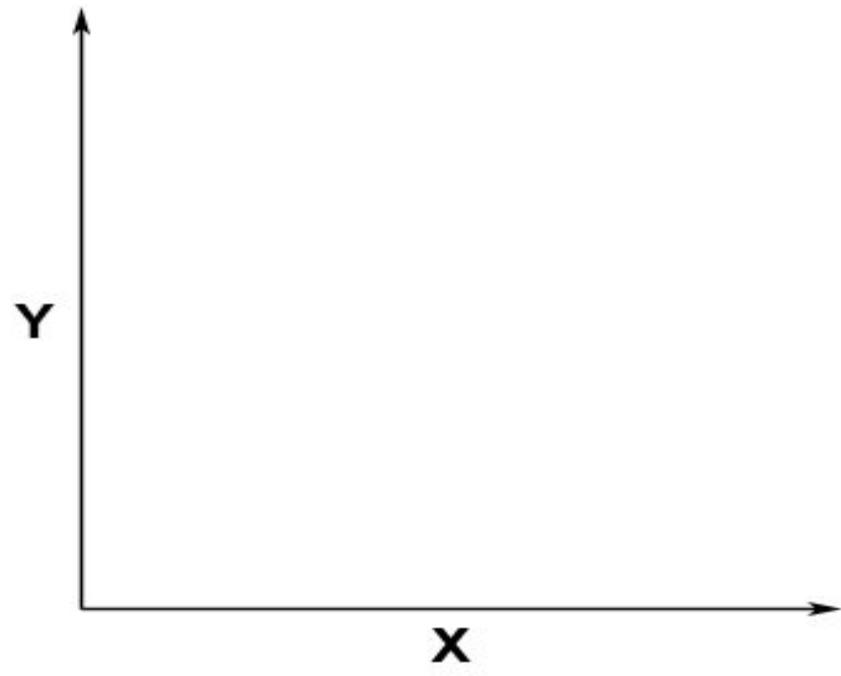
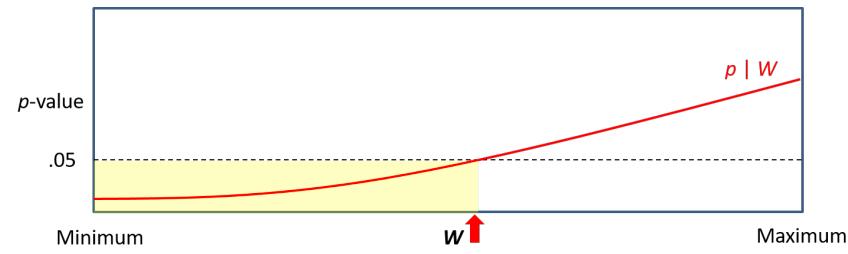
- If one value, this defines a single point of transition between a statistically significant and a statistically nonsignificant conditional effect of the focal predictor, such that $p \leq .05$ for either values of the moderator (1) equal to above W or (2) equal to and below W .
- If two values, this defines the two points of transition between a statistically significant and a statistically nonsignificant conditional effect of the focal predictor, such that the conditional effect is statistically significant for either (1) values of the moderator between the two values of W , or (2) values of the moderator at least as large as the larger W and at least as small as the smaller W .
- If no values, that means the conditional effect is statistically significant for ALL values of the moderator within the range of the data, or it NEVER is.

We would not attempt to do this by hand

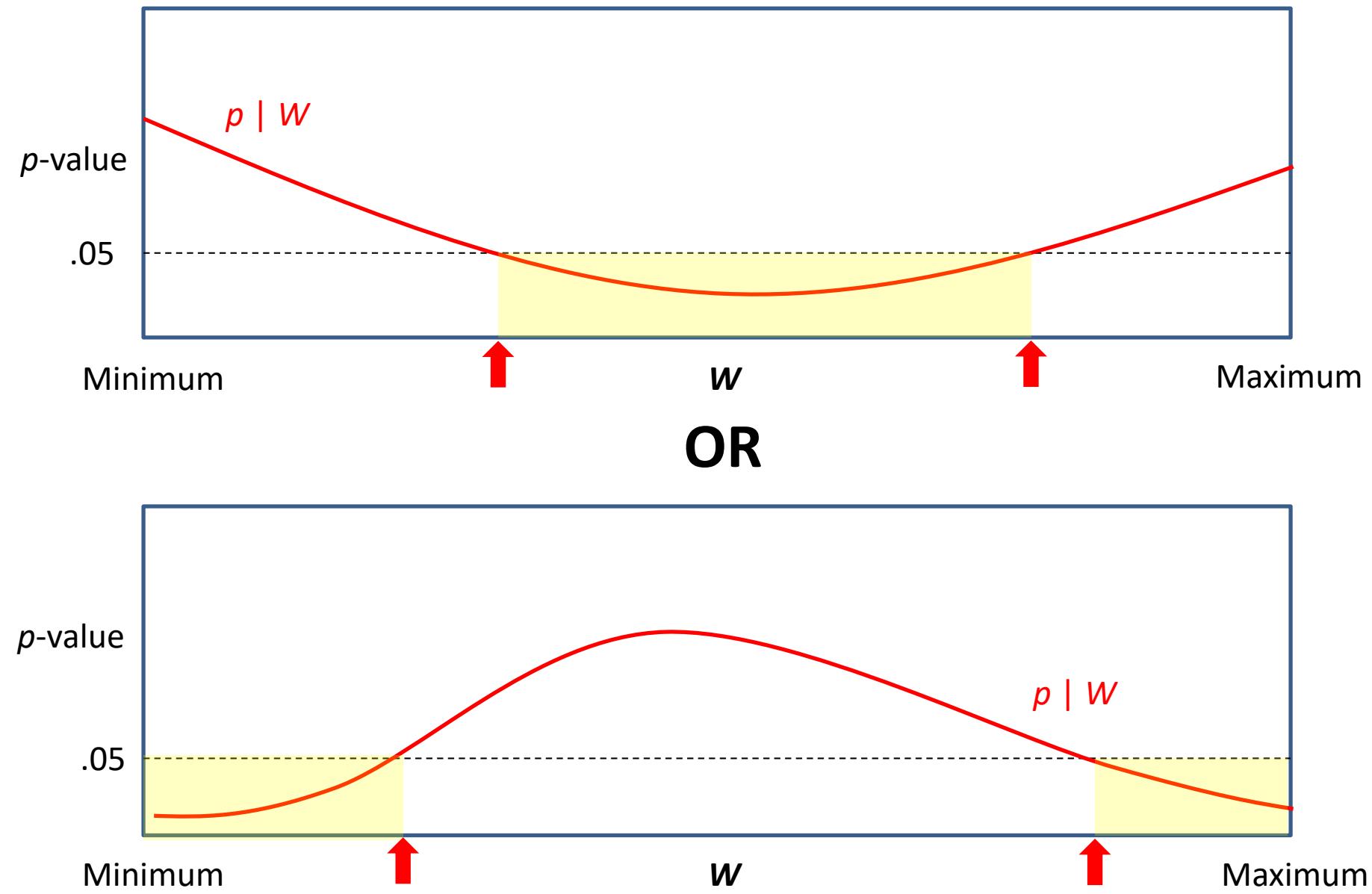
Examples of one solution



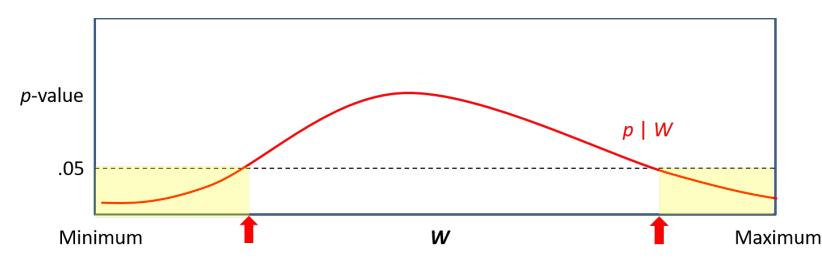
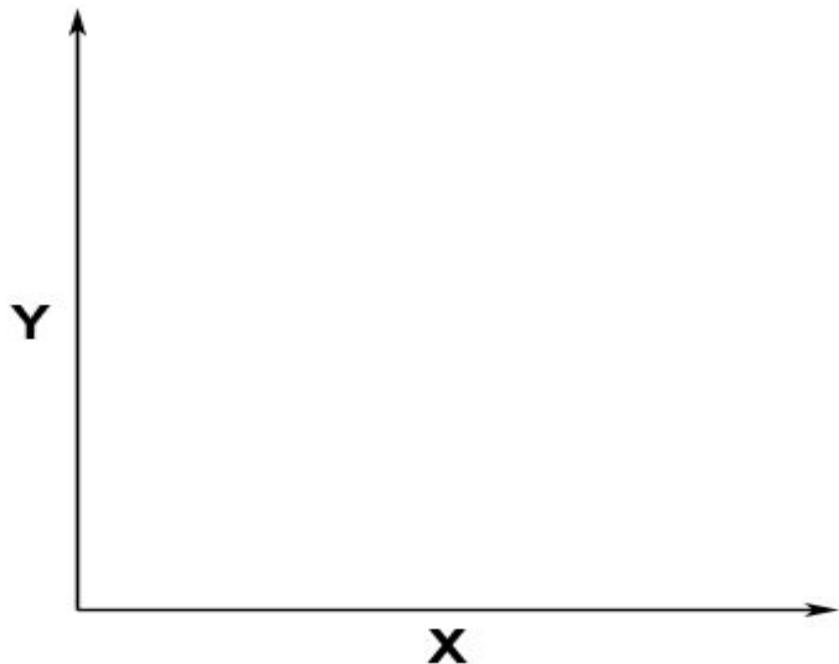
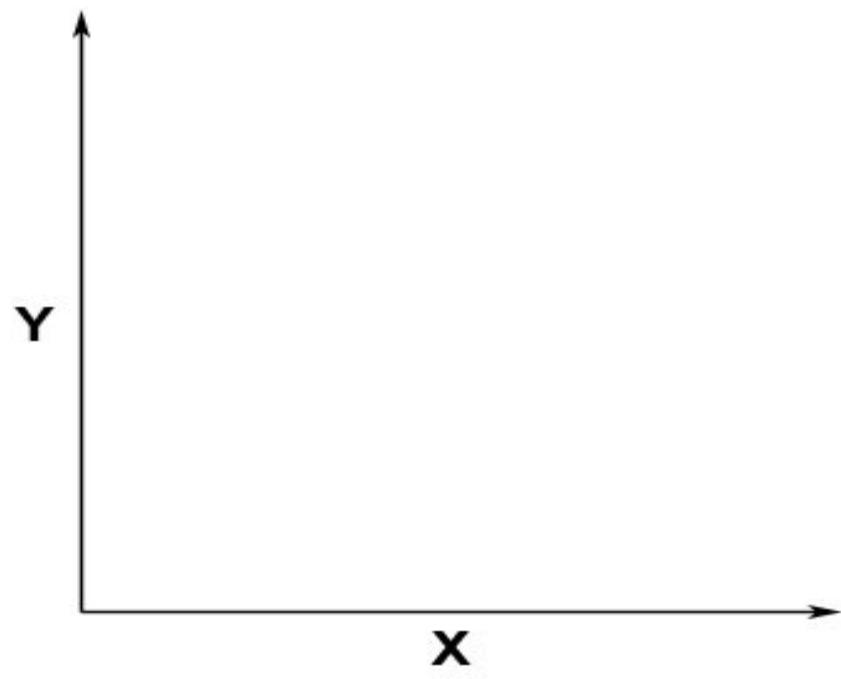
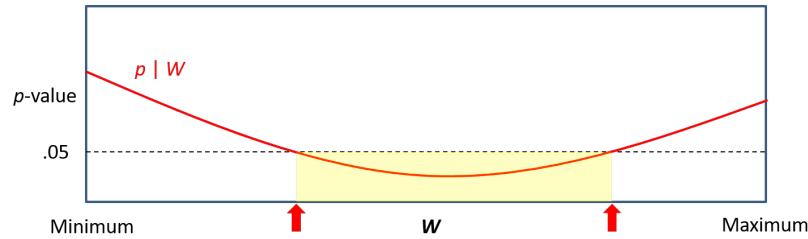
One Solution: What could graphs look like?



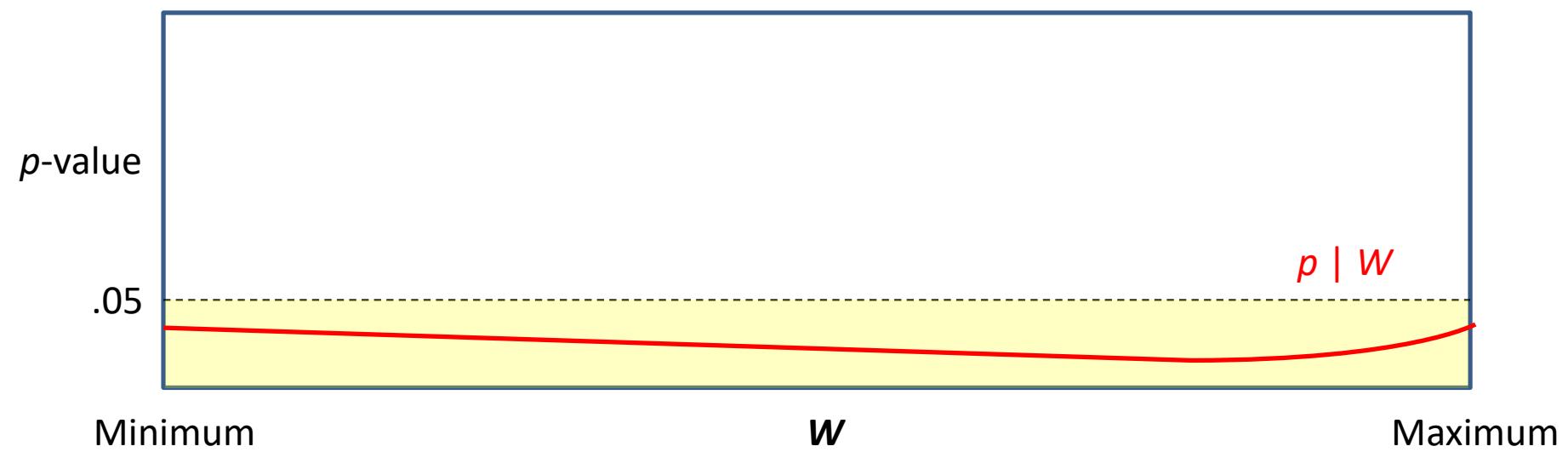
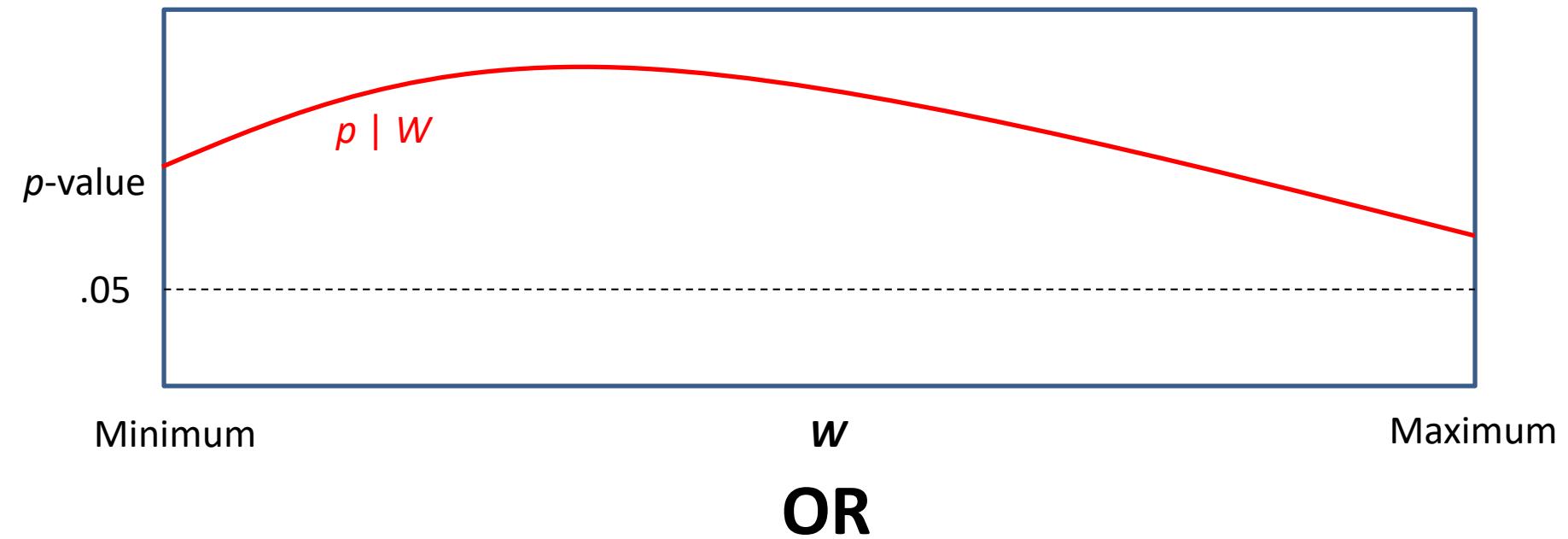
Examples of two solutions



Two solution graphs



Examples of no solutions



Johnson-Neyman output from PROCESS

```
process cov = treathrs crave0/y=crave2/x=mbrp/w=bdi0
  /model=1/jn=1/plot=1/moments=1.
```

```
%process (data=mbrp,cov=treathrs crave0,y=crave2,x=mbrp,
  w=bdi0,model=1,jn=1,plot=1,moments=1);
```

Moderator value(s) defining Johnson-Neyman significance region(s)

Value	% below	% above
.9681	21.4286	78.5714

Conditional effect of X on Y at values of the moderator:

bdi0	Effect	se	t	p	LLCI	ULCI
.0000	.5872	.5241	1.1204	.2642	-.4478	1.6222
.1070	.4858	.4806	1.0108	.3136	-.4632	1.4347
.2140	.3843	.4373	.8787	.3809	-.4793	1.2479
.3210	.2828	.3946	.7167	.4746	-.4964	1.0620
.4280	.1813	.3525	.5144	.6077	-.5147	.8773
.5350	.0798	.3112	.2565	.7979	-.5348	.6944
.6420	-.0217	.2713	-.0798	.9365	-.5574	.5141
.7490	-.1231	.2334	-.5276	.5985	-.5840	.3377
.8560	-.2246	.1986	-1.1312	.2596	-.6167	.1675
.9630	-.3261	.1688	-1.9318	.0551	-.6595	.0072
.9681	-.3309	.1676	-1.9747	.0500	-.6618	.0000
1.0700	-.4276	.1472	-2.9047	.0042	-.7183	-.1369
1.1770	-.5291	.1377	-3.8435	.0002	-.8009	-.2573
1.2840	-.6306	.1426	-4.4220	.0000	-.9122	-.3490
1.3910	-.7321	.1607	-4.5553	.0000	-1.0494	-.4147
1.4980	-.8335	.1882	-4.4288	.0000	-1.2052	-.4619
1.6050	-.9350	.2216	-4.2186	.0000	-1.3727	-.4973
1.7120	-1.0365	.2587	-4.0063	.0001	-1.5474	-.5256
1.8190	-1.1380	.2981	-3.8178	.0002	-1.7266	-.5494
1.9260	-1.2395	.3389	-3.6571	.0003	-1.9088	-.5702
2.0330	-1.3410	.3808	-3.5216	.0006	-2.0929	-.5890
2.1400	-1.4424	.4234	-3.4072	.0008	-2.2784	-.6064

$\theta_{X \rightarrow Y | W}$

W

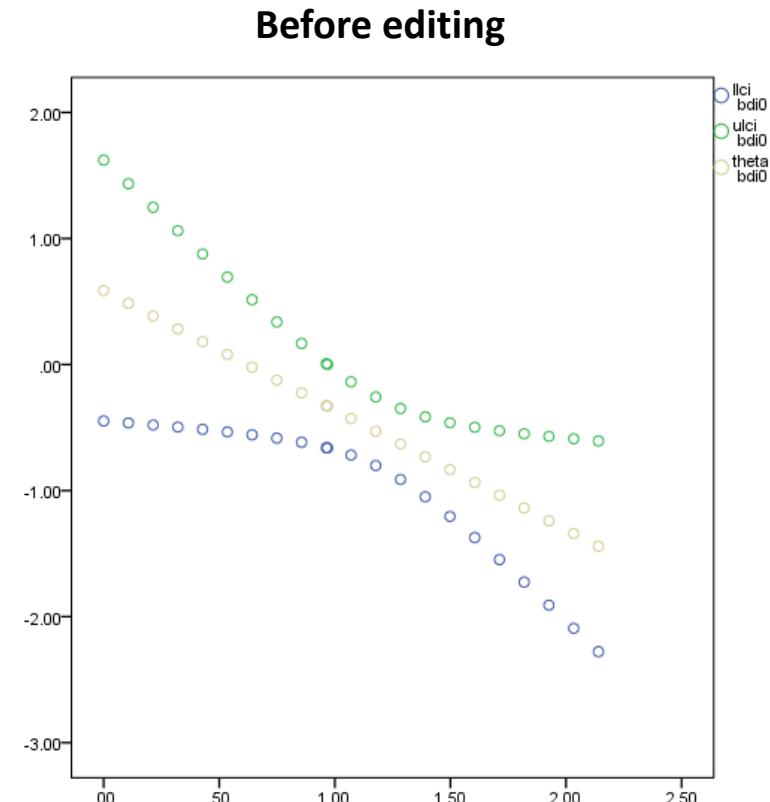
Output H

78.6%
of the
data are
up here

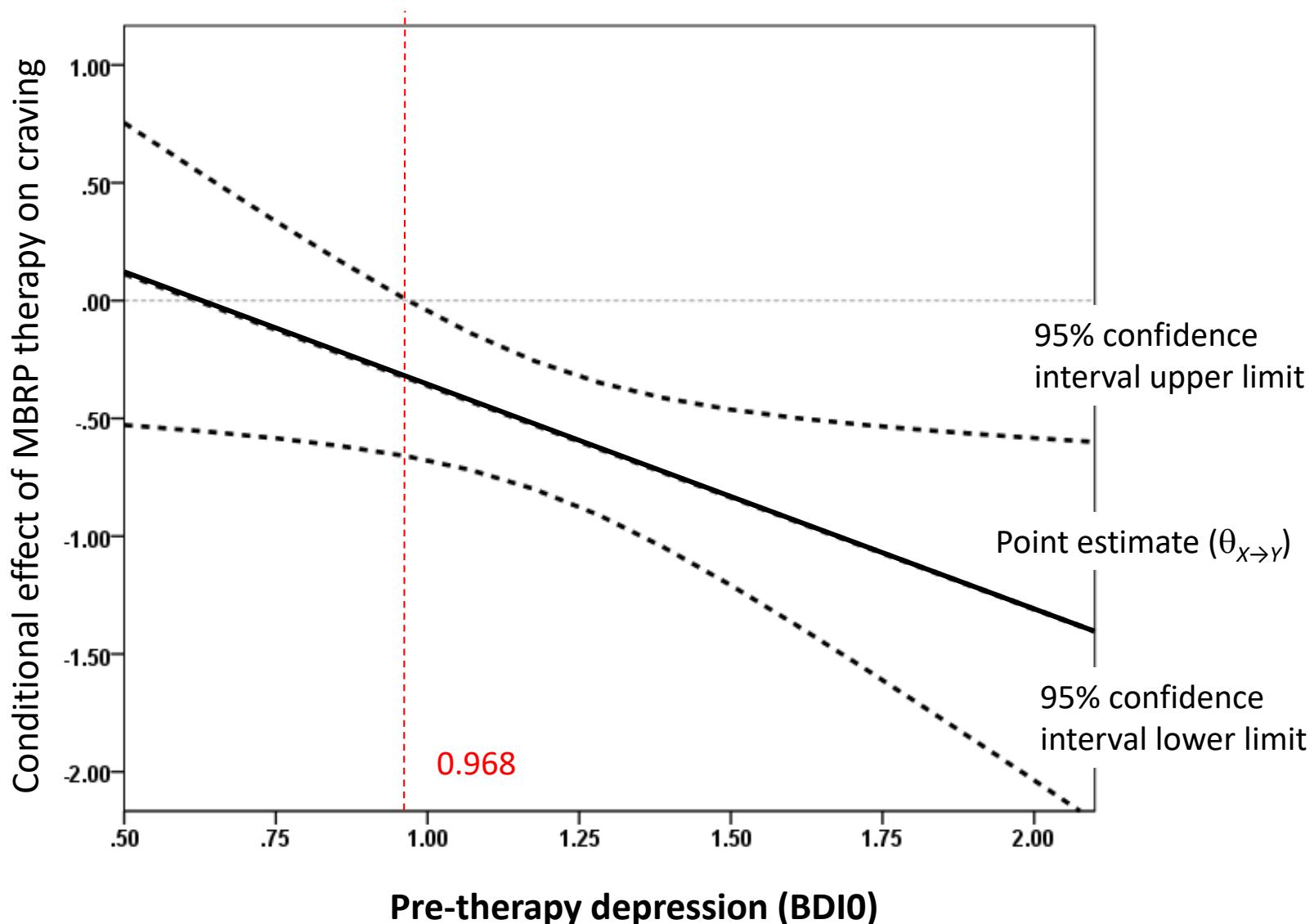
A visual representation (SPSS)

bdi0	effect	LLCI	ULCI	(from PROCESS JN output)
------	--------	------	------	--------------------------

```
data list free/bdi0 theta llci ulci.  
begin data.  
.0000 .5872 -.4478 1.6222  
.1070 .4858 -.4632 1.4347  
.2140 .3843 -.4793 1.2479  
.3210 .2828 -.4964 1.0620  
.4280 .1813 -.5147 .8773  
.5350 .0798 -.5348 .6944  
.6420 -.0217 -.5574 .5141  
.7490 -.1231 -.5840 .3377  
.8560 -.2246 -.6167 .1675  
.9630 -.3261 -.6595 .0072  
.9681 -.3309 -.6618 .0000  
1.0700 -.4276 -.7183 -.1369  
1.1770 -.5291 -.8009 -.2573  
1.2840 -.6306 -.9122 -.3490  
1.3910 -.7321 -1.0494 -.4147  
1.4980 -.8335 -1.2052 -.4619  
1.6050 -.9350 -1.3727 -.4973  
1.7120 -1.0365 -1.5474 -.5256  
1.8190 -1.1380 -1.7266 -.5494  
1.9260 -1.2395 -1.9088 -.5702  
2.0330 -1.3410 -2.0929 -.5890  
2.1400 -1.4424 -2.2784 -.6064  
end data.  
graph  
/scatterplot(overlay)=bdi0 bdi0 bdi0 WITH llci ulci theta (pair).
```

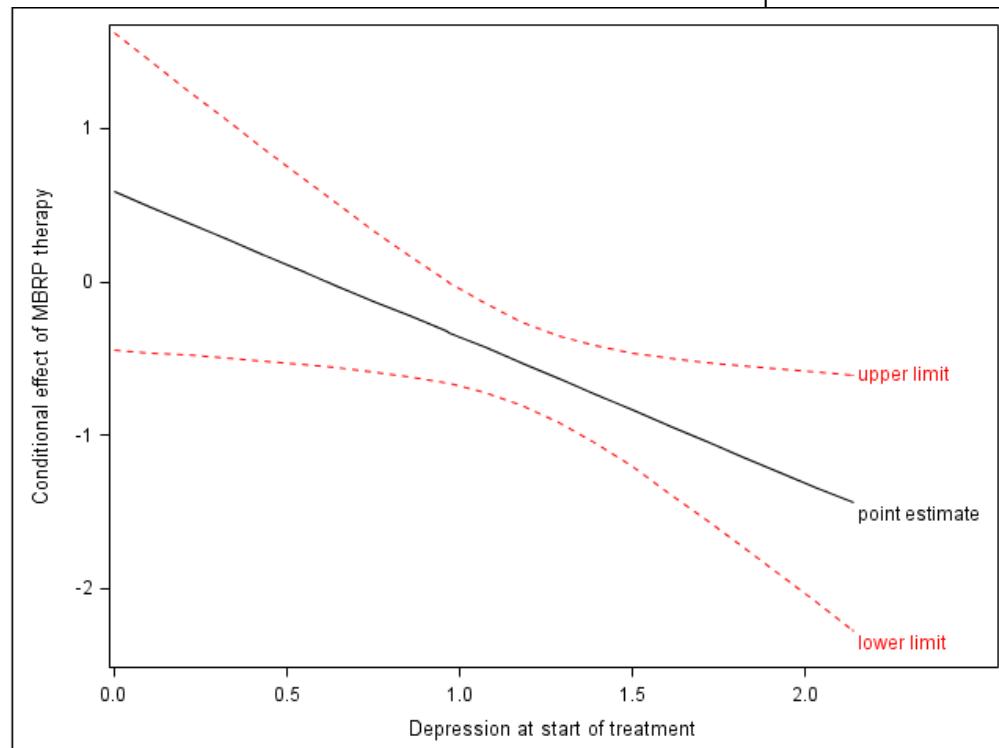


After some editing in SPSS



A visual representation (SAS)

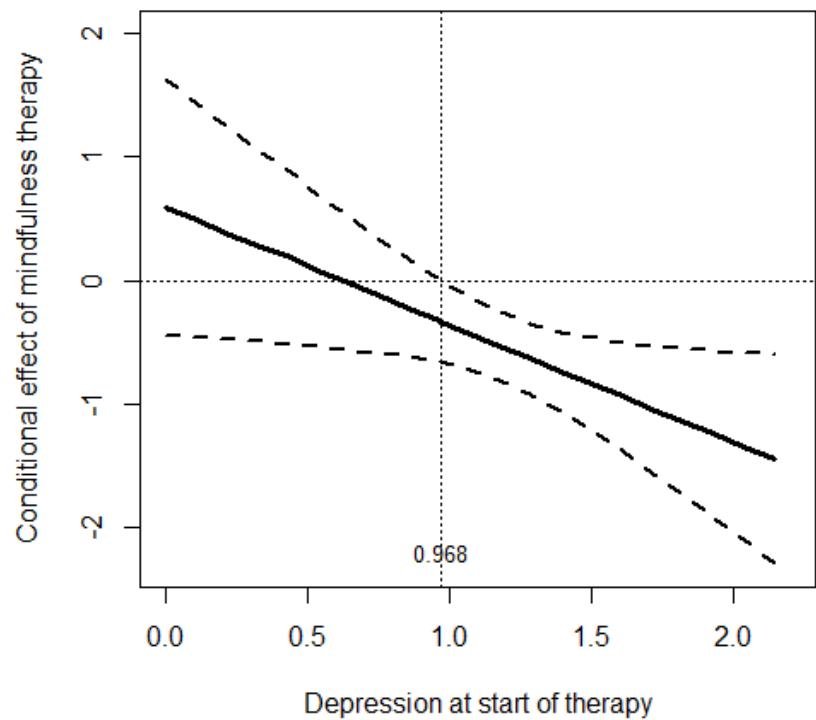
```
data;
input bdi0 effect llci ulci;
datalines;
.0000      .5872      -.4478      1.6222
.1070      .4858      -.4632      1.4347
.2140      .3843      -.4793      1.2479
.3210      .2828      -.4964      1.0620
.4280      .1813      -.5147      .8773
.5350      .0798      -.5348      .6944
.6420     -.0217      -.5574      .5141
.7490     -.1231      -.5840      .3377
.8560     -.2246      -.6167      .1675
.9630     -.3261      -.6595      .0072
.9681     -.3309      -.6618      .0000
1.0700    -.4276      -.7183     -.1369
1.1770    -.5291      -.8009     -.2573
1.2840    -.6306      -.9122     -.3490
1.3910    -.7321     -1.0494     -.4147
1.4980    -.8335     -1.2052     -.4619
1.6050    -.9350     -1.3727     -.4973
1.7120   -1.0365     -1.5474     -.5256
1.8190   -1.1380     -1.7266     -.5494
1.9260   -1.2395     -1.9088     -.5702
2.0330   -1.3410     -2.0929     -.5890
2.1400   -1.4424     -2.2784     -.6064
run;
proc sgplot;
  series x=bdi0 y=ulci/curvelabel = "upper limit" lineattrs=(color=red pattern=ShortDash);
  series x=bdi0 y=effect/curvelabel = "point estimate" lineattrs=(color=black pattern=Solid);
  series x=bdi0 y=llci/curvelabel = "lower limit" lineattrs=(color=red pattern=ShortDash);
  yaxis label = "Conditional effect of MBRP therapy";
  xaxis label = "Depression at start of treatment";
run;
```



A visual representation (R)

```
bdi0<-c(0,.107,.214,.321,.438,.535,.642,.749,.856,.963,.968,1.070,1.177,  
1.284,1.391,1.498,1.605,1.712,1.819,1.926,2.033,2.140)  
effect<-c(.587,.486,.384,.283,.181,.080,-.022,-.123,-.225,-.326,-.331,-.428,  
-.529,-.631,-.732,-.834,-.935,-1.037,-1.138,-1.240,-1.341,-1.442)  
llci<-c(-.448,-.463,-.479,-.496,-.515,-.535,-.557,-.584,-.617,-.660,-.662,  
-.718,-.801,-.912,-1.049,-1.205,-1.373,-1.547,-1.727,-1.909,-2.092,-2.278)  
ulci<-c(1.622,1.435,1.248,1.062,.877,.684,.515,.338,.168,.007,0,-.137,  
-.257,-.349,-.415,-.462,-.497,-.526,-.549,-.571,-.589,-.606)  
plot(x=bdi0,y=effect,type="l",pch=19,ylim=c(-2.3,2),xlim=c(0,2.2),lwd=3,  
ylab="Conditional effect of mindfulness therapy",  
xlab="Depression at start of therapy")  
points(bdi0,llci,lwd=2,lty=2,type="l")  
points(bdi0,ulci,lwd=2,lty=2,type="l")  
abline(h=0,untf = FALSE,lty=3,lwd=1)  
abline(v=0.968,untf=FALSE,lty=3,lwd=1)  
text(0.968,-2.2,"0.968",cex=0.8)
```

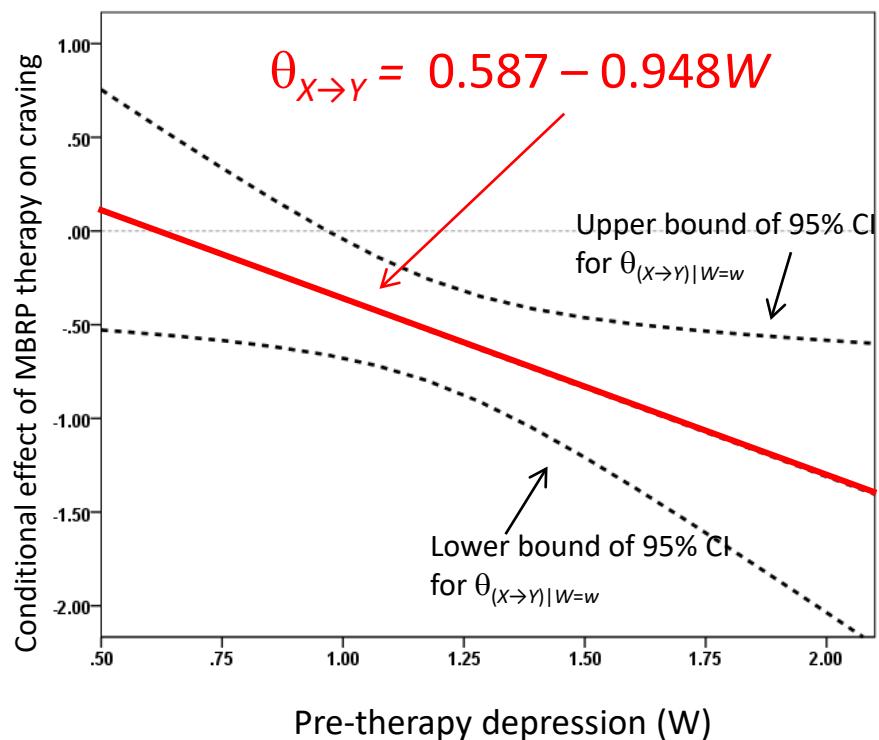
From the
JN option in
PROCESS.



Testing moderation versus testing a conditional effect

A test of moderation is a test as to whether the size of the effect of X on Y is related to the proposed moderator. This is different than asking whether a conditional effect is different from zero.

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \dots$$



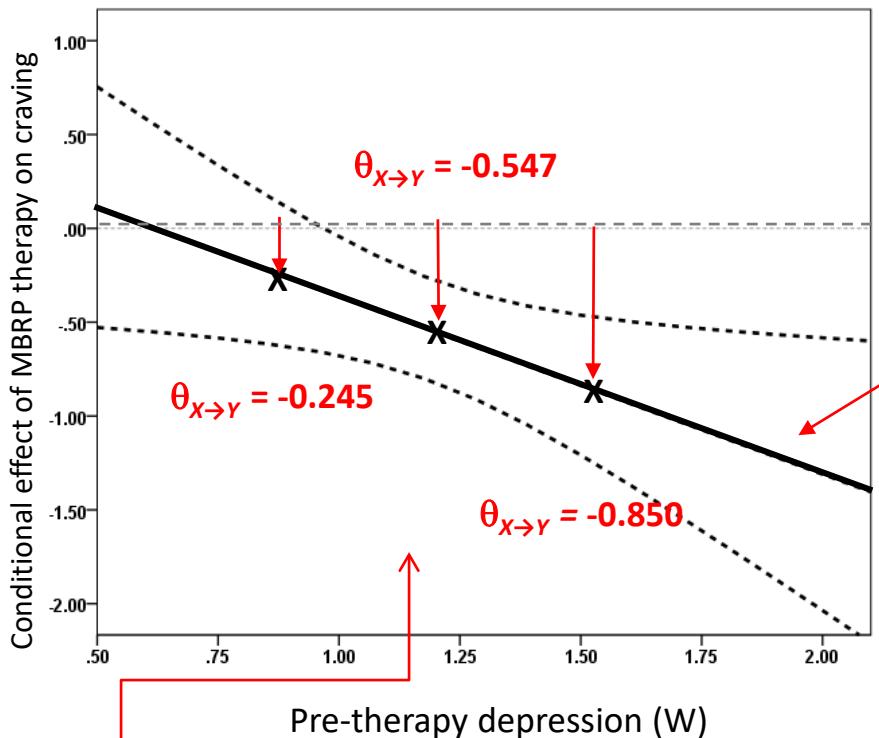
$$\theta_{X \rightarrow Y} = b_1 + b_3 W = 0.587 - 0.948W$$

b_3 is the slope of this line. It is statistically different from zero, meaning that the effect of X depends on W --moderation.

Moderation does **not** imply that the conditional effect of X is different from zero at some, any, or all specific values of the moderator that you choose. Often it will be, perhaps for some values of the moderator but not others. But this is not a requirement of moderation.

Testing moderation versus testing a conditional effect

A test of moderation is a test as to whether the size of the effect of X on Y is related to the proposed moderator. This is different than asking whether a conditional effect is different from zero.



When testing a conditional effect, we are asking whether the effect of X on Y at a specific value of W is statistically different from zero. This is the difference between the point estimate of $\theta_{X \rightarrow Y}$ and zero.

$$\theta_{X \rightarrow Y} = b_1 + b_3 W = 0.587 - 0.948W$$

Difference in significance does not imply significantly different. The pattern of significance or lack thereof across values of M does not say anything about moderation.

Conditional effect of X on Y at values of the moderator(s) :

bdi0	Effect	se	t	p	LLCI	ULCI
.8772	-.2447	.1922	-1.2733	.2047	-.6243	.1348
1.1963	-.5473	.1375	-3.9818	.0001	-.8188	-.2759
1.5153	-.8500	.1933	-4.3973	.0000	-1.2317	-.4683

Comparing conditional effects

We want to know whether the conditional effect of X on Y when $W = w_1$ is different from the conditional effect of X on Y when $W = w_2$.

$$\begin{aligned}\theta_{(X \rightarrow Y) | W=w_2} - \theta_{(X \rightarrow Y) | W=w_1} &= (b_1 + b_3 w_2) - (b_1 + b_3 w_1) \\ &= (w_2 - w_1)b_3\end{aligned}$$

and the standard error of the difference is $(w_2 - w_1) \times$ standard error of b_3 . Under the null hypothesis that the difference in conditional effects is zero, the ratio

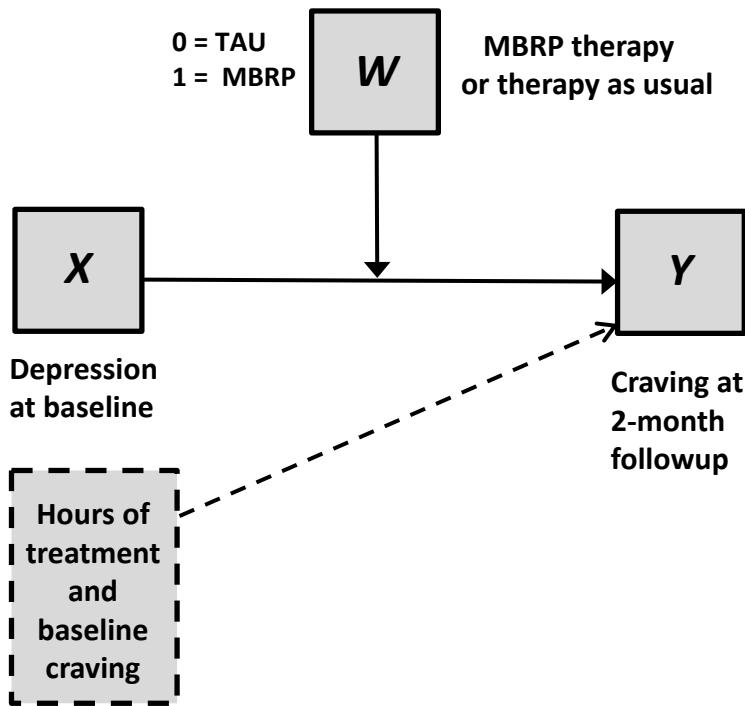
$$\frac{(w_2 - w_1)b_3}{(w_2 - w_1)\text{se}_{b_3}}$$

is distributed as $t(df_{\text{residual}})$. But notice that *regardless of the values of w_1 and w_2* , this ratio simplifies to b_3 / se_{b_3} . **We already have the p-value for this.** It is the p -value for b_3 from the regression model.

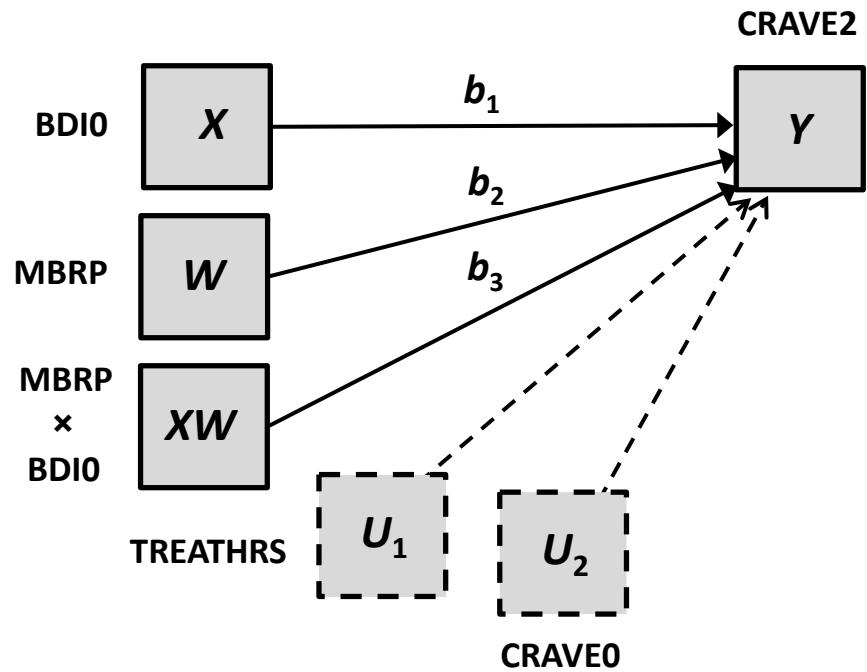
A test of linear moderation of X 's effect on Y by W is equivalent to a test of the difference between *any* two conditional effects of X . Moderation = any two conditional effects of X are different from each other. No moderation = no two conditional effects of X are different from each other. It doesn't matter what values of w_1 and w_2 you choose.

A Dichotomous Moderator

Conceptual diagram



Statistical model



Does the association between pre-treatment depression and later craving differ between those who receive MBRP therapy versus therapy as usual?

Probing the Interaction

When the moderator is dichotomous, the pick-a-point procedure is the only option available, as the Johnson-Neyman technique is meaningful only with a quantitative moderator. Typically, you'd want to estimate the effect of the focal predictor at the two values of the moderator and conduct an inferential test for each conditional effect.

$$\begin{aligned}\hat{Y} &= 1.038 + 1.122X + 0.587W - 0.948XW \\ &= 1.038 + (1.122 - 0.948W)X + 0.587W + \dots \\ \theta_{X \rightarrow Y} &= b_1 + b_3 W = 1.122 - 0.948W\end{aligned}$$

When one of the moderator categories is coded 0, we already have an estimate of $\theta_{X \rightarrow Y}$ when $W = 0$. That estimate is b_1 . And the regression output provides a test of significance.

Model	Coefficients ^a					
	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	
	B	Std. Error	Beta			
1	(Constant)	1.038	.470		.209	.029
	MBRP: Therapy as usual (0) or MBRP therapy (1)	.587	.524	.299	1.120	.264
	BDI0: Beck Depression Inventory baseline	1.122	.276	.366	4.063	.000
	mbrpdep	-.948	.423	-.598	-2.240	.026
	TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
	CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

Among those given therapy as usual, those who were relatively more depressed at the start of therapy had relatively higher craving at two months follow-up, $\theta_{X \rightarrow Y} = 1.122$, $t(162) = 4.063$, $p < .001$

a. Dependent Variable: CRAVE2: Craving at two month follow-up

Probing the Interaction

We already know effect of pre-therapy depression on later craving among those given MBRP therapy. That is $1.122 - 0.948(1) = 0.174$. We can use the regression centering approach, constructing $W' = W - 1$ and reestimating the model in order to get a test of significance.

```
compute mbrp_p = mbrp-1.  
compute mbrpdep = mbrp_p*bdi0.  
regression/dep = crave2/method = enter mbrp_p bdi0 mbrpdep treathrs crave0.
```

```
data mbrp; set mbrp; mbrp_p=mbrp-1; mbrpdep=mbrp_p*bdi0;  
proc reg data=mbrp; model crave2=mbrp_p bdi0 mbrpdep treathrs crave0; run;
```

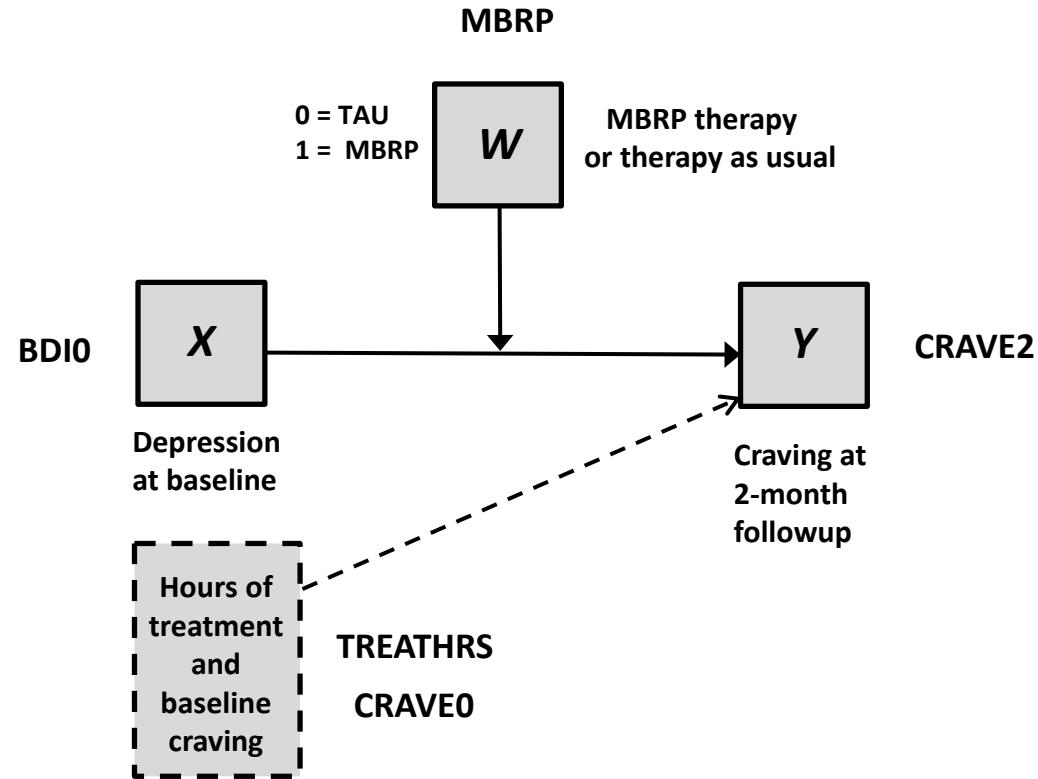
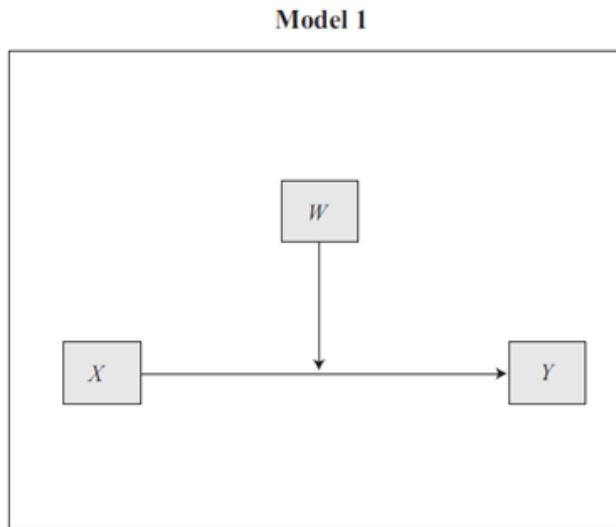
Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.
	B	Std. Error			
1 (Constant)	1.626	.535		3.041	.003
mbrp_p	.587	.524	.299	1.120	.264
BDI0: Beck Depression Inventory baseline	.174	.328	.057	.529	.597
mbrpdep	-.948	.423	-.639	-2.240	.026
TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

a. Dependent Variable: CRAVE2: Craving at two month follow-up

Among those given MBRP therapy, there was no statistically significant relationship between pre-therapy depression and later craving, $\theta_{X \rightarrow Y} = 0.174$, $t(162) = 0.529$, $p = .597$

Estimation Using PROCESS



```
process cov = treathrs crave0/y=crave2/x=bdi0/w=mbrp/model=1.
```

```
%process (data=mbrp,cov=treathrs crave0,y=crave2,x=bdi0,w=mbrp,model=1);
```

PROCESS Output

```
Model = 1
Y = crave2
X = bdi0
M = mbrp
```

Output I

```
Statistical Controls:
CONTROL= treathrs crave0
```

```
Sample size
168
```

```
*****
```

```
Outcome: crave2
```

Model Summary

R	R-sq	MSE	F	df1	df2	p
.5140	.2642	.7277	11.6319	5.0000	162.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	1.0385	.4701	2.2090	.0286	.1102	1.9668
mbrp	.5872	.5241	1.1204	.2642	-.4478	1.6222
bdi0	1.1221	.2762	4.0625	.0001	.5767	1.6675
int_1	-.9485	.4235	-2.2398	.0265	-1.7847	-.1122
treathrs	-.0177	.0103	-1.7190	.0875	-.0380	.0026
crave0	.1920	.0735	2.6138	.0098	.0470	.3371

Interactions:

```
int_1    bdi0      X     mbrp
```

R-square increase due to interaction(s):

	R2-chng	F	df1	df2	p
int_1	.0228	5.0166	1.0000	162.0000	.0265

```
*****
```

Conditional effect of X on Y at values of the moderator(s):

mbrp	Effect	se	t	p	LLCI	ULCI
.0000	1.1221	.2762	4.0625	.0001	.5767	1.6675
1.0000	.1736	.3281	.5291	.5974	-.4744	.8216

PROCESS detects
that the moderator
is dichotomous and
generates the
conditional effect of
the focal predictor
at the two values of
the moderator.

Myths and truths about mean centering

In a model such as

$$\widehat{Y}_i = b_0 + b_1 X_i + b_2 W_i + b_3 X_i W_i$$

there is much ado in the literature about the need to mean center X and W first, thereby estimating the following model instead:

$$\widehat{Y}_i = b'_0 + b'_1(X_i - \bar{X}) + b'_2(W_i - \bar{W}) + b_3(X_i - \bar{X})(W_i - \bar{W})$$

There are three reasons commonly offered for why this should be done.

- (1) Estimation accuracy and statistical power is increased by reducing collinearity. **A MYTH!**
- (2) The interpretations of b_1 and b_2 are more meaningful. **TYPICALLY TRUE!**
- (3) Rounding error is less likely to affect computations. **NOT A CONCERN THESE DAYS**

There is no need to mean center in this fashion, although you may do so if you choose. As standardization is a form of mean centering (combined with a rescaling), all these arguments apply to standardization as well.

Illustration

Without mean centering

```
compute mbrpdep = mbrp*bdi0.  
regression/dep = crave2/method = enter mbrp bdi0 mbrpdep treathrs crave0.
```

```
data mbrp; set mbrp; mbrpdep=mbrp*bdi0; run;  
proc reg data=mbrp; model crave2=mbrp bdi0 mbrpdep treathrs crave0; run;
```

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \dots$$

Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
1 (Constant)	1.038	.470		2.209	.029
MBRP: Therapy as usual (0) or MBRP therapy (1)	.587	.524	.299	1.120	.264
BDI0: Beck Depression Inventory baseline	1.122	.276	.366	4.063	.000
mbrpdep	-.948	.423	-.598	-2.240	.026
TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

a. Dependent Variable: CRAVE2: Craving at two month follow-up

$$b_3 = -0.948 \\ se_{b3} = 0.423 \\ t = -2.240, \\ p = 0.026$$

$b_1 = 0.587$. This is the effect of X (MBRP) when W (pre-therapy depression) = 0.

$b_2 = 1.122$. This is the effect of W (Pre-therapy depression) when $X = 0$ (therapy-as-usual condition)

Illustration

With mean centering

```
compute mbrp_c = mbrp-0.554.
compute bdi0_c = bdi0-1.196.
compute mbrpdep_c = mbrp_c*bdi0_c.
regression/dep = crave2/method = enter mbrp_c bdi0_c mbrpdep_c treathrs crave0.
```

```
data mbrp;set mbrp;bdi0_c=bdi0-1.196;mbrp_c=mbrp-0.554;mbrpdep_c=mbrp_c*bdi0_c;run;
proc reg data=mbrp;model crave2=mbrp_c bdi0_c mbrpdep_c treathrs crave0;run;
```

$$\hat{Y} = 2.077 - 0.547X + 0.597W - 0.948XW$$

Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.
	B	Std. Error			
1 (Constant)	2.077	.369		5.635	.000
mbrp_c	-.547	.137	-.279	-3.980	.000
bdi0_c	.597	.222	.194	2.685	.008
mbrpdep_c	-.948	.423	-.158	-2.240	.026
TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

a. Dependent Variable: CRAVE2: Craving at two month follow-up

$$b_3 = -0.948$$

$$se_{b_3} = 0.423$$

$$t = -2.240,$$

$$p = 0.026$$

Mean centering has had no effect on b_3 .

$b_1 = -0.547$. This is the effect of X (MBRP) among people average in W (pre-therapy depression)

$b_2 = 0.597$. This is the group-weighted average effect of W (Pre-therapy depression)

The interaction is unaffected by centering. b_1 and b_2 have changed because the meaning of "0" changes when X and W are mean centered. This change has nothing to do with multicollinearity being reduced by mean centering.

Collinearity and regression standard errors

The estimated standard error (s_{b_j}) for predictor variable j is

$$s_{b_j} = \sqrt{\frac{1}{1 - R_j^2}} \sqrt{\frac{MS_{\text{residual}}}{n(s_j^2)}} = \sqrt{\frac{MS_{\text{residual}} (\text{VIF})}{n(s_j^2)}}$$

where R_j^2 is the squared multiple correlation in a model estimating predictor variable j from the other predictor variables and s_j^2 is the variance of predictor j .

- $1 - R_j^2$ is called predictor variable j 's **tolerance**. It quantifies the proportion of the variance in variable j unexplained by the other predictor variables. Larger is better.
- The inverse of a variable's tolerance is its **variance inflation factor (VIF)**. It quantifies how much the sampling variance of predictor j 's regression coefficient is affected by the correlation between it and the other predictor variables (larger is worse)

In general, the weaker the correlation between predictor variables, the larger a variable's tolerance, the smaller its variation inflation factor, the smaller the standard error, and the more power the hypothesis test for that predictor. Thus, anything you can do to reduce the correlation between predictors would seem to be a good thing.

Collinearity and regression standard errors

In moderated multiple regression, some therefore argue that you should mean center X and M prior to computing their product, because this will lower the intercorrelation between X , M , and their product, reduce the standard error for the regression coefficient for the product, and therefore increase the power of the hypothesis test for interaction.

Correlations between variables in their original metric:

		Correlations			Tolerance	VIF
		MBRP: Therapy as usual (0) or MBRP therapy (1)	BDI0: Beck Depression Inventory baseline	mbrpdep		
MBRP: Therapy as usual (0) or MBRP therapy (1)	Pearson Correlation	1	-.091	.945	0.064	15.674
	Sig. (2-tailed)		.242	.000		
	N	168	168	168		
BDI0: Beck Depression Inventory baseline	Pearson Correlation	-.091	1	.123	0.561	1.782
	Sig. (2-tailed)	.242		.113		
	N	168	168	168		
mbrpdep	Pearson Correlation	.945	.123	1	0.064	15.704
	Sig. (2-tailed)	.000	.113			
	N	168	168	168		

Some say you should NEVER include two predictors in a model that are so highly correlated.

Collinearity and regression standard errors

In moderated multiple regression, some therefore argue that you should mean center X and M prior to computing their product, because this will lower the intercorrelation between X , M , and their product, reduce the standard error for the regression coefficient for the product, and therefore increase the power of the hypothesis test for interaction.

Correlations between variables after mean centering X and M :

		Correlations			Tol.	VIF
		mbrp_c	bdi0_c	mbrpdep_c		
mbrp_c	Pearson Correlation	1	-.091	.020	0.928	1.078
	Sig. (2-tailed)		.242	.798		
	N	168	168	168		
bdi0_c	Pearson Correlation	-.091	1	-.287	0.867	1.154
	Sig. (2-tailed)	.242		.000		
	N	168	168	168		
mbrpdep_c	Pearson Correlation	.020	-.287	1	0.915	1.093
	Sig. (2-tailed)	.798	.000			
	N	168	168	168		

The offensively large correlation has been reduced to near zero, and all the VIFs are near the minimum possible value of 1. Certainly this is a good thing, right?

But other things have changed too.

$$s_{b_j} = \sqrt{\frac{MS_{residual} (VIF)}{n(s_j^2)}}$$

Descriptive Statistics		
	N	Variance
mbrpdep	168	.3816
mbrpdep_c	168	.0266
Valid N (listwise)	168	

Without mean centering

$$\begin{aligned}s_{b_3} &= \sqrt{\frac{MS_{residual} (15.704)}{n(0.3816)}} \\&= \sqrt{\frac{15.704}{0.3816}} \sqrt{\frac{MS_{residual}}{n}} \\&= \sqrt{41.1} \sqrt{\frac{MS_{residual}}{n}}\end{aligned}$$

With mean centering

$$\begin{aligned}s_{b_3} &= \sqrt{\frac{MS_{residual} (1.093)}{n(0.0266)}} \\&= \sqrt{\frac{1.093}{0.0266}} \sqrt{\frac{MS_{residual}}{n}} \\&= \sqrt{41.1} \sqrt{\frac{MS_{residual}}{n}}\end{aligned}$$

The variance of the product changes by the same factor as the variance inflation factor is changed after mean centering. The result is no change in the standard error of the interaction. (The mean squared residual and sample size are unaffected by mean centering)

To mean center or not to mean center

- The choice is yours to make. It is not required for the purpose of estimation.
- Mean centering does nothing to the test of the interaction. Although mean centering does reduce multicollinearity, this has no consequence on the estimate of the interaction or its statistical test.
- If you mean center, you run no risk of interpreting the coefficients for the predictor and the moderator when they estimate quantities beyond the range of the data. This is a good reason for doing it.
- Mean centering does change the coefficient for the focal predictor and moderator. But this has nothing to do with reducing multicollinearity. Mean centering changes the conditioning from “0” to the sample mean.

All these arguments apply to standardization as well.

Mean centering in PROCESS

If you wish to mean center, you may do so before using PROCESS. But PROCESS has an option built in which does it for you. Use the **center = 1** option to automatically mean center variables which define product terms.

```
process cov = treachrs crave0  
 /y=crave2/x=mbrp /w=bdi0/model=1  
 /moments = 1/center=1.
```

```
%process (data=mbrp,cov=treachrs  
 crave0,y=crave2, x=mbrp,m=bdi0,  
 model=1, momens = 1, center=1);
```

Use the **center = 2** option to automatically mean center variables only continuous variables which define product terms. Dichotomous variables will be left in their original metric.

```
*****  
Outcome: crave2  
  
Model Summary
```

R	R-sq	MSE	F	df1	df2	p
.5140	.2642	.7277	11.6319	5.0000	162.0000	.0000

Model							
	coeff	se	t	p	LLCI	ULCI	
constant	2.0778	.3686	5.6363	.0000	1.3498	2.8057	
bdi0	.5970	.2221	2.6876	.0079	.1584	1.0357	
mbrp	-.5473	.1375	-3.9818	.0001	-.8188	-.2759	
int_1	-.9485	.4235	-2.2398	.0265	-1.7847	-.1122	
treachrs	-.0177	.0103	-1.7190	.0875	-.0380	.0026	
crave0	.1920	.0735	2.6138	.0098	.0470	.3371	

Interactions:

int_1	mbrp	x	bdi0
-------	------	---	------

R-square increase due to interaction(s):

R2-chng	F	df1	df2	p	
int_1	.0228	5.0166	1.0000	162.0000	.0265

```
*****
```

Conditional effect of X on Y at values of the moderator(s):

bdi0	Effect	se	t	p	LLCI	ULCI
-.3191	-.2447	.1922	-1.2733	.2047	-.6243	.1348
.0000	-.5473	.1375	-3.9818	.0001	-.8188	-.2759
.3191	-.8500	.1933	-4.3973	.0000	-1.2317	-.4683

Values for quantitative moderators are the mean and plus/minus one SD from mean.
Values for dichotomous moderators are the two values of the moderator.

```
***** ANALYSIS NOTES AND WARNINGS *****
```

Level of confidence for all confidence intervals in output:
95.00

NOTE: The following variables were mean centered prior to analysis:

mbrp	bdi0
------	------

Interactive: A tool for visualization

Another tool available for visualizing interactions is a shiny app called interActive by Conor McCabe

McCabe, C., Kim, D., & King, K. (2018). Improving Present Practices in the Visual Display of Interactions. *Advances in Methods and Practices in Psychological Science*, 1(2), 147-165.

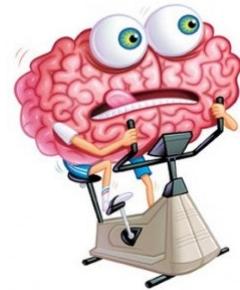
<https://connorjmccabe.shinyapps.io/interactive/>

I'll show you a little how it works, but I highly recommend you play around with it!

Moderation analysis summary

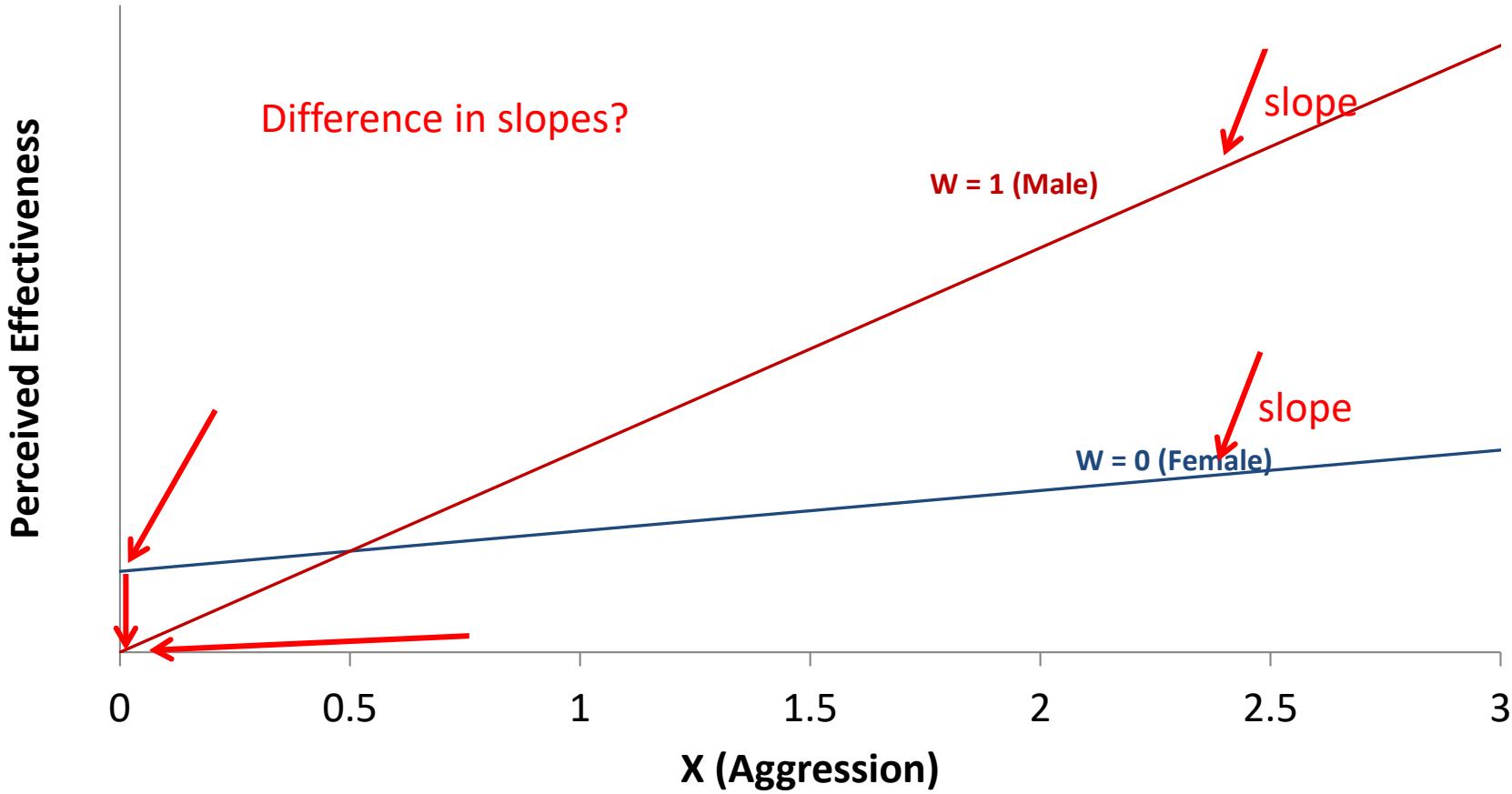
- ❑ A moderator of the effect of X on Y is a variable which influences or otherwise is related to the size of X 's effect on Y .
- ❑ Including a variable defined as the product XW to a regression model that includes X and W allows X 's effect on Y to be a linear function of W .
- ❑ The regression coefficient for XW in such a model is hard to interpret without a picture. Draw a picture of your model before attempting to interpret.
- ❑ We can dissect or “probe” interactions in a few different ways:
 - The pick-a-point approach requires us to select values of W at which to estimate the conditional effect of X on Y . Usually the selection is arbitrary.
 - The Johnson-Neyman technique avoids the need to choose values of the moderator arbitrarily.
- ❑ Care must be taken when interpreting the regression coefficients for X and W in a model that includes XW . They are not “main effects” and they may not have any substantive interpretation. Their interpretation will be influenced by their scaling and whether a value of zero is meaningful on the measurement scale. We can make it meaningful by centering.

Practice Interpreting Coefficients



Among upper level managers, does the relationship between aggression and perceived effectiveness depends on gender?

$$\hat{Y}_i = 1 + (.5 + 2W_i)X_i - 1W_i$$

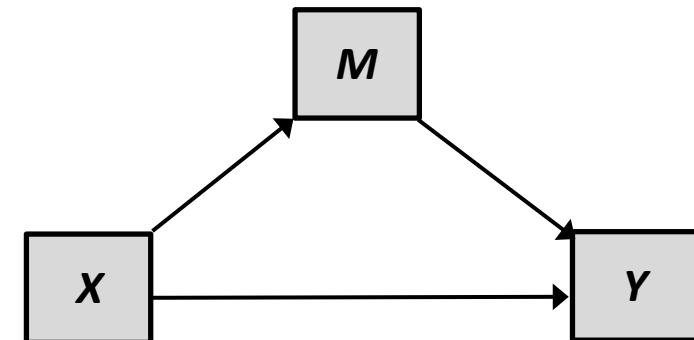


Combining moderation and mediation “Conditional Process Analysis”

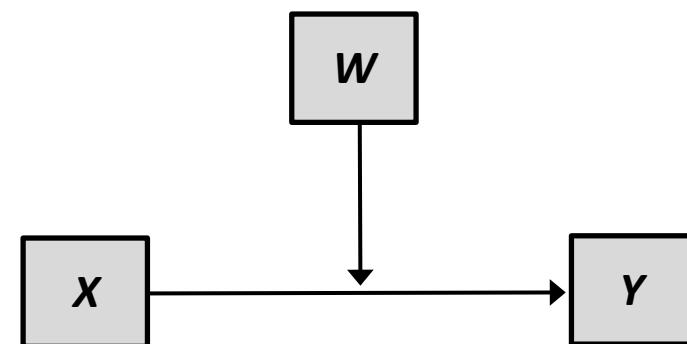
“Conditional process analysis” is a general modeling strategy undertaken with the goal of describing the *conditional* nature of the *mechanism(s)* by which a variable transmits its effect on another, and testing hypotheses about such contingent effects.

A merging of two ideas conceptually and analytically:

“Process analysis”, used to quantify and examine the direct and indirect pathways through which an antecedent variable X transmits its effect on a consequent variable Y through an intermediary M . Better known as “mediation analysis” these days.



“Moderation analysis” used to examine how the effect of an antecedent X on an consequent Y depends on a third moderator variable W (a.k.a. “interaction”)



History

Idea is not new (e.g., Judd & Kenny, 1981; James & Brett, 1984; Baron and Kenny, 1986). It goes by various names that often confuse, including “moderated mediation” and “mediated moderation.”

More recently:

Muller, Judd, and Yzerbyt (2005): Describe analytical models and steps for assessing when “mediation is moderated” and “moderation is mediated.”

Edwards and Lambert (2007): Take a path analysis perspective and show how various effects in a simple mediation model can be conditioned on a third variable.

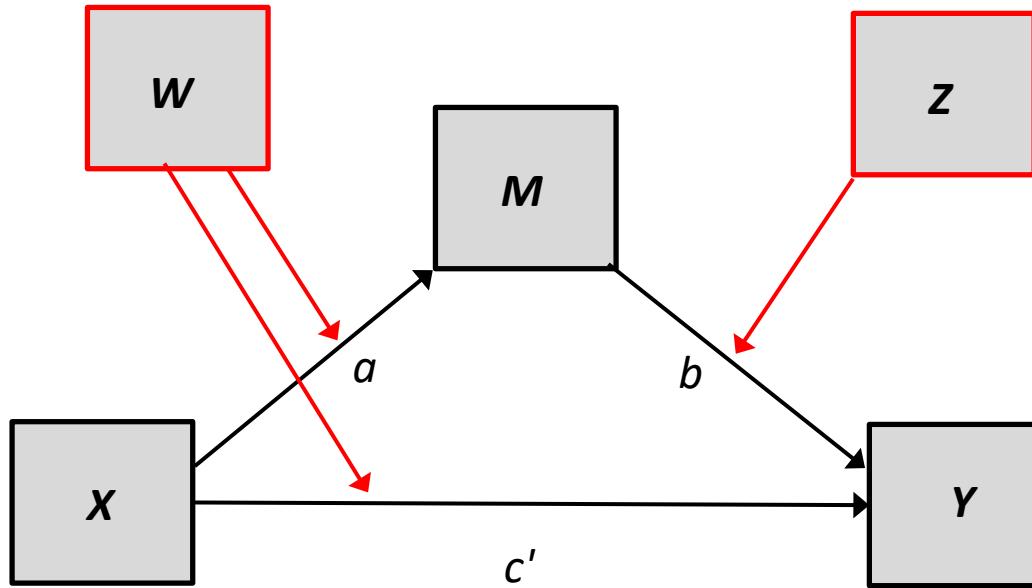
Preacher, Rucker, and Hayes (2007): Provide a formal definition of the *conditional indirect effect* and give formulas, standard errors, and a bootstrap approach for estimating and testing hypotheses about moderated mediation in five different models.

MacKinnon and colleagues (e.g., Fairchild & MacKinnon, 2009): Explicate various analytical approaches to testing hypotheses about mediated moderation and moderated mediation.

Hayes (2013): Introduces the term “conditional process modeling” (also see Hayes and Preacher, 2013) and provides tools for SPSS and SAS to make it easy to do.

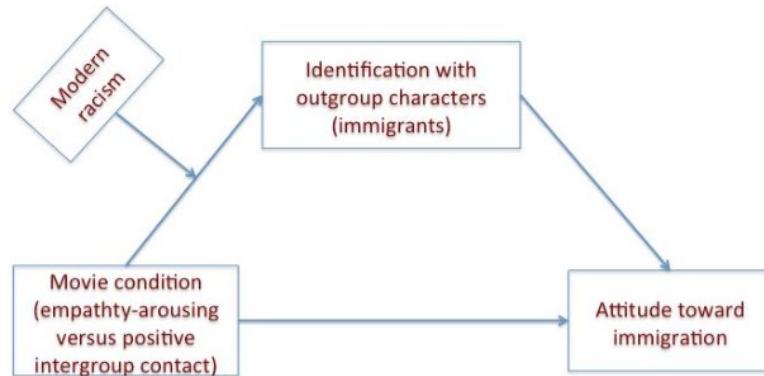
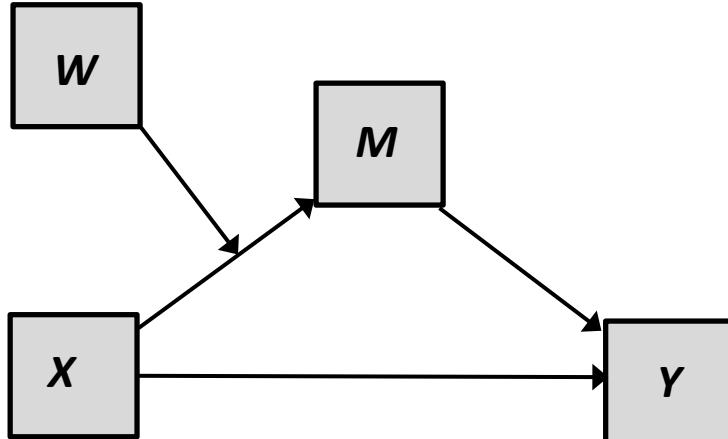
Hayes (2015): Introduces the *index of moderated mediation* which provides a formal test for moderated mediation in a variety of models.

“Moderated mediation”

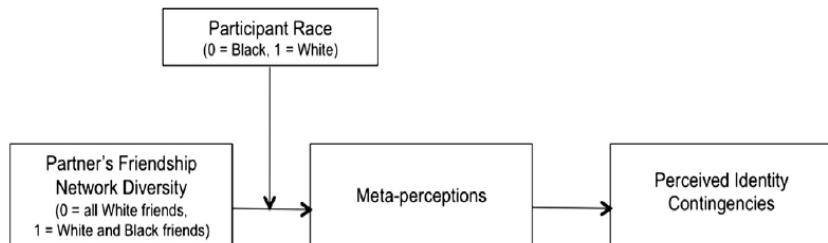


- The indirect effect of *X* on *Y* through *M* is estimated as the product of the *a* and *b* paths
- But what if the size of *a* or *b* (or both) depends on another variable (i.e., is moderated)?
- If so, then the magnitude of the indirect effect therefore depends on a third variable, meaning that “mediation is moderated”.
- When *a* or *b* is moderated, it is sensible then to estimate “conditional indirect effects”—values of indirect effect conditioned on values of the moderator variable that moderates *a* and/or *b*.
- Direct effects can also be conditional. For instance, above, *W* moderates *X*'s direct effect on *Y*.

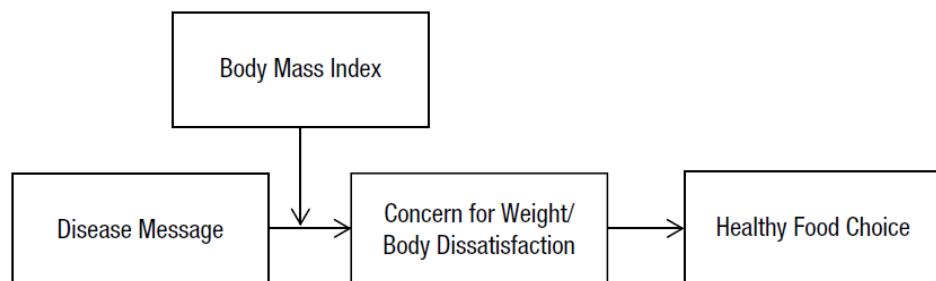
Examples: X to M path moderated by W



Igartua, J.-J., & Frutos, F. J. (2017). Enhancing attitudes toward stigmatized groups with movies. Mediating and moderating processes of narrative persuasion. *International Journal of Communication*, 11, 158-`77.

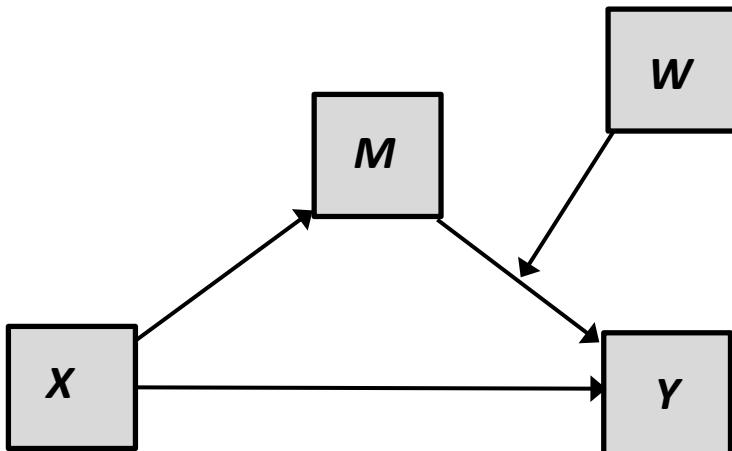


Wout, D. A., Murphy, M. C., & Steele, C. M. (2010). When your friends matter: The effect of White students' racial friendship networks on meta-perceptions and Perceived identity contingencies. *Journal of Experimental Social Psychology*, 46, 1035-1041.

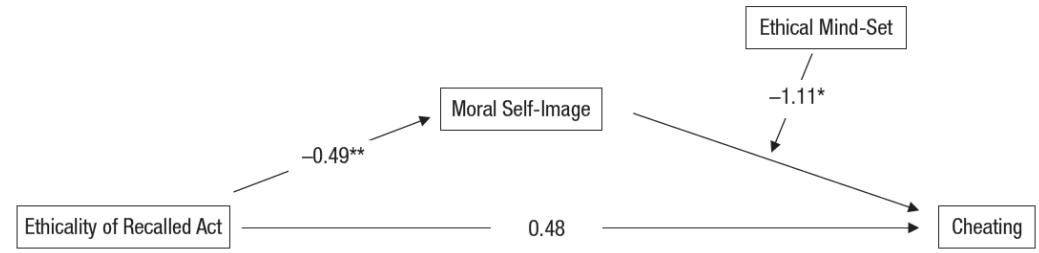


Hoyt, C. L., Burnette, J. L., & Auster-Gussman, L. (2014). "Obesity is a disease": Examining the self-regulatory impact of this public-health message. *Psychological Science*, 25, 997-1002.

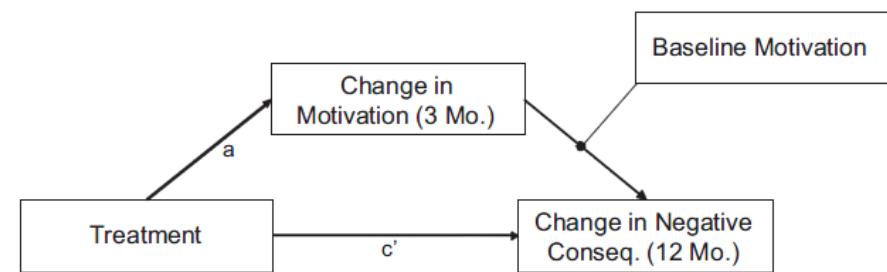
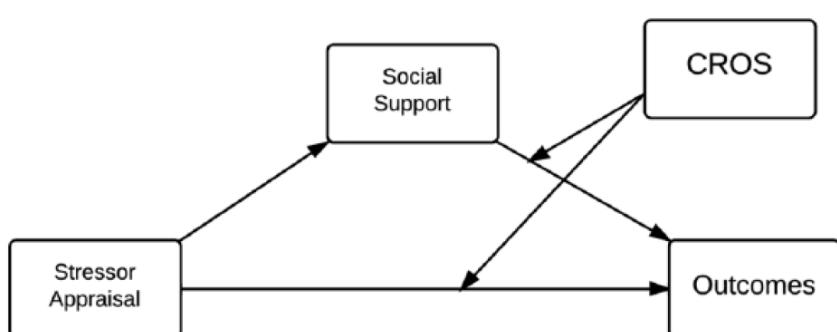
Examples: M to Y path moderated by W



Boren, J. P., & Veksler, A. E. (2015). Communicatively restricted organizational stress (CROS) I: Conceptualization and overview. *Management Communication Quarterly*, 29, 28-55.

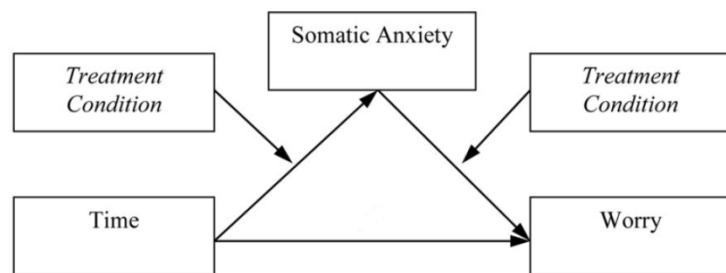
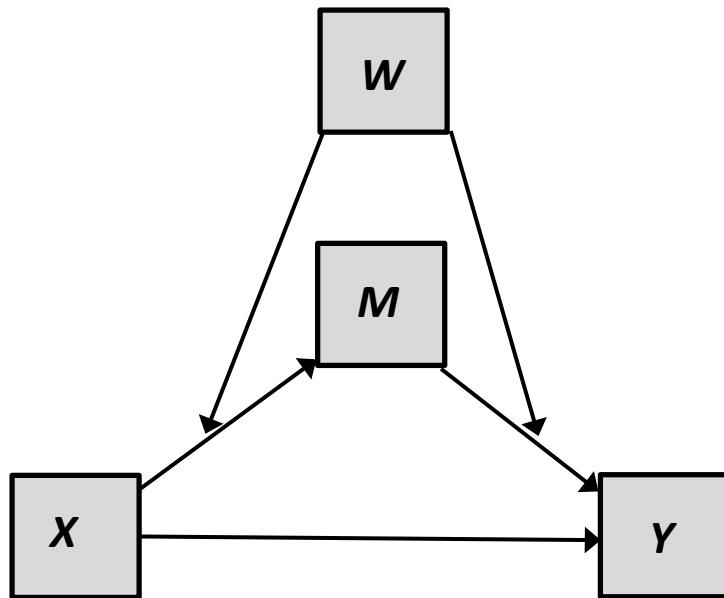


Cornelissen, G., Bashshur, M. R., Rode, J., & Le Menestrel, M. (2013). Rules or consequences? The role of ethical mind-sets in moral dynamics. *Psychological Science*, 24, 492-488.

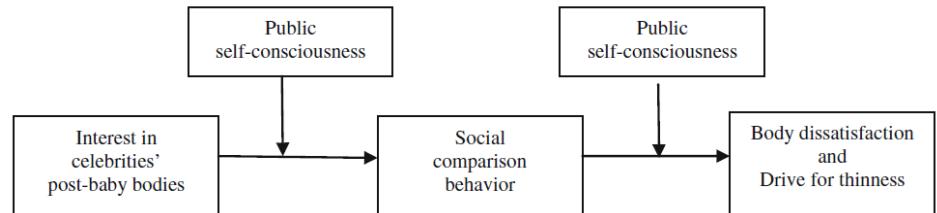


Stein, L. A. R., Minugh, P. A. et al. (2009). Readiness to change as a mediator of the effect of a brief motivational intervention on posttreatment alcohol-related consequences of injured emergency department hazardous drinkers. *Psychology of Addictive Behaviors*, 23, 185-195.

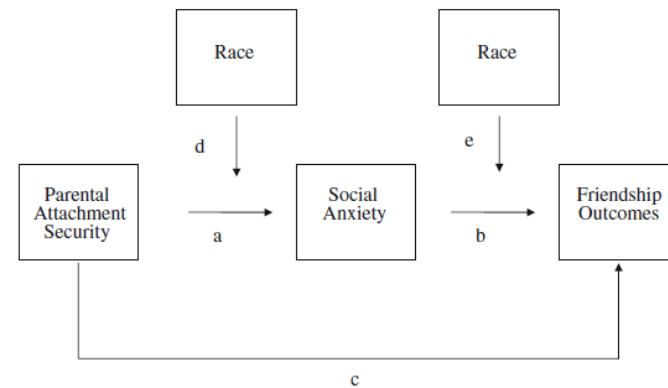
Examples: X to M and M to Y path moderated by W



Donegan, E., & Dugas, M. (2012). Generalized anxiety disorder: A comparison of symptom change in adults receiving cognitive-behavioral therapy or applied relaxation. *Journal of Consulting and Clinical Psychology*, 80, 490-496.



Chae, J. (2014). Interest in celebrities' post-baby bodies and Korean women's body image disturbance after childhood. *Sex Roles*, 71, 419-435.



Parade, S. H., Leerkes, E. M., & Blankson, A. (2010). Attachment to parents, social anxiety, and close relationships of female students over the transition to college. *Journal of Youth and Adolescence*, 39, 127-137.

“Conditional direct effect”

In a mediation model, the direct effect of X on Y quantifies X 's effect independent of the intervening variable or variables. If that direct effect is moderated, then the direct effect is conditional on the variable that moderates X 's effect. For example,

$$\hat{M} = a_0 + aX$$

$$\hat{Y} = c'_0 + c'_1 X + c'_2 W + c'_3 XW + bM$$

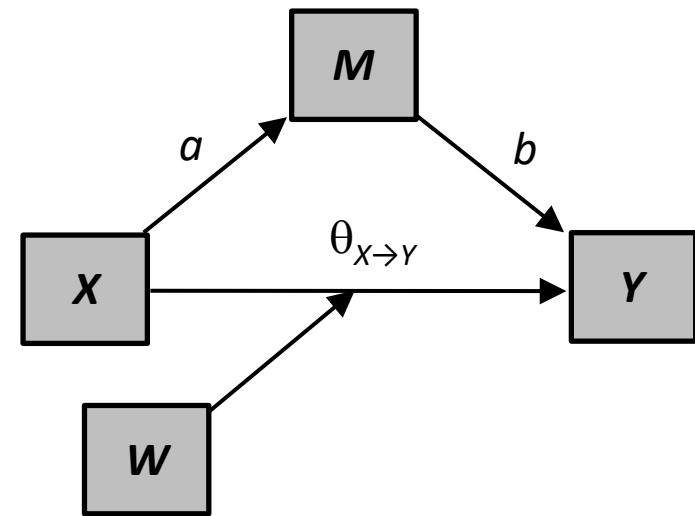
or, equivalently,

$$\hat{Y} = c'_0 + (c'_1 + c'_3 W)X + c'_2 W + bM$$

or, equivalently,

$$\hat{Y} = c'_0 + \theta_{X \rightarrow Y} X + c'_2 W + bM$$

$$\text{where } \theta_{X \rightarrow Y} = (c'_1 + c'_3 W)$$



In this model, $\theta_{X \rightarrow Y}$ is the **conditional direct effect of X** , which is defined by the function $c'_1 + c'_3 W$. Holding M constant, two cases that differ by one unit on X are estimated to differ by $c'_1 + c'_3 W$ units on Y .

This is a very basic conditional process model. It models two pathways through which X affects Y . One is unconditional and indirect via M , and the other is direct but conditional—the size of the direct effect depends on W .

“Conditional direct effect”

In a mediation model, the direct effect of X on Y quantifies X 's effect independent of the intervening variable or variables. If that direct effect is moderated, then the direct effect is conditional on the variable that moderates X 's effect. For example,

$$\hat{M} = a_0 + aX$$

$$\hat{Y} = c'_0 + c'_1 X + c'_2 W + c'_3 XW + bM$$

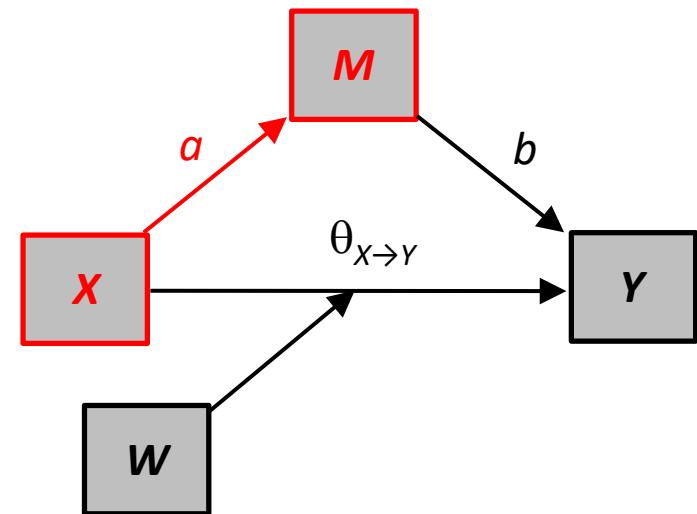
or, equivalently,

$$\hat{Y} = c'_0 + (c'_1 + c'_3 W)X + c'_2 W + bM$$

or, equivalently,

$$\hat{Y} = c'_0 + \theta_{X \rightarrow Y} X + c'_2 W + bM$$

$$\text{where } \theta_{X \rightarrow Y} = (c'_1 + c'_3 W)$$



In this model, $\theta_{X \rightarrow Y}$ is the **conditional direct effect of X** , which is defined by the function $c'_1 + c'_3 W$. Holding M constant, two cases that differ by one unit on X are estimated to differ by $c'_1 + c'_3 W$ units on Y .

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“Conditional direct effect”

In a mediation model, the direct effect of X on Y quantifies X 's effect independent of the intervening variable or variables. If that direct effect is moderated, then the direct effect is conditional on the variable that moderates X 's effect. For example,

$$\hat{M} = a_0 + aX$$

$$\hat{Y} = c'_0 + c'_1 X + c'_2 W + c'_3 XW + bM$$

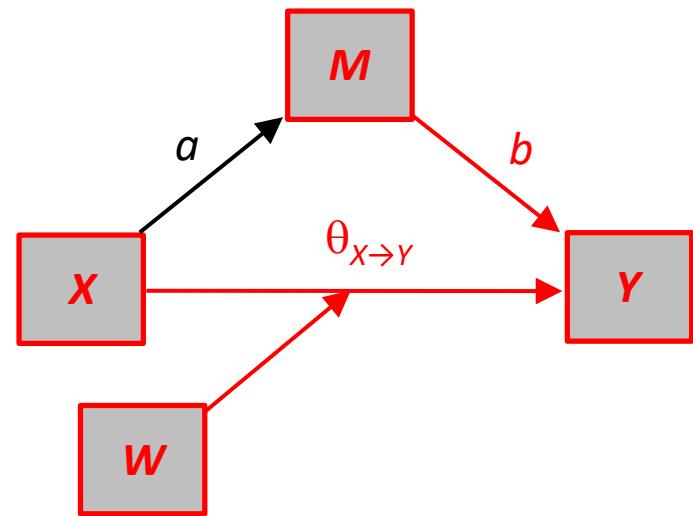
or, equivalently,

$$\hat{Y} = c'_0 + (c'_1 + c'_3 W)X + c'_2 W + bM$$

or, equivalently,

$$\hat{Y} = c'_0 + \theta_{X \rightarrow Y} X + c'_2 W + bM$$

where $\theta_{X \rightarrow Y} = (c'_1 + c'_3 W)$



In this model, $\theta_{X \rightarrow Y}$ is the **conditional direct effect of X** , which is defined by the function $c'_1 + c'_3 W$. Holding M constant, two cases that differ by one unit on X are estimated to differ by $c'_1 + c'_3 W$ units on Y .

This is a very basic conditional process model. It models two pathways through which X affects Y . One is unconditional and indirect via M , and the other is direct but conditional—the size of the direct effect depends on W .

“Conditional indirect effect”

An indirect effect is quantified as the product of paths linking X to Y via the intermediary variable. If one of those paths depends on a moderator, then so too does the indirect effect depend on that moderator. For example:

$$\hat{M} = a_0 + aX$$

$$\hat{Y} = c'_0 + c'_1 X + b_1 M + b_2 W + b_3 WM$$

or, equivalently,

$$\hat{Y} = c'_0 + c'_1 X + (b_1 + b_3 W)M + b_2 W$$

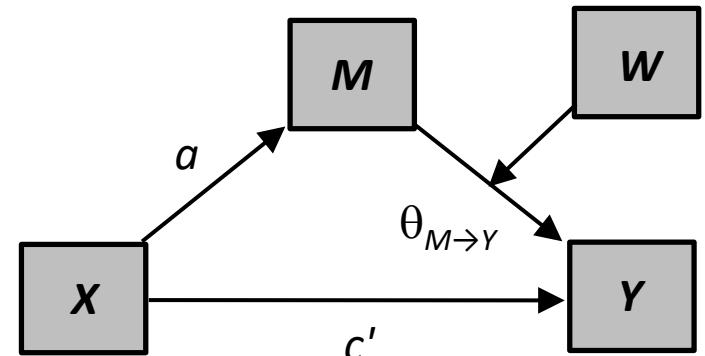
or, equivalently,

$$\hat{Y} = c'_0 + c'_1 X + \theta_{M \rightarrow Y} M + b_2 W$$

where $\theta_{M \rightarrow Y} = (b_1 + b_3 W)$

The indirect effect of X on Y via M is $a\theta_{M \rightarrow Y}$, but as $\theta_{M \rightarrow Y}$ is a conditional effect (the conditional effect of M), then $a\theta_{M \rightarrow Y}$ is the *conditional indirect effect* of X on Y via M : $a\theta_{M \rightarrow Y} = a(b_1 + b_3 W) = ab_1 + ab_3 W$. It depends on W .

This is also a basic conditional process model, and potentially more interesting one. It allows for the process or ‘mechanism’ linking X to Y via M to differ systematically as a function of W . This model allows “mediation to be moderated.”



“Conditional indirect effect”

An indirect effect is quantified as the product of paths linking X to Y via the intermediary variable. If one of those paths depends on a moderator, then so too does the indirect effect depend on that moderator. For example:

$$\hat{M} = a_0 + aX$$

$$\hat{Y} = c'_0 + c'_1 X + b_1 M + b_2 W + b_3 WM$$

or, equivalently,

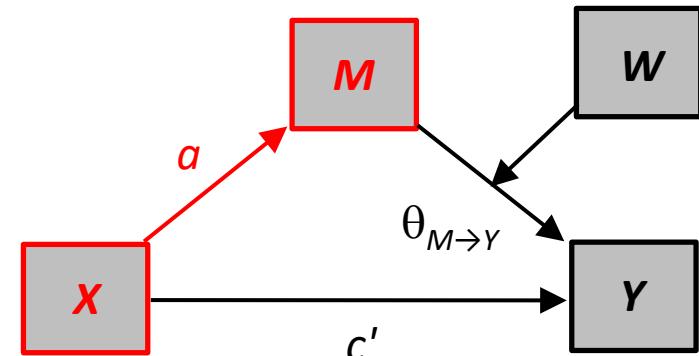
$$\hat{Y} = c'_0 + c'_1 X + (b_1 + b_3 W)M + b_2 W$$

or, equivalently,

$$\hat{Y} = c'_0 + c'_1 X + \theta_{M \rightarrow Y} M + b_2 W$$

$$\text{where } \theta_{M \rightarrow Y} = (b_1 + b_3 W)$$

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$$\hat{Y} = c'_0 + c'_1 X + b_1 M + b_2 W + b_3 WM$$

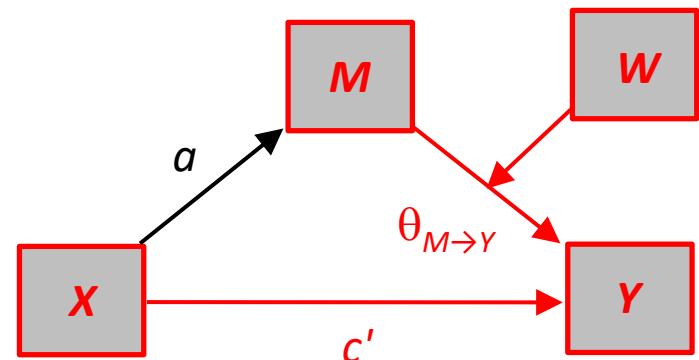
or, equivalently,

$$\hat{Y} = c'_0 + c'_1 X + (b_1 + b_3 W)M + b_2 W$$

or, equivalently,

$$\hat{Y} = c'_0 + c'_1 X + \theta_{M \rightarrow Y} M + b_2 W$$

where $\theta_{M \rightarrow Y} = (b_1 + b_3 W)$



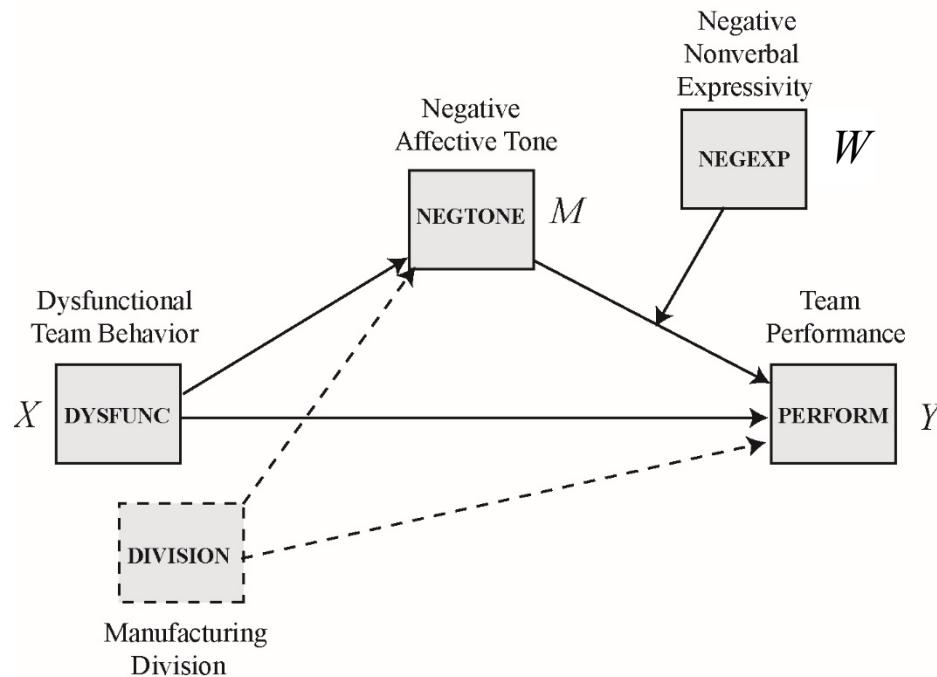
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This is also a basic conditional process model, and potentially more interesting one. It allows for the process or ‘mechanism’ linking X to Y via M to differ systematically as a function of W . This model allows “mediation to be moderated.”

Condition process analysis: Example 1



Condition process analysis: Example 1



This is a model of **negative affective tone (M)** as the mechanism by which **dysfunctional team behavior (X)** influences **performance (Y)**, with that mechanism being contingent on the extent to which **team members hide their negative feelings (W)** from the team. This “nonverbal expressivity” is postulated as moderating the effect of negative tone on performance. This is a “second stage” moderated mediation model.

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Affective Mechanisms Linking Dysfunctional Behavior to Performance in Work Teams: A Moderated Mediation Study

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University of St. Gallen

The present study examines the association between dysfunctional team behavior and team performance. Data included measures of teams' dysfunctional behavior and negative affective tone as well as supervisors' ratings of teams' (nonverbal) negative emotional expressivity and performance. Utilizing a field sample of 61 work teams, the authors tested the proposed relationships with robust data analytic techniques. Results were consistent with the hypothesized conceptual scheme, in that negative team affective tone mediated the relationship between dysfunctional team behavior and performance when teams' nonverbal negative expressivity was high but not when nonverbal expressivity was low. On the basis of the findings, the authors conclude that the connection between dysfunctional behavior and performance in team situations is more complex than was previously believed—thereby yielding a pattern of moderated mediation. In sum, the findings demonstrated that team members' collective emotions and emotional processing represent key mechanisms in determining how dysfunctional team behavior is associated with team performance.

Keywords: work teams, dysfunctional behavior, emotion, emotion regulation, performance

A body of research has recently emerged with an emphasis on “bad” employee behavior (e.g., Dunlop & Lee, 2004; Felps, Mitchell, & Byington, 2006; Griffin & Lopez, 2005; Robinson & O’Leary-Kelly, 1998). According to Griffin and Lopez (2005), bad employee behavior refers to any form of intentional act that has the potential to adversely affect organizations and their employees. In other words, bad behavior reflects employee conduct that an organization would otherwise prefer not to have displayed by its employees. Exemplars of these behaviors can range from employee theft and sabotage to social undermining and antisocial activity.

In their review on employee “bad” behavior, Lawrence and Robinson (2007) remarked that the prevalence and costs of such misconduct “make its study imperative” (p. 378). In the present instance, we focus on bad behavior occurring within a team con-

text (cf. Robinson & O’Leary-Kelly, 1998) and, as recommended by Griffin and Lopez (2005), dub these behaviors *dysfunctional team behavior*. As we suggested earlier, there is a range of possible forms that dysfunctional team behavior might take; however, we chose to focus on the readily observable but not illegal types. For our purposes, *dysfunctional team behavior* is defined as any observable, motivated (but not illegal) behavior by an employee or group of employees that is intended to impair team functioning. In accordance with this operational definition, dysfunctional behaviors within teams should encumber team processes and goals (Robinson & O’Leary-Kelly, 1998), violate norms that are necessary for effective team performance (Felps et al., 2006), and thus hold strong negative connotations for team members (Griffin & Lopez, 2005).

Whereas scholars have exerted considerable effort toward understanding the determinants of dysfunctional behavior (e.g., Dieendorff & Mehta, 2007; Duffy, Ganster, Shaw, Johnson, & Pagon, 2006; Mitchell & Ambrose, 2007), they have not devoted much attention to the associated consequences. Further, researchers have conducted the majority of existing studies at the individual level of analysis. Nevertheless, with the increasing use of teams in organizations (Kozlowski & Ilgen, 2006), there is mounting interest in dysfunctional behavior as a team-level construct (e.g., Felps et al., 2006). Research on this issue, however, is generally limited to investigating how individuals’ team context shapes their dysfunc-

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We thank Steven Brown, Hubert Feild, and Jochen Menges for vetting a draft manuscript and Silja Drack for her assistance with data collection. We also acknowledge the helpful and constructive feedback provided by three

The Data: TEAMS

	dysfunc	negtone	negexp	perform	division	d1	d2	d3
1	-.23	-.51	-.49	.12	1	1	0	0
2	-.13	.22	-.49	.52	1	1	0	0
3	.00	-.08	.84	-.08	1	1	0	0
4	-.33	-.11	.84	-.08	1	1	0	0
5	.39	-.48	.17	.12	1	1	0	0
6	1.02	.72	-.82	1.12	1	1	0	0
7	-.35	-.18	-.66	-.28	1	1	0	0
8	-.23	-.13	-.16	.32	2	0	1	0
9	.39	.52	-.16	-.108	2	0	1	0
10	-.08	-.26	-.16	-.28	2	0	1	0
11	-.23	1.08	-.16	-.108	2	0	1	0
12	.09	.53	.50	-.28	4	0	0	0
13	-.29	-.19	.84	-.28	4	0	0	0
14	-.06	.15	.50	-.88	3	0	0	1
15	.27	.73	-.16	-.08	3	0	0	1
16	.18	-.18	-.16	.32	3	0	0	1
17	.38	.22						
18	.43	.52						
19	.03	.20						
20	-.50	-.68						
21	.03	.31						
22	.41	.75						
23	-.08	-.48						


```
data teams;
input dysfunc negtone negexp perform division d1 d2 d3;
cards;
```

-.23 -.51 -.49 .12 1 1 0 0
-.13 .22 -.49 .52 1 1 0 0
.00 -.08 .84 -.08 1 1 0 0
-.33 -.11 .84 -.08 1 1 0 0
.39 -.48 .17 .12 1 1 0 0
1.02 .72 1.12 1 1 0 0
-.35 1.12 1 1 0 0
-.23 -.13 -.16 .32 2 0 1 0
.39 .52 -.16 1.08 2 0 1 0
-.08 -.26 -.16 -.28 2 0 1 0
-.23 1.08 -.16 -.108 2 0 1 0
.09 .53 .50 -.28 4 0 0 0
-.29 -.19 .84 -.28 4 0 0 0
-.06 .15 .50 1.45 -.23 1 1 0 0
.27 .73 -.16 -.08 3 0 0 1
.18 -.18 -.16 .32 3 0 0 1
.38 .22 -.116 .52 3 0 0 1
.43 .52 .50 -.68 3 0 0 1
.03 .20 .60 -.23 1 1 0 0
-.50 -.68 .26 .17 1 1 0 0
.03 .31 -.40 .17 1 1 0 0
.41 .75 1.06 -.23 1 1 0 0
-.08 -.48 -.06 -.23 1 1 0 0
.66 1.45 .60 -1.23 1 1 0 0

60 teams working in an automobile parts manufacturing facility.

DYSFUNC: Dysfunctional team behavior, i.e.,

How often members of the team do things to weaken the work of others or hinder change and innovation.

NEGTONE: Negative affective group tone.

How often team members report feeling negative emotions at work such as “angry”, “disgust”, etc.

NEGEXP: Negative nonverbal expressivity.

Supervisor's perception as to how easy it is to tell how team members are feeling.

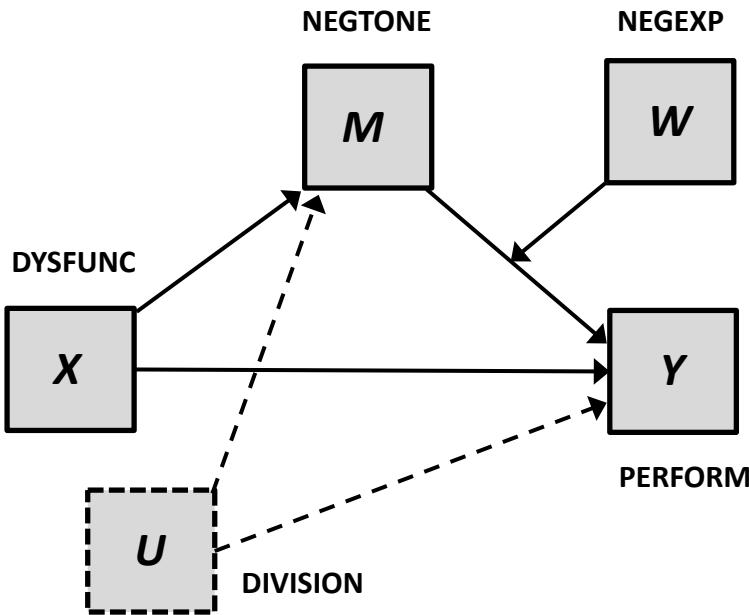
PERFORM: Team performance. Supervisor's judgment as to the team's efficiency, ability to get task done in a timely fashion, etc.

All variables are scaled arbitrarily, but higher = “more”

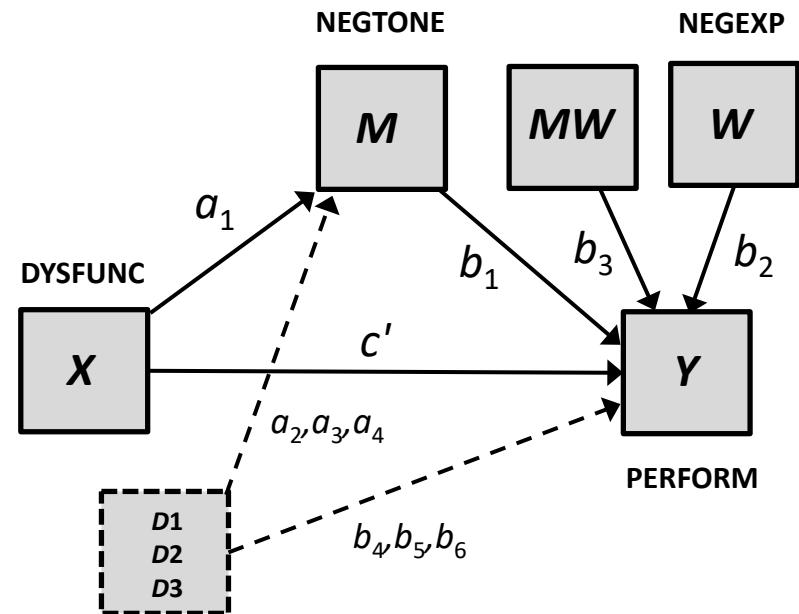
Also available is which of four parts divisions the team worked in, as a single categorical variable (**division**) as well as three dummy variables (**d1, d2, d3**).

Conceptual and statistical models

Conceptual Model



Statistical Model



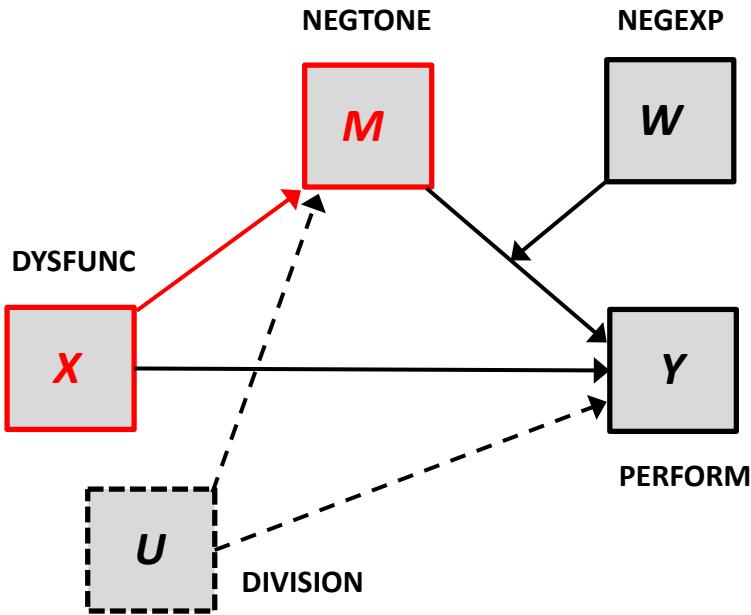
$$\hat{M} = a_0 + a_1 X + a_2 D_1 + a_3 D_2 + a_4 D_3$$

$$\hat{Y} = c'_0 + c' X + b_1 M + b_2 W + b_3 MW + b_4 D_1 + b_5 D_2 + b_6 D_3$$

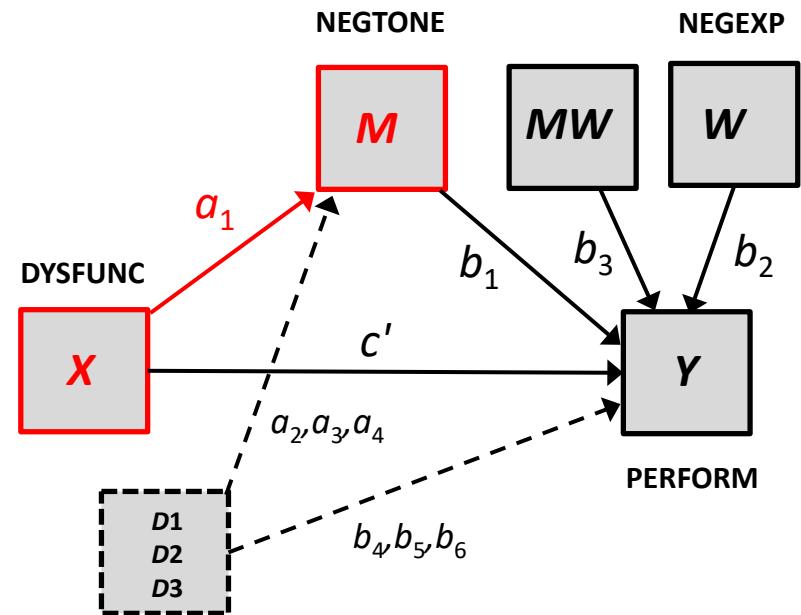
The number of equations needed is equal to the number of variables with arrows pointing at them in the conceptual or statistical diagram.

Conceptual and statistical models

Conceptual Model



Statistical Model



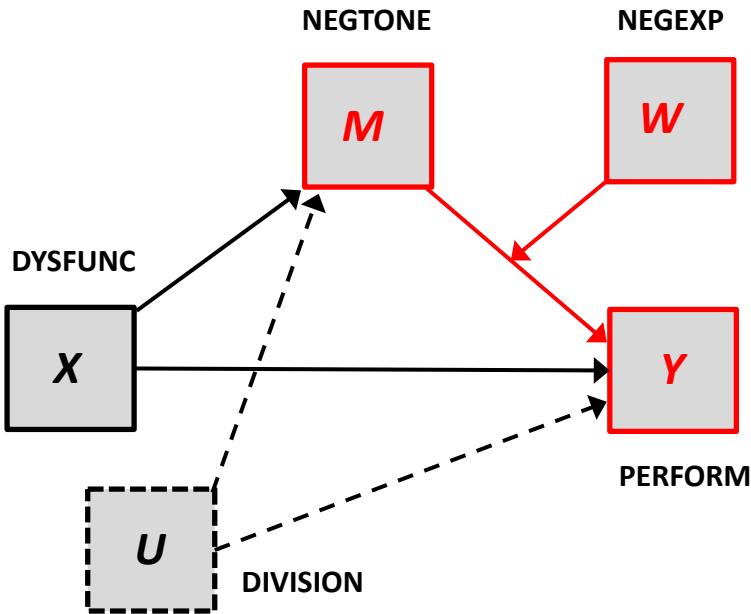
$$\hat{M} = a_0 + a_1 X + a_2 D_1 + a_3 D_2 + a_4 D_3$$

$$\hat{Y} = c'_0 + c' X + b_1 M + b_2 W + b_3 MW + b_4 D_1 + b_5 D_2 + b_6 D_3$$

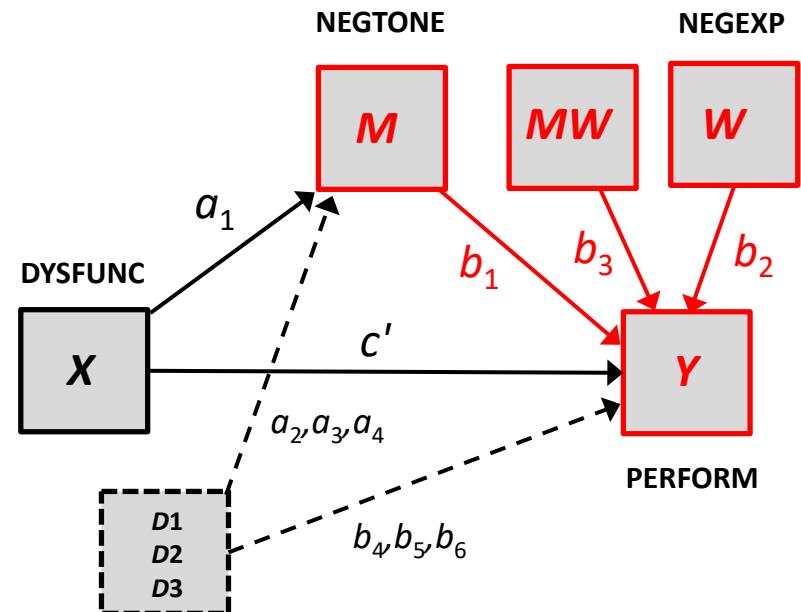
The effect of dysfunctional team behavior (X) on negative affective tone of the work environment (M).

Conceptual and statistical models

Conceptual Model



Statistical Model



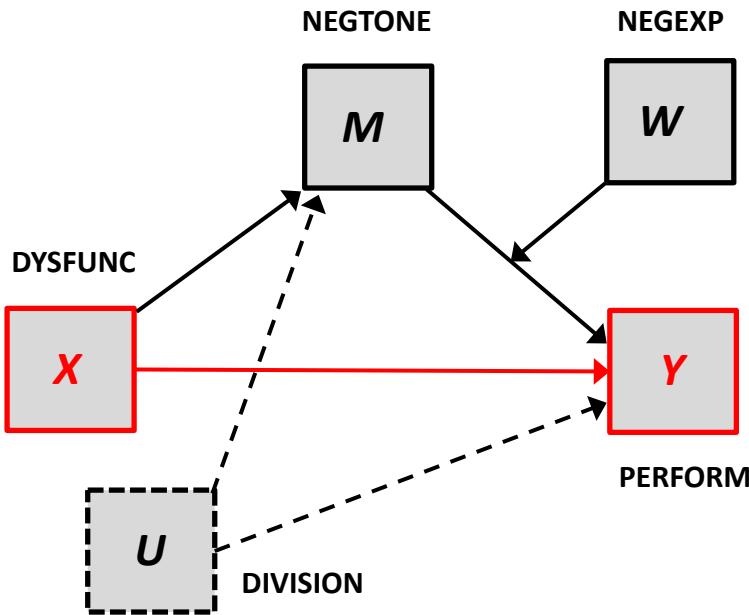
$$\hat{M} = a_0 + a_1 X + a_2 D_1 + a_3 D_2 + a_4 D_3$$

$$\hat{Y} = c'_0 + c' X + \boxed{b_1 M + b_2 W + b_3 MW} + b_4 D_1 + b_5 D_2 + b_6 D_3$$

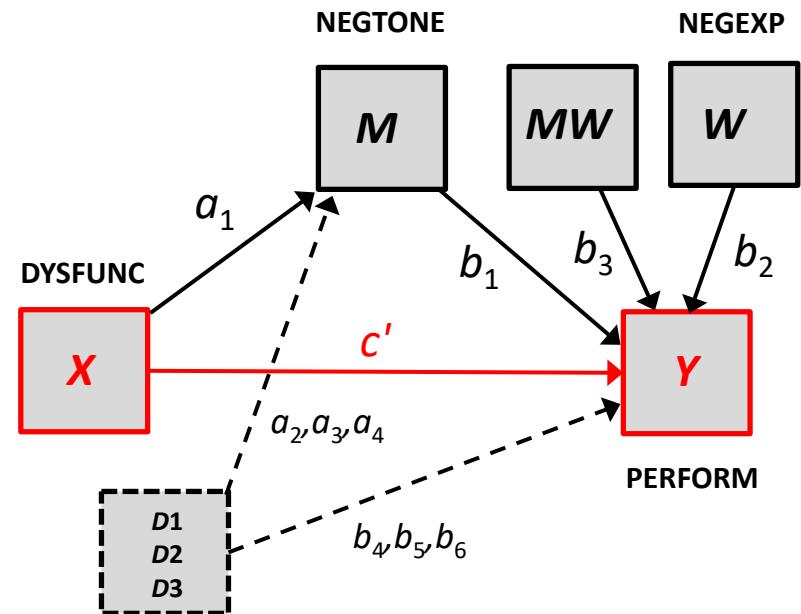
The moderation of the effect of the negative affective tone of the work environment (**M**) on team performance (**Y**) by negative nonverbal expressivity (**W**).

Conceptual and statistical models

Conceptual Model



Statistical Model

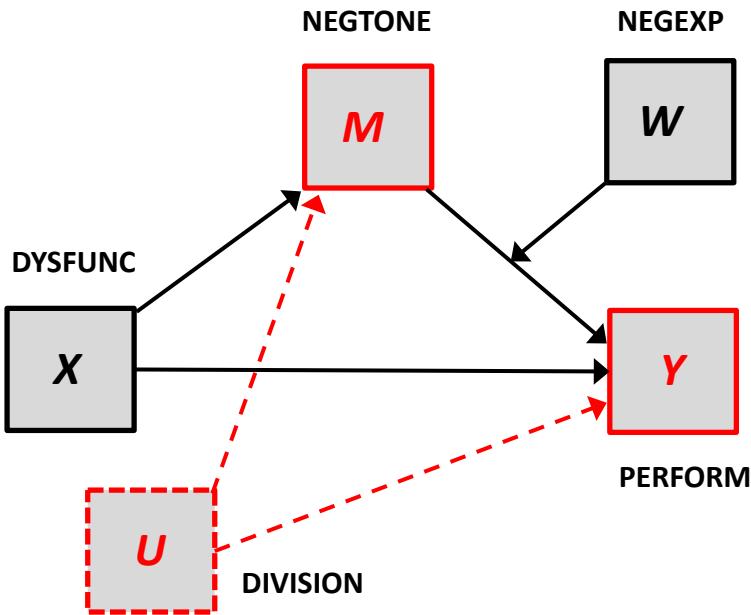


$$\begin{aligned}\hat{M} &= a_0 + a_1 X + a_2 D_1 + a_3 D_2 + a_4 D_3 \\ \hat{Y} &= c'_0 + c' X + b_1 M + b_2 W + b_3 MW + b_4 D_1 + b_5 D_2 + b_6 D_3\end{aligned}$$

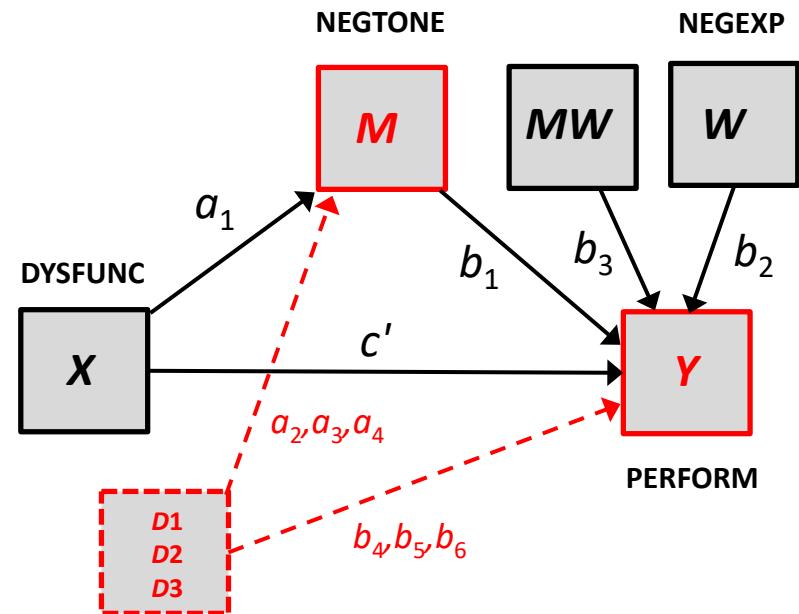
The direct effect of dysfunctional team behavior (**X**) on team performance (**Y**).

Conceptual and statistical models

Conceptual Model



Statistical Model



$$\hat{M} = a_0 + a_1 X + [a_2 D_1 + a_3 D_2 + a_4 D_3]$$

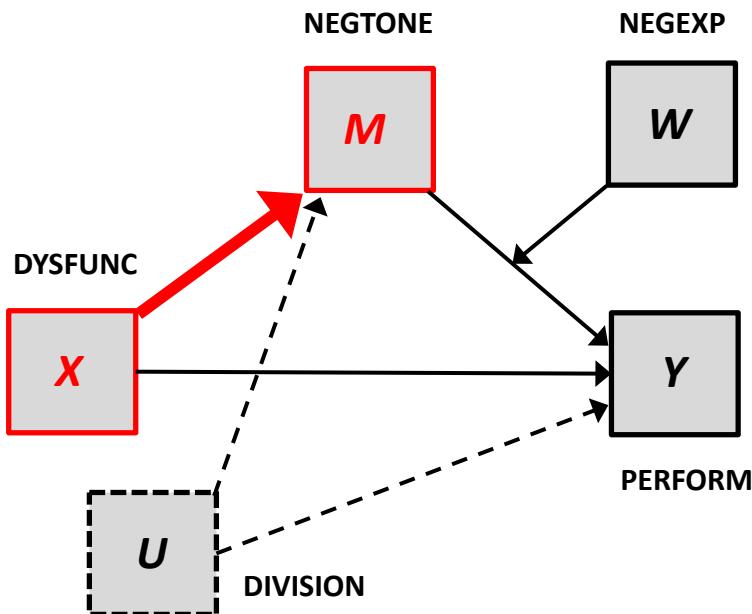
$$\hat{Y} = c'_0 + c' X + b_1 M + b_2 W + b_3 MW + [b_4 D_1 + b_5 D_2 + b_6 D_3]$$

Covariates to account for potential confounding by divisional differences (U) in negative tone of the work environment (M) and performance (Y)

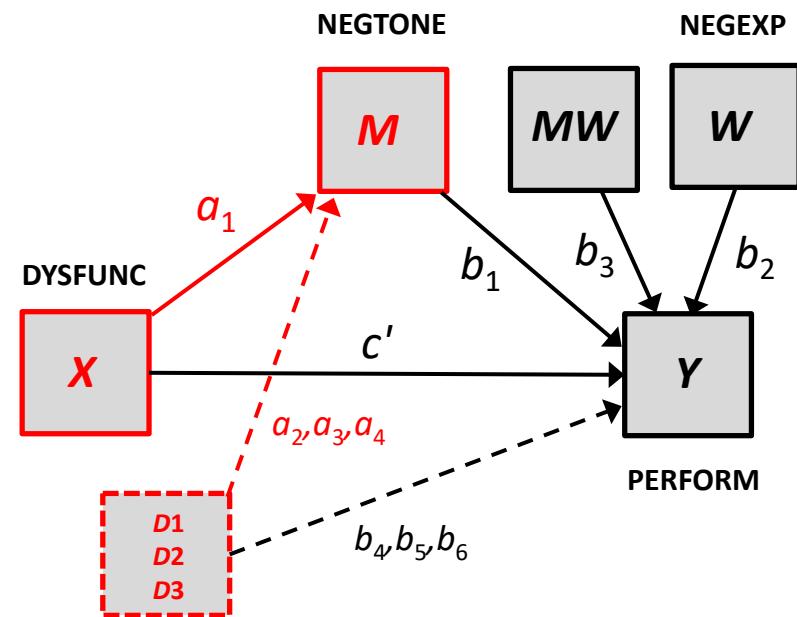
Estimating the a_1 path

Let's first estimate the effect of dysfunctional team behavior on the negative affective tone of the team environment: Path a_1 in the statistical model.

Conceptual Model



Statistical Model



$$\hat{M} = a_0 + a_1 X + a_2 D_1 + a_3 D_2 + a_4 D_3$$

Emphasis is not on statistical significance, as neither the direct or indirect effects of **X** are defined entirely in terms of a_1 .

Estimating the a_1 path

```
regression/dep=negtone/method=enter dysfunc d1 d2 d3.
```

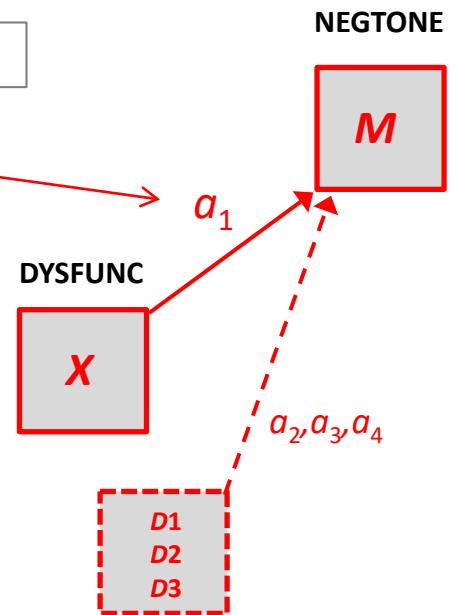
```
proc reg data=teams;model negtone=dysfunc d1 d2 d3;run;
```

Model	Coefficients ^a				
	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
1 (Constant)	- .206	.130		-1.576	.121
Dysfunctional team behavior	.609	.167	.431	3.655	.001
d1	.349	.171	.307	2.033	.047
d2	.295	.212	.193	1.391	.170
d3	.251	.166	.230	1.508	.137

a. Dependent Variable: Negative affective tone

$$a_1 = 0.609$$

$$\hat{M} = -0.206 + 0.609X + 0.349D_1 + 0.295D_2 + 0.251D_3$$

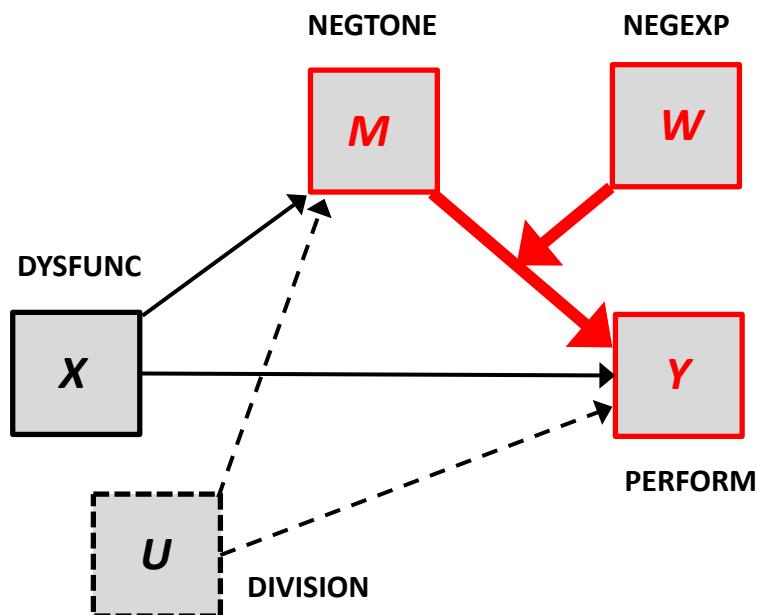


Teams whose members exhibit relatively more dysfunctional behavior tend to operate in a work environment characterized by relatively more negative affective tone (i.e., members report more negative affect)

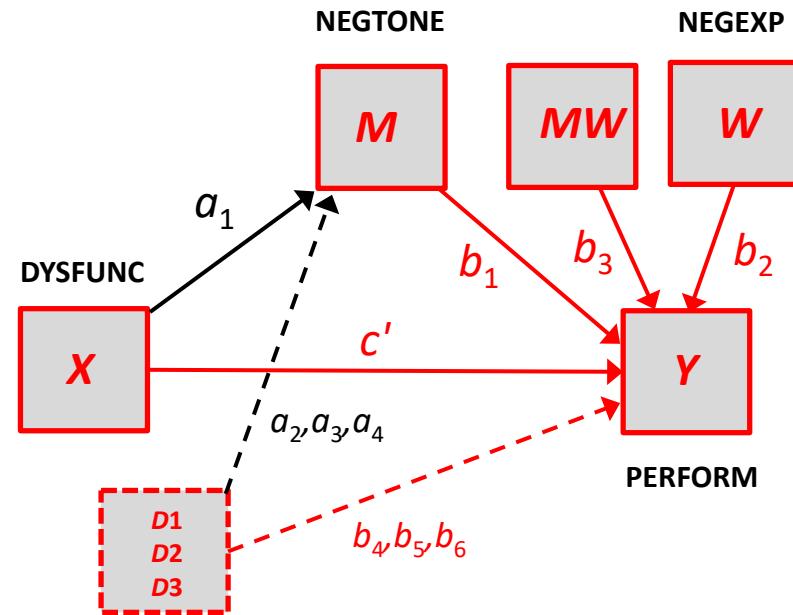
Estimating the moderation component of the model

The conceptual model proposes that the effect of negative work tone on performance depends on negative nonverbal expressivity. Let's see whether there is evidence of this.

Conceptual Model



Statistical Model



$$\hat{Y} = c'_0 + c'X + \boxed{b_1M + b_2W + b_3MW} + b_4D_1 + b_5D_2 + b_6D_3$$

We most care about the moderation components of the model of Y : b_1 , b_2 , and b_3 . But these must be estimated in the context of the complete model of Y , which includes X as well.

Estimating the moderation component of the model

```
compute toneexp=negtone*negexp.
regression/dep=perform/method=enter dysfunc negtone negexp toneexp d1 d2 d3.
```

```
data teams; set teams; toneexp=negtone*negexp; run;
proc reg data=teams; model perform=dysfunc negtone negexp toneexp d1 d2 d3; run;
```

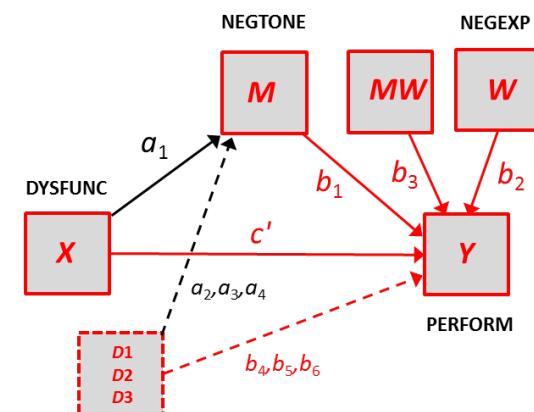
Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error			
1 (Constant)	-.175	.130		-1.344	.185
Dysfunctional team behavior	.373	.181	.265	2.062	.044
Negative affective tone	-.489	.138	-.491	-3.549	.001
Negative expressivity	-.022	.118	-.023	-.188	.852
toneexp	-.450	.245	-.240	-1.835	.072
d1	.182	.172	.161	1.056	.296
d2	.084	.210	.055	.400	.690
d3	.282	.165	.259	1.709	.093

a. Dependent Variable: Team performance

$$b_1 = -0.489, b_2 = -0.022, b_3 = -0.450$$

$$\hat{Y} = -0.175 + 0.373X - 0.489M - 0.022W - 0.450MW + 0.182D_1 + 0.084D_2 + 0.282D_3$$



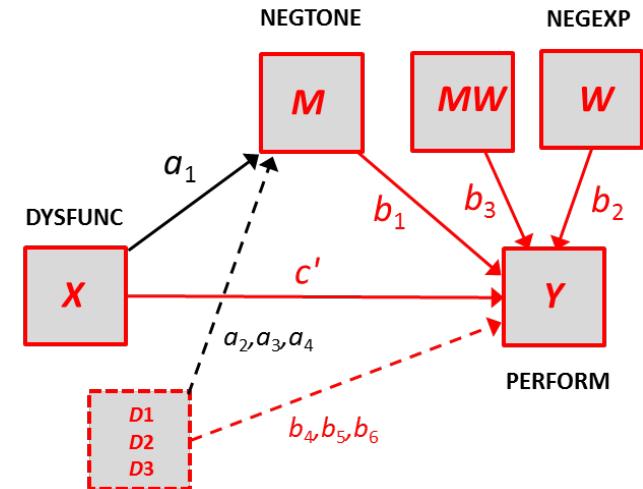
“Marginally significant” evidence that the effect of negative tone of the work environment on team performance depends on the negative nonverbal expressivity of team members. To better understand this, dissect this model.

Estimating the moderation component of the model

Model	Coefficients ^a				
	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
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d2	.084	.210	.055	.400	.690
d3	.282	.165	.259	1.709	.093

a. Dependent Variable: Team performance

$$b_1 = -0.489, b_3 = -0.450$$



$$\hat{Y} = -0.175 + 0.373X - 0.489M - 0.022W - 0.450MW + 0.182D_1 + 0.084D_2 + 0.282D_3$$

which can be written as

$$\hat{Y} = -0.175 + 0.373X + (-0.489 - 0.450W)M - 0.022W + 0.182D_1 + 0.084D_2 + 0.282D_3$$

or

$$\hat{Y} = -0.175 + 0.373X + \theta_{M \rightarrow Y}M - 0.022W + 0.182D_1 + 0.084D_2 + 0.282D_3$$

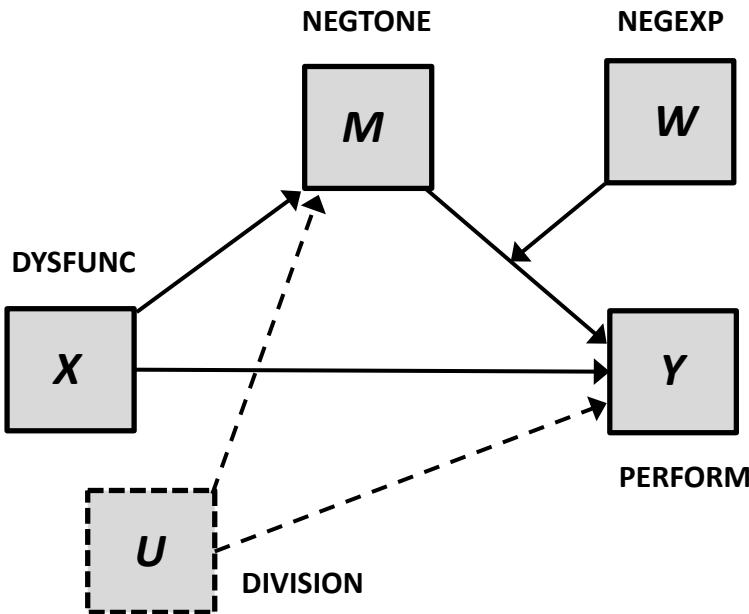
where

$$\theta_{M \rightarrow Y} = -0.489 - 0.450W = b_1 + b_3W$$

Let's visualize and probe this. PROCESS will take the work out of it.

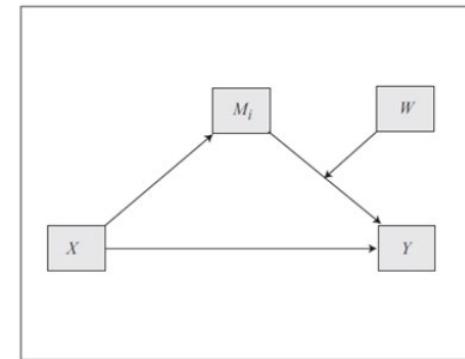
Estimation of the model in PROCESS

PROCESS takes most of the computational burden off our shoulders.

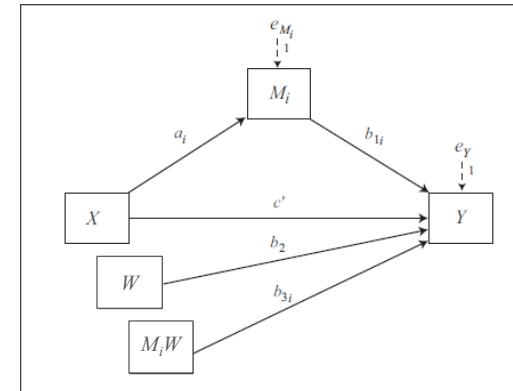


This is PROCESS model 14

Model 14



Statistical Diagram



```
process cov = d1 d2 d3/x=dysfunc/m=negtone/y=perform/w=negexp/boot=10000/model=14/plot  
= 1.
```

```
%process (data=teams,cov=d1 d2 d3,x=dysfunc,m=negtone,y=perform,w=negexp,  
boot=10000,model=14, plot = 1);
```

PROCESS output

Output J

OUTCOME VARIABLE:
perform

Model Summary

R	R-sq	MSE	F	df1	df2	p
.5937	.3524	.2006	4.0428	7.0000	52.0000	.0013

Model

	coeff	se	t	p	LLCI	ULCI
constant	-.1754	.1305	-1.3444	.1847	-.4373	.0864
dysfunc	.3729	.1808	2.0622	.0442	.0100	.7357
negtone	-.4886	.1377	-3.5485	.0008	-.7649	-.2123
negexp	-.0221	.1176	-.1875	.8520	-.2581	.2140
Int_1	-.4498	.2451	-1.8353	.0722	-.9417	.0420
d1	.1815	.1720	1.0556	.2960	-.1635	.5266
d2	.0841	.2099	.4004	.6905	-.3372	.5053
d3	.2816	.1648	1.7087	.0935	-.0491	.6123

Product terms key:

Int_1 : negtone x negexp

Test(s) of highest order unconditional interaction(s):

R2-chng	F	df1	df2	p
M*W	.0419	3.3684	1.0000	52.0000 .0722

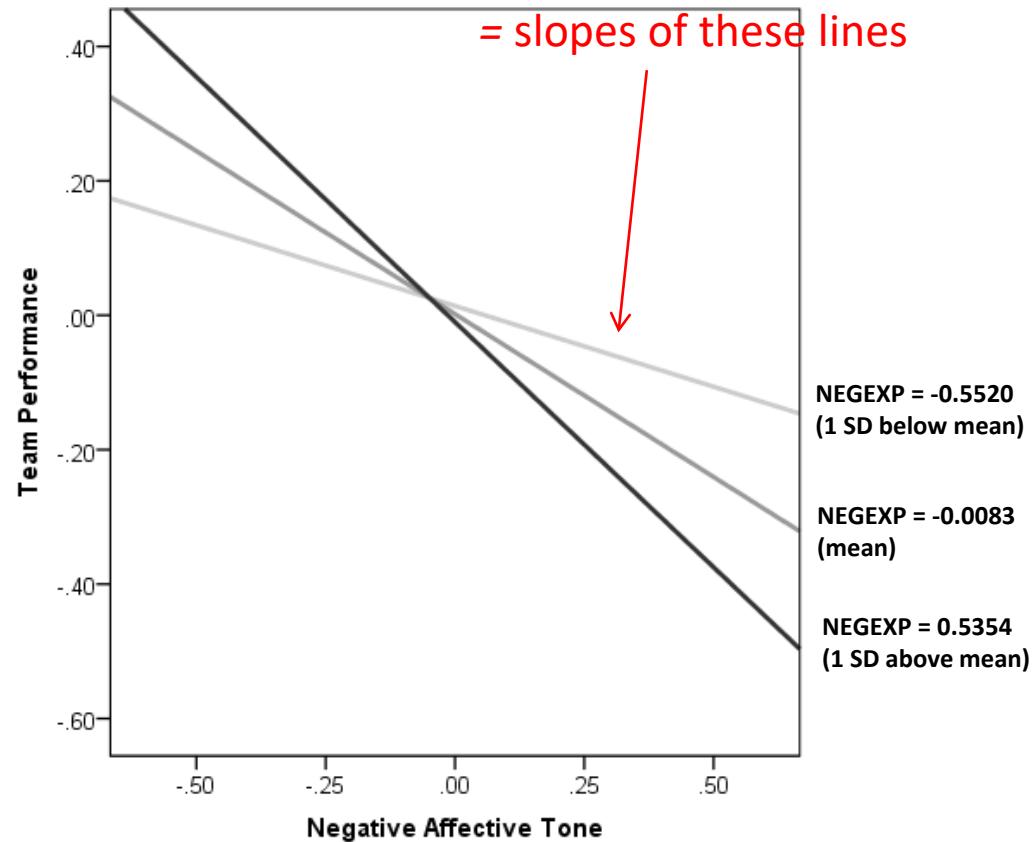
Visualizing the interaction

Use of the PLOT option in SPSS (`plot=1`) produces a table of estimated values of the outcome for various combinations of the focal predictor and moderator, and an SPSS program to generate a skeleton of the plot that can be edited.

M W \hat{Y}

```
DATA LIST FREE/negtone negexp perform.  
BEGIN DATA.  
-.4782 -.5520 .1288  
.0472 -.5520 .0026  
.5726 -.5520 -.1237  
-.4782 -.0083 .2338  
.0472 -.0083 -.0210  
.5726 -.0083 -.2757  
-.4782 .5354 .3387  
.0472 .5354 -.0445  
.5726 .5354 -.4278  
END DATA.  
GRAPH/SCATTERPLOT=negtone WITH perform  
BY negexp.
```

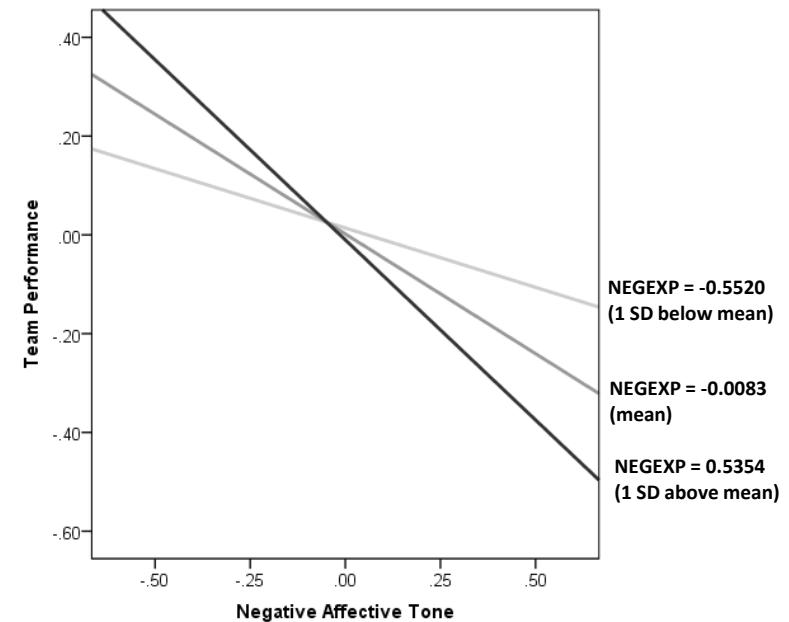
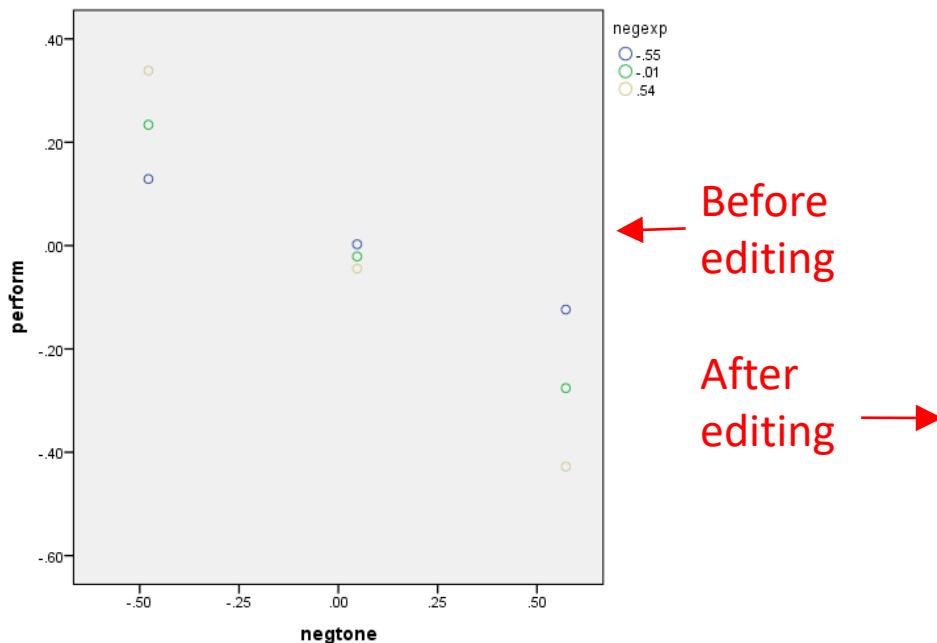
$$\begin{aligned}\theta_{M \rightarrow Y} &= b_1 + b_3 W = \\ &= -0.489 - 0.450W \\ &= \text{slopes of these lines}\end{aligned}$$



Visualizing the interaction

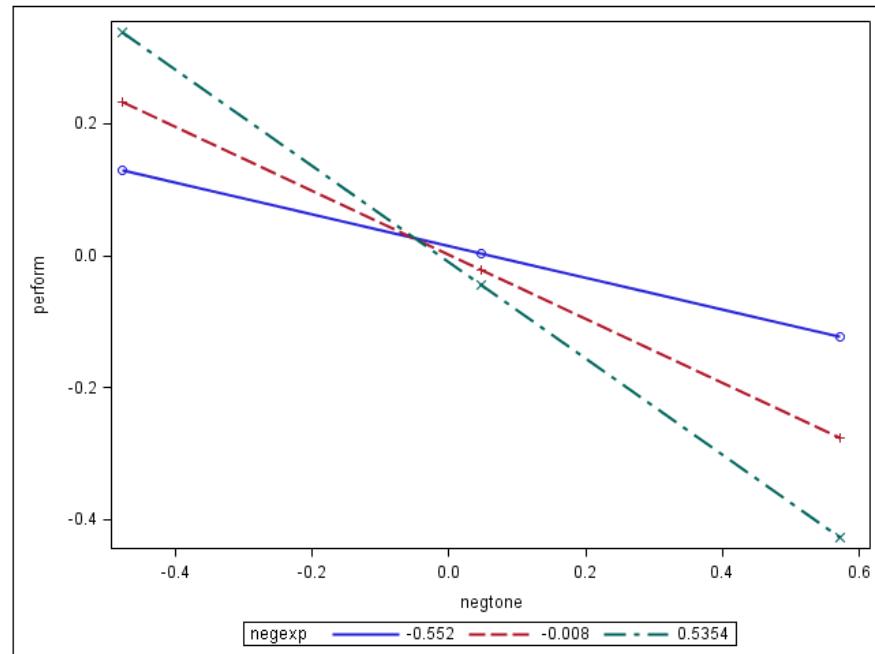
```
DATA LIST FREE/negtone negexp perform.  
BEGIN DATA.  
    -.4782    -.5520     .1288  
    .0472    -.5520     .0026  
    .5726    -.5520    -.1237  
    -.4782   -.0083     .2338  
    .0472   -.0083    -.0210  
    .5726   -.0083    -.2757  
    -.4782    .5354     .3387  
    .0472    .5354    -.0445  
    .5726    .5354    -.4278  
END DATA.  
GRAPH/SCATTERPLOT=negtone WITH perform BY negexp.
```

Use of the PLOT option (**plot=1**) produces a table of estimated values of the outcome for various combinations of the focal predictor and moderator.



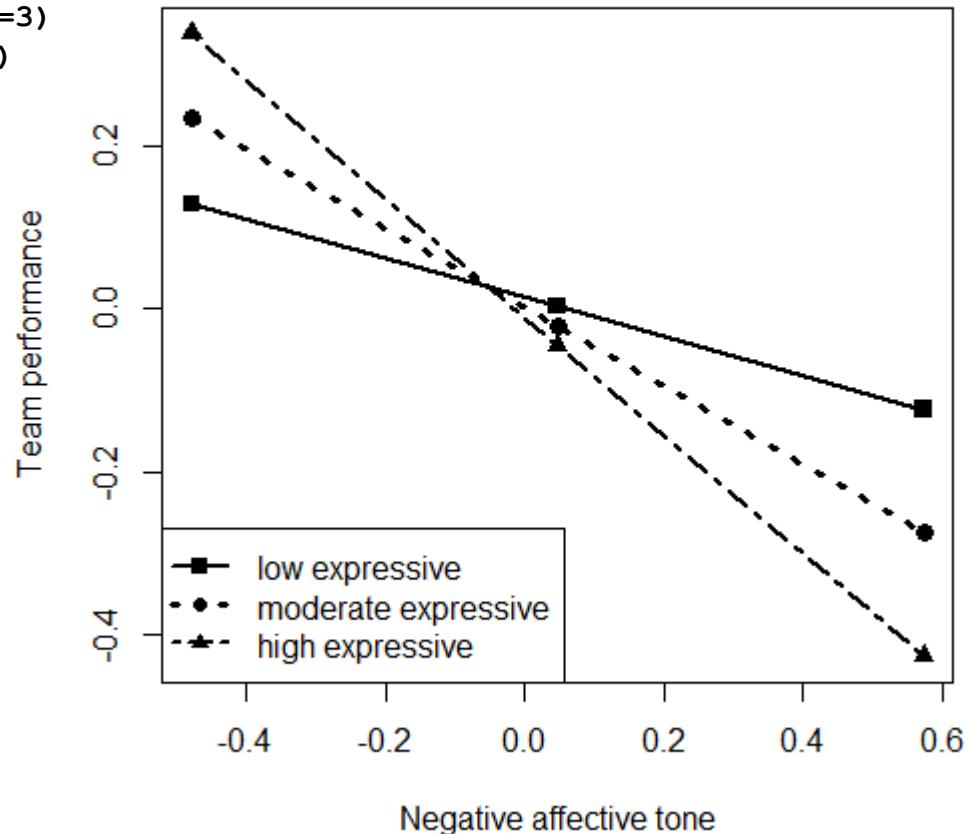
Example code in SAS

```
data;
input negtone negexp perform;
cards;
- .4782      -.5520       .1288
.0472       -.5520       .0026
.5726       -.5520      -.1237
-.4782      -.0083       .2338
.0472       -.0083      -.0210
.5726       -.0083      -.2757
-.4782      .5354        .3387
.0472       .5354        -.0445
.5726       .5354        -.4278
run;
proc sgplot; reg x=negtone y=perform/group=negexp;run;
```



Example code in R

```
m<-c(-.478,.047,.573,-.478,.047,.573,-.478,.047,.573)
w<-c(-.552,-.552,-.552,-.008,-.008,-.008,.535,.535,.535)
y<-c(.129,.003,-.124,.234,-.021,-.276,.339,-.045,-.428)
wmarker<-c(15,15,15,16,16,16,17,17,17)
plot(y=y,x=m,cex=1.2,pch=wmarker,xlab="Negative affective tone",
ylab="Team performance")
legend.txt<-c("low expressive","moderate expressive","high expressive")
legend("bottomleft", legend = legend.txt,cex=1,lty=c(1,3,6),lwd=c(2,3,2),pch=c(15,16,17))
lines(m[w==-.552],y[w==-.552],lwd=2)
lines(m[w==-.008],y[w==-.008],lwd=3,lty=3)
lines(m[w==.535],y[w==.535],lwd=2,lty=6)
```



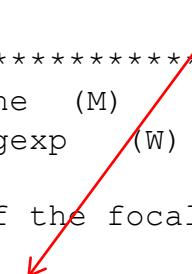
Probing the interaction: Pick-a-point

PROCESS sees that the moderator is quantitative (i.e., it has more than 2 values) so it implements the pick-a-point procedure with moderator values equal to 16th, 50th, and 84th percentile.

$$\theta_{M \rightarrow Y} = b_1 + b_3 W = -0.489 - 0.450W$$

```
*****
Focal predict: negtone (M)
Mod var: negexp (W)
```

Conditional effects of the focal predictor at values of the moderator(s) :

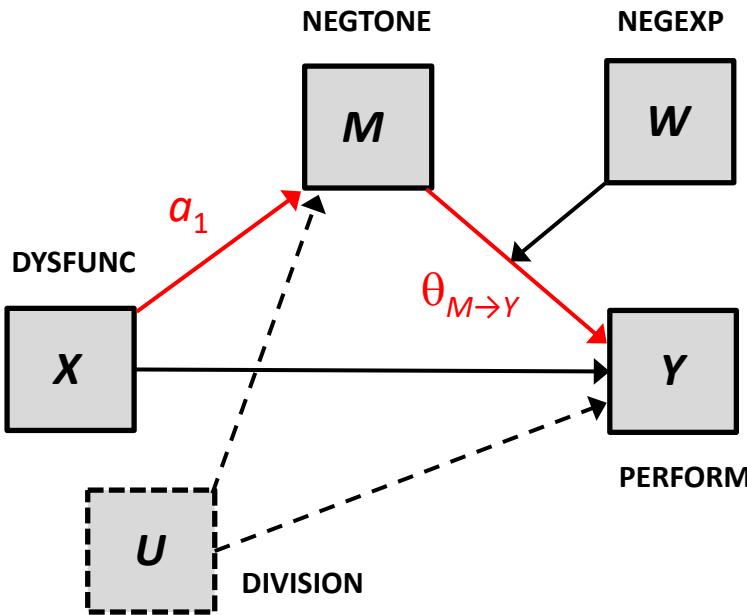


negexp	Effect	se	t	p	LLCI	ULCI
-.5308	-.2498	.2196	-1.1379	.2604	-.6904	.1907
-.0600	-.4616	.1434	-3.2188	.0022	-.7494	-.1738
.6000	-.7585	.1633	-4.6451	.0000	-1.0862	-.4308

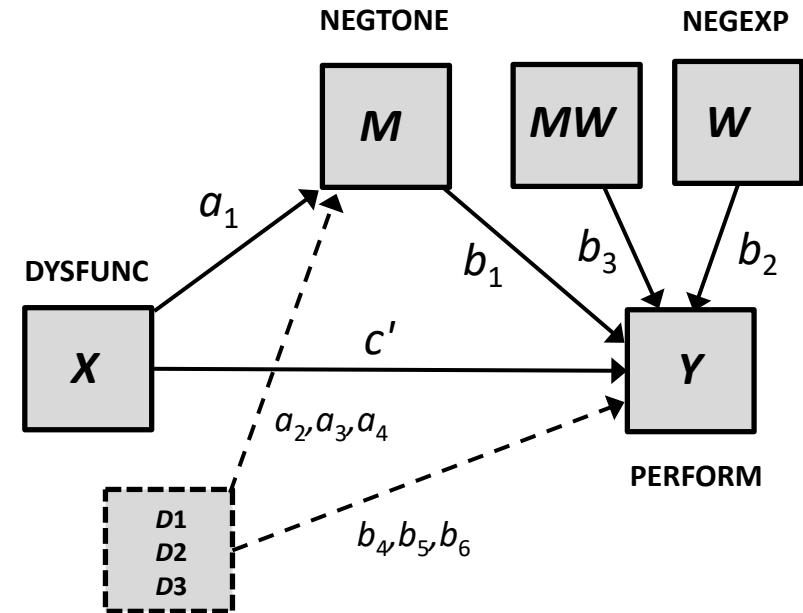
Negative affective tone is significantly negatively related to performance among teams relatively “moderate” and “relatively high” in negative nonverbal expressivity but not among teams “relatively low” in negative nonverbal expressivity.

The conditional indirect effect of X

Conceptual Model



Statistical Model

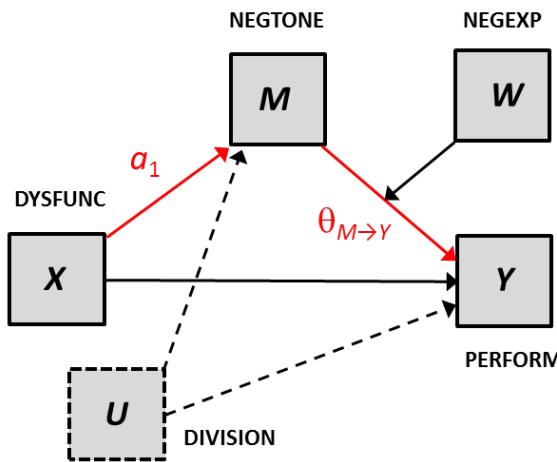


The conditional indirect effect of X on Y through M is the product of the effect of X on M (a_1) and the conditional effect of M on Y given W ($\theta_{M \rightarrow Y} = b_1 + b_3 W$):

$$a_1 \theta_{M \rightarrow Y} = a_1(b_1 + b_3 W) = 0.610(-0.489 - 0.450W)$$

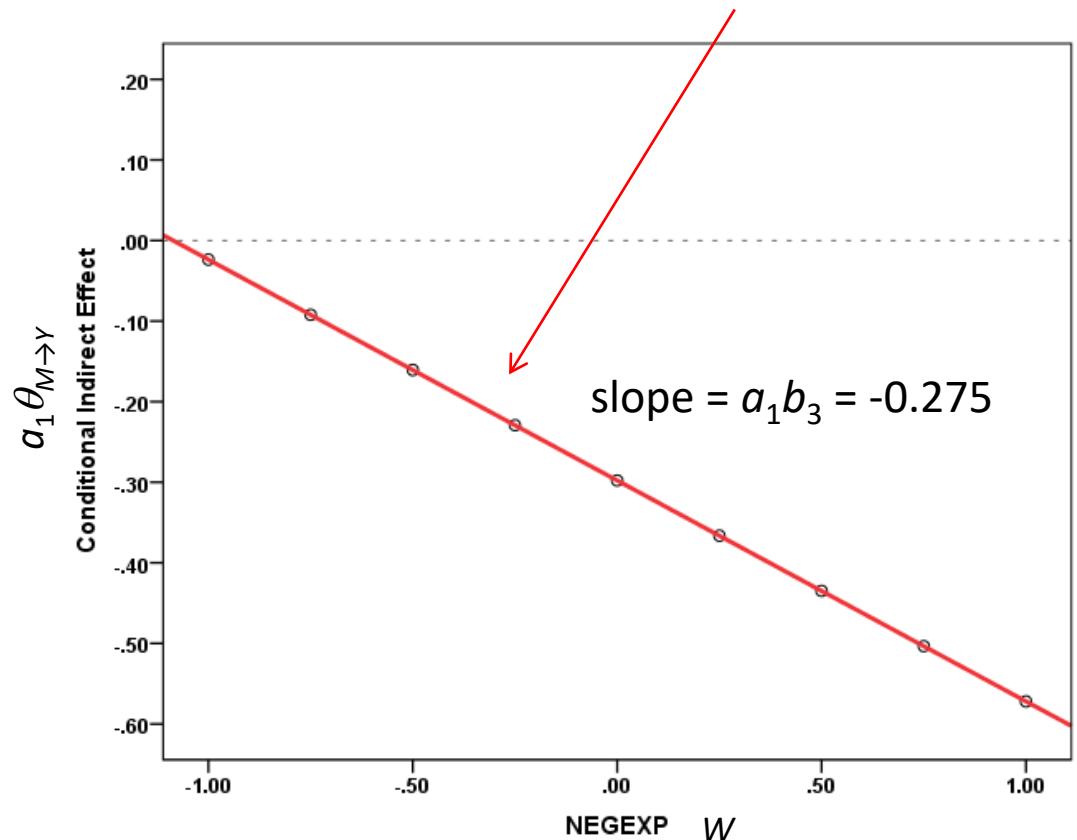
The indirect effect of dysfunctional team behavior on team performance through negative tone is allowed to be a function of negative nonverbal expressivity.

A visual representation of the indirect effect



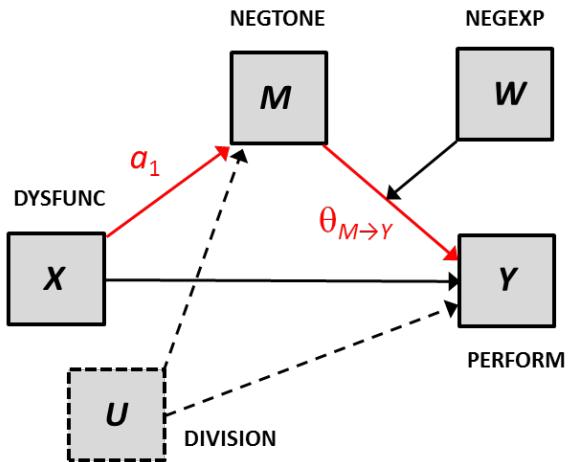
The indirect effect is more negative as negative nonverbal expressivity increases. The “**index of moderated mediation**” is $a_1 b_3 = -0.275$. It quantifies the relationship between the moderator and the indirect effect in this model.

$$a_1 \theta_{M \rightarrow Y} = a_1(b_1 + b_3 W) = 0.610(-0.489 - 0.450W) \\ = a_1 b_1 + a_1 b_3 W = -0.298 - 0.275W$$



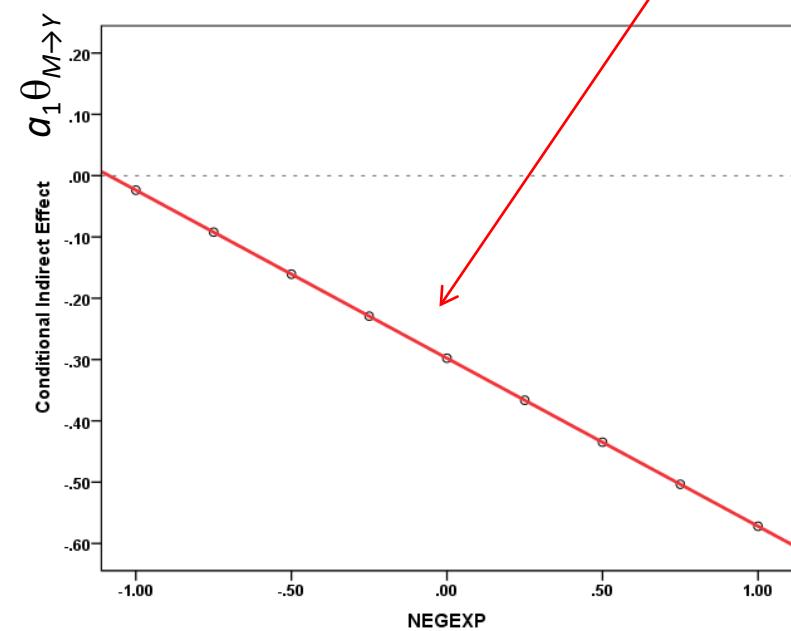
As will be seen, a hypothesis test that the slope of this function is equal to 0 is a formal test of moderated mediation....moderation of the indirect effect.

The conditional indirect effect of X



$$a_1 \theta_{M \rightarrow Y} = a_1(b_1 + b_3 W) = 0.610(-0.489 - 0.450W) \\ = a_1 b_1 + a_1 b_3 W = -0.298 - 0.275W$$

The indirect effect is more negative as negative nonverbal expressivity increases.



Using this function, we can estimate the indirect effect for any value of the moderator we choose:

If we had used moments = 1

NEGEXP (W)	a_1	$\theta_{M \rightarrow Y}$	$a_1 \theta_{M \rightarrow Y}$
$\bar{W} - SD_w$	0.610	-0.240	-0.146
\bar{W}	0.610	-0.485	-0.296
$\bar{W} + SD_w$	0.610	-0.730	-0.445

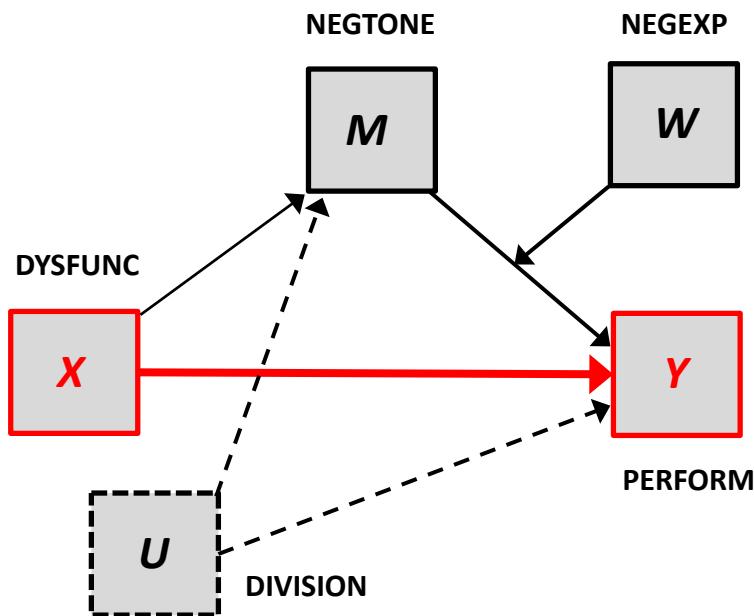
Conditional indirect effects



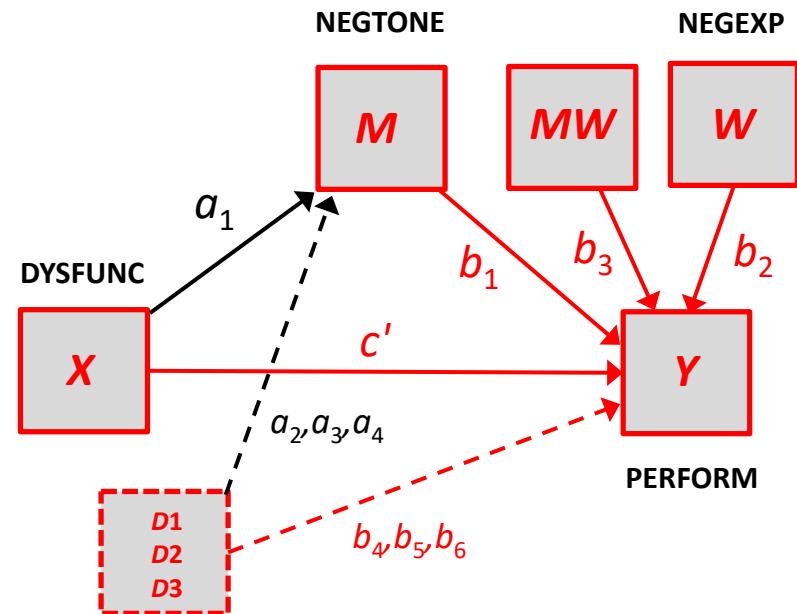
The direct effect of X

The direct effect of X is the effect of X of Y that does not operate through M .

Conceptual Model



Statistical Model



$$\hat{Y} = i_1 + c'X - b_1M + b_2W + b_3MW + b_4D_1 + b_5D_2 + b_6D_3$$

In this model, the direct effect is fixed to be unmoderated. It is a constant rather than a function of another variable in the model.

The direct effect of X (we estimated earlier)

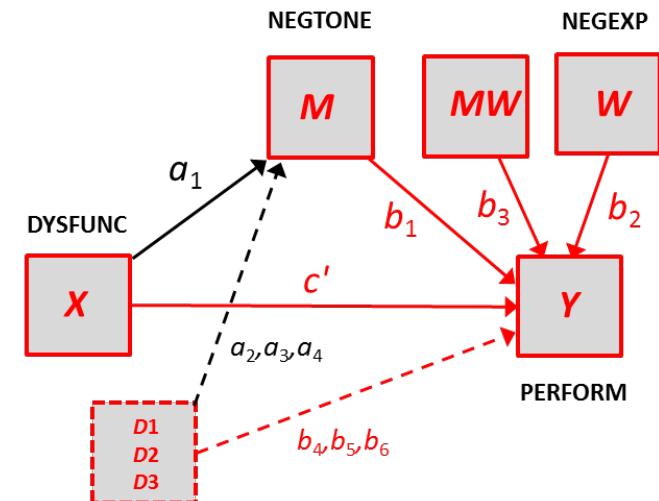
Model	Coefficients ^a				
	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
1 (Constant)	-.175	.130		-1.344	.185
Dysfunctional team behavior	.373	.181	.265	2.062	.044
Negative affective tone	-.489	.138	-.491	-3.549	.001
Negative expressivity	-.022	.118	-.023	-.188	.852
toneexp	-.450	.245	-.240	-1.835	.072
d1	.182	.172	.161	1.056	.296
d2	.084	.210	.055	.400	.690
d3	.282	.165	.259	1.709	.093

a. Dependent Variable: Team performance

$$c' = 0.373, t(55) = 2.062, p < .05.$$

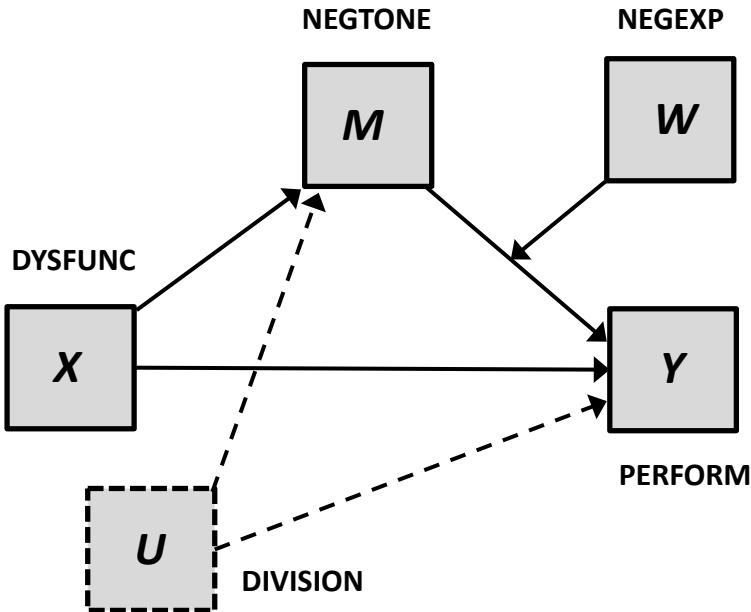
$$\hat{Y} = -0.175 + 0.373X - 0.489M - 0.022W - 0.450MW + 0.182D_1 + 0.084D_2 + 0.282D_3$$

Holding constant negative affective tone and negative nonverbal expressivity, teams that exhibit more dysfunctional behavior perform *better*.



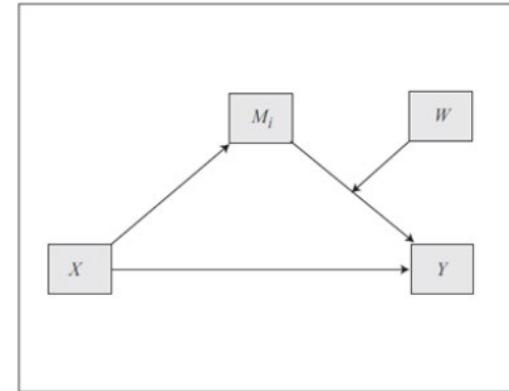
Estimation of the model in PROCESS

PROCESS takes most of the computational burden off our shoulders.

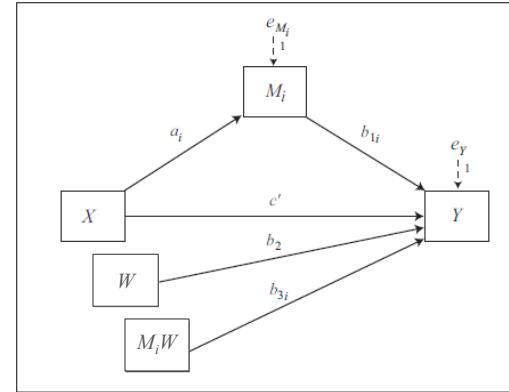


This is PROCESS model 14

Model 14



Statistical Diagram



```
process cov = d1 d2 d3/x=dysfunc/m=negtone/y=perform/w=negexp/boot=10000  
/model=14/plot = 1.
```

```
%process (data=teams,cov=d1 d2 d3,x=dysfunc,m=negtone,y=perform,w=negexp,  
boot=10000,model=14, plot = 1);
```

PROCESS output

```
Model = 14
Y = perform
X = dysfunc
M = negtone
W = negexp
```

Statistical Controls:
CONTROL= d1 d2 d3

Sample size
60

Outcome: negtone

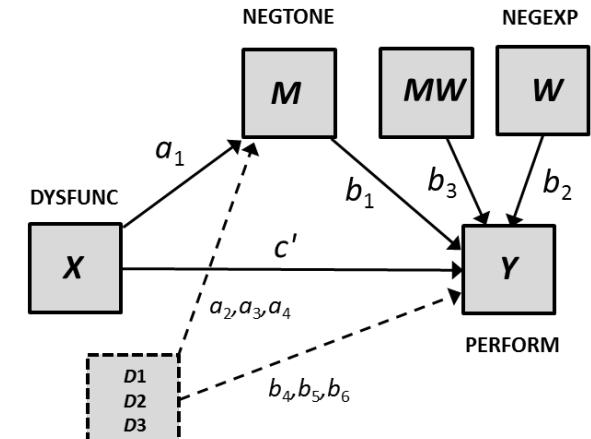
$$\hat{M} = -0.206 + 0.610X + \dots$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.5026	.2526	.2213	4.6462	4.0000	55.0000	.0027

Model

	coeff	se	t	p	LLCI	ULCI	
constant	-.2057	.1305	-1.5760	.1208	-.4672	.0559	
dysfunc	.6095	.1668	3.6546	.0006	.2753	.9437	$a_1 = 0.610$
d1	.3487	.1715	2.0332	.0469	.0050	.6923	
d2	.2951	.2122	1.3906	.1700	-.1302	.7204	
d3	.2507	.1663	1.5078	.1373	-.0825	.5840	



Output J

PROCESS output

$$\hat{Y} = -0.175 + 0.373X - 0.489M - 0.022W - 0.450MW + \dots$$

Outcome: perform

Model Summary

R	R-sq	MSE	F	df1	df2	p
.5937	.3524	.2006	4.0428	7.0000	52.0000	.0013

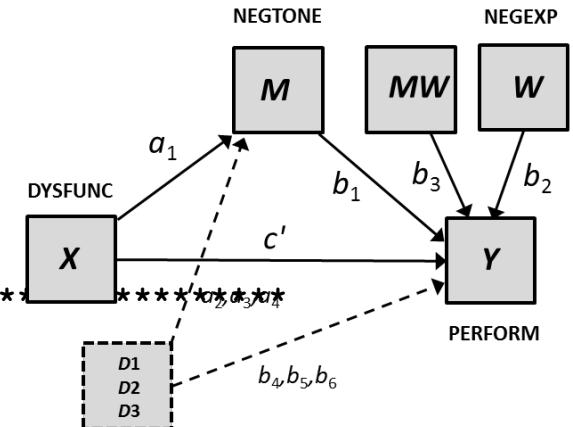
Model

	coeff	se	t	p	LLCI	ULCI
constant	-.1754	.1305	-1.3444	.1847	-.4373	.0864
negtone	-.4886	.1377	-3.5485	.0008	-.7649	-.2123
dysfunc	.3729	.1808	2.0622	.0442	.0100	.7357
negexp	-.0221	.1176	-.1875	.8520	-.2581	.2140
int_1	-.4498	.2451	-1.8353	.0722	-.9417	.0420
d1	.1815	.1720	1.0556	.2960	-.1635	.5266
d2	.0841	.2099	.4004	.6905	-.3372	.5053
d3	.2816	.1648	1.7087	.0935	-.0491	.6123

$$\begin{aligned}
 b_1 &= -0.489 \\
 c' &= 0.373 \\
 b_2 &= -0.022 \\
 b_3 &= -0.450
 \end{aligned}$$

Interactions:

int_1 negtone X negexp



Output J

PROCESS output

***** DIRECT AND INDIRECT EFFECTS *****

Direct effect of X on Y

Effect	SE	t	p	LLCI	ULCI	Direct effect $c' = .373, p < .05$
.3729	.1808	2.0622	.0442	.0100	.7357	

INDIRECT EFFECT:

dysfunc → negtone → perform

negexp	Effect	BootSE	BootLLCI	BootULCI
-.5308	-.1523	.1540	-.4335	.1943
-.0600	-.2813	.1249	-.5432	-.0549
.6000	-.4623	.1678	-.8095	-.1503

Output J

Index of moderated mediation:

	Index	BootSE	BootLLCI	BootULCI
negexp	-.2742	.1791	-.7172	-.0234

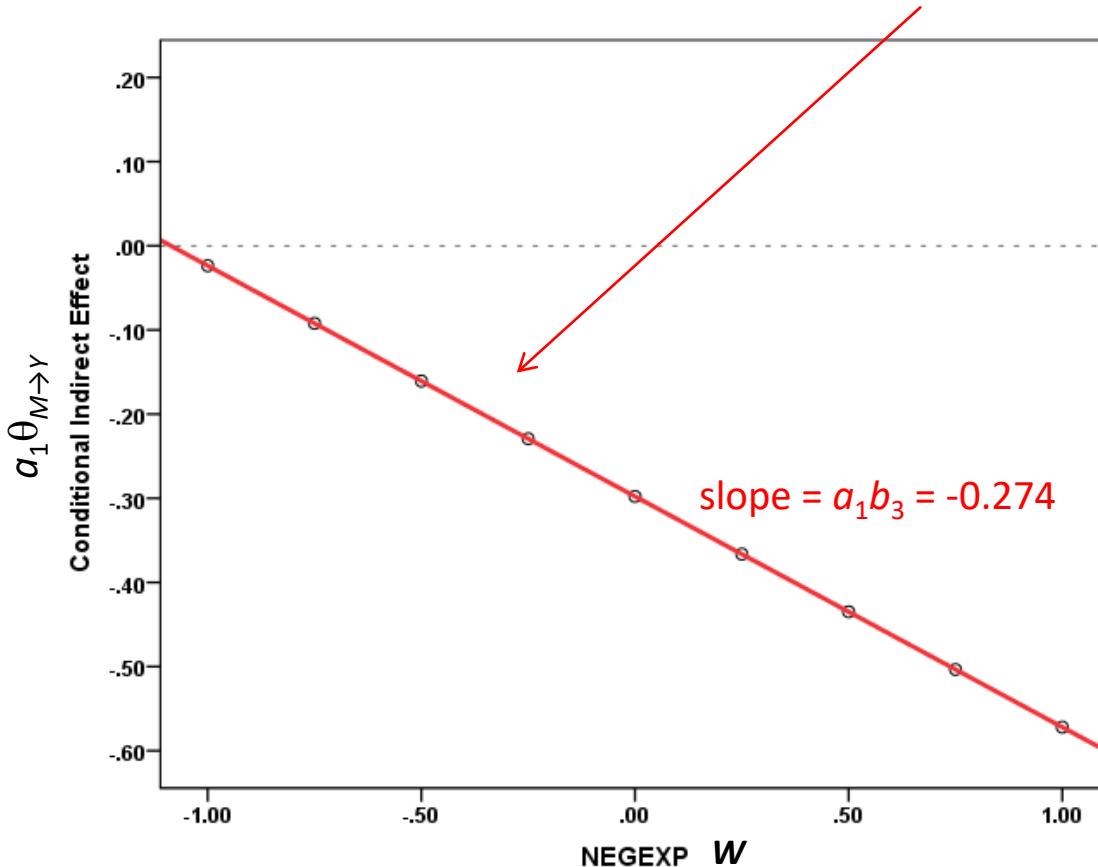
$$\begin{aligned}
 a_1\theta_{M \rightarrow Y} &= a_1(b_1 + b_3W) \\
 &= 0.610(-0.489 - 0.450W) \\
 &= a_1b_1 + a_1b_3W \\
 &= -0.298 - 0.274W
 \end{aligned}$$

W values in conditional tables are the 16th, 50th, and 84th percentiles.

PROCESS sees that the moderator is continuous so, without instruction otherwise, prints the conditional indirect effect at the 16th, 50th, and 84th percentiles. For the mean +/- 1 SD add moments = 1 to the command line.

A statistical test of moderated mediation in the second stage moderated mediation model

$$\begin{aligned}a_1 \theta_{M \rightarrow Y} &= a_1(b_1 + b_3 W) = 0.610(-0.489 - 0.450W) \\&= a_1 b_1 + a_1 b_3 W = -0.298 - 0.274W\end{aligned}$$



The indirect effect is a function of W (negative nonverbal expressivity) in our model. This function is a line.

$$\begin{aligned}a_1 \theta_{M \rightarrow Y} &= a_1(b_1 + b_3 W) \\&= a_1 b_1 + a_1 b_3 W \\&= -0.298 - 0.274W\end{aligned}$$

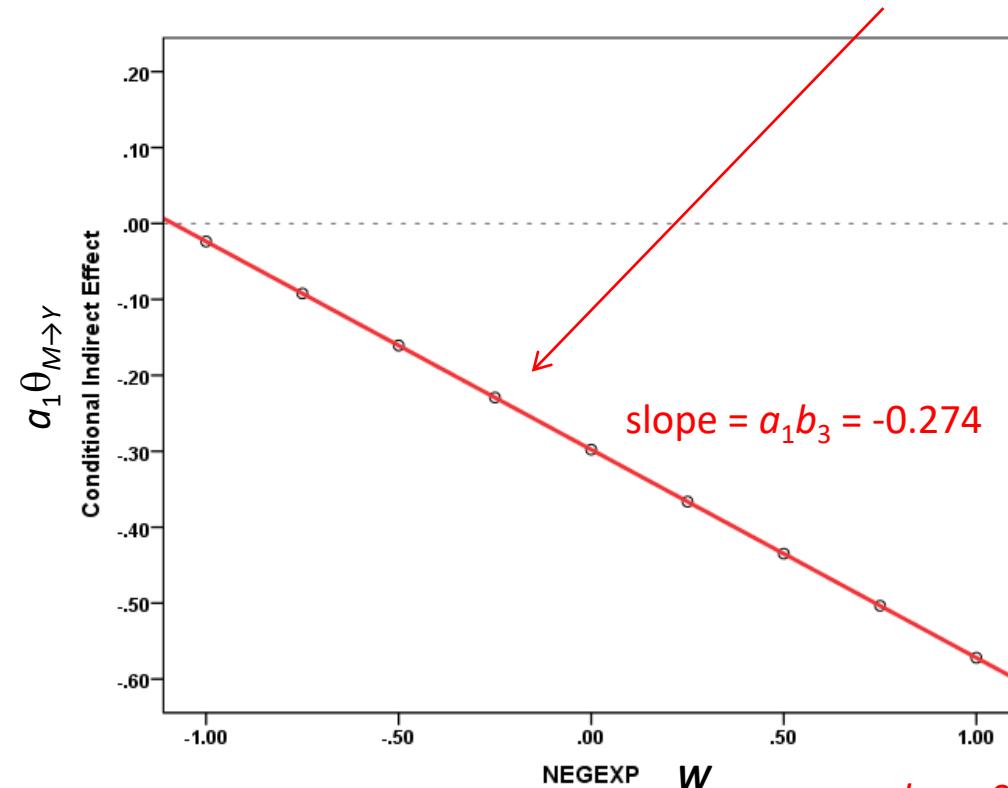
An inference about the slope of this line—the “index of moderated mediation”—is an inference about whether the indirect effect is moderated: Is this slope significantly different from zero? If so, that supports a claim of “moderated mediation.”

As $a_1 b_3$ is a product of regression coefficients, an inferential test should respect the nonnormality of the sampling distribution of the product. A bootstrap confidence interval is a good choice.

This test is provided automatically by PROCESS

$$a_1 \theta_{M \rightarrow Y} = a_1(b_1 + b_3 W) = 0.610(-0.489 - 0.450W)$$

$$= a_1 b_1 + a_1 b_3 W = -0.298 - 0.274W$$



The indirect effect is a function of V (negative nonverbal expressivity) in our model. This function is a line.

$$a\theta_{M \rightarrow Y} = a_1(b_1 + b_3 W)$$

$$= a_1 b_1 + a_1 b_3 W$$

$$= -0.298 - 0.274W$$

Output J

$a_1 b_3 = -0.274$, 95% bootstrap CI = -0.683 to -0.024

***** INDEX OF MODERATED MEDIATION *****

	Index	SE (Boot)	BootLLCI	BootULCI
Negexp	- .2742	.1727	- .6833	- .0243

This slope is statistically different from zero. The indirect effect depends on negative nonverbal expressivity.... The mediation is moderated.

Where is this test discussed?

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DOI: 10.1080/00273171.2014.962683



An Index and Test of Linear Moderated Mediation

Andrew F. Hayes
The Ohio State University

I describe a test of linear moderated mediation in path analysis based on an interval estimate of the parameter of a function linking the indirect effect to values of a moderator—a parameter that I call the *index of moderated mediation*. This test can be used for models that integrate moderation and mediation in which the relationship between the indirect effect and the moderator is estimated as linear, including many of the models described by Edwards and Lambert (2007) and Preacher, Rucker, and Hayes (2007) as well as extensions of these models to processes involving multiple mediators operating in parallel or in serial. Generalization of the method to latent variable models is straightforward. Three empirical examples describe the computation of the index and the test, and its implementation is illustrated using Mplus and the PROCESS macro for SPSS and SAS.

AN INDEX AND TEST OF LINEAR MODERATED MEDiation

Empirically substantiating the boundary conditions of one variable's causal effect on another and the mechanism(s) by which that effect operates are recognized as markers of deeper understanding than merely establishing that X affects Y . Successfully answering such questions as "Under what circumstance does X affect Y ?" and "How does X affect Y ?" adds much merit to one's science and can enhance its impact on a field.

Questions about the contingencies of an effect are often answered statistically through moderation analysis. Assuming continuous Y and dichotomous and/or continuous X and W (the only case considered in this paper), moderation of the effect of X on Y by W is popularly tested by estimating a linear model of the form

$$Y = i_Y + b_1 X + b_2 W + b_3 XW + e_Y$$

where b_1 , b_2 , and b_3 are estimated regression coefficients, i_Y is an error in estimation, and i_Y is a regression intercept. X 's effect on Y is linearly moderated by W if the regression coefficient for XW is different from zero by an inferential test or confidence interval.¹

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¹Unless otherwise stated, assume that all regression coefficients and functions thereof in my notation are estimates based on the data available.

Questions about the mechanism by which an effect operates are frequently answered with mediation analysis. In a mediation analysis, focus is on the estimation of the indirect effect of X on Y through an intermediary mediator variable M causally located between X and Y (i.e., a model of the form $X \rightarrow M \rightarrow Y$). Assuming continuous Y and M , the indirect effect of X on Y through M can be derived using two linear models:

$$\begin{aligned} M &= i_M + aX + e_M \\ Y &= i_Y + c'X + bM + e_Y \end{aligned}$$

where a , b , and c' are estimated regression coefficients, i_M and i_Y are regression intercepts, and e_M and e_Y are errors in estimation. The product of a and b quantifies the indirect effect of X on Y and estimates how much two cases that differ by one unit on X are estimated to differ on Y through the effect of X on M which in turn influences Y . Evidence that the indirect effect is different from zero by an inferential test or confidence interval bolsters a claim that the effect of X on Y is mediated at least in part by M .

Mediation and moderation analysis can be analytically integrated into a unified statistical model. Although not a new idea by any means—such terms as “mediated moderation” and “moderated mediation” appeared in the literature decades ago (e.g., Baron & Kenny, 1986; James & Brett, 1984; Judd & Kenny, 1981)—it is only recently that a handful of articles in the methodology literature have provided researchers the tools and systematic procedures for answering questions focused on the “when of the how” or the “how of the when.” Hayes (2013) introduces the term

402 Mediation, Moderation, and Conditional Process Analysis

in the causal model in order to claim M is a mediator. In addition, Kraemer et al. (2002, 2008) don't discuss formally quantifying the indirect effect. In this model, M 's effect on Y is not b but, rather, $b + c'_2 X$. Thus, the indirect effect of X on Y through M is $a(b + c'_2 X)$, meaning it is a function of X (see, e.g., Preacher et al., 2007).

In the model they recommend using to test for mediation, X is estimated to affect Y indirectly through M , as well as directly independent of M . But the direct effect of X in this model is not c'_1 as it might seem. Grouping terms in equation 12.2 involving X and then factoring out X yields the direct effect of X on Y :

$$O_{X \rightarrow Y} = c'_1 + c'_2 M$$

So the direct effect of X is conditioned on M . In other words, if c'_2 in equation 12.2 is statistically different from zero, M moderates X 's direct effect on Y . The MacArthur camp would reject this as a possibility, as a moderator can't be correlated with X . By their criteria, M can be deemed a mediator of X 's effect if a and c'_2 are both statistically different from zero, but that very circumstance implies that M is *not* uncorrelated with X . At the same time, a statistically significant c'_2 means that X 's direct effect on Y is moderated by M . Thus, in the model Kraemer et al. (2002, 2008) recommend as the best approach to testing mediation, meeting one subset of their criteria for establishing M as a mediator also means that M could be construed as a moderator of X 's effect, at least statistically or mathematically so.

Just because something is mathematically possible doesn't mean that it is sensible theoretically or substantively interpretable when it happens (as it does, as evidenced in some of the example studies cited on page 332). I will not take a firm position on whether construing M as a simultaneous mediator and moderator of a variable's effect could ever make substantive or theoretical sense. I am uncomfortable categorically ruling out the possibility that M could be a moderator just because it is correlated with X . My guess is that there are many real-life processes in which things caused by X also influence the size of the effect of X on Y measured well after X . But M would have to be causally prior to Y in order for this to be possible, implying that M could also be construed as a mediator if M is caused in part by X but also influences Y in some fashion.

12.3 Comparing Conditional Indirect Effects and a Formal Test of Moderated Mediation

If the indirect effect of X on Y through M depends on a particular moderator, that means that the indirect effect is a function of that moderator. A sensible question to ask is whether the conditional indirect effect when the

PROCESS output

We need an inferential test for these conditional indirect effects. Bootstrap confidence intervals are perfect for the job. PROCESS does this for us automatically.

***** DIRECT AND INDIRECT EFFECTS *****

Direct effect of X on Y

Effect	SE	t	p	LLCI	ULCI
.3729	.1808	2.0622	.0442	.0100	.7357

INDIRECT EFFECT:

dysfunc → negtone → perform

negexp	Effect	BootSE	BootLLCI	BootULCI
-.5308	-.1523	.1540	-.4335	.1943
-.0600	-.2813	.1249	-.5432	-.0549
.6000	-.4623	.1678	-.8095	-.1503

Conditional indirect effects with 95% bootstrap CIs based on 10,000 bootstrap samples.

Index of moderated mediation:

	Index	BootSE	BootLLCI	BootULCI
negexp	-.2742	.1791	-.7172	-.0234

The indirect effect of dysfunctional behavior on performance through negative tone is negative among teams relatively moderate (point estimate: -0.28, 95% CI from -0.54 to -0.05) and relatively high (point estimate: -0.46, 95% CI from -0.81 to -0.14) in negative nonverbal expressivity but not different from zero among those low in negative nonverbal expressivity (point estimate: -0.15, 95% CI from -0.43 to 0.19).

Comparing conditional indirect effects (2nd stage model)

A seemingly sensible question to ask is whether the conditional indirect effect of X when the moderator equals some value $W = w_1$ is different than the conditional indirect effect of X when the moderator is some different value $W = w_2$. For example, is the indirect effect among teams low in negative nonverbal expressivity different from the indirect effect among teams high in negative nonverbal expressivity?

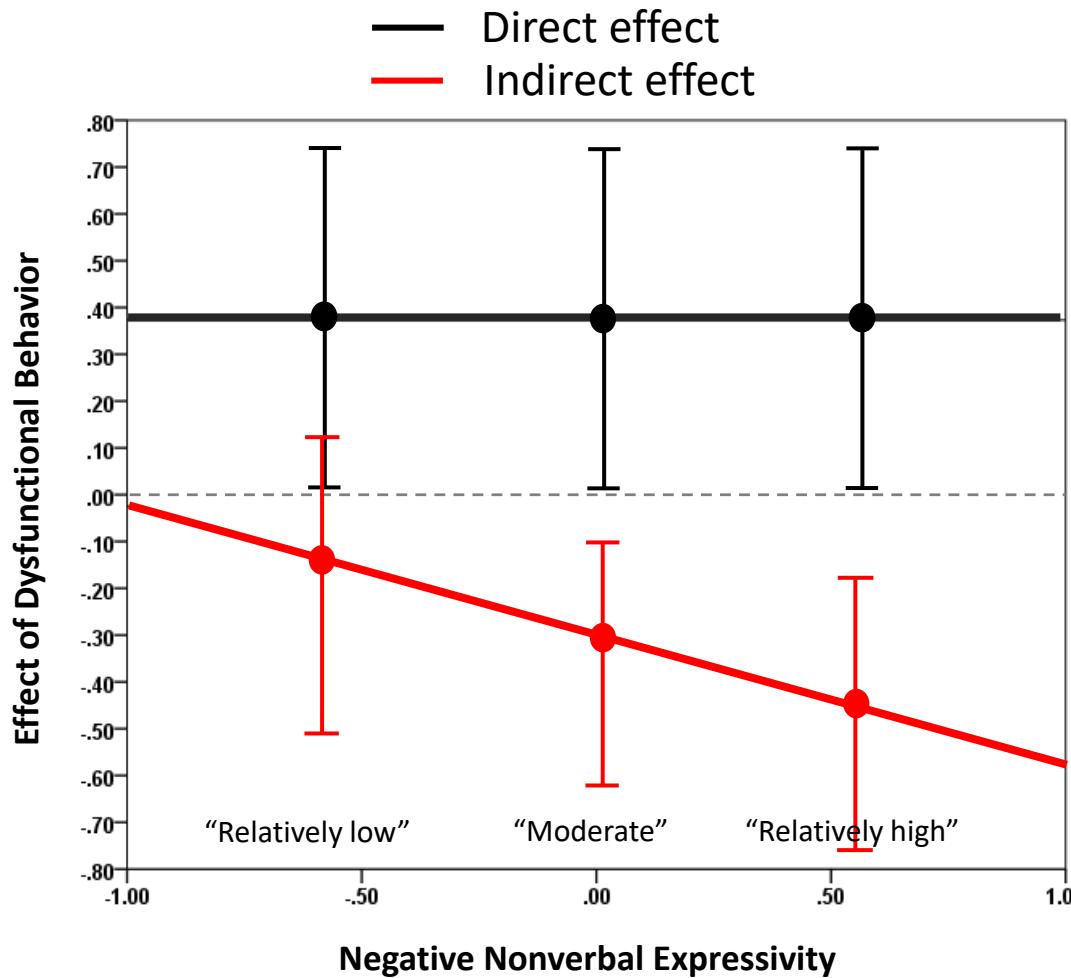
Rejection of the null hypothesis of no moderated mediation based on the index of moderated mediation implies that **any two conditional indirect effects are different!** No additional test is needed.

For example, for the second stage moderated mediation model just estimated:

$$\begin{aligned} a_1(b_1 + b_3 w_1) - a_1(b_1 + b_3 w_2) &= a_1(b_1 + b_3 w_1) - a_1(b_1 + b_3 w_2) \\ &= a_1 b_1 + a_1 b_3 w_1 - a_1 b_1 - a_1 b_3 w_2 \\ &= a_1 b_3 w_1 - a_1 b_3 w_2 \\ &= a_1 b_3 (w_1 - w_2) \end{aligned}$$

If a bootstrap confidence interval for $a_1 b_3$ does not contain zero, then neither will a confidence interval for $a_1 b_3 (w_1 - w_2)$, **regardless** of values of w_1 and w_2 chosen, so long as $w_1 \neq w_2$. And if a bootstrap confidence interval for $a_1 b_3$ contains zero, then so too will a confidence interval for $a_1 b_3 (w_1 - w_2)$, **for any two values** of w_1 and w_2 , ($w_1 \neq w_2$).

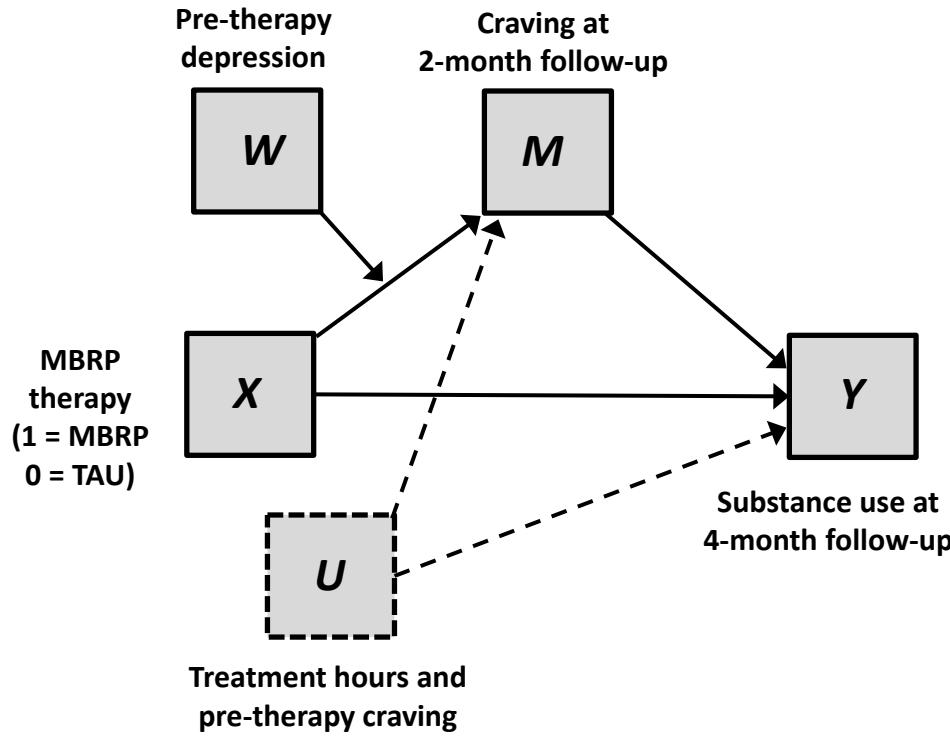
Putting it all together



Point estimates with 95% confidence intervals (OLS confidence intervals for direct effects, bootstrap confidence intervals for indirect effects).

More dysfunctional behavior tends to lead to more negative affective tone, yet this negative affective tone seems to lower performance only among teams that are more demonstrative of their negative feelings. Such a process does not operate among teams that hide their feelings. Independent of differences between teams in the negative affective tone of the work environment, teams that exhibit more dysfunctional behavior otherwise perform *better*.

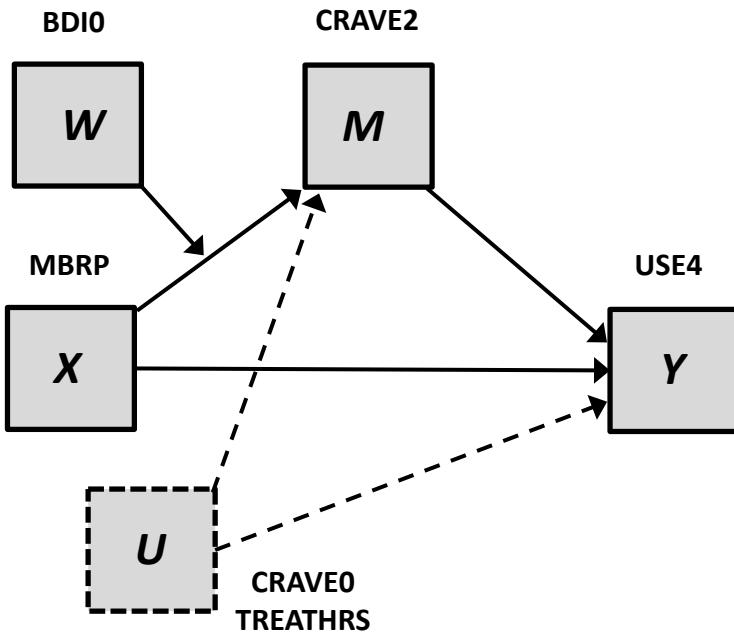
Conditional Process Modeling Example #2



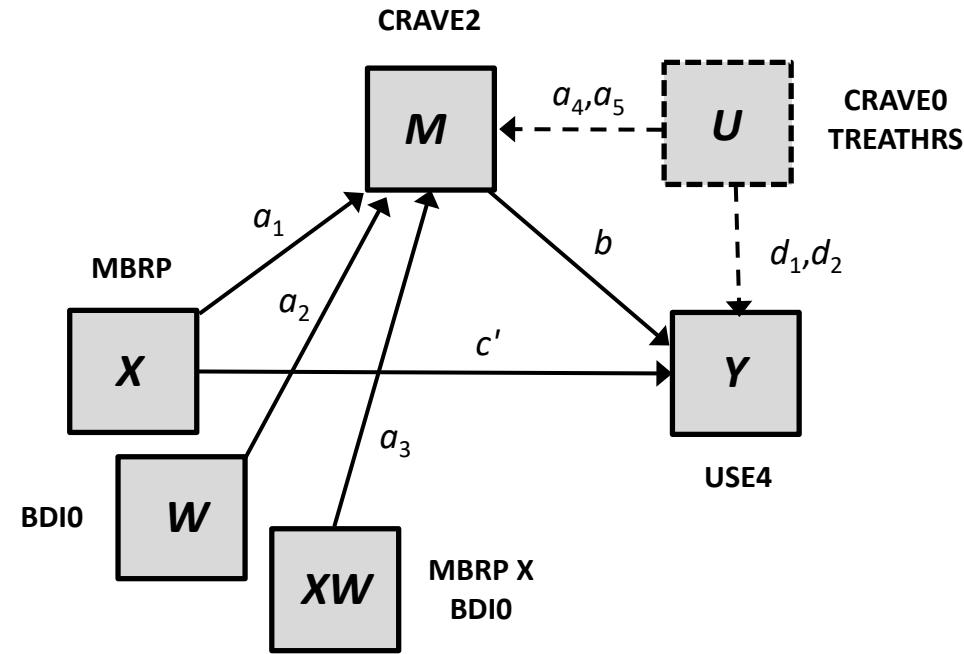
This is a model of **craving (M)** as the mechanism by which **mindfulness relapse prevention therapy (X)** affects **substance use (Y)** relative to therapy as usual. In this model, moderation of the mechanism is proposed as operating in the “first stage” of the mediation process via the moderation of the effect of mindfulness relapse prevention therapy on craving by **pre-therapy depression level (W)**.

Conceptual and Statistical Models

Conceptual model



Statistical model



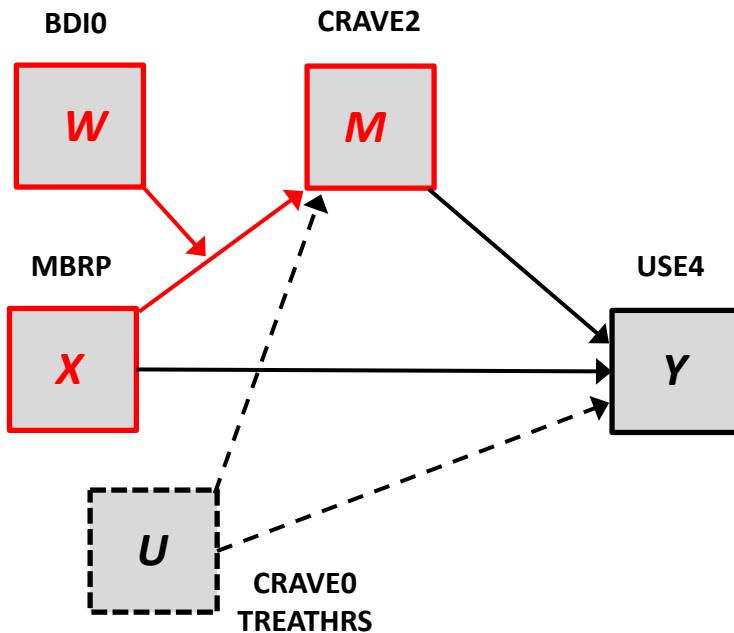
$$\hat{M} = a_0 + a_1 X + a_2 W + a_3 XW + a_4 U_1 + a_5 U_2$$

$$\hat{Y} = c'_0 + c' X + b M + d_1 U_1 + d_2 U_2$$

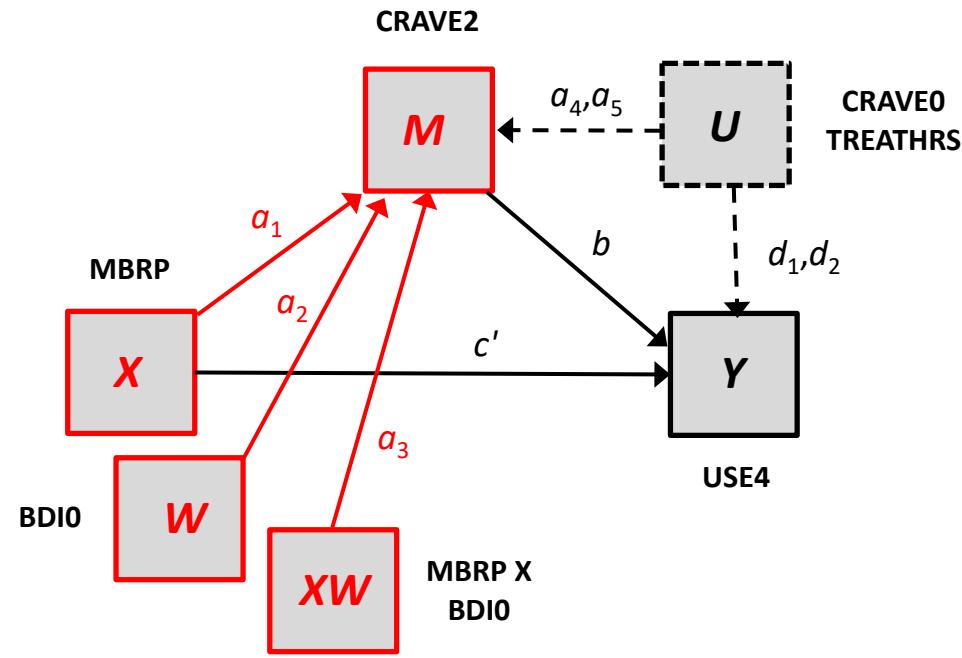
The number of equations needed is equal to the number of variables with arrows pointing at them in the conceptual or statistical diagram.

Conceptual and Statistical Models

Conceptual model



Statistical model



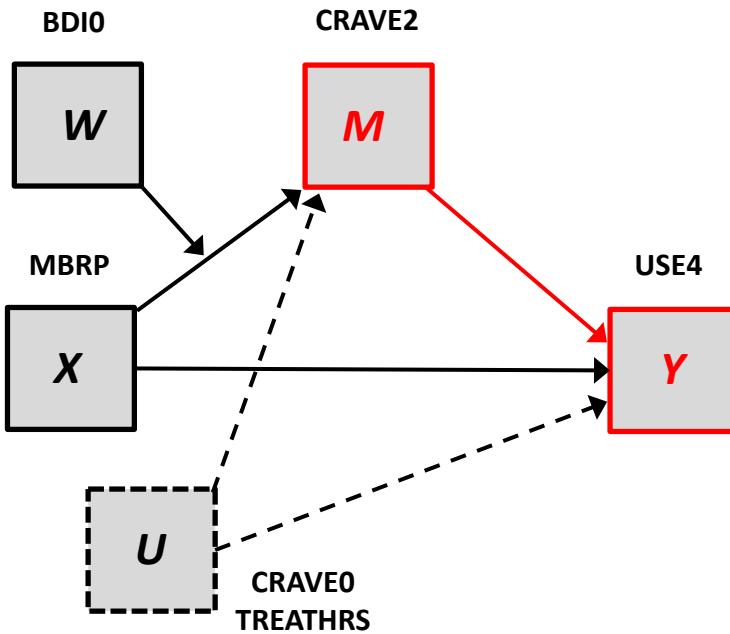
$$\hat{M} = a_0 + \boxed{a_1 X + a_2 W + a_3 XW} + a_4 U_1 + a_5 U_2$$

$$\hat{Y} = c'_0 + c' X + b M + d_1 U_1 + d_2 U_2$$

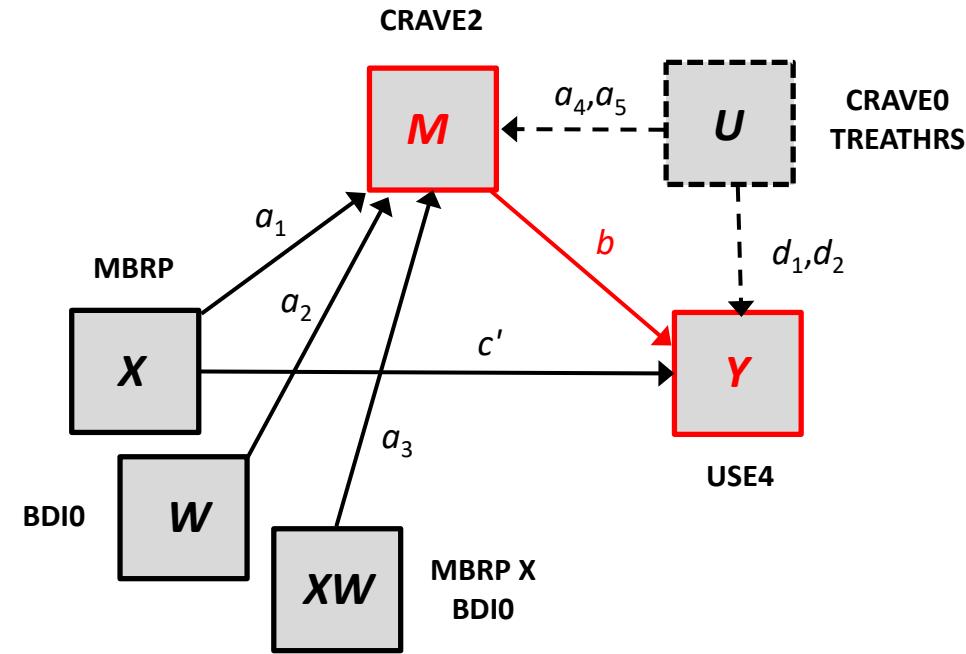
The moderation of the effect of mindfulness behavioral relapse prevention therapy relative to therapy as usual on craving by pre-therapy depression level.

Conceptual and Statistical Models

Conceptual model



Statistical model



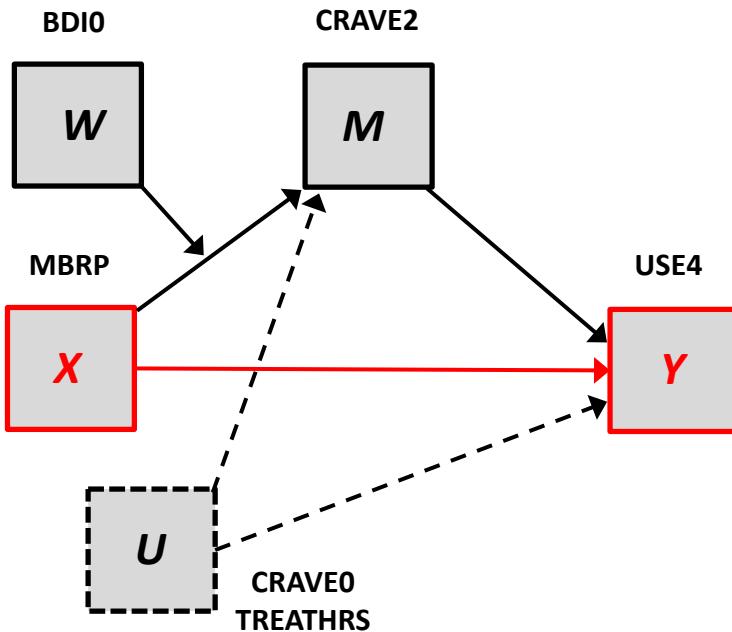
$$\hat{M} = a_0 + a_1 X + a_2 W + a_3 XW + a_4 U_1 + a_5 U_2$$

$$\hat{Y} = c'_0 + c' X + b M + d_1 U_1 + d_2 U_2$$

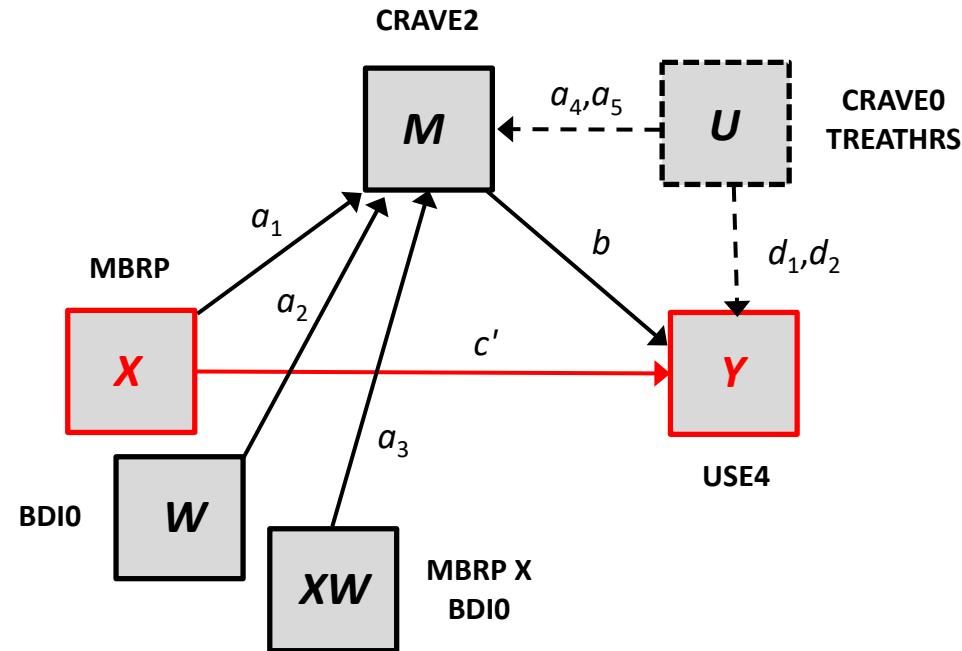
The effect of craving at two month follow-up on substance use after 4 months.

Conceptual and Statistical Models

Conceptual model



Statistical model



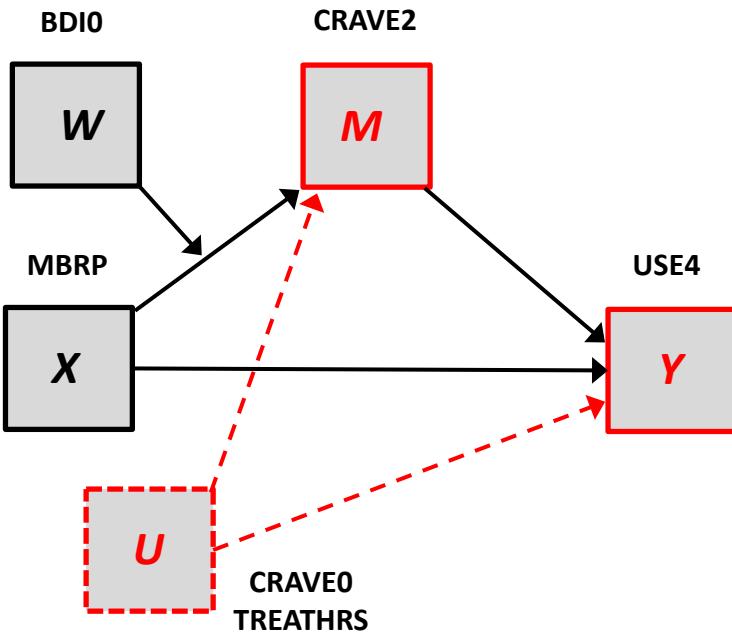
$$\hat{M} = a_0 + a_1 X + a_2 W + a_3 XW + a_4 U_1 + a_5 U_2$$

$$\hat{Y} = c'_0 + \boxed{c' X} + b M + d_1 U_1 + d_2 U_2$$

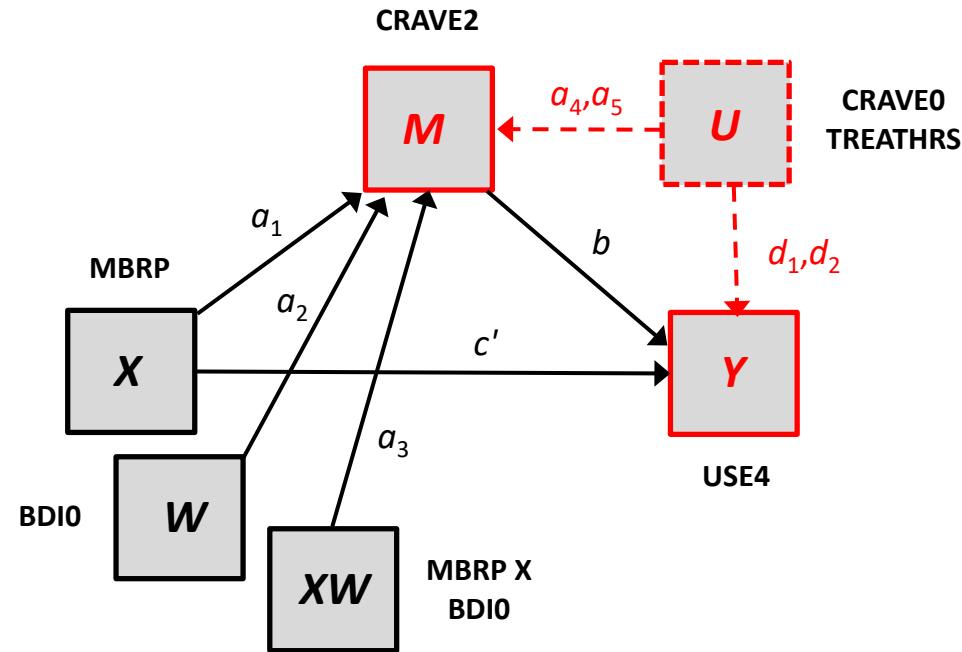
The direct effect of mindfulness behavioral relapse prevention therapy on substance use at four month follow up accounting for the mechanism through craving.

Conceptual and Statistical Models

Conceptual model



Statistical model



$$\hat{M} = a_0 + a_1 X + a_2 W + a_3 XW + a_4 U_1 + a_5 U_2$$

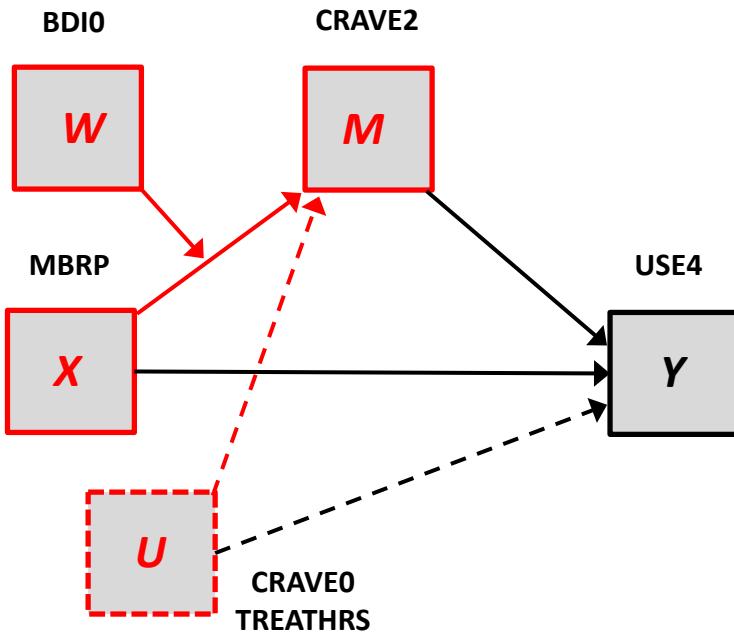
$$\hat{Y} = c'_0 + c' X + b M + d_1 U_1 + d_2 U_2$$

Covariates to account for potential confounding by treatment hours and pre-therapy craving.

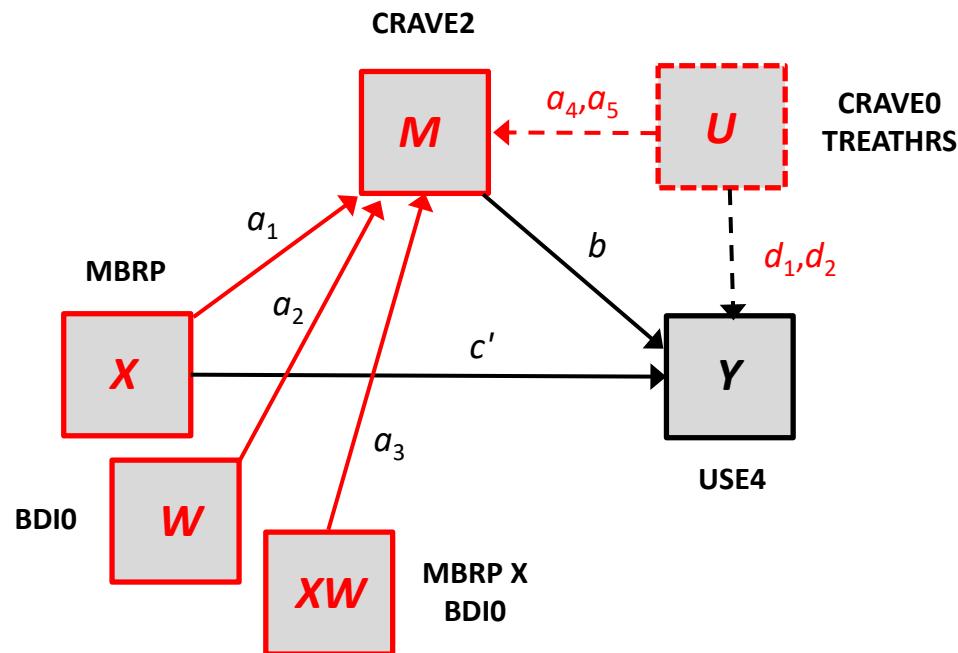
Estimating the moderation component of the model

The conceptual model proposes that the effect of mindfulness behavioral relapse prevention therapy depends on pre-therapy depression.

Conceptual model



Statistical model



$$\hat{M} = a_0 + a_1 X + a_2 W + a_3 XW + a_4 U_1 + a_5 U_2$$

We most care about the moderation components of the model of **M**: a_1 and a_3

We did this already

```
compute mbrpdep = mbrp*bdi0.
regression/dep = crave2/method = enter mbrp bdi0 mbrpdep treathrs crave0.
```

```
data mbrp;set mbrp;mbrpdep=mbrp*bdi0;run;
proc reg data=mbrp;model crave2=mbrp bdi0 mbrpdep treathrs crave0;run;
```

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \dots$$

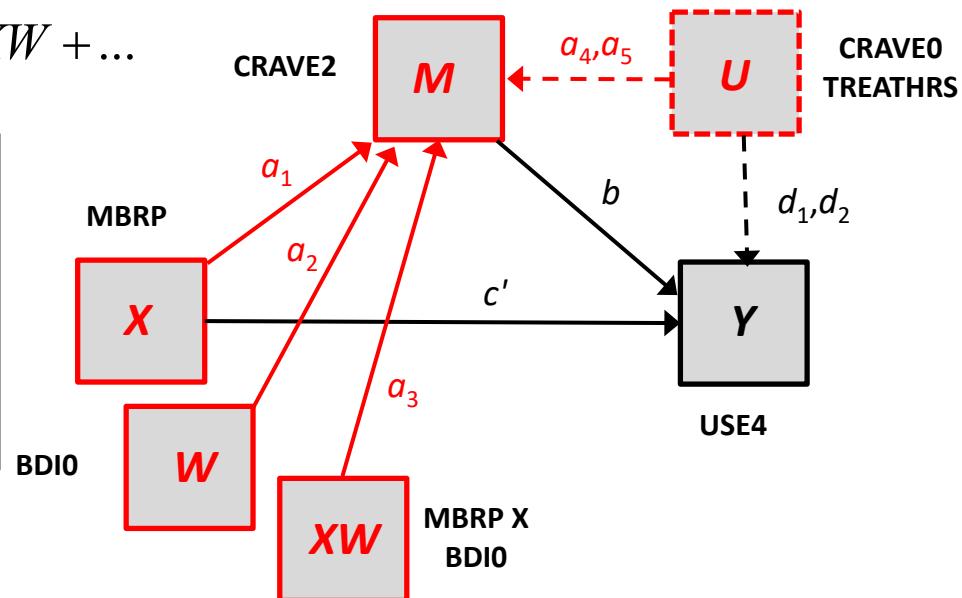
Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.
	B	Std. Error			
1 (Constant)	1.038	.470		2.209	.029
MBRP: Therapy as usual (0) or MBRP therapy (1)	.587	.524	.299	1.120	.264
BDI0: Beck Depression Inventory baseline	1.122	.276	.366	4.063	.000
mbrpdep	-.948	.423	-.598	-2.240	.026
TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

a. Dependent Variable: CRAVE2: Craving at two month follow-up

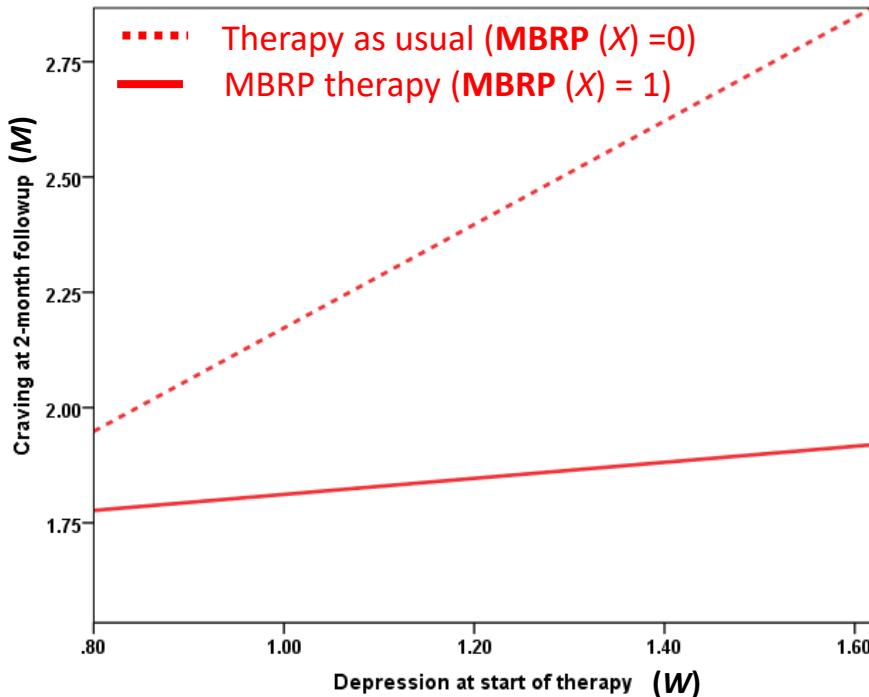
$$a_1 = 0.587, a_3 = -0.948$$

X = MBRP
W = BDI0
Y = CRAVE2



Pre-therapy depression moderates the effect of mindfulness behavioral relapse prevention therapy on craving. We can say that this moderation/interaction is statistically significant, but this doesn't matter for our purposes because neither the direct nor indirect effects in this model are determined entirely by a_3 , and it is the direct and indirect effects we care about. We need a_1 and a_3 to estimate the indirect effects.

Recall the pattern from the earlier analysis



$$\hat{M} = 1.038 + 0.587X + 1.122W - 0.948XW + \dots$$

which can be written as

$$\hat{M} = 1.038 + (0.587 - 0.948W)X + 1.122W + \dots$$

or

$$\hat{M} = 1.038 + \theta_{X \rightarrow M} X + 1.122W + \dots \text{ where } \theta_{X \rightarrow M} = 0.587 - 0.948W = a_1 + a_3 W$$

The conditional effect of MBRP therapy ($\theta_{X \rightarrow M}$) is defined by the function

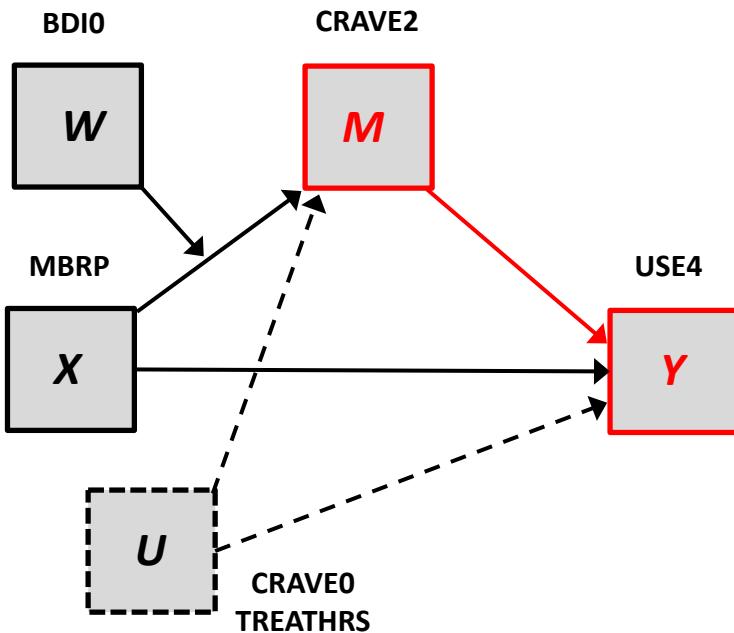
$$\theta_{X \rightarrow M} = 0.587 - 0.948W$$

BDI0 (W)	$\theta_{X \rightarrow M}$
0.877	-0.245
1.196	-0.547
1.515	-0.850

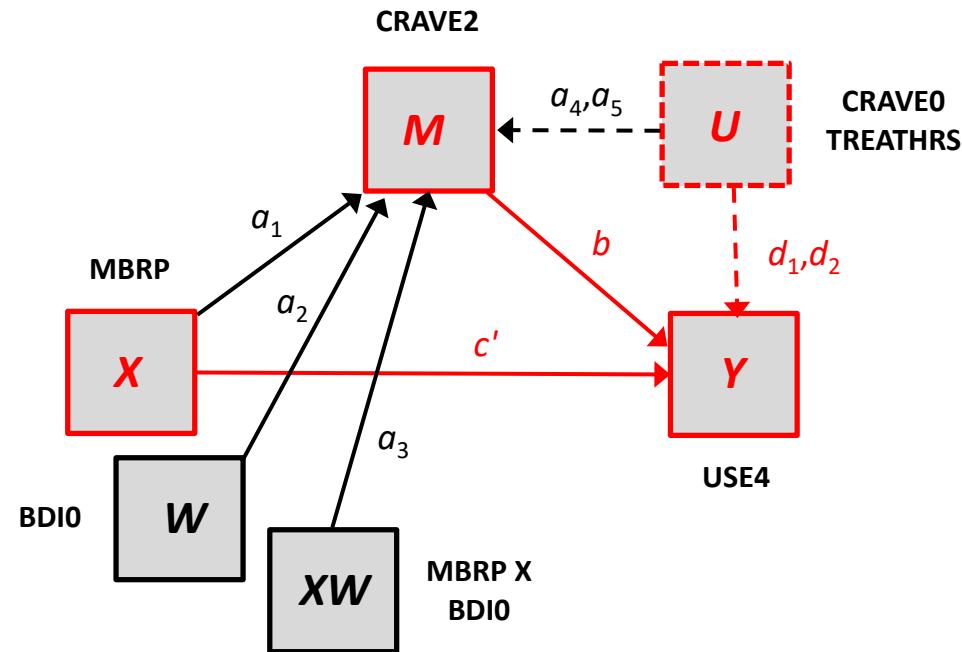
Recall these from our Implementation of the pick-a-point approach.

Estimating the b and c' paths

Conceptual model



Statistical model



$$\hat{M} = a_0 + a_1 X + a_2 W + a_3 XW + a_4 U_1 + a_5 U_2$$

$$\hat{Y} = c'_0 + c' X + b M + d_1 U_1 + d_2 U_2$$

Estimating the b and c' paths

```
regression/dep=use4/method=enter crave2 mbrp crave0 treathrs.
```

```
proc reg data=mbrp;model use4 = crave2 mbrp crave0 treathrs;run;
```

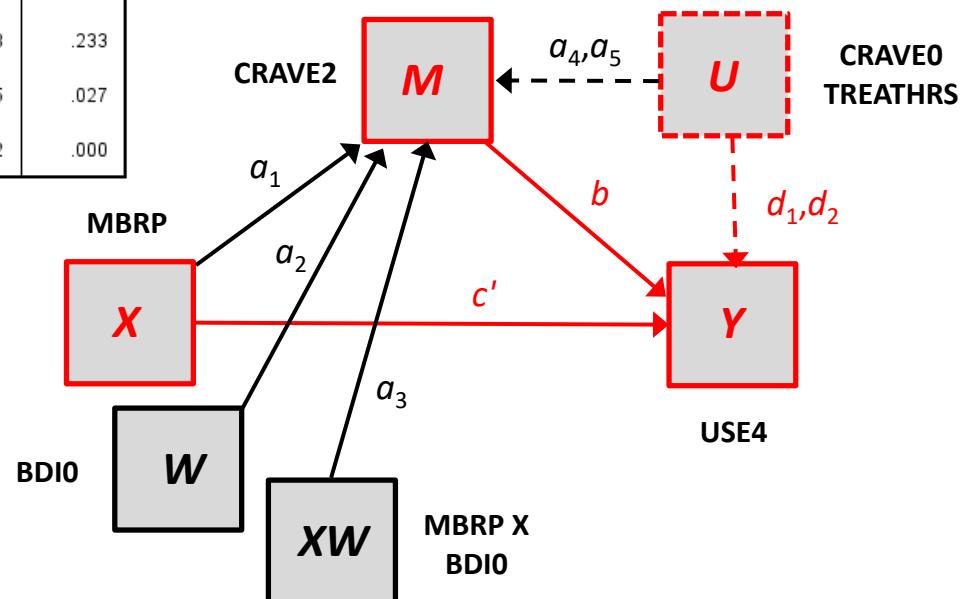
Model	Coefficients ^a				
	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
B	Std. Error	Beta			
1 (Constant)	1.130	.215		5.254	.000
CRAVE2: Craving at two month follow-up	.481	.040	.710	11.955	.000
MBRP: Therapy as usual (0) or MBRP therapy (1)	.093	.077	.070	1.198	.233
CRAVE0: Baseline craving	-.088	.040	-.124	-2.225	.027
TREATHRS: Hours of therapy	-.020	.006	-.200	-3.572	.000

a. Dependent Variable: USE4: Substance use at four month follow-up

$$\hat{Y} = 1.130 + 0.093X + 0.481M + \dots$$

$$b = 0.481, c' = 0.093$$

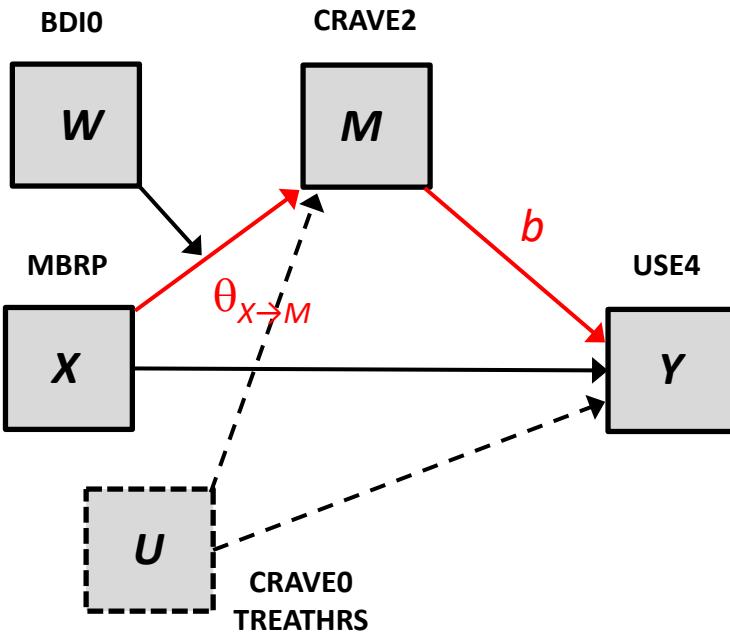
Statistical model



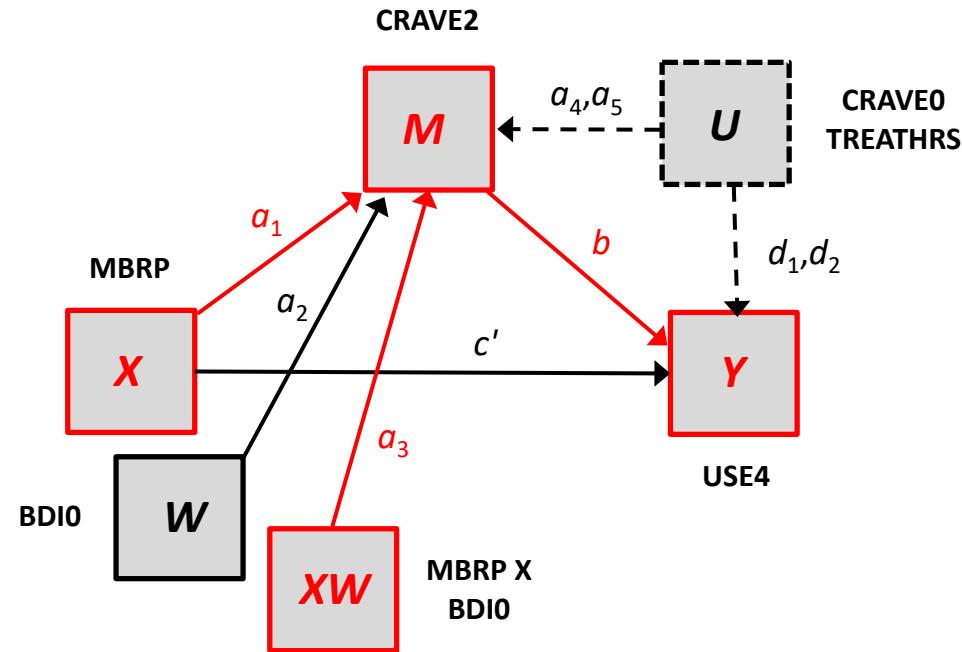
Emphasis is not on statistical significance of the b path, as the indirect effect of X is not defined entirely in terms of b . c' is the direct effect (discussed in a bit).

The conditional indirect effect of X

Conceptual model



Statistical model

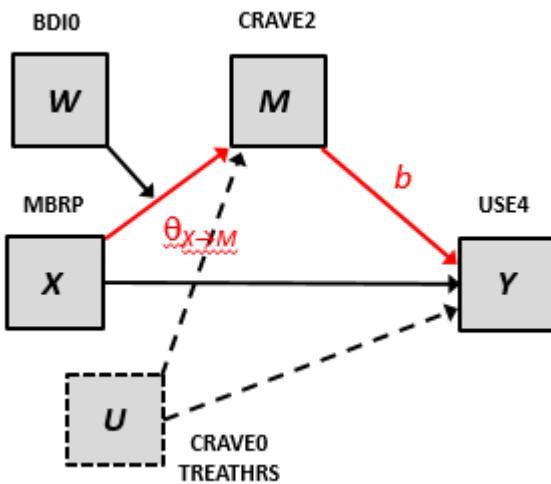


The conditional indirect effect of X on Y through M is the product of the conditional effect of X on M ($\theta_{X \rightarrow M} = a_1 + a_3 W$) and effect of M on Y (b):

$$\theta_{X \rightarrow M \rightarrow Y} = \theta_{X \rightarrow M} b = (a_1 + a_3 W)b = (0.587 - 0.948W)(0.481)$$

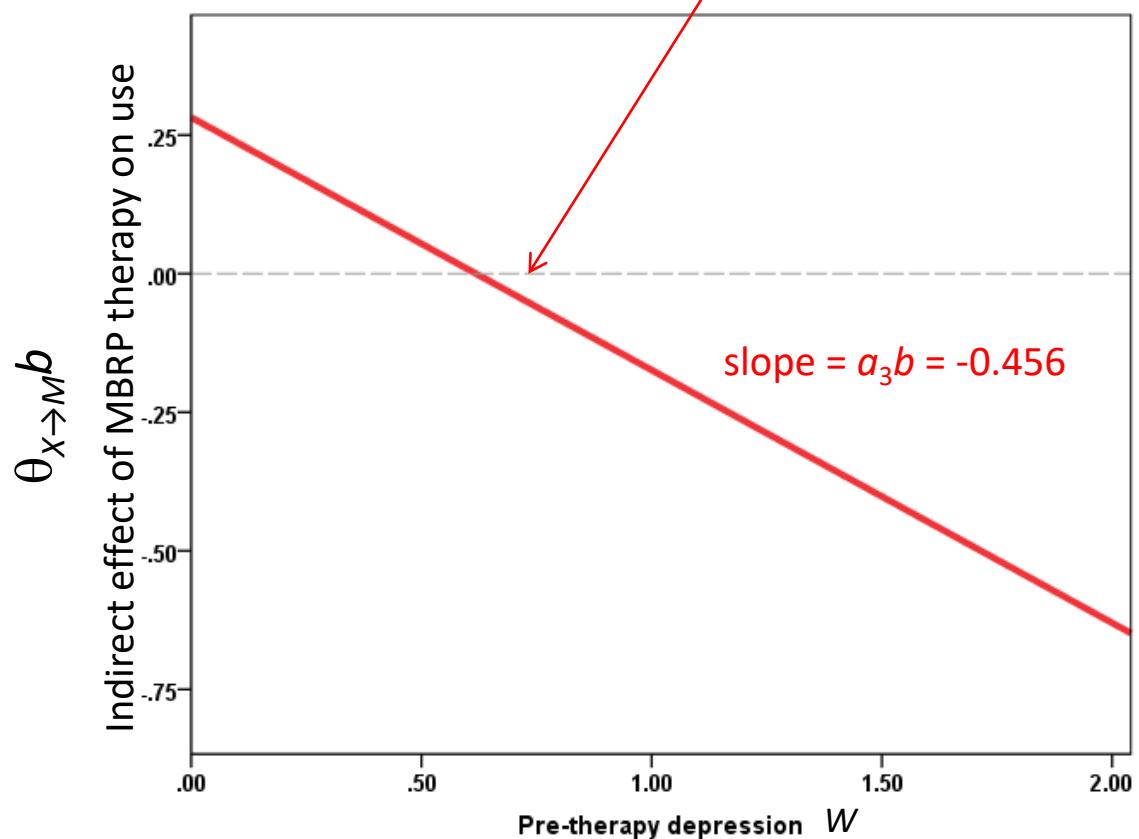
The indirect effect of MBRP therapy relative to therapy as usual on later substance use through craving is a function of pre-therapy depression.

A visual representation of the indirect effect



The indirect effect declines with increasing pre-therapy depression. The “**index of moderated mediation**” is $a_3b = -0.456$. It quantifies the relationship between the moderator and the indirect effect in this model.

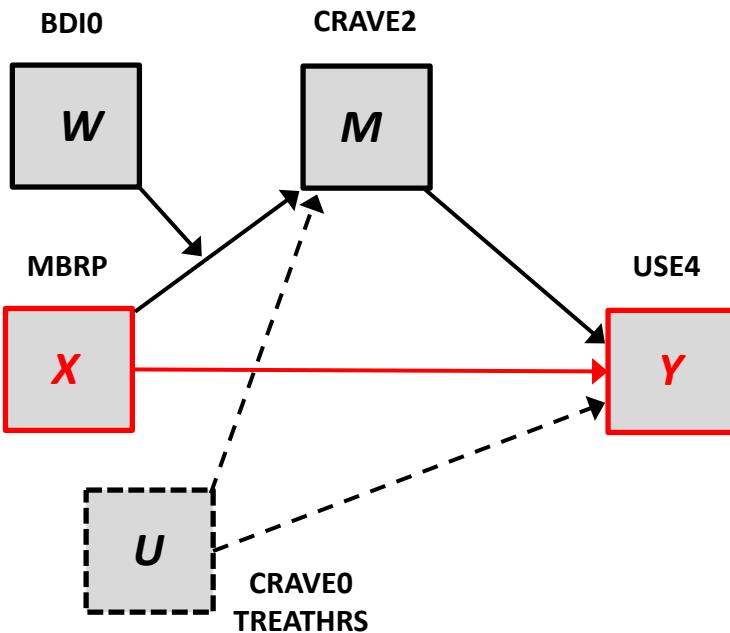
$$\begin{aligned}\theta_{X \rightarrow M} b &= (a_1 + a_3 W)b = (0.587 - 0.948W)(0.481) \\ &= a_1 b + a_3 b W = 0.282 - 0.456W\end{aligned}$$



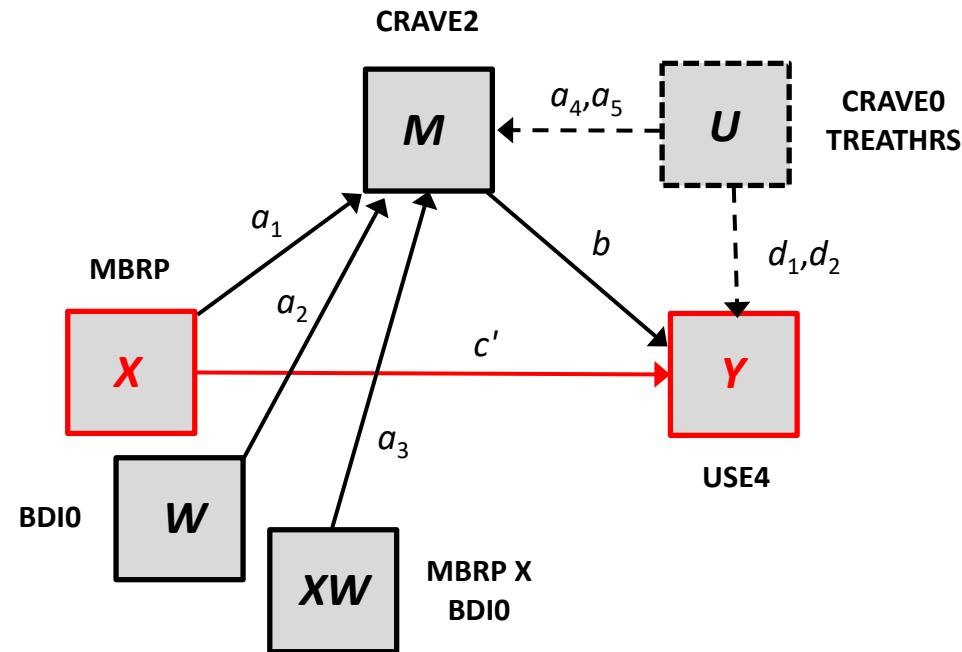
As will be seen, a hypothesis test that the slope of this function is equal to 0 is a formal test of moderated mediation....moderation of the indirect effect.

The direct effect of X

Conceptual model



Statistical model



$$\hat{M} = a_0 + a_1 X + a_2 W + a_3 XW + a_4 U_1 + a_5 U_2$$

$$\hat{Y} = c'_0 + c' X + b M + d_1 U_1 + d_2 U_2$$

In this model, the direct effect is fixed to be unmoderated. It is a constant rather than a function of another variable in the model. This is a modeling or theoretical decision, not a requirement.

The direct effect of X (estimated earlier)

```
regression/dep=use4/method=enter crave2 mbrp crave0 treathrs.
```

```
proc reg data=mbrp;model use4 = crave2 mbrp crave0 treathrs;run;
```

Coefficients^a

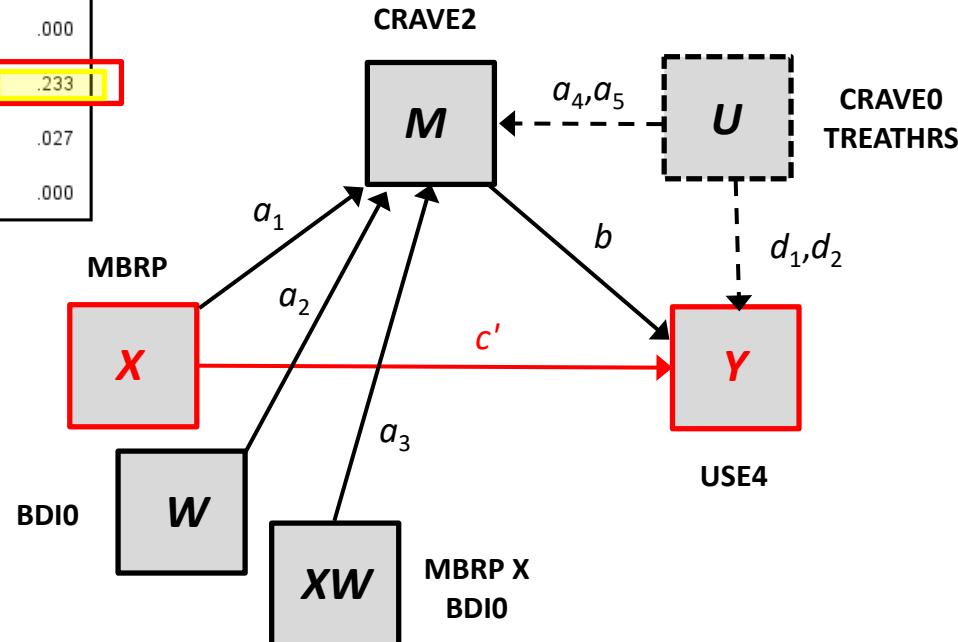
Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
1 (Constant)	1.130	.215		5.254	.000
CRAVE2: Craving at two month follow-up	.481	.040	.710	11.955	.000
MBRP: Therapy as usual (0) or MBRP therapy (1)	.093	.077	.070	1.198	.233
CRAVE0: Baseline craving	-.088	.040	-.124	-2.225	.027
TREATHRS: Hours of therapy	-.020	.006	-.200	-3.572	.000

a. Dependent Variable: USE4: Substance use at four month follow-up

$$\hat{Y} = 1.130 + 0.093X + 0.481M + \dots$$

$$c' = 0.093, t(163) = 1.198, p = 0.233$$

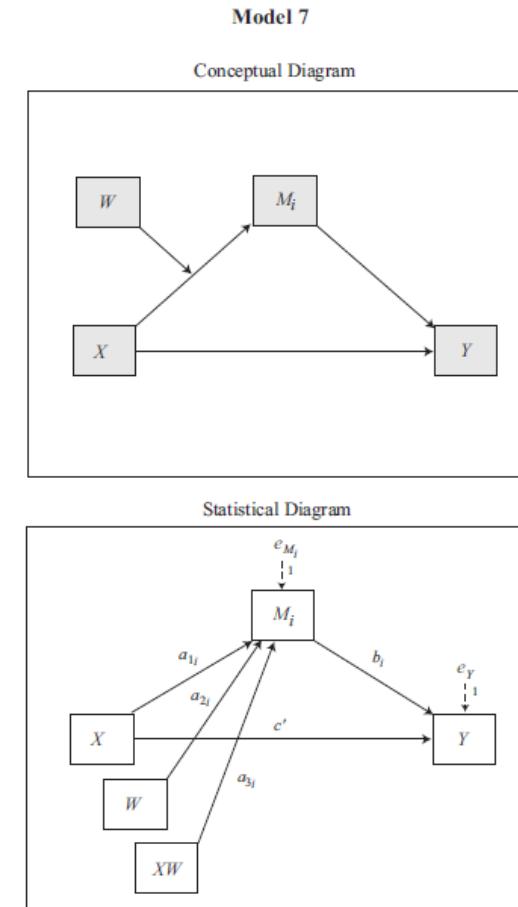
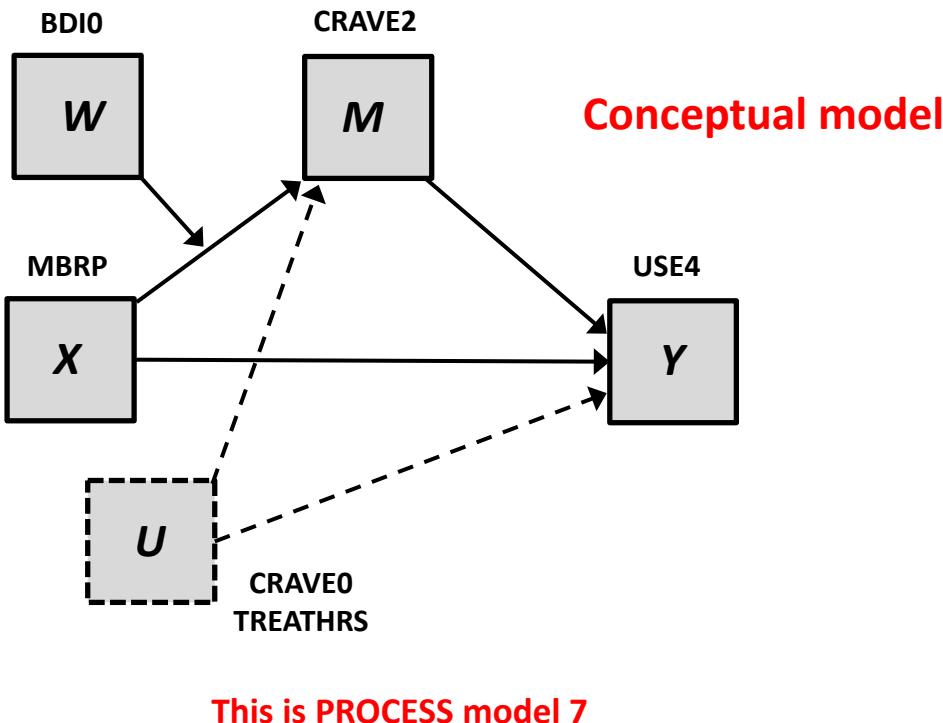
Statistical model



Mindfulness behavioral relapse prevention therapy has no apparent effect on later substance use relative to therapy as usual after accounting for the mechanism through craving.

Estimation of the model in PROCESS

PROCESS takes most of the computational burden off our shoulders.



```
process cov = crave0 treathrs/x=mbrp/m=crave2/y=use4/
w=bdi0/boot=10000/model=7.
```

```
%process (data=mbrp,cov=crave0 treathrs,x=mbrp,m=crave2,y=use4,w=bdi0,
boot=10000,model=7);
```

PROCESS output

Model = 7
 Y = use4
 X = mbrp
 M = crave2
 W = bdi0

Statistical Controls:
 CONTROL= crave0 treathrs

Sample size
 168

Outcome: crave2

$$\hat{M} = 1.039 + 0.587X + 1.122W - 0.948XW + \dots$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.5140	.2642	.7277	11.6319	5.0000	162.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	1.0385	.4701	2.2090	.0286	.1102	1.9668
mbrp	.5872	.5241	1.1204	.2642	-.4478	1.6222
bdi0	1.1221	.2762	4.0625	.0001	.5767	1.6675
int_1	-.9485	.4235	-2.2398	.0265	-1.7847	-.1122
crave0	.1920	.0735	2.6138	.0098	.0470	.3371
treathrs	-.0177	.0103	-1.7190	.0875	-.0380	.0026

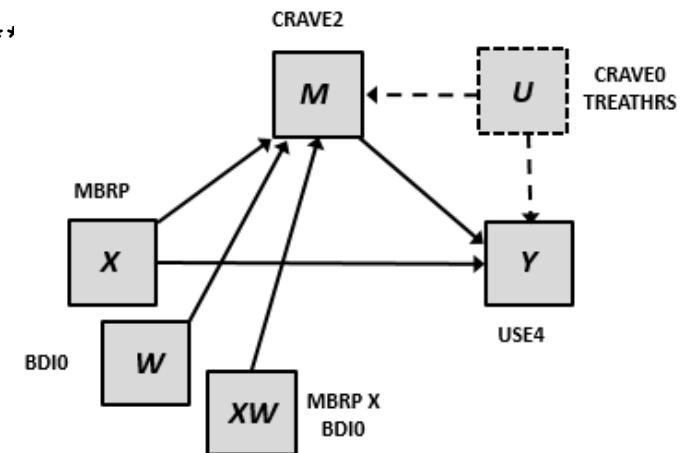
$a_1 = 0.587$

$a_2 = 1.122$

$a_3 = -0.948$

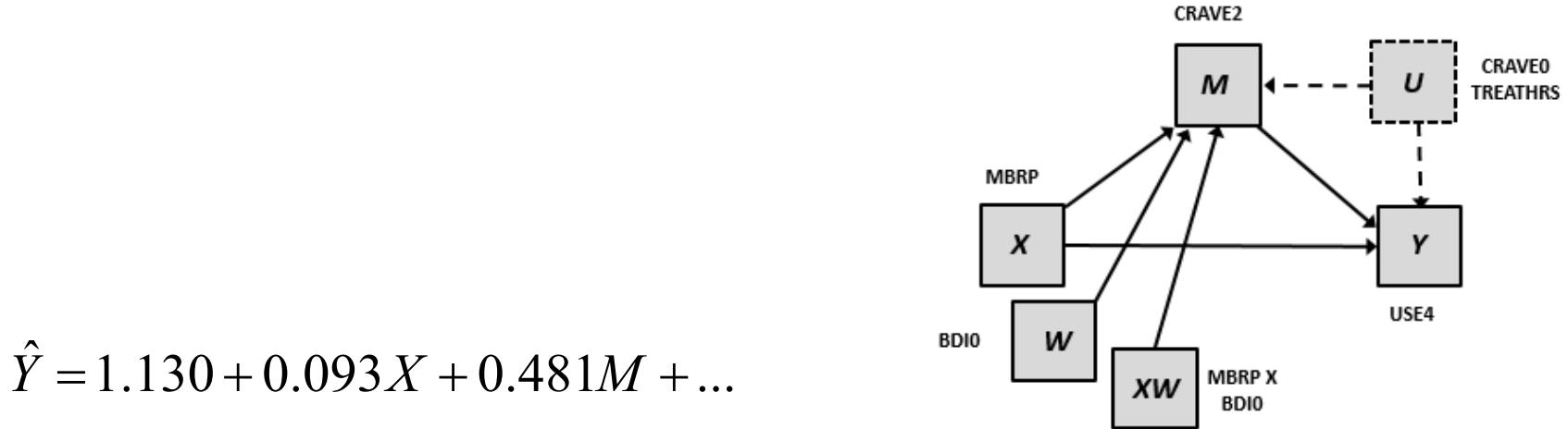
Interactions:

int_1 mbrp X bdi0



Output K

PROCESS output



Outcome: use4

Model Summary

R	R-sq	MSE	F	df1	df2	p
.7304	.5335	.2105	46.6070	4.0000	163.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	1.1298	.2150	5.2545	.0000	.7052	1.5544
crave2	.4810	.0402	11.9547	.0000	.4015	.5604
mbrp	.0926	.0773	1.1979	.2327	-.0601	.2453
crave0	-.0884	.0397	-2.2246	.0275	-.1668	-.0099
treathrs	-.0199	.0056	-3.5720	.0005	-.0309	-.0089

$b = 0.481$

$c' = 0.093$

Output K

PROCESS output

***** DIRECT AND INDIRECT EFFECTS *****

Direct effect of X on Y

Direct effect
 $c' = .093, p = .232$

Effect	SE	t	p	LLCI	ULCI
.0926	.0773	1.1979	.2327	-.0601	.2453

INDIRECT EFFECT:

mbrp -> crave2 -> use4

bdi0	Effect	BootSE	BootLLCI	BootULCI
.9020	-.1290	.0770	-.2869	.0183
1.1900	-.2604	.0862	-.4458	-.1090
1.5180	-.4100	.1367	-.7080	-.1767

Output K

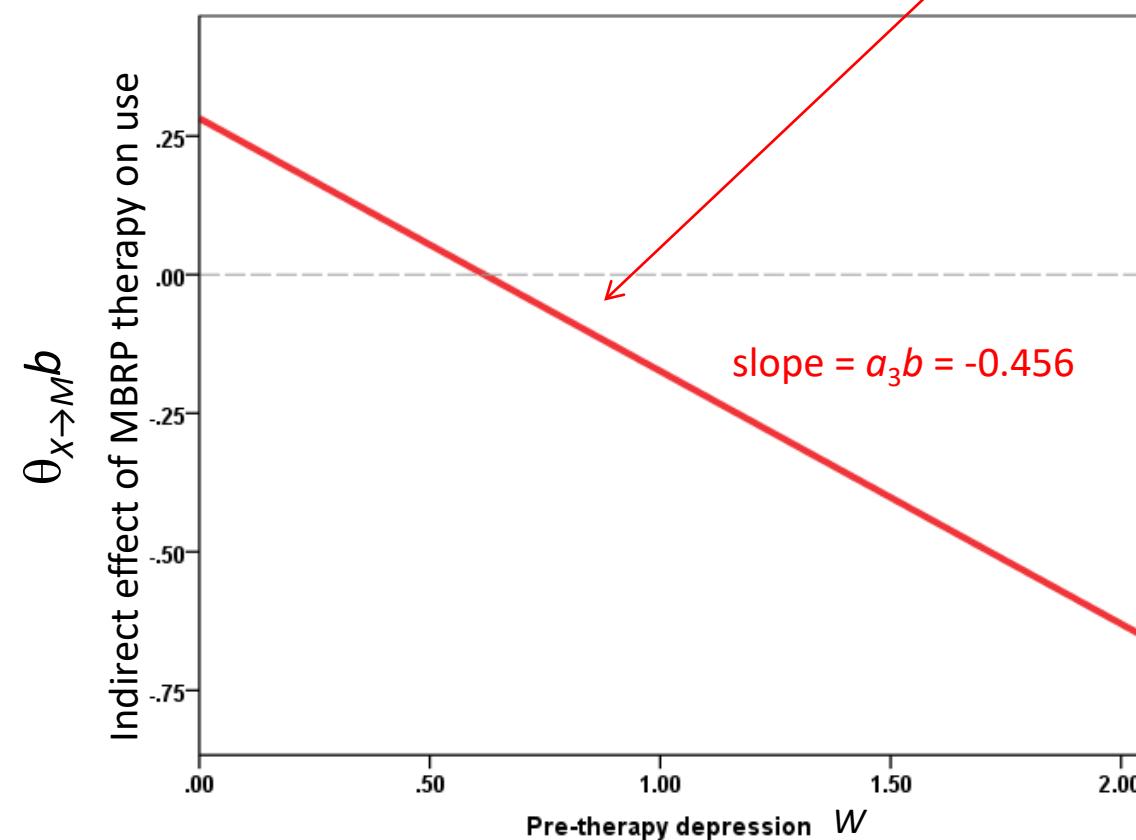
$$\begin{aligned}\theta_{X \rightarrow M} b &= (a_1 + a_3 W)b = (0.587 - 0.948W)(0.481) \\ &= a_1 b + a_3 b W = 0.282 - 0.456W\end{aligned}$$

PROCESS sees that the moderator is continuous so, without instruction otherwise, prints the conditional indirect effect at 16th, 50th, and 84th percentile of the moderator.

A statistical test of moderated mediation

in the first stage moderated mediation model

$$\begin{aligned}\theta_{X \rightarrow M} b &= (a_1 + a_3 W)b = (0.587 - 0.948W)(0.481) \\ &= a_1 b + a_3 b W = 0.282 - 0.456W\end{aligned}$$



The indirect effect is a function of W (pre-therapy depression) in our model. This function is a line.

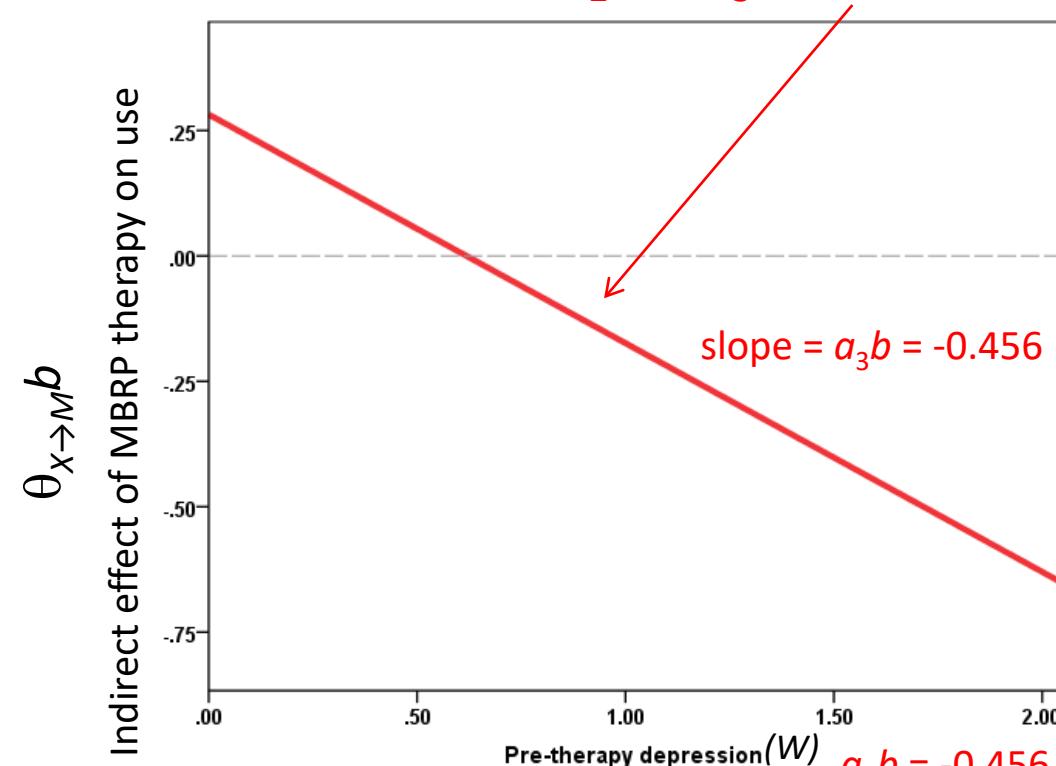
$$\begin{aligned}\theta_{X \rightarrow M} b &= (a_1 + a_3 W)b \\ &= a_1 b + a_3 b W \\ &= 0.282 - 0.456W\end{aligned}$$

An inference about the slope of this line—the “index of moderated mediation”—is an inference about whether the indirect effect is moderated: Is this slope significantly different from zero? If so, that supports a claim of “moderated mediation.”

As $a_3 b$ is a product of regression coefficients, an inferential test should respect the nonnormality of the sampling distribution of the product. A bootstrap confidence interval is a good choice.

This test is provided automatically by PROCESS

$$\begin{aligned}\theta_{X \rightarrow M} b &= (a_1 + a_3 W)b = (0.587 - 0.948W)(0.481) \\ &= a_1 b + a_3 b W = 0.282 - 0.456W\end{aligned}$$



The indirect effect is a function of W (pre-therapy depression) in our model. This function is a line.

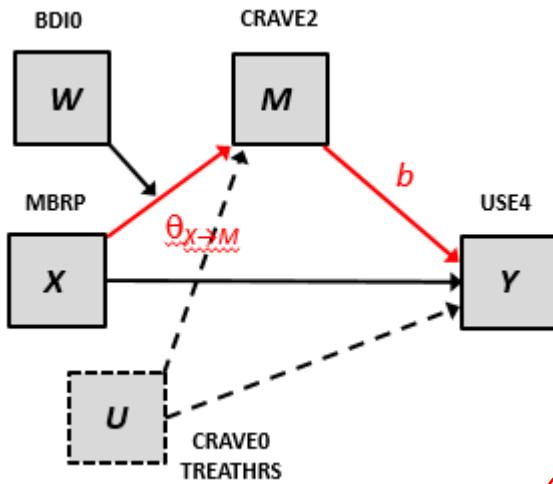
$$\begin{aligned}\theta_{X \rightarrow M} b &= (a_1 + a_3 W)b \\ &= a_1 b + a_3 b W \\ &= 0.282 - 0.456W\end{aligned}$$

Output K

***** INDEX OF MODERATED MEDIATION *****				
Mediator	Index	SE (Boot)	BootLLCI	BootULCI
crave2	-.4562	.2190	-.9596	-.1044

This slope is statistically different from zero. The indirect effect depends on pre-therapy depression....the mediation is moderated.

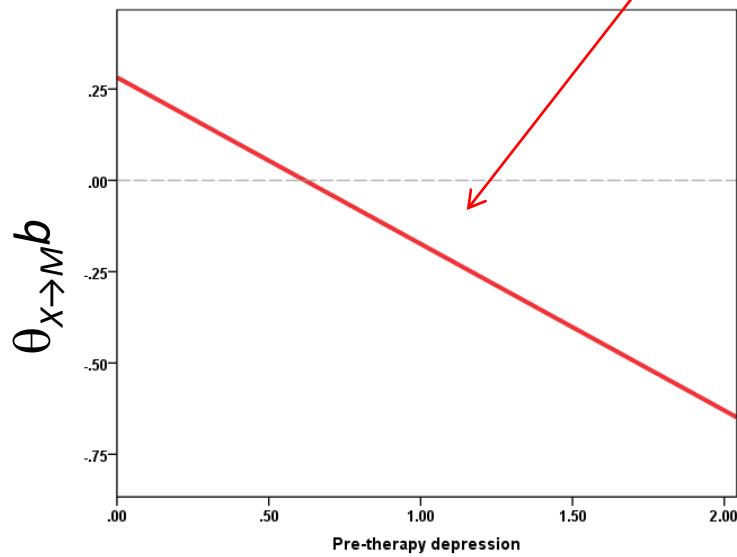
Probing the moderation of mediation



$$\theta_{X \rightarrow M} b = (a_1 + a_3 W)b = (0.587 - 0.948W)(0.481)$$

The indirect effect decreases with increased pre-therapy depression.

With evidence of moderation of the indirect effect, we can now probe this moderation of mediation through an analogue of the pick-a-point approach used in moderation analysis.



Pre-therapy depression (W)

	$\theta_{X \rightarrow M}$	b	$\theta_{X \rightarrow M} b$	
16 th %	.9020	-0.2681	0.481	-0.118
50 th %	1.1900	-0.5411	0.481	-0.263
84 th %	1.5180	-0.8521	0.481	-0.409

Conditional indirect effects

We need an inferential test for these conditional indirect effects. Bootstrap confidence intervals are perfect for the job. PROCESS does this for us automatically.

PROCESS output

***** DIRECT AND INDIRECT EFFECTS *****

Direct effect of X on Y

Effect	SE	t	p	LLCI	ULCI
.0926	.0773	1.1979	.2327	-.0601	.2453

Conditional indirect effect(s) of X on Y at values of the moderator(s) :

Mediator

bdi0	Effect	BootSE	BootLLCI	BootULCI
.9020	-.1290	.0770	-.2869	.0183
1.1900	-.2604	.0862	-.4458	-.1090
1.5180	-.4100	.1367	-.7080	-.1767

Conditional indirect effects with 95% bootstrap CIs based on 10,000 bootstrap samples.

Output K

The indirect effect of MBRP therapy relative to therapy as usual on substance use through craving is negative among the relatively moderate (point estimate: -0.260, 95% CI from -0.446 to -0.109) and relatively highly depressed (point estimate: -0.410, 95% CI from -0.708 to -0.177) but not different from zero among the relatively less depressed (point estimate: -0.129, 95% CI from -0.287 to 0.018).

Comparing conditional indirect effects (1st stage model)

A seemingly sensible question to ask is whether the conditional indirect effect of X when the moderator equals some value $W = w_1$ is different than the conditional indirect effect of X when the moderator is some different value $W = w_2$. For example, is the indirect effect of MBRP therapy through craving different for those relatively low in pre-therapy depression relative to those relatively high?

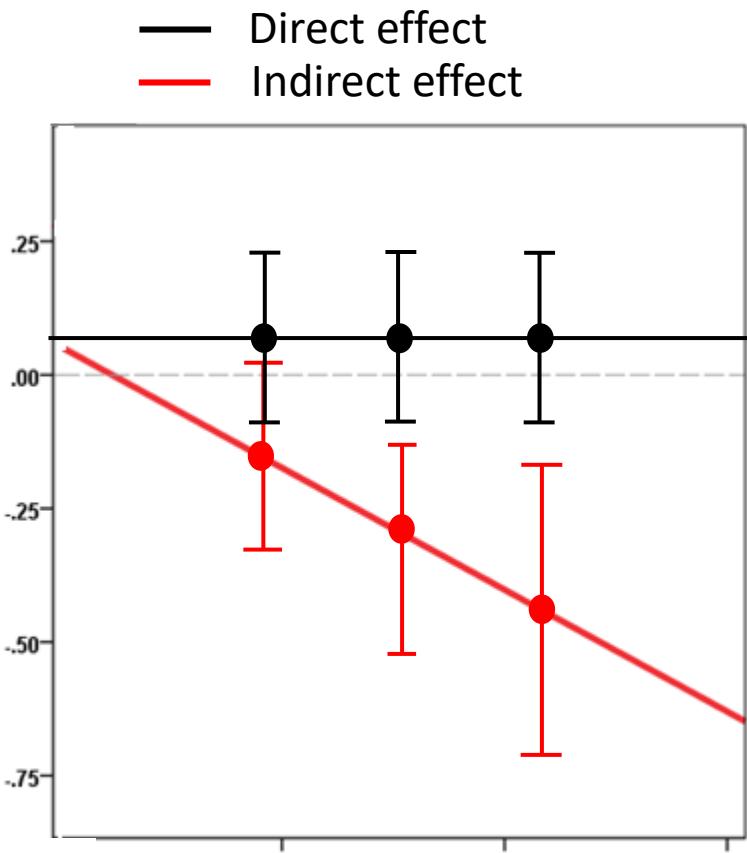
For this model, the difference between any two conditional indirect effects defined by different values of W equal to w_1 and w_2 is

$$\begin{aligned}(a_1 + a_3 w_1)b - (a_1 + a_3 w_2)b &= a_1 b + a_3 b w_1 - a_1 b - a_3 b w_2 \\&= a_3 b w_1 - a_3 b w_2 \\&= a_3 b (w_1 - w_2)\end{aligned}$$

Rejection of the null hypothesis of no moderated mediation based on the index of moderated mediation implies that **any** two conditional indirect effects are different! No additional test is needed.

If a bootstrap confidence interval for $a_3 b$ does not contain zero, then neither will a confidence interval for $a_3 b (w_1 - w_2)$, **regardless** of values of w_1 and w_2 chosen, so long as $w_1 \neq w_2$. And if a bootstrap confidence interval for $a_3 b$ contains zero, then so too will a confidence interval for $a_3 b (w_1 - w_2)$, **for any two values** of w_1 and w_2 , ($w_1 \neq w_2$).

Putting it all together



MBRP seems to reduce substance use through a reduction in craving which in turn lowers use, but more so among those who are more depressed at the start of therapy. Among those relatively lower in depression, we cannot say definitively that this mechanism is in operation. Independent of this mechanism, there is no evidence of an effect of MBRP therapy on later substance use.

Relatively least depressed” Relatively average depression Relatively most depressed

Point estimates with 95% confidence intervals (OLS confidence intervals for direct effects, bootstrap confidence intervals for indirect effects).

CPA Activity



The following slides have examples of conditional process analyses from the literature. Try to draw the path diagram for these models and write out the PROCESS code to analyze the data.

Mawritz, M. B., Mayer, D. M., Hoobler J. M., Wayne, S. J., & Marinova, S. V. (2012) A trickle-down model of abusive supervision, *Personnel Psychology*

“Specifically, we find that abusive manager behavior is positively related to abusive supervisor behavior, which in turn is positively related to work group interpersonal deviance. In addition, hostile climate moderates the relationship between abusive supervisor behavior and work group interpersonal deviance such that the relationship is stronger when hostile climate is high.”

HINT: You need to choose a model from the templates that matches the hypothesis

Activity:

- Draw a path diagram and label X , M , Y , and W with variable names (e.g., page 73)
- Variable Names: Abusive Manager Behavior (`Manage`), Abusive Supervisor Behavior (`Super`), Work Group Deviance (`Deviance`), and Hostile Climate (`Hostile`). What would the PROCESS command be to run this analysis? Use percentile bootstrapping with 7,000 bootstraps.
- Write the equations for the model of M and the model of Y . What would the conditional indirect effect be? What would the index of moderated mediation be? (e.g., page 69)
- Draw a graphical representation of the indirect effect across the moderator which fits the above hypotheses. (e.g., page 74)

CPA Wrap Up Activity



The following slides have examples of moderation from the literature. Try to draw the path diagram for these models and write out the PROCESS code to analyze the data.

Mawritz, M. B., Mayer, D. M., Hoobler J. M., Wayne, S. J., & Marinova, S. V. (2012) A trickle-down model of abusive supervision, *Personnel Psychology*

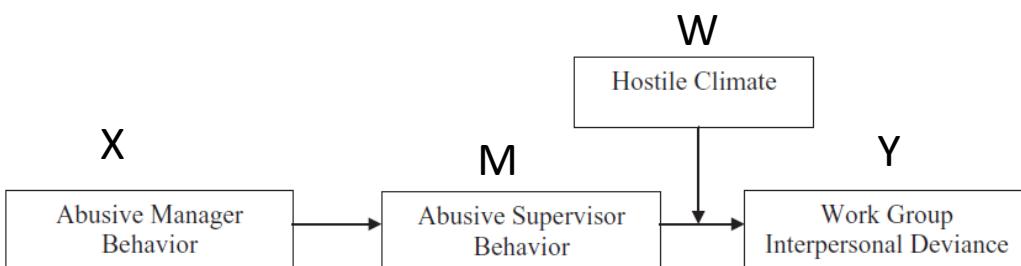


Figure 1: Theoretical Model of the Trickle-Down Effect of Abusive Supervision.

$$M_i = i_M + aX_i + \epsilon_{Mi}$$

$$Y_i = i_Y + c' X_i + b_1 M_i + b_2 W_i + b_3 W_i M_i + \epsilon_{Yi}$$

Effect of manager behavior on follower deviance through supervisor behavior when hostile climate is "low"?

Positive / Zero

Effect of manager behavior on follower deviance through supervisor behavior when hostile climate is "high"?

MORE Positive

How does an increase in hostile climate affect the indirect effect through supervisor behavior?

Positive (ab_3 is positive)

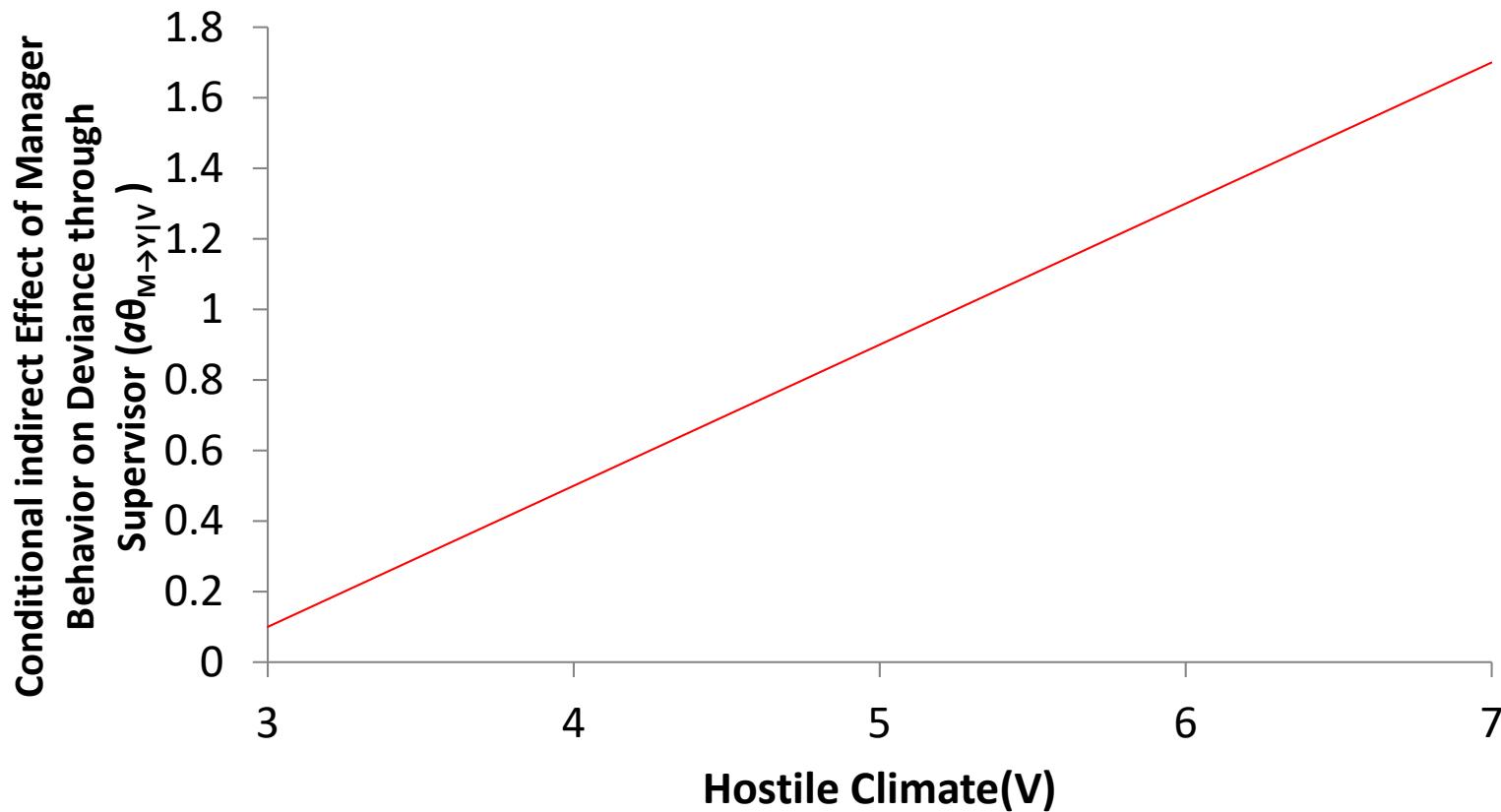
```
process y=Deviance/x=Manage/m=Super/w=Hostile/model=14/
boot=7000.
```

CPA Wrap Up Activity

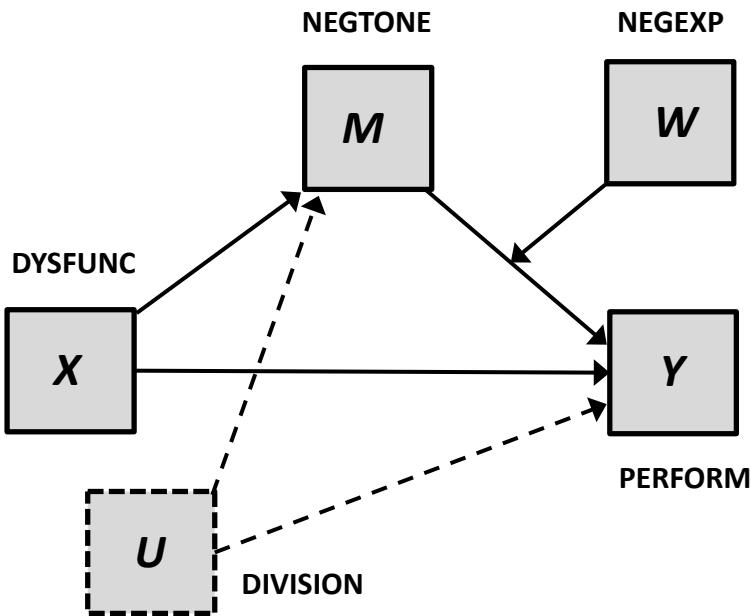


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Mawritz, M. B., Mayer, D. M., Hoobler J. M., Wayne, S. J., & Marinova, S. V. (2012) A trickle-down model of abusive supervision, *Personnel Psychology*

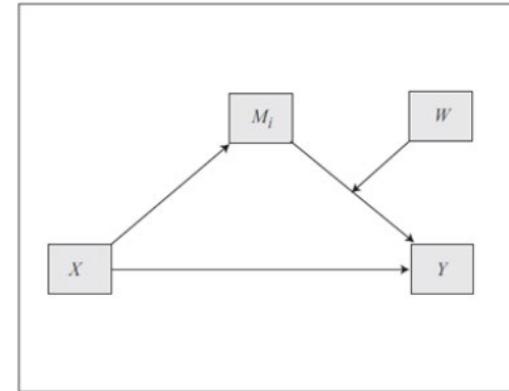


PROCESS versus SEM

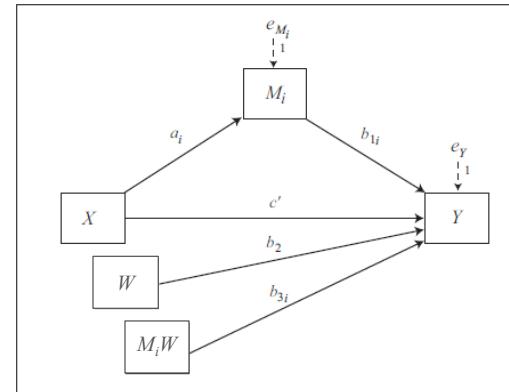


This is PROCESS model 14

Model 14



Statistical Diagram



```
process cov = d1 d2 d3/x=dysfunc/m=negtone/y=perform/w=negexp/boot=10000
      /model=14/plot = 1.
```

```
%process (data=teams,cov=d1 d2 d3,x=dysfunc,m=negtone,y=perform,w=negexp,
boot=10000,model=14, plot = 1);
```

In Mplus

```

DATA:
  file is 'c:\mplus\teams.csv';
VARIABLE:
  names are dysfunc negtone negexp perform division d1 d2 d3;
  usevariables are dysfunc negexp negtone perform d1 d2 d3 toneexp;
DEFINE:
  toneexp = negtone*negexp;
ANALYSIS:
  !bootstrap=10000;
MODEL:
  perform ON d1 d2 d3
    dysfunc (cp)
    negtone (b1)
    negexp (b2)
    toneexp (b3);
  negtone ON d1 d2 d3
    dysfunc (a1);
  d1;
  d2;
  d3;
  negexp;
  dysfunc;
  toneexp;
  negtone with negexp;
  negtone with toneexp;
MODEL CONSTRAINT:
  new (althetaL althetaM althetaH a1b3);
  althetaL=a1*(b1+b3*(-.5308));
  althetaM=a1*(b1+b3*(-0.0600));
  althetaH=a1*(b1+b3*(0.6000));
  a1b3=a1*b3;
OUTPUT:
  !cinterval(bootstrap);

```

COMPARE TO OUTPUT THAT
PROCESS GENERATES

		Estimate	S.E.	Est./S.E.	P-Value
PERFORM	ON				
	D1	0.182	0.147	1.231	0.218
	D2	0.084	0.252	0.333	0.739
	D3	0.282	0.170	1.659	0.097
	DYSFUNC	0.373	0.195	1.908	0.056
	NEGTONE	-0.489	0.131	-3.728	0.000
	NEGEXP	-0.022	0.102	-0.217	0.829
	TONEEXP	-0.450	0.243	-1.851	0.064
NEGTONE	ON				
	D1	0.349	0.167	2.093	0.036
	D2	0.295	0.206	1.436	0.151
	D3	0.251	0.116	2.167	0.030
	DYSFUNC	0.609	0.216	2.822	0.005
Intercepts					
	NEGTONE	-0.206	0.087	-2.368	0.018
	PERFORM	-0.175	0.127	-1.380	0.167
New/Additional Parameters					
	A1THETAL	-0.152	0.152	-1.003	0.316
	A1THETAM	-0.281	0.126	-2.235	0.025
	A1THETAH	-0.462	0.170	-2.722	0.006
	A1B3	-0.274	0.176	-1.557	0.119

In Mplus

Removing exclamation points from the code generates bootstrap confidence intervals for all parameter estimates, including conditional indirect effects.

CONFIDENCE INTERVALS OF MODEL RESULTS

COMPARE TO OUTPUT THAT
PROCESS GENERATES

	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
New/Additional Parameters							
A1THETAL	-0.568	-0.429	-0.377	-0.152	0.113	0.187	0.352
A1THETAM	-0.664	-0.544	-0.488	-0.281	-0.082	-0.052	0.010
A1THETAH	-0.954	-0.816	-0.753	-0.462	-0.194	-0.146	-0.064
A1B3	-0.886	-0.702	-0.617	-0.274	-0.051	-0.022	0.030



Point estimates

End points of a 95% bootstrap confidence intervals
for the conditional indirect effect based on 10,000 bootstrap samples.

Observe that the results using an SEM program are largely the same as the results that PROCESS produces.

Mplus vs. PROCESS output

CONFIDENCE INTERVALS OF MODEL RESULTS

	Lower 2.5%	Estimate	Upper 2.5%
New/Additional Parameters			
A1THETAL	-0.429	-0.152	0.187
A1THETAM	-0.544	-0.281	-0.052
A1THETAH	-0.816	-0.462	-0.146
A1B3	-0.702	-0.274	-0.022

Point estimates

INDIRECT EFFECT:

dysfunc → negtone → perform

negexp	Effect	BootSE	BootLLCI	BootULCI
-.5308	-.1523	.1540	-.4335	.1943
-.0600	-.2813	.1249	-.5432	-.0549
.6000	-.4623	.1678	-.8095	-.1503

Index of moderated mediation:

	Index	BootSE	BootLLCI	BootULCI
negexp	-.2742	.1791	-.7172	-.0234

Observe that the results using an SEM program are largely the same as the results that PROCESS produces.

PROCESS versus Structural Equation Modeling

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Commentary

The analysis of mechanisms and their contingencies:
PROCESS versus structural equation modeling

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ABSTRACT

Marketing, consumer, and organizational behavior researchers interested in studying the mechanisms by which effects operate and the conditions that enhance or inhibit such effects often rely on statistical moderation and conditional process analysis (also known as the analysis of "moderated mediation"). Model estimation is typically undertaken with ordinary least squares regression-based path analysis, such as implemented in the popular PROCESS macro for SPSS and SAS (Hayes, 2013), or using a structural equation modeling program. In this paper we answer a few frequently-asked questions about the difference between PROCESS and structural equation modeling and show by way of example that, for observed variable models, the choice of which to use is inconsequential, as the results are largely identical. We end by discussing considerations to ponder when making the choice between PROCESS and structural equation modeling.

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CHINESE ABSTRACT

营销、消费者和组织行为研究人员对研究这种影响机制的机制非常感兴趣，增强或抑制这种影响的条件通常依赖于统计调节和在一定条件下的处理分析（也叫做分析“调节中介”）。模型评估通常使用普通最小二乘法基于日的路径分析（例如，在SPSS和SAS深受青睐的PROCESS宏中实现(Hayes, 2013)）或采用结构方程模型方法。本文回答了一些有关PROCESS和结构方程模型之间常见的常见问题，并举例说明，对于观察变量模型，选择使用哪种模型都无关紧要，因为其结果相差不多。本文还讨论了在PROCESS和结构方程模型之间进行选择应考虑的因素。

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Marketing researchers and those who study organizational or consumer behavior strive to understand how marketing and other organizational effects operate, meaning the underlying cognitive, social, and biological processes that intervene between a stimulus (e.g., a particular kind of packaging or promotion, or the management style of a leader) and a response (e.g., the evaluation of a product, a decision or timing to purchase, or employee turnover at a company). Mediation analysis is a popular statistical procedure for testing hypotheses about the mechanisms by which a causal effect operates. A mediation model contains at least one mediator variable M that is causally between X and Y , such that X 's effect on Y is transmitted through the joint causal effect of X on M which in turn affects Y . Fig. 1, panel A, depicts a mediation model with two mediators. Some examples found in the pages of *Australasian Marketing Journal* include Kongcharapata and Shannon (2016), Baxter and Kleinaltenkamp (2015), and Schiele and Vos (2015). Such models are commonplace in the empirical literature.

Less common but growing in frequency are mediation models that allow for moderation of a mechanism, what Hayes (2013) calls a *conditional process model*. Fig. 1, panels B, C, and D, represent a few conditional process models, also known as *moderated mediation* models. Panel A is a *first stage* conditional process model that allows the effect of X on M in a mediation model to depend on variable W . The moderator, W , could be anything that influences or changes the effect of X on M . For some examples, see Voola et al. (2012), White et al. (2016), Shen et al. (2016), and Zenker et al. (2017). But if the moderation operates on the second stage of a mediation process (i.e., on the effect of M on Y), as in Cassar and Briner (2011) and Dubois et al. (2016), the result is a *second stage* conditional process model, as in Fig. 1, panel C. If the same moderator influences the relationship between X and M and M and Y (Fig. 1, panel D), this is a *first and second stage* conditional process model. Examples include Shenu-Fen et al. (2012) and Etkin and Sela (2016). These represent only three of the many ways that mediation and moderation can be integrated into a unified model.

Each of the models depicted in Fig. 1 looks like a path diagram, with variables connected with unidirectional arrows. Such diagrams, for most researchers, bring to mind structural equation

Hayes, A. F., Montoya, A. K., & Rockwood, N. J. (2017). The analysis of mechanisms and their contingencies: PROCESS versus structural equation modeling. *Australasian Marketing Journal*, 25, 76–81.

- For observed variable models, it makes no difference whether you use SEM or PROCESS. You get the same results and with a lot less effort.

Some reasons you might choose SEM:

- More options for dealing with missing data
- More sophisticated means of managing the effects of measurement error.
- Latent variables and blends of latent and observed variables.
- Greater flexibility for model specification.

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A study of sex discrimination in the workplace

Garcia, D. M., Schmitt, M. T., Branscombe, N. R., & Ellemers, N. (2010). Women's reactions to ingroup members who protest discriminatory treatment: The importance of beliefs about inequality and response appropriateness. *European Journal of Social Psychology*, 49, 733-745.

European Journal of Social Psychology
Eur. J. Soc. Psychol. 40, 733–745 (2010)
Published online 6 July 2009 in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/ejsp.644

Research article

Women's reactions to ingroup members who protest discriminatory treatment: The importance of beliefs about inequality and response appropriateness

DONNA M. GARCIA^{1*}, MICHAEL T. SCHMITT²,
NYLA R. BRANSCOMBE³ AND NAOMI ELLEMERS⁴

¹University of Guelph, Canada

²Simon Fraser University, Canada

³University of Kansas, USA

⁴Leiden University, The Netherlands

Abstract

Our goal was to identify factors that shape women's responses to ingroup members who protest gender discrimination. We predicted and found that women who perceived gender discrimination as pervasive regarded a protest response as being more appropriate than a no protest response and expressed greater liking and less anger towards a female lawyer who protested rather than did not protest an unfair promotion decision. Further, beliefs about the appropriateness of the response to discrimination contributed to evaluations of the protesting lawyer. Perceptions that the complaint was an appropriate response to the promotion decision led to more positive evaluations of an ingroup discrimination protester. Copyright © 2009 John Wiley & Sons, Ltd.

Protest can be an effective means of improving the plight of a devalued group. Historically, there are many examples of protest, even from a single individual, that have advanced a group's social position (e.g. *Meritor Savings Bank vs. Vinson*, 477 US 57, 1986; *Dekker vs. VTV-Centrum ECJ*, 1992). Despite the potential gains to be obtained by protesting illegitimate treatment, protestors might not always be appreciated by members of their own group. Whether disadvantaged group members respond positively or negatively to ingroup protestors will likely depend upon the *perceived* implications that the protestor's action has for the ingroup. Unless protest is seen as justified by the social circumstances and an effective means of bringing about positive change, a protestor might be seen as making the ingroup look like complainers. Such threat to the ingroup's reputation could evoke the ire and disdain of the disadvantaged group towards the protestor. Hence, perceptions of the justification for and likely consequences of protest will be critical to others' reactions to an ingroup discrimination claimer. We propose that protest by an ingroup member will be seen as appropriate and thus appreciated to the extent that observers perceive that their ingroup is targeted by pervasive discrimination.

SOCIAL CONSEQUENCES OF CLAIMING DISCRIMINATION

Gender discrimination continues to be widespread throughout Western employment settings (see Charles & Grusky, 2004). The continuation of gender discrimination has substantive negative implications for women's economic and

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Received 19 October 2008
Accepted 7 April 2009

All participants (129 females) read a narrative about a female attorney who lost a promotion at her firm to a less qualified male through unequivocally discriminatory actions of the senior partners. The participants were randomly assigned to one of three conditions:

"Individual protest": Participants were told that she protested by describing her qualifications for the job, how much she deserved it, and the unfairness and harm to her own career.

"Collective protest": Participants were told that she protested by describing how the firm has not been fair to women, women are just as qualified as men, and they should be treated equally.

"No protest" : These participants were told that although she was disappointed, she accepted the decision and continued working at the firm.

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After reading the narrative, the participants evaluated how **appropriate** they perceived her response to be for the situation. i.e., was it a positive response for dealing with the firm's discrimination.

They also responded to various questions about the attorney that were used to produce a measure of her evaluation which we'll simply call **liking**, such as "I like Catherine," and "Catherine has many positive traits."

Prior to the study, the participants filled out the **modern sexism scale**, used to score each participant with respect to how pervasive she believes sexism and sex discrimination are in society.

The Data: PROTEST

The screenshot shows the IBM SPSS Statistics Data Editor interface. On the left, a data view window displays 14 rows of data with columns labeled: subnum, cond, sexism, angry, liking, respappr, and protest. The data is as follows:

	subnum	cond	sexism	angry	liking	respappr	protest
1	209	2	4.87	2	4.83	4.25	1.00
2	44	0	4.25	1	4.50	5.75	.00
3	124	2	5.00	3	5.50	4.75	1.00
4	232	2	5.50	1	5.66	7.00	1.00
5	30	2	5.62	1	6.16	6.75	1.00
6	140	1	5.75	1	6.00	5.50	1.00
7	27	2	5.12	2	4.66	5.00	1.00
8	64	0	6.62	1	6.50	6.25	.00
9	67	0	5.75	6	1.00	3.00	.00
10	182	0	4.62	1	6.83	5.75	.00
11	85	2	4.75	2	5.00	5.25	.00
12	109	2	6.12	5	5.66	7.00	1.00
13	122	0	4.87	2	5.83	4.50	.00
14	69	1	5.87	1	6.50	6.25	1.00

On the right, a code editor window titled "protest" shows the SPSS syntax used to input the data:

```
data protest;
  input subnum cond sexism angry liking respappr protest;
cards;
209 2 4.87 2 4.83 4.25 1.00
44 0 4.25 1 4.50 5.75 .00
124 2 5.00 3 5.50 4.75 1.00
232 2 5.50 1 5.66 7.00 1.00
30 2 5.62 1 6.16 6.75 1.00
140 1 5.75 1 6.00 5.50 1.00
27 2 5.12 2 4.66 5.00 1.00
64 0 6.62 1 6.50 6.25 .00
67 0 5.75 6 1.00 3.00 .00
182 0 4.62 1 6.83 5.75 .00
85 2 4.75 2 5.00 5.25 1.00
109 2 6.12 5 5.66 7.00 1.00
122 0 4.87 2 5.83 4.50 .00
69 1 5.87 1 6.50 6.25 1.00

```

COND: Experimental condition (0 = no protest, 1 = individual protest, 2 = collective protest)

LIKING : Evaluation (liking) of the lawyer (higher = more positive evaluation, i.e. like more)

SEXISM: Score on the modern sexism scale: Beliefs about the pervasiveness of sex discrimination in society (higher = sex discrimination perceived as more pervasive in society)

RESPAPPR: A measure of how appropriate the lawyer's behavior in response to the action of the partners was perceived to be for the situation (higher = more appropriate)

Our objectives with these data

	Perceived Response Appropriateness (M)		Liking (Y)		
	M	SD	Y	SD	\bar{Y}^*
No protest ($n = 41$)	3.884	1.457	5.310	1.302	5.715
Individual protest ($n = 43$)	5.145	1.075	5.826	0.819	5.711
Collective protest ($n = 45$)	5.494	0.936	5.753	0.936	5.495
All groups combined	4.866	1.348	5.637	1.050	5.637

\bar{Y}^* = adjusted mean, adjusted to the sample mean of perceived response appropriateness.

We'll examine whether the effect of the attorney's response on how she is evaluated (3 conditions)...

... operates through the mechanism of how appropriate her response to the situation is perceived as being (mediation)

Course II....

... depends on perceived pervasiveness of sex discrimination in society (moderation).

... as mediated by the mechanism of perceived appropriateness of her response is moderated by perceived pervasiveness of sex discrimination (moderated mediation)

Single-factor (“one-way”) analysis of variance

Did the lawyer’s choice as to how to respond (not at all, individual protest, collective protest) influence how she was perceived? That is, is there a difference between conditions, on average, in how much she was liked? Most would answer this using a single-factor (a.k.a. “one-way”) analysis of variance (ANOVA).

```
means tables = liking by cond/statistics anova.
```

```
proc anova data=protest;
  class cond;model liking = cond;means cond;run;
```

Report

LIKING: liking of the target

COND: experimental condition	Mean	N	Std. Deviation
no protest	5.3102	41	1.30158
individual	5.8260	43	.81943
collective	5.7533	45	.93601
Total	5.6367	129	1.04970

H_0 : all means equal
 H_a : all means not equal

ANOVA Table

		Sum of Squares	df	Mean Square	F	Sig.
LIKING: liking of the target	Between Groups (Combined)	6.523	2	3.262	3.055	.051
* COND: experimental condition	Within Groups	134.515	126	1.068		
	Total	141.039	128			

The lawyer’s response to the discrimination affected how much she was liked on average, $F(2,126) = 3.055, p = .051$. She was most liked when she protested individually (Mean = 5.83, SD = 0.82), next most when protesting collectively (Mean = 5.75, SD = 0.94), and least when she didn’t protest at all (Mean = 5.31, SD = 1.30).

Mediation

Does perceived response appropriateness mediate this effect?

	Perceived Response Appropriateness (<i>M</i>)		Liking (<i>Y</i>)		
	<i>M</i>	<i>SD</i>	<i>Y</i>	<i>SD</i>	\bar{Y}^*
No protest (<i>n</i> = 41)	3.884	1.457	5.310	1.302	5.715
Individual protest (<i>n</i> = 43)	5.145	1.075	5.826	0.819	5.711
Collective protest (<i>n</i> = 45)	5.494	0.936	5.753	0.936	5.495
All groups combined	4.866	1.348	5.637	1.050	5.637

\bar{Y}^* = adjusted mean, adjusted to the sample mean of perceived response appropriateness.

Here, the presumed cause is a multicategorical variable with three levels. How does one conduct a mediation analysis in such a design?

Mediation of the effect of a multicategorical independent variable

Does perceived response appropriateness mediate this effect?

The causal steps approach

	Perceived Response Appropriateness (<i>M</i>)		Liking (<i>Y</i>)		
	<i>M</i>	<i>SD</i>	<i>Y</i>	<i>SD</i>	\bar{Y}^*
No protest (<i>n</i> = 41)	3.884	1.457	5.310	1.302	5.715
Individual protest (<i>n</i> = 43)	5.145	1.075	5.826	0.819	5.711
Collective protest (<i>n</i> = 45)	5.494	0.936	5.753	0.936	5.495
All groups combined	4.866	1.348	5.637	1.050	5.637

\bar{Y}^* = adjusted mean, adjusted to the sample mean of perceived response appropriateness.

From a single-factor ANOVA,
the effect of experimental condition

on...liking: $F(2,126) = 3.055, p = .051$ (**total effect**)

...response appropriateness: $F(2,126) = 22.219, p < .001$ ('**a**' effect)

From a single-factor ANCOVA...the relationship between perceived response appropriateness and liking is positive, $b = 0.412, p < 0.001$ ('**b**' effect), and...
...the effect of condition on liking disappears after controlling for response appropriateness
 $F(2,125) = 0.729, p = .485$ (**direct effect**)

This approach has all the problems of the causal steps ("Baron and Kenny") approach.
There is a better way.

Representing a multcategorical predictor in a linear model

Predictor variables in a linear model must be quantitative or dichotomous (i.e., categorical with only two values). What if we want to include multcategorical variables? **How do we proceed?**

Any multcategorical variable with k categories can be represented with $k - 1$ Variables in a regression model. For example, we can code membership in a category with a set of *dummy variables* and all of these $k - 1$ dummy variables in the model.

Indicator Coding (AKA “Dummy Coding”)

Set D_1 to 1 for cases in category 1, 0 otherwise

D_2 to 1 for cases in category 2, 0 otherwise

.

.

$D_{(k-1)}$ to 1 for cases in category $k - 1$, 0 otherwise

Category k is called the “reference category,” for reasons that will be clear soon. It is represented here in the coding system, but it doesn’t seem so.

Indicator coding condition

```
frequencies variables = cond.
```

```
proc freq data=protest;tables cond;run;
```

COND: experimental condition

		Frequency	Percent	Valid Percent	Cumulative Percent
cond = 0	Valid no protest	41	31.8	31.8	31.8
cond = 1	individual	43	33.3	33.3	65.1
cond = 2	collective	45	34.9	34.9	100.0
	Total	129	100.0	100.0	

One possible dummy variable coding system for condition ($k = 3$ categories):

	D_1	D_2
No protest	0	0
Individual	1	0
Collective	0	1

The “reference category” is the one with zeros on all $k - 1$ dummy variables. In this example, those told the lawyer did not protest are the reference category.

Constructing Indicator variables

There is a variety of ways of constructing indicator codes in a computing platform, each with its dangers, assumptions, and conveniences.

	D_1	D_2
cond = 0	No protest	0
cond = 1	Individual	1
cond = 2	Collective	0

Here is one way

```
compute d1 = 0.  
compute d2 = 0.  
if (cond = 1) d1 = 1.  
if (cond = 2) d2 = 1.  
execute.
```

```
data protest;set protest;  
d1=0;d2=0;  
if (cond=1) then d1 = 1;  
if (cond=2) then d2 = 1;  
run;
```

This approach can be dangerous. Any cases missing on condition will be coded as if they were assigned to the no protest condition. Use this approach with caution. As a general rule, know your data before you start manipulating it.

Constructing Indicator variables

There is a variety of ways of constructing indicator codes in a computing platform, each with its dangers, assumptions, and conveniences.

	D_1	D_2
cond = 0	No protest	0
cond = 1	Individual	1
cond = 2	Collective	0

Here is a safer approach:

```
compute d1 = (cond=1).  
compute d2 = (cond=2).  
execute.
```

```
data protest;set protest;  
d1 = (cond=1);  
d2 = (cond=2);  
if (cond=.) then d1=.;  
if (cond=.) then d2=.;  
run;
```

In SPSS, this is very efficient. SAS requires the explicit coding of missing data as such. The SPSS version will leave cases missing on cond missing on d1 and d2.

Estimating liking from experimental condition using regression

```
compute d1 = (cond=1).  
compute d2 = (cond=2).  
regression/dep = liking/method = enter d1 d2.
```

```
data protest;set protest;  
d1 = (cond=1);d2 = (cond=2);run;  
proc reg data=protest;model liking = d1 d2;run;
```

We know there are
no missing data on
cond, so this is ok.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.215 ^a	.046	.031	1.03324

a. Predictors: (Constant), d2, d1

$$\hat{Y} = 5.310 + 0.516D_1 + 0.443D_2$$

ANOVA^a

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	6.523	2	3.262	3.055	.051 ^b
	Residual	134.515	126	1.068		
	Total	141.039	128			

a. Dependent Variable: LIKING: liking of the target

b. Predictors: (Constant), d2, d1

Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.
	B	Std. Error			
1	(Constant)	5.310	.161	32.908	.000
	d1	.516	.226	.233	.024
	d2	.443	.223	.202	.049

a. Dependent Variable: LIKING: liking of the target

The model reproduces the group means

$$\hat{Y} = 5.310 + 0.516D_1 + 0.443D_2$$

	D_1	D_2
No protest	0	0
Individual	1	0
Collective	0	1

No protest $\hat{Y} = 5.310 + 0.516(0) + 0.443(0) = 5.310 = \bar{Y}_{NP}$

Individual protest $\hat{Y} = 5.310 + 0.516(1) + 0.443(0) = 5.826 = \bar{Y}_{IP}$

Collective protest $\hat{Y} = 5.310 + 0.516(0) + 0.443(1) = 5.753 = \bar{Y}_{CP}$

Report

LIKING: liking of the target

COND: experimental condition	Mean	N	Std. Deviation
no protest	5.3102	41	1.30158
individual	5.8260	43	.81943
collective	5.7533	45	.93601
Total	5.6367	129	1.04970

Interpretation of the coefficients

$$\hat{Y} = b_0 + b_1 D_1 + b_2 D_2$$

$$\hat{Y} = 5.310 + 0.516D_1 + 0.443D_2$$

	D_1	D_2
No protest	0	0
Individual	1	0
Collective	0	1

$b_0 = 5.310$ This is the mean liking among those assigned to the no protest condition ($D_1 = 0, D_2 = 0$).

$b_1 = 0.516$ This is the mean difference in liking between those in the individual protest condition ($D_1 = 1, D_2 = 0$) and those in the no protest condition ($D_1 = 0, D_2 = 0$)

$$b_1 = \bar{Y}_{IP} - \bar{Y}_{NP} = 5.826 - 5.310 = 0.516$$

$b_2 = 0.443$ This is the mean difference in liking between those in the collective protest condition ($D_1 = 0, D_2 = 1$) and those in the no protest condition ($D_1 = 0, D_2 = 0$)

$$b_2 = \bar{Y}_{CP} - \bar{Y}_{NP} = 5.753 - 5.310 = 0.443$$

When D_1 and D_2 are indicator codes constructed in this fashion, b_1 estimates the mean difference in Y between the group coded by D_1 and the reference group, and b_2 estimates the mean difference in Y between the group coded by D_2 and the reference group.

Statistical inference

We are estimating the coefficients of a model of the form

$$\hat{Y} = \tilde{b}_0 + \tilde{b}_1 D_1 + \tilde{b}_2 D_2$$

If there is no actual difference, on average, between these groups on Y , this implies that both “true” regression coefficients \tilde{b}_1 and \tilde{b}_2 are both equal to zero. The null hypothesis can be tested by converting the obtained R^2 to an F -ratio and then deriving a p -value.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.215 ^a	.046	.031	1.03324

a. Predictors: (Constant), d2, d1

$$H_0: \tilde{b}_1 = \tilde{b}_2 = 0$$

H_a : at least one is different from zero

ANOVA^a

Model	Sum of Squares	df	Mean Square	F	Sig.
1 Regression	6.523	2	3.262	3.055	.051 ^b
Residual	134.515	126	1.068		
Total	141.039	128			

a. Dependent Variable: LIKING: liking of the target

b. Predictors: (Constant), d2, d1

$$F(k-1, df_{\text{residual}}) = \frac{df_{\text{residual}} R^2}{(k-1)(1-R^2)}$$

$$F(2,126) = \frac{126(0.046)}{2(1-0.046)}$$

$$F(2,126) = 3.055$$

$F(2,126) = 3.055, p = .051$. Reject H_0 . The three group means differ from each other by more than can be explained by just ‘chance’. Compare this to the one-way ANOVA from earlier.

Statistical inference

We are estimating the coefficients of a model of the form

$$\hat{Y} = \tilde{b}_0 + \tilde{b}_1 D_1 + \tilde{b}_2 D_2$$

b_1 and b_2 can also be used to test hypotheses about differences between groups—specifically, between the group a dummy variable codes and the reference group.

Model	Coefficients ^a				
	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error			
1 (Constant)	5.310	.161		32.908	.000
d1	.516	.226	.233	2.287	.024
d2	.443	.223	.202	1.986	.049

a. Dependent Variable: LIKING: liking of the target

$$H_0: \tilde{b}_1 = 0$$

$$H_a: \tilde{b}_1 \neq 0$$

$$b_1 = 0.516, t(126) = 2.287, p = .024$$

$$H_0: \tilde{b}_2 = 0$$

$$H_a: \tilde{b}_2 \neq 0$$

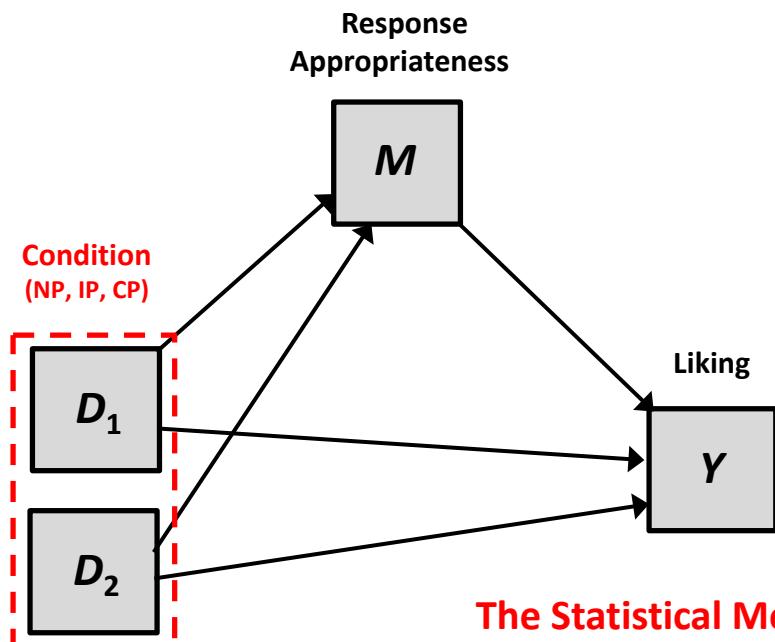
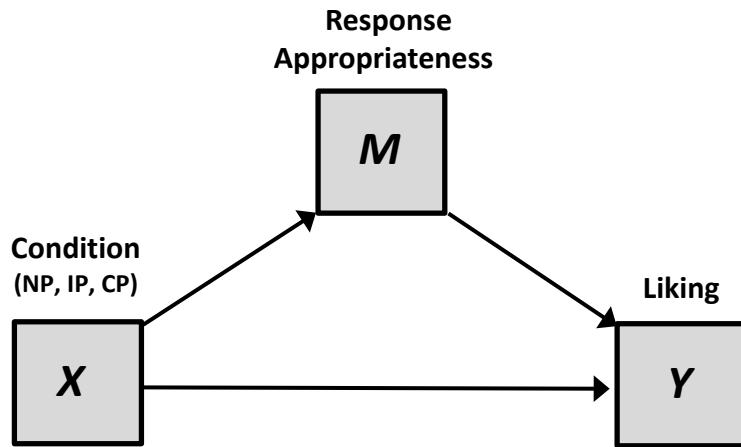
$$b_2 = 0.443, t(126) = 1.986, p = .049$$

Those told she individually protested liked her more on average than those told she did not protest.

Those told she collectively protested liked her more on average than those told she did not protest.

Mediation analysis with a multcategorical independent variable

The Conceptual Model



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Expert Tutorial

Statistical mediation analysis with a multcategorical independent variable

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Virtually all discussions and applications of statistical mediation analysis have been based on the condition that the independent variable is dichotomous or continuous, even though investigators frequently are interested in testing mediation hypotheses involving a multcategorical independent variable (such as two or more experimental conditions relative to a control group). We provide a tutorial illustrating an approach to estimation of and inference about direct, indirect, and total effects in statistical mediation analysis with a multcategorical independent variable. The approach is mathematically equivalent to analysis of (co)variance and reproduces the observed and adjusted group means while also generating effects having simple interpretations. Supplementary material available online includes extensions to this approach and Mplus, SPSS, and SAS code that implements it.

1. Introduction

Statistical mediation analysis is commonplace in psychological science (see, for example, Hayes & Scharkow, 2013). This may be because the concept of mediation gets to the heart of why social scientists become scientists in the first place – because they are curious and want to understand how things work. Establishing that independent variable X influences dependent variable Y while being able to describe and quantify the mechanism responsible for that effect is a lofty scientific accomplishment. Though hard to achieve convincingly (Bullock, Green, & Ha, 2010), documenting the process by which an effect operates is an important scientific goal.

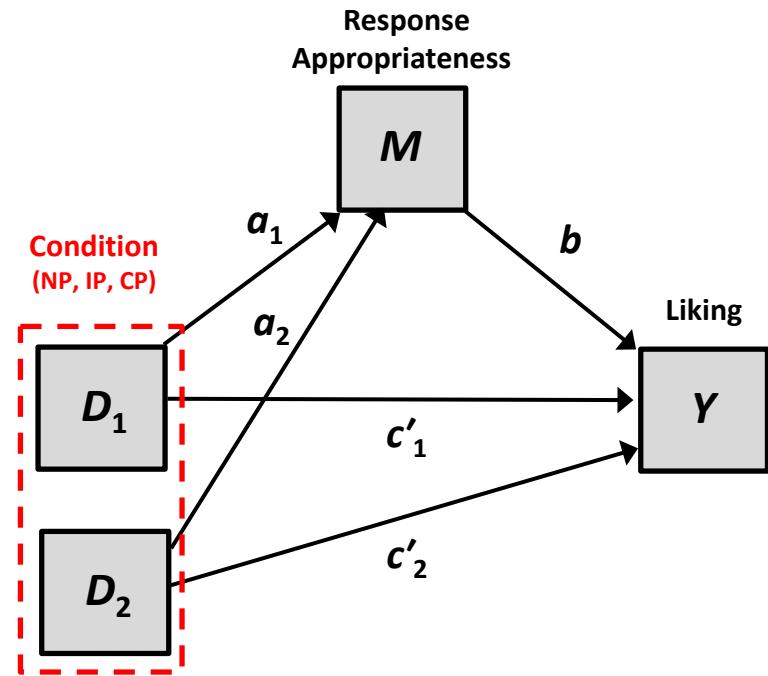
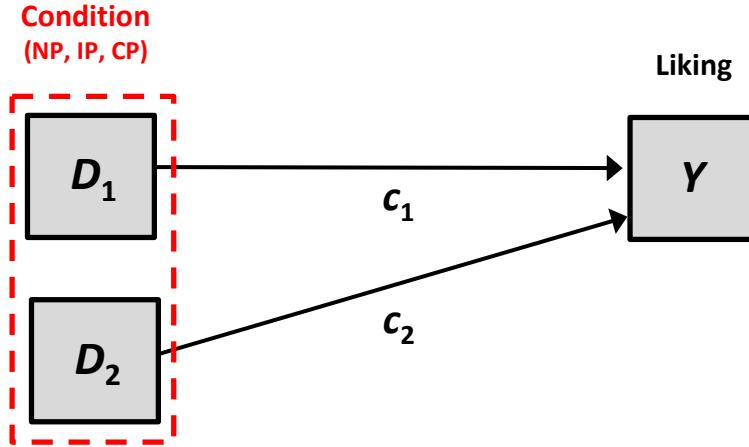
The simple mediation model, the focus of this paper, is diagrammed in Figure 1(b). This model reflects a causal sequence in which X affects Y indirectly through mediator M . In this model, X is postulated to affect M , and this effect then propagates causally to Y . This *indirect effect* represents the mechanism by which X transmits its effect on Y . According to this model, X can also affect Y directly – the *direct effect* of X – independent of X 's influence on M . Examples of such a model are found in abundance in psychological science (see Bearden, Feinstein, & Cohen, 2012; Johnson & Fujita, 2012).

The literature on statistical mediation analysis focuses predominantly on models with a dichotomous or continuous independent variable, for this is a requirement of the

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Hayes and Preacher (2014, BJMSP) available in your materials.

(Relative) total, direct, and indirect effects



c_1 and c_2 : Relative total effects of experimental condition on liking

c'_1 and c'_2 : Relative direct effects of experimental condition on liking

a_1b and a_2b : Relative indirect effects of condition on liking through perceived response appropriateness.

$$c_1 = c'_1 + a_1b; \text{ therefore, } a_1b = c_1 - c'_1$$

$$c_2 = c'_2 + a_2b; \text{ therefore, } a_2b = c_2 - c'_2$$

The relative total effects partition perfectly into relative direct and relative indirect effects. The relative indirect effects are the relative total effects minus the relative direct effects.

Coding the groups

We'll use indicator codes setting the no protest condition to the reference group. Condition (variable name COND) is coded 0 (no protest condition), 1 (individual protest condition), and 2 (collective protest condition).

Condition	D_1	D_2
No protest	0	0
Individual	1	0
Collective	0	1

```
compute d1 = (cond=1).  
compute d2 = (cond=2).  
execute.
```

```
data protest;set protest;  
d1 = (cond=1);  
d2 = (cond=2);  
if (cond=.) then d1=.;  
if (cond=.) then d2=.;  
run;
```

So effects for D_1 will compare individual protest to no protest, and effects for D_2 will compare collective protest to no protest.

The total effect of experimental condition on liking (c paths)

```
regression/dep = liking/method = enter d1 d2.
```

```
proc reg data=protest;model liking=d1 d2;run;
```

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.215 ^a	.046	.031	1.03324

a. Predictors: (Constant), d2, d1

ANOVA^a

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	6.523	2	3.262	3.055	.051 ^b
	Residual	134.515	126	1.068		
	Total	141.039	128			

a. Dependent Variable: LIKING: liking of the target

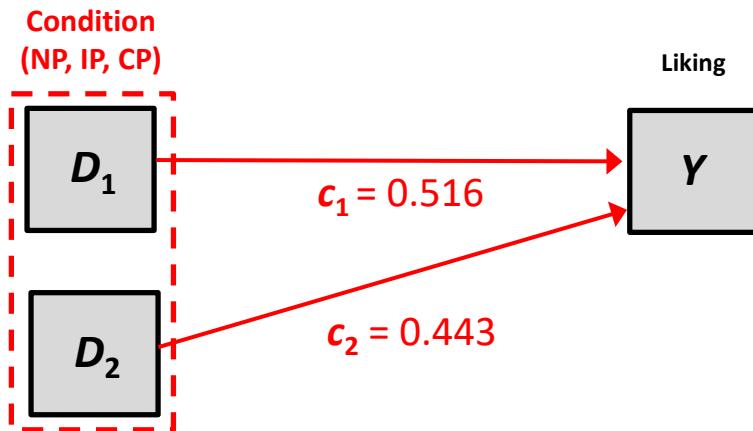
b. Predictors: (Constant), d2, d1

Coefficients^a

Model		Unstandardized Coefficients		Beta	t	Sig.
		B	Std. Error			
1	(Constant)	5.310	.161		32.908	.000
	d1	.516	.226	.233	2.287	.024
	d2	.443	.223	.202	1.986	.049

a. Dependent Variable: LIKING: liking of the target

We did this already!



Relative total effects

Relative to those told she did not protest, those told she individually protested liked her more on average ($c_1 = 0.516, p = .024$). Relative to those told she did not protest, those told she collectively protested also liked her more on average ($c_2 = 0.443, p = .049$).

The effect of experimental condition on perceived response appropriateness (a paths)

regression/dep = respappr/method = enter d1 d2.

proc reg data=protest;model respappr=d1 d2;run;

Model Summary				
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.511 ^a	.261	.249	1.16829

a. Predictors: (Constant), d2, d1

ANOVA^a

Model	Sum of Squares	df	Mean Square	F	Sig.
1 Regression	60.653	2	30.327	22.219	.000 ^b
Residual	171.977	126	1.365		
Total	232.631	128			

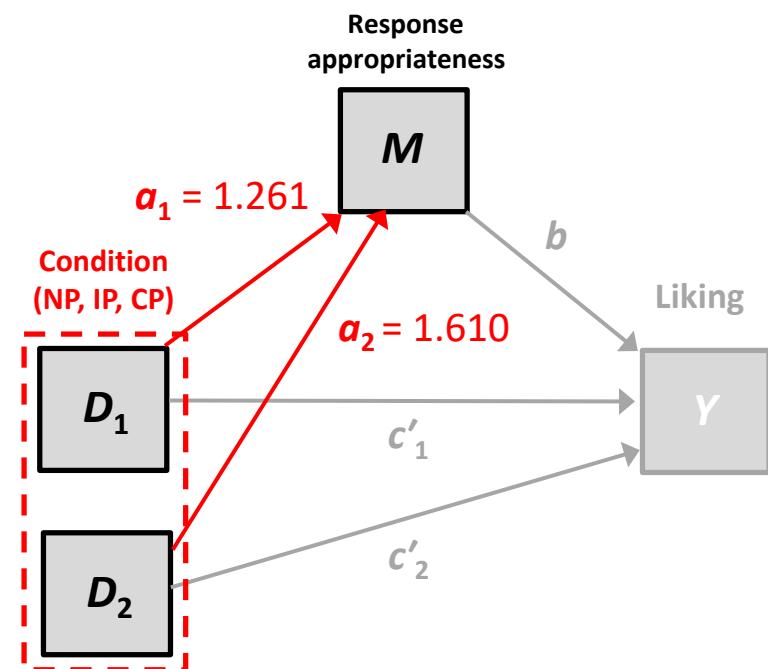
a. Dependent Variable: RESPAPPR: appropriateness of response

b. Predictors: (Constant), d2, d1

Coefficients^a

Model	Unstandardized Coefficients		Beta	t	Sig.
	B	Std. Error			
1 (Constant)	3.884	.182		21.288	.000
d1	1.261	.255	.443	4.946	.000
d2	1.610	.252	.572	6.384	.000

a. Dependent Variable: RESPAPPR: appropriateness of response



Relative to those told she did not protest, those told she individually protested felt her response was more appropriate on average ($a_1 = 1.261, p < .001$). Relative to those told she did not protest, those told she collectively protested felt her response was more appropriate on average ($a_2 = 1.610, p < .001$).

The direct effect of condition on liking (c' paths)

along with the effect of response appropriateness on liking (b path)

```
regression/dep = liking/method = enter respappr d1 d2.
```

```
proc reg data=protest;model liking=respappr d1 d2;run;
```

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.503 ^a	.253	.235	.91798

a. Predictors: (Constant), d2, RESPAPPR: appropriateness of response, d1

ANOVA^a

Model	Sum of Squares	df	Mean Square	F	Sig.
1 Regression	35.703	3	11.901	14.123	.000 ^b
Residual	105.336	125	.843		
Total	141.039	128			

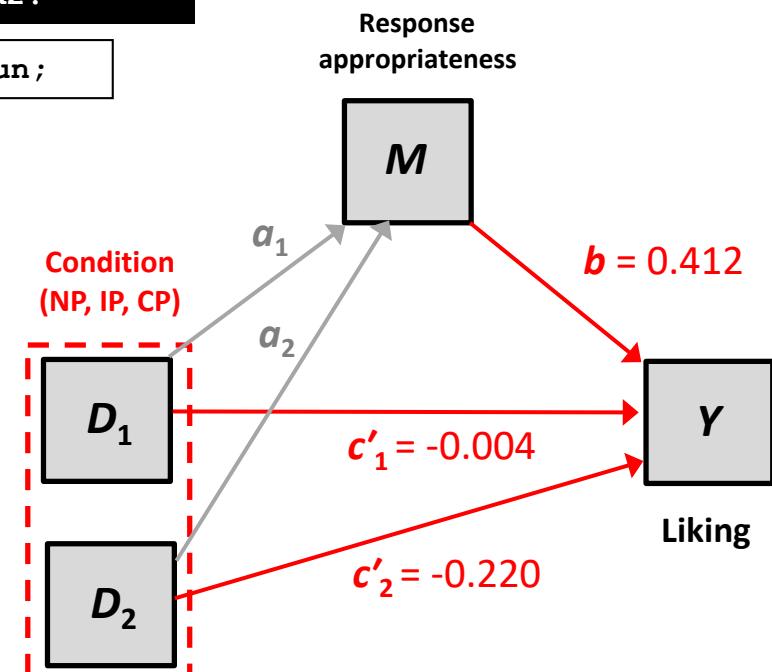
a. Dependent Variable: LIKING: liking of the target

b. Predictors: (Constant), d2, RESPAPPR: appropriateness of response, d1

Coefficients^a

Model	<i>b</i> path	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.
		B	Std. Error			
1	(Constant)	3.710	.307		12.071	.000
	RESPAPPR: appropriateness of response	.412	.070	.529	5.884	.000
	d1	-.004	.219	-.002	-.017	.987
	d2	-.220	.228	-.100	-.966	.336

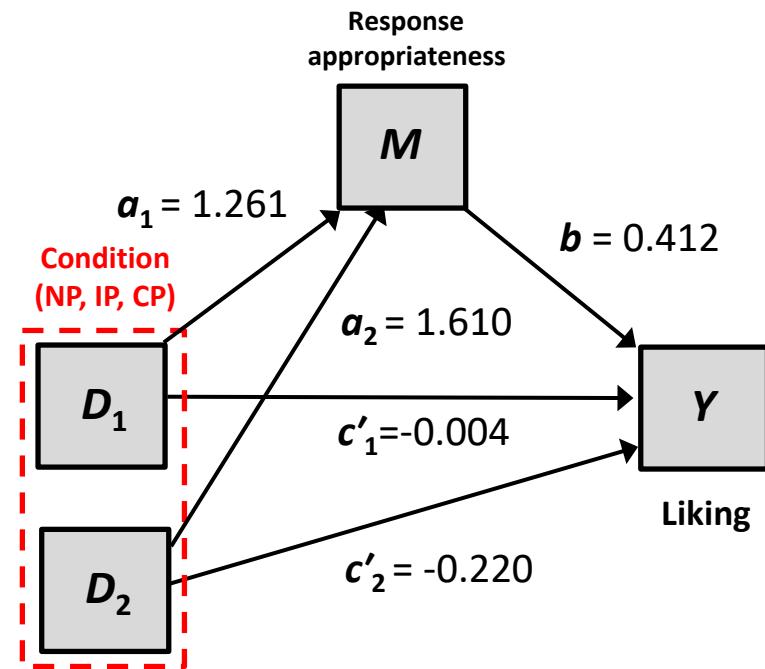
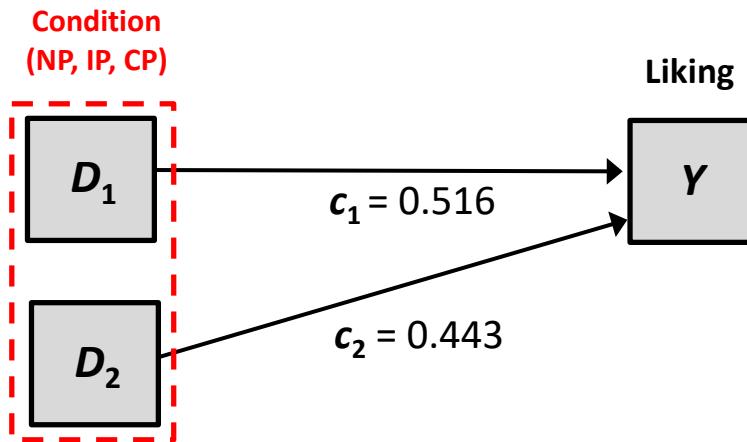
a. Dependent Variable: LIKING: liking of the target



Relative direct effects

Controlling for perceived responses appropriateness, those told she individually protested did not like her any more, on average, than those told she did not protest ($c'_1 = -0.004, p = .987$). And those told she collectively protested did not like her any more, on average, than those told she did not protest ($c'_2 = -0.220, p = .336$). Holding condition constant, those who perceived her behavior as relatively more appropriate likely her relatively more ($b = 0.412$).

(Relative) total, direct, and indirect effects



c_1 and c_2 : Relative total effects of condition on liking ($c_1 = 0.516$, $c_2 = 0.443$).

c'_1 and c'_2 : Relative direct effects of condition on liking ($c'_1 = -0.004$, $c'_2 = -0.220$).

a_1b and a_2b : Relative indirect effects of condition on liking through perceived response appropriateness

$$a_1b = 1.261(0.412) = 0.520, a_2b = 1.610(0.412) = 0.663$$

$$c_1 = c'_1 + a_1b: 0.516 = -0.004 + 1.261(0.412) = -0.004 + 0.520$$

$$c_2 = c'_2 + a_2b: 0.443 = -0.220 + 1.610(0.412) = -0.220 + 0.663$$

The relative total effects partition perfectly into relative direct and relative indirect effects. The relative indirect effects are the relative total effects minus the relative direct effects.

Relative total, direct, and indirect effects

	Perceived Response Appropriateness (M)		Liking (Y)		
	M	SD	Y	SD	\bar{Y}^*
No protest (<i>n</i> = 41)	$a_1 = 1.261$	{ 3.884 5.145 }	1.457 1.075	{ 5.310 5.826 }	1.302 0.819
Individual protest (<i>n</i> = 43)			$c_1 = 0.516$		$c'_1 = -0.004$
Collective protest (<i>n</i> = 45)	$a_2 = 1.610$	{ 5.494 }	0.936	{ 5.753 }	$c_2 = 0.443$
All groups combined	4.866	1.348	5.637	1.050	5.637

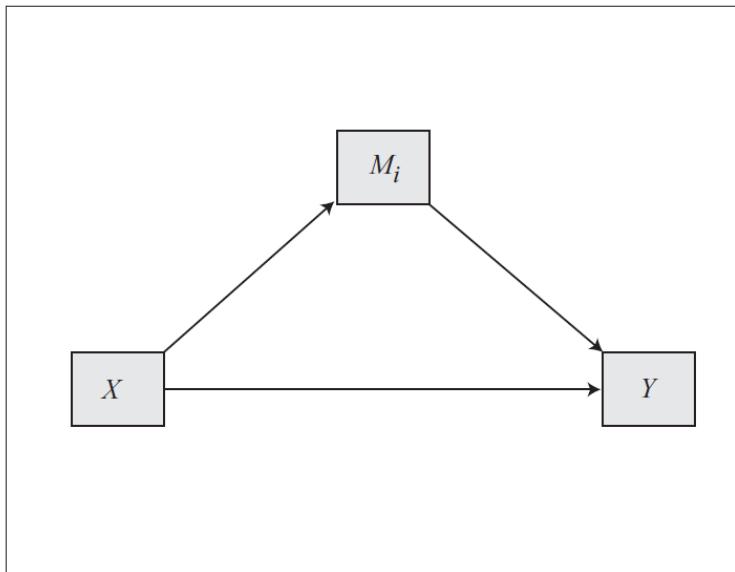
\bar{Y}^* = adjusted mean, adjusted to the sample mean of perceived response appropriateness.

$$c_1 = c'_1 + a_1 b: 0.516 = -0.004 + 1.261(0.412) = -0.004 + 0.520$$

$$c_2 = c'_2 + a_2 b: 0.443 = -0.220 + 1.610(0.412) = -0.220 + 0.663$$

Estimation using PROCESS

Model 4



PROCESS V3 has an option for specifying *X* as a mult categorial variable with up to 9 categories. Four options are available for coding the groups.

MCX=1 tells PROCESS that *X* is a multicategorical variable and to use dummy coding to represent the groups. Other coding options are available. See the PROCESS documentation.

MCX	Coding system
1	Simple dummy coding
2	Sequential ("adjacent categories") coding
3	Helmert coding
4	Effect coding

```
process y=liking/m=respappr/x=cond/mcx=1/model=4/total=1/boot=10000.
```

```
%process (data=protest,y=liking,m=respappr,x=cond,mcx=1,model=4,  
total=1,boot=10000);
```

PROCESS output

Model : 4
 Y : liking
 X : cond
 M : respappr

Sample
 Size: 129

Coding of categorical X variable for analysis:

cond	X1	X2
.000	.000	.000
1.000	1.000	.000
2.000	.000	1.000

X1 codes individual protest, X2 codes collective protest.
 No protest is the reference group. (The group with the numerically smallest value on the categorical variable is always the reference)

OUTCOME VARIABLE:
 respappr

$$\hat{M} = 3.884 + 1.261D_1 + 1.610D_2$$

Model Summary

R	R-sq	MSE	F	df1	df2	P
.5106	.2607	1.3649	22.2190	2.0000	126.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	3.8841	.1825	21.2881	.0000	3.5231	4.2452
X1	1.2612	.2550	4.9456	.0000	.7565	1.7659
X2	1.6103	.2522	6.3842	.0000	1.1111	2.1095

OUTCOME VARIABLE:
 liking

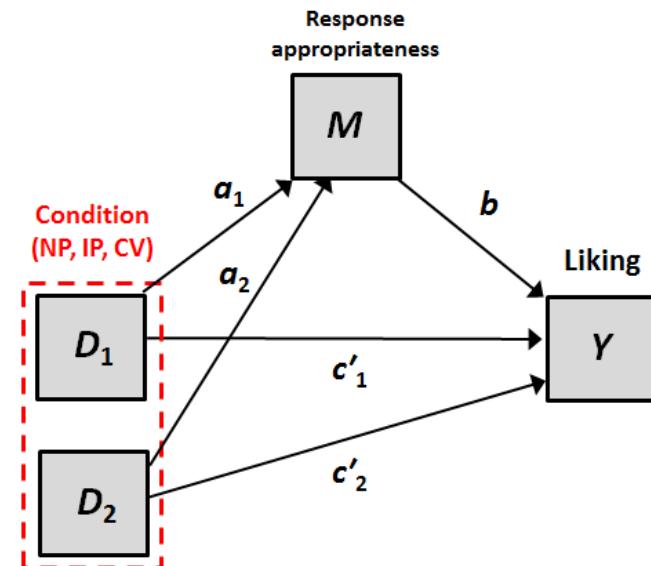
$$\hat{Y} = 3.710 - 0.004D_1 - 0.220D_2 + 0.412M$$

Model Summary

R	R-sq	MSE	F	df1	df2	P
.5031	.2531	.8427	14.1225	3.0000	125.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	3.7103	.3074	12.0711	.0000	3.1020	4.3187
X1	-.0037	.2190	-.0169	.9865	-.4371	.4297
X2	-.2202	.2280	-.9658	.3360	-.6715	.2310
respappr	.4119	.0700	5.8844	.0000	.2734	.5504

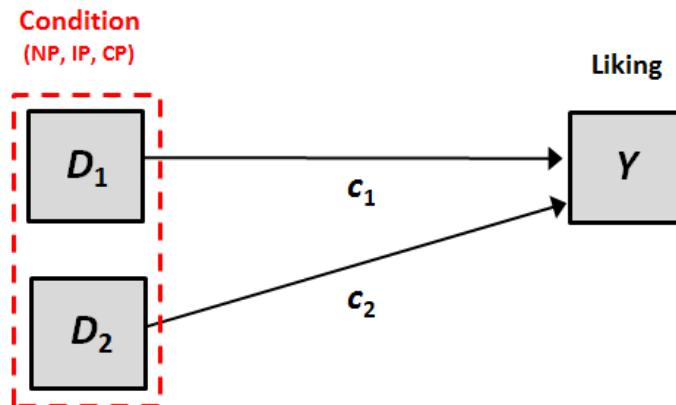


a_1 path
 a_2 path

c'_1 path
 c'_2 path
 b path

Output L

PROCESS output



***** TOTAL EFFECT MODEL *****

OUTCOME VARIABLE:

liking

Model Summary

R	R-sq	MSE	F	df1	df2	p
.2151	.0463	1.0676	3.0552	2.0000	126.0000	.0506

Model

	coeff	se	t	p	LLCI	ULCI
constant	5.3102	.1614	32.9083	.0000	4.9909	5.6296
x1	.5158	.2255	2.2870	.0239	.0695	.9621
x2	.4431	.2231	1.9863	.0492	.0016	.8845

c_1 path
 c_2 path

$$\hat{Y} = 5.310 + 0.516D_1 + 0.443D_2$$

Output M

PROCESS output

***** TOTAL, DIRECT, AND INDIRECT EFFECTS OF X ON Y *****

Relative total effects of X on Y:

	Effect	se	t	p	LLCI	ULCI
X1	.5158	.2255	2.2870	.0239	.0695	.9621
X2	.4431	.2231	1.9863	.0492	.0016	.8845

Omnibus test of total effect of X on Y:

R2-chng	F	df1	df2	p
.0463	3.0552	2.0000	126.0000	.0506

Relative direct effects of X on Y

	Effect	se	t	p	LLCI	ULCI
X1	-.0037	.2190	-.0169	.9865	-.4371	.4297
X2	-.2202	.2280	-.9658	.3360	-.6715	.2310

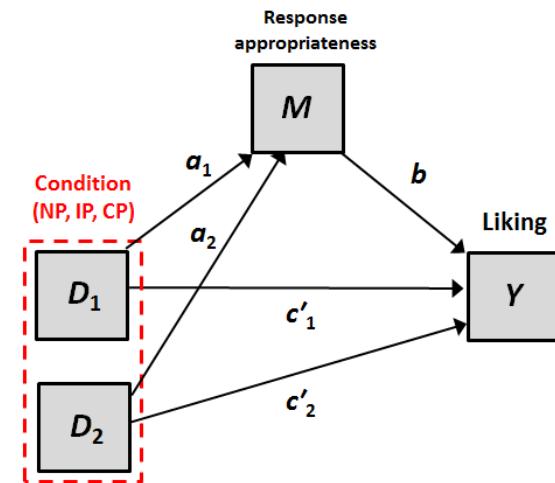
Omnibus test of direct effect of X on Y:

R2-chng	F	df1	df2	p
.0087	.7286	2.0000	125.0000	.4846

Relative indirect effects of X on Y

cond → respappr → liking

Effect	BootSE	BootLLCI	BootULCI
X1	.5195	.1524	.2590
X2	.6633	.1671	.3684



Indirect effect $a_1 b$ with
bootstrap confidence interval

Indirect effect $a_2 b$ with
bootstrap confidence interval

Output M

Those told she individually protested liked her more than those told she did not protest because protesting was perceived as more appropriate than not, which in turn enhanced liking (point estimate = 0.520, 95% CI: 0.259 to 0.854). There is no direct effect of individually protesting on liking. Those told she collectively protested liked her more than those told she did not protest because protesting was perceived as more appropriate than not, which in turn enhanced liking (point estimate= 0.663, 95% CI: 0.368 to 1.019). There is no direct effect of collectively protesting on liking.

Omnibus inference

PROCESS gives us tests of the $k-1$ relative total effects. It also provides a test of equality of the k group means on Y --the “omnibus” total effect. This is equivalent to a single-factor ANOVA.

***** TOTAL, DIRECT, AND INDIRECT EFFECTS OF X ON Y *****

Relative total effects of X on Y:

	Effect	se	t	p	LLCI	ULCI
X1	.5158	.2255	2.2870	.0239	.0695	.9621
X2	.4431	.2231	1.9863	.0492	.0016	.8845

Omnibus test of total effect of X on Y:

R2-chng	F	df1	df2	p
.0463	3.0552	2.0000	126.0000	.0506

Relative direct effects of X on Y

	Effect	se	t	p	LLCI	ULCI
X1	-.0037	.2190	-.0169	.9865	-.4371	.4297
X2	-.2202	.2280	-.9658	.3360	-.6715	.2310

Omnibus test of direct effect of X on Y:

R2-chng	F	df1	df2	p
.0087	.7286	2.0000	125.0000	.4846

Relative indirect effects of X on Y

cond -> respappr -> liking

	Effect	BootSE	BootLLCI	BootULCI
X1	.5195	.1524	.2590	.8536
X2	.6633	.1671	.3684	1.0187

Output M

Test of the “omnibus” total effect.

The three conditions differ on average in liking of the attorney , $F(2,126) = 3.055$, $p = .051$.

Omnibus inference

PROCESS gives us tests of the $k - 1$ relative direct effects. It also provides a test of equality of the k group adjusted means on Y when the mediator is held constant---the “omnibus” direct effect. This is equivalent to a single-factor ANCOVA.

Output M

***** TOTAL, DIRECT, AND INDIRECT EFFECTS OF X ON Y *****

Relative total effects of X on Y:

	Effect	se	t	p	LLCI	ULCI
X1	.5158	.2255	2.2870	.0239	.0695	.9621
X2	.4431	.2231	1.9863	.0492	.0016	.8845

Omnibus test of total effect of X on Y:

R2-chng	F	df1	df2	p
.0463	3.0552	2.0000	126.0000	.0506

Relative direct effects of X on Y

	Effect	se	t	p	LLCI	ULCI
X1	-.0037	.2190	-.0169	.9865	-.4371	.4297
X2	-.2202	.2280	-.9658	.3360	-.6715	.2310

Omnibus test of direct effect of X on Y:

R2-chng	F	df1	df2	p
.0087	.7286	2.0000	125.0000	.4846

Relative indirect effects of X on Y

cond	->	respappr	->	liking
	Effect	BootSE	BootLLCI	BootULCI
X1	.5195	.1524	.2590	.8536
X2	.6633	.1671	.3684	1.0187

Test of the “omnibus” direct effect.
The three conditions do not differ on average in how much they liked her after accounting for group differences in perceived response appropriateness $\Delta R^2 = 0.009$, $F(2,125) = 0.729$, $p = .485$. ΔR^2 is the change in R^2 when the $k - 1$ variables coding group are added to the model of Y that already contains the mediator.

Omnibus inference about the indirect effect

- The omnibus tests for the total and direct effect of X are not dependent on the system used for coding the groups, even though the relative direct and total effects are.
- The rule that X indirectly affects Y if at least one relative indirect effect is different from zero means our conclusion will depend on the system used for coding groups, since the relative indirect effects are dependent on that choice.
- If all of the bootstrap confidence intervals for the relative indirect effects straddle zero, that does NOT mean X does not indirectly affect Y . It could be that a different coding choice produces a different outcome.
- The rule can confirm that X indirectly affects Y if at least one relative indirect effect is different from zero. But a failure to meet this criterion does not disconfirm the existence of an indirect effect of X on Y through M .
- Moral: Choose your coding system wisely, so that it produces relative indirect effects you care about and that are sensitive to the question you are trying to answer.
- There are omnibus tests of the indirect effect that are not sensitive to the coding choice. This must be done in SEM and can require problematic assumptions.

A different coding system

Other systems for coding groups can be used. For instance, we might instead choose to estimate the direct and indirect effects of protesting (regardless of form) relative to not, and the effects of collectively protesting relative to individually protesting.

	Condition	D_1	D_2
cond = 0	No protest	-2/3	0
cond = 1	Individual	1/3	-1/2
cond = 2	Collective	1/3	1/2

You may recognize these as two *orthogonal contrasts*. It is also called “Helmert coding” when the mult categorial variable is ordinal.

```
if (cond = 0) d1 = -2/3.  
if (cond > 0) d1 = 1/3.  
if (cond = 0) d2 = 0.  
if (cond = 1) d2 = -1/2.  
if (cond = 2) d2 = 1/2.
```

```
data protest;set protest;  
  if (cond = 0) then d1 = -2/3;  
  if (cond > 0) then d1 = 1/3;  
  if (cond = 0) then d2 = 0;  
  if (cond = 1) then d2 = -1/2;  
  if (cond = 2) then d2 = 1/2;  
run;
```

Effects for D_1 will compare no protest to the average of the two protest conditions, and effects for D_2 will compare collective protest to individual protest.

The total effect of experimental condition on liking (c paths)

```
regression/dep = liking/method = enter d1 d2.
```

```
proc reg data=protest;model liking=d1 d2;run;
```

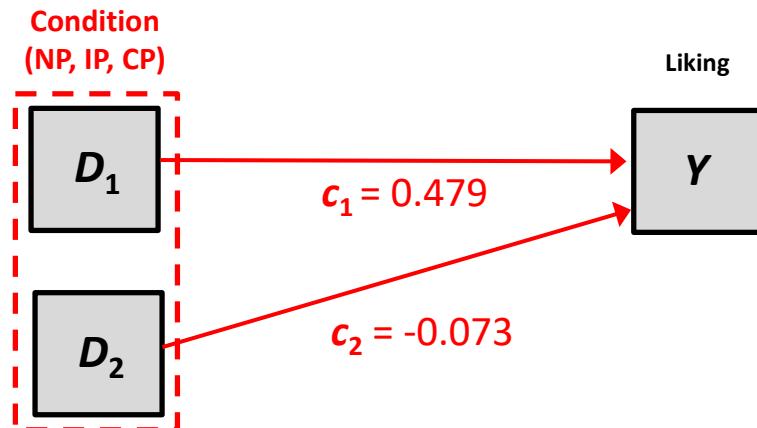
Model	Coefficients ^a					
	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	
	B	Std. Error	Beta			
1 (Constant)	5.630	.091		61.841	.000	
d1	.479	.195	.214	2.454	.016	
d2	-.073	.220	-.029	-.330	.742	

a. Dependent Variable: LIKING: liking of the target

Relative total effects

$$c_1 = \frac{(\bar{Y}_{IP} + \bar{Y}_{CP})}{2} - \bar{Y}_{NP} = \frac{(5.826 + 5.753)}{2} - 5.310 = 5.789 - 5.310 = 0.479$$

$$c_2 = \bar{Y}_{CP} - \bar{Y}_{IP} = 5.753 - 5.826 = -0.073$$



Relative to those told she did not protest, those told she protested liked her more on average ($c_1 = 0.479$, $p = .016$). Those told she collectively protested did not differ, on average, in how much they liked her relative to those told she individually protested ($c_2 = -0.073$, $p = .742$).

The effect of experimental condition on perceived response appropriateness (a paths)

regression/dep = respappr/method = enter d1 d2.

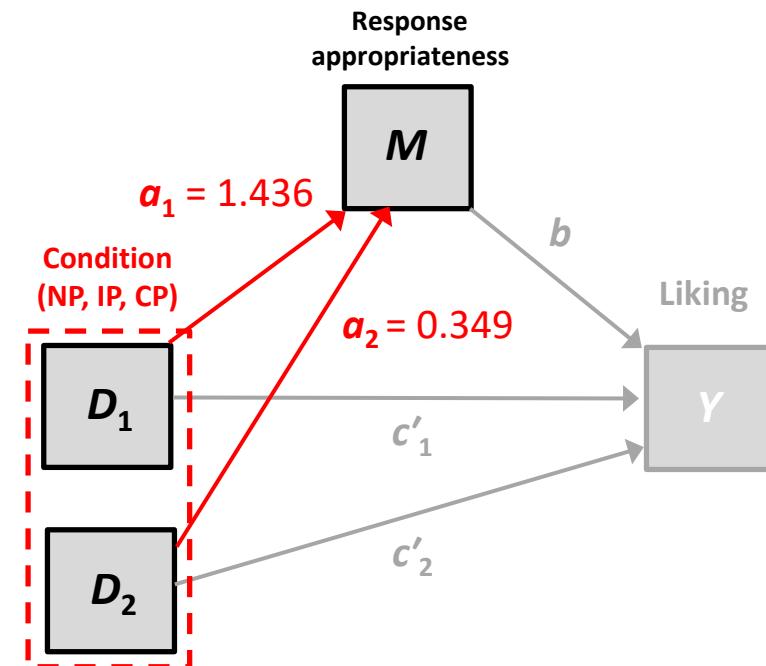
```
proc reg data=protest;model respappr=d1 d2;run;
```

Model	Coefficients ^a			t	Sig.
	B	Unstandardized Coefficients	Standardized Coefficients		
1 (Constant)	4.841	.103		47.032	.000
d1	1.436	.221	.498	6.499	.000
d2	.349	.249	.107	1.401	.164

a. Dependent Variable: RESPAPPR: appropriateness of response

$$a_1 = \frac{(\bar{M}_{IP} + \bar{M}_{CP})}{2} - \bar{M}_{NP} = \frac{(5.145 + 5.494)}{2} - 3.884 = 5.320 - 3.884 = 1.436$$

$$a_2 = \bar{M}_{CP} - \bar{M}_{IP} = 5.494 - 5.145 = 0.349$$



Relative to those told she did not protest, those told she protested felt this was a more appropriate response, on average ($a_1 = 1.436, p = .016$). Those told she collectively protested did not perceive this as any more or less appropriate, on average, relative to those told she individually protested ($a_2 = 0.349, p = .164$).

The direct effect of condition on liking (c' paths)

along with the effect of response appropriateness on liking (b path)

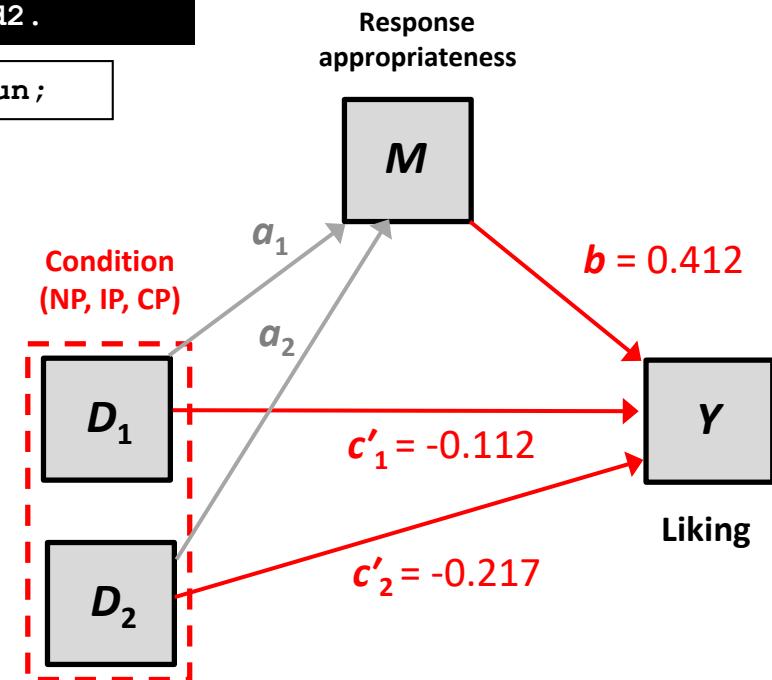
```
regression/dep = liking/method = enter respappr d1 d2.
```

```
proc reg data=protest;model liking=respappr d1 d2;run;
```

Model	<i>b</i> path	Coefficients ^a				
		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	3.636	.348	.529	10.435	.000
	RESPAPPR: appropriateness of response	.412	.070	.529	5.884	.000
	d1	-.112	.201	-.050	-.558	.578
	d2	-.217	.197	-.085	-1.097	.275

a. Dependent Variable: LIKING: liking of the target

Relative direct effects

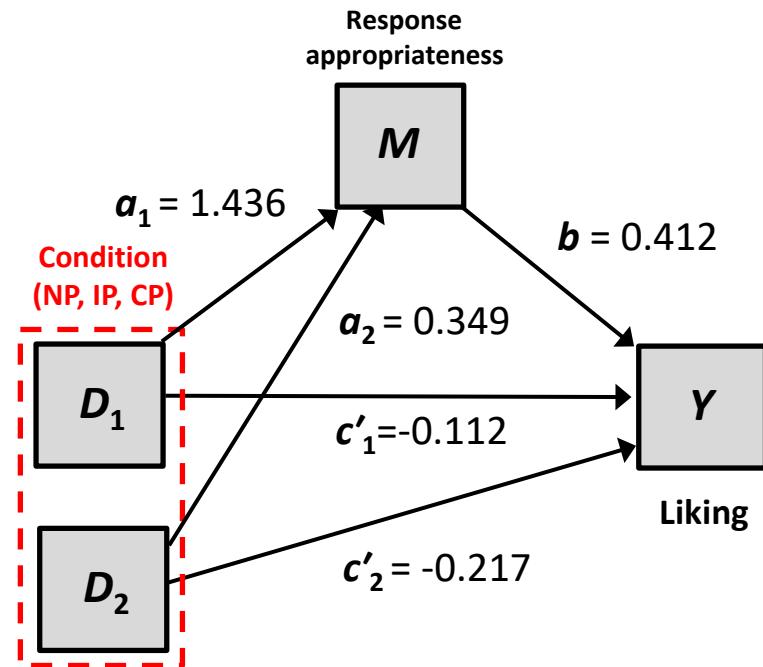
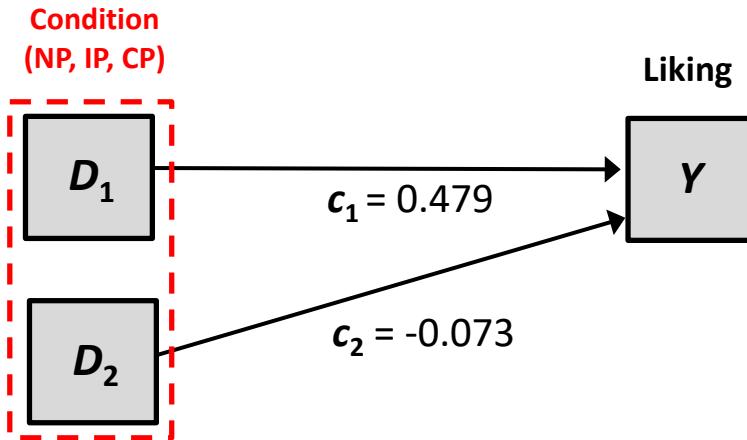


$$c'_1 = \frac{(\bar{Y}_{IP}^* + \bar{Y}_{CP}^*)}{2} - \bar{Y}_{NP}^* = \frac{(5.711 + 5.495)}{2} - 5.715 = 5.603 - 5.715 = -0.112$$

$$c'_2 = \bar{Y}_{CP}^* - \bar{Y}_{IP}^* = 5.495 - 5.711 = -0.217$$

Controlling for perceived responses appropriateness, on average, those told she protested did not like her any more than those told she did not protest ($c'_1 = -0.112$, $p = .578$). And those told she collectively protested did not like her any more, on average, than those told she individually protested ($c'_2 = -0.217$, $p = .275$). Holding condition constant, those who perceived her behavior as relatively more appropriate liked her relatively more ($b = 0.412$).

(Relative) total, direct, and indirect effects



c_1 and c_2 : Relative total effects of condition on liking ($c_1 = 0.479$, $c_2 = -0.073$).

c'_1 and c'_2 : Relative direct effects of condition on liking ($c'_1 = -0.112$, $c'_2 = -0.217$).

a_1b and a_2b : Relative indirect effects of condition on liking through perceived response appropriateness

$$a_1b = 1.436(0.412) = 0.591, a_2b = 0.349(0.412) = 0.144$$

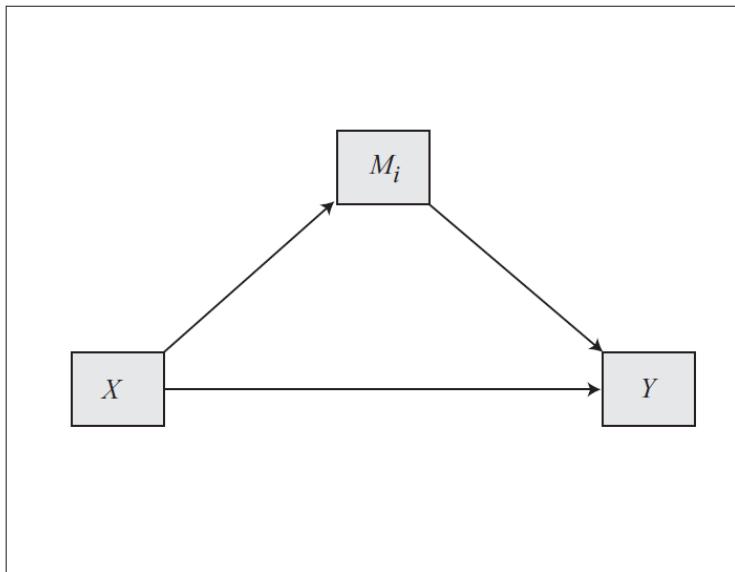
$$c_1 = c'_1 + a_1b: 0.479 = -0.112 + 1.436(0.412) = -0.112 + 0.591$$

$$c_2 = c'_2 + a_2b: -0.073 = -0.217 + 0.349(0.412) = -0.217 + 0.144$$

The relative total effects partition perfectly into relative direct and relative indirect effects. The relative indirect effects are the relative total effects minus the relative direct effects.

Estimation using PROCESS

Model 4



PROCESS has an option for specifying X as a mult categorial variable with up to 9 categories. Four options are available for coding the groups.

MCX=3 tells PROCESS that X is a multicategorical variable and to use “Helmert coding” to represent the groups. This is equivalent to the orthogonal contrasts we set up manually in this example.

MCX	Coding system
1	Simple dummy coding
2	Sequential (“adjacent categories”) coding
3	Helmert coding
4	Effect coding

```
process y=liking/m=respappr/x=cond/mcx=3/model=4/total=1/boot=10000 .
```

```
%process (data=protest,y=liking,m=respappr,x=cond,mcx=3,model=4,  
total=1,boot=10000) ;
```

PROCESS output

Model : 4
 Y : liking
 X : cond
 M : respappr

Sample
 Size: 129

Coding of categorical X variable for analysis:

cond	X1	X2
.000	-.667	.000
1.000	.333	-.500
2.000	.333	.500

X1 codes protest versus no protest. X2 codes collective versus individual protest.

 OUTCOME VARIABLE:
 respappr

$$\hat{M} = 4.841 + 1.436D_1 + 0.349D_2$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.5106	.2607	1.3649	22.2190	2.0000	126.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	4.8413	.1029	47.0321	.0000	4.6376	5.0450
x1	1.4358	.2209	6.4988	.0000	.9985	1.8730
x2	.3491	.2491	1.4012	.1636	-.1440	.8421

 OUTCOME VARIABLE:
 liking

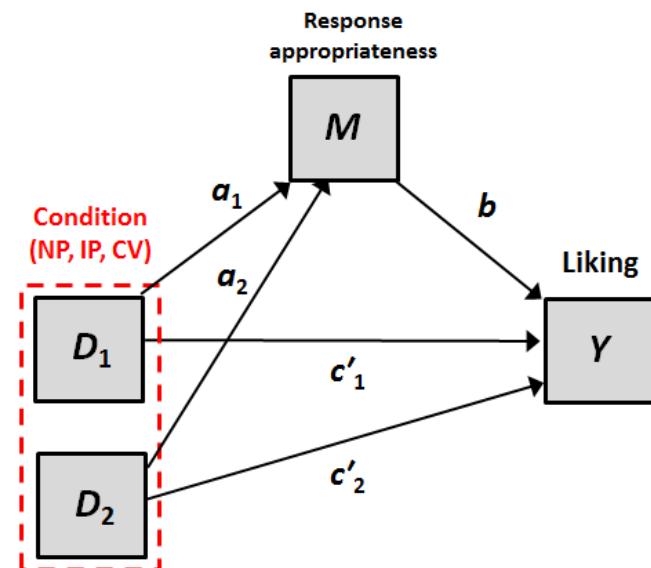
$$\hat{Y} = 3.636 - 0.112D_1 - 0.217D_2 + 0.412M$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.5031	.2531	.8427	14.1225	3.0000	125.0000	.0000

Model

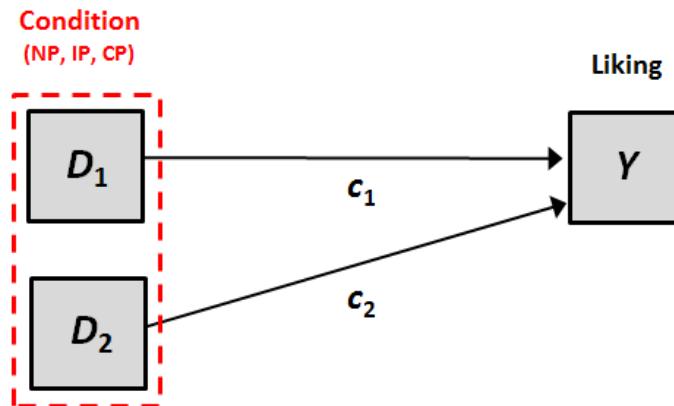
	coeff	se	t	p	LLCI	ULCI
constant	3.6357	.3484	10.4351	.0000	2.9461	4.3252
x1	-.1120	.2006	-.5581	.5778	-.5089	.2850
x2	-.2165	.1973	-1.0974	.2746	-.6070	.1739
respappr	.4119	.0700	5.8844	.0000	.2734	.5504



← a₁ path
 ← a₂ path

← c'₁ path
 ← c'₂ path
 ← b path

PROCESS output



***** TOTAL EFFECT MODEL *****

OUTCOME VARIABLE:

liking

Model Summary

R	R-sq	MSE	F	df1	df2	p
.2151	.0463	1.0676	3.0552	2.0000	126.0000	.0506

Model

	coeff	se	t	p	LLCI	ULCI
constant	5.6299	.0910	61.8414	.0000	5.4497	5.8100
x1	.4794	.1954	2.4538	.0155	.0928	.8661
x2	-.0727	.2203	-.3300	.7419	-.5088	.3633

c_1 path

c_2 path

$$\hat{Y} = 5.630 + 0.479D_1 - 0.073D_2$$

PROCESS output

***** TOTAL, DIRECT, AND INDIRECT EFFECTS OF X ON Y *****

Relative total effects of X on Y:

	Effect	se	t	p	LLCI	ULCI
X1	.4794	.1954	2.4538	.0155	.0928	.8661
X2	-.0727	.2203	-.3300	.7419	-.5088	.3633

Omnibus test of total effect of X on Y:

R2-chng	F	df1	df2	p
.0463	3.0552	2.0000	126.0000	.0506

Relative direct effects of X on Y

	Effect	se	t	p	LLCI	ULCI
X1	-.1120	.2006	-.5581	.5778	-.5089	.2850
X2	-.2165	.1973	-1.0974	.2746	-.6070	.1739

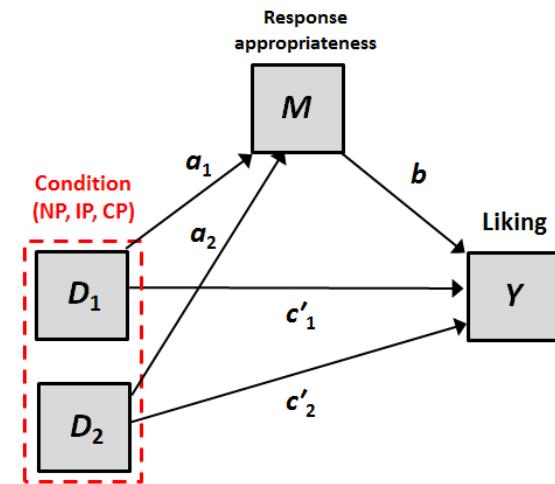
Omnibus test of direct effect of X on Y:

R2-chng	F	df1	df2	p
.0087	.7286	2.0000	125.0000	.4846

Relative indirect effects of X on Y

cond → respappr → liking

Effect	BootSE	BootLLCI	BootULCI
X1	.5914	.1529	.3273
X2	.1438	.0937	-.0261



Indirect effect $a_1 b$ with bootstrap confidence interval

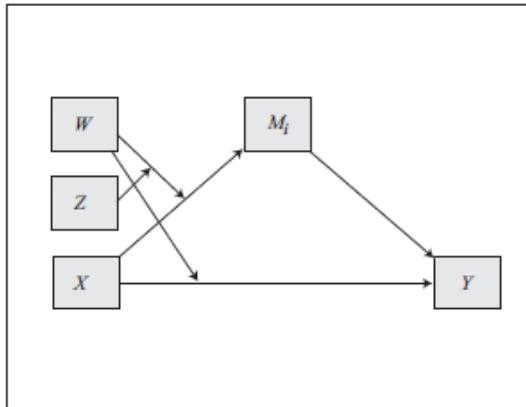
Indirect effect $a_2 b$ with bootstrap confidence interval

Those told she protested liked her more than those told she did not protest because protesting was perceived as more appropriate than not, which in turn enhanced liking (relative indirect effect = 0.591, 95% CI: 0.327 to 0.922). There was no direct effect of protesting relative to not on liking (relative direct effect = -0.112, $p = .578$). The relative direct and indirect effects of collectively protesting relative to individually protesting were not significantly different from zero (relative indirect effect = 0.144, 95% CI = -0.026 to 0.345; relative direct effect = -0.217, $p = .274$)

Constructing and editing models in PROCESS

Historically, PROCESS has operated by a model number system. The model numbers and the models those numbers represent can be found in the documentation. Choose the model number that corresponds to the model you would like to estimate.

Model 13



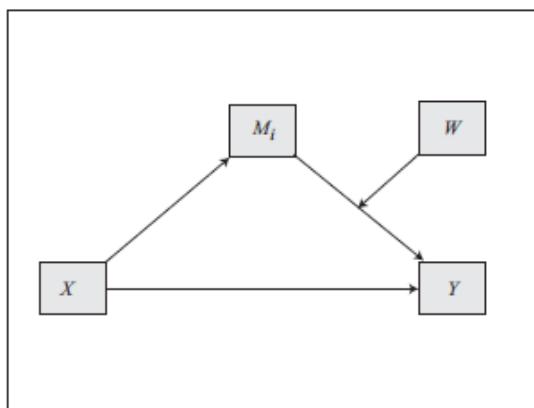
Many of the preprogrammed numbered models you will find useful.

But what if the model you want to estimate does not correspond to any preprogrammed model represented by a model number?

Version 2: Too bad. Nothing you can do about it (unless you know some tricks).

Version 3: Within certain constraints, you can create your own model from scratch, or edit an existing model number to make it correspond to the model you want to estimate.

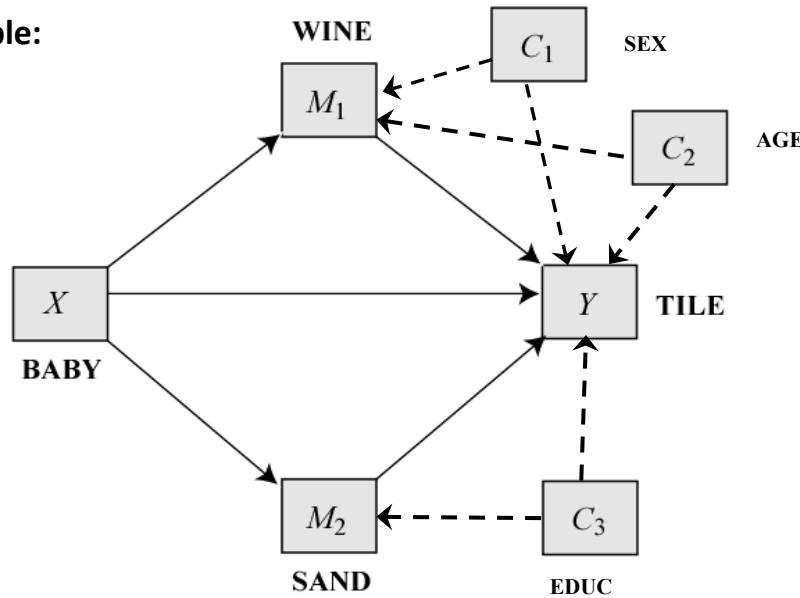
Model 14



Covariates are assignable to equations now

- Previous versions of PROCESS offer little flexibility in how covariates are assigned to equations. All covariates go in models of Y and mediator(s) M , or just M , or just Y . You can't split covariates up and assign them to different equations.
- With a new **cmatrix** option in version 3, covariates can now be assigned to different equations in whatever configuration you desire, rather than being forced to all be in the models of M s, Y , or both.

For example:



```
process y=tile/m=wine sand/x=baby/cov=sex age educ/model=4/  
cmatrix=1,1,0,0,0,1,1,1,1.
```

Customizing the assignment of covariates

By default, all covariates are assigned to all equations, but this can be overridden with the **cmatrix** option. The assignment of covariates to equations is internally represented with the *C* matrix.

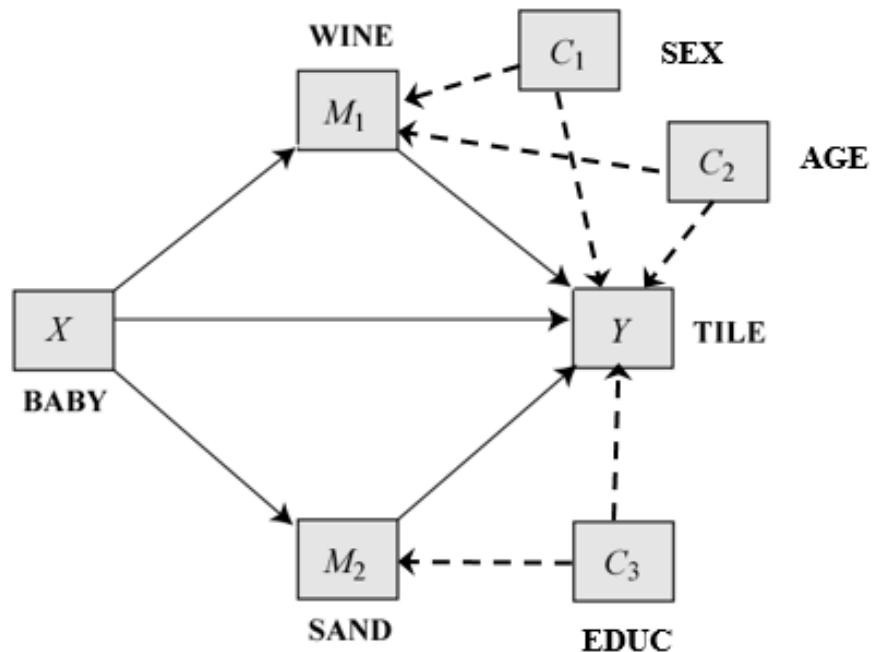
PROCESS y=yvar/x=xvar/m=med1 med2 ... medk/cov=cov1 cov2 ... covk/model= ...

	cov1	cov2	...	covk
med1	0/1	0/1	...	0/1
med2	0/1	0/1	...	0/1
.
.
.
medk	0/1	0/1	...	0/1
yvar	0/1	0/1	...	0/1

C matrix

A one in the cell means the covariate in that column is to be included in the model of the variable in that row. A zero means the covariate in that column is to be excluded from the model of the variable in that row. By default, all entries in the *C* matrix are set to one.

Customizing the assignment of covariates

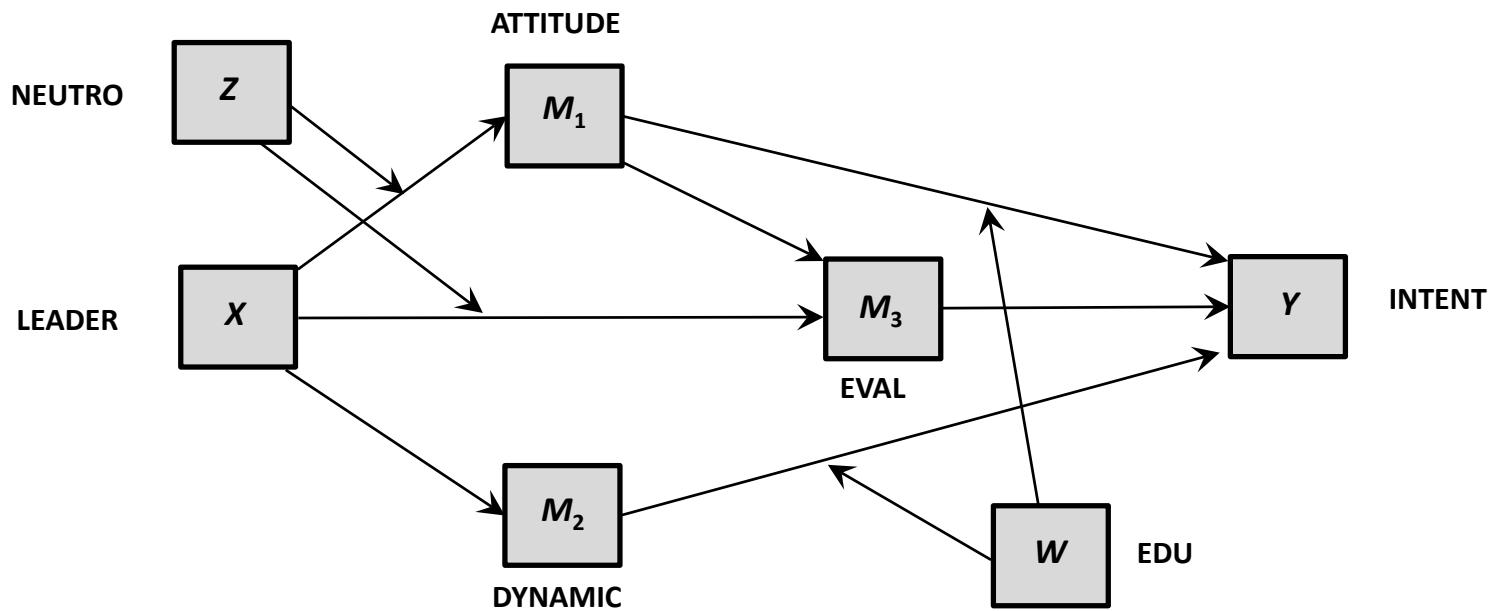


C matrix			
	sex	age	educ
wine	1 →	1 → 0	
sand	0 →	0 → 1	
tile	1 →	1 → 1	

The **cmatrix** option does the assignment of covariates to equations. Read the matrix left to right, top to bottom, assigning the zeros and ones. Separate by commas in SPSS.

```
process y=tile/x=baby/m=wine sand/cov=sex age educ  
/model=6 /cmatrix=1,1,0,0,0,1,1,1,1.
```

A complex model



Such a complex model is not preprogrammed into PROCESS as a model number. In version 2, this model could not be estimated. In less than one hour, you will know how to program this model in PROCESS v3.

The *B* matrix

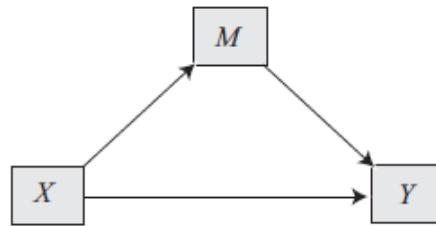
The *B* matrix is the heart of the representation of a model in PROCESS. It is a matrix of 0s and 1s specifying whether (1) or not (0) the variable in the column sends an effect to the variable in the row.

		Variables sending effects (i.e., arrow points away)				
		X	M ₁	M ₂	...	M _k
Variables receiving effects (an arrow points at)	M ₁	0/1	■	■	■	■
	M ₂	0/1	0/1	■	■	■
	...	0/1	0/1	0/1	■	■
	M _k	0/1	0/1	0/1	0/1	■
	Y	0/1	0/1	0/1	0/1	0/1

A model you program can contain one *X*, one *Y*, and up to 6 mediators. Certain cells are fixed to zero (the black squares above) to ensure the model is recursive (no feedback loops; PROCESS cannot estimate nonrecursive models).

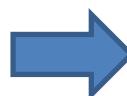
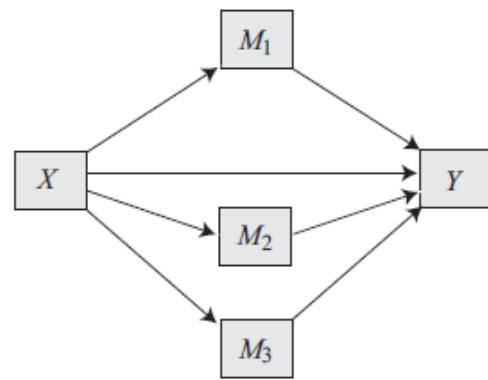
Examples

Model in conceptual form

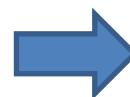
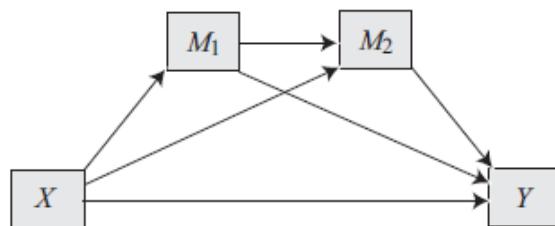


Model in B matrix form

	X	M
M	1	■
Y	1	1



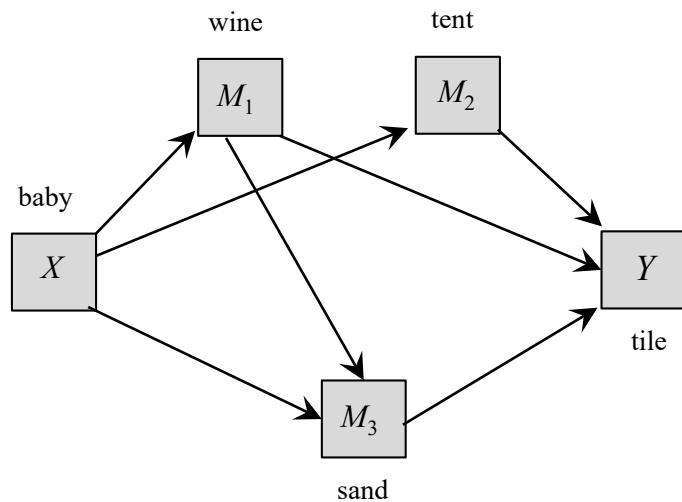
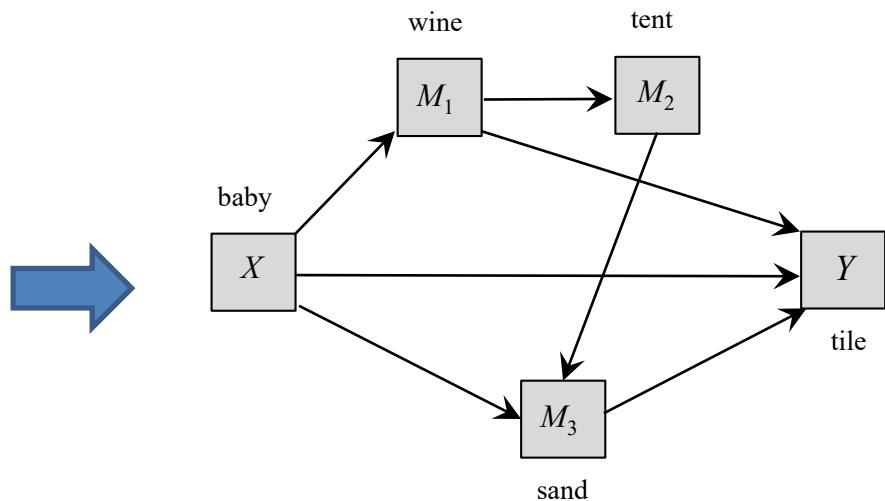
	X	M_1	M_2	M_3
M_1	1	■	■	■
M_2	1	0	■	■
M_3	1	0	0	■
Y	1	1	1	1



	X	M_1	M_2
M_1	1	■	■
M_2	1	1	■
Y	1	1	1

Examples

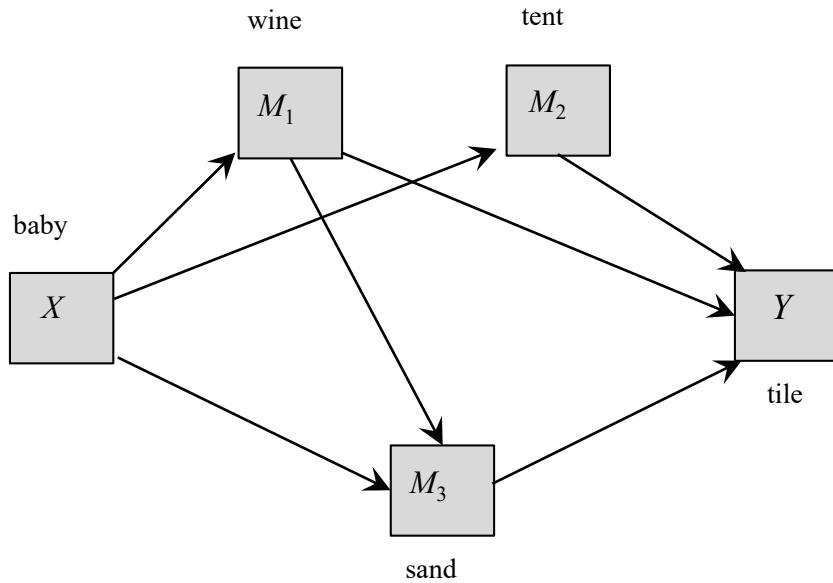
	X	M_1	M_2	M_3	
	baby	wine	tent	sand	
M_1	wine	1	■	■	■
M_2	tent	0	1	■	■
M_3	sand	1	0	1	■
Y	tile	1	1	0	1



	X	M_1	M_2	M_3	
	baby	wine	tent	sand	
M_1	wine	1	■	■	■
M_2	tent	1	0	■	■
M_3	sand	1	1	0	■
Y	tile	0	1	1	1

Programming the *B* matrix

The mediation component of a model is programmed using the **bmatrix=** statement followed by a sequence of zeros and ones.



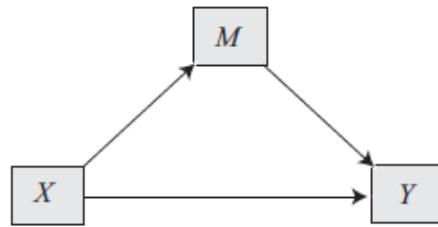
	<i>X</i>	<i>M₁</i>	<i>M₂</i>	<i>M₃</i>	
<i>M₁</i>	wine	1	■	■	■
<i>M₂</i>	tent	1	0	■	■
<i>M₃</i>	sand	1	1	0	■
<i>Y</i>	tile	0	1	1	1

Read the *B* matrix from left to right, top to bottom, skipping the black squares, and enter the zeros and ones in the sequence as they are encountered in the *B* matrix. Separate with commas in SPSS, but not in SAS.

```
process y=tile/x=baby/m=wine tent  
sand/bmatrix=1,1,0,1,1,0,0,1,1,1.
```

Examples

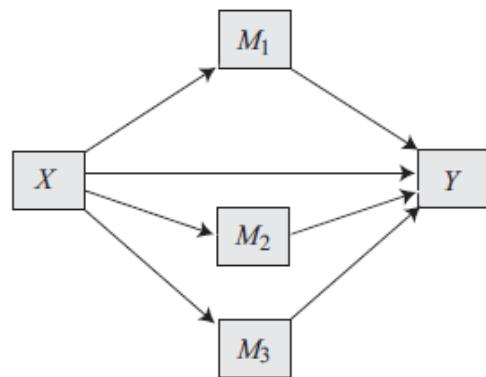
Model in conceptual form



Model in *B* matrix form

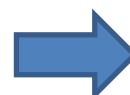
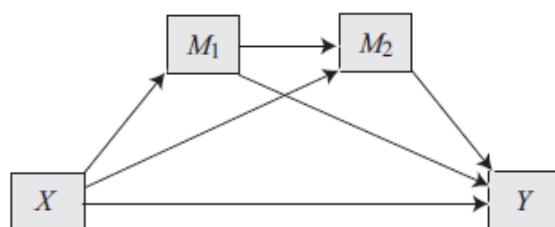
	X	M
M	1	■
Y	1	1

bmatrix=1,1,1



	X	<i>M</i> ₁	<i>M</i> ₂	<i>M</i> ₃
<i>M</i> ₁	1	■	■	■
<i>M</i> ₂	1	0	■	■
<i>M</i> ₃	1	0	0	■
Y	1	1	1	1

bmatrix=1,1,0,1,0,0,1,1,1,1



	X	<i>M</i> ₁	<i>M</i> ₂
<i>M</i> ₁	1	■	■
<i>M</i> ₂	1	1	■
Y	1	1	1

bmatrix=1,1,1,1,1,1

Moderation: The W and Z matrices

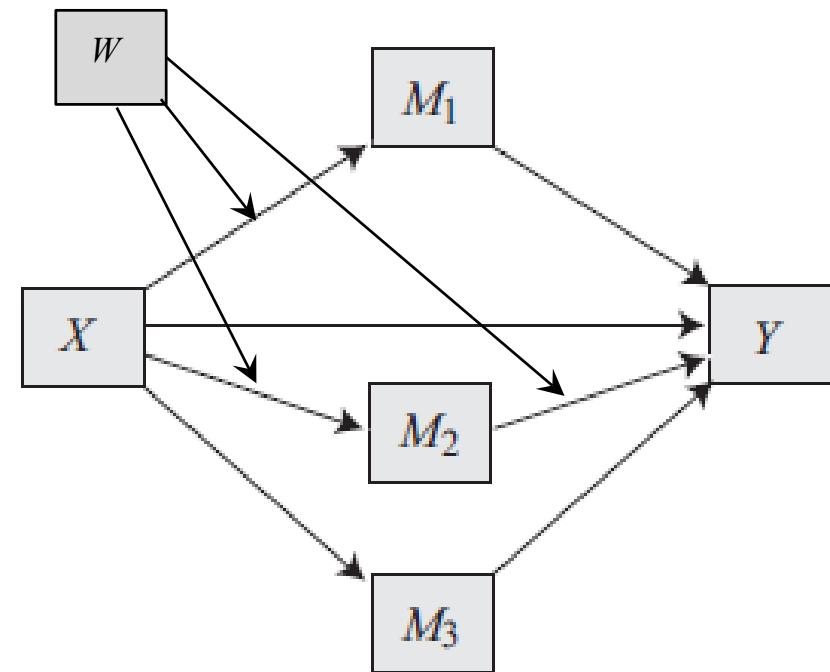
The W and Z matrices define which paths specified in the B matrix are moderated (1) and (0) not moderated. These matrices have the same form and size as the B matrix.

B matrix

	X	M_1	M_2	M_3
M_1	1	■	■	■
M_2	1	0	■	■
M_3	1	0	0	■
Y	1	1	1	1

W matrix

	X	M_1	M_2	M_3
M_1	1	■	■	■
M_2	1	0	■	■
M_3	0	0	0	■
Y	0	0	1	0



Note: A path fixed at zero in the B matrix cannot be moderated.

Moderation: The W and Z matrices

A second moderator Z can be specified. But Z can only be used if W has already been used. That is, if your model has only one moderator, it must be called W .

B matrix

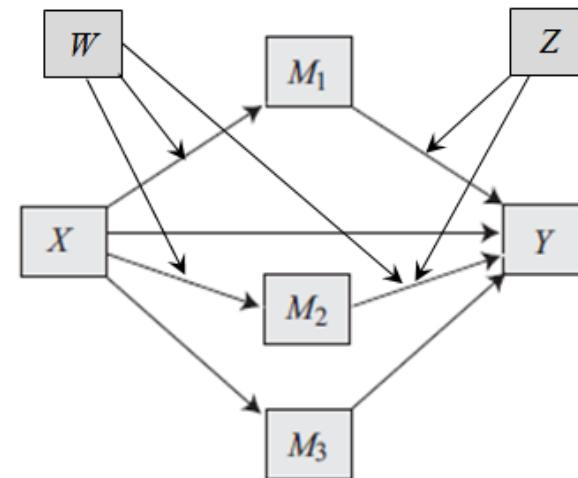
	X	M_1	M_2	M_3
M_1	1	■	■	■
M_2	1	0	■	■
M_3	1	0	0	■
Y	1	1	1	1

W matrix

	X	M_1	M_2	M_3
M_1	1	■	■	■
M_2	1	0	■	■
M_3	0	0	0	■
Y	0	0	1	0

Z matrix

	X	M_1	M_2	M_3
M_1	0	■	■	■
M_2	0	0	■	■
M_3	0	0	0	■
Y	0	1	1	0



Programming the W and Z matrices

B matrix

	X	M_1	M_2	M_3
M_1	1	■	■	■
M_2	1	0	■	■
M_3	1	0	0	■
Y	1	1	1	1

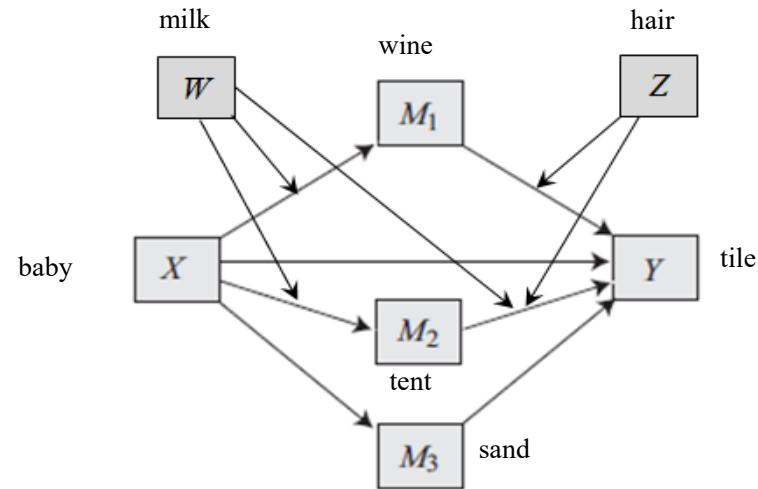
W matrix

	X	M_1	M_2	M_3
M_1	1	■	■	■
M_2	1	0	■	■
M_3	0	0	0	■
Y	0	0	1	0

Z matrix

	X	M_1	M_2	M_3
M_1	0	■	■	■
M_2	0	0	■	■
M_3	0	0	0	■
Y	0	1	1	0

The W and Z matrices are programmed with the **wmatrix=** and **zmatrix=** statements. Use the same procedure as for the B matrix.



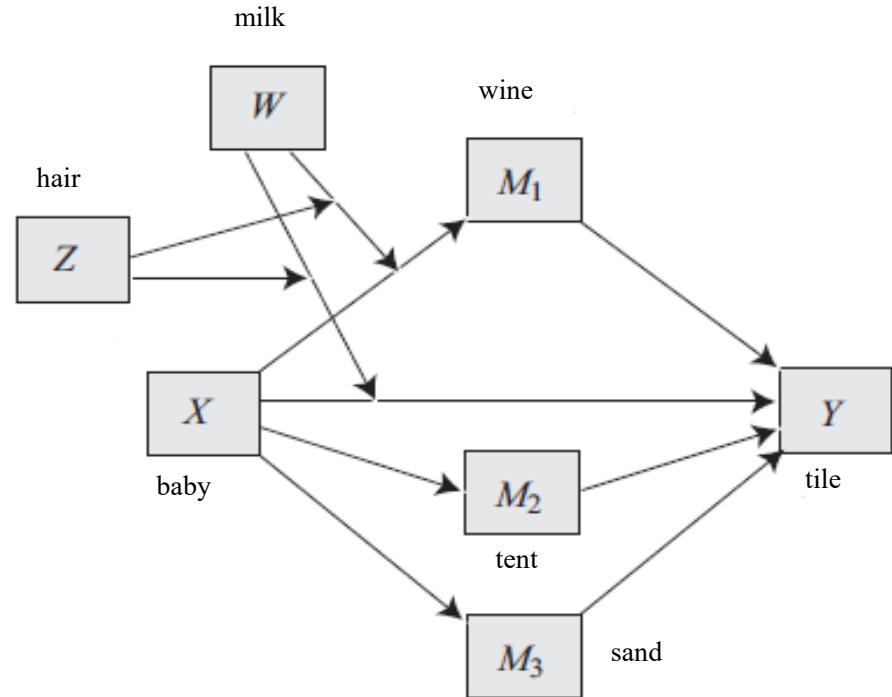
```
process y=tile/x=baby/m=wine tent sand/w=milk/z=hair  
/bmatrix=1,1,0,1,0,0,1,1,1/wmatrix=1,1,0,0,0,0,0,1,0  
/zmatrix=0,0,0,0,0,0,1,1,0.
```

Moderated moderation: The WZ matrix

Moderated moderation, also known as three-way interaction, is held in the *WZ* matrix. *W* is the primary moderator, and *Z* is the secondary moderator. Program using the **wzmatrix=** statement, using the same 0/1 system.

***WZ* matrix**

	<i>X</i>	<i>M</i> ₁	<i>M</i> ₂	<i>M</i> ₃
<i>M</i> ₁	1	■	■	■
<i>M</i> ₂	0	0	■	■
<i>M</i> ₃	0	0	0	■
<i>Y</i>	1	0	0	0



```
process y=tile/x=baby/m=wine tent sand/w=milk/z=hair/  
bmatrix=1,1,0,1,0,0,1,1,1/wzmatrix=1,0,0,0,0,1,0,0,0.
```

Note: if you don't use **wmatrix** or **zmatrix**, this implies all cells are zero

Putting it all together

This system allows for the construction of some very complex models.

B matrix

	<i>X</i>	<i>M</i> ₁	<i>M</i> ₂	<i>M</i> ₃
<i>M</i> ₁	1	■	■	■
<i>M</i> ₂	1	0	■	■
<i>M</i> ₃	1	0	0	■
<i>Y</i>	1	1	1	1

W matrix

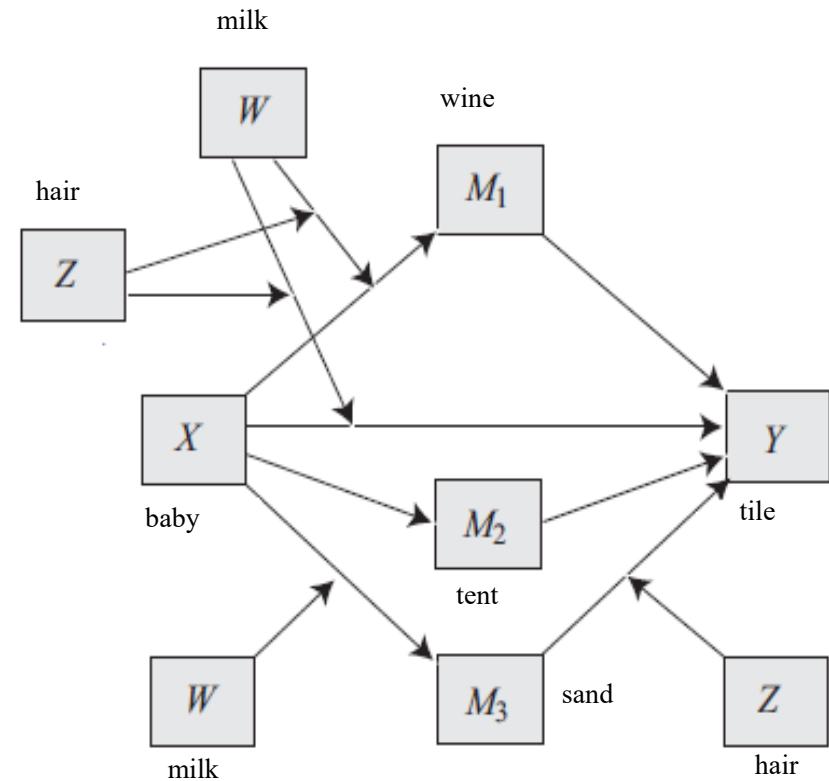
	<i>X</i>	<i>M</i> ₁	<i>M</i> ₂	<i>M</i> ₃
<i>M</i> ₁	0	■	■	■
<i>M</i> ₂	0	0	■	■
<i>M</i> ₃	1	0	0	■
<i>Y</i>	0	0	0	0

Z matrix

	<i>X</i>	<i>M</i> ₁	<i>M</i> ₂	<i>M</i> ₃
<i>M</i> ₁	0	■	■	■
<i>M</i> ₂	0	0	■	■
<i>M</i> ₃	0	0	0	■
<i>Y</i>	0	0	0	1

WZ matrix

	<i>X</i>	<i>M</i> ₁	<i>M</i> ₂	<i>M</i> ₃
<i>M</i> ₁	1	■	■	■
<i>M</i> ₂	0	0	■	■
<i>M</i> ₃	0	0	0	■
<i>Y</i>	1	0	0	0



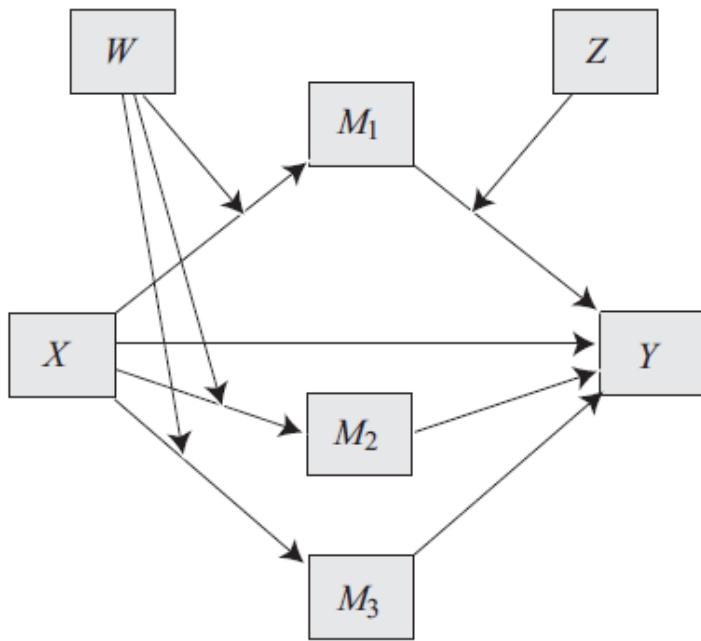
```

process y=tile/x=baby/m=wine tent sand/w=milk/z=hair
/bmatrix=1,1,0,1,0,0,1,1,1,1/wmatrix=0,0,0,1,0,0,0,0,0,0
/zmatrix=0,0,0,0,0,0,0,0,1/wzmatrix=1,0,0,0,0,1,0,0,0.
  
```

Editing a numbered model

Many preprogrammed numbered models are likely to be close to the model you want to estimate. You can edit a modeled number, adding a desired interaction, or removing one you don't want. This is done by reprogramming the W , Z , and/or WZ matrices.

Example

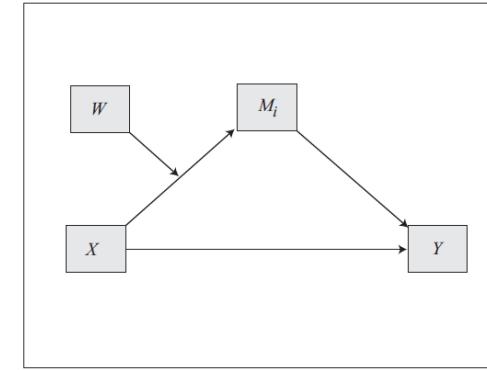


NOTE: When using a model number, the B matrix cannot be edited.

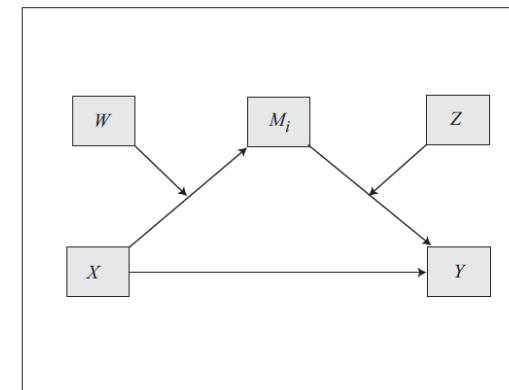
This is like model 7, except model 7 doesn't include moderation of any M to Y paths.

This is like model 21, except model 21 would include moderation by Z of the M_2 and M_3 to Y paths as well as the M_1 to Y path.

Model 7

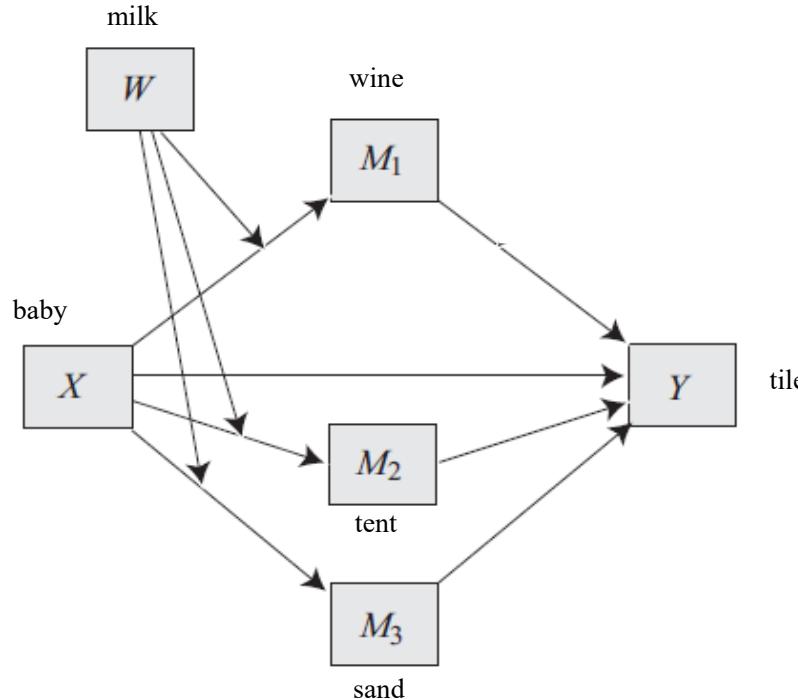


Model 21



Option 1: Edit model 7

In model 7, the Z matrix is all zeros because there is no Z in model 7. So program the Z matrix to include the moderation of the desired path by Z.



Preprogrammed Z matrix

	X	M ₁	M ₂	M ₃
M ₁	0	■	■	■
M ₂	0	0	■	■
M ₃	0	0	0	■
Y	0	0	0	0

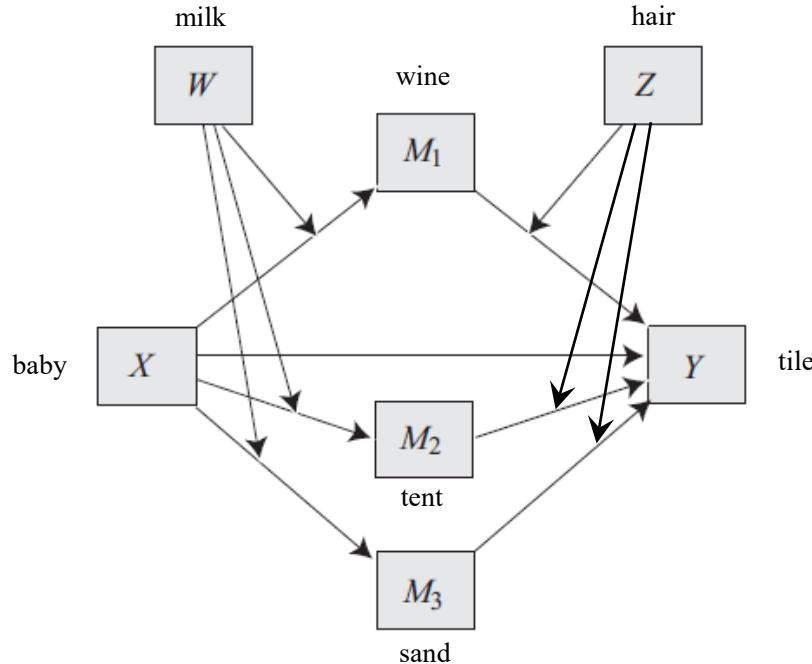
Desired Z matrix

	X	M ₁	M ₂	M ₃
M ₁	0	■	■	■
M ₂	0	0	■	■
M ₃	0	0	0	■
Y	0	1	0	0

```
process y=tile/x=baby/m=wine tent sand/w=milk/z=hair/model=7/  
zmatrix=0,0,0,0,0,0,0,0,1,0,0.
```

Option 2: Edit model 21

In model 21, the Z matrix contains ones in certain cells that allow the M_2 and M_3 paths to Y to be moderated by Z . We can reprogram the Z matrix, turning the offensives ones into zeros.



Preprogrammed Z matrix

	X	M_1	M_2	M_3
M_1	0	■	■	■
M_2	0	0	■	■
M_3	0	0	0	■
Y	0	1	1	1

Desired Z matrix

	X	M_1	M_2	M_3
M_1	0	■	■	■
M_2	0	0	■	■
M_3	0	0	0	■
Y	0	1	0	0

```
process y=tile/x=baby/m=wine tent sand/w=milk/z=hair/model=21/  
zmatrix=0,0,0,0,0,0,0,1,0,0.
```

The MATRICES statement

If you want to check to make sure you have programmed the matrices correctly, or you want to see what the matrices of a preprogrammed model look like, add **matrices=1** to a PROCESS command.

```
process y=tile/x=baby/m=wine tent  
sand/w=milk/z=hair/model=7/  
zmatrix=0,0,0,0,0,0,0,1,0,0  
/matrices=1.
```

Matrices that don't appear in the output have zeros in all cells. If your model includes covariates, the C matrix will appear here too.

```
***** MODEL DEFINITION MATRICES *****  
  
BMATRIX: Paths freely estimated (1) and fixed to zero (0):  
          baby wine tent sand  
wine    1  
tent   1     0  
sand   1     0     0  
tile   1     1     1     1  
  
WMATRIX: Paths moderated (1) and not moderated (0) by W:  
          baby wine tent sand  
wine    1  
tent   1     0  
sand   1     0     0  
tile   0     0     0     0  
  
ZMATRIX: Paths moderated (1) and not moderated (0) by Z:  
          baby wine tent sand  
wine    0  
tent   0     0  
sand   0     0     0  
tile   0     1     0     0
```

Exercise #1

What model does this set of matrices represent?
Draw a path diagram with all variables labeled

B Matrix

	X	M ₁	M ₂
M ₁	1	■	■
M ₂	1	1	■
Y	0	1	1

W Matrix

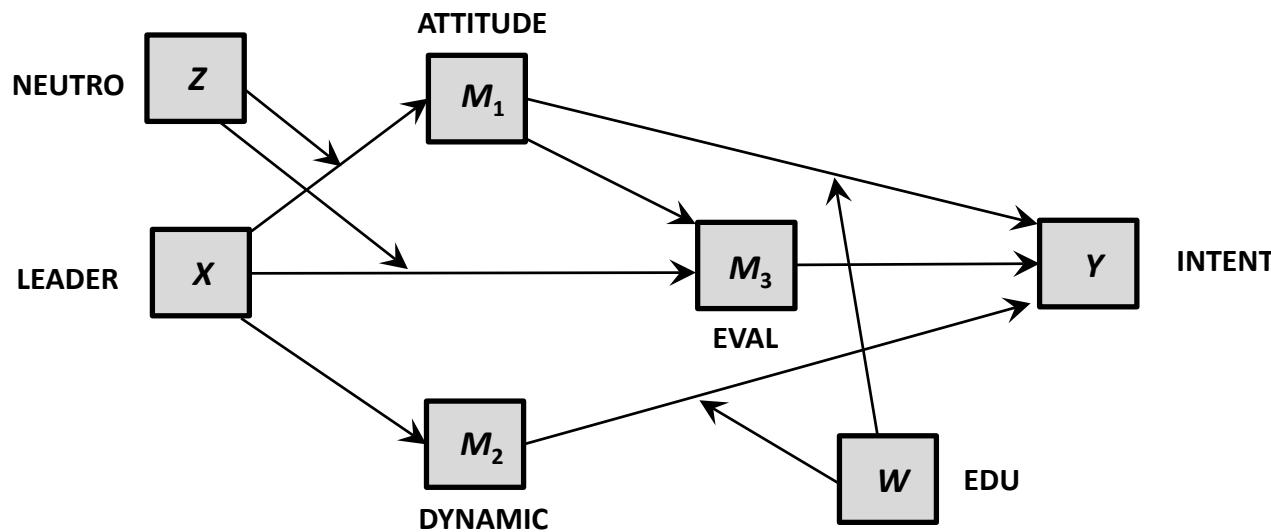
	X	M ₁	M ₂
M ₁	1	■	■
M ₂	0	1	■
Y	0	0	1

C Matrix

	U ₁	U ₂	U ₃
M ₁	1	0	0
M ₂	0	1	1
Y	1	1	0

Exercise #2

Write the PROCESS command that estimates this model:



Sample Size Planning

How many people/rats/observations do I need to collect for my study?

Two philosophies:

- Avoid making a Type II Error (Power)

Collect a sample which should have the power to detect a pre-specified effect up to a certain proportion (e.g., 80%, 95%)

Pros: Aligns with hypothesis testing mentality.

Cons: Requires an estimate of the effect of interest

- Have a very precise estimate (Precision)

Collect a sample which should have a precise estimate of the effect of interest (e.g., $SE \leq 0.5$)

Pros: Does not require an estimate of effect (kind of).

Cons: Doesn't necessarily guarantee a "high powered" study

Sample Size Planning

What effect size do I choose?

- Pilot Study
 - This method is often ineffective, because in order to estimate your effect size with enough precision to run a power analysis, you'd likely need to collect a larger sample than what you ultimately need. (See [Simonsohn, 2014](#) for a nice illustration)
- Published research
 - Published examples of “similar studies” can be useful
 - Meta-analyses in fields can be useful
 - Publication bias can cause problems
 - Meta-meta-analyses (if you just feel like you have no idea)
 - Typical effect size in psychology based on published findings is $d = .2$ (Hemphill (2003), Gignac & Szodorai, 2016; Haase, Waechter, & Solomon (1982)).
 - Doesn't take into account design
- Smallest effect size of interest (see Lakens, Scheel, Isager (2018) *AMPPS*)
 - Determine what size of an effect would be interesting / practically significant and then power for that effect.

Simulations as “Rules of Thumb”

Beware of simulations. Read carefully and understand the situations under which data is simulated.

Typically effect sizes can be interpreted as “standardized effect sizes.”

These simulations typically only reflect “best case scenarios”

Previous simulation work:

Fritz & MacKinnon (2007) – simple mediation

Hayes & Scharkow (2013) – simple mediation

Biesanz, Falk, & Savalei (2016) – mediation with normal and nonnormal data

Williams & MacKinnon (2008) – serial mediation models

TABLE 3

Empirical Estimates of Sample Sizes Needed for .8 Power

Test	Condition															
	SS	SH	SM	SL	HS	HH	HM	HL	MS	MH	MM	ML	LS	LH	LM	LL
Percentile bootstrap	558	412	406	398	414	162	126	122	404	124	78	59	401	123	59	36

Simulations for Power

Doing your own simulations:

You will need to make a variety of decisions about what's going on in your data, and those decisions **will definitely have an impact on your results**. If you're not sure about something, simulate under a variety of conditions.

Monte Carlo Simulations:

Thoemmes , MacKinnon & Reiser (2010) – specific to mediation

Sigal & Chalmers (2016) - Really great R package called ‘simdesign’ which can be used for Monte Carlo Simulations to understand power.

WebPower R package, Schoemann, et al., 2017

bmem R package (uses lavaan) allows for non-normal distributions and missing data

<https://davidakenny.shinyapps.io/MedPower/> simple mediation only

Sample Size Planning: Mediation

How many people do I need to have reasonable power to detect an indirect effect?

Need approximate estimates of all paths involved in the analysis (e.g., a , b , and c for mediation).

Previous simulation work:

Fritz & MacKinnon (2007)

Hayes & Scharkow (2013)

Biesanz, Falk, & Savalei (2016) normal and nonnormal data

Williams & MacKinnon (2008) serial models and

Monte Carlo Simulations:

Thoemmes , MacKinnon & Reiser (2010) – specific to mediation

Sigal & Chalmers (2016)

Really great R package called ‘simdesign’ which can be used for Monte Carlo Simulations to understand power.

Table: Tools available for power analysis in mediation

Tool Name	WebPower	WebPower	bmem	MedPower	pwr2ppl
Interface	Online/R	Online/R	R	Online-Shiny	R functions
Reference	Schoemann, Boulton, Short, 2017	Zhang & Yuan (2015)	Zhang, & Wang, 2013)	Kenny, 2017	Aberson
Inputs	Correlations, standard deviations)	Estimates of a and b, variances	Estimates of all paths, skew, kurtosis, N	Estimates of paths or partial correlations	Correlations, N
Outputs	Required N (Given Power), Estimated Power (Given N)	Required N (Given Power), Estimated Power (Given N)	Estimated Power (Given N)	Required N (Given Power), Estimated Power (Given N)	Estimated Power (Given N)
Standardized	No	No	Yes	Yes	No
Method of Estimation	Bootstrap, Monte Carlo	Sobel	Sobel, Bootstrap	Distribution of the Product	Sobel, Joint Significance
Model Complexity	Simple, Parallel, Serial	Simple	Simple, Parallel, Serial	Simple	Simple, Parallel
URL	https://schoemann.shinyapps.io/mc_power_med/	https://webpower.psychstat.org/models/med01/	https://cran.r-project.org/web/packages/bmem/bmem.pdf	https://davidakenny.shinyapps.io/MedPower/	https://github.com/chrisaberson/pwr2ppl/blob/master/R/med.R

Sample Size Planning: Moderation/CPA

There are a lot of rules of thumbs and some recommendation out there, but I'm not particularly happy with any of them.

Powering an interaction term is more complex than powering for a regression coefficient.

I recommend powering for simple effects (but this requires heavy thinking about what you expect).

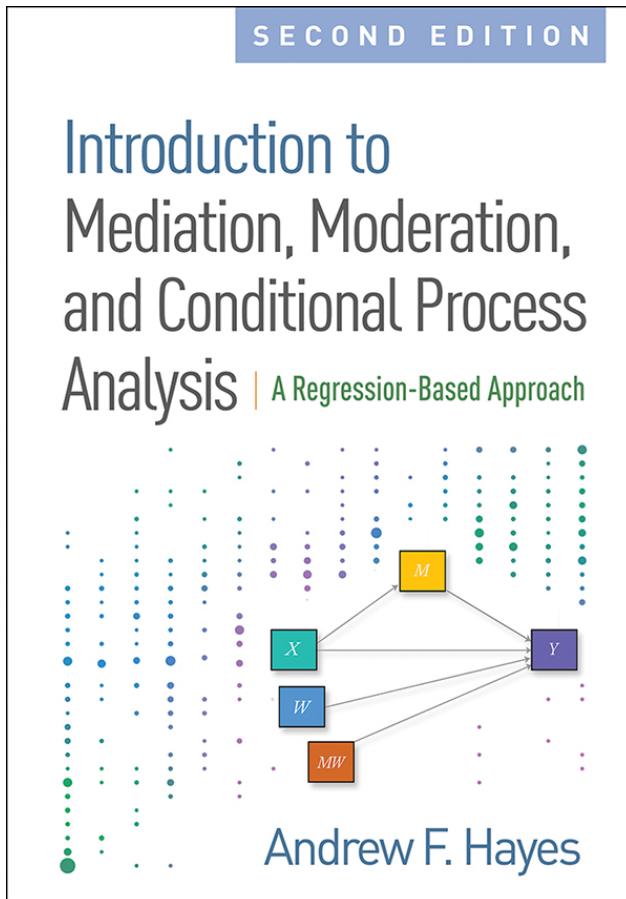
CPA: No specific tools I know of. Monte Carlo simulation seems to be the only way.

- Some shiny apps are in development for most common models in PROCESS
- One of my graduate students is working on this, so if you find yourself in need of a power analysis, feel free to reach out.

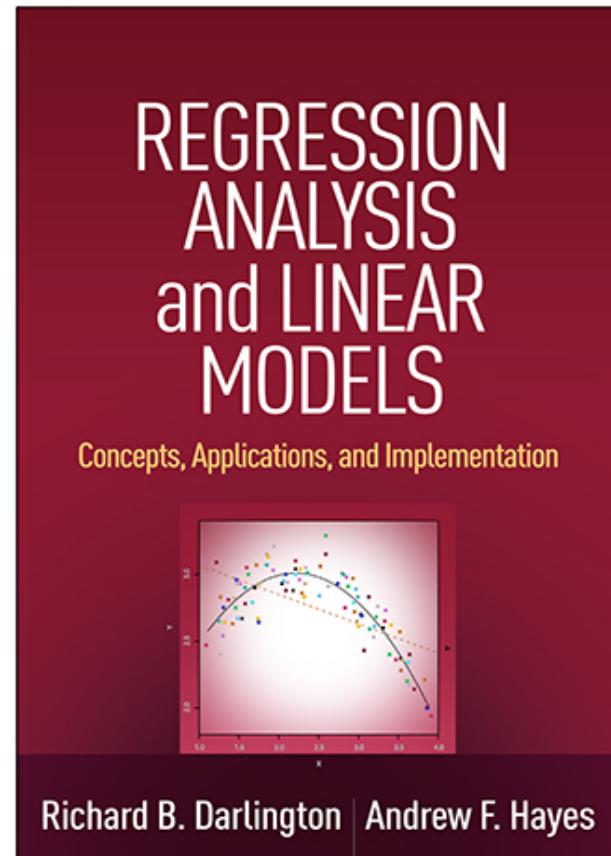
Amanda's Power Rule of Thumb: Collect enough people so that when you get a null result you believe it.

Where to learn more

The entire book



Chapters 13, 14, and 15



MMCIIA

Typically Taught in GSERM St. Gallen Summer

- Review of the fundamentals of mediation, moderation, and conditional process analysis.
- Testing whether an indirect effect is moderated and probing moderation of indirect effects.
- Partial and conditional moderated mediation.
- Mediation analysis with a mult categorial independent variable.
- Moderation analysis with a mult categorial (3 or more groups) independent variable or moderator.
- Conditional process analysis with a mult categorial independent variable
- Moderation of indirect effects in the serial mediation model.
- New features available in PROCESS v3.0, such as how to modify a numbered model or customize your own model.

Pertinent Publications

Rockwood, N. J., & Hayes, A. F. (2018). Mediation, moderation, and conditional process analysis: Regression-based approaches for clinical research. Draft submitted and to appear in A. G. C. Wright and M. N. Hallquist (Eds.) *Handbook of research methods in clinical psychology*. Cambridge University Press.

Rockwood, N. J., & Hayes, A. F. (2018). Multilevel mediation analysis. Draft submitted and to appear in A. A. O'Connell, D. B. McCoach, and B. Bell (Eds.). *Multilevel modeling methods with introductory and advanced applications*. Information Age Publishing.

Hayes, A. F. (2018). Partial, conditional, and moderated moderated mediation: Quantification, inference, and interpretation. *Communication Monographs*, 85, 4-40. [[PDF](#)]

Hayes, A. F., & Rockwood, N. J. (2017). Regression-based statistical mediation and moderation analysis in clinical research: Observations, recommendations, and implementation. *Behaviour Research and Therapy*, 98, 39-57. [[paper and data](#)]

Hayes, A. F., Montoya, A. K., & Rockwood, N. J. (2017). The analysis of mechanisms and their contingencies: PROCESS versus structural equation modeling. *Australasian Marketing Journal*, 25, 76-81. [[PDF and Mplus code](#)]

Hayes, A. F., & Montoya, A. K. (2017). A tutorial on testing, visualizing, and probing interaction involving a multcategorical variable in linear regression analysis. *Communication Methods and Measures*, 11, 1-30 [[paper and data](#)]

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Hayes, A. F., Preacher, K. J., & Myers, T. A. (2011). Mediation and the estimation of indirect effects in political communication research. In E. P. Bucy & R. L. Holbert (Eds), *Sourcebook for political communication research: Methods, measures, and analytical techniques*. (p. 434-465). New York: Routledge.

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Hayes, A. F. (2009). Beyond Baron and Kenny: Statistical mediation analysis in the new millennium. *Communication Monographs*, 76, 408-420.

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Preacher, K. J., & Hayes, A. F. (2004). SPSS and SAS procedures for estimating indirect effects in simple mediation models. *Behavior Research Methods, Instruments, and Computers*, 36, 717-731.

Running Example: Group Work in Computer Science (WS)

Montoya, A. K. (2013) Increasing Interest in Computer Science thought Group Work: A Goal Congruity Approach (Undergraduate Honors Thesis).

Within-Subjects Version (CompSci_WS.sav) :

Female participants (N = 51) read two syllabi for a different computer science classes.

One of the syllabi reported the class would have group projects throughout, and the other syllabi stated that individual project would be scheduled throughout.

- Syllabi also differed in professor's name (but not gender), and the primary programming language used in the class.

Measured Variables:

- Interest in each the class int_i int_g
- Perscom Personal Communal Goals ($\alpha = .87$)
 - Same as between subjects version
- Order
 - 1 = Group First; 2 = Individual First

Judd, McClelland, and Smith (1996)

Judd, C. M., McClelland, G. H., and Smith, E. R. (1996). Testing Treatment by Covariate Interactions When Treatment Varies Within Subjects. *Psychological Methods*, 1(4), 366-378.

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0898-2603/96/\$0.00 DOI: 10.1037/1062-9345.1.4.366

Testing Treatment by Covariate Interactions When Treatment Varies Within Subjects

Charles M. Judd and Gary H. McClelland
University of Colorado at Boulder

Eliot R. Smith
Purdue University

In contrast to situations when an independent or treatment variable is between subjects, procedures for testing treatment by covariate interactions are commonly understood when the treatment varies within subjects. The purpose of this article is to identify two approaches that test such interactions. Two design scenarios are considered. One is in which the treatment is measured only once for each subject and hence varies only between subjects, and the other is in which the covariate is measured at each level of the treatment variable and hence varies both within and between subjects. In each case, alternative analyses are identified and their efficiencies compared.

An issue that arises with some frequency in data analysis in psychological research concerns the relationship between some measured variable and the dependent variable and whether this relationship depends on or varies across levels of a manipulated or experimental independent variable. For instance, in a clinical intervention study, we might randomly assign patients to one of two conditions, either a treatment intervention or a control intervention, and measure the outcome variable. Prior to treatment, we measure a characteristic of the patient, probably focusing on the prior course and severity of their illness. Following the treatment, we again measure the same symptom severity. The primary question of interest, of course, is whether the outcome variable is affected by the manipulated treatment: Did the treatment create a difference in subsequent symptom severity? Additionally, however, we may well want to know whether the relationship between the treatment and posttreatment symptom severity depends on the patient's preintervention course of illness. It may be, for instance, that the treatment's effect is greater for patients whose preintervention symptoms were relatively severe. Equivalently, it may be that posttreatment symptom severity is less well explained by preintervention course of illness than are patients in the intervention condition than in the case of patients in the control condition.

The preintervention measure of course is typically a covariate. The analysis that is of interest is an analysis of covariance (ANCOVA), including the treatment by covariate interaction (Judd & McClelland, 1989). The two questions of interest are (a) does the treatment have an effect, and (b) is there a Treatment \times Covariate interaction? If the interaction is significant, it indicates that the covariate-moderated relationship depends on the treatment variable. Equivalently, it indicates that the effect of the treatment on the outcome variable depends on the level of the covariate.

The analysis can be conducted using multiple regression, making the standard assumption that errors or residuals are independently sampled from a single normally distributed population. Assume that Y is the outcome variable, X is the covariate, and Z is the contrast-coded (Judd & McClelland, 1989; Rosenthal & Rosnow, 1985) treatment variable. One estimates two least squares regression models:

$$Y_1 = \beta_0 + \beta_1 X_1 + \beta_2 Z_1 + e_1$$

and

$$Y_2 = \beta_0 + \beta_1 X_2 + \beta_2 Z_2 + e_2$$

In the first equation, β_1 represents the magnitude of

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Charles M. Judd and Gary H. McClelland, Department of Psychology, University of Colorado at Boulder; Eliot R. Smith, Department of Psychological Sciences, Purdue University.

This work was partially supported by National Institute of Mental Health Grant MH37010.

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366

A regression approach to considering a “cross level” interactions.

Approach is very simple:

1. Data should be a two-condition within-subjects design with a person level covariate.
2. Setup two regression equations, one for each condition
3. Take the difference between those two regression equations
4. Regression weight for person level covariate in Step 3 tests moderation.

Advantages of such a design...

Designs such as this are common:

- Participants might read two different scenarios that vary on some manipulated feature X and offer emotional reactions, make predictions about the own behavior, and so forth, in each.
- A therapist might measure certain symptoms and various outcomes when clients arrive in the office for the first time and after a few months of treatment. So X is “pre” or “post”, i.e., the passage of time.

Some advantages relative to “between-subjects” design and analysis:

- Fewer participants needed. Rather than having n people (n_1 in one condition, n_2 in the other), we need about $0.5n$ in total. That saves effort, time, money, labor, and so forth.
- Greater statistical power. Each person serves as his or her own control. “Noise” due to individual differences that increases standard errors in estimates of effects is reduced, sometimes substantially.
- Reduces (but doesn’t eliminate) the fundamental problem of causal inference. We don’t have to think counterfactually. We know how a person responds in *each* condition rather than having to make an assumption about he or she might have if assigned to the other condition instead.

Computer Science Within-Subjects Data Example

Montoya, A. K. (2013) Increasing Interest in Computer Science thought Group Work: A Goal Congruity Approach (Undergraduate Thesis).

1. Data should be a two-condition within-subjects design with a person level covariate.

Research Question: Does the degree to which preference for group work predicts interest in computer science depend on whether or not the class has group work?

Or

Does effect of group work on interest in computer science classes depend on an individual's preference for group work?

CompSci_WS.sav

Subject	int_I	int_G	grppref
300	1.50	4.00	6.67
301	2.75	3.25	6.33
325	5.75	2.50	2.67
342	3.50	5.75	6.00
349	2.25	2.00	4.00
350	1.50	1.75	3.67
305	2.50	4.25	4.00
348	6.00	1.75	2.33
318	3.00	2.00	4.67
320	4.00	5.25	4.00
332	5.00	5.00	3.67
338	2.00	1.75	3.00
310	1.00	1.75	3.00
304	1.25	4.50	5.67
306	5.75	4.50	4.00
308	3.25	4.75	4.00
315	2.75	2.25	4.33
322	5.50	2.00	2.33
343	1.75	5.25	6.00
314	4.00	5.50	3.00
319	2.25	4.00	5.00

Analysis using Judd et al. (1996)

2. Setup two regression equations, one for each condition

Setup a model of the outcome in each condition:

$$Y_{1i} = b_{10} + b_{11}M_i + \epsilon_{1i}$$

Is b_{11} different from b_{21} ?

$$Y_{2i} = b_{20} + b_{21}M_i + \epsilon_{2i}$$

3. Based on these models, setup a new model where you can directly estimate and conduct inference on what you are interested in (in this case $b_{11} - b_{21}$):

$$Y_{2i} - Y_{1i} = (b_{10} - b_{20}) + (b_{11} - b_{21})M_i + (\epsilon_{1i} - \epsilon_{2i}) = b_0 + b_1M_i + \epsilon_i$$

Use simple regression to conduct inference on $b_1 = b_{11} - b_{21}$

With the data: Does the relationship between preference for group work and interest depend on group work condition?

```
regression /dep = int_diff /method = enter grppref.
```

What sign do you expect b_1 to be? **Remember:** $\text{int_diff} = \text{int_G} - \text{int_i}$.



Analysis using Judd et al. (1996)

4. Regression weight for person level covariate in Step 3 tests moderation.

$$Y_{1i} = b_{10} + b_{11}M_i + \epsilon_{1i}$$

$$Y_{2i} = b_{20} + b_{21}M_i + \epsilon_{2i}$$

$$Y_{2i} - Y_{1i} = (b_{10} - b_{20}) + (b_{11} - b_{21})M_i + (\epsilon_{1i} - \epsilon_{2i}) = b_0 + b_1M_i + \epsilon_i$$

```
regression /dep = int_diff /method = enter grppref.
```

Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
1	(Constant) -3.550	.648		-5.474	.000
	grppref .994	.156	.674	6.388	.000

a. Dependent Variable: int_diff



What does it mean that b_1 is positive?

$$b_1 = b_{11} - b_{21} = .994$$

$$b_{11} > b_{21}$$

Practically, this means that the relationship between preference for group work and interest is significantly stronger (more positive) in the group work condition.

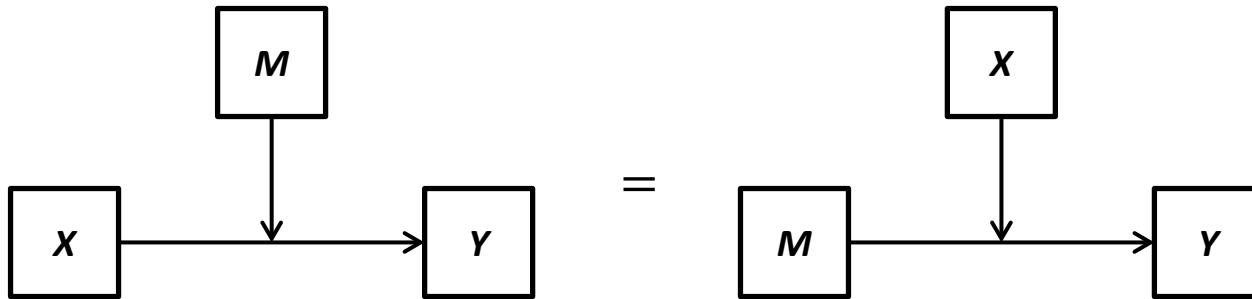
Symmetry in Within-Subjects Moderation

Does the effect of condition depend on M ?

$$Y_{2i} - Y_{1i} = (b_{10} - b_{20}) + (b_{11} - b_{21})M_i + (\epsilon_{1i} - \epsilon_{2i}) = b_0 + b_1 M_i + \epsilon_i$$

$Y_{2i} - Y_{1i}$ is a quantification of the effect of condition, which means that if M predicts $Y_{2i} - Y_{1i}$ then the effect of condition depends on M .

b_1 is a test of exactly that!



Conditional Effects in Within-Subjects Moderation

$$Y_{2i} - Y_{1i} = (b_{10} - b_{20}) + (b_{11} - b_{21})M_i + (\epsilon_{1i} - \epsilon_{2i}) = b_0 + b_1 M_i + \epsilon_i$$

Given a value of M what is the effect of condition on the outcome?

$Y_{2i} - Y_{1i}$ is a quantification of the effect of condition, which means that the conditional effect of condition $\theta_{c \rightarrow Y}(M) = b_0 + b_1 M$

Given a specific condition what is the effect of M on the outcome?

$$Y_{1i} = b_{10} + b_{11}M_i + \epsilon_{1i}$$

$$Y_{2i} = b_{20} + b_{21}M_i + \epsilon_{2i}$$

$$\theta_{X \rightarrow Y}(c) = b_{c1}$$

Conditional effects will become important when it comes to probing

Probing an Effect of Condition on Outcome: The “Pick-a-Point” Approach

$$\theta_{c \rightarrow Y}(X) = b_0 + b_1 M$$

Select a value of the moderator (M) at which you'd like to have an estimate of the condition's effect on Y . Then derive its standard error. The ratio of the effect to its standard error is distributed as $t(df_{residual})$ under the null hypothesis that the effect of condition is zero at that moderator value.

The estimated standard error of $\theta_{c \rightarrow Y}(M)$ is

$$S_{\theta_{c \rightarrow Y}(M)} = \sqrt{(s_{b_0}^2 + 2M s_{b_0 b_1} + M^2 s_{b_1}^2)}$$

Squared standard error of b_0

Covariance of b_0 and b_1

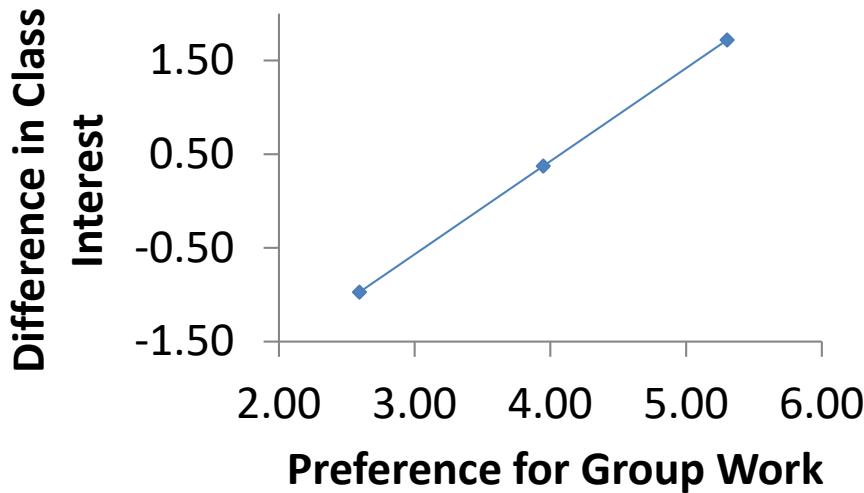
Squared standard error of b_1

Probing an Effect of Condition on Outcome: The “Pick-a-Point” Approach

You must choose the points along the moderator to “probe” the effect of condition on Y .

Let's look at an example with our computer science data:

$$Y_{Di} = -3.55 + .99M_i$$



M	$\theta_{C \rightarrow Y M}$	$s_{\theta_{C \rightarrow Y M}}$	p
2.59	-0.97	0.30	0.00
3.95	0.37	0.21	0.08
5.30	1.72	0.30	0.00

Participants relatively low in preference for group work are more interested in the individual work class, and those high in preference for group work are more interested in the class with group work.

The Johnson-Neyman Technique

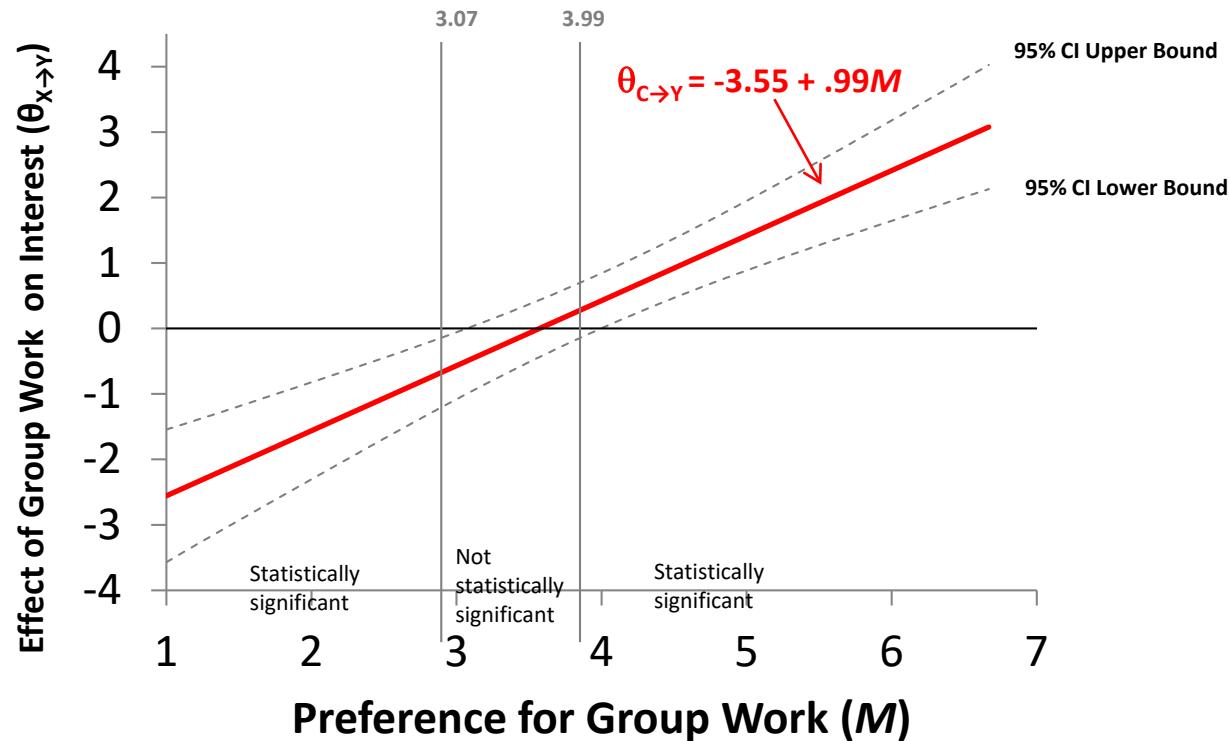
The Johnson-Neyman technique seeks to find the value or values of the moderator (M) within the data, if they exist, such that the p -value for the conditional effect of condition at that value or those values of M is exactly equal to some chosen level of significance α . Thus, no need to select values of M in advance.

To do so, we ask what value of M produces a ratio of $\theta_{C \rightarrow Y}(M)$ to its standard error exactly equal to the critical t value (t_{crit}) required to reject the null hypothesis that $\theta_{C \rightarrow Y}(M)$ is equal to zero at that value of M ?

$$t_{crit} = \frac{b_0 + b_1 M}{\sqrt{s_{b_0}^2 + 2Ms_{b_0}b_1 + M^2s_{b_1}^2}}$$

Isolating M yields to the solution in the form of a quadratic equation which always has two roots, though not always two that are interpretable.

A Plot of the “Region of Significance”



MEMORE

We can use MEMORE to estimate and probe this model.

Model Templates for MEMORE V2.Beta
©2017 Amanda K. Montoya

Model 2 Additive Moderation
Conceptual Diagram

The Conceptual Diagram shows a box labeled x with an arrow pointing down to a row of boxes labeled M_1, M_2, \dots, M_k . Arrows from each M_i point to a final box labeled y . Below the y box is the expression $Y_2 - Y_1$.

Statistical Diagram

The Statistical Diagram shows a triangle labeled 1 above a row of boxes labeled M_1, M_2, \dots, M_k . Arrows from each M_i point to a final box labeled y . Above the first arrow is the label b_0 , above the second b_1 , above the third b_2 , and above the last b_k . Below the y box is the expression $Y_2 - Y_1$.

```
MEMORE m = grppref /y = int_G int_I /model = 3  
/jn = 1 /plot = 1.
```

- List moderator(s) in the `m` list
- List outcomes in the `y` list
- Can use model 2 or model 3 when you have 1 moderator there is no difference.
- JN option calls the Johnson-Neyman technique
- PLOT option calls a table of values for making a nice plot.

Using MEMORE for CASC WS data

```
MEMORE m = grppref /y = int_G int_I /model = 3 /jn = 1 /plot = 1.
```

***** MEMORE Procedure for SPSS Version 2.Beta *****

Written by Amanda Montoya

Documentation available at akmontoya.com

Model:

3

Variables:

Y = int_G int_I
M = grppref

Computed Variables:

Ydiff = int_G - int_I

First part of output repeats what you told MEMORE to do. Always double check that this is correct!

Sample Size:

51

I double checked to make sure the order of subtraction was the same as when we did this by hand.

Using MEMORE for CASC WS data

```
MEMORE m = grppref /y = int_G int_I /model = 3 /jn = 1 /plot = 1.
```

Probing effect of condition on outcome at different values of the moderator

```
*****
```

Conditional Effect of 'X' on Y at values of moderator(s)

grppref	Effect	SE	t	p	LLCI	ULCI
2.5938	-.9728	.2964	-3.2823	.0019	-1.5684	-.3772
3.9478	.3725	.2085	1.7865	.0802	-.0465	.7916
5.3019	1.7179	.2964	5.7963	.0000	1.1223	2.3135

Degrees of freedom for all conditional effects:

49

Values for quantitative moderators are the mean and plus/minus one SD from the mean.

This is the default. You can change this to the 10th, 25th, 50th, 75th, and 90th quantiles by adding quantile =1 to the command line

Using MEMORE for CASC WS data

```
MEMORE m = grppref /y = int_G int_I /model = 3 /jn = 1 /plot = 1.
```

***** JOHNSON-NEYMAN PROCEDURE *****

Moderator value(s) defining Johnson-Neyman significance region(s) and percent of observed data above value:

Value	% Abv
3.0685	72.5490
3.9949	54.9020

Conditional Effect of 'X' on Y at values of moderator

grppref	Effect	SE	t	p	LLCI	ULCI
1.0000	-2.5564	.5037	-5.0752	.0000	-3.5687	-1.5442
1.2984	-2.2599	.4619	-4.8931	.0000	-3.1880	-1.3318
1.5968	-1.9634	.4210	-4.6641	.0000	-2.8094	-1.1174
1.8953	-1.6669	.3813	-4.3712	.0001	-2.4332	-.9006
2.1937	-1.3704	.3434	-3.9905	.0002	-2.0605	-.6803
2.4921	-1.0739	.3078	-3.4886	.0010	-1.6925	-.4553
2.7905	-.7774	.2755	-2.8218	.0069	-1.3310	-.2238
3.0685	-.5012	.2494	-2.0096	.0500	-1.0023	.0000
3.0889	-.4808	.2477	-1.9416	.0579	-.9785	.0168
3.3874	-.1843	.2260	-.8156	.4187	-.6385	.2699
3.6858	.1122	.2125	.5279	.5999	-.3148	.5392
3.9842	.4087	.2086	1.9591	.0558	-.0105	.8279
3.9949	.4193	.2087	2.0096	.0500	.0000	.8387
4.2826	.7052	.2149	3.2809	.0019	.2733	1.1371
4.5811	1.0017	.2306	4.3435	.0001	.5382	1.4652
4.8795	1.2982	.2539	5.1124	.0000	.7879	1.8085
5.1779	1.5947	.2830	5.6350	.0000	1.0260	2.1634
5.4763	1.8912	.3162	5.9804	.0000	1.2557	2.5267
5.7747	2.1877	.3525	6.2070	.0000	1.4794	2.8961
6.0732	2.4843	.3909	6.3560	.0000	1.6988	3.2697
6.3716	2.7808	.4308	6.4546	.0000	1.9150	3.6465
6.6700	3.0773	.4720	6.5200	.0000	2.1288	4.0258

Degrees of freedom for all conditional effects:

This will only print when we include jn =1 in the command line. JN technique does not work for multiple moderators.

Using MEMORE for CASC WS data

```
MEMORE m = grppref /y = int_G int_I /model = 3 /jn = 1 /plot = 1.
```

```
*****
```

Conditional Effect of Moderator(s) on Y in each Condition

Condition 1 Outcome:
int_G

Model Summary

R	R-sq	MSE	F	df1	df2	P
.4488	.2014	1.7964	12.3612	1.0000	49.0000	.0010

Model

	coeff	SE	t	p	LLCI	ULCI
constant	1.7874	.5836	3.0624	.0036	.6145	2.9603
grppref	.4922	.1400	3.5158	.0010	.2109	.7735

Degrees of freedom for all conditional effects:

49

Preference for group work
positively predicts interest in
class with group work

Condition 2 Outcome:
int_I

Model Summary

R	R-sq	MSE	F	df1	df2	P
.4710	.2218	1.6502	13.9671	1.0000	49.0000	.0005

Model

	coeff	SE	t	p	LLCI	ULCI
constant	5.3374	.5594	9.5415	.0000	4.2132	6.4615
grppref	-.5014	.1342	-3.7373	.0005	-.7710	-.2318

Degrees of freedom for all conditional effects:

49

and negatively predicts interest
in class with individual work.

Using MEMORE for CASC WS data

```
MEMORE m = grppref /y = int_G int_I /model = 3 /jn = 1 /plot = 1.
```

```
*****
```

Data for visualizing conditional effect of X on Y.
Paste text below into a SPSS syntax window and execute to produce plot.

```
DATA LIST FREE/grppref YdiffHAT int_GHAT int_IHAT.
```

```
BEGIN DATA.
```

2.5938	-.9728	3.0640	4.0368
3.9478	.3725	3.7304	3.3578
5.3019	1.7179	4.3968	2.6789

```
END DATA.
```

```
GRAPH/SCATTERPLOT = grppref WITH YdiffHAT.  
GRAPH/SCATTERPLOT = grppref WITH int_GHAT.  
GRAPH/SCATTERPLOT = grppref WITH int_IHAT.
```

Code for plotting. You'll get three plots each with the moderator on the X axis and a different outcome on the Y axis.

- 1) Predicted Differences between Y's
- 2) Predicted Y from first condition
- 3) Predicted Y from second condition

Writing up a Moderation Analysis

Tips:

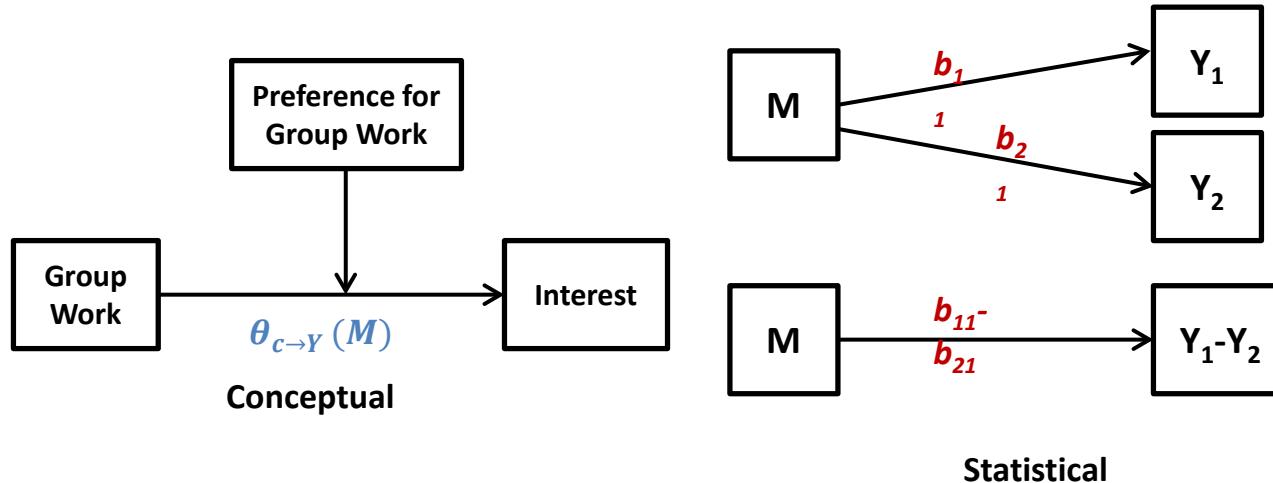
- Interpret the sign and the magnitude of the interaction coefficient with respect to X 's effect on Y (or M 's effect on Y ; or both).
- Provide probing results with interpretations
- Read the write ups of other's moderation analyses
- Provide a graphical representation of the effect of interest (like the ones we've done)

Does the effect of group work on interest in a computer science class depend on preference for group work?

Overall, the impact of including group work in a computer science class on interest in the class depends on an individual's general preference for group work ($b_1 = .49, p = .001$). As preference for group work increases relative interest in the class with group work compared to the class with individual work increases as well. (i.e. the group work class is more preferred as general preference for group work increases). Indeed we found that those who were relatively low in preference for group work preferred the individual work class over the class with group work ($\theta_{X \rightarrow Y}(M=2.59) = -.97, p = .002$). Whereas, those who were relatively moderate in preference for group work did not show a strong preference for one class over another, though they marginally preferred the class with group work ($\theta_{X \rightarrow Y}(M=3.97) = .37, p = .08$). Finally, those who showed a strong general preference for group work, unsurprisingly preferred the class with group work over the class with individual work ($\theta_{X \rightarrow Y}(M=5.30) = 1.72, p < .001$). The Johnson-Neyman procedure those whose preference for group work was less than 3.07 preferred the individual work class, and those who's preference for group work was greater than 3.99 preferred the group work class. Preference for group work was positively related to interest in the class with group work ($b = .49, p = .001$), and negatively related to interest in the class with individual work ($b = -0.50, p = .001$).

Visualizations

I recommend trying a number of different types of visualizations to decide what works best for your case.



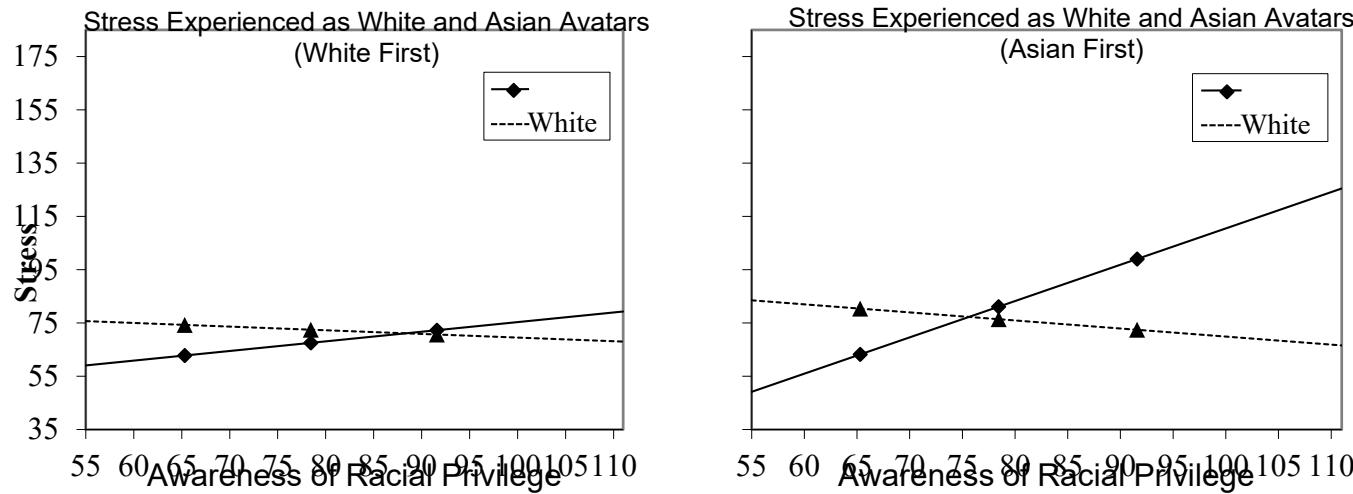
Tips:

- Try the different scales of the Y axis (difference vs. raw Y score with two lines for each condition)
- I do not like bar graphs with the effect of the moderator in each condition
- Provide path estimates on statistical diagram or in a table.

Visualizations: A Case Study

Tawa, J., & Montoya, A. K. (Under Review) White students' physiological stress while operating non-White avatars and the moderating role of awareness of racial privilege.

White participants operated avatars of three difference races (White, Black, and Asian) and wrote heart monitors to measure their stress while operating each avatar. We found that individual's awareness of racial privilege moderated the effect of avatar race on stress, and that this effect depended on the order of operating the avatars.

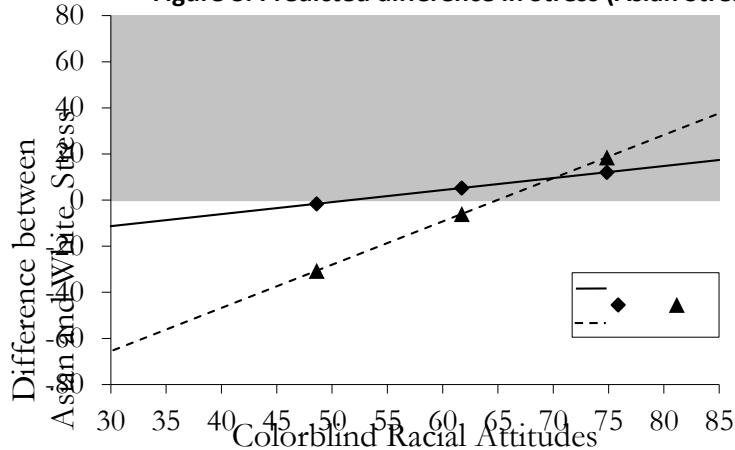


Visualizations: A Case Study

Tawa, J., & Montoya, A. K. (Under Review) White students' physiological stress while operating non-White avatars and the moderating role of awareness of racial privilege.

White participants operated avatars of three different races (White, Black, and Asian) and wrote heart monitors to measure their stress while operating each avatar. We found that individual's awareness of racial privilege moderated the effect of avatar race on stress, and that this effect depended on the order of operating the avatars.

Figure 3. Predicted difference in Stress (Asian Stress – White Stress), split by order.



Note. Scores above zero on the Y-axis represent greater predicted stress while piloting the Asian avatar than while piloting the White avatar. Points marked by shapes indicate predicted stress differences at the mean plus/minus one standard deviation on CBA.

Common Questions

- Can this method be used for more than two conditions?

YES! The same method for coming up with contrasts in Judd, Kenny, and McClelland (2001) describe a system for setting up contrasts among conditions can be used for moderation.

I recommend reading [Hayes & Montoya \(2017\)](#) on moderation analysis with a multcategorical IV if you want to try this out. I am happy to give instructions on how to get MEMORE to doing this.
- **ALTERNATIVES:** Some of the other repeated-measures mediation options are more appropriate if you have more than two conditions (especially longitudinal), so take a look at those when thinking about these options.
- Can I use multiple moderators?

YES! MEMORE models 2 and 3 accept up to 5 moderators. (See Documentation for instructions).
- How do I control for covariates?

All of MEMORE's mediation analyses are within-person models, so you do not need to control for any between subjects variables such as age, gender, big-5. But you can include them as additional moderators (likely using model 2).

Multiple Moderator Models

Model 2 vs. Model 3

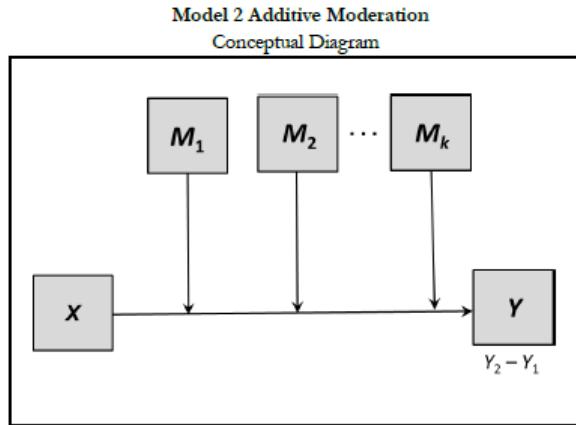
When you have multiple moderators you are interested, consider whether you think those moderators will themselves interact or not.

If you believe the moderators will interact **with each other** → Model 3

If you believe the moderators will **only interact with condition** → Model 2

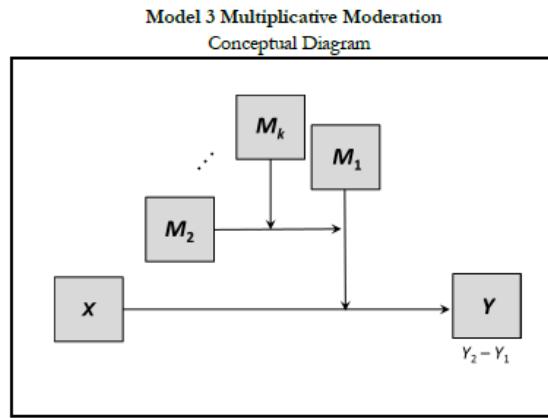
Model Templates for MEMORE V2.Beta

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Model Templates for MEMORE V2.Beta

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Multiple Moderator Models

```
MEMORE m = grppref order/y = int_G int_I /model = 2.
```

```
Model:  
2  
  
Variables:  
Y = int_G int_I  
M1 = grppref  
M2 = Order  
  
Computed Variables:  
Ydiff = int_G - int_I  
  
Sample Size:  
51  
  
*****  
Outcome: Ydiff = int_G - int_I  
  
Model Summary  
R R-sq MSE F df1 df2 p  
.7113 .5059 2.0502 24.5734 2.0000 48.0000 .0000  
  
Model  
coeff SE t p LLCI ULCI  
constant -4.8074 .8394 -5.7269 .0000 -6.4952 -3.1196  
grppref .9562 .1505 6.3542 .0000 .6536 1.2588  
Order .9071 .4055 2.2372 .0300 .0918 1.7223  
  
Degrees of freedom for all regression coefficient estimates:  
48
```

Think of it like 3 two-way interactions:
Condition x Group Preference
Condition x Order

Multiple Moderator Models

```
MEMORE m = grppref order/y = int_G int_I /model = 3.
```

```
Model:  
 3  
  
Variables:  
Y = int_G int_I  
M1 = grppref  
M2 = Order  
  
Computed Variables:  
Ydiff = int_G - int_I  
Int1 = grppref x Order  
  
Sample Size:  
 51  
  
*****  
Outcome: Ydiff = int_G - int_I  
  
Model Summary  
R      R-sq      MSE      F      df1      df2      p  
.7125  .5077  2.0862  16.1569  3.0000  47.0000  .0000  
  
Model  
coeff      SE      t      p      LLCI      ULCI  
constant  -5.5239  1.9247  -2.8700  .0061  -9.3960  -1.6518  
grppref   1.1401  .4690  2.4312  .0189  .1967  2.0836  
Order     1.4057  1.2704  1.1065  .2742  -1.1501  3.9615  
Int1     -.1263  .3048  -.4145  .6804  -.7395  .4868  
  
Degrees of freedom for all regression coefficient estimates:  
 47
```

Think of it like three-way interaction,
and three two-way interactions:
Condition x Group Preference
Condition x Order
Group Preference x Order
Condition x Group Preference x Order

Other Types of Repeated Measures Moderation

- Multilevel Models (Cross level interactions in particular)
 - Aguinis, Gottfredson, Culpepper (2013) *Journal of Management*
Very approachable article on estimating cross-level interactions
 - Bauer & Curran (2010) *Multivariate Behavioral Research*
Estimating and probing interactions in multilevel models
 - Many many others!
- Latent Growth Curve Models
 - Preacher, Curran, Bauer (2006) *Journal of Educational and Behavioral Statistics*
Also has MLM and regression
- Structural Equation Modeling (Can be used for a variety of data types)
 - Klein & Muthén (2007) *Multivariate Behavioral Research*
Methods for including latent interactions
- Multilevel SEM
 - Preacher, Zhang, Zyphur (2016) *Psychological Methods*
Very technical read, but deals with a lot of the issues of bias in MLM
 - Ryu (2015) *Structural Equation Modeling*
Impact of centering in MSEM

Mediation analysis in the 2-condition within-subject design

Data are from Judd et al. (2001). Estimating and testing mediation and moderation in within-subjects designs. *Psychological Methods*, 6, 115 –134.

20 participants with chronic pain symptoms participated in a pain drug trial. Each was measured twice, once after administration of a placebo **and** once after a administration of a pain inhibiting drug. Order is randomized.

Measurement 1 = Following placebo

Measurement 2 = Following drug

Y = pain sensations (0 to 100, higher = more)

M = pain enhancing hormone (0 to 100,
higher = *more*)

Analytical goal: Determine if the effect of the drug on pain experienced operates through the mechanism of reducing pain enhancing hormone levels.

		JUDD.sav	JUDD.sas
pain1	pain2	hormone1	hormone2
73.00	61.00	37.00	33.00
57.00	55.00	30.00	28.00
57.00	61.00	30.00	36.00
67.00	49.00	31.00	30.00
80.00	75.00	37.00	35.00
56.00	60.00	33.00	34.00
72.00	73.00	38.00	35.00
81.00	65.00	43.00	29.00
61.00	59.00	33.00	31.00
67.00	48.00	20.00	17.00
74.00	64.00	43.00	41.00
70.00	55.00	34.00	27.00
83.00	68.00	41.00	39.00
62.00	61.00	35.00	30.00
49.00	55.00	32.00	32.00
74.00	79.00	35.00	37.00
73.00	60.00	36.00	38.00
46.00	51.00	25.00	24.00
60.00	55.00	26.00	25.00
93.00	71.00	46.00	39.00

Advantages of such a design...

Designs such as this are common:

- Participants might read two different scenarios that vary on some manipulated feature X and offer emotional reactions, make predictions about the own behavior, and so forth, in each.
- A therapist might measure certain symptoms and various outcomes when clients arrive in the office for the first time and after a few months of treatment. So X is “pre” or “post”, i.e., the passage of time.

Some advantages relative to “between-subjects” design and analysis:

- Fewer participants needed. Rather than having n people (n_1 in one condition, n_2 in the other), we need about $0.5n$ in total. That saves effort, time, money, labor, and so forth.
- Greater statistical power. Each person serves as his or her own control. “Noise” due to individual differences that increases standard errors in estimates of effects is reduced, sometimes substantially.
- Reduces (but doesn’t eliminate) the fundamental problem of causal inference. We don’t have to think counterfactually. We know how a person responds in *each* condition rather than having to make an assumption about he or she might have if assigned to the other condition instead.

Judd, Kenny, and McClelland (2001)

Psychological Methods
2001, Vol. 6, No. 2, 115-134

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1082-989X/01/\$5.00 DOI: 10.1037/1082-989X.6.2.115

Estimating and Testing Mediation and Moderation in Within-Subject Designs

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University of Colorado at Boulder

Analyses designed to detect mediation and moderation of treatment effects are increasingly prevalent in research in psychology. The mediation question concerns the processes that produce a treatment effect. The moderation question concerns factors that affect the magnitude of that effect. Although analytic procedures have been reasonably well worked out in the case in which the treatment varies between participants, no systematic procedures for examining mediation and moderation have been developed in the case in which the treatment varies within participants. The authors present an analytic approach to these issues using ordinary least squares estimation.

The issues of mediation and moderation have received considerable attention in recent years in both basic and applied research (Baron & Kenny, 1986; James & Brett, 1984; Judd & Kenny, 1981b; MacKinnon & Dwyer, 1993). In addition to knowing whether a particular intervention has an effect, the researcher typically wants to know about factors that affect the magnitude of that effect (i.e., moderation) and mechanisms that produce the effect (i.e., mediation). Such knowledge helps in both theory development and intervention application.

To illustrate the difference between mediation and moderation, consider a design in which a researcher is interested in whether students who are taught with a new curriculum (the treatment condition) show higher performance on a subsequent standardized test than students taught under the old curriculum (the control

condition). Assuming that a performance difference is found, one might plausibly hypothesize different mediating mechanisms for this effect. The new curriculum might increase students' interest in the subject matter; it might cause students to study harder outside of class; or it might convey the material more clearly. These are alternative reasons why the performance difference is found, that is, alternative mediators of the treatment effect. The researcher might also be interested in factors that affect the magnitude of the difference between performance following the old curriculum and performance following the new one. That difference might be larger or smaller for different types of students or in different types of classrooms or when taught by different kinds of teachers. All of these then are potential moderators of the treatment effect.

It is possible that the same variable may serve as both a mediator and a moderator. For instance, study time might serve both roles. First, as a mediator, the new curriculum might lead to higher performance because it causes students to study more. Second, as a moderator, the treatment might be especially effective for students who spend more time studying.

Procedures for assessing mediation and moderation have been relatively well worked out through ordinary least squares regression and analysis of variance procedures. Mediation is assessed through a four-step procedure (Baron & Kenny, 1986; Judd & Kenny,

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One of the few treatments of mediation analysis in this common research design.

A "causal steps", Baron and Kenny type logic to determining whether M is functioning as a mediator of X 's effect on Y when both M and Y are measured twice in difference circumstances but on the same people.

Judd, Kenny, and McClelland (2001)

Data are from Judd et al. (2001). Estimating and testing mediation and moderation in within-subjects designs. *Psychological Methods*, 6, 115 –134.

Analytical goal. Determine if the effect of a pain-inhibiting drug on pain experienced operates through the mechanism of reducing pain-enhancing hormone levels.

- (1) On average, is pain following the drug lower than pain following the placebo?
- (2) On average, are there fewer pain enhancing hormones in the blood following the drug relative to the placebo?
- (3) Does difference in hormone level predict differences in pain experienced?
- (4) Do differences in hormone levels account for differences in pain?

Y_1	Y_2	M_1	M_2
pain1	pain2	hormone1	hormone2
73.00	61.00	37.00	33.00
57.00	55.00	30.00	28.00
57.00	61.00	30.00	36.00
67.00	49.00	31.00	30.00
80.00	75.00	37.00	35.00
56.00	60.00	33.00	34.00
72.00	73.00	38.00	35.00
81.00	65.00	43.00	29.00
61.00	59.00	33.00	31.00
67.00	48.00	20.00	17.00
74.00	64.00	43.00	41.00
70.00	55.00	34.00	27.00
83.00	68.00	41.00	39.00
62.00	61.00	35.00	30.00
49.00	55.00	32.00	32.00
74.00	79.00	35.00	37.00
73.00	60.00	36.00	38.00
46.00	51.00	25.00	24.00
60.00	55.00	26.00	25.00
93.00	71.00	46.00	39.00

Application of Judd et al. (2001)

- (1) On average, is pain following the drug significantly lower than pain following the placebo?

```
ttest pairs=pain2 pain1.
```

```
proc ttest data=judd;paired pain2*pain1;run;
```

$$\bar{Y}_{\text{drug}} = 61.250, SD = 8.583$$
$$\bar{Y}_{\text{placebo}} = 67.750, SD = 11.889$$

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	pain2	61.2500	20	8.58318	1.91926
	pain1	67.7500	20	11.88929	2.65853

Paired Samples Test

	Paired Differences					t	df	Sig. (2-tailed)			
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference							
				Lower	Upper						
Pair 1	pain2 - pain1	-6.50000	9.23665	2.06538	-10.82289	-2.17711	-3.147	.005			

Pain experienced following the administration of the drug was 6.500 units lower compared to pain experienced following the placebo, $t(19) = -3.147, p < .01$.

Application of Judd et al. (2001)

(2) On average, are there fewer pain-enhancing hormones in the blood following the drug relative to the placebo?

```
ttest pairs=hormone2 hormone1.
```

```
proc ttest data=judd;paired hormed2*hormone1;run;
```

$$\bar{M}_{\text{drug}} = 32.000, SD = 5.964$$
$$\bar{M}_{\text{placebo}} = 34.250, SD = 6.414$$

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	hormone2	32.0000	20	5.96481	1.33377
	hormone1	34.2500	20	6.41442	1.43431

Paired Samples Test

	Paired Differences					t	df	Sig. (2-tailed)			
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference							
				Lower	Upper						
Pair 1	hormone2 - hormone1	-2.25000	4.10231	.91730	-4.16994	-.33006	-2.453	.024			

Pain-enhancing hormone levels were 2.25 units lower on average following the administration of the drug compared to following the placebo, $t(19) = -2.453, p < .05$.

Application of Judd et al. (2001)

(3) Does difference in hormone level following the drug relative to placebo predict the difference in pain experienced following the drug relative to placebo?

Régress $Y_2 - Y_1$ on both $M_2 - M_1$ and mean centered ($M_2 + M_1$), as such:

$$Y_{2i} - Y_{1i} = b_0 + \textcircled{b}_1(M_{2i} - M_{1i}) + b_2 \left(M_{2i} + M_{1i} - \frac{\sum_{i=1}^n (M_{2i} + M_{1i})}{n} \right) + e_i$$

```
compute ydiff=pain2-pain1.  
compute mdiff=hormone2-hormone1.  
compute msumc=(hormone2+hormone1)-66.25.  
regression/dep=ydiff/method=enter mdiff msumc.
```

```
data judd;set judd;ydiff=pain2-pain1;mdiff=hormone2-hormone1;  
msumc=(hormone2+hormone1)-66.25;run;  
proc reg data=judd;model ydiff=mdiff msumc;run;
```

Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error			
1	(Constant)	-3.765	2.087	-1.804	.089
	mdiff	1.215	.457	2.658	.017
	msumc	-.065	.160	-.406	.690

a. Dependent Variable: ydiff

Relatively fewer pain-enhancing hormones following the drug relative to placebo is associated with less pain following the drug relative to placebo, $b_1 = 1.215, p < 0.05$.

Why include $(M_2 + M_1)$ in the model?

The impulse is to model $Y_2 - Y_1$ from $M_2 - M_1$ to assess the effect of difference in M on difference in Y .
This is appropriate only if the regression weight estimating Y_2 from M_2 is the same as the regression weight estimating Y_1 from M_1 .

If Y_1 and Y_2 are linked to M_1 to M_2 as

$$Y_1 = d_{01} + dM_1 + e_1$$

$$Y_2 = d_{02} + dM_2 + e_2, \text{ then}$$

$$\begin{aligned} Y_2 - Y_1 &= (d_{02} + dM_2 + e_2) - (d_{01} + dM_1 + e_1) \\ &= (d_{02} - d_{01}) + d(M_2 - M_1) + (e_2 - e_1) \\ &= d_0 + d(M_2 - M_1) + e_3 \end{aligned}$$

where $d_0 = d_{02} - d_{01}$ and $e_3 = e_2 - e_1$

$$\begin{aligned} Y_{2i} - Y_{1i} &= b_0 + b_1(M_{2i} - M_{1i}) \\ &\quad + b_2 \left(M_{2i} + M_{1i} - \frac{\sum_{i=1}^n (M_{2i} + M_{1i})}{n} \right) + e_i \end{aligned}$$

But if Y_1 and Y_2 are linked to M_1 and M_2 as

$$Y_1 = d_{01} + d_1 M_1 + e_1$$

$$Y_2 = d_{02} + d_2 M_2 + e_2, \text{ then}$$

$$\begin{aligned} Y_2 - Y_1 &= (d_{02} + d_2 M_2 + e_2) - (d_{01} + d_1 M_1 + e_1) \\ &= (d_{02} - d_{01}) + (d_2 M_2 - d_1 M_1) + (e_2 - e_1) \end{aligned}$$

It can be shown that this is equivalent to

$$\begin{aligned} Y_2 - Y_1 &= (d_{02} - d_{01}) + 0.5(d_2 - d_1)(M_2 + M_1) + \\ &\quad 0.5(d_2 + d_1)(M_2 - M_1) + (e_2 - e_1) \\ &= d_0 + 0.5(d_2 - d_1)(M_2 + M_1) + \\ &\quad \color{red}{0.5(d_2 + d_1)(M_2 - M_1)} + e_3 \end{aligned}$$

where $d_0 = d_{02} - d_{01}$ and $e_3 = e_2 - e_1$

These are nearly the same, with the exception of the centering, which has no effect on b_1 . $b_1 = 0.5(d_2 + d_1)$. So b_1 estimates the effect of $M_2 - M_1$ on $Y_2 - Y_1$ without assuming $d_1 = d_2$. It is also the average within period regression weight estimating Y from X . $\color{red}{b_1 = d \text{ if } d_1 = d_2}$. Otherwise, b_1 and d will likely be different.

Why mean center $M_2 + M_1$?

Mean centering ($M_2 + M_1$) yields an intercept that estimates the average difference in Y not attributable to differences in M .

$$Y_{2i} - Y_{1i} = \textcircled{b}_0 + b_1(M_{2i} - M_{1i}) + b_2\left(M_{2i} + M_{1i} - \frac{\sum_{i=1}^n (M_{2i} + M_{1i})}{n}\right) + e_i$$

Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients Beta	t	Sig.
	B	Std. Error			
1 (Constant)	-3.765	2.087		-1.804	.089
mdiff	1.215	.457	.540	2.658	.017
msumc	-.065	.160	-.082	-.406	.690

a. Dependent Variable: ydiff

(4) After accounting for differences in hormone levels, there is no statistically significant difference in pain experienced after the drug relative to the placebo, $i = -3.765$, $p = 0.089$.

Observations

(1) This method is squarely rooted in the causal steps tradition to mediation analysis that has been severely criticized. Compare it to the “Baron and Kenny” criteria:

- Is Y_2 statistically different than Y_1 ? This is like asking whether there is a total effect of X (drug) on Y (pain).
- Is M_2 statistically different than M_1 ? This is like asking whether X affects the mediator.
- Does difference in M significantly predict difference in Y ? This is like asking whether the mediator affects the outcome.
- Is there still evidence of a difference in Y after accounting for the mediator? This is like asking whether the mediator completely or partially accounts for the effect of X on Y .

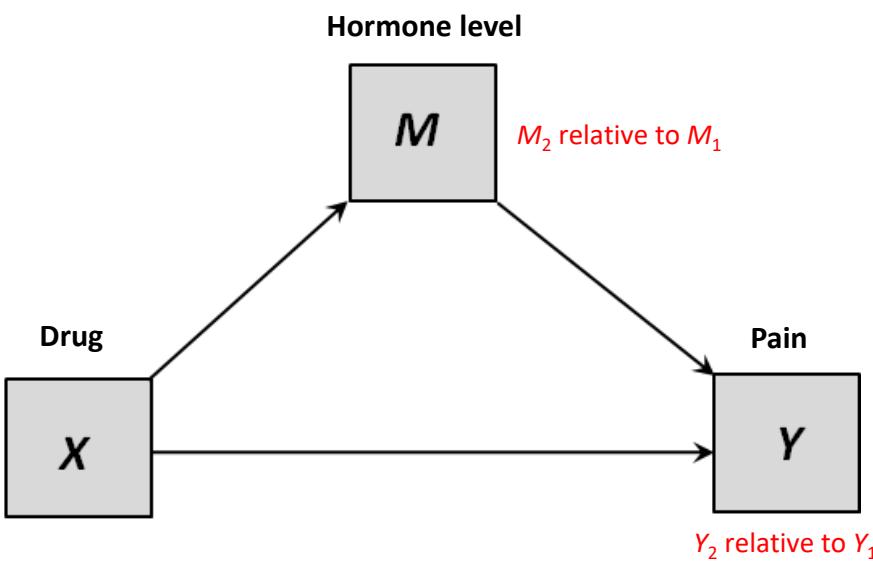
(2) There is no explicit quantification of the indirect effect, but it is the indirect effect that is the primary focus in 21st century mediation analysis.

All of the criticisms of the causal steps approach apply to the Judd, Kenny, and McClelland (2001) method of within-subject mediation analysis.

In a path analytic mediation framework

Data are from Judd et al. (2001). Estimating and testing mediation and moderation in within-subjects designs. *Psychological Methods*, 6, 115 –134.

Goal: Model the effect of the pain-facilitating drug on pain sensations, **directly** as well as **indirectly** through the effect of the drug on pain-enhancing hormone level.

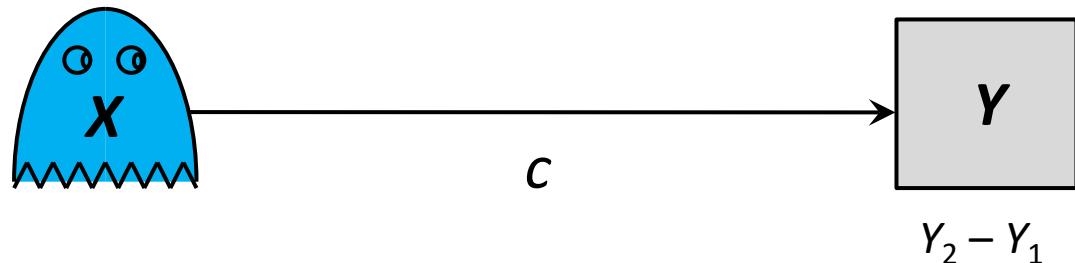


Where is X in the data?

Y_1	Y_2	M_1	M_2
pain1	pain2	hormone1	hormone2
73.00	61.00	37.00	33.00
57.00	55.00	30.00	28.00
57.00	61.00	30.00	36.00
67.00	49.00	31.00	30.00
80.00			35.00
56.00			34.00
72.00			35.00
81.00			29.00
61.00			31.00
67.00			17.00
74.00			41.00
70.00			27.00
83.00	68.00	41.00	39.00
62.00	61.00	35.00	30.00
49.00	55.00	32.00	32.00
74.00	79.00	35.00	37.00
73.00	60.00	36.00	38.00
46.00	51.00	25.00	24.00
60.00	55.00	26.00	25.00
93.00	71.00	46.00	39.00

A blue cartoon character with a large head, small eyes, and a wide mouth is overlaid on the data table. It has a jagged line at its base and a red 'X' mark on its chest.

In a path analytic mediation framework

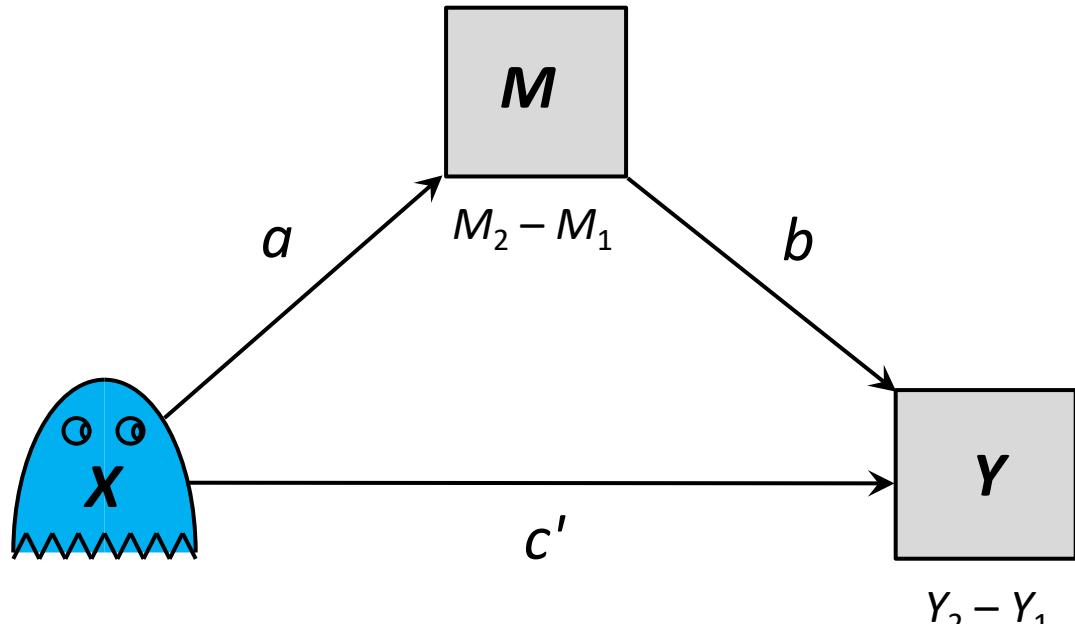


$$Y_1 = \text{pain1}$$

$$Y_2 = \text{pain2}$$

$$M_1 = \text{hormone1}$$

$$M_2 = \text{hormone2}$$



$$Y_2 - Y_1 = c + e_1$$

$$M_2 - M_1 = a + e_2$$

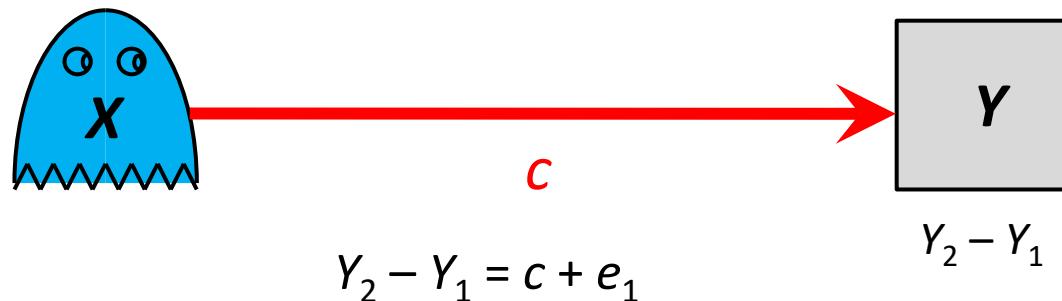
$$Y_2 - Y_1 = c' + b(M_2 - M_1) + b_2(M_2 + M_1)^* + e_3$$

$$c = c' + ab$$

$$ab = c - c'$$

* mean centered

Estimating the total effect (path c)



The total effect is the intercept in a “constant only” model of the difference between pain following the drug (Y_2) and pain following the placebo (Y_1). This is equivalent to the mean difference in pain experienced. No regression analysis needed. Just calculate the mean difference in Y (i.e., $Y_2 - Y_1$).

```
compute ydiff=pain2-pain1.  
descriptives variables=ydiff.
```

```
data judd;set judd;ydiff=pain2-pain1;run;  
proc means data=judd;var ydiff;run;
```

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
ydiff	20	-22.00	6.00	-6.5000	9.23665
Valid N (listwise)	20				

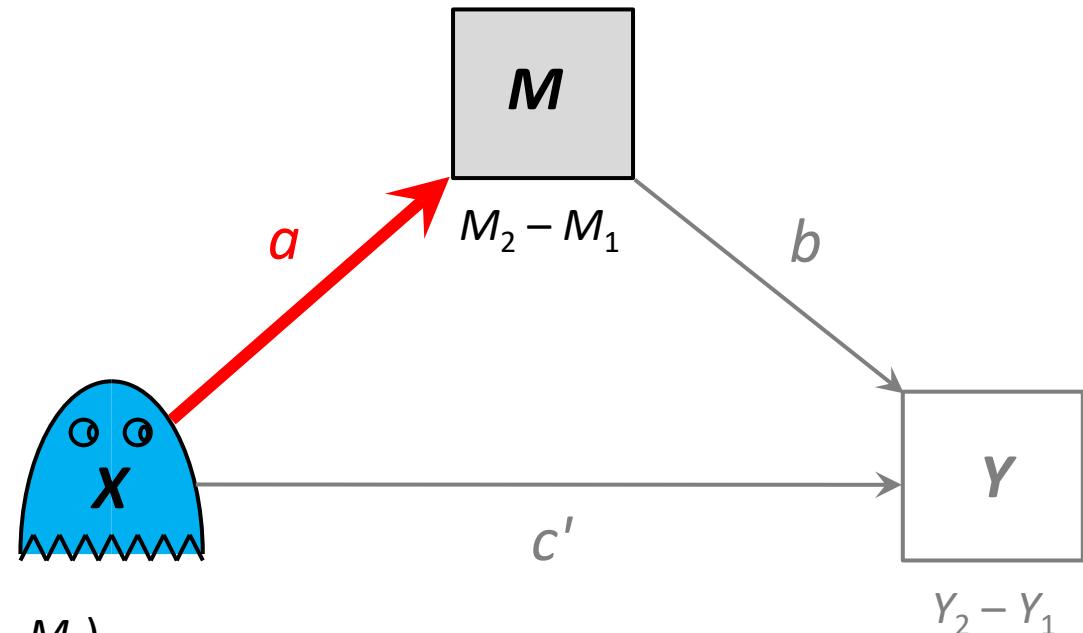
$$c = -6.500$$

6.50 units less pain following the drug compared to placebo

Estimating the a path

$$M_2 - M_1 = a + e_2$$

The a path is the intercept in a “constant only” model of the difference in hormone following the drug (M_2) and following the placebo (M_1). This is equivalent to the mean difference in hormone level. No regression analysis needed. Just calculate the mean difference in M (i.e., $M_2 - M_1$).



```
compute mdiff=hormone2-hormone1.  
descriptives variables= mdiff.
```

```
data judd;set judd;mdiff=hormone2-hormone1;run;  
proc means data=judd;var mdiff;run;
```

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
mdiff	20	-14.00	6.00	-2.2500	4.10231
Valid N (listwise)	20				

$$a = -2.250$$

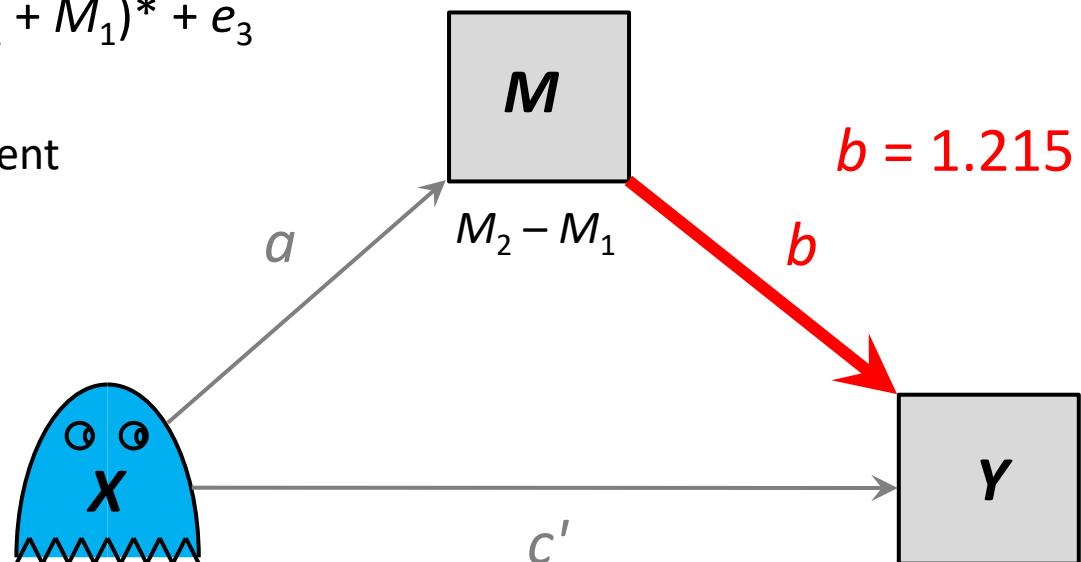
2.50 units less of the pain enhancing hormone following the drug compared to placebo

Estimating the b path

* mean centered

$$Y_2 - Y_1 = c' + b(M_2 - M_1) + b_2(M_2 + M_1)^* + e_3$$

The b path is the regression coefficient for the mean difference in M in a model of the mean difference in Y , including the mean centered sum of M_1 and M_2 as a covariate. In these data, the mean of the sum of M_1 and M_2 is 66.250.



```
compute msumc=(hormone2+hormone1)-66.250.
regression/dep=ydiff/method=enter mdiff msumc.
```

```
data judd;set judd;mdiff=hormone2-hormone1;msumc=(hormone2+hormone1)-66.250;run;
proc reg data=judd;model ydiff=mdiff msumc;run;
```

Model	Coefficients ^a					
	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	
	B	Std. Error	Beta			
1 (Constant)	-3.765	2.087		-1.804	.089	
mdiff	1.215	.457	.540	2.658	.017	
msumc	-.065	.160	-.082	-.406	.690	

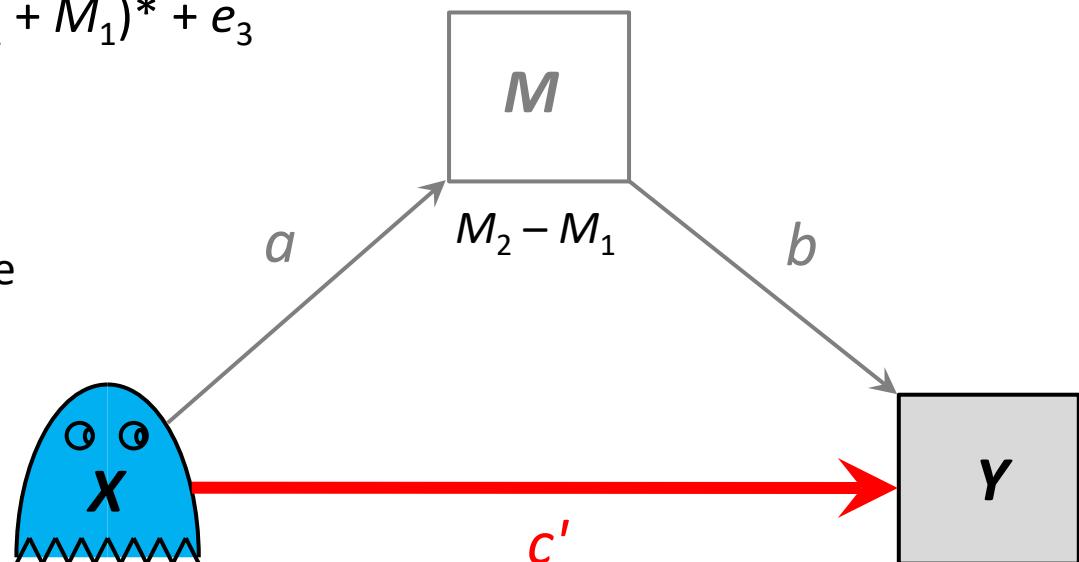
a. Dependent Variable: ydiff

On average, each fewer unit of pain enhancing hormone following drug relative to placebo is associated with 1.215 fewer units of pain following drug relative to placebo.

Estimating the direct effect (path c')

$$Y_2 - Y_1 = c' + b(M_2 - M_1) + d_2(M_2 + M_1)^* + e_3$$

The direct effect (path c') is the regression constant in the model of the mean difference in Y from the mean difference in M and the mean centered sum of M_1 and M_2 . This is the same model used to estimate path b .



```
compute msumc=(hormone2+hormone1)-66.250.
regression/dep=ydiff/method=enter mdiff msumc.
```

```
data judd;set judd;mdiff=hormone2-hormone1;msumc=(hormone2+hormone1)-66.250;run;
proc reg data=judd;model ydiff=mdiff msumc;run;
```

Coefficients^a

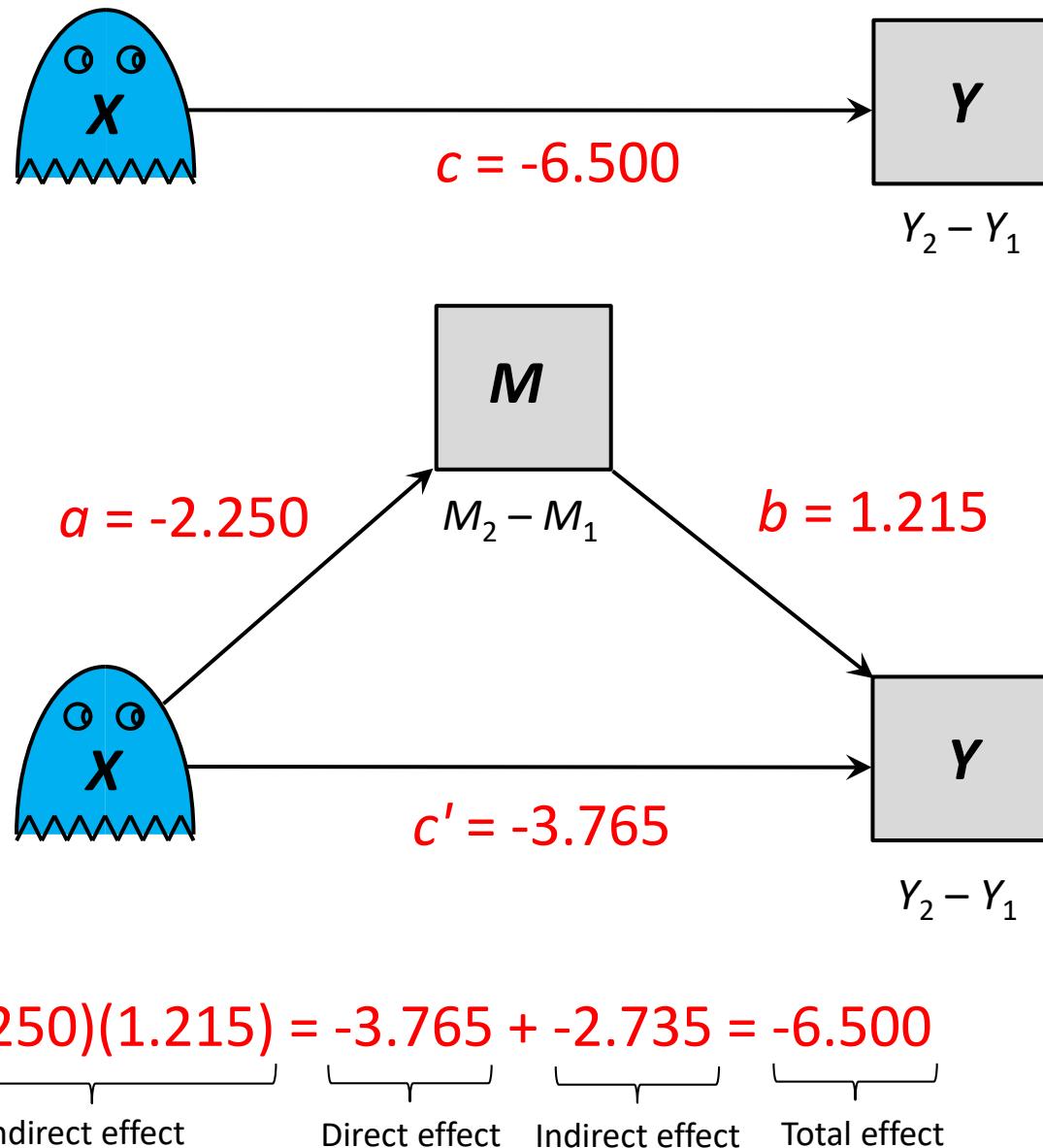
Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
1 (Constant)	-3.765	2.087		-1.804	.089
mdiff	1.215	.457	.540	2.658	.017
msumc	-.065	.160	-.082	-.406	.690

a. Dependent Variable: ydiff

Independent of the effect of drug vs. placebo hormone difference on pain difference, the drug results in 3.765 units less pain relative to placebo.

Putting it all together

In this form, it is clear that the effect of X partitions into two components direct and indirect in the usual way. We can conduct inferential tests on these estimates as in any mediation analysis.



Statistical inference

- The total effect is just the average difference between Y_2 and Y_1 . Inference can proceed in the usual way (paired t -test, a confidence interval for the difference, etc.)

In these data, $c = -6.500$, $t(19) = -3.147$, $p < 0.01$. On average, the drug appears to have an effect on the experience of pain. But we should not insist on evidence of a total effect to proceed with a mediation analysis, as we no longer do elsewhere in modern mediation analysis.

- The direct effect is just the average difference between Y_2 and Y_1 not accounted for by differences in M and their sum. A hypothesis test or confidence interval will do.

Here, $c' = -3.765$, $t(17) = -1.804$, $p = 0.089$. Accounting for the effect of change in M and individual differences in M , the remaining effect of the drug on pain is not statistically different from zero.

- Whereas the Judd et al. (2001) approach requires the joint significance of a and b to claim mediation, modern mediation analysis bases claims of mediation on a quantification of the indirect effect and inference about it. The indirect effect is ab , which is equivalent to $c - c'$. The sampling distribution of this difference is not normally distributed.

$ab = -2.250(1.215) = -2.785$. This is the reduction in pain due to the drug that results from the effect of the drug on reducing pain-enhancing hormones. A bootstrap CI for inference is a good choice.

MEMORE

This method is described in more detail in Montoya and Hayes (2017). The paper includes a description of a new macro (MEMORE, pronounced like “memory”).

- Single and multiple mediator models.
- Single and multiple moderator models.
- Various inferential methods for indirect effects
- Contrasts between indirect effects
- moderated mediation analysis functions coming soon.

```
memore y=pain2 pain1/m=hormone2 hormone1/samples=10000/model=1 .
```

```
%memore (data=judd,y=pain2 pain1,m=hormone2 hormone1,samples=10000, mode
```

Psychological Methods

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Two-Condition Within-Participant Statistical Mediation Analysis: A Path-Analytic Framework

AQ: au

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Researchers interested in testing mediation often use designs where participants are measured on a dependent variable Y and a mediator M in both of 2 different circumstances. The dominant approach to assessing mediation in such a design, proposed by Judd, Kenny, and McClelland (2001), relies on a series of hypothesis tests about components of the mediation model and is not based on an estimate of or formal inference about the indirect effect. In this article we recast Judd et al.'s approach in the path-analytic framework that is now commonly used in between-participant mediation analysis. By so doing, it is apparent how to estimate an indirect effect in a within-participant mediation analysis, some of which are through a mediator as the product of paths of influence. This path-analytic approach eliminates the need for discrete hypothesis tests about components of the model to support a claim of mediation, as Judd et al.'s method requires, because it relies only on an inference about the product of paths—the indirect effect. We generalize methods of inference for the indirect effect widely used in between-participant designs to this within participant version of mediation analysis, including bootstrapping confidence intervals and Monte Carlo confidence intervals. Using this path-analytic approach, we extend the method to models with multiple mediators operating in parallel and serially and discuss the comparison of indirect effects in these more complex models. We offer macros and code for SPSS, SAS, and Mplus that conduct these analyses.

AQ: 1

Keywords: mediation, indirect effect, path analysis, within-participant design, resampling methods

Statistical mediation analysis allows an investigator to answer questions about the process by which some presumed causal variable X operates to affect an outcome variable Y . Using simple principles of linear modeling (though other analytical approaches are possible; Imai, Keele, & Tingley, 2010; Pearl, 2010, 2012), mediation analysis is used to quantify and test the pathways of influence from X to Y . In a mediation process, one of those pathways consists of a sequence of causal steps in which X affects a mediator variable M , which in turn causally influences Y . This *indirect effect* of X —the conjunction of the effect of X on M and the effect of M on Y —quantifies the degree to which M acts as the “mechanism” by which X affects Y . An indirect effect that is different from zero by an inferential test is used to support (but by no means definitively establishes or proves) a claim of mediation of X 's effect on Y by M .

Mediation analysis is commonplace in the social sciences, business and medical research, and many other areas. For example,

White, Abu-Rayya, Bluci, and Faulkner (2015) investigated how long-term interaction with a member of the same religion or a different religion (X) influenced intergroup bias (Y) through five different emotions (e.g., anger and sadness; M). Littleton (2015) found that pregnant women who had a history of sexual victimization (X) had higher rates of depression (M), which predicted increased somatic complaints (e.g., back pain; Y). Schulte, Gullory, and Gay (2016) examined how the weight of a person recommending a recipe (X) influenced the perceived healthiness of the recipe (Y) through the perceived health of the recommender (M).

Discussions of mediation analysis and its application are most typically couched in terms of or conducted using data from research designs that are cross-sectional or “between-participant” in nature. Typically in these designs, participants are measured once on a proposed mediator M and dependent variable Y , as in the examples above. This may occur following random assignment of participants into one of two conditions (X) that vary via some manipulation (e.g., a “treatment” vs. a “control” group) that is presumed to cause differences in M and Y . Alternatively, measurement of M and Y may occur contemporaneously with the observation of X (rather than random assignment). For expositional convenience, we refer to designs of this sort (i.e., with or without random assignment to X) throughout this article as “between-participant” designs.

Less attention in the methodology literature has been dedicated to mediation analysis when the data come from repeated measurement of the same people on variables in the mediation process, even though such designs are common. In this article we address mediation analysis in a specific category of repeated measures designs. Researchers sometimes measure a dependent variable Y

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MEMORE Output

***** MEMORE Procedure for SPSS Version 1.1 *****

Written by Amanda Montoya

Documentation available at afhayes.com

Variables:

Y = pain2 pain1
M = hormone2 hormone1

MEMORE
constructs
differences
and averages
for you.

Computed Variables:

Ydiff = pain2 - pain1
Mdiff = hormone2 - hormone1
Mavg = (hormone2 + hormone1) /2 Centered

Sample Size:

20

Outcome: Ydiff = pain2 - pain1

Model

	Effect	SE	t	df	p	LLCI	ULCI	
c = -6.500	'x'	-6.5000	2.0654	-3.1471	19.0000	.0053	-10.8233	-2.1767

Outcome: Mdiff = hormone2 - hormone1

Model

	Effect	SE	t	df	p	LLCI	ULCI	
a = -2.250	'x'	-2.2500	.9173	-2.4528	19.0000	.0240	-4.1701	-.3299

MEMORE Output

 Outcome: Ydiff = pain2 - pain1

Model Summary

R	R-sq	MSE	F	df1	df2	p
.5554	.3085	65.9384	3.7918	2.0000	17.0000	.0435

Model

	coeff	SE	t	df	p	LLCI	ULCI
'X'	-3.7654	2.0869	-1.8043	17.0000	.0889	-8.1689	.6381
Mdiff	1.2154	.4572	2.6583	17.0000	.0166	.2506	2.1801
Mavg	-.1302	.3209	-.4057	17.0000	.6900	-.8074	.5470

***** TOTAL, DIRECT, AND INDIRECT EFFECTS *****

Total effect of X on Y

Effect	SE	t	df	p	LLCI	ULCI
-6.5000	2.0654	-3.1471	19.0000	.0053	-10.8233	-2.1767

Direct effect of X on Y

Effect	SE	t	df	p	LLCI	ULCI
-3.7654	2.0869	-1.8043	17.0000	.0889	-8.1689	.6381

Indirect Effect of X on Y through M

Effect	BootSE	BootLLCI	BootULCI
Ind1	-2.7346	1.3120	-5.6521

ab with 95%
 bootstrap
 confidence
 interval

Indirect Key

Ind1 X -> M1diff -> Ydiff

***** ANALYSIS NOTES AND WARNINGS *****

Bootstrap confidence interval method used: Percentile bootstrap.

Number of bootstrap samples for bootstrap confidence intervals: 10000